Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/1.1.1.2-a+b-x-^m-c+d-x-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1917]. This is test number [13].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (1917)	% 0. (0)
Mathematica	% 100. (1917)	% 0. (0)
Maple	% 81.22 (1557)	% 18.78 (360)
Maxima	% 50.23 (963)	% 49.77 (954)
Fricas	% 83.57 (1602)	% 16.43 (315)
Sympy	% 61.19 (1173)	% 38.81 (744)
Giac	% 65.62 (1258)	% 34.38 (659)

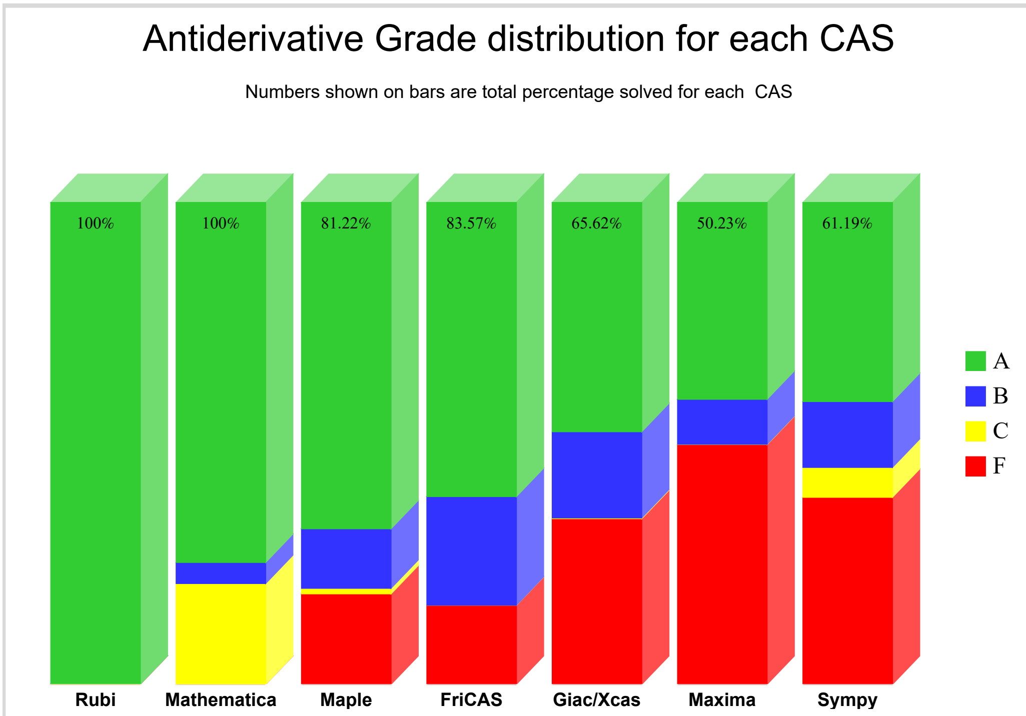
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

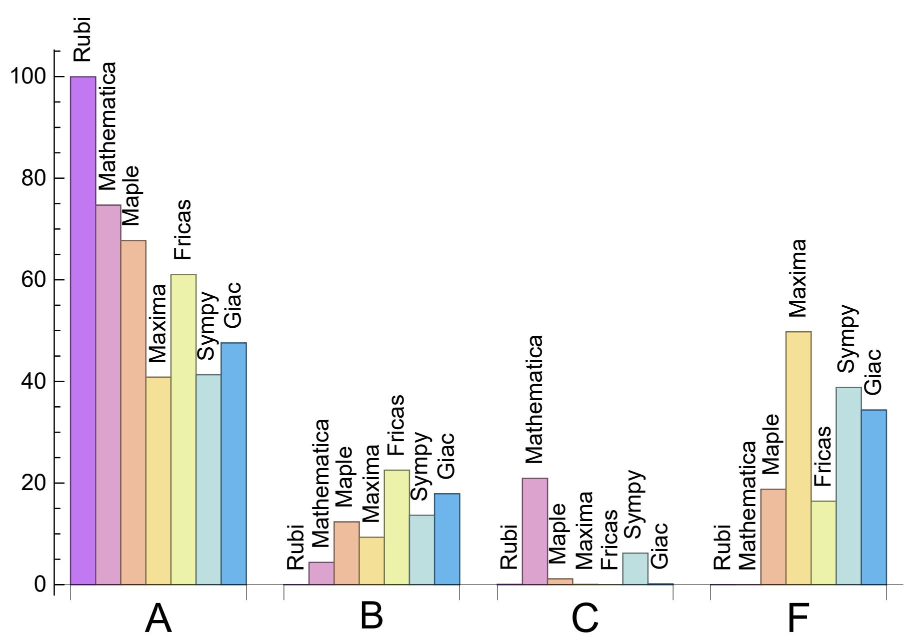
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.95	0.	0.05	0.
Mathematica	74.7	4.38	20.92	0.
Maple	67.71	12.36	1.15	18.78
Maxima	40.85	9.34	0.05	49.77
Fricas	61.03	22.54	0.	16.43
Sympy	41.31	13.67	6.21	38.81
Giac	47.57	17.89	0.16	34.38

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.08	108.1	1.	66.	1.
Mathematica	0.04	72.	0.93	48.	0.85
Maple	0.01	103.89	1.3	54.	0.93
Maxima	1.08	155.11	1.99	58.	1.2
Fricas	2.28	453.87	4.44	174.	3.2
Sympy	8.51	265.11	4.01	85.	1.5
Giac	1.7	203.91	2.47	82.	1.39

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

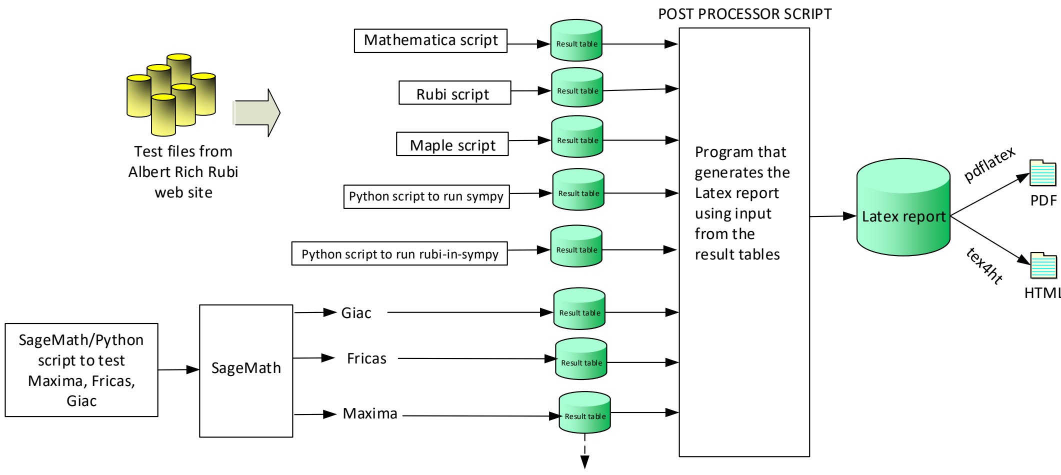
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 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1681, 1682, 1683, 1684, 1685, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1708, 1709, 1710, 1711, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1775, 1776, 1777, 1778, 1779, 1780, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1801, 1802, 1803, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1831, 1832, 1833, 1834, 1835, 1836, 1837, 1838, 1839, 1840, 1841, 1842, 1843, 1844, 1845, 1846, 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B grade: { }

C grade: { 369 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 230, 231, 232, 233, 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512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 527, 528, 529, 530, 533, 534, 535, 536, 539, 540, 541, 542, 545, 546, 547, 548, 551, 552, 553, 554, 557, 558, 559, 560, 563, 564, 565, 566, 569, 570, 571, 572, 573, 574, 575, 576, 579, 580, 581, 582, 583, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 598, 599, 600, 601, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 619, 620, 621, 622, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 637, 638, 639, 640, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 675, 677, 679, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 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961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1180, 1181, 1182, 1183, 1186, 1187, 1188, 1196, 1197, 1198, 1214, 1215, 1216, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1269, 1270, 1271, 1272, 1282, 1283, 1286, 1287, 1290, 1296, 1297, 1298, 1311, 1315, 1316, 1317, 1318, 1319, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1364, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1395, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1408, 1411, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1424, 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B grade: { 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 212, 226, 227, 243, 244, 1236, 1246, 1258, 1259, 1268, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1284, 1285, 1288, 1289, 1291, 1292, 1293, 1294, 1295, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1312, 1313, 1314, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1362, 1363, 1365, 1366, 1523, 1524, 1526, 1527, 1528, 1534, 1539, 1540, 1542, 1548, 1550, 1558 }

C grade: { 290, 291, 297, 299, 305, 306, 308, 318, 319, 320, 321, 323, 324, 327, 329, 332, 333, 341, 342, 348, 349, 350, 356, 357, 358, 361, 362, 363, 364, 365, 366, 367, 376, 377, 383, 384, 390, 391, 397, 398, 403, 404, 405, 411, 412, 417, 418, 419, 422, 423, 453, 454, 455, 456, 457, 460, 461, 462, 463, 465, 466, 467, 468, 473, 474, 475, 476, 477, 480, 481, 482, 483, 485, 486, 487, 488, 525, 526, 531, 532, 537, 538, 543, 544, 549, 550, 555, 556, 561, 562, 567, 568, 577, 578, 584, 596, 597, 602, 616, 617, 623, 635, 636, 641, 674, 676, 678, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 1081, 1082, 1095, 1096, 1097, 1116, 1117, 1118, 1126, 1127, 1128, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1184, 1185, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1383, 1384, 1385, 1386, 1394, 1396, 1397, 1398, 1406, 1407, 1409, 1410, 1412, 1421, 1422, 1423, 1430, 1431, 1432, 1433, 1440, 1441, 1442, 1443, 1475, 1476, 1485, 1486, 1487, 1502, 1503, 1504, 1512, 1513, 1514, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1772, 1773, 1774, 1779, 1780, 1781, 1798, 1799, 1800, 1805, 1806, 1807, 1824, 1825, 1826, 1831, 1832, 1833 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 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664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 702, 703, 719, 720, 724, 725, 728, 732, 733, 734, 735, 739, 740, 741, 750, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 930, 931, 932, 933, 934, 938, 939, 940, 941, 942, 943, 946, 947, 948, 949, 953, 954, 955, 956, 961, 962, 963, 964, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1080, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1106, 1107, 1108, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1139, 1140, 1141, 1142, 1143, 1144, 1149, 1150, 1151, 1152, 1153, 1157, 1158, 1159, 1160, 1161, 1163, 1164, 1165, 1166, 1167, 1168, 1180, 1181, 1182, 1183, 1186, 1187, 1188, 1196, 1197, 1198, 1214, 1215, 1216, 1226, 1227, 1228, 1229, 1230, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1248, 1249, 1250, 1251, 1252, 1253, 1255, 1256, 1257, 1263, 1264, 1270, 1271, 1272, 1282, 1311, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1346, 1347, 1348, 1349, 1350, 1351, 1356, 1357, 1358, 1359, 1360, 1361, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1388, 1389, 1390, 1391, 1392, 1395, 1396, 1397, 1398, 1400, 1401, 1402, 1403, 1404, 1408, 1409, 1410, 1411, 1412, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1454, 1455, 1456, 1457, 1458, 1459, 1464, 1466, 1467, 1468, 1469, 1470, 1477, 1478, 1479, 1480, 1488, 1489, 1490, 1491, 1495, 1497, 1498, 1499, 1500, 1501, 1506, 1507, 1508, 1509, 1510, 1516, 1517, 1518, 1519, 1520, 1530, 1541, 1544, 1578, 1579, 1580, 1581, 1590, 1591, 1592, 1593, 1604, 1605, 1606, 1607, 1618, 1619, 1620, 1621, 1682, 1683, 1684, 1685, 1696, 1697, 1698, 1699, 1708, 1709, 1710, 1711, 1719, 1720, 1721, 1722, 1775, 1776, 1777, 1778, 1782, 1783, 1784, 1785, 1801, 1802, 1803, 1804, 1808, 1809, 1810, 1811, 1827, 1828, 1829, 1830, 1834, 1835, 1836, 1837, 1844, 1848, 1854, 1855, 1860, 1861, 1868, 1869, 1874, 1875, 1881, 1882, 1883, 1884, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917 }

B grade: { 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 199, 212, 213, 226, 227, 228, 243, 244, 442, 516, 517, 543, 544, 572, 578, 584, 593, 597, 602, 611, 617, 623, 632, 635, 636, 641, 648, 649, 699, 700, 701, 1016, 1017, 1018, 1034, 1053, 1066, 1067, 1068, 1069, 1078, 1079, 1081, 1082, 1091, 1105, 1109, 1110, 1118, 1119, 1129, 1137, 1138, 1145, 1146, 1147, 1148, 1154, 1155, 1156, 1162, 1169, 1225, 1236, 1237, 1246, 1247, 1254, 1258, 1259, 1260, 1261, 1262, 1265, 1266, 1267, 1268, 1269, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1343, 1344, 1345, 1352, 1353, 1354, 1355, 1362, 1363, 1364, 1365, 1366, 1367, 1375, 1387, 1393, 1394, 1399, 1405, 1406, 1407, 1413, 1424, 1433, 1434, 1453, 1460, 1461, 1462, 1463, 1471, 1472, 1473, 1474, 1481, 1482, 1483, 1484, 1492, 1493, 1494, 1496, 1511, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1542, 1543, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1846, 1847, 1852, 1853, 1859, 1870, 1871, 1876, 1877 }

C grade: { 721, 722, 726, 727, 1171, 1172, 1174, 1175, 1176, 1177, 1206, 1207, 1208, 1209, 1210, 1211, 1217, 1218, 1219, 1220, 1222, 1223 }

F grade: { 369, 579, 585, 598, 603, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 723, 729, 730, 731, 736, 737, 738, 742, 743, 744, 745, 746, 747, 748, 749, 751, 752, 927, 928, 929, 935, 936, 937, 944, 945, 950, 951, 952, 957, 958, 959, 960, 965, 966, 967, 968, 981, 982, 983, 984, 985, 986, 997, 998, 1170, 1173, 1178, 1179, 1184, 1185, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1212, 1213, 1221, 1224, 1231, 1232, 1233, 1234, 1235, 1465, 1475, 1476, 1485, 1486, 1487, 1502, 1503, 1504, 1505, 1512, 1513, 1514, 1515, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1779, 1780, 1781, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1805, 1806, 1807, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1831, 1832, 1833, 1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1856, 1857, 1858, 1862, 1863, 1864, 1865, 1866, 1867, 1872, 1873, 1878, 1879, 1880, 1885, 1886, 1887, 1888, 1889, 1890 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 217, 218, 219, 220, 221, 222, 223, 224, 225, 230, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 292, 293, 294, 295, 300, 301, 302, 303, 309, 310, 311, 312, 313, 314, 315, 316, 334, 335, 336, 337, 338, 343, 344, 345, 346, 347, 351, 352, 353, 354, 355, 368, 369, 371, 372, 373, 374, 378, 379, 380, 381, 385, 386, 387, 388, 392, 393, 394, 395, 399, 400, 401, 402, 406, 407, 408, 409, 413, 414, 415, 416, 420, 421, 422, 423, 424, 425, 426, 427, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 494, 495, 496, 502, 503, 504, 510, 511, 512, 518, 519, 520, 573, 574, 575, 576, 580, 581, 582, 583, 586, 587, 588, 589, 594, 595, 599, 600, 601, 604, 605, 606, 607, 612, 613, 614, 615, 619, 620, 621, 622, 625, 626, 627, 628, 633, 634, 638, 639, 640, 643, 644, 645, 646, 647, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 698, 732, 733, 734, 753, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 828, 829, 831, 832, 833, 834, 835, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 857, 858, 859, 864, 865, 866, 867, 868, 874, 875, 876, 882, 883, 884, 892, 897, 898, 899, 900, 905, 906, 907, 908, 915, 916, 923, 924, 925, 926, 930, 931, 932, 933, 934, 938, 939, 940, 941, 942, 943, 946, 947, 948, 949, 953, 954, 955, 956, 961, 962, 963, 964, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 991, 1000, 1001, 1002, 1003, 1008, 1009, 1010, 1011, 1021, 1022, 1023, 1024, 1025, 1028, 1029, 1030, 1031, 1032, 1033, 1035, 1036, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1126, 1127, 1131, 1132, 1133, 1134, 1135, 1136, 1141, 1142, 1143, 1144, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1248, 1249, 1250, 1251, 1252, 1253, 1255, 1256, 1260, 1264, 1265, 1266, 1267, 1282, 1311, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1344, 1345, 1346, 1347, 1348, 1349, 1354, 1355, 1356, 1357, 1358, 1371, 1375, 1376, 1377, 1378, 1379, 1380, 1387, 1388, 1389, 1390, 1391, 1392, 1399, 1400, 1401, 1402, 1403, 1404, 1411, 1412, 1414, 1415, 1416, 1417, 1418, 1424, 1425, 1426, 1427, 1428, 1429, 1434, 1435, 1436, 1437, 1438, 1439, 1444, 1445, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1530, 1537, 1541, 1544, 1553, 1556, 1557, 1881, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917

B grade: { 68, 73, 82, 83, 90, 105, 115, 116, 132, 133, 146, 147, 148, 186, 199, 212, 213, 214, 215, 216, 226, 227, 228, 229, 231, 232, 233, 243, 244, 648, 830, 836, 999, 1004, 1005, 1006, 1007, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1020, 1026, 1034, 1037, 1045, 1053, 1070, 1071, 1072, 1073, 1074, 1082, 1083, 1084, 1085, 1086, 1087, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1124, 1125, 1128, 1129, 1130, 1236, 1237, 1246, 1247, 1254, 1257, 1258, 1259, 1261, 1262, 1263, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1342, 1343, 1350, 1351, 1352, 1353, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1372, 1373, 1374, 1413, 1446, 1527, 1534, 1548, 1550, 1552, 1555 }

C grade: { 1027 }

F grade: { 34, 35, 288, 289, 290, 291, 296, 297, 298, 299, 304, 305, 306, 307, 308, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 339, 340, 341, 342, 348, 349, 350, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 370, 375, 376, 377, 382, 383, 384, 389, 390, 391, 396, 397, 398, 403, 404, 405, 410, 411, 412, 417, 418, 419, 428, 435, 442, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 497, 498, 499, 500, 501, 505, 506, 507, 508, 509, 513, 514, 515, 516, 517, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 577, 578, 579, 584, 585, 590, 591, 592, 593, 596, 597, 598, 602, 603, 608, 609, 610, 611, 616, 617, 618, 623, 624, 629, 630, 631, 632, 635, 636, 637, 641, 642, 649, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 852, 853, 854, 855, 856, 860, 861, 862, 863, 869, 870, 871, 872, 873, 877, 878, 879, 880, 881, 885, 886, 887, 888, 889, 890, 891, 893, 894, 895, 896, 901, 902, 903, 904, 909, 910, 911, 912, 913, 914, 917, 918, 919, 920, 921, 922, 927, 928, 929, 935, 936, 937, 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1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1528, 1529, 1531, 1532, 1533, 1535, 1536, 1538, 1539, 1540, 1542, 1543, 1545, 1546, 1547, 1549, 1551, 1554, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1708, 1709, 1710, 1711, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1775, 1776, 1777, 1778, 1779, 1780, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1801, 1802, 1803, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1831, 1832, 1833, 1834, 1835, 1836, 1837, 1838, 1839, 1840, 1841, 1842, 1843, 1844, 1845, 1846, 1847, 1848, 1849, 1850, 1851, 1852, 1853, 1854, 1855, 1856, 1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1890 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 205, 206, 207, 208, 209, 210, 211, 222, 223, 230, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 412, 413, 414, 415, 416, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 688, 696, 698, 702, 703, 732, 733, 734, 735, 739, 740, 741, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 930, 931, 932, 933, 934, 938, 939, 940, 941, 942, 943, 946, 947, 948, 949, 953, 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1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917 }

B grade: { 37, 38, 42, 68, 73, 82, 83, 90, 105, 106, 115, 116, 132, 133, 134, 146, 147, 148, 186, 199, 201, 202, 203, 204, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 237, 238, 243, 244, 295, 303, 314, 315, 316, 355, 356, 388, 410, 411, 418, 442, 586, 604, 625, 643, 648, 684, 685, 686, 687, 689, 690, 691, 692, 693, 694, 695, 697, 699, 700, 701, 999, 1004, 1005, 1006, 1007, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1026, 1034, 1037, 1045, 1068, 1069, 1070, 1082, 1083, 1084, 1098, 1099, 1100, 1101, 1109, 1110, 1119, 1129, 1130, 1156, 1162, 1170, 1225, 1236, 1237, 1246, 1247, 1254, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1342, 1343, 1344, 1345, 1350, 1351, 1352, 1353, 1354, 1355, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1408, 1409, 1410, 1420, 1421, 1422, 1423, 1424, 1431, 1432, 1433, 1434, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1458, 1459, 1465, 1466, 1467, 1468, 1469, 1470, 1476, 1477, 1478, 1479, 1480, 1481, 1486, 1487, 1488, 1489, 1490, 1496, 1498, 1499, 1500, 1501, 1502, 1505, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1527, 1529, 1531, 1534, 1536, 1537, 1538, 1539, 1540, 1543, 1545, 1548, 1550, 1551, 1553, 1554, 1555, 1557, 1558, 1576, 1577, 1578, 1579, 1580, 1581, 1587, 1588, 1589, 1591, 1592, 1593, 1601, 1602, 1603, 1605, 1606, 1607, 1615, 1617, 1619, 1620, 1621, 1678, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1693, 1694, 1695, 1697, 1698, 1699, 1705, 1706, 1707, 1709, 1710, 1711, 1717, 1718, 1720, 1721, 1722, 1730, 1772, 1773, 1774, 1775, 1776, 1777, 1778, 1779, 1780, 1781, 1782, 1783, 1784, 1785, 1798, 1799, 1800, 1801, 1802, 1803, 1804, 1805, 1806, 1807, 1809, 1810, 1811, 1824, 1825, 1826, 1828, 1829, 1830, 1831, 1832, 1833, 1835, 1836, 1837, 1846, 1847, 1852, 1853, 1859, 1860, 1869, 1870, 1871, 1875, 1876, 1877 }

C grade: { }

F grade: { 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 736, 737, 738, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 927, 928, 929, 935, 936, 937, 944, 945, 950, 951, 952, 957, 958, 959, 960, 965, 966, 967, 968, 981, 982, 983, 984, 985, 986, 997, 998, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1189, 1190, 1191, 1192, 1193, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1231, 1232, 1233, 1234, 1235, 1491, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1582, 1583, 1584, 1585, 1586, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1622, 1623, 1624, 1625, 1626, 1627, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1700, 1701, 1702, 1703, 1704, 1712, 1713, 1714, 1715, 1716, 1723, 1724, 1725, 1726, 1727, 1728, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1856, 1857, 1858, 1862, 1863, 1864, 1865, 1866, 1867, 1872, 1873, 1878, 1879, 1880, 1885, 1886, 1887, 1888, 1889, 1890 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 218, 219, 220, 221, 222, 223, 224, 225, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 287, 289, 290, 291, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 330, 333, 338, 339, 340, 341, 342, 346, 347, 349, 350, 352, 353, 354, 355, 359, 361, 363, 364, 370, 374, 381, 387, 388, 395, 402, 409, 415, 416, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 464, 465, 466, 467, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 483, 484, 485, 486, 487, 489, 490, 491, 492, 493, 495, 497, 498, 499, 500, 501, 503, 505, 506, 507, 508, 509, 511, 513, 514, 515, 516, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 569, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 608, 609, 610, 611, 612, 613, 616, 617, 618, 619, 620, 625, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 666, 667, 668, 675, 676, 677, 678, 679, 680, 681, 685, 686, 699, 700, 701, 702, 703, 732, 733, 734, 735, 756, 757, 758, 759, 760, 763, 764, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 785, 786, 787, 788, 791, 792, 793, 794, 795, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 812, 813, 814, 815, 816, 817, 820, 821, 822, 823, 824, 825, 828, 829, 835, 841, 842, 843, 847, 848, 849, 850, 851, 897, 905, 913, 919, 991, 1000, 1001, 1002, 1003, 1008, 1009, 1010, 1011, 1015, 1022, 1023, 1024, 1025, 1028, 1029, 1030, 1031, 1032, 1033, 1035, 1038, 1039, 1040, 1042, 1043, 1044, 1046, 1049, 1050, 1051, 1052, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1064, 1065, 1069, 1070, 1076, 1080, 1081, 1093, 1094, 1095, 1104, 1105, 1106, 1107, 1108, 1111, 1112, 1116, 1117, 1118, 1120, 1121, 1130, 1131, 1132, 1156, 1160, 1161, 1162, 1163, 1165, 1167, 1173, 1191, 1209, 1229, 1230, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1248, 1249, 1251, 1252, 1253, 1255, 1264, 1265, 1266, 1267, 1334, 1335, 1336, 1337, 1338, 1339, 1343, 1344, 1345, 1346, 1347, 1348, 1354, 1355, 1356, 1357, 1376, 1377, 1378, 1379, 1380, 1381, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1456, 1457, 1525, 1530, 1532, 1544, 1546, 1552, 1553, 1555, 1556, 1557, 1642, 1846, 1847, 1848, 1852, 1853, 1854, 1855, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917 }

B grade: { 55, 68, 73, 82, 83, 90, 91, 104, 105, 106, 115, 116, 131, 132, 133, 134, 146, 147, 148, 186, 187, 199, 201, 212, 213, 214, 215, 216, 217, 226, 227, 228, 229, 230, 231, 232, 233, 234, 243, 244, 284, 285, 286, 288, 292, 293, 297, 325, 329, 334, 335, 336, 337, 343, 344, 345, 348, 351, 356, 357, 358, 360, 367, 371, 372, 373, 378, 379, 380, 385, 386, 392, 393, 394, 406, 407, 408, 413, 414, 494, 496, 502, 504, 510, 512, 518, 520, 575, 576, 582, 583, 584, 585, 586, 587, 588, 589, 601, 602, 603, 604, 605, 606, 607, 614, 615, 621, 622, 623, 624, 626, 627, 628, 640, 641, 642, 643, 644, 645, 646, 736, 737, 738, 808, 818, 826, 830, 834, 836, 840, 846, 999, 1004, 1005, 1006, 1007, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1020, 1021, 1026, 1034, 1036, 1037, 1041, 1045, 1047, 1048, 1053, 1066, 1067, 1068, 1071, 1072, 1077, 1078, 1079, 1082, 1083, 1084, 1091, 1092, 1096, 1097, 1098, 1109, 1110, 1113, 1114, 1119, 1122, 1123, 1133, 1134, 1155, 1164, 1236, 1237, 1245, 1246, 1247, 1250, 1254, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1340, 1341, 1342, 1349, 1350, 1351, 1352, 1353, 1358, 1359, 1360, 1361, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1382, 1383, 1394, 1406, 1844, 1881 }

C grade: { 328, 331, 332, 362, 365, 366, 369, 375, 376, 377, 382, 383, 384, 389, 390, 391, 396, 397, 398, 399, 400, 401, 403, 404, 405, 410, 411, 412, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 438, 445, 517, 532, 544, 556, 568, 661, 662, 663, 664, 665, 669, 670, 671, 672, 673, 698, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 743, 746, 747, 748, 750, 751, 752, 1027, 1127, 1128, 1129, 1140, 1141, 1142, 1148, 1149, 1150, 1166, 1169, 1233, 1234, 1235, 1455, 1527, 1534, 1537, 1539, 1541, 1548, 1550, 1628, 1885, 1886, 1889 }

F grade: { 368, 462, 463, 468, 482, 488, 674, 682, 683, 684, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 739, 740, 741, 742, 744, 745, 749, 753, 754, 755, 761, 762, 771, 783, 784, 789, 790, 796, 809, 810, 811, 819, 827, 831, 832, 833, 837, 838, 839, 844, 845, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 898, 899, 900, 901, 902, 903, 904, 906, 907, 908, 909, 910, 911, 912, 914, 915, 916, 917, 918, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 992, 993, 994, 995, 996, 997, 998, 1063, 1073, 1074, 1075, 1085, 1086, 1087, 1088, 1089, 1090, 1099, 1100, 1101, 1102, 1103, 1115, 1124, 1125, 1126, 1135, 1136, 1137, 1138, 1139, 1143, 1144, 1145, 1146, 1147, 1151, 1152, 1153, 1154, 1157, 1158, 1159, 1168, 1170, 1171, 1172, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1231, 1232, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1362, 1363, 1364, 1365, 1384, 1385, 1386, 1395, 1396, 1397, 1398, 1407, 1408, 1409, 1410, 1420, 1421, 1422, 1423, 1431, 1432, 1433, 1441, 1442, 1443, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1526, 1528, 1529, 1531, 1533, 1535, 1536, 1538, 1540, 1542, 1543, 1545, 1547, 1549, 1551, 1554, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1708, 1709, 1710, 1711, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1775, 1776, 1777, 1778, 1779, 1780, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1801, 1802, 1803, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1831, 1832, 1833, 1834, 1835, 1836, 1837, 1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1856, 1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1882, 1883, 1884, 1887, 1888, 1890 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 295, 296, 297, 298, 299, 304, 305, 306, 307, 308, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 495, 496, 503, 504, 511, 512, 519, 520, 574, 575, 576, 577, 582, 583, 595, 613, 614, 615, 634, 647, 648, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 703, 734, 735, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 788, 789, 790, 796, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 836, 837, 838, 844, 852, 853, 854, 855, 856, 860, 861, 862, 863, 864, 869, 870, 871, 872, 873, 874, 877, 878, 879, 880, 881, 882, 885, 886, 887, 888, 889, 890, 893, 894, 895, 896, 897, 901, 902, 903, 904, 905, 914, 926, 933, 934, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1020, 1021, 1022, 1024, 1025, 1026, 1028, 1029, 1030, 1031, 1032, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1054, 1055, 1056, 1057, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1076, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1123, 1124, 1125, 1126, 1127, 1136, 1139, 1140, 1153, 1154, 1155, 1156, 1160, 1161, 1162, 1163, 1164, 1178, 1184, 1213, 1225, 1230, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1248, 1249, 1250, 1251, 1252, 1253, 1255, 1256, 1257, 1263, 1264, 1266, 1267, 1270, 1271, 1272, 1282, 1311, 1335, 1336, 1337, 1338, 1339, 1341, 1346, 1347, 1348, 1349, 1350, 1354, 1355, 1356, 1357, 1358, 1368, 1369, 1370, 1371, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1392, 1393, 1394, 1395, 1396, 1397, 1405, 1406, 1407, 1408, 1409, 1411, 1412, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1426, 1427, 1428, 1429, 1430, 1431, 1436, 1437, 1438, 1439, 1440, 1451, 1452, 1454, 1456, 1457, 1458, 1459, 1463, 1464, 1492, 1493, 1494, 1495, 1496, 1504, 1505, 1506, 1522, 1523, 1524, 1525, 1526, 1528, 1529, 1530, 1531, 1532, 1533, 1535, 1536, 1537, 1538, 1539, 1540, 1543, 1544, 1545, 1546, 1547, 1549, 1552, 1555, 1556, 1557, 1558, 1855, 1881, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917 }

B grade: { 37, 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 212, 226, 227, 243, 244, 292, 293, 294, 300, 301, 302, 303, 309, 310, 311, 312, 313, 314, 315, 316, 385, 386, 387, 435, 442, 494, 502, 510, 518, 573, 578, 579, 580, 581, 584, 585, 586, 587, 588, 589, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 612, 618, 619, 620, 621, 622, 625, 626, 627, 628, 633, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 699, 700, 701, 702, 732, 733, 784, 923, 924, 925, 930, 931, 938, 987, 989, 999, 1007, 1015, 1016, 1017, 1018, 1019, 1023, 1033, 1053, 1058, 1075, 1077, 1088, 1089, 1090, 1091, 1092, 1110, 1118, 1119, 1120, 1121, 1122, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1137, 1138, 1141, 1142, 1143, 1144, 1149, 1150, 1151, 1152, 1157, 1158, 1159, 1165, 1166, 1167, 1168, 1229, 1236, 1237, 1246, 1247, 1254, 1258, 1259, 1260, 1261, 1262, 1265, 1268, 1269, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1342, 1343, 1344, 1345, 1351, 1352, 1353, 1359, 1360, 1362, 1363, 1364, 1365, 1366, 1367, 1372, 1373, 1374, 1375, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1410, 1413, 1423, 1424, 1425, 1432, 1433, 1434, 1435, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1453, 1460, 1461, 1462, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1527, 1534, 1548, 1550, 1551, 1554, 1844, 1846, 1847, 1848, 1852, 1853, 1854 }

C grade: { 1027, 1541, 1542 }

F grade: { 368, 369, 489, 490, 491, 492, 493, 497, 498, 499, 500, 501, 505, 506, 507, 508, 509, 513, 514, 515, 516, 517, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 590, 591, 592, 593, 608, 609, 610, 611, 616, 617, 623, 624, 629, 630, 631, 632, 635, 649, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 785, 786, 787, 791, 792, 793, 794, 795, 797, 798, 799, 800, 801, 802, 803, 833, 834, 835, 839, 840, 841, 842, 843, 845, 846, 847, 848, 849, 850, 851, 857, 858, 859, 865, 866, 867, 868, 875, 876, 883, 884, 891, 892, 898, 899, 900, 906, 907, 908, 909, 910, 911, 912, 913, 915, 916, 917, 918, 919, 920, 921, 922, 927, 928, 929, 932, 935, 936, 937, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 988, 990, 991, 992, 993, 994, 995, 996, 997, 998, 1145, 1146, 1147, 1148, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1179, 1180, 1181, 1182, 1183, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1226, 1227, 1228, 1231, 1232, 1233, 1234, 1235, 1340, 1361, 1455, 1553, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1708, 1709, 1710, 1711, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1775, 1776, 1777, 1778, 1779, 1780, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1801, 1802, 1803, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1831, 1832, 1833, 1834, 1835, 1836, 1837, 1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1856, 1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1890 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	4	0	1
normalized size	1	1.	1.	2.	1.	4.	0.	1.
time (sec)	N/A	0.	0.	0.	1.012	1.615	0.011	1.081

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	4	0	1
normalized size	1	1.	1.	2.	1.	4.	0.	1.
time (sec)	N/A	0.	0.	0.	0.94	1.565	0.017	1.094

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	7	2	4
normalized size	1	1.	1.	1.33	1.33	2.33	0.67	1.33
time (sec)	N/A	0.	0.	0.	0.944	1.615	0.016	1.09

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	8	3	4
normalized size	1	1.	1.	1.33	1.33	2.67	1.	1.33
time (sec)	N/A	0.	0.	0.	0.943	1.637	0.017	1.081

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	4	11	5	4
normalized size	1	1.	1.	0.8	0.8	2.2	1.	0.8
time (sec)	N/A	0.	0.	0.	0.961	1.659	0.016	1.124

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	8	2	4
normalized size	1	1.	1.	1.33	1.33	2.67	0.67	1.33
time (sec)	N/A	0.001	0.	0.001	0.94	1.473	0.016	1.125

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	7	2	4
normalized size	1	1.	1.	1.33	1.33	2.33	0.67	1.33
time (sec)	N/A	0.	0.	0.	0.924	1.289	0.016	1.102

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	9	3	5
normalized size	1	1.	1.	1.25	1.25	2.25	0.75	1.25
time (sec)	N/A	0.	0.	0.001	0.942	1.239	0.017	1.094

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	15	46	10	15
normalized size	1	1.	1.	0.86	1.07	3.29	0.71	1.07
time (sec)	N/A	0.008	0.	0.	0.944	1.547	0.045	1.112

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	18	3	7
normalized size	1	1.	1.	0.86	1.	2.57	0.43	1.
time (sec)	N/A	0.	0.	0.	1.012	1.291	0.049	1.092

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	12	3	7
normalized size	1	1.	1.	0.86	1.	1.71	0.43	1.
time (sec)	N/A	0.	0.	0.	0.948	1.262	0.048	1.089

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	12	3	7
normalized size	1	1.	1.	0.86	1.	1.71	0.43	1.
time (sec)	N/A	0.	0.	0.002	0.977	1.324	0.048	1.274

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	12	3	7
normalized size	1	1.	1.	0.86	1.	1.71	0.43	1.
time (sec)	N/A	0.	0.	0.	0.961	1.344	0.018	1.273

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	4	0	1
normalized size	1	1.	1.	2.	1.	4.	0.	1.
time (sec)	N/A	0.	0.	0.	0.968	1.376	0.017	1.094

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	11	2	4
normalized size	1	1.	1.	1.5	1.5	5.5	1.	2.
time (sec)	N/A	0.	0.	0.	0.973	1.624	0.05	1.102

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	8	3	7
normalized size	1	1.	1.	1.2	1.4	1.6	0.6	1.4
time (sec)	N/A	0.001	0.	0.001	0.97	1.451	0.051	1.107

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	14	7	7
normalized size	1	1.	1.	0.86	1.	2.	1.	1.
time (sec)	N/A	0.	0.	0.	0.971	1.457	0.052	1.089

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	14	7	7
normalized size	1	1.	1.	0.86	1.	2.	1.	1.
time (sec)	N/A	0.	0.	0.	0.971	1.457	0.051	1.157

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	16	7	7
normalized size	1	1.	1.	0.86	1.	2.29	1.	1.
time (sec)	N/A	0.	0.	0.	0.949	1.621	0.051	1.112

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	7	18	7	7
normalized size	1	1.	1.	0.67	0.78	2.	0.78	0.78
time (sec)	N/A	0.001	0.001	0.002	0.977	1.588	0.051	1.106

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	7	18	7	7
normalized size	1	1.	1.	0.67	0.78	2.	0.78	0.78
time (sec)	N/A	0.	0.002	0.001	0.972	1.375	0.051	1.094

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	7	18	7	7
normalized size	1	1.	1.	0.67	0.78	2.	0.78	0.78
time (sec)	N/A	0.	0.001	0.	1.038	1.551	0.051	1.123

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	15	5	7
normalized size	1	1.	1.	0.86	1.	2.14	0.71	1.
time (sec)	N/A	0.	0.001	0.002	1.059	1.518	0.052	1.111

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	16	7	7
normalized size	1	1.	1.	0.86	1.	2.29	1.	1.
time (sec)	N/A	0.	0.002	0.001	1.03	1.577	0.053	1.108

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	7	19	8	7
normalized size	1	1.	1.	0.67	0.78	2.11	0.89	0.78
time (sec)	N/A	0.	0.001	0.002	1.03	1.569	0.052	1.091

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	7	18	7	7
normalized size	1	1.	1.	0.67	0.78	2.	0.78	0.78
time (sec)	N/A	0.001	0.001	0.001	1.035	1.628	0.052	1.103

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	7	18	7	7
normalized size	1	1.	1.	0.67	0.78	2.	0.78	0.78
time (sec)	N/A	0.	0.001	0.002	0.996	1.585	0.051	1.125

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	7	18	7	7
normalized size	1	1.	1.	0.67	0.78	2.	0.78	0.78
time (sec)	N/A	0.	0.001	0.001	1.047	1.509	0.051	1.095

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	7	18	7	7
normalized size	1	1.	1.	0.67	0.78	2.	0.78	0.78
time (sec)	N/A	0.	0.002	0.002	1.023	1.538	0.051	1.096

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	7	18	7	7
normalized size	1	1.	1.	0.67	0.78	2.	0.78	0.78
time (sec)	N/A	0.	0.002	0.002	1.017	1.345	0.052	1.115

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	15	5	7
normalized size	1	1.	1.	0.86	1.	2.14	0.71	1.
time (sec)	N/A	0.	0.001	0.001	1.032	1.401	0.051	1.1

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	16	7	7
normalized size	1	1.	1.	0.86	1.	2.29	1.	1.
time (sec)	N/A	0.	0.001	0.002	0.998	1.431	0.054	1.101

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	7	19	8	7
normalized size	1	1.	1.	0.67	0.78	2.11	0.89	0.78
time (sec)	N/A	0.	0.001	0.002	0.983	1.586	0.052	1.197

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	20	12	15
normalized size	1	1.	1.	1.09	0.	1.82	1.09	1.36
time (sec)	N/A	0.002	0.001	0.001	0.	1.664	0.053	1.169

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	0	26	17	22
normalized size	1	1.	0.75	0.81	0.	1.62	1.06	1.38
time (sec)	N/A	0.003	0.007	0.003	0.	1.73	0.055	1.226

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	28	47	19	30
normalized size	1	1.	1.	0.96	1.22	2.04	0.83	1.3
time (sec)	N/A	0.014	0.009	0.001	1.027	1.765	0.07	1.183

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	220	267	263
normalized size	1	1.	1.	0.87	1.13	9.57	11.61	11.43
time (sec)	N/A	0.012	0.032	0.002	1.009	1.576	81.539	1.282

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	131	153	26
normalized size	1	1.	1.	0.87	1.13	5.7	6.65	1.13
time (sec)	N/A	0.011	0.015	0.001	1.027	1.605	5.32	1.167

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	47	78	26
normalized size	1	1.	1.	0.87	1.13	2.04	3.39	1.13
time (sec)	N/A	0.011	0.015	0.003	1.035	1.573	0.384	1.158

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	26	42	31	26
normalized size	1	1.	1.	0.95	1.24	2.	1.48	1.24
time (sec)	N/A	0.01	0.013	0.002	1.036	1.498	1.195	1.132

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	26	76	58	26
normalized size	1	1.	1.	0.95	1.24	3.62	2.76	1.24
time (sec)	N/A	0.011	0.013	0.001	1.093	1.485	1.393	1.167

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	146	102	26
normalized size	1	1.	1.	0.87	1.13	6.35	4.43	1.13
time (sec)	N/A	0.011	0.03	0.002	1.058	1.576	6.856	1.144

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.008	0.001	0.001	1.043	1.409	0.081	1.235

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.007	0.002	0.002	1.019	1.342	0.088	1.193

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.006	0.001	0.001	1.049	1.293	0.07	1.178

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	23	8	14
normalized size	1	1.	1.	0.92	1.17	1.92	0.67	1.17
time (sec)	N/A	0.002	0.	0.001	1.025	1.391	0.068	1.185

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	22	7	12
normalized size	1	1.	1.	1.12	1.38	2.75	0.88	1.5
time (sec)	N/A	0.003	0.001	0.002	1.034	1.554	0.117	1.151

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	27	7	16
normalized size	1	1.	1.	1.09	1.36	2.45	0.64	1.45
time (sec)	N/A	0.004	0.002	0.004	1.019	1.534	0.379	1.16

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	15	14	15	30	12	15
normalized size	1	1.	0.88	0.82	0.88	1.76	0.71	0.88
time (sec)	N/A	0.002	0.002	0.004	1.054	1.554	0.384	1.171

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	32	14	18
normalized size	1	1.	1.	0.82	1.06	1.88	0.82	1.06
time (sec)	N/A	0.005	0.002	0.005	1.036	1.627	0.424	1.18

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	34	14	18
normalized size	1	1.	1.	0.82	1.06	2.	0.82	1.06
time (sec)	N/A	0.005	0.002	0.006	1.014	1.364	0.354	1.212

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	26	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.87	1.07
time (sec)	N/A	0.012	0.002	0.	1.006	1.326	0.074	1.162

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	24	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.8	1.07
time (sec)	N/A	0.01	0.003	0.001	1.056	1.358	0.075	1.192

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	26	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.87	1.07
time (sec)	N/A	0.009	0.001	0.001	1.029	1.284	0.088	1.218

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	27	42	19	16
normalized size	1	1.	1.	0.93	1.93	3.	1.36	1.14
time (sec)	N/A	0.001	0.001	0.	1.031	1.328	0.067	1.213

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	49	20	28
normalized size	1	1.	1.	0.95	1.23	2.23	0.91	1.27
time (sec)	N/A	0.006	0.001	0.003	1.073	1.482	0.329	1.135

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	27	49	17	28
normalized size	1	1.	1.	1.05	1.35	2.45	0.85	1.4
time (sec)	N/A	0.008	0.001	0.006	1.024	1.596	0.304	1.158

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	28	59	20	30
normalized size	1	1.	1.	0.96	1.17	2.46	0.83	1.25
time (sec)	N/A	0.008	0.004	0.005	1.029	1.515	0.337	1.124

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	26	25	30	51	24	30
normalized size	1	1.	1.53	1.47	1.76	3.	1.41	1.76
time (sec)	N/A	0.002	0.007	0.005	1.	1.462	0.37	1.125

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	26	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.87	1.07
time (sec)	N/A	0.008	0.005	0.005	1.024	1.502	0.355	1.165

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	58	26	32
normalized size	1	1.	1.	0.83	1.07	1.93	0.87	1.07
time (sec)	N/A	0.008	0.008	0.005	1.01	1.496	0.464	1.199

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	59	26	32
normalized size	1	1.	1.	0.83	1.07	1.97	0.87	1.07
time (sec)	N/A	0.008	0.005	0.005	1.125	1.483	0.435	1.151

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	61	26	32
normalized size	1	1.	1.	0.83	1.07	2.03	0.87	1.07
time (sec)	N/A	0.008	0.011	0.005	1.07	1.488	0.462	1.185

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	80	37	47
normalized size	1	1.	1.	0.84	1.09	1.86	0.86	1.09
time (sec)	N/A	0.018	0.002	0.	1.017	1.3	0.073	1.177

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	80	37	47
normalized size	1	1.	1.	0.84	1.09	1.86	0.86	1.09
time (sec)	N/A	0.016	0.002	0.001	1.007	1.408	0.069	1.112

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	80	39	47
normalized size	1	1.	1.	0.84	1.09	1.86	0.91	1.09
time (sec)	N/A	0.015	0.002	0.001	1.035	1.364	0.067	1.163

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	40	35	46	74	36	46
normalized size	1	1.	1.33	1.17	1.53	2.47	1.2	1.53
time (sec)	N/A	0.009	0.002	0.	1.038	1.367	0.066	1.148

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	42	66	32	16
normalized size	1	1.	1.	0.93	3.	4.71	2.29	1.14
time (sec)	N/A	0.002	0.001	0.	1.051	1.335	0.069	1.167

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	42	73	34	43
normalized size	1	1.	1.	0.91	1.2	2.09	0.97	1.23
time (sec)	N/A	0.011	0.005	0.002	1.042	1.503	0.311	1.161

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	43	78	31	45
normalized size	1	1.	1.	0.97	1.26	2.29	0.91	1.32
time (sec)	N/A	0.013	0.006	0.005	1.071	1.508	0.316	1.173

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	32	41	81	31	42
normalized size	1	1.	1.	0.97	1.24	2.45	0.94	1.27
time (sec)	N/A	0.013	0.007	0.005	1.021	1.576	0.444	1.15

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	46	85	34	47
normalized size	1	1.	1.	0.92	1.24	2.3	0.92	1.27
time (sec)	N/A	0.012	0.004	0.006	1.043	1.586	0.564	1.197

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	39	36	45	73	36	45
normalized size	1	1.	2.29	2.12	2.65	4.29	2.12	2.65
time (sec)	N/A	0.002	0.005	0.005	1.011	1.613	0.448	1.19

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	41	36	47	81	37	47
normalized size	1	1.	1.14	1.	1.31	2.25	1.03	1.31
time (sec)	N/A	0.005	0.007	0.004	1.073	1.532	0.438	1.131

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	82	37	47
normalized size	1	1.	1.	0.84	1.09	1.91	0.86	1.09
time (sec)	N/A	0.013	0.004	0.006	1.044	1.518	0.553	1.107

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	84	37	47
normalized size	1	1.	1.	0.84	1.09	1.95	0.86	1.09
time (sec)	N/A	0.013	0.004	0.006	1.116	1.622	0.56	1.166

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	76	131	63	76
normalized size	1	1.	1.	0.86	1.15	1.98	0.95	1.15
time (sec)	N/A	0.032	0.003	0.001	1.012	1.29	0.087	1.186

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	134	65	77
normalized size	1	1.	1.	0.84	1.12	1.94	0.94	1.12
time (sec)	N/A	0.028	0.002	0.	1.029	1.325	0.085	1.143

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	132	66	77
normalized size	1	1.	1.	0.84	1.12	1.91	0.96	1.12
time (sec)	N/A	0.025	0.006	0.001	1.025	1.231	0.082	1.19

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	66	57	76	124	63	76
normalized size	1	1.	1.03	0.89	1.19	1.94	0.98	1.19
time (sec)	N/A	0.026	0.003	0.	1.071	1.35	0.077	1.156

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	67	58	77	126	65	77
normalized size	1	1.	1.43	1.23	1.64	2.68	1.38	1.64
time (sec)	N/A	0.021	0.005	0.001	1.007	1.399	0.082	1.178

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	67	58	77	126	65	77
normalized size	1	1.	2.23	1.93	2.57	4.2	2.17	2.57
time (sec)	N/A	0.008	0.002	0.	1.033	1.355	0.076	1.166

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	72	116	60	16
normalized size	1	1.	1.	0.93	5.14	8.29	4.29	1.14
time (sec)	N/A	0.002	0.006	0.	1.054	1.351	0.077	1.195

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	72	120	60	73
normalized size	1	1.	1.	0.92	1.22	2.03	1.02	1.24
time (sec)	N/A	0.018	0.005	0.002	1.058	1.586	0.36	1.196

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	55	73	134	56	74
normalized size	1	1.	1.	0.95	1.26	2.31	0.97	1.28
time (sec)	N/A	0.021	0.005	0.006	1.035	1.562	0.337	1.148

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	72	132	58	73
normalized size	1	1.	1.	0.92	1.2	2.2	0.97	1.22
time (sec)	N/A	0.022	0.006	0.006	1.039	1.477	0.383	1.183

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	74	132	58	76
normalized size	1	1.	1.	0.92	1.23	2.2	0.97	1.27
time (sec)	N/A	0.022	0.006	0.005	1.022	1.487	0.481	1.232

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	54	73	136	56	74
normalized size	1	1.	1.	0.95	1.28	2.39	0.98	1.3
time (sec)	N/A	0.02	0.006	0.006	1.084	1.565	0.522	1.194

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	76	140	58	77
normalized size	1	1.	1.	0.92	1.25	2.3	0.95	1.26
time (sec)	N/A	0.021	0.005	0.006	1.03	1.629	0.62	1.206

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	65	58	74	120	60	74
normalized size	1	1.	3.82	3.41	4.35	7.06	3.53	4.35
time (sec)	N/A	0.002	0.009	0.006	1.057	1.518	0.614	1.206

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	67	58	77	128	61	77
normalized size	1	1.	1.86	1.61	2.14	3.56	1.69	2.14
time (sec)	N/A	0.005	0.009	0.006	1.04	1.396	0.743	1.194

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	67	58	77	135	61	77
normalized size	1	1.	1.2	1.04	1.38	2.41	1.09	1.38
time (sec)	N/A	0.01	0.006	0.007	1.091	1.466	0.728	1.191

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	77	136	61	77
normalized size	1	1.	1.	0.87	1.15	2.03	0.91	1.15
time (sec)	N/A	0.022	0.012	0.006	1.051	1.574	0.758	1.188

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	144	61	77
normalized size	1	1.	1.	0.84	1.12	2.09	0.88	1.12
time (sec)	N/A	0.021	0.006	0.006	1.051	1.319	0.738	1.175

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	146	61	77
normalized size	1	1.	1.	0.84	1.12	2.12	0.88	1.12
time (sec)	N/A	0.021	0.004	0.006	1.075	1.563	0.827	1.171

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	77	146	61	77
normalized size	1	1.	1.	0.87	1.15	2.18	0.91	1.15
time (sec)	N/A	0.021	0.007	0.006	0.999	1.367	1.086	1.192

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	77	150	61	77
normalized size	1	1.	1.	0.87	1.15	2.24	0.91	1.15
time (sec)	N/A	0.022	0.004	0.006	1.061	1.43	0.93	1.207

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	107	198	94	107
normalized size	1	1.	1.	0.84	1.13	2.08	0.99	1.13
time (sec)	N/A	0.048	0.003	0.001	1.018	1.384	0.099	1.182

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	107	197	92	107
normalized size	1	1.	1.	0.84	1.13	2.07	0.97	1.13
time (sec)	N/A	0.04	0.004	0.002	1.031	1.292	0.087	1.188

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	107	189	94	107
normalized size	1	1.	1.	0.84	1.13	1.99	0.99	1.13
time (sec)	N/A	0.039	0.004	0.002	1.064	1.391	0.097	1.168

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	92	79	105	185	90	105
normalized size	1	1.	0.96	0.82	1.09	1.93	0.94	1.09
time (sec)	N/A	0.04	0.002	0.	1.069	1.353	0.082	1.223

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	93	80	107	186	92	107
normalized size	1	1.	1.15	0.99	1.32	2.3	1.14	1.32
time (sec)	N/A	0.036	0.005	0.001	1.045	1.362	0.087	1.227

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	93	80	107	181	92	107
normalized size	1	1.	1.45	1.25	1.67	2.83	1.44	1.67
time (sec)	N/A	0.03	0.003	0.001	1.019	1.499	0.088	1.168

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	93	80	107	181	92	107
normalized size	1	1.	1.98	1.7	2.28	3.85	1.96	2.28
time (sec)	N/A	0.024	0.003	0.	1.043	1.401	0.09	1.189

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	91	80	107	174	90	107
normalized size	1	1.	3.03	2.67	3.57	5.8	3.	3.57
time (sec)	N/A	0.008	0.003	0.002	1.02	1.381	0.082	1.21

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	159	83	16
normalized size	1	1.	1.	0.93	1.14	11.36	5.93	1.14
time (sec)	N/A	0.002	0.001	0.001	1.061	1.334	0.104	1.228

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	76	101	176	88	103
normalized size	1	1.	1.	0.87	1.16	2.02	1.01	1.18
time (sec)	N/A	0.027	0.004	0.004	1.032	1.491	0.352	1.198

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	77	103	189	85	104
normalized size	1	1.	1.	0.9	1.2	2.2	0.99	1.21
time (sec)	N/A	0.032	0.005	0.006	1.031	1.799	0.43	1.173

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	77	101	188	83	103
normalized size	1	1.	1.	0.92	1.2	2.24	0.99	1.23
time (sec)	N/A	0.033	0.006	0.004	1.027	1.809	0.448	1.163

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	77	104	185	85	105
normalized size	1	1.	1.	0.9	1.21	2.15	0.99	1.22
time (sec)	N/A	0.032	0.01	0.005	1.085	1.821	0.526	1.181

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	77	104	185	83	105
normalized size	1	1.	1.	0.9	1.21	2.15	0.97	1.22
time (sec)	N/A	0.032	0.015	0.008	1.008	1.768	0.592	1.171

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	77	104	188	82	105
normalized size	1	1.	1.	0.92	1.24	2.24	0.98	1.25
time (sec)	N/A	0.032	0.008	0.007	1.083	1.82	0.67	1.168

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	76	103	192	80	104
normalized size	1	1.	1.	0.89	1.21	2.26	0.94	1.22
time (sec)	N/A	0.031	0.009	0.007	1.01	1.707	0.734	1.162

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	78	105	200	82	107
normalized size	1	1.	1.	0.88	1.18	2.25	0.92	1.2
time (sec)	N/A	0.033	0.01	0.008	1.034	1.737	0.786	1.19

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	87	80	104	166	83	104
normalized size	1	1.	5.12	4.71	6.12	9.76	4.88	6.12
time (sec)	N/A	0.002	0.007	0.007	1.018	1.744	0.852	1.212

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	91	80	107	180	85	107
normalized size	1	1.	2.53	2.22	2.97	5.	2.36	2.97
time (sec)	N/A	0.005	0.004	0.005	1.04	1.73	0.826	1.194

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	93	80	107	190	85	107
normalized size	1	1.	1.66	1.43	1.91	3.39	1.52	1.91
time (sec)	N/A	0.01	0.005	0.005	1.059	1.776	0.95	1.207

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	93	80	107	196	85	107
normalized size	1	1.	1.22	1.05	1.41	2.58	1.12	1.41
time (sec)	N/A	0.016	0.008	0.006	1.109	1.789	0.953	1.212

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	93	80	107	201	85	107
normalized size	1	1.	0.97	0.83	1.11	2.09	0.89	1.11
time (sec)	N/A	0.026	0.006	0.006	1.018	1.753	0.959	1.187

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	80	107	207	85	107
normalized size	1	1.	1.	0.86	1.15	2.23	0.91	1.15
time (sec)	N/A	0.033	0.007	0.006	1.02	1.813	1.068	1.187

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	107	209	85	107
normalized size	1	1.	1.	0.84	1.13	2.2	0.89	1.13
time (sec)	N/A	0.03	0.005	0.008	1.067	1.653	1.118	1.222

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	107	213	85	107
normalized size	1	1.	1.	0.84	1.13	2.24	0.89	1.13
time (sec)	N/A	0.03	0.004	0.006	1.038	1.627	1.093	1.208

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	132	113	151	288	133	151
normalized size	1	1.	1.	0.86	1.14	2.18	1.01	1.14
time (sec)	N/A	0.076	0.006	0.001	1.064	1.509	0.1	1.145

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	132	113	151	282	131	151
normalized size	1	1.	1.	0.86	1.14	2.14	0.99	1.14
time (sec)	N/A	0.059	0.005	0.	1.06	1.508	0.121	1.178

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	132	113	151	288	133	151
normalized size	1	1.	1.	0.86	1.14	2.18	1.01	1.14
time (sec)	N/A	0.06	0.003	0.002	1.065	1.574	0.109	1.159

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	125	112	150	269	126	150
normalized size	1	1.	0.85	0.76	1.02	1.83	0.86	1.02
time (sec)	N/A	0.064	0.004	0.001	1.03	1.576	0.111	1.212

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	130	113	151	278	131	151
normalized size	1	1.	0.98	0.86	1.14	2.11	0.99	1.14
time (sec)	N/A	0.057	0.013	0.001	1.082	1.57	0.126	1.194

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	126	113	151	266	128	151
normalized size	1	1.	1.12	1.01	1.35	2.38	1.14	1.35
time (sec)	N/A	0.052	0.007	0.001	1.018	1.533	0.106	1.152

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	132	113	151	278	133	151
normalized size	1	1.	1.35	1.15	1.54	2.84	1.36	1.54
time (sec)	N/A	0.045	0.008	0.002	1.047	1.531	0.128	1.184

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	130	113	151	271	131	151
normalized size	1	1.	1.6	1.4	1.86	3.35	1.62	1.86
time (sec)	N/A	0.039	0.003	0.001	1.045	1.752	0.105	1.217

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	128	113	151	270	129	151
normalized size	1	1.	2.	1.77	2.36	4.22	2.02	2.36
time (sec)	N/A	0.036	0.003	0.001	1.022	1.537	0.106	1.162

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	126	113	151	258	128	151
normalized size	1	1.	2.68	2.4	3.21	5.49	2.72	3.21
time (sec)	N/A	0.03	0.003	0.	1.025	1.71	0.105	1.195

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	128	113	151	263	129	151
normalized size	1	1.	4.27	3.77	5.03	8.77	4.3	5.03
time (sec)	N/A	0.008	0.003	0.001	1.027	1.424	0.111	1.187

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	230	114	16
normalized size	1	1.	1.	0.93	1.14	16.43	8.14	1.14
time (sec)	N/A	0.002	0.002	0.	1.018	1.64	0.094	1.198

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	109	146	259	126	147
normalized size	1	1.	1.	0.89	1.2	2.12	1.03	1.2
time (sec)	N/A	0.043	0.005	0.003	1.043	1.887	0.553	1.131

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	115	110	147	285	117	149
normalized size	1	1.	1.	0.96	1.28	2.48	1.02	1.3
time (sec)	N/A	0.047	0.012	0.007	1.017	1.783	0.485	1.158

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	146	273	121	147
normalized size	1	1.	1.	0.92	1.23	2.29	1.02	1.24
time (sec)	N/A	0.049	0.009	0.006	1.012	1.699	0.514	1.186

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	115	110	146	269	117	147
normalized size	1	1.	1.	0.96	1.27	2.34	1.02	1.28
time (sec)	N/A	0.047	0.012	0.007	1.133	1.772	0.565	1.237

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	149	267	119	150
normalized size	1	1.	1.	0.92	1.25	2.24	1.	1.26
time (sec)	N/A	0.049	0.011	0.006	1.045	1.78	0.641	1.155

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	110	149	266	119	150
normalized size	1	1.	1.	0.94	1.27	2.27	1.02	1.28
time (sec)	N/A	0.052	0.013	0.007	1.02	1.758	0.731	1.187

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	149	267	121	150
normalized size	1	1.	1.	0.92	1.25	2.24	1.02	1.26
time (sec)	N/A	0.049	0.006	0.007	1.074	1.79	0.835	1.196

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	115	110	149	269	117	150
normalized size	1	1.	1.	0.96	1.3	2.34	1.02	1.3
time (sec)	N/A	0.05	0.016	0.007	1.055	1.593	0.996	1.233

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	149	273	117	150
normalized size	1	1.	1.	0.92	1.25	2.29	0.98	1.26
time (sec)	N/A	0.049	0.006	0.007	1.034	1.62	1.104	1.178

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	114	109	147	288	116	149
normalized size	1	1.	1.	0.96	1.29	2.53	1.02	1.31
time (sec)	N/A	0.053	0.008	0.008	1.032	1.909	1.11	1.239

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	124	111	150	302	117	151
normalized size	1	1.	1.	0.9	1.21	2.44	0.94	1.22
time (sec)	N/A	0.048	0.008	0.008	1.065	1.796	1.171	1.178

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	114	113	149	252	119	149
normalized size	1	1.	6.71	6.65	8.76	14.82	7.	8.76
time (sec)	N/A	0.002	0.011	0.007	0.992	1.539	1.314	1.165

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	128	113	151	270	121	151
normalized size	1	1.	3.56	3.14	4.19	7.5	3.36	4.19
time (sec)	N/A	0.005	0.006	0.007	1.029	1.565	1.347	1.247

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	126	113	151	281	121	151
normalized size	1	1.	2.25	2.02	2.7	5.02	2.16	2.7
time (sec)	N/A	0.01	0.016	0.007	1.01	1.496	1.818	1.183

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	128	113	151	292	121	151
normalized size	1	1.	1.68	1.49	1.99	3.84	1.59	1.99
time (sec)	N/A	0.017	0.011	0.006	1.009	1.401	1.874	1.15

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	130	113	151	301	121	151
normalized size	1	1.	1.35	1.18	1.57	3.14	1.26	1.57
time (sec)	N/A	0.025	0.014	0.007	1.087	1.47	1.844	1.157

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	132	113	151	306	121	151
normalized size	1	1.	1.14	0.97	1.3	2.64	1.04	1.3
time (sec)	N/A	0.035	0.005	0.007	1.088	1.569	2.1	1.126

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	126	113	151	315	121	151
normalized size	1	1.	0.93	0.83	1.11	2.32	0.89	1.11
time (sec)	N/A	0.049	0.017	0.009	1.024	1.454	1.923	1.159

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	130	113	151	319	121	151
normalized size	1	1.	1.	0.87	1.16	2.45	0.93	1.16
time (sec)	N/A	0.047	0.007	0.006	1.011	1.456	1.993	1.19

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	126	113	151	324	121	151
normalized size	1	1.	1.	0.9	1.2	2.57	0.96	1.2
time (sec)	N/A	0.049	0.007	0.007	1.069	1.502	2.032	1.192

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	14	13	18	28	12	18
normalized size	1	1.	0.93	0.87	1.2	1.87	0.8	1.2
time (sec)	N/A	0.002	0.001	0.001	0.997	1.263	0.074	1.176

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	18	24	58	22	23
normalized size	1	1.	0.95	0.9	1.2	2.9	1.1	1.15
time (sec)	N/A	0.004	0.001	0.001	1.047	1.539	0.069	1.18

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	63	86	144	61	88
normalized size	1	1.	1.	0.9	1.23	2.06	0.87	1.26
time (sec)	N/A	0.034	0.005	0.002	1.067	1.496	0.373	1.158

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	70	117	49	72
normalized size	1	1.	1.	0.91	1.23	2.05	0.86	1.26
time (sec)	N/A	0.023	0.007	0.001	1.079	1.427	0.368	1.139

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	41	57	92	37	58
normalized size	1	1.	1.	0.93	1.3	2.09	0.84	1.32
time (sec)	N/A	0.02	0.004	0.003	1.03	1.449	0.339	1.184

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	39	68	26	41
normalized size	1	1.	1.	0.97	1.26	2.19	0.84	1.32
time (sec)	N/A	0.014	0.003	0.002	1.048	1.427	0.345	1.173

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	38	14	26
normalized size	1	1.	1.	1.06	1.33	2.11	0.78	1.44
time (sec)	N/A	0.009	0.003	0.002	0.997	1.518	0.365	1.162

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	22	7	15
normalized size	1	1.	1.	1.1	1.4	2.2	0.7	1.5
time (sec)	N/A	0.002	0.001	0.001	1.045	1.462	0.08	1.177

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	38	10	27
normalized size	1	1.	1.	1.06	1.33	2.11	0.56	1.5
time (sec)	N/A	0.004	0.005	0.005	0.997	1.541	0.232	1.187

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	38	61	19	41
normalized size	1	1.	1.	1.04	1.36	2.18	0.68	1.46
time (sec)	N/A	0.013	0.005	0.009	1.035	1.528	0.433	1.185

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	54	103	31	61
normalized size	1	1.	1.	0.98	1.29	2.45	0.74	1.45
time (sec)	N/A	0.018	0.01	0.006	1.045	1.592	0.502	1.174

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	69	126	44	76
normalized size	1	1.	1.	0.95	1.23	2.25	0.79	1.36
time (sec)	N/A	0.021	0.006	0.007	1.072	1.52	0.529	1.213

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	63	84	154	56	90
normalized size	1	1.	1.	0.93	1.24	2.26	0.82	1.32
time (sec)	N/A	0.035	0.008	0.006	1.015	1.525	0.591	1.184

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	78	111	208	78	139
normalized size	1	1.	0.95	0.96	1.37	2.57	0.96	1.72
time (sec)	N/A	0.059	0.036	0.007	1.059	1.489	0.498	1.22

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	66	67	95	186	71	122
normalized size	1	1.	0.92	0.93	1.32	2.58	0.99	1.69
time (sec)	N/A	0.042	0.03	0.006	1.05	1.589	0.457	1.172

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	57	80	155	54	107
normalized size	1	1.	0.93	0.98	1.38	2.67	0.93	1.84
time (sec)	N/A	0.033	0.03	0.006	1.259	1.503	0.487	1.215

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	63	132	44	89
normalized size	1	1.	0.93	0.98	1.37	2.87	0.96	1.93
time (sec)	N/A	0.027	0.017	0.006	1.029	1.585	0.468	1.189

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	49	97	31	68
normalized size	1	1.	0.88	1.03	1.48	2.94	0.94	2.06
time (sec)	N/A	0.018	0.014	0.006	1.073	1.528	0.454	1.135

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	24	35	62	20	57
normalized size	1	1.	0.87	1.04	1.52	2.7	0.87	2.48
time (sec)	N/A	0.012	0.011	0.004	1.026	1.507	0.399	1.175

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	24	10	16
normalized size	1	1.	1.	1.08	1.33	2.	0.83	1.33
time (sec)	N/A	0.002	0.004	0.	1.087	1.468	0.342	1.168

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	30	38	89	22	51
normalized size	1	1.	0.83	1.03	1.31	3.07	0.76	1.76
time (sec)	N/A	0.014	0.015	0.007	1.052	1.398	0.52	1.177

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	61	138	36	70
normalized size	1	1.	0.83	1.02	1.45	3.29	0.86	1.67
time (sec)	N/A	0.021	0.08	0.009	1.08	1.629	0.589	1.23

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	53	57	86	177	54	100
normalized size	1	1.	0.91	0.98	1.48	3.05	0.93	1.72
time (sec)	N/A	0.028	0.081	0.01	1.034	1.639	0.638	1.2

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	68	99	204	66	122
normalized size	1	1.	0.96	0.99	1.43	2.96	0.96	1.77
time (sec)	N/A	0.035	0.092	0.01	1.056	1.704	0.66	1.235

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	79	116	231	80	140
normalized size	1	1.	0.94	0.94	1.38	2.75	0.95	1.67
time (sec)	N/A	0.043	0.079	0.01	1.059	1.498	0.705	1.193

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	89	94	139	284	107	128
normalized size	1	1.	0.9	0.95	1.4	2.87	1.08	1.29
time (sec)	N/A	0.07	0.057	0.007	1.061	1.499	0.731	1.166

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	77	83	123	247	92	112
normalized size	1	1.	0.9	0.97	1.43	2.87	1.07	1.3
time (sec)	N/A	0.053	0.04	0.007	0.994	1.514	0.652	1.206

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	67	72	109	227	83	99
normalized size	1	1.	0.87	0.94	1.42	2.95	1.08	1.29
time (sec)	N/A	0.046	0.035	0.006	1.088	1.392	0.575	1.256

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	55	61	93	200	70	82
normalized size	1	1.	0.86	0.95	1.45	3.12	1.09	1.28
time (sec)	N/A	0.036	0.028	0.007	1.096	1.623	0.622	1.138

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	40	49	77	176	56	59
normalized size	1	1.	0.8	0.98	1.54	3.52	1.12	1.18
time (sec)	N/A	0.026	0.061	0.007	1.062	1.517	0.601	1.155

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	33	40	65	132	46	50
normalized size	1	1.	0.8	0.98	1.59	3.22	1.12	1.22
time (sec)	N/A	0.02	0.019	0.005	1.02	1.567	0.534	1.144

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	20	27	43	68	32	24
normalized size	1	1.	1.18	1.59	2.53	4.	1.88	1.41
time (sec)	N/A	0.002	0.008	0.004	0.989	1.532	0.43	1.201

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	49	26	16
normalized size	1	1.	1.	0.93	1.14	3.5	1.86	1.14
time (sec)	N/A	0.001	0.003	0.	1.025	1.423	0.457	1.207

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	37	42	69	182	46	58
normalized size	1	1.	0.86	0.98	1.6	4.23	1.07	1.35
time (sec)	N/A	0.019	0.041	0.008	1.108	1.448	0.67	1.162

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	53	56	93	232	65	81
normalized size	1	1.	0.93	0.98	1.63	4.07	1.14	1.42
time (sec)	N/A	0.029	0.077	0.01	1.081	1.675	0.643	1.236

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	68	73	116	269	78	99
normalized size	1	1.	0.89	0.96	1.53	3.54	1.03	1.3
time (sec)	N/A	0.036	0.062	0.01	1.032	1.508	0.761	1.236

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	79	84	131	296	92	116
normalized size	1	1.	0.89	0.94	1.47	3.33	1.03	1.3
time (sec)	N/A	0.048	0.132	0.013	1.092	1.543	0.815	1.141

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	90	94	146	313	102	131
normalized size	1	1.	0.93	0.97	1.51	3.23	1.05	1.35
time (sec)	N/A	0.052	0.079	0.011	1.081	1.628	0.815	1.176

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	101	109	169	356	129	143
normalized size	1	1.	0.89	0.96	1.48	3.12	1.13	1.25
time (sec)	N/A	0.085	0.074	0.008	1.041	1.563	0.885	1.149

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	90	98	154	329	119	128
normalized size	1	1.	0.86	0.93	1.47	3.13	1.13	1.22
time (sec)	N/A	0.07	0.051	0.008	1.057	1.569	0.792	1.191

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	87	138	293	105	112
normalized size	1	1.	1.	0.97	1.53	3.26	1.17	1.24
time (sec)	N/A	0.057	0.028	0.006	1.113	1.604	0.758	1.204

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	68	76	123	271	94	97
normalized size	1	1.	0.84	0.94	1.52	3.35	1.16	1.2
time (sec)	N/A	0.048	0.041	0.007	1.055	1.434	0.73	1.213

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	64	107	244	80	74
normalized size	1	1.	0.78	0.98	1.65	3.75	1.23	1.14
time (sec)	N/A	0.036	0.067	0.008	1.08	1.439	0.709	1.198

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	44	55	95	201	70	62
normalized size	1	1.	0.76	0.95	1.64	3.47	1.21	1.07
time (sec)	N/A	0.03	0.019	0.005	1.061	1.516	0.553	1.196

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	31	41	73	111	56	39
normalized size	1	1.	1.82	2.41	4.29	6.53	3.29	2.29
time (sec)	N/A	0.002	0.017	0.004	1.082	1.482	0.516	1.138

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	20	27	58	89	44	24
normalized size	1	1.	0.67	0.9	1.93	2.97	1.47	0.8
time (sec)	N/A	0.013	0.007	0.005	1.034	1.517	0.54	1.204

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	70	37	16
normalized size	1	1.	1.	0.93	1.14	5.	2.64	1.14
time (sec)	N/A	0.002	0.003	0.001	1.071	1.39	0.45	1.166

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	48	54	99	271	70	73
normalized size	1	1.	0.84	0.95	1.74	4.75	1.23	1.28
time (sec)	N/A	0.028	0.057	0.006	1.072	1.578	0.77	1.169

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	64	69	123	325	88	96
normalized size	1	1.	0.91	0.99	1.76	4.64	1.26	1.37
time (sec)	N/A	0.039	0.102	0.009	1.042	1.39	0.793	1.18

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	79	88	146	363	104	116
normalized size	1	1.	0.85	0.95	1.57	3.9	1.12	1.25
time (sec)	N/A	0.048	0.106	0.01	1.042	1.612	0.864	1.195

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	88	99	158	385	114	126
normalized size	1	1.	0.86	0.97	1.55	3.77	1.12	1.24
time (sec)	N/A	0.053	0.097	0.012	1.065	1.617	1.002	1.224

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	101	110	176	419	128	146
normalized size	1	1.	0.86	0.94	1.5	3.58	1.09	1.25
time (sec)	N/A	0.07	0.104	0.012	1.007	1.601	1.021	1.169

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	139	143	243	568	190	173
normalized size	1	1.	0.93	0.95	1.62	3.79	1.27	1.15
time (sec)	N/A	0.135	0.036	0.012	1.082	1.46	1.401	1.187

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	128	132	228	541	178	158
normalized size	1	1.	0.92	0.95	1.64	3.89	1.28	1.14
time (sec)	N/A	0.114	0.049	0.008	1.085	1.434	1.406	1.216

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	104	121	212	514	165	142
normalized size	1	1.	0.81	0.95	1.66	4.02	1.29	1.11
time (sec)	N/A	0.089	0.072	0.01	1.083	1.475	1.305	1.262

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	104	109	196	479	151	119
normalized size	1	1.	0.88	0.92	1.66	4.06	1.28	1.01
time (sec)	N/A	0.071	0.049	0.009	1.084	1.56	1.23	1.192

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	77	100	184	428	141	107
normalized size	1	1.	0.71	0.92	1.69	3.93	1.29	0.98
time (sec)	N/A	0.064	0.038	0.007	1.081	1.542	1.05	1.208

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	64	87	162	252	128	84
normalized size	1	1.	3.76	5.12	9.53	14.82	7.53	4.94
time (sec)	N/A	0.002	0.018	0.007	1.063	1.497	0.99	1.188

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	53	72	147	231	116	69
normalized size	1	1.	1.51	2.06	4.2	6.6	3.31	1.97
time (sec)	N/A	0.005	0.016	0.006	1.073	1.541	0.899	1.154

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	64	42	57	132	207	104	54
normalized size	1	1.23	0.81	1.1	2.54	3.98	2.	1.04
time (sec)	N/A	0.029	0.018	0.005	1.044	1.473	0.873	1.219

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	31	42	117	182	92	39
normalized size	1	1.	0.66	0.89	2.49	3.87	1.96	0.83
time (sec)	N/A	0.021	0.014	0.004	1.072	1.506	0.791	1.157

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	20	27	103	159	80	24
normalized size	1	1.	0.67	0.9	3.43	5.3	2.67	0.8
time (sec)	N/A	0.013	0.011	0.004	1.083	1.51	0.777	1.204

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	139	73	16
normalized size	1	1.	1.	0.93	1.14	9.93	5.21	1.14
time (sec)	N/A	0.002	0.004	0.001	1.066	1.543	0.807	1.198

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	81	90	188	564	141	117
normalized size	1	1.	0.82	0.91	1.9	5.7	1.42	1.18
time (sec)	N/A	0.049	0.105	0.01	1.129	1.631	1.279	1.224

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	97	108	212	628	160	140
normalized size	1	1.	0.83	0.92	1.81	5.37	1.37	1.2
time (sec)	N/A	0.073	0.165	0.011	1.056	1.636	1.341	1.187

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	112	133	235	668	175	161
normalized size	1	1.	0.78	0.92	1.63	4.64	1.22	1.12
time (sec)	N/A	0.089	0.111	0.016	1.115	1.527	1.7	1.194

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	123	144	250	701	187	176
normalized size	1	1.	0.78	0.92	1.59	4.46	1.19	1.12
time (sec)	N/A	0.103	0.236	0.014	1.149	1.624	1.708	1.198

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	161	177	316	811	248	201
normalized size	1	1.	0.87	0.95	1.7	4.36	1.33	1.08
time (sec)	N/A	0.177	0.115	0.011	1.192	1.557	2.248	1.192

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	150	166	301	784	236	186
normalized size	1	1.	0.85	0.94	1.7	4.43	1.33	1.05
time (sec)	N/A	0.142	0.047	0.011	1.13	1.45	2.129	1.248

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	137	154	285	736	223	163
normalized size	1	1.	0.86	0.97	1.79	4.63	1.4	1.03
time (sec)	N/A	0.121	0.071	0.011	1.163	1.511	1.975	1.24

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	111	145	273	684	212	151
normalized size	1	1.	0.72	0.94	1.77	4.44	1.38	0.98
time (sec)	N/A	0.108	0.049	0.01	1.073	1.569	1.632	1.16

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	97	131	251	404	199	128
normalized size	1	1.	5.71	7.71	14.76	23.76	11.71	7.53
time (sec)	N/A	0.002	0.027	0.005	1.11	1.439	1.648	1.211

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	86	117	236	382	187	113
normalized size	1	1.	2.46	3.34	6.74	10.91	5.34	3.23
time (sec)	N/A	0.005	0.02	0.006	1.073	1.498	1.479	1.216

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	75	102	221	359	175	99
normalized size	1	1.	1.44	1.96	4.25	6.9	3.37	1.9
time (sec)	N/A	0.01	0.025	0.006	1.114	1.537	1.414	1.213

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	64	86	207	335	163	84
normalized size	1	1.	0.93	1.25	3.	4.86	2.36	1.22
time (sec)	N/A	0.016	0.023	0.005	1.152	1.55	1.326	1.184

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	53	72	192	309	151	69
normalized size	1	1.	0.65	0.89	2.37	3.81	1.86	0.85
time (sec)	N/A	0.04	0.024	0.005	1.156	1.437	1.249	1.181

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	42	57	177	284	139	54
normalized size	1	1.	0.66	0.89	2.77	4.44	2.17	0.84
time (sec)	N/A	0.03	0.018	0.006	1.099	1.474	1.104	1.217

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	31	42	162	259	128	39
normalized size	1	1.	0.66	0.89	3.45	5.51	2.72	0.83
time (sec)	N/A	0.021	0.015	0.005	1.048	1.48	1.108	1.206

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	20	27	147	234	116	24
normalized size	1	1.	0.67	0.9	4.9	7.8	3.87	0.8
time (sec)	N/A	0.014	0.009	0.005	1.172	1.381	1.072	1.209

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	212	109	16
normalized size	1	1.	1.	0.93	1.14	15.14	7.79	1.14
time (sec)	N/A	0.002	0.003	0.	1.018	1.453	1.051	1.193

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	127	126	277	896	212	162
normalized size	1	1.	0.9	0.89	1.96	6.35	1.5	1.15
time (sec)	N/A	0.075	0.149	0.011	1.138	1.577	2.11	1.196

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	130	147	301	956	231	185
normalized size	1	1.	0.82	0.93	1.91	6.05	1.46	1.17
time (sec)	N/A	0.125	0.162	0.014	1.101	1.644	2.646	1.184

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	145	178	324	1019	246	205
normalized size	1	1.	0.76	0.93	1.7	5.34	1.29	1.07
time (sec)	N/A	0.141	0.239	0.014	1.179	1.643	3.313	1.232

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	156	189	339	1054	258	220
normalized size	1	1.	0.79	0.95	1.71	5.32	1.3	1.11
time (sec)	N/A	0.165	0.171	0.016	1.196	1.678	4.142	1.181

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	141	132	178	346	141	180
normalized size	1	1.	1.	0.94	1.26	2.45	1.	1.28
time (sec)	N/A	0.079	0.019	0.007	1.092	1.553	1.369	1.196

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	132	121	163	320	129	165
normalized size	1	1.	1.	0.92	1.23	2.42	0.98	1.25
time (sec)	N/A	0.07	0.007	0.009	1.062	1.496	1.275	1.224

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	114	109	147	288	116	149
normalized size	1	1.	1.	0.96	1.29	2.53	1.02	1.31
time (sec)	N/A	0.058	0.007	0.	1.075	1.437	1.319	1.19

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	100	135	266	105	136
normalized size	1	1.	1.	0.92	1.24	2.44	0.96	1.25
time (sec)	N/A	0.052	0.007	0.008	1.065	1.462	1.27	1.222

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	96	91	119	192	95	119
normalized size	1	1.	5.65	5.35	7.	11.29	5.59	7.
time (sec)	N/A	0.002	0.015	0.005	1.06	1.457	1.097	1.198

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	91	80	107	180	85	107
normalized size	1	1.	2.53	2.22	2.97	5.	2.36	2.97
time (sec)	N/A	0.006	0.004	0.	1.062	1.477	0.941	1.2

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	80	69	92	159	73	92
normalized size	1	1.	1.43	1.23	1.64	2.84	1.3	1.64
time (sec)	N/A	0.01	0.009	0.005	1.09	1.56	0.863	1.151

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	77	136	61	77
normalized size	1	1.	1.	0.87	1.15	2.03	0.91	1.15
time (sec)	N/A	0.025	0.014	0.	1.043	1.676	0.864	1.192

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	62	112	49	62
normalized size	1	1.	1.	0.84	1.11	2.	0.88	1.11
time (sec)	N/A	0.019	0.011	0.006	1.103	1.639	0.711	1.19

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	86	37	47
normalized size	1	1.	1.	0.84	1.09	2.	0.86	1.09
time (sec)	N/A	0.014	0.004	0.005	1.106	1.705	0.54	1.143

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	61	26	32
normalized size	1	1.	1.	0.83	1.07	2.03	0.87	1.07
time (sec)	N/A	0.009	0.008	0.005	1.119	1.748	0.519	1.203

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	34	14	18
normalized size	1	1.	1.	0.82	1.06	2.	0.82	1.06
time (sec)	N/A	0.005	0.004	0.003	1.03	1.725	0.413	1.211

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	14	7	7
normalized size	1	1.	1.	0.86	1.	2.	1.	1.
time (sec)	N/A	0.	0.	0.	1.077	1.556	0.07	1.21

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	134	119	158	296	116	165
normalized size	1	1.	1.	0.89	1.18	2.21	0.87	1.23
time (sec)	N/A	0.061	0.006	0.01	1.079	1.786	1.006	1.194

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	134	135	190	377	139	243
normalized size	1	1.	0.92	0.92	1.3	2.58	0.95	1.66
time (sec)	N/A	0.09	0.15	0.013	1.114	1.797	1.257	1.232

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	145	150	220	490	163	205
normalized size	1	1.	0.89	0.92	1.35	3.01	1.	1.26
time (sec)	N/A	0.108	0.14	0.013	1.074	1.921	1.713	1.207

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	43	12	20
normalized size	1	1.	1.	0.82	1.06	2.53	0.71	1.18
time (sec)	N/A	0.002	0.005	0.004	1.038	1.724	0.123	1.224

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	43	12	20
normalized size	1	1.	1.	0.82	1.06	2.53	0.71	1.18
time (sec)	N/A	0.002	0.004	0.002	1.066	1.701	0.111	1.245

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	24	58	20	27
normalized size	1	1.	1.	0.79	1.	2.42	0.83	1.12
time (sec)	N/A	0.008	0.003	0.007	1.033	1.73	0.123	1.177

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	31	77	26	34
normalized size	1	1.	1.	0.77	1.	2.48	0.84	1.1
time (sec)	N/A	0.01	0.003	0.007	0.992	1.745	0.167	1.167

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	29	38	92	31	41
normalized size	1	1.	1.	0.76	1.	2.42	0.82	1.08
time (sec)	N/A	0.012	0.007	0.005	1.039	1.799	0.213	1.142

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	45	104	36	47
normalized size	1	1.	1.	0.76	1.	2.31	0.8	1.04
time (sec)	N/A	0.012	0.003	0.006	1.025	1.698	0.193	1.204

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	23	30	88	19	34
normalized size	1	1.	0.93	0.82	1.07	3.14	0.68	1.21
time (sec)	N/A	0.009	0.023	0.006	1.082	1.687	0.114	1.196

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	28	42	116	29	54
normalized size	1	1.	0.89	0.8	1.2	3.31	0.83	1.54
time (sec)	N/A	0.01	0.016	0.01	1.018	1.735	0.164	1.21

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	33	51	140	36	69
normalized size	1	1.	0.86	0.79	1.21	3.33	0.86	1.64
time (sec)	N/A	0.014	0.019	0.007	1.081	1.735	0.268	1.224

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	38	58	155	41	81
normalized size	1	1.	0.9	0.78	1.18	3.16	0.84	1.65
time (sec)	N/A	0.016	0.045	0.007	1.024	1.718	0.164	1.229

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	43	65	171	46	93
normalized size	1	1.	1.	0.77	1.16	3.05	0.82	1.66
time (sec)	N/A	0.02	0.018	0.01	1.045	1.836	0.173	1.16

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	32	41	132	27	36
normalized size	1	1.	0.74	0.82	1.05	3.38	0.69	0.92
time (sec)	N/A	0.012	0.035	0.007	1.055	1.667	0.213	1.196

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	39	37	55	169	39	50
normalized size	1	1.	0.85	0.8	1.2	3.67	0.85	1.09
time (sec)	N/A	0.015	0.034	0.009	1.06	1.762	0.165	1.236

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	44	42	65	193	46	58
normalized size	1	1.	0.83	0.79	1.23	3.64	0.87	1.09
time (sec)	N/A	0.016	0.037	0.011	1.044	1.802	0.239	1.233

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	49	47	72	215	51	63
normalized size	1	1.	0.82	0.78	1.2	3.58	0.85	1.05
time (sec)	N/A	0.021	0.032	0.01	1.009	1.795	0.204	1.198

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	52	78	232	56	70
normalized size	1	1.	0.81	0.78	1.16	3.46	0.84	1.04
time (sec)	N/A	0.022	0.032	0.01	1.073	1.552	0.189	1.182

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	10	9	8	22	7	9
normalized size	1	1.	1.25	1.12	1.	2.75	0.88	1.12
time (sec)	N/A	0.001	0.001	0.	1.024	1.535	0.066	1.164

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	26	8	12
normalized size	1	1.	1.	0.9	1.1	2.6	0.8	1.2
time (sec)	N/A	0.001	0.002	0.	1.029	1.491	0.086	1.15

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	16	35	14	18
normalized size	1	1.	1.14	0.93	1.14	2.5	1.	1.29
time (sec)	N/A	0.002	0.004	0.001	1.08	1.604	0.068	1.205

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	42	17	23
normalized size	1	1.	1.	0.85	1.1	2.1	0.85	1.15
time (sec)	N/A	0.007	0.006	0.001	1.061	1.424	0.07	1.179

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	46	19	26
normalized size	1	1.	1.	0.86	1.09	2.09	0.86	1.18
time (sec)	N/A	0.002	0.004	0.001	1.021	1.471	0.094	1.2

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	49	19	26
normalized size	1	1.	1.	0.86	1.09	2.23	0.86	1.18
time (sec)	N/A	0.002	0.009	0.	1.02	1.39	0.119	1.169

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	19	24	47	19	26
normalized size	1	1.	1.	0.9	1.14	2.24	0.9	1.24
time (sec)	N/A	0.004	0.013	0.	1.088	1.522	0.093	1.22

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	22	19	24	47	20	26
normalized size	1	1.	1.1	0.95	1.2	2.35	1.	1.3
time (sec)	N/A	0.004	0.018	0.001	1.036	1.454	0.11	1.273

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	32	8	18
normalized size	1	1.	1.	1.09	1.36	2.91	0.73	1.64
time (sec)	N/A	0.002	0.006	0.007	1.044	1.461	0.197	1.187

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	15	31	8	18
normalized size	1	1.	1.	1.	1.25	2.58	0.67	1.5
time (sec)	N/A	0.002	0.003	0.006	1.079	1.492	0.131	1.131

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	26	53	14	28
normalized size	1	1.	1.	1.05	1.37	2.79	0.74	1.47
time (sec)	N/A	0.008	0.004	0.007	1.099	1.505	0.333	1.171

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	23	53	14	26
normalized size	1	1.	1.	1.	1.28	2.94	0.78	1.44
time (sec)	N/A	0.01	0.003	0.008	1.099	1.525	0.166	1.236

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	35	10	20
normalized size	1	1.	1.	1.07	1.36	2.5	0.71	1.43
time (sec)	N/A	0.009	0.006	0.002	1.078	1.506	0.354	1.174

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	76	120	1742	66
normalized size	1	1.	0.64	0.6	1.06	1.67	24.19	0.92
time (sec)	N/A	0.018	0.032	0.004	1.09	1.468	3.27	1.188

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	55	97	666	50
normalized size	1	1.	0.66	0.6	1.04	1.83	12.57	0.94
time (sec)	N/A	0.013	0.025	0.005	1.053	1.532	2.235	1.177

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	35	70	202	34
normalized size	1	1.	0.71	0.62	1.03	2.06	5.94	1.
time (sec)	N/A	0.008	0.022	0.003	1.038	1.509	1.483	1.171

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	31	12	16
normalized size	1	1.	1.	0.81	1.	1.94	0.75	1.
time (sec)	N/A	0.001	0.004	0.003	1.058	1.595	0.088	1.174

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	0	188	68	43
normalized size	1	1.	1.	0.8	0.	5.37	1.94	1.23
time (sec)	N/A	0.01	0.014	0.	0.	1.655	1.941	1.28

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	47	37	0	225	44	55
normalized size	1	1.	1.21	0.95	0.	5.77	1.13	1.41
time (sec)	N/A	0.011	0.036	0.01	0.	1.646	2.607	1.211

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	35	53	0	292	97	89
normalized size	1	1.	0.54	0.82	0.	4.49	1.49	1.37
time (sec)	N/A	0.018	0.024	0.008	0.	1.551	4.867	1.169

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	35	65	0	347	122	113
normalized size	1	1.	0.4	0.75	0.	3.99	1.4	1.3
time (sec)	N/A	0.027	0.01	0.01	0.	1.455	7.492	1.224

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	76	147	1742	157
normalized size	1	1.	0.64	0.6	1.06	2.04	24.19	2.18
time (sec)	N/A	0.019	0.039	0.004	1.075	1.504	3.738	1.178

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	55	120	733	124
normalized size	1	1.	0.66	0.6	1.04	2.26	13.83	2.34
time (sec)	N/A	0.013	0.025	0.005	1.072	1.512	2.52	1.178

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	35	92	80	92
normalized size	1	1.	0.71	0.62	1.03	2.71	2.35	2.71
time (sec)	N/A	0.008	0.025	0.003	1.037	1.557	0.844	1.174

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	63	12	16
normalized size	1	1.	1.	0.81	1.	3.94	0.75	1.
time (sec)	N/A	0.001	0.01	0.002	1.061	1.451	0.069	1.208

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	38	0	228	71	59
normalized size	1	1.	0.9	0.78	0.	4.65	1.45	1.2
time (sec)	N/A	0.014	0.042	0.005	0.	1.646	2.724	1.133

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	33	47	0	247	92	76
normalized size	1	1.	0.65	0.92	0.	4.84	1.8	1.49
time (sec)	N/A	0.015	0.012	0.009	0.	1.661	3.386	1.21

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	68	51	0	296	76	86
normalized size	1	1.	1.1	0.82	0.	4.77	1.23	1.39
time (sec)	N/A	0.016	0.053	0.008	0.	1.554	4.14	1.237

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	35	63	0	351	124	113
normalized size	1	1.	0.42	0.75	0.	4.18	1.48	1.35
time (sec)	N/A	0.022	0.014	0.01	0.	1.529	7.225	1.198

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	76	171	146	261
normalized size	1	1.	0.64	0.6	1.06	2.38	2.03	3.62
time (sec)	N/A	0.017	0.039	0.005	1.083	1.547	5.894	1.193

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	55	146	124	211
normalized size	1	1.	0.66	0.6	1.04	2.75	2.34	3.98
time (sec)	N/A	0.012	0.026	0.004	1.027	1.51	4.741	1.182

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	35	116	102	162
normalized size	1	1.	0.71	0.62	1.03	3.41	3.	4.76
time (sec)	N/A	0.009	0.026	0.001	1.129	1.474	3.058	1.217

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	85	12	81
normalized size	1	1.	1.	0.81	1.	5.31	0.75	5.06
time (sec)	N/A	0.001	0.007	0.002	1.093	1.51	0.108	1.251

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	58	50	0	288	97	76
normalized size	1	1.	0.89	0.77	0.	4.43	1.49	1.17
time (sec)	N/A	0.021	0.089	0.003	0.	1.575	4.683	1.186

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	33	61	0	309	99	100
normalized size	1	1.	0.5	0.92	0.	4.68	1.5	1.52
time (sec)	N/A	0.02	0.012	0.009	0.	1.57	4.714	1.201

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	35	61	0	320	126	108
normalized size	1	1.	0.45	0.78	0.	4.1	1.62	1.38
time (sec)	N/A	0.021	0.021	0.009	0.	1.546	5.716	1.159

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	79	63	0	350	104	107
normalized size	1	1.	0.98	0.78	0.	4.32	1.28	1.32
time (sec)	N/A	0.023	0.057	0.009	0.	1.593	6.733	1.283

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	35	75	0	413	155	134
normalized size	1	1.	0.34	0.73	0.	4.01	1.5	1.3
time (sec)	N/A	0.031	0.011	0.012	0.	1.564	10.283	1.229

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	90	87	157	374	279	838
normalized size	1	1.	0.62	0.6	1.08	2.56	1.91	5.74
time (sec)	N/A	0.042	0.123	0.005	1.094	1.514	53.02	1.25

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	79	76	136	340	257	757
normalized size	1	1.	0.62	0.6	1.07	2.68	2.02	5.96
time (sec)	N/A	0.035	0.056	0.005	1.091	1.524	45.756	1.264

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	68	65	116	297	235	676
normalized size	1	1.	0.62	0.59	1.05	2.7	2.14	6.15
time (sec)	N/A	0.032	0.065	0.005	1.05	1.534	36.24	1.273

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	57	54	96	265	212	595
normalized size	1	1.	0.63	0.59	1.05	2.91	2.33	6.54
time (sec)	N/A	0.023	0.079	0.005	1.069	1.473	31.169	1.193

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	76	227	190	514
normalized size	1	1.	0.64	0.6	1.06	3.15	2.64	7.14
time (sec)	N/A	0.018	0.061	0.005	1.008	1.47	24.404	1.276

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	55	200	168	432
normalized size	1	1.	0.66	0.6	1.04	3.77	3.17	8.15
time (sec)	N/A	0.013	0.049	0.004	1.07	1.49	23.435	1.196

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	35	166	146	352
normalized size	1	1.	0.71	0.62	1.03	4.88	4.29	10.35
time (sec)	N/A	0.008	0.029	0.003	1.022	1.487	17.779	1.229

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	132	12	230
normalized size	1	1.	1.	0.81	1.	8.25	0.75	14.38
time (sec)	N/A	0.002	0.018	0.002	1.134	1.526	0.08	1.144

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	78	74	0	396	148	108
normalized size	1	1.	0.8	0.76	0.	4.08	1.53	1.11
time (sec)	N/A	0.034	0.209	0.004	0.	1.564	12.94	1.215

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	33	84	0	420	150	140
normalized size	1	1.	0.34	0.86	0.	4.29	1.53	1.43
time (sec)	N/A	0.034	0.026	0.007	0.	1.666	12.731	1.241

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	35	86	0	431	184	151
normalized size	1	1.	0.31	0.75	0.	3.78	1.61	1.32
time (sec)	N/A	0.035	0.139	0.01	0.	1.561	12.614	1.264

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	35	87	0	431	184	151
normalized size	1	1.	0.31	0.76	0.	3.78	1.61	1.32
time (sec)	N/A	0.036	0.026	0.01	0.	1.615	11.089	1.266

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	35	85	0	435	182	149
normalized size	1	1.	0.3	0.73	0.	3.75	1.57	1.28
time (sec)	N/A	0.037	0.047	0.012	0.	1.535	12.2	1.225

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	101	87	0	470	158	147
normalized size	1	1.	0.85	0.73	0.	3.95	1.33	1.24
time (sec)	N/A	0.039	0.131	0.013	0.	1.664	13.397	1.207

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	35	99	0	541	209	174
normalized size	1	1.	0.25	0.7	0.	3.84	1.48	1.23
time (sec)	N/A	0.051	0.025	0.013	0.	1.624	20.872	1.246

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	35	111	0	595	236	194
normalized size	1	1.	0.21	0.68	0.	3.65	1.45	1.19
time (sec)	N/A	0.068	0.019	0.011	0.	1.71	28.235	1.226

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	0	186	151	42
normalized size	1	1.	1.	0.82	0.	4.77	3.87	1.08
time (sec)	N/A	0.011	0.02	0.005	0.	1.52	2.164	1.181

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	52	35	0	224	124	55
normalized size	1	1.	1.24	0.83	0.	5.33	2.95	1.31
time (sec)	N/A	0.01	0.048	0.008	0.	1.533	2.733	1.197

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	38	55	0	289	211	89
normalized size	1	1.	0.54	0.77	0.	4.07	2.97	1.25
time (sec)	N/A	0.016	0.024	0.011	0.	1.609	4.888	1.215

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	48	44	0	225	189	58
normalized size	1	1.	0.87	0.8	0.	4.09	3.44	1.05
time (sec)	N/A	0.015	0.074	0.005	0.	1.579	3.193	1.163

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	36	48	0	246	201	78
normalized size	1	1.	0.63	0.84	0.	4.32	3.53	1.37
time (sec)	N/A	0.014	0.011	0.01	0.	1.604	3.562	1.24

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	72	53	0	294	194	89
normalized size	1	1.	1.06	0.78	0.	4.32	2.85	1.31
time (sec)	N/A	0.015	0.052	0.01	0.	1.557	4.502	1.219

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	58	0	286	241	77
normalized size	1	1.	0.82	0.79	0.	3.92	3.3	1.05
time (sec)	N/A	0.019	0.061	0.006	0.	1.546	4.973	1.191

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	36	64	0	306	246	101
normalized size	1	1.	0.49	0.86	0.	4.14	3.32	1.36
time (sec)	N/A	0.02	0.018	0.01	0.	1.568	4.855	1.143

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	38	70	0	319	270	112
normalized size	1	1.	0.44	0.81	0.	3.71	3.14	1.3
time (sec)	N/A	0.022	0.026	0.01	0.	1.624	5.674	1.195

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	57	54	96	126	3755	82
normalized size	1	1.	0.64	0.61	1.08	1.42	42.19	0.92
time (sec)	N/A	0.022	0.044	0.004	1.08	1.439	6.162	1.235

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	46	43	76	96	1640	66
normalized size	1	1.	0.68	0.63	1.12	1.41	24.12	0.97
time (sec)	N/A	0.018	0.039	0.004	1.038	1.522	3.227	1.199

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	35	32	55	73	600	50
normalized size	1	1.	0.69	0.63	1.08	1.43	11.76	0.98
time (sec)	N/A	0.012	0.02	0.003	1.057	1.48	2.183	1.167

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	23	21	35	47	162	31
normalized size	1	1.	0.72	0.66	1.09	1.47	5.06	0.97
time (sec)	N/A	0.008	0.016	0.003	1.069	1.502	1.433	1.19

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	26	10	16
normalized size	1	1.	1.	0.93	1.14	1.86	0.71	1.14
time (sec)	N/A	0.001	0.006	0.003	1.007	1.494	0.071	1.181

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	0	142	24	28
normalized size	1	1.	1.	0.78	0.	6.17	1.04	1.22
time (sec)	N/A	0.007	0.008	0.005	0.	1.536	1.631	1.213

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	47	40	0	232	44	63
normalized size	1	1.	1.15	0.98	0.	5.66	1.07	1.54
time (sec)	N/A	0.011	0.102	0.006	0.	1.565	3.184	1.195

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	33	66	0	301	102	93
normalized size	1	1.	0.49	0.97	0.	4.43	1.5	1.37
time (sec)	N/A	0.017	0.01	0.006	0.	1.559	6.009	1.217

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	33	90	0	356	129	113
normalized size	1	1.	0.37	1.	0.	3.96	1.43	1.26
time (sec)	N/A	0.025	0.008	0.006	0.	1.491	9.418	1.21

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	57	54	96	138	3606	104
normalized size	1	1.	0.67	0.64	1.13	1.62	42.42	1.22
time (sec)	N/A	0.024	0.042	0.005	1.058	1.537	5.739	1.142

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	45	42	76	108	1538	82
normalized size	1	1.	0.68	0.64	1.15	1.64	23.3	1.24
time (sec)	N/A	0.017	0.033	0.004	0.998	1.502	3.379	1.188

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	34	32	55	85	534	62
normalized size	1	1.	0.69	0.65	1.12	1.73	10.9	1.27
time (sec)	N/A	0.013	0.026	0.002	1.085	1.553	2.036	1.131

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	21	20	35	61	37	39
normalized size	1	1.	0.7	0.67	1.17	2.03	1.23	1.3
time (sec)	N/A	0.008	0.038	0.001	1.028	1.467	0.738	1.193

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	43	12	16
normalized size	1	1.	1.	0.93	1.14	3.07	0.86	1.14
time (sec)	N/A	0.001	0.005	0.003	1.071	1.513	0.065	1.203

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	30	31	0	266	146	50
normalized size	1	1.	0.79	0.82	0.	7.	3.84	1.32
time (sec)	N/A	0.011	0.013	0.007	0.	1.59	2.146	1.238

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	59	31	55	0	346	73	86
normalized size	1	1.04	0.54	0.96	0.	6.07	1.28	1.51
time (sec)	N/A	0.017	0.016	0.012	0.	1.634	4.144	1.188

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	85	33	67	0	420	107	108
normalized size	1	0.98	0.38	0.77	0.	4.83	1.23	1.24
time (sec)	N/A	0.023	0.009	0.01	0.	1.57	7.505	1.176

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	57	54	96	161	3456	101
normalized size	1	1.	0.66	0.62	1.1	1.85	39.72	1.16
time (sec)	N/A	0.022	0.047	0.005	1.07	1.547	5.721	1.184

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	45	43	76	131	163	80
normalized size	1	1.	0.66	0.63	1.12	1.93	2.4	1.18
time (sec)	N/A	0.017	0.037	0.006	1.069	1.549	1.723	1.203

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	35	32	55	111	121	53
normalized size	1	1.	0.71	0.65	1.12	2.27	2.47	1.08
time (sec)	N/A	0.013	0.018	0.006	1.149	1.557	1.542	1.153

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	24	21	35	89	80	27
normalized size	1	1.	0.75	0.66	1.09	2.78	2.5	0.84
time (sec)	N/A	0.008	0.012	0.003	1.044	1.602	1.404	1.127

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	68	14	16
normalized size	1	1.	1.	0.81	1.	4.25	0.88	1.
time (sec)	N/A	0.001	0.005	0.002	1.084	1.531	0.066	1.175

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	32	43	0	409	697	61
normalized size	1	1.	0.59	0.8	0.	7.57	12.91	1.13
time (sec)	N/A	0.015	0.006	0.009	0.	1.7	3.716	1.192

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	80	33	67	0	494	818	88
normalized size	1	1.08	0.45	0.91	0.	6.68	11.05	1.19
time (sec)	N/A	0.024	0.007	0.013	0.	1.477	7.264	1.196

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	35	80	0	566	464	126
normalized size	1	1.	0.33	0.75	0.	5.34	4.38	1.19
time (sec)	N/A	0.034	0.007	0.013	0.	1.593	12.109	1.16

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	0	140	58	26
normalized size	1	1.	1.	0.8	0.	5.6	2.32	1.04
time (sec)	N/A	0.006	0.005	0.005	0.	1.598	1.559	1.22

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	53	37	0	230	124	58
normalized size	1	1.	1.2	0.84	0.	5.23	2.82	1.32
time (sec)	N/A	0.011	0.138	0.007	0.	1.526	3.023	1.199

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	36	59	0	300	219	92
normalized size	1	1.	0.49	0.8	0.	4.05	2.96	1.24
time (sec)	N/A	0.017	0.006	0.006	0.	1.541	5.757	1.181

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	33	35	0	266	439	46
normalized size	1	1.	0.79	0.83	0.	6.33	10.45	1.1
time (sec)	N/A	0.01	0.008	0.007	0.	1.446	2.739	1.18

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	65	34	54	0	344	160	86
normalized size	1	1.05	0.55	0.87	0.	5.55	2.58	1.39
time (sec)	N/A	0.016	0.009	0.011	0.	1.628	4.731	1.184

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	93	36	75	0	420	230	109
normalized size	1	0.98	0.38	0.79	0.	4.42	2.42	1.15
time (sec)	N/A	0.023	0.011	0.011	0.	1.63	7.712	1.183

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	35	49	0	408	1952	57
normalized size	1	1.	0.58	0.82	0.	6.8	32.53	0.95
time (sec)	N/A	0.015	0.012	0.009	0.	1.586	4.296	1.193

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	88	36	68	0	491	2236	89
normalized size	1	1.09	0.44	0.84	0.	6.06	27.6	1.1
time (sec)	N/A	0.023	0.012	0.013	0.	1.896	7.725	1.17

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	38	92	0	564	1112	131
normalized size	1	1.	0.33	0.79	0.	4.86	9.59	1.13
time (sec)	N/A	0.033	0.013	0.016	0.	1.855	13.361	1.219

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	36	0	0
normalized size	1	1.	1.	0.92	1.15	2.77	0.	0.
time (sec)	N/A	0.006	0.079	0.006	1.257	1.964	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	92	13	0	15	26	73	0
normalized size	1	7.08	1.	0.	1.15	2.	5.62	0.
time (sec)	N/A	0.045	0.026	0.037	1.281	1.84	10.45	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	0	142	24	28
normalized size	1	1.	1.	0.78	0.	6.17	1.04	1.22
time (sec)	N/A	0.008	0.006	0.	0.	1.873	1.718	1.164

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	76	130	1742	66
normalized size	1	1.	0.64	0.6	1.06	1.81	24.19	0.92
time (sec)	N/A	0.019	0.052	0.004	1.074	1.654	3.54	1.17

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	55	100	666	50
normalized size	1	1.	0.66	0.6	1.04	1.89	12.57	0.94
time (sec)	N/A	0.012	0.028	0.003	1.018	1.788	2.473	1.13

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	35	73	202	34
normalized size	1	1.	0.71	0.62	1.03	2.15	5.94	1.
time (sec)	N/A	0.008	0.025	0.003	1.086	1.706	1.638	1.221

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	31	12	16
normalized size	1	1.	1.	0.81	1.	1.94	0.75	1.
time (sec)	N/A	0.001	0.021	0.002	1.025	1.779	0.075	1.206

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	113	85	0	286	180	117
normalized size	1	1.	1.24	0.93	0.	3.14	1.98	1.29
time (sec)	N/A	0.047	0.192	0.008	0.	1.897	2.781	2.288

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	33	92	0	410	643	142
normalized size	1	1.	0.34	0.95	0.	4.23	6.63	1.46
time (sec)	N/A	0.033	0.114	0.01	0.	1.883	3.258	2.203

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	35	113	0	481	2266	173
normalized size	1	1.	0.28	0.89	0.	3.79	17.84	1.36
time (sec)	N/A	0.049	0.019	0.01	0.	1.895	3.925	1.982

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	76	130	1742	66
normalized size	1	1.	0.64	0.6	1.06	1.81	24.19	0.92
time (sec)	N/A	0.018	0.053	0.005	1.096	1.853	3.739	1.239

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	55	100	666	50
normalized size	1	1.	0.66	0.6	1.04	1.89	12.57	0.94
time (sec)	N/A	0.013	0.038	0.005	1.078	1.745	2.655	1.205

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	35	76	202	34
normalized size	1	1.	0.71	0.62	1.03	2.24	5.94	1.
time (sec)	N/A	0.008	0.024	0.001	1.067	1.831	1.59	1.196

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	31	12	16
normalized size	1	1.	1.	0.81	1.	1.94	0.75	1.
time (sec)	N/A	0.002	0.004	0.002	1.106	1.838	0.073	1.146

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	86	84	0	333	182	116
normalized size	1	1.	0.93	0.91	0.	3.62	1.98	1.26
time (sec)	N/A	0.032	0.076	0.005	0.	1.911	2.771	1.731

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	33	92	0	760	643	143
normalized size	1	1.	0.35	0.98	0.	8.09	6.84	1.52
time (sec)	N/A	0.033	0.014	0.01	0.	1.916	3.176	2.162

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	35	113	0	950	2266	174
normalized size	1	1.	0.28	0.89	0.	7.48	17.84	1.37
time (sec)	N/A	0.047	0.014	0.011	0.	1.589	3.938	1.794

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	76	154	1844	157
normalized size	1	1.	0.64	0.6	1.06	2.14	25.61	2.18
time (sec)	N/A	0.018	0.076	0.004	1.041	1.475	4.667	1.185

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	55	123	733	124
normalized size	1	1.	0.66	0.6	1.04	2.32	13.83	2.34
time (sec)	N/A	0.013	0.046	0.004	1.014	1.479	2.98	1.188

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	35	96	80	92
normalized size	1	1.	0.71	0.62	1.03	2.82	2.35	2.71
time (sec)	N/A	0.009	0.042	0.003	1.054	1.545	1.941	1.246

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	66	12	16
normalized size	1	1.	1.	0.81	1.	4.12	0.75	1.
time (sec)	N/A	0.002	0.013	0.001	1.069	1.469	0.07	1.252

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	130	95	0	305	209	131
normalized size	1	1.	1.24	0.9	0.	2.9	1.99	1.25
time (sec)	N/A	0.041	0.069	0.004	0.	1.676	3.152	2.106

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	33	103	0	332	719	161
normalized size	1	1.	0.31	0.96	0.	3.1	6.72	1.5
time (sec)	N/A	0.042	0.017	0.009	0.	1.667	3.632	1.917

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	35	111	0	455	2266	171
normalized size	1	1.	0.28	0.9	0.	3.67	18.27	1.38
time (sec)	N/A	0.044	0.022	0.009	0.	1.648	4.112	1.888

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	76	104	1640	66
normalized size	1	1.	0.64	0.6	1.06	1.44	22.78	0.92
time (sec)	N/A	0.018	0.045	0.004	1.071	1.509	3.542	1.225

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	55	76	600	50
normalized size	1	1.	0.66	0.6	1.04	1.43	11.32	0.94
time (sec)	N/A	0.012	0.033	0.003	1.001	1.495	2.187	1.138

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	35	54	162	34
normalized size	1	1.	0.71	0.62	1.03	1.59	4.76	1.
time (sec)	N/A	0.008	0.027	0.003	1.084	1.647	1.445	1.159

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	31	12	16
normalized size	1	1.	1.	0.81	1.	1.94	0.75	1.
time (sec)	N/A	0.002	0.005	0.001	1.021	1.725	0.073	1.197

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	66	75	0	647	155	104
normalized size	1	1.	0.84	0.95	0.	8.19	1.96	1.32
time (sec)	N/A	0.025	0.023	0.004	0.	1.854	2.489	2.197

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	33	95	0	867	831	147
normalized size	1	1.	0.33	0.95	0.	8.67	8.31	1.47
time (sec)	N/A	0.033	0.008	0.007	0.	1.985	3.082	2.14

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	35	117	0	856	2730	176
normalized size	1	1.	0.27	0.9	0.	6.58	21.	1.35
time (sec)	N/A	0.048	0.011	0.006	0.	2.003	3.907	2.232

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	48	45	86	104	4976	77
normalized size	1	1.	0.6	0.56	1.08	1.3	62.2	0.96
time (sec)	N/A	0.019	0.05	0.004	1.054	1.755	4.128	1.189

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	37	34	63	76	1328	58
normalized size	1	1.	0.63	0.58	1.07	1.29	22.51	0.98
time (sec)	N/A	0.013	0.058	0.004	1.02	1.712	2.552	1.261

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	26	23	41	54	488	39
normalized size	1	1.	0.68	0.61	1.08	1.42	12.84	1.03
time (sec)	N/A	0.009	0.032	0.003	1.052	1.522	1.549	1.18

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	31	12	19
normalized size	1	1.	1.	0.83	1.06	1.72	0.67	1.06
time (sec)	N/A	0.001	0.008	0.002	0.987	1.438	0.068	1.196

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	35	83	0	747	160	151
normalized size	1	1.	0.43	1.01	0.	9.11	1.95	1.84
time (sec)	N/A	0.033	0.018	0.007	0.	1.724	2.748	1.778

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	36	103	0	865	838	194
normalized size	1	1.	0.35	1.	0.	8.4	8.14	1.88
time (sec)	N/A	0.034	0.011	0.006	0.	1.577	3.359	2.245

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	38	128	0	956	2744	225
normalized size	1	1.	0.28	0.94	0.	7.03	20.18	1.65
time (sec)	N/A	0.044	0.013	0.006	0.	1.648	4.014	1.757

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	46	43	76	104	1640	66
normalized size	1	1.	0.66	0.61	1.09	1.49	23.43	0.94
time (sec)	N/A	0.018	0.051	0.004	1.036	1.555	3.559	1.2

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	35	32	55	76	600	50
normalized size	1	1.	0.69	0.63	1.08	1.49	11.76	0.98
time (sec)	N/A	0.012	0.088	0.005	1.068	1.571	2.46	1.199

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	23	21	35	50	162	31
normalized size	1	1.	0.72	0.66	1.09	1.56	5.06	0.97
time (sec)	N/A	0.007	0.018	0.001	1.047	1.485	1.708	1.108

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	28	10	16
normalized size	1	1.	1.	0.93	1.14	2.	0.71	1.14
time (sec)	N/A	0.001	0.005	0.002	1.076	1.512	0.102	1.249

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	93	76	0	346	150	105
normalized size	1	1.	1.16	0.95	0.	4.32	1.88	1.31
time (sec)	N/A	0.024	0.137	0.004	0.	1.554	3.089	1.871

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	31	95	0	441	830	146
normalized size	1	1.	0.32	0.97	0.	4.5	8.47	1.49
time (sec)	N/A	0.033	0.032	0.006	0.	1.579	3.746	2.248

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	33	117	0	458	2728	176
normalized size	1	1.	0.25	0.9	0.	3.52	20.98	1.35
time (sec)	N/A	0.048	0.009	0.006	0.	1.575	4.254	2.354

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	46	43	76	116	1538	84
normalized size	1	1.	0.66	0.61	1.09	1.66	21.97	1.2
time (sec)	N/A	0.018	0.03	0.004	1.116	1.512	3.93	1.205

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	34	32	55	88	534	62
normalized size	1	1.	0.69	0.65	1.12	1.8	10.9	1.27
time (sec)	N/A	0.013	0.09	0.005	1.06	1.529	2.528	1.177

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	23	20	35	66	41	41
normalized size	1	1.	0.72	0.62	1.09	2.06	1.28	1.28
time (sec)	N/A	0.009	0.048	0.003	1.028	1.601	1.041	1.217

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	46	12	16
normalized size	1	1.	1.	0.93	1.14	3.29	0.86	1.14
time (sec)	N/A	0.001	0.012	0.002	1.017	1.442	0.107	1.204

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	30	87	0	834	184	120
normalized size	1	1.	0.32	0.94	0.	8.97	1.98	1.29
time (sec)	N/A	0.033	0.022	0.006	0.	1.638	2.922	2.311

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	115	31	108	0	1062	857	162
normalized size	1	1.02	0.27	0.96	0.	9.4	7.58	1.43
time (sec)	N/A	0.044	0.052	0.011	0.	1.686	3.283	1.812

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	147	33	131	0	1064	2793	189
normalized size	1	0.99	0.22	0.88	0.	7.14	18.74	1.27
time (sec)	N/A	0.058	0.029	0.012	0.	1.708	4.417	2.203

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	87	116	227	138	117
normalized size	1	1.	0.93	1.23	1.63	3.2	1.94	1.65
time (sec)	N/A	0.031	0.095	0.009	1.519	1.678	2.469	1.302

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	68	91	122	232	136	123
normalized size	1	1.	0.93	1.25	1.67	3.18	1.86	1.68
time (sec)	N/A	0.031	0.062	0.005	1.525	1.637	2.401	1.232

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	41	97	127	227	134	128
normalized size	1	1.	0.55	1.31	1.72	3.07	1.81	1.73
time (sec)	N/A	0.032	0.039	0.01	1.573	1.632	2.513	1.192

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	41	101	132	232	139	134
normalized size	1	1.	0.54	1.33	1.74	3.05	1.83	1.76
time (sec)	N/A	0.028	0.053	0.004	1.546	1.659	2.37	1.269

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	95	88	117	231	134	119
normalized size	1	1.	1.32	1.22	1.62	3.21	1.86	1.65
time (sec)	N/A	0.024	0.102	0.007	1.517	1.638	2.509	1.239

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	99	92	123	236	136	124
normalized size	1	1.	1.34	1.24	1.66	3.19	1.84	1.68
time (sec)	N/A	0.025	0.098	0.004	1.524	1.607	2.648	1.264

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	108	96	126	230	134	127
normalized size	1	1.	1.46	1.3	1.7	3.11	1.81	1.72
time (sec)	N/A	0.023	0.057	0.007	1.504	1.525	2.457	1.304

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	112	100	131	235	133	132
normalized size	1	1.	1.47	1.32	1.72	3.09	1.75	1.74
time (sec)	N/A	0.023	0.122	0.003	1.517	1.567	2.323	1.273

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	31	0	72	87	58
normalized size	1	1.	0.88	1.24	0.	2.88	3.48	2.32
time (sec)	N/A	0.008	0.014	0.002	0.	1.553	0.375	1.17

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	18	46	19	18
normalized size	1	1.	0.81	0.67	0.86	2.19	0.9	0.86
time (sec)	N/A	0.004	0.005	0.003	0.997	1.505	2.351	1.264

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	18	46	19	18
normalized size	1	1.	0.81	0.67	0.86	2.19	0.9	0.86
time (sec)	N/A	0.004	0.004	0.002	1.057	1.595	0.846	1.187

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	18	43	19	18
normalized size	1	1.	0.81	0.67	0.86	2.05	0.9	0.86
time (sec)	N/A	0.004	0.004	0.003	1.008	1.574	2.041	1.25

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	18	34	17	18
normalized size	1	1.	0.84	0.68	0.95	1.79	0.89	0.95
time (sec)	N/A	0.004	0.005	0.002	0.99	1.501	0.173	1.233

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	14	12	18	28	15	18
normalized size	1	1.	0.82	0.71	1.06	1.65	0.88	1.06
time (sec)	N/A	0.004	0.005	0.001	1.025	1.53	0.51	1.256

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	15	12	15	35	19	15
normalized size	1	1.	0.79	0.63	0.79	1.84	1.	0.79
time (sec)	N/A	0.004	0.005	0.002	0.992	1.553	0.958	1.238

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	38	87	0	178	299	158
normalized size	1	1.	0.88	2.02	0.	4.14	6.95	3.67
time (sec)	N/A	0.014	0.032	0.004	0.	1.736	0.719	1.234

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	32	74	34	32
normalized size	1	1.	0.78	0.69	0.89	2.06	0.94	0.89
time (sec)	N/A	0.007	0.008	0.003	0.991	1.448	3.749	1.225

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	32	73	34	32
normalized size	1	1.	0.78	0.69	0.89	2.03	0.94	0.89
time (sec)	N/A	0.007	0.008	0.004	1.041	1.508	1.315	1.228

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	32	70	1853	32
normalized size	1	1.	0.78	0.69	0.89	1.94	51.47	0.89
time (sec)	N/A	0.007	0.007	0.003	1.096	1.461	2.771	1.186

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	25	32	62	32	32
normalized size	1	1.	0.82	0.74	0.94	1.82	0.94	0.94
time (sec)	N/A	0.007	0.007	0.004	1.041	1.505	0.359	1.23

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	27	25	32	55	31	32
normalized size	1	1.	0.84	0.78	1.	1.72	0.97	1.
time (sec)	N/A	0.007	0.009	0.003	1.03	1.464	0.73	1.201

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	26	23	31	55	31	31
normalized size	1	1.	0.81	0.72	0.97	1.72	0.97	0.97
time (sec)	N/A	0.007	0.009	0.003	1.054	1.404	0.883	1.215

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	170	0	336	663	302
normalized size	1	1.	0.89	2.79	0.	5.51	10.87	4.95
time (sec)	N/A	0.02	0.032	0.004	0.	1.583	1.099	1.184

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	47	104	49	47
normalized size	1	1.	0.76	0.71	0.92	2.04	0.96	0.92
time (sec)	N/A	0.011	0.011	0.004	1.077	1.534	5.688	1.282

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	47	103	49	47
normalized size	1	1.	0.76	0.71	0.92	2.02	0.96	0.92
time (sec)	N/A	0.012	0.01	0.003	1.044	1.649	2.274	1.215

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	47	97	4886	47
normalized size	1	1.	0.76	0.71	0.92	1.9	95.8	0.92
time (sec)	N/A	0.011	0.011	0.003	1.061	1.495	4.843	1.203

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	39	36	47	85	46	47
normalized size	1	1.	0.83	0.77	1.	1.81	0.98	1.
time (sec)	N/A	0.011	0.01	0.004	1.026	1.448	0.644	1.193

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	38	36	47	78	44	47
normalized size	1	1.	0.84	0.8	1.04	1.73	0.98	1.04
time (sec)	N/A	0.011	0.011	0.004	1.096	1.512	0.897	1.228

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	38	34	46	74	46	46
normalized size	1	1.	0.81	0.72	0.98	1.57	0.98	0.98
time (sec)	N/A	0.011	0.015	0.003	1.109	1.543	1.199	1.206

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	54	0	308	121	80
normalized size	1	1.	0.9	0.79	0.	4.53	1.78	1.18
time (sec)	N/A	0.032	0.03	0.007	0.	1.529	16.361	1.197

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	49	43	0	244	105	61
normalized size	1	1.	0.92	0.81	0.	4.6	1.98	1.15
time (sec)	N/A	0.017	0.018	0.003	0.	1.633	3.925	1.165

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	0	189	92	42
normalized size	1	1.	1.	0.8	0.	4.72	2.3	1.05
time (sec)	N/A	0.012	0.009	0.005	0.	1.594	1.054	1.164

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	0	163	94	24
normalized size	1	1.	1.	0.66	0.	5.62	3.24	0.83
time (sec)	N/A	0.008	0.005	0.005	0.	1.555	2.08	1.256

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	25	32	0	207	102	42
normalized size	1	1.	0.62	0.8	0.	5.18	2.55	1.05
time (sec)	N/A	0.013	0.005	0.007	0.	1.636	5.387	1.197

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	27	43	0	275	121	55
normalized size	1	1.	0.51	0.81	0.	5.19	2.28	1.04
time (sec)	N/A	0.018	0.005	0.009	0.	1.616	15.891	1.26

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	27	54	0	336	139	70
normalized size	1	1.	0.4	0.79	0.	4.94	2.04	1.03
time (sec)	N/A	0.023	0.005	0.008	0.	1.559	75.785	1.24

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	27	61	0	366	479	88
normalized size	1	1.	0.39	0.87	0.	5.23	6.84	1.26
time (sec)	N/A	0.022	0.004	0.01	0.	1.637	74.524	1.214

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	27	47	0	300	411	62
normalized size	1	1.	0.47	0.82	0.	5.26	7.21	1.09
time (sec)	N/A	0.017	0.004	0.009	0.	1.491	19.207	1.194

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	0	277	337	49
normalized size	1	1.	1.	0.8	0.	6.02	7.33	1.07
time (sec)	N/A	0.013	0.02	0.008	0.	1.68	8.454	1.218

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	274	328	47
normalized size	1	1.	1.	0.8	0.	6.09	7.29	1.04
time (sec)	N/A	0.013	0.018	0.006	0.	1.612	15.836	1.203

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	25	48	0	323	434	66
normalized size	1	1.	0.45	0.86	0.	5.77	7.75	1.18
time (sec)	N/A	0.017	0.005	0.01	0.	1.565	38.774	1.243

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	27	60	0	402	507	78
normalized size	1	1.	0.39	0.87	0.	5.83	7.35	1.13
time (sec)	N/A	0.022	0.005	0.014	0.	1.336	116.113	1.238

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	27	79	0	509	0	104
normalized size	1	1.	0.28	0.83	0.	5.36	0.	1.09
time (sec)	N/A	0.033	0.004	0.012	0.	1.326	0.	1.258

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	27	66	0	443	0	80
normalized size	1	1.	0.33	0.8	0.	5.4	0.	0.98
time (sec)	N/A	0.023	0.005	0.011	0.	1.327	0.	1.259

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	59	50	0	423	726	63
normalized size	1	1.	0.84	0.71	0.	6.04	10.37	0.9
time (sec)	N/A	0.019	0.035	0.01	0.	1.396	89.485	1.184

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	27	52	0	412	721	70
normalized size	1	1.	0.37	0.71	0.	5.64	9.88	0.96
time (sec)	N/A	0.019	0.005	0.009	0.	1.389	35.094	1.219

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	25	53	0	423	712	63
normalized size	1	1.	0.36	0.76	0.	6.04	10.17	0.9
time (sec)	N/A	0.019	0.005	0.006	0.	1.384	61.433	1.224

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	25	66	0	466	865	80
normalized size	1	1.	0.3	0.8	0.	5.68	10.55	0.98
time (sec)	N/A	0.025	0.005	0.011	0.	1.448	155.787	1.227

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	27	79	0	545	0	96
normalized size	1	1.	0.28	0.83	0.	5.74	0.	1.01
time (sec)	N/A	0.029	0.005	0.015	0.	1.409	0.	1.174

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	54	0	306	116	82
normalized size	1	1.	0.9	0.79	0.	4.5	1.71	1.21
time (sec)	N/A	0.024	0.025	0.005	0.	1.309	16.802	1.177

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	49	43	0	244	100	63
normalized size	1	1.	0.92	0.81	0.	4.6	1.89	1.19
time (sec)	N/A	0.018	0.018	0.006	0.	1.315	3.956	1.208

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	0	188	87	45
normalized size	1	1.	1.	0.8	0.	4.7	2.17	1.12
time (sec)	N/A	0.014	0.01	0.006	0.	1.366	1.161	1.154

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	0	161	88	27
normalized size	1	1.	1.	0.66	0.	5.55	3.03	0.93
time (sec)	N/A	0.011	0.005	0.004	0.	1.587	2.11	1.223

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	24	32	0	207	94	45
normalized size	1	1.	0.6	0.8	0.	5.18	2.35	1.12
time (sec)	N/A	0.014	0.004	0.006	0.	1.65	6.077	1.185

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	26	43	0	274	112	55
normalized size	1	1.	0.49	0.81	0.	5.17	2.11	1.04
time (sec)	N/A	0.018	0.005	0.008	0.	1.605	17.828	1.199

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	26	54	0	336	131	73
normalized size	1	1.	0.38	0.79	0.	4.94	1.93	1.07
time (sec)	N/A	0.022	0.005	0.007	0.	1.55	76.126	1.259

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	26	61	0	366	444	93
normalized size	1	1.	0.37	0.87	0.	5.23	6.34	1.33
time (sec)	N/A	0.025	0.005	0.01	0.	1.6	85.475	1.214

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	26	49	0	298	381	69
normalized size	1	1.	0.46	0.86	0.	5.23	6.68	1.21
time (sec)	N/A	0.018	0.005	0.01	0.	1.652	21.079	1.19

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	61	40	0	277	311	54
normalized size	1	1.	1.3	0.85	0.	5.89	6.62	1.15
time (sec)	N/A	0.015	0.015	0.007	0.	1.651	9.048	1.2

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	0	277	303	55
normalized size	1	1.	1.	0.85	0.	6.02	6.59	1.2
time (sec)	N/A	0.014	0.021	0.007	0.	1.712	17.779	1.174

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	24	49	0	324	403	70
normalized size	1	1.	0.42	0.86	0.	5.68	7.07	1.23
time (sec)	N/A	0.018	0.006	0.01	0.	1.627	42.79	1.202

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	26	60	0	402	471	82
normalized size	1	1.	0.37	0.86	0.	5.74	6.73	1.17
time (sec)	N/A	0.022	0.006	0.011	0.	1.558	139.005	1.195

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	26	70	0	509	0	109
normalized size	1	1.	0.27	0.72	0.	5.25	0.	1.12
time (sec)	N/A	0.03	0.005	0.011	0.	1.572	0.	1.259

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	26	58	0	441	756	85
normalized size	1	1.	0.31	0.69	0.	5.25	9.	1.01
time (sec)	N/A	0.026	0.005	0.012	0.	1.607	174.976	1.209

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	52	0	420	673	69
normalized size	1	1.	0.83	0.72	0.	5.83	9.35	0.96
time (sec)	N/A	0.021	0.036	0.01	0.	1.719	80.473	1.226

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	26	54	0	410	668	74
normalized size	1	1.	0.35	0.72	0.	5.47	8.91	0.99
time (sec)	N/A	0.02	0.005	0.009	0.	1.729	33.367	1.208

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	24	63	0	420	660	69
normalized size	1	1.	0.33	0.88	0.	5.83	9.17	0.96
time (sec)	N/A	0.02	0.005	0.007	0.	1.663	56.536	1.211

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	24	58	0	466	802	85
normalized size	1	1.	0.29	0.69	0.	5.55	9.55	1.01
time (sec)	N/A	0.024	0.005	0.012	0.	1.664	132.681	1.145

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	26	69	0	544	0	99
normalized size	1	1.	0.27	0.71	0.	5.61	0.	1.02
time (sec)	N/A	0.03	0.005	0.014	0.	1.566	0.	1.255

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	96	120	0	417	153	0
normalized size	1	1.	0.79	0.98	0.	3.42	1.25	0.
time (sec)	N/A	0.042	0.181	0.006	0.	1.705	22.305	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	85	102	0	362	122	0
normalized size	1	1.	0.87	1.04	0.	3.69	1.24	0.
time (sec)	N/A	0.031	0.106	0.003	0.	1.693	7.251	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	72	81	0	304	97	0
normalized size	1	1.	0.97	1.09	0.	4.11	1.31	0.
time (sec)	N/A	0.023	0.114	0.004	0.	1.716	3.75	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	62	62	0	251	42	0
normalized size	1	1.	1.41	1.41	0.	5.7	0.95	0.
time (sec)	N/A	0.017	0.082	0.004	0.	1.625	2.125	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	64	61	0	243	68	0
normalized size	1	1.	1.42	1.36	0.	5.4	1.51	0.
time (sec)	N/A	0.017	0.098	0.017	0.	1.701	1.711	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	46	41	45
normalized size	1	1.	1.	0.76	0.95	2.19	1.95	2.14
time (sec)	N/A	0.002	0.006	0.005	1.041	1.495	2.202	1.441

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	29	24	42	84	65	68
normalized size	1	1.	0.66	0.55	0.95	1.91	1.48	1.55
time (sec)	N/A	0.005	0.009	0.003	1.098	1.576	20.81	1.304

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	40	35	62	112	347	89
normalized size	1	1.	0.59	0.51	0.91	1.65	5.1	1.31
time (sec)	N/A	0.01	0.011	0.005	1.014	1.557	120.955	1.448

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	98	127	0	424	325	0
normalized size	1	1.	0.77	1.	0.	3.34	2.56	0.
time (sec)	N/A	0.042	0.136	0.007	0.	1.621	21.375	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	87	108	0	367	262	0
normalized size	1	1.	0.85	1.06	0.	3.6	2.57	0.
time (sec)	N/A	0.03	0.114	0.005	0.	1.605	7.279	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	75	86	0	311	209	0
normalized size	1	1.	0.97	1.12	0.	4.04	2.71	0.
time (sec)	N/A	0.024	0.094	0.003	0.	1.678	3.733	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	65	66	0	257	121	0
normalized size	1	1.	1.41	1.43	0.	5.59	2.63	0.
time (sec)	N/A	0.017	0.09	0.004	0.	1.586	2.009	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	69	66	0	246	150	0
normalized size	1	1.	1.47	1.4	0.	5.23	3.19	0.
time (sec)	N/A	0.016	0.062	0.02	0.	1.666	1.82	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	57	92	57
normalized size	1	1.	1.	0.77	1.	2.59	4.18	2.59
time (sec)	N/A	0.002	0.006	0.003	1.022	1.594	2.135	1.241

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	30	25	45	85	245	82
normalized size	1	1.	0.65	0.54	0.98	1.85	5.33	1.78
time (sec)	N/A	0.005	0.009	0.003	1.055	1.638	20.014	1.234

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	41	36	66	112	711	107
normalized size	1	1.	0.58	0.51	0.93	1.58	10.01	1.51
time (sec)	N/A	0.01	0.011	0.003	1.032	1.515	114.542	1.22

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	70	108	0	363	117	0
normalized size	1	1.	0.65	1.	0.	3.36	1.08	0.
time (sec)	N/A	0.032	0.052	0.007	0.	1.7	18.057	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	58	93	0	317	90	0
normalized size	1	1.	0.69	1.11	0.	3.77	1.07	0.
time (sec)	N/A	0.02	0.043	0.003	0.	1.598	6.069	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	51	75	0	277	71	0
normalized size	1	1.	0.8	1.17	0.	4.33	1.11	0.
time (sec)	N/A	0.015	0.032	0.003	0.	1.718	3.149	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	58	0	234	37	0
normalized size	1	1.	1.	1.45	0.	5.85	0.92	0.
time (sec)	N/A	0.008	0.013	0.004	0.	1.732	1.786	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	59	0	238	48	0
normalized size	1	1.	1.	1.44	0.	5.8	1.17	0.
time (sec)	N/A	0.009	0.014	0.019	0.	1.507	1.581	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	16	41	37	39
normalized size	1	1.	1.	0.72	0.89	2.28	2.06	2.17
time (sec)	N/A	0.001	0.013	0.002	1.031	1.502	2.116	1.233

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	23	18	35	65	56	57
normalized size	1	1.	0.61	0.47	0.92	1.71	1.47	1.5
time (sec)	N/A	0.004	0.007	0.004	1.055	1.584	17.282	1.219

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	32	27	55	90	270	74
normalized size	1	1.	0.54	0.46	0.93	1.53	4.58	1.25
time (sec)	N/A	0.008	0.012	0.003	0.995	1.559	71.4	1.223

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	71	116	0	367	252	0
normalized size	1	1.	0.63	1.04	0.	3.28	2.25	0.
time (sec)	N/A	0.029	0.045	0.007	0.	1.512	17.609	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	60	100	0	321	196	0
normalized size	1	1.	0.69	1.15	0.	3.69	2.25	0.
time (sec)	N/A	0.023	0.059	0.004	0.	1.531	6.355	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	81	0	281	156	0
normalized size	1	1.	0.78	1.25	0.	4.32	2.4	0.
time (sec)	N/A	0.015	0.038	0.002	0.	1.601	3.313	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	63	0	238	121	0
normalized size	1	1.	1.	1.54	0.	5.8	2.95	0.
time (sec)	N/A	0.008	0.022	0.004	0.	1.622	1.931	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	90	0	240	136	0
normalized size	1	1.	1.	2.14	0.	5.71	3.24	0.
time (sec)	N/A	0.009	0.014	0.024	0.	1.545	1.744	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	51	82	47
normalized size	1	1.	1.	0.74	0.95	2.68	4.32	2.47
time (sec)	N/A	0.001	0.005	0.003	1.024	1.601	2.326	1.288

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	24	19	38	66	194	65
normalized size	1	1.	0.6	0.48	0.95	1.65	4.85	1.62
time (sec)	N/A	0.004	0.009	0.003	1.084	1.562	18.489	1.174

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	33	28	59	90	554	82
normalized size	1	1.	0.53	0.45	0.95	1.45	8.94	1.32
time (sec)	N/A	0.008	0.012	0.003	1.044	1.562	67.633	1.278

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	107	138	0	470	178	0
normalized size	1	1.	0.75	0.97	0.	3.29	1.24	0.
time (sec)	N/A	0.051	0.225	0.005	0.	1.59	52.044	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	96	120	0	412	153	0
normalized size	1	1.	0.81	1.01	0.	3.46	1.29	0.
time (sec)	N/A	0.038	0.123	0.004	0.	1.608	11.261	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	85	96	0	363	124	0
normalized size	1	1.	0.89	1.01	0.	3.82	1.31	0.
time (sec)	N/A	0.029	0.114	0.003	0.	1.647	6.187	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	69	78	0	311	75	0
normalized size	1	1.	0.97	1.1	0.	4.38	1.06	0.
time (sec)	N/A	0.022	0.097	0.003	0.	1.496	3.621	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	46	71	0	289	92	0
normalized size	1	1.	0.73	1.13	0.	4.59	1.46	0.
time (sec)	N/A	0.021	0.01	0.013	0.	1.654	3.336	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	48	67	0	302	71	0
normalized size	1	1.	0.75	1.05	0.	4.72	1.11	0.
time (sec)	N/A	0.021	0.01	0.015	0.	1.727	4.602	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	110	146	0	477	377	0
normalized size	1	1.	0.74	0.98	0.	3.2	2.53	0.
time (sec)	N/A	0.053	0.161	0.005	0.	1.958	56.42	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	99	127	0	417	325	0
normalized size	1	1.	0.8	1.02	0.	3.36	2.62	0.
time (sec)	N/A	0.041	0.139	0.003	0.	1.935	11.931	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	87	102	0	370	265	0
normalized size	1	1.	0.88	1.03	0.	3.74	2.68	0.
time (sec)	N/A	0.031	0.114	0.004	0.	1.971	6.477	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	71	83	0	316	192	0
normalized size	1	1.	0.96	1.12	0.	4.27	2.59	0.
time (sec)	N/A	0.021	0.114	0.006	0.	1.926	3.562	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	47	74	0	292	199	0
normalized size	1	1.	0.71	1.12	0.	4.42	3.02	0.
time (sec)	N/A	0.02	0.011	0.016	0.	1.9	3.212	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	49	71	0	306	190	0
normalized size	1	1.	0.73	1.06	0.	4.57	2.84	0.
time (sec)	N/A	0.022	0.01	0.016	0.	1.736	4.439	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	78	123	0	398	136	0
normalized size	1	1.	0.62	0.98	0.	3.16	1.08	0.
time (sec)	N/A	0.034	0.048	0.005	0.	1.848	34.567	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	70	108	0	350	117	0
normalized size	1	1.	0.67	1.03	0.	3.33	1.11	0.
time (sec)	N/A	0.026	0.035	0.005	0.	1.919	10.39	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	60	87	0	323	92	0
normalized size	1	1.	0.73	1.06	0.	3.94	1.12	0.
time (sec)	N/A	0.016	0.032	0.003	0.	1.795	5.322	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	48	72	0	279	76	0
normalized size	1	1.	0.79	1.18	0.	4.57	1.25	0.
time (sec)	N/A	0.011	0.024	0.004	0.	1.736	3.14	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	28	72	0	265	73	0
normalized size	1	1.	0.48	1.24	0.	4.57	1.26	0.
time (sec)	N/A	0.012	0.004	0.012	0.	1.888	2.846	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	30	73	0	300	70	0
normalized size	1	1.	0.5	1.22	0.	5.	1.17	0.
time (sec)	N/A	0.014	0.005	0.016	0.	1.897	4.732	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	79	132	0	405	291	0
normalized size	1	1.	0.6	1.01	0.	3.09	2.22	0.
time (sec)	N/A	0.04	0.062	0.005	0.	1.886	35.199	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	70	116	0	356	252	0
normalized size	1	1.	0.64	1.06	0.	3.27	2.31	0.
time (sec)	N/A	0.028	0.054	0.004	0.	1.83	10.743	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	60	94	0	329	199	0
normalized size	1	1.	0.71	1.12	0.	3.92	2.37	0.
time (sec)	N/A	0.016	0.041	0.004	0.	1.861	5.461	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	78	0	286	167	0
normalized size	1	1.	0.78	1.24	0.	4.54	2.65	0.
time (sec)	N/A	0.012	0.028	0.005	0.	1.839	3.249	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	28	97	0	267	160	0
normalized size	1	1.	0.47	1.62	0.	4.45	2.67	0.
time (sec)	N/A	0.012	0.005	0.018	0.	1.854	2.921	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	30	98	0	304	182	0
normalized size	1	1.	0.48	1.58	0.	4.9	2.94	0.
time (sec)	N/A	0.013	0.005	0.018	0.	1.921	4.929	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	118	156	0	520	207	0
normalized size	1	1.	0.72	0.95	0.	3.17	1.26	0.
time (sec)	N/A	0.063	0.234	0.003	0.	1.942	102.469	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	107	138	0	477	180	0
normalized size	1	1.	0.76	0.99	0.	3.41	1.29	0.
time (sec)	N/A	0.048	0.132	0.004	0.	1.948	50.755	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	96	111	0	425	155	0
normalized size	1	1.	0.83	0.96	0.	3.66	1.34	0.
time (sec)	N/A	0.038	0.179	0.005	0.	1.86	19.514	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	80	93	0	365	102	0
normalized size	1	1.	0.87	1.01	0.	3.97	1.11	0.
time (sec)	N/A	0.028	0.111	0.004	0.	1.852	11.78	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	48	84	0	351	126	0
normalized size	1	1.	0.54	0.94	0.	3.94	1.42	0.
time (sec)	N/A	0.028	0.012	0.014	0.	1.973	12.435	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	50	82	0	362	99	0
normalized size	1	1.	0.58	0.95	0.	4.21	1.15	0.
time (sec)	N/A	0.026	0.01	0.017	0.	1.836	11.836	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	120	165	0	527	437	0
normalized size	1	1.	0.7	0.96	0.	3.08	2.56	0.
time (sec)	N/A	0.061	0.187	0.003	0.	1.893	105.6	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	109	146	0	482	381	0
normalized size	1	1.	0.75	1.	0.	3.3	2.61	0.
time (sec)	N/A	0.051	0.143	0.005	0.	1.908	52.722	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	98	118	0	432	328	0
normalized size	1	1.	0.81	0.98	0.	3.57	2.71	0.
time (sec)	N/A	0.038	0.121	0.005	0.	1.789	19.061	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	82	99	0	370	248	0
normalized size	1	1.	0.85	1.03	0.	3.85	2.58	0.
time (sec)	N/A	0.03	0.117	0.004	0.	1.866	11.21	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	49	88	0	354	269	0
normalized size	1	1.	0.53	0.95	0.	3.81	2.89	0.
time (sec)	N/A	0.028	0.012	0.013	0.	1.862	12.606	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	51	86	0	366	248	0
normalized size	1	1.	0.57	0.96	0.	4.07	2.76	0.
time (sec)	N/A	0.03	0.011	0.017	0.	1.82	12.216	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	86	138	0	433	158	0
normalized size	1	1.	0.6	0.96	0.	3.01	1.1	0.
time (sec)	N/A	0.045	0.056	0.004	0.	1.901	67.264	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	78	123	0	401	138	0
normalized size	1	1.	0.63	1.	0.	3.26	1.12	0.
time (sec)	N/A	0.028	0.051	0.004	0.	1.937	34.215	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	70	99	0	369	119	0
normalized size	1	1.	0.69	0.97	0.	3.62	1.17	0.
time (sec)	N/A	0.022	0.04	0.003	0.	1.81	16.366	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	84	0	325	97	0
normalized size	1	1.	0.72	1.06	0.	4.11	1.23	0.
time (sec)	N/A	0.016	0.03	0.003	0.	1.752	10.18	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	28	81	0	313	94	0
normalized size	1	1.	0.35	1.03	0.	3.96	1.19	0.
time (sec)	N/A	0.016	0.005	0.015	0.	1.863	11.062	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	30	82	0	332	88	0
normalized size	1	1.	0.37	1.01	0.	4.1	1.09	0.
time (sec)	N/A	0.017	0.007	0.014	0.	1.808	10.9	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	87	148	0	437	337	0
normalized size	1	1.	0.58	0.99	0.	2.91	2.25	0.
time (sec)	N/A	0.045	0.065	0.004	0.	1.919	67.409	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	79	132	0	405	294	0
normalized size	1	1.	0.62	1.03	0.	3.16	2.3	0.
time (sec)	N/A	0.029	0.046	0.005	0.	1.896	33.345	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	71	107	0	373	255	0
normalized size	1	1.	0.67	1.01	0.	3.52	2.41	0.
time (sec)	N/A	0.023	0.044	0.004	0.	1.89	15.815	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	58	91	0	329	209	0
normalized size	1	1.	0.71	1.11	0.	4.01	2.55	0.
time (sec)	N/A	0.017	0.037	0.003	0.	1.848	9.954	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	28	106	0	316	202	0
normalized size	1	1.	0.34	1.29	0.	3.85	2.46	0.
time (sec)	N/A	0.017	0.005	0.018	0.	1.955	11.084	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	30	107	0	336	221	0
normalized size	1	1.	0.36	1.27	0.	4.	2.63	0.
time (sec)	N/A	0.017	0.007	0.017	0.	1.925	10.637	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	85	102	0	369	128	0
normalized size	1	1.	0.84	1.01	0.	3.65	1.27	0.
time (sec)	N/A	0.03	0.171	0.004	0.	1.948	13.054	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	85	84	0	316	100	0
normalized size	1	1.	1.1	1.09	0.	4.1	1.3	0.
time (sec)	N/A	0.022	0.059	0.003	0.	1.944	4.545	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	68	65	0	255	44	0
normalized size	1	1.	1.42	1.35	0.	5.31	0.92	0.
time (sec)	N/A	0.016	0.038	0.004	0.	1.845	2.252	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	50	48	0	162	22	0
normalized size	1	1.	1.79	1.71	0.	5.79	0.79	0.
time (sec)	N/A	0.013	0.013	0.002	0.	1.791	1.156	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	41	19	45
normalized size	1	1.	1.	0.84	1.05	2.16	1.	2.37
time (sec)	N/A	0.002	0.004	0.003	1.058	1.906	1.013	1.066

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	27	22	42	61	42	68
normalized size	1	1.	0.61	0.5	0.95	1.39	0.95	1.55
time (sec)	N/A	0.005	0.007	0.003	1.167	1.716	3.218	1.076

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	40	35	62	88	287	89
normalized size	1	1.	0.59	0.51	0.91	1.29	4.22	1.31
time (sec)	N/A	0.01	0.009	0.004	1.142	1.812	30.274	1.081

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	51	46	82	109	488	111
normalized size	1	1.	0.55	0.5	0.89	1.18	5.3	1.21
time (sec)	N/A	0.016	0.011	0.004	1.304	1.78	174.652	1.096

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	50	119	0	429	105	177
normalized size	1	1.	0.52	1.24	0.	4.47	1.09	1.84
time (sec)	N/A	0.029	0.01	0.023	0.	1.892	15.922	60.691

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	106	0	363	71	155
normalized size	1	1.	0.74	1.56	0.	5.34	1.04	2.28
time (sec)	N/A	0.022	0.01	0.02	0.	1.941	4.4	59.616

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	64	0	0	308	46	115
normalized size	1	1.	1.33	0.	0.	6.42	0.96	2.4
time (sec)	N/A	0.016	0.068	0.025	0.	1.872	2.106	59.649

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	53	17	61
normalized size	1	1.	1.	0.84	1.05	2.79	0.89	3.21
time (sec)	N/A	0.002	0.004	0.002	1.095	1.813	1.218	1.055

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	25	22	43	78	41	111
normalized size	1	1.	0.64	0.56	1.1	2.	1.05	2.85
time (sec)	N/A	0.005	0.01	0.003	1.121	1.952	2.797	1.071

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	38	33	68	104	219	126
normalized size	1	1.	0.6	0.52	1.08	1.65	3.48	2.
time (sec)	N/A	0.01	0.009	0.005	1.169	2.116	16.907	1.09

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	49	44	86	128	348	147
normalized size	1	1.	0.56	0.51	0.99	1.47	4.	1.69
time (sec)	N/A	0.017	0.011	0.006	1.034	2.049	106.382	1.102

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	50	147	0	512	396	266
normalized size	1	1.	0.55	1.62	0.	5.63	4.35	2.92
time (sec)	N/A	0.028	0.01	0.031	0.	2.118	15.803	59.562

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	80	0	0	451	328	223
normalized size	1	1.	1.16	0.	0.	6.54	4.75	3.23
time (sec)	N/A	0.022	0.13	0.025	0.	2.088	6.025	58.876

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	77	42	116
normalized size	1	1.	1.	0.76	0.95	3.67	2.	5.52
time (sec)	N/A	0.002	0.006	0.003	1.044	2.073	2.427	1.15

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	29	24	36	99	92	109
normalized size	1	1.	0.67	0.56	0.84	2.3	2.14	2.53
time (sec)	N/A	0.005	0.012	0.004	1.11	2.168	3.69	1.154

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	40	35	62	128	153	215
normalized size	1	1.	0.62	0.55	0.97	2.	2.39	3.36
time (sec)	N/A	0.009	0.014	0.004	1.123	2.107	17.404	1.113

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	49	44	86	150	337	234
normalized size	1	1.	0.58	0.52	1.02	1.79	4.01	2.79
time (sec)	N/A	0.015	0.023	0.003	1.127	2.059	40.057	1.155

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	88	108	0	375	272	0
normalized size	1	1.	0.84	1.03	0.	3.57	2.59	0.
time (sec)	N/A	0.03	0.174	0.003	0.	2.156	15.09	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	86	89	0	321	216	0
normalized size	1	1.	1.08	1.11	0.	4.01	2.7	0.
time (sec)	N/A	0.023	0.051	0.004	0.	2.085	5.286	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	71	70	0	262	122	0
normalized size	1	1.	1.42	1.4	0.	5.24	2.44	0.
time (sec)	N/A	0.017	0.043	0.004	0.	2.155	2.594	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	52	51	0	165	56	0
normalized size	1	1.	1.79	1.76	0.	5.69	1.93	0.
time (sec)	N/A	0.013	0.015	0.004	0.	2.131	1.359	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	42	49	47
normalized size	1	1.	1.	0.85	1.1	2.1	2.45	2.35
time (sec)	N/A	0.002	0.011	0.004	0.997	2.086	1.233	1.081

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	28	23	43	63	180	73
normalized size	1	1.	0.61	0.5	0.93	1.37	3.91	1.59
time (sec)	N/A	0.005	0.009	0.004	0.988	1.928	3.818	1.096

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	51	127	0	433	226	208
normalized size	1	1.	0.51	1.27	0.	4.33	2.26	2.08
time (sec)	N/A	0.03	0.013	0.023	0.	1.946	15.624	59.231

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	51	114	0	369	156	176
normalized size	1	1.	0.72	1.61	0.	5.2	2.2	2.48
time (sec)	N/A	0.022	0.011	0.02	0.	1.797	4.245	59.333

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	66	0	0	312	104	138
normalized size	1	1.	1.32	0.	0.	6.24	2.08	2.76
time (sec)	N/A	0.016	0.074	0.023	0.	1.762	2.11	59.564

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	55	48	72
normalized size	1	1.	1.	0.85	1.1	2.75	2.4	3.6
time (sec)	N/A	0.002	0.005	0.005	1.021	1.723	1.144	1.073

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	26	23	46	80	116	127
normalized size	1	1.	0.63	0.56	1.12	1.95	2.83	3.1
time (sec)	N/A	0.005	0.008	0.003	1.055	1.749	2.534	1.074

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	39	34	70	107	456	144
normalized size	1	1.	0.59	0.52	1.06	1.62	6.91	2.18
time (sec)	N/A	0.01	0.011	0.005	1.037	1.848	16.202	1.094

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	51	160	0	518	972	298
normalized size	1	1.	0.54	1.68	0.	5.45	10.23	3.14
time (sec)	N/A	0.029	0.012	0.032	0.	1.843	15.407	59.316

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	82	0	0	456	835	266
normalized size	1	1.	1.14	0.	0.	6.33	11.6	3.69
time (sec)	N/A	0.022	0.169	0.024	0.	1.903	6.045	59.183

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	78	97	138
normalized size	1	1.	1.	0.77	1.	3.55	4.41	6.27
time (sec)	N/A	0.002	0.006	0.003	1.057	1.775	2.259	1.117

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	30	25	41	101	201	130
normalized size	1	1.	0.67	0.56	0.91	2.24	4.47	2.89
time (sec)	N/A	0.005	0.009	0.003	1.029	1.871	3.565	1.096

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	41	36	68	130	318	255
normalized size	1	1.	0.61	0.54	1.01	1.94	4.75	3.81
time (sec)	N/A	0.01	0.012	0.004	1.118	1.855	16.007	1.16

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	50	45	92	153	692	277
normalized size	1	1.	0.57	0.51	1.05	1.74	7.86	3.15
time (sec)	N/A	0.016	0.014	0.003	1.025	1.85	36.542	1.156

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	60	93	0	328	95	0
normalized size	1	1.	0.68	1.06	0.	3.73	1.08	0.
time (sec)	N/A	0.022	0.041	0.005	0.	1.829	12.531	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	78	0	285	75	0
normalized size	1	1.	0.76	1.16	0.	4.25	1.12	0.
time (sec)	N/A	0.014	0.028	0.005	0.	1.924	4.222	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	62	0	239	54	0
normalized size	1	1.	1.	1.44	0.	5.56	1.26	0.
time (sec)	N/A	0.009	0.015	0.004	0.	1.835	2.258	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	46	0	157	24	0
normalized size	1	1.	1.	1.92	0.	6.54	1.	0.
time (sec)	N/A	0.006	0.004	0.002	0.	1.837	1.163	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	32	15	39
normalized size	1	1.	1.	0.81	1.	2.	0.94	2.44
time (sec)	N/A	0.001	0.003	0.003	1.024	1.81	1.121	1.075

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	23	18	35	50	34	57
normalized size	1	1.	0.61	0.47	0.92	1.32	0.89	1.5
time (sec)	N/A	0.004	0.005	0.003	1.194	1.869	4.147	1.075

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	32	27	55	72	224	74
normalized size	1	1.	0.54	0.46	0.93	1.22	3.8	1.25
time (sec)	N/A	0.007	0.008	0.003	1.029	1.771	28.007	1.098

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	40	35	76	86	374	92
normalized size	1	1.	0.5	0.44	0.95	1.08	4.68	1.15
time (sec)	N/A	0.012	0.009	0.005	1.086	1.755	104.571	1.106

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	30	106	0	383	80	0
normalized size	1	1.	0.35	1.23	0.	4.45	0.93	0.
time (sec)	N/A	0.02	0.006	0.02	0.	1.969	13.494	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	30	100	0	333	58	0
normalized size	1	1.	0.48	1.59	0.	5.29	0.92	0.
time (sec)	N/A	0.014	0.006	0.019	0.	1.743	3.785	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	48	0	302	41	111
normalized size	1	1.	1.	1.09	0.	6.86	0.93	2.52
time (sec)	N/A	0.009	0.029	0.034	0.	1.869	1.756	24.93

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	31	15	59
normalized size	1	1.	1.	0.8	1.	2.07	1.	3.93
time (sec)	N/A	0.001	0.003	0.003	1.087	1.487	1.112	1.073

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	21	18	35	65	34	100
normalized size	1	1.	0.66	0.56	1.09	2.03	1.06	3.12
time (sec)	N/A	0.003	0.006	0.002	1.043	1.562	2.527	1.089

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	32	27	55	90	170	116
normalized size	1	1.	0.6	0.51	1.04	1.7	3.21	2.19
time (sec)	N/A	0.007	0.007	0.003	1.072	1.485	13.931	1.08

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	39	35	76	105	269	144
normalized size	1	1.	0.53	0.47	1.03	1.42	3.64	1.95
time (sec)	N/A	0.014	0.01	0.004	1.027	1.587	55.999	1.091

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	30	136	0	455	308	0
normalized size	1	1.	0.35	1.58	0.	5.29	3.58	0.
time (sec)	N/A	0.02	0.006	0.028	0.	1.551	12.911	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	52	55	0	421	257	0
normalized size	1	1.	0.8	0.85	0.	6.48	3.95	0.
time (sec)	N/A	0.014	0.079	0.029	0.	1.658	5.864	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	16	66	27	111
normalized size	1	1.	1.	0.72	0.89	3.67	1.5	6.17
time (sec)	N/A	0.001	0.004	0.004	1.017	1.527	2.277	1.101

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	18	32	80	75	107
normalized size	1	1.	0.62	0.49	0.86	2.16	2.03	2.89
time (sec)	N/A	0.003	0.006	0.003	1.002	1.475	3.988	1.094

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	32	27	54	105	117	196
normalized size	1	1.	0.58	0.49	0.98	1.91	2.13	3.56
time (sec)	N/A	0.006	0.01	0.003	0.975	1.629	14.105	1.137

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	40	35	74	123	257	213
normalized size	1	1.	0.56	0.49	1.04	1.73	3.62	3.
time (sec)	N/A	0.009	0.011	0.003	0.977	1.515	31.472	1.139

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	61	100	0	335	206	0
normalized size	1	1.	0.67	1.1	0.	3.68	2.26	0.
time (sec)	N/A	0.021	0.041	0.005	0.	1.583	12.717	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	52	84	0	292	163	0
normalized size	1	1.	0.75	1.22	0.	4.23	2.36	0.
time (sec)	N/A	0.015	0.03	0.005	0.	1.564	4.372	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	67	0	246	121	0
normalized size	1	1.	1.	1.49	0.	5.47	2.69	0.
time (sec)	N/A	0.01	0.016	0.004	0.	1.565	2.268	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	50	0	159	58	0
normalized size	1	1.	1.	2.08	0.	6.62	2.42	0.
time (sec)	N/A	0.007	0.004	0.003	0.	1.488	1.221	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	34	39	41
normalized size	1	1.	1.	0.82	1.06	2.	2.29	2.41
time (sec)	N/A	0.001	0.004	0.003	1.	1.542	1.176	1.068

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	24	19	38	53	139	58
normalized size	1	1.	0.6	0.48	0.95	1.32	3.48	1.45
time (sec)	N/A	0.004	0.011	0.002	1.011	1.605	4.109	1.064

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	30	138	0	387	173	0
normalized size	1	1.	0.34	1.55	0.	4.35	1.94	0.
time (sec)	N/A	0.021	0.006	0.026	0.	1.537	12.808	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	30	133	0	339	128	162
normalized size	1	1.	0.46	2.05	0.	5.22	1.97	2.49
time (sec)	N/A	0.014	0.005	0.023	0.	1.601	3.581	17.237

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	67	0	306	92	124
normalized size	1	1.	1.	1.49	0.	6.8	2.04	2.76
time (sec)	N/A	0.009	0.039	0.028	0.	1.562	1.837	17.926

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	47	39	68
normalized size	1	1.	1.	0.81	1.	2.94	2.44	4.25
time (sec)	N/A	0.001	0.004	0.003	1.018	1.511	1.124	1.089

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	21	18	38	66	90	112
normalized size	1	1.	0.62	0.53	1.12	1.94	2.65	3.29
time (sec)	N/A	0.003	0.007	0.003	1.033	1.409	2.512	1.1

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	33	28	59	93	354	130
normalized size	1	1.	0.59	0.5	1.05	1.66	6.32	2.32
time (sec)	N/A	0.008	0.007	0.004	1.006	1.467	13.403	1.069

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	30	168	0	462	753	270
normalized size	1	1.	0.34	1.89	0.	5.19	8.46	3.03
time (sec)	N/A	0.022	0.006	0.029	0.	1.681	12.41	18.103

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	53	73	0	427	649	244
normalized size	1	1.	0.79	1.09	0.	6.37	9.69	3.64
time (sec)	N/A	0.015	0.055	0.032	0.	1.667	5.829	18.031

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	68	65	128
normalized size	1	1.	1.	0.74	0.95	3.58	3.42	6.74
time (sec)	N/A	0.001	0.004	0.002	1.078	1.652	2.268	1.094

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	24	19	34	82	163	122
normalized size	1	1.	0.62	0.49	0.87	2.1	4.18	3.13
time (sec)	N/A	0.004	0.007	0.004	1.056	1.621	4.082	1.089

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	33	28	57	107	243	230
normalized size	1	1.	0.57	0.48	0.98	1.84	4.19	3.97
time (sec)	N/A	0.006	0.011	0.003	1.021	1.589	13.958	1.111

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	41	36	78	126	529	247
normalized size	1	1.	0.55	0.48	1.04	1.68	7.05	3.29
time (sec)	N/A	0.01	0.013	0.003	0.991	1.632	30.869	1.164

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	41	50	73	54	23
normalized size	1	1.	0.93	1.52	1.85	2.7	2.	0.85
time (sec)	N/A	0.005	0.008	0.005	1.591	1.452	1.755	1.053

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	12	27	19	45	20	8
normalized size	1	1.	1.5	3.38	2.38	5.62	2.5	1.
time (sec)	N/A	0.003	0.008	0.003	1.55	1.53	1.005	1.07

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	48	0	165	42	0
normalized size	1	1.	1.	2.53	0.	8.68	2.21	0.
time (sec)	N/A	0.005	0.007	0.007	0.	1.534	1.155	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	18	47	19	18
normalized size	1	1.	0.81	0.67	0.86	2.24	0.9	0.86
time (sec)	N/A	0.004	0.006	0.002	1.072	1.39	2.212	1.049

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	18	47	19	18
normalized size	1	1.	0.81	0.67	0.86	2.24	0.9	0.86
time (sec)	N/A	0.004	0.004	0.002	1.11	1.532	1.469	1.054

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	18	43	19	18
normalized size	1	1.	0.81	0.67	0.86	2.05	0.9	0.86
time (sec)	N/A	0.004	0.005	0.001	1.049	1.544	0.485	1.056

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	18	43	19	18
normalized size	1	1.	0.81	0.67	0.86	2.05	0.9	0.86
time (sec)	N/A	0.004	0.004	0.002	1.053	1.501	1.245	1.05

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	18	38	19	18
normalized size	1	1.	0.81	0.67	0.86	1.81	0.9	0.86
time (sec)	N/A	0.004	0.005	0.002	1.063	1.53	1.32	1.073

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	18	34	17	18
normalized size	1	1.	0.84	0.68	0.95	1.79	0.89	0.95
time (sec)	N/A	0.003	0.005	0.002	1.017	1.538	1.143	1.055

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	14	18	34	17	18
normalized size	1	1.	0.84	0.74	0.95	1.79	0.89	0.95
time (sec)	N/A	0.004	0.005	0.003	1.072	1.537	0.437	1.064

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	12	18	34	17	18
normalized size	1	1.	1.	0.63	0.95	1.79	0.89	0.95
time (sec)	N/A	0.004	0.005	0.002	1.002	1.418	0.486	1.059

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	32	74	34	32
normalized size	1	1.	0.78	0.69	0.89	2.06	0.94	0.89
time (sec)	N/A	0.007	0.008	0.003	1.002	1.46	4.026	1.073

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	32	73	34	32
normalized size	1	1.	0.78	0.69	0.89	2.03	0.94	0.89
time (sec)	N/A	0.007	0.008	0.005	1.037	1.462	3.168	1.058

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	32	70	34	32
normalized size	1	1.	0.78	0.69	0.89	1.94	0.94	0.89
time (sec)	N/A	0.007	0.007	0.004	1.046	1.518	1.296	1.063

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	32	70	2635	32
normalized size	1	1.	0.78	0.69	0.89	1.94	73.19	0.89
time (sec)	N/A	0.007	0.007	0.004	0.995	1.546	2.695	1.045

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	32	62	1766	32
normalized size	1	1.	0.78	0.69	0.89	1.72	49.06	0.89
time (sec)	N/A	0.008	0.007	0.003	1.023	1.505	2.16	1.047

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	25	32	61	1742	32
normalized size	1	1.	0.82	0.74	0.94	1.79	51.24	0.94
time (sec)	N/A	0.007	0.007	0.003	1.501	1.506	2.398	1.064

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	27	25	32	55	1828	32
normalized size	1	1.	0.84	0.78	1.	1.72	57.12	1.
time (sec)	N/A	0.007	0.008	0.003	1.34	1.469	2.227	1.054

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	25	32	55	1958	32
normalized size	1	1.	0.79	0.74	0.94	1.62	57.59	0.94
time (sec)	N/A	0.007	0.007	0.004	1.561	1.46	2.185	1.057

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	47	108	49	47
normalized size	1	1.	0.76	0.71	0.92	2.12	0.96	0.92
time (sec)	N/A	0.01	0.012	0.004	1.42	1.55	7.315	1.079

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	47	107	49	47
normalized size	1	1.	0.76	0.71	0.92	2.1	0.96	0.92
time (sec)	N/A	0.011	0.011	0.004	1.462	1.476	5.237	1.051

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	47	101	49	47
normalized size	1	1.	0.76	0.71	0.92	1.98	0.96	0.92
time (sec)	N/A	0.011	0.01	0.004	1.187	1.458	2.505	1.056

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	47	100	5013	47
normalized size	1	1.	0.76	0.71	0.92	1.96	98.29	0.92
time (sec)	N/A	0.011	0.01	0.003	1.186	1.443	4.118	1.054

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	47	92	6248	47
normalized size	1	1.	0.76	0.71	0.92	1.8	122.51	0.92
time (sec)	N/A	0.01	0.01	0.004	1.104	1.509	3.64	1.061

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	39	36	47	90	6669	47
normalized size	1	1.	0.8	0.73	0.96	1.84	136.1	0.96
time (sec)	N/A	0.011	0.01	0.003	1.037	1.559	3.845	1.053

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	39	36	47	85	4005	47
normalized size	1	1.	0.8	0.73	0.96	1.73	81.73	0.96
time (sec)	N/A	0.011	0.01	0.004	1.036	1.522	3.627	1.059

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	39	36	47	85	3966	47
normalized size	1	1.	0.8	0.73	0.96	1.73	80.94	0.96
time (sec)	N/A	0.011	0.011	0.004	1.007	1.488	3.653	1.069

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	38	122	0	381	0	186
normalized size	1	1.	0.3	0.98	0.	3.05	0.	1.49
time (sec)	N/A	0.07	0.009	0.007	0.	1.509	0.	1.099

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	140	121	0	312	240	184
normalized size	1	1.	1.14	0.98	0.	2.54	1.95	1.5
time (sec)	N/A	0.056	0.058	0.005	0.	1.56	43.71	1.075

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	29	107	0	336	228	159
normalized size	1	1.	0.26	0.96	0.	3.03	2.05	1.43
time (sec)	N/A	0.04	0.007	0.005	0.	1.789	12.821	1.097

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	126	108	0	292	224	161
normalized size	1	1.	1.16	0.99	0.	2.68	2.06	1.48
time (sec)	N/A	0.039	0.028	0.004	0.	1.864	7.681	1.089

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	27	96	0	801	218	159
normalized size	1	1.	0.27	0.96	0.	8.01	2.18	1.59
time (sec)	N/A	0.028	0.005	0.004	0.	1.879	9.682	1.093

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	103	95	0	807	218	158
normalized size	1	1.	1.03	0.95	0.	8.07	2.18	1.58
time (sec)	N/A	0.028	0.023	0.003	0.	1.932	16.893	1.076

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	25	104	0	304	223	169
normalized size	1	1.	0.23	0.95	0.	2.79	2.05	1.55
time (sec)	N/A	0.038	0.005	0.006	0.	1.725	46.087	1.074

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	27	105	0	365	231	162
normalized size	1	1.	0.24	0.95	0.	3.29	2.08	1.46
time (sec)	N/A	0.039	0.005	0.005	0.	1.521	59.789	1.089

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	27	123	0	420	0	182
normalized size	1	1.	0.21	0.95	0.	3.26	0.	1.41
time (sec)	N/A	0.047	0.005	0.011	0.	1.559	0.	1.077

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	27	123	0	373	0	182
normalized size	1	1.	0.22	0.98	0.	2.98	0.	1.46
time (sec)	N/A	0.047	0.004	0.01	0.	1.623	0.	1.077

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	27	112	0	987	0	184
normalized size	1	1.	0.23	0.97	0.	8.58	0.	1.6
time (sec)	N/A	0.038	0.004	0.009	0.	1.4	0.	1.116

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	27	112	0	999	638	184
normalized size	1	1.	0.23	0.96	0.	8.54	5.45	1.57
time (sec)	N/A	0.038	0.005	0.008	0.	1.301	136.351	1.087

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	27	120	0	990	819	178
normalized size	1	1.	0.23	1.03	0.	8.53	7.06	1.53
time (sec)	N/A	0.041	0.005	0.006	0.	1.276	159.72	1.084

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	25	120	0	991	0	178
normalized size	1	1.	0.22	1.06	0.	8.77	0.	1.58
time (sec)	N/A	0.04	0.005	0.007	0.	1.336	0.	1.083

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	25	121	0	393	0	196
normalized size	1	1.	0.2	0.98	0.	3.17	0.	1.58
time (sec)	N/A	0.049	0.006	0.012	0.	1.353	0.	1.091

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	27	121	0	456	0	185
normalized size	1	1.	0.21	0.95	0.	3.56	0.	1.45
time (sec)	N/A	0.05	0.005	0.012	0.	1.451	0.	1.08

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	27	124	0	1219	0	197
normalized size	1	1.	0.19	0.89	0.	8.71	0.	1.41
time (sec)	N/A	0.049	0.004	0.011	0.	1.418	0.	1.088

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	27	124	0	1227	0	197
normalized size	1	1.	0.19	0.89	0.	8.76	0.	1.41
time (sec)	N/A	0.054	0.004	0.01	0.	1.476	0.	1.08

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	27	132	0	1208	0	201
normalized size	1	1.	0.19	0.92	0.	8.45	0.	1.41
time (sec)	N/A	0.05	0.005	0.01	0.	1.66	0.	1.095

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	27	132	0	1215	0	200
normalized size	1	1.	0.19	0.92	0.	8.5	0.	1.4
time (sec)	N/A	0.052	0.005	0.01	0.	1.727	0.	1.075

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	27	136	0	1220	0	193
normalized size	1	1.	0.19	0.97	0.	8.71	0.	1.38
time (sec)	N/A	0.053	0.005	0.006	0.	1.673	0.	1.084

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	25	136	0	1226	0	193
normalized size	1	1.	0.18	0.97	0.	8.76	0.	1.38
time (sec)	N/A	0.05	0.004	0.006	0.	1.685	0.	1.078

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	25	139	0	510	0	209
normalized size	1	1.	0.16	0.91	0.	3.36	0.	1.38
time (sec)	N/A	0.06	0.005	0.013	0.	1.742	0.	1.086

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	27	139	0	570	0	203
normalized size	1	1.	0.18	0.91	0.	3.75	0.	1.34
time (sec)	N/A	0.061	0.006	0.014	0.	1.568	0.	1.077

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	62	82	255	243	86
normalized size	1	1.	1.	1.07	1.41	4.4	4.19	1.48
time (sec)	N/A	0.021	0.016	0.007	1.55	1.668	2.202	1.095

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	166	1535	0	3479	9996	2599
normalized size	1	1.	0.89	8.21	0.	18.6	53.45	13.9
time (sec)	N/A	0.085	0.105	0.006	0.	1.781	6.459	1.105

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	118	782	0	1608	4257	1339
normalized size	1	1.	0.89	5.88	0.	12.09	32.01	10.07
time (sec)	N/A	0.05	0.067	0.005	0.	1.608	2.904	1.085

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	170	0	336	663	302
normalized size	1	1.	0.89	2.79	0.	5.51	10.87	4.95
time (sec)	N/A	0.018	0.03	0.	0.	1.584	0.769	1.073

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	38	87	0	178	299	158
normalized size	1	1.	0.88	2.02	0.	4.14	6.95	3.67
time (sec)	N/A	0.013	0.028	0.002	0.	1.562	0.476	1.055

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	31	0	72	87	58
normalized size	1	1.	0.88	1.24	0.	2.88	3.48	2.32
time (sec)	N/A	0.006	0.013	0.	0.	1.579	0.262	1.063

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	61	0
normalized size	1	1.	1.	0.	0.	0.	2.1	0.
time (sec)	N/A	0.006	0.006	0.026	0.	0.	0.63	0.

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	262	0
normalized size	1	1.	1.	0.	0.	0.	9.03	0.
time (sec)	N/A	0.005	0.005	0.032	0.	0.	0.842	0.

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	717	0
normalized size	1	1.	1.	0.	0.	0.	24.72	0.
time (sec)	N/A	0.005	0.005	0.041	0.	0.	1.168	0.

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	37	0
normalized size	1	1.	1.	0.	0.	0.	0.77	0.
time (sec)	N/A	0.011	0.063	0.02	0.	0.	53.269	0.

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	37	0
normalized size	1	1.	1.	0.	0.	0.	0.77	0.
time (sec)	N/A	0.012	0.04	0.02	0.	0.	5.222	0.

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	37	0
normalized size	1	1.	1.	0.	0.	0.	0.77	0.
time (sec)	N/A	0.011	0.03	0.02	0.	0.	1.444	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	36	0
normalized size	1	1.	1.	0.	0.	0.	0.78	0.
time (sec)	N/A	0.011	0.023	0.02	0.	0.	1.235	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	36	0
normalized size	1	1.	1.	0.	0.	0.	0.78	0.
time (sec)	N/A	0.011	0.035	0.018	0.	0.	1.953	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	36	0
normalized size	1	1.	1.	0.	0.	0.	0.75	0.
time (sec)	N/A	0.011	0.038	0.019	0.	0.	5.987	0.

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	37	0
normalized size	1	1.	1.	0.	0.	0.	0.73	0.
time (sec)	N/A	0.012	0.036	0.022	0.	0.	4.704	0.

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	37	0
normalized size	1	1.	1.	0.	0.	0.	0.76	0.
time (sec)	N/A	0.012	0.032	0.021	0.	0.	2.717	0.

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	36	0
normalized size	1	1.	1.	0.	0.	0.	0.78	0.
time (sec)	N/A	0.011	0.008	0.	0.	0.	1.256	0.

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	31	0
normalized size	1	1.	1.	0.	0.	0.	0.65	0.
time (sec)	N/A	0.012	0.029	0.021	0.	0.	5.56	0.

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	32	0
normalized size	1	1.	1.	0.	0.	0.	0.65	0.
time (sec)	N/A	0.012	0.031	0.02	0.	0.	34.981	0.

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	37	0
normalized size	1	1.	1.	0.	0.	0.	0.73	0.
time (sec)	N/A	0.013	0.032	0.02	0.	0.	137.449	0.

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	0	0	37	0
normalized size	1	1.	1.	0.94	0.	0.	1.19	0.
time (sec)	N/A	0.006	0.005	0.023	0.	0.	1.006	0.

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	0	0	46	0
normalized size	1	1.	1.	0.94	0.	0.	1.48	0.
time (sec)	N/A	0.004	0.006	0.026	0.	0.	1.015	0.

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	43	0	0	36	0
normalized size	1	1.	1.	1.19	0.	0.	1.	0.
time (sec)	N/A	0.008	0.006	0.033	0.	0.	1.016	0.

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	30	0	0	41	0
normalized size	1	1.	0.96	0.6	0.	0.	0.82	0.
time (sec)	N/A	0.011	0.023	0.016	0.	0.	1.018	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	42	0
normalized size	1	1.	1.	0.	0.	0.	0.88	0.
time (sec)	N/A	0.011	0.017	0.022	0.	0.	1.271	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	32	30	0	0	44	0
normalized size	1	1.	0.94	0.88	0.	0.	1.29	0.
time (sec)	N/A	0.006	0.005	0.017	0.	0.	0.989	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	32	30	0	0	53	0
normalized size	1	1.	0.94	0.88	0.	0.	1.56	0.
time (sec)	N/A	0.005	0.005	0.023	0.	0.	1.047	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	0	0	42	0
normalized size	1	1.	1.	0.9	0.	0.	0.86	0.
time (sec)	N/A	0.011	0.007	0.033	0.	0.	1.037	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	57	31	0	0	48	0
normalized size	1	1.	1.54	0.84	0.	0.	1.3	0.
time (sec)	N/A	0.005	0.014	0.017	0.	0.	1.058	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	0	26	0
normalized size	1	1.	1.	0.88	0.	0.	1.	0.
time (sec)	N/A	0.004	0.005	0.03	0.	0.	0.929	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	0	0	31	0
normalized size	1	1.	1.	0.	0.	0.	1.03	0.
time (sec)	N/A	0.004	0.007	0.023	0.	0.	0.996	0.

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	34	0
normalized size	1	1.	1.	0.	0.	0.	0.72	0.
time (sec)	N/A	0.011	0.01	0.054	0.	0.	2.699	0.

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	0	0	0	37	0
normalized size	1	1.	0.92	0.	0.	0.	0.71	0.
time (sec)	N/A	0.014	0.007	0.066	0.	0.	2.19	0.

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	126	136	292	1319	305
normalized size	1	1.	0.81	1.52	1.64	3.52	15.89	3.67
time (sec)	N/A	0.031	0.05	0.005	1.046	1.541	1.806	1.06

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	57	73	92	188	597	189
normalized size	1	1.	0.95	1.22	1.53	3.13	9.95	3.15
time (sec)	N/A	0.019	0.025	0.004	1.103	1.63	1.059	1.07

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	36	57	104	201	103
normalized size	1	1.	0.85	0.92	1.46	2.67	5.15	2.64
time (sec)	N/A	0.012	0.016	0.003	1.098	1.669	0.566	1.062

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	17	19	0	45	20	24
normalized size	1	1.	0.94	1.06	0.	2.5	1.11	1.33
time (sec)	N/A	0.003	0.009	0.001	0.	1.535	0.06	1.078

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	83	0
normalized size	1	1.	1.	0.	0.	0.	2.37	0.
time (sec)	N/A	0.006	0.011	0.021	0.	0.	1.428	0.

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	354	0
normalized size	1	1.	1.	0.	0.	0.	10.11	0.
time (sec)	N/A	0.007	0.006	0.027	0.	0.	1.901	0.

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	918	0
normalized size	1	1.	1.	0.	0.	0.	24.16	0.
time (sec)	N/A	0.008	0.006	0.033	0.	0.	2.507	0.

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	64	77	0	208	0	0
normalized size	1	1.	0.58	0.7	0.	1.89	0.	0.
time (sec)	N/A	0.036	0.026	0.007	0.	1.622	0.	0.

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	39	44	0	126	0	0
normalized size	1	1.	0.61	0.69	0.	1.97	0.	0.
time (sec)	N/A	0.009	0.016	0.004	0.	1.642	0.	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	29	0	66	0	0
normalized size	1	1.	0.89	1.04	0.	2.36	0.	0.
time (sec)	N/A	0.003	0.006	0.003	0.	1.659	0.	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.01	0.007	0.047	0.	0.	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	32	0
normalized size	1	1.	1.	0.	0.	0.	0.71	0.
time (sec)	N/A	0.01	0.007	0.048	0.	0.	41.13	0.

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.008	0.052	0.	0.	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.007	0.019	0.	0.	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	27	0
normalized size	1	1.	1.	0.	0.	0.	0.6	0.
time (sec)	N/A	0.009	0.007	0.019	0.	0.	13.821	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	26	0
normalized size	1	1.	1.	0.	0.	0.	0.6	0.
time (sec)	N/A	0.009	0.006	0.021	0.	0.	7.766	0.

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	29	0
normalized size	1	1.	1.	0.	0.	0.	0.67	0.
time (sec)	N/A	0.009	0.007	0.018	0.	0.	54.59	0.

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.007	0.019	0.	0.	0.	0.

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	32	0	0	37	0
normalized size	1	1.	0.89	0.91	0.	0.	1.06	0.
time (sec)	N/A	0.011	0.006	0.069	0.	0.	2.126	0.

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	44	0	0	0	37	0
normalized size	1	1.	1.1	0.	0.	0.	0.92	0.
time (sec)	N/A	0.009	0.01	0.092	0.	0.	2.14	0.

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	0	0	0	37	0
normalized size	1	1.	0.92	0.	0.	0.	0.71	0.
time (sec)	N/A	0.013	0.012	0.064	0.	0.	2.154	0.

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	30	68	0	0
normalized size	1	1.	1.	1.05	1.58	3.58	0.	0.
time (sec)	N/A	0.003	0.005	0.001	1.132	1.61	0.	0.

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	40	41	0	126	0	0
normalized size	1	1.	0.69	0.71	0.	2.17	0.	0.
time (sec)	N/A	0.013	0.014	0.005	0.	1.697	0.	0.

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	40	41	0	126	0	0
normalized size	1	1.	0.69	0.71	0.	2.17	0.	0.
time (sec)	N/A	0.011	0.002	0.	0.	1.562	0.	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	0	51	36	30
normalized size	1	1.	0.69	0.6	0.	1.46	1.03	0.86
time (sec)	N/A	0.011	0.005	0.004	0.	1.525	0.395	1.05

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	0	51	36	30
normalized size	1	1.	0.69	0.6	0.	1.46	1.03	0.86
time (sec)	N/A	0.01	0.004	0.003	0.	1.565	0.305	1.066

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	0	51	36	30
normalized size	1	1.	0.69	0.6	0.	1.46	1.03	0.86
time (sec)	N/A	0.009	0.003	0.002	0.	1.465	0.236	1.072

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	22	19	0	47	34	30
normalized size	1	1.	0.67	0.58	0.	1.42	1.03	0.91
time (sec)	N/A	0.008	0.003	0.004	0.	1.572	0.193	1.049

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	17	0	39	29	23
normalized size	1	1.	0.89	0.63	0.	1.44	1.07	0.85
time (sec)	N/A	0.004	0.005	0.002	0.	1.478	0.195	1.054

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	20	20	0	43	0	23
normalized size	1	1.	0.71	0.71	0.	1.54	0.	0.82
time (sec)	N/A	0.005	0.004	0.004	0.	1.529	0.	1.068

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	20	21	0	46	0	27
normalized size	1	1.	0.62	0.66	0.	1.44	0.	0.84
time (sec)	N/A	0.007	0.005	0.005	0.	1.521	0.	1.06

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	22	19	0	46	36	26
normalized size	1	1.	0.85	0.73	0.	1.77	1.38	1.
time (sec)	N/A	0.004	0.004	0.004	0.	1.459	0.482	1.067

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	24	21	0	57	36	30
normalized size	1	1.	0.65	0.57	0.	1.54	0.97	0.81
time (sec)	N/A	0.013	0.007	0.003	0.	1.496	1.096	1.064

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	24	21	0	57	36	30
normalized size	1	1.	0.65	0.57	0.	1.54	0.97	0.81
time (sec)	N/A	0.012	0.006	0.003	0.	1.545	0.833	1.051

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	24	21	0	57	36	30
normalized size	1	1.	0.65	0.57	0.	1.54	0.97	0.81
time (sec)	N/A	0.011	0.006	0.002	0.	1.516	0.651	1.065

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	22	19	0	57	34	30
normalized size	1	1.	0.59	0.51	0.	1.54	0.92	0.81
time (sec)	N/A	0.01	0.005	0.002	0.	1.548	0.485	1.063

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	25	18	0	57	31	30
normalized size	1	1.	0.68	0.49	0.	1.54	0.84	0.81
time (sec)	N/A	0.009	0.002	0.003	0.	1.396	0.521	1.058

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	23	21	0	53	31	30
normalized size	1	1.	0.66	0.6	0.	1.51	0.89	0.86
time (sec)	N/A	0.008	0.002	0.002	0.	1.524	0.512	1.071

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	21	20	0	45	32	23
normalized size	1	1.	0.72	0.69	0.	1.55	1.1	0.79
time (sec)	N/A	0.004	0.003	0.002	0.	1.497	0.696	1.066

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	21	20	0	49	0	23
normalized size	1	1.	0.7	0.67	0.	1.63	0.	0.77
time (sec)	N/A	0.005	0.004	0.004	0.	1.864	0.	1.06

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	0	63	36	38
normalized size	1	1.	0.59	0.51	0.	1.54	0.88	0.93
time (sec)	N/A	0.016	0.007	0.003	0.	1.825	2.872	1.057

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	0	62	36	38
normalized size	1	1.	0.59	0.51	0.	1.51	0.88	0.93
time (sec)	N/A	0.015	0.007	0.003	0.	1.665	2.036	1.073

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	0	62	36	38
normalized size	1	1.	0.59	0.51	0.	1.51	0.88	0.93
time (sec)	N/A	0.013	0.006	0.002	0.	1.753	1.732	1.048

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	22	19	0	62	34	38
normalized size	1	1.	0.54	0.46	0.	1.51	0.83	0.93
time (sec)	N/A	0.013	0.006	0.002	0.	1.733	1.323	1.061

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	25	18	0	62	31	38
normalized size	1	1.	0.61	0.44	0.	1.51	0.76	0.93
time (sec)	N/A	0.012	0.003	0.002	0.	1.801	1.334	1.069

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	23	21	0	62	31	38
normalized size	1	1.	0.56	0.51	0.	1.51	0.76	0.93
time (sec)	N/A	0.011	0.003	0.002	0.	1.844	1.359	1.042

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	27	21	0	62	34	38
normalized size	1	1.	0.66	0.51	0.	1.51	0.83	0.93
time (sec)	N/A	0.01	0.003	0.003	0.	1.885	1.527	1.055

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	25	21	0	58	36	38
normalized size	1	1.	0.64	0.54	0.	1.49	0.92	0.97
time (sec)	N/A	0.008	0.002	0.003	0.	1.827	1.523	1.07

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	45	54	36	35
normalized size	1	1.	0.69	0.6	1.29	1.54	1.03	1.
time (sec)	N/A	0.009	0.004	0.002	1.044	1.809	0.561	1.063

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	35	50	36	32
normalized size	1	1.	0.69	0.6	1.	1.43	1.03	0.91
time (sec)	N/A	0.008	0.004	0.001	1.044	1.691	0.527	1.066

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	23	20	30	42	34	30
normalized size	1	1.	0.72	0.62	0.94	1.31	1.06	0.94
time (sec)	N/A	0.004	0.001	0.002	1.006	1.665	0.438	1.078

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	19	18	27	49	0	47
normalized size	1	1.	0.66	0.62	0.93	1.69	0.	1.62
time (sec)	N/A	0.005	0.002	0.002	1.046	1.793	0.	1.076

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	18	23	51	0	63
normalized size	1	1.	0.85	0.67	0.85	1.89	0.	2.33
time (sec)	N/A	0.006	0.006	0.003	1.071	1.828	0.	1.073

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	23	19	26	51	31	0
normalized size	1	1.	0.88	0.73	1.	1.96	1.19	0.
time (sec)	N/A	0.004	0.006	0.002	1.05	1.692	0.505	0.

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	22	21	26	54	36	0
normalized size	1	1.	0.63	0.6	0.74	1.54	1.03	0.
time (sec)	N/A	0.007	0.006	0.003	1.022	1.544	0.593	0.

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	26	55	37	0
normalized size	1	1.	0.69	0.6	0.74	1.57	1.06	0.
time (sec)	N/A	0.007	0.005	0.003	1.039	1.581	0.729	0.

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	43	45	34	34
normalized size	1	1.	0.61	0.53	1.13	1.18	0.89	0.89
time (sec)	N/A	0.006	0.004	0.002	1.024	1.547	0.598	1.074

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	21	20	31	51	0	54
normalized size	1	1.	0.6	0.57	0.89	1.46	0.	1.54
time (sec)	N/A	0.005	0.004	0.002	1.073	1.603	0.	1.06

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	22	21	28	54	0	63
normalized size	1	1.	0.67	0.64	0.85	1.64	0.	1.91
time (sec)	N/A	0.007	0.003	0.003	1.069	1.619	0.	1.068

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	22	17	31	54	34	0
normalized size	1	1.	0.76	0.59	1.07	1.86	1.17	0.
time (sec)	N/A	0.004	0.003	0.003	1.062	1.629	0.506	0.

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	25	18	26	57	32	0
normalized size	1	1.	0.61	0.44	0.63	1.39	0.78	0.
time (sec)	N/A	0.008	0.009	0.003	1.123	1.455	0.637	0.

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	27	21	26	58	32	0
normalized size	1	1.	0.66	0.51	0.63	1.41	0.78	0.
time (sec)	N/A	0.007	0.008	0.003	1.044	1.58	0.712	0.

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	22	21	26	58	36	0
normalized size	1	1.	0.54	0.51	0.63	1.41	0.88	0.
time (sec)	N/A	0.008	0.008	0.003	1.031	1.495	0.86	0.

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	26	58	37	0
normalized size	1	1.	0.59	0.51	0.63	1.41	0.9	0.
time (sec)	N/A	0.007	0.007	0.003	1.063	1.497	1.002	0.

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	22	21	32	54	0	63
normalized size	1	1.	0.67	0.64	0.97	1.64	0.	1.91
time (sec)	N/A	0.008	0.006	0.003	1.104	1.501	0.	1.103

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	19	35	54	36	0
normalized size	1	1.	0.83	0.66	1.21	1.86	1.24	0.
time (sec)	N/A	0.005	0.006	0.003	1.014	1.467	0.855	0.

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	31	57	37	0
normalized size	1	1.	0.59	0.51	0.76	1.39	0.9	0.
time (sec)	N/A	0.008	0.004	0.001	1.069	1.591	0.853	0.

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	27	19	31	58	36	0
normalized size	1	1.	0.66	0.46	0.76	1.41	0.88	0.
time (sec)	N/A	0.007	0.003	0.002	1.065	1.544	0.879	0.

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	27	18	26	58	32	0
normalized size	1	1.	0.66	0.44	0.63	1.41	0.78	0.
time (sec)	N/A	0.008	0.008	0.004	1.122	1.512	0.994	0.

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	27	21	26	58	32	0
normalized size	1	1.	0.66	0.51	0.63	1.41	0.78	0.
time (sec)	N/A	0.008	0.007	0.002	1.034	1.572	1.234	0.

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	22	21	26	58	36	0
normalized size	1	1.	0.54	0.51	0.63	1.41	0.88	0.
time (sec)	N/A	0.009	0.008	0.003	1.087	1.606	1.492	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	26	58	37	0
normalized size	1	1.	0.59	0.51	0.63	1.41	0.9	0.
time (sec)	N/A	0.008	0.007	0.003	1.088	1.489	1.778	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	0	78	60	47
normalized size	1	1.	0.61	0.56	0.	1.37	1.05	0.82
time (sec)	N/A	0.015	0.006	0.002	0.	1.515	0.525	1.058

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	0	77	61	47
normalized size	1	1.	0.61	0.56	0.	1.35	1.07	0.82
time (sec)	N/A	0.015	0.005	0.003	0.	1.602	0.406	1.06

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	0	76	60	47
normalized size	1	1.	0.61	0.56	0.	1.33	1.05	0.82
time (sec)	N/A	0.013	0.005	0.003	0.	1.509	0.347	1.057

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	33	30	0	70	60	47
normalized size	1	1.	0.6	0.55	0.	1.27	1.09	0.85
time (sec)	N/A	0.014	0.006	0.001	0.	1.546	0.259	1.072

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	28	0	61	51	39
normalized size	1	1.	0.96	1.08	0.	2.35	1.96	1.5
time (sec)	N/A	0.004	0.005	0.003	0.	1.552	0.258	1.046

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	33	33	0	73	0	43
normalized size	1	1.	0.67	0.67	0.	1.49	0.	0.88
time (sec)	N/A	0.009	0.01	0.006	0.	1.581	0.	1.084

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	31	32	0	68	0	42
normalized size	1	1.	0.63	0.65	0.	1.39	0.	0.86
time (sec)	N/A	0.011	0.012	0.009	0.	1.732	0.	1.06

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	36	34	0	76	0	47
normalized size	1	1.	0.67	0.63	0.	1.41	0.	0.87
time (sec)	N/A	0.012	0.009	0.008	0.	1.6	0.	1.065

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	35	32	0	86	60	47
normalized size	1	1.	0.58	0.53	0.	1.43	1.	0.78
time (sec)	N/A	0.019	0.008	0.005	0.	1.487	1.386	1.059

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	35	32	0	86	61	47
normalized size	1	1.	0.58	0.53	0.	1.43	1.02	0.78
time (sec)	N/A	0.017	0.009	0.005	0.	1.458	1.138	1.051

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	35	32	0	86	60	47
normalized size	1	1.	0.58	0.53	0.	1.43	1.	0.78
time (sec)	N/A	0.016	0.008	0.004	0.	1.425	0.878	1.068

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	33	30	0	85	60	47
normalized size	1	1.	0.55	0.5	0.	1.42	1.	0.78
time (sec)	N/A	0.015	0.007	0.003	0.	1.544	0.732	1.048

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	36	29	0	84	54	47
normalized size	1	1.	0.6	0.48	0.	1.4	0.9	0.78
time (sec)	N/A	0.015	0.004	0.003	0.	1.512	0.688	1.081

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	34	32	0	78	54	47
normalized size	1	1.	0.59	0.55	0.	1.34	0.93	0.81
time (sec)	N/A	0.013	0.004	0.002	0.	1.416	0.709	1.056

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	31	0	69	51	39
normalized size	1	1.	0.96	1.15	0.	2.56	1.89	1.44
time (sec)	N/A	0.004	0.005	0.002	0.	1.438	0.855	1.06

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	34	33	0	81	0	43
normalized size	1	1.	0.65	0.63	0.	1.56	0.	0.83
time (sec)	N/A	0.01	0.008	0.004	0.	1.504	0.	1.05

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	35	32	0	95	60	59
normalized size	1	1.	0.53	0.48	0.	1.44	0.91	0.89
time (sec)	N/A	0.019	0.008	0.003	0.	1.538	2.087	1.058

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	33	30	0	95	60	59
normalized size	1	1.	0.5	0.45	0.	1.44	0.91	0.89
time (sec)	N/A	0.017	0.008	0.003	0.	1.51	1.679	1.049

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	36	29	0	95	54	59
normalized size	1	1.	0.55	0.44	0.	1.44	0.82	0.89
time (sec)	N/A	0.016	0.004	0.003	0.	1.56	1.697	1.099

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	34	32	0	93	54	59
normalized size	1	1.	0.52	0.48	0.	1.41	0.82	0.89
time (sec)	N/A	0.015	0.005	0.005	0.	1.559	1.753	1.058

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	38	32	0	92	54	59
normalized size	1	1.	0.58	0.48	0.	1.39	0.82	0.89
time (sec)	N/A	0.014	0.004	0.004	0.	1.466	1.858	1.051

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	32	0	86	60	59
normalized size	1	1.	0.56	0.5	0.	1.34	0.94	0.92
time (sec)	N/A	0.015	0.003	0.002	0.	1.443	1.878	1.045

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	31	0	77	56	55
normalized size	1	1.	0.9	1.07	0.	2.66	1.93	1.9
time (sec)	N/A	0.004	0.006	0.002	0.	1.494	1.905	1.074

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	35	33	0	89	0	55
normalized size	1	1.	0.6	0.57	0.	1.53	0.	0.95
time (sec)	N/A	0.011	0.01	0.005	0.	1.485	0.	1.048

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	73	78	60	55
normalized size	1	1.	0.61	0.56	1.28	1.37	1.05	0.96
time (sec)	N/A	0.013	0.006	0.003	1.031	1.443	0.696	1.068

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	63	73	61	51
normalized size	1	1.	0.61	0.56	1.11	1.28	1.07	0.89
time (sec)	N/A	0.012	0.005	0.003	1.05	1.525	0.594	1.088

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	31	57	63	56	49
normalized size	1	1.	1.	1.29	2.38	2.62	2.33	2.04
time (sec)	N/A	0.003	0.002	0.001	1.045	1.455	0.507	1.065

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	32	31	47	78	0	68
normalized size	1	1.	0.62	0.6	0.9	1.5	0.	1.31
time (sec)	N/A	0.01	0.003	0.004	1.035	1.568	0.	1.071

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	34	29	47	73	0	88
normalized size	1	1.	0.72	0.62	1.	1.55	0.	1.87
time (sec)	N/A	0.011	0.009	0.003	1.014	1.557	0.	1.075

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	35	34	42	81	0	0
normalized size	1	1.	0.71	0.69	0.86	1.65	0.	0.
time (sec)	N/A	0.011	0.008	0.004	1.084	1.465	0.	0.

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	33	30	45	73	53	0
normalized size	1	1.	1.27	1.15	1.73	2.81	2.04	0.
time (sec)	N/A	0.004	0.01	0.003	1.071	1.493	0.608	0.

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	45	77	61	0
normalized size	1	1.	0.61	0.56	0.79	1.35	1.07	0.
time (sec)	N/A	0.013	0.008	0.003	1.087	1.602	0.755	0.

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	31	70	66	56	53
normalized size	1	1.	0.96	1.15	2.59	2.44	2.07	1.96
time (sec)	N/A	0.004	0.004	0.004	1.066	1.496	0.759	1.07

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	34	33	61	81	0	74
normalized size	1	1.	0.56	0.54	1.	1.33	0.	1.21
time (sec)	N/A	0.011	0.007	0.005	1.048	1.545	0.	1.076

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	33	32	57	76	0	93
normalized size	1	1.	0.59	0.57	1.02	1.36	0.	1.66
time (sec)	N/A	0.012	0.006	0.003	1.065	1.54	0.	1.071

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	34	32	47	84	0	0
normalized size	1	1.	0.59	0.55	0.81	1.45	0.	0.
time (sec)	N/A	0.012	0.004	0.004	1.058	1.586	0.	0.

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	36	27	50	76	53	0
normalized size	1	1.	1.24	0.93	1.72	2.62	1.83	0.
time (sec)	N/A	0.004	0.012	0.003	1.053	1.483	0.611	0.

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	38	32	45	80	56	0
normalized size	1	1.	0.58	0.48	0.68	1.21	0.85	0.
time (sec)	N/A	0.013	0.01	0.004	1.046	1.51	0.733	0.

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	33	32	45	82	56	0
normalized size	1	1.	0.5	0.48	0.68	1.24	0.85	0.
time (sec)	N/A	0.013	0.012	0.003	1.148	1.467	0.905	0.

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	35	32	45	84	61	0
normalized size	1	1.	0.53	0.48	0.68	1.27	0.92	0.
time (sec)	N/A	0.014	0.008	0.003	1.014	1.463	1.062	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	33	32	61	76	0	88
normalized size	1	1.	0.59	0.57	1.09	1.36	0.	1.57
time (sec)	N/A	0.013	0.009	0.003	1.097	1.493	0.	1.072

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	36	34	51	84	0	0
normalized size	1	1.	0.62	0.59	0.88	1.45	0.	0.
time (sec)	N/A	0.012	0.009	0.004	1.105	1.574	0.	0.

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	35	30	59	76	58	0
normalized size	1	1.	1.21	1.03	2.03	2.62	2.	0.
time (sec)	N/A	0.004	0.006	0.004	1.065	1.489	0.868	0.

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	38	30	50	80	61	0
normalized size	1	1.	0.58	0.45	0.76	1.21	0.92	0.
time (sec)	N/A	0.013	0.004	0.004	1.066	1.538	0.901	0.

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	38	29	50	82	56	0
normalized size	1	1.	0.58	0.44	0.76	1.24	0.85	0.
time (sec)	N/A	0.012	0.012	0.005	1.088	1.472	1.031	0.

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	38	32	45	84	56	0
normalized size	1	1.	0.58	0.48	0.68	1.27	0.85	0.
time (sec)	N/A	0.013	0.009	0.004	1.087	1.502	1.301	0.

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	33	32	45	85	56	0
normalized size	1	1.	0.5	0.48	0.68	1.29	0.85	0.
time (sec)	N/A	0.013	0.013	0.004	1.069	1.511	1.537	0.

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	35	32	45	85	61	0
normalized size	1	1.	0.53	0.48	0.68	1.29	0.92	0.
time (sec)	N/A	0.013	0.009	0.005	1.01	1.562	1.836	0.

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	63	63	0	139	0	109
normalized size	1	1.	0.62	0.62	0.	1.36	0.	1.07
time (sec)	N/A	0.036	0.018	0.006	0.	1.496	0.	1.058

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	52	52	0	113	0	93
normalized size	1	1.	0.65	0.65	0.	1.41	0.	1.16
time (sec)	N/A	0.025	0.015	0.007	0.	1.5	0.	1.081

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	40	40	0	89	0	73
normalized size	1	1.	0.69	0.69	0.	1.53	0.	1.26
time (sec)	N/A	0.018	0.011	0.006	0.	1.608	0.	1.077

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	29	0	59	0	50
normalized size	1	1.	0.74	0.76	0.	1.55	0.	1.32
time (sec)	N/A	0.012	0.007	0.004	0.	1.554	0.	1.056

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	21	0	43	0	38
normalized size	1	1.	0.95	0.95	0.	1.95	0.	1.73
time (sec)	N/A	0.003	0.004	0.002	0.	1.465	0.	1.055

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	26	26	32	139	0	0
normalized size	1	1.	0.62	0.62	0.76	3.31	0.	0.
time (sec)	N/A	0.007	0.007	0.008	1.072	1.631	0.	0.

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	32	33	50	68	0	0
normalized size	1	1.	0.52	0.54	0.82	1.11	0.	0.
time (sec)	N/A	0.017	0.012	0.01	1.074	1.574	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	53	51	70	97	0	0
normalized size	1	1.	0.63	0.61	0.83	1.15	0.	0.
time (sec)	N/A	0.023	0.015	0.01	1.096	1.625	0.	0.

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	64	63	0	153	0	109
normalized size	1	1.	0.6	0.59	0.	1.43	0.	1.02
time (sec)	N/A	0.032	0.013	0.004	0.	1.606	0.	1.058

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	53	52	0	124	0	93
normalized size	1	1.	0.63	0.62	0.	1.48	0.	1.11
time (sec)	N/A	0.025	0.011	0.005	0.	1.596	0.	1.056

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	42	40	0	97	0	73
normalized size	1	1.	0.69	0.66	0.	1.59	0.	1.2
time (sec)	N/A	0.019	0.005	0.006	0.	1.516	0.	1.065

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	30	29	0	65	0	50
normalized size	1	1.	0.75	0.72	0.	1.62	0.	1.25
time (sec)	N/A	0.012	0.003	0.003	0.	1.635	0.	1.062

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	21	18	46	0	38
normalized size	1	1.	0.96	0.91	0.78	2.	0.	1.65
time (sec)	N/A	0.004	0.003	0.002	1.056	1.571	0.	1.056

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	27	26	32	144	0	0
normalized size	1	1.	0.61	0.59	0.73	3.27	0.	0.
time (sec)	N/A	0.007	0.008	0.004	1.031	1.593	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	34	33	50	73	0	0
normalized size	1	1.	0.53	0.52	0.78	1.14	0.	0.
time (sec)	N/A	0.016	0.01	0.005	1.052	1.551	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	53	51	70	105	0	0
normalized size	1	1.	0.6	0.58	0.8	1.19	0.	0.
time (sec)	N/A	0.024	0.014	0.003	1.112	1.614	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	65	62	89	132	0	0
normalized size	1	1.	0.58	0.55	0.79	1.18	0.	0.
time (sec)	N/A	0.031	0.025	0.01	1.049	1.668	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	76	74	0	198	0	157
normalized size	1	1.	0.54	0.52	0.	1.39	0.	1.11
time (sec)	N/A	0.045	0.022	0.005	0.	1.598	0.	1.049

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	65	63	0	166	0	134
normalized size	1	1.	0.56	0.54	0.	1.42	0.	1.15
time (sec)	N/A	0.051	0.006	0.004	0.	1.616	0.	1.11

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	54	52	0	135	0	113
normalized size	1	1.	0.59	0.57	0.	1.47	0.	1.23
time (sec)	N/A	0.03	0.005	0.004	0.	1.583	0.	1.054

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	42	40	0	105	0	89
normalized size	1	1.	0.63	0.6	0.	1.57	0.	1.33
time (sec)	N/A	0.02	0.005	0.005	0.	1.55	0.	1.077

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	30	29	0	70	0	62
normalized size	1	1.	0.68	0.66	0.	1.59	0.	1.41
time (sec)	N/A	0.012	0.004	0.002	0.	1.591	0.	1.06

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	21	18	49	0	46
normalized size	1	1.	0.88	0.84	0.72	1.96	0.	1.84
time (sec)	N/A	0.004	0.004	0.003	1.072	1.837	0.	1.065

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	28	26	32	150	0	0
normalized size	1	1.	0.58	0.54	0.67	3.12	0.	0.
time (sec)	N/A	0.008	0.009	0.003	1.05	1.723	0.	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	34	33	50	78	0	0
normalized size	1	1.	0.49	0.47	0.71	1.11	0.	0.
time (sec)	N/A	0.017	0.012	0.003	1.042	1.835	0.	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	51	50	0	116	0	108
normalized size	1	1.	0.61	0.6	0.	1.4	0.	1.3
time (sec)	N/A	0.024	0.01	0.004	0.	1.825	0.	1.106

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	39	38	0	92	0	89
normalized size	1	1.	0.64	0.62	0.	1.51	0.	1.46
time (sec)	N/A	0.017	0.012	0.004	0.	1.804	0.	1.069

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	27	27	0	62	0	68
normalized size	1	1.	0.69	0.69	0.	1.59	0.	1.74
time (sec)	N/A	0.012	0.008	0.003	0.	1.79	0.	1.085

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	46	0	47
normalized size	1	1.	1.	0.95	0.	2.3	0.	2.35
time (sec)	N/A	0.004	0.002	0.003	0.	1.814	0.	1.081

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	25	24	0	147	0	80
normalized size	1	1.	0.66	0.63	0.	3.87	0.	2.11
time (sec)	N/A	0.007	0.004	0.002	0.	1.821	0.	1.102

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	36	30	50	70	0	123
normalized size	1	1.	0.67	0.56	0.93	1.3	0.	2.28
time (sec)	N/A	0.016	0.011	0.004	1.026	1.669	0.	1.084

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	52	51	74	100	0	0
normalized size	1	1.	0.68	0.66	0.96	1.3	0.	0.
time (sec)	N/A	0.029	0.011	0.003	1.026	1.585	0.	0.

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	63	62	93	124	0	0
normalized size	1	1.	0.63	0.62	0.93	1.24	0.	0.
time (sec)	N/A	0.026	0.012	0.004	1.032	1.572	0.	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	53	52	0	119	0	115
normalized size	1	1.	0.56	0.55	0.	1.25	0.	1.21
time (sec)	N/A	0.027	0.011	0.004	0.	1.507	0.	1.086

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	41	40	0	95	0	95
normalized size	1	1.	0.59	0.57	0.	1.36	0.	1.36
time (sec)	N/A	0.02	0.008	0.003	0.	1.562	0.	1.074

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	29	29	0	65	0	73
normalized size	1	1.	0.64	0.64	0.	1.44	0.	1.62
time (sec)	N/A	0.013	0.007	0.003	0.	1.701	0.	1.082

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	21	0	49	0	47
normalized size	1	1.	0.96	0.91	0.	2.13	0.	2.04
time (sec)	N/A	0.004	0.004	0.001	0.	1.444	0.	1.079

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	27	26	0	153	0	85
normalized size	1	1.	0.61	0.59	0.	3.48	0.	1.93
time (sec)	N/A	0.008	0.007	0.004	0.	1.546	0.	1.081

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	35	33	0	73	0	123
normalized size	1	1.	0.56	0.52	0.	1.16	0.	1.95
time (sec)	N/A	0.017	0.006	0.002	0.	1.639	0.	1.086

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	51	49	0	103	0	0
normalized size	1	1.	0.57	0.55	0.	1.16	0.	0.
time (sec)	N/A	0.022	0.007	0.005	0.	1.496	0.	0.

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	66	59	93	127	0	0
normalized size	1	1.	0.57	0.51	0.81	1.1	0.	0.
time (sec)	N/A	0.028	0.015	0.004	1.043	1.603	0.	0.

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	81	88	0	177	0	130
normalized size	1	1.	0.76	0.83	0.	1.67	0.	1.23
time (sec)	N/A	0.039	0.028	0.009	0.	1.659	0.	1.053

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	70	76	0	154	0	108
normalized size	1	1.	0.82	0.89	0.	1.81	0.	1.27
time (sec)	N/A	0.032	0.02	0.009	0.	1.535	0.	1.05

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	53	62	0	119	0	78
normalized size	1	1.	0.82	0.95	0.	1.83	0.	1.2
time (sec)	N/A	0.022	0.02	0.009	0.	1.576	0.	1.078

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	36	41	0	84	0	62
normalized size	1	1.	0.77	0.87	0.	1.79	0.	1.32
time (sec)	N/A	0.016	0.012	0.008	0.	1.512	0.	1.079

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	23	23	22	43	39	39
normalized size	1	1.	0.96	0.96	0.92	1.79	1.62	1.62
time (sec)	N/A	0.004	0.006	0.003	1.032	1.265	0.802	1.062

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	52	51	89	0	0
normalized size	1	1.	0.69	0.8	0.78	1.37	0.	0.
time (sec)	N/A	0.018	0.014	0.01	1.032	1.432	0.	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	57	74	78	123	0	0
normalized size	1	1.	0.66	0.85	0.9	1.41	0.	0.
time (sec)	N/A	0.027	0.027	0.003	1.02	1.322	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	82	95	107	154	0	0
normalized size	1	1.	0.73	0.85	0.96	1.38	0.	0.
time (sec)	N/A	0.036	0.027	0.012	0.994	1.343	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	82	88	0	196	0	130
normalized size	1	1.	0.74	0.79	0.	1.77	0.	1.17
time (sec)	N/A	0.037	0.02	0.004	0.	1.377	0.	1.06

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	71	76	0	170	0	108
normalized size	1	1.	0.8	0.85	0.	1.91	0.	1.21
time (sec)	N/A	0.029	0.018	0.005	0.	1.27	0.	1.089

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	55	62	0	132	0	78
normalized size	1	1.	0.81	0.91	0.	1.94	0.	1.15
time (sec)	N/A	0.02	0.006	0.003	0.	1.215	0.	1.063

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	38	41	0	92	0	62
normalized size	1	1.	0.78	0.84	0.	1.88	0.	1.27
time (sec)	N/A	0.015	0.009	0.003	0.	1.213	0.	1.068

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	23	22	46	44	39
normalized size	1	1.	0.96	0.92	0.88	1.84	1.76	1.56
time (sec)	N/A	0.004	0.006	0.002	1.044	1.252	2.199	1.064

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	46	52	51	97	0	0
normalized size	1	1.	0.68	0.76	0.75	1.43	0.	0.
time (sec)	N/A	0.018	0.016	0.003	1.043	1.318	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	59	74	78	134	0	0
normalized size	1	1.	0.65	0.81	0.86	1.47	0.	0.
time (sec)	N/A	0.024	0.021	0.003	1.021	1.309	0.	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	82	95	107	167	0	0
normalized size	1	1.	0.7	0.81	0.91	1.43	0.	0.
time (sec)	N/A	0.032	0.021	0.004	1.055	1.371	0.	0.

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	80	86	0	182	0	0
normalized size	1	1.	0.75	0.8	0.	1.7	0.	0.
time (sec)	N/A	0.033	0.015	0.004	0.	1.256	0.	0.

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	69	74	0	159	0	0
normalized size	1	1.	0.8	0.86	0.	1.85	0.	0.
time (sec)	N/A	0.026	0.013	0.004	0.	1.168	0.	0.

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	52	60	0	124	0	0
normalized size	1	1.	0.81	0.94	0.	1.94	0.	0.
time (sec)	N/A	0.02	0.012	0.004	0.	1.343	0.	0.

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	35	39	0	89	0	0
normalized size	1	1.	0.81	0.91	0.	2.07	0.	0.
time (sec)	N/A	0.013	0.009	0.003	0.	1.294	0.	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	0	49	85	0
normalized size	1	1.	1.	0.95	0.	2.23	3.86	0.
time (sec)	N/A	0.003	0.003	0.001	0.	1.34	1.114	0.

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	44	50	0	95	0	116
normalized size	1	1.	0.75	0.85	0.	1.61	0.	1.97
time (sec)	N/A	0.016	0.005	0.004	0.	1.384	0.	1.123

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	60	71	77	128	0	0
normalized size	1	1.	0.77	0.91	0.99	1.64	0.	0.
time (sec)	N/A	0.022	0.025	0.003	1.069	1.345	0.	0.

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	81	95	103	159	0	0
normalized size	1	1.	0.79	0.92	1.	1.54	0.	0.
time (sec)	N/A	0.032	0.017	0.005	1.057	1.322	0.	0.

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	54	62	0	130	0	0
normalized size	1	1.	0.74	0.85	0.	1.78	0.	0.
time (sec)	N/A	0.021	0.014	0.004	0.	1.215	0.	0.

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	37	41	0	95	0	0
normalized size	1	1.	0.76	0.84	0.	1.94	0.	0.
time (sec)	N/A	0.014	0.011	0.003	0.	1.384	0.	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	23	0	54	90	0
normalized size	1	1.	0.96	0.92	0.	2.16	3.6	0.
time (sec)	N/A	0.004	0.006	0.001	0.	1.334	1.776	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	46	52	0	100	0	0
normalized size	1	1.	0.68	0.76	0.	1.47	0.	0.
time (sec)	N/A	0.017	0.013	0.005	0.	1.383	0.	0.

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	59	74	0	134	0	0
normalized size	1	1.	0.66	0.82	0.	1.49	0.	0.
time (sec)	N/A	0.024	0.011	0.004	0.	1.165	0.	0.

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	80	93	0	165	0	0
normalized size	1	1.	0.68	0.79	0.	1.4	0.	0.
time (sec)	N/A	0.032	0.009	0.006	0.	1.243	0.	0.

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	97	136	157	313	0	405
normalized size	1	1.	0.74	1.04	1.2	2.39	0.	3.09
time (sec)	N/A	0.037	0.066	0.004	1.061	1.311	0.	1.065

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	68	83	108	209	0	270
normalized size	1	1.	0.71	0.86	1.12	2.18	0.	2.81
time (sec)	N/A	0.029	0.046	0.004	1.027	1.442	0.	1.083

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	46	69	126	0	161
normalized size	1	1.	0.7	0.73	1.1	2.	0.	2.56
time (sec)	N/A	0.017	0.03	0.002	1.038	1.22	0.	1.062

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	29	38	66	0	57
normalized size	1	1.	0.97	0.97	1.27	2.2	0.	1.9
time (sec)	N/A	0.006	0.013	0.001	1.059	1.33	0.	1.064

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	46	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.01	0.022	0.	0.	0.	0.

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.014	0.022	0.	0.	0.	0.

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.016	0.026	0.	0.	0.	0.

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	132	199	212	494	0	575
normalized size	1	1.	0.78	1.18	1.25	2.92	0.	3.4
time (sec)	N/A	0.055	0.074	0.005	1.071	1.574	0.	1.073

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	98	136	157	343	0	405
normalized size	1	1.	0.73	1.01	1.16	2.54	0.	3.
time (sec)	N/A	0.039	0.056	0.005	1.016	1.532	0.	1.073

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	70	83	108	228	0	0
normalized size	1	1.	0.71	0.84	1.09	2.3	0.	0.
time (sec)	N/A	0.028	0.032	0.004	1.055	1.722	0.	0.

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	46	46	69	136	0	161
normalized size	1	1.	0.71	0.71	1.06	2.09	0.	2.48
time (sec)	N/A	0.018	0.008	0.001	1.022	1.606	0.	1.084

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	29	38	72	0	57
normalized size	1	1.	0.97	0.94	1.23	2.32	0.	1.84
time (sec)	N/A	0.006	0.014	0.001	0.992	1.611	0.	1.052

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.01	0.02	0.	0.	0.	0.

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.011	0.023	0.	0.	0.	0.

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.011	0.024	0.	0.	0.	0.

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	172	280	274	733	0	864
normalized size	1	1.	0.79	1.29	1.26	3.38	0.	3.98
time (sec)	N/A	0.074	0.109	0.006	1.123	1.717	0.	1.086

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	133	199	212	537	0	0
normalized size	1	1.	0.74	1.11	1.18	3.	0.	0.
time (sec)	N/A	0.051	0.022	0.006	1.009	1.596	0.	0.

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	99	136	157	373	0	0
normalized size	1	1.	0.69	0.95	1.1	2.61	0.	0.
time (sec)	N/A	0.042	0.019	0.004	1.045	1.626	0.	0.

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	70	83	108	247	0	0
normalized size	1	1.	0.67	0.79	1.03	2.35	0.	0.
time (sec)	N/A	0.029	0.046	0.005	1.011	1.692	0.	0.

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	46	46	69	147	0	0
normalized size	1	1.	0.67	0.67	1.	2.13	0.	0.
time (sec)	N/A	0.019	0.008	0.002	1.038	1.522	0.	0.

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	29	38	77	0	0
normalized size	1	1.	0.94	0.88	1.15	2.33	0.	0.
time (sec)	N/A	0.006	0.017	0.002	1.027	1.578	0.	0.

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	47	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.011	0.02	0.	0.	0.	0.

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	47	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.012	0.023	0.	0.	0.	0.

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	96	134	140	327	0	0
normalized size	1	1.	0.78	1.09	1.14	2.66	0.	0.
time (sec)	N/A	0.037	0.037	0.003	1.023	1.586	0.	0.

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	67	81	112	220	0	0
normalized size	1	1.	0.74	0.9	1.24	2.44	0.	0.
time (sec)	N/A	0.026	0.028	0.005	1.035	1.626	0.	0.

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	43	44	61	134	0	0
normalized size	1	1.	0.73	0.75	1.03	2.27	0.	0.
time (sec)	N/A	0.016	0.021	0.002	1.029	1.709	0.	0.

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	42	72	0	0
normalized size	1	1.	1.	0.96	1.5	2.57	0.	0.
time (sec)	N/A	0.005	0.011	0.	1.018	1.615	0.	0.

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.01	0.002	0.023	0.	0.	0.	0.

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	48	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.01	0.011	0.024	0.	0.	0.	0.

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.011	0.026	0.	0.	0.	0.

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	98	136	140	340	0	0
normalized size	1	1.	0.73	1.01	1.04	2.52	0.	0.
time (sec)	N/A	0.044	0.041	0.003	1.035	1.349	0.	0.

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	69	83	112	231	0	0
normalized size	1	1.	0.7	0.84	1.13	2.33	0.	0.
time (sec)	N/A	0.031	0.033	0.004	1.029	1.374	0.	0.

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	46	61	142	0	0
normalized size	1	1.	0.69	0.71	0.94	2.18	0.	0.
time (sec)	N/A	0.019	0.022	0.001	1.017	1.355	0.	0.

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	29	42	77	0	0
normalized size	1	1.	0.97	0.94	1.35	2.48	0.	0.
time (sec)	N/A	0.007	0.014	0.001	1.021	1.289	0.	0.

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.008	0.022	0.	0.	0.	0.

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.003	0.022	0.	0.	0.	0.

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.003	0.026	0.	0.	0.	0.

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.013	0.031	0.	0.	0.	0.

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	99	136	140	340	0	0
normalized size	1	1.	0.73	1.01	1.04	2.52	0.	0.
time (sec)	N/A	0.046	0.033	0.005	1.04	1.451	0.	0.

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	70	83	112	231	0	0
normalized size	1	1.	0.71	0.84	1.13	2.33	0.	0.
time (sec)	N/A	0.031	0.03	0.004	1.059	1.355	0.	0.

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	46	46	61	142	0	0
normalized size	1	1.	0.71	0.71	0.94	2.18	0.	0.
time (sec)	N/A	0.02	0.02	0.002	1.06	1.465	0.	0.

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	42	77	0	0
normalized size	1	1.	1.	0.94	1.35	2.48	0.	0.
time (sec)	N/A	0.007	0.012	0.002	1.039	1.251	0.	0.

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.008	0.022	0.	0.	0.	0.

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.009	0.022	0.	0.	0.	0.

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.01	0.026	0.	0.	0.	0.

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.003	0.028	0.	0.	0.	0.

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	38	40	53	123	0	0
normalized size	1	1.	0.58	0.62	0.82	1.89	0.	0.
time (sec)	N/A	0.03	0.027	0.004	1.066	1.355	0.	0.

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	38	40	53	111	0	0
normalized size	1	1.	0.62	0.66	0.87	1.82	0.	0.
time (sec)	N/A	0.029	0.03	0.002	1.058	1.375	0.	0.

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	38	40	53	96	0	0
normalized size	1	1.	0.64	0.68	0.9	1.63	0.	0.
time (sec)	N/A	0.027	0.02	0.002	1.044	1.298	0.	0.

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	33	32	43	77	0	0
normalized size	1	1.	0.69	0.67	0.9	1.6	0.	0.
time (sec)	N/A	0.019	0.016	0.002	1.059	1.358	0.	0.

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	38	40	53	109	0	0
normalized size	1	1.	0.58	0.62	0.82	1.68	0.	0.
time (sec)	N/A	0.036	0.026	0.003	1.061	1.361	0.	0.

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	38	40	53	113	0	0
normalized size	1	1.	0.57	0.6	0.79	1.69	0.	0.
time (sec)	N/A	0.037	0.055	0.002	1.049	1.321	0.	0.

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	48	95	86	265	0	0
normalized size	1	1.	0.47	0.92	0.83	2.57	0.	0.
time (sec)	N/A	0.049	0.07	0.005	1.072	1.373	0.	0.

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	48	95	86	238	0	0
normalized size	1	1.	0.49	0.98	0.89	2.45	0.	0.
time (sec)	N/A	0.045	0.056	0.004	1.069	1.231	0.	0.

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	72	95	86	203	0	0
normalized size	1	1.	0.77	1.01	0.91	2.16	0.	0.
time (sec)	N/A	0.041	0.05	0.005	1.089	1.358	0.	0.

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	62	79	77	174	0	0
normalized size	1	1.	0.77	0.98	0.95	2.15	0.	0.
time (sec)	N/A	0.036	0.033	0.004	1.088	1.424	0.	0.

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	62	83	80	185	0	0
normalized size	1	1.	0.67	0.89	0.86	1.99	0.	0.
time (sec)	N/A	0.045	0.08	0.003	1.094	1.308	0.	0.

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	72	95	86	224	0	0
normalized size	1	1.	0.69	0.9	0.82	2.13	0.	0.
time (sec)	N/A	0.054	0.055	0.005	1.095	1.346	0.	0.

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	57	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.02	0.038	0.	0.	0.	0.

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	57	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.023	0.034	0.	0.	0.	0.

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	57	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.014	0.033	0.	0.	0.	0.

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.01	0.033	0.	0.	0.	0.

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	57	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.015	0.034	0.	0.	0.	0.

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	57	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.016	0.035	0.	0.	0.	0.

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	32	0	86	0	100
normalized size	1	1.	0.97	0.97	0.	2.61	0.	3.03
time (sec)	N/A	0.01	0.018	0.003	0.	1.745	0.	1.098

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	34	33	0	84	0	0
normalized size	1	1.	1.06	1.03	0.	2.62	0.	0.
time (sec)	N/A	0.009	0.02	0.003	0.	1.59	0.	0.

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	32	0	84	0	97
normalized size	1	1.	0.97	0.97	0.	2.55	0.	2.94
time (sec)	N/A	0.01	0.016	0.003	0.	1.634	0.	1.095

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	31	0	78	0	0
normalized size	1	1.	0.93	1.03	0.	2.6	0.	0.
time (sec)	N/A	0.008	0.016	0.003	0.	1.561	0.	0.

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	36	70	264	0
normalized size	1	1.	1.	0.96	1.38	2.69	10.15	0.
time (sec)	N/A	0.006	0.007	0.003	1.068	1.721	70.408	0.

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	38	0	72	0	0
normalized size	1	1.	0.97	1.15	0.	2.18	0.	0.
time (sec)	N/A	0.011	0.01	0.003	0.	1.63	0.	0.

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	32	0	84	0	0
normalized size	1	1.	0.91	0.91	0.	2.4	0.	0.
time (sec)	N/A	0.011	0.011	0.002	0.	1.786	0.	0.

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	33	0	84	0	0
normalized size	1	1.	0.97	1.	0.	2.55	0.	0.
time (sec)	N/A	0.01	0.01	0.002	0.	1.615	0.	0.

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	39	0	119	0	0
normalized size	1	1.	1.	1.03	0.	3.13	0.	0.
time (sec)	N/A	0.01	0.029	0.003	0.	1.63	0.	0.

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	40	0	132	0	0
normalized size	1	1.	1.	1.03	0.	3.38	0.	0.
time (sec)	N/A	0.01	0.015	0.003	0.	1.594	0.	0.

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.013	0.102	0.	0.	0.	0.

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	64	64	0	0	0	0	0
normalized size	1	0.94	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.008	0.155	0.	0.	0.	0.

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	42	63	34	42
normalized size	1	1.	1.	0.94	2.47	3.71	2.	2.47
time (sec)	N/A	0.004	0.002	0.001	1.028	1.492	0.101	1.064

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	19	18	27	42	20	27
normalized size	1	1.	0.83	0.78	1.17	1.83	0.87	1.17
time (sec)	N/A	0.006	0.001	0.	1.017	1.561	0.096	1.06

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	15	7	11
normalized size	1	1.	1.	1.12	1.38	1.88	0.88	1.38
time (sec)	N/A	0.002	0.	0.002	1.002	1.5	0.086	1.065

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	30	19	19
normalized size	1	1.	1.	1.08	1.38	2.31	1.46	1.46
time (sec)	N/A	0.003	0.002	0.001	1.025	1.489	0.091	1.063

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	26	32	19	20
normalized size	1	1.	1.	1.07	1.73	2.13	1.27	1.33
time (sec)	N/A	0.003	0.004	0.001	0.98	1.504	0.309	1.064

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	63	92	53	20
normalized size	1	1.	1.	0.94	3.71	5.41	3.12	1.18
time (sec)	N/A	0.003	0.005	0.	0.995	1.644	0.404	1.07

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	82	119	68	20
normalized size	1	1.	1.	0.94	4.82	7.	4.	1.18
time (sec)	N/A	0.003	0.005	0.	1.004	1.633	0.475	1.053

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	101	149	83	20
normalized size	1	1.	1.	0.94	5.94	8.76	4.88	1.18
time (sec)	N/A	0.003	0.006	0.002	1.029	1.7	0.546	1.051

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	47	72	34	47
normalized size	1	1.	1.	0.94	2.76	4.24	2.	2.76
time (sec)	N/A	0.004	0.002	0.003	1.034	1.74	0.108	1.061

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	19	18	28	45	20	28
normalized size	1	1.	0.83	0.78	1.22	1.96	0.87	1.22
time (sec)	N/A	0.005	0.001	0.001	1.022	1.788	0.101	1.056

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	15	7	11
normalized size	1	1.	1.	1.12	1.38	1.88	0.88	1.38
time (sec)	N/A	0.001	0.	0.	1.01	1.642	0.088	1.084

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	30	17	19
normalized size	1	1.	1.	1.08	1.38	2.31	1.31	1.46
time (sec)	N/A	0.003	0.002	0.001	1.007	1.552	0.089	1.074

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	22	27	12	18
normalized size	1	1.	1.	1.08	1.69	2.08	0.92	1.38
time (sec)	N/A	0.003	0.004	0.001	0.987	1.459	0.285	1.059

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	49	76	44	16
normalized size	1	1.	1.	0.93	3.5	5.43	3.14	1.14
time (sec)	N/A	0.003	0.005	0.001	1.007	1.445	0.399	1.068

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	80	116	68	27
normalized size	1	1.	1.	0.93	5.33	7.73	4.53	1.8
time (sec)	N/A	0.003	0.005	0.001	1.001	1.45	0.483	1.067

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	101	149	83	20
normalized size	1	1.	1.	0.94	5.94	8.76	4.88	1.18
time (sec)	N/A	0.003	0.006	0.	1.053	1.396	0.571	1.058

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	25	27	0	165	212	190
normalized size	1	1.	1.04	1.12	0.	6.88	8.83	7.92
time (sec)	N/A	0.009	0.02	0.002	0.	1.681	1.766	1.072

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	114	153	231	124	153
normalized size	1	1.	1.	6.71	9.	13.59	7.29	9.
time (sec)	N/A	0.004	0.002	0.001	1.032	1.329	0.088	1.074

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	100	134	203	110	134
normalized size	1	1.	1.	5.88	7.88	11.94	6.47	7.88
time (sec)	N/A	0.004	0.002	0.002	1.007	1.376	0.085	1.072

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	72	96	147	78	96
normalized size	1	1.	1.	4.8	6.4	9.8	5.2	6.4
time (sec)	N/A	0.003	0.002	0.001	0.994	1.362	0.076	1.058

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	65	99	51	65
normalized size	1	1.	1.	0.94	3.82	5.82	3.	3.82
time (sec)	N/A	0.004	0.001	0.001	1.014	1.413	0.089	1.081

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	50	77	46	24
normalized size	1	1.	1.	0.94	2.94	4.53	2.71	1.41
time (sec)	N/A	0.004	0.001	0.	0.988	1.42	0.095	1.077

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	35	55	29	35
normalized size	1	1.	1.	0.94	2.06	3.24	1.71	2.06
time (sec)	N/A	0.004	0.001	0.001	1.011	1.459	0.094	1.058

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	16	15	20	34	15	20
normalized size	1	1.	0.89	0.83	1.11	1.89	0.83	1.11
time (sec)	N/A	0.005	0.001	0.	1.026	1.561	0.144	1.062

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	9	3	20
normalized size	1	1.	1.	1.2	1.4	1.8	0.6	4.
time (sec)	N/A	0.001	0.	0.002	1.035	1.456	0.099	1.07

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	30	17	19
normalized size	1	1.	1.	1.08	1.38	2.31	1.31	1.46
time (sec)	N/A	0.004	0.002	0.001	1.028	1.528	0.107	1.05

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	26	35	17	20
normalized size	1	1.	1.	1.07	1.73	2.33	1.13	1.33
time (sec)	N/A	0.004	0.002	0.002	1.033	1.505	0.422	1.051

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	45	65	36	20
normalized size	1	1.	1.	0.94	2.65	3.82	2.12	1.18
time (sec)	N/A	0.004	0.004	0.001	1.055	1.456	0.386	1.048

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	8	4	53	15
normalized size	1	1.	1.	0.82	0.29	0.14	1.89	0.54
time (sec)	N/A	0.003	0.006	0.002	1.555	1.487	1.405	1.064

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	45	59	89	44	59
normalized size	1	1.	1.05	1.18	1.55	2.34	1.16	1.55
time (sec)	N/A	0.012	0.003	0.	1.012	1.347	0.071	1.041

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	42	45	59	93	46	59
normalized size	1	1.	1.11	1.18	1.55	2.45	1.21	1.55
time (sec)	N/A	0.017	0.002	0.001	1.003	1.318	0.066	1.059

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	35	15	22
normalized size	1	1.	1.	0.94	1.22	1.94	0.83	1.22
time (sec)	N/A	0.004	0.002	0.	1.023	1.244	0.058	1.055

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	23	8	14
normalized size	1	1.	1.	0.92	1.17	1.92	0.67	1.17
time (sec)	N/A	0.002	0.	0.001	0.979	1.331	0.05	1.037

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	25	32	45	17	34
normalized size	1	1.	1.	1.09	1.39	1.96	0.74	1.48
time (sec)	N/A	0.012	0.005	0.002	1.021	1.478	0.286	1.053

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	28	35	50	73	29	109
normalized size	1	1.	0.88	1.09	1.56	2.28	0.91	3.41
time (sec)	N/A	0.017	0.017	0.005	1.021	1.41	0.338	1.07

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	33	41	55	27	19
normalized size	1	1.	1.	2.54	3.15	4.23	2.08	1.46
time (sec)	N/A	0.002	0.008	0.003	0.99	1.481	0.369	1.051

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	25	35	73	108	56	31
normalized size	1	1.	0.66	0.92	1.92	2.84	1.47	0.82
time (sec)	N/A	0.019	0.011	0.005	1.006	1.482	0.486	1.059

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	24	35	90	134	70	54
normalized size	1	1.	0.63	0.92	2.37	3.53	1.84	1.42
time (sec)	N/A	0.019	0.011	0.005	1.016	1.456	0.525	1.092

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	113	169	88	34
normalized size	1	1.	0.71	0.92	2.97	4.45	2.32	0.89
time (sec)	N/A	0.019	0.012	0.004	1.006	1.44	0.617	1.052

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	68	73	97	154	78	97
normalized size	1	1.	1.19	1.28	1.7	2.7	1.37	1.7
time (sec)	N/A	0.03	0.003	0.	0.994	1.34	0.077	1.06

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	46	69	36	46
normalized size	1	1.	1.	0.92	1.21	1.82	0.95	1.21
time (sec)	N/A	0.017	0.002	0.001	1.024	1.289	0.068	1.06

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	37	49	84	39	49
normalized size	1	1.	1.25	1.16	1.53	2.62	1.22	1.53
time (sec)	N/A	0.015	0.002	0.	1.013	1.35	0.067	1.054

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	27	42	19	16
normalized size	1	1.	1.	0.93	1.93	3.	1.36	1.14
time (sec)	N/A	0.001	0.001	0.	1.02	1.261	0.057	1.139

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	37	37	47	72	31	62
normalized size	1	1.	0.86	0.86	1.09	1.67	0.72	1.44
time (sec)	N/A	0.014	0.007	0.002	1.01	1.512	0.298	1.046

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	35	44	62	108	39	107
normalized size	1	1.	0.85	1.07	1.51	2.63	0.95	2.61
time (sec)	N/A	0.022	0.03	0.005	1.009	1.513	0.359	1.082

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	33	56	82	138	53	62
normalized size	1	1.	0.63	1.08	1.58	2.65	1.02	1.19
time (sec)	N/A	0.028	0.024	0.005	1.036	1.575	0.447	1.056

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	31	52	81	116	61	39
normalized size	1	1.	1.11	1.86	2.89	4.14	2.18	1.39
time (sec)	N/A	0.005	0.018	0.004	1.037	1.557	0.508	1.065

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	35	51	105	155	82	86
normalized size	1	1.	0.62	0.91	1.88	2.77	1.46	1.54
time (sec)	N/A	0.024	0.012	0.004	1.061	1.487	0.585	1.058

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	38	52	128	190	100	49
normalized size	1	1.	0.67	0.91	2.25	3.33	1.75	0.86
time (sec)	N/A	0.025	0.018	0.005	1.034	1.503	0.652	1.061

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	37	52	146	220	114	49
normalized size	1	1.	0.63	0.88	2.47	3.73	1.93	0.83
time (sec)	N/A	0.029	0.015	0.004	1.055	1.457	0.76	1.06

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	42	49	65	112	49	80
normalized size	1	1.	0.69	0.8	1.07	1.84	0.8	1.31
time (sec)	N/A	0.021	0.006	0.002	1.052	1.561	0.318	1.048

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	35	46	81	34	61
normalized size	1	1.	0.72	0.81	1.07	1.88	0.79	1.42
time (sec)	N/A	0.014	0.006	0.001	1.024	1.494	0.296	1.057

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	45	15	26
normalized size	1	1.	1.	1.06	1.33	2.5	0.83	1.44
time (sec)	N/A	0.01	0.004	0.001	1.029	1.411	0.277	1.049

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	22	7	15
normalized size	1	1.	1.	1.1	1.4	2.2	0.7	1.5
time (sec)	N/A	0.002	0.001	0.001	1.061	1.428	0.059	1.065

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	38	50	58	22	53
normalized size	1	1.	1.	2.24	2.94	3.41	1.29	3.12
time (sec)	N/A	0.009	0.008	0.004	1.04	1.422	0.159	1.065

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	53	58	81	120	48	72
normalized size	1	1.	1.26	1.38	1.93	2.86	1.14	1.71
time (sec)	N/A	0.029	0.015	0.007	1.065	1.535	0.469	1.08

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	65	78	111	208	71	93
normalized size	1	1.	1.03	1.24	1.76	3.3	1.13	1.48
time (sec)	N/A	0.039	0.021	0.005	1.026	1.513	0.522	1.07

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	46	53	72	167	51	108
normalized size	1	1.	0.85	0.98	1.33	3.09	0.94	2.
time (sec)	N/A	0.031	0.019	0.005	1.019	1.518	0.389	1.053

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	40	54	124	36	80
normalized size	1	1.	0.85	1.03	1.38	3.18	0.92	2.05
time (sec)	N/A	0.021	0.018	0.005	1.047	1.532	0.345	1.08

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	28	38	72	24	73
normalized size	1	1.	0.85	1.04	1.41	2.67	0.89	2.7
time (sec)	N/A	0.013	0.009	0.004	1.03	1.535	0.318	1.053

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	24	10	16
normalized size	1	1.	1.	1.08	1.33	2.	0.83	1.33
time (sec)	N/A	0.001	0.002	0.	1.012	1.402	0.304	1.047

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	50	56	74	115	44	59
normalized size	1	1.	1.22	1.37	1.8	2.8	1.07	1.44
time (sec)	N/A	0.028	0.014	0.009	1.017	1.539	0.421	1.062

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	74	76	86	146	49	112
normalized size	1	1.	1.61	1.65	1.87	3.17	1.07	2.43
time (sec)	N/A	0.017	0.023	0.01	1.033	1.642	0.407	1.058

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	68	96	146	290	104	109
normalized size	1	1.	0.82	1.16	1.76	3.49	1.25	1.31
time (sec)	N/A	0.05	0.036	0.01	1.079	1.665	0.656	1.062

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	60	113	92	178	0	201
normalized size	1	1.	0.56	1.05	0.85	1.65	0.	1.86
time (sec)	N/A	0.021	0.051	0.005	1.571	1.601	0.	1.129

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	56	99	73	163	253	143
normalized size	1	1.	0.64	1.12	0.83	1.85	2.88	1.62
time (sec)	N/A	0.015	0.053	0.003	1.539	1.541	74.634	1.102

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	85	54	143	218	103
normalized size	1	1.	0.74	1.25	0.79	2.1	3.21	1.51
time (sec)	N/A	0.011	0.04	0.005	1.541	1.581	15.673	1.089

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	44	71	38	124	168	59
normalized size	1	1.	0.92	1.48	0.79	2.58	3.5	1.23
time (sec)	N/A	0.006	0.038	0.003	1.552	1.535	4.925	1.068

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	20	57	23	101	133	36
normalized size	1	1.	0.71	2.04	0.82	3.61	4.75	1.29
time (sec)	N/A	0.004	0.006	0.003	1.547	1.485	2.672	1.099

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	32	42	19	97	100	38
normalized size	1	1.	1.52	2.	0.9	4.62	4.76	1.81
time (sec)	N/A	0.004	0.007	0.003	1.525	1.634	1.844	1.066

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	36	64	28	131	71	45
normalized size	1	1.	1.57	2.78	1.22	5.7	3.09	1.96
time (sec)	N/A	0.004	0.013	0.022	1.569	1.5	1.578	1.075

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	51	89	61	26
normalized size	1	1.	1.	0.75	2.55	4.45	3.05	1.3
time (sec)	N/A	0.002	0.006	0.002	1.025	1.538	2.361	1.083

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	23	18	86	136	173	30
normalized size	1	1.	0.56	0.44	2.1	3.32	4.22	0.73
time (sec)	N/A	0.004	0.008	0.002	1.031	1.582	23.105	1.098

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	30	25	128	181	568	39
normalized size	1	1.	0.49	0.41	2.1	2.97	9.31	0.64
time (sec)	N/A	0.008	0.011	0.004	1.033	1.529	87.159	1.072

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	35	30	177	224	0	47
normalized size	1	1.	0.43	0.37	2.19	2.77	0.	0.58
time (sec)	N/A	0.013	0.014	0.002	1.032	1.55	0.	1.084

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	40	35	232	279	0	57
normalized size	1	1.	0.4	0.35	2.3	2.76	0.	0.56
time (sec)	N/A	0.019	0.017	0.002	1.043	1.534	0.	1.151

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	66	127	89	192	0	259
normalized size	1	1.	0.61	1.17	0.82	1.76	0.	2.38
time (sec)	N/A	0.02	0.057	0.004	1.534	1.577	0.	1.211

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	61	113	70	176	289	161
normalized size	1	1.	0.69	1.27	0.79	1.98	3.25	1.81
time (sec)	N/A	0.013	0.054	0.005	1.538	1.554	137.498	1.097

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	55	99	54	155	250	143
normalized size	1	1.	0.8	1.43	0.78	2.25	3.62	2.07
time (sec)	N/A	0.008	0.044	0.005	1.58	1.49	37.343	1.095

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	29	85	39	124	214	80
normalized size	1	1.	0.59	1.73	0.8	2.53	4.37	1.63
time (sec)	N/A	0.007	0.011	0.004	1.53	1.556	9.936	1.096

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	44	71	38	123	165	59
normalized size	1	1.	0.92	1.48	0.79	2.56	3.44	1.23
time (sec)	N/A	0.006	0.034	0.003	1.522	1.527	5.359	1.073

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	57	38	113	136	42
normalized size	1	1.	0.79	1.21	0.81	2.4	2.89	0.89
time (sec)	N/A	0.007	0.012	0.004	1.547	1.546	3.6	1.064

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	35	72	57	144	100	47
normalized size	1	1.	0.85	1.76	1.39	3.51	2.44	1.15
time (sec)	N/A	0.007	0.006	0.014	1.53	1.478	3.278	1.083

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	37	76	89	189	500	51
normalized size	1	1.	0.9	1.85	2.17	4.61	12.2	1.24
time (sec)	N/A	0.005	0.006	0.016	1.518	1.469	5.584	1.071

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	127	130	88	26
normalized size	1	1.	1.	0.75	6.35	6.5	4.4	1.3
time (sec)	N/A	0.002	0.007	0.002	1.049	1.558	23.435	1.079

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	23	18	177	171	228	30
normalized size	1	1.	0.56	0.44	4.32	4.17	5.56	0.73
time (sec)	N/A	0.004	0.011	0.002	1.023	1.677	86.921	1.085

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	30	25	232	224	0	39
normalized size	1	1.	0.49	0.41	3.8	3.67	0.	0.64
time (sec)	N/A	0.008	0.012	0.004	1.003	1.693	0.	1.094

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	35	30	294	273	0	47
normalized size	1	1.	0.43	0.37	3.63	3.37	0.	0.58
time (sec)	N/A	0.013	0.023	0.003	1.045	1.543	0.	1.099

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	40	35	363	333	0	57
normalized size	1	1.	0.4	0.35	3.59	3.3	0.	0.56
time (sec)	N/A	0.019	0.019	0.002	1.035	1.609	0.	1.121

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	75	155	105	238	0	409
normalized size	1	1.	0.58	1.19	0.81	1.83	0.	3.15
time (sec)	N/A	0.025	0.071	0.003	1.554	1.546	0.	1.211

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	70	141	86	209	0	335
normalized size	1	1.	0.64	1.28	0.78	1.9	0.	3.05
time (sec)	N/A	0.019	0.066	0.004	1.527	1.63	0.	1.149

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	66	127	70	190	0	259
normalized size	1	1.	0.73	1.41	0.78	2.11	0.	2.88
time (sec)	N/A	0.012	0.063	0.003	1.527	1.561	0.	1.17

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	34	113	55	138	286	138
normalized size	1	1.	0.49	1.61	0.79	1.97	4.09	1.97
time (sec)	N/A	0.011	0.015	0.003	1.492	1.502	79.347	1.128

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	55	99	54	157	246	143
normalized size	1	1.	0.8	1.43	0.78	2.28	3.57	2.07
time (sec)	N/A	0.008	0.046	0.005	1.494	1.472	38.543	1.096

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	85	54	143	214	103
normalized size	1	1.	0.74	1.25	0.79	2.1	3.15	1.51
time (sec)	N/A	0.009	0.038	0.003	1.511	1.776	17.104	1.104

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	44	71	57	128	172	53
normalized size	1	1.	0.66	1.06	0.85	1.91	2.57	0.79
time (sec)	N/A	0.01	0.022	0.004	1.56	1.631	11.565	1.06

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	35	77	76	166	139	57
normalized size	1	1.	0.54	1.18	1.17	2.55	2.14	0.88
time (sec)	N/A	0.01	0.007	0.014	1.497	1.563	14.384	1.088

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	37	84	134	205	576	59
normalized size	1	1.	0.59	1.33	2.13	3.25	9.14	0.94
time (sec)	N/A	0.01	0.008	0.017	1.489	1.579	14.24	1.09

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	37	84	216	240	1608	59
normalized size	1	1.	0.59	1.33	3.43	3.81	25.52	0.94
time (sec)	N/A	0.007	0.008	0.017	1.551	1.605	37.589	1.066

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	231	162	116	26
normalized size	1	1.	1.	0.75	11.55	8.1	5.8	1.3
time (sec)	N/A	0.002	0.007	0.001	1.027	1.585	86.554	1.088

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	23	18	294	209	0	30
normalized size	1	1.	0.56	0.44	7.17	5.1	0.	0.73
time (sec)	N/A	0.004	0.015	0.002	1.029	1.676	0.	1.097

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	30	25	363	269	0	39
normalized size	1	1.	0.49	0.41	5.95	4.41	0.	0.64
time (sec)	N/A	0.008	0.019	0.003	1.022	1.657	0.	1.103

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	35	30	439	319	0	47
normalized size	1	1.	0.43	0.37	5.42	3.94	0.	0.58
time (sec)	N/A	0.013	0.017	0.003	1.01	1.495	0.	1.121

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	40	35	521	385	0	57
normalized size	1	1.	0.4	0.35	5.16	3.81	0.	0.56
time (sec)	N/A	0.019	0.021	0.003	1.016	1.657	0.	1.124

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	45	40	610	448	0	65
normalized size	1	1.	0.37	0.33	5.04	3.7	0.	0.54
time (sec)	N/A	0.025	0.021	0.003	1.031	1.564	0.	1.163

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	47	98	69	138	75	57
normalized size	1	1.	0.73	1.53	1.08	2.16	1.17	0.89
time (sec)	N/A	0.012	0.04	0.01	1.477	1.649	24.935	1.073

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	91	118	57	111	76	46
normalized size	1	1.	1.47	1.9	0.92	1.79	1.23	0.74
time (sec)	N/A	0.028	0.104	0.011	1.487	1.579	4.492	1.075

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	61	85	76	149	201	136
normalized size	1	1.	0.7	0.98	0.87	1.71	2.31	1.56
time (sec)	N/A	0.018	0.027	0.001	1.519	1.679	50.551	1.128

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	71	57	127	175	93
normalized size	1	1.	0.81	1.06	0.85	1.9	2.61	1.39
time (sec)	N/A	0.011	0.026	0.005	1.504	1.546	10.003	1.078

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	57	38	113	139	59
normalized size	1	1.	1.	1.21	0.81	2.4	2.96	1.26
time (sec)	N/A	0.007	0.02	0.003	1.502	1.552	2.951	1.075

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	30	41	16	96	100	36
normalized size	1	1.	1.5	2.05	0.8	4.8	5.	1.8
time (sec)	N/A	0.003	0.014	0.002	1.523	1.892	1.61	1.06

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	27	3	61	41	18
normalized size	1	1.	1.	13.5	1.5	30.5	20.5	9.
time (sec)	N/A	0.001	0.004	0.002	1.531	1.814	1.097	1.059

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	22	59	29	26
normalized size	1	1.	1.	0.82	1.29	3.47	1.71	1.53
time (sec)	N/A	0.003	0.003	0.002	1.486	1.761	1.049	1.085

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	23	18	51	100	126	30
normalized size	1	1.	0.56	0.44	1.24	2.44	3.07	0.73
time (sec)	N/A	0.004	0.006	0.003	1.492	1.724	4.178	1.07

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	30	25	86	139	301	39
normalized size	1	1.	0.49	0.41	1.41	2.28	4.93	0.64
time (sec)	N/A	0.008	0.009	0.003	1.483	1.868	35.362	1.08

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	35	30	128	178	541	47
normalized size	1	1.	0.43	0.37	1.58	2.2	6.68	0.58
time (sec)	N/A	0.014	0.011	0.003	1.495	1.836	126.932	1.086

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	40	35	177	225	0	57
normalized size	1	1.	0.4	0.35	1.75	2.23	0.	0.56
time (sec)	N/A	0.018	0.011	0.003	1.509	1.844	0.	1.082

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	37	84	95	189	207	109
normalized size	1	1.	0.44	0.99	1.12	2.22	2.44	1.28
time (sec)	N/A	0.016	0.012	0.015	1.544	1.859	62.	1.307

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	37	77	76	166	168	99
normalized size	1	1.	0.57	1.18	1.17	2.55	2.58	1.52
time (sec)	N/A	0.011	0.01	0.012	1.48	1.791	13.622	1.105

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	37	71	55	146	133	95
normalized size	1	1.	0.9	1.73	1.34	3.56	3.24	2.32
time (sec)	N/A	0.007	0.009	0.013	1.577	1.852	2.82	1.077

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	34	67	28	131	104	74
normalized size	1	1.	1.48	2.91	1.22	5.7	4.52	3.22
time (sec)	N/A	0.003	0.037	0.011	1.537	1.77	1.469	1.07

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	22	61	29	58
normalized size	1	1.	1.	0.83	1.22	3.39	1.61	3.22
time (sec)	N/A	0.002	0.003	0.002	1.55	1.772	1.076	1.079

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	13	15	15	53	65	84
normalized size	1	1.	0.72	0.83	0.83	2.94	3.61	4.67
time (sec)	N/A	0.002	0.003	0.002	1.019	1.747	2.718	1.064

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	30	25	54	122	158	90
normalized size	1	1.	0.71	0.6	1.29	2.9	3.76	2.14
time (sec)	N/A	0.005	0.007	0.002	1.042	1.746	17.529	1.07

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	33	28	107	143	282	99
normalized size	1	1.	0.53	0.45	1.73	2.31	4.55	1.6
time (sec)	N/A	0.008	0.008	0.003	1.063	1.949	70.182	1.07

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	40	35	181	213	0	107
normalized size	1	1.	0.49	0.43	2.21	2.6	0.	1.3
time (sec)	N/A	0.013	0.01	0.003	1.009	1.801	0.	1.086

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	45	40	271	230	0	115
normalized size	1	1.	0.44	0.39	2.66	2.25	0.	1.13
time (sec)	N/A	0.02	0.012	0.003	1.032	1.797	0.	1.061

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	37	89	169	238	0	171
normalized size	1	1.	0.36	0.86	1.64	2.31	0.	1.66
time (sec)	N/A	0.021	0.014	0.018	1.483	1.911	0.	1.193

Problem 1127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	37	84	150	224	207	161
normalized size	1	1.	0.43	0.97	1.72	2.57	2.38	1.85
time (sec)	N/A	0.015	0.012	0.017	1.524	1.824	60.253	1.14

Problem 1128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	37	79	132	204	160	155
normalized size	1	1.	0.59	1.25	2.1	3.24	2.54	2.46
time (sec)	N/A	0.011	0.01	0.019	1.501	1.86	12.969	1.113

Problem 1129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	49	73	89	188	126	138
normalized size	1	1.	1.2	1.78	2.17	4.59	3.07	3.37
time (sec)	N/A	0.006	0.054	0.015	1.5	1.82	5.097	1.14

Problem 1130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	51	99	65	120
normalized size	1	1.	1.	0.75	2.55	4.95	3.25	6.
time (sec)	N/A	0.002	0.004	0.002	0.983	1.851	2.417	1.078

Problem 1131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	23	18	51	101	65	120
normalized size	1	1.	0.56	0.44	1.24	2.46	1.59	2.93
time (sec)	N/A	0.004	0.011	0.001	1.493	1.922	4.233	1.071

Problem 1132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	30	25	51	123	165	146
normalized size	1	1.	0.52	0.43	0.88	2.12	2.84	2.52
time (sec)	N/A	0.008	0.006	0.004	0.991	1.904	17.469	1.085

Problem 1133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	23	23	34	85	279	153
normalized size	1	1.	0.53	0.53	0.79	1.98	6.49	3.56
time (sec)	N/A	0.005	0.006	0.002	0.992	1.587	38.473	1.082

Problem 1134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	40	35	70	198	423	161
normalized size	1	1.	0.63	0.56	1.11	3.14	6.71	2.56
time (sec)	N/A	0.008	0.011	0.003	0.999	1.504	149.631	1.096

Problem 1135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	45	40	123	235	0	169
normalized size	1	1.	0.54	0.48	1.48	2.83	0.	2.04
time (sec)	N/A	0.014	0.013	0.001	0.994	1.559	0.	1.089

Problem 1136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	50	45	197	279	0	177
normalized size	1	1.	0.49	0.44	1.91	2.71	0.	1.72
time (sec)	N/A	0.019	0.013	0.002	1.006	1.532	0.	1.082

Problem 1137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	114	193	0	478	0	478
normalized size	1	1.	0.9	1.53	0.	3.79	0.	3.79
time (sec)	N/A	0.055	0.098	0.013	0.	1.667	0.	1.352

Problem 1138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	104	143	0	393	0	277
normalized size	1	1.	1.08	1.49	0.	4.09	0.	2.89
time (sec)	N/A	0.037	0.079	0.004	0.	1.64	0.	1.249

Problem 1139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	69	98	0	325	0	115
normalized size	1	1.	1.03	1.46	0.	4.85	0.	1.72
time (sec)	N/A	0.029	0.059	0.004	0.	1.619	0.	1.137

Problem 1140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	47	57	0	239	85	66
normalized size	1	1.	1.09	1.33	0.	5.56	1.98	1.53
time (sec)	N/A	0.024	0.016	0.005	0.	1.662	3.179	1.143

Problem 1141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	25	28	77	82	157
normalized size	1	1.	1.	0.93	1.04	2.85	3.04	5.81
time (sec)	N/A	0.003	0.016	0.001	1.002	1.576	5.445	1.089

Problem 1142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	42	32	61	120	82	320
normalized size	1	1.	0.69	0.52	1.	1.97	1.34	5.25
time (sec)	N/A	0.01	0.027	0.001	0.986	1.533	53.739	1.217

Problem 1143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	49	37	90	157	0	450
normalized size	1	1.	0.54	0.41	0.99	1.73	0.	4.95
time (sec)	N/A	0.018	0.036	0.004	0.982	1.608	0.	1.571

Problem 1144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	54	42	120	192	0	590
normalized size	1	1.	0.45	0.35	0.99	1.59	0.	4.88
time (sec)	N/A	0.028	0.039	0.003	1.	1.593	0.	1.817

Problem 1145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	120	243	0	524	0	0
normalized size	1	1.	0.89	1.8	0.	3.88	0.	0.
time (sec)	N/A	0.052	0.147	0.01	0.	1.704	0.	0.

Problem 1146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	109	185	0	447	0	0
normalized size	1	1.	1.07	1.81	0.	4.38	0.	0.
time (sec)	N/A	0.036	0.122	0.004	0.	1.623	0.	0.

Problem 1147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	95	127	0	373	0	0
normalized size	1	1.	1.4	1.87	0.	5.49	0.	0.
time (sec)	N/A	0.027	0.118	0.004	0.	1.669	0.	0.

Problem 1148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	48	71	0	252	90	0
normalized size	1	1.	1.26	1.87	0.	6.63	2.37	0.
time (sec)	N/A	0.02	0.015	0.005	0.	1.574	3.43	0.

Problem 1149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	30	34	88	94	155
normalized size	1	1.	0.97	1.	1.13	2.93	3.13	5.17
time (sec)	N/A	0.004	0.013	0.002	1.002	1.568	5.546	1.12

Problem 1150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	46	45	72	147	94	339
normalized size	1	1.	0.69	0.67	1.07	2.19	1.4	5.06
time (sec)	N/A	0.011	0.024	0.001	0.981	1.647	47.532	1.253

Problem 1151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	57	56	107	203	0	494
normalized size	1	1.	0.57	0.56	1.07	2.03	0.	4.94
time (sec)	N/A	0.02	0.03	0.004	0.997	1.76	0.	1.422

Problem 1152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	76	67	142	257	0	657
normalized size	1	1.	0.57	0.5	1.07	1.93	0.	4.94
time (sec)	N/A	0.034	0.041	0.004	0.971	1.88	0.	2.051

Problem 1153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	44	134	62	194	0	174
normalized size	1	1.	0.44	1.34	0.62	1.94	0.	1.74
time (sec)	N/A	0.017	0.033	0.006	1.463	1.653	0.	1.104

Problem 1154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	39	102	46	177	0	103
normalized size	1	1.	0.53	1.38	0.62	2.39	0.	1.39
time (sec)	N/A	0.01	0.034	0.003	1.503	1.558	0.	1.094

Problem 1155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	30	70	30	159	187	51
normalized size	1	1.	0.7	1.63	0.7	3.7	4.35	1.19
time (sec)	N/A	0.006	0.01	0.003	1.481	1.463	5.524	1.063

Problem 1156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	37	12	90	41	20
normalized size	1	1.	1.	2.85	0.92	6.92	3.15	1.54
time (sec)	N/A	0.003	0.015	0.003	1.433	1.465	5.284	1.073

Problem 1157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	16	28	16	68	0	96
normalized size	1	1.	0.57	1.	0.57	2.43	0.	3.43
time (sec)	N/A	0.002	0.016	0.003	0.964	1.557	0.	1.054

Problem 1158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	37	35	34	97	0	174
normalized size	1	1.	0.65	0.61	0.6	1.7	0.	3.05
time (sec)	N/A	0.006	0.024	0.004	0.969	1.559	0.	1.083

Problem 1159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	42	40	50	130	0	248
normalized size	1	1.	0.49	0.47	0.59	1.53	0.	2.92
time (sec)	N/A	0.011	0.03	0.002	1.016	1.548	0.	1.111

Problem 1160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	80	89	90	184	199	117
normalized size	1	1.	0.88	0.98	0.99	2.02	2.19	1.29
time (sec)	N/A	0.022	0.047	0.005	1.476	1.579	10.278	1.099

Problem 1161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	69	61	51	147	124	38
normalized size	1	1.	1.35	1.2	1.	2.88	2.43	0.75
time (sec)	N/A	0.01	0.023	0.004	1.423	1.499	3.007	1.097

Problem 1162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	12	31	8	88	26	11
normalized size	1	1.	1.5	3.88	1.	11.	3.25	1.38
time (sec)	N/A	0.004	0.01	0.003	1.487	1.588	1.62	1.081

Problem 1163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	21	20	41	74	100	72
normalized size	1	1.	0.57	0.54	1.11	2.	2.7	1.95
time (sec)	N/A	0.004	0.007	0.003	0.978	1.522	3.259	1.069

Problem 1164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	33	30	80	135	282	131
normalized size	1	1.	0.42	0.38	1.01	1.71	3.57	1.66
time (sec)	N/A	0.013	0.013	0.003	0.99	1.589	39.378	1.084

Problem 1165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	16	16	16	58	73	84
normalized size	1	1.	0.76	0.76	0.76	2.76	3.48	4.
time (sec)	N/A	0.002	0.004	0.002	0.984	1.48	2.846	1.075

Problem 1166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	19	19	20	69	73	111
normalized size	1	1.	0.79	0.79	0.83	2.88	3.04	4.62
time (sec)	N/A	0.003	0.008	0.003	0.995	1.55	5.329	1.063

Problem 1167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	21	19	16	62	90	96
normalized size	1	1.	0.81	0.73	0.62	2.38	3.46	3.69
time (sec)	N/A	0.002	0.012	0.001	1.003	1.488	123.053	1.073

Problem 1168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	19	24	20	73	0	123
normalized size	1	1.	0.66	0.83	0.69	2.52	0.	4.24
time (sec)	N/A	0.003	0.018	0.002	0.97	1.594	0.	1.073

Problem 1169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	76	0	248	88	0
normalized size	1	1.	1.	1.95	0.	6.36	2.26	0.
time (sec)	N/A	0.024	0.021	0.007	0.	1.583	3.358	0.

Problem 1170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	42	0	0	1565	0	0
normalized size	1	1.	0.17	0.	0.	6.49	0.	0.
time (sec)	N/A	0.254	0.021	0.059	0.	1.759	0.	0.

Problem 1171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	70	104	0	0	0	0
normalized size	1	1.	0.49	0.72	0.	0.	0.	0.
time (sec)	N/A	0.034	0.037	0.059	0.	0.	0.	0.

Problem 1172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	70	94	0	0	0	0
normalized size	1	1.	0.66	0.89	0.	0.	0.	0.
time (sec)	N/A	0.022	0.023	0.039	0.	0.	0.	0.

Problem 1173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	102	0
normalized size	1	1.	0.99	0.	0.	0.	1.44	0.
time (sec)	N/A	0.012	0.022	0.039	0.	0.	3.696	0.

Problem 1174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	68	94	0	0	0	0
normalized size	1	1.	0.87	1.21	0.	0.	0.	0.
time (sec)	N/A	0.016	0.022	0.05	0.	0.	0.	0.

Problem 1175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	70	105	0	0	0	0
normalized size	1	1.	0.85	1.28	0.	0.	0.	0.
time (sec)	N/A	0.017	0.026	0.053	0.	0.	0.	0.

Problem 1176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	70	113	0	0	0	0
normalized size	1	1.	0.61	0.98	0.	0.	0.	0.
time (sec)	N/A	0.026	0.028	0.059	0.	0.	0.	0.

Problem 1177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	70	114	0	0	0	0
normalized size	1	1.	0.47	0.77	0.	0.	0.	0.
time (sec)	N/A	0.038	0.028	0.059	0.	0.	0.	0.

Problem 1178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	70	0	0	540	0	252
normalized size	1	1.	0.27	0.	0.	2.11	0.	0.98
time (sec)	N/A	0.173	0.021	0.049	0.	1.679	0.	1.253

Problem 1179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	68	0	0	601	0	0
normalized size	1	1.	0.29	0.	0.	2.58	0.	0.
time (sec)	N/A	0.131	0.022	0.042	0.	1.657	0.	0.

Problem 1180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	0	78	0	0
normalized size	1	1.	1.	0.94	0.	2.36	0.	0.
time (sec)	N/A	0.003	0.014	0.03	0.	1.532	0.	0.

Problem 1181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	45	44	0	116	0	0
normalized size	1	1.	0.67	0.66	0.	1.73	0.	0.
time (sec)	N/A	0.009	0.021	0.035	0.	1.637	0.	0.

Problem 1182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	52	50	0	157	0	0
normalized size	1	1.	0.52	0.5	0.	1.57	0.	0.
time (sec)	N/A	0.018	0.025	0.038	0.	1.552	0.	0.

Problem 1183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	57	55	0	200	0	0
normalized size	1	1.	0.43	0.41	0.	1.5	0.	0.
time (sec)	N/A	0.028	0.028	0.039	0.	1.52	0.	0.

Problem 1184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	70	0	0	594	0	242
normalized size	1	1.	0.27	0.	0.	2.32	0.	0.95
time (sec)	N/A	0.158	0.025	0.043	0.	1.588	0.	1.185

Problem 1185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	70	0	0	601	0	0
normalized size	1	1.	0.3	0.	0.	2.58	0.	0.
time (sec)	N/A	0.135	0.022	0.039	0.	1.627	0.	0.

Problem 1186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	0	76	0	0
normalized size	1	1.	1.	1.	0.	2.45	0.	0.
time (sec)	N/A	0.003	0.013	0.029	0.	1.542	0.	0.

Problem 1187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	45	44	0	112	0	0
normalized size	1	1.	0.67	0.66	0.	1.67	0.	0.
time (sec)	N/A	0.01	0.018	0.034	0.	1.581	0.	0.

Problem 1188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	52	50	0	154	0	0
normalized size	1	1.	0.52	0.5	0.	1.54	0.	0.
time (sec)	N/A	0.018	0.024	0.038	0.	1.451	0.	0.

Problem 1189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	70	0	0	0	0	0
normalized size	1	1.	0.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.026	0.054	0.	0.	0.	0.

Problem 1190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.021	0.031	0.	0.	0.	0.

Problem 1191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	68	0	0	0	100	0
normalized size	1	1.	1.58	0.	0.	0.	2.33	0.
time (sec)	N/A	0.008	0.02	0.039	0.	0.	10.274	0.

Problem 1192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	70	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.021	0.043	0.	0.	0.	0.

Problem 1193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	70	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.024	0.047	0.	0.	0.	0.

Problem 1194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	70	0	0	701	0	0
normalized size	1	1.	0.24	0.	0.	2.41	0.	0.
time (sec)	N/A	0.181	0.027	0.04	0.	1.704	0.	0.

Problem 1195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	70	0	0	763	0	0
normalized size	1	1.	0.26	0.	0.	2.87	0.	0.
time (sec)	N/A	0.14	0.022	0.038	0.	1.675	0.	0.

Problem 1196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	0	78	0	0
normalized size	1	1.	1.	0.94	0.	2.36	0.	0.
time (sec)	N/A	0.003	0.011	0.03	0.	1.557	0.	0.

Problem 1197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	38	33	0	95	0	0
normalized size	1	1.	0.58	0.51	0.	1.46	0.	0.
time (sec)	N/A	0.009	0.016	0.039	0.	1.567	0.	0.

Problem 1198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	50	44	0	149	0	0
normalized size	1	1.	0.5	0.44	0.	1.49	0.	0.
time (sec)	N/A	0.017	0.021	0.044	0.	1.574	0.	0.

Problem 1199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	70	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.029	0.043	0.	0.	0.	0.

Problem 1200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	70	0	0	0	0	0
normalized size	1	1.	0.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.025	0.035	0.	0.	0.	0.

Problem 1201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	70	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.021	0.036	0.	0.	0.	0.

Problem 1202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	68	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.02	0.035	0.	0.	0.	0.

Problem 1203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.022	0.041	0.	0.	0.	0.

Problem 1204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	70	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.024	0.055	0.	0.	0.	0.

Problem 1205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	70	0	0	0	0	0
normalized size	1	1.	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.028	0.069	0.	0.	0.	0.

Problem 1206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	70	96	0	0	0	0
normalized size	1	1.	0.51	0.7	0.	0.	0.	0.
time (sec)	N/A	0.032	0.028	0.051	0.	0.	0.	0.

Problem 1207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	70	88	0	0	0	0
normalized size	1	1.	0.69	0.86	0.	0.	0.	0.
time (sec)	N/A	0.02	0.022	0.042	0.	0.	0.	0.

Problem 1208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	94	0	0	0	0
normalized size	1	1.	0.9	1.21	0.	0.	0.	0.
time (sec)	N/A	0.016	0.022	0.033	0.	0.	0.	0.

Problem 1209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	68	91	0	0	97	0
normalized size	1	1.	1.48	1.98	0.	0.	2.11	0.
time (sec)	N/A	0.009	0.023	0.038	0.	0.	54.012	0.

Problem 1210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	70	107	0	0	0	0
normalized size	1	1.	0.85	1.3	0.	0.	0.	0.
time (sec)	N/A	0.016	0.024	0.059	0.	0.	0.	0.

Problem 1211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	70	113	0	0	0	0
normalized size	1	1.	0.61	0.98	0.	0.	0.	0.
time (sec)	N/A	0.026	0.026	0.066	0.	0.	0.	0.

Problem 1212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	70	0	0	686	0	0
normalized size	1	1.	0.24	0.	0.	2.39	0.	0.
time (sec)	N/A	0.178	0.027	0.04	0.	1.811	0.	0.

Problem 1213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	70	0	0	752	0	244
normalized size	1	1.	0.27	0.	0.	2.85	0.	0.92
time (sec)	N/A	0.139	0.022	0.037	0.	1.973	0.	1.138

Problem 1214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	0	76	0	0
normalized size	1	1.	1.	1.	0.	2.45	0.	0.
time (sec)	N/A	0.003	0.011	0.028	0.	1.773	0.	0.

Problem 1215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	38	33	0	95	0	0
normalized size	1	1.	0.57	0.49	0.	1.42	0.	0.
time (sec)	N/A	0.009	0.017	0.033	0.	1.739	0.	0.

Problem 1216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	50	44	0	150	0	0
normalized size	1	1.	0.5	0.44	0.	1.5	0.	0.
time (sec)	N/A	0.018	0.02	0.041	0.	1.917	0.	0.

Problem 1217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	70	101	0	0	0	0
normalized size	1	1.	0.5	0.72	0.	0.	0.	0.
time (sec)	N/A	0.03	0.038	0.05	0.	0.	0.	0.

Problem 1218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	70	107	0	0	0	0
normalized size	1	1.	0.61	0.93	0.	0.	0.	0.
time (sec)	N/A	0.025	0.023	0.034	0.	0.	0.	0.

Problem 1219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	70	105	0	0	0	0
normalized size	1	1.	0.85	1.28	0.	0.	0.	0.
time (sec)	N/A	0.016	0.023	0.035	0.	0.	0.	0.

Problem 1220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	68	107	0	0	0	0
normalized size	1	1.	0.83	1.3	0.	0.	0.	0.
time (sec)	N/A	0.025	0.026	0.056	0.	0.	0.	0.

Problem 1221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	70	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.028	0.042	0.	0.	0.	0.

Problem 1222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	70	124	0	0	0	0
normalized size	1	1.	0.58	1.02	0.	0.	0.	0.
time (sec)	N/A	0.026	0.028	0.077	0.	0.	0.	0.

Problem 1223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	70	130	0	0	0	0
normalized size	1	1.	0.45	0.84	0.	0.	0.	0.
time (sec)	N/A	0.04	0.034	0.086	0.	0.	0.	0.

Problem 1224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	70	0	0	879	0	0
normalized size	1	1.	0.24	0.	0.	2.96	0.	0.
time (sec)	N/A	0.14	0.024	0.043	0.	2.19	0.	0.

Problem 1225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	50	0	113	0	46
normalized size	1	1.	1.	1.52	0.	3.42	0.	1.39
time (sec)	N/A	0.003	0.011	0.034	0.	1.985	0.	1.118

Problem 1226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	45	44	0	112	0	0
normalized size	1	1.	0.67	0.66	0.	1.67	0.	0.
time (sec)	N/A	0.01	0.019	0.036	0.	2.17	0.	0.

Problem 1227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	50	44	0	149	0	0
normalized size	1	1.	0.5	0.44	0.	1.49	0.	0.
time (sec)	N/A	0.018	0.021	0.041	0.	2.211	0.	0.

Problem 1228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	57	56	0	142	0	0
normalized size	1	1.	0.43	0.42	0.	1.07	0.	0.
time (sec)	N/A	0.029	0.03	0.048	0.	2.05	0.	0.

Problem 1229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	77	103	0	261	785	346
normalized size	1	1.	0.93	1.24	0.	3.14	9.46	4.17
time (sec)	N/A	0.029	0.038	0.006	0.	2.246	1.074	1.073

Problem 1230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	47	0	117	245	139
normalized size	1	1.	0.81	0.89	0.	2.21	4.62	2.62
time (sec)	N/A	0.017	0.021	0.002	0.	2.138	0.616	1.053

Problem 1231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.013	0.04	0.	0.	0.	0.

Problem 1232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.011	0.047	0.	0.	0.	0.

Problem 1233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	53	0	0	0	124	0
normalized size	1	1.	1.29	0.	0.	0.	3.02	0.
time (sec)	N/A	0.012	0.022	0.1	0.	0.	4.376	0.

Problem 1234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	72	0	0	0	146	0
normalized size	1	1.	1.26	0.	0.	0.	2.56	0.
time (sec)	N/A	0.018	0.024	0.084	0.	0.	5.122	0.

Problem 1235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	0	42	0
normalized size	1	1.	1.	0.	0.	0.	2.1	0.
time (sec)	N/A	0.006	0.005	0.073	0.	0.	4.617	0.

Problem 1236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	84	97	130	217	100	131
normalized size	1	1.	2.21	2.55	3.42	5.71	2.63	3.45
time (sec)	N/A	0.015	0.018	0.001	0.962	1.718	0.076	1.063

Problem 1237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	67	73	93	163	73	97
normalized size	1	1.	1.76	1.92	2.45	4.29	1.92	2.55
time (sec)	N/A	0.012	0.01	0.	0.978	1.712	0.07	1.068

Problem 1238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	46	49	65	115	49	66
normalized size	1	1.	1.21	1.29	1.71	3.03	1.29	1.74
time (sec)	N/A	0.027	0.008	0.	0.958	1.759	0.079	1.067

Problem 1239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	66	26	35
normalized size	1	1.	1.	0.89	1.14	2.36	0.93	1.25
time (sec)	N/A	0.015	0.004	0.001	0.97	1.734	0.056	1.05

Problem 1240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	23	8	14
normalized size	1	1.	1.	0.92	1.17	1.92	0.67	1.17
time (sec)	N/A	0.002	0.	0.001	0.941	1.746	0.05	1.054

Problem 1241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	34	54	20	35
normalized size	1	1.	1.	1.28	1.36	2.16	0.8	1.4
time (sec)	N/A	0.017	0.008	0.002	0.983	1.933	0.293	1.078

Problem 1242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	39	47	80	27	77
normalized size	1	1.	0.97	1.22	1.47	2.5	0.84	2.41
time (sec)	N/A	0.019	0.011	0.006	1.051	1.92	0.342	1.066

Problem 1243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	35	51	81	39	32
normalized size	1	1.	0.93	1.25	1.82	2.89	1.39	1.14
time (sec)	N/A	0.004	0.009	0.006	0.965	1.968	0.429	1.064

Problem 1244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	68	105	53	34
normalized size	1	1.	0.71	0.92	1.79	2.76	1.39	0.89
time (sec)	N/A	0.02	0.009	0.004	0.957	1.95	0.501	1.078

Problem 1245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	82	128	65	55
normalized size	1	1.	0.71	0.92	2.16	3.37	1.71	1.45
time (sec)	N/A	0.02	0.01	0.005	0.949	2.012	0.593	1.051

Problem 1246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	148	163	211	363	168	230
normalized size	1	1.	2.28	2.51	3.25	5.58	2.58	3.54
time (sec)	N/A	0.088	0.026	0.002	0.969	1.72	0.089	1.051

Problem 1247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	122	125	167	285	133	176
normalized size	1	1.	1.88	1.92	2.57	4.38	2.05	2.71
time (sec)	N/A	0.064	0.015	0.001	0.948	1.734	0.081	1.053

Problem 1248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	79	87	109	198	87	120
normalized size	1	1.	1.22	1.34	1.68	3.05	1.34	1.85
time (sec)	N/A	0.045	0.01	0.	0.952	1.707	0.075	1.051

Problem 1249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	47	49	65	115	49	66
normalized size	1	1.	1.24	1.29	1.71	3.03	1.29	1.74
time (sec)	N/A	0.027	0.01	0.	0.949	1.79	0.067	1.059

Problem 1250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	27	42	19	16
normalized size	1	1.	1.	0.93	1.93	3.	1.36	1.14
time (sec)	N/A	0.002	0.001	0.	0.951	1.764	0.057	1.074

Problem 1251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	43	74	82	135	44	81
normalized size	1	1.	0.88	1.51	1.67	2.76	0.9	1.65
time (sec)	N/A	0.019	0.016	0.003	0.977	2.069	0.379	1.067

Problem 1252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	86	90	184	60	132
normalized size	1	1.	0.92	1.69	1.76	3.61	1.18	2.59
time (sec)	N/A	0.035	0.036	0.005	0.961	1.933	0.562	1.067

Problem 1253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	92	107	207	80	92
normalized size	1	1.	0.83	1.56	1.81	3.51	1.36	1.56
time (sec)	N/A	0.035	0.023	0.005	0.964	1.886	0.631	1.056

Problem 1254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	53	70	113	170	88	80
normalized size	1	1.	1.89	2.5	4.04	6.07	3.14	2.86
time (sec)	N/A	0.004	0.022	0.004	0.963	1.985	0.773	1.058

Problem 1255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	56	71	132	201	104	130
normalized size	1	1.	0.86	1.09	2.03	3.09	1.6	2.
time (sec)	N/A	0.033	0.019	0.005	0.962	2.01	0.939	1.061

Problem 1256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	57	71	147	227	116	82
normalized size	1	1.	0.88	1.09	2.26	3.49	1.78	1.26
time (sec)	N/A	0.033	0.025	0.005	1.102	1.991	1.106	1.081

Problem 1257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	58	71	162	251	128	82
normalized size	1	1.	0.89	1.09	2.49	3.86	1.97	1.26
time (sec)	N/A	0.033	0.02	0.006	1.152	1.913	1.343	1.072

Problem 1258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	235	281	374	651	308	409
normalized size	1	1.	2.55	3.05	4.07	7.08	3.35	4.45
time (sec)	N/A	0.157	0.074	0.001	0.948	1.725	0.101	1.052

Problem 1259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	217	229	304	520	243	331
normalized size	1	1.	2.36	2.49	3.3	5.65	2.64	3.6
time (sec)	N/A	0.114	0.027	0.001	0.967	1.735	0.094	1.066

Problem 1260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	161	177	225	409	190	254
normalized size	1	1.	1.75	1.92	2.45	4.45	2.07	2.76
time (sec)	N/A	0.083	0.021	0.	0.961	1.803	0.086	1.096

Problem 1261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	122	125	167	285	133	176
normalized size	1	1.	1.88	1.92	2.57	4.38	2.05	2.71
time (sec)	N/A	0.064	0.014	0.	1.005	1.746	0.082	1.052

Problem 1262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	67	73	93	163	73	97
normalized size	1	1.	1.76	1.92	2.45	4.29	1.92	2.55
time (sec)	N/A	0.015	0.008	0.	0.974	1.778	0.071	1.049

Problem 1263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	42	66	32	16
normalized size	1	1.	1.	0.93	3.	4.71	2.29	1.14
time (sec)	N/A	0.002	0.001	0.	0.967	1.764	0.06	1.06

Problem 1264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	74	133	154	238	82	155
normalized size	1	1.	1.01	1.82	2.11	3.26	1.12	2.12
time (sec)	N/A	0.028	0.03	0.003	0.993	1.939	0.458	1.048

Problem 1265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	72	149	159	354	100	225
normalized size	1	1.	0.96	1.99	2.12	4.72	1.33	3.
time (sec)	N/A	0.055	0.053	0.006	0.964	2.245	0.7	1.055

Problem 1266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	114	160	169	375	128	151
normalized size	1	1.	1.46	2.05	2.17	4.81	1.64	1.94
time (sec)	N/A	0.052	0.043	0.006	0.971	2.205	1.045	1.08

Problem 1267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	80	166	192	360	148	159
normalized size	1	1.	0.93	1.93	2.23	4.19	1.72	1.85
time (sec)	N/A	0.05	0.042	0.006	0.976	2.245	1.388	1.056

Problem 1268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	91	122	193	284	153	215
normalized size	1	1.	3.25	4.36	6.89	10.14	5.46	7.68
time (sec)	N/A	0.003	0.032	0.005	0.98	2.29	1.764	1.055

Problem 1269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	97	121	216	328	170	154
normalized size	1	1.	1.67	2.09	3.72	5.66	2.93	2.66
time (sec)	N/A	0.01	0.036	0.005	0.963	2.331	2.299	1.056

Problem 1270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	97	122	231	354	182	154
normalized size	1	1.	1.05	1.33	2.51	3.85	1.98	1.67
time (sec)	N/A	0.05	0.035	0.006	0.971	2.24	3.054	1.058

Problem 1271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	97	122	246	382	194	154
normalized size	1	1.	1.05	1.33	2.67	4.15	2.11	1.67
time (sec)	N/A	0.049	0.031	0.004	1.015	2.119	3.793	1.069

Problem 1272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	97	122	261	406	206	154
normalized size	1	1.	1.05	1.33	2.84	4.41	2.24	1.67
time (sec)	N/A	0.046	0.035	0.006	1.031	2.255	4.839	1.06

Problem 1273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	993	1033	1381	2626	1163	1586
normalized size	1	1.	4.96	5.16	6.9	13.13	5.82	7.93
time (sec)	N/A	0.676	0.14	0.003	0.981	1.926	0.204	1.075

Problem 1274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	897	925	1243	2337	1046	1418
normalized size	1	1.	4.48	4.62	6.22	11.68	5.23	7.09
time (sec)	N/A	0.574	0.104	0.001	1.012	1.794	0.19	1.057

Problem 1275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	785	817	1089	2079	935	1247
normalized size	1	1.	3.92	4.08	5.44	10.4	4.68	6.24
time (sec)	N/A	0.454	0.08	0.002	0.994	1.951	0.195	1.059

Problem 1276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	684	709	953	1736	796	1077
normalized size	1	1.	3.95	4.1	5.51	10.03	4.6	6.23
time (sec)	N/A	0.435	0.079	0.001	0.98	2.099	0.159	1.056

Problem 1277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	574	601	802	1467	673	905
normalized size	1	1.	3.99	4.17	5.57	10.19	4.67	6.28
time (sec)	N/A	0.36	0.068	0.001	0.979	2.077	0.141	1.06

Problem 1278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	473	493	660	1184	549	737
normalized size	1	1.	3.97	4.14	5.55	9.95	4.61	6.19
time (sec)	N/A	0.279	0.05	0.002	0.983	1.879	0.13	1.078

Problem 1279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	360	385	508	913	427	567
normalized size	1	1.	3.91	4.18	5.52	9.92	4.64	6.16
time (sec)	N/A	0.218	0.041	0.002	0.965	2.018	0.138	1.065

Problem 1280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	261	277	369	644	303	397
normalized size	1	1.	4.02	4.26	5.68	9.91	4.66	6.11
time (sec)	N/A	0.159	0.027	0.002	0.986	1.948	0.106	1.046

Problem 1281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	151	169	220	378	178	228
normalized size	1	1.	3.97	4.45	5.79	9.95	4.68	6.
time (sec)	N/A	0.016	0.017	0.001	0.945	1.857	0.09	1.075

Problem 1282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	159	83	16
normalized size	1	1.	1.	0.93	1.14	11.36	5.93	1.14
time (sec)	N/A	0.002	0.002	0.	0.944	1.978	0.071	1.044

Problem 1283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	304	539	621	952	384	671
normalized size	1	1.	1.8	3.19	3.67	5.63	2.27	3.97
time (sec)	N/A	0.073	0.142	0.006	0.967	2.32	0.968	1.044

Problem 1284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	388	571	630	1310	410	765
normalized size	1	1.	2.07	3.05	3.37	7.01	2.19	4.09
time (sec)	N/A	0.235	0.12	0.011	0.978	2.294	1.74	1.075

Problem 1285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	389	599	639	1445	437	644
normalized size	1	1.	2.1	3.24	3.45	7.81	2.36	3.48
time (sec)	N/A	0.217	0.13	0.011	1.021	2.389	3.637	1.068

Problem 1286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	199	622	653	1520	468	635
normalized size	1	1.	1.06	3.33	3.49	8.13	2.5	3.4
time (sec)	N/A	0.213	0.105	0.011	1.026	2.337	8.268	1.055

Problem 1287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	173	641	667	1551	495	891
normalized size	1	1.	0.93	3.43	3.57	8.29	2.65	4.76
time (sec)	N/A	0.198	0.114	0.011	1.114	2.328	26.055	1.081

Problem 1288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	389	656	680	1517	522	625
normalized size	1	1.	2.15	3.62	3.76	8.38	2.88	3.45
time (sec)	N/A	0.191	0.149	0.013	1.125	2.241	91.875	1.061

Problem 1289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	390	666	697	1436	0	620
normalized size	1	1.	2.1	3.58	3.75	7.72	0.	3.33
time (sec)	N/A	0.172	0.206	0.012	1.117	2.039	0.	1.064

Problem 1290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	308	672	721	1320	0	629
normalized size	1	1.	1.59	3.46	3.72	6.8	0.	3.24
time (sec)	N/A	0.156	0.16	0.01	1.071	2.157	0.	1.06

Problem 1291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	353	464	687	1003	0	660
normalized size	1	1.	12.61	16.57	24.54	35.82	0.	23.57
time (sec)	N/A	0.003	0.119	0.007	1.056	2.11	0.	1.064

Problem 1292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	367	464	740	1111	0	670
normalized size	1	1.	6.33	8.	12.76	19.16	0.	11.55
time (sec)	N/A	0.009	0.119	0.008	1.072	2.039	0.	1.061

Problem 1293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	371	464	755	1172	0	670
normalized size	1	1.	4.17	5.21	8.48	13.17	0.	7.53
time (sec)	N/A	0.022	0.118	0.006	1.085	2.031	0.	1.058

Problem 1294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	369	464	770	1211	0	670
normalized size	1	1.	3.08	3.87	6.42	10.09	0.	5.58
time (sec)	N/A	0.033	0.116	0.008	1.071	1.753	0.	1.072

Problem 1295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	371	464	784	1246	0	670
normalized size	1	1.	2.46	3.07	5.19	8.25	0.	4.44
time (sec)	N/A	0.047	0.12	0.007	1.117	1.848	0.	1.081

Problem 1296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	369	463	799	1291	0	670
normalized size	1	1.	1.86	2.34	4.04	6.52	0.	3.38
time (sec)	N/A	0.153	0.122	0.008	1.116	1.763	0.	1.064

Problem 1297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	371	464	814	1326	0	670
normalized size	1	1.	1.86	2.32	4.07	6.63	0.	3.35
time (sec)	N/A	0.143	0.12	0.008	1.139	1.853	0.	1.067

Problem 1298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	371	464	829	1369	0	670
normalized size	1	1.	1.86	2.32	4.14	6.84	0.	3.35
time (sec)	N/A	0.14	0.121	0.008	1.131	1.849	0.	1.063

Problem 1299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	1817	1891	2534	5079	2088	2951
normalized size	1	1.	6.61	6.88	9.21	18.47	7.59	10.73
time (sec)	N/A	1.465	0.262	0.004	1.013	1.602	0.296	1.072

Problem 1300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	1702	1741	2349	4709	1965	2714
normalized size	1	1.	6.1	6.24	8.42	16.88	7.04	9.73
time (sec)	N/A	1.275	0.213	0.004	1.031	1.734	0.281	1.088

Problem 1301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	1539	1591	2134	4215	1775	2475
normalized size	1	1.	5.52	5.7	7.65	15.11	6.36	8.87
time (sec)	N/A	1.112	0.172	0.001	1.014	1.571	0.257	1.069

Problem 1302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	1397	1441	1940	3740	1598	2236
normalized size	1	1.	5.59	5.76	7.76	14.96	6.39	8.94
time (sec)	N/A	1.042	0.173	0.003	1.001	1.605	0.238	1.06

Problem 1303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	1241	1291	1732	3305	1428	1995
normalized size	1	1.	5.52	5.74	7.7	14.69	6.35	8.87
time (sec)	N/A	0.899	0.154	0.002	0.997	1.659	0.227	1.075

Problem 1304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	1105	1141	1532	2931	1280	1758
normalized size	1	1.	5.52	5.7	7.66	14.66	6.4	8.79
time (sec)	N/A	0.766	0.125	0.003	0.98	1.614	0.233	1.063

Problem 1305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	939	991	1319	2484	1088	1517
normalized size	1	1.	5.52	5.83	7.76	14.61	6.4	8.92
time (sec)	N/A	0.673	0.112	0.003	1.004	1.542	0.187	1.055

Problem 1306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	811	841	1127	2132	940	1280
normalized size	1	1.	5.55	5.76	7.72	14.6	6.44	8.77
time (sec)	N/A	0.529	0.083	0.003	0.987	1.499	0.173	1.071

Problem 1307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	660	691	926	1689	748	1041
normalized size	1	1.	5.55	5.81	7.78	14.19	6.29	8.75
time (sec)	N/A	0.437	0.075	0.002	1.241	1.731	0.156	1.053

Problem 1308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	511	541	722	1318	586	802
normalized size	1	1.	5.55	5.88	7.85	14.33	6.37	8.72
time (sec)	N/A	0.349	0.061	0.	0.975	1.541	0.139	1.067

Problem 1309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	358	391	518	922	415	563
normalized size	1	1.	5.51	6.02	7.97	14.18	6.38	8.66
time (sec)	N/A	0.251	0.04	0.001	0.984	1.561	0.122	1.062

Problem 1310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	220	241	324	554	248	325
normalized size	1	1.	5.79	6.34	8.53	14.58	6.53	8.55
time (sec)	N/A	0.016	0.028	0.002	0.971	1.515	0.105	1.075

Problem 1311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	230	114	16
normalized size	1	1.	1.	0.93	1.14	16.43	8.14	1.14
time (sec)	N/A	0.002	0.001	0.	0.957	1.591	0.077	1.067

Problem 1312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	591	1022	1169	1879	772	1297
normalized size	1	1.	2.45	4.24	4.85	7.8	3.2	5.38
time (sec)	N/A	0.098	0.298	0.007	0.979	1.761	1.438	1.058

Problem 1313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	708	1066	1180	2402	796	1366
normalized size	1	1.	2.74	4.13	4.57	9.31	3.09	5.29
time (sec)	N/A	0.473	0.232	0.014	0.985	1.957	2.837	1.087

Problem 1314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	708	1105	1189	2612	828	1247
normalized size	1	1.	2.7	4.22	4.54	9.97	3.16	4.76
time (sec)	N/A	0.444	0.236	0.016	1.023	1.949	6.579	1.049

Problem 1315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	427	1141	1203	2786	853	1224
normalized size	1	1.	1.66	4.42	4.66	10.8	3.31	4.74
time (sec)	N/A	0.441	0.18	0.018	1.113	1.969	33.521	1.067

Problem 1316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	359	1172	1219	2925	0	1577
normalized size	1	1.	1.37	4.47	4.65	11.16	0.	6.02
time (sec)	N/A	0.422	0.197	0.018	1.174	1.929	0.	1.127

Problem 1317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	305	1199	1231	2981	0	1192
normalized size	1	1.	1.17	4.61	4.73	11.47	0.	4.58
time (sec)	N/A	0.421	0.223	0.02	1.276	1.841	0.	1.068

Problem 1318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	265	1222	1249	2954	0	1185
normalized size	1	1.	1.01	4.66	4.77	11.27	0.	4.52
time (sec)	N/A	0.387	0.236	0.02	1.334	1.95	0.	1.055

Problem 1319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	239	1241	1261	2878	0	1177
normalized size	1	1.	0.93	4.81	4.89	11.16	0.	4.56
time (sec)	N/A	0.365	0.265	0.02	1.348	1.957	0.	1.064

Problem 1320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	712	1256	1276	2763	0	1176
normalized size	1	1.	2.76	4.87	4.95	10.71	0.	4.56
time (sec)	N/A	0.341	0.314	0.017	1.378	1.869	0.	1.069

Problem 1321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	708	1266	1292	2597	0	1170
normalized size	1	1.	2.75	4.93	5.03	10.11	0.	4.55
time (sec)	N/A	0.311	0.411	0.017	1.305	1.966	0.	1.065

Problem 1322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	591	1271	1316	2456	0	1180
normalized size	1	1.	2.18	4.69	4.86	9.06	0.	4.35
time (sec)	N/A	0.287	0.358	0.012	1.192	2.011	0.	1.077

Problem 1323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	665	866	1242	1858	0	1284
normalized size	1	1.	23.75	30.93	44.36	66.36	0.	45.86
time (sec)	N/A	0.003	0.322	0.01	1.21	1.646	0.	1.06

Problem 1324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	684	867	1331	2041	0	1297
normalized size	1	1.	11.79	14.95	22.95	35.19	0.	22.36
time (sec)	N/A	0.01	0.301	0.011	1.243	1.865	0.	1.072

Problem 1325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	690	867	1346	2126	0	1297
normalized size	1	1.	7.75	9.74	15.12	23.89	0.	14.57
time (sec)	N/A	0.02	0.288	0.008	1.268	1.781	0.	1.06

Problem 1326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	692	867	1361	2190	0	1297
normalized size	1	1.	5.77	7.22	11.34	18.25	0.	10.81
time (sec)	N/A	0.03	0.278	0.01	1.261	1.894	0.	1.065

Problem 1327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	690	867	1376	2241	0	1297
normalized size	1	1.	4.57	5.74	9.11	14.84	0.	8.59
time (sec)	N/A	0.044	0.282	0.01	1.254	1.9	0.	1.085

Problem 1328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	694	867	1391	2295	0	1297
normalized size	1	1.	3.81	4.76	7.64	12.61	0.	7.13
time (sec)	N/A	0.063	0.269	0.009	1.263	1.836	0.	1.06

Problem 1329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	690	867	1405	2338	0	1297
normalized size	1	1.	3.24	4.07	6.6	10.98	0.	6.09
time (sec)	N/A	0.079	0.36	0.01	1.302	1.934	0.	1.068

Problem 1330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	694	867	1420	2391	0	1297
normalized size	1	1.	2.84	3.55	5.82	9.8	0.	5.32
time (sec)	N/A	0.105	0.288	0.01	1.312	1.946	0.	1.059

Problem 1331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	692	866	1435	2438	0	1297
normalized size	1	1.	2.53	3.17	5.26	8.93	0.	4.75
time (sec)	N/A	0.284	0.279	0.009	1.338	1.957	0.	1.072

Problem 1332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	692	867	1450	2504	0	1297
normalized size	1	1.	2.48	3.11	5.2	8.97	0.	4.65
time (sec)	N/A	0.272	0.271	0.01	1.334	1.916	0.	1.062

Problem 1333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	692	867	1465	2552	0	1297
normalized size	1	1.	2.48	3.11	5.25	9.15	0.	4.65
time (sec)	N/A	0.271	0.287	0.009	1.383	1.964	0.	1.069

Problem 1334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	167	302	348	537	202	369
normalized size	1	1.	1.37	2.48	2.85	4.4	1.66	3.02
time (sec)	N/A	0.053	0.066	0.004	0.974	1.847	0.662	1.058

Problem 1335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	115	209	239	369	134	248
normalized size	1	1.	1.17	2.13	2.44	3.77	1.37	2.53
time (sec)	N/A	0.038	0.042	0.003	0.995	1.675	0.535	1.054

Problem 1336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	133	154	238	82	157
normalized size	1	1.	1.	1.8	2.08	3.22	1.11	2.12
time (sec)	N/A	0.03	0.028	0.002	0.965	1.744	0.448	1.051

Problem 1337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	43	74	81	135	44	81
normalized size	1	1.	0.86	1.48	1.62	2.7	0.88	1.62
time (sec)	N/A	0.021	0.017	0.002	0.976	1.779	0.361	1.057

Problem 1338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	32	35	54	20	36
normalized size	1	1.	0.96	1.23	1.35	2.08	0.77	1.38
time (sec)	N/A	0.018	0.007	0.003	0.968	1.759	0.288	1.062

Problem 1339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	22	7	15
normalized size	1	1.	1.	1.1	1.4	2.2	0.7	1.5
time (sec)	N/A	0.001	0.001	0.001	0.956	1.733	0.055	1.059

Problem 1340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	26	37	49	58	128	0
normalized size	1	1.	0.72	1.03	1.36	1.61	3.56	0.
time (sec)	N/A	0.008	0.011	0.006	0.98	1.712	0.306	0.

Problem 1341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	53	57	124	200	233	105
normalized size	1	1.	0.93	1.	2.18	3.51	4.09	1.84
time (sec)	N/A	0.032	0.025	0.014	0.97	1.87	0.779	1.079

Problem 1342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	67	81	273	491	381	223
normalized size	1	1.	0.82	0.99	3.33	5.99	4.65	2.72
time (sec)	N/A	0.045	0.064	0.007	1.01	1.831	1.212	1.068

Problem 1343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	228	326	356	767	224	458
normalized size	1	1.	1.75	2.51	2.74	5.9	1.72	3.52
time (sec)	N/A	0.139	0.072	0.009	0.973	1.758	1.142	1.076

Problem 1344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	165	230	247	540	151	331
normalized size	1	1.	1.59	2.21	2.38	5.19	1.45	3.18
time (sec)	N/A	0.1	0.056	0.009	0.956	1.728	0.903	1.07

Problem 1345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	114	149	158	354	100	224
normalized size	1	1.	1.52	1.99	2.11	4.72	1.33	2.99
time (sec)	N/A	0.062	0.034	0.006	0.974	1.831	0.682	1.074

Problem 1346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	86	90	184	60	132
normalized size	1	1.	0.92	1.69	1.76	3.61	1.18	2.59
time (sec)	N/A	0.039	0.035	0.005	0.965	1.717	0.503	1.07

Problem 1347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	39	46	78	27	77
normalized size	1	1.	1.	1.26	1.48	2.52	0.87	2.48
time (sec)	N/A	0.021	0.01	0.004	0.943	1.623	0.337	1.05

Problem 1348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	24	10	16
normalized size	1	1.	1.	1.08	1.33	2.	0.83	1.33
time (sec)	N/A	0.002	0.002	0.001	0.961	1.564	0.292	1.083

Problem 1349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	53	58	122	198	233	104
normalized size	1	1.	0.95	1.04	2.18	3.54	4.16	1.86
time (sec)	N/A	0.032	0.025	0.01	0.964	1.796	0.774	1.072

Problem 1350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	66	82	281	486	405	207
normalized size	1	1.	0.81	1.01	3.47	6.	5.	2.56
time (sec)	N/A	0.05	0.07	0.009	0.986	1.858	1.238	1.095

Problem 1351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	98	109	521	991	632	292
normalized size	1	1.	0.9	1.	4.78	9.09	5.8	2.68
time (sec)	N/A	0.076	0.075	0.01	1.015	1.883	1.941	1.095

Problem 1352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	303	464	491	1110	335	489
normalized size	1	1.	1.92	2.94	3.11	7.03	2.12	3.09
time (sec)	N/A	0.202	0.112	0.01	0.987	1.78	2.633	1.063

Problem 1353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	230	346	366	840	253	356
normalized size	1	1.	1.73	2.6	2.75	6.32	1.9	2.68
time (sec)	N/A	0.121	0.07	0.01	0.997	1.785	2.014	1.076

Problem 1354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	167	245	258	586	184	247
normalized size	1	1.	1.62	2.38	2.5	5.69	1.79	2.4
time (sec)	N/A	0.087	0.053	0.007	0.995	1.8	1.44	1.056

Problem 1355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	114	160	169	375	128	151
normalized size	1	1.	1.46	2.05	2.17	4.81	1.64	1.94
time (sec)	N/A	0.055	0.039	0.007	0.971	1.795	1.034	1.063

Problem 1356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	92	108	205	80	93
normalized size	1	1.	0.81	1.56	1.83	3.47	1.36	1.58
time (sec)	N/A	0.038	0.023	0.006	0.978	1.868	0.605	1.057

Problem 1357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	35	51	81	39	32
normalized size	1	1.	0.93	1.25	1.82	2.89	1.39	1.14
time (sec)	N/A	0.003	0.009	0.004	0.959	1.711	0.408	1.064

Problem 1358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	49	26	16
normalized size	1	1.	1.	0.93	1.14	3.5	1.86	1.14
time (sec)	N/A	0.002	0.002	0.	0.958	1.62	0.309	1.075

Problem 1359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	67	81	273	490	381	223
normalized size	1	1.	0.82	0.99	3.33	5.98	4.65	2.72
time (sec)	N/A	0.046	0.048	0.009	0.974	1.694	1.216	1.074

Problem 1360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	108	521	991	632	293
normalized size	1	1.	0.88	0.98	4.74	9.01	5.75	2.66
time (sec)	N/A	0.074	0.097	0.012	1.029	1.848	1.974	1.095

Problem 1361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	128	140	802	1488	881	0
normalized size	1	1.	0.9	0.98	5.61	10.41	6.16	0.
time (sec)	N/A	0.102	0.109	0.011	1.021	1.983	2.713	0.

Problem 1362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	584	1035	1061	2298	0	976
normalized size	1	1.	2.52	4.46	4.57	9.91	0.	4.21
time (sec)	N/A	0.357	0.26	0.015	1.264	1.891	0.	1.062

Problem 1363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	474	845	876	1770	0	784
normalized size	1	1.	2.27	4.04	4.19	8.47	0.	3.75
time (sec)	N/A	0.278	0.195	0.013	1.148	2.019	0.	1.06

Problem 1364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	308	672	722	1319	0	630
normalized size	1	1.	1.59	3.46	3.72	6.8	0.	3.25
time (sec)	N/A	0.209	0.159	0.008	1.047	1.829	0.	1.073

Problem 1365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	271	357	537	787	0	498
normalized size	1	1.	9.68	12.75	19.18	28.11	0.	17.79
time (sec)	N/A	0.003	0.089	0.006	1.05	1.7	0.	1.064

Problem 1366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	205	265	440	663	348	366
normalized size	1	1.	3.53	4.57	7.59	11.43	6.	6.31
time (sec)	N/A	0.011	0.058	0.005	1.018	1.748	52.101	1.066

Problem 1367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	144	186	333	512	264	248
normalized size	1	1.	1.62	2.09	3.74	5.75	2.97	2.79
time (sec)	N/A	0.019	0.048	0.005	1.004	1.483	11.613	1.068

Problem 1368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	94	122	246	382	194	154
normalized size	1	1.	1.02	1.33	2.67	4.15	2.11	1.67
time (sec)	N/A	0.057	0.029	0.005	1.003	1.77	3.489	1.079

Problem 1369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	55	71	177	277	139	82
normalized size	1	1.	0.85	1.09	2.72	4.26	2.14	1.26
time (sec)	N/A	0.04	0.024	0.003	0.982	1.767	1.518	1.058

Problem 1370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	127	198	100	34
normalized size	1	1.	0.71	0.92	3.34	5.21	2.63	0.89
time (sec)	N/A	0.022	0.009	0.004	0.975	1.787	0.88	1.066

Problem 1371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	162	85	16
normalized size	1	1.	1.	0.93	1.14	11.57	6.07	1.14
time (sec)	N/A	0.002	0.003	0.	0.961	1.759	0.61	1.065

Problem 1372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	196	192	1914	3332	1776	949
normalized size	1	1.	0.97	0.95	9.48	16.5	8.79	4.7
time (sec)	N/A	0.169	0.093	0.016	1.504	2.201	10.643	1.068

Problem 1373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	213	223	2539	4788	2334	964
normalized size	1	1.	0.92	0.97	10.99	20.73	10.1	4.17
time (sec)	N/A	0.27	0.232	0.02	1.773	2.55	22.206	1.135

Problem 1374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	254	265	3239	6507	2914	1389
normalized size	1	1.	0.92	0.96	11.74	23.58	10.56	5.03
time (sec)	N/A	0.356	0.199	0.02	2.187	3.055	43.525	1.087

Problem 1375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	123	273	350	761	314	385
normalized size	1	1.	0.79	1.75	2.24	4.88	2.01	2.47
time (sec)	N/A	0.063	0.138	0.005	0.972	2.102	3.752	1.097

Problem 1376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	101	186	244	547	223	278
normalized size	1	1.	0.78	1.44	1.89	4.24	1.73	2.16
time (sec)	N/A	0.052	0.087	0.005	0.956	1.957	3.069	1.098

Problem 1377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	79	116	159	359	146	188
normalized size	1	1.	0.79	1.16	1.59	3.59	1.46	1.88
time (sec)	N/A	0.036	0.06	0.004	0.967	2.058	2.363	1.064

Problem 1378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	61	63	92	220	85	113
normalized size	1	1.	0.86	0.89	1.3	3.1	1.2	1.59
time (sec)	N/A	0.025	0.035	0.004	0.979	1.993	1.9	1.061

Problem 1379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	30	27	45	108	36	55
normalized size	1	1.	0.71	0.64	1.07	2.57	0.86	1.31
time (sec)	N/A	0.014	0.017	0.002	0.942	2.106	1.488	1.054

Problem 1380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	31	12	16
normalized size	1	1.	1.	0.81	1.	1.94	0.75	1.
time (sec)	N/A	0.002	0.004	0.001	0.96	1.948	0.053	1.09

Problem 1381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	92	0	306	61	84
normalized size	1	1.	1.	1.48	0.	4.94	0.98	1.35
time (sec)	N/A	0.053	0.034	0.01	0.	2.152	3.063	1.064

Problem 1382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	69	64	0	498	573	97
normalized size	1	1.	0.99	0.91	0.	7.11	8.19	1.39
time (sec)	N/A	0.03	0.074	0.011	0.	2.156	28.585	1.089

Problem 1383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	52	111	0	949	1658	170
normalized size	1	1.	0.47	1.01	0.	8.63	15.07	1.55
time (sec)	N/A	0.078	0.017	0.012	0.	2.169	92.782	1.074

Problem 1384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	52	170	0	1581	0	279
normalized size	1	1.	0.36	1.16	0.	10.83	0.	1.91
time (sec)	N/A	0.098	0.013	0.012	0.	2.277	0.	1.08

Problem 1385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	52	248	0	2404	0	420
normalized size	1	1.	0.29	1.36	0.	13.21	0.	2.31
time (sec)	N/A	0.123	0.014	0.013	0.	2.33	0.	1.101

Problem 1386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	52	337	0	3452	0	583
normalized size	1	1.	0.24	1.55	0.	15.83	0.	2.67
time (sec)	N/A	0.151	0.015	0.016	0.	2.432	0.	1.101

Problem 1387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	123	273	350	953	763	869
normalized size	1	1.	0.78	1.73	2.22	6.03	4.83	5.5
time (sec)	N/A	0.053	0.143	0.006	0.962	2.109	20.25	1.105

Problem 1388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	101	186	244	702	559	640
normalized size	1	1.	0.78	1.44	1.89	5.44	4.33	4.96
time (sec)	N/A	0.042	0.095	0.006	0.977	2.132	15.525	1.082

Problem 1389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	79	116	159	474	386	440
normalized size	1	1.	0.79	1.16	1.59	4.74	3.86	4.4
time (sec)	N/A	0.034	0.066	0.006	0.949	1.985	10.98	1.069

Problem 1390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	61	63	92	300	240	274
normalized size	1	1.	0.86	0.89	1.3	4.23	3.38	3.86
time (sec)	N/A	0.024	0.041	0.005	0.974	1.955	7.673	1.065

Problem 1391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	30	27	45	155	146	140
normalized size	1	1.	0.71	0.64	1.07	3.69	3.48	3.33
time (sec)	N/A	0.014	0.02	0.002	0.949	1.947	0.682	1.049

Problem 1392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	63	12	16
normalized size	1	1.	1.	0.81	1.	3.94	0.75	1.
time (sec)	N/A	0.001	0.005	0.003	0.964	1.818	0.057	1.061

Problem 1393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	77	167	0	424	82	142
normalized size	1	1.	0.9	1.94	0.	4.93	0.95	1.65
time (sec)	N/A	0.046	0.072	0.006	0.	1.892	10.78	1.066

Problem 1394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	50	148	0	455	923	153
normalized size	1	1.	0.59	1.74	0.	5.35	10.86	1.8
time (sec)	N/A	0.038	0.014	0.011	0.	1.81	83.982	1.083

Problem 1395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	90	121	0	795	0	146
normalized size	1	1.	0.9	1.21	0.	7.95	0.	1.46
time (sec)	N/A	0.048	0.098	0.011	0.	1.81	0.	1.091

Problem 1396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	52	163	0	1368	0	250
normalized size	1	1.	0.38	1.2	0.	10.06	0.	1.84
time (sec)	N/A	0.057	0.017	0.013	0.	1.856	0.	1.089

Problem 1397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	52	222	0	2101	0	385
normalized size	1	1.	0.3	1.29	0.	12.22	0.	2.24
time (sec)	N/A	0.074	0.017	0.013	0.	2.154	0.	1.128

Problem 1398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	52	300	0	3065	0	554
normalized size	1	1.	0.25	1.44	0.	14.74	0.	2.66
time (sec)	N/A	0.093	0.018	0.015	0.	2.06	0.	1.122

Problem 1399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	123	273	350	1148	1292	1469
normalized size	1	1.	0.78	1.73	2.22	7.27	8.18	9.3
time (sec)	N/A	0.051	0.111	0.005	0.962	1.861	32.865	1.142

Problem 1400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	101	186	244	855	960	1094
normalized size	1	1.	0.78	1.44	1.89	6.63	7.44	8.48
time (sec)	N/A	0.043	0.127	0.005	0.967	1.943	25.162	1.123

Problem 1401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	79	116	159	595	549	770
normalized size	1	1.	0.79	1.16	1.59	5.95	5.49	7.7
time (sec)	N/A	0.031	0.07	0.005	0.965	1.77	4.373	1.085

Problem 1402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	61	63	92	382	355	491
normalized size	1	1.	0.86	0.89	1.3	5.38	5.	6.92
time (sec)	N/A	0.023	0.055	0.006	0.953	1.847	3.215	1.09

Problem 1403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	30	27	45	205	194	263
normalized size	1	1.	0.71	0.64	1.07	4.88	4.62	6.26
time (sec)	N/A	0.015	0.022	0.002	0.95	1.757	2.178	1.071

Problem 1404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	85	12	81
normalized size	1	1.	1.	0.81	1.	5.31	0.75	5.06
time (sec)	N/A	0.001	0.006	0.002	0.952	1.73	0.059	1.073

Problem 1405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	105	263	0	644	121	231
normalized size	1	1.	0.94	2.35	0.	5.75	1.08	2.06
time (sec)	N/A	0.058	0.15	0.007	0.	1.897	19.904	1.075

Problem 1406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	50	258	0	707	1312	244
normalized size	1	1.	0.45	2.35	0.	6.43	11.93	2.22
time (sec)	N/A	0.055	0.016	0.011	0.	1.834	148.11	1.082

Problem 1407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	52	238	0	728	0	231
normalized size	1	1.	0.44	2.	0.	6.12	0.	1.94
time (sec)	N/A	0.049	0.016	0.015	0.	1.873	0.	1.286

Problem 1408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	119	204	0	1162	0	217
normalized size	1	1.	0.94	1.62	0.	9.22	0.	1.72
time (sec)	N/A	0.05	0.148	0.013	0.	1.914	0.	1.088

Problem 1409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	52	246	0	1859	0	350
normalized size	1	1.	0.32	1.52	0.	11.48	0.	2.16
time (sec)	N/A	0.07	0.018	0.014	0.	2.062	0.	1.116

Problem 1410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	52	305	0	2743	0	513
normalized size	1	1.	0.26	1.54	0.	13.85	0.	2.59
time (sec)	N/A	0.092	0.017	0.015	0.	2.051	0.	1.135

Problem 1411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	51	30	39	107	104	39
normalized size	1	1.	1.46	0.86	1.11	3.06	2.97	1.11
time (sec)	N/A	0.008	0.028	0.009	1.436	1.757	1.482	1.069

Problem 1412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	28	40	58	140	167	50
normalized size	1	1.	0.5	0.71	1.04	2.5	2.98	0.89
time (sec)	N/A	0.013	0.005	0.007	1.437	1.838	2.494	1.081

Problem 1413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	123	273	382	586	728	382
normalized size	1	1.	0.8	1.77	2.48	3.81	4.73	2.48
time (sec)	N/A	0.051	0.082	0.006	0.984	1.825	55.899	1.083

Problem 1414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	101	186	275	405	532	275
normalized size	1	1.	0.8	1.46	2.17	3.19	4.19	2.17
time (sec)	N/A	0.041	0.083	0.005	0.966	1.797	39.681	1.064

Problem 1415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	79	116	185	251	366	185
normalized size	1	1.	0.82	1.21	1.93	2.61	3.81	1.93
time (sec)	N/A	0.031	0.067	0.005	0.982	1.787	25.739	1.08

Problem 1416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	60	63	111	146	231	111
normalized size	1	1.	0.87	0.91	1.61	2.12	3.35	1.61
time (sec)	N/A	0.021	0.035	0.004	0.964	1.762	14.482	1.104

Problem 1417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	29	26	53	63	121	53
normalized size	1	1.	0.72	0.65	1.32	1.58	3.02	1.32
time (sec)	N/A	0.013	0.017	0.002	0.956	1.862	3.274	1.056

Problem 1418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	26	10	16
normalized size	1	1.	1.	0.93	1.14	1.86	0.71	1.14
time (sec)	N/A	0.001	0.003	0.002	0.946	1.774	0.054	1.046

Problem 1419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	37	0	266	44	51
normalized size	1	1.	1.	0.79	0.	5.66	0.94	1.09
time (sec)	N/A	0.02	0.013	0.004	0.	1.869	3.001	1.06

Problem 1420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	77	0	603	0	117
normalized size	1	1.	1.	1.01	0.	7.93	0.	1.54
time (sec)	N/A	0.027	0.069	0.008	0.	2.12	0.	1.076

Problem 1421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	50	115	0	1119	0	200
normalized size	1	1.	0.44	1.01	0.	9.82	0.	1.75
time (sec)	N/A	0.037	0.011	0.008	0.	2.355	0.	1.065

Problem 1422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	50	147	0	1805	0	312
normalized size	1	1.	0.34	1.	0.	12.28	0.	2.12
time (sec)	N/A	0.05	0.012	0.007	0.	2.26	0.	1.072

Problem 1423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	50	179	0	2709	0	447
normalized size	1	1.	0.28	0.99	0.	15.05	0.	2.48
time (sec)	N/A	0.065	0.012	0.006	0.	2.396	0.	1.071

Problem 1424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	123	273	360	590	243	473
normalized size	1	1.	0.81	1.8	2.37	3.88	1.6	3.11
time (sec)	N/A	0.049	0.116	0.005	0.97	2.112	30.554	1.079

Problem 1425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	101	186	255	412	168	324
normalized size	1	1.	0.82	1.51	2.07	3.35	1.37	2.63
time (sec)	N/A	0.037	0.076	0.006	0.952	2.015	20.223	1.085

Problem 1426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	78	116	169	259	109	205
normalized size	1	1.	0.83	1.23	1.8	2.76	1.16	2.18
time (sec)	N/A	0.03	0.054	0.006	0.966	2.112	13.058	1.068

Problem 1427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	59	63	101	157	65	113
normalized size	1	1.	0.88	0.94	1.51	2.34	0.97	1.69
time (sec)	N/A	0.021	0.034	0.005	0.946	2.03	7.995	1.053

Problem 1428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	26	50	74	60	46
normalized size	1	1.	0.71	0.68	1.32	1.95	1.58	1.21
time (sec)	N/A	0.014	0.019	0.003	0.947	1.98	0.577	1.067

Problem 1429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	43	12	16
normalized size	1	1.	1.	0.93	1.14	3.07	0.86	1.14
time (sec)	N/A	0.002	0.004	0.002	0.932	2.108	0.057	1.061

Problem 1430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	46	68	0	456	60	93
normalized size	1	1.	0.67	0.99	0.	6.61	0.87	1.35
time (sec)	N/A	0.027	0.01	0.009	0.	2.102	5.064	1.062

Problem 1431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	48	101	0	883	0	193
normalized size	1	1.	0.48	1.02	0.	8.92	0.	1.95
time (sec)	N/A	0.038	0.012	0.013	0.	2.127	0.	1.055

Problem 1432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	50	179	0	1582	0	316
normalized size	1	1.	0.36	1.28	0.	11.3	0.	2.26
time (sec)	N/A	0.052	0.013	0.014	0.	2.26	0.	1.083

Problem 1433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	50	292	0	2441	0	440
normalized size	1	1.	0.29	1.69	0.	14.11	0.	2.54
time (sec)	N/A	0.066	0.016	0.018	0.	2.375	0.	1.079

Problem 1434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	123	273	358	603	196	452
normalized size	1	1.	0.81	1.8	2.36	3.97	1.29	2.97
time (sec)	N/A	0.048	0.113	0.006	0.961	2.088	38.868	1.072

Problem 1435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	101	186	252	431	136	309
normalized size	1	1.	0.81	1.49	2.02	3.45	1.09	2.47
time (sec)	N/A	0.039	0.082	0.005	0.974	2.085	28.054	1.077

Problem 1436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	76	115	165	281	461	190
normalized size	1	1.	0.79	1.2	1.72	2.93	4.8	1.98
time (sec)	N/A	0.031	0.056	0.005	0.961	1.988	1.361	1.065

Problem 1437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	62	62	97	174	265	97
normalized size	1	1.	0.93	0.93	1.45	2.6	3.96	1.45
time (sec)	N/A	0.021	0.032	0.004	0.957	2.084	1.201	1.086

Problem 1438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	29	26	38	103	124	38
normalized size	1	1.	0.72	0.65	0.95	2.58	3.1	0.95
time (sec)	N/A	0.014	0.028	0.003	0.95	2.001	1.073	1.055

Problem 1439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	68	14	16
normalized size	1	1.	1.	0.81	1.	4.25	0.88	1.
time (sec)	N/A	0.001	0.007	0.002	0.96	2.046	0.061	1.081

Problem 1440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	48	90	0	841	83	153
normalized size	1	1.	0.52	0.97	0.	9.04	0.89	1.65
time (sec)	N/A	0.039	0.011	0.009	0.	2.145	6.623	1.07

Problem 1441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	50	125	0	1592	0	292
normalized size	1	1.	0.4	1.01	0.	12.84	0.	2.35
time (sec)	N/A	0.05	0.014	0.016	0.	2.296	0.	1.07

Problem 1442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	52	206	0	2472	0	402
normalized size	1	1.	0.31	1.23	0.	14.8	0.	2.41
time (sec)	N/A	0.062	0.016	0.016	0.	2.421	0.	1.084

Problem 1443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	52	319	0	3717	0	583
normalized size	1	1.	0.26	1.6	0.	18.58	0.	2.92
time (sec)	N/A	0.134	0.021	0.019	0.	2.559	0.	1.067

Problem 1444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	25	23	24	205	66	668
normalized size	1	1.	1.14	1.05	1.09	9.32	3.	30.36
time (sec)	N/A	0.005	0.02	0.002	0.956	2.104	1.1	1.092

Problem 1445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	25	23	24	161	66	505
normalized size	1	1.	1.14	1.05	1.09	7.32	3.	22.95
time (sec)	N/A	0.005	0.011	0.003	0.958	1.936	0.877	1.095

Problem 1446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	25	23	505	143	73	505
normalized size	1	1.	1.14	1.05	22.95	6.5	3.32	22.95
time (sec)	N/A	0.004	0.013	0.001	0.965	2.026	1.296	1.068

Problem 1447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	25	23	24	120	73	359
normalized size	1	1.	1.14	1.05	1.09	5.45	3.32	16.32
time (sec)	N/A	0.004	0.015	0.001	0.977	2.079	1.344	1.061

Problem 1448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	25	23	24	99	73	240
normalized size	1	1.	1.14	1.05	1.09	4.5	3.32	10.91
time (sec)	N/A	0.004	0.013	0.002	0.945	2.114	1.387	1.077

Problem 1449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	25	23	24	77	80	143
normalized size	1	1.	1.14	1.05	1.09	3.5	3.64	6.5
time (sec)	N/A	0.004	0.013	0.002	0.98	2.002	3.49	1.076

Problem 1450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	26	23	24	55	53	73
normalized size	1	1.	1.18	1.05	1.09	2.5	2.41	3.32
time (sec)	N/A	0.004	0.013	0.001	0.967	1.965	7.325	1.072

Problem 1451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	24	23	24	39	29	24
normalized size	1	1.	1.2	1.15	1.2	1.95	1.45	1.2
time (sec)	N/A	0.004	0.007	0.002	0.964	2.028	14.415	1.054

Problem 1452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	24	23	24	59	48	24
normalized size	1	1.	1.2	1.15	1.2	2.95	2.4	1.2
time (sec)	N/A	0.005	0.009	0.001	0.956	1.971	39.472	1.055

Problem 1453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	22	28	73	27	30
normalized size	1	1.	1.	1.57	2.	5.21	1.93	2.14
time (sec)	N/A	0.004	0.003	0.006	0.941	2.023	0.494	1.047

Problem 1454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	19	24	68	61	24
normalized size	1	1.	1.	0.76	0.96	2.72	2.44	0.96
time (sec)	N/A	0.009	0.009	0.005	1.443	2.06	0.991	1.075

Problem 1455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	104	84	116	275	170	0
normalized size	1	1.	1.24	1.	1.38	3.27	2.02	0.
time (sec)	N/A	0.038	0.047	0.005	1.462	2.19	1.924	0.

Problem 1456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	26	63	114	35
normalized size	1	1.	0.67	0.56	0.96	2.33	4.22	1.3
time (sec)	N/A	0.005	0.008	0.002	0.954	1.991	0.888	1.055

Problem 1457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	38	68	146	51
normalized size	1	1.	0.61	0.53	1.	1.79	3.84	1.34
time (sec)	N/A	0.007	0.009	0.003	0.954	2.05	1.309	1.058

Problem 1458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	106	161	0	1289	0	265
normalized size	1	1.	0.76	1.16	0.	9.27	0.	1.91
time (sec)	N/A	0.108	0.073	0.008	0.	2.283	0.	1.115

Problem 1459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	154	160	0	1995	0	279
normalized size	1	1.	1.1	1.14	0.	14.25	0.	1.99
time (sec)	N/A	0.074	0.07	0.005	0.	2.427	0.	1.123

Problem 1460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	194	858	0	1573	0	1335
normalized size	1	1.	0.84	3.73	0.	6.84	0.	5.8
time (sec)	N/A	0.16	1.52	0.008	0.	2.604	0.	1.384

Problem 1461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	190	645	0	1203	0	842
normalized size	1	1.	0.99	3.36	0.	6.27	0.	4.39
time (sec)	N/A	0.095	0.571	0.006	0.	2.367	0.	1.351

Problem 1462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	151	460	0	919	0	456
normalized size	1	1.	0.98	2.99	0.	5.97	0.	2.96
time (sec)	N/A	0.072	0.419	0.006	0.	2.361	0.	1.233

Problem 1463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	118	305	0	699	0	189
normalized size	1	1.	1.02	2.63	0.	6.03	0.	1.63
time (sec)	N/A	0.055	0.269	0.004	0.	2.278	0.	1.111

Problem 1464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	117	107	0	558	0	126
normalized size	1	1.	1.62	1.49	0.	7.75	0.	1.75
time (sec)	N/A	0.038	0.1	0.004	0.	2.211	0.	1.098

Problem 1465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	99	0	0	551	0	177
normalized size	1	1.	1.5	0.	0.	8.35	0.	2.68
time (sec)	N/A	0.033	0.218	0.033	0.	2.61	0.	1.127

Problem 1466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	140	0	205
normalized size	1	1.	1.	0.84	0.	4.38	0.	6.41
time (sec)	N/A	0.003	0.011	0.002	0.	2.344	0.	1.137

Problem 1467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	365	0	603
normalized size	1	1.	0.7	0.82	0.	5.53	0.	9.14
time (sec)	N/A	0.009	0.02	0.005	0.	4.313	0.	1.223

Problem 1468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	682	0	930
normalized size	1	1.	0.76	1.04	0.	6.75	0.	9.21
time (sec)	N/A	0.016	0.039	0.006	0.	10.185	0.	1.285

Problem 1469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	171	0	1080	0	1335
normalized size	1	1.	0.87	1.26	0.	7.94	0.	9.82
time (sec)	N/A	0.029	0.053	0.009	0.	42.659	0.	1.416

Problem 1470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	170	256	0	1613	0	1816
normalized size	1	1.	0.99	1.5	0.	9.43	0.	10.62
time (sec)	N/A	0.041	0.078	0.009	0.	90.364	0.	1.59

Problem 1471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	187	853	0	1547	0	1987
normalized size	1	1.	0.82	3.76	0.	6.81	0.	8.75
time (sec)	N/A	0.13	1.756	0.007	0.	2.189	0.	1.432

Problem 1472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	193	640	0	1177	0	1107
normalized size	1	1.	1.02	3.39	0.	6.23	0.	5.86
time (sec)	N/A	0.09	0.536	0.004	0.	2.175	0.	1.284

Problem 1473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	152	459	0	918	0	458
normalized size	1	1.	1.01	3.04	0.	6.08	0.	3.03
time (sec)	N/A	0.068	0.428	0.006	0.	2.016	0.	1.177

Problem 1474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	109	308	0	711	0	327
normalized size	1	1.	0.96	2.73	0.	6.29	0.	2.89
time (sec)	N/A	0.051	0.289	0.007	0.	1.915	0.	1.15

Problem 1475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	71	0	0	703	0	275
normalized size	1	1.	0.72	0.	0.	7.17	0.	2.81
time (sec)	N/A	0.047	0.052	0.034	0.	2.676	0.	1.219

Problem 1476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	73	0	0	734	0	614
normalized size	1	1.	0.79	0.	0.	7.98	0.	6.67
time (sec)	N/A	0.04	0.045	0.033	0.	3.581	0.	1.306

Problem 1477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	216	0	505
normalized size	1	1.	1.	0.84	0.	6.75	0.	15.78
time (sec)	N/A	0.003	0.014	0.004	0.	4.337	0.	1.532

Problem 1478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	483	0	1382
normalized size	1	1.	0.7	0.82	0.	7.32	0.	20.94
time (sec)	N/A	0.008	0.025	0.005	0.	11.806	0.	1.536

Problem 1479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	860	0	1882
normalized size	1	1.	0.76	1.04	0.	8.51	0.	18.63
time (sec)	N/A	0.016	0.057	0.006	0.	48.118	0.	1.745

Problem 1480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	171	0	1328	0	2461
normalized size	1	1.	0.87	1.26	0.	9.76	0.	18.1
time (sec)	N/A	0.028	0.063	0.009	0.	100.004	0.	2.096

Problem 1481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	209	1089	0	1924	0	3542
normalized size	1	1.	0.8	4.16	0.	7.34	0.	13.52
time (sec)	N/A	0.149	2.529	0.006	0.	2.502	0.	1.654

Problem 1482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	187	848	0	1548	0	2002
normalized size	1	1.	0.83	3.79	0.	6.91	0.	8.94
time (sec)	N/A	0.119	1.575	0.005	0.	2.209	0.	1.461

Problem 1483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	191	641	0	1203	0	852
normalized size	1	1.	1.03	3.45	0.	6.47	0.	4.58
time (sec)	N/A	0.086	0.56	0.005	0.	2.077	0.	1.346

Problem 1484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	139	465	0	933	0	614
normalized size	1	1.	0.94	3.14	0.	6.3	0.	4.15
time (sec)	N/A	0.066	0.406	0.006	0.	1.969	0.	1.327

Problem 1485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	71	0	0	971	0	387
normalized size	1	1.	0.51	0.	0.	7.04	0.	2.8
time (sec)	N/A	0.063	0.065	0.035	0.	3.273	0.	1.316

Problem 1486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	73	0	0	1034	0	878
normalized size	1	1.	0.57	0.	0.	8.08	0.	6.86
time (sec)	N/A	0.062	0.066	0.033	0.	4.811	0.	1.47

Problem 1487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	73	0	0	1015	0	1384
normalized size	1	1.	0.61	0.	0.	8.46	0.	11.53
time (sec)	N/A	0.052	0.069	0.035	0.	7.973	0.	1.716

Problem 1488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	281	0	953
normalized size	1	1.	1.	0.84	0.	8.78	0.	29.78
time (sec)	N/A	0.003	0.02	0.004	0.	10.883	0.	1.937

Problem 1489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	605	0	2465
normalized size	1	1.	0.7	0.82	0.	9.17	0.	37.35
time (sec)	N/A	0.009	0.031	0.005	0.	44.933	0.	2.155

Problem 1490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	1045	0	3127
normalized size	1	1.	0.76	1.04	0.	10.35	0.	30.96
time (sec)	N/A	0.016	0.061	0.007	0.	112.612	0.	2.653

Problem 1491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	171	0	0	0	3872
normalized size	1	1.	0.87	1.26	0.	0.	0.	28.47
time (sec)	N/A	0.028	0.079	0.008	0.	0.	0.	3.246

Problem 1492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	189	650	0	1226	0	362
normalized size	1	1.	1.03	3.55	0.	6.7	0.	1.98
time (sec)	N/A	0.097	0.651	0.005	0.	2.965	0.	1.122

Problem 1493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	150	465	0	932	0	267
normalized size	1	1.	1.01	3.14	0.	6.3	0.	1.8
time (sec)	N/A	0.072	0.498	0.007	0.	2.523	0.	1.136

Problem 1494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	119	308	0	711	0	188
normalized size	1	1.	1.05	2.73	0.	6.29	0.	1.66
time (sec)	N/A	0.05	0.363	0.004	0.	2.506	0.	1.097

Problem 1495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	103	107	0	558	0	131
normalized size	1	1.	1.41	1.47	0.	7.64	0.	1.79
time (sec)	N/A	0.035	0.256	0.005	0.	2.599	0.	1.165

Problem 1496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	77	76	0	413	0	68
normalized size	1	1.	1.83	1.81	0.	9.83	0.	1.62
time (sec)	N/A	0.026	0.061	0.003	0.	2.368	0.	1.094

Problem 1497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	92	0	89
normalized size	1	1.	1.	0.9	0.	3.07	0.	2.97
time (sec)	N/A	0.003	0.008	0.004	0.	2.293	0.	1.071

Problem 1498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	250	0	163
normalized size	1	1.	0.7	0.82	0.	3.79	0.	2.47
time (sec)	N/A	0.008	0.015	0.005	0.	3.044	0.	1.099

Problem 1499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	105	0	509	0	306
normalized size	1	1.	0.74	1.04	0.	5.04	0.	3.03
time (sec)	N/A	0.016	0.03	0.005	0.	5.724	0.	1.141

Problem 1500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	116	171	0	848	0	521
normalized size	1	1.	0.85	1.26	0.	6.24	0.	3.83
time (sec)	N/A	0.028	0.046	0.008	0.	12.349	0.	1.184

Problem 1501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	168	256	0	1301	0	805
normalized size	1	1.	0.98	1.5	0.	7.61	0.	4.71
time (sec)	N/A	0.041	0.063	0.009	0.	48.815	0.	1.266

Problem 1502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	73	0	0	1312	0	423
normalized size	1	1.	0.42	0.	0.	7.54	0.	2.43
time (sec)	N/A	0.089	0.067	0.033	0.	7.127	0.	1.167

Problem 1503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	73	0	0	971	0	302
normalized size	1	1.	0.53	0.	0.	7.04	0.	2.19
time (sec)	N/A	0.067	0.055	0.033	0.	4.456	0.	1.177

Problem 1504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	73	0	0	702	0	207
normalized size	1	1.	0.74	0.	0.	7.16	0.	2.11
time (sec)	N/A	0.047	0.048	0.034	0.	3.501	0.	1.138

Problem 1505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	95	0	0	551	0	130
normalized size	1	1.	1.44	0.	0.	8.35	0.	1.97
time (sec)	N/A	0.032	0.334	0.033	0.	3.573	0.	1.135

Problem 1506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	90	0	63
normalized size	1	1.	1.	0.9	0.	3.	0.	2.1
time (sec)	N/A	0.003	0.007	0.003	0.	2.872	0.	1.073

Problem 1507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	42	52	0	259	0	192
normalized size	1	1.	0.68	0.84	0.	4.18	0.	3.1
time (sec)	N/A	0.009	0.018	0.004	0.	2.993	0.	1.135

Problem 1508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	105	0	544	0	497
normalized size	1	1.	0.74	1.04	0.	5.39	0.	4.92
time (sec)	N/A	0.018	0.029	0.006	0.	4.707	0.	1.323

Problem 1509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	114	170	0	915	0	1121
normalized size	1	1.	0.84	1.25	0.	6.73	0.	8.24
time (sec)	N/A	0.027	0.042	0.009	0.	9.545	0.	1.828

Problem 1510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	166	256	0	1400	0	2049
normalized size	1	1.	0.97	1.5	0.	8.19	0.	11.98
time (sec)	N/A	0.043	0.058	0.009	0.	28.402	0.	2.838

Problem 1511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	226	356	0	1960	0	3291
normalized size	1	1.	1.1	1.73	0.	9.51	0.	15.98
time (sec)	N/A	0.057	0.078	0.011	0.	88.242	0.	4.5

Problem 1512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	73	0	0	1874	0	675
normalized size	1	1.	0.36	0.	0.	9.19	0.	3.31
time (sec)	N/A	0.109	0.097	0.035	0.	16.191	0.	1.25

Problem 1513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	73	0	0	1423	0	513
normalized size	1	1.	0.43	0.	0.	8.37	0.	3.02
time (sec)	N/A	0.079	0.079	0.033	0.	9.688	0.	1.227

Problem 1514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	73	0	0	1033	0	373
normalized size	1	1.	0.57	0.	0.	8.07	0.	2.91
time (sec)	N/A	0.058	0.071	0.035	0.	6.472	0.	1.189

Problem 1515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	111	0	0	734	0	296
normalized size	1	1.	1.21	0.	0.	7.98	0.	3.22
time (sec)	N/A	0.04	0.522	0.034	0.	5.149	0.	1.151

Problem 1516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	139	0	90
normalized size	1	1.	1.	0.84	0.	4.34	0.	2.81
time (sec)	N/A	0.003	0.009	0.003	0.	3.196	0.	1.148

Problem 1517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	53	0	250	0	173
normalized size	1	1.	0.7	0.8	0.	3.79	0.	2.62
time (sec)	N/A	0.008	0.015	0.006	0.	3.659	0.	1.084

Problem 1518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	78	104	0	545	0	393
normalized size	1	1.	0.8	1.06	0.	5.56	0.	4.01
time (sec)	N/A	0.017	0.032	0.006	0.	6.791	0.	1.193

Problem 1519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	118	169	0	886	0	718
normalized size	1	1.	0.87	1.25	0.	6.56	0.	5.32
time (sec)	N/A	0.028	0.051	0.007	0.	11.761	0.	1.771

Problem 1520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	170	256	0	1440	0	1362
normalized size	1	1.	0.99	1.49	0.	8.37	0.	7.92
time (sec)	N/A	0.044	0.062	0.01	0.	43.612	0.	3.226

Problem 1521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	233	356	0	2045	0	2314
normalized size	1	1.	1.13	1.72	0.	9.88	0.	11.18
time (sec)	N/A	0.061	0.079	0.011	0.	97.443	0.	6.447

Problem 1522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	86	0	76	0	34
normalized size	1	1.	1.	4.53	0.	4.	0.	1.79
time (sec)	N/A	0.006	0.009	0.008	0.	2.104	0.	1.246

Problem 1523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	39	66	0	65	0	32
normalized size	1	1.	2.05	3.47	0.	3.42	0.	1.68
time (sec)	N/A	0.005	0.011	0.006	0.	2.058	0.	1.143

Problem 1524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	39	66	0	65	0	32
normalized size	1	1.	2.05	3.47	0.	3.42	0.	1.68
time (sec)	N/A	0.005	0.011	0.007	0.	2.133	0.	1.144

Problem 1525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	34	60	0	59	15	30
normalized size	1	1.	2.	3.53	0.	3.47	0.88	1.76
time (sec)	N/A	0.004	0.01	0.006	0.	2.115	1.366	1.159

Problem 1526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	39	64	0	65	0	32
normalized size	1	1.	2.05	3.37	0.	3.42	0.	1.68
time (sec)	N/A	0.005	0.012	0.006	0.	1.955	0.	1.142

Problem 1527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	57	43	59	75	32
normalized size	1	1.	2.27	5.18	3.91	5.36	6.82	2.91
time (sec)	N/A	0.003	0.004	0.004	0.957	2.002	3.148	1.11

Problem 1528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	39	66	0	65	0	32
normalized size	1	1.	2.05	3.47	0.	3.42	0.	1.68
time (sec)	N/A	0.005	0.01	0.007	0.	1.98	0.	1.154

Problem 1529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	66	0	70	0	32
normalized size	1	1.	1.	4.4	0.	4.67	0.	2.13
time (sec)	N/A	0.004	0.004	0.006	0.	2.025	0.	1.136

Problem 1530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	22	7	15
normalized size	1	1.	1.	1.1	1.4	2.2	0.7	1.5
time (sec)	N/A	0.001	0.001	0.001	0.948	1.903	0.058	1.061

Problem 1531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	66	0	70	0	32
normalized size	1	1.	1.	4.4	0.	4.67	0.	2.13
time (sec)	N/A	0.004	0.004	0.005	0.	2.032	0.	1.129

Problem 1532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	36	58	0	59	20	30
normalized size	1	1.	1.89	3.05	0.	3.11	1.05	1.58
time (sec)	N/A	0.005	0.008	0.005	0.	1.964	1.421	1.139

Problem 1533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	41	65	0	70	0	32
normalized size	1	1.	1.95	3.1	0.	3.33	0.	1.52
time (sec)	N/A	0.006	0.009	0.006	0.	1.951	0.	1.152

Problem 1534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	57	43	59	75	32
normalized size	1	1.	2.27	5.18	3.91	5.36	6.82	2.91
time (sec)	N/A	0.002	0.002	0.	0.982	2.048	3.182	1.103

Problem 1535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	41	66	0	70	0	32
normalized size	1	1.	1.95	3.14	0.	3.33	0.	1.52
time (sec)	N/A	0.006	0.01	0.007	0.	1.989	0.	1.148

Problem 1536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	22	65	0	104	0	24
normalized size	1	1.	1.38	4.06	0.	6.5	0.	1.5
time (sec)	N/A	0.014	0.011	0.008	0.	2.014	0.	1.079

Problem 1537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	56	24	74	76	20
normalized size	1	1.	1.	5.09	2.18	6.73	6.91	1.82
time (sec)	N/A	0.003	0.008	0.007	1.429	2.03	3.182	1.073

Problem 1538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	22	66	0	104	0	24
normalized size	1	1.	1.38	4.12	0.	6.5	0.	1.5
time (sec)	N/A	0.013	0.012	0.007	0.	2.011	0.	1.076

Problem 1539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	51	58	0	61	24	24
normalized size	1	1.	5.1	5.8	0.	6.1	2.4	2.4
time (sec)	N/A	0.01	0.012	0.005	0.	2.001	1.417	1.061

Problem 1540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	49	66	0	107	0	18
normalized size	1	1.	4.45	6.	0.	9.73	0.	1.64
time (sec)	N/A	0.01	0.014	0.006	0.	2.055	0.	1.065

Problem 1541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	28	26	22	4	53	16
normalized size	1	1.	0.97	0.9	0.76	0.14	1.83	0.55
time (sec)	N/A	0.004	0.008	0.003	0.928	2.048	1.877	1.055

Problem 1542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	53	66	0	107	0	20
normalized size	1	1.	2.04	2.54	0.	4.12	0.	0.77
time (sec)	N/A	0.014	0.015	0.006	0.	2.041	0.	1.063

Problem 1543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	70	0	73	0	35
normalized size	1	1.	1.	4.38	0.	4.56	0.	2.19
time (sec)	N/A	0.005	0.007	0.005	0.	2.021	0.	1.146

Problem 1544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	15	23	8	16
normalized size	1	1.	1.	1.08	1.25	1.92	0.67	1.33
time (sec)	N/A	0.001	0.001	0.001	0.955	1.935	0.061	1.054

Problem 1545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	70	0	73	0	35
normalized size	1	1.	1.	4.38	0.	4.56	0.	2.19
time (sec)	N/A	0.005	0.007	0.007	0.	1.946	0.	1.144

Problem 1546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	37	64	0	62	53	32
normalized size	1	1.	1.85	3.2	0.	3.1	2.65	1.6
time (sec)	N/A	0.005	0.009	0.004	0.	1.991	1.488	1.128

Problem 1547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	40	70	0	73	0	35
normalized size	1	1.	1.82	3.18	0.	3.32	0.	1.59
time (sec)	N/A	0.007	0.01	0.006	0.	1.985	0.	1.151

Problem 1548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	27	61	43	62	78	35
normalized size	1	1.	2.25	5.08	3.58	5.17	6.5	2.92
time (sec)	N/A	0.003	0.005	0.005	0.952	2.015	3.251	1.105

Problem 1549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	40	69	0	73	0	35
normalized size	1	1.	1.82	3.14	0.	3.32	0.	1.59
time (sec)	N/A	0.007	0.01	0.005	0.	1.99	0.	1.152

Problem 1550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	57	43	59	75	32
normalized size	1	1.	2.27	5.18	3.91	5.36	6.82	2.91
time (sec)	N/A	0.002	0.005	0.005	0.95	1.988	3.211	1.103

Problem 1551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	41	100	0	405	0	84
normalized size	1	1.	0.95	2.33	0.	9.42	0.	1.95
time (sec)	N/A	0.015	0.025	0.014	0.	2.11	0.	1.063

Problem 1552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	31	48	55	82	44	31
normalized size	1	1.	1.41	2.18	2.5	3.73	2.	1.41
time (sec)	N/A	0.004	0.008	0.005	1.465	1.987	1.054	1.069

Problem 1553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	57	38	146	58	0
normalized size	1	1.	1.	2.19	1.46	5.62	2.23	0.
time (sec)	N/A	0.008	0.008	0.005	1.442	1.987	1.099	0.

Problem 1554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	67	118	0	409	0	90
normalized size	1	1.	1.6	2.81	0.	9.74	0.	2.14
time (sec)	N/A	0.016	0.05	0.023	0.	2.116	0.	1.076

Problem 1555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	14	27	19	45	26	11
normalized size	1	1.	1.4	2.7	1.9	4.5	2.6	1.1
time (sec)	N/A	0.007	0.01	0.004	1.427	2.025	1.009	1.07

Problem 1556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	31	28	72	44	18
normalized size	1	1.	1.	1.55	1.4	3.6	2.2	0.9
time (sec)	N/A	0.007	0.004	0.005	1.434	1.989	1.022	1.063

Problem 1557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	39	15	140	58	28
normalized size	1	1.	1.04	1.5	0.58	5.38	2.23	1.08
time (sec)	N/A	0.009	0.009	0.007	1.426	1.99	1.097	1.062

Problem 1558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	103	84	0	417	0	73
normalized size	1	1.	2.4	1.95	0.	9.7	0.	1.7
time (sec)	N/A	0.028	0.079	0.007	0.	2.122	0.	1.083

Problem 1559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	73	0	0	0	0	0
normalized size	1	1.	0.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.627	0.038	0.03	0.	0.	0.	0.

Problem 1560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	73	0	0	0	0	0
normalized size	1	1.	0.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.374	0.026	0.017	0.	0.	0.	0.

Problem 1561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	71	0	0	0	0	0
normalized size	1	1.	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.272	0.022	0.02	0.	0.	0.	0.

Problem 1562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	71	0	0	0	0	0
normalized size	1	1.	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.262	0.023	0.039	0.	0.	0.	0.

Problem 1563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	73	0	0	0	0	0
normalized size	1	1.	0.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.366	0.023	0.041	0.	0.	0.	0.

Problem 1564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	73	0	0	0	0	0
normalized size	1	1.	0.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.447	0.026	0.041	0.	0.	0.	0.

Problem 1565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	839	839	73	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.938	0.028	0.029	0.	0.	0.	0.

Problem 1566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	804	804	73	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.668	0.025	0.015	0.	0.	0.	0.

Problem 1567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	762	762	71	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.578	0.02	0.039	0.	0.	0.	0.

Problem 1568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	796	796	71	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.69	0.023	0.043	0.	0.	0.	0.

Problem 1569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	842	842	73	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.834	0.022	0.044	0.	0.	0.	0.

Problem 1570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	73	0	0	0	0	0
normalized size	1	1.	0.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.385	0.036	0.021	0.	0.	0.	0.

Problem 1571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	73	0	0	0	0	0
normalized size	1	1.	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.296	0.025	0.016	0.	0.	0.	0.

Problem 1572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	71	0	0	0	0	0
normalized size	1	1.	0.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.223	0.022	0.039	0.	0.	0.	0.

Problem 1573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	71	0	0	0	0	0
normalized size	1	1.	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.294	0.022	0.04	0.	0.	0.	0.

Problem 1574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	421	73	0	0	0	0	0
normalized size	1	1.	0.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.368	0.022	0.043	0.	0.	0.	0.

Problem 1575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	73	0	0	1800	0	0
normalized size	1	1.	0.33	0.	0.	8.22	0.	0.
time (sec)	N/A	0.088	0.031	0.024	0.	2.069	0.	0.

Problem 1576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	73	0	0	1547	0	0
normalized size	1	1.	0.42	0.	0.	8.99	0.	0.
time (sec)	N/A	0.046	0.024	0.015	0.	2.009	0.	0.

Problem 1577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	71	0	0	603	0	0
normalized size	1	1.	0.48	0.	0.	4.05	0.	0.
time (sec)	N/A	0.033	0.027	0.04	0.	1.82	0.	0.

Problem 1578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	143	0	0
normalized size	1	1.	1.	0.84	0.	4.47	0.	0.
time (sec)	N/A	0.003	0.012	0.004	0.	1.724	0.	0.

Problem 1579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	370	0	0
normalized size	1	1.	0.7	0.82	0.	5.61	0.	0.
time (sec)	N/A	0.009	0.023	0.005	0.	1.72	0.	0.

Problem 1580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	689	0	0
normalized size	1	1.	0.76	1.04	0.	6.82	0.	0.
time (sec)	N/A	0.017	0.04	0.005	0.	1.752	0.	0.

Problem 1581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	171	0	1098	0	0
normalized size	1	1.	0.87	1.26	0.	8.07	0.	0.
time (sec)	N/A	0.028	0.059	0.009	0.	1.774	0.	0.

Problem 1582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	655	655	73	0	0	0	0	0
normalized size	1	1.	0.11	0.	0.	0.	0.	0.
time (sec)	N/A	1.416	0.03	0.024	0.	0.	0.	0.

Problem 1583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	617	617	73	0	0	0	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.849	0.029	0.014	0.	0.	0.	0.

Problem 1584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	576	576	71	0	0	0	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.563	0.024	0.015	0.	0.	0.	0.

Problem 1585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	568	568	73	0	0	0	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.594	0.024	0.037	0.	0.	0.	0.

Problem 1586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	617	617	73	0	0	0	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.843	0.025	0.038	0.	0.	0.	0.

Problem 1587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	73	0	0	1835	0	0
normalized size	1	1.	0.34	0.	0.	8.5	0.	0.
time (sec)	N/A	0.09	0.03	0.025	0.	2.002	0.	0.

Problem 1588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	73	0	0	1571	0	0
normalized size	1	1.	0.43	0.	0.	9.19	0.	0.
time (sec)	N/A	0.041	0.026	0.012	0.	1.992	0.	0.

Problem 1589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	71	0	0	1334	0	0
normalized size	1	1.	0.56	0.	0.	10.59	0.	0.
time (sec)	N/A	0.014	0.027	0.037	0.	1.918	0.	0.

Problem 1590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	100	0	0
normalized size	1	1.	1.	0.84	0.	3.12	0.	0.
time (sec)	N/A	0.003	0.012	0.005	0.	1.763	0.	0.

Problem 1591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	259	0	0
normalized size	1	1.	0.7	0.82	0.	3.92	0.	0.
time (sec)	N/A	0.009	0.017	0.006	0.	1.871	0.	0.

Problem 1592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	514	0	0
normalized size	1	1.	0.76	1.04	0.	5.09	0.	0.
time (sec)	N/A	0.018	0.034	0.005	0.	2.099	0.	0.

Problem 1593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	171	0	865	0	0
normalized size	1	1.	0.87	1.26	0.	6.36	0.	0.
time (sec)	N/A	0.028	0.047	0.007	0.	2.747	0.	0.

Problem 1594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1365	1365	73	0	0	0	0	0
normalized size	1	1.	0.05	0.	0.	0.	0.	0.
time (sec)	N/A	2.803	0.032	0.036	0.	0.	0.	0.

Problem 1595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1330	1330	73	0	0	0	0	0
normalized size	1	1.	0.05	0.	0.	0.	0.	0.
time (sec)	N/A	2.01	0.029	0.033	0.	0.	0.	0.

Problem 1596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1293	1293	73	0	0	0	0	0
normalized size	1	1.	0.06	0.	0.	0.	0.	0.
time (sec)	N/A	1.563	0.028	0.03	0.	0.	0.	0.

Problem 1597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1257	1257	73	0	0	0	0	0
normalized size	1	1.	0.06	0.	0.	0.	0.	0.
time (sec)	N/A	1.179	0.023	0.034	0.	0.	0.	0.

Problem 1598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1297	1297	71	0	0	0	0	0
normalized size	1	1.	0.05	0.	0.	0.	0.	0.
time (sec)	N/A	1.586	0.023	0.038	0.	0.	0.	0.

Problem 1599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1335	1335	73	0	0	0	0	0
normalized size	1	1.	0.05	0.	0.	0.	0.	0.
time (sec)	N/A	2.012	0.022	0.039	0.	0.	0.	0.

Problem 1600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1372	1372	73	0	0	0	0	0
normalized size	1	1.	0.05	0.	0.	0.	0.	0.
time (sec)	N/A	2.442	0.025	0.04	0.	0.	0.	0.

Problem 1601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	73	0	0	1835	0	0
normalized size	1	1.	0.34	0.	0.	8.5	0.	0.
time (sec)	N/A	0.08	0.035	0.018	0.	2.093	0.	0.

Problem 1602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	73	0	0	1567	0	0
normalized size	1	1.	0.43	0.	0.	9.27	0.	0.
time (sec)	N/A	0.041	0.026	0.013	0.	2.261	0.	0.

Problem 1603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	73	0	0	1335	0	0
normalized size	1	1.	0.58	0.	0.	10.6	0.	0.
time (sec)	N/A	0.015	0.028	0.037	0.	2.394	0.	0.

Problem 1604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	97	0	0
normalized size	1	1.	1.	0.9	0.	3.23	0.	0.
time (sec)	N/A	0.003	0.009	0.004	0.	2.478	0.	0.

Problem 1605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	255	0	0
normalized size	1	1.	0.7	0.82	0.	3.86	0.	0.
time (sec)	N/A	0.01	0.017	0.003	0.	2.363	0.	0.

Problem 1606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	105	0	513	0	0
normalized size	1	1.	0.74	1.04	0.	5.08	0.	0.
time (sec)	N/A	0.017	0.031	0.006	0.	2.443	0.	0.

Problem 1607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	116	171	0	861	0	0
normalized size	1	1.	0.85	1.26	0.	6.33	0.	0.
time (sec)	N/A	0.03	0.048	0.005	0.	2.239	0.	0.

Problem 1608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	649	649	73	0	0	0	0	0
normalized size	1	1.	0.11	0.	0.	0.	0.	0.
time (sec)	N/A	1.118	0.031	0.018	0.	0.	0.	0.

Problem 1609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	614	73	0	0	0	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.817	0.029	0.018	0.	0.	0.	0.

Problem 1610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	577	577	73	0	0	0	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.547	0.026	0.015	0.	0.	0.	0.

Problem 1611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	542	71	0	0	0	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.43	0.023	0.036	0.	0.	0.	0.

Problem 1612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	586	586	73	0	0	0	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.55	0.024	0.041	0.	0.	0.	0.

Problem 1613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	621	621	73	0	0	0	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.812	0.023	0.04	0.	0.	0.	0.

Problem 1614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	656	656	73	0	0	0	0	0
normalized size	1	1.	0.11	0.	0.	0.	0.	0.
time (sec)	N/A	1.09	0.023	0.044	0.	0.	0.	0.

Problem 1615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	73	0	0	995	0	0
normalized size	1	1.	0.3	0.	0.	4.13	0.	0.
time (sec)	N/A	0.107	0.058	0.049	0.	2.453	0.	0.

Problem 1616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	73	0	0	745	0	0
normalized size	1	1.	0.37	0.	0.	3.82	0.	0.
time (sec)	N/A	0.071	0.048	0.042	0.	2.394	0.	0.

Problem 1617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	73	0	0	603	0	0
normalized size	1	1.	0.49	0.	0.	4.05	0.	0.
time (sec)	N/A	0.031	0.044	0.036	0.	2.309	0.	0.

Problem 1618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	96	0	0
normalized size	1	1.	1.	0.9	0.	3.2	0.	0.
time (sec)	N/A	0.003	0.008	0.004	0.	2.295	0.	0.

Problem 1619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	45	53	0	270	0	0
normalized size	1	1.	0.68	0.8	0.	4.09	0.	0.
time (sec)	N/A	0.009	0.016	0.004	0.	2.318	0.	0.

Problem 1620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	105	0	549	0	0
normalized size	1	1.	0.74	1.04	0.	5.44	0.	0.
time (sec)	N/A	0.018	0.029	0.004	0.	2.371	0.	0.

Problem 1621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	116	171	0	927	0	0
normalized size	1	1.	0.85	1.26	0.	6.82	0.	0.
time (sec)	N/A	0.03	0.047	0.008	0.	2.958	0.	0.

Problem 1622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1355	1355	73	0	0	0	0	0
normalized size	1	1.	0.05	0.	0.	0.	0.	0.
time (sec)	N/A	2.511	0.058	0.037	0.	0.	0.	0.

Problem 1623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1317	1317	73	0	0	0	0	0
normalized size	1	1.	0.06	0.	0.	0.	0.	0.
time (sec)	N/A	1.998	0.048	0.033	0.	0.	0.	0.

Problem 1624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1279	1279	73	0	0	0	0	0
normalized size	1	1.	0.06	0.	0.	0.	0.	0.
time (sec)	N/A	1.541	0.041	0.03	0.	0.	0.	0.

Problem 1625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1298	1298	73	0	0	0	0	0
normalized size	1	1.	0.06	0.	0.	0.	0.	0.
time (sec)	N/A	1.524	0.035	0.037	0.	0.	0.	0.

Problem 1626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1327	1327	71	0	0	0	0	0
normalized size	1	1.	0.05	0.	0.	0.	0.	0.
time (sec)	N/A	1.958	0.035	0.038	0.	0.	0.	0.

Problem 1627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1370	1370	73	0	0	0	0	0
normalized size	1	1.	0.05	0.	0.	0.	0.	0.
time (sec)	N/A	2.418	0.038	0.037	0.	0.	0.	0.

Problem 1628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	48	0	0	359	39	0
normalized size	1	1.	0.62	0.	0.	4.66	0.51	0.
time (sec)	N/A	0.014	0.019	0.017	0.	2.04	2.483	0.

Problem 1629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	73	0	0	0	0	0
normalized size	1	1.	0.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.036	0.028	0.	0.	0.	0.

Problem 1630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	73	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.026	0.015	0.	0.	0.	0.

Problem 1631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	71	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.023	0.016	0.	0.	0.	0.

Problem 1632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	71	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.023	0.038	0.	0.	0.	0.

Problem 1633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	73	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.024	0.04	0.	0.	0.	0.

Problem 1634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	73	0	0	0	0	0
normalized size	1	1.	0.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.024	0.039	0.	0.	0.	0.

Problem 1635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	73	0	0	0	0	0
normalized size	1	1.	0.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.365	0.036	0.026	0.	0.	0.	0.

Problem 1636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	73	0	0	0	0	0
normalized size	1	1.	0.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.246	0.027	0.016	0.	0.	0.	0.

Problem 1637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	71	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.222	0.023	0.018	0.	0.	0.	0.

Problem 1638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	71	0	0	0	0	0
normalized size	1	1.	0.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.215	0.022	0.038	0.	0.	0.	0.

Problem 1639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	73	0	0	0	0	0
normalized size	1	1.	0.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.236	0.024	0.037	0.	0.	0.	0.

Problem 1640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	73	0	0	0	0	0
normalized size	1	1.	0.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.285	0.027	0.041	0.	0.	0.	0.

Problem 1641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	73	0	0	0	0	0
normalized size	1	1.	0.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	0.068	0.022	0.	0.	0.	0.

Problem 1642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	73	0	0	0	218	0
normalized size	1	1.	0.4	0.	0.	0.	1.2	0.
time (sec)	N/A	0.104	0.047	0.017	0.	0.	70.52	0.

Problem 1643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	71	0	0	0	0	0
normalized size	1	1.	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.039	0.02	0.	0.	0.	0.

Problem 1644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	71	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.038	0.043	0.	0.	0.	0.

Problem 1645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	73	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.041	0.04	0.	0.	0.	0.

Problem 1646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	73	0	0	0	0	0
normalized size	1	1.	0.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.046	0.04	0.	0.	0.	0.

Problem 1647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	73	0	0	0	0	0
normalized size	1	1.	0.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.05	0.045	0.	0.	0.	0.

Problem 1648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	73	0	0	0	0	0
normalized size	1	1.	0.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.298	0.033	0.02	0.	0.	0.	0.

Problem 1649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	73	0	0	0	0	0
normalized size	1	1.	0.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.245	0.028	0.018	0.	0.	0.	0.

Problem 1650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	73	0	0	0	0	0
normalized size	1	1.	0.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.216	0.025	0.015	0.	0.	0.	0.

Problem 1651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	71	0	0	0	0	0
normalized size	1	1.	0.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	0.022	0.038	0.	0.	0.	0.

Problem 1652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	71	0	0	0	0	0
normalized size	1	1.	0.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	0.022	0.04	0.	0.	0.	0.

Problem 1653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	73	0	0	0	0	0
normalized size	1	1.	0.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.262	0.021	0.043	0.	0.	0.	0.

Problem 1654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	73	0	0	0	0	0
normalized size	1	1.	0.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.03	0.017	0.	0.	0.	0.

Problem 1655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	73	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.026	0.016	0.	0.	0.	0.

Problem 1656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	71	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.023	0.038	0.	0.	0.	0.

Problem 1657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	71	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.022	0.04	0.	0.	0.	0.

Problem 1658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	73	0	0	0	0	0
normalized size	1	1.	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.024	0.041	0.	0.	0.	0.

Problem 1659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	73	0	0	0	0	0
normalized size	1	1.	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.262	0.063	0.047	0.	0.	0.	0.

Problem 1660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	73	0	0	0	0	0
normalized size	1	1.	0.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.235	0.052	0.041	0.	0.	0.	0.

Problem 1661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	73	0	0	0	0	0
normalized size	1	1.	0.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	0.041	0.038	0.	0.	0.	0.

Problem 1662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	71	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	0.035	0.042	0.	0.	0.	0.

Problem 1663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	71	0	0	0	0	0
normalized size	1	1.	0.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.231	0.038	0.042	0.	0.	0.	0.

Problem 1664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	73	0	0	0	0	0
normalized size	1	1.	0.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.26	0.038	0.043	0.	0.	0.	0.

Problem 1665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	73	0	0	0	0	0
normalized size	1	1.	0.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	0.071	0.053	0.	0.	0.	0.

Problem 1666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	73	0	0	0	0	0
normalized size	1	1.	0.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.049	0.044	0.	0.	0.	0.

Problem 1667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	73	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.044	0.037	0.	0.	0.	0.

Problem 1668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	71	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.037	0.042	0.	0.	0.	0.

Problem 1669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	71	0	0	0	0	0
normalized size	1	1.	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.036	0.042	0.	0.	0.	0.

Problem 1670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	73	0	0	0	0	0
normalized size	1	1.	0.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.038	0.042	0.	0.	0.	0.

Problem 1671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	73	0	0	0	0	0
normalized size	1	1.	0.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.297	0.088	0.092	0.	0.	0.	0.

Problem 1672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	73	0	0	0	0	0
normalized size	1	1.	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.257	0.073	0.079	0.	0.	0.	0.

Problem 1673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	73	0	0	0	0	0
normalized size	1	1.	0.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.235	0.064	0.04	0.	0.	0.	0.

Problem 1674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	73	0	0	0	0	0
normalized size	1	1.	0.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	0.054	0.037	0.	0.	0.	0.

Problem 1675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	71	0	0	0	0	0
normalized size	1	1.	0.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.236	0.047	0.044	0.	0.	0.	0.

Problem 1676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	71	0	0	0	0	0
normalized size	1	1.	0.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.257	0.046	0.041	0.	0.	0.	0.

Problem 1677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	73	0	0	0	0	0
normalized size	1	1.	0.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.29	0.048	0.044	0.	0.	0.	0.

Problem 1678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	73	0	0	4761	0	0
normalized size	1	1.	0.36	0.	0.	23.22	0.	0.
time (sec)	N/A	0.139	0.06	0.028	0.	4.423	0.	0.

Problem 1679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	73	0	0	3090	0	0
normalized size	1	1.	0.44	0.	0.	18.5	0.	0.
time (sec)	N/A	0.102	0.044	0.017	0.	3.64	0.	0.

Problem 1680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	71	0	0	1817	0	0
normalized size	1	1.	0.47	0.	0.	11.95	0.	0.
time (sec)	N/A	0.098	0.044	0.042	0.	3.108	0.	0.

Problem 1681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	73	0	0	842	0	0
normalized size	1	1.	0.54	0.	0.	6.28	0.	0.
time (sec)	N/A	0.088	0.058	0.042	0.	2.788	0.	0.

Problem 1682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	221	0	0
normalized size	1	1.	1.	0.84	0.	6.91	0.	0.
time (sec)	N/A	0.003	0.015	0.005	0.	2.632	0.	0.

Problem 1683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	494	0	0
normalized size	1	1.	0.7	0.82	0.	7.48	0.	0.
time (sec)	N/A	0.009	0.026	0.005	0.	2.985	0.	0.

Problem 1684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	879	0	0
normalized size	1	1.	0.76	1.04	0.	8.7	0.	0.
time (sec)	N/A	0.017	0.047	0.006	0.	3.576	0.	0.

Problem 1685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	171	0	1354	0	0
normalized size	1	1.	0.87	1.26	0.	9.96	0.	0.
time (sec)	N/A	0.029	0.064	0.008	0.	4.626	0.	0.

Problem 1686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	73	0	0	0	0	0
normalized size	1	1.	0.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.59	0.058	0.029	0.	0.	0.	0.

Problem 1687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	73	0	0	0	0	0
normalized size	1	1.	0.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.378	0.05	0.015	0.	0.	0.	0.

Problem 1688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	71	0	0	0	0	0
normalized size	1	1.	0.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.287	0.044	0.018	0.	0.	0.	0.

Problem 1689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	73	0	0	0	0	0
normalized size	1	1.	0.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.278	0.041	0.046	0.	0.	0.	0.

Problem 1690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	73	0	0	0	0	0
normalized size	1	1.	0.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.299	0.046	0.042	0.	0.	0.	0.

Problem 1691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	73	0	0	0	0	0
normalized size	1	1.	0.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.352	0.053	0.042	0.	0.	0.	0.

Problem 1692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	73	0	0	0	0	0
normalized size	1	1.	0.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.425	0.052	0.041	0.	0.	0.	0.

Problem 1693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	73	0	0	3090	0	0
normalized size	1	1.	0.44	0.	0.	18.5	0.	0.
time (sec)	N/A	0.102	0.029	0.026	0.	3.194	0.	0.

Problem 1694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	73	0	0	1744	0	0
normalized size	1	1.	0.57	0.	0.	13.73	0.	0.
time (sec)	N/A	0.074	0.026	0.013	0.	2.588	0.	0.

Problem 1695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	71	0	0	581	0	0
normalized size	1	1.	0.84	0.	0.	6.84	0.	0.
time (sec)	N/A	0.058	0.029	0.037	0.	2.171	0.	0.

Problem 1696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	100	0	0
normalized size	1	1.	1.	0.84	0.	3.12	0.	0.
time (sec)	N/A	0.003	0.011	0.004	0.	1.936	0.	0.

Problem 1697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	259	0	0
normalized size	1	1.	0.7	0.82	0.	3.92	0.	0.
time (sec)	N/A	0.009	0.016	0.003	0.	3.194	0.	0.

Problem 1698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	522	0	0
normalized size	1	1.	0.76	1.04	0.	5.17	0.	0.
time (sec)	N/A	0.017	0.031	0.005	0.	6.774	0.	0.

Problem 1699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	171	0	865	0	0
normalized size	1	1.	0.87	1.26	0.	6.36	0.	0.
time (sec)	N/A	0.031	0.048	0.008	0.	17.795	0.	0.

Problem 1700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	751	751	73	0	0	0	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.789	0.029	0.034	0.	0.	0.	0.

Problem 1701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	705	705	73	0	0	0	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.617	0.026	0.03	0.	0.	0.	0.

Problem 1702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	688	688	73	0	0	0	0	0
normalized size	1	1.	0.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.495	0.022	0.034	0.	0.	0.	0.

Problem 1703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	718	71	0	0	0	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.609	0.023	0.039	0.	0.	0.	0.

Problem 1704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	760	760	73	0	0	0	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.726	0.025	0.036	0.	0.	0.	0.

Problem 1705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	73	0	0	3069	0	0
normalized size	1	1.	0.44	0.	0.	18.38	0.	0.
time (sec)	N/A	0.105	0.032	0.018	0.	3.75	0.	0.

Problem 1706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	73	0	0	1721	0	0
normalized size	1	1.	0.57	0.	0.	13.55	0.	0.
time (sec)	N/A	0.081	0.027	0.014	0.	3.116	0.	0.

Problem 1707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	73	0	0	581	0	0
normalized size	1	1.	0.86	0.	0.	6.84	0.	0.
time (sec)	N/A	0.067	0.029	0.036	0.	2.435	0.	0.

Problem 1708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	97	0	0
normalized size	1	1.	1.	0.9	0.	3.23	0.	0.
time (sec)	N/A	0.003	0.007	0.004	0.	2.319	0.	0.

Problem 1709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	255	0	0
normalized size	1	1.	0.7	0.82	0.	3.86	0.	0.
time (sec)	N/A	0.009	0.015	0.005	0.	2.236	0.	0.

Problem 1710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	105	0	516	0	0
normalized size	1	1.	0.74	1.04	0.	5.11	0.	0.
time (sec)	N/A	0.017	0.032	0.005	0.	2.367	0.	0.

Problem 1711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	116	171	0	864	0	0
normalized size	1	1.	0.85	1.26	0.	6.35	0.	0.
time (sec)	N/A	0.028	0.047	0.007	0.	2.596	0.	0.

Problem 1712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	73	0	0	0	0	0
normalized size	1	1.	0.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.286	0.031	0.016	0.	0.	0.	0.

Problem 1713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	73	0	0	0	0	0
normalized size	1	1.	0.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	0.025	0.015	0.	0.	0.	0.

Problem 1714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	71	0	0	0	0	0
normalized size	1	1.	0.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.175	0.022	0.035	0.	0.	0.	0.

Problem 1715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	73	0	0	0	0	0
normalized size	1	1.	0.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.215	0.024	0.039	0.	0.	0.	0.

Problem 1716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	339	73	0	0	0	0	0
normalized size	1	1.	0.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.28	0.025	0.038	0.	0.	0.	0.

Problem 1717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	73	0	0	1818	0	0
normalized size	1	1.	0.48	0.	0.	11.96	0.	0.
time (sec)	N/A	0.087	0.053	0.049	0.	3.16	0.	0.

Problem 1718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	73	0	0	656	0	0
normalized size	1	1.	0.68	0.	0.	6.07	0.	0.
time (sec)	N/A	0.065	0.043	0.035	0.	2.473	0.	0.

Problem 1719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	96	0	0
normalized size	1	1.	1.	0.9	0.	3.2	0.	0.
time (sec)	N/A	0.003	0.007	0.005	0.	2.193	0.	0.

Problem 1720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	45	53	0	270	0	0
normalized size	1	1.	0.68	0.8	0.	4.09	0.	0.
time (sec)	N/A	0.008	0.017	0.005	0.	2.681	0.	0.

Problem 1721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	76	105	0	558	0	0
normalized size	1	1.	0.75	1.04	0.	5.52	0.	0.
time (sec)	N/A	0.017	0.031	0.006	0.	4.197	0.	0.

Problem 1722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	116	171	0	934	0	0
normalized size	1	1.	0.85	1.26	0.	6.87	0.	0.
time (sec)	N/A	0.027	0.042	0.008	0.	7.97	0.	0.

Problem 1723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	776	776	73	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.88	0.061	0.037	0.	0.	0.	0.

Problem 1724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	730	730	73	0	0	0	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.725	0.05	0.034	0.	0.	0.	0.

Problem 1725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	712	712	73	0	0	0	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.609	0.042	0.03	0.	0.	0.	0.

Problem 1726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	719	719	73	0	0	0	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.612	0.035	0.035	0.	0.	0.	0.

Problem 1727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	750	750	71	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.74	0.041	0.036	0.	0.	0.	0.

Problem 1728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	795	795	73	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.865	0.04	0.036	0.	0.	0.	0.

Problem 1729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	65	0	0	586	0	0
normalized size	1	1.	0.23	0.	0.	2.1	0.	0.
time (sec)	N/A	0.304	0.035	0.051	0.	2.565	0.	0.

Problem 1730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	42	0	0	1235	0	0
normalized size	1	1.	0.22	0.	0.	6.4	0.	0.
time (sec)	N/A	0.138	0.008	0.045	0.	2.595	0.	0.

Problem 1731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.032	0.03	0.	0.	0.	0.

Problem 1732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.026	0.015	0.	0.	0.	0.

Problem 1733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.021	0.039	0.	0.	0.	0.

Problem 1734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.021	0.041	0.	0.	0.	0.

Problem 1735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.025	0.044	0.	0.	0.	0.

Problem 1736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	73	0	0	0	0	0
normalized size	1	1.	0.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.522	0.043	0.032	0.	0.	0.	0.

Problem 1737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	73	0	0	0	0	0
normalized size	1	1.	0.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.336	0.03	0.018	0.	0.	0.	0.

Problem 1738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	73	0	0	0	0	0
normalized size	1	1.	0.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.284	0.025	0.014	0.	0.	0.	0.

Problem 1739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	71	0	0	0	0	0
normalized size	1	1.	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	0.021	0.018	0.	0.	0.	0.

Problem 1740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	71	0	0	0	0	0
normalized size	1	1.	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.229	0.022	0.038	0.	0.	0.	0.

Problem 1741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	73	0	0	0	0	0
normalized size	1	1.	0.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.28	0.023	0.038	0.	0.	0.	0.

Problem 1742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	896	896	73	0	0	0	0	0
normalized size	1	1.	0.08	0.	0.	0.	0.	0.
time (sec)	N/A	1.078	0.037	0.025	0.	0.	0.	0.

Problem 1743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	858	858	73	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.823	0.031	0.015	0.	0.	0.	0.

Problem 1744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	817	817	71	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.71	0.022	0.019	0.	0.	0.	0.

Problem 1745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	798	798	71	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.684	0.023	0.038	0.	0.	0.	0.

Problem 1746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	854	854	73	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.798	0.025	0.04	0.	0.	0.	0.

Problem 1747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	896	896	73	0	0	0	0	0
normalized size	1	1.	0.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.901	0.025	0.039	0.	0.	0.	0.

Problem 1748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	890	890	73	0	0	0	0	0
normalized size	1	1.	0.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.952	0.035	0.02	0.	0.	0.	0.

Problem 1749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	855	855	73	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.79	0.028	0.016	0.	0.	0.	0.

Problem 1750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	820	820	73	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.69	0.024	0.015	0.	0.	0.	0.

Problem 1751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	780	780	71	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.582	0.021	0.036	0.	0.	0.	0.

Problem 1752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	813	813	71	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.677	0.023	0.037	0.	0.	0.	0.

Problem 1753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	858	858	73	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.752	0.021	0.038	0.	0.	0.	0.

Problem 1754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	440	73	0	0	0	0	0
normalized size	1	1.	0.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.327	0.039	0.022	0.	0.	0.	0.

Problem 1755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	73	0	0	0	0	0
normalized size	1	1.	0.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.279	0.031	0.019	0.	0.	0.	0.

Problem 1756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	73	0	0	0	0	0
normalized size	1	1.	0.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.236	0.025	0.016	0.	0.	0.	0.

Problem 1757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	71	0	0	0	0	0
normalized size	1	1.	0.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	0.023	0.038	0.	0.	0.	0.

Problem 1758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	71	0	0	0	0	0
normalized size	1	1.	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.231	0.023	0.041	0.	0.	0.	0.

Problem 1759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	73	0	0	0	0	0
normalized size	1	1.	0.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.268	0.024	0.041	0.	0.	0.	0.

Problem 1760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	880	880	73	0	0	0	0	0
normalized size	1	1.	0.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.901	0.061	0.047	0.	0.	0.	0.

Problem 1761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	844	844	73	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.773	0.048	0.043	0.	0.	0.	0.

Problem 1762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	806	806	73	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.668	0.041	0.034	0.	0.	0.	0.

Problem 1763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	817	817	71	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.672	0.033	0.043	0.	0.	0.	0.

Problem 1764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	844	844	71	0	0	0	0	0
normalized size	1	1.	0.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.759	0.035	0.04	0.	0.	0.	0.

Problem 1765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	893	893	73	0	0	0	0	0
normalized size	1	1.	0.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.868	0.036	0.042	0.	0.	0.	0.

Problem 1766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	73	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.089	0.03	0.	0.	0.	0.

Problem 1767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.049	0.019	0.	0.	0.	0.

Problem 1768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.026	0.017	0.	0.	0.	0.

Problem 1769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.026	0.018	0.	0.	0.	0.

Problem 1770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.048	0.038	0.	0.	0.	0.

Problem 1771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	81	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.031	0.039	0.	0.	0.	0.

Problem 1772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	427	73	0	0	12593	0	0
normalized size	1	1.	0.17	0.	0.	29.49	0.	0.
time (sec)	N/A	0.643	0.03	0.024	0.	2.892	0.	0.

Problem 1773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	73	0	0	6461	0	0
normalized size	1	1.	0.19	0.	0.	17.09	0.	0.
time (sec)	N/A	0.497	0.025	0.017	0.	2.404	0.	0.

Problem 1774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	73	0	0	1704	0	0
normalized size	1	1.	0.22	0.	0.	5.13	0.	0.
time (sec)	N/A	0.487	0.041	0.038	0.	1.805	0.	0.

Problem 1775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	142	0	0
normalized size	1	1.	1.	0.84	0.	4.44	0.	0.
time (sec)	N/A	0.003	0.01	0.003	0.	1.491	0.	0.

Problem 1776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	373	0	0
normalized size	1	1.	0.7	0.82	0.	5.65	0.	0.
time (sec)	N/A	0.009	0.022	0.005	0.	1.558	0.	0.

Problem 1777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	695	0	0
normalized size	1	1.	0.76	1.04	0.	6.88	0.	0.
time (sec)	N/A	0.021	0.042	0.007	0.	1.501	0.	0.

Problem 1778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	171	0	1115	0	0
normalized size	1	1.	0.87	1.26	0.	8.2	0.	0.
time (sec)	N/A	0.034	0.06	0.008	0.	1.641	0.	0.

Problem 1779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	427	73	0	0	12593	0	0
normalized size	1	1.	0.17	0.	0.	29.49	0.	0.
time (sec)	N/A	0.642	0.033	0.025	0.	3.305	0.	0.

Problem 1780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	73	0	0	6407	0	0
normalized size	1	1.	0.19	0.	0.	16.95	0.	0.
time (sec)	N/A	0.557	0.029	0.016	0.	2.829	0.	0.

Problem 1781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	73	0	0	1887	0	0
normalized size	1	1.	0.22	0.	0.	5.65	0.	0.
time (sec)	N/A	0.561	0.051	0.037	0.	2.19	0.	0.

Problem 1782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	144	0	0
normalized size	1	1.	1.	0.84	0.	4.5	0.	0.
time (sec)	N/A	0.003	0.011	0.005	0.	1.817	0.	0.

Problem 1783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	378	0	0
normalized size	1	1.	0.7	0.82	0.	5.73	0.	0.
time (sec)	N/A	0.009	0.023	0.003	0.	1.838	0.	0.

Problem 1784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	702	0	0
normalized size	1	1.	0.76	1.04	0.	6.95	0.	0.
time (sec)	N/A	0.02	0.043	0.005	0.	1.906	0.	0.

Problem 1785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	171	0	1131	0	0
normalized size	1	1.	0.87	1.26	0.	8.32	0.	0.
time (sec)	N/A	0.033	0.06	0.008	0.	1.909	0.	0.

Problem 1786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.058	0.034	0.	0.	0.	0.

Problem 1787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.027	0.03	0.	0.	0.	0.

Problem 1788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.026	0.033	0.	0.	0.	0.

Problem 1789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.04	0.033	0.	0.	0.	0.

Problem 1790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.055	0.035	0.	0.	0.	0.

Problem 1791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	81	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.039	0.035	0.	0.	0.	0.

Problem 1792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	73	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.086	0.022	0.	0.	0.	0.

Problem 1793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.057	0.018	0.	0.	0.	0.

Problem 1794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.03	0.017	0.	0.	0.	0.

Problem 1795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.031	0.018	0.	0.	0.	0.

Problem 1796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.05	0.047	0.	0.	0.	0.

Problem 1797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	81	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.035	0.042	0.	0.	0.	0.

Problem 1798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	73	0	0	12598	0	0
normalized size	1	1.	0.17	0.	0.	29.71	0.	0.
time (sec)	N/A	0.552	0.028	0.017	0.	4.278	0.	0.

Problem 1799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	73	0	0	6580	0	0
normalized size	1	1.	0.18	0.	0.	16.33	0.	0.
time (sec)	N/A	0.526	0.05	0.044	0.	3.768	0.	0.

Problem 1800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	73	0	0	2087	0	0
normalized size	1	1.	0.2	0.	0.	5.83	0.	0.
time (sec)	N/A	0.501	0.064	0.043	0.	2.269	0.	0.

Problem 1801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	221	0	0
normalized size	1	1.	1.	0.84	0.	6.91	0.	0.
time (sec)	N/A	0.003	0.013	0.004	0.	1.761	0.	0.

Problem 1802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	497	0	0
normalized size	1	1.	0.7	0.82	0.	7.53	0.	0.
time (sec)	N/A	0.009	0.027	0.005	0.	1.926	0.	0.

Problem 1803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	880	0	0
normalized size	1	1.	0.76	1.04	0.	8.71	0.	0.
time (sec)	N/A	0.018	0.044	0.006	0.	1.862	0.	0.

Problem 1804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	171	0	1373	0	0
normalized size	1	1.	0.87	1.26	0.	10.1	0.	0.
time (sec)	N/A	0.03	0.067	0.008	0.	1.889	0.	0.

Problem 1805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	73	0	0	12598	0	0
normalized size	1	1.	0.17	0.	0.	29.71	0.	0.
time (sec)	N/A	0.61	0.04	0.02	0.	3.46	0.	0.

Problem 1806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	73	0	0	6460	0	0
normalized size	1	1.	0.19	0.	0.	17.09	0.	0.
time (sec)	N/A	0.56	0.022	0.013	0.	2.65	0.	0.

Problem 1807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	73	0	0	1638	0	0
normalized size	1	1.	0.24	0.	0.	5.3	0.	0.
time (sec)	N/A	0.509	0.027	0.036	0.	2.053	0.	0.

Problem 1808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	99	0	0
normalized size	1	1.	1.	0.84	0.	3.09	0.	0.
time (sec)	N/A	0.003	0.011	0.004	0.	1.86	0.	0.

Problem 1809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	261	0	0
normalized size	1	1.	0.7	0.82	0.	3.95	0.	0.
time (sec)	N/A	0.01	0.017	0.006	0.	1.87	0.	0.

Problem 1810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	525	0	0
normalized size	1	1.	0.76	1.04	0.	5.2	0.	0.
time (sec)	N/A	0.02	0.034	0.006	0.	1.844	0.	0.

Problem 1811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	171	0	883	0	0
normalized size	1	1.	0.87	1.26	0.	6.49	0.	0.
time (sec)	N/A	0.03	0.051	0.007	0.	1.918	0.	0.

Problem 1812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.05	0.034	0.	0.	0.	0.

Problem 1813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.025	0.031	0.	0.	0.	0.

Problem 1814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.022	0.033	0.	0.	0.	0.

Problem 1815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.034	0.036	0.	0.	0.	0.

Problem 1816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.047	0.038	0.	0.	0.	0.

Problem 1817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	81	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.033	0.039	0.	0.	0.	0.

Problem 1818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	71	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.064	0.023	0.	0.	0.	0.

Problem 1819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.039	0.017	0.	0.	0.	0.

Problem 1820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.024	0.016	0.	0.	0.	0.

Problem 1821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.023	0.034	0.	0.	0.	0.

Problem 1822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.038	0.044	0.	0.	0.	0.

Problem 1823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	79	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.025	0.042	0.	0.	0.	0.

Problem 1824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	71	0	0	12492	0	0
normalized size	1	1.	0.17	0.	0.	29.46	0.	0.
time (sec)	N/A	0.549	0.043	0.019	0.	2.936	0.	0.

Problem 1825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	71	0	0	6406	0	0
normalized size	1	1.	0.19	0.	0.	16.95	0.	0.
time (sec)	N/A	0.479	0.024	0.015	0.	2.354	0.	0.

Problem 1826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	71	0	0	1638	0	0
normalized size	1	1.	0.23	0.	0.	5.3	0.	0.
time (sec)	N/A	0.444	0.027	0.037	0.	1.793	0.	0.

Problem 1827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	96	0	0
normalized size	1	1.	1.	0.9	0.	3.2	0.	0.
time (sec)	N/A	0.003	0.007	0.004	0.	1.461	0.	0.

Problem 1828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	53	0	255	0	0
normalized size	1	1.	0.7	0.8	0.	3.86	0.	0.
time (sec)	N/A	0.009	0.016	0.004	0.	1.522	0.	0.

Problem 1829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	517	0	0
normalized size	1	1.	0.76	1.04	0.	5.12	0.	0.
time (sec)	N/A	0.018	0.032	0.006	0.	1.56	0.	0.

Problem 1830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	171	0	872	0	0
normalized size	1	1.	0.87	1.26	0.	6.41	0.	0.
time (sec)	N/A	0.031	0.048	0.008	0.	1.622	0.	0.

Problem 1831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	71	0	0	12721	0	0
normalized size	1	1.	0.16	0.	0.	28.33	0.	0.
time (sec)	N/A	0.66	0.06	0.051	0.	2.982	0.	0.

Problem 1832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	71	0	0	6579	0	0
normalized size	1	1.	0.18	0.	0.	16.33	0.	0.
time (sec)	N/A	0.596	0.041	0.042	0.	2.501	0.	0.

Problem 1833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	71	0	0	1704	0	0
normalized size	1	1.	0.21	0.	0.	5.13	0.	0.
time (sec)	N/A	0.541	0.026	0.036	0.	1.888	0.	0.

Problem 1834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	97	0	0
normalized size	1	1.	1.	0.9	0.	3.23	0.	0.
time (sec)	N/A	0.003	0.008	0.003	0.	1.472	0.	0.

Problem 1835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	45	53	0	270	0	0
normalized size	1	1.	0.7	0.83	0.	4.22	0.	0.
time (sec)	N/A	0.011	0.016	0.005	0.	1.607	0.	0.

Problem 1836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	77	105	0	562	0	0
normalized size	1	1.	0.79	1.07	0.	5.73	0.	0.
time (sec)	N/A	0.02	0.03	0.006	0.	1.671	0.	0.

Problem 1837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	118	171	0	946	0	0
normalized size	1	1.	0.88	1.28	0.	7.06	0.	0.
time (sec)	N/A	0.033	0.047	0.007	0.	1.68	0.	0.

Problem 1838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.048	0.036	0.	0.	0.	0.

Problem 1839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.022	0.031	0.	0.	0.	0.

Problem 1840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.023	0.036	0.	0.	0.	0.

Problem 1841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.035	0.036	0.	0.	0.	0.

Problem 1842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.045	0.039	0.	0.	0.	0.

Problem 1843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	79	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.034	0.039	0.	0.	0.	0.

Problem 1844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	36	20	31
normalized size	1	1.	1.	1.09	0.	3.27	1.82	2.82
time (sec)	N/A	0.002	0.011	0.003	0.	1.711	0.252	1.07

Problem 1845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	74	73	0	0	0	0	0
normalized size	1	1.21	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.026	0.083	0.	0.	0.	0.

Problem 1846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	94	389	0	1011	4056	1125
normalized size	1	1.	0.85	3.54	0.	9.19	36.87	10.23
time (sec)	N/A	0.055	0.075	0.009	0.	1.958	4.211	1.076

Problem 1847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	67	159	0	478	1504	520
normalized size	1	1.	0.86	2.04	0.	6.13	19.28	6.67
time (sec)	N/A	0.032	0.074	0.006	0.	1.841	1.879	1.054

Problem 1848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	41	49	0	171	377	178
normalized size	1	1.	0.89	1.07	0.	3.72	8.2	3.87
time (sec)	N/A	0.018	0.03	0.002	0.	1.791	0.804	1.082

Problem 1849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.01	0.041	0.	0.	0.	0.

Problem 1850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.014	0.048	0.	0.	0.	0.

Problem 1851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.014	0.071	0.	0.	0.	0.

Problem 1852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	95	386	0	1011	4056	1125
normalized size	1	1.	0.86	3.48	0.	9.11	36.54	10.14
time (sec)	N/A	0.057	0.075	0.008	0.	1.767	4.446	1.058

Problem 1853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	67	159	0	478	1504	520
normalized size	1	1.	0.86	2.04	0.	6.13	19.28	6.67
time (sec)	N/A	0.032	0.068	0.007	0.	1.932	1.996	1.073

Problem 1854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	46	0	171	377	178
normalized size	1	1.	0.87	0.98	0.	3.64	8.02	3.79
time (sec)	N/A	0.018	0.028	0.002	0.	1.885	0.841	1.062

Problem 1855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	17	19	0	45	20	24
normalized size	1	1.	0.94	1.06	0.	2.5	1.11	1.33
time (sec)	N/A	0.003	0.009	0.001	0.	1.853	0.07	1.049

Problem 1856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.008	0.039	0.	0.	0.	0.

Problem 1857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	52	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.013	0.048	0.	0.	0.	0.

Problem 1858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.014	0.071	0.	0.	0.	0.

Problem 1859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	112	322	0	1023	0	0
normalized size	1	1.	0.78	2.25	0.	7.15	0.	0.
time (sec)	N/A	0.063	0.065	0.006	0.	2.044	0.	0.

Problem 1860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	59	127	0	417	0	0
normalized size	1	1.	0.69	1.48	0.	4.85	0.	0.
time (sec)	N/A	0.012	0.031	0.006	0.	1.952	0.	0.

Problem 1861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	45	0	127	0	0
normalized size	1	1.	0.92	1.15	0.	3.26	0.	0.
time (sec)	N/A	0.004	0.012	0.002	0.	1.821	0.	0.

Problem 1862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.02	0.074	0.	0.	0.	0.

Problem 1863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.018	0.07	0.	0.	0.	0.

Problem 1864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	89	0	0	0	0	0
normalized size	1	1.	1.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.043	0.076	0.	0.	0.	0.

Problem 1865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	92	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.037	0.077	0.	0.	0.	0.

Problem 1866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.02	0.069	0.	0.	0.	0.

Problem 1867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	74	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.02	0.069	0.	0.	0.	0.

Problem 1868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	38	41	0	126	0	0
normalized size	1	1.	1.03	1.11	0.	3.41	0.	0.
time (sec)	N/A	0.006	0.013	0.003	0.	1.919	0.	0.

Problem 1869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	60	123	0	416	0	0
normalized size	1	1.	0.75	1.54	0.	5.2	0.	0.
time (sec)	N/A	0.02	0.029	0.004	0.	2.269	0.	0.

Problem 1870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	113	318	0	1022	0	0
normalized size	1	1.	0.86	2.43	0.	7.8	0.	0.
time (sec)	N/A	0.046	0.057	0.006	0.	2.239	0.	0.

Problem 1871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	195	661	0	1945	0	0
normalized size	1	1.	1.05	3.55	0.	10.46	0.	0.
time (sec)	N/A	0.086	0.087	0.008	0.	2.365	0.	0.

Problem 1872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.004	0.	0.	0.	0.	0.

Problem 1873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	74	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.023	0.069	0.	0.	0.	0.

Problem 1874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	42	0	124	0	0
normalized size	1	1.	1.	1.17	0.	3.44	0.	0.
time (sec)	N/A	0.005	0.01	0.004	0.	2.255	0.	0.

Problem 1875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	59	124	0	416	0	0
normalized size	1	1.	0.75	1.57	0.	5.27	0.	0.
time (sec)	N/A	0.017	0.029	0.003	0.	2.19	0.	0.

Problem 1876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	112	319	0	1021	0	0
normalized size	1	1.	0.86	2.45	0.	7.85	0.	0.
time (sec)	N/A	0.037	0.058	0.006	0.	2.262	0.	0.

Problem 1877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	195	662	0	1945	0	0
normalized size	1	1.	1.05	3.58	0.	10.51	0.	0.
time (sec)	N/A	0.062	0.091	0.007	0.	2.412	0.	0.

Problem 1878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	75	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.042	0.073	0.	0.	0.	0.

Problem 1879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	83	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.032	0.072	0.	0.	0.	0.

Problem 1880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.004	0.	0.	0.	0.	0.

Problem 1881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	53	57	124	200	233	105
normalized size	1	1.	0.93	1.	2.18	3.51	4.09	1.84
time (sec)	N/A	0.03	0.026	0.007	0.99	2.288	0.815	1.067

Problem 1882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	54	57	0	181	0	0
normalized size	1	1.	0.57	0.6	0.	1.91	0.	0.
time (sec)	N/A	0.036	0.052	0.005	0.	2.612	0.	0.

Problem 1883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	46	66	0	161	0	0
normalized size	1	1.	0.47	0.68	0.	1.66	0.	0.
time (sec)	N/A	0.022	0.029	0.004	0.	2.526	0.	0.

Problem 1884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	46	66	0	161	0	0
normalized size	1	1.	0.47	0.68	0.	1.66	0.	0.
time (sec)	N/A	0.018	0.046	0.004	0.	2.436	0.	0.

Problem 1885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	0	0	29	0
normalized size	1	1.	1.	0.	0.	0.	0.97	0.
time (sec)	N/A	0.007	0.006	0.031	0.	0.	2.013	0.

Problem 1886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	31	0
normalized size	1	1.	1.	0.	0.	0.	0.89	0.
time (sec)	N/A	0.006	0.006	0.028	0.	0.	2.049	0.

Problem 1887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.007	0.012	0.029	0.	0.	0.	0.

Problem 1888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	0.012	0.026	0.	0.	0.	0.

Problem 1889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	42	0
normalized size	1	1.	1.	0.	0.	0.	0.89	0.
time (sec)	N/A	0.014	0.013	0.06	0.	0.	61.154	0.

Problem 1890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	88	0	0	0	0	0
normalized size	1	1.	1.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.033	0.118	0.	0.	0.	0.

Problem 1891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	30	55	22	30
normalized size	1	1.	1.	0.82	1.07	1.96	0.79	1.07
time (sec)	N/A	0.005	0.	0.	0.99	1.669	0.054	1.052

Problem 1892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	26	8	15
normalized size	1	1.	1.	0.8	1.	1.73	0.53	1.
time (sec)	N/A	0.002	0.	0.002	0.972	1.754	0.05	1.065

Problem 1893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	18	5	12
normalized size	1	1.	1.	0.91	1.09	1.64	0.45	1.09
time (sec)	N/A	0.002	0.	0.	0.984	1.69	0.049	1.075

Problem 1894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	18	7	12
normalized size	1	1.	1.	1.11	1.33	2.	0.78	1.33
time (sec)	N/A	0.001	0.	0.002	0.988	1.72	0.048	1.074

Problem 1895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	27	10	16
normalized size	1	1.	1.	0.93	1.14	1.93	0.71	1.14
time (sec)	N/A	0.002	0.	0.	0.986	2.015	0.053	1.058

Problem 1896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	20	8	12
normalized size	1	1.	1.	0.91	1.09	1.82	0.73	1.09
time (sec)	N/A	0.001	0.	0.001	0.969	1.964	0.048	1.054

Problem 1897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	27	8	15
normalized size	1	1.	1.	0.8	1.	1.8	0.53	1.
time (sec)	N/A	0.002	0.	0.001	0.974	1.899	0.05	1.097

Problem 1898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	34	15	19
normalized size	1	1.	1.	0.83	1.06	1.89	0.83	1.06
time (sec)	N/A	0.003	0.	0.001	0.95	1.863	0.052	1.057

Problem 1899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	34	12	22
normalized size	1	1.	1.	0.85	1.1	1.7	0.6	1.1
time (sec)	N/A	0.003	0.	0.	0.985	1.676	0.051	1.049

Problem 1900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	32	10	16
normalized size	1	1.	1.	0.81	1.	2.	0.62	1.
time (sec)	N/A	0.002	0.	0.001	0.983	1.693	0.052	1.05

Problem 1901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	31	12	18
normalized size	1	1.	1.	1.08	1.38	2.38	0.92	1.38
time (sec)	N/A	0.002	0.	0.	0.985	1.689	0.053	1.074

Problem 1902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	65	19	28
normalized size	1	1.	1.	0.95	1.23	2.95	0.86	1.27
time (sec)	N/A	0.004	0.006	0.001	0.984	2.041	0.305	1.078

Problem 1903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	42	15	22
normalized size	1	1.	1.	0.77	1.	1.91	0.68	1.
time (sec)	N/A	0.003	0.001	0.001	1.032	1.894	0.077	1.06

Problem 1904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	46	12	19
normalized size	1	1.	1.	0.93	1.2	3.07	0.8	1.27
time (sec)	N/A	0.002	0.002	0.001	0.956	1.948	0.08	1.051

Problem 1905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	27	7	15
normalized size	1	1.	1.	1.1	1.4	2.7	0.7	1.5
time (sec)	N/A	0.002	0.001	0.001	0.948	1.95	0.076	1.054

Problem 1906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	31	10	15
normalized size	1	1.	1.	0.8	1.	2.07	0.67	1.
time (sec)	N/A	0.002	0.001	0.002	0.956	1.878	0.081	1.059

Problem 1907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	30	8	14
normalized size	1	1.	1.	0.91	1.09	2.73	0.73	1.27
time (sec)	N/A	0.001	0.001	0.	0.953	1.951	0.07	1.043

Problem 1908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	27	10	15
normalized size	1	1.	1.	0.92	1.15	2.08	0.77	1.15
time (sec)	N/A	0.002	0.001	0.	0.989	1.826	0.074	1.051

Problem 1909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	32	10	16
normalized size	1	1.	1.	0.92	1.15	2.46	0.77	1.23
time (sec)	N/A	0.002	0.001	0.001	0.986	1.973	0.074	1.056

Problem 1910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	15	35	12	15
normalized size	1	1.	1.	0.71	0.88	2.06	0.71	0.88
time (sec)	N/A	0.002	0.002	0.	0.97	1.892	0.055	1.05

Problem 1911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	32	10	12
normalized size	1	1.	1.	0.77	0.92	2.46	0.77	0.92
time (sec)	N/A	0.002	0.001	0.001	0.956	1.915	0.052	1.078

Problem 1912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	14	11	12	31	12	12
normalized size	1	1.	0.93	0.73	0.8	2.07	0.8	0.8
time (sec)	N/A	0.001	0.003	0.002	0.966	2.125	0.054	1.048

Problem 1913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	53	14	19
normalized size	1	1.	1.	0.93	1.2	3.53	0.93	1.27
time (sec)	N/A	0.002	0.009	0.001	0.961	2.369	0.056	1.053

Problem 1914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	14	11	15	31	14	15
normalized size	1	1.	0.82	0.65	0.88	1.82	0.82	0.88
time (sec)	N/A	0.002	0.004	0.003	0.97	2.181	0.054	1.048

Problem 1915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	10	9	15	31	12	15
normalized size	1	1.	0.67	0.6	1.	2.07	0.8	1.
time (sec)	N/A	0.002	0.003	0.004	0.973	2.21	0.054	1.053

Problem 1916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	22	43	19	22
normalized size	1	1.	1.	0.71	0.92	1.79	0.79	0.92
time (sec)	N/A	0.002	0.004	0.001	0.973	2.348	0.057	1.055

Problem 1917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	16	20	58	20	22
normalized size	1	1.	1.	0.7	0.87	2.52	0.87	0.96
time (sec)	N/A	0.003	0.007	0.	0.962	2.169	0.058	1.065

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [1] had the largest ratio of [1.]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	1	1.
2	A	1	1	1.	1	1.
3	A	1	1	1.	1	1.
4	A	1	1	1.	1	1.
5	A	1	1	1.	3	0.333
6	A	1	1	1.	1	1.
7	A	1	1	1.	1	1.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	1	1	1.	3	0.333
9	A	1	1	1.	13	0.077
10	A	1	1	1.	3	0.333
11	A	1	1	1.	3	0.333
12	A	1	1	1.	3	0.333
13	A	1	1	1.	1	1.
14	A	1	1	1.	1	1.
15	A	1	1	1.	3	0.333
16	A	1	1	1.	3	0.333
17	A	1	1	1.	3	0.333
18	A	1	1	1.	3	0.333
19	A	1	1	1.	3	0.333
20	A	1	1	1.	5	0.2
21	A	1	1	1.	5	0.2
22	A	1	1	1.	5	0.2
23	A	1	1	1.	5	0.2
24	A	1	1	1.	5	0.2
25	A	1	1	1.	5	0.2
26	A	1	1	1.	5	0.2
27	A	1	1	1.	5	0.2
28	A	1	1	1.	5	0.2
29	A	1	1	1.	5	0.2
30	A	1	1	1.	5	0.2
31	A	1	1	1.	5	0.2
32	A	1	1	1.	5	0.2
33	A	1	1	1.	5	0.2
34	A	1	1	1.	3	0.333
35	A	1	1	1.	5	0.2
36	A	2	2	1.	17	0.118
37	A	2	2	1.	13	0.154
38	A	2	2	1.	13	0.154
39	A	2	2	1.	13	0.154
40	A	2	2	1.	13	0.154
41	A	2	2	1.	13	0.154
42	A	2	2	1.	13	0.154
43	A	2	1	1.	9	0.111
44	A	2	1	1.	9	0.111
45	A	2	1	1.	7	0.143
46	A	1	0	1.	5	0.
47	A	2	1	1.	9	0.111
48	A	2	1	1.	9	0.111
49	A	1	1	1.	9	0.111
50	A	2	1	1.	9	0.111
51	A	2	1	1.	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	2	1	1.	11	0.091
53	A	2	1	1.	11	0.091
54	A	2	1	1.	9	0.111
55	A	1	1	1.	7	0.143
56	A	2	1	1.	11	0.091
57	A	2	1	1.	11	0.091
58	A	2	1	1.	11	0.091
59	A	1	1	1.	11	0.091
60	A	2	1	1.	11	0.091
61	A	2	1	1.	11	0.091
62	A	2	1	1.	11	0.091
63	A	2	1	1.	11	0.091
64	A	2	1	1.	11	0.091
65	A	2	1	1.	11	0.091
66	A	2	1	1.	11	0.091
67	A	2	1	1.	9	0.111
68	A	1	1	1.	7	0.143
69	A	2	1	1.	11	0.091
70	A	2	1	1.	11	0.091
71	A	2	1	1.	11	0.091
72	A	2	1	1.	11	0.091
73	A	1	1	1.	11	0.091
74	A	2	2	1.	11	0.182
75	A	2	1	1.	11	0.091
76	A	2	1	1.	11	0.091
77	A	2	1	1.	11	0.091
78	A	2	1	1.	11	0.091
79	A	2	1	1.	11	0.091
80	A	2	1	1.	11	0.091
81	A	2	1	1.	11	0.091
82	A	2	1	1.	9	0.111
83	A	1	1	1.	7	0.143
84	A	2	1	1.	11	0.091
85	A	2	1	1.	11	0.091
86	A	2	1	1.	11	0.091
87	A	2	1	1.	11	0.091
88	A	2	1	1.	11	0.091
89	A	2	1	1.	11	0.091
90	A	1	1	1.	11	0.091
91	A	2	2	1.	11	0.182
92	A	3	2	1.	11	0.182
93	A	2	1	1.	11	0.091
94	A	2	1	1.	11	0.091
95	A	2	1	1.	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	1	1.	11	0.091
97	A	2	1	1.	11	0.091
98	A	2	1	1.	11	0.091
99	A	2	1	1.	11	0.091
100	A	2	1	1.	11	0.091
101	A	2	1	1.	11	0.091
102	A	2	1	1.	11	0.091
103	A	2	1	1.	11	0.091
104	A	2	1	1.	11	0.091
105	A	2	1	1.	9	0.111
106	A	1	1	1.	7	0.143
107	A	2	1	1.	11	0.091
108	A	2	1	1.	11	0.091
109	A	2	1	1.	11	0.091
110	A	2	1	1.	11	0.091
111	A	2	1	1.	11	0.091
112	A	2	1	1.	11	0.091
113	A	2	1	1.	11	0.091
114	A	2	1	1.	11	0.091
115	A	1	1	1.	11	0.091
116	A	2	2	1.	11	0.182
117	A	3	2	1.	11	0.182
118	A	4	2	1.	11	0.182
119	A	5	2	1.	11	0.182
120	A	2	1	1.	11	0.091
121	A	2	1	1.	11	0.091
122	A	2	1	1.	11	0.091
123	A	2	1	1.	11	0.091
124	A	2	1	1.	11	0.091
125	A	2	1	1.	11	0.091
126	A	2	1	1.	11	0.091
127	A	2	1	1.	11	0.091
128	A	2	1	1.	11	0.091
129	A	2	1	1.	11	0.091
130	A	2	1	1.	11	0.091
131	A	2	1	1.	11	0.091
132	A	2	1	1.	11	0.091
133	A	2	1	1.	9	0.111
134	A	1	1	1.	7	0.143
135	A	2	1	1.	11	0.091
136	A	2	1	1.	11	0.091
137	A	2	1	1.	11	0.091
138	A	2	1	1.	11	0.091
139	A	2	1	1.	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
140	A	2	1	1.	11	0.091
141	A	2	1	1.	11	0.091
142	A	2	1	1.	11	0.091
143	A	2	1	1.	11	0.091
144	A	2	1	1.	11	0.091
145	A	2	1	1.	11	0.091
146	A	1	1	1.	11	0.091
147	A	2	2	1.	11	0.182
148	A	3	2	1.	11	0.182
149	A	4	2	1.	11	0.182
150	A	5	2	1.	11	0.182
151	A	6	2	1.	11	0.182
152	A	7	2	1.	11	0.182
153	A	2	1	1.	11	0.091
154	A	2	1	1.	11	0.091
155	A	1	1	1.	7	0.143
156	A	1	1	1.	12	0.083
157	A	2	1	1.	11	0.091
158	A	2	1	1.	11	0.091
159	A	2	1	1.	11	0.091
160	A	2	1	1.	11	0.091
161	A	2	1	1.	9	0.111
162	A	1	1	1.	7	0.143
163	A	3	3	1.	11	0.273
164	A	2	1	1.	11	0.091
165	A	2	1	1.	11	0.091
166	A	2	1	1.	11	0.091
167	A	2	1	1.	11	0.091
168	A	2	1	1.	11	0.091
169	A	2	1	1.	11	0.091
170	A	2	1	1.	11	0.091
171	A	2	1	1.	11	0.091
172	A	2	1	1.	11	0.091
173	A	2	1	1.	9	0.111
174	A	1	1	1.	7	0.143
175	A	2	1	1.	11	0.091
176	A	2	1	1.	11	0.091
177	A	2	1	1.	11	0.091
178	A	2	1	1.	11	0.091
179	A	2	1	1.	11	0.091
180	A	2	1	1.	11	0.091
181	A	2	1	1.	11	0.091
182	A	2	1	1.	11	0.091
183	A	2	1	1.	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
184	A	2	1	1.	11	0.091
185	A	2	1	1.	11	0.091
186	A	1	1	1.	9	0.111
187	A	1	1	1.	7	0.143
188	A	2	1	1.	11	0.091
189	A	2	1	1.	11	0.091
190	A	2	1	1.	11	0.091
191	A	2	1	1.	11	0.091
192	A	2	1	1.	11	0.091
193	A	2	1	1.	11	0.091
194	A	2	1	1.	11	0.091
195	A	2	1	1.	11	0.091
196	A	2	1	1.	11	0.091
197	A	2	1	1.	11	0.091
198	A	2	1	1.	11	0.091
199	A	1	1	1.	11	0.091
200	A	2	1	1.	9	0.111
201	A	1	1	1.	7	0.143
202	A	2	1	1.	11	0.091
203	A	2	1	1.	11	0.091
204	A	2	1	1.	11	0.091
205	A	2	1	1.	11	0.091
206	A	2	1	1.	11	0.091
207	A	2	1	1.	11	0.091
208	A	2	1	1.	11	0.091
209	A	2	1	1.	11	0.091
210	A	2	1	1.	11	0.091
211	A	2	1	1.	11	0.091
212	A	1	1	1.	11	0.091
213	A	2	2	1.	11	0.182
214	A	2	1	1.23	11	0.091
215	A	2	1	1.	11	0.091
216	A	2	1	1.	9	0.111
217	A	1	1	1.	7	0.143
218	A	2	1	1.	11	0.091
219	A	2	1	1.	11	0.091
220	A	2	1	1.	11	0.091
221	A	2	1	1.	11	0.091
222	A	2	1	1.	11	0.091
223	A	2	1	1.	11	0.091
224	A	2	1	1.	11	0.091
225	A	2	1	1.	11	0.091
226	A	1	1	1.	11	0.091
227	A	2	2	1.	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
228	A	3	2	1.	11	0.182
229	A	4	2	1.	11	0.182
230	A	2	1	1.	11	0.091
231	A	2	1	1.	11	0.091
232	A	2	1	1.	11	0.091
233	A	2	1	1.	9	0.111
234	A	1	1	1.	7	0.143
235	A	2	1	1.	11	0.091
236	A	2	1	1.	11	0.091
237	A	2	1	1.	11	0.091
238	A	2	1	1.	11	0.091
239	A	2	1	1.	11	0.091
240	A	2	1	1.	11	0.091
241	A	2	1	1.	11	0.091
242	A	2	1	1.	11	0.091
243	A	1	1	1.	11	0.091
244	A	2	2	1.	11	0.182
245	A	3	2	1.	11	0.182
246	A	2	1	1.	11	0.091
247	A	2	1	1.	11	0.091
248	A	2	1	1.	11	0.091
249	A	2	1	1.	11	0.091
250	A	2	1	1.	9	0.111
251	A	1	1	1.	3	0.333
252	A	2	1	1.	11	0.091
253	A	2	1	1.	11	0.091
254	A	2	1	1.	11	0.091
255	A	3	3	1.	11	0.273
256	A	3	3	1.	11	0.273
257	A	2	1	1.	11	0.091
258	A	2	1	1.	11	0.091
259	A	2	1	1.	11	0.091
260	A	2	1	1.	11	0.091
261	A	2	1	1.	11	0.091
262	A	2	1	1.	11	0.091
263	A	2	1	1.	11	0.091
264	A	2	1	1.	11	0.091
265	A	2	1	1.	11	0.091
266	A	2	1	1.	11	0.091
267	A	2	1	1.	11	0.091
268	A	2	1	1.	11	0.091
269	A	2	1	1.	11	0.091
270	A	2	1	1.	11	0.091
271	A	1	1	1.	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	1	1	1.	7	0.143
273	A	1	1	1.	11	0.091
274	A	1	1	1.	13	0.077
275	A	1	1	1.	15	0.067
276	A	1	1	1.	15	0.067
277	A	1	1	1.	15	0.067
278	A	1	1	1.	15	0.067
279	A	3	3	1.	11	0.273
280	A	3	3	1.	11	0.273
281	A	2	1	1.	11	0.091
282	A	2	1	1.	11	0.091
283	A	3	1	1.	17	0.059
284	A	2	1	1.	13	0.077
285	A	2	1	1.	13	0.077
286	A	2	1	1.	11	0.091
287	A	1	1	1.	9	0.111
288	A	3	3	1.	13	0.231
289	A	3	3	1.	13	0.231
290	A	4	4	1.	13	0.308
291	A	5	4	1.	13	0.308
292	A	2	1	1.	13	0.077
293	A	2	1	1.	13	0.077
294	A	2	1	1.	11	0.091
295	A	1	1	1.	9	0.111
296	A	4	3	1.	13	0.231
297	A	4	4	1.	13	0.308
298	A	4	3	1.	13	0.231
299	A	5	4	1.	13	0.308
300	A	2	1	1.	13	0.077
301	A	2	1	1.	13	0.077
302	A	2	1	1.	11	0.091
303	A	1	1	1.	9	0.111
304	A	5	3	1.	13	0.231
305	A	5	4	1.	13	0.308
306	A	5	4	1.	13	0.308
307	A	5	3	1.	13	0.231
308	A	6	4	1.	13	0.308
309	A	2	1	1.	13	0.077
310	A	2	1	1.	13	0.077
311	A	2	1	1.	13	0.077
312	A	2	1	1.	13	0.077
313	A	2	1	1.	13	0.077
314	A	2	1	1.	13	0.077
315	A	2	1	1.	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
316	A	1	1	1.	9	0.111
317	A	7	3	1.	13	0.231
318	A	7	4	1.	13	0.308
319	A	7	4	1.	13	0.308
320	A	7	4	1.	13	0.308
321	A	7	4	1.	13	0.308
322	A	7	3	1.	13	0.231
323	A	8	4	1.	13	0.308
324	A	9	4	1.	13	0.308
325	A	3	3	1.	15	0.2
326	A	3	3	1.	15	0.2
327	A	4	4	1.	15	0.267
328	A	4	3	1.	15	0.2
329	A	4	4	1.	15	0.267
330	A	4	3	1.	15	0.2
331	A	5	3	1.	15	0.2
332	A	5	4	1.	15	0.267
333	A	5	4	1.	15	0.267
334	A	2	1	1.	13	0.077
335	A	2	1	1.	13	0.077
336	A	2	1	1.	13	0.077
337	A	2	1	1.	11	0.091
338	A	1	1	1.	9	0.111
339	A	2	2	1.	13	0.154
340	A	3	3	1.	13	0.231
341	A	4	3	1.	13	0.231
342	A	5	3	1.	13	0.231
343	A	2	1	1.	13	0.077
344	A	2	1	1.	13	0.077
345	A	2	1	1.	13	0.077
346	A	2	1	1.	11	0.091
347	A	1	1	1.	9	0.111
348	A	3	3	1.	13	0.231
349	A	4	3	1.04	13	0.231
350	A	5	3	0.98	13	0.231
351	A	2	1	1.	13	0.077
352	A	2	1	1.	13	0.077
353	A	2	1	1.	13	0.077
354	A	2	1	1.	11	0.091
355	A	1	1	1.	9	0.111
356	A	4	3	1.	13	0.231
357	A	5	3	1.08	13	0.231
358	A	6	3	1.	13	0.231
359	A	2	2	1.	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
360	A	3	3	1.	15	0.2
361	A	4	3	1.	15	0.2
362	A	3	3	1.	15	0.2
363	A	4	3	1.05	15	0.2
364	A	5	3	0.98	15	0.2
365	A	4	3	1.	15	0.2
366	A	5	3	1.09	15	0.2
367	A	6	3	1.	15	0.2
368	A	2	2	1.	31	0.065
369	C	5	2	7.08	34	0.059
370	A	3	3	1.	29	0.103
371	A	2	1	1.	13	0.077
372	A	2	1	1.	13	0.077
373	A	2	1	1.	11	0.091
374	A	1	1	1.	9	0.111
375	A	5	5	1.	13	0.385
376	A	5	5	1.	13	0.385
377	A	6	6	1.	13	0.462
378	A	2	1	1.	13	0.077
379	A	2	1	1.	13	0.077
380	A	2	1	1.	11	0.091
381	A	1	1	1.	9	0.111
382	A	5	5	1.	13	0.385
383	A	5	5	1.	13	0.385
384	A	6	6	1.	13	0.462
385	A	2	1	1.	13	0.077
386	A	2	1	1.	13	0.077
387	A	2	1	1.	11	0.091
388	A	1	1	1.	9	0.111
389	A	6	5	1.	13	0.385
390	A	6	6	1.	13	0.462
391	A	6	5	1.	13	0.385
392	A	2	1	1.	13	0.077
393	A	2	1	1.	13	0.077
394	A	2	1	1.	11	0.091
395	A	1	1	1.	9	0.111
396	A	4	4	1.	13	0.308
397	A	5	5	1.	13	0.385
398	A	6	5	1.	13	0.385
399	A	2	1	1.	15	0.067
400	A	2	1	1.	15	0.067
401	A	2	1	1.	13	0.077
402	A	1	1	1.	11	0.091
403	A	4	4	1.	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
404	A	5	5	1.	15	0.333
405	A	6	5	1.	15	0.333
406	A	2	1	1.	13	0.077
407	A	2	1	1.	13	0.077
408	A	2	1	1.	11	0.091
409	A	1	1	1.	9	0.111
410	A	4	4	1.	13	0.308
411	A	5	5	1.	13	0.385
412	A	6	5	1.	13	0.385
413	A	2	1	1.	13	0.077
414	A	2	1	1.	13	0.077
415	A	2	1	1.	11	0.091
416	A	1	1	1.	9	0.111
417	A	5	5	1.	13	0.385
418	A	6	5	1.02	13	0.385
419	A	7	5	0.99	13	0.385
420	A	4	4	1.	17	0.235
421	A	4	4	1.	18	0.222
422	A	4	4	1.	19	0.21
423	A	4	4	1.	20	0.2
424	A	4	4	1.	17	0.235
425	A	4	4	1.	18	0.222
426	A	4	4	1.	19	0.21
427	A	4	4	1.	20	0.2
428	A	2	1	1.	9	0.111
429	A	2	1	1.	11	0.091
430	A	2	1	1.	11	0.091
431	A	2	1	1.	11	0.091
432	A	2	1	1.	11	0.091
433	A	2	1	1.	11	0.091
434	A	2	1	1.	11	0.091
435	A	2	1	1.	11	0.091
436	A	2	1	1.	13	0.077
437	A	2	1	1.	13	0.077
438	A	2	1	1.	13	0.077
439	A	2	1	1.	13	0.077
440	A	2	1	1.	13	0.077
441	A	2	1	1.	13	0.077
442	A	2	1	1.	11	0.091
443	A	2	1	1.	13	0.077
444	A	2	1	1.	13	0.077
445	A	2	1	1.	13	0.077
446	A	2	1	1.	13	0.077
447	A	2	1	1.	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
448	A	2	1	1.	13	0.077
449	A	5	3	1.	13	0.231
450	A	4	3	1.	13	0.231
451	A	3	3	1.	13	0.231
452	A	2	2	1.	13	0.154
453	A	3	3	1.	13	0.231
454	A	4	3	1.	13	0.231
455	A	5	3	1.	13	0.231
456	A	5	4	1.	13	0.308
457	A	4	4	1.	13	0.308
458	A	3	3	1.	13	0.231
459	A	3	3	1.	13	0.231
460	A	4	3	1.	13	0.231
461	A	5	3	1.	13	0.231
462	A	6	4	1.	13	0.308
463	A	5	4	1.	13	0.308
464	A	4	3	1.	13	0.231
465	A	4	4	1.	13	0.308
466	A	4	3	1.	13	0.231
467	A	5	3	1.	13	0.231
468	A	6	3	1.	13	0.231
469	A	5	3	1.	15	0.2
470	A	4	3	1.	15	0.2
471	A	3	3	1.	15	0.2
472	A	2	2	1.	15	0.133
473	A	3	3	1.	15	0.2
474	A	4	3	1.	15	0.2
475	A	5	3	1.	15	0.2
476	A	5	4	1.	15	0.267
477	A	4	4	1.	15	0.267
478	A	3	3	1.	15	0.2
479	A	3	3	1.	15	0.2
480	A	4	3	1.	15	0.2
481	A	5	3	1.	15	0.2
482	A	6	4	1.	15	0.267
483	A	5	4	1.	15	0.267
484	A	4	3	1.	15	0.2
485	A	4	4	1.	15	0.267
486	A	4	3	1.	15	0.2
487	A	5	3	1.	15	0.2
488	A	6	3	1.	15	0.2
489	A	7	4	1.	15	0.267
490	A	6	4	1.	15	0.267
491	A	5	4	1.	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
492	A	4	4	1.	15	0.267
493	A	4	4	1.	15	0.267
494	A	1	1	1.	15	0.067
495	A	2	2	1.	15	0.133
496	A	3	2	1.	15	0.133
497	A	7	4	1.	16	0.25
498	A	6	4	1.	16	0.25
499	A	5	4	1.	16	0.25
500	A	4	4	1.	16	0.25
501	A	4	4	1.	16	0.25
502	A	1	1	1.	16	0.062
503	A	2	2	1.	16	0.125
504	A	3	2	1.	16	0.125
505	A	6	3	1.	15	0.2
506	A	5	3	1.	15	0.2
507	A	4	3	1.	15	0.2
508	A	3	3	1.	15	0.2
509	A	3	3	1.	15	0.2
510	A	1	1	1.	15	0.067
511	A	2	2	1.	15	0.133
512	A	3	2	1.	15	0.133
513	A	6	3	1.	16	0.188
514	A	5	3	1.	16	0.188
515	A	4	3	1.	16	0.188
516	A	3	3	1.	16	0.188
517	A	3	3	1.	16	0.188
518	A	1	1	1.	16	0.062
519	A	2	2	1.	16	0.125
520	A	3	2	1.	16	0.125
521	A	8	4	1.	15	0.267
522	A	7	4	1.	15	0.267
523	A	6	4	1.	15	0.267
524	A	5	4	1.	15	0.267
525	A	5	5	1.	15	0.333
526	A	5	4	1.	15	0.267
527	A	8	4	1.	16	0.25
528	A	7	4	1.	16	0.25
529	A	6	4	1.	16	0.25
530	A	5	4	1.	16	0.25
531	A	5	5	1.	16	0.312
532	A	5	4	1.	16	0.25
533	A	7	3	1.	15	0.2
534	A	6	3	1.	15	0.2
535	A	5	3	1.	15	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
536	A	4	3	1.	15	0.2
537	A	4	4	1.	15	0.267
538	A	4	3	1.	15	0.2
539	A	7	3	1.	16	0.188
540	A	6	3	1.	16	0.188
541	A	5	3	1.	16	0.188
542	A	4	3	1.	16	0.188
543	A	4	4	1.	16	0.25
544	A	4	3	1.	16	0.188
545	A	9	4	1.	15	0.267
546	A	8	4	1.	15	0.267
547	A	7	4	1.	15	0.267
548	A	6	4	1.	15	0.267
549	A	6	5	1.	15	0.333
550	A	6	5	1.	15	0.333
551	A	9	4	1.	16	0.25
552	A	8	4	1.	16	0.25
553	A	7	4	1.	16	0.25
554	A	6	4	1.	16	0.25
555	A	6	5	1.	16	0.312
556	A	6	5	1.	16	0.312
557	A	8	3	1.	15	0.2
558	A	7	3	1.	15	0.2
559	A	6	3	1.	15	0.2
560	A	5	3	1.	15	0.2
561	A	5	4	1.	15	0.267
562	A	5	4	1.	15	0.267
563	A	8	3	1.	16	0.188
564	A	7	3	1.	16	0.188
565	A	6	3	1.	16	0.188
566	A	5	3	1.	16	0.188
567	A	5	4	1.	16	0.25
568	A	5	4	1.	16	0.25
569	A	6	4	1.	15	0.267
570	A	5	4	1.	15	0.267
571	A	4	4	1.	15	0.267
572	A	3	3	1.	15	0.2
573	A	1	1	1.	15	0.067
574	A	2	2	1.	15	0.133
575	A	3	2	1.	15	0.133
576	A	4	2	1.	15	0.133
577	A	6	5	1.	15	0.333
578	A	5	5	1.	15	0.333
579	A	4	4	1.	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
580	A	1	1	1.	15	0.067
581	A	2	2	1.	15	0.133
582	A	3	2	1.	15	0.133
583	A	4	2	1.	15	0.133
584	A	6	5	1.	15	0.333
585	A	5	4	1.	15	0.267
586	A	1	1	1.	15	0.067
587	A	2	2	1.	15	0.133
588	A	3	2	1.	15	0.133
589	A	4	2	1.	15	0.133
590	A	6	4	1.	16	0.25
591	A	5	4	1.	16	0.25
592	A	4	4	1.	16	0.25
593	A	3	3	1.	16	0.188
594	A	1	1	1.	16	0.062
595	A	2	2	1.	16	0.125
596	A	6	5	1.	16	0.312
597	A	5	5	1.	16	0.312
598	A	4	4	1.	16	0.25
599	A	1	1	1.	16	0.062
600	A	2	2	1.	16	0.125
601	A	3	2	1.	16	0.125
602	A	6	5	1.	16	0.312
603	A	5	4	1.	16	0.25
604	A	1	1	1.	16	0.062
605	A	2	2	1.	16	0.125
606	A	3	2	1.	16	0.125
607	A	4	2	1.	16	0.125
608	A	5	3	1.	15	0.2
609	A	4	3	1.	15	0.2
610	A	3	3	1.	15	0.2
611	A	2	2	1.	15	0.133
612	A	1	1	1.	15	0.067
613	A	2	2	1.	15	0.133
614	A	3	2	1.	15	0.133
615	A	4	2	1.	15	0.133
616	A	5	4	1.	15	0.267
617	A	4	4	1.	15	0.267
618	A	3	3	1.	15	0.2
619	A	1	1	1.	15	0.067
620	A	2	2	1.	15	0.133
621	A	3	2	1.	15	0.133
622	A	4	2	1.	15	0.133
623	A	5	4	1.	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
624	A	4	3	1.	15	0.2
625	A	1	1	1.	15	0.067
626	A	2	2	1.	15	0.133
627	A	3	2	1.	15	0.133
628	A	4	2	1.	15	0.133
629	A	5	3	1.	16	0.188
630	A	4	3	1.	16	0.188
631	A	3	3	1.	16	0.188
632	A	2	2	1.	16	0.125
633	A	1	1	1.	16	0.062
634	A	2	2	1.	16	0.125
635	A	5	4	1.	16	0.25
636	A	4	4	1.	16	0.25
637	A	3	3	1.	16	0.188
638	A	1	1	1.	16	0.062
639	A	2	2	1.	16	0.125
640	A	3	2	1.	16	0.125
641	A	5	4	1.	16	0.25
642	A	4	3	1.	16	0.188
643	A	1	1	1.	16	0.062
644	A	2	2	1.	16	0.125
645	A	3	2	1.	16	0.125
646	A	4	2	1.	16	0.125
647	A	4	4	1.	15	0.267
648	A	3	3	1.	15	0.2
649	A	2	2	1.	16	0.125
650	A	2	1	1.	11	0.091
651	A	2	1	1.	11	0.091
652	A	2	1	1.	11	0.091
653	A	2	1	1.	11	0.091
654	A	2	1	1.	11	0.091
655	A	2	1	1.	11	0.091
656	A	2	1	1.	11	0.091
657	A	2	1	1.	11	0.091
658	A	2	1	1.	13	0.077
659	A	2	1	1.	13	0.077
660	A	2	1	1.	13	0.077
661	A	2	1	1.	13	0.077
662	A	2	1	1.	13	0.077
663	A	2	1	1.	13	0.077
664	A	2	1	1.	13	0.077
665	A	2	1	1.	13	0.077
666	A	2	1	1.	13	0.077
667	A	2	1	1.	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
668	A	2	1	1.	13	0.077
669	A	2	1	1.	13	0.077
670	A	2	1	1.	13	0.077
671	A	2	1	1.	13	0.077
672	A	2	1	1.	13	0.077
673	A	2	1	1.	13	0.077
674	A	6	5	1.	13	0.385
675	A	6	5	1.	13	0.385
676	A	5	5	1.	13	0.385
677	A	5	5	1.	13	0.385
678	A	4	4	1.	13	0.308
679	A	4	4	1.	13	0.308
680	A	5	5	1.	13	0.385
681	A	5	5	1.	13	0.385
682	A	6	6	1.	13	0.462
683	A	6	6	1.	13	0.462
684	A	5	5	1.	13	0.385
685	A	5	5	1.	13	0.385
686	A	5	5	1.	13	0.385
687	A	5	5	1.	13	0.385
688	A	6	5	1.	13	0.385
689	A	6	5	1.	13	0.385
690	A	6	5	1.	13	0.385
691	A	6	5	1.	13	0.385
692	A	6	6	1.	13	0.462
693	A	6	6	1.	13	0.462
694	A	6	5	1.	13	0.385
695	A	6	5	1.	13	0.385
696	A	7	5	1.	13	0.385
697	A	7	5	1.	13	0.385
698	A	5	5	1.	15	0.333
699	A	2	1	1.	11	0.091
700	A	2	1	1.	11	0.091
701	A	2	1	1.	11	0.091
702	A	2	1	1.	11	0.091
703	A	2	1	1.	9	0.111
704	A	1	1	1.	11	0.091
705	A	1	1	1.	11	0.091
706	A	1	1	1.	11	0.091
707	A	2	2	1.	13	0.154
708	A	2	2	1.	13	0.154
709	A	2	2	1.	13	0.154
710	A	2	2	1.	13	0.154
711	A	2	2	1.	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
712	A	2	2	1.	13	0.154
713	A	2	2	1.	15	0.133
714	A	2	2	1.	15	0.133
715	A	2	2	1.	13	0.154
716	A	2	2	1.	15	0.133
717	A	2	2	1.	15	0.133
718	A	2	2	1.	15	0.133
719	A	1	1	1.	13	0.077
720	A	1	1	1.	13	0.077
721	A	1	1	1.	13	0.077
722	A	3	3	1.	13	0.231
723	A	2	2	1.	15	0.133
724	A	1	1	1.	15	0.067
725	A	1	1	1.	15	0.067
726	A	3	3	1.	15	0.2
727	A	1	1	1.	15	0.067
728	A	1	1	1.	13	0.077
729	A	1	1	1.	14	0.071
730	A	2	2	1.	11	0.182
731	A	2	2	1.	13	0.154
732	A	2	1	1.	11	0.091
733	A	2	1	1.	11	0.091
734	A	2	1	1.	9	0.111
735	A	1	1	1.	7	0.143
736	A	1	1	1.	11	0.091
737	A	1	1	1.	11	0.091
738	A	1	1	1.	11	0.091
739	A	3	2	1.	15	0.133
740	A	2	2	1.	15	0.133
741	A	1	1	1.	15	0.067
742	A	2	2	1.	15	0.133
743	A	2	2	1.	13	0.154
744	A	2	2	1.	15	0.133
745	A	2	2	1.	13	0.154
746	A	2	2	1.	13	0.154
747	A	2	2	1.	13	0.154
748	A	2	2	1.	13	0.154
749	A	2	2	1.	13	0.154
750	A	1	1	1.	13	0.077
751	A	1	1	1.	15	0.067
752	A	2	2	1.	13	0.154
753	A	1	1	1.	17	0.059
754	A	2	2	1.	15	0.133
755	A	2	2	1.	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
756	A	3	2	1.	18	0.111
757	A	3	2	1.	18	0.111
758	A	3	2	1.	16	0.125
759	A	3	2	1.	15	0.133
760	A	2	1	1.	18	0.056
761	A	3	2	1.	18	0.111
762	A	3	2	1.	18	0.111
763	A	2	2	1.	18	0.111
764	A	3	2	1.	18	0.111
765	A	3	2	1.	18	0.111
766	A	3	2	1.	16	0.125
767	A	3	2	1.	15	0.133
768	A	3	2	1.	18	0.111
769	A	3	2	1.	18	0.111
770	A	2	1	1.	18	0.056
771	A	3	2	1.	18	0.111
772	A	3	2	1.	18	0.111
773	A	3	2	1.	18	0.111
774	A	3	2	1.	16	0.125
775	A	3	2	1.	15	0.133
776	A	3	2	1.	18	0.111
777	A	3	2	1.	18	0.111
778	A	3	2	1.	18	0.111
779	A	3	2	1.	18	0.111
780	A	3	2	1.	18	0.111
781	A	3	2	1.	18	0.111
782	A	2	1	1.	16	0.062
783	A	3	2	1.	15	0.133
784	A	3	2	1.	18	0.111
785	A	2	2	1.	18	0.111
786	A	3	2	1.	18	0.111
787	A	3	2	1.	18	0.111
788	A	2	1	1.	18	0.056
789	A	3	2	1.	18	0.111
790	A	3	2	1.	16	0.125
791	A	2	2	1.	15	0.133
792	A	3	2	1.	18	0.111
793	A	3	2	1.	18	0.111
794	A	3	2	1.	18	0.111
795	A	3	2	1.	18	0.111
796	A	3	2	1.	18	0.111
797	A	2	2	1.	18	0.111
798	A	3	2	1.	16	0.125
799	A	3	2	1.	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
800	A	3	2	1.	18	0.111
801	A	3	2	1.	18	0.111
802	A	3	2	1.	18	0.111
803	A	3	2	1.	18	0.111
804	A	3	2	1.	20	0.1
805	A	3	2	1.	20	0.1
806	A	3	2	1.	18	0.111
807	A	3	2	1.	17	0.118
808	A	2	2	1.	20	0.1
809	A	3	2	1.	20	0.1
810	A	3	2	1.	20	0.1
811	A	3	2	1.	20	0.1
812	A	3	2	1.	20	0.1
813	A	3	2	1.	20	0.1
814	A	3	2	1.	18	0.111
815	A	3	2	1.	17	0.118
816	A	3	2	1.	20	0.1
817	A	3	2	1.	20	0.1
818	A	2	2	1.	20	0.1
819	A	3	2	1.	20	0.1
820	A	3	2	1.	18	0.111
821	A	3	2	1.	17	0.118
822	A	3	2	1.	20	0.1
823	A	3	2	1.	20	0.1
824	A	3	2	1.	20	0.1
825	A	3	2	1.	20	0.1
826	A	2	2	1.	20	0.1
827	A	3	2	1.	20	0.1
828	A	3	2	1.	20	0.1
829	A	3	2	1.	20	0.1
830	A	2	2	1.	18	0.111
831	A	3	2	1.	17	0.118
832	A	3	2	1.	20	0.1
833	A	3	2	1.	20	0.1
834	A	2	2	1.	20	0.1
835	A	3	2	1.	20	0.1
836	A	2	2	1.	20	0.1
837	A	3	2	1.	20	0.1
838	A	3	2	1.	18	0.111
839	A	3	2	1.	17	0.118
840	A	2	2	1.	20	0.1
841	A	3	2	1.	20	0.1
842	A	3	2	1.	20	0.1
843	A	3	2	1.	20	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
844	A	3	2	1.	20	0.1
845	A	3	2	1.	20	0.1
846	A	2	2	1.	18	0.111
847	A	3	2	1.	17	0.118
848	A	3	2	1.	20	0.1
849	A	3	2	1.	20	0.1
850	A	3	2	1.	20	0.1
851	A	3	2	1.	20	0.1
852	A	3	2	1.	20	0.1
853	A	3	2	1.	20	0.1
854	A	3	2	1.	18	0.111
855	A	3	2	1.	17	0.118
856	A	2	2	1.	20	0.1
857	A	4	4	1.	20	0.2
858	A	3	2	1.	20	0.1
859	A	3	2	1.	20	0.1
860	A	3	2	1.	18	0.111
861	A	3	2	1.	17	0.118
862	A	3	2	1.	20	0.1
863	A	3	2	1.	20	0.1
864	A	2	2	1.	20	0.1
865	A	4	4	1.	20	0.2
866	A	3	2	1.	20	0.1
867	A	3	2	1.	20	0.1
868	A	3	2	1.	20	0.1
869	A	3	2	1.	17	0.118
870	A	3	2	1.	20	0.1
871	A	3	2	1.	20	0.1
872	A	3	2	1.	20	0.1
873	A	3	2	1.	20	0.1
874	A	2	2	1.	20	0.1
875	A	4	4	1.	20	0.2
876	A	3	2	1.	20	0.1
877	A	3	2	1.	20	0.1
878	A	3	2	1.	20	0.1
879	A	3	2	1.	20	0.1
880	A	2	2	1.	18	0.111
881	A	4	4	1.	17	0.235
882	A	3	2	1.	20	0.1
883	A	3	2	1.	20	0.1
884	A	3	2	1.	20	0.1
885	A	3	2	1.	20	0.1
886	A	3	2	1.	20	0.1
887	A	3	2	1.	20	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
888	A	2	2	1.	20	0.1
889	A	4	4	1.	20	0.2
890	A	3	2	1.	18	0.111
891	A	3	2	1.	17	0.118
892	A	3	2	1.	20	0.1
893	A	3	2	1.	20	0.1
894	A	3	2	1.	20	0.1
895	A	3	2	1.	18	0.111
896	A	3	2	1.	17	0.118
897	A	2	2	1.	20	0.1
898	A	3	2	1.	20	0.1
899	A	3	2	1.	20	0.1
900	A	3	2	1.	20	0.1
901	A	3	2	1.	18	0.111
902	A	3	2	1.	17	0.118
903	A	3	2	1.	20	0.1
904	A	3	2	1.	20	0.1
905	A	2	2	1.	20	0.1
906	A	3	2	1.	20	0.1
907	A	3	2	1.	20	0.1
908	A	3	2	1.	20	0.1
909	A	3	2	1.	20	0.1
910	A	3	2	1.	20	0.1
911	A	3	2	1.	20	0.1
912	A	3	2	1.	20	0.1
913	A	2	2	1.	18	0.111
914	A	3	2	1.	17	0.118
915	A	3	2	1.	20	0.1
916	A	3	2	1.	20	0.1
917	A	3	2	1.	20	0.1
918	A	3	2	1.	20	0.1
919	A	2	2	1.	20	0.1
920	A	3	2	1.	20	0.1
921	A	3	2	1.	18	0.111
922	A	3	2	1.	17	0.118
923	A	3	2	1.	20	0.1
924	A	3	2	1.	18	0.111
925	A	3	2	1.	17	0.118
926	A	2	2	1.	20	0.1
927	A	2	2	1.	20	0.1
928	A	2	2	1.	20	0.1
929	A	2	2	1.	20	0.1
930	A	3	2	1.	18	0.111
931	A	3	2	1.	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
932	A	3	2	1.	20	0.1
933	A	3	2	1.	20	0.1
934	A	2	2	1.	20	0.1
935	A	2	2	1.	20	0.1
936	A	2	2	1.	20	0.1
937	A	2	2	1.	20	0.1
938	A	3	2	1.	17	0.118
939	A	3	2	1.	20	0.1
940	A	3	2	1.	20	0.1
941	A	3	2	1.	20	0.1
942	A	3	2	1.	20	0.1
943	A	2	2	1.	20	0.1
944	A	2	2	1.	20	0.1
945	A	2	2	1.	20	0.1
946	A	3	2	1.	20	0.1
947	A	3	2	1.	20	0.1
948	A	3	2	1.	20	0.1
949	A	2	2	1.	18	0.111
950	A	2	2	1.	17	0.118
951	A	2	2	1.	20	0.1
952	A	2	2	1.	20	0.1
953	A	3	2	1.	20	0.1
954	A	3	2	1.	20	0.1
955	A	3	2	1.	20	0.1
956	A	2	2	1.	20	0.1
957	A	2	2	1.	20	0.1
958	A	2	2	1.	18	0.111
959	A	2	2	1.	17	0.118
960	A	2	2	1.	20	0.1
961	A	3	2	1.	20	0.1
962	A	3	2	1.	20	0.1
963	A	3	2	1.	20	0.1
964	A	2	2	1.	20	0.1
965	A	2	2	1.	20	0.1
966	A	2	2	1.	20	0.1
967	A	2	2	1.	20	0.1
968	A	2	2	1.	18	0.111
969	A	4	3	1.	20	0.15
970	A	4	3	1.	20	0.15
971	A	4	3	1.	20	0.15
972	A	4	3	1.	20	0.15
973	A	4	3	1.	20	0.15
974	A	4	3	1.	20	0.15
975	A	4	3	1.	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
976	A	4	3	1.	22	0.136
977	A	4	3	1.	22	0.136
978	A	4	3	1.	22	0.136
979	A	4	3	1.	22	0.136
980	A	4	3	1.	22	0.136
981	A	4	4	1.	22	0.182
982	A	4	4	1.	22	0.182
983	A	4	4	1.	22	0.182
984	A	4	4	1.	22	0.182
985	A	4	4	1.	22	0.182
986	A	4	4	1.	22	0.182
987	A	2	2	1.	22	0.091
988	A	2	2	1.	22	0.091
989	A	2	2	1.	20	0.1
990	A	2	2	1.	19	0.105
991	A	2	2	1.	22	0.091
992	A	2	2	1.	20	0.1
993	A	2	2	1.	22	0.091
994	A	2	2	1.	22	0.091
995	A	2	2	1.	25	0.08
996	A	3	3	1.	27	0.111
997	A	3	3	1.	18	0.167
998	A	4	4	0.94	20	0.2
999	A	2	2	1.	20	0.1
1000	A	2	1	1.	20	0.05
1001	A	2	2	1.	20	0.1
1002	A	2	2	1.	20	0.1
1003	A	2	2	1.	18	0.111
1004	A	2	2	1.	20	0.1
1005	A	2	2	1.	20	0.1
1006	A	2	2	1.	20	0.1
1007	A	2	2	1.	20	0.1
1008	A	2	1	1.	20	0.05
1009	A	2	2	1.	20	0.1
1010	A	2	2	1.	20	0.1
1011	A	2	2	1.	18	0.111
1012	A	2	2	1.	20	0.1
1013	A	2	2	1.	20	0.1
1014	A	2	2	1.	20	0.1
1015	A	2	2	1.	18	0.111
1016	A	2	2	1.	18	0.111
1017	A	2	2	1.	18	0.111
1018	A	2	2	1.	16	0.125
1019	A	2	2	1.	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1020	A	2	2	1.	18	0.111
1021	A	2	2	1.	18	0.111
1022	A	2	1	1.	18	0.056
1023	A	2	2	1.	18	0.111
1024	A	2	2	1.	18	0.111
1025	A	2	2	1.	18	0.111
1026	A	2	2	1.	18	0.111
1027	A	2	2	1.	19	0.105
1028	A	2	1	1.	17	0.059
1029	A	2	1	1.	17	0.059
1030	A	2	1	1.	15	0.067
1031	A	1	0	1.	5	0.
1032	A	2	1	1.	17	0.059
1033	A	2	1	1.	17	0.059
1034	A	1	1	1.	17	0.059
1035	A	2	1	1.	17	0.059
1036	A	2	1	1.	17	0.059
1037	A	2	1	1.	17	0.059
1038	A	2	1	1.	19	0.053
1039	A	3	2	1.	19	0.105
1040	A	2	1	1.	17	0.059
1041	A	1	1	1.	7	0.143
1042	A	2	1	1.	19	0.053
1043	A	2	1	1.	19	0.053
1044	A	2	1	1.	19	0.053
1045	A	1	1	1.	19	0.053
1046	A	2	1	1.	19	0.053
1047	A	2	1	1.	19	0.053
1048	A	2	1	1.	19	0.053
1049	A	2	1	1.	19	0.053
1050	A	2	1	1.	19	0.053
1051	A	2	1	1.	17	0.059
1052	A	1	1	1.	7	0.143
1053	A	2	2	1.	19	0.105
1054	A	3	2	1.	19	0.105
1055	A	3	2	1.	19	0.105
1056	A	2	1	1.	19	0.053
1057	A	2	1	1.	19	0.053
1058	A	2	1	1.	17	0.059
1059	A	1	1	1.	7	0.143
1060	A	3	2	1.	19	0.105
1061	A	3	3	1.	19	0.158
1062	A	3	2	1.	19	0.105
1063	A	7	4	1.	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1064	A	6	4	1.	17	0.235
1065	A	5	4	1.	17	0.235
1066	A	4	4	1.	17	0.235
1067	A	3	3	1.	17	0.176
1068	A	3	3	1.	17	0.176
1069	A	3	3	1.	17	0.176
1070	A	1	1	1.	17	0.059
1071	A	2	2	1.	17	0.118
1072	A	3	2	1.	17	0.118
1073	A	4	2	1.	17	0.118
1074	A	5	2	1.	17	0.118
1075	A	7	4	1.	17	0.235
1076	A	6	4	1.	17	0.235
1077	A	5	4	1.	17	0.235
1078	A	4	3	1.	17	0.176
1079	A	4	4	1.	17	0.235
1080	A	4	3	1.	17	0.176
1081	A	4	4	1.	17	0.235
1082	A	4	3	1.	17	0.176
1083	A	1	1	1.	17	0.059
1084	A	2	2	1.	17	0.118
1085	A	3	2	1.	17	0.118
1086	A	4	2	1.	17	0.118
1087	A	5	2	1.	17	0.118
1088	A	8	4	1.	17	0.235
1089	A	7	4	1.	17	0.235
1090	A	6	4	1.	17	0.235
1091	A	5	3	1.	17	0.176
1092	A	5	4	1.	17	0.235
1093	A	5	4	1.	17	0.235
1094	A	5	3	1.	17	0.176
1095	A	5	4	1.	17	0.235
1096	A	5	4	1.	17	0.235
1097	A	5	3	1.	17	0.176
1098	A	1	1	1.	17	0.059
1099	A	2	2	1.	17	0.118
1100	A	3	2	1.	17	0.118
1101	A	4	2	1.	17	0.118
1102	A	5	2	1.	17	0.118
1103	A	6	2	1.	17	0.118
1104	A	4	3	1.	20	0.15
1105	A	3	3	1.	28	0.107
1106	A	6	3	1.	17	0.176
1107	A	5	3	1.	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1108	A	4	3	1.	17	0.176
1109	A	3	3	1.	17	0.176
1110	A	2	2	1.	17	0.118
1111	A	1	1	1.	17	0.059
1112	A	2	2	1.	17	0.118
1113	A	3	2	1.	17	0.118
1114	A	4	2	1.	17	0.118
1115	A	5	2	1.	17	0.118
1116	A	6	4	1.	17	0.235
1117	A	5	4	1.	17	0.235
1118	A	4	4	1.	17	0.235
1119	A	3	3	1.	17	0.176
1120	A	1	1	1.	17	0.059
1121	A	1	1	1.	17	0.059
1122	A	2	2	1.	17	0.118
1123	A	3	2	1.	17	0.118
1124	A	4	2	1.	17	0.118
1125	A	5	2	1.	17	0.118
1126	A	7	4	1.	17	0.235
1127	A	6	4	1.	17	0.235
1128	A	5	4	1.	17	0.235
1129	A	4	3	1.	17	0.176
1130	A	1	1	1.	17	0.059
1131	A	2	2	1.	17	0.118
1132	A	3	2	1.	17	0.118
1133	A	2	2	1.	17	0.118
1134	A	3	3	1.	17	0.176
1135	A	4	3	1.	17	0.176
1136	A	5	3	1.	17	0.176
1137	A	6	4	1.	20	0.2
1138	A	5	4	1.	20	0.2
1139	A	4	4	1.	20	0.2
1140	A	3	3	1.	20	0.15
1141	A	1	1	1.	20	0.05
1142	A	2	2	1.	20	0.1
1143	A	3	2	1.	20	0.1
1144	A	4	2	1.	20	0.1
1145	A	6	4	1.	23	0.174
1146	A	5	4	1.	23	0.174
1147	A	4	4	1.	23	0.174
1148	A	3	3	1.	23	0.13
1149	A	1	1	1.	23	0.043
1150	A	2	2	1.	23	0.087
1151	A	3	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1152	A	4	2	1.	23	0.087
1153	A	5	3	1.	19	0.158
1154	A	4	3	1.	19	0.158
1155	A	3	3	1.	19	0.158
1156	A	2	2	1.	19	0.105
1157	A	1	1	1.	19	0.053
1158	A	2	2	1.	19	0.105
1159	A	3	2	1.	19	0.105
1160	A	7	4	1.	17	0.235
1161	A	5	4	1.	17	0.235
1162	A	3	3	1.	17	0.176
1163	A	2	2	1.	17	0.118
1164	A	4	2	1.	17	0.118
1165	A	1	1	1.	17	0.059
1166	A	1	1	1.	20	0.05
1167	A	1	1	1.	17	0.059
1168	A	1	1	1.	20	0.05
1169	A	3	3	1.	23	0.13
1170	A	11	8	1.	20	0.4
1171	A	6	5	1.	25	0.2
1172	A	5	5	1.	25	0.2
1173	A	4	4	1.	25	0.16
1174	A	4	4	1.	25	0.16
1175	A	4	4	1.	25	0.16
1176	A	5	5	1.	25	0.2
1177	A	6	5	1.	25	0.2
1178	A	12	9	1.	25	0.36
1179	A	11	8	1.	25	0.32
1180	A	1	1	1.	25	0.04
1181	A	2	2	1.	25	0.08
1182	A	3	2	1.	25	0.08
1183	A	4	2	1.	25	0.08
1184	A	12	9	1.	25	0.36
1185	A	11	8	1.	25	0.32
1186	A	1	1	1.	25	0.04
1187	A	2	2	1.	25	0.08
1188	A	3	2	1.	25	0.08
1189	A	5	4	1.	25	0.16
1190	A	4	4	1.	25	0.16
1191	A	3	3	1.	25	0.12
1192	A	4	4	1.	25	0.16
1193	A	5	4	1.	25	0.16
1194	A	13	10	1.	25	0.4
1195	A	12	9	1.	25	0.36

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1196	A	1	1	1.	25	0.04
1197	A	2	2	1.	25	0.08
1198	A	3	2	1.	25	0.08
1199	A	6	5	1.	25	0.2
1200	A	5	5	1.	25	0.2
1201	A	4	4	1.	25	0.16
1202	A	4	4	1.	25	0.16
1203	A	4	4	1.	25	0.16
1204	A	5	5	1.	25	0.2
1205	A	6	5	1.	25	0.2
1206	A	6	6	1.	25	0.24
1207	A	5	5	1.	25	0.2
1208	A	4	4	1.	25	0.16
1209	A	3	3	1.	25	0.12
1210	A	4	4	1.	25	0.16
1211	A	5	4	1.	25	0.16
1212	A	13	10	1.	25	0.4
1213	A	12	9	1.	25	0.36
1214	A	1	1	1.	25	0.04
1215	A	2	2	1.	25	0.08
1216	A	3	2	1.	25	0.08
1217	A	6	5	1.	25	0.2
1218	A	5	5	1.	25	0.2
1219	A	4	4	1.	25	0.16
1220	A	5	5	1.	25	0.2
1221	A	4	4	1.	25	0.16
1222	A	5	5	1.	25	0.2
1223	A	6	5	1.	25	0.2
1224	A	13	9	1.	25	0.36
1225	A	1	1	1.	25	0.04
1226	A	2	2	1.	25	0.08
1227	A	3	2	1.	25	0.08
1228	A	4	2	1.	25	0.08
1229	A	2	1	1.	19	0.053
1230	A	2	1	1.	17	0.059
1231	A	1	1	1.	19	0.053
1232	A	1	1	1.	19	0.053
1233	A	3	3	1.	16	0.188
1234	A	3	3	1.	19	0.158
1235	A	2	2	1.	15	0.133
1236	A	2	1	1.	13	0.077
1237	A	2	1	1.	13	0.077
1238	A	2	1	1.	13	0.077
1239	A	2	1	1.	11	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1240	A	1	0	1.	5	0.
1241	A	2	1	1.	13	0.077
1242	A	2	1	1.	13	0.077
1243	A	1	1	1.	13	0.077
1244	A	2	1	1.	13	0.077
1245	A	2	1	1.	13	0.077
1246	A	2	1	1.	15	0.067
1247	A	2	1	1.	15	0.067
1248	A	2	1	1.	15	0.067
1249	A	2	1	1.	13	0.077
1250	A	1	1	1.	7	0.143
1251	A	2	1	1.	15	0.067
1252	A	2	1	1.	15	0.067
1253	A	2	1	1.	15	0.067
1254	A	1	1	1.	15	0.067
1255	A	2	1	1.	15	0.067
1256	A	2	1	1.	15	0.067
1257	A	2	1	1.	15	0.067
1258	A	2	1	1.	15	0.067
1259	A	2	1	1.	15	0.067
1260	A	2	1	1.	15	0.067
1261	A	2	1	1.	15	0.067
1262	A	2	1	1.	13	0.077
1263	A	1	1	1.	7	0.143
1264	A	2	1	1.	15	0.067
1265	A	2	1	1.	15	0.067
1266	A	2	1	1.	15	0.067
1267	A	2	1	1.	15	0.067
1268	A	1	1	1.	15	0.067
1269	A	2	2	1.	15	0.133
1270	A	2	1	1.	15	0.067
1271	A	2	1	1.	15	0.067
1272	A	2	1	1.	15	0.067
1273	A	2	1	1.	15	0.067
1274	A	2	1	1.	15	0.067
1275	A	2	1	1.	15	0.067
1276	A	2	1	1.	15	0.067
1277	A	2	1	1.	15	0.067
1278	A	2	1	1.	15	0.067
1279	A	2	1	1.	15	0.067
1280	A	2	1	1.	15	0.067
1281	A	2	1	1.	13	0.077
1282	A	1	1	1.	7	0.143
1283	A	2	1	1.	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1284	A	2	1	1.	15	0.067
1285	A	2	1	1.	15	0.067
1286	A	2	1	1.	15	0.067
1287	A	2	1	1.	15	0.067
1288	A	2	1	1.	15	0.067
1289	A	2	1	1.	15	0.067
1290	A	2	1	1.	15	0.067
1291	A	1	1	1.	15	0.067
1292	A	2	2	1.	15	0.133
1293	A	3	2	1.	15	0.133
1294	A	4	2	1.	15	0.133
1295	A	5	2	1.	15	0.133
1296	A	2	1	1.	15	0.067
1297	A	2	1	1.	15	0.067
1298	A	2	1	1.	15	0.067
1299	A	2	1	1.	15	0.067
1300	A	2	1	1.	15	0.067
1301	A	2	1	1.	15	0.067
1302	A	2	1	1.	15	0.067
1303	A	2	1	1.	15	0.067
1304	A	2	1	1.	15	0.067
1305	A	2	1	1.	15	0.067
1306	A	2	1	1.	15	0.067
1307	A	2	1	1.	15	0.067
1308	A	2	1	1.	15	0.067
1309	A	2	1	1.	15	0.067
1310	A	2	1	1.	13	0.077
1311	A	1	1	1.	7	0.143
1312	A	2	1	1.	15	0.067
1313	A	2	1	1.	15	0.067
1314	A	2	1	1.	15	0.067
1315	A	2	1	1.	15	0.067
1316	A	2	1	1.	15	0.067
1317	A	2	1	1.	15	0.067
1318	A	2	1	1.	15	0.067
1319	A	2	1	1.	15	0.067
1320	A	2	1	1.	15	0.067
1321	A	2	1	1.	15	0.067
1322	A	2	1	1.	15	0.067
1323	A	1	1	1.	15	0.067
1324	A	2	2	1.	15	0.133
1325	A	3	2	1.	15	0.133
1326	A	4	2	1.	15	0.133
1327	A	5	2	1.	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1328	A	6	2	1.	15	0.133
1329	A	7	2	1.	15	0.133
1330	A	8	2	1.	15	0.133
1331	A	2	1	1.	15	0.067
1332	A	2	1	1.	15	0.067
1333	A	2	1	1.	15	0.067
1334	A	2	1	1.	15	0.067
1335	A	2	1	1.	15	0.067
1336	A	2	1	1.	15	0.067
1337	A	2	1	1.	15	0.067
1338	A	2	1	1.	13	0.077
1339	A	1	1	1.	7	0.143
1340	A	3	2	1.	15	0.133
1341	A	2	1	1.	15	0.067
1342	A	2	1	1.	15	0.067
1343	A	2	1	1.	15	0.067
1344	A	2	1	1.	15	0.067
1345	A	2	1	1.	15	0.067
1346	A	2	1	1.	15	0.067
1347	A	2	1	1.	13	0.077
1348	A	1	1	1.	7	0.143
1349	A	2	1	1.	15	0.067
1350	A	2	1	1.	15	0.067
1351	A	2	1	1.	15	0.067
1352	A	2	1	1.	15	0.067
1353	A	2	1	1.	15	0.067
1354	A	2	1	1.	15	0.067
1355	A	2	1	1.	15	0.067
1356	A	2	1	1.	15	0.067
1357	A	1	1	1.	13	0.077
1358	A	1	1	1.	7	0.143
1359	A	2	1	1.	15	0.067
1360	A	2	1	1.	15	0.067
1361	A	2	1	1.	15	0.067
1362	A	2	1	1.	15	0.067
1363	A	2	1	1.	15	0.067
1364	A	2	1	1.	15	0.067
1365	A	1	1	1.	15	0.067
1366	A	2	2	1.	15	0.133
1367	A	3	2	1.	15	0.133
1368	A	2	1	1.	15	0.067
1369	A	2	1	1.	15	0.067
1370	A	2	1	1.	13	0.077
1371	A	1	1	1.	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1372	A	2	1	1.	15	0.067
1373	A	2	1	1.	15	0.067
1374	A	2	1	1.	15	0.067
1375	A	2	1	1.	17	0.059
1376	A	2	1	1.	17	0.059
1377	A	2	1	1.	17	0.059
1378	A	2	1	1.	17	0.059
1379	A	2	1	1.	15	0.067
1380	A	1	1	1.	9	0.111
1381	A	3	3	1.	17	0.176
1382	A	3	3	1.	17	0.176
1383	A	4	4	1.	17	0.235
1384	A	5	4	1.	17	0.235
1385	A	6	4	1.	17	0.235
1386	A	7	4	1.	17	0.235
1387	A	2	1	1.	17	0.059
1388	A	2	1	1.	17	0.059
1389	A	2	1	1.	17	0.059
1390	A	2	1	1.	17	0.059
1391	A	2	1	1.	15	0.067
1392	A	1	1	1.	9	0.111
1393	A	4	3	1.	17	0.176
1394	A	4	4	1.	17	0.235
1395	A	4	3	1.	17	0.176
1396	A	5	4	1.	17	0.235
1397	A	6	4	1.	17	0.235
1398	A	7	4	1.	17	0.235
1399	A	2	1	1.	17	0.059
1400	A	2	1	1.	17	0.059
1401	A	2	1	1.	17	0.059
1402	A	2	1	1.	17	0.059
1403	A	2	1	1.	15	0.067
1404	A	1	1	1.	9	0.111
1405	A	5	3	1.	17	0.176
1406	A	5	4	1.	17	0.235
1407	A	5	4	1.	17	0.235
1408	A	5	3	1.	17	0.176
1409	A	6	4	1.	17	0.235
1410	A	7	4	1.	17	0.235
1411	A	3	3	1.	13	0.231
1412	A	4	4	1.	13	0.308
1413	A	2	1	1.	17	0.059
1414	A	2	1	1.	17	0.059
1415	A	2	1	1.	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1416	A	2	1	1.	17	0.059
1417	A	2	1	1.	15	0.067
1418	A	1	1	1.	9	0.111
1419	A	2	2	1.	17	0.118
1420	A	3	3	1.	17	0.176
1421	A	4	3	1.	17	0.176
1422	A	5	3	1.	17	0.176
1423	A	6	3	1.	17	0.176
1424	A	2	1	1.	17	0.059
1425	A	2	1	1.	17	0.059
1426	A	2	1	1.	17	0.059
1427	A	2	1	1.	17	0.059
1428	A	2	1	1.	15	0.067
1429	A	1	1	1.	9	0.111
1430	A	3	3	1.	17	0.176
1431	A	4	3	1.	17	0.176
1432	A	5	3	1.	17	0.176
1433	A	6	3	1.	17	0.176
1434	A	2	1	1.	17	0.059
1435	A	2	1	1.	17	0.059
1436	A	2	1	1.	17	0.059
1437	A	2	1	1.	17	0.059
1438	A	2	1	1.	15	0.067
1439	A	1	1	1.	9	0.111
1440	A	4	3	1.	17	0.176
1441	A	5	3	1.	17	0.176
1442	A	6	3	1.	17	0.176
1443	A	7	3	1.	17	0.176
1444	A	2	2	1.	20	0.1
1445	A	2	2	1.	20	0.1
1446	A	2	2	1.	20	0.1
1447	A	2	2	1.	20	0.1
1448	A	2	2	1.	20	0.1
1449	A	2	2	1.	20	0.1
1450	A	2	2	1.	20	0.1
1451	A	2	2	1.	20	0.1
1452	A	2	2	1.	20	0.1
1453	A	2	2	1.	13	0.154
1454	A	2	2	1.	17	0.118
1455	A	5	5	1.	15	0.333
1456	A	2	1	1.	13	0.077
1457	A	2	1	1.	15	0.067
1458	A	4	4	1.	17	0.235
1459	A	4	4	1.	17	0.235

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1460	A	8	4	1.	19	0.21
1461	A	7	4	1.	19	0.21
1462	A	6	4	1.	19	0.21
1463	A	5	4	1.	19	0.21
1464	A	4	4	1.	19	0.21
1465	A	4	4	1.	19	0.21
1466	A	1	1	1.	19	0.053
1467	A	2	2	1.	19	0.105
1468	A	3	2	1.	19	0.105
1469	A	4	2	1.	19	0.105
1470	A	5	2	1.	19	0.105
1471	A	8	4	1.	19	0.21
1472	A	7	4	1.	19	0.21
1473	A	6	4	1.	19	0.21
1474	A	5	4	1.	19	0.21
1475	A	5	5	1.	19	0.263
1476	A	5	4	1.	19	0.21
1477	A	1	1	1.	19	0.053
1478	A	2	2	1.	19	0.105
1479	A	3	2	1.	19	0.105
1480	A	4	2	1.	19	0.105
1481	A	9	4	1.	19	0.21
1482	A	8	4	1.	19	0.21
1483	A	7	4	1.	19	0.21
1484	A	6	4	1.	19	0.21
1485	A	6	5	1.	19	0.263
1486	A	6	5	1.	19	0.263
1487	A	6	4	1.	19	0.21
1488	A	1	1	1.	19	0.053
1489	A	2	2	1.	19	0.105
1490	A	3	2	1.	19	0.105
1491	A	4	2	1.	19	0.105
1492	A	7	4	1.	19	0.21
1493	A	6	4	1.	19	0.21
1494	A	5	4	1.	19	0.21
1495	A	4	4	1.	19	0.21
1496	A	3	3	1.	19	0.158
1497	A	1	1	1.	19	0.053
1498	A	2	2	1.	19	0.105
1499	A	3	2	1.	19	0.105
1500	A	4	2	1.	19	0.105
1501	A	5	2	1.	19	0.105
1502	A	7	5	1.	19	0.263
1503	A	6	5	1.	19	0.263

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1504	A	5	5	1.	19	0.263
1505	A	4	4	1.	19	0.21
1506	A	1	1	1.	19	0.053
1507	A	2	2	1.	19	0.105
1508	A	3	2	1.	19	0.105
1509	A	4	2	1.	19	0.105
1510	A	5	2	1.	19	0.105
1511	A	6	2	1.	19	0.105
1512	A	8	5	1.	19	0.263
1513	A	7	5	1.	19	0.263
1514	A	6	5	1.	19	0.263
1515	A	5	4	1.	19	0.21
1516	A	1	1	1.	19	0.053
1517	A	2	2	1.	19	0.105
1518	A	3	2	1.	19	0.105
1519	A	4	2	1.	19	0.105
1520	A	5	2	1.	19	0.105
1521	A	6	2	1.	19	0.105
1522	A	2	2	1.	20	0.1
1523	A	2	2	1.	19	0.105
1524	A	2	2	1.	19	0.105
1525	A	2	2	1.	17	0.118
1526	A	2	2	1.	19	0.105
1527	A	1	1	1.	19	0.053
1528	A	2	2	1.	19	0.105
1529	A	2	2	1.	19	0.105
1530	A	1	1	1.	7	0.143
1531	A	2	2	1.	19	0.105
1532	A	2	2	1.	17	0.118
1533	A	2	2	1.	19	0.105
1534	A	1	1	1.	19	0.053
1535	A	2	2	1.	19	0.105
1536	A	3	3	1.	20	0.15
1537	A	2	2	1.	20	0.1
1538	A	3	3	1.	20	0.15
1539	A	3	3	1.	18	0.167
1540	A	3	3	1.	20	0.15
1541	A	2	2	1.	20	0.1
1542	A	3	3	1.	20	0.15
1543	A	2	2	1.	21	0.095
1544	A	1	1	1.	8	0.125
1545	A	2	2	1.	21	0.095
1546	A	2	2	1.	19	0.105
1547	A	2	2	1.	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1548	A	1	1	1.	21	0.048
1549	A	2	2	1.	21	0.095
1550	A	1	1	1.	19	0.053
1551	A	2	2	1.	29	0.069
1552	A	2	2	1.	15	0.133
1553	A	2	2	1.	19	0.105
1554	A	2	2	1.	29	0.069
1555	A	3	3	1.	15	0.2
1556	A	2	2	1.	15	0.133
1557	A	2	2	1.	19	0.105
1558	A	3	3	1.	20	0.15
1559	A	5	3	1.	19	0.158
1560	A	4	3	1.	19	0.158
1561	A	3	3	1.	19	0.158
1562	A	3	3	1.	19	0.158
1563	A	4	4	1.	19	0.21
1564	A	5	4	1.	19	0.21
1565	A	6	5	1.	19	0.263
1566	A	5	5	1.	19	0.263
1567	A	4	4	1.	19	0.21
1568	A	5	5	1.	19	0.263
1569	A	6	5	1.	19	0.263
1570	A	4	3	1.	19	0.158
1571	A	3	3	1.	19	0.158
1572	A	2	2	1.	19	0.105
1573	A	3	3	1.	19	0.158
1574	A	4	3	1.	19	0.158
1575	A	3	2	1.	19	0.105
1576	A	2	2	1.	19	0.105
1577	A	2	2	1.	19	0.105
1578	A	1	1	1.	19	0.053
1579	A	2	2	1.	19	0.105
1580	A	3	2	1.	19	0.105
1581	A	4	2	1.	19	0.105
1582	A	6	4	1.	19	0.21
1583	A	5	4	1.	19	0.21
1584	A	4	4	1.	19	0.21
1585	A	4	4	1.	19	0.21
1586	A	5	5	1.	19	0.263
1587	A	3	2	1.	19	0.105
1588	A	2	2	1.	19	0.105
1589	A	1	1	1.	19	0.053
1590	A	1	1	1.	19	0.053
1591	A	2	2	1.	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1592	A	3	2	1.	19	0.105
1593	A	4	2	1.	19	0.105
1594	A	8	6	1.	19	0.316
1595	A	7	6	1.	19	0.316
1596	A	6	6	1.	19	0.316
1597	A	5	5	1.	19	0.263
1598	A	6	6	1.	19	0.316
1599	A	7	6	1.	19	0.316
1600	A	8	6	1.	19	0.316
1601	A	3	2	1.	19	0.105
1602	A	2	2	1.	19	0.105
1603	A	1	1	1.	19	0.053
1604	A	1	1	1.	19	0.053
1605	A	2	2	1.	19	0.105
1606	A	3	2	1.	19	0.105
1607	A	4	2	1.	19	0.105
1608	A	6	4	1.	19	0.21
1609	A	5	4	1.	19	0.21
1610	A	4	4	1.	19	0.21
1611	A	3	3	1.	19	0.158
1612	A	4	4	1.	19	0.21
1613	A	5	4	1.	19	0.21
1614	A	6	4	1.	19	0.21
1615	A	4	3	1.	19	0.158
1616	A	3	3	1.	19	0.158
1617	A	2	2	1.	19	0.105
1618	A	1	1	1.	19	0.053
1619	A	2	2	1.	19	0.105
1620	A	3	2	1.	19	0.105
1621	A	4	2	1.	19	0.105
1622	A	8	7	1.	19	0.368
1623	A	7	7	1.	19	0.368
1624	A	6	6	1.	19	0.316
1625	A	6	6	1.	19	0.316
1626	A	7	6	1.	19	0.316
1627	A	8	6	1.	19	0.316
1628	A	2	2	1.	15	0.133
1629	A	6	4	1.	19	0.21
1630	A	5	4	1.	19	0.21
1631	A	4	4	1.	19	0.21
1632	A	4	4	1.	19	0.21
1633	A	5	5	1.	19	0.263
1634	A	6	5	1.	19	0.263
1635	A	10	8	1.	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1636	A	9	8	1.	19	0.421
1637	A	8	8	1.	19	0.421
1638	A	8	8	1.	19	0.421
1639	A	9	9	1.	19	0.474
1640	A	10	9	1.	19	0.474
1641	A	7	4	1.	19	0.21
1642	A	6	4	1.	19	0.21
1643	A	5	4	1.	19	0.21
1644	A	5	5	1.	19	0.263
1645	A	5	4	1.	19	0.21
1646	A	6	5	1.	19	0.263
1647	A	7	5	1.	19	0.263
1648	A	10	8	1.	19	0.421
1649	A	9	8	1.	19	0.421
1650	A	8	8	1.	19	0.421
1651	A	7	7	1.	19	0.368
1652	A	8	8	1.	19	0.421
1653	A	9	8	1.	19	0.421
1654	A	5	4	1.	19	0.21
1655	A	4	4	1.	19	0.21
1656	A	3	3	1.	19	0.158
1657	A	4	4	1.	19	0.21
1658	A	5	4	1.	19	0.21
1659	A	10	9	1.	19	0.474
1660	A	9	9	1.	19	0.474
1661	A	8	8	1.	19	0.421
1662	A	8	8	1.	19	0.421
1663	A	9	8	1.	19	0.421
1664	A	10	8	1.	19	0.421
1665	A	7	5	1.	19	0.263
1666	A	5	5	1.	19	0.263
1667	A	4	4	1.	19	0.21
1668	A	4	4	1.	19	0.21
1669	A	5	4	1.	19	0.21
1670	A	6	4	1.	19	0.21
1671	A	11	9	1.	19	0.474
1672	A	10	9	1.	19	0.474
1673	A	9	8	1.	19	0.421
1674	A	9	9	1.	19	0.474
1675	A	9	8	1.	19	0.421
1676	A	10	8	1.	19	0.421
1677	A	11	8	1.	19	0.421
1678	A	8	6	1.	19	0.316
1679	A	7	6	1.	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1680	A	7	7	1.	19	0.368
1681	A	7	6	1.	19	0.316
1682	A	1	1	1.	19	0.053
1683	A	2	2	1.	19	0.105
1684	A	3	2	1.	19	0.105
1685	A	4	2	1.	19	0.105
1686	A	7	4	1.	19	0.21
1687	A	6	4	1.	19	0.21
1688	A	5	4	1.	19	0.21
1689	A	5	5	1.	19	0.263
1690	A	5	4	1.	19	0.21
1691	A	6	5	1.	19	0.263
1692	A	7	5	1.	19	0.263
1693	A	7	6	1.	19	0.316
1694	A	6	6	1.	19	0.316
1695	A	5	5	1.	19	0.263
1696	A	1	1	1.	19	0.053
1697	A	2	2	1.	19	0.105
1698	A	3	2	1.	19	0.105
1699	A	4	2	1.	19	0.105
1700	A	7	6	1.	19	0.316
1701	A	6	6	1.	19	0.316
1702	A	5	5	1.	19	0.263
1703	A	6	6	1.	19	0.316
1704	A	7	6	1.	19	0.316
1705	A	7	6	1.	19	0.316
1706	A	6	6	1.	19	0.316
1707	A	5	5	1.	19	0.263
1708	A	1	1	1.	19	0.053
1709	A	2	2	1.	19	0.105
1710	A	3	2	1.	19	0.105
1711	A	4	2	1.	19	0.105
1712	A	5	4	1.	19	0.21
1713	A	4	4	1.	19	0.21
1714	A	3	3	1.	19	0.158
1715	A	4	4	1.	19	0.21
1716	A	5	4	1.	19	0.21
1717	A	7	7	1.	19	0.368
1718	A	6	6	1.	19	0.316
1719	A	1	1	1.	19	0.053
1720	A	2	2	1.	19	0.105
1721	A	3	2	1.	19	0.105
1722	A	4	2	1.	19	0.105
1723	A	8	7	1.	19	0.368

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1724	A	7	7	1.	19	0.368
1725	A	6	6	1.	19	0.316
1726	A	6	6	1.	19	0.316
1727	A	7	6	1.	19	0.316
1728	A	8	6	1.	19	0.316
1729	A	11	8	1.	20	0.4
1730	A	11	8	1.	20	0.4
1731	A	2	2	1.	19	0.105
1732	A	2	2	1.	19	0.105
1733	A	2	2	1.	19	0.105
1734	A	2	2	1.	19	0.105
1735	A	2	2	1.	19	0.105
1736	A	6	3	1.	19	0.158
1737	A	5	3	1.	19	0.158
1738	A	4	3	1.	19	0.158
1739	A	3	3	1.	19	0.158
1740	A	3	3	1.	19	0.158
1741	A	4	4	1.	19	0.21
1742	A	7	5	1.	19	0.263
1743	A	6	5	1.	19	0.263
1744	A	5	5	1.	19	0.263
1745	A	5	5	1.	19	0.263
1746	A	6	6	1.	19	0.316
1747	A	7	6	1.	19	0.316
1748	A	7	5	1.	19	0.263
1749	A	6	5	1.	19	0.263
1750	A	5	5	1.	19	0.263
1751	A	4	4	1.	19	0.21
1752	A	5	5	1.	19	0.263
1753	A	6	5	1.	19	0.263
1754	A	5	3	1.	19	0.158
1755	A	4	3	1.	19	0.158
1756	A	3	3	1.	19	0.158
1757	A	2	2	1.	19	0.105
1758	A	3	3	1.	19	0.158
1759	A	4	3	1.	19	0.158
1760	A	7	6	1.	19	0.316
1761	A	6	6	1.	19	0.316
1762	A	5	5	1.	19	0.263
1763	A	5	5	1.	19	0.263
1764	A	6	5	1.	19	0.263
1765	A	7	5	1.	19	0.263
1766	A	2	2	1.	19	0.105
1767	A	2	2	1.	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1768	A	2	2	1.	19	0.105
1769	A	2	2	1.	19	0.105
1770	A	2	2	1.	19	0.105
1771	A	2	2	1.	19	0.105
1772	A	14	9	1.	19	0.474
1773	A	13	9	1.	19	0.474
1774	A	13	9	1.	19	0.474
1775	A	1	1	1.	19	0.053
1776	A	2	2	1.	19	0.105
1777	A	3	2	1.	19	0.105
1778	A	4	2	1.	19	0.105
1779	A	14	9	1.	19	0.474
1780	A	13	9	1.	19	0.474
1781	A	13	9	1.	19	0.474
1782	A	1	1	1.	19	0.053
1783	A	2	2	1.	19	0.105
1784	A	3	2	1.	19	0.105
1785	A	4	2	1.	19	0.105
1786	A	2	2	1.	19	0.105
1787	A	2	2	1.	19	0.105
1788	A	2	2	1.	19	0.105
1789	A	2	2	1.	19	0.105
1790	A	2	2	1.	19	0.105
1791	A	2	2	1.	19	0.105
1792	A	2	2	1.	19	0.105
1793	A	2	2	1.	19	0.105
1794	A	2	2	1.	19	0.105
1795	A	2	2	1.	19	0.105
1796	A	2	2	1.	19	0.105
1797	A	2	2	1.	19	0.105
1798	A	14	9	1.	19	0.474
1799	A	14	10	1.	19	0.526
1800	A	14	9	1.	19	0.474
1801	A	1	1	1.	19	0.053
1802	A	2	2	1.	19	0.105
1803	A	3	2	1.	19	0.105
1804	A	4	2	1.	19	0.105
1805	A	14	9	1.	19	0.474
1806	A	13	9	1.	19	0.474
1807	A	12	8	1.	19	0.421
1808	A	1	1	1.	19	0.053
1809	A	2	2	1.	19	0.105
1810	A	3	2	1.	19	0.105
1811	A	4	2	1.	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1812	A	2	2	1.	19	0.105
1813	A	2	2	1.	19	0.105
1814	A	2	2	1.	19	0.105
1815	A	2	2	1.	19	0.105
1816	A	2	2	1.	19	0.105
1817	A	2	2	1.	19	0.105
1818	A	2	2	1.	19	0.105
1819	A	2	2	1.	19	0.105
1820	A	2	2	1.	19	0.105
1821	A	2	2	1.	19	0.105
1822	A	2	2	1.	19	0.105
1823	A	2	2	1.	19	0.105
1824	A	14	9	1.	19	0.474
1825	A	13	9	1.	19	0.474
1826	A	12	8	1.	19	0.421
1827	A	1	1	1.	19	0.053
1828	A	2	2	1.	19	0.105
1829	A	3	2	1.	19	0.105
1830	A	4	2	1.	19	0.105
1831	A	15	10	1.	19	0.526
1832	A	14	10	1.	19	0.526
1833	A	13	9	1.	19	0.474
1834	A	1	1	1.	19	0.053
1835	A	2	2	1.	19	0.105
1836	A	3	2	1.	19	0.105
1837	A	4	2	1.	19	0.105
1838	A	2	2	1.	19	0.105
1839	A	2	2	1.	19	0.105
1840	A	2	2	1.	19	0.105
1841	A	2	2	1.	19	0.105
1842	A	2	2	1.	19	0.105
1843	A	2	2	1.	19	0.105
1844	A	1	1	1.	16	0.062
1845	A	2	2	1.21	15	0.133
1846	A	2	1	1.	15	0.067
1847	A	2	1	1.	15	0.067
1848	A	2	1	1.	13	0.077
1849	A	1	1	1.	15	0.067
1850	A	1	1	1.	15	0.067
1851	A	1	1	1.	15	0.067
1852	A	2	1	1.	15	0.067
1853	A	2	1	1.	15	0.067
1854	A	2	1	1.	13	0.077
1855	A	1	1	1.	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1856	A	1	1	1.	15	0.067
1857	A	1	1	1.	15	0.067
1858	A	1	1	1.	15	0.067
1859	A	3	2	1.	19	0.105
1860	A	2	2	1.	19	0.105
1861	A	1	1	1.	19	0.053
1862	A	2	2	1.	19	0.105
1863	A	2	2	1.	17	0.118
1864	A	2	2	1.	19	0.105
1865	A	2	2	1.	19	0.105
1866	A	2	2	1.	17	0.118
1867	A	2	2	1.	19	0.105
1868	A	1	1	1.	19	0.053
1869	A	2	2	1.	19	0.105
1870	A	3	2	1.	19	0.105
1871	A	4	2	1.	19	0.105
1872	A	2	2	1.	17	0.118
1873	A	2	2	1.	19	0.105
1874	A	1	1	1.	19	0.053
1875	A	2	2	1.	19	0.105
1876	A	3	2	1.	19	0.105
1877	A	4	2	1.	19	0.105
1878	A	2	2	1.	21	0.095
1879	A	2	2	1.	21	0.095
1880	A	2	2	1.	24	0.083
1881	A	3	2	1.	24	0.083
1882	A	2	2	1.	28	0.071
1883	A	2	2	1.	44	0.045
1884	A	2	2	1.	51	0.039
1885	A	1	1	1.	15	0.067
1886	A	1	1	1.	15	0.067
1887	A	1	1	1.	15	0.067
1888	A	1	1	1.	15	0.067
1889	A	1	1	1.	17	0.059
1890	A	2	2	1.	27	0.074
1891	A	1	0	1.	15	0.
1892	A	1	0	1.	9	0.
1893	A	1	0	1.	5	0.
1894	A	1	0	1.	5	0.
1895	A	1	0	1.	9	0.
1896	A	1	0	1.	9	0.
1897	A	1	0	1.	15	0.
1898	A	1	0	1.	10	0.
1899	A	1	0	1.	10	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1900	A	1	0	1.	12	0.
1901	A	1	0	1.	15	0.
1902	A	1	0	1.	17	0.
1903	A	1	0	1.	8	0.
1904	A	1	0	1.	10	0.
1905	A	1	0	1.	11	0.
1906	A	1	0	1.	11	0.
1907	A	1	0	1.	6	0.
1908	A	1	0	1.	11	0.
1909	A	1	0	1.	10	0.
1910	A	1	0	1.	11	0.
1911	A	1	0	1.	7	0.
1912	A	1	0	1.	17	0.
1913	A	1	0	1.	18	0.
1914	A	1	0	1.	11	0.
1915	A	1	0	1.	15	0.
1916	A	1	0	1.	18	0.
1917	A	1	0	1.	20	0.

Chapter 3

Listing of integrals

3.1 $\int 0\,dx$

Optimal. Leaf size=1

$$0$$

```
[Out] 0
```

Rubi [A] time = 0.0000911, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {8}

$$0$$

Antiderivative was successfully verified.

```
[In] Int[0,x]

[Out] 0
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 0\,dx=0$$

Mathematica [A] time = 0.0000228, size = 1, normalized size = 1.

$$0$$

Antiderivative was successfully verified.

```
[In] Integrate[0,x]
```

```
[Out] 0
```

Maple [A] time = 0., size = 2, normalized size = 2.

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(0,x)
```

```
[Out] 0
```

Maxima [A] time = 1.01215, size = 1, normalized size = 1.

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="maxima")
```

```
[Out] 0
```

Fricas [A] time = 1.61545, size = 4, normalized size = 4.

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [A] time = 0.011118, size = 0, normalized size = 0.

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x)
```

```
[Out] 0
```

Giac [A] time = 1.0815, size = 1, normalized size = 1.

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="giac")
```

```
[Out] 0
```

3.2 $\int 1\,dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A] time = 0.0000688, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {8}

$$x$$

Antiderivative was successfully verified.

[In] Int[1,x]

[Out] x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 1\,dx = x$$

Mathematica [A] time = 0.000033, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

[In] Integrate[1,x]

[Out] x

Maple [A] time = 0., size = 2, normalized size = 2.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1,x)

[Out] x

Maxima [A] time = 0.940239, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.56468, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [A] time = 0.01688, size = 0, normalized size = 0.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x)
```

```
[Out] x
```

Giac [A] time = 1.09401, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="giac")
```

```
[Out] x
```

3.3 $\int 5\,dx$

Optimal. Leaf size=3

$$5x$$

```
[Out] 5*x
```

Rubi [A] time = 0.0002996, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {8}

$$5x$$

Antiderivative was successfully verified.

```
[In] Int[5,x]
```

```
[Out] 5*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 5\,dx = 5x$$

Mathematica [A] time = 0.0001866, size = 3, normalized size = 1.

$$5x$$

Antiderivative was successfully verified.

```
[In] Integrate[5,x]
```

```
[Out] 5*x
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5,x)
```

```
[Out] 5*x
```

Maxima [A] time = 0.944133, size = 4, normalized size = 1.33

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x, algorithm="maxima")
```

```
[Out] 5*x
```

Fricas [A] time = 1.6147, size = 7, normalized size = 2.33

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x, algorithm="fricas")
```

```
[Out] 5*x
```

Sympy [A] time = 0.016212, size = 2, normalized size = 0.67

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x)
```

```
[Out] 5*x
```

Giac [A] time = 1.08952, size = 4, normalized size = 1.33

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x, algorithm="giac")
```

```
[Out] 5*x
```

3.4 $\int -2\,dx$

Optimal. Leaf size=3

$$-2x$$

```
[Out] -2*x
```

Rubi [A] time = 0.0002742, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {8}

$$-2x$$

Antiderivative was successfully verified.

```
[In] Int[-2,x]
```

```
[Out] -2*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int -2\,dx = -2x$$

Mathematica [A] time = 0.0000265, size = 3, normalized size = 1.

$$-2x$$

Antiderivative was successfully verified.

```
[In] Integrate[-2,x]
```

```
[Out] -2*x
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2,x)
```

```
[Out] -2*x
```

Maxima [A] time = 0.94299, size = 4, normalized size = 1.33

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="maxima")
```

```
[Out] -2*x
```

Fricas [A] time = 1.63651, size = 8, normalized size = 2.67

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="fricas")
```

```
[Out] -2*x
```

Sympy [A] time = 0.016675, size = 3, normalized size = 1.

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x)
```

```
[Out] -2*x
```

Giac [A] time = 1.08113, size = 4, normalized size = 1.33

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="giac")
```

```
[Out] -2*x
```

3.5 $\int -\frac{3}{2}\,dx$

Optimal. Leaf size=5

$$-\frac{3x}{2}$$

```
[Out] (-3*x)/2
```

Rubi [A] time = 0.0002704, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {8}

$$-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[-3/2,x]
```

```
[Out] (-3*x)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int -\frac{3}{2}\,dx = -\frac{3x}{2}$$

Mathematica [A] time = 0.0001681, size = 5, normalized size = 1.

$$-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/2,x]
```

```
[Out] (-3*x)/2
```

Maple [A] time = 0., size = 4, normalized size = 0.8

$$-\frac{3\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/2,x)
```

```
[Out] -3/2*x
```

Maxima [A] time = 0.960652, size = 4, normalized size = 0.8

$$-\frac{3}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="maxima")
```

```
[Out] -3/2*x
```

Fricas [A] time = 1.65863, size = 11, normalized size = 2.2

$$-\frac{3}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="fricas")
```

```
[Out] -3/2*x
```

Sympy [A] time = 0.016378, size = 5, normalized size = 1.

$$-\frac{3x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x)
```

```
[Out] -3*x/2
```

Giac [A] time = 1.12385, size = 4, normalized size = 0.8

$$-\frac{3}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="giac")
```

```
[Out] -3/2*x
```

3.6 $\int \pi \, dx$

Optimal. Leaf size=3

$$\pi x$$

```
[Out] Pi*x
```

Rubi [A] time = 0.0005006, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {8}

$$\pi x$$

Antiderivative was successfully verified.

```
[In] Int[Pi,x]
```

```
[Out] Pi*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \pi \, dx = \pi x$$

Mathematica [A] time = 0.0001817, size = 3, normalized size = 1.

$$\pi x$$

Antiderivative was successfully verified.

```
[In] Integrate[Pi,x]
```

```
[Out] Pi*x
```

Maple [A] time = 0.001, size = 4, normalized size = 1.3

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi,x)
```

```
[Out] Pi*x
```

Maxima [A] time = 0.939773, size = 4, normalized size = 1.33

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x, algorithm="maxima")
```

```
[Out] pi*x
```

Fricas [A] time = 1.47275, size = 8, normalized size = 2.67

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x, algorithm="fricas")
```

```
[Out] pi*x
```

Sympy [A] time = 0.016236, size = 2, normalized size = 0.67

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x)
```

```
[Out] pi*x
```

Giac [A] time = 1.12539, size = 4, normalized size = 1.33

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x, algorithm="giac")
```

```
[Out] pi*x
```

3.7 $\int a\,dx$

Optimal. Leaf size=3

$$ax$$

```
[Out] a*x
```

Rubi [A] time = 0.0003791, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {8}

$$ax$$

Antiderivative was successfully verified.

```
[In] Int[a,x]
```

```
[Out] a*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int a\,dx = ax$$

Mathematica [A] time = 0.0001677, size = 3, normalized size = 1.

$$ax$$

Antiderivative was successfully verified.

```
[In] Integrate[a,x]
```

```
[Out] a*x
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a,x)
```

```
[Out] a*x
```

Maxima [A] time = 0.923964, size = 4, normalized size = 1.33

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x, algorithm="maxima")
```

```
[Out] a*x
```

Fricas [A] time = 1.28907, size = 7, normalized size = 2.33

$$xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x, algorithm="fricas")
```

```
[Out] x*a
```

Sympy [A] time = 0.016325, size = 2, normalized size = 0.67

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x)
```

```
[Out] a*x
```

Giac [A] time = 1.10167, size = 4, normalized size = 1.33

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x, algorithm="giac")
```

```
[Out] a*x
```

3.8 $\int 3a\,dx$

Optimal. Leaf size=4

$$3ax$$

[Out] 3*a*x

Rubi [A] time = 0.0004491, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {8}

$$3ax$$

Antiderivative was successfully verified.

[In] Int[3*a,x]

[Out] 3*a*x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 3a\,dx = 3ax$$

Mathematica [A] time = 0.0001987, size = 4, normalized size = 1.

$$3ax$$

Antiderivative was successfully verified.

[In] Integrate[3*a,x]

[Out] 3*a*x

Maple [A] time = 0.001, size = 5, normalized size = 1.3

$$3\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(3*a,x)

[Out] 3*a*x

Maxima [A] time = 0.942301, size = 5, normalized size = 1.25

$$3\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x, algorithm="maxima")
```

```
[Out] 3*a*x
```

Fricas [A] time = 1.23851, size = 9, normalized size = 2.25

$$3xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x, algorithm="fricas")
```

```
[Out] 3*x*a
```

Sympy [A] time = 0.017083, size = 3, normalized size = 0.75

$$3ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x)
```

```
[Out] 3*a*x
```

Giac [A] time = 1.09375, size = 5, normalized size = 1.25

$$3\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x, algorithm="giac")
```

```
[Out] 3*a*x
```

3.9 $\int \frac{\pi}{\sqrt{16-e^2}} \, dx$

Optimal. Leaf size=14

$$\frac{\pi x}{\sqrt{16-e^2}}$$

```
[Out] (Pi*x)/Sqrt[16 - E^2]
```

Rubi [A] time = 0.0077652, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {8}

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Pi/Sqrt[16 - E^2],x]

[Out] (Pi*x)/Sqrt[16 - E^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\pi}{\sqrt{16-e^2}} \, dx = \frac{\pi x}{\sqrt{16-e^2}}$$

Mathematica [A] time = 0.0000276, size = 14, normalized size = 1.

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Pi/Sqrt[16 - E^2],x]

[Out] (Pi*x)/Sqrt[16 - E^2]
```

Maple [A] time = 0., size = 12, normalized size = 0.9

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/(16-exp(2))^(1/2),x)
```

```
[Out] Pi*x/(16-exp(2))^(1/2)
```

Maxima [A] time = 0.943684, size = 15, normalized size = 1.07

$$\frac{\pi x}{\sqrt{-e^2+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="maxima")
```

```
[Out] pi*x/sqrt(-e^2 + 16)
```

Fricas [A] time = 1.54692, size = 46, normalized size = 3.29

$$-\frac{\pi x\sqrt{-e^2+16}}{e^2-16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="fricas")
```

```
[Out] -pi*x*sqrt(-e^2 + 16)/(e^2 - 16)
```

Sympy [A] time = 0.045001, size = 10, normalized size = 0.71

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))**(1/2),x)
```

```
[Out] pi*x/sqrt(16 - exp(2))
```

Giac [A] time = 1.11218, size = 15, normalized size = 1.07

$$\frac{\pi x}{\sqrt{-e^2+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="giac")
```

```
[Out] pi*x/sqrt(-e^2 + 16)
```

3.10 $\int x^{100}\,dx$

Optimal. Leaf size=7

$$\frac{x^{101}}{101}$$

```
[Out] x^101/101
```

Rubi [A] time = 0.0004215, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\frac{x^{101}}{101}$$

Antiderivative was successfully verified.

```
[In] Int[x^100,x]
```

```
[Out] x^101/101
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{100}\,dx = \frac{x^{101}}{101}$$

Mathematica [A] time = 0.0002596, size = 7, normalized size = 1.

$$\frac{x^{101}}{101}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^100,x]
```

```
[Out] x^101/101
```

Maple [A] time = 0., size = 6, normalized size = 0.9

$$\frac{x^{101}}{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^100,x)
```

```
[Out] 1/101*x^101
```

Maxima [A] time = 1.01248, size = 7, normalized size = 1.

$$\frac{1}{101}\,x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x, algorithm="maxima")
```

```
[Out] 1/101*x^101
```

Fricas [A] time = 1.29079, size = 18, normalized size = 2.57

$$\frac{1}{101}x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x, algorithm="fricas")
```

```
[Out] 1/101*x^101
```

Sympy [A] time = 0.048739, size = 3, normalized size = 0.43

$$\frac{x^{101}}{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**100,x)
```

```
[Out] x**101/101
```

Giac [A] time = 1.09151, size = 7, normalized size = 1.

$$\frac{1}{101}\,x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x, algorithm="giac")
```

```
[Out] 1/101*x^101
```

3.11 $\int x^3\,dx$

Optimal. Leaf size=7

$$\frac{x^4}{4}$$

```
[Out] x^4/4
```

Rubi [A] time = 0.0003908, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3,x]
```

```
[Out] x^4/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3\,dx=\frac{x^4}{4}$$

Mathematica [A] time = 0.0001082, size = 7, normalized size = 1.

$$\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3,x]
```

```
[Out] x^4/4
```

Maple [A] time = 0., size = 6, normalized size = 0.9

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3,x)
```

```
[Out] 1/4*x^4
```

Maxima [A] time = 0.948067, size = 7, normalized size = 1.

$$\frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4
```

Fricas [A] time = 1.2617, size = 12, normalized size = 1.71

$$\frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4
```

Sympy [A] time = 0.04775, size = 3, normalized size = 0.43

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3,x)
```

```
[Out] x**4/4
```

Giac [A] time = 1.08909, size = 7, normalized size = 1.

$$\frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4
```

3.12 $\int x^2 \, dx$

Optimal. Leaf size=7

$$\frac{x^3}{3}$$

```
[Out] x^3/3
```

Rubi [A] time = 0.0004105, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2,x]

[Out] x^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2 \, dx = \frac{x^3}{3}$$

Mathematica [A] time = 0.0000218, size = 7, normalized size = 1.

$$\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2,x]

[Out] x^3/3
```

Maple [A] time = 0.002, size = 6, normalized size = 0.9

$$\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2,x)
```

```
[Out] 1/3*x^3
```

Maxima [A] time = 0.976844, size = 7, normalized size = 1.

$$\frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3
```

Fricas [A] time = 1.32392, size = 12, normalized size = 1.71

$$\frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3
```

Sympy [A] time = 0.047793, size = 3, normalized size = 0.43

$$\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2,x)
```

```
[Out] x**3/3
```

Giac [A] time = 1.27413, size = 7, normalized size = 1.

$$\frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3
```

3.13 $\int x\,dx$

Optimal. Leaf size=7

$$\frac{x^2}{2}$$

[Out] x^2/2

Rubi [A] time = 0.0004007, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {30}

$$\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[x,x]

[Out] x^2/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x\,dx = \frac{x^2}{2}$$

Mathematica [A] time = 0.0002987, size = 7, normalized size = 1.

$$\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x,x]

[Out] x^2/2

Maple [A] time = 0., size = 6, normalized size = 0.9

$$\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x,x)

```
[Out] 1/2*x^2
```

Maxima [A] time = 0.960682, size = 7, normalized size = 1.

$$\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2
```

Fricas [A] time = 1.34425, size = 12, normalized size = 1.71

$$\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2
```

Sympy [A] time = 0.017683, size = 3, normalized size = 0.43

$$\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x)
```

```
[Out] x**2/2
```

Giac [A] time = 1.27298, size = 7, normalized size = 1.

$$\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="giac")
```

```
[Out] 1/2*x^2
```

3.14 $\int 1\,dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A] time = 0.0000875, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {8}

$$x$$

Antiderivative was successfully verified.

[In] Int[1,x]

[Out] x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 1\,dx = x$$

Mathematica [A] time = 0.0000176, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

[In] Integrate[1,x]

[Out] x

Maple [A] time = 0., size = 2, normalized size = 2.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1,x)

[Out] x

Maxima [A] time = 0.967602, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.37615, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [A] time = 0.016652, size = 0, normalized size = 0.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x)
```

```
[Out] x
```

Giac [A] time = 1.09408, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="giac")
```

```
[Out] x
```

3.15 $\int \frac{1}{x} \, dx$

Optimal. Leaf size=2

$$\log(x)$$

[Out] Log[x]

Rubi [A] time = 0.0001859, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {29}

$$\log(x)$$

Antiderivative was successfully verified.

[In] Int[x^(-1),x]

[Out] Log[x]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x} \, dx = \log(x)$$

Mathematica [A] time = 0.0000923, size = 2, normalized size = 1.

$$\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1),x]

[Out] Log[x]

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x,x)

[Out] ln(x)

Maxima [A] time = 0.972831, size = 3, normalized size = 1.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

Fricas [A] time = 1.62448, size = 11, normalized size = 5.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
```

```
[Out] log(x)
```

Sympy [A] time = 0.050127, size = 2, normalized size = 1.

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
```

```
[Out] log(x)
```

Giac [A] time = 1.10212, size = 4, normalized size = 2.

$$\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="giac")
```

```
[Out] log(abs(x))
```

3.16 $\int \frac{1}{x^2} dx$

Optimal. Leaf size=5

$$-\frac{1}{x}$$

[Out] -x^(-1)

Rubi [A] time = 0.0005682, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[x^(-2),x]

[Out] -x^(-1)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2} dx = -\frac{1}{x}$$

Mathematica [A] time = 0.0004322, size = 5, normalized size = 1.

$$-\frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-2),x]

[Out] -x^(-1)

Maple [A] time = 0.001, size = 6, normalized size = 1.2

$$-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2,x)

```
[Out] -1/x
```

Maxima [A] time = 0.970355, size = 7, normalized size = 1.4

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="maxima")
```

```
[Out] -1/x
```

Fricas [A] time = 1.45116, size = 8, normalized size = 1.6

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="fricas")
```

```
[Out] -1/x
```

Sympy [A] time = 0.051356, size = 3, normalized size = 0.6

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2,x)
```

```
[Out] -1/x
```

Giac [A] time = 1.1073, size = 7, normalized size = 1.4

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="giac")
```

```
[Out] -1/x
```

3.17 $\int \frac{1}{x^3} dx$

Optimal. Leaf size=7

$$-\frac{1}{2x^2}$$

```
[Out] -1/(2*x^2)
```

Rubi [A] time = 0.0004021, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3),x]

[Out] -1/(2*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3} dx = -\frac{1}{2x^2}$$

Mathematica [A] time = 0.0001609, size = 7, normalized size = 1.

$$-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3),x]

[Out] -1/(2*x^2)
```

Maple [A] time = 0., size = 6, normalized size = 0.9

$$-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3,x)
```

```
[Out] -1/2/x^2
```

Maxima [A] time = 0.97134, size = 7, normalized size = 1.

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="maxima")
```

```
[Out] -1/2/x^2
```

Fricas [A] time = 1.45694, size = 14, normalized size = 2.

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="fricas")
```

```
[Out] -1/2/x^2
```

Sympy [A] time = 0.051958, size = 7, normalized size = 1.

$$-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3,x)
```

```
[Out] -1/(2*x**2)
```

Giac [A] time = 1.089, size = 7, normalized size = 1.

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="giac")
```

```
[Out] -1/2/x^2
```

3.18 $\int \frac{1}{x^4} \, dx$

Optimal. Leaf size=7

$$-\frac{1}{3x^3}$$

```
[Out] -1/(3*x^3)
```

Rubi [A] time = 0.0004275, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4),x]

[Out] -1/(3*x^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4} \, dx = -\frac{1}{3x^3}$$

Mathematica [A] time = 0.0001591, size = 7, normalized size = 1.

$$-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4),x]

[Out] -1/(3*x^3)
```

Maple [A] time = 0., size = 6, normalized size = 0.9

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4,x)
```

```
[Out] -1/3/x^3
```

Maxima [A] time = 0.970812, size = 7, normalized size = 1.

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x, algorithm="maxima")
```

```
[Out] -1/3/x^3
```

Fricas [A] time = 1.45742, size = 14, normalized size = 2.

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x, algorithm="fricas")
```

```
[Out] -1/3/x^3
```

Sympy [A] time = 0.050997, size = 7, normalized size = 1.

$$-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4,x)
```

```
[Out] -1/(3*x**3)
```

Giac [A] time = 1.15655, size = 7, normalized size = 1.

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x, algorithm="giac")
```

```
[Out] -1/3/x^3
```

3.19 $\int \frac{1}{x^{100}}\,dx$

Optimal. Leaf size=7

$$-\frac{1}{99x^{99}}$$

```
[Out] -1/(99*x^99)
```

Rubi [A] time = 0.0004328, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{99x^{99}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-100),x]

[Out] -1/(99*x^99)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{100}}\,dx = -\frac{1}{99x^{99}}$$

Mathematica [A] time = 0.0001873, size = 7, normalized size = 1.

$$-\frac{1}{99x^{99}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-100),x]

[Out] -1/(99*x^99)
```

Maple [A] time = 0., size = 6, normalized size = 0.9

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^100,x)
```

```
[Out] -1/99/x^99
```

Maxima [A] time = 0.948583, size = 7, normalized size = 1.

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="maxima")
```

```
[Out] -1/99/x^99
```

Fricas [A] time = 1.62135, size = 16, normalized size = 2.29

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="fricas")
```

```
[Out] -1/99/x^99
```

Sympy [A] time = 0.051427, size = 7, normalized size = 1.

$$-\frac{1}{99x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**100,x)
```

```
[Out] -1/(99*x**99)
```

Giac [A] time = 1.11222, size = 7, normalized size = 1.

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="giac")
```

```
[Out] -1/99/x^99
```

3.20 $\int x^{5/2} dx$

Optimal. Leaf size=9

$$\frac{2x^{7/2}}{7}$$

[Out] (2*x^(7/2))/7

Rubi [A] time = 0.000528, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$\frac{2x^{7/2}}{7}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2),x]

[Out] (2*x^(7/2))/7

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{5/2} \, dx = \frac{2x^{7/2}}{7}$$

Mathematica [A] time = 0.000643, size = 9, normalized size = 1.

$$\frac{2x^{7/2}}{7}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2),x]

[Out] (2*x^(7/2))/7

Maple [A] time = 0.002, size = 6, normalized size = 0.7

$$\frac{2}{7}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2),x)

```
[Out] 2/7*x^(7/2)
```

Maxima [A] time = 0.977138, size = 7, normalized size = 0.78

$$\frac{2}{7}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*x^(7/2)
```

Fricas [A] time = 1.58773, size = 18, normalized size = 2.

$$\frac{2}{7}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*x^(7/2)
```

Sympy [A] time = 0.050961, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2),x)
```

```
[Out] 2*x**(7/2)/7
```

Giac [A] time = 1.1065, size = 7, normalized size = 0.78

$$\frac{2}{7}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="giac")
```

```
[Out] 2/7*x^(7/2)
```

3.21 $\int x^{3/2}\,dx$

Optimal. Leaf size=9

$$\frac{2x^{5/2}}{5}$$

[Out] (2*x^(5/2))/5

Rubi [A] time = 0.0004273, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2),x]

[Out] (2*x^(5/2))/5

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{3/2}\,dx=\frac{2x^{5/2}}{5}$$

Mathematica [A] time = 0.0023685, size = 9, normalized size = 1.

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2),x]

[Out] (2*x^(5/2))/5

Maple [A] time = 0.001, size = 6, normalized size = 0.7

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2),x)

```
[Out] 2/5*x^(5/2)
```

Maxima [A] time = 0.97159, size = 7, normalized size = 0.78

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)
```

Fricas [A] time = 1.3753, size = 18, normalized size = 2.

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*x^(5/2)
```

Sympy [A] time = 0.050773, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2),x)
```

```
[Out] 2*x**(5/2)/5
```

Giac [A] time = 1.09434, size = 7, normalized size = 0.78

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2)
```

3.22 $\int \sqrt{x}\,dx$

Optimal. Leaf size=9

$$\frac{2x^{3/2}}{3}$$

```
[Out] (2*x^(3/2))/3
```

Rubi [A] time = 0.000424, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{x}\,dx = \frac{2x^{3/2}}{3}$$

Mathematica [A] time = 0.0008817, size = 9, normalized size = 1.

$$\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3
```

Maple [A] time = 0., size = 6, normalized size = 0.7

$$\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)
```

Maxima [A] time = 1.03761, size = 7, normalized size = 0.78

$$\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)
```

Fricas [A] time = 1.55057, size = 18, normalized size = 2.

$$\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2)
```

Sympy [A] time = 0.051497, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2),x)
```

```
[Out] 2*x**(3/2)/3
```

Giac [A] time = 1.12254, size = 7, normalized size = 0.78

$$\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)
```

3.23 $\int \frac{1}{\sqrt{x}}\, dx$

Optimal. Leaf size=7

$$2\sqrt{x}$$

```
[Out] 2*Sqrt[x]
```

Rubi [A] time = 0.0004059, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x],x]

[Out] 2*Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}}\, dx = 2\sqrt{x}$$

Mathematica [A] time = 0.0006325, size = 7, normalized size = 1.

$$2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x],x]

[Out] 2*Sqrt[x]
```

Maple [A] time = 0.002, size = 6, normalized size = 0.9

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2),x)
```

```
[Out] 2*x^(1/2)
```

Maxima [A] time = 1.05939, size = 7, normalized size = 1.

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)
```

Fricas [A] time = 1.51773, size = 15, normalized size = 2.14

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)
```

Sympy [A] time = 0.051554, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2),x)
```

```
[Out] 2*sqrt(x)
```

Giac [A] time = 1.11068, size = 7, normalized size = 1.

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)
```

3.24 $\int \frac{1}{x^{3/2}} \, dx$

Optimal. Leaf size=7

$$-\frac{2}{\sqrt{x}}$$

```
[Out] -2/Sqrt[x]
```

Rubi [A] time = 0.0003881, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3/2),x]
```

```
[Out] -2/Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}} \, dx = -\frac{2}{\sqrt{x}}$$

Mathematica [A] time = 0.0019054, size = 7, normalized size = 1.

$$-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3/2),x]
```

```
[Out] -2/Sqrt[x]
```

Maple [A] time = 0.001, size = 6, normalized size = 0.9

$$-2\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2),x)
```

```
[Out] -2/x^(1/2)
```

Maxima [A] time = 1.02961, size = 7, normalized size = 1.

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2/sqrt(x)
```

Fricas [A] time = 1.57674, size = 16, normalized size = 2.29

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2/sqrt(x)
```

Sympy [A] time = 0.053295, size = 7, normalized size = 1.

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2),x)
```

```
[Out] -2/sqrt(x)
```

Giac [A] time = 1.10779, size = 7, normalized size = 1.

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="giac")
```

```
[Out] -2/sqrt(x)
```

3.25 $\int \frac{1}{x^{5/2}}\,dx$

Optimal. Leaf size=9

$$-\frac{2}{3x^{3/2}}$$

[Out] -2/(3*x^(3/2))

Rubi [A] time = 0.0003914, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$-\frac{2}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^(-5/2),x]

[Out] -2/(3*x^(3/2))

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/2}}\,dx = -\frac{2}{3x^{3/2}}$$

Mathematica [A] time = 0.0007702, size = 9, normalized size = 1.

$$-\frac{2}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-5/2),x]

[Out] -2/(3*x^(3/2))

Maple [A] time = 0.002, size = 6, normalized size = 0.7

$$-\frac{2}{3}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2),x)

```
[Out] -2/3/x^(3/2)
```

Maxima [A] time = 1.02999, size = 7, normalized size = 0.78

$$-\frac{2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/x^(3/2)
```

Fricas [A] time = 1.56867, size = 19, normalized size = 2.11

$$-\frac{2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3/x^(3/2)
```

Sympy [A] time = 0.051968, size = 8, normalized size = 0.89

$$-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2),x)
```

```
[Out] -2/(3*x**(3/2))
```

Giac [A] time = 1.09085, size = 7, normalized size = 0.78

$$-\frac{2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/x^(3/2)
```

3.26 $\int x^{5/3}\,dx$

Optimal. Leaf size=9

$$\frac{3x^{8/3}}{8}$$

```
[Out] (3*x^(8/3))/8
```

Rubi [A] time = 0.0006694, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$\frac{3x^{8/3}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3),x]
```

```
[Out] (3*x^(8/3))/8
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{5/3}\,dx = \frac{3x^{8/3}}{8}$$

Mathematica [A] time = 0.0008471, size = 9, normalized size = 1.

$$\frac{3x^{8/3}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3),x]
```

```
[Out] (3*x^(8/3))/8
```

Maple [A] time = 0.001, size = 6, normalized size = 0.7

$$\frac{3}{8}x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3),x)
```

```
[Out] 3/8*x^(8/3)
```

Maxima [A] time = 1.03534, size = 7, normalized size = 0.78

$$\frac{3}{8}x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="maxima")
```

```
[Out] 3/8*x^(8/3)
```

Fricas [A] time = 1.62781, size = 18, normalized size = 2.

$$\frac{3}{8}x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="fricas")
```

```
[Out] 3/8*x^(8/3)
```

Sympy [A] time = 0.051743, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3),x)
```

```
[Out] 3*x**(8/3)/8
```

Giac [A] time = 1.10271, size = 7, normalized size = 0.78

$$\frac{3}{8}x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="giac")
```

```
[Out] 3/8*x^(8/3)
```

3.27 $\int x^{4/3}\,dx$

Optimal. Leaf size=9

$$\frac{3x^{7/3}}{7}$$

[Out] (3*x^(7/3))/7

Rubi [A] time = 0.0004124, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$\frac{3x^{7/3}}{7}$$

Antiderivative was successfully verified.

[In] Int[x^(4/3),x]

[Out] (3*x^(7/3))/7

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{4/3}\,dx=\frac{3x^{7/3}}{7}$$

Mathematica [A] time = 0.0008281, size = 9, normalized size = 1.

$$\frac{3x^{7/3}}{7}$$

Antiderivative was successfully verified.

[In] Integrate[x^(4/3),x]

[Out] (3*x^(7/3))/7

Maple [A] time = 0.002, size = 6, normalized size = 0.7

$$\frac{3}{7}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(4/3),x)

```
[Out] 3/7*x^(7/3)
```

Maxima [A] time = 0.995995, size = 7, normalized size = 0.78

$$\frac{3}{7}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="maxima")
```

```
[Out] 3/7*x^(7/3)
```

Fricas [A] time = 1.58468, size = 18, normalized size = 2.

$$\frac{3}{7}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="fricas")
```

```
[Out] 3/7*x^(7/3)
```

Sympy [A] time = 0.051116, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3),x)
```

```
[Out] 3*x**(7/3)/7
```

Giac [A] time = 1.1249, size = 7, normalized size = 0.78

$$\frac{3}{7}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="giac")
```

```
[Out] 3/7*x^(7/3)
```

3.28 $\int x^{2/3} \, dx$

Optimal. Leaf size=9

$$\frac{3x^{5/3}}{5}$$

```
[Out] (3*x^(5/3))/5
```

Rubi [A] time = 0.0004024, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$\frac{3x^{5/3}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3),x]
```

```
[Out] (3*x^(5/3))/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{2/3} \, dx = \frac{3x^{5/3}}{5}$$

Mathematica [A] time = 0.000563, size = 9, normalized size = 1.

$$\frac{3x^{5/3}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3),x]
```

```
[Out] (3*x^(5/3))/5
```

Maple [A] time = 0.001, size = 6, normalized size = 0.7

$$\frac{3}{5}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3),x)
```

```
[Out] 3/5*x^(5/3)
```

Maxima [A] time = 1.04663, size = 7, normalized size = 0.78

$$\frac{3}{5}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/5*x^(5/3)
```

Fricas [A] time = 1.50939, size = 18, normalized size = 2.

$$\frac{3}{5}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="fricas")
```

```
[Out] 3/5*x^(5/3)
```

Sympy [A] time = 0.051212, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3),x)
```

```
[Out] 3*x**(5/3)/5
```

Giac [A] time = 1.095, size = 7, normalized size = 0.78

$$\frac{3}{5}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="giac")
```

```
[Out] 3/5*x^(5/3)
```

3.29 $\int \sqrt[3]{x}\,dx$

Optimal. Leaf size=9

$$\frac{3x^{4/3}}{4}$$

```
[Out] (3*x^(4/3))/4
```

Rubi [A] time = 0.0004372, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$\frac{3x^{4/3}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3),x]
```

```
[Out] (3*x^(4/3))/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{x}\,dx = \frac{3x^{4/3}}{4}$$

Mathematica [A] time = 0.0024849, size = 9, normalized size = 1.

$$\frac{3x^{4/3}}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3),x]
```

```
[Out] (3*x^(4/3))/4
```

Maple [A] time = 0.002, size = 6, normalized size = 0.7

$$\frac{3}{4}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3),x)
```

```
[Out] 3/4*x^(4/3)
```

Maxima [A] time = 1.02265, size = 7, normalized size = 0.78

$$\frac{3}{4}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*x^(4/3)
```

Fricas [A] time = 1.53806, size = 18, normalized size = 2.

$$\frac{3}{4}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*x^(4/3)
```

Sympy [A] time = 0.051377, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3),x)
```

```
[Out] 3*x**(4/3)/4
```

Giac [A] time = 1.09591, size = 7, normalized size = 0.78

$$\frac{3}{4}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*x^(4/3)
```

3.30 $\int \frac{1}{\sqrt[3]{x}} \, dx$

Optimal. Leaf size=9

$$\frac{3x^{2/3}}{2}$$

```
[Out] (3*x^(2/3))/2
```

Rubi [A] time = 0.0004063, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$\frac{3x^{2/3}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1/3),x]
```

```
[Out] (3*x^(2/3))/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}} \, dx = \frac{3x^{2/3}}{2}$$

Mathematica [A] time = 0.0015627, size = 9, normalized size = 1.

$$\frac{3x^{2/3}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1/3),x]
```

```
[Out] (3*x^(2/3))/2
```

Maple [A] time = 0.002, size = 6, normalized size = 0.7

$$\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3),x)
```

```
[Out] 3/2*x^(2/3)
```

Maxima [A] time = 1.01689, size = 7, normalized size = 0.78

$$\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3)
```

Fricas [A] time = 1.34488, size = 18, normalized size = 2.

$$\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3)
```

Sympy [A] time = 0.051862, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3),x)
```

```
[Out] 3*x**(2/3)/2
```

Giac [A] time = 1.11525, size = 7, normalized size = 0.78

$$\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3)
```

3.31 $\int \frac{1}{x^{2/3}} dx$

Optimal. Leaf size=7

$$3\sqrt[3]{x}$$

[Out] 3*x^(1/3)

Rubi [A] time = 0.0004344, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$3\sqrt[3]{x}$$

Antiderivative was successfully verified.

[In] Int[x^(-2/3),x]

[Out] 3*x^(1/3)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{2/3}} dx = 3\sqrt[3]{x}$$

Mathematica [A] time = 0.0007684, size = 7, normalized size = 1.

$$3\sqrt[3]{x}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-2/3),x]

[Out] 3*x^(1/3)

Maple [A] time = 0.001, size = 6, normalized size = 0.9

$$3\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(2/3),x)

```
[Out] 3*x^(1/3)
```

Maxima [A] time = 1.03224, size = 7, normalized size = 1.

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3*x^(1/3)
```

Fricas [A] time = 1.4012, size = 15, normalized size = 2.14

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3*x^(1/3)
```

Sympy [A] time = 0.051347, size = 5, normalized size = 0.71

$$3\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3),x)
```

```
[Out] 3*x**(1/3)
```

Giac [A] time = 1.09975, size = 7, normalized size = 1.

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="giac")
```

```
[Out] 3*x^(1/3)
```

3.32 $\int \frac{1}{x^{4/3}} dx$

Optimal. Leaf size=7

$$-\frac{3}{\sqrt[3]{x}}$$

[Out] -3/x^(1/3)

Rubi [A] time = 0.0004932, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$-\frac{3}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Int[x^(-4/3),x]

[Out] -3/x^(1/3)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{4/3}} dx = -\frac{3}{\sqrt[3]{x}}$$

Mathematica [A] time = 0.0006004, size = 7, normalized size = 1.

$$-\frac{3}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-4/3),x]

[Out] -3/x^(1/3)

Maple [A] time = 0.002, size = 6, normalized size = 0.9

$$-3\frac{1}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3),x)
```

```
[Out] -3/x^(1/3)
```

Maxima [A] time = 0.99825, size = 7, normalized size = 1.

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x, algorithm="maxima")
```

```
[Out] -3/x^(1/3)
```

Fricas [A] time = 1.43113, size = 16, normalized size = 2.29

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x, algorithm="fricas")
```

```
[Out] -3/x^(1/3)
```

Sympy [A] time = 0.053594, size = 7, normalized size = 1.

$$-\frac{3}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3),x)
```

```
[Out] -3/x**(1/3)
```

Giac [A] time = 1.10112, size = 7, normalized size = 1.

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x, algorithm="giac")
```

```
[Out] -3/x^(1/3)
```

3.33 $\int \frac{1}{x^{5/3}} \, dx$

Optimal. Leaf size=9

$$-\frac{3}{2x^{2/3}}$$

```
[Out] -3/(2*x^(2/3))
```

Rubi [A] time = 0.0004151, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$-\frac{3}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5/3),x]
```

```
[Out] -3/(2*x^(2/3))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/3}} \, dx = -\frac{3}{2x^{2/3}}$$

Mathematica [A] time = 0.0008142, size = 9, normalized size = 1.

$$-\frac{3}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5/3),x]
```

```
[Out] -3/(2*x^(2/3))
```

Maple [A] time = 0.002, size = 6, normalized size = 0.7

$$-\frac{3}{2}x^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3),x)
```

```
[Out] -3/2/x^(2/3)
```

Maxima [A] time = 0.98278, size = 7, normalized size = 0.78

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="maxima")
```

```
[Out] -3/2/x^(2/3)
```

Fricas [A] time = 1.58644, size = 19, normalized size = 2.11

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="fricas")
```

```
[Out] -3/2/x^(2/3)
```

Sympy [A] time = 0.051818, size = 8, normalized size = 0.89

$$-\frac{3}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3),x)
```

```
[Out] -3/(2*x**(2/3))
```

Giac [A] time = 1.19723, size = 7, normalized size = 0.78

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="giac")
```

```
[Out] -3/2/x^(2/3)
```

3.34 $\int x^n \, dx$

Optimal. Leaf size=11

$$\frac{x^{n+1}}{n+1}$$

```
[Out] x^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0020883, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^n \, dx = \frac{x^{1+n}}{1+n}$$

Mathematica [A] time = 0.0011644, size = 11, normalized size = 1.

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

Maple [A] time = 0.001, size = 12, normalized size = 1.1

$$\frac{x^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n,x)
```

```
[Out] x^(1+n)/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6637, size = 20, normalized size = 1.82

$$\frac{xx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="fricas")
```

```
[Out] x*x^n/(n + 1)
```

Sympy [A] time = 0.052567, size = 12, normalized size = 1.09

$$\begin{cases} \frac{x^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n,x)
```

```
[Out] Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True))
```

Giac [A] time = 1.16934, size = 15, normalized size = 1.36

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="giac")
```

```
[Out] x^(n + 1)/(n + 1)
```

3.35 $\int (bx)^n \, dx$

Optimal. Leaf size=16

$$\frac{(bx)^{n+1}}{b(n+1)}$$

```
[Out] (b*x)^(1 + n)/(b*(1 + n))
```

Rubi [A] time = 0.00327, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {32}

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^n,x]
```

```
[Out] (b*x)^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (bx)^n \, dx = \frac{(bx)^{1+n}}{b(1+n)}$$

Mathematica [A] time = 0.0065322, size = 12, normalized size = 0.75

$$\frac{x(bx)^n}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^n,x]
```

```
[Out] (x*(b*x)^n)/(1 + n)
```

Maple [A] time = 0.003, size = 13, normalized size = 0.8

$$\frac{x\,(bx)^n}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^n,x)
```

```
[Out] x/(1+n)*(b*x)^n
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73015, size = 26, normalized size = 1.62

$$\frac{(bx)^n x}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^n,x, algorithm="fricas")
```

```
[Out] (b*x)^n*x/(n + 1)
```

Sympy [A] time = 0.054687, size = 17, normalized size = 1.06

$$\frac{\begin{cases} \frac{(bx)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**n,x)
```

```
[Out] Piecewise(((b*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(b*x), True))/b
```

Giac [A] time = 1.22592, size = 22, normalized size = 1.38

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^n,x, algorithm="giac")
```

```
[Out] (b*x)^(n + 1)/(b*(n + 1))
```

3.36 $\int \frac{1}{\sqrt{-a}+e(c+dx)}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

```
[Out] Log[Sqrt[-a] + c*e + d*e*x]/(d*e)
```

Rubi [A] time = 0.0140041, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {33, 31}

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-a] + e*(c + d*x))^(-1),x]
```

```
[Out] Log[Sqrt[-a] + c*e + d*e*x]/(d*e)
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a}+e(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}+ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.0088897, size = 23, normalized size = 1.

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-a] + e*(c + d*x))^(-1),x]
```

```
[Out] Log[Sqrt[-a] + c*e + d*e*x]/(d*e)
```

Maple [A] time = 0.001, size = 22, normalized size = 1.

$$\frac{1}{ed}\ln\left(ce+dex+\sqrt{-a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*(d*x+c)+(-a)^(1/2)),x)
```

```
[Out] ln(c*e+d*e*x+(-a)^(1/2))/d/e
```

Maxima [A] time = 1.02679, size = 28, normalized size = 1.22

$$\frac{\log\left((dx+c)e+\sqrt{-a}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log((d*x + c)*e + sqrt(-a))/(d*e)
```

Fricas [A] time = 1.76498, size = 47, normalized size = 2.04

$$\frac{\log\left(dex+ce+\sqrt{-a}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] log(d*e*x + c*e + sqrt(-a))/(d*e)
```

Sympy [A] time = 0.069685, size = 19, normalized size = 0.83

$$\frac{\log\left(ce+dex+\sqrt{-a}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)**(1/2)),x)
```

```
[Out] log(c*e + d*e*x + sqrt(-a))/(d*e)
```

Giac [A] time = 1.18278, size = 30, normalized size = 1.3

$$\frac{e^{(-1)}\log\left(\left|(dx+c)e+\sqrt{-a}\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] e^(-1)*log(abs((d*x + c)*e + sqrt(-a)))/d
```

3.37 $\int (c+d(a+bx))^{5/2}\,dx$

Optimal. Leaf size=23

$$\frac{2(d(a+bx)+c)^{7/2}}{7bd}$$

```
[Out] (2*(c + d*(a + b*x))^(7/2))/(7*b*d)
```

Rubi [A] time = 0.0118938, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(5/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(7/2))/(7*b*d)
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (c+d(a+bx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (c+dx)^{5/2}\,dx, x, a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{7/2}}{7bd}\end{aligned}$$

Mathematica [A] time = 0.0318637, size = 23, normalized size = 1.

$$\frac{2(d(a+bx)+c)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(5/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(7/2))/(7*b*d)
```

Maple [A] time = 0.002, size = 20, normalized size = 0.9

$$\frac{2}{7\,bd}(bdx+ad+c)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(5/2),x)
```

```
[Out] 2/7*(b*d*x+a*d+c)^(7/2)/b/d
```

Maxima [A] time = 1.00917, size = 26, normalized size = 1.13

$$\frac{2\,((bx+a)d+c)^{\frac{7}{2}}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*((b*x + a)*d + c)^(7/2)/(b*d)
```

Fricas [B] time = 1.57616, size = 220, normalized size = 9.57

$$\frac{2\left(b^3d^3x^3+a^3d^3+3\,a^2cd^2+3\,ac^2d+c^3+3\left(ab^2d^3+b^2cd^2\right)x^2+3\left(a^2bd^3+2\,abcd^2+bc^2d\right)x\right)\sqrt{bdx+ad+c}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*d^3*x^3 + a^3*d^3 + 3*a^2*c*d^2 + 3*a*c^2*d + c^3 + 3*(a*b^2*d^3 +
 b^2*c*d^2)*x^2 + 3*(a^2*b*d^3 + 2*a*b*c*d^2 + b*c^2*d)*x)*sqrt(b*d*x + a*d
 + c)/(b*d)
```

Sympy [A] time = 81.5393, size = 267, normalized size = 11.61

$$\begin{cases} c^{\frac{5}{2}}x \\ x\,(ad+c)^{\frac{5}{2}} \\ \frac{2a^3d^2\sqrt{ad+bdx+c}}{7b}+\frac{6a^2d^2x\sqrt{ad+bdx+c}}{7}+\frac{6a^2cd\sqrt{ad+bdx+c}}{7b}+\frac{6abd^2x^2\sqrt{ad+bdx+c}}{7}+\frac{12acdx\sqrt{ad+bdx+c}}{7}+\frac{6ac^2\sqrt{ad+bdx+c}}{7b}+\frac{2b^2d^2x^3\sqrt{ad+bdx+c}}{7}+ \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((c**(5/2)*x, Eq(d, 0) & (Eq(b, 0) | Eq(d, 0))), (x*(a*d + c)**(5/
2), Eq(b, 0)), (2*a**3*d**2*sqrt(a*d + b*d*x + c)/(7*b) + 6*a**2*d**2*x*sqr
t(a*d + b*d*x + c)/7 + 6*a**2*c*d*sqrt(a*d + b*d*x + c)/(7*b) + 6*a*b*d**2*
x**2*sqrt(a*d + b*d*x + c)/7 + 12*a*c*d*x*sqrt(a*d + b*d*x + c)/7 + 6*a*c**
2*sqrt(a*d + b*d*x + c)/(7*b) + 2*b**2*d**2*x**3*sqrt(a*d + b*d*x + c)/7 +
```

```
6*b*c*d*x**2*sqrt(a*d + b*d*x + c)/7 + 6*c**2*x*sqrt(a*d + b*d*x + c)/7 + 2
*c**3*sqrt(a*d + b*d*x + c)/(7*b*d), True))
```

Giac [B] time = 1.28241, size = 263, normalized size = 11.43

$$2\left(70\,(bdx+ad+c)^{\frac{3}{2}}a^2d^2-42\,(bdx+ad+c)^{\frac{5}{2}}ad+140\,(bdx+ad+c)^{\frac{3}{2}}acd+15\,(bdx+ad+c)^{\frac{7}{2}}-42\,(bdx+ad+c)^{\frac{5}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/105*(70*(b*d*x + a*d + c)^(3/2)*a^2*d^2 - 42*(b*d*x + a*d + c)^(5/2)*a*d
+ 140*(b*d*x + a*d + c)^(3/2)*a*c*d + 15*(b*d*x + a*d + c)^(7/2) - 42*(b*d*
x + a*d + c)^(5/2)*c + 70*(b*d*x + a*d + c)^(3/2)*c^2 - 14*(5*(b*d*x + a*d
+ c)^(3/2)*a*d - 3*(b*d*x + a*d + c)^(5/2) + 5*(b*d*x + a*d + c)^(3/2)*c)*a
*d - 14*(5*(b*d*x + a*d + c)^(3/2)*a*d - 3*(b*d*x + a*d + c)^(5/2) + 5*(b*d
*x + a*d + c)^(3/2)*c)*c)/(b*d)
```

3.38 $\int (c+d(a+bx))^{3/2}\,dx$

Optimal. Leaf size=23

$$\frac{2(d(a+bx)+c)^{5/2}}{5bd}$$

```
[Out] (2*(c + d*(a + b*x))^(5/2))/(5*b*d)
```

Rubi [A] time = 0.0109621, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(3/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(5/2))/(5*b*d)
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (c+d(a+bx))^{3/2}\,dx &= \frac{\operatorname{Subst}\left(\int (c+dx)^{3/2}\,dx, x, a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{5/2}}{5bd}\end{aligned}$$

Mathematica [A] time = 0.0146494, size = 23, normalized size = 1.

$$\frac{2(d(a+bx)+c)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(3/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(5/2))/(5*b*d)
```

Maple [A] time = 0.001, size = 20, normalized size = 0.9

$$\frac{2 (bdx + ad + c)^{\frac{5}{2}}}{5 bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(3/2),x)
```

```
[Out] 2/5*(b*d*x+a*d+c)^(5/2)/b/d
```

Maxima [A] time = 1.02746, size = 26, normalized size = 1.13

$$\frac{2 ((bx + a)d + c)^{\frac{5}{2}}}{5 bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*((b*x + a)*d + c)^(5/2)/(b*d)
```

Fricas [B] time = 1.60548, size = 131, normalized size = 5.7

$$\frac{2 \left(b^2 d^2 x^2 + a^2 d^2 + 2 acd + c^2 + 2 \left(abd^2 + bcd\right)x\right)\sqrt{bdx + ad + c}}{5 bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*d^2*x^2 + a^2*d^2 + 2*a*c*d + c^2 + 2*(a*b*d^2 + b*c*d)*x)*sqrt(b*
d*x + a*d + c)/(b*d)
```

Sympy [A] time = 5.31987, size = 153, normalized size = 6.65

$$\begin{cases} c^{\frac{3}{2}} x & \text{for}\ d = 0 \wedge (b = 0 \vee d \\ x (ad + c)^{\frac{3}{2}} & \text{for}\ b = 0 \\ \frac{2a^2 d\sqrt{ad+bdx+c}}{5b} + \frac{4adx\sqrt{ad+bdx+c}}{5} + \frac{4ac\sqrt{ad+bdx+c}}{5b} + \frac{2bdx^2\sqrt{ad+bdx+c}}{5} + \frac{4cx\sqrt{ad+bdx+c}}{5} + \frac{2c^2\sqrt{ad+bdx+c}}{5bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((c**(3/2)*x, Eq(d, 0) & (Eq(b, 0) | Eq(d, 0))), (x*(a*d + c)**(3/
2), Eq(b, 0)), (2*a**2*d*sqrt(a*d + b*d*x + c)/(5*b) + 4*a*d*x*sqrt(a*d + b
*d*x + c)/5 + 4*a*c*sqrt(a*d + b*d*x + c)/(5*b) + 2*b*d*x**2*sqrt(a*d + b*d
*x + c)/5 + 4*c*x*sqrt(a*d + b*d*x + c)/5 + 2*c**2*sqrt(a*d + b*d*x + c)/(5
*b*d), True))
```

Giac [A] time = 1.16723, size = 26, normalized size = 1.13

$$\frac{2\,(bdx+ad+c)^{\frac{5}{2}}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*(b*d*x + a*d + c)^(5/2)/(b*d)
```

3.39 $\int \sqrt{c+d(a+bx)}\,dx$

Optimal. Leaf size=23

$$\frac{2(d(a+bx)+c)^{3/2}}{3bd}$$

```
[Out] (2*(c + d*(a + b*x))^(3/2))/(3*b*d)
```

Rubi [A] time = 0.0108698, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*(c + d*(a + b*x))^(3/2))/(3*b*d)
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c+d(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{c+dx}\,dx, x, a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{3/2}}{3bd}\end{aligned}$$

Mathematica [A] time = 0.0154463, size = 23, normalized size = 1.

$$\frac{2(d(a+bx)+c)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*(c + d*(a + b*x))^(3/2))/(3*b*d)
```

Maple [A] time = 0.003, size = 20, normalized size = 0.9

$$\frac{2}{3\,bd}\,(bdx + ad + c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(1/2),x)
```

```
[Out] 2/3*(b*d*x+a*d+c)^(3/2)/b/d
```

Maxima [A] time = 1.03498, size = 26, normalized size = 1.13

$$\frac{2\,((bx + a)d + c)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*((b*x + a)*d + c)^(3/2)/(b*d)
```

Fricas [A] time = 1.57262, size = 47, normalized size = 2.04

$$\frac{2\,(bdx + ad + c)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*d*x + a*d + c)^(3/2)/(b*d)
```

Sympy [A] time = 0.383804, size = 78, normalized size = 3.39

$$\begin{cases} \sqrt{c}x & \text{for}\ d = 0 \wedge (b = 0 \vee d = 0) \\ x\sqrt{ad + c} & \text{for}\ b = 0 \\ \frac{2a\sqrt{ad+bdx+c}}{3b} + \frac{2x\sqrt{ad+bdx+c}}{3} + \frac{2c\sqrt{ad+bdx+c}}{3bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((sqrt(c)*x, Eq(d, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sqrt(a*d + c),
Eq(b, 0)), (2*a*sqrt(a*d + b*d*x + c)/(3*b) + 2*x*sqrt(a*d + b*d*x + c)/3 +
 2*c*sqrt(a*d + b*d*x + c)/(3*b*d), True))
```

Giac [A] time = 1.15791, size = 26, normalized size = 1.13

$$\frac{2\,(bdx+ad+c)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*d*x + a*d + c)^(3/2)/(b*d)
```

3.40 $\int \frac{1}{\sqrt{c+d(a+bx)}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sqrt{d(a+bx)+c}}{bd}$$

```
[Out] (2*Sqrt[c + d*(a + b*x)])/(b*d)
```

Rubi [A] time = 0.0103886, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$\frac{2\sqrt{d(a+bx)+c}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*Sqrt[c + d*(a + b*x)])/(b*d)
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c+d(a+bx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}}\,dx, x, a+bx\right)}{b}\\ &= \frac{2\sqrt{c+d(a+bx)}}{bd}\end{aligned}$$

Mathematica [A] time = 0.0127891, size = 21, normalized size = 1.

$$\frac{2\sqrt{d(a+bx)+c}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*Sqrt[c + d*(a + b*x)])/(b*d)
```

Maple [A] time = 0.002, size = 20, normalized size = 1.

$$2\frac{\sqrt{bdx+ad+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(1/2),x)
```

```
[Out] 2*(b*d*x+a*d+c)^(1/2)/b/d
```

Maxima [A] time = 1.03585, size = 26, normalized size = 1.24

$$\frac{2\sqrt{(bx+a)d+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt((b*x + a)*d + c)/(b*d)
```

Fricas [A] time = 1.49795, size = 42, normalized size = 2.

$$\frac{2\sqrt{bdx+ad+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*d*x + a*d + c)/(b*d)
```

Sympy [A] time = 1.19525, size = 31, normalized size = 1.48

$$\begin{cases} \frac{x}{\sqrt{ad+c}} & \text{for}\: b = 0 \\ \frac{x}{\sqrt{c}} & \text{for}\: d = 0 \\ \frac{2\sqrt{c+d(a+bx)}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((x/sqrt(a*d + c), Eq(b, 0)), (x/sqrt(c), Eq(d, 0)), (2*sqrt(c + d
*(a + b*x))/(b*d), True))
```

Giac [A] time = 1.13198, size = 26, normalized size = 1.24

$$\frac{2\sqrt{bdx + ad + c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*d*x + a*d + c)/(b*d)
```

3.41 $\int \frac{1}{(c+d(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{2}{bd\sqrt{d(a+bx)+c}}$$

```
[Out] -2/(b*d*Sqrt[c + d*(a + b*x)])
```

Rubi [A] time = 0.0105183, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$-\frac{2}{bd\sqrt{d(a+bx)+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(-3/2),x]
```

```
[Out] -2/(b*d*Sqrt[c + d*(a + b*x)])
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+d(a+bx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(c+dx)^{3/2}}\,dx, x, a+bx\right)}{b}\\ &= -\frac{2}{bd\sqrt{c+d(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0130418, size = 21, normalized size = 1.

$$-\frac{2}{bd\sqrt{d(a+bx)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(-3/2),x]
```

```
[Out] -2/(b*d*Sqrt[c + d*(a + b*x)])
```

Maple [A] time = 0.001, size = 20, normalized size = 1.

$$-2\frac{1}{\sqrt{bdx+ad+cbd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(3/2),x)
```

```
[Out] -2/(b*d*x+a*d+c)^(1/2)/b/d
```

Maxima [A] time = 1.09262, size = 26, normalized size = 1.24

$$-\frac{2}{\sqrt{(bx+a)d+c}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt((b*x + a)*d + c)*b*d)
```

Fricas [A] time = 1.48526, size = 76, normalized size = 3.62

$$-\frac{2\sqrt{bdx+ad+c}}{b^2d^2x+abd^2+bcd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*d*x + a*d + c)/(b^2*d^2*x + a*b*d^2 + b*c*d)
```

Sympy [A] time = 1.39265, size = 58, normalized size = 2.76

$$\begin{cases} \frac{x}{c^{\frac{3}{2}}} & \text{for } b=0 \wedge d=0 \\ \frac{x}{(ad+c)^{\frac{3}{2}}} & \text{for } b=0 \\ \frac{x}{c^{\frac{3}{2}}} & \text{for } d=0 \\ -\frac{2\sqrt{ad+bdx+c}}{abd^2+b^2d^2x+bcd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((x/c**(3/2), Eq(b, 0) & Eq(d, 0)), (x/(a*d + c)**(3/2), Eq(b, 0))
, (x/c**(3/2), Eq(d, 0)), (-2*sqrt(a*d + b*d*x + c)/(a*b*d**2 + b**2*d**2*x
 + b*c*d), True))
```

Giac [A] time = 1.16669, size = 26, normalized size = 1.24

$$-\frac{2}{\sqrt{(bx+a)d+c}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt((b*x + a)*d + c)*b*d)
```

3.42 $\int \frac{1}{(c+d(a+bx))^{5/2}} dx$

Optimal. Leaf size=23

$$-\frac{2}{3bd(d(a+bx)+c)^{3/2}}$$

```
[Out] -2/(3*b*d*(c + d*(a + b*x))^(3/2))
```

Rubi [A] time = 0.0108823, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$-\frac{2}{3bd(d(a+bx)+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(-5/2),x]
```

```
[Out] -2/(3*b*d*(c + d*(a + b*x))^(3/2))
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+d(a+bx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(c+dx)^{5/2}}\,dx, x, a+bx\right)}{b}\\ &= -\frac{2}{3bd(c+d(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0304856, size = 23, normalized size = 1.

$$-\frac{2}{3bd(d(a+bx)+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(-5/2),x]
```

```
[Out] -2/(3*b*d*(c + d*(a + b*x))^(3/2))
```

Maple [A] time = 0.002, size = 20, normalized size = 0.9

$$-\frac{2}{3\,bd}\left(bdx+ad+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(5/2),x)
```

```
[Out] -2/3/(b*d*x+a*d+c)^(3/2)/b/d
```

Maxima [A] time = 1.05801, size = 26, normalized size = 1.13

$$-\frac{2}{3\,\left((bx+a)d+c\right)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/(((b*x + a)*d + c)^(3/2)*b*d)
```

Fricas [B] time = 1.57646, size = 146, normalized size = 6.35

$$-\frac{2\,\sqrt{bdx+ad+c}}{3\left(b^3d^3x^2+a^2bd^3+2\,abcd^2+bc^2d+2\left(ab^2d^3+b^2cd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*d*x + a*d + c)/(b^3*d^3*x^2 + a^2*b*d^3 + 2*a*b*c*d^2 + b*c^2*d
 + 2*(a*b^2*d^3 + b^2*c*d^2)*x)
```

Sympy [A] time = 6.85587, size = 102, normalized size = 4.43

$$\begin{cases} \frac{x}{c^{\frac{5}{2}}} & \text{for}\: b = 0 \wedge d = 0 \\ \frac{x}{(ad+c)^{\frac{5}{2}}} & \text{for}\: b = 0 \\ \frac{x}{c^{\frac{5}{2}}} & \text{for}\: d = 0 \\ -\frac{2\sqrt{ad+bdx+c}}{3a^2bd^3+6ab^2d^3x+6abcd^2+3b^3d^3x^2+6b^2cd^2x+3bc^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((x/c**(5/2), Eq(b, 0) & Eq(d, 0)), (x/(a*d + c)**(5/2), Eq(b, 0))
, (x/c**(5/2), Eq(d, 0)), (-2*sqrt(a*d + b*d*x + c)/(3*a**2*b*d**3 + 6*a*b*
```

```
*2*d**3*x + 6*a*b*c*d**2 + 3*b**3*d**3*x**2 + 6*b**2*c*d**2*x + 3*b*c**2*d)
, True))
```

Giac [A] time = 1.14405, size = 26, normalized size = 1.13

$$-\frac{2}{3\left((bx+a)d+c\right)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/(((b*x + a)*d + c)^(3/2)*b*d)
```

3.43 $\int x^3(a + bx)\,dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

Rubi [A] time = 0.0081869, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a + bx)\,dx &= \int \left(ax^3 + bx^4\right)\,dx\\ &= \frac{ax^4}{4} + \frac{bx^5}{5}\end{aligned}$$

Mathematica [A] time = 0.0010632, size = 17, normalized size = 1.

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a),x)
```

```
[Out] 1/4*a*x^4+1/5*b*x^5
```

Maxima [A] time = 1.04301, size = 18, normalized size = 1.06

$$\frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Fricas [A] time = 1.40915, size = 31, normalized size = 1.82

$$\frac{1}{5}x^5b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b + 1/4*x^4*a
```

Sympy [A] time = 0.08146, size = 12, normalized size = 0.71

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a),x)
```

```
[Out] a*x**4/4 + b*x**5/5
```

Giac [A] time = 1.23466, size = 18, normalized size = 1.06

$$\frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

3.44 $\int x^2(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

Rubi [A] time = 0.0067273, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x),x]

[Out] (a*x^3)/3 + (b*x^4)/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)\,dx &= \int \left(ax^2+bx^3\right)\,dx\\ &= \frac{ax^3}{3}+\frac{bx^4}{4}\end{aligned}$$

Mathematica [A] time = 0.0015769, size = 17, normalized size = 1.

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x),x]

[Out] (a*x^3)/3 + (b*x^4)/4
```

Maple [A] time = 0.002, size = 14, normalized size = 0.8

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a),x)
```

```
[Out] 1/3*a*x^3+1/4*b*x^4
```

Maxima [A] time = 1.01881, size = 18, normalized size = 1.06

$$\frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Fricas [A] time = 1.34153, size = 31, normalized size = 1.82

$$\frac{1}{4}x^4b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b + 1/3*x^3*a
```

Sympy [A] time = 0.088192, size = 12, normalized size = 0.71

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a),x)
```

```
[Out] a*x**3/3 + b*x**4/4
```

Giac [A] time = 1.1926, size = 18, normalized size = 1.06

$$\frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

3.45 $\int x(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

Rubi [A] time = 0.0060435, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {43}

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x),x]

[Out] (a*x^2)/2 + (b*x^3)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)\,dx &= \int \left(ax+bx^2\right)\,dx\\ &= \frac{ax^2}{2}+\frac{bx^3}{3}\end{aligned}$$

Mathematica [A] time = 0.0008597, size = 17, normalized size = 1.

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x),x]

[Out] (a*x^2)/2 + (b*x^3)/3
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a),x)
```

```
[Out] 1/2*a*x^2+1/3*b*x^3
```

Maxima [A] time = 1.04867, size = 18, normalized size = 1.06

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Fricas [A] time = 1.2934, size = 31, normalized size = 1.82

$$\frac{1}{3}x^3b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*b + 1/2*x^2*a
```

Sympy [A] time = 0.069685, size = 12, normalized size = 0.71

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x)
```

```
[Out] a*x**2/2 + b*x**3/3
```

Giac [A] time = 1.17788, size = 18, normalized size = 1.06

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

3.46 $\int (a+bx)\,dx$

Optimal. Leaf size=12

$$ax+\frac{bx^2}{2}$$

```
[Out] a*x + (b*x^2)/2
```

Rubi [A] time = 0.0022218, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x,x]

[Out] a*x + (b*x^2)/2
```

Rubi steps

$$\int (a+bx)\,dx=ax+\frac{bx^2}{2}$$

Mathematica [A] time = 0.0000907, size = 12, normalized size = 1.

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x,x]

[Out] a*x + (b*x^2)/2
```

Maple [A] time = 0.001, size = 11, normalized size = 0.9

$$ax+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a,x)

[Out] a*x+1/2*b*x^2
```

Maxima [A] time = 1.02483, size = 14, normalized size = 1.17

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

Fricas [A] time = 1.39129, size = 23, normalized size = 1.92

$$\frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*b + x*a
```

Sympy [A] time = 0.067836, size = 8, normalized size = 0.67

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x)
```

```
[Out] a*x + b*x**2/2
```

Giac [A] time = 1.18463, size = 14, normalized size = 1.17

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

3.47 $\int \frac{a+bx}{x} dx$

Optimal. Leaf size=8

$$a\log(x) + bx$$

```
[Out] b*x + a*Log[x]
```

Rubi [A] time = 0.0030192, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$a\log(x) + bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x,x]
```

```
[Out] b*x + a*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x}\,dx &= \int \left(b+\frac{a}{x}\right)dx\\ &= bx + a\log(x)\end{aligned}$$

Mathematica [A] time = 0.0008298, size = 8, normalized size = 1.

$$a\log(x) + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x,x]
```

```
[Out] b*x + a*Log[x]
```

Maple [A] time = 0.002, size = 9, normalized size = 1.1

$$bx + a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x,x)
```

```
[Out] b*x+a*ln(x)
```

Maxima [A] time = 1.0344, size = 11, normalized size = 1.38

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="maxima")
```

```
[Out] b*x + a*log(x)
```

Fricas [A] time = 1.55437, size = 22, normalized size = 2.75

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="fricas")
```

```
[Out] b*x + a*log(x)
```

Sympy [A] time = 0.116574, size = 7, normalized size = 0.88

$$a\log(x) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x)
```

```
[Out] a*log(x) + b*x
```

Giac [A] time = 1.15128, size = 12, normalized size = 1.5

$$bx + a\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="giac")
```

```
[Out] b*x + a*log(abs(x))
```

3.48 $\int \frac{a+bx}{x^2} dx$

Optimal. Leaf size=11

$$b\log(x) - \frac{a}{x}$$

```
[Out] -(a/x) + b*Log[x]
```

Rubi [A] time = 0.0043358, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$b\log(x) - \frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^2,x]
```

```
[Out] -(a/x) + b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{x^2} dx &= \int \left(\frac{a}{x^2} + \frac{b}{x}\right) dx \\ &= -\frac{a}{x} + b\log(x) \end{aligned}$$

Mathematica [A] time = 0.0019118, size = 11, normalized size = 1.

$$b\log(x) - \frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^2,x]
```

```
[Out] -(a/x) + b*Log[x]
```

Maple [A] time = 0.004, size = 12, normalized size = 1.1

$$-\frac{a}{x} + b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2,x)
```

```
[Out] -a/x+b*ln(x)
```

Maxima [A] time = 1.01865, size = 15, normalized size = 1.36

$$b \log (x) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x) - a/x
```

Fricas [A] time = 1.53353, size = 27, normalized size = 2.45

$$\frac{bx \log (x) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - a)/x
```

Sympy [A] time = 0.379399, size = 7, normalized size = 0.64

$$-\frac{a}{x} + b \log (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2,x)
```

```
[Out] -a/x + b*log(x)
```

Giac [A] time = 1.15994, size = 16, normalized size = 1.45

$$b \log (|x|) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(x)) - a/x
```

3.49 $\int \frac{a+bx}{x^3} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^2}{2ax^2}$$

```
[Out] -(a + b*x)^2/(2*a*x^2)
```

Rubi [A] time = 0.0017435, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {37}

$$-\frac{(a+bx)^2}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^3,x]

[Out] -(a + b*x)^2/(2*a*x^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{a+bx}{x^3} dx = -\frac{(a+bx)^2}{2ax^2}$$

Mathematica [A] time = 0.0015521, size = 15, normalized size = 0.88

$$-\frac{a}{2x^2} - \frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^3,x]

[Out] -a/(2*x^2) - b/x
```

Maple [A] time = 0.004, size = 14, normalized size = 0.8

$$-\frac{a}{2x^2} - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3,x)
```

```
[Out] -1/2/x^2*a-b/x
```

Maxima [A] time = 1.0543, size = 15, normalized size = 0.88

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Fricas [A] time = 1.55443, size = 30, normalized size = 1.76

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Sympy [A] time = 0.383781, size = 12, normalized size = 0.71

$$-\frac{a+2bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3,x)
```

```
[Out] -(a + 2*b*x)/(2*x**2)
```

Giac [A] time = 1.1712, size = 15, normalized size = 0.88

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

3.50 $\int \frac{a+bx}{x^4} dx$

Optimal. Leaf size=17

$$-\frac{a}{3x^3} - \frac{b}{2x^2}$$

[Out] -a/(3*x^3) - b/(2*x^2)

Rubi [A] time = 0.0047191, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$-\frac{a}{3x^3} - \frac{b}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/x^4,x]

[Out] -a/(3*x^3) - b/(2*x^2)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{a+bx}{x^4} dx = \int \left(\frac{a}{x^4} + \frac{b}{x^3}\right) dx$$
$$= -\frac{a}{3x^3} - \frac{b}{2x^2}$$

Mathematica [A] time = 0.0020062, size = 17, normalized size = 1.

$$-\frac{a}{3x^3} - \frac{b}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/x^4,x]

[Out] -a/(3*x^3) - b/(2*x^2)

Maple [A] time = 0.005, size = 14, normalized size = 0.8

$$-\frac{a}{3\,x^3} - \frac{b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4,x)
```

```
[Out] -1/3*a/x^3-1/2/x^2*b
```

Maxima [A] time = 1.0356, size = 18, normalized size = 1.06

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Fricas [A] time = 1.62695, size = 32, normalized size = 1.88

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Sympy [A] time = 0.424286, size = 14, normalized size = 0.82

$$-\frac{2a+3bx}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4,x)
```

```
[Out] -(2*a + 3*b*x)/(6*x**3)
```

Giac [A] time = 1.17997, size = 18, normalized size = 1.06

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

3.51 $\int \frac{a+bx}{x^5} dx$

Optimal. Leaf size=17

$$-\frac{a}{4x^4} - \frac{b}{3x^3}$$

```
[Out] -a/(4*x^4) - b/(3*x^3)
```

Rubi [A] time = 0.0047952, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$-\frac{a}{4x^4} - \frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^5,x]

[Out] -a/(4*x^4) - b/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{x^5} dx &= \int \left(\frac{a}{x^5} + \frac{b}{x^4}\right) dx \\ &= -\frac{a}{4x^4} - \frac{b}{3x^3} \end{aligned}$$

Mathematica [A] time = 0.0021174, size = 17, normalized size = 1.

$$-\frac{a}{4x^4} - \frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^5,x]

[Out] -a/(4*x^4) - b/(3*x^3)
```

Maple [A] time = 0.006, size = 14, normalized size = 0.8

$$-\frac{a}{4\,x^4} - \frac{b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^5,x)
```

```
[Out] -1/4*a/x^4-1/3*b/x^3
```

Maxima [A] time = 1.01408, size = 18, normalized size = 1.06

$$-\frac{4\,bx + 3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(4*b*x + 3*a)/x^4
```

Fricas [A] time = 1.36441, size = 34, normalized size = 2.

$$-\frac{4\,bx + 3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*x + 3*a)/x^4
```

Sympy [A] time = 0.354097, size = 14, normalized size = 0.82

$$-\frac{3a + 4bx}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**5,x)
```

```
[Out] -(3*a + 4*b*x)/(12*x**4)
```

Giac [A] time = 1.212, size = 18, normalized size = 1.06

$$-\frac{4\,bx + 3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(4*b*x + 3*a)/x^4
```

3.52 $\int x^3(a+bx)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Rubi [A] time = 0.011715, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^2,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^2\,dx &= \int \left(a^2x^3+2abx^4+b^2x^5\right)\,dx\\ &= \frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.0020881, size = 30, normalized size = 1.

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^2,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Maple [A] time = 0., size = 25, normalized size = 0.8

$$\frac{a^2x^4}{4}+\frac{2\,abx^5}{5}+\frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2,x)
```

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

Maxima [A] time = 1.00635, size = 32, normalized size = 1.07

$$\frac{1}{6}b^2x^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Fricas [A] time = 1.32588, size = 55, normalized size = 1.83

$$\frac{1}{6}x^6b^2 + \frac{2}{5}x^5ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^2 + 2/5*x^5*b*a + 1/4*x^4*a^2
```

Sympy [A] time = 0.073641, size = 26, normalized size = 0.87

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2,x)
```

```
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6
```

Giac [A] time = 1.16219, size = 32, normalized size = 1.07

$$\frac{1}{6}b^2x^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

3.53 $\int x^2(a+bx)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rubi [A] time = 0.0099083, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^2\,dx&=\int\left(a^2x^2+2abx^3+b^2x^4\right)dx\\&=\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.0029569, size = 30, normalized size = 1.

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Maple [A] time = 0.001, size = 25, normalized size = 0.8

$$\frac{a^2x^3}{3}+\frac{abx^4}{2}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2,x)
```

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

Maxima [A] time = 1.05648, size = 32, normalized size = 1.07

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Fricas [A] time = 1.35776, size = 55, normalized size = 1.83

$$\frac{1}{5}x^5b^2 + \frac{1}{2}x^4ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b^2 + 1/2*x^4*b*a + 1/3*x^3*a^2
```

Sympy [A] time = 0.075105, size = 24, normalized size = 0.8

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2,x)
```

```
[Out] a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

Giac [A] time = 1.1918, size = 32, normalized size = 1.07

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

3.54 $\int x(a+bx)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

```
[Out] (a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

Rubi [A] time = 0.0094348, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^2\,dx &= \int\left(a^2x+2abx^2+b^2x^3\right)dx\\ &= \frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}\end{aligned}$$

Mathematica [A] time = 0.0013115, size = 30, normalized size = 1.

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

Maple [A] time = 0.001, size = 25, normalized size = 0.8

$$\frac{a^2x^2}{2}+\frac{2\,abx^3}{3}+\frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2,x)
```

```
[Out] 1/2*a^2*x^2+2/3*a*b*x^3+1/4*b^2*x^4
```

Maxima [A] time = 1.02898, size = 32, normalized size = 1.07

$$\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Fricas [A] time = 1.28362, size = 55, normalized size = 1.83

$$\frac{1}{4}x^4b^2+\frac{2}{3}x^3ba+\frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b^2 + 2/3*x^3*b*a + 1/2*x^2*a^2
```

Sympy [A] time = 0.087937, size = 26, normalized size = 0.87

$$\frac{a^2x^2}{2}+\frac{2abx^3}{3}+\frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2,x)
```

```
[Out] a**2*x**2/2 + 2*a*b*x**3/3 + b**2*x**4/4
```

Giac [A] time = 1.21765, size = 32, normalized size = 1.07

$$\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

3.55 $\int (a+bx)^2\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^3}{3b}$$

```
[Out] (a + b*x)^3/(3*b)
```

Rubi [A] time = 0.0014801, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2,x]
```

```
[Out] (a + b*x)^3/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^2\,dx=\frac{(a+bx)^3}{3b}$$

Mathematica [A] time = 0.0012379, size = 14, normalized size = 1.

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2,x]
```

```
[Out] (a + b*x)^3/(3*b)
```

Maple [A] time = 0., size = 13, normalized size = 0.9

$$\frac{(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2,x)
```

```
[Out] 1/3*(b*x+a)^3/b
```

Maxima [A] time = 1.03112, size = 27, normalized size = 1.93

$$\frac{1}{3}b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Fricas [A] time = 1.32787, size = 42, normalized size = 3.

$$\frac{1}{3}x^3b^2 + x^2ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*b^2 + x^2*b*a + x*a^2
```

Sympy [B] time = 0.067018, size = 19, normalized size = 1.36

$$a^2x + abx^2 + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2,x)
```

```
[Out] a**2*x + a*b*x**2 + b**2*x**3/3
```

Giac [A] time = 1.21265, size = 16, normalized size = 1.14

$$\frac{(bx + a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a)^3/b
```

3.56 $\int \frac{(a+bx)^2}{x} \, dx$

Optimal. Leaf size=22

$$a^2 \log(x) + 2abx + \frac{b^2x^2}{2}$$

```
[Out] 2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

Rubi [A] time = 0.006339, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$a^2 \log(x) + 2abx + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x,x]

[Out] 2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x} \, dx &= \int \left(2ab + \frac{a^2}{x} + b^2x\right) dx \\ &= 2abx + \frac{b^2x^2}{2} + a^2 \log(x)\end{aligned}$$

Mathematica [A] time = 0.0007976, size = 22, normalized size = 1.

$$a^2 \log(x) + 2abx + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x,x]

[Out] 2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

Maple [A] time = 0.003, size = 21, normalized size = 1.

$$2\,abx + \frac{b^2x^2}{2} + a^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x,x)
```

```
[Out] 2*a*b*x+1/2*b^2*x^2+a^2*ln(x)
```

Maxima [A] time = 1.07302, size = 27, normalized size = 1.23

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Fricas [A] time = 1.48218, size = 49, normalized size = 2.23

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Sympy [A] time = 0.328913, size = 20, normalized size = 0.91

$$a^2\log\left(x\right) + 2abx + \frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x,x)
```

```
[Out] a**2*log(x) + 2*a*b*x + b**2*x**2/2
```

Giac [A] time = 1.13502, size = 28, normalized size = 1.27

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(abs(x))
```

3.57 $\int \frac{(a+bx)^2}{x^2} dx$

Optimal. Leaf size=20

$$-\frac{a^2}{x} + 2ab\log(x) + b^2x$$

```
[Out] -(a^2/x) + b^2*x + 2*a*b*Log[x]
```

Rubi [A] time = 0.0081317, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{x} + 2ab\log(x) + b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^2,x]

[Out] -(a^2/x) + b^2*x + 2*a*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^2}\,dx &= \int \left(b^2 + \frac{a^2}{x^2} + \frac{2ab}{x}\right) dx\\ &= -\frac{a^2}{x} + b^2x + 2ab\log(x)\end{aligned}$$

Mathematica [A] time = 0.0010358, size = 20, normalized size = 1.

$$-\frac{a^2}{x} + 2ab\log(x) + b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^2,x]

[Out] -(a^2/x) + b^2*x + 2*a*b*Log[x]
```

Maple [A] time = 0.006, size = 21, normalized size = 1.1

$$-\frac{a^2}{x} + b^2x + 2\,ab\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2,x)
```

```
[Out] -a^2/x+b^2*x+2*a*b*ln(x)
```

Maxima [A] time = 1.02448, size = 27, normalized size = 1.35

$$b^2x + 2\,ab\log\left(x\right) - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] b^2*x + 2*a*b*log(x) - a^2/x
```

Fricas [A] time = 1.59555, size = 49, normalized size = 2.45

$$\frac{b^2x^2 + 2\,abx\log\left(x\right) - a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)/x
```

Sympy [A] time = 0.30444, size = 17, normalized size = 0.85

$$-\frac{a^2}{x} + 2ab\log\left(x\right) + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*log(x) + b**2*x
```

Giac [A] time = 1.15834, size = 28, normalized size = 1.4

$$b^2x + 2\,ab\log\left(|x|\right) - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] b^2*x + 2*a*b*log(abs(x)) - a^2/x
```

3.58 $\int \frac{(a+bx)^2}{x^3} dx$

Optimal. Leaf size=24

$$-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)$$

```
[Out] -a^2/(2*x^2) - (2*a*b)/x + b^2*Log[x]
```

Rubi [A] time = 0.0081578, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) - (2*a*b)/x + b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^2}{x^3} dx &= \int \left(\frac{a^2}{x^3} + \frac{2ab}{x^2} + \frac{b^2}{x}\right) dx \\ &= -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x) \end{aligned}$$

Mathematica [A] time = 0.004359, size = 24, normalized size = 1.

$$-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) - (2*a*b)/x + b^2*Log[x]
```

Maple [A] time = 0.005, size = 23, normalized size = 1.

$$-\frac{a^2}{2\,x^2} - 2\,\frac{ab}{x} + b^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3,x)
```

```
[Out] -1/2/x^2*a^2-2*a*b/x+b^2*ln(x)
```

Maxima [A] time = 1.02885, size = 28, normalized size = 1.17

$$b^2 \log(x) - \frac{4\,abx + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] b^2*log(x) - 1/2*(4*a*b*x + a^2)/x^2
```

Fricas [A] time = 1.51537, size = 59, normalized size = 2.46

$$\frac{2\,b^2x^2 \log(x) - 4\,abx - a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)/x^2
```

Sympy [A] time = 0.336594, size = 20, normalized size = 0.83

$$b^2 \log(x) - \frac{a^2 + 4abx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3,x)
```

```
[Out] b**2*log(x) - (a**2 + 4*a*b*x)/(2*x**2)
```

Giac [A] time = 1.12442, size = 30, normalized size = 1.25

$$b^2 \log(|x|) - \frac{4\,abx + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] b^2*log(abs(x)) - 1/2*(4*a*b*x + a^2)/x^2
```

3.59 $\int \frac{(a+bx)^2}{x^4} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^3}{3ax^3}$$

```
[Out] -(a + b*x)^3/(3*a*x^3)
```

Rubi [A] time = 0.0015496, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^3}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^4,x]
```

```
[Out] -(a + b*x)^3/(3*a*x^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4} dx = -\frac{(a+bx)^3}{3ax^3}$$

Mathematica [A] time = 0.0073309, size = 26, normalized size = 1.53

$$-\frac{a^2}{3x^3} - \frac{ab}{x^2} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (a*b)/x^2 - b^2/x
```

Maple [A] time = 0.005, size = 25, normalized size = 1.5

$$-\frac{a^2}{3\,x^3} - \frac{ab}{x^2} - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4,x)
```

```
[Out] -1/3*a^2/x^3-1/x^2*a*b-b^2/x
```

Maxima [A] time = 1.00007, size = 30, normalized size = 1.76

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Fricas [A] time = 1.46217, size = 51, normalized size = 3.

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Sympy [A] time = 0.369987, size = 24, normalized size = 1.41

$$-\frac{a^2+3abx+3b^2x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4,x)
```

```
[Out] -(a**2 + 3*a*b*x + 3*b**2*x**2)/(3*x**3)
```

Giac [A] time = 1.12523, size = 30, normalized size = 1.76

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

3.60 $\int \frac{(a+bx)^2}{x^5} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$$

```
[Out] -a^2/(4*x^4) - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

Rubi [A] time = 0.0078972, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^5}\,dx &= \int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx\\ &= -\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0052963, size = 30, normalized size = 1.

$$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

Maple [A] time = 0.005, size = 25, normalized size = 0.8

$$-\frac{a^2}{4\,x^4}-\frac{2\,ab}{3\,x^3}-\frac{b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^5,x)
```

```
[Out] -1/4*a^2/x^4-2/3*a*b/x^3-1/2*b^2/x^2
```

Maxima [A] time = 1.02448, size = 32, normalized size = 1.07

$$-\frac{6\,b^2x^2 + 8\,abx + 3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Fricas [A] time = 1.50164, size = 55, normalized size = 1.83

$$-\frac{6\,b^2x^2 + 8\,abx + 3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Sympy [A] time = 0.355127, size = 26, normalized size = 0.87

$$-\frac{3a^2 + 8abx + 6b^2x^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**5,x)
```

```
[Out] -(3*a**2 + 8*a*b*x + 6*b**2*x**2)/(12*x**4)
```

Giac [A] time = 1.16497, size = 32, normalized size = 1.07

$$-\frac{6\,b^2x^2 + 8\,abx + 3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

3.61 $\int \frac{(a+bx)^2}{x^6} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$$

```
[Out] -a^2/(5*x^5) - (a*b)/(2*x^4) - b^2/(3*x^3)
```

Rubi [A] time = 0.0080537, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (a*b)/(2*x^4) - b^2/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^6} dx &= \int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right) dx\\ &= -\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0078579, size = 30, normalized size = 1.

$$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (a*b)/(2*x^4) - b^2/(3*x^3)
```

Maple [A] time = 0.005, size = 25, normalized size = 0.8

$$-\frac{a^2}{5\,x^5}-\frac{ab}{2\,x^4}-\frac{b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^6,x)
```

```
[Out] -1/5*a^2/x^5-1/2*a*b/x^4-1/3*b^2/x^3
```

Maxima [A] time = 1.0103, size = 32, normalized size = 1.07

$$-\frac{10 b^2 x^2 + 15 abx + 6 a^2}{30 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Fricas [A] time = 1.49608, size = 58, normalized size = 1.93

$$-\frac{10 b^2 x^2 + 15 abx + 6 a^2}{30 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Sympy [A] time = 0.464478, size = 26, normalized size = 0.87

$$-\frac{6a^2 + 15abx + 10b^2x^2}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**6,x)
```

```
[Out] -(6*a**2 + 15*a*b*x + 10*b**2*x**2)/(30*x**5)
```

Giac [A] time = 1.19931, size = 32, normalized size = 1.07

$$-\frac{10 b^2 x^2 + 15 abx + 6 a^2}{30 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="giac")
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

3.62 $\int \frac{(a+bx)^2}{x^7} \, dx$

Optimal. Leaf size=30

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

```
[Out] -a^2/(6*x^6) - (2*a*b)/(5*x^5) - b^2/(4*x^4)
```

Rubi [A] time = 0.0080024, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^7,x]
```

```
[Out] -a^2/(6*x^6) - (2*a*b)/(5*x^5) - b^2/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^7}\,dx &= \int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx\\ &= -\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.0048049, size = 30, normalized size = 1.

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^7,x]
```

```
[Out] -a^2/(6*x^6) - (2*a*b)/(5*x^5) - b^2/(4*x^4)
```

Maple [A] time = 0.005, size = 25, normalized size = 0.8

$$-\frac{a^2}{6\,x^6}-\frac{2\,ab}{5\,x^5}-\frac{b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^7,x)
```

```
[Out] -1/6*a^2/x^6-2/5*a*b/x^5-1/4*b^2/x^4
```

Maxima [A] time = 1.12473, size = 32, normalized size = 1.07

$$-\frac{15\,b^2x^2 + 24\,abx + 10\,a^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

Fricas [A] time = 1.48331, size = 59, normalized size = 1.97

$$-\frac{15\,b^2x^2 + 24\,abx + 10\,a^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^7,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

Sympy [A] time = 0.435002, size = 26, normalized size = 0.87

$$-\frac{10a^2 + 24abx + 15b^2x^2}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**7,x)
```

```
[Out] -(10*a**2 + 24*a*b*x + 15*b**2*x**2)/(60*x**6)
```

Giac [A] time = 1.1513, size = 32, normalized size = 1.07

$$-\frac{15\,b^2x^2 + 24\,abx + 10\,a^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^7,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

3.63 $\int \frac{(a+bx)^2}{x^8} \, dx$

Optimal. Leaf size=30

$$-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}$$

```
[Out] -a^2/(7*x^7) - (a*b)/(3*x^6) - b^2/(5*x^5)
```

Rubi [A] time = 0.0081628, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (a*b)/(3*x^6) - b^2/(5*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^8}\,dx &= \int \left(\frac{a^2}{x^8}+\frac{2ab}{x^7}+\frac{b^2}{x^6}\right)dx\\ &= -\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.0109021, size = 30, normalized size = 1.

$$-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (a*b)/(3*x^6) - b^2/(5*x^5)
```

Maple [A] time = 0.005, size = 25, normalized size = 0.8

$$-\frac{a^2}{7\,x^7}-\frac{ab}{3\,x^6}-\frac{b^2}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^8,x)
```

```
[Out] -1/7*a^2/x^7-1/3*a*b/x^6-1/5*b^2/x^5
```

Maxima [A] time = 1.07029, size = 32, normalized size = 1.07

$$-\frac{21\,b^2x^2 + 35\,abx + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

Fricas [A] time = 1.48804, size = 61, normalized size = 2.03

$$-\frac{21\,b^2x^2 + 35\,abx + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

Sympy [A] time = 0.462055, size = 26, normalized size = 0.87

$$-\frac{15a^2 + 35abx + 21b^2x^2}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**8,x)
```

```
[Out] -(15*a**2 + 35*a*b*x + 21*b**2*x**2)/(105*x**7)
```

Giac [A] time = 1.18508, size = 32, normalized size = 1.07

$$-\frac{21\,b^2x^2 + 35\,abx + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

3.64 $\int x^4(a+bx)^3\,dx$

Optimal. Leaf size=43

$$\frac{1}{2}a^2bx^6+\frac{a^3x^5}{5}+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

```
[Out] (a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7 + (b^3*x^8)/8
```

Rubi [A] time = 0.0181752, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{1}{2}a^2bx^6+\frac{a^3x^5}{5}+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^3,x]
```

```
[Out] (a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7 + (b^3*x^8)/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^3\,dx&=\int\left(a^3x^4+3a^2bx^5+3ab^2x^6+b^3x^7\right)dx\\&=\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.0021594, size = 43, normalized size = 1.

$$\frac{1}{2}a^2bx^6+\frac{a^3x^5}{5}+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^3,x]
```

```
[Out] (a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7 + (b^3*x^8)/8
```

Maple [A] time = 0., size = 36, normalized size = 0.8

$$\frac{a^3x^5}{5}+\frac{a^2bx^6}{2}+\frac{3\,ab^2x^7}{7}+\frac{b^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^3,x)
```

```
[Out] 1/5*a^3*x^5+1/2*a^2*b*x^6+3/7*a*b^2*x^7+1/8*b^3*x^8
```

Maxima [A] time = 1.01717, size = 47, normalized size = 1.09

$$\frac{1}{8}b^3x^8 + \frac{3}{7}ab^2x^7 + \frac{1}{2}a^2bx^6 + \frac{1}{5}a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

Fricas [A] time = 1.29981, size = 80, normalized size = 1.86

$$\frac{1}{8}x^8b^3 + \frac{3}{7}x^7b^2a + \frac{1}{2}x^6ba^2 + \frac{1}{5}x^5a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^3 + 3/7*x^7*b^2*a + 1/2*x^6*b*a^2 + 1/5*x^5*a^3
```

Sympy [A] time = 0.072973, size = 37, normalized size = 0.86

$$\frac{a^3x^5}{5} + \frac{a^2bx^6}{2} + \frac{3ab^2x^7}{7} + \frac{b^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**3,x)
```

```
[Out] a**3*x**5/5 + a**2*b*x**6/2 + 3*a*b**2*x**7/7 + b**3*x**8/8
```

Giac [A] time = 1.17695, size = 47, normalized size = 1.09

$$\frac{1}{8}b^3x^8 + \frac{3}{7}ab^2x^7 + \frac{1}{2}a^2bx^6 + \frac{1}{5}a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

3.65 $\int x^3(a+bx)^3\,dx$

Optimal. Leaf size=43

$$\frac{3}{5}a^2bx^5+\frac{a^3x^4}{4}+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

```
[Out] (a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2 + (b^3*x^7)/7
```

Rubi [A] time = 0.0161845, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{5}a^2bx^5+\frac{a^3x^4}{4}+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^3,x]
```

```
[Out] (a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2 + (b^3*x^7)/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^3\,dx&=\int\left(a^3x^3+3a^2bx^4+3ab^2x^5+b^3x^6\right)dx\\&=\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0019028, size = 43, normalized size = 1.

$$\frac{3}{5}a^2bx^5+\frac{a^3x^4}{4}+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^3,x]
```

```
[Out] (a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2 + (b^3*x^7)/7
```

Maple [A] time = 0.001, size = 36, normalized size = 0.8

$$\frac{a^3x^4}{4}+\frac{3\,a^2bx^5}{5}+\frac{ab^2x^6}{2}+\frac{b^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^3,x)
```

```
[Out] 1/4*a^3*x^4+3/5*a^2*b*x^5+1/2*a*b^2*x^6+1/7*b^3*x^7
```

Maxima [A] time = 1.00693, size = 47, normalized size = 1.09

$$\frac{1}{7}b^3x^7 + \frac{1}{2}ab^2x^6 + \frac{3}{5}a^2bx^5 + \frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

Fricas [A] time = 1.40786, size = 80, normalized size = 1.86

$$\frac{1}{7}x^7b^3 + \frac{1}{2}x^6b^2a + \frac{3}{5}x^5ba^2 + \frac{1}{4}x^4a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^3 + 1/2*x^6*b^2*a + 3/5*x^5*b*a^2 + 1/4*x^4*a^3
```

Sympy [A] time = 0.068522, size = 37, normalized size = 0.86

$$\frac{a^3x^4}{4} + \frac{3a^2bx^5}{5} + \frac{ab^2x^6}{2} + \frac{b^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**3,x)
```

```
[Out] a**3*x**4/4 + 3*a**2*b*x**5/5 + a*b**2*x**6/2 + b**3*x**7/7
```

Giac [A] time = 1.11211, size = 47, normalized size = 1.09

$$\frac{1}{7}b^3x^7 + \frac{1}{2}ab^2x^6 + \frac{3}{5}a^2bx^5 + \frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

3.66 $\int x^2(a+bx)^3\,dx$

Optimal. Leaf size=43

$$\frac{3}{4}a^2bx^4+\frac{a^3x^3}{3}+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5 + (b^3*x^6)/6
```

Rubi [A] time = 0.0146646, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{4}a^2bx^4+\frac{a^3x^3}{3}+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5 + (b^3*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^3\,dx&=\int\left(a^3x^2+3a^2bx^3+3ab^2x^4+b^3x^5\right)dx\\&=\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.0017854, size = 43, normalized size = 1.

$$\frac{3}{4}a^2bx^4+\frac{a^3x^3}{3}+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5 + (b^3*x^6)/6
```

Maple [A] time = 0.001, size = 36, normalized size = 0.8

$$\frac{a^3x^3}{3}+\frac{3\,a^2bx^4}{4}+\frac{3\,ab^2x^5}{5}+\frac{b^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^3,x)
```

```
[Out] 1/3*a^3*x^3+3/4*a^2*b*x^4+3/5*a*b^2*x^5+1/6*b^3*x^6
```

Maxima [A] time = 1.03505, size = 47, normalized size = 1.09

$$\frac{1}{6}b^3x^6 + \frac{3}{5}ab^2x^5 + \frac{3}{4}a^2bx^4 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Fricas [A] time = 1.36393, size = 80, normalized size = 1.86

$$\frac{1}{6}x^6b^3 + \frac{3}{5}x^5b^2a + \frac{3}{4}x^4ba^2 + \frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^3 + 3/5*x^5*b^2*a + 3/4*x^4*b*a^2 + 1/3*x^3*a^3
```

Sympy [A] time = 0.067495, size = 39, normalized size = 0.91

$$\frac{a^3x^3}{3} + \frac{3a^2bx^4}{4} + \frac{3ab^2x^5}{5} + \frac{b^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**4/4 + 3*a*b**2*x**5/5 + b**3*x**6/6
```

Giac [A] time = 1.16336, size = 47, normalized size = 1.09

$$\frac{1}{6}b^3x^6 + \frac{3}{5}ab^2x^5 + \frac{3}{4}a^2bx^4 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

3.67 $\int x(a+bx)^3\,dx$

Optimal. Leaf size=30

$$\frac{(a+bx)^5}{5b^2}-\frac{a(a+bx)^4}{4b^2}$$

```
[Out] -(a*(a + b*x)^4)/(4*b^2) + (a + b*x)^5/(5*b^2)
```

Rubi [A] time = 0.008693, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^5}{5b^2}-\frac{a(a+bx)^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^3,x]
```

```
[Out] -(a*(a + b*x)^4)/(4*b^2) + (a + b*x)^5/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^3\,dx &= \int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx\\ &= -\frac{a(a+bx)^4}{4b^2}+\frac{(a+bx)^5}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.0015622, size = 40, normalized size = 1.33

$$a^2bx^3+\frac{a^3x^2}{2}+\frac{3}{4}ab^2x^4+\frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^3,x]
```

```
[Out] (a^3*x^2)/2 + a^2*b*x^3 + (3*a*b^2*x^4)/4 + (b^3*x^5)/5
```

Maple [A] time = 0., size = 35, normalized size = 1.2

$$\frac{b^3x^5}{5}+\frac{3\,b^2ax^4}{4}+a^2bx^3+\frac{a^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^3,x)
```

```
[Out] 1/5*b^3*x^5+3/4*b^2*a*x^4+a^2*b*x^3+1/2*a^3*x^2
```

Maxima [A] time = 1.03847, size = 46, normalized size = 1.53

$$\frac{1}{5}\,b^{3}x^{5}+\frac{3}{4}\,ab^{2}x^{4}+a^{2}bx^{3}+\frac{1}{2}\,a^{3}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Fricas [A] time = 1.36675, size = 74, normalized size = 2.47

$$\frac{1}{5}x^{5}b^{3}+\frac{3}{4}x^{4}b^{2}a+x^{3}ba^{2}+\frac{1}{2}x^{2}a^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b^3 + 3/4*x^4*b^2*a + x^3*b*a^2 + 1/2*x^2*a^3
```

Sympy [A] time = 0.065796, size = 36, normalized size = 1.2

$$\frac{a^{3}x^{2}}{2}+a^{2}bx^{3}+\frac{3ab^{2}x^{4}}{4}+\frac{b^{3}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**3,x)
```

```
[Out] a**3*x**2/2 + a**2*b*x**3 + 3*a*b**2*x**4/4 + b**3*x**5/5
```

Giac [A] time = 1.14779, size = 46, normalized size = 1.53

$$\frac{1}{5}\,b^{3}x^{5}+\frac{3}{4}\,ab^{2}x^{4}+a^{2}bx^{3}+\frac{1}{2}\,a^{3}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

3.68 $\int (a+bx)^3 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^4}{4b}$$

[Out] (a + b*x)^4/(4*b)

Rubi [A] time = 0.0016024, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^3,x]

[Out] (a + b*x)^4/(4*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^3 \, dx = \frac{(a+bx)^4}{4b}$$

Mathematica [A] time = 0.0012879, size = 14, normalized size = 1.

$$\frac{(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^3,x]

[Out] (a + b*x)^4/(4*b)

Maple [A] time = 0., size = 13, normalized size = 0.9

$$\frac{(bx+a)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^3,x)

```
[Out] 1/4*(b*x+a)^4/b
```

Maxima [B] time = 1.0509, size = 42, normalized size = 3.

$$\frac{1}{4}b^3x^4 + ab^2x^3 + \frac{3}{2}a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Fricas [B] time = 1.33451, size = 66, normalized size = 4.71

$$\frac{1}{4}x^4b^3 + x^3b^2a + \frac{3}{2}x^2ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b^3 + x^3*b^2*a + 3/2*x^2*b*a^2 + x*a^3
```

Sympy [B] time = 0.069013, size = 32, normalized size = 2.29

$$a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3,x)
```

```
[Out] a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4
```

Giac [A] time = 1.16738, size = 16, normalized size = 1.14

$$\frac{(bx + a)^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b*x + a)^4/b
```

3.69 $\int \frac{(a+bx)^3}{x}\,dx$

Optimal. Leaf size=35

$$3a^2bx + a^3\log(x) + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3}$$

```
[Out] 3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x]
```

Rubi [A] time = 0.0105865, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$3a^2bx + a^3\log(x) + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x,x]
```

```
[Out] 3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x}\,dx &= \int \left(3a^2b + \frac{a^3}{x} + 3ab^2x + b^3x^2\right)dx\\ &= 3a^2bx + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3} + a^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.0050428, size = 35, normalized size = 1.

$$3a^2bx + a^3\log(x) + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x,x]
```

```
[Out] 3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x]
```

Maple [A] time = 0.002, size = 32, normalized size = 0.9

$$3\,a^2bx + \frac{3\,ab^2x^2}{2} + \frac{b^3x^3}{3} + a^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x,x)
```

```
[Out] 3*a^2*b*x+3/2*a*b^2*x^2+1/3*b^3*x^3+a^3*ln(x)
```

Maxima [A] time = 1.04152, size = 42, normalized size = 1.2

$$\frac{1}{3}\,b^{3}x^{3}+\frac{3}{2}\,ab^{2}x^{2}+3\,a^{2}bx+a^{3}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Fricas [A] time = 1.50323, size = 73, normalized size = 2.09

$$\frac{1}{3}\,b^{3}x^{3}+\frac{3}{2}\,ab^{2}x^{2}+3\,a^{2}bx+a^{3}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Sympy [A] time = 0.310695, size = 34, normalized size = 0.97

$$a^{3}\log\left(x\right)+3a^{2}bx+\frac{3ab^{2}x^{2}}{2}+\frac{b^{3}x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x,x)
```

```
[Out] a**3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3
```

Giac [A] time = 1.16073, size = 43, normalized size = 1.23

$$\frac{1}{3}\,b^{3}x^{3}+\frac{3}{2}\,ab^{2}x^{2}+3\,a^{2}bx+a^{3}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(abs(x))
```

3.70 $\int \frac{(a+bx)^3}{x^2} dx$

Optimal. Leaf size=34

$$3a^2b\log(x) - \frac{a^3}{x} + 3ab^2x + \frac{b^3x^2}{2}$$

```
[Out] -(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2*b*Log[x]
```

Rubi [A] time = 0.0132317, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$3a^2b\log(x) - \frac{a^3}{x} + 3ab^2x + \frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^2} dx &= \int \left(3ab^2 + \frac{a^3}{x^2} + \frac{3a^2b}{x} + b^3x\right) dx \\ &= -\frac{a^3}{x} + 3ab^2x + \frac{b^3x^2}{2} + 3a^2b\log(x)\end{aligned}$$

Mathematica [A] time = 0.0064276, size = 34, normalized size = 1.

$$3a^2b\log(x) - \frac{a^3}{x} + 3ab^2x + \frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2*b*Log[x]
```

Maple [A] time = 0.005, size = 33, normalized size = 1.

$$-\frac{a^3}{x} + 3\,ab^2x + \frac{b^3x^2}{2} + 3\,a^2b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^2,x)
```

```
[Out] -a^3/x+3*a*b^2*x+1/2*b^3*x^2+3*a^2*b*ln(x)
```

Maxima [A] time = 1.07062, size = 43, normalized size = 1.26

$$\frac{1}{2}\,b^3x^2 + 3\,ab^2x + 3\,a^2b\log\left(x\right) - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(x) - a^3/x
```

Fricas [A] time = 1.50798, size = 78, normalized size = 2.29

$$\frac{b^3x^3 + 6\,ab^2x^2 + 6\,a^2bx\log\left(x\right) - 2\,a^3}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + 6*a*b^2*x^2 + 6*a^2*b*x*log(x) - 2*a^3)/x
```

Sympy [A] time = 0.316417, size = 31, normalized size = 0.91

$$-\frac{a^3}{x} + 3a^2b\log\left(x\right) + 3ab^2x + \frac{b^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**2,x)
```

```
[Out] -a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2
```

Giac [A] time = 1.1732, size = 45, normalized size = 1.32

$$\frac{1}{2}\,b^3x^2 + 3\,ab^2x + 3\,a^2b\log\left(|x|\right) - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(abs(x)) - a^3/x
```

3.71 $\int \frac{(a+bx)^3}{x^3}\,dx$

Optimal. Leaf size=33

$$-\frac{3a^2b}{x}-\frac{a^3}{2x^2}+3ab^2\log(x)+b^3x$$

```
[Out] -a^3/(2*x^2) - (3*a^2*b)/x + b^3*x + 3*a*b^2*Log[x]
```

Rubi [A] time = 0.0126738, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{3a^2b}{x}-\frac{a^3}{2x^2}+3ab^2\log(x)+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) - (3*a^2*b)/x + b^3*x + 3*a*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^3}\,dx &= \int \left(b^3+\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3ab^2}{x}\right)dx\\ &= -\frac{a^3}{2x^2}-\frac{3a^2b}{x}+b^3x+3ab^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0071219, size = 33, normalized size = 1.

$$-\frac{3a^2b}{x}-\frac{a^3}{2x^2}+3ab^2\log(x)+b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) - (3*a^2*b)/x + b^3*x + 3*a*b^2*Log[x]
```

Maple [A] time = 0.005, size = 32, normalized size = 1.

$$-\frac{a^3}{2\,x^2}-3\,\frac{a^2b}{x}+b^3x+3\,ab^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^3,x)
```

```
[Out] -1/2*a^3/x^2-3*a^2*b/x+b^3*x+3*a*b^2*ln(x)
```

Maxima [A] time = 1.0208, size = 41, normalized size = 1.24

$$b^3x + 3\,ab^2\log\left(x\right) - \frac{6\,a^2bx + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] b^3*x + 3*a*b^2*log(x) - 1/2*(6*a^2*b*x + a^3)/x^2
```

Fricas [A] time = 1.57627, size = 81, normalized size = 2.45

$$\frac{2\,b^3x^3 + 6\,ab^2x^2\log\left(x\right) - 6\,a^2bx - a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 6*a*b^2*x^2*log(x) - 6*a^2*b*x - a^3)/x^2
```

Sympy [A] time = 0.44382, size = 31, normalized size = 0.94

$$3ab^2\log\left(x\right) + b^3x - \frac{a^3 + 6a^2bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**3,x)
```

```
[Out] 3*a*b**2*log(x) + b**3*x - (a**3 + 6*a**2*b*x)/(2*x**2)
```

Giac [A] time = 1.14986, size = 42, normalized size = 1.27

$$b^3x + 3\,ab^2\log\left(|x|\right) - \frac{6\,a^2bx + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="giac")
```

```
[Out] b^3*x + 3*a*b^2*log(abs(x)) - 1/2*(6*a^2*b*x + a^3)/x^2
```

3.72 $\int \frac{(a+bx)^3}{x^4} dx$

Optimal. Leaf size=37

$$-\frac{3a^2b}{2x^2} - \frac{a^3}{3x^3} - \frac{3ab^2}{x} + b^3 \log(x)$$

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x]
```

Rubi [A] time = 0.0122868, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{3a^2b}{2x^2} - \frac{a^3}{3x^3} - \frac{3ab^2}{x} + b^3 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^3}{x^4} dx &= \int \left(\frac{a^3}{x^4} + \frac{3a^2b}{x^3} + \frac{3ab^2}{x^2} + \frac{b^3}{x}\right) dx \\ &= -\frac{a^3}{3x^3} - \frac{3a^2b}{2x^2} - \frac{3ab^2}{x} + b^3 \log(x) \end{aligned}$$

Mathematica [A] time = 0.0037367, size = 37, normalized size = 1.

$$-\frac{3a^2b}{2x^2} - \frac{a^3}{3x^3} - \frac{3ab^2}{x} + b^3 \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x]
```

Maple [A] time = 0.006, size = 34, normalized size = 0.9

$$-\frac{a^3}{3x^3} - \frac{3a^2b}{2x^2} - 3\frac{b^2a}{x} + b^3 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^4,x)
```

```
[Out] -1/3*a^3/x^3-3/2*a^2*b/x^2-3*a*b^2/x+b^3*ln(x)
```

Maxima [A] time = 1.04267, size = 46, normalized size = 1.24

$$b^3 \log(x) - \frac{18\,ab^2x^2 + 9\,a^2bx + 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] b^3*log(x) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

Fricas [A] time = 1.58624, size = 85, normalized size = 2.3

$$\frac{6\,b^3x^3\log(x) - 18\,ab^2x^2 - 9\,a^2bx - 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(x) - 18*a*b^2*x^2 - 9*a^2*b*x - 2*a^3)/x^3
```

Sympy [A] time = 0.56435, size = 34, normalized size = 0.92

$$b^3 \log(x) - \frac{2a^3 + 9a^2bx + 18ab^2x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**4,x)
```

```
[Out] b**3*log(x) - (2*a**3 + 9*a**2*b*x + 18*a*b**2*x**2)/(6*x**3)
```

Giac [A] time = 1.19726, size = 47, normalized size = 1.27

$$b^3 \log(|x|) - \frac{18\,ab^2x^2 + 9\,a^2bx + 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x)) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

3.73 $\int \frac{(a+bx)^3}{x^5} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^4}{4ax^4}$$

```
[Out] -(a + b*x)^4/(4*a*x^4)
```

Rubi [A] time = 0.0016722, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^4}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^5,x]

[Out] -(a + b*x)^4/(4*a*x^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^3}{x^5} dx = -\frac{(a+bx)^4}{4ax^4}$$

Mathematica [B] time = 0.0046872, size = 39, normalized size = 2.29

$$-\frac{a^2b}{x^3} - \frac{a^3}{4x^4} - \frac{3ab^2}{2x^2} - \frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^5,x]

[Out] -a^3/(4*x^4) - (a^2*b)/x^3 - (3*a*b^2)/(2*x^2) - b^3/x
```

Maple [B] time = 0.005, size = 36, normalized size = 2.1

$$-\frac{a^2b}{x^3} - \frac{a^3}{4x^4} - \frac{3b^2a}{2x^2} - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^5,x)
```

```
[Out] -a^2*b/x^3-1/4*a^3/x^4-3/2*b^2*a/x^2-b^3/x
```

Maxima [B] time = 1.01065, size = 45, normalized size = 2.65

$$-\frac{4\,b^3x^3 + 6\,ab^2x^2 + 4\,a^2bx + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Fricas [B] time = 1.61322, size = 73, normalized size = 4.29

$$-\frac{4\,b^3x^3 + 6\,ab^2x^2 + 4\,a^2bx + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Sympy [B] time = 0.448039, size = 36, normalized size = 2.12

$$-\frac{a^3 + 4a^2bx + 6ab^2x^2 + 4b^3x^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**5,x)
```

```
[Out] -(a**3 + 4*a**2*b*x + 6*a*b**2*x**2 + 4*b**3*x**3)/(4*x**4)
```

Giac [B] time = 1.1902, size = 45, normalized size = 2.65

$$-\frac{4\,b^3x^3 + 6\,ab^2x^2 + 4\,a^2bx + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

3.74 $\int \frac{(a+bx)^3}{x^6} dx$

Optimal. Leaf size=36

$$\frac{b(a+bx)^4}{20a^2x^4} - \frac{(a+bx)^4}{5ax^5}$$

```
[Out] -(a + b*x)^4/(5*a*x^5) + (b*(a + b*x)^4)/(20*a^2*x^4)
```

Rubi [A] time = 0.005332, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b(a+bx)^4}{20a^2x^4} - \frac{(a+bx)^4}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^6,x]
```

```
[Out] -(a + b*x)^4/(5*a*x^5) + (b*(a + b*x)^4)/(20*a^2*x^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^3}{x^6} dx &= -\frac{(a+bx)^4}{5ax^5} - \frac{b \int \frac{(a+bx)^3}{x^5} dx}{5a} \\ &= -\frac{(a+bx)^4}{5ax^5} + \frac{b(a+bx)^4}{20a^2x^4} \end{aligned}$$

Mathematica [A] time = 0.006572, size = 41, normalized size = 1.14

$$-\frac{3a^2b}{4x^4} - \frac{a^3}{5x^5} - \frac{ab^2}{x^3} - \frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^6,x]
```

```
[Out] -a^3/(5*x^5) - (3*a^2*b)/(4*x^4) - (a*b^2)/x^3 - b^3/(2*x^2)
```

Maple [A] time = 0.004, size = 36, normalized size = 1.

$$-\frac{b^2a}{x^3}-\frac{a^3}{5\,x^5}-\frac{3\,a^2b}{4\,x^4}-\frac{b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^6,x)
```

```
[Out] -b^2*a/x^3-1/5*a^3/x^5-3/4*a^2*b/x^4-1/2*b^3/x^2
```

Maxima [A] time = 1.07295, size = 47, normalized size = 1.31

$$-\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Fricas [A] time = 1.5316, size = 81, normalized size = 2.25

$$-\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Sympy [A] time = 0.438314, size = 37, normalized size = 1.03

$$-\frac{4a^3+15a^2bx+20ab^2x^2+10b^3x^3}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**6,x)
```

```
[Out] -(4*a**3 + 15*a**2*b*x + 20*a*b**2*x**2 + 10*b**3*x**3)/(20*x**5)
```

Giac [A] time = 1.13052, size = 47, normalized size = 1.31

$$-\frac{10\,b^3x^3 + 20\,ab^2x^2 + 15\,a^2bx + 4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="giac")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

3.75 $\int \frac{(a+bx)^3}{x^7} dx$

Optimal. Leaf size=43

$$-\frac{3a^2b}{5x^5}-\frac{a^3}{6x^6}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

```
[Out] -a^3/(6*x^6) - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3)
```

Rubi [A] time = 0.0126824, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{3a^2b}{5x^5}-\frac{a^3}{6x^6}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^7,x]

[Out] -a^3/(6*x^6) - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^7}\,dx &= \int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right)dx\\ &= -\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.003698, size = 43, normalized size = 1.

$$-\frac{3a^2b}{5x^5}-\frac{a^3}{6x^6}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^7,x]

[Out] -a^3/(6*x^6) - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3)
```

Maple [A] time = 0.006, size = 36, normalized size = 0.8

$$-\frac{a^3}{6\,x^6}-\frac{3\,a^2b}{5\,x^5}-\frac{3\,b^2a}{4\,x^4}-\frac{b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^7,x)
```

```
[Out] -1/6*a^3/x^6-3/5*a^2*b/x^5-3/4*a*b^2/x^4-1/3*b^3/x^3
```

Maxima [A] time = 1.04416, size = 47, normalized size = 1.09

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="maxima")
```

```
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

Fricas [A] time = 1.51823, size = 82, normalized size = 1.91

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

Sympy [A] time = 0.552533, size = 37, normalized size = 0.86

$$-\frac{10a^3+36a^2bx+45ab^2x^2+20b^3x^3}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**7,x)
```

```
[Out] -(10*a**3 + 36*a**2*b*x + 45*a*b**2*x**2 + 20*b**3*x**3)/(60*x**6)
```

Giac [A] time = 1.10676, size = 47, normalized size = 1.09

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="giac")
```

```
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

3.76 $\int \frac{(a+bx)^3}{x^8}\,dx$

Optimal. Leaf size=43

$$-\frac{a^2b}{2x^6}-\frac{a^3}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

```
[Out] -a^3/(7*x^7) - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4)
```

Rubi [A] time = 0.0129022, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2b}{2x^6}-\frac{a^3}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^8,x]
```

```
[Out] -a^3/(7*x^7) - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^8}\,dx &= \int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^7}+\frac{3ab^2}{x^6}+\frac{b^3}{x^5}\right)dx\\ &= -\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.0037817, size = 43, normalized size = 1.

$$-\frac{a^2b}{2x^6}-\frac{a^3}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^8,x]
```

```
[Out] -a^3/(7*x^7) - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4)
```

Maple [A] time = 0.006, size = 36, normalized size = 0.8

$$-\frac{a^3}{7\,x^7}-\frac{a^2b}{2\,x^6}-\frac{3\,b^2a}{5\,x^5}-\frac{b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^8,x)
```

```
[Out] -1/7*a^3/x^7-1/2*a^2*b/x^6-3/5*a*b^2/x^5-1/4*b^3/x^4
```

Maxima [A] time = 1.11632, size = 47, normalized size = 1.09

$$-\frac{35\,b^3x^3+84\,ab^2x^2+70\,a^2bx+20\,a^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

Fricas [A] time = 1.62208, size = 84, normalized size = 1.95

$$-\frac{35\,b^3x^3+84\,ab^2x^2+70\,a^2bx+20\,a^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

Sympy [A] time = 0.559754, size = 37, normalized size = 0.86

$$-\frac{20a^3+70a^2bx+84ab^2x^2+35b^3x^3}{140x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**8,x)
```

```
[Out] -(20*a**3 + 70*a**2*b*x + 84*a*b**2*x**2 + 35*b**3*x**3)/(140*x**7)
```

Giac [A] time = 1.16648, size = 47, normalized size = 1.09

$$-\frac{35\,b^3x^3+84\,ab^2x^2+70\,a^2bx+20\,a^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

3.77 $\int x^6(a+bx)^5\,dx$

Optimal. Leaf size=66

$$a^2b^3x^{10}+\frac{10}{9}a^3b^2x^9+\frac{5}{8}a^4bx^8+\frac{a^5x^7}{7}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^9)/9 + a^2*b^3*x^10 + (5*a*b^
4*x^11)/11 + (b^5*x^12)/12
```

Rubi [A] time = 0.031675, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$a^2b^3x^{10}+\frac{10}{9}a^3b^2x^9+\frac{5}{8}a^4bx^8+\frac{a^5x^7}{7}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^9)/9 + a^2*b^3*x^10 + (5*a*b^
4*x^11)/11 + (b^5*x^12)/12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^5\,dx &= \int \left(a^5x^6+5a^4bx^7+10a^3b^2x^8+10a^2b^3x^9+5ab^4x^{10}+b^5x^{11}\right)dx\\ &= \frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}\end{aligned}$$

Mathematica [A] time = 0.0028657, size = 66, normalized size = 1.

$$a^2b^3x^{10}+\frac{10}{9}a^3b^2x^9+\frac{5}{8}a^4bx^8+\frac{a^5x^7}{7}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^9)/9 + a^2*b^3*x^10 + (5*a*b^
4*x^11)/11 + (b^5*x^12)/12
```

Maple [A] time = 0.001, size = 57, normalized size = 0.9

$$\frac{a^5x^7}{7}+\frac{5\,a^4bx^8}{8}+\frac{10\,a^3b^2x^9}{9}+a^2b^3x^{10}+\frac{5\,ab^4x^{11}}{11}+\frac{b^5x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^5,x)
```

```
[Out] 1/7*a^5*x^7+5/8*a^4*b*x^8+10/9*a^3*b^2*x^9+a^2*b^3*x^10+5/11*a*b^4*x^11+1/1
2*b^5*x^12
```

Maxima [A] time = 1.01192, size = 76, normalized size = 1.15

$$\frac{1}{12}\,b^5x^{12}+\frac{5}{11}\,ab^4x^{11}+a^2b^3x^{10}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{8}\,a^4bx^8+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^4
*b*x^8 + 1/7*a^5*x^7
```

Fricas [A] time = 1.29006, size = 131, normalized size = 1.98

$$\frac{1}{12}x^{12}b^5+\frac{5}{11}x^{11}b^4a+x^{10}b^3a^2+\frac{10}{9}x^9b^2a^3+\frac{5}{8}x^8ba^4+\frac{1}{7}x^7a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^5 + 5/11*x^11*b^4*a + x^10*b^3*a^2 + 10/9*x^9*b^2*a^3 + 5/8*x^8
*b*a^4 + 1/7*x^7*a^5
```

Sympy [A] time = 0.087194, size = 63, normalized size = 0.95

$$\frac{a^5x^7}{7}+\frac{5a^4bx^8}{8}+\frac{10a^3b^2x^9}{9}+a^2b^3x^{10}+\frac{5ab^4x^{11}}{11}+\frac{b^5x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**5,x)
```

```
[Out] a**5*x**7/7 + 5*a**4*b*x**8/8 + 10*a**3*b**2*x**9/9 + a**2*b**3*x**10 + 5*a
*b**4*x**11/11 + b**5*x**12/12
```

Giac [A] time = 1.18553, size = 76, normalized size = 1.15

$$\frac{1}{12}\,b^5x^{12}+\frac{5}{11}\,ab^4x^{11}+a^2b^3x^{10}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{8}\,a^4bx^8+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^4
*b*x^8 + 1/7*a^5*x^7
```

3.78 $\int x^5(a+bx)^5\,dx$

Optimal. Leaf size=69

$$\frac{10}{9}a^2b^3x^9+\frac{5}{4}a^3b^2x^8+\frac{5}{7}a^4bx^7+\frac{a^5x^6}{6}+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^9)/9 + (a
*b^4*x^10)/2 + (b^5*x^11)/11
```

Rubi [A] time = 0.0276943, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{10}{9}a^2b^3x^9+\frac{5}{4}a^3b^2x^8+\frac{5}{7}a^4bx^7+\frac{a^5x^6}{6}+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^9)/9 + (a
*b^4*x^10)/2 + (b^5*x^11)/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^5\,dx &= \int \left(a^5x^5+5a^4bx^6+10a^3b^2x^7+10a^2b^3x^8+5ab^4x^9+b^5x^{10}\right)dx\\ &= \frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.002325, size = 69, normalized size = 1.

$$\frac{10}{9}a^2b^3x^9+\frac{5}{4}a^3b^2x^8+\frac{5}{7}a^4bx^7+\frac{a^5x^6}{6}+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^9)/9 + (a
*b^4*x^10)/2 + (b^5*x^11)/11
```

Maple [A] time = 0., size = 58, normalized size = 0.8

$$\frac{a^5x^6}{6} + \frac{5\,a^4bx^7}{7} + \frac{5\,a^3b^2x^8}{4} + \frac{10\,a^2b^3x^9}{9} + \frac{ab^4x^{10}}{2} + \frac{b^5x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^5,x)
```

```
[Out] 1/6*a^5*x^6+5/7*a^4*b*x^7+5/4*a^3*b^2*x^8+10/9*a^2*b^3*x^9+1/2*a*b^4*x^10+1
/11*b^5*x^11
```

Maxima [A] time = 1.02887, size = 77, normalized size = 1.12

$$\frac{1}{11}\,b^5x^{11} + \frac{1}{2}\,ab^4x^{10} + \frac{10}{9}\,a^2b^3x^9 + \frac{5}{4}\,a^3b^2x^8 + \frac{5}{7}\,a^4bx^7 + \frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*a
^4*b*x^7 + 1/6*a^5*x^6
```

Fricas [A] time = 1.32468, size = 134, normalized size = 1.94

$$\frac{1}{11}x^{11}b^5 + \frac{1}{2}x^{10}b^4a + \frac{10}{9}x^9b^3a^2 + \frac{5}{4}x^8b^2a^3 + \frac{5}{7}x^7ba^4 + \frac{1}{6}x^6a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^5 + 1/2*x^10*b^4*a + 10/9*x^9*b^3*a^2 + 5/4*x^8*b^2*a^3 + 5/7*x
^7*b*a^4 + 1/6*x^6*a^5
```

Sympy [A] time = 0.084904, size = 65, normalized size = 0.94

$$\frac{a^5x^6}{6} + \frac{5a^4bx^7}{7} + \frac{5a^3b^2x^8}{4} + \frac{10a^2b^3x^9}{9} + \frac{ab^4x^{10}}{2} + \frac{b^5x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**5,x)
```

```
[Out] a**5*x**6/6 + 5*a**4*b*x**7/7 + 5*a**3*b**2*x**8/4 + 10*a**2*b**3*x**9/9 +
a*b**4*x**10/2 + b**5*x**11/11
```

Giac [A] time = 1.14324, size = 77, normalized size = 1.12

$$\frac{1}{11}\,b^5x^{11} + \frac{1}{2}\,ab^4x^{10} + \frac{10}{9}\,a^2b^3x^9 + \frac{5}{4}\,a^3b^2x^8 + \frac{5}{7}\,a^4bx^7 + \frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*a
^4*b*x^7 + 1/6*a^5*x^6
```

3.79 $\int x^4(a+bx)^5\,dx$

Optimal. Leaf size=69

$$\frac{5}{4}a^2b^3x^8+\frac{10}{7}a^3b^2x^7+\frac{5}{6}a^4bx^6+\frac{a^5x^5}{5}+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^7)/7 + (5*a^2*b^3*x^8)/4 + (5
*a*b^4*x^9)/9 + (b^5*x^10)/10
```

Rubi [A] time = 0.0250605, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{5}{4}a^2b^3x^8+\frac{10}{7}a^3b^2x^7+\frac{5}{6}a^4bx^6+\frac{a^5x^5}{5}+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^7)/7 + (5*a^2*b^3*x^8)/4 + (5
*a*b^4*x^9)/9 + (b^5*x^10)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^5\,dx&=\int\left(a^5x^4+5a^4bx^5+10a^3b^2x^6+10a^2b^3x^7+5ab^4x^8+b^5x^9\right)dx\\&=\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.0059638, size = 69, normalized size = 1.

$$\frac{5}{4}a^2b^3x^8+\frac{10}{7}a^3b^2x^7+\frac{5}{6}a^4bx^6+\frac{a^5x^5}{5}+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^7)/7 + (5*a^2*b^3*x^8)/4 + (5
*a*b^4*x^9)/9 + (b^5*x^10)/10
```

Maple [A] time = 0.001, size = 58, normalized size = 0.8

$$\frac{a^5x^5}{5}+\frac{5\,a^4bx^6}{6}+\frac{10\,a^3b^2x^7}{7}+\frac{5\,a^2b^3x^8}{4}+\frac{5\,ab^4x^9}{9}+\frac{b^5x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^5,x)
```

```
[Out] 1/5*a^5*x^5+5/6*a^4*b*x^6+10/7*a^3*b^2*x^7+5/4*a^2*b^3*x^8+5/9*a*b^4*x^9+1/
10*b^5*x^10
```

Maxima [A] time = 1.02455, size = 77, normalized size = 1.12

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a^
4*b*x^6 + 1/5*a^5*x^5
```

Fricas [A] time = 1.23133, size = 132, normalized size = 1.91

$$\frac{1}{10}x^{10}b^5+\frac{5}{9}x^9b^4a+\frac{5}{4}x^8b^3a^2+\frac{10}{7}x^7b^2a^3+\frac{5}{6}x^6ba^4+\frac{1}{5}x^5a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^5 + 5/9*x^9*b^4*a + 5/4*x^8*b^3*a^2 + 10/7*x^7*b^2*a^3 + 5/6*x^
6*b*a^4 + 1/5*x^5*a^5
```

Sympy [A] time = 0.082492, size = 66, normalized size = 0.96

$$\frac{a^5x^5}{5}+\frac{5a^4bx^6}{6}+\frac{10a^3b^2x^7}{7}+\frac{5a^2b^3x^8}{4}+\frac{5ab^4x^9}{9}+\frac{b^5x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**5,x)
```

```
[Out] a**5*x**5/5 + 5*a**4*b*x**6/6 + 10*a**3*b**2*x**7/7 + 5*a**2*b**3*x**8/4 +
5*a*b**4*x**9/9 + b**5*x**10/10
```

Giac [A] time = 1.19006, size = 77, normalized size = 1.12

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a^
4*b*x^6 + 1/5*a^5*x^5
```

3.80 $\int x^3(a+bx)^5\,dx$

Optimal. Leaf size=64

$$\frac{3a^2(a+bx)^7}{7b^4}-\frac{a^3(a+bx)^6}{6b^4}+\frac{(a+bx)^9}{9b^4}-\frac{3a(a+bx)^8}{8b^4}$$

```
[Out] -(a^3*(a + b*x)^6)/(6*b^4) + (3*a^2*(a + b*x)^7)/(7*b^4) - (3*a*(a + b*x)^8
)/(8*b^4) + (a + b*x)^9/(9*b^4)
```

Rubi [A] time = 0.0264511, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2(a+bx)^7}{7b^4}-\frac{a^3(a+bx)^6}{6b^4}+\frac{(a+bx)^9}{9b^4}-\frac{3a(a+bx)^8}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^5,x]
```

```
[Out] -(a^3*(a + b*x)^6)/(6*b^4) + (3*a^2*(a + b*x)^7)/(7*b^4) - (3*a*(a + b*x)^8
)/(8*b^4) + (a + b*x)^9/(9*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^5\,dx&=\int\left(-\frac{a^3(a+bx)^5}{b^3}+\frac{3a^2(a+bx)^6}{b^3}-\frac{3a(a+bx)^7}{b^3}+\frac{(a+bx)^8}{b^3}\right)dx\\&=-\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}-\frac{3a(a+bx)^8}{8b^4}+\frac{(a+bx)^9}{9b^4}\end{aligned}$$

Mathematica [A] time = 0.0025793, size = 66, normalized size = 1.03

$$\frac{10}{7}a^2b^3x^7+\frac{5}{3}a^3b^2x^6+a^4bx^5+\frac{a^5x^4}{4}+\frac{5}{8}ab^4x^8+\frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^5,x]
```

```
[Out] (a^5*x^4)/4 + a^4*b*x^5 + (5*a^3*b^2*x^6)/3 + (10*a^2*b^3*x^7)/7 + (5*a*b^4
*x^8)/8 + (b^5*x^9)/9
```

Maple [A] time = 0., size = 57, normalized size = 0.9

$$\frac{b^5x^9}{9}+\frac{5\,ab^4x^8}{8}+\frac{10\,a^2b^3x^7}{7}+\frac{5\,a^3b^2x^6}{3}+a^4bx^5+\frac{a^5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^5,x)
```

```
[Out] 1/9*b^5*x^9+5/8*a*b^4*x^8+10/7*a^2*b^3*x^7+5/3*a^3*b^2*x^6+a^4*b*x^5+1/4*a^
5*x^4
```

Maxima [A] time = 1.07099, size = 76, normalized size = 1.19

$$\frac{1}{9}\,b^5x^9+\frac{5}{8}\,ab^4x^8+\frac{10}{7}\,a^2b^3x^7+\frac{5}{3}\,a^3b^2x^6+a^4bx^5+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x^
5 + 1/4*a^5*x^4
```

Fricas [A] time = 1.35036, size = 124, normalized size = 1.94

$$\frac{1}{9}x^9b^5+\frac{5}{8}x^8b^4a+\frac{10}{7}x^7b^3a^2+\frac{5}{3}x^6b^2a^3+x^5ba^4+\frac{1}{4}x^4a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^5 + 5/8*x^8*b^4*a + 10/7*x^7*b^3*a^2 + 5/3*x^6*b^2*a^3 + x^5*b*a^
4 + 1/4*x^4*a^5
```

Sympy [A] time = 0.076684, size = 63, normalized size = 0.98

$$\frac{a^5x^4}{4}+a^4bx^5+\frac{5a^3b^2x^6}{3}+\frac{10a^2b^3x^7}{7}+\frac{5ab^4x^8}{8}+\frac{b^5x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**5,x)
```

```
[Out] a**5*x**4/4 + a**4*b*x**5 + 5*a**3*b**2*x**6/3 + 10*a**2*b**3*x**7/7 + 5*a*
b**4*x**8/8 + b**5*x**9/9
```

Giac [A] time = 1.15571, size = 76, normalized size = 1.19

$$\frac{1}{9}\,b^5x^9+\frac{5}{8}\,ab^4x^8+\frac{10}{7}\,a^2b^3x^7+\frac{5}{3}\,a^3b^2x^6+a^4bx^5+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x^
5 + 1/4*a^5*x^4
```

3.81 $\int x^2(a+bx)^5\,dx$

Optimal. Leaf size=47

$$\frac{a^2(a+bx)^6}{6b^3}+\frac{(a+bx)^8}{8b^3}-\frac{2a(a+bx)^7}{7b^3}$$

```
[Out] (a^2*(a + b*x)^6)/(6*b^3) - (2*a*(a + b*x)^7)/(7*b^3) + (a + b*x)^8/(8*b^3)
```

Rubi [A] time = 0.0205076, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^6}{6b^3}+\frac{(a+bx)^8}{8b^3}-\frac{2a(a+bx)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^5,x]
```

```
[Out] (a^2*(a + b*x)^6)/(6*b^3) - (2*a*(a + b*x)^7)/(7*b^3) + (a + b*x)^8/(8*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^5\,dx&=\int\left(\frac{a^2(a+bx)^5}{b^2}-\frac{2a(a+bx)^6}{b^2}+\frac{(a+bx)^7}{b^2}\right)dx\\&=\frac{a^2(a+bx)^6}{6b^3}-\frac{2a(a+bx)^7}{7b^3}+\frac{(a+bx)^8}{8b^3}\end{aligned}$$

Mathematica [A] time = 0.0048094, size = 67, normalized size = 1.43

$$\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4bx^4+\frac{a^5x^3}{3}+\frac{5}{7}ab^4x^7+\frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^5,x]
```

```
[Out] (a^5*x^3)/3 + (5*a^4*b*x^4)/4 + 2*a^3*b^2*x^5 + (5*a^2*b^3*x^6)/3 + (5*a*b^
4*x^7)/7 + (b^5*x^8)/8
```

Maple [A] time = 0.001, size = 58, normalized size = 1.2

$$\frac{b^5x^8}{8}+\frac{5\,ab^4x^7}{7}+\frac{5\,a^2b^3x^6}{3}+2\,a^3b^2x^5+\frac{5\,a^4bx^4}{4}+\frac{a^5x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^5,x)
```

```
[Out] 1/8*b^5*x^8+5/7*a*b^4*x^7+5/3*a^2*b^3*x^6+2*a^3*b^2*x^5+5/4*a^4*b*x^4+1/3*a
^5*x^3
```

Maxima [A] time = 1.0072, size = 77, normalized size = 1.64

$$\frac{1}{8}\,b^5x^8+\frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^2b^3x^6+2\,a^3b^2x^5+\frac{5}{4}\,a^4bx^4+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*x
^4 + 1/3*a^5*x^3
```

Fricas [A] time = 1.39859, size = 126, normalized size = 2.68

$$\frac{1}{8}x^8b^5+\frac{5}{7}x^7b^4a+\frac{5}{3}x^6b^3a^2+2x^5b^2a^3+\frac{5}{4}x^4ba^4+\frac{1}{3}x^3a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^5 + 5/7*x^7*b^4*a + 5/3*x^6*b^3*a^2 + 2*x^5*b^2*a^3 + 5/4*x^4*b*a
^4 + 1/3*x^3*a^5
```

Sympy [A] time = 0.081626, size = 65, normalized size = 1.38

$$\frac{a^5x^3}{3}+\frac{5a^4bx^4}{4}+2a^3b^2x^5+\frac{5a^2b^3x^6}{3}+\frac{5ab^4x^7}{7}+\frac{b^5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**5,x)
```

```
[Out] a**5*x**3/3 + 5*a**4*b*x**4/4 + 2*a**3*b**2*x**5 + 5*a**2*b**3*x**6/3 + 5*a
*b**4*x**7/7 + b**5*x**8/8
```

Giac [A] time = 1.17811, size = 77, normalized size = 1.64

$$\frac{1}{8}\,b^5x^8+\frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^2b^3x^6+2\,a^3b^2x^5+\frac{5}{4}\,a^4bx^4+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*x
^4 + 1/3*a^5*x^3
```

3.82 $\int x(a+bx)^5\,dx$

Optimal. Leaf size=30

$$\frac{(a+bx)^7}{7b^2}-\frac{a(a+bx)^6}{6b^2}$$

```
[Out] -(a*(a + b*x)^6)/(6*b^2) + (a + b*x)^7/(7*b^2)
```

Rubi [A] time = 0.0079793, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^7}{7b^2}-\frac{a(a+bx)^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^5,x]
```

```
[Out] -(a*(a + b*x)^6)/(6*b^2) + (a + b*x)^7/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^5\,dx&=\int\left(-\frac{a(a+bx)^5}{b}+\frac{(a+bx)^6}{b}\right)dx\\&=-\frac{a(a+bx)^6}{6b^2}+\frac{(a+bx)^7}{7b^2}\end{aligned}$$

Mathematica [B] time = 0.0020969, size = 67, normalized size = 2.23

$$2a^2b^3x^5+\frac{5}{2}a^3b^2x^4+\frac{5}{3}a^4bx^3+\frac{a^5x^2}{2}+\frac{5}{6}ab^4x^6+\frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^5,x]
```

```
[Out] (a^5*x^2)/2 + (5*a^4*b*x^3)/3 + (5*a^3*b^2*x^4)/2 + 2*a^2*b^3*x^5 + (5*a*b^
4*x^6)/6 + (b^5*x^7)/7
```

Maple [B] time = 0., size = 58, normalized size = 1.9

$$\frac{b^5x^7}{7}+\frac{5\,ab^4x^6}{6}+2\,a^2b^3x^5+\frac{5\,a^3b^2x^4}{2}+\frac{5\,a^4bx^3}{3}+\frac{a^5x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^5,x)
```

```
[Out] 1/7*b^5*x^7+5/6*a*b^4*x^6+2*a^2*b^3*x^5+5/2*a^3*b^2*x^4+5/3*a^4*b*x^3+1/2*a
^5*x^2
```

Maxima [B] time = 1.03267, size = 77, normalized size = 2.57

$$\frac{1}{7}\,b^5x^7 + \frac{5}{6}\,ab^4x^6 + 2\,a^2b^3x^5 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{3}\,a^4bx^3 + \frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*x
^3 + 1/2*a^5*x^2
```

Fricas [B] time = 1.35533, size = 126, normalized size = 4.2

$$\frac{1}{7}x^7b^5 + \frac{5}{6}x^6b^4a + 2x^5b^3a^2 + \frac{5}{2}x^4b^2a^3 + \frac{5}{3}x^3ba^4 + \frac{1}{2}x^2a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^5 + 5/6*x^6*b^4*a + 2*x^5*b^3*a^2 + 5/2*x^4*b^2*a^3 + 5/3*x^3*b*a
^4 + 1/2*x^2*a^5
```

Sympy [B] time = 0.076103, size = 65, normalized size = 2.17

$$\frac{a^5x^2}{2} + \frac{5a^4bx^3}{3} + \frac{5a^3b^2x^4}{2} + 2a^2b^3x^5 + \frac{5ab^4x^6}{6} + \frac{b^5x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**5,x)
```

```
[Out] a**5*x**2/2 + 5*a**4*b*x**3/3 + 5*a**3*b**2*x**4/2 + 2*a**2*b**3*x**5 + 5*a
*b**4*x**6/6 + b**5*x**7/7
```

Giac [B] time = 1.16592, size = 77, normalized size = 2.57

$$\frac{1}{7}\,b^5x^7 + \frac{5}{6}\,ab^4x^6 + 2\,a^2b^3x^5 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{3}\,a^4bx^3 + \frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*x
^3 + 1/2*a^5*x^2
```

3.83 $\int (a+bx)^5\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^6}{6b}$$

```
[Out] (a + b*x)^6/(6*b)
```

Rubi [A] time = 0.0017074, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5,x]
```

```
[Out] (a + b*x)^6/(6*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^5\,dx = \frac{(a+bx)^6}{6b}$$

Mathematica [A] time = 0.0058494, size = 14, normalized size = 1.

$$\frac{(a+bx)^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5,x]
```

```
[Out] (a + b*x)^6/(6*b)
```

Maple [A] time = 0., size = 13, normalized size = 0.9

$$\frac{(bx+a)^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5,x)
```

```
[Out] 1/6*(b*x+a)^6/b
```

Maxima [B] time = 1.05434, size = 72, normalized size = 5.14

$$\frac{1}{6}b^5x^6 + ab^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Fricas [B] time = 1.35106, size = 116, normalized size = 8.29

$$\frac{1}{6}x^6b^5 + x^5b^4a + \frac{5}{2}x^4b^3a^2 + \frac{10}{3}x^3b^2a^3 + \frac{5}{2}x^2ba^4 + xa^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^5 + x^5*b^4*a + 5/2*x^4*b^3*a^2 + 10/3*x^3*b^2*a^3 + 5/2*x^2*b*a^
4 + x*a^5
```

Sympy [B] time = 0.076919, size = 60, normalized size = 4.29

$$a^5x + \frac{5a^4bx^2}{2} + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^4}{2} + ab^4x^5 + \frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/2 + a*b**
4*x**5 + b**5*x**6/6
```

Giac [A] time = 1.19522, size = 16, normalized size = 1.14

$$\frac{(bx+a)^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/6*(b*x + a)^6/b
```

3.84 $\int \frac{(a+bx)^5}{x} \, dx$

Optimal. Leaf size=59

$$5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+5a^4bx+a^5\log(x)+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}$$

```
[Out] 5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^5
)/5 + a^5*Log[x]
```

Rubi [A] time = 0.0175937, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+5a^4bx+a^5\log(x)+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x,x]
```

```
[Out] 5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^5
)/5 + a^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x}\,dx &= \int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right)dx\\ &= 5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}+a^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.0048367, size = 59, normalized size = 1.

$$5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+5a^4bx+a^5\log(x)+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x,x]
```

```
[Out] 5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^5
)/5 + a^5*Log[x]
```

Maple [A] time = 0.002, size = 54, normalized size = 0.9

$$5\,a^4bx+5\,a^3b^2x^2+\frac{10\,a^2b^3x^3}{3}+\frac{5\,ab^4x^4}{4}+\frac{b^5x^5}{5}+a^5\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x,x)
```

```
[Out] 5*a^4*b*x+5*a^3*b^2*x^2+10/3*a^2*b^3*x^3+5/4*a*b^4*x^4+1/5*b^5*x^5+a^5*ln(x
)
```

Maxima [A] time = 1.05842, size = 72, normalized size = 1.22

$$\frac{1}{5}\,b^5x^5+\frac{5}{4}\,ab^4x^4+\frac{10}{3}\,a^2b^3x^3+5\,a^3b^2x^2+5\,a^4bx+a^5\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(x)
```

Fricas [A] time = 1.58551, size = 120, normalized size = 2.03

$$\frac{1}{5}\,b^5x^5+\frac{5}{4}\,ab^4x^4+\frac{10}{3}\,a^2b^3x^3+5\,a^3b^2x^2+5\,a^4bx+a^5\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(x)
```

Sympy [A] time = 0.360342, size = 60, normalized size = 1.02

$$a^5\log\left(x\right)+5a^4bx+5a^3b^2x^2+\frac{10a^2b^3x^3}{3}+\frac{5ab^4x^4}{4}+\frac{b^5x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x,x)
```

```
[Out] a**5*log(x) + 5*a**4*b*x + 5*a**3*b**2*x**2 + 10*a**2*b**3*x**3/3 + 5*a*b**
4*x**4/4 + b**5*x**5/5
```

Giac [A] time = 1.19554, size = 73, normalized size = 1.24

$$\frac{1}{5}\,b^5x^5+\frac{5}{4}\,ab^4x^4+\frac{10}{3}\,a^2b^3x^3+5\,a^3b^2x^2+5\,a^4bx+a^5\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(abs(x))
```

3.85 $\int \frac{(a+bx)^5}{x^2}\,dx$

Optimal. Leaf size=58

$$5a^2b^3x^2+10a^3b^2x+5a^4b\log(x)-\frac{a^5}{x}+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}$$

```
[Out] -(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 + (5*a*b^4*x^3)/3 + (b^5*x^4)/4 + 5
*a^4*b*Log[x]
```

Rubi [A] time = 0.0207801, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$5a^2b^3x^2+10a^3b^2x+5a^4b\log(x)-\frac{a^5}{x}+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^2,x]
```

```
[Out] -(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 + (5*a*b^4*x^3)/3 + (b^5*x^4)/4 + 5
*a^4*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^2}\,dx &= \int \left(10a^3b^2+\frac{a^5}{x^2}+\frac{5a^4b}{x}+10a^2b^3x+5ab^4x^2+b^5x^3\right)dx\\ &= -\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}+5a^4b\log(x)\end{aligned}$$

Mathematica [A] time = 0.0050512, size = 58, normalized size = 1.

$$5a^2b^3x^2+10a^3b^2x+5a^4b\log(x)-\frac{a^5}{x}+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^2,x]
```

```
[Out] -(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 + (5*a*b^4*x^3)/3 + (b^5*x^4)/4 + 5
*a^4*b*Log[x]
```

Maple [A] time = 0.006, size = 55, normalized size = 1.

$$-\frac{a^5}{x}+10\,a^3b^2x+5\,a^2b^3x^2+\frac{5\,ab^4x^3}{3}+\frac{b^5x^4}{4}+5\,a^4b\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^2,x)
```

```
[Out] -a^5/x+10*a^3*b^2*x+5*a^2*b^3*x^2+5/3*a*b^4*x^3+1/4*b^5*x^4+5*a^4*b*ln(x)
```

Maxima [A] time = 1.03527, size = 73, normalized size = 1.26

$$\frac{1}{4}\,b^5x^4+\frac{5}{3}\,ab^4x^3+5\,a^2b^3x^2+10\,a^3b^2x+5\,a^4b\log\left(x\right)-\frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^5*x^4 + 5/3*a*b^4*x^3 + 5*a^2*b^3*x^2 + 10*a^3*b^2*x + 5*a^4*b*log(x)
 - a^5/x
```

Fricas [A] time = 1.56236, size = 134, normalized size = 2.31

$$\frac{3\,b^5x^5+20\,ab^4x^4+60\,a^2b^3x^3+120\,a^3b^2x^2+60\,a^4bx\log\left(x\right)-12\,a^5}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^5 + 20*a*b^4*x^4 + 60*a^2*b^3*x^3 + 120*a^3*b^2*x^2 + 60*a^4*
b*x*log(x) - 12*a^5)/x
```

Sympy [A] time = 0.33742, size = 56, normalized size = 0.97

$$-\frac{a^5}{x}+5a^4b\log\left(x\right)+10a^3b^2x+5a^2b^3x^2+\frac{5ab^4x^3}{3}+\frac{b^5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**2,x)
```

```
[Out] -a**5/x + 5*a**4*b*log(x) + 10*a**3*b**2*x + 5*a**2*b**3*x**2 + 5*a*b**4*x*
*3/3 + b**5*x**4/4
```

Giac [A] time = 1.14791, size = 74, normalized size = 1.28

$$\frac{1}{4}\,b^5x^4+\frac{5}{3}\,ab^4x^3+5\,a^2b^3x^2+10\,a^3b^2x+5\,a^4b\log\left(|x|\right)-\frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4 + 5/3*a*b^4*x^3 + 5*a^2*b^3*x^2 + 10*a^3*b^2*x + 5*a^4*b*log(ab
s(x)) - a^5/x
```

3.86 $\int \frac{(a+bx)^5}{x^3} dx$

Optimal. Leaf size=60

$$10a^2b^3x + 10a^3b^2\log(x) - \frac{5a^4b}{x} - \frac{a^5}{2x^2} + \frac{5}{2}ab^4x^2 + \frac{b^5x^3}{3}$$

```
[Out] -a^5/(2*x^2) - (5*a^4*b)/x + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + (b^5*x^3)/3 +
 10*a^3*b^2*Log[x]
```

Rubi [A] time = 0.0216892, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$10a^2b^3x + 10a^3b^2\log(x) - \frac{5a^4b}{x} - \frac{a^5}{2x^2} + \frac{5}{2}ab^4x^2 + \frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^3,x]
```

```
[Out] -a^5/(2*x^2) - (5*a^4*b)/x + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + (b^5*x^3)/3 +
 10*a^3*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^3} dx &= \int \left(10a^2b^3 + \frac{a^5}{x^3} + \frac{5a^4b}{x^2} + \frac{10a^3b^2}{x} + 5ab^4x + b^5x^2\right) dx\\ &= -\frac{a^5}{2x^2} - \frac{5a^4b}{x} + 10a^2b^3x + \frac{5}{2}ab^4x^2 + \frac{b^5x^3}{3} + 10a^3b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0059594, size = 60, normalized size = 1.

$$10a^2b^3x + 10a^3b^2\log(x) - \frac{5a^4b}{x} - \frac{a^5}{2x^2} + \frac{5}{2}ab^4x^2 + \frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^3,x]
```

```
[Out] -a^5/(2*x^2) - (5*a^4*b)/x + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + (b^5*x^3)/3 +
 10*a^3*b^2*Log[x]
```

Maple [A] time = 0.006, size = 55, normalized size = 0.9

$$-\frac{a^5}{2x^2}-5\frac{a^4b}{x}+10a^2b^3x+\frac{5ab^4x^2}{2}+\frac{b^5x^3}{3}+10a^3b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^3,x)
```

```
[Out] -1/2*a^5/x^2-5*a^4*b/x+10*a^2*b^3*x+5/2*a*b^4*x^2+1/3*b^5*x^3+10*a^3*b^2*ln
(x)
```

Maxima [A] time = 1.03899, size = 72, normalized size = 1.2

$$\frac{1}{3}b^5x^3+\frac{5}{2}ab^4x^2+10a^2b^3x+10a^3b^2\log(x)-\frac{10a^4bx+a^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*x^3 + 5/2*a*b^4*x^2 + 10*a^2*b^3*x + 10*a^3*b^2*log(x) - 1/2*(10*a^
4*b*x + a^5)/x^2
```

Fricas [A] time = 1.47676, size = 132, normalized size = 2.2

$$\frac{2b^5x^5+15ab^4x^4+60a^2b^3x^3+60a^3b^2x^2\log(x)-30a^4bx-3a^5}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*x^5 + 15*a*b^4*x^4 + 60*a^2*b^3*x^3 + 60*a^3*b^2*x^2*log(x) - 30
*a^4*b*x - 3*a^5)/x^2
```

Sympy [A] time = 0.382662, size = 58, normalized size = 0.97

$$10a^3b^2\log(x)+10a^2b^3x+\frac{5ab^4x^2}{2}+\frac{b^5x^3}{3}-\frac{a^5+10a^4bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**3,x)
```

```
[Out] 10*a**3*b**2*log(x) + 10*a**2*b**3*x + 5*a*b**4*x**2/2 + b**5*x**3/3 - (a**
5 + 10*a**4*b*x)/(2*x**2)
```

Giac [A] time = 1.18271, size = 73, normalized size = 1.22

$$\frac{1}{3}b^5x^3+\frac{5}{2}ab^4x^2+10a^2b^3x+10a^3b^2\log(|x|)-\frac{10a^4bx+a^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3 + 5/2*a*b^4*x^2 + 10*a^2*b^3*x + 10*a^3*b^2*log(abs(x)) - 1/2*(
10*a^4*b*x + a^5)/x^2
```

3.87 $\int \frac{(a+bx)^5}{x^4} dx$

Optimal. Leaf size=60

$$-\frac{10a^3b^2}{x} + 10a^2b^3\log(x) - \frac{5a^4b}{2x^2} - \frac{a^5}{3x^3} + 5ab^4x + \frac{b^5x^2}{2}$$

```
[Out] -a^5/(3*x^3) - (5*a^4*b)/(2*x^2) - (10*a^3*b^2)/x + 5*a*b^4*x + (b^5*x^2)/2
 + 10*a^2*b^3*Log[x]
```

Rubi [A] time = 0.0216694, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{x} + 10a^2b^3\log(x) - \frac{5a^4b}{2x^2} - \frac{a^5}{3x^3} + 5ab^4x + \frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^4,x]
```

```
[Out] -a^5/(3*x^3) - (5*a^4*b)/(2*x^2) - (10*a^3*b^2)/x + 5*a*b^4*x + (b^5*x^2)/2
 + 10*a^2*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^4} dx &= \int \left(5ab^4 + \frac{a^5}{x^4} + \frac{5a^4b}{x^3} + \frac{10a^3b^2}{x^2} + \frac{10a^2b^3}{x} + b^5x\right) dx \\ &= -\frac{a^5}{3x^3} - \frac{5a^4b}{2x^2} - \frac{10a^3b^2}{x} + 5ab^4x + \frac{b^5x^2}{2} + 10a^2b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.0058558, size = 60, normalized size = 1.

$$-\frac{10a^3b^2}{x} + 10a^2b^3\log(x) - \frac{5a^4b}{2x^2} - \frac{a^5}{3x^3} + 5ab^4x + \frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^4,x]
```

```
[Out] -a^5/(3*x^3) - (5*a^4*b)/(2*x^2) - (10*a^3*b^2)/x + 5*a*b^4*x + (b^5*x^2)/2
 + 10*a^2*b^3*Log[x]
```

Maple [A] time = 0.005, size = 55, normalized size = 0.9

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-10\frac{a^3b^2}{x}+5ab^4x+\frac{b^5x^2}{2}+10a^2b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^4,x)
```

```
[Out] -1/3*a^5/x^3-5/2*a^4*b/x^2-10*a^3*b^2/x+5*a*b^4*x+1/2*b^5*x^2+10*a^2*b^3*ln
(x)
```

Maxima [A] time = 1.02227, size = 74, normalized size = 1.23

$$\frac{1}{2}b^5x^2+5ab^4x+10a^2b^3\log(x)-\frac{60a^3b^2x^2+15a^4bx+2a^5}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*x^2 + 5*a*b^4*x + 10*a^2*b^3*log(x) - 1/6*(60*a^3*b^2*x^2 + 15*a^4*
b*x + 2*a^5)/x^3
```

Fricas [A] time = 1.48722, size = 132, normalized size = 2.2

$$\frac{3b^5x^5+30ab^4x^4+60a^2b^3x^3\log(x)-60a^3b^2x^2-15a^4bx-2a^5}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*x^5 + 30*a*b^4*x^4 + 60*a^2*b^3*x^3*log(x) - 60*a^3*b^2*x^2 - 15
*a^4*b*x - 2*a^5)/x^3
```

Sympy [A] time = 0.481205, size = 58, normalized size = 0.97

$$10a^2b^3\log(x)+5ab^4x+\frac{b^5x^2}{2}-\frac{2a^5+15a^4bx+60a^3b^2x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**4,x)
```

```
[Out] 10*a**2*b**3*log(x) + 5*a*b**4*x + b**5*x**2/2 - (2*a**5 + 15*a**4*b*x + 60
*a**3*b**2*x**2)/(6*x**3)
```

Giac [A] time = 1.2318, size = 76, normalized size = 1.27

$$\frac{1}{2}b^5x^2+5ab^4x+10a^2b^3\log(|x|)-\frac{60a^3b^2x^2+15a^4bx+2a^5}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2 + 5*a*b^4*x + 10*a^2*b^3*log(abs(x)) - 1/6*(60*a^3*b^2*x^2 + 15
*a^4*b*x + 2*a^5)/x^3
```

3.88 $\int \frac{(a+bx)^5}{x^5} \, dx$

Optimal. Leaf size=57

$$-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}-\frac{5a^4b}{3x^3}-\frac{a^5}{4x^4}+5ab^4\log(x)+b^5x$$

```
[Out] -a^5/(4*x^4) - (5*a^4*b)/(3*x^3) - (5*a^3*b^2)/x^2 - (10*a^2*b^3)/x + b^5*x
 + 5*a*b^4*Log[x]
```

Rubi [A] time = 0.0201166, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}-\frac{5a^4b}{3x^3}-\frac{a^5}{4x^4}+5ab^4\log(x)+b^5x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^5,x]
```

```
[Out] -a^5/(4*x^4) - (5*a^4*b)/(3*x^3) - (5*a^3*b^2)/x^2 - (10*a^2*b^3)/x + b^5*x
 + 5*a*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^5}\,dx &= \int \left(b^5+\frac{a^5}{x^5}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^3}+\frac{10a^2b^3}{x^2}+\frac{5ab^4}{x}\right)dx\\ &= -\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+b^5x+5ab^4\log(x)\end{aligned}$$

Mathematica [A] time = 0.0057439, size = 57, normalized size = 1.

$$-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}-\frac{5a^4b}{3x^3}-\frac{a^5}{4x^4}+5ab^4\log(x)+b^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^5,x]
```

```
[Out] -a^5/(4*x^4) - (5*a^4*b)/(3*x^3) - (5*a^3*b^2)/x^2 - (10*a^2*b^3)/x + b^5*x
 + 5*a*b^4*Log[x]
```

Maple [A] time = 0.006, size = 54, normalized size = 1.

$$-\frac{a^{5}}{4\,x^{4}}-\frac{5\,a^{4}b}{3\,x^{3}}-5\,\frac{a^{3}b^{2}}{x^{2}}-10\,\frac{a^{2}b^{3}}{x}+b^{5}x+5\,ab^{4}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^5,x)
```

```
[Out] -1/4*a^5/x^4-5/3*a^4*b/x^3-5*a^3*b^2/x^2-10*a^2*b^3/x+b^5*x+5*a*b^4*ln(x)
```

Maxima [A] time = 1.08368, size = 73, normalized size = 1.28

$$b^{5}x+5\,ab^{4}\log\left(x\right)-\frac{120\,a^{2}b^{3}x^{3}+60\,a^{3}b^{2}x^{2}+20\,a^{4}bx+3\,a^{5}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x, algorithm="maxima")
```

```
[Out] b^5*x + 5*a*b^4*log(x) - 1/12*(120*a^2*b^3*x^3 + 60*a^3*b^2*x^2 + 20*a^4*b*
x + 3*a^5)/x^4
```

Fricas [A] time = 1.56456, size = 136, normalized size = 2.39

$$\frac{12\,b^{5}x^{5}+60\,ab^{4}x^{4}\log\left(x\right)-120\,a^{2}b^{3}x^{3}-60\,a^{3}b^{2}x^{2}-20\,a^{4}bx-3\,a^{5}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^5*x^5 + 60*a*b^4*x^4*log(x) - 120*a^2*b^3*x^3 - 60*a^3*b^2*x^2 -
 20*a^4*b*x - 3*a^5)/x^4
```

Sympy [A] time = 0.522321, size = 56, normalized size = 0.98

$$5ab^{4}\log\left(x\right)+b^{5}x-\frac{3a^{5}+20a^{4}bx+60a^{3}b^{2}x^{2}+120a^{2}b^{3}x^{3}}{12x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**5,x)
```

```
[Out] 5*a*b**4*log(x) + b**5*x - (3*a**5 + 20*a**4*b*x + 60*a**3*b**2*x**2 + 120*
a**2*b**3*x**3)/(12*x**4)
```

Giac [A] time = 1.19398, size = 74, normalized size = 1.3

$$b^{5}x+5\,ab^{4}\log\left(|x|\right)-\frac{120\,a^{2}b^{3}x^{3}+60\,a^{3}b^{2}x^{2}+20\,a^{4}bx+3\,a^{5}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x, algorithm="giac")
```

```
[Out] b^5*x + 5*a*b^4*log(abs(x)) - 1/12*(120*a^2*b^3*x^3 + 60*a^3*b^2*x^2 + 20*a
^4*b*x + 3*a^5)/x^4
```

3.89 $\int \frac{(a+bx)^5}{x^6} dx$

Optimal. Leaf size=61

$$-\frac{10a^3b^2}{3x^3} - \frac{5a^2b^3}{x^2} - \frac{5a^4b}{4x^4} - \frac{a^5}{5x^5} - \frac{5ab^4}{x} + b^5 \log(x)$$

```
[Out] -a^5/(5*x^5) - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 -
 (5*a*b^4)/x + b^5*Log[x]
```

Rubi [A] time = 0.0209423, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{3x^3} - \frac{5a^2b^3}{x^2} - \frac{5a^4b}{4x^4} - \frac{a^5}{5x^5} - \frac{5ab^4}{x} + b^5 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^6,x]
```

```
[Out] -a^5/(5*x^5) - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 -
 (5*a*b^4)/x + b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^6} dx &= \int \left(\frac{a^5}{x^6} + \frac{5a^4b}{x^5} + \frac{10a^3b^2}{x^4} + \frac{10a^2b^3}{x^3} + \frac{5ab^4}{x^2} + \frac{b^5}{x}\right) dx \\ &= -\frac{a^5}{5x^5} - \frac{5a^4b}{4x^4} - \frac{10a^3b^2}{3x^3} - \frac{5a^2b^3}{x^2} - \frac{5ab^4}{x} + b^5 \log(x)\end{aligned}$$

Mathematica [A] time = 0.0053105, size = 61, normalized size = 1.

$$-\frac{10a^3b^2}{3x^3} - \frac{5a^2b^3}{x^2} - \frac{5a^4b}{4x^4} - \frac{a^5}{5x^5} - \frac{5ab^4}{x} + b^5 \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^6,x]
```

```
[Out] -a^5/(5*x^5) - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 -
 (5*a*b^4)/x + b^5*Log[x]
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$-\frac{a^5}{5\,x^5}-\frac{5\,a^4b}{4\,x^4}-\frac{10\,a^3b^2}{3\,x^3}-5\,\frac{a^2b^3}{x^2}-5\,\frac{ab^4}{x}+b^5\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^6,x)
```

```
[Out] -1/5*a^5/x^5-5/4*a^4*b/x^4-10/3*a^3*b^2/x^3-5*a^2*b^3/x^2-5*a*b^4/x+b^5*ln(
x)
```

Maxima [A] time = 1.02984, size = 76, normalized size = 1.25

$$b^5\log\left(x\right)-\frac{300\,ab^4x^4+300\,a^2b^3x^3+200\,a^3b^2x^2+75\,a^4bx+12\,a^5}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="maxima")
```

```
[Out] b^5*log(x) - 1/60*(300*a*b^4*x^4 + 300*a^2*b^3*x^3 + 200*a^3*b^2*x^2 + 75*a
^4*b*x + 12*a^5)/x^5
```

Fricas [A] time = 1.62891, size = 140, normalized size = 2.3

$$\frac{60\,b^5x^5\log\left(x\right)-300\,ab^4x^4-300\,a^2b^3x^3-200\,a^3b^2x^2-75\,a^4bx-12\,a^5}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^5*x^5*log(x) - 300*a*b^4*x^4 - 300*a^2*b^3*x^3 - 200*a^3*b^2*x^2
 - 75*a^4*b*x - 12*a^5)/x^5
```

Sympy [A] time = 0.620298, size = 58, normalized size = 0.95

$$b^5\log\left(x\right)-\frac{12a^5+75a^4bx+200a^3b^2x^2+300a^2b^3x^3+300ab^4x^4}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**6,x)
```

```
[Out] b**5*log(x) - (12*a**5 + 75*a**4*b*x + 200*a**3*b**2*x**2 + 300*a**2*b**3*x
**3 + 300*a*b**4*x**4)/(60*x**5)
```

Giac [A] time = 1.20627, size = 77, normalized size = 1.26

$$b^5\log\left(|x|\right)-\frac{300\,ab^4x^4+300\,a^2b^3x^3+200\,a^3b^2x^2+75\,a^4bx+12\,a^5}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="giac")
```

```
[Out] b^5*log(abs(x)) - 1/60*(300*a*b^4*x^4 + 300*a^2*b^3*x^3 + 200*a^3*b^2*x^2 +
 75*a^4*b*x + 12*a^5)/x^5
```

3.90 $\int \frac{(a+bx)^5}{x^7} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^6}{6ax^6}$$

```
[Out] -(a + b*x)^6/(6*a*x^6)
```

Rubi [A] time = 0.0016275, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^6}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^7,x]
```

```
[Out] -(a + b*x)^6/(6*a*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^5}{x^7} dx = -\frac{(a+bx)^6}{6ax^6}$$

Mathematica [B] time = 0.0087569, size = 65, normalized size = 3.82

$$-\frac{5a^3b^2}{2x^4} - \frac{10a^2b^3}{3x^3} - \frac{a^4b}{x^5} - \frac{a^5}{6x^6} - \frac{5ab^4}{2x^2} - \frac{b^5}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^7,x]
```

```
[Out] -a^5/(6*x^6) - (a^4*b)/x^5 - (5*a^3*b^2)/(2*x^4) - (10*a^2*b^3)/(3*x^3) - (
5*a*b^4)/(2*x^2) - b^5/x
```

Maple [B] time = 0.006, size = 58, normalized size = 3.4

$$-\frac{10\,a^2b^3}{3\,x^3} - \frac{a^4b}{x^5} - \frac{5\,a^3b^2}{2\,x^4} - \frac{a^5}{6\,x^6} - \frac{5\,ab^4}{2\,x^2} - \frac{b^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^7,x)
```

```
[Out] -10/3*a^2*b^3/x^3-a^4*b/x^5-5/2*a^3*b^2/x^4-1/6*a^5/x^6-5/2*a*b^4/x^2-b^5/x
```

Maxima [B] time = 1.05717, size = 74, normalized size = 4.35

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

Fricas [B] time = 1.51836, size = 120, normalized size = 7.06

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

Sympy [B] time = 0.613611, size = 60, normalized size = 3.53

$$-\frac{a^5+6a^4bx+15a^3b^2x^2+20a^2b^3x^3+15ab^4x^4+6b^5x^5}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**7,x)
```

```
[Out] -(a**5 + 6*a**4*b*x + 15*a**3*b**2*x**2 + 20*a**2*b**3*x**3 + 15*a*b**4*x**
4 + 6*b**5*x**5)/(6*x**6)
```

Giac [B] time = 1.20617, size = 74, normalized size = 4.35

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

3.91 $\int \frac{(a+bx)^5}{x^8} dx$

Optimal. Leaf size=36

$$\frac{b(a+bx)^6}{42a^2x^6} - \frac{(a+bx)^6}{7ax^7}$$

```
[Out] -(a + b*x)^6/(7*a*x^7) + (b*(a + b*x)^6)/(42*a^2*x^6)
```

Rubi [A] time = 0.0045536, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b(a+bx)^6}{42a^2x^6} - \frac{(a+bx)^6}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^8,x]
```

```
[Out] -(a + b*x)^6/(7*a*x^7) + (b*(a + b*x)^6)/(42*a^2*x^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^8} dx &= -\frac{(a+bx)^6}{7ax^7} - \frac{b\int \frac{(a+bx)^5}{x^7} dx}{7a}\\ &= -\frac{(a+bx)^6}{7ax^7} + \frac{b(a+bx)^6}{42a^2x^6}\end{aligned}$$

Mathematica [A] time = 0.0085953, size = 67, normalized size = 1.86

$$-\frac{2a^3b^2}{x^5} - \frac{5a^2b^3}{2x^4} - \frac{5a^4b}{6x^6} - \frac{a^5}{7x^7} - \frac{5ab^4}{3x^3} - \frac{b^5}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^8,x]
```

```
[Out] -a^5/(7*x^7) - (5*a^4*b)/(6*x^6) - (2*a^3*b^2)/x^5 - (5*a^2*b^3)/(2*x^4) -
(5*a*b^4)/(3*x^3) - b^5/(2*x^2)
```

Maple [A] time = 0.006, size = 58, normalized size = 1.6

$$-\frac{5\,ab^4}{3\,x^3}-2\,\frac{a^3b^2}{x^5}-\frac{5\,a^2b^3}{2\,x^4}-\frac{5\,a^4b}{6\,x^6}-\frac{b^5}{2\,x^2}-\frac{a^5}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^8,x)
```

```
[Out] -5/3*a*b^4/x^3-2*a^3*b^2/x^5-5/2*a^2*b^3/x^4-5/6*a^4*b/x^6-1/2*b^5/x^2-1/7*
a^5/x^7
```

Maxima [A] time = 1.03991, size = 77, normalized size = 2.14

$$-\frac{21\,b^5x^5+70\,ab^4x^4+105\,a^2b^3x^3+84\,a^3b^2x^2+35\,a^4bx+6\,a^5}{42\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="maxima")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

Fricas [A] time = 1.39562, size = 128, normalized size = 3.56

$$-\frac{21\,b^5x^5+70\,ab^4x^4+105\,a^2b^3x^3+84\,a^3b^2x^2+35\,a^4bx+6\,a^5}{42\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="fricas")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

Sympy [B] time = 0.742721, size = 61, normalized size = 1.69

$$-\frac{6a^5+35a^4bx+84a^3b^2x^2+105a^2b^3x^3+70ab^4x^4+21b^5x^5}{42x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**8,x)
```

```
[Out] -(6*a**5 + 35*a**4*b*x + 84*a**3*b**2*x**2 + 105*a**2*b**3*x**3 + 70*a*b**4
*x**4 + 21*b**5*x**5)/(42*x**7)
```

Giac [A] time = 1.19444, size = 77, normalized size = 2.14

$$-\frac{21\,b^5x^5 + 70\,ab^4x^4 + 105\,a^2b^3x^3 + 84\,a^3b^2x^2 + 35\,a^4bx + 6\,a^5}{42\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="giac")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

3.92 $\int \frac{(a+bx)^5}{x^9} dx$

Optimal. Leaf size=56

$$-\frac{b^2(a+bx)^6}{168a^3x^6}+\frac{b(a+bx)^6}{28a^2x^7}-\frac{(a+bx)^6}{8ax^8}$$

```
[Out] -(a + b*x)^6/(8*a*x^8) + (b*(a + b*x)^6)/(28*a^2*x^7) - (b^2*(a + b*x)^6)/(
168*a^3*x^6)
```

Rubi [A] time = 0.0096565, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^2(a+bx)^6}{168a^3x^6}+\frac{b(a+bx)^6}{28a^2x^7}-\frac{(a+bx)^6}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^9,x]
```

```
[Out] -(a + b*x)^6/(8*a*x^8) + (b*(a + b*x)^6)/(28*a^2*x^7) - (b^2*(a + b*x)^6)/(
168*a^3*x^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^5}{x^9} dx &= -\frac{(a+bx)^6}{8ax^8}-\frac{b\int \frac{(a+bx)^5}{x^8} dx}{4a}\\
&= -\frac{(a+bx)^6}{8ax^8}+\frac{b(a+bx)^6}{28a^2x^7}+\frac{b^2\int \frac{(a+bx)^5}{x^7} dx}{28a^2}\\
&= -\frac{(a+bx)^6}{8ax^8}+\frac{b(a+bx)^6}{28a^2x^7}-\frac{b^2(a+bx)^6}{168a^3x^6}
\end{aligned}$$

Mathematica [A] time = 0.006202, size = 67, normalized size = 1.2

$$-\frac{5a^3b^2}{3x^6}-\frac{2a^2b^3}{x^5}-\frac{5a^4b}{7x^7}-\frac{a^5}{8x^8}-\frac{5ab^4}{4x^4}-\frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^9,x]
```

```
[Out] -a^5/(8*x^8) - (5*a^4*b)/(7*x^7) - (5*a^3*b^2)/(3*x^6) - (2*a^2*b^3)/x^5 -
(5*a*b^4)/(4*x^4) - b^5/(3*x^3)
```

Maple [A] time = 0.007, size = 58, normalized size = 1.

$$-\frac{b^5}{3\,x^3}-2\,\frac{a^2b^3}{x^5}-\frac{5\,ab^4}{4\,x^4}-\frac{5\,a^3b^2}{3\,x^6}-\frac{a^5}{8\,x^8}-\frac{5\,a^4b}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^9,x)
```

```
[Out] -1/3*b^5/x^3-2*a^2*b^3/x^5-5/4*a*b^4/x^4-5/3*a^3*b^2/x^6-1/8*a^5/x^8-5/7*a^
4*b/x^7
```

Maxima [A] time = 1.09121, size = 77, normalized size = 1.38

$$-\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="maxima")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

Fricas [A] time = 1.46598, size = 135, normalized size = 2.41

$$-\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="fricas")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

Sympy [A] time = 0.727871, size = 61, normalized size = 1.09

$$-\frac{21a^5+120a^4bx+280a^3b^2x^2+336a^2b^3x^3+210ab^4x^4+56b^5x^5}{168x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**9,x)
```

```
[Out] -(21*a**5 + 120*a**4*b*x + 280*a**3*b**2*x**2 + 336*a**2*b**3*x**3 + 210*a*
b**4*x**4 + 56*b**5*x**5)/(168*x**8)
```

Giac [A] time = 1.19054, size = 77, normalized size = 1.38

$$-\frac{56\,b^{5}x^{5}+210\,ab^{4}x^{4}+336\,a^{2}b^{3}x^{3}+280\,a^{3}b^{2}x^{2}+120\,a^{4}bx+21\,a^{5}}{168\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="giac")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

3.93 $\int \frac{(a+bx)^5}{x^{10}} dx$

Optimal. Leaf size=67

$$-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rubi [A] time = 0.022074, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^10,x]
```

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{10}}\,dx &= \int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx\\ &= -\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.0116235, size = 67, normalized size = 1.

$$-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^10,x]
```

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Maple [A] time = 0.006, size = 58, normalized size = 0.9

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^10,x)
```

```
[Out] -1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*b
^5/x^4
```

Maxima [A] time = 1.05083, size = 77, normalized size = 1.15

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Fricas [A] time = 1.57443, size = 136, normalized size = 2.03

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Sympy [A] time = 0.757735, size = 61, normalized size = 0.91

$$-\frac{56a^5+315a^4bx+720a^3b^2x^2+840a^2b^3x^3+504ab^4x^4+126b^5x^5}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**10,x)
```

```
[Out] -(56*a**5 + 315*a**4*b*x + 720*a**3*b**2*x**2 + 840*a**2*b**3*x**3 + 504*a*
b**4*x**4 + 126*b**5*x**5)/(504*x**9)
```

Giac [A] time = 1.18753, size = 77, normalized size = 1.15

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

3.94 $\int \frac{(a+bx)^5}{x^{11}} \, dx$

Optimal. Leaf size=69

$$-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5a^4b}{9x^9}-\frac{a^5}{10x^{10}}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

```
[Out] -a^5/(10*x^10) - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/(7*
x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5)
```

Rubi [A] time = 0.0205153, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5a^4b}{9x^9}-\frac{a^5}{10x^{10}}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^11,x]
```

```
[Out] -a^5/(10*x^10) - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/(7*
x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{11}} \, dx &= \int \left(\frac{a^5}{x^{11}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^9}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^7}+\frac{b^5}{x^6}\right) dx\\ &= -\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.0061435, size = 69, normalized size = 1.

$$-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5a^4b}{9x^9}-\frac{a^5}{10x^{10}}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^11,x]
```

```
[Out] -a^5/(10*x^10) - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/(7*
x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5)
```

Maple [A] time = 0.006, size = 58, normalized size = 0.8

$$-\frac{a^{5}}{10\,x^{10}}-\frac{5\,a^{4}b}{9\,x^{9}}-\frac{5\,a^{3}b^{2}}{4\,x^{8}}-\frac{10\,a^{2}b^{3}}{7\,x^{7}}-\frac{5\,ab^{4}}{6\,x^{6}}-\frac{b^{5}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^11,x)
```

```
[Out] -1/10*a^5/x^10-5/9*a^4*b/x^9-5/4*a^3*b^2/x^8-10/7*a^2*b^3/x^7-5/6*a*b^4/x^6
-1/5*b^5/x^5
```

Maxima [A] time = 1.05087, size = 77, normalized size = 1.12

$$-\frac{252\,b^{5}x^{5}+1050\,ab^{4}x^{4}+1800\,a^{2}b^{3}x^{3}+1575\,a^{3}b^{2}x^{2}+700\,a^{4}bx+126\,a^{5}}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="maxima")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

Fricas [A] time = 1.31939, size = 144, normalized size = 2.09

$$-\frac{252\,b^{5}x^{5}+1050\,ab^{4}x^{4}+1800\,a^{2}b^{3}x^{3}+1575\,a^{3}b^{2}x^{2}+700\,a^{4}bx+126\,a^{5}}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="fricas")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

Sympy [A] time = 0.738283, size = 61, normalized size = 0.88

$$-\frac{126a^{5}+700a^{4}bx+1575a^{3}b^{2}x^{2}+1800a^{2}b^{3}x^{3}+1050ab^{4}x^{4}+252b^{5}x^{5}}{1260x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**11,x)
```

```
[Out] -(126*a**5 + 700*a**4*b*x + 1575*a**3*b**2*x**2 + 1800*a**2*b**3*x**3 + 105
0*a*b**4*x**4 + 252*b**5*x**5)/(1260*x**10)
```

Giac [A] time = 1.17538, size = 77, normalized size = 1.12

$$-\frac{252\,b^{5}x^{5}+1050\,ab^{4}x^{4}+1800\,a^{2}b^{3}x^{3}+1575\,a^{3}b^{2}x^{2}+700\,a^{4}bx+126\,a^{5}}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="giac")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

3.95 $\int \frac{(a+bx)^5}{x^{12}} dx$

Optimal. Leaf size=69

$$-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{a^4b}{2x^{10}}-\frac{a^5}{11x^{11}}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

```
[Out] -a^5/(11*x^11) - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(4*x
^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6)
```

Rubi [A] time = 0.020542, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{a^4b}{2x^{10}}-\frac{a^5}{11x^{11}}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^12,x]
```

```
[Out] -a^5/(11*x^11) - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(4*x
^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{12}}\,dx &= \int \left(\frac{a^5}{x^{12}}+\frac{5a^4b}{x^{11}}+\frac{10a^3b^2}{x^{10}}+\frac{10a^2b^3}{x^9}+\frac{5ab^4}{x^8}+\frac{b^5}{x^7}\right)dx\\ &= -\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}\end{aligned}$$

Mathematica [A] time = 0.0041838, size = 69, normalized size = 1.

$$-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{a^4b}{2x^{10}}-\frac{a^5}{11x^{11}}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^12,x]
```

```
[Out] -a^5/(11*x^11) - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(4*x
^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6)
```

Maple [A] time = 0.006, size = 58, normalized size = 0.8

$$-\frac{a^5}{11\,x^{11}}-\frac{a^4b}{2\,x^{10}}-\frac{10\,a^3b^2}{9\,x^9}-\frac{5\,a^2b^3}{4\,x^8}-\frac{5\,ab^4}{7\,x^7}-\frac{b^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^12,x)
```

```
[Out] -1/11*a^5/x^11-1/2*a^4*b/x^10-10/9*a^3*b^2/x^9-5/4*a^2*b^3/x^8-5/7*a*b^4/x^
7-1/6*b^5/x^6
```

Maxima [A] time = 1.07538, size = 77, normalized size = 1.12

$$-\frac{462\,b^5x^5+1980\,ab^4x^4+3465\,a^2b^3x^3+3080\,a^3b^2x^2+1386\,a^4bx+252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="maxima")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

Fricas [A] time = 1.56253, size = 146, normalized size = 2.12

$$-\frac{462\,b^5x^5+1980\,ab^4x^4+3465\,a^2b^3x^3+3080\,a^3b^2x^2+1386\,a^4bx+252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="fricas")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

Sympy [A] time = 0.826751, size = 61, normalized size = 0.88

$$-\frac{252a^5+1386a^4bx+3080a^3b^2x^2+3465a^2b^3x^3+1980ab^4x^4+462b^5x^5}{2772x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**12,x)
```

```
[Out] -(252*a**5 + 1386*a**4*b*x + 3080*a**3*b**2*x**2 + 3465*a**2*b**3*x**3 + 19
80*a*b**4*x**4 + 462*b**5*x**5)/(2772*x**11)
```

Giac [A] time = 1.17135, size = 77, normalized size = 1.12

$$-\frac{462\,b^5x^5+1980\,ab^4x^4+3465\,a^2b^3x^3+3080\,a^3b^2x^2+1386\,a^4bx+252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="giac")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

3.96 $\int \frac{(a+bx)^5}{x^{13}}\,dx$

Optimal. Leaf size=67

$$-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5a^4b}{11x^{11}}-\frac{a^5}{12x^{12}}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

```
[Out] -a^5/(12*x^12) - (5*a^4*b)/(11*x^11) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(9*x^9
) - (5*a*b^4)/(8*x^8) - b^5/(7*x^7)
```

Rubi [A] time = 0.0207173, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5a^4b}{11x^{11}}-\frac{a^5}{12x^{12}}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^13,x]
```

```
[Out] -a^5/(12*x^12) - (5*a^4*b)/(11*x^11) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(9*x^9
) - (5*a*b^4)/(8*x^8) - b^5/(7*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{13}}\,dx &= \int \left(\frac{a^5}{x^{13}}+\frac{5a^4b}{x^{12}}+\frac{10a^3b^2}{x^{11}}+\frac{10a^2b^3}{x^{10}}+\frac{5ab^4}{x^9}+\frac{b^5}{x^8}\right)dx\\ &= -\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}\end{aligned}$$

Mathematica [A] time = 0.0073379, size = 67, normalized size = 1.

$$-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5a^4b}{11x^{11}}-\frac{a^5}{12x^{12}}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^13,x]
```

```
[Out] -a^5/(12*x^12) - (5*a^4*b)/(11*x^11) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(9*x^9
) - (5*a*b^4)/(8*x^8) - b^5/(7*x^7)
```

Maple [A] time = 0.006, size = 58, normalized size = 0.9

$$-\frac{a^5}{12\,x^{12}}-\frac{5\,a^4b}{11\,x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10\,a^2b^3}{9\,x^9}-\frac{5\,ab^4}{8\,x^8}-\frac{b^5}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^13,x)
```

```
[Out] -1/12*a^5/x^12-5/11*a^4*b/x^11-a^3*b^2/x^10-10/9*a^2*b^3/x^9-5/8*a*b^4/x^8-
1/7*b^5/x^7
```

Maxima [A] time = 0.999046, size = 77, normalized size = 1.15

$$-\frac{792\,b^5x^5+3465\,ab^4x^4+6160\,a^2b^3x^3+5544\,a^3b^2x^2+2520\,a^4bx+462\,a^5}{5544\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="maxima")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

Fricas [A] time = 1.36745, size = 146, normalized size = 2.18

$$-\frac{792\,b^5x^5+3465\,ab^4x^4+6160\,a^2b^3x^3+5544\,a^3b^2x^2+2520\,a^4bx+462\,a^5}{5544\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="fricas")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

Sympy [A] time = 1.08598, size = 61, normalized size = 0.91

$$-\frac{462a^5+2520a^4bx+5544a^3b^2x^2+6160a^2b^3x^3+3465ab^4x^4+792b^5x^5}{5544x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**13,x)
```

```
[Out] -(462*a**5 + 2520*a**4*b*x + 5544*a**3*b**2*x**2 + 6160*a**2*b**3*x**3 + 34
65*a*b**4*x**4 + 792*b**5*x**5)/(5544*x**12)
```

Giac [A] time = 1.19159, size = 77, normalized size = 1.15

$$-\frac{792\,b^5x^5+3465\,ab^4x^4+6160\,a^2b^3x^3+5544\,a^3b^2x^2+2520\,a^4bx+462\,a^5}{5544\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="giac")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

3.97 $\int \frac{(a+bx)^5}{x^{14}} dx$

Optimal. Leaf size=67

$$-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5a^4b}{12x^{12}}-\frac{a^5}{13x^{13}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

```
[Out] -a^5/(13*x^13) - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(11*x^11) - (a^2*b^3)/x
^10 - (5*a*b^4)/(9*x^9) - b^5/(8*x^8)
```

Rubi [A] time = 0.0215932, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5a^4b}{12x^{12}}-\frac{a^5}{13x^{13}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^14,x]
```

```
[Out] -a^5/(13*x^13) - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(11*x^11) - (a^2*b^3)/x
^10 - (5*a*b^4)/(9*x^9) - b^5/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{14}}\,dx &= \int \left(\frac{a^5}{x^{14}}+\frac{5a^4b}{x^{13}}+\frac{10a^3b^2}{x^{12}}+\frac{10a^2b^3}{x^{11}}+\frac{5ab^4}{x^{10}}+\frac{b^5}{x^9}\right)dx\\ &= -\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}\end{aligned}$$

Mathematica [A] time = 0.0044323, size = 67, normalized size = 1.

$$-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5a^4b}{12x^{12}}-\frac{a^5}{13x^{13}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^14,x]
```

```
[Out] -a^5/(13*x^13) - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(11*x^11) - (a^2*b^3)/x
^10 - (5*a*b^4)/(9*x^9) - b^5/(8*x^8)
```

Maple [A] time = 0.006, size = 58, normalized size = 0.9

$$-\frac{a^5}{13\,x^{13}}-\frac{5\,a^4b}{12\,x^{12}}-\frac{10\,a^3b^2}{11\,x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5\,ab^4}{9\,x^9}-\frac{b^5}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^14,x)
```

```
[Out] -1/13*a^5/x^13-5/12*a^4*b/x^12-10/11*a^3*b^2/x^11-a^2*b^3/x^10-5/9*a*b^4/x^
9-1/8*b^5/x^8
```

Maxima [A] time = 1.06149, size = 77, normalized size = 1.15

$$-\frac{1287\,b^5x^5+5720\,ab^4x^4+10296\,a^2b^3x^3+9360\,a^3b^2x^2+4290\,a^4bx+792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

Fricas [A] time = 1.42976, size = 150, normalized size = 2.24

$$-\frac{1287\,b^5x^5+5720\,ab^4x^4+10296\,a^2b^3x^3+9360\,a^3b^2x^2+4290\,a^4bx+792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

Sympy [A] time = 0.930171, size = 61, normalized size = 0.91

$$-\frac{792a^5+4290a^4bx+9360a^3b^2x^2+10296a^2b^3x^3+5720ab^4x^4+1287b^5x^5}{10296x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**14,x)
```

```
[Out] -(792*a**5 + 4290*a**4*b*x + 9360*a**3*b**2*x**2 + 10296*a**2*b**3*x**3 + 5
720*a*b**4*x**4 + 1287*b**5*x**5)/(10296*x**13)
```

Giac [A] time = 1.20667, size = 77, normalized size = 1.15

$$-\frac{1287\,b^5x^5+5720\,ab^4x^4+10296\,a^2b^3x^3+9360\,a^3b^2x^2+4290\,a^4bx+792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="giac")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

3.98 $\int x^8(a+bx)^7\,dx$

Optimal. Leaf size=95

$$\frac{3}{2}a^2b^5x^{14}+\frac{35}{13}a^3b^4x^{13}+\frac{35}{12}a^4b^3x^{12}+\frac{21}{11}a^5b^2x^{11}+\frac{7}{10}a^6bx^{10}+\frac{a^7x^9}{9}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

```
[Out] (a^7*x^9)/9 + (7*a^6*b*x^10)/10 + (21*a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/
12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*x^14)/2 + (7*a*b^6*x^15)/15 + (b^7*x
^16)/16
```

Rubi [A] time = 0.0480825, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{2}a^2b^5x^{14}+\frac{35}{13}a^3b^4x^{13}+\frac{35}{12}a^4b^3x^{12}+\frac{21}{11}a^5b^2x^{11}+\frac{7}{10}a^6bx^{10}+\frac{a^7x^9}{9}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x)^7,x]
```

```
[Out] (a^7*x^9)/9 + (7*a^6*b*x^10)/10 + (21*a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/
12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*x^14)/2 + (7*a*b^6*x^15)/15 + (b^7*x
^16)/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^8(a+bx)^7\,dx&=\int\left(a^7x^8+7a^6bx^9+21a^5b^2x^{10}+35a^4b^3x^{11}+35a^3b^4x^{12}+21a^2b^5x^{13}+7ab^6x^{14}+b^7x^{15}\right)dx\\&=\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}\end{aligned}$$

Mathematica [A] time = 0.0031677, size = 95, normalized size = 1.

$$\frac{3}{2}a^2b^5x^{14}+\frac{35}{13}a^3b^4x^{13}+\frac{35}{12}a^4b^3x^{12}+\frac{21}{11}a^5b^2x^{11}+\frac{7}{10}a^6bx^{10}+\frac{a^7x^9}{9}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x)^7,x]
```

```
[Out] (a^7*x^9)/9 + (7*a^6*b*x^10)/10 + (21*a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/
12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*x^14)/2 + (7*a*b^6*x^15)/15 + (b^7*x
^16)/16
```

Maple [A] time = 0.001, size = 80, normalized size = 0.8

$$\frac{a^7x^9}{9}+\frac{7\,a^6bx^{10}}{10}+\frac{21\,a^5b^2x^{11}}{11}+\frac{35\,a^4b^3x^{12}}{12}+\frac{35\,a^3b^4x^{13}}{13}+\frac{3\,a^2b^5x^{14}}{2}+\frac{7\,ab^6x^{15}}{15}+\frac{b^7x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^7,x)
```

```
[Out] 1/9*a^7*x^9+7/10*a^6*b*x^10+21/11*a^5*b^2*x^11+35/12*a^4*b^3*x^12+35/13*a^3
*b^4*x^13+3/2*a^2*b^5*x^14+7/15*a*b^6*x^15+1/16*b^7*x^16
```

Maxima [A] time = 1.01829, size = 107, normalized size = 1.13

$$\frac{1}{16}\,b^7x^{16}+\frac{7}{15}\,ab^6x^{15}+\frac{3}{2}\,a^2b^5x^{14}+\frac{35}{13}\,a^3b^4x^{13}+\frac{35}{12}\,a^4b^3x^{12}+\frac{21}{11}\,a^5b^2x^{11}+\frac{7}{10}\,a^6bx^{10}+\frac{1}{9}\,a^7x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 + 3
5/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

Fricas [A] time = 1.38447, size = 198, normalized size = 2.08

$$\frac{1}{16}x^{16}b^7+\frac{7}{15}x^{15}b^6a+\frac{3}{2}x^{14}b^5a^2+\frac{35}{13}x^{13}b^4a^3+\frac{35}{12}x^{12}b^3a^4+\frac{21}{11}x^{11}b^2a^5+\frac{7}{10}x^{10}ba^6+\frac{1}{9}x^9a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^7 + 7/15*x^15*b^6*a + 3/2*x^14*b^5*a^2 + 35/13*x^13*b^4*a^3 + 3
5/12*x^12*b^3*a^4 + 21/11*x^11*b^2*a^5 + 7/10*x^10*b*a^6 + 1/9*x^9*a^7
```

Sympy [A] time = 0.09891, size = 94, normalized size = 0.99

$$\frac{a^7x^9}{9}+\frac{7a^6bx^{10}}{10}+\frac{21a^5b^2x^{11}}{11}+\frac{35a^4b^3x^{12}}{12}+\frac{35a^3b^4x^{13}}{13}+\frac{3a^2b^5x^{14}}{2}+\frac{7ab^6x^{15}}{15}+\frac{b^7x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**7,x)
```

```
[Out] a**7*x**9/9 + 7*a**6*b*x**10/10 + 21*a**5*b**2*x**11/11 + 35*a**4*b**3*x**1
2/12 + 35*a**3*b**4*x**13/13 + 3*a**2*b**5*x**14/2 + 7*a*b**6*x**15/15 + b*
*7*x**16/16
```

Giac [A] time = 1.18201, size = 107, normalized size = 1.13

$$\frac{1}{16}\,b^{7}x^{16}+\frac{7}{15}\,ab^{6}x^{15}+\frac{3}{2}\,a^{2}b^{5}x^{14}+\frac{35}{13}\,a^{3}b^{4}x^{13}+\frac{35}{12}\,a^{4}b^{3}x^{12}+\frac{21}{11}\,a^{5}b^{2}x^{11}+\frac{7}{10}\,a^{6}bx^{10}+\frac{1}{9}\,a^{7}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 + 3
5/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

3.99 $\int x^7(a+bx)^7\,dx$

Optimal. Leaf size=95

$$\frac{21}{13}a^2b^5x^{13}+\frac{35}{12}a^3b^4x^{12}+\frac{35}{11}a^4b^3x^{11}+\frac{21}{10}a^5b^2x^{10}+\frac{7}{9}a^6bx^9+\frac{a^7x^8}{8}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

```
[Out] (a^7*x^8)/8 + (7*a^6*b*x^9)/9 + (21*a^5*b^2*x^10)/10 + (35*a^4*b^3*x^11)/11
 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13)/13 + (a*b^6*x^14)/2 + (b^7*x^15
)/15
```

Rubi [A] time = 0.0396834, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21}{13}a^2b^5x^{13}+\frac{35}{12}a^3b^4x^{12}+\frac{35}{11}a^4b^3x^{11}+\frac{21}{10}a^5b^2x^{10}+\frac{7}{9}a^6bx^9+\frac{a^7x^8}{8}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^7,x]
```

```
[Out] (a^7*x^8)/8 + (7*a^6*b*x^9)/9 + (21*a^5*b^2*x^10)/10 + (35*a^4*b^3*x^11)/11
 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13)/13 + (a*b^6*x^14)/2 + (b^7*x^15
)/15
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7(a+bx)^7\,dx &= \int \left(a^7x^7+7a^6bx^8+21a^5b^2x^9+35a^4b^3x^{10}+35a^3b^4x^{11}+21a^2b^5x^{12}+7ab^6x^{13}+b^7x^{14}\right)dx\\ &= \frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}\end{aligned}$$

Mathematica [A] time = 0.0042165, size = 95, normalized size = 1.

$$\frac{21}{13}a^2b^5x^{13}+\frac{35}{12}a^3b^4x^{12}+\frac{35}{11}a^4b^3x^{11}+\frac{21}{10}a^5b^2x^{10}+\frac{7}{9}a^6bx^9+\frac{a^7x^8}{8}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^7,x]
```

```
[Out] (a^7*x^8)/8 + (7*a^6*b*x^9)/9 + (21*a^5*b^2*x^10)/10 + (35*a^4*b^3*x^11)/11
 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13)/13 + (a*b^6*x^14)/2 + (b^7*x^15
)/15
```

Maple [A] time = 0.002, size = 80, normalized size = 0.8

$$\frac{a^7x^8}{8}+\frac{7\,a^6bx^9}{9}+\frac{21\,a^5b^2x^{10}}{10}+\frac{35\,a^4b^3x^{11}}{11}+\frac{35\,a^3b^4x^{12}}{12}+\frac{21\,a^2b^5x^{13}}{13}+\frac{ab^6x^{14}}{2}+\frac{b^7x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^7,x)
```

```
[Out] 1/8*a^7*x^8+7/9*a^6*b*x^9+21/10*a^5*b^2*x^10+35/11*a^4*b^3*x^11+35/12*a^3*b
^4*x^12+21/13*a^2*b^5*x^13+1/2*a*b^6*x^14+1/15*b^7*x^15
```

Maxima [A] time = 1.03137, size = 107, normalized size = 1.13

$$\frac{1}{15}\,b^7x^{15}+\frac{1}{2}\,ab^6x^{14}+\frac{21}{13}\,a^2b^5x^{13}+\frac{35}{12}\,a^3b^4x^{12}+\frac{35}{11}\,a^4b^3x^{11}+\frac{21}{10}\,a^5b^2x^{10}+\frac{7}{9}\,a^6bx^9+\frac{1}{8}\,a^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

Fricas [A] time = 1.29183, size = 197, normalized size = 2.07

$$\frac{1}{15}x^{15}b^7+\frac{1}{2}x^{14}b^6a+\frac{21}{13}x^{13}b^5a^2+\frac{35}{12}x^{12}b^4a^3+\frac{35}{11}x^{11}b^3a^4+\frac{21}{10}x^{10}b^2a^5+\frac{7}{9}x^9ba^6+\frac{1}{8}x^8a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^7 + 1/2*x^14*b^6*a + 21/13*x^13*b^5*a^2 + 35/12*x^12*b^4*a^3 +
35/11*x^11*b^3*a^4 + 21/10*x^10*b^2*a^5 + 7/9*x^9*b*a^6 + 1/8*x^8*a^7
```

Sympy [A] time = 0.087307, size = 92, normalized size = 0.97

$$\frac{a^7x^8}{8}+\frac{7a^6bx^9}{9}+\frac{21a^5b^2x^{10}}{10}+\frac{35a^4b^3x^{11}}{11}+\frac{35a^3b^4x^{12}}{12}+\frac{21a^2b^5x^{13}}{13}+\frac{ab^6x^{14}}{2}+\frac{b^7x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**7,x)
```

```
[Out] a**7*x**8/8 + 7*a**6*b*x**9/9 + 21*a**5*b**2*x**10/10 + 35*a**4*b**3*x**11/
11 + 35*a**3*b**4*x**12/12 + 21*a**2*b**5*x**13/13 + a*b**6*x**14/2 + b**7*
x**15/15
```

Giac [A] time = 1.18826, size = 107, normalized size = 1.13

$$\frac{1}{15}b^7x^{15} + \frac{1}{2}ab^6x^{14} + \frac{21}{13}a^2b^5x^{13} + \frac{35}{12}a^3b^4x^{12} + \frac{35}{11}a^4b^3x^{11} + \frac{21}{10}a^5b^2x^{10} + \frac{7}{9}a^6bx^9 + \frac{1}{8}a^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

3.100 $\int x^6(a+bx)^7\,dx$

Optimal. Leaf size=95

$$\frac{7}{4}a^2b^5x^{12}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{2}a^4b^3x^{10}+\frac{7}{3}a^5b^2x^9+\frac{7}{8}a^6bx^8+\frac{a^7x^7}{7}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

```
[Out] (a^7*x^7)/7 + (7*a^6*b*x^8)/8 + (7*a^5*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (3
5*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4 + (7*a*b^6*x^13)/13 + (b^7*x^14)/14
```

Rubi [A] time = 0.0385288, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{7}{4}a^2b^5x^{12}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{2}a^4b^3x^{10}+\frac{7}{3}a^5b^2x^9+\frac{7}{8}a^6bx^8+\frac{a^7x^7}{7}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^7,x]
```

```
[Out] (a^7*x^7)/7 + (7*a^6*b*x^8)/8 + (7*a^5*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (3
5*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4 + (7*a*b^6*x^13)/13 + (b^7*x^14)/14
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^7\,dx&=\int\left(a^7x^6+7a^6bx^7+21a^5b^2x^8+35a^4b^3x^9+35a^3b^4x^{10}+21a^2b^5x^{11}+7ab^6x^{12}+b^7x^{13}\right)dx\\&=\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}\end{aligned}$$

Mathematica [A] time = 0.0035383, size = 95, normalized size = 1.

$$\frac{7}{4}a^2b^5x^{12}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{2}a^4b^3x^{10}+\frac{7}{3}a^5b^2x^9+\frac{7}{8}a^6bx^8+\frac{a^7x^7}{7}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^7,x]
```

```
[Out] (a^7*x^7)/7 + (7*a^6*b*x^8)/8 + (7*a^5*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (3
5*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4 + (7*a*b^6*x^13)/13 + (b^7*x^14)/14
```

Maple [A] time = 0.002, size = 80, normalized size = 0.8

$$\frac{a^7x^7}{7}+\frac{7\,a^6bx^8}{8}+\frac{7\,a^5b^2x^9}{3}+\frac{7\,a^4b^3x^{10}}{2}+\frac{35\,a^3b^4x^{11}}{11}+\frac{7\,a^2b^5x^{12}}{4}+\frac{7\,ab^6x^{13}}{13}+\frac{b^7x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^7,x)
```

```
[Out] 1/7*a^7*x^7+7/8*a^6*b*x^8+7/3*a^5*b^2*x^9+7/2*a^4*b^3*x^10+35/11*a^3*b^4*x^
11+7/4*a^2*b^5*x^12+7/13*a*b^6*x^13+1/14*b^7*x^14
```

Maxima [A] time = 1.0641, size = 107, normalized size = 1.13

$$\frac{1}{14}\,b^7x^{14}+\frac{7}{13}\,ab^6x^{13}+\frac{7}{4}\,a^2b^5x^{12}+\frac{35}{11}\,a^3b^4x^{11}+\frac{7}{2}\,a^4b^3x^{10}+\frac{7}{3}\,a^5b^2x^9+\frac{7}{8}\,a^6bx^8+\frac{1}{7}\,a^7x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 + 7
/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

Fricas [A] time = 1.39136, size = 189, normalized size = 1.99

$$\frac{1}{14}x^{14}b^7+\frac{7}{13}x^{13}b^6a+\frac{7}{4}x^{12}b^5a^2+\frac{35}{11}x^{11}b^4a^3+\frac{7}{2}x^{10}b^3a^4+\frac{7}{3}x^9b^2a^5+\frac{7}{8}x^8ba^6+\frac{1}{7}x^7a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^7 + 7/13*x^13*b^6*a + 7/4*x^12*b^5*a^2 + 35/11*x^11*b^4*a^3 + 7
/2*x^10*b^3*a^4 + 7/3*x^9*b^2*a^5 + 7/8*x^8*b*a^6 + 1/7*x^7*a^7
```

Sympy [A] time = 0.09656, size = 94, normalized size = 0.99

$$\frac{a^7x^7}{7}+\frac{7a^6bx^8}{8}+\frac{7a^5b^2x^9}{3}+\frac{7a^4b^3x^{10}}{2}+\frac{35a^3b^4x^{11}}{11}+\frac{7a^2b^5x^{12}}{4}+\frac{7ab^6x^{13}}{13}+\frac{b^7x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**7,x)
```

```
[Out] a**7*x**7/7 + 7*a**6*b*x**8/8 + 7*a**5*b**2*x**9/3 + 7*a**4*b**3*x**10/2 +
35*a**3*b**4*x**11/11 + 7*a**2*b**5*x**12/4 + 7*a*b**6*x**13/13 + b**7*x**1
4/14
```

<u>Giac</u> [A] time = 1.16793, size = 107, normalized size = 1.13

$$\frac{1}{14}b^7x^{14} + \frac{7}{13}ab^6x^{13} + \frac{7}{4}a^2b^5x^{12} + \frac{35}{11}a^3b^4x^{11} + \frac{7}{2}a^4b^3x^{10} + \frac{7}{3}a^5b^2x^9 + \frac{7}{8}a^6bx^8 + \frac{1}{7}a^7x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 + 7
/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

3.101 $\int x^5(a+bx)^7\,dx$

Optimal. Leaf size=96

$$\frac{10a^2(a+bx)^{11}}{11b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^5(a+bx)^8}{8b^6}+\frac{(a+bx)^{13}}{13b^6}-\frac{5a(a+bx)^{12}}{12b^6}$$

```
[Out] -(a^5*(a + b*x)^8)/(8*b^6) + (5*a^4*(a + b*x)^9)/(9*b^6) - (a^3*(a + b*x)^1
0)/b^6 + (10*a^2*(a + b*x)^11)/(11*b^6) - (5*a*(a + b*x)^12)/(12*b^6) + (a
+ b*x)^13/(13*b^6)
```

Rubi [A] time = 0.0402941, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{10a^2(a+bx)^{11}}{11b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^5(a+bx)^8}{8b^6}+\frac{(a+bx)^{13}}{13b^6}-\frac{5a(a+bx)^{12}}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^7,x]
```

```
[Out] -(a^5*(a + b*x)^8)/(8*b^6) + (5*a^4*(a + b*x)^9)/(9*b^6) - (a^3*(a + b*x)^1
0)/b^6 + (10*a^2*(a + b*x)^11)/(11*b^6) - (5*a*(a + b*x)^12)/(12*b^6) + (a
+ b*x)^13/(13*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^7\,dx&=\int\left(-\frac{a^5(a+bx)^7}{b^5}+\frac{5a^4(a+bx)^8}{b^5}-\frac{10a^3(a+bx)^9}{b^5}+\frac{10a^2(a+bx)^{10}}{b^5}-\frac{5a(a+bx)^{11}}{b^5}+\frac{(a+bx)^{12}}{b^5}\right)\\&=-\frac{a^5(a+bx)^8}{8b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{10a^2(a+bx)^{11}}{11b^6}-\frac{5a(a+bx)^{12}}{12b^6}+\frac{(a+bx)^{13}}{13b^6}\end{aligned}$$

Mathematica [A] time = 0.0024743, size = 92, normalized size = 0.96

$$\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6bx^7+\frac{a^7x^6}{6}+\frac{7}{12}ab^6x^{12}+\frac{b^7x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^7,x]
```

```
[Out] (a^7*x^6)/6 + a^6*b*x^7 + (21*a^5*b^2*x^8)/8 + (35*a^4*b^3*x^9)/9 + (7*a^3*
b^4*x^10)/2 + (21*a^2*b^5*x^11)/11 + (7*a*b^6*x^12)/12 + (b^7*x^13)/13
```

Maple [A] time = 0., size = 79, normalized size = 0.8

$$\frac{b^7x^{13}}{13}+\frac{7\,ab^6x^{12}}{12}+\frac{21\,a^2b^5x^{11}}{11}+\frac{7\,a^3b^4x^{10}}{2}+\frac{35\,a^4b^3x^9}{9}+\frac{21\,a^5b^2x^8}{8}+a^6bx^7+\frac{a^7x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^7,x)
```

```
[Out] 1/13*b^7*x^13+7/12*a*b^6*x^12+21/11*a^2*b^5*x^11+7/2*a^3*b^4*x^10+35/9*a^4*
b^3*x^9+21/8*a^5*b^2*x^8+a^6*b*x^7+1/6*a^7*x^6
```

Maxima [A] time = 1.06887, size = 105, normalized size = 1.09

$$\frac{1}{13}\,b^7x^{13}+\frac{7}{12}\,ab^6x^{12}+\frac{21}{11}\,a^2b^5x^{11}+\frac{7}{2}\,a^3b^4x^{10}+\frac{35}{9}\,a^4b^3x^9+\frac{21}{8}\,a^5b^2x^8+a^6bx^7+\frac{1}{6}\,a^7x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 + 3
5/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

Fricas [A] time = 1.35283, size = 185, normalized size = 1.93

$$\frac{1}{13}x^{13}b^7+\frac{7}{12}x^{12}b^6a+\frac{21}{11}x^{11}b^5a^2+\frac{7}{2}x^{10}b^4a^3+\frac{35}{9}x^9b^3a^4+\frac{21}{8}x^8b^2a^5+x^7ba^6+\frac{1}{6}x^6a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^7 + 7/12*x^12*b^6*a + 21/11*x^11*b^5*a^2 + 7/2*x^10*b^4*a^3 + 3
5/9*x^9*b^3*a^4 + 21/8*x^8*b^2*a^5 + x^7*b*a^6 + 1/6*x^6*a^7
```

Sympy [A] time = 0.08211, size = 90, normalized size = 0.94

$$\frac{a^7x^6}{6}+a^6bx^7+\frac{21a^5b^2x^8}{8}+\frac{35a^4b^3x^9}{9}+\frac{7a^3b^4x^{10}}{2}+\frac{21a^2b^5x^{11}}{11}+\frac{7ab^6x^{12}}{12}+\frac{b^7x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**7,x)
```

```
[Out] a**7*x**6/6 + a**6*b*x**7 + 21*a**5*b**2*x**8/8 + 35*a**4*b**3*x**9/9 + 7*a
**3*b**4*x**10/2 + 21*a**2*b**5*x**11/11 + 7*a*b**6*x**12/12 + b**7*x**13/1
3
```

Giac [A] time = 1.22312, size = 105, normalized size = 1.09

$$\frac{1}{13}b^7x^{13} + \frac{7}{12}ab^6x^{12} + \frac{21}{11}a^2b^5x^{11} + \frac{7}{2}a^3b^4x^{10} + \frac{35}{9}a^4b^3x^9 + \frac{21}{8}a^5b^2x^8 + a^6bx^7 + \frac{1}{6}a^7x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 + 3
5/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

3.102 $\int x^4(a+bx)^7\,dx$

Optimal. Leaf size=81

$$\frac{3a^2(a+bx)^{10}}{5b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{a^4(a+bx)^8}{8b^5}+\frac{(a+bx)^{12}}{12b^5}-\frac{4a(a+bx)^{11}}{11b^5}$$

```
[Out] (a^4*(a + b*x)^8)/(8*b^5) - (4*a^3*(a + b*x)^9)/(9*b^5) + (3*a^2*(a + b*x)^
10)/(5*b^5) - (4*a*(a + b*x)^11)/(11*b^5) + (a + b*x)^12/(12*b^5)
```

Rubi [A] time = 0.0364159, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2(a+bx)^{10}}{5b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{a^4(a+bx)^8}{8b^5}+\frac{(a+bx)^{12}}{12b^5}-\frac{4a(a+bx)^{11}}{11b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^7,x]
```

```
[Out] (a^4*(a + b*x)^8)/(8*b^5) - (4*a^3*(a + b*x)^9)/(9*b^5) + (3*a^2*(a + b*x)^
10)/(5*b^5) - (4*a*(a + b*x)^11)/(11*b^5) + (a + b*x)^12/(12*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^7\,dx&=\int\left(\frac{a^4(a+bx)^7}{b^4}-\frac{4a^3(a+bx)^8}{b^4}+\frac{6a^2(a+bx)^9}{b^4}-\frac{4a(a+bx)^{10}}{b^4}+\frac{(a+bx)^{11}}{b^4}\right)dx\\&=\frac{a^4(a+bx)^8}{8b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{3a^2(a+bx)^{10}}{5b^5}-\frac{4a(a+bx)^{11}}{11b^5}+\frac{(a+bx)^{12}}{12b^5}\end{aligned}$$

Mathematica [A] time = 0.0048541, size = 93, normalized size = 1.15

$$\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6bx^6+\frac{a^7x^5}{5}+\frac{7}{11}ab^6x^{11}+\frac{b^7x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^7,x]
```

```
[Out] (a^7*x^5)/5 + (7*a^6*b*x^6)/6 + 3*a^5*b^2*x^7 + (35*a^4*b^3*x^8)/8 + (35*a^
3*b^4*x^9)/9 + (21*a^2*b^5*x^10)/10 + (7*a*b^6*x^11)/11 + (b^7*x^12)/12
```

Maple [A] time = 0.001, size = 80, normalized size = 1.

$$\frac{b^7x^{12}}{12}+\frac{7\,ab^6x^{11}}{11}+\frac{21\,a^2b^5x^{10}}{10}+\frac{35\,a^3b^4x^9}{9}+\frac{35\,a^4b^3x^8}{8}+3\,a^5b^2x^7+\frac{7\,a^6bx^6}{6}+\frac{a^7x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^7,x)
```

```
[Out] 1/12*b^7*x^12+7/11*a*b^6*x^11+21/10*a^2*b^5*x^10+35/9*a^3*b^4*x^9+35/8*a^4*
b^3*x^8+3*a^5*b^2*x^7+7/6*a^6*b*x^6+1/5*a^7*x^5
```

Maxima [A] time = 1.04485, size = 107, normalized size = 1.32

$$\frac{1}{12}\,b^7x^{12}+\frac{7}{11}\,ab^6x^{11}+\frac{21}{10}\,a^2b^5x^{10}+\frac{35}{9}\,a^3b^4x^9+\frac{35}{8}\,a^4b^3x^8+3\,a^5b^2x^7+\frac{7}{6}\,a^6bx^6+\frac{1}{5}\,a^7x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 + 3
5/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

Fricas [A] time = 1.36228, size = 186, normalized size = 2.3

$$\frac{1}{12}x^{12}b^7+\frac{7}{11}x^{11}b^6a+\frac{21}{10}x^{10}b^5a^2+\frac{35}{9}x^9b^4a^3+\frac{35}{8}x^8b^3a^4+3x^7b^2a^5+\frac{7}{6}x^6ba^6+\frac{1}{5}x^5a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^7 + 7/11*x^11*b^6*a + 21/10*x^10*b^5*a^2 + 35/9*x^9*b^4*a^3 + 3
5/8*x^8*b^3*a^4 + 3*x^7*b^2*a^5 + 7/6*x^6*b*a^6 + 1/5*x^5*a^7
```

Sympy [A] time = 0.086854, size = 92, normalized size = 1.14

$$\frac{a^7x^5}{5}+\frac{7a^6bx^6}{6}+3a^5b^2x^7+\frac{35a^4b^3x^8}{8}+\frac{35a^3b^4x^9}{9}+\frac{21a^2b^5x^{10}}{10}+\frac{7ab^6x^{11}}{11}+\frac{b^7x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**7,x)
```

```
[Out] a**7*x**5/5 + 7*a**6*b*x**6/6 + 3*a**5*b**2*x**7 + 35*a**4*b**3*x**8/8 + 35
*a**3*b**4*x**9/9 + 21*a**2*b**5*x**10/10 + 7*a*b**6*x**11/11 + b**7*x**12/
12
```

Giac [A] time = 1.2272, size = 107, normalized size = 1.32

$$\frac{1}{12}b^7x^{12} + \frac{7}{11}ab^6x^{11} + \frac{21}{10}a^2b^5x^{10} + \frac{35}{9}a^3b^4x^9 + \frac{35}{8}a^4b^3x^8 + 3a^5b^2x^7 + \frac{7}{6}a^6bx^6 + \frac{1}{5}a^7x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 + 3
5/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

3.103 $\int x^3(a+bx)^7\,dx$

Optimal. Leaf size=64

$$\frac{a^2(a+bx)^9}{3b^4}-\frac{a^3(a+bx)^8}{8b^4}+\frac{(a+bx)^{11}}{11b^4}-\frac{3a(a+bx)^{10}}{10b^4}$$

```
[Out] -(a^3*(a + b*x)^8)/(8*b^4) + (a^2*(a + b*x)^9)/(3*b^4) - (3*a*(a + b*x)^10)
/(10*b^4) + (a + b*x)^11/(11*b^4)
```

Rubi [A] time = 0.0299538, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^9}{3b^4}-\frac{a^3(a+bx)^8}{8b^4}+\frac{(a+bx)^{11}}{11b^4}-\frac{3a(a+bx)^{10}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^7,x]
```

```
[Out] -(a^3*(a + b*x)^8)/(8*b^4) + (a^2*(a + b*x)^9)/(3*b^4) - (3*a*(a + b*x)^10)
/(10*b^4) + (a + b*x)^11/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^7\,dx&=\int\left(-\frac{a^3(a+bx)^7}{b^3}+\frac{3a^2(a+bx)^8}{b^3}-\frac{3a(a+bx)^9}{b^3}+\frac{(a+bx)^{10}}{b^3}\right)dx\\&=-\frac{a^3(a+bx)^8}{8b^4}+\frac{a^2(a+bx)^9}{3b^4}-\frac{3a(a+bx)^{10}}{10b^4}+\frac{(a+bx)^{11}}{11b^4}\end{aligned}$$

Mathematica [A] time = 0.0025907, size = 93, normalized size = 1.45

$$\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6bx^5+\frac{a^7x^4}{4}+\frac{7}{10}ab^6x^{10}+\frac{b^7x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^7,x]
```

```
[Out] (a^7*x^4)/4 + (7*a^6*b*x^5)/5 + (7*a^5*b^2*x^6)/2 + 5*a^4*b^3*x^7 + (35*a^3
*b^4*x^8)/8 + (7*a^2*b^5*x^9)/3 + (7*a*b^6*x^10)/10 + (b^7*x^11)/11
```

Maple [A] time = 0.001, size = 80, normalized size = 1.3

$$\frac{b^7x^{11}}{11}+\frac{7\,ab^6x^{10}}{10}+\frac{7\,a^2b^5x^9}{3}+\frac{35\,a^3b^4x^8}{8}+5\,a^4b^3x^7+\frac{7\,a^5b^2x^6}{2}+\frac{7\,a^6bx^5}{5}+\frac{a^7x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^7,x)
```

```
[Out] 1/11*b^7*x^11+7/10*a*b^6*x^10+7/3*a^2*b^5*x^9+35/8*a^3*b^4*x^8+5*a^4*b^3*x^
7+7/2*a^5*b^2*x^6+7/5*a^6*b*x^5+1/4*a^7*x^4
```

Maxima [A] time = 1.01883, size = 107, normalized size = 1.67

$$\frac{1}{11}\,b^7x^{11}+\frac{7}{10}\,ab^6x^{10}+\frac{7}{3}\,a^2b^5x^9+\frac{35}{8}\,a^3b^4x^8+5\,a^4b^3x^7+\frac{7}{2}\,a^5b^2x^6+\frac{7}{5}\,a^6bx^5+\frac{1}{4}\,a^7x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a^
4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4
```

Fricas [A] time = 1.49904, size = 181, normalized size = 2.83

$$\frac{1}{11}x^{11}b^7+\frac{7}{10}x^{10}b^6a+\frac{7}{3}x^9b^5a^2+\frac{35}{8}x^8b^4a^3+5x^7b^3a^4+\frac{7}{2}x^6b^2a^5+\frac{7}{5}x^5ba^6+\frac{1}{4}x^4a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^7 + 7/10*x^10*b^6*a + 7/3*x^9*b^5*a^2 + 35/8*x^8*b^4*a^3 + 5*x^
7*b^3*a^4 + 7/2*x^6*b^2*a^5 + 7/5*x^5*b*a^6 + 1/4*x^4*a^7
```

Sympy [A] time = 0.087945, size = 92, normalized size = 1.44

$$\frac{a^7x^4}{4}+\frac{7a^6bx^5}{5}+\frac{7a^5b^2x^6}{2}+5a^4b^3x^7+\frac{35a^3b^4x^8}{8}+\frac{7a^2b^5x^9}{3}+\frac{7ab^6x^{10}}{10}+\frac{b^7x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**7,x)
```

```
[Out] a**7*x**4/4 + 7*a**6*b*x**5/5 + 7*a**5*b**2*x**6/2 + 5*a**4*b**3*x**7 + 35*
a**3*b**4*x**8/8 + 7*a**2*b**5*x**9/3 + 7*a*b**6*x**10/10 + b**7*x**11/11
```

Giac [A] time = 1.16786, size = 107, normalized size = 1.67

$$\frac{1}{11}\,b^7x^{11}+\frac{7}{10}\,ab^6x^{10}+\frac{7}{3}\,a^2b^5x^9+\frac{35}{8}\,a^3b^4x^8+5\,a^4b^3x^7+\frac{7}{2}\,a^5b^2x^6+\frac{7}{5}\,a^6bx^5+\frac{1}{4}\,a^7x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a^
4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4
```

3.104 $\int x^2(a+bx)^7\,dx$

Optimal. Leaf size=47

$$\frac{a^2(a+bx)^8}{8b^3}+\frac{(a+bx)^{10}}{10b^3}-\frac{2a(a+bx)^9}{9b^3}$$

```
[Out] (a^2*(a + b*x)^8)/(8*b^3) - (2*a*(a + b*x)^9)/(9*b^3) + (a + b*x)^10/(10*b^
3)
```

Rubi [A] time = 0.0236088, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^8}{8b^3}+\frac{(a+bx)^{10}}{10b^3}-\frac{2a(a+bx)^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^7,x]
```

```
[Out] (a^2*(a + b*x)^8)/(8*b^3) - (2*a*(a + b*x)^9)/(9*b^3) + (a + b*x)^10/(10*b^
3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^7\,dx&=\int\left(\frac{a^2(a+bx)^7}{b^2}-\frac{2a(a+bx)^8}{b^2}+\frac{(a+bx)^9}{b^2}\right)dx\\&=\frac{a^2(a+bx)^8}{8b^3}-\frac{2a(a+bx)^9}{9b^3}+\frac{(a+bx)^{10}}{10b^3}\end{aligned}$$

Mathematica [A] time = 0.0026457, size = 93, normalized size = 1.98

$$\frac{21}{8}a^2b^5x^8+5a^3b^4x^7+\frac{35}{6}a^4b^3x^6+\frac{21}{5}a^5b^2x^5+\frac{7}{4}a^6bx^4+\frac{a^7x^3}{3}+\frac{7}{9}ab^6x^9+\frac{b^7x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^7,x]
```

```
[Out] (a^7*x^3)/3 + (7*a^6*b*x^4)/4 + (21*a^5*b^2*x^5)/5 + (35*a^4*b^3*x^6)/6 + 5
*a^3*b^4*x^7 + (21*a^2*b^5*x^8)/8 + (7*a*b^6*x^9)/9 + (b^7*x^10)/10
```

Maple [A] time = 0., size = 80, normalized size = 1.7

$$\frac{b^7x^{10}}{10}+\frac{7\,ab^6x^9}{9}+\frac{21\,a^2b^5x^8}{8}+5\,a^3b^4x^7+\frac{35\,a^4b^3x^6}{6}+\frac{21\,a^5b^2x^5}{5}+\frac{7\,a^6bx^4}{4}+\frac{a^7x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^7,x)
```

```
[Out] 1/10*b^7*x^10+7/9*a*b^6*x^9+21/8*a^2*b^5*x^8+5*a^3*b^4*x^7+35/6*a^4*b^3*x^6
+21/5*a^5*b^2*x^5+7/4*a^6*b*x^4+1/3*a^7*x^3
```

Maxima [A] time = 1.04321, size = 107, normalized size = 2.28

$$\frac{1}{10}\,b^7x^{10}+\frac{7}{9}\,ab^6x^9+\frac{21}{8}\,a^2b^5x^8+5\,a^3b^4x^7+\frac{35}{6}\,a^4b^3x^6+\frac{21}{5}\,a^5b^2x^5+\frac{7}{4}\,a^6bx^4+\frac{1}{3}\,a^7x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^4
*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

Fricas [A] time = 1.40097, size = 181, normalized size = 3.85

$$\frac{1}{10}x^{10}b^7+\frac{7}{9}x^9b^6a+\frac{21}{8}x^8b^5a^2+5x^7b^4a^3+\frac{35}{6}x^6b^3a^4+\frac{21}{5}x^5b^2a^5+\frac{7}{4}x^4ba^6+\frac{1}{3}x^3a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^7 + 7/9*x^9*b^6*a + 21/8*x^8*b^5*a^2 + 5*x^7*b^4*a^3 + 35/6*x^6
*b^3*a^4 + 21/5*x^5*b^2*a^5 + 7/4*x^4*b*a^6 + 1/3*x^3*a^7
```

Sympy [B] time = 0.089851, size = 92, normalized size = 1.96

$$\frac{a^7x^3}{3}+\frac{7a^6bx^4}{4}+\frac{21a^5b^2x^5}{5}+\frac{35a^4b^3x^6}{6}+5a^3b^4x^7+\frac{21a^2b^5x^8}{8}+\frac{7ab^6x^9}{9}+\frac{b^7x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**7,x)
```

```
[Out] a**7*x**3/3 + 7*a**6*b*x**4/4 + 21*a**5*b**2*x**5/5 + 35*a**4*b**3*x**6/6 +
 5*a**3*b**4*x**7 + 21*a**2*b**5*x**8/8 + 7*a*b**6*x**9/9 + b**7*x**10/10
```

Giac [A] time = 1.18946, size = 107, normalized size = 2.28

$$\frac{1}{10}\,b^7x^{10}+\frac{7}{9}\,ab^6x^9+\frac{21}{8}\,a^2b^5x^8+5\,a^3b^4x^7+\frac{35}{6}\,a^4b^3x^6+\frac{21}{5}\,a^5b^2x^5+\frac{7}{4}\,a^6bx^4+\frac{1}{3}\,a^7x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^4
*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

3.105 $\int x(a+bx)^7 \, dx$

Optimal. Leaf size=30

$$\frac{(a+bx)^9}{9b^2}-\frac{a(a+bx)^8}{8b^2}$$

```
[Out] -(a*(a + b*x)^8)/(8*b^2) + (a + b*x)^9/(9*b^2)
```

Rubi [A] time = 0.0081933, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^9}{9b^2}-\frac{a(a+bx)^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^7,x]

[Out] -(a*(a + b*x)^8)/(8*b^2) + (a + b*x)^9/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^7\,dx &= \int \left(-\frac{a(a+bx)^7}{b}+\frac{(a+bx)^8}{b}\right)dx\\ &= -\frac{a(a+bx)^8}{8b^2}+\frac{(a+bx)^9}{9b^2}\end{aligned}$$

Mathematica [B] time = 0.0025274, size = 91, normalized size = 3.03

$$3a^2b^5x^7+\frac{35}{6}a^3b^4x^6+7a^4b^3x^5+\frac{21}{4}a^5b^2x^4+\frac{7}{3}a^6bx^3+\frac{a^7x^2}{2}+\frac{7}{8}ab^6x^8+\frac{b^7x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^7,x]

[Out] (a^7*x^2)/2 + (7*a^6*b*x^3)/3 + (21*a^5*b^2*x^4)/4 + 7*a^4*b^3*x^5 + (35*a^
3*b^4*x^6)/6 + 3*a^2*b^5*x^7 + (7*a*b^6*x^8)/8 + (b^7*x^9)/9
```

Maple [B] time = 0.002, size = 80, normalized size = 2.7

$$\frac{b^7x^9}{9}+\frac{7\,ab^6x^8}{8}+3\,a^2b^5x^7+\frac{35\,a^3b^4x^6}{6}+7\,a^4b^3x^5+\frac{21\,a^5b^2x^4}{4}+\frac{7\,a^6bx^3}{3}+\frac{a^7x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^7,x)
```

```
[Out] 1/9*b^7*x^9+7/8*a*b^6*x^8+3*a^2*b^5*x^7+35/6*a^3*b^4*x^6+7*a^4*b^3*x^5+21/4
*a^5*b^2*x^4+7/3*a^6*b*x^3+1/2*a^7*x^2
```

Maxima [B] time = 1.01952, size = 107, normalized size = 3.57

$$\frac{1}{9}\,b^7x^9 + \frac{7}{8}\,ab^6x^8 + 3\,a^2b^5x^7 + \frac{35}{6}\,a^3b^4x^6 + 7\,a^4b^3x^5 + \frac{21}{4}\,a^5b^2x^4 + \frac{7}{3}\,a^6bx^3 + \frac{1}{2}\,a^7x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3*
x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2
```

Fricas [B] time = 1.38148, size = 174, normalized size = 5.8

$$\frac{1}{9}x^9b^7 + \frac{7}{8}x^8b^6a + 3x^7b^5a^2 + \frac{35}{6}x^6b^4a^3 + 7x^5b^3a^4 + \frac{21}{4}x^4b^2a^5 + \frac{7}{3}x^3ba^6 + \frac{1}{2}x^2a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^7 + 7/8*x^8*b^6*a + 3*x^7*b^5*a^2 + 35/6*x^6*b^4*a^3 + 7*x^5*b^3*
a^4 + 21/4*x^4*b^2*a^5 + 7/3*x^3*b*a^6 + 1/2*x^2*a^7
```

Sympy [B] time = 0.0818, size = 90, normalized size = 3.

$$\frac{a^7x^2}{2} + \frac{7a^6bx^3}{3} + \frac{21a^5b^2x^4}{4} + 7a^4b^3x^5 + \frac{35a^3b^4x^6}{6} + 3a^2b^5x^7 + \frac{7ab^6x^8}{8} + \frac{b^7x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**7,x)
```

```
[Out] a**7*x**2/2 + 7*a**6*b*x**3/3 + 21*a**5*b**2*x**4/4 + 7*a**4*b**3*x**5 + 35
*a**3*b**4*x**6/6 + 3*a**2*b**5*x**7 + 7*a*b**6*x**8/8 + b**7*x**9/9
```

Giac [B] time = 1.21016, size = 107, normalized size = 3.57

$$\frac{1}{9}\,b^7x^9 + \frac{7}{8}\,ab^6x^8 + 3\,a^2b^5x^7 + \frac{35}{6}\,a^3b^4x^6 + 7\,a^4b^3x^5 + \frac{21}{4}\,a^5b^2x^4 + \frac{7}{3}\,a^6bx^3 + \frac{1}{2}\,a^7x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3*
x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2
```

3.106 $\int (a+bx)^7 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^8}{8b}$$

```
[Out] (a + b*x)^8/(8*b)
```

Rubi [A] time = 0.0017025, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7,x]
```

```
[Out] (a + b*x)^8/(8*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^7 \, dx = \frac{(a+bx)^8}{8b}$$

Mathematica [A] time = 0.000834, size = 14, normalized size = 1.

$$\frac{(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7,x]
```

```
[Out] (a + b*x)^8/(8*b)
```

Maple [A] time = 0.001, size = 13, normalized size = 0.9

$$\frac{(bx+a)^8}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7,x)
```

```
[Out] 1/8*(b*x+a)^8/b
```

Maxima [A] time = 1.06137, size = 16, normalized size = 1.14

$$\frac{(bx+a)^8}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x + a)^8/b
```

Fricas [B] time = 1.33431, size = 159, normalized size = 11.36

$$\frac{1}{8}x^8b^7 + x^7b^6a + \frac{7}{2}x^6b^5a^2 + 7x^5b^4a^3 + \frac{35}{4}x^4b^3a^4 + 7x^3b^2a^5 + \frac{7}{2}x^2ba^6 + xa^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^7 + x^7*b^6*a + 7/2*x^6*b^5*a^2 + 7*x^5*b^4*a^3 + 35/4*x^4*b^3*a^
4 + 7*x^3*b^2*a^5 + 7/2*x^2*b*a^6 + x*a^7
```

Sympy [B] time = 0.103633, size = 83, normalized size = 5.93

$$a^7x + \frac{7a^6bx^2}{2} + 7a^5b^2x^3 + \frac{35a^4b^3x^4}{4} + 7a^3b^4x^5 + \frac{7a^2b^5x^6}{2} + ab^6x^7 + \frac{b^7x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7,x)
```

```
[Out] a**7*x + 7*a**6*b*x**2/2 + 7*a**5*b**2*x**3 + 35*a**4*b**3*x**4/4 + 7*a**3*
b**4*x**5 + 7*a**2*b**5*x**6/2 + a*b**6*x**7 + b**7*x**8/8
```

Giac [A] time = 1.22762, size = 16, normalized size = 1.14

$$\frac{(bx+a)^8}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(b*x + a)^8/b
```

3.107 $\int \frac{(a+bx)^7}{x} \, dx$

Optimal. Leaf size=87

$$\frac{21}{2}a^5b^2x^2+\frac{35}{3}a^4b^3x^3+\frac{35}{4}a^3b^4x^4+\frac{21}{5}a^2b^5x^5+7a^6bx+a^7\log(x)+\frac{7}{6}ab^6x^6+\frac{b^7x^7}{7}$$

```
[Out] 7*a^6*b*x + (21*a^5*b^2*x^2)/2 + (35*a^4*b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 +
(21*a^2*b^5*x^5)/5 + (7*a*b^6*x^6)/6 + (b^7*x^7)/7 + a^7*Log[x]
```

Rubi [A] time = 0.0273506, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21}{2}a^5b^2x^2+\frac{35}{3}a^4b^3x^3+\frac{35}{4}a^3b^4x^4+\frac{21}{5}a^2b^5x^5+7a^6bx+a^7\log(x)+\frac{7}{6}ab^6x^6+\frac{b^7x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x,x]
```

```
[Out] 7*a^6*b*x + (21*a^5*b^2*x^2)/2 + (35*a^4*b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 +
(21*a^2*b^5*x^5)/5 + (7*a*b^6*x^6)/6 + (b^7*x^7)/7 + a^7*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x}\,dx &= \int \left(7a^6b+\frac{a^7}{x}+21a^5b^2x+35a^4b^3x^2+35a^3b^4x^3+21a^2b^5x^4+7ab^6x^5+b^7x^6\right)dx\\ &= 7a^6bx+\frac{21}{2}a^5b^2x^2+\frac{35}{3}a^4b^3x^3+\frac{35}{4}a^3b^4x^4+\frac{21}{5}a^2b^5x^5+\frac{7}{6}ab^6x^6+\frac{b^7x^7}{7}+a^7\log(x)\end{aligned}$$

Mathematica [A] time = 0.0040599, size = 87, normalized size = 1.

$$\frac{21}{2}a^5b^2x^2+\frac{35}{3}a^4b^3x^3+\frac{35}{4}a^3b^4x^4+\frac{21}{5}a^2b^5x^5+7a^6bx+a^7\log(x)+\frac{7}{6}ab^6x^6+\frac{b^7x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x,x]
```

```
[Out] 7*a^6*b*x + (21*a^5*b^2*x^2)/2 + (35*a^4*b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 +
(21*a^2*b^5*x^5)/5 + (7*a*b^6*x^6)/6 + (b^7*x^7)/7 + a^7*Log[x]
```

Maple [A] time = 0.004, size = 76, normalized size = 0.9

$$7\,a^{6}bx+\frac{21\,a^{5}b^{2}x^{2}}{2}+\frac{35\,a^{4}b^{3}x^{3}}{3}+\frac{35\,a^{3}b^{4}x^{4}}{4}+\frac{21\,a^{2}b^{5}x^{5}}{5}+\frac{7\,ab^{6}x^{6}}{6}+\frac{b^{7}x^{7}}{7}+a^{7}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x,x)
```

```
[Out] 7*a^6*b*x+21/2*a^5*b^2*x^2+35/3*a^4*b^3*x^3+35/4*a^3*b^4*x^4+21/5*a^2*b^5*x
^5+7/6*a*b^6*x^6+1/7*b^7*x^7+a^7*ln(x)
```

Maxima [A] time = 1.03238, size = 101, normalized size = 1.16

$$\frac{1}{7}\,b^{7}x^{7}+\frac{7}{6}\,ab^{6}x^{6}+\frac{21}{5}\,a^{2}b^{5}x^{5}+\frac{35}{4}\,a^{3}b^{4}x^{4}+\frac{35}{3}\,a^{4}b^{3}x^{3}+\frac{21}{2}\,a^{5}b^{2}x^{2}+7\,a^{6}bx+a^{7}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="maxima")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(x)
```

Fricas [A] time = 1.49098, size = 176, normalized size = 2.02

$$\frac{1}{7}\,b^{7}x^{7}+\frac{7}{6}\,ab^{6}x^{6}+\frac{21}{5}\,a^{2}b^{5}x^{5}+\frac{35}{4}\,a^{3}b^{4}x^{4}+\frac{35}{3}\,a^{4}b^{3}x^{3}+\frac{21}{2}\,a^{5}b^{2}x^{2}+7\,a^{6}bx+a^{7}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="fricas")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(x)
```

Sympy [A] time = 0.352165, size = 88, normalized size = 1.01

$$a^{7}\log\left(x\right)+7a^{6}bx+\frac{21a^{5}b^{2}x^{2}}{2}+\frac{35a^{4}b^{3}x^{3}}{3}+\frac{35a^{3}b^{4}x^{4}}{4}+\frac{21a^{2}b^{5}x^{5}}{5}+\frac{7ab^{6}x^{6}}{6}+\frac{b^{7}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x,x)
```

```
[Out] a**7*log(x) + 7*a**6*b*x + 21*a**5*b**2*x**2/2 + 35*a**4*b**3*x**3/3 + 35*a
**3*b**4*x**4/4 + 21*a**2*b**5*x**5/5 + 7*a*b**6*x**6/6 + b**7*x**7/7
```

Giac [A] time = 1.19813, size = 103, normalized size = 1.18

$$\frac{1}{7}\,b^{7}x^{7}+\frac{7}{6}\,ab^{6}x^{6}+\frac{21}{5}\,a^{2}b^{5}x^{5}+\frac{35}{4}\,a^{3}b^{4}x^{4}+\frac{35}{3}\,a^{4}b^{3}x^{3}+\frac{21}{2}\,a^{5}b^{2}x^{2}+7\,a^{6}bx+a^{7}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="giac")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(abs(x))
```

3.108 $\int \frac{(a+bx)^7}{x^2}\,dx$

Optimal. Leaf size=86

$$\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+21a^5b^2x+7a^6b\log(x)-\frac{a^7}{x}+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}$$

```
[Out] -(a^7/x) + 21*a^5*b^2*x + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2
*b^5*x^4)/4 + (7*a*b^6*x^5)/5 + (b^7*x^6)/6 + 7*a^6*b*Log[x]
```

Rubi [A] time = 0.0321975, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+21a^5b^2x+7a^6b\log(x)-\frac{a^7}{x}+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^2,x]
```

```
[Out] -(a^7/x) + 21*a^5*b^2*x + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2
*b^5*x^4)/4 + (7*a*b^6*x^5)/5 + (b^7*x^6)/6 + 7*a^6*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^2}\,dx &= \int \left(21a^5b^2+\frac{a^7}{x^2}+\frac{7a^6b}{x}+35a^4b^3x+35a^3b^4x^2+21a^2b^5x^3+7ab^6x^4+b^7x^5\right)dx\\ &= -\frac{a^7}{x}+21a^5b^2x+\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}+7a^6b\log(x)\end{aligned}$$

Mathematica [A] time = 0.0048845, size = 86, normalized size = 1.

$$\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+21a^5b^2x+7a^6b\log(x)-\frac{a^7}{x}+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^2,x]
```

```
[Out] -(a^7/x) + 21*a^5*b^2*x + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2
*b^5*x^4)/4 + (7*a*b^6*x^5)/5 + (b^7*x^6)/6 + 7*a^6*b*Log[x]
```

Maple [A] time = 0.006, size = 77, normalized size = 0.9

$$-\frac{a^{7}}{x}+21\,a^{5}b^{2}x+\frac{35\,a^{4}b^{3}x^{2}}{2}+\frac{35\,a^{3}b^{4}x^{3}}{3}+\frac{21\,a^{2}b^{5}x^{4}}{4}+\frac{7\,ab^{6}x^{5}}{5}+\frac{b^{7}x^{6}}{6}+7\,a^{6}b\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^2,x)
```

```
[Out] -a^7/x+21*a^5*b^2*x+35/2*a^4*b^3*x^2+35/3*a^3*b^4*x^3+21/4*a^2*b^5*x^4+7/5*
a*b^6*x^5+1/6*b^7*x^6+7*a^6*b*ln(x)
```

Maxima [A] time = 1.03052, size = 103, normalized size = 1.2

$$\frac{1}{6}\,b^{7}x^{6}+\frac{7}{5}\,ab^{6}x^{5}+\frac{21}{4}\,a^{2}b^{5}x^{4}+\frac{35}{3}\,a^{3}b^{4}x^{3}+\frac{35}{2}\,a^{4}b^{3}x^{2}+21\,a^{5}b^{2}x+7\,a^{6}b\log\left(x\right)-\frac{a^{7}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^7*x^6 + 7/5*a*b^6*x^5 + 21/4*a^2*b^5*x^4 + 35/3*a^3*b^4*x^3 + 35/2*a^
4*b^3*x^2 + 21*a^5*b^2*x + 7*a^6*b*log(x) - a^7/x
```

Fricas [A] time = 1.79872, size = 189, normalized size = 2.2

$$\frac{10\,b^{7}x^{7}+84\,ab^{6}x^{6}+315\,a^{2}b^{5}x^{5}+700\,a^{3}b^{4}x^{4}+1050\,a^{4}b^{3}x^{3}+1260\,a^{5}b^{2}x^{2}+420\,a^{6}bx\log\left(x\right)-60\,a^{7}}{60\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^7*x^7 + 84*a*b^6*x^6 + 315*a^2*b^5*x^5 + 700*a^3*b^4*x^4 + 1050*
a^4*b^3*x^3 + 1260*a^5*b^2*x^2 + 420*a^6*b*x*log(x) - 60*a^7)/x
```

Sympy [A] time = 0.430041, size = 85, normalized size = 0.99

$$-\frac{a^{7}}{x}+7a^{6}b\log\left(x\right)+21a^{5}b^{2}x+\frac{35a^{4}b^{3}x^{2}}{2}+\frac{35a^{3}b^{4}x^{3}}{3}+\frac{21a^{2}b^{5}x^{4}}{4}+\frac{7ab^{6}x^{5}}{5}+\frac{b^{7}x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**2,x)
```

```
[Out] -a**7/x + 7*a**6*b*log(x) + 21*a**5*b**2*x + 35*a**4*b**3*x**2/2 + 35*a**3*
b**4*x**3/3 + 21*a**2*b**5*x**4/4 + 7*a*b**6*x**5/5 + b**7*x**6/6
```

Giac [A] time = 1.17345, size = 104, normalized size = 1.21

$$\frac{1}{6}\,b^{7}x^{6}+\frac{7}{5}\,ab^{6}x^{5}+\frac{21}{4}\,a^{2}b^{5}x^{4}+\frac{35}{3}\,a^{3}b^{4}x^{3}+\frac{35}{2}\,a^{4}b^{3}x^{2}+21\,a^{5}b^{2}x+7\,a^{6}b\log\left(|x|\right)-\frac{a^{7}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x, algorithm="giac")
```

```
[Out] 1/6*b^7*x^6 + 7/5*a*b^6*x^5 + 21/4*a^2*b^5*x^4 + 35/3*a^3*b^4*x^3 + 35/2*a^
4*b^3*x^2 + 21*a^5*b^2*x + 7*a^6*b*log(abs(x)) - a^7/x
```

3.109 $\int \frac{(a+bx)^7}{x^3} \, dx$

Optimal. Leaf size=84

$$\frac{35}{2}a^3b^4x^2 + 7a^2b^5x^3 + 35a^4b^3x + 21a^5b^2\log(x) - \frac{7a^6b}{x} - \frac{a^7}{2x^2} + \frac{7}{4}ab^6x^4 + \frac{b^7x^5}{5}$$

```
[Out] -a^7/(2*x^2) - (7*a^6*b)/x + 35*a^4*b^3*x + (35*a^3*b^4*x^2)/2 + 7*a^2*b^5*
x^3 + (7*a*b^6*x^4)/4 + (b^7*x^5)/5 + 21*a^5*b^2*Log[x]
```

Rubi [A] time = 0.0327259, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{35}{2}a^3b^4x^2 + 7a^2b^5x^3 + 35a^4b^3x + 21a^5b^2\log(x) - \frac{7a^6b}{x} - \frac{a^7}{2x^2} + \frac{7}{4}ab^6x^4 + \frac{b^7x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^3,x]
```

```
[Out] -a^7/(2*x^2) - (7*a^6*b)/x + 35*a^4*b^3*x + (35*a^3*b^4*x^2)/2 + 7*a^2*b^5*
x^3 + (7*a*b^6*x^4)/4 + (b^7*x^5)/5 + 21*a^5*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^7}{x^3} \, dx &= \int \left(35a^4b^3 + \frac{a^7}{x^3} + \frac{7a^6b}{x^2} + \frac{21a^5b^2}{x} + 35a^3b^4x + 21a^2b^5x^2 + 7ab^6x^3 + b^7x^4\right) dx \\ &= -\frac{a^7}{2x^2} - \frac{7a^6b}{x} + 35a^4b^3x + \frac{35}{2}a^3b^4x^2 + 7a^2b^5x^3 + \frac{7}{4}ab^6x^4 + \frac{b^7x^5}{5} + 21a^5b^2\log(x) \end{aligned}$$

Mathematica [A] time = 0.0060226, size = 84, normalized size = 1.

$$\frac{35}{2}a^3b^4x^2 + 7a^2b^5x^3 + 35a^4b^3x + 21a^5b^2\log(x) - \frac{7a^6b}{x} - \frac{a^7}{2x^2} + \frac{7}{4}ab^6x^4 + \frac{b^7x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^3,x]
```

```
[Out] -a^7/(2*x^2) - (7*a^6*b)/x + 35*a^4*b^3*x + (35*a^3*b^4*x^2)/2 + 7*a^2*b^5*
x^3 + (7*a*b^6*x^4)/4 + (b^7*x^5)/5 + 21*a^5*b^2*Log[x]
```

Maple [A] time = 0.004, size = 77, normalized size = 0.9

$$-\frac{a^{7}}{2 x^{2}}-7 \frac{a^{6} b}{x}+35 a^{4} b^{3} x+\frac{35 a^{3} b^{4} x^{2}}{2}+7 a^{2} b^{5} x^{3}+\frac{7 a b^{6} x^{4}}{4}+\frac{b^{7} x^{5}}{5}+21 a^{5} b^{2} \ln (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^3,x)
```

```
[Out] -1/2*a^7/x^2-7*a^6*b/x+35*a^4*b^3*x+35/2*a^3*b^4*x^2+7*a^2*b^5*x^3+7/4*a*b^
6*x^4+1/5*b^7*x^5+21*a^5*b^2*ln(x)
```

Maxima [A] time = 1.02658, size = 101, normalized size = 1.2

$$\frac{1}{5} b^{7} x^{5}+\frac{7}{4} a b^{6} x^{4}+7 a^{2} b^{5} x^{3}+\frac{35}{2} a^{3} b^{4} x^{2}+35 a^{4} b^{3} x+21 a^{5} b^{2} \log (x)-\frac{14 a^{6} b x+a^{7}}{2 x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="maxima")
```

```
[Out] 1/5*b^7*x^5 + 7/4*a*b^6*x^4 + 7*a^2*b^5*x^3 + 35/2*a^3*b^4*x^2 + 35*a^4*b^3
*x + 21*a^5*b^2*log(x) - 1/2*(14*a^6*b*x + a^7)/x^2
```

Fricas [A] time = 1.80903, size = 188, normalized size = 2.24

$$\frac{4 b^{7} x^{7}+35 a b^{6} x^{6}+140 a^{2} b^{5} x^{5}+350 a^{3} b^{4} x^{4}+700 a^{4} b^{3} x^{3}+420 a^{5} b^{2} x^{2} \log (x)-140 a^{6} b x-10 a^{7}}{20 x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*x^7 + 35*a*b^6*x^6 + 140*a^2*b^5*x^5 + 350*a^3*b^4*x^4 + 700*a^
4*b^3*x^3 + 420*a^5*b^2*x^2*log(x) - 140*a^6*b*x - 10*a^7)/x^2
```

Sympy [A] time = 0.448175, size = 83, normalized size = 0.99

$$21 a^{5} b^{2} \log (x)+35 a^{4} b^{3} x+\frac{35 a^{3} b^{4} x^{2}}{2}+7 a^{2} b^{5} x^{3}+\frac{7 a b^{6} x^{4}}{4}+\frac{b^{7} x^{5}}{5}-\frac{a^{7}+14 a^{6} b x}{2 x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**3,x)
```

```
[Out] 21*a**5*b**2*log(x) + 35*a**4*b**3*x + 35*a**3*b**4*x**2/2 + 7*a**2*b**5*x*
*3 + 7*a*b**6*x**4/4 + b**7*x**5/5 - (a**7 + 14*a**6*b*x)/(2*x**2)
```

Giac [A] time = 1.16313, size = 103, normalized size = 1.23

$$\frac{1}{5} b^{7} x^{5}+\frac{7}{4} a b^{6} x^{4}+7 a^{2} b^{5} x^{3}+\frac{35}{2} a^{3} b^{4} x^{2}+35 a^{4} b^{3} x+21 a^{5} b^{2} \log (|x|)-\frac{14 a^{6} b x+a^{7}}{2 x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="giac")
```

```
[Out] 1/5*b^7*x^5 + 7/4*a*b^6*x^4 + 7*a^2*b^5*x^3 + 35/2*a^3*b^4*x^2 + 35*a^4*b^3
*x + 21*a^5*b^2*log(abs(x)) - 1/2*(14*a^6*b*x + a^7)/x^2
```

3.110 $\int \frac{(a+bx)^7}{x^4}\,dx$

Optimal. Leaf size=86

$$\frac{21}{2}a^2b^5x^2-\frac{21a^5b^2}{x}+35a^3b^4x+35a^4b^3\log(x)-\frac{7a^6b}{2x^2}-\frac{a^7}{3x^3}+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}$$

```
[Out] -a^7/(3*x^3) - (7*a^6*b)/(2*x^2) - (21*a^5*b^2)/x + 35*a^3*b^4*x + (21*a^2*
b^5*x^2)/2 + (7*a*b^6*x^3)/3 + (b^7*x^4)/4 + 35*a^4*b^3*Log[x]
```

Rubi [A] time = 0.0316721, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21}{2}a^2b^5x^2-\frac{21a^5b^2}{x}+35a^3b^4x+35a^4b^3\log(x)-\frac{7a^6b}{2x^2}-\frac{a^7}{3x^3}+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^4,x]
```

```
[Out] -a^7/(3*x^3) - (7*a^6*b)/(2*x^2) - (21*a^5*b^2)/x + 35*a^3*b^4*x + (21*a^2*
b^5*x^2)/2 + (7*a*b^6*x^3)/3 + (b^7*x^4)/4 + 35*a^4*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^4}\,dx &= \int \left(35a^3b^4+\frac{a^7}{x^4}+\frac{7a^6b}{x^3}+\frac{21a^5b^2}{x^2}+\frac{35a^4b^3}{x}+21a^2b^5x+7ab^6x^2+b^7x^3\right)dx\\ &= -\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}+35a^4b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.0100841, size = 86, normalized size = 1.

$$\frac{21}{2}a^2b^5x^2-\frac{21a^5b^2}{x}+35a^3b^4x+35a^4b^3\log(x)-\frac{7a^6b}{2x^2}-\frac{a^7}{3x^3}+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^4,x]
```

```
[Out] -a^7/(3*x^3) - (7*a^6*b)/(2*x^2) - (21*a^5*b^2)/x + 35*a^3*b^4*x + (21*a^2*
b^5*x^2)/2 + (7*a*b^6*x^3)/3 + (b^7*x^4)/4 + 35*a^4*b^3*Log[x]
```

Maple [A] time = 0.005, size = 77, normalized size = 0.9

$$-\frac{a^7}{3\,x^3}-\frac{7\,a^6b}{2\,x^2}-21\,\frac{a^5b^2}{x}+35\,a^3b^4x+\frac{21\,a^2b^5x^2}{2}+\frac{7\,ab^6x^3}{3}+\frac{b^7x^4}{4}+35\,a^4b^3\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^4,x)
```

```
[Out] -1/3*a^7/x^3-7/2*a^6*b/x^2-21*a^5*b^2/x+35*a^3*b^4*x+21/2*a^2*b^5*x^2+7/3*a
*b^6*x^3+1/4*b^7*x^4+35*a^4*b^3*ln(x)
```

Maxima [A] time = 1.08463, size = 104, normalized size = 1.21

$$\frac{1}{4}\,b^7x^4+\frac{7}{3}\,ab^6x^3+\frac{21}{2}\,a^2b^5x^2+35\,a^3b^4x+35\,a^4b^3\log\left(x\right)-\frac{126\,a^5b^2x^2+21\,a^6bx+2\,a^7}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*b^7*x^4 + 7/3*a*b^6*x^3 + 21/2*a^2*b^5*x^2 + 35*a^3*b^4*x + 35*a^4*b^3*
log(x) - 1/6*(126*a^5*b^2*x^2 + 21*a^6*b*x + 2*a^7)/x^3
```

Fricas [A] time = 1.82103, size = 185, normalized size = 2.15

$$\frac{3\,b^7x^7+28\,ab^6x^6+126\,a^2b^5x^5+420\,a^3b^4x^4+420\,a^4b^3x^3\log\left(x\right)-252\,a^5b^2x^2-42\,a^6bx-4\,a^7}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*x^7 + 28*a*b^6*x^6 + 126*a^2*b^5*x^5 + 420*a^3*b^4*x^4 + 420*a^
4*b^3*x^3*log(x) - 252*a^5*b^2*x^2 - 42*a^6*b*x - 4*a^7)/x^3
```

Sympy [A] time = 0.525546, size = 85, normalized size = 0.99

$$35a^4b^3\log\left(x\right)+35a^3b^4x+\frac{21a^2b^5x^2}{2}+\frac{7ab^6x^3}{3}+\frac{b^7x^4}{4}-\frac{2a^7+21a^6bx+126a^5b^2x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**4,x)
```

```
[Out] 35*a**4*b**3*log(x) + 35*a**3*b**4*x + 21*a**2*b**5*x**2/2 + 7*a*b**6*x**3/
3 + b**7*x**4/4 - (2*a**7 + 21*a**6*b*x + 126*a**5*b**2*x**2)/(6*x**3)
```

Giac [A] time = 1.18138, size = 105, normalized size = 1.22

$$\frac{1}{4}\,b^7x^4+\frac{7}{3}\,ab^6x^3+\frac{21}{2}\,a^2b^5x^2+35\,a^3b^4x+35\,a^4b^3\log\left(|x|\right)-\frac{126\,a^5b^2x^2+21\,a^6bx+2\,a^7}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="giac")
```

```
[Out] 1/4*b^7*x^4 + 7/3*a*b^6*x^3 + 21/2*a^2*b^5*x^2 + 35*a^3*b^4*x + 35*a^4*b^3*
log(abs(x)) - 1/6*(126*a^5*b^2*x^2 + 21*a^6*b*x + 2*a^7)/x^3
```

3.111 $\int \frac{(a+bx)^7}{x^5}\,dx$

Optimal. Leaf size=86

$$-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+35a^3b^4\log(x)-\frac{7a^6b}{3x^3}-\frac{a^7}{4x^4}+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}$$

```
[Out] -a^7/(4*x^4) - (7*a^6*b)/(3*x^3) - (21*a^5*b^2)/(2*x^2) - (35*a^4*b^3)/x +
21*a^2*b^5*x + (7*a*b^6*x^2)/2 + (b^7*x^3)/3 + 35*a^3*b^4*Log[x]
```

Rubi [A] time = 0.0320514, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+35a^3b^4\log(x)-\frac{7a^6b}{3x^3}-\frac{a^7}{4x^4}+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^5,x]
```

```
[Out] -a^7/(4*x^4) - (7*a^6*b)/(3*x^3) - (21*a^5*b^2)/(2*x^2) - (35*a^4*b^3)/x +
21*a^2*b^5*x + (7*a*b^6*x^2)/2 + (b^7*x^3)/3 + 35*a^3*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^5}\,dx &= \int \left(21a^2b^5+\frac{a^7}{x^5}+\frac{7a^6b}{x^4}+\frac{21a^5b^2}{x^3}+\frac{35a^4b^3}{x^2}+\frac{35a^3b^4}{x}+7ab^6x+b^7x^2\right)dx\\ &= -\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}+35a^3b^4\log(x)\end{aligned}$$

Mathematica [A] time = 0.0154708, size = 86, normalized size = 1.

$$-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+35a^3b^4\log(x)-\frac{7a^6b}{3x^3}-\frac{a^7}{4x^4}+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^5,x]
```

```
[Out] -a^7/(4*x^4) - (7*a^6*b)/(3*x^3) - (21*a^5*b^2)/(2*x^2) - (35*a^4*b^3)/x +
21*a^2*b^5*x + (7*a*b^6*x^2)/2 + (b^7*x^3)/3 + 35*a^3*b^4*Log[x]
```

Maple [A] time = 0.008, size = 77, normalized size = 0.9

$$-\frac{a^{7}}{4\,x^{4}}-\frac{7\,a^{6}b}{3\,x^{3}}-\frac{21\,a^{5}b^{2}}{2\,x^{2}}-35\,\frac{a^{4}b^{3}}{x}+21\,a^{2}b^{5}x+\frac{7\,ab^{6}x^{2}}{2}+\frac{b^{7}x^{3}}{3}+35\,a^{3}b^{4}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^5,x)
```

```
[Out] -1/4*a^7/x^4-7/3*a^6*b/x^3-21/2*a^5*b^2/x^2-35*a^4*b^3/x+21*a^2*b^5*x+7/2*a
*b^6*x^2+1/3*b^7*x^3+35*a^3*b^4*ln(x)
```

Maxima [A] time = 1.00811, size = 104, normalized size = 1.21

$$\frac{1}{3}\,b^{7}x^{3}+\frac{7}{2}\,ab^{6}x^{2}+21\,a^{2}b^{5}x+35\,a^{3}b^{4}\log\left(x\right)-\frac{420\,a^{4}b^{3}x^{3}+126\,a^{5}b^{2}x^{2}+28\,a^{6}bx+3\,a^{7}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x, algorithm="maxima")
```

```
[Out] 1/3*b^7*x^3 + 7/2*a*b^6*x^2 + 21*a^2*b^5*x + 35*a^3*b^4*log(x) - 1/12*(420*
a^4*b^3*x^3 + 126*a^5*b^2*x^2 + 28*a^6*b*x + 3*a^7)/x^4
```

Fricas [A] time = 1.76785, size = 185, normalized size = 2.15

$$\frac{4\,b^{7}x^{7}+42\,ab^{6}x^{6}+252\,a^{2}b^{5}x^{5}+420\,a^{3}b^{4}x^{4}\log\left(x\right)-420\,a^{4}b^{3}x^{3}-126\,a^{5}b^{2}x^{2}-28\,a^{6}bx-3\,a^{7}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^7*x^7 + 42*a*b^6*x^6 + 252*a^2*b^5*x^5 + 420*a^3*b^4*x^4*log(x) -
 420*a^4*b^3*x^3 - 126*a^5*b^2*x^2 - 28*a^6*b*x - 3*a^7)/x^4
```

Sympy [A] time = 0.592147, size = 83, normalized size = 0.97

$$35a^{3}b^{4}\log\left(x\right)+21a^{2}b^{5}x+\frac{7ab^{6}x^{2}}{2}+\frac{b^{7}x^{3}}{3}-\frac{3a^{7}+28a^{6}bx+126a^{5}b^{2}x^{2}+420a^{4}b^{3}x^{3}}{12x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**5,x)
```

```
[Out] 35*a**3*b**4*log(x) + 21*a**2*b**5*x + 7*a*b**6*x**2/2 + b**7*x**3/3 - (3*a
**7 + 28*a**6*b*x + 126*a**5*b**2*x**2 + 420*a**4*b**3*x**3)/(12*x**4)
```

Giac [A] time = 1.17055, size = 105, normalized size = 1.22

$$\frac{1}{3}\,b^{7}x^{3}+\frac{7}{2}\,ab^{6}x^{2}+21\,a^{2}b^{5}x+35\,a^{3}b^{4}\log\left(|x|\right)-\frac{420\,a^{4}b^{3}x^{3}+126\,a^{5}b^{2}x^{2}+28\,a^{6}bx+3\,a^{7}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x, algorithm="giac")
```

```
[Out] 1/3*b^7*x^3 + 7/2*a*b^6*x^2 + 21*a^2*b^5*x + 35*a^3*b^4*log(abs(x)) - 1/12*
(420*a^4*b^3*x^3 + 126*a^5*b^2*x^2 + 28*a^6*b*x + 3*a^7)/x^4
```

3.112 $\int \frac{(a+bx)^7}{x^6} dx$

Optimal. Leaf size=84

$$-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+21a^2b^5\log(x)-\frac{7a^6b}{4x^4}-\frac{a^7}{5x^5}+7ab^6x+\frac{b^7x^2}{2}$$

```
[Out] -a^7/(5*x^5) - (7*a^6*b)/(4*x^4) - (7*a^5*b^2)/x^3 - (35*a^4*b^3)/(2*x^2) -
 (35*a^3*b^4)/x + 7*a*b^6*x + (b^7*x^2)/2 + 21*a^2*b^5*Log[x]
```

Rubi [A] time = 0.0321007, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+21a^2b^5\log(x)-\frac{7a^6b}{4x^4}-\frac{a^7}{5x^5}+7ab^6x+\frac{b^7x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^6,x]
```

```
[Out] -a^7/(5*x^5) - (7*a^6*b)/(4*x^4) - (7*a^5*b^2)/x^3 - (35*a^4*b^3)/(2*x^2) -
 (35*a^3*b^4)/x + 7*a*b^6*x + (b^7*x^2)/2 + 21*a^2*b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^6}\,dx &= \int \left(7ab^6+\frac{a^7}{x^6}+\frac{7a^6b}{x^5}+\frac{21a^5b^2}{x^4}+\frac{35a^4b^3}{x^3}+\frac{35a^3b^4}{x^2}+\frac{21a^2b^5}{x}+b^7x\right)dx\\ &= -\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+7ab^6x+\frac{b^7x^2}{2}+21a^2b^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.0084338, size = 84, normalized size = 1.

$$-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+21a^2b^5\log(x)-\frac{7a^6b}{4x^4}-\frac{a^7}{5x^5}+7ab^6x+\frac{b^7x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^6,x]
```

```
[Out] -a^7/(5*x^5) - (7*a^6*b)/(4*x^4) - (7*a^5*b^2)/x^3 - (35*a^4*b^3)/(2*x^2) -
 (35*a^3*b^4)/x + 7*a*b^6*x + (b^7*x^2)/2 + 21*a^2*b^5*Log[x]
```

Maple [A] time = 0.007, size = 77, normalized size = 0.9

$$-\frac{a^7}{5\,x^5}-\frac{7\,a^6b}{4\,x^4}-7\,\frac{a^5b^2}{x^3}-\frac{35\,a^4b^3}{2\,x^2}-35\,\frac{a^3b^4}{x}+7\,ab^6x+\frac{b^7x^2}{2}+21\,a^2b^5\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^6,x)
```

```
[Out] -1/5*a^7/x^5-7/4*a^6*b/x^4-7*a^5*b^2/x^3-35/2*a^4*b^3/x^2-35*a^3*b^4/x+7*a*
b^6*x+1/2*b^7*x^2+21*a^2*b^5*ln(x)
```

Maxima [A] time = 1.08316, size = 104, normalized size = 1.24

$$\frac{1}{2}\,b^7x^2+7\,ab^6x+21\,a^2b^5\log(x)-\frac{700\,a^3b^4x^4+350\,a^4b^3x^3+140\,a^5b^2x^2+35\,a^6bx+4\,a^7}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="maxima")
```

```
[Out] 1/2*b^7*x^2 + 7*a*b^6*x + 21*a^2*b^5*log(x) - 1/20*(700*a^3*b^4*x^4 + 350*a
^4*b^3*x^3 + 140*a^5*b^2*x^2 + 35*a^6*b*x + 4*a^7)/x^5
```

Fricas [A] time = 1.81999, size = 188, normalized size = 2.24

$$\frac{10\,b^7x^7+140\,ab^6x^6+420\,a^2b^5x^5\log(x)-700\,a^3b^4x^4-350\,a^4b^3x^3-140\,a^5b^2x^2-35\,a^6bx-4\,a^7}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^7*x^7 + 140*a*b^6*x^6 + 420*a^2*b^5*x^5*log(x) - 700*a^3*b^4*x^4
 - 350*a^4*b^3*x^3 - 140*a^5*b^2*x^2 - 35*a^6*b*x - 4*a^7)/x^5
```

Sympy [A] time = 0.670363, size = 82, normalized size = 0.98

$$21a^2b^5\log(x)+7ab^6x+\frac{b^7x^2}{2}-\frac{4a^7+35a^6bx+140a^5b^2x^2+350a^4b^3x^3+700a^3b^4x^4}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**6,x)
```

```
[Out] 21*a**2*b**5*log(x) + 7*a*b**6*x + b**7*x**2/2 - (4*a**7 + 35*a**6*b*x + 14
0*a**5*b**2*x**2 + 350*a**4*b**3*x**3 + 700*a**3*b**4*x**4)/(20*x**5)
```

Giac [A] time = 1.16802, size = 105, normalized size = 1.25

$$\frac{1}{2}\,b^7x^2+7\,ab^6x+21\,a^2b^5\log(|x|)-\frac{700\,a^3b^4x^4+350\,a^4b^3x^3+140\,a^5b^2x^2+35\,a^6bx+4\,a^7}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="giac")
```

```
[Out] 1/2*b^7*x^2 + 7*a*b^6*x + 21*a^2*b^5*log(abs(x)) - 1/20*(700*a^3*b^4*x^4 +
350*a^4*b^3*x^3 + 140*a^5*b^2*x^2 + 35*a^6*b*x + 4*a^7)/x^5
```

3.113 $\int \frac{(a+bx)^7}{x^7}\,dx$

Optimal. Leaf size=85

$$-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}-\frac{7a^6b}{5x^5}-\frac{a^7}{6x^6}+7ab^6\log(x)+b^7x$$

```
[Out] -a^7/(6*x^6) - (7*a^6*b)/(5*x^5) - (21*a^5*b^2)/(4*x^4) - (35*a^4*b^3)/(3*x
^3) - (35*a^3*b^4)/(2*x^2) - (21*a^2*b^5)/x + b^7*x + 7*a*b^6*Log[x]
```

Rubi [A] time = 0.0313059, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}-\frac{7a^6b}{5x^5}-\frac{a^7}{6x^6}+7ab^6\log(x)+b^7x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^7,x]
```

```
[Out] -a^7/(6*x^6) - (7*a^6*b)/(5*x^5) - (21*a^5*b^2)/(4*x^4) - (35*a^4*b^3)/(3*x
^3) - (35*a^3*b^4)/(2*x^2) - (21*a^2*b^5)/x + b^7*x + 7*a*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^7}\,dx &= \int \left(b^7+\frac{a^7}{x^7}+\frac{7a^6b}{x^6}+\frac{21a^5b^2}{x^5}+\frac{35a^4b^3}{x^4}+\frac{35a^3b^4}{x^3}+\frac{21a^2b^5}{x^2}+\frac{7ab^6}{x}\right)dx\\ &= -\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+b^7x+7ab^6\log(x)\end{aligned}$$

Mathematica [A] time = 0.0094263, size = 85, normalized size = 1.

$$-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}-\frac{7a^6b}{5x^5}-\frac{a^7}{6x^6}+7ab^6\log(x)+b^7x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^7,x]
```

```
[Out] -a^7/(6*x^6) - (7*a^6*b)/(5*x^5) - (21*a^5*b^2)/(4*x^4) - (35*a^4*b^3)/(3*x
^3) - (35*a^3*b^4)/(2*x^2) - (21*a^2*b^5)/x + b^7*x + 7*a*b^6*Log[x]
```

Maple [A] time = 0.007, size = 76, normalized size = 0.9

$$-\frac{a^{7}}{6\,x^{6}}-\frac{7\,a^{6}b}{5\,x^{5}}-\frac{21\,a^{5}b^{2}}{4\,x^{4}}-\frac{35\,a^{4}b^{3}}{3\,x^{3}}-\frac{35\,a^{3}b^{4}}{2\,x^{2}}-21\,\frac{a^{2}b^{5}}{x}+b^{7}x+7\,ab^{6}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^7,x)
```

```
[Out] -1/6*a^7/x^6-7/5*a^6*b/x^5-21/4*a^5*b^2/x^4-35/3*a^4*b^3/x^3-35/2*a^3*b^4/x
^2-21*a^2*b^5/x+b^7*x+7*a*b^6*ln(x)
```

Maxima [A] time = 1.01042, size = 103, normalized size = 1.21

$$b^{7}x+7\,ab^{6}\log\left(x\right)-\frac{1260\,a^{2}b^{5}x^{5}+1050\,a^{3}b^{4}x^{4}+700\,a^{4}b^{3}x^{3}+315\,a^{5}b^{2}x^{2}+84\,a^{6}bx+10\,a^{7}}{60\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="maxima")
```

```
[Out] b^7*x + 7*a*b^6*log(x) - 1/60*(1260*a^2*b^5*x^5 + 1050*a^3*b^4*x^4 + 700*a^
4*b^3*x^3 + 315*a^5*b^2*x^2 + 84*a^6*b*x + 10*a^7)/x^6
```

Fricas [A] time = 1.70654, size = 192, normalized size = 2.26

$$\frac{60\,b^{7}x^{7}+420\,ab^{6}x^{6}\log\left(x\right)-1260\,a^{2}b^{5}x^{5}-1050\,a^{3}b^{4}x^{4}-700\,a^{4}b^{3}x^{3}-315\,a^{5}b^{2}x^{2}-84\,a^{6}bx-10\,a^{7}}{60\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*x^7 + 420*a*b^6*x^6*log(x) - 1260*a^2*b^5*x^5 - 1050*a^3*b^4*x
^4 - 700*a^4*b^3*x^3 - 315*a^5*b^2*x^2 - 84*a^6*b*x - 10*a^7)/x^6
```

Sympy [A] time = 0.734266, size = 80, normalized size = 0.94

$$7ab^{6}\log\left(x\right)+b^{7}x-\frac{10a^{7}+84a^{6}bx+315a^{5}b^{2}x^{2}+700a^{4}b^{3}x^{3}+1050a^{3}b^{4}x^{4}+1260a^{2}b^{5}x^{5}}{60x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**7,x)
```

```
[Out] 7*a*b**6*log(x) + b**7*x - (10*a**7 + 84*a**6*b*x + 315*a**5*b**2*x**2 + 70
0*a**4*b**3*x**3 + 1050*a**3*b**4*x**4 + 1260*a**2*b**5*x**5)/(60*x**6)
```

Giac [A] time = 1.16218, size = 104, normalized size = 1.22

$$b^{7}x+7\,ab^{6}\log\left(|x|\right)-\frac{1260\,a^{2}b^{5}x^{5}+1050\,a^{3}b^{4}x^{4}+700\,a^{4}b^{3}x^{3}+315\,a^{5}b^{2}x^{2}+84\,a^{6}bx+10\,a^{7}}{60\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="giac")
```

```
[Out] b^7*x + 7*a*b^6*log(abs(x)) - 1/60*(1260*a^2*b^5*x^5 + 1050*a^3*b^4*x^4 + 7
00*a^4*b^3*x^3 + 315*a^5*b^2*x^2 + 84*a^6*b*x + 10*a^7)/x^6
```

3.114 $\int \frac{(a+bx)^7}{x^8}\,dx$

Optimal. Leaf size=89

$$-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7a^6b}{6x^6}-\frac{a^7}{7x^7}-\frac{7ab^6}{x}+b^7\log(x)$$

```
[Out] -a^7/(7*x^7) - (7*a^6*b)/(6*x^6) - (21*a^5*b^2)/(5*x^5) - (35*a^4*b^3)/(4*x
^4) - (35*a^3*b^4)/(3*x^3) - (21*a^2*b^5)/(2*x^2) - (7*a*b^6)/x + b^7*Log[x
]
```

Rubi [A] time = 0.0332283, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7a^6b}{6x^6}-\frac{a^7}{7x^7}-\frac{7ab^6}{x}+b^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^8,x]
```

```
[Out] -a^7/(7*x^7) - (7*a^6*b)/(6*x^6) - (21*a^5*b^2)/(5*x^5) - (35*a^4*b^3)/(4*x
^4) - (35*a^3*b^4)/(3*x^3) - (21*a^2*b^5)/(2*x^2) - (7*a*b^6)/x + b^7*Log[x
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^8}\,dx &= \int \left(\frac{a^7}{x^8}+\frac{7a^6b}{x^7}+\frac{21a^5b^2}{x^6}+\frac{35a^4b^3}{x^5}+\frac{35a^3b^4}{x^4}+\frac{21a^2b^5}{x^3}+\frac{7ab^6}{x^2}+\frac{b^7}{x}\right)dx\\ &= -\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)\end{aligned}$$

Mathematica [A] time = 0.009578, size = 89, normalized size = 1.

$$-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7a^6b}{6x^6}-\frac{a^7}{7x^7}-\frac{7ab^6}{x}+b^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^8,x]
```

```
[Out] -a^7/(7*x^7) - (7*a^6*b)/(6*x^6) - (21*a^5*b^2)/(5*x^5) - (35*a^4*b^3)/(4*x
^4) - (35*a^3*b^4)/(3*x^3) - (21*a^2*b^5)/(2*x^2) - (7*a*b^6)/x + b^7*Log[x
```

]

Maple [A] time = 0.008, size = 78, normalized size = 0.9

$$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-7\frac{ab^6}{x}+b^7\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^8,x)
```

```
[Out] -1/7*a^7/x^7-7/6*a^6*b/x^6-21/5*a^5*b^2/x^5-35/4*a^4*b^3/x^4-35/3*a^3*b^4/x
^3-21/2*a^2*b^5/x^2-7*a*b^6/x+b^7*ln(x)
```

Maxima [A] time = 1.03378, size = 105, normalized size = 1.18

$$b^7\log(x)-\frac{2940\,ab^6x^6+4410\,a^2b^5x^5+4900\,a^3b^4x^4+3675\,a^4b^3x^3+1764\,a^5b^2x^2+490\,a^6bx+60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="maxima")
```

```
[Out] b^7*log(x) - 1/420*(2940*a*b^6*x^6 + 4410*a^2*b^5*x^5 + 4900*a^3*b^4*x^4 +
3675*a^4*b^3*x^3 + 1764*a^5*b^2*x^2 + 490*a^6*b*x + 60*a^7)/x^7
```

Fricas [A] time = 1.73739, size = 200, normalized size = 2.25

$$\frac{420\,b^7x^7\log(x)-2940\,ab^6x^6-4410\,a^2b^5x^5-4900\,a^3b^4x^4-3675\,a^4b^3x^3-1764\,a^5b^2x^2-490\,a^6bx-60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="fricas")
```

```
[Out] 1/420*(420*b^7*x^7*log(x) - 2940*a*b^6*x^6 - 4410*a^2*b^5*x^5 - 4900*a^3*b^
4*x^4 - 3675*a^4*b^3*x^3 - 1764*a^5*b^2*x^2 - 490*a^6*b*x - 60*a^7)/x^7
```

Sympy [A] time = 0.78602, size = 82, normalized size = 0.92

$$b^7\log(x)-\frac{60a^7+490a^6bx+1764a^5b^2x^2+3675a^4b^3x^3+4900a^3b^4x^4+4410a^2b^5x^5+2940ab^6x^6}{420x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**8,x)
```

```
[Out] b**7*log(x) - (60*a**7 + 490*a**6*b*x + 1764*a**5*b**2*x**2 + 3675*a**4*b**
3*x**3 + 4900*a**3*b**4*x**4 + 4410*a**2*b**5*x**5 + 2940*a*b**6*x**6)/(420
```

```
*x**7)
```

Giac [A] time = 1.18989, size = 107, normalized size = 1.2

$$b^7 \log(|x|) - \frac{2940\,ab^6x^6 + 4410\,a^2b^5x^5 + 4900\,a^3b^4x^4 + 3675\,a^4b^3x^3 + 1764\,a^5b^2x^2 + 490\,a^6bx + 60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="giac")
```

```
[Out] b^7*log(abs(x)) - 1/420*(2940*a*b^6*x^6 + 4410*a^2*b^5*x^5 + 4900*a^3*b^4*x
^4 + 3675*a^4*b^3*x^3 + 1764*a^5*b^2*x^2 + 490*a^6*b*x + 60*a^7)/x^7
```

3.115 $\int \frac{(a+bx)^7}{x^9} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^8}{8ax^8}$$

```
[Out] -(a + b*x)^8/(8*a*x^8)
```

Rubi [A] time = 0.0016742, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^8}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^9,x]
```

```
[Out] -(a + b*x)^8/(8*a*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^7}{x^9} dx = -\frac{(a+bx)^8}{8ax^8}$$

Mathematica [B] time = 0.0069037, size = 87, normalized size = 5.12

$$-\frac{7a^5b^2}{2x^6}-\frac{7a^4b^3}{x^5}-\frac{35a^3b^4}{4x^4}-\frac{7a^2b^5}{x^3}-\frac{a^6b}{x^7}-\frac{a^7}{8x^8}-\frac{7ab^6}{2x^2}-\frac{b^7}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^9,x]
```

```
[Out] -a^7/(8*x^8) - (a^6*b)/x^7 - (7*a^5*b^2)/(2*x^6) - (7*a^4*b^3)/x^5 - (35*a^
3*b^4)/(4*x^4) - (7*a^2*b^5)/x^3 - (7*a*b^6)/(2*x^2) - b^7/x
```

Maple [B] time = 0.007, size = 80, normalized size = 4.7

$$-7\,\frac{a^2b^5}{x^3}-7\,\frac{a^4b^3}{x^5}-\frac{35\,a^3b^4}{4\,x^4}-\frac{7\,a^5b^2}{2\,x^6}-\frac{a^7}{8\,x^8}-\frac{7\,ab^6}{2\,x^2}-\frac{a^6b}{x^7}-\frac{b^7}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^9,x)
```

```
[Out] -7*a^2*b^5/x^3-7*a^4*b^3/x^5-35/4*a^3*b^4/x^4-7/2*a^5*b^2/x^6-1/8*a^7/x^8-7
/2*a*b^6/x^2-a^6*b/x^7-b^7/x
```

Maxima [B] time = 1.01849, size = 104, normalized size = 6.12

$$-\frac{8\,b^7x^7+28\,ab^6x^6+56\,a^2b^5x^5+70\,a^3b^4x^4+56\,a^4b^3x^3+28\,a^5b^2x^2+8\,a^6bx+a^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^9,x, algorithm="maxima")
```

```
[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b
^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

Fricas [B] time = 1.74426, size = 166, normalized size = 9.76

$$-\frac{8\,b^7x^7+28\,ab^6x^6+56\,a^2b^5x^5+70\,a^3b^4x^4+56\,a^4b^3x^3+28\,a^5b^2x^2+8\,a^6bx+a^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^9,x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b
^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

Sympy [B] time = 0.851885, size = 83, normalized size = 4.88

$$-\frac{a^7+8a^6bx+28a^5b^2x^2+56a^4b^3x^3+70a^3b^4x^4+56a^2b^5x^5+28ab^6x^6+8b^7x^7}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**9,x)
```

```
[Out] -(a**7 + 8*a**6*b*x + 28*a**5*b**2*x**2 + 56*a**4*b**3*x**3 + 70*a**3*b**4*
x**4 + 56*a**2*b**5*x**5 + 28*a*b**6*x**6 + 8*b**7*x**7)/(8*x**8)
```

Giac [B] time = 1.21237, size = 104, normalized size = 6.12

$$-\frac{8\,b^7x^7+28\,ab^6x^6+56\,a^2b^5x^5+70\,a^3b^4x^4+56\,a^4b^3x^3+28\,a^5b^2x^2+8\,a^6bx+a^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^9,x, algorithm="giac")
```

```
[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b
^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

3.116 $\int \frac{(a+bx)^7}{x^{10}} dx$

Optimal. Leaf size=36

$$\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}$$

```
[Out] -(a + b*x)^8/(9*a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Rubi [A] time = 0.0048768, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^10,x]
```

```
[Out] -(a + b*x)^8/(9*a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{10}}\,dx &= -\frac{(a+bx)^8}{9ax^9}-\frac{b\int \frac{(a+bx)^7}{x^9}\,dx}{9a}\\ &= -\frac{(a+bx)^8}{9ax^9}+\frac{b(a+bx)^8}{72a^2x^8}\end{aligned}$$

Mathematica [B] time = 0.0039336, size = 91, normalized size = 2.53

$$-\frac{3a^5b^2}{x^7}-\frac{35a^4b^3}{6x^6}-\frac{7a^3b^4}{x^5}-\frac{21a^2b^5}{4x^4}-\frac{7a^6b}{8x^8}-\frac{a^7}{9x^9}-\frac{7ab^6}{3x^3}-\frac{b^7}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^10,x]
```

```
[Out] -a^7/(9*x^9) - (7*a^6*b)/(8*x^8) - (3*a^5*b^2)/x^7 - (35*a^4*b^3)/(6*x^6) -
 (7*a^3*b^4)/x^5 - (21*a^2*b^5)/(4*x^4) - (7*a*b^6)/(3*x^3) - b^7/(2*x^2)
```

Maple [B] time = 0.005, size = 80, normalized size = 2.2

$$-\frac{7\,ab^{6}}{3\,x^{3}}-7\,\frac{a^{3}b^{4}}{x^{5}}-\frac{21\,a^{2}b^{5}}{4\,x^{4}}-\frac{35\,a^{4}b^{3}}{6\,x^{6}}-\frac{7\,a^{6}b}{8\,x^{8}}-\frac{b^{7}}{2\,x^{2}}-3\,\frac{a^{5}b^{2}}{x^{7}}-\frac{a^{7}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^10,x)
```

```
[Out] -7/3*a*b^6/x^3-7*a^3*b^4/x^5-21/4*a^2*b^5/x^4-35/6*a^4*b^3/x^6-7/8*a^6*b/x^
8-1/2*b^7/x^2-3*a^5*b^2/x^7-1/9*a^7/x^9
```

Maxima [B] time = 1.03955, size = 107, normalized size = 2.97

$$-\frac{36\,b^{7}x^{7}+168\,ab^{6}x^{6}+378\,a^{2}b^{5}x^{5}+504\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}+216\,a^{5}b^{2}x^{2}+63\,a^{6}bx+8\,a^{7}}{72\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Fricas [B] time = 1.72973, size = 180, normalized size = 5.

$$-\frac{36\,b^{7}x^{7}+168\,ab^{6}x^{6}+378\,a^{2}b^{5}x^{5}+504\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}+216\,a^{5}b^{2}x^{2}+63\,a^{6}bx+8\,a^{7}}{72\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Sympy [B] time = 0.826221, size = 85, normalized size = 2.36

$$-\frac{8a^{7}+63a^{6}bx+216a^{5}b^{2}x^{2}+420a^{4}b^{3}x^{3}+504a^{3}b^{4}x^{4}+378a^{2}b^{5}x^{5}+168ab^{6}x^{6}+36b^{7}x^{7}}{72x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**10,x)
```

```
[Out] -(8*a**7 + 63*a**6*b*x + 216*a**5*b**2*x**2 + 420*a**4*b**3*x**3 + 504*a**3
*b**4*x**4 + 378*a**2*b**5*x**5 + 168*a*b**6*x**6 + 36*b**7*x**7)/(72*x**9)
```

Giac [B] time = 1.19416, size = 107, normalized size = 2.97

$$-\frac{36\,b^7x^7+168\,ab^6x^6+378\,a^2b^5x^5+504\,a^3b^4x^4+420\,a^4b^3x^3+216\,a^5b^2x^2+63\,a^6bx+8\,a^7}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

3.117 $\int \frac{(a+bx)^7}{x^{11}}\,dx$

Optimal. Leaf size=56

$$-\frac{b^2(a+bx)^8}{360a^3x^8}+\frac{b(a+bx)^8}{45a^2x^9}-\frac{(a+bx)^8}{10ax^{10}}$$

```
[Out] -(a + b*x)^8/(10*a*x^10) + (b*(a + b*x)^8)/(45*a^2*x^9) - (b^2*(a + b*x)^8)
/(360*a^3*x^8)
```

Rubi [A] time = 0.0098383, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^2(a+bx)^8}{360a^3x^8}+\frac{b(a+bx)^8}{45a^2x^9}-\frac{(a+bx)^8}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^11,x]
```

```
[Out] -(a + b*x)^8/(10*a*x^10) + (b*(a + b*x)^8)/(45*a^2*x^9) - (b^2*(a + b*x)^8)
/(360*a^3*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{11}}\,dx &= -\frac{(a+bx)^8}{10ax^{10}}-\frac{b\int \frac{(a+bx)^7}{x^{10}}\,dx}{5a}\\ &= -\frac{(a+bx)^8}{10ax^{10}}+\frac{b(a+bx)^8}{45a^2x^9}+\frac{b^2\int \frac{(a+bx)^7}{x^9}\,dx}{45a^2}\\ &= -\frac{(a+bx)^8}{10ax^{10}}+\frac{b(a+bx)^8}{45a^2x^9}-\frac{b^2(a+bx)^8}{360a^3x^8}\end{aligned}$$

Mathematica [A] time = 0.00455, size = 93, normalized size = 1.66

$$-\frac{21a^5b^2}{8x^8}-\frac{5a^4b^3}{x^7}-\frac{35a^3b^4}{6x^6}-\frac{21a^2b^5}{5x^5}-\frac{7a^6b}{9x^9}-\frac{a^7}{10x^{10}}-\frac{7ab^6}{4x^4}-\frac{b^7}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^11,x]
```

```
[Out] -a^7/(10*x^10) - (7*a^6*b)/(9*x^9) - (21*a^5*b^2)/(8*x^8) - (5*a^4*b^3)/x^7
 - (35*a^3*b^4)/(6*x^6) - (21*a^2*b^5)/(5*x^5) - (7*a*b^6)/(4*x^4) - b^7/(3
*x^3)
```

Maple [A] time = 0.005, size = 80, normalized size = 1.4

$$-\frac{a^7}{10\,x^{10}}-\frac{b^7}{3\,x^3}-\frac{21\,a^2b^5}{5\,x^5}-\frac{7\,ab^6}{4\,x^4}-\frac{35\,a^3b^4}{6\,x^6}-\frac{21\,a^5b^2}{8\,x^8}-5\,\frac{a^4b^3}{x^7}-\frac{7\,a^6b}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^11,x)
```

```
[Out] -1/10*a^7/x^10-1/3*b^7/x^3-21/5*a^2*b^5/x^5-7/4*a*b^6/x^4-35/6*a^3*b^4/x^6-
21/8*a^5*b^2/x^8-5*a^4*b^3/x^7-7/9*a^6*b/x^9
```

Maxima [A] time = 1.05918, size = 107, normalized size = 1.91

$$-\frac{120\,b^7x^7+630\,ab^6x^6+1512\,a^2b^5x^5+2100\,a^3b^4x^4+1800\,a^4b^3x^3+945\,a^5b^2x^2+280\,a^6bx+36\,a^7}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="maxima")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

Fricas [A] time = 1.77579, size = 190, normalized size = 3.39

$$-\frac{120\,b^7x^7+630\,ab^6x^6+1512\,a^2b^5x^5+2100\,a^3b^4x^4+1800\,a^4b^3x^3+945\,a^5b^2x^2+280\,a^6bx+36\,a^7}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="fricas")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

Sympy [A] time = 0.949686, size = 85, normalized size = 1.52

$$-\frac{36a^7 + 280a^6bx + 945a^5b^2x^2 + 1800a^4b^3x^3 + 2100a^3b^4x^4 + 1512a^2b^5x^5 + 630ab^6x^6 + 120b^7x^7}{360x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**11,x)
```

```
[Out] -(36*a**7 + 280*a**6*b*x + 945*a**5*b**2*x**2 + 1800*a**4*b**3*x**3 + 2100*
a**3*b**4*x**4 + 1512*a**2*b**5*x**5 + 630*a*b**6*x**6 + 120*b**7*x**7)/(36
0*x**10)
```

Giac [A] time = 1.20727, size = 107, normalized size = 1.91

$$-\frac{120\,b^7x^7 + 630\,ab^6x^6 + 1512\,a^2b^5x^5 + 2100\,a^3b^4x^4 + 1800\,a^4b^3x^3 + 945\,a^5b^2x^2 + 280\,a^6bx + 36\,a^7}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="giac")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

3.118 $\int \frac{(a+bx)^7}{x^{12}} dx$

Optimal. Leaf size=76

$$\frac{b^3(a+bx)^8}{1320a^4x^8}-\frac{b^2(a+bx)^8}{165a^3x^9}+\frac{3b(a+bx)^8}{110a^2x^{10}}-\frac{(a+bx)^8}{11ax^{11}}$$

```
[Out] -(a + b*x)^8/(11*a*x^11) + (3*b*(a + b*x)^8)/(110*a^2*x^10) - (b^2*(a + b*x
)^8)/(165*a^3*x^9) + (b^3*(a + b*x)^8)/(1320*a^4*x^8)
```

Rubi [A] time = 0.0163322, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b^3(a+bx)^8}{1320a^4x^8}-\frac{b^2(a+bx)^8}{165a^3x^9}+\frac{3b(a+bx)^8}{110a^2x^{10}}-\frac{(a+bx)^8}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^12,x]
```

```
[Out] -(a + b*x)^8/(11*a*x^11) + (3*b*(a + b*x)^8)/(110*a^2*x^10) - (b^2*(a + b*x
)^8)/(165*a^3*x^9) + (b^3*(a + b*x)^8)/(1320*a^4*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^7}{x^{12}} dx &= -\frac{(a+bx)^8}{11ax^{11}}-\frac{(3b)\int \frac{(a+bx)^7}{x^{11}} dx}{11a}\\
&= -\frac{(a+bx)^8}{11ax^{11}}+\frac{3b(a+bx)^8}{110a^2x^{10}}+\frac{\left(3b^2\right)\int \frac{(a+bx)^7}{x^{10}} dx}{55a^2}\\
&= -\frac{(a+bx)^8}{11ax^{11}}+\frac{3b(a+bx)^8}{110a^2x^{10}}-\frac{b^2(a+bx)^8}{165a^3x^9}-\frac{b^3\int \frac{(a+bx)^7}{x^9} dx}{165a^3}\\
&= -\frac{(a+bx)^8}{11ax^{11}}+\frac{3b(a+bx)^8}{110a^2x^{10}}-\frac{b^2(a+bx)^8}{165a^3x^9}+\frac{b^3(a+bx)^8}{1320a^4x^8}
\end{aligned}$$

Mathematica [A] time = 0.0081419, size = 93, normalized size = 1.22

$$-\frac{7a^5b^2}{3x^9}-\frac{35a^4b^3}{8x^8}-\frac{5a^3b^4}{x^7}-\frac{7a^2b^5}{2x^6}-\frac{7a^6b}{10x^{10}}-\frac{a^7}{11x^{11}}-\frac{7ab^6}{5x^5}-\frac{b^7}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^12,x]
```

```
[Out] -a^7/(11*x^11) - (7*a^6*b)/(10*x^10) - (7*a^5*b^2)/(3*x^9) - (35*a^4*b^3)/(
8*x^8) - (5*a^3*b^4)/x^7 - (7*a^2*b^5)/(2*x^6) - (7*a*b^6)/(5*x^5) - b^7/(4
*x^4)
```

Maple [A] time = 0.006, size = 80, normalized size = 1.1

$$-\frac{7\,a^6b}{10\,x^{10}}-\frac{7\,ab^6}{5\,x^5}-\frac{a^7}{11\,x^{11}}-\frac{b^7}{4\,x^4}-\frac{7\,a^2b^5}{2\,x^6}-\frac{35\,a^4b^3}{8\,x^8}-5\,\frac{a^3b^4}{x^7}-\frac{7\,a^5b^2}{3\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^12,x)
```

```
[Out] -7/10*a^6*b/x^10-7/5*a*b^6/x^5-1/11*a^7/x^11-1/4*b^7/x^4-7/2*a^2*b^5/x^6-35
/8*a^4*b^3/x^8-5*a^3*b^4/x^7-7/3*a^5*b^2/x^9
```

Maxima [A] time = 1.10873, size = 107, normalized size = 1.41

$$-\frac{330\,b^7x^7+1848\,ab^6x^6+4620\,a^2b^5x^5+6600\,a^3b^4x^4+5775\,a^4b^3x^3+3080\,a^5b^2x^2+924\,a^6bx+120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="maxima")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

Fricas [A] time = 1.78941, size = 196, normalized size = 2.58

$$-\frac{330\,b^7x^7+1848\,ab^6x^6+4620\,a^2b^5x^5+6600\,a^3b^4x^4+5775\,a^4b^3x^3+3080\,a^5b^2x^2+924\,a^6bx+120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="fricas")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

Sympy [A] time = 0.952636, size = 85, normalized size = 1.12

$$-\frac{120a^7 + 924a^6bx + 3080a^5b^2x^2 + 5775a^4b^3x^3 + 6600a^3b^4x^4 + 4620a^2b^5x^5 + 1848ab^6x^6 + 330b^7x^7}{1320x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**12,x)
```

```
[Out] -(120*a**7 + 924*a**6*b*x + 3080*a**5*b**2*x**2 + 5775*a**4*b**3*x**3 + 660
0*a**3*b**4*x**4 + 4620*a**2*b**5*x**5 + 1848*a*b**6*x**6 + 330*b**7*x**7)/
(1320*x**11)
```

Giac [A] time = 1.21208, size = 107, normalized size = 1.41

$$-\frac{330\,b^7x^7 + 1848\,ab^6x^6 + 4620\,a^2b^5x^5 + 6600\,a^3b^4x^4 + 5775\,a^4b^3x^3 + 3080\,a^5b^2x^2 + 924\,a^6bx + 120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="giac")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

3.119 $\int \frac{(a+bx)^7}{x^{13}}\,dx$

Optimal. Leaf size=96

$$-\frac{b^4(a+bx)^8}{3960a^5x^8}+\frac{b^3(a+bx)^8}{495a^4x^9}-\frac{b^2(a+bx)^8}{110a^3x^{10}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{(a+bx)^8}{12ax^{12}}$$

```
[Out] -(a + b*x)^8/(12*a*x^12) + (b*(a + b*x)^8)/(33*a^2*x^11) - (b^2*(a + b*x)^8
)/(110*a^3*x^10) + (b^3*(a + b*x)^8)/(495*a^4*x^9) - (b^4*(a + b*x)^8)/(396
0*a^5*x^8)
```

Rubi [A] time = 0.0255781, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^4(a+bx)^8}{3960a^5x^8}+\frac{b^3(a+bx)^8}{495a^4x^9}-\frac{b^2(a+bx)^8}{110a^3x^{10}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{(a+bx)^8}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^13,x]
```

```
[Out] -(a + b*x)^8/(12*a*x^12) + (b*(a + b*x)^8)/(33*a^2*x^11) - (b^2*(a + b*x)^8
)/(110*a^3*x^10) + (b^3*(a + b*x)^8)/(495*a^4*x^9) - (b^4*(a + b*x)^8)/(396
0*a^5*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{13}}\,dx &= -\frac{(a+bx)^8}{12ax^{12}} - \frac{b\int \frac{(a+bx)^7}{x^{12}}\,dx}{3a}\\ &= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} + \frac{b^2\int \frac{(a+bx)^7}{x^{11}}\,dx}{11a^2}\\ &= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} - \frac{b^2(a+bx)^8}{110a^3x^{10}} - \frac{b^3\int \frac{(a+bx)^7}{x^{10}}\,dx}{55a^3}\\ &= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} - \frac{b^2(a+bx)^8}{110a^3x^{10}} + \frac{b^3(a+bx)^8}{495a^4x^9} + \frac{b^4\int \frac{(a+bx)^7}{x^9}\,dx}{495a^4}\\ &= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} - \frac{b^2(a+bx)^8}{110a^3x^{10}} + \frac{b^3(a+bx)^8}{495a^4x^9} - \frac{b^4(a+bx)^8}{3960a^5x^8}\end{aligned}$$

Mathematica [A] time = 0.0059421, size = 93, normalized size = 0.97

$$-\frac{21a^5b^2}{10x^{10}} - \frac{35a^4b^3}{9x^9} - \frac{35a^3b^4}{8x^8} - \frac{3a^2b^5}{x^7} - \frac{7a^6b}{11x^{11}} - \frac{a^7}{12x^{12}} - \frac{7ab^6}{6x^6} - \frac{b^7}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^13,x]
```

```
[Out] -a^7/(12*x^12) - (7*a^6*b)/(11*x^11) - (21*a^5*b^2)/(10*x^10) - (35*a^4*b^3
)/(9*x^9) - (35*a^3*b^4)/(8*x^8) - (3*a^2*b^5)/x^7 - (7*a*b^6)/(6*x^6) - b^
7/(5*x^5)
```

Maple [A] time = 0.006, size = 80, normalized size = 0.8

$$-\frac{21\,a^5b^2}{10\,x^{10}} - \frac{a^7}{12\,x^{12}} - \frac{b^7}{5\,x^5} - \frac{7\,a^6b}{11\,x^{11}} - \frac{7\,ab^6}{6\,x^6} - \frac{35\,a^3b^4}{8\,x^8} - 3\,\frac{a^2b^5}{x^7} - \frac{35\,a^4b^3}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^13,x)
```

```
[Out] -21/10*a^5*b^2/x^10-1/12*a^7/x^12-1/5*b^7/x^5-7/11*a^6*b/x^11-7/6*a*b^6/x^6
-35/8*a^3*b^4/x^8-3*a^2*b^5/x^7-35/9*a^4*b^3/x^9
```

Maxima [A] time = 1.01831, size = 107, normalized size = 1.11

$$-\frac{792\,b^7x^7 + 4620\,ab^6x^6 + 11880\,a^2b^5x^5 + 17325\,a^3b^4x^4 + 15400\,a^4b^3x^3 + 8316\,a^5b^2x^2 + 2520\,a^6bx + 330\,a^7}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="maxima")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

Fricas [A] time = 1.75261, size = 201, normalized size = 2.09

$$-\frac{792\,b^7x^7+4620\,ab^6x^6+11880\,a^2b^5x^5+17325\,a^3b^4x^4+15400\,a^4b^3x^3+8316\,a^5b^2x^2+2520\,a^6bx+330\,a^7}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="fricas")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

Sympy [A] time = 0.958756, size = 85, normalized size = 0.89

$$-\frac{330a^7+2520a^6bx+8316a^5b^2x^2+15400a^4b^3x^3+17325a^3b^4x^4+11880a^2b^5x^5+4620ab^6x^6+792b^7x^7}{3960x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**13,x)
```

```
[Out] -(330*a**7 + 2520*a**6*b*x + 8316*a**5*b**2*x**2 + 15400*a**4*b**3*x**3 + 1
7325*a**3*b**4*x**4 + 11880*a**2*b**5*x**5 + 4620*a*b**6*x**6 + 792*b**7*x*
*7)/(3960*x**12)
```

Giac [A] time = 1.18725, size = 107, normalized size = 1.11

$$-\frac{792\,b^7x^7+4620\,ab^6x^6+11880\,a^2b^5x^5+17325\,a^3b^4x^4+15400\,a^4b^3x^3+8316\,a^5b^2x^2+2520\,a^6bx+330\,a^7}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="giac")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

3.120 $\int \frac{(a+bx)^7}{x^{14}}\,dx$

Optimal. Leaf size=93

$$-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{7a^6b}{12x^{12}}-\frac{a^7}{13x^{13}}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

```
[Out] -a^7/(13*x^13) - (7*a^6*b)/(12*x^12) - (21*a^5*b^2)/(11*x^11) - (7*a^4*b^3)
/(2*x^10) - (35*a^3*b^4)/(9*x^9) - (21*a^2*b^5)/(8*x^8) - (a*b^6)/x^7 - b^7
/(6*x^6)
```

Rubi [A] time = 0.0328734, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{7a^6b}{12x^{12}}-\frac{a^7}{13x^{13}}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^14,x]
```

```
[Out] -a^7/(13*x^13) - (7*a^6*b)/(12*x^12) - (21*a^5*b^2)/(11*x^11) - (7*a^4*b^3)
/(2*x^10) - (35*a^3*b^4)/(9*x^9) - (21*a^2*b^5)/(8*x^8) - (a*b^6)/x^7 - b^7
/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{14}}\,dx &= \int \left(\frac{a^7}{x^{14}}+\frac{7a^6b}{x^{13}}+\frac{21a^5b^2}{x^{12}}+\frac{35a^4b^3}{x^{11}}+\frac{35a^3b^4}{x^{10}}+\frac{21a^2b^5}{x^9}+\frac{7ab^6}{x^8}+\frac{b^7}{x^7}\right)dx\\ &= -\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}\end{aligned}$$

Mathematica [A] time = 0.0072827, size = 93, normalized size = 1.

$$-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{7a^6b}{12x^{12}}-\frac{a^7}{13x^{13}}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^14,x]
```

```
[Out] -a^7/(13*x^13) - (7*a^6*b)/(12*x^12) - (21*a^5*b^2)/(11*x^11) - (7*a^4*b^3)
/(2*x^10) - (35*a^3*b^4)/(9*x^9) - (21*a^2*b^5)/(8*x^8) - (a*b^6)/x^7 - b^7
```

```
/(6*x^6)
```

Maple [A] time = 0.006, size = 80, normalized size = 0.9

$$-\frac{a^{7}}{13\,x^{13}}-\frac{7\,a^{6}b}{12\,x^{12}}-\frac{21\,a^{5}b^{2}}{11\,x^{11}}-\frac{7\,a^{4}b^{3}}{2\,x^{10}}-\frac{35\,a^{3}b^{4}}{9\,x^{9}}-\frac{21\,a^{2}b^{5}}{8\,x^{8}}-\frac{ab^{6}}{x^{7}}-\frac{b^{7}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^14,x)
```

```
[Out] -1/13*a^7/x^13-7/12*a^6*b/x^12-21/11*a^5*b^2/x^11-7/2*a^4*b^3/x^10-35/9*a^3
*b^4/x^9-21/8*a^2*b^5/x^8-a*b^6/x^7-1/6*b^7/x^6
```

Maxima [A] time = 1.01975, size = 107, normalized size = 1.15

$$-\frac{1716\,b^{7}x^{7}+10296\,ab^{6}x^{6}+27027\,a^{2}b^{5}x^{5}+40040\,a^{3}b^{4}x^{4}+36036\,a^{4}b^{3}x^{3}+19656\,a^{5}b^{2}x^{2}+6006\,a^{6}bx+792\,a^{7}}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

Fricas [A] time = 1.81349, size = 207, normalized size = 2.23

$$-\frac{1716\,b^{7}x^{7}+10296\,ab^{6}x^{6}+27027\,a^{2}b^{5}x^{5}+40040\,a^{3}b^{4}x^{4}+36036\,a^{4}b^{3}x^{3}+19656\,a^{5}b^{2}x^{2}+6006\,a^{6}bx+792\,a^{7}}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

Sympy [A] time = 1.06813, size = 85, normalized size = 0.91

$$-\frac{792a^{7}+6006a^{6}bx+19656a^{5}b^{2}x^{2}+36036a^{4}b^{3}x^{3}+40040a^{3}b^{4}x^{4}+27027a^{2}b^{5}x^{5}+10296ab^{6}x^{6}+1716b^{7}x^{7}}{10296x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**14,x)
```

```
[Out] -(792*a**7 + 6006*a**6*b*x + 19656*a**5*b**2*x**2 + 36036*a**4*b**3*x**3 +
40040*a**3*b**4*x**4 + 27027*a**2*b**5*x**5 + 10296*a*b**6*x**6 + 1716*b**7
```

```
*x**7)/(10296*x**13)
```

Giac [A] time = 1.18739, size = 107, normalized size = 1.15

$$-\frac{1716\,b^7x^7+10296\,ab^6x^6+27027\,a^2b^5x^5+40040\,a^3b^4x^4+36036\,a^4b^3x^3+19656\,a^5b^2x^2+6006\,a^6bx+792\,a^7}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^14,x, algorithm="giac")
```

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

3.121 $\int \frac{(a+bx)^7}{x^{15}} dx$

Optimal. Leaf size=95

$$-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7a^6b}{13x^{13}}-\frac{a^7}{14x^{14}}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

```
[Out] -a^7/(14*x^14) - (7*a^6*b)/(13*x^13) - (7*a^5*b^2)/(4*x^12) - (35*a^4*b^3)/
(11*x^11) - (7*a^3*b^4)/(2*x^10) - (7*a^2*b^5)/(3*x^9) - (7*a*b^6)/(8*x^8)
- b^7/(7*x^7)
```

Rubi [A] time = 0.0302973, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7a^6b}{13x^{13}}-\frac{a^7}{14x^{14}}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^15,x]
```

```
[Out] -a^7/(14*x^14) - (7*a^6*b)/(13*x^13) - (7*a^5*b^2)/(4*x^12) - (35*a^4*b^3)/
(11*x^11) - (7*a^3*b^4)/(2*x^10) - (7*a^2*b^5)/(3*x^9) - (7*a*b^6)/(8*x^8)
- b^7/(7*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{15}}\,dx &= \int \left(\frac{a^7}{x^{15}}+\frac{7a^6b}{x^{14}}+\frac{21a^5b^2}{x^{13}}+\frac{35a^4b^3}{x^{12}}+\frac{35a^3b^4}{x^{11}}+\frac{21a^2b^5}{x^{10}}+\frac{7ab^6}{x^9}+\frac{b^7}{x^8}\right)dx\\ &= -\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}\end{aligned}$$

Mathematica [A] time = 0.0053063, size = 95, normalized size = 1.

$$-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7a^6b}{13x^{13}}-\frac{a^7}{14x^{14}}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^15,x]
```

```
[Out] -a^7/(14*x^14) - (7*a^6*b)/(13*x^13) - (7*a^5*b^2)/(4*x^12) - (35*a^4*b^3)/
(11*x^11) - (7*a^3*b^4)/(2*x^10) - (7*a^2*b^5)/(3*x^9) - (7*a*b^6)/(8*x^8)
```

```
- b^7/(7*x^7)
```

Maple [A] time = 0.008, size = 80, normalized size = 0.8

$$-\frac{a^{7}}{14\,x^{14}}-\frac{7\,a^{6}b}{13\,x^{13}}-\frac{7\,a^{5}b^{2}}{4\,x^{12}}-\frac{35\,a^{4}b^{3}}{11\,x^{11}}-\frac{7\,a^{3}b^{4}}{2\,x^{10}}-\frac{7\,a^{2}b^{5}}{3\,x^{9}}-\frac{7\,ab^{6}}{8\,x^{8}}-\frac{b^{7}}{7\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^15,x)
```

```
[Out] -1/14*a^7/x^14-7/13*a^6*b/x^13-7/4*a^5*b^2/x^12-35/11*a^4*b^3/x^11-7/2*a^3*
b^4/x^10-7/3*a^2*b^5/x^9-7/8*a*b^6/x^8-1/7*b^7/x^7
```

Maxima [A] time = 1.06713, size = 107, normalized size = 1.13

$$-\frac{3432\,b^{7}x^{7}+21021\,ab^{6}x^{6}+56056\,a^{2}b^{5}x^{5}+84084\,a^{3}b^{4}x^{4}+76440\,a^{4}b^{3}x^{3}+42042\,a^{5}b^{2}x^{2}+12936\,a^{6}bx+1716\,a^{7}}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="maxima")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

Fricas [A] time = 1.65273, size = 209, normalized size = 2.2

$$-\frac{3432\,b^{7}x^{7}+21021\,ab^{6}x^{6}+56056\,a^{2}b^{5}x^{5}+84084\,a^{3}b^{4}x^{4}+76440\,a^{4}b^{3}x^{3}+42042\,a^{5}b^{2}x^{2}+12936\,a^{6}bx+1716\,a^{7}}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="fricas")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

Sympy [A] time = 1.11824, size = 85, normalized size = 0.89

$$-\frac{1716a^{7}+12936a^{6}bx+42042a^{5}b^{2}x^{2}+76440a^{4}b^{3}x^{3}+84084a^{3}b^{4}x^{4}+56056a^{2}b^{5}x^{5}+21021ab^{6}x^{6}+3432b^{7}x^{7}}{24024x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**15,x)
```

```
[Out] -(1716*a**7 + 12936*a**6*b*x + 42042*a**5*b**2*x**2 + 76440*a**4*b**3*x**3
+ 84084*a**3*b**4*x**4 + 56056*a**2*b**5*x**5 + 21021*a*b**6*x**6 + 3432*b*
```

```
*7*x**7)/(24024*x**14)
```

Giac [A] time = 1.22169, size = 107, normalized size = 1.13

$$-\frac{3432\,b^7x^7 + 21021\,ab^6x^6 + 56056\,a^2b^5x^5 + 84084\,a^3b^4x^4 + 76440\,a^4b^3x^3 + 42042\,a^5b^2x^2 + 12936\,a^6bx + 1716\,a^7}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="giac")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

3.122 $\int \frac{(a+bx)^7}{x^{16}}\,dx$

Optimal. Leaf size=95

$$-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{a^6b}{2x^{14}}-\frac{a^7}{15x^{15}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

```
[Out] -a^7/(15*x^15) - (a^6*b)/(2*x^14) - (21*a^5*b^2)/(13*x^13) - (35*a^4*b^3)/(
12*x^12) - (35*a^3*b^4)/(11*x^11) - (21*a^2*b^5)/(10*x^10) - (7*a*b^6)/(9*x
^9) - b^7/(8*x^8)
```

Rubi [A] time = 0.0304734, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{a^6b}{2x^{14}}-\frac{a^7}{15x^{15}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^16,x]
```

```
[Out] -a^7/(15*x^15) - (a^6*b)/(2*x^14) - (21*a^5*b^2)/(13*x^13) - (35*a^4*b^3)/(
12*x^12) - (35*a^3*b^4)/(11*x^11) - (21*a^2*b^5)/(10*x^10) - (7*a*b^6)/(9*x
^9) - b^7/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{16}}\,dx &= \int \left(\frac{a^7}{x^{16}}+\frac{7a^6b}{x^{15}}+\frac{21a^5b^2}{x^{14}}+\frac{35a^4b^3}{x^{13}}+\frac{35a^3b^4}{x^{12}}+\frac{21a^2b^5}{x^{11}}+\frac{7ab^6}{x^{10}}+\frac{b^7}{x^9}\right)dx\\ &= -\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}\end{aligned}$$

Mathematica [A] time = 0.0036613, size = 95, normalized size = 1.

$$-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{a^6b}{2x^{14}}-\frac{a^7}{15x^{15}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^16,x]
```

```
[Out] -a^7/(15*x^15) - (a^6*b)/(2*x^14) - (21*a^5*b^2)/(13*x^13) - (35*a^4*b^3)/(
12*x^12) - (35*a^3*b^4)/(11*x^11) - (21*a^2*b^5)/(10*x^10) - (7*a*b^6)/(9*x
```

```
^9) - b^7/(8*x^8)
```

Maple [A] time = 0.006, size = 80, normalized size = 0.8

$$-\frac{a^7}{15\,x^{15}}-\frac{a^6b}{2\,x^{14}}-\frac{21\,a^5b^2}{13\,x^{13}}-\frac{35\,a^4b^3}{12\,x^{12}}-\frac{35\,a^3b^4}{11\,x^{11}}-\frac{21\,a^2b^5}{10\,x^{10}}-\frac{7\,ab^6}{9\,x^9}-\frac{b^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^16,x)
```

```
[Out] -1/15*a^7/x^15-1/2*a^6*b/x^14-21/13*a^5*b^2/x^13-35/12*a^4*b^3/x^12-35/11*a
^3*b^4/x^11-21/10*a^2*b^5/x^10-7/9*a*b^6/x^9-1/8*b^7/x^8
```

Maxima [A] time = 1.03825, size = 107, normalized size = 1.13

$$-\frac{6435\,b^7x^7+40040\,ab^6x^6+108108\,a^2b^5x^5+163800\,a^3b^4x^4+150150\,a^4b^3x^3+83160\,a^5b^2x^2+25740\,a^6bx+3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="maxima")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

Fricas [A] time = 1.62739, size = 213, normalized size = 2.24

$$-\frac{6435\,b^7x^7+40040\,ab^6x^6+108108\,a^2b^5x^5+163800\,a^3b^4x^4+150150\,a^4b^3x^3+83160\,a^5b^2x^2+25740\,a^6bx+3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="fricas")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

Sympy [A] time = 1.09255, size = 85, normalized size = 0.89

$$-\frac{3432a^7+25740a^6bx+83160a^5b^2x^2+150150a^4b^3x^3+163800a^3b^4x^4+108108a^2b^5x^5+40040ab^6x^6+6435b^7x^7}{51480x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**16,x)
```

```
[Out] -(3432*a**7 + 25740*a**6*b*x + 83160*a**5*b**2*x**2 + 150150*a**4*b**3*x**3
 + 163800*a**3*b**4*x**4 + 108108*a**2*b**5*x**5 + 40040*a*b**6*x**6 + 6435
```

```
*b**7*x**7)/(51480*x**15)
```

Giac [A] time = 1.20802, size = 107, normalized size = 1.13

$$-\frac{6435\,b^7x^7+40040\,ab^6x^6+108108\,a^2b^5x^5+163800\,a^3b^4x^4+150150\,a^4b^3x^3+83160\,a^5b^2x^2+25740\,a^6bx+3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="giac")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

3.123 $\int x^{11}(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$\frac{9}{4}a^2b^8x^{20}+\frac{120}{19}a^3b^7x^{19}+\frac{35}{3}a^4b^6x^{18}+\frac{252}{17}a^5b^5x^{17}+\frac{105}{8}a^6b^4x^{16}+8a^7b^3x^{15}+\frac{45}{14}a^8b^2x^{14}+\frac{10}{13}a^9bx^{13}+\frac{a^{10}x^{12}}{12}+\frac{10}{21}ab$$

```
[Out] (a^10*x^12)/12 + (10*a^9*b*x^13)/13 + (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15
 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (12
0*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)/4 + (10*a*b^9*x^21)/21 + (b^10*x^22)/
22
```

Rubi [A] time = 0.0759259, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{9}{4}a^2b^8x^{20}+\frac{120}{19}a^3b^7x^{19}+\frac{35}{3}a^4b^6x^{18}+\frac{252}{17}a^5b^5x^{17}+\frac{105}{8}a^6b^4x^{16}+8a^7b^3x^{15}+\frac{45}{14}a^8b^2x^{14}+\frac{10}{13}a^9bx^{13}+\frac{a^{10}x^{12}}{12}+\frac{10}{21}ab$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x)^10,x]
```

```
[Out] (a^10*x^12)/12 + (10*a^9*b*x^13)/13 + (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15
 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (12
0*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)/4 + (10*a*b^9*x^21)/21 + (b^10*x^22)/
22
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^{11}(a+bx)^{10}\,dx=\int\left(a^{10}x^{11}+10a^9bx^{12}+45a^8b^2x^{13}+120a^7b^3x^{14}+210a^6b^4x^{15}+252a^5b^5x^{16}+210a^4b^6x^{17}+120a^3\right.$$
$$=\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x$$

Mathematica [A] time = 0.0064768, size = 132, normalized size = 1.

$$\frac{9}{4}a^2b^8x^{20}+\frac{120}{19}a^3b^7x^{19}+\frac{35}{3}a^4b^6x^{18}+\frac{252}{17}a^5b^5x^{17}+\frac{105}{8}a^6b^4x^{16}+8a^7b^3x^{15}+\frac{45}{14}a^8b^2x^{14}+\frac{10}{13}a^9bx^{13}+\frac{a^{10}x^{12}}{12}+\frac{10}{21}ab$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x)^10,x]
```

```
[Out] (a^10*x^12)/12 + (10*a^9*b*x^13)/13 + (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15
 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (12
```

```
0*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)/4 + (10*a*b^9*x^21)/21 + (b^10*x^22)/
22
```

Maple [A] time = 0.001, size = 113, normalized size = 0.9

$$\frac{a^{10}x^{12}}{12}+\frac{10\,a^{9}bx^{13}}{13}+\frac{45\,a^{8}b^{2}x^{14}}{14}+8\,a^{7}b^{3}x^{15}+\frac{105\,a^{6}b^{4}x^{16}}{8}+\frac{252\,a^{5}b^{5}x^{17}}{17}+\frac{35\,a^{4}b^{6}x^{18}}{3}+\frac{120\,a^{3}b^{7}x^{19}}{19}+\frac{9\,a^{2}b^{8}x^{20}}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x+a)^10,x)
```

```
[Out] 1/12*a^10*x^12+10/13*a^9*b*x^13+45/14*a^8*b^2*x^14+8*a^7*b^3*x^15+105/8*a^6
*b^4*x^16+252/17*a^5*b^5*x^17+35/3*a^4*b^6*x^18+120/19*a^3*b^7*x^19+9/4*a^2
*b^8*x^20+10/21*a*b^9*x^21+1/22*b^10*x^22
```

Maxima [A] time = 1.06356, size = 151, normalized size = 1.14

$$\frac{1}{22}\,b^{10}x^{22}+\frac{10}{21}\,ab^{9}x^{21}+\frac{9}{4}\,a^{2}b^{8}x^{20}+\frac{120}{19}\,a^{3}b^{7}x^{19}+\frac{35}{3}\,a^{4}b^{6}x^{18}+\frac{252}{17}\,a^{5}b^{5}x^{17}+\frac{105}{8}\,a^{6}b^{4}x^{16}+8\,a^{7}b^{3}x^{15}+\frac{45}{14}\,a^{8}b^{2}x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
+ 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^3*
x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

Fricas [A] time = 1.50884, size = 288, normalized size = 2.18

$$\frac{1}{22}x^{22}b^{10}+\frac{10}{21}x^{21}b^{9}a+\frac{9}{4}x^{20}b^{8}a^{2}+\frac{120}{19}x^{19}b^{7}a^{3}+\frac{35}{3}x^{18}b^{6}a^{4}+\frac{252}{17}x^{17}b^{5}a^{5}+\frac{105}{8}x^{16}b^{4}a^{6}+8x^{15}b^{3}a^{7}+\frac{45}{14}x^{14}b^{2}a^{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^10 + 10/21*x^21*b^9*a + 9/4*x^20*b^8*a^2 + 120/19*x^19*b^7*a^3
+ 35/3*x^18*b^6*a^4 + 252/17*x^17*b^5*a^5 + 105/8*x^16*b^4*a^6 + 8*x^15*b^3
*a^7 + 45/14*x^14*b^2*a^8 + 10/13*x^13*b*a^9 + 1/12*x^12*a^10
```

Sympy [A] time = 0.099574, size = 133, normalized size = 1.01

$$\frac{a^{10}x^{12}}{12}+\frac{10a^{9}bx^{13}}{13}+\frac{45a^{8}b^{2}x^{14}}{14}+8a^{7}b^{3}x^{15}+\frac{105a^{6}b^{4}x^{16}}{8}+\frac{252a^{5}b^{5}x^{17}}{17}+\frac{35a^{4}b^{6}x^{18}}{3}+\frac{120a^{3}b^{7}x^{19}}{19}+\frac{9a^{2}b^{8}x^{20}}{4}+1($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x+a)**10,x)
```

```
[Out] a**10*x**12/12 + 10*a**9*b*x**13/13 + 45*a**8*b**2*x**14/14 + 8*a**7*b**3*x
**15 + 105*a**6*b**4*x**16/8 + 252*a**5*b**5*x**17/17 + 35*a**4*b**6*x**18/
3 + 120*a**3*b**7*x**19/19 + 9*a**2*b**8*x**20/4 + 10*a*b**9*x**21/21 + b**
10*x**22/22
```

Giac [A] time = 1.14462, size = 151, normalized size = 1.14

$$\frac{1}{22}\,b^{10}x^{22}+\frac{10}{21}\,ab^{9}x^{21}+\frac{9}{4}\,a^{2}b^{8}x^{20}+\frac{120}{19}\,a^{3}b^{7}x^{19}+\frac{35}{3}\,a^{4}b^{6}x^{18}+\frac{252}{17}\,a^{5}b^{5}x^{17}+\frac{105}{8}\,a^{6}b^{4}x^{16}+8\,a^{7}b^{3}x^{15}+\frac{45}{14}\,a^{8}b^{2}x^{14}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
+ 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^3*
x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

3.124 $\int x^{10}(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$\frac{45}{19}a^2b^8x^{19}+\frac{20}{3}a^3b^7x^{18}+\frac{210}{17}a^4b^6x^{17}+\frac{63}{4}a^5b^5x^{16}+14a^6b^4x^{15}+\frac{60}{7}a^7b^3x^{14}+\frac{45}{13}a^8b^2x^{13}+\frac{5}{6}a^9bx^{12}+\frac{a^{10}x^{11}}{11}+\frac{1}{2}a$$

```
[Out] (a^10*x^11)/11 + (5*a^9*b*x^12)/6 + (45*a^8*b^2*x^13)/13 + (60*a^7*b^3*x^14
)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)/4 + (210*a^4*b^6*x^17)/17 + (20*a
^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19 + (a*b^9*x^20)/2 + (b^10*x^21)/21
```

Rubi [A] time = 0.0592057, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{45}{19}a^2b^8x^{19}+\frac{20}{3}a^3b^7x^{18}+\frac{210}{17}a^4b^6x^{17}+\frac{63}{4}a^5b^5x^{16}+14a^6b^4x^{15}+\frac{60}{7}a^7b^3x^{14}+\frac{45}{13}a^8b^2x^{13}+\frac{5}{6}a^9bx^{12}+\frac{a^{10}x^{11}}{11}+\frac{1}{2}a$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a + b*x)^10,x]
```

```
[Out] (a^10*x^11)/11 + (5*a^9*b*x^12)/6 + (45*a^8*b^2*x^13)/13 + (60*a^7*b^3*x^14
)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)/4 + (210*a^4*b^6*x^17)/17 + (20*a
^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19 + (a*b^9*x^20)/2 + (b^10*x^21)/21
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^{10}(a+bx)^{10}\,dx=\int\left(a^{10}x^{10}+10a^9bx^{11}+45a^8b^2x^{12}+120a^7b^3x^{13}+210a^6b^4x^{14}+252a^5b^5x^{15}+210a^4b^6x^{16}+12\right.$$
$$=\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7$$

Mathematica [A] time = 0.0045776, size = 132, normalized size = 1.

$$\frac{45}{19}a^2b^8x^{19}+\frac{20}{3}a^3b^7x^{18}+\frac{210}{17}a^4b^6x^{17}+\frac{63}{4}a^5b^5x^{16}+14a^6b^4x^{15}+\frac{60}{7}a^7b^3x^{14}+\frac{45}{13}a^8b^2x^{13}+\frac{5}{6}a^9bx^{12}+\frac{a^{10}x^{11}}{11}+\frac{1}{2}a$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a + b*x)^10,x]
```

```
[Out] (a^10*x^11)/11 + (5*a^9*b*x^12)/6 + (45*a^8*b^2*x^13)/13 + (60*a^7*b^3*x^14
)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)/4 + (210*a^4*b^6*x^17)/17 + (20*a
^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19 + (a*b^9*x^20)/2 + (b^10*x^21)/21
```

Maple [A] time = 0., size = 113, normalized size = 0.9

$$\frac{a^{10}x^{11}}{11}+\frac{5\,a^{9}b\,x^{12}}{6}+\frac{45\,a^{8}b^{2}x^{13}}{13}+\frac{60\,a^{7}b^{3}x^{14}}{7}+14\,a^{6}b^{4}x^{15}+\frac{63\,a^{5}b^{5}x^{16}}{4}+\frac{210\,a^{4}b^{6}x^{17}}{17}+\frac{20\,a^{3}b^{7}x^{18}}{3}+\frac{45\,a^{2}b^{8}x^{19}}{19}+\frac{ab^{9}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b*x+a)^10,x)
```

```
[Out] 1/11*a^10*x^11+5/6*a^9*b*x^12+45/13*a^8*b^2*x^13+60/7*a^7*b^3*x^14+14*a^6*b
^4*x^15+63/4*a^5*b^5*x^16+210/17*a^4*b^6*x^17+20/3*a^3*b^7*x^18+45/19*a^2*b
^8*x^19+1/2*a*b^9*x^20+1/21*b^10*x^21
```

Maxima [A] time = 1.05995, size = 151, normalized size = 1.14

$$\frac{1}{21}\,b^{10}x^{21}+\frac{1}{2}\,ab^{9}x^{20}+\frac{45}{19}\,a^{2}b^{8}x^{19}+\frac{20}{3}\,a^{3}b^{7}x^{18}+\frac{210}{17}\,a^{4}b^{6}x^{17}+\frac{63}{4}\,a^{5}b^{5}x^{16}+14\,a^{6}b^{4}x^{15}+\frac{60}{7}\,a^{7}b^{3}x^{14}+\frac{45}{13}\,a^{8}b^{2}x^{13}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 +
210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*x^
14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11
```

Fricas [A] time = 1.50767, size = 282, normalized size = 2.14

$$\frac{1}{21}x^{21}b^{10}+\frac{1}{2}x^{20}b^{9}a+\frac{45}{19}x^{19}b^{8}a^{2}+\frac{20}{3}x^{18}b^{7}a^{3}+\frac{210}{17}x^{17}b^{6}a^{4}+\frac{63}{4}x^{16}b^{5}a^{5}+14x^{15}b^{4}a^{6}+\frac{60}{7}x^{14}b^{3}a^{7}+\frac{45}{13}x^{13}b^{2}a^{8}+\frac{5}{6}x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^10 + 1/2*x^20*b^9*a + 45/19*x^19*b^8*a^2 + 20/3*x^18*b^7*a^3 +
210/17*x^17*b^6*a^4 + 63/4*x^16*b^5*a^5 + 14*x^15*b^4*a^6 + 60/7*x^14*b^3*a
^7 + 45/13*x^13*b^2*a^8 + 5/6*x^12*b*a^9 + 1/11*x^11*a^10
```

Sympy [A] time = 0.12072, size = 131, normalized size = 0.99

$$\frac{a^{10}x^{11}}{11}+\frac{5a^{9}bx^{12}}{6}+\frac{45a^{8}b^{2}x^{13}}{13}+\frac{60a^{7}b^{3}x^{14}}{7}+14a^{6}b^{4}x^{15}+\frac{63a^{5}b^{5}x^{16}}{4}+\frac{210a^{4}b^{6}x^{17}}{17}+\frac{20a^{3}b^{7}x^{18}}{3}+\frac{45a^{2}b^{8}x^{19}}{19}+\frac{ab^{9}x^{20}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b*x+a)**10,x)
```

```
[Out] a**10*x**11/11 + 5*a**9*b*x**12/6 + 45*a**8*b**2*x**13/13 + 60*a**7*b**3*x*
*14/7 + 14*a**6*b**4*x**15 + 63*a**5*b**5*x**16/4 + 210*a**4*b**6*x**17/17
+ 20*a**3*b**7*x**18/3 + 45*a**2*b**8*x**19/19 + a*b**9*x**20/2 + b**10*x**
```

21/21

Giac [A] time = 1.17783, size = 151, normalized size = 1.14

$$\frac{1}{21}\,b^{10}x^{21}+\frac{1}{2}\,ab^{9}x^{20}+\frac{45}{19}\,a^{2}b^{8}x^{19}+\frac{20}{3}\,a^{3}b^{7}x^{18}+\frac{210}{17}\,a^{4}b^{6}x^{17}+\frac{63}{4}\,a^{5}b^{5}x^{16}+14\,a^{6}b^{4}x^{15}+\frac{60}{7}\,a^{7}b^{3}x^{14}+\frac{45}{13}\,a^{8}b^{2}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 +
210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*x^
14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11
```

3.125 $\int x^9(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$\frac{5}{2}a^2b^8x^{18}+\frac{120}{17}a^3b^7x^{17}+\frac{105}{8}a^4b^6x^{16}+\frac{84}{5}a^5b^5x^{15}+15a^6b^4x^{14}+\frac{120}{13}a^7b^3x^{13}+\frac{15}{4}a^8b^2x^{12}+\frac{10}{11}a^9bx^{11}+\frac{a^{10}x^{10}}{10}+\frac{10}{19}ab$$

```
[Out] (a^10*x^10)/10 + (10*a^9*b*x^11)/11 + (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^
13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (12
0*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)/2 + (10*a*b^9*x^19)/19 + (b^10*x^20)/
20
```

Rubi [A] time = 0.0595646, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{5}{2}a^2b^8x^{18}+\frac{120}{17}a^3b^7x^{17}+\frac{105}{8}a^4b^6x^{16}+\frac{84}{5}a^5b^5x^{15}+15a^6b^4x^{14}+\frac{120}{13}a^7b^3x^{13}+\frac{15}{4}a^8b^2x^{12}+\frac{10}{11}a^9bx^{11}+\frac{a^{10}x^{10}}{10}+\frac{10}{19}ab$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x)^10,x]
```

```
[Out] (a^10*x^10)/10 + (10*a^9*b*x^11)/11 + (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^
13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (12
0*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)/2 + (10*a*b^9*x^19)/19 + (b^10*x^20)/
20
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^9(a+bx)^{10}\,dx &= \int \left(a^{10}x^9+10a^9bx^{10}+45a^8b^2x^{11}+120a^7b^3x^{12}+210a^6b^4x^{13}+252a^5b^5x^{14}+210a^4b^6x^{15}+120a^3b^7\right.\\ &= \frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x\end{aligned}$$

Mathematica [A] time = 0.003226, size = 132, normalized size = 1.

$$\frac{5}{2}a^2b^8x^{18}+\frac{120}{17}a^3b^7x^{17}+\frac{105}{8}a^4b^6x^{16}+\frac{84}{5}a^5b^5x^{15}+15a^6b^4x^{14}+\frac{120}{13}a^7b^3x^{13}+\frac{15}{4}a^8b^2x^{12}+\frac{10}{11}a^9bx^{11}+\frac{a^{10}x^{10}}{10}+\frac{10}{19}ab$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x)^10,x]
```

```
[Out] (a^10*x^10)/10 + (10*a^9*b*x^11)/11 + (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^
13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (12
```

```
0*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)/2 + (10*a*b^9*x^19)/19 + (b^10*x^20)/
20
```

Maple [A] time = 0.002, size = 113, normalized size = 0.9

$$\frac{a^{10}x^{10}}{10}+\frac{10\,a^{9}bx^{11}}{11}+\frac{15\,a^{8}b^{2}x^{12}}{4}+\frac{120\,a^{7}b^{3}x^{13}}{13}+15\,a^{6}b^{4}x^{14}+\frac{84\,a^{5}b^{5}x^{15}}{5}+\frac{105\,a^{4}b^{6}x^{16}}{8}+\frac{120\,a^{3}b^{7}x^{17}}{17}+\frac{5\,a^{2}b^{8}x^{18}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x+a)^10,x)
```

```
[Out] 1/10*a^10*x^10+10/11*a^9*b*x^11+15/4*a^8*b^2*x^12+120/13*a^7*b^3*x^13+15*a^
6*b^4*x^14+84/5*a^5*b^5*x^15+105/8*a^4*b^6*x^16+120/17*a^3*b^7*x^17+5/2*a^2
*b^8*x^18+10/19*a*b^9*x^19+1/20*b^10*x^20
```

Maxima [A] time = 1.06536, size = 151, normalized size = 1.14

$$\frac{1}{20}\,b^{10}x^{20}+\frac{10}{19}\,ab^{9}x^{19}+\frac{5}{2}\,a^{2}b^{8}x^{18}+\frac{120}{17}\,a^{3}b^{7}x^{17}+\frac{105}{8}\,a^{4}b^{6}x^{16}+\frac{84}{5}\,a^{5}b^{5}x^{15}+15\,a^{6}b^{4}x^{14}+\frac{120}{13}\,a^{7}b^{3}x^{13}+\frac{15}{4}\,a^{8}b^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
+ 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b^3
*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

Fricas [A] time = 1.57353, size = 288, normalized size = 2.18

$$\frac{1}{20}x^{20}b^{10}+\frac{10}{19}x^{19}b^{9}a+\frac{5}{2}x^{18}b^{8}a^{2}+\frac{120}{17}x^{17}b^{7}a^{3}+\frac{105}{8}x^{16}b^{6}a^{4}+\frac{84}{5}x^{15}b^{5}a^{5}+15x^{14}b^{4}a^{6}+\frac{120}{13}x^{13}b^{3}a^{7}+\frac{15}{4}x^{12}b^{2}a^{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^10 + 10/19*x^19*b^9*a + 5/2*x^18*b^8*a^2 + 120/17*x^17*b^7*a^3
+ 105/8*x^16*b^6*a^4 + 84/5*x^15*b^5*a^5 + 15*x^14*b^4*a^6 + 120/13*x^13*b^
3*a^7 + 15/4*x^12*b^2*a^8 + 10/11*x^11*b*a^9 + 1/10*x^10*a^10
```

Sympy [A] time = 0.108687, size = 133, normalized size = 1.01

$$\frac{a^{10}x^{10}}{10}+\frac{10a^{9}bx^{11}}{11}+\frac{15a^{8}b^{2}x^{12}}{4}+\frac{120a^{7}b^{3}x^{13}}{13}+15a^{6}b^{4}x^{14}+\frac{84a^{5}b^{5}x^{15}}{5}+\frac{105a^{4}b^{6}x^{16}}{8}+\frac{120a^{3}b^{7}x^{17}}{17}+\frac{5a^{2}b^{8}x^{18}}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x+a)**10,x)
```

```
[Out] a**10*x**10/10 + 10*a**9*b*x**11/11 + 15*a**8*b**2*x**12/4 + 120*a**7*b**3*
x**13/13 + 15*a**6*b**4*x**14 + 84*a**5*b**5*x**15/5 + 105*a**4*b**6*x**16/
8 + 120*a**3*b**7*x**17/17 + 5*a**2*b**8*x**18/2 + 10*a*b**9*x**19/19 + b**
10*x**20/20
```

Giac [A] time = 1.15854, size = 151, normalized size = 1.14

$$\frac{1}{20}b^{10}x^{20} + \frac{10}{19}ab^9x^{19} + \frac{5}{2}a^2b^8x^{18} + \frac{120}{17}a^3b^7x^{17} + \frac{105}{8}a^4b^6x^{16} + \frac{84}{5}a^5b^5x^{15} + 15\,a^6b^4x^{14} + \frac{120}{13}a^7b^3x^{13} + \frac{15}{4}a^8b^2x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
+ 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b^3
*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

3.126 $\int x^8(a+bx)^{10}\,dx$

Optimal. Leaf size=147

$$\frac{28a^2(a+bx)^{17}}{17b^9}-\frac{7a^3(a+bx)^{16}}{2b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{a^8(a+bx)^{11}}{11b^9}+$$

```
[Out] (a^8*(a + b*x)^11)/(11*b^9) - (2*a^7*(a + b*x)^12)/(3*b^9) + (28*a^6*(a + b
*x)^13)/(13*b^9) - (4*a^5*(a + b*x)^14)/b^9 + (14*a^4*(a + b*x)^15)/(3*b^9)
 - (7*a^3*(a + b*x)^16)/(2*b^9) + (28*a^2*(a + b*x)^17)/(17*b^9) - (4*a*(a
+ b*x)^18)/(9*b^9) + (a + b*x)^19/(19*b^9)
```

Rubi [A] time = 0.0642461, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{28a^2(a+bx)^{17}}{17b^9}-\frac{7a^3(a+bx)^{16}}{2b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{a^8(a+bx)^{11}}{11b^9}+$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x)^10,x]
```

```
[Out] (a^8*(a + b*x)^11)/(11*b^9) - (2*a^7*(a + b*x)^12)/(3*b^9) + (28*a^6*(a + b
*x)^13)/(13*b^9) - (4*a^5*(a + b*x)^14)/b^9 + (14*a^4*(a + b*x)^15)/(3*b^9)
 - (7*a^3*(a + b*x)^16)/(2*b^9) + (28*a^2*(a + b*x)^17)/(17*b^9) - (4*a*(a
+ b*x)^18)/(9*b^9) + (a + b*x)^19/(19*b^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^8(a+bx)^{10}\,dx=\int\left(\frac{a^8(a+bx)^{10}}{b^8}-\frac{8a^7(a+bx)^{11}}{b^8}+\frac{28a^6(a+bx)^{12}}{b^8}-\frac{56a^5(a+bx)^{13}}{b^8}+\frac{70a^4(a+bx)^{14}}{b^8}-\frac{56a^3(a$$

$$=\frac{a^8(a+bx)^{11}}{11b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{7a^3(a+bx)^1}{2b^9}$$

Mathematica [A] time = 0.0035812, size = 125, normalized size = 0.85

$$\frac{45}{17}a^2b^8x^{17}+\frac{15}{2}a^3b^7x^{16}+14a^4b^6x^{15}+18a^5b^5x^{14}+\frac{210}{13}a^6b^4x^{13}+10a^7b^3x^{12}+\frac{45}{11}a^8b^2x^{11}+a^9bx^{10}+\frac{a^{10}x^9}{9}+\frac{5}{9}ab^9x^1$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x)^10,x]
```

```
[Out] (a^10*x^9)/9 + a^9*b*x^10 + (45*a^8*b^2*x^11)/11 + 10*a^7*b^3*x^12 + (210*a
^6*b^4*x^13)/13 + 18*a^5*b^5*x^14 + 14*a^4*b^6*x^15 + (15*a^3*b^7*x^16)/2 +
```

```
(45*a^2*b^8*x^17)/17 + (5*a*b^9*x^18)/9 + (b^10*x^19)/19
```

Maple [A] time = 0.001, size = 112, normalized size = 0.8

$$\frac{b^{10}x^{19}}{19}+\frac{5\,ab^{9}x^{18}}{9}+\frac{45\,a^{2}b^{8}x^{17}}{17}+\frac{15\,a^{3}b^{7}x^{16}}{2}+14\,a^{4}b^{6}x^{15}+18\,a^{5}b^{5}x^{14}+\frac{210\,a^{6}b^{4}x^{13}}{13}+10\,a^{7}b^{3}x^{12}+\frac{45\,a^{8}b^{2}x^{11}}{11}+a^{9}bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^10,x)
```

```
[Out] 1/19*b^10*x^19+5/9*a*b^9*x^18+45/17*a^2*b^8*x^17+15/2*a^3*b^7*x^16+14*a^4*b
^6*x^15+18*a^5*b^5*x^14+210/13*a^6*b^4*x^13+10*a^7*b^3*x^12+45/11*a^8*b^2*x
^11+a^9*b*x^10+1/9*a^10*x^9
```

Maxima [A] time = 1.03017, size = 150, normalized size = 1.02

$$\frac{1}{19}\,b^{10}x^{19}+\frac{5}{9}\,ab^{9}x^{18}+\frac{45}{17}\,a^{2}b^{8}x^{17}+\frac{15}{2}\,a^{3}b^{7}x^{16}+14\,a^{4}b^{6}x^{15}+18\,a^{5}b^{5}x^{14}+\frac{210}{13}\,a^{6}b^{4}x^{13}+10\,a^{7}b^{3}x^{12}+\frac{45}{11}\,a^{8}b^{2}x^{11}+a^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12 +
 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

Fricas [A] time = 1.57643, size = 269, normalized size = 1.83

$$\frac{1}{19}x^{19}b^{10}+\frac{5}{9}x^{18}b^{9}a+\frac{45}{17}x^{17}b^{8}a^{2}+\frac{15}{2}x^{16}b^{7}a^{3}+14x^{15}b^{6}a^{4}+18x^{14}b^{5}a^{5}+\frac{210}{13}x^{13}b^{4}a^{6}+10x^{12}b^{3}a^{7}+\frac{45}{11}x^{11}b^{2}a^{8}+x^{10}ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^10 + 5/9*x^18*b^9*a + 45/17*x^17*b^8*a^2 + 15/2*x^16*b^7*a^3 +
14*x^15*b^6*a^4 + 18*x^14*b^5*a^5 + 210/13*x^13*b^4*a^6 + 10*x^12*b^3*a^7 +
 45/11*x^11*b^2*a^8 + x^10*b*a^9 + 1/9*x^9*a^10
```

Sympy [A] time = 0.111013, size = 126, normalized size = 0.86

$$\frac{a^{10}x^{9}}{9}+a^{9}bx^{10}+\frac{45a^{8}b^{2}x^{11}}{11}+10a^{7}b^{3}x^{12}+\frac{210a^{6}b^{4}x^{13}}{13}+18a^{5}b^{5}x^{14}+14a^{4}b^{6}x^{15}+\frac{15a^{3}b^{7}x^{16}}{2}+\frac{45a^{2}b^{8}x^{17}}{17}+\frac{5ab^{9}x^{18}}{9}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**10,x)
```

```
[Out] a**10*x**9/9 + a**9*b*x**10 + 45*a**8*b**2*x**11/11 + 10*a**7*b**3*x**12 +
210*a**6*b**4*x**13/13 + 18*a**5*b**5*x**14 + 14*a**4*b**6*x**15 + 15*a**3*
b**7*x**16/2 + 45*a**2*b**8*x**17/17 + 5*a*b**9*x**18/9 + b**10*x**19/19
```

Giac [A] time = 1.21193, size = 150, normalized size = 1.02

$$\frac{1}{19}b^{10}x^{19}+\frac{5}{9}ab^{9}x^{18}+\frac{45}{17}a^{2}b^{8}x^{17}+\frac{15}{2}a^{3}b^{7}x^{16}+14\,a^{4}b^{6}x^{15}+18\,a^{5}b^{5}x^{14}+\frac{210}{13}a^{6}b^{4}x^{13}+10\,a^{7}b^{3}x^{12}+\frac{45}{11}a^{8}b^{2}x^{11}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12 +
 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

3.127 $\int x^7(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$\frac{21a^2(a+bx)^{16}}{16b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{a^7(a+bx)^{11}}{11b^8}+\frac{(a+bx)^{18}}{18b^8}-\frac{7a(a+}{17b}$$

```
[Out] -(a^7*(a + b*x)^11)/(11*b^8) + (7*a^6*(a + b*x)^12)/(12*b^8) - (21*a^5*(a +
 b*x)^13)/(13*b^8) + (5*a^4*(a + b*x)^14)/(2*b^8) - (7*a^3*(a + b*x)^15)/(3
*b^8) + (21*a^2*(a + b*x)^16)/(16*b^8) - (7*a*(a + b*x)^17)/(17*b^8) + (a +
 b*x)^18/(18*b^8)
```

Rubi [A] time = 0.0566174, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21a^2(a+bx)^{16}}{16b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{a^7(a+bx)^{11}}{11b^8}+\frac{(a+bx)^{18}}{18b^8}-\frac{7a(a+}{17b}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^10,x]
```

```
[Out] -(a^7*(a + b*x)^11)/(11*b^8) + (7*a^6*(a + b*x)^12)/(12*b^8) - (21*a^5*(a +
 b*x)^13)/(13*b^8) + (5*a^4*(a + b*x)^14)/(2*b^8) - (7*a^3*(a + b*x)^15)/(3
*b^8) + (21*a^2*(a + b*x)^16)/(16*b^8) - (7*a*(a + b*x)^17)/(17*b^8) + (a +
 b*x)^18/(18*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7(a+bx)^{10}\,dx &= \int\left(-\frac{a^7(a+bx)^{10}}{b^7}+\frac{7a^6(a+bx)^{11}}{b^7}-\frac{21a^5(a+bx)^{12}}{b^7}+\frac{35a^4(a+bx)^{13}}{b^7}-\frac{35a^3(a+bx)^{14}}{b^7}+\frac{21a^2(a+}{b^7}\right.\\ &= -\frac{a^7(a+bx)^{11}}{11b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{21a^2(a+bx)^{16}}{16b^8}\end{aligned}$$

Mathematica [A] time = 0.0134137, size = 130, normalized size = 0.98

$$\frac{45}{16}a^2b^8x^{16}+8a^3b^7x^{15}+15a^4b^6x^{14}+\frac{252}{13}a^5b^5x^{13}+\frac{35}{2}a^6b^4x^{12}+\frac{120}{11}a^7b^3x^{11}+\frac{9}{2}a^8b^2x^{10}+\frac{10}{9}a^9bx^9+\frac{a^{10}x^8}{8}+\frac{10}{17}ab^9x^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^10,x]
```

```
[Out] (a^10*x^8)/8 + (10*a^9*b*x^9)/9 + (9*a^8*b^2*x^10)/2 + (120*a^7*b^3*x^11)/1
1 + (35*a^6*b^4*x^12)/2 + (252*a^5*b^5*x^13)/13 + 15*a^4*b^6*x^14 + 8*a^3*b
```

```
^7*x^15 + (45*a^2*b^8*x^16)/16 + (10*a*b^9*x^17)/17 + (b^10*x^18)/18
```

Maple [A] time = 0.001, size = 113, normalized size = 0.9

$$\frac{b^{10}x^{18}}{18}+\frac{10\,ab^{9}x^{17}}{17}+\frac{45\,a^{2}b^{8}x^{16}}{16}+8\,a^{3}b^{7}x^{15}+15\,a^{4}b^{6}x^{14}+\frac{252\,a^{5}b^{5}x^{13}}{13}+\frac{35\,a^{6}b^{4}x^{12}}{2}+\frac{120\,a^{7}b^{3}x^{11}}{11}+\frac{9\,a^{8}b^{2}x^{10}}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^10,x)
```

```
[Out] 1/18*b^10*x^18+10/17*a*b^9*x^17+45/16*a^2*b^8*x^16+8*a^3*b^7*x^15+15*a^4*b^
6*x^14+252/13*a^5*b^5*x^13+35/2*a^6*b^4*x^12+120/11*a^7*b^3*x^11+9/2*a^8*b^
2*x^10+10/9*a^9*b*x^9+1/8*a^10*x^8
```

Maxima [A] time = 1.08172, size = 151, normalized size = 1.14

$$\frac{1}{18}\,b^{10}x^{18}+\frac{10}{17}\,ab^{9}x^{17}+\frac{45}{16}\,a^{2}b^{8}x^{16}+8\,a^{3}b^{7}x^{15}+15\,a^{4}b^{6}x^{14}+\frac{252}{13}\,a^{5}b^{5}x^{13}+\frac{35}{2}\,a^{6}b^{4}x^{12}+\frac{120}{11}\,a^{7}b^{3}x^{11}+\frac{9}{2}\,a^{8}b^{2}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 + 1
5*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3*x
^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8
```

Fricas [A] time = 1.56997, size = 278, normalized size = 2.11

$$\frac{1}{18}x^{18}b^{10}+\frac{10}{17}x^{17}b^{9}a+\frac{45}{16}x^{16}b^{8}a^{2}+8x^{15}b^{7}a^{3}+15x^{14}b^{6}a^{4}+\frac{252}{13}x^{13}b^{5}a^{5}+\frac{35}{2}x^{12}b^{4}a^{6}+\frac{120}{11}x^{11}b^{3}a^{7}+\frac{9}{2}x^{10}b^{2}a^{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^10 + 10/17*x^17*b^9*a + 45/16*x^16*b^8*a^2 + 8*x^15*b^7*a^3 + 1
5*x^14*b^6*a^4 + 252/13*x^13*b^5*a^5 + 35/2*x^12*b^4*a^6 + 120/11*x^11*b^3*
a^7 + 9/2*x^10*b^2*a^8 + 10/9*x^9*b*a^9 + 1/8*x^8*a^10
```

Sympy [A] time = 0.125765, size = 131, normalized size = 0.99

$$\frac{a^{10}x^{8}}{8}+\frac{10a^{9}bx^{9}}{9}+\frac{9a^{8}b^{2}x^{10}}{2}+\frac{120a^{7}b^{3}x^{11}}{11}+\frac{35a^{6}b^{4}x^{12}}{2}+\frac{252a^{5}b^{5}x^{13}}{13}+15a^{4}b^{6}x^{14}+8a^{3}b^{7}x^{15}+\frac{45a^{2}b^{8}x^{16}}{16}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**10,x)
```

```
[Out] a**10*x**8/8 + 10*a**9*b*x**9/9 + 9*a**8*b**2*x**10/2 + 120*a**7*b**3*x**11
/11 + 35*a**6*b**4*x**12/2 + 252*a**5*b**5*x**13/13 + 15*a**4*b**6*x**14 +
8*a**3*b**7*x**15 + 45*a**2*b**8*x**16/16 + 10*a*b**9*x**17/17 + b**10*x**1
8/18
```

Giac [A] time = 1.19444, size = 151, normalized size = 1.14

$$\frac{1}{18}b^{10}x^{18} + \frac{10}{17}ab^{9}x^{17} + \frac{45}{16}a^{2}b^{8}x^{16} + 8\,a^{3}b^{7}x^{15} + 15\,a^{4}b^{6}x^{14} + \frac{252}{13}a^{5}b^{5}x^{13} + \frac{35}{2}a^{6}b^{4}x^{12} + \frac{120}{11}a^{7}b^{3}x^{11} + \frac{9}{2}a^{8}b^{2}x^{10} + \frac{1}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 + 1
5*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3*x
^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8
```

3.128 $\int x^6(a+bx)^{10}\,dx$

Optimal. Leaf size=112

$$\frac{a^2(a+bx)^{15}}{b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{a^6(a+bx)^{11}}{11b^7}+\frac{(a+bx)^{17}}{17b^7}-\frac{3a(a+bx)^{16}}{8b^7}$$

```
[Out] (a^6*(a + b*x)^11)/(11*b^7) - (a^5*(a + b*x)^12)/(2*b^7) + (15*a^4*(a + b*x
)^13)/(13*b^7) - (10*a^3*(a + b*x)^14)/(7*b^7) + (a^2*(a + b*x)^15)/b^7 - (
3*a*(a + b*x)^16)/(8*b^7) + (a + b*x)^17/(17*b^7)
```

Rubi [A] time = 0.0524065, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{15}}{b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{a^6(a+bx)^{11}}{11b^7}+\frac{(a+bx)^{17}}{17b^7}-\frac{3a(a+bx)^{16}}{8b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^10,x]
```

```
[Out] (a^6*(a + b*x)^11)/(11*b^7) - (a^5*(a + b*x)^12)/(2*b^7) + (15*a^4*(a + b*x
)^13)/(13*b^7) - (10*a^3*(a + b*x)^14)/(7*b^7) + (a^2*(a + b*x)^15)/b^7 - (
3*a*(a + b*x)^16)/(8*b^7) + (a + b*x)^17/(17*b^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^{10}\,dx &= \int\left(\frac{a^6(a+bx)^{10}}{b^6}-\frac{6a^5(a+bx)^{11}}{b^6}+\frac{15a^4(a+bx)^{12}}{b^6}-\frac{20a^3(a+bx)^{13}}{b^6}+\frac{15a^2(a+bx)^{14}}{b^6}-\frac{6a(a+}{b^6}\right.\\ &= \frac{a^6(a+bx)^{11}}{11b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{a^2(a+bx)^{15}}{b^7}-\frac{3a(a+bx)^{16}}{8b^7}+\end{aligned}$$

Mathematica [A] time = 0.0074339, size = 126, normalized size = 1.12

$$3a^2b^8x^{15}+\frac{60}{7}a^3b^7x^{14}+\frac{210}{13}a^4b^6x^{13}+21a^5b^5x^{12}+\frac{210}{11}a^6b^4x^{11}+12a^7b^3x^{10}+5a^8b^2x^9+\frac{5}{4}a^9bx^8+\frac{a^{10}x^7}{7}+\frac{5}{8}ab^9x^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^10,x]
```

```
[Out] (a^10*x^7)/7 + (5*a^9*b*x^8)/4 + 5*a^8*b^2*x^9 + 12*a^7*b^3*x^10 + (210*a^6
*b^4*x^11)/11 + 21*a^5*b^5*x^12 + (210*a^4*b^6*x^13)/13 + (60*a^3*b^7*x^14)
/7 + 3*a^2*b^8*x^15 + (5*a*b^9*x^16)/8 + (b^10*x^17)/17
```

Maple [A] time = 0.001, size = 113, normalized size = 1.

$$\frac{b^{10}x^{17}}{17}+\frac{5\,ab^{9}x^{16}}{8}+3\,a^{2}b^{8}x^{15}+\frac{60\,a^{3}b^{7}x^{14}}{7}+\frac{210\,a^{4}b^{6}x^{13}}{13}+21\,a^{5}b^{5}x^{12}+\frac{210\,a^{6}b^{4}x^{11}}{11}+12\,a^{7}b^{3}x^{10}+5\,a^{8}b^{2}x^{9}+\frac{5\,a^{9}bx^{8}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^10,x)
```

```
[Out] 1/17*b^10*x^17+5/8*a*b^9*x^16+3*a^2*b^8*x^15+60/7*a^3*b^7*x^14+210/13*a^4*b
^6*x^13+21*a^5*b^5*x^12+210/11*a^6*b^4*x^11+12*a^7*b^3*x^10+5*a^8*b^2*x^9+5
/4*a^9*b*x^8+1/7*a^10*x^7
```

Maxima [A] time = 1.01775, size = 151, normalized size = 1.35

$$\frac{1}{17}\,b^{10}x^{17}+\frac{5}{8}\,ab^{9}x^{16}+3\,a^{2}b^{8}x^{15}+\frac{60}{7}\,a^{3}b^{7}x^{14}+\frac{210}{13}\,a^{4}b^{6}x^{13}+21\,a^{5}b^{5}x^{12}+\frac{210}{11}\,a^{6}b^{4}x^{11}+12\,a^{7}b^{3}x^{10}+5\,a^{8}b^{2}x^{9}+\frac{5}{4}\,a^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210/
13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10 +
 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

Fricas [A] time = 1.53279, size = 266, normalized size = 2.38

$$\frac{1}{17}x^{17}b^{10}+\frac{5}{8}x^{16}b^{9}a+3x^{15}b^{8}a^{2}+\frac{60}{7}x^{14}b^{7}a^{3}+\frac{210}{13}x^{13}b^{6}a^{4}+21x^{12}b^{5}a^{5}+\frac{210}{11}x^{11}b^{4}a^{6}+12x^{10}b^{3}a^{7}+5x^{9}b^{2}a^{8}+\frac{5}{4}x^{8}ba^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^10 + 5/8*x^16*b^9*a + 3*x^15*b^8*a^2 + 60/7*x^14*b^7*a^3 + 210/
13*x^13*b^6*a^4 + 21*x^12*b^5*a^5 + 210/11*x^11*b^4*a^6 + 12*x^10*b^3*a^7 +
 5*x^9*b^2*a^8 + 5/4*x^8*b*a^9 + 1/7*x^7*a^10
```

Sympy [A] time = 0.106189, size = 128, normalized size = 1.14

$$\frac{a^{10}x^{7}}{7}+\frac{5a^{9}bx^{8}}{4}+5a^{8}b^{2}x^{9}+12a^{7}b^{3}x^{10}+\frac{210a^{6}b^{4}x^{11}}{11}+21a^{5}b^{5}x^{12}+\frac{210a^{4}b^{6}x^{13}}{13}+\frac{60a^{3}b^{7}x^{14}}{7}+3a^{2}b^{8}x^{15}+\frac{5ab^{9}x^{16}}{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**10,x)
```

```
[Out] a**10*x**7/7 + 5*a**9*b*x**8/4 + 5*a**8*b**2*x**9 + 12*a**7*b**3*x**10 + 21
0*a**6*b**4*x**11/11 + 21*a**5*b**5*x**12 + 210*a**4*b**6*x**13/13 + 60*a**
```

```
3*b**7*x**14/7 + 3*a**2*b**8*x**15 + 5*a*b**9*x**16/8 + b**10*x**17/17
```

Giac [A] time = 1.1522, size = 151, normalized size = 1.35

$$\frac{1}{17}\,b^{10}x^{17} + \frac{5}{8}\,ab^{9}x^{16} + 3\,a^{2}b^{8}x^{15} + \frac{60}{7}\,a^{3}b^{7}x^{14} + \frac{210}{13}\,a^{4}b^{6}x^{13} + 21\,a^{5}b^{5}x^{12} + \frac{210}{11}\,a^{6}b^{4}x^{11} + 12\,a^{7}b^{3}x^{10} + 5\,a^{8}b^{2}x^{9} + \frac{5}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210/
13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10 +
 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

3.129 $\int x^5(a+bx)^{10}\,dx$

Optimal. Leaf size=98

$$\frac{5a^2(a+bx)^{14}}{7b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{a^5(a+bx)^{11}}{11b^6}+\frac{(a+bx)^{16}}{16b^6}-\frac{a(a+bx)^{15}}{3b^6}$$

```
[Out] -(a^5*(a + b*x)^11)/(11*b^6) + (5*a^4*(a + b*x)^12)/(12*b^6) - (10*a^3*(a +
 b*x)^13)/(13*b^6) + (5*a^2*(a + b*x)^14)/(7*b^6) - (a*(a + b*x)^15)/(3*b^6
) + (a + b*x)^16/(16*b^6)
```

Rubi [A] time = 0.0452189, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{5a^2(a+bx)^{14}}{7b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{a^5(a+bx)^{11}}{11b^6}+\frac{(a+bx)^{16}}{16b^6}-\frac{a(a+bx)^{15}}{3b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^10,x]
```

```
[Out] -(a^5*(a + b*x)^11)/(11*b^6) + (5*a^4*(a + b*x)^12)/(12*b^6) - (10*a^3*(a +
 b*x)^13)/(13*b^6) + (5*a^2*(a + b*x)^14)/(7*b^6) - (a*(a + b*x)^15)/(3*b^6
) + (a + b*x)^16/(16*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^{10}\,dx&=\int\left(-\frac{a^5(a+bx)^{10}}{b^5}+\frac{5a^4(a+bx)^{11}}{b^5}-\frac{10a^3(a+bx)^{12}}{b^5}+\frac{10a^2(a+bx)^{13}}{b^5}-\frac{5a(a+bx)^{14}}{b^5}+\frac{(a+bx)^{15}}{b^5}\right)\\&=-\frac{a^5(a+bx)^{11}}{11b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^2(a+bx)^{14}}{7b^6}-\frac{a(a+bx)^{15}}{3b^6}+\frac{(a+bx)^{16}}{16b^6}\end{aligned}$$

Mathematica [A] time = 0.0080195, size = 132, normalized size = 1.35

$$\frac{45}{14}a^2b^8x^{14}+\frac{120}{13}a^3b^7x^{13}+\frac{35}{2}a^4b^6x^{12}+\frac{252}{11}a^5b^5x^{11}+21a^6b^4x^{10}+\frac{40}{3}a^7b^3x^9+\frac{45}{8}a^8b^2x^8+\frac{10}{7}a^9bx^7+\frac{a^{10}x^6}{6}+\frac{2}{3}ab^9x^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^10,x]
```

```
[Out] (a^10*x^6)/6 + (10*a^9*b*x^7)/7 + (45*a^8*b^2*x^8)/8 + (40*a^7*b^3*x^9)/3 +
 21*a^6*b^4*x^10 + (252*a^5*b^5*x^11)/11 + (35*a^4*b^6*x^12)/2 + (120*a^3*b
^7*x^13)/13 + (45*a^2*b^8*x^14)/14 + (2*a*b^9*x^15)/3 + (b^10*x^16)/16
```

Maple [A] time = 0.002, size = 113, normalized size = 1.2

$$\frac{b^{10}x^{16}}{16}+\frac{2\,ab^{9}x^{15}}{3}+\frac{45\,a^{2}b^{8}x^{14}}{14}+\frac{120\,a^{3}b^{7}x^{13}}{13}+\frac{35\,a^{4}b^{6}x^{12}}{2}+\frac{252\,a^{5}b^{5}x^{11}}{11}+21\,a^{6}b^{4}x^{10}+\frac{40\,a^{7}b^{3}x^{9}}{3}+\frac{45\,a^{8}b^{2}x^{8}}{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^10,x)
```

```
[Out] 1/16*b^10*x^16+2/3*a*b^9*x^15+45/14*a^2*b^8*x^14+120/13*a^3*b^7*x^13+35/2*a
^4*b^6*x^12+252/11*a^5*b^5*x^11+21*a^6*b^4*x^10+40/3*a^7*b^3*x^9+45/8*a^8*b
^2*x^8+10/7*a^9*b*x^7+1/6*a^10*x^6
```

Maxima [A] time = 1.04719, size = 151, normalized size = 1.54

$$\frac{1}{16}\,b^{10}x^{16}+\frac{2}{3}\,ab^{9}x^{15}+\frac{45}{14}\,a^{2}b^{8}x^{14}+\frac{120}{13}\,a^{3}b^{7}x^{13}+\frac{35}{2}\,a^{4}b^{6}x^{12}+\frac{252}{11}\,a^{5}b^{5}x^{11}+21\,a^{6}b^{4}x^{10}+\frac{40}{3}\,a^{7}b^{3}x^{9}+\frac{45}{8}\,a^{8}b^{2}x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
+ 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^3*
x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

Fricas [A] time = 1.5311, size = 278, normalized size = 2.84

$$\frac{1}{16}x^{16}b^{10}+\frac{2}{3}x^{15}b^{9}a+\frac{45}{14}x^{14}b^{8}a^{2}+\frac{120}{13}x^{13}b^{7}a^{3}+\frac{35}{2}x^{12}b^{6}a^{4}+\frac{252}{11}x^{11}b^{5}a^{5}+21x^{10}b^{4}a^{6}+\frac{40}{3}x^{9}b^{3}a^{7}+\frac{45}{8}x^{8}b^{2}a^{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^10 + 2/3*x^15*b^9*a + 45/14*x^14*b^8*a^2 + 120/13*x^13*b^7*a^3
+ 35/2*x^12*b^6*a^4 + 252/11*x^11*b^5*a^5 + 21*x^10*b^4*a^6 + 40/3*x^9*b^3*
a^7 + 45/8*x^8*b^2*a^8 + 10/7*x^7*b*a^9 + 1/6*x^6*a^10
```

Sympy [A] time = 0.127557, size = 133, normalized size = 1.36

$$\frac{a^{10}x^{6}}{6}+\frac{10a^{9}bx^{7}}{7}+\frac{45a^{8}b^{2}x^{8}}{8}+\frac{40a^{7}b^{3}x^{9}}{3}+21a^{6}b^{4}x^{10}+\frac{252a^{5}b^{5}x^{11}}{11}+\frac{35a^{4}b^{6}x^{12}}{2}+\frac{120a^{3}b^{7}x^{13}}{13}+\frac{45a^{2}b^{8}x^{14}}{14}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**10,x)
```

```
[Out] a**10*x**6/6 + 10*a**9*b*x**7/7 + 45*a**8*b**2*x**8/8 + 40*a**7*b**3*x**9/3
 + 21*a**6*b**4*x**10 + 252*a**5*b**5*x**11/11 + 35*a**4*b**6*x**12/2 + 120
*a**3*b**7*x**13/13 + 45*a**2*b**8*x**14/14 + 2*a*b**9*x**15/3 + b**10*x**1
```

6/16

Giac [A] time = 1.18422, size = 151, normalized size = 1.54

$$\frac{1}{16}\,b^{10}x^{16}+\frac{2}{3}\,ab^{9}x^{15}+\frac{45}{14}\,a^{2}b^{8}x^{14}+\frac{120}{13}\,a^{3}b^{7}x^{13}+\frac{35}{2}\,a^{4}b^{6}x^{12}+\frac{252}{11}\,a^{5}b^{5}x^{11}+21\,a^{6}b^{4}x^{10}+\frac{40}{3}\,a^{7}b^{3}x^{9}+\frac{45}{8}\,a^{8}b^{2}x^{8}+\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
+ 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^3*
x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

3.130 $\int x^4(a+bx)^{10}\,dx$

Optimal. Leaf size=81

$$\frac{6a^2(a+bx)^{13}}{13b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{a^4(a+bx)^{11}}{11b^5}+\frac{(a+bx)^{15}}{15b^5}-\frac{2a(a+bx)^{14}}{7b^5}$$

```
[Out] (a^4*(a + b*x)^11)/(11*b^5) - (a^3*(a + b*x)^12)/(3*b^5) + (6*a^2*(a + b*x)
^13)/(13*b^5) - (2*a*(a + b*x)^14)/(7*b^5) + (a + b*x)^15/(15*b^5)
```

Rubi [A] time = 0.0390572, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{6a^2(a+bx)^{13}}{13b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{a^4(a+bx)^{11}}{11b^5}+\frac{(a+bx)^{15}}{15b^5}-\frac{2a(a+bx)^{14}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^10,x]
```

```
[Out] (a^4*(a + b*x)^11)/(11*b^5) - (a^3*(a + b*x)^12)/(3*b^5) + (6*a^2*(a + b*x)
^13)/(13*b^5) - (2*a*(a + b*x)^14)/(7*b^5) + (a + b*x)^15/(15*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^{10}\,dx&=\int\left(\frac{a^4(a+bx)^{10}}{b^4}-\frac{4a^3(a+bx)^{11}}{b^4}+\frac{6a^2(a+bx)^{12}}{b^4}-\frac{4a(a+bx)^{13}}{b^4}+\frac{(a+bx)^{14}}{b^4}\right)dx\\&=\frac{a^4(a+bx)^{11}}{11b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{6a^2(a+bx)^{13}}{13b^5}-\frac{2a(a+bx)^{14}}{7b^5}+\frac{(a+bx)^{15}}{15b^5}\end{aligned}$$

Mathematica [A] time = 0.0033182, size = 130, normalized size = 1.6

$$\frac{45}{13}a^2b^8x^{13}+10a^3b^7x^{12}+\frac{210}{11}a^4b^6x^{11}+\frac{126}{5}a^5b^5x^{10}+\frac{70}{3}a^6b^4x^9+15a^7b^3x^8+\frac{45}{7}a^8b^2x^7+\frac{5}{3}a^9bx^6+\frac{a^{10}x^5}{5}+\frac{5}{7}ab^9x^1$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^10,x]
```

```
[Out] (a^10*x^5)/5 + (5*a^9*b*x^6)/3 + (45*a^8*b^2*x^7)/7 + 15*a^7*b^3*x^8 + (70*
a^6*b^4*x^9)/3 + (126*a^5*b^5*x^10)/5 + (210*a^4*b^6*x^11)/11 + 10*a^3*b^7*
x^12 + (45*a^2*b^8*x^13)/13 + (5*a*b^9*x^14)/7 + (b^10*x^15)/15
```

Maple [A] time = 0.001, size = 113, normalized size = 1.4

$$\frac{b^{10}x^{15}}{15}+\frac{5\,ab^{9}x^{14}}{7}+\frac{45\,a^{2}b^{8}x^{13}}{13}+10\,a^{3}b^{7}x^{12}+\frac{210\,a^{4}b^{6}x^{11}}{11}+\frac{126\,a^{5}b^{5}x^{10}}{5}+\frac{70\,a^{6}b^{4}x^{9}}{3}+15\,a^{7}b^{3}x^{8}+\frac{45\,a^{8}b^{2}x^{7}}{7}+\frac{5\,a^{9}b}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^10,x)
```

```
[Out] 1/15*b^10*x^15+5/7*a*b^9*x^14+45/13*a^2*b^8*x^13+10*a^3*b^7*x^12+210/11*a^4
*b^6*x^11+126/5*a^5*b^5*x^10+70/3*a^6*b^4*x^9+15*a^7*b^3*x^8+45/7*a^8*b^2*x
^7+5/3*a^9*b*x^6+1/5*a^10*x^5
```

Maxima [A] time = 1.04508, size = 151, normalized size = 1.86

$$\frac{1}{15}\,b^{10}x^{15}+\frac{5}{7}\,ab^{9}x^{14}+\frac{45}{13}\,a^{2}b^{8}x^{13}+10\,a^{3}b^{7}x^{12}+\frac{210}{11}\,a^{4}b^{6}x^{11}+\frac{126}{5}\,a^{5}b^{5}x^{10}+\frac{70}{3}\,a^{6}b^{4}x^{9}+15\,a^{7}b^{3}x^{8}+\frac{45}{7}\,a^{8}b^{2}x^{7}+\frac{5}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 21
0/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^8
+ 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

Fricas [A] time = 1.752, size = 271, normalized size = 3.35

$$\frac{1}{15}x^{15}b^{10}+\frac{5}{7}x^{14}b^{9}a+\frac{45}{13}x^{13}b^{8}a^{2}+10x^{12}b^{7}a^{3}+\frac{210}{11}x^{11}b^{6}a^{4}+\frac{126}{5}x^{10}b^{5}a^{5}+\frac{70}{3}x^{9}b^{4}a^{6}+15x^{8}b^{3}a^{7}+\frac{45}{7}x^{7}b^{2}a^{8}+\frac{5}{3}x^{6}ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^10 + 5/7*x^14*b^9*a + 45/13*x^13*b^8*a^2 + 10*x^12*b^7*a^3 + 21
0/11*x^11*b^6*a^4 + 126/5*x^10*b^5*a^5 + 70/3*x^9*b^4*a^6 + 15*x^8*b^3*a^7
+ 45/7*x^7*b^2*a^8 + 5/3*x^6*b*a^9 + 1/5*x^5*a^10
```

Sympy [A] time = 0.104979, size = 131, normalized size = 1.62

$$\frac{a^{10}x^{5}}{5}+\frac{5a^{9}bx^{6}}{3}+\frac{45a^{8}b^{2}x^{7}}{7}+15a^{7}b^{3}x^{8}+\frac{70a^{6}b^{4}x^{9}}{3}+\frac{126a^{5}b^{5}x^{10}}{5}+\frac{210a^{4}b^{6}x^{11}}{11}+10a^{3}b^{7}x^{12}+\frac{45a^{2}b^{8}x^{13}}{13}+\frac{5ab^{9}x^{14}}{7}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**10,x)
```

```
[Out] a**10*x**5/5 + 5*a**9*b*x**6/3 + 45*a**8*b**2*x**7/7 + 15*a**7*b**3*x**8 +
70*a**6*b**4*x**9/3 + 126*a**5*b**5*x**10/5 + 210*a**4*b**6*x**11/11 + 10*a
**3*b**7*x**12 + 45*a**2*b**8*x**13/13 + 5*a*b**9*x**14/7 + b**10*x**15/15
```

Giac [A] time = 1.21715, size = 151, normalized size = 1.86

$$\frac{1}{15}\,b^{10}x^{15} + \frac{5}{7}\,ab^{9}x^{14} + \frac{45}{13}\,a^{2}b^{8}x^{13} + 10\,a^{3}b^{7}x^{12} + \frac{210}{11}\,a^{4}b^{6}x^{11} + \frac{126}{5}\,a^{5}b^{5}x^{10} + \frac{70}{3}\,a^{6}b^{4}x^{9} + 15\,a^{7}b^{3}x^{8} + \frac{45}{7}\,a^{8}b^{2}x^{7} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 21
0/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^8
+ 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

3.131 $\int x^3(a+bx)^{10}\,dx$

Optimal. Leaf size=64

$$\frac{a^2(a+bx)^{12}}{4b^4}-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{(a+bx)^{14}}{14b^4}-\frac{3a(a+bx)^{13}}{13b^4}$$

```
[Out] -(a^3*(a + b*x)^11)/(11*b^4) + (a^2*(a + b*x)^12)/(4*b^4) - (3*a*(a + b*x)^
13)/(13*b^4) + (a + b*x)^14/(14*b^4)
```

Rubi [A] time = 0.035897, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{12}}{4b^4}-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{(a+bx)^{14}}{14b^4}-\frac{3a(a+bx)^{13}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^10,x]
```

```
[Out] -(a^3*(a + b*x)^11)/(11*b^4) + (a^2*(a + b*x)^12)/(4*b^4) - (3*a*(a + b*x)^
13)/(13*b^4) + (a + b*x)^14/(14*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{10}\,dx &= \int \left(-\frac{a^3(a+bx)^{10}}{b^3}+\frac{3a^2(a+bx)^{11}}{b^3}-\frac{3a(a+bx)^{12}}{b^3}+\frac{(a+bx)^{13}}{b^3}\right)dx\\ &= -\frac{a^3(a+bx)^{11}}{11b^4}+\frac{a^2(a+bx)^{12}}{4b^4}-\frac{3a(a+bx)^{13}}{13b^4}+\frac{(a+bx)^{14}}{14b^4}\end{aligned}$$

Mathematica [A] time = 0.003124, size = 128, normalized size = 2.

$$\frac{15}{4}a^2b^8x^{12}+\frac{120}{11}a^3b^7x^{11}+21a^4b^6x^{10}+28a^5b^5x^9+\frac{105}{4}a^6b^4x^8+\frac{120}{7}a^7b^3x^7+\frac{15}{2}a^8b^2x^6+2a^9bx^5+\frac{a^{10}x^4}{4}+\frac{10}{13}ab^9x^{13}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^10,x]
```

```
[Out] (a^10*x^4)/4 + 2*a^9*b*x^5 + (15*a^8*b^2*x^6)/2 + (120*a^7*b^3*x^7)/7 + (10
5*a^6*b^4*x^8)/4 + 28*a^5*b^5*x^9 + 21*a^4*b^6*x^10 + (120*a^3*b^7*x^11)/11
 + (15*a^2*b^8*x^12)/4 + (10*a*b^9*x^13)/13 + (b^10*x^14)/14
```

Maple [A] time = 0.001, size = 113, normalized size = 1.8

$$\frac{b^{10}x^{14}}{14}+\frac{10\,ab^{9}x^{13}}{13}+\frac{15\,a^{2}b^{8}x^{12}}{4}+\frac{120\,a^{3}b^{7}x^{11}}{11}+21\,a^{4}b^{6}x^{10}+28\,a^{5}b^{5}x^{9}+\frac{105\,a^{6}b^{4}x^{8}}{4}+\frac{120\,a^{7}b^{3}x^{7}}{7}+\frac{15\,a^{8}b^{2}x^{6}}{2}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^10,x)
```

```
[Out] 1/14*b^10*x^14+10/13*a*b^9*x^13+15/4*a^2*b^8*x^12+120/11*a^3*b^7*x^11+21*a^
4*b^6*x^10+28*a^5*b^5*x^9+105/4*a^6*b^4*x^8+120/7*a^7*b^3*x^7+15/2*a^8*b^2*
x^6+2*a^9*b*x^5+1/4*a^10*x^4
```

Maxima [A] time = 1.0224, size = 151, normalized size = 2.36

$$\frac{1}{14}\,b^{10}x^{14}+\frac{10}{13}\,ab^{9}x^{13}+\frac{15}{4}\,a^{2}b^{8}x^{12}+\frac{120}{11}\,a^{3}b^{7}x^{11}+21\,a^{4}b^{6}x^{10}+28\,a^{5}b^{5}x^{9}+\frac{105}{4}\,a^{6}b^{4}x^{8}+\frac{120}{7}\,a^{7}b^{3}x^{7}+\frac{15}{2}\,a^{8}b^{2}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^11
 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x^7
 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

Fricas [A] time = 1.53716, size = 270, normalized size = 4.22

$$\frac{1}{14}x^{14}b^{10}+\frac{10}{13}x^{13}b^{9}a+\frac{15}{4}x^{12}b^{8}a^{2}+\frac{120}{11}x^{11}b^{7}a^{3}+21x^{10}b^{6}a^{4}+28x^{9}b^{5}a^{5}+\frac{105}{4}x^{8}b^{4}a^{6}+\frac{120}{7}x^{7}b^{3}a^{7}+\frac{15}{2}x^{6}b^{2}a^{8}+2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^10 + 10/13*x^13*b^9*a + 15/4*x^12*b^8*a^2 + 120/11*x^11*b^7*a^3
 + 21*x^10*b^6*a^4 + 28*x^9*b^5*a^5 + 105/4*x^8*b^4*a^6 + 120/7*x^7*b^3*a^7
 + 15/2*x^6*b^2*a^8 + 2*x^5*b*a^9 + 1/4*x^4*a^10
```

Sympy [B] time = 0.105953, size = 129, normalized size = 2.02

$$\frac{a^{10}x^{4}}{4}+2a^{9}bx^{5}+\frac{15a^{8}b^{2}x^{6}}{2}+\frac{120a^{7}b^{3}x^{7}}{7}+\frac{105a^{6}b^{4}x^{8}}{4}+28a^{5}b^{5}x^{9}+21a^{4}b^{6}x^{10}+\frac{120a^{3}b^{7}x^{11}}{11}+\frac{15a^{2}b^{8}x^{12}}{4}+\frac{10ab^{9}x^{1}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**10,x)
```

```
[Out] a**10*x**4/4 + 2*a**9*b*x**5 + 15*a**8*b**2*x**6/2 + 120*a**7*b**3*x**7/7 +
 105*a**6*b**4*x**8/4 + 28*a**5*b**5*x**9 + 21*a**4*b**6*x**10 + 120*a**3*b
**7*x**11/11 + 15*a**2*b**8*x**12/4 + 10*a*b**9*x**13/13 + b**10*x**14/14
```

Giac [A] time = 1.16197, size = 151, normalized size = 2.36

$$\frac{1}{14}\,b^{10}x^{14} + \frac{10}{13}\,ab^{9}x^{13} + \frac{15}{4}\,a^{2}b^{8}x^{12} + \frac{120}{11}\,a^{3}b^{7}x^{11} + 21\,a^{4}b^{6}x^{10} + 28\,a^{5}b^{5}x^{9} + \frac{105}{4}\,a^{6}b^{4}x^{8} + \frac{120}{7}\,a^{7}b^{3}x^{7} + \frac{15}{2}\,a^{8}b^{2}x^{6} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^11
 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x^7
 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

3.132 $\int x^2(a+bx)^{10}\,dx$

Optimal. Leaf size=47

$$\frac{a^2(a+bx)^{11}}{11b^3}+\frac{(a+bx)^{13}}{13b^3}-\frac{a(a+bx)^{12}}{6b^3}$$

```
[Out] (a^2*(a + b*x)^11)/(11*b^3) - (a*(a + b*x)^12)/(6*b^3) + (a + b*x)^13/(13*b
^3)
```

Rubi [A] time = 0.0296192, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{11}}{11b^3}+\frac{(a+bx)^{13}}{13b^3}-\frac{a(a+bx)^{12}}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^10,x]
```

```
[Out] (a^2*(a + b*x)^11)/(11*b^3) - (a*(a + b*x)^12)/(6*b^3) + (a + b*x)^13/(13*b
^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{10}\,dx &= \int\left(\frac{a^2(a+bx)^{10}}{b^2}-\frac{2a(a+bx)^{11}}{b^2}+\frac{(a+bx)^{12}}{b^2}\right)dx\\ &= \frac{a^2(a+bx)^{11}}{11b^3}-\frac{a(a+bx)^{12}}{6b^3}+\frac{(a+bx)^{13}}{13b^3}\end{aligned}$$

Mathematica [B] time = 0.0030878, size = 126, normalized size = 2.68

$$\frac{45}{11}a^2b^8x^{11}+12a^3b^7x^{10}+\frac{70}{3}a^4b^6x^9+\frac{63}{2}a^5b^5x^8+30a^6b^4x^7+20a^7b^3x^6+9a^8b^2x^5+\frac{5}{2}a^9bx^4+\frac{a^{10}x^3}{3}+\frac{5}{6}ab^9x^{12}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^10,x]
```

```
[Out] (a^10*x^3)/3 + (5*a^9*b*x^4)/2 + 9*a^8*b^2*x^5 + 20*a^7*b^3*x^6 + 30*a^6*b^
4*x^7 + (63*a^5*b^5*x^8)/2 + (70*a^4*b^6*x^9)/3 + 12*a^3*b^7*x^10 + (45*a^2
*b^8*x^11)/11 + (5*a*b^9*x^12)/6 + (b^10*x^13)/13
```

Maple [B] time = 0., size = 113, normalized size = 2.4

$$\frac{b^{10}x^{13}}{13}+\frac{5\,ab^{9}x^{12}}{6}+\frac{45\,a^{2}b^{8}x^{11}}{11}+12\,a^{3}b^{7}x^{10}+\frac{70\,a^{4}b^{6}x^{9}}{3}+\frac{63\,a^{5}b^{5}x^{8}}{2}+30\,a^{6}b^{4}x^{7}+20\,a^{7}b^{3}x^{6}+9\,a^{8}b^{2}x^{5}+\frac{5\,a^{9}bx^{4}}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^10,x)
```

```
[Out] 1/13*b^10*x^13+5/6*a*b^9*x^12+45/11*a^2*b^8*x^11+12*a^3*b^7*x^10+70/3*a^4*b
^6*x^9+63/2*a^5*b^5*x^8+30*a^6*b^4*x^7+20*a^7*b^3*x^6+9*a^8*b^2*x^5+5/2*a^9
*b*x^4+1/3*a^10*x^3
```

Maxima [B] time = 1.02456, size = 151, normalized size = 3.21

$$\frac{1}{13}\,b^{10}x^{13}+\frac{5}{6}\,ab^{9}x^{12}+\frac{45}{11}\,a^{2}b^{8}x^{11}+12\,a^{3}b^{7}x^{10}+\frac{70}{3}\,a^{4}b^{6}x^{9}+\frac{63}{2}\,a^{5}b^{5}x^{8}+30\,a^{6}b^{4}x^{7}+20\,a^{7}b^{3}x^{6}+9\,a^{8}b^{2}x^{5}+\frac{5}{2}\,a^{9}bx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 70
/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a^8
*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

Fricas [B] time = 1.70969, size = 258, normalized size = 5.49

$$\frac{1}{13}x^{13}b^{10}+\frac{5}{6}x^{12}b^{9}a+\frac{45}{11}x^{11}b^{8}a^{2}+12x^{10}b^{7}a^{3}+\frac{70}{3}x^{9}b^{6}a^{4}+\frac{63}{2}x^{8}b^{5}a^{5}+30x^{7}b^{4}a^{6}+20x^{6}b^{3}a^{7}+9x^{5}b^{2}a^{8}+\frac{5}{2}x^{4}ba^{9}+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^10 + 5/6*x^12*b^9*a + 45/11*x^11*b^8*a^2 + 12*x^10*b^7*a^3 + 70
/3*x^9*b^6*a^4 + 63/2*x^8*b^5*a^5 + 30*x^7*b^4*a^6 + 20*x^6*b^3*a^7 + 9*x^5
*b^2*a^8 + 5/2*x^4*b*a^9 + 1/3*x^3*a^10
```

Sympy [B] time = 0.105326, size = 128, normalized size = 2.72

$$\frac{a^{10}x^{3}}{3}+\frac{5a^{9}bx^{4}}{2}+9a^{8}b^{2}x^{5}+20a^{7}b^{3}x^{6}+30a^{6}b^{4}x^{7}+\frac{63a^{5}b^{5}x^{8}}{2}+\frac{70a^{4}b^{6}x^{9}}{3}+12a^{3}b^{7}x^{10}+\frac{45a^{2}b^{8}x^{11}}{11}+\frac{5ab^{9}x^{12}}{6}+\frac{b^{10}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**10,x)
```

```
[Out] a**10*x**3/3 + 5*a**9*b*x**4/2 + 9*a**8*b**2*x**5 + 20*a**7*b**3*x**6 + 30*
a**6*b**4*x**7 + 63*a**5*b**5*x**8/2 + 70*a**4*b**6*x**9/3 + 12*a**3*b**7*x
**10 + 45*a**2*b**8*x**11/11 + 5*a*b**9*x**12/6 + b**10*x**13/13
```

Giac [B] time = 1.19456, size = 151, normalized size = 3.21

$$\frac{1}{13}\,b^{10}x^{13} + \frac{5}{6}\,ab^{9}x^{12} + \frac{45}{11}\,a^{2}b^{8}x^{11} + 12\,a^{3}b^{7}x^{10} + \frac{70}{3}\,a^{4}b^{6}x^{9} + \frac{63}{2}\,a^{5}b^{5}x^{8} + 30\,a^{6}b^{4}x^{7} + 20\,a^{7}b^{3}x^{6} + 9\,a^{8}b^{2}x^{5} + \frac{5}{2}\,a^{9}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 70
/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a^8
*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

3.133 $\int x(a+bx)^{10}\,dx$

Optimal. Leaf size=30

$$\frac{(a+bx)^{12}}{12b^2}-\frac{a(a+bx)^{11}}{11b^2}$$

```
[Out] -(a*(a + b*x)^11)/(11*b^2) + (a + b*x)^12/(12*b^2)
```

Rubi [A] time = 0.0082832, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^{12}}{12b^2}-\frac{a(a+bx)^{11}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^10,x]
```

```
[Out] -(a*(a + b*x)^11)/(11*b^2) + (a + b*x)^12/(12*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{10}\,dx &= \int\left(-\frac{a(a+bx)^{10}}{b}+\frac{(a+bx)^{11}}{b}\right)dx\\ &= -\frac{a(a+bx)^{11}}{11b^2}+\frac{(a+bx)^{12}}{12b^2}\end{aligned}$$

Mathematica [B] time = 0.002549, size = 128, normalized size = 4.27

$$\frac{9}{2}a^2b^8x^{10}+\frac{40}{3}a^3b^7x^9+\frac{105}{4}a^4b^6x^8+36a^5b^5x^7+35a^6b^4x^6+24a^7b^3x^5+\frac{45}{4}a^8b^2x^4+\frac{10}{3}a^9bx^3+\frac{a^{10}x^2}{2}+\frac{10}{11}ab^9x^{11}+\frac{b^{1}}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^10,x]
```

```
[Out] (a^10*x^2)/2 + (10*a^9*b*x^3)/3 + (45*a^8*b^2*x^4)/4 + 24*a^7*b^3*x^5 + 35*
a^6*b^4*x^6 + 36*a^5*b^5*x^7 + (105*a^4*b^6*x^8)/4 + (40*a^3*b^7*x^9)/3 + (
9*a^2*b^8*x^10)/2 + (10*a*b^9*x^11)/11 + (b^10*x^12)/12
```

Maple [B] time = 0.001, size = 113, normalized size = 3.8

$$\frac{b^{10}x^{12}}{12}+\frac{10\,ab^{9}x^{11}}{11}+\frac{9\,a^{2}b^{8}x^{10}}{2}+\frac{40\,a^{3}b^{7}x^{9}}{3}+\frac{105\,a^{4}b^{6}x^{8}}{4}+36\,a^{5}b^{5}x^{7}+35\,a^{6}b^{4}x^{6}+24\,a^{7}b^{3}x^{5}+\frac{45\,a^{8}b^{2}x^{4}}{4}+\frac{10\,a^{9}b}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^10,x)
```

```
[Out] 1/12*b^10*x^12+10/11*a*b^9*x^11+9/2*a^2*b^8*x^10+40/3*a^3*b^7*x^9+105/4*a^4
*b^6*x^8+36*a^5*b^5*x^7+35*a^6*b^4*x^6+24*a^7*b^3*x^5+45/4*a^8*b^2*x^4+10/3
*a^9*b*x^3+1/2*a^10*x^2
```

Maxima [B] time = 1.02693, size = 151, normalized size = 5.03

$$\frac{1}{12}\,b^{10}x^{12}+\frac{10}{11}\,ab^{9}x^{11}+\frac{9}{2}\,a^{2}b^{8}x^{10}+\frac{40}{3}\,a^{3}b^{7}x^{9}+\frac{105}{4}\,a^{4}b^{6}x^{8}+36\,a^{5}b^{5}x^{7}+35\,a^{6}b^{4}x^{6}+24\,a^{7}b^{3}x^{5}+\frac{45}{4}\,a^{8}b^{2}x^{4}+\frac{10}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 + 1
05/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/4*
a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

Fricas [B] time = 1.42404, size = 263, normalized size = 8.77

$$\frac{1}{12}x^{12}b^{10}+\frac{10}{11}x^{11}b^{9}a+\frac{9}{2}x^{10}b^{8}a^{2}+\frac{40}{3}x^{9}b^{7}a^{3}+\frac{105}{4}x^{8}b^{6}a^{4}+36x^{7}b^{5}a^{5}+35x^{6}b^{4}a^{6}+24x^{5}b^{3}a^{7}+\frac{45}{4}x^{4}b^{2}a^{8}+\frac{10}{3}x^{3}ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^10 + 10/11*x^11*b^9*a + 9/2*x^10*b^8*a^2 + 40/3*x^9*b^7*a^3 + 1
05/4*x^8*b^6*a^4 + 36*x^7*b^5*a^5 + 35*x^6*b^4*a^6 + 24*x^5*b^3*a^7 + 45/4*
x^4*b^2*a^8 + 10/3*x^3*b*a^9 + 1/2*x^2*a^10
```

Sympy [B] time = 0.111035, size = 129, normalized size = 4.3

$$\frac{a^{10}x^{2}}{2}+\frac{10a^{9}bx^{3}}{3}+\frac{45a^{8}b^{2}x^{4}}{4}+24a^{7}b^{3}x^{5}+35a^{6}b^{4}x^{6}+36a^{5}b^{5}x^{7}+\frac{105a^{4}b^{6}x^{8}}{4}+\frac{40a^{3}b^{7}x^{9}}{3}+\frac{9a^{2}b^{8}x^{10}}{2}+\frac{10ab^{9}x^{11}}{11}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**10,x)
```

```
[Out] a**10*x**2/2 + 10*a**9*b*x**3/3 + 45*a**8*b**2*x**4/4 + 24*a**7*b**3*x**5 +
 35*a**6*b**4*x**6 + 36*a**5*b**5*x**7 + 105*a**4*b**6*x**8/4 + 40*a**3*b**
7*x**9/3 + 9*a**2*b**8*x**10/2 + 10*a*b**9*x**11/11 + b**10*x**12/12
```

Giac [B] time = 1.18686, size = 151, normalized size = 5.03

$$\frac{1}{12}b^{10}x^{12} + \frac{10}{11}ab^9x^{11} + \frac{9}{2}a^2b^8x^{10} + \frac{40}{3}a^3b^7x^9 + \frac{105}{4}a^4b^6x^8 + 36a^5b^5x^7 + 35a^6b^4x^6 + 24a^7b^3x^5 + \frac{45}{4}a^8b^2x^4 + \frac{10}{3}a^9b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 + 1
05/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/4*
a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

3.134 $\int (a+bx)^{10}\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^{11}}{11b}$$

```
[Out] (a + b*x)^11/(11*b)
```

Rubi [A] time = 0.0015093, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{10}\,dx = \frac{(a+bx)^{11}}{11b}$$

Mathematica [A] time = 0.0015456, size = 14, normalized size = 1.

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

Maple [A] time = 0., size = 13, normalized size = 0.9

$$\frac{(bx+a)^{11}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10,x)
```

```
[Out] 1/11*(b*x+a)^11/b
```

Maxima [A] time = 1.01827, size = 16, normalized size = 1.14

$$\frac{(bx+a)^{11}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/11*(b*x + a)^11/b
```

Fricas [B] time = 1.64011, size = 230, normalized size = 16.43

$$\frac{1}{11}x^{11}b^{10} + x^{10}b^9a + 5x^9b^8a^2 + 15x^8b^7a^3 + 30x^7b^6a^4 + 42x^6b^5a^5 + 42x^5b^4a^6 + 30x^4b^3a^7 + 15x^3b^2a^8 + 5x^2ba^9 + xa^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^10 + x^10*b^9*a + 5*x^9*b^8*a^2 + 15*x^8*b^7*a^3 + 30*x^7*b^6*a
^4 + 42*x^6*b^5*a^5 + 42*x^5*b^4*a^6 + 30*x^4*b^3*a^7 + 15*x^3*b^2*a^8 + 5*
x^2*b*a^9 + x*a^10
```

Sympy [B] time = 0.093853, size = 114, normalized size = 8.14

$$a^{10}x + 5a^9bx^2 + 15a^8b^2x^3 + 30a^7b^3x^4 + 42a^6b^4x^5 + 42a^5b^5x^6 + 30a^4b^6x^7 + 15a^3b^7x^8 + 5a^2b^8x^9 + ab^9x^{10} + \frac{b^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10,x)
```

```
[Out] a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 + 42*a**6*b
**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a**3*b**7*x**8 + 5*a*
*2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11
```

Giac [A] time = 1.19818, size = 16, normalized size = 1.14

$$\frac{(bx+a)^{11}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/11*(b*x + a)^11/b
```

3.135 $\int \frac{(a+bx)^{10}}{x} \, dx$

Optimal. Leaf size=122

$$\frac{45}{2}a^8b^2x^2 + 40a^7b^3x^3 + \frac{105}{2}a^6b^4x^4 + \frac{252}{5}a^5b^5x^5 + 35a^4b^6x^6 + \frac{120}{7}a^3b^7x^7 + \frac{45}{8}a^2b^8x^8 + 10a^9bx + a^{10}\log(x) + \frac{10}{9}ab^9$$

```
[Out] 10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (2
52*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)
/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^10*Log[x]
```

Rubi [A] time = 0.0433098, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{45}{2}a^8b^2x^2 + 40a^7b^3x^3 + \frac{105}{2}a^6b^4x^4 + \frac{252}{5}a^5b^5x^5 + 35a^4b^6x^6 + \frac{120}{7}a^3b^7x^7 + \frac{45}{8}a^2b^8x^8 + 10a^9bx + a^{10}\log(x) + \frac{10}{9}ab^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x,x]
```

```
[Out] 10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (2
52*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)
/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^10*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x} \, dx = \int \left(10a^9b + \frac{a^{10}}{x} + 45a^8b^2x + 120a^7b^3x^2 + 210a^6b^4x^3 + 252a^5b^5x^4 + 210a^4b^6x^5 + 120a^3b^7x^6 + 45a\right.$$
$$= 10a^9bx + \frac{45}{2}a^8b^2x^2 + 40a^7b^3x^3 + \frac{105}{2}a^6b^4x^4 + \frac{252}{5}a^5b^5x^5 + 35a^4b^6x^6 + \frac{120}{7}a^3b^7x^7 + \frac{45}{8}a^2b^8x^8 +$$

Mathematica [A] time = 0.0048686, size = 122, normalized size = 1.

$$\frac{45}{2}a^8b^2x^2 + 40a^7b^3x^3 + \frac{105}{2}a^6b^4x^4 + \frac{252}{5}a^5b^5x^5 + 35a^4b^6x^6 + \frac{120}{7}a^3b^7x^7 + \frac{45}{8}a^2b^8x^8 + 10a^9bx + a^{10}\log(x) + \frac{10}{9}ab^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x,x]
```

```
[Out] 10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (2
52*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)
```

```
/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^10*Log[x]
```

Maple [A] time = 0.003, size = 109, normalized size = 0.9

$$10\,a^{9}bx+\frac{45\,a^{8}b^{2}x^{2}}{2}+40\,a^{7}b^{3}x^{3}+\frac{105\,a^{6}b^{4}x^{4}}{2}+\frac{252\,a^{5}b^{5}x^{5}}{5}+35\,a^{4}b^{6}x^{6}+\frac{120\,a^{3}b^{7}x^{7}}{7}+\frac{45\,a^{2}b^{8}x^{8}}{8}+\frac{10\,ab^{9}x^{9}}{9}+\frac{b^{10}x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x,x)
```

```
[Out] 10*a^9*b*x+45/2*a^8*b^2*x^2+40*a^7*b^3*x^3+105/2*a^6*b^4*x^4+252/5*a^5*b^5*
x^5+35*a^4*b^6*x^6+120/7*a^3*b^7*x^7+45/8*a^2*b^8*x^8+10/9*a*b^9*x^9+1/10*b
^10*x^10+a^10*ln(x)
```

Maxima [A] time = 1.04289, size = 146, normalized size = 1.2

$$\frac{1}{10}\,b^{10}x^{10}+\frac{10}{9}\,ab^{9}x^{9}+\frac{45}{8}\,a^{2}b^{8}x^{8}+\frac{120}{7}\,a^{3}b^{7}x^{7}+35\,a^{4}b^{6}x^{6}+\frac{252}{5}\,a^{5}b^{5}x^{5}+\frac{105}{2}\,a^{6}b^{4}x^{4}+40\,a^{7}b^{3}x^{3}+\frac{45}{2}\,a^{8}b^{2}x^{2}+10\,a^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(x)
```

Fricas [A] time = 1.8874, size = 259, normalized size = 2.12

$$\frac{1}{10}\,b^{10}x^{10}+\frac{10}{9}\,ab^{9}x^{9}+\frac{45}{8}\,a^{2}b^{8}x^{8}+\frac{120}{7}\,a^{3}b^{7}x^{7}+35\,a^{4}b^{6}x^{6}+\frac{252}{5}\,a^{5}b^{5}x^{5}+\frac{105}{2}\,a^{6}b^{4}x^{4}+40\,a^{7}b^{3}x^{3}+\frac{45}{2}\,a^{8}b^{2}x^{2}+10\,a^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(x)
```

Sympy [A] time = 0.553125, size = 126, normalized size = 1.03

$$a^{10}\log{(x)}+10a^{9}bx+\frac{45a^{8}b^{2}x^{2}}{2}+40a^{7}b^{3}x^{3}+\frac{105a^{6}b^{4}x^{4}}{2}+\frac{252a^{5}b^{5}x^{5}}{5}+35a^{4}b^{6}x^{6}+\frac{120a^{3}b^{7}x^{7}}{7}+\frac{45a^{2}b^{8}x^{8}}{8}+\frac{10ab^{9}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x,x)
```

```
[Out] a**10*log(x) + 10*a**9*b*x + 45*a**8*b**2*x**2/2 + 40*a**7*b**3*x**3 + 105*
a**6*b**4*x**4/2 + 252*a**5*b**5*x**5/5 + 35*a**4*b**6*x**6 + 120*a**3*b**7
*x**7/7 + 45*a**2*b**8*x**8/8 + 10*a*b**9*x**9/9 + b**10*x**10/10
```

Giac [A] time = 1.13069, size = 147, normalized size = 1.2

$$\frac{1}{10}b^{10}x^{10} + \frac{10}{9}ab^9x^9 + \frac{45}{8}a^2b^8x^8 + \frac{120}{7}a^3b^7x^7 + 35\,a^4b^6x^6 + \frac{252}{5}a^5b^5x^5 + \frac{105}{2}a^6b^4x^4 + 40\,a^7b^3x^3 + \frac{45}{2}a^8b^2x^2 + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="giac")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(abs(x))
```

3.136 $\int \frac{(a+bx)^{10}}{x^2}\,dx$

Optimal. Leaf size=115

$$60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+45a^8b^2x+10a^9b\log(x)-\frac{a^{10}}{x}+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}$$

```
[Out] -(a^10/x) + 45*a^8*b^2*x + 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4
 + 42*a^4*b^6*x^5 + 20*a^3*b^7*x^6 + (45*a^2*b^8*x^7)/7 + (5*a*b^9*x^8)/4 +
 (b^10*x^9)/9 + 10*a^9*b*Log[x]
```

Rubi [A] time = 0.0468345, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+45a^8b^2x+10a^9b\log(x)-\frac{a^{10}}{x}+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^2,x]
```

```
[Out] -(a^10/x) + 45*a^8*b^2*x + 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4
 + 42*a^4*b^6*x^5 + 20*a^3*b^7*x^6 + (45*a^2*b^8*x^7)/7 + (5*a*b^9*x^8)/4 +
 (b^10*x^9)/9 + 10*a^9*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^2}\,dx=\int \left(45a^8b^2+\frac{a^{10}}{x^2}+\frac{10a^9b}{x}+120a^7b^3x+210a^6b^4x^2+252a^5b^5x^3+210a^4b^6x^4+120a^3b^7x^5+45a^2b^8x\right.$$
$$=-\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+$$

Mathematica [A] time = 0.0115629, size = 115, normalized size = 1.

$$60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+45a^8b^2x+10a^9b\log(x)-\frac{a^{10}}{x}+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^2,x]
```

```
[Out] -(a^10/x) + 45*a^8*b^2*x + 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4
 + 42*a^4*b^6*x^5 + 20*a^3*b^7*x^6 + (45*a^2*b^8*x^7)/7 + (5*a*b^9*x^8)/4 +
```

```
(b^10*x^9)/9 + 10*a^9*b*Log[x]
```

Maple [A] time = 0.007, size = 110, normalized size = 1.

$$-\frac{a^{10}}{x}+45\,a^{8}b^{2}x+60\,a^{7}b^{3}x^{2}+70\,a^{6}b^{4}x^{3}+63\,a^{5}b^{5}x^{4}+42\,a^{4}b^{6}x^{5}+20\,a^{3}b^{7}x^{6}+\frac{45\,a^{2}b^{8}x^{7}}{7}+\frac{5\,ab^{9}x^{8}}{4}+\frac{b^{10}x^{9}}{9}+10\,a^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^2,x)
```

```
[Out] -a^10/x+45*a^8*b^2*x+60*a^7*b^3*x^2+70*a^6*b^4*x^3+63*a^5*b^5*x^4+42*a^4*b^
6*x^5+20*a^3*b^7*x^6+45/7*a^2*b^8*x^7+5/4*a*b^9*x^8+1/9*b^10*x^9+10*a^9*b*l
n(x)
```

Maxima [A] time = 1.01666, size = 147, normalized size = 1.28

$$\frac{1}{9}\,b^{10}x^{9}+\frac{5}{4}\,ab^{9}x^{8}+\frac{45}{7}\,a^{2}b^{8}x^{7}+20\,a^{3}b^{7}x^{6}+42\,a^{4}b^{6}x^{5}+63\,a^{5}b^{5}x^{4}+70\,a^{6}b^{4}x^{3}+60\,a^{7}b^{3}x^{2}+45\,a^{8}b^{2}x+10\,a^{9}b\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^10*x^9 + 5/4*a*b^9*x^8 + 45/7*a^2*b^8*x^7 + 20*a^3*b^7*x^6 + 42*a^4*b
^6*x^5 + 63*a^5*b^5*x^4 + 70*a^6*b^4*x^3 + 60*a^7*b^3*x^2 + 45*a^8*b^2*x +
10*a^9*b*log(x) - a^10/x
```

Fricas [A] time = 1.7833, size = 285, normalized size = 2.48

$$\frac{28\,b^{10}x^{10}+315\,ab^{9}x^{9}+1620\,a^{2}b^{8}x^{8}+5040\,a^{3}b^{7}x^{7}+10584\,a^{4}b^{6}x^{6}+15876\,a^{5}b^{5}x^{5}+17640\,a^{6}b^{4}x^{4}+15120\,a^{7}b^{3}x^{3}+}{252\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^10*x^10 + 315*a*b^9*x^9 + 1620*a^2*b^8*x^8 + 5040*a^3*b^7*x^7 +
 10584*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 17640*a^6*b^4*x^4 + 15120*a^7*b^3*
x^3 + 11340*a^8*b^2*x^2 + 2520*a^9*b*x*log(x) - 252*a^10)/x
```

Sympy [A] time = 0.484801, size = 117, normalized size = 1.02

$$-\frac{a^{10}}{x}+10a^{9}b\log{(x)}+45a^{8}b^{2}x+60a^{7}b^{3}x^{2}+70a^{6}b^{4}x^{3}+63a^{5}b^{5}x^{4}+42a^{4}b^{6}x^{5}+20a^{3}b^{7}x^{6}+\frac{45a^{2}b^{8}x^{7}}{7}+\frac{5ab^{9}x^{8}}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**2,x)
```

```
[Out] -a**10/x + 10*a**9*b*log(x) + 45*a**8*b**2*x + 60*a**7*b**3*x**2 + 70*a**6*
b**4*x**3 + 63*a**5*b**5*x**4 + 42*a**4*b**6*x**5 + 20*a**3*b**7*x**6 + 45*
a**2*b**8*x**7/7 + 5*a*b**9*x**8/4 + b**10*x**9/9
```

Giac [A] time = 1.15766, size = 149, normalized size = 1.3

$$\frac{1}{9}b^{10}x^9 + \frac{5}{4}ab^9x^8 + \frac{45}{7}a^2b^8x^7 + 20\,a^3b^7x^6 + 42\,a^4b^6x^5 + 63\,a^5b^5x^4 + 70\,a^6b^4x^3 + 60\,a^7b^3x^2 + 45\,a^8b^2x + 10\,a^9b\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^10*x^9 + 5/4*a*b^9*x^8 + 45/7*a^2*b^8*x^7 + 20*a^3*b^7*x^6 + 42*a^4*b
^6*x^5 + 63*a^5*b^5*x^4 + 70*a^6*b^4*x^3 + 60*a^7*b^3*x^2 + 45*a^8*b^2*x +
10*a^9*b*log(abs(x)) - a^10/x
```

3.137 $\int \frac{(a+bx)^{10}}{x^3} \, dx$

Optimal. Leaf size=119

$$105a^6b^4x^2 + 84a^5b^5x^3 + \frac{105}{2}a^4b^6x^4 + 24a^3b^7x^5 + \frac{15}{2}a^2b^8x^6 + 120a^7b^3x + 45a^8b^2\log(x) - \frac{10a^9b}{x} - \frac{a^{10}}{2x^2} + \frac{10}{7}ab^9x^7 +$$

```
[Out] -a^10/(2*x^2) - (10*a^9*b)/x + 120*a^7*b^3*x + 105*a^6*b^4*x^2 + 84*a^5*b^5
*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5 + (15*a^2*b^8*x^6)/2 + (10*a*b^
9*x^7)/7 + (b^10*x^8)/8 + 45*a^8*b^2*Log[x]
```

Rubi [A] time = 0.0490299, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$105a^6b^4x^2 + 84a^5b^5x^3 + \frac{105}{2}a^4b^6x^4 + 24a^3b^7x^5 + \frac{15}{2}a^2b^8x^6 + 120a^7b^3x + 45a^8b^2\log(x) - \frac{10a^9b}{x} - \frac{a^{10}}{2x^2} + \frac{10}{7}ab^9x^7 +$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^3,x]
```

```
[Out] -a^10/(2*x^2) - (10*a^9*b)/x + 120*a^7*b^3*x + 105*a^6*b^4*x^2 + 84*a^5*b^5
*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5 + (15*a^2*b^8*x^6)/2 + (10*a*b^
9*x^7)/7 + (b^10*x^8)/8 + 45*a^8*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^3} \, dx &= \int \left(120a^7b^3 + \frac{a^{10}}{x^3} + \frac{10a^9b}{x^2} + \frac{45a^8b^2}{x} + 210a^6b^4x + 252a^5b^5x^2 + 210a^4b^6x^3 + 120a^3b^7x^4 + 45a^2b^8\right.\\ &= -\frac{a^{10}}{2x^2} - \frac{10a^9b}{x} + 120a^7b^3x + 105a^6b^4x^2 + 84a^5b^5x^3 + \frac{105}{2}a^4b^6x^4 + 24a^3b^7x^5 + \frac{15}{2}a^2b^8x^6 + \frac{10}{7}ab^9\end{aligned}$$

Mathematica [A] time = 0.0091698, size = 119, normalized size = 1.

$$105a^6b^4x^2 + 84a^5b^5x^3 + \frac{105}{2}a^4b^6x^4 + 24a^3b^7x^5 + \frac{15}{2}a^2b^8x^6 + 120a^7b^3x + 45a^8b^2\log(x) - \frac{10a^9b}{x} - \frac{a^{10}}{2x^2} + \frac{10}{7}ab^9x^7 +$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^3,x]
```

```
[Out] -a^10/(2*x^2) - (10*a^9*b)/x + 120*a^7*b^3*x + 105*a^6*b^4*x^2 + 84*a^5*b^5
*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5 + (15*a^2*b^8*x^6)/2 + (10*a*b^
```

```
9*x^7)/7 + (b^10*x^8)/8 + 45*a^8*b^2*Log[x]
```

Maple [A] time = 0.006, size = 110, normalized size = 0.9

$$-\frac{a^{10}}{2\,x^{2}}-10\,\frac{a^{9}b}{x}+120\,a^{7}b^{3}x+105\,a^{6}b^{4}x^{2}+84\,a^{5}b^{5}x^{3}+\frac{105\,a^{4}b^{6}x^{4}}{2}+24\,a^{3}b^{7}x^{5}+\frac{15\,a^{2}b^{8}x^{6}}{2}+\frac{10\,ab^{9}x^{7}}{7}+\frac{b^{10}x^{8}}{8}+45\,a^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^3,x)
```

```
[Out] -1/2*a^10/x^2-10*a^9*b/x+120*a^7*b^3*x+105*a^6*b^4*x^2+84*a^5*b^5*x^3+105/2
*a^4*b^6*x^4+24*a^3*b^7*x^5+15/2*a^2*b^8*x^6+10/7*a*b^9*x^7+1/8*b^10*x^8+45
*a^8*b^2*ln(x)
```

Maxima [A] time = 1.01244, size = 146, normalized size = 1.23

$$\frac{1}{8}\,b^{10}x^{8}+\frac{10}{7}\,ab^{9}x^{7}+\frac{15}{2}\,a^{2}b^{8}x^{6}+24\,a^{3}b^{7}x^{5}+\frac{105}{2}\,a^{4}b^{6}x^{4}+84\,a^{5}b^{5}x^{3}+105\,a^{6}b^{4}x^{2}+120\,a^{7}b^{3}x+45\,a^{8}b^{2}\log\left(x\right)-\frac{20\,a^{9}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^10*x^8 + 10/7*a*b^9*x^7 + 15/2*a^2*b^8*x^6 + 24*a^3*b^7*x^5 + 105/2*a
^4*b^6*x^4 + 84*a^5*b^5*x^3 + 105*a^6*b^4*x^2 + 120*a^7*b^3*x + 45*a^8*b^2*
log(x) - 1/2*(20*a^9*b*x + a^10)/x^2
```

Fricas [A] time = 1.6992, size = 273, normalized size = 2.29

$$\frac{7\,b^{10}x^{10}+80\,ab^{9}x^{9}+420\,a^{2}b^{8}x^{8}+1344\,a^{3}b^{7}x^{7}+2940\,a^{4}b^{6}x^{6}+4704\,a^{5}b^{5}x^{5}+5880\,a^{6}b^{4}x^{4}+6720\,a^{7}b^{3}x^{3}+2520\,a^{8}b^{2}x^{2}}{56\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^10*x^10 + 80*a*b^9*x^9 + 420*a^2*b^8*x^8 + 1344*a^3*b^7*x^7 + 294
0*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 5880*a^6*b^4*x^4 + 6720*a^7*b^3*x^3 + 25
20*a^8*b^2*x^2*log(x) - 560*a^9*b*x - 28*a^10)/x^2
```

Sympy [A] time = 0.51387, size = 121, normalized size = 1.02

$$45a^{8}b^{2}\log\left(x\right)+120a^{7}b^{3}x+105a^{6}b^{4}x^{2}+84a^{5}b^{5}x^{3}+\frac{105a^{4}b^{6}x^{4}}{2}+24a^{3}b^{7}x^{5}+\frac{15a^{2}b^{8}x^{6}}{2}+\frac{10ab^{9}x^{7}}{7}+\frac{b^{10}x^{8}}{8}-\frac{a^{10}+20a}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**3,x)
```

```
[Out] 45*a**8*b**2*log(x) + 120*a**7*b**3*x + 105*a**6*b**4*x**2 + 84*a**5*b**5*x
**3 + 105*a**4*b**6*x**4/2 + 24*a**3*b**7*x**5 + 15*a**2*b**8*x**6/2 + 10*a
*b**9*x**7/7 + b**10*x**8/8 - (a**10 + 20*a**9*b*x)/(2*x**2)
```

Giac [A] time = 1.18559, size = 147, normalized size = 1.24

$$\frac{1}{8}\,b^{10}x^{8} + \frac{10}{7}\,ab^{9}x^{7} + \frac{15}{2}\,a^{2}b^{8}x^{6} + 24\,a^{3}b^{7}x^{5} + \frac{105}{2}\,a^{4}b^{6}x^{4} + 84\,a^{5}b^{5}x^{3} + 105\,a^{6}b^{4}x^{2} + 120\,a^{7}b^{3}x + 45\,a^{8}b^{2}\log\left(|x|\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^10*x^8 + 10/7*a*b^9*x^7 + 15/2*a^2*b^8*x^6 + 24*a^3*b^7*x^5 + 105/2*a
^4*b^6*x^4 + 84*a^5*b^5*x^3 + 105*a^6*b^4*x^2 + 120*a^7*b^3*x + 45*a^8*b^2*
log(abs(x)) - 1/2*(20*a^9*b*x + a^10)/x^2
```

3.138 $\int \frac{(a+bx)^{10}}{x^4} dx$

Optimal. Leaf size=115

$$126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5-\frac{45a^8b^2}{x}+210a^6b^4x+120a^7b^3\log(x)-\frac{5a^9b}{x^2}-\frac{a^{10}}{3x^3}+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}$$

```
[Out] -a^10/(3*x^3) - (5*a^9*b)/x^2 - (45*a^8*b^2)/x + 210*a^6*b^4*x + 126*a^5*b^
5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2*b^8*x^5 + (5*a*b^9*x^6)/3 +
 (b^10*x^7)/7 + 120*a^7*b^3*Log[x]
```

Rubi [A] time = 0.047252, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5-\frac{45a^8b^2}{x}+210a^6b^4x+120a^7b^3\log(x)-\frac{5a^9b}{x^2}-\frac{a^{10}}{3x^3}+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^4,x]
```

```
[Out] -a^10/(3*x^3) - (5*a^9*b)/x^2 - (45*a^8*b^2)/x + 210*a^6*b^4*x + 126*a^5*b^
5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2*b^8*x^5 + (5*a*b^9*x^6)/3 +
 (b^10*x^7)/7 + 120*a^7*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^4}dx = \int \left(210a^6b^4+\frac{a^{10}}{x^4}+\frac{10a^9b}{x^3}+\frac{45a^8b^2}{x^2}+\frac{120a^7b^3}{x}+252a^5b^5x+210a^4b^6x^2+120a^3b^7x^3+45a^2b^8x^4+\right.$$
$$=-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}$$

Mathematica [A] time = 0.0121197, size = 115, normalized size = 1.

$$126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5-\frac{45a^8b^2}{x}+210a^6b^4x+120a^7b^3\log(x)-\frac{5a^9b}{x^2}-\frac{a^{10}}{3x^3}+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^4,x]
```

```
[Out] -a^10/(3*x^3) - (5*a^9*b)/x^2 - (45*a^8*b^2)/x + 210*a^6*b^4*x + 126*a^5*b^
5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2*b^8*x^5 + (5*a*b^9*x^6)/3 +
```

```
(b^10*x^7)/7 + 120*a^7*b^3*Log[x]
```

Maple [A] time = 0.007, size = 110, normalized size = 1.

$$-\frac{a^{10}}{3x^3}-5\frac{a^9b}{x^2}-45\frac{a^8b^2}{x}+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5ab^9x^6}{3}+\frac{b^{10}x^7}{7}+120a^7b^3\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^4,x)
```

```
[Out] -1/3*a^10/x^3-5*a^9*b/x^2-45*a^8*b^2/x+210*a^6*b^4*x+126*a^5*b^5*x^2+70*a^4
*b^6*x^3+30*a^3*b^7*x^4+9*a^2*b^8*x^5+5/3*a*b^9*x^6+1/7*b^10*x^7+120*a^7*b^
3*ln(x)
```

Maxima [A] time = 1.13313, size = 146, normalized size = 1.27

$$\frac{1}{7}b^{10}x^7+\frac{5}{3}ab^9x^6+9a^2b^8x^5+30a^3b^7x^4+70a^4b^6x^3+126a^5b^5x^2+210a^6b^4x+120a^7b^3\log(x)-\frac{135a^8b^2x^2+15a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b^10*x^7 + 5/3*a*b^9*x^6 + 9*a^2*b^8*x^5 + 30*a^3*b^7*x^4 + 70*a^4*b^6*
x^3 + 126*a^5*b^5*x^2 + 210*a^6*b^4*x + 120*a^7*b^3*log(x) - 1/3*(135*a^8*b
^2*x^2 + 15*a^9*b*x + a^10)/x^3
```

Fricas [A] time = 1.77161, size = 269, normalized size = 2.34

$$\frac{3b^{10}x^{10}+35ab^9x^9+189a^2b^8x^8+630a^3b^7x^7+1470a^4b^6x^6+2646a^5b^5x^5+4410a^6b^4x^4+2520a^7b^3x^3\log(x)-94}{21x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^10*x^10 + 35*a*b^9*x^9 + 189*a^2*b^8*x^8 + 630*a^3*b^7*x^7 + 1470
*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 4410*a^6*b^4*x^4 + 2520*a^7*b^3*x^3*log(x
) - 945*a^8*b^2*x^2 - 105*a^9*b*x - 7*a^10)/x^3
```

Sympy [A] time = 0.565141, size = 117, normalized size = 1.02

$$120a^7b^3\log(x)+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5ab^9x^6}{3}+\frac{b^{10}x^7}{7}-\frac{a^{10}+15a^9bx+135a^8}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**4,x)
```

```
[Out] 120*a**7*b**3*log(x) + 210*a**6*b**4*x + 126*a**5*b**5*x**2 + 70*a**4*b**6*
x**3 + 30*a**3*b**7*x**4 + 9*a**2*b**8*x**5 + 5*a*b**9*x**6/3 + b**10*x**7/
7 - (a**10 + 15*a**9*b*x + 135*a**8*b**2*x**2)/(3*x**3)
```

Giac [A] time = 1.23735, size = 147, normalized size = 1.28

$$\frac{1}{7} b^{10} x^7 + \frac{5}{3} a b^9 x^6 + 9 a^2 b^8 x^5 + 30 a^3 b^7 x^4 + 70 a^4 b^6 x^3 + 126 a^5 b^5 x^2 + 210 a^6 b^4 x + 120 a^7 b^3 \log(|x|) - \frac{135 a^8 b^2 x^2 + 15 a^9 b}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^10*x^7 + 5/3*a*b^9*x^6 + 9*a^2*b^8*x^5 + 30*a^3*b^7*x^4 + 70*a^4*b^6*
x^3 + 126*a^5*b^5*x^2 + 210*a^6*b^4*x + 120*a^7*b^3*log(abs(x)) - 1/3*(135*
a^8*b^2*x^2 + 15*a^9*b*x + a^10)/x^3
```

3.139 $\int \frac{(a+bx)^{10}}{x^5}\, dx$

Optimal. Leaf size=119

$$-\frac{45a^8b^2}{2x^2} + 105a^4b^6x^2 + 40a^3b^7x^3 + \frac{45}{4}a^2b^8x^4 - \frac{120a^7b^3}{x} + 252a^5b^5x + 210a^6b^4\log(x) - \frac{10a^9b}{3x^3} - \frac{a^{10}}{4x^4} + 2ab^9x^5 + \frac{b^{10}}{6}$$

```
[Out] -a^10/(4*x^4) - (10*a^9*b)/(3*x^3) - (45*a^8*b^2)/(2*x^2) - (120*a^7*b^3)/x
 + 252*a^5*b^5*x + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 + (45*a^2*b^8*x^4)/4 +
2*a*b^9*x^5 + (b^10*x^6)/6 + 210*a^6*b^4*Log[x]
```

Rubi [A] time = 0.0494338, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{2x^2} + 105a^4b^6x^2 + 40a^3b^7x^3 + \frac{45}{4}a^2b^8x^4 - \frac{120a^7b^3}{x} + 252a^5b^5x + 210a^6b^4\log(x) - \frac{10a^9b}{3x^3} - \frac{a^{10}}{4x^4} + 2ab^9x^5 + \frac{b^{10}}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^5,x]
```

```
[Out] -a^10/(4*x^4) - (10*a^9*b)/(3*x^3) - (45*a^8*b^2)/(2*x^2) - (120*a^7*b^3)/x
 + 252*a^5*b^5*x + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 + (45*a^2*b^8*x^4)/4 +
2*a*b^9*x^5 + (b^10*x^6)/6 + 210*a^6*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^5}\, dx &= \int \left(252a^5b^5 + \frac{a^{10}}{x^5} + \frac{10a^9b}{x^4} + \frac{45a^8b^2}{x^3} + \frac{120a^7b^3}{x^2} + \frac{210a^6b^4}{x} + 210a^4b^6x + 120a^3b^7x^2 + 45a^2b^8x^3 \ldots\right.\\ &= -\frac{a^{10}}{4x^4} - \frac{10a^9b}{3x^3} - \frac{45a^8b^2}{2x^2} - \frac{120a^7b^3}{x} + 252a^5b^5x + 105a^4b^6x^2 + 40a^3b^7x^3 + \frac{45}{4}a^2b^8x^4 + 2ab^9x^5 + \ldots\end{aligned}$$

Mathematica [A] time = 0.0114508, size = 119, normalized size = 1.

$$-\frac{45a^8b^2}{2x^2} + 105a^4b^6x^2 + 40a^3b^7x^3 + \frac{45}{4}a^2b^8x^4 - \frac{120a^7b^3}{x} + 252a^5b^5x + 210a^6b^4\log(x) - \frac{10a^9b}{3x^3} - \frac{a^{10}}{4x^4} + 2ab^9x^5 + \frac{b^{10}}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^5,x]
```

```
[Out] -a^10/(4*x^4) - (10*a^9*b)/(3*x^3) - (45*a^8*b^2)/(2*x^2) - (120*a^7*b^3)/x
 + 252*a^5*b^5*x + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 + (45*a^2*b^8*x^4)/4 +
```

```
2*a*b^9*x^5 + (b^10*x^6)/6 + 210*a^6*b^4*Log[x]
```

Maple [A] time = 0.006, size = 110, normalized size = 0.9

$$-\frac{a^{10}}{4\,x^{4}}-\frac{10\,a^{9}b}{3\,x^{3}}-\frac{45\,a^{8}b^{2}}{2\,x^{2}}-120\,\frac{a^{7}b^{3}}{x}+252\,a^{5}b^{5}x+105\,a^{4}b^{6}x^{2}+40\,a^{3}b^{7}x^{3}+\frac{45\,a^{2}b^{8}x^{4}}{4}+2\,ab^{9}x^{5}+\frac{b^{10}x^{6}}{6}+210\,a^{6}b^{4}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^5,x)
```

```
[Out] -1/4*a^10/x^4-10/3*a^9*b/x^3-45/2*a^8*b^2/x^2-120*a^7*b^3/x+252*a^5*b^5*x+1
05*a^4*b^6*x^2+40*a^3*b^7*x^3+45/4*a^2*b^8*x^4+2*a*b^9*x^5+1/6*b^10*x^6+210
*a^6*b^4*ln(x)
```

Maxima [A] time = 1.04482, size = 149, normalized size = 1.25

$$\frac{1}{6}\,b^{10}x^{6}+2\,ab^{9}x^{5}+\frac{45}{4}\,a^{2}b^{8}x^{4}+40\,a^{3}b^{7}x^{3}+105\,a^{4}b^{6}x^{2}+252\,a^{5}b^{5}x+210\,a^{6}b^{4}\log\left(x\right)-\frac{1440\,a^{7}b^{3}x^{3}+270\,a^{8}b^{2}x^{2}+40}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^10*x^6 + 2*a*b^9*x^5 + 45/4*a^2*b^8*x^4 + 40*a^3*b^7*x^3 + 105*a^4*b^
6*x^2 + 252*a^5*b^5*x + 210*a^6*b^4*log(x) - 1/12*(1440*a^7*b^3*x^3 + 270*a
^8*b^2*x^2 + 40*a^9*b*x + 3*a^10)/x^4
```

Fricas [A] time = 1.78006, size = 267, normalized size = 2.24

$$\frac{2\,b^{10}x^{10}+24\,ab^{9}x^{9}+135\,a^{2}b^{8}x^{8}+480\,a^{3}b^{7}x^{7}+1260\,a^{4}b^{6}x^{6}+3024\,a^{5}b^{5}x^{5}+2520\,a^{6}b^{4}x^{4}\log\left(x\right)-1440\,a^{7}b^{3}x^{3}-270\,a^{8}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^10*x^10 + 24*a*b^9*x^9 + 135*a^2*b^8*x^8 + 480*a^3*b^7*x^7 + 1260
*a^4*b^6*x^6 + 3024*a^5*b^5*x^5 + 2520*a^6*b^4*x^4*log(x) - 1440*a^7*b^3*x^
3 - 270*a^8*b^2*x^2 - 40*a^9*b*x - 3*a^10)/x^4
```

Sympy [A] time = 0.641291, size = 119, normalized size = 1.

$$210a^{6}b^{4}\log\left(x\right)+252a^{5}b^{5}x+105a^{4}b^{6}x^{2}+40a^{3}b^{7}x^{3}+\frac{45a^{2}b^{8}x^{4}}{4}+2ab^{9}x^{5}+\frac{b^{10}x^{6}}{6}-\frac{3a^{10}+40a^{9}bx+270a^{8}b^{2}x^{2}+1440}{12x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**5,x)
```

```
[Out] 210*a**6*b**4*log(x) + 252*a**5*b**5*x + 105*a**4*b**6*x**2 + 40*a**3*b**7*
x**3 + 45*a**2*b**8*x**4/4 + 2*a*b**9*x**5 + b**10*x**6/6 - (3*a**10 + 40*a
**9*b*x + 270*a**8*b**2*x**2 + 1440*a**7*b**3*x**3)/(12*x**4)
```

Giac [A] time = 1.15546, size = 150, normalized size = 1.26

$$\frac{1}{6}\,b^{10}x^6 + 2\,ab^9x^5 + \frac{45}{4}\,a^2b^8x^4 + 40\,a^3b^7x^3 + 105\,a^4b^6x^2 + 252\,a^5b^5x + 210\,a^6b^4\log\left(|x|\right) - \frac{1440\,a^7b^3x^3 + 270\,a^8b^2x^2 +}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="giac")
```

```
[Out] 1/6*b^10*x^6 + 2*a*b^9*x^5 + 45/4*a^2*b^8*x^4 + 40*a^3*b^7*x^3 + 105*a^4*b^
6*x^2 + 252*a^5*b^5*x + 210*a^6*b^4*log(abs(x)) - 1/12*(1440*a^7*b^3*x^3 +
270*a^8*b^2*x^2 + 40*a^9*b*x + 3*a^10)/x^4
```

3.140 $\int \frac{(a+bx)^{10}}{x^6} dx$

Optimal. Leaf size=117

$$-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}+60a^3b^7x^2+15a^2b^8x^3-\frac{210a^6b^4}{x}+210a^4b^6x+252a^5b^5\log(x)-\frac{5a^9b}{2x^4}-\frac{a^{10}}{5x^5}+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}$$

```
[Out] -a^10/(5*x^5) - (5*a^9*b)/(2*x^4) - (15*a^8*b^2)/x^3 - (60*a^7*b^3)/x^2 - (
210*a^6*b^4)/x + 210*a^4*b^6*x + 60*a^3*b^7*x^2 + 15*a^2*b^8*x^3 + (5*a*b^9
*x^4)/2 + (b^10*x^5)/5 + 252*a^5*b^5*Log[x]
```

Rubi [A] time = 0.0518441, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}+60a^3b^7x^2+15a^2b^8x^3-\frac{210a^6b^4}{x}+210a^4b^6x+252a^5b^5\log(x)-\frac{5a^9b}{2x^4}-\frac{a^{10}}{5x^5}+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^6,x]
```

```
[Out] -a^10/(5*x^5) - (5*a^9*b)/(2*x^4) - (15*a^8*b^2)/x^3 - (60*a^7*b^3)/x^2 - (
210*a^6*b^4)/x + 210*a^4*b^6*x + 60*a^3*b^7*x^2 + 15*a^2*b^8*x^3 + (5*a*b^9
*x^4)/2 + (b^10*x^5)/5 + 252*a^5*b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^6}\,dx = \int \left(210a^4b^6+\frac{a^{10}}{x^6}+\frac{10a^9b}{x^5}+\frac{45a^8b^2}{x^4}+\frac{120a^7b^3}{x^3}+\frac{210a^6b^4}{x^2}+\frac{252a^5b^5}{x}+120a^3b^7x+45a^2b^8x^2+10a\right.$$
$$= -\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}+$$

Mathematica [A] time = 0.0125019, size = 117, normalized size = 1.

$$-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}+60a^3b^7x^2+15a^2b^8x^3-\frac{210a^6b^4}{x}+210a^4b^6x+252a^5b^5\log(x)-\frac{5a^9b}{2x^4}-\frac{a^{10}}{5x^5}+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^6,x]
```

```
[Out] -a^10/(5*x^5) - (5*a^9*b)/(2*x^4) - (15*a^8*b^2)/x^3 - (60*a^7*b^3)/x^2 - (
210*a^6*b^4)/x + 210*a^4*b^6*x + 60*a^3*b^7*x^2 + 15*a^2*b^8*x^3 + (5*a*b^9
```

```
*x^4)/2 + (b^10*x^5)/5 + 252*a^5*b^5*Log[x]
```

Maple [A] time = 0.007, size = 110, normalized size = 0.9

$$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-15\frac{a^8b^2}{x^3}-60\frac{a^7b^3}{x^2}-210\frac{a^6b^4}{x}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5ab^9x^4}{2}+\frac{b^{10}x^5}{5}+252a^5b^5\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^6,x)
```

```
[Out] -1/5*a^10/x^5-5/2*a^9*b/x^4-15*a^8*b^2/x^3-60*a^7*b^3/x^2-210*a^6*b^4/x+210
*a^4*b^6*x+60*a^3*b^7*x^2+15*a^2*b^8*x^3+5/2*a*b^9*x^4+1/5*b^10*x^5+252*a^5
*b^5*ln(x)
```

Maxima [A] time = 1.02008, size = 149, normalized size = 1.27

$$\frac{1}{5}b^{10}x^5+\frac{5}{2}ab^9x^4+15a^2b^8x^3+60a^3b^7x^2+210a^4b^6x+252a^5b^5\log(x)-\frac{2100a^6b^4x^4+600a^7b^3x^3+150a^8b^2x^2+}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b^10*x^5 + 5/2*a*b^9*x^4 + 15*a^2*b^8*x^3 + 60*a^3*b^7*x^2 + 210*a^4*b^
6*x + 252*a^5*b^5*log(x) - 1/10*(2100*a^6*b^4*x^4 + 600*a^7*b^3*x^3 + 150*a
^8*b^2*x^2 + 25*a^9*b*x + 2*a^10)/x^5
```

Fricas [A] time = 1.75786, size = 266, normalized size = 2.27

$$\frac{2b^{10}x^{10}+25ab^9x^9+150a^2b^8x^8+600a^3b^7x^7+2100a^4b^6x^6+2520a^5b^5x^5\log(x)-2100a^6b^4x^4-600a^7b^3x^3-150}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^10*x^10 + 25*a*b^9*x^9 + 150*a^2*b^8*x^8 + 600*a^3*b^7*x^7 + 2100
*a^4*b^6*x^6 + 2520*a^5*b^5*x^5*log(x) - 2100*a^6*b^4*x^4 - 600*a^7*b^3*x^3
 - 150*a^8*b^2*x^2 - 25*a^9*b*x - 2*a^10)/x^5
```

Sympy [A] time = 0.730664, size = 119, normalized size = 1.02

$$252a^5b^5\log(x)+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5ab^9x^4}{2}+\frac{b^{10}x^5}{5}-\frac{2a^{10}+25a^9bx+150a^8b^2x^2+600a^7b^3x^3+21}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**6,x)
```

```
[Out] 252*a**5*b**5*log(x) + 210*a**4*b**6*x + 60*a**3*b**7*x**2 + 15*a**2*b**8*x
**3 + 5*a*b**9*x**4/2 + b**10*x**5/5 - (2*a**10 + 25*a**9*b*x + 150*a**8*b*
*2*x**2 + 600*a**7*b**3*x**3 + 2100*a**6*b**4*x**4)/(10*x**5)
```

Giac [A] time = 1.18703, size = 150, normalized size = 1.28

$$\frac{1}{5}b^{10}x^5+\frac{5}{2}ab^9x^4+15a^2b^8x^3+60a^3b^7x^2+210a^4b^6x+252a^5b^5\log(|x|)-\frac{2100a^6b^4x^4+600a^7b^3x^3+150a^8b^2x^2+25}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^10*x^5 + 5/2*a*b^9*x^4 + 15*a^2*b^8*x^3 + 60*a^3*b^7*x^2 + 210*a^4*b^
6*x + 252*a^5*b^5*log(abs(x)) - 1/10*(2100*a^6*b^4*x^4 + 600*a^7*b^3*x^3 +
150*a^8*b^2*x^2 + 25*a^9*b*x + 2*a^10)/x^5
```

3.141 $\int \frac{(a+bx)^{10}}{x^7} \, dx$

Optimal. Leaf size=119

$$-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}+\frac{45}{2}a^2b^8x^2-\frac{252a^5b^5}{x}+120a^3b^7x+210a^4b^6\log(x)-\frac{2a^9b}{x^5}-\frac{a^{10}}{6x^6}+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}$$

```
[Out] -a^10/(6*x^6) - (2*a^9*b)/x^5 - (45*a^8*b^2)/(4*x^4) - (40*a^7*b^3)/x^3 - (
105*a^6*b^4)/x^2 - (252*a^5*b^5)/x + 120*a^3*b^7*x + (45*a^2*b^8*x^2)/2 + (
10*a*b^9*x^3)/3 + (b^10*x^4)/4 + 210*a^4*b^6*Log[x]
```

Rubi [A] time = 0.0488931, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}+\frac{45}{2}a^2b^8x^2-\frac{252a^5b^5}{x}+120a^3b^7x+210a^4b^6\log(x)-\frac{2a^9b}{x^5}-\frac{a^{10}}{6x^6}+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^7,x]
```

```
[Out] -a^10/(6*x^6) - (2*a^9*b)/x^5 - (45*a^8*b^2)/(4*x^4) - (40*a^7*b^3)/x^3 - (
105*a^6*b^4)/x^2 - (252*a^5*b^5)/x + 120*a^3*b^7*x + (45*a^2*b^8*x^2)/2 + (
10*a*b^9*x^3)/3 + (b^10*x^4)/4 + 210*a^4*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^7}\,dx = \int \left(120a^3b^7+\frac{a^{10}}{x^7}+\frac{10a^9b}{x^6}+\frac{45a^8b^2}{x^5}+\frac{120a^7b^3}{x^4}+\frac{210a^6b^4}{x^3}+\frac{252a^5b^5}{x^2}+\frac{210a^4b^6}{x}+45a^2b^8x+1\right.$$

$$= -\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3+\frac{b^{10}x}{4}$$

Mathematica [A] time = 0.0058564, size = 119, normalized size = 1.

$$-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}+\frac{45}{2}a^2b^8x^2-\frac{252a^5b^5}{x}+120a^3b^7x+210a^4b^6\log(x)-\frac{2a^9b}{x^5}-\frac{a^{10}}{6x^6}+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^7,x]
```

```
[Out] -a^10/(6*x^6) - (2*a^9*b)/x^5 - (45*a^8*b^2)/(4*x^4) - (40*a^7*b^3)/x^3 - (
105*a^6*b^4)/x^2 - (252*a^5*b^5)/x + 120*a^3*b^7*x + (45*a^2*b^8*x^2)/2 + (
```

```
10*a*b^9*x^3)/3 + (b^10*x^4)/4 + 210*a^4*b^6*Log[x]
```

Maple [A] time = 0.007, size = 110, normalized size = 0.9

$$-\frac{a^{10}}{6\,x^{6}}-2\,\frac{a^{9}b}{x^{5}}-\frac{45\,a^{8}b^{2}}{4\,x^{4}}-40\,\frac{a^{7}b^{3}}{x^{3}}-105\,\frac{a^{6}b^{4}}{x^{2}}-252\,\frac{a^{5}b^{5}}{x}+120\,a^{3}b^{7}x+\frac{45\,a^{2}b^{8}x^{2}}{2}+\frac{10\,ab^{9}x^{3}}{3}+\frac{b^{10}x^{4}}{4}+210\,a^{4}b^{6}\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^7,x)
```

```
[Out] -1/6*a^10/x^6-2*a^9*b/x^5-45/4*a^8*b^2/x^4-40*a^7*b^3/x^3-105*a^6*b^4/x^2-2
52*a^5*b^5/x+120*a^3*b^7*x+45/2*a^2*b^8*x^2+10/3*a*b^9*x^3+1/4*b^10*x^4+210
*a^4*b^6*ln(x)
```

Maxima [A] time = 1.07376, size = 149, normalized size = 1.25

$$\frac{1}{4}\,b^{10}x^{4}+\frac{10}{3}\,ab^{9}x^{3}+\frac{45}{2}\,a^{2}b^{8}x^{2}+120\,a^{3}b^{7}x+210\,a^{4}b^{6}\log(x)-\frac{3024\,a^{5}b^{5}x^{5}+1260\,a^{6}b^{4}x^{4}+480\,a^{7}b^{3}x^{3}+135\,a^{8}b^{2}x^{2}+}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*b^10*x^4 + 10/3*a*b^9*x^3 + 45/2*a^2*b^8*x^2 + 120*a^3*b^7*x + 210*a^4*
b^6*log(x) - 1/12*(3024*a^5*b^5*x^5 + 1260*a^6*b^4*x^4 + 480*a^7*b^3*x^3 +
135*a^8*b^2*x^2 + 24*a^9*b*x + 2*a^10)/x^6
```

Fricas [A] time = 1.79039, size = 267, normalized size = 2.24

$$\frac{3\,b^{10}x^{10}+40\,ab^{9}x^{9}+270\,a^{2}b^{8}x^{8}+1440\,a^{3}b^{7}x^{7}+2520\,a^{4}b^{6}x^{6}\log(x)-3024\,a^{5}b^{5}x^{5}-1260\,a^{6}b^{4}x^{4}-480\,a^{7}b^{3}x^{3}-135\,a^{8}}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*x^10 + 40*a*b^9*x^9 + 270*a^2*b^8*x^8 + 1440*a^3*b^7*x^7 + 252
0*a^4*b^6*x^6*log(x) - 3024*a^5*b^5*x^5 - 1260*a^6*b^4*x^4 - 480*a^7*b^3*x^
3 - 135*a^8*b^2*x^2 - 24*a^9*b*x - 2*a^10)/x^6
```

Sympy [A] time = 0.835287, size = 121, normalized size = 1.02

$$210a^{4}b^{6}\log(x)+120a^{3}b^{7}x+\frac{45a^{2}b^{8}x^{2}}{2}+\frac{10ab^{9}x^{3}}{3}+\frac{b^{10}x^{4}}{4}-\frac{2a^{10}+24a^{9}bx+135a^{8}b^{2}x^{2}+480a^{7}b^{3}x^{3}+1260a^{6}b^{4}x^{4}+3}{12x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**7,x)
```

```
[Out] 210*a**4*b**6*log(x) + 120*a**3*b**7*x + 45*a**2*b**8*x**2/2 + 10*a*b**9*x*
*3/3 + b**10*x**4/4 - (2*a**10 + 24*a**9*b*x + 135*a**8*b**2*x**2 + 480*a**
7*b**3*x**3 + 1260*a**6*b**4*x**4 + 3024*a**5*b**5*x**5)/(12*x**6)
```

Giac [A] time = 1.1962, size = 150, normalized size = 1.26

$$\frac{1}{4}\,b^{10}x^4 + \frac{10}{3}\,ab^9x^3 + \frac{45}{2}\,a^2b^8x^2 + 120\,a^3b^7x + 210\,a^4b^6\log\left(|x|\right) - \frac{3024\,a^5b^5x^5 + 1260\,a^6b^4x^4 + 480\,a^7b^3x^3 + 135\,a^8b^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="giac")
```

```
[Out] 1/4*b^10*x^4 + 10/3*a*b^9*x^3 + 45/2*a^2*b^8*x^2 + 120*a^3*b^7*x + 210*a^4*
b^6*log(abs(x)) - 1/12*(3024*a^5*b^5*x^5 + 1260*a^6*b^4*x^4 + 480*a^7*b^3*x
^3 + 135*a^8*b^2*x^2 + 24*a^9*b*x + 2*a^10)/x^6
```

3.142 $\int \frac{(a+bx)^{10}}{x^8} dx$

Optimal. Leaf size=115

$$-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+120a^3b^7\log(x)-\frac{5a^9b}{3x^6}-\frac{a^{10}}{7x^7}+5ab^9x^2+\frac{b^{10}x^3}{3}$$

```
[Out] -a^10/(7*x^7) - (5*a^9*b)/(3*x^6) - (9*a^8*b^2)/x^5 - (30*a^7*b^3)/x^4 - (7
0*a^6*b^4)/x^3 - (126*a^5*b^5)/x^2 - (210*a^4*b^6)/x + 45*a^2*b^8*x + 5*a*b
^9*x^2 + (b^10*x^3)/3 + 120*a^3*b^7*Log[x]
```

Rubi [A] time = 0.0496511, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+120a^3b^7\log(x)-\frac{5a^9b}{3x^6}-\frac{a^{10}}{7x^7}+5ab^9x^2+\frac{b^{10}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^8,x]
```

```
[Out] -a^10/(7*x^7) - (5*a^9*b)/(3*x^6) - (9*a^8*b^2)/x^5 - (30*a^7*b^3)/x^4 - (7
0*a^6*b^4)/x^3 - (126*a^5*b^5)/x^2 - (210*a^4*b^6)/x + 45*a^2*b^8*x + 5*a*b
^9*x^2 + (b^10*x^3)/3 + 120*a^3*b^7*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^8}dx &= \int \left(45a^2b^8+\frac{a^{10}}{x^8}+\frac{10a^9b}{x^7}+\frac{45a^8b^2}{x^6}+\frac{120a^7b^3}{x^5}+\frac{210a^6b^4}{x^4}+\frac{252a^5b^5}{x^3}+\frac{210a^4b^6}{x^2}+\frac{120a^3b^7}{x}+10ab^9\right.\\ &= -\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}+120a\end{aligned}$$

Mathematica [A] time = 0.0159778, size = 115, normalized size = 1.

$$-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+120a^3b^7\log(x)-\frac{5a^9b}{3x^6}-\frac{a^{10}}{7x^7}+5ab^9x^2+\frac{b^{10}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^8,x]
```

```
[Out] -a^10/(7*x^7) - (5*a^9*b)/(3*x^6) - (9*a^8*b^2)/x^5 - (30*a^7*b^3)/x^4 - (7
0*a^6*b^4)/x^3 - (126*a^5*b^5)/x^2 - (210*a^4*b^6)/x + 45*a^2*b^8*x + 5*a*b
```

```
^9*x^2 + (b^10*x^3)/3 + 120*a^3*b^7*Log[x]
```

Maple [A] time = 0.007, size = 110, normalized size = 1.

$$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-9\frac{a^8b^2}{x^5}-30\frac{a^7b^3}{x^4}-70\frac{a^6b^4}{x^3}-126\frac{a^5b^5}{x^2}-210\frac{a^4b^6}{x}+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}+120a^3b^7\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^8,x)
```

```
[Out] -1/7*a^10/x^7-5/3*a^9*b/x^6-9*a^8*b^2/x^5-30*a^7*b^3/x^4-70*a^6*b^4/x^3-126
*a^5*b^5/x^2-210*a^4*b^6/x+45*a^2*b^8*x+5*a*b^9*x^2+1/3*b^10*x^3+120*a^3*b^
7*ln(x)
```

Maxima [A] time = 1.05544, size = 149, normalized size = 1.3

$$\frac{1}{3}b^{10}x^3+5ab^9x^2+45a^2b^8x+120a^3b^7\log(x)-\frac{4410a^4b^6x^6+2646a^5b^5x^5+1470a^6b^4x^4+630a^7b^3x^3+189a^8b^2x^2}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*b^10*x^3 + 5*a*b^9*x^2 + 45*a^2*b^8*x + 120*a^3*b^7*log(x) - 1/21*(4410
*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 1470*a^6*b^4*x^4 + 630*a^7*b^3*x^3 + 189*
a^8*b^2*x^2 + 35*a^9*b*x + 3*a^10)/x^7
```

Fricas [A] time = 1.59277, size = 269, normalized size = 2.34

$$\frac{7b^{10}x^{10}+105ab^9x^9+945a^2b^8x^8+2520a^3b^7x^7\log(x)-4410a^4b^6x^6-2646a^5b^5x^5-1470a^6b^4x^4-630a^7b^3x^3-18}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^10*x^10 + 105*a*b^9*x^9 + 945*a^2*b^8*x^8 + 2520*a^3*b^7*x^7*log(
x) - 4410*a^4*b^6*x^6 - 2646*a^5*b^5*x^5 - 1470*a^6*b^4*x^4 - 630*a^7*b^3*x
^3 - 189*a^8*b^2*x^2 - 35*a^9*b*x - 3*a^10)/x^7
```

Sympy [A] time = 0.99561, size = 117, normalized size = 1.02

$$120a^3b^7\log(x)+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}-\frac{3a^{10}+35a^9bx+189a^8b^2x^2+630a^7b^3x^3+1470a^6b^4x^4+2646a^5b^5x^5+}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**8,x)
```

```
[Out] 120*a**3*b**7*log(x) + 45*a**2*b**8*x + 5*a*b**9*x**2 + b**10*x**3/3 - (3*a
**10 + 35*a**9*b*x + 189*a**8*b**2*x**2 + 630*a**7*b**3*x**3 + 1470*a**6*b*
*4*x**4 + 2646*a**5*b**5*x**5 + 4410*a**4*b**6*x**6)/(21*x**7)
```

Giac [A] time = 1.23328, size = 150, normalized size = 1.3

$$\frac{1}{3}\,b^{10}x^3 + 5\,ab^9x^2 + 45\,a^2b^8x + 120\,a^3b^7\log\left(|x|\right) - \frac{4410\,a^4b^6x^6 + 2646\,a^5b^5x^5 + 1470\,a^6b^4x^4 + 630\,a^7b^3x^3 + 189\,a^8b^2x^2 +}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="giac")
```

```
[Out] 1/3*b^10*x^3 + 5*a*b^9*x^2 + 45*a^2*b^8*x + 120*a^3*b^7*log(abs(x)) - 1/21*
(4410*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 1470*a^6*b^4*x^4 + 630*a^7*b^3*x^3 +
 189*a^8*b^2*x^2 + 35*a^9*b*x + 3*a^10)/x^7
```

3.143 $\int \frac{(a+bx)^{10}}{x^9}\,dx$

Optimal. Leaf size=119

$$-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+45a^2b^8\log(x)-\frac{10a^9b}{7x^7}-\frac{a^{10}}{8x^8}+10ab^9x+\frac{b^{10}x^2}{2}$$

```
[Out] -a^10/(8*x^8) - (10*a^9*b)/(7*x^7) - (15*a^8*b^2)/(2*x^6) - (24*a^7*b^3)/x^
5 - (105*a^6*b^4)/(2*x^4) - (84*a^5*b^5)/x^3 - (105*a^4*b^6)/x^2 - (120*a^3
*b^7)/x + 10*a*b^9*x + (b^10*x^2)/2 + 45*a^2*b^8*Log[x]
```

Rubi [A] time = 0.0489553, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+45a^2b^8\log(x)-\frac{10a^9b}{7x^7}-\frac{a^{10}}{8x^8}+10ab^9x+\frac{b^{10}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^9,x]
```

```
[Out] -a^10/(8*x^8) - (10*a^9*b)/(7*x^7) - (15*a^8*b^2)/(2*x^6) - (24*a^7*b^3)/x^
5 - (105*a^6*b^4)/(2*x^4) - (84*a^5*b^5)/x^3 - (105*a^4*b^6)/x^2 - (120*a^3
*b^7)/x + 10*a*b^9*x + (b^10*x^2)/2 + 45*a^2*b^8*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^9}\,dx=\int \left(10ab^9+\frac{a^{10}}{x^9}+\frac{10a^9b}{x^8}+\frac{45a^8b^2}{x^7}+\frac{120a^7b^3}{x^6}+\frac{210a^6b^4}{x^5}+\frac{252a^5b^5}{x^4}+\frac{210a^4b^6}{x^3}+\frac{120a^3b^7}{x^2}+\frac{45a}{x}\right.$$
$$=-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+10ab^9x+\frac{b^{10}x^2}{2}+$$

Mathematica [A] time = 0.0058256, size = 119, normalized size = 1.

$$-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+45a^2b^8\log(x)-\frac{10a^9b}{7x^7}-\frac{a^{10}}{8x^8}+10ab^9x+\frac{b^{10}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^9,x]
```

```
[Out] -a^10/(8*x^8) - (10*a^9*b)/(7*x^7) - (15*a^8*b^2)/(2*x^6) - (24*a^7*b^3)/x^
5 - (105*a^6*b^4)/(2*x^4) - (84*a^5*b^5)/x^3 - (105*a^4*b^6)/x^2 - (120*a^3
```

```
*b^7)/x + 10*a*b^9*x + (b^10*x^2)/2 + 45*a^2*b^8*Log[x]
```

Maple [A] time = 0.007, size = 110, normalized size = 0.9

$$-\frac{a^{10}}{8\,x^{8}}-\frac{10\,a^{9}b}{7\,x^{7}}-\frac{15\,a^{8}b^{2}}{2\,x^{6}}-24\,\frac{a^{7}b^{3}}{x^{5}}-\frac{105\,a^{6}b^{4}}{2\,x^{4}}-84\,\frac{a^{5}b^{5}}{x^{3}}-105\,\frac{a^{4}b^{6}}{x^{2}}-120\,\frac{a^{3}b^{7}}{x}+10\,ab^{9}x+\frac{b^{10}x^{2}}{2}+45\,a^{2}b^{8}\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^9,x)
```

```
[Out] -1/8*a^10/x^8-10/7*a^9*b/x^7-15/2*a^8*b^2/x^6-24*a^7*b^3/x^5-105/2*a^6*b^4/
x^4-84*a^5*b^5/x^3-105*a^4*b^6/x^2-120*a^3*b^7/x+10*a*b^9*x+1/2*b^10*x^2+45
*a^2*b^8*ln(x)
```

Maxima [A] time = 1.03382, size = 149, normalized size = 1.25

$$\frac{1}{2}\,b^{10}x^{2}+10\,ab^{9}x+45\,a^{2}b^{8}\log(x)-\frac{6720\,a^{3}b^{7}x^{7}+5880\,a^{4}b^{6}x^{6}+4704\,a^{5}b^{5}x^{5}+2940\,a^{6}b^{4}x^{4}+1344\,a^{7}b^{3}x^{3}+420\,a^{8}b^{2}x}{56\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^10*x^2 + 10*a*b^9*x + 45*a^2*b^8*log(x) - 1/56*(6720*a^3*b^7*x^7 + 58
80*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 2940*a^6*b^4*x^4 + 1344*a^7*b^3*x^3 + 4
20*a^8*b^2*x^2 + 80*a^9*b*x + 7*a^10)/x^8
```

Fricas [A] time = 1.61997, size = 273, normalized size = 2.29

$$\frac{28\,b^{10}x^{10}+560\,ab^{9}x^{9}+2520\,a^{2}b^{8}x^{8}\log(x)-6720\,a^{3}b^{7}x^{7}-5880\,a^{4}b^{6}x^{6}-4704\,a^{5}b^{5}x^{5}-2940\,a^{6}b^{4}x^{4}-1344\,a^{7}b^{3}x^{3}-4}{56\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="fricas")
```

```
[Out] 1/56*(28*b^10*x^10 + 560*a*b^9*x^9 + 2520*a^2*b^8*x^8*log(x) - 6720*a^3*b^7
*x^7 - 5880*a^4*b^6*x^6 - 4704*a^5*b^5*x^5 - 2940*a^6*b^4*x^4 - 1344*a^7*b^
3*x^3 - 420*a^8*b^2*x^2 - 80*a^9*b*x - 7*a^10)/x^8
```

Sympy [A] time = 1.10422, size = 117, normalized size = 0.98

$$45a^{2}b^{8}\log(x)+10ab^{9}x+\frac{b^{10}x^{2}}{2}-\frac{7a^{10}+80a^{9}bx+420a^{8}b^{2}x^{2}+1344a^{7}b^{3}x^{3}+2940a^{6}b^{4}x^{4}+4704a^{5}b^{5}x^{5}+5880a^{4}b^{6}x^{6}}{56x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**9,x)
```

```
[Out] 45*a**2*b**8*log(x) + 10*a*b**9*x + b**10*x**2/2 - (7*a**10 + 80*a**9*b*x +
 420*a**8*b**2*x**2 + 1344*a**7*b**3*x**3 + 2940*a**6*b**4*x**4 + 4704*a**5
*b**5*x**5 + 5880*a**4*b**6*x**6 + 6720*a**3*b**7*x**7)/(56*x**8)
```

Giac [A] time = 1.17808, size = 150, normalized size = 1.26

$$\frac{1}{2}b^{10}x^2 + 10\,ab^9x + 45\,a^2b^8\log\left(|x|\right) - \frac{6720\,a^3b^7x^7 + 5880\,a^4b^6x^6 + 4704\,a^5b^5x^5 + 2940\,a^6b^4x^4 + 1344\,a^7b^3x^3 + 420\,a}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^10*x^2 + 10*a*b^9*x + 45*a^2*b^8*log(abs(x)) - 1/56*(6720*a^3*b^7*x^7
 + 5880*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 2940*a^6*b^4*x^4 + 1344*a^7*b^3*x^
3 + 420*a^8*b^2*x^2 + 80*a^9*b*x + 7*a^10)/x^8
```

3.144 $\int \frac{(a+bx)^{10}}{x^{10}}\,dx$

Optimal. Leaf size=114

$$-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}-\frac{5a^9b}{4x^8}-\frac{a^{10}}{9x^9}+10ab^9\log(x)+b^{10}x$$

```
[Out] -a^10/(9*x^9) - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rubi [A] time = 0.0530741, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}-\frac{5a^9b}{4x^8}-\frac{a^{10}}{9x^9}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^10,x]
```

```
[Out] -a^10/(9*x^9) - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{10}}\,dx &= \int \left(b^{10}+\frac{a^{10}}{x^{10}}+\frac{10a^9b}{x^9}+\frac{45a^8b^2}{x^8}+\frac{120a^7b^3}{x^7}+\frac{210a^6b^4}{x^6}+\frac{252a^5b^5}{x^5}+\frac{210a^4b^6}{x^4}+\frac{120a^3b^7}{x^3}+\frac{45a^2b^8}{x^2}+\right.\\ &= -\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10ab^9\,\text{lo}\end{aligned}$$

Mathematica [A] time = 0.0077861, size = 114, normalized size = 1.

$$-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}-\frac{5a^9b}{4x^8}-\frac{a^{10}}{9x^9}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^10,x]
```

```
[Out] -a^10/(9*x^9) - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
```

```
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Maple [A] time = 0.008, size = 109, normalized size = 1.

$$-\frac{a^{10}}{9\,x^{9}}-\frac{5\,a^{9}b}{4\,x^{8}}-\frac{45\,a^{8}b^{2}}{7\,x^{7}}-20\,\frac{a^{7}b^{3}}{x^{6}}-42\,\frac{a^{6}b^{4}}{x^{5}}-63\,\frac{a^{5}b^{5}}{x^{4}}-70\,\frac{a^{4}b^{6}}{x^{3}}-60\,\frac{a^{3}b^{7}}{x^{2}}-45\,\frac{a^{2}b^{8}}{x}+b^{10}x+10\,ab^{9}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^10,x)
```

```
[Out] -1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5-
63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9*l
n(x)
```

Maxima [A] time = 1.03206, size = 147, normalized size = 1.29

$$b^{10}x+10\,ab^{9}\log\left(x\right)-\frac{11340\,a^{2}b^{8}x^{8}+15120\,a^{3}b^{7}x^{7}+17640\,a^{4}b^{6}x^{6}+15876\,a^{5}b^{5}x^{5}+10584\,a^{6}b^{4}x^{4}+5040\,a^{7}b^{3}x^{3}+}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="maxima")
```

```
[Out] b^10*x + 10*a*b^9*log(x) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^7 + 1
7640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^3*x^3
 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Fricas [A] time = 1.9087, size = 288, normalized size = 2.53

$$\frac{252\,b^{10}x^{10}+2520\,ab^{9}x^{9}\log\left(x\right)-11340\,a^{2}b^{8}x^{8}-15120\,a^{3}b^{7}x^{7}-17640\,a^{4}b^{6}x^{6}-15876\,a^{5}b^{5}x^{5}-10584\,a^{6}b^{4}x^{4}-5040}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2520*a*b^9*x^9*log(x) - 11340*a^2*b^8*x^8 - 15120*a^
3*b^7*x^7 - 17640*a^4*b^6*x^6 - 15876*a^5*b^5*x^5 - 10584*a^6*b^4*x^4 - 504
0*a^7*b^3*x^3 - 1620*a^8*b^2*x^2 - 315*a^9*b*x - 28*a^10)/x^9
```

Sympy [A] time = 1.11039, size = 116, normalized size = 1.02

$$10ab^{9}\log\left(x\right)+b^{10}x-\frac{28a^{10}+315a^{9}bx+1620a^{8}b^{2}x^{2}+5040a^{7}b^{3}x^{3}+10584a^{6}b^{4}x^{4}+15876a^{5}b^{5}x^{5}+17640a^{4}b^{6}x^{6}+}{252x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**10,x)
```

```
[Out] 10*a*b**9*log(x) + b**10*x - (28*a**10 + 315*a**9*b*x + 1620*a**8*b**2*x**2
 + 5040*a**7*b**3*x**3 + 10584*a**6*b**4*x**4 + 15876*a**5*b**5*x**5 + 1764
0*a**4*b**6*x**6 + 15120*a**3*b**7*x**7 + 11340*a**2*b**8*x**8)/(252*x**9)
```

Giac [A] time = 1.23941, size = 149, normalized size = 1.31

$$b^{10}x + 10\,ab^9\log\left(|x|\right) - \frac{11340\,a^2b^8x^8 + 15120\,a^3b^7x^7 + 17640\,a^4b^6x^6 + 15876\,a^5b^5x^5 + 10584\,a^6b^4x^4 + 5040\,a^7b^3x^3 + 16}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="giac")
```

```
[Out] b^10*x + 10*a*b^9*log(abs(x)) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^
7 + 17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^
3*x^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

3.145 $\int \frac{(a+bx)^{10}}{x^{11}}\,dx$

Optimal. Leaf size=124

$$-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10a^9b}{9x^9}-\frac{a^{10}}{10x^{10}}-\frac{10ab^9}{x}+b^{10}\log(x)$$

```
[Out] -a^10/(10*x^10) - (10*a^9*b)/(9*x^9) - (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3)
/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^4)
 - (40*a^3*b^7)/x^3 - (45*a^2*b^8)/(2*x^2) - (10*a*b^9)/x + b^10*Log[x]
```

Rubi [A] time = 0.0484612, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10a^9b}{9x^9}-\frac{a^{10}}{10x^{10}}-\frac{10ab^9}{x}+b^{10}\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^11,x]
```

```
[Out] -a^10/(10*x^10) - (10*a^9*b)/(9*x^9) - (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3)
/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^4)
 - (40*a^3*b^7)/x^3 - (45*a^2*b^8)/(2*x^2) - (10*a*b^9)/x + b^10*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{11}}\,dx=\int \left(\frac{a^{10}}{x^{11}}+\frac{10a^9b}{x^{10}}+\frac{45a^8b^2}{x^9}+\frac{120a^7b^3}{x^8}+\frac{210a^6b^4}{x^7}+\frac{252a^5b^5}{x^6}+\frac{210a^4b^6}{x^5}+\frac{120a^3b^7}{x^4}+\frac{45a^2b^8}{x^3}+\frac{1\cdots}{}\right.$$
$$=-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10ab^9}{x}$$

Mathematica [A] time = 0.0082178, size = 124, normalized size = 1.

$$-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10a^9b}{9x^9}-\frac{a^{10}}{10x^{10}}-\frac{10ab^9}{x}+b^{10}\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^11,x]
```

```
[Out] -a^10/(10*x^10) - (10*a^9*b)/(9*x^9) - (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3)
/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^4)
```

```
- (40*a^3*b^7)/x^3 - (45*a^2*b^8)/(2*x^2) - (10*a*b^9)/x + b^10*Log[x]
```

Maple [A] time = 0.008, size = 111, normalized size = 0.9

$$-\frac{a^{10}}{10\,x^{10}}-\frac{10\,a^9b}{9\,x^9}-\frac{45\,a^8b^2}{8\,x^8}-\frac{120\,a^7b^3}{7\,x^7}-35\,\frac{a^6b^4}{x^6}-\frac{252\,a^5b^5}{5\,x^5}-\frac{105\,a^4b^6}{2\,x^4}-40\,\frac{a^3b^7}{x^3}-\frac{45\,a^2b^8}{2\,x^2}-10\,\frac{ab^9}{x}+b^{10}\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^11,x)
```

```
[Out] -1/10*a^10/x^10-10/9*a^9*b/x^9-45/8*a^8*b^2/x^8-120/7*a^7*b^3/x^7-35*a^6*b^
4/x^6-252/5*a^5*b^5/x^5-105/2*a^4*b^6/x^4-40*a^3*b^7/x^3-45/2*a^2*b^8/x^2-1
0*a*b^9/x+b^10*ln(x)
```

Maxima [A] time = 1.0649, size = 150, normalized size = 1.21

$$b^{10}\log(x)-\frac{25200\,ab^9x^9+56700\,a^2b^8x^8+100800\,a^3b^7x^7+132300\,a^4b^6x^6+127008\,a^5b^5x^5+88200\,a^6b^4x^4+43200\,a^7}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^11,x, algorithm="maxima")
```

```
[Out] b^10*log(x) - 1/2520*(25200*a*b^9*x^9 + 56700*a^2*b^8*x^8 + 100800*a^3*b^7*
x^7 + 132300*a^4*b^6*x^6 + 127008*a^5*b^5*x^5 + 88200*a^6*b^4*x^4 + 43200*a
^7*b^3*x^3 + 14175*a^8*b^2*x^2 + 2800*a^9*b*x + 252*a^10)/x^10
```

Fricas [A] time = 1.79562, size = 302, normalized size = 2.44

$$\frac{2520\,b^{10}x^{10}\log(x)-25200\,ab^9x^9-56700\,a^2b^8x^8-100800\,a^3b^7x^7-132300\,a^4b^6x^6-127008\,a^5b^5x^5-88200\,a^6b^4x^4-4}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^11,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^10*x^10*log(x) - 25200*a*b^9*x^9 - 56700*a^2*b^8*x^8 - 10080
0*a^3*b^7*x^7 - 132300*a^4*b^6*x^6 - 127008*a^5*b^5*x^5 - 88200*a^6*b^4*x^4
 - 43200*a^7*b^3*x^3 - 14175*a^8*b^2*x^2 - 2800*a^9*b*x - 252*a^10)/x^10
```

Sympy [A] time = 1.17134, size = 117, normalized size = 0.94

$$b^{10}\log(x)-\frac{252a^{10}+2800a^9bx+14175a^8b^2x^2+43200a^7b^3x^3+88200a^6b^4x^4+127008a^5b^5x^5+132300a^4b^6x^6+10080}{2520x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**11,x)
```

```
[Out] b**10*log(x) - (252*a**10 + 2800*a**9*b*x + 14175*a**8*b**2*x**2 + 43200*a*
*7*b**3*x**3 + 88200*a**6*b**4*x**4 + 127008*a**5*b**5*x**5 + 132300*a**4*b
**6*x**6 + 100800*a**3*b**7*x**7 + 56700*a**2*b**8*x**8 + 25200*a*b**9*x**9
)/(2520*x**10)
```

Giac [A] time = 1.17817, size = 151, normalized size = 1.22

$$b^{10}\log\left(|x|\right) - \frac{25200\,ab^{9}x^{9} + 56700\,a^{2}b^{8}x^{8} + 100800\,a^{3}b^{7}x^{7} + 132300\,a^{4}b^{6}x^{6} + 127008\,a^{5}b^{5}x^{5} + 88200\,a^{6}b^{4}x^{4} + 4320}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^11,x, algorithm="giac")
```

```
[Out] b^10*log(abs(x)) - 1/2520*(25200*a*b^9*x^9 + 56700*a^2*b^8*x^8 + 100800*a^3
*b^7*x^7 + 132300*a^4*b^6*x^6 + 127008*a^5*b^5*x^5 + 88200*a^6*b^4*x^4 + 43
200*a^7*b^3*x^3 + 14175*a^8*b^2*x^2 + 2800*a^9*b*x + 252*a^10)/x^10
```

3.146 $\int \frac{(a+bx)^{10}}{x^{12}}\,dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^{11}}{11ax^{11}}$$

```
[Out] -(a + b*x)^11/(11*a*x^11)
```

Rubi [A] time = 0.0016535, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^{11}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^12,x]
```

```
[Out] -(a + b*x)^11/(11*a*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{12}}\,dx = -\frac{(a+bx)^{11}}{11ax^{11}}$$

Mathematica [B] time = 0.0112869, size = 114, normalized size = 6.71

$$-\frac{5a^8b^2}{x^9}-\frac{15a^7b^3}{x^8}-\frac{30a^6b^4}{x^7}-\frac{42a^5b^5}{x^6}-\frac{42a^4b^6}{x^5}-\frac{30a^3b^7}{x^4}-\frac{15a^2b^8}{x^3}-\frac{a^9b}{x^{10}}-\frac{a^{10}}{11x^{11}}-\frac{5ab^9}{x^2}-\frac{b^{10}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^12,x]
```

```
[Out] -a^10/(11*x^11) - (a^9*b)/x^10 - (5*a^8*b^2)/x^9 - (15*a^7*b^3)/x^8 - (30*a
^6*b^4)/x^7 - (42*a^5*b^5)/x^6 - (42*a^4*b^6)/x^5 - (30*a^3*b^7)/x^4 - (15*
a^2*b^8)/x^3 - (5*a*b^9)/x^2 - b^10/x
```

Maple [B] time = 0.007, size = 113, normalized size = 6.7

$$-\frac{a^9b}{x^{10}}-15\frac{a^2b^8}{x^3}-42\frac{a^4b^6}{x^5}-\frac{a^{10}}{11x^{11}}-30\frac{a^3b^7}{x^4}-42\frac{a^5b^5}{x^6}-15\frac{a^7b^3}{x^8}-5\frac{ab^9}{x^2}-30\frac{a^6b^4}{x^7}-\frac{b^{10}}{x}-5\frac{a^8b^2}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^12,x)
```

```
[Out] -a^9*b/x^10-15*a^2*b^8/x^3-42*a^4*b^6/x^5-1/11*a^10/x^11-30*a^3*b^7/x^4-42*
a^5*b^5/x^6-15*a^7*b^3/x^8-5*a*b^9/x^2-30*a^6*b^4/x^7-b^10/x-5*a^8*b^2/x^9
```

Maxima [B] time = 0.992049, size = 149, normalized size = 8.76

$$-\frac{11\,b^{10}x^{10} + 55\,ab^9x^9 + 165\,a^2b^8x^8 + 330\,a^3b^7x^7 + 462\,a^4b^6x^6 + 462\,a^5b^5x^5 + 330\,a^6b^4x^4 + 165\,a^7b^3x^3 + 55\,a^8b^2x^2 +}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="maxima")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

Fricas [B] time = 1.53949, size = 252, normalized size = 14.82

$$-\frac{11\,b^{10}x^{10} + 55\,ab^9x^9 + 165\,a^2b^8x^8 + 330\,a^3b^7x^7 + 462\,a^4b^6x^6 + 462\,a^5b^5x^5 + 330\,a^6b^4x^4 + 165\,a^7b^3x^3 + 55\,a^8b^2x^2 +}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="fricas")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

Sympy [B] time = 1.31416, size = 119, normalized size = 7.

$$-\frac{a^{10} + 11a^9bx + 55a^8b^2x^2 + 165a^7b^3x^3 + 330a^6b^4x^4 + 462a^5b^5x^5 + 462a^4b^6x^6 + 330a^3b^7x^7 + 165a^2b^8x^8 + 55ab^9x^9 +}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**12,x)
```

```
[Out] -(a**10 + 11*a**9*b*x + 55*a**8*b**2*x**2 + 165*a**7*b**3*x**3 + 330*a**6*b
**4*x**4 + 462*a**5*b**5*x**5 + 462*a**4*b**6*x**6 + 330*a**3*b**7*x**7 + 1
65*a**2*b**8*x**8 + 55*a*b**9*x**9 + 11*b**10*x**10)/(11*x**11)
```

Giac [B] time = 1.16533, size = 149, normalized size = 8.76

$$-\frac{11\,b^{10}x^{10} + 55\,ab^9x^9 + 165\,a^2b^8x^8 + 330\,a^3b^7x^7 + 462\,a^4b^6x^6 + 462\,a^5b^5x^5 + 330\,a^6b^4x^4 + 165\,a^7b^3x^3 + 55\,a^8b^2x^2 +}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="giac")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

3.147 $\int \frac{(a+bx)^{10}}{x^{13}} dx$

Optimal. Leaf size=36

$$\frac{b(a+bx)^{11}}{132a^2x^{11}} - \frac{(a+bx)^{11}}{12ax^{12}}$$

```
[Out] -(a + b*x)^11/(12*a*x^12) + (b*(a + b*x)^11)/(132*a^2*x^11)
```

Rubi [A] time = 0.0048916, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b(a+bx)^{11}}{132a^2x^{11}} - \frac{(a+bx)^{11}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^13,x]

[Out] -(a + b*x)^11/(12*a*x^12) + (b*(a + b*x)^11)/(132*a^2*x^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{13}} dx &= -\frac{(a+bx)^{11}}{12ax^{12}} - \frac{b\int \frac{(a+bx)^{10}}{x^{12}} dx}{12a} \\ &= -\frac{(a+bx)^{11}}{12ax^{12}} + \frac{b(a+bx)^{11}}{132a^2x^{11}}\end{aligned}$$

Mathematica [B] time = 0.0059036, size = 128, normalized size = 3.56

$$-\frac{9a^8b^2}{2x^{10}} - \frac{40a^7b^3}{3x^9} - \frac{105a^6b^4}{4x^8} - \frac{36a^5b^5}{x^7} - \frac{35a^4b^6}{x^6} - \frac{24a^3b^7}{x^5} - \frac{45a^2b^8}{4x^4} - \frac{10a^9b}{11x^{11}} - \frac{a^{10}}{12x^{12}} - \frac{10ab^9}{3x^3} - \frac{b^{10}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^13,x]
```

```
[Out] -a^10/(12*x^12) - (10*a^9*b)/(11*x^11) - (9*a^8*b^2)/(2*x^10) - (40*a^7*b^3
)/(3*x^9) - (105*a^6*b^4)/(4*x^8) - (36*a^5*b^5)/x^7 - (35*a^4*b^6)/x^6 - (
24*a^3*b^7)/x^5 - (45*a^2*b^8)/(4*x^4) - (10*a*b^9)/(3*x^3) - b^10/(2*x^2)
```

Maple [B] time = 0.007, size = 113, normalized size = 3.1

$$-\frac{9\,a^8b^2}{2\,x^{10}}-\frac{a^{10}}{12\,x^{12}}-\frac{10\,ab^9}{3\,x^3}-24\,\frac{a^3b^7}{x^5}-\frac{10\,a^9b}{11\,x^{11}}-\frac{45\,a^2b^8}{4\,x^4}-35\,\frac{a^4b^6}{x^6}-\frac{105\,a^6b^4}{4\,x^8}-\frac{b^{10}}{2\,x^2}-36\,\frac{a^5b^5}{x^7}-\frac{40\,a^7b^3}{3\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^13,x)
```

```
[Out] -9/2*a^8*b^2/x^10-1/12*a^10/x^12-10/3*a*b^9/x^3-24*a^3*b^7/x^5-10/11*a^9*b/
x^11-45/4*a^2*b^8/x^4-35*a^4*b^6/x^6-105/4*a^6*b^4/x^8-1/2*b^10/x^2-36*a^5*
b^5/x^7-40/3*a^7*b^3/x^9
```

Maxima [B] time = 1.02893, size = 151, normalized size = 4.19

$$-\frac{66\,b^{10}x^{10}+440\,ab^9x^9+1485\,a^2b^8x^8+3168\,a^3b^7x^7+4620\,a^4b^6x^6+4752\,a^5b^5x^5+3465\,a^6b^4x^4+1760\,a^7b^3x^3+594\,a^8}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="maxima")
```

```
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

Fricas [B] time = 1.56472, size = 270, normalized size = 7.5

$$-\frac{66\,b^{10}x^{10}+440\,ab^9x^9+1485\,a^2b^8x^8+3168\,a^3b^7x^7+4620\,a^4b^6x^6+4752\,a^5b^5x^5+3465\,a^6b^4x^4+1760\,a^7b^3x^3+594\,a^8}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="fricas")
```

```
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

Sympy [B] time = 1.34703, size = 121, normalized size = 3.36

$$-\frac{11a^{10}+120a^9bx+594a^8b^2x^2+1760a^7b^3x^3+3465a^6b^4x^4+4752a^5b^5x^5+4620a^4b^6x^6+3168a^3b^7x^7+1485a^2b^8x^8+4}{132x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**13,x)
```

```
[Out] -(11*a**10 + 120*a**9*b*x + 594*a**8*b**2*x**2 + 1760*a**7*b**3*x**3 + 3465
*a**6*b**4*x**4 + 4752*a**5*b**5*x**5 + 4620*a**4*b**6*x**6 + 3168*a**3*b**
7*x**7 + 1485*a**2*b**8*x**8 + 440*a*b**9*x**9 + 66*b**10*x**10)/(132*x**12
)
```

Giac [B] time = 1.24711, size = 151, normalized size = 4.19

$$-\frac{66\,b^{10}x^{10}+440\,ab^{9}x^{9}+1485\,a^{2}b^{8}x^{8}+3168\,a^{3}b^{7}x^{7}+4620\,a^{4}b^{6}x^{6}+4752\,a^{5}b^{5}x^{5}+3465\,a^{6}b^{4}x^{4}+1760\,a^{7}b^{3}x^{3}+594}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="giac")
```

```
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

3.148 $\int \frac{(a+bx)^{10}}{x^{14}}\,dx$

Optimal. Leaf size=56

$$-\frac{b^2(a+bx)^{11}}{858a^3x^{11}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}-\frac{(a+bx)^{11}}{13ax^{13}}$$

```
[Out] -(a + b*x)^11/(13*a*x^13) + (b*(a + b*x)^11)/(78*a^2*x^12) - (b^2*(a + b*x)
^11)/(858*a^3*x^11)
```

Rubi [A] time = 0.0099815, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^2(a+bx)^{11}}{858a^3x^{11}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}-\frac{(a+bx)^{11}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^14,x]
```

```
[Out] -(a + b*x)^11/(13*a*x^13) + (b*(a + b*x)^11)/(78*a^2*x^12) - (b^2*(a + b*x)
^11)/(858*a^3*x^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{10}}{x^{14}}\,dx &= -\frac{(a+bx)^{11}}{13ax^{13}}-\frac{(2b)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{13a}\\
&= -\frac{(a+bx)^{11}}{13ax^{13}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}+\frac{b^2\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{78a^2}\\
&= -\frac{(a+bx)^{11}}{13ax^{13}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}-\frac{b^2(a+bx)^{11}}{858a^3x^{11}}
\end{aligned}$$

Mathematica [B] time = 0.0158769, size = 126, normalized size = 2.25

$$-\frac{45a^8b^2}{11x^{11}}-\frac{12a^7b^3}{x^{10}}-\frac{70a^6b^4}{3x^9}-\frac{63a^5b^5}{2x^8}-\frac{30a^4b^6}{x^7}-\frac{20a^3b^7}{x^6}-\frac{9a^2b^8}{x^5}-\frac{5a^9b}{6x^{12}}-\frac{a^{10}}{13x^{13}}-\frac{5ab^9}{2x^4}-\frac{b^{10}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^14,x]
```

```
[Out] -a^10/(13*x^13) - (5*a^9*b)/(6*x^12) - (45*a^8*b^2)/(11*x^11) - (12*a^7*b^3
)/x^10 - (70*a^6*b^4)/(3*x^9) - (63*a^5*b^5)/(2*x^8) - (30*a^4*b^6)/x^7 - (
20*a^3*b^7)/x^6 - (9*a^2*b^8)/x^5 - (5*a*b^9)/(2*x^4) - b^10/(3*x^3)
```

Maple [B] time = 0.007, size = 113, normalized size = 2.

$$-12\,\frac{a^7b^3}{x^{10}}-\frac{5\,a^9b}{6\,x^{12}}-\frac{b^{10}}{3\,x^3}-9\,\frac{a^2b^8}{x^5}-\frac{45\,a^8b^2}{11\,x^{11}}-\frac{5\,ab^9}{2\,x^4}-20\,\frac{a^3b^7}{x^6}-\frac{63\,a^5b^5}{2\,x^8}-\frac{a^{10}}{13\,x^{13}}-30\,\frac{a^4b^6}{x^7}-\frac{70\,a^6b^4}{3\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^14,x)
```

```
[Out] -12*a^7*b^3/x^10-5/6*a^9*b/x^12-1/3*b^10/x^3-9*a^2*b^8/x^5-45/11*a^8*b^2/x^
11-5/2*a*b^9/x^4-20*a^3*b^7/x^6-63/2*a^5*b^5/x^8-1/13*a^10/x^13-30*a^4*b^6/
x^7-70/3*a^6*b^4/x^9
```

Maxima [B] time = 1.00961, size = 151, normalized size = 2.7

$$-\frac{286\,b^{10}x^{10}+2145\,ab^9x^9+7722\,a^2b^8x^8+17160\,a^3b^7x^7+25740\,a^4b^6x^6+27027\,a^5b^5x^5+20020\,a^6b^4x^4+10296\,a^7b^3}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="maxima")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

Fricas [B] time = 1.49559, size = 281, normalized size = 5.02

$$-\frac{286\,b^{10}x^{10}+2145\,ab^9x^9+7722\,a^2b^8x^8+17160\,a^3b^7x^7+25740\,a^4b^6x^6+27027\,a^5b^5x^5+20020\,a^6b^4x^4+10296\,a^7b^3}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="fricas")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

Sympy [B] time = 1.81789, size = 121, normalized size = 2.16

$$-\frac{66a^{10}+715a^9bx+3510a^8b^2x^2+10296a^7b^3x^3+20020a^6b^4x^4+27027a^5b^5x^5+25740a^4b^6x^6+17160a^3b^7x^7+7722a^2b^8x^8+\ldots}{858x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**14,x)
```

```
[Out] -(66*a**10 + 715*a**9*b*x + 3510*a**8*b**2*x**2 + 10296*a**7*b**3*x**3 + 20
020*a**6*b**4*x**4 + 27027*a**5*b**5*x**5 + 25740*a**4*b**6*x**6 + 17160*a*
*3*b**7*x**7 + 7722*a**2*b**8*x**8 + 2145*a*b**9*x**9 + 286*b**10*x**10)/(8
58*x**13)
```

Giac [B] time = 1.1832, size = 151, normalized size = 2.7

$$-\frac{286\,b^{10}x^{10}+2145\,ab^9x^9+7722\,a^2b^8x^8+17160\,a^3b^7x^7+25740\,a^4b^6x^6+27027\,a^5b^5x^5+20020\,a^6b^4x^4+10296\,a^7b^3x^3+\ldots}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="giac")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

3.149 $\int \frac{(a+bx)^{10}}{x^{15}}\,dx$

Optimal. Leaf size=76

$$\frac{b^3(a+bx)^{11}}{4004a^4x^{11}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{(a+bx)^{11}}{14ax^{14}}$$

```
[Out] -(a + b*x)^11/(14*a*x^14) + (3*b*(a + b*x)^11)/(182*a^2*x^13) - (b^2*(a + b
*x)^11)/(364*a^3*x^12) + (b^3*(a + b*x)^11)/(4004*a^4*x^11)
```

Rubi [A] time = 0.0170879, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b^3(a+bx)^{11}}{4004a^4x^{11}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{(a+bx)^{11}}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^15,x]
```

```
[Out] -(a + b*x)^11/(14*a*x^14) + (3*b*(a + b*x)^11)/(182*a^2*x^13) - (b^2*(a + b
*x)^11)/(364*a^3*x^12) + (b^3*(a + b*x)^11)/(4004*a^4*x^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{10}}{x^{15}}\,dx &= -\frac{(a+bx)^{11}}{14ax^{14}}-\frac{(3b)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{14a}\\
&= -\frac{(a+bx)^{11}}{14ax^{14}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}+\frac{\left(3b^2\right)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{91a^2}\\
&= -\frac{(a+bx)^{11}}{14ax^{14}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}-\frac{b^3\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{364a^3}\\
&= -\frac{(a+bx)^{11}}{14ax^{14}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}+\frac{b^3(a+bx)^{11}}{4004a^4x^{11}}
\end{aligned}$$

Mathematica [A] time = 0.0112619, size = 128, normalized size = 1.68

$$-\frac{15a^8b^2}{4x^{12}}-\frac{120a^7b^3}{11x^{11}}-\frac{21a^6b^4}{x^{10}}-\frac{28a^5b^5}{x^9}-\frac{105a^4b^6}{4x^8}-\frac{120a^3b^7}{7x^7}-\frac{15a^2b^8}{2x^6}-\frac{10a^9b}{13x^{13}}-\frac{a^{10}}{14x^{14}}-\frac{2ab^9}{x^5}-\frac{b^{10}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^15,x]
```

```
[Out] -a^10/(14*x^14) - (10*a^9*b)/(13*x^13) - (15*a^8*b^2)/(4*x^12) - (120*a^7*b
^3)/(11*x^11) - (21*a^6*b^4)/x^10 - (28*a^5*b^5)/x^9 - (105*a^4*b^6)/(4*x^8
) - (120*a^3*b^7)/(7*x^7) - (15*a^2*b^8)/(2*x^6) - (2*a*b^9)/x^5 - b^10/(4*
x^4)
```

Maple [A] time = 0.006, size = 113, normalized size = 1.5

$$-21\,{\frac{a^6b^4}{x^{10}}}-{\frac{15\,a^8b^2}{4\,x^{12}}}-2\,{\frac{ab^9}{x^5}}-{\frac{120\,a^7b^3}{11\,x^{11}}}-{\frac{b^{10}}{4\,x^4}}-{\frac{15\,a^2b^8}{2\,x^6}}-{\frac{105\,a^4b^6}{4\,x^8}}-{\frac{10\,a^9b}{13\,x^{13}}}-{\frac{120\,a^3b^7}{7\,x^7}}-28\,{\frac{a^5b^5}{x^9}}-{\frac{a^{10}}{14\,x^{14}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^15,x)
```

```
[Out] -21*a^6*b^4/x^10-15/4*a^8*b^2/x^12-2*a*b^9/x^5-120/11*a^7*b^3/x^11-1/4*b^10
/x^4-15/2*a^2*b^8/x^6-105/4*a^4*b^6/x^8-10/13*a^9*b/x^13-120/7*a^3*b^7/x^7-
28*a^5*b^5/x^9-1/14*a^10/x^14
```

Maxima [A] time = 1.00864, size = 151, normalized size = 1.99

$$-\frac{1001\,b^{10}x^{10}+8008\,ab^9x^9+30030\,a^2b^8x^8+68640\,a^3b^7x^7+105105\,a^4b^6x^6+112112\,a^5b^5x^5+84084\,a^6b^4x^4+43680\,a^7b}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="maxima")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

Fricas [A] time = 1.40059, size = 292, normalized size = 3.84

$$-\frac{1001\,b^{10}x^{10}+8008\,ab^9x^9+30030\,a^2b^8x^8+68640\,a^3b^7x^7+105105\,a^4b^6x^6+112112\,a^5b^5x^5+84084\,a^6b^4x^4+43680\,a^7b}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="fricas")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

Sympy [A] time = 1.87377, size = 121, normalized size = 1.59

$$-\frac{286a^{10} + 3080a^9bx + 15015a^8b^2x^2 + 43680a^7b^3x^3 + 84084a^6b^4x^4 + 112112a^5b^5x^5 + 105105a^4b^6x^6 + 68640a^3b^7x^7 +}{4004x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**15,x)
```

```
[Out] -(286*a**10 + 3080*a**9*b*x + 15015*a**8*b**2*x**2 + 43680*a**7*b**3*x**3 +
 84084*a**6*b**4*x**4 + 112112*a**5*b**5*x**5 + 105105*a**4*b**6*x**6 + 686
40*a**3*b**7*x**7 + 30030*a**2*b**8*x**8 + 8008*a*b**9*x**9 + 1001*b**10*x*
*10)/(4004*x**14)
```

Giac [A] time = 1.14972, size = 151, normalized size = 1.99

$$-\frac{1001\,b^{10}x^{10} + 8008\,ab^9x^9 + 30030\,a^2b^8x^8 + 68640\,a^3b^7x^7 + 105105\,a^4b^6x^6 + 112112\,a^5b^5x^5 + 84084\,a^6b^4x^4 + 43680}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="giac")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

3.150 $\int \frac{(a+bx)^{10}}{x^{16}} dx$

Optimal. Leaf size=96

$$-\frac{b^4(a+bx)^{11}}{15015a^5x^{11}}+\frac{b^3(a+bx)^{11}}{1365a^4x^{12}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{(a+bx)^{11}}{15ax^{15}}$$

```
[Out] -(a + b*x)^11/(15*a*x^15) + (2*b*(a + b*x)^11)/(105*a^2*x^14) - (2*b^2*(a +
 b*x)^11)/(455*a^3*x^13) + (b^3*(a + b*x)^11)/(1365*a^4*x^12) - (b^4*(a + b
*x)^11)/(15015*a^5*x^11)
```

Rubi [A] time = 0.0250919, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^4(a+bx)^{11}}{15015a^5x^{11}}+\frac{b^3(a+bx)^{11}}{1365a^4x^{12}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{(a+bx)^{11}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^16,x]
```

```
[Out] -(a + b*x)^11/(15*a*x^15) + (2*b*(a + b*x)^11)/(105*a^2*x^14) - (2*b^2*(a +
 b*x)^11)/(455*a^3*x^13) + (b^3*(a + b*x)^11)/(1365*a^4*x^12) - (b^4*(a + b
*x)^11)/(15015*a^5*x^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{16}}\,dx = -\frac{(a+bx)^{11}}{15ax^{15}} - \frac{(4b)\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{15a}$$
$$= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} + \frac{\left(2b^2\right)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{35a^2}$$
$$= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} - \frac{2b^2(a+bx)^{11}}{455a^3x^{13}} - \frac{\left(4b^3\right)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{455a^3}$$
$$= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} - \frac{2b^2(a+bx)^{11}}{455a^3x^{13}} + \frac{b^3(a+bx)^{11}}{1365a^4x^{12}} + \frac{b^4\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{1365a^4}$$
$$= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} - \frac{2b^2(a+bx)^{11}}{455a^3x^{13}} + \frac{b^3(a+bx)^{11}}{1365a^4x^{12}} - \frac{b^4(a+bx)^{11}}{15015a^5x^{11}}$$

Mathematica [A] time = 0.0136676, size = 130, normalized size = 1.35

$$-\frac{45a^8b^2}{13x^{13}} - \frac{10a^7b^3}{x^{12}} - \frac{210a^6b^4}{11x^{11}} - \frac{126a^5b^5}{5x^{10}} - \frac{70a^4b^6}{3x^9} - \frac{15a^3b^7}{x^8} - \frac{45a^2b^8}{7x^7} - \frac{5a^9b}{7x^{14}} - \frac{a^{10}}{15x^{15}} - \frac{5ab^9}{3x^6} - \frac{b^{10}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^16,x]
```

```
[Out] -a^10/(15*x^15) - (5*a^9*b)/(7*x^14) - (45*a^8*b^2)/(13*x^13) - (10*a^7*b^3
)/x^12 - (210*a^6*b^4)/(11*x^11) - (126*a^5*b^5)/(5*x^10) - (70*a^4*b^6)/(3
*x^9) - (15*a^3*b^7)/x^8 - (45*a^2*b^8)/(7*x^7) - (5*a*b^9)/(3*x^6) - b^10/
(5*x^5)
```

Maple [A] time = 0.007, size = 113, normalized size = 1.2

$$-\frac{126\,a^5b^5}{5\,x^{10}} - 10\,\frac{a^7b^3}{x^{12}} - \frac{b^{10}}{5\,x^5} - \frac{a^{10}}{15\,x^{15}} - \frac{210\,a^6b^4}{11\,x^{11}} - \frac{5\,ab^9}{3\,x^6} - 15\,\frac{a^3b^7}{x^8} - \frac{45\,a^8b^2}{13\,x^{13}} - \frac{45\,a^2b^8}{7\,x^7} - \frac{70\,a^4b^6}{3\,x^9} - \frac{5\,a^9b}{7\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^16,x)
```

```
[Out] -126/5*a^5*b^5/x^10-10*a^7*b^3/x^12-1/5*b^10/x^5-1/15*a^10/x^15-210/11*a^6*
b^4/x^11-5/3*a*b^9/x^6-15*a^3*b^7/x^8-45/13*a^8*b^2/x^13-45/7*a^2*b^8/x^7-7
0/3*a^4*b^6/x^9-5/7*a^9*b/x^14
```

Maxima [A] time = 1.08695, size = 151, normalized size = 1.57

$$-\frac{3003\,b^{10}x^{10} + 25025\,ab^9x^9 + 96525\,a^2b^8x^8 + 225225\,a^3b^7x^7 + 350350\,a^4b^6x^6 + 378378\,a^5b^5x^5 + 286650\,a^6b^4x^4 + 1}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
```

```
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

Fricas [A] time = 1.47006, size = 301, normalized size = 3.14

$$-\frac{3003\,b^{10}x^{10} + 25025\,ab^9x^9 + 96525\,a^2b^8x^8 + 225225\,a^3b^7x^7 + 350350\,a^4b^6x^6 + 378378\,a^5b^5x^5 + 286650\,a^6b^4x^4 + 1501}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="fricas")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

Sympy [A] time = 1.84367, size = 121, normalized size = 1.26

$$-\frac{1001a^{10} + 10725a^9bx + 51975a^8b^2x^2 + 150150a^7b^3x^3 + 286650a^6b^4x^4 + 378378a^5b^5x^5 + 350350a^4b^6x^6 + 225225a^3b^7}{15015x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**16,x)
```

```
[Out] -(1001*a**10 + 10725*a**9*b*x + 51975*a**8*b**2*x**2 + 150150*a**7*b**3*x**
3 + 286650*a**6*b**4*x**4 + 378378*a**5*b**5*x**5 + 350350*a**4*b**6*x**6 +
 225225*a**3*b**7*x**7 + 96525*a**2*b**8*x**8 + 25025*a*b**9*x**9 + 3003*b*
*10*x**10)/(15015*x**15)
```

Giac [A] time = 1.15674, size = 151, normalized size = 1.57

$$-\frac{3003\,b^{10}x^{10} + 25025\,ab^9x^9 + 96525\,a^2b^8x^8 + 225225\,a^3b^7x^7 + 350350\,a^4b^6x^6 + 378378\,a^5b^5x^5 + 286650\,a^6b^4x^4 + 1501}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="giac")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

3.151 $\int \frac{(a+bx)^{10}}{x^{17}} dx$

Optimal. Leaf size=116

$$\frac{b^5(a+bx)^{11}}{48048a^6x^{11}}-\frac{b^4(a+bx)^{11}}{4368a^5x^{12}}+\frac{b^3(a+bx)^{11}}{728a^4x^{13}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{(a+bx)^{11}}{16ax^{16}}$$

```
[Out] -(a + b*x)^11/(16*a*x^16) + (b*(a + b*x)^11)/(48*a^2*x^15) - (b^2*(a + b*x)
^11)/(168*a^3*x^14) + (b^3*(a + b*x)^11)/(728*a^4*x^13) - (b^4*(a + b*x)^11
)/(4368*a^5*x^12) + (b^5*(a + b*x)^11)/(48048*a^6*x^11)
```

Rubi [A] time = 0.0351052, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b^5(a+bx)^{11}}{48048a^6x^{11}}-\frac{b^4(a+bx)^{11}}{4368a^5x^{12}}+\frac{b^3(a+bx)^{11}}{728a^4x^{13}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{(a+bx)^{11}}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^17,x]
```

```
[Out] -(a + b*x)^11/(16*a*x^16) + (b*(a + b*x)^11)/(48*a^2*x^15) - (b^2*(a + b*x)
^11)/(168*a^3*x^14) + (b^3*(a + b*x)^11)/(728*a^4*x^13) - (b^4*(a + b*x)^11
)/(4368*a^5*x^12) + (b^5*(a + b*x)^11)/(48048*a^6*x^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{10}}{x^{17}}\,dx &= -\frac{(a+bx)^{11}}{16ax^{16}} - \frac{(5b)\int \frac{(a+bx)^{10}}{x^{16}}\,dx}{16a} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} + \frac{b^2\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{12a^2} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} - \frac{b^3\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{56a^3} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} + \frac{b^3(a+bx)^{11}}{728a^4x^{13}} + \frac{b^4\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{364a^4} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} + \frac{b^3(a+bx)^{11}}{728a^4x^{13}} - \frac{b^4(a+bx)^{11}}{4368a^5x^{12}} - \frac{b^5\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{4368a^5} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} + \frac{b^3(a+bx)^{11}}{728a^4x^{13}} - \frac{b^4(a+bx)^{11}}{4368a^5x^{12}} + \frac{b^5(a+bx)^{11}}{48048a^6x^{11}}
\end{aligned}
$$

Mathematica [A] time = 0.0049497, size = 132, normalized size = 1.14

$$
-\frac{45a^8b^2}{14x^{14}} - \frac{120a^7b^3}{13x^{13}} - \frac{35a^6b^4}{2x^{12}} - \frac{252a^5b^5}{11x^{11}} - \frac{21a^4b^6}{x^{10}} - \frac{40a^3b^7}{3x^9} - \frac{45a^2b^8}{8x^8} - \frac{2a^9b}{3x^{15}} - \frac{a^{10}}{16x^{16}} - \frac{10ab^9}{7x^7} - \frac{b^{10}}{6x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^17,x]
```

```
[Out] -a^10/(16*x^16) - (2*a^9*b)/(3*x^15) - (45*a^8*b^2)/(14*x^14) - (120*a^7*b^
3)/(13*x^13) - (35*a^6*b^4)/(2*x^12) - (252*a^5*b^5)/(11*x^11) - (21*a^4*b^
6)/x^10 - (40*a^3*b^7)/(3*x^9) - (45*a^2*b^8)/(8*x^8) - (10*a*b^9)/(7*x^7)
- b^10/(6*x^6)
```

Maple [A] time = 0.007, size = 113, normalized size = 1.

$$
-21\,\frac{a^4b^6}{x^{10}} - \frac{35\,a^6b^4}{2\,x^{12}} - \frac{2\,a^9b}{3\,x^{15}} - \frac{252\,a^5b^5}{11\,x^{11}} - \frac{b^{10}}{6\,x^6} - \frac{45\,a^2b^8}{8\,x^8} - \frac{120\,a^7b^3}{13\,x^{13}} - \frac{10\,ab^9}{7\,x^7} - \frac{a^{10}}{16\,x^{16}} - \frac{40\,a^3b^7}{3\,x^9} - \frac{45\,a^8b^2}{14\,x^{14}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^17,x)
```

```
[Out] -21*a^4*b^6/x^10-35/2*a^6*b^4/x^12-2/3*a^9*b/x^15-252/11*a^5*b^5/x^11-1/6*b
^10/x^6-45/8*a^2*b^8/x^8-120/13*a^7*b^3/x^13-10/7*a*b^9/x^7-1/16*a^10/x^16-
40/3*a^3*b^7/x^9-45/14*a^8*b^2/x^14
```

Maxima [A] time = 1.08816, size = 151, normalized size = 1.3

$$
-\frac{8008\,b^{10}x^{10} + 68640\,ab^9x^9 + 270270\,a^2b^8x^8 + 640640\,a^3b^7x^7 + 1009008\,a^4b^6x^6 + 1100736\,a^5b^5x^5 + 840840\,a^6b^4x^4 + 4}{48048\,x^{16}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="maxima")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

Fricas [A] time = 1.56903, size = 306, normalized size = 2.64

$$-\frac{8008\, b^{10}x^{10} + 68640\, ab^9x^9 + 270270\, a^2b^8x^8 + 640640\, a^3b^7x^7 + 1009008\, a^4b^6x^6 + 1100736\, a^5b^5x^5 + 840840\, a^6b^4x^4}{48048\, x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="fricas")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

Sympy [A] time = 2.09986, size = 121, normalized size = 1.04

$$-\frac{3003a^{10} + 32032a^9bx + 154440a^8b^2x^2 + 443520a^7b^3x^3 + 840840a^6b^4x^4 + 1100736a^5b^5x^5 + 1009008a^4b^6x^6 + 6406}{48048x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**17,x)
```

```
[Out] -(3003*a**10 + 32032*a**9*b*x + 154440*a**8*b**2*x**2 + 443520*a**7*b**3*x*
*3 + 840840*a**6*b**4*x**4 + 1100736*a**5*b**5*x**5 + 1009008*a**4*b**6*x**
6 + 640640*a**3*b**7*x**7 + 270270*a**2*b**8*x**8 + 68640*a*b**9*x**9 + 800
8*b**10*x**10)/(48048*x**16)
```

Giac [A] time = 1.12558, size = 151, normalized size = 1.3

$$-\frac{8008\, b^{10}x^{10} + 68640\, ab^9x^9 + 270270\, a^2b^8x^8 + 640640\, a^3b^7x^7 + 1009008\, a^4b^6x^6 + 1100736\, a^5b^5x^5 + 840840\, a^6b^4x^4}{48048\, x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="giac")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

3.152 $\int \frac{(a+bx)^{10}}{x^{18}}\,dx$

Optimal. Leaf size=136

$$-\frac{b^6(a+bx)^{11}}{136136a^7x^{11}}+\frac{b^5(a+bx)^{11}}{12376a^6x^{12}}-\frac{3b^4(a+bx)^{11}}{6188a^5x^{13}}+\frac{b^3(a+bx)^{11}}{476a^4x^{14}}-\frac{b^2(a+bx)^{11}}{136a^3x^{15}}+\frac{3b(a+bx)^{11}}{136a^2x^{16}}-\frac{(a+bx)^{11}}{17ax^{17}}$$

```
[Out] -(a + b*x)^11/(17*a*x^17) + (3*b*(a + b*x)^11)/(136*a^2*x^16) - (b^2*(a + b
*x)^11)/(136*a^3*x^15) + (b^3*(a + b*x)^11)/(476*a^4*x^14) - (3*b^4*(a + b*
x)^11)/(6188*a^5*x^13) + (b^5*(a + b*x)^11)/(12376*a^6*x^12) - (b^6*(a + b*
x)^11)/(136136*a^7*x^11)
```

Rubi [A] time = 0.0487079, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^6(a+bx)^{11}}{136136a^7x^{11}}+\frac{b^5(a+bx)^{11}}{12376a^6x^{12}}-\frac{3b^4(a+bx)^{11}}{6188a^5x^{13}}+\frac{b^3(a+bx)^{11}}{476a^4x^{14}}-\frac{b^2(a+bx)^{11}}{136a^3x^{15}}+\frac{3b(a+bx)^{11}}{136a^2x^{16}}-\frac{(a+bx)^{11}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^18,x]
```

```
[Out] -(a + b*x)^11/(17*a*x^17) + (3*b*(a + b*x)^11)/(136*a^2*x^16) - (b^2*(a + b
*x)^11)/(136*a^3*x^15) + (b^3*(a + b*x)^11)/(476*a^4*x^14) - (3*b^4*(a + b*
x)^11)/(6188*a^5*x^13) + (b^5*(a + b*x)^11)/(12376*a^6*x^12) - (b^6*(a + b*
x)^11)/(136136*a^7*x^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{18}}\,dx = -\frac{(a+bx)^{11}}{17ax^{17}} - \frac{(6b)\int \frac{(a+bx)^{10}}{x^{17}}\,dx}{17a}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} + \frac{\left(15b^2\right)\int \frac{(a+bx)^{10}}{x^{16}}\,dx}{136a^2}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} - \frac{b^3\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{34a^3}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} + \frac{\left(3b^4\right)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{476a^4}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} - \frac{\left(3b^5\right)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{3094a^5}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} + \frac{b^5(a+bx)^{11}}{12376a^6x^{12}} + \frac{b^6\int}{12}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} + \frac{b^5(a+bx)^{11}}{12376a^6x^{12}} - \frac{b^6(a}{1361}$$

Mathematica [A] time = 0.017388, size = 126, normalized size = 0.93

$$-\frac{3a^8b^2}{x^{15}} - \frac{60a^7b^3}{7x^{14}} - \frac{210a^6b^4}{13x^{13}} - \frac{21a^5b^5}{x^{12}} - \frac{210a^4b^6}{11x^{11}} - \frac{12a^3b^7}{x^{10}} - \frac{5a^2b^8}{x^9} - \frac{5a^9b}{8x^{16}} - \frac{a^{10}}{17x^{17}} - \frac{5ab^9}{4x^8} - \frac{b^{10}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^18,x]
```

```
[Out] -a^10/(17*x^17) - (5*a^9*b)/(8*x^16) - (3*a^8*b^2)/x^15 - (60*a^7*b^3)/(7*x
^14) - (210*a^6*b^4)/(13*x^13) - (21*a^5*b^5)/x^12 - (210*a^4*b^6)/(11*x^11
) - (12*a^3*b^7)/x^10 - (5*a^2*b^8)/x^9 - (5*a*b^9)/(4*x^8) - b^10/(7*x^7)
```

Maple [A] time = 0.009, size = 113, normalized size = 0.8

$$-12\,\frac{a^3b^7}{x^{10}} - 21\,\frac{a^5b^5}{x^{12}} - 3\,\frac{a^8b^2}{x^{15}} - \frac{210\,a^4b^6}{11\,x^{11}} - \frac{a^{10}}{17\,x^{17}} - \frac{5\,ab^9}{4\,x^8} - \frac{210\,a^6b^4}{13\,x^{13}} - \frac{b^{10}}{7\,x^7} - \frac{5\,a^9b}{8\,x^{16}} - 5\,\frac{a^2b^8}{x^9} - \frac{60\,a^7b^3}{7\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^18,x)
```

```
[Out] -12*a^3*b^7/x^10-21*a^5*b^5/x^12-3*a^8*b^2/x^15-210/11*a^4*b^6/x^11-1/17*a^
10/x^17-5/4*a*b^9/x^8-210/13*a^6*b^4/x^13-1/7*b^10/x^7-5/8*a^9*b/x^16-5*a^2
*b^8/x^9-60/7*a^7*b^3/x^14
```

Maxima [A] time = 1.02435, size = 151, normalized size = 1.11

$$-\frac{19448\,b^{10}x^{10} + 170170\,ab^9x^9 + 680680\,a^2b^8x^8 + 1633632\,a^3b^7x^7 + 2598960\,a^4b^6x^6 + 2858856\,a^5b^5x^5 + 2199120\,a^6b}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="maxima")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

Fricas [A] time = 1.45369, size = 315, normalized size = 2.32

$$-\frac{19448\,b^{10}x^{10} + 170170\,ab^{9}x^{9} + 680680\,a^{2}b^{8}x^{8} + 1633632\,a^{3}b^{7}x^{7} + 2598960\,a^{4}b^{6}x^{6} + 2858856\,a^{5}b^{5}x^{5} + 2199120\,a^{6}b^{4}x^{4}}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="fricas")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

Sympy [A] time = 1.92296, size = 121, normalized size = 0.89

$$-\frac{8008a^{10} + 85085a^{9}bx + 408408a^{8}b^{2}x^{2} + 1166880a^{7}b^{3}x^{3} + 2199120a^{6}b^{4}x^{4} + 2858856a^{5}b^{5}x^{5} + 2598960a^{4}b^{6}x^{6} + 16336}{136136x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**18,x)
```

```
[Out] -(8008*a**10 + 85085*a**9*b*x + 408408*a**8*b**2*x**2 + 1166880*a**7*b**3*x
**3 + 2199120*a**6*b**4*x**4 + 2858856*a**5*b**5*x**5 + 2598960*a**4*b**6*x
**6 + 1633632*a**3*b**7*x**7 + 680680*a**2*b**8*x**8 + 170170*a*b**9*x**9 +
 19448*b**10*x**10)/(136136*x**17)
```

Giac [A] time = 1.1591, size = 151, normalized size = 1.11

$$-\frac{19448\,b^{10}x^{10} + 170170\,ab^{9}x^{9} + 680680\,a^{2}b^{8}x^{8} + 1633632\,a^{3}b^{7}x^{7} + 2598960\,a^{4}b^{6}x^{6} + 2858856\,a^{5}b^{5}x^{5} + 2199120\,a^{6}b^{4}x^{4}}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="giac")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

3.153 $\int \frac{(a+bx)^{10}}{x^{19}} dx$

Optimal. Leaf size=130

$$-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10a^9b}{17x^{17}}-\frac{a^{10}}{18x^{18}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

```
[Out] -a^10/(18*x^18) - (10*a^9*b)/(17*x^17) - (45*a^8*b^2)/(16*x^16) - (8*a^7*b^
3)/x^15 - (15*a^6*b^4)/x^14 - (252*a^5*b^5)/(13*x^13) - (35*a^4*b^6)/(2*x^1
2) - (120*a^3*b^7)/(11*x^11) - (9*a^2*b^8)/(2*x^10) - (10*a*b^9)/(9*x^9) -
b^10/(8*x^8)
```

Rubi [A] time = 0.047175, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10a^9b}{17x^{17}}-\frac{a^{10}}{18x^{18}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^19,x]
```

```
[Out] -a^10/(18*x^18) - (10*a^9*b)/(17*x^17) - (45*a^8*b^2)/(16*x^16) - (8*a^7*b^
3)/x^15 - (15*a^6*b^4)/x^14 - (252*a^5*b^5)/(13*x^13) - (35*a^4*b^6)/(2*x^1
2) - (120*a^3*b^7)/(11*x^11) - (9*a^2*b^8)/(2*x^10) - (10*a*b^9)/(9*x^9) -
b^10/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{19}} dx = \int \left(\frac{a^{10}}{x^{19}}+\frac{10a^9b}{x^{18}}+\frac{45a^8b^2}{x^{17}}+\frac{120a^7b^3}{x^{16}}+\frac{210a^6b^4}{x^{15}}+\frac{252a^5b^5}{x^{14}}+\frac{210a^4b^6}{x^{13}}+\frac{120a^3b^7}{x^{12}}+\frac{45a^2b^8}{x^{11}}+\right.$$

$$=-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab^9}{9x^9}-$$

Mathematica [A] time = 0.0065784, size = 130, normalized size = 1.

$$-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10a^9b}{17x^{17}}-\frac{a^{10}}{18x^{18}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^19,x]
```

```
[Out] -a^10/(18*x^18) - (10*a^9*b)/(17*x^17) - (45*a^8*b^2)/(16*x^16) - (8*a^7*b^
3)/x^15 - (15*a^6*b^4)/x^14 - (252*a^5*b^5)/(13*x^13) - (35*a^4*b^6)/(2*x^1
```

```
2) - (120*a^3*b^7)/(11*x^11) - (9*a^2*b^8)/(2*x^10) - (10*a*b^9)/(9*x^9) -
b^10/(8*x^8)
```

Maple [A] time = 0.006, size = 113, normalized size = 0.9

$$-\frac{a^{10}}{18\,x^{18}}-\frac{10\,a^{9}b}{17\,x^{17}}-\frac{45\,a^{8}b^{2}}{16\,x^{16}}-8\,\frac{a^{7}b^{3}}{x^{15}}-15\,\frac{a^{6}b^{4}}{x^{14}}-\frac{252\,a^{5}b^{5}}{13\,x^{13}}-\frac{35\,a^{4}b^{6}}{2\,x^{12}}-\frac{120\,a^{3}b^{7}}{11\,x^{11}}-\frac{9\,a^{2}b^{8}}{2\,x^{10}}-\frac{10\,ab^{9}}{9\,x^{9}}-\frac{b^{10}}{8\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^19,x)
```

```
[Out] -1/18*a^10/x^18-10/17*a^9*b/x^17-45/16*a^8*b^2/x^16-8*a^7*b^3/x^15-15*a^6*b
^4/x^14-252/13*a^5*b^5/x^13-35/2*a^4*b^6/x^12-120/11*a^3*b^7/x^11-9/2*a^2*b
^8/x^10-10/9*a*b^9/x^9-1/8*b^10/x^8
```

Maxima [A] time = 1.011, size = 151, normalized size = 1.16

$$-\frac{43758\,b^{10}x^{10}+388960\,ab^{9}x^{9}+1575288\,a^{2}b^{8}x^{8}+3818880\,a^{3}b^{7}x^{7}+6126120\,a^{4}b^{6}x^{6}+6785856\,a^{5}b^{5}x^{5}+5250960\,a^{6}b^{4}}{350064\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^19,x, algorithm="maxima")
```

```
[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18
```

Fricas [A] time = 1.45553, size = 319, normalized size = 2.45

$$-\frac{43758\,b^{10}x^{10}+388960\,ab^{9}x^{9}+1575288\,a^{2}b^{8}x^{8}+3818880\,a^{3}b^{7}x^{7}+6126120\,a^{4}b^{6}x^{6}+6785856\,a^{5}b^{5}x^{5}+5250960\,a^{6}b^{4}}{350064\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^19,x, algorithm="fricas")
```

```
[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18
```

Sympy [A] time = 1.99317, size = 121, normalized size = 0.93

$$-\frac{19448a^{10}+205920a^{9}bx+984555a^{8}b^{2}x^{2}+2800512a^{7}b^{3}x^{3}+5250960a^{6}b^{4}x^{4}+6785856a^{5}b^{5}x^{5}+6126120a^{4}b^{6}x^{6}+381}{350064x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**19,x)
```

```
[Out] -(19448*a**10 + 205920*a**9*b*x + 984555*a**8*b**2*x**2 + 2800512*a**7*b**3
*x**3 + 5250960*a**6*b**4*x**4 + 6785856*a**5*b**5*x**5 + 6126120*a**4*b**6
*x**6 + 3818880*a**3*b**7*x**7 + 1575288*a**2*b**8*x**8 + 388960*a*b**9*x**
9 + 43758*b**10*x**10)/(350064*x**18)
```

Giac [A] time = 1.18986, size = 151, normalized size = 1.16

$$-\frac{43758\, b^{10}x^{10} + 388960\, ab^{9}x^{9} + 1575288\, a^{2}b^{8}x^{8} + 3818880\, a^{3}b^{7}x^{7} + 6126120\, a^{4}b^{6}x^{6} + 6785856\, a^{5}b^{5}x^{5} + 5250960\, a}{350064\, x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^19,x, algorithm="giac")
```

```
[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18
```

3.154 $\int \frac{(a+bx)^{10}}{x^{20}}\,dx$

Optimal. Leaf size=126

$$-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{5a^9b}{9x^{18}}-\frac{a^{10}}{19x^{19}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

```
[Out] -a^10/(19*x^19) - (5*a^9*b)/(9*x^18) - (45*a^8*b^2)/(17*x^17) - (15*a^7*b^3
)/(2*x^16) - (14*a^6*b^4)/x^15 - (18*a^5*b^5)/x^14 - (210*a^4*b^6)/(13*x^13
) - (10*a^3*b^7)/x^12 - (45*a^2*b^8)/(11*x^11) - (a*b^9)/x^10 - b^10/(9*x^9
)
```

Rubi [A] time = 0.0493592, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{5a^9b}{9x^{18}}-\frac{a^{10}}{19x^{19}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^20,x]
```

```
[Out] -a^10/(19*x^19) - (5*a^9*b)/(9*x^18) - (45*a^8*b^2)/(17*x^17) - (15*a^7*b^3
)/(2*x^16) - (14*a^6*b^4)/x^15 - (18*a^5*b^5)/x^14 - (210*a^4*b^6)/(13*x^13
) - (10*a^3*b^7)/x^12 - (45*a^2*b^8)/(11*x^11) - (a*b^9)/x^10 - b^10/(9*x^9
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{20}}\,dx &= \int \left(\frac{a^{10}}{x^{20}}+\frac{10a^9b}{x^{19}}+\frac{45a^8b^2}{x^{18}}+\frac{120a^7b^3}{x^{17}}+\frac{210a^6b^4}{x^{16}}+\frac{252a^5b^5}{x^{15}}+\frac{210a^4b^6}{x^{14}}+\frac{120a^3b^7}{x^{13}}+\frac{45a^2b^8}{x^{12}}+\frac{10ab^9}{x^{11}}\right.\\ &= -\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}\end{aligned}$$

Mathematica [A] time = 0.0072441, size = 126, normalized size = 1.

$$-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{5a^9b}{9x^{18}}-\frac{a^{10}}{19x^{19}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^20,x]
```

```
[Out] -a^10/(19*x^19) - (5*a^9*b)/(9*x^18) - (45*a^8*b^2)/(17*x^17) - (15*a^7*b^3
)/(2*x^16) - (14*a^6*b^4)/x^15 - (18*a^5*b^5)/x^14 - (210*a^4*b^6)/(13*x^13
```

```
) - (10*a^3*b^7)/x^12 - (45*a^2*b^8)/(11*x^11) - (a*b^9)/x^10 - b^10/(9*x^9
)
```

Maple [A] time = 0.007, size = 113, normalized size = 0.9

$$-\frac{a^{10}}{19\,x^{19}}-\frac{5\,a^{9}b}{9\,x^{18}}-\frac{45\,a^{8}b^{2}}{17\,x^{17}}-\frac{15\,a^{7}b^{3}}{2\,x^{16}}-14\,\frac{a^{6}b^{4}}{x^{15}}-18\,\frac{a^{5}b^{5}}{x^{14}}-\frac{210\,a^{4}b^{6}}{13\,x^{13}}-10\,\frac{a^{3}b^{7}}{x^{12}}-\frac{45\,a^{2}b^{8}}{11\,x^{11}}-\frac{ab^{9}}{x^{10}}-\frac{b^{10}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^20,x)
```

```
[Out] -1/19*a^10/x^19-5/9*a^9*b/x^18-45/17*a^8*b^2/x^17-15/2*a^7*b^3/x^16-14*a^6*
b^4/x^15-18*a^5*b^5/x^14-210/13*a^4*b^6/x^13-10*a^3*b^7/x^12-45/11*a^2*b^8/
x^11-a*b^9/x^10-1/9*b^10/x^9
```

Maxima [A] time = 1.0694, size = 151, normalized size = 1.2

$$-\frac{92378\,b^{10}x^{10}+831402\,ab^{9}x^{9}+3401190\,a^{2}b^{8}x^{8}+8314020\,a^{3}b^{7}x^{7}+13430340\,a^{4}b^{6}x^{6}+14965236\,a^{5}b^{5}x^{5}+1163962}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="maxima")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

Fricas [A] time = 1.50204, size = 324, normalized size = 2.57

$$-\frac{92378\,b^{10}x^{10}+831402\,ab^{9}x^{9}+3401190\,a^{2}b^{8}x^{8}+8314020\,a^{3}b^{7}x^{7}+13430340\,a^{4}b^{6}x^{6}+14965236\,a^{5}b^{5}x^{5}+1163962}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="fricas")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

Sympy [A] time = 2.03177, size = 121, normalized size = 0.96

$$-\frac{43758a^{10}+461890a^{9}bx+2200770a^{8}b^{2}x^{2}+6235515a^{7}b^{3}x^{3}+11639628a^{6}b^{4}x^{4}+14965236a^{5}b^{5}x^{5}+13430340a^{4}b^{6}}{831402x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**20,x)
```

```
[Out] -(43758*a**10 + 461890*a**9*b*x + 2200770*a**8*b**2*x**2 + 6235515*a**7*b**
3*x**3 + 11639628*a**6*b**4*x**4 + 14965236*a**5*b**5*x**5 + 13430340*a**4*
b**6*x**6 + 8314020*a**3*b**7*x**7 + 3401190*a**2*b**8*x**8 + 831402*a*b**9
*x**9 + 92378*b**10*x**10)/(831402*x**19)
```

Giac [A] time = 1.19186, size = 151, normalized size = 1.2

$$-\frac{92378\,b^{10}x^{10} + 831402\,ab^{9}x^{9} + 3401190\,a^{2}b^{8}x^{8} + 8314020\,a^{3}b^{7}x^{7} + 13430340\,a^{4}b^{6}x^{6} + 14965236\,a^{5}b^{5}x^{5} + 11639628\,a}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="giac")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

3.155 $\int c(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{c(a+bx)^2}{2b}$$

```
[Out] (c*(a + b*x)^2)/(2*b)
```

Rubi [A] time = 0.0016425, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {9}

$$\frac{c(a+bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[c*(a + b*x),x]
```

```
[Out] (c*(a + b*x)^2)/(2*b)
```

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[(a*(b + c*x)^2)/(2*c), x] /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int c(a+bx)\,dx = \frac{c(a+bx)^2}{2b}$$

Mathematica [A] time = 0.001267, size = 14, normalized size = 0.93

$$c\left(ax+\frac{bx^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[c*(a + b*x),x]
```

```
[Out] c*(a*x + (b*x^2)/2)
```

Maple [A] time = 0.001, size = 13, normalized size = 0.9

$$c\left(ax+\frac{bx^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*(b*x+a),x)
```

```
[Out] c*(a*x+1/2*b*x^2)
```

Maxima [A] time = 0.996894, size = 18, normalized size = 1.2

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*c
```

Fricas [A] time = 1.26331, size = 28, normalized size = 1.87

$$\frac{1}{2}x^2cb + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*c*b + x*c*a
```

Sympy [A] time = 0.074225, size = 12, normalized size = 0.8

$$acx + \frac{bcx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x)
```

```
[Out] a*c*x + b*c*x**2/2
```

Giac [A] time = 1.17623, size = 18, normalized size = 1.2

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*c
```

3.156 $\int \frac{(c+d)(a+bx)}{e} \, dx$

Optimal. Leaf size=20

$$\frac{(c+d)(a+bx)^2}{2be}$$

```
[Out] ((c + d)*(a + b*x)^2)/(2*b*e)
```

Rubi [A] time = 0.0036228, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {9}

$$\frac{(c+d)(a+bx)^2}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d)*(a + b*x))/e,x]
```

```
[Out] ((c + d)*(a + b*x)^2)/(2*b*e)
```

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[(a*(b + c*x)^2)/(2*c), x] /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(c+d)(a+bx)}{e} \, dx = \frac{(c+d)(a+bx)^2}{2be}$$

Mathematica [A] time = 0.0007703, size = 19, normalized size = 0.95

$$\frac{(c+d)\left(ax+\frac{bx^2}{2}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d)*(a + b*x))/e,x]
```

```
[Out] ((c + d)*(a*x + (b*x^2)/2))/e
```

Maple [A] time = 0.001, size = 18, normalized size = 0.9

$$\frac{c+d}{e}\left(ax+\frac{bx^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d)*(b*x+a)/e,x)
```

```
[Out] (c+d)/e*(a*x+1/2*b*x^2)
```

Maxima [A] time = 1.04736, size = 24, normalized size = 1.2

$$\frac{\left(bx^2 + 2\,ax\right)(c + d)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*(c + d)/e
```

Fricas [A] time = 1.53868, size = 58, normalized size = 2.9

$$\frac{(bc + bd)x^2 + 2\,(ac + ad)x}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="fricas")
```

```
[Out] 1/2*((b*c + b*d)*x^2 + 2*(a*c + a*d)*x)/e
```

Sympy [A] time = 0.068977, size = 22, normalized size = 1.1

$$\frac{x^2\,(bc + bd)}{2e} + \frac{x\,(ac + ad)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x)
```

```
[Out] x**2*(b*c + b*d)/(2*e) + x*(a*c + a*d)/e
```

Giac [A] time = 1.17999, size = 23, normalized size = 1.15

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)(c + d)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*(c + d)*e^(-1)
```

3.157 $\int \frac{x^5}{a+bx} dx$

Optimal. Leaf size=70

$$-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}+\frac{a^4x}{b^5}-\frac{a^5\log(a+bx)}{b^6}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}$$

```
[Out] (a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a*x^4)/(4*b^2) + x^5
/(5*b) - (a^5*Log[a + b*x])/b^6
```

Rubi [A] time = 0.0344801, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}+\frac{a^4x}{b^5}-\frac{a^5\log(a+bx)}{b^6}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a*x^4)/(4*b^2) + x^5
/(5*b) - (a^5*Log[a + b*x])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{a+bx}\,dx &= \int \left(\frac{a^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}-\frac{a^5}{b^5(a+bx)}\right)dx\\ &= \frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.0053479, size = 70, normalized size = 1.

$$-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}+\frac{a^4x}{b^5}-\frac{a^5\log(a+bx)}{b^6}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a*x^4)/(4*b^2) + x^5
/(5*b) - (a^5*Log[a + b*x])/b^6
```

Maple [A] time = 0.002, size = 63, normalized size = 0.9

$$\frac{a^4 x}{b^5} - \frac{a^3 x^2}{2\, b^4} + \frac{a^2 x^3}{3\, b^3} - \frac{a x^4}{4\, b^2} + \frac{x^5}{5\, b} - \frac{a^5 \ln (bx+a)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a),x)
```

```
[Out] a^4*x/b^5-1/2*a^3*x^2/b^4+1/3*a^2*x^3/b^3-1/4*a*x^4/b^2+1/5*x^5/b-a^5*ln(b*
x+a)/b^6
```

Maxima [A] time = 1.06738, size = 86, normalized size = 1.23

$$-\frac{a^5 \log (bx+a)}{b^6} + \frac{12\, b^4 x^5 - 15\, ab^3 x^4 + 20\, a^2 b^2 x^3 - 30\, a^3 b x^2 + 60\, a^4 x}{60\, b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] -a^5*log(b*x + a)/b^6 + 1/60*(12*b^4*x^5 - 15*a*b^3*x^4 + 20*a^2*b^2*x^3 -
30*a^3*b*x^2 + 60*a^4*x)/b^5
```

Fricas [A] time = 1.49602, size = 144, normalized size = 2.06

$$\frac{12\, b^5 x^5 - 15\, ab^4 x^4 + 20\, a^2 b^3 x^3 - 30\, a^3 b^2 x^2 + 60\, a^4 bx - 60\, a^5 \log (bx+a)}{60\, b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*x^5 - 15*a*b^4*x^4 + 20*a^2*b^3*x^3 - 30*a^3*b^2*x^2 + 60*a^4*
b*x - 60*a^5*log(b*x + a))/b^6
```

Sympy [A] time = 0.373252, size = 61, normalized size = 0.87

$$-\frac{a^5 \log (a+bx)}{b^6} + \frac{a^4 x}{b^5} - \frac{a^3 x^2}{2b^4} + \frac{a^2 x^3}{3b^3} - \frac{a x^4}{4b^2} + \frac{x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a),x)
```

```
[Out] -a**5*log(a + b*x)/b**6 + a**4*x/b**5 - a**3*x**2/(2*b**4) + a**2*x**3/(3*b
**3) - a*x**4/(4*b**2) + x**5/(5*b)
```

Giac [A] time = 1.15778, size = 88, normalized size = 1.26

$$-\frac{a^5 \log (|bx+a|)}{b^6} + \frac{12\, b^4 x^5 - 15\, ab^3 x^4 + 20\, a^2 b^2 x^3 - 30\, a^3 b x^2 + 60\, a^4 x}{60\, b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] -a^5*log(abs(b*x + a))/b^6 + 1/60*(12*b^4*x^5 - 15*a*b^3*x^4 + 20*a^2*b^2*x
^3 - 30*a^3*b*x^2 + 60*a^4*x)/b^5
```

3.158 $\int \frac{x^4}{a+bx}\,dx$

Optimal. Leaf size=57

$$\frac{a^2x^2}{2b^3}-\frac{a^3x}{b^4}+\frac{a^4\log(a+bx)}{b^5}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Rubi [A] time = 0.0231751, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x^2}{2b^3}-\frac{a^3x}{b^4}+\frac{a^4\log(a+bx)}{b^5}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a+bx}\,dx&=\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx\\&=-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.0072726, size = 57, normalized size = 1.

$$\frac{a^2x^2}{2b^3}-\frac{a^3x}{b^4}+\frac{a^4\log(a+bx)}{b^5}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Maple [A] time = 0.001, size = 52, normalized size = 0.9

$$-\frac{a^{3}x}{b^{4}}+\frac{a^{2}x^{2}}{2\,b^{3}}-\frac{a\,x^{3}}{3\,b^{2}}+\frac{x^{4}}{4\,b}+\frac{a^{4}\ln\left(bx+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a),x)
```

```
[Out] -a^3*x/b^4+1/2*a^2*x^2/b^3-1/3*a*x^3/b^2+1/4*x^4/b+a^4*ln(b*x+a)/b^5
```

Maxima [A] time = 1.07898, size = 70, normalized size = 1.23

$$\frac{a^{4}\log\left(bx+a\right)}{b^{5}}+\frac{3\,b^{3}x^{4}-4\,ab^{2}x^{3}+6\,a^{2}bx^{2}-12\,a^{3}x}{12\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3
*x)/b^4
```

Fricas [A] time = 1.42708, size = 117, normalized size = 2.05

$$\frac{3\,b^{4}x^{4}-4\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}-12\,a^{3}bx+12\,a^{4}\log\left(bx+a\right)}{12\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))/b^5
```

Sympy [A] time = 0.367903, size = 49, normalized size = 0.86

$$\frac{a^{4}\log\left(a+bx\right)}{b^{5}}-\frac{a^{3}x}{b^{4}}+\frac{a^{2}x^{2}}{2b^{3}}-\frac{ax^{3}}{3b^{2}}+\frac{x^{4}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a),x)
```

```
[Out] a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2)
 + x**4/(4*b)
```

Giac [A] time = 1.13922, size = 72, normalized size = 1.26

$$\frac{a^{4}\log\left(\left|bx+a\right|\right)}{b^{5}}+\frac{3\,b^{3}x^{4}-4\,ab^{2}x^{3}+6\,a^{2}bx^{2}-12\,a^{3}x}{12\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] a^4*log(abs(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 1
2*a^3*x)/b^4
```

3.159 $\int \frac{x^3}{a+bx} \, dx$

Optimal. Leaf size=44

$$\frac{a^2x}{b^3} - \frac{a^3\log(a+bx)}{b^4} - \frac{ax^2}{2b^2} + \frac{x^3}{3b}$$

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Rubi [A] time = 0.0196606, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x}{b^3} - \frac{a^3\log(a+bx)}{b^4} - \frac{ax^2}{2b^2} + \frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+bx} \, dx &= \int \left(\frac{a^2}{b^3} - \frac{ax}{b^2} + \frac{x^2}{b} - \frac{a^3}{b^3(a+bx)}\right) dx \\ &= \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b} - \frac{a^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0040877, size = 44, normalized size = 1.

$$\frac{a^2x}{b^3} - \frac{a^3\log(a+bx)}{b^4} - \frac{ax^2}{2b^2} + \frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Maple [A] time = 0.003, size = 41, normalized size = 0.9

$$\frac{a^2x}{b^3} - \frac{ax^2}{2\,b^2} + \frac{x^3}{3\,b} - \frac{a^3\ln(bx+a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a),x)
```

```
[Out] a^2*x/b^3-1/2*a*x^2/b^2+1/3*x^3/b-a^3*ln(b*x+a)/b^4
```

Maxima [A] time = 1.02967, size = 57, normalized size = 1.3

$$-\frac{a^3 \log(bx+a)}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] -a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Fricas [A] time = 1.44858, size = 92, normalized size = 2.09

$$\frac{2\,b^3x^3 - 3\,ab^2x^2 + 6\,a^2bx - 6\,a^3 \log(bx+a)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

Sympy [A] time = 0.339452, size = 37, normalized size = 0.84

$$-\frac{a^3 \log(a+bx)}{b^4} + \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a),x)
```

```
[Out] -a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

Giac [A] time = 1.18386, size = 58, normalized size = 1.32

$$-\frac{a^3 \log(|bx+a|)}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

3.160 $\int \frac{x^2}{a+bx} dx$

Optimal. Leaf size=31

$$\frac{a^2 \log(a + bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rubi [A] time = 0.013926, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2 \log(a + bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a + bx} dx &= \int \left(-\frac{a}{b^2} + \frac{x}{b} + \frac{a^2}{b^2(a + bx)}\right) dx \\ &= -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a + bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0034313, size = 31, normalized size = 1.

$$\frac{a^2 \log(a + bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Maple [A] time = 0.002, size = 30, normalized size = 1.

$$-\frac{ax}{b^2} + \frac{x^2}{2\,b} + \frac{a^2 \ln(bx + a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a),x)
```

```
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

Maxima [A] time = 1.04779, size = 39, normalized size = 1.26

$$\frac{a^2 \log (bx + a)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Fricas [A] time = 1.42654, size = 68, normalized size = 2.19

$$\frac{b^2x^2 - 2\,abx + 2\,a^2 \log (bx + a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [A] time = 0.345305, size = 26, normalized size = 0.84

$$\frac{a^2 \log (a + bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a),x)
```

```
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

Giac [A] time = 1.17318, size = 41, normalized size = 1.32

$$\frac{a^2 \log (|bx + a|)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

3.161 $\int \frac{x}{a+bx} dx$

Optimal. Leaf size=18

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rubi [A] time = 0.0092203, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{a + bx} dx &= \int \left(\frac{1}{b} - \frac{a}{b(a + bx)}\right) dx \\ &= \frac{x}{b} - \frac{a \log(a + bx)}{b^2} \end{aligned}$$

Mathematica [A] time = 0.002952, size = 18, normalized size = 1.

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Maple [A] time = 0.002, size = 19, normalized size = 1.1

$$\frac{x}{b} - \frac{a \ln(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a),x)
```

```
[Out] x/b-a*ln(b*x+a)/b^2
```

Maxima [A] time = 0.996971, size = 24, normalized size = 1.33

$$\frac{x}{b} - \frac{a\log\left(bx + a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="maxima")
```

```
[Out] x/b - a*log(b*x + a)/b^2
```

Fricas [A] time = 1.51795, size = 38, normalized size = 2.11

$$\frac{bx - a\log\left(bx + a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(b*x + a))/b^2
```

Sympy [A] time = 0.364722, size = 14, normalized size = 0.78

$$-\frac{a\log\left(a + bx\right)}{b^2} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x)
```

```
[Out] -a*log(a + b*x)/b**2 + x/b
```

Giac [A] time = 1.16234, size = 26, normalized size = 1.44

$$\frac{x}{b} - \frac{a\log\left(|bx + a|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="giac")
```

```
[Out] x/b - a*log(abs(b*x + a))/b^2
```

3.162 $\int \frac{1}{a+bx} dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

```
[Out] Log[a + b*x]/b
```

Rubi [A] time = 0.0016758, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+bx} dx = \frac{\log(a+bx)}{b}$$

Mathematica [A] time = 0.0008798, size = 10, normalized size = 1.

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Maple [A] time = 0.001, size = 11, normalized size = 1.1

$$\frac{\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a),x)
```

[Out] ln(b*x+a)/b

Maxima [A] time = 1.04484, size = 14, normalized size = 1.4

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a),x, algorithm="maxima")

[Out] log(b*x + a)/b

Fricas [A] time = 1.46162, size = 22, normalized size = 2.2

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a),x, algorithm="fricas")

[Out] log(b*x + a)/b

Sympy [A] time = 0.079588, size = 7, normalized size = 0.7

$$\frac{\log(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a),x)

[Out] log(a + b*x)/b

Giac [A] time = 1.17694, size = 15, normalized size = 1.5

$$\frac{\log(|bx+a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a),x, algorithm="giac")

[Out] log(abs(b*x + a))/b

3.163 $\int \frac{1}{x(a+bx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rubi [A] time = 0.0038605, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)}\,dx &= \frac{\int \frac{1}{x}\,dx}{a}-\frac{b\int \frac{1}{a+bx}\,dx}{a}\\ &= \frac{\log(x)}{a}-\frac{\log(a+bx)}{a}\end{aligned}$$

Mathematica [A] time = 0.0054579, size = 18, normalized size = 1.

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Maple [A] time = 0.005, size = 19, normalized size = 1.1

$$\frac{\ln(x)}{a} - \frac{\ln(bx+a)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a),x)
```

```
[Out] ln(x)/a-ln(b*x+a)/a
```

Maxima [A] time = 0.996568, size = 24, normalized size = 1.33

$$-\frac{\log(bx+a)}{a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] -log(b*x + a)/a + log(x)/a
```

Fricas [A] time = 1.54129, size = 38, normalized size = 2.11

$$-\frac{\log(bx+a) - \log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) - log(x))/a
```

Sympy [A] time = 0.232074, size = 10, normalized size = 0.56

$$\frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x)
```

```
[Out] (log(x) - log(a/b + x))/a
```

Giac [A] time = 1.18659, size = 27, normalized size = 1.5

$$-\frac{\log(|bx+a|)}{a} + \frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

3.164 $\int \frac{1}{x^2(a+bx)}\,dx$

Optimal. Leaf size=28

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rubi [A] time = 0.012679, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)}\,dx &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0051245, size = 28, normalized size = 1.

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Maple [A] time = 0.009, size = 29, normalized size = 1.

$$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a),x)
```

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Maxima [A] time = 1.035, size = 38, normalized size = 1.36

$$\frac{b\log\left(bx+a\right)}{a^2}-\frac{b\log\left(x\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Fricas [A] time = 1.52783, size = 61, normalized size = 2.18

$$\frac{bx\log\left(bx+a\right)-bx\log\left(x\right)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A] time = 0.433244, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a),x)
```

```
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Giac [A] time = 1.18548, size = 41, normalized size = 1.46

$$\frac{b\log\left(|bx+a|\right)}{a^2}-\frac{b\log\left(|x|\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

3.165 $\int \frac{1}{x^3(a+bx)} dx$

Optimal. Leaf size=42

$$\frac{b^2\log(x)}{a^3} - \frac{b^2\log(a+bx)}{a^3} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rubi [A] time = 0.0175658, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{b^2\log(x)}{a^3} - \frac{b^2\log(a+bx)}{a^3} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)} dx &= \int \left(\frac{1}{ax^3} - \frac{b}{a^2x^2} + \frac{b^2}{a^3x} - \frac{b^3}{a^3(a+bx)}\right) dx\\ &= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b^2\log(x)}{a^3} - \frac{b^2\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0097045, size = 42, normalized size = 1.

$$\frac{b^2\log(x)}{a^3} - \frac{b^2\log(a+bx)}{a^3} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Maple [A] time = 0.006, size = 41, normalized size = 1.

$$-\frac{1}{2\,ax^2} + \frac{b}{a^2x} + \frac{b^2\ln(x)}{a^3} - \frac{b^2\ln(bx+a)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a),x)
```

```
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Maxima [A] time = 1.04497, size = 54, normalized size = 1.29

$$-\frac{b^2 \log(bx+a)}{a^3} + \frac{b^2 \log(x)}{a^3} + \frac{2\,bx - a}{2\,a^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

Fricas [A] time = 1.59182, size = 103, normalized size = 2.45

$$-\frac{2\,b^2x^2 \log(bx+a) - 2\,b^2x^2 \log(x) - 2\,abx + a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [A] time = 0.50212, size = 31, normalized size = 0.74

$$\frac{-a + 2bx}{2a^2x^2} + \frac{b^2\left(\log(x) - \log\left(\frac{a}{b} + x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a),x)
```

```
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

Giac [A] time = 1.1739, size = 61, normalized size = 1.45

$$-\frac{b^2 \log(|bx+a|)}{a^3} + \frac{b^2 \log(|x|)}{a^3} + \frac{2\,abx - a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

3.166 $\int \frac{1}{x^4(a+bx)}\,dx$

Optimal. Leaf size=56

$$-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[a
+ b*x])/a^4
```

Rubi [A] time = 0.0214565, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[a
+ b*x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)}\,dx &= \int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.0061387, size = 56, normalized size = 1.

$$-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[a
+ b*x])/a^4
```

Maple [A] time = 0.007, size = 53, normalized size = 1.

$$-\frac{1}{3\,a x^{3}}+\frac{b}{2\,a^{2} x^{2}}-\frac{b^{2}}{a^{3} x}-\frac{b^{3} \ln \left(x\right)}{a^{4}}+\frac{b^{3} \ln \left(b x +a \right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a),x)
```

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Maxima [A] time = 1.0718, size = 69, normalized size = 1.23

$$\frac{b^{3} \log \left(b x + a\right)}{a^{4}} - \frac{b^{3} \log \left(x\right)}{a^{4}} - \frac{6 \, b^{2} x^{2} - 3 \, a b x + 2 \, a^{2}}{6 \, a^{3} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/a^4 - b^3*log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/(
a^3*x^3)
```

Fricas [A] time = 1.51962, size = 126, normalized size = 2.25

$$\frac{6 \, b^{3} x^{3} \log \left(b x + a\right) - 6 \, b^{3} x^{3} \log \left(x\right) - 6 \, a b^{2} x^{2} + 3 \, a^{2} b x - 2 \, a^{3}}{6 \, a^{4} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
2*a^3)/(a^4*x^3)
```

Sympy [A] time = 0.529138, size = 44, normalized size = 0.79

$$- \frac{2 a^{2} - 3 a b x + 6 b^{2} x^{2}}{6 a^{3} x^{3}} + \frac{b^{3} \left(- \log{\left(x \right)} + \log{\left(\frac{a}{b} + x \right)}\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a),x)
```

```
[Out] -(2*a**2 - 3*a*b*x + 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b +
 x))/a**4
```

Giac [A] time = 1.21334, size = 76, normalized size = 1.36

$$\frac{b^{3} \log \left({\left| b x + a \right|}\right)}{a^{4}} - \frac{b^{3} \log \left({\left| x \right|}\right)}{a^{4}} - \frac{6 \, a b^{2} x^{2} - 3 \, a^{2} b x + 2 \, a^{3}}{6 \, a^{4} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2*
b*x + 2*a^3)/(a^4*x^3)
```

3.167 $\int \frac{1}{x^5(a+bx)}\,dx$

Optimal. Leaf size=68

$$-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}+\frac{b}{3a^2x^3}-\frac{1}{4ax^4}$$

```
[Out] -1/(4*a*x^4) + b/(3*a^2*x^3) - b^2/(2*a^3*x^2) + b^3/(a^4*x) + (b^4*Log[x])
/a^5 - (b^4*Log[a + b*x])/a^5
```

Rubi [A] time = 0.0354385, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}+\frac{b}{3a^2x^3}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)),x]
```

```
[Out] -1/(4*a*x^4) + b/(3*a^2*x^3) - b^2/(2*a^3*x^2) + b^3/(a^4*x) + (b^4*Log[x])
/a^5 - (b^4*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)}\,dx &= \int \left(\frac{1}{ax^5}-\frac{b}{a^2x^4}+\frac{b^2}{a^3x^3}-\frac{b^3}{a^4x^2}+\frac{b^4}{a^5x}-\frac{b^5}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{4ax^4}+\frac{b}{3a^2x^3}-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.0075066, size = 68, normalized size = 1.

$$-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}+\frac{b}{3a^2x^3}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)),x]
```

```
[Out] -1/(4*a*x^4) + b/(3*a^2*x^3) - b^2/(2*a^3*x^2) + b^3/(a^4*x) + (b^4*Log[x])
/a^5 - (b^4*Log[a + b*x])/a^5
```

Maple [A] time = 0.006, size = 63, normalized size = 0.9

$$-\frac{1}{4\,a x^{4}}+\frac{b}{3\,a^{2}x^{3}}-\frac{b^{2}}{2\,a^{3}x^{2}}+\frac{b^{3}}{a^{4}x}+\frac{b^{4}\ln\left(x\right)}{a^{5}}-\frac{b^{4}\ln\left(bx+a\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a),x)
```

```
[Out] -1/4/a/x^4+1/3*b/a^2/x^3-1/2*b^2/a^3/x^2+b^3/a^4/x+b^4*ln(x)/a^5-b^4*ln(b*x
+a)/a^5
```

Maxima [A] time = 1.01518, size = 84, normalized size = 1.24

$$-\frac{b^{4}\log\left(bx+a\right)}{a^{5}}+\frac{b^{4}\log\left(x\right)}{a^{5}}+\frac{12\,b^{3}x^{3}-6\,ab^{2}x^{2}+4\,a^{2}bx-3\,a^{3}}{12\,a^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^4*log(b*x + a)/a^5 + b^4*log(x)/a^5 + 1/12*(12*b^3*x^3 - 6*a*b^2*x^2 + 4
*a^2*b*x - 3*a^3)/(a^4*x^4)
```

Fricas [A] time = 1.5254, size = 154, normalized size = 2.26

$$-\frac{12\,b^{4}x^{4}\log\left(bx+a\right)-12\,b^{4}x^{4}\log\left(x\right)-12\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}-4\,a^{3}bx+3\,a^{4}}{12\,a^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(12*b^4*x^4*log(b*x + a) - 12*b^4*x^4*log(x) - 12*a*b^3*x^3 + 6*a^2*b
^2*x^2 - 4*a^3*b*x + 3*a^4)/(a^5*x^4)
```

Sympy [A] time = 0.591385, size = 56, normalized size = 0.82

$$\frac{-3a^{3}+4a^{2}bx-6ab^{2}x^{2}+12b^{3}x^{3}}{12a^{4}x^{4}}+\frac{b^{4}\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a),x)
```

```
[Out] (-3*a**3 + 4*a**2*b*x - 6*a*b**2*x**2 + 12*b**3*x**3)/(12*a**4*x**4) + b**4
*(log(x) - log(a/b + x))/a**5
```

Giac [A] time = 1.18353, size = 90, normalized size = 1.32

$$-\frac{b^4\log(|bx+a|)}{a^5}+\frac{b^4\log(|x|)}{a^5}+\frac{12\,ab^3x^3-6\,a^2b^2x^2+4\,a^3bx-3\,a^4}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] -b^4*log(abs(b*x + a))/a^5 + b^4*log(abs(x))/a^5 + 1/12*(12*a*b^3*x^3 - 6*a
^2*b^2*x^2 + 4*a^3*b*x - 3*a^4)/(a^5*x^4)
```

3.168 $\int \frac{x^6}{(a+bx)^2} \, dx$

Optimal. Leaf size=81

$$-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{a^6}{b^7(a+bx)}+\frac{5a^4x}{b^6}-\frac{6a^5\log(a+bx)}{b^7}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}$$

```
[Out] (5*a^4*x)/b^6 - (2*a^3*x^2)/b^5 + (a^2*x^3)/b^4 - (a*x^4)/(2*b^3) + x^5/(5*
b^2) - a^6/(b^7*(a + b*x)) - (6*a^5*Log[a + b*x])/b^7
```

Rubi [A] time = 0.0592848, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{a^6}{b^7(a+bx)}+\frac{5a^4x}{b^6}-\frac{6a^5\log(a+bx)}{b^7}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^2,x]
```

```
[Out] (5*a^4*x)/b^6 - (2*a^3*x^2)/b^5 + (a^2*x^3)/b^4 - (a*x^4)/(2*b^3) + x^5/(5*
b^2) - a^6/(b^7*(a + b*x)) - (6*a^5*Log[a + b*x])/b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^2} \, dx &= \int \left(\frac{5a^4}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{b^4}-\frac{2ax^3}{b^3}+\frac{x^4}{b^2}+\frac{a^6}{b^6(a+bx)^2}-\frac{6a^5}{b^6(a+bx)}\right) dx\\ &= \frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}-\frac{a^6}{b^7(a+bx)}-\frac{6a^5\log(a+bx)}{b^7}\end{aligned}$$

Mathematica [A] time = 0.0356202, size = 77, normalized size = 0.95

$$\frac{-20a^3b^2x^2+10a^2b^3x^3-\frac{10a^6}{a+bx}+50a^4bx-60a^5\log(a+bx)-5ab^4x^4+2b^5x^5}{10b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^2,x]
```

```
[Out] (50*a^4*b*x - 20*a^3*b^2*x^2 + 10*a^2*b^3*x^3 - 5*a*b^4*x^4 + 2*b^5*x^5 - (
10*a^6)/(a + b*x) - 60*a^5*Log[a + b*x])/(10*b^7)
```

Maple [A] time = 0.007, size = 78, normalized size = 1.

$$5\frac{a^4x}{b^6}-2\frac{a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}-\frac{a^6}{b^7(bx+a)}-6\frac{a^5\ln(bx+a)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^2,x)
```

```
[Out] 5*a^4*x/b^6-2*a^3*x^2/b^5+a^2*x^3/b^4-1/2*a*x^4/b^3+1/5*x^5/b^2-a^6/b^7/(b*
x+a)-6*a^5*ln(b*x+a)/b^7
```

Maxima [A] time = 1.05872, size = 111, normalized size = 1.37

$$-\frac{a^6}{b^8x+ab^7}-\frac{6a^5\log(bx+a)}{b^7}+\frac{2b^4x^5-5ab^3x^4+10a^2b^2x^3-20a^3bx^2+50a^4x}{10b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^6/(b^8*x + a*b^7) - 6*a^5*log(b*x + a)/b^7 + 1/10*(2*b^4*x^5 - 5*a*b^3*x
^4 + 10*a^2*b^2*x^3 - 20*a^3*b*x^2 + 50*a^4*x)/b^6
```

Fricas [A] time = 1.48875, size = 208, normalized size = 2.57

$$\frac{2b^6x^6-3ab^5x^5+5a^2b^4x^4-10a^3b^3x^3+30a^4b^2x^2+50a^5bx-10a^6-60\left(a^5bx+a^6\right)\log(bx+a)}{10\left(b^8x+ab^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^6*x^6 - 3*a*b^5*x^5 + 5*a^2*b^4*x^4 - 10*a^3*b^3*x^3 + 30*a^4*b^2
*x^2 + 50*a^5*b*x - 10*a^6 - 60*(a^5*b*x + a^6)*log(b*x + a))/(b^8*x + a*b^
7)
```

Sympy [A] time = 0.497526, size = 78, normalized size = 0.96

$$-\frac{a^6}{ab^7+b^8x}-\frac{6a^5\log(a+bx)}{b^7}+\frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**2,x)
```

```
[Out] -a**6/(a*b**7 + b**8*x) - 6*a**5*log(a + b*x)/b**7 + 5*a**4*x/b**6 - 2*a**3
*x**2/b**5 + a**2*x**3/b**4 - a*x**4/(2*b**3) + x**5/(5*b**2)
```

Giac [A] time = 1.22041, size = 139, normalized size = 1.72

$$-\frac{(bx+a)^5\left(\frac{15\,a}{bx+a}-\frac{50\,a^2}{(bx+a)^2}+\frac{100\,a^3}{(bx+a)^3}-\frac{150\,a^4}{(bx+a)^4}-2\right)}{10\,b^7}+\frac{6\,a^5\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^7}-\frac{a^6}{(bx+a)b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/10*(b*x + a)^5*(15*a/(b*x + a) - 50*a^2/(b*x + a)^2 + 100*a^3/(b*x + a)^
3 - 150*a^4/(b*x + a)^4 - 2)/b^7 + 6*a^5*log(abs(b*x + a)/((b*x + a)^2*abs(
b)))/b^7 - a^6/((b*x + a)*b^7)
```

3.169 $\int \frac{x^5}{(a+bx)^2} \, dx$

Optimal. Leaf size=72

$$\frac{3a^2x^2}{2b^4}+\frac{a^5}{b^6(a+bx)}-\frac{4a^3x}{b^5}+\frac{5a^4\log(a+bx)}{b^6}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}$$

```
[Out] (-4*a^3*x)/b^5 + (3*a^2*x^2)/(2*b^4) - (2*a*x^3)/(3*b^3) + x^4/(4*b^2) + a^
5/(b^6*(a + b*x)) + (5*a^4*Log[a + b*x])/b^6
```

Rubi [A] time = 0.0423157, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2x^2}{2b^4}+\frac{a^5}{b^6(a+bx)}-\frac{4a^3x}{b^5}+\frac{5a^4\log(a+bx)}{b^6}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^2,x]
```

```
[Out] (-4*a^3*x)/b^5 + (3*a^2*x^2)/(2*b^4) - (2*a*x^3)/(3*b^3) + x^4/(4*b^2) + a^
5/(b^6*(a + b*x)) + (5*a^4*Log[a + b*x])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^2}\,dx &= \int \left(-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a^5}{b^5(a+bx)^2}+\frac{5a^4}{b^5(a+bx)}\right)dx\\ &= -\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}+\frac{a^5}{b^6(a+bx)}+\frac{5a^4\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.0295879, size = 66, normalized size = 0.92

$$\frac{18a^2b^2x^2+\frac{12a^5}{a+bx}-48a^3bx+60a^4\log(a+bx)-8ab^3x^3+3b^4x^4}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^2,x]
```

```
[Out] (-48*a^3*b*x + 18*a^2*b^2*x^2 - 8*a*b^3*x^3 + 3*b^4*x^4 + (12*a^5)/(a + b*x
) + 60*a^4*Log[a + b*x])/(12*b^6)
```

Maple [A] time = 0.006, size = 67, normalized size = 0.9

$$-4\frac{a^{3}x}{b^{5}}+\frac{3a^{2}x^{2}}{2b^{4}}-\frac{2ax^{3}}{3b^{3}}+\frac{x^{4}}{4b^{2}}+\frac{a^{5}}{b^{6}(bx+a)}+5\frac{a^{4}\ln(bx+a)}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^2,x)
```

```
[Out] -4*a^3*x/b^5+3/2*a^2*x^2/b^4-2/3*a*x^3/b^3+1/4*x^4/b^2+a^5/b^6/(b*x+a)+5*a^
4*ln(b*x+a)/b^6
```

Maxima [A] time = 1.04987, size = 95, normalized size = 1.32

$$\frac{a^{5}}{b^{7}x+ab^{6}}+\frac{5a^{4}\log(bx+a)}{b^{6}}+\frac{3b^{3}x^{4}-8ab^{2}x^{3}+18a^{2}bx^{2}-48a^{3}x}{12b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a^5/(b^7*x + a*b^6) + 5*a^4*log(b*x + a)/b^6 + 1/12*(3*b^3*x^4 - 8*a*b^2*x^
3 + 18*a^2*b*x^2 - 48*a^3*x)/b^5
```

Fricas [A] time = 1.58896, size = 186, normalized size = 2.58

$$\frac{3b^{5}x^{5}-5ab^{4}x^{4}+10a^{2}b^{3}x^{3}-30a^{3}b^{2}x^{2}-48a^{4}bx+12a^{5}+60\left(a^{4}bx+a^{5}\right)\log(bx+a)}{12\left(b^{7}x+ab^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^5 - 5*a*b^4*x^4 + 10*a^2*b^3*x^3 - 30*a^3*b^2*x^2 - 48*a^4*b*
x + 12*a^5 + 60*(a^4*b*x + a^5)*log(b*x + a))/(b^7*x + a*b^6)
```

Sympy [A] time = 0.45716, size = 71, normalized size = 0.99

$$\frac{a^{5}}{ab^{6}+b^{7}x}+\frac{5a^{4}\log(a+bx)}{b^{6}}-\frac{4a^{3}x}{b^{5}}+\frac{3a^{2}x^{2}}{2b^{4}}-\frac{2ax^{3}}{3b^{3}}+\frac{x^{4}}{4b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**2,x)
```

```
[Out] a**5/(a*b**6 + b**7*x) + 5*a**4*log(a + b*x)/b**6 - 4*a**3*x/b**5 + 3*a**2*
x**2/(2*b**4) - 2*a*x**3/(3*b**3) + x**4/(4*b**2)
```

Giac [A] time = 1.17184, size = 122, normalized size = 1.69

$$-\frac{(bx+a)^4\left(\frac{20\,a}{bx+a}-\frac{60\,a^2}{(bx+a)^2}+\frac{120\,a^3}{(bx+a)^3}-3\right)}{12\,b^6}-\frac{5\,a^4\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^6}+\frac{a^5}{(bx+a)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/12*(b*x + a)^4*(20*a/(b*x + a) - 60*a^2/(b*x + a)^2 + 120*a^3/(b*x + a)^
3 - 3)/b^6 - 5*a^4*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^6 + a^5/((b*x +
 a)*b^6)
```

3.170 $\int \frac{x^4}{(a+bx)^2} \, dx$

Optimal. Leaf size=58

$$-\frac{a^4}{b^5(a+bx)}+\frac{3a^2x}{b^4}-\frac{4a^3\log(a+bx)}{b^5}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

```
[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Lo
g[a + b*x])/b^5
```

Rubi [A] time = 0.0329027, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^4}{b^5(a+bx)}+\frac{3a^2x}{b^4}-\frac{4a^3\log(a+bx)}{b^5}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^2,x]
```

```
[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Lo
g[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^2}\,dx &= \int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx\\ &= \frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.0301503, size = 54, normalized size = 0.93

$$\frac{-\frac{3a^4}{a+bx}+9a^2bx-12a^3\log(a+bx)-3ab^2x^2+b^3x^3}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^2,x]
```

```
[Out] (9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*x
])/(3*b^5)
```

Maple [A] time = 0.006, size = 57, normalized size = 1.

$$3\,\frac{a^{2}x}{b^{4}}-\frac{ax^{2}}{b^{3}}+\frac{x^{3}}{3\,b^{2}}-\frac{a^{4}}{b^{5}\left(bx+a\right)}-4\,\frac{a^{3}\ln\left(bx+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^2,x)
```

```
[Out] 3*a^2*x/b^4-a*x^2/b^3+1/3*x^3/b^2-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

Maxima [A] time = 1.25861, size = 80, normalized size = 1.38

$$-\frac{a^{4}}{b^{6}x+ab^{5}}-\frac{4\,a^{3}\log\left(bx+a\right)}{b^{5}}+\frac{b^{2}x^{3}-3\,abx^{2}+9\,a^{2}x}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^4/(b^6*x + a*b^5) - 4*a^3*log(b*x + a)/b^5 + 1/3*(b^2*x^3 - 3*a*b*x^2 +
9*a^2*x)/b^4
```

Fricas [A] time = 1.50341, size = 155, normalized size = 2.67

$$\frac{b^{4}x^{4}-2\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}+9\,a^{3}bx-3\,a^{4}-12\,\left(a^{3}bx+a^{4}\right)\log\left(bx+a\right)}{3\,\left(b^{6}x+ab^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

Sympy [A] time = 0.487386, size = 54, normalized size = 0.93

$$-\frac{a^{4}}{ab^{5}+b^{6}x}-\frac{4a^{3}\log\left(a+bx\right)}{b^{5}}+\frac{3a^{2}x}{b^{4}}-\frac{ax^{2}}{b^{3}}+\frac{x^{3}}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**2,x)
```

```
[Out] -a**4/(a*b**5 + b**6*x) - 4*a**3*log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**2
/b**3 + x**3/(3*b**2)
```

Giac [A] time = 1.21519, size = 107, normalized size = 1.84

$$-\frac{(bx+a)^3\left(\frac{6\,a}{bx+a}-\frac{18\,a^2}{(bx+a)^2}-1\right)}{3\,b^5}+\frac{4\,a^3\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^5}-\frac{a^4}{(bx+a)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(b*x + a)^3*(6*a/(b*x + a) - 18*a^2/(b*x + a)^2 - 1)/b^5 + 4*a^3*log(a
bs(b*x + a)/((b*x + a)^2*abs(b)))/b^5 - a^4/((b*x + a)*b^5)
```

3.171 $\int \frac{x^3}{(a+bx)^2} \, dx$

Optimal. Leaf size=46

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

```
[Out] (-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4
```

Rubi [A] time = 0.0268024, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^2,x]
```

```
[Out] (-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^2}\,dx &= \int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx\\ &= -\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.016625, size = 43, normalized size = 0.93

$$\frac{\frac{2a^3}{a+bx}+6a^2\log(a+bx)-4abx+b^2x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^2,x]
```

```
[Out] (-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)
```

Maple [A] time = 0.006, size = 45, normalized size = 1.

$$-2\,\frac{ax}{b^3}+\frac{x^2}{2\,b^2}+\frac{a^3}{b^4\,(bx+a)}+3\,\frac{a^2\ln(bx+a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^2,x)
```

```
[Out] -2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Maxima [A] time = 1.02949, size = 63, normalized size = 1.37

$$\frac{a^3}{b^5x + ab^4} + \frac{3\,a^2 \log(bx + a)}{b^4} + \frac{bx^2 - 4\,ax}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3
```

Fricas [A] time = 1.58452, size = 132, normalized size = 2.87

$$\frac{b^3x^3 - 3\,ab^2x^2 - 4\,a^2bx + 2\,a^3 + 6\left(a^2bx + a^3\right)\log(bx + a)}{2\left(b^5x + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))/(b^5*x + a*b^4)
```

Sympy [A] time = 0.467723, size = 44, normalized size = 0.96

$$\frac{a^3}{ab^4 + b^5x} + \frac{3a^2 \log(a + bx)}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**2,x)
```

```
[Out] a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b*
*2)
```

Giac [A] time = 1.18912, size = 89, normalized size = 1.93

$$-\frac{(bx + a)^2\left(\frac{6\,a}{bx+a} - 1\right)}{2\,b^4} - \frac{3\,a^2 \log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4} + \frac{a^3}{(bx + a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*x + a)^2*(6*a/(b*x + a) - 1)/b^4 - 3*a^2*log(abs(b*x + a)/((b*x + a
)^2*abs(b)))/b^4 + a^3/((b*x + a)*b^4)
```

3.172 $\int \frac{x^2}{(a+bx)^2} dx$

Optimal. Leaf size=33

$$-\frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3} + \frac{x}{b^2}$$

```
[Out] x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Rubi [A] time = 0.0179673, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3} + \frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^2,x]
```

```
[Out] x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+bx)^2} dx &= \int \left(\frac{1}{b^2} + \frac{a^2}{b^2(a+bx)^2} - \frac{2a}{b^2(a+bx)}\right) dx \\
&= \frac{x}{b^2} - \frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.0141772, size = 29, normalized size = 0.88

$$\frac{-\frac{a^2}{a+bx} - 2a\log(a+bx) + bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^2,x]
```

```
[Out] (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

Maple [A] time = 0.006, size = 34, normalized size = 1.

$$\frac{x}{b^2} - \frac{a^2}{b^3(bx+a)} - 2\frac{a\ln(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^2,x)
```

```
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Maxima [A] time = 1.07287, size = 49, normalized size = 1.48

$$-\frac{a^2}{b^4x+ab^3}+\frac{x}{b^2}-\frac{2\,a\log\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

Fricas [A] time = 1.52762, size = 97, normalized size = 2.94

$$\frac{b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)}{b^4x+ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A] time = 0.453715, size = 31, normalized size = 0.94

$$-\frac{a^2}{ab^3+b^4x}-\frac{2a\log\left(a+bx\right)}{b^3}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**2,x)
```

```
[Out] -a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

Giac [A] time = 1.13514, size = 68, normalized size = 2.06

$$\frac{2\,a\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3}+\frac{bx+a}{b^3}-\frac{a^2}{(bx+a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*a*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^3 + (b*x + a)/b^3 - a^2/((b*x
+ a)*b^3)
```

3.173 $\int \frac{x}{(a+bx)^2} dx$

Optimal. Leaf size=23

$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

```
[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Rubi [A] time = 0.0119928, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^2,x]

[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^2}\,dx &= \int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx\\ &= \frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0105572, size = 20, normalized size = 0.87

$$\frac{\frac{a}{a+bx}+\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^2,x]

[Out] (a/(a + b*x) + Log[a + b*x])/b^2
```

Maple [A] time = 0.004, size = 24, normalized size = 1.

$$\frac{a}{b^2\,(bx+a)}+\frac{\ln(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^2,x)
```

```
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Maxima [A] time = 1.02621, size = 35, normalized size = 1.52

$$\frac{a}{b^3x + ab^2} + \frac{\log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

Fricas [A] time = 1.50717, size = 62, normalized size = 2.7

$$\frac{(bx + a)\log(bx + a) + a}{b^3x + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

Sympy [A] time = 0.399283, size = 20, normalized size = 0.87

$$\frac{a}{ab^2 + b^3x} + \frac{\log(a + bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**2,x)
```

```
[Out] a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

Giac [A] time = 1.17541, size = 57, normalized size = 2.48

$$-\frac{\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} - \frac{a}{(bx+a)b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b))/b
```

3.174 $\int \frac{1}{(a+bx)^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

```
[Out] -(1/(b*(a + b*x)))
```

Rubi [A] time = 0.0017349, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

Mathematica [A] time = 0.003512, size = 12, normalized size = 1.

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Maple [A] time = 0., size = 13, normalized size = 1.1

$$-\frac{1}{b(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2,x)
```

```
[Out] -1/b/(b*x+a)
```

Maxima [A] time = 1.08683, size = 16, normalized size = 1.33

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

Fricas [A] time = 1.46787, size = 24, normalized size = 2.

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

Sympy [A] time = 0.34207, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2,x)
```

```
[Out] -1/(a*b + b**2*x)
```

Giac [A] time = 1.16801, size = 16, normalized size = 1.33

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b)
```

3.175 $\int \frac{1}{x(a+bx)^2}\,dx$

Optimal. Leaf size=29

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

```
[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Rubi [A] time = 0.014131, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^2),x]
```

```
[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx\\ &= \frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.014973, size = 24, normalized size = 0.83

$$\frac{\frac{a}{a+bx}-\log(a+bx)+\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^2),x]
```

```
[Out] (a/(a + b*x) + Log[x] - Log[a + b*x])/a^2
```

Maple [A] time = 0.007, size = 30, normalized size = 1.

$$\frac{1}{a\,(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^2,x)
```

```
[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Maxima [A] time = 1.05159, size = 38, normalized size = 1.31

$$\frac{1}{abx + a^2} - \frac{\log (bx + a)}{a^2} + \frac{\log (x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

Fricas [A] time = 1.39837, size = 89, normalized size = 3.07

$$-\frac{(bx + a) \log (bx + a) - (bx + a) \log (x) - a}{a^2bx + a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

Sympy [A] time = 0.519681, size = 22, normalized size = 0.76

$$\frac{1}{a^2 + abx} + \frac{\log (x) - \log \left(\frac{a}{b} + x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**2,x)
```

```
[Out] 1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

Giac [A] time = 1.17666, size = 51, normalized size = 1.76

$$b\left(\frac{\log \left(\left|-\frac{a}{bx+a} + 1\right|\right)}{a^2b} + \frac{1}{(bx + a)ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] b*(log(abs(-a/(b*x + a) + 1))/(a^2*b) + 1/((b*x + a)*a*b))
```

3.176 $\int \frac{1}{x^2(a+bx)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{1}{a^2x}$$

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rubi [A] time = 0.0208581, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.080398, size = 35, normalized size = 0.83

$$-\frac{a\left(\frac{b}{a+bx}+\frac{1}{x}\right)-2b\log(a+bx)+2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

Maple [A] time = 0.009, size = 43, normalized size = 1.

$$-\frac{1}{a^2x}-\frac{b}{a^2\left(bx+a\right)}-2\frac{b\ln\left(x\right)}{a^3}+2\frac{b\ln\left(bx+a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^2,x)
```

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Maxima [A] time = 1.07965, size = 61, normalized size = 1.45

$$-\frac{2\,bx+a}{a^2bx^2+a^3x}+\frac{2\,b\log\left(bx+a\right)}{a^3}-\frac{2\,b\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

Fricas [A] time = 1.62949, size = 138, normalized size = 3.29

$$-\frac{2\,abx+a^2-2\left(b^2x^2+abx\right)\log\left(bx+a\right)+2\left(b^2x^2+abx\right)\log\left(x\right)}{a^3bx^2+a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [A] time = 0.588879, size = 36, normalized size = 0.86

$$-\frac{a+2bx}{a^3x+a^2bx^2}+\frac{2b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**2,x)
```

```
[Out] -(a + 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

Giac [A] time = 1.22982, size = 70, normalized size = 1.67

$$-\frac{2\,b\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3}-\frac{b}{(bx+a)a^2}+\frac{b}{a^3\left(\frac{a}{bx+a}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*b*log(abs(-a/(b*x + a) + 1))/a^3 - b/((b*x + a)*a^2) + b/(a^3*(a/(b*x +
a) - 1))
```

3.177 $\int \frac{1}{x^3(a+bx)^2}\,dx$

Optimal. Leaf size=58

$$\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

```
[Out] -1/(2*a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4 -
 (3*b^2*Log[a + b*x])/a^4
```

Rubi [A] time = 0.0276379, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^2),x]
```

```
[Out] -1/(2*a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4 -
 (3*b^2*Log[a + b*x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.0812477, size = 53, normalized size = 0.91

$$\frac{a\left(\frac{2b^2}{a+bx}-\frac{a}{x^2}+\frac{4b}{x}\right)-6b^2\log(a+bx)+6b^2\log(x)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^2),x]
```

```
[Out] (a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a +
b*x])/(2*a^4)
```

Maple [A] time = 0.01, size = 57, normalized size = 1.

$$-\frac{1}{2\,a^{2}x^{2}}+2\,\frac{b}{a^{3}x}+\frac{b^{2}}{a^{3}\left(bx+a\right)}+3\,\frac{b^{2}\ln\left(x\right)}{a^{4}}-3\,\frac{b^{2}\ln\left(bx+a\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^2,x)
```

```
[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Maxima [A] time = 1.03414, size = 86, normalized size = 1.48

$$\frac{6\,b^{2}x^{2}+3\,abx-a^{2}}{2\left(a^{3}bx^{3}+a^{4}x^{2}\right)}-\frac{3\,b^{2}\log\left(bx+a\right)}{a^{4}}+\frac{3\,b^{2}\log\left(x\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/
a^4 + 3*b^2*log(x)/a^4
```

Fricas [A] time = 1.63911, size = 177, normalized size = 3.05

$$\frac{6\,ab^{2}x^{2}+3\,a^{2}bx-a^{3}-6\left(b^{3}x^{3}+ab^{2}x^{2}\right)\log\left(bx+a\right)+6\left(b^{3}x^{3}+ab^{2}x^{2}\right)\log\left(x\right)}{2\left(a^{4}bx^{3}+a^{5}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) +
 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

Sympy [A] time = 0.637608, size = 54, normalized size = 0.93

$$\frac{-a^{2}+3abx+6b^{2}x^{2}}{2a^{4}x^{2}+2a^{3}bx^{3}}+\frac{3b^{2}\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(log
(x) - log(a/b + x))/a**4
```

Giac [A] time = 1.19975, size = 100, normalized size = 1.72

$$\frac{3\,b^2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^4}+\frac{b^2}{(bx+a)a^3}-\frac{\frac{6\,ab^2}{bx+a}-5\,b^2}{2\,a^4\left(\frac{a}{bx+a}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3*b^2*log(abs(-a/(b*x + a) + 1))/a^4 + b^2/((b*x + a)*a^3) - 1/2*(6*a*b^2/(
b*x + a) - 5*b^2)/(a^4*(a/(b*x + a) - 1)^2)
```

3.178 $\int \frac{1}{x^4(a+bx)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

```
[Out] -1/(3*a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4*b
^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Rubi [A] time = 0.0347626, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^2),x]
```

```
[Out] -1/(3*a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4*b
^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.0924556, size = 66, normalized size = 0.96

$$-\frac{\frac{a\left(-2a^2bx+a^3+6ab^2x^2+12b^3x^3\right)}{x^3(a+bx)}-12b^3\log(a+bx)+12b^3\log(x)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^2),x]
```

```
[Out] -((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 12*b^3
*Log[x] - 12*b^3*Log[a + b*x])/(3*a^5)
```

Maple [A] time = 0.01, size = 68, normalized size = 1.

$$-\frac{1}{3\,a^{2}x^{3}}+\frac{b}{a^{3}x^{2}}-3\,\frac{b^{2}}{a^{4}x}-\frac{b^{3}}{a^{4}\left(bx+a\right)}-4\,\frac{b^{3}\ln\left(x\right)}{a^{5}}+4\,\frac{b^{3}\ln\left(bx+a\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^2,x)
```

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln
(b*x+a)/a^5
```

Maxima [A] time = 1.05616, size = 99, normalized size = 1.43

$$-\frac{12\,b^{3}x^{3}+6\,ab^{2}x^{2}-2\,a^{2}bx+a^{3}}{3\left(a^{4}bx^{4}+a^{5}x^{3}\right)}+\frac{4\,b^{3}\log\left(bx+a\right)}{a^{5}}-\frac{4\,b^{3}\log\left(x\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) + 4
*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

Fricas [A] time = 1.70433, size = 204, normalized size = 2.96

$$-\frac{12\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}-2\,a^{3}bx+a^{4}-12\left(b^{4}x^{4}+ab^{3}x^{3}\right)\log\left(bx+a\right)+12\left(b^{4}x^{4}+ab^{3}x^{3}\right)\log\left(x\right)}{3\left(a^{5}bx^{4}+a^{6}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3*
x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3)
```

Sympy [A] time = 0.660023, size = 66, normalized size = 0.96

$$-\frac{a^{3}-2a^{2}bx+6ab^{2}x^{2}+12b^{3}x^{3}}{3a^{5}x^{3}+3a^{4}bx^{4}}+\frac{4b^{3}\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**2,x)
```

```
[Out] -(a**3 - 2*a**2*b*x + 6*a*b**2*x**2 + 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*b
*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

Giac [A] time = 1.23453, size = 122, normalized size = 1.77

$$-\frac{4\,b^3\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^5}-\frac{b^3}{(bx+a)a^4}-\frac{\frac{30\,ab^3}{bx+a}-\frac{18\,a^2b^3}{(bx+a)^2}-13\,b^3}{3\,a^5\left(\frac{a}{bx+a}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4*b^3*log(abs(-a/(b*x + a) + 1))/a^5 - b^3/((b*x + a)*a^4) - 1/3*(30*a*b^3
/(b*x + a) - 18*a^2*b^3/(b*x + a)^2 - 13*b^3)/(a^5*(a/(b*x + a) - 1)^3)
```

3.179 $\int \frac{1}{x^5(a+bx)^2} dx$

Optimal. Leaf size=84

$$-\frac{3b^2}{2a^4x^2}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

```
[Out] -1/(4*a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x)
+ b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

Rubi [A] time = 0.0428869, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{3b^2}{2a^4x^2}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^2),x]
```

```
[Out] -1/(4*a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x)
+ b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)^2}dx &= \int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right)dx\\ &= -\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}\end{aligned}$$

Mathematica [A] time = 0.0792331, size = 79, normalized size = 0.94

$$\frac{\frac{a\left(-10a^2b^2x^2+5a^3bx-3a^4+30ab^3x^3+60b^4x^4\right)}{x^4(a+bx)}-60b^4\log(a+bx)+60b^4\log(x)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^2),x]
```

```
[Out] ((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^4
*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)
```

Maple [A] time = 0.01, size = 79, normalized size = 0.9

$$-\frac{1}{4\,a^{2}x^{4}}+\frac{2\,b}{3\,a^{3}x^{3}}-\frac{3\,b^{2}}{2\,a^{4}x^{2}}+4\,\frac{b^{3}}{a^{5}x}+\frac{b^{4}}{a^{5}\,(bx+a)}+5\,\frac{b^{4}\ln(x)}{a^{6}}-5\,\frac{b^{4}\ln(bx+a)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^2,x)
```

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^
4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Maxima [A] time = 1.05882, size = 116, normalized size = 1.38

$$\frac{60\,b^{4}x^{4}+30\,ab^{3}x^{3}-10\,a^{2}b^{2}x^{2}+5\,a^{3}bx-3\,a^{4}}{12\left(a^{5}bx^{5}+a^{6}x^{4}\right)}-\frac{5\,b^{4}\log(bx+a)}{a^{6}}+\frac{5\,b^{4}\log(x)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5*
b*x^5 + a^6*x^4) - 5*b^4*log(b*x + a)/a^6 + 5*b^4*log(x)/a^6
```

Fricas [A] time = 1.4977, size = 231, normalized size = 2.75

$$\frac{60\,ab^{4}x^{4}+30\,a^{2}b^{3}x^{3}-10\,a^{3}b^{2}x^{2}+5\,a^{4}bx-3\,a^{5}-60\left(b^{5}x^{5}+ab^{4}x^{4}\right)\log(bx+a)+60\left(b^{5}x^{5}+ab^{4}x^{4}\right)\log(x)}{12\left(a^{6}bx^{5}+a^{7}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 -
60*(b^5*x^5 + a*b^4*x^4)*log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*log(x))/(a
^6*b*x^5 + a^7*x^4)
```

Sympy [A] time = 0.704667, size = 80, normalized size = 0.95

$$\frac{-3a^{4}+5a^{3}bx-10a^{2}b^{2}x^{2}+30ab^{3}x^{3}+60b^{4}x^{4}}{12a^{6}x^{4}+12a^{5}bx^{5}}+\frac{5b^{4}\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)/
(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(log(x) - log(a/b + x))/a**6
```

Giac [A] time = 1.19309, size = 140, normalized size = 1.67

$$\frac{5\,b^{4}\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^{6}}+\frac{b^{4}}{(bx+a)a^{5}}-\frac{\frac{260\,ab^{4}}{bx+a}-\frac{300\,a^{2}b^{4}}{(bx+a)^{2}}+\frac{120\,a^{3}b^{4}}{(bx+a)^{3}}-77\,b^{4}}{12\,a^{6}\left(\frac{a}{bx+a}-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*b^4*log(abs(-a/(b*x + a) + 1))/a^6 + b^4/((b*x + a)*a^5) - 1/12*(260*a*b^
4/(b*x + a) - 300*a^2*b^4/(b*x + a)^2 + 120*a^3*b^4/(b*x + a)^3 - 77*b^4)/(
a^6*(a/(b*x + a) - 1)^4)
```

3.180 $\int \frac{x^7}{(a+bx)^3} dx$

Optimal. Leaf size=99

$$-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}+\frac{a^7}{2b^8(a+bx)^2}-\frac{7a^6}{b^8(a+bx)}+\frac{15a^4x}{b^7}-\frac{21a^5\log(a+bx)}{b^8}-\frac{3ax^4}{4b^4}+\frac{x^5}{5b^3}$$

```
[Out] (15*a^4*x)/b^7 - (5*a^3*x^2)/b^6 + (2*a^2*x^3)/b^5 - (3*a*x^4)/(4*b^4) + x^
5/(5*b^3) + a^7/(2*b^8*(a + b*x)^2) - (7*a^6)/(b^8*(a + b*x)) - (21*a^5*Log
[a + b*x])/b^8
```

Rubi [A] time = 0.0703669, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}+\frac{a^7}{2b^8(a+bx)^2}-\frac{7a^6}{b^8(a+bx)}+\frac{15a^4x}{b^7}-\frac{21a^5\log(a+bx)}{b^8}-\frac{3ax^4}{4b^4}+\frac{x^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^3,x]
```

```
[Out] (15*a^4*x)/b^7 - (5*a^3*x^2)/b^6 + (2*a^2*x^3)/b^5 - (3*a*x^4)/(4*b^4) + x^
5/(5*b^3) + a^7/(2*b^8*(a + b*x)^2) - (7*a^6)/(b^8*(a + b*x)) - (21*a^5*Log
[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^3}\,dx &= \int \left(\frac{15a^4}{b^7}-\frac{10a^3x}{b^6}+\frac{6a^2x^2}{b^5}-\frac{3ax^3}{b^4}+\frac{x^4}{b^3}-\frac{a^7}{b^7(a+bx)^3}+\frac{7a^6}{b^7(a+bx)^2}-\frac{21a^5}{b^7(a+bx)}\right)dx\\ &= \frac{15a^4x}{b^7}-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^5}{5b^3}+\frac{a^7}{2b^8(a+bx)^2}-\frac{7a^6}{b^8(a+bx)}-\frac{21a^5\log(a+bx)}{b^8}\end{aligned}$$

Mathematica [A] time = 0.0574518, size = 89, normalized size = 0.9

$$\frac{-100a^3b^2x^2+40a^2b^3x^3+\frac{10a^7}{(a+bx)^2}-\frac{140a^6}{a+bx}+300a^4bx-420a^5\log(a+bx)-15ab^4x^4+4b^5x^5}{20b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^3,x]
```

```
[Out] (300*a^4*b*x - 100*a^3*b^2*x^2 + 40*a^2*b^3*x^3 - 15*a*b^4*x^4 + 4*b^5*x^5
+ (10*a^7)/(a + b*x)^2 - (140*a^6)/(a + b*x) - 420*a^5*Log[a + b*x])/(20*b^
```

```
8)
```

Maple [A] time = 0.007, size = 94, normalized size = 1.

$$15\frac{a^4x}{b^7}-5\frac{a^3x^2}{b^6}+2\frac{a^2x^3}{b^5}-\frac{3\,ax^4}{4\,b^4}+\frac{x^5}{5\,b^3}+\frac{a^7}{2\,b^8\,(bx+a)^2}-7\frac{a^6}{b^8\,(bx+a)}-21\frac{a^5\ln(bx+a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^3,x)
```

```
[Out] 15*a^4*x/b^7-5*a^3*x^2/b^6+2*a^2*x^3/b^5-3/4*a*x^4/b^4+1/5*x^5/b^3+1/2*a^7/
b^8/(b*x+a)^2-7*a^6/b^8/(b*x+a)-21*a^5*ln(b*x+a)/b^8
```

Maxima [A] time = 1.06132, size = 139, normalized size = 1.4

$$-\frac{14\,a^6bx+13\,a^7}{2\left(b^{10}x^2+2\,ab^9x+a^2b^8\right)}-\frac{21\,a^5\log(bx+a)}{b^8}+\frac{4\,b^4x^5-15\,ab^3x^4+40\,a^2b^2x^3-100\,a^3bx^2+300\,a^4x}{20\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(14*a^6*b*x + 13*a^7)/(b^10*x^2 + 2*a*b^9*x + a^2*b^8) - 21*a^5*log(b*
x + a)/b^8 + 1/20*(4*b^4*x^5 - 15*a*b^3*x^4 + 40*a^2*b^2*x^3 - 100*a^3*b*x^
2 + 300*a^4*x)/b^7
```

Fricas [A] time = 1.49915, size = 284, normalized size = 2.87

$$\frac{4\,b^7x^7-7\,ab^6x^6+14\,a^2b^5x^5-35\,a^3b^4x^4+140\,a^4b^3x^3+500\,a^5b^2x^2+160\,a^6bx-130\,a^7-420\left(a^5b^2x^2+2\,a^6bx+a^7\right)}{20\left(b^{10}x^2+2\,ab^9x+a^2b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*x^7 - 7*a*b^6*x^6 + 14*a^2*b^5*x^5 - 35*a^3*b^4*x^4 + 140*a^4*b
^3*x^3 + 500*a^5*b^2*x^2 + 160*a^6*b*x - 130*a^7 - 420*(a^5*b^2*x^2 + 2*a^6
*b*x + a^7)*log(b*x + a))/(b^10*x^2 + 2*a*b^9*x + a^2*b^8)
```

Sympy [A] time = 0.730562, size = 107, normalized size = 1.08

$$-\frac{21a^5\log(a+bx)}{b^8}+\frac{15a^4x}{b^7}-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^4}{4b^4}-\frac{13a^7+14a^6bx}{2a^2b^8+4ab^9x+2b^{10}x^2}+\frac{x^5}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**3,x)
```

```
[Out] -21*a**5*log(a + b*x)/b**8 + 15*a**4*x/b**7 - 5*a**3*x**2/b**6 + 2*a**2*x**
3/b**5 - 3*a*x**4/(4*b**4) - (13*a**7 + 14*a**6*b*x)/(2*a**2*b**8 + 4*a*b**
9*x + 2*b**10*x**2) + x**5/(5*b**3)
```

Giac [A] time = 1.16591, size = 128, normalized size = 1.29

$$-\frac{21\,a^5\log\left(|bx+a|\right)}{b^8}-\frac{14\,a^6bx+13\,a^7}{2\,(bx+a)^2b^8}+\frac{4\,b^{12}x^5-15\,ab^{11}x^4+40\,a^2b^{10}x^3-100\,a^3b^9x^2+300\,a^4b^8x}{20\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -21*a^5*log(abs(b*x + a))/b^8 - 1/2*(14*a^6*b*x + 13*a^7)/((b*x + a)^2*b^8)
 + 1/20*(4*b^12*x^5 - 15*a*b^11*x^4 + 40*a^2*b^10*x^3 - 100*a^3*b^9*x^2 + 3
00*a^4*b^8*x)/b^15
```

3.181 $\int \frac{x^6}{(a+bx)^3} dx$

Optimal. Leaf size=86

$$\frac{3a^2x^2}{b^5} - \frac{a^6}{2b^7(a+bx)^2} + \frac{6a^5}{b^7(a+bx)} - \frac{10a^3x}{b^6} + \frac{15a^4\log(a+bx)}{b^7} - \frac{ax^3}{b^4} + \frac{x^4}{4b^3}$$

```
[Out] (-10*a^3*x)/b^6 + (3*a^2*x^2)/b^5 - (a*x^3)/b^4 + x^4/(4*b^3) - a^6/(2*b^7*
(a + b*x)^2) + (6*a^5)/(b^7*(a + b*x)) + (15*a^4*Log[a + b*x])/b^7
```

Rubi [A] time = 0.0525685, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2x^2}{b^5} - \frac{a^6}{2b^7(a+bx)^2} + \frac{6a^5}{b^7(a+bx)} - \frac{10a^3x}{b^6} + \frac{15a^4\log(a+bx)}{b^7} - \frac{ax^3}{b^4} + \frac{x^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^3,x]
```

```
[Out] (-10*a^3*x)/b^6 + (3*a^2*x^2)/b^5 - (a*x^3)/b^4 + x^4/(4*b^3) - a^6/(2*b^7*
(a + b*x)^2) + (6*a^5)/(b^7*(a + b*x)) + (15*a^4*Log[a + b*x])/b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(a+bx)^3} dx &= \int \left(-\frac{10a^3}{b^6} + \frac{6a^2x}{b^5} - \frac{3ax^2}{b^4} + \frac{x^3}{b^3} + \frac{a^6}{b^6(a+bx)^3} - \frac{6a^5}{b^6(a+bx)^2} + \frac{15a^4}{b^6(a+bx)}\right) dx \\
&= -\frac{10a^3x}{b^6} + \frac{3a^2x^2}{b^5} - \frac{ax^3}{b^4} + \frac{x^4}{4b^3} - \frac{a^6}{2b^7(a+bx)^2} + \frac{6a^5}{b^7(a+bx)} + \frac{15a^4\log(a+bx)}{b^7}
\end{aligned}$$

Mathematica [A] time = 0.0398734, size = 77, normalized size = 0.9

$$\frac{12a^2b^2x^2 - \frac{2a^6}{(a+bx)^2} + \frac{24a^5}{a+bx} - 40a^3bx + 60a^4\log(a+bx) - 4ab^3x^3 + b^4x^4}{4b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^3,x]
```

```
[Out] (-40*a^3*b*x + 12*a^2*b^2*x^2 - 4*a*b^3*x^3 + b^4*x^4 - (2*a^6)/(a + b*x)^2
 + (24*a^5)/(a + b*x) + 60*a^4*Log[a + b*x])/(4*b^7)
```

Maple [A] time = 0.007, size = 83, normalized size = 1.

$$-10\frac{a^3x}{b^6}+3\frac{a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}-\frac{a^6}{2b^7(bx+a)^2}+6\frac{a^5}{b^7(bx+a)}+15\frac{a^4\ln(bx+a)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^3,x)
```

```
[Out] -10*a^3*x/b^6+3*a^2*x^2/b^5-a*x^3/b^4+1/4*x^4/b^3-1/2*a^6/b^7/(b*x+a)^2+6*a
^5/b^7/(b*x+a)+15*a^4*ln(b*x+a)/b^7
```

Maxima [A] time = 0.994151, size = 123, normalized size = 1.43

$$\frac{12a^5bx+11a^6}{2(b^9x^2+2ab^8x+a^2b^7)}+\frac{15a^4\log(bx+a)}{b^7}+\frac{b^3x^4-4ab^2x^3+12a^2bx^2-40a^3x}{4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*a^5*b*x + 11*a^6)/(b^9*x^2 + 2*a*b^8*x + a^2*b^7) + 15*a^4*log(b*x
+ a)/b^7 + 1/4*(b^3*x^4 - 4*a*b^2*x^3 + 12*a^2*b*x^2 - 40*a^3*x)/b^6
```

Fricas [A] time = 1.51429, size = 247, normalized size = 2.87

$$\frac{b^6x^6-2ab^5x^5+5a^2b^4x^4-20a^3b^3x^3-68a^4b^2x^2-16a^5bx+22a^6+60(a^4b^2x^2+2a^5bx+a^6)\log(bx+a)}{4(b^9x^2+2ab^8x+a^2b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^6*x^6 - 2*a*b^5*x^5 + 5*a^2*b^4*x^4 - 20*a^3*b^3*x^3 - 68*a^4*b^2*x^
2 - 16*a^5*b*x + 22*a^6 + 60*(a^4*b^2*x^2 + 2*a^5*b*x + a^6)*log(b*x + a))/
(b^9*x^2 + 2*a*b^8*x + a^2*b^7)
```

Sympy [A] time = 0.651687, size = 92, normalized size = 1.07

$$\frac{15a^4\log(a+bx)}{b^7}-\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{11a^6+12a^5bx}{2a^2b^7+4ab^8x+2b^9x^2}+\frac{x^4}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**3,x)
```

```
[Out] 15*a**4*log(a + b*x)/b**7 - 10*a**3*x/b**6 + 3*a**2*x**2/b**5 - a*x**3/b**4
 + (11*a**6 + 12*a**5*b*x)/(2*a**2*b**7 + 4*a*b**8*x + 2*b**9*x**2) + x**4/
(4*b**3)
```

Giac [A] time = 1.20584, size = 112, normalized size = 1.3

$$\frac{15\,a^4\log\left(|bx+a|\right)}{b^7}+\frac{12\,a^5bx+11\,a^6}{2\,(bx+a)^2b^7}+\frac{b^9x^4-4\,ab^8x^3+12\,a^2b^7x^2-40\,a^3b^6x}{4\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 15*a^4*log(abs(b*x + a))/b^7 + 1/2*(12*a^5*b*x + 11*a^6)/((b*x + a)^2*b^7)
+ 1/4*(b^9*x^4 - 4*a*b^8*x^3 + 12*a^2*b^7*x^2 - 40*a^3*b^6*x)/b^12
```

3.182 $\int \frac{x^5}{(a+bx)^3}\,dx$

Optimal. Leaf size=77

$$\frac{a^5}{2b^6(a+bx)^2}-\frac{5a^4}{b^6(a+bx)}+\frac{6a^2x}{b^5}-\frac{10a^3\log(a+bx)}{b^6}-\frac{3ax^2}{2b^4}+\frac{x^3}{3b^3}$$

```
[Out] (6*a^2*x)/b^5 - (3*a*x^2)/(2*b^4) + x^3/(3*b^3) + a^5/(2*b^6*(a + b*x)^2) -
 (5*a^4)/(b^6*(a + b*x)) - (10*a^3*Log[a + b*x])/b^6
```

Rubi [A] time = 0.0460517, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^5}{2b^6(a+bx)^2}-\frac{5a^4}{b^6(a+bx)}+\frac{6a^2x}{b^5}-\frac{10a^3\log(a+bx)}{b^6}-\frac{3ax^2}{2b^4}+\frac{x^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^3,x]
```

```
[Out] (6*a^2*x)/b^5 - (3*a*x^2)/(2*b^4) + x^3/(3*b^3) + a^5/(2*b^6*(a + b*x)^2) -
 (5*a^4)/(b^6*(a + b*x)) - (10*a^3*Log[a + b*x])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^3}\,dx &= \int \left(\frac{6a^2}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{b^3}-\frac{a^5}{b^5(a+bx)^3}+\frac{5a^4}{b^5(a+bx)^2}-\frac{10a^3}{b^5(a+bx)}\right)dx\\ &= \frac{6a^2x}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^3}{3b^3}+\frac{a^5}{2b^6(a+bx)^2}-\frac{5a^4}{b^6(a+bx)}-\frac{10a^3\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.0352137, size = 67, normalized size = 0.87

$$\frac{\frac{3a^5}{(a+bx)^2}-\frac{30a^4}{a+bx}+36a^2bx-60a^3\log(a+bx)-9ab^2x^2+2b^3x^3}{6b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^3,x]
```

```
[Out] (36*a^2*b*x - 9*a*b^2*x^2 + 2*b^3*x^3 + (3*a^5)/(a + b*x)^2 - (30*a^4)/(a +
 b*x) - 60*a^3*Log[a + b*x])/(6*b^6)
```

Maple [A] time = 0.006, size = 72, normalized size = 0.9

$$6\frac{a^2x}{b^5}-\frac{3\,ax^2}{2\,b^4}+\frac{x^3}{3\,b^3}+\frac{a^5}{2\,b^6\,(bx+a)^2}-5\,\frac{a^4}{b^6\,(bx+a)}-10\,\frac{a^3\ln(bx+a)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^3,x)
```

```
[Out] 6*a^2*x/b^5-3/2*a*x^2/b^4+1/3*x^3/b^3+1/2*a^5/b^6/(b*x+a)^2-5*a^4/b^6/(b*x+
a)-10*a^3*ln(b*x+a)/b^6
```

Maxima [A] time = 1.08787, size = 109, normalized size = 1.42

$$-\frac{10\,a^4bx+9\,a^5}{2\left(b^8x^2+2\,ab^7x+a^2b^6\right)}-\frac{10\,a^3\log(bx+a)}{b^6}+\frac{2\,b^2x^3-9\,abx^2+36\,a^2x}{6\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(10*a^4*b*x + 9*a^5)/(b^8*x^2 + 2*a*b^7*x + a^2*b^6) - 10*a^3*log(b*x
+ a)/b^6 + 1/6*(2*b^2*x^3 - 9*a*b*x^2 + 36*a^2*x)/b^5
```

Fricas [A] time = 1.39161, size = 227, normalized size = 2.95

$$\frac{2\,b^5x^5-5\,ab^4x^4+20\,a^2b^3x^3+63\,a^3b^2x^2+6\,a^4bx-27\,a^5-60\left(a^3b^2x^2+2\,a^4bx+a^5\right)\log(bx+a)}{6\left(b^8x^2+2\,ab^7x+a^2b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*x^5 - 5*a*b^4*x^4 + 20*a^2*b^3*x^3 + 63*a^3*b^2*x^2 + 6*a^4*b*x
- 27*a^5 - 60*(a^3*b^2*x^2 + 2*a^4*b*x + a^5)*log(b*x + a))/(b^8*x^2 + 2*a*
b^7*x + a^2*b^6)
```

Sympy [A] time = 0.574858, size = 83, normalized size = 1.08

$$-\frac{10a^3\log(a+bx)}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{2b^4}-\frac{9a^5+10a^4bx}{2a^2b^6+4ab^7x+2b^8x^2}+\frac{x^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**3,x)
```

```
[Out] -10*a**3*log(a + b*x)/b**6 + 6*a**2*x/b**5 - 3*a*x**2/(2*b**4) - (9*a**5 +
10*a**4*b*x)/(2*a**2*b**6 + 4*a*b**7*x + 2*b**8*x**2) + x**3/(3*b**3)
```

Giac [A] time = 1.25648, size = 99, normalized size = 1.29

$$-\frac{10\,a^3\log\left(|bx+a|\right)}{b^6}-\frac{10\,a^4bx+9\,a^5}{2\,(bx+a)^2b^6}+\frac{2\,b^6x^3-9\,ab^5x^2+36\,a^2b^4x}{6\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -10*a^3*log(abs(b*x + a))/b^6 - 1/2*(10*a^4*b*x + 9*a^5)/((b*x + a)^2*b^6)
+ 1/6*(2*b^6*x^3 - 9*a*b^5*x^2 + 36*a^2*b^4*x)/b^9
```

3.183 $\int \frac{x^4}{(a+bx)^3} dx$

Optimal. Leaf size=64

$$-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{2b^3}$$

```
[Out] (-3*a*x)/b^4 + x^2/(2*b^3) - a^4/(2*b^5*(a + b*x)^2) + (4*a^3)/(b^5*(a + b*
x)) + (6*a^2*Log[a + b*x])/b^5
```

Rubi [A] time = 0.0361858, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^3,x]
```

```
[Out] (-3*a*x)/b^4 + x^2/(2*b^3) - a^4/(2*b^5*(a + b*x)^2) + (4*a^3)/(b^5*(a + b*
x)) + (6*a^2*Log[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^3}\,dx &= \int \left(-\frac{3a}{b^4}+\frac{x}{b^3}+\frac{a^4}{b^4(a+bx)^3}-\frac{4a^3}{b^4(a+bx)^2}+\frac{6a^2}{b^4(a+bx)}\right)dx\\ &= -\frac{3ax}{b^4}+\frac{x^2}{2b^3}-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.027626, size = 55, normalized size = 0.86

$$\frac{-\frac{a^4}{(a+bx)^2}+\frac{8a^3}{a+bx}+12a^2\log(a+bx)-6abx+b^2x^2}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^3,x]
```

```
[Out] (-6*a*b*x + b^2*x^2 - a^4/(a + b*x)^2 + (8*a^3)/(a + b*x) + 12*a^2*Log[a +
b*x])/(2*b^5)
```

Maple [A] time = 0.007, size = 61, normalized size = 1.

$$-3\,{\frac {ax}{{b}^{4}}}+{\frac {{x}^{2}}{2\,{b}^{3}}}-{\frac {{a}^{4}}{2\,{b}^{5} \left(bx+a \right) ^{2}}}+4\,{\frac {{a}^{3}}{{b}^{5} \left(bx+a \right) }}+6\,{\frac {{a}^{2}\ln \left(bx+a \right) }{{b}^{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^3,x)
```

```
[Out] -3*a*x/b^4+1/2*x^2/b^3-1/2*a^4/b^5/(b*x+a)^2+4*a^3/b^5/(b*x+a)+6*a^2*ln(b*x
+a)/b^5
```

Maxima [A] time = 1.0957, size = 93, normalized size = 1.45

$$\frac{8\,a^{3} b x + 7\,a^{4}}{2\,{\left(b^{7} x^{2} + 2\,a b^{6} x + a^{2} b^{5}\right)}} + \frac{6\,a^{2} \log\left(b x + a\right)}{b^{5}} + \frac{b x^{2} - 6\,a x}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a^3*b*x + 7*a^4)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5) + 6*a^2*log(b*x + a
)/b^5 + 1/2*(b*x^2 - 6*a*x)/b^4
```

Fricas [A] time = 1.62256, size = 200, normalized size = 3.12

$$\frac{b^{4} x^{4} - 4\,a b^{3} x^{3} - 11\,a^{2} b^{2} x^{2} + 2\,a^{3} b x + 7\,a^{4} + 12\,{\left(a^{2} b^{2} x^{2} + 2\,a^{3} b x + a^{4}\right)} \log\left(b x + a\right)}{2\,{\left(b^{7} x^{2} + 2\,a b^{6} x + a^{2} b^{5}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4 + 12*(a^2*b
^2*x^2 + 2*a^3*b*x + a^4)*log(b*x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [A] time = 0.622383, size = 70, normalized size = 1.09

$$\frac{6a^{2} \log{\left (a + bx \right)}}{b^{5}} - \frac{3ax}{b^{4}} + \frac{7a^{4} + 8a^{3}bx}{2a^{2}b^{5} + 4ab^{6}x + 2b^{7}x^{2}} + \frac{x^{2}}{2b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**3,x)
```

```
[Out] 6*a**2*log(a + b*x)/b**5 - 3*a*x/b**4 + (7*a**4 + 8*a**3*b*x)/(2*a**2*b**5
+ 4*a*b**6*x + 2*b**7*x**2) + x**2/(2*b**3)
```

Giac [A] time = 1.13793, size = 82, normalized size = 1.28

$$\frac{6\,a^{2}\log\left(|bx+a|\right)}{b^{5}}+\frac{b^{3}x^{2}-6\,ab^{2}x}{2\,b^{6}}+\frac{8\,a^{3}bx+7\,a^{4}}{2\,(bx+a)^{2}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 6*a^2*log(abs(b*x + a))/b^5 + 1/2*(b^3*x^2 - 6*a*b^2*x)/b^6 + 1/2*(8*a^3*b*
x + 7*a^4)/((b*x + a)^2*b^5)
```

3.184 $\int \frac{x^3}{(a+bx)^3} dx$

Optimal. Leaf size=50

$$\frac{a^3}{2b^4(a+bx)^2} - \frac{3a^2}{b^4(a+bx)} - \frac{3a\log(a+bx)}{b^4} + \frac{x}{b^3}$$

```
[Out] x/b^3 + a^3/(2*b^4*(a + b*x)^2) - (3*a^2)/(b^4*(a + b*x)) - (3*a*Log[a + b*
x])/b^4
```

Rubi [A] time = 0.0262389, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{2b^4(a+bx)^2} - \frac{3a^2}{b^4(a+bx)} - \frac{3a\log(a+bx)}{b^4} + \frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^3,x]
```

```
[Out] x/b^3 + a^3/(2*b^4*(a + b*x)^2) - (3*a^2)/(b^4*(a + b*x)) - (3*a*Log[a + b*
x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^3} dx &= \int \left(\frac{1}{b^3} - \frac{a^3}{b^3(a+bx)^3} + \frac{3a^2}{b^3(a+bx)^2} - \frac{3a}{b^3(a+bx)}\right) dx \\ &= \frac{x}{b^3} + \frac{a^3}{2b^4(a+bx)^2} - \frac{3a^2}{b^4(a+bx)} - \frac{3a\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0613447, size = 40, normalized size = 0.8

$$-\frac{\frac{a^2(5a+6bx)}{(a+bx)^2} + 6a\log(a+bx) - 2bx}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^3,x]
```

```
[Out] -(-2*b*x + (a^2*(5*a + 6*b*x))/(a + b*x)^2 + 6*a*Log[a + b*x])/(2*b^4)
```

Maple [A] time = 0.007, size = 49, normalized size = 1.

$$\frac{x}{b^{3}}+\frac{a^{3}}{2\,b^{4}\left(bx+a\right)^{2}}-3\,\frac{a^{2}}{b^{4}\left(bx+a\right)}-3\,\frac{a\ln\left(bx+a\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^3,x)
```

```
[Out] x/b^3+1/2*a^3/b^4/(b*x+a)^2-3*a^2/b^4/(b*x+a)-3*a*ln(b*x+a)/b^4
```

Maxima [A] time = 1.0621, size = 77, normalized size = 1.54

$$-\frac{6\,a^{2}bx+5\,a^{3}}{2\left(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4}\right)}+\frac{x}{b^{3}}-\frac{3\,a\log\left(bx+a\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*a^2*b*x + 5*a^3)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) + x/b^3 - 3*a*log(
b*x + a)/b^4
```

Fricas [A] time = 1.51677, size = 176, normalized size = 3.52

$$\frac{2\,b^{3}x^{3}+4\,ab^{2}x^{2}-4\,a^{2}bx-5\,a^{3}-6\left(ab^{2}x^{2}+2\,a^{2}bx+a^{3}\right)\log\left(bx+a\right)}{2\left(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3 - 6*(a*b^2*x^2 + 2*a^2*b*x
 + a^3)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [A] time = 0.600725, size = 56, normalized size = 1.12

$$-\frac{3a\log\left(a+bx\right)}{b^{4}}-\frac{5a^{3}+6a^{2}bx}{2a^{2}b^{4}+4ab^{5}x+2b^{6}x^{2}}+\frac{x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**3,x)
```

```
[Out] -3*a*log(a + b*x)/b**4 - (5*a**3 + 6*a**2*b*x)/(2*a**2*b**4 + 4*a*b**5*x +
2*b**6*x**2) + x/b**3
```

Giac [A] time = 1.15452, size = 59, normalized size = 1.18

$$\frac{x}{b^3} - \frac{3\,a\log\left(|bx + a|\right)}{b^4} - \frac{6\,a^2bx + 5\,a^3}{2\,(bx + a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] x/b^3 - 3*a*log(abs(b*x + a))/b^4 - 1/2*(6*a^2*b*x + 5*a^3)/((b*x + a)^2*b^
4)
```

3.185 $\int \frac{x^2}{(a+bx)^3} dx$

Optimal. Leaf size=41

$$-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}$$

```
[Out] -a^2/(2*b^3*(a + b*x)^2) + (2*a)/(b^3*(a + b*x)) + Log[a + b*x]/b^3
```

Rubi [A] time = 0.020299, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^3,x]
```

```
[Out] -a^2/(2*b^3*(a + b*x)^2) + (2*a)/(b^3*(a + b*x)) + Log[a + b*x]/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^3} dx &= \int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right) dx\\ &= -\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0194683, size = 33, normalized size = 0.8

$$\frac{\frac{a(3a+4bx)}{(a+bx)^2}+2\log(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^3,x]
```

```
[Out] ((a*(3*a + 4*b*x))/(a + b*x)^2 + 2*Log[a + b*x])/(2*b^3)
```

Maple [A] time = 0.005, size = 40, normalized size = 1.

$$-\frac{a^2}{2\,b^3\,(bx+a)^2}+2\,\frac{a}{b^3\,(bx+a)}+\frac{\ln(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^3,x)
```

```
[Out] -1/2*a^2/b^3/(b*x+a)^2+2*a/b^3/(b*x+a)+ln(b*x+a)/b^3
```

Maxima [A] time = 1.01963, size = 65, normalized size = 1.59

$$\frac{4\,abx + 3\,a^2}{2\,\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)} + \frac{\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) + log(b*x + a)/b^3
```

Fricas [A] time = 1.56722, size = 132, normalized size = 3.22

$$\frac{4\,abx + 3\,a^2 + 2\,\left(b^2x^2 + 2\,abx + a^2\right)\log\left(bx + a\right)}{2\,\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3)
```

Sympy [A] time = 0.533647, size = 46, normalized size = 1.12

$$\frac{3a^2 + 4abx}{2a^2b^3 + 4ab^4x + 2b^5x^2} + \frac{\log\left(a + bx\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**3,x)
```

```
[Out] (3*a**2 + 4*a*b*x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + log(a + b*x)/
b**3
```

Giac [A] time = 1.14395, size = 50, normalized size = 1.22

$$\frac{\log\left(|bx + a|\right)}{b^3} + \frac{4\,ax + \frac{3\,a^2}{b}}{2\,(bx + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="giac")

[Out] log(abs(b*x + a))/b^3 + 1/2*(4*a*x + 3*a^2/b)/((b*x + a)^2*b^2)
```

3.186 $\int \frac{x}{(a+bx)^3} \, dx$

Optimal. Leaf size=17

$$\frac{x^2}{2a(a+bx)^2}$$

[Out] x^2/(2*a*(a + b*x)^2)

Rubi [A] time = 0.0016454, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {37}

$$\frac{x^2}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x)^3,x]

[Out] x^2/(2*a*(a + b*x)^2)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x}{(a+bx)^3} \, dx = \frac{x^2}{2a(a+bx)^2}$$

Mathematica [A] time = 0.0078628, size = 20, normalized size = 1.18

$$-\frac{a+2bx}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x)^3,x]

[Out] -(a + 2*b*x)/(2*b^2*(a + b*x)^2)

Maple [A] time = 0.004, size = 27, normalized size = 1.6

$$-\frac{1}{b^2(bx+a)} + \frac{a}{2b^2(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^3,x)
```

```
[Out] -1/b^2/(b*x+a)+1/2/b^2*a/(b*x+a)^2
```

Maxima [B] time = 0.989177, size = 43, normalized size = 2.53

$$-\frac{2\,bx+a}{2\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Fricas [B] time = 1.53172, size = 68, normalized size = 4.

$$-\frac{2\,bx+a}{2\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [B] time = 0.429561, size = 32, normalized size = 1.88

$$-\frac{a+2bx}{2a^2b^2+4ab^3x+2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**3,x)
```

```
[Out] -(a + 2*b*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

Giac [A] time = 1.2015, size = 24, normalized size = 1.41

$$-\frac{2\,bx+a}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/((b*x + a)^2*b^2)
```

3.187 $\int \frac{1}{(a+bx)^3} dx$

Optimal. Leaf size=14

$$-\frac{1}{2b(a+bx)^2}$$

[Out] -1/(2*b*(a + b*x)^2)

Rubi [A] time = 0.0014833, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-3),x]

[Out] -1/(2*b*(a + b*x)^2)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3} dx = -\frac{1}{2b(a+bx)^2}$$

Mathematica [A] time = 0.003185, size = 14, normalized size = 1.

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-3),x]

[Out] -1/(2*b*(a + b*x)^2)

Maple [A] time = 0., size = 13, normalized size = 0.9

$$-\frac{1}{2b(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3,x)
```

```
[Out] -1/2/b/(b*x+a)^2
```

Maxima [A] time = 1.02536, size = 16, normalized size = 1.14

$$-\frac{1}{2\,(bx+a)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x + a)^2*b)
```

Fricas [A] time = 1.42328, size = 49, normalized size = 3.5

$$-\frac{1}{2\left(b^3x^2 + 2\,ab^2x + a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [B] time = 0.457129, size = 26, normalized size = 1.86

$$-\frac{1}{2a^2b + 4ab^2x + 2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3,x)
```

```
[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

Giac [A] time = 1.20683, size = 16, normalized size = 1.14

$$-\frac{1}{2\,(bx+a)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*x + a)^2*b)
```

3.188 $\int \frac{1}{x(a+bx)^3}\,dx$

Optimal. Leaf size=43

$$\frac{1}{a^2(a+bx)}-\frac{\log(a+bx)}{a^3}+\frac{\log(x)}{a^3}+\frac{1}{2a(a+bx)^2}$$

```
[Out] 1/(2*a*(a + b*x)^2) + 1/(a^2*(a + b*x)) + Log[x]/a^3 - Log[a + b*x]/a^3
```

Rubi [A] time = 0.0194961, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{a^2(a+bx)}-\frac{\log(a+bx)}{a^3}+\frac{\log(x)}{a^3}+\frac{1}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^3),x]
```

```
[Out] 1/(2*a*(a + b*x)^2) + 1/(a^2*(a + b*x)) + Log[x]/a^3 - Log[a + b*x]/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)dx\\ &= \frac{1}{2a(a+bx)^2}+\frac{1}{a^2(a+bx)}+\frac{\log(x)}{a^3}-\frac{\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0405402, size = 37, normalized size = 0.86

$$\frac{\frac{a(3a+2bx)}{(a+bx)^2}-2\log(a+bx)+2\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^3),x]
```

```
[Out] ((a*(3*a + 2*b*x))/(a + b*x)^2 + 2*Log[x] - 2*Log[a + b*x])/(2*a^3)
```

Maple [A] time = 0.008, size = 42, normalized size = 1.

$$\frac{1}{2\,a\,(bx+a)^2}+\frac{1}{a^2\,(bx+a)}+\frac{\ln(x)}{a^3}-\frac{\ln(bx+a)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^3,x)
```

```
[Out] 1/2/a/(b*x+a)^2+1/a^2/(b*x+a)+ln(x)/a^3-ln(b*x+a)/a^3
```

Maxima [A] time = 1.10759, size = 69, normalized size = 1.6

$$\frac{2\,bx+3\,a}{2\,\left(a^2b^2x^2+2\,a^3bx+a^4\right)}-\frac{\log\left(bx+a\right)}{a^3}+\frac{\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x + 3*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) - log(b*x + a)/a^3 + log(
x)/a^3
```

Fricas [A] time = 1.44843, size = 182, normalized size = 4.23

$$\frac{2\,abx+3\,a^2-2\,\left(b^2x^2+2\,abx+a^2\right)\log\left(bx+a\right)+2\,\left(b^2x^2+2\,abx+a^2\right)\log\left(x\right)}{2\,\left(a^3b^2x^2+2\,a^4bx+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x + 3*a^2 - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + 2*(b^2*x^
2 + 2*a*b*x + a^2)*log(x))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
```

Sympy [A] time = 0.670261, size = 46, normalized size = 1.07

$$\frac{3a+2bx}{2a^4+4a^3bx+2a^2b^2x^2}+\frac{\log\left(x\right)-\log\left(\frac{a}{b}+x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**3,x)
```

```
[Out] (3*a + 2*b*x)/(2*a**4 + 4*a**3*b*x + 2*a**2*b**2*x**2) + (log(x) - log(a/b
+ x))/a**3
```

Giac [A] time = 1.16206, size = 58, normalized size = 1.35

$$-\frac{\log\left(|bx+a|\right)}{a^3}+\frac{\log\left(|x|\right)}{a^3}+\frac{2\,abx+3\,a^2}{2\,\left(bx+a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^3 + log(abs(x))/a^3 + 1/2*(2*a*b*x + 3*a^2)/((b*x + a)
^2*a^3)
```

3.189 $\int \frac{1}{x^2(a+bx)^3} dx$

Optimal. Leaf size=57

$$-\frac{2b}{a^3(a+bx)}-\frac{b}{2a^2(a+bx)^2}-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}-\frac{1}{a^3x}$$

```
[Out] -(1/(a^3*x)) - b/(2*a^2*(a + b*x)^2) - (2*b)/(a^3*(a + b*x)) - (3*b*Log[x])
/a^4 + (3*b*Log[a + b*x])/a^4
```

Rubi [A] time = 0.0292758, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{2b}{a^3(a+bx)}-\frac{b}{2a^2(a+bx)^2}-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}-\frac{1}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^3),x]
```

```
[Out] -(1/(a^3*x)) - b/(2*a^2*(a + b*x)^2) - (2*b)/(a^3*(a + b*x)) - (3*b*Log[x])
/a^4 + (3*b*Log[a + b*x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^2}-\frac{3b}{a^4x}+\frac{b^2}{a^2(a+bx)^3}+\frac{2b^2}{a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{a^3x}-\frac{b}{2a^2(a+bx)^2}-\frac{2b}{a^3(a+bx)}-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.0767011, size = 53, normalized size = 0.93

$$-\frac{\frac{a(2a^2+9abx+6b^2x^2)}{x(a+bx)^2}-6b\log(a+bx)+6b\log(x)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^3),x]
```

```
[Out] -((a*(2*a^2 + 9*a*b*x + 6*b^2*x^2))/(x*(a + b*x)^2) + 6*b*Log[x] - 6*b*Log[
a + b*x])/(2*a^4)
```

Maple [A] time = 0.01, size = 56, normalized size = 1.

$$-\frac{1}{a^3x}-\frac{b}{2\,a^2\left(bx+a\right)^2}-2\,\frac{b}{a^3\left(bx+a\right)}-3\,\frac{b\ln\left(x\right)}{a^4}+3\,\frac{b\ln\left(bx+a\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^3,x)
```

```
[Out] -1/a^3/x-1/2*b/a^2/(b*x+a)^2-2*b/a^3/(b*x+a)-3*b*ln(x)/a^4+3*b*ln(b*x+a)/a^
4
```

Maxima [A] time = 1.08097, size = 93, normalized size = 1.63

$$-\frac{6\,b^2x^2+9\,abx+2\,a^2}{2\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}+\frac{3\,b\log\left(bx+a\right)}{a^4}-\frac{3\,b\log\left(x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b^2*x^2 + 9*a*b*x + 2*a^2)/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x) + 3*
b*log(b*x + a)/a^4 - 3*b*log(x)/a^4
```

Fricas [A] time = 1.67475, size = 232, normalized size = 4.07

$$-\frac{6\,ab^2x^2+9\,a^2bx+2\,a^3-6\left(b^3x^3+2\,ab^2x^2+a^2bx\right)\log\left(bx+a\right)+6\left(b^3x^3+2\,ab^2x^2+a^2bx\right)\log\left(x\right)}{2\left(a^4b^2x^3+2\,a^5bx^2+a^6x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3 - 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)
*log(b*x + a) + 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*log(x))/(a^4*b^2*x^3 +
2*a^5*b*x^2 + a^6*x)
```

Sympy [A] time = 0.643188, size = 65, normalized size = 1.14

$$-\frac{2a^2+9abx+6b^2x^2}{2a^5x+4a^4bx^2+2a^3b^2x^3}+\frac{3b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**3,x)
```

```
[Out] -(2*a**2 + 9*a*b*x + 6*b**2*x**2)/(2*a**5*x + 4*a**4*b*x**2 + 2*a**3*b**2*x
**3) + 3*b*(-log(x) + log(a/b + x))/a**4
```

Giac [A] time = 1.23607, size = 81, normalized size = 1.42

$$\frac{3\,b\log\left(|bx+a|\right)}{a^4}-\frac{3\,b\log\left(|x|\right)}{a^4}-\frac{6\,ab^2x^2+9\,a^2bx+2\,a^3}{2\,(bx+a)^2a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3*b*log(abs(b*x + a))/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(6*a*b^2*x^2 + 9*a^2*
b*x + 2*a^3)/((b*x + a)^2*a^4*x)
```

3.190 $\int \frac{1}{x^3(a+bx)^3} dx$

Optimal. Leaf size=76

$$\frac{3b^2}{a^4(a+bx)} + \frac{b^2}{2a^3(a+bx)^2} + \frac{6b^2\log(x)}{a^5} - \frac{6b^2\log(a+bx)}{a^5} + \frac{3b}{a^4x} - \frac{1}{2a^3x^2}$$

```
[Out] -1/(2*a^3*x^2) + (3*b)/(a^4*x) + b^2/(2*a^3*(a + b*x)^2) + (3*b^2)/(a^4*(a
+ b*x)) + (6*b^2*Log[x])/a^5 - (6*b^2*Log[a + b*x])/a^5
```

Rubi [A] time = 0.0359509, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{3b^2}{a^4(a+bx)} + \frac{b^2}{2a^3(a+bx)^2} + \frac{6b^2\log(x)}{a^5} - \frac{6b^2\log(a+bx)}{a^5} + \frac{3b}{a^4x} - \frac{1}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^3),x]
```

```
[Out] -1/(2*a^3*x^2) + (3*b)/(a^4*x) + b^2/(2*a^3*(a + b*x)^2) + (3*b^2)/(a^4*(a
+ b*x)) + (6*b^2*Log[x])/a^5 - (6*b^2*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^3} dx &= \int \left(\frac{1}{a^3x^3} - \frac{3b}{a^4x^2} + \frac{6b^2}{a^5x} - \frac{b^3}{a^3(a+bx)^3} - \frac{3b^3}{a^4(a+bx)^2} - \frac{6b^3}{a^5(a+bx)}\right) dx\\ &= -\frac{1}{2a^3x^2} + \frac{3b}{a^4x} + \frac{b^2}{2a^3(a+bx)^2} + \frac{3b^2}{a^4(a+bx)} + \frac{6b^2\log(x)}{a^5} - \frac{6b^2\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.0622125, size = 68, normalized size = 0.89

$$\frac{\frac{a\left(4a^2bx - a^3 + 18ab^2x^2 + 12b^3x^3\right)}{x^2(a+bx)^2} - 12b^2\log(a+bx) + 12b^2\log(x)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^3),x]
```

```
[Out] ((a*(-a^3 + 4*a^2*b*x + 18*a*b^2*x^2 + 12*b^3*x^3))/(x^2*(a + b*x)^2) + 12*
b^2*Log[x] - 12*b^2*Log[a + b*x])/(2*a^5)
```

Maple [A] time = 0.01, size = 73, normalized size = 1.

$$-\frac{1}{2\,a^{3}x^{2}}+3\,\frac{b}{a^{4}x}+\frac{b^{2}}{2\,a^{3}\left(bx+a\right)^{2}}+3\,\frac{b^{2}}{a^{4}\left(bx+a\right)}+6\,\frac{b^{2}\ln\left(x\right)}{a^{5}}-6\,\frac{b^{2}\ln\left(bx+a\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^3,x)
```

```
[Out] -1/2/a^3/x^2+3*b/a^4/x+1/2*b^2/a^3/(b*x+a)^2+3*b^2/a^4/(b*x+a)+6*b^2*ln(x)/
a^5-6*b^2*ln(b*x+a)/a^5
```

Maxima [A] time = 1.03154, size = 116, normalized size = 1.53

$$\frac{12\,b^{3}x^{3}+18\,ab^{2}x^{2}+4\,a^{2}bx-a^{3}}{2\left(a^{4}b^{2}x^{4}+2\,a^{5}bx^{3}+a^{6}x^{2}\right)}-\frac{6\,b^{2}\log\left(bx+a\right)}{a^{5}}+\frac{6\,b^{2}\log\left(x\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^3*x^3 + 18*a*b^2*x^2 + 4*a^2*b*x - a^3)/(a^4*b^2*x^4 + 2*a^5*b*x^
3 + a^6*x^2) - 6*b^2*log(b*x + a)/a^5 + 6*b^2*log(x)/a^5
```

Fricas [A] time = 1.50816, size = 269, normalized size = 3.54

$$\frac{12\,ab^{3}x^{3}+18\,a^{2}b^{2}x^{2}+4\,a^{3}bx-a^{4}-12\left(b^{4}x^{4}+2\,ab^{3}x^{3}+a^{2}b^{2}x^{2}\right)\log\left(bx+a\right)+12\left(b^{4}x^{4}+2\,ab^{3}x^{3}+a^{2}b^{2}x^{2}\right)\log\left(x\right)}{2\left(a^{5}b^{2}x^{4}+2\,a^{6}bx^{3}+a^{7}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4 - 12*(b^4*x^4 + 2*a*b^
3*x^3 + a^2*b^2*x^2)*log(b*x + a) + 12*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2
)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

Sympy [A] time = 0.761018, size = 78, normalized size = 1.03

$$\frac{-a^{3}+4a^{2}bx+18ab^{2}x^{2}+12b^{3}x^{3}}{2a^{6}x^{2}+4a^{5}bx^{3}+2a^{4}b^{2}x^{4}}+\frac{6b^{2}\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**3,x)
```

```
[Out] (-a**3 + 4*a**2*b*x + 18*a*b**2*x**2 + 12*b**3*x**3)/(2*a**6*x**2 + 4*a**5*
b*x**3 + 2*a**4*b**2*x**4) + 6*b**2*(log(x) - log(a/b + x))/a**5
```

Giac [A] time = 1.23645, size = 99, normalized size = 1.3

$$-\frac{6\,b^2\log\left(|bx+a|\right)}{a^5}+\frac{6\,b^2\log\left(|x|\right)}{a^5}+\frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\left(bx^2+ax\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -6*b^2*log(abs(b*x + a))/a^5 + 6*b^2*log(abs(x))/a^5 + 1/2*(12*b^3*x^3 + 18
*a*b^2*x^2 + 4*a^2*b*x - a^3)/((b*x^2 + a*x)^2*a^4)
```

3.191 $\int \frac{1}{x^4(a+bx)^3}\,dx$

Optimal. Leaf size=89

$$-\frac{4b^3}{a^5(a+bx)}-\frac{b^3}{2a^4(a+bx)^2}-\frac{6b^2}{a^5x}-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}+\frac{3b}{2a^4x^2}-\frac{1}{3a^3x^3}$$

```
[Out] -1/(3*a^3*x^3) + (3*b)/(2*a^4*x^2) - (6*b^2)/(a^5*x) - b^3/(2*a^4*(a + b*x)
^2) - (4*b^3)/(a^5*(a + b*x)) - (10*b^3*Log[x])/a^6 + (10*b^3*Log[a + b*x])
/a^6
```

Rubi [A] time = 0.0477162, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{4b^3}{a^5(a+bx)}-\frac{b^3}{2a^4(a+bx)^2}-\frac{6b^2}{a^5x}-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}+\frac{3b}{2a^4x^2}-\frac{1}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^3),x]
```

```
[Out] -1/(3*a^3*x^3) + (3*b)/(2*a^4*x^2) - (6*b^2)/(a^5*x) - b^3/(2*a^4*(a + b*x)
^2) - (4*b^3)/(a^5*(a + b*x)) - (10*b^3*Log[x])/a^6 + (10*b^3*Log[a + b*x])
/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^4}-\frac{3b}{a^4x^3}+\frac{6b^2}{a^5x^2}-\frac{10b^3}{a^6x}+\frac{b^4}{a^4(a+bx)^3}+\frac{4b^4}{a^5(a+bx)^2}+\frac{10b^4}{a^6(a+bx)}\right)dx\\ &= -\frac{1}{3a^3x^3}+\frac{3b}{2a^4x^2}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(a+bx)^2}-\frac{4b^3}{a^5(a+bx)}-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}\end{aligned}$$

Mathematica [A] time = 0.132316, size = 79, normalized size = 0.89

$$-\frac{\frac{a\left(20a^2b^2x^2-5a^3bx+2a^4+90ab^3x^3+60b^4x^4\right)}{x^3(a+bx)^2}-60b^3\log(a+bx)+60b^3\log(x)}{6a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^3),x]
```

```
[Out] -((a*(2*a^4 - 5*a^3*b*x + 20*a^2*b^2*x^2 + 90*a*b^3*x^3 + 60*b^4*x^4))/(x^3
*(a + b*x)^2) + 60*b^3*Log[x] - 60*b^3*Log[a + b*x])/(6*a^6)
```

Maple [A] time = 0.013, size = 84, normalized size = 0.9

$$-\frac{1}{3\,a^{3}x^{3}}+\frac{3\,b}{2\,a^{4}x^{2}}-6\,\frac{b^{2}}{a^{5}x}-\frac{b^{3}}{2\,a^{4}\left(bx+a\right)^{2}}-4\,\frac{b^{3}}{a^{5}\left(bx+a\right)}-10\,\frac{b^{3}\ln\left(x\right)}{a^{6}}+10\,\frac{b^{3}\ln\left(bx+a\right)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^3,x)
```

```
[Out] -1/3/a^3/x^3+3/2*b/a^4/x^2-6*b^2/a^5/x-1/2*b^3/a^4/(b*x+a)^2-4*b^3/a^5/(b*x
+a)-10*b^3*ln(x)/a^6+10*b^3*ln(b*x+a)/a^6
```

Maxima [A] time = 1.0925, size = 131, normalized size = 1.47

$$-\frac{60\,b^{4}x^{4}+90\,ab^{3}x^{3}+20\,a^{2}b^{2}x^{2}-5\,a^{3}bx+2\,a^{4}}{6\left(a^{5}b^{2}x^{5}+2\,a^{6}bx^{4}+a^{7}x^{3}\right)}+\frac{10\,b^{3}\log\left(bx+a\right)}{a^{6}}-\frac{10\,b^{3}\log\left(x\right)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(60*b^4*x^4 + 90*a*b^3*x^3 + 20*a^2*b^2*x^2 - 5*a^3*b*x + 2*a^4)/(a^5*
b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3) + 10*b^3*log(b*x + a)/a^6 - 10*b^3*log(x)/
a^6
```

Fricas [A] time = 1.54313, size = 296, normalized size = 3.33

$$-\frac{60\,ab^{4}x^{4}+90\,a^{2}b^{3}x^{3}+20\,a^{3}b^{2}x^{2}-5\,a^{4}bx+2\,a^{5}-60\left(b^{5}x^{5}+2\,ab^{4}x^{4}+a^{2}b^{3}x^{3}\right)\log\left(bx+a\right)+60\left(b^{5}x^{5}+2\,ab^{4}x^{4}+a^{2}b\right.}{6\left(a^{6}b^{2}x^{5}+2\,a^{7}bx^{4}+a^{8}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(60*a*b^4*x^4 + 90*a^2*b^3*x^3 + 20*a^3*b^2*x^2 - 5*a^4*b*x + 2*a^5 -
60*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*log(b*x + a) + 60*(b^5*x^5 + 2*a*b
^4*x^4 + a^2*b^3*x^3)*log(x))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8*x^3)
```

Sympy [A] time = 0.814525, size = 92, normalized size = 1.03

$$-\frac{2a^{4}-5a^{3}bx+20a^{2}b^{2}x^{2}+90ab^{3}x^{3}+60b^{4}x^{4}}{6a^{7}x^{3}+12a^{6}bx^{4}+6a^{5}b^{2}x^{5}}+\frac{10b^{3}\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**3,x)
```

```
[Out] -(2*a**4 - 5*a**3*b*x + 20*a**2*b**2*x**2 + 90*a*b**3*x**3 + 60*b**4*x**4)/
(6*a**7*x**3 + 12*a**6*b*x**4 + 6*a**5*b**2*x**5) + 10*b**3*(-log(x) + log(
```

```
a/b + x))/a**6
```

Giac [A] time = 1.14114, size = 116, normalized size = 1.3

$$\frac{10\,b^3\log\left(|bx+a|\right)}{a^6}-\frac{10\,b^3\log\left(|x|\right)}{a^6}-\frac{60\,ab^4x^4+90\,a^2b^3x^3+20\,a^3b^2x^2-5\,a^4bx+2\,a^5}{6\,(bx+a)^2a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 10*b^3*log(abs(b*x + a))/a^6 - 10*b^3*log(abs(x))/a^6 - 1/6*(60*a*b^4*x^4 +
 90*a^2*b^3*x^3 + 20*a^3*b^2*x^2 - 5*a^4*b*x + 2*a^5)/((b*x + a)^2*a^6*x^3)
```

3.192 $\int \frac{1}{x^5(a+bx)^3} \, dx$

Optimal. Leaf size=97

$$-\frac{3b^2}{a^5x^2}+\frac{5b^4}{a^6(a+bx)}+\frac{b^4}{2a^5(a+bx)^2}+\frac{10b^3}{a^6x}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx)}{a^7}+\frac{b}{a^4x^3}-\frac{1}{4a^3x^4}$$

```
[Out] -1/(4*a^3*x^4) + b/(a^4*x^3) - (3*b^2)/(a^5*x^2) + (10*b^3)/(a^6*x) + b^4/(
2*a^5*(a + b*x)^2) + (5*b^4)/(a^6*(a + b*x)) + (15*b^4*Log[x])/a^7 - (15*b^
4*Log[a + b*x])/a^7
```

Rubi [A] time = 0.052223, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{3b^2}{a^5x^2}+\frac{5b^4}{a^6(a+bx)}+\frac{b^4}{2a^5(a+bx)^2}+\frac{10b^3}{a^6x}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx)}{a^7}+\frac{b}{a^4x^3}-\frac{1}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^3),x]
```

```
[Out] -1/(4*a^3*x^4) + b/(a^4*x^3) - (3*b^2)/(a^5*x^2) + (10*b^3)/(a^6*x) + b^4/(
2*a^5*(a + b*x)^2) + (5*b^4)/(a^6*(a + b*x)) + (15*b^4*Log[x])/a^7 - (15*b^
4*Log[a + b*x])/a^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^5}-\frac{3b}{a^4x^4}+\frac{6b^2}{a^5x^3}-\frac{10b^3}{a^6x^2}+\frac{15b^4}{a^7x}-\frac{b^5}{a^5(a+bx)^3}-\frac{5b^5}{a^6(a+bx)^2}-\frac{15b^5}{a^7(a+bx)}\right)dx\\ &= -\frac{1}{4a^3x^4}+\frac{b}{a^4x^3}-\frac{3b^2}{a^5x^2}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(a+bx)^2}+\frac{5b^4}{a^6(a+bx)}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx)}{a^7}\end{aligned}$$

Mathematica [A] time = 0.0793184, size = 90, normalized size = 0.93

$$\frac{\frac{a\left(-5a^3b^2x^2+20a^2b^3x^3+2a^4bx-a^5+90ab^4x^4+60b^5x^5\right)}{x^4(a+bx)^2}-60b^4\log(a+bx)+60b^4\log(x)}{4a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^3),x]
```

```
[Out] ((a*(-a^5 + 2*a^4*b*x - 5*a^3*b^2*x^2 + 20*a^2*b^3*x^3 + 90*a*b^4*x^4 + 60*
b^5*x^5))/(x^4*(a + b*x)^2) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(4*a^7)
```

Maple [A] time = 0.011, size = 94, normalized size = 1.

$$-\frac{1}{4\,a^{3}x^{4}}+\frac{b}{a^{4}x^{3}}-3\,\frac{b^{2}}{a^{5}x^{2}}+10\,\frac{b^{3}}{a^{6}x}+\frac{b^{4}}{2\,a^{5}\left(bx+a\right)^{2}}+5\,\frac{b^{4}}{a^{6}\left(bx+a\right)}+15\,\frac{b^{4}\ln\left(x\right)}{a^{7}}-15\,\frac{b^{4}\ln\left(bx+a\right)}{a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^3,x)
```

```
[Out] -1/4/a^3/x^4+b/a^4/x^3-3*b^2/a^5/x^2+10*b^3/a^6/x+1/2*b^4/a^5/(b*x+a)^2+5*b
^4/a^6/(b*x+a)+15*b^4*ln(x)/a^7-15*b^4*ln(b*x+a)/a^7
```

Maxima [A] time = 1.08144, size = 146, normalized size = 1.51

$$\frac{60\,b^{5}x^{5}+90\,ab^{4}x^{4}+20\,a^{2}b^{3}x^{3}-5\,a^{3}b^{2}x^{2}+2\,a^{4}bx-a^{5}}{4\left(a^{6}b^{2}x^{6}+2\,a^{7}bx^{5}+a^{8}x^{4}\right)}-\frac{15\,b^{4}\log\left(bx+a\right)}{a^{7}}+\frac{15\,b^{4}\log\left(x\right)}{a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(60*b^5*x^5 + 90*a*b^4*x^4 + 20*a^2*b^3*x^3 - 5*a^3*b^2*x^2 + 2*a^4*b*x
 - a^5)/(a^6*b^2*x^6 + 2*a^7*b*x^5 + a^8*x^4) - 15*b^4*log(b*x + a)/a^7 + 1
5*b^4*log(x)/a^7
```

Fricas [A] time = 1.62782, size = 313, normalized size = 3.23

$$\frac{60\,ab^{5}x^{5}+90\,a^{2}b^{4}x^{4}+20\,a^{3}b^{3}x^{3}-5\,a^{4}b^{2}x^{2}+2\,a^{5}bx-a^{6}-60\left(b^{6}x^{6}+2\,ab^{5}x^{5}+a^{2}b^{4}x^{4}\right)\log\left(bx+a\right)+60\left(b^{6}x^{6}+2\,\cdots\right.}{4\left(a^{7}b^{2}x^{6}+2\,a^{8}bx^{5}+a^{9}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(60*a*b^5*x^5 + 90*a^2*b^4*x^4 + 20*a^3*b^3*x^3 - 5*a^4*b^2*x^2 + 2*a^5
*b*x - a^6 - 60*(b^6*x^6 + 2*a*b^5*x^5 + a^2*b^4*x^4)*log(b*x + a) + 60*(b^
6*x^6 + 2*a*b^5*x^5 + a^2*b^4*x^4)*log(x))/(a^7*b^2*x^6 + 2*a^8*b*x^5 + a^9
*x^4)
```

Sympy [A] time = 0.814769, size = 102, normalized size = 1.05

$$\frac{-a^{5}+2a^{4}bx-5a^{3}b^{2}x^{2}+20a^{2}b^{3}x^{3}+90ab^{4}x^{4}+60b^{5}x^{5}}{4a^{8}x^{4}+8a^{7}bx^{5}+4a^{6}b^{2}x^{6}}+\frac{15b^{4}\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a)**3,x)
```

```
[Out] (-a**5 + 2*a**4*b*x - 5*a**3*b**2*x**2 + 20*a**2*b**3*x**3 + 90*a*b**4*x**4
 + 60*b**5*x**5)/(4*a**8*x**4 + 8*a**7*b*x**5 + 4*a**6*b**2*x**6) + 15*b**4
*(log(x) - log(a/b + x))/a**7
```

Giac [A] time = 1.17565, size = 131, normalized size = 1.35

$$-\frac{15\,b^4\log\left(|bx+a|\right)}{a^7}+\frac{15\,b^4\log\left(|x|\right)}{a^7}+\frac{60\,ab^5x^5+90\,a^2b^4x^4+20\,a^3b^3x^3-5\,a^4b^2x^2+2\,a^5bx-a^6}{4\,(bx+a)^2a^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -15*b^4*log(abs(b*x + a))/a^7 + 15*b^4*log(abs(x))/a^7 + 1/4*(60*a*b^5*x^5
+ 90*a^2*b^4*x^4 + 20*a^3*b^3*x^3 - 5*a^4*b^2*x^2 + 2*a^5*b*x - a^6)/((b*x
+ a)^2*a^7*x^4)
```

3.193 $\int \frac{x^8}{(a+bx)^4}\,dx$

Optimal. Leaf size=114

$$-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}+\frac{35a^4x}{b^8}-\frac{56a^5\log(a+bx)}{b^9}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}$$

```
[Out] (35*a^4*x)/b^8 - (10*a^3*x^2)/b^7 + (10*a^2*x^3)/(3*b^6) - (a*x^4)/b^5 + x^
5/(5*b^4) - a^8/(3*b^9*(a + b*x)^3) + (4*a^7)/(b^9*(a + b*x)^2) - (28*a^6)/
(b^9*(a + b*x)) - (56*a^5*Log[a + b*x])/b^9
```

Rubi [A] time = 0.08529, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}+\frac{35a^4x}{b^8}-\frac{56a^5\log(a+bx)}{b^9}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^4,x]
```

```
[Out] (35*a^4*x)/b^8 - (10*a^3*x^2)/b^7 + (10*a^2*x^3)/(3*b^6) - (a*x^4)/b^5 + x^
5/(5*b^4) - a^8/(3*b^9*(a + b*x)^3) + (4*a^7)/(b^9*(a + b*x)^2) - (28*a^6)/
(b^9*(a + b*x)) - (56*a^5*Log[a + b*x])/b^9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{(a+bx)^4}\,dx &= \int \left(\frac{35a^4}{b^8}-\frac{20a^3x}{b^7}+\frac{10a^2x^2}{b^6}-\frac{4ax^3}{b^5}+\frac{x^4}{b^4}+\frac{a^8}{b^8(a+bx)^4}-\frac{8a^7}{b^8(a+bx)^3}+\frac{28a^6}{b^8(a+bx)^2}-\frac{56a^5}{b^8(a+bx)}\right)dx\\ &= \frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}-\frac{56a^5\log(a+bx)}{b^9}\end{aligned}$$

Mathematica [A] time = 0.0735772, size = 101, normalized size = 0.89

$$\frac{-150a^3b^2x^2+50a^2b^3x^3-\frac{5a^8}{(a+bx)^3}+\frac{60a^7}{(a+bx)^2}-\frac{420a^6}{a+bx}+525a^4bx-840a^5\log(a+bx)-15ab^4x^4+3b^5x^5}{15b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^4,x]
```

```
[Out] (525*a^4*b*x - 150*a^3*b^2*x^2 + 50*a^2*b^3*x^3 - 15*a*b^4*x^4 + 3*b^5*x^5
- (5*a^8)/(a + b*x)^3 + (60*a^7)/(a + b*x)^2 - (420*a^6)/(a + b*x) - 840*a^
```

```
5*Log[a + b*x])/(15*b^9)
```

Maple [A] time = 0.008, size = 109, normalized size = 1.

$$35\,{\frac {{a}^{4}x}{{b}^{8}}}-10\,{\frac {{a}^{3}{x}^{2}}{{b}^{7}}}+{\frac {10\,{a}^{2}{x}^{3}}{3\,{b}^{6}}}-{\frac {a{x}^{4}}{{b}^{5}}}+{\frac {{x}^{5}}{5\,{b}^{4}}}-{\frac {{a}^{8}}{3\,{b}^{9} \left(bx+a \right) ^{3}}}+4\,{\frac {{a}^{7}}{{b}^{9} \left(bx+a \right) ^{2}}}-28\,{\frac {{a}^{6}}{{b}^{9} \left(bx+a \right) }}-56\,{\frac {{a}^{5}\ln \left(bx+a \right) }{{b}^{9}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^4,x)
```

```
[Out] 35*a^4*x/b^8-10*a^3*x^2/b^7+10/3*a^2*x^3/b^6-a*x^4/b^5+1/5*x^5/b^4-1/3*a^8/
b^9/(b*x+a)^3+4*a^7/b^9/(b*x+a)^2-28*a^6/b^9/(b*x+a)-56*a^5*ln(b*x+a)/b^9
```

Maxima [A] time = 1.04087, size = 169, normalized size = 1.48

$$-\frac{84 \, a^{6} b^{2} x^{2} + 156 \, a^{7} b x + 73 \, a^{8}}{3 \, {\left(b^{12} x^{3} + 3 \, a b^{11} x^{2} + 3 \, a^{2} b^{10} x + a^{3} b^{9}\right)}} - \frac{56 \, a^{5} \log\left(b x + a\right)}{b^{9}} + \frac{3 \, b^{4} x^{5} - 15 \, a b^{3} x^{4} + 50 \, a^{2} b^{2} x^{3} - 150 \, a^{3} b x^{2} + 525 \, a^{4} x}{15 \, b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(84*a^6*b^2*x^2 + 156*a^7*b*x + 73*a^8)/(b^12*x^3 + 3*a*b^11*x^2 + 3*a
^2*b^10*x + a^3*b^9) - 56*a^5*log(b*x + a)/b^9 + 1/15*(3*b^4*x^5 - 15*a*b^3
*x^4 + 50*a^2*b^2*x^3 - 150*a^3*b*x^2 + 525*a^4*x)/b^8
```

Fricas [A] time = 1.56293, size = 356, normalized size = 3.12

$$\frac{3 \, b^{8} x^{8} - 6 \, a b^{7} x^{7} + 14 \, a^{2} b^{6} x^{6} - 42 \, a^{3} b^{5} x^{5} + 210 \, a^{4} b^{4} x^{4} + 1175 \, a^{5} b^{3} x^{3} + 1005 \, a^{6} b^{2} x^{2} - 255 \, a^{7} b x - 365 \, a^{8} - 840 \, {\left(a^{5} b^{3} x^{3} + \right.}}{15 \, {\left(b^{12} x^{3} + 3 \, a b^{11} x^{2} + 3 \, a^{2} b^{10} x + a^{3} b^{9}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^8*x^8 - 6*a*b^7*x^7 + 14*a^2*b^6*x^6 - 42*a^3*b^5*x^5 + 210*a^4*b
^4*x^4 + 1175*a^5*b^3*x^3 + 1005*a^6*b^2*x^2 - 255*a^7*b*x - 365*a^8 - 840*
(a^5*b^3*x^3 + 3*a^6*b^2*x^2 + 3*a^7*b*x + a^8)*log(b*x + a))/(b^12*x^3 + 3
*a*b^11*x^2 + 3*a^2*b^10*x + a^3*b^9)
```

Sympy [A] time = 0.885249, size = 129, normalized size = 1.13

$$- \frac{56 a^{5} \log{\left (a + b x \right)}}{b^{9}} + \frac{35 a^{4} x}{b^{8}} - \frac{10 a^{3} x^{2}}{b^{7}} + \frac{10 a^{2} x^{3}}{3 b^{6}} - \frac{a x^{4}}{b^{5}} - \frac{73 a^{8} + 156 a^{7} b x + 84 a^{6} b^{2} x^{2}}{3 a^{3} b^{9} + 9 a^{2} b^{10} x + 9 a b^{11} x^{2} + 3 b^{12} x^{3}} + \frac{x^{5}}{5 b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**4,x)
```

```
[Out] -56*a**5*log(a + b*x)/b**9 + 35*a**4*x/b**8 - 10*a**3*x**2/b**7 + 10*a**2*x
**3/(3*b**6) - a*x**4/b**5 - (73*a**8 + 156*a**7*b*x + 84*a**6*b**2*x**2)/(
3*a**3*b**9 + 9*a**2*b**10*x + 9*a*b**11*x**2 + 3*b**12*x**3) + x**5/(5*b**
4)
```

Giac [A] time = 1.14936, size = 143, normalized size = 1.25

$$-\frac{56\,a^5\log\left(|bx+a|\right)}{b^9}-\frac{84\,a^6b^2x^2+156\,a^7bx+73\,a^8}{3\,(bx+a)^3b^9}+\frac{3\,b^{16}x^5-15\,ab^{15}x^4+50\,a^2b^{14}x^3-150\,a^3b^{13}x^2+525\,a^4b^{12}x}{15\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -56*a^5*log(abs(b*x + a))/b^9 - 1/3*(84*a^6*b^2*x^2 + 156*a^7*b*x + 73*a^8)
/((b*x + a)^3*b^9) + 1/15*(3*b^16*x^5 - 15*a*b^15*x^4 + 50*a^2*b^14*x^3 - 1
50*a^3*b^13*x^2 + 525*a^4*b^12*x)/b^20
```

3.194 $\int \frac{x^7}{(a+bx)^4}\,dx$

Optimal. Leaf size=105

$$\frac{5a^2x^2}{b^6}+\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}-\frac{20a^3x}{b^7}+\frac{35a^4\log(a+bx)}{b^8}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}$$

```
[Out] (-20*a^3*x)/b^7 + (5*a^2*x^2)/b^6 - (4*a*x^3)/(3*b^5) + x^4/(4*b^4) + a^7/(
3*b^8*(a + b*x)^3) - (7*a^6)/(2*b^8*(a + b*x)^2) + (21*a^5)/(b^8*(a + b*x))
 + (35*a^4*Log[a + b*x])/b^8
```

Rubi [A] time = 0.0701249, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{5a^2x^2}{b^6}+\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}-\frac{20a^3x}{b^7}+\frac{35a^4\log(a+bx)}{b^8}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^4,x]
```

```
[Out] (-20*a^3*x)/b^7 + (5*a^2*x^2)/b^6 - (4*a*x^3)/(3*b^5) + x^4/(4*b^4) + a^7/(
3*b^8*(a + b*x)^3) - (7*a^6)/(2*b^8*(a + b*x)^2) + (21*a^5)/(b^8*(a + b*x))
 + (35*a^4*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^4}\,dx &= \int \left(-\frac{20a^3}{b^7}+\frac{10a^2x}{b^6}-\frac{4ax^2}{b^5}+\frac{x^3}{b^4}-\frac{a^7}{b^7(a+bx)^4}+\frac{7a^6}{b^7(a+bx)^3}-\frac{21a^5}{b^7(a+bx)^2}+\frac{35a^4}{b^7(a+bx)}\right)dx\\ &= -\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}+\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}+\frac{35a^4\log(a+bx)}{b^8}\end{aligned}$$

Mathematica [A] time = 0.0513451, size = 90, normalized size = 0.86

$$\frac{60a^2b^2x^2+\frac{4a^7}{(a+bx)^3}-\frac{42a^6}{(a+bx)^2}+\frac{252a^5}{a+bx}-240a^3bx+420a^4\log(a+bx)-16ab^3x^3+3b^4x^4}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^4,x]
```

```
[Out] (-240*a^3*b*x + 60*a^2*b^2*x^2 - 16*a*b^3*x^3 + 3*b^4*x^4 + (4*a^7)/(a + b*
x)^3 - (42*a^6)/(a + b*x)^2 + (252*a^5)/(a + b*x) + 420*a^4*Log[a + b*x])/(
```

```
12*b^8)
```

Maple [A] time = 0.008, size = 98, normalized size = 0.9

$$-20\,\frac{a^{3}x}{b^{7}}+5\,\frac{a^{2}x^{2}}{b^{6}}-\frac{4\,ax^{3}}{3\,b^{5}}+\frac{x^{4}}{4\,b^{4}}+\frac{a^{7}}{3\,b^{8}\left(bx+a\right)^{3}}-\frac{7\,a^{6}}{2\,b^{8}\left(bx+a\right)^{2}}+21\,\frac{a^{5}}{b^{8}\left(bx+a\right)}+35\,\frac{a^{4}\ln\left(bx+a\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^4,x)
```

```
[Out] -20*a^3*x/b^7+5*a^2*x^2/b^6-4/3*a*x^3/b^5+1/4*x^4/b^4+1/3*a^7/b^8/(b*x+a)^3
-7/2*a^6/b^8/(b*x+a)^2+21*a^5/b^8/(b*x+a)+35*a^4*ln(b*x+a)/b^8
```

Maxima [A] time = 1.05678, size = 154, normalized size = 1.47

$$\frac{126\,a^{5}b^{2}x^{2}+231\,a^{6}bx+107\,a^{7}}{6\left(b^{11}x^{3}+3\,ab^{10}x^{2}+3\,a^{2}b^{9}x+a^{3}b^{8}\right)}+\frac{35\,a^{4}\log\left(bx+a\right)}{b^{8}}+\frac{3\,b^{3}x^{4}-16\,ab^{2}x^{3}+60\,a^{2}bx^{2}-240\,a^{3}x}{12\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(126*a^5*b^2*x^2 + 231*a^6*b*x + 107*a^7)/(b^11*x^3 + 3*a*b^10*x^2 + 3*
a^2*b^9*x + a^3*b^8) + 35*a^4*log(b*x + a)/b^8 + 1/12*(3*b^3*x^4 - 16*a*b^2
*x^3 + 60*a^2*b*x^2 - 240*a^3*x)/b^7
```

Fricas [A] time = 1.56855, size = 329, normalized size = 3.13

$$\frac{3\,b^{7}x^{7}-7\,ab^{6}x^{6}+21\,a^{2}b^{5}x^{5}-105\,a^{3}b^{4}x^{4}-556\,a^{4}b^{3}x^{3}-408\,a^{5}b^{2}x^{2}+222\,a^{6}bx+214\,a^{7}+420\left(a^{4}b^{3}x^{3}+3\,a^{5}b^{2}x^{2}+\right.}{12\left(b^{11}x^{3}+3\,ab^{10}x^{2}+3\,a^{2}b^{9}x+a^{3}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*x^7 - 7*a*b^6*x^6 + 21*a^2*b^5*x^5 - 105*a^3*b^4*x^4 - 556*a^4*
b^3*x^3 - 408*a^5*b^2*x^2 + 222*a^6*b*x + 214*a^7 + 420*(a^4*b^3*x^3 + 3*a^
5*b^2*x^2 + 3*a^6*b*x + a^7)*log(b*x + a))/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2
*b^9*x + a^3*b^8)
```

Sympy [A] time = 0.792181, size = 119, normalized size = 1.13

$$\frac{35a^{4}\log\left(a+bx\right)}{b^{8}}-\frac{20a^{3}x}{b^{7}}+\frac{5a^{2}x^{2}}{b^{6}}-\frac{4ax^{3}}{3b^{5}}+\frac{107a^{7}+231a^{6}bx+126a^{5}b^{2}x^{2}}{6a^{3}b^{8}+18a^{2}b^{9}x+18ab^{10}x^{2}+6b^{11}x^{3}}+\frac{x^{4}}{4b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**4,x)
```

```
[Out] 35*a**4*log(a + b*x)/b**8 - 20*a**3*x/b**7 + 5*a**2*x**2/b**6 - 4*a*x**3/(3
*b**5) + (107*a**7 + 231*a**6*b*x + 126*a**5*b**2*x**2)/(6*a**3*b**8 + 18*a
**2*b**9*x + 18*a*b**10*x**2 + 6*b**11*x**3) + x**4/(4*b**4)
```

Giac [A] time = 1.19132, size = 128, normalized size = 1.22

$$\frac{35\,a^4\log\left(|bx+a|\right)}{b^8}+\frac{126\,a^5b^2x^2+231\,a^6bx+107\,a^7}{6\,(bx+a)^3b^8}+\frac{3\,b^{12}x^4-16\,ab^{11}x^3+60\,a^2b^{10}x^2-240\,a^3b^9x}{12\,b^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 35*a^4*log(abs(b*x + a))/b^8 + 1/6*(126*a^5*b^2*x^2 + 231*a^6*b*x + 107*a^7
)/((b*x + a)^3*b^8) + 1/12*(3*b^12*x^4 - 16*a*b^11*x^3 + 60*a^2*b^10*x^2 -
240*a^3*b^9*x)/b^16
```

3.195 $\int \frac{x^6}{(a+bx)^4}\,dx$

Optimal. Leaf size=90

$$-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}+\frac{10a^2x}{b^6}-\frac{20a^3\log(a+bx)}{b^7}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}$$

```
[Out] (10*a^2*x)/b^6 - (2*a*x^2)/b^5 + x^3/(3*b^4) - a^6/(3*b^7*(a + b*x)^3) + (3
*a^5)/(b^7*(a + b*x)^2) - (15*a^4)/(b^7*(a + b*x)) - (20*a^3*Log[a + b*x])/
b^7
```

Rubi [A] time = 0.0565142, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}+\frac{10a^2x}{b^6}-\frac{20a^3\log(a+bx)}{b^7}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^4,x]
```

```
[Out] (10*a^2*x)/b^6 - (2*a*x^2)/b^5 + x^3/(3*b^4) - a^6/(3*b^7*(a + b*x)^3) + (3
*a^5)/(b^7*(a + b*x)^2) - (15*a^4)/(b^7*(a + b*x)) - (20*a^3*Log[a + b*x])/
b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^4}\,dx &= \int \left(\frac{10a^2}{b^6}-\frac{4ax}{b^5}+\frac{x^2}{b^4}+\frac{a^6}{b^6(a+bx)^4}-\frac{6a^5}{b^6(a+bx)^3}+\frac{15a^4}{b^6(a+bx)^2}-\frac{20a^3}{b^6(a+bx)}\right)dx\\ &= \frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}-\frac{20a^3\log(a+bx)}{b^7}\end{aligned}$$

Mathematica [A] time = 0.0281285, size = 90, normalized size = 1.

$$-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}+\frac{10a^2x}{b^6}-\frac{20a^3\log(a+bx)}{b^7}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^4,x]
```

```
[Out] (10*a^2*x)/b^6 - (2*a*x^2)/b^5 + x^3/(3*b^4) - a^6/(3*b^7*(a + b*x)^3) + (3
*a^5)/(b^7*(a + b*x)^2) - (15*a^4)/(b^7*(a + b*x)) - (20*a^3*Log[a + b*x])/
```

```
b^7
```

Maple [A] time = 0.006, size = 87, normalized size = 1.

$$10\frac{a^2x}{b^6}-2\frac{ax^2}{b^5}+\frac{x^3}{3\,b^4}-\frac{a^6}{3\,b^7\,(bx+a)^3}+3\frac{a^5}{b^7\,(bx+a)^2}-15\frac{a^4}{b^7\,(bx+a)}-20\frac{a^3\ln(bx+a)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^4,x)
```

```
[Out] 10*a^2*x/b^6-2*a*x^2/b^5+1/3*x^3/b^4-1/3*a^6/b^7/(b*x+a)^3+3*a^5/b^7/(b*x+a
)^2-15*a^4/b^7/(b*x+a)-20*a^3*ln(b*x+a)/b^7
```

Maxima [A] time = 1.11261, size = 138, normalized size = 1.53

$$-\frac{45\,a^4b^2x^2+81\,a^5bx+37\,a^6}{3\left(b^{10}x^3+3\,ab^9x^2+3\,a^2b^8x+a^3b^7\right)}-\frac{20\,a^3\log(bx+a)}{b^7}+\frac{b^2x^3-6\,abx^2+30\,a^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(45*a^4*b^2*x^2 + 81*a^5*b*x + 37*a^6)/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^2
*b^8*x + a^3*b^7) - 20*a^3*log(b*x + a)/b^7 + 1/3*(b^2*x^3 - 6*a*b*x^2 + 30
*a^2*x)/b^6
```

Fricas [A] time = 1.60368, size = 293, normalized size = 3.26

$$\frac{b^6x^6-3\,ab^5x^5+15\,a^2b^4x^4+73\,a^3b^3x^3+39\,a^4b^2x^2-51\,a^5bx-37\,a^6-60\left(a^3b^3x^3+3\,a^4b^2x^2+3\,a^5bx+a^6\right)\log(bx+a)}{3\left(b^{10}x^3+3\,ab^9x^2+3\,a^2b^8x+a^3b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^6*x^6 - 3*a*b^5*x^5 + 15*a^2*b^4*x^4 + 73*a^3*b^3*x^3 + 39*a^4*b^2*x
^2 - 51*a^5*b*x - 37*a^6 - 60*(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b*x + a^
6)*log(b*x + a))/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^2*b^8*x + a^3*b^7)
```

Sympy [A] time = 0.758466, size = 105, normalized size = 1.17

$$-\frac{20a^3\log(a+bx)}{b^7}+\frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}-\frac{37a^6+81a^5bx+45a^4b^2x^2}{3a^3b^7+9a^2b^8x+9ab^9x^2+3b^{10}x^3}+\frac{x^3}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**4,x)
```

```
[Out] -20*a**3*log(a + b*x)/b**7 + 10*a**2*x/b**6 - 2*a*x**2/b**5 - (37*a**6 + 81
*a**5*b*x + 45*a**4*b**2*x**2)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2
 + 3*b**10*x**3) + x**3/(3*b**4)
```

Giac [A] time = 1.20424, size = 112, normalized size = 1.24

$$-\frac{20\,a^3\log\left(|bx+a|\right)}{b^7}-\frac{45\,a^4b^2x^2+81\,a^5bx+37\,a^6}{3\,(bx+a)^3b^7}+\frac{b^8x^3-6\,ab^7x^2+30\,a^2b^6x}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -20*a^3*log(abs(b*x + a))/b^7 - 1/3*(45*a^4*b^2*x^2 + 81*a^5*b*x + 37*a^6)/
((b*x + a)^3*b^7) + 1/3*(b^8*x^3 - 6*a*b^7*x^2 + 30*a^2*b^6*x)/b^12
```

3.196 $\int \frac{x^5}{(a+bx)^4} \, dx$

Optimal. Leaf size=81

$$\frac{a^5}{3b^6(a+bx)^3} - \frac{5a^4}{2b^6(a+bx)^2} + \frac{10a^3}{b^6(a+bx)} + \frac{10a^2\log(a+bx)}{b^6} - \frac{4ax}{b^5} + \frac{x^2}{2b^4}$$

```
[Out] (-4*a*x)/b^5 + x^2/(2*b^4) + a^5/(3*b^6*(a + b*x)^3) - (5*a^4)/(2*b^6*(a + b*x)^2) + (10*a^3)/(b^6*(a + b*x)) + (10*a^2*Log[a + b*x])/b^6
```

Rubi [A] time = 0.0476785, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^5}{3b^6(a+bx)^3} - \frac{5a^4}{2b^6(a+bx)^2} + \frac{10a^3}{b^6(a+bx)} + \frac{10a^2\log(a+bx)}{b^6} - \frac{4ax}{b^5} + \frac{x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^4,x]
```

```
[Out] (-4*a*x)/b^5 + x^2/(2*b^4) + a^5/(3*b^6*(a + b*x)^3) - (5*a^4)/(2*b^6*(a + b*x)^2) + (10*a^3)/(b^6*(a + b*x)) + (10*a^2*Log[a + b*x])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a+bx)^4} \, dx &= \int \left(-\frac{4a}{b^5} + \frac{x}{b^4} - \frac{a^5}{b^5(a+bx)^4} + \frac{5a^4}{b^5(a+bx)^3} - \frac{10a^3}{b^5(a+bx)^2} + \frac{10a^2}{b^5(a+bx)}\right) dx \\
&= -\frac{4ax}{b^5} + \frac{x^2}{2b^4} + \frac{a^5}{3b^6(a+bx)^3} - \frac{5a^4}{2b^6(a+bx)^2} + \frac{10a^3}{b^6(a+bx)} + \frac{10a^2\log(a+bx)}{b^6}
\end{aligned}$$

Mathematica [A] time = 0.041163, size = 68, normalized size = 0.84

$$\frac{\frac{2a^5}{(a+bx)^3} - \frac{15a^4}{(a+bx)^2} + \frac{60a^3}{a+bx} + 60a^2\log(a+bx) - 24abx + 3b^2x^2}{6b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^4,x]
```

```
[Out] (-24*a*b*x + 3*b^2*x^2 + (2*a^5)/(a + b*x)^3 - (15*a^4)/(a + b*x)^2 + (60*a^3)/(a + b*x) + 60*a^2*Log[a + b*x])/(6*b^6)
```

Maple [A] time = 0.007, size = 76, normalized size = 0.9

$$-4\frac{ax}{b^5}+\frac{x^2}{2\,b^4}+\frac{a^5}{3\,b^6\,(bx+a)^3}-\frac{5\,a^4}{2\,b^6\,(bx+a)^2}+10\,\frac{a^3}{b^6\,(bx+a)}+10\,\frac{a^2\ln(bx+a)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^4,x)
```

```
[Out] -4*a*x/b^5+1/2*x^2/b^4+1/3*a^5/b^6/(b*x+a)^3-5/2*a^4/b^6/(b*x+a)^2+10*a^3/b
^6/(b*x+a)+10*a^2*ln(b*x+a)/b^6
```

Maxima [A] time = 1.05502, size = 123, normalized size = 1.52

$$\frac{60\,a^3b^2x^2+105\,a^4bx+47\,a^5}{6\left(b^9x^3+3\,ab^8x^2+3\,a^2b^7x+a^3b^6\right)}+\frac{10\,a^2\log(bx+a)}{b^6}+\frac{bx^2-8\,ax}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*a^3*b^2*x^2 + 105*a^4*b*x + 47*a^5)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*
b^7*x + a^3*b^6) + 10*a^2*log(b*x + a)/b^6 + 1/2*(b*x^2 - 8*a*x)/b^5
```

Fricas [A] time = 1.43358, size = 271, normalized size = 3.35

$$\frac{3\,b^5x^5-15\,ab^4x^4-63\,a^2b^3x^3-9\,a^3b^2x^2+81\,a^4bx+47\,a^5+60\left(a^2b^3x^3+3\,a^3b^2x^2+3\,a^4bx+a^5\right)\log(bx+a)}{6\left(b^9x^3+3\,ab^8x^2+3\,a^2b^7x+a^3b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*x^5 - 15*a*b^4*x^4 - 63*a^2*b^3*x^3 - 9*a^3*b^2*x^2 + 81*a^4*b*x
 + 47*a^5 + 60*(a^2*b^3*x^3 + 3*a^3*b^2*x^2 + 3*a^4*b*x + a^5)*log(b*x + a)
)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b^6)
```

Sympy [A] time = 0.730283, size = 94, normalized size = 1.16

$$\frac{10a^2\log(a+bx)}{b^6}-\frac{4ax}{b^5}+\frac{47a^5+105a^4bx+60a^3b^2x^2}{6a^3b^6+18a^2b^7x+18ab^8x^2+6b^9x^3}+\frac{x^2}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**4,x)
```

```
[Out] 10*a**2*log(a + b*x)/b**6 - 4*a*x/b**5 + (47*a**5 + 105*a**4*b*x + 60*a**3*
b**2*x**2)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) +
x**2/(2*b**4)
```

Giac [A] time = 1.21348, size = 97, normalized size = 1.2

$$\frac{10\,a^{2}\log\left(|bx+a|\right)}{b^{6}}+\frac{b^{4}x^{2}-8\,ab^{3}x}{2\,b^{8}}+\frac{60\,a^{3}b^{2}x^{2}+105\,a^{4}bx+47\,a^{5}}{6\,(bx+a)^{3}b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 10*a^2*log(abs(b*x + a))/b^6 + 1/2*(b^4*x^2 - 8*a*b^3*x)/b^8 + 1/6*(60*a^3*
b^2*x^2 + 105*a^4*b*x + 47*a^5)/((b*x + a)^3*b^6)
```

3.197 $\int \frac{x^4}{(a+bx)^4} dx$

Optimal. Leaf size=65

$$-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}+\frac{x}{b^4}$$

```
[Out] x/b^4 - a^4/(3*b^5*(a + b*x)^3) + (2*a^3)/(b^5*(a + b*x)^2) - (6*a^2)/(b^5*
(a + b*x)) - (4*a*Log[a + b*x])/b^5
```

Rubi [A] time = 0.0363399, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}+\frac{x}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^4,x]
```

```
[Out] x/b^4 - a^4/(3*b^5*(a + b*x)^3) + (2*a^3)/(b^5*(a + b*x)^2) - (6*a^2)/(b^5*
(a + b*x)) - (4*a*Log[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^4}\,dx &= \int \left(\frac{1}{b^4}+\frac{a^4}{b^4(a+bx)^4}-\frac{4a^3}{b^4(a+bx)^3}+\frac{6a^2}{b^4(a+bx)^2}-\frac{4a}{b^4(a+bx)}\right)dx\\ &= \frac{x}{b^4}-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.0674403, size = 51, normalized size = 0.78

$$-\frac{\frac{a^2(13a^2+30abx+18b^2x^2)}{(a+bx)^3}+12a\log(a+bx)-3bx}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^4,x]
```

```
[Out] -(-3*b*x + (a^2*(13*a^2 + 30*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 12*a*Log[a
+ b*x])/(3*b^5)
```

Maple [A] time = 0.008, size = 64, normalized size = 1.

$$\frac{x}{b^4}-\frac{a^4}{3\,b^5\,(bx+a)^3}+2\,\frac{a^3}{b^5\,(bx+a)^2}-6\,\frac{a^2}{b^5\,(bx+a)}-4\,\frac{a\ln(bx+a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^4,x)
```

```
[Out] x/b^4-1/3*a^4/b^5/(b*x+a)^3+2*a^3/b^5/(b*x+a)^2-6*a^2/b^5/(b*x+a)-4*a*ln(b*
x+a)/b^5
```

Maxima [A] time = 1.08027, size = 107, normalized size = 1.65

$$-\frac{18\,a^2b^2x^2+30\,a^3bx+13\,a^4}{3\left(b^8x^3+3\,ab^7x^2+3\,a^2b^6x+a^3b^5\right)}+\frac{x}{b^4}-\frac{4\,a\log(bx+a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(18*a^2*b^2*x^2 + 30*a^3*b*x + 13*a^4)/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*
b^6*x + a^3*b^5) + x/b^4 - 4*a*log(b*x + a)/b^5
```

Fricas [A] time = 1.43883, size = 244, normalized size = 3.75

$$\frac{3\,b^4x^4+9\,ab^3x^3-9\,a^2b^2x^2-27\,a^3bx-13\,a^4-12\left(ab^3x^3+3\,a^2b^2x^2+3\,a^3bx+a^4\right)\log(bx+a)}{3\left(b^8x^3+3\,ab^7x^2+3\,a^2b^6x+a^3b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*x^4 + 9*a*b^3*x^3 - 9*a^2*b^2*x^2 - 27*a^3*b*x - 13*a^4 - 12*(a*
b^3*x^3 + 3*a^2*b^2*x^2 + 3*a^3*b*x + a^4)*log(b*x + a))/(b^8*x^3 + 3*a*b^7
*x^2 + 3*a^2*b^6*x + a^3*b^5)
```

Sympy [A] time = 0.708941, size = 80, normalized size = 1.23

$$-\frac{4a\log(a+bx)}{b^5}-\frac{13a^4+30a^3bx+18a^2b^2x^2}{3a^3b^5+9a^2b^6x+9ab^7x^2+3b^8x^3}+\frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**4,x)
```

```
[Out] -4*a*log(a + b*x)/b**5 - (13*a**4 + 30*a**3*b*x + 18*a**2*b**2*x**2)/(3*a**
3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) + x/b**4
```

Giac [A] time = 1.19794, size = 74, normalized size = 1.14

$$\frac{x}{b^4} - \frac{4\,a\log\left(|bx+a|\right)}{b^5} - \frac{18\,a^2b^2x^2 + 30\,a^3bx + 13\,a^4}{3\,(bx+a)^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] x/b^4 - 4*a*log(abs(b*x + a))/b^5 - 1/3*(18*a^2*b^2*x^2 + 30*a^3*b*x + 13*a
^4)/((b*x + a)^3*b^5)
```

3.198 $\int \frac{x^3}{(a+bx)^4} \, dx$

Optimal. Leaf size=58

$$\frac{a^3}{3b^4(a+bx)^3} - \frac{3a^2}{2b^4(a+bx)^2} + \frac{3a}{b^4(a+bx)} + \frac{\log(a+bx)}{b^4}$$

```
[Out] a^3/(3*b^4*(a + b*x)^3) - (3*a^2)/(2*b^4*(a + b*x)^2) + (3*a)/(b^4*(a + b*x
)) + Log[a + b*x]/b^4
```

Rubi [A] time = 0.0303948, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{3b^4(a+bx)^3} - \frac{3a^2}{2b^4(a+bx)^2} + \frac{3a}{b^4(a+bx)} + \frac{\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^4,x]
```

```
[Out] a^3/(3*b^4*(a + b*x)^3) - (3*a^2)/(2*b^4*(a + b*x)^2) + (3*a)/(b^4*(a + b*x
)) + Log[a + b*x]/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(a+bx)^4} \, dx &= \int \left(-\frac{a^3}{b^3(a+bx)^4} + \frac{3a^2}{b^3(a+bx)^3} - \frac{3a}{b^3(a+bx)^2} + \frac{1}{b^3(a+bx)}\right) dx \\ &= \frac{a^3}{3b^4(a+bx)^3} - \frac{3a^2}{2b^4(a+bx)^2} + \frac{3a}{b^4(a+bx)} + \frac{\log(a+bx)}{b^4} \end{aligned}$$

Mathematica [A] time = 0.0189906, size = 44, normalized size = 0.76

$$\frac{\frac{a(11a^2+27abx+18b^2x^2)}{(a+bx)^3} + 6\log(a+bx)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^4,x]
```

```
[Out] ((a*(11*a^2 + 27*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 6*Log[a + b*x])/(6*b^4)
```

Maple [A] time = 0.005, size = 55, normalized size = 1.

$$\frac{a^{3}}{3 b^{4} \left(b x +a \right)^{3}}-\frac{3 a^{2}}{2 b^{4} \left(b x +a \right)^{2}}+3 \frac{a}{b^{4} \left(b x +a \right)}+\frac{\ln \left(b x +a \right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^4,x)
```

```
[Out] 1/3*a^3/b^4/(b*x+a)^3-3/2*a^2/b^4/(b*x+a)^2+3*a/b^4/(b*x+a)+ln(b*x+a)/b^4
```

Maxima [A] time = 1.06126, size = 95, normalized size = 1.64

$$\frac{18 \, a b^{2} x^{2} + 27 \, a^{2} b x + 11 \, a^{3}}{6 \, {\left(b^{7} x^{3} + 3 \, a b^{6} x^{2} + 3 \, a^{2} b^{5} x + a^{3} b^{4}\right)}} + \frac{\log \left(b x + a\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3)/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5
*x + a^3*b^4) + log(b*x + a)/b^4
```

Fricas [A] time = 1.51631, size = 201, normalized size = 3.47

$$\frac{18 \, a b^{2} x^{2} + 27 \, a^{2} b x + 11 \, a^{3} + 6 \, {\left(b^{3} x^{3} + 3 \, a b^{2} x^{2} + 3 \, a^{2} b x + a^{3}\right)} \log \left(b x + a\right)}{6 \, {\left(b^{7} x^{3} + 3 \, a b^{6} x^{2} + 3 \, a^{2} b^{5} x + a^{3} b^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3 + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*
b*x + a^3)*log(b*x + a))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

Sympy [A] time = 0.552948, size = 70, normalized size = 1.21

$$\frac{11 a^{3} + 27 a^{2} b x + 18 a b^{2} x^{2}}{6 a^{3} b^{4} + 18 a^{2} b^{5} x + 18 a b^{6} x^{2} + 6 b^{7} x^{3}} + \frac{\log{\left(a + b x \right)}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**4,x)
```

```
[Out] (11*a**3 + 27*a**2*b*x + 18*a*b**2*x**2)/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) + log(a + b*x)/b**4
```

Giac [A] time = 1.19585, size = 62, normalized size = 1.07

$$\frac{\log\left(|bx+a|\right)}{b^4}+\frac{18\,abx^2+27\,a^2x+\frac{11\,a^3}{b}}{6\,(bx+a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^4 + 1/6*(18*a*b*x^2 + 27*a^2*x + 11*a^3/b)/((b*x + a)^3
*b^3)
```

3.199 $\int \frac{x^2}{(a+bx)^4}\,dx$

Optimal. Leaf size=17

$$\frac{x^3}{3a(a+bx)^3}$$

```
[Out] x^3/(3*a*(a + b*x)^3)
```

Rubi [A] time = 0.0016848, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$\frac{x^3}{3a(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^4,x]

[Out] x^3/(3*a*(a + b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^2}{(a+bx)^4}\,dx = \frac{x^3}{3a(a+bx)^3}$$

Mathematica [A] time = 0.0171861, size = 31, normalized size = 1.82

$$-\frac{a^2+3abx+3b^2x^2}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^4,x]

[Out] -(a^2 + 3*a*b*x + 3*b^2*x^2)/(3*b^3*(a + b*x)^3)
```

Maple [B] time = 0.004, size = 41, normalized size = 2.4

$$-\frac{1}{b^3\,(bx+a)}+\frac{a}{b^3\,(bx+a)^2}-\frac{a^2}{3\,b^3\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^4,x)
```

```
[Out] -1/b^3/(b*x+a)+1/b^3*a/(b*x+a)^2-1/3/b^3*a^2/(b*x+a)^3
```

Maxima [B] time = 1.08234, size = 73, normalized size = 4.29

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3
*b^3)
```

Fricas [B] time = 1.48234, size = 111, normalized size = 6.53

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3
*b^3)
```

Sympy [B] time = 0.516355, size = 56, normalized size = 3.29

$$-\frac{a^2+3abx+3b^2x^2}{3a^3b^3+9a^2b^4x+9ab^5x^2+3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**4,x)
```

```
[Out] -(a**2 + 3*a*b*x + 3*b**2*x**2)/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**
2 + 3*b**6*x**3)
```

Giac [A] time = 1.13832, size = 39, normalized size = 2.29

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,(bx+a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/((b*x + a)^3*b^3)
```

3.200 $\int \frac{x}{(a+bx)^4} dx$

Optimal. Leaf size=30

$$\frac{a}{3b^2(a+bx)^3} - \frac{1}{2b^2(a+bx)^2}$$

```
[Out] a/(3*b^2*(a + b*x)^3) - 1/(2*b^2*(a + b*x)^2)
```

Rubi [A] time = 0.0128605, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a}{3b^2(a+bx)^3} - \frac{1}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^4,x]
```

```
[Out] a/(3*b^2*(a + b*x)^3) - 1/(2*b^2*(a + b*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^4}\,dx &= \int \left(-\frac{a}{b(a+bx)^4}+\frac{1}{b(a+bx)^3}\right)dx\\ &= \frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}\end{aligned}$$

Mathematica [A] time = 0.0065877, size = 20, normalized size = 0.67

$$-\frac{a+3bx}{6b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^4,x]
```

```
[Out] -(a + 3*b*x)/(6*b^2*(a + b*x)^3)
```

Maple [A] time = 0.005, size = 27, normalized size = 0.9

$$\frac{a}{3\,b^2\,(bx+a)^3} - \frac{1}{2\,b^2\,(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^4,x)
```

```
[Out] 1/3*a/b^2/(b*x+a)^3-1/2/b^2/(b*x+a)^2
```

Maxima [A] time = 1.03351, size = 58, normalized size = 1.93

$$-\frac{3\,bx+a}{6\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

Fricas [A] time = 1.51679, size = 89, normalized size = 2.97

$$-\frac{3\,bx+a}{6\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

Sympy [A] time = 0.539855, size = 44, normalized size = 1.47

$$-\frac{a+3bx}{6a^3b^2+18a^2b^3x+18ab^4x^2+6b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**4,x)
```

```
[Out] -(a + 3*b*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6*b**5*x**3)
```

Giac [A] time = 1.20428, size = 24, normalized size = 0.8

$$-\frac{3\,bx+a}{6\,(bx+a)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*x + a)/((b*x + a)^3*b^2)
```

3.201 $\int \frac{1}{(a+bx)^4} dx$

Optimal. Leaf size=14

$$-\frac{1}{3b(a+bx)^3}$$

[Out] -1/(3*b*(a + b*x)^3)

Rubi [A] time = 0.0015438, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-4),x]

[Out] -1/(3*b*(a + b*x)^3)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4} dx = -\frac{1}{3b(a+bx)^3}$$

Mathematica [A] time = 0.0030585, size = 14, normalized size = 1.

$$-\frac{1}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-4),x]

[Out] -1/(3*b*(a + b*x)^3)

Maple [A] time = 0.001, size = 13, normalized size = 0.9

$$-\frac{1}{3\,b\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4,x)
```

```
[Out] -1/3/b/(b*x+a)^3
```

Maxima [A] time = 1.07077, size = 16, normalized size = 1.14

$$-\frac{1}{3\,(bx+a)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3/((b*x + a)^3*b)
```

Fricas [B] time = 1.38961, size = 70, normalized size = 5.

$$-\frac{1}{3\left(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

Sympy [B] time = 0.449615, size = 37, normalized size = 2.64

$$-\frac{1}{3a^3b+9a^2b^2x+9ab^3x^2+3b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4,x)
```

```
[Out] -1/(3*a**3*b + 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**4*x**3)
```

Giac [A] time = 1.16565, size = 16, normalized size = 1.14

$$-\frac{1}{3\,(bx+a)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3/((b*x + a)^3*b)
```

3.202 $\int \frac{1}{x(a+bx)^4}\,dx$

Optimal. Leaf size=57

$$\frac{1}{a^3(a+bx)}+\frac{1}{2a^2(a+bx)^2}-\frac{\log(a+bx)}{a^4}+\frac{\log(x)}{a^4}+\frac{1}{3a(a+bx)^3}$$

```
[Out] 1/(3*a*(a + b*x)^3) + 1/(2*a^2*(a + b*x)^2) + 1/(a^3*(a + b*x)) + Log[x]/a^
4 - Log[a + b*x]/a^4
```

Rubi [A] time = 0.0283706, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{a^3(a+bx)}+\frac{1}{2a^2(a+bx)^2}-\frac{\log(a+bx)}{a^4}+\frac{\log(x)}{a^4}+\frac{1}{3a(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^4),x]
```

```
[Out] 1/(3*a*(a + b*x)^3) + 1/(2*a^2*(a + b*x)^2) + 1/(a^3*(a + b*x)) + Log[x]/a^
4 - Log[a + b*x]/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x}-\frac{b}{a(a+bx)^4}-\frac{b}{a^2(a+bx)^3}-\frac{b}{a^3(a+bx)^2}-\frac{b}{a^4(a+bx)}\right)dx\\ &= \frac{1}{3a(a+bx)^3}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{a^3(a+bx)}+\frac{\log(x)}{a^4}-\frac{\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.0566188, size = 48, normalized size = 0.84

$$\frac{\frac{a(11a^2+15abx+6b^2x^2)}{(a+bx)^3}-6\log(a+bx)+6\log(x)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^4),x]
```

```
[Out] ((a*(11*a^2 + 15*a*b*x + 6*b^2*x^2))/(a + b*x)^3 + 6*Log[x] - 6*Log[a + b*x
])/(6*a^4)
```

Maple [A] time = 0.006, size = 54, normalized size = 1.

$$\frac{1}{3 a (bx+a)^3}+\frac{1}{2 a^2 (bx+a)^2}+\frac{1}{a^3 (bx+a)}+\frac{\ln (x)}{a^4}-\frac{\ln (bx+a)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^4,x)
```

```
[Out] 1/3/a/(b*x+a)^3+1/2/a^2/(b*x+a)^2+1/a^3/(b*x+a)+ln(x)/a^4-ln(b*x+a)/a^4
```

Maxima [A] time = 1.0721, size = 99, normalized size = 1.74

$$\frac{6 b^2x^2+15 abx+11 a^2}{6 \left(a^3b^3x^3+3 a^4b^2x^2+3 a^5bx+a^6\right)}-\frac{\log (bx+a)}{a^4}+\frac{\log (x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*b^2*x^2 + 15*a*b*x + 11*a^2)/(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b*
x + a^6) - log(b*x + a)/a^4 + log(x)/a^4
```

Fricas [B] time = 1.57789, size = 271, normalized size = 4.75

$$\frac{6 ab^2x^2+15 a^2bx+11 a^3-6 \left(b^3x^3+3 ab^2x^2+3 a^2bx+a^3\right)\log (bx+a)+6 \left(b^3x^3+3 ab^2x^2+3 a^2bx+a^3\right)\log (x)}{6 \left(a^4b^3x^3+3 a^5b^2x^2+3 a^6bx+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*b^2*x^2 + 15*a^2*b*x + 11*a^3 - 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b
*x + a^3)*log(b*x + a) + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(x)
)/(a^4*b^3*x^3 + 3*a^5*b^2*x^2 + 3*a^6*b*x + a^7)
```

Sympy [A] time = 0.769741, size = 70, normalized size = 1.23

$$\frac{11a^2+15abx+6b^2x^2}{6a^6+18a^5bx+18a^4b^2x^2+6a^3b^3x^3}+\frac{\log (x)-\log \left(\frac{a}{b}+x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**4,x)
```

```
[Out] (11*a**2 + 15*a*b*x + 6*b**2*x**2)/(6*a**6 + 18*a**5*b*x + 18*a**4*b**2*x**
2 + 6*a**3*b**3*x**3) + (log(x) - log(a/b + x))/a**4
```

Giac [A] time = 1.16938, size = 73, normalized size = 1.28

$$-\frac{\log\left(|bx+a|\right)}{a^4}+\frac{\log\left(|x|\right)}{a^4}+\frac{6\,ab^2x^2+15\,a^2bx+11\,a^3}{6\,(bx+a)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^4 + log(abs(x))/a^4 + 1/6*(6*a*b^2*x^2 + 15*a^2*b*x +
11*a^3)/((b*x + a)^3*a^4)
```

3.203 $\int \frac{1}{x^2(a+bx)^4} dx$

Optimal. Leaf size=70

$$-\frac{3b}{a^4(a+bx)}-\frac{b}{a^3(a+bx)^2}-\frac{b}{3a^2(a+bx)^3}-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}-\frac{1}{a^4x}$$

```
[Out] -(1/(a^4*x)) - b/(3*a^2*(a + b*x)^3) - b/(a^3*(a + b*x)^2) - (3*b)/(a^4*(a
+ b*x)) - (4*b*Log[x])/a^5 + (4*b*Log[a + b*x])/a^5
```

Rubi [A] time = 0.0388295, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{3b}{a^4(a+bx)}-\frac{b}{a^3(a+bx)^2}-\frac{b}{3a^2(a+bx)^3}-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}-\frac{1}{a^4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^4),x]
```

```
[Out] -(1/(a^4*x)) - b/(3*a^2*(a + b*x)^3) - b/(a^3*(a + b*x)^2) - (3*b)/(a^4*(a
+ b*x)) - (4*b*Log[x])/a^5 + (4*b*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^4} dx &= \int \left(\frac{1}{a^4x^2}-\frac{4b}{a^5x}+\frac{b^2}{a^2(a+bx)^4}+\frac{2b^2}{a^3(a+bx)^3}+\frac{3b^2}{a^4(a+bx)^2}+\frac{4b^2}{a^5(a+bx)}\right) dx\\ &= -\frac{1}{a^4x}-\frac{b}{3a^2(a+bx)^3}-\frac{b}{a^3(a+bx)^2}-\frac{3b}{a^4(a+bx)}-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.102326, size = 64, normalized size = 0.91

$$-\frac{\frac{a(22a^2bx+3a^3+30ab^2x^2+12b^3x^3)}{x(a+bx)^3}-12b\log(a+bx)+12b\log(x)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^4),x]
```

```
[Out] -((a*(3*a^3 + 22*a^2*b*x + 30*a*b^2*x^2 + 12*b^3*x^3))/(x*(a + b*x)^3) + 12
*b*Log[x] - 12*b*Log[a + b*x])/(3*a^5)
```

Maple [A] time = 0.009, size = 69, normalized size = 1.

$$-\frac{1}{a^4x}-\frac{b}{3\,a^2\,(bx+a)^3}-\frac{b}{a^3\,(bx+a)^2}-3\,\frac{b}{a^4\,(bx+a)}-4\,\frac{b\ln(x)}{a^5}+4\,\frac{b\ln(bx+a)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^4,x)
```

```
[Out] -1/a^4/x-1/3*b/a^2/(b*x+a)^3-b/a^3/(b*x+a)^2-3*b/a^4/(b*x+a)-4*b*ln(x)/a^5+
4*b*ln(b*x+a)/a^5
```

Maxima [A] time = 1.04235, size = 123, normalized size = 1.76

$$-\frac{12\,b^3x^3+30\,ab^2x^2+22\,a^2bx+3\,a^3}{3\left(a^4b^3x^4+3\,a^5b^2x^3+3\,a^6bx^2+a^7x\right)}+\frac{4\,b\log(bx+a)}{a^5}-\frac{4\,b\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 30*a*b^2*x^2 + 22*a^2*b*x + 3*a^3)/(a^4*b^3*x^4 + 3*a^5*
b^2*x^3 + 3*a^6*b*x^2 + a^7*x) + 4*b*log(b*x + a)/a^5 - 4*b*log(x)/a^5
```

Fricas [B] time = 1.39027, size = 325, normalized size = 4.64

$$-\frac{12\,ab^3x^3+30\,a^2b^2x^2+22\,a^3bx+3\,a^4-12\left(b^4x^4+3\,ab^3x^3+3\,a^2b^2x^2+a^3bx\right)\log(bx+a)+12\left(b^4x^4+3\,ab^3x^3+3\,a^2b\right.}{3\left(a^5b^3x^4+3\,a^6b^2x^3+3\,a^7bx^2+a^8x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 30*a^2*b^2*x^2 + 22*a^3*b*x + 3*a^4 - 12*(b^4*x^4 + 3*
a*b^3*x^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(b*x + a) + 12*(b^4*x^4 + 3*a*b^3*x
^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(x))/(a^5*b^3*x^4 + 3*a^6*b^2*x^3 + 3*a^7*
b*x^2 + a^8*x)
```

Sympy [A] time = 0.793333, size = 88, normalized size = 1.26

$$-\frac{3a^3+22a^2bx+30ab^2x^2+12b^3x^3}{3a^7x+9a^6bx^2+9a^5b^2x^3+3a^4b^3x^4}+\frac{4b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**4,x)
```

```
[Out] -(3*a**3 + 22*a**2*b*x + 30*a*b**2*x**2 + 12*b**3*x**3)/(3*a**7*x + 9*a**6*
b*x**2 + 9*a**5*b**2*x**3 + 3*a**4*b**3*x**4) + 4*b*(-log(x) + log(a/b + x)
)/a**5
```

Giac [A] time = 1.1801, size = 96, normalized size = 1.37

$$\frac{4\,b\log\left(|bx+a|\right)}{a^5}-\frac{4\,b\log\left(|x|\right)}{a^5}-\frac{12\,ab^3x^3+30\,a^2b^2x^2+22\,a^3bx+3\,a^4}{3\,(bx+a)^3a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 4*b*log(abs(b*x + a))/a^5 - 4*b*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 30*a^
2*b^2*x^2 + 22*a^3*b*x + 3*a^4)/((b*x + a)^3*a^5*x)
```

3.204 $\int \frac{1}{x^3(a+bx)^4}\,dx$

Optimal. Leaf size=93

$$\frac{6b^2}{a^5(a+bx)}+\frac{3b^2}{2a^4(a+bx)^2}+\frac{b^2}{3a^3(a+bx)^3}+\frac{10b^2\log(x)}{a^6}-\frac{10b^2\log(a+bx)}{a^6}+\frac{4b}{a^5x}-\frac{1}{2a^4x^2}$$

```
[Out] -1/(2*a^4*x^2) + (4*b)/(a^5*x) + b^2/(3*a^3*(a + b*x)^3) + (3*b^2)/(2*a^4*(
a + b*x)^2) + (6*b^2)/(a^5*(a + b*x)) + (10*b^2*Log[x])/a^6 - (10*b^2*Log[a
 + b*x])/a^6
```

Rubi [A] time = 0.0481338, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{6b^2}{a^5(a+bx)}+\frac{3b^2}{2a^4(a+bx)^2}+\frac{b^2}{3a^3(a+bx)^3}+\frac{10b^2\log(x)}{a^6}-\frac{10b^2\log(a+bx)}{a^6}+\frac{4b}{a^5x}-\frac{1}{2a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^4),x]
```

```
[Out] -1/(2*a^4*x^2) + (4*b)/(a^5*x) + b^2/(3*a^3*(a + b*x)^3) + (3*b^2)/(2*a^4*(
a + b*x)^2) + (6*b^2)/(a^5*(a + b*x)) + (10*b^2*Log[x])/a^6 - (10*b^2*Log[a
 + b*x])/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^3}-\frac{4b}{a^5x^2}+\frac{10b^2}{a^6x}-\frac{b^3}{a^3(a+bx)^4}-\frac{3b^3}{a^4(a+bx)^3}-\frac{6b^3}{a^5(a+bx)^2}-\frac{10b^3}{a^6(a+bx)}\right)dx\\ &= -\frac{1}{2a^4x^2}+\frac{4b}{a^5x}+\frac{b^2}{3a^3(a+bx)^3}+\frac{3b^2}{2a^4(a+bx)^2}+\frac{6b^2}{a^5(a+bx)}+\frac{10b^2\log(x)}{a^6}-\frac{10b^2\log(a+bx)}{a^6}\end{aligned}$$

Mathematica [A] time = 0.10632, size = 79, normalized size = 0.85

$$\frac{\frac{a\left(110a^2b^2x^2+15a^3bx-3a^4+150ab^3x^3+60b^4x^4\right)}{x^2(a+bx)^3}-60b^2\log(a+bx)+60b^2\log(x)}{6a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^4),x]
```

```
[Out] ((a*(-3*a^4 + 15*a^3*b*x + 110*a^2*b^2*x^2 + 150*a*b^3*x^3 + 60*b^4*x^4))/(
x^2*(a + b*x)^3) + 60*b^2*Log[x] - 60*b^2*Log[a + b*x])/(6*a^6)
```

Maple [A] time = 0.01, size = 88, normalized size = 1.

$$-\frac{1}{2\,a^4x^2}+4\,\frac{b}{a^5x}+\frac{b^2}{3\,a^3\,(bx+a)^3}+\frac{3\,b^2}{2\,a^4\,(bx+a)^2}+6\,\frac{b^2}{a^5\,(bx+a)}+10\,\frac{b^2\ln(x)}{a^6}-10\,\frac{b^2\ln(bx+a)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^4,x)
```

```
[Out] -1/2/a^4/x^2+4*b/a^5/x+1/3*b^2/a^3/(b*x+a)^3+3/2*b^2/a^4/(b*x+a)^2+6*b^2/a^
5/(b*x+a)+10*b^2*ln(x)/a^6-10*b^2*ln(b*x+a)/a^6
```

Maxima [A] time = 1.04163, size = 146, normalized size = 1.57

$$\frac{60\,b^4x^4+150\,ab^3x^3+110\,a^2b^2x^2+15\,a^3bx-3\,a^4}{6\left(a^5b^3x^5+3\,a^6b^2x^4+3\,a^7bx^3+a^8x^2\right)}-\frac{10\,b^2\log(bx+a)}{a^6}+\frac{10\,b^2\log(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*b^4*x^4 + 150*a*b^3*x^3 + 110*a^2*b^2*x^2 + 15*a^3*b*x - 3*a^4)/(a^
5*b^3*x^5 + 3*a^6*b^2*x^4 + 3*a^7*b*x^3 + a^8*x^2) - 10*b^2*log(b*x + a)/a^
6 + 10*b^2*log(x)/a^6
```

Fricas [B] time = 1.61244, size = 363, normalized size = 3.9

$$\frac{60\,ab^4x^4+150\,a^2b^3x^3+110\,a^3b^2x^2+15\,a^4bx-3\,a^5-60\left(b^5x^5+3\,ab^4x^4+3\,a^2b^3x^3+a^3b^2x^2\right)\log(bx+a)+60\left(b^5x^5\right.}{6\left(a^6b^3x^5+3\,a^7b^2x^4+3\,a^8bx^3+a^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*a*b^4*x^4 + 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5
- 60*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(b*x + a) + 6
0*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(x))/(a^6*b^3*x^
5 + 3*a^7*b^2*x^4 + 3*a^8*b*x^3 + a^9*x^2)
```

Sympy [A] time = 0.8636, size = 104, normalized size = 1.12

$$\frac{-3a^4+15a^3bx+110a^2b^2x^2+150ab^3x^3+60b^4x^4}{6a^8x^2+18a^7bx^3+18a^6b^2x^4+6a^5b^3x^5}+\frac{10b^2\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**4,x)
```

```
[Out] (-3*a**4 + 15*a**3*b*x + 110*a**2*b**2*x**2 + 150*a*b**3*x**3 + 60*b**4*x**
4)/(6*a**8*x**2 + 18*a**7*b*x**3 + 18*a**6*b**2*x**4 + 6*a**5*b**3*x**5) +
10*b**2*(log(x) - log(a/b + x))/a**6
```

Giac [A] time = 1.19504, size = 116, normalized size = 1.25

$$-\frac{10\,b^2\log\left(|bx+a|\right)}{a^6}+\frac{10\,b^2\log\left(|x|\right)}{a^6}+\frac{60\,ab^4x^4+150\,a^2b^3x^3+110\,a^3b^2x^2+15\,a^4bx-3\,a^5}{6\,(bx+a)^3a^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -10*b^2*log(abs(b*x + a))/a^6 + 10*b^2*log(abs(x))/a^6 + 1/6*(60*a*b^4*x^4
+ 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5)/((b*x + a)^3*a^6*
x^2)
```

3.205 $\int \frac{1}{x^4(a+bx)^4} dx$

Optimal. Leaf size=102

$$-\frac{10b^3}{a^6(a+bx)}-\frac{2b^3}{a^5(a+bx)^2}-\frac{b^3}{3a^4(a+bx)^3}-\frac{10b^2}{a^6x}-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+bx)}{a^7}+\frac{2b}{a^5x^2}-\frac{1}{3a^4x^3}$$

```
[Out] -1/(3*a^4*x^3) + (2*b)/(a^5*x^2) - (10*b^2)/(a^6*x) - b^3/(3*a^4*(a + b*x)^
3) - (2*b^3)/(a^5*(a + b*x)^2) - (10*b^3)/(a^6*(a + b*x)) - (20*b^3*Log[x])
/a^7 + (20*b^3*Log[a + b*x])/a^7
```

Rubi [A] time = 0.0529369, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{10b^3}{a^6(a+bx)}-\frac{2b^3}{a^5(a+bx)^2}-\frac{b^3}{3a^4(a+bx)^3}-\frac{10b^2}{a^6x}-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+bx)}{a^7}+\frac{2b}{a^5x^2}-\frac{1}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^4),x]
```

```
[Out] -1/(3*a^4*x^3) + (2*b)/(a^5*x^2) - (10*b^2)/(a^6*x) - b^3/(3*a^4*(a + b*x)^
3) - (2*b^3)/(a^5*(a + b*x)^2) - (10*b^3)/(a^6*(a + b*x)) - (20*b^3*Log[x])
/a^7 + (20*b^3*Log[a + b*x])/a^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^4} dx &= \int \left(\frac{1}{a^4x^4}-\frac{4b}{a^5x^3}+\frac{10b^2}{a^6x^2}-\frac{20b^3}{a^7x}+\frac{b^4}{a^4(a+bx)^4}+\frac{4b^4}{a^5(a+bx)^3}+\frac{10b^4}{a^6(a+bx)^2}+\frac{20b^4}{a^7(a+bx)}\right) dx\\ &= -\frac{1}{3a^4x^3}+\frac{2b}{a^5x^2}-\frac{10b^2}{a^6x}-\frac{b^3}{3a^4(a+bx)^3}-\frac{2b^3}{a^5(a+bx)^2}-\frac{10b^3}{a^6(a+bx)}-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+}{a^7}\end{aligned}$$

Mathematica [A] time = 0.0965544, size = 88, normalized size = 0.86

$$-\frac{\frac{a\left(15a^3b^2x^2+110a^2b^3x^3-3a^4bx+a^5+150ab^4x^4+60b^5x^5\right)}{x^3(a+bx)^3}-60b^3\log(a+bx)+60b^3\log(x)}{3a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^4),x]
```

```
[Out] -((a*(a^5 - 3*a^4*b*x + 15*a^3*b^2*x^2 + 110*a^2*b^3*x^3 + 150*a*b^4*x^4 +
60*b^5*x^5))/(x^3*(a + b*x)^3) + 60*b^3*Log[x] - 60*b^3*Log[a + b*x])/(3*a^
```

```
7)
```

Maple [A] time = 0.012, size = 99, normalized size = 1.

$$-\frac{1}{3\,a^{4}x^{3}}+2\,\frac{b}{a^{5}x^{2}}-10\,\frac{b^{2}}{a^{6}x}-\frac{b^{3}}{3\,a^{4}\left(bx+a\right)^{3}}-2\,\frac{b^{3}}{a^{5}\left(bx+a\right)^{2}}-10\,\frac{b^{3}}{a^{6}\left(bx+a\right)}-20\,\frac{b^{3}\ln\left(x\right)}{a^{7}}+20\,\frac{b^{3}\ln\left(bx+a\right)}{a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^4,x)
```

```
[Out] -1/3/a^4/x^3+2*b/a^5/x^2-10*b^2/a^6/x-1/3*b^3/a^4/(b*x+a)^3-2*b^3/a^5/(b*x+
a)^2-10*b^3/a^6/(b*x+a)-20*b^3*ln(x)/a^7+20*b^3*ln(b*x+a)/a^7
```

Maxima [A] time = 1.06512, size = 158, normalized size = 1.55

$$-\frac{60\,b^{5}x^{5}+150\,ab^{4}x^{4}+110\,a^{2}b^{3}x^{3}+15\,a^{3}b^{2}x^{2}-3\,a^{4}bx+a^{5}}{3\left(a^{6}b^{3}x^{6}+3\,a^{7}b^{2}x^{5}+3\,a^{8}bx^{4}+a^{9}x^{3}\right)}+\frac{20\,b^{3}\log\left(bx+a\right)}{a^{7}}-\frac{20\,b^{3}\log\left(x\right)}{a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(60*b^5*x^5 + 150*a*b^4*x^4 + 110*a^2*b^3*x^3 + 15*a^3*b^2*x^2 - 3*a^4
*b*x + a^5)/(a^6*b^3*x^6 + 3*a^7*b^2*x^5 + 3*a^8*b*x^4 + a^9*x^3) + 20*b^3*
log(b*x + a)/a^7 - 20*b^3*log(x)/a^7
```

Fricas [A] time = 1.61725, size = 385, normalized size = 3.77

$$-\frac{60\,ab^{5}x^{5}+150\,a^{2}b^{4}x^{4}+110\,a^{3}b^{3}x^{3}+15\,a^{4}b^{2}x^{2}-3\,a^{5}bx+a^{6}-60\left(b^{6}x^{6}+3\,ab^{5}x^{5}+3\,a^{2}b^{4}x^{4}+a^{3}b^{3}x^{3}\right)\log\left(bx+a\right)+}{3\left(a^{7}b^{3}x^{6}+3\,a^{8}b^{2}x^{5}+3\,a^{9}bx^{4}+a^{10}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(60*a*b^5*x^5 + 150*a^2*b^4*x^4 + 110*a^3*b^3*x^3 + 15*a^4*b^2*x^2 - 3
*a^5*b*x + a^6 - 60*(b^6*x^6 + 3*a*b^5*x^5 + 3*a^2*b^4*x^4 + a^3*b^3*x^3)*l
og(b*x + a) + 60*(b^6*x^6 + 3*a*b^5*x^5 + 3*a^2*b^4*x^4 + a^3*b^3*x^3)*log(
x))/(a^7*b^3*x^6 + 3*a^8*b^2*x^5 + 3*a^9*b*x^4 + a^10*x^3)
```

Sympy [A] time = 1.00234, size = 114, normalized size = 1.12

$$-\frac{a^{5}-3a^{4}bx+15a^{3}b^{2}x^{2}+110a^{2}b^{3}x^{3}+150ab^{4}x^{4}+60b^{5}x^{5}}{3a^{9}x^{3}+9a^{8}bx^{4}+9a^{7}b^{2}x^{5}+3a^{6}b^{3}x^{6}}+\frac{20b^{3}\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**4,x)
```

```
[Out] -(a**5 - 3*a**4*b*x + 15*a**3*b**2*x**2 + 110*a**2*b**3*x**3 + 150*a*b**4*x
**4 + 60*b**5*x**5)/(3*a**9*x**3 + 9*a**8*b*x**4 + 9*a**7*b**2*x**5 + 3*a**
6*b**3*x**6) + 20*b**3*(-log(x) + log(a/b + x))/a**7
```

Giac [A] time = 1.22387, size = 126, normalized size = 1.24

$$\frac{20\,b^3\log\left(|bx+a|\right)}{a^7}-\frac{20\,b^3\log\left(|x|\right)}{a^7}-\frac{60\,b^5x^5+150\,ab^4x^4+110\,a^2b^3x^3+15\,a^3b^2x^2-3\,a^4bx+a^5}{3\left(bx^2+ax\right)^3a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 20*b^3*log(abs(b*x + a))/a^7 - 20*b^3*log(abs(x))/a^7 - 1/3*(60*b^5*x^5 + 1
50*a*b^4*x^4 + 110*a^2*b^3*x^3 + 15*a^3*b^2*x^2 - 3*a^4*b*x + a^5)/((b*x^2
+ a*x)^3*a^6)
```

3.206 $\int \frac{1}{x^5(a+bx)^4}\,dx$

Optimal. Leaf size=117

$$-\frac{5b^2}{a^6x^2}+\frac{15b^4}{a^7(a+bx)}+\frac{5b^4}{2a^6(a+bx)^2}+\frac{b^4}{3a^5(a+bx)^3}+\frac{20b^3}{a^7x}+\frac{35b^4\log(x)}{a^8}-\frac{35b^4\log(a+bx)}{a^8}+\frac{4b}{3a^5x^3}-\frac{1}{4a^4x^4}$$

```
[Out] -1/(4*a^4*x^4) + (4*b)/(3*a^5*x^3) - (5*b^2)/(a^6*x^2) + (20*b^3)/(a^7*x) +
 b^4/(3*a^5*(a + b*x)^3) + (5*b^4)/(2*a^6*(a + b*x)^2) + (15*b^4)/(a^7*(a +
 b*x)) + (35*b^4*Log[x])/a^8 - (35*b^4*Log[a + b*x])/a^8
```

Rubi [A] time = 0.0702539, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{5b^2}{a^6x^2}+\frac{15b^4}{a^7(a+bx)}+\frac{5b^4}{2a^6(a+bx)^2}+\frac{b^4}{3a^5(a+bx)^3}+\frac{20b^3}{a^7x}+\frac{35b^4\log(x)}{a^8}-\frac{35b^4\log(a+bx)}{a^8}+\frac{4b}{3a^5x^3}-\frac{1}{4a^4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^4),x]
```

```
[Out] -1/(4*a^4*x^4) + (4*b)/(3*a^5*x^3) - (5*b^2)/(a^6*x^2) + (20*b^3)/(a^7*x) +
 b^4/(3*a^5*(a + b*x)^3) + (5*b^4)/(2*a^6*(a + b*x)^2) + (15*b^4)/(a^7*(a +
 b*x)) + (35*b^4*Log[x])/a^8 - (35*b^4*Log[a + b*x])/a^8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^5}-\frac{4b}{a^5x^4}+\frac{10b^2}{a^6x^3}-\frac{20b^3}{a^7x^2}+\frac{35b^4}{a^8x}-\frac{b^5}{a^5(a+bx)^4}-\frac{5b^5}{a^6(a+bx)^3}-\frac{15b^5}{a^7(a+bx)^2}-\frac{35b^5}{a^8(a+bx)}\right) d\\ &= -\frac{1}{4a^4x^4}+\frac{4b}{3a^5x^3}-\frac{5b^2}{a^6x^2}+\frac{20b^3}{a^7x}+\frac{b^4}{3a^5(a+bx)^3}+\frac{5b^4}{2a^6(a+bx)^2}+\frac{15b^4}{a^7(a+bx)}+\frac{35b^4\log(x)}{a^8}-\frac{35b^4}{}\end{aligned}$$

Mathematica [A] time = 0.10412, size = 101, normalized size = 0.86

$$\frac{\frac{a\left(-21a^4b^2x^2+105a^3b^3x^3+770a^2b^4x^4+7a^5bx-3a^6+1050ab^5x^5+420b^6x^6\right)}{x^4(a+bx)^3}-420b^4\log(a+bx)+420b^4\log(x)}{12a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^4),x]
```

```
[Out] ((a*(-3*a^6 + 7*a^5*b*x - 21*a^4*b^2*x^2 + 105*a^3*b^3*x^3 + 770*a^2*b^4*x^
4 + 1050*a*b^5*x^5 + 420*b^6*x^6))/(x^4*(a + b*x)^3) + 420*b^4*Log[x] - 420
```

```
*b^4*Log[a + b*x])/(12*a^8)
```

Maple [A] time = 0.012, size = 110, normalized size = 0.9

$$-\frac{1}{4a^4x^4}+\frac{4b}{3a^5x^3}-5\frac{b^2}{a^6x^2}+20\frac{b^3}{a^7x}+\frac{b^4}{3a^5(bx+a)^3}+\frac{5b^4}{2a^6(bx+a)^2}+15\frac{b^4}{a^7(bx+a)}+35\frac{b^4\ln(x)}{a^8}-35\frac{b^4\ln(bx+}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^4,x)
```

```
[Out] -1/4/a^4/x^4+4/3*b/a^5/x^3-5*b^2/a^6/x^2+20*b^3/a^7/x+1/3*b^4/a^5/(b*x+a)^3
+5/2*b^4/a^6/(b*x+a)^2+15*b^4/a^7/(b*x+a)+35*b^4*ln(x)/a^8-35*b^4*ln(b*x+a)
/a^8
```

Maxima [A] time = 1.00662, size = 176, normalized size = 1.5

$$\frac{420b^6x^6+1050ab^5x^5+770a^2b^4x^4+105a^3b^3x^3-21a^4b^2x^2+7a^5bx-3a^6}{12\left(a^7b^3x^7+3a^8b^2x^6+3a^9bx^5+a^{10}x^4\right)}-\frac{35b^4\log(bx+a)}{a^8}+\frac{35b^4\log(x)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(420*b^6*x^6 + 1050*a*b^5*x^5 + 770*a^2*b^4*x^4 + 105*a^3*b^3*x^3 - 21
*a^4*b^2*x^2 + 7*a^5*b*x - 3*a^6)/(a^7*b^3*x^7 + 3*a^8*b^2*x^6 + 3*a^9*b*x^
5 + a^10*x^4) - 35*b^4*log(b*x + a)/a^8 + 35*b^4*log(x)/a^8
```

Fricas [A] time = 1.60078, size = 419, normalized size = 3.58

$$\frac{420ab^6x^6+1050a^2b^5x^5+770a^3b^4x^4+105a^4b^3x^3-21a^5b^2x^2+7a^6bx-3a^7-420\left(b^7x^7+3ab^6x^6+3a^2b^5x^5+a^3\right.}{12\left(a^8b^3x^7+3a^9b^2x^6+3a^{10}bx^5+a^{11}x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(420*a*b^6*x^6 + 1050*a^2*b^5*x^5 + 770*a^3*b^4*x^4 + 105*a^4*b^3*x^3
- 21*a^5*b^2*x^2 + 7*a^6*b*x - 3*a^7 - 420*(b^7*x^7 + 3*a*b^6*x^6 + 3*a^2*b
^5*x^5 + a^3*b^4*x^4)*log(b*x + a) + 420*(b^7*x^7 + 3*a*b^6*x^6 + 3*a^2*b^5
*x^5 + a^3*b^4*x^4)*log(x))/(a^8*b^3*x^7 + 3*a^9*b^2*x^6 + 3*a^10*b*x^5 + a
^11*x^4)
```

Sympy [A] time = 1.02134, size = 128, normalized size = 1.09

$$\frac{-3a^6+7a^5bx-21a^4b^2x^2+105a^3b^3x^3+770a^2b^4x^4+1050ab^5x^5+420b^6x^6}{12a^{10}x^4+36a^9bx^5+36a^8b^2x^6+12a^7b^3x^7}+\frac{35b^4\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a)**4,x)
```

```
[Out] (-3*a**6 + 7*a**5*b*x - 21*a**4*b**2*x**2 + 105*a**3*b**3*x**3 + 770*a**2*b
**4*x**4 + 1050*a*b**5*x**5 + 420*b**6*x**6)/(12*a**10*x**4 + 36*a**9*b*x**
5 + 36*a**8*b**2*x**6 + 12*a**7*b**3*x**7) + 35*b**4*(log(x) - log(a/b + x)
)/a**8
```

Giac [A] time = 1.16894, size = 146, normalized size = 1.25

$$-\frac{35\,b^4\log\left(|bx+a|\right)}{a^8}+\frac{35\,b^4\log\left(|x|\right)}{a^8}+\frac{420\,ab^6x^6+1050\,a^2b^5x^5+770\,a^3b^4x^4+105\,a^4b^3x^3-21\,a^5b^2x^2+7\,a^6bx-3\,a^7}{12\,(bx+a)^3a^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -35*b^4*log(abs(b*x + a))/a^8 + 35*b^4*log(abs(x))/a^8 + 1/12*(420*a*b^6*x^
6 + 1050*a^2*b^5*x^5 + 770*a^3*b^4*x^4 + 105*a^4*b^3*x^3 - 21*a^5*b^2*x^2 +
 7*a^6*b*x - 3*a^7)/((b*x + a)^3*a^8*x^4)
```

3.207 $\int \frac{x^{10}}{(a+bx)^7} dx$

Optimal. Leaf size=150

$$\frac{14a^2x^2}{b^9} - \frac{a^{10}}{6b^{11}(a+bx)^6} + \frac{2a^9}{b^{11}(a+bx)^5} - \frac{45a^8}{4b^{11}(a+bx)^4} + \frac{40a^7}{b^{11}(a+bx)^3} - \frac{105a^6}{b^{11}(a+bx)^2} + \frac{252a^5}{b^{11}(a+bx)} - \frac{84a^3x}{b^{10}} + \frac{210a}{}$$

```
[Out] (-84*a^3*x)/b^10 + (14*a^2*x^2)/b^9 - (7*a*x^3)/(3*b^8) + x^4/(4*b^7) - a^1
0/(6*b^11*(a + b*x)^6) + (2*a^9)/(b^11*(a + b*x)^5) - (45*a^8)/(4*b^11*(a +
 b*x)^4) + (40*a^7)/(b^11*(a + b*x)^3) - (105*a^6)/(b^11*(a + b*x)^2) + (25
2*a^5)/(b^11*(a + b*x)) + (210*a^4*Log[a + b*x])/b^11
```

Rubi [A] time = 0.13534, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{14a^2x^2}{b^9} - \frac{a^{10}}{6b^{11}(a+bx)^6} + \frac{2a^9}{b^{11}(a+bx)^5} - \frac{45a^8}{4b^{11}(a+bx)^4} + \frac{40a^7}{b^{11}(a+bx)^3} - \frac{105a^6}{b^{11}(a+bx)^2} + \frac{252a^5}{b^{11}(a+bx)} - \frac{84a^3x}{b^{10}} + \frac{210a}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x)^7,x]
```

```
[Out] (-84*a^3*x)/b^10 + (14*a^2*x^2)/b^9 - (7*a*x^3)/(3*b^8) + x^4/(4*b^7) - a^1
0/(6*b^11*(a + b*x)^6) + (2*a^9)/(b^11*(a + b*x)^5) - (45*a^8)/(4*b^11*(a +
 b*x)^4) + (40*a^7)/(b^11*(a + b*x)^3) - (105*a^6)/(b^11*(a + b*x)^2) + (25
2*a^5)/(b^11*(a + b*x)) + (210*a^4*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{(a+bx)^7} dx &= \int \left(-\frac{84a^3}{b^{10}} + \frac{28a^2x}{b^9} - \frac{7ax^2}{b^8} + \frac{x^3}{b^7} + \frac{a^{10}}{b^{10}(a+bx)^7} - \frac{10a^9}{b^{10}(a+bx)^6} + \frac{45a^8}{b^{10}(a+bx)^5} - \frac{120a^7}{b^{10}(a+bx)^4} + \frac{}{b} \right.\\ &= -\frac{84a^3x}{b^{10}} + \frac{14a^2x^2}{b^9} - \frac{7ax^3}{3b^8} + \frac{x^4}{4b^7} - \frac{a^{10}}{6b^{11}(a+bx)^6} + \frac{2a^9}{b^{11}(a+bx)^5} - \frac{45a^8}{4b^{11}(a+bx)^4} + \frac{40a^7}{b^{11}(a+bx)^3} - \end{aligned}$$

Mathematica [A] time = 0.0355748, size = 139, normalized size = 0.93

$$\frac{18105a^8b^2x^2 + 11540a^7b^3x^3 - 3945a^6b^4x^4 - 9138a^5b^5x^5 - 4043a^4b^6x^6 - 360a^3b^7x^7 + 45a^2b^8x^8 + 10266a^9bx + 2520a}{12b^{11}(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x)^7,x]
```

```
[Out] (2131*a^10 + 10266*a^9*b*x + 18105*a^8*b^2*x^2 + 11540*a^7*b^3*x^3 - 3945*a
^6*b^4*x^4 - 9138*a^5*b^5*x^5 - 4043*a^4*b^6*x^6 - 360*a^3*b^7*x^7 + 45*a^2
*b^8*x^8 - 10*a*b^9*x^9 + 3*b^10*x^10 + 2520*a^4*(a + b*x)^6*Log[a + b*x])/
(12*b^11*(a + b*x)^6)
```

Maple [A] time = 0.012, size = 143, normalized size = 1.

$$-84\,{\frac{{a}^{3}x}{{b}^{10}}}+14\,{\frac{{a}^{2}{x}^{2}}{{b}^{9}}}-{\frac{7\,a{x}^{3}}{3\,{b}^{8}}}+{\frac{{x}^{4}}{4\,{b}^{7}}}-{\frac{{a}^{10}}{6\,{b}^{11} \left(bx+a \right) ^{6}}}+2\,{\frac{{a}^{9}}{{b}^{11} \left(bx+a \right) ^{5}}}-{\frac{45\,{a}^{8}}{4\,{b}^{11} \left(bx+a \right) ^{4}}}+40\,{\frac{{a}^{7}}{{b}^{11} \left(bx+a \right) ^{3}}}-105\,{\frac{{a}^{6}}{{b}^{11} \left(bx+ \right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x+a)^7,x)
```

```
[Out] -84*a^3*x/b^10+14*a^2*x^2/b^9-7/3*a*x^3/b^8+1/4*x^4/b^7-1/6*a^10/b^11/(b*x+
a)^6+2*a^9/b^11/(b*x+a)^5-45/4*a^8/b^11/(b*x+a)^4+40*a^7/b^11/(b*x+a)^3-105
*a^6/b^11/(b*x+a)^2+252*a^5/b^11/(b*x+a)+210*a^4*ln(b*x+a)/b^11
```

Maxima [A] time = 1.08163, size = 243, normalized size = 1.62

$$\frac{3024\,a^{5}b^{5}x^{5}+13860\,a^{6}b^{4}x^{4}+25680\,a^{7}b^{3}x^{3}+23985\,a^{8}b^{2}x^{2}+11274\,a^{9}bx+2131\,a^{10}}{12\left(b^{17}x^{6}+6\,ab^{16}x^{5}+15\,a^{2}b^{15}x^{4}+20\,a^{3}b^{14}x^{3}+15\,a^{4}b^{13}x^{2}+6\,a^{5}b^{12}x+a^{6}b^{11}\right)}+\frac{210\,a^{4}\log\left(bx+a\right)}{b^{11}}+\frac{3\,b^{3}x^{4}-28\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/12*(3024*a^5*b^5*x^5 + 13860*a^6*b^4*x^4 + 25680*a^7*b^3*x^3 + 23985*a^8*
b^2*x^2 + 11274*a^9*b*x + 2131*a^10)/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15
*x^4 + 20*a^3*b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11) + 210*a
^4*log(b*x + a)/b^11 + 1/12*(3*b^3*x^4 - 28*a*b^2*x^3 + 168*a^2*b*x^2 - 100
8*a^3*x)/b^10
```

Fricas [A] time = 1.45993, size = 568, normalized size = 3.79

$$\frac{3\,b^{10}x^{10}-10\,ab^{9}x^{9}+45\,a^{2}b^{8}x^{8}-360\,a^{3}b^{7}x^{7}-4043\,a^{4}b^{6}x^{6}-9138\,a^{5}b^{5}x^{5}-3945\,a^{6}b^{4}x^{4}+11540\,a^{7}b^{3}x^{3}+18105\,a^{8}b^{2}x^{2}\cdots}{12\left(b^{17}x^{6}+6\,ab^{16}x^{5}+15\,a^{2}b^{15}x^{4}+20\,a^{3}b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*x^10 - 10*a*b^9*x^9 + 45*a^2*b^8*x^8 - 360*a^3*b^7*x^7 - 4043*
a^4*b^6*x^6 - 9138*a^5*b^5*x^5 - 3945*a^6*b^4*x^4 + 11540*a^7*b^3*x^3 + 181
05*a^8*b^2*x^2 + 10266*a^9*b*x + 2131*a^10 + 2520*(a^4*b^6*x^6 + 6*a^5*b^5*
x^5 + 15*a^6*b^4*x^4 + 20*a^7*b^3*x^3 + 15*a^8*b^2*x^2 + 6*a^9*b*x + a^10)*
log(b*x + a))/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15*x^4 + 20*a^3*b^14*x^3
+ 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11)
```

Sympy [A] time = 1.40053, size = 190, normalized size = 1.27

$$\frac{210a^4\log(a+bx)}{b^{11}} - \frac{84a^3x}{b^{10}} + \frac{14a^2x^2}{b^9} - \frac{7ax^3}{3b^8} + \frac{2131a^{10}+11274a^9bx+23985a^8b^2x^2+25680a^7b^3x^3+13860a^6b^4x^4}{12a^6b^{11}+72a^5b^{12}x+180a^4b^{13}x^2+240a^3b^{14}x^3+180a^2b^{15}x^4+72ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x+a)**7,x)
```

```
[Out] 210*a**4*log(a + b*x)/b**11 - 84*a**3*x/b**10 + 14*a**2*x**2/b**9 - 7*a*x**
3/(3*b**8) + (2131*a**10 + 11274*a**9*b*x + 23985*a**8*b**2*x**2 + 25680*a*
*7*b**3*x**3 + 13860*a**6*b**4*x**4 + 3024*a**5*b**5*x**5)/(12*a**6*b**11 +
 72*a**5*b**12*x + 180*a**4*b**13*x**2 + 240*a**3*b**14*x**3 + 180*a**2*b**
15*x**4 + 72*a*b**16*x**5 + 12*b**17*x**6) + x**4/(4*b**7)
```

Giac [A] time = 1.18732, size = 173, normalized size = 1.15

$$\frac{210\,a^4\log(|bx+a|)}{b^{11}} + \frac{3024\,a^5b^5x^5+13860\,a^6b^4x^4+25680\,a^7b^3x^3+23985\,a^8b^2x^2+11274\,a^9bx+2131\,a^{10}}{12\,(bx+a)^6b^{11}} + \frac{3\,b^{21}x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 210*a^4*log(abs(b*x + a))/b^11 + 1/12*(3024*a^5*b^5*x^5 + 13860*a^6*b^4*x^4
 + 25680*a^7*b^3*x^3 + 23985*a^8*b^2*x^2 + 11274*a^9*b*x + 2131*a^10)/((b*x
 + a)^6*b^11) + 1/12*(3*b^21*x^4 - 28*a*b^20*x^3 + 168*a^2*b^19*x^2 - 1008*
a^3*b^18*x)/b^28
```

3.208 $\int \frac{x^9}{(a+bx)^7}\,dx$

Optimal. Leaf size=139

$$\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{63a^5}{b^{10}(a+bx)^2}-\frac{126a^4}{b^{10}(a+bx)}+\frac{28a^2x}{b^9}-\frac{84a^3\log(a+bx)}{b^{10}}-$$

```
[Out] (28*a^2*x)/b^9 - (7*a*x^2)/(2*b^8) + x^3/(3*b^7) + a^9/(6*b^10*(a + b*x)^6)
 - (9*a^8)/(5*b^10*(a + b*x)^5) + (9*a^7)/(b^10*(a + b*x)^4) - (28*a^6)/(b^
10*(a + b*x)^3) + (63*a^5)/(b^10*(a + b*x)^2) - (126*a^4)/(b^10*(a + b*x))
- (84*a^3*Log[a + b*x])/b^10
```

Rubi [A] time = 0.113718, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{63a^5}{b^{10}(a+bx)^2}-\frac{126a^4}{b^{10}(a+bx)}+\frac{28a^2x}{b^9}-\frac{84a^3\log(a+bx)}{b^{10}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x)^7,x]
```

```
[Out] (28*a^2*x)/b^9 - (7*a*x^2)/(2*b^8) + x^3/(3*b^7) + a^9/(6*b^10*(a + b*x)^6)
 - (9*a^8)/(5*b^10*(a + b*x)^5) + (9*a^7)/(b^10*(a + b*x)^4) - (28*a^6)/(b^
10*(a + b*x)^3) + (63*a^5)/(b^10*(a + b*x)^2) - (126*a^4)/(b^10*(a + b*x))
- (84*a^3*Log[a + b*x])/b^10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(a+bx)^7}\,dx &= \int \left(\frac{28a^2}{b^9}-\frac{7ax}{b^8}+\frac{x^2}{b^7}-\frac{a^9}{b^9(a+bx)^7}+\frac{9a^8}{b^9(a+bx)^6}-\frac{36a^7}{b^9(a+bx)^5}+\frac{84a^6}{b^9(a+bx)^4}-\frac{126a^5}{b^9(a+bx)^3}+\frac{12\ldots}{b^9(a+\ldots}\right.\\ &= \frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}+\frac{x^3}{3b^7}+\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{63a^5}{b^{10}(a+bx)^2}-\end{aligned}$$

Mathematica [A] time = 0.0488204, size = 128, normalized size = 0.92

$$-\frac{23775a^7b^2x^2+19100a^6b^3x^3+1725a^5b^4x^4-6870a^4b^5x^5-3665a^3b^6x^6-360a^2b^7x^7+12534a^8bx+2520a^3(a+bx)^6\,\text{lo}\ldots}{30b^{10}(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x)^7,x]
```

```
[Out] -(2509*a^9 + 12534*a^8*b*x + 23775*a^7*b^2*x^2 + 19100*a^6*b^3*x^3 + 1725*a
^5*b^4*x^4 - 6870*a^4*b^5*x^5 - 3665*a^3*b^6*x^6 - 360*a^2*b^7*x^7 + 45*a*b
^8*x^8 - 10*b^9*x^9 + 2520*a^3*(a + b*x)^6*Log[a + b*x])/(30*b^10*(a + b*x)
^6)
```

Maple [A] time = 0.008, size = 132, normalized size = 1.

$$28\,\frac{a^2x}{b^9}-\frac{7\,ax^2}{2\,b^8}+\frac{x^3}{3\,b^7}+\frac{a^9}{6\,b^{10}\,(bx+a)^6}-\frac{9\,a^8}{5\,b^{10}\,(bx+a)^5}+9\,\frac{a^7}{b^{10}\,(bx+a)^4}-28\,\frac{a^6}{b^{10}\,(bx+a)^3}+63\,\frac{a^5}{b^{10}\,(bx+a)^2}-126$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x+a)^7,x)
```

```
[Out] 28*a^2*x/b^9-7/2*a*x^2/b^8+1/3*x^3/b^7+1/6*a^9/b^10/(b*x+a)^6-9/5*a^8/b^10/
(b*x+a)^5+9*a^7/b^10/(b*x+a)^4-28*a^6/b^10/(b*x+a)^3+63*a^5/b^10/(b*x+a)^2-
126*a^4/b^10/(b*x+a)-84*a^3*ln(b*x+a)/b^10
```

Maxima [A] time = 1.08471, size = 228, normalized size = 1.64

$$-\frac{3780\,a^4b^5x^5+17010\,a^5b^4x^4+31080\,a^6b^3x^3+28710\,a^7b^2x^2+13374\,a^8bx+2509\,a^9}{30\left(b^{16}x^6+6\,ab^{15}x^5+15\,a^2b^{14}x^4+20\,a^3b^{13}x^3+15\,a^4b^{12}x^2+6\,a^5b^{11}x+a^6b^{10}\right)}-\frac{84\,a^3\log\left(bx+a\right)}{b^{10}}+\frac{2\,b^2x^3-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(3780*a^4*b^5*x^5 + 17010*a^5*b^4*x^4 + 31080*a^6*b^3*x^3 + 28710*a^7
*b^2*x^2 + 13374*a^8*b*x + 2509*a^9)/(b^16*x^6 + 6*a*b^15*x^5 + 15*a^2*b^14
*x^4 + 20*a^3*b^13*x^3 + 15*a^4*b^12*x^2 + 6*a^5*b^11*x + a^6*b^10) - 84*a^
3*log(b*x + a)/b^10 + 1/6*(2*b^2*x^3 - 21*a*b*x^2 + 168*a^2*x)/b^9
```

Fricas [A] time = 1.43355, size = 541, normalized size = 3.89

$$\frac{10\,b^9x^9-45\,ab^8x^8+360\,a^2b^7x^7+3665\,a^3b^6x^6+6870\,a^4b^5x^5-1725\,a^5b^4x^4-19100\,a^6b^3x^3-23775\,a^7b^2x^2-12534\,a}{30\left(b^{16}x^6+6\,ab^{15}x^5+15\,a^2b^{14}x^4+20\,a^3b^{13}x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^9*x^9 - 45*a*b^8*x^8 + 360*a^2*b^7*x^7 + 3665*a^3*b^6*x^6 + 6870
*a^4*b^5*x^5 - 1725*a^5*b^4*x^4 - 19100*a^6*b^3*x^3 - 23775*a^7*b^2*x^2 - 1
2534*a^8*b*x - 2509*a^9 - 2520*(a^3*b^6*x^6 + 6*a^4*b^5*x^5 + 15*a^5*b^4*x^
4 + 20*a^6*b^3*x^3 + 15*a^7*b^2*x^2 + 6*a^8*b*x + a^9)*log(b*x + a))/(b^16*
x^6 + 6*a*b^15*x^5 + 15*a^2*b^14*x^4 + 20*a^3*b^13*x^3 + 15*a^4*b^12*x^2 +
6*a^5*b^11*x + a^6*b^10)
```

Sympy [A] time = 1.40578, size = 178, normalized size = 1.28

$$-\frac{84a^3\log(a+bx)}{b^{10}}+\frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}-\frac{2509a^9+13374a^8bx+28710a^7b^2x^2+31080a^6b^3x^3+17010a^5b^4x^4+3780a^4b^5x^5}{30a^6b^{10}+180a^5b^{11}x+450a^4b^{12}x^2+600a^3b^{13}x^3+450a^2b^{14}x^4+180ab^{15}x^5+30b^{16}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x+a)**7,x)
```

```
[Out] -84*a**3*log(a + b*x)/b**10 + 28*a**2*x/b**9 - 7*a*x**2/(2*b**8) - (2509*a*
*9 + 13374*a**8*b*x + 28710*a**7*b**2*x**2 + 31080*a**6*b**3*x**3 + 17010*a
**5*b**4*x**4 + 3780*a**4*b**5*x**5)/(30*a**6*b**10 + 180*a**5*b**11*x + 45
0*a**4*b**12*x**2 + 600*a**3*b**13*x**3 + 450*a**2*b**14*x**4 + 180*a*b**15
*x**5 + 30*b**16*x**6) + x**3/(3*b**7)
```

Giac [A] time = 1.21574, size = 158, normalized size = 1.14

$$-\frac{84\,a^3\log(|bx+a|)}{b^{10}}-\frac{3780\,a^4b^5x^5+17010\,a^5b^4x^4+31080\,a^6b^3x^3+28710\,a^7b^2x^2+13374\,a^8bx+2509\,a^9}{30\,(bx+a)^6b^{10}}+\frac{2\,b^{14}x^3-21\,ab^{13}x^2+168\,a^2b^{12}x}{6\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -84*a^3*log(abs(b*x + a))/b^10 - 1/30*(3780*a^4*b^5*x^5 + 17010*a^5*b^4*x^4
 + 31080*a^6*b^3*x^3 + 28710*a^7*b^2*x^2 + 13374*a^8*b*x + 2509*a^9)/((b*x
+ a)^6*b^10) + 1/6*(2*b^14*x^3 - 21*a*b^13*x^2 + 168*a^2*b^12*x)/b^21
```

3.209 $\int \frac{x^8}{(a+bx)^7} dx$

Optimal. Leaf size=128

$$-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{56a^3}{b^9(a+bx)}+\frac{28a^2\log(a+bx)}{b^9}-\frac{7ax}{b^8}+$$

```
[Out] (-7*a*x)/b^8 + x^2/(2*b^7) - a^8/(6*b^9*(a + b*x)^6) + (8*a^7)/(5*b^9*(a +
b*x)^5) - (7*a^6)/(b^9*(a + b*x)^4) + (56*a^5)/(3*b^9*(a + b*x)^3) - (35*a^
4)/(b^9*(a + b*x)^2) + (56*a^3)/(b^9*(a + b*x)) + (28*a^2*Log[a + b*x])/b^9
```

Rubi [A] time = 0.0889429, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{56a^3}{b^9(a+bx)}+\frac{28a^2\log(a+bx)}{b^9}-\frac{7ax}{b^8}+$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^7,x]
```

```
[Out] (-7*a*x)/b^8 + x^2/(2*b^7) - a^8/(6*b^9*(a + b*x)^6) + (8*a^7)/(5*b^9*(a +
b*x)^5) - (7*a^6)/(b^9*(a + b*x)^4) + (56*a^5)/(3*b^9*(a + b*x)^3) - (35*a^
4)/(b^9*(a + b*x)^2) + (56*a^3)/(b^9*(a + b*x)) + (28*a^2*Log[a + b*x])/b^9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^8}{(a+bx)^7}dx=\int \left(-\frac{7a}{b^8}+\frac{x}{b^7}+\frac{a^8}{b^8(a+bx)^7}-\frac{8a^7}{b^8(a+bx)^6}+\frac{28a^6}{b^8(a+bx)^5}-\frac{56a^5}{b^8(a+bx)^4}+\frac{70a^4}{b^8(a+bx)^3}-\frac{56a^3}{b^8(a+bx)}\right.$$
$$=-\frac{7ax}{b^8}+\frac{x^2}{2b^7}-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{56a^3}{b^9(a+b}$$

Mathematica [A] time = 0.0718687, size = 104, normalized size = 0.81

$$\frac{-\frac{5a^8}{(a+bx)^6}+\frac{48a^7}{(a+bx)^5}-\frac{210a^6}{(a+bx)^4}+\frac{560a^5}{(a+bx)^3}-\frac{1050a^4}{(a+bx)^2}+\frac{1680a^3}{a+bx}+840a^2\log(a+bx)-210abx+15b^2x^2}{30b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^7,x]
```

```
[Out] (-210*a*b*x + 15*b^2*x^2 - (5*a^8)/(a + b*x)^6 + (48*a^7)/(a + b*x)^5 - (21
0*a^6)/(a + b*x)^4 + (560*a^5)/(a + b*x)^3 - (1050*a^4)/(a + b*x)^2 + (1680
```

```
*a^3)/(a + b*x) + 840*a^2*Log[a + b*x])/(30*b^9)
```

Maple [A] time = 0.01, size = 121, normalized size = 1.

$$-7\,\frac{ax}{b^8}+\frac{x^2}{2\,b^7}-\frac{a^8}{6\,b^9\,(bx+a)^6}+\frac{8\,a^7}{5\,b^9\,(bx+a)^5}-7\,\frac{a^6}{b^9\,(bx+a)^4}+\frac{56\,a^5}{3\,b^9\,(bx+a)^3}-35\,\frac{a^4}{b^9\,(bx+a)^2}+56\,\frac{a^3}{b^9\,(bx+a)}+28\,\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^7,x)
```

```
[Out] -7*a*x/b^8+1/2*x^2/b^7-1/6*a^8/b^9/(b*x+a)^6+8/5*a^7/b^9/(b*x+a)^5-7*a^6/b^
9/(b*x+a)^4+56/3*a^5/b^9/(b*x+a)^3-35*a^4/b^9/(b*x+a)^2+56*a^3/b^9/(b*x+a)+
28*a^2*ln(b*x+a)/b^9
```

Maxima [A] time = 1.08347, size = 212, normalized size = 1.66

$$\frac{1680\,a^3b^5x^5+7350\,a^4b^4x^4+13160\,a^5b^3x^3+11970\,a^6b^2x^2+5508\,a^7bx+1023\,a^8}{30\left(b^{15}x^6+6\,ab^{14}x^5+15\,a^2b^{13}x^4+20\,a^3b^{12}x^3+15\,a^4b^{11}x^2+6\,a^5b^{10}x+a^6b^9\right)}+\frac{28\,a^2\log(bx+a)}{b^9}+\frac{bx^2-14\,ax}{2\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/30*(1680*a^3*b^5*x^5 + 7350*a^4*b^4*x^4 + 13160*a^5*b^3*x^3 + 11970*a^6*b
^2*x^2 + 5508*a^7*b*x + 1023*a^8)/(b^15*x^6 + 6*a*b^14*x^5 + 15*a^2*b^13*x^
4 + 20*a^3*b^12*x^3 + 15*a^4*b^11*x^2 + 6*a^5*b^10*x + a^6*b^9) + 28*a^2*lo
g(b*x + a)/b^9 + 1/2*(b*x^2 - 14*a*x)/b^8
```

Fricas [A] time = 1.47469, size = 514, normalized size = 4.02

$$\frac{15\,b^8x^8-120\,ab^7x^7-1035\,a^2b^6x^6-1170\,a^3b^5x^5+3375\,a^4b^4x^4+10100\,a^5b^3x^3+10725\,a^6b^2x^2+5298\,a^7bx+1023\,a^8+}{30\left(b^{15}x^6+6\,ab^{14}x^5+15\,a^2b^{13}x^4+20\,a^3b^{12}x^3+15\,a^4b^1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(15*b^8*x^8 - 120*a*b^7*x^7 - 1035*a^2*b^6*x^6 - 1170*a^3*b^5*x^5 + 33
75*a^4*b^4*x^4 + 10100*a^5*b^3*x^3 + 10725*a^6*b^2*x^2 + 5298*a^7*b*x + 102
3*a^8 + 840*(a^2*b^6*x^6 + 6*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 20*a^5*b^3*x^3
+ 15*a^6*b^2*x^2 + 6*a^7*b*x + a^8)*log(b*x + a))/(b^15*x^6 + 6*a*b^14*x^5
+ 15*a^2*b^13*x^4 + 20*a^3*b^12*x^3 + 15*a^4*b^11*x^2 + 6*a^5*b^10*x + a^6*
b^9)
```

Sympy [A] time = 1.3054, size = 165, normalized size = 1.29

$$\frac{28a^2\log(a+bx)}{b^9}-\frac{7ax}{b^8}+\frac{1023a^8+5508a^7bx+11970a^6b^2x^2+13160a^5b^3x^3+7350a^4b^4x^4+1680a^3b^5x^5}{30a^6b^9+180a^5b^{10}x+450a^4b^{11}x^2+600a^3b^{12}x^3+450a^2b^{13}x^4+180ab^{14}x^5+30b^{15}x^6}+\frac{x^2}{2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**7,x)
```

```
[Out] 28*a**2*log(a + b*x)/b**9 - 7*a*x/b**8 + (1023*a**8 + 5508*a**7*b*x + 11970
*a**6*b**2*x**2 + 13160*a**5*b**3*x**3 + 7350*a**4*b**4*x**4 + 1680*a**3*b*
*5*x**5)/(30*a**6*b**9 + 180*a**5*b**10*x + 450*a**4*b**11*x**2 + 600*a**3*
b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*b**14*x**5 + 30*b**15*x**6) + x**2
/(2*b**7)
```

Giac [A] time = 1.26158, size = 142, normalized size = 1.11

$$\frac{28\,a^2\log\left(|bx+a|\right)}{b^9}+\frac{b^7x^2-14\,ab^6x}{2\,b^{14}}+\frac{1680\,a^3b^5x^5+7350\,a^4b^4x^4+13160\,a^5b^3x^3+11970\,a^6b^2x^2+5508\,a^7bx+1023}{30\,(bx+a)^6b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 28*a^2*log(abs(b*x + a))/b^9 + 1/2*(b^7*x^2 - 14*a*b^6*x)/b^14 + 1/30*(1680
*a^3*b^5*x^5 + 7350*a^4*b^4*x^4 + 13160*a^5*b^3*x^3 + 11970*a^6*b^2*x^2 + 5
508*a^7*b*x + 1023*a^8)/((b*x + a)^6*b^9)
```

3.210 $\int \frac{x^7}{(a+bx)^7} dx$

Optimal. Leaf size=118

$$\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+bx)}-\frac{7a\log(a+bx)}{b^8}+\frac{x}{b^7}$$

```
[Out] x/b^7 + a^7/(6*b^8*(a + b*x)^6) - (7*a^6)/(5*b^8*(a + b*x)^5) + (21*a^5)/(4
*b^8*(a + b*x)^4) - (35*a^4)/(3*b^8*(a + b*x)^3) + (35*a^3)/(2*b^8*(a + b*x
)^2) - (21*a^2)/(b^8*(a + b*x)) - (7*a*Log[a + b*x])/b^8
```

Rubi [A] time = 0.0705354, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+bx)}-\frac{7a\log(a+bx)}{b^8}+\frac{x}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^7,x]
```

```
[Out] x/b^7 + a^7/(6*b^8*(a + b*x)^6) - (7*a^6)/(5*b^8*(a + b*x)^5) + (21*a^5)/(4
*b^8*(a + b*x)^4) - (35*a^4)/(3*b^8*(a + b*x)^3) + (35*a^3)/(2*b^8*(a + b*x
)^2) - (21*a^2)/(b^8*(a + b*x)) - (7*a*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^7}dx &= \int \left(\frac{1}{b^7}-\frac{a^7}{b^7(a+bx)^7}+\frac{7a^6}{b^7(a+bx)^6}-\frac{21a^5}{b^7(a+bx)^5}+\frac{35a^4}{b^7(a+bx)^4}-\frac{35a^3}{b^7(a+bx)^3}+\frac{21a^2}{b^7(a+bx)^2}-\frac{7a}{b^7(a+\ldots}\right.\\ &= \frac{x}{b^7}+\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+bx)}-\frac{7a\log\ldots}{}\end{aligned}$$

Mathematica [A] time = 0.0485582, size = 104, normalized size = 0.88

$$-\frac{7725a^5b^2x^2+8200a^4b^3x^3+4050a^3b^4x^4+360a^2b^5x^5+3594a^6bx+669a^7-360ab^6x^6+420a(a+bx)^6\log(a+bx)-6\ldots}{60b^8(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^7,x]
```

```
[Out] -(669*a^7 + 3594*a^6*b*x + 7725*a^5*b^2*x^2 + 8200*a^4*b^3*x^3 + 4050*a^3*b
^4*x^4 + 360*a^2*b^5*x^5 - 360*a*b^6*x^6 - 60*b^7*x^7 + 420*a*(a + b*x)^6*L
```

```
og[a + b*x])/(60*b^8*(a + b*x)^6)
```

Maple [A] time = 0.009, size = 109, normalized size = 0.9

$$\frac{x}{b^7}+\frac{a^7}{6\,b^8\,(bx+a)^6}-\frac{7\,a^6}{5\,b^8\,(bx+a)^5}+\frac{21\,a^5}{4\,b^8\,(bx+a)^4}-\frac{35\,a^4}{3\,b^8\,(bx+a)^3}+\frac{35\,a^3}{2\,b^8\,(bx+a)^2}-21\,\frac{a^2}{b^8\,(bx+a)}-7\,\frac{a\ln(bx+a}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^7,x)
```

```
[Out] x/b^7+1/6*a^7/b^8/(b*x+a)^6-7/5*a^6/b^8/(b*x+a)^5+21/4*a^5/b^8/(b*x+a)^4-35
/3*a^4/b^8/(b*x+a)^3+35/2*a^3/b^8/(b*x+a)^2-21*a^2/b^8/(b*x+a)-7*a*ln(b*x+a
)/b^8
```

Maxima [A] time = 1.08428, size = 196, normalized size = 1.66

$$-\frac{1260\,a^2b^5x^5+5250\,a^3b^4x^4+9100\,a^4b^3x^3+8085\,a^5b^2x^2+3654\,a^6bx+669\,a^7}{60\left(b^{14}x^6+6\,ab^{13}x^5+15\,a^2b^{12}x^4+20\,a^3b^{11}x^3+15\,a^4b^{10}x^2+6\,a^5b^9x+a^6b^8\right)}+\frac{x}{b^7}-\frac{7\,a\log(bx+a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(1260*a^2*b^5*x^5 + 5250*a^3*b^4*x^4 + 9100*a^4*b^3*x^3 + 8085*a^5*b^
2*x^2 + 3654*a^6*b*x + 669*a^7)/(b^14*x^6 + 6*a*b^13*x^5 + 15*a^2*b^12*x^4
+ 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a^6*b^8) + x/b^7 - 7*a*
log(b*x + a)/b^8
```

Fricas [A] time = 1.55993, size = 479, normalized size = 4.06

$$\frac{60\,b^7x^7+360\,ab^6x^6-360\,a^2b^5x^5-4050\,a^3b^4x^4-8200\,a^4b^3x^3-7725\,a^5b^2x^2-3594\,a^6bx-669\,a^7-420\left(ab^6x^6+6\,a\right.}{60\left(b^{14}x^6+6\,ab^{13}x^5+15\,a^2b^{12}x^4+20\,a^3b^{11}x^3+15\,a^4b^{10}x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*x^7 + 360*a*b^6*x^6 - 360*a^2*b^5*x^5 - 4050*a^3*b^4*x^4 - 820
0*a^4*b^3*x^3 - 7725*a^5*b^2*x^2 - 3594*a^6*b*x - 669*a^7 - 420*(a*b^6*x^6
+ 6*a^2*b^5*x^5 + 15*a^3*b^4*x^4 + 20*a^4*b^3*x^3 + 15*a^5*b^2*x^2 + 6*a^6*
b*x + a^7)*log(b*x + a))/(b^14*x^6 + 6*a*b^13*x^5 + 15*a^2*b^12*x^4 + 20*a^
3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a^6*b^8)
```

Sympy [A] time = 1.22954, size = 151, normalized size = 1.28

$$-\frac{7a\log(a+bx)}{b^8}-\frac{669a^7+3654a^6bx+8085a^5b^2x^2+9100a^4b^3x^3+5250a^3b^4x^4+1260a^2b^5x^5}{60a^6b^8+360a^5b^9x+900a^4b^{10}x^2+1200a^3b^{11}x^3+900a^2b^{12}x^4+360ab^{13}x^5+60b^{14}x^6}+\frac{x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**7,x)
```

```
[Out] -7*a*log(a + b*x)/b**8 - (669*a**7 + 3654*a**6*b*x + 8085*a**5*b**2*x**2 +
9100*a**4*b**3*x**3 + 5250*a**3*b**4*x**4 + 1260*a**2*b**5*x**5)/(60*a**6*b
**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a*
*2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) + x/b**7
```

Giac [A] time = 1.19203, size = 119, normalized size = 1.01

$$\frac{x}{b^7} - \frac{7\,a\log\left(|bx + a|\right)}{b^8} - \frac{1260\,a^2b^5x^5 + 5250\,a^3b^4x^4 + 9100\,a^4b^3x^3 + 8085\,a^5b^2x^2 + 3654\,a^6bx + 669\,a^7}{60\,(bx + a)^6b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] x/b^7 - 7*a*log(abs(b*x + a))/b^8 - 1/60*(1260*a^2*b^5*x^5 + 5250*a^3*b^4*x
^4 + 9100*a^4*b^3*x^3 + 8085*a^5*b^2*x^2 + 3654*a^6*b*x + 669*a^7)/((b*x +
a)^6*b^8)
```

3.211 $\int \frac{x^6}{(a+bx)^7} \, dx$

Optimal. Leaf size=109

$$-\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{\log(a+bx)}{b^7}$$

```
[Out] -a^6/(6*b^7*(a + b*x)^6) + (6*a^5)/(5*b^7*(a + b*x)^5) - (15*a^4)/(4*b^7*(a
 + b*x)^4) + (20*a^3)/(3*b^7*(a + b*x)^3) - (15*a^2)/(2*b^7*(a + b*x)^2) +
(6*a)/(b^7*(a + b*x)) + Log[a + b*x]/b^7
```

Rubi [A] time = 0.0642931, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{\log(a+bx)}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^7,x]
```

```
[Out] -a^6/(6*b^7*(a + b*x)^6) + (6*a^5)/(5*b^7*(a + b*x)^5) - (15*a^4)/(4*b^7*(a
 + b*x)^4) + (20*a^3)/(3*b^7*(a + b*x)^3) - (15*a^2)/(2*b^7*(a + b*x)^2) +
(6*a)/(b^7*(a + b*x)) + Log[a + b*x]/b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^7}\,dx &= \int \left(\frac{a^6}{b^6(a+bx)^7}-\frac{6a^5}{b^6(a+bx)^6}+\frac{15a^4}{b^6(a+bx)^5}-\frac{20a^3}{b^6(a+bx)^4}+\frac{15a^2}{b^6(a+bx)^3}-\frac{6a}{b^6(a+bx)^2}+\frac{1}{b^6(a+b\right.\\ &= -\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{\log(a}{b}\end{aligned}$$

Mathematica [A] time = 0.0377205, size = 77, normalized size = 0.71

$$\frac{\frac{a\left(1875a^3b^2x^2+2200a^2b^3x^3+822a^4bx+147a^5+1350ab^4x^4+360b^5x^5\right)}{(a+bx)^6}+60\log(a+bx)}{60b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^7,x]
```

```
[Out] ((a*(147*a^5 + 822*a^4*b*x + 1875*a^3*b^2*x^2 + 2200*a^2*b^3*x^3 + 1350*a*b
^4*x^4 + 360*b^5*x^5))/(a + b*x)^6 + 60*Log[a + b*x])/(60*b^7)
```

Maple [A] time = 0.007, size = 100, normalized size = 0.9

$$-\frac{a^6}{6\,b^7\,(bx+a)^6}+\frac{6\,a^5}{5\,b^7\,(bx+a)^5}-\frac{15\,a^4}{4\,b^7\,(bx+a)^4}+\frac{20\,a^3}{3\,b^7\,(bx+a)^3}-\frac{15\,a^2}{2\,b^7\,(bx+a)^2}+6\,\frac{a}{b^7\,(bx+a)}+\frac{\ln(bx+a)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^7,x)
```

```
[Out] -1/6*a^6/b^7/(b*x+a)^6+6/5*a^5/b^7/(b*x+a)^5-15/4*a^4/b^7/(b*x+a)^4+20/3*a^
3/b^7/(b*x+a)^3-15/2*a^2/b^7/(b*x+a)^2+6*a/b^7/(b*x+a)+ln(b*x+a)/b^7
```

Maxima [A] time = 1.08102, size = 184, normalized size = 1.69

$$\frac{360\,ab^5x^5+1350\,a^2b^4x^4+2200\,a^3b^3x^3+1875\,a^4b^2x^2+822\,a^5bx+147\,a^6}{60\left(b^{13}x^6+6\,ab^{12}x^5+15\,a^2b^{11}x^4+20\,a^3b^{10}x^3+15\,a^4b^9x^2+6\,a^5b^8x+a^6b^7\right)}+\frac{\log(bx+a)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(360*a*b^5*x^5 + 1350*a^2*b^4*x^4 + 2200*a^3*b^3*x^3 + 1875*a^4*b^2*x^
2 + 822*a^5*b*x + 147*a^6)/(b^13*x^6 + 6*a*b^12*x^5 + 15*a^2*b^11*x^4 + 20*
a^3*b^10*x^3 + 15*a^4*b^9*x^2 + 6*a^5*b^8*x + a^6*b^7) + log(b*x + a)/b^7
```

Fricas [A] time = 1.54159, size = 428, normalized size = 3.93

$$\frac{360\,ab^5x^5+1350\,a^2b^4x^4+2200\,a^3b^3x^3+1875\,a^4b^2x^2+822\,a^5bx+147\,a^6+60\left(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3\right.}{60\left(b^{13}x^6+6\,ab^{12}x^5+15\,a^2b^{11}x^4+20\,a^3b^{10}x^3+15\,a^4b^9x^2+6\,a^5b^8x+a^6b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(360*a*b^5*x^5 + 1350*a^2*b^4*x^4 + 2200*a^3*b^3*x^3 + 1875*a^4*b^2*x^
2 + 822*a^5*b*x + 147*a^6 + 60*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20
*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6)*log(b*x + a))/(b^13*x^6 +
6*a*b^12*x^5 + 15*a^2*b^11*x^4 + 20*a^3*b^10*x^3 + 15*a^4*b^9*x^2 + 6*a^5*b
^8*x + a^6*b^7)
```

Sympy [A] time = 1.04979, size = 141, normalized size = 1.29

$$\frac{147a^6+822a^5bx+1875a^4b^2x^2+2200a^3b^3x^3+1350a^2b^4x^4+360ab^5x^5}{60a^6b^7+360a^5b^8x+900a^4b^9x^2+1200a^3b^{10}x^3+900a^2b^{11}x^4+360ab^{12}x^5+60b^{13}x^6}+\frac{\log(a+bx)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**7,x)
```

```
[Out] (147*a**6 + 822*a**5*b*x + 1875*a**4*b**2*x**2 + 2200*a**3*b**3*x**3 + 1350
*a**2*b**4*x**4 + 360*a*b**5*x**5)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a*
*4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**
5 + 60*b**13*x**6) + log(a + b*x)/b**7
```

Giac [A] time = 1.20754, size = 107, normalized size = 0.98

$$\frac{\log\left(|bx+a|\right)}{b^7}+\frac{360\,ab^4x^5+1350\,a^2b^3x^4+2200\,a^3b^2x^3+1875\,a^4bx^2+822\,a^5x+\frac{147\,a^6}{b}}{60\,(bx+a)^6b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^7 + 1/60*(360*a*b^4*x^5 + 1350*a^2*b^3*x^4 + 2200*a^3*b
^2*x^3 + 1875*a^4*b*x^2 + 822*a^5*x + 147*a^6/b)/((b*x + a)^6*b^6)
```

3.212 $\int \frac{x^5}{(a+bx)^7} dx$

Optimal. Leaf size=17

$$\frac{x^6}{6a(a+bx)^6}$$

```
[Out] x^6/(6*a*(a + b*x)^6)
```

Rubi [A] time = 0.0018992, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$\frac{x^6}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^7,x]

[Out] x^6/(6*a*(a + b*x)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^5}{(a+bx)^7} dx = \frac{x^6}{6a(a+bx)^6}$$

Mathematica [B] time = 0.0184956, size = 64, normalized size = 3.76

$$-\frac{15a^3b^2x^2+20a^2b^3x^3+6a^4bx+a^5+15ab^4x^4+6b^5x^5}{6b^6(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^7,x]

[Out] -(a^5 + 6*a^4*b*x + 15*a^3*b^2*x^2 + 20*a^2*b^3*x^3 + 15*a*b^4*x^4 + 6*b^5*
x^5)/(6*b^6*(a + b*x)^6)
```

Maple [B] time = 0.007, size = 87, normalized size = 5.1

$$-\frac{a^4}{b^6(bx+a)^5}-\frac{1}{b^6(bx+a)}+\frac{5a^3}{2b^6(bx+a)^4}+\frac{5a}{2b^6(bx+a)^2}+\frac{a^5}{6b^6(bx+a)^6}-\frac{10a^2}{3b^6(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^7,x)
```

```
[Out] -1/b^6*a^4/(b*x+a)^5-1/b^6/(b*x+a)+5/2/b^6*a^3/(b*x+a)^4+5/2/b^6*a/(b*x+a)^
2+1/6/b^6*a^5/(b*x+a)^6-10/3/b^6*a^2/(b*x+a)^3
```

Maxima [B] time = 1.06284, size = 162, normalized size = 9.53

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\left(b^{12}x^6+6\,ab^{11}x^5+15\,a^2b^{10}x^4+20\,a^3b^9x^3+15\,a^4b^8x^2+6\,a^5b^7x+a^6b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 + 15*a
^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

Fricas [B] time = 1.4968, size = 252, normalized size = 14.82

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\left(b^{12}x^6+6\,ab^{11}x^5+15\,a^2b^{10}x^4+20\,a^3b^9x^3+15\,a^4b^8x^2+6\,a^5b^7x+a^6b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 + 15*a
^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

Sympy [B] time = 0.99007, size = 128, normalized size = 7.53

$$-\frac{a^5+6a^4bx+15a^3b^2x^2+20a^2b^3x^3+15ab^4x^4+6b^5x^5}{6a^6b^6+36a^5b^7x+90a^4b^8x^2+120a^3b^9x^3+90a^2b^{10}x^4+36ab^{11}x^5+6b^{12}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**7,x)
```

```
[Out] -(a**5 + 6*a**4*b*x + 15*a**3*b**2*x**2 + 20*a**2*b**3*x**3 + 15*a*b**4*x**
4 + 6*b**5*x**5)/(6*a**6*b**6 + 36*a**5*b**7*x + 90*a**4*b**8*x**2 + 120*a*
*3*b**9*x**3 + 90*a**2*b**10*x**4 + 36*a*b**11*x**5 + 6*b**12*x**6)
```

Giac [B] time = 1.18808, size = 84, normalized size = 4.94

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,(bx+a)^6b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/((b*x + a)^6*b^6)
```

3.213 $\int \frac{x^4}{(a+bx)^7} \, dx$

Optimal. Leaf size=35

$$\frac{x^5}{30a^2(a+bx)^5}+\frac{x^5}{6a(a+bx)^6}$$

```
[Out] x^5/(6*a*(a + b*x)^6) + x^5/(30*a^2*(a + b*x)^5)
```

Rubi [A] time = 0.0049189, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{x^5}{30a^2(a+bx)^5}+\frac{x^5}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^7,x]
```

```
[Out] x^5/(6*a*(a + b*x)^6) + x^5/(30*a^2*(a + b*x)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^7} \, dx &= \frac{x^5}{6a(a+bx)^6}+\frac{\int \frac{x^4}{(a+bx)^6} \, dx}{6a}\\ &= \frac{x^5}{6a(a+bx)^6}+\frac{x^5}{30a^2(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.0155113, size = 53, normalized size = 1.51

$$-\frac{15a^2b^2x^2+6a^3bx+a^4+20ab^3x^3+15b^4x^4}{30b^5(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^7,x]
```

```
[Out] -(a^4 + 6*a^3*b*x + 15*a^2*b^2*x^2 + 20*a*b^3*x^3 + 15*b^4*x^4)/(30*b^5*(a
+ b*x)^6)
```

Maple [B] time = 0.006, size = 72, normalized size = 2.1

$$-\frac{3\,a^{2}}{2\,b^{5}\left(bx+a\right)^{4}}-\frac{1}{2\,b^{5}\left(bx+a\right)^{2}}-\frac{a^{4}}{6\,b^{5}\left(bx+a\right)^{6}}+\frac{4\,a}{3\,b^{5}\left(bx+a\right)^{3}}+\frac{4\,a^{3}}{5\,b^{5}\left(bx+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^7,x)
```

```
[Out] -3/2/b^5*a^2/(b*x+a)^4-1/2/b^5/(b*x+a)^2-1/6/b^5*a^4/(b*x+a)^6+4/3/b^5*a/(b
*x+a)^3+4/5/b^5*a^3/(b*x+a)^5
```

Maxima [B] time = 1.07266, size = 147, normalized size = 4.2

$$-\frac{15\,b^{4}x^{4}+20\,ab^{3}x^{3}+15\,a^{2}b^{2}x^{2}+6\,a^{3}bx+a^{4}}{30\left(b^{11}x^{6}+6\,ab^{10}x^{5}+15\,a^{2}b^{9}x^{4}+20\,a^{3}b^{8}x^{3}+15\,a^{4}b^{7}x^{2}+6\,a^{5}b^{6}x+a^{6}b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/(b^11*
x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3*b^8*x^3 + 15*a^4*b^7*x^2 + 6*a
^5*b^6*x + a^6*b^5)
```

Fricas [B] time = 1.54071, size = 231, normalized size = 6.6

$$-\frac{15\,b^{4}x^{4}+20\,ab^{3}x^{3}+15\,a^{2}b^{2}x^{2}+6\,a^{3}bx+a^{4}}{30\left(b^{11}x^{6}+6\,ab^{10}x^{5}+15\,a^{2}b^{9}x^{4}+20\,a^{3}b^{8}x^{3}+15\,a^{4}b^{7}x^{2}+6\,a^{5}b^{6}x+a^{6}b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/(b^11*
x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3*b^8*x^3 + 15*a^4*b^7*x^2 + 6*a
^5*b^6*x + a^6*b^5)
```

Sympy [B] time = 0.899177, size = 116, normalized size = 3.31

$$-\frac{a^{4}+6a^{3}bx+15a^{2}b^{2}x^{2}+20ab^{3}x^{3}+15b^{4}x^{4}}{30a^{6}b^{5}+180a^{5}b^{6}x+450a^{4}b^{7}x^{2}+600a^{3}b^{8}x^{3}+450a^{2}b^{9}x^{4}+180ab^{10}x^{5}+30b^{11}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**7,x)
```

```
[Out] -(a**4 + 6*a**3*b*x + 15*a**2*b**2*x**2 + 20*a*b**3*x**3 + 15*b**4*x**4)/(3
0*a**6*b**5 + 180*a**5*b**6*x + 450*a**4*b**7*x**2 + 600*a**3*b**8*x**3 + 4
50*a**2*b**9*x**4 + 180*a*b**10*x**5 + 30*b**11*x**6)
```

Giac [A] time = 1.15434, size = 69, normalized size = 1.97

$$-\frac{15\,b^4x^4 + 20\,ab^3x^3 + 15\,a^2b^2x^2 + 6\,a^3bx + a^4}{30\,(bx+a)^6b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/((b*x
+ a)^6*b^5)
```

3.214 $\int \frac{x^3}{(a+bx)^7} \, dx$

Optimal. Leaf size=52

$$\frac{x^4}{60a^3(a+bx)^4}+\frac{x^4}{15a^2(a+bx)^5}+\frac{x^4}{6a(a+bx)^6}$$

```
[Out] x^4/(6*a*(a + b*x)^6) + x^4/(15*a^2*(a + b*x)^5) + x^4/(60*a^3*(a + b*x)^4)
```

Rubi [A] time = 0.0293148, antiderivative size = 64, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{6b^4(a+bx)^6}-\frac{3a^2}{5b^4(a+bx)^5}+\frac{3a}{4b^4(a+bx)^4}-\frac{1}{3b^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^7,x]
```

```
[Out] a^3/(6*b^4*(a + b*x)^6) - (3*a^2)/(5*b^4*(a + b*x)^5) + (3*a)/(4*b^4*(a + b
*x)^4) - 1/(3*b^4*(a + b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^7} \, dx &= \int \left(-\frac{a^3}{b^3(a+bx)^7}+\frac{3a^2}{b^3(a+bx)^6}-\frac{3a}{b^3(a+bx)^5}+\frac{1}{b^3(a+bx)^4}\right) dx\\ &= \frac{a^3}{6b^4(a+bx)^6}-\frac{3a^2}{5b^4(a+bx)^5}+\frac{3a}{4b^4(a+bx)^4}-\frac{1}{3b^4(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.0182702, size = 42, normalized size = 0.81

$$-\frac{6a^2bx+a^3+15ab^2x^2+20b^3x^3}{60b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^7,x]
```

```
[Out] -(a^3 + 6*a^2*b*x + 15*a*b^2*x^2 + 20*b^3*x^3)/(60*b^4*(a + b*x)^6)
```

Maple [A] time = 0.005, size = 57, normalized size = 1.1

$$\frac{a^3}{6\,b^4\,(bx+a)^6}-\frac{3\,a^2}{5\,b^4\,(bx+a)^5}+\frac{3\,a}{4\,b^4\,(bx+a)^4}-\frac{1}{3\,b^4\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^7,x)
```

```
[Out] 1/6*a^3/b^4/(b*x+a)^6-3/5*a^2/b^4/(b*x+a)^5+3/4*a/b^4/(b*x+a)^4-1/3/b^4/(b*
x+a)^3
```

Maxima [B] time = 1.04443, size = 132, normalized size = 2.54

$$-\frac{20\,b^3x^3+15\,ab^2x^2+6\,a^2bx+a^3}{60\,\left(b^{10}x^6+6\,ab^9x^5+15\,a^2b^8x^4+20\,a^3b^7x^3+15\,a^4b^6x^2+6\,a^5b^5x+a^6b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/(b^10*x^6 + 6*a*b^9*x^5
 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4
)
```

Fricas [B] time = 1.47325, size = 207, normalized size = 3.98

$$-\frac{20\,b^3x^3+15\,ab^2x^2+6\,a^2bx+a^3}{60\,\left(b^{10}x^6+6\,ab^9x^5+15\,a^2b^8x^4+20\,a^3b^7x^3+15\,a^4b^6x^2+6\,a^5b^5x+a^6b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/(b^10*x^6 + 6*a*b^9*x^5
 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4
)
```

Sympy [B] time = 0.873424, size = 104, normalized size = 2.

$$-\frac{a^3+6a^2bx+15ab^2x^2+20b^3x^3}{60a^6b^4+360a^5b^5x+900a^4b^6x^2+1200a^3b^7x^3+900a^2b^8x^4+360ab^9x^5+60b^{10}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**7,x)
```

```
[Out] -(a**3 + 6*a**2*b*x + 15*a*b**2*x**2 + 20*b**3*x**3)/(60*a**6*b**4 + 360*a*
*5*b**5*x + 900*a**4*b**6*x**2 + 1200*a**3*b**7*x**3 + 900*a**2*b**8*x**4 +
 360*a*b**9*x**5 + 60*b**10*x**6)
```

Giac [A] time = 1.21901, size = 54, normalized size = 1.04

$$-\frac{20 b^3 x^3 + 15 a b^2 x^2 + 6 a^2 b x + a^3}{60 (b x + a)^6 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/((b*x + a)^6*b^4)
```

3.215 $\int \frac{x^2}{(a+bx)^7} dx$

Optimal. Leaf size=47

$$-\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}$$

```
[Out] -a^2/(6*b^3*(a + b*x)^6) + (2*a)/(5*b^3*(a + b*x)^5) - 1/(4*b^3*(a + b*x)^4
)
```

Rubi [A] time = 0.0209128, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^7,x]
```

```
[Out] -a^2/(6*b^3*(a + b*x)^6) + (2*a)/(5*b^3*(a + b*x)^5) - 1/(4*b^3*(a + b*x)^4
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^7}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^7}-\frac{2a}{b^2(a+bx)^6}+\frac{1}{b^2(a+bx)^5}\right)dx\\ &= -\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.0144736, size = 31, normalized size = 0.66

$$-\frac{a^2+6abx+15b^2x^2}{60b^3(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^7,x]
```

```
[Out] -(a^2 + 6*a*b*x + 15*b^2*x^2)/(60*b^3*(a + b*x)^6)
```

Maple [A] time = 0.004, size = 42, normalized size = 0.9

$$-\frac{a^2}{6\,b^3\,(bx+a)^6}+\frac{2\,a}{5\,b^3\,(bx+a)^5}-\frac{1}{4\,b^3\,(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^7,x)
```

```
[Out] -1/6*a^2/b^3/(b*x+a)^6+2/5*a/b^3/(b*x+a)^5-1/4/b^3/(b*x+a)^4
```

Maxima [B] time = 1.07224, size = 117, normalized size = 2.49

$$-\frac{15\,b^2x^2+6\,abx+a^2}{60\,\left(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4
+ 20*a^3*b^6*x^3 + 15*a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Fricas [B] time = 1.50643, size = 182, normalized size = 3.87

$$-\frac{15\,b^2x^2+6\,abx+a^2}{60\,\left(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4
+ 20*a^3*b^6*x^3 + 15*a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Sympy [B] time = 0.791499, size = 92, normalized size = 1.96

$$-\frac{a^2+6abx+15b^2x^2}{60a^6b^3+360a^5b^4x+900a^4b^5x^2+1200a^3b^6x^3+900a^2b^7x^4+360ab^8x^5+60b^9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**7,x)
```

```
[Out] -(a**2 + 6*a*b*x + 15*b**2*x**2)/(60*a**6*b**3 + 360*a**5*b**4*x + 900*a**4
*b**5*x**2 + 1200*a**3*b**6*x**3 + 900*a**2*b**7*x**4 + 360*a*b**8*x**5 + 6
0*b**9*x**6)
```

Giac [A] time = 1.15681, size = 39, normalized size = 0.83

$$-\frac{15\,b^2x^2+6\,abx+a^2}{60\,(bx+a)^6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/((b*x + a)^6*b^3)
```

3.216 $\int \frac{x}{(a+bx)^7} dx$

Optimal. Leaf size=30

$$\frac{a}{6b^2(a+bx)^6} - \frac{1}{5b^2(a+bx)^5}$$

```
[Out] a/(6*b^2*(a + b*x)^6) - 1/(5*b^2*(a + b*x)^5)
```

Rubi [A] time = 0.0134989, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a}{6b^2(a+bx)^6} - \frac{1}{5b^2(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^7,x]
```

```
[Out] a/(6*b^2*(a + b*x)^6) - 1/(5*b^2*(a + b*x)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^7} dx &= \int \left(-\frac{a}{b(a+bx)^7} + \frac{1}{b(a+bx)^6}\right) dx\\ &= \frac{a}{6b^2(a+bx)^6} - \frac{1}{5b^2(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.0105081, size = 20, normalized size = 0.67

$$-\frac{a+6bx}{30b^2(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^7,x]
```

```
[Out] -(a + 6*b*x)/(30*b^2*(a + b*x)^6)
```

Maple [A] time = 0.004, size = 27, normalized size = 0.9

$$\frac{a}{6\,b^2\,(bx+a)^6} - \frac{1}{5\,b^2\,(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^7,x)
```

```
[Out] 1/6*a/b^2/(b*x+a)^6-1/5/b^2/(b*x+a)^5
```

Maxima [B] time = 1.08307, size = 103, normalized size = 3.43

$$-\frac{6\,bx+a}{30\,\left(b^{8}x^{6}+6\,ab^{7}x^{5}+15\,a^{2}b^{6}x^{4}+20\,a^{3}b^{5}x^{3}+15\,a^{4}b^{4}x^{2}+6\,a^{5}b^{3}x+a^{6}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(6*b*x + a)/(b^8*x^6 + 6*a*b^7*x^5 + 15*a^2*b^6*x^4 + 20*a^3*b^5*x^3
+ 15*a^4*b^4*x^2 + 6*a^5*b^3*x + a^6*b^2)
```

Fricas [B] time = 1.51044, size = 159, normalized size = 5.3

$$-\frac{6\,bx+a}{30\,\left(b^{8}x^{6}+6\,ab^{7}x^{5}+15\,a^{2}b^{6}x^{4}+20\,a^{3}b^{5}x^{3}+15\,a^{4}b^{4}x^{2}+6\,a^{5}b^{3}x+a^{6}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(6*b*x + a)/(b^8*x^6 + 6*a*b^7*x^5 + 15*a^2*b^6*x^4 + 20*a^3*b^5*x^3
+ 15*a^4*b^4*x^2 + 6*a^5*b^3*x + a^6*b^2)
```

Sympy [B] time = 0.77669, size = 80, normalized size = 2.67

$$-\frac{a+6bx}{30a^{6}b^{2}+180a^{5}b^{3}x+450a^{4}b^{4}x^{2}+600a^{3}b^{5}x^{3}+450a^{2}b^{6}x^{4}+180ab^{7}x^{5}+30b^{8}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**7,x)
```

```
[Out] -(a + 6*b*x)/(30*a**6*b**2 + 180*a**5*b**3*x + 450*a**4*b**4*x**2 + 600*a**
3*b**5*x**3 + 450*a**2*b**6*x**4 + 180*a*b**7*x**5 + 30*b**8*x**6)
```

Giac [A] time = 1.20376, size = 24, normalized size = 0.8

$$-\frac{6\,bx+a}{30\,(bx+a)^{6}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/30*(6*b*x + a)/((b*x + a)^6*b^2)
```

3.217 $\int \frac{1}{(a+bx)^7} dx$

Optimal. Leaf size=14

$$-\frac{1}{6b(a+bx)^6}$$

```
[Out] -1/(6*b*(a + b*x)^6)
```

Rubi [A] time = 0.0017102, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{6b(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-7),x]

[Out] -1/(6*b*(a + b*x)^6)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^7} dx = -\frac{1}{6b(a+bx)^6}$$

Mathematica [A] time = 0.0044064, size = 14, normalized size = 1.

$$-\frac{1}{6b(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-7),x]

[Out] -1/(6*b*(a + b*x)^6)
```

Maple [A] time = 0.001, size = 13, normalized size = 0.9

$$-\frac{1}{6\,b\,(bx+a)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^7,x)
```

```
[Out] -1/6/b/(b*x+a)^6
```

Maxima [A] time = 1.06624, size = 16, normalized size = 1.14

$$-\frac{1}{6\,(bx+a)^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/6/((b*x + a)^6*b)
```

Fricas [B] time = 1.54331, size = 139, normalized size = 9.93

$$-\frac{1}{6\left(b^7x^6+6\,ab^6x^5+15\,a^2b^5x^4+20\,a^3b^4x^3+15\,a^4b^3x^2+6\,a^5b^2x+a^6b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/6/(b^7*x^6 + 6*a*b^6*x^5 + 15*a^2*b^5*x^4 + 20*a^3*b^4*x^3 + 15*a^4*b^3*
x^2 + 6*a^5*b^2*x + a^6*b)
```

Sympy [B] time = 0.806741, size = 73, normalized size = 5.21

$$-\frac{1}{6a^6b+36a^5b^2x+90a^4b^3x^2+120a^3b^4x^3+90a^2b^5x^4+36ab^6x^5+6b^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**7,x)
```

```
[Out] -1/(6*a**6*b + 36*a**5*b**2*x + 90*a**4*b**3*x**2 + 120*a**3*b**4*x**3 + 90
*a**2*b**5*x**4 + 36*a*b**6*x**5 + 6*b**7*x**6)
```

Giac [A] time = 1.19839, size = 16, normalized size = 1.14

$$-\frac{1}{6\,(bx+a)^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/6/((b*x + a)^6*b)
```

3.218 $\int \frac{1}{x(a+bx)^7} dx$

Optimal. Leaf size=99

$$\frac{1}{a^6(a+bx)}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{5a^2(a+bx)^5}-\frac{\log(a+bx)}{a^7}+\frac{\log(x)}{a^7}+\frac{1}{6a(a+bx)^6}$$

```
[Out] 1/(6*a*(a + b*x)^6) + 1/(5*a^2*(a + b*x)^5) + 1/(4*a^3*(a + b*x)^4) + 1/(3*
a^4*(a + b*x)^3) + 1/(2*a^5*(a + b*x)^2) + 1/(a^6*(a + b*x)) + Log[x]/a^7 -
 Log[a + b*x]/a^7
```

Rubi [A] time = 0.0488924, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{a^6(a+bx)}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{5a^2(a+bx)^5}-\frac{\log(a+bx)}{a^7}+\frac{\log(x)}{a^7}+\frac{1}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^7),x]
```

```
[Out] 1/(6*a*(a + b*x)^6) + 1/(5*a^2*(a + b*x)^5) + 1/(4*a^3*(a + b*x)^4) + 1/(3*
a^4*(a + b*x)^3) + 1/(2*a^5*(a + b*x)^2) + 1/(a^6*(a + b*x)) + Log[x]/a^7 -
 Log[a + b*x]/a^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^7}\,dx &= \int \left(\frac{1}{a^7x}-\frac{b}{a(a+bx)^7}-\frac{b}{a^2(a+bx)^6}-\frac{b}{a^3(a+bx)^5}-\frac{b}{a^4(a+bx)^4}-\frac{b}{a^5(a+bx)^3}-\frac{b}{a^6(a+bx)^2}-\frac{b}{a^7}\right.\\ &= \frac{1}{6a(a+bx)^6}+\frac{1}{5a^2(a+bx)^5}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{a^6(a+bx)}+\frac{\log(x)}{a^7}\end{aligned}$$

Mathematica [A] time = 0.104877, size = 81, normalized size = 0.82

$$\frac{\frac{a\left(855a^3b^2x^2+740a^2b^3x^3+522a^4bx+147a^5+330ab^4x^4+60b^5x^5\right)}{(a+bx)^6}-60\log(a+bx)+60\log(x)}{60a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^7),x]
```

```
[Out] ((a*(147*a^5 + 522*a^4*b*x + 855*a^3*b^2*x^2 + 740*a^2*b^3*x^3 + 330*a*b^4*
x^4 + 60*b^5*x^5))/(a + b*x)^6 + 60*Log[x] - 60*Log[a + b*x])/(60*a^7)
```

Maple [A] time = 0.01, size = 90, normalized size = 0.9

$$\frac{1}{6\,a\,(bx+a)^{6}}+\frac{1}{5\,a^{2}\,(bx+a)^{5}}+\frac{1}{4\,a^{3}\,(bx+a)^{4}}+\frac{1}{3\,a^{4}\,(bx+a)^{3}}+\frac{1}{2\,a^{5}\,(bx+a)^{2}}+\frac{1}{a^{6}\,(bx+a)}+\frac{\ln(x)}{a^{7}}-\frac{\ln(bx+a)}{a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^7,x)
```

```
[Out] 1/6/a/(b*x+a)^6+1/5/a^2/(b*x+a)^5+1/4/a^3/(b*x+a)^4+1/3/a^4/(b*x+a)^3+1/2/a
^5/(b*x+a)^2+1/a^6/(b*x+a)+ln(x)/a^7-ln(b*x+a)/a^7
```

Maxima [A] time = 1.12927, size = 188, normalized size = 1.9

$$\frac{60\,b^{5}x^{5}+330\,ab^{4}x^{4}+740\,a^{2}b^{3}x^{3}+855\,a^{3}b^{2}x^{2}+522\,a^{4}bx+147\,a^{5}}{60\left(a^{6}b^{6}x^{6}+6\,a^{7}b^{5}x^{5}+15\,a^{8}b^{4}x^{4}+20\,a^{9}b^{3}x^{3}+15\,a^{10}b^{2}x^{2}+6\,a^{11}bx+a^{12}\right)}-\frac{\log(bx+a)}{a^{7}}+\frac{\log(x)}{a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(60*b^5*x^5 + 330*a*b^4*x^4 + 740*a^2*b^3*x^3 + 855*a^3*b^2*x^2 + 522*
a^4*b*x + 147*a^5)/(a^6*b^6*x^6 + 6*a^7*b^5*x^5 + 15*a^8*b^4*x^4 + 20*a^9*b
^3*x^3 + 15*a^10*b^2*x^2 + 6*a^11*b*x + a^12) - log(b*x + a)/a^7 + log(x)/a
^7
```

Fricas [B] time = 1.63107, size = 564, normalized size = 5.7

$$\frac{60\,ab^{5}x^{5}+330\,a^{2}b^{4}x^{4}+740\,a^{3}b^{3}x^{3}+855\,a^{4}b^{2}x^{2}+522\,a^{5}bx+147\,a^{6}-60\left(b^{6}x^{6}+6\,ab^{5}x^{5}+15\,a^{2}b^{4}x^{4}+20\,a^{3}b^{3}x^{3}+15\ldots\right.}{60\left(a^{7}b^{6}x^{6}+6\,a^{8}b^{5}x^{5}+15\,a^{9}b^{4}x^{4}+20\,a^{1\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a*b^5*x^5 + 330*a^2*b^4*x^4 + 740*a^3*b^3*x^3 + 855*a^4*b^2*x^2 +
522*a^5*b*x + 147*a^6 - 60*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3
*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6)*log(b*x + a) + 60*(b^6*x^6 + 6
*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x +
 a^6)*log(x))/(a^7*b^6*x^6 + 6*a^8*b^5*x^5 + 15*a^9*b^4*x^4 + 20*a^10*b^3*x
^3 + 15*a^11*b^2*x^2 + 6*a^12*b*x + a^13)
```

Sympy [A] time = 1.279, size = 141, normalized size = 1.42

$$\frac{147a^{5}+522a^{4}bx+855a^{3}b^{2}x^{2}+740a^{2}b^{3}x^{3}+330ab^{4}x^{4}+60b^{5}x^{5}}{60a^{12}+360a^{11}bx+900a^{10}b^{2}x^{2}+1200a^{9}b^{3}x^{3}+900a^{8}b^{4}x^{4}+360a^{7}b^{5}x^{5}+60a^{6}b^{6}x^{6}}+\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**7,x)
```

```
[Out] (147*a**5 + 522*a**4*b*x + 855*a**3*b**2*x**2 + 740*a**2*b**3*x**3 + 330*a*
b**4*x**4 + 60*b**5*x**5)/(60*a**12 + 360*a**11*b*x + 900*a**10*b**2*x**2 +
 1200*a**9*b**3*x**3 + 900*a**8*b**4*x**4 + 360*a**7*b**5*x**5 + 60*a**6*b*
*6*x**6) + (log(x) - log(a/b + x))/a**7
```

Giac [A] time = 1.22369, size = 117, normalized size = 1.18

$$-\frac{\log\left(|bx+a|\right)}{a^7}+\frac{\log\left(|x|\right)}{a^7}+\frac{60\,ab^5x^5+330\,a^2b^4x^4+740\,a^3b^3x^3+855\,a^4b^2x^2+522\,a^5bx+147\,a^6}{60\,(bx+a)^6a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^7 + log(abs(x))/a^7 + 1/60*(60*a*b^5*x^5 + 330*a^2*b^4
*x^4 + 740*a^3*b^3*x^3 + 855*a^4*b^2*x^2 + 522*a^5*b*x + 147*a^6)/((b*x + a
)^6*a^7)
```

3.219 $\int \frac{1}{x^2(a+bx)^7} \, dx$

Optimal. Leaf size=117

$$-\frac{6b}{a^7(a+bx)}-\frac{5b}{2a^6(a+bx)^2}-\frac{4b}{3a^5(a+bx)^3}-\frac{3b}{4a^4(a+bx)^4}-\frac{2b}{5a^3(a+bx)^5}-\frac{b}{6a^2(a+bx)^6}-\frac{7b\log(x)}{a^8}+\frac{7b\log(a+bx)}{a^8}$$

```
[Out] -(1/(a^7*x)) - b/(6*a^2*(a + b*x)^6) - (2*b)/(5*a^3*(a + b*x)^5) - (3*b)/(4
*a^4*(a + b*x)^4) - (4*b)/(3*a^5*(a + b*x)^3) - (5*b)/(2*a^6*(a + b*x)^2) -
 (6*b)/(a^7*(a + b*x)) - (7*b*Log[x])/a^8 + (7*b*Log[a + b*x])/a^8
```

Rubi [A] time = 0.0732826, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{6b}{a^7(a+bx)}-\frac{5b}{2a^6(a+bx)^2}-\frac{4b}{3a^5(a+bx)^3}-\frac{3b}{4a^4(a+bx)^4}-\frac{2b}{5a^3(a+bx)^5}-\frac{b}{6a^2(a+bx)^6}-\frac{7b\log(x)}{a^8}+\frac{7b\log(a+bx)}{a^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^7),x]
```

```
[Out] -(1/(a^7*x)) - b/(6*a^2*(a + b*x)^6) - (2*b)/(5*a^3*(a + b*x)^5) - (3*b)/(4
*a^4*(a + b*x)^4) - (4*b)/(3*a^5*(a + b*x)^3) - (5*b)/(2*a^6*(a + b*x)^2) -
 (6*b)/(a^7*(a + b*x)) - (7*b*Log[x])/a^8 + (7*b*Log[a + b*x])/a^8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^2(a+bx)^7} \, dx = \int \left(\frac{1}{a^7x^2}-\frac{7b}{a^8x}+\frac{b^2}{a^2(a+bx)^7}+\frac{2b^2}{a^3(a+bx)^6}+\frac{3b^2}{a^4(a+bx)^5}+\frac{4b^2}{a^5(a+bx)^4}+\frac{5b^2}{a^6(a+bx)^3}+\frac{6b^2}{a^7(a+b\ldots}\right.$$

$$=-\frac{1}{a^7x}-\frac{b}{6a^2(a+bx)^6}-\frac{2b}{5a^3(a+bx)^5}-\frac{3b}{4a^4(a+bx)^4}-\frac{4b}{3a^5(a+bx)^3}-\frac{5b}{2a^6(a+bx)^2}-\frac{6b}{a^7(a+bx)}-\ldots$$

Mathematica [A] time = 0.164784, size = 97, normalized size = 0.83

$$-\frac{\frac{a\left(3654a^4b^2x^2+5985a^3b^3x^3+5180a^2b^4x^4+1029a^5bx+60a^6+2310ab^5x^5+420b^6x^6\right)}{x(a+bx)^6}-420b\log(a+bx)+420b\log(x)}{60a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^7),x]
```

```
[Out] -((a*(60*a^6 + 1029*a^5*b*x + 3654*a^4*b^2*x^2 + 5985*a^3*b^3*x^3 + 5180*a^
2*b^4*x^4 + 2310*a*b^5*x^5 + 420*b^6*x^6))/(x*(a + b*x)^6) + 420*b*Log[x] -
```

```
420*b*Log[a + b*x])/(60*a^8)
```

Maple [A] time = 0.011, size = 108, normalized size = 0.9

$$-\frac{1}{a^7 x}-\frac{b}{6\,a^2\,(bx+a)^6}-\frac{2\,b}{5\,a^3\,(bx+a)^5}-\frac{3\,b}{4\,a^4\,(bx+a)^4}-\frac{4\,b}{3\,a^5\,(bx+a)^3}-\frac{5\,b}{2\,a^6\,(bx+a)^2}-6\,\frac{b}{a^7\,(bx+a)}-7\,\frac{b\ln(x)}{a^8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^7,x)
```

```
[Out] -1/a^7/x-1/6*b/a^2/(b*x+a)^6-2/5*b/a^3/(b*x+a)^5-3/4*b/a^4/(b*x+a)^4-4/3*b/
a^5/(b*x+a)^3-5/2*b/a^6/(b*x+a)^2-6*b/a^7/(b*x+a)-7*b*ln(x)/a^8+7*b*ln(b*x+
a)/a^8
```

Maxima [A] time = 1.05639, size = 212, normalized size = 1.81

$$-\frac{420\,b^6x^6+2310\,ab^5x^5+5180\,a^2b^4x^4+5985\,a^3b^3x^3+3654\,a^4b^2x^2+1029\,a^5bx+60\,a^6}{60\left(a^7b^6x^7+6\,a^8b^5x^6+15\,a^9b^4x^5+20\,a^{10}b^3x^4+15\,a^{11}b^2x^3+6\,a^{12}bx^2+a^{13}x\right)}+\frac{7\,b\log(bx+a)}{a^8}-\frac{7\,b\log(}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(420*b^6*x^6 + 2310*a*b^5*x^5 + 5180*a^2*b^4*x^4 + 5985*a^3*b^3*x^3 +
 3654*a^4*b^2*x^2 + 1029*a^5*b*x + 60*a^6)/(a^7*b^6*x^7 + 6*a^8*b^5*x^6 + 1
5*a^9*b^4*x^5 + 20*a^10*b^3*x^4 + 15*a^11*b^2*x^3 + 6*a^12*b*x^2 + a^13*x)
+ 7*b*log(b*x + a)/a^8 - 7*b*log(x)/a^8
```

Fricas [B] time = 1.63639, size = 628, normalized size = 5.37

$$-\frac{420\,ab^6x^6+2310\,a^2b^5x^5+5180\,a^3b^4x^4+5985\,a^4b^3x^3+3654\,a^5b^2x^2+1029\,a^6bx+60\,a^7-420\left(b^7x^7+6\,ab^6x^6+15\right.}{60\left(a^8b^6x^7+6\,a^9b^5x^6+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(420*a*b^6*x^6 + 2310*a^2*b^5*x^5 + 5180*a^3*b^4*x^4 + 5985*a^4*b^3*x
^3 + 3654*a^5*b^2*x^2 + 1029*a^6*b*x + 60*a^7 - 420*(b^7*x^7 + 6*a*b^6*x^6
+ 15*a^2*b^5*x^5 + 20*a^3*b^4*x^4 + 15*a^4*b^3*x^3 + 6*a^5*b^2*x^2 + a^6*b*
x)*log(b*x + a) + 420*(b^7*x^7 + 6*a*b^6*x^6 + 15*a^2*b^5*x^5 + 20*a^3*b^4*
x^4 + 15*a^4*b^3*x^3 + 6*a^5*b^2*x^2 + a^6*b*x)*log(x))/(a^8*b^6*x^7 + 6*a^
9*b^5*x^6 + 15*a^10*b^4*x^5 + 20*a^11*b^3*x^4 + 15*a^12*b^2*x^3 + 6*a^13*b*
x^2 + a^14*x)
```

Sympy [A] time = 1.34076, size = 160, normalized size = 1.37

$$-\frac{60a^6+1029a^5bx+3654a^4b^2x^2+5985a^3b^3x^3+5180a^2b^4x^4+2310ab^5x^5+420b^6x^6}{60a^{13}x+360a^{12}bx^2+900a^{11}b^2x^3+1200a^{10}b^3x^4+900a^9b^4x^5+360a^8b^5x^6+60a^7b^6x^7}+\frac{7b\left(-\log(x)+\log\left(\frac{a}{b}+x\right.\right.}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**7,x)
```

```
[Out] -(60*a**6 + 1029*a**5*b*x + 3654*a**4*b**2*x**2 + 5985*a**3*b**3*x**3 + 518
0*a**2*b**4*x**4 + 2310*a*b**5*x**5 + 420*b**6*x**6)/(60*a**13*x + 360*a**1
2*b*x**2 + 900*a**11*b**2*x**3 + 1200*a**10*b**3*x**4 + 900*a**9*b**4*x**5
+ 360*a**8*b**5*x**6 + 60*a**7*b**6*x**7) + 7*b*(-log(x) + log(a/b + x))/a*
*8
```

Giac [A] time = 1.1868, size = 140, normalized size = 1.2

$$\frac{7\,b\log\left(|bx+a|\right)}{a^8}-\frac{7\,b\log\left(|x|\right)}{a^8}-\frac{420\,ab^6x^6+2310\,a^2b^5x^5+5180\,a^3b^4x^4+5985\,a^4b^3x^3+3654\,a^5b^2x^2+1029\,a^6bx+60}{60\,(bx+a)^6a^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 7*b*log(abs(b*x + a))/a^8 - 7*b*log(abs(x))/a^8 - 1/60*(420*a*b^6*x^6 + 231
0*a^2*b^5*x^5 + 5180*a^3*b^4*x^4 + 5985*a^4*b^3*x^3 + 3654*a^5*b^2*x^2 + 10
29*a^6*b*x + 60*a^7)/((b*x + a)^6*a^8*x)
```

3.220 $\int \frac{1}{x^3(a+bx)^7} dx$

Optimal. Leaf size=144

$$\frac{21b^2}{a^8(a+bx)}+\frac{15b^2}{2a^7(a+bx)^2}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{b^2}{6a^3(a+bx)^6}+\frac{28b^2\log(x)}{a^9}-\frac{28b^2\log(a}{a^9}$$

```
[Out] -1/(2*a^7*x^2) + (7*b)/(a^8*x) + b^2/(6*a^3*(a + b*x)^6) + (3*b^2)/(5*a^4*(
a + b*x)^5) + (3*b^2)/(2*a^5*(a + b*x)^4) + (10*b^2)/(3*a^6*(a + b*x)^3) +
(15*b^2)/(2*a^7*(a + b*x)^2) + (21*b^2)/(a^8*(a + b*x)) + (28*b^2*Log[x])/a
^9 - (28*b^2*Log[a + b*x])/a^9
```

Rubi [A] time = 0.0892933, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{21b^2}{a^8(a+bx)}+\frac{15b^2}{2a^7(a+bx)^2}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{b^2}{6a^3(a+bx)^6}+\frac{28b^2\log(x)}{a^9}-\frac{28b^2\log(a}{a^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^7),x]
```

```
[Out] -1/(2*a^7*x^2) + (7*b)/(a^8*x) + b^2/(6*a^3*(a + b*x)^6) + (3*b^2)/(5*a^4*(
a + b*x)^5) + (3*b^2)/(2*a^5*(a + b*x)^4) + (10*b^2)/(3*a^6*(a + b*x)^3) +
(15*b^2)/(2*a^7*(a + b*x)^2) + (21*b^2)/(a^8*(a + b*x)) + (28*b^2*Log[x])/a
^9 - (28*b^2*Log[a + b*x])/a^9
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3(a+bx)^7}dx=\int \left(\frac{1}{a^7x^3}-\frac{7b}{a^8x^2}+\frac{28b^2}{a^9x}-\frac{b^3}{a^3(a+bx)^7}-\frac{3b^3}{a^4(a+bx)^6}-\frac{6b^3}{a^5(a+bx)^5}-\frac{10b^3}{a^6(a+bx)^4}-\frac{15b^3}{a^7(a+bx)^3}\right.$$
$$=-\frac{1}{2a^7x^2}+\frac{7b}{a^8x}+\frac{b^2}{6a^3(a+bx)^6}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{15b^2}{2a^7(a+bx)^2}+$$

Mathematica [A] time = 0.11095, size = 112, normalized size = 0.78

$$\frac{\frac{a\left(2058a^5b^2x^2+7308a^4b^3x^3+11970a^3b^4x^4+10360a^2b^5x^5+120a^6bx-15a^7+4620ab^6x^6+840b^7x^7\right)}{x^2(a+bx)^6}-840b^2\log(a+bx)+840b^2\log(x)}{30a^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^7),x]
```

```
[Out] ((a*(-15*a^7 + 120*a^6*b*x + 2058*a^5*b^2*x^2 + 7308*a^4*b^3*x^3 + 11970*a^
3*b^4*x^4 + 10360*a^2*b^5*x^5 + 4620*a*b^6*x^6 + 840*b^7*x^7))/(x^2*(a + b*
x)^6) + 840*b^2*Log[x] - 840*b^2*Log[a + b*x])/(30*a^9)
```

Maple [A] time = 0.016, size = 133, normalized size = 0.9

$$-\frac{1}{2\,a^7x^2}+7\,\frac{b}{a^8x}+\frac{b^2}{6\,a^3\,(bx+a)^6}+\frac{3\,b^2}{5\,a^4\,(bx+a)^5}+\frac{3\,b^2}{2\,a^5\,(bx+a)^4}+\frac{10\,b^2}{3\,a^6\,(bx+a)^3}+\frac{15\,b^2}{2\,a^7\,(bx+a)^2}+21\,\frac{b^2}{a^8\,(bx+a)}+28$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^7,x)
```

```
[Out] -1/2/a^7/x^2+7*b/a^8/x+1/6*b^2/a^3/(b*x+a)^6+3/5*b^2/a^4/(b*x+a)^5+3/2*b^2/
a^5/(b*x+a)^4+10/3*b^2/a^6/(b*x+a)^3+15/2*b^2/a^7/(b*x+a)^2+21*b^2/a^8/(b*x
+a)+28*b^2*ln(x)/a^9-28*b^2*ln(b*x+a)/a^9
```

Maxima [A] time = 1.1152, size = 235, normalized size = 1.63

$$\frac{840\,b^7x^7+4620\,ab^6x^6+10360\,a^2b^5x^5+11970\,a^3b^4x^4+7308\,a^4b^3x^3+2058\,a^5b^2x^2+120\,a^6bx-15\,a^7}{30\left(a^8b^6x^8+6\,a^9b^5x^7+15\,a^{10}b^4x^6+20\,a^{11}b^3x^5+15\,a^{12}b^2x^4+6\,a^{13}bx^3+a^{14}x^2\right)}-\frac{28\,b^2\log(bx+}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/30*(840*b^7*x^7 + 4620*a*b^6*x^6 + 10360*a^2*b^5*x^5 + 11970*a^3*b^4*x^4
+ 7308*a^4*b^3*x^3 + 2058*a^5*b^2*x^2 + 120*a^6*b*x - 15*a^7)/(a^8*b^6*x^8
+ 6*a^9*b^5*x^7 + 15*a^10*b^4*x^6 + 20*a^11*b^3*x^5 + 15*a^12*b^2*x^4 + 6*a
^13*b*x^3 + a^14*x^2) - 28*b^2*log(b*x + a)/a^9 + 28*b^2*log(x)/a^9
```

Fricas [B] time = 1.52653, size = 668, normalized size = 4.64

$$\frac{840\,ab^7x^7+4620\,a^2b^6x^6+10360\,a^3b^5x^5+11970\,a^4b^4x^4+7308\,a^5b^3x^3+2058\,a^6b^2x^2+120\,a^7bx-15\,a^8-840\left(b^8x^8+6\right.}{30\left(a^9b^6x^8+6\,a^{10}b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(840*a*b^7*x^7 + 4620*a^2*b^6*x^6 + 10360*a^3*b^5*x^5 + 11970*a^4*b^4*
x^4 + 7308*a^5*b^3*x^3 + 2058*a^6*b^2*x^2 + 120*a^7*b*x - 15*a^8 - 840*(b^8
*x^8 + 6*a*b^7*x^7 + 15*a^2*b^6*x^6 + 20*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 6*a
^5*b^3*x^3 + a^6*b^2*x^2)*log(b*x + a) + 840*(b^8*x^8 + 6*a*b^7*x^7 + 15*a^
2*b^6*x^6 + 20*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 6*a^5*b^3*x^3 + a^6*b^2*x^2)*
log(x))/(a^9*b^6*x^8 + 6*a^10*b^5*x^7 + 15*a^11*b^4*x^6 + 20*a^12*b^3*x^5 +
 15*a^13*b^2*x^4 + 6*a^14*b*x^3 + a^15*x^2)
```

Sympy [A] time = 1.69975, size = 175, normalized size = 1.22

$$\frac{-15a^7+120a^6bx+2058a^5b^2x^2+7308a^4b^3x^3+11970a^3b^4x^4+10360a^2b^5x^5+4620ab^6x^6+840b^7x^7}{30a^{14}x^2+180a^{13}bx^3+450a^{12}b^2x^4+600a^{11}b^3x^5+450a^{10}b^4x^6+180a^9b^5x^7+30a^8b^6x^8}+\frac{28b^2\left(\log(x)\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**7,x)
```

```
[Out] (-15*a**7 + 120*a**6*b*x + 2058*a**5*b**2*x**2 + 7308*a**4*b**3*x**3 + 1197
0*a**3*b**4*x**4 + 10360*a**2*b**5*x**5 + 4620*a*b**6*x**6 + 840*b**7*x**7)
/(30*a**14*x**2 + 180*a**13*b*x**3 + 450*a**12*b**2*x**4 + 600*a**11*b**3*x
**5 + 450*a**10*b**4*x**6 + 180*a**9*b**5*x**7 + 30*a**8*b**6*x**8) + 28*b*
*2*(log(x) - log(a/b + x))/a**9
```

Giac [A] time = 1.19445, size = 161, normalized size = 1.12

$$-\frac{28\,b^2\log\left(|bx+a|\right)}{a^9}+\frac{28\,b^2\log\left(|x|\right)}{a^9}+\frac{840\,ab^7x^7+4620\,a^2b^6x^6+10360\,a^3b^5x^5+11970\,a^4b^4x^4+7308\,a^5b^3x^3+205}{30\,(bx+a)^6a^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -28*b^2*log(abs(b*x + a))/a^9 + 28*b^2*log(abs(x))/a^9 + 1/30*(840*a*b^7*x^
7 + 4620*a^2*b^6*x^6 + 10360*a^3*b^5*x^5 + 11970*a^4*b^4*x^4 + 7308*a^5*b^3
*x^3 + 2058*a^6*b^2*x^2 + 120*a^7*b*x - 15*a^8)/((b*x + a)^6*a^9*x^2)
```

3.221 $\int \frac{1}{x^4(a+bx)^7} dx$

Optimal. Leaf size=157

$$-\frac{56b^3}{a^9(a+bx)}-\frac{35b^3}{2a^8(a+bx)^2}-\frac{20b^3}{3a^7(a+bx)^3}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{b^3}{6a^4(a+bx)^6}-\frac{28b^2}{a^9x}-\frac{84b^3\log(x)}{a^{10}}+\frac{84b^3}{}$$

```
[Out] -1/(3*a^7*x^3) + (7*b)/(2*a^8*x^2) - (28*b^2)/(a^9*x) - b^3/(6*a^4*(a + b*x
)^6) - (4*b^3)/(5*a^5*(a + b*x)^5) - (5*b^3)/(2*a^6*(a + b*x)^4) - (20*b^3)
/(3*a^7*(a + b*x)^3) - (35*b^3)/(2*a^8*(a + b*x)^2) - (56*b^3)/(a^9*(a + b*
x)) - (84*b^3*Log[x])/a^10 + (84*b^3*Log[a + b*x])/a^10
```

Rubi [A] time = 0.102711, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{56b^3}{a^9(a+bx)}-\frac{35b^3}{2a^8(a+bx)^2}-\frac{20b^3}{3a^7(a+bx)^3}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{b^3}{6a^4(a+bx)^6}-\frac{28b^2}{a^9x}-\frac{84b^3\log(x)}{a^{10}}+\frac{84b^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^7),x]
```

```
[Out] -1/(3*a^7*x^3) + (7*b)/(2*a^8*x^2) - (28*b^2)/(a^9*x) - b^3/(6*a^4*(a + b*x
)^6) - (4*b^3)/(5*a^5*(a + b*x)^5) - (5*b^3)/(2*a^6*(a + b*x)^4) - (20*b^3)
/(3*a^7*(a + b*x)^3) - (35*b^3)/(2*a^8*(a + b*x)^2) - (56*b^3)/(a^9*(a + b*
x)) - (84*b^3*Log[x])/a^10 + (84*b^3*Log[a + b*x])/a^10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^7}dx &= \int \left(\frac{1}{a^7x^4}-\frac{7b}{a^8x^3}+\frac{28b^2}{a^9x^2}-\frac{84b^3}{a^{10}x}+\frac{b^4}{a^4(a+bx)^7}+\frac{4b^4}{a^5(a+bx)^6}+\frac{10b^4}{a^6(a+bx)^5}+\frac{20b^4}{a^7(a+bx)^4}+\frac{35b}{a^8(a+}\right.\\ &= -\frac{1}{3a^7x^3}+\frac{7b}{2a^8x^2}-\frac{28b^2}{a^9x}-\frac{b^3}{6a^4(a+bx)^6}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{20b^3}{3a^7(a+bx)^3}-\frac{35b^3}{2a^8(a+bx}\end{aligned}$$

Mathematica [A] time = 0.236212, size = 123, normalized size = 0.78

$$-\frac{\frac{a\left(360a^6b^2x^2+6174a^5b^3x^3+21924a^4b^4x^4+35910a^3b^5x^5+31080a^2b^6x^6-45a^7bx+10a^8+13860ab^7x^7+2520b^8x^8\right)}{x^3(a+bx)^6}-2520b^3\log(a+bx)+2520b^3\log(}{30a^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^7),x]
```

```
[Out] -((a*(10*a^8 - 45*a^7*b*x + 360*a^6*b^2*x^2 + 6174*a^5*b^3*x^3 + 21924*a^4*
b^4*x^4 + 35910*a^3*b^5*x^5 + 31080*a^2*b^6*x^6 + 13860*a*b^7*x^7 + 2520*b^
8*x^8))/(x^3*(a + b*x)^6) + 2520*b^3*Log[x] - 2520*b^3*Log[a + b*x])/(30*a^
10)
```

Maple [A] time = 0.014, size = 144, normalized size = 0.9

$$-\frac{1}{3\,a^{7}x^{3}}+\frac{7\,b}{2\,a^{8}x^{2}}-28\,\frac{b^{2}}{a^{9}x}-\frac{b^{3}}{6\,a^{4}\left(bx+a\right)^{6}}-\frac{4\,b^{3}}{5\,a^{5}\left(bx+a\right)^{5}}-\frac{5\,b^{3}}{2\,a^{6}\left(bx+a\right)^{4}}-\frac{20\,b^{3}}{3\,a^{7}\left(bx+a\right)^{3}}-\frac{35\,b^{3}}{2\,a^{8}\left(bx+a\right)^{2}}-56\,\frac{}{a^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^7,x)
```

```
[Out] -1/3/a^7/x^3+7/2*b/a^8/x^2-28*b^2/a^9/x-1/6*b^3/a^4/(b*x+a)^6-4/5*b^3/a^5/(
b*x+a)^5-5/2*b^3/a^6/(b*x+a)^4-20/3*b^3/a^7/(b*x+a)^3-35/2*b^3/a^8/(b*x+a)^
2-56*b^3/a^9/(b*x+a)-84*b^3*ln(x)/a^10+84*b^3*ln(b*x+a)/a^10
```

Maxima [A] time = 1.1492, size = 250, normalized size = 1.59

$$-\frac{2520\,b^{8}x^{8}+13860\,ab^{7}x^{7}+31080\,a^{2}b^{6}x^{6}+35910\,a^{3}b^{5}x^{5}+21924\,a^{4}b^{4}x^{4}+6174\,a^{5}b^{3}x^{3}+360\,a^{6}b^{2}x^{2}-45\,a^{7}bx+10}{30\left(a^{9}b^{6}x^{9}+6\,a^{10}b^{5}x^{8}+15\,a^{11}b^{4}x^{7}+20\,a^{12}b^{3}x^{6}+15\,a^{13}b^{2}x^{5}+6\,a^{14}bx^{4}+a^{15}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(2520*b^8*x^8 + 13860*a*b^7*x^7 + 31080*a^2*b^6*x^6 + 35910*a^3*b^5*x
^5 + 21924*a^4*b^4*x^4 + 6174*a^5*b^3*x^3 + 360*a^6*b^2*x^2 - 45*a^7*b*x +
10*a^8)/(a^9*b^6*x^9 + 6*a^10*b^5*x^8 + 15*a^11*b^4*x^7 + 20*a^12*b^3*x^6 +
 15*a^13*b^2*x^5 + 6*a^14*b*x^4 + a^15*x^3) + 84*b^3*log(b*x + a)/a^10 - 84
*b^3*log(x)/a^10
```

Fricas [B] time = 1.62415, size = 701, normalized size = 4.46

$$-\frac{2520\,ab^{8}x^{8}+13860\,a^{2}b^{7}x^{7}+31080\,a^{3}b^{6}x^{6}+35910\,a^{4}b^{5}x^{5}+21924\,a^{5}b^{4}x^{4}+6174\,a^{6}b^{3}x^{3}+360\,a^{7}b^{2}x^{2}-45\,a^{8}bx+}{30\left(a^{1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(2520*a*b^8*x^8 + 13860*a^2*b^7*x^7 + 31080*a^3*b^6*x^6 + 35910*a^4*b
^5*x^5 + 21924*a^5*b^4*x^4 + 6174*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 45*a^8*b*
x + 10*a^9 - 2520*(b^9*x^9 + 6*a*b^8*x^8 + 15*a^2*b^7*x^7 + 20*a^3*b^6*x^6
+ 15*a^4*b^5*x^5 + 6*a^5*b^4*x^4 + a^6*b^3*x^3)*log(b*x + a) + 2520*(b^9*x^
9 + 6*a*b^8*x^8 + 15*a^2*b^7*x^7 + 20*a^3*b^6*x^6 + 15*a^4*b^5*x^5 + 6*a^5*
b^4*x^4 + a^6*b^3*x^3)*log(x))/(a^10*b^6*x^9 + 6*a^11*b^5*x^8 + 15*a^12*b^4
*x^7 + 20*a^13*b^3*x^6 + 15*a^14*b^2*x^5 + 6*a^15*b*x^4 + a^16*x^3)
```

Sympy [A] time = 1.70848, size = 187, normalized size = 1.19

$$-\frac{10a^8 - 45a^7bx + 360a^6b^2x^2 + 6174a^5b^3x^3 + 21924a^4b^4x^4 + 35910a^3b^5x^5 + 31080a^2b^6x^6 + 13860ab^7x^7 + 2520b^8x^8}{30a^{15}x^3 + 180a^{14}bx^4 + 450a^{13}b^2x^5 + 600a^{12}b^3x^6 + 450a^{11}b^4x^7 + 180a^{10}b^5x^8 + 30a^9b^6x^9} + \frac{8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**7,x)
```

```
[Out] -(10*a**8 - 45*a**7*b*x + 360*a**6*b**2*x**2 + 6174*a**5*b**3*x**3 + 21924*
a**4*b**4*x**4 + 35910*a**3*b**5*x**5 + 31080*a**2*b**6*x**6 + 13860*a*b**7
*x**7 + 2520*b**8*x**8)/(30*a**15*x**3 + 180*a**14*b*x**4 + 450*a**13*b**2*
x**5 + 600*a**12*b**3*x**6 + 450*a**11*b**4*x**7 + 180*a**10*b**5*x**8 + 30
*a**9*b**6*x**9) + 84*b**3*(-log(x) + log(a/b + x))/a**10
```

Giac [A] time = 1.19823, size = 176, normalized size = 1.12

$$\frac{84\, b^3 \log\left(|bx + a|\right)}{a^{10}} - \frac{84\, b^3 \log\left(|x|\right)}{a^{10}} - \frac{2520\, ab^8x^8 + 13860\, a^2b^7x^7 + 31080\, a^3b^6x^6 + 35910\, a^4b^5x^5 + 21924\, a^5b^4x^4 + 6174}{30\, (bx + a)^6 a^{10}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 84*b^3*log(abs(b*x + a))/a^10 - 84*b^3*log(abs(x))/a^10 - 1/30*(2520*a*b^8*
x^8 + 13860*a^2*b^7*x^7 + 31080*a^3*b^6*x^6 + 35910*a^4*b^5*x^5 + 21924*a^5
*b^4*x^4 + 6174*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 45*a^8*b*x + 10*a^9)/((b*x
+ a)^6*a^10*x^3)
```

3.222 $\int \frac{x^{12}}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=186

$$-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\frac{99a^8}{b^{13}(a+bx)^5}+\frac{198a^7}{b^{13}(a+bx)^4}-\frac{308a^6}{b^{13}(a+bx)^3}+\frac{3}{b^{13}(}$$

```
[Out] (55*a^2*x)/b^12 - (5*a*x^2)/b^11 + x^3/(3*b^10) - a^12/(9*b^13*(a + b*x)^9)
 + (3*a^11)/(2*b^13*(a + b*x)^8) - (66*a^10)/(7*b^13*(a + b*x)^7) + (110*a^
9)/(3*b^13*(a + b*x)^6) - (99*a^8)/(b^13*(a + b*x)^5) + (198*a^7)/(b^13*(a
+ b*x)^4) - (308*a^6)/(b^13*(a + b*x)^3) + (396*a^5)/(b^13*(a + b*x)^2) - (
495*a^4)/(b^13*(a + b*x)) - (220*a^3*Log[a + b*x])/b^13
```

Rubi [A] time = 0.176762, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\frac{99a^8}{b^{13}(a+bx)^5}+\frac{198a^7}{b^{13}(a+bx)^4}-\frac{308a^6}{b^{13}(a+bx)^3}+\frac{3}{b^{13}(}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x)^10,x]
```

```
[Out] (55*a^2*x)/b^12 - (5*a*x^2)/b^11 + x^3/(3*b^10) - a^12/(9*b^13*(a + b*x)^9)
 + (3*a^11)/(2*b^13*(a + b*x)^8) - (66*a^10)/(7*b^13*(a + b*x)^7) + (110*a^
9)/(3*b^13*(a + b*x)^6) - (99*a^8)/(b^13*(a + b*x)^5) + (198*a^7)/(b^13*(a
+ b*x)^4) - (308*a^6)/(b^13*(a + b*x)^3) + (396*a^5)/(b^13*(a + b*x)^2) - (
495*a^4)/(b^13*(a + b*x)) - (220*a^3*Log[a + b*x])/b^13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{12}}{(a+bx)^{10}}\,dx&=\int \left(\frac{55a^2}{b^{12}}-\frac{10ax}{b^{11}}+\frac{x^2}{b^{10}}+\frac{a^{12}}{b^{12}(a+bx)^{10}}-\frac{12a^{11}}{b^{12}(a+bx)^9}+\frac{66a^{10}}{b^{12}(a+bx)^8}-\frac{220a^9}{b^{12}(a+bx)^7}+\frac{495a^8}{b^{12}(a+b}\right.\\&=\frac{55a^2x}{b^{12}}-\frac{5ax^2}{b^{11}}+\frac{x^3}{3b^{10}}-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\frac{99a}{b^{13}(a+}\end{aligned}$$

Mathematica [A] time = 0.115452, size = 161, normalized size = 0.87

$$-\frac{1031616a^{10}b^2x^2+2074464a^9b^3x^3+2529576a^8b^4x^4+1831032a^7b^5x^5+638568a^6b^6x^6-58968a^5b^7x^7-139482a^4b^}{126b^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x)^10,x]
```

```
[Out] -(35201*a^12 + 289089*a^11*b*x + 1031616*a^10*b^2*x^2 + 2074464*a^9*b^3*x^3
 + 2529576*a^8*b^4*x^4 + 1831032*a^7*b^5*x^5 + 638568*a^6*b^6*x^6 - 58968*a
^5*b^7*x^7 - 139482*a^4*b^8*x^8 - 43218*a^3*b^9*x^9 - 2772*a^2*b^10*x^10 +
252*a*b^11*x^11 - 42*b^12*x^12 + 27720*a^3*(a + b*x)^9*Log[a + b*x])/(126*b
^13*(a + b*x)^9)
```

Maple [A] time = 0.011, size = 177, normalized size = 1.

$$55\,{\frac {{a}^{2}x}{{b}^{12}}}-5\,{\frac {a{x}^{2}}{{b}^{11}}}+{\frac {{x}^{3}}{3\,{b}^{10}}}-{\frac {{a}^{12}}{9\,{b}^{13} \left(bx+a \right) ^{9}}}+{\frac {3\,{a}^{11}}{2\,{b}^{13} \left(bx+a \right) ^{8}}}-{\frac {66\,{a}^{10}}{7\,{b}^{13} \left(bx+a \right) ^{7}}}+{\frac {110\,{a}^{9}}{3\,{b}^{13} \left(bx+a \right) ^{6}}}-99\,{\frac {{a}^{8}}{{b}^{13} \left(bx+a \right) ^{5}}}+198\,\frac{\ldots}{b^{13}\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x+a)^10,x)
```

```
[Out] 55*a^2*x/b^12-5*a*x^2/b^11+1/3*x^3/b^10-1/9*a^12/b^13/(b*x+a)^9+3/2*a^11/b^
13/(b*x+a)^8-66/7*a^10/b^13/(b*x+a)^7+110/3*a^9/b^13/(b*x+a)^6-99*a^8/b^13/
(b*x+a)^5+198*a^7/b^13/(b*x+a)^4-308*a^6/b^13/(b*x+a)^3+396*a^5/b^13/(b*x+a
)^2-495*a^4/b^13/(b*x+a)-220*a^3*ln(b*x+a)/b^13
```

Maple [A] time = 1.19224, size = 316, normalized size = 1.7

$$-\frac{62370\,a^{4}b^{8}x^{8}+449064\,a^{5}b^{7}x^{7}+1435896\,a^{6}b^{6}x^{6}+2652804\,a^{7}b^{5}x^{5}+3089394\,a^{8}b^{4}x^{4}+2318316\,a^{9}b^{3}x^{3}+1093356\,a^{10}\ldots}{126\left(b^{22}x^{9}+9\,ab^{21}x^{8}+36\,a^{2}b^{20}x^{7}+84\,a^{3}b^{19}x^{6}+126\,a^{4}b^{18}x^{5}+126\,a^{5}b^{17}x^{4}+84\,a^{6}b^{16}x^{3}+36\,a^{7}b^{15}x^{2}+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/126*(62370*a^4*b^8*x^8 + 449064*a^5*b^7*x^7 + 1435896*a^6*b^6*x^6 + 2652
804*a^7*b^5*x^5 + 3089394*a^8*b^4*x^4 + 2318316*a^9*b^3*x^3 + 1093356*a^10*
b^2*x^2 + 296019*a^11*b*x + 35201*a^12)/(b^22*x^9 + 9*a*b^21*x^8 + 36*a^2*b
^20*x^7 + 84*a^3*b^19*x^6 + 126*a^4*b^18*x^5 + 126*a^5*b^17*x^4 + 84*a^6*b^
16*x^3 + 36*a^7*b^15*x^2 + 9*a^8*b^14*x + a^9*b^13) - 220*a^3*log(b*x + a)/
b^13 + 1/3*(b^2*x^3 - 15*a*b*x^2 + 165*a^2*x)/b^12
```

Fricas [A] time = 1.55658, size = 811, normalized size = 4.36

$$\frac{42\,b^{12}x^{12}-252\,ab^{11}x^{11}+2772\,a^{2}b^{10}x^{10}+43218\,a^{3}b^{9}x^{9}+139482\,a^{4}b^{8}x^{8}+58968\,a^{5}b^{7}x^{7}-638568\,a^{6}b^{6}x^{6}-1831032\,a^{7}b\ldots}{126\left(b^{22}x^{9}\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/126*(42*b^12*x^12 - 252*a*b^11*x^11 + 2772*a^2*b^10*x^10 + 43218*a^3*b^9*
x^9 + 139482*a^4*b^8*x^8 + 58968*a^5*b^7*x^7 - 638568*a^6*b^6*x^6 - 1831032
*a^7*b^5*x^5 - 2529576*a^8*b^4*x^4 - 2074464*a^9*b^3*x^3 - 1031616*a^10*b^2
*x^2 - 289089*a^11*b*x - 35201*a^12 - 27720*(a^3*b^9*x^9 + 9*a^4*b^8*x^8 +
36*a^5*b^7*x^7 + 84*a^6*b^6*x^6 + 126*a^7*b^5*x^5 + 126*a^8*b^4*x^4 + 84*a^
9*b^3*x^3 + 36*a^10*b^2*x^2 + 9*a^11*b*x + a^12)*log(b*x + a))/(b^22*x^9 +
```

```
9*a*b^21*x^8 + 36*a^2*b^20*x^7 + 84*a^3*b^19*x^6 + 126*a^4*b^18*x^5 + 126*a
^5*b^17*x^4 + 84*a^6*b^16*x^3 + 36*a^7*b^15*x^2 + 9*a^8*b^14*x + a^9*b^13)
```

Sympy [A] time = 2.24824, size = 248, normalized size = 1.33

$$-\frac{220a^3\log(a+bx)}{b^{13}}+\frac{55a^2x}{b^{12}}-\frac{5ax^2}{b^{11}}-\frac{35201a^{12}+296019a^{11}bx+1093356a^{10}b^2x^2+2318316a^9b^3x^3+3089394a^8b^4}{126a^9b^{13}+1134a^8b^{14}x+4536a^7b^{15}x^2+10584a^6b^{16}x^3+15876a^5b^{17}x^4+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x+a)**10,x)
```

```
[Out] -220*a**3*log(a + b*x)/b**13 + 55*a**2*x/b**12 - 5*a*x**2/b**11 - (35201*a*
*12 + 296019*a**11*b*x + 1093356*a**10*b**2*x**2 + 2318316*a**9*b**3*x**3 +
 3089394*a**8*b**4*x**4 + 2652804*a**7*b**5*x**5 + 1435896*a**6*b**6*x**6 +
 449064*a**5*b**7*x**7 + 62370*a**4*b**8*x**8)/(126*a**9*b**13 + 1134*a**8*
b**14*x + 4536*a**7*b**15*x**2 + 10584*a**6*b**16*x**3 + 15876*a**5*b**17*x
**4 + 15876*a**4*b**18*x**5 + 10584*a**3*b**19*x**6 + 4536*a**2*b**20*x**7
+ 1134*a*b**21*x**8 + 126*b**22*x**9) + x**3/(3*b**10)
```

Giac [A] time = 1.19218, size = 201, normalized size = 1.08

$$-\frac{220\,a^3\log(|bx+a|)}{b^{13}}-\frac{62370\,a^4b^8x^8+449064\,a^5b^7x^7+1435896\,a^6b^6x^6+2652804\,a^7b^5x^5+3089394\,a^8b^4x^4+2318}{126\,(bx+a)^9b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -220*a^3*log(abs(b*x + a))/b^13 - 1/126*(62370*a^4*b^8*x^8 + 449064*a^5*b^7
*x^7 + 1435896*a^6*b^6*x^6 + 2652804*a^7*b^5*x^5 + 3089394*a^8*b^4*x^4 + 23
18316*a^9*b^3*x^3 + 1093356*a^10*b^2*x^2 + 296019*a^11*b*x + 35201*a^12)/((
b*x + a)^9*b^13) + 1/3*(b^20*x^3 - 15*a*b^19*x^2 + 165*a^2*b^18*x)/b^30
```

3.223 $\int \frac{x^{11}}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=177

$$\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+bx)^5}-\frac{231a^6}{2b^{12}(a+bx)^4}+\frac{154a^5}{b^{12}(a+bx)^3}-\frac{165a^4}{b^{12}(a+bx)^2}$$

```
[Out] (-10*a*x)/b^11 + x^2/(2*b^10) + a^11/(9*b^12*(a + b*x)^9) - (11*a^10)/(8*b^
12*(a + b*x)^8) + (55*a^9)/(7*b^12*(a + b*x)^7) - (55*a^8)/(2*b^12*(a + b*x
)^6) + (66*a^7)/(b^12*(a + b*x)^5) - (231*a^6)/(2*b^12*(a + b*x)^4) + (154*
a^5)/(b^12*(a + b*x)^3) - (165*a^4)/(b^12*(a + b*x)^2) + (165*a^3)/(b^12*(a
 + b*x)) + (55*a^2*Log[a + b*x])/b^12
```

Rubi [A] time = 0.142378, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+bx)^5}-\frac{231a^6}{2b^{12}(a+bx)^4}+\frac{154a^5}{b^{12}(a+bx)^3}-\frac{165a^4}{b^{12}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x)^10,x]
```

```
[Out] (-10*a*x)/b^11 + x^2/(2*b^10) + a^11/(9*b^12*(a + b*x)^9) - (11*a^10)/(8*b^
12*(a + b*x)^8) + (55*a^9)/(7*b^12*(a + b*x)^7) - (55*a^8)/(2*b^12*(a + b*x
)^6) + (66*a^7)/(b^12*(a + b*x)^5) - (231*a^6)/(2*b^12*(a + b*x)^4) + (154*
a^5)/(b^12*(a + b*x)^3) - (165*a^4)/(b^12*(a + b*x)^2) + (165*a^3)/(b^12*(a
 + b*x)) + (55*a^2*Log[a + b*x])/b^12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{(a+bx)^{10}}\,dx &= \int \left(-\frac{10a}{b^{11}}+\frac{x}{b^{10}}-\frac{a^{11}}{b^{11}(a+bx)^{10}}+\frac{11a^{10}}{b^{11}(a+bx)^9}-\frac{55a^9}{b^{11}(a+bx)^8}+\frac{165a^8}{b^{11}(a+bx)^7}-\frac{330a^7}{b^{11}(a+bx)^6}+\frac{46}{b^{11}(a}\right.\\ &= -\frac{10ax}{b^{11}}+\frac{x^2}{2b^{10}}+\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+bx)^5}-\frac{}{2b}\end{aligned}$$

Mathematica [A] time = 0.0468254, size = 150, normalized size = 0.85

$$\frac{1281096a^9b^2x^2+2656584a^8b^3x^3+3402756a^7b^4x^4+2704212a^6b^5x^5+1220688a^5b^6x^6+190512a^4b^7x^7-77112a^3b^8x^8-}{504b^{12}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x)^10,x]
```

```
[Out] (42131*a^11 + 351459*a^10*b*x + 1281096*a^9*b^2*x^2 + 2656584*a^8*b^3*x^3 +
 3402756*a^7*b^4*x^4 + 2704212*a^6*b^5*x^5 + 1220688*a^5*b^6*x^6 + 190512*a
^4*b^7*x^7 - 77112*a^3*b^8*x^8 - 36288*a^2*b^9*x^9 - 2772*a*b^10*x^10 + 252
*b^11*x^11 + 27720*a^2*(a + b*x)^9*Log[a + b*x])/(504*b^12*(a + b*x)^9)
```

Maple [A] time = 0.011, size = 166, normalized size = 0.9

$$-10\,\frac{ax}{b^{11}}+\frac{x^2}{2\,b^{10}}+\frac{a^{11}}{9\,b^{12}\,(bx+a)^9}-\frac{11\,a^{10}}{8\,b^{12}\,(bx+a)^8}+\frac{55\,a^9}{7\,b^{12}\,(bx+a)^7}-\frac{55\,a^8}{2\,b^{12}\,(bx+a)^6}+66\,\frac{a^7}{b^{12}\,(bx+a)^5}-\frac{231\,a^6}{2\,b^{12}\,(bx+ \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x+a)^10,x)
```

```
[Out] -10*a*x/b^11+1/2*x^2/b^10+1/9*a^11/b^12/(b*x+a)^9-11/8*a^10/b^12/(b*x+a)^8+
55/7*a^9/b^12/(b*x+a)^7-55/2*a^8/b^12/(b*x+a)^6+66*a^7/b^12/(b*x+a)^5-231/2
*a^6/b^12/(b*x+a)^4+154*a^5/b^12/(b*x+a)^3-165*a^4/b^12/(b*x+a)^2+165*a^3/b
^12/(b*x+a)+55*a^2*ln(b*x+a)/b^12
```

Maxima [A] time = 1.12964, size = 301, normalized size = 1.7

$$\frac{83160\,a^3b^8x^8+582120\,a^4b^7x^7+1823976\,a^5b^6x^6+3318084\,a^6b^5x^5+3817044\,a^7b^4x^4+2835756\,a^8b^3x^3+1326204\,a^{9} \cdots}{504\left(b^{21}x^9+9\,ab^{20}x^8+36\,a^2b^{19}x^7+84\,a^3b^{18}x^6+126\,a^4b^{17}x^5+126\,a^5b^{16}x^4+84\,a^6b^{15}x^3+36\,a^7b^{14}x^2 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/504*(83160*a^3*b^8*x^8 + 582120*a^4*b^7*x^7 + 1823976*a^5*b^6*x^6 + 33180
84*a^6*b^5*x^5 + 3817044*a^7*b^4*x^4 + 2835756*a^8*b^3*x^3 + 1326204*a^9*b^
2*x^2 + 356499*a^10*b*x + 42131*a^11)/(b^21*x^9 + 9*a*b^20*x^8 + 36*a^2*b^1
9*x^7 + 84*a^3*b^18*x^6 + 126*a^4*b^17*x^5 + 126*a^5*b^16*x^4 + 84*a^6*b^15
*x^3 + 36*a^7*b^14*x^2 + 9*a^8*b^13*x + a^9*b^12) + 55*a^2*log(b*x + a)/b^1
2 + 1/2*(b*x^2 - 20*a*x)/b^11
```

Fricas [A] time = 1.44952, size = 784, normalized size = 4.43

$$\frac{252\,b^{11}x^{11}-2772\,ab^{10}x^{10}-36288\,a^2b^9x^9-77112\,a^3b^8x^8+190512\,a^4b^7x^7+1220688\,a^5b^6x^6+2704212\,a^6b^5x^5+340 \cdots}{504\left(b^{21}x^9+9 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^11*x^11 - 2772*a*b^10*x^10 - 36288*a^2*b^9*x^9 - 77112*a^3*b^8
*x^8 + 190512*a^4*b^7*x^7 + 1220688*a^5*b^6*x^6 + 2704212*a^6*b^5*x^5 + 340
2756*a^7*b^4*x^4 + 2656584*a^8*b^3*x^3 + 1281096*a^9*b^2*x^2 + 351459*a^10*
b*x + 42131*a^11 + 27720*(a^2*b^9*x^9 + 9*a^3*b^8*x^8 + 36*a^4*b^7*x^7 + 84
*a^5*b^6*x^6 + 126*a^6*b^5*x^5 + 126*a^7*b^4*x^4 + 84*a^8*b^3*x^3 + 36*a^9*
b^2*x^2 + 9*a^10*b*x + a^11)*log(b*x + a))/(b^21*x^9 + 9*a*b^20*x^8 + 36*a^
2*b^19*x^7 + 84*a^3*b^18*x^6 + 126*a^4*b^17*x^5 + 126*a^5*b^16*x^4 + 84*a^6
```

```
*b^15*x^3 + 36*a^7*b^14*x^2 + 9*a^8*b^13*x + a^9*b^12)
```

Sympy [A] time = 2.12916, size = 236, normalized size = 1.33

$$\frac{55a^2\log{(a+bx)}}{b^{12}} - \frac{10ax}{b^{11}} + \frac{42131a^{11} + 356499a^{10}bx + 1326204a^9b^2x^2 + 2835756a^8b^3x^3 + 3817044a^7b^4x^4 + 3318084a^6 \cdots}{504a^9b^{12} + 4536a^8b^{13}x + 18144a^7b^{14}x^2 + 42336a^6b^{15}x^3 + 63504a^5b^{16}x^4 + 63504a^4b^{17}x^5 + \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x+a)**10,x)
```

```
[Out] 55*a**2*log(a + b*x)/b**12 - 10*a*x/b**11 + (42131*a**11 + 356499*a**10*b*x
 + 1326204*a**9*b**2*x**2 + 2835756*a**8*b**3*x**3 + 3817044*a**7*b**4*x**4
 + 3318084*a**6*b**5*x**5 + 1823976*a**5*b**6*x**6 + 582120*a**4*b**7*x**7
+ 83160*a**3*b**8*x**8)/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b*
*14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17
*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 +
 504*b**21*x**9) + x**2/(2*b**10)
```

Giac [A] time = 1.2484, size = 186, normalized size = 1.05

$$\frac{55\,a^2\log{(|bx+a|)}}{b^{12}} + \frac{b^{10}x^2 - 20\,ab^9x}{2\,b^{20}} + \frac{83160\,a^3b^8x^8 + 582120\,a^4b^7x^7 + 1823976\,a^5b^6x^6 + 3318084\,a^6b^5x^5 + 3817044\,a \cdots}{504\,(bx+a)^9 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 55*a^2*log(abs(b*x + a))/b^12 + 1/2*(b^10*x^2 - 20*a*b^9*x)/b^20 + 1/504*(8
3160*a^3*b^8*x^8 + 582120*a^4*b^7*x^7 + 1823976*a^5*b^6*x^6 + 3318084*a^6*b
^5*x^5 + 3817044*a^7*b^4*x^4 + 2835756*a^8*b^3*x^3 + 1326204*a^9*b^2*x^2 +
356499*a^10*b*x + 42131*a^11)/((b*x + a)^9*b^12)
```

3.224 $\int \frac{x^{10}}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=159

$$-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{63a^5}{b^{11}(a+bx)^4}-\frac{70a^4}{b^{11}(a+bx)^3}+\frac{6}{b^{11}(a}$$

```
[Out] x/b^10 - a^10/(9*b^11*(a + b*x)^9) + (5*a^9)/(4*b^11*(a + b*x)^8) - (45*a^8
)/(7*b^11*(a + b*x)^7) + (20*a^7)/(b^11*(a + b*x)^6) - (42*a^6)/(b^11*(a +
b*x)^5) + (63*a^5)/(b^11*(a + b*x)^4) - (70*a^4)/(b^11*(a + b*x)^3) + (60*a
^3)/(b^11*(a + b*x)^2) - (45*a^2)/(b^11*(a + b*x)) - (10*a*Log[a + b*x])/b^
11
```

Rubi [A] time = 0.120925, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{63a^5}{b^{11}(a+bx)^4}-\frac{70a^4}{b^{11}(a+bx)^3}+\frac{6}{b^{11}(a}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x)^10,x]
```

```
[Out] x/b^10 - a^10/(9*b^11*(a + b*x)^9) + (5*a^9)/(4*b^11*(a + b*x)^8) - (45*a^8
)/(7*b^11*(a + b*x)^7) + (20*a^7)/(b^11*(a + b*x)^6) - (42*a^6)/(b^11*(a +
b*x)^5) + (63*a^5)/(b^11*(a + b*x)^4) - (70*a^4)/(b^11*(a + b*x)^3) + (60*a
^3)/(b^11*(a + b*x)^2) - (45*a^2)/(b^11*(a + b*x)) - (10*a*Log[a + b*x])/b^
11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{(a+bx)^{10}}\,dx &= \int \left(\frac{1}{b^{10}}+\frac{a^{10}}{b^{10}(a+bx)^{10}}-\frac{10a^9}{b^{10}(a+bx)^9}+\frac{45a^8}{b^{10}(a+bx)^8}-\frac{120a^7}{b^{10}(a+bx)^7}+\frac{210a^6}{b^{10}(a+bx)^6}-\frac{252a^5}{b^{10}(a+bx}\right.\\ &= \frac{x}{b^{10}}-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{63a^5}{b^{11}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0707504, size = 137, normalized size = 0.86

$$-\frac{153576a^8b^2x^2+328104a^7b^3x^3+439236a^6b^4x^4+375732a^5b^5x^5+197568a^4b^6x^6+54432a^3b^7x^7+2268a^2b^8x^8+41}{252b^{11}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x)^10,x]
```

```
[Out] -(4861*a^10 + 41229*a^9*b*x + 153576*a^8*b^2*x^2 + 328104*a^7*b^3*x^3 + 439
236*a^6*b^4*x^4 + 375732*a^5*b^5*x^5 + 197568*a^4*b^6*x^6 + 54432*a^3*b^7*x
^7 + 2268*a^2*b^8*x^8 - 2268*a*b^9*x^9 - 252*b^10*x^10 + 2520*a*(a + b*x)^9
*Log[a + b*x])/(252*b^11*(a + b*x)^9)
```

Maple [A] time = 0.011, size = 154, normalized size = 1.

$$\frac{x}{b^{10}} - \frac{a^{10}}{9\,b^{11}\,(bx+a)^{9}} + \frac{5\,a^{9}}{4\,b^{11}\,(bx+a)^{8}} - \frac{45\,a^{8}}{7\,b^{11}\,(bx+a)^{7}} + 20\,\frac{a^{7}}{b^{11}\,(bx+a)^{6}} - 42\,\frac{a^{6}}{b^{11}\,(bx+a)^{5}} + 63\,\frac{a^{5}}{b^{11}\,(bx+a)^{4}} - 70\,\frac{}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x+a)^10,x)
```

```
[Out] x/b^10-1/9*a^10/b^11/(b*x+a)^9+5/4*a^9/b^11/(b*x+a)^8-45/7*a^8/b^11/(b*x+a)
^7+20*a^7/b^11/(b*x+a)^6-42*a^6/b^11/(b*x+a)^5+63*a^5/b^11/(b*x+a)^4-70*a^4
/b^11/(b*x+a)^3+60*a^3/b^11/(b*x+a)^2-45*a^2/b^11/(b*x+a)-10*a*ln(b*x+a)/b^
11
```

Maxima [A] time = 1.16287, size = 285, normalized size = 1.79

$$-\frac{11340\,a^{2}b^{8}x^{8} + 75600\,a^{3}b^{7}x^{7} + 229320\,a^{4}b^{6}x^{6} + 407484\,a^{5}b^{5}x^{5} + 460404\,a^{6}b^{4}x^{4} + 337176\,a^{7}b^{3}x^{3} + 155844\,a^{8}b^{2}x^{2} + 4}{252\,\left(b^{20}x^{9} + 9\,ab^{19}x^{8} + 36\,a^{2}b^{18}x^{7} + 84\,a^{3}b^{17}x^{6} + 126\,a^{4}b^{16}x^{5} + 126\,a^{5}b^{15}x^{4} + 84\,a^{6}b^{14}x^{3} + 36\,a^{7}b^{13}x^{2} + 9\,a^{8}b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(11340*a^2*b^8*x^8 + 75600*a^3*b^7*x^7 + 229320*a^4*b^6*x^6 + 407484
*a^5*b^5*x^5 + 460404*a^6*b^4*x^4 + 337176*a^7*b^3*x^3 + 155844*a^8*b^2*x^2
 + 41481*a^9*b*x + 4861*a^10)/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*x^7 +
84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x^3 + 3
6*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11) + x/b^10 - 10*a*log(b*x + a)/b^11
```

Fricas [B] time = 1.51115, size = 736, normalized size = 4.63

$$\frac{252\,b^{10}x^{10} + 2268\,ab^{9}x^{9} - 2268\,a^{2}b^{8}x^{8} - 54432\,a^{3}b^{7}x^{7} - 197568\,a^{4}b^{6}x^{6} - 375732\,a^{5}b^{5}x^{5} - 439236\,a^{6}b^{4}x^{4} - 328104\,a^{7}b^{3}}{252\,\left(b^{20}x^{9} + 9\,ab^{19}x^{8} + 36\,a^{2}b^{18}x^{7}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2268*a*b^9*x^9 - 2268*a^2*b^8*x^8 - 54432*a^3*b^7*x^
7 - 197568*a^4*b^6*x^6 - 375732*a^5*b^5*x^5 - 439236*a^6*b^4*x^4 - 328104*a
^7*b^3*x^3 - 153576*a^8*b^2*x^2 - 41229*a^9*b*x - 4861*a^10 - 2520*(a*b^9*x
^9 + 9*a^2*b^8*x^8 + 36*a^3*b^7*x^7 + 84*a^4*b^6*x^6 + 126*a^5*b^5*x^5 + 12
6*a^6*b^4*x^4 + 84*a^7*b^3*x^3 + 36*a^8*b^2*x^2 + 9*a^9*b*x + a^10)*log(b*x
 + a))/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*x^7 + 84*a^3*b^17*x^6 + 126*a
^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x^3 + 36*a^7*b^13*x^2 + 9*a^8*
b^12*x + a^9*b^11)
```

Sympy [A] time = 1.97515, size = 223, normalized size = 1.4

$$-\frac{10a\log{(a+bx)}}{b^{11}}-\frac{4861a^{10}+41481a^{9}bx+155844a^{8}b^{2}x^{2}+337176a^{7}b^{3}x^{3}+460404a^{6}b^{4}x^{4}+407484a^{5}b^{5}x^{5}+2}{252a^{9}b^{11}+2268a^{8}b^{12}x+9072a^{7}b^{13}x^{2}+21168a^{6}b^{14}x^{3}+31752a^{5}b^{15}x^{4}+31752a^{4}b^{16}x^{5}+21168}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x+a)**10,x)
```

```
[Out] -10*a*log(a + b*x)/b**11 - (4861*a**10 + 41481*a**9*b*x + 155844*a**8*b**2*
x**2 + 337176*a**7*b**3*x**3 + 460404*a**6*b**4*x**4 + 407484*a**5*b**5*x**
5 + 229320*a**4*b**6*x**6 + 75600*a**3*b**7*x**7 + 11340*a**2*b**8*x**8)/(2
52*a**9*b**11 + 2268*a**8*b**12*x + 9072*a**7*b**13*x**2 + 21168*a**6*b**14
*x**3 + 31752*a**5*b**15*x**4 + 31752*a**4*b**16*x**5 + 21168*a**3*b**17*x*
*6 + 9072*a**2*b**18*x**7 + 2268*a*b**19*x**8 + 252*b**20*x**9) + x/b**10
```

Giac [A] time = 1.24039, size = 163, normalized size = 1.03

$$\frac{x}{b^{10}}-\frac{10\,a\log\left(|bx+a|\right)}{b^{11}}-\frac{11340\,a^{2}b^{8}x^{8}+75600\,a^{3}b^{7}x^{7}+229320\,a^{4}b^{6}x^{6}+407484\,a^{5}b^{5}x^{5}+460404\,a^{6}b^{4}x^{4}+337176}{252\,(bx+a)^{9}b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] x/b^10 - 10*a*log(abs(b*x + a))/b^11 - 1/252*(11340*a^2*b^8*x^8 + 75600*a^3
*b^7*x^7 + 229320*a^4*b^6*x^6 + 407484*a^5*b^5*x^5 + 460404*a^6*b^4*x^4 + 3
37176*a^7*b^3*x^3 + 155844*a^8*b^2*x^2 + 41481*a^9*b*x + 4861*a^10)/((b*x +
 a)^9*b^11)
```

3.225 $\int \frac{x^9}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=154

$$\frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+bx)^4}+\frac{28a^3}{b^{10}(a+bx)^3}-\frac{18a}{b^{10}(a+}$$

```
[Out] a^9/(9*b^10*(a + b*x)^9) - (9*a^8)/(8*b^10*(a + b*x)^8) + (36*a^7)/(7*b^10*
(a + b*x)^7) - (14*a^6)/(b^10*(a + b*x)^6) + (126*a^5)/(5*b^10*(a + b*x)^5)
 - (63*a^4)/(2*b^10*(a + b*x)^4) + (28*a^3)/(b^10*(a + b*x)^3) - (18*a^2)/(
b^10*(a + b*x)^2) + (9*a)/(b^10*(a + b*x)) + Log[a + b*x]/b^10
```

Rubi [A] time = 0.108229, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+bx)^4}+\frac{28a^3}{b^{10}(a+bx)^3}-\frac{18a}{b^{10}(a+}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x)^10,x]
```

```
[Out] a^9/(9*b^10*(a + b*x)^9) - (9*a^8)/(8*b^10*(a + b*x)^8) + (36*a^7)/(7*b^10*
(a + b*x)^7) - (14*a^6)/(b^10*(a + b*x)^6) + (126*a^5)/(5*b^10*(a + b*x)^5)
 - (63*a^4)/(2*b^10*(a + b*x)^4) + (28*a^3)/(b^10*(a + b*x)^3) - (18*a^2)/(
b^10*(a + b*x)^2) + (9*a)/(b^10*(a + b*x)) + Log[a + b*x]/b^10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^9}{(a+bx)^{10}}\,dx=\int \left(-\frac{a^9}{b^9(a+bx)^{10}}+\frac{9a^8}{b^9(a+bx)^9}-\frac{36a^7}{b^9(a+bx)^8}+\frac{84a^6}{b^9(a+bx)^7}-\frac{126a^5}{b^9(a+bx)^6}+\frac{126a^4}{b^9(a+bx)^5}-\frac{84a^3}{b^9(a+b}\right.$$

$$=\frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+bx)^4}+\frac{}{b^{10}}$$

Mathematica [A] time = 0.0487159, size = 111, normalized size = 0.72

$$\frac{a\left(235224a^6b^2x^2+518616a^5b^3x^3+725004a^4b^4x^4+661500a^3b^5x^5+388080a^2b^6x^6+61641a^7bx+7129a^8+136080ab^7\right.}{2520b^{10}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x)^10,x]
```

```
[Out] (a*(7129*a^8 + 61641*a^7*b*x + 235224*a^6*b^2*x^2 + 518616*a^5*b^3*x^3 + 72
5004*a^4*b^4*x^4 + 661500*a^3*b^5*x^5 + 388080*a^2*b^6*x^6 + 136080*a*b^7*x
^7 + 22680*b^8*x^8))/(2520*b^10*(a + b*x)^9) + Log[a + b*x]/b^10
```

Maple [A] time = 0.01, size = 145, normalized size = 0.9

$$\frac{a^9}{9\,b^{10}\,(bx+a)^9} - \frac{9\,a^8}{8\,b^{10}\,(bx+a)^8} + \frac{36\,a^7}{7\,b^{10}\,(bx+a)^7} - 14\,\frac{a^6}{b^{10}\,(bx+a)^6} + \frac{126\,a^5}{5\,b^{10}\,(bx+a)^5} - \frac{63\,a^4}{2\,b^{10}\,(bx+a)^4} + 28\,\frac{a^3}{b^{10}\,(bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x+a)^10,x)
```

```
[Out] 1/9*a^9/b^10/(b*x+a)^9-9/8*a^8/b^10/(b*x+a)^8+36/7*a^7/b^10/(b*x+a)^7-14*a^
6/b^10/(b*x+a)^6+126/5*a^5/b^10/(b*x+a)^5-63/2*a^4/b^10/(b*x+a)^4+28*a^3/b^
10/(b*x+a)^3-18*a^2/b^10/(b*x+a)^2+9*a/b^10/(b*x+a)+ln(b*x+a)/b^10
```

Maxima [A] time = 1.07296, size = 273, normalized size = 1.77

$$\frac{22680\,ab^8x^8 + 136080\,a^2b^7x^7 + 388080\,a^3b^6x^6 + 661500\,a^4b^5x^5 + 725004\,a^5b^4x^4 + 518616\,a^6b^3x^3 + 235224\,a^7b^2x^2 +}{2520\left(b^{19}x^9 + 9\,ab^{18}x^8 + 36\,a^2b^{17}x^7 + 84\,a^3b^{16}x^6 + 126\,a^4b^{15}x^5 + 126\,a^5b^{14}x^4 + 84\,a^6b^{13}x^3 + 36\,a^7b^{12}x^2 + 9\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/2520*(22680*a*b^8*x^8 + 136080*a^2*b^7*x^7 + 388080*a^3*b^6*x^6 + 661500*
a^4*b^5*x^5 + 725004*a^5*b^4*x^4 + 518616*a^6*b^3*x^3 + 235224*a^7*b^2*x^2
+ 61641*a^8*b*x + 7129*a^9)/(b^19*x^9 + 9*a*b^18*x^8 + 36*a^2*b^17*x^7 + 84
*a^3*b^16*x^6 + 126*a^4*b^15*x^5 + 126*a^5*b^14*x^4 + 84*a^6*b^13*x^3 + 36*
a^7*b^12*x^2 + 9*a^8*b^11*x + a^9*b^10) + log(b*x + a)/b^10
```

Fricas [B] time = 1.56912, size = 684, normalized size = 4.44

$$\frac{22680\,ab^8x^8 + 136080\,a^2b^7x^7 + 388080\,a^3b^6x^6 + 661500\,a^4b^5x^5 + 725004\,a^5b^4x^4 + 518616\,a^6b^3x^3 + 235224\,a^7b^2x^2 +}{2520\left(b^{19}x^9 + 9\,ab^{18}x^8 + 36\,a^2b^{17}x^7 + 84\,a^3b^{16}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(22680*a*b^8*x^8 + 136080*a^2*b^7*x^7 + 388080*a^3*b^6*x^6 + 661500*
a^4*b^5*x^5 + 725004*a^5*b^4*x^4 + 518616*a^6*b^3*x^3 + 235224*a^7*b^2*x^2
+ 61641*a^8*b*x + 7129*a^9 + 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*a^2*b^7*x^7 +
 84*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b^3*x^3 + 36*a
^7*b^2*x^2 + 9*a^8*b*x + a^9)*log(b*x + a))/(b^19*x^9 + 9*a*b^18*x^8 + 36*a
^2*b^17*x^7 + 84*a^3*b^16*x^6 + 126*a^4*b^15*x^5 + 126*a^5*b^14*x^4 + 84*a^
6*b^13*x^3 + 36*a^7*b^12*x^2 + 9*a^8*b^11*x + a^9*b^10)
```

Sympy [A] time = 1.63164, size = 212, normalized size = 1.38

$$\frac{7129a^9 + 61641a^8bx + 235224a^7b^2x^2 + 518616a^6b^3x^3 + 725004a^5b^4x^4 + 661500a^4b^5x^5 + 388080a^3b^6x^6 + 13}{2520a^9b^{10} + 22680a^8b^{11}x + 90720a^7b^{12}x^2 + 211680a^6b^{13}x^3 + 317520a^5b^{14}x^4 + 317520a^4b^{15}x^5 + 211680a^3b^{16}x^6 + 9072}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x+a)**10,x)
```

```
[Out] (7129*a**9 + 61641*a**8*b*x + 235224*a**7*b**2*x**2 + 518616*a**6*b**3*x**3
 + 725004*a**5*b**4*x**4 + 661500*a**4*b**5*x**5 + 388080*a**3*b**6*x**6 +
136080*a**2*b**7*x**7 + 22680*a*b**8*x**8)/(2520*a**9*b**10 + 22680*a**8*b*
*11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*
x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x
**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + log(a + b*x)/b**10
```

Giac [A] time = 1.1597, size = 151, normalized size = 0.98

$$\frac{\log\left(|bx + a|\right)}{b^{10}} + \frac{22680\, ab^7x^8 + 136080\, a^2b^6x^7 + 388080\, a^3b^5x^6 + 661500\, a^4b^4x^5 + 725004\, a^5b^3x^4 + 518616\, a^6b^2x^3 + 235}{2520\,(bx + a)^9 b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^10 + 1/2520*(22680*a*b^7*x^8 + 136080*a^2*b^6*x^7 + 388
080*a^3*b^5*x^6 + 661500*a^4*b^4*x^5 + 725004*a^5*b^3*x^4 + 518616*a^6*b^2*
x^3 + 235224*a^7*b*x^2 + 61641*a^8*x + 7129*a^9/b)/((b*x + a)^9*b^9)
```

3.226 $\int \frac{x^8}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=17

$$\frac{x^9}{9a(a+bx)^9}$$

```
[Out] x^9/(9*a*(a + b*x)^9)
```

Rubi [A] time = 0.0016771, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$\frac{x^9}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^10,x]
```

```
[Out] x^9/(9*a*(a + b*x)^9)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^8}{(a+bx)^{10}}\,dx = \frac{x^9}{9a(a+bx)^9}$$

Mathematica [B] time = 0.0272101, size = 97, normalized size = 5.71

$$-\frac{36a^6b^2x^2+84a^5b^3x^3+126a^4b^4x^4+126a^3b^5x^5+84a^2b^6x^6+9a^7bx+a^8+36ab^7x^7+9b^8x^8}{9b^9(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^10,x]
```

```
[Out] -(a^8 + 9*a^7*b*x + 36*a^6*b^2*x^2 + 84*a^5*b^3*x^3 + 126*a^4*b^4*x^4 + 126
*a^3*b^5*x^5 + 84*a^2*b^6*x^6 + 36*a*b^7*x^7 + 9*b^8*x^8)/(9*b^9*(a + b*x)^
9)
```

Maple [B] time = 0.005, size = 131, normalized size = 7.7

$$-14\,\frac{a^4}{b^9(bx+a)^5}-\frac{a^8}{9\,b^9(bx+a)^9}-\frac{1}{b^9(bx+a)}+4\,\frac{a}{b^9(bx+a)^2}-4\,\frac{a^6}{b^9(bx+a)^7}+\frac{28\,a^5}{3\,b^9(bx+a)^6}+14\,\frac{a^3}{b^9(bx+a)^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^10,x)
```

```
[Out] -14/b^9*a^4/(b*x+a)^5-1/9/b^9*a^8/(b*x+a)^9-1/b^9/(b*x+a)+4/b^9*a/(b*x+a)^2
-4/b^9*a^6/(b*x+a)^7+28/3/b^9*a^5/(b*x+a)^6+14/b^9*a^3/(b*x+a)^4-28/3/b^9*a
^2/(b*x+a)^3+1/b^9*a^7/(b*x+a)^8
```

Maxima [B] time = 1.10988, size = 251, normalized size = 14.76

$$-\frac{9\,b^{8}x^{8}+36\,ab^{7}x^{7}+84\,a^{2}b^{6}x^{6}+126\,a^{3}b^{5}x^{5}+126\,a^{4}b^{4}x^{4}+84\,a^{5}b^{3}x^{3}+36\,a^{6}b^{2}x^{2}+9\,a^{7}bx+a^{8}}{9\left(b^{18}x^{9}+9\,ab^{17}x^{8}+36\,a^{2}b^{16}x^{7}+84\,a^{3}b^{15}x^{6}+126\,a^{4}b^{14}x^{5}+126\,a^{5}b^{13}x^{4}+84\,a^{6}b^{12}x^{3}+36\,a^{7}b^{11}x^{2}+9\,a^{8}b^{10}x+a^{9}b^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/(b^18*x^9 + 9
*a*b^17*x^8 + 36*a^2*b^16*x^7 + 84*a^3*b^15*x^6 + 126*a^4*b^14*x^5 + 126*a^
5*b^13*x^4 + 84*a^6*b^12*x^3 + 36*a^7*b^11*x^2 + 9*a^8*b^10*x + a^9*b^9)
```

Fricas [B] time = 1.43883, size = 404, normalized size = 23.76

$$-\frac{9\,b^{8}x^{8}+36\,ab^{7}x^{7}+84\,a^{2}b^{6}x^{6}+126\,a^{3}b^{5}x^{5}+126\,a^{4}b^{4}x^{4}+84\,a^{5}b^{3}x^{3}+36\,a^{6}b^{2}x^{2}+9\,a^{7}bx+a^{8}}{9\left(b^{18}x^{9}+9\,ab^{17}x^{8}+36\,a^{2}b^{16}x^{7}+84\,a^{3}b^{15}x^{6}+126\,a^{4}b^{14}x^{5}+126\,a^{5}b^{13}x^{4}+84\,a^{6}b^{12}x^{3}+36\,a^{7}b^{11}x^{2}+9\,a^{8}b^{10}x+a^{9}b^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/(b^18*x^9 + 9
*a*b^17*x^8 + 36*a^2*b^16*x^7 + 84*a^3*b^15*x^6 + 126*a^4*b^14*x^5 + 126*a^
5*b^13*x^4 + 84*a^6*b^12*x^3 + 36*a^7*b^11*x^2 + 9*a^8*b^10*x + a^9*b^9)
```

Sympy [B] time = 1.64783, size = 199, normalized size = 11.71

$$-\frac{a^{8}+9a^{7}bx+36a^{6}b^{2}x^{2}+84a^{5}b^{3}x^{3}+126a^{4}b^{4}x^{4}+126a^{3}b^{5}x^{5}+84a^{2}b^{6}x^{6}+36ab^{7}x^{7}+9b^{8}x^{8}}{9a^{9}b^{9}+81a^{8}b^{10}x+324a^{7}b^{11}x^{2}+756a^{6}b^{12}x^{3}+1134a^{5}b^{13}x^{4}+1134a^{4}b^{14}x^{5}+756a^{3}b^{15}x^{6}+324a^{2}b^{16}x^{7}+81ab^{17}x^{8}+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**10,x)
```

```
[Out] -(a**8 + 9*a**7*b*x + 36*a**6*b**2*x**2 + 84*a**5*b**3*x**3 + 126*a**4*b**4
*x**4 + 126*a**3*b**5*x**5 + 84*a**2*b**6*x**6 + 36*a*b**7*x**7 + 9*b**8*x*
*8)/(9*a**9*b**9 + 81*a**8*b**10*x + 324*a**7*b**11*x**2 + 756*a**6*b**12*x
**3 + 1134*a**5*b**13*x**4 + 1134*a**4*b**14*x**5 + 756*a**3*b**15*x**6 + 3
24*a**2*b**16*x**7 + 81*a*b**17*x**8 + 9*b**18*x**9)
```

Giac [B] time = 1.21145, size = 128, normalized size = 7.53

$$-\frac{9\,b^{8}x^{8}+36\,ab^{7}x^{7}+84\,a^{2}b^{6}x^{6}+126\,a^{3}b^{5}x^{5}+126\,a^{4}b^{4}x^{4}+84\,a^{5}b^{3}x^{3}+36\,a^{6}b^{2}x^{2}+9\,a^{7}bx+a^{8}}{9\,(bx+a)^{9}b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/((b*x + a)^9*
b^9)
```

3.227 $\int \frac{x^7}{(a+bx)^{10}} dx$

Optimal. Leaf size=35

$$\frac{x^8}{72a^2(a+bx)^8}+\frac{x^8}{9a(a+bx)^9}$$

```
[Out] x^8/(9*a*(a + b*x)^9) + x^8/(72*a^2*(a + b*x)^8)
```

Rubi [A] time = 0.0051056, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{x^8}{72a^2(a+bx)^8}+\frac{x^8}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^10,x]
```

```
[Out] x^8/(9*a*(a + b*x)^9) + x^8/(72*a^2*(a + b*x)^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^{10}}\,dx &= \frac{x^8}{9a(a+bx)^9}+\frac{\int \frac{x^7}{(a+bx)^9}\,dx}{9a}\\ &= \frac{x^8}{9a(a+bx)^9}+\frac{x^8}{72a^2(a+bx)^8}\end{aligned}$$

Mathematica [B] time = 0.0198964, size = 86, normalized size = 2.46

$$-\frac{36a^5b^2x^2+84a^4b^3x^3+126a^3b^4x^4+126a^2b^5x^5+9a^6bx+a^7+84ab^6x^6+36b^7x^7}{72b^8(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^10,x]
```

```
[Out] -(a^7 + 9*a^6*b*x + 36*a^5*b^2*x^2 + 84*a^4*b^3*x^3 + 126*a^3*b^4*x^4 + 126
*a^2*b^5*x^5 + 84*a*b^6*x^6 + 36*b^7*x^7)/(72*b^8*(a + b*x)^9)
```

Maple [B] time = 0.006, size = 117, normalized size = 3.3

$$7\,{\frac{{a}^{3}}{{b}^{8} \left(bx+a \right) ^{5}}}+{\frac{{a}^{7}}{9\,{b}^{8} \left(bx+a \right) ^{9}}}-{\frac{7\,{a}^{6}}{8\,{b}^{8} \left(bx+a \right) ^{8}}}-{\frac{21\,{a}^{2}}{4\,{b}^{8} \left(bx+a \right) ^{4}}}+3\,{\frac{{a}^{5}}{{b}^{8} \left(bx+a \right) ^{7}}}-{\frac{1}{2\,{b}^{8} \left(bx+a \right) ^{2}}}-{\frac{35\,{a}^{4}}{6\,{b}^{8} \left(bx+a \right) ^{6}}}+ \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^10,x)
```

```
[Out] 7/b^8*a^3/(b*x+a)^5+1/9/b^8*a^7/(b*x+a)^9-7/8/b^8*a^6/(b*x+a)^8-21/4/b^8*a^
2/(b*x+a)^4+3/b^8*a^5/(b*x+a)^7-1/2/b^8/(b*x+a)^2-35/6/b^8*a^4/(b*x+a)^6+7/
3/b^8*a/(b*x+a)^3
```

Maxima [B] time = 1.07251, size = 236, normalized size = 6.74

$$-\frac{36\,b^{7}x^{7}+84\,ab^{6}x^{6}+126\,a^{2}b^{5}x^{5}+126\,a^{3}b^{4}x^{4}+84\,a^{4}b^{3}x^{3}+36\,a^{5}b^{2}x^{2}+9\,a^{6}bx+a^{7}}{72\left(b^{17}x^{9}+9\,ab^{16}x^{8}+36\,a^{2}b^{15}x^{7}+84\,a^{3}b^{14}x^{6}+126\,a^{4}b^{13}x^{5}+126\,a^{5}b^{12}x^{4}+84\,a^{6}b^{11}x^{3}+36\,a^{7}b^{10}x^{2}+9\,a^{8}b^{9}x+a \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/(b^17*x^9 + 9*a*b^16*x^8 + 3
6*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^5*b^12*x^4 + 84
*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

Fricas [B] time = 1.49776, size = 382, normalized size = 10.91

$$-\frac{36\,b^{7}x^{7}+84\,ab^{6}x^{6}+126\,a^{2}b^{5}x^{5}+126\,a^{3}b^{4}x^{4}+84\,a^{4}b^{3}x^{3}+36\,a^{5}b^{2}x^{2}+9\,a^{6}bx+a^{7}}{72\left(b^{17}x^{9}+9\,ab^{16}x^{8}+36\,a^{2}b^{15}x^{7}+84\,a^{3}b^{14}x^{6}+126\,a^{4}b^{13}x^{5}+126\,a^{5}b^{12}x^{4}+84\,a^{6}b^{11}x^{3}+36\,a^{7}b^{10}x^{2}+9\,a^{8}b^{9}x+a \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/(b^17*x^9 + 9*a*b^16*x^8 + 3
6*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^5*b^12*x^4 + 84
*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

Sympy [B] time = 1.47864, size = 187, normalized size = 5.34

$$-\frac{a^{7}+9a^{6}bx+36a^{5}b^{2}x^{2}+84a^{4}b^{3}x^{3}+126a^{3}b^{4}x^{4}+126a^{2}b^{5}x^{5}+84ab^{6}x^{6}+36b^{7}x^{7}}{72a^{9}b^{8}+648a^{8}b^{9}x+2592a^{7}b^{10}x^{2}+6048a^{6}b^{11}x^{3}+9072a^{5}b^{12}x^{4}+9072a^{4}b^{13}x^{5}+6048a^{3}b^{14}x^{6}+2592a^{2}b^{15}x^{7}+64 \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**10,x)
```

```
[Out] -(a**7 + 9*a**6*b*x + 36*a**5*b**2*x**2 + 84*a**4*b**3*x**3 + 126*a**3*b**4
*x**4 + 126*a**2*b**5*x**5 + 84*a*b**6*x**6 + 36*b**7*x**7)/(72*a**9*b**8 +
 648*a**8*b**9*x + 2592*a**7*b**10*x**2 + 6048*a**6*b**11*x**3 + 9072*a**5*
b**12*x**4 + 9072*a**4*b**13*x**5 + 6048*a**3*b**14*x**6 + 2592*a**2*b**15*
x**7 + 648*a*b**16*x**8 + 72*b**17*x**9)
```

Giac [B] time = 1.21552, size = 113, normalized size = 3.23

$$-\frac{36 b^7 x^7 + 84 a b^6 x^6 + 126 a^2 b^5 x^5 + 126 a^3 b^4 x^4 + 84 a^4 b^3 x^3 + 36 a^5 b^2 x^2 + 9 a^6 b x + a^7}{72 (b x + a)^9 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/((b*x + a)^9*b^8)
```

3.228 $\int \frac{x^6}{(a+bx)^{10}} dx$

Optimal. Leaf size=52

$$\frac{x^7}{252a^3(a+bx)^7} + \frac{x^7}{36a^2(a+bx)^8} + \frac{x^7}{9a(a+bx)^9}$$

```
[Out] x^7/(9*a*(a + b*x)^9) + x^7/(36*a^2*(a + b*x)^8) + x^7/(252*a^3*(a + b*x)^7
)
```

Rubi [A] time = 0.0096319, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{x^7}{252a^3(a+bx)^7} + \frac{x^7}{36a^2(a+bx)^8} + \frac{x^7}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^10,x]
```

```
[Out] x^7/(9*a*(a + b*x)^9) + x^7/(36*a^2*(a + b*x)^8) + x^7/(252*a^3*(a + b*x)^7
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(a+bx)^{10}} dx &= \frac{x^7}{9a(a+bx)^9} + \frac{2\int \frac{x^6}{(a+bx)^9} dx}{9a} \\
&= \frac{x^7}{9a(a+bx)^9} + \frac{x^7}{36a^2(a+bx)^8} + \frac{\int \frac{x^6}{(a+bx)^8} dx}{36a^2} \\
&= \frac{x^7}{9a(a+bx)^9} + \frac{x^7}{36a^2(a+bx)^8} + \frac{x^7}{252a^3(a+bx)^7}
\end{aligned}$$

Mathematica [A] time = 0.0254138, size = 75, normalized size = 1.44

$$-\frac{36a^4b^2x^2+84a^3b^3x^3+126a^2b^4x^4+9a^5bx+a^6+126ab^5x^5+84b^6x^6}{252b^7(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^10,x]
```

```
[Out] -(a^6 + 9*a^5*b*x + 36*a^4*b^2*x^2 + 84*a^3*b^3*x^3 + 126*a^2*b^4*x^4 + 126
*a*b^5*x^5 + 84*b^6*x^6)/(252*b^7*(a + b*x)^9)
```

Maple [B] time = 0.006, size = 102, normalized size = 2.

$$-\frac{15\,a^4}{7\,b^7\,(bx+a)^7}-\frac{a^6}{9\,b^7\,(bx+a)^9}+\frac{3\,a^5}{4\,b^7\,(bx+a)^8}+\frac{3\,a}{2\,b^7\,(bx+a)^4}+\frac{10\,a^3}{3\,b^7\,(bx+a)^6}-\frac{1}{3\,b^7\,(bx+a)^3}-3\,\frac{a^2}{b^7\,(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^10,x)
```

```
[Out] -15/7/b^7*a^4/(b*x+a)^7-1/9/b^7*a^6/(b*x+a)^9+3/4/b^7*a^5/(b*x+a)^8+3/2/b^7
*a/(b*x+a)^4+10/3/b^7*a^3/(b*x+a)^6-1/3/b^7/(b*x+a)^3-3/b^7*a^2/(b*x+a)^5
```

Maxima [B] time = 1.11422, size = 221, normalized size = 4.25

$$-\frac{84\,b^6x^6+126\,ab^5x^5+126\,a^2b^4x^4+84\,a^3b^3x^3+36\,a^4b^2x^2+9\,a^5bx+a^6}{252\,(b^{16}x^9+9\,ab^{15}x^8+36\,a^2b^{14}x^7+84\,a^3b^{13}x^6+126\,a^4b^{12}x^5+126\,a^5b^{11}x^4+84\,a^6b^{10}x^3+36\,a^7b^9x^2+9\,a^8b^8x+a^9b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/(b^16*x^9 + 9*a*b^15*x^8 + 36*a^2*b^14*x^7 +
 84*a^3*b^13*x^6 + 126*a^4*b^12*x^5 + 126*a^5*b^11*x^4 + 84*a^6*b^10*x^3 +
36*a^7*b^9*x^2 + 9*a^8*b^8*x + a^9*b^7)
```

Fricas [B] time = 1.53719, size = 359, normalized size = 6.9

$$-\frac{84\,b^6x^6+126\,ab^5x^5+126\,a^2b^4x^4+84\,a^3b^3x^3+36\,a^4b^2x^2+9\,a^5bx+a^6}{252\,(b^{16}x^9+9\,ab^{15}x^8+36\,a^2b^{14}x^7+84\,a^3b^{13}x^6+126\,a^4b^{12}x^5+126\,a^5b^{11}x^4+84\,a^6b^{10}x^3+36\,a^7b^9x^2+9\,a^8b^8x+a^9b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/(b^16*x^9 + 9*a*b^15*x^8 + 36*a^2*b^14*x^7 +
 84*a^3*b^13*x^6 + 126*a^4*b^12*x^5 + 126*a^5*b^11*x^4 + 84*a^6*b^10*x^3 +
```

```
36*a^7*b^9*x^2 + 9*a^8*b^8*x + a^9*b^7)
```

Sympy [B] time = 1.41402, size = 175, normalized size = 3.37

$$-\frac{a^6+9a^5bx+36a^4b^2x^2+84a^3b^3x^3+126a^2b^4x^4+126ab^5x^5+84b^6x^6}{252a^9b^7+2268a^8b^8x+9072a^7b^9x^2+21168a^6b^{10}x^3+31752a^5b^{11}x^4+31752a^4b^{12}x^5+21168a^3b^{13}x^6+9072a^2b^{14}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**10,x)
```

```
[Out] -(a**6 + 9*a**5*b*x + 36*a**4*b**2*x**2 + 84*a**3*b**3*x**3 + 126*a**2*b**4
*x**4 + 126*a*b**5*x**5 + 84*b**6*x**6)/(252*a**9*b**7 + 2268*a**8*b**8*x +
 9072*a**7*b**9*x**2 + 21168*a**6*b**10*x**3 + 31752*a**5*b**11*x**4 + 3175
2*a**4*b**12*x**5 + 21168*a**3*b**13*x**6 + 9072*a**2*b**14*x**7 + 2268*a*b
**15*x**8 + 252*b**16*x**9)
```

Giac [A] time = 1.21344, size = 99, normalized size = 1.9

$$-\frac{84\,b^6x^6+126\,ab^5x^5+126\,a^2b^4x^4+84\,a^3b^3x^3+36\,a^4b^2x^2+9\,a^5bx+a^6}{252\,(bx+a)^9b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/((b*x + a)^9*b^7)
```

3.229 $\int \frac{x^5}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=69

$$\frac{x^6}{504a^4(a+bx)^6}+\frac{x^6}{84a^3(a+bx)^7}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{9a(a+bx)^9}$$

```
[Out] x^6/(9*a*(a + b*x)^9) + x^6/(24*a^2*(a + b*x)^8) + x^6/(84*a^3*(a + b*x)^7)
 + x^6/(504*a^4*(a + b*x)^6)
```

Rubi [A] time = 0.0155327, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{x^6}{504a^4(a+bx)^6}+\frac{x^6}{84a^3(a+bx)^7}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^10,x]
```

```
[Out] x^6/(9*a*(a + b*x)^9) + x^6/(24*a^2*(a + b*x)^8) + x^6/(84*a^3*(a + b*x)^7)
 + x^6/(504*a^4*(a + b*x)^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a+bx)^{10}}\,dx &= \frac{x^6}{9a(a+bx)^9}+\frac{\int \frac{x^5}{(a+bx)^9}\,dx}{3a}\\
&= \frac{x^6}{9a(a+bx)^9}+\frac{x^6}{24a^2(a+bx)^8}+\frac{\int \frac{x^5}{(a+bx)^8}\,dx}{12a^2}\\
&= \frac{x^6}{9a(a+bx)^9}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{84a^3(a+bx)^7}+\frac{\int \frac{x^5}{(a+bx)^7}\,dx}{84a^3}\\
&= \frac{x^6}{9a(a+bx)^9}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{84a^3(a+bx)^7}+\frac{x^6}{504a^4(a+bx)^6}
\end{aligned}$$

Mathematica [A] time = 0.0227998, size = 64, normalized size = 0.93

$$-\frac{36a^3b^2x^2 + 84a^2b^3x^3 + 9a^4bx + a^5 + 126ab^4x^4 + 126b^5x^5}{504b^6(a + bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^10,x]
```

```
[Out] -(a^5 + 9*a^4*b*x + 36*a^3*b^2*x^2 + 84*a^2*b^3*x^3 + 126*a*b^4*x^4 + 126*b
^5*x^5)/(504*b^6*(a + b*x)^9)
```

Maple [A] time = 0.005, size = 86, normalized size = 1.3

$$-\frac{5a^4}{8b^6(bx+a)^8} + \frac{10a^3}{7b^6(bx+a)^7} + \frac{a^5}{9b^6(bx+a)^9} - \frac{1}{4b^6(bx+a)^4} - \frac{5a^2}{3b^6(bx+a)^6} + \frac{a}{b^6(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^10,x)
```

```
[Out] -5/8/b^6*a^4/(b*x+a)^8+10/7/b^6*a^3/(b*x+a)^7+1/9/b^6*a^5/(b*x+a)^9-1/4/b^6
/(b*x+a)^4-5/3/b^6*a^2/(b*x+a)^6+1/b^6*a/(b*x+a)^5
```

Maxima [B] time = 1.1517, size = 207, normalized size = 3.

$$-\frac{126\,b^5x^5 + 126\,ab^4x^4 + 84\,a^2b^3x^3 + 36\,a^3b^2x^2 + 9\,a^4bx + a^5}{504\left(b^{15}x^9 + 9\,ab^{14}x^8 + 36\,a^2b^{13}x^7 + 84\,a^3b^{12}x^6 + 126\,a^4b^{11}x^5 + 126\,a^5b^{10}x^4 + 84\,a^6b^9x^3 + 36\,a^7b^8x^2 + 9\,a^8b^7x + a^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/(b^15*x^9 + 9*a*b^14*x^8 + 36*a^2*b^13*x^7 + 84*a^3*b^12*x^6
+ 126*a^4*b^11*x^5 + 126*a^5*b^10*x^4 + 84*a^6*b^9*x^3 + 36*a^7*b^8*x^2 + 9
*a^8*b^7*x + a^9*b^6)
```

Fricas [B] time = 1.54984, size = 335, normalized size = 4.86

$$-\frac{126\,b^5x^5 + 126\,ab^4x^4 + 84\,a^2b^3x^3 + 36\,a^3b^2x^2 + 9\,a^4bx + a^5}{504\left(b^{15}x^9 + 9\,ab^{14}x^8 + 36\,a^2b^{13}x^7 + 84\,a^3b^{12}x^6 + 126\,a^4b^{11}x^5 + 126\,a^5b^{10}x^4 + 84\,a^6b^9x^3 + 36\,a^7b^8x^2 + 9\,a^8b^7x + a^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/(b^15*x^9 + 9*a*b^14*x^8 + 36*a^2*b^13*x^7 + 84*a^3*b^12*x^6
```

```
+ 126*a^4*b^11*x^5 + 126*a^5*b^10*x^4 + 84*a^6*b^9*x^3 + 36*a^7*b^8*x^2 + 9
*a^8*b^7*x + a^9*b^6)
```

Sympy [B] time = 1.32557, size = 163, normalized size = 2.36

$$-\frac{a^5+9a^4bx+36a^3b^2x^2+84a^2b^3x^3+126ab^4x^4+126b^5x^5}{504a^9b^6+4536a^8b^7x+18144a^7b^8x^2+42336a^6b^9x^3+63504a^5b^{10}x^4+63504a^4b^{11}x^5+42336a^3b^{12}x^6+18144a^2b^{13}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**10,x)
```

```
[Out] -(a**5 + 9*a**4*b*x + 36*a**3*b**2*x**2 + 84*a**2*b**3*x**3 + 126*a*b**4*x*
*4 + 126*b**5*x**5)/(504*a**9*b**6 + 4536*a**8*b**7*x + 18144*a**7*b**8*x**
2 + 42336*a**6*b**9*x**3 + 63504*a**5*b**10*x**4 + 63504*a**4*b**11*x**5 +
42336*a**3*b**12*x**6 + 18144*a**2*b**13*x**7 + 4536*a*b**14*x**8 + 504*b**
15*x**9)
```

Giac [A] time = 1.18393, size = 84, normalized size = 1.22

$$-\frac{126\,b^5x^5+126\,ab^4x^4+84\,a^2b^3x^3+36\,a^3b^2x^2+9\,a^4bx+a^5}{504\,(bx+a)^9b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/((b*x + a)^9*b^6)
```

3.230 $\int \frac{x^4}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=81

$$-\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}$$

```
[Out] -a^4/(9*b^5*(a + b*x)^9) + a^3/(2*b^5*(a + b*x)^8) - (6*a^2)/(7*b^5*(a + b*
x)^7) + (2*a)/(3*b^5*(a + b*x)^6) - 1/(5*b^5*(a + b*x)^5)
```

Rubi [A] time = 0.0401836, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^10,x]
```

```
[Out] -a^4/(9*b^5*(a + b*x)^9) + a^3/(2*b^5*(a + b*x)^8) - (6*a^2)/(7*b^5*(a + b*
x)^7) + (2*a)/(3*b^5*(a + b*x)^6) - 1/(5*b^5*(a + b*x)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{10}}\,dx &= \int \left(\frac{a^4}{b^4(a+bx)^{10}}-\frac{4a^3}{b^4(a+bx)^9}+\frac{6a^2}{b^4(a+bx)^8}-\frac{4a}{b^4(a+bx)^7}+\frac{1}{b^4(a+bx)^6}\right)dx\\ &= -\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.0235085, size = 53, normalized size = 0.65

$$-\frac{36a^2b^2x^2+9a^3bx+a^4+84ab^3x^3+126b^4x^4}{630b^5(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^10,x]
```

```
[Out] -(a^4 + 9*a^3*b*x + 36*a^2*b^2*x^2 + 84*a*b^3*x^3 + 126*b^4*x^4)/(630*b^5*(
a + b*x)^9)
```

Maple [A] time = 0.005, size = 72, normalized size = 0.9

$$-\frac{a^{4}}{9 b^{5} \left(b x +a \right)^{9}}+\frac{a^{3}}{2 b^{5} \left(b x +a \right)^{8}}-\frac{6 a^{2}}{7 b^{5} \left(b x +a \right)^{7}}+\frac{2 a}{3 b^{5} \left(b x +a \right)^{6}}-\frac{1}{5 b^{5} \left(b x +a \right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^10,x)
```

```
[Out] -1/9*a^4/b^5/(b*x+a)^9+1/2*a^3/b^5/(b*x+a)^8-6/7*a^2/b^5/(b*x+a)^7+2/3*a/b^
5/(b*x+a)^6-1/5/b^5/(b*x+a)^5
```

Maxima [A] time = 1.15553, size = 192, normalized size = 2.37

$$-\frac{126 b^{4} x^{4} + 84 a b^{3} x^{3} + 36 a^{2} b^{2} x^{2} + 9 a^{3} b x + a^{4}}{630 \left(b^{14} x^{9} + 9 a b^{13} x^{8} + 36 a^{2} b^{12} x^{7} + 84 a^{3} b^{11} x^{6} + 126 a^{4} b^{10} x^{5} + 126 a^{5} b^{9} x^{4} + 84 a^{6} b^{8} x^{3} + 36 a^{7} b^{7} x^{2} + 9 a^{8} b^{6} x + a^{9} b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/(b^1
4*x^9 + 9*a*b^13*x^8 + 36*a^2*b^12*x^7 + 84*a^3*b^11*x^6 + 126*a^4*b^10*x^5
 + 126*a^5*b^9*x^4 + 84*a^6*b^8*x^3 + 36*a^7*b^7*x^2 + 9*a^8*b^6*x + a^9*b^
5)
```

Fricas [A] time = 1.4368, size = 309, normalized size = 3.81

$$-\frac{126 b^{4} x^{4} + 84 a b^{3} x^{3} + 36 a^{2} b^{2} x^{2} + 9 a^{3} b x + a^{4}}{630 \left(b^{14} x^{9} + 9 a b^{13} x^{8} + 36 a^{2} b^{12} x^{7} + 84 a^{3} b^{11} x^{6} + 126 a^{4} b^{10} x^{5} + 126 a^{5} b^{9} x^{4} + 84 a^{6} b^{8} x^{3} + 36 a^{7} b^{7} x^{2} + 9 a^{8} b^{6} x + a^{9} b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/(b^1
4*x^9 + 9*a*b^13*x^8 + 36*a^2*b^12*x^7 + 84*a^3*b^11*x^6 + 126*a^4*b^10*x^5
 + 126*a^5*b^9*x^4 + 84*a^6*b^8*x^3 + 36*a^7*b^7*x^2 + 9*a^8*b^6*x + a^9*b^
5)
```

Sympy [B] time = 1.24915, size = 151, normalized size = 1.86

$$-\frac{a^{4} + 9a^{3}bx + 36a^{2}b^{2}x^{2} + 84ab^{3}x^{3} + 126b^{4}x^{4}}{630a^{9}b^{5} + 5670a^{8}b^{6}x + 22680a^{7}b^{7}x^{2} + 52920a^{6}b^{8}x^{3} + 79380a^{5}b^{9}x^{4} + 79380a^{4}b^{10}x^{5} + 52920a^{3}b^{11}x^{6} + 22680a^{2}b^{12}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**10,x)
```

```
[Out] -(a**4 + 9*a**3*b*x + 36*a**2*b**2*x**2 + 84*a*b**3*x**3 + 126*b**4*x**4)/(
630*a**9*b**5 + 5670*a**8*b**6*x + 22680*a**7*b**7*x**2 + 52920*a**6*b**8*x
```

```
**3 + 79380*a**5*b**9*x**4 + 79380*a**4*b**10*x**5 + 52920*a**3*b**11*x**6
+ 22680*a**2*b**12*x**7 + 5670*a*b**13*x**8 + 630*b**14*x**9)
```

Giac [A] time = 1.1815, size = 69, normalized size = 0.85

$$-\frac{126\,b^4x^4+84\,ab^3x^3+36\,a^2b^2x^2+9\,a^3bx+a^4}{630\,(bx+a)^9b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/((b*
x + a)^9*b^5)
```

3.231 $\int \frac{x^3}{(a+bx)^{10}} dx$

Optimal. Leaf size=64

$$\frac{a^3}{9b^4(a+bx)^9} - \frac{3a^2}{8b^4(a+bx)^8} + \frac{3a}{7b^4(a+bx)^7} - \frac{1}{6b^4(a+bx)^6}$$

```
[Out] a^3/(9*b^4*(a + b*x)^9) - (3*a^2)/(8*b^4*(a + b*x)^8) + (3*a)/(7*b^4*(a + b
*x)^7) - 1/(6*b^4*(a + b*x)^6)
```

Rubi [A] time = 0.0297847, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{9b^4(a+bx)^9} - \frac{3a^2}{8b^4(a+bx)^8} + \frac{3a}{7b^4(a+bx)^7} - \frac{1}{6b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^10,x]
```

```
[Out] a^3/(9*b^4*(a + b*x)^9) - (3*a^2)/(8*b^4*(a + b*x)^8) + (3*a)/(7*b^4*(a + b
*x)^7) - 1/(6*b^4*(a + b*x)^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{10}} dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{10}} + \frac{3a^2}{b^3(a+bx)^9} - \frac{3a}{b^3(a+bx)^8} + \frac{1}{b^3(a+bx)^7}\right) dx \\ &= \frac{a^3}{9b^4(a+bx)^9} - \frac{3a^2}{8b^4(a+bx)^8} + \frac{3a}{7b^4(a+bx)^7} - \frac{1}{6b^4(a+bx)^6}\end{aligned}$$

Mathematica [A] time = 0.0178355, size = 42, normalized size = 0.66

$$-\frac{9a^2bx + a^3 + 36ab^2x^2 + 84b^3x^3}{504b^4(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^10,x]
```

```
[Out] -(a^3 + 9*a^2*b*x + 36*a*b^2*x^2 + 84*b^3*x^3)/(504*b^4*(a + b*x)^9)
```

Maple [A] time = 0.006, size = 57, normalized size = 0.9

$$\frac{a^3}{9\,b^4\,(bx+a)^9}-\frac{3\,a^2}{8\,b^4\,(bx+a)^8}+\frac{3\,a}{7\,b^4\,(bx+a)^7}-\frac{1}{6\,b^4\,(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^10,x)
```

```
[Out] 1/9*a^3/b^4/(b*x+a)^9-3/8*a^2/b^4/(b*x+a)^8+3/7*a/b^4/(b*x+a)^7-1/6/b^4/(b*
x+a)^6
```

Maxima [B] time = 1.09933, size = 177, normalized size = 2.77

$$-\frac{84\,b^3x^3+36\,ab^2x^2+9\,a^2bx+a^3}{504\,\left(b^{13}x^9+9\,ab^{12}x^8+36\,a^2b^{11}x^7+84\,a^3b^{10}x^6+126\,a^4b^9x^5+126\,a^5b^8x^4+84\,a^6b^7x^3+36\,a^7b^6x^2+9\,a^8b^5x+a^9b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/(b^13*x^9 + 9*a*b^12*x
^8 + 36*a^2*b^11*x^7 + 84*a^3*b^10*x^6 + 126*a^4*b^9*x^5 + 126*a^5*b^8*x^4
+ 84*a^6*b^7*x^3 + 36*a^7*b^6*x^2 + 9*a^8*b^5*x + a^9*b^4)
```

Fricas [B] time = 1.47435, size = 284, normalized size = 4.44

$$-\frac{84\,b^3x^3+36\,ab^2x^2+9\,a^2bx+a^3}{504\,\left(b^{13}x^9+9\,ab^{12}x^8+36\,a^2b^{11}x^7+84\,a^3b^{10}x^6+126\,a^4b^9x^5+126\,a^5b^8x^4+84\,a^6b^7x^3+36\,a^7b^6x^2+9\,a^8b^5x+a^9b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/(b^13*x^9 + 9*a*b^12*x
^8 + 36*a^2*b^11*x^7 + 84*a^3*b^10*x^6 + 126*a^4*b^9*x^5 + 126*a^5*b^8*x^4
+ 84*a^6*b^7*x^3 + 36*a^7*b^6*x^2 + 9*a^8*b^5*x + a^9*b^4)
```

Sympy [B] time = 1.10376, size = 139, normalized size = 2.17

$$-\frac{a^3+9a^2bx+36ab^2x^2+84b^3x^3}{504a^9b^4+4536a^8b^5x+18144a^7b^6x^2+42336a^6b^7x^3+63504a^5b^8x^4+63504a^4b^9x^5+42336a^3b^{10}x^6+18144a^2b^{11}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**10,x)
```

```
[Out] -(a**3 + 9*a**2*b*x + 36*a*b**2*x**2 + 84*b**3*x**3)/(504*a**9*b**4 + 4536*
a**8*b**5*x + 18144*a**7*b**6*x**2 + 42336*a**6*b**7*x**3 + 63504*a**5*b**8
*x**4 + 63504*a**4*b**9*x**5 + 42336*a**3*b**10*x**6 + 18144*a**2*b**11*x**
```

```
7 + 4536*a*b**12*x**8 + 504*b**13*x**9)
```

Giac [A] time = 1.21656, size = 54, normalized size = 0.84

$$-\frac{84\,b^3x^3 + 36\,ab^2x^2 + 9\,a^2bx + a^3}{504\,(bx+a)^9b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/((b*x + a)^9*b^4)
```

3.232 $\int \frac{x^2}{(a+bx)^{10}} \, dx$

Optimal. Leaf size=47

$$-\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}$$

```
[Out] -a^2/(9*b^3*(a + b*x)^9) + a/(4*b^3*(a + b*x)^8) - 1/(7*b^3*(a + b*x)^7)
```

Rubi [A] time = 0.0207929, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^10,x]
```

```
[Out] -a^2/(9*b^3*(a + b*x)^9) + a/(4*b^3*(a + b*x)^8) - 1/(7*b^3*(a + b*x)^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{10}} \, dx &= \int \left(\frac{a^2}{b^2(a+bx)^{10}}-\frac{2a}{b^2(a+bx)^9}+\frac{1}{b^2(a+bx)^8}\right) dx\\ &= -\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}\end{aligned}$$

Mathematica [A] time = 0.0146774, size = 31, normalized size = 0.66

$$-\frac{a^2+9abx+36b^2x^2}{252b^3(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^10,x]
```

```
[Out] -(a^2 + 9*a*b*x + 36*b^2*x^2)/(252*b^3*(a + b*x)^9)
```

Maple [A] time = 0.005, size = 42, normalized size = 0.9

$$-\frac{a^2}{9\,b^3\,(bx+a)^9}+\frac{a}{4\,b^3\,(bx+a)^8}-\frac{1}{7\,b^3\,(bx+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^10,x)
```

```
[Out] -1/9*a^2/b^3/(b*x+a)^9+1/4*a/b^3/(b*x+a)^8-1/7/b^3/(b*x+a)^7
```

Maxima [B] time = 1.04763, size = 162, normalized size = 3.45

$$-\frac{36\,b^2x^2 + 9\,abx + a^2}{252\left(b^{12}x^9 + 9\,ab^{11}x^8 + 36\,a^2b^{10}x^7 + 84\,a^3b^9x^6 + 126\,a^4b^8x^5 + 126\,a^5b^7x^4 + 84\,a^6b^6x^3 + 36\,a^7b^5x^2 + 9\,a^8b^4x + a^9b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/(b^12*x^9 + 9*a*b^11*x^8 + 36*a^2*b^10*
x^7 + 84*a^3*b^9*x^6 + 126*a^4*b^8*x^5 + 126*a^5*b^7*x^4 + 84*a^6*b^6*x^3 +
 36*a^7*b^5*x^2 + 9*a^8*b^4*x + a^9*b^3)
```

Fricas [B] time = 1.47967, size = 259, normalized size = 5.51

$$-\frac{36\,b^2x^2 + 9\,abx + a^2}{252\left(b^{12}x^9 + 9\,ab^{11}x^8 + 36\,a^2b^{10}x^7 + 84\,a^3b^9x^6 + 126\,a^4b^8x^5 + 126\,a^5b^7x^4 + 84\,a^6b^6x^3 + 36\,a^7b^5x^2 + 9\,a^8b^4x + a^9b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/(b^12*x^9 + 9*a*b^11*x^8 + 36*a^2*b^10*
x^7 + 84*a^3*b^9*x^6 + 126*a^4*b^8*x^5 + 126*a^5*b^7*x^4 + 84*a^6*b^6*x^3 +
 36*a^7*b^5*x^2 + 9*a^8*b^4*x + a^9*b^3)
```

Sympy [B] time = 1.10751, size = 128, normalized size = 2.72

$$-\frac{a^2 + 9abx + 36b^2x^2}{252a^9b^3 + 2268a^8b^4x + 9072a^7b^5x^2 + 21168a^6b^6x^3 + 31752a^5b^7x^4 + 31752a^4b^8x^5 + 21168a^3b^9x^6 + 9072a^2b^{10}x^7 + 226}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**10,x)
```

```
[Out] -(a**2 + 9*a*b*x + 36*b**2*x**2)/(252*a**9*b**3 + 2268*a**8*b**4*x + 9072*a
**7*b**5*x**2 + 21168*a**6*b**6*x**3 + 31752*a**5*b**7*x**4 + 31752*a**4*b*
*8*x**5 + 21168*a**3*b**9*x**6 + 9072*a**2*b**10*x**7 + 2268*a*b**11*x**8 +
 252*b**12*x**9)
```

Giac [A] time = 1.20627, size = 39, normalized size = 0.83

$$-\frac{36\,b^2x^2 + 9\,abx + a^2}{252\,(bx + a)^9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/((b*x + a)^9*b^3)
```

3.233 $\int \frac{x}{(a+bx)^{10}} dx$

Optimal. Leaf size=30

$$\frac{a}{9b^2(a+bx)^9} - \frac{1}{8b^2(a+bx)^8}$$

```
[Out] a/(9*b^2*(a + b*x)^9) - 1/(8*b^2*(a + b*x)^8)
```

Rubi [A] time = 0.0135037, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a}{9b^2(a+bx)^9} - \frac{1}{8b^2(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^10,x]
```

```
[Out] a/(9*b^2*(a + b*x)^9) - 1/(8*b^2*(a + b*x)^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(a+bx)^{10}} dx &= \int \left(-\frac{a}{b(a+bx)^{10}} + \frac{1}{b(a+bx)^9}\right) dx \\ &= \frac{a}{9b^2(a+bx)^9} - \frac{1}{8b^2(a+bx)^8} \end{aligned}$$

Mathematica [A] time = 0.0091609, size = 20, normalized size = 0.67

$$-\frac{a+9bx}{72b^2(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^10,x]
```

```
[Out] -(a + 9*b*x)/(72*b^2*(a + b*x)^9)
```

Maple [A] time = 0.005, size = 27, normalized size = 0.9

$$\frac{a}{9\,b^2\,(bx+a)^9} - \frac{1}{8\,b^2\,(bx+a)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^10,x)
```

```
[Out] 1/9*a/b^2/(b*x+a)^9-1/8/b^2/(b*x+a)^8
```

Maxima [B] time = 1.17228, size = 147, normalized size = 4.9

$$-\frac{9\,bx+a}{72\left(b^{11}x^{9}+9\,ab^{10}x^{8}+36\,a^{2}b^{9}x^{7}+84\,a^{3}b^{8}x^{6}+126\,a^{4}b^{7}x^{5}+126\,a^{5}b^{6}x^{4}+84\,a^{6}b^{5}x^{3}+36\,a^{7}b^{4}x^{2}+9\,a^{8}b^{3}x+a^{9}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(9*b*x + a)/(b^11*x^9 + 9*a*b^10*x^8 + 36*a^2*b^9*x^7 + 84*a^3*b^8*x^
6 + 126*a^4*b^7*x^5 + 126*a^5*b^6*x^4 + 84*a^6*b^5*x^3 + 36*a^7*b^4*x^2 + 9
*a^8*b^3*x + a^9*b^2)
```

Fricas [B] time = 1.38064, size = 234, normalized size = 7.8

$$-\frac{9\,bx+a}{72\left(b^{11}x^{9}+9\,ab^{10}x^{8}+36\,a^{2}b^{9}x^{7}+84\,a^{3}b^{8}x^{6}+126\,a^{4}b^{7}x^{5}+126\,a^{5}b^{6}x^{4}+84\,a^{6}b^{5}x^{3}+36\,a^{7}b^{4}x^{2}+9\,a^{8}b^{3}x+a^{9}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(9*b*x + a)/(b^11*x^9 + 9*a*b^10*x^8 + 36*a^2*b^9*x^7 + 84*a^3*b^8*x^
6 + 126*a^4*b^7*x^5 + 126*a^5*b^6*x^4 + 84*a^6*b^5*x^3 + 36*a^7*b^4*x^2 + 9
*a^8*b^3*x + a^9*b^2)
```

Sympy [B] time = 1.07181, size = 116, normalized size = 3.87

$$-\frac{a+9bx}{72a^{9}b^{2}+648a^{8}b^{3}x+2592a^{7}b^{4}x^{2}+6048a^{6}b^{5}x^{3}+9072a^{5}b^{6}x^{4}+9072a^{4}b^{7}x^{5}+6048a^{3}b^{8}x^{6}+2592a^{2}b^{9}x^{7}+648ab^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**10,x)
```

```
[Out] -(a + 9*b*x)/(72*a**9*b**2 + 648*a**8*b**3*x + 2592*a**7*b**4*x**2 + 6048*a
**6*b**5*x**3 + 9072*a**5*b**6*x**4 + 9072*a**4*b**7*x**5 + 6048*a**3*b**8*
x**6 + 2592*a**2*b**9*x**7 + 648*a*b**10*x**8 + 72*b**11*x**9)
```

Giac [A] time = 1.20915, size = 24, normalized size = 0.8

$$-\frac{9\,bx+a}{72\,(bx+a)^{9}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/72*(9*b*x + a)/((b*x + a)^9*b^2)
```

3.234 $\int \frac{1}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{9b(a+bx)^9}$$

```
[Out] -1/(9*b*(a + b*x)^9)
```

Rubi [A] time = 0.0015978, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-10),x]
```

```
[Out] -1/(9*b*(a + b*x)^9)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{10}}\,dx = -\frac{1}{9b(a+bx)^9}$$

Mathematica [A] time = 0.0033223, size = 14, normalized size = 1.

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-10),x]
```

```
[Out] -1/(9*b*(a + b*x)^9)
```

Maple [A] time = 0., size = 13, normalized size = 0.9

$$-\frac{1}{9\,b\,(bx+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^10,x)
```

```
[Out] -1/9/b/(b*x+a)^9
```

Maxima [A] time = 1.01816, size = 16, normalized size = 1.14

$$-\frac{1}{9\,(bx+a)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

Fricas [B] time = 1.45276, size = 212, normalized size = 15.14

$$-\frac{1}{9\left(b^{10}x^9+9\,ab^9x^8+36\,a^2b^8x^7+84\,a^3b^7x^6+126\,a^4b^6x^5+126\,a^5b^5x^4+84\,a^6b^4x^3+36\,a^7b^3x^2+9\,a^8b^2x+a^9b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

Sympy [B] time = 1.05076, size = 109, normalized size = 7.79

$$-\frac{1}{9a^9b+81a^8b^2x+324a^7b^3x^2+756a^6b^4x^3+1134a^5b^5x^4+1134a^4b^6x^5+756a^3b^7x^6+324a^2b^8x^7+81ab^9x^8+9b^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**10,x)
```

```
[Out] -1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 + 1
134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*b*
*8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

Giac [A] time = 1.19332, size = 16, normalized size = 1.14

$$-\frac{1}{9\,(bx+a)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

3.235 $\int \frac{1}{x(a+bx)^{10}} dx$

Optimal. Leaf size=141

$$\frac{1}{a^9(a+bx)} + \frac{1}{2a^8(a+bx)^2} + \frac{1}{3a^7(a+bx)^3} + \frac{1}{4a^6(a+bx)^4} + \frac{1}{5a^5(a+bx)^5} + \frac{1}{6a^4(a+bx)^6} + \frac{1}{7a^3(a+bx)^7} + \frac{1}{8a^2(a+b}$$

```
[Out] 1/(9*a*(a + b*x)^9) + 1/(8*a^2*(a + b*x)^8) + 1/(7*a^3*(a + b*x)^7) + 1/(6*
a^4*(a + b*x)^6) + 1/(5*a^5*(a + b*x)^5) + 1/(4*a^6*(a + b*x)^4) + 1/(3*a^7
*(a + b*x)^3) + 1/(2*a^8*(a + b*x)^2) + 1/(a^9*(a + b*x)) + Log[x]/a^10 - L
og[a + b*x]/a^10
```

Rubi [A] time = 0.0748266, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{a^9(a+bx)} + \frac{1}{2a^8(a+bx)^2} + \frac{1}{3a^7(a+bx)^3} + \frac{1}{4a^6(a+bx)^4} + \frac{1}{5a^5(a+bx)^5} + \frac{1}{6a^4(a+bx)^6} + \frac{1}{7a^3(a+bx)^7} + \frac{1}{8a^2(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^10),x]
```

```
[Out] 1/(9*a*(a + b*x)^9) + 1/(8*a^2*(a + b*x)^8) + 1/(7*a^3*(a + b*x)^7) + 1/(6*
a^4*(a + b*x)^6) + 1/(5*a^5*(a + b*x)^5) + 1/(4*a^6*(a + b*x)^4) + 1/(3*a^7
*(a + b*x)^3) + 1/(2*a^8*(a + b*x)^2) + 1/(a^9*(a + b*x)) + Log[x]/a^10 - L
og[a + b*x]/a^10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x(a+bx)^{10}} dx = \int \left(\frac{1}{a^{10}x} - \frac{b}{a(a+bx)^{10}} - \frac{b}{a^2(a+bx)^9} - \frac{b}{a^3(a+bx)^8} - \frac{b}{a^4(a+bx)^7} - \frac{b}{a^5(a+bx)^6} - \frac{b}{a^6(a+bx)^5} -$$

$$= \frac{1}{9a(a+bx)^9} + \frac{1}{8a^2(a+bx)^8} + \frac{1}{7a^3(a+bx)^7} + \frac{1}{6a^4(a+bx)^6} + \frac{1}{5a^5(a+bx)^5} + \frac{1}{4a^6(a+bx)^4} + \frac{}{3a^7}$$

Mathematica [A] time = 0.148634, size = 127, normalized size = 0.9

$$\frac{315a^7(a+bx) + 360a^6(a+bx)^2 + 420a^5(a+bx)^3 + 504a^4(a+bx)^4 + 630a^3(a+bx)^5 + 840a^2(a+bx)^6 + 280a^8 + 126}{2520a^9(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^10),x]
```

```
[Out] (280*a^8 + 315*a^7*(a + b*x) + 360*a^6*(a + b*x)^2 + 420*a^5*(a + b*x)^3 +
504*a^4*(a + b*x)^4 + 630*a^3*(a + b*x)^5 + 840*a^2*(a + b*x)^6 + 1260*a*(a
 + b*x)^7 + 2520*(a + b*x)^8)/(2520*a^9*(a + b*x)^9) + Log[x]/a^10 - Log[a
+ b*x]/a^10
```

Maple [A] time = 0.011, size = 126, normalized size = 0.9

$$\frac{1}{9\,a\,(bx+a)^{9}}+\frac{1}{8\,a^{2}\,(bx+a)^{8}}+\frac{1}{7\,a^{3}\,(bx+a)^{7}}+\frac{1}{6\,a^{4}\,(bx+a)^{6}}+\frac{1}{5\,a^{5}\,(bx+a)^{5}}+\frac{1}{4\,a^{6}\,(bx+a)^{4}}+\frac{1}{3\,a^{7}\,(bx+a)^{3}}+\frac{}{2\,a^{8}\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^10,x)
```

```
[Out] 1/9/a/(b*x+a)^9+1/8/a^2/(b*x+a)^8+1/7/a^3/(b*x+a)^7+1/6/a^4/(b*x+a)^6+1/5/a
^5/(b*x+a)^5+1/4/a^6/(b*x+a)^4+1/3/a^7/(b*x+a)^3+1/2/a^8/(b*x+a)^2+1/a^9/(b
*x+a)+ln(x)/a^10-ln(b*x+a)/a^10
```

Maxima [A] time = 1.13768, size = 277, normalized size = 1.96

$$\frac{2520\,b^{8}x^{8}+21420\,ab^{7}x^{7}+80220\,a^{2}b^{6}x^{6}+173250\,a^{3}b^{5}x^{5}+236754\,a^{4}b^{4}x^{4}+210756\,a^{5}b^{3}x^{3}+120564\,a^{6}b^{2}x^{2}+41481\,a^{7}}{2520\,\left(a^{9}b^{9}x^{9}+9\,a^{10}b^{8}x^{8}+36\,a^{11}b^{7}x^{7}+84\,a^{12}b^{6}x^{6}+126\,a^{13}b^{5}x^{5}+126\,a^{14}b^{4}x^{4}+84\,a^{15}b^{3}x^{3}+36\,a^{16}b^{2}x^{2}+9\,a^{17}b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/2520*(2520*b^8*x^8 + 21420*a*b^7*x^7 + 80220*a^2*b^6*x^6 + 173250*a^3*b^5
*x^5 + 236754*a^4*b^4*x^4 + 210756*a^5*b^3*x^3 + 120564*a^6*b^2*x^2 + 41481
*a^7*b*x + 7129*a^8)/(a^9*b^9*x^9 + 9*a^10*b^8*x^8 + 36*a^11*b^7*x^7 + 84*a
^12*b^6*x^6 + 126*a^13*b^5*x^5 + 126*a^14*b^4*x^4 + 84*a^15*b^3*x^3 + 36*a^
16*b^2*x^2 + 9*a^17*b*x + a^18) - log(b*x + a)/a^10 + log(x)/a^10
```

Fricas [B] time = 1.57749, size = 896, normalized size = 6.35

$$\frac{2520\,ab^{8}x^{8}+21420\,a^{2}b^{7}x^{7}+80220\,a^{3}b^{6}x^{6}+173250\,a^{4}b^{5}x^{5}+236754\,a^{5}b^{4}x^{4}+210756\,a^{6}b^{3}x^{3}+120564\,a^{7}b^{2}x^{2}+41481}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*a*b^8*x^8 + 21420*a^2*b^7*x^7 + 80220*a^3*b^6*x^6 + 173250*a^4
*b^5*x^5 + 236754*a^5*b^4*x^4 + 210756*a^6*b^3*x^3 + 120564*a^7*b^2*x^2 + 4
1481*a^8*b*x + 7129*a^9 - 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*a^2*b^7*x^7 + 84
*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b^3*x^3 + 36*a^7*
b^2*x^2 + 9*a^8*b*x + a^9)*log(b*x + a) + 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*
a^2*b^7*x^7 + 84*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b
^3*x^3 + 36*a^7*b^2*x^2 + 9*a^8*b*x + a^9)*log(x))/(a^10*b^9*x^9 + 9*a^11*b
^8*x^8 + 36*a^12*b^7*x^7 + 84*a^13*b^6*x^6 + 126*a^14*b^5*x^5 + 126*a^15*b^
4*x^4 + 84*a^16*b^3*x^3 + 36*a^17*b^2*x^2 + 9*a^18*b*x + a^19)
```

Sympy [A] time = 2.11026, size = 212, normalized size = 1.5

$$\frac{7129a^8 + 41481a^7bx + 120564a^6b^2x^2 + 210756a^5b^3x^3 + 236754a^4b^4x^4 + 173250a^3b^5x^5 + 80220a^2b^6x^6}{2520a^{18} + 22680a^{17}bx + 90720a^{16}b^2x^2 + 211680a^{15}b^3x^3 + 317520a^{14}b^4x^4 + 317520a^{13}b^5x^5 + 211680a^{12}b^6x^6 + 9072}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**10,x)
```

```
[Out] (7129*a**8 + 41481*a**7*b*x + 120564*a**6*b**2*x**2 + 210756*a**5*b**3*x**3
 + 236754*a**4*b**4*x**4 + 173250*a**3*b**5*x**5 + 80220*a**2*b**6*x**6 + 2
1420*a*b**7*x**7 + 2520*b**8*x**8)/(2520*a**18 + 22680*a**17*b*x + 90720*a*
*16*b**2*x**2 + 211680*a**15*b**3*x**3 + 317520*a**14*b**4*x**4 + 317520*a*
*13*b**5*x**5 + 211680*a**12*b**6*x**6 + 90720*a**11*b**7*x**7 + 22680*a**1
0*b**8*x**8 + 2520*a**9*b**9*x**9) + (log(x) - log(a/b + x))/a**10
```

Giac [A] time = 1.19564, size = 162, normalized size = 1.15

$$-\frac{\log(|bx+a|)}{a^{10}} + \frac{\log(|x|)}{a^{10}} + \frac{2520\,ab^8x^8 + 21420\,a^2b^7x^7 + 80220\,a^3b^6x^6 + 173250\,a^4b^5x^5 + 236754\,a^5b^4x^4 + 210756\,a}{2520\,(bx+a)^9a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^10 + log(abs(x))/a^10 + 1/2520*(2520*a*b^8*x^8 + 21420
*a^2*b^7*x^7 + 80220*a^3*b^6*x^6 + 173250*a^4*b^5*x^5 + 236754*a^5*b^4*x^4
+ 210756*a^6*b^3*x^3 + 120564*a^7*b^2*x^2 + 41481*a^8*b*x + 7129*a^9)/((b*x
 + a)^9*a^10)
```

3.236 $\int \frac{1}{x^2(a+bx)^{10}}\,dx$

Optimal. Leaf size=158

$$-\frac{9b}{a^{10}(a+bx)}-\frac{4b}{a^9(a+bx)^2}-\frac{7b}{3a^8(a+bx)^3}-\frac{3b}{2a^7(a+bx)^4}-\frac{b}{a^6(a+bx)^5}-\frac{2b}{3a^5(a+bx)^6}-\frac{3b}{7a^4(a+bx)^7}-\frac{b}{4a^3(a+bx)^8}$$

```
[Out] -(1/(a^10*x)) - b/(9*a^2*(a + b*x)^9) - b/(4*a^3*(a + b*x)^8) - (3*b)/(7*a^
4*(a + b*x)^7) - (2*b)/(3*a^5*(a + b*x)^6) - b/(a^6*(a + b*x)^5) - (3*b)/(2
*a^7*(a + b*x)^4) - (7*b)/(3*a^8*(a + b*x)^3) - (4*b)/(a^9*(a + b*x)^2) - (
9*b)/(a^10*(a + b*x)) - (10*b*Log[x])/a^11 + (10*b*Log[a + b*x])/a^11
```

Rubi [A] time = 0.124898, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{9b}{a^{10}(a+bx)}-\frac{4b}{a^9(a+bx)^2}-\frac{7b}{3a^8(a+bx)^3}-\frac{3b}{2a^7(a+bx)^4}-\frac{b}{a^6(a+bx)^5}-\frac{2b}{3a^5(a+bx)^6}-\frac{3b}{7a^4(a+bx)^7}-\frac{b}{4a^3(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^10),x]
```

```
[Out] -(1/(a^10*x)) - b/(9*a^2*(a + b*x)^9) - b/(4*a^3*(a + b*x)^8) - (3*b)/(7*a^
4*(a + b*x)^7) - (2*b)/(3*a^5*(a + b*x)^6) - b/(a^6*(a + b*x)^5) - (3*b)/(2
*a^7*(a + b*x)^4) - (7*b)/(3*a^8*(a + b*x)^3) - (4*b)/(a^9*(a + b*x)^2) - (
9*b)/(a^10*(a + b*x)) - (10*b*Log[x])/a^11 + (10*b*Log[a + b*x])/a^11
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^2(a+bx)^{10}}\,dx = \int \left(\frac{1}{a^{10}x^2}-\frac{10b}{a^{11}x}+\frac{b^2}{a^2(a+bx)^{10}}+\frac{2b^2}{a^3(a+bx)^9}+\frac{3b^2}{a^4(a+bx)^8}+\frac{4b^2}{a^5(a+bx)^7}+\frac{5b^2}{a^6(a+bx)^6}+\frac{6}{a^7(a}\right.$$
$$=-\frac{1}{a^{10}x}-\frac{b}{9a^2(a+bx)^9}-\frac{b}{4a^3(a+bx)^8}-\frac{3b}{7a^4(a+bx)^7}-\frac{2b}{3a^5(a+bx)^6}-\frac{b}{a^6(a+bx)^5}-\frac{3b}{2a^7(a+bx)^4}$$

Mathematica [A] time = 0.162486, size = 130, normalized size = 0.82

$$-\frac{\frac{a\left(41481a^7b^2x^2+120564a^6b^3x^3+210756a^5b^4x^4+236754a^4b^5x^5+173250a^3b^6x^6+80220a^2b^7x^7+7129a^8bx+252a^9+21420ab^8x^8+2520b^9x^9\right)}{x(a+bx)^9}-2520b\log(a+}{252a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^10),x]
```

```
[Out] -((a*(252*a^9 + 7129*a^8*b*x + 41481*a^7*b^2*x^2 + 120564*a^6*b^3*x^3 + 210
756*a^5*b^4*x^4 + 236754*a^4*b^5*x^5 + 173250*a^3*b^6*x^6 + 80220*a^2*b^7*x
^7 + 21420*a*b^8*x^8 + 2520*b^9*x^9))/(x*(a + b*x)^9) + 2520*b*Log[x] - 252
0*b*Log[a + b*x])/(252*a^11)
```

Maple [A] time = 0.014, size = 147, normalized size = 0.9

$$-\frac{1}{a^{10}x}-\frac{b}{9\,a^{2}\,(bx+a)^{9}}-\frac{b}{4\,a^{3}\,(bx+a)^{8}}-\frac{3\,b}{7\,a^{4}\,(bx+a)^{7}}-\frac{2\,b}{3\,a^{5}\,(bx+a)^{6}}-\frac{b}{a^{6}\,(bx+a)^{5}}-\frac{3\,b}{2\,a^{7}\,(bx+a)^{4}}-\frac{7\,b}{3\,a^{8}\,(bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^10,x)
```

```
[Out] -1/a^10/x-1/9*b/a^2/(b*x+a)^9-1/4*b/a^3/(b*x+a)^8-3/7*b/a^4/(b*x+a)^7-2/3*b
/a^5/(b*x+a)^6-b/a^6/(b*x+a)^5-3/2*b/a^7/(b*x+a)^4-7/3*b/a^8/(b*x+a)^3-4*b/
a^9/(b*x+a)^2-9*b/a^10/(b*x+a)-10*b*ln(x)/a^11+10*b*ln(b*x+a)/a^11
```

Maxima [A] time = 1.10074, size = 301, normalized size = 1.91

$$-\frac{2520\,b^{9}x^{9}+21420\,ab^{8}x^{8}+80220\,a^{2}b^{7}x^{7}+173250\,a^{3}b^{6}x^{6}+236754\,a^{4}b^{5}x^{5}+210756\,a^{5}b^{4}x^{4}+120564\,a^{6}b^{3}x^{3}+414}{252\left(a^{10}b^{9}x^{10}+9\,a^{11}b^{8}x^{9}+36\,a^{12}b^{7}x^{8}+84\,a^{13}b^{6}x^{7}+126\,a^{14}b^{5}x^{6}+126\,a^{15}b^{4}x^{5}+84\,a^{16}b^{3}x^{4}+36\,a^{17}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(2520*b^9*x^9 + 21420*a*b^8*x^8 + 80220*a^2*b^7*x^7 + 173250*a^3*b^6
*x^6 + 236754*a^4*b^5*x^5 + 210756*a^5*b^4*x^4 + 120564*a^6*b^3*x^3 + 41481
*a^7*b^2*x^2 + 7129*a^8*b*x + 252*a^9)/(a^10*b^9*x^10 + 9*a^11*b^8*x^9 + 36
*a^12*b^7*x^8 + 84*a^13*b^6*x^7 + 126*a^14*b^5*x^6 + 126*a^15*b^4*x^5 + 84*
a^16*b^3*x^4 + 36*a^17*b^2*x^3 + 9*a^18*b*x^2 + a^19*x) + 10*b*log(b*x + a)
/a^11 - 10*b*log(x)/a^11
```

Fricas [B] time = 1.64411, size = 956, normalized size = 6.05

$$-\frac{2520\,ab^{9}x^{9}+21420\,a^{2}b^{8}x^{8}+80220\,a^{3}b^{7}x^{7}+173250\,a^{4}b^{6}x^{6}+236754\,a^{5}b^{5}x^{5}+210756\,a^{6}b^{4}x^{4}+120564\,a^{7}b^{3}x^{3}+4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(2520*a*b^9*x^9 + 21420*a^2*b^8*x^8 + 80220*a^3*b^7*x^7 + 173250*a^4
*b^6*x^6 + 236754*a^5*b^5*x^5 + 210756*a^6*b^4*x^4 + 120564*a^7*b^3*x^3 + 4
1481*a^8*b^2*x^2 + 7129*a^9*b*x + 252*a^10 - 2520*(b^10*x^10 + 9*a*b^9*x^9
+ 36*a^2*b^8*x^8 + 84*a^3*b^7*x^7 + 126*a^4*b^6*x^6 + 126*a^5*b^5*x^5 + 84*
a^6*b^4*x^4 + 36*a^7*b^3*x^3 + 9*a^8*b^2*x^2 + a^9*b*x)*log(b*x + a) + 2520
*(b^10*x^10 + 9*a*b^9*x^9 + 36*a^2*b^8*x^8 + 84*a^3*b^7*x^7 + 126*a^4*b^6*x
^6 + 126*a^5*b^5*x^5 + 84*a^6*b^4*x^4 + 36*a^7*b^3*x^3 + 9*a^8*b^2*x^2 + a^
9*b*x)*log(x))/(a^11*b^9*x^10 + 9*a^12*b^8*x^9 + 36*a^13*b^7*x^8 + 84*a^14*
```

```
b^6*x^7 + 126*a^15*b^5*x^6 + 126*a^16*b^4*x^5 + 84*a^17*b^3*x^4 + 36*a^18*b
^2*x^3 + 9*a^19*b*x^2 + a^20*x)
```

Sympy [A] time = 2.64632, size = 231, normalized size = 1.46

$$-\frac{252a^9 + 7129a^8bx + 41481a^7b^2x^2 + 120564a^6b^3x^3 + 210756a^5b^4x^4 + 236754a^4b^5x^5 + 173250a^3b^6x^6 + 80220a^2b^7x^7}{252a^{19}x + 2268a^{18}bx^2 + 9072a^{17}b^2x^3 + 21168a^{16}b^3x^4 + 31752a^{15}b^4x^5 + 31752a^{14}b^5x^6 + 21168a^{13}b^6x^7 + 9072a^{12}b^7x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**10,x)
```

```
[Out] -(252*a**9 + 7129*a**8*b*x + 41481*a**7*b**2*x**2 + 120564*a**6*b**3*x**3 +
 210756*a**5*b**4*x**4 + 236754*a**4*b**5*x**5 + 173250*a**3*b**6*x**6 + 80
220*a**2*b**7*x**7 + 21420*a*b**8*x**8 + 2520*b**9*x**9)/(252*a**19*x + 226
8*a**18*b*x**2 + 9072*a**17*b**2*x**3 + 21168*a**16*b**3*x**4 + 31752*a**15
*b**4*x**5 + 31752*a**14*b**5*x**6 + 21168*a**13*b**6*x**7 + 9072*a**12*b**
7*x**8 + 2268*a**11*b**8*x**9 + 252*a**10*b**9*x**10) + 10*b*(-log(x) + log
(a/b + x))/a**11
```

Giac [A] time = 1.18441, size = 185, normalized size = 1.17

$$\frac{10\, b \log\left(|bx + a|\right)}{a^{11}} - \frac{10\, b \log\left(|x|\right)}{a^{11}} - \frac{2520\, ab^9x^9 + 21420\, a^2b^8x^8 + 80220\, a^3b^7x^7 + 173250\, a^4b^6x^6 + 236754\, a^5b^5x^5 + 2107}{252\, (bx + a)^9 a^{11} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 10*b*log(abs(b*x + a))/a^11 - 10*b*log(abs(x))/a^11 - 1/252*(2520*a*b^9*x^9
 + 21420*a^2*b^8*x^8 + 80220*a^3*b^7*x^7 + 173250*a^4*b^6*x^6 + 236754*a^5*
b^5*x^5 + 210756*a^6*b^4*x^4 + 120564*a^7*b^3*x^3 + 41481*a^8*b^2*x^2 + 712
9*a^9*b*x + 252*a^10)/((b*x + a)^9*a^11*x)
```

3.237 $\int \frac{1}{x^3(a+bx)^{10}}\,dx$

Optimal. Leaf size=191

$$\frac{45b^2}{a^{11}(a+bx)}+\frac{18b^2}{a^{10}(a+bx)^2}+\frac{28b^2}{3a^9(a+bx)^3}+\frac{21b^2}{4a^8(a+bx)^4}+\frac{3b^2}{a^7(a+bx)^5}+\frac{5b^2}{3a^6(a+bx)^6}+\frac{6b^2}{7a^5(a+bx)^7}+\frac{3b^2}{8a^4(a+b}$$

```
[Out] -1/(2*a^10*x^2) + (10*b)/(a^11*x) + b^2/(9*a^3*(a + b*x)^9) + (3*b^2)/(8*a^
4*(a + b*x)^8) + (6*b^2)/(7*a^5*(a + b*x)^7) + (5*b^2)/(3*a^6*(a + b*x)^6)
+ (3*b^2)/(a^7*(a + b*x)^5) + (21*b^2)/(4*a^8*(a + b*x)^4) + (28*b^2)/(3*a^
9*(a + b*x)^3) + (18*b^2)/(a^10*(a + b*x)^2) + (45*b^2)/(a^11*(a + b*x)) +
(55*b^2*Log[x])/a^12 - (55*b^2*Log[a + b*x])/a^12
```

Rubi [A] time = 0.141398, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{45b^2}{a^{11}(a+bx)}+\frac{18b^2}{a^{10}(a+bx)^2}+\frac{28b^2}{3a^9(a+bx)^3}+\frac{21b^2}{4a^8(a+bx)^4}+\frac{3b^2}{a^7(a+bx)^5}+\frac{5b^2}{3a^6(a+bx)^6}+\frac{6b^2}{7a^5(a+bx)^7}+\frac{3b^2}{8a^4(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^10),x]
```

```
[Out] -1/(2*a^10*x^2) + (10*b)/(a^11*x) + b^2/(9*a^3*(a + b*x)^9) + (3*b^2)/(8*a^
4*(a + b*x)^8) + (6*b^2)/(7*a^5*(a + b*x)^7) + (5*b^2)/(3*a^6*(a + b*x)^6)
+ (3*b^2)/(a^7*(a + b*x)^5) + (21*b^2)/(4*a^8*(a + b*x)^4) + (28*b^2)/(3*a^
9*(a + b*x)^3) + (18*b^2)/(a^10*(a + b*x)^2) + (45*b^2)/(a^11*(a + b*x)) +
(55*b^2*Log[x])/a^12 - (55*b^2*Log[a + b*x])/a^12
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3(a+bx)^{10}}\,dx=\int \left(\frac{1}{a^{10}x^3}-\frac{10b}{a^{11}x^2}+\frac{55b^2}{a^{12}x}-\frac{b^3}{a^3(a+bx)^{10}}-\frac{3b^3}{a^4(a+bx)^9}-\frac{6b^3}{a^5(a+bx)^8}-\frac{10b^3}{a^6(a+bx)^7}-\frac{15b^3}{a^7(a+b}\right.$$

$$=-\frac{1}{2a^{10}x^2}+\frac{10b}{a^{11}x}+\frac{b^2}{9a^3(a+bx)^9}+\frac{3b^2}{8a^4(a+bx)^8}+\frac{6b^2}{7a^5(a+bx)^7}+\frac{5b^2}{3a^6(a+bx)^6}+\frac{3b^2}{a^7(a+bx)^5}+$$

Mathematica [A] time = 0.239428, size = 145, normalized size = 0.76

$$\frac{\frac{a\left(78419a^8b^2x^2+456291a^7b^3x^3+1326204a^6b^4x^4+2318316a^5b^5x^5+2604294a^4b^6x^6+1905750a^3b^7x^7+882420a^2b^8x^8+2772a^9bx-252a^{10}+235620ab^9x^9+27720b\right.}{x^2(a+bx)^9}}{504a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^10),x]
```

```
[Out] ((a*(-252*a^10 + 2772*a^9*b*x + 78419*a^8*b^2*x^2 + 456291*a^7*b^3*x^3 + 13
26204*a^6*b^4*x^4 + 2318316*a^5*b^5*x^5 + 2604294*a^4*b^6*x^6 + 1905750*a^3
*b^7*x^7 + 882420*a^2*b^8*x^8 + 235620*a*b^9*x^9 + 27720*b^10*x^10))/(x^2*(
a + b*x)^9) + 27720*b^2*Log[x] - 27720*b^2*Log[a + b*x])/(504*a^12)
```

Maple [A] time = 0.014, size = 178, normalized size = 0.9

$$-\frac{1}{2\,a^{10}x^{2}}+10\,\frac{b}{a^{11}x}+\frac{b^{2}}{9\,a^{3}\,(bx+a)^{9}}+\frac{3\,b^{2}}{8\,a^{4}\,(bx+a)^{8}}+\frac{6\,b^{2}}{7\,a^{5}\,(bx+a)^{7}}+\frac{5\,b^{2}}{3\,a^{6}\,(bx+a)^{6}}+3\,\frac{b^{2}}{a^{7}\,(bx+a)^{5}}+\frac{21\,b^{2}}{4\,a^{8}\,(bx+a)^{4}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^10,x)
```

```
[Out] -1/2/a^10/x^2+10*b/a^11/x+1/9*b^2/a^3/(b*x+a)^9+3/8*b^2/a^4/(b*x+a)^8+6/7*b
^2/a^5/(b*x+a)^7+5/3*b^2/a^6/(b*x+a)^6+3*b^2/a^7/(b*x+a)^5+21/4*b^2/a^8/(b*
x+a)^4+28/3*b^2/a^9/(b*x+a)^3+18*b^2/a^10/(b*x+a)^2+45*b^2/a^11/(b*x+a)+55*
b^2*ln(x)/a^12-55*b^2*ln(b*x+a)/a^12
```

Maxima [A] time = 1.17859, size = 324, normalized size = 1.7

$$\frac{27720\,b^{10}x^{10}+235620\,ab^{9}x^{9}+882420\,a^{2}b^{8}x^{8}+1905750\,a^{3}b^{7}x^{7}+2604294\,a^{4}b^{6}x^{6}+2318316\,a^{5}b^{5}x^{5}+1326204\,a^{6}b^{4}x^{4}}{504\left(a^{11}b^{9}x^{11}+9\,a^{12}b^{8}x^{10}+36\,a^{13}b^{7}x^{9}+84\,a^{14}b^{6}x^{8}+126\,a^{15}b^{5}x^{7}+126\,a^{16}b^{4}x^{6}+84\,a^{17}b^{3}x^{5}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/504*(27720*b^10*x^10 + 235620*a*b^9*x^9 + 882420*a^2*b^8*x^8 + 1905750*a^
3*b^7*x^7 + 2604294*a^4*b^6*x^6 + 2318316*a^5*b^5*x^5 + 1326204*a^6*b^4*x^4
 + 456291*a^7*b^3*x^3 + 78419*a^8*b^2*x^2 + 2772*a^9*b*x - 252*a^10)/(a^11*
b^9*x^11 + 9*a^12*b^8*x^10 + 36*a^13*b^7*x^9 + 84*a^14*b^6*x^8 + 126*a^15*b
^5*x^7 + 126*a^16*b^4*x^6 + 84*a^17*b^3*x^5 + 36*a^18*b^2*x^4 + 9*a^19*b*x^
3 + a^20*x^2) - 55*b^2*log(b*x + a)/a^12 + 55*b^2*log(x)/a^12
```

Fricas [B] time = 1.64254, size = 1019, normalized size = 5.34

$$\frac{27720\,ab^{10}x^{10}+235620\,a^{2}b^{9}x^{9}+882420\,a^{3}b^{8}x^{8}+1905750\,a^{4}b^{7}x^{7}+2604294\,a^{5}b^{6}x^{6}+2318316\,a^{6}b^{5}x^{5}+1326204\,a^{7}b^{4}x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(27720*a*b^10*x^10 + 235620*a^2*b^9*x^9 + 882420*a^3*b^8*x^8 + 190575
0*a^4*b^7*x^7 + 2604294*a^5*b^6*x^6 + 2318316*a^6*b^5*x^5 + 1326204*a^7*b^4
*x^4 + 456291*a^8*b^3*x^3 + 78419*a^9*b^2*x^2 + 2772*a^10*b*x - 252*a^11 -
27720*(b^11*x^11 + 9*a*b^10*x^10 + 36*a^2*b^9*x^9 + 84*a^3*b^8*x^8 + 126*a^
4*b^7*x^7 + 126*a^5*b^6*x^6 + 84*a^6*b^5*x^5 + 36*a^7*b^4*x^4 + 9*a^8*b^3*x
```

```
^3 + a^9*b^2*x^2)*log(b*x + a) + 27720*(b^11*x^11 + 9*a*b^10*x^10 + 36*a^2*
b^9*x^9 + 84*a^3*b^8*x^8 + 126*a^4*b^7*x^7 + 126*a^5*b^6*x^6 + 84*a^6*b^5*x
^5 + 36*a^7*b^4*x^4 + 9*a^8*b^3*x^3 + a^9*b^2*x^2)*log(x))/(a^12*b^9*x^11 +
 9*a^13*b^8*x^10 + 36*a^14*b^7*x^9 + 84*a^15*b^6*x^8 + 126*a^16*b^5*x^7 + 1
26*a^17*b^4*x^6 + 84*a^18*b^3*x^5 + 36*a^19*b^2*x^4 + 9*a^20*b*x^3 + a^21*x
^2)
```

Sympy [A] time = 3.31266, size = 246, normalized size = 1.29

$$\frac{-252a^{10} + 2772a^9bx + 78419a^8b^2x^2 + 456291a^7b^3x^3 + 1326204a^6b^4x^4 + 2318316a^5b^5x^5 + 2604294a^4b^6x^6 + 190575}{504a^{20}x^2 + 4536a^{19}bx^3 + 18144a^{18}b^2x^4 + 42336a^{17}b^3x^5 + 63504a^{16}b^4x^6 + 63504a^{15}b^5x^7 + 42336a^{14}b^6x^8 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**10,x)
```

```
[Out] (-252*a**10 + 2772*a**9*b*x + 78419*a**8*b**2*x**2 + 456291*a**7*b**3*x**3
+ 1326204*a**6*b**4*x**4 + 2318316*a**5*b**5*x**5 + 2604294*a**4*b**6*x**6
+ 1905750*a**3*b**7*x**7 + 882420*a**2*b**8*x**8 + 235620*a*b**9*x**9 + 277
20*b**10*x**10)/(504*a**20*x**2 + 4536*a**19*b*x**3 + 18144*a**18*b**2*x**4
 + 42336*a**17*b**3*x**5 + 63504*a**16*b**4*x**6 + 63504*a**15*b**5*x**7 +
42336*a**14*b**6*x**8 + 18144*a**13*b**7*x**9 + 4536*a**12*b**8*x**10 + 504
*a**11*b**9*x**11) + 55*b**2*(log(x) - log(a/b + x))/a**12
```

Giac [A] time = 1.23231, size = 205, normalized size = 1.07

$$-\frac{55\,b^2\log\left(|bx+a|\right)}{a^{12}} + \frac{55\,b^2\log\left(|x|\right)}{a^{12}} + \frac{27720\,ab^{10}x^{10} + 235620\,a^2b^9x^9 + 882420\,a^3b^8x^8 + 1905750\,a^4b^7x^7 + 260429}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -55*b^2*log(abs(b*x + a))/a^12 + 55*b^2*log(abs(x))/a^12 + 1/504*(27720*a*b
^10*x^10 + 235620*a^2*b^9*x^9 + 882420*a^3*b^8*x^8 + 1905750*a^4*b^7*x^7 +
2604294*a^5*b^6*x^6 + 2318316*a^6*b^5*x^5 + 1326204*a^7*b^4*x^4 + 456291*a^
8*b^3*x^3 + 78419*a^9*b^2*x^2 + 2772*a^10*b*x - 252*a^11)/((b*x + a)^9*a^12
*x^2)
```

3.238 $\int \frac{1}{x^4(a+bx)^{10}}\,dx$

Optimal. Leaf size=198

$$-\frac{165b^3}{a^{12}(a+bx)}-\frac{60b^3}{a^{11}(a+bx)^2}-\frac{28b^3}{a^{10}(a+bx)^3}-\frac{14b^3}{a^9(a+bx)^4}-\frac{7b^3}{a^8(a+bx)^5}-\frac{10b^3}{3a^7(a+bx)^6}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{b^3}{2a^5(a+bx)^8}-$$

```
[Out] -1/(3*a^10*x^3) + (5*b)/(a^11*x^2) - (55*b^2)/(a^12*x) - b^3/(9*a^4*(a + b*
x)^9) - b^3/(2*a^5*(a + b*x)^8) - (10*b^3)/(7*a^6*(a + b*x)^7) - (10*b^3)/(
3*a^7*(a + b*x)^6) - (7*b^3)/(a^8*(a + b*x)^5) - (14*b^3)/(a^9*(a + b*x)^4)
 - (28*b^3)/(a^10*(a + b*x)^3) - (60*b^3)/(a^11*(a + b*x)^2) - (165*b^3)/(a
^12*(a + b*x)) - (220*b^3*Log[x])/a^13 + (220*b^3*Log[a + b*x])/a^13
```

Rubi [A] time = 0.164663, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{165b^3}{a^{12}(a+bx)}-\frac{60b^3}{a^{11}(a+bx)^2}-\frac{28b^3}{a^{10}(a+bx)^3}-\frac{14b^3}{a^9(a+bx)^4}-\frac{7b^3}{a^8(a+bx)^5}-\frac{10b^3}{3a^7(a+bx)^6}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{b^3}{2a^5(a+bx)^8}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^10),x]
```

```
[Out] -1/(3*a^10*x^3) + (5*b)/(a^11*x^2) - (55*b^2)/(a^12*x) - b^3/(9*a^4*(a + b*
x)^9) - b^3/(2*a^5*(a + b*x)^8) - (10*b^3)/(7*a^6*(a + b*x)^7) - (10*b^3)/(
3*a^7*(a + b*x)^6) - (7*b^3)/(a^8*(a + b*x)^5) - (14*b^3)/(a^9*(a + b*x)^4)
 - (28*b^3)/(a^10*(a + b*x)^3) - (60*b^3)/(a^11*(a + b*x)^2) - (165*b^3)/(a
^12*(a + b*x)) - (220*b^3*Log[x])/a^13 + (220*b^3*Log[a + b*x])/a^13
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^4(a+bx)^{10}}\,dx=\int \left(\frac{1}{a^{10}x^4}-\frac{10b}{a^{11}x^3}+\frac{55b^2}{a^{12}x^2}-\frac{220b^3}{a^{13}x}+\frac{b^4}{a^4(a+bx)^{10}}+\frac{4b^4}{a^5(a+bx)^9}+\frac{10b^4}{a^6(a+bx)^8}+\frac{20b^4}{a^7(a+bx)^7}+\right.$$
$$=-\frac{1}{3a^{10}x^3}+\frac{5b}{a^{11}x^2}-\frac{55b^2}{a^{12}x}-\frac{b^3}{9a^4(a+bx)^9}-\frac{b^3}{2a^5(a+bx)^8}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{10b^3}{3a^7(a+bx)^6}-\frac{7b^3}{a^8(a+b}$$

Mathematica [A] time = 0.170529, size = 156, normalized size = 0.79

$$-\frac{\frac{a\left(2772a^9b^2x^2+78419a^8b^3x^3+456291a^7b^4x^4+1326204a^6b^5x^5+2318316a^5b^6x^6+2604294a^4b^7x^7+1905750a^3b^8x^8+882420a^2b^9x^9-252a^{10}bx+42a^{11}+235620ab^{10}\right.}{x^3(a+bx)^9}}{126a^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^10),x]
```

```
[Out] -((a*(42*a^11 - 252*a^10*b*x + 2772*a^9*b^2*x^2 + 78419*a^8*b^3*x^3 + 45629
1*a^7*b^4*x^4 + 1326204*a^6*b^5*x^5 + 2318316*a^5*b^6*x^6 + 2604294*a^4*b^7
*x^7 + 1905750*a^3*b^8*x^8 + 882420*a^2*b^9*x^9 + 235620*a*b^10*x^10 + 2772
0*b^11*x^11))/(x^3*(a + b*x)^9) + 27720*b^3*Log[x] - 27720*b^3*Log[a + b*x]
)/(126*a^13)
```

Maple [A] time = 0.016, size = 189, normalized size = 1.

$$-\frac{1}{3\,a^{10}x^{3}}+5\,\frac{b}{a^{11}x^{2}}-55\,\frac{b^{2}}{a^{12}x}-\frac{b^{3}}{9\,a^{4}\left(bx+a\right)^{9}}-\frac{b^{3}}{2\,a^{5}\left(bx+a\right)^{8}}-\frac{10\,b^{3}}{7\,a^{6}\left(bx+a\right)^{7}}-\frac{10\,b^{3}}{3\,a^{7}\left(bx+a\right)^{6}}-7\,\frac{b^{3}}{a^{8}\left(bx+a\right)^{5}}-14$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^10,x)
```

```
[Out] -1/3/a^10/x^3+5*b/a^11/x^2-55*b^2/a^12/x-1/9*b^3/a^4/(b*x+a)^9-1/2*b^3/a^5/
(b*x+a)^8-10/7*b^3/a^6/(b*x+a)^7-10/3*b^3/a^7/(b*x+a)^6-7*b^3/a^8/(b*x+a)^5
-14*b^3/a^9/(b*x+a)^4-28*b^3/a^10/(b*x+a)^3-60*b^3/a^11/(b*x+a)^2-165*b^3/a
^12/(b*x+a)-220*b^3*ln(x)/a^13+220*b^3*ln(b*x+a)/a^13
```

Maxima [A] time = 1.19602, size = 339, normalized size = 1.71

$$-\frac{27720\,b^{11}x^{11}+235620\,ab^{10}x^{10}+882420\,a^{2}b^{9}x^{9}+1905750\,a^{3}b^{8}x^{8}+2604294\,a^{4}b^{7}x^{7}+2318316\,a^{5}b^{6}x^{6}+1326204\,a}{126\left(a^{12}b^{9}x^{12}+9\,a^{13}b^{8}x^{11}+36\,a^{14}b^{7}x^{10}+84\,a^{15}b^{6}x^{9}+126\,a^{16}b^{5}x^{8}+126\,a^{17}b^{4}x^{7}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/126*(27720*b^11*x^11 + 235620*a*b^10*x^10 + 882420*a^2*b^9*x^9 + 1905750
*a^3*b^8*x^8 + 2604294*a^4*b^7*x^7 + 2318316*a^5*b^6*x^6 + 1326204*a^6*b^5*
x^5 + 456291*a^7*b^4*x^4 + 78419*a^8*b^3*x^3 + 2772*a^9*b^2*x^2 - 252*a^10*
b*x + 42*a^11)/(a^12*b^9*x^12 + 9*a^13*b^8*x^11 + 36*a^14*b^7*x^10 + 84*a^1
5*b^6*x^9 + 126*a^16*b^5*x^8 + 126*a^17*b^4*x^7 + 84*a^18*b^3*x^6 + 36*a^19
*b^2*x^5 + 9*a^20*b*x^4 + a^21*x^3) + 220*b^3*log(b*x + a)/a^13 - 220*b^3*l
og(x)/a^13
```

Fricas [B] time = 1.67768, size = 1054, normalized size = 5.32

$$-\frac{27720\,ab^{11}x^{11}+235620\,a^{2}b^{10}x^{10}+882420\,a^{3}b^{9}x^{9}+1905750\,a^{4}b^{8}x^{8}+2604294\,a^{5}b^{7}x^{7}+2318316\,a^{6}b^{6}x^{6}+132620}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/126*(27720*a*b^11*x^11 + 235620*a^2*b^10*x^10 + 882420*a^3*b^9*x^9 + 190
5750*a^4*b^8*x^8 + 2604294*a^5*b^7*x^7 + 2318316*a^6*b^6*x^6 + 1326204*a^7*
b^5*x^5 + 456291*a^8*b^4*x^4 + 78419*a^9*b^3*x^3 + 2772*a^10*b^2*x^2 - 252*
```

```
a^11*b*x + 42*a^12 - 27720*(b^12*x^12 + 9*a*b^11*x^11 + 36*a^2*b^10*x^10 +
84*a^3*b^9*x^9 + 126*a^4*b^8*x^8 + 126*a^5*b^7*x^7 + 84*a^6*b^6*x^6 + 36*a^
7*b^5*x^5 + 9*a^8*b^4*x^4 + a^9*b^3*x^3)*log(b*x + a) + 27720*(b^12*x^12 +
9*a*b^11*x^11 + 36*a^2*b^10*x^10 + 84*a^3*b^9*x^9 + 126*a^4*b^8*x^8 + 126*a
^5*b^7*x^7 + 84*a^6*b^6*x^6 + 36*a^7*b^5*x^5 + 9*a^8*b^4*x^4 + a^9*b^3*x^3)
*log(x))/(a^13*b^9*x^12 + 9*a^14*b^8*x^11 + 36*a^15*b^7*x^10 + 84*a^16*b^6*
x^9 + 126*a^17*b^5*x^8 + 126*a^18*b^4*x^7 + 84*a^19*b^3*x^6 + 36*a^20*b^2*x
^5 + 9*a^21*b*x^4 + a^22*x^3)
```

Sympy [A] time = 4.14182, size = 258, normalized size = 1.3

$$-\frac{42a^{11}-252a^{10}bx+2772a^9b^2x^2+78419a^8b^3x^3+456291a^7b^4x^4+1326204a^6b^5x^5+2318316a^5b^6x^6+2604294a^4b^7x^7}{126a^{21}x^3+1134a^{20}bx^4+4536a^{19}b^2x^5+10584a^{18}b^3x^6+15876a^{17}b^4x^7+15876a^{16}b^5x^8+10584a^{15}b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**10,x)
```

```
[Out] -(42*a**11 - 252*a**10*b*x + 2772*a**9*b**2*x**2 + 78419*a**8*b**3*x**3 + 4
56291*a**7*b**4*x**4 + 1326204*a**6*b**5*x**5 + 2318316*a**5*b**6*x**6 + 26
04294*a**4*b**7*x**7 + 1905750*a**3*b**8*x**8 + 882420*a**2*b**9*x**9 + 235
620*a*b**10*x**10 + 27720*b**11*x**11)/(126*a**21*x**3 + 1134*a**20*b*x**4
+ 4536*a**19*b**2*x**5 + 10584*a**18*b**3*x**6 + 15876*a**17*b**4*x**7 + 15
876*a**16*b**5*x**8 + 10584*a**15*b**6*x**9 + 4536*a**14*b**7*x**10 + 1134*
a**13*b**8*x**11 + 126*a**12*b**9*x**12) + 220*b**3*(-log(x) + log(a/b + x)
)/a**13
```

Giac [A] time = 1.18127, size = 220, normalized size = 1.11

$$\frac{220\,b^3\log\left(|bx+a|\right)}{a^{13}}-\frac{220\,b^3\log\left(|x|\right)}{a^{13}}-\frac{27720\,ab^{11}x^{11}+235620\,a^2b^{10}x^{10}+882420\,a^3b^9x^9+1905750\,a^4b^8x^8+2604294}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 220*b^3*log(abs(b*x + a))/a^13 - 220*b^3*log(abs(x))/a^13 - 1/126*(27720*a*
b^11*x^11 + 235620*a^2*b^10*x^10 + 882420*a^3*b^9*x^9 + 1905750*a^4*b^8*x^8
 + 2604294*a^5*b^7*x^7 + 2318316*a^6*b^6*x^6 + 1326204*a^7*b^5*x^5 + 456291
*a^8*b^4*x^4 + 78419*a^9*b^3*x^3 + 2772*a^10*b^2*x^2 - 252*a^11*b*x + 42*a^
12)/((b*x + a)^9*a^13*x^3)
```

3.239 $\int \frac{(a+bx)^{12}}{x^{10}}\,dx$

Optimal. Leaf size=141

$$-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+220a^3b^9\log(x)-\frac{3a^{11}b}{2x^8}-\frac{a}{9}$$

```
[Out] -a^12/(9*x^9) - (3*a^11*b)/(2*x^8) - (66*a^10*b^2)/(7*x^7) - (110*a^9*b^3)/
(3*x^6) - (99*a^8*b^4)/x^5 - (198*a^7*b^5)/x^4 - (308*a^6*b^6)/x^3 - (396*a
^5*b^7)/x^2 - (495*a^4*b^8)/x + 66*a^2*b^10*x + 6*a*b^11*x^2 + (b^12*x^3)/3
 + 220*a^3*b^9*Log[x]
```

Rubi [A] time = 0.078976, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+220a^3b^9\log(x)-\frac{3a^{11}b}{2x^8}-\frac{a}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^12/x^10,x]
```

```
[Out] -a^12/(9*x^9) - (3*a^11*b)/(2*x^8) - (66*a^10*b^2)/(7*x^7) - (110*a^9*b^3)/
(3*x^6) - (99*a^8*b^4)/x^5 - (198*a^7*b^5)/x^4 - (308*a^6*b^6)/x^3 - (396*a
^5*b^7)/x^2 - (495*a^4*b^8)/x + 66*a^2*b^10*x + 6*a*b^11*x^2 + (b^12*x^3)/3
 + 220*a^3*b^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{12}}{x^{10}}\,dx=\int \left(66a^2b^{10}+\frac{a^{12}}{x^{10}}+\frac{12a^{11}b}{x^9}+\frac{66a^{10}b^2}{x^8}+\frac{220a^9b^3}{x^7}+\frac{495a^8b^4}{x^6}+\frac{792a^7b^5}{x^5}+\frac{924a^6b^6}{x^4}+\frac{792a^5b^7}{x^3}+\right.$$
$$=-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2$$

Mathematica [A] time = 0.0187305, size = 141, normalized size = 1.

$$-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+220a^3b^9\log(x)-\frac{3a^{11}b}{2x^8}-\frac{a}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^12/x^10,x]
```

```
[Out] -a^12/(9*x^9) - (3*a^11*b)/(2*x^8) - (66*a^10*b^2)/(7*x^7) - (110*a^9*b^3)/
(3*x^6) - (99*a^8*b^4)/x^5 - (198*a^7*b^5)/x^4 - (308*a^6*b^6)/x^3 - (396*a
```

```
^5*b^7)/x^2 - (495*a^4*b^8)/x + 66*a^2*b^10*x + 6*a*b^11*x^2 + (b^12*x^3)/3
 + 220*a^3*b^9*Log[x]
```

Maple [A] time = 0.007, size = 132, normalized size = 0.9

$$-\frac{a^{12}}{9\,x^9}-\frac{3\,a^{11}b}{2\,x^8}-\frac{66\,a^{10}b^2}{7\,x^7}-\frac{110\,a^9b^3}{3\,x^6}-99\,\frac{a^8b^4}{x^5}-198\,\frac{a^7b^5}{x^4}-308\,\frac{a^6b^6}{x^3}-396\,\frac{a^5b^7}{x^2}-495\,\frac{a^4b^8}{x}+66\,a^2b^{10}x+6\,ab^{11}x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^12/x^10,x)
```

```
[Out] -1/9*a^12/x^9-3/2*a^11*b/x^8-66/7*a^10*b^2/x^7-110/3*a^9*b^3/x^6-99*a^8*b^4
/x^5-198*a^7*b^5/x^4-308*a^6*b^6/x^3-396*a^5*b^7/x^2-495*a^4*b^8/x+66*a^2*b
^10*x+6*a*b^11*x^2+1/3*b^12*x^3+220*a^3*b^9*ln(x)
```

Maxima [A] time = 1.09162, size = 178, normalized size = 1.26

$$\frac{1}{3}\,b^{12}x^3+6\,ab^{11}x^2+66\,a^2b^{10}x+220\,a^3b^9\log\left(x\right)-\frac{62370\,a^4b^8x^8+49896\,a^5b^7x^7+38808\,a^6b^6x^6+24948\,a^7b^5x^5+1247}{126\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*b^12*x^3 + 6*a*b^11*x^2 + 66*a^2*b^10*x + 220*a^3*b^9*log(x) - 1/126*(6
2370*a^4*b^8*x^8 + 49896*a^5*b^7*x^7 + 38808*a^6*b^6*x^6 + 24948*a^7*b^5*x^
5 + 12474*a^8*b^4*x^4 + 4620*a^9*b^3*x^3 + 1188*a^10*b^2*x^2 + 189*a^11*b*x
 + 14*a^12)/x^9
```

Fricas [A] time = 1.55274, size = 346, normalized size = 2.45

$$\frac{42\,b^{12}x^{12}+756\,ab^{11}x^{11}+8316\,a^2b^{10}x^{10}+27720\,a^3b^9x^9\log\left(x\right)-62370\,a^4b^8x^8-49896\,a^5b^7x^7-38808\,a^6b^6x^6-24948\,a}{126\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="fricas")
```

```
[Out] 1/126*(42*b^12*x^12 + 756*a*b^11*x^11 + 8316*a^2*b^10*x^10 + 27720*a^3*b^9*
x^9*log(x) - 62370*a^4*b^8*x^8 - 49896*a^5*b^7*x^7 - 38808*a^6*b^6*x^6 - 24
948*a^7*b^5*x^5 - 12474*a^8*b^4*x^4 - 4620*a^9*b^3*x^3 - 1188*a^10*b^2*x^2
- 189*a^11*b*x - 14*a^12)/x^9
```

Sympy [A] time = 1.36934, size = 141, normalized size = 1.

$$220a^3b^9\log\left(x\right)+66a^2b^{10}x+6ab^{11}x^2+\frac{b^{12}x^3}{3}-\frac{14a^{12}+189a^{11}bx+1188a^{10}b^2x^2+4620a^9b^3x^3+12474a^8b^4x^4+24948a}{126x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**12/x**10,x)
```

```
[Out] 220*a**3*b**9*log(x) + 66*a**2*b**10*x + 6*a*b**11*x**2 + b**12*x**3/3 - (1
4*a**12 + 189*a**11*b*x + 1188*a**10*b**2*x**2 + 4620*a**9*b**3*x**3 + 1247
4*a**8*b**4*x**4 + 24948*a**7*b**5*x**5 + 38808*a**6*b**6*x**6 + 49896*a**5
*b**7*x**7 + 62370*a**4*b**8*x**8)/(126*x**9)
```

Giac [A] time = 1.19574, size = 180, normalized size = 1.28

$$\frac{1}{3}b^{12}x^3 + 6\,ab^{11}x^2 + 66\,a^2b^{10}x + 220\,a^3b^9\log\left(|x|\right) - \frac{62370\,a^4b^8x^8 + 49896\,a^5b^7x^7 + 38808\,a^6b^6x^6 + 24948\,a^7b^5x^5 + 1}{126}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="giac")
```

```
[Out] 1/3*b^12*x^3 + 6*a*b^11*x^2 + 66*a^2*b^10*x + 220*a^3*b^9*log(abs(x)) - 1/1
26*(62370*a^4*b^8*x^8 + 49896*a^5*b^7*x^7 + 38808*a^6*b^6*x^6 + 24948*a^7*b
^5*x^5 + 12474*a^8*b^4*x^4 + 4620*a^9*b^3*x^3 + 1188*a^10*b^2*x^2 + 189*a^1
1*b*x + 14*a^12)/x^9
```

3.240 $\int \frac{(a+bx)^{11}}{x^{10}} dx$

Optimal. Leaf size=132

$$-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+55a^2b^9\log(x)-\frac{11a^{10}b}{8x^8}-\frac{a^{11}}{9x^9}+11ab^{10}x+\frac{b}{}$$

```
[Out] -a^11/(9*x^9) - (11*a^10*b)/(8*x^8) - (55*a^9*b^2)/(7*x^7) - (55*a^8*b^3)/(
2*x^6) - (66*a^7*b^4)/x^5 - (231*a^6*b^5)/(2*x^4) - (154*a^5*b^6)/x^3 - (16
5*a^4*b^7)/x^2 - (165*a^3*b^8)/x + 11*a*b^10*x + (b^11*x^2)/2 + 55*a^2*b^9*
Log[x]
```

Rubi [A] time = 0.0695941, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+55a^2b^9\log(x)-\frac{11a^{10}b}{8x^8}-\frac{a^{11}}{9x^9}+11ab^{10}x+\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^11/x^10,x]
```

```
[Out] -a^11/(9*x^9) - (11*a^10*b)/(8*x^8) - (55*a^9*b^2)/(7*x^7) - (55*a^8*b^3)/(
2*x^6) - (66*a^7*b^4)/x^5 - (231*a^6*b^5)/(2*x^4) - (154*a^5*b^6)/x^3 - (16
5*a^4*b^7)/x^2 - (165*a^3*b^8)/x + 11*a*b^10*x + (b^11*x^2)/2 + 55*a^2*b^9*
Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{11}}{x^{10}}\,dx &= \int \left(11ab^{10}+\frac{a^{11}}{x^{10}}+\frac{11a^{10}b}{x^9}+\frac{55a^9b^2}{x^8}+\frac{165a^8b^3}{x^7}+\frac{330a^7b^4}{x^6}+\frac{462a^6b^5}{x^5}+\frac{462a^5b^6}{x^4}+\frac{330a^4b^7}{x^3}+\frac{165a}{x^2}\right.\\ &= -\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+11ab^{10}x-\end{aligned}$$

Mathematica [A] time = 0.0068515, size = 132, normalized size = 1.

$$-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+55a^2b^9\log(x)-\frac{11a^{10}b}{8x^8}-\frac{a^{11}}{9x^9}+11ab^{10}x+\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^11/x^10,x]
```

```
[Out] -a^11/(9*x^9) - (11*a^10*b)/(8*x^8) - (55*a^9*b^2)/(7*x^7) - (55*a^8*b^3)/(
2*x^6) - (66*a^7*b^4)/x^5 - (231*a^6*b^5)/(2*x^4) - (154*a^5*b^6)/x^3 - (16
```

```
5*a^4*b^7)/x^2 - (165*a^3*b^8)/x + 11*a*b^10*x + (b^11*x^2)/2 + 55*a^2*b^9*
Log[x]
```

Maple [A] time = 0.009, size = 121, normalized size = 0.9

$$-\frac{a^{11}}{9\,x^{9}}-\frac{11\,a^{10}b}{8\,x^{8}}-\frac{55\,a^{9}b^{2}}{7\,x^{7}}-\frac{55\,a^{8}b^{3}}{2\,x^{6}}-66\,\frac{a^{7}b^{4}}{x^{5}}-\frac{231\,a^{6}b^{5}}{2\,x^{4}}-154\,\frac{a^{5}b^{6}}{x^{3}}-165\,\frac{a^{4}b^{7}}{x^{2}}-165\,\frac{a^{3}b^{8}}{x}+11\,ab^{10}x+\frac{b^{11}x^{2}}{2}+55$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^11/x^10,x)
```

```
[Out] -1/9*a^11/x^9-11/8*a^10*b/x^8-55/7*a^9*b^2/x^7-55/2*a^8*b^3/x^6-66*a^7*b^4/
x^5-231/2*a^6*b^5/x^4-154*a^5*b^6/x^3-165*a^4*b^7/x^2-165*a^3*b^8/x+11*a*b^
10*x+1/2*b^11*x^2+55*a^2*b^9*ln(x)
```

Maxima [A] time = 1.06214, size = 163, normalized size = 1.23

$$\frac{1}{2}\,b^{11}x^{2}+11\,ab^{10}x+55\,a^{2}b^{9}\log\left(x\right)-\frac{83160\,a^{3}b^{8}x^{8}+83160\,a^{4}b^{7}x^{7}+77616\,a^{5}b^{6}x^{6}+58212\,a^{6}b^{5}x^{5}+33264\,a^{7}b^{4}x^{4}+}{504\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="maxima")
```

```
[Out] 1/2*b^11*x^2 + 11*a*b^10*x + 55*a^2*b^9*log(x) - 1/504*(83160*a^3*b^8*x^8 +
 83160*a^4*b^7*x^7 + 77616*a^5*b^6*x^6 + 58212*a^6*b^5*x^5 + 33264*a^7*b^4*
x^4 + 13860*a^8*b^3*x^3 + 3960*a^9*b^2*x^2 + 693*a^10*b*x + 56*a^11)/x^9
```

Fricas [A] time = 1.49566, size = 320, normalized size = 2.42

$$\frac{252\,b^{11}x^{11}+5544\,ab^{10}x^{10}+27720\,a^{2}b^{9}x^{9}\log\left(x\right)-83160\,a^{3}b^{8}x^{8}-83160\,a^{4}b^{7}x^{7}-77616\,a^{5}b^{6}x^{6}-58212\,a^{6}b^{5}x^{5}-33}{504\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^11*x^11 + 5544*a*b^10*x^10 + 27720*a^2*b^9*x^9*log(x) - 83160*
a^3*b^8*x^8 - 83160*a^4*b^7*x^7 - 77616*a^5*b^6*x^6 - 58212*a^6*b^5*x^5 - 3
3264*a^7*b^4*x^4 - 13860*a^8*b^3*x^3 - 3960*a^9*b^2*x^2 - 693*a^10*b*x - 56
*a^11)/x^9
```

Sympy [A] time = 1.2748, size = 129, normalized size = 0.98

$$55a^{2}b^{9}\log\left(x\right)+11ab^{10}x+\frac{b^{11}x^{2}}{2}-\frac{56a^{11}+693a^{10}bx+3960a^{9}b^{2}x^{2}+13860a^{8}b^{3}x^{3}+33264a^{7}b^{4}x^{4}+58212a^{6}b^{5}x^{5}+}{504x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**11/x**10,x)
```

```
[Out] 55*a**2*b**9*log(x) + 11*a*b**10*x + b**11*x**2/2 - (56*a**11 + 693*a**10*b
*x + 3960*a**9*b**2*x**2 + 13860*a**8*b**3*x**3 + 33264*a**7*b**4*x**4 + 58
212*a**6*b**5*x**5 + 77616*a**5*b**6*x**6 + 83160*a**4*b**7*x**7 + 83160*a*
*3*b**8*x**8)/(504*x**9)
```

Giac [A] time = 1.22444, size = 165, normalized size = 1.25

$$\frac{1}{2}b^{11}x^2 + 11\,ab^{10}x + 55\,a^2b^9\log(|x|) - \frac{83160\,a^3b^8x^8 + 83160\,a^4b^7x^7 + 77616\,a^5b^6x^6 + 58212\,a^6b^5x^5 + 33264\,a^7b^4x^4 + 13}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="giac")
```

```
[Out] 1/2*b^11*x^2 + 11*a*b^10*x + 55*a^2*b^9*log(abs(x)) - 1/504*(83160*a^3*b^8*
x^8 + 83160*a^4*b^7*x^7 + 77616*a^5*b^6*x^6 + 58212*a^6*b^5*x^5 + 33264*a^7
*b^4*x^4 + 13860*a^8*b^3*x^3 + 3960*a^9*b^2*x^2 + 693*a^10*b*x + 56*a^11)/x
^9
```

3.241 $\int \frac{(a+bx)^{10}}{x^{10}} \, dx$

Optimal. Leaf size=114

$$-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}-\frac{5a^9b}{4x^8}-\frac{a^{10}}{9x^9}+10ab^9\log(x)+b^{10}x$$

```
[Out] -a^10/(9*x^9) - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rubi [A] time = 0.0577027, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}-\frac{5a^9b}{4x^8}-\frac{a^{10}}{9x^9}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^10,x]
```

```
[Out] -a^10/(9*x^9) - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{10}} \, dx = \int \left(b^{10}+\frac{a^{10}}{x^{10}}+\frac{10a^9b}{x^9}+\frac{45a^8b^2}{x^8}+\frac{120a^7b^3}{x^7}+\frac{210a^6b^4}{x^6}+\frac{252a^5b^5}{x^5}+\frac{210a^4b^6}{x^4}+\frac{120a^3b^7}{x^3}+\frac{45a^2b}{x^2}\right.$$

$$= -\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10ab$$

Mathematica [A] time = 0.007026, size = 114, normalized size = 1.

$$-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}-\frac{5a^9b}{4x^8}-\frac{a^{10}}{9x^9}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^10,x]
```

```
[Out] -a^10/(9*x^9) - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
```

```
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Maple [A] time = 0., size = 109, normalized size = 1.

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-20\frac{a^7b^3}{x^6}-42\frac{a^6b^4}{x^5}-63\frac{a^5b^5}{x^4}-70\frac{a^4b^6}{x^3}-60\frac{a^3b^7}{x^2}-45\frac{a^2b^8}{x}+b^{10}x+10ab^9\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^10,x)
```

```
[Out] -1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5-
63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9*l
n(x)
```

Maxima [A] time = 1.07468, size = 147, normalized size = 1.29

$$b^{10}x+10ab^9\log(x)-\frac{11340a^2b^8x^8+15120a^3b^7x^7+17640a^4b^6x^6+15876a^5b^5x^5+10584a^6b^4x^4+5040a^7b^3x^3+162}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="maxima")
```

```
[Out] b^10*x + 10*a*b^9*log(x) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^7 + 1
7640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^3*x^3
 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Fricas [A] time = 1.43675, size = 288, normalized size = 2.53

$$\frac{252b^{10}x^{10}+2520ab^9x^9\log(x)-11340a^2b^8x^8-15120a^3b^7x^7-17640a^4b^6x^6-15876a^5b^5x^5-10584a^6b^4x^4-5040a^7b}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2520*a*b^9*x^9*log(x) - 11340*a^2*b^8*x^8 - 15120*a^
3*b^7*x^7 - 17640*a^4*b^6*x^6 - 15876*a^5*b^5*x^5 - 10584*a^6*b^4*x^4 - 504
0*a^7*b^3*x^3 - 1620*a^8*b^2*x^2 - 315*a^9*b*x - 28*a^10)/x^9
```

Sympy [A] time = 1.31935, size = 116, normalized size = 1.02

$$10ab^9\log(x)+b^{10}x-\frac{28a^{10}+315a^9bx+1620a^8b^2x^2+5040a^7b^3x^3+10584a^6b^4x^4+15876a^5b^5x^5+17640a^4b^6x^6+151}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**10,x)
```

```
[Out] 10*a*b**9*log(x) + b**10*x - (28*a**10 + 315*a**9*b*x + 1620*a**8*b**2*x**2
 + 5040*a**7*b**3*x**3 + 10584*a**6*b**4*x**4 + 15876*a**5*b**5*x**5 + 1764
0*a**4*b**6*x**6 + 15120*a**3*b**7*x**7 + 11340*a**2*b**8*x**8)/(252*x**9)
```

Giac [A] time = 1.18988, size = 149, normalized size = 1.31

$$b^{10}x + 10\,ab^9\log\left(|x|\right) - \frac{11340\,a^2b^8x^8 + 15120\,a^3b^7x^7 + 17640\,a^4b^6x^6 + 15876\,a^5b^5x^5 + 10584\,a^6b^4x^4 + 5040\,a^7b^3x^3 + }{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="giac")
```

```
[Out] b^10*x + 10*a*b^9*log(abs(x)) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^
7 + 17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^
3*x^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

3.242 $\int \frac{(a+bx)^9}{x^{10}}\,dx$

Optimal. Leaf size=109

$$-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9a^8b}{8x^8}-\frac{a^9}{9x^9}-\frac{9ab^8}{x}+b^9\log(x)$$

```
[Out] -a^9/(9*x^9) - (9*a^8*b)/(8*x^8) - (36*a^7*b^2)/(7*x^7) - (14*a^6*b^3)/x^6
- (126*a^5*b^4)/(5*x^5) - (63*a^4*b^5)/(2*x^4) - (28*a^3*b^6)/x^3 - (18*a^2
*b^7)/x^2 - (9*a*b^8)/x + b^9*Log[x]
```

Rubi [A] time = 0.0516384, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9a^8b}{8x^8}-\frac{a^9}{9x^9}-\frac{9ab^8}{x}+b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9/x^10,x]
```

```
[Out] -a^9/(9*x^9) - (9*a^8*b)/(8*x^8) - (36*a^7*b^2)/(7*x^7) - (14*a^6*b^3)/x^6
- (126*a^5*b^4)/(5*x^5) - (63*a^4*b^5)/(2*x^4) - (28*a^3*b^6)/x^3 - (18*a^2
*b^7)/x^2 - (9*a*b^8)/x + b^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^9}{x^{10}}\,dx &= \int \left(\frac{a^9}{x^{10}}+\frac{9a^8b}{x^9}+\frac{36a^7b^2}{x^8}+\frac{84a^6b^3}{x^7}+\frac{126a^5b^4}{x^6}+\frac{126a^4b^5}{x^5}+\frac{84a^3b^6}{x^4}+\frac{36a^2b^7}{x^3}+\frac{9ab^8}{x^2}+\frac{b^9}{x}\right)dx\\ &= -\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)\end{aligned}$$

Mathematica [A] time = 0.007086, size = 109, normalized size = 1.

$$-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9a^8b}{8x^8}-\frac{a^9}{9x^9}-\frac{9ab^8}{x}+b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9/x^10,x]
```

```
[Out] -a^9/(9*x^9) - (9*a^8*b)/(8*x^8) - (36*a^7*b^2)/(7*x^7) - (14*a^6*b^3)/x^6
- (126*a^5*b^4)/(5*x^5) - (63*a^4*b^5)/(2*x^4) - (28*a^3*b^6)/x^3 - (18*a^2
```

```
*b^7)/x^2 - (9*a*b^8)/x + b^9*Log[x]
```

Maple [A] time = 0.008, size = 100, normalized size = 0.9

$$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-14\frac{a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-28\frac{a^3b^6}{x^3}-18\frac{a^2b^7}{x^2}-9\frac{ab^8}{x}+b^9\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9/x^10,x)
```

```
[Out] -1/9*a^9/x^9-9/8*a^8*b/x^8-36/7*a^7*b^2/x^7-14*a^6*b^3/x^6-126/5*a^5*b^4/x^
5-63/2*a^4*b^5/x^4-28*a^3*b^6/x^3-18*a^2*b^7/x^2-9*a*b^8/x+b^9*ln(x)
```

Maxima [A] time = 1.06486, size = 135, normalized size = 1.24

$$b^9\log(x)-\frac{22680\,ab^8x^8+45360\,a^2b^7x^7+70560\,a^3b^6x^6+79380\,a^4b^5x^5+63504\,a^5b^4x^4+35280\,a^6b^3x^3+12960\,a^7b^2}{2520\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="maxima")
```

```
[Out] b^9*log(x) - 1/2520*(22680*a*b^8*x^8 + 45360*a^2*b^7*x^7 + 70560*a^3*b^6*x^
6 + 79380*a^4*b^5*x^5 + 63504*a^5*b^4*x^4 + 35280*a^6*b^3*x^3 + 12960*a^7*b
^2*x^2 + 2835*a^8*b*x + 280*a^9)/x^9
```

Fricas [A] time = 1.46236, size = 266, normalized size = 2.44

$$\frac{2520\,b^9x^9\log(x)-22680\,ab^8x^8-45360\,a^2b^7x^7-70560\,a^3b^6x^6-79380\,a^4b^5x^5-63504\,a^5b^4x^4-35280\,a^6b^3x^3-129}{2520\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^9*x^9*log(x) - 22680*a*b^8*x^8 - 45360*a^2*b^7*x^7 - 70560*a
^3*b^6*x^6 - 79380*a^4*b^5*x^5 - 63504*a^5*b^4*x^4 - 35280*a^6*b^3*x^3 - 12
960*a^7*b^2*x^2 - 2835*a^8*b*x - 280*a^9)/x^9
```

Sympy [A] time = 1.27001, size = 105, normalized size = 0.96

$$b^9\log(x)-\frac{280a^9+2835a^8bx+12960a^7b^2x^2+35280a^6b^3x^3+63504a^5b^4x^4+79380a^4b^5x^5+70560a^3b^6x^6+45360}{2520x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9/x**10,x)
```

```
[Out] b**9*log(x) - (280*a**9 + 2835*a**8*b*x + 12960*a**7*b**2*x**2 + 35280*a**6
*b**3*x**3 + 63504*a**5*b**4*x**4 + 79380*a**4*b**5*x**5 + 70560*a**3*b**6*
```

```
x**6 + 45360*a**2*b**7*x**7 + 22680*a*b**8*x**8)/(2520*x**9)
```

Giac [A] time = 1.22176, size = 136, normalized size = 1.25

$$b^9 \log(|x|) - \frac{22680\, ab^8x^8 + 45360\, a^2b^7x^7 + 70560\, a^3b^6x^6 + 79380\, a^4b^5x^5 + 63504\, a^5b^4x^4 + 35280\, a^6b^3x^3 + 12960\, a^7b^2x^2}{2520\, x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="giac")
```

```
[Out] b^9*log(abs(x)) - 1/2520*(22680*a*b^8*x^8 + 45360*a^2*b^7*x^7 + 70560*a^3*b
^6*x^6 + 79380*a^4*b^5*x^5 + 63504*a^5*b^4*x^4 + 35280*a^6*b^3*x^3 + 12960*
a^7*b^2*x^2 + 2835*a^8*b*x + 280*a^9)/x^9
```

3.243 $\int \frac{(a+bx)^8}{x^{10}} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^9}{9ax^9}$$

```
[Out] -(a + b*x)^9/(9*a*x^9)
```

Rubi [A] time = 0.0016689, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^9}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8/x^10,x]

[Out] -(a + b*x)^9/(9*a*x^9)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^8}{x^{10}} dx = -\frac{(a+bx)^9}{9ax^9}$$

Mathematica [B] time = 0.0154863, size = 96, normalized size = 5.65

$$-\frac{4a^6b^2}{x^7}-\frac{28a^5b^3}{3x^6}-\frac{14a^4b^4}{x^5}-\frac{14a^3b^5}{x^4}-\frac{28a^2b^6}{3x^3}-\frac{a^7b}{x^8}-\frac{a^8}{9x^9}-\frac{4ab^7}{x^2}-\frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8/x^10,x]

[Out] -a^8/(9*x^9) - (a^7*b)/x^8 - (4*a^6*b^2)/x^7 - (28*a^5*b^3)/(3*x^6) - (14*a
^4*b^4)/x^5 - (14*a^3*b^5)/x^4 - (28*a^2*b^6)/(3*x^3) - (4*a*b^7)/x^2 - b^8
/x
```

Maple [B] time = 0.005, size = 91, normalized size = 5.4

$$-\frac{28\,a^2b^6}{3\,x^3}-14\,\frac{a^4b^4}{x^5}-14\,\frac{a^3b^5}{x^4}-\frac{28\,a^5b^3}{3\,x^6}-\frac{a^7b}{x^8}-4\,\frac{ab^7}{x^2}-4\,\frac{a^6b^2}{x^7}-\frac{b^8}{x}-\frac{a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8/x^10,x)
```

```
[Out] -28/3*a^2*b^6/x^3-14*a^4*b^4/x^5-14*a^3*b^5/x^4-28/3*a^5*b^3/x^6-a^7*b/x^8-
4*a*b^7/x^2-4*a^6*b^2/x^7-b^8/x-1/9*a^8/x^9
```

Maxima [B] time = 1.06012, size = 119, normalized size = 7.

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="maxima")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

Fricas [B] time = 1.45661, size = 192, normalized size = 11.29

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="fricas")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

Sympy [B] time = 1.09683, size = 95, normalized size = 5.59

$$-\frac{a^8+9a^7bx+36a^6b^2x^2+84a^5b^3x^3+126a^4b^4x^4+126a^3b^5x^5+84a^2b^6x^6+36ab^7x^7+9b^8x^8}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8/x**10,x)
```

```
[Out] -(a**8 + 9*a**7*b*x + 36*a**6*b**2*x**2 + 84*a**5*b**3*x**3 + 126*a**4*b**4
*x**4 + 126*a**3*b**5*x**5 + 84*a**2*b**6*x**6 + 36*a*b**7*x**7 + 9*b**8*x*
*8)/(9*x**9)
```

Giac [B] time = 1.19798, size = 119, normalized size = 7.

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="giac")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

3.244 $\int \frac{(a+bx)^7}{x^{10}} dx$

Optimal. Leaf size=36

$$\frac{b(a+bx)^8}{72a^2x^8} - \frac{(a+bx)^8}{9ax^9}$$

```
[Out] -(a + b*x)^8/(9*a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Rubi [A] time = 0.0055383, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b(a+bx)^8}{72a^2x^8} - \frac{(a+bx)^8}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^10,x]
```

```
[Out] -(a + b*x)^8/(9*a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{10}} dx &= -\frac{(a+bx)^8}{9ax^9} - \frac{b\int \frac{(a+bx)^7}{x^9} dx}{9a} \\ &= -\frac{(a+bx)^8}{9ax^9} + \frac{b(a+bx)^8}{72a^2x^8}\end{aligned}$$

Mathematica [B] time = 0.0041562, size = 91, normalized size = 2.53

$$-\frac{3a^5b^2}{x^7} - \frac{35a^4b^3}{6x^6} - \frac{7a^3b^4}{x^5} - \frac{21a^2b^5}{4x^4} - \frac{7a^6b}{8x^8} - \frac{a^7}{9x^9} - \frac{7ab^6}{3x^3} - \frac{b^7}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^10,x]
```

```
[Out] -a^7/(9*x^9) - (7*a^6*b)/(8*x^8) - (3*a^5*b^2)/x^7 - (35*a^4*b^3)/(6*x^6) -
 (7*a^3*b^4)/x^5 - (21*a^2*b^5)/(4*x^4) - (7*a*b^6)/(3*x^3) - b^7/(2*x^2)
```

Maple [B] time = 0., size = 80, normalized size = 2.2

$$-\frac{7\,ab^{6}}{3\,x^{3}}-7\,\frac{a^{3}b^{4}}{x^{5}}-\frac{21\,a^{2}b^{5}}{4\,x^{4}}-\frac{35\,a^{4}b^{3}}{6\,x^{6}}-\frac{7\,a^{6}b}{8\,x^{8}}-\frac{b^{7}}{2\,x^{2}}-3\,\frac{a^{5}b^{2}}{x^{7}}-\frac{a^{7}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^10,x)
```

```
[Out] -7/3*a*b^6/x^3-7*a^3*b^4/x^5-21/4*a^2*b^5/x^4-35/6*a^4*b^3/x^6-7/8*a^6*b/x^
8-1/2*b^7/x^2-3*a^5*b^2/x^7-1/9*a^7/x^9
```

Maxima [B] time = 1.06176, size = 107, normalized size = 2.97

$$-\frac{36\,b^{7}x^{7}+168\,ab^{6}x^{6}+378\,a^{2}b^{5}x^{5}+504\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}+216\,a^{5}b^{2}x^{2}+63\,a^{6}bx+8\,a^{7}}{72\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Fricas [B] time = 1.47686, size = 180, normalized size = 5.

$$-\frac{36\,b^{7}x^{7}+168\,ab^{6}x^{6}+378\,a^{2}b^{5}x^{5}+504\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}+216\,a^{5}b^{2}x^{2}+63\,a^{6}bx+8\,a^{7}}{72\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Sympy [B] time = 0.940685, size = 85, normalized size = 2.36

$$-\frac{8a^{7}+63a^{6}bx+216a^{5}b^{2}x^{2}+420a^{4}b^{3}x^{3}+504a^{3}b^{4}x^{4}+378a^{2}b^{5}x^{5}+168ab^{6}x^{6}+36b^{7}x^{7}}{72x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**10,x)
```

```
[Out] -(8*a**7 + 63*a**6*b*x + 216*a**5*b**2*x**2 + 420*a**4*b**3*x**3 + 504*a**3
*b**4*x**4 + 378*a**2*b**5*x**5 + 168*a*b**6*x**6 + 36*b**7*x**7)/(72*x**9)
```

Giac [B] time = 1.19962, size = 107, normalized size = 2.97

$$-\frac{36\,b^7x^7 + 168\,ab^6x^6 + 378\,a^2b^5x^5 + 504\,a^3b^4x^4 + 420\,a^4b^3x^3 + 216\,a^5b^2x^2 + 63\,a^6bx + 8\,a^7}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

3.245 $\int \frac{(a+bx)^6}{x^{10}} dx$

Optimal. Leaf size=56

$$-\frac{b^2(a+bx)^7}{252a^3x^7}+\frac{b(a+bx)^7}{36a^2x^8}-\frac{(a+bx)^7}{9ax^9}$$

```
[Out] -(a + b*x)^7/(9*a*x^9) + (b*(a + b*x)^7)/(36*a^2*x^8) - (b^2*(a + b*x)^7)/(
252*a^3*x^7)
```

Rubi [A] time = 0.0101443, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^2(a+bx)^7}{252a^3x^7}+\frac{b(a+bx)^7}{36a^2x^8}-\frac{(a+bx)^7}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/x^10,x]
```

```
[Out] -(a + b*x)^7/(9*a*x^9) + (b*(a + b*x)^7)/(36*a^2*x^8) - (b^2*(a + b*x)^7)/(
252*a^3*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^6}{x^{10}} dx &= -\frac{(a+bx)^7}{9ax^9}-\frac{(2b)\int \frac{(a+bx)^6}{x^9} dx}{9a}\\
&= -\frac{(a+bx)^7}{9ax^9}+\frac{b(a+bx)^7}{36a^2x^8}+\frac{b^2\int \frac{(a+bx)^6}{x^8} dx}{36a^2}\\
&= -\frac{(a+bx)^7}{9ax^9}+\frac{b(a+bx)^7}{36a^2x^8}-\frac{b^2(a+bx)^7}{252a^3x^7}
\end{aligned}$$

Mathematica [A] time = 0.0093772, size = 80, normalized size = 1.43

$$-\frac{15a^4b^2}{7x^7}-\frac{10a^3b^3}{3x^6}-\frac{3a^2b^4}{x^5}-\frac{3a^5b}{4x^8}-\frac{a^6}{9x^9}-\frac{3ab^5}{2x^4}-\frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/x^10,x]
```

```
[Out] -a^6/(9*x^9) - (3*a^5*b)/(4*x^8) - (15*a^4*b^2)/(7*x^7) - (10*a^3*b^3)/(3*x
^6) - (3*a^2*b^4)/x^5 - (3*a*b^5)/(2*x^4) - b^6/(3*x^3)
```

Maple [A] time = 0.005, size = 69, normalized size = 1.2

$$-\frac{b^6}{3x^3}-3\frac{a^2b^4}{x^5}-\frac{3ab^5}{2x^4}-\frac{10a^3b^3}{3x^6}-\frac{3a^5b}{4x^8}-\frac{15a^4b^2}{7x^7}-\frac{a^6}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/x^10,x)
```

```
[Out] -1/3*b^6/x^3-3*a^2*b^4/x^5-3/2*a*b^5/x^4-10/3*a^3*b^3/x^6-3/4*a^5*b/x^8-15/
7*a^4*b^2/x^7-1/9*a^6/x^9
```

Maxima [A] time = 1.08964, size = 92, normalized size = 1.64

$$-\frac{84\,b^6x^6+378\,ab^5x^5+756\,a^2b^4x^4+840\,a^3b^3x^3+540\,a^4b^2x^2+189\,a^5bx+28\,a^6}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="maxima")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

Fricas [A] time = 1.56025, size = 159, normalized size = 2.84

$$-\frac{84\,b^6x^6+378\,ab^5x^5+756\,a^2b^4x^4+840\,a^3b^3x^3+540\,a^4b^2x^2+189\,a^5bx+28\,a^6}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="fricas")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

Sympy [A] time = 0.863066, size = 73, normalized size = 1.3

$$-\frac{28a^6+189a^5bx+540a^4b^2x^2+840a^3b^3x^3+756a^2b^4x^4+378ab^5x^5+84b^6x^6}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/x**10,x)
```

```
[Out] -(28*a**6 + 189*a**5*b*x + 540*a**4*b**2*x**2 + 840*a**3*b**3*x**3 + 756*a*
*2*b**4*x**4 + 378*a*b**5*x**5 + 84*b**6*x**6)/(252*x**9)
```

Giac [A] time = 1.15094, size = 92, normalized size = 1.64

$$-\frac{84\,b^6x^6 + 378\,ab^5x^5 + 756\,a^2b^4x^4 + 840\,a^3b^3x^3 + 540\,a^4b^2x^2 + 189\,a^5bx + 28\,a^6}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="giac")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

3.246 $\int \frac{(a+bx)^5}{x^{10}}\,dx$

Optimal. Leaf size=67

$$-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rubi [A] time = 0.024606, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^10,x]
```

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{10}}\,dx &= \int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx\\ &= -\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.0139479, size = 67, normalized size = 1.

$$-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^10,x]
```

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Maple [A] time = 0., size = 58, normalized size = 0.9

$$-\frac{a^5}{9\,x^9}-\frac{5\,a^4b}{8\,x^8}-\frac{10\,a^3b^2}{7\,x^7}-\frac{5\,a^2b^3}{3\,x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^10,x)
```

```
[Out] -1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*b
^5/x^4
```

Maxima [A] time = 1.04288, size = 77, normalized size = 1.15

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Fricas [A] time = 1.67649, size = 136, normalized size = 2.03

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Sympy [A] time = 0.863862, size = 61, normalized size = 0.91

$$-\frac{56a^5+315a^4bx+720a^3b^2x^2+840a^2b^3x^3+504ab^4x^4+126b^5x^5}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**10,x)
```

```
[Out] -(56*a**5 + 315*a**4*b*x + 720*a**3*b**2*x**2 + 840*a**2*b**3*x**3 + 504*a*
b**4*x**4 + 126*b**5*x**5)/(504*x**9)
```

Giac [A] time = 1.1921, size = 77, normalized size = 1.15

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

3.247 $\int \frac{(a+bx)^4}{x^{10}} \, dx$

Optimal. Leaf size=56

$$-\frac{6a^2b^2}{7x^7}-\frac{a^3b}{2x^8}-\frac{a^4}{9x^9}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

```
[Out] -a^4/(9*x^9) - (a^3*b)/(2*x^8) - (6*a^2*b^2)/(7*x^7) - (2*a*b^3)/(3*x^6) -
b^4/(5*x^5)
```

Rubi [A] time = 0.0191013, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{6a^2b^2}{7x^7}-\frac{a^3b}{2x^8}-\frac{a^4}{9x^9}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/x^10,x]
```

```
[Out] -a^4/(9*x^9) - (a^3*b)/(2*x^8) - (6*a^2*b^2)/(7*x^7) - (2*a*b^3)/(3*x^6) -
b^4/(5*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{x^{10}} \, dx &= \int \left(\frac{a^4}{x^{10}}+\frac{4a^3b}{x^9}+\frac{6a^2b^2}{x^8}+\frac{4ab^3}{x^7}+\frac{b^4}{x^6}\right) dx\\ &= -\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.0110654, size = 56, normalized size = 1.

$$-\frac{6a^2b^2}{7x^7}-\frac{a^3b}{2x^8}-\frac{a^4}{9x^9}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/x^10,x]
```

```
[Out] -a^4/(9*x^9) - (a^3*b)/(2*x^8) - (6*a^2*b^2)/(7*x^7) - (2*a*b^3)/(3*x^6) -
b^4/(5*x^5)
```

Maple [A] time = 0.006, size = 47, normalized size = 0.8

$$-\frac{a^4}{9\,x^9}-\frac{a^3b}{2\,x^8}-\frac{6\,b^2a^2}{7\,x^7}-\frac{2\,ab^3}{3\,x^6}-\frac{b^4}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/x^10,x)
```

```
[Out] -1/9*a^4/x^9-1/2*a^3*b/x^8-6/7*a^2*b^2/x^7-2/3*a*b^3/x^6-1/5*b^4/x^5
```

Maxima [A] time = 1.10312, size = 62, normalized size = 1.11

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/x^10,x, algorithm="maxima")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

Fricas [A] time = 1.63862, size = 112, normalized size = 2.

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/x^10,x, algorithm="fricas")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

Sympy [A] time = 0.710616, size = 49, normalized size = 0.88

$$-\frac{70a^4+315a^3bx+540a^2b^2x^2+420ab^3x^3+126b^4x^4}{630x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/x**10,x)
```

```
[Out] -(70*a**4 + 315*a**3*b*x + 540*a**2*b**2*x**2 + 420*a*b**3*x**3 + 126*b**4*
x**4)/(630*x**9)
```

Giac [A] time = 1.19042, size = 62, normalized size = 1.11

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/x^10,x, algorithm="giac")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

3.248 $\int \frac{(a+bx)^3}{x^{10}}\,dx$

Optimal. Leaf size=43

$$-\frac{3a^2b}{8x^8}-\frac{a^3}{9x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

```
[Out] -a^3/(9*x^9) - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(7*x^7) - b^3/(6*x^6)
```

Rubi [A] time = 0.0140346, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{3a^2b}{8x^8}-\frac{a^3}{9x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^10,x]
```

```
[Out] -a^3/(9*x^9) - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(7*x^7) - b^3/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{10}}\,dx &= \int \left(\frac{a^3}{x^{10}}+\frac{3a^2b}{x^9}+\frac{3ab^2}{x^8}+\frac{b^3}{x^7}\right)dx\\ &= -\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}\end{aligned}$$

Mathematica [A] time = 0.0036274, size = 43, normalized size = 1.

$$-\frac{3a^2b}{8x^8}-\frac{a^3}{9x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^10,x]
```

```
[Out] -a^3/(9*x^9) - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(7*x^7) - b^3/(6*x^6)
```

Maple [A] time = 0.005, size = 36, normalized size = 0.8

$$-\frac{a^3}{9\,x^9}-\frac{3\,a^2b}{8\,x^8}-\frac{3\,b^2a}{7\,x^7}-\frac{b^3}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^10,x)
```

```
[Out] -1/9*a^3/x^9-3/8*a^2*b/x^8-3/7*a*b^2/x^7-1/6*b^3/x^6
```

Maxima [A] time = 1.10647, size = 47, normalized size = 1.09

$$-\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

Fricas [A] time = 1.70502, size = 86, normalized size = 2.

$$-\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

Sympy [A] time = 0.540495, size = 37, normalized size = 0.86

$$-\frac{56a^3+189a^2bx+216ab^2x^2+84b^3x^3}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**10,x)
```

```
[Out] -(56*a**3 + 189*a**2*b*x + 216*a*b**2*x**2 + 84*b**3*x**3)/(504*x**9)
```

Giac [A] time = 1.14324, size = 47, normalized size = 1.09

$$-\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

3.249 $\int \frac{(a+bx)^2}{x^{10}}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

```
[Out] -a^2/(9*x^9) - (a*b)/(4*x^8) - b^2/(7*x^7)
```

Rubi [A] time = 0.009093, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^10,x]
```

```
[Out] -a^2/(9*x^9) - (a*b)/(4*x^8) - b^2/(7*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{10}}\,dx &= \int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^9}+\frac{b^2}{x^8}\right)dx\\ &= -\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}\end{aligned}$$

Mathematica [A] time = 0.0077923, size = 30, normalized size = 1.

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^10,x]
```

```
[Out] -a^2/(9*x^9) - (a*b)/(4*x^8) - b^2/(7*x^7)
```

Maple [A] time = 0.005, size = 25, normalized size = 0.8

$$-\frac{a^2}{9\,x^9}-\frac{ab}{4\,x^8}-\frac{b^2}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^10,x)
```

```
[Out] -1/9*a^2/x^9-1/4*a*b/x^8-1/7*b^2/x^7
```

Maxima [A] time = 1.11856, size = 32, normalized size = 1.07

$$-\frac{36\,b^{2}x^{2}+63\,abx+28\,a^{2}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

Fricas [A] time = 1.74829, size = 61, normalized size = 2.03

$$-\frac{36\,b^{2}x^{2}+63\,abx+28\,a^{2}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

Sympy [A] time = 0.51904, size = 26, normalized size = 0.87

$$-\frac{28a^{2}+63abx+36b^{2}x^{2}}{252x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**10,x)
```

```
[Out] -(28*a**2 + 63*a*b*x + 36*b**2*x**2)/(252*x**9)
```

Giac [A] time = 1.20263, size = 32, normalized size = 1.07

$$-\frac{36\,b^{2}x^{2}+63\,abx+28\,a^{2}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

3.250 $\int \frac{a+bx}{x^{10}} \, dx$

Optimal. Leaf size=17

$$-\frac{a}{9x^9}-\frac{b}{8x^8}$$

```
[Out] -a/(9*x^9) - b/(8*x^8)
```

Rubi [A] time = 0.0047767, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$-\frac{a}{9x^9}-\frac{b}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^10,x]
```

```
[Out] -a/(9*x^9) - b/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{10}} \, dx &= \int \left(\frac{a}{x^{10}}+\frac{b}{x^9}\right) dx\\ &= -\frac{a}{9x^9}-\frac{b}{8x^8}\end{aligned}$$

Mathematica [A] time = 0.0036081, size = 17, normalized size = 1.

$$-\frac{a}{9x^9}-\frac{b}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^10,x]
```

```
[Out] -a/(9*x^9) - b/(8*x^8)
```

Maple [A] time = 0.003, size = 14, normalized size = 0.8

$$-\frac{a}{9\,x^9}-\frac{b}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^10,x)
```

```
[Out] -1/9*a/x^9-1/8*b/x^8
```

Maxima [A] time = 1.03029, size = 18, normalized size = 1.06

$$-\frac{9\,bx+8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="maxima")
```

```
[Out] -1/72*(9*b*x + 8*a)/x^9
```

Fricas [A] time = 1.72476, size = 34, normalized size = 2.

$$-\frac{9\,bx+8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="fricas")
```

```
[Out] -1/72*(9*b*x + 8*a)/x^9
```

Sympy [A] time = 0.412731, size = 14, normalized size = 0.82

$$-\frac{8a+9bx}{72x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**10,x)
```

```
[Out] -(8*a + 9*b*x)/(72*x**9)
```

Giac [A] time = 1.21106, size = 18, normalized size = 1.06

$$-\frac{9\,bx+8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="giac")
```

```
[Out] -1/72*(9*b*x + 8*a)/x^9
```

3.251 $\int \frac{1}{x^{10}} \, dx$

Optimal. Leaf size=7

$$-\frac{1}{9x^9}$$

```
[Out] -1/(9*x^9)
```

Rubi [A] time = 0.0004806, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-10),x]
```

```
[Out] -1/(9*x^9)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{10}} \, dx = -\frac{1}{9x^9}$$

Mathematica [A] time = 0.0001022, size = 7, normalized size = 1.

$$-\frac{1}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-10),x]
```

```
[Out] -1/(9*x^9)
```

Maple [A] time = 0., size = 6, normalized size = 0.9

$$-\frac{1}{9 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10,x)
```

```
[Out] -1/9/x^9
```

Maxima [A] time = 1.07673, size = 7, normalized size = 1.

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="maxima")
```

```
[Out] -1/9/x^9
```

Fricas [A] time = 1.55583, size = 14, normalized size = 2.

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="fricas")
```

```
[Out] -1/9/x^9
```

Sympy [A] time = 0.070336, size = 7, normalized size = 1.

$$-\frac{1}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10,x)
```

```
[Out] -1/(9*x**9)
```

Giac [A] time = 1.20951, size = 7, normalized size = 1.

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="giac")
```

```
[Out] -1/9/x^9
```

3.252 $\int \frac{1}{x^{10}(a+bx)} dx$

Optimal. Leaf size=134

$$\frac{b^7}{2a^8x^2} - \frac{b^6}{3a^7x^3} + \frac{b^5}{4a^6x^4} - \frac{b^4}{5a^5x^5} + \frac{b^3}{6a^4x^6} - \frac{b^2}{7a^3x^7} - \frac{b^8}{a^9x} - \frac{b^9\log(x)}{a^{10}} + \frac{b^9\log(a+bx)}{a^{10}} + \frac{b}{8a^2x^8} - \frac{1}{9ax^9}$$

```
[Out] -1/(9*a*x^9) + b/(8*a^2*x^8) - b^2/(7*a^3*x^7) + b^3/(6*a^4*x^6) - b^4/(5*a
^5*x^5) + b^5/(4*a^6*x^4) - b^6/(3*a^7*x^3) + b^7/(2*a^8*x^2) - b^8/(a^9*x)
 - (b^9*Log[x])/a^10 + (b^9*Log[a + b*x])/a^10
```

Rubi [A] time = 0.0608035, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{b^7}{2a^8x^2} - \frac{b^6}{3a^7x^3} + \frac{b^5}{4a^6x^4} - \frac{b^4}{5a^5x^5} + \frac{b^3}{6a^4x^6} - \frac{b^2}{7a^3x^7} - \frac{b^8}{a^9x} - \frac{b^9\log(x)}{a^{10}} + \frac{b^9\log(a+bx)}{a^{10}} + \frac{b}{8a^2x^8} - \frac{1}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)),x]
```

```
[Out] -1/(9*a*x^9) + b/(8*a^2*x^8) - b^2/(7*a^3*x^7) + b^3/(6*a^4*x^6) - b^4/(5*a
^5*x^5) + b^5/(4*a^6*x^4) - b^6/(3*a^7*x^3) + b^7/(2*a^8*x^2) - b^8/(a^9*x)
 - (b^9*Log[x])/a^10 + (b^9*Log[a + b*x])/a^10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^{10}(a+bx)} dx = \int \left(\frac{1}{ax^{10}} - \frac{b}{a^2x^9} + \frac{b^2}{a^3x^8} - \frac{b^3}{a^4x^7} + \frac{b^4}{a^5x^6} - \frac{b^5}{a^6x^5} + \frac{b^6}{a^7x^4} - \frac{b^7}{a^8x^3} + \frac{b^8}{a^9x^2} - \frac{b^9}{a^{10}x} + \frac{b^{10}}{a^{10}(a+bx)}\right) dx$$
$$= -\frac{1}{9ax^9} + \frac{b}{8a^2x^8} - \frac{b^2}{7a^3x^7} + \frac{b^3}{6a^4x^6} - \frac{b^4}{5a^5x^5} + \frac{b^5}{4a^6x^4} - \frac{b^6}{3a^7x^3} + \frac{b^7}{2a^8x^2} - \frac{b^8}{a^9x} - \frac{b^9\log(x)}{a^{10}} + \frac{b^9\log(a-}{a^{10}}$$

Mathematica [A] time = 0.0060532, size = 134, normalized size = 1.

$$\frac{b^7}{2a^8x^2} - \frac{b^6}{3a^7x^3} + \frac{b^5}{4a^6x^4} - \frac{b^4}{5a^5x^5} + \frac{b^3}{6a^4x^6} - \frac{b^2}{7a^3x^7} - \frac{b^8}{a^9x} - \frac{b^9\log(x)}{a^{10}} + \frac{b^9\log(a+bx)}{a^{10}} + \frac{b}{8a^2x^8} - \frac{1}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)),x]
```

```
[Out] -1/(9*a*x^9) + b/(8*a^2*x^8) - b^2/(7*a^3*x^7) + b^3/(6*a^4*x^6) - b^4/(5*a
^5*x^5) + b^5/(4*a^6*x^4) - b^6/(3*a^7*x^3) + b^7/(2*a^8*x^2) - b^8/(a^9*x)
 - (b^9*Log[x])/a^10 + (b^9*Log[a + b*x])/a^10
```

Maple [A] time = 0.01, size = 119, normalized size = 0.9

$$-\frac{1}{9\,a x^{9}}+\frac{b}{8\,a^{2}x^{8}}-\frac{b^{2}}{7\,a^{3}x^{7}}+\frac{b^{3}}{6\,a^{4}x^{6}}-\frac{b^{4}}{5\,a^{5}x^{5}}+\frac{b^{5}}{4\,a^{6}x^{4}}-\frac{b^{6}}{3\,a^{7}x^{3}}+\frac{b^{7}}{2\,a^{8}x^{2}}-\frac{b^{8}}{a^{9}x}-\frac{b^{9}\ln\left(x\right)}{a^{10}}+\frac{b^{9}\ln\left(bx+a\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a),x)
```

```
[Out] -1/9/a/x^9+1/8*b/a^2/x^8-1/7*b^2/a^3/x^7+1/6*b^3/a^4/x^6-1/5*b^4/a^5/x^5+1/
4*b^5/a^6/x^4-1/3*b^6/a^7/x^3+1/2*b^7/a^8/x^2-b^8/a^9/x-b^9*ln(x)/a^10+b^9*
ln(b*x+a)/a^10
```

Maxima [A] time = 1.0785, size = 158, normalized size = 1.18

$$\frac{b^{9}\log\left(bx+a\right)}{a^{10}}-\frac{b^{9}\log\left(x\right)}{a^{10}}-\frac{2520\,b^{8}x^{8}-1260\,ab^{7}x^{7}+840\,a^{2}b^{6}x^{6}-630\,a^{3}b^{5}x^{5}+504\,a^{4}b^{4}x^{4}-420\,a^{5}b^{3}x^{3}+360\,a^{6}b}{2520\,a^{9}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^9*log(b*x + a)/a^10 - b^9*log(x)/a^10 - 1/2520*(2520*b^8*x^8 - 1260*a*b^7
*x^7 + 840*a^2*b^6*x^6 - 630*a^3*b^5*x^5 + 504*a^4*b^4*x^4 - 420*a^5*b^3*x^
3 + 360*a^6*b^2*x^2 - 315*a^7*b*x + 280*a^8)/(a^9*x^9)
```

Fricas [A] time = 1.78598, size = 296, normalized size = 2.21

$$\frac{2520\,b^{9}x^{9}\log\left(bx+a\right)-2520\,b^{9}x^{9}\log\left(x\right)-2520\,ab^{8}x^{8}+1260\,a^{2}b^{7}x^{7}-840\,a^{3}b^{6}x^{6}+630\,a^{4}b^{5}x^{5}-504\,a^{5}b^{4}x^{4}+420}{2520\,a^{10}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^9*x^9*log(b*x + a) - 2520*b^9*x^9*log(x) - 2520*a*b^8*x^8 +
1260*a^2*b^7*x^7 - 840*a^3*b^6*x^6 + 630*a^4*b^5*x^5 - 504*a^5*b^4*x^4 + 42
0*a^6*b^3*x^3 - 360*a^7*b^2*x^2 + 315*a^8*b*x - 280*a^9)/(a^10*x^9)
```

Sympy [A] time = 1.00563, size = 116, normalized size = 0.87

$$-\frac{280a^{8}-315a^{7}bx+360a^{6}b^{2}x^{2}-420a^{5}b^{3}x^{3}+504a^{4}b^{4}x^{4}-630a^{3}b^{5}x^{5}+840a^{2}b^{6}x^{6}-1260ab^{7}x^{7}+2520b^{8}x^{8}}{2520a^{9}x^{9}}+\frac{b^{9}\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x+a),x)
```

```
[Out] -(280*a**8 - 315*a**7*b*x + 360*a**6*b**2*x**2 - 420*a**5*b**3*x**3 + 504*a
**4*b**4*x**4 - 630*a**3*b**5*x**5 + 840*a**2*b**6*x**6 - 1260*a*b**7*x**7
```

```
+ 2520*b**8*x**8)/(2520*a**9*x**9) + b**9*(-log(x) + log(a/b + x))/a**10
```

Giac [A] time = 1.19382, size = 165, normalized size = 1.23

$$\frac{b^9 \log(|bx+a|)}{a^{10}} - \frac{b^9 \log(|x|)}{a^{10}} - \frac{2520\,ab^8x^8 - 1260\,a^2b^7x^7 + 840\,a^3b^6x^6 - 630\,a^4b^5x^5 + 504\,a^5b^4x^4 - 420\,a^6b^3x^3 + 360\,a^7}{2520\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a),x, algorithm="giac")
```

```
[Out] b^9*log(abs(b*x + a))/a^10 - b^9*log(abs(x))/a^10 - 1/2520*(2520*a*b^8*x^8
- 1260*a^2*b^7*x^7 + 840*a^3*b^6*x^6 - 630*a^4*b^5*x^5 + 504*a^5*b^4*x^4 -
420*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 315*a^8*b*x + 280*a^9)/(a^10*x^9)
```

3.253 $\int \frac{1}{x^{10}(a+bx)^2}\,dx$

Optimal. Leaf size=146

$$\frac{4b^7}{a^9x^2}-\frac{7b^6}{3a^8x^3}+\frac{3b^5}{2a^7x^4}-\frac{b^4}{a^6x^5}+\frac{2b^3}{3a^5x^6}-\frac{3b^2}{7a^4x^7}-\frac{b^9}{a^{10}(a+bx)}-\frac{9b^8}{a^{10}x}-\frac{10b^9\log(x)}{a^{11}}+\frac{10b^9\log(a+bx)}{a^{11}}+\frac{b}{4a^3x^8}-\frac{9}{}$$

```
[Out] -1/(9*a^2*x^9) + b/(4*a^3*x^8) - (3*b^2)/(7*a^4*x^7) + (2*b^3)/(3*a^5*x^6)
- b^4/(a^6*x^5) + (3*b^5)/(2*a^7*x^4) - (7*b^6)/(3*a^8*x^3) + (4*b^7)/(a^9*
x^2) - (9*b^8)/(a^10*x) - b^9/(a^10*(a + b*x)) - (10*b^9*Log[x])/a^11 + (10
*b^9*Log[a + b*x])/a^11
```

Rubi [A] time = 0.0896469, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{4b^7}{a^9x^2}-\frac{7b^6}{3a^8x^3}+\frac{3b^5}{2a^7x^4}-\frac{b^4}{a^6x^5}+\frac{2b^3}{3a^5x^6}-\frac{3b^2}{7a^4x^7}-\frac{b^9}{a^{10}(a+bx)}-\frac{9b^8}{a^{10}x}-\frac{10b^9\log(x)}{a^{11}}+\frac{10b^9\log(a+bx)}{a^{11}}+\frac{b}{4a^3x^8}-\frac{9}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)^2),x]
```

```
[Out] -1/(9*a^2*x^9) + b/(4*a^3*x^8) - (3*b^2)/(7*a^4*x^7) + (2*b^3)/(3*a^5*x^6)
- b^4/(a^6*x^5) + (3*b^5)/(2*a^7*x^4) - (7*b^6)/(3*a^8*x^3) + (4*b^7)/(a^9*
x^2) - (9*b^8)/(a^10*x) - b^9/(a^10*(a + b*x)) - (10*b^9*Log[x])/a^11 + (10
*b^9*Log[a + b*x])/a^11
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^{10}}-\frac{2b}{a^3x^9}+\frac{3b^2}{a^4x^8}-\frac{4b^3}{a^5x^7}+\frac{5b^4}{a^6x^6}-\frac{6b^5}{a^7x^5}+\frac{7b^6}{a^8x^4}-\frac{8b^7}{a^9x^3}+\frac{9b^8}{a^{10}x^2}-\frac{10b^9}{a^{11}x}+\frac{b^{10}}{a^{10}(a+bx)^2}\right.\\ &= -\frac{1}{9a^2x^9}+\frac{b}{4a^3x^8}-\frac{3b^2}{7a^4x^7}+\frac{2b^3}{3a^5x^6}-\frac{b^4}{a^6x^5}+\frac{3b^5}{2a^7x^4}-\frac{7b^6}{3a^8x^3}+\frac{4b^7}{a^9x^2}-\frac{9b^8}{a^{10}x}-\frac{b^9}{a^{10}(a+bx)}-\frac{10b}{}\end{aligned}$$

Mathematica [A] time = 0.150389, size = 134, normalized size = 0.92

$$-\frac{\frac{a\left(45a^7b^2x^2-60a^6b^3x^3+84a^5b^4x^4-126a^4b^5x^5+210a^3b^6x^6-420a^2b^7x^7-35a^8bx+28a^9+1260ab^8x^8+2520b^9x^9\right)}{x^9(a+bx)}-2520b^9\log(a+bx)+2520b^9\log}{252a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)^2),x]
```

```
[Out] -((a*(28*a^9 - 35*a^8*b*x + 45*a^7*b^2*x^2 - 60*a^6*b^3*x^3 + 84*a^5*b^4*x^
4 - 126*a^4*b^5*x^5 + 210*a^3*b^6*x^6 - 420*a^2*b^7*x^7 + 1260*a*b^8*x^8 +
2520*b^9*x^9))/(x^9*(a + b*x)) + 2520*b^9*Log[x] - 2520*b^9*Log[a + b*x])/(
252*a^11)
```

Maple [A] time = 0.013, size = 135, normalized size = 0.9

$$-\frac{1}{9\,a^2x^9}+\frac{b}{4\,a^3x^8}-\frac{3\,b^2}{7\,a^4x^7}+\frac{2\,b^3}{3\,a^5x^6}-\frac{b^4}{a^6x^5}+\frac{3\,b^5}{2\,a^7x^4}-\frac{7\,b^6}{3\,a^8x^3}+4\,\frac{b^7}{a^9x^2}-9\,\frac{b^8}{a^{10}x}-\frac{b^9}{a^{10}\,(bx+a)}-10\,\frac{b^9\ln(x)}{a^{11}}+10\,\frac{b^9\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a)^2,x)
```

```
[Out] -1/9/a^2/x^9+1/4*b/a^3/x^8-3/7*b^2/a^4/x^7+2/3*b^3/a^5/x^6-b^4/a^6/x^5+3/2*
b^5/a^7/x^4-7/3*b^6/a^8/x^3+4*b^7/a^9/x^2-9*b^8/a^10/x-b^9/a^10/(b*x+a)-10*
b^9*ln(x)/a^11+10*b^9*ln(b*x+a)/a^11
```

Maxima [A] time = 1.1136, size = 190, normalized size = 1.3

$$-\frac{2520\,b^9x^9+1260\,ab^8x^8-420\,a^2b^7x^7+210\,a^3b^6x^6-126\,a^4b^5x^5+84\,a^5b^4x^4-60\,a^6b^3x^3+45\,a^7b^2x^2-35\,a^8bx+28\,a^9}{252\left(a^{10}bx^{10}+a^{11}x^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/252*(2520*b^9*x^9 + 1260*a*b^8*x^8 - 420*a^2*b^7*x^7 + 210*a^3*b^6*x^6 -
 126*a^4*b^5*x^5 + 84*a^5*b^4*x^4 - 60*a^6*b^3*x^3 + 45*a^7*b^2*x^2 - 35*a^
8*b*x + 28*a^9)/(a^10*b*x^10 + a^11*x^9) + 10*b^9*log(b*x + a)/a^11 - 10*b^
9*log(x)/a^11
```

Fricas [A] time = 1.79665, size = 377, normalized size = 2.58

$$-\frac{2520\,ab^9x^9+1260\,a^2b^8x^8-420\,a^3b^7x^7+210\,a^4b^6x^6-126\,a^5b^5x^5+84\,a^6b^4x^4-60\,a^7b^3x^3+45\,a^8b^2x^2-35\,a^9bx+28\,a}{252\left(a^{11}bx^{10}+a^{12}x^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/252*(2520*a*b^9*x^9 + 1260*a^2*b^8*x^8 - 420*a^3*b^7*x^7 + 210*a^4*b^6*x
^6 - 126*a^5*b^5*x^5 + 84*a^6*b^4*x^4 - 60*a^7*b^3*x^3 + 45*a^8*b^2*x^2 - 3
5*a^9*b*x + 28*a^10 - 2520*(b^10*x^10 + a*b^9*x^9)*log(b*x + a) + 2520*(b^1
0*x^10 + a*b^9*x^9)*log(x))/(a^11*b*x^10 + a^12*x^9)
```

Sympy [A] time = 1.25739, size = 139, normalized size = 0.95

$$-\frac{28a^9-35a^8bx+45a^7b^2x^2-60a^6b^3x^3+84a^5b^4x^4-126a^4b^5x^5+210a^3b^6x^6-420a^2b^7x^7+1260ab^8x^8+2520b^9x^9}{252a^{11}x^9+252a^{10}bx^{10}}+\frac{10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x+a)**2,x)
```

```
[Out] -(28*a**9 - 35*a**8*b*x + 45*a**7*b**2*x**2 - 60*a**6*b**3*x**3 + 84*a**5*b
**4*x**4 - 126*a**4*b**5*x**5 + 210*a**3*b**6*x**6 - 420*a**2*b**7*x**7 + 1
260*a*b**8*x**8 + 2520*b**9*x**9)/(252*a**11*x**9 + 252*a**10*b*x**10) + 10
*b**9*(-log(x) + log(a/b + x))/a**11
```

Giac [A] time = 1.23207, size = 243, normalized size = 1.66

$$-\frac{10\,b^9\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^{11}}-\frac{b^9}{(bx+a)a^{10}}-\frac{\frac{41481\,ab^9}{bx+a}-\frac{155844\,a^2b^9}{(bx+a)^2}+\frac{337176\,a^3b^9}{(bx+a)^3}-\frac{460404\,a^4b^9}{(bx+a)^4}+\frac{407484\,a^5b^9}{(bx+a)^5}-\frac{229320\,a^6b^9}{(bx+a)^6}+\frac{7560\ldots}{(bx\ldots}}{252\,a^{11}\left(\frac{a}{bx+a}-1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -10*b^9*log(abs(-a/(b*x + a) + 1))/a^11 - b^9/((b*x + a)*a^10) - 1/252*(414
81*a*b^9/(b*x + a) - 155844*a^2*b^9/(b*x + a)^2 + 337176*a^3*b^9/(b*x + a)^
3 - 460404*a^4*b^9/(b*x + a)^4 + 407484*a^5*b^9/(b*x + a)^5 - 229320*a^6*b^
9/(b*x + a)^6 + 75600*a^7*b^9/(b*x + a)^7 - 11340*a^8*b^9/(b*x + a)^8 - 486
1*b^9)/(a^11*(a/(b*x + a) - 1)^9)
```

3.254 $\int \frac{1}{x^{10}(a+bx)^3}\,dx$

Optimal. Leaf size=163

$$\frac{18b^7}{a^{10}x^2}-\frac{28b^6}{3a^9x^3}+\frac{21b^5}{4a^8x^4}-\frac{3b^4}{a^7x^5}+\frac{5b^3}{3a^6x^6}-\frac{6b^2}{7a^5x^7}-\frac{10b^9}{a^{11}(a+bx)}-\frac{b^9}{2a^{10}(a+bx)^2}-\frac{45b^8}{a^{11}x}-\frac{55b^9\log(x)}{a^{12}}+\frac{55b^9\log(a+bx}{a^{12}}$$

```
[Out] -1/(9*a^3*x^9) + (3*b)/(8*a^4*x^8) - (6*b^2)/(7*a^5*x^7) + (5*b^3)/(3*a^6*x
^6) - (3*b^4)/(a^7*x^5) + (21*b^5)/(4*a^8*x^4) - (28*b^6)/(3*a^9*x^3) + (18
*b^7)/(a^10*x^2) - (45*b^8)/(a^11*x) - b^9/(2*a^10*(a + b*x)^2) - (10*b^9)/
(a^11*(a + b*x)) - (55*b^9*Log[x])/a^12 + (55*b^9*Log[a + b*x])/a^12
```

Rubi [A] time = 0.108238, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{18b^7}{a^{10}x^2}-\frac{28b^6}{3a^9x^3}+\frac{21b^5}{4a^8x^4}-\frac{3b^4}{a^7x^5}+\frac{5b^3}{3a^6x^6}-\frac{6b^2}{7a^5x^7}-\frac{10b^9}{a^{11}(a+bx)}-\frac{b^9}{2a^{10}(a+bx)^2}-\frac{45b^8}{a^{11}x}-\frac{55b^9\log(x)}{a^{12}}+\frac{55b^9\log(a+bx}{a^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)^3),x]
```



```
[Out] -1/(9*a^3*x^9) + (3*b)/(8*a^4*x^8) - (6*b^2)/(7*a^5*x^7) + (5*b^3)/(3*a^6*x
^6) - (3*b^4)/(a^7*x^5) + (21*b^5)/(4*a^8*x^4) - (28*b^6)/(3*a^9*x^3) + (18
*b^7)/(a^10*x^2) - (45*b^8)/(a^11*x) - b^9/(2*a^10*(a + b*x)^2) - (10*b^9)/
(a^11*(a + b*x)) - (55*b^9*Log[x])/a^12 + (55*b^9*Log[a + b*x])/a^12
```


Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^{10}(a+bx)^3}\,dx = \int \left(\frac{1}{a^3x^{10}}-\frac{3b}{a^4x^9}+\frac{6b^2}{a^5x^8}-\frac{10b^3}{a^6x^7}+\frac{15b^4}{a^7x^6}-\frac{21b^5}{a^8x^5}+\frac{28b^6}{a^9x^4}-\frac{36b^7}{a^{10}x^3}+\frac{45b^8}{a^{11}x^2}-\frac{55b^9}{a^{12}x}+\frac{b^{10}}{a^{10}(a+bx)^3}+\right.$$

$$=-\frac{1}{9a^3x^9}+\frac{3b}{8a^4x^8}-\frac{6b^2}{7a^5x^7}+\frac{5b^3}{3a^6x^6}-\frac{3b^4}{a^7x^5}+\frac{21b^5}{4a^8x^4}-\frac{28b^6}{3a^9x^3}+\frac{18b^7}{a^{10}x^2}-\frac{45b^8}{a^{11}x}-\frac{b^9}{2a^{10}(a+bx)^2}-\frac{}{a^{11}}$$

Mathematica [A] time = 0.139999, size = 145, normalized size = 0.89

$$-\frac{\frac{a\left(110a^8b^2x^2-165a^7b^3x^3+264a^6b^4x^4-462a^5b^5x^5+924a^4b^6x^6-2310a^3b^7x^7+9240a^2b^8x^8-77a^9bx+56a^{10}+41580ab^9x^9+27720b^{10}x^{10}\right)}{x^9(a+bx)^2}-27720b^9\log(a+b}{504a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)^3),x]
```

```
[Out] -((a*(56*a^10 - 77*a^9*b*x + 110*a^8*b^2*x^2 - 165*a^7*b^3*x^3 + 264*a^6*b^
4*x^4 - 462*a^5*b^5*x^5 + 924*a^4*b^6*x^6 - 2310*a^3*b^7*x^7 + 9240*a^2*b^8
*x^8 + 41580*a*b^9*x^9 + 27720*b^10*x^10))/(x^9*(a + b*x)^2) + 27720*b^9*Lo
g[x] - 27720*b^9*Log[a + b*x])/(504*a^12)
```

Maple [A] time = 0.013, size = 150, normalized size = 0.9

$$-\frac{1}{9\,a^3x^9}+\frac{3\,b}{8\,a^4x^8}-\frac{6\,b^2}{7\,a^5x^7}+\frac{5\,b^3}{3\,a^6x^6}-3\,\frac{b^4}{a^7x^5}+\frac{21\,b^5}{4\,a^8x^4}-\frac{28\,b^6}{3\,a^9x^3}+18\,\frac{b^7}{a^{10}x^2}-45\,\frac{b^8}{a^{11}x}-\frac{b^9}{2\,a^{10}\,(bx+a)^2}-10\,\frac{b^9}{a^{11}\,(bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a)^3,x)
```

```
[Out] -1/9/a^3/x^9+3/8*b/a^4/x^8-6/7*b^2/a^5/x^7+5/3*b^3/a^6/x^6-3*b^4/a^7/x^5+21
/4*b^5/a^8/x^4-28/3*b^6/a^9/x^3+18*b^7/a^10/x^2-45*b^8/a^11/x-1/2*b^9/a^10/
(b*x+a)^2-10*b^9/a^11/(b*x+a)-55*b^9*ln(x)/a^12+55*b^9*ln(b*x+a)/a^12
```

Maxima [A] time = 1.07396, size = 220, normalized size = 1.35

$$-\frac{27720\,b^{10}x^{10}+41580\,ab^9x^9+9240\,a^2b^8x^8-2310\,a^3b^7x^7+924\,a^4b^6x^6-462\,a^5b^5x^5+264\,a^6b^4x^4-165\,a^7b^3x^3+11}{504\left(a^{11}b^2x^{11}+2\,a^{12}bx^{10}+a^{13}x^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/504*(27720*b^10*x^10 + 41580*a*b^9*x^9 + 9240*a^2*b^8*x^8 - 2310*a^3*b^7
*x^7 + 924*a^4*b^6*x^6 - 462*a^5*b^5*x^5 + 264*a^6*b^4*x^4 - 165*a^7*b^3*x^
3 + 110*a^8*b^2*x^2 - 77*a^9*b*x + 56*a^10)/(a^11*b^2*x^11 + 2*a^12*b*x^10
+ a^13*x^9) + 55*b^9*log(b*x + a)/a^12 - 55*b^9*log(x)/a^12
```

Fricas [A] time = 1.92144, size = 490, normalized size = 3.01

$$-\frac{27720\,ab^{10}x^{10}+41580\,a^2b^9x^9+9240\,a^3b^8x^8-2310\,a^4b^7x^7+924\,a^5b^6x^6-462\,a^6b^5x^5+264\,a^7b^4x^4-165\,a^8b^3x^3+}{504\left(a^{12}b^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/504*(27720*a*b^10*x^10 + 41580*a^2*b^9*x^9 + 9240*a^3*b^8*x^8 - 2310*a^4
*b^7*x^7 + 924*a^5*b^6*x^6 - 462*a^6*b^5*x^5 + 264*a^7*b^4*x^4 - 165*a^8*b^
3*x^3 + 110*a^9*b^2*x^2 - 77*a^10*b*x + 56*a^11 - 27720*(b^11*x^11 + 2*a*b^
10*x^10 + a^2*b^9*x^9)*log(b*x + a) + 27720*(b^11*x^11 + 2*a*b^10*x^10 + a^
2*b^9*x^9)*log(x))/(a^12*b^2*x^11 + 2*a^13*b*x^10 + a^14*x^9)
```

Sympy [A] time = 1.71289, size = 163, normalized size = 1.

$$-\frac{56a^{10}-77a^9bx+110a^8b^2x^2-165a^7b^3x^3+264a^6b^4x^4-462a^5b^5x^5+924a^4b^6x^6-2310a^3b^7x^7+9240a^2b^8x^8+4158}{504a^{13}x^9+1008a^{12}bx^{10}+504a^{11}b^2x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x+a)**3,x)
```

```
[Out] -(56*a**10 - 77*a**9*b*x + 110*a**8*b**2*x**2 - 165*a**7*b**3*x**3 + 264*a*
*6*b**4*x**4 - 462*a**5*b**5*x**5 + 924*a**4*b**6*x**6 - 2310*a**3*b**7*x**
7 + 9240*a**2*b**8*x**8 + 41580*a*b**9*x**9 + 27720*b**10*x**10)/(504*a**13
*x**9 + 1008*a**12*b*x**10 + 504*a**11*b**2*x**11) + 55*b**9*(-log(x) + log
(a/b + x))/a**12
```

Giac [A] time = 1.20668, size = 205, normalized size = 1.26

$$\frac{55\,b^9\log\left(|bx+a|\right)}{a^{12}} - \frac{55\,b^9\log\left(|x|\right)}{a^{12}} - \frac{27720\,ab^{10}x^{10} + 41580\,a^2b^9x^9 + 9240\,a^3b^8x^8 - 2310\,a^4b^7x^7 + 924\,a^5b^6x^6 - 462\,a^6b}{504\,(bx+a)^2a^{12}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 55*b^9*log(abs(b*x + a))/a^12 - 55*b^9*log(abs(x))/a^12 - 1/504*(27720*a*b^
10*x^10 + 41580*a^2*b^9*x^9 + 9240*a^3*b^8*x^8 - 2310*a^4*b^7*x^7 + 924*a^5
*b^6*x^6 - 462*a^6*b^5*x^5 + 264*a^7*b^4*x^4 - 165*a^8*b^3*x^3 + 110*a^9*b^
2*x^2 - 77*a^10*b*x + 56*a^11)/((b*x + a)^2*a^12*x^9)
```

3.255 $\int \frac{1}{x(2+3x)} \, dx$

Optimal. Leaf size=17

$$\frac{\log(x)}{2} - \frac{1}{2}\log(3x+2)$$

```
[Out] Log[x]/2 - Log[2 + 3*x]/2
```

Rubi [A] time = 0.002297, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\frac{\log(x)}{2} - \frac{1}{2}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 + 3*x)),x]

[Out] Log[x]/2 - Log[2 + 3*x]/2
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(2+3x)} \, dx &= \frac{1}{2}\int \frac{1}{x} \, dx - \frac{3}{2}\int \frac{1}{2+3x} \, dx \\ &= \frac{\log(x)}{2} - \frac{1}{2}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0053061, size = 17, normalized size = 1.

$$\frac{\log(x)}{2} - \frac{1}{2}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 + 3*x)),x]
```

```
[Out] Log[x]/2 - Log[2 + 3*x]/2
```

Maple [A] time = 0.004, size = 14, normalized size = 0.8

$$\frac{\ln(x)}{2} - \frac{\ln(2+3x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2+3*x),x)
```

```
[Out] 1/2*ln(x)-1/2*ln(2+3*x)
```

Maxima [A] time = 1.03794, size = 18, normalized size = 1.06

$$-\frac{1}{2}\log(3x+2) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(3*x + 2) + 1/2*log(x)
```

Fricas [A] time = 1.72443, size = 43, normalized size = 2.53

$$-\frac{1}{2}\log(3x+2) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] time = 0.12257, size = 12, normalized size = 0.71

$$\frac{\log(x)}{2} - \frac{\log\left(x+\frac{2}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x)
```

```
[Out] log(x)/2 - log(x + 2/3)/2
```

Giac [A] time = 1.22379, size = 20, normalized size = 1.18

$$-\frac{1}{2}\log(|3x+2|)+\frac{1}{2}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

3.256 $\int \frac{1}{x(4+6x)} dx$

Optimal. Leaf size=17

$$\frac{\log(x)}{4} - \frac{1}{4}\log(3x+2)$$

[Out] Log[x]/4 - Log[2 + 3*x]/4

Rubi [A] time = 0.0022819, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\frac{\log(x)}{4} - \frac{1}{4}\log(3x+2)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(4 + 6*x)),x]

[Out] Log[x]/4 - Log[2 + 3*x]/4

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x(4+6x)} dx &= \frac{1}{4}\int \frac{1}{x} dx - \frac{3}{2}\int \frac{1}{4+6x} dx \\ &= \frac{\log(x)}{4} - \frac{1}{4}\log(2+3x) \end{aligned}$$

Mathematica [A] time = 0.0035897, size = 17, normalized size = 1.

$$\frac{\log(x)}{4} - \frac{1}{4}\log(3x+2)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(4 + 6*x)),x]

```
[Out] Log[x]/4 - Log[2 + 3*x]/4
```

Maple [A] time = 0.002, size = 14, normalized size = 0.8

$$\frac{\ln(x)}{4} - \frac{\ln(2+3x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x),x)
```

```
[Out] 1/4*ln(x)-1/4*ln(2+3*x)
```

Maxima [A] time = 1.06633, size = 18, normalized size = 1.06

$$-\frac{1}{4}\log(3x+2) + \frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(3*x + 2) + 1/4*log(x)
```

Fricas [A] time = 1.70136, size = 43, normalized size = 2.53

$$-\frac{1}{4}\log(3x+2) + \frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(3*x + 2) + 1/4*log(x)
```

Sympy [A] time = 0.110987, size = 12, normalized size = 0.71

$$\frac{\log(x)}{4} - \frac{\log\left(x+\frac{2}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x)
```

```
[Out] log(x)/4 - log(x + 2/3)/4
```

Giac [A] time = 1.24486, size = 20, normalized size = 1.18

$$-\frac{1}{4}\log(|3x+2|)+\frac{1}{4}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(3*x + 2)) + 1/4*log(abs(x))
```

3.257 $\int \frac{1}{x^2(4+6x)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(3x+2)$$

```
[Out] -1/(4*x) - (3*Log[x])/8 + (3*Log[2 + 3*x])/8
```

Rubi [A] time = 0.0078345, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)),x]
```

```
[Out] -1/(4*x) - (3*Log[x])/8 + (3*Log[2 + 3*x])/8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)}\,dx &= \int \left(\frac{1}{4x^2}-\frac{3}{8x}+\frac{9}{8(2+3x)}\right)dx\\ &= -\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0026833, size = 24, normalized size = 1.

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)),x]
```

```
[Out] -1/(4*x) - (3*Log[x])/8 + (3*Log[2 + 3*x])/8
```

Maple [A] time = 0.007, size = 19, normalized size = 0.8

$$-\frac{1}{4\,x}-\frac{3\,\ln(x)}{8}+\frac{3\,\ln(2+3\,x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x),x)
```

```
[Out] -1/4/x-3/8*ln(x)+3/8*ln(2+3*x)
```

Maxima [A] time = 1.03344, size = 24, normalized size = 1.

$$-\frac{1}{4\,x}+\frac{3}{8}\,\log\left(3\,x+2\right)-\frac{3}{8}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="maxima")
```

```
[Out] -1/4/x + 3/8*log(3*x + 2) - 3/8*log(x)
```

Fricas [A] time = 1.7301, size = 58, normalized size = 2.42

$$\frac{3\,x\log\left(3\,x+2\right)-3\,x\log\left(x\right)-2}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="fricas")
```

```
[Out] 1/8*(3*x*log(3*x + 2) - 3*x*log(x) - 2)/x
```

Sympy [A] time = 0.123414, size = 20, normalized size = 0.83

$$-\frac{3\log\left(x\right)}{8}+\frac{3\log\left(x+\frac{2}{3}\right)}{8}-\frac{1}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x),x)
```

```
[Out] -3*log(x)/8 + 3*log(x + 2/3)/8 - 1/(4*x)
```

Giac [A] time = 1.17741, size = 27, normalized size = 1.12

$$-\frac{1}{4\,x}+\frac{3}{8}\,\log\left(|3\,x+2|\right)-\frac{3}{8}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="giac")
```

```
[Out] -1/4/x + 3/8*log(abs(3*x + 2)) - 3/8*log(abs(x))
```

3.258 $\int \frac{1}{x^3(4+6x)}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(3x+2)$$

```
[Out] -1/(8*x^2) + 3/(8*x) + (9*Log[x])/16 - (9*Log[2 + 3*x])/16
```

Rubi [A] time = 0.0100174, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)),x]

[Out] -1/(8*x^2) + 3/(8*x) + (9*Log[x])/16 - (9*Log[2 + 3*x])/16
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)}\,dx &= \int \left(\frac{1}{4x^3}-\frac{3}{8x^2}+\frac{9}{16x}-\frac{27}{16(2+3x)}\right)dx\\ &= -\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0028378, size = 31, normalized size = 1.

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)),x]

[Out] -1/(8*x^2) + 3/(8*x) + (9*Log[x])/16 - (9*Log[2 + 3*x])/16
```

Maple [A] time = 0.007, size = 24, normalized size = 0.8

$$-\frac{1}{8\,x^2}+\frac{3}{8\,x}+\frac{9\,\ln(x)}{16}-\frac{9\,\ln(2+3\,x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x),x)
```

```
[Out] -1/8/x^2+3/8/x+9/16*ln(x)-9/16*ln(2+3*x)
```

Maxima [A] time = 0.991965, size = 31, normalized size = 1.

$$\frac{3x-1}{8x^2}-\frac{9}{16}\log(3x+2)+\frac{9}{16}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="maxima")
```

```
[Out] 1/8*(3*x - 1)/x^2 - 9/16*log(3*x + 2) + 9/16*log(x)
```

Fricas [A] time = 1.74511, size = 77, normalized size = 2.48

$$-\frac{9x^2\log(3x+2)-9x^2\log(x)-6x+2}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="fricas")
```

```
[Out] -1/16*(9*x^2*log(3*x + 2) - 9*x^2*log(x) - 6*x + 2)/x^2
```

Sympy [A] time = 0.167408, size = 26, normalized size = 0.84

$$\frac{9\log(x)}{16}-\frac{9\log\left(x+\frac{2}{3}\right)}{16}+\frac{3x-1}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x),x)
```

```
[Out] 9*log(x)/16 - 9*log(x + 2/3)/16 + (3*x - 1)/(8*x**2)
```

Giac [A] time = 1.16679, size = 34, normalized size = 1.1

$$\frac{3x-1}{8x^2}-\frac{9}{16}\log(|3x+2|)+\frac{9}{16}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="giac")
```

```
[Out] 1/8*(3*x - 1)/x^2 - 9/16*log(abs(3*x + 2)) + 9/16*log(abs(x))
```

3.259 $\int \frac{1}{x^4(4+6x)}\,dx$

Optimal. Leaf size=38

$$\frac{3}{16x^2}-\frac{1}{12x^3}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

```
[Out] -1/(12*x^3) + 3/(16*x^2) - 9/(16*x) - (27*Log[x])/32 + (27*Log[2 + 3*x])/32
```

Rubi [A] time = 0.0117281, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{3}{16x^2}-\frac{1}{12x^3}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)),x]
```

```
[Out] -1/(12*x^3) + 3/(16*x^2) - 9/(16*x) - (27*Log[x])/32 + (27*Log[2 + 3*x])/32
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)}\,dx &= \int \left(\frac{1}{4x^4}-\frac{3}{8x^3}+\frac{9}{16x^2}-\frac{27}{32x}+\frac{81}{32(2+3x)}\right)dx\\ &= -\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0065298, size = 38, normalized size = 1.

$$\frac{3}{16x^2}-\frac{1}{12x^3}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)),x]
```

```
[Out] -1/(12*x^3) + 3/(16*x^2) - 9/(16*x) - (27*Log[x])/32 + (27*Log[2 + 3*x])/32
```

Maple [A] time = 0.005, size = 29, normalized size = 0.8

$$-\frac{1}{12\,x^3}+\frac{3}{16\,x^2}-\frac{9}{16\,x}-\frac{27\,\ln(x)}{32}+\frac{27\,\ln(2+3\,x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4+6*x),x)
```

```
[Out] -1/12/x^3+3/16/x^2-9/16/x-27/32*ln(x)+27/32*ln(2+3*x)
```

Maxima [A] time = 1.03935, size = 38, normalized size = 1.

$$-\frac{27\,x^2-9\,x+4}{48\,x^3}+\frac{27}{32}\,\log\left(3\,x+2\right)-\frac{27}{32}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x),x, algorithm="maxima")
```

```
[Out] -1/48*(27*x^2 - 9*x + 4)/x^3 + 27/32*log(3*x + 2) - 27/32*log(x)
```

Fricas [A] time = 1.79884, size = 92, normalized size = 2.42

$$\frac{81\,x^3\log\left(3\,x+2\right)-81\,x^3\log\left(x\right)-54\,x^2+18\,x-8}{96\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x),x, algorithm="fricas")
```

```
[Out] 1/96*(81*x^3*log(3*x + 2) - 81*x^3*log(x) - 54*x^2 + 18*x - 8)/x^3
```

Sympy [A] time = 0.21336, size = 31, normalized size = 0.82

$$-\frac{27\log\left(x\right)}{32}+\frac{27\log\left(x+\frac{2}{3}\right)}{32}-\frac{27x^2-9x+4}{48x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x),x)
```

```
[Out] -27*log(x)/32 + 27*log(x + 2/3)/32 - (27*x**2 - 9*x + 4)/(48*x**3)
```

Giac [A] time = 1.14213, size = 41, normalized size = 1.08

$$-\frac{27\,x^2-9\,x+4}{48\,x^3}+\frac{27}{32}\,\log\left(|3\,x+2|\right)-\frac{27}{32}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x),x, algorithm="giac")
```

```
[Out] -1/48*(27*x^2 - 9*x + 4)/x^3 + 27/32*log(abs(3*x + 2)) - 27/32*log(abs(x))
```

3.260 $\int \frac{1}{x^5(4+6x)}\,dx$

Optimal. Leaf size=45

$$-\frac{9}{32x^2}+\frac{1}{8x^3}-\frac{1}{16x^4}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(3x+2)$$

```
[Out] -1/(16*x^4) + 1/(8*x^3) - 9/(32*x^2) + 27/(32*x) + (81*Log[x])/64 - (81*Log
[2 + 3*x])/64
```

Rubi [A] time = 0.0117359, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{9}{32x^2}+\frac{1}{8x^3}-\frac{1}{16x^4}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)),x]
```

```
[Out] -1/(16*x^4) + 1/(8*x^3) - 9/(32*x^2) + 27/(32*x) + (81*Log[x])/64 - (81*Log
[2 + 3*x])/64
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)}\,dx &= \int \left(\frac{1}{4x^5}-\frac{3}{8x^4}+\frac{9}{16x^3}-\frac{27}{32x^2}+\frac{81}{64x}-\frac{243}{64(2+3x)}\right)dx\\ &= -\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0031462, size = 45, normalized size = 1.

$$-\frac{9}{32x^2}+\frac{1}{8x^3}-\frac{1}{16x^4}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)),x]
```

```
[Out] -1/(16*x^4) + 1/(8*x^3) - 9/(32*x^2) + 27/(32*x) + (81*Log[x])/64 - (81*Log
[2 + 3*x])/64
```

Maple [A] time = 0.006, size = 34, normalized size = 0.8

$$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\ln(x)}{64}-\frac{81\ln(2+3x)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x),x)
```

```
[Out] -1/16/x^4+1/8/x^3-9/32/x^2+27/32/x+81/64*ln(x)-81/64*ln(2+3*x)
```

Maxima [A] time = 1.02507, size = 45, normalized size = 1.

$$\frac{27x^3-9x^2+4x-2}{32x^4}-\frac{81}{64}\log(3x+2)+\frac{81}{64}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="maxima")
```

```
[Out] 1/32*(27*x^3 - 9*x^2 + 4*x - 2)/x^4 - 81/64*log(3*x + 2) + 81/64*log(x)
```

Fricas [A] time = 1.69802, size = 104, normalized size = 2.31

$$-\frac{81x^4\log(3x+2)-81x^4\log(x)-54x^3+18x^2-8x+4}{64x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="fricas")
```

```
[Out] -1/64*(81*x^4*log(3*x + 2) - 81*x^4*log(x) - 54*x^3 + 18*x^2 - 8*x + 4)/x^4
```

Sympy [A] time = 0.193303, size = 36, normalized size = 0.8

$$\frac{81\log(x)}{64}-\frac{81\log\left(x+\frac{2}{3}\right)}{64}+\frac{27x^3-9x^2+4x-2}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x),x)
```

```
[Out] 81*log(x)/64 - 81*log(x + 2/3)/64 + (27*x**3 - 9*x**2 + 4*x - 2)/(32*x**4)
```

Giac [A] time = 1.2037, size = 47, normalized size = 1.04

$$\frac{27x^3-9x^2+4x-2}{32x^4}-\frac{81}{64}\log(|3x+2|)+\frac{81}{64}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="giac")
```

```
[Out] 1/32*(27*x^3 - 9*x^2 + 4*x - 2)/x^4 - 81/64*log(abs(3*x + 2)) + 81/64*log(a
bs(x))
```

3.261 $\int \frac{1}{x(4+6x)^2} dx$

Optimal. Leaf size=28

$$\frac{1}{8(3x+2)}+\frac{\log(x)}{16}-\frac{1}{16}\log(3x+2)$$

```
[Out] 1/(8*(2 + 3*x)) + Log[x]/16 - Log[2 + 3*x]/16
```

Rubi [A] time = 0.0089128, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{8(3x+2)}+\frac{\log(x)}{16}-\frac{1}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 + 6*x)^2),x]

[Out] 1/(8*(2 + 3*x)) + Log[x]/16 - Log[2 + 3*x]/16
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)^2}\,dx &= \int \left(\frac{1}{16x}-\frac{3}{8(2+3x)^2}-\frac{3}{16(2+3x)}\right)dx\\ &= \frac{1}{8(2+3x)}+\frac{\log(x)}{16}-\frac{1}{16}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0226824, size = 26, normalized size = 0.93

$$\frac{1}{16}\left(\frac{2}{3x+2}+\log(-6x)-\log(6x+4)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 + 6*x)^2),x]

[Out] (2/(2 + 3*x) + Log[-6*x] - Log[4 + 6*x])/16
```

Maple [A] time = 0.006, size = 23, normalized size = 0.8

$$\frac{1}{16+24\,x}+\frac{\ln(x)}{16}-\frac{\ln(2+3\,x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x)^2,x)
```

```
[Out] 1/8/(2+3*x)+1/16*ln(x)-1/16*ln(2+3*x)
```

Maxima [A] time = 1.08183, size = 30, normalized size = 1.07

$$\frac{1}{8\,(3\,x+2)}-\frac{1}{16}\,\log\left(3\,x+2\right)+\frac{1}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] 1/8/(3*x + 2) - 1/16*log(3*x + 2) + 1/16*log(x)
```

Fricas [A] time = 1.68688, size = 88, normalized size = 3.14

$$-\frac{(3\,x+2)\log\left(3\,x+2\right)-(3\,x+2)\log\left(x\right)-2}{16\,(3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] -1/16*((3*x + 2)*log(3*x + 2) - (3*x + 2)*log(x) - 2)/(3*x + 2)
```

Sympy [A] time = 0.113678, size = 19, normalized size = 0.68

$$\frac{\log\left(x\right)}{16}-\frac{\log\left(x+\frac{2}{3}\right)}{16}+\frac{1}{24x+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)**2,x)
```

```
[Out] log(x)/16 - log(x + 2/3)/16 + 1/(24*x + 16)
```

Giac [A] time = 1.19607, size = 34, normalized size = 1.21

$$\frac{1}{8\,(3\,x+2)}+\frac{1}{16}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] 1/8/(3*x + 2) + 1/16*log(abs(-2/(3*x + 2) + 1))
```

3.262 $\int \frac{1}{x^2(4+6x)^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{16x}-\frac{3}{16(3x+2)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(3x+2)$$

```
[Out] -1/(16*x) - 3/(16*(2 + 3*x)) - (3*Log[x])/16 + (3*Log[2 + 3*x])/16
```

Rubi [A] time = 0.0099562, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{16x}-\frac{3}{16(3x+2)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)^2),x]
```

```
[Out] -1/(16*x) - 3/(16*(2 + 3*x)) - (3*Log[x])/16 + (3*Log[2 + 3*x])/16
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^2}-\frac{3}{16x}+\frac{9}{16(2+3x)^2}+\frac{9}{16(2+3x)}\right)dx\\ &= -\frac{1}{16x}-\frac{3}{16(2+3x)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0163434, size = 31, normalized size = 0.89

$$\frac{1}{16}\left(-\frac{1}{x}-\frac{3}{3x+2}-3\log(x)+3\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)^2),x]
```

```
[Out] (-x^(-1) - 3/(2 + 3*x) - 3*Log[x] + 3*Log[2 + 3*x])/16
```

Maple [A] time = 0.01, size = 28, normalized size = 0.8

$$-\frac{1}{16\,x}-\frac{3}{32+48\,x}-\frac{3\,\ln(x)}{16}+\frac{3\,\ln(2+3\,x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x)^2,x)
```

```
[Out] -1/16/x-3/16/(2+3*x)-3/16*ln(x)+3/16*ln(2+3*x)
```

Maxima [A] time = 1.01809, size = 42, normalized size = 1.2

$$-\frac{3\,x+1}{8\left(3\,x^2+2\,x\right)}+\frac{3}{16}\,\log\left(3\,x+2\right)-\frac{3}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(3*x + 1)/(3*x^2 + 2*x) + 3/16*log(3*x + 2) - 3/16*log(x)
```

Fricas [A] time = 1.73492, size = 116, normalized size = 3.31

$$\frac{3\left(3\,x^2+2\,x\right)\log\left(3\,x+2\right)-3\left(3\,x^2+2\,x\right)\log\left(x\right)-6\,x-2}{16\left(3\,x^2+2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(3*x^2 + 2*x)*log(3*x + 2) - 3*(3*x^2 + 2*x)*log(x) - 6*x - 2)/(3*x
^2 + 2*x)
```

Sympy [A] time = 0.164169, size = 29, normalized size = 0.83

$$-\frac{3x+1}{24x^2+16x}-\frac{3\log\left(x\right)}{16}+\frac{3\log\left(x+\frac{2}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x)**2,x)
```

```
[Out] -(3*x + 1)/(24*x**2 + 16*x) - 3*log(x)/16 + 3*log(x + 2/3)/16
```

Giac [A] time = 1.2096, size = 54, normalized size = 1.54

$$-\frac{3}{16\left(3\,x+2\right)}+\frac{3}{32\left(\frac{2}{3\,x+2}-1\right)}-\frac{3}{16}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="giac")

[Out] -3/16/(3*x + 2) + 3/32/(2/(3*x + 2) - 1) - 3/16*log(abs(-2/(3*x + 2) + 1))
```

3.263 $\int \frac{1}{x^3(4+6x)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(3x+2)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(3x+2)$$

```
[Out] -1/(32*x^2) + 3/(16*x) + 9/(32*(2 + 3*x)) + (27*Log[x])/64 - (27*Log[2 + 3*
x])/64
```

Rubi [A] time = 0.0135602, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(3x+2)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)^2),x]
```

```
[Out] -1/(32*x^2) + 3/(16*x) + 9/(32*(2 + 3*x)) + (27*Log[x])/64 - (27*Log[2 + 3*
x])/64
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^3}-\frac{3}{16x^2}+\frac{27}{64x}-\frac{27}{32(2+3x)^2}-\frac{81}{64(2+3x)}\right)dx\\ &= -\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(2+3x)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0192639, size = 36, normalized size = 0.86

$$\frac{1}{64}\left(-\frac{2}{x^2}+\frac{12}{x}+\frac{18}{3x+2}+27\log(x)-27\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)^2),x]
```

```
[Out] (-2/x^2 + 12/x + 18/(2 + 3*x) + 27*Log[x] - 27*Log[2 + 3*x])/64
```

Maple [A] time = 0.007, size = 33, normalized size = 0.8

$$-\frac{1}{32 x^{2}}+\frac{3}{16 x}+\frac{9}{64+96 x}+\frac{27 \ln \left(x\right)}{64}-\frac{27 \ln \left(2+3 x\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x)^2,x)
```

```
[Out] -1/32/x^2+3/16/x+9/32/(2+3*x)+27/64*ln(x)-27/64*ln(2+3*x)
```

Maxima [A] time = 1.08064, size = 51, normalized size = 1.21

$$\frac{27 x^{2}+9 x-2}{32 \left(3 x^{3}+2 x^{2}\right)}-\frac{27}{64} \log \left(3 x+2\right)+\frac{27}{64} \log \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(27*x^2 + 9*x - 2)/(3*x^3 + 2*x^2) - 27/64*log(3*x + 2) + 27/64*log(x)
```

Fricas [A] time = 1.73493, size = 140, normalized size = 3.33

$$\frac{54 x^{2}-27 \left(3 x^{3}+2 x^{2}\right) \log \left(3 x+2\right)+27 \left(3 x^{3}+2 x^{2}\right) \log \left(x\right)+18 x-4}{64 \left(3 x^{3}+2 x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(54*x^2 - 27*(3*x^3 + 2*x^2)*log(3*x + 2) + 27*(3*x^3 + 2*x^2)*log(x)
+ 18*x - 4)/(3*x^3 + 2*x^2)
```

Sympy [A] time = 0.268462, size = 36, normalized size = 0.86

$$\frac{27 \log \left(x\right)}{64}-\frac{27 \log \left(x+\frac{2}{3}\right)}{64}+\frac{27x^{2}+9x-2}{96x^{3}+64x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x)**2,x)
```

```
[Out] 27*log(x)/64 - 27*log(x + 2/3)/64 + (27*x**2 + 9*x - 2)/(96*x**3 + 64*x**2)
```

Giac [A] time = 1.22409, size = 69, normalized size = 1.64

$$\frac{9}{32 \left(3 x+2\right)}-\frac{9 \left(\frac{12}{3 x+2}-5\right)}{128 \left(\frac{2}{3 x+2}-1\right)^{2}}+\frac{27}{64} \log \left(\left|-\frac{2}{3 x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] 9/32/(3*x + 2) - 9/128*(12/(3*x + 2) - 5)/(2/(3*x + 2) - 1)^2 + 27/64*log(a
bs(-2/(3*x + 2) + 1))
```

3.264 $\int \frac{1}{x^4(4+6x)^2}\,dx$

Optimal. Leaf size=49

$$\frac{3}{32x^2}-\frac{1}{48x^3}-\frac{27}{64x}-\frac{27}{64(3x+2)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

```
[Out] -1/(48*x^3) + 3/(32*x^2) - 27/(64*x) - 27/(64*(2 + 3*x)) - (27*Log[x])/32 +
 (27*Log[2 + 3*x])/32
```

Rubi [A] time = 0.0164746, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{3}{32x^2}-\frac{1}{48x^3}-\frac{27}{64x}-\frac{27}{64(3x+2)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)^2),x]
```

```
[Out] -1/(48*x^3) + 3/(32*x^2) - 27/(64*x) - 27/(64*(2 + 3*x)) - (27*Log[x])/32 +
 (27*Log[2 + 3*x])/32
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^4}-\frac{3}{16x^3}+\frac{27}{64x^2}-\frac{27}{32x}+\frac{81}{64(2+3x)^2}+\frac{81}{32(2+3x)}\right)dx\\ &= -\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(2+3x)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0453362, size = 44, normalized size = 0.9

$$\frac{1}{192}\left(-\frac{4\left(81x^3+27x^2-6x+2\right)}{x^3(3x+2)}-162\log(x)+162\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)^2),x]
```

```
[Out] ((-4*(2 - 6*x + 27*x^2 + 81*x^3))/(x^3*(2 + 3*x)) - 162*Log[x] + 162*Log[2
+ 3*x])/192
```

Maple [A] time = 0.007, size = 38, normalized size = 0.8

$$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{128+192x}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4+6*x)^2,x)
```

```
[Out] -1/48/x^3+3/32/x^2-27/64/x-27/64/(2+3*x)-27/32*ln(x)+27/32*ln(2+3*x)
```

Maxima [A] time = 1.0237, size = 58, normalized size = 1.18

$$-\frac{81x^3+27x^2-6x+2}{48(3x^4+2x^3)}+\frac{27}{32}\log(3x+2)-\frac{27}{32}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(81*x^3 + 27*x^2 - 6*x + 2)/(3*x^4 + 2*x^3) + 27/32*log(3*x + 2) - 27
/32*log(x)
```

Fricas [A] time = 1.71794, size = 155, normalized size = 3.16

$$-\frac{162x^3+54x^2-81(3x^4+2x^3)\log(3x+2)+81(3x^4+2x^3)\log(x)-12x+4}{96(3x^4+2x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] -1/96*(162*x^3 + 54*x^2 - 81*(3*x^4 + 2*x^3)*log(3*x + 2) + 81*(3*x^4 + 2*x
^3)*log(x) - 12*x + 4)/(3*x^4 + 2*x^3)
```

Sympy [A] time = 0.164463, size = 41, normalized size = 0.84

$$-\frac{27\log(x)}{32}+\frac{27\log\left(x+\frac{2}{3}\right)}{32}-\frac{81x^3+27x^2-6x+2}{144x^4+96x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x)**2,x)
```

```
[Out] -27*log(x)/32 + 27*log(x + 2/3)/32 - (81*x**3 + 27*x**2 - 6*x + 2)/(144*x**
4 + 96*x**3)
```

Giac [A] time = 1.22874, size = 81, normalized size = 1.65

$$-\frac{27}{64\,(3\,x+2)}-\frac{9\left(\frac{60}{3\,x+2}-\frac{72}{(3\,x+2)^{2}}-13\right)}{128\left(\frac{2}{3\,x+2}-1\right)^{3}}-\frac{27}{32}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] -27/64/(3*x + 2) - 9/128*(60/(3*x + 2) - 72/(3*x + 2)^2 - 13)/(2/(3*x + 2)
- 1)^3 - 27/32*log(abs(-2/(3*x + 2) + 1))
```

3.265 $\int \frac{1}{x^5(4+6x)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{27}{128x^2}+\frac{1}{16x^3}-\frac{1}{64x^4}+\frac{27}{32x}+\frac{81}{128(3x+2)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(3x+2)$$

```
[Out] -1/(64*x^4) + 1/(16*x^3) - 27/(128*x^2) + 27/(32*x) + 81/(128*(2 + 3*x)) +
(405*Log[x])/256 - (405*Log[2 + 3*x])/256
```

Rubi [A] time = 0.019642, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{27}{128x^2}+\frac{1}{16x^3}-\frac{1}{64x^4}+\frac{27}{32x}+\frac{81}{128(3x+2)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)^2),x]
```

```
[Out] -1/(64*x^4) + 1/(16*x^3) - 27/(128*x^2) + 27/(32*x) + 81/(128*(2 + 3*x)) +
(405*Log[x])/256 - (405*Log[2 + 3*x])/256
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^5}-\frac{3}{16x^4}+\frac{27}{64x^3}-\frac{27}{32x^2}+\frac{405}{256x}-\frac{243}{128(2+3x)^2}-\frac{1215}{256(2+3x)}\right)dx\\ &= -\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(2+3x)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0178101, size = 56, normalized size = 1.

$$-\frac{27}{128x^2}+\frac{1}{16x^3}-\frac{1}{64x^4}+\frac{27}{32x}+\frac{81}{128(3x+2)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)^2),x]
```

```
[Out] -1/(64*x^4) + 1/(16*x^3) - 27/(128*x^2) + 27/(32*x) + 81/(128*(2 + 3*x)) +
(405*Log[x])/256 - (405*Log[2 + 3*x])/256
```

Maple [A] time = 0.01, size = 43, normalized size = 0.8

$$-\frac{1}{64\,x^{4}}+\frac{1}{16\,x^{3}}-\frac{27}{128\,x^{2}}+\frac{27}{32\,x}+\frac{81}{256+384\,x}+\frac{405\,\ln\left(x\right)}{256}-\frac{405\,\ln\left(2+3\,x\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x)^2,x)
```

```
[Out] -1/64/x^4+1/16/x^3-27/128/x^2+27/32/x+81/128/(2+3*x)+405/256*ln(x)-405/256*
ln(2+3*x)
```

Maxima [A] time = 1.04453, size = 65, normalized size = 1.16

$$\frac{405\,x^{4}+135\,x^{3}-30\,x^{2}+10\,x-4}{128\,\left(3\,x^{5}+2\,x^{4}\right)}-\frac{405}{256}\,\log\left(3\,x+2\right)+\frac{405}{256}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] 1/128*(405*x^4 + 135*x^3 - 30*x^2 + 10*x - 4)/(3*x^5 + 2*x^4) - 405/256*log
(3*x + 2) + 405/256*log(x)
```

Fricas [A] time = 1.83577, size = 171, normalized size = 3.05

$$\frac{810\,x^{4}+270\,x^{3}-60\,x^{2}-405\,\left(3\,x^{5}+2\,x^{4}\right)\log\left(3\,x+2\right)+405\,\left(3\,x^{5}+2\,x^{4}\right)\log\left(x\right)+20\,x-8}{256\,\left(3\,x^{5}+2\,x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] 1/256*(810*x^4 + 270*x^3 - 60*x^2 - 405*(3*x^5 + 2*x^4)*log(3*x + 2) + 405*
(3*x^5 + 2*x^4)*log(x) + 20*x - 8)/(3*x^5 + 2*x^4)
```

Sympy [A] time = 0.172982, size = 46, normalized size = 0.82

$$\frac{405\log\left(x\right)}{256}-\frac{405\log\left(x+\frac{2}{3}\right)}{256}+\frac{405x^{4}+135x^{3}-30x^{2}+10x-4}{384x^{5}+256x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x)**2,x)
```

```
[Out] 405*log(x)/256 - 405*log(x + 2/3)/256 + (405*x**4 + 135*x**3 - 30*x**2 + 10
*x - 4)/(384*x**5 + 256*x**4)
```

Giac [A] time = 1.16018, size = 93, normalized size = 1.66

$$\frac{81}{128\,(3\,x+2)}-\frac{27\,\left(\frac{520}{3\,x+2}-\frac{1200}{(3\,x+2)^2}+\frac{960}{(3\,x+2)^3}-77\right)}{1024\,\left(\frac{2}{3\,x+2}-1\right)^4}+\frac{405}{256}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] 81/128/(3*x + 2) - 27/1024*(520/(3*x + 2) - 1200/(3*x + 2)^2 + 960/(3*x + 2
)^3 - 77)/(2/(3*x + 2) - 1)^4 + 405/256*log(abs(-2/(3*x + 2) + 1))
```

3.266 $\int \frac{1}{x(4+6x)^3} dx$

Optimal. Leaf size=39

$$\frac{1}{32(3x+2)}+\frac{1}{32(3x+2)^2}+\frac{\log(x)}{64}-\frac{1}{64}\log(3x+2)$$

```
[Out] 1/(32*(2 + 3*x)^2) + 1/(32*(2 + 3*x)) + Log[x]/64 - Log[2 + 3*x]/64
```

Rubi [A] time = 0.0119353, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{32(3x+2)}+\frac{1}{32(3x+2)^2}+\frac{\log(x)}{64}-\frac{1}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 + 6*x)^3),x]
```

```
[Out] 1/(32*(2 + 3*x)^2) + 1/(32*(2 + 3*x)) + Log[x]/64 - Log[2 + 3*x]/64
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)^3}\,dx &= \int \left(\frac{1}{64x}-\frac{3}{16(2+3x)^3}-\frac{3}{32(2+3x)^2}-\frac{3}{64(2+3x)}\right)dx\\ &= \frac{1}{32(2+3x)^2}+\frac{1}{32(2+3x)}+\frac{\log(x)}{64}-\frac{1}{64}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0346237, size = 29, normalized size = 0.74

$$\frac{1}{64}\left(\frac{6(x+1)}{(3x+2)^2}+\log(-6x)-\log(6x+4)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 + 6*x)^3),x]
```

```
[Out] ((6*(1 + x))/(2 + 3*x)^2 + Log[-6*x] - Log[4 + 6*x])/64
```

Maple [A] time = 0.007, size = 32, normalized size = 0.8

$$\frac{1}{32\,(2+3\,x)^2}+\frac{1}{64+96\,x}+\frac{\ln(x)}{64}-\frac{\ln(2+3\,x)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x)^3,x)
```

```
[Out] 1/32/(2+3*x)^2+1/32/(2+3*x)+1/64*ln(x)-1/64*ln(2+3*x)
```

Maxima [A] time = 1.05525, size = 41, normalized size = 1.05

$$\frac{3(x+1)}{32\left(9x^2+12x+4\right)}-\frac{1}{64}\log(3x+2)+\frac{1}{64}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] 3/32*(x + 1)/(9*x^2 + 12*x + 4) - 1/64*log(3*x + 2) + 1/64*log(x)
```

Fricas [A] time = 1.66655, size = 132, normalized size = 3.38

$$-\frac{\left(9x^2+12x+4\right)\log(3x+2)-\left(9x^2+12x+4\right)\log(x)-6x-6}{64\left(9x^2+12x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/64*((9*x^2 + 12*x + 4)*log(3*x + 2) - (9*x^2 + 12*x + 4)*log(x) - 6*x -
6)/(9*x^2 + 12*x + 4)
```

Sympy [A] time = 0.212665, size = 27, normalized size = 0.69

$$\frac{3x+3}{288x^2+384x+128}+\frac{\log(x)}{64}-\frac{\log\left(x+\frac{2}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)**3,x)
```

```
[Out] (3*x + 3)/(288*x**2 + 384*x + 128) + log(x)/64 - log(x + 2/3)/64
```

Giac [A] time = 1.19562, size = 36, normalized size = 0.92

$$\frac{3(x+1)}{32(3x+2)^2}-\frac{1}{64}\log(|3x+2|)+\frac{1}{64}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] 3/32*(x + 1)/(3*x + 2)^2 - 1/64*log(abs(3*x + 2)) + 1/64*log(abs(x))
```

3.267 $\int \frac{1}{x^2(4+6x)^3} \, dx$

Optimal. Leaf size=46

$$-\frac{1}{64x}-\frac{3}{32(3x+2)}-\frac{3}{64(3x+2)^2}-\frac{9\log(x)}{128}+\frac{9}{128}\log(3x+2)$$

```
[Out] -1/(64*x) - 3/(64*(2 + 3*x)^2) - 3/(32*(2 + 3*x)) - (9*Log[x])/128 + (9*Log
[2 + 3*x])/128
```

Rubi [A] time = 0.0150952, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{64x}-\frac{3}{32(3x+2)}-\frac{3}{64(3x+2)^2}-\frac{9\log(x)}{128}+\frac{9}{128}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)^3),x]
```

```
[Out] -1/(64*x) - 3/(64*(2 + 3*x)^2) - 3/(32*(2 + 3*x)) - (9*Log[x])/128 + (9*Log
[2 + 3*x])/128
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)^3} \, dx &= \int \left(\frac{1}{64x^2}-\frac{9}{128x}+\frac{9}{32(2+3x)^3}+\frac{9}{32(2+3x)^2}+\frac{27}{128(2+3x)}\right) dx\\ &= -\frac{1}{64x}-\frac{3}{64(2+3x)^2}-\frac{3}{32(2+3x)}-\frac{9\log(x)}{128}+\frac{9}{128}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0343409, size = 39, normalized size = 0.85

$$\frac{1}{128}\left(-\frac{2\left(27x^2+27x+4\right)}{x(3x+2)^2}-9\log(x)+9\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)^3),x]
```

```
[Out] ((-2*(4 + 27*x + 27*x^2))/(x*(2 + 3*x)^2) - 9*Log[x] + 9*Log[2 + 3*x])/128
```

Maple [A] time = 0.009, size = 37, normalized size = 0.8

$$-\frac{1}{64x}-\frac{3}{64\left(2+3x\right)^{2}}-\frac{3}{64+96x}-\frac{9\ln\left(x\right)}{128}+\frac{9\ln\left(2+3x\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x)^3,x)
```

```
[Out] -1/64/x-3/64/(2+3*x)^2-3/32/(2+3*x)-9/128*ln(x)+9/128*ln(2+3*x)
```

Maxima [A] time = 1.06005, size = 55, normalized size = 1.2

$$-\frac{27\,x^{2}+27\,x+4}{64\left(9\,x^{3}+12\,x^{2}+4\,x\right)}+\frac{9}{128}\log\left(3\,x+2\right)-\frac{9}{128}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(27*x^2 + 27*x + 4)/(9*x^3 + 12*x^2 + 4*x) + 9/128*log(3*x + 2) - 9/1
28*log(x)
```

Fricas [A] time = 1.76173, size = 169, normalized size = 3.67

$$-\frac{54\,x^{2}-9\left(9\,x^{3}+12\,x^{2}+4\,x\right)\log\left(3\,x+2\right)+9\left(9\,x^{3}+12\,x^{2}+4\,x\right)\log\left(x\right)+54\,x+8}{128\left(9\,x^{3}+12\,x^{2}+4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(54*x^2 - 9*(9*x^3 + 12*x^2 + 4*x)*log(3*x + 2) + 9*(9*x^3 + 12*x^2
+ 4*x)*log(x) + 54*x + 8)/(9*x^3 + 12*x^2 + 4*x)
```

Sympy [A] time = 0.16527, size = 39, normalized size = 0.85

$$-\frac{27x^{2}+27x+4}{576x^{3}+768x^{2}+256x}-\frac{9\log\left(x\right)}{128}+\frac{9\log\left(x+\frac{2}{3}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x)**3,x)
```

```
[Out] -(27*x**2 + 27*x + 4)/(576*x**3 + 768*x**2 + 256*x) - 9*log(x)/128 + 9*log(
x + 2/3)/128
```

Giac [A] time = 1.23613, size = 50, normalized size = 1.09

$$-\frac{27\,x^2+27\,x+4}{64\,(3\,x+2)^2x}+\frac{9}{128}\,\log\left(|3\,x+2|\right)-\frac{9}{128}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] -1/64*(27*x^2 + 27*x + 4)/((3*x + 2)^2*x) + 9/128*log(abs(3*x + 2)) - 9/128
*log(abs(x))
```

3.268 $\int \frac{1}{x^3(4+6x)^3}\,dx$

Optimal. Leaf size=53

$$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{27}{128(3x+2)}+\frac{9}{128(3x+2)^2}+\frac{27\log(x)}{128}-\frac{27}{128}\log(3x+2)$$

```
[Out] -1/(128*x^2) + 9/(128*x) + 9/(128*(2 + 3*x)^2) + 27/(128*(2 + 3*x)) + (27*L
og[x])/128 - (27*Log[2 + 3*x])/128
```

Rubi [A] time = 0.015826, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{27}{128(3x+2)}+\frac{9}{128(3x+2)^2}+\frac{27\log(x)}{128}-\frac{27}{128}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)^3),x]
```

```
[Out] -1/(128*x^2) + 9/(128*x) + 9/(128*(2 + 3*x)^2) + 27/(128*(2 + 3*x)) + (27*L
og[x])/128 - (27*Log[2 + 3*x])/128
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)^3}\,dx &= \int \left(\frac{1}{64x^3}-\frac{9}{128x^2}+\frac{27}{128x}-\frac{27}{64(2+3x)^3}-\frac{81}{128(2+3x)^2}-\frac{81}{128(2+3x)}\right)dx\\ &= -\frac{1}{128x^2}+\frac{9}{128x}+\frac{9}{128(2+3x)^2}+\frac{27}{128(2+3x)}+\frac{27\log(x)}{128}-\frac{27}{128}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0369721, size = 44, normalized size = 0.83

$$\frac{1}{128}\left(\frac{2\left(81x^3+81x^2+12x-2\right)}{x^2(3x+2)^2}+27\log(x)-27\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)^3),x]
```

```
[Out] ((2*(-2 + 12*x + 81*x^2 + 81*x^3))/(x^2*(2 + 3*x)^2) + 27*Log[x] - 27*Log[2
 + 3*x])/128
```

Maple [A] time = 0.011, size = 42, normalized size = 0.8

$$-\frac{1}{128\,x^{2}}+\frac{9}{128\,x}+\frac{9}{128\,\left(2+3\,x\right)^{2}}+\frac{27}{256+384\,x}+\frac{27\,\ln\left(x\right)}{128}-\frac{27\,\ln\left(2+3\,x\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x)^3,x)
```

```
[Out] -1/128/x^2+9/128/x+9/128/(2+3*x)^2+27/128/(2+3*x)+27/128*ln(x)-27/128*ln(2+
3*x)
```

Maxima [A] time = 1.0443, size = 65, normalized size = 1.23

$$\frac{81\,x^{3}+81\,x^{2}+12\,x-2}{64\,\left(9\,x^{4}+12\,x^{3}+4\,x^{2}\right)}-\frac{27}{128}\,\log\left(3\,x+2\right)+\frac{27}{128}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(81*x^3 + 81*x^2 + 12*x - 2)/(9*x^4 + 12*x^3 + 4*x^2) - 27/128*log(3*x
 + 2) + 27/128*log(x)
```

Fricas [A] time = 1.80194, size = 193, normalized size = 3.64

$$\frac{162\,x^{3}+162\,x^{2}-27\,\left(9\,x^{4}+12\,x^{3}+4\,x^{2}\right)\log\left(3\,x+2\right)+27\,\left(9\,x^{4}+12\,x^{3}+4\,x^{2}\right)\log\left(x\right)+24\,x-4}{128\,\left(9\,x^{4}+12\,x^{3}+4\,x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(162*x^3 + 162*x^2 - 27*(9*x^4 + 12*x^3 + 4*x^2)*log(3*x + 2) + 27*(9
*x^4 + 12*x^3 + 4*x^2)*log(x) + 24*x - 4)/(9*x^4 + 12*x^3 + 4*x^2)
```

Sympy [A] time = 0.239392, size = 46, normalized size = 0.87

$$\frac{27\log\left(x\right)}{128}-\frac{27\log\left(x+\frac{2}{3}\right)}{128}+\frac{81x^{3}+81x^{2}+12x-2}{576x^{4}+768x^{3}+256x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x)**3,x)
```

```
[Out] 27*log(x)/128 - 27*log(x + 2/3)/128 + (81*x**3 + 81*x**2 + 12*x - 2)/(576*x
**4 + 768*x**3 + 256*x**2)
```

Giac [A] time = 1.23302, size = 58, normalized size = 1.09

$$\frac{81\,x^3+81\,x^2+12\,x-2}{64\,\left(3\,x^2+2\,x\right)^2}-\frac{27}{128}\,\log\left(|3\,x+2|\right)+\frac{27}{128}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] 1/64*(81*x^3 + 81*x^2 + 12*x - 2)/(3*x^2 + 2*x)^2 - 27/128*log(abs(3*x + 2)
) + 27/128*log(abs(x))
```

3.269 $\int \frac{1}{x^4(4+6x)^3} dx$

Optimal. Leaf size=60

$$\frac{9}{256x^2} - \frac{1}{192x^3} - \frac{27}{128x} - \frac{27}{64(3x+2)} - \frac{27}{256(3x+2)^2} - \frac{135\log(x)}{256} + \frac{135}{256}\log(3x+2)$$

```
[Out] -1/(192*x^3) + 9/(256*x^2) - 27/(128*x) - 27/(256*(2 + 3*x)^2) - 27/(64*(2
+ 3*x)) - (135*Log[x])/256 + (135*Log[2 + 3*x])/256
```

Rubi [A] time = 0.0207767, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{9}{256x^2} - \frac{1}{192x^3} - \frac{27}{128x} - \frac{27}{64(3x+2)} - \frac{27}{256(3x+2)^2} - \frac{135\log(x)}{256} + \frac{135}{256}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)^3),x]
```

```
[Out] -1/(192*x^3) + 9/(256*x^2) - 27/(128*x) - 27/(256*(2 + 3*x)^2) - 27/(64*(2
+ 3*x)) - (135*Log[x])/256 + (135*Log[2 + 3*x])/256
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)^3} dx &= \int \left(\frac{1}{64x^4} - \frac{9}{128x^3} + \frac{27}{128x^2} - \frac{135}{256x} + \frac{81}{128(2+3x)^3} + \frac{81}{64(2+3x)^2} + \frac{405}{256(2+3x)}\right) dx\\ &= -\frac{1}{192x^3} + \frac{9}{256x^2} - \frac{27}{128x} - \frac{27}{256(2+3x)^2} - \frac{27}{64(2+3x)} - \frac{135\log(x)}{256} + \frac{135}{256}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.0320831, size = 49, normalized size = 0.82

$$\frac{1}{768}\left(-\frac{2\left(1215x^4+1215x^3+180x^2-30x+8\right)}{x^3(3x+2)^2} - 405\log(x) + 405\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)^3),x]
```

```
[Out] ((-2*(8 - 30*x + 180*x^2 + 1215*x^3 + 1215*x^4))/(x^3*(2 + 3*x)^2) - 405*Lo
g[x] + 405*Log[2 + 3*x])/768
```

Maple [A] time = 0.01, size = 47, normalized size = 0.8

$$-\frac{1}{192 x^{3}}+\frac{9}{256 x^{2}}-\frac{27}{128 x}-\frac{27}{256 (2+3 x)^{2}}-\frac{27}{128+192 x}-\frac{135 \ln (x)}{256}+\frac{135 \ln (2+3 x)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4+6*x)^3,x)
```

```
[Out] -1/192/x^3+9/256/x^2-27/128/x-27/256/(2+3*x)^2-27/64/(2+3*x)-135/256*ln(x)+
135/256*ln(2+3*x)
```

Maxima [A] time = 1.00904, size = 72, normalized size = 1.2

$$-\frac{1215 x^{4}+1215 x^{3}+180 x^{2}-30 x+8}{384 \left(9 x^{5}+12 x^{4}+4 x^{3}\right)}+\frac{135}{256} \log (3 x+2)-\frac{135}{256} \log (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] -1/384*(1215*x^4 + 1215*x^3 + 180*x^2 - 30*x + 8)/(9*x^5 + 12*x^4 + 4*x^3)
+ 135/256*log(3*x + 2) - 135/256*log(x)
```

Fricas [A] time = 1.79541, size = 215, normalized size = 3.58

$$-\frac{2430 x^{4}+2430 x^{3}+360 x^{2}-405 \left(9 x^{5}+12 x^{4}+4 x^{3}\right) \log (3 x+2)+405 \left(9 x^{5}+12 x^{4}+4 x^{3}\right) \log (x)-60 x+16}{768 \left(9 x^{5}+12 x^{4}+4 x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/768*(2430*x^4 + 2430*x^3 + 360*x^2 - 405*(9*x^5 + 12*x^4 + 4*x^3)*log(3*
x + 2) + 405*(9*x^5 + 12*x^4 + 4*x^3)*log(x) - 60*x + 16)/(9*x^5 + 12*x^4 +
 4*x^3)
```

Sympy [A] time = 0.204437, size = 51, normalized size = 0.85

$$-\frac{135 \log (x)}{256}+\frac{135 \log \left(x+\frac{2}{3}\right)}{256}-\frac{1215x^{4}+1215x^{3}+180x^{2}-30x+8}{3456x^{5}+4608x^{4}+1536x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x)**3,x)
```

```
[Out] -135*log(x)/256 + 135*log(x + 2/3)/256 - (1215*x**4 + 1215*x**3 + 180*x**2
- 30*x + 8)/(3456*x**5 + 4608*x**4 + 1536*x**3)
```

Giac [A] time = 1.19775, size = 63, normalized size = 1.05

$$-\frac{1215\,x^4+1215\,x^3+180\,x^2-30\,x+8}{384\,(3\,x+2)^2x^3}+\frac{135}{256}\,\log\left(|3\,x+2|\right)-\frac{135}{256}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] -1/384*(1215*x^4 + 1215*x^3 + 180*x^2 - 30*x + 8)/((3*x + 2)^2*x^3) + 135/2
56*log(abs(3*x + 2)) - 135/256*log(abs(x))
```

3.270 $\int \frac{1}{x^5(4+6x)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{27}{256x^2}+\frac{3}{128x^3}-\frac{1}{256x^4}+\frac{135}{256x}+\frac{405}{512(3x+2)}+\frac{81}{512(3x+2)^2}+\frac{1215\log(x)}{1024}-\frac{1215\log(3x+2)}{1024}$$

```
[Out] -1/(256*x^4) + 3/(128*x^3) - 27/(256*x^2) + 135/(256*x) + 81/(512*(2 + 3*x)
^2) + 405/(512*(2 + 3*x)) + (1215*Log[x])/1024 - (1215*Log[2 + 3*x])/1024
```

Rubi [A] time = 0.0223792, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{27}{256x^2}+\frac{3}{128x^3}-\frac{1}{256x^4}+\frac{135}{256x}+\frac{405}{512(3x+2)}+\frac{81}{512(3x+2)^2}+\frac{1215\log(x)}{1024}-\frac{1215\log(3x+2)}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)^3),x]
```

```
[Out] -1/(256*x^4) + 3/(128*x^3) - 27/(256*x^2) + 135/(256*x) + 81/(512*(2 + 3*x)
^2) + 405/(512*(2 + 3*x)) + (1215*Log[x])/1024 - (1215*Log[2 + 3*x])/1024
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)^3}\,dx &= \int \left(\frac{1}{64x^5}-\frac{9}{128x^4}+\frac{27}{128x^3}-\frac{135}{256x^2}+\frac{1215}{1024x}-\frac{243}{256(2+3x)^3}-\frac{1215}{512(2+3x)^2}-\frac{3645}{1024(2+3x)}\right) d\\ &= -\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{81}{512(2+3x)^2}+\frac{405}{512(2+3x)}+\frac{1215\log(x)}{1024}-\frac{1215\log(2+}{1024}\end{aligned}$$

Mathematica [A] time = 0.032343, size = 54, normalized size = 0.81

$$\frac{\frac{2\left(3645x^5+3645x^4+540x^3-90x^2+24x-8\right)}{x^4(3x+2)^2}+1215\log(x)-1215\log(3x+2)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)^3),x]
```

```
[Out] ((2*(-8 + 24*x - 90*x^2 + 540*x^3 + 3645*x^4 + 3645*x^5))/(x^4*(2 + 3*x)^2)
 + 1215*Log[x] - 1215*Log[2 + 3*x])/1024
```

Maple [A] time = 0.01, size = 52, normalized size = 0.8

$$-\frac{1}{256\,x^{4}}+\frac{3}{128\,x^{3}}-\frac{27}{256\,x^{2}}+\frac{135}{256\,x}+\frac{81}{512\,\left(2+3\,x\right)^{2}}+\frac{405}{1024+1536\,x}+\frac{1215\,\ln\left(x\right)}{1024}-\frac{1215\,\ln\left(2+3\,x\right)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x)^3,x)
```

```
[Out] -1/256/x^4+3/128/x^3-27/256/x^2+135/256/x+81/512/(2+3*x)^2+405/512/(2+3*x)+
1215/1024*ln(x)-1215/1024*ln(2+3*x)
```

Maxima [A] time = 1.07264, size = 78, normalized size = 1.16

$$\frac{3645\,x^{5}+3645\,x^{4}+540\,x^{3}-90\,x^{2}+24\,x-8}{512\,\left(9\,x^{6}+12\,x^{5}+4\,x^{4}\right)}-\frac{1215}{1024}\,\log\left(3\,x+2\right)+\frac{1215}{1024}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/512*(3645*x^5 + 3645*x^4 + 540*x^3 - 90*x^2 + 24*x - 8)/(9*x^6 + 12*x^5 +
 4*x^4) - 1215/1024*log(3*x + 2) + 1215/1024*log(x)
```

Fricas [A] time = 1.55242, size = 232, normalized size = 3.46

$$\frac{7290\,x^{5}+7290\,x^{4}+1080\,x^{3}-180\,x^{2}-1215\,\left(9\,x^{6}+12\,x^{5}+4\,x^{4}\right)\log\left(3\,x+2\right)+1215\,\left(9\,x^{6}+12\,x^{5}+4\,x^{4}\right)\log\left(x\right)+48\,x}{1024\,\left(9\,x^{6}+12\,x^{5}+4\,x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(7290*x^5 + 7290*x^4 + 1080*x^3 - 180*x^2 - 1215*(9*x^6 + 12*x^5 + 4
*x^4)*log(3*x + 2) + 1215*(9*x^6 + 12*x^5 + 4*x^4)*log(x) + 48*x - 16)/(9*x
^6 + 12*x^5 + 4*x^4)
```

Sympy [A] time = 0.18851, size = 56, normalized size = 0.84

$$\frac{1215\log\left(x\right)}{1024}-\frac{1215\log\left(x+\frac{2}{3}\right)}{1024}+\frac{3645x^{5}+3645x^{4}+540x^{3}-90x^{2}+24x-8}{4608x^{6}+6144x^{5}+2048x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x)**3,x)
```

```
[Out] 1215*log(x)/1024 - 1215*log(x + 2/3)/1024 + (3645*x**5 + 3645*x**4 + 540*x*
*3 - 90*x**2 + 24*x - 8)/(4608*x**6 + 6144*x**5 + 2048*x**4)
```

Giac [A] time = 1.18215, size = 70, normalized size = 1.04

$$\frac{3645\,x^5+3645\,x^4+540\,x^3-90\,x^2+24\,x-8}{512\,(3\,x+2)^2x^4}-\frac{1215}{1024}\,\log\left(|3\,x+2|\right)+\frac{1215}{1024}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] 1/512*(3645*x^5 + 3645*x^4 + 540*x^3 - 90*x^2 + 24*x - 8)/((3*x + 2)^2*x^4)
 - 1215/1024*log(abs(3*x + 2)) + 1215/1024*log(abs(x))
```

3.271 $\int \frac{1}{2+2x}\,dx$

Optimal. Leaf size=8

$$\frac{1}{2}\log(x+1)$$

```
[Out] Log[1 + x]/2
```

Rubi [A] time = 0.0008358, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x)^(-1),x]
```

```
[Out] Log[1 + x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+2x}\,dx=\frac{1}{2}\log(1+x)$$

Mathematica [A] time = 0.0011513, size = 10, normalized size = 1.25

$$\frac{1}{2}\log(2x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x)^(-1),x]
```

```
[Out] Log[2 + 2*x]/2
```

Maple [A] time = 0., size = 9, normalized size = 1.1

$$\frac{\ln(2+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+2*x),x)
```

```
[Out] 1/2*ln(2+2*x)
```

Maxima [A] time = 1.0238, size = 8, normalized size = 1.

$$\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="maxima")
```

```
[Out] 1/2*log(x + 1)
```

Fricas [A] time = 1.53517, size = 22, normalized size = 2.75

$$\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 1)
```

Sympy [A] time = 0.066004, size = 7, normalized size = 0.88

$$\frac{\log(2x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x)
```

```
[Out] log(2*x + 2)/2
```

Giac [A] time = 1.16433, size = 9, normalized size = 1.12

$$\frac{1}{2}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x + 1))
```

3.272 $\int \frac{1}{4-6x}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{6}\log(2-3x)$$

```
[Out] -Log[2 - 3*x]/6
```

Rubi [A] time = 0.0009693, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$-\frac{1}{6}\log(2-3x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 6*x)^(-1),x]

[Out] -Log[2 - 3*x]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{4-6x}\,dx = -\frac{1}{6}\log(2-3x)$$

Mathematica [A] time = 0.0015938, size = 10, normalized size = 1.

$$-\frac{1}{6}\log(4-6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 6*x)^(-1),x]

[Out] -Log[4 - 6*x]/6
```

Maple [A] time = 0., size = 9, normalized size = 0.9

$$-\frac{\ln(4-6x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-6*x),x)
```

```
[Out] -1/6*ln(4-6*x)
```

Maxima [A] time = 1.02901, size = 11, normalized size = 1.1

$$-\frac{1}{6}\log(3x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="maxima")
```

```
[Out] -1/6*log(3*x - 2)
```

Fricas [A] time = 1.4907, size = 26, normalized size = 2.6

$$-\frac{1}{6}\log(3x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="fricas")
```

```
[Out] -1/6*log(3*x - 2)
```

Sympy [A] time = 0.086162, size = 8, normalized size = 0.8

$$-\frac{\log(6x-4)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x)
```

```
[Out] -log(6*x - 4)/6
```

Giac [A] time = 1.15047, size = 12, normalized size = 1.2

$$-\frac{1}{6}\log(|3x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="giac")
```

```
[Out] -1/6*log(abs(3*x - 2))
```

3.273 $\int \frac{1}{a+\sqrt{a}x}\,dx$

Optimal. Leaf size=14

$$\frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

```
[Out] Log[Sqrt[a] + x]/Sqrt[a]
```

Rubi [A] time = 0.0021261, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {31}

$$\frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + Sqrt[a]*x)^(-1),x]

[Out] Log[Sqrt[a] + x]/Sqrt[a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+\sqrt{a}x}\,dx = \frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.0039626, size = 16, normalized size = 1.14

$$\frac{\log\left(\sqrt{a}x+a\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + Sqrt[a]*x)^(-1),x]

[Out] Log[a + Sqrt[a]*x]/Sqrt[a]
```

Maple [A] time = 0.001, size = 13, normalized size = 0.9

$$\ln\left(a+x\sqrt{a}\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+x*a^(1/2)),x)
```

```
[Out] ln(a+x*a^(1/2))/a^(1/2)
```

Maxima [A] time = 1.08035, size = 16, normalized size = 1.14

$$\frac{\log\left(\sqrt{a}x + a\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(a)*x + a)/sqrt(a)
```

Fricas [A] time = 1.60401, size = 35, normalized size = 2.5

$$\frac{\log\left(x + \sqrt{a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="fricas")
```

```
[Out] log(x + sqrt(a))/sqrt(a)
```

Sympy [A] time = 0.068242, size = 14, normalized size = 1.

$$\frac{\log\left(\sqrt{a}x + a\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a**(1/2)),x)
```

```
[Out] log(sqrt(a)*x + a)/sqrt(a)
```

Giac [A] time = 1.20454, size = 18, normalized size = 1.29

$$\frac{\log\left(\left|\sqrt{a}x + a\right|\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(sqrt(a)*x + a))/sqrt(a)
```

3.274 $\int \frac{1}{a+\sqrt{-a}x}\,dx$

Optimal. Leaf size=20

$$\frac{\log\left(\sqrt{-a}x+a\right)}{\sqrt{-a}}$$

```
[Out] Log[a + Sqrt[-a]*x]/Sqrt[-a]
```

Rubi [A] time = 0.0074811, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {31}

$$\frac{\log\left(\sqrt{-a}x+a\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+\sqrt{-a}x}\,dx=\frac{\log\left(a+\sqrt{-a}x\right)}{\sqrt{-a}}$$

Mathematica [A] time = 0.0061207, size = 20, normalized size = 1.

$$\frac{\log\left(\sqrt{-a}x+a\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a + Sqrt[-a]*x]/Sqrt[-a]
```

Maple [A] time = 0.001, size = 17, normalized size = 0.9

$$\ln\left(a+x\sqrt{-a}\right)\frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+x*(-a)^(1/2)),x)

[Out] ln(a+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A] time = 1.06086, size = 22, normalized size = 1.1

$$\frac{\log\left(\sqrt{-a}x + a\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="maxima")

[Out] log(sqrt(-a)*x + a)/sqrt(-a)
```

Fricas [A] time = 1.42402, size = 42, normalized size = 2.1

$$-\frac{\sqrt{-a}\log\left(x - \sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="fricas")

[Out] -sqrt(-a)*log(x - sqrt(-a))/a
```

Sympy [A] time = 0.070181, size = 17, normalized size = 0.85

$$\frac{\log\left(a + x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)**(1/2)),x)

[Out] log(a + x*sqrt(-a))/sqrt(-a)
```

Giac [A] time = 1.17921, size = 23, normalized size = 1.15

$$\frac{\log\left(\left|\sqrt{-a}x + a\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="giac")

[Out] log(abs(sqrt(-a)*x + a))/sqrt(-a)
```

3.275 $\int \frac{1}{a^2+\sqrt{-a}x} \, dx$

Optimal. Leaf size=22

$$\frac{\log\left(a^2+\sqrt{-a}x\right)}{\sqrt{-a}}$$

```
[Out] Log[a^2 + Sqrt[-a]*x]/Sqrt[-a]
```

Rubi [A] time = 0.0023876, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log\left(a^2+\sqrt{-a}x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + Sqrt[-a]*x)^(-1),x]

[Out] Log[a^2 + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a^2+\sqrt{-a}x} \, dx = \frac{\log\left(a^2+\sqrt{-a}x\right)}{\sqrt{-a}}$$

Mathematica [A] time = 0.0036597, size = 22, normalized size = 1.

$$\frac{\log\left(a^2+\sqrt{-a}x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + Sqrt[-a]*x)^(-1),x]

[Out] Log[a^2 + Sqrt[-a]*x]/Sqrt[-a]
```

Maple [A] time = 0.001, size = 19, normalized size = 0.9

$$\ln\left(a^2+x\sqrt{-a}\right)\frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+x*(-a)^(1/2)),x)
```

```
[Out] ln(a^2+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A] time = 1.02064, size = 24, normalized size = 1.09

$$\frac{\log\left(a^2+\sqrt{-a}x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(a^2 + sqrt(-a)*x)/sqrt(-a)
```

Fricas [A] time = 1.47102, size = 46, normalized size = 2.09

$$-\frac{\sqrt{-a}\log\left(-\sqrt{-a}a+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(-sqrt(-a)*a + x)/a
```

Sympy [A] time = 0.09407, size = 19, normalized size = 0.86

$$\frac{\log\left(a^2+x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+x*(-a)**(1/2)),x)
```

```
[Out] log(a**2 + x*sqrt(-a))/sqrt(-a)
```

Giac [A] time = 1.20031, size = 26, normalized size = 1.18

$$\frac{\log\left(\left|a^2+\sqrt{-a}x\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(a^2 + sqrt(-a)*x))/sqrt(-a)
```

3.276 $\int \frac{1}{a^3+\sqrt{-a}x}\,dx$

Optimal. Leaf size=22

$$\frac{\log\left(a^3+\sqrt{-a}x\right)}{\sqrt{-a}}$$

```
[Out] Log[a^3 + Sqrt[-a]*x]/Sqrt[-a]
```

Rubi [A] time = 0.0023425, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log\left(a^3+\sqrt{-a}x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + Sqrt[-a]*x)^(-1),x]

[Out] Log[a^3 + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a^3+\sqrt{-a}x}\,dx=\frac{\log\left(a^3+\sqrt{-a}x\right)}{\sqrt{-a}}$$

Mathematica [A] time = 0.0093667, size = 22, normalized size = 1.

$$\frac{\log\left(a^3+\sqrt{-a}x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + Sqrt[-a]*x)^(-1),x]

[Out] Log[a^3 + Sqrt[-a]*x]/Sqrt[-a]
```

Maple [A] time = 0., size = 19, normalized size = 0.9

$$\ln\left(a^3+x\sqrt{-a}\right)\frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3+x*(-a)^(1/2)),x)
```

```
[Out] ln(a^3+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A] time = 1.02045, size = 24, normalized size = 1.09

$$\frac{\log\left(a^3 + \sqrt{-a}x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(a^3 + sqrt(-a)*x)/sqrt(-a)
```

Fricas [A] time = 1.38995, size = 49, normalized size = 2.23

$$-\frac{\sqrt{-a}\log\left(-\sqrt{-a}a^2 + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(-sqrt(-a)*a^2 + x)/a
```

Sympy [A] time = 0.118619, size = 19, normalized size = 0.86

$$\frac{\log\left(a^3 + x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**3+x*(-a)**(1/2)),x)
```

```
[Out] log(a**3 + x*sqrt(-a))/sqrt(-a)
```

Giac [A] time = 1.16913, size = 26, normalized size = 1.18

$$\frac{\log\left(\left|a^3 + \sqrt{-a}x\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(a^3 + sqrt(-a)*x))/sqrt(-a)
```

3.277 $\int \frac{1}{\frac{1}{a}+\sqrt{-a}x}\,dx$

Optimal. Leaf size=21

$$\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

```
[Out] Log[1 - (-a)^(3/2)*x]/Sqrt[-a]
```

Rubi [A] time = 0.0037768, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(-1) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[1 - (-a)^(3/2)*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\frac{1}{a}+\sqrt{-a}x}\,dx=\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

Mathematica [A] time = 0.0131803, size = 21, normalized size = 1.

$$\frac{\log\left(\sqrt{-a}ax+1\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(-1) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[1 + Sqrt[-a]*a*x]/Sqrt[-a]
```

Maple [A] time = 0., size = 19, normalized size = 0.9

$$\ln\left(a^{-1}+x\sqrt{-a}\right)\frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a+x*(-a)^(1/2)),x)
```

```
[Out] ln(1/a+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A] time = 1.08819, size = 24, normalized size = 1.14

$$\frac{\log\left(\sqrt{-a}x+\frac{1}{a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(-a)*x + 1/a)/sqrt(-a)
```

Fricas [A] time = 1.52163, size = 47, normalized size = 2.24

$$-\frac{\sqrt{-a}\log\left(a^2x-\sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(a^2*x - sqrt(-a))/a
```

Sympy [A] time = 0.092976, size = 19, normalized size = 0.9

$$\frac{\log\left(ax\sqrt{-a}+1\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)**(1/2)),x)
```

```
[Out] log(a*x*sqrt(-a) + 1)/sqrt(-a)
```

Giac [A] time = 1.22001, size = 26, normalized size = 1.24

$$\frac{\log\left(\left|\sqrt{-a}x+\frac{1}{a}\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(sqrt(-a)*x + 1/a))/sqrt(-a)
```

3.278 $\int \frac{1}{\frac{1}{a^2}+\sqrt{-a}x}\,dx$

Optimal. Leaf size=20

$$\frac{\log\left((-a)^{5/2}x+1\right)}{\sqrt{-a}}$$

```
[Out] Log[1 + (-a)^(5/2)*x]/Sqrt[-a]
```

Rubi [A] time = 0.0042515, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log\left((-a)^{5/2}x+1\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(-2) + Sqrt[-a]*x)^(-1),x]

[Out] Log[1 + (-a)^(5/2)*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\frac{1}{a^2}+\sqrt{-a}x}\,dx=\frac{\log\left(1+(-a)^{5/2}x\right)}{\sqrt{-a}}$$

Mathematica [A] time = 0.0183021, size = 22, normalized size = 1.1

$$\frac{\log\left(\frac{1}{a^2}+\sqrt{-a}x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(-2) + Sqrt[-a]*x)^(-1),x]

[Out] Log[a^(-2) + Sqrt[-a]*x]/Sqrt[-a]
```

Maple [A] time = 0.001, size = 19, normalized size = 1.

$$\ln\left(a^{-2}+x\sqrt{-a}\right)\frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a^2+x*(-a)^(1/2)),x)
```

```
[Out] ln(1/a^2+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A] time = 1.03575, size = 24, normalized size = 1.2

$$\frac{\log\left(\sqrt{-a}x + \frac{1}{a^2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(-a)*x + 1/a^2)/sqrt(-a)
```

Fricas [A] time = 1.45429, size = 47, normalized size = 2.35

$$-\frac{\sqrt{-a}\log\left(a^3x - \sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(a^3*x - sqrt(-a))/a
```

Sympy [A] time = 0.11012, size = 20, normalized size = 1.

$$\frac{\log\left(a^2x\sqrt{-a} + 1\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a**2+x*(-a)**(1/2)),x)
```

```
[Out] log(a**2*x*sqrt(-a) + 1)/sqrt(-a)
```

Giac [A] time = 1.2728, size = 26, normalized size = 1.3

$$\frac{\log\left(\left|\sqrt{-a}x + \frac{1}{a^2}\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(sqrt(-a)*x + 1/a^2))/sqrt(-a)
```

3.279 $\int \frac{1}{x(1+bx)}\,dx$

Optimal. Leaf size=11

$$\log(x) - \log(bx+1)$$

```
[Out] Log[x] - Log[1 + b*x]
```

Rubi [A] time = 0.0023277, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\log(x) - \log(bx+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + b*x)),x]
```

```
[Out] Log[x] - Log[1 + b*x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+bx)}\,dx &= -\left(b\int \frac{1}{1+bx}\,dx\right)+\int \frac{1}{x}\,dx\\ &= \log(x)-\log(1+bx)\end{aligned}$$

Mathematica [A] time = 0.0055922, size = 11, normalized size = 1.

$$\log(x) - \log(bx+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + b*x)),x]
```

```
[Out] Log[x] - Log[1 + b*x]
```

Maple [A] time = 0.007, size = 12, normalized size = 1.1

$$\ln(x) - \ln(bx + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+1),x)
```

```
[Out] ln(x)-ln(b*x+1)
```

Maxima [A] time = 1.0438, size = 15, normalized size = 1.36

$$-\log(bx + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="maxima")
```

```
[Out] -log(b*x + 1) + log(x)
```

Fricas [A] time = 1.46126, size = 32, normalized size = 2.91

$$-\log(bx + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="fricas")
```

```
[Out] -log(b*x + 1) + log(x)
```

Sympy [A] time = 0.197343, size = 8, normalized size = 0.73

$$\log(x) - \log\left(x + \frac{1}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x)
```

```
[Out] log(x) - log(x + 1/b)
```

Giac [A] time = 1.18662, size = 18, normalized size = 1.64

$$-\log(|bx + 1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="giac")
```

```
[Out] -log(abs(b*x + 1)) + log(abs(x))
```

3.280 $\int \frac{1}{x(-1+bx)} dx$

Optimal. Leaf size=12

$$\log(1-bx)-\log(x)$$

```
[Out] -Log[x] + Log[1 - b*x]
```

Rubi [A] time = 0.0023875, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\log(1-bx)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + b*x)),x]

[Out] -Log[x] + Log[1 - b*x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x(-1+bx)} dx &= b\int \frac{1}{-1+bx} dx - \int \frac{1}{x} dx \\ &= -\log(x)+\log(1-bx) \end{aligned}$$

Mathematica [A] time = 0.0028893, size = 12, normalized size = 1.

$$\log(1-bx)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + b*x)),x]

[Out] -Log[x] + Log[1 - b*x]
```

Maple [A] time = 0.006, size = 12, normalized size = 1.

$$\ln(bx-1)-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-1),x)
```

```
[Out] ln(b*x-1)-ln(x)
```

Maxima [A] time = 1.07908, size = 15, normalized size = 1.25

$$\log(bx-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="maxima")
```

```
[Out] log(b*x - 1) - log(x)
```

Fricas [A] time = 1.4917, size = 31, normalized size = 2.58

$$\log(bx-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="fricas")
```

```
[Out] log(b*x - 1) - log(x)
```

Sympy [A] time = 0.131059, size = 8, normalized size = 0.67

$$-\log(x)+\log\left(x-\frac{1}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x)
```

```
[Out] -log(x) + log(x - 1/b)
```

Giac [A] time = 1.13137, size = 18, normalized size = 1.5

$$\log(|bx-1|)-\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="giac")
```

```
[Out] log(abs(b*x - 1)) - log(abs(x))
```

3.281 $\int \frac{1}{x^2(1+bx)}\,dx$

Optimal. Leaf size=19

$$-b\log(x)+b\log(bx+1)-\frac{1}{x}$$

```
[Out] -x^(-1) - b*Log[x] + b*Log[1 + b*x]
```

Rubi [A] time = 0.0080161, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-b\log(x)+b\log(bx+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) - b*Log[x] + b*Log[1 + b*x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(1+bx)}\,dx &= \int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right)dx\\ &= -\frac{1}{x}-b\log(x)+b\log(1+bx)\end{aligned}$$

Mathematica [A] time = 0.0041456, size = 19, normalized size = 1.

$$-b\log(x)+b\log(bx+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) - b*Log[x] + b*Log[1 + b*x]
```

Maple [A] time = 0.007, size = 20, normalized size = 1.1

$$-x^{-1}-b\ln(x)+b\ln(bx+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+1),x)
```

```
[Out] -1/x-b*ln(x)+b*ln(b*x+1)
```

Maxima [A] time = 1.09871, size = 26, normalized size = 1.37

$$b\log\left(bx+1\right)-b\log\left(x\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="maxima")
```

```
[Out] b*log(b*x + 1) - b*log(x) - 1/x
```

Fricas [A] time = 1.50508, size = 53, normalized size = 2.79

$$\frac{bx\log\left(bx+1\right)-bx\log\left(x\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + 1) - b*x*log(x) - 1)/x
```

Sympy [A] time = 0.333195, size = 14, normalized size = 0.74

$$b\left(-\log\left(x\right)+\log\left(x+\frac{1}{b}\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+1),x)
```

```
[Out] b*(-log(x) + log(x + 1/b)) - 1/x
```

Giac [A] time = 1.17108, size = 28, normalized size = 1.47

$$b\log\left(|bx+1|\right)-b\log\left(|x|\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + 1)) - b*log(abs(x)) - 1/x
```

3.282 $\int \frac{1}{x^2(-1+bx)} dx$

Optimal. Leaf size=18

$$-b\log(x) + b\log(1 - bx) + \frac{1}{x}$$

```
[Out] x^(-1) - b*Log[x] + b*Log[1 - b*x]
```

Rubi [A] time = 0.0099948, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-b\log(x) + b\log(1 - bx) + \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-1 + b*x)),x]
```

```
[Out] x^(-1) - b*Log[x] + b*Log[1 - b*x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(-1+bx)} dx &= \int \left(-\frac{1}{x^2} - \frac{b}{x} + \frac{b^2}{-1+bx}\right) dx \\ &= \frac{1}{x} - b\log(x) + b\log(1 - bx)\end{aligned}$$

Mathematica [A] time = 0.0034923, size = 18, normalized size = 1.

$$-b\log(x) + b\log(1 - bx) + \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-1 + b*x)),x]
```

```
[Out] x^(-1) - b*Log[x] + b*Log[1 - b*x]
```

Maple [A] time = 0.008, size = 18, normalized size = 1.

$$b\ln(bx - 1) + x^{-1} - b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-1),x)
```

```
[Out] b*ln(b*x-1)+1/x-b*ln(x)
```

Maxima [A] time = 1.09852, size = 23, normalized size = 1.28

$$b\log\left(bx-1\right)-b\log\left(x\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x, algorithm="maxima")
```

```
[Out] b*log(b*x - 1) - b*log(x) + 1/x
```

Fricas [A] time = 1.52477, size = 53, normalized size = 2.94

$$\frac{bx\log\left(bx-1\right)-bx\log\left(x\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x - 1) - b*x*log(x) + 1)/x
```

Sympy [A] time = 0.165637, size = 14, normalized size = 0.78

$$b\left(-\log\left(x\right)+\log\left(x-\frac{1}{b}\right)\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-1),x)
```

```
[Out] b*(-log(x) + log(x - 1/b)) + 1/x
```

Giac [A] time = 1.23621, size = 26, normalized size = 1.44

$$b\log\left(|bx-1|\right)-b\log\left(|x|\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x - 1)) - b*log(abs(x)) + 1/x
```

3.283 $\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right) dx$

Optimal. Leaf size=14

$$b\log(bx+1)-\frac{1}{x}$$

```
[Out] -x^(-1) + b*Log[1 + b*x]
```

Rubi [A] time = 0.0094408, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {44}

$$b\log(bx+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[b/x + 1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) + b*Log[1 + b*x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right) dx &= b\log(x)+\int \frac{1}{x^2(1+bx)}\,dx\\ &= b\log(x)+\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right) dx\\ &= -\frac{1}{x}+b\log(1+bx)\end{aligned}$$

Mathematica [A] time = 0.0062296, size = 14, normalized size = 1.

$$b\log(bx+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[b/x + 1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) + b*Log[1 + b*x]
```

Maple [A] time = 0.002, size = 15, normalized size = 1.1

$$-x^{-1} + b\ln(bx + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b/x+1/x^2/(b*x+1),x)
```

```
[Out] -1/x+b*ln(b*x+1)
```

Maxima [A] time = 1.07829, size = 19, normalized size = 1.36

$$b\log(bx + 1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="maxima")
```

```
[Out] b*log(b*x + 1) - 1/x
```

Fricas [A] time = 1.50597, size = 35, normalized size = 2.5

$$\frac{bx\log(bx + 1) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + 1) - 1)/x
```

Sympy [A] time = 0.354426, size = 10, normalized size = 0.71

$$b\log(bx + 1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x**2/(b*x+1),x)
```

```
[Out] b*log(b*x + 1) - 1/x
```

Giac [A] time = 1.17364, size = 20, normalized size = 1.43

$$b\log(|bx + 1|) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + 1)) - 1/x
```

3.284 $\int x^3\sqrt{a+bx}\,dx$

Optimal. Leaf size=72

$$\frac{6a^2(a+bx)^{5/2}}{5b^4}-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{2(a+bx)^{9/2}}{9b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}$$

```
[Out] (-2*a^3*(a + b*x)^(3/2))/(3*b^4) + (6*a^2*(a + b*x)^(5/2))/(5*b^4) - (6*a*(
a + b*x)^(7/2))/(7*b^4) + (2*(a + b*x)^(9/2))/(9*b^4)
```

Rubi [A] time = 0.018334, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6a^2(a+bx)^{5/2}}{5b^4}-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{2(a+bx)^{9/2}}{9b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x],x]
```

```
[Out] (-2*a^3*(a + b*x)^(3/2))/(3*b^4) + (6*a^2*(a + b*x)^(5/2))/(5*b^4) - (6*a*(
a + b*x)^(7/2))/(7*b^4) + (2*(a + b*x)^(9/2))/(9*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a+bx}\,dx &= \int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx\\ &= -\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{6a^2(a+bx)^{5/2}}{5b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{9/2}}{9b^4}\end{aligned}$$

Mathematica [A] time = 0.0315592, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{3/2}\left(24a^2bx-16a^3-30ab^2x^2+35b^3x^3\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3))/(315
*b^4)
```

Maple [A] time = 0.004, size = 43, normalized size = 0.6

$$-\frac{-70\,b^{3}x^{3}+60\,a b^{2}x^{2}-48\,a^{2}b x+32\,a^{3}}{315\,b^{4}}\,(bx+a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(1/2),x)
```

```
[Out] -2/315*(b*x+a)^(3/2)*(-35*b^3*x^3+30*a*b^2*x^2-24*a^2*b*x+16*a^3)/b^4
```

Maxima [A] time = 1.09047, size = 76, normalized size = 1.06

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^{4}}-\frac{6\,(bx+a)^{\frac{7}{2}}a}{7\,b^{4}}+\frac{6\,(bx+a)^{\frac{5}{2}}a^{2}}{5\,b^{4}}-\frac{2\,(bx+a)^{\frac{3}{2}}a^{3}}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^4 - 6/7*(b*x + a)^(7/2)*a/b^4 + 6/5*(b*x + a)^(5/2)*a
^2/b^4 - 2/3*(b*x + a)^(3/2)*a^3/b^4
```

Fricas [A] time = 1.46788, size = 120, normalized size = 1.67

$$\frac{2\left(35\,b^{4}x^{4}+5\,a b^{3}x^{3}-6\,a^{2}b^{2}x^{2}+8\,a^{3}b x-16\,a^{4}\right)\sqrt{bx+a}}{315\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

Sympy [B] time = 3.26982, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(1/2),x)
```

```
[Out] -32*a**(49/2)*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a*
*18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b*
*9*x**5 + 315*a**14*b**10*x**6) + 32*a**(49/2)/(315*a**20*b**4 + 1890*a**19
*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**
4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) - 176*a**(47/2)*b*x*sqrt(1
 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300
*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*
b**10*x**6) + 192*a**(47/2)*b*x/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*
```

```
a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*
b**9*x**5 + 315*a**14*b**10*x**6) - 396*a**(45/2)*b**2*x**2*sqrt(1 + b*x/a)
/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b*
*7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**
6) + 480*a**(45/2)*b**2*x**2/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**
18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**
9*x**5 + 315*a**14*b**10*x**6) - 462*a**(43/2)*b**3*x**3*sqrt(1 + b*x/a)/(3
15*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*
x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6)
+ 640*a**(43/2)*b**3*x**3/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*
b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x
**5 + 315*a**14*b**10*x**6) - 210*a**(41/2)*b**4*x**4*sqrt(1 + b*x/a)/(315*
a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**
3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 4
80*a**(41/2)*b**4*x**4/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**
6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5
 + 315*a**14*b**10*x**6) + 378*a**(39/2)*b**5*x**5*sqrt(1 + b*x/a)/(315*a**
20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 +
 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 192*
a**(39/2)*b**5*x**5/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x
**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 +
315*a**14*b**10*x**6) + 1134*a**(37/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*a**20
*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4
725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 32*a**
(37/2)*b**6*x**6/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2
 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315
*a**14*b**10*x**6) + 1494*a**(35/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**20*b*
*4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725
*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 1098*a**(
33/2)*b**8*x**8*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*
a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*
b**9*x**5 + 315*a**14*b**10*x**6) + 430*a**(31/2)*b**9*x**9*sqrt(1 + b*x/a)
/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b*
*7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**
6) + 70*a**(29/2)*b**10*x**10*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*
b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4
 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6)
```

Giac [A] time = 1.18806, size = 66, normalized size = 0.92

$$\frac{2\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^2-105\,(bx+a)^{\frac{3}{2}}a^3\right)}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x + a)^(5/2)*a^2
 - 105*(b*x + a)^(3/2)*a^3)/b^4
```

3.285 $\int x^2\sqrt{a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a + b
*x)^(7/2))/(7*b^3)
```

Rubi [A] time = 0.0129626, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a + b
*x)^(7/2))/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx}\,dx &= \int \left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx\\ &= \frac{2a^2(a+bx)^{3/2}}{3b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{7/2}}{7b^3}\end{aligned}$$

Mathematica [A] time = 0.0246084, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{3/2}\left(8a^2-12abx+15b^2x^2\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3)
```

Maple [A] time = 0.005, size = 32, normalized size = 0.6

$$\frac{30\,b^2x^2-24\,abx+16\,a^2}{105\,b^3}\,(bx+a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(1/2),x)
```

```
[Out] 2/105*(b*x+a)^(3/2)*(15*b^2*x^2-12*a*b*x+8*a^2)/b^3
```

Maxima [A] time = 1.05255, size = 55, normalized size = 1.04

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^3}-\frac{4\,(bx+a)^{\frac{5}{2}}a}{5\,b^3}+\frac{2\,(bx+a)^{\frac{3}{2}}a^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^3 - 4/5*(b*x + a)^(5/2)*a/b^3 + 2/3*(b*x + a)^(3/2)*a
^2/b^3
```

Fricas [A] time = 1.53216, size = 97, normalized size = 1.83

$$\frac{2\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Sympy [B] time = 2.23526, size = 666, normalized size = 12.57

$$\frac{16a^{\frac{23}{2}}\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{16a^{\frac{23}{2}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{40a^{\frac{21}{2}}b}{105a^8b^3+315a^7b^4x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(23/2)*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b*
*5*x**2 + 105*a**5*b**6*x**3) - 16*a**(23/2)/(105*a**8*b**3 + 315*a**7*b**4
*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 40*a**(21/2)*b*x*sqrt(1 + b
*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6
*x**3) - 48*a**(21/2)*b*x/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*
x**2 + 105*a**5*b**6*x**3) + 30*a**(19/2)*b**2*x**2*sqrt(1 + b*x/a)/(105*a*
*8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 48*a
**(19/2)*b**2*x**2/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 +
```

```
105*a**5*b**6*x**3) + 40*a**(17/2)*b**3*x**3*sqrt(1 + b*x/a)/(105*a**8*b**3
 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 16*a**(17/2
)*b**3*x**3/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**
5*b**6*x**3) + 100*a**(15/2)*b**4*x**4*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315
*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 96*a**(13/2)*b**5
*x**5*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2
 + 105*a**5*b**6*x**3) + 30*a**(11/2)*b**6*x**6*sqrt(1 + b*x/a)/(105*a**8*b
**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3)
```

Giac [A] time = 1.17686, size = 50, normalized size = 0.94

$$\frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}\right)}{105\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)/
b^3
```

3.286 $\int x\sqrt{a+bx}\,dx$

Optimal. Leaf size=34

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

Rubi [A] time = 0.0077808, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x],x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx}\,dx&=\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx\\&=-\frac{2a(a+bx)^{3/2}}{3b^2}+\frac{2(a+bx)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.0218594, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{3/2}(3bx-2a)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(-2*a + 3*b*x))/(15*b^2)
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$-\frac{-6\,bx+4\,a}{15\,b^2}(bx+a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)^(3/2)*(-3*b*x+2*a)/b^2
```

Maxima [A] time = 1.03846, size = 35, normalized size = 1.03

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^2}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^2 - 2/3*(b*x + a)^(3/2)*a/b^2
```

Fricas [A] time = 1.5094, size = 70, normalized size = 2.06

$$\frac{2\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Sympy [B] time = 1.48306, size = 202, normalized size = 5.94

$$-\frac{4a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{9}{2}}}{15a^2b^2+15ab^3x}-\frac{2a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{7}{2}}bx}{15a^2b^2+15ab^3x}+\frac{8a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{6a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(9/2)*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(9/2)/(15*a
**2*b**2 + 15*a*b**3*x) - 2*a**(7/2)*b*x*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15
*a*b**3*x) + 4*a**(7/2)*b*x/(15*a**2*b**2 + 15*a*b**3*x) + 8*a**(5/2)*b**2*
x**2*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 6*a**(3/2)*b**3*x**3*sq
rt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x)
```

Giac [A] time = 1.17056, size = 34, normalized size = 1.

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/b^2
```

3.287 $\int \sqrt{a+bx}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{3/2}}{3b}$$

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Rubi [A] time = 0.0014043, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+bx}\,dx=\frac{2(a+bx)^{3/2}}{3b}$$

Mathematica [A] time = 0.0040628, size = 16, normalized size = 1.

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Maple [A] time = 0.003, size = 13, normalized size = 0.8

$$\frac{2}{3\,b}(bx+a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2),x)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b
```

Maxima [A] time = 1.05832, size = 16, normalized size = 1.

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Fricas [A] time = 1.59457, size = 31, normalized size = 1.94

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Sympy [A] time = 0.087623, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2),x)
```

```
[Out] 2*(a + b*x)**(3/2)/(3*b)
```

Giac [A] time = 1.17373, size = 16, normalized size = 1.

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

3.288 $\int \frac{\sqrt{a+bx}}{x}\,dx$

Optimal. Leaf size=35

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.0104539, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x}\,dx &= 2\sqrt{a+bx}+a\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= 2\sqrt{a+bx}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b}\\ &= 2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.0138259, size = 35, normalized size = 1.

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A] time = 0., size = 28, normalized size = 0.8

$$-2\operatorname{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x,x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65474, size = 188, normalized size = 5.37

$$\left[\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}, 2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+2\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2*
sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

Sympy [B] time = 1.94101, size = 68, normalized size = 1.94

$$-2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2a}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*a/(sqrt(b)*sqrt(x)*sqrt(a/(
b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(a/(b*x) + 1)
```

Giac [A] time = 1.28033, size = 43, normalized size = 1.23

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\,\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)
```

3.289 $\int \frac{\sqrt{a+bx}}{x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0106631, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = -\frac{\sqrt{a+bx}}{x} + \frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= -\frac{\sqrt{a+bx}}{x} + \text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)$$
$$= -\frac{\sqrt{a+bx}}{x} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.0363316, size = 47, normalized size = 1.21

$$-\frac{bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx}{x\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -((a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/(x*Sqrt[a +
b*x]))
```

Maple [A] time = 0.01, size = 37, normalized size = 1.

$$2\,b\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-1/2\,\frac{1}{\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^2,x)
```

```
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64647, size = 225, normalized size = 5.77

$$\left[\frac{\sqrt{a}bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\sqrt{bx+a}a}{2\,ax}, \frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{bx+a}a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x +
 a)*a)/(a*x)]
```

Sympy [A] time = 2.60729, size = 44, normalized size = 1.13

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}}-\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**2,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqr
t(a)
```

Giac [A] time = 1.21076, size = 55, normalized size = 1.41

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx+a}b}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)/b
```

3.290 $\int \frac{\sqrt{a+bx}}{x^3} dx$

Optimal. Leaf size=65

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{\sqrt{a+bx}}{2x^2} - \frac{b\sqrt{a+bx}}{4ax}$$

```
[Out] -Sqrt[a + b*x]/(2*x^2) - (b*Sqrt[a + b*x])/(4*a*x) + (b^2*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rubi [A] time = 0.0183526, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{\sqrt{a+bx}}{2x^2} - \frac{b\sqrt{a+bx}}{4ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^3,x]
```

```
[Out] -Sqrt[a + b*x]/(2*x^2) - (b*Sqrt[a + b*x])/(4*a*x) + (b^2*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx = -\frac{\sqrt{a+bx}}{2x^2} + \frac{1}{4}b\int \frac{1}{x^2\sqrt{a+bx}}\,dx$$
$$= -\frac{\sqrt{a+bx}}{2x^2} - \frac{b\sqrt{a+bx}}{4ax} - \frac{b^2\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a}$$
$$= -\frac{\sqrt{a+bx}}{2x^2} - \frac{b\sqrt{a+bx}}{4ax} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{4a}$$
$$= -\frac{\sqrt{a+bx}}{2x^2} - \frac{b\sqrt{a+bx}}{4ax} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

Mathematica [C] time = 0.0243131, size = 35, normalized size = 0.54

$$-\frac{2b^2(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^3,x]
```

```
[Out] (-2*b^2*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/(3*a^3
)
```

Maple [A] time = 0.008, size = 53, normalized size = 0.8

$$2\,b^2\left(\frac{1}{b^2x^2}\left(-1/8\,\frac{(bx+a)^{3/2}}{a} - 1/8\,\sqrt{bx+a}\right) + 1/8\,\frac{1}{a^{3/2}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^3,x)
```

```
[Out] 2*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/b^2/x^2+1/8*arctanh((b*x+a)
^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5508, size = 292, normalized size = 4.49

$$\left[\frac{\sqrt{a}b^2x^2\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(abx+2a^2\right)\sqrt{bx+a}}{8a^2x^2},-\frac{\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(abx+2a^2\right)\sqrt{bx+a}}{4a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b^2*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(a*b
*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2), -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*
x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2)]
```

Sympy [A] time = 4.86688, size = 97, normalized size = 1.49

$$-\frac{a}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{b^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**3,x)
```

```
[Out] -a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(4*x**(3/2)*sqrt(a/(b
*x) + 1)) - b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) + 1)) + b**2*asinh(sqrt(a)/(
sqrt(b)*sqrt(x)))/(4*a**(3/2))
```

Giac [A] time = 1.1688, size = 89, normalized size = 1.37

$$-\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{(bx+a)^{\frac{3}{2}}b^3+\sqrt{bx+a}ab^3}{ab^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x + a)^(3/2)*b^
3 + sqrt(b*x + a)*a*b^3)/(a*b^2*x^2))/b
```

3.291 $\int \frac{\sqrt{a+bx}}{x^4}\,dx$

Optimal. Leaf size=87

$$\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}-\frac{b\sqrt{a+bx}}{12ax^2}-\frac{\sqrt{a+bx}}{3x^3}$$

```
[Out] -Sqrt[a + b*x]/(3*x^3) - (b*Sqrt[a + b*x])/(12*a*x^2) + (b^2*Sqrt[a + b*x])
/(8*a^2*x) - (b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(5/2))
```

Rubi [A] time = 0.0269141, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}-\frac{b\sqrt{a+bx}}{12ax^2}-\frac{\sqrt{a+bx}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^4,x]
```

```
[Out] -Sqrt[a + b*x]/(3*x^3) - (b*Sqrt[a + b*x])/(12*a*x^2) + (b^2*Sqrt[a + b*x])
/(8*a^2*x) - (b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx}}{x^4}\,dx &= -\frac{\sqrt{a+bx}}{3x^3}+\frac{1}{6}b\int \frac{1}{x^3\sqrt{a+bx}}\,dx\\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}-\frac{b^2\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{8a}\\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}+\frac{b^3\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a^2}\\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{8a^2}\\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0104371, size = 35, normalized size = 0.4

$$\frac{2b^3(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^4,x]
```

```
[Out] (2*b^3*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (b*x)/a])/(3*a^4)
```

Maple [A] time = 0.01, size = 65, normalized size = 0.8

$$2\,b^3\left(\frac{1}{b^3x^3}\left(1/16\,\frac{(bx+a)^{5/2}}{a^2}-1/6\,\frac{(bx+a)^{3/2}}{a}-1/16\,\sqrt{bx+a}\right)-1/16\,\frac{1}{a^{5/2}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^4,x)
```

```
[Out] 2*b^3*((1/16/a^2*(b*x+a)^(5/2)-1/6/a*(b*x+a)^(3/2)-1/16*(b*x+a)^(1/2))/b^3/
x^3-1/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45495, size = 347, normalized size = 3.99

$$\left[\frac{3\,\sqrt{a}b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(3\,ab^2x^2-2\,a^2bx-8\,a^3\right)\sqrt{bx+a}}{48\,a^3x^3},\frac{3\,\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3\,ab^2x^2-2\,a^2bx\right.}{24\,a^3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x + a))/(a^3*x^3), 1/24*(3*sqrt(-a)
*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 - 2*a^2*b*x - 8*a^
3)*sqrt(b*x + a))/(a^3*x^3)]
```

Sympy [A] time = 7.4923, size = 122, normalized size = 1.4

$$-\frac{a}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{b^{\frac{3}{2}}}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{b^{\frac{5}{2}}}{8a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**4,x)
```

```
[Out] -a/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 5*sqrt(b)/(12*x**(5/2)*sqrt(a/(
b*x) + 1)) + b**(3/2)/(24*a*x**(3/2)*sqrt(a/(b*x) + 1)) + b**(5/2)/(8*a**2*
sqrt(x)*sqrt(a/(b*x) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*a**(5
/2))
```

Giac [A] time = 1.22449, size = 113, normalized size = 1.3

$$\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\,(bx+a)^{\frac{5}{2}}b^4-8\,(bx+a)^{\frac{3}{2}}ab^4-3\,\sqrt{bx+a}a^2b^4}{a^2b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(5
/2)*b^4 - 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*a^2*b^4)/(a^2*b^3*x^3))
/b
```

3.292 $\int x^3(a+bx)^{3/2}\,dx$

Optimal. Leaf size=72

$$\frac{6a^2(a+bx)^{7/2}}{7b^4}-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{11/2}}{11b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}$$

```
[Out] (-2*a^3*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a + b*x)^(7/2))/(7*b^4) - (2*a*(
a + b*x)^(9/2))/(3*b^4) + (2*(a + b*x)^(11/2))/(11*b^4)
```

Rubi [A] time = 0.0188042, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6a^2(a+bx)^{7/2}}{7b^4}-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{11/2}}{11b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a + b*x)^(7/2))/(7*b^4) - (2*a*(
a + b*x)^(9/2))/(3*b^4) + (2*(a + b*x)^(11/2))/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{3/2}\,dx&=\int\left(-\frac{a^3(a+bx)^{3/2}}{b^3}+\frac{3a^2(a+bx)^{5/2}}{b^3}-\frac{3a(a+bx)^{7/2}}{b^3}+\frac{(a+bx)^{9/2}}{b^3}\right)dx\\&=-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{6a^2(a+bx)^{7/2}}{7b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}+\frac{2(a+bx)^{11/2}}{11b^4}\end{aligned}$$

Mathematica [A] time = 0.0390633, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{5/2}\left(40a^2bx-16a^3-70ab^2x^2+105b^3x^3\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(11
55*b^4)
```

Maple [A] time = 0.004, size = 43, normalized size = 0.6

$$-\frac{-210\,b^3x^3+140\,ab^2x^2-80\,a^2bx+32\,a^3}{1155\,b^4}(bx+a)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(3/2),x)
```

```
[Out] -2/1155*(b*x+a)^(5/2)*(-105*b^3*x^3+70*a*b^2*x^2-40*a^2*b*x+16*a^3)/b^4
```

Maxima [A] time = 1.07472, size = 76, normalized size = 1.06

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b^4}-\frac{2\,(bx+a)^{\frac{9}{2}}a}{3\,b^4}+\frac{6\,(bx+a)^{\frac{7}{2}}a^2}{7\,b^4}-\frac{2\,(bx+a)^{\frac{5}{2}}a^3}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b*x + a)^(11/2)/b^4 - 2/3*(b*x + a)^(9/2)*a/b^4 + 6/7*(b*x + a)^(7/2)
*a^2/b^4 - 2/5*(b*x + a)^(5/2)*a^3/b^4
```

Fricas [A] time = 1.50403, size = 147, normalized size = 2.04

$$\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx+a}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

Sympy [B] time = 3.73798, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(3/2),x)
```

```
[Out] -32*a**(51/2)*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*
a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**1
5*b**9*x**5 + 1155*a**14*b**10*x**6) + 32*a**(51/2)/(1155*a**20*b**4 + 6930
*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16
*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 176*a**(49/2)*
b*x*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6
*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**
5 + 1155*a**14*b**10*x**6) + 192*a**(49/2)*b*x/(1155*a**20*b**4 + 6930*a**1
```

```
9*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8
*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 396*a**(47/2)*b**2*
x**2*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**
6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x*
*5 + 1155*a**14*b**10*x**6) + 480*a**(47/2)*b**2*x**2/(1155*a**20*b**4 + 69
30*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**
16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 462*a**(45/2
)*b**3*x**3*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a*
*18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*
b**9*x**5 + 1155*a**14*b**10*x**6) + 640*a**(45/2)*b**3*x**3/(1155*a**20*b*
*4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17
325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 480*a
**(43/2)*b**4*x**4/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*
x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5
 + 1155*a**14*b**10*x**6) + 1848*a**(41/2)*b**5*x**5*sqrt(1 + b*x/a)/(1155*
a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x
**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6)
 + 192*a**(41/2)*b**5*x**5/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**
18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b
**9*x**5 + 1155*a**14*b**10*x**6) + 5544*a**(39/2)*b**6*x**6*sqrt(1 + b*x/a
)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**1
7*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**
10*x**6) + 32*a**(39/2)*b**6*x**6/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17
325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*
a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 8844*a**(37/2)*b**7*x**7*sqrt(1
+ b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 231
00*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a*
*14*b**10*x**6) + 8448*a**(35/2)*b**8*x**8*sqrt(1 + b*x/a)/(1155*a**20*b**4
 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 1732
5*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 4840*a*
*(33/2)*b**9*x**9*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17
325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*
a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 1540*a**(31/2)*b**10*x**10*sqrt(
1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 2
3100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*
a**14*b**10*x**6) + 210*a**(29/2)*b**11*x**11*sqrt(1 + b*x/a)/(1155*a**20*b
**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 1
7325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6)
```

Giac [B] time = 1.17763, size = 157, normalized size = 2.18

$$\frac{2\left(\frac{11\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^{2}-105\,(bx+a)^{\frac{3}{2}}a^{3}\right)a}{b^{3}}+\frac{315\,(bx+a)^{\frac{11}{2}}-1540\,(bx+a)^{\frac{9}{2}}a+2970\,(bx+a)^{\frac{7}{2}}a^{2}-2772\,(bx+a)^{\frac{5}{2}}a^{3}+1155\,(bx+a)^{\frac{3}{2}}a^{4}}{b^{3}}\right)}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3465*(11*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x + a)^(5/2
)*a^2 - 105*(b*x + a)^(3/2)*a^3)*a/b^3 + (315*(b*x + a)^(11/2) - 1540*(b*x
+ a)^(9/2)*a + 2970*(b*x + a)^(7/2)*a^2 - 2772*(b*x + a)^(5/2)*a^3 + 1155*(
b*x + a)^(3/2)*a^4)/b^3)/b
```

3.293 $\int x^2(a+bx)^{3/2}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{9/2}}{9b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}$$

```
[Out] (2*a^2*(a + b*x)^(5/2))/(5*b^3) - (4*a*(a + b*x)^(7/2))/(7*b^3) + (2*(a + b
*x)^(9/2))/(9*b^3)
```

Rubi [A] time = 0.0128408, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{9/2}}{9b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(3/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(5/2))/(5*b^3) - (4*a*(a + b*x)^(7/2))/(7*b^3) + (2*(a + b
*x)^(9/2))/(9*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{3/2}\,dx &= \int\left(\frac{a^2(a+bx)^{3/2}}{b^2}-\frac{2a(a+bx)^{5/2}}{b^2}+\frac{(a+bx)^{7/2}}{b^2}\right)dx\\ &= \frac{2a^2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{9/2}}{9b^3}\end{aligned}$$

Mathematica [A] time = 0.025302, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{5/2}\left(8a^2-20abx+35b^2x^2\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3)
```

Maple [A] time = 0.005, size = 32, normalized size = 0.6

$$\frac{70 b^2 x^2 - 40 abx + 16 a^2}{315 b^3} (bx + a)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(3/2),x)
```

```
[Out] 2/315*(b*x+a)^(5/2)*(35*b^2*x^2-20*a*b*x+8*a^2)/b^3
```

Maxima [A] time = 1.07241, size = 55, normalized size = 1.04

$$\frac{2 (bx + a)^{\frac{9}{2}}}{9 b^3} - \frac{4 (bx + a)^{\frac{7}{2}} a}{7 b^3} + \frac{2 (bx + a)^{\frac{5}{2}} a^2}{5 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^3 - 4/7*(b*x + a)^(7/2)*a/b^3 + 2/5*(b*x + a)^(5/2)*a
^2/b^3
```

Fricas [A] time = 1.51169, size = 120, normalized size = 2.26

$$\frac{2 \left(35 b^4 x^4 + 50 ab^3 x^3 + 3 a^2 b^2 x^2 - 4 a^3 bx + 8 a^4\right) \sqrt{bx + a}}{315 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x + a)/b^3
```

Sympy [B] time = 2.51985, size = 733, normalized size = 13.83

$$\frac{16a^{\frac{25}{2}} \sqrt{1 + \frac{bx}{a}}}{315a^8b^3 + 945a^7b^4x + 945a^6b^5x^2 + 315a^5b^6x^3} - \frac{16a^{\frac{25}{2}}}{315a^8b^3 + 945a^7b^4x + 945a^6b^5x^2 + 315a^5b^6x^3} + \frac{40a}{315a^8b^3 + 945a^7b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(3/2),x)
```

```
[Out] 16*a**(25/2)*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b*
*5*x**2 + 315*a**5*b**6*x**3) - 16*a**(25/2)/(315*a**8*b**3 + 945*a**7*b**4
*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 40*a**(23/2)*b*x*sqrt(1 + b
*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6
*x**3) - 48*a**(23/2)*b*x/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*
x**2 + 315*a**5*b**6*x**3) + 30*a**(21/2)*b**2*x**2*sqrt(1 + b*x/a)/(315*a*
*8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) - 48*a
```

```
**(21/2)*b**2*x**2/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 +
315*a**5*b**6*x**3) + 110*a**(19/2)*b**3*x**3*sqrt(1 + b*x/a)/(315*a**8*b**
3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) - 16*a**(19/
2)*b**3*x**3/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a*
*5*b**6*x**3) + 380*a**(17/2)*b**4*x**4*sqrt(1 + b*x/a)/(315*a**8*b**3 + 94
5*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 516*a**(15/2)*b*
*5*x**5*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x*
*2 + 315*a**5*b**6*x**3) + 310*a**(13/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*a**
8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 70*a*
*(11/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a*
*6*b**5*x**2 + 315*a**5*b**6*x**3)
```

<u>Giac</u> [B] time = 1.17771, size = 124, normalized size = 2.34

$$\frac{2\left(\frac{3\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)a}{b^2}+\frac{35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^2-105\,(bx+a)^{\frac{3}{2}}a^3}{b^2}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/315*(3*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^
2)*a/b^2 + (35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x + a)^(5/2
)*a^2 - 105*(b*x + a)^(3/2)*a^3)/b^2)/b
```

3.294 $\int x(a+bx)^{3/2}\,dx$

Optimal. Leaf size=34

$$\frac{2(a+bx)^{7/2}}{7b^2}-\frac{2a(a+bx)^{5/2}}{5b^2}$$

```
[Out] (-2*a*(a + b*x)^(5/2))/(5*b^2) + (2*(a + b*x)^(7/2))/(7*b^2)
```

Rubi [A] time = 0.0077795, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{7/2}}{7b^2}-\frac{2a(a+bx)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(3/2),x]
```

```
[Out] (-2*a*(a + b*x)^(5/2))/(5*b^2) + (2*(a + b*x)^(7/2))/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{3/2}\,dx &= \int\left(-\frac{a(a+bx)^{3/2}}{b}+\frac{(a+bx)^{5/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{5/2}}{5b^2}+\frac{2(a+bx)^{7/2}}{7b^2}\end{aligned}$$

Mathematica [A] time = 0.0248003, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{5/2}(5bx-2a)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(-2*a + 5*b*x))/(35*b^2)
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$-\frac{-10\,bx+4\,a}{35\,b^2}(bx+a)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(3/2),x)
```

```
[Out] -2/35*(b*x+a)^(5/2)*(-5*b*x+2*a)/b^2
```

Maxima [A] time = 1.0367, size = 35, normalized size = 1.03

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^2}-\frac{2\,(bx+a)^{\frac{5}{2}}a}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^2 - 2/5*(b*x + a)^(5/2)*a/b^2
```

Fricas [A] time = 1.55714, size = 92, normalized size = 2.71

$$\frac{2\left(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3\right)\sqrt{bx+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

Sympy [A] time = 0.843876, size = 80, normalized size = 2.35

$$\begin{cases}-\frac{4a^3\sqrt{a+bx}}{35b^2}+\frac{2a^2x\sqrt{a+bx}}{35b}+\frac{16ax^2\sqrt{a+bx}}{35}+\frac{2bx^3\sqrt{a+bx}}{7} & \text{for } b\neq 0\\ \frac{a^{\frac{3}{2}}x^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-4*a**3*sqrt(a + b*x)/(35*b**2) + 2*a**2*x*sqrt(a + b*x)/(35*b)
+ 16*a*x**2*sqrt(a + b*x)/35 + 2*b*x**3*sqrt(a + b*x)/7, Ne(b, 0)), (a**(3/
2)*x**2/2, True))
```

Giac [B] time = 1.17375, size = 92, normalized size = 2.71

$$\frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)a}{b}+\frac{15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2}{b}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)*a/b + (15*(b*x + a)^(7/2
) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)/b)/b
```

3.295 $\int (a+bx)^{3/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{5/2}}{5b}$$

```
[Out] (2*(a + b*x)^(5/2))/(5*b)
```

Rubi [A] time = 0.0013847, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{3/2}\,dx = \frac{2(a+bx)^{5/2}}{5b}$$

Mathematica [A] time = 0.0101099, size = 16, normalized size = 1.

$$\frac{2(a+bx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b)
```

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$\frac{2}{5\,b}(bx+a)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2),x)
```

```
[Out] 2/5*(b*x+a)^(5/2)/b
```

Maxima [A] time = 1.06051, size = 16, normalized size = 1.

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b
```

Fricas [B] time = 1.45074, size = 63, normalized size = 3.94

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

Sympy [A] time = 0.069356, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2),x)
```

```
[Out] 2*(a + b*x)**(5/2)/(5*b)
```

Giac [A] time = 1.20759, size = 16, normalized size = 1.

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b
```

3.296 $\int \frac{(a+bx)^{3/2}}{x} dx$

Optimal. Leaf size=49

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}$$

```
[Out] 2*a*Sqrt[a + b*x] + (2*(a + b*x)^(3/2))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*x]
/Sqrt[a]]
```

Rubi [A] time = 0.0144803, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x,x]
```

```
[Out] 2*a*Sqrt[a + b*x] + (2*(a + b*x)^(3/2))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*x]
/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{x}\,dx = \frac{2}{3}(a+bx)^{3/2} + a\int \frac{\sqrt{a+bx}}{x}\,dx$$
$$= 2a\sqrt{a+bx} + \frac{2}{3}(a+bx)^{3/2} + a^2\int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= 2a\sqrt{a+bx} + \frac{2}{3}(a+bx)^{3/2} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= 2a\sqrt{a+bx} + \frac{2}{3}(a+bx)^{3/2} - 2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Mathematica [A] time = 0.0420943, size = 44, normalized size = 0.9

$$\frac{2}{3}\sqrt{a+bx}(4a+bx) - 2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(4*a + b*x))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A] time = 0.005, size = 38, normalized size = 0.8

$$\frac{2}{3}(bx+a)^{\frac{3}{2}} - 2a^{3/2}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) + 2a\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x,x)
```

```
[Out] 2/3*(b*x+a)^(3/2)-2*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a*(b*x+a)^(1/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64594, size = 228, normalized size = 4.65

$$\left[a^{\frac{3}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) + \frac{2}{3}(bx+4a)\sqrt{bx+a}, 2\sqrt{-a}a\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \frac{2}{3}(bx+4a)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(3/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/3*(b*x + 4*a)*sqr
t(b*x + a), 2*sqrt(-a)*a*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/3*(b*x + 4*a)
*sqrt(b*x + a)]
```

Sympy [A] time = 2.72374, size = 71, normalized size = 1.45

$$\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{3}+a^{\frac{3}{2}}\log\left(\frac{bx}{a}\right)-2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{2\sqrt{a}bx\sqrt{1+\frac{bx}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x,x)
```

```
[Out] 8*a**(3/2)*sqrt(1 + b*x/a)/3 + a**(3/2)*log(b*x/a) - 2*a**(3/2)*log(sqrt(1
+ b*x/a) + 1) + 2*sqrt(a)*b*x*sqrt(1 + b*x/a)/3
```

Giac [A] time = 1.13304, size = 59, normalized size = 1.2

$$\frac{2\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}+2\,\sqrt{bx+a}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/3*(b*x + a)^(3/2) + 2*sqr
t(b*x + a)*a
```

3.297 $\int \frac{(a+bx)^{3/2}}{x^2} dx$

Optimal. Leaf size=51

$$-\frac{(a+bx)^{3/2}}{x}+3b\sqrt{a+bx}-3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 3*b*Sqrt[a + b*x] - (a + b*x)^(3/2)/x - 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/S
qrt[a]]
```

Rubi [A] time = 0.0147685, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$-\frac{(a+bx)^{3/2}}{x}+3b\sqrt{a+bx}-3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^2,x]
```

```
[Out] 3*b*Sqrt[a + b*x] - (a + b*x)^(3/2)/x - 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/S
qrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x^2}\,dx &= -\frac{(a+bx)^{3/2}}{x}+\frac{1}{2}(3b)\int \frac{\sqrt{a+bx}}{x}\,dx\\
&= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}+\frac{1}{2}(3ab)\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}+(3a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\
&= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}-3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0118689, size = 33, normalized size = 0.65

$$\frac{2b(a+bx)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx}{a}+1\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^2,x]
```

```
[Out] (2*b*(a + b*x)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x)/a])/(5*a^2)
```

Maple [A] time = 0.009, size = 47, normalized size = 0.9

$$2\,b\left(\sqrt{bx+a}+a\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-3/2\,\frac{1}{\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^2,x)
```

```
[Out] 2*b*((b*x+a)^(1/2)+a*(-1/2*(b*x+a)^(1/2)/b/x-3/2*arctanh((b*x+a)^(1/2)/a^(1
/2))/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66102, size = 247, normalized size = 4.84

$$\left[\frac{3\sqrt{a}bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2(2bx-a)\sqrt{bx+a}}{2x},\ \frac{3\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(2bx-a)\sqrt{bx+a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b*x
 - a)*sqrt(b*x + a))/x, (3*sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) +
(2*b*x - a)*sqrt(b*x + a))/x]
```

Sympy [B] time = 3.38634, size = 92, normalized size = 1.8

$$-3\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)-\frac{a^2}{\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{a\sqrt{b}}{\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{3}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**2,x)
```

```
[Out] -3*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) - a**2/(sqrt(b)*x**(3/2)*sqrt
(a/(b*x) + 1)) + a*sqrt(b)/(sqrt(x)*sqrt(a/(b*x) + 1)) + 2*b**(3/2)*sqrt(x)
/sqrt(a/(b*x) + 1)
```

Giac [A] time = 1.21026, size = 76, normalized size = 1.49

$$\frac{\frac{3ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{bx+a}b^2-\frac{\sqrt{bx+a}ab}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] (3*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)*b^2 - sq
rt(b*x + a)*a*b/x)/b
```

3.298 $\int \frac{(a+bx)^{3/2}}{x^3} dx$

Optimal. Leaf size=62

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{(a+bx)^{3/2}}{2x^2}-\frac{3b\sqrt{a+bx}}{4x}$$

```
[Out] (-3*b*Sqrt[a + b*x])/(4*x) - (a + b*x)^(3/2)/(2*x^2) - (3*b^2*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(4*Sqrt[a])
```

Rubi [A] time = 0.0160989, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{(a+bx)^{3/2}}{2x^2}-\frac{3b\sqrt{a+bx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^3,x]
```

```
[Out] (-3*b*Sqrt[a + b*x])/(4*x) - (a + b*x)^(3/2)/(2*x^2) - (3*b^2*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{x^3}dx = -\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\int \frac{\sqrt{a+bx}}{x^2}dx$$
$$= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{x\sqrt{a+bx}}dx$$
$$= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx,x,\sqrt{a+bx}\right)$$
$$= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

Mathematica [A] time = 0.0531582, size = 68, normalized size = 1.1

$$-\frac{2a^2+3b^2x^2\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+7abx+5b^2x^2}{4x^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^3,x]
```

```
[Out] -(2*a^2 + 7*a*b*x + 5*b^2*x^2 + 3*b^2*x^2*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1
+ (b*x)/a]])/(4*x^2*Sqrt[a + b*x])
```

Maple [A] time = 0.008, size = 51, normalized size = 0.8

$$2\,b^2\left(\frac{-5/8\,(bx+a)^{3/2}+3/8\,a\sqrt{bx+a}}{b^2x^2}-3/8\,\frac{1}{\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^3,x)
```

```
[Out] 2*b^2*((-5/8*(b*x+a)^(3/2)+3/8*a*(b*x+a)^(1/2))/b^2/x^2-3/8*arctanh((b*x+a)
^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55413, size = 296, normalized size = 4.77

$$\left[\frac{3\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(5abx+2a^2\right)\sqrt{bx+a}}{8ax^2}, \frac{3\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\left(5abx+2a^2\right)\sqrt{bx+a}}{4ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(5
*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sqrt
(b*x + a)*sqrt(-a)/a) - (5*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2)]
```

Sympy [A] time = 4.1397, size = 76, normalized size = 1.23

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx}+1}}{2x^{\frac{3}{2}}}-\frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{4\sqrt{x}}-\frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**3,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x) + 1)/(2*x**(3/2)) - 5*b**(3/2)*sqrt(a/(b*x) + 1)/(4
*sqrt(x)) - 3*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a))
```

Giac [A] time = 1.23718, size = 86, normalized size = 1.39

$$\frac{\frac{3b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5(bx+a)^{\frac{3}{2}}b^3-3\sqrt{bx+a}ab^3}{b^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (5*(b*x + a)^(3/2)*b^3
 - 3*sqrt(b*x + a)*a*b^3)/(b^2*x^2))/b
```

3.299 $\int \frac{(a+bx)^{3/2}}{x^4} dx$

Optimal. Leaf size=84

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}} - \frac{b^2\sqrt{a+bx}}{8ax} - \frac{b\sqrt{a+bx}}{4x^2} - \frac{(a+bx)^{3/2}}{3x^3}$$

```
[Out] -(b*Sqrt[a + b*x])/(4*x^2) - (b^2*Sqrt[a + b*x])/(8*a*x) - (a + b*x)^(3/2)/
(3*x^3) + (b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(3/2))
```

Rubi [A] time = 0.0222849, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}} - \frac{b^2\sqrt{a+bx}}{8ax} - \frac{b\sqrt{a+bx}}{4x^2} - \frac{(a+bx)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^4,x]
```

```
[Out] -(b*Sqrt[a + b*x])/(4*x^2) - (b^2*Sqrt[a + b*x])/(8*a*x) - (a + b*x)^(3/2)/
(3*x^3) + (b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x^4}\,dx &= -\frac{(a+bx)^{3/2}}{3x^3}+\frac{1}{2}b\int \frac{\sqrt{a+bx}}{x^3}\,dx\\
&= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{(a+bx)^{3/2}}{3x^3}+\frac{1}{8}b^2\int \frac{1}{x^2\sqrt{a+bx}}\,dx\\
&= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}-\frac{b^3\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a}\\
&= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}-\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{8a}\\
&= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0136337, size = 35, normalized size = 0.42

$$\frac{2b^3(a+bx)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{bx}{a}+1\right)}{5a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^4,x]
```

```
[Out] (2*b^3*(a + b*x)^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x)/a])/(5*a^4)
```

Maple [A] time = 0.01, size = 63, normalized size = 0.8

$$2\,b^3\left(\frac{1}{b^3x^3}\left(-1/16\,\frac{(bx+a)^{5/2}}{a}-1/6\,(bx+a)^{3/2}+1/16\,a\sqrt{bx+a}\right)+1/16\,\frac{1}{a^{3/2}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^4,x)
```

```
[Out] 2*b^3*((-1/16/a*(b*x+a)^(5/2)-1/6*(b*x+a)^(3/2)+1/16*a*(b*x+a)^(1/2))/b^3/x
^3+1/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52887, size = 351, normalized size = 4.18

$$\left[\frac{3\sqrt{a}b^3x^3\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(3ab^2x^2+14a^2bx+8a^3\right)\sqrt{bx+a}}{48a^2x^3}, -\frac{3\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3ab^2x^2+\right.}{24a^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(
3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^2*x^3), -1/24*(3*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 + 14*a^2*b*x + 8
*a^3)*sqrt(b*x + a))/(a^2*x^3)]
```

Sympy [A] time = 7.22523, size = 124, normalized size = 1.48

$$-\frac{a^2}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{11a\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{17b^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{b^{\frac{5}{2}}}{8a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**4,x)
```

```
[Out] -a**2/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 11*a*sqrt(b)/(12*x**(5/2)*sq
rt(a/(b*x) + 1)) - 17*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) - b**(5/2)/(
8*a*sqrt(x)*sqrt(a/(b*x) + 1)) + b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*a
**(3/2))
```

Giac [A] time = 1.19848, size = 113, normalized size = 1.35

$$-\frac{\frac{3b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{3(bx+a)^{\frac{5}{2}}b^4+8(bx+a)^{\frac{3}{2}}ab^4-3\sqrt{bx+a}a^2b^4}{ab^3x^3}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (3*(b*x + a)^(5/
2)*b^4 + 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*a^2*b^4)/(a*b^3*x^3))/b
```

3.300 $\int x^3(a+bx)^{5/2}\,dx$

Optimal. Leaf size=72

$$\frac{2a^2(a+bx)^{9/2}}{3b^4}-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{13/2}}{13b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}$$

```
[Out] (-2*a^3*(a + b*x)^(7/2))/(7*b^4) + (2*a^2*(a + b*x)^(9/2))/(3*b^4) - (6*a*(
a + b*x)^(11/2))/(11*b^4) + (2*(a + b*x)^(13/2))/(13*b^4)
```

Rubi [A] time = 0.0171271, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{9/2}}{3b^4}-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{13/2}}{13b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(7/2))/(7*b^4) + (2*a^2*(a + b*x)^(9/2))/(3*b^4) - (6*a*(
a + b*x)^(11/2))/(11*b^4) + (2*(a + b*x)^(13/2))/(13*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{5/2}\,dx&=\int\left(-\frac{a^3(a+bx)^{5/2}}{b^3}+\frac{3a^2(a+bx)^{7/2}}{b^3}-\frac{3a(a+bx)^{9/2}}{b^3}+\frac{(a+bx)^{11/2}}{b^3}\right)dx\\&=-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2a^2(a+bx)^{9/2}}{3b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}+\frac{2(a+bx)^{13/2}}{13b^4}\end{aligned}$$

Mathematica [A] time = 0.0385916, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{7/2}\left(56a^2bx-16a^3-126ab^2x^2+231b^3x^3\right)}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(-16*a^3 + 56*a^2*b*x - 126*a*b^2*x^2 + 231*b^3*x^3))/(3
003*b^4)
```

Maple [A] time = 0.005, size = 43, normalized size = 0.6

$$-\frac{-462 b^3 x^3 + 252 a b^2 x^2 - 112 a^2 b x + 32 a^3}{3003 b^4} (bx + a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(5/2),x)
```

```
[Out] -2/3003*(b*x+a)^(7/2)*(-231*b^3*x^3+126*a*b^2*x^2-56*a^2*b*x+16*a^3)/b^4
```

Maxima [A] time = 1.08305, size = 76, normalized size = 1.06

$$\frac{2 (bx + a)^{\frac{13}{2}}}{13 b^4} - \frac{6 (bx + a)^{\frac{11}{2}} a}{11 b^4} + \frac{2 (bx + a)^{\frac{9}{2}} a^2}{3 b^4} - \frac{2 (bx + a)^{\frac{7}{2}} a^3}{7 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*x + a)^(13/2)/b^4 - 6/11*(b*x + a)^(11/2)*a/b^4 + 2/3*(b*x + a)^(9/
2)*a^2/b^4 - 2/7*(b*x + a)^(7/2)*a^3/b^4
```

Fricas [A] time = 1.54731, size = 171, normalized size = 2.38

$$\frac{2 \left(231 b^6 x^6 + 567 a b^5 x^5 + 371 a^2 b^4 x^4 + 5 a^3 b^3 x^3 - 6 a^4 b^2 x^2 + 8 a^5 b x - 16 a^6\right) \sqrt{bx + a}}{3003 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 + 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*a
^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqrt(b*x + a)/b^4
```

Sympy [A] time = 5.89415, size = 146, normalized size = 2.03

$$\begin{cases} -\frac{32a^6\sqrt{a+bx}}{3003b^4} + \frac{16a^5x\sqrt{a+bx}}{3003b^3} - \frac{4a^4x^2\sqrt{a+bx}}{1001b^2} + \frac{10a^3x^3\sqrt{a+bx}}{3003b} + \frac{106a^2x^4\sqrt{a+bx}}{429} + \frac{54abx^5\sqrt{a+bx}}{143} + \frac{2b^2x^6\sqrt{a+bx}}{13} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}} x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-32*a**6*sqrt(a + b*x)/(3003*b**4) + 16*a**5*x*sqrt(a + b*x)/(30
03*b**3) - 4*a**4*x**2*sqrt(a + b*x)/(1001*b**2) + 10*a**3*x**3*sqrt(a + b*
x)/(3003*b) + 106*a**2*x**4*sqrt(a + b*x)/429 + 54*a*b*x**5*sqrt(a + b*x)/1
43 + 2*b**2*x**6*sqrt(a + b*x)/13, Ne(b, 0)), (a**(5/2)*x**4/4, True))
```

Giac [B] time = 1.19315, size = 261, normalized size = 3.62

$$\frac{2\left(\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^2-105\,(bx+a)^{\frac{3}{2}}a^3\right)a^2}{b^3}+\frac{26\left(315\,(bx+a)^{\frac{11}{2}}-1540\,(bx+a)^{\frac{9}{2}}a+2970\,(bx+a)^{\frac{7}{2}}a^2-2772\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}\cdots\right.}{b^3}\right.}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/45045*(143*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x + a)^(5
/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*a^2/b^3 + 26*(315*(b*x + a)^(11/2) - 154
0*(b*x + a)^(9/2)*a + 2970*(b*x + a)^(7/2)*a^2 - 2772*(b*x + a)^(5/2)*a^3 +
 1155*(b*x + a)^(3/2)*a^4)*a/b^3 + 5*(693*(b*x + a)^(13/2) - 4095*(b*x + a)
^(11/2)*a + 10010*(b*x + a)^(9/2)*a^2 - 12870*(b*x + a)^(7/2)*a^3 + 9009*(b
*x + a)^(5/2)*a^4 - 3003*(b*x + a)^(3/2)*a^5)/b^3)/b
```

3.301 $\int x^2(a+bx)^{5/2}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}$$

```
[Out] (2*a^2*(a + b*x)^(7/2))/(7*b^3) - (4*a*(a + b*x)^(9/2))/(9*b^3) + (2*(a + b
*x)^(11/2))/(11*b^3)
```

Rubi [A] time = 0.0124758, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(5/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(7/2))/(7*b^3) - (4*a*(a + b*x)^(9/2))/(9*b^3) + (2*(a + b
*x)^(11/2))/(11*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{5/2}\,dx&=\int\left(\frac{a^2(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{7/2}}{b^2}+\frac{(a+bx)^{9/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}+\frac{2(a+bx)^{11/2}}{11b^3}\end{aligned}$$

Mathematica [A] time = 0.0264435, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{7/2}\left(8a^2-28abx+63b^2x^2\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(8*a^2 - 28*a*b*x + 63*b^2*x^2))/(693*b^3)
```

Maple [A] time = 0.004, size = 32, normalized size = 0.6

$$\frac{126 b^2 x^2 - 56 abx + 16 a^2}{693 b^3} (bx + a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(5/2),x)
```

```
[Out] 2/693*(b*x+a)^(7/2)*(63*b^2*x^2-28*a*b*x+8*a^2)/b^3
```

Maxima [A] time = 1.02675, size = 55, normalized size = 1.04

$$\frac{2 (bx + a)^{\frac{11}{2}}}{11 b^3} - \frac{4 (bx + a)^{\frac{9}{2}} a}{9 b^3} + \frac{2 (bx + a)^{\frac{7}{2}} a^2}{7 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b*x + a)^(11/2)/b^3 - 4/9*(b*x + a)^(9/2)*a/b^3 + 2/7*(b*x + a)^(7/2)
*a^2/b^3
```

Fricas [A] time = 1.50952, size = 146, normalized size = 2.75

$$\frac{2 \left(63 b^5 x^5 + 161 ab^4 x^4 + 113 a^2 b^3 x^3 + 3 a^3 b^2 x^2 - 4 a^4 bx + 8 a^5\right) \sqrt{bx + a}}{693 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3*b^2*x^2 - 4*a^4
*b*x + 8*a^5)*sqrt(b*x + a)/b^3
```

Sympy [A] time = 4.74117, size = 124, normalized size = 2.34

$$\begin{cases} \frac{16a^5\sqrt{a+bx}}{693b^3} - \frac{8a^4x\sqrt{a+bx}}{693b^2} + \frac{2a^3x^2\sqrt{a+bx}}{231b} + \frac{226a^2x^3\sqrt{a+bx}}{693} + \frac{46abx^4\sqrt{a+bx}}{99} + \frac{2b^2x^5\sqrt{a+bx}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((16*a**5*sqrt(a + b*x)/(693*b**3) - 8*a**4*x*sqrt(a + b*x)/(693*b
**2) + 2*a**3*x**2*sqrt(a + b*x)/(231*b) + 226*a**2*x**3*sqrt(a + b*x)/693
+ 46*a*b*x**4*sqrt(a + b*x)/99 + 2*b**2*x**5*sqrt(a + b*x)/11, Ne(b, 0)), (
a**(5/2)*x**3/3, True))
```

Giac [B] time = 1.18155, size = 211, normalized size = 3.98

$$\frac{2\left(\frac{33\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}\right)a^{2}}{b^{2}}+\frac{22\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^{2}-105\,(bx+a)^{\frac{3}{2}}a^{3}\right)a}{b^{2}}+\frac{315\,(bx+a)^{\frac{11}{2}}-1540\,(bx+a)^{\frac{9}{2}}a\cdots}{\cdots}\right.}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3465*(33*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*
a^2)*a^2/b^2 + 22*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x +
a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*a/b^2 + (315*(b*x + a)^(11/2) - 154
0*(b*x + a)^(9/2)*a + 2970*(b*x + a)^(7/2)*a^2 - 2772*(b*x + a)^(5/2)*a^3 +
 1155*(b*x + a)^(3/2)*a^4)/b^2)/b
```

3.302 $\int x(a+bx)^{5/2}\,dx$

Optimal. Leaf size=34

$$\frac{2(a+bx)^{9/2}}{9b^2}-\frac{2a(a+bx)^{7/2}}{7b^2}$$

```
[Out] (-2*a*(a + b*x)^(7/2))/(7*b^2) + (2*(a + b*x)^(9/2))/(9*b^2)
```

Rubi [A] time = 0.0086827, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{9/2}}{9b^2}-\frac{2a(a+bx)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(5/2),x]
```

```
[Out] (-2*a*(a + b*x)^(7/2))/(7*b^2) + (2*(a + b*x)^(9/2))/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{5/2}\,dx &= \int\left(-\frac{a(a+bx)^{5/2}}{b}+\frac{(a+bx)^{7/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{7/2}}{7b^2}+\frac{2(a+bx)^{9/2}}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.0257153, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{7/2}(7bx-2a)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(-2*a + 7*b*x))/(63*b^2)
```

Maple [A] time = 0.001, size = 21, normalized size = 0.6

$$-\frac{-14\,bx+4\,a}{63\,b^2}(bx+a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(5/2),x)
```

```
[Out] -2/63*(b*x+a)^(7/2)*(-7*b*x+2*a)/b^2
```

Maxima [A] time = 1.12851, size = 35, normalized size = 1.03

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^2}-\frac{2\,(bx+a)^{\frac{7}{2}}a}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^2 - 2/7*(b*x + a)^(7/2)*a/b^2
```

Fricas [A] time = 1.47407, size = 116, normalized size = 3.41

$$\frac{2\left(7\,b^4x^4+19\,ab^3x^3+15\,a^2b^2x^2+a^3bx-2\,a^4\right)\sqrt{bx+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*x
 + a)/b^2
```

Sympy [A] time = 3.0582, size = 102, normalized size = 3.

$$\begin{cases} -\frac{4a^4\sqrt{a+bx}}{63b^2}+\frac{2a^3x\sqrt{a+bx}}{63b}+\frac{10a^2x^2\sqrt{a+bx}}{21}+\frac{38abx^3\sqrt{a+bx}}{63}+\frac{2b^2x^4\sqrt{a+bx}}{9} & \text{for } b\neq 0 \\ \frac{a^{\frac{5}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-4*a**4*sqrt(a + b*x)/(63*b**2) + 2*a**3*x*sqrt(a + b*x)/(63*b)
+ 10*a**2*x**2*sqrt(a + b*x)/21 + 38*a*b*x**3*sqrt(a + b*x)/63 + 2*b**2*x**
4*sqrt(a + b*x)/9, Ne(b, 0)), (a**(5/2)*x**2/2, True))
```

Giac [B] time = 1.21709, size = 162, normalized size = 4.76

$$\frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)a^2}{b}+\frac{6\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)a}{b}+\frac{35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^2-105\,(bx+a)^{\frac{3}{2}}a^3}{b}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/315*(21*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)*a^2/b + 6*(15*(b*x + a)
^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)*a/b + (35*(b*x + a)
^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x + a)^(5/2)*a^2 - 105*(b*x + a)^(3
/2)*a^3)/b)/b
```

3.303 $\int (a+bx)^{5/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{7/2}}{7b}$$

```
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

Rubi [A] time = 0.0013822, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{5/2}\,dx=\frac{2(a+bx)^{7/2}}{7b}$$

Mathematica [A] time = 0.0072039, size = 16, normalized size = 1.

$$\frac{2(a+bx)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$\frac{2}{7\,b}\,(bx+a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2),x)
```

```
[Out] 2/7*(b*x+a)^(7/2)/b
```

Maxima [A] time = 1.09342, size = 16, normalized size = 1.

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b
```

Fricas [B] time = 1.5099, size = 85, normalized size = 5.31

$$\frac{2\left(b^{3}x^{3}+3\,ab^{2}x^{2}+3\,a^{2}bx+a^{3}\right)\sqrt{bx+a}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/b
```

Sympy [A] time = 0.107884, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2),x)
```

```
[Out] 2*(a + b*x)**(7/2)/(7*b)
```

Giac [B] time = 1.25108, size = 81, normalized size = 5.06

$$\frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+70\,(bx+a)^{\frac{3}{2}}a^{2}+14\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)a\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/105*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 70*(b*x + a)^(3/2)*a^2 +
 14*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)*a)/b
```

3.304 $\int \frac{(a+bx)^{5/2}}{x} dx$

Optimal. Leaf size=65

$$2a^2\sqrt{a+bx} - 2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + \frac{2}{3}a(a+bx)^{3/2} + \frac{2}{5}(a+bx)^{5/2}$$

```
[Out] 2*a^2*Sqrt[a + b*x] + (2*a*(a + b*x)^(3/2))/3 + (2*(a + b*x)^(5/2))/5 - 2*a
^(5/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.0208395, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$2a^2\sqrt{a+bx} - 2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + \frac{2}{3}a(a+bx)^{3/2} + \frac{2}{5}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x,x]
```

```
[Out] 2*a^2*Sqrt[a + b*x] + (2*a*(a + b*x)^(3/2))/3 + (2*(a + b*x)^(5/2))/5 - 2*a
^(5/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{x}\,dx = \frac{2}{5}(a+bx)^{5/2} + a\int \frac{(a+bx)^{3/2}}{x}\,dx$$
$$= \frac{2}{3}a(a+bx)^{3/2} + \frac{2}{5}(a+bx)^{5/2} + a^2\int \frac{\sqrt{a+bx}}{x}\,dx$$
$$= 2a^2\sqrt{a+bx} + \frac{2}{3}a(a+bx)^{3/2} + \frac{2}{5}(a+bx)^{5/2} + a^3\int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= 2a^2\sqrt{a+bx} + \frac{2}{3}a(a+bx)^{3/2} + \frac{2}{5}(a+bx)^{5/2} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= 2a^2\sqrt{a+bx} + \frac{2}{3}a(a+bx)^{3/2} + \frac{2}{5}(a+bx)^{5/2} - 2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Mathematica [A] time = 0.0887927, size = 58, normalized size = 0.89

$$-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + \frac{2}{3}a\sqrt{a+bx}(4a+bx) + \frac{2}{5}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x,x]
```

```
[Out] (2*(a + b*x)^(5/2))/5 + (2*a*Sqrt[a + b*x]*(4*a + b*x))/3 - 2*a^(5/2)*ArcTa
nh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A] time = 0.003, size = 50, normalized size = 0.8

$$\frac{2a}{3}(bx+a)^{\frac{3}{2}} + \frac{2}{5}(bx+a)^{\frac{5}{2}} - 2a^{5/2}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) + 2a^2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x,x)
```

```
[Out] 2/3*a*(b*x+a)^(3/2)+2/5*(b*x+a)^(5/2)-2*a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/
2))+2*a^2*(b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57475, size = 288, normalized size = 4.43

$$\left[a^{\frac{5}{2}} \log\left(\frac{bx - 2\sqrt{bx+a}\sqrt{a} + 2a}{x}\right) + \frac{2}{15}\left(3b^2x^2 + 11abx + 23a^2\right)\sqrt{bx+a}, 2\sqrt{-a}a^2 \arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \frac{2}{15}\left(3b^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(5/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/15*(3*b^2*x^2 + 1
1*a*b*x + 23*a^2)*sqrt(b*x + a), 2*sqrt(-a)*a^2*arctan(sqrt(b*x + a)*sqrt(-
a)/a) + 2/15*(3*b^2*x^2 + 11*a*b*x + 23*a^2)*sqrt(b*x + a)]
```

Sympy [A] time = 4.68259, size = 97, normalized size = 1.49

$$\frac{46a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{15} + a^{\frac{5}{2}}\log\left(\frac{bx}{a}\right) - 2a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right) + \frac{22a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}}{15} + \frac{2\sqrt{a}b^2x^2\sqrt{1+\frac{bx}{a}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x,x)
```

```
[Out] 46*a**(5/2)*sqrt(1 + b*x/a)/15 + a**(5/2)*log(b*x/a) - 2*a**(5/2)*log(sqrt(
1 + b*x/a) + 1) + 22*a**(3/2)*b*x*sqrt(1 + b*x/a)/15 + 2*sqrt(a)*b**2*x**2*
sqrt(1 + b*x/a)/5
```

Giac [A] time = 1.18573, size = 76, normalized size = 1.17

$$\frac{2a^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2}{5}(bx+a)^{\frac{5}{2}} + \frac{2}{3}(bx+a)^{\frac{3}{2}}a + 2\sqrt{bx+a}a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/5*(b*x + a)^(5/2) + 2/3*(
b*x + a)^(3/2)*a + 2*sqrt(b*x + a)*a^2
```

3.305 $\int \frac{(a+bx)^{5/2}}{x^2} dx$

Optimal. Leaf size=66

$$-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{x}+\frac{5}{3}b(a+bx)^{3/2}+5ab\sqrt{a+bx}$$

```
[Out] 5*a*b*Sqrt[a + b*x] + (5*b*(a + b*x)^(3/2))/3 - (a + b*x)^(5/2)/x - 5*a^(3/
2)*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.0204844, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{x}+\frac{5}{3}b(a+bx)^{3/2}+5ab\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^2,x]
```

```
[Out] 5*a*b*Sqrt[a + b*x] + (5*b*(a + b*x)^(3/2))/3 - (a + b*x)^(5/2)/x - 5*a^(3/
2)*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^2}\,dx &= -\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}(5b)\int \frac{(a+bx)^{3/2}}{x}\,dx\\
&= \frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}(5ab)\int \frac{\sqrt{a+bx}}{x}\,dx\\
&= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}\left(5a^2b\right)\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\left(5a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\
&= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.012207, size = 33, normalized size = 0.5

$$\frac{2b(a+bx)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx}{a}+1\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^2,x]
```

```
[Out] (2*b*(a + b*x)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 + (b*x)/a])/(7*a^2)
```

Maple [A] time = 0.009, size = 61, normalized size = 0.9

$$2\,b\left(1/3\,(bx+a)^{3/2}+2\,a\sqrt{bx+a}+a^2\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-5/2\,\frac{1}{\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^2,x)
```

```
[Out] 2*b*(1/3*(b*x+a)^(3/2)+2*a*(b*x+a)^(1/2)+a^2*(-1/2*(b*x+a)^(1/2)/b/x-5/2*ar
ctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57004, size = 309, normalized size = 4.68

$$\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(2\,b^2x^2+14\,abx-3\,a^2\right)\sqrt{bx+a}}{6\,x},\ \frac{15\,\sqrt{-a}abx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(2\,b^2x^2+14\,abx-3\,a^2\right)\sqrt{bx+a}}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*a^(3/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b^
2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x, 1/3*(15*sqrt(-a)*a*b*x*arctan(s
qrt(b*x + a)*sqrt(-a)/a) + (2*b^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x]
```

Sympy [A] time = 4.71397, size = 99, normalized size = 1.5

$$-\frac{a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{x}+\frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{bx}{a}}}{3}+\frac{5a^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2}-5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{2\sqrt{a}b^2x\sqrt{1+\frac{bx}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**2,x)
```

```
[Out] -a**(5/2)*sqrt(1 + b*x/a)/x + 14*a**(3/2)*b*sqrt(1 + b*x/a)/3 + 5*a**(3/2)*
b*log(b*x/a)/2 - 5*a**(3/2)*b*log(sqrt(1 + b*x/a) + 1) + 2*sqrt(a)*b**2*x*s
qrt(1 + b*x/a)/3
```

Giac [A] time = 1.20062, size = 100, normalized size = 1.52

$$\frac{\frac{15\,a^2b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\,(bx+a)^{\frac{3}{2}}b^2+12\,\sqrt{bx+a}ab^2-\frac{3\,\sqrt{bx+a}a^2b}{x}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*a^2*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*(b*x + a)^(3/2)
*b^2 + 12*sqrt(b*x + a)*a*b^2 - 3*sqrt(b*x + a)*a^2*b/x)/b
```

3.306 $\int \frac{(a+bx)^{5/2}}{x^3} dx$

Optimal. Leaf size=78

$$\frac{15}{4}b^2\sqrt{a+bx}-\frac{15}{4}\sqrt{a}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{2x^2}-\frac{5b(a+bx)^{3/2}}{4x}$$

```
[Out] (15*b^2*Sqrt[a + b*x])/4 - (5*b*(a + b*x)^(3/2))/(4*x) - (a + b*x)^(5/2)/(2
*x^2) - (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rubi [A] time = 0.0211749, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$\frac{15}{4}b^2\sqrt{a+bx}-\frac{15}{4}\sqrt{a}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{2x^2}-\frac{5b(a+bx)^{3/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^3,x]
```

```
[Out] (15*b^2*Sqrt[a + b*x])/4 - (5*b*(a + b*x)^(3/2))/(4*x) - (a + b*x)^(5/2)/(2
*x^2) - (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^3}\,dx &= -\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{4}(5b)\int \frac{(a+bx)^{3/2}}{x^2}\,dx\\
&= -\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15b^2\right)\int \frac{\sqrt{a+bx}}{x}\,dx\\
&= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15ab^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{4}(15ab)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\
&= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}-\frac{15}{4}\sqrt{a}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0207901, size = 35, normalized size = 0.45

$$
-\frac{2b^2(a+bx)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{bx}{a}+1\right)}{7a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^3,x]
```

```
[Out] (-2*b^2*(a + b*x)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 + (b*x)/a])/(7*a^3
)
```

Maple [A] time = 0.009, size = 61, normalized size = 0.8

$$
2\,b^2\left(\sqrt{bx+a}+a\left(\frac{1}{b^2x^2}\left(-\frac{9\,(bx+a)^{3/2}}{8}+\frac{7\,a\sqrt{bx+a}}{8}\right)-\frac{15}{8\,\sqrt{a}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^3,x)
```

```
[Out] 2*b^2*((b*x+a)^(1/2)+a*((-9/8*(b*x+a)^(3/2)+7/8*a*(b*x+a)^(1/2))/b^2/x^2-15
/8*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54592, size = 320, normalized size = 4.1

$$\left[\frac{15\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(8b^2x^2-9abx-2a^2\right)\sqrt{bx+a}}{8x^2}, \frac{15\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(8b^2x^2-9ab\right.}{4x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x + a))/x^2, 1/4*(15*sqrt(-a)*b^2*x^2*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x +
a))/x^2]
```

Sympy [A] time = 5.71615, size = 126, normalized size = 1.62

$$-\frac{15\sqrt{a}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4}-\frac{a^3}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{11a^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{ab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{5}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**3,x)
```

```
[Out] -15*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - a**3/(2*sqrt(b)*x**(5
/2)*sqrt(a/(b*x) + 1)) - 11*a**2*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) - a
*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) + 1)) + 2*b**(5/2)*sqrt(x)/sqrt(a/(b*x) +
 1)
```

Giac [A] time = 1.15919, size = 108, normalized size = 1.38

$$\frac{\frac{15ab^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\sqrt{bx+a}b^3-\frac{9(bx+a)^{\frac{3}{2}}ab^3-7\sqrt{bx+a}a^2b^3}{b^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(15*a*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*sqrt(b*x + a)*b^3
 - (9*(b*x + a)^(3/2)*a*b^3 - 7*sqrt(b*x + a)*a^2*b^3)/(b^2*x^2))/b
```

3.307 $\int \frac{(a+bx)^{5/2}}{x^4} dx$

Optimal. Leaf size=81

$$-\frac{5b^2\sqrt{a+bx}}{8x} - \frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}} - \frac{5b(a+bx)^{3/2}}{12x^2} - \frac{(a+bx)^{5/2}}{3x^3}$$

```
[Out] (-5*b^2*Sqrt[a + b*x])/(8*x) - (5*b*(a + b*x)^(3/2))/(12*x^2) - (a + b*x)^(
5/2)/(3*x^3) - (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*Sqrt[a])
```

Rubi [A] time = 0.0226134, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{5b^2\sqrt{a+bx}}{8x} - \frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}} - \frac{5b(a+bx)^{3/2}}{12x^2} - \frac{(a+bx)^{5/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^4,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x])/(8*x) - (5*b*(a + b*x)^(3/2))/(12*x^2) - (a + b*x)^(
5/2)/(3*x^3) - (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^4}\,dx &= -\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{6}(5b)\int \frac{(a+bx)^{3/2}}{x^3}\,dx\\
&= -\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{8}\left(5b^2\right)\int \frac{\sqrt{a+bx}}{x^2}\,dx\\
&= -\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{16}\left(5b^3\right)\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= -\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{8}\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\
&= -\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.0569685, size = 79, normalized size = 0.98

$$
-\frac{34a^2bx+8a^3+59ab^2x^2+15b^3x^3\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+33b^3x^3}{24x^3\sqrt{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^4,x]
```

```
[Out] -(8*a^3 + 34*a^2*b*x + 59*a*b^2*x^2 + 33*b^3*x^3 + 15*b^3*x^3*Sqrt[1 + (b*x
)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/(24*x^3*Sqrt[a + b*x])
```

Maple [A] time = 0.009, size = 63, normalized size = 0.8

$$
2\,b^3\left(\frac{1}{b^3x^3}\left(-\frac{11\,(bx+a)^{5/2}}{16}+5/6\,a\,(bx+a)^{3/2}-\frac{5\,a^2\sqrt{bx+a}}{16}\right)-\frac{5}{16\,\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^4,x)
```

```
[Out] 2*b^3*((-11/16*(b*x+a)^(5/2)+5/6*a*(b*x+a)^(3/2)-5/16*a^2*(b*x+a)^(1/2))/b^
3/x^3-5/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59282, size = 350, normalized size = 4.32

$$\left[\frac{15\sqrt{a}b^3x^3\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(33ab^2x^2+26a^2bx+8a^3\right)\sqrt{bx+a}}{48ax^3}, \frac{15\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\left(33ab^2x^2+\right.}{24ax^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(33*a*b^2*x^2 + 26*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a*x^3), 1/24*(15*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) - (33*a*b^2*x^2 + 26*a^2*b*x +
8*a^3)*sqrt(b*x + a))/(a*x^3)]
```

Sympy [A] time = 6.73303, size = 104, normalized size = 1.28

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x^{\frac{5}{2}}}-\frac{13ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{12x^{\frac{3}{2}}}-\frac{11b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{8\sqrt{x}}-\frac{5b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**4,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x**(5/2)) - 13*a*b**(3/2)*sqrt(a/(b*x) +
 1)/(12*x**(3/2)) - 11*b**(5/2)*sqrt(a/(b*x) + 1)/(8*sqrt(x)) - 5*b**3*asin
h(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*sqrt(a))
```

Giac [A] time = 1.28311, size = 107, normalized size = 1.32

$$\frac{\frac{15b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{33(bx+a)^{\frac{5}{2}}b^4-40(bx+a)^{\frac{3}{2}}ab^4+15\sqrt{bx+a}a^2b^4}{b^3x^3}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (33*(b*x + a)^(5/2)*
b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 15*sqrt(b*x + a)*a^2*b^4)/(b^3*x^3))/b
```

3.308 $\int \frac{(a+bx)^{5/2}}{x^5} dx$

Optimal. Leaf size=103

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}-\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}$$

```
[Out] (-5*b^2*Sqrt[a + b*x])/(32*x^2) - (5*b^3*Sqrt[a + b*x])/(64*a*x) - (5*b*(a
+ b*x)^(3/2))/(24*x^3) - (a + b*x)^(5/2)/(4*x^4) + (5*b^4*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(64*a^(3/2))
```

Rubi [A] time = 0.0309203, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}-\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^5,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x])/(32*x^2) - (5*b^3*Sqrt[a + b*x])/(64*a*x) - (5*b*(a
+ b*x)^(3/2))/(24*x^3) - (a + b*x)^(5/2)/(4*x^4) + (5*b^4*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(64*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^5}\,dx &= -\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{8}(5b)\int \frac{(a+bx)^{3/2}}{x^4}\,dx\\
&= -\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{16}\left(5b^2\right)\int \frac{\sqrt{a+bx}}{x^3}\,dx\\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{64}\left(5b^3\right)\int \frac{1}{x^2\sqrt{a+bx}}\,dx\\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}-\frac{\left(5b^4\right)\int \frac{1}{x\sqrt{a+bx}}\,dx}{128a}\\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}-\frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{64a}\\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0112828, size = 35, normalized size = 0.34

$$-\frac{2b^4(a+bx)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{bx}{a}+1\right)}{7a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^5,x]
```

```
[Out] (-2*b^4*(a + b*x)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, 1 + (b*x)/a])/(7*a^5
)
```

Maple [A] time = 0.012, size = 75, normalized size = 0.7

$$2\,b^4\left(\frac{1}{b^4x^4}\left(-\frac{5\,(bx+a)^{7/2}}{128\,a}-\frac{73\,(bx+a)^{5/2}}{384}+\frac{55\,a\,(bx+a)^{3/2}}{384}-\frac{5\,a^2\sqrt{bx+a}}{128}\right)+\frac{5}{128\,a^{3/2}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^5,x)
```

```
[Out] 2*b^4*((-5/128/a*(b*x+a)^(7/2)-73/384*(b*x+a)^(5/2)+55/384*a*(b*x+a)^(3/2)-
5/128*a^2*(b*x+a)^(1/2))/b^4/x^4+5/128*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56401, size = 413, normalized size = 4.01

$$\left[\frac{15\sqrt{a}b^4x^4\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(15ab^3x^3+118a^2b^2x^2+136a^3bx+48a^4\right)\sqrt{bx+a}}{384a^2x^4}, -\frac{15\sqrt{-a}b^4x^4\arctan\left(\frac{\sqrt{bx+a}}{a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/384*(15*sqrt(a)*b^4*x^4*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2
*(15*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^
2*x^4), -1/192*(15*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*
a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^2*x^4
)]
```

Sympy [A] time = 10.2826, size = 155, normalized size = 1.5

$$-\frac{a^3}{4\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{23a^2\sqrt{b}}{24x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{127ab^{\frac{3}{2}}}{96x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{133b^{\frac{5}{2}}}{192x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{7}{2}}}{64a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**5,x)
```

```
[Out] -a**3/(4*sqrt(b)*x**(9/2)*sqrt(a/(b*x) + 1)) - 23*a**2*sqrt(b)/(24*x**(7/2)
*sqrt(a/(b*x) + 1)) - 127*a*b**(3/2)/(96*x**(5/2)*sqrt(a/(b*x) + 1)) - 133*
b**(5/2)/(192*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*b**(7/2)/(64*a*sqrt(x)*sqrt(a
/(b*x) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(64*a**(3/2))
```

Giac [A] time = 1.2294, size = 134, normalized size = 1.3

$$-\frac{\frac{15b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{15(bx+a)^{\frac{7}{2}}b^5+73(bx+a)^{\frac{5}{2}}ab^5-55(bx+a)^{\frac{3}{2}}a^2b^5+15\sqrt{bx+a}a^3b^5}{ab^4x^4}}{192b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/192*(15*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (15*(b*x + a)^
(7/2)*b^5 + 73*(b*x + a)^(5/2)*a*b^5 - 55*(b*x + a)^(3/2)*a^2*b^5 + 15*sqrt
(b*x + a)*a^3*b^5)/(a*b^4*x^4))/b
```

3.309 $\int x^7(a+bx)^{9/2}\,dx$

Optimal. Leaf size=146

$$\frac{2a^2(a+bx)^{21/2}}{b^8}-\frac{70a^3(a+bx)^{19/2}}{19b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{2(a+b}{25b}$$

```
[Out] (-2*a^7*(a + b*x)^(11/2))/(11*b^8) + (14*a^6*(a + b*x)^(13/2))/(13*b^8) - (
14*a^5*(a + b*x)^(15/2))/(5*b^8) + (70*a^4*(a + b*x)^(17/2))/(17*b^8) - (70
*a^3*(a + b*x)^(19/2))/(19*b^8) + (2*a^2*(a + b*x)^(21/2))/b^8 - (14*a*(a +
 b*x)^(23/2))/(23*b^8) + (2*(a + b*x)^(25/2))/(25*b^8)
```

Rubi [A] time = 0.0424934, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{21/2}}{b^8}-\frac{70a^3(a+bx)^{19/2}}{19b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{2(a+b}{25b}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^7*(a + b*x)^(11/2))/(11*b^8) + (14*a^6*(a + b*x)^(13/2))/(13*b^8) - (
14*a^5*(a + b*x)^(15/2))/(5*b^8) + (70*a^4*(a + b*x)^(17/2))/(17*b^8) - (70
*a^3*(a + b*x)^(19/2))/(19*b^8) + (2*a^2*(a + b*x)^(21/2))/b^8 - (14*a*(a +
 b*x)^(23/2))/(23*b^8) + (2*(a + b*x)^(25/2))/(25*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7(a+bx)^{9/2}\,dx &= \int\left(-\frac{a^7(a+bx)^{9/2}}{b^7}+\frac{7a^6(a+bx)^{11/2}}{b^7}-\frac{21a^5(a+bx)^{13/2}}{b^7}+\frac{35a^4(a+bx)^{15/2}}{b^7}-\frac{35a^3(a+bx)^{17/2}}{b^7}+\frac{2\right.\\ &= -\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{70a^3(a+bx)^{19/2}}{19b^8}+\frac{2a}{}\end{aligned}$$

Mathematica [A] time = 0.12302, size = 90, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(-36608a^5b^2x^2+91520a^4b^3x^3-194480a^3b^4x^4+369512a^2b^5x^5+11264a^6bx-2048a^7-646646ab^6x^6+10\right.}{26558675b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-2048*a^7 + 11264*a^6*b*x - 36608*a^5*b^2*x^2 + 91520*
a^4*b^3*x^3 - 194480*a^3*b^4*x^4 + 369512*a^2*b^5*x^5 - 646646*a*b^6*x^6 +
```

```
1062347*b^7*x^7))/(26558675*b^8)
```

Maple [A] time = 0.005, size = 87, normalized size = 0.6

$$-\frac{-2124694\,b^{7}x^{7}+1293292\,ab^{6}x^{6}-739024\,a^{2}b^{5}x^{5}+388960\,a^{3}b^{4}x^{4}-183040\,a^{4}b^{3}x^{3}+73216\,a^{5}b^{2}x^{2}-22528\,a^{6}bx+}{26558675\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^(9/2),x)
```

```
[Out] -2/26558675*(b*x+a)^(11/2)*(-1062347*b^7*x^7+646646*a*b^6*x^6-369512*a^2*b^
5*x^5+194480*a^3*b^4*x^4-91520*a^4*b^3*x^3+36608*a^5*b^2*x^2-11264*a^6*b*x+
2048*a^7)/b^8
```

Maxima [A] time = 1.09388, size = 157, normalized size = 1.08

$$\frac{2\,(bx+a)^{\frac{25}{2}}}{25\,b^{8}}-\frac{14\,(bx+a)^{\frac{23}{2}}a}{23\,b^{8}}+\frac{2\,(bx+a)^{\frac{21}{2}}a^{2}}{b^{8}}-\frac{70\,(bx+a)^{\frac{19}{2}}a^{3}}{19\,b^{8}}+\frac{70\,(bx+a)^{\frac{17}{2}}a^{4}}{17\,b^{8}}-\frac{14\,(bx+a)^{\frac{15}{2}}a^{5}}{5\,b^{8}}+\frac{14\,(bx+a)^{\frac{13}{2}}}{13\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/25*(b*x + a)^(25/2)/b^8 - 14/23*(b*x + a)^(23/2)*a/b^8 + 2*(b*x + a)^(21/
2)*a^2/b^8 - 70/19*(b*x + a)^(19/2)*a^3/b^8 + 70/17*(b*x + a)^(17/2)*a^4/b^
8 - 14/5*(b*x + a)^(15/2)*a^5/b^8 + 14/13*(b*x + a)^(13/2)*a^6/b^8 - 2/11*(
b*x + a)^(11/2)*a^7/b^8
```

Fricas [A] time = 1.51393, size = 374, normalized size = 2.56

$$\frac{2\left(1062347\,b^{12}x^{12}+4665089\,ab^{11}x^{11}+7759752\,a^{2}b^{10}x^{10}+5810090\,a^{3}b^{9}x^{9}+1659515\,a^{4}b^{8}x^{8}+429\,a^{5}b^{7}x^{7}-462\,a^{6}b\right.}{26558675\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/26558675*(1062347*b^12*x^12 + 4665089*a*b^11*x^11 + 7759752*a^2*b^10*x^10
 + 5810090*a^3*b^9*x^9 + 1659515*a^4*b^8*x^8 + 429*a^5*b^7*x^7 - 462*a^6*b^
6*x^6 + 504*a^7*b^5*x^5 - 560*a^8*b^4*x^4 + 640*a^9*b^3*x^3 - 768*a^10*b^2*
x^2 + 1024*a^11*b*x - 2048*a^12)*sqrt(b*x + a)/b^8
```

Sympy [A] time = 53.0196, size = 279, normalized size = 1.91

$$\begin{cases} -\frac{4096a^{12}\sqrt{a+bx}}{26558675b^{8}}+\frac{2048a^{11}x\sqrt{a+bx}}{26558675b^{7}}-\frac{1536a^{10}x^{2}\sqrt{a+bx}}{26558675b^{6}}+\frac{256a^{9}x^{3}\sqrt{a+bx}}{5311735b^{5}}-\frac{224a^{8}x^{4}\sqrt{a+bx}}{5311735b^{4}}+\frac{1008a^{7}x^{5}\sqrt{a+bx}}{26558675b^{3}}-\frac{84a^{6}x^{6}\sqrt{a+bx}}{2414425b^{2}}+\frac{6a^{5}x^{7}\sqrt{a+bx}}{185725b} \\ \frac{a^{\frac{9}{2}}x^{8}}{8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-4096*a**12*sqrt(a + b*x)/(26558675*b**8) + 2048*a**11*x*sqrt(a
+ b*x)/(26558675*b**7) - 1536*a**10*x**2*sqrt(a + b*x)/(26558675*b**6) + 25
6*a**9*x**3*sqrt(a + b*x)/(5311735*b**5) - 224*a**8*x**4*sqrt(a + b*x)/(531
1735*b**4) + 1008*a**7*x**5*sqrt(a + b*x)/(26558675*b**3) - 84*a**6*x**6*sq
rt(a + b*x)/(2414425*b**2) + 6*a**5*x**7*sqrt(a + b*x)/(185725*b) + 4642*a*
*4*x**8*sqrt(a + b*x)/37145 + 956*a**3*b*x**9*sqrt(a + b*x)/2185 + 336*a**2
*b**2*x**10*sqrt(a + b*x)/575 + 202*a*b**3*x**11*sqrt(a + b*x)/575 + 2*b**4
*x**12*sqrt(a + b*x)/25, Ne(b, 0)), (a**(9/2)*x**8/8, True))
```

Giac [B] time = 1.24997, size = 838, normalized size = 5.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/1673196525*(15295*(6435*(b*x + a)^(17/2) - 51051*(b*x + a)^(15/2)*a + 176
715*(b*x + a)^(13/2)*a^2 - 348075*(b*x + a)^(11/2)*a^3 + 425425*(b*x + a)^(
9/2)*a^4 - 328185*(b*x + a)^(7/2)*a^5 + 153153*(b*x + a)^(5/2)*a^6 - 36465*
(b*x + a)^(3/2)*a^7)*a^4/b^7 + 3220*(109395*(b*x + a)^(19/2) - 978120*(b*x
+ a)^(17/2)*a + 3879876*(b*x + a)^(15/2)*a^2 - 8953560*(b*x + a)^(13/2)*a^3
 + 13226850*(b*x + a)^(11/2)*a^4 - 12932920*(b*x + a)^(9/2)*a^5 + 8314020*(
b*x + a)^(7/2)*a^6 - 3325608*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a
^8)*a^3/b^7 + 2070*(230945*(b*x + a)^(21/2) - 2297295*(b*x + a)^(19/2)*a +
10270260*(b*x + a)^(17/2)*a^2 - 27159132*(b*x + a)^(15/2)*a^3 + 47006190*(b
*x + a)^(13/2)*a^4 - 55552770*(b*x + a)^(11/2)*a^5 + 45265220*(b*x + a)^(9/
2)*a^6 - 24942060*(b*x + a)^(7/2)*a^7 + 8729721*(b*x + a)^(5/2)*a^8 - 16166
15*(b*x + a)^(3/2)*a^9)*a^2/b^7 + 300*(969969*(b*x + a)^(23/2) - 10623470*(
b*x + a)^(21/2)*a + 52837785*(b*x + a)^(19/2)*a^2 - 157477320*(b*x + a)^(17
/2)*a^3 + 312330018*(b*x + a)^(15/2)*a^4 - 432456948*(b*x + a)^(13/2)*a^5 +
 425904570*(b*x + a)^(11/2)*a^6 - 297457160*(b*x + a)^(9/2)*a^7 + 143416845
*(b*x + a)^(7/2)*a^8 - 44618574*(b*x + a)^(5/2)*a^9 + 7436429*(b*x + a)^(3/
2)*a^10)*a/b^7 + 33*(2028117*(b*x + a)^(25/2) - 24249225*(b*x + a)^(23/2)*a
 + 132793375*(b*x + a)^(21/2)*a^2 - 440314875*(b*x + a)^(19/2)*a^3 + 984233
250*(b*x + a)^(17/2)*a^4 - 1561650090*(b*x + a)^(15/2)*a^5 + 1801903950*(b*
x + a)^(13/2)*a^6 - 1521087750*(b*x + a)^(11/2)*a^7 + 929553625*(b*x + a)^(
9/2)*a^8 - 398380125*(b*x + a)^(7/2)*a^9 + 111546435*(b*x + a)^(5/2)*a^10 -
 16900975*(b*x + a)^(3/2)*a^11)/b^7)/b
```

3.310 $\int x^6(a+bx)^{9/2}\,dx$

Optimal. Leaf size=127

$$\frac{30a^2(a+bx)^{19/2}}{19b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^6(a+bx)^{11/2}}{11b^7}+\frac{2(a+bx)^{23/2}}{23b^7}-\frac{4a(a+bx)^{21/2}}{7b^7}$$

```
[Out] (2*a^6*(a + b*x)^(11/2))/(11*b^7) - (12*a^5*(a + b*x)^(13/2))/(13*b^7) + (2
*a^4*(a + b*x)^(15/2))/b^7 - (40*a^3*(a + b*x)^(17/2))/(17*b^7) + (30*a^2*(
a + b*x)^(19/2))/(19*b^7) - (4*a*(a + b*x)^(21/2))/(7*b^7) + (2*(a + b*x)^(
23/2))/(23*b^7)
```

Rubi [A] time = 0.0350039, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{30a^2(a+bx)^{19/2}}{19b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^6(a+bx)^{11/2}}{11b^7}+\frac{2(a+bx)^{23/2}}{23b^7}-\frac{4a(a+bx)^{21/2}}{7b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^6*(a + b*x)^(11/2))/(11*b^7) - (12*a^5*(a + b*x)^(13/2))/(13*b^7) + (2
*a^4*(a + b*x)^(15/2))/b^7 - (40*a^3*(a + b*x)^(17/2))/(17*b^7) + (30*a^2*(
a + b*x)^(19/2))/(19*b^7) - (4*a*(a + b*x)^(21/2))/(7*b^7) + (2*(a + b*x)^(
23/2))/(23*b^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^{9/2}\,dx &= \int\left(\frac{a^6(a+bx)^{9/2}}{b^6}-\frac{6a^5(a+bx)^{11/2}}{b^6}+\frac{15a^4(a+bx)^{13/2}}{b^6}-\frac{20a^3(a+bx)^{15/2}}{b^6}+\frac{15a^2(a+bx)^{17/2}}{b^6}-\right.\\ &= \frac{2a^6(a+bx)^{11/2}}{11b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{30a^2(a+bx)^{19/2}}{19b^7}-\end{aligned}$$

Mathematica [A] time = 0.0558602, size = 79, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(18304a^4b^2x^2-45760a^3b^3x^3+97240a^2b^4x^4-5632a^5bx+1024a^6-184756ab^5x^5+323323b^6x^6\right)}{7436429b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(1024*a^6 - 5632*a^5*b*x + 18304*a^4*b^2*x^2 - 45760*a^
3*b^3*x^3 + 97240*a^2*b^4*x^4 - 184756*a*b^5*x^5 + 323323*b^6*x^6))/(743642
```

```
9*b^7)
```

Maple [A] time = 0.005, size = 76, normalized size = 0.6

$$\frac{646646\,x^{6}b^{6}-369512\,a x^{5}b^{5}+194480\,a^{2}x^{4}b^{4}-91520\,a^{3}x^{3}b^{3}+36608\,a^{4}x^{2}b^{2}-11264\,a^{5}x b+2048\,a^{6}}{7436429\,b^{7}}\,(bx+a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^(9/2),x)
```

```
[Out] 2/7436429*(b*x+a)^(11/2)*(323323*b^6*x^6-184756*a*b^5*x^5+97240*a^2*b^4*x^4
-45760*a^3*b^3*x^3+18304*a^4*b^2*x^2-5632*a^5*b*x+1024*a^6)/b^7
```

Maxima [A] time = 1.0907, size = 136, normalized size = 1.07

$$\frac{2\,(bx+a)^{\frac{23}{2}}}{23\,b^{7}}-\frac{4\,(bx+a)^{\frac{21}{2}}\,a}{7\,b^{7}}+\frac{30\,(bx+a)^{\frac{19}{2}}\,a^{2}}{19\,b^{7}}-\frac{40\,(bx+a)^{\frac{17}{2}}\,a^{3}}{17\,b^{7}}+\frac{2\,(bx+a)^{\frac{15}{2}}\,a^{4}}{b^{7}}-\frac{12\,(bx+a)^{\frac{13}{2}}\,a^{5}}{13\,b^{7}}+\frac{2\,(bx+a)^{\frac{11}{2}}\,a^{6}}{11\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/23*(b*x + a)^(23/2)/b^7 - 4/7*(b*x + a)^(21/2)*a/b^7 + 30/19*(b*x + a)^(1
9/2)*a^2/b^7 - 40/17*(b*x + a)^(17/2)*a^3/b^7 + 2*(b*x + a)^(15/2)*a^4/b^7
- 12/13*(b*x + a)^(13/2)*a^5/b^7 + 2/11*(b*x + a)^(11/2)*a^6/b^7
```

Fricas [A] time = 1.52381, size = 340, normalized size = 2.68

$$\frac{2\left(323323\,b^{11}x^{11}+1431859\,ab^{10}x^{10}+2406690\,a^{2}b^{9}x^{9}+1826110\,a^{3}b^{8}x^{8}+530959\,a^{4}b^{7}x^{7}+231\,a^{5}b^{6}x^{6}-252\,a^{6}b^{5}x^{5}+2\right.}{7436429\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/7436429*(323323*b^11*x^11 + 1431859*a*b^10*x^10 + 2406690*a^2*b^9*x^9 + 1
826110*a^3*b^8*x^8 + 530959*a^4*b^7*x^7 + 231*a^5*b^6*x^6 - 252*a^6*b^5*x^5
 + 280*a^7*b^4*x^4 - 320*a^8*b^3*x^3 + 384*a^9*b^2*x^2 - 512*a^10*b*x + 102
4*a^11)*sqrt(b*x + a)/b^7
```

Sympy [A] time = 45.7565, size = 257, normalized size = 2.02

$$\begin{cases} \frac{2048a^{11}\sqrt{a+bx}}{7436429b^{7}}-\frac{1024a^{10}x\sqrt{a+bx}}{7436429b^{6}}+\frac{768a^{9}x^{2}\sqrt{a+bx}}{7436429b^{5}}-\frac{640a^{8}x^{3}\sqrt{a+bx}}{7436429b^{4}}+\frac{80a^{7}x^{4}\sqrt{a+bx}}{1062347b^{3}}-\frac{72a^{6}x^{5}\sqrt{a+bx}}{1062347b^{2}}+\frac{6a^{5}x^{6}\sqrt{a+bx}}{96577b}+\frac{7426a^{4}x^{7}\sqrt{a+bx}}{52003}+\frac{2554 \\ \frac{a^{\frac{9}{2}}x^{7}}{7} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((2048*a**11*sqrt(a + b*x)/(7436429*b**7) - 1024*a**10*x*sqrt(a +
b*x)/(7436429*b**6) + 768*a**9*x**2*sqrt(a + b*x)/(7436429*b**5) - 640*a**8
*x**3*sqrt(a + b*x)/(7436429*b**4) + 80*a**7*x**4*sqrt(a + b*x)/(1062347*b*
*3) - 72*a**6*x**5*sqrt(a + b*x)/(1062347*b**2) + 6*a**5*x**6*sqrt(a + b*x)
/(96577*b) + 7426*a**4*x**7*sqrt(a + b*x)/52003 + 25540*a**3*b*x**8*sqrt(a
+ b*x)/52003 + 1980*a**2*b**2*x**9*sqrt(a + b*x)/3059 + 62*a*b**3*x**10*sqr
t(a + b*x)/161 + 2*b**4*x**11*sqrt(a + b*x)/23, Ne(b, 0)), (a**(9/2)*x**7/7
, True))
```

Giac [B] time = 1.26437, size = 757, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/334639305*(7429*(3003*(b*x + a)^(15/2) - 20790*(b*x + a)^(13/2)*a + 61425
*(b*x + a)^(11/2)*a^2 - 100100*(b*x + a)^(9/2)*a^3 + 96525*(b*x + a)^(7/2)*
a^4 - 54054*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6)*a^4/b^6 + 1223
6*(6435*(b*x + a)^(17/2) - 51051*(b*x + a)^(15/2)*a + 176715*(b*x + a)^(13/
2)*a^2 - 348075*(b*x + a)^(11/2)*a^3 + 425425*(b*x + a)^(9/2)*a^4 - 328185*
(b*x + a)^(7/2)*a^5 + 153153*(b*x + a)^(5/2)*a^6 - 36465*(b*x + a)^(3/2)*a^
7)*a^3/b^6 + 966*(109395*(b*x + a)^(19/2) - 978120*(b*x + a)^(17/2)*a + 387
9876*(b*x + a)^(15/2)*a^2 - 8953560*(b*x + a)^(13/2)*a^3 + 13226850*(b*x +
a)^(11/2)*a^4 - 12932920*(b*x + a)^(9/2)*a^5 + 8314020*(b*x + a)^(7/2)*a^6
- 3325608*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^8)*a^2/b^6 + 276*(
230945*(b*x + a)^(21/2) - 2297295*(b*x + a)^(19/2)*a + 10270260*(b*x + a)^(
17/2)*a^2 - 27159132*(b*x + a)^(15/2)*a^3 + 47006190*(b*x + a)^(13/2)*a^4 -
 55552770*(b*x + a)^(11/2)*a^5 + 45265220*(b*x + a)^(9/2)*a^6 - 24942060*(b
*x + a)^(7/2)*a^7 + 8729721*(b*x + a)^(5/2)*a^8 - 1616615*(b*x + a)^(3/2)*a
^9)*a/b^6 + 15*(969969*(b*x + a)^(23/2) - 10623470*(b*x + a)^(21/2)*a + 528
37785*(b*x + a)^(19/2)*a^2 - 157477320*(b*x + a)^(17/2)*a^3 + 312330018*(b*
x + a)^(15/2)*a^4 - 432456948*(b*x + a)^(13/2)*a^5 + 425904570*(b*x + a)^(1
1/2)*a^6 - 297457160*(b*x + a)^(9/2)*a^7 + 143416845*(b*x + a)^(7/2)*a^8 -
44618574*(b*x + a)^(5/2)*a^9 + 7436429*(b*x + a)^(3/2)*a^10)/b^6)/b
```

3.311 $\int x^5(a+bx)^{9/2}\,dx$

Optimal. Leaf size=110

$$\frac{20a^2(a+bx)^{17/2}}{17b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{2(a+bx)^{21/2}}{21b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}$$

```
[Out] (-2*a^5*(a + b*x)^(11/2))/(11*b^6) + (10*a^4*(a + b*x)^(13/2))/(13*b^6) - (
4*a^3*(a + b*x)^(15/2))/(3*b^6) + (20*a^2*(a + b*x)^(17/2))/(17*b^6) - (10*
a*(a + b*x)^(19/2))/(19*b^6) + (2*(a + b*x)^(21/2))/(21*b^6)
```

Rubi [A] time = 0.0321136, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{20a^2(a+bx)^{17/2}}{17b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{2(a+bx)^{21/2}}{21b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^5*(a + b*x)^(11/2))/(11*b^6) + (10*a^4*(a + b*x)^(13/2))/(13*b^6) - (
4*a^3*(a + b*x)^(15/2))/(3*b^6) + (20*a^2*(a + b*x)^(17/2))/(17*b^6) - (10*
a*(a + b*x)^(19/2))/(19*b^6) + (2*(a + b*x)^(21/2))/(21*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^5(a+bx)^{9/2}\,dx=\int\left(-\frac{a^5(a+bx)^{9/2}}{b^5}+\frac{5a^4(a+bx)^{11/2}}{b^5}-\frac{10a^3(a+bx)^{13/2}}{b^5}+\frac{10a^2(a+bx)^{15/2}}{b^5}-\frac{5a(a+bx)^{17/2}}{b^5}+\frac{(a+$$

$$=-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{20a^2(a+bx)^{17/2}}{17b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}+\frac{2(a+$$

Mathematica [A] time = 0.0646423, size = 68, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(-4576a^3b^2x^2+11440a^2b^3x^3+1408a^4bx-256a^5-24310ab^4x^4+46189b^5x^5\right)}{969969b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-256*a^5 + 1408*a^4*b*x - 4576*a^3*b^2*x^2 + 11440*a^2
*b^3*x^3 - 24310*a*b^4*x^4 + 46189*b^5*x^5))/(969969*b^6)
```

Maple [A] time = 0.005, size = 65, normalized size = 0.6

$$-\frac{-92378\,b^{5}x^{5}+48620\,a b^{4}x^{4}-22880\,a^{2}b^{3}x^{3}+9152\,a^{3}b^{2}x^{2}-2816\,a^{4}bx+512\,a^{5}}{969969\,b^{6}}\left(bx+a\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^(9/2),x)
```

```
[Out] -2/969969*(b*x+a)^(11/2)*(-46189*b^5*x^5+24310*a*b^4*x^4-11440*a^2*b^3*x^3+
4576*a^3*b^2*x^2-1408*a^4*b*x+256*a^5)/b^6
```

Maxima [A] time = 1.05034, size = 116, normalized size = 1.05

$$\frac{2\left(bx+a\right)^{\frac{21}{2}}}{21\,b^{6}}-\frac{10\left(bx+a\right)^{\frac{19}{2}}a}{19\,b^{6}}+\frac{20\left(bx+a\right)^{\frac{17}{2}}a^{2}}{17\,b^{6}}-\frac{4\left(bx+a\right)^{\frac{15}{2}}a^{3}}{3\,b^{6}}+\frac{10\left(bx+a\right)^{\frac{13}{2}}a^{4}}{13\,b^{6}}-\frac{2\left(bx+a\right)^{\frac{11}{2}}a^{5}}{11\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/21*(b*x + a)^(21/2)/b^6 - 10/19*(b*x + a)^(19/2)*a/b^6 + 20/17*(b*x + a)^
(17/2)*a^2/b^6 - 4/3*(b*x + a)^(15/2)*a^3/b^6 + 10/13*(b*x + a)^(13/2)*a^4/
b^6 - 2/11*(b*x + a)^(11/2)*a^5/b^6
```

Fricas [A] time = 1.53445, size = 297, normalized size = 2.7

$$\frac{2\left(46189\,b^{10}x^{10}+206635\,a b^{9}x^{9}+351780\,a^{2}b^{8}x^{8}+271414\,a^{3}b^{7}x^{7}+80773\,a^{4}b^{6}x^{6}+63\,a^{5}b^{5}x^{5}-70\,a^{6}b^{4}x^{4}+80\,a^{7}b^{3}x\right.}{969969\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/969969*(46189*b^10*x^10 + 206635*a*b^9*x^9 + 351780*a^2*b^8*x^8 + 271414*
a^3*b^7*x^7 + 80773*a^4*b^6*x^6 + 63*a^5*b^5*x^5 - 70*a^6*b^4*x^4 + 80*a^7*
b^3*x^3 - 96*a^8*b^2*x^2 + 128*a^9*b*x - 256*a^10)*sqrt(b*x + a)/b^6
```

Sympy [A] time = 36.2401, size = 235, normalized size = 2.14

$$\begin{cases} -\frac{512a^{10}\sqrt{a+bx}}{969969b^{6}}+\frac{256a^{9}x\sqrt{a+bx}}{969969b^{5}}-\frac{64a^{8}x^{2}\sqrt{a+bx}}{323323b^{4}}+\frac{160a^{7}x^{3}\sqrt{a+bx}}{969969b^{3}}-\frac{20a^{6}x^{4}\sqrt{a+bx}}{138567b^{2}}+\frac{6a^{5}x^{5}\sqrt{a+bx}}{46189b}+\frac{2098a^{4}x^{6}\sqrt{a+bx}}{12597}+\frac{3796a^{3}bx^{7}\sqrt{a+bx}}{6783}+ \\ \frac{a^{\frac{9}{2}}x^{6}}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-512*a**10*sqrt(a + b*x)/(969969*b**6) + 256*a**9*x*sqrt(a + b*x
)/(969969*b**5) - 64*a**8*x**2*sqrt(a + b*x)/(323323*b**4) + 160*a**7*x**3*
```

```
sqrt(a + b*x)/(969969*b**3) - 20*a**6*x**4*sqrt(a + b*x)/(138567*b**2) + 6*
a**5*x**5*sqrt(a + b*x)/(46189*b) + 2098*a**4*x**6*sqrt(a + b*x)/12597 + 37
96*a**3*b*x**7*sqrt(a + b*x)/6783 + 1640*a**2*b**2*x**8*sqrt(a + b*x)/2261
+ 170*a*b**3*x**9*sqrt(a + b*x)/399 + 2*b**4*x**10*sqrt(a + b*x)/21, Ne(b,
0)), (a**(9/2)*x**6/6, True))
```

Giac [B] time = 1.27336, size = 676, normalized size = 6.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/14549535*(1615*(693*(b*x + a)^(13/2) - 4095*(b*x + a)^(11/2)*a + 10010*(b
*x + a)^(9/2)*a^2 - 12870*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 -
3003*(b*x + a)^(3/2)*a^5)*a^4/b^5 + 1292*(3003*(b*x + a)^(15/2) - 20790*(b*
x + a)^(13/2)*a + 61425*(b*x + a)^(11/2)*a^2 - 100100*(b*x + a)^(9/2)*a^3 +
 96525*(b*x + a)^(7/2)*a^4 - 54054*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3
/2)*a^6)*a^3/b^5 + 798*(6435*(b*x + a)^(17/2) - 51051*(b*x + a)^(15/2)*a +
176715*(b*x + a)^(13/2)*a^2 - 348075*(b*x + a)^(11/2)*a^3 + 425425*(b*x + a
)^(9/2)*a^4 - 328185*(b*x + a)^(7/2)*a^5 + 153153*(b*x + a)^(5/2)*a^6 - 364
65*(b*x + a)^(3/2)*a^7)*a^2/b^5 + 28*(109395*(b*x + a)^(19/2) - 978120*(b*x
 + a)^(17/2)*a + 3879876*(b*x + a)^(15/2)*a^2 - 8953560*(b*x + a)^(13/2)*a^
3 + 13226850*(b*x + a)^(11/2)*a^4 - 12932920*(b*x + a)^(9/2)*a^5 + 8314020*
(b*x + a)^(7/2)*a^6 - 3325608*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*
a^8)*a/b^5 + 3*(230945*(b*x + a)^(21/2) - 2297295*(b*x + a)^(19/2)*a + 1027
0260*(b*x + a)^(17/2)*a^2 - 27159132*(b*x + a)^(15/2)*a^3 + 47006190*(b*x +
 a)^(13/2)*a^4 - 55552770*(b*x + a)^(11/2)*a^5 + 45265220*(b*x + a)^(9/2)*a
^6 - 24942060*(b*x + a)^(7/2)*a^7 + 8729721*(b*x + a)^(5/2)*a^8 - 1616615*(
b*x + a)^(3/2)*a^9)/b^5)/b
```

3.312 $\int x^4(a+bx)^{9/2}\,dx$

Optimal. Leaf size=91

$$\frac{4a^2(a+bx)^{15/2}}{5b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{2a^4(a+bx)^{11/2}}{11b^5}+\frac{2(a+bx)^{19/2}}{19b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}$$

```
[Out] (2*a^4*(a + b*x)^(11/2))/(11*b^5) - (8*a^3*(a + b*x)^(13/2))/(13*b^5) + (4*
a^2*(a + b*x)^(15/2))/(5*b^5) - (8*a*(a + b*x)^(17/2))/(17*b^5) + (2*(a + b
*x)^(19/2))/(19*b^5)
```

Rubi [A] time = 0.023489, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{4a^2(a+bx)^{15/2}}{5b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{2a^4(a+bx)^{11/2}}{11b^5}+\frac{2(a+bx)^{19/2}}{19b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^4*(a + b*x)^(11/2))/(11*b^5) - (8*a^3*(a + b*x)^(13/2))/(13*b^5) + (4*
a^2*(a + b*x)^(15/2))/(5*b^5) - (8*a*(a + b*x)^(17/2))/(17*b^5) + (2*(a + b
*x)^(19/2))/(19*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^{9/2}\,dx&=\int\left(\frac{a^4(a+bx)^{9/2}}{b^4}-\frac{4a^3(a+bx)^{11/2}}{b^4}+\frac{6a^2(a+bx)^{13/2}}{b^4}-\frac{4a(a+bx)^{15/2}}{b^4}+\frac{(a+bx)^{17/2}}{b^4}\right)dx\\&=\frac{2a^4(a+bx)^{11/2}}{11b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{4a^2(a+bx)^{15/2}}{5b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}+\frac{2(a+bx)^{19/2}}{19b^5}\end{aligned}$$

Mathematica [A] time = 0.0790657, size = 57, normalized size = 0.63

$$\frac{2(a+bx)^{11/2}\left(2288a^2b^2x^2-704a^3bx+128a^4-5720ab^3x^3+12155b^4x^4\right)}{230945b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(128*a^4 - 704*a^3*b*x + 2288*a^2*b^2*x^2 - 5720*a*b^3*
x^3 + 12155*b^4*x^4))/(230945*b^5)
```

Maple [A] time = 0.005, size = 54, normalized size = 0.6

$$\frac{24310\,x^4b^4 - 11440\,ax^3b^3 + 4576\,a^2x^2b^2 - 1408\,a^3xb + 256\,a^4}{230945\,b^5}(bx+a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^(9/2),x)
```

```
[Out] 2/230945*(b*x+a)^(11/2)*(12155*b^4*x^4-5720*a*b^3*x^3+2288*a^2*b^2*x^2-704*
a^3*b*x+128*a^4)/b^5
```

Maxima [A] time = 1.06945, size = 96, normalized size = 1.05

$$\frac{2\,(bx+a)^{\frac{19}{2}}}{19\,b^5} - \frac{8\,(bx+a)^{\frac{17}{2}}a}{17\,b^5} + \frac{4\,(bx+a)^{\frac{15}{2}}a^2}{5\,b^5} - \frac{8\,(bx+a)^{\frac{13}{2}}a^3}{13\,b^5} + \frac{2\,(bx+a)^{\frac{11}{2}}a^4}{11\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/19*(b*x + a)^(19/2)/b^5 - 8/17*(b*x + a)^(17/2)*a/b^5 + 4/5*(b*x + a)^(15
/2)*a^2/b^5 - 8/13*(b*x + a)^(13/2)*a^3/b^5 + 2/11*(b*x + a)^(11/2)*a^4/b^5
```

Fricas [A] time = 1.47349, size = 265, normalized size = 2.91

$$\frac{2\left(12155\,b^9x^9 + 55055\,ab^8x^8 + 95238\,a^2b^7x^7 + 75086\,a^3b^6x^6 + 23063\,a^4b^5x^5 + 35\,a^5b^4x^4 - 40\,a^6b^3x^3 + 48\,a^7b^2x^2 - 64\,a^8\right.}{230945\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/230945*(12155*b^9*x^9 + 55055*a*b^8*x^8 + 95238*a^2*b^7*x^7 + 75086*a^3*b
^6*x^6 + 23063*a^4*b^5*x^5 + 35*a^5*b^4*x^4 - 40*a^6*b^3*x^3 + 48*a^7*b^2*x
^2 - 64*a^8*b*x + 128*a^9)*sqrt(b*x + a)/b^5
```

Sympy [A] time = 31.1687, size = 212, normalized size = 2.33

$$\begin{cases} \frac{256a^9\sqrt{a+bx}}{230945b^5} - \frac{128a^8x\sqrt{a+bx}}{230945b^4} + \frac{96a^7x^2\sqrt{a+bx}}{230945b^3} - \frac{16a^6x^3\sqrt{a+bx}}{46189b^2} + \frac{14a^5x^4\sqrt{a+bx}}{46189b} + \frac{46126a^4x^5\sqrt{a+bx}}{230945} + \frac{13652a^3bx^6\sqrt{a+bx}}{20995} + \frac{1332a^2b^2x^7\sqrt{a+bx}}{1615} + \\ \frac{a^{\frac{9}{2}}x^5}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((256*a**9*sqrt(a + b*x)/(230945*b**5) - 128*a**8*x*sqrt(a + b*x)/
(230945*b**4) + 96*a**7*x**2*sqrt(a + b*x)/(230945*b**3) - 16*a**6*x**3*sqr
```

```
t(a + b*x)/(46189*b**2) + 14*a**5*x**4*sqrt(a + b*x)/(46189*b) + 46126*a**4
*x**5*sqrt(a + b*x)/230945 + 13652*a**3*b*x**6*sqrt(a + b*x)/20995 + 1332*a
**2*b**2*x**7*sqrt(a + b*x)/1615 + 154*a*b**3*x**8*sqrt(a + b*x)/323 + 2*b*
*4*x**9*sqrt(a + b*x)/19, Ne(b, 0)), (a**(9/2)*x**5/5, True))
```

Giac [B] time = 1.19301, size = 595, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/14549535*(4199*(315*(b*x + a)^(11/2) - 1540*(b*x + a)^(9/2)*a + 2970*(b*x
 + a)^(7/2)*a^2 - 2772*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4)*a^4/
b^4 + 6460*(693*(b*x + a)^(13/2) - 4095*(b*x + a)^(11/2)*a + 10010*(b*x + a
)^(9/2)*a^2 - 12870*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 3003*(
b*x + a)^(3/2)*a^5)*a^3/b^4 + 1938*(3003*(b*x + a)^(15/2) - 20790*(b*x + a)
^(13/2)*a + 61425*(b*x + a)^(11/2)*a^2 - 100100*(b*x + a)^(9/2)*a^3 + 96525
*(b*x + a)^(7/2)*a^4 - 54054*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^
6)*a^2/b^4 + 532*(6435*(b*x + a)^(17/2) - 51051*(b*x + a)^(15/2)*a + 176715
*(b*x + a)^(13/2)*a^2 - 348075*(b*x + a)^(11/2)*a^3 + 425425*(b*x + a)^(9/2
)*a^4 - 328185*(b*x + a)^(7/2)*a^5 + 153153*(b*x + a)^(5/2)*a^6 - 36465*(b*
x + a)^(3/2)*a^7)*a/b^4 + 7*(109395*(b*x + a)^(19/2) - 978120*(b*x + a)^(17
/2)*a + 3879876*(b*x + a)^(15/2)*a^2 - 8953560*(b*x + a)^(13/2)*a^3 + 13226
850*(b*x + a)^(11/2)*a^4 - 12932920*(b*x + a)^(9/2)*a^5 + 8314020*(b*x + a)
^(7/2)*a^6 - 3325608*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^8)/b^4)
/b
```

3.313 $\int x^3(a+bx)^{9/2}\,dx$

Optimal. Leaf size=72

$$\frac{6a^2(a+bx)^{13/2}}{13b^4}-\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{2(a+bx)^{17/2}}{17b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}$$

```
[Out] (-2*a^3*(a + b*x)^(11/2))/(11*b^4) + (6*a^2*(a + b*x)^(13/2))/(13*b^4) - (2
*a*(a + b*x)^(15/2))/(5*b^4) + (2*(a + b*x)^(17/2))/(17*b^4)
```

Rubi [A] time = 0.0177268, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6a^2(a+bx)^{13/2}}{13b^4}-\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{2(a+bx)^{17/2}}{17b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(11/2))/(11*b^4) + (6*a^2*(a + b*x)^(13/2))/(13*b^4) - (2
*a*(a + b*x)^(15/2))/(5*b^4) + (2*(a + b*x)^(17/2))/(17*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3(a+bx)^{9/2}\,dx &= \int\left(-\frac{a^3(a+bx)^{9/2}}{b^3}+\frac{3a^2(a+bx)^{11/2}}{b^3}-\frac{3a(a+bx)^{13/2}}{b^3}+\frac{(a+bx)^{15/2}}{b^3}\right)dx\\
&= -\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{6a^2(a+bx)^{13/2}}{13b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}+\frac{2(a+bx)^{17/2}}{17b^4}
\end{aligned}$$

Mathematica [A] time = 0.0611463, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{11/2}\left(88a^2bx-16a^3-286ab^2x^2+715b^3x^3\right)}{12155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-16*a^3 + 88*a^2*b*x - 286*a*b^2*x^2 + 715*b^3*x^3))/(
12155*b^4)
```

Maple [A] time = 0.005, size = 43, normalized size = 0.6

$$-\frac{-1430\,b^3x^3+572\,ab^2x^2-176\,a^2bx+32\,a^3}{12155\,b^4}(bx+a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(9/2),x)
```

```
[Out] -2/12155*(b*x+a)^(11/2)*(-715*b^3*x^3+286*a*b^2*x^2-88*a^2*b*x+16*a^3)/b^4
```

Maxima [A] time = 1.00772, size = 76, normalized size = 1.06

$$\frac{2\,(bx+a)^{\frac{17}{2}}}{17\,b^4}-\frac{2\,(bx+a)^{\frac{15}{2}}a}{5\,b^4}+\frac{6\,(bx+a)^{\frac{13}{2}}a^2}{13\,b^4}-\frac{2\,(bx+a)^{\frac{11}{2}}a^3}{11\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/17*(b*x + a)^(17/2)/b^4 - 2/5*(b*x + a)^(15/2)*a/b^4 + 6/13*(b*x + a)^(13
/2)*a^2/b^4 - 2/11*(b*x + a)^(11/2)*a^3/b^4
```

Fricas [A] time = 1.46976, size = 227, normalized size = 3.15

$$\frac{2\left(715\,b^8x^8+3289\,ab^7x^7+5808\,a^2b^6x^6+4714\,a^3b^5x^5+1515\,a^4b^4x^4+5\,a^5b^3x^3-6\,a^6b^2x^2+8\,a^7bx-16\,a^8\right)\sqrt{bx+a}}{12155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/12155*(715*b^8*x^8 + 3289*a*b^7*x^7 + 5808*a^2*b^6*x^6 + 4714*a^3*b^5*x^5
 + 1515*a^4*b^4*x^4 + 5*a^5*b^3*x^3 - 6*a^6*b^2*x^2 + 8*a^7*b*x - 16*a^8)*s
qrt(b*x + a)/b^4
```

Sympy [A] time = 24.4038, size = 190, normalized size = 2.64

$$\begin{cases}-\frac{32a^8\sqrt{a+bx}}{12155b^4}+\frac{16a^7x\sqrt{a+bx}}{12155b^3}-\frac{12a^6x^2\sqrt{a+bx}}{12155b^2}+\frac{2a^5x^3\sqrt{a+bx}}{2431b}+\frac{606a^4x^4\sqrt{a+bx}}{2431}+\frac{9428a^3bx^5\sqrt{a+bx}}{12155}+\frac{1056a^2b^2x^6\sqrt{a+bx}}{1105}+\frac{46ab^3x^7\sqrt{a+bx}}{85}+ \\ \frac{a^{\frac{9}{2}}x^4}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-32*a**8*sqrt(a + b*x)/(12155*b**4) + 16*a**7*x*sqrt(a + b*x)/(1
2155*b**3) - 12*a**6*x**2*sqrt(a + b*x)/(12155*b**2) + 2*a**5*x**3*sqrt(a +
 b*x)/(2431*b) + 606*a**4*x**4*sqrt(a + b*x)/2431 + 9428*a**3*b*x**5*sqrt(a
 + b*x)/12155 + 1056*a**2*b**2*x**6*sqrt(a + b*x)/1105 + 46*a*b**3*x**7*sqr
t(a + b*x)/85 + 2*b**4*x**8*sqrt(a + b*x)/17, Ne(b, 0)), (a**(9/2)*x**4/4,
```

```
True))
```

Giac [B] time = 1.27581, size = 514, normalized size = 7.14

$$2\left(\frac{2431\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^2-105\,(bx+a)^{\frac{3}{2}}a^3\right)a^4}{b^3}+\frac{884\left(315\,(bx+a)^{\frac{11}{2}}-1540\,(bx+a)^{\frac{9}{2}}a+2970\,(bx+a)^{\frac{7}{2}}a^2-2772\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)\right.}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/765765*(2431*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x + a)^
(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*a^4/b^3 + 884*(315*(b*x + a)^(11/2) -
1540*(b*x + a)^(9/2)*a + 2970*(b*x + a)^(7/2)*a^2 - 2772*(b*x + a)^(5/2)*a^
3 + 1155*(b*x + a)^(3/2)*a^4)*a^3/b^3 + 510*(693*(b*x + a)^(13/2) - 4095*(b
*x + a)^(11/2)*a + 10010*(b*x + a)^(9/2)*a^2 - 12870*(b*x + a)^(7/2)*a^3 +
9009*(b*x + a)^(5/2)*a^4 - 3003*(b*x + a)^(3/2)*a^5)*a^2/b^3 + 68*(3003*(b*
x + a)^(15/2) - 20790*(b*x + a)^(13/2)*a + 61425*(b*x + a)^(11/2)*a^2 - 100
100*(b*x + a)^(9/2)*a^3 + 96525*(b*x + a)^(7/2)*a^4 - 54054*(b*x + a)^(5/2)
*a^5 + 15015*(b*x + a)^(3/2)*a^6)*a/b^3 + 7*(6435*(b*x + a)^(17/2) - 51051*
(b*x + a)^(15/2)*a + 176715*(b*x + a)^(13/2)*a^2 - 348075*(b*x + a)^(11/2)*
a^3 + 425425*(b*x + a)^(9/2)*a^4 - 328185*(b*x + a)^(7/2)*a^5 + 153153*(b*x
 + a)^(5/2)*a^6 - 36465*(b*x + a)^(3/2)*a^7)/b^3)/b
```

3.314 $\int x^2(a+bx)^{9/2}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{11/2}}{11b^3}+\frac{2(a+bx)^{15/2}}{15b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}$$

```
[Out] (2*a^2*(a + b*x)^(11/2))/(11*b^3) - (4*a*(a + b*x)^(13/2))/(13*b^3) + (2*(a
 + b*x)^(15/2))/(15*b^3)
```

Rubi [A] time = 0.0134202, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{11/2}}{11b^3}+\frac{2(a+bx)^{15/2}}{15b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(9/2),x]
```


```
[Out] (2*a^2*(a + b*x)^(11/2))/(11*b^3) - (4*a*(a + b*x)^(13/2))/(13*b^3) + (2*(a
 + b*x)^(15/2))/(15*b^3)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{9/2}\,dx &= \int\left(\frac{a^2(a+bx)^{9/2}}{b^2}-\frac{2a(a+bx)^{11/2}}{b^2}+\frac{(a+bx)^{13/2}}{b^2}\right)dx\\ &= \frac{2a^2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}+\frac{2(a+bx)^{15/2}}{15b^3}\end{aligned}$$

Mathematica [A] time = 0.0493958, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{11/2}\left(8a^2-44abx+143b^2x^2\right)}{2145b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(8*a^2 - 44*a*b*x + 143*b^2*x^2))/(2145*b^3)
```

Maple [A] time = 0.004, size = 32, normalized size = 0.6

$$\frac{286\,b^2x^2-88\,abx+16\,a^2}{2145\,b^3}\,(bx+a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(9/2),x)
```

```
[Out] 2/2145*(b*x+a)^(11/2)*(143*b^2*x^2-44*a*b*x+8*a^2)/b^3
```

Maxima [A] time = 1.06994, size = 55, normalized size = 1.04

$$\frac{2\,(bx+a)^{\frac{15}{2}}}{15\,b^3}-\frac{4\,(bx+a)^{\frac{13}{2}}\,a}{13\,b^3}+\frac{2\,(bx+a)^{\frac{11}{2}}\,a^2}{11\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/15*(b*x + a)^(15/2)/b^3 - 4/13*(b*x + a)^(13/2)*a/b^3 + 2/11*(b*x + a)^(1
1/2)*a^2/b^3
```

Fricas [B] time = 1.49029, size = 200, normalized size = 3.77

$$\frac{2\left(143\,b^7x^7+671\,ab^6x^6+1218\,a^2b^5x^5+1030\,a^3b^4x^4+355\,a^4b^3x^3+3\,a^5b^2x^2-4\,a^6bx+8\,a^7\right)\sqrt{bx+a}}{2145\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/2145*(143*b^7*x^7 + 671*a*b^6*x^6 + 1218*a^2*b^5*x^5 + 1030*a^3*b^4*x^4 +
 355*a^4*b^3*x^3 + 3*a^5*b^2*x^2 - 4*a^6*b*x + 8*a^7)*sqrt(b*x + a)/b^3
```

Sympy [A] time = 23.4346, size = 168, normalized size = 3.17

$$\begin{cases} \frac{16a^7\sqrt{a+bx}}{2145b^3}-\frac{8a^6x\sqrt{a+bx}}{2145b^2}+\frac{2a^5x^2\sqrt{a+bx}}{715b}+\frac{142a^4x^3\sqrt{a+bx}}{429}+\frac{412a^3bx^4\sqrt{a+bx}}{429}+\frac{812a^2b^2x^5\sqrt{a+bx}}{715}+\frac{122ab^3x^6\sqrt{a+bx}}{195}+\frac{2b^4x^7\sqrt{a+bx}}{15} & \text{for}\ b\neq \\ \frac{a^{\frac{9}{2}}x^3}{3} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((16*a**7*sqrt(a + b*x)/(2145*b**3) - 8*a**6*x*sqrt(a + b*x)/(2145
*b**2) + 2*a**5*x**2*sqrt(a + b*x)/(715*b) + 142*a**4*x**3*sqrt(a + b*x)/42
9 + 412*a**3*b*x**4*sqrt(a + b*x)/429 + 812*a**2*b**2*x**5*sqrt(a + b*x)/71
5 + 122*a*b**3*x**6*sqrt(a + b*x)/195 + 2*b**4*x**7*sqrt(a + b*x)/15, Ne(b,
 0)), (a**(9/2)*x**3/3, True))
```

Giac [B] time = 1.19632, size = 432, normalized size = 8.15

$$2\left(\frac{429\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)a^4}{b^2}+\frac{572\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^2-105\,(bx+a)^{\frac{3}{2}}a^3\right)a^3}{b^2}+\frac{78\left(315\,(bx+a)^{\frac{11}{2}}-1540\,(bx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/45045*(429*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2
)*a^2)*a^4/b^2 + 572*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x
 + a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*a^3/b^2 + 78*(315*(b*x + a)^(11/
2) - 1540*(b*x + a)^(9/2)*a + 2970*(b*x + a)^(7/2)*a^2 - 2772*(b*x + a)^(5/
2)*a^3 + 1155*(b*x + a)^(3/2)*a^4)*a^2/b^2 + 20*(693*(b*x + a)^(13/2) - 409
5*(b*x + a)^(11/2)*a + 10010*(b*x + a)^(9/2)*a^2 - 12870*(b*x + a)^(7/2)*a^
3 + 9009*(b*x + a)^(5/2)*a^4 - 3003*(b*x + a)^(3/2)*a^5)*a/b^2 + (3003*(b*x
 + a)^(15/2) - 20790*(b*x + a)^(13/2)*a + 61425*(b*x + a)^(11/2)*a^2 - 1001
00*(b*x + a)^(9/2)*a^3 + 96525*(b*x + a)^(7/2)*a^4 - 54054*(b*x + a)^(5/2)*
a^5 + 15015*(b*x + a)^(3/2)*a^6)/b^2)/b
```

3.315 $\int x(a+bx)^{9/2}\,dx$

Optimal. Leaf size=34

$$\frac{2(a+bx)^{13/2}}{13b^2}-\frac{2a(a+bx)^{11/2}}{11b^2}$$

```
[Out] (-2*a*(a + b*x)^(11/2))/(11*b^2) + (2*(a + b*x)^(13/2))/(13*b^2)
```

Rubi [A] time = 0.0080125, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{13/2}}{13b^2}-\frac{2a(a+bx)^{11/2}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a*(a + b*x)^(11/2))/(11*b^2) + (2*(a + b*x)^(13/2))/(13*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{9/2}\,dx &= \int \left(-\frac{a(a+bx)^{9/2}}{b}+\frac{(a+bx)^{11/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{11/2}}{11b^2}+\frac{2(a+bx)^{13/2}}{13b^2}\end{aligned}$$

Mathematica [A] time = 0.0289238, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{11/2}(11bx-2a)}{143b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-2*a + 11*b*x))/(143*b^2)
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$-\frac{-22\,bx+4\,a}{143\,b^2}\,(bx+a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(9/2),x)
```

```
[Out] -2/143*(b*x+a)^(11/2)*(-11*b*x+2*a)/b^2
```

Maxima [A] time = 1.02162, size = 35, normalized size = 1.03

$$\frac{2\,(bx+a)^{\frac{13}{2}}}{13\,b^2}-\frac{2\,(bx+a)^{\frac{11}{2}}a}{11\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*x + a)^(13/2)/b^2 - 2/11*(b*x + a)^(11/2)*a/b^2
```

Fricas [B] time = 1.48734, size = 166, normalized size = 4.88

$$\frac{2\left(11\,b^6x^6+53\,ab^5x^5+100\,a^2b^4x^4+90\,a^3b^3x^3+35\,a^4b^2x^2+a^5bx-2\,a^6\right)\sqrt{bx+a}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/143*(11*b^6*x^6 + 53*a*b^5*x^5 + 100*a^2*b^4*x^4 + 90*a^3*b^3*x^3 + 35*a^
4*b^2*x^2 + a^5*b*x - 2*a^6)*sqrt(b*x + a)/b^2
```

Sympy [A] time = 17.7795, size = 146, normalized size = 4.29

$$\begin{cases}-\frac{4a^6\sqrt{a+bx}}{143b^2}+\frac{2a^5x\sqrt{a+bx}}{143b}+\frac{70a^4x^2\sqrt{a+bx}}{143}+\frac{180a^3bx^3\sqrt{a+bx}}{143}+\frac{200a^2b^2x^4\sqrt{a+bx}}{143}+\frac{106ab^3x^5\sqrt{a+bx}}{143}+\frac{2b^4x^6\sqrt{a+bx}}{13} & \text{for } b\neq 0\\ \frac{a^{\frac{9}{2}}x^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-4*a**6*sqrt(a + b*x)/(143*b**2) + 2*a**5*x*sqrt(a + b*x)/(143*b
) + 70*a**4*x**2*sqrt(a + b*x)/143 + 180*a**3*b*x**3*sqrt(a + b*x)/143 + 20
0*a**2*b**2*x**4*sqrt(a + b*x)/143 + 106*a*b**3*x**5*sqrt(a + b*x)/143 + 2*
b**4*x**6*sqrt(a + b*x)/13, Ne(b, 0)), (a**(9/2)*x**2/2, True))
```

Giac [B] time = 1.22893, size = 352, normalized size = 10.35

$$2\left(\frac{3003\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)a^4}{b}+\frac{1716\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)a^3}{b}+\frac{858\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^2-105\,(bx+a)\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/45045*(3003*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)*a^4/b + 1716*(15*(b
*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)*a^3/b + 858*
(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x + a)^(5/2)*a^2 - 105
*(b*x + a)^(3/2)*a^3)*a^2/b + 52*(315*(b*x + a)^(11/2) - 1540*(b*x + a)^(9/
2)*a + 2970*(b*x + a)^(7/2)*a^2 - 2772*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)
^(3/2)*a^4)*a/b + 5*(693*(b*x + a)^(13/2) - 4095*(b*x + a)^(11/2)*a + 10010
*(b*x + a)^(9/2)*a^2 - 12870*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4
 - 3003*(b*x + a)^(3/2)*a^5)/b)/b
```

3.316 $\int (a+bx)^{9/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{11/2}}{11b}$$

```
[Out] (2*(a + b*x)^(11/2))/(11*b)
```

Rubi [A] time = 0.0015142, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2),x]

[Out] (2*(a + b*x)^(11/2))/(11*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{9/2}\,dx = \frac{2(a+bx)^{11/2}}{11b}$$

Mathematica [A] time = 0.0177189, size = 16, normalized size = 1.

$$\frac{2(a+bx)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2),x]

[Out] (2*(a + b*x)^(11/2))/(11*b)
```

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$\frac{2}{11\,b}(bx+a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2),x)
```

```
[Out] 2/11*(b*x+a)^(11/2)/b
```

Maxima [A] time = 1.13427, size = 16, normalized size = 1.

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b*x + a)^(11/2)/b
```

Fricas [B] time = 1.52622, size = 132, normalized size = 8.25

$$\frac{2\left(b^{5}x^{5}+5\,ab^{4}x^{4}+10\,a^{2}b^{3}x^{3}+10\,a^{3}b^{2}x^{2}+5\,a^{4}bx+a^{5}\right)\sqrt{bx+a}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/11*(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x +
 a^5)*sqrt(b*x + a)/b
```

Sympy [A] time = 0.080241, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{11}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2),x)
```

```
[Out] 2*(a + b*x)**(11/2)/(11*b)
```

Giac [B] time = 1.14426, size = 230, normalized size = 14.38

$$2\left(315\,(bx+a)^{\frac{11}{2}}-1540\,(bx+a)^{\frac{9}{2}}a+2970\,(bx+a)^{\frac{7}{2}}a^{2}-2772\,(bx+a)^{\frac{5}{2}}a^{3}+2310\,(bx+a)^{\frac{3}{2}}a^{4}+924\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/3465*(315*(b*x + a)^(11/2) - 1540*(b*x + a)^(9/2)*a + 2970*(b*x + a)^(7/2
)*a^2 - 2772*(b*x + a)^(5/2)*a^3 + 2310*(b*x + a)^(3/2)*a^4 + 924*(3*(b*x +
 a)^(5/2) - 5*(b*x + a)^(3/2)*a)*a^3 + 198*(15*(b*x + a)^(7/2) - 42*(b*x +
a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)*a^2 + 44*(35*(b*x + a)^(9/2) - 135*(b*
x + a)^(7/2)*a + 189*(b*x + a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*a)/b
```

3.317 $\int \frac{(a+bx)^{9/2}}{x} dx$

Optimal. Leaf size=97

$$2a^4\sqrt{a+bx}+\frac{2}{3}a^3(a+bx)^{3/2}+\frac{2}{5}a^2(a+bx)^{5/2}-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{2}{7}a(a+bx)^{7/2}+\frac{2}{9}(a+bx)^{9/2}$$

```
[Out] 2*a^4*Sqrt[a + b*x] + (2*a^3*(a + b*x)^(3/2))/3 + (2*a^2*(a + b*x)^(5/2))/5
 + (2*a*(a + b*x)^(7/2))/7 + (2*(a + b*x)^(9/2))/9 - 2*a^(9/2)*ArcTanh[Sqrt
[a + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.0342687, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$2a^4\sqrt{a+bx}+\frac{2}{3}a^3(a+bx)^{3/2}+\frac{2}{5}a^2(a+bx)^{5/2}-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{2}{7}a(a+bx)^{7/2}+\frac{2}{9}(a+bx)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x,x]
```

```
[Out] 2*a^4*Sqrt[a + b*x] + (2*a^3*(a + b*x)^(3/2))/3 + (2*a^2*(a + b*x)^(5/2))/5
 + (2*a*(a + b*x)^(7/2))/7 + (2*(a + b*x)^(9/2))/9 - 2*a^(9/2)*ArcTanh[Sqrt
[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x}\,dx &= \frac{2}{9}(a+bx)^{9/2} + a\int \frac{(a+bx)^{7/2}}{x}\,dx \\
&= \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^2\int \frac{(a+bx)^{5/2}}{x}\,dx \\
&= \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^3\int \frac{(a+bx)^{3/2}}{x}\,dx \\
&= \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^4\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^5\int \frac{1}{x\sqrt{a+bx}}\,dx \\
&= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + \frac{(2a^5)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\ldots}\right.}{b} \\
&= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} - 2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+b\ldots}}{\sqrt{a}}\right.
\end{aligned}
$$

Mathematica [A] time = 0.208892, size = 78, normalized size = 0.8

$$\frac{2}{315}\sqrt{a+bx}\left(408a^2b^2x^2+506a^3bx+563a^4+185ab^3x^3+35b^4x^4\right)-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(563*a^4 + 506*a^3*b*x + 408*a^2*b^2*x^2 + 185*a*b^3*x^3 +
 35*b^4*x^4))/315 - 2*a^(9/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A] time = 0.004, size = 74, normalized size = 0.8

$$\frac{2a^3}{3}(bx+a)^{\frac{3}{2}}+\frac{2a^2}{5}(bx+a)^{\frac{5}{2}}+\frac{2a}{7}(bx+a)^{\frac{7}{2}}+\frac{2}{9}(bx+a)^{\frac{9}{2}}-2a^{9/2}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2a^4\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x,x)
```

```
[Out] 2/3*a^3*(b*x+a)^(3/2)+2/5*a^2*(b*x+a)^(5/2)+2/7*a*(b*x+a)^(7/2)+2/9*(b*x+a)
^(9/2)-2*a^(9/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a^4*(b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56437, size = 396, normalized size = 4.08

$$\left[a^{\frac{9}{2}}\log\left(\frac{bx - 2\sqrt{bx+a}\sqrt{a} + 2a}{x}\right) + \frac{2}{315}\left(35b^4x^4 + 185ab^3x^3 + 408a^2b^2x^2 + 506a^3bx + 563a^4\right)\sqrt{bx+a}, 2\sqrt{-a}a^4 \text{ ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(9/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/315*(35*b^4*x^4 +
 185*a*b^3*x^3 + 408*a^2*b^2*x^2 + 506*a^3*b*x + 563*a^4)*sqrt(b*x + a), 2*
sqrt(-a)*a^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/315*(35*b^4*x^4 + 185*a*b
^3*x^3 + 408*a^2*b^2*x^2 + 506*a^3*b*x + 563*a^4)*sqrt(b*x + a)]
```

Sympy [A] time = 12.9398, size = 148, normalized size = 1.53

$$\frac{1126a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{315} + a^{\frac{9}{2}}\log\left(\frac{bx}{a}\right) - 2a^{\frac{9}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right) + \frac{1012a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{315} + \frac{272a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{105} + \frac{74a^{\frac{3}{2}}b^3x^3\sqrt{1+}}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x,x)
```

```
[Out] 1126*a**(9/2)*sqrt(1 + b*x/a)/315 + a**(9/2)*log(b*x/a) - 2*a**(9/2)*log(sq
rt(1 + b*x/a) + 1) + 1012*a**(7/2)*b*x*sqrt(1 + b*x/a)/315 + 272*a**(5/2)*b
**2*x**2*sqrt(1 + b*x/a)/105 + 74*a**(3/2)*b**3*x**3*sqrt(1 + b*x/a)/63 + 2
*sqrt(a)*b**4*x**4*sqrt(1 + b*x/a)/9
```

Giac [A] time = 1.21461, size = 108, normalized size = 1.11

$$\frac{2a^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2}{9}(bx+a)^{\frac{9}{2}} + \frac{2}{7}(bx+a)^{\frac{7}{2}}a + \frac{2}{5}(bx+a)^{\frac{5}{2}}a^2 + \frac{2}{3}(bx+a)^{\frac{3}{2}}a^3 + 2\sqrt{bx+a}a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^5*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/9*(b*x + a)^(9/2) + 2/7*(
b*x + a)^(7/2)*a + 2/5*(b*x + a)^(5/2)*a^2 + 2/3*(b*x + a)^(3/2)*a^3 + 2*sq
rt(b*x + a)*a^4
```

3.318 $\int \frac{(a+bx)^{9/2}}{x^2}\,dx$

Optimal. Leaf size=98

$$3a^2b(a+bx)^{3/2}+9a^3b\sqrt{a+bx}-9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{9/2}}{x}+\frac{9}{7}b(a+bx)^{7/2}+\frac{9}{5}ab(a+bx)^{5/2}$$

```
[Out] 9*a^3*b*Sqrt[a + b*x] + 3*a^2*b*(a + b*x)^(3/2) + (9*a*b*(a + b*x)^(5/2))/5
 + (9*b*(a + b*x)^(7/2))/7 - (a + b*x)^(9/2)/x - 9*a^(7/2)*b*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.0340381, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$3a^2b(a+bx)^{3/2}+9a^3b\sqrt{a+bx}-9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{9/2}}{x}+\frac{9}{7}b(a+bx)^{7/2}+\frac{9}{5}ab(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^2,x]
```


```
[Out] 9*a^3*b*Sqrt[a + b*x] + 3*a^2*b*(a + b*x)^(3/2) + (9*a*b*(a + b*x)^(5/2))/5
 + (9*b*(a + b*x)^(7/2))/7 - (a + b*x)^(9/2)/x - 9*a^(7/2)*b*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^2}\,dx &= -\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}(9b)\int \frac{(a+bx)^{7/2}}{x}\,dx \\
&= \frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}(9ab)\int \frac{(a+bx)^{5/2}}{x}\,dx \\
&= \frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}\left(9a^2b\right)\int \frac{(a+bx)^{3/2}}{x}\,dx \\
&= 3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}\left(9a^3b\right)\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= 9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}\left(9a^4b\right)\int \frac{1}{x\sqrt{a}
\\
&= 9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\left(9a^4\right)\mathrm{Subst}\left(\int
\\
&= 9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}-9a^{7/2}b\tanh^{-1}\left(\sqrt{}\right.
\end{aligned}$$

Mathematica [C] time = 0.0257217, size = 33, normalized size = 0.34

$$\frac{2b(a+bx)^{11/2}\,{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};\frac{bx}{a}+1\right)}{11a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^2,x]
```

```
[Out] (2*b*(a + b*x)^(11/2)*Hypergeometric2F1[2, 11/2, 13/2, 1 + (b*x)/a])/(11*a^
2)
```

Maple [A] time = 0.007, size = 84, normalized size = 0.9

$$2\,b\left(1/7\,(bx+a)^{7/2}+2/5\,a\,(bx+a)^{5/2}+a^2\,(bx+a)^{3/2}+4\,\sqrt{bx+a}a^3+a^4\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-9/2\,\frac{1}{\sqrt{a}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^2,x)
```

```
[Out] 2*b*(1/7*(b*x+a)^(7/2)+2/5*a*(b*x+a)^(5/2)+a^2*(b*x+a)^(3/2)+4*(b*x+a)^(1/2
)*a^3+a^4*(-1/2*(b*x+a)^(1/2)/b/x-9/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66573, size = 420, normalized size = 4.29

$$\left[\frac{315\,a^{\frac{7}{2}}bx\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(10\,b^4x^4+58\,ab^3x^3+156\,a^2b^2x^2+388\,a^3bx-35\,a^4\right)\sqrt{bx+a}}{70\,x}, \frac{315\,\sqrt{-a}a^3bx\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/70*(315*a^(7/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(10
*b^4*x^4 + 58*a*b^3*x^3 + 156*a^2*b^2*x^2 + 388*a^3*b*x - 35*a^4)*sqrt(b*x
+ a))/x, 1/35*(315*sqrt(-a)*a^3*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (10*
b^4*x^4 + 58*a*b^3*x^3 + 156*a^2*b^2*x^2 + 388*a^3*b*x - 35*a^4)*sqrt(b*x +
 a))/x]
```

Sympy [A] time = 12.7312, size = 150, normalized size = 1.53

$$-\frac{a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{x}+\frac{388a^{\frac{7}{2}}b\sqrt{1+\frac{bx}{a}}}{35}+\frac{9a^{\frac{7}{2}}b\log\left(\frac{bx}{a}\right)}{2}-9a^{\frac{7}{2}}b\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{156a^{\frac{5}{2}}b^2x\sqrt{1+\frac{bx}{a}}}{35}+\frac{58a^{\frac{3}{2}}b^3x^2\sqrt{1+\frac{bx}{a}}}{35}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**2,x)
```

```
[Out] -a**(9/2)*sqrt(1 + b*x/a)/x + 388*a**(7/2)*b*sqrt(1 + b*x/a)/35 + 9*a**(7/2
)*b*log(b*x/a)/2 - 9*a**(7/2)*b*log(sqrt(1 + b*x/a) + 1) + 156*a**(5/2)*b**
2*x*sqrt(1 + b*x/a)/35 + 58*a**(3/2)*b**3*x**2*sqrt(1 + b*x/a)/35 + 2*sqrt(
a)*b**4*x**3*sqrt(1 + b*x/a)/7
```

Giac [A] time = 1.24112, size = 140, normalized size = 1.43

$$\frac{\frac{315\,a^4b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+10\,(bx+a)^{\frac{7}{2}}b^2+28\,(bx+a)^{\frac{5}{2}}ab^2+70\,(bx+a)^{\frac{3}{2}}a^2b^2+280\,\sqrt{bx+a}a^3b^2-\frac{35\,\sqrt{bx+a}a^4b}{x}}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/35*(315*a^4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 10*(b*x + a)^(7
/2)*b^2 + 28*(b*x + a)^(5/2)*a*b^2 + 70*(b*x + a)^(3/2)*a^2*b^2 + 280*sqrt(
b*x + a)*a^3*b^2 - 35*sqrt(b*x + a)*a^4*b/x)/b
```

3.319 $\int \frac{(a+bx)^{9/2}}{x^3}\,dx$

Optimal. Leaf size=114

$$\frac{63}{4}a^2b^2\sqrt{a+bx}-\frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{63}{20}b^2(a+bx)^{5/2}+\frac{21}{4}ab^2(a+bx)^{3/2}-\frac{(a+bx)^{9/2}}{2x^2}-\frac{9b(a+bx)^{7/2}}{4x}$$

```
[Out] (63*a^2*b^2*Sqrt[a + b*x])/4 + (21*a*b^2*(a + b*x)^(3/2))/4 + (63*b^2*(a +
b*x)^(5/2))/20 - (9*b*(a + b*x)^(7/2))/(4*x) - (a + b*x)^(9/2)/(2*x^2) - (6
3*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rubi [A] time = 0.0350025, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$\frac{63}{4}a^2b^2\sqrt{a+bx}-\frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{63}{20}b^2(a+bx)^{5/2}+\frac{21}{4}ab^2(a+bx)^{3/2}-\frac{(a+bx)^{9/2}}{2x^2}-\frac{9b(a+bx)^{7/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^3,x]
```

```
[Out] (63*a^2*b^2*Sqrt[a + b*x])/4 + (21*a*b^2*(a + b*x)^(3/2))/4 + (63*b^2*(a +
b*x)^(5/2))/20 - (9*b*(a + b*x)^(7/2))/(4*x) - (a + b*x)^(9/2)/(2*x^2) - (6
3*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^3}\,dx &= -\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{4}(9b)\int \frac{(a+bx)^{7/2}}{x^2}\,dx \\
&= -\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63b^2\right)\int \frac{(a+bx)^{5/2}}{x}\,dx \\
&= \frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63ab^2\right)\int \frac{(a+bx)^{3/2}}{x}\,dx \\
&= \frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63a^2b^2\right)\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= \frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63a^3b^2\right)\int \\
&= \frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{4}\left(63a^3b\right)\text{Sub} \\
&= \frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}-\frac{63}{4}a^{5/2}b^2\tanh^{-}
\end{aligned}$$

Mathematica [C] time = 0.139489, size = 35, normalized size = 0.31

$$-\frac{2b^2(a+bx)^{11/2}\,{}_2F_1\left(3,\frac{11}{2};\frac{13}{2};\frac{bx}{a}+1\right)}{11a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^3,x]
```

```
[Out] (-2*b^2*(a + b*x)^(11/2)*Hypergeometric2F1[3, 11/2, 13/2, 1 + (b*x)/a])/(11
*a^3)
```

Maple [A] time = 0.01, size = 86, normalized size = 0.8

$$2\,b^2\left(1/5\,(bx+a)^{5/2}+a\,(bx+a)^{3/2}+6\,a^2\sqrt{bx+a}+a^3\left(\frac{1}{b^2x^2}\left(-\frac{17\,(bx+a)^{3/2}}{8}+\frac{15\,a\sqrt{bx+a}}{8}\right)-\frac{63}{8\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^3,x)
```

```
[Out] 2*b^2*(1/5*(b*x+a)^(5/2)+a*(b*x+a)^(3/2)+6*a^2*(b*x+a)^(1/2)+a^3*((-17/8*(b
*x+a)^(3/2)+15/8*a*(b*x+a)^(1/2))/b^2/x^2-63/8*arctanh((b*x+a)^(1/2)/a^(1/2
))/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56112, size = 431, normalized size = 3.78

$$\left[\frac{315\,a^{\frac{5}{2}}b^2x^2\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(8\,b^4x^4+56\,ab^3x^3+288\,a^2b^2x^2-85\,a^3bx-10\,a^4\right)\sqrt{bx+a}}{40\,x^2}, \frac{315\,\sqrt{-a}a^2b^2x^2\,\mathrm{arc}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/40*(315*a^(5/2)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2
*(8*b^4*x^4 + 56*a*b^3*x^3 + 288*a^2*b^2*x^2 - 85*a^3*b*x - 10*a^4)*sqrt(b*
x + a))/x^2, 1/20*(315*sqrt(-a)*a^2*b^2*x^2*arctan(sqrt(b*x + a)*sqrt(-a)/a
) + (8*b^4*x^4 + 56*a*b^3*x^3 + 288*a^2*b^2*x^2 - 85*a^3*b*x - 10*a^4)*sqrt
(b*x + a))/x^2]
```

Sympy [A] time = 12.614, size = 184, normalized size = 1.61

$$-\frac{63a^{\frac{5}{2}}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4}-\frac{a^5}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{19a^4\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{203a^3b^{\frac{3}{2}}}{20\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{86a^2b^{\frac{5}{2}}\sqrt{x}}{5\sqrt{\frac{a}{bx}+1}}+\frac{16ab^{\frac{7}{2}}x^{\frac{3}{2}}}{5\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}x^{\frac{5}{2}}}{5\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**3,x)
```

```
[Out] -63*a**(5/2)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - a**5/(2*sqrt(b)*x**(
5/2)*sqrt(a/(b*x) + 1)) - 19*a**4*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) +
203*a**3*b**(3/2)/(20*sqrt(x)*sqrt(a/(b*x) + 1)) + 86*a**2*b**(5/2)*sqrt(x)
/(5*sqrt(a/(b*x) + 1)) + 16*a*b**(7/2)*x**(3/2)/(5*sqrt(a/(b*x) + 1)) + 2*b
**(9/2)*x**(5/2)/(5*sqrt(a/(b*x) + 1))
```

Giac [A] time = 1.26392, size = 151, normalized size = 1.32

$$\frac{\frac{315\,a^3b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\,(bx+a)^{\frac{5}{2}}b^3+40\,(bx+a)^{\frac{3}{2}}ab^3+240\,\sqrt{bx+a}a^2b^3-\frac{5\left(17\,(bx+a)^{\frac{3}{2}}a^3b^3-15\,\sqrt{bx+a}a^4b^3\right)}{b^2x^2}}{20\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/20*(315*a^3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*(b*x + a)^(5/
2)*b^3 + 40*(b*x + a)^(3/2)*a*b^3 + 240*sqrt(b*x + a)*a^2*b^3 - 5*(17*(b*x
+ a)^(3/2)*a^3*b^3 - 15*sqrt(b*x + a)*a^4*b^3)/(b^2*x^2))/b
```

3.320 $\int \frac{(a+bx)^{9/2}}{x^4} dx$

Optimal. Leaf size=114

$$-\frac{105}{8}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{21b^2(a+bx)^{5/2}}{8x}+\frac{35}{8}b^3(a+bx)^{3/2}+\frac{105}{8}ab^3\sqrt{a+bx}-\frac{(a+bx)^{9/2}}{3x^3}-\frac{3b(a+bx)^{7/2}}{4x^2}$$

```
[Out] (105*a*b^3*Sqrt[a + b*x])/8 + (35*b^3*(a + b*x)^(3/2))/8 - (21*b^2*(a + b*x
)^(5/2))/(8*x) - (3*b*(a + b*x)^(7/2))/(4*x^2) - (a + b*x)^(9/2)/(3*x^3) -
(105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/8
```

Rubi [A] time = 0.0357739, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$-\frac{105}{8}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{21b^2(a+bx)^{5/2}}{8x}+\frac{35}{8}b^3(a+bx)^{3/2}+\frac{105}{8}ab^3\sqrt{a+bx}-\frac{(a+bx)^{9/2}}{3x^3}-\frac{3b(a+bx)^{7/2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^4,x]
```

```
[Out] (105*a*b^3*Sqrt[a + b*x])/8 + (35*b^3*(a + b*x)^(3/2))/8 - (21*b^2*(a + b*x
)^(5/2))/(8*x) - (3*b*(a + b*x)^(7/2))/(4*x^2) - (a + b*x)^(9/2)/(3*x^3) -
(105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^4}\,dx &= -\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{2}(3b)\int \frac{(a+bx)^{7/2}}{x^3}\,dx \\
&= -\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{8}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^2}\,dx \\
&= -\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{16}\left(105b^3\right)\int \frac{(a+bx)^{3/2}}{x}\,dx \\
&= \frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{16}\left(105ab^3\right)\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= \frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{16}\left(105a^2b^3\right) \\
&= \frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{8}\left(105a^2b^2\right) \\
&= \frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}-\frac{105}{8}a^{3/2}b^3\tan
\end{aligned}$$

Mathematica [C] time = 0.0257829, size = 35, normalized size = 0.31

$$\frac{2b^3(a+bx)^{11/2}\,{}_2F_1\left(4,\frac{11}{2};\frac{13}{2};\frac{bx}{a}+1\right)}{11a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^4,x]
```

```
[Out] (2*b^3*(a + b*x)^(11/2)*Hypergeometric2F1[4, 11/2, 13/2, 1 + (b*x)/a])/(11*
a^4)
```

Maple [A] time = 0.01, size = 87, normalized size = 0.8

$$2\,b^3\left(1/3\,(bx+a)^{3/2}+4\,a\sqrt{bx+a}+a^2\left(\frac{1}{b^3x^3}\left(-\frac{55\,(bx+a)^{5/2}}{16}+\frac{35\,a\,(bx+a)^{3/2}}{6}-\frac{41\,a^2\sqrt{bx+a}}{16}\right)-\frac{105}{16\sqrt{a}}\mathrm{Artanh}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^4,x)
```

```
[Out] 2*b^3*(1/3*(b*x+a)^(3/2)+4*a*(b*x+a)^(1/2)+a^2*((-55/16*(b*x+a)^(5/2)+35/6*
a*(b*x+a)^(3/2)-41/16*a^2*(b*x+a)^(1/2))/b^3/x^3-105/16*arctanh((b*x+a)^(1/
2)/a^(1/2))/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61518, size = 431, normalized size = 3.78

$$\left[\frac{315\,a^{\frac{3}{2}}b^{3}x^{3}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(16\,b^{4}x^{4}+208\,ab^{3}x^{3}-165\,a^{2}b^{2}x^{2}-50\,a^{3}bx-8\,a^{4}\right)\sqrt{bx+a}}{48\,x^{3}},\ \frac{315\,\sqrt{-a}ab^{3}x^{3}\arctan\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(315*a^(3/2)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2
*(16*b^4*x^4 + 208*a*b^3*x^3 - 165*a^2*b^2*x^2 - 50*a^3*b*x - 8*a^4)*sqrt(b
*x + a))/x^3, 1/24*(315*sqrt(-a)*a*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a)
 + (16*b^4*x^4 + 208*a*b^3*x^3 - 165*a^2*b^2*x^2 - 50*a^3*b*x - 8*a^4)*sqrt
(b*x + a))/x^3]
```

Sympy [A] time = 11.0889, size = 184, normalized size = 1.61

$$-\frac{105a^{\frac{3}{2}}b^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8}-\frac{a^{5}}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{29a^{4}\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{215a^{3}b^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{43a^{2}b^{\frac{5}{2}}}{24\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{28ab^{\frac{7}{2}}\sqrt{x}}{3\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**4,x)
```

```
[Out] -105*a**(3/2)*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/8 - a**5/(3*sqrt(b)*x**
(7/2)*sqrt(a/(b*x) + 1)) - 29*a**4*sqrt(b)/(12*x**(5/2)*sqrt(a/(b*x) + 1))
- 215*a**3*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) + 43*a**2*b**(5/2)/(24*
sqrt(x)*sqrt(a/(b*x) + 1)) + 28*a*b**(7/2)*sqrt(x)/(3*sqrt(a/(b*x) + 1)) +
2*b**(9/2)*x**(3/2)/(3*sqrt(a/(b*x) + 1))
```

Giac [A] time = 1.26577, size = 151, normalized size = 1.32

$$\frac{\frac{315\,a^{2}b^{4}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+16\,(bx+a)^{\frac{3}{2}}b^{4}+192\,\sqrt{bx+a}ab^{4}-\frac{165\,(bx+a)^{\frac{5}{2}}a^{2}b^{4}-280\,(bx+a)^{\frac{3}{2}}a^{3}b^{4}+123\,\sqrt{bx+a}a^{4}b^{4}}{b^{3}x^{3}}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(315*a^2*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 16*(b*x + a)^(3
/2)*b^4 + 192*sqrt(b*x + a)*a*b^4 - (165*(b*x + a)^(5/2)*a^2*b^4 - 280*(b*x
 + a)^(3/2)*a^3*b^4 + 123*sqrt(b*x + a)*a^4*b^4)/(b^3*x^3))/b
```

3.321 $\int \frac{(a+bx)^{9/2}}{x^5} dx$

Optimal. Leaf size=116

$$-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{105b^3(a+bx)^{3/2}}{64x}+\frac{315}{64}b^4\sqrt{a+bx}-\frac{315}{64}\sqrt{a}b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{9/2}}{4x^4}-\frac{3b(a+bx)^{7/2}}{8x^3}$$

```
[Out] (315*b^4*Sqrt[a + b*x])/64 - (105*b^3*(a + b*x)^(3/2))/(64*x) - (21*b^2*(a
+ b*x)^(5/2))/(32*x^2) - (3*b*(a + b*x)^(7/2))/(8*x^3) - (a + b*x)^(9/2)/(4
*x^4) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/64
```

Rubi [A] time = 0.0366599, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{105b^3(a+bx)^{3/2}}{64x}+\frac{315}{64}b^4\sqrt{a+bx}-\frac{315}{64}\sqrt{a}b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{9/2}}{4x^4}-\frac{3b(a+bx)^{7/2}}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^5,x]
```

```
[Out] (315*b^4*Sqrt[a + b*x])/64 - (105*b^3*(a + b*x)^(3/2))/(64*x) - (21*b^2*(a
+ b*x)^(5/2))/(32*x^2) - (3*b*(a + b*x)^(7/2))/(8*x^3) - (a + b*x)^(9/2)/(4
*x^4) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/64
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^5}\,dx &= -\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{8}(9b)\int \frac{(a+bx)^{7/2}}{x^4}\,dx \\
&= -\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{16}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^3}\,dx \\
&= -\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{64}\left(105b^3\right)\int \frac{(a+bx)^{3/2}}{x^2}\,dx \\
&= -\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{128}\left(315b^4\right)\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= \frac{315}{64}b^4\sqrt{a+bx}-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{128}\left(315ab^4\right)\int \\
&= \frac{315}{64}b^4\sqrt{a+bx}-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{64}\left(315ab^3\right)\mathrm{Sub} \\
&= \frac{315}{64}b^4\sqrt{a+bx}-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}-\frac{315}{64}\sqrt{a}b^4\tanh^{-1}
\end{aligned}$$

Mathematica [C] time = 0.0468802, size = 35, normalized size = 0.3

$$-\frac{2b^4(a+bx)^{11/2}\,{}_2F_1\left(5,\frac{11}{2};\frac{13}{2};\frac{bx}{a}+1\right)}{11a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^5,x]
```

```
[Out] (-2*b^4*(a + b*x)^(11/2)*Hypergeometric2F1[5, 11/2, 13/2, 1 + (b*x)/a])/(11
*a^5)
```

Maple [A] time = 0.012, size = 85, normalized size = 0.7

$$2\,b^4\left(\sqrt{bx+a}+a\left(\frac{1}{b^4x^4}\left(-\frac{325\,(bx+a)^{7/2}}{128}+\frac{765\,a\,(bx+a)^{5/2}}{128}-\frac{643\,a^2\,(bx+a)^{3/2}}{128}+\frac{187\,\sqrt{bx+a}a^3}{128}\right)-\frac{315}{128\,\sqrt{a}}\mathrm{Artanh}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^5,x)
```

```
[Out] 2*b^4*((b*x+a)^(1/2)+a*((-325/128*(b*x+a)^(7/2)+765/128*a*(b*x+a)^(5/2)-643
/128*a^2*(b*x+a)^(3/2)+187/128*(b*x+a)^(1/2)*a^3)/b^4/x^4-315/128*arctanh((
b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53474, size = 435, normalized size = 3.75

$$\left[\frac{315\,\sqrt{a}b^4x^4\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(128\,b^4x^4-325\,ab^3x^3-210\,a^2b^2x^2-88\,a^3bx-16\,a^4\right)\sqrt{bx+a}}{128\,x^4}, \frac{315\,\sqrt{-a}b^4x^4\,\mathrm{ar}\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/128*(315*sqrt(a)*b^4*x^4*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
2*(128*b^4*x^4 - 325*a*b^3*x^3 - 210*a^2*b^2*x^2 - 88*a^3*b*x - 16*a^4)*sqr
t(b*x + a))/x^4, 1/64*(315*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a
) + (128*b^4*x^4 - 325*a*b^3*x^3 - 210*a^2*b^2*x^2 - 88*a^3*b*x - 16*a^4)*s
qrt(b*x + a))/x^4]
```

Sympy [A] time = 12.2001, size = 182, normalized size = 1.57

$$-\frac{315\sqrt{a}b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64}-\frac{a^5}{4\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{13a^4\sqrt{b}}{8x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{149a^3b^{\frac{3}{2}}}{32x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{535a^2b^{\frac{5}{2}}}{64x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{197ab^{\frac{7}{2}}}{64\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}\ldots}{\sqrt{\frac{a}{bx}\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**5,x)
```

```
[Out] -315*sqrt(a)*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/64 - a**5/(4*sqrt(b)*x**
(9/2)*sqrt(a/(b*x) + 1)) - 13*a**4*sqrt(b)/(8*x**(7/2)*sqrt(a/(b*x) + 1)) -
 149*a**3*b**(3/2)/(32*x**(5/2)*sqrt(a/(b*x) + 1)) - 535*a**2*b**(5/2)/(64*
x**(3/2)*sqrt(a/(b*x) + 1)) - 197*a*b**(7/2)/(64*sqrt(x)*sqrt(a/(b*x) + 1))
 + 2*b**(9/2)*sqrt(x)/sqrt(a/(b*x) + 1)
```

Giac [A] time = 1.22471, size = 149, normalized size = 1.28

$$\frac{\frac{315\,ab^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+128\,\sqrt{bx+a}b^5-\frac{325\,(bx+a)^{\frac{7}{2}}ab^5-765\,(bx+a)^{\frac{5}{2}}a^2b^5+643\,(bx+a)^{\frac{3}{2}}a^3b^5-187\,\sqrt{bx+a}a^4b^5}{b^4x^4}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/64*(315*a*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 128*sqrt(b*x + a)
*b^5 - (325*(b*x + a)^(7/2)*a*b^5 - 765*(b*x + a)^(5/2)*a^2*b^5 + 643*(b*x
+ a)^(3/2)*a^3*b^5 - 187*sqrt(b*x + a)*a^4*b^5)/(b^4*x^4))/b
```

3.322 $\int \frac{(a+bx)^{9/2}}{x^6} dx$

Optimal. Leaf size=119

$$-\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{63b^4\sqrt{a+bx}}{128x}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128\sqrt{a}}-\frac{9b(a+bx)^{7/2}}{40x^4}-\frac{(a+bx)^{9/2}}{5x^5}$$

```
[Out] (-63*b^4*Sqrt[a + b*x])/(128*x) - (21*b^3*(a + b*x)^(3/2))/(64*x^2) - (21*b
^2*(a + b*x)^(5/2))/(80*x^3) - (9*b*(a + b*x)^(7/2))/(40*x^4) - (a + b*x)^(
9/2)/(5*x^5) - (63*b^5*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(128*Sqrt[a])
```

Rubi [A] time = 0.0385725, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{63b^4\sqrt{a+bx}}{128x}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128\sqrt{a}}-\frac{9b(a+bx)^{7/2}}{40x^4}-\frac{(a+bx)^{9/2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^6,x]
```

```
[Out] (-63*b^4*Sqrt[a + b*x])/(128*x) - (21*b^3*(a + b*x)^(3/2))/(64*x^2) - (21*b
^2*(a + b*x)^(5/2))/(80*x^3) - (9*b*(a + b*x)^(7/2))/(40*x^4) - (a + b*x)^(
9/2)/(5*x^5) - (63*b^5*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(128*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x^6}\,dx = -\frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{10}(9b)\int \frac{(a+bx)^{7/2}}{x^5}\,dx$$
$$= -\frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{80}\left(63b^2\right)\int \frac{(a+bx)^{5/2}}{x^4}\,dx$$
$$= -\frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{32}\left(21b^3\right)\int \frac{(a+bx)^{3/2}}{x^3}\,dx$$
$$= -\frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{128}\left(63b^4\right)\int \frac{\sqrt{a+bx}}{x^2}\,dx$$
$$= -\frac{63b^4\sqrt{a+bx}}{128x} - \frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{256}\left(63b^5\right)\int$$
$$= -\frac{63b^4\sqrt{a+bx}}{128x} - \frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{128}\left(63b^4\right)\text{Su}$$
$$= -\frac{63b^4\sqrt{a+bx}}{128x} - \frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} - \frac{63b^5\tanh^{-1}\left(\right.}{128\sqrt{a}}$$

Mathematica [A] time = 0.130866, size = 101, normalized size = 0.85

$$-\frac{2024a^3b^2x^2 + 2858a^2b^3x^3 + 784a^4bx + 128a^5 + 2455ab^4x^4 + 315b^5x^5\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right) + 965b^5x^5}{640x^5\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^6,x]
```

```
[Out] -(128*a^5 + 784*a^4*b*x + 2024*a^3*b^2*x^2 + 2858*a^2*b^3*x^3 + 2455*a*b^4*
x^4 + 965*b^5*x^5 + 315*b^5*x^5*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]
])/(640*x^5*Sqrt[a + b*x])
```

Maple [A] time = 0.013, size = 87, normalized size = 0.7

$$2\,b^5\left(\frac{1}{b^5x^5}\left(-\frac{193\,(bx+a)^{9/2}}{256} + \frac{237\,a\,(bx+a)^{7/2}}{128} - \frac{21\,a^2\,(bx+a)^{5/2}}{10} + \frac{147\,a^3\,(bx+a)^{3/2}}{128} - \frac{63\,a^4\sqrt{bx+a}}{256}\right) - \frac{63}{256\,\sqrt{a}}\text{A}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^6,x)
```

```
[Out] 2*b^5*((-193/256*(b*x+a)^(9/2)+237/128*a*(b*x+a)^(7/2)-21/10*a^2*(b*x+a)^(5
/2)+147/128*a^3*(b*x+a)^(3/2)-63/256*a^4*(b*x+a)^(1/2))/b^5/x^5-63/256*arct
anh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6636, size = 470, normalized size = 3.95

$$\left[\frac{315\sqrt{a}b^5x^5\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(965ab^4x^4+1490a^2b^3x^3+1368a^3b^2x^2+656a^4bx+128a^5\right)\sqrt{bx+a}}{1280ax^5}, \frac{315\sqrt{-a}b^5}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/1280*(315*sqrt(a)*b^5*x^5*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
 2*(965*a*b^4*x^4 + 1490*a^2*b^3*x^3 + 1368*a^3*b^2*x^2 + 656*a^4*b*x + 128
*a^5)*sqrt(b*x + a))/(a*x^5), 1/640*(315*sqrt(-a)*b^5*x^5*arctan(sqrt(b*x +
 a)*sqrt(-a)/a) - (965*a*b^4*x^4 + 1490*a^2*b^3*x^3 + 1368*a^3*b^2*x^2 + 65
6*a^4*b*x + 128*a^5)*sqrt(b*x + a))/(a*x^5)]
```

Sympy [A] time = 13.3973, size = 158, normalized size = 1.33

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^{\frac{9}{2}}}-\frac{41a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{40x^{\frac{7}{2}}}-\frac{171a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{80x^{\frac{5}{2}}}-\frac{149ab^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}{64x^{\frac{3}{2}}}-\frac{193b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{128\sqrt{x}}-\frac{63b^5\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{128\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**6,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**(9/2)) - 41*a**3*b**(3/2)*sqrt(a/(b*x
) + 1)/(40*x**(7/2)) - 171*a**2*b**(5/2)*sqrt(a/(b*x) + 1)/(80*x**(5/2)) -
149*a*b**(7/2)*sqrt(a/(b*x) + 1)/(64*x**(3/2)) - 193*b**(9/2)*sqrt(a/(b*x)
+ 1)/(128*sqrt(x)) - 63*b**5*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(128*sqrt(a))
```

Giac [A] time = 1.20668, size = 147, normalized size = 1.24

$$\frac{\frac{315\,b^6\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{965\,(bx+a)^{\frac{9}{2}}b^6-2370\,(bx+a)^{\frac{7}{2}}ab^6+2688\,(bx+a)^{\frac{5}{2}}a^2b^6-1470\,(bx+a)^{\frac{3}{2}}a^3b^6+315\,\sqrt{bx+a}a^4b^6}{b^5x^5}}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/640*(315*b^6*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (965*(b*x + a)^(9/
2)*b^6 - 2370*(b*x + a)^(7/2)*a*b^6 + 2688*(b*x + a)^(5/2)*a^2*b^6 - 1470*(
b*x + a)^(3/2)*a^3*b^6 + 315*sqrt(b*x + a)*a^4*b^6)/(b^5*x^5))/b
```

3.323 $\int \frac{(a+bx)^{9/2}}{x^7}\,dx$

Optimal. Leaf size=141

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}}-\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}$$

```
[Out] (-21*b^4*Sqrt[a + b*x])/(256*x^2) - (21*b^5*Sqrt[a + b*x])/(512*a*x) - (7*b
^3*(a + b*x)^(3/2))/(64*x^3) - (21*b^2*(a + b*x)^(5/2))/(160*x^4) - (3*b*(a
 + b*x)^(7/2))/(20*x^5) - (a + b*x)^(9/2)/(6*x^6) + (21*b^6*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]])/(512*a^(3/2))
```

Rubi [A] time = 0.0510884, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}}-\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^7,x]
```

```
[Out] (-21*b^4*Sqrt[a + b*x])/(256*x^2) - (21*b^5*Sqrt[a + b*x])/(512*a*x) - (7*b
^3*(a + b*x)^(3/2))/(64*x^3) - (21*b^2*(a + b*x)^(5/2))/(160*x^4) - (3*b*(a
 + b*x)^(7/2))/(20*x^5) - (a + b*x)^(9/2)/(6*x^6) + (21*b^6*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]])/(512*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x^7}\,dx = -\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{4}(3b)\int \frac{(a+bx)^{7/2}}{x^6}\,dx$$
$$= -\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{40}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^5}\,dx$$
$$= -\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{64}\left(21b^3\right)\int \frac{(a+bx)^{3/2}}{x^4}\,dx$$
$$= -\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{128}\left(21b^4\right)\int \frac{\sqrt{a+bx}}{x^3}\,dx$$
$$= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{512}\left(21b^5\right)\int \frac{}{x^2\sqrt{}}$$
$$= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}-\frac{(2}{}$$
$$= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}-\frac{(2}{}$$
$$= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{21}{}$$

Mathematica [C] time = 0.0248076, size = 35, normalized size = 0.25

$$-\frac{2b^6(a+bx)^{11/2}\,{}_2F_1\left(\frac{11}{2},7;\frac{13}{2};\frac{bx}{a}+1\right)}{11a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^7,x]
```

```
[Out] (-2*b^6*(a + b*x)^(11/2)*Hypergeometric2F1[11/2, 7, 13/2, 1 + (b*x)/a])/(11
*a^7)
```

Maple [A] time = 0.013, size = 99, normalized size = 0.7

$$2\,b^6\left(\frac{1}{b^6x^6}\left(-\frac{21\,(bx+a)^{11/2}}{1024\,a}-\frac{667\,(bx+a)^{9/2}}{3072}+\frac{843\,a\,(bx+a)^{7/2}}{2560}-\frac{693\,a^2\,(bx+a)^{5/2}}{2560}+\frac{119\,a^3\,(bx+a)^{3/2}}{1024}-\frac{21\,a^4\sqrt{bx+}}{1024}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^7,x)
```

```
[Out] 2*b^6*((-21/1024/a*(b*x+a)^(11/2)-667/3072*(b*x+a)^(9/2)+843/2560*a*(b*x+a)
^(7/2)-693/2560*a^2*(b*x+a)^(5/2)+119/1024*a^3*(b*x+a)^(3/2)-21/1024*a^4*(b
*x+a)^(1/2))/b^6/x^6+21/1024*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62402, size = 541, normalized size = 3.84

$$\left[\frac{315\sqrt{a}b^6x^6\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(315ab^5x^5+4910a^2b^4x^4+11432a^3b^3x^3+12144a^4b^2x^2+6272a^5bx+1280a\right.}{15360a^2x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/15360*(315*sqrt(a)*b^6*x^6*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
- 2*(315*a*b^5*x^5 + 4910*a^2*b^4*x^4 + 11432*a^3*b^3*x^3 + 12144*a^4*b^2*x
^2 + 6272*a^5*b*x + 1280*a^6)*sqrt(b*x + a))/(a^2*x^6), -1/7680*(315*sqrt(-
a)*b^6*x^6*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (315*a*b^5*x^5 + 4910*a^2*b^4
*x^4 + 11432*a^3*b^3*x^3 + 12144*a^4*b^2*x^2 + 6272*a^5*b*x + 1280*a^6)*sqr
t(b*x + a))/(a^2*x^6)]
```

Sympy [A] time = 20.8717, size = 209, normalized size = 1.48

$$-\frac{a^5}{6\sqrt{b}x^{\frac{13}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{59a^4\sqrt{b}}{60x^{\frac{11}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{1151a^3b^{\frac{3}{2}}}{480x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{2947a^2b^{\frac{5}{2}}}{960x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{8171ab^{\frac{7}{2}}}{3840x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{1045b^{\frac{9}{2}}}{1536x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{2}{512a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**7,x)
```

```
[Out] -a**5/(6*sqrt(b)*x**(13/2)*sqrt(a/(b*x) + 1)) - 59*a**4*sqrt(b)/(60*x**(11/
2)*sqrt(a/(b*x) + 1)) - 1151*a**3*b**(3/2)/(480*x**(9/2)*sqrt(a/(b*x) + 1))
 - 2947*a**2*b**(5/2)/(960*x**(7/2)*sqrt(a/(b*x) + 1)) - 8171*a*b**(7/2)/(3
840*x**(5/2)*sqrt(a/(b*x) + 1)) - 1045*b**(9/2)/(1536*x**(3/2)*sqrt(a/(b*x)
 + 1)) - 21*b**(11/2)/(512*a*sqrt(x)*sqrt(a/(b*x) + 1)) + 21*b**6*asinh(sqr
t(a)/(sqrt(b)*sqrt(x)))/(512*a**(3/2))
```

Giac [A] time = 1.246, size = 174, normalized size = 1.23

$$-\frac{\frac{315b^7\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{315(bx+a)^{\frac{11}{2}}b^7+3335(bx+a)^{\frac{9}{2}}ab^7-5058(bx+a)^{\frac{7}{2}}a^2b^7+4158(bx+a)^{\frac{5}{2}}a^3b^7-1785(bx+a)^{\frac{3}{2}}a^4b^7+315\sqrt{bx+a}a^5b^7}{ab^6x^6}}{7680b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/7680*(315*b^7*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (315*(b*x +
a)^(11/2)*b^7 + 3335*(b*x + a)^(9/2)*a*b^7 - 5058*(b*x + a)^(7/2)*a^2*b^7 +
 4158*(b*x + a)^(5/2)*a^3*b^7 - 1785*(b*x + a)^(3/2)*a^4*b^7 + 315*sqrt(b*x
 + a)*a^5*b^7)/(a*b^6*x^6))/b
```

3.324 $\int \frac{(a+bx)^{9/2}}{x^8} \, dx$

Optimal. Leaf size=163

$$\frac{9b^6\sqrt{a+bx}}{1024a^2x} - \frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}} - \frac{3b^5\sqrt{a+bx}}{512ax^2} - \frac{3b^4\sqrt{a+bx}}{128x^3} - \frac{3b^3(a+bx)^{3/2}}{64x^4} - \frac{3b^2(a+bx)^{5/2}}{40x^5} - \frac{3b(a+bx)^{7/2}}{28x^6} - $$

```
[Out] (-3*b^4*Sqrt[a + b*x])/(128*x^3) - (3*b^5*Sqrt[a + b*x])/(512*a*x^2) + (9*b
^6*Sqrt[a + b*x])/(1024*a^2*x) - (3*b^3*(a + b*x)^(3/2))/(64*x^4) - (3*b^2*
(a + b*x)^(5/2))/(40*x^5) - (3*b*(a + b*x)^(7/2))/(28*x^6) - (a + b*x)^(9/2
)/(7*x^7) - (9*b^7*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(1024*a^(5/2))
```

Rubi [A] time = 0.0675368, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{9b^6\sqrt{a+bx}}{1024a^2x} - \frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}} - \frac{3b^5\sqrt{a+bx}}{512ax^2} - \frac{3b^4\sqrt{a+bx}}{128x^3} - \frac{3b^3(a+bx)^{3/2}}{64x^4} - \frac{3b^2(a+bx)^{5/2}}{40x^5} - \frac{3b(a+bx)^{7/2}}{28x^6} - $$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^8,x]
```

```
[Out] (-3*b^4*Sqrt[a + b*x])/(128*x^3) - (3*b^5*Sqrt[a + b*x])/(512*a*x^2) + (9*b
^6*Sqrt[a + b*x])/(1024*a^2*x) - (3*b^3*(a + b*x)^(3/2))/(64*x^4) - (3*b^2*
(a + b*x)^(5/2))/(40*x^5) - (3*b*(a + b*x)^(7/2))/(28*x^6) - (a + b*x)^(9/2
)/(7*x^7) - (9*b^7*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(1024*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x^8}\,dx = -\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{14}(9b)\int \frac{(a+bx)^{7/2}}{x^7}\,dx$$
$$= -\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{8}\left(3b^2\right)\int \frac{(a+bx)^{5/2}}{x^6}\,dx$$
$$= -\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{16}\left(3b^3\right)\int \frac{(a+bx)^{3/2}}{x^5}\,dx$$
$$= -\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{128}\left(9b^4\right)\int \frac{\sqrt{a+bx}}{x^4}\,dx$$
$$= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{256}\left(3b^5\right)\int \frac{1}{x^3\sqrt{a+}}$$
$$= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}-\frac{\left(9b^6\right)}{}$$
$$= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+}{7}$$
$$= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+}{7}$$
$$= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+}{7}$$

Mathematica [C] time = 0.0187144, size = 35, normalized size = 0.21

$$\frac{2b^7(a+bx)^{11/2}\,{}_2F_1\left(\frac{11}{2},8;\frac{13}{2};\frac{bx}{a}+1\right)}{11a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^8,x]
```

```
[Out] (2*b^7*(a + b*x)^(11/2)*Hypergeometric2F1[11/2, 8, 13/2, 1 + (b*x)/a])/(11*
a^8)
```

Maple [A] time = 0.011, size = 111, normalized size = 0.7

$$2\,b^7\left(\frac{1}{b^7x^7}\left(\frac{9\,(bx+a)^{13/2}}{2048\,a^2}-\frac{15\,(bx+a)^{11/2}}{512\,a}-\frac{1199\,(bx+a)^{9/2}}{10240}+\frac{9\,a\,(bx+a)^{7/2}}{70}-\frac{849\,a^2\,(bx+a)^{5/2}}{10240}+\frac{15\,a^3\,(bx+a)^{3/2}}{512}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^8,x)
```

```
[Out] 2*b^7*((9/2048/a^2*(b*x+a)^(13/2)-15/512/a*(b*x+a)^(11/2)-1199/10240*(b*x+a
)^(9/2)+9/70*a*(b*x+a)^(7/2)-849/10240*a^2*(b*x+a)^(5/2)+15/512*a^3*(b*x+a)
```

```
^(3/2)-9/2048*a^4*(b*x+a)^(1/2))/b^7/x^7-9/2048*arctanh((b*x+a)^(1/2)/a^(1/
2))/a^(5/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71013, size = 595, normalized size = 3.65

$$\left[\frac{315\,\sqrt{a}b^7x^7\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(315\,ab^6x^6-210\,a^2b^5x^5-14168\,a^3b^4x^4-39056\,a^4b^3x^3-44928\,a^5b^2x^2-2432\right.}{71680\,a^3x^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/71680*(315*sqrt(a)*b^7*x^7*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(315*a*b^6*x^6 - 210*a^2*b^5*x^5 - 14168*a^3*b^4*x^4 - 39056*a^4*b^3*x^
3 - 44928*a^5*b^2*x^2 - 24320*a^6*b*x - 5120*a^7)*sqrt(b*x + a))/(a^3*x^7),
 1/35840*(315*sqrt(-a)*b^7*x^7*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (315*a*b^
6*x^6 - 210*a^2*b^5*x^5 - 14168*a^3*b^4*x^4 - 39056*a^4*b^3*x^3 - 44928*a^5
*b^2*x^2 - 24320*a^6*b*x - 5120*a^7)*sqrt(b*x + a))/(a^3*x^7)]
```

Sympy [A] time = 28.2355, size = 236, normalized size = 1.45

$$-\frac{a^5}{7\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{23a^4\sqrt{b}}{28x^{\frac{13}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{541a^3b^{\frac{3}{2}}}{280x^{\frac{11}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5249a^2b^{\frac{5}{2}}}{2240x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{6653ab^{\frac{7}{2}}}{4480x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{1027b^{\frac{9}{2}}}{2560x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{}{102}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**8,x)
```

```
[Out] -a**5/(7*sqrt(b)*x**(15/2)*sqrt(a/(b*x) + 1)) - 23*a**4*sqrt(b)/(28*x**(13/
2)*sqrt(a/(b*x) + 1)) - 541*a**3*b**(3/2)/(280*x**(11/2)*sqrt(a/(b*x) + 1))
 - 5249*a**2*b**(5/2)/(2240*x**(9/2)*sqrt(a/(b*x) + 1)) - 6653*a*b**(7/2)/(
4480*x**(7/2)*sqrt(a/(b*x) + 1)) - 1027*b**(9/2)/(2560*x**(5/2)*sqrt(a/(b*x
) + 1)) + 3*b**(11/2)/(1024*a*x**(3/2)*sqrt(a/(b*x) + 1)) + 9*b**(13/2)/(10
24*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - 9*b**7*asinh(sqrt(a)/(sqrt(b)*sqrt(x))
)/(1024*a**(5/2))
```

Giac [A] time = 1.22627, size = 194, normalized size = 1.19

$$\frac{\frac{315\,b^8\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{315\,(bx+a)^{\frac{13}{2}}b^8-2100\,(bx+a)^{\frac{11}{2}}ab^8-8393\,(bx+a)^{\frac{9}{2}}a^2b^8+9216\,(bx+a)^{\frac{7}{2}}a^3b^8-5943\,(bx+a)^{\frac{5}{2}}a^4b^8+2100\,(bx+a)^{\frac{3}{2}}a^5b^8-315\,\sqrt{bx+a}a^6b^8}{a^2b^7x^7}}{35840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/35840*(315*b^8*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (315*(b*x
+ a)^(13/2)*b^8 - 2100*(b*x + a)^(11/2)*a*b^8 - 8393*(b*x + a)^(9/2)*a^2*b^
8 + 9216*(b*x + a)^(7/2)*a^3*b^8 - 5943*(b*x + a)^(5/2)*a^4*b^8 + 2100*(b*x
 + a)^(3/2)*a^5*b^8 - 315*sqrt(b*x + a)*a^6*b^8)/(a^2*b^7*x^7))/b
```

3.325 $\int \frac{\sqrt{-a+bx}}{x} dx$

Optimal. Leaf size=39

$$2\sqrt{bx-a} - 2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

```
[Out] 2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.0105244, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 63, 205}

$$2\sqrt{bx-a} - 2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x,x]

[Out] 2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-a+bx}}{x} dx &= 2\sqrt{-a+bx} - a\int \frac{1}{x\sqrt{-a+bx}} dx \\
&= 2\sqrt{-a+bx} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{-a+bx}\right)}{b} \\
&= 2\sqrt{-a+bx} - 2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0204661, size = 39, normalized size = 1.

$$2\sqrt{bx-a}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x,x]
```

```
[Out] 2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Maple [A] time = 0.005, size = 32, normalized size = 0.8

$$-2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x,x)
```

```
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x-a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52038, size = 186, normalized size = 4.77

$$\left[\sqrt{-a}\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a},-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqrt(b*x - a),
-2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)]
```

Sympy [B] time = 2.16425, size = 151, normalized size = 3.87

$$\begin{cases} -2i\sqrt{a}\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{2ia}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}-1}} - \frac{2i\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for } \frac{|a|}{|b||x|} > 1 \\ 2\sqrt{a}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) - \frac{2a}{\sqrt{b}\sqrt{x}\sqrt{-\frac{a}{bx}+1}} + \frac{2\sqrt{b}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x,x)
```

```
[Out] Piecewise((-2*I*sqrt(a)*acosh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*I*a/(sqrt(b)*s
qrt(x)*sqrt(a/(b*x) - 1)) - 2*I*sqrt(b)*sqrt(x)/sqrt(a/(b*x) - 1), Abs(a)/(
Abs(b)*Abs(x)) > 1), (2*sqrt(a)*asin(sqrt(a)/(sqrt(b)*sqrt(x))) - 2*a/(sqrt
(b)*sqrt(x)*sqrt(-a/(b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(-a/(b*x) + 1), Tru
e))
```

Giac [A] time = 1.18125, size = 42, normalized size = 1.08

$$-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + 2\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)
```

3.326 $\int \frac{\sqrt{-a+bx}}{x^2} dx$

Optimal. Leaf size=42

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}$$

```
[Out] -(Sqrt[-a + b*x]/x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0103159, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {47, 63, 205}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[-a + b*x]/x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{-a+bx}}{x^2}\,dx = -\frac{\sqrt{-a+bx}}{x} + \frac{1}{2}b\int \frac{1}{x\sqrt{-a+bx}}\,dx$$
$$= -\frac{\sqrt{-a+bx}}{x} + \text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)$$
$$= -\frac{\sqrt{-a+bx}}{x} + \frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.0477617, size = 52, normalized size = 1.24

$$\frac{-bx\sqrt{1-\frac{bx}{a}}\tanh^{-1}\left(\sqrt{1-\frac{bx}{a}}\right)+a-bx}{x\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x^2,x]
```

```
[Out] (a - b*x - b*x*Sqrt[1 - (b*x)/a]*ArcTanh[Sqrt[1 - (b*x)/a]])/(x*Sqrt[-a + b
*x])
```

Maple [A] time = 0.008, size = 35, normalized size = 0.8

$$b\arctan\left(\sqrt{bx-a}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}} - \frac{1}{x}\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x^2,x)
```

```
[Out] b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)-(b*x-a)^(1/2)/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53313, size = 224, normalized size = 5.33

$$\left[-\frac{\sqrt{-a}bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a}a}{2ax}, \frac{\sqrt{a}bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\sqrt{bx-a}a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqrt(
b*x - a)*a)/(a*x), (sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) - sqrt(b*x -
a)*a)/(a*x)]
```

Sympy [A] time = 2.73324, size = 124, normalized size = 2.95

$$\begin{cases} -\frac{ia}{\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{i\sqrt{b}}{\sqrt{x}\sqrt{\frac{a}{bx}-1}} + \frac{ib\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{for } \frac{|a|}{|b||x|} > 1 \\ -\frac{\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{\sqrt{x}} - \frac{b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x**2,x)
```

```
[Out] Piecewise((-I*a/(sqrt(b)*x**(3/2)*sqrt(a/(b*x) - 1)) + I*sqrt(b)/(sqrt(x)*s
qrt(a/(b*x) - 1)) + I*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), Abs(a)/(A
bs(b)*Abs(x)) > 1), (-sqrt(b)*sqrt(-a/(b*x) + 1)/sqrt(x) - b*asin(sqrt(a)/(
sqrt(b)*sqrt(x)))/sqrt(a), True))
```

Giac [A] time = 1.1966, size = 55, normalized size = 1.31

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{bx-a}b}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - sqrt(b*x - a)*b/x)/b
```

3.327 $\int \frac{\sqrt{-a+bx}}{x^3}\,dx$

Optimal. Leaf size=71

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt{bx-a}}{2x^2}+\frac{b\sqrt{bx-a}}{4ax}$$

```
[Out] -Sqrt[-a + b*x]/(2*x^2) + (b*Sqrt[-a + b*x])/(4*a*x) + (b^2*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rubi [A] time = 0.015946, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 51, 63, 205}

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt{bx-a}}{2x^2}+\frac{b\sqrt{bx-a}}{4ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x^3,x]
```

```
[Out] -Sqrt[-a + b*x]/(2*x^2) + (b*Sqrt[-a + b*x])/(4*a*x) + (b^2*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-a+bx}}{x^3}\,dx &= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{1}{4}b\int \frac{1}{x^2\sqrt{-a+bx}}\,dx\\
&= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b^2\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a}\\
&= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{4a}\\
&= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0241265, size = 38, normalized size = 0.54

$$\frac{2b^2(bx-a)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-\frac{bx}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x^3,x]
```

```
[Out] (2*b^2*(-a + b*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 - (b*x)/a])/(3*a^3
)
```

Maple [A] time = 0.011, size = 55, normalized size = 0.8

$$\frac{1}{4\,a{x}^{2}}\left(bx-a\right)^{\frac{3}{2}}-\frac{1}{4\,{x}^{2}}\sqrt{bx-a}+\frac{b^2}{4}\arctan\left(\sqrt{bx-a}\frac{1}{\sqrt{a}}\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x^3,x)
```

```
[Out] 1/4/x^2/a*(b*x-a)^(3/2)-1/4*(b*x-a)^(1/2)/x^2+1/4*b^2*arctan((b*x-a)^(1/2)/
a^(1/2))/a^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60857, size = 289, normalized size = 4.07

$$\left[-\frac{\sqrt{-a}b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)-2\left(abx-2a^2\right)\sqrt{bx-a}}{8a^2x^2},\frac{\sqrt{a}b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(abx-2a^2\right)\sqrt{bx-a}}{4a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2*(
a*b*x - 2*a^2)*sqrt(b*x - a))/(a^2*x^2), 1/4*(sqrt(a)*b^2*x^2*arctan(sqrt(b
*x - a)/sqrt(a)) + (a*b*x - 2*a^2)*sqrt(b*x - a))/(a^2*x^2)]
```

Sympy [A] time = 4.88827, size = 211, normalized size = 2.97

$$\begin{cases}-\frac{ia}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{3i\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{ib^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{\frac{a}{bx}-1}}+\frac{ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}} & \text{for}\ \frac{|a|}{|b||x|}>1\\ \frac{a}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{b^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x**3,x)
```

```
[Out] Piecewise((-I*a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + 3*I*sqrt(b)/(4*x**
(3/2)*sqrt(a/(b*x) - 1)) - I*b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) - 1)) + I*b
**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(3/2)), Abs(a)/(Abs(b)*Abs(x)) >
 1), (a/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - 3*sqrt(b)/(4*x**(3/2)*sqr
t(-a/(b*x) + 1)) + b**(3/2)/(4*a*sqrt(x)*sqrt(-a/(b*x) + 1)) - b**2*asin(sq
rt(a)/(sqrt(b)*sqrt(x)))/(4*a**(3/2)), True))
```

Giac [A] time = 1.21496, size = 89, normalized size = 1.25

$$\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{(bx-a)^{\frac{3}{2}}b^3-\sqrt{bx-a}ab^3}{ab^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + ((b*x - a)^(3/2)*b^3 - sqr
t(b*x - a)*a*b^3)/(a*b^2*x^2))/b
```

3.328 $\int \frac{(-a+bx)^{3/2}}{x}\,dx$

Optimal. Leaf size=55

$$2a^{3/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2a\sqrt{bx-a}+\frac{2}{3}(bx-a)^{3/2}$$

```
[Out] -2*a*Sqrt[-a + b*x] + (2*(-a + b*x)^(3/2))/3 + 2*a^(3/2)*ArcTan[Sqrt[-a + b
*x]/Sqrt[a]]
```

Rubi [A] time = 0.0145422, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 63, 205}

$$2a^{3/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2a\sqrt{bx-a}+\frac{2}{3}(bx-a)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x,x]
```


```
[Out] -2*a*Sqrt[-a + b*x] + (2*(-a + b*x)^(3/2))/3 + 2*a^(3/2)*ArcTan[Sqrt[-a + b
*x]/Sqrt[a]]
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-a+bx)^{3/2}}{x}\,dx &= \frac{2}{3}(-a+bx)^{3/2} - a\int \frac{\sqrt{-a+bx}}{x}\,dx \\
&= -2a\sqrt{-a+bx} + \frac{2}{3}(-a+bx)^{3/2} + a^2\int \frac{1}{x\sqrt{-a+bx}}\,dx \\
&= -2a\sqrt{-a+bx} + \frac{2}{3}(-a+bx)^{3/2} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b} \\
&= -2a\sqrt{-a+bx} + \frac{2}{3}(-a+bx)^{3/2} + 2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0742335, size = 48, normalized size = 0.87

$$
2a^{3/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + \frac{2}{3}(bx-4a)\sqrt{bx-a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x,x]
```

```
[Out] (2*(-4*a + b*x)*Sqrt[-a + b*x])/3 + 2*a^(3/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a]
]
```

Maple [A] time = 0.005, size = 44, normalized size = 0.8

$$
\frac{2}{3}(bx-a)^{\frac{3}{2}} + 2\,a^{3/2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) - 2\,a\sqrt{bx-a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x,x)
```

```
[Out] 2/3*(b*x-a)^(3/2)+2*a^(3/2)*arctan((b*x-a)^(1/2)/a^(1/2))-2*a*(b*x-a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57924, size = 225, normalized size = 4.09

$$
\left[\sqrt{-a}a\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right) + \frac{2}{3}\sqrt{bx-a}(bx-4a), 2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + \frac{2}{3}\sqrt{bx-a}(bx-4a)\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*a*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2/3*sqrt(b*x -
a)*(b*x - 4*a), 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*sqrt(b*x - a)
*(b*x - 4*a)]
```

Sympy [C] time = 3.19287, size = 189, normalized size = 3.44

$$\begin{cases} -\frac{8a^{\frac{3}{2}}\sqrt{-1+\frac{bx}{a}}}{3} - ia^{\frac{3}{2}}\log\left(\frac{bx}{a}\right) + 2ia^{\frac{3}{2}}\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - 2a^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{2\sqrt{a}bx\sqrt{-1+\frac{bx}{a}}}{3} & \text{for } \frac{|bx|}{|a|} > 1 \\ -\frac{8ia^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{3} - ia^{\frac{3}{2}}\log\left(\frac{bx}{a}\right) + 2ia^{\frac{3}{2}}\log\left(\sqrt{1-\frac{bx}{a}}+1\right) + \frac{2i\sqrt{a}bx\sqrt{1-\frac{bx}{a}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x,x)
```

```
[Out] Piecewise((-8*a**(3/2)*sqrt(-1 + b*x/a)/3 - I*a**(3/2)*log(b*x/a) + 2*I*a**
(3/2)*log(sqrt(b)*sqrt(x)/sqrt(a)) - 2*a**(3/2)*asin(sqrt(a)/(sqrt(b)*sqrt(
x))) + 2*sqrt(a)*b*x*sqrt(-1 + b*x/a)/3, Abs(b*x)/Abs(a) > 1), (-8*I*a**(3/
2)*sqrt(1 - b*x/a)/3 - I*a**(3/2)*log(b*x/a) + 2*I*a**(3/2)*log(sqrt(1 - b*
x/a) + 1) + 2*I*sqrt(a)*b*x*sqrt(1 - b*x/a)/3, True))
```

Giac [A] time = 1.16273, size = 58, normalized size = 1.05

$$2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + \frac{2}{3}\,(bx-a)^{\frac{3}{2}} - 2\,\sqrt{bx-a}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2) - 2*sqrt(b*x
- a)*a
```

3.329 $\int \frac{(-a+bx)^{3/2}}{x^2} dx$

Optimal. Leaf size=57

$$-\frac{(bx-a)^{3/2}}{x}+3b\sqrt{bx-a}-3\sqrt{a}b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

```
[Out] 3*b*Sqrt[-a + b*x] - (-a + b*x)^(3/2)/x - 3*Sqrt[a]*b*ArcTan[Sqrt[-a + b*x]
/Sqrt[a]]
```

Rubi [A] time = 0.0143466, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 205}

$$-\frac{(bx-a)^{3/2}}{x}+3b\sqrt{bx-a}-3\sqrt{a}b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x^2,x]
```

```
[Out] 3*b*Sqrt[-a + b*x] - (-a + b*x)^(3/2)/x - 3*Sqrt[a]*b*ArcTan[Sqrt[-a + b*x]
/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-a+bx)^{3/2}}{x^2}\,dx &= -\frac{(-a+bx)^{3/2}}{x}+\frac{1}{2}(3b)\int \frac{\sqrt{-a+bx}}{x}\,dx\\
&= 3b\sqrt{-a+bx}-\frac{(-a+bx)^{3/2}}{x}-\frac{1}{2}(3ab)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\
&= 3b\sqrt{-a+bx}-\frac{(-a+bx)^{3/2}}{x}-(3a)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)\\
&= 3b\sqrt{-a+bx}-\frac{(-a+bx)^{3/2}}{x}-3\sqrt{a}b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0112193, size = 36, normalized size = 0.63

$$
\frac{2b(bx-a)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1-\frac{bx}{a}\right)}{5a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x^2,x]
```

```
[Out] (2*b*(-a + b*x)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 - (b*x)/a])/(5*a^2)
```

Maple [A] time = 0.01, size = 48, normalized size = 0.8

$$
2\,b\sqrt{bx-a}+\frac{a}{x}\sqrt{bx-a}-3\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\sqrt{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x^2,x)
```

```
[Out] 2*b*(b*x-a)^(1/2)+a*(b*x-a)^(1/2)/x-3*b*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60363, size = 246, normalized size = 4.32

$$\left[\frac{3\sqrt{-a}bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2(2bx+a)\sqrt{bx-a}}{2x}, -\frac{3\sqrt{a}bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-(2bx+a)\sqrt{bx-a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*(2*b
*x + a)*sqrt(b*x - a))/x, -(3*sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) - (
2*b*x + a)*sqrt(b*x - a))/x]
```

Sympy [B] time = 3.56156, size = 201, normalized size = 3.53

$$\begin{cases} -3i\sqrt{a}b\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{ia^2}{\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{ia\sqrt{b}}{\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{2ib^{\frac{3}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for}\ \frac{|a|}{|b||x|}>1 \\ 3\sqrt{a}b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)-\frac{a^2}{\sqrt{b}x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{a\sqrt{b}}{\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{2b^{\frac{3}{2}}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x**2,x)
```

```
[Out] Piecewise((-3*I*sqrt(a)*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x))) + I*a**2/(sqrt(b
)*x**(3/2)*sqrt(a/(b*x) - 1)) + I*a*sqrt(b)/(sqrt(x)*sqrt(a/(b*x) - 1)) - 2
*I*b**(3/2)*sqrt(x)/sqrt(a/(b*x) - 1), Abs(a)/(Abs(b)*Abs(x)) > 1), (3*sqrt
(a)*b*asin(sqrt(a)/(sqrt(b)*sqrt(x))) - a**2/(sqrt(b)*x**(3/2)*sqrt(-a/(b*x
) + 1)) - a*sqrt(b)/(sqrt(x)*sqrt(-a/(b*x) + 1)) + 2*b**(3/2)*sqrt(x)/sqrt(
-a/(b*x) + 1), True))
```

Giac [A] time = 1.23978, size = 78, normalized size = 1.37

$$-\frac{3\sqrt{a}b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2\sqrt{bx-a}b^2-\frac{\sqrt{bx-a}ab}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -(3*sqrt(a)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) - 2*sqrt(b*x - a)*b^2 - sqrt(
b*x - a)*a*b/x)/b
```

3.330 $\int \frac{(-a+bx)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=68

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{(bx-a)^{3/2}}{2x^2}-\frac{3b\sqrt{bx-a}}{4x}$$

```
[Out] (-3*b*Sqrt[-a + b*x])/(4*x) - (-a + b*x)^(3/2)/(2*x^2) + (3*b^2*ArcTan[Sqrt
[-a + b*x]/Sqrt[a]])/(4*Sqrt[a])
```

Rubi [A] time = 0.0148347, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {47, 63, 205}

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{(bx-a)^{3/2}}{2x^2}-\frac{3b\sqrt{bx-a}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x^3,x]
```

```
[Out] (-3*b*Sqrt[-a + b*x])/(4*x) - (-a + b*x)^(3/2)/(2*x^2) + (3*b^2*ArcTan[Sqrt
[-a + b*x]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-a+bx)^{3/2}}{x^3}\,dx &= -\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\int \frac{\sqrt{-a+bx}}{x^2}\,dx\\
&= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\
&= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)\\
&= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.0521995, size = 72, normalized size = 1.06

$$
-\frac{2a^2+3b^2x^2\sqrt{1-\frac{bx}{a}}\tanh^{-1}\left(\sqrt{1-\frac{bx}{a}}\right)-7abx+5b^2x^2}{4x^2\sqrt{bx-a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x^3,x]
```

```
[Out] -(2*a^2 - 7*a*b*x + 5*b^2*x^2 + 3*b^2*x^2*Sqrt[1 - (b*x)/a]*ArcTanh[Sqrt[1
- (b*x)/a]])/(4*x^2*Sqrt[-a + b*x])
```

Maple [A] time = 0.01, size = 53, normalized size = 0.8

$$
-\frac{5}{4\,x^2}\,(bx-a)^{\frac{3}{2}}-\frac{3\,a}{4\,x^2}\sqrt{bx-a}+\frac{3\,b^2}{4}\arctan\left(\sqrt{bx-a}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x^3,x)
```

```
[Out] -5/4*(b*x-a)^(3/2)/x^2-3/4/x^2*a*(b*x-a)^(1/2)+3/4*b^2*arctan((b*x-a)^(1/2)
/a^(1/2))/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.557, size = 294, normalized size = 4.32

$$\left[-\frac{3\sqrt{-a}b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(5abx-2a^2\right)\sqrt{bx-a}}{8ax^2},\ \frac{3\sqrt{a}b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\left(5abx-2a^2\right)\sqrt{bx-a}}{4ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2
*(5*a*b*x - 2*a^2)*sqrt(b*x - a))/(a*x^2), 1/4*(3*sqrt(a)*b^2*x^2*arctan(sq
rt(b*x - a)/sqrt(a)) - (5*a*b*x - 2*a^2)*sqrt(b*x - a))/(a*x^2)]
```

Sympy [A] time = 4.50158, size = 194, normalized size = 2.85

$$\begin{cases} \frac{ia^2}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{7ia\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{5ib^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}-1}}+\frac{3ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}} & \text{for}\ \frac{|a|}{|b||x|}>1 \\ \frac{a\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{2x^{\frac{3}{2}}}-\frac{5b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{4\sqrt{x}}-\frac{3b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x**3,x)
```

```
[Out] Piecewise((I*a**2/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) - 7*I*a*sqrt(b)/(4
*x**(3/2)*sqrt(a/(b*x) - 1)) + 5*I*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) - 1)) +
 3*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a)), Abs(a)/(Abs(b)*Abs(
x)) > 1), (a*sqrt(b)*sqrt(-a/(b*x) + 1)/(2*x**(3/2)) - 5*b**(3/2)*sqrt(-a/(
b*x) + 1)/(4*sqrt(x)) - 3*b**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a)),
 True))
```

Giac [A] time = 1.21901, size = 89, normalized size = 1.31

$$\frac{\frac{3b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{5(bx-a)^{\frac{3}{2}}b^3+3\sqrt{bx-a}ab^3}{b^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - (5*(b*x - a)^(3/2)*b^3 +
 3*sqrt(b*x - a)*a*b^3)/(b^2*x^2))/b
```

3.331 $\int \frac{(-a+bx)^{5/2}}{x} dx$

Optimal. Leaf size=73

$$2a^2\sqrt{bx-a}-2a^{5/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{2}{3}a(bx-a)^{3/2}+\frac{2}{5}(bx-a)^{5/2}$$

```
[Out] 2*a^2*Sqrt[-a + b*x] - (2*a*(-a + b*x)^(3/2))/3 + (2*(-a + b*x)^(5/2))/5 -
2*a^(5/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.0191096, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 63, 205}

$$2a^2\sqrt{bx-a}-2a^{5/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{2}{3}a(bx-a)^{3/2}+\frac{2}{5}(bx-a)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x,x]
```

```
[Out] 2*a^2*Sqrt[-a + b*x] - (2*a*(-a + b*x)^(3/2))/3 + (2*(-a + b*x)^(5/2))/5 -
2*a^(5/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-a+bx)^{5/2}}{x}\,dx &= \frac{2}{5}(-a+bx)^{5/2} - a\int \frac{(-a+bx)^{3/2}}{x}\,dx \\
&= -\frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} + a^2\int \frac{\sqrt{-a+bx}}{x}\,dx \\
&= 2a^2\sqrt{-a+bx} - \frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} - a^3\int \frac{1}{x\sqrt{-a+bx}}\,dx \\
&= 2a^2\sqrt{-a+bx} - \frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} - \frac{(2a^3)\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b} \\
&= 2a^2\sqrt{-a+bx} - \frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} - 2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0607656, size = 60, normalized size = 0.82

$$
\frac{2}{15}\sqrt{bx-a}\left(23a^2-11abx+3b^2x^2\right) - 2a^{5/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[-a + b*x]*(23*a^2 - 11*a*b*x + 3*b^2*x^2))/15 - 2*a^(5/2)*ArcTan[Sq
rt[-a + b*x]/Sqrt[a]]
```

Maple [A] time = 0.006, size = 58, normalized size = 0.8

$$
-\frac{2\,a}{3}\,(bx-a)^{\frac{3}{2}} + \frac{2}{5}\,(bx-a)^{\frac{5}{2}} - 2\,a^{5/2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + 2\,a^2\sqrt{bx-a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x,x)
```

```
[Out] -2/3*a*(b*x-a)^(3/2)+2/5*(b*x-a)^(5/2)-2*a^(5/2)*arctan((b*x-a)^(1/2)/a^(1/
2))+2*a^2*(b*x-a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54555, size = 286, normalized size = 3.92

$$\left[\sqrt{-a}a^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+\frac{2}{15}\left(3b^2x^2-11abx+23a^2\right)\sqrt{bx-a},-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{15}\left(3b^2x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*a^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2/15*(3*b^2*x
^2 - 11*a*b*x + 23*a^2)*sqrt(b*x - a), -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt
(a)) + 2/15*(3*b^2*x^2 - 11*a*b*x + 23*a^2)*sqrt(b*x - a)]
```

Sympy [C] time = 4.97312, size = 241, normalized size = 3.3

$$\begin{cases} \frac{46a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}}{15}+ia^{\frac{5}{2}}\log\left(\frac{bx}{a}\right)-2ia^{\frac{5}{2}}\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+2a^{\frac{5}{2}}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)-\frac{22a^{\frac{3}{2}}bx\sqrt{-1+\frac{bx}{a}}}{15}+\frac{2\sqrt{a}b^2x^2\sqrt{-1+\frac{bx}{a}}}{5} & \text{for}\ \frac{|bx|}{|a|}>1 \\ \frac{46ia^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{15}+ia^{\frac{5}{2}}\log\left(\frac{bx}{a}\right)-2ia^{\frac{5}{2}}\log\left(\sqrt{1-\frac{bx}{a}}+1\right)-\frac{22ia^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}}{15}+\frac{2i\sqrt{a}b^2x^2\sqrt{1-\frac{bx}{a}}}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x,x)
```

```
[Out] Piecewise((46*a**(5/2)*sqrt(-1 + b*x/a)/15 + I*a**(5/2)*log(b*x/a) - 2*I*a*
*(5/2)*log(sqrt(b)*sqrt(x)/sqrt(a)) + 2*a**(5/2)*asin(sqrt(a)/(sqrt(b)*sqrt
(x))) - 22*a**(3/2)*b*x*sqrt(-1 + b*x/a)/15 + 2*sqrt(a)*b**2*x**2*sqrt(-1 +
 b*x/a)/5, Abs(b*x)/Abs(a) > 1), (46*I*a**(5/2)*sqrt(1 - b*x/a)/15 + I*a**(
5/2)*log(b*x/a) - 2*I*a**(5/2)*log(sqrt(1 - b*x/a) + 1) - 22*I*a**(3/2)*b*x
*sqrt(1 - b*x/a)/15 + 2*I*sqrt(a)*b**2*x**2*sqrt(1 - b*x/a)/5, True))
```

Giac [A] time = 1.19064, size = 77, normalized size = 1.05

$$-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{5}(bx-a)^{\frac{5}{2}}-\frac{2}{3}(bx-a)^{\frac{3}{2}}a+2\sqrt{bx-a}a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/5*(b*x - a)^(5/2) - 2/3*(b*x -
 a)^(3/2)*a + 2*sqrt(b*x - a)*a^2
```

3.332 $\int \frac{(-a+bx)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=74

$$5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{(bx-a)^{5/2}}{x}+\frac{5}{3}b(bx-a)^{3/2}-5ab\sqrt{bx-a}$$

```
[Out] -5*a*b*Sqrt[-a + b*x] + (5*b*(-a + b*x)^(3/2))/3 - (-a + b*x)^(5/2)/x + 5*a
^(3/2)*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.0197625, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 205}

$$5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{(bx-a)^{5/2}}{x}+\frac{5}{3}b(bx-a)^{3/2}-5ab\sqrt{bx-a}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x^2,x]
```

```
[Out] -5*a*b*Sqrt[-a + b*x] + (5*b*(-a + b*x)^(3/2))/3 - (-a + b*x)^(5/2)/x + 5*a
^(3/2)*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-a+bx)^{5/2}}{x^2}\,dx &= -\frac{(-a+bx)^{5/2}}{x}+\frac{1}{2}(5b)\int \frac{(-a+bx)^{3/2}}{x}\,dx\\
&= \frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}-\frac{1}{2}(5ab)\int \frac{\sqrt{-a+bx}}{x}\,dx\\
&= -5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+\frac{1}{2}\left(5a^2b\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\
&= -5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+\left(5a^2\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)\\
&= -5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0175202, size = 36, normalized size = 0.49

$$\frac{2b(bx-a)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{bx}{a}\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x^2,x]
```

```
[Out] (2*b*(-a + b*x)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 - (b*x)/a])/(7*a^2)
```

Maple [A] time = 0.01, size = 64, normalized size = 0.9

$$\frac{2b}{3}(bx-a)^{\frac{3}{2}}-4ab\sqrt{bx-a}-\frac{a^2}{x}\sqrt{bx-a}+5a^{3/2}b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x^2,x)
```

```
[Out] 2/3*b*(b*x-a)^(3/2)-4*a*b*(b*x-a)^(1/2)-a^2*(b*x-a)^(1/2)/x+5*a^(3/2)*b*arc
tan((b*x-a)^(1/2)/a^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5684, size = 306, normalized size = 4.14

$$\left[\frac{15\sqrt{-a}abx\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(2b^2x^2-14abx-3a^2\right)\sqrt{bx-a}}{6x}, \frac{15a^{\frac{3}{2}}bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(2b^2x^2-14abx-3a^2\right)\sqrt{bx-a}}{3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*sqrt(-a)*a*b*x*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*(
2*b^2*x^2 - 14*a*b*x - 3*a^2)*sqrt(b*x - a))/x, 1/3*(15*a^(3/2)*b*x*arctan(
sqrt(b*x - a)/sqrt(a)) + (2*b^2*x^2 - 14*a*b*x - 3*a^2)*sqrt(b*x - a))/x]
```

Sympy [C] time = 4.85502, size = 246, normalized size = 3.32

$$\begin{cases} -\frac{a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}}{x} - \frac{14a^{\frac{3}{2}}b\sqrt{-1+\frac{bx}{a}}}{3} - \frac{5ia^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2} + 5ia^{\frac{3}{2}}b\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - 5a^{\frac{3}{2}}b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{2\sqrt{a}b^2x\sqrt{-1+\frac{bx}{a}}}{3} & \text{for}\ \frac{|bx|}{|a|} > 1 \\ -\frac{ia^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{x} - \frac{14ia^{\frac{3}{2}}b\sqrt{1-\frac{bx}{a}}}{3} - \frac{5ia^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2} + 5ia^{\frac{3}{2}}b\log\left(\sqrt{1-\frac{bx}{a}}+1\right) + \frac{2i\sqrt{a}b^2x\sqrt{1-\frac{bx}{a}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x**2,x)
```

```
[Out] Piecewise((-a**(5/2)*sqrt(-1 + b*x/a)/x - 14*a**(3/2)*b*sqrt(-1 + b*x/a)/3
- 5*I*a**(3/2)*b*log(b*x/a)/2 + 5*I*a**(3/2)*b*log(sqrt(b)*sqrt(x)/sqrt(a))
 - 5*a**(3/2)*b*asin(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*sqrt(a)*b**2*x*sqrt(-1
+ b*x/a)/3, Abs(b*x)/Abs(a) > 1), (-I*a**(5/2)*sqrt(1 - b*x/a)/x - 14*I*a**
(3/2)*b*sqrt(1 - b*x/a)/3 - 5*I*a**(3/2)*b*log(b*x/a)/2 + 5*I*a**(3/2)*b*lo
g(sqrt(1 - b*x/a) + 1) + 2*I*sqrt(a)*b**2*x*sqrt(1 - b*x/a)/3, True))
```

Giac [A] time = 1.14342, size = 101, normalized size = 1.36

$$\frac{15a^{\frac{3}{2}}b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2(bx-a)^{\frac{3}{2}}b^2-12\sqrt{bx-a}ab^2-\frac{3\sqrt{bx-a}a^2b}{x}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*a^(3/2)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) + 2*(b*x - a)^(3/2)*b^2 -
 12*sqrt(b*x - a)*a*b^2 - 3*sqrt(b*x - a)*a^2*b/x)/b
```

3.333 $\int \frac{(-a+bx)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=86

$$\frac{15}{4}b^2\sqrt{bx-a}-\frac{15}{4}\sqrt{a}b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{(bx-a)^{5/2}}{2x^2}-\frac{5b(bx-a)^{3/2}}{4x}$$

```
[Out] (15*b^2*Sqrt[-a + b*x])/4 - (5*b*(-a + b*x)^(3/2))/(4*x) - (-a + b*x)^(5/2)
/(2*x^2) - (15*Sqrt[a]*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/4
```

Rubi [A] time = 0.022194, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 205}

$$\frac{15}{4}b^2\sqrt{bx-a}-\frac{15}{4}\sqrt{a}b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{(bx-a)^{5/2}}{2x^2}-\frac{5b(bx-a)^{3/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x^3,x]
```

```
[Out] (15*b^2*Sqrt[-a + b*x])/4 - (5*b*(-a + b*x)^(3/2))/(4*x) - (-a + b*x)^(5/2)
/(2*x^2) - (15*Sqrt[a]*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-a+bx)^{5/2}}{x^3}\,dx &= -\frac{(-a+bx)^{5/2}}{2x^2}+\frac{1}{4}(5b)\int \frac{(-a+bx)^{3/2}}{x^2}\,dx\\
&= -\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15b^2\right)\int \frac{\sqrt{-a+bx}}{x}\,dx\\
&= \frac{15}{4}b^2\sqrt{-a+bx}-\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}-\frac{1}{8}\left(15ab^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\
&= \frac{15}{4}b^2\sqrt{-a+bx}-\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}-\frac{1}{4}(15ab)\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)\\
&= \frac{15}{4}b^2\sqrt{-a+bx}-\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}-\frac{15}{4}\sqrt{a}b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.026345, size = 38, normalized size = 0.44

$$
\frac{2b^2(bx-a)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\frac{bx}{a}\right)}{7a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x^3,x]

[Out] (2*b^2*(-a + b*x)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 - (b*x)/a])/(7*a^3
)
```

Maple [A] time = 0.01, size = 70, normalized size = 0.8

$$
2\,b^2\sqrt{bx-a}+\frac{9\,a}{4\,x^2}\,(bx-a)^{\frac{3}{2}}+\frac{7\,a^2}{4\,x^2}\sqrt{bx-a}-\frac{15\,b^2}{4}\,\mathrm{arctan}\left(\sqrt{bx-a}\frac{1}{\sqrt{a}}\right)\sqrt{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x^3,x)

[Out] 2*b^2*(b*x-a)^(1/2)+9/4*a/x^2*(b*x-a)^(3/2)+7/4/x^2*a^2*(b*x-a)^(1/2)-15/4*
b^2*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62388, size = 319, normalized size = 3.71

$$\left[\frac{15\sqrt{-a}b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(8b^2x^2+9abx-2a^2\right)\sqrt{bx-a}}{8x^2}, -\frac{15\sqrt{a}b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\left(8b^2x^2+9abx\right.}{4x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2
*(8*b^2*x^2 + 9*a*b*x - 2*a^2)*sqrt(b*x - a))/x^2, -1/4*(15*sqrt(a)*b^2*x^2
*arctan(sqrt(b*x - a)/sqrt(a)) - (8*b^2*x^2 + 9*a*b*x - 2*a^2)*sqrt(b*x - a
))/x^2]
```

Sympy [A] time = 5.67359, size = 270, normalized size = 3.14

$$\begin{cases} -\frac{15i\sqrt{a}b^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4}-\frac{ia^3}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{11ia^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{iab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{2ib^{\frac{5}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for}\ \frac{|a|}{|b||x|}>1 \\ \frac{15\sqrt{a}b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4}+\frac{a^3}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{11a^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{ab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{2b^{\frac{5}{2}}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x**3,x)
```

```
[Out] Piecewise((-15*I*sqrt(a)*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - I*a**3/(
2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + 11*I*a**2*sqrt(b)/(4*x**(3/2)*sqrt(
a/(b*x) - 1)) - I*a*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) - 1)) - 2*I*b**(5/2)*s
qrt(x)/sqrt(a/(b*x) - 1), Abs(a)/(Abs(b)*Abs(x)) > 1), (15*sqrt(a)*b**2*asi
n(sqrt(a)/(sqrt(b)*sqrt(x)))/4 + a**3/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1
)) - 11*a**2*sqrt(b)/(4*x**(3/2)*sqrt(-a/(b*x) + 1)) + a*b**(3/2)/(4*sqrt(x
)*sqrt(-a/(b*x) + 1)) + 2*b**(5/2)*sqrt(x)/sqrt(-a/(b*x) + 1), True))
```

Giac [A] time = 1.19511, size = 112, normalized size = 1.3

$$-\frac{15\sqrt{a}b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-8\sqrt{bx-a}b^3-\frac{9(bx-a)^{\frac{3}{2}}ab^3+7\sqrt{bx-a}a^2b^3}{b^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(15*sqrt(a)*b^3*arctan(sqrt(b*x - a)/sqrt(a)) - 8*sqrt(b*x - a)*b^3 -
(9*(b*x - a)^(3/2)*a*b^3 + 7*sqrt(b*x - a)*a^2*b^3)/(b^2*x^2))/b
```

3.334 $\int \frac{x^4}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=89

$$\frac{12a^2(a+bx)^{5/2}}{5b^5}-\frac{8a^3(a+bx)^{3/2}}{3b^5}+\frac{2a^4\sqrt{a+bx}}{b^5}+\frac{2(a+bx)^{9/2}}{9b^5}-\frac{8a(a+bx)^{7/2}}{7b^5}$$

```
[Out] (2*a^4*Sqrt[a + b*x])/b^5 - (8*a^3*(a + b*x)^(3/2))/(3*b^5) + (12*a^2*(a +
b*x)^(5/2))/(5*b^5) - (8*a*(a + b*x)^(7/2))/(7*b^5) + (2*(a + b*x)^(9/2))/(
9*b^5)
```

Rubi [A] time = 0.0222366, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{12a^2(a+bx)^{5/2}}{5b^5}-\frac{8a^3(a+bx)^{3/2}}{3b^5}+\frac{2a^4\sqrt{a+bx}}{b^5}+\frac{2(a+bx)^{9/2}}{9b^5}-\frac{8a(a+bx)^{7/2}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x],x]
```

```
[Out] (2*a^4*Sqrt[a + b*x])/b^5 - (8*a^3*(a + b*x)^(3/2))/(3*b^5) + (12*a^2*(a +
b*x)^(5/2))/(5*b^5) - (8*a*(a + b*x)^(7/2))/(7*b^5) + (2*(a + b*x)^(9/2))/(
9*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+bx}}\,dx &= \int \left(\frac{a^4}{b^4\sqrt{a+bx}}-\frac{4a^3\sqrt{a+bx}}{b^4}+\frac{6a^2(a+bx)^{3/2}}{b^4}-\frac{4a(a+bx)^{5/2}}{b^4}+\frac{(a+bx)^{7/2}}{b^4}\right)dx\\ &= \frac{2a^4\sqrt{a+bx}}{b^5}-\frac{8a^3(a+bx)^{3/2}}{3b^5}+\frac{12a^2(a+bx)^{5/2}}{5b^5}-\frac{8a(a+bx)^{7/2}}{7b^5}+\frac{2(a+bx)^{9/2}}{9b^5}\end{aligned}$$

Mathematica [A] time = 0.0437078, size = 57, normalized size = 0.64

$$\frac{2\sqrt{a+bx}\left(48a^2b^2x^2-64a^3bx+128a^4-40ab^3x^3+35b^4x^4\right)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(128*a^4 - 64*a^3*b*x + 48*a^2*b^2*x^2 - 40*a*b^3*x^3 + 35
*b^4*x^4))/(315*b^5)
```

Maple [A] time = 0.004, size = 54, normalized size = 0.6

$$\frac{70 x^4 b^4 - 80 a x^3 b^3 + 96 a^2 x^2 b^2 - 128 a^3 x b + 256 a^4}{315 b^5} \sqrt{bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^(1/2),x)
```

```
[Out] 2/315*(b*x+a)^(1/2)*(35*b^4*x^4-40*a*b^3*x^3+48*a^2*b^2*x^2-64*a^3*b*x+128*
a^4)/b^5
```

Maxima [A] time = 1.07997, size = 96, normalized size = 1.08

$$\frac{2 (bx + a)^{\frac{9}{2}}}{9 b^5} - \frac{8 (bx + a)^{\frac{7}{2}} a}{7 b^5} + \frac{12 (bx + a)^{\frac{5}{2}} a^2}{5 b^5} - \frac{8 (bx + a)^{\frac{3}{2}} a^3}{3 b^5} + \frac{2 \sqrt{bx + a} a^4}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^5 - 8/7*(b*x + a)^(7/2)*a/b^5 + 12/5*(b*x + a)^(5/2)*
a^2/b^5 - 8/3*(b*x + a)^(3/2)*a^3/b^5 + 2*sqrt(b*x + a)*a^4/b^5
```

Fricas [A] time = 1.43878, size = 126, normalized size = 1.42

$$\frac{2 \left(35 b^4 x^4 - 40 a b^3 x^3 + 48 a^2 b^2 x^2 - 64 a^3 b x + 128 a^4\right) \sqrt{bx + a}}{315 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 - 40*a*b^3*x^3 + 48*a^2*b^2*x^2 - 64*a^3*b*x + 128*a^4)*s
qrt(b*x + a)/b^5
```

Sympy [B] time = 6.16229, size = 3755, normalized size = 42.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(1/2),x)
```

```
[Out] 256*a**(89/2)*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a
**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**3
5*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**3
2*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 256*a**(89/
2)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**3
7*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*
```

b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b
14*x9 + 315*a**30*b**15*x**10) + 2432*a**(87/2)*b*x*sqrt(1 + b*x/a)/(31
5*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8
*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*
x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x
9 + 315*a30*b**15*x**10) - 2560*a**(87/2)*b*x/(315*a**40*b**5 + 3150*a*
*39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b*
*9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**
12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*
x**10) + 10336*a**(85/2)*b**2*x**2*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a
39*b6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b
9*x4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b*
*12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15
*x**10) - 11520*a**(85/2)*b**2*x**2/(315*a**40*b**5 + 3150*a**39*b**6*x + 1
4175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 7938
0*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 1417
5*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 25840
*a**(83/2)*b**3*x**3*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x +
14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 793
80*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 141
75*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 3072
0*a**(83/2)*b**3*x**3/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**
7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*
x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*
x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 41990*a**(81/2)*b**
4*x**4*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b*
*7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10
*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13
*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 53760*a**(81/2)*b*
*4*x**4/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800
*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a
34*b11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a*
*31*b**14*x**9 + 315*a**30*b**15*x**10) + 46252*a**(79/2)*b**5*x**5*sqrt(1
+ b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 3780
0*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*
a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a
31*b14*x**9 + 315*a**30*b**15*x**10) - 64512*a**(79/2)*b**5*x**5/(315*a
40*b5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x*
*3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**
6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9
+ 315*a**30*b**15*x**10) + 35214*a**(77/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*
a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x
3 + 66150*a36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x*
*6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**
9 + 315*a**30*b**15*x**10) - 53760*a**(77/2)*b**6*x**6/(315*a**40*b**5 + 31
50*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**
36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**3
3*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b
15*x10) + 19632*a**(75/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3
150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a*
*36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**
33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*
b**15*x**10) - 30720*a**(75/2)*b**7*x**7/(315*a**40*b**5 + 3150*a**39*b**6*
x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 +
79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 +
14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) +
10860*a**(73/2)*b**8*x**8*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6
*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4
+ 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7

```
+ 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) -
 11520*a**(73/2)*b**8*x**8/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**3
8*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b
**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b
**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 9160*a**(71/2)
*b**9*x**9*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**3
8*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b
**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b
**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 2560*a**(71/2)
*b**9*x**9/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37
800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 6615
0*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150
*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 8396*a**(69/2)*b**10*x**10*sqr
t(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 +
37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66
150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 31
50*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 256*a**(69/2)*b**10*x**10/(3
15*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**
8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11
*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*
x**9 + 315*a**30*b**15*x**10) + 5632*a**(67/2)*b**11*x**11*sqrt(1 + b*x/a)/
(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b
**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**
11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**1
4*x**9 + 315*a**30*b**15*x**10) + 2446*a**(65/2)*b**12*x**12*sqrt(1 + b*x/a
)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37
*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b
**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b*
*14*x**9 + 315*a**30*b**15*x**10) + 620*a**(63/2)*b**13*x**13*sqrt(1 + b*x/
a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**3
7*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*
b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b
**14*x**9 + 315*a**30*b**15*x**10) + 70*a**(61/2)*b**14*x**14*sqrt(1 + b*x/
a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**3
7*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*
b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b
**14*x**9 + 315*a**30*b**15*x**10)
```

Giac [A] time = 1.23519, size = 82, normalized size = 0.92

$$\frac{2\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}a^4\right)}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b^5
```

3.335 $\int \frac{x^3}{\sqrt{a+bx}} dx$

Optimal. Leaf size=68

$$\frac{2a^2(a+bx)^{3/2}}{b^4} - \frac{2a^3\sqrt{a+bx}}{b^4} + \frac{2(a+bx)^{7/2}}{7b^4} - \frac{6a(a+bx)^{5/2}}{5b^4}$$

```
[Out] (-2*a^3*Sqrt[a + b*x])/b^4 + (2*a^2*(a + b*x)^(3/2))/b^4 - (6*a*(a + b*x)^(
5/2))/(5*b^4) + (2*(a + b*x)^(7/2))/(7*b^4)
```

Rubi [A] time = 0.0178664, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{3/2}}{b^4} - \frac{2a^3\sqrt{a+bx}}{b^4} + \frac{2(a+bx)^{7/2}}{7b^4} - \frac{6a(a+bx)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x],x]
```

```
[Out] (-2*a^3*Sqrt[a + b*x])/b^4 + (2*a^2*(a + b*x)^(3/2))/b^4 - (6*a*(a + b*x)^(
5/2))/(5*b^4) + (2*(a + b*x)^(7/2))/(7*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx}} dx &= \int \left(-\frac{a^3}{b^3\sqrt{a+bx}} + \frac{3a^2\sqrt{a+bx}}{b^3} - \frac{3a(a+bx)^{3/2}}{b^3} + \frac{(a+bx)^{5/2}}{b^3}\right) dx \\ &= -\frac{2a^3\sqrt{a+bx}}{b^4} + \frac{2a^2(a+bx)^{3/2}}{b^4} - \frac{6a(a+bx)^{5/2}}{5b^4} + \frac{2(a+bx)^{7/2}}{7b^4}\end{aligned}$$

Mathematica [A] time = 0.0385522, size = 46, normalized size = 0.68

$$\frac{2\sqrt{a+bx}\left(8a^2bx - 16a^3 - 6ab^2x^2 + 5b^3x^3\right)}{35b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(35*b^4)
```

Maple [A] time = 0.004, size = 43, normalized size = 0.6

$$-\frac{-10\,b^{3}x^{3}+12\,ab^{2}x^{2}-16\,a^{2}bx+32\,a^{3}}{35\,b^{4}}\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)^(1/2)*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*x+16*a^3)/b^4
```

Maxima [A] time = 1.03788, size = 76, normalized size = 1.12

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^{4}}-\frac{6\,(bx+a)^{\frac{5}{2}}a}{5\,b^{4}}+\frac{2\,(bx+a)^{\frac{3}{2}}a^{2}}{b^{4}}-\frac{2\,\sqrt{bx+a}a^{3}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^4 - 6/5*(b*x + a)^(5/2)*a/b^4 + 2*(b*x + a)^(3/2)*a^2
/b^4 - 2*sqrt(b*x + a)*a^3/b^4
```

Fricas [A] time = 1.52225, size = 96, normalized size = 1.41

$$\frac{2\left(5\,b^{3}x^{3}-6\,ab^{2}x^{2}+8\,a^{2}bx-16\,a^{3}\right)\sqrt{bx+a}}{35\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x + a)/b^4
```

Sympy [B] time = 3.22659, size = 1640, normalized size = 24.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(1/2),x)
```

```
[Out] -32*a**(47/2)*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18
*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**
5 + 35*a**14*b**10*x**6) + 32*a**(47/2)/(35*a**20*b**4 + 210*a**19*b**5*x +
 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**1
5*b**9*x**5 + 35*a**14*b**10*x**6) - 176*a**(45/2)*b*x*sqrt(1 + b*x/a)/(35*
a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 192*a**
(45/2)*b*x/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a*
*17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*
```

```
x**6) - 396*a**(43/2)*b**2*x**2*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*
b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 +
210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 480*a**(43/2)*b**2*x**2/(35*a*
*20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 5
25*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) - 462*a**(4
1/2)*b**3*x**3*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**1
8*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x*
*5 + 35*a**14*b**10*x**6) + 640*a**(41/2)*b**3*x**3/(35*a**20*b**4 + 210*a*
*19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**
4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) - 280*a**(39/2)*b**4*x**4*sq
rt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700
*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**
10*x**6) + 480*a**(39/2)*b**4*x**4/(35*a**20*b**4 + 210*a**19*b**5*x + 525*
a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**
9*x**5 + 35*a**14*b**10*x**6) - 42*a**(37/2)*b**5*x**5*sqrt(1 + b*x/a)/(35*
a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 192*a**
(37/2)*b**5*x**5/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 +
700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*
b**10*x**6) + 84*a**(35/2)*b**6*x**6*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a
**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x*
*4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 32*a**(35/2)*b**6*x**6/(3
5*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3
 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 94*a*
*(33/2)*b**7*x**7*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a
**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9
*x**5 + 35*a**14*b**10*x**6) + 48*a**(31/2)*b**8*x**8*sqrt(1 + b*x/a)/(35*a
**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 10*a**(2
9/2)*b**9*x**9*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**1
8*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x*
*5 + 35*a**14*b**10*x**6)
```

Giac [A] time = 1.19916, size = 66, normalized size = 0.97

$$\frac{2\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}a^3\right)}{35b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 3
5*sqrt(b*x + a)*a^3)/b^4
```

3.336 $\int \frac{x^2}{\sqrt{a+bx}} dx$

Optimal. Leaf size=51

$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

```
[Out] (2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(5
/2))/(5*b^3)
```

Rubi [A] time = 0.0120363, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(5
/2))/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx}}\,dx &= \int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx\\ &= \frac{2a^2\sqrt{a+bx}}{b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.0198487, size = 35, normalized size = 0.69

$$\frac{2\sqrt{a+bx}\left(8a^2-4abx+3b^2x^2\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)
```

Maple [A] time = 0.003, size = 32, normalized size = 0.6

$$\frac{6 b^2 x^2 - 8 abx + 16 a^2}{15 b^3} \sqrt{bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(1/2),x)
```

```
[Out] 2/15*(b*x+a)^(1/2)*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3
```

Maxima [A] time = 1.0567, size = 55, normalized size = 1.08

$$\frac{2 (bx + a)^{\frac{5}{2}}}{5 b^3} - \frac{4 (bx + a)^{\frac{3}{2}} a}{3 b^3} + \frac{2 \sqrt{bx + a} a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^3 - 4/3*(b*x + a)^(3/2)*a/b^3 + 2*sqrt(b*x + a)*a^2/b
^3
```

Fricas [A] time = 1.48006, size = 73, normalized size = 1.43

$$\frac{2 \left(3 b^2 x^2 - 4 abx + 8 a^2\right) \sqrt{bx + a}}{15 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

Sympy [B] time = 2.18259, size = 600, normalized size = 11.76

$$\frac{16 a^{\frac{21}{2}} \sqrt{1 + \frac{bx}{a}}}{15 a^8 b^3 + 45 a^7 b^4 x + 45 a^6 b^5 x^2 + 15 a^5 b^6 x^3} - \frac{16 a^{\frac{21}{2}}}{15 a^8 b^3 + 45 a^7 b^4 x + 45 a^6 b^5 x^2 + 15 a^5 b^6 x^3} + \frac{40 a^{\frac{19}{2}} bx \sqrt{1 + \frac{bx}{a}}}{15 a^8 b^3 + 45 a^7 b^4 x + 45 a^6 b^5 x^2 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(21/2)*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*
x**2 + 15*a**5*b**6*x**3) - 16*a**(21/2)/(15*a**8*b**3 + 45*a**7*b**4*x + 4
5*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 40*a**(19/2)*b*x*sqrt(1 + b*x/a)/(1
5*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*
a**(19/2)*b*x/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*
b**6*x**3) + 30*a**(17/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7
*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(17/2)*b**2*x**2/(
15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10
```

```
*a**(15/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a*
*6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a**(15/2)*b**3*x**3/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10*a**(13/2)*b**4
*x**4*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 +
15*a**5*b**6*x**3) + 6*a**(11/2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3)
```

Giac [A] time = 1.16703, size = 50, normalized size = 0.98

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

3.337 $\int \frac{x}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=32

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

```
[Out] (-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Rubi [A] time = 0.0082382, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}}\,dx &= \int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx\\ &= -\frac{2a\sqrt{a+bx}}{b^2}+\frac{2(a+bx)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.0155484, size = 23, normalized size = 0.72

$$\frac{2(bx-2a)\sqrt{a+bx}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x],x]
```

```
[Out] (2*(-2*a + b*x)*Sqrt[a + b*x])/(3*b^2)
```

Maple [A] time = 0.003, size = 21, normalized size = 0.7

$$-\frac{-2\,bx+4\,a}{3\,b^2}\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(1/2),x)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2
```

Maxima [A] time = 1.06938, size = 35, normalized size = 1.09

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^2}-\frac{2\,\sqrt{bx+a}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^2 - 2*sqrt(b*x + a)*a/b^2
```

Fricas [A] time = 1.50168, size = 47, normalized size = 1.47

$$\frac{2\,\sqrt{bx+a}(bx-2\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2
```

Sympy [B] time = 1.43331, size = 162, normalized size = 5.06

$$-\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}+\frac{4a^{\frac{7}{2}}}{3a^2b^2+3ab^3x}-\frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}+\frac{4a^{\frac{5}{2}}bx}{3a^2b^2+3ab^3x}+\frac{2a^{\frac{3}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(7/2)*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(7/2)/(3*a**2
*b**2 + 3*a*b**3*x) - 2*a**(5/2)*b*x*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**
3*x) + 4*a**(5/2)*b*x/(3*a**2*b**2 + 3*a*b**3*x) + 2*a**(3/2)*b**2*x**2*sqr
t(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x)
```

Giac [A] time = 1.18997, size = 31, normalized size = 0.97

$$\frac{2\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

3.338 $\int \frac{1}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=14

$$\frac{2\sqrt{a+bx}}{b}$$

```
[Out] (2*Sqrt[a + b*x])/b
```

Rubi [A] time = 0.0013527, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x],x]

[Out] (2*Sqrt[a + b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}}{b}$$

Mathematica [A] time = 0.0056035, size = 14, normalized size = 1.

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x],x]

[Out] (2*Sqrt[a + b*x])/b
```

Maple [A] time = 0.003, size = 13, normalized size = 0.9

$$2\,\frac{\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2),x)
```

```
[Out] 2*(b*x+a)^(1/2)/b
```

Maxima [A] time = 1.00684, size = 16, normalized size = 1.14

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Fricas [A] time = 1.49353, size = 26, normalized size = 1.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Sympy [A] time = 0.07112, size = 10, normalized size = 0.71

$$\frac{2\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2),x)
```

```
[Out] 2*sqrt(a + b*x)/b
```

Giac [A] time = 1.18106, size = 16, normalized size = 1.14

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b
```

3.339 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0072444, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0075859, size = 23, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.005, size = 18, normalized size = 0.8

$$-2\frac{1}{\sqrt{a}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53609, size = 142, normalized size = 6.17

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [A] time = 1.63057, size = 24, normalized size = 1.04

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Giac [A] time = 1.21298, size = 28, normalized size = 1.22

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

3.340 $\int \frac{1}{x^2\sqrt{a+bx}}\,dx$

Optimal. Leaf size=41

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.01085, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{ax} - \frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a}$$
$$= -\frac{\sqrt{a+bx}}{ax} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{a}$$
$$= -\frac{\sqrt{a+bx}}{ax} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.102085, size = 47, normalized size = 1.15

$$\frac{\sqrt{a+bx}\left(\frac{b\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)}{\sqrt{\frac{bx}{a}+1}} - \frac{a}{x}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] (Sqrt[a + b*x]*(-(a/x) + (b*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[1 + (b*x)/a]))
/a^2
```

Maple [A] time = 0.006, size = 40, normalized size = 1.

$$2\,b\left(-1/2\,\frac{\sqrt{bx+a}}{abx} + 1/2\,\frac{1}{a^{3/2}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(1/2),x)
```

```
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56466, size = 232, normalized size = 5.66

$$\left[\frac{\sqrt{a}bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right) - 2\sqrt{bx+a}a}{2\,a^2x}, -\frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \sqrt{bx+a}a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(b*
x + a)*a)/(a^2*x)]
```

Sympy [A] time = 3.18364, size = 44, normalized size = 1.07

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/a**(3/2)
```

Giac [A] time = 1.1949, size = 63, normalized size = 1.54

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx+a}b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
b
```

3.341 $\int \frac{1}{x^3\sqrt{a+bx}}\,dx$

Optimal. Leaf size=68

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{\sqrt{a+bx}}{2ax^2}$$

```
[Out] -Sqrt[a + b*x]/(2*a*x^2) + (3*b*Sqrt[a + b*x])/(4*a^2*x) - (3*b^2*ArcTanh[S
qrt[a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rubi [A] time = 0.0165941, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{\sqrt{a+bx}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x]),x]
```

```
[Out] -Sqrt[a + b*x]/(2*a*x^2) + (3*b*Sqrt[a + b*x])/(4*a^2*x) - (3*b^2*ArcTanh[S
qrt[a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{2ax^2} - \frac{(3b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a}$$
$$= -\frac{\sqrt{a+bx}}{2ax^2} + \frac{3b\sqrt{a+bx}}{4a^2x} + \frac{\left(3b^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^2}$$
$$= -\frac{\sqrt{a+bx}}{2ax^2} + \frac{3b\sqrt{a+bx}}{4a^2x} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{4a^2}$$
$$= -\frac{\sqrt{a+bx}}{2ax^2} + \frac{3b\sqrt{a+bx}}{4a^2x} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

Mathematica [C] time = 0.010191, size = 33, normalized size = 0.49

$$-\frac{2b^2\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x]),x]
```

```
[Out] (-2*b^2*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x)/a])/a^3
```

Maple [A] time = 0.006, size = 66, normalized size = 1.

$$2\,b^2\left(-1/4\,\frac{\sqrt{bx+a}}{ab^2x^2} - 3/4\,\frac{1}{a}\left(-1/2\,\frac{\sqrt{bx+a}}{abx} + 1/2\,\frac{1}{a^{3/2}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(1/2),x)
```

```
[Out] 2*b^2*(-1/4*(b*x+a)^(1/2)/a/b^2/x^2-3/4/a*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2*arc
tanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55904, size = 301, normalized size = 4.43

$$\left[\frac{3\,\sqrt{a}b^2x^2\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(3\,abx-2\,a^2\right)\sqrt{bx+a}}{8\,a^3x^2},\ \frac{3\,\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3\,abx-2\,a^2\right)\sqrt{bx+a}}{4\,a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(3
*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sq
rt(b*x + a)*sqrt(-a)/a) + (3*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2)]
```

Sympy [A] time = 6.00912, size = 102, normalized size = 1.5

$$-\frac{1}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(1/2),x)
```

```
[Out] -1/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + sqrt(b)/(4*a*x**(3/2)*sqrt(a/(b
*x) + 1)) + 3*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - 3*b**2*asinh(sq
rt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2))
```

Giac [A] time = 1.2173, size = 93, normalized size = 1.37

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\,(bx+a)^{\frac{3}{2}}b^3-5\,\sqrt{bx+a}ab^3}{a^2b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3/
2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/b
```

3.342 $\int \frac{1}{x^4\sqrt{a+bx}}\,dx$

Optimal. Leaf size=90

$$-\frac{5b^2\sqrt{a+bx}}{8a^3x}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}+\frac{5b\sqrt{a+bx}}{12a^2x^2}-\frac{\sqrt{a+bx}}{3ax^3}$$

```
[Out] -Sqrt[a + b*x]/(3*a*x^3) + (5*b*Sqrt[a + b*x])/(12*a^2*x^2) - (5*b^2*Sqrt[a
 + b*x])/(8*a^3*x) + (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(7/2))
```

Rubi [A] time = 0.0253039, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{5b^2\sqrt{a+bx}}{8a^3x}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}+\frac{5b\sqrt{a+bx}}{12a^2x^2}-\frac{\sqrt{a+bx}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x]),x]
```


```
[Out] -Sqrt[a + b*x]/(3*a*x^3) + (5*b*Sqrt[a + b*x])/(12*a^2*x^2) - (5*b^2*Sqrt[a
 + b*x])/(8*a^3*x) + (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(7/2))
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+bx}}\,dx &= -\frac{\sqrt{a+bx}}{3ax^3} - \frac{(5b)\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{6a} \\ &= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} + \frac{(5b^2)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{8a^2} \\ &= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{5b^2\sqrt{a+bx}}{8a^3x} - \frac{(5b^3)\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a^3} \\ &= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{5b^2\sqrt{a+bx}}{8a^3x} - \frac{(5b^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{8a^3} \\ &= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{5b^2\sqrt{a+bx}}{8a^3x} + \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}\end{aligned}$$

Mathematica [C] time = 0.0084621, size = 33, normalized size = 0.37

$$\frac{2b^3\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{bx}{a}+1\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x]),x]
```

```
[Out] (2*b^3*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x)/a])/a^4
```

Maple [A] time = 0.006, size = 90, normalized size = 1.

$$2\,b^3\left(-1/6\,\frac{\sqrt{bx+a}}{ab^3x^3}-5/6\,\frac{1}{a}\left(-1/4\,\frac{\sqrt{bx+a}}{ab^2x^2}-3/4\,\frac{1}{a}\left(-1/2\,\frac{\sqrt{bx+a}}{abx}+1/2\,\frac{1}{a^{3/2}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^(1/2),x)
```

```
[Out] 2*b^3*(-1/6*(b*x+a)^(1/2)/a/b^3/x^3-5/6/a*(-1/4*(b*x+a)^(1/2)/a/b^2/x^2-3/4
/a*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49112, size = 356, normalized size = 3.96

$$\left[\frac{15\,\sqrt{a}b^3x^3\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\left(15\,ab^2x^2-10\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{48\,a^4x^3},-\frac{15\,\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(15\,ab^2x^2-\right.}{24\,a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^4*x^3), -1/24*(15*sqr
t(-a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 - 10*a^2*b*x
 + 8*a^3)*sqrt(b*x + a))/(a^4*x^3)]
```

Sympy [A] time = 9.4177, size = 129, normalized size = 1.43

$$-\frac{1}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{\sqrt{b}}{12ax^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{3}{2}}}{24a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{5}{2}}}{8a^3\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{5b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**(1/2),x)
```

```
[Out] -1/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) + sqrt(b)/(12*a*x**(5/2)*sqrt(a/(
b*x) + 1)) - 5*b**(3/2)/(24*a**2*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*b**(5/2)/(
8*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) + 5*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/(8*a**(7/2))
```

Giac [A] time = 1.20976, size = 113, normalized size = 1.26

$$-\frac{\frac{15\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{15\,(bx+a)^{\frac{5}{2}}b^4-40\,(bx+a)^{\frac{3}{2}}ab^4+33\,\sqrt{bx+a}a^2b^4}{a^3b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a)
^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 33*sqrt(b*x + a)*a^2*b^4)/(a^3*b^3*
x^3))/b
```

3.343 $\int \frac{x^4}{(a+bx)^{3/2}} \, dx$

Optimal. Leaf size=85

$$-\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}$$

```
[Out] (-2*a^4)/(b^5*Sqrt[a + b*x]) - (8*a^3*Sqrt[a + b*x])/b^5 + (4*a^2*(a + b*x)
^(3/2))/b^5 - (8*a*(a + b*x)^(5/2))/(5*b^5) + (2*(a + b*x)^(7/2))/(7*b^5)
```

Rubi [A] time = 0.0235158, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^4)/(b^5*Sqrt[a + b*x]) - (8*a^3*Sqrt[a + b*x])/b^5 + (4*a^2*(a + b*x)
^(3/2))/b^5 - (8*a*(a + b*x)^(5/2))/(5*b^5) + (2*(a + b*x)^(7/2))/(7*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{3/2}} \, dx &= \int \left(\frac{a^4}{b^4(a+bx)^{3/2}}-\frac{4a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{4a(a+bx)^{3/2}}{b^4}+\frac{(a+bx)^{5/2}}{b^4}\right) dx\\ &= -\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}\end{aligned}$$

Mathematica [A] time = 0.0424042, size = 57, normalized size = 0.67

$$\frac{2\left(16a^2b^2x^2-64a^3bx-128a^4-8ab^3x^3+5b^4x^4\right)}{35b^5\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-128*a^4 - 64*a^3*b*x + 16*a^2*b^2*x^2 - 8*a*b^3*x^3 + 5*b^4*x^4))/(35*
b^5*Sqrt[a + b*x])
```

Maple [A] time = 0.005, size = 54, normalized size = 0.6

$$-\frac{-10 x^4 b^4 + 16 a x^3 b^3 - 32 a^2 x^2 b^2 + 128 a^3 x b + 256 a^4}{35 b^5} \frac{1}{\sqrt{bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^(3/2),x)
```

```
[Out] -2/35/(b*x+a)^(1/2)*(-5*b^4*x^4+8*a*b^3*x^3-16*a^2*b^2*x^2+64*a^3*b*x+128*a
^4)/b^5
```

Maxima [A] time = 1.05825, size = 96, normalized size = 1.13

$$\frac{2 (bx + a)^{\frac{7}{2}}}{7 b^5} - \frac{8 (bx + a)^{\frac{5}{2}} a}{5 b^5} + \frac{4 (bx + a)^{\frac{3}{2}} a^2}{b^5} - \frac{8 \sqrt{bx + a} a^3}{b^5} - \frac{2 a^4}{\sqrt{bx + a} b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^5 - 8/5*(b*x + a)^(5/2)*a/b^5 + 4*(b*x + a)^(3/2)*a^2
/b^5 - 8*sqrt(b*x + a)*a^3/b^5 - 2*a^4/(sqrt(b*x + a)*b^5)
```

Fricas [A] time = 1.53654, size = 138, normalized size = 1.62

$$\frac{2 \left(5 b^4 x^4 - 8 a b^3 x^3 + 16 a^2 b^2 x^2 - 64 a^3 b x - 128 a^4\right) \sqrt{bx + a}}{35 \left(b^6 x + a b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*x^4 - 8*a*b^3*x^3 + 16*a^2*b^2*x^2 - 64*a^3*b*x - 128*a^4)*sqrt
(b*x + a)/(b^6*x + a*b^5)
```

Sympy [B] time = 5.73927, size = 3606, normalized size = 42.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(3/2),x)
```

```
[Out] -256*a**(87/2)*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**
38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**
10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*
x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 256*a**(87/2)/(35*a**
40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 +
```

```
7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200
*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30
*b**15*x**10) - 2432*a**(85/2)*b*x*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**
39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x
**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7
 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 2
560*a**(85/2)*b*x/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2
+ 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 735
0*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a*
*31*b**14*x**9 + 35*a**30*b**15*x**10) - 10336*a**(83/2)*b**2*x**2*sqrt(1 +
 b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**
37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b*
*11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*
x**9 + 35*a**30*b**15*x**10) + 11520*a**(83/2)*b**2*x**2/(35*a**40*b**5 + 3
50*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*
b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**1
2*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**1
0) - 25840*a**(81/2)*b**3*x**3*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b
**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4
+ 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1
575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 30720
*a**(81/2)*b**3*x**3/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x*
*2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 +
7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350
*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 41990*a**(79/2)*b**4*x**4*sqrt(
1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*
a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34
*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**
14*x**9 + 35*a**30*b**15*x**10) + 53760*a**(79/2)*b**4*x**4/(35*a**40*b**5
+ 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**
36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b
**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x
**10) - 46182*a**(77/2)*b**5*x**5*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**3
9*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x*
*4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7
+ 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 64
512*a**(77/2)*b**5*x**5/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7
*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5
 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 +
350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 34584*a**(75/2)*b**6*x**6*sq
rt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 42
00*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a*
*34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*
b**14*x**9 + 35*a**30*b**15*x**10) + 53760*a**(75/2)*b**6*x**6/(35*a**40*b*
*5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*
a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**3
3*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**1
5*x**10) - 17112*a**(73/2)*b**7*x**7*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a
**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9
*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x*
*7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) +
 30720*a**(73/2)*b**7*x**7/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b
**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x
**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8
 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 4980*a**(71/2)*b**8*x**8*
sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 +
4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*
a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**3
1*b**14*x**9 + 35*a**30*b**15*x**10) + 11520*a**(71/2)*b**8*x**8/(35*a**40*
```

```
b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 735
0*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a*
*33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b*
*15*x**10) - 340*a**(69/2)*b**9*x**9*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a
**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9
*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x*
*7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) +
 2560*a**(69/2)*b**9*x**9/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b*
*7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x*
*5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8
+ 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 424*a**(67/2)*b**10*x**10*
sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 +
4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*
a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**3
1*b**14*x**9 + 35*a**30*b**15*x**10) + 256*a**(67/2)*b**10*x**10/(35*a**40*
b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 735
0*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a*
*33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b*
*15*x**10) + 248*a**(65/2)*b**11*x**11*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350
*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b*
*9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*
x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10)
 + 74*a**(63/2)*b**12*x**12*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6
*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8
820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575
*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 10*a**(6
1/2)*b**13*x**13*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a
**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b
**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**1
3*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10)
```

Giac [A] time = 1.14232, size = 104, normalized size = 1.22

$$-\frac{2\,a^4}{\sqrt{bx+a}b^5}+\frac{2\left(5\,(bx+a)^{\frac{7}{2}}b^{30}-28\,(bx+a)^{\frac{5}{2}}ab^{30}+70\,(bx+a)^{\frac{3}{2}}a^2b^{30}-140\,\sqrt{bx+a}a^3b^{30}\right)}{35\,b^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^4/(sqrt(b*x + a)*b^5) + 2/35*(5*(b*x + a)^(7/2)*b^30 - 28*(b*x + a)^(5
/2)*a*b^30 + 70*(b*x + a)^(3/2)*a^2*b^30 - 140*sqrt(b*x + a)*a^3*b^30)/b^35
```

3.344 $\int \frac{x^3}{(a+bx)^{3/2}} \, dx$

Optimal. Leaf size=66

$$\frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}$$

```
[Out] (2*a^3)/(b^4*Sqrt[a + b*x]) + (6*a^2*Sqrt[a + b*x])/b^4 - (2*a*(a + b*x)^(3
/2))/b^4 + (2*(a + b*x)^(5/2))/(5*b^4)
```

Rubi [A] time = 0.0172614, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(3/2),x]
```

```
[Out] (2*a^3)/(b^4*Sqrt[a + b*x]) + (6*a^2*Sqrt[a + b*x])/b^4 - (2*a*(a + b*x)^(3
/2))/b^4 + (2*(a + b*x)^(5/2))/(5*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{3/2}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{3/2}}+\frac{3a^2}{b^3\sqrt{a+bx}}-\frac{3a\sqrt{a+bx}}{b^3}+\frac{(a+bx)^{3/2}}{b^3}\right)dx\\ &= \frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}\end{aligned}$$

Mathematica [A] time = 0.0332129, size = 45, normalized size = 0.68

$$\frac{2\left(8a^2bx+16a^3-2ab^2x^2+b^3x^3\right)}{5b^4\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(3/2),x]
```

```
[Out] (2*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[a + b*x])
```

Maple [A] time = 0.004, size = 42, normalized size = 0.6

$$\frac{2 b^3x^3 - 4 ab^2x^2 + 16 a^2bx + 32 a^3}{5 b^4} \frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(3/2),x)
```

```
[Out] 2/5/(b*x+a)^(1/2)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)/b^4
```

Maxima [A] time = 0.99771, size = 76, normalized size = 1.15

$$\frac{2 (bx+a)^{\frac{5}{2}}}{5 b^4} - \frac{2 (bx+a)^{\frac{3}{2}} a}{b^4} + \frac{6 \sqrt{bx+a} a^2}{b^4} + \frac{2 a^3}{\sqrt{bx+a} b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^4 - 2*(b*x + a)^(3/2)*a/b^4 + 6*sqrt(b*x + a)*a^2/b^4
 + 2*a^3/(sqrt(b*x + a)*b^4)
```

Fricas [A] time = 1.50216, size = 108, normalized size = 1.64

$$\frac{2 \left(b^3x^3 - 2 ab^2x^2 + 8 a^2bx + 16 a^3\right)\sqrt{bx+a}}{5 \left(b^5x + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x + a)/(b^5*x + a*b
^4)
```

Sympy [B] time = 3.37869, size = 1538, normalized size = 23.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(3/2),x)
```

```
[Out] 32*a**(45/2)*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**
6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*
a**14*b**10*x**6) - 32*a**(45/2)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18
*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5
+ 5*a**14*b**10*x**6) + 176*a**(43/2)*b*x*sqrt(1 + b*x/a)/(5*a**20*b**4 + 3
0*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x
**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 192*a**(43/2)*b*x/(5*a**20
```

```
*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**
16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 396*a**(41/2)*b**
2*x**2*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2
 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*
b**10*x**6) - 480*a**(41/2)*b**2*x**2/(5*a**20*b**4 + 30*a**19*b**5*x + 75*
a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*
x**5 + 5*a**14*b**10*x**6) + 462*a**(39/2)*b**3*x**3*sqrt(1 + b*x/a)/(5*a**
20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a
**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 640*a**(39/2)*b
**3*x**3/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b
**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) +
290*a**(37/2)*b**4*x**4*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 7
5*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**
9*x**5 + 5*a**14*b**10*x**6) - 480*a**(37/2)*b**4*x**4/(5*a**20*b**4 + 30*a
**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4
 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 92*a**(35/2)*b**5*x**5*sqrt(1
 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*
b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) -
 192*a**(35/2)*b**5*x**5/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x*
*2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**1
4*b**10*x**6) + 16*a**(33/2)*b**6*x**6*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a
**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4
 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 32*a**(33/2)*b**6*x**6/(5*a**
20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a
**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 6*a**(31/2)*b**
7*x**7*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2
 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*
b**10*x**6) + 2*a**(29/2)*b**8*x**8*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**1
9*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 +
30*a**15*b**9*x**5 + 5*a**14*b**10*x**6)
```

Giac [A] time = 1.18833, size = 82, normalized size = 1.24

$$\frac{2\,a^3}{\sqrt{bx+a}b^4}+\frac{2\left((bx+a)^{\frac{5}{2}}b^{16}-5\,(bx+a)^{\frac{3}{2}}ab^{16}+15\,\sqrt{bx+a}a^2b^{16}\right)}{5\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*a^3/(sqrt(b*x + a)*b^4) + 2/5*((b*x + a)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*a
*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/b^20
```

3.345 $\int \frac{x^2}{(a+bx)^{3/2}} \, dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{b^3\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{3/2}}{3b^3}$$

```
[Out] (-2*a^2)/(b^3*Sqrt[a + b*x]) - (4*a*Sqrt[a + b*x])/b^3 + (2*(a + b*x)^(3/2)
)/(3*b^3)
```

Rubi [A] time = 0.0130437, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^2}{b^3\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^2)/(b^3*Sqrt[a + b*x]) - (4*a*Sqrt[a + b*x])/b^3 + (2*(a + b*x)^(3/2)
)/(3*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{3/2}} \, dx &= \int \left(\frac{a^2}{b^2(a+bx)^{3/2}}-\frac{2a}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^2}\right) dx\\ &= -\frac{2a^2}{b^3\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{3/2}}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.026362, size = 34, normalized size = 0.69

$$\frac{2\left(-8a^2-4abx+b^2x^2\right)}{3b^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[a + b*x])
```

Maple [A] time = 0.002, size = 32, normalized size = 0.7

$$-\frac{-2b^2x^2+8abx+16a^2}{3b^3}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(3/2),x)
```

```
[Out] -2/3/(b*x+a)^(1/2)*(-b^2*x^2+4*a*b*x+8*a^2)/b^3
```

Maxima [A] time = 1.08489, size = 55, normalized size = 1.12

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^3}-\frac{4\,\sqrt{bx+a}a}{b^3}-\frac{2\,a^2}{\sqrt{bx+a}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^3 - 4*sqrt(b*x + a)*a/b^3 - 2*a^2/(sqrt(b*x + a)*b^3)
```

Fricas [A] time = 1.55304, size = 85, normalized size = 1.73

$$\frac{2\left(b^2x^2-4\,abx-8\,a^2\right)\sqrt{bx+a}}{3\left(b^4x+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x + a)/(b^4*x + a*b^3)
```

Sympy [B] time = 2.03609, size = 534, normalized size = 10.9

$$-\frac{16a^{\frac{19}{2}}\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}+\frac{16a^{\frac{19}{2}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}-\frac{40a^{\frac{17}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(3/2),x)
```

```
[Out] -16*a**(19/2)*sqrt(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x*
*2 + 3*a**5*b**6*x**3) + 16*a**(19/2)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6
*b**5*x**2 + 3*a**5*b**6*x**3) - 40*a**(17/2)*b*x*sqrt(1 + b*x/a)/(3*a**8*b
**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) + 48*a**(17/2)*b
*x/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) - 30
*a**(15/2)*b**2*x**2*sqrt(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*
b**5*x**2 + 3*a**5*b**6*x**3) + 48*a**(15/2)*b**2*x**2/(3*a**8*b**3 + 9*a**
7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) - 4*a**(13/2)*b**3*x**3*sqr
```

```
t(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*
x**3) + 16*a**(13/2)*b**3*x**3/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x
**2 + 3*a**5*b**6*x**3) + 2*a**(11/2)*b**4*x**4*sqrt(1 + b*x/a)/(3*a**8*b**
3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3)
```

Giac [A] time = 1.13146, size = 62, normalized size = 1.27

$$-\frac{2\,a^2}{\sqrt{bx+a}b^3}+\frac{2\left((bx+a)^{\frac{3}{2}}b^6-6\,\sqrt{bx+a}ab^6\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^2/(sqrt(b*x + a)*b^3) + 2/3*((b*x + a)^(3/2)*b^6 - 6*sqrt(b*x + a)*a*b
^6)/b^9
```

3.346 $\int \frac{x}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}$$

```
[Out] (2*a)/(b^2*Sqrt[a + b*x]) + (2*Sqrt[a + b*x])/b^2
```

Rubi [A] time = 0.0076305, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(3/2),x]
```

```
[Out] (2*a)/(b^2*Sqrt[a + b*x]) + (2*Sqrt[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{3/2}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right)dx\\ &= \frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0379939, size = 21, normalized size = 0.7

$$\frac{2(2a+bx)}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(3/2),x]
```

```
[Out] (2*(2*a + b*x))/(b^2*Sqrt[a + b*x])
```

Maple [A] time = 0.001, size = 20, normalized size = 0.7

$$2\,\frac{bx+2\,a}{b^{2}\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(3/2),x)
```

```
[Out] 2/(b*x+a)^(1/2)*(b*x+2*a)/b^2
```

Maxima [A] time = 1.02808, size = 35, normalized size = 1.17

$$\frac{2\,\sqrt{bx+a}}{b^{2}}+\frac{2\,a}{\sqrt{bx+a}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b^2 + 2*a/(sqrt(b*x + a)*b^2)
```

Fricas [A] time = 1.46682, size = 61, normalized size = 2.03

$$\frac{2\,(bx+2\,a)\sqrt{bx+a}}{b^{3}x+ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*x + 2*a)*sqrt(b*x + a)/(b^3*x + a*b^2)
```

Sympy [A] time = 0.737788, size = 37, normalized size = 1.23

$$\begin{cases} \frac{4a}{b^{2}\sqrt{a+bx}}+\frac{2x}{b\sqrt{a+bx}} & \text{for } b\neq 0 \\ \frac{x^{2}}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(3/2),x)
```

```
[Out] Piecewise((4*a/(b**2*sqrt(a + b*x)) + 2*x/(b*sqrt(a + b*x)), Ne(b, 0)), (x*
*2/(2*a**(3/2)), True))
```

Giac [A] time = 1.1926, size = 39, normalized size = 1.3

$$\frac{2\left(\frac{\sqrt{bx+a}}{b}+\frac{a}{\sqrt{bx+a}b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(b*x + a)/b + a/(sqrt(b*x + a)*b))/b
```

3.347 $\int \frac{1}{(a+bx)^{3/2}} dx$

Optimal. Leaf size=14

$$-\frac{2}{b\sqrt{a+bx}}$$

```
[Out] -2/(b*Sqrt[a + b*x])
```

Rubi [A] time = 0.0013755, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3/2),x]
```

```
[Out] -2/(b*Sqrt[a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}} dx = -\frac{2}{b\sqrt{a+bx}}$$

Mathematica [A] time = 0.0048989, size = 14, normalized size = 1.

$$-\frac{2}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3/2),x]
```

```
[Out] -2/(b*Sqrt[a + b*x])
```

Maple [A] time = 0.003, size = 13, normalized size = 0.9

$$-2\frac{1}{b\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2),x)
```

```
[Out] -2/b/(b*x+a)^(1/2)
```

Maxima [A] time = 1.0712, size = 16, normalized size = 1.14

$$-\frac{2}{\sqrt{bx+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

Fricas [A] time = 1.51328, size = 43, normalized size = 3.07

$$-\frac{2\sqrt{bx+a}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)/(b^2*x + a*b)
```

Sympy [A] time = 0.0651, size = 12, normalized size = 0.86

$$-\frac{2}{b\sqrt{a+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2),x)
```

```
[Out] -2/(b*sqrt(a + b*x))
```

Giac [A] time = 1.20343, size = 16, normalized size = 1.14

$$-\frac{2}{\sqrt{bx+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

3.348 $\int \frac{1}{x(a+bx)^{3/2}} dx$

Optimal. Leaf size=38

$$\frac{2}{a\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] 2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0110024, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$\frac{2}{a\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(a+bx)^{3/2}}\,dx = \frac{2}{a\sqrt{a+bx}} + \frac{\int \frac{1}{x\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{a\sqrt{a+bx}} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{ab}$$
$$= \frac{2}{a\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [C] time = 0.0127394, size = 30, normalized size = 0.79

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx}{a}+1\right)}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x)/a])/(a*Sqrt[a + b*x])
```

Maple [A] time = 0.007, size = 31, normalized size = 0.8

$$-2\,\frac{1}{a^{3/2}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\,\frac{1}{a\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(3/2),x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)+2/a/(b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59034, size = 266, normalized size = 7.

$$\left[\frac{(bx+a)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}a}{a^2bx+a^3}, \frac{2\left((bx+a)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}a\right)}{a^2bx+a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + a)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b
*x + a)*a)/(a^2*b*x + a^3), 2*((b*x + a)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + sqrt(b*x + a)*a)/(a^2*b*x + a^3)]
```

Sympy [B] time = 2.14611, size = 146, normalized size = 3.84

$$\frac{2a^3\sqrt{1+\frac{bx}{a}}}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{a^3\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{a^2bx\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^2bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(3/2),x)
```

```
[Out] 2*a**3*sqrt(1 + b*x/a)/(a**(9/2) + a**(7/2)*b*x) + a**3*log(b*x/a)/(a**(9/2
) + a**(7/2)*b*x) - 2*a**3*log(sqrt(1 + b*x/a) + 1)/(a**(9/2) + a**(7/2)*b*
x) + a**2*b*x*log(b*x/a)/(a**(9/2) + a**(7/2)*b*x) - 2*a**2*b*x*log(sqrt(1
+ b*x/a) + 1)/(a**(9/2) + a**(7/2)*b*x)
```

Giac [A] time = 1.23816, size = 50, normalized size = 1.32

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{2}{\sqrt{bx+a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + 2/(sqrt(b*x + a)*a)
```

3.349 $\int \frac{1}{x^2(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{3b}{a^2\sqrt{a+bx}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{1}{ax\sqrt{a+bx}}$$

```
[Out] (-3*b)/(a^2*Sqrt[a + b*x]) - 1/(a*x*Sqrt[a + b*x]) + (3*b*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0165216, antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{3\sqrt{a+bx}}{a^2x}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{ax\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x*Sqrt[a + b*x]) - (3*Sqrt[a + b*x])/(a^2*x) + (3*b*ArcTanh[Sqrt[a + b
*x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx)^{3/2}}\,dx = \frac{2}{ax\sqrt{a+bx}} + \frac{3\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{ax\sqrt{a+bx}} - \frac{3\sqrt{a+bx}}{a^2x} - \frac{(3b)\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a^2}$$
$$= \frac{2}{ax\sqrt{a+bx}} - \frac{3\sqrt{a+bx}}{a^2x} - \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{a^2}$$
$$= \frac{2}{ax\sqrt{a+bx}} - \frac{3\sqrt{a+bx}}{a^2x} + \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.0156457, size = 31, normalized size = 0.54

$$-\frac{2b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx}{a}+1\right)}{a^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*b*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x)/a])/(a^2*Sqrt[a + b*x])
```

Maple [A] time = 0.012, size = 55, normalized size = 1.

$$2\,b\left(-\frac{1}{a^2}\left(1/2\,\frac{\sqrt{bx+a}}{bx}-3/2\,\frac{1}{\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-\frac{1}{a^2\sqrt{bx+a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(3/2),x)
```

```
[Out] 2*b*(-1/a^2*(1/2*(b*x+a)^(1/2)/b/x-3/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/
2))-1/a^2/(b*x+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63446, size = 346, normalized size = 6.07

$$\left[\frac{3\left(b^2x^2+abx\right)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(3abx+a^2\right)\sqrt{bx+a}}{2\left(a^3bx^2+a^4x\right)},-\frac{3\left(b^2x^2+abx\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3abx+\right.}{a^3bx^2+a^4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^2 + a*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)
/x) - 2*(3*a*b*x + a^2)*sqrt(b*x + a))/(a^3*b*x^2 + a^4*x), -(3*(b^2*x^2 +
a*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b*x + a^2)*sqrt(b*x
 + a))/(a^3*b*x^2 + a^4*x)]
```

Sympy [A] time = 4.14375, size = 73, normalized size = 1.28

$$-\frac{1}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(3/2),x)
```

```
[Out] -1/(a*sqrt(b)*x**(3/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(a**2*sqrt(x)*sqrt(a/
(b*x) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2)
```

Giac [A] time = 1.18841, size = 86, normalized size = 1.51

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}-\frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) - (3*(b*x + a)*b - 2*a*b
)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2)
```

3.350 $\int \frac{1}{x^3(a+bx)^{3/2}} \, dx$

Optimal. Leaf size=87

$$\frac{15b^2}{4a^3\sqrt{a+bx}} - \frac{15b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}} + \frac{5b}{4a^2x\sqrt{a+bx}} - \frac{1}{2ax^2\sqrt{a+bx}}$$

```
[Out] (15*b^2)/(4*a^3*Sqrt[a + b*x]) - 1/(2*a*x^2*Sqrt[a + b*x]) + (5*b)/(4*a^2*x
*Sqrt[a + b*x]) - (15*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rubi [A] time = 0.0231892, antiderivative size = 85, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{15b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}} - \frac{5\sqrt{a+bx}}{2a^2x^2} + \frac{15b\sqrt{a+bx}}{4a^3x} + \frac{2}{ax^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^2*Sqrt[a + b*x]) - (5*Sqrt[a + b*x])/(2*a^2*x^2) + (15*b*Sqrt[a + b*
x])/(4*a^3*x) - (15*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx)^{3/2}}\,dx = \frac{2}{ax^2\sqrt{a+bx}}+\frac{5\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{ax^2\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{2a^2x^2}-\frac{(15b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a^2}$$
$$= \frac{2}{ax^2\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{2a^2x^2}+\frac{15b\sqrt{a+bx}}{4a^3x}+\frac{\left(15b^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^3}$$
$$= \frac{2}{ax^2\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{2a^2x^2}+\frac{15b\sqrt{a+bx}}{4a^3x}+\frac{(15b)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{4a^3}$$
$$= \frac{2}{ax^2\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{2a^2x^2}+\frac{15b\sqrt{a+bx}}{4a^3x}-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Mathematica [C] time = 0.0093782, size = 33, normalized size = 0.38

$$\frac{2b^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{bx}{a}+1\right)}{a^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(3/2)),x]
```

```
[Out] (2*b^2*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (b*x)/a])/(a^3*Sqrt[a + b*x])
```

Maple [A] time = 0.01, size = 67, normalized size = 0.8

$$2\,b^2\left(\frac{1}{a^3}\left(\frac{1}{b^2x^2}\left(\frac{7\,(bx+a)^{3/2}}{8}-\frac{9\,a\sqrt{bx+a}}{8}\right)-\frac{15}{8\,\sqrt{a}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)+\frac{1}{\sqrt{bx+a}a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(3/2),x)
```

```
[Out] 2*b^2*(1/a^3*((7/8*(b*x+a)^(3/2)-9/8*a*(b*x+a)^(1/2))/b^2/x^2-15/8*arctanh(
(b*x+a)^(1/2)/a^(1/2))/a^(1/2))+1/a^3/(b*x+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57, size = 420, normalized size = 4.83

$$\left[\frac{15\left(b^3x^3+ab^2x^2\right)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(15\,ab^2x^2+5\,a^2bx-2\,a^3\right)\sqrt{bx+a}}{8\left(a^4bx^3+a^5x^2\right)},\frac{15\left(b^3x^3+ab^2x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\cdots}{a}\right)\cdots}{4\left(a^4b\cdots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) +
 2*a)/x) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 +
 a^5*x^2), 1/4*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 + a^
5*x^2)]
```

Sympy [A] time = 7.50486, size = 107, normalized size = 1.23

$$-\frac{1}{2a\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{5\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{15b^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(3/2),x)
```

```
[Out] -1/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + 5*sqrt(b)/(4*a**2*x**(3/2)*sq
rt(a/(b*x) + 1)) + 15*b**(3/2)/(4*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) - 15*b**2
*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2))
```

Giac [A] time = 1.17616, size = 108, normalized size = 1.24

$$\frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}a^3}+\frac{2\,b^2}{\sqrt{bx+a}a^3}+\frac{7\,(bx+a)^{\frac{3}{2}}b^2-9\sqrt{bx+a}ab^2}{4\,a^3b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2*b^2/(sqrt(b*x +
a)*a^3) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^2)/(a^3*b^2*x^2)
```

3.351 $\int \frac{x^4}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}$$

```
[Out] (-2*a^4)/(3*b^5*(a + b*x)^(3/2)) + (8*a^3)/(b^5*Sqrt[a + b*x]) + (12*a^2*Sq
rt[a + b*x])/b^5 - (8*a*(a + b*x)^(3/2))/(3*b^5) + (2*(a + b*x)^(5/2))/(5*b
^5)
```

Rubi [A] time = 0.0217943, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^4)/(3*b^5*(a + b*x)^(3/2)) + (8*a^3)/(b^5*Sqrt[a + b*x]) + (12*a^2*Sq
rt[a + b*x])/b^5 - (8*a*(a + b*x)^(3/2))/(3*b^5) + (2*(a + b*x)^(5/2))/(5*b
^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{5/2}}\,dx &= \int \left(\frac{a^4}{b^4(a+bx)^{5/2}}-\frac{4a^3}{b^4(a+bx)^{3/2}}+\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^4}+\frac{(a+bx)^{3/2}}{b^4}\right)dx\\ &= -\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}\end{aligned}$$

Mathematica [A] time = 0.0472755, size = 57, normalized size = 0.66

$$\frac{2\left(48a^2b^2x^2+192a^3bx+128a^4-8ab^3x^3+3b^4x^4\right)}{15b^5(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^(5/2),x]
```

```
[Out] (2*(128*a^4 + 192*a^3*b*x + 48*a^2*b^2*x^2 - 8*a*b^3*x^3 + 3*b^4*x^4))/(15*
b^5*(a + b*x)^(3/2))
```

Maple [A] time = 0.005, size = 54, normalized size = 0.6

$$\frac{6\,x^4b^4 - 16\,ax^3b^3 + 96\,a^2x^2b^2 + 384\,a^3xb + 256\,a^4}{15\,b^5}(bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^(5/2),x)
```

```
[Out] 2/15/(b*x+a)^(3/2)*(3*b^4*x^4-8*a*b^3*x^3+48*a^2*b^2*x^2+192*a^3*b*x+128*a^
4)/b^5
```

Maxima [A] time = 1.06957, size = 96, normalized size = 1.1

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^5} - \frac{8\,(bx+a)^{\frac{3}{2}}a}{3\,b^5} + \frac{12\,\sqrt{bx+a}a^2}{b^5} + \frac{8\,a^3}{\sqrt{bx+a}b^5} - \frac{2\,a^4}{3\,(bx+a)^{\frac{3}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^5 - 8/3*(b*x + a)^(3/2)*a/b^5 + 12*sqrt(b*x + a)*a^2/
b^5 + 8*a^3/(sqrt(b*x + a)*b^5) - 2/3*a^4/((b*x + a)^(3/2)*b^5)
```

Fricas [A] time = 1.54716, size = 161, normalized size = 1.85

$$\frac{2\left(3\,b^4x^4 - 8\,ab^3x^3 + 48\,a^2b^2x^2 + 192\,a^3bx + 128\,a^4\right)\sqrt{bx+a}}{15\left(b^7x^2 + 2\,ab^6x + a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*x^4 - 8*a*b^3*x^3 + 48*a^2*b^2*x^2 + 192*a^3*b*x + 128*a^4)*sqr
t(b*x + a)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [B] time = 5.72088, size = 3456, normalized size = 39.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(5/2),x)
```

```
[Out] 256*a**(85/2)*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38
*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10
```

*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**
8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 256*a**(85/2)/(15*a**40*
b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150
*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**
33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**1
5*x**10) + 2432*a**(83/2)*b*x*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b*
*6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 +
3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675
*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 2560*a**
(83/2)*b*x/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a
37*b8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*
b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14
*x**9 + 15*a**30*b**15*x**10) + 10336*a**(81/2)*b**2*x**2*sqrt(1 + b*x/a)/(
15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x*
*3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 +
 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a
30*b15*x**10) - 11520*a**(81/2)*b**2*x**2/(15*a**40*b**5 + 150*a**39*b*
*6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 +
3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675
*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 25840*a*
*(79/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a
38*b7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b
10*x5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13
*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 30720*a**(79/2)*b**3
*x**3/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*
b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11
*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9
 + 15*a**30*b**15*x**10) + 41990*a**(77/2)*b**4*x**4*sqrt(1 + b*x/a)/(15*a*
*40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 +
3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800
*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*
b**15*x**10) - 53760*a**(77/2)*b**4*x**4/(15*a**40*b**5 + 150*a**39*b**6*x
+ 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*
a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**3
2*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 46192*a**(75/
2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*
b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*
x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8
 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 64512*a**(75/2)*b**5*x**5
/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*
x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6
 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15
*a**30*b**15*x**10) + 34664*a**(73/2)*b**6*x**6*sqrt(1 + b*x/a)/(15*a**40*b
5 + 150*a39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*
a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**3
3*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15
*x**10) - 53760*a**(73/2)*b**6*x**6/(15*a**40*b**5 + 150*a**39*b**6*x + 675
*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35
*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**
13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 17392*a**(71/2)*b*
*7*x**7*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*
x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5
+ 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 15
0*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 30720*a**(71/2)*b**7*x**7/(15*
a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3
+ 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 18
00*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**3
0*b**15*x**10) + 5540*a**(69/2)*b**8*x**8*sqrt(1 + b*x/a)/(15*a**40*b**5 +
150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*

```
b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**1
2*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10
) - 11520*a**(69/2)*b**8*x**8/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38
*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10
*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**
8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 1040*a**(67/2)*b**9*x**9
*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 +
1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*
a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31
*b**14*x**9 + 15*a**30*b**15*x**10) - 2560*a**(67/2)*b**9*x**9/(15*a**40*b*
*5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a
**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33
*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*
x**10) + 136*a**(65/2)*b**10*x**10*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**
39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x*
*4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7
+ 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 256
*a**(65/2)*b**10*x**10/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x
**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 +
 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150
*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 32*a**(63/2)*b**11*x**11*sqrt(1
 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a*
*37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b
**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*
x**9 + 15*a**30*b**15*x**10) + 6*a**(61/2)*b**12*x**12*sqrt(1 + b*x/a)/(15*
a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3
+ 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 18
00*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**3
0*b**15*x**10)
```

Giac [A] time = 1.1844, size = 101, normalized size = 1.16

$$\frac{2\left(12\,(bx+a)a^3-a^4\right)}{3\,(bx+a)^{\frac{3}{2}}b^5}+\frac{2\left(3\,(bx+a)^{\frac{5}{2}}b^{20}-20\,(bx+a)^{\frac{3}{2}}ab^{20}+90\,\sqrt{bx+a}a^2b^{20}\right)}{15\,b^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(12*(b*x + a)*a^3 - a^4)/((b*x + a)^(3/2)*b^5) + 2/15*(3*(b*x + a)^(5/2
)*b^20 - 20*(b*x + a)^(3/2)*a*b^20 + 90*sqrt(b*x + a)*a^2*b^20)/b^25
```

3.352 $\int \frac{x^3}{(a+bx)^{5/2}} \, dx$

Optimal. Leaf size=68

$$\frac{2a^3}{3b^4(a+bx)^{3/2}} - \frac{6a^2}{b^4\sqrt{a+bx}} - \frac{6a\sqrt{a+bx}}{b^4} + \frac{2(a+bx)^{3/2}}{3b^4}$$

```
[Out] (2*a^3)/(3*b^4*(a + b*x)^(3/2)) - (6*a^2)/(b^4*Sqrt[a + b*x]) - (6*a*Sqrt[a
 + b*x])/b^4 + (2*(a + b*x)^(3/2))/(3*b^4)
```

Rubi [A] time = 0.0169521, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^3}{3b^4(a+bx)^{3/2}} - \frac{6a^2}{b^4\sqrt{a+bx}} - \frac{6a\sqrt{a+bx}}{b^4} + \frac{2(a+bx)^{3/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(5/2),x]
```

```
[Out] (2*a^3)/(3*b^4*(a + b*x)^(3/2)) - (6*a^2)/(b^4*Sqrt[a + b*x]) - (6*a*Sqrt[a
 + b*x])/b^4 + (2*(a + b*x)^(3/2))/(3*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(a+bx)^{5/2}} \, dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{5/2}} + \frac{3a^2}{b^3(a+bx)^{3/2}} - \frac{3a}{b^3\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b^3}\right) dx \\ &= \frac{2a^3}{3b^4(a+bx)^{3/2}} - \frac{6a^2}{b^4\sqrt{a+bx}} - \frac{6a\sqrt{a+bx}}{b^4} + \frac{2(a+bx)^{3/2}}{3b^4} \end{aligned}$$

Mathematica [A] time = 0.0371734, size = 45, normalized size = 0.66

$$\frac{2\left(-24a^2bx - 16a^3 - 6ab^2x^2 + b^3x^3\right)}{3b^4(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(5/2),x]
```

```
[Out] (2*(-16*a^3 - 24*a^2*b*x - 6*a*b^2*x^2 + b^3*x^3))/(3*b^4*(a + b*x)^(3/2))
```

Maple [A] time = 0.006, size = 43, normalized size = 0.6

$$-\frac{-2\,b^3x^3+12\,ab^2x^2+48\,a^2bx+32\,a^3}{3\,b^4}(bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(5/2),x)
```

```
[Out] -2/3/(b*x+a)^(3/2)*(-b^3*x^3+6*a*b^2*x^2+24*a^2*b*x+16*a^3)/b^4
```

Maxima [A] time = 1.06944, size = 76, normalized size = 1.12

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^4}-\frac{6\,\sqrt{bx+a}a}{b^4}-\frac{6\,a^2}{\sqrt{bx+a}b^4}+\frac{2\,a^3}{3\,(bx+a)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^4 - 6*sqrt(b*x + a)*a/b^4 - 6*a^2/(sqrt(b*x + a)*b^4)
 + 2/3*a^3/((b*x + a)^(3/2)*b^4)
```

Fricas [A] time = 1.54887, size = 131, normalized size = 1.93

$$\frac{2\left(b^3x^3-6\,ab^2x^2-24\,a^2bx-16\,a^3\right)\sqrt{bx+a}}{3\left(b^6x^2+2\,ab^5x+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)*sqrt(b*x + a)/(b^6*x^2 +
2*a*b^5*x + a^2*b^4)
```

Sympy [A] time = 1.72283, size = 163, normalized size = 2.4

$$\begin{cases} -\frac{32a^3}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}}-\frac{48a^2bx}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}}-\frac{12ab^2x^2}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}}+\frac{2b^3x^3}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-32*a**3/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 48*
a**2*b*x/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 12*a*b**2*x**2
/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) + 2*b**3*x**3/(3*a*b**4*
sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)), Ne(b, 0)), (x**4/(4*a**(5/2)), Tru
```

```
e))
```

Giac [A] time = 1.20256, size = 80, normalized size = 1.18

$$-\frac{2\left(9\,(bx+a)a^2-a^3\right)}{3\,(bx+a)^{\frac{3}{2}}b^4}+\frac{2\left((bx+a)^{\frac{3}{2}}b^8-9\,\sqrt{bx+a}ab^8\right)}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(9*(b*x + a)*a^2 - a^3)/((b*x + a)^(3/2)*b^4) + 2/3*((b*x + a)^(3/2)*b
^8 - 9*sqrt(b*x + a)*a*b^8)/b^12
```

3.353 $\int \frac{x^2}{(a+bx)^{5/2}} dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}$$

```
[Out] (-2*a^2)/(3*b^3*(a + b*x)^(3/2)) + (4*a)/(b^3*Sqrt[a + b*x]) + (2*Sqrt[a + b*x])/b^3
```

Rubi [A] time = 0.0130616, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^2)/(3*b^3*(a + b*x)^(3/2)) + (4*a)/(b^3*Sqrt[a + b*x]) + (2*Sqrt[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{5/2}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{5/2}}-\frac{2a}{b^2(a+bx)^{3/2}}+\frac{1}{b^2\sqrt{a+bx}}\right)dx\\ &= -\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0176182, size = 35, normalized size = 0.71

$$\frac{2\left(8a^2+12abx+3b^2x^2\right)}{3b^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(5/2),x]
```

```
[Out] (2*(8*a^2 + 12*a*b*x + 3*b^2*x^2))/(3*b^3*(a + b*x)^(3/2))
```

Maple [A] time = 0.006, size = 32, normalized size = 0.7

$$\frac{6\,b^{2}x^{2}+24\,abx+16\,a^{2}}{3\,b^{3}}\,(bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(5/2),x)
```

```
[Out] 2/3/(b*x+a)^(3/2)*(3*b^2*x^2+12*a*b*x+8*a^2)/b^3
```

Maxima [A] time = 1.14883, size = 55, normalized size = 1.12

$$\frac{2\,\sqrt{bx+a}}{b^{3}}+\frac{4\,a}{\sqrt{bx+a}b^{3}}-\frac{2\,a^{2}}{3\,(bx+a)^{\frac{3}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b^3 + 4*a/(sqrt(b*x + a)*b^3) - 2/3*a^2/((b*x + a)^(3/2)*b^
3)
```

Fricas [A] time = 1.55674, size = 111, normalized size = 2.27

$$\frac{2\,\left(3\,b^{2}x^{2}+12\,abx+8\,a^{2}\right)\sqrt{bx+a}}{3\,\left(b^{5}x^{2}+2\,ab^{4}x+a^{2}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*x^2 + 12*a*b*x + 8*a^2)*sqrt(b*x + a)/(b^5*x^2 + 2*a*b^4*x + a^2
*b^3)
```

Sympy [A] time = 1.54181, size = 121, normalized size = 2.47

$$\begin{cases} \frac{16a^{2}}{3ab^{3}\sqrt{a+bx}+3b^{4}x\sqrt{a+bx}}+\frac{24abx}{3ab^{3}\sqrt{a+bx}+3b^{4}x\sqrt{a+bx}}+\frac{6b^{2}x^{2}}{3ab^{3}\sqrt{a+bx}+3b^{4}x\sqrt{a+bx}} & \text{for } b\neq 0 \\ \frac{x^{3}}{3a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((16*a**2/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 24*a
*b*x/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 6*b**2*x**2/(3*a*b
**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)), Ne(b, 0)), (x**3/(3*a**(5/2)),
 True))
```

Giac [A] time = 1.15255, size = 53, normalized size = 1.08

$$\frac{2\sqrt{bx+a}}{b^3}+\frac{2\left(6\,(bx+a)a-a^2\right)}{3\,(bx+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b^3 + 2/3*(6*(b*x + a)*a - a^2)/((b*x + a)^(3/2)*b^3)
```

3.354 $\int \frac{x}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=32

$$\frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}$$

```
[Out] (2*a)/(3*b^2*(a + b*x)^(3/2)) - 2/(b^2*Sqrt[a + b*x])
```

Rubi [A] time = 0.0084376, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(5/2),x]
```

```
[Out] (2*a)/(3*b^2*(a + b*x)^(3/2)) - 2/(b^2*Sqrt[a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{5/2}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{5/2}}+\frac{1}{b(a+bx)^{3/2}}\right)dx\\ &= \frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}\end{aligned}$$

Mathematica [A] time = 0.0117096, size = 24, normalized size = 0.75

$$-\frac{2(2a+3bx)}{3b^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(2*a + 3*b*x))/(3*b^2*(a + b*x)^(3/2))
```

Maple [A] time = 0.003, size = 21, normalized size = 0.7

$$-\frac{6\,bx+4\,a}{3\,b^2}(bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(5/2),x)
```

```
[Out] -2/3/(b*x+a)^(3/2)*(3*b*x+2*a)/b^2
```

Maxima [A] time = 1.04362, size = 35, normalized size = 1.09

$$-\frac{2}{\sqrt{bx+a}b^2}+\frac{2\,a}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x + a)*b^2) + 2/3*a/((b*x + a)^(3/2)*b^2)
```

Fricas [A] time = 1.60219, size = 89, normalized size = 2.78

$$-\frac{2\,(3\,bx+2\,a)\sqrt{bx+a}}{3\,\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b*x + 2*a)*sqrt(b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [A] time = 1.40403, size = 80, normalized size = 2.5

$$\begin{cases} -\frac{4a}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}}-\frac{6bx}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-4*a/(3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)) - 6*b*x/(
3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)), Ne(b, 0)), (x**2/(2*a**(5
/2)), True))
```

Giac [A] time = 1.12735, size = 27, normalized size = 0.84

$$-\frac{2\,(3\,bx+2\,a)}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*x + 2*a)/((b*x + a)^(3/2)*b^2)
```

3.355 $\int \frac{1}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=16

$$-\frac{2}{3b(a+bx)^{3/2}}$$

```
[Out] -2/(3*b*(a + b*x)^(3/2))
```

Rubi [A] time = 0.0014716, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-5/2),x]
```

```
[Out] -2/(3*b*(a + b*x)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}}\,dx = -\frac{2}{3b(a+bx)^{3/2}}$$

Mathematica [A] time = 0.0045605, size = 16, normalized size = 1.

$$-\frac{2}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-5/2),x]
```

```
[Out] -2/(3*b*(a + b*x)^(3/2))
```

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$-\frac{2}{3\,b}\,(bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2),x)
```

```
[Out] -2/3/b/(b*x+a)^(3/2)
```

Maxima [A] time = 1.08387, size = 16, normalized size = 1.

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((b*x + a)^(3/2)*b)
```

Fricas [B] time = 1.53132, size = 68, normalized size = 4.25

$$-\frac{2\,\sqrt{bx+a}}{3\,\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x + a)/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [A] time = 0.066401, size = 14, normalized size = 0.88

$$-\frac{2}{3b\,(a+bx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2),x)
```

```
[Out] -2/(3*b*(a + b*x)**(3/2))
```

Giac [A] time = 1.17457, size = 16, normalized size = 1.

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/((b*x + a)^(3/2)*b)
```

3.356 $\int \frac{1}{x(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2}{a^2\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2}{3a(a+bx)^{3/2}}$$

```
[Out] 2/(3*a*(a + b*x)^(3/2)) + 2/(a^2*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0152049, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$\frac{2}{a^2\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*(a + b*x)^(3/2)) + 2/(a^2*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(a+bx)^{5/2}}\,dx = \frac{2}{3a(a+bx)^{3/2}} + \frac{\int \frac{1}{x(a+bx)^{3/2}}\,dx}{a}$$
$$= \frac{2}{3a(a+bx)^{3/2}} + \frac{2}{a^2\sqrt{a+bx}} + \frac{\int \frac{1}{x\sqrt{a+bx}}\,dx}{a^2}$$
$$= \frac{2}{3a(a+bx)^{3/2}} + \frac{2}{a^2\sqrt{a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{a^2 b}$$
$$= \frac{2}{3a(a+bx)^{3/2}} + \frac{2}{a^2\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.0058095, size = 32, normalized size = 0.59

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx}{a}+1\right)}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(5/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*x)/a])/(3*a*(a + b*x)^(3/2))
```

Maple [A] time = 0.009, size = 43, normalized size = 0.8

$$\frac{2}{3\,a}(bx+a)^{-\frac{3}{2}} - 2\,\frac{1}{a^{5/2}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) + 2\,\frac{1}{a^2\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(5/2),x)
```

```
[Out] 2/3/a/(b*x+a)^(3/2)-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)+2/a^2/(b*x+a)^
(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.69967, size = 409, normalized size = 7.57

$$\left[\frac{3\,\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{a}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,\left(3\,abx+4\,a^{2}\right)\sqrt{bx+a}}{3\,\left(a^{3}b^{2}x^{2}+2\,a^{4}bx+a^{5}\right)},\ \frac{2\,\left(3\,\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-}}{a}\right.}{3\,\left(a^{3}b^{2}x^{2}+2\,a^{4}bx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a
) + 2*a)/x) + 2*(3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x +
 a^5), 2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(
-a)/a) + (3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)]
```

Sympy [B] time = 3.71646, size = 697, normalized size = 12.91

$$\frac{8a^{7}\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}+\frac{3a^{7}\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}-\frac{6a^{7}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(5/2),x)
```

```
[Out] 8*a**7*sqrt(1 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x*
*2 + 3*a**(13/2)*b**3*x**3) + 3*a**7*log(b*x/a)/(3*a**(19/2) + 9*a**(17/2)*
b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*a**7*log(sqrt(1 +
b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(
13/2)*b**3*x**3) + 14*a**6*b*x*sqrt(1 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b
*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 9*a**6*b*x*log(b*x/a)
/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*
x**3) - 18*a**6*b*x*log(sqrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x
 + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**5*b**2*x**2*sqrt(1
 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13
/2)*b**3*x**3) + 9*a**5*b**2*x**2*log(b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x
 + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*a**5*b**2*x**2*log(s
qrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2
+ 3*a**(13/2)*b**3*x**3) + 3*a**4*b**3*x**3*log(b*x/a)/(3*a**(19/2) + 9*a**
(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*a**4*b**3*x
**3*log(sqrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b
**2*x**2 + 3*a**(13/2)*b**3*x**3)
```

Giac [A] time = 1.19167, size = 61, normalized size = 1.13

$$\frac{2\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{2}}+\frac{2\,(3\,bx+4\,a)}{3\,(bx+a)^{\frac{3}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + 2/3*(3*b*x + 4*a)/((b*x +
 a)^(3/2)*a^2)
```

3.357 $\int \frac{1}{x^2(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{5b}{a^3\sqrt{a+bx}}-\frac{5b}{3a^2(a+bx)^{3/2}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{1}{ax(a+bx)^{3/2}}$$

```
[Out] (-5*b)/(3*a^2*(a + b*x)^(3/2)) - 1/(a*x*(a + b*x)^(3/2)) - (5*b)/(a^3*Sqrt[
a + b*x]) + (5*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(7/2)
```

Rubi [A] time = 0.0242279, antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{5\sqrt{a+bx}}{a^3x}+\frac{10}{3a^2x\sqrt{a+bx}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2}{3ax(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x*(a + b*x)^(3/2)) + 10/(3*a^2*x*Sqrt[a + b*x]) - (5*Sqrt[a + b*x])/
(a^3*x) + (5*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2(a+bx)^{5/2}}\,dx &= \frac{2}{3ax(a+bx)^{3/2}}+\frac{5\int \frac{1}{x^2(a+bx)^{3/2}}\,dx}{3a}\\
&= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}+\frac{5\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{a^2}\\
&= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{a^3x}-\frac{(5b)\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a^3}\\
&= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{a^3x}-\frac{5\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a^3}\\
&= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{a^3x}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0065211, size = 33, normalized size = 0.45

$$
-\frac{2b\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{bx}{a}+1\right)}{3a^2(a+bx)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(5/2)),x]
```

```
[Out] (-2*b*Hypergeometric2F1[-3/2, 2, -1/2, 1 + (b*x)/a])/(3*a^2*(a + b*x)^(3/2)
)
```

Maple [A] time = 0.013, size = 67, normalized size = 0.9

$$
2\,b\left(-\frac{1}{a^3}\left(1/2\,\frac{\sqrt{bx+a}}{bx}-5/2\,\frac{1}{\sqrt{a}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-1/3\,\frac{1}{a^2\,(bx+a)^{3/2}}-2\,\frac{1}{\sqrt{bx+a}a^3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(5/2),x)
```

```
[Out] 2*b*(-1/a^3*(1/2*(b*x+a)^(1/2)/b/x-5/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/
2))-1/3/a^2/(b*x+a)^(3/2)-2/a^3/(b*x+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47718, size = 494, normalized size = 6.68

$$\left[\frac{15\left(b^3x^3+2\,ab^2x^2+a^2bx\right)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\left(15\,ab^2x^2+20\,a^2bx+3\,a^3\right)\sqrt{bx+a}}{6\left(a^4b^2x^3+2\,a^5bx^2+a^6x\right)},-\frac{15\left(b^3x^3+2\,ab^2x^2+a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x +
a)*sqrt(a) + 2*a)/x) - 2*(15*a*b^2*x^2 + 20*a^2*b*x + 3*a^3)*sqrt(b*x + a))
/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x), -1/3*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2
*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 + 20*a^2*b*
x + 3*a^3)*sqrt(b*x + a))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x)]
```

Sympy [B] time = 7.26392, size = 818, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**17*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2
)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 46*a**16*b*x*sqrt(1 + b*x/a)/(6*a**(
39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x
**4) - 15*a**16*b*x*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a*
*(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**16*b*x*log(sqrt(1 + b*x/
a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a
**(33/2)*b**3*x**4) - 70*a**15*b**2*x**2*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 1
8*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 45*a
**15*b**2*x**2*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/
2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**15*b**2*x**2*log(sqrt(1 + b*x
/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*
a**(33/2)*b**3*x**4) - 30*a**14*b**3*x**3*sqrt(1 + b*x/a)/(6*a**(39/2)*x +
18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 45*
a**14*b**3*x**3*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35
/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**14*b**3*x**3*log(sqrt(1 + b*
x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6
*a**(33/2)*b**3*x**4) - 15*a**13*b**4*x**4*log(b*x/a)/(6*a**(39/2)*x + 18*a
**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**1
3*b**4*x**4*log(sqrt(1 + b*x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 +
 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4)
```

Giac [A] time = 1.19638, size = 88, normalized size = 1.19

$$-\frac{5\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}-\frac{2\left(6\left(bx+a\right)b+ab\right)}{3\left(bx+a\right)^{\frac{3}{2}}a^3}-\frac{\sqrt{bx+a}}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) - 2/3*(6*(b*x + a)*b + a
*b)/((b*x + a)^(3/2)*a^3) - sqrt(b*x + a)/(a^3*x)
```

3.358 $\int \frac{1}{x^3(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=106

$$\frac{35b^2}{4a^4\sqrt{a+bx}}+\frac{35b^2}{12a^3(a+bx)^{3/2}}-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{7b}{4a^2x(a+bx)^{3/2}}-\frac{1}{2ax^2(a+bx)^{3/2}}$$

```
[Out] (35*b^2)/(12*a^3*(a + b*x)^(3/2)) - 1/(2*a*x^2*(a + b*x)^(3/2)) + (7*b)/(4*
a^2*x*(a + b*x)^(3/2)) + (35*b^2)/(4*a^4*Sqrt[a + b*x]) - (35*b^2*ArcTanh[S
qrt[a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rubi [A] time = 0.0339514, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}-\frac{35\sqrt{a+bx}}{6a^3x^2}+\frac{14}{3a^2x^2\sqrt{a+bx}}+\frac{35b\sqrt{a+bx}}{4a^4x}+\frac{2}{3ax^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x^2*(a + b*x)^(3/2)) + 14/(3*a^2*x^2*Sqrt[a + b*x]) - (35*Sqrt[a + b
*x])/(6*a^3*x^2) + (35*b*Sqrt[a + b*x])/(4*a^4*x) - (35*b^2*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx)^{5/2}}\,dx = \frac{2}{3ax^2(a+bx)^{3/2}} + \frac{7\int \frac{1}{x^3(a+bx)^{3/2}}\,dx}{3a}$$
$$= \frac{2}{3ax^2(a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{a+bx}} + \frac{35\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{3a^2}$$
$$= \frac{2}{3ax^2(a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{a+bx}} - \frac{35\sqrt{a+bx}}{6a^3x^2} - \frac{(35b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a^3}$$
$$= \frac{2}{3ax^2(a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{a+bx}} - \frac{35\sqrt{a+bx}}{6a^3x^2} + \frac{35b\sqrt{a+bx}}{4a^4x} + \frac{(35b^2)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^4}$$
$$= \frac{2}{3ax^2(a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{a+bx}} - \frac{35\sqrt{a+bx}}{6a^3x^2} + \frac{35b\sqrt{a+bx}}{4a^4x} + \frac{(35b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+}\right.}{4a^4}$$
$$= \frac{2}{3ax^2(a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{a+bx}} - \frac{35\sqrt{a+bx}}{6a^3x^2} + \frac{35b\sqrt{a+bx}}{4a^4x} - \frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Mathematica [C] time = 0.0067251, size = 35, normalized size = 0.33

$$\frac{2b^2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\frac{bx}{a}+1\right)}{3a^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(5/2)),x]
```

```
[Out] (2*b^2*Hypergeometric2F1[-3/2, 3, -1/2, 1 + (b*x)/a])/(3*a^3*(a + b*x)^(3/2
))
```

Maple [A] time = 0.013, size = 80, normalized size = 0.8

$$2\,b^2\left(\frac{1}{a^4}\left(\frac{1}{b^2x^2}\left(\frac{11\,(bx+a)^{3/2}}{8}-\frac{13\,a\sqrt{bx+a}}{8}\right)-\frac{35}{8\,\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)+3\,\frac{1}{a^4\sqrt{bx+a}}+1/3\,\frac{1}{a^3\,(bx+a)^{3/2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(5/2),x)
```

```
[Out] 2*b^2*(1/a^4*((11/8*(b*x+a)^(3/2)-13/8*a*(b*x+a)^(1/2))/b^2/x^2-35/8*arctan
h((b*x+a)^(1/2)/a^(1/2))/a^(1/2))+3/a^4/(b*x+a)^(1/2)+1/3/a^3/(b*x+a)^(3/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59303, size = 566, normalized size = 5.34

$$\left[\frac{105\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\sqrt{a}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(105\,ab^3x^3+140\,a^2b^2x^2+21\,a^3bx-6\,a^4\right)\sqrt{bx+a}}{24\left(a^5b^2x^4+2\,a^6bx^3+a^7x^2\right)},\ \frac{105\left(b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(
b*x + a)*sqrt(a) + 2*a)/x) + 2*(105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*
x - 6*a^4)*sqrt(b*x + a))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2), 1/12*(105*
(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a
)/a) + (105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x + a)
)/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)]
```

Sympy [B] time = 12.1094, size = 464, normalized size = 4.38

$$-\frac{6a^{\frac{89}{2}}b^{75}x^{75}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{21a^{\frac{87}{2}}b^{76}x^{76}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**(89/2)*b**75*x**75/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) +
 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 21*a**(87/2)*
b**76*x**76/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(
91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 140*a**(85/2)*b**77*x**77/
(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(15
3/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 105*a**(83/2)*b**78*x**78/(12*a**(93/2
)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157
/2)*sqrt(a/(b*x) + 1)) - 105*a**42*b**(155/2)*x**(155/2)*sqrt(a/(b*x) + 1)*
asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a
/(b*x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) - 105*a
**41*b**(157/2)*x**(157/2)*sqrt(a/(b*x) + 1)*asinh(sqrt(a)/(sqrt(b)*sqrt(x)
))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**
(153/2)*x**(157/2)*sqrt(a/(b*x) + 1))
```

Giac [A] time = 1.15996, size = 126, normalized size = 1.19

$$\frac{35\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^4}+\frac{2\left(9\,(bx+a)b^2+ab^2\right)}{3\,(bx+a)^{\frac{3}{2}}a^4}+\frac{11\,(bx+a)^{\frac{3}{2}}b^2-13\,\sqrt{bx+a}ab^2}{4\,a^4b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) + 2/3*(9*(b*x + a)*b
^2 + a*b^2)/((b*x + a)^(3/2)*a^4) + 1/4*(11*(b*x + a)^(3/2)*b^2 - 13*sqrt(b
*x + a)*a*b^2)/(a^4*b^2*x^2)
```

3.359 $\int \frac{1}{x\sqrt{-a+bx}}\,dx$

Optimal. Leaf size=25

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0064869, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {63, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a + b*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-a+bx}}\,dx &= \frac{2\operatorname{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.004967, size = 25, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a + b*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.005, size = 20, normalized size = 0.8

$$2\frac{1}{\sqrt{a}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(1/2),x)
```

```
[Out] 2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59789, size = 140, normalized size = 5.6

$$\left[-\frac{\sqrt{-a}\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)}{a}, \frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x)/a, 2*arctan(sqrt(b
*x - a)/sqrt(a))/sqrt(a)]
```

Sympy [A] time = 1.55866, size = 58, normalized size = 2.32

$$\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{for}\ \frac{|a|}{|b||x|} > 1 \\ -\frac{2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), Abs(a)/(Abs(b)*Abs
(x)) > 1), (-2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), True))
```

Giac [A] time = 1.22039, size = 26, normalized size = 1.04

$$\frac{2 \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a)
```

3.360 $\int \frac{1}{x^2\sqrt{-a+bx}}\,dx$

Optimal. Leaf size=44

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{bx-a}}{ax}$$

```
[Out] Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0108631, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 205}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{bx-a}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-a + b*x]),x]
```

```
[Out] Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-a+bx}}\,dx = \frac{\sqrt{-a+bx}}{ax} + \frac{b\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a}$$
$$= \frac{\sqrt{-a+bx}}{ax} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{a}$$
$$= \frac{\sqrt{-a+bx}}{ax} + \frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.137626, size = 53, normalized size = 1.2

$$\frac{b\sqrt{bx-a}\left(\frac{a}{bx}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{bx}{a}}\right)}{\sqrt{1-\frac{bx}{a}}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-a + b*x]),x]
```

```
[Out] (b*Sqrt[-a + b*x]*(a/(b*x) + ArcTanh[Sqrt[1 - (b*x)/a]]/Sqrt[1 - (b*x)/a]))
/a^2
```

Maple [A] time = 0.007, size = 37, normalized size = 0.8

$$b\arctan\left(\sqrt{bx-a}\frac{1}{\sqrt{a}}\right)a^{-\frac{3}{2}}+\frac{1}{ax}\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(1/2),x)
```

```
[Out] b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)+(b*x-a)^(1/2)/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52628, size = 230, normalized size = 5.23

$$\left[-\frac{\sqrt{-a}bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)-2\sqrt{bx-a}a}{2a^2x}, \frac{\sqrt{a}bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2*sqrt(
b*x - a)*a)/(a^2*x), (sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) + sqrt(b*x
- a)*a)/(a^2*x)]
```

Sympy [B] time = 3.02319, size = 124, normalized size = 2.82

$$\begin{cases} \frac{i\sqrt{b}\sqrt{\frac{a}{bx}-1}}{a\sqrt{x}} + \frac{ib\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}} & \text{for}\ \frac{|a|}{|b||x|} > 1 \\ -\frac{1}{\sqrt{b}x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} + \frac{\sqrt{b}}{a\sqrt{x}\sqrt{-\frac{a}{bx}+1}} - \frac{b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((I*sqrt(b)*sqrt(a/(b*x) - 1)/(a*sqrt(x)) + I*b*acosh(sqrt(a)/(sqr
t(b)*sqrt(x)))/a**(3/2), Abs(a)/(Abs(b)*Abs(x)) > 1), (-1/(sqrt(b)*x**(3/2)
*sqrt(-a/(b*x) + 1)) + sqrt(b)/(a*sqrt(x)*sqrt(-a/(b*x) + 1)) - b*asin(sqrt
(a)/(sqrt(b)*sqrt(x)))/a**(3/2), True))
```

Giac [A] time = 1.19898, size = 58, normalized size = 1.32

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{\sqrt{bx-a}b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + sqrt(b*x - a)*b/(a*x))/b
```

3.361 $\int \frac{1}{x^3\sqrt{-a+bx}}\,dx$

Optimal. Leaf size=74

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{bx-a}}{4a^2x}+\frac{\sqrt{bx-a}}{2ax^2}$$

```
[Out] Sqrt[-a + b*x]/(2*a*x^2) + (3*b*Sqrt[-a + b*x])/(4*a^2*x) + (3*b^2*ArcTan[S
qrt[-a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rubi [A] time = 0.0170277, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 205}

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{bx-a}}{4a^2x}+\frac{\sqrt{bx-a}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-a + b*x]),x]
```

```
[Out] Sqrt[-a + b*x]/(2*a*x^2) + (3*b*Sqrt[-a + b*x])/(4*a^2*x) + (3*b^2*ArcTan[S
qrt[-a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{-a+bx}}\,dx = \frac{\sqrt{-a+bx}}{2ax^2} + \frac{(3b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a}$$
$$= \frac{\sqrt{-a+bx}}{2ax^2} + \frac{3b\sqrt{-a+bx}}{4a^2x} + \frac{\left(3b^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a^2}$$
$$= \frac{\sqrt{-a+bx}}{2ax^2} + \frac{3b\sqrt{-a+bx}}{4a^2x} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{4a^2}$$
$$= \frac{\sqrt{-a+bx}}{2ax^2} + \frac{3b\sqrt{-a+bx}}{4a^2x} + \frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

Mathematica [C] time = 0.0061519, size = 36, normalized size = 0.49

$$\frac{2b^2\sqrt{bx-a}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{bx}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-a + b*x]),x]
```

```
[Out] (2*b^2*Sqrt[-a + b*x]*Hypergeometric2F1[1/2, 3, 3/2, 1 - (b*x)/a])/a^3
```

Maple [A] time = 0.006, size = 59, normalized size = 0.8

$$\frac{3\,b^2}{4}\arctan\left(\sqrt{bx-a}\frac{1}{\sqrt{a}}\right)a^{-\frac{5}{2}} + \frac{1}{2\,ax^2}\sqrt{bx-a} + \frac{3\,b}{4\,a^2x}\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(1/2),x)
```

```
[Out] 3/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)+1/2*(b*x-a)^(1/2)/a/x^2+3/4*b
*(b*x-a)^(1/2)/a^2/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54129, size = 300, normalized size = 4.05

$$\left[-\frac{3\sqrt{-a}b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)-2\left(3abx+2a^2\right)\sqrt{bx-a}}{8a^3x^2}, \frac{3\sqrt{a}b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(3abx+2a^2\right)\sqrt{bx-a}}{4a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2
*(3*a*b*x + 2*a^2)*sqrt(b*x - a))/(a^3*x^2), 1/4*(3*sqrt(a)*b^2*x^2*arctan(
sqrt(b*x - a)/sqrt(a)) + (3*a*b*x + 2*a^2)*sqrt(b*x - a))/(a^3*x^2)]
```

Sympy [A] time = 5.75695, size = 219, normalized size = 2.96

$$\begin{cases} \frac{i}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{i\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{3ib^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{\frac{a}{bx}-1}}+\frac{3ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}} & \text{for}\ \frac{|a|}{|b||x|}>1 \\ -\frac{1}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{3b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((I/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + I*sqrt(b)/(4*a*x**(3/
2)*sqrt(a/(b*x) - 1)) - 3*I*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) - 1)) + 3
*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2)), Abs(a)/(Abs(b)*Abs(x
)) > 1), (-1/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - sqrt(b)/(4*a*x**(3/2
)*sqrt(-a/(b*x) + 1)) + 3*b**(3/2)/(4*a**2*sqrt(x)*sqrt(-a/(b*x) + 1)) - 3*
b**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2)), True))
```

Giac [A] time = 1.18143, size = 92, normalized size = 1.24

$$\frac{\frac{3b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{3(bx-a)^{\frac{3}{2}}b^3+5\sqrt{bx-a}ab^3}{a^2b^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + (3*(b*x - a)^(3/2)*b^3 +
 5*sqrt(b*x - a)*a*b^3)/(a^2*b^2*x^2))/b
```

3.362 $\int \frac{1}{x(-a+bx)^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}$$

```
[Out] -2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0099389, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 205}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(3/2)),x]
```

```
[Out] -2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(-a+bx)^{3/2}}\,dx = -\frac{2}{a\sqrt{-a+bx}} - \frac{\int \frac{1}{x\sqrt{-a+bx}}\,dx}{a}$$

$$= -\frac{2}{a\sqrt{-a+bx}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{ab}$$

$$= -\frac{2}{a\sqrt{-a+bx}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [C] time = 0.0082974, size = 33, normalized size = 0.79

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{bx}{a}\right)}{a\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (b*x)/a])/(a*Sqrt[-a + b*x])
```

Maple [A] time = 0.007, size = 35, normalized size = 0.8

$$-2\,\frac{1}{a^{3/2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2\,\frac{1}{a\sqrt{bx-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(3/2),x)
```

```
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)-2/a/(b*x-a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44623, size = 266, normalized size = 6.33

$$\left[-\frac{(bx-a)\sqrt{-a}\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a}a}{a^2bx-a^3}, -\frac{2\left((bx-a)\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}a\right)}{a^2bx-a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((b*x - a)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqr
t(b*x - a)*a)/(a^2*b*x - a^3), -2*((b*x - a)*sqrt(a)*arctan(sqrt(b*x - a)/s
qrt(a)) + sqrt(b*x - a)*a)/(a^2*b*x - a^3)]
```

Sympy [C] time = 2.73861, size = 439, normalized size = 10.45

$$\begin{cases} -\frac{2a^3\sqrt{-1+\frac{bx}{a}}}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{ia^3\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{2ia^3\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{2a^3\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{ia^2bx\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{2ia^2bx\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{2a^2bx\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} & \text{for } \frac{|bx|}{|a|} > 1 \\ -\frac{2ia^3\sqrt{1-\frac{bx}{a}}}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{ia^3\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{2ia^3\log\left(\sqrt{1-\frac{bx}{a}}+1\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{\pi a^3}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{ia^2bx\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{2ia^2bx\log\left(\sqrt{1-\frac{bx}{a}}+1\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{\pi a^2bx}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((-2*a**3*sqrt(-1 + b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - I*a**3*log
(b*x/a)/(-a**(9/2) + a**(7/2)*b*x) + 2*I*a**3*log(sqrt(b)*sqrt(x)/sqrt(a))/
(-a**(9/2) + a**(7/2)*b*x) - 2*a**3*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(-a**(9
/2) + a**(7/2)*b*x) + I*a**2*b*x*log(b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - 2*
I*a**2*b*x*log(sqrt(b)*sqrt(x)/sqrt(a))/(-a**(9/2) + a**(7/2)*b*x) + 2*a**2
*b*x*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(-a**(9/2) + a**(7/2)*b*x), Abs(b*x)/A
bs(a) > 1), (-2*I*a**3*sqrt(1 - b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - I*a**3*
log(b*x/a)/(-a**(9/2) + a**(7/2)*b*x) + 2*I*a**3*log(sqrt(1 - b*x/a) + 1)/(
-a**(9/2) + a**(7/2)*b*x) - pi*a**3/(-a**(9/2) + a**(7/2)*b*x) + I*a**2*b*x
*log(b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - 2*I*a**2*b*x*log(sqrt(1 - b*x/a) +
 1)/(-a**(9/2) + a**(7/2)*b*x) + pi*a**2*b*x/(-a**(9/2) + a**(7/2)*b*x), Tr
ue))
```

Giac [A] time = 1.17981, size = 46, normalized size = 1.1

$$-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2}{\sqrt{bx-a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) - 2/(sqrt(b*x - a)*a)
```

3.363 $\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{3b}{a^2\sqrt{bx-a}}-\frac{3b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{1}{ax\sqrt{bx-a}}$$

```
[Out] (-3*b)/(a^2*Sqrt[-a + b*x]) + 1/(a*x*Sqrt[-a + b*x]) - (3*b*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0157637, antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 205}

$$-\frac{3\sqrt{bx-a}}{a^2x}-\frac{3b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2}{ax\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(3/2)),x]
```

```
[Out] -2/(a*x*Sqrt[-a + b*x]) - (3*Sqrt[-a + b*x])/(a^2*x) - (3*b*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx = -\frac{2}{ax\sqrt{-a+bx}} - \frac{3\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{a}$$
$$= -\frac{2}{ax\sqrt{-a+bx}} - \frac{3\sqrt{-a+bx}}{a^2x} - \frac{(3b)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a^2}$$
$$= -\frac{2}{ax\sqrt{-a+bx}} - \frac{3\sqrt{-a+bx}}{a^2x} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{a^2}$$
$$= -\frac{2}{ax\sqrt{-a+bx}} - \frac{3\sqrt{-a+bx}}{a^2x} - \frac{3b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.0087758, size = 34, normalized size = 0.55

$$-\frac{2b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};1-\frac{bx}{a}\right)}{a^2\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(3/2)),x]
```

```
[Out] (-2*b*Hypergeometric2F1[-1/2, 2, 1/2, 1 - (b*x)/a])/(a^2*Sqrt[-a + b*x])
```

Maple [A] time = 0.011, size = 54, normalized size = 0.9

$$-2\frac{b}{a^2\sqrt{bx-a}} - \frac{1}{a^2x}\sqrt{bx-a} - 3\frac{b}{a^{5/2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(3/2),x)
```

```
[Out] -2*b/a^2/(b*x-a)^(1/2)-1/a^2*(b*x-a)^(1/2)/x-3*b*arctan((b*x-a)^(1/2)/a^(1/
2))/a^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62799, size = 344, normalized size = 5.55

$$\left[-\frac{3\left(b^2x^2-abx\right)\sqrt{-a}\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(3abx-a^2\right)\sqrt{bx-a}}{2\left(a^3bx^2-a^4x\right)},-\frac{3\left(b^2x^2-abx\right)\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(3abx-a\right.}{a^3bx^2-a^4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(b^2*x^2 - a*b*x)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2
*a)/x) + 2*(3*a*b*x - a^2)*sqrt(b*x - a))/(a^3*b*x^2 - a^4*x), -(3*(b^2*x^2
 - a*b*x)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + (3*a*b*x - a^2)*sqrt(b*x
- a))/(a^3*b*x^2 - a^4*x)]
```

Sympy [A] time = 4.73075, size = 160, normalized size = 2.58

$$\begin{cases} -\frac{i}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{3i\sqrt{b}}{a^2\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{3ib\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}} & \text{for}\ \frac{|a|}{|b||x|}>1 \\ \frac{1}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{a^2\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{3b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((-I/(a*sqrt(b)*x**(3/2)*sqrt(a/(b*x) - 1)) + 3*I*sqrt(b)/(a**2*sq
rt(x)*sqrt(a/(b*x) - 1)) - 3*I*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2),
 Abs(a)/(Abs(b)*Abs(x)) > 1), (1/(a*sqrt(b)*x**(3/2)*sqrt(-a/(b*x) + 1)) -
3*sqrt(b)/(a**2*sqrt(x)*sqrt(-a/(b*x) + 1)) + 3*b*asin(sqrt(a)/(sqrt(b)*sqr
t(x)))/a**(5/2), True))
```

Giac [A] time = 1.18359, size = 86, normalized size = 1.39

$$-\frac{3b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{3(bx-a)b+2ab}{\left((bx-a)^{\frac{3}{2}}+\sqrt{bx-a}a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) - (3*(b*x - a)*b + 2*a*b)/(((b*x
 - a)^(3/2) + sqrt(b*x - a)*a)*a^2)
```

3.364 $\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{15b^2}{4a^3\sqrt{bx-a}}-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{5b}{4a^2x\sqrt{bx-a}}+\frac{1}{2ax^2\sqrt{bx-a}}$$

```
[Out] (-15*b^2)/(4*a^3*Sqrt[-a + b*x]) + 1/(2*a*x^2*Sqrt[-a + b*x]) + (5*b)/(4*a^
2*x*Sqrt[-a + b*x]) - (15*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rubi [A] time = 0.0231135, antiderivative size = 93, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 205}

$$-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{5\sqrt{bx-a}}{2a^2x^2}-\frac{15b\sqrt{bx-a}}{4a^3x}-\frac{2}{ax^2\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(3/2)),x]
```

```
[Out] -2/(a*x^2*Sqrt[-a + b*x]) - (5*Sqrt[-a + b*x])/(2*a^2*x^2) - (15*b*Sqrt[-a
+ b*x])/(4*a^3*x) - (15*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx &= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\int \frac{1}{x^3\sqrt{-a+bx}}\,dx}{a} \\
&= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\sqrt{-a+bx}}{2a^2x^2} - \frac{(15b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a^2} \\
&= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\sqrt{-a+bx}}{2a^2x^2} - \frac{15b\sqrt{-a+bx}}{4a^3x} - \frac{(15b^2)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a^3} \\
&= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\sqrt{-a+bx}}{2a^2x^2} - \frac{15b\sqrt{-a+bx}}{4a^3x} - \frac{(15b)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{4a^3} \\
&= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\sqrt{-a+bx}}{2a^2x^2} - \frac{15b\sqrt{-a+bx}}{4a^3x} - \frac{15b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0108433, size = 36, normalized size = 0.38

$$
-\frac{2b^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};1-\frac{bx}{a}\right)}{a^3\sqrt{bx-a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(3/2)),x]
```

```
[Out] (-2*b^2*Hypergeometric2F1[-1/2, 3, 1/2, 1 - (b*x)/a])/(a^3*Sqrt[-a + b*x])
```

Maple [A] time = 0.011, size = 75, normalized size = 0.8

$$
-2\,\frac{b^2}{a^3\sqrt{bx-a}} - \frac{7}{4\,a^3x^2}\,(bx-a)^{\frac{3}{2}} - \frac{9}{4\,a^2x^2}\sqrt{bx-a} - \frac{15\,b^2}{4}\arctan\left(\sqrt{bx-a}\frac{1}{\sqrt{a}}\right)a^{-\frac{7}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(3/2),x)
```

```
[Out] -2*b^2/a^3/(b*x-a)^(1/2)-7/4/a^3/x^2*(b*x-a)^(3/2)-9/4/a^2/x^2*(b*x-a)^(1/2
)-15/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(7/2)
```

Maximа [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62984, size = 420, normalized size = 4.42

$$\left[-\frac{15\left(b^3x^3-ab^2x^2\right)\sqrt{-a}\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(15ab^2x^2-5a^2bx-2a^3\right)\sqrt{bx-a}}{8\left(a^4bx^3-a^5x^2\right)},-\frac{15\left(b^3x^3-ab^2x^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{4\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(b^3*x^3 - a*b^2*x^2)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a
) - 2*a)/x) + 2*(15*a*b^2*x^2 - 5*a^2*b*x - 2*a^3)*sqrt(b*x - a))/(a^4*b*x^
3 - a^5*x^2), -1/4*(15*(b^3*x^3 - a*b^2*x^2)*sqrt(a)*arctan(sqrt(b*x - a)/s
qrt(a)) + (15*a*b^2*x^2 - 5*a^2*b*x - 2*a^3)*sqrt(b*x - a))/(a^4*b*x^3 - a^
5*x^2)]
```

Sympy [A] time = 7.71199, size = 230, normalized size = 2.42

$$\begin{cases} -\frac{i}{2a\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{5i\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{15ib^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{15ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}} & \text{for}\ \frac{|a|}{|b||x|}>1 \\ \frac{1}{2a\sqrt{b}x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{5\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{15b^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{15b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((-I/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) - 5*I*sqrt(b)/(4*a**
2*x**(3/2)*sqrt(a/(b*x) - 1)) + 15*I*b**(3/2)/(4*a**3*sqrt(x)*sqrt(a/(b*x)
- 1)) - 15*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2)), Abs(a)/(Ab
s(b)*Abs(x)) > 1), (1/(2*a*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) + 5*sqrt(b)
/(4*a**2*x**(3/2)*sqrt(-a/(b*x) + 1)) - 15*b**(3/2)/(4*a**3*sqrt(x)*sqrt(-a
/(b*x) + 1)) + 15*b**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2)), True))
```

Giac [A] time = 1.18303, size = 109, normalized size = 1.15

$$-\frac{15\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{7}{2}}}-\frac{2\,b^2}{\sqrt{bx-a}a^3}-\frac{7\,(bx-a)^{\frac{3}{2}}b^2+9\,\sqrt{bx-a}ab^2}{4\,a^3b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="giac")
```

```
[Out] -15/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2) - 2*b^2/(sqrt(b*x - a)*a^3)
 - 1/4*(7*(b*x - a)^(3/2)*b^2 + 9*sqrt(b*x - a)*a*b^2)/(a^3*b^2*x^2)
```

3.365 $\int \frac{1}{x(-a+bx)^{5/2}}\,dx$

Optimal. Leaf size=60

$$\frac{2}{a^2\sqrt{bx-a}}+\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2}{3a(bx-a)^{3/2}}$$

```
[Out] -2/(3*a*(-a + b*x)^(3/2)) + 2/(a^2*Sqrt[-a + b*x]) + (2*ArcTan[Sqrt[-a + b*
x]/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0151089, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 205}

$$\frac{2}{a^2\sqrt{bx-a}}+\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2}{3a(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(5/2)),x]
```

```
[Out] -2/(3*a*(-a + b*x)^(3/2)) + 2/(a^2*Sqrt[-a + b*x]) + (2*ArcTan[Sqrt[-a + b*
x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(-a+bx)^{5/2}}\,dx = -\frac{2}{3a(-a+bx)^{3/2}} - \frac{\int \frac{1}{x(-a+bx)^{3/2}}\,dx}{a}$$
$$= -\frac{2}{3a(-a+bx)^{3/2}} + \frac{2}{a^2\sqrt{-a+bx}} + \frac{\int \frac{1}{x\sqrt{-a+bx}}\,dx}{a^2}$$
$$= -\frac{2}{3a(-a+bx)^{3/2}} + \frac{2}{a^2\sqrt{-a+bx}} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{a^2 b}$$
$$= -\frac{2}{3a(-a+bx)^{3/2}} + \frac{2}{a^2\sqrt{-a+bx}} + \frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.0123223, size = 35, normalized size = 0.58

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{bx}{a}\right)}{3a(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(5/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, 1 - (b*x)/a])/(3*a*(-a + b*x)^(3/2))
```

Maple [A] time = 0.009, size = 49, normalized size = 0.8

$$-\frac{2}{3\,a}(bx-a)^{-\frac{3}{2}} + 2\,\frac{1}{a^{5/2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + 2\,\frac{1}{a^2\sqrt{bx-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(5/2),x)
```

```
[Out] -2/3/a/(b*x-a)^(3/2)+2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)+2/a^2/(b*x-a)^
(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58615, size = 408, normalized size = 6.8

$$\left[-\frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{-a}\log\left(\frac{bx-2\,\sqrt{bx-a}\sqrt{-a}-2\,a}{x}\right)-2\left(3\,abx-4\,a^2\right)\sqrt{bx-a}}{3\left(a^3b^2x^2-2\,a^4bx+a^5\right)}, \frac{2\left(3\left(b^2x^2-2\,abx+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{bx}}{\sqrt{}}\right.\right.}{3\left(a^3b^2x^2-2\,a^4b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt
(-a) - 2*a)/x) - 2*(3*a*b*x - 4*a^2)*sqrt(b*x - a))/(a^3*b^2*x^2 - 2*a^4*b*
x + a^5), 2/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a)*arctan(sqrt(b*x - a)/sqr
t(a)) + (3*a*b*x - 4*a^2)*sqrt(b*x - a))/(a^3*b^2*x^2 - 2*a^4*b*x + a^5)]
```

Sympy [C] time = 4.29591, size = 1952, normalized size = 32.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(5/2),x)
```

```
[Out] Piecewise((8*a**7*sqrt(-1 + b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(
15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 3*I*a**7*log(b*x/a)/(-3*a**(19/2
) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*I*
a**7*log(sqrt(b)*sqrt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(1
5/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**7*asin(sqrt(a)/(sqrt(b)*sqrt
(x)))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)
*b**3*x**3) - 14*a**6*b*x*sqrt(-1 + b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x
- 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 9*I*a**6*b*x*log(b*x/a)/
(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*
x**3) + 18*I*a**6*b*x*log(sqrt(b)*sqrt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17
/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*a**6*b*x*asin
(sqrt(a)/(sqrt(b)*sqrt(x)))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b
**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**5*b**2*x**2*sqrt(-1 + b*x/a)/(-3*a
**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3)
 + 9*I*a**5*b**2*x**2*log(b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15
/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*I*a**5*b**2*x**2*log(sqrt(b)*sq
rt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*
a**(13/2)*b**3*x**3) + 18*a**5*b**2*x**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(-
3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x*
*3) - 3*I*a**4*b**3*x**3*log(b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**
(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*I*a**4*b**3*x**3*log(sqrt(b)*
sqrt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 +
3*a**(13/2)*b**3*x**3) - 6*a**4*b**3*x**3*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(
-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x
**3), Abs(b*x)/Abs(a) > 1), (8*I*a**7*sqrt(1 - b*x/a)/(-3*a**(19/2) + 9*a**
(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 3*I*a**7*log(
b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2
)*b**3*x**3) - 6*I*a**7*log(sqrt(1 - b*x/a) + 1)/(-3*a**(19/2) + 9*a**(17/2
)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 3*pi*a**7/(-3*a**(
19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) -
14*I*a**6*b*x*sqrt(1 - b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)
*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 9*I*a**6*b*x*log(b*x/a)/(-3*a**(19/2)
 + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 18*I*
```

```
a**6*b*x*log(sqrt(1 - b*x/a) + 1)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(1
5/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 9*pi*a**6*b*x/(-3*a**(19/2) + 9*a
**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*I*a**5*b*
*2*x**2*sqrt(1 - b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*
x**2 + 3*a**(13/2)*b**3*x**3) + 9*I*a**5*b**2*x**2*log(b*x/a)/(-3*a**(19/2)
 + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*I*
a**5*b**2*x**2*log(sqrt(1 - b*x/a) + 1)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9
*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 9*pi*a**5*b**2*x**2/(-3*a**
(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) -
 3*I*a**4*b**3*x**3*log(b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2
)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*I*a**4*b**3*x**3*log(sqrt(1 - b*x/
a) + 1)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/
2)*b**3*x**3) - 3*pi*a**4*b**3*x**3/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**
(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3), True))
```

Giac [A] time = 1.19288, size = 57, normalized size = 0.95

$$\frac{2 \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2 (3 bx - 4 a)}{3 (bx - a)^{\frac{3}{2}} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 2/3*(3*b*x - 4*a)/((b*x - a)^(3/2
)*a^2)
```

3.366 $\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx$

Optimal. Leaf size=81

$$\frac{5b}{a^3\sqrt{bx-a}}-\frac{5b}{3a^2(bx-a)^{3/2}}+\frac{5b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{1}{ax(bx-a)^{3/2}}$$

```
[Out] (-5*b)/(3*a^2*(-a + b*x)^(3/2)) + 1/(a*x*(-a + b*x)^(3/2)) + (5*b)/(a^3*Sqr
t[-a + b*x]) + (5*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(7/2)
```

Rubi [A] time = 0.0234259, antiderivative size = 88, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 205}

$$\frac{5\sqrt{bx-a}}{a^3x}+\frac{10}{3a^2x\sqrt{bx-a}}+\frac{5b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{2}{3ax(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(5/2)),x]
```

```
[Out] -2/(3*a*x*(-a + b*x)^(3/2)) + 10/(3*a^2*x*Sqrt[-a + b*x]) + (5*Sqrt[-a + b*
x])/(a^3*x) + (5*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx &= -\frac{2}{3ax(-a+bx)^{3/2}} - \frac{5\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx}{3a} \\
&= -\frac{2}{3ax(-a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{-a+bx}} + \frac{5\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{a^2} \\
&= -\frac{2}{3ax(-a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{-a+bx}} + \frac{5\sqrt{-a+bx}}{a^3x} + \frac{(5b)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a^3} \\
&= -\frac{2}{3ax(-a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{-a+bx}} + \frac{5\sqrt{-a+bx}}{a^3x} + \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{a^3} \\
&= -\frac{2}{3ax(-a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{-a+bx}} + \frac{5\sqrt{-a+bx}}{a^3x} + \frac{5b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0124766, size = 36, normalized size = 0.44

$$-\frac{2b\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};1-\frac{bx}{a}\right)}{3a^2(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(5/2)),x]
```

```
[Out] (-2*b*Hypergeometric2F1[-3/2, 2, -1/2, 1 - (b*x)/a])/(3*a^2*(-a + b*x)^(3/2
))
```

Maple [A] time = 0.013, size = 68, normalized size = 0.8

$$-\frac{2\,b}{3\,a^2}\,(bx-a)^{-\frac{3}{2}}+4\,\frac{b}{a^3\sqrt{bx-a}}+\frac{1}{a^3x}\sqrt{bx-a}+5\,\frac{b}{a^{7/2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(5/2),x)
```

```
[Out] -2/3*b/a^2/(b*x-a)^(3/2)+4*b/a^3/(b*x-a)^(1/2)+1/a^3*(b*x-a)^(1/2)/x+5*b*ar
ctan((b*x-a)^(1/2)/a^(1/2))/a^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89575, size = 491, normalized size = 6.06

$$\left[-\frac{15\left(b^3x^3-2\,ab^2x^2+a^2bx\right)\sqrt{-a}\log\left(\frac{bx-2\,\sqrt{bx-a}\sqrt{-a}-2\,a}{x}\right)-2\left(15\,ab^2x^2-20\,a^2bx+3\,a^3\right)\sqrt{bx-a}}{6\left(a^4b^2x^3-2\,a^5bx^2+a^6x\right)},\ \frac{15\left(b^3x^3-2\,ab^2x^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(b^3*x^3 - 2*a*b^2*x^2 + a^2*b*x)*sqrt(-a)*log((b*x - 2*sqrt(b*x
- a)*sqrt(-a) - 2*a)/x) - 2*(15*a*b^2*x^2 - 20*a^2*b*x + 3*a^3)*sqrt(b*x -
a))/(a^4*b^2*x^3 - 2*a^5*b*x^2 + a^6*x), 1/3*(15*(b^3*x^3 - 2*a*b^2*x^2 + a
^2*b*x)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + (15*a*b^2*x^2 - 20*a^2*b*x
+ 3*a^3)*sqrt(b*x - a))/(a^4*b^2*x^3 - 2*a^5*b*x^2 + a^6*x)]
```

Sympy [C] time = 7.7245, size = 2236, normalized size = 27.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(5/2),x)
```

```
[Out] Piecewise((-6*a**17*sqrt(-1 + b*x/a)/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2
- 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 46*a**16*b*x*sqrt(-1 +
b*x/a)/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a
**(33/2)*b**3*x**4) + 15*I*a**16*b*x*log(b*x/a)/(-6*a**(39/2)*x + 18*a**(37
/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 30*I*a**16*b
*x*log(sqrt(b)*sqrt(x)/sqrt(a))/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*
a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**16*b*x*asin(sqrt(a)/(s
qrt(b)*sqrt(x)))/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*
x**3 + 6*a**(33/2)*b**3*x**4) - 70*a**15*b**2*x**2*sqrt(-1 + b*x/a)/(-6*a**
(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*
x**4) - 45*I*a**15*b**2*x**2*log(b*x/a)/(-6*a**(39/2)*x + 18*a**(37/2)*b*x*
*2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*I*a**15*b**2*x**2
*log(sqrt(b)*sqrt(x)/sqrt(a))/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a*
*(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 90*a**15*b**2*x**2*asin(sqrt(a
)/(sqrt(b)*sqrt(x)))/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b
**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**14*b**3*x**3*sqrt(-1 + b*x/a)/(-6
*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b
**3*x**4) + 45*I*a**14*b**3*x**3*log(b*x/a)/(-6*a**(39/2)*x + 18*a**(37/2)*
b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 90*I*a**14*b**3*
x**3*log(sqrt(b)*sqrt(x)/sqrt(a))/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 1
8*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**14*b**3*x**3*asin(sq
rt(a)/(sqrt(b)*sqrt(x)))/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/
2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 15*I*a**13*b**4*x**4*log(b*x/a)/(-6
*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b
**3*x**4) + 30*I*a**13*b**4*x**4*log(sqrt(b)*sqrt(x)/sqrt(a))/(-6*a**(39/2)
*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4)
- 30*a**13*b**4*x**4*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(-6*a**(39/2)*x + 18*a
**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4), Abs(b*x)
/Abs(a) > 1), (-6*I*a**17*sqrt(1 - b*x/a)/(-6*a**(39/2)*x + 18*a**(37/2)*b*
x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 46*I*a**16*b*x*sqr
t(1 - b*x/a)/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3
```

```
 + 6*a**(33/2)*b**3*x**4) + 15*I*a**16*b*x*log(b*x/a)/(-6*a**(39/2)*x + 18*
a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 30*I*a
**16*b*x*log(sqrt(1 - b*x/a) + 1)/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 1
8*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 15*pi*a**16*b*x/(-6*a**(39
/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**
4) - 70*I*a**15*b**2*x**2*sqrt(1 - b*x/a)/(-6*a**(39/2)*x + 18*a**(37/2)*b*
x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 45*I*a**15*b**2*x*
*2*log(b*x/a)/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**
3 + 6*a**(33/2)*b**3*x**4) + 90*I*a**15*b**2*x**2*log(sqrt(1 - b*x/a) + 1)/
(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2
)*b**3*x**4) - 45*pi*a**15*b**2*x**2/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2
- 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*I*a**14*b**3*x**3*sq
rt(1 - b*x/a)/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**
3 + 6*a**(33/2)*b**3*x**4) + 45*I*a**14*b**3*x**3*log(b*x/a)/(-6*a**(39/2)*
x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) -
 90*I*a**14*b**3*x**3*log(sqrt(1 - b*x/a) + 1)/(-6*a**(39/2)*x + 18*a**(37/
2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 45*pi*a**14*b
**3*x**3/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6
*a**(33/2)*b**3*x**4) - 15*I*a**13*b**4*x**4*log(b*x/a)/(-6*a**(39/2)*x + 1
8*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*I
*a**13*b**4*x**4*log(sqrt(1 - b*x/a) + 1)/(-6*a**(39/2)*x + 18*a**(37/2)*b*
x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 15*pi*a**13*b**4*x
**4/(-6*a**(39/2)*x + 18*a**(37/2)*b*x**2 - 18*a**(35/2)*b**2*x**3 + 6*a**(
33/2)*b**3*x**4), True))
```

Giac [A] time = 1.17017, size = 89, normalized size = 1.1

$$\frac{5\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}+\frac{2\,(6\,(bx-a)b-ab)}{3\,(bx-a)^{\frac{3}{2}}a^{3}}+\frac{\sqrt{bx-a}}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="giac")
```

```
[Out] 5*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2) + 2/3*(6*(b*x - a)*b - a*b)/((b*x
 - a)^(3/2)*a^3) + sqrt(b*x - a)/(a^3*x)
```

3.367 $\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{35b^2}{4a^4\sqrt{bx-a}}-\frac{35b^2}{12a^3(bx-a)^{3/2}}+\frac{35b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{7b}{4a^2x(bx-a)^{3/2}}+\frac{1}{2ax^2(bx-a)^{3/2}}$$

```
[Out] (-35*b^2)/(12*a^3*(-a + b*x)^(3/2)) + 1/(2*a*x^2*(-a + b*x)^(3/2)) + (7*b)/
(4*a^2*x*(-a + b*x)^(3/2)) + (35*b^2)/(4*a^4*Sqrt[-a + b*x]) + (35*b^2*ArcT
an[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rubi [A] time = 0.0331239, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 205}

$$\frac{35b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35\sqrt{bx-a}}{6a^3x^2}+\frac{14}{3a^2x^2\sqrt{bx-a}}+\frac{35b\sqrt{bx-a}}{4a^4x}-\frac{2}{3ax^2(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(5/2)),x]
```

```
[Out] -2/(3*a*x^2*(-a + b*x)^(3/2)) + 14/(3*a^2*x^2*Sqrt[-a + b*x]) + (35*Sqrt[-a
 + b*x])/(6*a^3*x^2) + (35*b*Sqrt[-a + b*x])/(4*a^4*x) + (35*b^2*ArcTan[Sqr
t[-a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx = -\frac{2}{3ax^2(-a+bx)^{3/2}} - \frac{7\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx}{3a}$$
$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\int \frac{1}{x^3\sqrt{-a+bx}}\,dx}{3a^2}$$
$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{(35b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a^3}$$
$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{35b\sqrt{-a+bx}}{4a^4x} + \frac{(35b^2)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a^4}$$
$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{35b\sqrt{-a+bx}}{4a^4x} + \frac{(35b)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{4a^4}$$
$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{35b\sqrt{-a+bx}}{4a^4x} + \frac{35b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Mathematica [C] time = 0.0133301, size = 38, normalized size = 0.33

$$-\frac{2b^2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};1-\frac{bx}{a}\right)}{3a^3(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(5/2)),x]
```

```
[Out] (-2*b^2*Hypergeometric2F1[-3/2, 3, -1/2, 1 - (b*x)/a])/(3*a^3*(-a + b*x)^(3
/2))
```

Maple [A] time = 0.016, size = 92, normalized size = 0.8

$$-\frac{2\,b^2}{3\,a^3}(bx-a)^{-\frac{3}{2}} + 6\,\frac{b^2}{a^4\sqrt{bx-a}} + \frac{11}{4\,a^4x^2}(bx-a)^{\frac{3}{2}} + \frac{13}{4\,a^3x^2}\sqrt{bx-a} + \frac{35\,b^2}{4}\arctan\left(\sqrt{bx-a}\frac{1}{\sqrt{a}}\right)a^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(5/2),x)
```

```
[Out] -2/3*b^2/a^3/(b*x-a)^(3/2)+6*b^2/a^4/(b*x-a)^(1/2)+11/4/a^4/x^2*(b*x-a)^(3/
2)+13/4/a^3/x^2*(b*x-a)^(1/2)+35/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(9/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85456, size = 564, normalized size = 4.86

$$\left[-\frac{105\left(b^{4}x^{4}-2\,ab^{3}x^{3}+a^{2}b^{2}x^{2}\right)\sqrt{-a}\log\left(\frac{bx-2\,\sqrt{bx-a}\sqrt{-a}-2\,a}{x}\right)-2\left(105\,ab^{3}x^{3}-140\,a^{2}b^{2}x^{2}+21\,a^{3}bx+6\,a^{4}\right)\sqrt{bx-a}}{24\left(a^{5}b^{2}x^{4}-2\,a^{6}bx^{3}+a^{7}x^{2}\right)}, 10\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(105*(b^4*x^4 - 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*log((b*x - 2*sqr
t(b*x - a)*sqrt(-a) - 2*a)/x) - 2*(105*a*b^3*x^3 - 140*a^2*b^2*x^2 + 21*a^3
*b*x + 6*a^4)*sqrt(b*x - a))/(a^5*b^2*x^4 - 2*a^6*b*x^3 + a^7*x^2), 1/12*(1
05*(b^4*x^4 - 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(
a)) + (105*a*b^3*x^3 - 140*a^2*b^2*x^2 + 21*a^3*b*x + 6*a^4)*sqrt(b*x - a))
/(a^5*b^2*x^4 - 2*a^6*b*x^3 + a^7*x^2)]
```

Sympy [B] time = 13.3606, size = 1112, normalized size = 9.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(5/2),x)
```

```
[Out] Piecewise((12*I*a**(89/2)*b**75*x**75/(24*a**(93/2)*b**(151/2)*x**(155/2)*s
qrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) +
42*I*a**(87/2)*b**76*x**76/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x)
 - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) - 280*I*a**(8
5/2)*b**77*x**77/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24
*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) + 210*I*a**(83/2)*b**78
*x**78/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)
*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) + 210*I*a**42*b**(155/2)*x**(155/
2)*sqrt(a/(b*x) - 1)*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(93/2)*b**(151
/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(
a/(b*x) - 1)) - 105*pi*a**42*b**(155/2)*x**(155/2)*sqrt(a/(b*x) - 1)/(24*a*
*(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x
**(157/2)*sqrt(a/(b*x) - 1)) - 210*I*a**41*b**(157/2)*x**(157/2)*sqrt(a/(b*
x) - 1)*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(93/2)*b**(151/2)*x**(155/2
)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1))
 + 105*pi*a**41*b**(157/2)*x**(157/2)*sqrt(a/(b*x) - 1)/(24*a**(93/2)*b**(1
51/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqr
t(a/(b*x) - 1)), Abs(a)/(Abs(b)*Abs(x)) > 1), (-6*a**(89/2)*b**75*x**75/(12
*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/
2)*x**(157/2)*sqrt(-a/(b*x) + 1)) - 21*a**(87/2)*b**76*x**76/(12*a**(93/2)*
b**(151/2)*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/
2)*sqrt(-a/(b*x) + 1)) + 140*a**(85/2)*b**77*x**77/(12*a**(93/2)*b**(151/2)
*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a
/(b*x) + 1)) - 105*a**(83/2)*b**78*x**78/(12*a**(93/2)*b**(151/2)*x**(155/2
)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1
```

```
)) - 105*a**42*b**(155/2)*x**(155/2)*sqrt(-a/(b*x) + 1)*asin(sqrt(a)/(sqrt(
b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a*
*(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1)) + 105*a**41*b**(157/2)*x*
*(157/2)*sqrt(-a/(b*x) + 1)*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(12*a**(93/2)*b
**(151/2)*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2
)*sqrt(-a/(b*x) + 1)), True))
```

Giac [A] time = 1.2193, size = 131, normalized size = 1.13

$$\frac{35\, b^2 \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\, a^{\frac{9}{2}}} + \frac{2\left(9\,(bx-a)b^2 - ab^2\right)}{3\,(bx-a)^{\frac{3}{2}}a^4} + \frac{11\,(bx-a)^{\frac{3}{2}}b^2 + 13\,\sqrt{bx-a}ab^2}{4\,a^4b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(9/2) + 2/3*(9*(b*x - a)*b^2 - a*b
^2)/((b*x - a)^(3/2)*a^4) + 1/4*(11*(b*x - a)^(3/2)*b^2 + 13*sqrt(b*x - a)*
a*b^2)/(a^4*b^2*x^2)
```

3.368 $\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=13

$$\frac{x^m}{\sqrt{a+bx}}$$

```
[Out] x^m/Sqrt[a + b*x]
```

Rubi [A] time = 0.0062557, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {12, 74}

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx &= \frac{1}{2}\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{(a+bx)^{3/2}}\,dx\\ &= \frac{x^m}{\sqrt{a+bx}}\end{aligned}$$

Mathematica [A] time = 0.0793584, size = 13, normalized size = 1.

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

Maple [A] time = 0.006, size = 12, normalized size = 0.9

$$x^{m}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x)
```

```
[Out] x^m/(b*x+a)^(1/2)
```

Maxima [A] time = 1.25709, size = 15, normalized size = 1.15

$$\frac{x^{m}}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="max
ima")
```

```
[Out] x^m/sqrt(b*x + a)
```

Fricas [A] time = 1.96447, size = 36, normalized size = 2.77

$$\frac{xx^{m-1}}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="fri
cas")
```

```
[Out] x*x^(m - 1)/sqrt(b*x + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b(2m-1)x+2am)x^{m-1}}{2(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/2*(b*(2*m - 1)*x + 2*a*m)*x^(m - 1)/(b*x + a)^(3/2), x)
```

3.369 $\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right) dx$

Optimal. Leaf size=13

$$\frac{x^m}{\sqrt{a+bx}}$$

```
[Out] x^m/Sqrt[a + b*x]
```

Rubi [C] time = 0.0446209, antiderivative size = 92, normalized size of antiderivative = 7.08, number of steps used = 5, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {67, 65}

$$\frac{x^m\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};\frac{bx}{a}+1\right)}{\sqrt{a+bx}}-\frac{2mx^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[-(b*x^m)/(2*(a + b*x)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x],x]
```

```
[Out] (x^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/((-((b*x)/a))^m*Sqrt[a
+ b*x]) - (2*m*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*
x)/a])/(a*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right) dx &= -\left(\frac{1}{2}b\int \frac{x^m}{(a+bx)^{3/2}}\,dx\right)+m\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx\\ &= -\left(\frac{1}{2}\left(bx^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{(a+bx)^{3/2}}\,dx\right)-\frac{\left(bmx^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{-1+m}}{\sqrt{a+bx}}\,dx}{a}\\ &= \frac{x^m\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};1+\frac{bx}{a}\right)}{\sqrt{a+bx}}-\frac{2mx^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0259762, size = 13, normalized size = 1.

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(b*x^m)/(2*(a + b*x)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x],x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int -\frac{bx^m}{2}(bx+a)^{-\frac{3}{2}} + mx^{-1+m}\frac{1}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x)
```

```
[Out] int(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x)
```

Maxima [A] time = 1.28101, size = 15, normalized size = 1.15

$$\frac{x^m}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] x^m/sqrt(b*x + a)
```

Fricas [A] time = 1.84039, size = 26, normalized size = 2.

$$\frac{x^m}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] x^m/sqrt(b*x + a)
```

Sympy [C] time = 10.4497, size = 73, normalized size = 5.62

$$\frac{mx^m\Gamma(m)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m\\ m+1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\Gamma(m+1)} - \frac{bxx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{3}{2}, m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x**m/(b*x+a)**(3/2)+m*x**(-1+m)/(b*x+a)**(1/2),x)
```

```
[Out] m*x**m*gamma(m)*hyper((1/2, m), (m + 1,), b*x*exp_polar(I*pi)/a)/(sqrt(a)*g
amma(m + 1)) - b*x*x**m*gamma(m + 1)*hyper((3/2, m + 1), (m + 2,), b*x*exp_
polar(I*pi)/a)/(2*a**(3/2)*gamma(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{mx^{m-1}}{\sqrt{bx+a}} - \frac{bx^m}{2\,(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(m*x^(m - 1)/sqrt(b*x + a) - 1/2*b*x^m/(b*x + a)^(3/2), x)
```

3.370 $\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0077666, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {7, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx &= \int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.0063637, size = 23, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0., size = 18, normalized size = 0.8

$$-2\frac{1}{\sqrt{a}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87295, size = 142, normalized size = 6.17

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [A] time = 1.7182, size = 24, normalized size = 1.04

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Giac [A] time = 1.16434, size = 28, normalized size = 1.22

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

3.371 $\int x^3 \sqrt[3]{a+bx}\,dx$

Optimal. Leaf size=72

$$\frac{9a^2(a+bx)^{7/3}}{7b^4}-\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{3(a+bx)^{13/3}}{13b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}$$

```
[Out] (-3*a^3*(a + b*x)^(4/3))/(4*b^4) + (9*a^2*(a + b*x)^(7/3))/(7*b^4) - (9*a*(
a + b*x)^(10/3))/(10*b^4) + (3*(a + b*x)^(13/3))/(13*b^4)
```

Rubi [A] time = 0.0186468, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{7/3}}{7b^4}-\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{3(a+bx)^{13/3}}{13b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(4/3))/(4*b^4) + (9*a^2*(a + b*x)^(7/3))/(7*b^4) - (9*a*(
a + b*x)^(10/3))/(10*b^4) + (3*(a + b*x)^(13/3))/(13*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt[3]{a+bx}\,dx &= \int\left(-\frac{a^3\sqrt[3]{a+bx}}{b^3}+\frac{3a^2(a+bx)^{4/3}}{b^3}-\frac{3a(a+bx)^{7/3}}{b^3}+\frac{(a+bx)^{10/3}}{b^3}\right)dx\\ &= -\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{9a^2(a+bx)^{7/3}}{7b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}+\frac{3(a+bx)^{13/3}}{13b^4}\end{aligned}$$

Mathematica [A] time = 0.052127, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{4/3}\left(108a^2bx-81a^3-126ab^2x^2+140b^3x^3\right)}{1820b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(-81*a^3 + 108*a^2*b*x - 126*a*b^2*x^2 + 140*b^3*x^3))/(
1820*b^4)
```

Maple [A] time = 0.004, size = 43, normalized size = 0.6

$$-\frac{-420\,b^3x^3+378\,ab^2x^2-324\,a^2bx+243\,a^3}{1820\,b^4}\,(bx+a)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(1/3),x)
```

```
[Out] -3/1820*(b*x+a)^(4/3)*(-140*b^3*x^3+126*a*b^2*x^2-108*a^2*b*x+81*a^3)/b^4
```

Maxima [A] time = 1.07404, size = 76, normalized size = 1.06

$$\frac{3\,(bx+a)^{\frac{13}{3}}}{13\,b^4}-\frac{9\,(bx+a)^{\frac{10}{3}}a}{10\,b^4}+\frac{9\,(bx+a)^{\frac{7}{3}}a^2}{7\,b^4}-\frac{3\,(bx+a)^{\frac{4}{3}}a^3}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/13*(b*x + a)^(13/3)/b^4 - 9/10*(b*x + a)^(10/3)*a/b^4 + 9/7*(b*x + a)^(7/
3)*a^2/b^4 - 3/4*(b*x + a)^(4/3)*a^3/b^4
```

Fricas [A] time = 1.65421, size = 130, normalized size = 1.81

$$\frac{3\left(140\,b^4x^4+14\,ab^3x^3-18\,a^2b^2x^2+27\,a^3bx-81\,a^4\right)(bx+a)^{\frac{1}{3}}}{1820\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/1820*(140*b^4*x^4 + 14*a*b^3*x^3 - 18*a^2*b^2*x^2 + 27*a^3*b*x - 81*a^4)*
(b*x + a)^(1/3)/b^4
```

Sympy [B] time = 3.53965, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(1/3),x)
```

```
[Out] -243*a**(73/3)*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 2
7300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 1092
0*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 243*a**(73/3)/(1820*a**20*b**4
 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 273
00*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) - 1377*
a**(70/3)*b*x*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27
300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920
*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 1458*a**(70/3)*b*x/(1820*a**20*
```

```
b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 +
 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) - 3
213*a**(67/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b
**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x*
*4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 3645*a**(67/3)*b**2*x
**2/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a
**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14
*b**10*x**6) - 3927*a**(64/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(1820*a**20*b**4
 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 273
00*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 4860*
a**(64/3)*b**3*x**3/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**
6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x
**5 + 1820*a**14*b**10*x**6) - 2163*a**(61/3)*b**4*x**4*(1 + b*x/a)**(1/3)/
(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17
*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**
10*x**6) + 3645*a**(61/3)*b**4*x**4/(1820*a**20*b**4 + 10920*a**19*b**5*x +
 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10
920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 1827*a**(58/3)*b**5*x**5*(1
+ b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x*
*2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5
+ 1820*a**14*b**10*x**6) + 1458*a**(58/3)*b**5*x**5/(1820*a**20*b**4 + 1092
0*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**1
6*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 6573*a**(55/
3)*b**6*x**6*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 273
00*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*
a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 243*a**(55/3)*b**6*x**6/(1820*a*
*20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x*
*3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6)
 + 8787*a**(52/3)*b**7*x**7*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**
19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**
8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 6498*a**(49/3)*b*
*8*x**8*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a*
*18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15
*b**9*x**5 + 1820*a**14*b**10*x**6) + 2562*a**(46/3)*b**9*x**9*(1 + b*x/a)*
*(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 3640
0*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a*
*14*b**10*x**6) + 420*a**(43/3)*b**10*x**10*(1 + b*x/a)**(1/3)/(1820*a**20*
b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 +
 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6)
```

Giac [A] time = 1.1697, size = 66, normalized size = 0.92

$$\frac{3\left(140\,(bx+a)^{\frac{13}{3}}-546\,(bx+a)^{\frac{10}{3}}a+780\,(bx+a)^{\frac{7}{3}}a^{2}-455\,(bx+a)^{\frac{4}{3}}a^{3}\right)}{1820\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/1820*(140*(b*x + a)^(13/3) - 546*(b*x + a)^(10/3)*a + 780*(b*x + a)^(7/3)
*a^2 - 455*(b*x + a)^(4/3)*a^3)/b^4
```

3.372 $\int x^2 \sqrt[3]{a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{4/3}}{4b^3}+\frac{3(a+bx)^{10/3}}{10b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}$$

```
[Out] (3*a^2*(a + b*x)^(4/3))/(4*b^3) - (6*a*(a + b*x)^(7/3))/(7*b^3) + (3*(a + b
*x)^(10/3))/(10*b^3)
```

Rubi [A] time = 0.0123199, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2(a+bx)^{4/3}}{4b^3}+\frac{3(a+bx)^{10/3}}{10b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(4/3))/(4*b^3) - (6*a*(a + b*x)^(7/3))/(7*b^3) + (3*(a + b
*x)^(10/3))/(10*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt[3]{a+bx}\,dx &= \int\left(\frac{a^2\sqrt[3]{a+bx}}{b^2}-\frac{2a(a+bx)^{4/3}}{b^2}+\frac{(a+bx)^{7/3}}{b^2}\right)dx\\ &= \frac{3a^2(a+bx)^{4/3}}{4b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{10/3}}{10b^3}\end{aligned}$$

Mathematica [A] time = 0.0277879, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{4/3}\left(9a^2-12abx+14b^2x^2\right)}{140b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(9*a^2 - 12*a*b*x + 14*b^2*x^2))/(140*b^3)
```

Maple [A] time = 0.003, size = 32, normalized size = 0.6

$$\frac{42 b^2 x^2 - 36 abx + 27 a^2}{140 b^3} (bx+a)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(1/3),x)
```

```
[Out] 3/140*(b*x+a)^(4/3)*(14*b^2*x^2-12*a*b*x+9*a^2)/b^3
```

Maxima [A] time = 1.01767, size = 55, normalized size = 1.04

$$\frac{3 (bx+a)^{\frac{10}{3}}}{10 b^3} - \frac{6 (bx+a)^{\frac{7}{3}} a}{7 b^3} + \frac{3 (bx+a)^{\frac{4}{3}} a^2}{4 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x + a)^(10/3)/b^3 - 6/7*(b*x + a)^(7/3)*a/b^3 + 3/4*(b*x + a)^(4/3)
*a^2/b^3
```

Fricas [A] time = 1.78826, size = 100, normalized size = 1.89

$$\frac{3 \left(14 b^3 x^3 + 2 ab^2 x^2 - 3 a^2 bx + 9 a^3\right)(bx+a)^{\frac{1}{3}}}{140 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/140*(14*b^3*x^3 + 2*a*b^2*x^2 - 3*a^2*b*x + 9*a^3)*(b*x + a)^(1/3)/b^3
```

Sympy [B] time = 2.47257, size = 666, normalized size = 12.57

$$\frac{27a^{\frac{34}{3}} \sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3 + 420a^7b^4x + 420a^6b^5x^2 + 140a^5b^6x^3} - \frac{27a^{\frac{34}{3}}}{140a^8b^3 + 420a^7b^4x + 420a^6b^5x^2 + 140a^5b^6x^3} + \frac{72a^{\frac{31}{3}} b}{140a^8b^3 + 420a^7b^4x +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(1/3),x)
```

```
[Out] 27*a**(34/3)*(1 + b*x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6
*b**5*x**2 + 140*a**5*b**6*x**3) - 27*a**(34/3)/(140*a**8*b**3 + 420*a**7*b
**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 72*a**(31/3)*b*x*(1 + b*
x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**
5*b**6*x**3) - 81*a**(31/3)*b*x/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6
*b**5*x**2 + 140*a**5*b**6*x**3) + 60*a**(28/3)*b**2*x**2*(1 + b*x/a)**(1/3
)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**
3) - 81*a**(28/3)*b**2*x**2/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**
```

```
5*x**2 + 140*a**5*b**6*x**3) + 60*a**(25/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(1
40*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) -
 27*a**(25/3)*b**3*x**3/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x*
*2 + 140*a**5*b**6*x**3) + 135*a**(22/3)*b**4*x**4*(1 + b*x/a)**(1/3)/(140*
a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 13
2*a**(19/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x +
 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 42*a**(16/3)*b**6*x**6*(1 + b*x
/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5
*b**6*x**3)
```

Giac [A] time = 1.12951, size = 50, normalized size = 0.94

$$\frac{3\left(14\,(bx+a)^{\frac{10}{3}}-40\,(bx+a)^{\frac{7}{3}}a+35\,(bx+a)^{\frac{4}{3}}a^{2}\right)}{140\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/140*(14*(b*x + a)^(10/3) - 40*(b*x + a)^(7/3)*a + 35*(b*x + a)^(4/3)*a^2)
/b^3
```

3.373 $\int x\sqrt[3]{a+bx}\,dx$

Optimal. Leaf size=34

$$\frac{3(a+bx)^{7/3}}{7b^2}-\frac{3a(a+bx)^{4/3}}{4b^2}$$

```
[Out] (-3*a*(a + b*x)^(4/3))/(4*b^2) + (3*(a + b*x)^(7/3))/(7*b^2)
```

Rubi [A] time = 0.0082427, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{7/3}}{7b^2}-\frac{3a(a+bx)^{4/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x)^(4/3))/(4*b^2) + (3*(a + b*x)^(7/3))/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt[3]{a+bx}\,dx&=\int\left(-\frac{a\sqrt[3]{a+bx}}{b}+\frac{(a+bx)^{4/3}}{b}\right)dx\\&=-\frac{3a(a+bx)^{4/3}}{4b^2}+\frac{3(a+bx)^{7/3}}{7b^2}\end{aligned}$$

Mathematica [A] time = 0.0245567, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{4/3}(4bx-3a)}{28b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(-3*a + 4*b*x))/(28*b^2)
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$-\frac{-12\,bx+9\,a}{28\,b^2}(bx+a)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(1/3),x)
```

```
[Out] -3/28*(b*x+a)^(4/3)*(-4*b*x+3*a)/b^2
```

Maxima [A] time = 1.08618, size = 35, normalized size = 1.03

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b^2}-\frac{3\,(bx+a)^{\frac{4}{3}}a}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/7*(b*x + a)^(7/3)/b^2 - 3/4*(b*x + a)^(4/3)*a/b^2
```

Fricas [A] time = 1.70608, size = 73, normalized size = 2.15

$$\frac{3\left(4\,b^2x^2+abx-3\,a^2\right)(bx+a)^{\frac{1}{3}}}{28\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/28*(4*b^2*x^2 + a*b*x - 3*a^2)*(b*x + a)^(1/3)/b^2
```

Sympy [B] time = 1.63821, size = 202, normalized size = 5.94

$$-\frac{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}+\frac{9a^{\frac{13}{3}}}{28a^2b^2+28ab^3x}-\frac{6a^{\frac{10}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}+\frac{9a^{\frac{10}{3}}bx}{28a^2b^2+28ab^3x}+\frac{15a^{\frac{7}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}+\frac{12a^{\frac{4}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(1/3),x)
```

```
[Out] -9*a**(13/3)*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x) + 9*a**(13/3)/
(28*a**2*b**2 + 28*a*b**3*x) - 6*a**(10/3)*b*x*(1 + b*x/a)**(1/3)/(28*a**2*
b**2 + 28*a*b**3*x) + 9*a**(10/3)*b*x/(28*a**2*b**2 + 28*a*b**3*x) + 15*a**
(7/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x) + 12*a**(4/
3)*b**3*x**3*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x)
```

Giac [A] time = 1.22147, size = 34, normalized size = 1.

$$\frac{3\left(4\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a\right)}{28\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/28*(4*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a)/b^2
```

3.374 $\int \sqrt[3]{a+bx}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{4/3}}{4b}$$

```
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

Rubi [A] time = 0.001431, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3(a+bx)^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{a+bx}\,dx=\frac{3(a+bx)^{4/3}}{4b}$$

Mathematica [A] time = 0.0210137, size = 16, normalized size = 1.

$$\frac{3(a+bx)^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$\frac{3}{4\,b}\left(bx+a\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3),x)
```

```
[Out] 3/4*(b*x+a)^(4/3)/b
```

Maxima [A] time = 1.02454, size = 16, normalized size = 1.

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b
```

Fricas [A] time = 1.77857, size = 31, normalized size = 1.94

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b
```

Sympy [A] time = 0.074832, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{4}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3),x)
```

```
[Out] 3*(a + b*x)**(4/3)/(4*b)
```

Giac [A] time = 1.2059, size = 16, normalized size = 1.

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b
```

3.375 $\int \frac{\sqrt[3]{a+bx}}{x}\,dx$

Optimal. Leaf size=91

$$3\sqrt[3]{a+bx}+\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\log(x)$$

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(S
qrt[3]*a^(1/3))] - (a^(1/3)*Log[x])/2 + (3*a^(1/3)*Log[a^(1/3) - (a + b*x)^
(1/3)])/2
```

Rubi [A] time = 0.0470565, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 57, 617, 204, 31}

$$3\sqrt[3]{a+bx}+\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x,x]
```

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(S
qrt[3]*a^(1/3))] - (a^(1/3)*Log[x])/2 + (3*a^(1/3)*Log[a^(1/3) - (a + b*x)^
(1/3)])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = 3\sqrt[3]{a+bx}+a\int \frac{1}{x(a+bx)^{2/3}}\,dx$$
$$= 3\sqrt[3]{a+bx}-\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{1}{2}\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)-\frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x}\right.$$
$$= 3\sqrt[3]{a+bx}-\frac{1}{2}\sqrt[3]{a}\log(x)+\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)$$
$$= 3\sqrt[3]{a+bx}-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}\sqrt[3]{a}\log(x)+\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

Mathematica [A] time = 0.191845, size = 113, normalized size = 1.24

$$-\frac{1}{2}\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)+3\sqrt[3]{a+bx}+\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x,x]
```

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3)
)/Sqrt[3]] + a^(1/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(1/3)*Log[a^(2/3)
+ a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

Maple [A] time = 0.008, size = 85, normalized size = 0.9

$$3\sqrt[3]{bx+a}+\sqrt[3]{a}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)-\frac{1}{2}\sqrt[3]{a}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)-\sqrt[3]{a}\sqrt{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{bx+a}}{\sqrt[3]{a}}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/x,x)
```

```
[Out] 3*(b*x+a)^(1/3)+a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2*a^(1/3)*ln((b*x+a)^(2
/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-a^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^
(1/3)*(b*x+a)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89735, size = 286, normalized size = 3.14

$$-\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)-\frac{1}{2}a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) + sqrt(3)*a)
/a) - 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
+ a^(1/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)
```

Sympy [C] time = 2.78096, size = 180, normalized size = 1.98

$$\frac{4\sqrt[3]{a}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}}{\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x,x)
```

```
[Out] 4*a**(1/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(3*gamma(
7/3)) + 4*a**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pol
ar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*a**(1/3)*exp(2*I*pi/3)
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)
/(3*gamma(7/3)) + 4*b**(1/3)*(a/b + x)**(1/3)*gamma(4/3)/gamma(7/3)
```

Giac [A] time = 2.28773, size = 117, normalized size = 1.29

$$-\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{2}a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{1}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+3(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(1/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3*(b*x + a)^(1/3)
```

3.376 $\int \frac{\sqrt[3]{a+bx}}{x^2} \, dx$

Optimal. Leaf size=97

$$-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}}-\frac{\sqrt[3]{a+bx}}{x}$$

```
[Out] -((a + b*x)^(1/3)/x) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(2/3)) - (b*Log[x])/(6*a^(2/3)) + (b*Log[a^(1/3) - (a +
b*x)^(1/3)])/(2*a^(2/3))
```

Rubi [A] time = 0.0334369, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 57, 617, 204, 31}

$$-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}}-\frac{\sqrt[3]{a+bx}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x^2,x]
```

```
[Out] -((a + b*x)^(1/3)/x) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(2/3)) - (b*Log[x])/(6*a^(2/3)) + (b*Log[a^(1/3) - (a +
b*x)^(1/3)])/(2*a^(2/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx &= -\frac{\sqrt[3]{a+bx}}{x}+\frac{1}{3}b\int \frac{1}{x(a+bx)^{2/3}}\,dx\\
&= -\frac{\sqrt[3]{a+bx}}{x}-\frac{b\log(x)}{6a^{2/3}}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx,x,\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}\\
&= -\frac{\sqrt[3]{a+bx}}{x}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{2/3}}\\
&= -\frac{\sqrt[3]{a+bx}}{x}-\frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}
\end{aligned}
$$

Mathematica [C] time = 0.113986, size = 33, normalized size = 0.34

$$\frac{3b(a+bx)^{4/3}\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{bx}{a}+1\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x^2,x]
```

```
[Out] (3*b*(a + b*x)^(4/3)*Hypergeometric2F1[4/3, 2, 7/3, 1 + (b*x)/a])/(4*a^2)
```

Maple [A] time = 0.01, size = 92, normalized size = 1.

$$-\frac{1}{x}\sqrt[3]{bx+a}+\frac{b}{3}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)a^{-\frac{2}{3}}-\frac{b}{6}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)a^{-\frac{2}{3}}-\frac{b\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{bx+a}}{\sqrt[3]{a}}+1\right)\right)a^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/x^2,x)
```

```
[Out] -(b*x+a)^(1/3)/x+1/3*b/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/6*b/a^(2/3)*ln((
b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-1/3*b/a^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88343, size = 410, normalized size = 4.23

$$-\frac{2\sqrt{3}(a^2)^{\frac{1}{6}} abx \arctan\left(\frac{(a^2)^{\frac{1}{6}}\left(\sqrt{3}(a^2)^{\frac{1}{3}}a+2\sqrt{3}(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^2}\right) + (a^2)^{\frac{2}{3}} bx\log\left((bx+a)^{\frac{2}{3}}a + (a^2)^{\frac{1}{3}}a + (a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right) - 2(a^2)^{\frac{2}{3}}}{6a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(a^2)^(1/6)*a*b*x*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/
3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + (a^2)^(2/3)*b*x*log((b
*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 2*(a^2)^(2
/3)*b*x*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x + a)^(1/3)*a^2)/(a^2*
x)
```

Sympy [C] time = 3.25836, size = 643, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x**2,x)
```

```
[Out] 4*a**(7/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamm
a(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*
gamma(7/3)) + 4*a**(7/3)*b*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*
pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b
 + x)*exp(2*I*pi/3)*gamma(7/3)) + 4*a**(7/3)*b*exp(-2*I*pi/3)*log(1 - b**(1
/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2
*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(
4/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3
))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I
*pi/3)*gamma(7/3)) - 4*a**(4/3)*b**2*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**
(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(
7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(4/3)*b**2*(a/b
+ x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a
**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*e
xp(2*I*pi/3)*gamma(7/3)) + 12*a**2*b**(4/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*
gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi
/3)*gamma(7/3))
```

Giac [A] time = 2.20314, size = 142, normalized size = 1.46

$$-\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{6(bx+a)^{\frac{1}{3}}b}{x}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] -1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(2/3) + b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(2/3) - 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3) + 6*(b*x + a)^(1
/3)*b/x)/b
```

3.377 $\int \frac{\sqrt[3]{a+bx}}{x^3} dx$

Optimal. Leaf size=127

$$\frac{b^2\log(x)}{18a^{5/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}} + \frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}} - \frac{\sqrt[3]{a+bx}}{2x^2} - \frac{b\sqrt[3]{a+bx}}{6ax}$$

```
[Out] -(a + b*x)^(1/3)/(2*x^2) - (b*(a + b*x)^(1/3))/(6*a*x) + (b^2*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)) + (b^2*Log[
x])/(18*a^(5/3)) - (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(5/3))
```

Rubi [A] time = 0.0494358, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 51, 57, 617, 204, 31}

$$\frac{b^2\log(x)}{18a^{5/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}} + \frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}} - \frac{\sqrt[3]{a+bx}}{2x^2} - \frac{b\sqrt[3]{a+bx}}{6ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x^3,x]
```

```
[Out] -(a + b*x)^(1/3)/(2*x^2) - (b*(a + b*x)^(1/3))/(6*a*x) + (b^2*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)) + (b^2*Log[
x])/(18*a^(5/3)) - (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(5/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx &= -\frac{\sqrt[3]{a+bx}}{2x^2}+\frac{1}{6}b\int \frac{1}{x^2(a+bx)^{2/3}}\,dx\\ &= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}-\frac{b^2\int \frac{1}{x(a+bx)^{2/3}}\,dx}{9a}\\ &= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\log(x)}{18a^{5/3}}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{5/3}}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,d\right.}{6a^{4/3}}\\ &= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{3a^{5/3}}\\ &= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}\end{aligned}$$

Mathematica [C] time = 0.0191414, size = 35, normalized size = 0.28

$$-\frac{3b^2(a+bx)^{4/3}\,{}_2F_1\left(\frac{4}{3},3;\frac{7}{3};\frac{bx}{a}+1\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x^3,x]
```

```
[Out] (-3*b^2*(a + b*x)^(4/3)*Hypergeometric2F1[4/3, 3, 7/3, 1 + (b*x)/a])/(4*a^3
)
```

Maple [A] time = 0.01, size = 113, normalized size = 0.9

$$-\frac{1}{6\,ax^2}(bx+a)^{\frac{4}{3}}-\frac{1}{3\,x^2}\sqrt[3]{bx+a}-\frac{b^2}{9}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)a^{-\frac{5}{3}}+\frac{b^2}{18}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)a^{-\frac{5}{3}}+\frac{b^2\sqrt{3}}{9}\,\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/x^3,x)
```

```
[Out] -1/6/x^2/a*(b*x+a)^(4/3)-1/3*(b*x+a)^(1/3)/x^2-1/9*b^2/a^(5/3)*ln((b*x+a)^(
1/3)-a^(1/3))+1/18*b^2/a^(5/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/
3))+1/9*b^2/a^(5/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89489, size = 481, normalized size = 3.79

$$\frac{2\sqrt{3}ab^2x^2\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(-a^2\right)^{\frac{2}{3}}\right.}{18a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*a*b^2*x^2*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(
1/3)*a - 2*sqrt(3)*(-a^2)^(2/3)*(b*x + a)^(1/3))*sqrt(-(-a^2)^(1/3))/a^2) +
 (-a^2)^(2/3)*b^2*x^2*log((b*x + a)^(2/3)*a - (-a^2)^(1/3)*a + (-a^2)^(2/3)
*(b*x + a)^(1/3)) - 2*(-a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (-a^2)^(
2/3)) - 3*(a^2*b*x + 3*a^3)*(b*x + a)^(1/3))/(a^3*x^2)
```

Sympy [C] time = 3.92463, size = 2266, normalized size = 17.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x**3,x)
```

```
[Out] -4*a**(16/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))
*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I
*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27
*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*log(1
- b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a*
*7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)
+ 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b +
x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*exp(-2*I*pi/3)*log(1 - b
**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*
exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 8
```

```
1*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)*
*3*exp(2*I*pi/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)
*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(
a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi
/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(
7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x
)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gam
ma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b
 + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3
)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*
(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*p
i/3)*gamma(7/3)) - 12*a**(10/3)*b**4*(a/b + x)**2*exp(2*I*pi/3)*log(1 - b**
(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/
3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)*
*2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma
(7/3)) - 12*a**(10/3)*b**4*(a/b + x)**2*log(1 - b**(1/3)*(a/b + x)**(1/3)*e
xp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*e
xp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3
)) - 12*a**(10/3)*b**4*(a/b + x)**2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b +
x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*g
amma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/
b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3
)*gamma(7/3)) + 4*a**(7/3)*b**5*(a/b + x)**3*exp(2*I*pi/3)*log(1 - b**(1/3)
*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*ex
p(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)
) + 4*a**(7/3)*b**5*(a/b + x)**3*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**
6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*
pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) + 4*
a**(7/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)
*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3)
 - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2
*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7
/3)) - 12*a**5*b**(7/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(4/3)/(27*a**7*
exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 8
1*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)*
*3*exp(2*I*pi/3)*gamma(7/3)) + 6*a**4*b**(10/3)*(a/b + x)**(4/3)*exp(2*I*pi
/3)*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(
2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) -
 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) + 6*a**3*b**(13/3)*(a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*ex
p(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)
)
```

Giac [A] time = 1.98182, size = 173, normalized size = 1.36

$$\frac{\frac{2\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left((bx+a)^{\frac{4}{3}}b^3+2(bx+a)^{\frac{1}{3}}ab^3\right)}{ab^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] 1/18*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(5/3) + b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(5/3) - 2*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*((b*x + a)^(
4/3)*b^3 + 2*(b*x + a)^(1/3)*a*b^3)/(a*b^2*x^2))/b
```

3.378 $\int x^3(a+bx)^{2/3}\,dx$

Optimal. Leaf size=72

$$\frac{9a^2(a+bx)^{8/3}}{8b^4}-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{14/3}}{14b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}$$

```
[Out] (-3*a^3*(a + b*x)^(5/3))/(5*b^4) + (9*a^2*(a + b*x)^(8/3))/(8*b^4) - (9*a*(
a + b*x)^(11/3))/(11*b^4) + (3*(a + b*x)^(14/3))/(14*b^4)
```

Rubi [A] time = 0.018402, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{8/3}}{8b^4}-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{14/3}}{14b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(5/3))/(5*b^4) + (9*a^2*(a + b*x)^(8/3))/(8*b^4) - (9*a*(
a + b*x)^(11/3))/(11*b^4) + (3*(a + b*x)^(14/3))/(14*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{2/3}\,dx&=\int\left(-\frac{a^3(a+bx)^{2/3}}{b^3}+\frac{3a^2(a+bx)^{5/3}}{b^3}-\frac{3a(a+bx)^{8/3}}{b^3}+\frac{(a+bx)^{11/3}}{b^3}\right)dx\\&=-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{9a^2(a+bx)^{8/3}}{8b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}+\frac{3(a+bx)^{14/3}}{14b^4}\end{aligned}$$

Mathematica [A] time = 0.0534829, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{5/3}\left(135a^2bx-81a^3-180ab^2x^2+220b^3x^3\right)}{3080b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(-81*a^3 + 135*a^2*b*x - 180*a*b^2*x^2 + 220*b^3*x^3))/(
3080*b^4)
```

Maple [A] time = 0.005, size = 43, normalized size = 0.6

$$-\frac{-660\,b^{3}x^{3}+540\,ab^{2}x^{2}-405\,a^{2}bx+243\,a^{3}}{3080\,b^{4}}(bx+a)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(2/3),x)
```

```
[Out] -3/3080*(b*x+a)^(5/3)*(-220*b^3*x^3+180*a*b^2*x^2-135*a^2*b*x+81*a^3)/b^4
```

Maxima [A] time = 1.09555, size = 76, normalized size = 1.06

$$\frac{3\,(bx+a)^{\frac{14}{3}}}{14\,b^{4}}-\frac{9\,(bx+a)^{\frac{11}{3}}a}{11\,b^{4}}+\frac{9\,(bx+a)^{\frac{8}{3}}a^{2}}{8\,b^{4}}-\frac{3\,(bx+a)^{\frac{5}{3}}a^{3}}{5\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x + a)^(14/3)/b^4 - 9/11*(b*x + a)^(11/3)*a/b^4 + 9/8*(b*x + a)^(8/
3)*a^2/b^4 - 3/5*(b*x + a)^(5/3)*a^3/b^4
```

Fricas [A] time = 1.85305, size = 130, normalized size = 1.81

$$\frac{3\left(220\,b^{4}x^{4}+40\,ab^{3}x^{3}-45\,a^{2}b^{2}x^{2}+54\,a^{3}bx-81\,a^{4}\right)(bx+a)^{\frac{2}{3}}}{3080\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/3080*(220*b^4*x^4 + 40*a*b^3*x^3 - 45*a^2*b^2*x^2 + 54*a^3*b*x - 81*a^4)*
(b*x + a)^(2/3)/b^4
```

Sympy [B] time = 3.7386, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(2/3),x)
```

```
[Out] -243*a**(74/3)*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 4
6200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 1848
0*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 243*a**(74/3)/(3080*a**20*b**4
 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 462
00*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) - 1296*
a**(71/3)*b*x*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46
200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480
*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 1458*a**(71/3)*b*x/(3080*a**20*
```

```
b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 +
 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) - 2
808*a**(68/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b
**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x*
*4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 3645*a**(68/3)*b**2*x
**2/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a
**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14
*b**10*x**6) - 3120*a**(65/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(3080*a**20*b**4
 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 462
00*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4860*
a**(65/3)*b**3*x**3/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**
6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x
**5 + 3080*a**14*b**10*x**6) - 1050*a**(62/3)*b**4*x**4*(1 + b*x/a)**(2/3)/
(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17
*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**
10*x**6) + 3645*a**(62/3)*b**4*x**4/(3080*a**20*b**4 + 18480*a**19*b**5*x +
 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18
480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4032*a**(59/3)*b**5*x**5*(1
+ b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x*
*2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5
+ 3080*a**14*b**10*x**6) + 1458*a**(59/3)*b**5*x**5/(3080*a**20*b**4 + 1848
0*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**1
6*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 11004*a**(56
/3)*b**6*x**6*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46
200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480
*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 243*a**(56/3)*b**6*x**6/(3080*a
**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x
**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6
) + 14352*a**(53/3)*b**7*x**7*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a
**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b
**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 10485*a**(50/3)
*b**8*x**8*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200
*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a*
*15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4080*a**(47/3)*b**9*x**9*(1 + b*x/
a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 6
1600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080
*a**14*b**10*x**6) + 660*a**(44/3)*b**10*x**10*(1 + b*x/a)**(2/3)/(3080*a**
20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**
3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6)
```

Giac [A] time = 1.23861, size = 66, normalized size = 0.92

$$\frac{3\left(220\,(bx+a)^{\frac{14}{3}}-840\,(bx+a)^{\frac{11}{3}}a+1155\,(bx+a)^{\frac{8}{3}}a^2-616\,(bx+a)^{\frac{5}{3}}a^3\right)}{3080\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/3080*(220*(b*x + a)^(14/3) - 840*(b*x + a)^(11/3)*a + 1155*(b*x + a)^(8/3
)*a^2 - 616*(b*x + a)^(5/3)*a^3)/b^4
```

3.379 $\int x^2(a+bx)^{2/3}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{11/3}}{11b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}$$

```
[Out] (3*a^2*(a + b*x)^(5/3))/(5*b^3) - (3*a*(a + b*x)^(8/3))/(4*b^3) + (3*(a + b
*x)^(11/3))/(11*b^3)
```

Rubi [A] time = 0.0126484, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{11/3}}{11b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(5/3))/(5*b^3) - (3*a*(a + b*x)^(8/3))/(4*b^3) + (3*(a + b
*x)^(11/3))/(11*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{2/3}\,dx &= \int\left(\frac{a^2(a+bx)^{2/3}}{b^2}-\frac{2a(a+bx)^{5/3}}{b^2}+\frac{(a+bx)^{8/3}}{b^2}\right)dx\\ &= \frac{3a^2(a+bx)^{5/3}}{5b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}+\frac{3(a+bx)^{11/3}}{11b^3}\end{aligned}$$

Mathematica [A] time = 0.0376232, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{5/3}\left(9a^2-15abx+20b^2x^2\right)}{220b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(9*a^2 - 15*a*b*x + 20*b^2*x^2))/(220*b^3)
```

Maple [A] time = 0.005, size = 32, normalized size = 0.6

$$\frac{60\,b^{2}x^{2}-45\,abx+27\,a^{2}}{220\,b^{3}}\left(bx+a\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(2/3),x)
```

```
[Out] 3/220*(b*x+a)^(5/3)*(20*b^2*x^2-15*a*b*x+9*a^2)/b^3
```

Maxima [A] time = 1.07781, size = 55, normalized size = 1.04

$$\frac{3\left(bx+a\right)^{\frac{11}{3}}}{11\,b^{3}}-\frac{3\left(bx+a\right)^{\frac{8}{3}}a}{4\,b^{3}}+\frac{3\left(bx+a\right)^{\frac{5}{3}}a^{2}}{5\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x + a)^(11/3)/b^3 - 3/4*(b*x + a)^(8/3)*a/b^3 + 3/5*(b*x + a)^(5/3)
*a^2/b^3
```

Fricas [A] time = 1.74525, size = 100, normalized size = 1.89

$$\frac{3\left(20\,b^{3}x^{3}+5\,ab^{2}x^{2}-6\,a^{2}bx+9\,a^{3}\right)\left(bx+a\right)^{\frac{2}{3}}}{220\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/220*(20*b^3*x^3 + 5*a*b^2*x^2 - 6*a^2*b*x + 9*a^3)*(b*x + a)^(2/3)/b^3
```

Sympy [B] time = 2.65505, size = 666, normalized size = 12.57

$$\frac{27a^{\frac{35}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^{8}b^{3}+660a^{7}b^{4}x+660a^{6}b^{5}x^{2}+220a^{5}b^{6}x^{3}}-\frac{27a^{\frac{35}{3}}}{220a^{8}b^{3}+660a^{7}b^{4}x+660a^{6}b^{5}x^{2}+220a^{5}b^{6}x^{3}}+\frac{63a}{220a^{8}b^{3}+660a^{7}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(2/3),x)
```

```
[Out] 27*a**(35/3)*(1 + b*x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6
*b**5*x**2 + 220*a**5*b**6*x**3) - 27*a**(35/3)/(220*a**8*b**3 + 660*a**7*b
**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 63*a**(32/3)*b*x*(1 + b*
x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**
5*b**6*x**3) - 81*a**(32/3)*b*x/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6
*b**5*x**2 + 220*a**5*b**6*x**3) + 42*a**(29/3)*b**2*x**2*(1 + b*x/a)**(2/3
)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**
```

```
3) - 81*a**(29/3)*b**2*x**2/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**
5*x**2 + 220*a**5*b**6*x**3) + 78*a**(26/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(2
20*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) -
 27*a**(26/3)*b**3*x**3/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x*
*2 + 220*a**5*b**6*x**3) + 207*a**(23/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(220*
a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 19
5*a**(20/3)*b**5*x**5*(1 + b*x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x +
 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 60*a**(17/3)*b**6*x**6*(1 + b*x
/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5
*b**6*x**3)
```

Giac [A] time = 1.20521, size = 50, normalized size = 0.94

$$\frac{3\left(20\,(bx+a)^{\frac{11}{3}}-55\,(bx+a)^{\frac{8}{3}}a+44\,(bx+a)^{\frac{5}{3}}a^{2}\right)}{220\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/220*(20*(b*x + a)^(11/3) - 55*(b*x + a)^(8/3)*a + 44*(b*x + a)^(5/3)*a^2)
/b^3
```

3.380 $\int x(a+bx)^{2/3}\,dx$

Optimal. Leaf size=34

$$\frac{3(a+bx)^{8/3}}{8b^2}-\frac{3a(a+bx)^{5/3}}{5b^2}$$

```
[Out] (-3*a*(a + b*x)^(5/3))/(5*b^2) + (3*(a + b*x)^(8/3))/(8*b^2)
```

Rubi [A] time = 0.0078079, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{8/3}}{8b^2}-\frac{3a(a+bx)^{5/3}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x)^(5/3))/(5*b^2) + (3*(a + b*x)^(8/3))/(8*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{2/3}\,dx &= \int\left(-\frac{a(a+bx)^{2/3}}{b}+\frac{(a+bx)^{5/3}}{b}\right)dx\\ &= -\frac{3a(a+bx)^{5/3}}{5b^2}+\frac{3(a+bx)^{8/3}}{8b^2}\end{aligned}$$

Mathematica [A] time = 0.023607, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{5/3}(5bx-3a)}{40b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(-3*a + 5*b*x))/(40*b^2)
```

Maple [A] time = 0.001, size = 21, normalized size = 0.6

$$-\frac{-15\,bx+9\,a}{40\,b^2}(bx+a)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(2/3),x)
```

```
[Out] -3/40*(b*x+a)^(5/3)*(-5*b*x+3*a)/b^2
```

Maxima [A] time = 1.06682, size = 35, normalized size = 1.03

$$\frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^2}-\frac{3\,(bx+a)^{\frac{5}{3}}a}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)^(8/3)/b^2 - 3/5*(b*x + a)^(5/3)*a/b^2
```

Fricas [A] time = 1.83083, size = 76, normalized size = 2.24

$$\frac{3\left(5\,b^2x^2+2\,abx-3\,a^2\right)(bx+a)^{\frac{2}{3}}}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^2*x^2 + 2*a*b*x - 3*a^2)*(b*x + a)^(2/3)/b^2
```

Sympy [B] time = 1.5898, size = 202, normalized size = 5.94

$$-\frac{9a^{\frac{14}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}+\frac{9a^{\frac{14}{3}}}{40a^2b^2+40ab^3x}-\frac{3a^{\frac{11}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}+\frac{9a^{\frac{11}{3}}bx}{40a^2b^2+40ab^3x}+\frac{21a^{\frac{8}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}+\frac{15a^{\frac{5}{3}}b^3x^3\left(1+\frac{b\cdots}{\cdots}\right.}{40a^2b^2+40ab\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(2/3),x)
```

```
[Out] -9*a**(14/3)*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x) + 9*a**(14/3)/
(40*a**2*b**2 + 40*a*b**3*x) - 3*a**(11/3)*b*x*(1 + b*x/a)**(2/3)/(40*a**2*
b**2 + 40*a*b**3*x) + 9*a**(11/3)*b*x/(40*a**2*b**2 + 40*a*b**3*x) + 21*a**
(8/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x) + 15*a**(5/
3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x)
```

Giac [A] time = 1.19636, size = 34, normalized size = 1.

$$\frac{3\left(5\,(bx+a)^{\frac{8}{3}}-8\,(bx+a)^{\frac{5}{3}}a\right)}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/40*(5*(b*x + a)^(8/3) - 8*(b*x + a)^(5/3)*a)/b^2
```

3.381 $\int (a+bx)^{2/3}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{5/3}}{5b}$$

```
[Out] (3*(a + b*x)^(5/3))/(5*b)
```

Rubi [A] time = 0.0015048, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3(a+bx)^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3),x]

[Out] (3*(a + b*x)^(5/3))/(5*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{2/3}\,dx = \frac{3(a+bx)^{5/3}}{5b}$$

Mathematica [A] time = 0.0041166, size = 16, normalized size = 1.

$$\frac{3(a+bx)^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3),x]

[Out] (3*(a + b*x)^(5/3))/(5*b)
```

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$\frac{3}{5\,b}(bx+a)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3),x)
```

```
[Out] 3/5*(b*x+a)^(5/3)/b
```

Maxima [A] time = 1.1062, size = 16, normalized size = 1.

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b
```

Fricas [A] time = 1.83823, size = 31, normalized size = 1.94

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b
```

Sympy [A] time = 0.073232, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{5}{3}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3),x)
```

```
[Out] 3*(a + b*x)**(5/3)/(5*b)
```

Giac [A] time = 1.14559, size = 16, normalized size = 1.

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b
```

3.382 $\int \frac{(a+bx)^{2/3}}{x} \, dx$

Optimal. Leaf size=92

$$\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}(a+bx)^{2/3}$$

```
[Out] (3*(a + b*x)^(2/3))/2 + Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3)
)/(Sqrt[3]*a^(1/3))] - (a^(2/3)*Log[x])/2 + (3*a^(2/3)*Log[a^(1/3) - (a + b
*x)^(1/3)])/2
```

Rubi [A] time = 0.0320364, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 55, 617, 204, 31}

$$\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}(a+bx)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x)^(2/3))/2 + Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3)
)/(Sqrt[3]*a^(1/3))] - (a^(2/3)*Log[x])/2 + (3*a^(2/3)*Log[a^(1/3) - (a + b
*x)^(1/3)])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{2/3}}{x}\,dx &= \frac{3}{2}(a+bx)^{2/3}+a\int \frac{1}{x\sqrt[3]{a+bx}}\,dx\\ &= \frac{3}{2}(a+bx)^{2/3}-\frac{1}{2}a^{2/3}\log(x)-\frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)+\frac{1}{2}(3a)\,\text{Subst}\left(\int \frac{}{a^{2/3}}\right.\\ &= \frac{3}{2}(a+bx)^{2/3}-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{}}{}\right.\\ &= \frac{3}{2}(a+bx)^{2/3}+\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)\end{aligned}$$

Mathematica [A] time = 0.0761639, size = 86, normalized size = 0.93

$$\frac{3}{2}\left(a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+(a+bx)^{2/3}\right)+\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-\frac{1}{2}a^{2/3}\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x,x]
```

```
[Out] Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - (a^(2/3
)*Log[x])/2 + (3*((a + b*x)^(2/3) + a^(2/3)*Log[a^(1/3) - (a + b*x)^(1/3)])
)/2
```

Maple [A] time = 0.005, size = 84, normalized size = 0.9

$$\frac{3}{2}(bx+a)^{\frac{2}{3}}+a^{\frac{2}{3}}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)-\frac{1}{2}a^{\frac{2}{3}}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)+a^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{bx+a}}{\sqrt[3]{a}}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x,x)
```

```
[Out] 3/2*(b*x+a)^(2/3)+a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2*a^(2/3)*ln((b*x+a)^
(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
a^(1/3)*(b*x+a)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9108, size = 333, normalized size = 3.62

$$\sqrt{3}\left(a^{2}\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}a + 2\sqrt{3}\left(a^{2}\right)^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}}{3\,a}\right) - \frac{1}{2}\left(a^{2}\right)^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}a + \left(a^{2}\right)^{\frac{1}{3}}a + \left(a^{2}\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right) + \left(a^{2}\right)^{\frac{1}{3}}\log\left((bx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(a^2)^(1/3)*(b*x + a)
^(1/3))/a) - 1/2*(a^2)^(1/3)*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^
(2/3)*(b*x + a)^(1/3)) + (a^2)^(1/3)*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) +
 3/2*(b*x + a)^(2/3)
```

Sympy [C] time = 2.77062, size = 182, normalized size = 1.98

$$\frac{5a^{\frac{2}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{5a^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{5a^{\frac{2}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{5b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}}{2\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x,x)
```

```
[Out] 5*a**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(3*gamma(
8/3)) + 5*a**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(5/3)/(3*gamma(8/3)) + 5*a**(2/3)*exp(-2*I*pi/3)
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)
/(3*gamma(8/3)) + 5*b**(2/3)*(a/b + x)**(2/3)*gamma(5/3)/(2*gamma(8/3))
```

Giac [A] time = 1.73107, size = 116, normalized size = 1.26

$$\sqrt{3}a^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{2}a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}} + (bx+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right) + a^{\frac{2}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right) + \frac{3}{2}(bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="giac")
```

```
[Out] sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) -
 1/2*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^(
2/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3/2*(b*x + a)^(2/3)
```

3.383 $\int \frac{(a+bx)^{2/3}}{x^2} \, dx$

Optimal. Leaf size=94

$$-\frac{(a+bx)^{2/3}}{x}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

```
[Out] -((a + b*x)^(2/3)/x) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a
^(1/3))])/(Sqrt[3]*a^(1/3)) - (b*Log[x])/(3*a^(1/3)) + (b*Log[a^(1/3) - (a
+ b*x)^(1/3)])/a^(1/3)
```

Rubi [A] time = 0.0332543, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 55, 617, 204, 31}

$$-\frac{(a+bx)^{2/3}}{x}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x^2,x]
```

```
[Out] -((a + b*x)^(2/3)/x) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a
^(1/3))])/(Sqrt[3]*a^(1/3)) - (b*Log[x])/(3*a^(1/3)) + (b*Log[a^(1/3) - (a
+ b*x)^(1/3)])/a^(1/3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{x^2}\,dx = -\frac{(a+bx)^{2/3}}{x}+\frac{1}{3}(2b)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$$
$$= -\frac{(a+bx)^{2/3}}{x}-\frac{b\log(x)}{3\sqrt[3]{a}}+b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx,x,\sqrt[3]{a+bx}\right)-\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}$$
$$= -\frac{(a+bx)^{2/3}}{x}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$
$$= -\frac{(a+bx)^{2/3}}{x}+\frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}$$

Mathematica [C] time = 0.0140435, size = 33, normalized size = 0.35

$$\frac{3b(a+bx)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};\frac{bx}{a}+1\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x^2,x]
```

```
[Out] (3*b*(a + b*x)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + (b*x)/a])/(5*a^2)
```

Maple [A] time = 0.01, size = 92, normalized size = 1.

$$-\frac{1}{x}(bx+a)^{\frac{2}{3}}+\frac{2b}{3}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)\frac{1}{\sqrt[3]{a}}-\frac{b}{3}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{a}}+\frac{2b\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{bx+a}}{\sqrt[3]{a}}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x^2,x)
```

```
[Out] -(b*x+a)^(2/3)/x+2/3*b/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/3*b/a^(1/3)*ln((
b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+2/3*b*3^(1/2)/a^(1/3)*arctan(1/
3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91553, size = 760, normalized size = 8.09

$$\left[\frac{3\sqrt{\frac{1}{3}}abx\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x}\right)-a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3ax}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(1/3)*a*b*x*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*(b*x +
 a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x
+ a)^(1/3)*a^(2/3) + 3*a)/x) - a^(2/3)*b*x*log((b*x + a)^(2/3) + (b*x + a)^
(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x*log((b*x + a)^(1/3) - a^(1/3)) - 3
*(b*x + a)^(2/3)*a)/(a*x), 1/3*(6*sqrt(1/3)*a^(2/3)*b*x*arctan(sqrt(1/3)*(2
*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3)*b*x*log((b*x + a)^(2/3) + (b
*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x*log((b*x + a)^(1/3) - a^(1
/3)) - 3*(b*x + a)^(2/3)*a)/(a*x)]
```

Sympy [C] time = 3.17599, size = 643, normalized size = 6.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x**2,x)
```

```
[Out] 10*a**(8/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gam
ma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)
*gamma(8/3)) + 10*a**(8/3)*b*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/
3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3
) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*ex
p(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) - 10*
a**(5/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**
(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp
(2*I*pi/3)*gamma(8/3)) - 10*a**(5/3)*b**2*(a/b + x)*exp(-2*I*pi/3)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*
exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) - 1
0*a**(5/3)*b**2*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(8/3)) + 15*a**2*b**(5/3)*(a/b + x)**(2/3)*exp(2*I*
pi/3)*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(
```

```
2*I*pi/3)*gamma(8/3))
```

Giac [A] time = 2.16197, size = 143, normalized size = 1.52

$$\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{2b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}-\frac{3(bx+a)^{\frac{2}{3}}b}{x}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(1/3) - b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(1/3) + 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(1/3) - 3*(b*x + a)^(2/
3)*b/x)/b
```

3.384 $\int \frac{(a+bx)^{2/3}}{x^3} \, dx$

Optimal. Leaf size=127

$$\frac{b^2 \log(x)}{18a^{4/3}} - \frac{b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{6a^{4/3}} - \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}} - \frac{(a+bx)^{2/3}}{2x^2} - \frac{b(a+bx)^{2/3}}{3ax}$$

```
[Out] -(a + b*x)^(2/3)/(2*x^2) - (b*(a + b*x)^(2/3))/(3*a*x) - (b^2*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)) + (b^2*Log[
x])/(18*a^(4/3)) - (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(4/3))
```

Rubi [A] time = 0.0470729, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 51, 55, 617, 204, 31}

$$\frac{b^2 \log(x)}{18a^{4/3}} - \frac{b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{6a^{4/3}} - \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}} - \frac{(a+bx)^{2/3}}{2x^2} - \frac{b(a+bx)^{2/3}}{3ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x^3,x]
```

```
[Out] -(a + b*x)^(2/3)/(2*x^2) - (b*(a + b*x)^(2/3))/(3*a*x) - (b^2*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)) + (b^2*Log[
x])/(18*a^(4/3)) - (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(4/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{2/3}}{x^3}\,dx &= -\frac{(a+bx)^{2/3}}{2x^2}+\frac{1}{3}b\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx\\
&= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}-\frac{b^2\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a}\\
&= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}+\frac{b^2\log(x)}{18a^{4/3}}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{4/3}}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x}\right.}{6a}\\
&= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+}}{\sqrt[3]{a}}\right.}{3a^{4/3}}\\
&= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}-\frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{4/3}}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}
\end{aligned}
$$

Mathematica [C] time = 0.0135362, size = 35, normalized size = 0.28

$$
-\frac{3b^2(a+bx)^{5/3}\,{}_2F_1\left(\frac{5}{3},3;\frac{8}{3};\frac{bx}{a}+1\right)}{5a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x^3,x]
```

```
[Out] (-3*b^2*(a + b*x)^(5/3)*Hypergeometric2F1[5/3, 3, 8/3, 1 + (b*x)/a])/(5*a^3
)
```

Maple [A] time = 0.011, size = 113, normalized size = 0.9

$$
-\frac{1}{3\,ax^2}(bx+a)^{\frac{5}{3}}-\frac{1}{6\,x^2}(bx+a)^{\frac{2}{3}}-\frac{b^2}{9}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)a^{-\frac{4}{3}}+\frac{b^2}{18}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)a^{-\frac{4}{3}}-\frac{b^2\sqrt{3}}{9}\text{arcta}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x^3,x)
```

```
[Out] -1/3/x^2/a*(b*x+a)^(5/3)-1/6*(b*x+a)^(2/3)/x^2-1/9*b^2/a^(4/3)*ln((b*x+a)^(
1/3)-a^(1/3))+1/18*b^2/a^(4/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/
3))-1/9*b^2/a^(4/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58869, size = 950, normalized size = 7.48

$$\left[\frac{3\sqrt{\frac{1}{3}}ab^2x^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx-3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x}\right)+(-a)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}-(\cdots\right.}{18a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*a*b^2*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*sqrt(1/3)*(2
*(b*x + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(
1/3)/a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + (-a)^(2/3)*b^2*x^2*log((
b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b^
2*x^2*log((b*x + a)^(1/3) + (-a)^(1/3)) - 3*(2*a*b*x + 3*a^2)*(b*x + a)^(2/
3))/(a^2*x^2), -1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sqrt
(1/3)*(2*(b*x + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b^
2*x^2*log((b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-
a)^(2/3)*b^2*x^2*log((b*x + a)^(1/3) + (-a)^(1/3)) + 3*(2*a*b*x + 3*a^2)*(b
*x + a)^(2/3))/(a^2*x^2)]
```

Sympy [C] time = 3.93751, size = 2266, normalized size = 17.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x**3,x)
```

```
[Out] -10*a**(17/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3)
)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) -
 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 10*a**(17/3)*b**2*ex
p(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3)
)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) -
 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 10*a**(17/3)*b**2*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(
a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*exp(2*I
*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(
2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*
a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3
*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(54
*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8
/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a
/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a**7
*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) +
 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b +
x)**3*exp(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*exp(2*I*pi
/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(2*I
*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**
5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*ex
p(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(54
*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8
/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a
/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*log(1
 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a
**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3
) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b
 + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*exp(2*I*
pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(2
*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a
**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*
exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(
a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*log(1
 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a
**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3
) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b
 + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 15*a**5*b**(8/3)*(a/b + x)**(2/3)*exp(
2*I*pi/3)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b +
x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamm
a(8/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 15*a**4*b**(
11/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamm
a(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b
 + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)
*gamma(8/3)) + 30*a**3*b**(14/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(5/3)/
(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamm
a(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3
*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3))
```

Giac [A] time = 1.79425, size = 174, normalized size = 1.37

$$-\frac{\frac{2\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3\left(2(bx+a)^{\frac{5}{3}}b^3+(bx+a)^{\frac{2}{3}}ab^3\right)}{ab^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/18*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(4/3) - b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(4/3) + 2*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 3*(2*(b*x + a
)^(5/3)*b^3 + (b*x + a)^(2/3)*a*b^3)/(a*b^2*x^2))/b
```

3.385 $\int x^3(a+bx)^{4/3}\,dx$

Optimal. Leaf size=72

$$\frac{9a^2(a+bx)^{10/3}}{10b^4}-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{16/3}}{16b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}$$

```
[Out] (-3*a^3*(a + b*x)^(7/3))/(7*b^4) + (9*a^2*(a + b*x)^(10/3))/(10*b^4) - (9*a
*(a + b*x)^(13/3))/(13*b^4) + (3*(a + b*x)^(16/3))/(16*b^4)
```

Rubi [A] time = 0.0179075, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{10/3}}{10b^4}-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{16/3}}{16b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(4/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(7/3))/(7*b^4) + (9*a^2*(a + b*x)^(10/3))/(10*b^4) - (9*a
*(a + b*x)^(13/3))/(13*b^4) + (3*(a + b*x)^(16/3))/(16*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{4/3}\,dx &= \int \left(-\frac{a^3(a+bx)^{4/3}}{b^3}+\frac{3a^2(a+bx)^{7/3}}{b^3}-\frac{3a(a+bx)^{10/3}}{b^3}+\frac{(a+bx)^{13/3}}{b^3}\right)dx\\ &= -\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{9a^2(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}+\frac{3(a+bx)^{16/3}}{16b^4}\end{aligned}$$

Mathematica [A] time = 0.0758572, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{7/3}\left(189a^2bx-81a^3-315ab^2x^2+455b^3x^3\right)}{7280b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(-81*a^3 + 189*a^2*b*x - 315*a*b^2*x^2 + 455*b^3*x^3))/(
7280*b^4)
```

Maple [A] time = 0.004, size = 43, normalized size = 0.6

$$-\frac{-1365\,b^3x^3+945\,ab^2x^2-567\,a^2bx+243\,a^3}{7280\,b^4}(bx+a)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(4/3),x)
```

```
[Out] -3/7280*(b*x+a)^(7/3)*(-455*b^3*x^3+315*a*b^2*x^2-189*a^2*b*x+81*a^3)/b^4
```

Maxima [A] time = 1.04098, size = 76, normalized size = 1.06

$$\frac{3\,(bx+a)^{\frac{16}{3}}}{16\,b^4}-\frac{9\,(bx+a)^{\frac{13}{3}}a}{13\,b^4}+\frac{9\,(bx+a)^{\frac{10}{3}}a^2}{10\,b^4}-\frac{3\,(bx+a)^{\frac{7}{3}}a^3}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/16*(b*x + a)^(16/3)/b^4 - 9/13*(b*x + a)^(13/3)*a/b^4 + 9/10*(b*x + a)^(1
0/3)*a^2/b^4 - 3/7*(b*x + a)^(7/3)*a^3/b^4
```

Fricas [A] time = 1.47519, size = 154, normalized size = 2.14

$$\frac{3\left(455\,b^5x^5+595\,ab^4x^4+14\,a^2b^3x^3-18\,a^3b^2x^2+27\,a^4bx-81\,a^5\right)(bx+a)^{\frac{1}{3}}}{7280\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/7280*(455*b^5*x^5 + 595*a*b^4*x^4 + 14*a^2*b^3*x^3 - 18*a^3*b^2*x^2 + 27*
a^4*b*x - 81*a^5)*(b*x + a)^(1/3)/b^4
```

Sympy [B] time = 4.66662, size = 1844, normalized size = 25.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(4/3),x)
```

```
[Out] -243*a**(76/3)*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 1
09200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 4
3680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 243*a**(76/3)/(7280*a**20*b
**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3
+ 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) -
 1377*a**(73/3)*b*x*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*
x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**
4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1458*a**(73/3)*b*x/(72
```

```
80*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*
b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**
10*x**6) - 3213*a**(70/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 4
3680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 10920
0*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 3645*a
**(70/3)*b**2*x**2/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**
6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9
*x**5 + 7280*a**14*b**10*x**6) - 3927*a**(67/3)*b**3*x**3*(1 + b*x/a)**(1/3
)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a
**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**1
4*b**10*x**6) + 4860*a**(67/3)*b**3*x**3/(7280*a**20*b**4 + 43680*a**19*b**
5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x
**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) - 798*a**(64/3)*b**4*x
**4*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18
*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*
b**9*x**5 + 7280*a**14*b**10*x**6) + 3645*a**(64/3)*b**4*x**4/(7280*a**20*b
**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3
+ 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) +
 11382*a**(61/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**1
9*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b
**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1458*a**(61/3)*
b**5*x**5/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 +
145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7
280*a**14*b**10*x**6) + 35238*a**(58/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(7280*
a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**
7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*
x**6) + 243*a**(58/3)*b**6*x**6/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109
200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 436
80*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 56562*a**(55/3)*b**7*x**7*(1
+ b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x
**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x*
*5 + 7280*a**14*b**10*x**6) + 54273*a**(52/3)*b**8*x**8*(1 + b*x/a)**(1/3)/
(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**
17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*
b**10*x**6) + 31227*a**(49/3)*b**9*x**9*(1 + b*x/a)**(1/3)/(7280*a**20*b**4
 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 1
09200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 99
75*a**(46/3)*b**10*x**10*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*
b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**
8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1365*a**(43/3)*b*
*11*x**11*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200
*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*
a**15*b**9*x**5 + 7280*a**14*b**10*x**6)
```

Giac [B] time = 1.18533, size = 157, normalized size = 2.18

$$\frac{3\left(\frac{4\left(140\,(bx+a)^{\frac{13}{3}}-546\,(bx+a)^{\frac{10}{3}}a+780\,(bx+a)^{\frac{7}{3}}a^2-455\,(bx+a)^{\frac{4}{3}}a^3\right)a}{b^3}+\frac{455\,(bx+a)^{\frac{16}{3}}-2240\,(bx+a)^{\frac{13}{3}}a+4368\,(bx+a)^{\frac{10}{3}}a^2-4160\,(bx+a)^{\frac{7}{3}}a^3+1820\,(bx+a)^{\frac{4}{3}}a^4}{b^3}\right.}{7280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/7280*(4*(140*(b*x + a)^(13/3) - 546*(b*x + a)^(10/3)*a + 780*(b*x + a)^(7
/3)*a^2 - 455*(b*x + a)^(4/3)*a^3)*a/b^3 + (455*(b*x + a)^(16/3) - 2240*(b*
```

```
x + a)^(13/3)*a + 4368*(b*x + a)^(10/3)*a^2 - 4160*(b*x + a)^(7/3)*a^3 + 18
20*(b*x + a)^(4/3)*a^4)/b^3)/b
```

3.386 $\int x^2(a+bx)^{4/3}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{13/3}}{13b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}$$

```
[Out] (3*a^2*(a + b*x)^(7/3))/(7*b^3) - (3*a*(a + b*x)^(10/3))/(5*b^3) + (3*(a +
b*x)^(13/3))/(13*b^3)
```

Rubi [A] time = 0.0128579, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{13/3}}{13b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(4/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(7/3))/(7*b^3) - (3*a*(a + b*x)^(10/3))/(5*b^3) + (3*(a +
b*x)^(13/3))/(13*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{4/3}\,dx&=\int\left(\frac{a^2(a+bx)^{4/3}}{b^2}-\frac{2a(a+bx)^{7/3}}{b^2}+\frac{(a+bx)^{10/3}}{b^2}\right)dx\\&=\frac{3a^2(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}+\frac{3(a+bx)^{13/3}}{13b^3}\end{aligned}$$

Mathematica [A] time = 0.0457376, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{7/3}\left(9a^2-21abx+35b^2x^2\right)}{455b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(9*a^2 - 21*a*b*x + 35*b^2*x^2))/(455*b^3)
```

Maple [A] time = 0.004, size = 32, normalized size = 0.6

$$\frac{105\,b^2x^2-63\,abx+27\,a^2}{455\,b^3}\,(bx+a)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(4/3),x)
```

```
[Out] 3/455*(b*x+a)^(7/3)*(35*b^2*x^2-21*a*b*x+9*a^2)/b^3
```

Maxima [A] time = 1.01368, size = 55, normalized size = 1.04

$$\frac{3\,(bx+a)^{\frac{13}{3}}}{13\,b^3}-\frac{3\,(bx+a)^{\frac{10}{3}}a}{5\,b^3}+\frac{3\,(bx+a)^{\frac{7}{3}}a^2}{7\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/13*(b*x + a)^(13/3)/b^3 - 3/5*(b*x + a)^(10/3)*a/b^3 + 3/7*(b*x + a)^(7/3
)*a^2/b^3
```

Fricas [A] time = 1.47854, size = 123, normalized size = 2.32

$$\frac{3\left(35\,b^4x^4+49\,ab^3x^3+2\,a^2b^2x^2-3\,a^3bx+9\,a^4\right)(bx+a)^{\frac{1}{3}}}{455\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/455*(35*b^4*x^4 + 49*a*b^3*x^3 + 2*a^2*b^2*x^2 - 3*a^3*b*x + 9*a^4)*(b*x
+ a)^(1/3)/b^3
```

Sympy [B] time = 2.97957, size = 733, normalized size = 13.83

$$\frac{27a^{\frac{37}{3}}\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}-\frac{27a^{\frac{37}{3}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{}{455a^8b^3+13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(4/3),x)
```

```
[Out] 27*a**(37/3)*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a*
*6*b**5*x**2 + 455*a**5*b**6*x**3) - 27*a**(37/3)/(455*a**8*b**3 + 1365*a**
7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 72*a**(34/3)*b*x*(1
+ b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 4
55*a**5*b**6*x**3) - 81*a**(34/3)*b*x/(455*a**8*b**3 + 1365*a**7*b**4*x + 1
365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 60*a**(31/3)*b**2*x**2*(1 + b*x/
a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**
```

```
5*b**6*x**3) - 81*a**(31/3)*b**2*x**2/(455*a**8*b**3 + 1365*a**7*b**4*x + 1
365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 165*a**(28/3)*b**3*x**3*(1 + b*x
/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a*
*5*b**6*x**3) - 27*a**(28/3)*b**3*x**3/(455*a**8*b**3 + 1365*a**7*b**4*x +
1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 555*a**(25/3)*b**4*x**4*(1 + b*
x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a
**5*b**6*x**3) + 762*a**(22/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(455*a**8*b**3
+ 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 462*a**(19
/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a
**6*b**5*x**2 + 455*a**5*b**6*x**3) + 105*a**(16/3)*b**7*x**7*(1 + b*x/a)**
(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b*
*6*x**3)
```

Giac [B] time = 1.18807, size = 124, normalized size = 2.34

$$\frac{3\left(\frac{13\left(14(bx+a)^{\frac{10}{3}}-40(bx+a)^{\frac{7}{3}}a+35(bx+a)^{\frac{4}{3}}a^2\right)a}{b^2}+\frac{140(bx+a)^{\frac{13}{3}}-546(bx+a)^{\frac{10}{3}}a+780(bx+a)^{\frac{7}{3}}a^2-455(bx+a)^{\frac{4}{3}}a^3}{b^2}\right)}{1820\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/1820*(13*(14*(b*x + a)^(10/3) - 40*(b*x + a)^(7/3)*a + 35*(b*x + a)^(4/3)
*a^2)*a/b^2 + (140*(b*x + a)^(13/3) - 546*(b*x + a)^(10/3)*a + 780*(b*x + a
)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3)/b^2)/b
```

3.387 $\int x(a+bx)^{4/3}\,dx$

Optimal. Leaf size=34

$$\frac{3(a+bx)^{10/3}}{10b^2}-\frac{3a(a+bx)^{7/3}}{7b^2}$$

```
[Out] (-3*a*(a + b*x)^(7/3))/(7*b^2) + (3*(a + b*x)^(10/3))/(10*b^2)
```

Rubi [A] time = 0.0087666, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{10/3}}{10b^2}-\frac{3a(a+bx)^{7/3}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(4/3),x]
```

```
[Out] (-3*a*(a + b*x)^(7/3))/(7*b^2) + (3*(a + b*x)^(10/3))/(10*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{4/3}\,dx&=\int\left(-\frac{a(a+bx)^{4/3}}{b}+\frac{(a+bx)^{7/3}}{b}\right)dx\\&=-\frac{3a(a+bx)^{7/3}}{7b^2}+\frac{3(a+bx)^{10/3}}{10b^2}\end{aligned}$$

Mathematica [A] time = 0.0420025, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{7/3}(7bx-3a)}{70b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(-3*a + 7*b*x))/(70*b^2)
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$-{\frac{-21\,bx+9\,a}{70\,{b}^{2}}} \left(bx+a \right) ^{{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(4/3),x)
```

```
[Out] -3/70*(b*x+a)^(7/3)*(-7*b*x+3*a)/b^2
```

Maxima [A] time = 1.05417, size = 35, normalized size = 1.03

$$\frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^2}-\frac{3\,(bx+a)^{\frac{7}{3}}a}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x + a)^(10/3)/b^2 - 3/7*(b*x + a)^(7/3)*a/b^2
```

Fricas [A] time = 1.54511, size = 96, normalized size = 2.82

$$\frac{3\left(7\,b^3x^3+11\,ab^2x^2+a^2bx-3\,a^3\right)(bx+a)^{\frac{1}{3}}}{70\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/70*(7*b^3*x^3 + 11*a*b^2*x^2 + a^2*b*x - 3*a^3)*(b*x + a)^(1/3)/b^2
```

Sympy [A] time = 1.94136, size = 80, normalized size = 2.35

$$\begin{cases} -\frac{9a^3\sqrt[3]{a+bx}}{70b^2}+\frac{3a^2x\sqrt[3]{a+bx}}{70b}+\frac{33ax^2\sqrt[3]{a+bx}}{70}+\frac{3bx^3\sqrt[3]{a+bx}}{10} & \text{for } b\neq 0 \\ \frac{a^{\frac{4}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(4/3),x)
```

```
[Out] Piecewise((-9*a**3*(a + b*x)**(1/3)/(70*b**2) + 3*a**2*x*(a + b*x)**(1/3)/(
70*b) + 33*a*x**2*(a + b*x)**(1/3)/70 + 3*b*x**3*(a + b*x)**(1/3)/10, Ne(b,
 0)), (a**(4/3)*x**2/2, True))
```

Giac [B] time = 1.24592, size = 92, normalized size = 2.71

$$\frac{3\left(\frac{5\left(4\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a\right)a}{b}+\frac{14\,(bx+a)^{\frac{10}{3}}-40\,(bx+a)^{\frac{7}{3}}a+35\,(bx+a)^{\frac{4}{3}}a^2}{b}\right)}{140\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/140*(5*(4*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a)*a/b + (14*(b*x + a)^(10/
3) - 40*(b*x + a)^(7/3)*a + 35*(b*x + a)^(4/3)*a^2)/b)/b
```

3.388 $\int (a+bx)^{4/3}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{7/3}}{7b}$$

```
[Out] (3*(a + b*x)^(7/3))/(7*b)
```

Rubi [A] time = 0.0015413, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3(a+bx)^{7/3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3),x]

[Out] (3*(a + b*x)^(7/3))/(7*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{4/3}\,dx = \frac{3(a+bx)^{7/3}}{7b}$$

Mathematica [A] time = 0.012641, size = 16, normalized size = 1.

$$\frac{3(a+bx)^{7/3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3),x]

[Out] (3*(a + b*x)^(7/3))/(7*b)
```

Maple [A] time = 0.001, size = 13, normalized size = 0.8

$$\frac{3}{7\,b}(bx+a)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3),x)
```

```
[Out] 3/7*(b*x+a)^(7/3)/b
```

Maxima [A] time = 1.06903, size = 16, normalized size = 1.

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/7*(b*x + a)^(7/3)/b
```

Fricas [B] time = 1.46872, size = 66, normalized size = 4.12

$$\frac{3\left(b^2x^2+2\,abx+a^2\right)(bx+a)^{\frac{1}{3}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/7*(b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/3)/b
```

Sympy [A] time = 0.069693, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{7}{3}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3),x)
```

```
[Out] 3*(a + b*x)**(7/3)/(7*b)
```

Giac [A] time = 1.25241, size = 16, normalized size = 1.

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/7*(b*x + a)^(7/3)/b
```

3.389 $\int \frac{(a+bx)^{4/3}}{x} dx$

Optimal. Leaf size=105

$$\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}$$

```
[Out] 3*a*(a + b*x)^(1/3) + (3*(a + b*x)^(4/3))/4 - Sqrt[3]*a^(4/3)*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))] - (a^(4/3)*Log[x])/2 + (3*a^(4/3
)*Log[a^(1/3) - (a + b*x)^(1/3)])/2
```

Rubi [A] time = 0.0406306, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 57, 617, 204, 31}

$$\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x,x]
```

```
[Out] 3*a*(a + b*x)^(1/3) + (3*(a + b*x)^(4/3))/4 - Sqrt[3]*a^(4/3)*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))] - (a^(4/3)*Log[x])/2 + (3*a^(4/3
)*Log[a^(1/3) - (a + b*x)^(1/3)])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{4/3}}{x}\,dx &= \frac{3}{4}(a+bx)^{4/3}+a\int \frac{\sqrt[3]{a+bx}}{x}\,dx\\ &= 3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}+a^2\int \frac{1}{x(a+bx)^{2/3}}\,dx\\ &= 3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}-\frac{1}{2}a^{4/3}\log(x)-\frac{1}{2}\left(3a^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)-\frac{1}{2}\left(3a^{5/3}\right.\\ &= 3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\left(3a^{4/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\right.\\ &= 3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}-\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right.\end{aligned}$$

Mathematica [A] time = 0.0687591, size = 130, normalized size = 1.24

$$\frac{1}{4}\left(4a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-2a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)-4\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)+15a\sqrt[3]{a+b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x,x]
```

```
[Out] (15*a*(a + b*x)^(1/3) + 3*b*x*(a + b*x)^(1/3) - 4*Sqrt[3]*a^(4/3)*ArcTan[(1
 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[a^(1/3) - (a + b*x
)^(1/3)] - 2*a^(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3
)])/4
```

Maple [A] time = 0.004, size = 95, normalized size = 0.9

$$\frac{3}{4}(bx+a)^{\frac{4}{3}}+3a\sqrt[3]{bx+a}+a^{\frac{4}{3}}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)-\frac{1}{2}a^{\frac{4}{3}}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)-a^{\frac{4}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x,x)
```

```
[Out] 3/4*(b*x+a)^(4/3)+3*a*(b*x+a)^(1/3)+a^(4/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2*a
^(4/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-a^(4/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67614, size = 305, normalized size = 2.9

$$-\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)-\frac{1}{2}a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{4}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{4}(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) + sqrt(3)*a)
/a) - 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
+ a^(4/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/4*(b*x + 5*a)*(b*x + a)^(1/3)
```

Sympy [C] time = 3.15185, size = 209, normalized size = 1.99

$$\frac{7a^{\frac{4}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a^{\frac{4}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+xe^{\frac{2i\pi}{3}}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+xe^{\frac{4i\pi}{3}}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x,x)
```

```
[Out] 7*a**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(7/3)/(3*gamma(
10/3)) + 7*a**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(3*gamma(10/3)) + 7*a**(4/3)*exp(2*I*pi/
3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/
3)/(3*gamma(10/3)) + 7*a*b**(1/3)*(a/b + x)**(1/3)*gamma(7/3)/gamma(10/3) +
 7*b**(4/3)*(a/b + x)**(4/3)*gamma(7/3)/(4*gamma(10/3))
```

Giac [A] time = 2.10626, size = 131, normalized size = 1.25

$$-\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{2}a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{4}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{4}(bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(4/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3/4*(b*x + a)^(4/3) + 3*(b*x +
a)^(1/3)*a
```

3.390 $\int \frac{(a+bx)^{4/3}}{x^2} dx$

Optimal. Leaf size=107

$$-\frac{(a+bx)^{4/3}}{x}+4b\sqrt[3]{a+bx}-\frac{2}{3}\sqrt[3]{a}b\log(x)+2\sqrt[3]{a}b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{4\sqrt[3]{a}b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

```
[Out] 4*b*(a + b*x)^(1/3) - (a + b*x)^(4/3)/x - (4*a^(1/3)*b*ArcTan[(a^(1/3) + 2*
(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (2*a^(1/3)*b*Log[x])/3 + 2*a
^(1/3)*b*Log[a^(1/3) - (a + b*x)^(1/3)]
```

Rubi [A] time = 0.0416443, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 50, 57, 617, 204, 31}

$$-\frac{(a+bx)^{4/3}}{x}+4b\sqrt[3]{a+bx}-\frac{2}{3}\sqrt[3]{a}b\log(x)+2\sqrt[3]{a}b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{4\sqrt[3]{a}b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x^2,x]
```

```
[Out] 4*b*(a + b*x)^(1/3) - (a + b*x)^(4/3)/x - (4*a^(1/3)*b*ArcTan[(a^(1/3) + 2*
(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (2*a^(1/3)*b*Log[x])/3 + 2*a
^(1/3)*b*Log[a^(1/3) - (a + b*x)^(1/3)]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{4/3}}{x^2}\,dx &= -\frac{(a+bx)^{4/3}}{x}+\frac{1}{3}(4b)\int \frac{\sqrt[3]{a+bx}}{x}\,dx\\
&= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}+\frac{1}{3}(4ab)\int \frac{1}{x(a+bx)^{2/3}}\,dx\\
&= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{2}{3}\sqrt[3]{a}b\log(x)-\left(2\sqrt[3]{a}b\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)-\left(2a^{2/3}b\right)\operatorname{Su}\\
&= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{2}{3}\sqrt[3]{a}b\log(x)+2\sqrt[3]{a}b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\left(4\sqrt[3]{a}b\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\right.\\
&= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{4\sqrt[3]{a}b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}b\log(x)+2\sqrt[3]{a}b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0166116, size = 33, normalized size = 0.31

$$\frac{3b(a+bx)^{7/3}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{bx}{a}+1\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x^2,x]
```

```
[Out] (3*b*(a + b*x)^(7/3)*Hypergeometric2F1[2, 7/3, 10/3, 1 + (b*x)/a])/(7*a^2)
```

Maple [A] time = 0.009, size = 103, normalized size = 1.

$$3\,b\sqrt[3]{bx+a}-\frac{a}{x}\sqrt[3]{bx+a}+\frac{4\,b}{3}\sqrt[3]{a}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)-\frac{2\,b}{3}\sqrt[3]{a}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)-\frac{4\,b\sqrt{3}}{3}\sqrt[3]{a}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x^2,x)
```

```
[Out] 3*b*(b*x+a)^(1/3)-a*(b*x+a)^(1/3)/x+4/3*b*a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))
-2/3*b*a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-4/3*b*a^(1/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66737, size = 332, normalized size = 3.1

$$-\frac{4\sqrt{3}a^{\frac{1}{3}}bx\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+2a^{\frac{1}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4a^{\frac{1}{3}}bx\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3(3}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*sqrt(3)*a^(1/3)*b*x*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) +
 sqrt(3)*a)/a) + 2*a^(1/3)*b*x*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3
) + a^(2/3)) - 4*a^(1/3)*b*x*log((b*x + a)^(1/3) - a^(1/3)) - 3*(3*b*x - a)
*(b*x + a)^(1/3))/x
```

Sympy [C] time = 3.63229, size = 719, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x**2,x)
```

```
[Out] 28*a**(10/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*ga
mma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/
3)*gamma(10/3)) + 28*a**(10/3)*b*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2
*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(10/3)*b*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(9*
a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/
3)) - 28*a**(7/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**
(1/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/
b + x)*exp(2*I*pi/3)*gamma(10/3)) - 28*a**(7/3)*b**2*(a/b + x)*log(1 - b**(
1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(
2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) - 28*
a**(7/3)*b**2*(a/b + x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*ex
p_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) -
9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) + 84*a**3*b**(4/3)*(a/b + x)*
```

```
*(1/3)*exp(2*I*pi/3)*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*
b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) - 63*a**2*b**(7/3)*(a/b + x)**(4/3)*
exp(2*I*pi/3)*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(10/3))
```

Giac [A] time = 1.91699, size = 161, normalized size = 1.5

$$-\frac{4\sqrt{3}a^{\frac{1}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)+2a^{\frac{1}{3}}b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4a^{\frac{1}{3}}b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)-9}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="giac")
```

```
[Out] -1/3*(4*sqrt(3)*a^(1/3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3)
)/a^(1/3)) + 2*a^(1/3)*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
a^(2/3)) - 4*a^(1/3)*b^2*log(abs((b*x + a)^(1/3) - a^(1/3))) - 9*(b*x + a)^
(1/3)*b^2 + 3*(b*x + a)^(1/3)*a*b/x)/b
```

3.391 $\int \frac{(a+bx)^{4/3}}{x^3} dx$

Optimal. Leaf size=124

$$-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}-\frac{2b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b\sqrt[3]{a+bx}}{3x}$$

```
[Out] (-2*b*(a + b*x)^(1/3))/(3*x) - (a + b*x)^(4/3)/(2*x^2) - (2*b^2*ArcTan[(a^(
1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) - (b^2*Lo
g[x])/(9*a^(2/3)) + (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(2/3))
```

Rubi [A] time = 0.0443007, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 57, 617, 204, 31}

$$-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}-\frac{2b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b\sqrt[3]{a+bx}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x^3,x]
```

```
[Out] (-2*b*(a + b*x)^(1/3))/(3*x) - (a + b*x)^(4/3)/(2*x^2) - (2*b^2*ArcTan[(a^(
1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) - (b^2*Lo
g[x])/(9*a^(2/3)) + (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(2/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{x^3}\,dx = -\frac{(a+bx)^{4/3}}{2x^2}+\frac{1}{3}(2b)\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx$$
$$= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}+\frac{1}{9}\left(2b^2\right)\int \frac{1}{x(a+bx)^{2/3}}\,dx$$
$$= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{b^2\log(x)}{9a^{2/3}}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{3a^{2/3}}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x}\right.}{3\sqrt[3]{}}$$
$$= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}+\frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\right.}{3a^{2/3}}$$
$$= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}$$

Mathematica [C] time = 0.0217566, size = 35, normalized size = 0.28

$$-\frac{3b^2(a+bx)^{7/3}\,{}_2F_1\left(\frac{7}{3},3;\frac{10}{3};\frac{bx}{a}+1\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x^3,x]
```

```
[Out] (-3*b^2*(a + b*x)^(7/3)*Hypergeometric2F1[7/3, 3, 10/3, 1 + (b*x)/a])/(7*a^
3)
```

Maple [A] time = 0.009, size = 111, normalized size = 0.9

$$-\frac{7}{6x^2}(bx+a)^{\frac{4}{3}}+\frac{2a}{3x^2}\sqrt[3]{bx+a}+\frac{2b^2}{9}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)a^{-\frac{2}{3}}-\frac{b^2}{9}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)a^{-\frac{2}{3}}-\frac{2b^2\sqrt{3}}{9}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x^3,x)
```

```
[Out] -7/6*(b*x+a)^(4/3)/x^2+2/3/x^2*a*(b*x+a)^(1/3)+2/9*b^2/a^(2/3)*ln((b*x+a)^(
1/3)-a^(1/3))-1/9*b^2/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3
))-2/9*b^2/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6479, size = 455, normalized size = 3.67

$$-\frac{4\sqrt{3}\left(a^2\right)^{\frac{1}{6}}ab^2x^2\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^2}\right)+2\left(a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-4}{18a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/18*(4*sqrt(3)*(a^2)^(1/6)*a*b^2*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2
)^(1/3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + 2*(a^2)^(2/3)*b^2
*x^2*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) -
 4*(a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2*b*x
+ 3*a^3)*(b*x + a)^(1/3))/(a^2*x^2)
```

Sympy [C] time = 4.11215, size = 2266, normalized size = 18.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x**3,x)
```

```
[Out] 28*a**(19/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))
*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3)
 - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(19/3)*b**2
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)
/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*ga
mma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*
b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(19/3)*b**2*exp(-2*I*p
i/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3
)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a
**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b +
 x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(7/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma
(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**
3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b + x)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(54
*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
```

```
10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3
*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b + x)*exp(
-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*
gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*
I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3)
- 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*
(a/b + x)**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamm
a(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi
/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54
*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*(a/b
 + x)**2*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*ga
mma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*
pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) -
54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*(a
/b + x)**2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(
a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/
3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*
a**(10/3)*b**5*(a/b + x)**3*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)
/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*g
amma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*a**(
10/3)*b**5*(a/b + x)**3*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/
3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/
b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)
*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*a*
*(10/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*
exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3)
 - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)
**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gam
ma(10/3)) + 84*a**6*b**(7/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(7/3)/(54*
a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(1
0/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*
(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 231*a**5*b**(10/3)*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b
*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*p
i/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 1
47*a**4*b**(13/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(7/3)/(54*a**7*exp(2*
I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*
a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**
3*exp(2*I*pi/3)*gamma(10/3))
```

Giac [A] time = 1.88783, size = 171, normalized size = 1.38

$$-\frac{\frac{4\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{2b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{4b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{3\left(7(bx+a)^{\frac{4}{3}}b^3-4(bx+a)^{\frac{1}{3}}ab^3\right)}{b^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(2/3) + 2*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(2/3) - 4*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3) + 3*(7*(b*x +
```

```
a)^(4/3)*b^3 - 4*(b*x + a)^(1/3)*a*b^3)/(b^2*x^2))/b
```

3.392 $\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=72

$$\frac{9a^2(a+bx)^{5/3}}{5b^4}-\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{3(a+bx)^{11/3}}{11b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}$$

```
[Out] (-3*a^3*(a + b*x)^(2/3))/(2*b^4) + (9*a^2*(a + b*x)^(5/3))/(5*b^4) - (9*a*(
a + b*x)^(8/3))/(8*b^4) + (3*(a + b*x)^(11/3))/(11*b^4)
```

Rubi [A] time = 0.0179759, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{5/3}}{5b^4}-\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{3(a+bx)^{11/3}}{11b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(2/3))/(2*b^4) + (9*a^2*(a + b*x)^(5/3))/(5*b^4) - (9*a*(
a + b*x)^(8/3))/(8*b^4) + (3*(a + b*x)^(11/3))/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx &= \int \left(-\frac{a^3}{b^3\sqrt[3]{a+bx}}+\frac{3a^2(a+bx)^{2/3}}{b^3}-\frac{3a(a+bx)^{5/3}}{b^3}+\frac{(a+bx)^{8/3}}{b^3}\right)dx\\ &= -\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{9a^2(a+bx)^{5/3}}{5b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}+\frac{3(a+bx)^{11/3}}{11b^4}\end{aligned}$$

Mathematica [A] time = 0.0449445, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{2/3}\left(54a^2bx-81a^3-45ab^2x^2+40b^3x^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(-81*a^3 + 54*a^2*b*x - 45*a*b^2*x^2 + 40*b^3*x^3))/(440
*b^4)
```

Maple [A] time = 0.004, size = 43, normalized size = 0.6

$$-\frac{-120\,b^3x^3+135\,ab^2x^2-162\,a^2bx+243\,a^3}{440\,b^4}(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(1/3),x)
```

```
[Out] -3/440*(b*x+a)^(2/3)*(-40*b^3*x^3+45*a*b^2*x^2-54*a^2*b*x+81*a^3)/b^4
```

Maxima [A] time = 1.07116, size = 76, normalized size = 1.06

$$\frac{3\,(bx+a)^{\frac{11}{3}}}{11\,b^4}-\frac{9\,(bx+a)^{\frac{8}{3}}a}{8\,b^4}+\frac{9\,(bx+a)^{\frac{5}{3}}a^2}{5\,b^4}-\frac{3\,(bx+a)^{\frac{2}{3}}a^3}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x + a)^(11/3)/b^4 - 9/8*(b*x + a)^(8/3)*a/b^4 + 9/5*(b*x + a)^(5/3)
*a^2/b^4 - 3/2*(b*x + a)^(2/3)*a^3/b^4
```

Fricas [A] time = 1.50877, size = 104, normalized size = 1.44

$$\frac{3\left(40\,b^3x^3-45\,ab^2x^2+54\,a^2bx-81\,a^3\right)(bx+a)^{\frac{2}{3}}}{440\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(40*b^3*x^3 - 45*a*b^2*x^2 + 54*a^2*b*x - 81*a^3)*(b*x + a)^(2/3)/b^4
```

Sympy [B] time = 3.54217, size = 1640, normalized size = 22.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(1/3),x)
```

```
[Out] -243*a**(71/3)*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 660
0*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**1
5*b**9*x**5 + 440*a**14*b**10*x**6) + 243*a**(71/3)/(440*a**20*b**4 + 2640*
a**19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**
8*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 1296*a**(68/3)*b*x*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 1458*a**(68/3)*b*x/(440*a**20*b**4 + 2640*a**19*b**5
*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2
```

```
640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 2808*a**(65/3)*b**2*x**2*(1 +
 b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2 +
 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440*a
**14*b**10*x**6) + 3645*a**(65/3)*b**2*x**2/(440*a**20*b**4 + 2640*a**19*b*
*5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 +
 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 3120*a**(62/3)*b**3*x**3*(1
 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2
 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440
*a**14*b**10*x**6) + 4860*a**(62/3)*b**3*x**3/(440*a**20*b**4 + 2640*a**19*
b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4
 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 1710*a**(59/3)*b**4*x**4*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4 + 2640*a**1
9*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x*
*4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 72*a**(56/3)*b**5*x**5*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 1458*a**(56/3)*b**5*x**5/(440*a**20*b**4 + 2640*a**1
9*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x*
*4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 1104*a**(53/3)*b**6*x**
6*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*
x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 +
 440*a**14*b**10*x**6) + 243*a**(53/3)*b**6*x**6/(440*a**20*b**4 + 2640*a**
19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x
**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 1152*a**(50/3)*b**7*x*
*7*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6
*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5
+ 440*a**14*b**10*x**6) + 585*a**(47/3)*b**8*x**8*(1 + b*x/a)**(2/3)/(440*a
**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3
 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 12
0*a**(44/3)*b**9*x**9*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*
x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 26
40*a**15*b**9*x**5 + 440*a**14*b**10*x**6)
```

Giac [A] time = 1.22544, size = 66, normalized size = 0.92

$$\frac{3\left(40\,(bx+a)^{\frac{11}{3}}-165\,(bx+a)^{\frac{8}{3}}a+264\,(bx+a)^{\frac{5}{3}}a^2-220\,(bx+a)^{\frac{2}{3}}a^3\right)}{440\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/440*(40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/3)*a^
2 - 220*(b*x + a)^(2/3)*a^3)/b^4
```

3.393 $\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{2/3}}{2b^3}+\frac{3(a+bx)^{8/3}}{8b^3}-\frac{6a(a+bx)^{5/3}}{5b^3}$$

```
[Out] (3*a^2*(a + b*x)^(2/3))/(2*b^3) - (6*a*(a + b*x)^(5/3))/(5*b^3) + (3*(a + b
*x)^(8/3))/(8*b^3)
```

Rubi [A] time = 0.0124392, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2(a+bx)^{2/3}}{2b^3}+\frac{3(a+bx)^{8/3}}{8b^3}-\frac{6a(a+bx)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(2/3))/(2*b^3) - (6*a*(a + b*x)^(5/3))/(5*b^3) + (3*(a + b
*x)^(8/3))/(8*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx &= \int \left(\frac{a^2}{b^2\sqrt[3]{a+bx}}-\frac{2a(a+bx)^{2/3}}{b^2}+\frac{(a+bx)^{5/3}}{b^2}\right)dx\\ &= \frac{3a^2(a+bx)^{2/3}}{2b^3}-\frac{6a(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{8/3}}{8b^3}\end{aligned}$$

Mathematica [A] time = 0.0333041, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{2/3}\left(9a^2-6abx+5b^2x^2\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(9*a^2 - 6*a*b*x + 5*b^2*x^2))/(40*b^3)
```

Maple [A] time = 0.003, size = 32, normalized size = 0.6

$$\frac{15 b^2 x^2 - 18 abx + 27 a^2}{40 b^3} (bx + a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(1/3),x)
```

```
[Out] 3/40*(b*x+a)^(2/3)*(5*b^2*x^2-6*a*b*x+9*a^2)/b^3
```

Maxima [A] time = 1.00102, size = 55, normalized size = 1.04

$$\frac{3 (bx + a)^{\frac{8}{3}}}{8 b^3} - \frac{6 (bx + a)^{\frac{5}{3}} a}{5 b^3} + \frac{3 (bx + a)^{\frac{2}{3}} a^2}{2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)^(8/3)/b^3 - 6/5*(b*x + a)^(5/3)*a/b^3 + 3/2*(b*x + a)^(2/3)*a
^2/b^3
```

Fricas [A] time = 1.49519, size = 76, normalized size = 1.43

$$\frac{3 \left(5 b^2 x^2 - 6 abx + 9 a^2\right)(bx + a)^{\frac{2}{3}}}{40 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^2*x^2 - 6*a*b*x + 9*a^2)*(b*x + a)^(2/3)/b^3
```

Sympy [B] time = 2.18696, size = 600, normalized size = 11.32

$$\frac{27a^{\frac{32}{3}} \left(1 + \frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3 + 120a^7b^4x + 120a^6b^5x^2 + 40a^5b^6x^3} - \frac{27a^{\frac{32}{3}}}{40a^8b^3 + 120a^7b^4x + 120a^6b^5x^2 + 40a^5b^6x^3} + \frac{63a^{\frac{29}{3}} bx \left(1\right.}{40a^8b^3 + 120a^7b^4x + 12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(1/3),x)
```

```
[Out] 27*a**(32/3)*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*
b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(32/3)/(40*a**8*b**3 + 120*a**7*b**4
*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 63*a**(29/3)*b*x*(1 + b*x/a)
**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6
*x**3) - 81*a**(29/3)*b*x/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x
**2 + 40*a**5*b**6*x**3) + 42*a**(26/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a*
*8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 81*a*
```

```
*(26/3)*b**2*x**2/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40
*a**5*b**6*x**3) + 18*a**(23/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**8*b**3
+ 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23/3)*
b**3*x**3/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b*
*6*x**3) + 27*a**(20/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a*
*7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 15*a**(17/3)*b**5*x**
5*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 +
 40*a**5*b**6*x**3)
```

Giac [A] time = 1.13827, size = 50, normalized size = 0.94

$$\frac{3\left(5\,(bx+a)^{\frac{8}{3}} - 16\,(bx+a)^{\frac{5}{3}}a + 20\,(bx+a)^{\frac{2}{3}}a^2\right)}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/40*(5*(b*x + a)^(8/3) - 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^2)/b^
3
```

3.394 $\int \frac{x}{\sqrt[3]{a+bx}} \, dx$

Optimal. Leaf size=34

$$\frac{3(a+bx)^{5/3}}{5b^2} - \frac{3a(a+bx)^{2/3}}{2b^2}$$

```
[Out] (-3*a*(a + b*x)^(2/3))/(2*b^2) + (3*(a + b*x)^(5/3))/(5*b^2)
```

Rubi [A] time = 0.0078093, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{5/3}}{5b^2} - \frac{3a(a+bx)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x)^(2/3))/(2*b^2) + (3*(a + b*x)^(5/3))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt[3]{a+bx}} \, dx &= \int \left(-\frac{a}{b\sqrt[3]{a+bx}} + \frac{(a+bx)^{2/3}}{b} \right) dx \\ &= -\frac{3a(a+bx)^{2/3}}{2b^2} + \frac{3(a+bx)^{5/3}}{5b^2} \end{aligned}$$

Mathematica [A] time = 0.0273163, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{2/3}(2bx-3a)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(-3*a + 2*b*x))/(10*b^2)
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$-\frac{-6\,bx+9\,a}{10\,b^2}\,(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(1/3),x)
```

```
[Out] -3/10*(b*x+a)^(2/3)*(-2*b*x+3*a)/b^2
```

Maxima [A] time = 1.08437, size = 35, normalized size = 1.03

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b^2}-\frac{3\,(bx+a)^{\frac{2}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b^2 - 3/2*(b*x + a)^(2/3)*a/b^2
```

Fricas [A] time = 1.64726, size = 54, normalized size = 1.59

$$\frac{3\,(2\,bx-3\,a)(bx+a)^{\frac{2}{3}}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(2*b*x - 3*a)*(b*x + a)^(2/3)/b^2
```

Sympy [B] time = 1.44533, size = 162, normalized size = 4.76

$$-\frac{9a^{\frac{11}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}+\frac{9a^{\frac{11}{3}}}{10a^2b^2+10ab^3x}-\frac{3a^{\frac{8}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}+\frac{9a^{\frac{8}{3}}bx}{10a^2b^2+10ab^3x}+\frac{6a^{\frac{5}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(1/3),x)
```

```
[Out] -9*a**(11/3)*(1 + b*x/a)**(2/3)/(10*a**2*b**2 + 10*a*b**3*x) + 9*a**(11/3)/
(10*a**2*b**2 + 10*a*b**3*x) - 3*a**(8/3)*b*x*(1 + b*x/a)**(2/3)/(10*a**2*b
**2 + 10*a*b**3*x) + 9*a**(8/3)*b*x/(10*a**2*b**2 + 10*a*b**3*x) + 6*a**(5/
3)*b**2*x**2*(1 + b*x/a)**(2/3)/(10*a**2*b**2 + 10*a*b**3*x)
```

Giac [A] time = 1.15929, size = 34, normalized size = 1.

$$\frac{3\left(2\,(bx+a)^{\frac{5}{3}}-5\,(bx+a)^{\frac{2}{3}}a\right)}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="giac")

[Out] 3/10*(2*(b*x + a)^(5/3) - 5*(b*x + a)^(2/3)*a)/b^2
```

3.395 $\int \frac{1}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{2/3}}{2b}$$

```
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

Rubi [A] time = 0.0017111, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3(a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1/3),x]

[Out] (3*(a + b*x)^(2/3))/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}}\,dx = \frac{3(a+bx)^{2/3}}{2b}$$

Mathematica [A] time = 0.0052223, size = 16, normalized size = 1.

$$\frac{3(a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1/3),x]

[Out] (3*(a + b*x)^(2/3))/(2*b)
```

Maple [A] time = 0.001, size = 13, normalized size = 0.8

$$\frac{3}{2\,b}\,(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3),x)
```

```
[Out] 3/2*(b*x+a)^(2/3)/b
```

Maxima [A] time = 1.02125, size = 16, normalized size = 1.

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b
```

Fricas [A] time = 1.72467, size = 31, normalized size = 1.94

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b
```

Sympy [A] time = 0.072691, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3),x)
```

```
[Out] 3*(a + b*x)**(2/3)/(2*b)
```

Giac [A] time = 1.19731, size = 16, normalized size = 1.

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b
```

3.396 $\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=79

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

```
[Out] (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3) -
 Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))
```

Rubi [A] time = 0.0250843, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {55, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3) -
 Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx}\right) - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} - \frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

Mathematica [A] time = 0.0229908, size = 66, normalized size = 0.84

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - Log[x] + 3*L
og[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))
```

Maple [A] time = 0.004, size = 75, normalized size = 1.

$$\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)\frac{1}{\sqrt[3]{a}} - \frac{1}{2}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{a}} + \sqrt{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{bx+a}}{\sqrt[3]{a}}+1\right)\right)\frac{1}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/3),x)
```

```
[Out] 1/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b
*x+a)^(1/3)+a^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^
(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85358, size = 647, normalized size = 8.19

$$\left[\frac{\sqrt{3}a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left(\right.}{2\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*sqrt(-1/a^(2/3))*log((2*b*x + sqrt(3)*(2*(b*x + a)^(2/3)*a^
(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x + a)^(1/3)*a
^(2/3) + 3*a)/x) - a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
a^(2/3)) + 2*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)))/a, 1/2*(2*sqrt(3)*a^(2
/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3)*log
((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x
+ a)^(1/3) - a^(1/3)))/a]
```

Sympy [C] time = 2.48862, size = 155, normalized size = 1.96

$$\frac{2\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}+\frac{2e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}+\frac{2e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/3),x)
```

```
[Out] 2*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(
5/3)) + 2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/
3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) + 2*exp(-2*I*pi/3)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(3*a**(1
/3)*gamma(5/3))
```

Giac [A] time = 2.1971, size = 104, normalized size = 1.32

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}}+\frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + log
(abs((b*x + a)^(1/3) - a^(1/3)))/a^(1/3)
```

3.397 $\int \frac{1}{x^2 \sqrt[3]{a+bx}} dx$

Optimal. Leaf size=100

$$\frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{2a^{4/3}} - \frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}} - \frac{(a+bx)^{2/3}}{ax}$$

```
[Out] -((a + b*x)^(2/3)/(a*x)) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(1/3) - (
a + b*x)^(1/3)])/(2*a^(4/3))
```

Rubi [A] time = 0.0334924, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$\frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{2a^{4/3}} - \frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}} - \frac{(a+bx)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] -((a + b*x)^(2/3)/(a*x)) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(1/3) - (
a + b*x)^(1/3)])/(2*a^(4/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = -\frac{(a+bx)^{2/3}}{ax} - \frac{b\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{3a}$$

$$= -\frac{(a+bx)^{2/3}}{ax} + \frac{b\log(x)}{6a^{4/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2a^{4/3}} - \frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+b\right.}}{2a}$$

$$= -\frac{(a+bx)^{2/3}}{ax} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{4/3}}$$

$$= -\frac{(a+bx)^{2/3}}{ax} - \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}$$

Mathematica [C] time = 0.0078275, size = 33, normalized size = 0.33

$$\frac{3b(a+bx)^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{bx}{a}+1\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] (3*b*(a + b*x)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, 1 + (b*x)/a])/(2*a^2)
```

Maple [A] time = 0.007, size = 95, normalized size = 1.

$$-\frac{1}{ax}(bx+a)^{\frac{2}{3}} - \frac{b}{3}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)a^{-\frac{4}{3}} + \frac{b}{6}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)a^{-\frac{4}{3}} - \frac{b\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{bx+a}}{\sqrt[3]{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(1/3),x)
```

```
[Out] -(b*x+a)^(2/3)/a/x-1/3*b/a^(4/3)*ln((b*x+a)^(1/3)-a^(1/3))+1/6*b/a^(4/3)*ln
((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-1/3*b/a^(4/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98536, size = 867, normalized size = 8.67

$$\left[\frac{3\sqrt{\frac{1}{3}}abx\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx-3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x}\right)+(-a)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}-(bx+a)\right.}{6a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*sqrt(1/3)*(2*(b*x
 + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/
a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + (-a)^(2/3)*b*x*log((b*x + a)^
(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*
x + a)^(1/3) + (-a)^(1/3)) - 6*(b*x + a)^(2/3)*a)/(a^2*x), -1/6*(6*sqrt(1/3
)*a*b*x*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) - (-a)^(1/3
))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b*x*log((b*x + a)^(2/3) - (b*x + a)^(1
/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-a)^(2/3)*b*x*log((b*x + a)^(1/3) + (-a)^
(1/3)) + 6*(b*x + a)^(2/3)*a)/(a^2*x)]
```

Sympy [C] time = 3.08162, size = 831, normalized size = 8.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(1/3),x)
```

```
[Out] -2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*p
i/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)
) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a
/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a
/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*
exp(2*I*pi/3)*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b*
*(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x
)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*(a/b + x)**(7/3)*
exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(9*a**
3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b
 + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*(a/b + x)**(7
/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a*
```

```
*(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/
3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3
)*b**(10/3)*(a/b + x)**(7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*
I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3
)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) +
 6*a*b**3*(a/b + x)**2*exp(2*I*pi/3)*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)*
*(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*
pi/3)*gamma(5/3))
```

Giac [A] time = 2.13989, size = 147, normalized size = 1.47

$$-\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{6(bx+a)^{\frac{2}{3}}b}{ax}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] -1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(4/3) - b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(4/3) + 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*x + a)^(2
/3)*b/(a*x))/b
```

3.398 $\int \frac{1}{x^3 \sqrt[3]{a+bx}} dx$

Optimal. Leaf size=130

$$-\frac{b^2 \log(x)}{9a^{7/3}} + \frac{b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{3a^{7/3}} + \frac{2b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}} + \frac{2b(a+bx)^{2/3}}{3a^2x} - \frac{(a+bx)^{2/3}}{2ax^2}$$

```
[Out] -(a + b*x)^(2/3)/(2*a*x^2) + (2*b*(a + b*x)^(2/3))/(3*a^2*x) + (2*b^2*ArcTa
n[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) - (
b^2*Log[x])/(9*a^(7/3)) + (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(7/3))
```

Rubi [A] time = 0.047821, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$-\frac{b^2 \log(x)}{9a^{7/3}} + \frac{b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{3a^{7/3}} + \frac{2b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}} + \frac{2b(a+bx)^{2/3}}{3a^2x} - \frac{(a+bx)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] -(a + b*x)^(2/3)/(2*a*x^2) + (2*b*(a + b*x)^(2/3))/(3*a^2*x) + (2*b^2*ArcTa
n[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) - (
b^2*Log[x])/(9*a^(7/3)) + (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(7/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx = -\frac{(a+bx)^{2/3}}{2ax^2}-\frac{(2b)\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{3a}$$
$$= -\frac{(a+bx)^{2/3}}{2ax^2}+\frac{2b(a+bx)^{2/3}}{3a^2x}+\frac{\left(2b^2\right)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a^2}$$
$$= -\frac{(a+bx)^{2/3}}{2ax^2}+\frac{2b(a+bx)^{2/3}}{3a^2x}-\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{3a^{7/3}}+\frac{b^2\,\text{Subst}\left(\int \frac{}{a^{2/3}+}\right.}{}$$
$$= -\frac{(a+bx)^{2/3}}{2ax^2}+\frac{2b(a+bx)^{2/3}}{3a^2x}-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}-\frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1\right.}{3a^{7/3}}$$
$$= -\frac{(a+bx)^{2/3}}{2ax^2}+\frac{2b(a+bx)^{2/3}}{3a^2x}+\frac{2b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{7/3}}-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}$$

Mathematica [C] time = 0.0113681, size = 35, normalized size = 0.27

$$-\frac{3b^2(a+bx)^{2/3}\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};\frac{bx}{a}+1\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] (-3*b^2*(a + b*x)^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, 1 + (b*x)/a])/(2*a^3
)
```

Maple [A] time = 0.006, size = 117, normalized size = 0.9

$$-\frac{1}{2\,ax^2}(bx+a)^{\frac{2}{3}}+\frac{2\,b}{3\,a^2x}(bx+a)^{\frac{2}{3}}+\frac{2\,b^2}{9}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)a^{-\frac{7}{3}}-\frac{b^2}{9}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)a^{-\frac{7}{3}}+\frac{2\,b^2\sqrt{}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(1/3),x)
```

```
[Out] -1/2*(b*x+a)^(2/3)/a/x^2+2/3*b*(b*x+a)^(2/3)/a^2/x+2/9*b^2/a^(7/3)*ln((b*x+
a)^(1/3)-a^(1/3))-1/9*b^2/a^(7/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^
(2/3))+2/9*b^2/a^(7/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+
1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00321, size = 856, normalized size = 6.58

$$\left[\frac{6\sqrt{\frac{1}{3}}ab^2x^2\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x}\right)-2a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a\right.}{18a^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*(
b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*
(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) +
 (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3)
 - a^(1/3)) + 3*(4*a*b*x - 3*a^2)*(b*x + a)^(2/3))/(a^3*x^2), 1/18*(12*sqrt
(1/3)*a^(2/3)*b^2*x^2*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
)) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2
/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3) - a^(1/3)) + 3*(4*a*b*x - 3*a^
2)*(b*x + a)^(2/3))/(a^3*x^2)]
```

Sympy [C] time = 3.90696, size = 2730, normalized size = 21.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(1/3),x)
```

```
[Out] 4*a**(14/3)*b**(10/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I
*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5
/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**
4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a**(14/3)*b**(1
0/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27
```

```
*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a**(14/3)*b
**(10/3)*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 8
1*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(1
3/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(
a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*g
amma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*
a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)
*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**
(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(a/b + x)**(7/3)*
exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/
3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3)
- 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(
10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b +
 x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(a/b + x)**(
7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*
b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b
 + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3
)*exp(2*I*pi/3)*gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*exp(2
*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b*
*(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b +
x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*ex
p(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*
gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*exp(-2*I*pi/3)*log(1
- b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a*
*7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a
/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*p
i/3)*gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*log(1 - b**(1/3)
*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3
)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(
7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I
*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 4*a**(5/3)*b**(19/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*log(1 - b**(1
/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3
)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3
)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3)
 - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/
3)*b**(19/3)*(a/b + x)**(13/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**
(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*
pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(
5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a*
*(5/3)*b**(19/3)*(a/b + x)**(13/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2
*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma
(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a
**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 21*a**4*b**4*(a
/b + x)**2*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(
2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamm
a(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*
a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 33*a**3*b**5*(
a/b + x)**3*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27
*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**2*b**6*
(a/b + x)**4*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*ex
```

```
p(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 2
7*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
```

Giac [A] time = 2.23204, size = 176, normalized size = 1.35

$$\frac{\frac{4\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}}-\frac{2b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}}+\frac{4b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{7}{3}}}+\frac{3\left(4(bx+a)^{\frac{5}{3}}b^3-7(bx+a)^{\frac{2}{3}}ab^3\right)}{a^2b^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(7/3) - 2*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(7/3) + 4*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(7/3) + 3*(4*(b*x +
a)^(5/3)*b^3 - 7*(b*x + a)^(2/3)*a*b^3)/(a^2*b^2*x^2))/b
```

3.399 $\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=80

$$\frac{9a^2(bx-a)^{5/3}}{5b^4}+\frac{3a^3(bx-a)^{2/3}}{2b^4}+\frac{3(bx-a)^{11/3}}{11b^4}+\frac{9a(bx-a)^{8/3}}{8b^4}$$

```
[Out] (3*a^3*(-a + b*x)^(2/3))/(2*b^4) + (9*a^2*(-a + b*x)^(5/3))/(5*b^4) + (9*a*
(-a + b*x)^(8/3))/(8*b^4) + (3*(-a + b*x)^(11/3))/(11*b^4)
```

Rubi [A] time = 0.018856, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{9a^2(bx-a)^{5/3}}{5b^4}+\frac{3a^3(bx-a)^{2/3}}{2b^4}+\frac{3(bx-a)^{11/3}}{11b^4}+\frac{9a(bx-a)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a^3*(-a + b*x)^(2/3))/(2*b^4) + (9*a^2*(-a + b*x)^(5/3))/(5*b^4) + (9*a*
(-a + b*x)^(8/3))/(8*b^4) + (3*(-a + b*x)^(11/3))/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx &= \int \left(\frac{a^3}{b^3\sqrt[3]{-a+bx}}+\frac{3a^2(-a+bx)^{2/3}}{b^3}+\frac{3a(-a+bx)^{5/3}}{b^3}+\frac{(-a+bx)^{8/3}}{b^3}\right)dx\\ &= \frac{3a^3(-a+bx)^{2/3}}{2b^4}+\frac{9a^2(-a+bx)^{5/3}}{5b^4}+\frac{9a(-a+bx)^{8/3}}{8b^4}+\frac{3(-a+bx)^{11/3}}{11b^4}\end{aligned}$$

Mathematica [A] time = 0.0495671, size = 48, normalized size = 0.6

$$\frac{3(bx-a)^{2/3}\left(54a^2bx+81a^3+45ab^2x^2+40b^3x^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(81*a^3 + 54*a^2*b*x + 45*a*b^2*x^2 + 40*b^3*x^3))/(440
*b^4)
```

Maple [A] time = 0.004, size = 45, normalized size = 0.6

$$\frac{120\,b^{3}x^{3}+135\,ab^{2}x^{2}+162\,a^{2}bx+243\,a^{3}}{440\,b^{4}}\left(bx-a\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x-a)^(1/3),x)
```

```
[Out] 3/440*(40*b^3*x^3+45*a*b^2*x^2+54*a^2*b*x+81*a^3)/b^4*(b*x-a)^(2/3)
```

Maxima [A] time = 1.05447, size = 86, normalized size = 1.08

$$\frac{3\,(bx-a)^{\frac{11}{3}}}{11\,b^{4}}+\frac{9\,(bx-a)^{\frac{8}{3}}a}{8\,b^{4}}+\frac{9\,(bx-a)^{\frac{5}{3}}a^{2}}{5\,b^{4}}+\frac{3\,(bx-a)^{\frac{2}{3}}a^{3}}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x - a)^(11/3)/b^4 + 9/8*(b*x - a)^(8/3)*a/b^4 + 9/5*(b*x - a)^(5/3)
*a^2/b^4 + 3/2*(b*x - a)^(2/3)*a^3/b^4
```

Fricas [A] time = 1.75451, size = 104, normalized size = 1.3

$$\frac{3\left(40\,b^{3}x^{3}+45\,ab^{2}x^{2}+54\,a^{2}bx+81\,a^{3}\right)(bx-a)^{\frac{2}{3}}}{440\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(40*b^3*x^3 + 45*a*b^2*x^2 + 54*a^2*b*x + 81*a^3)*(b*x - a)^(2/3)/b^4
```

Sympy [C] time = 4.1281, size = 4976, normalized size = 62.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((243*a**(71/3)*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*ex
p(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3
) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2
640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 243*a
**(71/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600
*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**1
6*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**1
0*x**6*exp(I*pi/3)) - 1296*a**(68/3)*b*x*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(4
40*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6
```

```
*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4
*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(
I*pi/3)) - 1458*a**(68/3)*b*x/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5
*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*ex
p(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*p
i/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 2808*a**(65/3)*b**2*x**2*(-1 + b
*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*ex
p(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*p
i/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3)
+ 440*a**14*b**10*x**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*x**2/(440*a**20*b
**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(
I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/
3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) -
 3120*a**(62/3)*b**3*x**3*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*e
xp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/
3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) -
2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 4860
*a**(62/3)*b**3*x**3/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*
pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3)
 + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 44
0*a**14*b**10*x**6*exp(I*pi/3)) + 1710*a**(59/3)*b**4*x**4*(-1 + b*x/a)**(2
/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3)
 + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 66
00*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**
14*b**10*x**6*exp(I*pi/3)) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4*exp(I
*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) -
 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640
*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 72*a**(5
6/3)*b**5*x**5*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3)
- 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a
**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*
b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 1458*a**(56/3)*
b**5*x**5/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 660
0*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**
16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**
10*x**6*exp(I*pi/3)) - 1104*a**(53/3)*b**6*x**6*(-1 + b*x/a)**(2/3)*exp(I*p
i/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**
18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b*
*8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x*
*6*exp(I*pi/3)) + 243*a**(53/3)*b**6*x**6/(440*a**20*b**4*exp(I*pi/3) - 264
0*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*
b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*
x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 1152*a**(50/3)*b**7*
x**7*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**
19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*
x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*
exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 585*a**(47/3)*b**8*x**8*(
-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**
5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*e
xp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*
pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 120*a**(44/3)*b**9*x**9*(-1 + b
*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*ex
p(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*p
i/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3)
+ 440*a**14*b**10*x**6*exp(I*pi/3)), Abs(b*x)/Abs(a) > 1), (-243*a**(71/3)*
(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi
/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) +
 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*
a**14*b**10*x**6*exp(I*pi/3)) + 243*a**(71/3)/(440*a**20*b**4*exp(I*pi/3) -
```

```
 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a*
*17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b
**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 1296*a**(68/3)*b
*x*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I
*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3
) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 4
40*a**14*b**10*x**6*exp(I*pi/3)) - 1458*a**(68/3)*b*x/(440*a**20*b**4*exp(I
*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) -
 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640
*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 2808*a**
(65/3)*b**2*x**2*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**1
9*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x
**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*e
xp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*x**2/(
440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**
6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**
4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp
(I*pi/3)) + 3120*a**(62/3)*b**3*x**3*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp
(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3)
 - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 26
40*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 4860*a
**(62/3)*b**3*x**3/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi
/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) +
 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*
a**14*b**10*x**6*exp(I*pi/3)) - 1710*a**(59/3)*b**4*x**4*(1 - b*x/a)**(2/3)
/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b
**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x
**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*e
xp(I*pi/3)) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4*exp(I*pi/3) - 2640*a
**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**
7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**
5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 72*a**(56/3)*b**5*x**5*
(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi
/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) +
 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*
a**14*b**10*x**6*exp(I*pi/3)) - 1458*a**(56/3)*b**5*x**5/(440*a**20*b**4*ex
p(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3
) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2
640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 1104*
a**(53/3)*b**6*x**6*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a
**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**
7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**
5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 243*a**(53/3)*b**6*x**6
/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b
**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x
**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*e
xp(I*pi/3)) - 1152*a**(50/3)*b**7*x**7*(1 - b*x/a)**(2/3)/(440*a**20*b**4*e
xp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/
3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) -
2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 585*
a**(47/3)*b**8*x**8*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a
**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**
7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**
5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 120*a**(44/3)*b**9*x**9
*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*p
i/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3)
+ 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440
*a**14*b**10*x**6*exp(I*pi/3)), True))
```

Giac [A] time = 1.18941, size = 77, normalized size = 0.96

$$\frac{3\left(40\,(bx-a)^{\frac{11}{3}}+165\,(bx-a)^{\frac{8}{3}}a+264\,(bx-a)^{\frac{5}{3}}a^{2}+220\,(bx-a)^{\frac{2}{3}}a^{3}\right)}{440\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/440*(40*(b*x - a)^(11/3) + 165*(b*x - a)^(8/3)*a + 264*(b*x - a)^(5/3)*a^
2 + 220*(b*x - a)^(2/3)*a^3)/b^4
```

3.400 $\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=59

$$\frac{3a^2(bx-a)^{2/3}}{2b^3}+\frac{3(bx-a)^{8/3}}{8b^3}+\frac{6a(bx-a)^{5/3}}{5b^3}$$

```
[Out] (3*a^2*(-a + b*x)^(2/3))/(2*b^3) + (6*a*(-a + b*x)^(5/3))/(5*b^3) + (3*(-a
+ b*x)^(8/3))/(8*b^3)
```

Rubi [A] time = 0.0129849, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3a^2(bx-a)^{2/3}}{2b^3}+\frac{3(bx-a)^{8/3}}{8b^3}+\frac{6a(bx-a)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(-a + b*x)^(2/3))/(2*b^3) + (6*a*(-a + b*x)^(5/3))/(5*b^3) + (3*(-a
+ b*x)^(8/3))/(8*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx &= \int \left(\frac{a^2}{b^2\sqrt[3]{-a+bx}}+\frac{2a(-a+bx)^{2/3}}{b^2}+\frac{(-a+bx)^{5/3}}{b^2}\right)dx\\ &= \frac{3a^2(-a+bx)^{2/3}}{2b^3}+\frac{6a(-a+bx)^{5/3}}{5b^3}+\frac{3(-a+bx)^{8/3}}{8b^3}\end{aligned}$$

Mathematica [A] time = 0.0579486, size = 37, normalized size = 0.63

$$\frac{3(bx-a)^{2/3}\left(9a^2+6abx+5b^2x^2\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(9*a^2 + 6*a*b*x + 5*b^2*x^2))/(40*b^3)
```

Maple [A] time = 0.004, size = 34, normalized size = 0.6

$$\frac{15\,b^{2}x^{2}+18\,abx+27\,a^{2}}{40\,b^{3}}\,(bx-a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x-a)^(1/3),x)
```

```
[Out] 3/40*(5*b^2*x^2+6*a*b*x+9*a^2)/b^3*(b*x-a)^(2/3)
```

Maxima [A] time = 1.02014, size = 63, normalized size = 1.07

$$\frac{3\,(bx-a)^{\frac{8}{3}}}{8\,b^{3}}+\frac{6\,(bx-a)^{\frac{5}{3}}a}{5\,b^{3}}+\frac{3\,(bx-a)^{\frac{2}{3}}a^{2}}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x - a)^(8/3)/b^3 + 6/5*(b*x - a)^(5/3)*a/b^3 + 3/2*(b*x - a)^(2/3)*a
^2/b^3
```

Fricas [A] time = 1.71182, size = 76, normalized size = 1.29

$$\frac{3\left(5\,b^{2}x^{2}+6\,abx+9\,a^{2}\right)(bx-a)^{\frac{2}{3}}}{40\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^2*x^2 + 6*a*b*x + 9*a^2)*(b*x - a)^(2/3)/b^3
```

Sympy [C] time = 2.55204, size = 1328, normalized size = 22.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((-27*a**(32/3)*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4
*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 27*a**(32/3)*exp(2*I*pi/3)/(
-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) +
 63*a**(29/3)*b*x*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 12
0*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 81*a**(29/3)*b*x*exp(2*I*pi/3)/(-40
*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 42
*a**(26/3)*b**2*x**2*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x -
 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 81*a**(26/3)*b**2*x**2*exp(2*I*p
i/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x
```

```
**3) + 18*a**(23/3)*b**3*x**3*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7
*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23/3)*b**3*x**3*
exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a*
*5*b**6*x**3) - 27*a**(20/3)*b**4*x**4*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 +
 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 15*a**(17/3)*b
**5*x**5*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b*
*5*x**2 + 40*a**5*b**6*x**3), Abs(b*x)/Abs(a) > 1), (-27*a**(32/3)*(1 - b*x
/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x
**2 + 40*a**5*b**6*x**3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*
a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 63*a**(29/3)*b*x*(1
 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*
b**5*x**2 + 40*a**5*b**6*x**3) - 81*a**(29/3)*b*x*exp(2*I*pi/3)/(-40*a**8*b
**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 42*a**(26
/3)*b**2*x**2*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b*
*4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 81*a**(26/3)*b**2*x**2*exp
(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*
b**6*x**3) + 18*a**(23/3)*b**3*x**3*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a
**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a
**(23/3)*b**3*x**3*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**
6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(20/3)*b**4*x**4*(1 - b*x/a)**(2/3
)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*
a**5*b**6*x**3) + 15*a**(17/3)*b**5*x**5*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(
-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3),
True))
```

Giac [A] time = 1.26082, size = 58, normalized size = 0.98

$$\frac{3\left(5\,(bx-a)^{\frac{8}{3}}+16\,(bx-a)^{\frac{5}{3}}a+20\,(bx-a)^{\frac{2}{3}}a^2\right)}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/40*(5*(b*x - a)^(8/3) + 16*(b*x - a)^(5/3)*a + 20*(b*x - a)^(2/3)*a^2)/b^
3
```

3.401 $\int \frac{x}{\sqrt[3]{-a+bx}} \, dx$

Optimal. Leaf size=38

$$\frac{3(bx-a)^{5/3}}{5b^2}+\frac{3a(bx-a)^{2/3}}{2b^2}$$

```
[Out] (3*a*(-a + b*x)^(2/3))/(2*b^2) + (3*(-a + b*x)^(5/3))/(5*b^2)
```

Rubi [A] time = 0.0088401, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3(bx-a)^{5/3}}{5b^2}+\frac{3a(bx-a)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a*(-a + b*x)^(2/3))/(2*b^2) + (3*(-a + b*x)^(5/3))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[3]{-a+bx}} \, dx &= \int \left(\frac{a}{b\sqrt[3]{-a+bx}}+\frac{(-a+bx)^{2/3}}{b}\right) dx\\ &= \frac{3a(-a+bx)^{2/3}}{2b^2}+\frac{3(-a+bx)^{5/3}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.0316602, size = 26, normalized size = 0.68

$$\frac{3(bx-a)^{2/3}(3a+2bx)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(3*a + 2*b*x))/(10*b^2)
```

Maple [A] time = 0.003, size = 23, normalized size = 0.6

$$\frac{6\,bx+9\,a}{10\,b^2}(bx-a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x-a)^(1/3),x)
```

```
[Out] 3/10*(2*b*x+3*a)/b^2*(b*x-a)^(2/3)
```

Maxima [A] time = 1.05181, size = 41, normalized size = 1.08

$$\frac{3\,(bx-a)^{\frac{5}{3}}}{5\,b^2}+\frac{3\,(bx-a)^{\frac{2}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x - a)^(5/3)/b^2 + 3/2*(b*x - a)^(2/3)*a/b^2
```

Fricas [A] time = 1.5218, size = 54, normalized size = 1.42

$$\frac{3\,(2\,bx+3\,a)(bx-a)^{\frac{2}{3}}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(2*b*x + 3*a)*(b*x - a)^(2/3)/b^2
```

Sympy [C] time = 1.54925, size = 488, normalized size = 12.84

$$\begin{cases} -\frac{9a^{\frac{11}{3}}\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{9a^{\frac{11}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{3a^{\frac{8}{3}}bx\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{9a^{\frac{8}{3}}bx}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{6a^{\frac{5}{3}}b^2x^2\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ \frac{9a^{\frac{11}{3}}\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{9a^{\frac{11}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{3a^{\frac{8}{3}}bx\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{9a^{\frac{8}{3}}bx}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{6a^{\frac{5}{3}}b^2x^2\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((-9*a**(11/3)*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(-10*a**2*b**2*exp(
I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 9*a**(11/3)/(-10*a**2*b**2*exp(I*pi/3)
 + 10*a*b**3*x*exp(I*pi/3)) + 3*a**(8/3)*b*x*(-1 + b*x/a)**(2/3)*exp(I*pi/3
)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 9*a**(8/3)*b*x/(-
10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 6*a**(5/3)*b**2*x**2*
(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*ex
p(I*pi/3)), Abs(b*x)/Abs(a) > 1), (9*a**(11/3)*(1 - b*x/a)**(2/3)/(-10*a**2
*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 9*a**(11/3)/(-10*a**2*b**2*e
xp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 3*a**(8/3)*b*x*(1 - b*x/a)**(2/3)/(
-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 9*a**(8/3)*b*x/(-10*
```

```
a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 6*a**(5/3)*b**2*x**2*(1
- b*x/a)**(2/3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)), True
))
```

Giac [A] time = 1.17993, size = 39, normalized size = 1.03

$$\frac{3\left(2\,(bx-a)^{\frac{5}{3}}+5\,(bx-a)^{\frac{2}{3}}a\right)}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/10*(2*(b*x - a)^(5/3) + 5*(b*x - a)^(2/3)*a)/b^2
```

3.402 $\int \frac{1}{\sqrt[3]{-a+bx}} dx$

Optimal. Leaf size=18

$$\frac{3(bx-a)^{2/3}}{2b}$$

```
[Out] (3*(-a + b*x)^(2/3))/(2*b)
```

Rubi [A] time = 0.0014808, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {32}

$$\frac{3(bx-a)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(-1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3))/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-a+bx}} dx = \frac{3(-a+bx)^{2/3}}{2b}$$

Mathematica [A] time = 0.0078637, size = 18, normalized size = 1.

$$\frac{3(bx-a)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(-1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3))/(2*b)
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$\frac{3}{2b}(bx-a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^(1/3),x)

[Out] 3/2*(b*x-a)^(2/3)/b
```

Maxima [A] time = 0.986556, size = 19, normalized size = 1.06

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="maxima")

[Out] 3/2*(b*x - a)^(2/3)/b
```

Fricas [A] time = 1.43817, size = 31, normalized size = 1.72

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="fricas")

[Out] 3/2*(b*x - a)^(2/3)/b
```

Sympy [A] time = 0.068408, size = 12, normalized size = 0.67

$$\frac{3\,(-a+bx)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**(1/3),x)

[Out] 3*(-a + b*x)**(2/3)/(2*b)
```

Giac [A] time = 1.19638, size = 19, normalized size = 1.06

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="giac")

[Out] 3/2*(b*x - a)^(2/3)/b
```

3.403 $\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=82

$$-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}$$

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3
)) + Log[x]/(2*a^(1/3)) - (3*Log[a^(1/3) + (-a + b*x)^(1/3)])/(2*a^(1/3))
```

Rubi [A] time = 0.0325087, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {56, 617, 204, 31}

$$-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3
)) + Log[x]/(2*a^(1/3)) - (3*Log[a^(1/3) + (-a + b*x)^(1/3)])/(2*a^(1/3))
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx = \frac{\log(x)}{2\sqrt[3]{a}} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{-a+bx}\right) - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx, x, \sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}}$$

$$= \frac{\log(x)}{2\sqrt[3]{a}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{\log(x)}{2\sqrt[3]{a}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}}$$

Mathematica [C] time = 0.0175193, size = 35, normalized size = 0.43

$$\frac{3(bx-a)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-\frac{bx}{a}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(1/3)),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 - (b*x)/a])/(2*a)
```

Maple [A] time = 0.007, size = 83, normalized size = 1.

$$-\ln\left(\sqrt[3]{a}+\sqrt[3]{bx-a}\right)\frac{1}{\sqrt[3]{a}} + \frac{1}{2}\ln\left((bx-a)^{\frac{2}{3}} - \sqrt[3]{a}\sqrt[3]{bx-a} + a^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{a}} + \sqrt{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{bx-a}}{\sqrt[3]{a}}-1\right)\right)\frac{1}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(1/3),x)
```

```
[Out] -ln(a^(1/3)+(b*x-a)^(1/3))/a^(1/3)+1/2/a^(1/3)*ln((b*x-a)^(2/3)-a^(1/3)*(b*
x-a)^(1/3)+a^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x-a)^(
1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72384, size = 747, normalized size = 9.11

$$\left[\frac{\sqrt{3}a\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3\,a}{x}\right)+(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+\right.}{2\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*sqrt((-a)^(1/3)/a)*log((2*b*x + sqrt(3)*(2*(b*x - a)^(2/3)*
(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x
- a)^(1/3)*(-a)^(2/3) - 3*a)/x) + (-a)^(2/3)*log((b*x - a)^(2/3) + (b*x - a
)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*log((b*x - a)^(1/3) - (-a)^
(1/3)))/a, 1/2*(2*sqrt(3)*a*sqrt(-(-a)^(1/3)/a)*arctan(1/3*sqrt(3)*(2*(b*x
- a)^(1/3) + (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + (-a)^(2/3)*log((b*x - a)^(2
/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*log((b*x - a)
^(1/3) - (-a)^(1/3)))/a]
```

Sympy [C] time = 2.74842, size = 160, normalized size = 1.95

$$-\frac{2e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}-\frac{2\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}-\frac{2e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(1/3),x)
```

```
[Out] -2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(
1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) - 2*log(1 - b**(1/3)*(-a/b + x)**(
1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) - 2*exp(2
*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*g
amma(2/3)/(3*a**(1/3)*gamma(5/3))
```

Giac [A] time = 1.77774, size = 151, normalized size = 1.84

$$-\frac{\sqrt{3}\,(-a)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)}{a}+\frac{(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{2\,a}-\frac{(-a)^{\frac{2}{3}}\log\left(\left|(bx-a)^{\frac{1}{3}}-(-a\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*(-a)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-a
)^(1/3))/a + 1/2*(-a)^(2/3)*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3
) + (-a)^(2/3))/a - (-a)^(2/3)*log(abs((b*x - a)^(1/3) - (-a)^(1/3)))/a
```

3.404 $\int \frac{1}{x^2 \sqrt[3]{-a+bx}} \, dx$

Optimal. Leaf size=103

$$\frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2a^{4/3}} - \frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}} + \frac{(bx-a)^{2/3}}{ax}$$

```
[Out] (-a + b*x)^(2/3)/(a*x) - (b*ArcTan[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(1/3) + (-
a + b*x)^(1/3)])/(2*a^(4/3))
```

Rubi [A] time = 0.0340923, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {51, 56, 617, 204, 31}

$$\frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2a^{4/3}} - \frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}} + \frac{(bx-a)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(1/3)),x]
```

```
[Out] (-a + b*x)^(2/3)/(a*x) - (b*ArcTan[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(1/3) + (-
a + b*x)^(1/3)])/(2*a^(4/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx &= \frac{(-a+bx)^{2/3}}{ax} + \frac{b\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx}{3a} \\
&= \frac{(-a+bx)^{2/3}}{ax} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx, x, \sqrt[3]{-a+bx}\right)}{2a^{4/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{-
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(-a+bx)^{2/3}}{ax} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2a^{4/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}\right)}{a^{4/3}} \\
&= \frac{(-a+bx)^{2/3}}{ax} - \frac{b\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2a^{4/3}}
\end{aligned}
$$

Mathematica [C] time = 0.0113463, size = 36, normalized size = 0.35

$$\frac{3b(bx-a)^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};1-\frac{bx}{a}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(1/3)),x]
```

```
[Out] (3*b*(-a + b*x)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, 1 - (b*x)/a])/(2*a^2)
```

Maple [A] time = 0.006, size = 103, normalized size = 1.

$$\frac{1}{ax}(bx-a)^{\frac{2}{3}} - \frac{b}{3}\ln\left(\sqrt[3]{a}+\sqrt[3]{bx-a}\right)a^{-\frac{4}{3}} + \frac{b}{6}\ln\left((bx-a)^{\frac{2}{3}} - \sqrt[3]{a}\sqrt[3]{bx-a} + a^{\frac{2}{3}}\right)a^{-\frac{4}{3}} + \frac{b\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{bx-a}}{\sqrt[3]{a}} - \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(1/3),x)
```

```
[Out] (b*x-a)^(2/3)/a/x-1/3*b*ln(a^(1/3)+(b*x-a)^(1/3))/a^(4/3)+1/6*b/a^(4/3)*ln(
(b*x-a)^(2/3)-a^(1/3)*(b*x-a)^(1/3)+a^(2/3))+1/3*b/a^(4/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/a^(1/3)*(b*x-a)^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57728, size = 865, normalized size = 8.4

$$\left[\frac{3\sqrt{\frac{1}{3}}abx\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3a}{x}\right)+(-a)^{\frac{2}{3}}bx\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}\right.}{6a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x*sqrt((-a)^(1/3)/a)*log((2*b*x + 3*sqrt(1/3)*(2*(b*x
 - a)^(2/3)*(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/
a) - 3*(b*x - a)^(1/3)*(-a)^(2/3) - 3*a)/x) + (-a)^(2/3)*b*x*log((b*x - a)^
(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*
x - a)^(1/3) - (-a)^(1/3)) + 6*(b*x - a)^(2/3)*a)/(a^2*x), 1/6*(6*sqrt(1/3)
*a*b*x*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x - a)^(1/3) + (-a)^(1/3)
)*sqrt(-(-a)^(1/3)/a)) + (-a)^(2/3)*b*x*log((b*x - a)^(2/3) + (b*x - a)^(1/
3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*x - a)^(1/3) - (-a)^(
1/3)) + 6*(b*x - a)^(2/3)*a)/(a^2*x)]
```

Sympy [C] time = 3.35895, size = 838, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(1/3),x)
```

```
[Out] -2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*e
xp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*ex
p(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(5/3)) - 2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**
(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/
3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)*
*(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*ex
p(-2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3
))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) +
 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*a**(2/3)*b
**(10/3)*(-a/b + x)**(7/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*p
i/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) -
2*a**(2/3)*b**(10/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(-a/b
```

```
 + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(
2*I*pi/3)*gamma(5/3)) - 2*a**(2/3)*b**(10/3)*(-a/b + x)**(7/3)*exp(-2*I*pi/
3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2
/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b*
*(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 6*a*b**3*(-a/b + x)**2
*exp(2*I*pi/3)*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3))
```

Giac [A] time = 2.24496, size = 194, normalized size = 1.88

$$-\frac{\frac{2\sqrt{3}(-a)^{\frac{2}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3(-a)^{\frac{1}{3}}}\right)}{a^2}-\frac{(-a)^{\frac{2}{3}}b^2\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{a^2}+\frac{2(-a)^{\frac{2}{3}}b^2\log\left(\left|(bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|\right)}{a^2}-\frac{6(bx-a)^{\frac{2}{3}}b}{ax}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] -1/6*(2*sqrt(3)*(-a)^(2/3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)
^(1/3))/(-a)^(1/3))/a^2 - (-a)^(2/3)*b^2*log((b*x - a)^(2/3) + (b*x - a)^(1
/3)*(-a)^(1/3) + (-a)^(2/3))/a^2 + 2*(-a)^(2/3)*b^2*log(abs((b*x - a)^(1/3)
 - (-a)^(1/3)))/a^2 - 6*(b*x - a)^(2/3)*b/(a*x))/b
```

3.405 $\int \frac{1}{x^3 \sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=136

$$\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{3a^{7/3}}-\frac{2b^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}}+\frac{2b(bx-a)^{2/3}}{3a^2x}+\frac{(bx-a)^{2/3}}{2ax^2}$$

```
[Out] (-a + b*x)^(2/3)/(2*a*x^2) + (2*b*(-a + b*x)^(2/3))/(3*a^2*x) - (2*b^2*ArcT
an[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) +
 (b^2*Log[x])/(9*a^(7/3)) - (b^2*Log[a^(1/3) + (-a + b*x)^(1/3)])/(3*a^(7/3
))
```

Rubi [A] time = 0.0440761, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {51, 56, 617, 204, 31}

$$\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{3a^{7/3}}-\frac{2b^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}}+\frac{2b(bx-a)^{2/3}}{3a^2x}+\frac{(bx-a)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(1/3)),x]
```

```
[Out] (-a + b*x)^(2/3)/(2*a*x^2) + (2*b*(-a + b*x)^(2/3))/(3*a^2*x) - (2*b^2*ArcT
an[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) +
 (b^2*Log[x])/(9*a^(7/3)) - (b^2*Log[a^(1/3) + (-a + b*x)^(1/3)])/(3*a^(7/3
))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{-a+bx}}\,dx = \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{(2b)\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx}{3a}$$
$$= \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{2b(-a+bx)^{2/3}}{3a^2x} + \frac{\left(2b^2\right)\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx}{9a^2}$$
$$= \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{2b(-a+bx)^{2/3}}{3a^2x} + \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx, x, \sqrt[3]{-a+bx}\right)}{3a^{7/3}} + \frac{b^2\,\mathrm{Subst}\left(\int\right.}{}$$
$$= \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{2b(-a+bx)^{2/3}}{3a^2x} + \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{3a^{7/3}} + \frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right.}{3a^{7/3}}$$
$$= \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{2b(-a+bx)^{2/3}}{3a^2x} - \frac{2b^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{7/3}} + \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{3a^{7/3}}$$

Mathematica [C] time = 0.0125004, size = 38, normalized size = 0.28

$$\frac{3b^2(bx-a)^{2/3}\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};1-\frac{bx}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(1/3)),x]
```

```
[Out] (3*b^2*(-a + b*x)^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, 1 - (b*x)/a])/(2*a^3
)
```

Maple [A] time = 0.006, size = 128, normalized size = 0.9

$$\frac{1}{2\,ax^2}(bx-a)^{\frac{2}{3}} + \frac{2\,b}{3\,a^2x}(bx-a)^{\frac{2}{3}} - \frac{2\,b^2}{9}\ln\left(\sqrt[3]{a}+\sqrt[3]{bx-a}\right)a^{-\frac{7}{3}} + \frac{b^2}{9}\ln\left((bx-a)^{\frac{2}{3}} - \sqrt[3]{a}\sqrt[3]{bx-a} + a^{\frac{2}{3}}\right)a^{-\frac{7}{3}} + \frac{2\,b^2\sqrt{3}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(1/3),x)
```

```
[Out] 1/2*(b*x-a)^(2/3)/a/x^2+2/3*b*(b*x-a)^(2/3)/a^2/x-2/9*b^2*ln(a^(1/3)+(b*x-a
)^(1/3))/a^(7/3)+1/9*b^2/a^(7/3)*ln((b*x-a)^(2/3)-a^(1/3)*(b*x-a)^(1/3)+a^(
2/3))+2/9*b^2/a^(7/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x-a)^(1/3)-1
```

```
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64782, size = 956, normalized size = 7.03

$$\left[\frac{6\sqrt{\frac{1}{3}}ab^2x^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx+3\sqrt{\frac{1}{3}}\left(2\,(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3\,a}{x}\right)+2\,(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{2}{3}}+(b\ldots\right.}{18\,a^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x + 3*sqrt(1/3)*(2
*(b*x - a)^(2/3)*(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(
1/3)/a) - 3*(b*x - a)^(1/3)*(-a)^(2/3) - 3*a)/x) + 2*(-a)^(2/3)*b^2*x^2*log
((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 4*(-a)^(2/3)*
b^2*x^2*log((b*x - a)^(1/3) - (-a)^(1/3)) + 3*(4*a*b*x + 3*a^2)*(b*x - a)^(
2/3))/(a^3*x^2), 1/18*(12*sqrt(1/3)*a*b^2*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sq
rt(1/3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 2*(-a)^(2/3
)*b^2*x^2*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) -
4*(-a)^(2/3)*b^2*x^2*log((b*x - a)^(1/3) - (-a)^(1/3)) + 3*(4*a*b*x + 3*a^2
)*(b*x - a)^(2/3))/(a^3*x^2)]
```

Sympy [C] time = 4.01392, size = 2744, normalized size = 20.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(1/3),x)
```

```
[Out] -4*a**(14/3)*b**(10/3)*(-a/b + x)**(4/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)
*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)
*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3
)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3
) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(
```

```
14/3)*b**(10/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)
**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**
(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I
*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamm
a(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4
*a**(14/3)*b**(10/3)*(-a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(-a/
b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-
a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/
3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*
pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*log(1 - b**(1/3)*(-a/b +
x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp
(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*
gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
 - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*
(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-
a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/
3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*
pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*exp(-2*I*pi/3)*log(1 - b*
*(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*
b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*
pi/3)*gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*log(1 - b**(1/
3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/
3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)
**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp
(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*
gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*log(1
- b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*
b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*
pi/3)*gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*exp(-2*I*pi/3)
*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3
)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b*
*(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b
 + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/
3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*log(
1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a
**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*
(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**
(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(
2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*exp(2*I*pi/
3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/
(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(
7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b +
 x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)
*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*exp(-2
*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*g
amma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81
*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/
3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b +
x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 21*a**4*b**4*(-a/b + x)**2*exp(2*I*p
i/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3
) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b
**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-
```

```
a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 33*a**3*b**5*(-a/b + x)**3*exp
(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*
a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13
/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 12*a**2*b**6*(-a/b + x)*
*4*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/
3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)
 + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*
b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
```

Giac [A] time = 1.75651, size = 225, normalized size = 1.65

$$-\frac{\frac{4\sqrt{3}(-a)^{\frac{2}{3}}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3(-a)^{\frac{1}{3}}}\right)}{a^3}-\frac{2(-a)^{\frac{2}{3}}b^3\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{a^3}+\frac{4(-a)^{\frac{2}{3}}b^3\log\left(\left|(bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|\right)}{a^3}-\frac{3\left(4(bx-a)^{\frac{5}{3}}b^3+7(bx-\right.}{a^2b^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] -1/18*(4*sqrt(3)*(-a)^(2/3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a
)^(1/3))/(-a)^(1/3))/a^3 - 2*(-a)^(2/3)*b^3*log((b*x - a)^(2/3) + (b*x - a)
^(1/3)*(-a)^(1/3) + (-a)^(2/3))/a^3 + 4*(-a)^(2/3)*b^3*log(abs((b*x - a)^(1
/3) - (-a)^(1/3)))/a^3 - 3*(4*(b*x - a)^(5/3)*b^3 + 7*(b*x - a)^(2/3)*a*b^3
)/(a^2*b^2*x^2))/b
```

3.406 $\int \frac{x^3}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=70

$$\frac{9a^2(a+bx)^{4/3}}{4b^4}-\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{3(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}$$

```
[Out] (-3*a^3*(a + b*x)^(1/3))/b^4 + (9*a^2*(a + b*x)^(4/3))/(4*b^4) - (9*a*(a +
b*x)^(7/3))/(7*b^4) + (3*(a + b*x)^(10/3))/(10*b^4)
```

Rubi [A] time = 0.0182285, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{4/3}}{4b^4}-\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{3(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(1/3))/b^4 + (9*a^2*(a + b*x)^(4/3))/(4*b^4) - (9*a*(a +
b*x)^(7/3))/(7*b^4) + (3*(a + b*x)^(10/3))/(10*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{2/3}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{2/3}}+\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{b^3}+\frac{(a+bx)^{7/3}}{b^3}\right)dx\\ &= -\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{9a^2(a+bx)^{4/3}}{4b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{10/3}}{10b^4}\end{aligned}$$

Mathematica [A] time = 0.0506735, size = 46, normalized size = 0.66

$$\frac{3\sqrt[3]{a+bx}\left(27a^2bx-81a^3-18ab^2x^2+14b^3x^3\right)}{140b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(-81*a^3 + 27*a^2*b*x - 18*a*b^2*x^2 + 14*b^3*x^3))/(140
*b^4)
```

Maple [A] time = 0.004, size = 43, normalized size = 0.6

$$-\frac{-42\,b^3x^3+54\,ab^2x^2-81\,a^2bx+243\,a^3}{140\,b^4}\sqrt[3]{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(2/3),x)
```

```
[Out] -3/140*(b*x+a)^(1/3)*(-14*b^3*x^3+18*a*b^2*x^2-27*a^2*b*x+81*a^3)/b^4
```

Maxima [A] time = 1.03641, size = 76, normalized size = 1.09

$$\frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^4}-\frac{9\,(bx+a)^{\frac{7}{3}}a}{7\,b^4}+\frac{9\,(bx+a)^{\frac{4}{3}}a^2}{4\,b^4}-\frac{3\,(bx+a)^{\frac{1}{3}}a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x + a)^(10/3)/b^4 - 9/7*(b*x + a)^(7/3)*a/b^4 + 9/4*(b*x + a)^(4/3)
*a^2/b^4 - 3*(b*x + a)^(1/3)*a^3/b^4
```

Fricas [A] time = 1.55516, size = 104, normalized size = 1.49

$$\frac{3\left(14\,b^3x^3-18\,ab^2x^2+27\,a^2bx-81\,a^3\right)(bx+a)^{\frac{1}{3}}}{140\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/140*(14*b^3*x^3 - 18*a*b^2*x^2 + 27*a^2*b*x - 81*a^3)*(b*x + a)^(1/3)/b^4
```

Sympy [B] time = 3.55856, size = 1640, normalized size = 23.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(2/3),x)
```

```
[Out] -243*a**(70/3)*(1 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100
*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*
b**9*x**5 + 140*a**14*b**10*x**6) + 243*a**(70/3)/(140*a**20*b**4 + 840*a**
19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x
**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) - 1377*a**(67/3)*b*x*(1 +
 b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 +
2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**
14*b**10*x**6) + 1458*a**(67/3)*b*x/(140*a**20*b**4 + 840*a**19*b**5*x + 21
00*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**1
```

```
5*b**9*x**5 + 140*a**14*b**10*x**6) - 3213*a**(64/3)*b**2*x**2*(1 + b*x/a)*
*(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**
17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10
*x**6) + 3645*a**(64/3)*b**2*x**2/(140*a**20*b**4 + 840*a**19*b**5*x + 2100
*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*
b**9*x**5 + 140*a**14*b**10*x**6) - 3927*a**(61/3)*b**3*x**3*(1 + b*x/a)**(
1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17
*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x
**6) + 4860*a**(61/3)*b**3*x**3/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a
**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b*
*9*x**5 + 140*a**14*b**10*x**6) - 2583*a**(58/3)*b**4*x**4*(1 + b*x/a)**(1/
3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b
**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**
6) + 3645*a**(58/3)*b**4*x**4/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**
18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9
*x**5 + 140*a**14*b**10*x**6) - 693*a**(55/3)*b**5*x**5*(1 + b*x/a)**(1/3)/
(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7
*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6)
+ 1458*a**(55/3)*b**5*x**5/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*
b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x*
*5 + 140*a**14*b**10*x**6) + 273*a**(52/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(14
0*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x*
*3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) + 2
43*a**(52/3)*b**6*x**6/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6
*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 +
 140*a**14*b**10*x**6) + 387*a**(49/3)*b**7*x**7*(1 + b*x/a)**(1/3)/(140*a*
*20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 +
 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) + 198*a
**(46/3)*b**8*x**8*(1 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x +
2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a*
*15*b**9*x**5 + 140*a**14*b**10*x**6) + 42*a**(43/3)*b**9*x**9*(1 + b*x/a)*
*(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**
17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10
*x**6)
```

Giac [A] time = 1.20013, size = 66, normalized size = 0.94

$$\frac{3\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3\right)}{140\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/140*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)*a^2
 - 140*(b*x + a)^(1/3)*a^3)/b^4
```

3.407 $\int \frac{x^2}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=51

$$\frac{3a^2\sqrt[3]{a+bx}}{b^3}+\frac{3(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}$$

```
[Out] (3*a^2*(a + b*x)^(1/3))/b^3 - (3*a*(a + b*x)^(4/3))/(2*b^3) + (3*(a + b*x)^
(7/3))/(7*b^3)
```

Rubi [A] time = 0.0119599, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2\sqrt[3]{a+bx}}{b^3}+\frac{3(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(1/3))/b^3 - (3*a*(a + b*x)^(4/3))/(2*b^3) + (3*(a + b*x)^
(7/3))/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{2/3}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{2/3}}-\frac{2a\sqrt[3]{a+bx}}{b^2}+\frac{(a+bx)^{4/3}}{b^2}\right)dx\\ &= \frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}+\frac{3(a+bx)^{7/3}}{7b^3}\end{aligned}$$

Mathematica [A] time = 0.0884474, size = 35, normalized size = 0.69

$$\frac{3\sqrt[3]{a+bx}\left(9a^2-3abx+2b^2x^2\right)}{14b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(9*a^2 - 3*a*b*x + 2*b^2*x^2))/(14*b^3)
```

Maple [A] time = 0.005, size = 32, normalized size = 0.6

$$\frac{6 b^{2} x^{2}-9 a b x+27 a^{2}}{14 b^{3}} \sqrt[3]{b x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(2/3),x)
```

```
[Out] 3/14*(b*x+a)^(1/3)*(2*b^2*x^2-3*a*b*x+9*a^2)/b^3
```

Maxima [A] time = 1.06806, size = 55, normalized size = 1.08

$$\frac{3 (b x+a)^{\frac{7}{3}}}{7 b^{3}}-\frac{3 (b x+a)^{\frac{4}{3}} a}{2 b^{3}}+\frac{3 (b x+a)^{\frac{1}{3}} a^{2}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/7*(b*x + a)^(7/3)/b^3 - 3/2*(b*x + a)^(4/3)*a/b^3 + 3*(b*x + a)^(1/3)*a^2
/b^3
```

Fricas [A] time = 1.57083, size = 76, normalized size = 1.49

$$\frac{3 \left(2 b^{2} x^{2}-3 a b x+9 a^{2}\right)(b x+a)^{\frac{1}{3}}}{14 b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/14*(2*b^2*x^2 - 3*a*b*x + 9*a^2)*(b*x + a)^(1/3)/b^3
```

Sympy [B] time = 2.45954, size = 600, normalized size = 11.76

$$\frac{27 a^{\frac{31}{3}} \sqrt[3]{1+\frac{b x}{a}}}{14 a^{8} b^{3}+42 a^{7} b^{4} x+42 a^{6} b^{5} x^{2}+14 a^{5} b^{6} x^{3}}-\frac{27 a^{\frac{31}{3}}}{14 a^{8} b^{3}+42 a^{7} b^{4} x+42 a^{6} b^{5} x^{2}+14 a^{5} b^{6} x^{3}}+\frac{72 a^{\frac{28}{3}} b x \sqrt[3]{1+\frac{bx}{a}}}{14 a^{8} b^{3}+42 a^{7} b^{4} x+42 a^{6} b^{5} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(2/3),x)
```

```
[Out] 27*a**(31/3)*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b*
*5*x**2 + 14*a**5*b**6*x**3) - 27*a**(31/3)/(14*a**8*b**3 + 42*a**7*b**4*x
+ 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) + 72*a**(28/3)*b*x*(1 + b*x/a)**(1
/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3)
 - 81*a**(28/3)*b*x/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14
*a**5*b**6*x**3) + 60*a**(25/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(14*a**8*b**3
+ 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) - 81*a**(25/3)*b*
*2*x**2/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x
```

```
**3) + 18*a**(22/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b*
*4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) - 27*a**(22/3)*b**3*x**3/(14*
a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) + 9*a**
(19/3)*b**4*x**4*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**
6*b**5*x**2 + 14*a**5*b**6*x**3) + 6*a**(16/3)*b**5*x**5*(1 + b*x/a)**(1/3)
/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3)
```

Giac [A] time = 1.19898, size = 50, normalized size = 0.98

$$\frac{3\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^{2}\right)}{14\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/14*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)/b^3
```

3.408 $\int \frac{x}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=32

$$\frac{3(a+bx)^{4/3}}{4b^2}-\frac{3a\sqrt[3]{a+bx}}{b^2}$$

```
[Out] (-3*a*(a + b*x)^(1/3))/b^2 + (3*(a + b*x)^(4/3))/(4*b^2)
```

Rubi [A] time = 0.0074868, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{4/3}}{4b^2}-\frac{3a\sqrt[3]{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x)^(1/3))/b^2 + (3*(a + b*x)^(4/3))/(4*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{2/3}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{2/3}}+\frac{\sqrt[3]{a+bx}}{b}\right)dx\\ &= -\frac{3a\sqrt[3]{a+bx}}{b^2}+\frac{3(a+bx)^{4/3}}{4b^2}\end{aligned}$$

Mathematica [A] time = 0.0179814, size = 23, normalized size = 0.72

$$\frac{3(bx-3a)\sqrt[3]{a+bx}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(2/3),x]
```

```
[Out] (3*(-3*a + b*x)*(a + b*x)^(1/3))/(4*b^2)
```

Maple [A] time = 0.001, size = 21, normalized size = 0.7

$$-\frac{-3\,bx+9\,a}{4\,b^2}\sqrt[3]{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(2/3),x)
```

```
[Out] -3/4*(b*x+a)^(1/3)*(-b*x+3*a)/b^2
```

Maxima [A] time = 1.04654, size = 35, normalized size = 1.09

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b^2}-\frac{3\,(bx+a)^{\frac{1}{3}}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b^2 - 3*(b*x + a)^(1/3)*a/b^2
```

Fricas [A] time = 1.48452, size = 50, normalized size = 1.56

$$\frac{3\,(bx+a)^{\frac{1}{3}}(bx-3\,a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x + a)^(1/3)*(b*x - 3*a)/b^2
```

Sympy [B] time = 1.70781, size = 162, normalized size = 5.06

$$-\frac{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}+\frac{9a^{\frac{10}{3}}}{4a^2b^2+4ab^3x}-\frac{6a^{\frac{7}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}+\frac{9a^{\frac{7}{3}}bx}{4a^2b^2+4ab^3x}+\frac{3a^{\frac{4}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(2/3),x)
```

```
[Out] -9*a**(10/3)*(1 + b*x/a)**(1/3)/(4*a**2*b**2 + 4*a*b**3*x) + 9*a**(10/3)/(4
*a**2*b**2 + 4*a*b**3*x) - 6*a**(7/3)*b*x*(1 + b*x/a)**(1/3)/(4*a**2*b**2 +
 4*a*b**3*x) + 9*a**(7/3)*b*x/(4*a**2*b**2 + 4*a*b**3*x) + 3*a**(4/3)*b**2*
x**2*(1 + b*x/a)**(1/3)/(4*a**2*b**2 + 4*a*b**3*x)
```

Giac [A] time = 1.10816, size = 31, normalized size = 0.97

$$\frac{3\left((bx+a)^{\frac{4}{3}}-4\,(bx+a)^{\frac{1}{3}}a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/4*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)/b^2
```

3.409 $\int \frac{1}{(a+bx)^{2/3}} dx$

Optimal. Leaf size=14

$$\frac{3\sqrt[3]{a+bx}}{b}$$

```
[Out] (3*(a + b*x)^(1/3))/b
```

Rubi [A] time = 0.001359, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3))/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}} dx = \frac{3\sqrt[3]{a+bx}}{b}$$

Mathematica [A] time = 0.0052719, size = 14, normalized size = 1.

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3))/b
```

Maple [A] time = 0.002, size = 13, normalized size = 0.9

$$3\,\frac{\sqrt[3]{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3),x)
```

```
[Out] 3*(b*x+a)^(1/3)/b
```

Maxima [A] time = 1.07606, size = 16, normalized size = 1.14

$$\frac{3\,(bx+a)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3*(b*x + a)^(1/3)/b
```

Fricas [A] time = 1.51201, size = 28, normalized size = 2.

$$\frac{3\,(bx+a)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3*(b*x + a)^(1/3)/b
```

Sympy [A] time = 0.102487, size = 10, normalized size = 0.71

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3),x)
```

```
[Out] 3*(a + b*x)**(1/3)/b
```

Giac [A] time = 1.24887, size = 16, normalized size = 1.14

$$\frac{3\,(bx+a)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3*(b*x + a)^(1/3)/b
```

3.410 $\int \frac{1}{x(a+bx)^{2/3}} dx$

Optimal. Leaf size=80

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(2/3)
) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(2/3))
```

Rubi [A] time = 0.0238317, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {57, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(2/3)
) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(2/3))
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x(a+bx)^{2/3}}\,dx = -\frac{\log(x)}{2a^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2a^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{2/3}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}$$

Mathematica [A] time = 0.136673, size = 93, normalized size = 1.16

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(2/3)),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/
3) - (a + b*x)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(
2/3)])/(2*a^(2/3))
```

Maple [A] time = 0.004, size = 76, normalized size = 1.

$$\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)a^{-\frac{2}{3}} - \frac{1}{2}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)a^{-\frac{2}{3}} - \sqrt{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{bx+a}}{\sqrt[3]{a}}+1\right)\right)a^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(2/3),x)
```

```
[Out] 1/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b
*x+a)^(1/3)+a^(2/3))-1/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a
)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.55426, size = 346, normalized size = 4.32

$$-\frac{2\sqrt{3}\left(a^2\right)^{\frac{1}{6}} a \arctan\left(\frac{\sqrt{3}\left(a^2\right)^{\frac{1}{6}}\left(\left(a^2\right)^{\frac{1}{3}} a+2\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^2}\right)+\left(a^2\right)^{\frac{2}{3}} \log\left((bx+a)^{\frac{2}{3}} a+\left(a^2\right)^{\frac{1}{3}} a+\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\left(a^2\right)^{\frac{2}{3}} \log\left((b\right.}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*sqrt(3)*(a^2)^(1/6)*((a^2)^(1/3)*a
 + 2*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + (a^2)^(2/3)*log((b*x + a)^(2/3)*a
+ (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 2*(a^2)^(2/3)*log((b*x + a
)^(1/3)*a - (a^2)^(2/3)))/a^2
```

Sympy [C] time = 3.08918, size = 150, normalized size = 1.88

$$\frac{\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(2/3),x)
```

```
[Out] log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(3*a**(2/3)*gamma(4/
3)) + exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/
a**(1/3))*gamma(1/3)/(3*a**(2/3)*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b**(1/
3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(3*a**(2/3)*ga
mma(4/3))
```

Giac [A] time = 1.87057, size = 105, normalized size = 1.31

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{2}{3}}}+\frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3)
- 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) + lo
g(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3)
```

3.411 $\int \frac{1}{x^2(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=98

$$\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}-\frac{\sqrt[3]{a+bx}}{ax}$$

```
[Out] -((a + b*x)^(1/3)/(a*x)) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (b*Log[x])/(3*a^(5/3)) - (b*Log[a^(1/3) -
 (a + b*x)^(1/3)])/a^(5/3)
```

Rubi [A] time = 0.0328974, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 57, 617, 204, 31}

$$\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}-\frac{\sqrt[3]{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(2/3)),x]
```

```
[Out] -((a + b*x)^(1/3)/(a*x)) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (b*Log[x])/(3*a^(5/3)) - (b*Log[a^(1/3) -
 (a + b*x)^(1/3)])/a^(5/3)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(a+bx)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx}}{ax} - \frac{(2b)\int \frac{1}{x(a+bx)^{2/3}}\,dx}{3a} \\
&= -\frac{\sqrt[3]{a+bx}}{ax} + \frac{b\log(x)}{3a^{5/3}} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{a^{5/3}} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx}\right)}{a^{4/3}} \\
&= -\frac{\sqrt[3]{a+bx}}{ax} + \frac{b\log(x)}{3a^{5/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{5/3}} \\
&= -\frac{\sqrt[3]{a+bx}}{ax} + \frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{5/3}} + \frac{b\log(x)}{3a^{5/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.0322353, size = 31, normalized size = 0.32

$$\frac{3b\sqrt[3]{a+bx}\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{bx}{a}+1\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(2/3)),x]
```

```
[Out] (3*b*(a + b*x)^(1/3)*Hypergeometric2F1[1/3, 2, 4/3, 1 + (b*x)/a])/a^2
```

Maple [A] time = 0.006, size = 95, normalized size = 1.

$$-\frac{1}{ax}\sqrt[3]{bx+a} - \frac{2b}{3}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)a^{-\frac{5}{3}} + \frac{b}{3}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)a^{-\frac{5}{3}} + \frac{2b\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{bx+a}}{\sqrt[3]{a}}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(2/3),x)
```

```
[Out] -(b*x+a)^(1/3)/a/x-2/3*b/a^(5/3)*ln((b*x+a)^(1/3)-a^(1/3))+1/3*b/a^(5/3)*ln
((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+2/3*b/a^(5/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.57891, size = 441, normalized size = 4.5

$$\frac{2\sqrt{3}abx\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(-a^2\right)^{\frac{2}{3}}\right.}{3a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*a*b*x*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3)*
a - 2*sqrt(3)*(-a^2)^(2/3)*(b*x + a)^(1/3))*sqrt(-(-a^2)^(1/3))/a^2) + (-a^
2)^(2/3)*b*x*log((b*x + a)^(2/3)*a - (-a^2)^(1/3)*a + (-a^2)^(2/3)*(b*x + a
)^(1/3)) - 2*(-a^2)^(2/3)*b*x*log((b*x + a)^(1/3)*a - (-a^2)^(2/3)) - 3*(b*
x + a)^(1/3)*a^2)/(a^3*x)
```

Sympy [C] time = 3.7463, size = 830, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(2/3),x)
```

```
[Out] -2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*p
i/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)
) - 2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*
exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*
exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*g
amma(4/3)) - 2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*exp(-2*I*pi/3)*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b*
*(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x
)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b + x)**(5/3)*e
xp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(9*a**3
*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b
+ x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b + x)**(5/3
)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3
)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5
/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b +
 x)**(5/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*g
amma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 3*
a*b**2*(a/b + x)*exp(2*I*pi/3)*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)
*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*
gamma(4/3))
```

Giac [A] time = 2.24808, size = 146, normalized size = 1.49

$$\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3(bx+a)^{\frac{1}{3}}b}{ax}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 1/3*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(5/3) + b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(5/3) - 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(b*x + a)^(1/
3)*b/(a*x))/b
```

3.412 $\int \frac{1}{x^3(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=130

$$-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

```
[Out] -(a + b*x)^(1/3)/(2*a*x^2) + (5*b*(a + b*x)^(1/3))/(6*a^2*x) - (5*b^2*ArcTa
n[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) - (
5*b^2*Log[x])/(18*a^(8/3)) + (5*b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(8
/3))
```

Rubi [A] time = 0.0475448, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 57, 617, 204, 31}

$$-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(2/3)),x]
```

```
[Out] -(a + b*x)^(1/3)/(2*a*x^2) + (5*b*(a + b*x)^(1/3))/(6*a^2*x) - (5*b^2*ArcTa
n[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) - (
5*b^2*Log[x])/(18*a^(8/3)) + (5*b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(8
/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx}}{2ax^2}-\frac{(5b)\int \frac{1}{x^2(a+bx)^{2/3}}\,dx}{6a}\\ &= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}+\frac{\left(5b^2\right)\int \frac{1}{x(a+bx)^{2/3}}\,dx}{9a^2}\\ &= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\log(x)}{18a^{8/3}}-\frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{8/3}}-\frac{\left(5b^2\right)\text{Subst}\left(\int \frac{}{a^{2/3}}\right.}{}\\ &= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}+\frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\right.}{3a^{8/3}}\\ &= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}\end{aligned}$$

Mathematica [C] time = 0.0092028, size = 33, normalized size = 0.25

$$-\frac{3b^2\sqrt[3]{a+bx}\,{}_2F_1\left(\frac{1}{3},3;\frac{4}{3};\frac{bx}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(2/3)),x]
```

```
[Out] (-3*b^2*(a + b*x)^(1/3)*Hypergeometric2F1[1/3, 3, 4/3, 1 + (b*x)/a])/a^3
```

Maple [A] time = 0.006, size = 117, normalized size = 0.9

$$-\frac{1}{2\,ax^2}\sqrt[3]{bx+a}+\frac{5\,b}{6\,a^2x}\sqrt[3]{bx+a}+\frac{5\,b^2}{9}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)a^{-\frac{8}{3}}-\frac{5\,b^2}{18}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)a^{-\frac{8}{3}}-\frac{5\,b^2\sqrt{3}}{9}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(2/3),x)
```

```
[Out] -1/2*(b*x+a)^(1/3)/a/x^2+5/6*b*(b*x+a)^(1/3)/a^2/x+5/9*b^2/a^(8/3)*ln((b*x+
a)^(1/3)-a^(1/3))-5/18*b^2/a^(8/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a
^(2/3))-5/9*b^2/a^(8/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)
+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57479, size = 458, normalized size = 3.52

$$-\frac{10\sqrt{3}\left(a^2\right)^{\frac{1}{6}}ab^2x^2\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^2}\right)+5\left(a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{18a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/18*(10*sqrt(3)*(a^2)^(1/6)*a*b^2*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^
2)^(1/3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + 5*(a^2)^(2/3)*b^
2*x^2*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3))
- 10*(a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) - 3*(5*a^2*b*
x - 3*a^3)*(b*x + a)^(1/3))/(a^4*x^2)
```

Sympy [C] time = 4.2537, size = 2728, normalized size = 20.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(2/3),x)
```

```
[Out] 10*a**(13/3)*b**(8/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I
*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(
4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**
4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 10*a**(13/3)*b**(
8/3)*(a/b + x)**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)
/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamm
a(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*
a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)
*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 10*a**(13/3)*b**(8/3)*(a/b +
 x)**(2/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*
gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) +
162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(1
1/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(
a/b + x)**(5/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*g
amma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162
```

```
*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3
)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**
(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(a/b + x)**(5/3)*
log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/
(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(
5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x
)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp
(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(a/b + x)**(5/3)*exp(-2*I*p
i/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6
*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/
b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3
)*exp(2*I*pi/3)*gamma(4/3)) + 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*exp(2*
I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**
(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b +
x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp
(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*g
amma(4/3)) + 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b
+ x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*e
xp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*
gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3))
+ 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a
/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(
a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/
3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi
/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/
3)) - 10*a**(4/3)*b**(17/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*log(1 - b**(1/3
)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*
exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)
*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) -
 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 10*a**(4/3
)*b**(17/3)*(a/b + x)**(11/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2
*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi
/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3
) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b
**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 10*a**(4/3)*b**(17/3
)*(a/b + x)**(11/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2
*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamm
a(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a
**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 24*a**4*b**3*(a
/b + x)*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I
*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(
4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**
4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 39*a**3*b**4*(a/b
 + x)**2*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*
I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma
(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a*
*4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 15*a**2*b**5*(a/
b + x)**3*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2
*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamm
a(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a
**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3))
```

Giac [A] time = 2.35361, size = 176, normalized size = 1.35

$$-\frac{\frac{10\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{8}{3}}}+\frac{5b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{8}{3}}}-\frac{10b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{8}{3}}}-\frac{3\left(5(bx+a)^{\frac{4}{3}}b^3-8(bx+a)^{\frac{1}{3}}ab^3\right)}{a^2b^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] -1/18*(10*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(8/3) + 5*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(8/3) - 10*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(8/3) - 3*(5*(b*x
 + a)^(4/3)*b^3 - 8*(b*x + a)^(1/3)*a*b^3)/(a^2*b^2*x^2))/b
```

3.413 $\int \frac{x^3}{(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=70

$$\frac{3a^3}{b^4\sqrt[3]{a+bx}}+\frac{9a^2(a+bx)^{2/3}}{2b^4}-\frac{9a(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{8/3}}{8b^4}$$

```
[Out] (3*a^3)/(b^4*(a + b*x)^(1/3)) + (9*a^2*(a + b*x)^(2/3))/(2*b^4) - (9*a*(a + b*x)^(5/3))/(5*b^4) + (3*(a + b*x)^(8/3))/(8*b^4)
```

Rubi [A] time = 0.0178824, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^3}{b^4\sqrt[3]{a+bx}}+\frac{9a^2(a+bx)^{2/3}}{2b^4}-\frac{9a(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(4/3),x]
```

```
[Out] (3*a^3)/(b^4*(a + b*x)^(1/3)) + (9*a^2*(a + b*x)^(2/3))/(2*b^4) - (9*a*(a + b*x)^(5/3))/(5*b^4) + (3*(a + b*x)^(8/3))/(8*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{4/3}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{4/3}}+\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{(a+bx)^{5/3}}{b^3}\right)dx\\ &= \frac{3a^3}{b^4\sqrt[3]{a+bx}}+\frac{9a^2(a+bx)^{2/3}}{2b^4}-\frac{9a(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{8/3}}{8b^4}\end{aligned}$$

Mathematica [A] time = 0.0301732, size = 46, normalized size = 0.66

$$\frac{3\left(27a^2bx+81a^3-9ab^2x^2+5b^3x^3\right)}{40b^4\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(4/3),x]
```

```
[Out] (3*(81*a^3 + 27*a^2*b*x - 9*a*b^2*x^2 + 5*b^3*x^3))/(40*b^4*(a + b*x)^(1/3))
```

Maple [A] time = 0.004, size = 43, normalized size = 0.6

$$\frac{15\,b^{3}x^{3}-27\,ab^{2}x^{2}+81\,a^{2}bx+243\,a^{3}}{40\,b^{4}}\frac{1}{\sqrt[3]{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(4/3),x)
```

```
[Out] 3/40/(b*x+a)^(1/3)*(5*b^3*x^3-9*a*b^2*x^2+27*a^2*b*x+81*a^3)/b^4
```

Maxima [A] time = 1.11649, size = 76, normalized size = 1.09

$$\frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^{4}}-\frac{9\,(bx+a)^{\frac{5}{3}}a}{5\,b^{4}}+\frac{9\,(bx+a)^{\frac{2}{3}}a^{2}}{2\,b^{4}}+\frac{3\,a^{3}}{(bx+a)^{\frac{1}{3}}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)^(8/3)/b^4 - 9/5*(b*x + a)^(5/3)*a/b^4 + 9/2*(b*x + a)^(2/3)*a
^2/b^4 + 3*a^3/((b*x + a)^(1/3)*b^4)
```

Fricas [A] time = 1.512, size = 116, normalized size = 1.66

$$\frac{3\left(5\,b^{3}x^{3}-9\,ab^{2}x^{2}+27\,a^{2}bx+81\,a^{3}\right)(bx+a)^{\frac{2}{3}}}{40\left(b^{5}x+ab^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^3*x^3 - 9*a*b^2*x^2 + 27*a^2*b*x + 81*a^3)*(b*x + a)^(2/3)/(b^5*x
 + a*b^4)
```

Sympy [B] time = 3.92976, size = 1538, normalized size = 21.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(4/3),x)
```

```
[Out] 243*a**(68/3)*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a*
*18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*
x**5 + 40*a**14*b**10*x**6) - 243*a**(68/3)/(40*a**20*b**4 + 240*a**19*b**5
*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*
a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 1296*a**(65/3)*b*x*(1 + b*x/a)**(2
```

```
/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**
7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) -
 1458*a**(65/3)*b*x/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2
 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**
14*b**10*x**6) + 2808*a**(62/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a**20*b**4
 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16
*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) - 3645*a**(62/3)*b*
*2*x**2/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17
*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**
6) + 3120*a**(59/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19
*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 +
 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) - 4860*a**(59/3)*b**3*x**3/(40*
a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 +
 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 1830*a*
*(56/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 60
0*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b
**9*x**5 + 40*a**14*b**10*x**6) - 3645*a**(56/3)*b**4*x**4/(40*a**20*b**4 +
 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b
**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 528*a**(53/3)*b**5*
x**5*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*
x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40
*a**14*b**10*x**6) - 1458*a**(53/3)*b**5*x**5/(40*a**20*b**4 + 240*a**19*b*
*5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 24
0*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 96*a**(50/3)*b**6*x**6*(1 + b*x/
a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**
17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x
**6) - 243*a**(50/3)*b**6*x**6/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**1
8*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x*
*5 + 40*a**14*b**10*x**6) + 48*a**(47/3)*b**7*x**7*(1 + b*x/a)**(2/3)/(40*a
**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 +
600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 15*a**(4
4/3)*b**8*x**8*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a
**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9
*x**5 + 40*a**14*b**10*x**6)
```

Giac [A] time = 1.20517, size = 84, normalized size = 1.2

$$\frac{3\,a^3}{(bx+a)^{\frac{1}{3}}b^4}+\frac{3\left(5\,(bx+a)^{\frac{8}{3}}b^{28}-24\,(bx+a)^{\frac{5}{3}}ab^{28}+60\,(bx+a)^{\frac{2}{3}}a^2b^{28}\right)}{40\,b^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3*a^3/((b*x + a)^(1/3)*b^4) + 3/40*(5*(b*x + a)^(8/3)*b^28 - 24*(b*x + a)^(
5/3)*a*b^28 + 60*(b*x + a)^(2/3)*a^2*b^28)/b^32
```

3.414 $\int \frac{x^2}{(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=49

$$-\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}$$

```
[Out] (-3*a^2)/(b^3*(a + b*x)^(1/3)) - (3*a*(a + b*x)^(2/3))/b^3 + (3*(a + b*x)^(
5/3))/(5*b^3)
```

Rubi [A] time = 0.0133071, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(4/3),x]
```

```
[Out] (-3*a^2)/(b^3*(a + b*x)^(1/3)) - (3*a*(a + b*x)^(2/3))/b^3 + (3*(a + b*x)^(
5/3))/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{4/3}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{4/3}}-\frac{2a}{b^2\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b^2}\right)dx\\ &= -\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.0900145, size = 34, normalized size = 0.69

$$\frac{3\left(-9a^2-3abx+b^2x^2\right)}{5b^3\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(4/3),x]
```

```
[Out] (3*(-9*a^2 - 3*a*b*x + b^2*x^2))/(5*b^3*(a + b*x)^(1/3))
```

Maple [A] time = 0.005, size = 32, normalized size = 0.7

$$-\frac{-3 b^2x^2+9 abx+27 a^2}{5 b^3}\frac{1}{\sqrt[3]{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(4/3),x)
```

```
[Out] -3/5/(b*x+a)^(1/3)*(-b^2*x^2+3*a*b*x+9*a^2)/b^3
```

Maxima [A] time = 1.06022, size = 55, normalized size = 1.12

$$\frac{3 (bx+a)^{\frac{5}{3}}}{5 b^3}-\frac{3 (bx+a)^{\frac{2}{3}} a}{b^3}-\frac{3 a^2}{(bx+a)^{\frac{1}{3}} b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b^3 - 3*(b*x + a)^(2/3)*a/b^3 - 3*a^2/((b*x + a)^(1/3)*
b^3)
```

Fricas [A] time = 1.52898, size = 88, normalized size = 1.8

$$\frac{3 \left(b^2x^2-3 abx-9 a^2\right)(bx+a)^{\frac{2}{3}}}{5 \left(b^4x+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/5*(b^2*x^2 - 3*a*b*x - 9*a^2)*(b*x + a)^(2/3)/(b^4*x + a*b^3)
```

Sympy [B] time = 2.52779, size = 534, normalized size = 10.9

$$-\frac{27a^{\frac{29}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}+\frac{27a^{\frac{29}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}-\frac{63a^{\frac{26}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(4/3),x)
```

```
[Out] -27*a**(29/3)*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b*
*5*x**2 + 5*a**5*b**6*x**3) + 27*a**(29/3)/(5*a**8*b**3 + 15*a**7*b**4*x +
15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) - 63*a**(26/3)*b*x*(1 + b*x/a)**(2/3)
/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 81
*a**(26/3)*b*x/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b
**6*x**3) - 42*a**(23/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**
```

```
7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 81*a**(23/3)*b**2*x**2/(
5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) - 3*a*
*(20/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**
6*b**5*x**2 + 5*a**5*b**6*x**3) + 27*a**(20/3)*b**3*x**3/(5*a**8*b**3 + 15*
a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 3*a**(17/3)*b**4*x**4
*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a
**5*b**6*x**3)
```

Giac [A] time = 1.17738, size = 62, normalized size = 1.27

$$-\frac{3\,a^2}{(bx+a)^{\frac{1}{3}}b^3}+\frac{3\left((bx+a)^{\frac{5}{3}}b^{12}-5\,(bx+a)^{\frac{2}{3}}ab^{12}\right)}{5\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] -3*a^2/((b*x + a)^(1/3)*b^3) + 3/5*((b*x + a)^(5/3)*b^12 - 5*(b*x + a)^(2/3
)*a*b^12)/b^15
```

3.415 $\int \frac{x}{(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=32

$$\frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}$$

```
[Out] (3*a)/(b^2*(a + b*x)^(1/3)) + (3*(a + b*x)^(2/3))/(2*b^2)
```

Rubi [A] time = 0.0085798, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(4/3),x]
```

```
[Out] (3*a)/(b^2*(a + b*x)^(1/3)) + (3*(a + b*x)^(2/3))/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{4/3}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{4/3}}+\frac{1}{b\sqrt[3]{a+bx}}\right)dx\\ &= \frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.04836, size = 23, normalized size = 0.72

$$\frac{3(3a+bx)}{2b^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(4/3),x]
```

```
[Out] (3*(3*a + b*x))/(2*b^2*(a + b*x)^(1/3))
```

Maple [A] time = 0.003, size = 20, normalized size = 0.6

$$\frac{3\,bx+9\,a}{2\,b^2}\frac{1}{\sqrt[3]{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(4/3),x)
```

```
[Out] 3/2/(b*x+a)^(1/3)*(b*x+3*a)/b^2
```

Maxima [A] time = 1.02825, size = 35, normalized size = 1.09

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b^2}+\frac{3\,a}{(bx+a)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b^2 + 3*a/((b*x + a)^(1/3)*b^2)
```

Fricas [A] time = 1.60112, size = 66, normalized size = 2.06

$$\frac{3\,(bx+3\,a)(bx+a)^{\frac{2}{3}}}{2\left(b^3x+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x + 3*a)*(b*x + a)^(2/3)/(b^3*x + a*b^2)
```

Sympy [A] time = 1.04138, size = 41, normalized size = 1.28

$$\begin{cases} \frac{9a}{2b^2\sqrt[3]{a+bx}}+\frac{3x}{2b\sqrt[3]{a+bx}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(4/3),x)
```

```
[Out] Piecewise((9*a/(2*b**2*(a + b*x)**(1/3)) + 3*x/(2*b*(a + b*x)**(1/3)), Ne(b
, 0)), (x**2/(2*a**(4/3)), True))
```

Giac [A] time = 1.21717, size = 41, normalized size = 1.28

$$\frac{3\left(\frac{(bx+a)^{\frac{2}{3}}}{b}+\frac{2\,a}{(bx+a)^{\frac{1}{3}}b}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/2*((b*x + a)^(2/3)/b + 2*a/((b*x + a)^(1/3)*b))/b
```

3.416 $\int \frac{1}{(a+bx)^{4/3}} dx$

Optimal. Leaf size=14

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

```
[Out] -3/(b*(a + b*x)^(1/3))
```

Rubi [A] time = 0.0013637, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4/3),x]
```

```
[Out] -3/(b*(a + b*x)^(1/3))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}} dx = -\frac{3}{b\sqrt[3]{a+bx}}$$

Mathematica [A] time = 0.0122369, size = 14, normalized size = 1.

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4/3),x]
```

```
[Out] -3/(b*(a + b*x)^(1/3))
```

Maple [A] time = 0.002, size = 13, normalized size = 0.9

$$-3\frac{1}{b\sqrt[3]{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3),x)

[Out] -3/b/(b*x+a)^(1/3)
```

Maxima [A] time = 1.01713, size = 16, normalized size = 1.14

$$-\frac{3}{(bx+a)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="maxima")

[Out] -3/((b*x + a)^(1/3)*b)
```

Fricas [A] time = 1.44181, size = 46, normalized size = 3.29

$$-\frac{3(bx+a)^{\frac{2}{3}}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="fricas")

[Out] -3*(b*x + a)^(2/3)/(b^2*x + a*b)
```

Sympy [A] time = 0.107064, size = 12, normalized size = 0.86

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3),x)

[Out] -3/(b*(a + b*x)**(1/3))
```

Giac [A] time = 1.20436, size = 16, normalized size = 1.14

$$-\frac{3}{(bx+a)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="giac")

[Out] -3/((b*x + a)^(1/3)*b)
```

3.417 $\int \frac{1}{x(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=93

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{a\sqrt[3]{a+bx}}$$

```
[Out] 3/(a*(a + b*x)^(1/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt
[3]*a^(1/3))])/a^(4/3) - Log[x]/(2*a^(4/3)) + (3*Log[a^(1/3) - (a + b*x)^(1
/3)])/(2*a^(4/3))
```

Rubi [A] time = 0.0331465, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{a\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(4/3)),x]
```

```
[Out] 3/(a*(a + b*x)^(1/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt
[3]*a^(1/3))])/a^(4/3) - Log[x]/(2*a^(4/3)) + (3*Log[a^(1/3) - (a + b*x)^(1
/3)])/(2*a^(4/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx)^{4/3}}\,dx &= \frac{3}{a\sqrt[3]{a+bx}}+\frac{\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{a}\\
&= \frac{3}{a\sqrt[3]{a+bx}}-\frac{\log(x)}{2a^{4/3}}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx,x,\sqrt[3]{a+bx}\right)}{2a}\\
&= \frac{3}{a\sqrt[3]{a+bx}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{4/3}}\\
&= \frac{3}{a\sqrt[3]{a+bx}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.0222593, size = 30, normalized size = 0.32

$$\frac{3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{bx}{a}+1\right)}{a\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(4/3)),x]
```

```
[Out] (3*Hypergeometric2F1[-1/3, 1, 2/3, 1 + (b*x)/a])/(a*(a + b*x)^(1/3))
```

Maple [A] time = 0.006, size = 87, normalized size = 0.9

$$\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)a^{-\frac{4}{3}}-\frac{1}{2}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)a^{-\frac{4}{3}}+\sqrt{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{bx+a}}{\sqrt[3]{a}}+1\right)\right)a^{-\frac{4}{3}}+3\frac{1}{a\sqrt[3]{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(4/3),x)
```

```
[Out] 1/a^(4/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2/a^(4/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b
*x+a)^(1/3)+a^(2/3))+1/a^(4/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a
)^(1/3)+1))+3/a/(b*x+a)^(1/3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63811, size = 834, normalized size = 8.97

$$\left[\frac{\sqrt{3}\left(abx+a^{2}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}\right.}{2\left(a^{2}bx+a^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*(a*b*x + a^2)*sqrt(-1/a^(2/3))*log((2*b*x + sqrt(3)*(2*(b*x +
 a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x
+ a)^(1/3)*a^(2/3) + 3*a)/x) - (b*x + a)*a^(2/3)*log((b*x + a)^(2/3) + (b*x
 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b*x + a)*a^(2/3)*log((b*x + a)^(1/3) -
a^(1/3)) + 6*(b*x + a)^(2/3)*a)/(a^2*b*x + a^3), -1/2*((b*x + a)*a^(2/3)*lo
g((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*(b*x + a)*a^(2/3
)*log((b*x + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*(a*b*x + a^2)*arctan(1/3*sqrt(
3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 6*(b*x + a)^(2/3)*a)/(a
^2*b*x + a^3)]
```

Sympy [C] time = 2.92168, size = 184, normalized size = 1.98

$$-\frac{\Gamma\left(-\frac{1}{3}\right)}{a\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}-\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right.}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(4/3),x)
```

```
[Out] -gamma(-1/3)/(a*b**(1/3)*(a/b + x)**(1/3)*gamma(2/3)) - log(1 - b**(1/3)*(a
/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*gamma(2/3)) - exp(2*I*pi/3
)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/
3)/(3*a**(4/3)*gamma(2/3)) - exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/
3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*gamma(2/3))
```

Giac [A] time = 2.31149, size = 120, normalized size = 1.29

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{4}{3}}}+\frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3}{(bx+a)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) + log
(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 3/((b*x + a)^(1/3)*a)
```

3.418 $\int \frac{1}{x^2(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=113

$$-\frac{4b}{a^2\sqrt[3]{a+bx}}+\frac{2b\log(x)}{3a^{7/3}}-\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}}-\frac{1}{ax\sqrt[3]{a+bx}}$$

```
[Out] (-4*b)/(a^2*(a + b*x)^(1/3)) - 1/(a*x*(a + b*x)^(1/3)) - (4*b*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (2*b*Log[x]
)/(3*a^(7/3)) - (2*b*Log[a^(1/3) - (a + b*x)^(1/3)])/a^(7/3)
```

Rubi [A] time = 0.0437349, antiderivative size = 115, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$-\frac{4(a+bx)^{2/3}}{a^2x}+\frac{2b\log(x)}{3a^{7/3}}-\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}}+\frac{3}{ax\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(4/3)),x]
```

```
[Out] 3/(a*x*(a + b*x)^(1/3)) - (4*(a + b*x)^(2/3))/(a^2*x) - (4*b*ArcTan[(a^(1/3
) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (2*b*Log[x])
/(3*a^(7/3)) - (2*b*Log[a^(1/3) - (a + b*x)^(1/3)])/a^(7/3)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(a+bx)^{4/3}}\,dx &= \frac{3}{ax\sqrt[3]{a+bx}} + \frac{4\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{a}\\
&= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} - \frac{(4b)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{3a^2}\\
&= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} + \frac{2b\log(x)}{3a^{7/3}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{a^{7/3}} - \frac{(2b)\,\text{Subst}\left(\int \frac{}{a^{2/3}+}\right.}{}\\
&= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} + \frac{2b\log(x)}{3a^{7/3}} - \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}} + \frac{(4b)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2}{}\right.}{a^{7/3}}\\
&= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} - \frac{4b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{7/3}} + \frac{2b\log(x)}{3a^{7/3}} - \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}
\end{aligned}$$

Mathematica [C] time = 0.0518632, size = 31, normalized size = 0.27

$$-\frac{3b\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{bx}{a}+1\right)}{a^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(4/3)),x]
```

```
[Out] (-3*b*Hypergeometric2F1[-1/3, 2, 2/3, 1 + (b*x)/a])/(a^2*(a + b*x)^(1/3))
```

Maple [A] time = 0.011, size = 108, normalized size = 1.

$$-3\,\frac{b}{a^2\sqrt[3]{bx+a}} - \frac{1}{a^2x}(bx+a)^{\frac{2}{3}} - \frac{4\,b}{3}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)a^{-\frac{7}{3}} + \frac{2\,b}{3}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+a^{\frac{2}{3}}\right)a^{-\frac{7}{3}} - \frac{4\,b\sqrt{3}}{3}\,\text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(4/3),x)
```

```
[Out] -3*b/a^2/(b*x+a)^(1/3)-1/a^2*(b*x+a)^(2/3)/x-4/3*b/a^(7/3)*ln((b*x+a)^(1/3)
-a^(1/3))+2/3*b/a^(7/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-4/3
*b/a^(7/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.6863, size = 1062, normalized size = 9.4

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(ab^2x^2 + a^2bx\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx-3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3\,a}{x}\right) + 2\left(b^2x^2 + abx\right)(-a)}{3\left(a^3bx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/3*(6*sqrt(1/3)*(a*b^2*x^2 + a^2*b*x)*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*s
qrt(1/3)*(2*(b*x + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*
sqrt((-a)^(1/3)/a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + 2*(b^2*x^2 +
a*b*x)*(-a)^(2/3)*log((b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(
2/3)) - 4*(b^2*x^2 + a*b*x)*(-a)^(2/3)*log((b*x + a)^(1/3) + (-a)^(1/3)) -
3*(4*a*b*x + a^2)*(b*x + a)^(2/3))/(a^3*b*x^2 + a^4*x), -1/3*(12*sqrt(1/3)*
(a*b^2*x^2 + a^2*b*x)*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/
3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - 2*(b^2*x^2 + a*b*x)*(-a)^(2/3)*log(
(b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 4*(b^2*x^2 + a
*b*x)*(-a)^(2/3)*log((b*x + a)^(1/3) + (-a)^(1/3)) + 3*(4*a*b*x + a^2)*(b*x
 + a)^(2/3))/(a^3*b*x^2 + a^4*x)]
```

Sympy [C] time = 3.28327, size = 857, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(4/3),x)
```

```
[Out] -9*a**(4/3)*b**(2/3)*exp(2*I*pi/3)*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/
3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*g
amma(2/3)) + 12*a**(1/3)*b**(5/3)*(a/b + x)*exp(2*I*pi/3)*gamma(-1/3)/(-9*a
**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)
**(4/3)*exp(2*I*pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b +
x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*
```

```
pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*
(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a
/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*log(1 - b**(1/3)*(a/b + x)*
*(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(
1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)
*gamma(2/3)) + 4*b**2*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi
/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3)) +
4*b**2*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*ex
p_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(
2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/
3)) + 4*b**2*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4
*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)
*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3))
```

Giac [A] time = 1.81187, size = 162, normalized size = 1.43

$$-\frac{4\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{7}{3}}}+\frac{2b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{7}{3}}}-\frac{4b\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3a^{\frac{7}{3}}}-\frac{4(bx+a}{\left((bx+a)^{\frac{4}{3}}-(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(7/3) + 2/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(7
/3) - 4/3*b*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(7/3) - (4*(b*x + a)*b -
3*a*b)/(((b*x + a)^(4/3) - (b*x + a)^(1/3)*a)*a^2)
```

3.419 $\int \frac{1}{x^3(a+bx)^{4/3}} dx$

Optimal. Leaf size=149

$$\frac{14b^2}{3a^3\sqrt[3]{a+bx}} - \frac{7b^2\log(x)}{9a^{10/3}} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{10/3}} + \frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}} + \frac{7b}{6a^2x\sqrt[3]{a+bx}} - \frac{1}{2ax^2\sqrt[3]{a+bx}}$$

```
[Out] (14*b^2)/(3*a^3*(a + b*x)^(1/3)) - 1/(2*a*x^2*(a + b*x)^(1/3)) + (7*b)/(6*a
^2*x*(a + b*x)^(1/3)) + (14*b^2*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[
3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (7*b^2*Log[x])/(9*a^(10/3)) + (7*b^2*L
og[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(10/3))
```

Rubi [A] time = 0.0579336, antiderivative size = 147, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$-\frac{7b^2\log(x)}{9a^{10/3}} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{10/3}} + \frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} + \frac{3}{ax^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(4/3)),x]
```

```
[Out] 3/(a*x^2*(a + b*x)^(1/3)) - (7*(a + b*x)^(2/3))/(2*a^2*x^2) + (14*b*(a + b*
x)^(2/3))/(3*a^3*x) + (14*b^2*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]
*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (7*b^2*Log[x])/(9*a^(10/3)) + (7*b^2*Log
[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(10/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^{4/3}}\,dx &= \frac{3}{ax^2\sqrt[3]{a+bx}}+\frac{7\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx}{a}\\ &= \frac{3}{ax^2\sqrt[3]{a+bx}}-\frac{7(a+bx)^{2/3}}{2a^2x^2}-\frac{(14b)\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{3a^2}\\ &= \frac{3}{ax^2\sqrt[3]{a+bx}}-\frac{7(a+bx)^{2/3}}{2a^2x^2}+\frac{14b(a+bx)^{2/3}}{3a^3x}+\frac{(14b^2)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a^3}\\ &= \frac{3}{ax^2\sqrt[3]{a+bx}}-\frac{7(a+bx)^{2/3}}{2a^2x^2}+\frac{14b(a+bx)^{2/3}}{3a^3x}-\frac{7b^2\log(x)}{9a^{10/3}}-\frac{(7b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{3a^{10/3}}+\\ &= \frac{3}{ax^2\sqrt[3]{a+bx}}-\frac{7(a+bx)^{2/3}}{2a^2x^2}+\frac{14b(a+bx)^{2/3}}{3a^3x}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{10/3}}-\frac{(14b^2)\,\text{Sub}}{}\\ &= \frac{3}{ax^2\sqrt[3]{a+bx}}-\frac{7(a+bx)^{2/3}}{2a^2x^2}+\frac{14b(a+bx)^{2/3}}{3a^3x}+\frac{14b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{10/3}}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2\log\left(\sqrt[3]{a}-\right.}{3a^{1}}\end{aligned}$$

Mathematica [C] time = 0.0289249, size = 33, normalized size = 0.22

$$\frac{3b^2\,{}_2F_1\left(-\frac{1}{3},3;\frac{2}{3};\frac{bx}{a}+1\right)}{a^3\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(4/3)),x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-1/3, 3, 2/3, 1 + (b*x)/a])/(a^3*(a + b*x)^(1/3))
```

Maple [A] time = 0.012, size = 131, normalized size = 0.9

$$3\,\frac{b^2}{a^3\sqrt[3]{bx+a}}+\frac{5}{3\,a^3x^2}(bx+a)^{\frac{5}{3}}-\frac{13}{6\,a^2x^2}(bx+a)^{\frac{2}{3}}+\frac{14\,b^2}{9}\ln\left(\sqrt[3]{bx+a}-\sqrt[3]{a}\right)a^{-\frac{10}{3}}-\frac{7\,b^2}{9}\ln\left((bx+a)^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{bx+a}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(4/3),x)
```

```
[Out] 3*b^2/a^3/(b*x+a)^(1/3)+5/3/a^3/x^2*(b*x+a)^(5/3)-13/6/a^2/x^2*(b*x+a)^(2/3
)+14/9*b^2/a^(10/3)*ln((b*x+a)^(1/3)-a^(1/3))-7/9*b^2/a^(10/3)*ln((b*x+a)^(
2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+14/9*b^2/a^(10/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70796, size = 1064, normalized size = 7.14

$$\left[\frac{42\sqrt{\frac{1}{3}}\left(ab^3x^3+a^2b^2x^2\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x}\right)-14\left(b^3x^3+ab^2x^2\right)a^{\frac{2}{3}}\log\left(\right.}{18\left(a^4bx^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/18*(42*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-1/a^(2/3))*log((2*b*x +
 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt
(-1/a^(2/3)) - 3*(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - 14*(b^3*x^3 + a*b^2*x^
2)*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 28*(b
^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(28*a*b^2*x^
2 + 7*a^2*b*x - 3*a^3)*(b*x + a)^(2/3))/(a^4*b*x^3 + a^5*x^2), -1/18*(14*(b
^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
 a^(2/3)) - 28*(b^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3))
 - 84*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2*x^2)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/
3) + a^(1/3))/a^(1/3))/a^(1/3) - 3*(28*a*b^2*x^2 + 7*a^2*b*x - 3*a^3)*(b*x
+ a)^(2/3))/(a^4*b*x^3 + a^5*x^2)]
```

Sympy [C] time = 4.41727, size = 2793, normalized size = 18.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(4/3),x)
```

```
[Out] 54*a**(13/3)*b**(5/3)*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(
1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi
```

```
/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/
3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 201*a*
*(10/3)*b**(8/3)*(a/b + x)*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)*(a/b +
x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2
*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 2
31*a**(7/3)*b**(11/3)*(a/b + x)**2*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)
*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*p
i/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2
/3)) - 84*a**(4/3)*b**(14/3)*(a/b + x)**3*exp(2*I*pi/3)*gamma(-1/3)/(-54*a*
*(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*ex
p(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*
gamma(2/3)) + 28*a**4*b**2*(a/b + x)**(1/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*
(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(
2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma
(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a
**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 28*a**4*b**2*(a
/b + x)**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2
*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3
)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) -
162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)
*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 28*a**4*b**2*(a/b + x)*
*(1/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamm
a(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(
19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b
 + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)
*exp(2*I*pi/3)*gamma(2/3)) - 84*a**3*b**3*(a/b + x)**(4/3)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b +
 x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(
2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) -
84*a**3*b**3*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1
/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/
3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3
)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3)
 + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 84*a**3*
b**3*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)
/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(
2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(
16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a
/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*e
xp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a
**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b +
 x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*e
xp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)
*gamma(2/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*exp(-2*I*pi/3)*log(1 - b**(1/3
)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)
*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*p
i/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2
/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2
*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(
2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a*
*(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 28*a*b**5*(a/b +
 x)**(10/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma
(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(1
```

```
9/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b
+ x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*
exp(2*I*pi/3)*gamma(2/3)) - 28*a*b**5*(a/b + x)**(10/3)*exp(-2*I*pi/3)*log(
1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-5
4*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/
b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3
)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi
/3)*gamma(2/3)) - 28*a*b**5*(a/b + x)**(10/3)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1
/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/
3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3
) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3))
```

Giac [A] time = 2.20288, size = 189, normalized size = 1.27

$$\frac{14\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{10}{3}}} - \frac{7b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{10}{3}}} + \frac{14b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9a^{\frac{10}{3}}} + \frac{3b^{}}{(bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 14/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(10/3) - 7/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(10/3) + 14/9*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(10/3) + 3*b^2/(
(b*x + a)^(1/3)*a^3) + 1/6*(10*(b*x + a)^(5/3)*b^2 - 13*(b*x + a)^(2/3)*a*b
^2)/(a^3*b^2*x^2)
```

3.420 $\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx$

Optimal. Leaf size=71

$$\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}$$

```
[Out] (Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a - Log[x]/(2*a)
+ (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a)
```

Rubi [A] time = 0.0310501, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {55, 617, 204, 31}

$$\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a - Log[x]/(2*a)
+ (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a)
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx = -\frac{\log(x)}{2a} + \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx, x, \sqrt[3]{a^3+b^3x}\right) - \frac{3\operatorname{Subst}\left(\int \frac{1}{a-x}\,dx, x, \sqrt[3]{a^3+b^3x}\right)}{2a}$$
$$= -\frac{\log(x)}{2a} + \frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a} - \frac{3\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a^3+b^3x}}{a}\right)}{a}$$
$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a} - \frac{\log(x)}{2a} + \frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}$$

Mathematica [A] time = 0.0947209, size = 66, normalized size = 0.93

$$\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)-\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - Log[x] + 3*Log
[a - (a^3 + b^3*x)^(1/3)])/(2*a)
```

Maple [A] time = 0.009, size = 87, normalized size = 1.2

$$\frac{1}{a}\ln\left(-a+\sqrt[3]{b^3x+a^3}\right)-\frac{1}{2a}\ln\left(\left(b^3x+a^3\right)^{\frac{2}{3}}+a\sqrt[3]{b^3x+a^3}+a^2\right)+\frac{\sqrt{3}}{a}\arctan\left(\frac{\sqrt{3}}{3a}\left(a+2\sqrt[3]{b^3x+a^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x+a^3)^(1/3),x)
```

```
[Out] 1/a*ln(-a+(b^3*x+a^3)^(1/3))-1/2/a*ln((b^3*x+a^3)^(2/3)+a*(b^3*x+a^3)^(1/3)
+a^2)+arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))/a*3^(1/2))*3^(1/2)/a
```

Maxima [A] time = 1.51902, size = 116, normalized size = 1.63

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}-\frac{\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a}+\frac{\log\left(-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2 +
 (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a + log(-a + (b^3*x + a^3)^(1
```

```
/3))/a
```

Fricas [A] time = 1.67824, size = 227, normalized size = 3.2

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^{2}+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)+2\log\left(-a+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b^3*x + a^3)^(1/3))/a) -
log(a^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3)) + 2*log(-a + (b^3*x
+ a^3)^(1/3)))/a
```

Sympy [C] time = 2.46933, size = 138, normalized size = 1.94

$$\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{2i\pi}{3}}}{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{4i\pi}{3}}}{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(-\frac{ae^{2i\pi}}{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x+a**3)**(1/3),x)
```

```
[Out] exp(I*pi/3)*log(-a*exp_polar(2*I*pi/3)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamm
a(-1/3)/(3*a*gamma(2/3)) + exp(-I*pi/3)*log(-a*exp_polar(4*I*pi/3)/(b*(a**3
/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - log(-a*exp_polar(2*I
*pi)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Giac [A] time = 1.30183, size = 117, normalized size = 1.65

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}-\frac{\log\left(a^{2}+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2a}+\frac{\log\left(\left|-a+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2 +
 (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a + log(abs(-a + (b^3*x + a^3
)^(1/3)))/a
```

3.421 $\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx$

Optimal. Leaf size=73

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}$$

```
[Out] (Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a - Log[x]/(2*a)
+ (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a)
```

Rubi [A] time = 0.030922, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {55, 617, 204, 31}

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a - Log[x]/(2*a)
+ (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a)
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx = -\frac{\log(x)}{2a} + \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx, x, \sqrt[3]{a^3-b^3x}\right) - \frac{3\,\text{Subst}\left(\int \frac{1}{a-x}\,dx, x, \sqrt[3]{a^3-b^3x}\right)}{2a}$$
$$= -\frac{\log(x)}{2a} + \frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a^3-b^3x}}{a}\right)}{a}$$
$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a} - \frac{\log(x)}{2a} + \frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}$$

Mathematica [A] time = 0.0618446, size = 68, normalized size = 0.93

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right) + 2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right) - \log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - Log[x] + 3*Log
[a - (a^3 - b^3*x)^(1/3)])/(2*a)
```

Maple [A] time = 0.005, size = 91, normalized size = 1.3

$$\frac{1}{a}\ln\left(-a+\sqrt[3]{-b^3x+a^3}\right) - \frac{1}{2\,a}\ln\left(\left(-b^3x+a^3\right)^{\frac{2}{3}} + a\sqrt[3]{-b^3x+a^3} + a^2\right) + \frac{\sqrt{3}}{a}\arctan\left(\frac{\sqrt{3}}{3\,a}\left(a+2\sqrt[3]{-b^3x+a^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x+a^3)^(1/3),x)
```

```
[Out] 1/a*ln(-a+(-b^3*x+a^3)^(1/3))-1/2/a*ln((-b^3*x+a^3)^(2/3)+a*(-b^3*x+a^3)^(1
/3)+a^2)+arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))/a*3^(1/2))*3^(1/2)/a
```

Maxima [A] time = 1.52501, size = 122, normalized size = 1.67

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a} - \frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2\,a} + \frac{\log\left(-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
+ (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a + log(-a + (-b^3*x + a^3
```

```
)^(1/3))/a
```

Fricas [A] time = 1.63676, size = 232, normalized size = 3.18

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(-b^3x+a^3\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)+2\log\left(-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(-b^3*x + a^3)^(1/3))/a) -
 log(a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3)) + 2*log(-a + (-b^
3*x + a^3)^(1/3)))/a
```

Sympy [C] time = 2.40131, size = 136, normalized size = 1.86

$$-\frac{e^{-\frac{2i\pi}{3}}\log\left(-\frac{ae^{\frac{i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{i\pi}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(-\frac{ae^{\frac{5i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x+a**3)**(1/3),x)
```

```
[Out] -exp(-2*I*pi/3)*log(-a*exp_polar(I*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*g
amma(-1/3)/(3*a*gamma(2/3)) + exp(-I*pi/3)*log(-a*exp_polar(I*pi)/(b*(-a**3
/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - log(-a*exp_polar(5*I
*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Giac [A] time = 1.23209, size = 123, normalized size = 1.68

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}-\frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a}+\frac{\log\left(\left|-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
+ (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a + log(abs(-a + (-b^3*x +
 a^3)^(1/3)))/a
```

3.422 $\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx$

Optimal. Leaf size=74

$$-\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a) + Log[x]/(2
*a) - (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a)
```

Rubi [A] time = 0.0316239, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {56, 617, 204, 31}

$$-\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 + b^3*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a) + Log[x]/(2
*a) - (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a)
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx = \frac{\log(x)}{2a} + \frac{3}{2}\,\mathrm{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx, x, \sqrt[3]{-a^3+b^3x}\right) - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x, \sqrt[3]{-a^3+b^3x}\right)}{2a}$$

$$= \frac{\log(x)}{2a} - \frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}\right)}{a}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a} + \frac{\log(x)}{2a} - \frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a}$$

Mathematica [C] time = 0.0392213, size = 41, normalized size = 0.55

$$\frac{3\left(b^3x-a^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-\frac{b^3x}{a^3}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (3*(-a^3 + b^3*x)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 - (b^3*x)/a^3])/(2
*a^3)
```

Maple [A] time = 0.01, size = 97, normalized size = 1.3

$$-\frac{1}{a}\ln\left(a+\sqrt[3]{b^3x-a^3}\right) + \frac{1}{2a}\ln\left(\left(b^3x-a^3\right)^{\frac{2}{3}} - a\sqrt[3]{b^3x-a^3} + a^2\right) + \frac{\sqrt{3}}{a}\arctan\left(\frac{\sqrt{3}}{3a}\left(2\sqrt[3]{b^3x-a^3}-a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x-a^3)^(1/3),x)
```

```
[Out] -ln(a+(b^3*x-a^3)^(1/3))/a+1/2/a*ln((b^3*x-a^3)^(2/3)-a*(b^3*x-a^3)^(1/3)+a
^2)+1/a*3^(1/2)*arctan(1/3*(2*(b^3*x-a^3)^(1/3)-a)*3^(1/2)/a)
```

Maxima [A] time = 1.57294, size = 127, normalized size = 1.72

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a} + \frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a} - \frac{\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
- (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a - log(a + (b^3*x - a^3)^(1
```

```
/3))/a
```

Fricas [A] time = 1.63243, size = 227, normalized size = 3.07

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)-2\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(b^3*x - a^3)^(1/3))/a) +
 log(a^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3)) - 2*log(a + (b^3*x
- a^3)^(1/3)))/a
```

Sympy [C] time = 2.51294, size = 134, normalized size = 1.81

$$-\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{\log\left(-\frac{ae^{i\pi}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{5i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x-a**3)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*log(-a*exp_polar(I*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gam
ma(-1/3)/(3*a*gamma(2/3)) + log(-a*exp_polar(I*pi)/(b*(-a**3/b**3 + x)**(1/
3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - exp(I*pi/3)*log(-a*exp_polar(5*I*pi
/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Giac [A] time = 1.19175, size = 128, normalized size = 1.73

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}+\frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a}-\frac{\log\left(\left|a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
- (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a - log(abs(a + (b^3*x - a^3
)^(1/3)))/a
```

3.423 $\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx$

Optimal. Leaf size=76

$$-\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a) + Log[x]/(2
*a) - (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a)
```

Rubi [A] time = 0.0283408, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {56, 617, 204, 31}

$$-\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```


```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a) + Log[x]/(2
*a) - (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a)
```


Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx = \frac{\log(x)}{2a}+\frac{3}{2}\,\mathrm{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx,x,\sqrt[3]{-a^3-b^3x}\right)-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,\sqrt[3]{-a^3-b^3x}\right)}{2a}$$
$$=\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}\right)}{a}$$
$$=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a}+\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a}$$

Mathematica [C] time = 0.0531762, size = 41, normalized size = 0.54

$$\frac{3\left(-a^3-b^3x\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{xb^3}{a^3}+1\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (3*(-a^3 - b^3*x)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 + (b^3*x)/a^3])/(2
*a^3)
```

Maple [A] time = 0.004, size = 101, normalized size = 1.3

$$-\frac{1}{a}\ln\left(a+\sqrt[3]{-b^3x-a^3}\right)+\frac{1}{2\,a}\ln\left(\left(-b^3x-a^3\right)^{\frac{2}{3}}-a\sqrt[3]{-b^3x-a^3}+a^2\right)+\frac{\sqrt{3}}{a}\arctan\left(\frac{\sqrt{3}}{3\,a}\left(2\sqrt[3]{-b^3x-a^3}-a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x-a^3)^(1/3),x)
```

```
[Out] -ln(a+(-b^3*x-a^3)^(1/3))/a+1/2/a*ln((-b^3*x-a^3)^(2/3)-a*(-b^3*x-a^3)^(1/3
)+a^2)+1/a*3^(1/2)*arctan(1/3*(2*(-b^3*x-a^3)^(1/3)-a)*3^(1/2)/a)
```

Maxima [A] time = 1.54618, size = 132, normalized size = 1.74

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a}+\frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2\,a}-\frac{\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a - log(a + (-b^3*x - a^3
```

```
)^(1/3))/a
```

Fricas [A] time = 1.65885, size = 232, normalized size = 3.05

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(-b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)-2\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-b^3*x - a^3)^(1/3))/a)
+ log(a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3)) - 2*log(a + (-b^
3*x - a^3)^(1/3)))/a
```

Sympy [C] time = 2.37006, size = 139, normalized size = 1.83

$$\frac{\log\left(-\frac{ae^{\frac{2i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{4i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(-\frac{ae^{2i\pi}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x-a**3)**(1/3),x)
```

```
[Out] log(-a*exp_polar(2*I*pi/3)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a
*gamma(2/3)) - exp(I*pi/3)*log(-a*exp_polar(4*I*pi/3)/(b*(a**3/b**3 + x)**(
1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) + exp(2*I*pi/3)*log(-a*exp_polar(2*
I*pi)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Giac [A] time = 1.26894, size = 134, normalized size = 1.76

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}+\frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a}-\frac{\log\left(\left|a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a - log(abs(a + (-b^3*x -
 a^3)^(1/3)))/a
```

3.424 $\int \frac{1}{x(a^3+b^3x)^{2/3}}\,dx$

Optimal. Leaf size=72

$$\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/(
2*a^2) + (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Rubi [A] time = 0.0238681, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {57, 617, 204, 31}

$$\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/(
2*a^2) + (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{a-x}\,dx, x, \sqrt[3]{a^3+b^3x}\right)}{2a^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx, x, \sqrt[3]{a^3+b^3x}\right)}{2a}$$

$$= -\frac{\log(x)}{2a^2} + \frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a^3+b^3x}}{a}\right)}{a^2}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a^2} - \frac{\log(x)}{2a^2} + \frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}$$

Mathematica [A] time = 0.101855, size = 95, normalized size = 1.32

$$-\frac{-2\log\left(a-\sqrt[3]{a^3+b^3x}\right)+\log\left(a\sqrt[3]{a^3+b^3x}+\left(a^3+b^3x\right)^{2/3}+a^2\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a - (a^
3 + b^3*x)^(1/3)] + Log[a^2 + a*(a^3 + b^3*x)^(1/3) + (a^3 + b^3*x)^(2/3)])
/(2*a^2)
```

Maple [A] time = 0.007, size = 88, normalized size = 1.2

$$\frac{1}{a^2}\ln\left(-a+\sqrt[3]{b^3x+a^3}\right)-\frac{1}{2\,a^2}\ln\left(\left(b^3x+a^3\right)^{\frac{2}{3}}+a\sqrt[3]{b^3x+a^3}+a^2\right)-\frac{\sqrt{3}}{a^2}\arctan\left(\frac{\sqrt{3}}{3\,a}\left(a+2\sqrt[3]{b^3x+a^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x+a^3)^(2/3),x)
```

```
[Out] 1/a^2*ln(-a+(b^3*x+a^3)^(1/3))-1/2/a^2*ln((b^3*x+a^3)^(2/3)+a*(b^3*x+a^3)^(
1/3)+a^2)-arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))/a*3^(1/2))*3^(1/2)/a^2
```

Maxima [A] time = 1.51738, size = 117, normalized size = 1.62

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^2}-\frac{\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2\,a^2}+\frac{\log\left(-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a^2 + log(-a + (b^3*x + a^
```

```
3)^(1/3))/a^2
```

Fricas [A] time = 1.63793, size = 231, normalized size = 3.21

$$-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(b^3x+a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)-2\log\left(-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b^3*x + a^3)^(1/3))/a) +
 log(a^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3)) - 2*log(-a + (b^3*x
 + a^3)^(1/3)))/a^2
```

Sympy [C] time = 2.50924, size = 134, normalized size = 1.86

$$\frac{\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}e^{\frac{2i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}e^{\frac{4i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x+a**3)**(2/3),x)
```

```
[Out] log(1 - b*(a**3/b**3 + x)**(1/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) + exp(-2
*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_polar(2*I*pi/3)/a)*gamma(1/3)
/(3*a**2*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_p
olar(4*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Giac [A] time = 1.23923, size = 119, normalized size = 1.65

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(\left|-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a^2 + log(abs(-a + (b^3*x
+ a^3)^(1/3)))/a^2
```

3.425 $\int \frac{1}{x(a^3-b^3x)^{2/3}}\,dx$

Optimal. Leaf size=74

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/(
2*a^2) + (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Rubi [A] time = 0.0248979, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {57, 617, 204, 31}

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/(
2*a^2) + (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{a-x}\,dx,x,\sqrt[3]{a^3-b^3x}\right)}{2a^2}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx,x,\sqrt[3]{a^3-b^3x}\right)}{2a}$$

$$= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a^3-b^3x}}{a}\right)}{a^2}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}$$

Mathematica [A] time = 0.0979139, size = 99, normalized size = 1.34

$$-\frac{-2\log\left(a-\sqrt[3]{a^3-b^3x}\right)+\log\left(a\sqrt[3]{a^3-b^3x}+\left(a^3-b^3x\right)^{2/3}+a^2\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a - (a^
3 - b^3*x)^(1/3)] + Log[a^2 + a*(a^3 - b^3*x)^(1/3) + (a^3 - b^3*x)^(2/3)])
/(2*a^2)
```

Maple [A] time = 0.004, size = 92, normalized size = 1.2

$$\frac{1}{a^2}\ln\left(-a+\sqrt[3]{-b^3x+a^3}\right)-\frac{1}{2\,a^2}\ln\left(\left(-b^3x+a^3\right)^{\frac{2}{3}}+a\sqrt[3]{-b^3x+a^3}+a^2\right)-\frac{\sqrt{3}}{a^2}\arctan\left(\frac{\sqrt{3}}{3\,a}\left(a+2\sqrt[3]{-b^3x+a^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x+a^3)^(2/3),x)
```

```
[Out] 1/a^2*ln(-a+(-b^3*x+a^3)^(1/3))-1/2/a^2*ln((-b^3*x+a^3)^(2/3)+a*(-b^3*x+a^3
)^(1/3)+a^2)-arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))/a*3^(1/2))*3^(1/2)/a^2
```

Maxima [A] time = 1.52431, size = 123, normalized size = 1.66

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^2}-\frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2\,a^2}+\frac{\log\left(-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a^2 + log(-a + (-b^3*x
```

```
+ a^3)^(1/3))/a^2
```

Fricas [A] time = 1.60652, size = 236, normalized size = 3.19

$$-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(-b^3x+a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)-2\log\left(-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(-b^3*x + a^3)^(1/3))/a)
+ log(a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3)) - 2*log(-a + (-b
^3*x + a^3)^(1/3)))/a^2
```

Sympy [C] time = 2.6476, size = 136, normalized size = 1.84

$$\frac{\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+xe^{\frac{i\pi}{3}}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+xe^{i\pi}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+xe^{\frac{5i\pi}{3}}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x+a**3)**(2/3),x)
```

```
[Out] log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi/3)/a)*gamma(1/3)/(3*a**2*g
amma(4/3)) - exp(I*pi/3)*log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi)/
a)*gamma(1/3)/(3*a**2*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b*(-a**3/b**3 + x
)**(1/3)*exp_polar(5*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Giac [A] time = 1.26415, size = 124, normalized size = 1.68

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(\left|-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a^2 + log(abs(-a + (-b^
3*x + a^3)^(1/3)))/a^2
```

3.426 $\int \frac{1}{x(-a^3+b^3x)^{2/3}}\,dx$

Optimal. Leaf size=74

$$\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/
(2*a^2) + (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Rubi [A] time = 0.0232063, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {58, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/
(2*a^2) + (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\left(-a^3+b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, \sqrt[3]{-a^3+b^3x}\right)}{2a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx, x, \sqrt[3]{-a^3+b^3x}\right)}{2a}$$

$$= -\frac{\log(x)}{2a^2} + \frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}\right)}{a^2}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a^2} - \frac{\log(x)}{2a^2} + \frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a^2}$$

Mathematica [A] time = 0.0570846, size = 108, normalized size = 1.46

$$\frac{\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{a^2} - \frac{\log\left(-a\sqrt[3]{b^3x-a^3}+\left(b^3x-a^3\right)^{2/3}+a^2\right)}{2a^2} + \frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b^3x-a^3}-a}{\sqrt{3}a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(-a + 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2 + Log[a + (
-a^3 + b^3*x)^(1/3)]/a^2 - Log[a^2 - a*(-a^3 + b^3*x)^(1/3) + (-a^3 + b^3*x
)^(2/3)]/(2*a^2)
```

Maple [A] time = 0.007, size = 96, normalized size = 1.3

$$\frac{1}{a^2}\ln\left(a+\sqrt[3]{b^3x-a^3}\right) - \frac{1}{2a^2}\ln\left(\left(b^3x-a^3\right)^{\frac{2}{3}} - a\sqrt[3]{b^3x-a^3}+a^2\right) + \frac{\sqrt{3}}{a^2}\arctan\left(\frac{\sqrt{3}}{3a}\left(2\sqrt[3]{b^3x-a^3}-a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x-a^3)^(2/3),x)
```

```
[Out] ln(a+(b^3*x-a^3)^(1/3))/a^2-1/2/a^2*ln((b^3*x-a^3)^(2/3)-a*(b^3*x-a^3)^(1/3
)+a^2)+1/a^2*3^(1/2)*arctan(1/3*(2*(b^3*x-a^3)^(1/3)-a)*3^(1/2)/a)
```

Maxima [A] time = 1.50449, size = 126, normalized size = 1.7

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2} - \frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2} + \frac{\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a^2 + log(a + (b^3*x - a^3
```

```
)^(1/3))/a^2
```

Fricas [A] time = 1.52455, size = 230, normalized size = 3.11

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}(b^3x-a^3)^{\frac{1}{3}}}{3a}\right)-\log\left(a^2-(b^3x-a^3)^{\frac{1}{3}}a+(b^3x-a^3)^{\frac{2}{3}}\right)+2\log\left(a+(b^3x-a^3)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(b^3*x - a^3)^(1/3))/a) -
 log(a^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3)) + 2*log(a + (b^3*x
- a^3)^(1/3)))/a^2
```

Sympy [C] time = 2.45744, size = 134, normalized size = 1.81

$$-\frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}e^{\frac{i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}e^{i\pi}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}e^{\frac{5i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x-a**3)**(2/3),x)
```

```
[Out] -exp(-I*pi/3)*log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi/3)/a)*gamma(
1/3)/(3*a**2*gamma(4/3)) + log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi
)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) - exp(I*pi/3)*log(1 - b*(-a**3/b**3 + x
)**(1/3)*exp_polar(5*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Giac [A] time = 1.30449, size = 127, normalized size = 1.72

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2(b^3x-a^3)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2-(b^3x-a^3)^{\frac{1}{3}}a+(b^3x-a^3)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(\left|a+(b^3x-a^3)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a^2 + log(abs(a + (b^3*x -
 a^3)^(1/3)))/a^2
```

3.427 $\int \frac{1}{x(-a^3-b^3x)^{2/3}}\,dx$

Optimal. Leaf size=76

$$\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/
(2*a^2) + (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Rubi [A] time = 0.0233589, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {58, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/
(2*a^2) + (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,\sqrt[3]{-a^3-b^3x}\right)}{2a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx,x,\sqrt[3]{-a^3-b^3x}\right)}{2a}$$
$$= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}\right)}{a^2}$$
$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a^2}$$

Mathematica [A] time = 0.122299, size = 112, normalized size = 1.47

$$\frac{\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{a^2}-\frac{\log\left(-a\sqrt[3]{-a^3-b^3x}+\left(-a^3-b^3x\right)^{2/3}+a^2\right)}{2a^2}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{-a^3-b^3x}-a}{\sqrt{3}a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 - b^3*x)^(2/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(-a + 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2 + Log[a + (
-a^3 - b^3*x)^(1/3)]/a^2 - Log[a^2 - a*(-a^3 - b^3*x)^(1/3) + (-a^3 - b^3*x
)^(2/3)]/(2*a^2)
```

Maple [A] time = 0.003, size = 100, normalized size = 1.3

$$\frac{1}{a^2}\ln\left(a+\sqrt[3]{-b^3x-a^3}\right)-\frac{1}{2\,a^2}\ln\left(\left(-b^3x-a^3\right)^{\frac{2}{3}}-a\sqrt[3]{-b^3x-a^3}+a^2\right)+\frac{\sqrt{3}}{a^2}\arctan\left(\frac{\sqrt{3}}{3\,a}\left(2\sqrt[3]{-b^3x-a^3}-a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x-a^3)^(2/3),x)
```

```
[Out] ln(a+(-b^3*x-a^3)^(1/3))/a^2-1/2/a^2*ln((-b^3*x-a^3)^(2/3)-a*(-b^3*x-a^3)^(
1/3)+a^2)+1/a^2*3^(1/2)*arctan(1/3*(2*(-b^3*x-a^3)^(1/3)-a)*3^(1/2)/a)
```

Maxima [A] time = 1.51693, size = 131, normalized size = 1.72

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^2}-\frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2\,a^2}+\frac{\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a^2 + log(a + (-b^3*x -
```

```
a^3)^(1/3))/a^2
```

Fricas [A] time = 1.56733, size = 235, normalized size = 3.09

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(-b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)+2\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-b^3*x - a^3)^(1/3))/a)
- log(a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3)) + 2*log(a + (-b^
3*x - a^3)^(1/3)))/a^2
```

Sympy [C] time = 2.32263, size = 133, normalized size = 1.75

$$\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}-\frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}e^{\frac{2i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}e^{\frac{4i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x-a**3)**(2/3),x)
```

```
[Out] exp(-2*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)/a)*gamma(1/3)/(3*a**2*gamma
(4/3)) - exp(-I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_polar(2*I*pi/3)/
a)*gamma(1/3)/(3*a**2*gamma(4/3)) + log(1 - b*(a**3/b**3 + x)**(1/3)*exp_po
lar(4*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Giac [A] time = 1.27258, size = 132, normalized size = 1.74

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(\left|a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a^2 + log(abs(a + (-b^3
*x - a^3)^(1/3)))/a^2
```

3.428 $\int x^m(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(2 + m))/(2 + m)
```

Rubi [A] time = 0.0083796, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(2 + m))/(2 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)\,dx &= \int \left(ax^m+bx^{1+m}\right)\,dx\\ &= \frac{ax^{1+m}}{1+m}+\frac{bx^{2+m}}{2+m}\end{aligned}$$

Mathematica [A] time = 0.0143177, size = 22, normalized size = 0.88

$$x^{m+1}\left(\frac{a}{m+1}+\frac{bx}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x),x]
```

```
[Out] x^(1 + m)*(a/(1 + m) + (b*x)/(2 + m))
```

Maple [A] time = 0.002, size = 31, normalized size = 1.2

$$\frac{x^{1+m}\left(bmx+am+bx+2\,a\right)}{(2+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a),x)
```

```
[Out] x^(1+m)*(b*m*x+a*m+b*x+2*a)/(2+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55342, size = 72, normalized size = 2.88

$$\frac{\left((bm+b)x^2+(am+2a)x\right)x^m}{m^2+3m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [A] time = 0.374517, size = 87, normalized size = 3.48

$$\begin{cases} -\frac{a}{x}+b\log(x) & \text{for } m=-2 \\ a\log(x)+bx & \text{for } m=-1 \\ \frac{amxx^m}{m^2+3m+2}+\frac{2axx^m}{m^2+3m+2}+\frac{bmx^2x^m}{m^2+3m+2}+\frac{bx^2x^m}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a),x)
```

```
[Out] Piecewise((-a/x + b*log(x), Eq(m, -2)), (a*log(x) + b*x, Eq(m, -1)), (a*m*x
*x**m/(m**2 + 3*m + 2) + 2*a*x*x**m/(m**2 + 3*m + 2) + b*m*x**2*x**m/(m**2
+ 3*m + 2) + b*x**2*x**m/(m**2 + 3*m + 2), True))
```

Giac [A] time = 1.17033, size = 58, normalized size = 2.32

$$\frac{bmx^2x^m+amxx^m+bx^2x^m+2axx^m}{m^2+3m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="giac")
```

```
[Out] (b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

3.429 $\int x^{5/2}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}$$

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(9/2))/9
```

Rubi [A] time = 0.0040576, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x),x]

[Out] (2*a*x^(7/2))/7 + (2*b*x^(9/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)\,dx &= \int \left(ax^{5/2}+bx^{7/2}\right)dx\\ &= \frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.0051564, size = 17, normalized size = 0.81

$$\frac{2}{63}x^{7/2}(9a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x),x]

[Out] (2*x^(7/2)*(9*a + 7*b*x))/63
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$\frac{14\,bx+18\,a}{63}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a),x)
```

```
[Out] 2/63*x^(7/2)*(7*b*x+9*a)
```

Maxima [A] time = 0.997184, size = 18, normalized size = 0.86

$$\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

Fricas [A] time = 1.50522, size = 46, normalized size = 2.19

$$\frac{2}{63}\left(7\,bx^{4}+9\,ax^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b*x^4 + 9*a*x^3)*sqrt(x)
```

Sympy [A] time = 2.3505, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a),x)
```

```
[Out] 2*a*x**(7/2)/7 + 2*b*x**(9/2)/9
```

Giac [A] time = 1.26386, size = 18, normalized size = 0.86

$$\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

3.430 $\int x^{3/2}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}$$

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(7/2))/7
```

Rubi [A] time = 0.0036704, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)\,dx &= \int \left(ax^{3/2}+bx^{5/2}\right)dx\\ &= \frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0041991, size = 17, normalized size = 0.81

$$\frac{2}{35}x^{5/2}(7a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x),x]
```

```
[Out] (2*x^(5/2)*(7*a + 5*b*x))/35
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$\frac{10\,bx+14\,a}{35}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a),x)
```

```
[Out] 2/35*x^(5/2)*(5*b*x+7*a)
```

Maxima [A] time = 1.05742, size = 18, normalized size = 0.86

$$\frac{2}{7}\,bx^{\frac{7}{2}} + \frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

Fricas [A] time = 1.59495, size = 46, normalized size = 2.19

$$\frac{2}{35}\left(5\,bx^{3} + 7\,ax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b*x^3 + 7*a*x^2)*sqrt(x)
```

Sympy [A] time = 0.846176, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{5}{2}}}{5} + \frac{2bx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a),x)
```

```
[Out] 2*a*x**(5/2)/5 + 2*b*x**(7/2)/7
```

Giac [A] time = 1.18716, size = 18, normalized size = 0.86

$$\frac{2}{7}\,bx^{\frac{7}{2}} + \frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

3.431 $\int \sqrt{x}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}$$

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(5/2))/5
```

Rubi [A] time = 0.0041047, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(a+bx)\,dx &= \int \left(a\sqrt{x}+bx^{3/2}\right)\,dx\\ &= \frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.004014, size = 17, normalized size = 0.81

$$\frac{2}{15}x^{3/2}(5a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x),x]
```

```
[Out] (2*x^(3/2)*(5*a + 3*b*x))/15
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$\frac{6\,bx+10\,a}{15}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*x^(1/2),x)
```

```
[Out] 2/15*x^(3/2)*(3*b*x+5*a)
```

Maxima [A] time = 1.00839, size = 18, normalized size = 0.86

$$\frac{2}{5}\,bx^{\frac{5}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

Fricas [A] time = 1.57422, size = 43, normalized size = 2.05

$$\frac{2}{15}\left(3\,bx^{2}+5\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b*x^2 + 5*a*x)*sqrt(x)
```

Sympy [A] time = 2.04054, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{3}{2}}}{3}+\frac{2bx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x**(1/2),x)
```

```
[Out] 2*a*x**(3/2)/3 + 2*b*x**(5/2)/5
```

Giac [A] time = 1.25034, size = 18, normalized size = 0.86

$$\frac{2}{5}\,bx^{\frac{5}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

3.432 $\int \frac{a+bx}{\sqrt{x}}\,dx$

Optimal. Leaf size=19

$$2a\sqrt{x}+\frac{2}{3}bx^{3/2}$$

```
[Out] 2*a*Sqrt[x] + (2*b*x^(3/2))/3
```

Rubi [A] time = 0.0036023, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$2a\sqrt{x}+\frac{2}{3}bx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*x^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt{x}}\,dx &= \int \left(\frac{a}{\sqrt{x}}+b\sqrt{x}\right)dx\\ &= 2a\sqrt{x}+\frac{2}{3}bx^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0047361, size = 16, normalized size = 0.84

$$\frac{2}{3}\sqrt{x}(3a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(3*a + b*x))/3
```

Maple [A] time = 0.002, size = 13, normalized size = 0.7

$$\frac{2\,bx+6\,a}{3}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(1/2),x)
```

```
[Out] 2/3*x^(1/2)*(b*x+3*a)
```

Maxima [A] time = 0.989671, size = 18, normalized size = 0.95

$$\frac{2}{3}\,bx^{\frac{3}{2}} + 2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*b*x^(3/2) + 2*a*sqrt(x)
```

Fricas [A] time = 1.50122, size = 34, normalized size = 1.79

$$\frac{2}{3}\,(bx + 3\,a)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + 3*a)*sqrt(x)
```

Sympy [A] time = 0.17349, size = 17, normalized size = 0.89

$$2a\sqrt{x} + \frac{2bx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(1/2),x)
```

```
[Out] 2*a*sqrt(x) + 2*b*x**(3/2)/3
```

Giac [A] time = 1.2328, size = 18, normalized size = 0.95

$$\frac{2}{3}\,bx^{\frac{3}{2}} + 2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*b*x^(3/2) + 2*a*sqrt(x)
```

3.433 $\int \frac{a+bx}{x^{3/2}} dx$

Optimal. Leaf size=17

$$2b\sqrt{x} - \frac{2a}{\sqrt{x}}$$

```
[Out] (-2*a)/Sqrt[x] + 2*b*Sqrt[x]
```

Rubi [A] time = 0.0040094, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$2b\sqrt{x} - \frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + 2*b*Sqrt[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{a+bx}{x^{3/2}}\,dx = \int \left(\frac{a}{x^{3/2}} + \frac{b}{\sqrt{x}}\right)dx$$
$$= -\frac{2a}{\sqrt{x}} + 2b\sqrt{x}$$

Mathematica [A] time = 0.0051799, size = 14, normalized size = 0.82

$$\frac{2(bx-a)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(3/2),x]
```

```
[Out] (2*(-a + b*x))/Sqrt[x]
```

Maple [A] time = 0.001, size = 12, normalized size = 0.7

$$-2\,\frac{-bx+a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(3/2),x)
```

```
[Out] -2*(-b*x+a)/x^(1/2)
```

Maxima [A] time = 1.02495, size = 18, normalized size = 1.06

$$2\,b\sqrt{x}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b*sqrt(x) - 2*a/sqrt(x)
```

Fricas [A] time = 1.52989, size = 28, normalized size = 1.65

$$\frac{2\,(bx-a)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*x - a)/sqrt(x)
```

Sympy [A] time = 0.510208, size = 15, normalized size = 0.88

$$-\frac{2a}{\sqrt{x}}+2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(3/2),x)
```

```
[Out] -2*a/sqrt(x) + 2*b*sqrt(x)
```

Giac [A] time = 1.25551, size = 18, normalized size = 1.06

$$2\,b\sqrt{x}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2*b*sqrt(x) - 2*a/sqrt(x)
```

3.434 $\int \frac{a+bx}{x^{5/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}$$

```
[Out] (-2*a)/(3*x^(3/2)) - (2*b)/Sqrt[x]
```

Rubi [A] time = 0.0036581, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) - (2*b)/Sqrt[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{5/2}}\,dx &= \int \left(\frac{a}{x^{5/2}}+\frac{b}{x^{3/2}}\right)dx\\ &= -\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0049646, size = 15, normalized size = 0.79

$$-\frac{2(a+3bx)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(5/2),x]
```

```
[Out] (-2*(a + 3*b*x))/(3*x^(3/2))
```

Maple [A] time = 0.002, size = 12, normalized size = 0.6

$$-\frac{6\,bx+2\,a}{3}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(5/2),x)
```

```
[Out] -2/3*(3*b*x+a)/x^(3/2)
```

Maxima [A] time = 0.992358, size = 15, normalized size = 0.79

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

Fricas [A] time = 1.55319, size = 35, normalized size = 1.84

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

Sympy [A] time = 0.957543, size = 19, normalized size = 1.

$$-\frac{2a}{3x^{\frac{3}{2}}}-\frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(5/2),x)
```

```
[Out] -2*a/(3*x**(3/2)) - 2*b/sqrt(x)
```

Giac [A] time = 1.23794, size = 15, normalized size = 0.79

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

3.435 $\int x^m(a+bx)^2\,dx$

Optimal. Leaf size=43

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(2 + m))/(2 + m) + (b^2*x^(3 + m))/(3 +
m)
```

Rubi [A] time = 0.0142616, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(2 + m))/(2 + m) + (b^2*x^(3 + m))/(3 +
m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^2\,dx&=\int \left(a^2x^m+2abx^{1+m}+b^2x^{2+m}\right)\,dx\\&=\frac{a^2x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2x^{3+m}}{3+m}\end{aligned}$$

Mathematica [A] time = 0.0315192, size = 38, normalized size = 0.88

$$x^{m+1}\left(\frac{a^2}{m+1}+\frac{2abx}{m+2}+\frac{b^2x^2}{m+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))
```

Maple [A] time = 0.004, size = 87, normalized size = 2.

$$\frac{x^{1+m}\left(b^{2}m^{2}x^{2}+2\,ab\,m^{2}x+3\,b^{2}m\,x^{2}+a^{2}m^{2}+8\,abmx+2\,b^{2}x^{2}+5\,a^{2}m+6\,abx+6\,a^{2}\right)}{(3+m)\,(2+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^2,x)
```

```
[Out] x^(1+m)*(b^2*m^2*x^2+2*a*b*m^2*x+3*b^2*m*x^2+a^2*m^2+8*a*b*m*x+2*b^2*x^2+5*
a^2*m+6*a*b*x+6*a^2)/(3+m)/(2+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73606, size = 178, normalized size = 4.14

$$\frac{\left(\left(b^{2}m^{2}+3\,b^{2}m+2\,b^{2}\right)x^{3}+2\left(abm^{2}+4\,abm+3\,ab\right)x^{2}+\left(a^{2}m^{2}+5\,a^{2}m+6\,a^{2}\right)x\right)x^{m}}{m^{3}+6\,m^{2}+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2
*m^2 + 5*a^2*m + 6*a^2)*x)*x^m/(m^3 + 6*m^2 + 11*m + 6)
```

Sympy [A] time = 0.719145, size = 299, normalized size = 6.95

$$\begin{cases} -\frac{a^{2}}{2x^{2}}-\frac{2ab}{x}+b^{2}\log(x) \\ -\frac{a^{2}}{x}+2ab\log(x)+b^{2}x \\ a^{2}\log(x)+2abx+\frac{b^{2}x^{2}}{2} \\ \frac{a^{2}m^{2}xx^{m}}{m^{3}+6m^{2}+11m+6}+\frac{5a^{2}mxx^{m}}{m^{3}+6m^{2}+11m+6}+\frac{6a^{2}xx^{m}}{m^{3}+6m^{2}+11m+6}+\frac{2abm^{2}x^{2}x^{m}}{m^{3}+6m^{2}+11m+6}+\frac{8abmx^{2}x^{m}}{m^{3}+6m^{2}+11m+6}+\frac{6abx^{2}x^{m}}{m^{3}+6m^{2}+11m+6}+\frac{b^{2}m^{2}x^{3}x^{m}}{m^{3}+6m^{2}+11m+6}+ \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x), Eq(m, -3)), (-a**2/x + 2
*a*b*log(x) + b**2*x, Eq(m, -2)), (a**2*log(x) + 2*a*b*x + b**2*x**2/2, Eq(
m, -1)), (a**2*m**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 5*a**2*m*x*x**m/(m*
*3 + 6*m**2 + 11*m + 6) + 6*a**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 2*a*b*
```

```
m**2*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 8*a*b*m*x**2*x**m/(m**3 + 6*m**
2 + 11*m + 6) + 6*a*b*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + b**2*m**2*x**3
*x**m/(m**3 + 6*m**2 + 11*m + 6) + 3*b**2*m*x**3*x**m/(m**3 + 6*m**2 + 11*m
 + 6) + 2*b**2*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

Giac [B] time = 1.23421, size = 158, normalized size = 3.67

$$\frac{b^2m^2x^3x^m + 2\,abm^2x^2x^m + 3\,b^2mx^3x^m + a^2m^2xx^m + 8\,abmx^2x^m + 2\,b^2x^3x^m + 5\,a^2mxx^m + 6\,abx^2x^m + 6\,a^2xx^m}{m^3 + 6\,m^2 + 11\,m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^3*x^m + 2*a*b*m^2*x^2*x^m + 3*b^2*m*x^3*x^m + a^2*m^2*x*x^m + 8*
a*b*m*x^2*x^m + 2*b^2*x^3*x^m + 5*a^2*m*x*x^m + 6*a*b*x^2*x^m + 6*a^2*x*x^m
)/(m^3 + 6*m^2 + 11*m + 6)
```

3.436 $\int x^{5/2}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}$$

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(9/2))/9 + (2*b^2*x^(11/2))/11
```

Rubi [A] time = 0.0071863, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(9/2))/9 + (2*b^2*x^(11/2))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)^2\,dx &= \int \left(a^2x^{5/2}+2abx^{7/2}+b^2x^{9/2}\right)dx\\ &= \frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.0077778, size = 28, normalized size = 0.78

$$\frac{2}{693}x^{7/2}\left(99a^2+154abx+63b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^2,x]
```

```
[Out] (2*x^(7/2)*(99*a^2 + 154*a*b*x + 63*b^2*x^2))/693
```

Maple [A] time = 0.003, size = 25, normalized size = 0.7

$$\frac{126\,b^2x^2+308\,abx+198\,a^2}{693}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^2,x)
```

```
[Out] 2/693*x^(7/2)*(63*b^2*x^2+154*a*b*x+99*a^2)
```

Maxima [A] time = 0.991102, size = 32, normalized size = 0.89

$$\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

Fricas [A] time = 1.44803, size = 74, normalized size = 2.06

$$\frac{2}{693}\left(63\,b^2x^5+154\,abx^4+99\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^2*x^5 + 154*a*b*x^4 + 99*a^2*x^3)*sqrt(x)
```

Sympy [A] time = 3.74924, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(11/2)/11
```

Giac [A] time = 1.2254, size = 32, normalized size = 0.89

$$\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

3.437 $\int x^{3/2}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}$$

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(7/2))/7 + (2*b^2*x^(9/2))/9
```

Rubi [A] time = 0.007302, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(7/2))/7 + (2*b^2*x^(9/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^2\,dx &= \int \left(a^2x^{3/2}+2abx^{5/2}+b^2x^{7/2}\right)dx\\ &= \frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}\end{aligned}$$

Mathematica [A] time = 0.0077449, size = 28, normalized size = 0.78

$$\frac{2}{315}x^{5/2}\left(63a^2+90abx+35b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^2,x]
```

```
[Out] (2*x^(5/2)*(63*a^2 + 90*a*b*x + 35*b^2*x^2))/315
```

Maple [A] time = 0.004, size = 25, normalized size = 0.7

$$\frac{70\,b^2x^2+180\,abx+126\,a^2}{315}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^2,x)
```

```
[Out] 2/315*x^(5/2)*(35*b^2*x^2+90*a*b*x+63*a^2)
```

Maxima [A] time = 1.04069, size = 32, normalized size = 0.89

$$\frac{2}{9}\, b^2 x^{\frac{9}{2}} + \frac{4}{7}\, a b x^{\frac{7}{2}} + \frac{2}{5}\, a^2 x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

Fricas [A] time = 1.50846, size = 73, normalized size = 2.03

$$\frac{2}{315} \left(35\, b^2 x^4 + 90\, a b x^3 + 63\, a^2 x^2\right) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^2*x^4 + 90*a*b*x^3 + 63*a^2*x^2)*sqrt(x)
```

Sympy [A] time = 1.31535, size = 34, normalized size = 0.94

$$\frac{2a^2 x^{\frac{5}{2}}}{5} + \frac{4abx^{\frac{7}{2}}}{7} + \frac{2b^2 x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(7/2)/7 + 2*b**2*x**(9/2)/9
```

Giac [A] time = 1.22786, size = 32, normalized size = 0.89

$$\frac{2}{9}\, b^2 x^{\frac{9}{2}} + \frac{4}{7}\, a b x^{\frac{7}{2}} + \frac{2}{5}\, a^2 x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

3.438 $\int \sqrt{x}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}$$

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(5/2))/5 + (2*b^2*x^(7/2))/7
```

Rubi [A] time = 0.0073302, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(5/2))/5 + (2*b^2*x^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(a+bx)^2\,dx &= \int \left(a^2\sqrt{x}+2abx^{3/2}+b^2x^{5/2}\right)dx\\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0072735, size = 28, normalized size = 0.78

$$\frac{2}{105}x^{3/2}\left(35a^2+42abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^2,x]
```

```
[Out] (2*x^(3/2)*(35*a^2 + 42*a*b*x + 15*b^2*x^2))/105
```

Maple [A] time = 0.003, size = 25, normalized size = 0.7

$$\frac{30\,b^2x^2+84\,abx+70\,a^2}{105}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*x^(1/2),x)
```

```
[Out] 2/105*x^(3/2)*(15*b^2*x^2+42*a*b*x+35*a^2)
```

Maxima [A] time = 1.09635, size = 32, normalized size = 0.89

$$\frac{2}{7}\,b^2x^{\frac{7}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + \frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

Fricas [A] time = 1.46079, size = 70, normalized size = 1.94

$$\frac{2}{105}\left(15\,b^2x^3 + 42\,abx^2 + 35\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*x^3 + 42*a*b*x^2 + 35*a^2*x)*sqrt(x)
```

Sympy [C] time = 2.77118, size = 1853, normalized size = 51.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*x**(1/2),x)
```

```
[Out] Piecewise((16*a**(23/2)*sqrt(-1 + b*(a/b + x)/a)/(-105*a**8*b**(3/2) + 315*
a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2
)*(a/b + x)**3) - 16*I*a**(23/2)/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a
/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3)
- 40*a**(21/2)*b*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)/(-105*a**8*b**(3/2) + 3
15*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(
9/2)*(a/b + x)**3) + 48*I*a**(21/2)*b*(a/b + x)/(-105*a**8*b**(3/2) + 315*a
**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)
*(a/b + x)**3) + 30*a**(19/2)*b**2*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)**2/(-
105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b +
x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 48*I*a**(19/2)*b**2*(a/b + x)**2/
(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b
+ x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 40*a**(17/2)*b**3*sqrt(-1 + b*(
a/b + x)/a)*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x)
- 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) + 16*I*a
**(17/2)*b**3*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x
```

```
) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) + 100*
a**(15/2)*b**4*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)**4/(-105*a**8*b**(3/2) +
315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**
(9/2)*(a/b + x)**3) - 96*a**(13/2)*b**5*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)*
*5/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a
/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) + 30*a**(11/2)*b**6*sqrt(-1 +
b*(a/b + x)/a)*(a/b + x)**6/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b +
x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3), Abs(
b*(a/b + x))/Abs(a) > 1), (16*I*a**(23/2)*sqrt(1 - b*(a/b + x)/a)/(-105*a**
8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 +
 105*a**5*b**(9/2)*(a/b + x)**3) - 16*I*a**(23/2)/(-105*a**8*b**(3/2) + 315
*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/
2)*(a/b + x)**3) - 40*I*a**(21/2)*b*sqrt(1 - b*(a/b + x)/a)*(a/b + x)/(-105
*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)*
*2 + 105*a**5*b**(9/2)*(a/b + x)**3) + 48*I*a**(21/2)*b*(a/b + x)/(-105*a**
8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 +
 105*a**5*b**(9/2)*(a/b + x)**3) + 30*I*a**(19/2)*b**2*sqrt(1 - b*(a/b + x)
/a)*(a/b + x)**2/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a*
*6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 48*I*a**(19/2)
*b**2*(a/b + x)**2/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*
a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 40*I*a**(17/
2)*b**3*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a**7
*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a
/b + x)**3) + 16*I*a**(17/2)*b**3*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a*
*7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*
(a/b + x)**3) + 100*I*a**(15/2)*b**4*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**4/(
-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b +
 x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 96*I*a**(13/2)*b**5*sqrt(1 - b*(
a/b + x)/a)*(a/b + x)**5/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x)
- 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) + 30*I*a
**(11/2)*b**6*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**6/(-105*a**8*b**(3/2) + 31
5*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9
/2)*(a/b + x)**3), True))
```

Giac [A] time = 1.18563, size = 32, normalized size = 0.89

$$\frac{2}{7}b^2x^{\frac{7}{2}} + \frac{4}{5}abx^{\frac{5}{2}} + \frac{2}{3}a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

3.439 $\int \frac{(a+bx)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=34

$$2a^2\sqrt{x}+\frac{4}{3}abx^{3/2}+\frac{2}{5}b^2x^{5/2}$$

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*b^2*x^(5/2))/5
```

Rubi [A] time = 0.0069611, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$2a^2\sqrt{x}+\frac{4}{3}abx^{3/2}+\frac{2}{5}b^2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[x],x]

[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*b^2*x^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{x}}\,dx &= \int \left(\frac{a^2}{\sqrt{x}}+2ab\sqrt{x}+b^2x^{3/2}\right)dx\\ &= 2a^2\sqrt{x}+\frac{4}{3}abx^{3/2}+\frac{2}{5}b^2x^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0073289, size = 28, normalized size = 0.82

$$\frac{2}{15}\sqrt{x}\left(15a^2+10abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[x],x]

[Out] (2*Sqrt[x]*(15*a^2 + 10*a*b*x + 3*b^2*x^2))/15
```

Maple [A] time = 0.004, size = 25, normalized size = 0.7

$$\frac{6\,b^2x^2+20\,abx+30\,a^2}{15}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(1/2),x)
```

```
[Out] 2/15*x^(1/2)*(3*b^2*x^2+10*a*b*x+15*a^2)
```

Maxima [A] time = 1.04053, size = 32, normalized size = 0.94

$$\frac{2}{5}\,b^2 x^{\frac{5}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} + 2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

Fricas [A] time = 1.50465, size = 62, normalized size = 1.82

$$\frac{2}{15}\left(3\,b^2x^2 + 10\,abx + 15\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 + 10*a*b*x + 15*a^2)*sqrt(x)
```

Sympy [A] time = 0.359422, size = 32, normalized size = 0.94

$$2a^2\sqrt{x} + \frac{4abx^{\frac{3}{2}}}{3} + \frac{2b^2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(3/2)/3 + 2*b**2*x**(5/2)/5
```

Giac [A] time = 1.23031, size = 32, normalized size = 0.94

$$\frac{2}{5}\,b^2 x^{\frac{5}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} + 2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

3.440 $\int \frac{(a+bx)^2}{x^{3/2}} \, dx$

Optimal. Leaf size=32

$$-\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}b^2x^{3/2}$$

```
[Out] (-2*a^2)/Sqrt[x] + 4*a*b*Sqrt[x] + (2*b^2*x^(3/2))/3
```

Rubi [A] time = 0.0067661, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + 4*a*b*Sqrt[x] + (2*b^2*x^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{3/2}} \, dx &= \int \left(\frac{a^2}{x^{3/2}}+\frac{2ab}{\sqrt{x}}+b^2\sqrt{x}\right) dx \\ &= -\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}b^2x^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0094845, size = 27, normalized size = 0.84

$$\frac{2\left(-3a^2+6abx+b^2x^2\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(3/2),x]
```

```
[Out] (2*(-3*a^2 + 6*a*b*x + b^2*x^2))/(3*Sqrt[x])
```

Maple [A] time = 0.003, size = 25, normalized size = 0.8

$$-\frac{-2\,b^{2}x^{2}-12\,abx+6\,a^{2}}{3}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(3/2),x)
```

```
[Out] -2/3*(-b^2*x^2-6*a*b*x+3*a^2)/x^(1/2)
```

Maxima [A] time = 1.02959, size = 32, normalized size = 1.

$$\frac{2}{3}\,b^{2}x^{\frac{3}{2}}+4\,ab\sqrt{x}-\frac{2\,a^{2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

Fricas [A] time = 1.46386, size = 55, normalized size = 1.72

$$\frac{2\left(b^{2}x^{2}+6\,abx-3\,a^{2}\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*x^2 + 6*a*b*x - 3*a^2)/sqrt(x)
```

Sympy [A] time = 0.730087, size = 31, normalized size = 0.97

$$-\frac{2a^{2}}{\sqrt{x}}+4ab\sqrt{x}+\frac{2b^{2}x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(3/2),x)
```

```
[Out] -2*a**2/sqrt(x) + 4*a*b*sqrt(x) + 2*b**2*x**(3/2)/3
```

Giac [A] time = 1.20122, size = 32, normalized size = 1.

$$\frac{2}{3}\,b^{2}x^{\frac{3}{2}}+4\,ab\sqrt{x}-\frac{2\,a^{2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

3.441 $\int \frac{(a+bx)^2}{x^{5/2}} \, dx$

Optimal. Leaf size=32

$$-\frac{2a^2}{3x^{3/2}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$$

```
[Out] (-2*a^2)/(3*x^(3/2)) - (4*a*b)/Sqrt[x] + 2*b^2*Sqrt[x]
```

Rubi [A] time = 0.0074047, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^2}{3x^{3/2}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(5/2),x]
```

```
[Out] (-2*a^2)/(3*x^(3/2)) - (4*a*b)/Sqrt[x] + 2*b^2*Sqrt[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{5/2}}\,dx &= \int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{x^{3/2}}+\frac{b^2}{\sqrt{x}}\right)dx\\ &= -\frac{2a^2}{3x^{3/2}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}\end{aligned}$$

Mathematica [A] time = 0.0087331, size = 26, normalized size = 0.81

$$-\frac{2\left(a^2+6abx-3b^2x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(5/2),x]
```

```
[Out] (-2*(a^2 + 6*a*b*x - 3*b^2*x^2))/(3*x^(3/2))
```

Maple [A] time = 0.003, size = 23, normalized size = 0.7

$$-\frac{-6\,b^2x^2+12\,abx+2\,a^2}{3}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(5/2),x)
```

```
[Out] -2/3*(-3*b^2*x^2+6*a*b*x+a^2)/x^(3/2)
```

Maxima [A] time = 1.05362, size = 31, normalized size = 0.97

$$2\,b^2\sqrt{x} - \frac{2\left(6\,abx + a^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

Fricas [A] time = 1.40446, size = 55, normalized size = 1.72

$$\frac{2\left(3\,b^2x^2 - 6\,abx - a^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*x^2 - 6*a*b*x - a^2)/x^(3/2)
```

Sympy [A] time = 0.883137, size = 31, normalized size = 0.97

$$-\frac{2a^2}{3x^{\frac{3}{2}}} - \frac{4ab}{\sqrt{x}} + 2b^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(5/2),x)
```

```
[Out] -2*a**2/(3*x**(3/2)) - 4*a*b/sqrt(x) + 2*b**2*sqrt(x)
```

Giac [A] time = 1.21539, size = 31, normalized size = 0.97

$$2\,b^2\sqrt{x} - \frac{2\left(6\,abx + a^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

3.442 $\int x^m(a+bx)^3\,dx$

Optimal. Leaf size=61

$$\frac{3a^2bx^{m+2}}{m+2}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(2 + m))/(2 + m) + (3*a*b^2*x^(3 + m))
/(3 + m) + (b^3*x^(4 + m))/(4 + m)
```

Rubi [A] time = 0.0199992, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2bx^{m+2}}{m+2}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(2 + m))/(2 + m) + (3*a*b^2*x^(3 + m))
/(3 + m) + (b^3*x^(4 + m))/(4 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^3\,dx &= \int \left(a^3x^m+3a^2bx^{1+m}+3ab^2x^{2+m}+b^3x^{3+m}\right)dx\\ &= \frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m}\end{aligned}$$

Mathematica [A] time = 0.0320955, size = 54, normalized size = 0.89

$$x^{m+1}\left(\frac{3a^2bx}{m+2}+\frac{a^3}{m+1}+\frac{3ab^2x^2}{m+3}+\frac{b^3x^3}{m+4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x)/(2 + m) + (3*a*b^2*x^2)/(3 + m) + (b^3
*x^3)/(4 + m))
```

Maple [B] time = 0.004, size = 170, normalized size = 2.8

$$\frac{x^{1+m}\left(b^3m^3x^3+3\,ab^2m^3x^2+6\,b^3m^2x^3+3\,a^2bm^3x+21\,ab^2m^2x^2+11\,b^3mx^3+a^3m^3+24\,a^2bm^2x+42\,ab^2mx^2+6\,b^3x^3\right.}{(4+m)\,(3+m)\,(2+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^3,x)
```

```
[Out] x^(1+m)*(b^3*m^3*x^3+3*a*b^2*m^3*x^2+6*b^3*m^2*x^3+3*a^2*b*m^3*x+21*a*b^2*m
^2*x^2+11*b^3*m*x^3+a^3*m^3+24*a^2*b*m^2*x+42*a*b^2*m*x^2+6*b^3*x^3+9*a^3*m
^2+57*a^2*b*m*x+24*a*b^2*x^2+26*a^3*m+36*a^2*b*x+24*a^3)/(4+m)/(3+m)/(2+m)/
(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.58277, size = 336, normalized size = 5.51

$$\frac{\left(\left(b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3\right)x^4+3\,\left(ab^2m^3+7\,ab^2m^2+14\,ab^2m+8\,ab^2\right)x^3+3\,\left(a^2bm^3+8\,a^2bm^2+19\,a^2bm+12\,a\right.\right.}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
+ 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*
a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*x^m/(m^4 + 10*m^3
 + 35*m^2 + 50*m + 24)
```

Sympy [A] time = 1.09904, size = 663, normalized size = 10.87

$$\begin{cases} -\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x) \\ -\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log(x)+b^3x \\ -\frac{a^3}{x}+3a^2b\log(x)+3ab^2x+\frac{b^3x^2}{2} \\ a^3\log(x)+3a^2bx+\frac{3ab^2x^2}{2}+\frac{b^3x^3}{3} \\ \frac{a^3m^3xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{9a^3m^2xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{26a^3mxx^m}{m^4+10m^3+35m^2+50m+24}+\frac{24a^3xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{3a^2bm^3x^2x^m}{m^4+10m^3+35m^2+50m+24}+\ldots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x), E
q(m, -4)), (-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**3*x, Eq(m, -
3)), (-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2, Eq(m, -2)), (a*
*3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3, Eq(m, -1)), (a**3*m
**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 9*a**3*m**2*x*x**m/(m**
4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*a**3*m*x*x**m/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24) + 24*a**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
 3*a**2*b*m**3*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*a**2*b
*m**2*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 57*a**2*b*m*x**2*x
**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 36*a**2*b*x**2*x**m/(m**4 + 10
*m**3 + 35*m**2 + 50*m + 24) + 3*a*b**2*m**3*x**3*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 21*a*b**2*m**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 5
0*m + 24) + 42*a*b**2*m*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
24*a*b**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + b**3*m**3*x**4
*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**3*m**2*x**4*x**m/(m**4
+ 10*m**3 + 35*m**2 + 50*m + 24) + 11*b**3*m*x**4*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24
), True))
```

Giac [B] time = 1.18397, size = 302, normalized size = 4.95

$$\frac{b^3m^3x^4x^m + 3\,ab^2m^3x^3x^m + 6\,b^3m^2x^4x^m + 3\,a^2bm^3x^2x^m + 21\,ab^2m^2x^3x^m + 11\,b^3mx^4x^m + a^3m^3xx^m + 24\,a^2bm^2x^2x^m}{m^4 + 10\,m^3 + 35\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^4*x^m + 3*a*b^2*m^3*x^3*x^m + 6*b^3*m^2*x^4*x^m + 3*a^2*b*m^3*x^
2*x^m + 21*a*b^2*m^2*x^3*x^m + 11*b^3*m*x^4*x^m + a^3*m^3*x*x^m + 24*a^2*b*
m^2*x^2*x^m + 42*a*b^2*m*x^3*x^m + 6*b^3*x^4*x^m + 9*a^3*m^2*x*x^m + 57*a^2
*b*m*x^2*x^m + 24*a*b^2*x^3*x^m + 26*a^3*m*x*x^m + 36*a^2*b*x^2*x^m + 24*a^
3*x*x^m)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

3.443 $\int x^{5/2}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{3}a^2bx^{9/2}+\frac{2}{7}a^3x^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}$$

```
[Out] (2*a^3*x^(7/2))/7 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^(11/2))/11 + (2*b^3*x^
(13/2))/13
```

Rubi [A] time = 0.0106187, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2}{3}a^2bx^{9/2}+\frac{2}{7}a^3x^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^(11/2))/11 + (2*b^3*x^
(13/2))/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)^3\,dx &= \int \left(a^3x^{5/2}+3a^2bx^{7/2}+3ab^2x^{9/2}+b^3x^{11/2}\right)dx\\ &= \frac{2}{7}a^3x^{7/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}\end{aligned}$$

Mathematica [A] time = 0.0112686, size = 39, normalized size = 0.76

$$\frac{2x^{7/2}\left(1001a^2bx+429a^3+819ab^2x^2+231b^3x^3\right)}{3003}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^3,x]
```

```
[Out] (2*x^(7/2)*(429*a^3 + 1001*a^2*b*x + 819*a*b^2*x^2 + 231*b^3*x^3))/3003
```

Maple [A] time = 0.004, size = 36, normalized size = 0.7

$$\frac{462\,b^3x^3+1638\,ab^2x^2+2002\,a^2bx+858\,a^3}{3003}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^3,x)
```

```
[Out] 2/3003*x^(7/2)*(231*b^3*x^3+819*a*b^2*x^2+1001*a^2*b*x+429*a^3)
```

Maxima [A] time = 1.07651, size = 47, normalized size = 0.92

$$\frac{2}{13}\,b^3x^{\frac{13}{2}} + \frac{6}{11}\,ab^2x^{\frac{11}{2}} + \frac{2}{3}\,a^2bx^{\frac{9}{2}} + \frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7/
2)
```

Fricas [A] time = 1.53442, size = 104, normalized size = 2.04

$$\frac{2}{3003}\left(231\,b^3x^6 + 819\,ab^2x^5 + 1001\,a^2bx^4 + 429\,a^3x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^3*x^6 + 819*a*b^2*x^5 + 1001*a^2*b*x^4 + 429*a^3*x^3)*sqrt(x)
```

Sympy [A] time = 5.68782, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{7}{2}}}{7} + \frac{2a^2bx^{\frac{9}{2}}}{3} + \frac{6ab^2x^{\frac{11}{2}}}{11} + \frac{2b^3x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 2*a**2*b*x**(9/2)/3 + 6*a*b**2*x**(11/2)/11 + 2*b**3*x*
*(13/2)/13
```

Giac [A] time = 1.28215, size = 47, normalized size = 0.92

$$\frac{2}{13}\,b^3x^{\frac{13}{2}} + \frac{6}{11}\,ab^2x^{\frac{11}{2}} + \frac{2}{3}\,a^2bx^{\frac{9}{2}} + \frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7/
2)
```

3.444 $\int x^{3/2}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{6}{7}a^2bx^{7/2}+\frac{2}{5}a^3x^{5/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}$$

```
[Out] (2*a^3*x^(5/2))/5 + (6*a^2*b*x^(7/2))/7 + (2*a*b^2*x^(9/2))/3 + (2*b^3*x^(1
1/2))/11
```

Rubi [A] time = 0.0115775, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6}{7}a^2bx^{7/2}+\frac{2}{5}a^3x^{5/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (6*a^2*b*x^(7/2))/7 + (2*a*b^2*x^(9/2))/3 + (2*b^3*x^(1
1/2))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^3\,dx &= \int \left(a^3x^{3/2}+3a^2bx^{5/2}+3ab^2x^{7/2}+b^3x^{9/2}\right)dx\\ &= \frac{2}{5}a^3x^{5/2}+\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.0099204, size = 39, normalized size = 0.76

$$\frac{2x^{5/2}\left(495a^2bx+231a^3+385ab^2x^2+105b^3x^3\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^3,x]
```

```
[Out] (2*x^(5/2)*(231*a^3 + 495*a^2*b*x + 385*a*b^2*x^2 + 105*b^3*x^3))/1155
```

Maple [A] time = 0.003, size = 36, normalized size = 0.7

$$\frac{210\,b^3x^3+770\,ab^2x^2+990\,a^2bx+462\,a^3}{1155}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^3,x)
```

```
[Out] 2/1155*x^(5/2)*(105*b^3*x^3+385*a*b^2*x^2+495*a^2*b*x+231*a^3)
```

Maxima [A] time = 1.04363, size = 47, normalized size = 0.92

$$\frac{2}{11}\,b^3x^{\frac{11}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2)
```

Fricas [A] time = 1.64924, size = 103, normalized size = 2.02

$$\frac{2}{1155}\left(105\,b^3x^5+385\,ab^2x^4+495\,a^2bx^3+231\,a^3x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^3*x^5 + 385*a*b^2*x^4 + 495*a^2*b*x^3 + 231*a^3*x^2)*sqrt(x)
```

Sympy [A] time = 2.27367, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{6a^2bx^{\frac{7}{2}}}{7}+\frac{2ab^2x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 6*a**2*b*x**(7/2)/7 + 2*a*b**2*x**(9/2)/3 + 2*b**3*x**(
11/2)/11
```

Giac [A] time = 1.21508, size = 47, normalized size = 0.92

$$\frac{2}{11}\,b^3x^{\frac{11}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2)
```

3.445 $\int \sqrt{x}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}a^3x^{3/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}$$

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(5/2))/5 + (6*a*b^2*x^(7/2))/7 + (2*b^3*x^(9
/2))/9
```

Rubi [A] time = 0.0111421, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}a^3x^{3/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(5/2))/5 + (6*a*b^2*x^(7/2))/7 + (2*b^3*x^(9
/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(a+bx)^3\,dx &= \int \left(a^3\sqrt{x}+3a^2bx^{3/2}+3ab^2x^{5/2}+b^3x^{7/2}\right)dx\\ &= \frac{2}{3}a^3x^{3/2}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}\end{aligned}$$

Mathematica [A] time = 0.0105123, size = 39, normalized size = 0.76

$$\frac{2}{315}x^{3/2}\left(189a^2bx+105a^3+135ab^2x^2+35b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^3,x]
```

```
[Out] (2*x^(3/2)*(105*a^3 + 189*a^2*b*x + 135*a*b^2*x^2 + 35*b^3*x^3))/315
```

Maple [A] time = 0.003, size = 36, normalized size = 0.7

$$\frac{70\,b^3x^3+270\,ab^2x^2+378\,a^2bx+210\,a^3}{315}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*x^(1/2),x)
```

```
[Out] 2/315*x^(3/2)*(35*b^3*x^3+135*a*b^2*x^2+189*a^2*b*x+105*a^3)
```

Maxima [A] time = 1.06095, size = 47, normalized size = 0.92

$$\frac{2}{9}b^3x^{\frac{9}{2}} + \frac{6}{7}ab^2x^{\frac{7}{2}} + \frac{6}{5}a^2bx^{\frac{5}{2}} + \frac{2}{3}a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

Fricas [A] time = 1.49454, size = 97, normalized size = 1.9

$$\frac{2}{315}\left(35\,b^3x^4 + 135\,ab^2x^3 + 189\,a^2bx^2 + 105\,a^3x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^3*x^4 + 135*a*b^2*x^3 + 189*a^2*b*x^2 + 105*a^3*x)*sqrt(x)
```

Sympy [C] time = 4.84307, size = 4886, normalized size = 95.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*x**(1/2),x)
```

```
[Out] Piecewise((-32*a**(49/2)*sqrt(-1 + b*(a/b + x)/a)/(315*a**20*b**(3/2) - 189
0*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*
b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(
13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) + 32*I*a**(49/2)/(31
5*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b
 + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x
)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6
) + 176*a**(47/2)*b*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)/(315*a**20*b**(3/2)
- 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300*a
**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**15
*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) - 192*I*a**(47/
2)*b*(a/b + x)/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a
**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*
b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(
15/2)*(a/b + x)**6) - 396*a**(45/2)*b**2*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)
**2/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/
```

```
2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(
a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b
+ x)**6) + 480*I*a**(45/2)*b**2*(a/b + x)**2/(315*a**20*b**(3/2) - 1890*a**
19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b**(9
/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(13/2)
*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) + 462*a**(43/2)*b**3*sqrt
(-1 + b*(a/b + x)/a)*(a/b + x)**3/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)
*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b +
x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)*
*5 + 315*a**14*b**(15/2)*(a/b + x)**6) - 640*I*a**(43/2)*b**3*(a/b + x)**3/
(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(
a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b
+ x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)
**6) - 210*a**(41/2)*b**4*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)**4/(315*a**20*
b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2
 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1
890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) + 480*
I*a**(41/2)*b**4*(a/b + x)**4/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/
b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**
3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 +
 315*a**14*b**(15/2)*(a/b + x)**6) - 378*a**(39/2)*b**5*sqrt(-1 + b*(a/b +
x)/a)*(a/b + x)**5/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 47
25*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a*
*16*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*
b**(15/2)*(a/b + x)**6) - 192*I*a**(39/2)*b**5*(a/b + x)**5/(315*a**20*b**(
3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6
300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*
a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) + 1134*a**
(37/2)*b**6*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)**6/(315*a**20*b**(3/2) - 189
0*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*
b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(
13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) + 32*I*a**(37/2)*b**
6*(a/b + x)**6/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a
**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*
b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(
15/2)*(a/b + x)**6) - 1494*a**(35/2)*b**7*sqrt(-1 + b*(a/b + x)/a)*(a/b + x
)**7/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7
/2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*
(a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b
 + x)**6) + 1098*a**(33/2)*b**8*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)**8/(315*
a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b +
 x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)*
*4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6)
- 430*a**(31/2)*b**9*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)**9/(315*a**20*b**(3
/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 63
00*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a
**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) + 70*a**(29
/2)*b**10*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)**10/(315*a**20*b**(3/2) - 1890
*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b
**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(1
3/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6), Abs(b*(a/b + x))/Abs
(a) > 1), (-32*I*a**(49/2)*sqrt(1 - b*(a/b + x)/a)/(315*a**20*b**(3/2) - 18
90*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17
*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**
(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) + 32*I*a**(49/2)/(3
15*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/
b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b +
x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**
6) + 176*I*a**(47/2)*b*sqrt(1 - b*(a/b + x)/a)*(a/b + x)/(315*a**20*b**(3/2
```

```
) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300
*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**
15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) - 192*I*a**(4
7/2)*b*(a/b + x)/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725
*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**1
6*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b*
*(15/2)*(a/b + x)**6) - 396*I*a**(45/2)*b**2*sqrt(1 - b*(a/b + x)/a)*(a/b +
 x)**2/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**
(7/2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2
)*(a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a
/b + x)**6) + 480*I*a**(45/2)*b**2*(a/b + x)**2/(315*a**20*b**(3/2) - 1890*
a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b*
*(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(13
/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) + 462*I*a**(43/2)*b**3
*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**3/(315*a**20*b**(3/2) - 1890*a**19*b**(
5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/
b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b +
 x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) - 640*I*a**(43/2)*b**3*(a/b + x)
**3/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/
2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(
a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b
+ x)**6) - 210*I*a**(41/2)*b**4*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**4/(315*a
**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b +
x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**
4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) +
 480*I*a**(41/2)*b**4*(a/b + x)**4/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2
)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b +
 x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)
**5 + 315*a**14*b**(15/2)*(a/b + x)**6) - 378*I*a**(39/2)*b**5*sqrt(1 - b*(
a/b + x)/a)*(a/b + x)**5/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x
) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4
725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*
a**14*b**(15/2)*(a/b + x)**6) - 192*I*a**(39/2)*b**5*(a/b + x)**5/(315*a**2
0*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)*
*2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 -
 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) + 11
34*I*a**(37/2)*b**6*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**6/(315*a**20*b**(3/2
) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300
*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**
15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) + 32*I*a**(37
/2)*b**6*(a/b + x)**6/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) +
 4725*a**18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725
*a**16*b**(11/2)*(a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**
14*b**(15/2)*(a/b + x)**6) - 1494*I*a**(35/2)*b**7*sqrt(1 - b*(a/b + x)/a)*
(a/b + x)**7/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**
18*b**(7/2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b*
*(11/2)*(a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15
/2)*(a/b + x)**6) + 1098*I*a**(33/2)*b**8*sqrt(1 - b*(a/b + x)/a)*(a/b + x)
**8/(315*a**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/
2)*(a/b + x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(
a/b + x)**4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b
+ x)**6) - 430*I*a**(31/2)*b**9*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**9/(315*a
**20*b**(3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b +
x)**2 - 6300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**
4 - 1890*a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6) +
 70*I*a**(29/2)*b**10*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**10/(315*a**20*b**(
3/2) - 1890*a**19*b**(5/2)*(a/b + x) + 4725*a**18*b**(7/2)*(a/b + x)**2 - 6
300*a**17*b**(9/2)*(a/b + x)**3 + 4725*a**16*b**(11/2)*(a/b + x)**4 - 1890*
a**15*b**(13/2)*(a/b + x)**5 + 315*a**14*b**(15/2)*(a/b + x)**6), True))
```

Giac [A] time = 1.20288, size = 47, normalized size = 0.92

$$\frac{2}{9}\,b^{3}x^{\frac{9}{2}}+\frac{6}{7}\,ab^{2}x^{\frac{7}{2}}+\frac{6}{5}\,a^{2}bx^{\frac{5}{2}}+\frac{2}{3}\,a^{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

3.446 $\int \frac{(a+bx)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=47

$$2a^2bx^{3/2}+2a^3\sqrt{x}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}$$

```
[Out] 2*a^3*Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(7/2))/7
```

Rubi [A] time = 0.0114421, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$2a^2bx^{3/2}+2a^3\sqrt{x}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^3}{\sqrt{x}}+3a^2b\sqrt{x}+3ab^2x^{3/2}+b^3x^{5/2}\right)dx\\ &= 2a^3\sqrt{x}+2a^2bx^{3/2}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0102448, size = 39, normalized size = 0.83

$$\frac{2}{35}\sqrt{x}\left(35a^2bx+35a^3+21ab^2x^2+5b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(35*a^3 + 35*a^2*b*x + 21*a*b^2*x^2 + 5*b^3*x^3))/35
```

Maple [A] time = 0.004, size = 36, normalized size = 0.8

$$\frac{10\,b^3x^3+42\,ab^2x^2+70\,a^2bx+70\,a^3}{35}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(1/2),x)
```

```
[Out] 2/35*x^(1/2)*(5*b^3*x^3+21*a*b^2*x^2+35*a^2*b*x+35*a^3)
```

Maxima [A] time = 1.02586, size = 47, normalized size = 1.

$$\frac{2}{7}\,b^3x^{\frac{7}{2}} + \frac{6}{5}\,ab^2x^{\frac{5}{2}} + 2\,a^2bx^{\frac{3}{2}} + 2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

Fricas [A] time = 1.44814, size = 85, normalized size = 1.81

$$\frac{2}{35}\left(5\,b^3x^3 + 21\,ab^2x^2 + 35\,a^2bx + 35\,a^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 21*a*b^2*x^2 + 35*a^2*b*x + 35*a^3)*sqrt(x)
```

Sympy [A] time = 0.644315, size = 46, normalized size = 0.98

$$2a^3\sqrt{x} + 2a^2bx^{\frac{3}{2}} + \frac{6ab^2x^{\frac{5}{2}}}{5} + \frac{2b^3x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 2*a**2*b*x**(3/2) + 6*a*b**2*x**(5/2)/5 + 2*b**3*x**(7/2)/
7
```

Giac [A] time = 1.19342, size = 47, normalized size = 1.

$$\frac{2}{7}\,b^3x^{\frac{7}{2}} + \frac{6}{5}\,ab^2x^{\frac{5}{2}} + 2\,a^2bx^{\frac{3}{2}} + 2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

3.447 $\int \frac{(a+bx)^3}{x^{3/2}} dx$

Optimal. Leaf size=45

$$6a^2b\sqrt{x} - \frac{2a^3}{\sqrt{x}} + 2ab^2x^{3/2} + \frac{2}{5}b^3x^{5/2}$$

```
[Out] (-2*a^3)/Sqrt[x] + 6*a^2*b*Sqrt[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(5/2))/5
```

Rubi [A] time = 0.0111342, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$6a^2b\sqrt{x} - \frac{2a^3}{\sqrt{x}} + 2ab^2x^{3/2} + \frac{2}{5}b^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 6*a^2*b*Sqrt[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^3}{x^{3/2}} dx &= \int \left(\frac{a^3}{x^{3/2}} + \frac{3a^2b}{\sqrt{x}} + 3ab^2\sqrt{x} + b^3x^{3/2}\right) dx \\ &= -\frac{2a^3}{\sqrt{x}} + 6a^2b\sqrt{x} + 2ab^2x^{3/2} + \frac{2}{5}b^3x^{5/2} \end{aligned}$$

Mathematica [A] time = 0.0114963, size = 38, normalized size = 0.84

$$\frac{2\left(15a^2bx - 5a^3 + 5ab^2x^2 + b^3x^3\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(3/2),x]
```

```
[Out] (2*(-5*a^3 + 15*a^2*b*x + 5*a*b^2*x^2 + b^3*x^3))/(5*Sqrt[x])
```

Maple [A] time = 0.004, size = 36, normalized size = 0.8

$$-\frac{-2\,b^{3}x^{3}-10\,ab^{2}x^{2}-30\,a^{2}bx+10\,a^{3}}{5}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(3/2),x)
```

```
[Out] -2/5*(-b^3*x^3-5*a*b^2*x^2-15*a^2*b*x+5*a^3)/x^(1/2)
```

Maxima [A] time = 1.09566, size = 47, normalized size = 1.04

$$\frac{2}{5}\,b^{3}x^{\frac{5}{2}}+2\,ab^{2}x^{\frac{3}{2}}+6\,a^{2}b\sqrt{x}-\frac{2\,a^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

Fricas [A] time = 1.51226, size = 78, normalized size = 1.73

$$\frac{2\left(b^{3}x^{3}+5\,ab^{2}x^{2}+15\,a^{2}bx-5\,a^{3}\right)}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 + 5*a*b^2*x^2 + 15*a^2*b*x - 5*a^3)/sqrt(x)
```

Sympy [A] time = 0.897178, size = 44, normalized size = 0.98

$$-\frac{2a^{3}}{\sqrt{x}}+6a^{2}b\sqrt{x}+2ab^{2}x^{\frac{3}{2}}+\frac{2b^{3}x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(3/2),x)
```

```
[Out] -2*a**3/sqrt(x) + 6*a**2*b*sqrt(x) + 2*a*b**2*x**(3/2) + 2*b**3*x**(5/2)/5
```

Giac [A] time = 1.22818, size = 47, normalized size = 1.04

$$\frac{2}{5}\,b^{3}x^{\frac{5}{2}}+2\,ab^{2}x^{\frac{3}{2}}+6\,a^{2}b\sqrt{x}-\frac{2\,a^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

3.448 $\int \frac{(a+bx)^3}{x^{5/2}} dx$

Optimal. Leaf size=47

$$-\frac{6a^2b}{\sqrt{x}}-\frac{2a^3}{3x^{3/2}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}$$

```
[Out] (-2*a^3)/(3*x^(3/2)) - (6*a^2*b)/Sqrt[x] + 6*a*b^2*Sqrt[x] + (2*b^3*x^(3/2)
)/3
```

Rubi [A] time = 0.0107336, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{6a^2b}{\sqrt{x}}-\frac{2a^3}{3x^{3/2}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) - (6*a^2*b)/Sqrt[x] + 6*a*b^2*Sqrt[x] + (2*b^3*x^(3/2)
)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{5/2}}\,dx &= \int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{x^{3/2}}+\frac{3ab^2}{\sqrt{x}}+b^3\sqrt{x}\right)dx\\ &= -\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{\sqrt{x}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0145666, size = 38, normalized size = 0.81

$$\frac{2\left(-9a^2bx-a^3+9ab^2x^2+b^3x^3\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(5/2),x]
```

```
[Out] (2*(-a^3 - 9*a^2*b*x + 9*a*b^2*x^2 + b^3*x^3))/(3*x^(3/2))
```

Maple [A] time = 0.003, size = 34, normalized size = 0.7

$$-\frac{-2 b^3x^3 - 18 ab^2x^2 + 18 a^2bx + 2 a^3}{3}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(5/2),x)
```

```
[Out] -2/3*(-b^3*x^3-9*a*b^2*x^2+9*a^2*b*x+a^3)/x^(3/2)
```

Maxima [A] time = 1.10933, size = 46, normalized size = 0.98

$$\frac{2}{3} b^3x^{\frac{3}{2}} + 6 ab^2\sqrt{x} - \frac{2 \left(9 a^2bx + a^3\right)}{3 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

Fricas [A] time = 1.54299, size = 74, normalized size = 1.57

$$\frac{2 \left(b^3x^3 + 9 ab^2x^2 - 9 a^2bx - a^3\right)}{3 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*x^3 + 9*a*b^2*x^2 - 9*a^2*b*x - a^3)/x^(3/2)
```

Sympy [A] time = 1.19895, size = 46, normalized size = 0.98

$$-\frac{2a^3}{3x^{\frac{3}{2}}} - \frac{6a^2b}{\sqrt{x}} + 6ab^2\sqrt{x} + \frac{2b^3x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) - 6*a**2*b/sqrt(x) + 6*a*b**2*sqrt(x) + 2*b**3*x**(3/2
)/3
```

Giac [A] time = 1.20573, size = 46, normalized size = 0.98

$$\frac{2}{3} b^3x^{\frac{3}{2}} + 6 ab^2\sqrt{x} - \frac{2 \left(9 a^2bx + a^3\right)}{3 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

3.449 $\int \frac{x^{5/2}}{a+bx} \, dx$

Optimal. Leaf size=68

$$\frac{2a^2\sqrt{x}}{b^3}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}-\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

```
[Out] (2*a^2*Sqrt[x])/b^3 - (2*a*x^(3/2))/(3*b^2) + (2*x^(5/2))/(5*b) - (2*a^(5/2
)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rubi [A] time = 0.0319405, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 205}

$$\frac{2a^2\sqrt{x}}{b^3}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}-\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x),x]
```

```
[Out] (2*a^2*Sqrt[x])/b^3 - (2*a*x^(3/2))/(3*b^2) + (2*x^(5/2))/(5*b) - (2*a^(5/2
)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{5/2}}{a+bx}\,dx = \frac{2x^{5/2}}{5b} - \frac{a\int \frac{x^{3/2}}{a+bx}\,dx}{b}$$
$$= -\frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} + \frac{a^2\int \frac{\sqrt{x}}{a+bx}\,dx}{b^2}$$
$$= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{a^3\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{b^3}$$
$$= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b^3}$$
$$= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Mathematica [A] time = 0.0296076, size = 61, normalized size = 0.9

$$\frac{2\sqrt{x}\left(15a^2 - 5abx + 3b^2x^2\right)}{15b^3} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(15*a^2 - 5*a*b*x + 3*b^2*x^2))/(15*b^3) - (2*a^(5/2)*ArcTan[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Maple [A] time = 0.007, size = 54, normalized size = 0.8

$$\frac{2}{5b}x^{\frac{5}{2}} - \frac{2a}{3b^2}x^{\frac{3}{2}} + 2\frac{a^2\sqrt{x}}{b^3} - 2\frac{a^3}{b^3\sqrt{ab}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a),x)
```

```
[Out] 2/5*x^(5/2)/b-2/3*a*x^(3/2)/b^2+2*a^2*x^(1/2)/b^3-2*a^3/b^3/(a*b)^(1/2)*arc
tan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52884, size = 308, normalized size = 4.53

$$\left[\frac{15\,a^{2}\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(3\,b^{2}x^{2}-5\,abx+15\,a^{2}\right)\sqrt{x}}{15\,b^{3}},-\frac{2\left(15\,a^{2}\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\left(3\,b^{2}x^{2}-5\,abx+15\,a\right.\right.}{15\,b^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a))
+ 2*(3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3, -2/15*(15*a^2*sqrt(a/b)*ar
ctan(b*sqrt(x)*sqrt(a/b)/a) - (3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

Sympy [A] time = 16.3615, size = 121, normalized size = 1.78

$$\begin{cases} \frac{ia^{\frac{5}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^{4}\sqrt{\frac{1}{b}}}-\frac{ia^{\frac{5}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^{4}\sqrt{\frac{1}{b}}}+\frac{2a^{2}\sqrt{x}}{b^{3}}-\frac{2ax^{\frac{3}{2}}}{3b^{2}}+\frac{2x^{\frac{5}{2}}}{5b} & \text{for}\ b\neq 0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a),x)
```

```
[Out] Piecewise((I*a**(5/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**4*sqrt(1/b))
- I*a**(5/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**4*sqrt(1/b)) + 2*a**2*s
qrt(x)/b**3 - 2*a*x**(3/2)/(3*b**2) + 2*x**(5/2)/(5*b), Ne(b, 0)), (2*x**(7
/2)/(7*a), True))
```

Giac [A] time = 1.19747, size = 80, normalized size = 1.18

$$-\frac{2\,a^{3}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^{3}}+\frac{2\left(3\,b^{4}x^{\frac{5}{2}}-5\,ab^{3}x^{\frac{3}{2}}+15\,a^{2}b^{2}\sqrt{x}\right)}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^4*x^(5/2) -
5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

3.450 $\int \frac{x^{3/2}}{a+bx} \, dx$

Optimal. Leaf size=53

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

```
[Out] (-2*a*Sqrt[x])/b^2 + (2*x^(3/2))/(3*b) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

Rubi [A] time = 0.0166034, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 205}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x),x]
```

```
[Out] (-2*a*Sqrt[x])/b^2 + (2*x^(3/2))/(3*b) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{3/2}}{a+bx}\,dx = \frac{2x^{3/2}}{3b} - \frac{a\int \frac{\sqrt{x}}{a+bx}\,dx}{b}$$
$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{a^2\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{b^2}$$
$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.018496, size = 49, normalized size = 0.92

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}} + \frac{2\sqrt{x}(bx-3a)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(-3*a + b*x))/(3*b^2) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/b^(5/2)
```

Maple [A] time = 0.003, size = 43, normalized size = 0.8

$$\frac{2}{3\,b}x^{\frac{3}{2}} - 2\,\frac{a\sqrt{x}}{b^2} + 2\,\frac{a^2}{b^2\sqrt{ab}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a),x)
```

```
[Out] 2/3*x^(3/2)/b-2*a*x^(1/2)/b^2+2*a^2/b^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^
(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63256, size = 244, normalized size = 4.6

$$\left[\frac{3\,a\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(bx-3\,a)\sqrt{x}}{3\,b^2}, \frac{2\left(3\,a\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+(bx-3\,a)\sqrt{x}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*a*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*
(b*x - 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a)
+ (b*x - 3*a)*sqrt(x))/b^2]
```

Sympy [A] time = 3.92548, size = 105, normalized size = 1.98

$$\begin{cases} -\frac{ia^{\frac{3}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^3\sqrt{\frac{1}{b}}} + \frac{ia^{\frac{3}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^3\sqrt{\frac{1}{b}}} - \frac{2a\sqrt{x}}{b^2} + \frac{2x^{\frac{3}{2}}}{3b} & \text{for}\ b \neq 0 \\ \frac{2x^{\frac{5}{2}}}{5a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a),x)
```

```
[Out] Piecewise((-I*a**(3/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**3*sqrt(1/b))
 + I*a**(3/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**3*sqrt(1/b)) - 2*a*sqr
t(x)/b**2 + 2*x**(3/2)/(3*b), Ne(b, 0)), (2*x**(5/2)/(5*a), True))
```

Giac [A] time = 1.16497, size = 61, normalized size = 1.15

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\left(b^2x^{\frac{3}{2}} - 3\,ab\sqrt{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b^2*x^(3/2) - 3*a*
b*sqrt(x))/b^3
```

3.451 $\int \frac{\sqrt{x}}{a+bx}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rubi [A] time = 0.0117361, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 205}

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{a+bx}\,dx &= \frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{b}\\
&= \frac{2\sqrt{x}}{b}-\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b}\\
&= \frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0092629, size = 40, normalized size = 1.

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Maple [A] time = 0.005, size = 32, normalized size = 0.8

$$2\,\frac{\sqrt{x}}{b}-2\,\frac{a}{b\sqrt{ab}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a),x)
```

```
[Out] 2*x^(1/2)/b-2/b*a/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59417, size = 189, normalized size = 4.72

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,\sqrt{x}}{b},-\frac{2\left(\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{x}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*sqrt(x))
/b, -2*(sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - sqrt(x))/b]
```

Sympy [A] time = 1.05438, size = 92, normalized size = 2.3

$$\begin{cases} \frac{i\sqrt{a}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^2\sqrt{\frac{1}{b}}} - \frac{i\sqrt{a}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^2\sqrt{\frac{1}{b}}} + \frac{2\sqrt{x}}{b} & \text{for } b \neq 0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a),x)
```

```
[Out] Piecewise((I*sqrt(a)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**2*sqrt(1/b)) -
 I*sqrt(a)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**2*sqrt(1/b)) + 2*sqrt(x)/
b, Ne(b, 0)), (2*x**(3/2)/(3*a), True))
```

Giac [A] time = 1.16379, size = 42, normalized size = 1.05

$$-\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

3.452 $\int \frac{1}{\sqrt{x}(a+bx)} dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0082801, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(a+bx)} dx &= 2\,\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sqrt{x}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0047345, size = 29, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.005, size = 19, normalized size = 0.7

$$2\frac{1}{\sqrt{ab}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/x^(1/2),x)
```

```
[Out] 2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55453, size = 163, normalized size = 5.62

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a))/(a*b), -2*sqrt
(a*b)*arctan(sqrt(a*b)/(b*sqrt(x)))/(a*b)]
```

Sympy [A] time = 2.07992, size = 94, normalized size = 3.24

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for}\: a = 0 \\ \frac{2\sqrt{x}}{a} & \text{for}\: b = 0 \\ -\frac{i\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{\sqrt{a}b\sqrt{\frac{1}{b}}} + \frac{i\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{\sqrt{a}b\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)), (
2*sqrt(x)/a, Eq(b, 0)), (-I*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(sqrt(a)*b*
sqrt(1/b)) + I*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(sqrt(a)*b*sqrt(1/b)), Tr
ue))
```

Giac [A] time = 1.25566, size = 24, normalized size = 0.83

$$\frac{2 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

3.453 $\int \frac{1}{x^{3/2}(a+bx)} \, dx$

Optimal. Leaf size=40

$$-\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}$$

```
[Out] -2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0129225, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$-\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)),x]
```

```
[Out] -2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx)}\,dx = -\frac{2}{a\sqrt{x}} - \frac{b\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{a}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [C] time = 0.0045101, size = 25, normalized size = 0.62

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{bx}{a}\right)}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x)/a)])/(a*Sqrt[x])
```

Maple [A] time = 0.007, size = 32, normalized size = 0.8

$$-2\,\frac{b}{a\sqrt{ab}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right) - 2\,\frac{1}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a),x)
```

```
[Out] -2*b/a/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))-2/a/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63636, size = 207, normalized size = 5.18

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right) - 2\,\sqrt{x}}{ax}, \frac{2\left(x\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right) - \sqrt{x}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(x*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*sqrt(x
))/(a*x), 2*(x*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - sqrt(x))/(a*x)]
```

Sympy [A] time = 5.38717, size = 102, normalized size = 2.55

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a = 0 \\ -\frac{2}{a\sqrt{x}} + \frac{i\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{3}{2}}\sqrt{\frac{1}{b}}} - \frac{i\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{3}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(a*sqrt(x)), Eq(b, 0)),
(-2/(3*b*x**(3/2)), Eq(a, 0)), (-2/(a*sqrt(x)) + I*log(-I*sqrt(a)*sqrt(1/b)
 + sqrt(x))/(a**(3/2)*sqrt(1/b)) - I*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(a*
*(3/2)*sqrt(1/b)), True))
```

Giac [A] time = 1.19662, size = 42, normalized size = 1.05

$$-\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

3.454 $\int \frac{1}{x^{5/2}(a+bx)}\,dx$

Optimal. Leaf size=53

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{3ax^{3/2}}$$

```
[Out] -2/(3*a*x^(3/2)) + (2*b)/(a^2*Sqrt[x]) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0176221, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (2*b)/(a^2*Sqrt[x]) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/2}(a+bx)}\,dx &= -\frac{2}{3ax^{3/2}}-\frac{b\int \frac{1}{x^{3/2}(a+bx)}\,dx}{a}\\
&= -\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{b^2\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{a^2}\\
&= -\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\
&= -\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0046637, size = 27, normalized size = 0.51

$$
-\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{bx}{a}\right)}{3ax^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x)/a)])/(3*a*x^(3/2))
```

Maple [A] time = 0.009, size = 43, normalized size = 0.8

$$
-\frac{2}{3\,a}x^{-\frac{3}{2}}+2\,\frac{b}{a^2\sqrt{x}}+2\,\frac{b^2}{a^2\sqrt{ab}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a),x)
```

```
[Out] -2/3/a/x^(3/2)+2*b/a^2/x^(1/2)+2*b^2/a^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61631, size = 275, normalized size = 5.19

$$\left[\frac{3\,bx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(3\,bx-a)\sqrt{x}}{3\,a^2x^2},\,-\frac{2\left(3\,bx^2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx-a)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*x^2*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a))
+ 2*(3*b*x - a)*sqrt(x))/(a^2*x^2), -2/3*(3*b*x^2*sqrt(b/a)*arctan(a*sqrt(b
/a)/(b*sqrt(x))) - (3*b*x - a)*sqrt(x))/(a^2*x^2)]
```

Sympy [A] time = 15.8911, size = 121, normalized size = 2.28

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for}\: a = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}} & \text{for}\: b = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}} + \frac{2b}{a^{2}\sqrt{x}} - \frac{ib\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{5}{2}}\sqrt{\frac{1}{b}}} + \frac{ib\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{5}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)
), (-2/(3*a*x**(3/2)), Eq(b, 0)), (-2/(3*a*x**(3/2)) + 2*b/(a**2*sqrt(x)) -
 I*b*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(5/2)*sqrt(1/b)) + I*b*log(I*s
qrt(a)*sqrt(1/b) + sqrt(x))/(a**(5/2)*sqrt(1/b)), True))
```

Giac [A] time = 1.2596, size = 55, normalized size = 1.04

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{2\,(3\,bx-a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

3.455 $\int \frac{1}{x^{7/2}(a+bx)}\,dx$

Optimal. Leaf size=68

$$-\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2b}{3a^2x^{3/2}}-\frac{2}{5ax^{5/2}}$$

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(3*a^2*x^(3/2)) - (2*b^2)/(a^3*Sqrt[x]) - (2*b^(5/
2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rubi [A] time = 0.0232084, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$-\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2b}{3a^2x^{3/2}}-\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x)),x]
```

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(3*a^2*x^(3/2)) - (2*b^2)/(a^3*Sqrt[x]) - (2*b^(5/
2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}(a+bx)}\,dx &= -\frac{2}{5ax^{5/2}} - \frac{b\int \frac{1}{x^{5/2}(a+bx)}\,dx}{a} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} + \frac{b^2\int \frac{1}{x^{3/2}(a+bx)}\,dx}{a^2} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} - \frac{2b^2}{a^3\sqrt{x}} - \frac{b^3\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{a^3} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} - \frac{2b^2}{a^3\sqrt{x}} - \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a^3} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} - \frac{2b^2}{a^3\sqrt{x}} - \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.004815, size = 27, normalized size = 0.4

$$-\frac{2\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\frac{bx}{a}\right)}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x)),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/2, 1, -3/2, -((b*x)/a)])/(5*a*x^(5/2))
```

Maple [A] time = 0.008, size = 54, normalized size = 0.8

$$-\frac{2}{5\,a}x^{-\frac{5}{2}} - 2\,\frac{b^2}{a^3\sqrt{x}} + \frac{2\,b}{3\,a^2}x^{-\frac{3}{2}} - 2\,\frac{b^3}{a^3\sqrt{ab}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a),x)
```

```
[Out] -2/5/a/x^(5/2)-2*b^2/a^3/x^(1/2)+2/3*b/a^2/x^(3/2)-2*b^3/a^3/(a*b)^(1/2)*ar
ctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55926, size = 336, normalized size = 4.94

$$\left[\frac{15\,b^2x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(15\,b^2x^2-5\,abx+3\,a^2\right)\sqrt{x}}{15\,a^3x^3}, \frac{2\left(15\,b^2x^3\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(15\,b^2x^2-5\,abx+3\,a^2\right)\sqrt{x}\right)}{15\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^2*x^3*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x +
a)) - 2*(15*b^2*x^2 - 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3
*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 - 5*a*b*x + 3*a^2)
*sqrt(x))/(a^3*x^3)]
```

Sympy [A] time = 75.7853, size = 139, normalized size = 2.04

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for}\: a = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for}\: b = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} + \frac{2b}{3a^2x^{\frac{3}{2}}} - \frac{2b^2}{a^3\sqrt{x}} + \frac{ib^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{7}{2}}\sqrt{\frac{1}{b}}} - \frac{ib^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{7}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0)
), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + 2*b/(3*a**2*x**(3/2)
) - 2*b**2/(a**3*sqrt(x)) + I*b**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(a**
(7/2)*sqrt(1/b)) - I*b**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(7/2)*sqrt
(1/b)), True))
```

Giac [A] time = 1.23981, size = 70, normalized size = 1.03

$$-\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{2\left(15\,b^2x^2-5\,abx+3\,a^2\right)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

3.456 $\int \frac{x^{5/2}}{(a+bx)^2} dx$

Optimal. Leaf size=70

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}-\frac{5a\sqrt{x}}{b^3}-\frac{x^{5/2}}{b(a+bx)}+\frac{5x^{3/2}}{3b^2}$$

```
[Out] (-5*a*Sqrt[x])/b^3 + (5*x^(3/2))/(3*b^2) - x^(5/2)/(b*(a + b*x)) + (5*a^(3/
2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rubi [A] time = 0.0221569, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 205}

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}-\frac{5a\sqrt{x}}{b^3}-\frac{x^{5/2}}{b(a+bx)}+\frac{5x^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^2,x]
```

```
[Out] (-5*a*Sqrt[x])/b^3 + (5*x^(3/2))/(3*b^2) - x^(5/2)/(b*(a + b*x)) + (5*a^(3/
2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^2}\,dx &= -\frac{x^{5/2}}{b(a+bx)}+\frac{5\int \frac{x^{3/2}}{a+bx}\,dx}{2b}\\
&= \frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}-\frac{(5a)\int \frac{\sqrt{x}}{a+bx}\,dx}{2b^2}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{\left(5a^2\right)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2b^3}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{\left(5a^2\right)\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0043399, size = 27, normalized size = 0.39

$$
\frac{2x^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{bx}{a}\right)}{7a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^2,x]
```

```
[Out] (2*x^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*x)/a)])/(7*a^2)
```

Maple [A] time = 0.01, size = 61, normalized size = 0.9

$$
\frac{2}{3\,b^2}x^{\frac{3}{2}}-4\,\frac{a\sqrt{x}}{b^3}-\frac{a^2}{b^3\,(bx+a)}\sqrt{x}+5\,\frac{a^2}{b^3\sqrt{ab}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^2,x)
```

```
[Out] 2/3*x^(3/2)/b^2-4*a*x^(1/2)/b^3-1/b^3*a^2*x^(1/2)/(b*x+a)+5/b^3*a^2/(a*b)^(
1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63717, size = 366, normalized size = 5.23

$$\left[\frac{15\left(abx+a^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(2b^2x^2-10abx-15a^2\right)\sqrt{x}}{6\left(b^4x+ab^3\right)}, \frac{15\left(abx+a^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+\left(2b^2x^2-1\right.}{3\left(b^4x+ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b*x + a^2)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b
*x + a)) + 2*(2*b^2*x^2 - 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3), 1/3*
(15*(a*b*x + a^2)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (2*b^2*x^2 - 10
*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3)]
```

Sympy [A] time = 74.5236, size = 479, normalized size = 6.84

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} \\ \frac{2x^{\frac{3}{2}}}{3b^2} \\ \frac{2x^{\frac{7}{2}}}{7a^2} \\ -\frac{30ia^{\frac{5}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}} - \frac{20ia^{\frac{3}{2}}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}} + \frac{4i\sqrt{a}b^3x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}} + \frac{15a^3\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}} - \frac{15a^3\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}} + \frac{15a^2bx\log\left(\right.}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*b**2), Eq(a,
0)), (2*x**(7/2)/(7*a**2), Eq(b, 0)), (-30*I*a**(5/2)*b*sqrt(x)*sqrt(1/b)/(
6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)*b**5*x*sqrt(1/b)) - 20*I*a**(3/2)
*b**2*x**(3/2)*sqrt(1/b)/(6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)*b**5*x*
sqrt(1/b)) + 4*I*sqrt(a)*b**3*x**(5/2)*sqrt(1/b)/(6*I*a**(3/2)*b**4*sqrt(1/
b) + 6*I*sqrt(a)*b**5*x*sqrt(1/b)) + 15*a**3*log(-I*sqrt(a)*sqrt(1/b) + sqr
t(x))/(6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)*b**5*x*sqrt(1/b)) - 15*a**
3*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqr
t(a)*b**5*x*sqrt(1/b)) + 15*a**2*b*x*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6
*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)*b**5*x*sqrt(1/b)) - 15*a**2*b*x*lo
g(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)
*b**5*x*sqrt(1/b)), True))
```

Giac [A] time = 1.21441, size = 88, normalized size = 1.26

$$\frac{5a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{a^2\sqrt{x}}{(bx+a)b^3} + \frac{2\left(b^4x^{\frac{3}{2}} - 6ab^3\sqrt{x}\right)}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - a^2*sqrt(x)/((b*x + a)*
b^3) + 2/3*(b^4*x^(3/2) - 6*a*b^3*sqrt(x))/b^6
```

3.457 $\int \frac{x^{3/2}}{(a+bx)^2} dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{x^{3/2}}{b(a+bx)}+\frac{3\sqrt{x}}{b^2}$$

```
[Out] (3*Sqrt[x])/b^2 - x^(3/2)/(b*(a + b*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/b^(5/2)
```

Rubi [A] time = 0.0168536, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 205}

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{x^{3/2}}{b(a+bx)}+\frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 - x^(3/2)/(b*(a + b*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(a+bx)^2}\,dx &= -\frac{x^{3/2}}{b(a+bx)} + \frac{3\int \frac{\sqrt{x}}{a+bx}\,dx}{2b} \\
&= \frac{3\sqrt{x}}{b^2} - \frac{x^{3/2}}{b(a+bx)} - \frac{(3a)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2b^2} \\
&= \frac{3\sqrt{x}}{b^2} - \frac{x^{3/2}}{b(a+bx)} - \frac{(3a)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{3\sqrt{x}}{b^2} - \frac{x^{3/2}}{b(a+bx)} - \frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0039348, size = 27, normalized size = 0.47

$$\frac{2x^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{bx}{a}\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^2,x]
```

```
[Out] (2*x^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*x)/a)])/(5*a^2)
```

Maple [A] time = 0.009, size = 47, normalized size = 0.8

$$2\frac{\sqrt{x}}{b^2} + \frac{a}{b^2(bx+a)}\sqrt{x} - 3\frac{a}{b^2\sqrt{ab}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^2,x)
```

```
[Out] 2*x^(1/2)/b^2+1/b^2*a*x^(1/2)/(b*x+a)-3/b^2*a/(a*b)^(1/2)*arctan(b*x^(1/2)/
(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49069, size = 300, normalized size = 5.26

$$\left[\frac{3\,(bx+a)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,bx+3\,a)\sqrt{x}}{2\,\left(b^3x+ab^2\right)},-\frac{3\,(bx+a)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(2\,bx+3\,a)\sqrt{x}}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x + a)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x +
a)) + 2*(2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2), -(3*(b*x + a)*sqrt(a/b)*arc
tan(b*sqrt(x)*sqrt(a/b)/a) - (2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2)]
```

Sympy [A] time = 19.2066, size = 411, normalized size = 7.21

$$\begin{cases} \tilde{\infty}\sqrt{x} \\ \frac{2x^{\frac{5}{2}}}{5a^2} \\ \frac{2\sqrt{x}}{b^2} \\ \frac{6ia^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2i\sqrt{a}b^4x\sqrt{\frac{1}{b}}}+\frac{4i\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2i\sqrt{a}b^4x\sqrt{\frac{1}{b}}}-\frac{3a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2i\sqrt{a}b^4x\sqrt{\frac{1}{b}}}+\frac{3a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2i\sqrt{a}b^4x\sqrt{\frac{1}{b}}}-\frac{3abx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2i\sqrt{a}b^4x\sqrt{\frac{1}{b}}}+\frac{3abx\log\left(i\sqrt{a}\ldots\right.}{2ia^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\ldots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**2), Eq(b, 0
)), (2*sqrt(x)/b**2, Eq(a, 0)), (6*I*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(2*I*a**(
3/2)*b**3*sqrt(1/b) + 2*I*sqrt(a)*b**4*x*sqrt(1/b)) + 4*I*sqrt(a)*b**2*x**(
3/2)*sqrt(1/b)/(2*I*a**(3/2)*b**3*sqrt(1/b) + 2*I*sqrt(a)*b**4*x*sqrt(1/b))
 - 3*a**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**3*sqrt(1/b)
+ 2*I*sqrt(a)*b**4*x*sqrt(1/b)) + 3*a**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))
/(2*I*a**(3/2)*b**3*sqrt(1/b) + 2*I*sqrt(a)*b**4*x*sqrt(1/b)) - 3*a*b*x*log
(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**3*sqrt(1/b) + 2*I*sqrt(a)
*b**4*x*sqrt(1/b)) + 3*a*b*x*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/
2)*b**3*sqrt(1/b) + 2*I*sqrt(a)*b**4*x*sqrt(1/b)), True))
```

Giac [A] time = 1.19443, size = 62, normalized size = 1.09

$$-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}+\frac{a\sqrt{x}}{(bx+a)b^2}+\frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + a*sqrt(x)/((b*x + a)*b^2
) + 2*sqrt(x)/b^2
```

3.458 $\int \frac{\sqrt{x}}{(a+bx)^2} dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}$$

```
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

Rubi [A] time = 0.0133927, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = -\frac{\sqrt{x}}{b(a+bx)} + \frac{\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2b}$$
$$= -\frac{\sqrt{x}}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{\sqrt{x}}{b(a+bx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

Mathematica [A] time = 0.019689, size = 46, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}} - \frac{\sqrt{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

Maple [A] time = 0.008, size = 37, normalized size = 0.8

$$-\frac{1}{b\,(bx+a)}\sqrt{x} + \frac{1}{b}\arctan\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^2,x)
```

```
[Out] -x^(1/2)/b/(b*x+a)+1/b/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68023, size = 277, normalized size = 6.02

$$\left[-\frac{2\,ab\sqrt{x} + \sqrt{-ab}(bx+a)\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\left(ab^3x + a^2b^2\right)}, -\frac{ab\sqrt{x} + \sqrt{ab}(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab^3x + a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) + sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)))/(a*b^3*x + a^2*b^2), -(a*b*sqrt(x) + sqrt(a*b)*(b*x + a)*
arctan(sqrt(a*b)/(b*sqrt(x))))/(a*b^3*x + a^2*b^2)]
```

Sympy [A] time = 8.45385, size = 337, normalized size = 7.33

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{for } b = \\ -\frac{2}{b^2\sqrt{x}} & \text{for } a = \\ -\frac{2i\sqrt{a}b\sqrt{x}\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}b^3x\sqrt{\frac{1}{b}}} + \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}b^3x\sqrt{\frac{1}{b}}} - \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}b^3x\sqrt{\frac{1}{b}}} + \frac{bx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}b^3x\sqrt{\frac{1}{b}}} - \frac{bx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}b^3x\sqrt{\frac{1}{b}}} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0
)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (-2*I*sqrt(a)*b*sqrt(x)*sqrt(1/b)/(2*I*a
**(3/2)*b**2*sqrt(1/b) + 2*I*sqrt(a)*b**3*x*sqrt(1/b)) + a*log(-I*sqrt(a)*s
qrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**2*sqrt(1/b) + 2*I*sqrt(a)*b**3*x*sqrt(
1/b)) - a*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**2*sqrt(1/b) +
 2*I*sqrt(a)*b**3*x*sqrt(1/b)) + b*x*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2
*I*a**(3/2)*b**2*sqrt(1/b) + 2*I*sqrt(a)*b**3*x*sqrt(1/b)) - b*x*log(I*sqrt
(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**2*sqrt(1/b) + 2*I*sqrt(a)*b**3*x*
sqrt(1/b)), True))
```

Giac [A] time = 1.21819, size = 49, normalized size = 1.07

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{\sqrt{x}}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) - sqrt(x)/((b*x + a)*b)
```

3.459 $\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}$$

```
[Out] Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.0128271, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \frac{\sqrt{x}}{a(a+bx)} + \frac{\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2a}$$
$$= \frac{\sqrt{x}}{a(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{\sqrt{x}}{a(a+bx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Mathematica [A] time = 0.0181418, size = 45, normalized size = 1.

$$\frac{\sqrt{x}}{a^2+abx} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a^2 + a*b*x) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

Maple [A] time = 0.006, size = 36, normalized size = 0.8

$$\frac{1}{a\,(bx+a)}\sqrt{x} + \frac{1}{a}\arctan\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/x^(1/2),x)
```

```
[Out] x^(1/2)/a/(b*x+a)+1/a/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6122, size = 274, normalized size = 6.09

$$\left[\frac{2\,ab\sqrt{x} - \sqrt{-ab}(bx+a)\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\left(a^2b^2x+a^3b\right)}, \frac{ab\sqrt{x} - \sqrt{ab}(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{a^2b^2x+a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*sqrt(x) - sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqrt
(x))/(b*x + a)))/(a^2*b^2*x + a^3*b), (a*b*sqrt(x) - sqrt(a*b)*(b*x + a)*ar
ctan(sqrt(a*b)/(b*sqrt(x))))/(a^2*b^2*x + a^3*b)]
```

Sympy [A] time = 15.8363, size = 328, normalized size = 7.29

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = \\ -\frac{2}{3b^{2}x^{\frac{3}{2}}} & \text{for } a = 0 \\ \frac{2\sqrt{x}}{a^{2}} & \text{for } b = 0 \\ \frac{2i\sqrt{a}b\sqrt{x}\sqrt{\frac{1}{b}}}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^{2}x\sqrt{\frac{1}{b}}} + \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^{2}x\sqrt{\frac{1}{b}}} - \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^{2}x\sqrt{\frac{1}{b}}} + \frac{bx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^{2}x\sqrt{\frac{1}{b}}} - \frac{bx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^{2}x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b**2*x**(3/2)), Eq(a,
 0)), (2*sqrt(x)/a**2, Eq(b, 0)), (2*I*sqrt(a)*b*sqrt(x)*sqrt(1/b)/(2*I*a**
(5/2)*b*sqrt(1/b) + 2*I*a**(3/2)*b**2*x*sqrt(1/b)) + a*log(-I*sqrt(a)*sqrt(
1/b) + sqrt(x))/(2*I*a**(5/2)*b*sqrt(1/b) + 2*I*a**(3/2)*b**2*x*sqrt(1/b))
- a*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(5/2)*b*sqrt(1/b) + 2*I*a**(
3/2)*b**2*x*sqrt(1/b)) + b*x*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(5
/2)*b*sqrt(1/b) + 2*I*a**(3/2)*b**2*x*sqrt(1/b)) - b*x*log(I*sqrt(a)*sqrt(1
/b) + sqrt(x))/(2*I*a**(5/2)*b*sqrt(1/b) + 2*I*a**(3/2)*b**2*x*sqrt(1/b)),
True))
```

Giac [A] time = 1.2027, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{\sqrt{x}}{(bx+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) + sqrt(x)/((b*x + a)*a)
```

3.460 $\int \frac{1}{x^{3/2}(a+bx)^2} dx$

Optimal. Leaf size=56

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}(a+bx)}$$

```
[Out] -3/(a^2*Sqrt[x]) + 1/(a*Sqrt[x]*(a + b*x)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0168896, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^2),x]
```

```
[Out] -3/(a^2*Sqrt[x]) + 1/(a*Sqrt[x]*(a + b*x)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx)^2}\,dx = \frac{1}{a\sqrt{x}(a+bx)} + \frac{3\int \frac{1}{x^{3/2}(a+bx)}\,dx}{2a}$$
$$= -\frac{3}{a^2\sqrt{x}} + \frac{1}{a\sqrt{x}(a+bx)} - \frac{(3b)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2a^2}$$
$$= -\frac{3}{a^2\sqrt{x}} + \frac{1}{a\sqrt{x}(a+bx)} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a^2}$$
$$= -\frac{3}{a^2\sqrt{x}} + \frac{1}{a\sqrt{x}(a+bx)} - \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.0047055, size = 25, normalized size = 0.45

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{bx}{a}\right)}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 2, 1/2, -((b*x)/a)])/(a^2*Sqrt[x])
```

Maple [A] time = 0.01, size = 48, normalized size = 0.9

$$-2\frac{1}{a^2\sqrt{x}} - \frac{b}{a^2(bx+a)}\sqrt{x} - 3\frac{b}{a^2\sqrt{ab}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^2,x)
```

```
[Out] -2/a^2/x^(1/2)-b/a^2*x^(1/2)/(b*x+a)-3*b/a^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(
a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56498, size = 323, normalized size = 5.77

$$\left[\frac{3\left(bx^2+ax\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(3bx+2a\right)\sqrt{x}}{2\left(a^2bx^2+a^3x\right)}, \frac{3\left(bx^2+ax\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(3bx+2a\right)\sqrt{x}}{a^2bx^2+a^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x^2 + a*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*
x + a)) - 2*(3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2 + a^3*x), (3*(b*x^2 + a*x)*sq
rt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2
 + a^3*x)]
```

Sympy [A] time = 38.7742, size = 434, normalized size = 7.75

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ -\frac{2}{5b^2x^{\frac{5}{2}}} \\ -\frac{2}{a^2\sqrt{x}} \\ -\frac{4ia^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{2ia^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+2ia^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}} - \frac{6i\sqrt{a}bx\sqrt{\frac{1}{b}}}{2ia^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+2ia^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}} - \frac{3a\sqrt{x}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+2ia^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}} + \frac{3a\sqrt{x}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+2ia^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}} - \frac{3bx^{\frac{3}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+2ia^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}} + \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**2*x**(5/2)), Eq(a,
 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)), (-4*I*a**(3/2)*sqrt(1/b)/(2*I*a**(7/2)
*sqrt(x)*sqrt(1/b) + 2*I*a**(5/2)*b*x**(3/2)*sqrt(1/b)) - 6*I*sqrt(a)*b*x*s
qrt(1/b)/(2*I*a**(7/2)*sqrt(x)*sqrt(1/b) + 2*I*a**(5/2)*b*x**(3/2)*sqrt(1/b
)) - 3*a*sqrt(x)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(7/2)*sqrt(x)*
sqrt(1/b) + 2*I*a**(5/2)*b*x**(3/2)*sqrt(1/b)) + 3*a*sqrt(x)*log(I*sqrt(a)*
sqrt(1/b) + sqrt(x))/(2*I*a**(7/2)*sqrt(x)*sqrt(1/b) + 2*I*a**(5/2)*b*x**(3
/2)*sqrt(1/b)) - 3*b*x**(3/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(
7/2)*sqrt(x)*sqrt(1/b) + 2*I*a**(5/2)*b*x**(3/2)*sqrt(1/b)) + 3*b*x**(3/2)*
log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(7/2)*sqrt(x)*sqrt(1/b) + 2*I*a*
*(5/2)*b*x**(3/2)*sqrt(1/b)), True))
```

Giac [A] time = 1.24275, size = 66, normalized size = 1.18

$$-\frac{3b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} - \frac{3bx+2a}{\left(bx^{\frac{3}{2}}+a\sqrt{x}\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -3*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - (3*b*x + 2*a)/((b*x^(3/2
) + a*sqrt(x))*a^2)
```

3.461 $\int \frac{1}{x^{5/2}(a+bx)^2}\,dx$

Optimal. Leaf size=69

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a+bx)}$$

```
[Out] -5/(3*a^2*x^(3/2)) + (5*b)/(a^3*Sqrt[x]) + 1/(a*x^(3/2)*(a + b*x)) + (5*b^(
3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rubi [A] time = 0.0218529, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^2),x]
```

```
[Out] -5/(3*a^2*x^(3/2)) + (5*b)/(a^3*Sqrt[x]) + 1/(a*x^(3/2)*(a + b*x)) + (5*b^(
3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx)^2}\,dx = \frac{1}{ax^{3/2}(a+bx)} + \frac{5\int \frac{1}{x^{5/2}(a+bx)}\,dx}{2a}$$
$$= -\frac{5}{3a^2x^{3/2}} + \frac{1}{ax^{3/2}(a+bx)} - \frac{(5b)\int \frac{1}{x^{3/2}(a+bx)}\,dx}{2a^2}$$
$$= -\frac{5}{3a^2x^{3/2}} + \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a+bx)} + \frac{(5b^2)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2a^3}$$
$$= -\frac{5}{3a^2x^{3/2}} + \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a+bx)} + \frac{(5b^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a^3}$$
$$= -\frac{5}{3a^2x^{3/2}} + \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a+bx)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [C] time = 0.0049991, size = 27, normalized size = 0.39

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};-\frac{bx}{a}\right)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 2, -1/2, -((b*x)/a)])/(3*a^2*x^(3/2))
```

Maple [A] time = 0.014, size = 60, normalized size = 0.9

$$-\frac{2}{3\,a^2}x^{-\frac{3}{2}} + 4\,\frac{b}{a^3\sqrt{x}} + \frac{b^2}{a^3\,(bx+a)}\sqrt{x} + 5\,\frac{b^2}{a^3\sqrt{ab}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^2,x)
```

```
[Out] -2/3/a^2/x^(3/2)+4*b/a^3/x^(1/2)+b^2/a^3*x^(1/2)/(b*x+a)+5*b^2/a^3/(a*b)^(1
/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33587, size = 402, normalized size = 5.83

$$\left[\frac{15\left(b^2x^3+abx^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\left(15b^2x^2+10abx-2a^2\right)\sqrt{x}}{6\left(a^3bx^3+a^4x^2\right)},-\frac{15\left(b^2x^3+abx^2\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(1}{3\left(a^3bx^3+a^4x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^2*x^3 + a*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) -
 a)/(b*x + a)) + 2*(15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^
4*x^2), -1/3*(15*(b^2*x^3 + a*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x
))) - (15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^4*x^2)]
```

Sympy [A] time = 116.113, size = 507, normalized size = 7.35

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} \\ -\frac{2}{7b^2x^{\frac{7}{2}}} \\ -\frac{2}{3a^2x^{\frac{3}{2}}} \\ -\frac{4ia^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{6ia^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}+6ia^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{20ia^{\frac{3}{2}}bx\sqrt{\frac{1}{b}}}{6ia^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}+6ia^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{30i\sqrt{a}b^2x^2\sqrt{\frac{1}{b}}}{6ia^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}+6ia^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{15abx^{\frac{3}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}+6ia^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}-\frac{15abx^{\frac{3}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}+6ia^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+ \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**2*x**(7/2)), Eq(a,
 0)), (-2/(3*a**2*x**(3/2)), Eq(b, 0)), (-4*I*a**(5/2)*sqrt(1/b)/(6*I*a**(9
/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 20*I*a**(3/2)
*b*x*sqrt(1/b)/(6*I*a**(9/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/2)*s
qrt(1/b)) + 30*I*sqrt(a)*b**2*x**2*sqrt(1/b)/(6*I*a**(9/2)*x**(3/2)*sqrt(1/
b) + 6*I*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 15*a*b*x**(3/2)*log(-I*sqrt(a)*sq
rt(1/b) + sqrt(x))/(6*I*a**(9/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/
2)*sqrt(1/b)) - 15*a*b*x**(3/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6*I*a**
(9/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 15*b**2*x**
(5/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6*I*a**(9/2)*x**(3/2)*sqrt(1/b)
+ 6*I*a**(7/2)*b*x**(5/2)*sqrt(1/b)) - 15*b**2*x**(5/2)*log(I*sqrt(a)*sqrt(
1/b) + sqrt(x))/(6*I*a**(9/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/2)*
sqrt(1/b)), True))
```

Giac [A] time = 1.23766, size = 78, normalized size = 1.13

$$\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}+\frac{b^2\sqrt{x}}{(bx+a)a^3}+\frac{2\,(6\,bx-a)}{3\,a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + b^2*sqrt(x)/((b*x + a)*
a^3) + 2/3*(6*b*x - a)/(a^3*x^(3/2))
```

3.462 $\int \frac{x^{7/2}}{(a+bx)^3} \, dx$

Optimal. Leaf size=95

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}-\frac{7x^{5/2}}{4b^2(a+bx)}-\frac{35a\sqrt{x}}{4b^4}-\frac{x^{7/2}}{2b(a+bx)^2}+\frac{35x^{3/2}}{12b^3}$$

```
[Out] (-35*a*Sqrt[x])/(4*b^4) + (35*x^(3/2))/(12*b^3) - x^(7/2)/(2*b*(a + b*x)^2)
 - (7*x^(5/2))/(4*b^2*(a + b*x)) + (35*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/(4*b^(9/2))
```

Rubi [A] time = 0.0326997, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 205}

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}-\frac{7x^{5/2}}{4b^2(a+bx)}-\frac{35a\sqrt{x}}{4b^4}-\frac{x^{7/2}}{2b(a+bx)^2}+\frac{35x^{3/2}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x)^3,x]
```

```
[Out] (-35*a*Sqrt[x])/(4*b^4) + (35*x^(3/2))/(12*b^3) - x^(7/2)/(2*b*(a + b*x)^2)
 - (7*x^(5/2))/(4*b^2*(a + b*x)) + (35*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/(4*b^(9/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}}{(a+bx)^3}\,dx &= -\frac{x^{7/2}}{2b(a+bx)^2}+\frac{7\int \frac{x^{5/2}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{35\int \frac{x^{3/2}}{a+bx}\,dx}{8b^2}\\
&= \frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}-\frac{(35a)\int \frac{\sqrt{x}}{a+bx}\,dx}{8b^3}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{\left(35a^2\right)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8b^4}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{\left(35a^2\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^4}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0041032, size = 27, normalized size = 0.28

$$\frac{2x^{9/2}\,{}_2F_1\left(3,\frac{9}{2};\frac{11}{2};-\frac{bx}{a}\right)}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x)^3,x]
```

```
[Out] (2*x^(9/2)*Hypergeometric2F1[3, 9/2, 11/2, -((b*x)/a)])/(9*a^3)
```

Maple [A] time = 0.012, size = 79, normalized size = 0.8

$$\frac{2}{3\,b^3}x^{\frac{3}{2}}-6\,\frac{a\sqrt{x}}{b^4}-\frac{13\,a^2}{4\,b^3\,(bx+a)^2}x^{\frac{3}{2}}-\frac{11\,a^3}{4\,b^4\,(bx+a)^2}\sqrt{x}+\frac{35\,a^2}{4\,b^4}\arctan\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x+a)^3,x)
```

```
[Out] 2/3*x^(3/2)/b^3-6*a*x^(1/2)/b^4-13/4/b^3*a^2/(b*x+a)^2*x^(3/2)-11/4/b^4*a^3
/(b*x+a)^2*x^(1/2)+35/4/b^4*a^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32586, size = 509, normalized size = 5.36

$$\left[\frac{105\left(ab^2x^2+2a^2bx+a^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(8b^3x^3-56ab^2x^2-175a^2bx-105a^3\right)\sqrt{x}}{24\left(b^6x^2+2ab^5x+a^2b^4\right)}, \frac{105\left(ab^2x^2+2a^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*
sqrt(-a/b) - a)/(b*x + a)) + 2*(8*b^3*x^3 - 56*a*b^2*x^2 - 175*a^2*b*x - 10
5*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 + 2*a
^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (8*b^3*x^3 - 56*a*b
^2*x^2 - 175*a^2*b*x - 105*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25841, size = 104, normalized size = 1.09

$$\frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}b^4}-\frac{13\,a^2bx^{\frac{3}{2}}+11\,a^3\sqrt{x}}{4\,(bx+a)^2b^4}+\frac{2\left(b^6x^{\frac{3}{2}}-9\,ab^5\sqrt{x}\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/4*(13*a^2*b*x^(3/2
) + 11*a^3*sqrt(x))/((b*x + a)^2*b^4) + 2/3*(b^6*x^(3/2) - 9*a*b^5*sqrt(x))
/b^9
```

3.463 $\int \frac{x^{5/2}}{(a+bx)^3}\,dx$

Optimal. Leaf size=82

$$-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}-\frac{x^{5/2}}{2b(a+bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

```
[Out] (15*Sqrt[x])/(4*b^3) - x^(5/2)/(2*b*(a + b*x)^2) - (5*x^(3/2))/(4*b^2*(a +
b*x)) - (15*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rubi [A] time = 0.0234385, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 205}

$$-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}-\frac{x^{5/2}}{2b(a+bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^3,x]
```

```
[Out] (15*Sqrt[x])/(4*b^3) - x^(5/2)/(2*b*(a + b*x)^2) - (5*x^(3/2))/(4*b^2*(a +
b*x)) - (15*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^3}\,dx &= -\frac{x^{5/2}}{2b(a+bx)^2}+\frac{5\int \frac{x^{3/2}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}+\frac{15\int \frac{\sqrt{x}}{a+bx}\,dx}{8b^2}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{(15a)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{(15a)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0047717, size = 27, normalized size = 0.33

$$\frac{2x^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};-\frac{bx}{a}\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^3,x]
```

```
[Out] (2*x^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, -((b*x)/a)])/(7*a^3)
```

Maple [A] time = 0.011, size = 66, normalized size = 0.8

$$2\frac{\sqrt{x}}{b^3}+\frac{9a}{4b^2(bx+a)^2}x^{\frac{3}{2}}+\frac{7a^2}{4b^3(bx+a)^2}\sqrt{x}-\frac{15a}{4b^3}\arctan\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^3,x)
```

```
[Out] 2*x^(1/2)/b^3+9/4/b^2*a/(b*x+a)^2*x^(3/2)+7/4/b^3*a^2/(b*x+a)^2*x^(1/2)-15/
4/b^3*a/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32697, size = 443, normalized size = 5.4

$$\left[\frac{15\left(b^2x^2+2\,abx+a^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(8\,b^2x^2+25\,abx+15\,a^2\right)\sqrt{x}}{8\left(b^5x^2+2\,ab^4x+a^2b^3\right)},-\frac{15\left(b^2x^2+2\,abx+a^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\left(8\,b^2x^2+25\,abx+15\,a^2\right)\sqrt{x}}{4\left(b^5x^2+2\,ab^4x+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-
a/b) - a)/(b*x + a)) + 2*(8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a/b)*arctan
(b*sqrt(x)*sqrt(a/b)/a) - (8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25916, size = 80, normalized size = 0.98

$$-\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3}+\frac{2\,\sqrt{x}}{b^3}+\frac{9\,abx^{\frac{3}{2}}+7\,a^2\sqrt{x}}{4\,(bx+a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3 + 1/4*(
9*a*b*x^(3/2) + 7*a^2*sqrt(x))/((b*x + a)^2*b^3)
```

3.464 $\int \frac{x^{3/2}}{(a+bx)^3} \, dx$

Optimal. Leaf size=70

$$-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}-\frac{x^{3/2}}{2b(a+bx)^2}$$

```
[Out] -x^(3/2)/(2*b*(a + b*x)^2) - (3*Sqrt[x])/(4*b^2*(a + b*x)) + (3*ArcTan[(Sqr
t[b]*Sqrt[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rubi [A] time = 0.0192098, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 205}

$$-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}-\frac{x^{3/2}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^3,x]
```

```
[Out] -x^(3/2)/(2*b*(a + b*x)^2) - (3*Sqrt[x])/(4*b^2*(a + b*x)) + (3*ArcTan[(Sqr
t[b]*Sqrt[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = -\frac{x^{3/2}}{2b(a+bx)^2} + \frac{3\int \frac{\sqrt{x}}{(a+bx)^2}\,dx}{4b}$$
$$= -\frac{x^{3/2}}{2b(a+bx)^2} - \frac{3\sqrt{x}}{4b^2(a+bx)} + \frac{3\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8b^2}$$
$$= -\frac{x^{3/2}}{2b(a+bx)^2} - \frac{3\sqrt{x}}{4b^2(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{4b^2}$$
$$= -\frac{x^{3/2}}{2b(a+bx)^2} - \frac{3\sqrt{x}}{4b^2(a+bx)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}$$

Mathematica [A] time = 0.0352336, size = 59, normalized size = 0.84

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}} - \frac{\sqrt{x}(3a+5bx)}{4b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^3,x]
```

```
[Out] -(Sqrt[x]*(3*a + 5*b*x))/(4*b^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[x])/
Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Maple [A] time = 0.01, size = 50, normalized size = 0.7

$$2\frac{1}{(bx+a)^2}\left(-5/8\frac{x^{3/2}}{b} - 3/8\frac{a\sqrt{x}}{b^2}\right) + \frac{3}{4b^2}\arctan\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^3,x)
```

```
[Out] 2*(-5/8*x^(3/2)/b-3/8*a*x^(1/2)/b^2)/(b*x+a)^2+3/4/b^2/(a*b)^(1/2)*arctan(b
*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39583, size = 423, normalized size = 6.04

$$\left[-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)+2\left(5\,ab^2x+3\,a^2b\right)\sqrt{x}}{8\left(ab^5x^2+2\,a^2b^4x+a^3b^3\right)},-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+\ldots}{4\left(ab^5x^2+2\,a^2b^4x+a^3b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) + 2*(5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^
4*x + a^3*b^3), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) + (5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^4*x +
a^3*b^3)]
```

Sympy [A] time = 89.4851, size = 726, normalized size = 10.37

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} \\ -\frac{2}{b^3\sqrt{x}} \\ \frac{2x^{\frac{5}{2}}}{5a^3} \\ -\frac{6ia^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8ia^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} - \frac{10i\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8ia^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} + \frac{3a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} - \frac{3a^2\log(\ldots}{8ia^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}\ldots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b**3*sqrt(x)), Eq(a, 0))
, (2*x**(5/2)/(5*a**3), Eq(b, 0)), (-6*I*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8*I*
a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5
*x**2*sqrt(1/b)) - 10*I*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*I*a**(5/2)*b**3*
sqrt(1/b) + 16*I*a**(3/2)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b
)) + 3*a**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b
) + 16*I*a**(3/2)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) - 3*a
**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*
a**(3/2)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) + 6*a*b*x*log(
-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2
)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) - 6*a*b*x*log(I*sqrt(
a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2)*b**4*x
*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) + 3*b**2*x**2*log(-I*sqrt(a)*
sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2)*b**4*x*sq
rt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) - 3*b**2*x**2*log(I*sqrt(a)*sqrt
(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2)*b**4*x*sqrt(1
/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)), True))
```

Giac [A] time = 1.18405, size = 63, normalized size = 0.9

$$\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}b^2}-\frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/4*(5*b*x^(3/2) + 3*a*sq
rt(x))/((b*x + a)^2*b^2)
```

3.465 $\int \frac{\sqrt{x}}{(a+bx)^3} dx$

Optimal. Leaf size=73

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a+bx)}-\frac{\sqrt{x}}{2b(a+bx)^2}$$

```
[Out] -Sqrt[x]/(2*b*(a + b*x)^2) + Sqrt[x]/(4*a*b*(a + b*x)) + ArcTan[(Sqrt[b]*Sq
rt[x])/Sqrt[a]]/(4*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.0194928, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a+bx)}-\frac{\sqrt{x}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^3,x]
```

```
[Out] -Sqrt[x]/(2*b*(a + b*x)^2) + Sqrt[x]/(4*a*b*(a + b*x)) + ArcTan[(Sqrt[b]*Sq
rt[x])/Sqrt[a]]/(4*a^(3/2)*b^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{(a+bx)^3}\,dx &= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx}{4b}\\
&= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8ab}\\
&= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4ab}\\
&= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0052862, size = 27, normalized size = 0.37

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};-\frac{bx}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^3,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, -((b*x)/a)])/(3*a^3)
```

Maple [A] time = 0.009, size = 52, normalized size = 0.7

$$2\frac{1}{(bx+a)^2}\left(1/8\frac{x^{3/2}}{a}-1/8\frac{\sqrt{x}}{b}\right)+\frac{1}{4\,ab}\arctan\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^3,x)
```

```
[Out] 2*(1/8/a*x^(3/2)-1/8*x^(1/2)/b)/(b*x+a)^2+1/4/b/a/(a*b)^(1/2)*arctan(b*x^(1
/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38871, size = 412, normalized size = 5.64

$$\left[-\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)-2\left(ab^2x-a^2b\right)\sqrt{x}}{8\left(a^2b^4x^2+2\,a^3b^3x+a^4b^2\right)},-\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-\left(ab^2x-\right.}{4\left(a^2b^4x^2+2\,a^3b^3x+a^4b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)) - 2*(a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x
+ a^4*b^2), -1/4*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)/(b*s
qrt(x))) - (a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x + a^4*b^2)
]
```

Sympy [A] time = 35.0942, size = 721, normalized size = 9.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b**3*x**(3/2)), Eq(a,
 0)), (2*x**(3/2)/(3*a**3), Eq(b, 0)), (-2*I*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(
8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)
*b**4*x**2*sqrt(1/b)) + 2*I*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*I*a**(7/2)*b
**2*sqrt(1/b) + 16*I*a**(5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqr
t(1/b)) + a**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(
1/b) + 16*I*a**(5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) -
 a**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*
I*a**(5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) + 2*a*b*x*l
og(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(
5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) - 2*a*b*x*log(I*s
qrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(5/2)*b*
*3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) + b**2*x**2*log(-I*sqrt(
a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(5/2)*b**3*x
*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) - b**2*x**2*log(I*sqrt(a)*sq
rt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(5/2)*b**3*x*sqrt
(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)), True))
```

Giac [A] time = 1.21856, size = 70, normalized size = 0.96

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}ab}+\frac{bx^{\frac{3}{2}}-a\sqrt{x}}{4\,(bx+a)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/4*(b*x^(3/2) - a*sqrt(x
))/((b*x + a)^2*a*b)
```

3.466 $\int \frac{1}{\sqrt{x}(a+bx)^3} dx$

Optimal. Leaf size=70

$$\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}+\frac{\sqrt{x}}{2a(a+bx)^2}$$

```
[Out] Sqrt[x]/(2*a*(a + b*x)^2) + (3*Sqrt[x])/(4*a^2*(a + b*x)) + (3*ArcTan[(Sqrt
[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.0191969, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}+\frac{\sqrt{x}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^3),x]
```

```
[Out] Sqrt[x]/(2*a*(a + b*x)^2) + (3*Sqrt[x])/(4*a^2*(a + b*x)) + (3*ArcTan[(Sqrt
[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \frac{\sqrt{x}}{2a(a+bx)^2} + \frac{3\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx}{4a}$$
$$= \frac{\sqrt{x}}{2a(a+bx)^2} + \frac{3\sqrt{x}}{4a^2(a+bx)} + \frac{3\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8a^2}$$
$$= \frac{\sqrt{x}}{2a(a+bx)^2} + \frac{3\sqrt{x}}{4a^2(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{4a^2}$$
$$= \frac{\sqrt{x}}{2a(a+bx)^2} + \frac{3\sqrt{x}}{4a^2(a+bx)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

Mathematica [C] time = 0.0047083, size = 25, normalized size = 0.36

$$\frac{2\sqrt{x}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};-\frac{bx}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^3),x]
```

```
[Out] (2*Sqrt[x]*Hypergeometric2F1[1/2, 3, 3/2, -((b*x)/a)])/a^3
```

Maple [A] time = 0.006, size = 53, normalized size = 0.8

$$\frac{1}{2\,a\,(bx+a)^2}\sqrt{x} + \frac{3}{4\,a^2\,(bx+a)}\sqrt{x} + \frac{3}{4\,a^2}\arctan\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/x^(1/2),x)
```

```
[Out] 1/2*x^(1/2)/a/(b*x+a)^2+3/4*x^(1/2)/a^2/(b*x+a)+3/4/a^2/(a*b)^(1/2)*arctan(
b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38426, size = 423, normalized size = 6.04

$$\left[-\frac{3\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)-2\left(3\,ab^{2}x+5\,a^{2}b\right)\sqrt{x}}{8\left(a^{3}b^{3}x^{2}+2\,a^{4}b^{2}x+a^{5}b\right)},-\frac{3\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-(3\,a\ldots}{4\left(a^{3}b^{3}x^{2}+2\,a^{4}b^{2}x+a^{5}b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) - 2*(3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*
b^2*x + a^5*b), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) - (3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*b^2*x
+ a^5*b)]
```

Sympy [A] time = 61.4325, size = 712, normalized size = 10.17

$$\begin{cases}\frac{\tilde{\infty}}{x^{\frac{5}{2}}}\\ -\frac{2}{5b^{3}x^{\frac{5}{2}}}\\ \frac{2\sqrt{x}}{a^{3}}\\ \frac{10ia^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8ia^{\frac{9}{2}}b\sqrt{\frac{1}{b}}+16ia^{\frac{7}{2}}b^{2}x\sqrt{\frac{1}{b}}+8ia^{\frac{5}{2}}b^{3}x^{2}\sqrt{\frac{1}{b}}}+\frac{6i\sqrt{a}b^{2}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8ia^{\frac{9}{2}}b\sqrt{\frac{1}{b}}+16ia^{\frac{7}{2}}b^{2}x\sqrt{\frac{1}{b}}+8ia^{\frac{5}{2}}b^{3}x^{2}\sqrt{\frac{1}{b}}}+\frac{3a^{2}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{9}{2}}b\sqrt{\frac{1}{b}}+16ia^{\frac{7}{2}}b^{2}x\sqrt{\frac{1}{b}}+8ia^{\frac{5}{2}}b^{3}x^{2}\sqrt{\frac{1}{b}}}-\frac{3a^{2}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\ldots\right.}{8ia^{\frac{9}{2}}b\sqrt{\frac{1}{b}}+16ia^{\frac{7}{2}}b^{2}x\sqrt{\frac{1}{b}}+\ldots}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**3*x**(5/2)), Eq(a,
 0)), (2*sqrt(x)/a**3, Eq(b, 0)), (10*I*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8*I*a
**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**(5/2)*b**3*x*
*2*sqrt(1/b)) + 6*I*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*I*a**(9/2)*b*sqrt(1/
b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**(5/2)*b**3*x**2*sqrt(1/b)) + 3
*a**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*
a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**(5/2)*b**3*x**2*sqrt(1/b)) - 3*a**2*log(
I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b*
*2*x*sqrt(1/b) + 8*I*a**(5/2)*b**3*x**2*sqrt(1/b)) + 6*a*b*x*log(-I*sqrt(a)
*sqrt(1/b) + sqrt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt
(1/b) + 8*I*a**(5/2)*b**3*x**2*sqrt(1/b)) - 6*a*b*x*log(I*sqrt(a)*sqrt(1/b)
 + sqrt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*
I*a**(5/2)*b**3*x**2*sqrt(1/b)) + 3*b**2*x**2*log(-I*sqrt(a)*sqrt(1/b) + sq
rt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**
(5/2)*b**3*x**2*sqrt(1/b)) - 3*b**2*x**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))
/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**(5/2)*
b**3*x**2*sqrt(1/b)), True))
```

Giac [A] time = 1.22448, size = 63, normalized size = 0.9

$$\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}a^{2}}+\frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\,(bx+a)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/4*(3*b*x^(3/2) + 5*a*sq
rt(x))/((b*x + a)^2*a^2)
```

3.467 $\int \frac{1}{x^{3/2}(a+bx)^3} dx$

Optimal. Leaf size=82

$$\frac{5}{4a^2\sqrt{x}(a+bx)} - \frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}} - \frac{15}{4a^3\sqrt{x}} + \frac{1}{2a\sqrt{x}(a+bx)^2}$$

```
[Out] -15/(4*a^3*Sqrt[x]) + 1/(2*a*Sqrt[x]*(a + b*x)^2) + 5/(4*a^2*Sqrt[x]*(a + b
*x)) - (15*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rubi [A] time = 0.0251132, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{5}{4a^2\sqrt{x}(a+bx)} - \frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}} - \frac{15}{4a^3\sqrt{x}} + \frac{1}{2a\sqrt{x}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^3),x]
```

```
[Out] -15/(4*a^3*Sqrt[x]) + 1/(2*a*Sqrt[x]*(a + b*x)^2) + 5/(4*a^2*Sqrt[x]*(a + b
*x)) - (15*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx)^3}\,dx = \frac{1}{2a\sqrt{x}(a+bx)^2} + \frac{5\int \frac{1}{x^{3/2}(a+bx)^2}\,dx}{4a}$$
$$= \frac{1}{2a\sqrt{x}(a+bx)^2} + \frac{5}{4a^2\sqrt{x}(a+bx)} + \frac{15\int \frac{1}{x^{3/2}(a+bx)}\,dx}{8a^2}$$
$$= -\frac{15}{4a^3\sqrt{x}} + \frac{1}{2a\sqrt{x}(a+bx)^2} + \frac{5}{4a^2\sqrt{x}(a+bx)} - \frac{(15b)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8a^3}$$
$$= -\frac{15}{4a^3\sqrt{x}} + \frac{1}{2a\sqrt{x}(a+bx)^2} + \frac{5}{4a^2\sqrt{x}(a+bx)} - \frac{(15b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{4a^3}$$
$$= -\frac{15}{4a^3\sqrt{x}} + \frac{1}{2a\sqrt{x}(a+bx)^2} + \frac{5}{4a^2\sqrt{x}(a+bx)} - \frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Mathematica [C] time = 0.0049071, size = 25, normalized size = 0.3

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};-\frac{bx}{a}\right)}{a^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 3, 1/2, -((b*x)/a)])/(a^3*Sqrt[x])
```

Maple [A] time = 0.011, size = 66, normalized size = 0.8

$$-2\frac{1}{a^3\sqrt{x}} - \frac{7b^2}{4a^3(bx+a)^2}x^{\frac{3}{2}} - \frac{9b}{4a^2(bx+a)^2}\sqrt{x} - \frac{15b}{4a^3}\arctan\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^3,x)
```

```
[Out] -2/a^3/x^(1/2)-7/4/a^3*b^2/(b*x+a)^2*x^(3/2)-9/4/a^2*b/(b*x+a)^2*x^(1/2)-15
/4/a^3*b/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44803, size = 466, normalized size = 5.68

$$\left[\frac{15\left(b^{2}x^{3}+2\,abx^{2}+a^{2}x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(15\,b^{2}x^{2}+25\,abx+8\,a^{2}\right)\sqrt{x}}{8\left(a^{3}b^{2}x^{3}+2\,a^{4}bx^{2}+a^{5}x\right)},\frac{15\left(b^{2}x^{3}+2\,abx^{2}+a^{2}x\right)\sqrt{\frac{b}{a}}\arctan\right.}{4\left(a^{3}b^{2}x^{3}+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sq
rt(-b/a) - a)/(b*x + a)) - 2*(15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*
b^2*x^3 + 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(
b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt
(x))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)]
```

Sympy [A] time = 155.787, size = 865, normalized size = 10.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**3*sqrt(x)), Eq(b, 0)
), (-2/(7*b**3*x**(7/2)), Eq(a, 0)), (-16*I*a**(5/2)*sqrt(1/b)/(8*I*a**(11/
2)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b*
*2*x**(5/2)*sqrt(1/b)) - 50*I*a**(3/2)*b*x*sqrt(1/b)/(8*I*a**(11/2)*sqrt(x)
*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2
)*sqrt(1/b)) - 30*I*sqrt(a)*b**2*x**2*sqrt(1/b)/(8*I*a**(11/2)*sqrt(x)*sqrt
(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2)*sqr
t(1/b)) - 15*a**2*sqrt(x)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(11/2
)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**
2*x**(5/2)*sqrt(1/b)) + 15*a**2*sqrt(x)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/
(8*I*a**(11/2)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I
*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) - 30*a*b*x**(3/2)*log(-I*sqrt(a)*sqrt(1/
b) + sqrt(x))/(8*I*a**(11/2)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*s
qrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) + 30*a*b*x**(3/2)*log(I*sq
rt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(11/2)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)
*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) - 15*b**2*x**
(5/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(11/2)*sqrt(x)*sqrt(1/b)
+ 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)
) + 15*b**2*x**(5/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(11/2)*sqrt
(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(
5/2)*sqrt(1/b)), True))
```

Giac [A] time = 1.22685, size = 80, normalized size = 0.98

$$-\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^{3}}-\frac{2}{a^{3}\sqrt{x}}-\frac{7\,b^{2}x^{\frac{3}{2}}+9\,ab\sqrt{x}}{4\,(bx+a)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/(a^3*sqrt(x)) - 1/4
*(7*b^2*x^(3/2) + 9*a*b*sqrt(x))/((b*x + a)^2*a^3)
```

3.468 $\int \frac{1}{x^{5/2}(a+bx)^3}\,dx$

Optimal. Leaf size=95

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35b}{4a^4\sqrt{x}}-\frac{35}{12a^3x^{3/2}}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

```
[Out] -35/(12*a^3*x^(3/2)) + (35*b)/(4*a^4*Sqrt[x]) + 1/(2*a*x^(3/2)*(a + b*x)^2)
 + 7/(4*a^2*x^(3/2)*(a + b*x)) + (35*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
a]])/(4*a^(9/2))
```

Rubi [A] time = 0.0289663, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35b}{4a^4\sqrt{x}}-\frac{35}{12a^3x^{3/2}}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^3),x]
```

```
[Out] -35/(12*a^3*x^(3/2)) + (35*b)/(4*a^4*Sqrt[x]) + 1/(2*a*x^(3/2)*(a + b*x)^2)
 + 7/(4*a^2*x^(3/2)*(a + b*x)) + (35*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
a]])/(4*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(a+bx)^3}\,dx &= \frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7\int \frac{1}{x^{5/2}(a+bx)^2}\,dx}{4a}\\ &= \frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35\int \frac{1}{x^{5/2}(a+bx)}\,dx}{8a^2}\\ &= -\frac{35}{12a^3x^{3/2}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}-\frac{(35b)\int \frac{1}{x^{3/2}(a+bx)}\,dx}{8a^3}\\ &= -\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{(35b^2)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8a^4}\\ &= -\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{(35b^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4a^4}\\ &= -\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}\end{aligned}$$

Mathematica [C] time = 0.0051931, size = 27, normalized size = 0.28

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};-\frac{bx}{a}\right)}{3a^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 3, -1/2, -((b*x)/a)])/(3*a^3*x^(3/2))
```

Maple [A] time = 0.015, size = 79, normalized size = 0.8

$$-\frac{2}{3\,a^3}x^{-\frac{3}{2}}+6\,\frac{b}{a^4\sqrt{x}}+\frac{11\,b^3}{4\,a^4\,(bx+a)^2}x^{\frac{3}{2}}+\frac{13\,b^2}{4\,a^3\,(bx+a)^2}\sqrt{x}+\frac{35\,b^2}{4\,a^4}\arctan\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^3,x)
```

```
[Out] -2/3/a^3/x^(3/2)+6*b/a^4/x^(1/2)+11/4/a^4*b^3/(b*x+a)^2*x^(3/2)+13/4/a^3*b^
2/(b*x+a)^2*x^(1/2)+35/4/a^4*b^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40856, size = 545, normalized size = 5.74

$$\left[\frac{105\left(b^3x^4+2\,ab^2x^3+a^2bx^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\left(105\,b^3x^3+175\,ab^2x^2+56\,a^2bx-8\,a^3\right)\sqrt{x}}{24\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)},-\frac{105\left(b^3x^4+2\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sq
rt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*
b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2), -1/12*(105*(b^
3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x)))
- (105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4
+ 2*a^5*b*x^3 + a^6*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17401, size = 96, normalized size = 1.01

$$\frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^4}+\frac{2\,(9\,bx-a)}{3\,a^4x^{\frac{3}{2}}}+\frac{11\,b^3x^{\frac{3}{2}}+13\,ab^2\sqrt{x}}{4\,(bx+a)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 2/3*(9*b*x - a)/(a^4
*x^(3/2)) + 1/4*(11*b^3*x^(3/2) + 13*a*b^2*sqrt(x))/((b*x + a)^2*a^4)
```

3.469 $\int \frac{x^{5/2}}{-a+bx}\,dx$

Optimal. Leaf size=68

$$\frac{2a^2\sqrt{x}}{b^3}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

```
[Out] (2*a^2*Sqrt[x])/b^3 + (2*a*x^(3/2))/(3*b^2) + (2*x^(5/2))/(5*b) - (2*a^(5/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rubi [A] time = 0.02392, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 63, 208}

$$\frac{2a^2\sqrt{x}}{b^3}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x),x]
```

```
[Out] (2*a^2*Sqrt[x])/b^3 + (2*a*x^(3/2))/(3*b^2) + (2*x^(5/2))/(5*b) - (2*a^(5/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{5/2}}{-a+bx}\,dx = \frac{2x^{5/2}}{5b} + \frac{a\int \frac{x^{3/2}}{-a+bx}\,dx}{b}$$
$$= \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} + \frac{a^2\int \frac{\sqrt{x}}{-a+bx}\,dx}{b^2}$$
$$= \frac{2a^2\sqrt{x}}{b^3} + \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} + \frac{a^3\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{b^3}$$
$$= \frac{2a^2\sqrt{x}}{b^3} + \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b^3}$$
$$= \frac{2a^2\sqrt{x}}{b^3} + \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Mathematica [A] time = 0.0250639, size = 61, normalized size = 0.9

$$\frac{2\sqrt{x}\left(15a^2+5abx+3b^2x^2\right)}{15b^3} - \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(15*a^2 + 5*a*b*x + 3*b^2*x^2))/(15*b^3) - (2*a^(5/2)*ArcTanh[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Maple [A] time = 0.005, size = 54, normalized size = 0.8

$$2\,\frac{1/5\,b^2x^{5/2}+1/3\,abx^{3/2}+a^2\sqrt{x}}{b^3} - 2\,\frac{a^3}{b^3\sqrt{ab}}\text{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a),x)
```

```
[Out] 2/b^3*(1/5*b^2*x^(5/2)+1/3*a*b*x^(3/2)+a^2*x^(1/2))-2*a^3/b^3/(a*b)^(1/2)*a
rctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30866, size = 306, normalized size = 4.5

$$\left[\frac{15\,a^2\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\left(3\,b^2x^2+5\,abx+15\,a^2\right)\sqrt{x}}{15\,b^3},\frac{2\left(15\,a^2\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\left(3\,b^2x^2+5\,abx+15\,a^2\right.\right.}{15\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) +
2*(3*b^2*x^2 + 5*a*b*x + 15*a^2)*sqrt(x))/b^3, 2/15*(15*a^2*sqrt(-a/b)*arct
an(b*sqrt(x)*sqrt(-a/b)/a) + (3*b^2*x^2 + 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

Sympy [A] time = 16.8025, size = 116, normalized size = 1.71

$$\begin{cases}\frac{a^{\frac{5}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^4\sqrt{\frac{1}{b}}}-\frac{a^{\frac{5}{2}}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^4\sqrt{\frac{1}{b}}}+\frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{\frac{3}{2}}}{3b^2}+\frac{2x^{\frac{5}{2}}}{5b} & \text{for } b\neq 0\\ -\frac{2x^{\frac{7}{2}}}{7a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a),x)
```

```
[Out] Piecewise((a**(5/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(b**4*sqrt(1/b)) - a*
*(5/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(b**4*sqrt(1/b)) + 2*a**2*sqrt(x)/b
**3 + 2*a*x**(3/2)/(3*b**2) + 2*x**(5/2)/(5*b), Ne(b, 0)), (-2*x**(7/2)/(7*
a), True))
```

Giac [A] time = 1.17681, size = 82, normalized size = 1.21

$$\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}b^3}+\frac{2\left(3\,b^4x^{\frac{5}{2}}+5\,ab^3x^{\frac{3}{2}}+15\,a^2b^2\sqrt{x}\right)}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*a^3*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) + 2/15*(3*b^4*x^(5/2) +
 5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

3.470 $\int \frac{x^{3/2}}{-a+bx} dx$

Optimal. Leaf size=53

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

```
[Out] (2*a*Sqrt[x])/b^2 + (2*x^(3/2))/(3*b) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

Rubi [A] time = 0.017739, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 63, 208}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x),x]
```

```
[Out] (2*a*Sqrt[x])/b^2 + (2*x^(3/2))/(3*b) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{3/2}}{-a+bx}\,dx = \frac{2x^{3/2}}{3b} + \frac{a\int \frac{\sqrt{x}}{-a+bx}\,dx}{b}$$
$$= \frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{a^2\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{b^2}$$
$$= \frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= \frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.0184775, size = 49, normalized size = 0.92

$$\frac{2\sqrt{x}(3a+bx)}{3b^2} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(3*a + b*x))/(3*b^2) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/b^(5/2)
```

Maple [A] time = 0.006, size = 43, normalized size = 0.8

$$2\,\frac{1/3\,bx^{3/2}+a\sqrt{x}}{b^2} - 2\,\frac{a^2}{b^2\sqrt{ab}}\mathrm{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a),x)
```

```
[Out] 2/b^2*(1/3*b*x^(3/2)+a*x^(1/2))-2*a^2/b^2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*
b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31468, size = 244, normalized size = 4.6

$$\left[\frac{3\,a\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(bx+3\,a)\sqrt{x}}{3\,b^2},\ \frac{2\left(3\,a\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+(bx+3\,a)\sqrt{x}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*a*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*(b
*x + 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a)
+ (b*x + 3*a)*sqrt(x))/b^2]
```

Sympy [A] time = 3.95647, size = 100, normalized size = 1.89

$$\begin{cases} \frac{a^{\frac{3}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^3\sqrt{\frac{1}{b}}}-\frac{a^{\frac{3}{2}}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^3\sqrt{\frac{1}{b}}}+\frac{2a\sqrt{x}}{b^2}+\frac{2x^{\frac{3}{2}}}{3b} & \text{for } b\neq 0 \\ -\frac{2x^{\frac{5}{2}}}{5a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a),x)
```

```
[Out] Piecewise((a**(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(b**3*sqrt(1/b)) - a*
*(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(b**3*sqrt(1/b)) + 2*a*sqrt(x)/b**2
 + 2*x**(3/2)/(3*b), Ne(b, 0)), (-2*x**(5/2)/(5*a), True))
```

Giac [A] time = 1.20798, size = 63, normalized size = 1.19

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}b^2}+\frac{2\left(b^2x^{\frac{3}{2}}+3\,ab\sqrt{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) + 2/3*(b^2*x^(3/2) + 3*
a*b*sqrt(x))/b^3
```

3.471 $\int \frac{\sqrt{x}}{-a+bx} dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rubi [A] time = 0.0138221, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 63, 208}

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{-a+bx} dx &= \frac{2\sqrt{x}}{b} + \frac{a\int \frac{1}{\sqrt{x}(-a+bx)} dx}{b} \\
&= \frac{2\sqrt{x}}{b} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-a+bx^2} dx, x, \sqrt{x}\right)}{b} \\
&= \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0099759, size = 40, normalized size = 1.

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Maple [A] time = 0.006, size = 32, normalized size = 0.8

$$2\frac{\sqrt{x}}{b}-2\frac{a}{b\sqrt{ab}}\mathrm{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a),x)
```

```
[Out] 2*x^(1/2)/b-2/b*a/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36639, size = 188, normalized size = 4.7

$$\left[\frac{\sqrt{\frac{a}{b}}\log\left(\frac{bx-2b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\sqrt{x}}{b}, \frac{2\left(\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\sqrt{x}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [(sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*sqrt(x))/b
, 2*(sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + sqrt(x))/b]
```

Sympy [A] time = 1.16132, size = 87, normalized size = 2.17

$$\begin{cases} \frac{\sqrt{a}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^2\sqrt{\frac{1}{b}}} - \frac{\sqrt{a}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^2\sqrt{\frac{1}{b}}} + \frac{2\sqrt{x}}{b} & \text{for } b \neq 0 \\ -\frac{2x^{\frac{3}{2}}}{3a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a),x)
```

```
[Out] Piecewise((sqrt(a)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(b**2*sqrt(1/b)) - sqr
t(a)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(b**2*sqrt(1/b)) + 2*sqrt(x)/b, Ne(b,
 0)), (-2*x**(3/2)/(3*a), True))
```

Giac [A] time = 1.15392, size = 45, normalized size = 1.12

$$\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}b} + \frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b) + 2*sqrt(x)/b
```

3.472 $\int \frac{1}{\sqrt{x}(-a+bx)}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0106771, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(-a+bx)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0050798, size = 29, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)),x]

[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.004, size = 19, normalized size = 0.7

$$-2\frac{1}{\sqrt{ab}}\operatorname{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)/x^(1/2),x)

[Out] -2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58672, size = 161, normalized size = 5.55

$$\left[\frac{\sqrt{ab}\log\left(\frac{bx+a-2\sqrt{ab}\sqrt{x}}{bx-a}\right)}{ab}, \frac{2\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="fricas")

[Out] [sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt(x))/(b*x - a))/(a*b), 2*sqrt(-a*
b)*arctan(sqrt(-a*b)/(b*sqrt(x)))/(a*b)]
```

Sympy [A] time = 2.10954, size = 88, normalized size = 3.03

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a = 0 \\ -\frac{2\sqrt{x}}{a} & \text{for } b = 0 \\ \frac{\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{\sqrt{ab}\sqrt{\frac{1}{b}}} - \frac{\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{\sqrt{ab}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)), (
-2*sqrt(x)/a, Eq(b, 0)), (log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(sqrt(a)*b*sqrt
(1/b)) - log(sqrt(a)*sqrt(1/b) + sqrt(x))/(sqrt(a)*b*sqrt(1/b)), True))
```

Giac [A] time = 1.22255, size = 27, normalized size = 0.93

$$\frac{2 \arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(-a*b))/sqrt(-a*b)
```

3.473 $\int \frac{1}{x^{3/2}(-a+bx)} dx$

Optimal. Leaf size=40

$$\frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] 2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0141089, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 208}

$$\frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)),x]
```

```
[Out] 2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(-a+bx)}\,dx = \frac{2}{a\sqrt{x}} + \frac{b\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{a}$$
$$= \frac{2}{a\sqrt{x}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [C] time = 0.0042176, size = 24, normalized size = 0.6

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx}{a}\right)}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, (b*x)/a])/(a*Sqrt[x])
```

Maple [A] time = 0.006, size = 32, normalized size = 0.8

$$-2\,\frac{b}{a\sqrt{ab}}\text{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)+2\,\frac{1}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a),x)
```

```
[Out] -2*b/a/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))+2/a/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64952, size = 207, normalized size = 5.18

$$\left[\frac{x\sqrt{\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,\sqrt{x}}{ax}, \frac{2\left(x\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\sqrt{x}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [(x*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)) + 2*sqrt(x))
/(a*x), 2*(x*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + sqrt(x))/(a*x)]
```

Sympy [A] time = 6.07702, size = 94, normalized size = 2.35

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ \frac{2}{a\sqrt{x}} & \text{for}\ b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for}\ a = 0 \\ \frac{2}{a\sqrt{x}} + \frac{\log\left(-\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{a^{\frac{3}{2}}\sqrt{\frac{1}{b}}} - \frac{\log\left(\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{a^{\frac{3}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2/(a*sqrt(x)), Eq(b, 0)), (
-2/(3*b*x**(3/2)), Eq(a, 0)), (2/(a*sqrt(x)) + log(-sqrt(a)*sqrt(1/b) + sqr
t(x))/(a**(3/2)*sqrt(1/b)) - log(sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(3/2)*sqr
t(1/b)), True))
```

Giac [A] time = 1.18499, size = 45, normalized size = 1.12

$$\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}a} + \frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a) + 2/(a*sqrt(x))
```

3.474 $\int \frac{1}{x^{5/2}(-a+bx)} dx$

Optimal. Leaf size=53

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2}{3ax^{3/2}}$$

```
[Out] 2/(3*a*x^(3/2)) + (2*b)/(a^2*Sqrt[x]) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0183376, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 208}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)),x]
```

```
[Out] 2/(3*a*x^(3/2)) + (2*b)/(a^2*Sqrt[x]) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/2}(-a+bx)}\,dx &= \frac{2}{3ax^{3/2}}+\frac{b\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{a}\\
&= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{b^2\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{a^2}\\
&= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\
&= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0045749, size = 26, normalized size = 0.49

$$
\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx}{a}\right)}{3ax^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, (b*x)/a])/(3*a*x^(3/2))
```

Maple [A] time = 0.008, size = 43, normalized size = 0.8

$$
\frac{2}{3\,a}x^{-\frac{3}{2}}+2\,\frac{b}{a^2\sqrt{x}}-2\,\frac{b^2}{a^2\sqrt{ab}}\mathrm{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a),x)
```

```
[Out] 2/3/a/x^(3/2)+2*b/a^2/x^(1/2)-2*b^2/a^2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60511, size = 274, normalized size = 5.17

$$\left[\frac{3\,bx^2\sqrt{\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,(3\,bx+a)\sqrt{x}}{3\,a^2x^2},\ \frac{2\left(3\,bx^2\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(3\,bx+a)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*x^2*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)) +
2*(3*b*x + a)*sqrt(x))/(a^2*x^2), 2/3*(3*b*x^2*sqrt(-b/a)*arctan(a*sqrt(-b/
a)/(b*sqrt(x))) + (3*b*x + a)*sqrt(x))/(a^2*x^2)]
```

Sympy [A] time = 17.8283, size = 112, normalized size = 2.11

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a = 0 \\ \frac{2}{3ax^{\frac{3}{2}}} & \text{for } b = 0 \\ \frac{2}{3ax^{\frac{3}{2}}} + \frac{2b}{a^2\sqrt{x}} + \frac{b\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{5}{2}}\sqrt{\frac{1}{b}}} - \frac{b\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{5}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)
), (2/(3*a*x**(3/2)), Eq(b, 0)), (2/(3*a*x**(3/2)) + 2*b/(a**2*sqrt(x)) + b
*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(5/2)*sqrt(1/b)) - b*log(sqrt(a)*sqr
t(1/b) + sqrt(x))/(a**(5/2)*sqrt(1/b)), True))
```

Giac [A] time = 1.1986, size = 55, normalized size = 1.04

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}a^2}+\frac{2\,(3\,bx+a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) + 2/3*(3*b*x + a)/(a^2*
x^(3/2))
```

3.475 $\int \frac{1}{x^{7/2}(-a+bx)}\,dx$

Optimal. Leaf size=68

$$\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2}{5ax^{5/2}}$$

```
[Out] 2/(5*a*x^(5/2)) + (2*b)/(3*a^2*x^(3/2)) + (2*b^2)/(a^3*Sqrt[x]) - (2*b^(5/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rubi [A] time = 0.0224059, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 208}

$$\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(-a + b*x)),x]
```

```
[Out] 2/(5*a*x^(5/2)) + (2*b)/(3*a^2*x^(3/2)) + (2*b^2)/(a^3*Sqrt[x]) - (2*b^(5/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{7/2}(-a+bx)}\,dx = \frac{2}{5ax^{5/2}}+\frac{b\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{a}$$
$$= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{b^2\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{a^2}$$
$$= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{b^3\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{a^3}$$
$$= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a^3}$$
$$= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [C] time = 0.005193, size = 26, normalized size = 0.38

$$\frac{2\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{bx}{a}\right)}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(-a + b*x)),x]
```

```
[Out] (2*Hypergeometric2F1[-5/2, 1, -3/2, (b*x)/a])/(5*a*x^(5/2))
```

Maple [A] time = 0.007, size = 54, normalized size = 0.8

$$\frac{2}{5\,a}x^{-\frac{5}{2}}+\frac{2\,b}{3\,a^2}x^{-\frac{3}{2}}+2\,\frac{b^2}{a^3\sqrt{x}}-2\,\frac{b^3}{a^3\sqrt{ab}}\text{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x-a),x)
```

```
[Out] 2/5/a/x^(5/2)+2/3*b/a^2/x^(3/2)+2*b^2/a^3/x^(1/2)-2*b^3/a^3/(a*b)^(1/2)*arc
tanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54962, size = 336, normalized size = 4.94

$$\left[\frac{15\,b^2x^3\sqrt{\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\left(15\,b^2x^2+5\,abx+3\,a^2\right)\sqrt{x}}{15\,a^3x^3},\ \frac{2\left(15\,b^2x^3\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\left(15\,b^2x^2+5\,abx+3\right.\right.}{15\,a^3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^2*x^3*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)
) + 2*(15*b^2*x^2 + 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3*s
qrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (15*b^2*x^2 + 5*a*b*x + 3*a^2)
*sqrt(x))/(a^3*x^3)]
```

Sympy [A] time = 76.1262, size = 131, normalized size = 1.93

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for}\ a = 0 \\ \frac{2}{5ax^{\frac{5}{2}}} & \text{for}\ b = 0 \\ \frac{2}{5ax^{\frac{5}{2}}} + \frac{2b}{3a^2x^{\frac{3}{2}}} + \frac{2b^2}{a^3\sqrt{x}} + \frac{b^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{7}{2}}\sqrt{\frac{1}{b}}} - \frac{b^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{7}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0)
), (2/(5*a*x**(5/2)), Eq(b, 0)), (2/(5*a*x**(5/2)) + 2*b/(3*a**2*x**(3/2))
+ 2*b**2/(a**3*sqrt(x)) + b**2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(7/2)*
sqrt(1/b)) - b**2*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(7/2)*sqrt(1/b)), Tr
ue))
```

Giac [A] time = 1.25937, size = 73, normalized size = 1.07

$$\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}a^3}+\frac{2\left(15\,b^2x^2+5\,abx+3\,a^2\right)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*b^3*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) + 2/15*(15*b^2*x^2 + 5*
a*b*x + 3*a^2)/(a^3*x^(5/2))
```

3.476 $\int \frac{x^{5/2}}{(-a+bx)^2} dx$

Optimal. Leaf size=70

$$-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{5a\sqrt{x}}{b^3}+\frac{x^{5/2}}{b(a-bx)}+\frac{5x^{3/2}}{3b^2}$$

```
[Out] (5*a*Sqrt[x])/b^3 + (5*x^(3/2))/(3*b^2) + x^(5/2)/(b*(a - b*x)) - (5*a^(3/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rubi [A] time = 0.0246949, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 208}

$$-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{5a\sqrt{x}}{b^3}+\frac{x^{5/2}}{b(a-bx)}+\frac{5x^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x)^2,x]
```

```
[Out] (5*a*Sqrt[x])/b^3 + (5*x^(3/2))/(3*b^2) + x^(5/2)/(b*(a - b*x)) - (5*a^(3/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(-a+bx)^2}\,dx &= \frac{x^{5/2}}{b(a-bx)} + \frac{5\int \frac{x^{3/2}}{-a+bx}\,dx}{2b}\\
&= \frac{5x^{3/2}}{3b^2} + \frac{x^{5/2}}{b(a-bx)} + \frac{(5a)\int \frac{\sqrt{x}}{-a+bx}\,dx}{2b^2}\\
&= \frac{5a\sqrt{x}}{b^3} + \frac{5x^{3/2}}{3b^2} + \frac{x^{5/2}}{b(a-bx)} + \frac{\left(5a^2\right)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{2b^3}\\
&= \frac{5a\sqrt{x}}{b^3} + \frac{5x^{3/2}}{3b^2} + \frac{x^{5/2}}{b(a-bx)} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b^3}\\
&= \frac{5a\sqrt{x}}{b^3} + \frac{5x^{3/2}}{3b^2} + \frac{x^{5/2}}{b(a-bx)} - \frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0050997, size = 26, normalized size = 0.37

$$\frac{2x^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx}{a}\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x)^2,x]
```

```
[Out] (2*x^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, (b*x)/a])/(7*a^2)
```

Maple [A] time = 0.01, size = 61, normalized size = 0.9

$$2\,\frac{1/3\,bx^{3/2}+2\,a\sqrt{x}}{b^3}+2\,\frac{a^2}{b^3}\left(-1/2\,\frac{\sqrt{x}}{bx-a}-5/2\,\frac{1}{\sqrt{ab}}\text{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a)^2,x)
```

```
[Out] 2/b^3*(1/3*b*x^(3/2)+2*a*x^(1/2))+2/b^3*a^2*(-1/2*x^(1/2)/(b*x-a)-5/2/(a*b)
^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60024, size = 366, normalized size = 5.23

$$\left[\frac{15\left(abx-a^{2}\right)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\left(2b^{2}x^{2}+10abx-15a^{2}\right)\sqrt{x}}{6\left(b^{4}x-ab^{3}\right)}, \frac{15\left(abx-a^{2}\right)\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\left(2b^{2}x^{2}+1\right.}{3\left(b^{4}x-ab^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b*x - a^2)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x
 - a)) + 2*(2*b^2*x^2 + 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x - a*b^3), 1/3*(1
5*(a*b*x - a^2)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (2*b^2*x^2 + 10
*a*b*x - 15*a^2)*sqrt(x))/(b^4*x - a*b^3)]
```

Sympy [A] time = 85.4747, size = 444, normalized size = 6.34

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} \\ \frac{2x^{\frac{7}{2}}}{7a^{2}} \\ \frac{2x^{\frac{3}{2}}}{3b^{2}} \\ -\frac{30a^{\frac{5}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{-6a^{\frac{3}{2}}b^{4}\sqrt{\frac{1}{b}}+6\sqrt{a}b^{5}x\sqrt{\frac{1}{b}}}+\frac{20a^{\frac{3}{2}}b^{2}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{-6a^{\frac{3}{2}}b^{4}\sqrt{\frac{1}{b}}+6\sqrt{a}b^{5}x\sqrt{\frac{1}{b}}}+\frac{4\sqrt{a}b^{3}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{-6a^{\frac{3}{2}}b^{4}\sqrt{\frac{1}{b}}+6\sqrt{a}b^{5}x\sqrt{\frac{1}{b}}}-\frac{15a^{3}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-6a^{\frac{3}{2}}b^{4}\sqrt{\frac{1}{b}}+6\sqrt{a}b^{5}x\sqrt{\frac{1}{b}}}+\frac{15a^{3}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-6a^{\frac{3}{2}}b^{4}\sqrt{\frac{1}{b}}+6\sqrt{a}b^{5}x\sqrt{\frac{1}{b}}}+\frac{15a^{2}bx\log}{-6a^{\frac{3}{2}}b^{4}\sqrt{}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**2), Eq(b,
0)), (2*x**(3/2)/(3*b**2), Eq(a, 0)), (-30*a**(5/2)*b*sqrt(x)*sqrt(1/b)/(-6
*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b)) + 20*a**(3/2)*b**2*x
**(3/2)*sqrt(1/b)/(-6*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b))
 + 4*sqrt(a)*b**3*x**(5/2)*sqrt(1/b)/(-6*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a
)*b**5*x*sqrt(1/b)) - 15*a**3*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-6*a**(3/2
)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b)) + 15*a**3*log(sqrt(a)*sqrt(1
/b) + sqrt(x))/(-6*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b)) +
15*a**2*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-6*a**(3/2)*b**4*sqrt(1/b) +
 6*sqrt(a)*b**5*x*sqrt(1/b)) - 15*a**2*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))
/(-6*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b)), True))
```

Giac [A] time = 1.2141, size = 93, normalized size = 1.33

$$\frac{5a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}b^{3}}-\frac{a^{2}\sqrt{x}}{(bx-a)b^{3}}+\frac{2\left(b^{4}x^{\frac{3}{2}}+6ab^{3}\sqrt{x}\right)}{3b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) - a^2*sqrt(x)/((b*x - a
)*b^3) + 2/3*(b^4*x^(3/2) + 6*a*b^3*sqrt(x))/b^6
```

3.477 $\int \frac{x^{3/2}}{(-a+bx)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x^{3/2}}{b(a-bx)}+\frac{3\sqrt{x}}{b^2}$$

```
[Out] (3*Sqrt[x])/b^2 + x^(3/2)/(b*(a - b*x)) - (3*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[
x])/Sqrt[a]])/b^(5/2)
```

Rubi [A] time = 0.0175417, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 208}

$$-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x^{3/2}}{b(a-bx)}+\frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x)^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 + x^(3/2)/(b*(a - b*x)) - (3*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[
x])/Sqrt[a]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(-a+bx)^2}\,dx &= \frac{x^{3/2}}{b(a-bx)}+\frac{3\int \frac{\sqrt{x}}{-a+bx}\,dx}{2b}\\ &= \frac{3\sqrt{x}}{b^2}+\frac{x^{3/2}}{b(a-bx)}+\frac{(3a)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{2b^2}\\ &= \frac{3\sqrt{x}}{b^2}+\frac{x^{3/2}}{b(a-bx)}+\frac{(3a)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{b^2}\\ &= \frac{3\sqrt{x}}{b^2}+\frac{x^{3/2}}{b(a-bx)}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.00483, size = 26, normalized size = 0.46

$$\frac{2x^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx}{a}\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x)^2,x]
```

```
[Out] (2*x^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (b*x)/a])/(5*a^2)
```

Maple [A] time = 0.01, size = 49, normalized size = 0.9

$$2\,\frac{\sqrt{x}}{b^2}+2\,\frac{a}{b^2}\left(-1/2\,\frac{\sqrt{x}}{bx-a}-3/2\,\frac{1}{\sqrt{ab}}\text{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a)^2,x)
```

```
[Out] 2*x^(1/2)/b^2+2/b^2*a*(-1/2*x^(1/2)/(b*x-a)-3/2/(a*b)^(1/2)*arctanh(b*x^(1/
2)/(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65208, size = 298, normalized size = 5.23

$$\left[\frac{3\,(bx-a)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(2\,bx-3\,a)\sqrt{x}}{2\left(b^3x-ab^2\right)},\ \frac{3\,(bx-a)\sqrt{-\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+(2\,bx-3\,a)\sqrt{x}}{b^3x-ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x - a)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)
) + 2*(2*b*x - 3*a)*sqrt(x))/(b^3*x - a*b^2), (3*(b*x - a)*sqrt(-a/b)*arcta
n(b*sqrt(x)*sqrt(-a/b)/a) + (2*b*x - 3*a)*sqrt(x))/(b^3*x - a*b^2)]
```

Sympy [A] time = 21.0795, size = 381, normalized size = 6.68

$$\begin{cases} \tilde{\infty}\sqrt{x} \\ \frac{2\sqrt{x}}{b^2} \\ \frac{2x^{\frac{5}{2}}}{5a^2} \\ -\frac{6a^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}+\frac{4\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}-\frac{3a^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}+\frac{3a^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}+\frac{3abx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}-\frac{3abx\log\left(\right.}{-2a^{\frac{3}{2}}b^3\sqrt{\right.}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b**2, Eq(a, 0)), (
2*x**(5/2)/(5*a**2), Eq(b, 0)), (-6*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(-2*a**(3/
2)*b**3*sqrt(1/b) + 2*sqrt(a)*b**4*x*sqrt(1/b)) + 4*sqrt(a)*b**2*x**(3/2)*s
qrt(1/b)/(-2*a**(3/2)*b**3*sqrt(1/b) + 2*sqrt(a)*b**4*x*sqrt(1/b)) - 3*a**2
*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b**3*sqrt(1/b) + 2*sqrt(a)*
b**4*x*sqrt(1/b)) + 3*a**2*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b*
*3*sqrt(1/b) + 2*sqrt(a)*b**4*x*sqrt(1/b)) + 3*a*b*x*log(-sqrt(a)*sqrt(1/b)
 + sqrt(x))/(-2*a**(3/2)*b**3*sqrt(1/b) + 2*sqrt(a)*b**4*x*sqrt(1/b)) - 3*a
*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b**3*sqrt(1/b) + 2*sqrt(
a)*b**4*x*sqrt(1/b)), True))
```

Giac [A] time = 1.19042, size = 69, normalized size = 1.21

$$\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}b^2}-\frac{a\sqrt{x}}{(bx-a)b^2}+\frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] 3*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) - a*sqrt(x)/((b*x - a)*b^
2) + 2*sqrt(x)/b^2
```

3.478 $\int \frac{\sqrt{x}}{(-a+bx)^2} dx$

Optimal. Leaf size=47

$$\frac{\sqrt{x}}{b(a-bx)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

```
[Out] Sqrt[x]/(b*(a - b*x)) - ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/2)
)
```

Rubi [A] time = 0.0147455, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {47, 63, 208}

$$\frac{\sqrt{x}}{b(a-bx)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x)^2,x]
```

```
[Out] Sqrt[x]/(b*(a - b*x)) - ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/2)
)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx = \frac{\sqrt{x}}{b(a-bx)} + \frac{\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{2b}$$
$$= \frac{\sqrt{x}}{b(a-bx)} + \frac{\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{\sqrt{x}}{b(a-bx)} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

Mathematica [A] time = 0.0152502, size = 61, normalized size = 1.3

$$\frac{\sqrt{a}\sqrt{b}\sqrt{x} + (bx-a)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x)^2,x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[x] + (-a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(
Sqrt[a]*b^(3/2)*(a - b*x))
```

Maple [A] time = 0.007, size = 40, normalized size = 0.9

$$-\frac{1}{b(bx-a)}\sqrt{x} - \frac{1}{b}\text{Artanh}\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a)^2,x)
```

```
[Out] -1/b*x^(1/2)/(b*x-a)-1/b/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65146, size = 277, normalized size = 5.89

$$\left[-\frac{2\,ab\sqrt{x} - \sqrt{ab}(bx-a)\log\left(\frac{bx+a-2\sqrt{ab}\sqrt{x}}{bx-a}\right)}{2\left(ab^3x - a^2b^2\right)}, -\frac{ab\sqrt{x} - \sqrt{-ab}(bx-a)\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{ab^3x - a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) - sqrt(a*b)*(b*x - a)*log((b*x + a - 2*sqrt(a*b)*sqrt(
x))/(b*x - a)))/(a*b^3*x - a^2*b^2), -(a*b*sqrt(x) - sqrt(-a*b)*(b*x - a)*a
rctan(sqrt(-a*b)/(b*sqrt(x))))/(a*b^3*x - a^2*b^2)]
```

Sympy [A] time = 9.04806, size = 311, normalized size = 6.62

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a \\ -\frac{2}{b^2\sqrt{x}} & \text{for } a \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{for } b \\ -\frac{2\sqrt{a}b\sqrt{x}\sqrt{\frac{1}{b}}}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}b^3x\sqrt{\frac{1}{b}}} - \frac{a\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}b^3x\sqrt{\frac{1}{b}}} + \frac{a\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}b^3x\sqrt{\frac{1}{b}}} + \frac{bx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}b^3x\sqrt{\frac{1}{b}}} - \frac{bx\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}b^3x\sqrt{\frac{1}{b}}} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b**2*sqrt(x)), Eq(a, 0))
, (2*x**(3/2)/(3*a**2), Eq(b, 0)), (-2*sqrt(a)*b*sqrt(x)*sqrt(1/b)/(-2*a**(
3/2)*b**2*sqrt(1/b) + 2*sqrt(a)*b**3*x*sqrt(1/b)) - a*log(-sqrt(a)*sqrt(1/b
) + sqrt(x))/(-2*a**(3/2)*b**2*sqrt(1/b) + 2*sqrt(a)*b**3*x*sqrt(1/b)) + a*
log(sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b**2*sqrt(1/b) + 2*sqrt(a)*b*
*3*x*sqrt(1/b)) + b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b**2*s
qrt(1/b) + 2*sqrt(a)*b**3*x*sqrt(1/b)) - b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x
))/(-2*a**(3/2)*b**2*sqrt(1/b) + 2*sqrt(a)*b**3*x*sqrt(1/b)), True))
```

Giac [A] time = 1.20035, size = 54, normalized size = 1.15

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}b} - \frac{\sqrt{x}}{(bx-a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b) - sqrt(x)/((b*x - a)*b)
```

3.479 $\int \frac{1}{\sqrt{x}(-a+bx)^2} dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a-bx)}$$

```
[Out] Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

Rubi [A] time = 0.0142779, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx = \frac{\sqrt{x}}{a(a-bx)} - \frac{\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{2a}$$
$$= \frac{\sqrt{x}}{a(a-bx)} - \frac{\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{\sqrt{x}}{a(a-bx)} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Mathematica [A] time = 0.0209973, size = 46, normalized size = 1.

$$\frac{\sqrt{x}}{a^2-abx} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a^2 - a*b*x) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

Maple [A] time = 0.007, size = 39, normalized size = 0.9

$$-\frac{1}{a(bx-a)}\sqrt{x} + \frac{1}{a}\text{Artanh}\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^2/x^(1/2),x)
```

```
[Out] -x^(1/2)/a/(b*x-a)+1/a/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71234, size = 277, normalized size = 6.02

$$\left[-\frac{2\,ab\sqrt{x} - \sqrt{ab}(bx-a)\log\left(\frac{bx+a+2\sqrt{ab}\sqrt{x}}{bx-a}\right)}{2\left(a^2b^2x-a^3b\right)}, -\frac{ab\sqrt{x} + \sqrt{-ab}(bx-a)\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{a^2b^2x-a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) - sqrt(a*b)*(b*x - a)*log((b*x + a + 2*sqrt(a*b)*sqrt(
x))/(b*x - a)))/(a^2*b^2*x - a^3*b), -(a*b*sqrt(x) + sqrt(-a*b)*(b*x - a)*a
rctan(sqrt(-a*b)/(b*sqrt(x))))/(a^2*b^2*x - a^3*b)]
```

Sympy [A] time = 17.7792, size = 303, normalized size = 6.59

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a = 0 \wedge b \\ -\frac{2}{3b^2x^{\frac{3}{2}}} & \text{for } a = 0 \\ \frac{2\sqrt{x}}{a^2} & \text{for } b = 0 \\ -\frac{2\sqrt{a}b\sqrt{x}\sqrt{\frac{1}{b}}}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} + \frac{a\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} - \frac{a\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} - \frac{bx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} + \frac{bx\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b**2*x**(3/2)), Eq(a,
 0)), (2*sqrt(x)/a**2, Eq(b, 0)), (-2*sqrt(a)*b*sqrt(x)*sqrt(1/b)/(-2*a**(5
/2)*b*sqrt(1/b) + 2*a**(3/2)*b**2*x*sqrt(1/b)) + a*log(-sqrt(a)*sqrt(1/b) +
 sqrt(x))/(-2*a**(5/2)*b*sqrt(1/b) + 2*a**(3/2)*b**2*x*sqrt(1/b)) - a*log(s
qrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(5/2)*b*sqrt(1/b) + 2*a**(3/2)*b**2*x*sq
rt(1/b)) - b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(5/2)*b*sqrt(1/b) +
 2*a**(3/2)*b**2*x*sqrt(1/b)) + b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a*
*(5/2)*b*sqrt(1/b) + 2*a**(3/2)*b**2*x*sqrt(1/b)), True))
```

Giac [A] time = 1.17384, size = 55, normalized size = 1.2

$$-\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}a} - \frac{\sqrt{x}}{(bx-a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a) - sqrt(x)/((b*x - a)*a)
```

3.480 $\int \frac{1}{x^{3/2}(-a+bx)^2} dx$

Optimal. Leaf size=57

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}(a-bx)}$$

```
[Out] -3/(a^2*Sqrt[x]) + 1/(a*Sqrt[x]*(a - b*x)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[x])/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0179159, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 208}

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)^2),x]
```

```
[Out] -3/(a^2*Sqrt[x]) + 1/(a*Sqrt[x]*(a - b*x)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[x])/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx = \frac{1}{a\sqrt{x}(a-bx)} - \frac{3\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{2a}$$
$$= -\frac{3}{a^2\sqrt{x}} + \frac{1}{a\sqrt{x}(a-bx)} - \frac{(3b)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{2a^2}$$
$$= -\frac{3}{a^2\sqrt{x}} + \frac{1}{a\sqrt{x}(a-bx)} - \frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a^2}$$
$$= -\frac{3}{a^2\sqrt{x}} + \frac{1}{a\sqrt{x}(a-bx)} + \frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.0058549, size = 24, normalized size = 0.42

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx}{a}\right)}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 2, 1/2, (b*x)/a])/(a^2*Sqrt[x])
```

Maple [A] time = 0.01, size = 49, normalized size = 0.9

$$-2\,\frac{1}{a^2\sqrt{x}} - 2\,\frac{b}{a^2}\left(1/2\,\frac{\sqrt{x}}{bx-a} - 3/2\,\frac{1}{\sqrt{ab}}\mathrm{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a)^2,x)
```

```
[Out] -2/a^2/x^(1/2)-2*b/a^2*(1/2*x^(1/2)/(b*x-a)-3/2/(a*b)^(1/2)*arctanh(b*x^(1/
2)/(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62697, size = 324, normalized size = 5.68

$$\left[\frac{3\left(bx^2-ax\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx+2a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)-2(3bx-2a)\sqrt{x}}{2\left(a^2bx^2-a^3x\right)},-\frac{3\left(bx^2-ax\right)\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(3bx-2a)\sqrt{x}}{a^2bx^2-a^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x^2 - a*x)*sqrt(b/a)*log((b*x + 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x
- a)) - 2*(3*b*x - 2*a)*sqrt(x))/(a^2*b*x^2 - a^3*x), -(3*(b*x^2 - a*x)*sqr
t(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (3*b*x - 2*a)*sqrt(x))/(a^2*b*x^
2 - a^3*x)]
```

Sympy [A] time = 42.7898, size = 403, normalized size = 7.07

$$\begin{cases}\frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ -\frac{2}{5b^2x^{\frac{5}{2}}} \\ -\frac{2}{a^2\sqrt{x}} \\ -\frac{4a^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{2a^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-2a^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}}+\frac{6\sqrt{a}bx\sqrt{\frac{1}{b}}}{2a^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-2a^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}}-\frac{3a\sqrt{x}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-2a^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}}+\frac{3a\sqrt{x}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-2a^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}}+\frac{3bx^{\frac{3}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-2a^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}}-\frac{3bx^{\frac{3}{2}}\log}{2a^{\frac{7}{2}}\sqrt{x}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**2*x**(5/2)), Eq(a,
 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)), (-4*a**(3/2)*sqrt(1/b)/(2*a**(7/2)*sqr
t(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)) + 6*sqrt(a)*b*x*sqrt(1/b)
/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)) - 3*a*sqr
t(x)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a*
*(5/2)*b*x**(3/2)*sqrt(1/b)) + 3*a*sqrt(x)*log(sqrt(a)*sqrt(1/b) + sqrt(x))
/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)) + 3*b*x**
(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a
**(5/2)*b*x**(3/2)*sqrt(1/b)) - 3*b*x**(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x
))/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)), True))
```

Giac [A] time = 1.20185, size = 70, normalized size = 1.23

$$-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}a^2}-\frac{3\,bx-2\,a}{\left(bx^{\frac{3}{2}}-a\sqrt{x}\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] -3*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) - (3*b*x - 2*a)/((b*x^(3
/2) - a*sqrt(x))*a^2)
```

3.481 $\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx$

Optimal. Leaf size=70

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a-bx)}$$

```
[Out] -5/(3*a^2*x^(3/2)) - (5*b)/(a^3*Sqrt[x]) + 1/(a*x^(3/2)*(a - b*x)) + (5*b^(
3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rubi [A] time = 0.0224064, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 208}

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)^2),x]
```

```
[Out] -5/(3*a^2*x^(3/2)) - (5*b)/(a^3*Sqrt[x]) + 1/(a*x^(3/2)*(a - b*x)) + (5*b^(
3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx = \frac{1}{ax^{3/2}(a-bx)} - \frac{5\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{2a}$$
$$= -\frac{5}{3a^2x^{3/2}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{(5b)\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{2a^2}$$
$$= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{(5b^2)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{2a^3}$$
$$= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{(5b^2)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a^3}$$
$$= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} + \frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [C] time = 0.0058593, size = 26, normalized size = 0.37

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{bx}{a}\right)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 2, -1/2, (b*x)/a])/(3*a^2*x^(3/2))
```

Maple [A] time = 0.011, size = 60, normalized size = 0.9

$$-\frac{2}{3\,a^2}x^{-\frac{3}{2}} - 4\,\frac{b}{a^3\sqrt{x}} - 2\,\frac{b^2}{a^3}\left(1/2\,\frac{\sqrt{x}}{bx-a} - 5/2\,\frac{1}{\sqrt{ab}}\text{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a)^2,x)
```

```
[Out] -2/3/a^2/x^(3/2)-4*b/a^3/x^(1/2)-2*b^2/a^3*(1/2*x^(1/2)/(b*x-a)-5/2/(a*b)^(
1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55767, size = 402, normalized size = 5.74

$$\left[\frac{15\left(b^2x^3-abx^2\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx+2a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)-2\left(15b^2x^2-10abx-2a^2\right)\sqrt{x}}{6\left(a^3bx^3-a^4x^2\right)},-\frac{15\left(b^2x^3-abx^2\right)\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\left(15b^2\right.}{3\left(a^3bx^3-a^4x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^2*x^3 - a*b*x^2)*sqrt(b/a)*log((b*x + 2*a*sqrt(x)*sqrt(b/a) + a
)/(b*x - a)) - 2*(15*b^2*x^2 - 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 - a^4*
x^2), -1/3*(15*(b^2*x^3 - a*b*x^2)*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x
))) + (15*b^2*x^2 - 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 - a^4*x^2)]
```

Sympy [A] time = 139.005, size = 471, normalized size = 6.73

$$\begin{cases}\frac{\tilde{\infty}}{x^{\frac{7}{2}}}\\ -\frac{2}{7b^2x^{\frac{7}{2}}}\\ -\frac{2}{3a^2x^{\frac{3}{2}}}\\ -\frac{4a^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{6a^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-6a^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}-\frac{20a^{\frac{3}{2}}bx\sqrt{\frac{1}{b}}}{6a^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-6a^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{30\sqrt{a}b^2x^2\sqrt{\frac{1}{b}}}{6a^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-6a^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}-\frac{15abx^{\frac{3}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6a^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-6a^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{15abx^{\frac{3}{2}}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6a^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-6a^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{15b^2x^{\frac{5}{2}}\,\mathrm{l}}{6a^{\frac{9}{2}}x^{\frac{3}{2}}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**2*x**(7/2)), Eq(a,
 0)), (-2/(3*a**2*x**(3/2)), Eq(b, 0)), (-4*a**(5/2)*sqrt(1/b)/(6*a**(9/2)*
x**(3/2)*sqrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b)) - 20*a**(3/2)*b*x*sqr
t(1/b)/(6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b)) +
30*sqrt(a)*b**2*x**2*sqrt(1/b)/(6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6*a**(7/2)*
b*x**(5/2)*sqrt(1/b)) - 15*a*b*x**(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(
6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 15*a*b*x
**(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6
*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 15*b**2*x**(5/2)*log(-sqrt(a)*sqrt(1/b) +
 sqrt(x))/(6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b))
 - 15*b**2*x**(5/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(6*a**(9/2)*x**(3/2)*s
qrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b)), True))
```

Giac [A] time = 1.19526, size = 82, normalized size = 1.17

$$-\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}a^3}-\frac{b^2\sqrt{x}}{(bx-a)a^3}-\frac{2\,(6\,bx+a)}{3\,a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] -5*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) - b^2*sqrt(x)/((b*x -
a)*a^3) - 2/3*(6*b*x + a)/(a^3*x^(3/2))
```

3.482 $\int \frac{x^{7/2}}{(-a+bx)^3}\,dx$

Optimal. Leaf size=97

$$-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{35a\sqrt{x}}{4b^4}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{35x^{3/2}}{12b^3}$$

```
[Out] (35*a*Sqrt[x])/(4*b^4) + (35*x^(3/2))/(12*b^3) - x^(7/2)/(2*b*(a - b*x)^2)
+ (7*x^(5/2))/(4*b^2*(a - b*x)) - (35*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/(4*b^(9/2))
```

Rubi [A] time = 0.0301179, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 208}

$$-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{35a\sqrt{x}}{4b^4}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{35x^{3/2}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(-a + b*x)^3,x]
```

```
[Out] (35*a*Sqrt[x])/(4*b^4) + (35*x^(3/2))/(12*b^3) - x^(7/2)/(2*b*(a - b*x)^2)
+ (7*x^(5/2))/(4*b^2*(a - b*x)) - (35*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/(4*b^(9/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{(-a+bx)^3}\,dx &= -\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7\int \frac{x^{5/2}}{(-a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{35\int \frac{x^{3/2}}{-a+bx}\,dx}{8b^2}\\
&= \frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{(35a)\int \frac{\sqrt{x}}{-a+bx}\,dx}{8b^3}\\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{\left(35a^2\right)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8b^4}\\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{\left(35a^2\right)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^4}\\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}
\end{aligned}$$

Mathematica [C] time = 0.0052807, size = 26, normalized size = 0.27

$$-\frac{2x^{9/2}\,{}_2F_1\left(3,\frac{9}{2};\frac{11}{2};\frac{bx}{a}\right)}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(-a + b*x)^3,x]
```

```
[Out] (-2*x^(9/2)*Hypergeometric2F1[3, 9/2, 11/2, (b*x)/a])/(9*a^3)
```

Maple [A] time = 0.011, size = 70, normalized size = 0.7

$$2\,\frac{1/3\,bx^{3/2}+3\,a\sqrt{x}}{b^4}+2\,\frac{a^2}{b^4}\left(\frac{1}{(bx-a)^2}\left(-\frac{13\,bx^{3/2}}{8}+\frac{11\,a\sqrt{x}}{8}\right)-\frac{35}{8\sqrt{ab}}\text{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x-a)^3,x)
```

```
[Out] 2/b^4*(1/3*b*x^(3/2)+3*a*x^(1/2))+2/b^4*a^2*((-13/8*b*x^(3/2)+11/8*a*x^(1/2
))/(b*x-a)^2-35/8/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5717, size = 509, normalized size = 5.25

$$\left[\frac{105\left(ab^2x^2-2\,a^2bx+a^3\right)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\left(8\,b^3x^3+56\,ab^2x^2-175\,a^2bx+105\,a^3\right)\sqrt{x}}{24\left(b^6x^2-2\,ab^5x+a^2b^4\right)},\ \frac{105\left(ab^2x^2-2\,a^2bx+a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*s
qrt(a/b) + a)/(b*x - a)) + 2*(8*b^3*x^3 + 56*a*b^2*x^2 - 175*a^2*b*x + 105*
a^3)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 - 2*a^2
*b*x + a^3)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (8*b^3*x^3 + 56*a*b
^2*x^2 - 175*a^2*b*x + 105*a^3)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x-a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25906, size = 109, normalized size = 1.12

$$\frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}b^4}-\frac{13\,a^2bx^{\frac{3}{2}}-11\,a^3\sqrt{x}}{4\,(bx-a)^2b^4}+\frac{2\left(b^6x^{\frac{3}{2}}+9\,ab^5\sqrt{x}\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 35/4*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^4) - 1/4*(13*a^2*b*x^(3
/2) - 11*a^3*sqrt(x))/((b*x - a)^2*b^4) + 2/3*(b^6*x^(3/2) + 9*a*b^5*sqrt(x
))/b^9
```

3.483 $\int \frac{x^{5/2}}{(-a+bx)^3}\,dx$

Optimal. Leaf size=84

$$\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{15\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

```
[Out] (15*Sqrt[x])/(4*b^3) - x^(5/2)/(2*b*(a - b*x)^2) + (5*x^(3/2))/(4*b^2*(a - b*x)) - (15*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rubi [A] time = 0.0260807, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 208}

$$\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{15\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x)^3,x]
```

```
[Out] (15*Sqrt[x])/(4*b^3) - x^(5/2)/(2*b*(a - b*x)^2) + (5*x^(3/2))/(4*b^2*(a - b*x)) - (15*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(-a+bx)^3}\,dx &= -\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5\int \frac{x^{3/2}}{(-a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}+\frac{15\int \frac{\sqrt{x}}{-a+bx}\,dx}{8b^2}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}+\frac{(15a)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}+\frac{(15a)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{15\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0050722, size = 26, normalized size = 0.31

$$
-\frac{2x^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{bx}{a}\right)}{7a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x)^3,x]
```

```
[Out] (-2*x^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, (b*x)/a])/(7*a^3)
```

Maple [A] time = 0.012, size = 58, normalized size = 0.7

$$
2\,\frac{\sqrt{x}}{b^3}+2\,\frac{a}{b^3}\left(\frac{1}{(bx-a)^2}\left(-\frac{9\,bx^{3/2}}{8}+\frac{7\,a\sqrt{x}}{8}\right)-\frac{15}{8\,\sqrt{ab}}\mathrm{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a)^3,x)
```

```
[Out] 2*x^(1/2)/b^3+2/b^3*a*((-9/8*b*x^(3/2)+7/8*a*x^(1/2))/(b*x-a)^2-15/8/(a*b)^
(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60673, size = 441, normalized size = 5.25

$$\left[\frac{15\left(b^{2}x^{2}-2\,abx+a^{2}\right)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\left(8\,b^{2}x^{2}-25\,abx+15\,a^{2}\right)\sqrt{x}}{8\left(b^{5}x^{2}-2\,ab^{4}x+a^{2}b^{3}\right)},\ \frac{15\left(b^{2}x^{2}-2\,abx+a^{2}\right)\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\cdots}{\cdots}\right)\cdots}{4\left(b^{5}x^{2}-2\,\cdots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/
b) + a)/(b*x - a)) + 2*(8*b^2*x^2 - 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2 -
2*a*b^4*x + a^2*b^3), 1/4*(15*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a/b)*arctan(b
*sqrt(x)*sqrt(-a/b)/a) + (8*b^2*x^2 - 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
- 2*a*b^4*x + a^2*b^3)]
```

Sympy [A] time = 174.976, size = 756, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b**3, Eq(a, 0)), (
-2*x**(7/2)/(7*a**3), Eq(b, 0)), (30*a**(5/2)*b*sqrt(x)*sqrt(1/b)/(8*a**(5/
2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt
(1/b)) - 50*a**(3/2)*b**2*x**(3/2)*sqrt(1/b)/(8*a**(5/2)*b**4*sqrt(1/b) - 1
6*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) + 16*sqrt(a)*b
**3*x**(5/2)*sqrt(1/b)/(8*a**(5/2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*sqrt
(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) + 15*a**3*log(-sqrt(a)*sqrt(1/b) + s
qrt(x))/(8*a**(5/2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(
a)*b**6*x**2*sqrt(1/b)) - 15*a**3*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5
/2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqr
t(1/b)) - 30*a**2*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**4*sq
rt(1/b) - 16*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) + 3
0*a**2*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**4*sqrt(1/b) - 16
*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) + 15*a*b**2*x**
2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**4*sqrt(1/b) - 16*a**(3/2
)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) - 15*a*b**2*x**2*log(sq
rt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*
sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)), True))
```

Giac [A] time = 1.20904, size = 85, normalized size = 1.01

$$\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}b^{3}}+\frac{2\,\sqrt{x}}{b^{3}}-\frac{9\,abx^{\frac{3}{2}}-7\,a^{2}\sqrt{x}}{4\,(bx-a)^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 15/4*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) + 2*sqrt(x)/b^3 - 1/4*
(9*a*b*x^(3/2) - 7*a^2*sqrt(x))/((b*x - a)^2*b^3)
```

3.484 $\int \frac{x^{3/2}}{(-a+bx)^3}\,dx$

Optimal. Leaf size=72

$$\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}-\frac{x^{3/2}}{2b(a-bx)^2}$$

```
[Out] -x^(3/2)/(2*b*(a - b*x)^2) + (3*Sqrt[x])/(4*b^2*(a - b*x)) - (3*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rubi [A] time = 0.0208706, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {47, 63, 208}

$$\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}-\frac{x^{3/2}}{2b(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x)^3,x]
```

```
[Out] -x^(3/2)/(2*b*(a - b*x)^2) + (3*Sqrt[x])/(4*b^2*(a - b*x)) - (3*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{3/2}}{(-a+bx)^3}\,dx = -\frac{x^{3/2}}{2b(a-bx)^2} + \frac{3\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx}{4b}$$
$$= -\frac{x^{3/2}}{2b(a-bx)^2} + \frac{3\sqrt{x}}{4b^2(a-bx)} + \frac{3\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8b^2}$$
$$= -\frac{x^{3/2}}{2b(a-bx)^2} + \frac{3\sqrt{x}}{4b^2(a-bx)} + \frac{3\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{4b^2}$$
$$= -\frac{x^{3/2}}{2b(a-bx)^2} + \frac{3\sqrt{x}}{4b^2(a-bx)} - \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}$$

Mathematica [A] time = 0.0357522, size = 60, normalized size = 0.83

$$\frac{\sqrt{x}(3a-5bx)}{4b^2(a-bx)^2} - \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x)^3,x]
```

```
[Out] (Sqrt[x]*(3*a - 5*b*x))/(4*b^2*(a - b*x)^2) - (3*ArcTanh[(Sqrt[b]*Sqrt[x])/
Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Maple [A] time = 0.01, size = 52, normalized size = 0.7

$$2\frac{1}{(bx-a)^2}\left(-5/8\,\frac{x^{3/2}}{b} + 3/8\,\frac{a\sqrt{x}}{b^2}\right) - \frac{3}{4\,b^2}\text{Artanh}\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a)^3,x)
```

```
[Out] 2*(-5/8*x^(3/2)/b+3/8*a*x^(1/2)/b^2)/(b*x-a)^2-3/4/b^2/(a*b)^(1/2)*arctanh(
b*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71914, size = 420, normalized size = 5.83

$$\left[\frac{3\left(b^{2}x^{2}-2\,abx+a^{2}\right)\sqrt{ab}\log\left(\frac{bx+a-2\sqrt{ab}\sqrt{x}}{bx-a}\right)-2\left(5\,ab^{2}x-3\,a^{2}b\right)\sqrt{x}}{8\left(ab^{5}x^{2}-2\,a^{2}b^{4}x+a^{3}b^{3}\right)},\frac{3\left(b^{2}x^{2}-2\,abx+a^{2}\right)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)-\left(5\,a\right.}{4\left(ab^{5}x^{2}-2\,a^{2}b^{4}x+a^{3}b^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt
(x))/(b*x - a)) - 2*(5*a*b^2*x - 3*a^2*b)*sqrt(x))/(a*b^5*x^2 - 2*a^2*b^4*x
 + a^3*b^3), 1/4*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/
(b*sqrt(x))) - (5*a*b^2*x - 3*a^2*b)*sqrt(x))/(a*b^5*x^2 - 2*a^2*b^4*x + a^
3*b^3)]
```

Sympy [A] time = 80.4734, size = 673, normalized size = 9.35

$$\begin{cases}\frac{\tilde{\infty}}{\sqrt{x}}\\ -\frac{2}{b^{3}\sqrt{x}}\\ -\frac{2x^{\frac{5}{2}}}{5a^{3}}\\ \frac{6a^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8a^{\frac{5}{2}}b^{3}\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^{4}x\sqrt{\frac{1}{b}}+8\sqrt{a}b^{5}x^{2}\sqrt{\frac{1}{b}}}-\frac{10\sqrt{a}b^{2}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8a^{\frac{5}{2}}b^{3}\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^{4}x\sqrt{\frac{1}{b}}+8\sqrt{a}b^{5}x^{2}\sqrt{\frac{1}{b}}}+\frac{3a^{2}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{5}{2}}b^{3}\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^{4}x\sqrt{\frac{1}{b}}+8\sqrt{a}b^{5}x^{2}\sqrt{\frac{1}{b}}}-\frac{3a^{2}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\right.}{8a^{\frac{5}{2}}b^{3}\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^{4}x\sqrt{\frac{1}{b}}+}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b**3*sqrt(x)), Eq(a, 0))
, (-2*x**(5/2)/(5*a**3), Eq(b, 0)), (6*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8*a**(
5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sq
rt(1/b)) - 10*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*a**(5/2)*b**3*sqrt(1/b) -
16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)) + 3*a**2*log(
-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*b**4
*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)) - 3*a**2*log(sqrt(a)*sqrt(1/b
) + sqrt(x))/(8*a**(5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*b**4*x*sqrt(1/b) + 8*
sqrt(a)*b**5*x**2*sqrt(1/b)) - 6*a*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8
*a**(5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x*
*2*sqrt(1/b)) + 6*a*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**3*s
qrt(1/b) - 16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)) +
3*b**2*x**2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**3*sqrt(1/b) -
16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)) - 3*b**2*x**2
*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*
b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)), True))
```

Giac [A] time = 1.22572, size = 69, normalized size = 0.96

$$\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}b^{2}}-\frac{5\,bx^{\frac{3}{2}}-3\,a\sqrt{x}}{4\,(bx-a)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) - 1/4*(5*b*x^(3/2) - 3*a*
sqrt(x))/((b*x - a)^2*b^2)
```

3.485 $\int \frac{\sqrt{x}}{(-a+bx)^3} dx$

Optimal. Leaf size=75

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\sqrt{x}}{2b(a-bx)^2}$$

```
[Out] -Sqrt[x]/(2*b*(a - b*x)^2) + Sqrt[x]/(4*a*b*(a - b*x)) + ArcTanh[(Sqrt[b]*S
qrt[x])/Sqrt[a]]/(4*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.0199319, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\sqrt{x}}{2b(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x)^3,x]
```

```
[Out] -Sqrt[x]/(2*b*(a - b*x)^2) + Sqrt[x]/(4*a*b*(a - b*x)) + ArcTanh[(Sqrt[b]*S
qrt[x])/Sqrt[a]]/(4*a^(3/2)*b^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx &= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx}{4b}\\
&= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8ab}\\
&= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4ab}\\
&= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0050309, size = 26, normalized size = 0.35

$$
-\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}\right)}{3a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x)^3,x]
```

```
[Out] (-2*x^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, (b*x)/a])/(3*a^3)
```

Maple [A] time = 0.009, size = 54, normalized size = 0.7

$$
2\,\frac{1}{(bx-a)^2}\left(-1/8\,\frac{x^{3/2}}{a}-1/8\,\frac{\sqrt{x}}{b}\right)+\frac{1}{4\,ab}\text{Artanh}\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a)^3,x)
```

```
[Out] 2*(-1/8/a*x^(3/2)-1/8*x^(1/2)/b)/(b*x-a)^2+1/4/b/a/(a*b)^(1/2)*arctanh(b*x^
(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72903, size = 410, normalized size = 5.47

$$\left[\frac{\left(b^2x^2-2\,abx+a^2\right)\sqrt{ab}\log\left(\frac{bx+a+2\,\sqrt{ab}\sqrt{x}}{bx-a}\right)-2\left(ab^2x+a^2b\right)\sqrt{x}}{8\left(a^2b^4x^2-2\,a^3b^3x+a^4b^2\right)},-\frac{\left(b^2x^2-2\,abx+a^2\right)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)+\left(ab^2x+\right.}{4\left(a^2b^4x^2-2\,a^3b^3x+a^4b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*((b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a + 2*sqrt(a*b)*sqrt(x
))/(b*x - a)) - 2*(a*b^2*x + a^2*b)*sqrt(x))/(a^2*b^4*x^2 - 2*a^3*b^3*x + a
^4*b^2), -1/4*((b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/(b*sq
rt(x))) + (a*b^2*x + a^2*b)*sqrt(x))/(a^2*b^4*x^2 - 2*a^3*b^3*x + a^4*b^2)]
```

Sympy [A] time = 33.3675, size = 668, normalized size = 8.91

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} \\ -\frac{2}{3b^3x^{\frac{3}{2}}} \\ -\frac{2x^{\frac{3}{2}}}{3a^3} \\ -\frac{2a^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}-16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}}-\frac{2\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}-16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}}-\frac{a^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}-16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}}+\frac{a^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\right.}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}-16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b**3*x**(3/2)), Eq(a,
 0)), (-2*x**(3/2)/(3*a**3), Eq(b, 0)), (-2*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8
*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x
**2*sqrt(1/b)) - 2*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*a**(7/2)*b**2*sqrt(1/
b) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/b)) - a**2*
log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/2)*
b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/b)) + a**2*log(sqrt(a)*sqrt(
1/b) + sqrt(x))/(8*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/2)*b**3*x*sqrt(1/b) +
 8*a**(3/2)*b**4*x**2*sqrt(1/b)) + 2*a*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x)
)/(8*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b*
*4*x**2*sqrt(1/b)) - 2*a*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(7/2)*b
**2*sqrt(1/b) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/
b)) - b**2*x**2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(7/2)*b**2*sqrt(1/b
) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/b)) + b**2*x
**2*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/
2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/b)), True))
```

Giac [A] time = 1.20825, size = 74, normalized size = 0.99

$$-\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\sqrt{-ab}ab}-\frac{bx^{\frac{3}{2}}+a\sqrt{x}}{4(bx-a)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] -1/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a*b) - 1/4*(b*x^(3/2) + a*sqr
t(x))/((b*x - a)^2*a*b)
```

3.486 $\int \frac{1}{\sqrt{x}(-a+bx)^3} dx$

Optimal. Leaf size=72

$$-\frac{3\sqrt{x}}{4a^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}-\frac{\sqrt{x}}{2a(a-bx)^2}$$

```
[Out] -Sqrt[x]/(2*a*(a - b*x)^2) - (3*Sqrt[x])/(4*a^2*(a - b*x)) - (3*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.0196717, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 208}

$$-\frac{3\sqrt{x}}{4a^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}-\frac{\sqrt{x}}{2a(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)^3),x]
```

```
[Out] -Sqrt[x]/(2*a*(a - b*x)^2) - (3*Sqrt[x])/(4*a^2*(a - b*x)) - (3*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(-a+bx)^3}\,dx = -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx}{4a}$$
$$= -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\sqrt{x}}{4a^2(a-bx)} + \frac{3\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8a^2}$$
$$= -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\sqrt{x}}{4a^2(a-bx)} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{4a^2}$$
$$= -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\sqrt{x}}{4a^2(a-bx)} - \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

Mathematica [C] time = 0.0049799, size = 24, normalized size = 0.33

$$-\frac{2\sqrt{x}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)^3),x]
```

```
[Out] (-2*Sqrt[x]*Hypergeometric2F1[1/2, 3, 3/2, (b*x)/a])/a^3
```

Maple [A] time = 0.007, size = 63, normalized size = 0.9

$$-\frac{1}{2\,a\,(bx-a)^2}\sqrt{x} - \frac{3}{2\,a}\left(-\frac{1}{2\,a\,(bx-a)}\sqrt{x} + \frac{1}{2\,a}\mathrm{Artanh}\left(b\sqrt{x}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^3/x^(1/2),x)
```

```
[Out] -1/2*x^(1/2)/a/(b*x-a)^2-3/2/a*(-1/2*x^(1/2)/a/(b*x-a)+1/2/a/(a*b)^(1/2)*ar
ctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66265, size = 420, normalized size = 5.83

$$\left[\frac{3\left(b^2x^2 - 2\,abx + a^2\right)\sqrt{ab}\log\left(\frac{bx+a-2\,\sqrt{ab}\sqrt{x}}{bx-a}\right) + 2\left(3\,ab^2x - 5\,a^2b\right)\sqrt{x}}{8\left(a^3b^3x^2 - 2\,a^4b^2x + a^5b\right)}, \frac{3\left(b^2x^2 - 2\,abx + a^2\right)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right) + \left(3\,a\right.}{4\left(a^3b^3x^2 - 2\,a^4b^2x + a^5b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt
(x))/(b*x - a)) + 2*(3*a*b^2*x - 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 - 2*a^4*b^2
*x + a^5*b), 1/4*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/
(b*sqrt(x))) + (3*a*b^2*x - 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 - 2*a^4*b^2*x +
a^5*b)]
```

Sympy [A] time = 56.5358, size = 660, normalized size = 9.17

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ -\frac{2}{5b^3x^{\frac{5}{2}}} \\ -\frac{2\sqrt{x}}{a^3} \\ -\frac{10a^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}}-16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}}+8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} + \frac{6\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}}-16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}}+8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} + \frac{3a^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}}-16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}}+8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} - \frac{3a^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}}-16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}}+8a\ldots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**3/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**3*x**(5/2)), Eq(a,
 0)), (-2*sqrt(x)/a**3, Eq(b, 0)), (-10*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8*a**
(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqr
t(1/b)) + 6*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*a**(9/2)*b*sqrt(1/b) - 16*a*
*(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/b)) + 3*a**2*log(-sqr
t(a)*sqrt(1/b) + sqrt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt
(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/b)) - 3*a**2*log(sqrt(a)*sqrt(1/b) + sq
rt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*
b**3*x**2*sqrt(1/b)) - 6*a*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(9/2
)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/
b)) + 6*a*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16
*a**(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/b)) + 3*b**2*x**2*
log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**
2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/b)) - 3*b**2*x**2*log(sqrt(a)*s
qrt(1/b) + sqrt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt(1/b)
+ 8*a**(5/2)*b**3*x**2*sqrt(1/b)), True))
```

Giac [A] time = 1.21071, size = 69, normalized size = 0.96

$$\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}a^2} + \frac{3\,bx^{\frac{3}{2}} - 5\,a\sqrt{x}}{4\,(bx-a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) + 1/4*(3*b*x^(3/2) - 5*a*
sqrt(x))/((b*x - a)^2*a^2)
```

3.487 $\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx$

Optimal. Leaf size=84

$$-\frac{5}{4a^2\sqrt{x}(a-bx)}-\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{15}{4a^3\sqrt{x}}-\frac{1}{2a\sqrt{x}(a-bx)^2}$$

```
[Out] 15/(4*a^3*Sqrt[x]) - 1/(2*a*Sqrt[x]*(a - b*x)^2) - 5/(4*a^2*Sqrt[x]*(a - b*
x)) - (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rubi [A] time = 0.0239046, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 208}

$$-\frac{5}{4a^2\sqrt{x}(a-bx)}-\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{15}{4a^3\sqrt{x}}-\frac{1}{2a\sqrt{x}(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)^3),x]
```

```
[Out] 15/(4*a^3*Sqrt[x]) - 1/(2*a*Sqrt[x]*(a - b*x)^2) - 5/(4*a^2*Sqrt[x]*(a - b*
x)) - (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx = -\frac{1}{2a\sqrt{x}(a-bx)^2} - \frac{5\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx}{4a}$$
$$= -\frac{1}{2a\sqrt{x}(a-bx)^2} - \frac{5}{4a^2\sqrt{x}(a-bx)} + \frac{15\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{8a^2}$$
$$= \frac{15}{4a^3\sqrt{x}} - \frac{1}{2a\sqrt{x}(a-bx)^2} - \frac{5}{4a^2\sqrt{x}(a-bx)} + \frac{(15b)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8a^3}$$
$$= \frac{15}{4a^3\sqrt{x}} - \frac{1}{2a\sqrt{x}(a-bx)^2} - \frac{5}{4a^2\sqrt{x}(a-bx)} + \frac{(15b)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{4a^3}$$
$$= \frac{15}{4a^3\sqrt{x}} - \frac{1}{2a\sqrt{x}(a-bx)^2} - \frac{5}{4a^2\sqrt{x}(a-bx)} - \frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Mathematica [C] time = 0.0053577, size = 24, normalized size = 0.29

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{bx}{a}\right)}{a^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)^3),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 3, 1/2, (b*x)/a])/(a^3*Sqrt[x])
```

Maple [A] time = 0.012, size = 58, normalized size = 0.7

$$2\frac{1}{a^3\sqrt{x}} + 2\frac{b}{a^3}\left(\frac{1}{(bx-a)^2}\left(\frac{7\,bx^{3/2}}{8} - \frac{9\,a\sqrt{x}}{8}\right) - \frac{15}{8\sqrt{ab}}\text{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a)^3,x)
```

```
[Out] 2/a^3/x^(1/2)+2/a^3*b*((7/8*b*x^(3/2)-9/8*a*x^(1/2))/(b*x-a)^2-15/8/(a*b)^(
1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66448, size = 466, normalized size = 5.55

$$\left[\frac{15\left(b^2x^3-2\,abx^2+a^2x\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\left(15\,b^2x^2-25\,abx+8\,a^2\right)\sqrt{x}}{8\left(a^3b^2x^3-2\,a^4bx^2+a^5x\right)},\ \frac{15\left(b^2x^3-2\,abx^2+a^2x\right)\sqrt{-\frac{b}{a}}\,\mathrm{arcta}}{4\left(a^3b^2x^3-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^3 - 2*a*b*x^2 + a^2*x)*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqr
t(b/a) + a)/(b*x - a)) + 2*(15*b^2*x^2 - 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*b^
2*x^3 - 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 - 2*a*b*x^2 + a^2*x)*sqrt(-b
/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (15*b^2*x^2 - 25*a*b*x + 8*a^2)*sqrt
(x))/(a^3*b^2*x^3 - 2*a^4*b*x^2 + a^5*x)]
```

Sympy [A] time = 132.681, size = 802, normalized size = 9.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**3*x**(7/2)), Eq(a,
 0)), (2/(a**3*sqrt(x)), Eq(b, 0)), (16*a**(5/2)*sqrt(1/b)/(8*a**(11/2)*sqr
t(x)*sqrt(1/b) - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2
)*sqrt(1/b)) - 50*a**(3/2)*b*x*sqrt(1/b)/(8*a**(11/2)*sqrt(x)*sqrt(1/b) - 1
6*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) + 30*
sqrt(a)*b**2*x**2*sqrt(1/b)/(8*a**(11/2)*sqrt(x)*sqrt(1/b) - 16*a**(9/2)*b*
x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) + 15*a**2*sqrt(x)*
log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(x)*sqrt(1/b) - 16*a**(9
/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) - 15*a**2*sq
rt(x)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(x)*sqrt(1/b) - 16*
a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) - 30*a*
b*x**(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(x)*sqrt(1/b)
 - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) +
 30*a*b*x**(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(x)*sqrt
(1/b) - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/
b)) + 15*b**2*x**(5/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(
x)*sqrt(1/b) - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*
sqrt(1/b)) - 15*b**2*x**(5/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)
*sqrt(x)*sqrt(1/b) - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**
(5/2)*sqrt(1/b)), True))
```

Giac [A] time = 1.1446, size = 85, normalized size = 1.01

$$\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}a^3}+\frac{2}{a^3\sqrt{x}}+\frac{7\,b^2x^{\frac{3}{2}}-9\,ab\sqrt{x}}{4\,(bx-a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 15/4*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) + 2/(a^3*sqrt(x)) + 1/
4*(7*b^2*x^(3/2) - 9*a*b*sqrt(x))/((b*x - a)^2*a^3)
```

3.488 $\int \frac{1}{x^{5/2}(-a+bx)^3} dx$

Optimal. Leaf size=97

$$-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}-\frac{7}{4a^2x^{3/2}(a-bx)}+\frac{35b}{4a^4\sqrt{x}}+\frac{35}{12a^3x^{3/2}}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

```
[Out] 35/(12*a^3*x^(3/2)) + (35*b)/(4*a^4*Sqrt[x]) - 1/(2*a*x^(3/2)*(a - b*x)^2)
- 7/(4*a^2*x^(3/2)*(a - b*x)) - (35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[
a]])/(4*a^(9/2))
```

Rubi [A] time = 0.0303405, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 63, 208}

$$-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}-\frac{7}{4a^2x^{3/2}(a-bx)}+\frac{35b}{4a^4\sqrt{x}}+\frac{35}{12a^3x^{3/2}}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)^3),x]
```

```
[Out] 35/(12*a^3*x^(3/2)) + (35*b)/(4*a^4*Sqrt[x]) - 1/(2*a*x^(3/2)*(a - b*x)^2)
- 7/(4*a^2*x^(3/2)*(a - b*x)) - (35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[
a]])/(4*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx &= -\frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx}{4a} \\
&= -\frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} + \frac{35\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{8a^2} \\
&= \frac{35}{12a^3x^{3/2}} - \frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} + \frac{(35b)\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{8a^3} \\
&= \frac{35}{12a^3x^{3/2}} + \frac{35b}{4a^4\sqrt{x}} - \frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} + \frac{\left(35b^2\right)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8a^4} \\
&= \frac{35}{12a^3x^{3/2}} + \frac{35b}{4a^4\sqrt{x}} - \frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} + \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{4a^4} \\
&= \frac{35}{12a^3x^{3/2}} + \frac{35b}{4a^4\sqrt{x}} - \frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} - \frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}
\end{aligned}
$$

Mathematica [C] time = 0.005165, size = 26, normalized size = 0.27

$$
\frac{2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\frac{bx}{a}\right)}{3a^3x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)^3),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 3, -1/2, (b*x)/a])/(3*a^3*x^(3/2))
```

Maple [A] time = 0.014, size = 69, normalized size = 0.7

$$
\frac{2}{3\,a^3}x^{-\frac{3}{2}} + 6\,\frac{b}{a^4\sqrt{x}} + 2\,\frac{b^2}{a^4}\left(\frac{1}{(bx-a)^2}\left(\frac{11\,bx^{3/2}}{8} - \frac{13\,a\sqrt{x}}{8}\right) - \frac{35}{8\sqrt{ab}}\text{Artanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a)^3,x)
```

```
[Out] 2/3/a^3/x^(3/2)+6*b/a^4/x^(1/2)+2/a^4*b^2*((11/8*b*x^(3/2)-13/8*a*x^(1/2))/
(b*x-a)^2-35/8/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.56587, size = 544, normalized size = 5.61

$$\left[\frac{105\left(b^3x^4 - 2\,ab^2x^3 + a^2bx^2\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx - 2\,a\sqrt{x}\sqrt{\frac{b}{a}} + a}{bx - a}\right) + 2\left(105\,b^3x^3 - 175\,ab^2x^2 + 56\,a^2bx + 8\,a^3\right)\sqrt{x}}{24\left(a^4b^2x^4 - 2\,a^5bx^3 + a^6x^2\right)}, \frac{105\left(b^3x^4 - 2\,ab^2 \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*x^4 - 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*log((b*x - 2*a*sqr
t(x)*sqrt(b/a) + a)/(b*x - a)) + 2*(105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*
x + 8*a^3)*sqrt(x))/(a^4*b^2*x^4 - 2*a^5*b*x^3 + a^6*x^2), 1/12*(105*(b^3*x
^4 - 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) +
 (105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*x + 8*a^3)*sqrt(x))/(a^4*b^2*x^4 -
 2*a^5*b*x^3 + a^6*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25547, size = 99, normalized size = 1.02

$$\frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}a^4} + \frac{2\,(9\,bx + a)}{3\,a^4x^{\frac{3}{2}}} + \frac{11\,b^3x^{\frac{3}{2}} - 13\,ab^2\sqrt{x}}{4\,(bx - a)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^4) + 2/3*(9*b*x + a)/(a
^4*x^(3/2)) + 1/4*(11*b^3*x^(3/2) - 13*a*b^2*sqrt(x))/((b*x - a)^2*a^4)
```

3.489 $\int x^{5/2}\sqrt{a+bx}\,dx$

Optimal. Leaf size=122

$$-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

```
[Out] (5*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b^3) - (5*a^2*x^(3/2)*Sqrt[a + b*x])/(96*
b^2) + (a*x^(5/2)*Sqrt[a + b*x])/(24*b) + (x^(7/2)*Sqrt[a + b*x])/4 - (5*a^
4*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(7/2))
```

Rubi [A] time = 0.0417613, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[a + b*x],x]
```

```
[Out] (5*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b^3) - (5*a^2*x^(3/2)*Sqrt[a + b*x])/(96*
b^2) + (a*x^(5/2)*Sqrt[a + b*x])/(24*b) + (x^(7/2)*Sqrt[a + b*x])/4 - (5*a^
4*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^{5/2}\sqrt{a+bx}\,dx = \frac{1}{4}x^{7/2}\sqrt{a+bx}+\frac{1}{8}a\int\frac{x^{5/2}}{\sqrt{a+bx}}\,dx$$
$$=\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{(5a^2)\int\frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{48b}$$
$$=-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}+\frac{(5a^3)\int\frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{64b^2}$$
$$=\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{(5a^4)\int\frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{128b^3}$$
$$=\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{(5a^4)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x\right.}{64b^3}$$
$$=\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{(5a^4)\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\right.}{64b^3}$$
$$=\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}$$

Mathematica [A] time = 0.180964, size = 96, normalized size = 0.79

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(-10a^2bx+15a^3+8ab^2x^2+48b^3x^3\right)-\frac{15a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{192b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^3 - 10*a^2*b*x + 8*a*b^2*x^2 + 48*b^3
*x^3) - (15*a^(7/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))
/(192*b^(7/2))
```

Maple [A] time = 0.006, size = 120, normalized size = 1.

$$\frac{1}{4b}x^{\frac{5}{2}}(bx+a)^{\frac{3}{2}}-\frac{5a}{24b^2}x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}+\frac{5a^2}{32b^3}(bx+a)^{\frac{3}{2}}\sqrt{x}-\frac{5a^3}{64b^3}\sqrt{x}\sqrt{bx+a}-\frac{5a^4}{128}\sqrt{x(bx+a)}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(1/2),x)
```

```
[Out] 1/4/b*x^(5/2)*(b*x+a)^(3/2)-5/24/b^2*a*x^(3/2)*(b*x+a)^(3/2)+5/32/b^3*a^2*x
^(1/2)*(b*x+a)^(3/2)-5/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b^3-5/128/b^(7/2)*a^4*(
x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7051, size = 417, normalized size = 3.42

$$\left[\frac{15\,a^4\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(48\,b^4x^3+8\,ab^3x^2-10\,a^2b^2x+15\,a^3b\right)\sqrt{bx+a}\sqrt{x}}{384\,b^4}, \frac{15\,a^4\sqrt{-b}\arctan\left(\frac{\sqrt{bx}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(48*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt(x)
)/b^4, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) +
(48*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt(x))
/b^4]
```

Sympy [A] time = 22.3046, size = 153, normalized size = 1.25

$$\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1+\frac{bx}{a}}}+\frac{7\sqrt{a}x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}}-\frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}}+\frac{bx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(1/2),x)
```

```
[Out] 5*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 + b*x/a)) + 5*a**(5/2)*x**(3/2)/(192*b**
2*sqrt(1 + b*x/a)) - a**(3/2)*x**(5/2)/(96*b*sqrt(1 + b*x/a)) + 7*sqrt(a)*x
**(7/2)/(24*sqrt(1 + b*x/a)) - 5*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(64*b*
*(7/2)) + b*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.490 $\int x^{3/2}\sqrt{a+bx}\,dx$

Optimal. Leaf size=98

$$-\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}$$

```
[Out] -(a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b^2) + (a*x^(3/2)*Sqrt[a + b*x])/(12*b) + (
x^(5/2)*Sqrt[a + b*x])/3 + (a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(
8*b^(5/2))
```

Rubi [A] time = 0.0307894, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[a + b*x],x]
```

```
[Out] -(a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b^2) + (a*x^(3/2)*Sqrt[a + b*x])/(12*b) + (
x^(5/2)*Sqrt[a + b*x])/3 + (a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(
8*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}\sqrt{a+bx}\,dx &= \frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{1}{6}a\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx\\
&= \frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}-\frac{a^2\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{8b}\\
&= -\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{16b^2}\\
&= -\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{8b^2}\\
&= -\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^2}\\
&= -\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.105517, size = 85, normalized size = 0.87

$$
\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(-3a^2+2abx+8b^2x^2\right)+\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{24b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(-3*a^2 + 2*a*b*x + 8*b^2*x^2) + (3*a^(5/2)
*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(24*b^(5/2))
```

Maple [A] time = 0.003, size = 102, normalized size = 1.

$$
\frac{1}{3\,b}x^{\frac{3}{2}}\left(bx+a\right)^{\frac{3}{2}}-\frac{a}{4\,b^2}\left(bx+a\right)^{\frac{3}{2}}\sqrt{x}+\frac{a^2}{8\,b^2}\sqrt{x}\sqrt{bx+a}+\frac{a^3}{16}\sqrt{x\left(bx+a\right)}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)b^{-\frac{5}{2}}\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{x}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(1/2),x)
```

```
[Out] 1/3/b*x^(3/2)*(b*x+a)^(3/2)-1/4/b^2*a*x^(1/2)*(b*x+a)^(3/2)+1/8*a^2*x^(1/2)
*(b*x+a)^(1/2)/b^2+1/16/b^(5/2)*a^3*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)
*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69255, size = 362, normalized size = 3.69

$$\left[\frac{3\,a^3\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(8\,b^3x^2+2\,ab^2x-3\,a^2b\right)\sqrt{bx+a}\sqrt{x}}{48\,b^3},-\frac{3\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-(8\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 2*a*b^2*x - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 2*a*b^2*x
 - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

Sympy [A] time = 7.25147, size = 122, normalized size = 1.24

$$-\frac{a^{\frac{5}{2}}\sqrt{x}}{8b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{1+\frac{bx}{a}}}+\frac{5\sqrt{a}x^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}}+\frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{bx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(1/2),x)
```

```
[Out] -a**(5/2)*sqrt(x)/(8*b**2*sqrt(1 + b*x/a)) - a**(3/2)*x**(3/2)/(24*b*sqrt(1
 + b*x/a)) + 5*sqrt(a)*x**(5/2)/(12*sqrt(1 + b*x/a)) + a**3*asinh(sqrt(b)*s
qrt(x)/sqrt(a))/(8*b**(5/2)) + b*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.491 $\int \sqrt{x}\sqrt{a+bx}\,dx$

Optimal. Leaf size=74

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a+bx}+\frac{a\sqrt{x}\sqrt{a+bx}}{4b}$$

```
[Out] (a*Sqrt[x]*Sqrt[a + b*x])/(4*b) + (x^(3/2)*Sqrt[a + b*x])/2 - (a^2*ArcTanh[
(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*b^(3/2))
```

Rubi [A] time = 0.0226927, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a+bx}+\frac{a\sqrt{x}\sqrt{a+bx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[a + b*x],x]
```

```
[Out] (a*Sqrt[x]*Sqrt[a + b*x])/(4*b) + (x^(3/2)*Sqrt[a + b*x])/2 - (a^2*ArcTanh[
(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{x}\sqrt{a+bx}\,dx = \frac{1}{2}x^{3/2}\sqrt{a+bx} + \frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx$$
$$= \frac{a\sqrt{x}\sqrt{a+bx}}{4b} + \frac{1}{2}x^{3/2}\sqrt{a+bx} - \frac{a^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{8b}$$
$$= \frac{a\sqrt{x}\sqrt{a+bx}}{4b} + \frac{1}{2}x^{3/2}\sqrt{a+bx} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{4b}$$
$$= \frac{a\sqrt{x}\sqrt{a+bx}}{4b} + \frac{1}{2}x^{3/2}\sqrt{a+bx} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{4b}$$
$$= \frac{a\sqrt{x}\sqrt{a+bx}}{4b} + \frac{1}{2}x^{3/2}\sqrt{a+bx} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}$$

Mathematica [A] time = 0.113867, size = 72, normalized size = 0.97

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}(a+2bx) - \frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(a + 2*b*x) - (a^(3/2)*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(4*b^(3/2))
```

Maple [A] time = 0.004, size = 81, normalized size = 1.1

$$\frac{1}{2}x^{\frac{3}{2}}\sqrt{bx+a} + \frac{a}{4b}\sqrt{x}\sqrt{bx+a} - \frac{a^2}{8}\sqrt{x(bx+a)}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}} + \sqrt{bx^2+ax}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x+a)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(b*x+a)^(1/2)+1/4*a*x^(1/2)*(b*x+a)^(1/2)/b-1/8*a^2/b^(3/2)*(x*
(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.71643, size = 304, normalized size = 4.11

$$\left[\frac{a^2\sqrt{b}\log\left(2\,bx - 2\,\sqrt{bx+a}\sqrt{b}\sqrt{x} + a\right) + 2\left(2\,b^2x + ab\right)\sqrt{bx+a}\sqrt{x}}{8\,b^2}, \frac{a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right) + \left(2\,b^2x + ab\right)\sqrt{bx+a}\sqrt{x}}{4\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(a^2*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2*b
^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/4*(a^2*sqrt(-b)*arctan(sqrt(b*x +
 a)*sqrt(-b)/(b*sqrt(x))) + (2*b^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [A] time = 3.74969, size = 97, normalized size = 1.31

$$\frac{a^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{1+\frac{bx}{a}}} + \frac{3\sqrt{a}x^{\frac{3}{2}}}{4\sqrt{1+\frac{bx}{a}}} - \frac{a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + \frac{bx^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x+a)**(1/2),x)
```

```
[Out] a**(3/2)*sqrt(x)/(4*b*sqrt(1 + b*x/a)) + 3*sqrt(a)*x**(3/2)/(4*sqrt(1 + b*x
/a)) - a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2)) + b*x**(5/2)/(2*sqr
t(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.492 $\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=44

$$\sqrt{x}\sqrt{a+bx}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

```
[Out] Sqrt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b
]
```

Rubi [A] time = 0.0171004, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\sqrt{x}\sqrt{a+bx}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b
]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{a+bx} + \frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx$$
$$= \sqrt{x}\sqrt{a+bx} + a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\sqrt{a+bx} + a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)$$
$$= \sqrt{x}\sqrt{a+bx} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0817941, size = 62, normalized size = 1.41

$$\frac{\frac{a^{3/2}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} + \sqrt{x}(a+bx)}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*(a + b*x) + (a^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/S
qrt[a]])/Sqrt[b])/Sqrt[a + b*x]
```

Maple [A] time = 0.004, size = 62, normalized size = 1.4

$$\sqrt{x}\sqrt{bx+a} + \frac{a}{2}\sqrt{x\,(bx+a)}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}} + \sqrt{bx^2+ax}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2
*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62505, size = 251, normalized size = 5.7

$$\left[\frac{a\sqrt{b}\log\left(2\,bx + 2\,\sqrt{bx+a}\sqrt{b}\sqrt{x} + a\right) + 2\,\sqrt{bx+a}b\sqrt{x}}{2\,b}, -\frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right) - \sqrt{bx+a}b\sqrt{x}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b, -(a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)
)) - sqrt(b*x + a)*b*sqrt(x))/b]
```

Sympy [A] time = 2.12503, size = 42, normalized size = 0.95

$$\sqrt{a}\sqrt{x}\sqrt{1+\frac{bx}{a}} + \frac{a\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x)*sqrt(1 + b*x/a) + a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.493 $\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx$

Optimal. Leaf size=45

$$2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}$$

```
[Out] (-2*Sqrt[a + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b
*x]]
```

Rubi [A] time = 0.0166056, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 63, 217, 206}

$$2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b
*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{a+bx}}{\sqrt{x}} + b\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx\\ &= -\frac{2\sqrt{a+bx}}{\sqrt{x}} + (2b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2\sqrt{a+bx}}{\sqrt{x}} + (2b)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\ &= -\frac{2\sqrt{a+bx}}{\sqrt{x}} + 2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\end{aligned}$$

Mathematica [A] time = 0.0980414, size = 64, normalized size = 1.42

$$\frac{2\left(\sqrt{a}\sqrt{b}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-\frac{a+bx}{\sqrt{x}}\right)}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(3/2),x]
```

```
[Out] (2*(-((a + b*x)/Sqrt[x]) + Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[
b]*Sqrt[x])/Sqrt[a]]))/Sqrt[a + b*x]
```

Maple [A] time = 0.017, size = 61, normalized size = 1.4

$$-2\,\frac{\sqrt{bx+a}}{\sqrt{x}}+\sqrt{b}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\sqrt{x\,(bx+a)}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(3/2),x)
```

```
[Out] -2*(b*x+a)^(1/2)/x^(1/2)+b^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*
(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70109, size = 243, normalized size = 5.4

$$\left[\frac{\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,\sqrt{bx+a}\sqrt{x}}{x},\,-\frac{2\left(\sqrt{-b}x\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sqrt(b*x +
 a)*sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) +
 sqrt(b*x + a)*sqrt(x))/x]
```

Sympy [A] time = 1.71108, size = 68, normalized size = 1.51

$$-\frac{2\sqrt{a}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}}+2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-\frac{2b\sqrt{x}}{\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(3/2),x)
```

```
[Out] -2*sqrt(a)/(sqrt(x)*sqrt(1 + b*x/a)) + 2*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqrt
(a)) - 2*b*sqrt(x)/(sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.494 $\int \frac{\sqrt{a+bx}}{x^{5/2}} dx$

Optimal. Leaf size=21

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

```
[Out] (-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

Rubi [A] time = 0.0019194, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx = -\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Mathematica [A] time = 0.0061355, size = 21, normalized size = 1.

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

Maple [A] time = 0.005, size = 16, normalized size = 0.8

$$-\frac{2}{3\,a}(bx+a)^{\frac{3}{2}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(5/2),x)
```

```
[Out] -2/3*(b*x+a)^(3/2)/a/x^(3/2)
```

Maxima [A] time = 1.04145, size = 20, normalized size = 0.95

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

Fricas [A] time = 1.49494, size = 46, normalized size = 2.19

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

Sympy [B] time = 2.20224, size = 41, normalized size = 1.95

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(5/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 2*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a)
```

Giac [B] time = 1.44095, size = 45, normalized size = 2.14

$$-\frac{2\,(bx+a)^{\frac{3}{2}}b^4}{3\,((bx+a)b-ab)^{\frac{3}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(b*x + a)^(3/2)*b^4/(((b*x + a)*b - a*b)^(3/2)*a*abs(b))
```

3.495 $\int \frac{\sqrt{a+bx}}{x^{7/2}} dx$

Optimal. Leaf size=44

$$\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}$$

```
[Out] (-2*(a + b*x)^(3/2))/(5*a*x^(5/2)) + (4*b*(a + b*x)^(3/2))/(15*a^2*x^(3/2))
```

Rubi [A] time = 0.0053334, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(7/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(5*a*x^(5/2)) + (4*b*(a + b*x)^(3/2))/(15*a^2*x^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^{7/2}} dx &= -\frac{2(a+bx)^{3/2}}{5ax^{5/2}}-\frac{(2b)\int \frac{\sqrt{a+bx}}{x^{5/2}} dx}{5a}\\ &= -\frac{2(a+bx)^{3/2}}{5ax^{5/2}}+\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0086478, size = 29, normalized size = 0.66

$$-\frac{2(3a-2bx)(a+bx)^{3/2}}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(7/2),x]
```

```
[Out] (-2*(3*a - 2*b*x)*(a + b*x)^(3/2))/(15*a^2*x^(5/2))
```

Maple [A] time = 0.003, size = 24, normalized size = 0.6

$$-\frac{-4\,bx+6\,a}{15\,a^{2}}\,(bx+a)^{\frac{3}{2}}\,x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(7/2),x)
```

```
[Out] -2/15*(b*x+a)^(3/2)*(-2*b*x+3*a)/x^(5/2)/a^2
```

Maxima [A] time = 1.09814, size = 42, normalized size = 0.95

$$\frac{2\left(\frac{5\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*(b*x + a)^(3/2)*b/x^(3/2) - 3*(b*x + a)^(5/2)/x^(5/2))/a^2
```

Fricas [A] time = 1.57641, size = 84, normalized size = 1.91

$$\frac{2\left(2\,b^{2}x^{2}-abx-3\,a^{2}\right)\sqrt{bx+a}}{15\,a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

Sympy [A] time = 20.8099, size = 65, normalized size = 1.48

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^{2}}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{15ax}+\frac{4b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{15a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(7/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**2) - 2*b**(3/2)*sqrt(a/(b*x) + 1)/(15*a*
x) + 4*b**(5/2)*sqrt(a/(b*x) + 1)/(15*a**2)
```

Giac [A] time = 1.30399, size = 68, normalized size = 1.55

$$-\frac{(bx+a)^{\frac{3}{2}}b\left(\frac{2(bx+a)}{a^3b^4}-\frac{5}{a^2b^4}\right)}{480\,((bx+a)b-ab)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] -1/480*(b*x + a)^(3/2)*b*(2*(b*x + a)/(a^3*b^4) - 5/(a^2*b^4))/(((b*x + a)*
b - a*b)^(5/2)*abs(b))
```

3.496 $\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}$$

```
[Out] (-2*(a + b*x)^(3/2))/(7*a*x^(7/2)) + (8*b*(a + b*x)^(3/2))/(35*a^2*x^(5/2))
 - (16*b^2*(a + b*x)^(3/2))/(105*a^3*x^(3/2))
```

Rubi [A] time = 0.0096716, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(9/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(7*a*x^(7/2)) + (8*b*(a + b*x)^(3/2))/(35*a^2*x^(5/2))
 - (16*b^2*(a + b*x)^(3/2))/(105*a^3*x^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{7ax^{7/2}}-\frac{(4b)\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx}{7a}\\ &= -\frac{2(a+bx)^{3/2}}{7ax^{7/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}+\frac{\left(8b^2\right)\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx}{35a^2}\\ &= -\frac{2(a+bx)^{3/2}}{7ax^{7/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}-\frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0107707, size = 40, normalized size = 0.59

$$-\frac{2(a+bx)^{3/2}\left(15a^2-12abx+8b^2x^2\right)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(9/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(15*a^2 - 12*a*b*x + 8*b^2*x^2))/(105*a^3*x^(7/2))
```

Maple [A] time = 0.005, size = 35, normalized size = 0.5

$$-\frac{16\,b^2x^2-24\,abx+30\,a^2}{105\,a^3}(bx+a)^{\frac{3}{2}}x^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(9/2),x)
```

```
[Out] -2/105*(b*x+a)^(3/2)*(8*b^2*x^2-12*a*b*x+15*a^2)/x^(7/2)/a^3
```

Maxima [A] time = 1.01434, size = 62, normalized size = 0.91

$$-\frac{2\left(\frac{35\,(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{42\,(bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(35*(b*x + a)^(3/2)*b^2/x^(3/2) - 42*(b*x + a)^(5/2)*b/x^(5/2) + 15*
(b*x + a)^(7/2)/x^(7/2))/a^3
```

Fricas [A] time = 1.55729, size = 112, normalized size = 1.65

$$-\frac{2\left(8\,b^3x^3-4\,ab^2x^2+3\,a^2bx+15\,a^3\right)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x^
(7/2))
```

Sympy [B] time = 120.955, size = 347, normalized size = 5.1

$$-\frac{30a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{66a^4b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{34a^3b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(9/2),x)
```

```
[Out] -30*a**5*b**(9/2)*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**
4 + 105*a**3*b**6*x**5) - 66*a**4*b**(11/2)*x*sqrt(a/(b*x) + 1)/(105*a**5*b
**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*a**3*b**(13/2)*x**
2*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**
6*x**5) - 6*a**2*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210
*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*a*b**(17/2)*x**4*sqrt(a/(b*x) +
1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 16*b**(
19/2)*x**5*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105
*a**3*b**6*x**5)
```

Giac [A] time = 1.4484, size = 89, normalized size = 1.31

$$\frac{(bx+a)^{\frac{3}{2}}\left(4\,(bx+a)\left(\frac{2\,(bx+a)}{a^4b^5}-\frac{7}{a^3b^5}\right)+\frac{35}{a^2b^5}\right)b}{40320\,((bx+a)b-ab)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 1/40320*(b*x + a)^(3/2)*(4*(b*x + a)*(2*(b*x + a)/(a^4*b^5) - 7/(a^3*b^5))
+ 35/(a^2*b^5))*b/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

3.497 $\int x^{5/2}\sqrt{a-bx}\,dx$

Optimal. Leaf size=127

$$-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}+\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

```
[Out] (-5*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b^3) - (5*a^2*x^(3/2)*Sqrt[a - b*x])/(96
*b^2) - (a*x^(5/2)*Sqrt[a - b*x])/(24*b) + (x^(7/2)*Sqrt[a - b*x])/4 + (5*a
^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(7/2))
```

Rubi [A] time = 0.0422133, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}+\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[a - b*x],x]
```

```
[Out] (-5*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b^3) - (5*a^2*x^(3/2)*Sqrt[a - b*x])/(96
*b^2) - (a*x^(5/2)*Sqrt[a - b*x])/(24*b) + (x^(7/2)*Sqrt[a - b*x])/4 + (5*a
^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^{5/2}\sqrt{a-bx}\,dx = \frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{1}{8}a\int\frac{x^{5/2}}{\sqrt{a-bx}}\,dx$$

$$= -\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{\left(5a^2\right)\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{48b}$$

$$= -\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{\left(5a^3\right)\int\frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{64b^2}$$

$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{\left(5a^4\right)\int\frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{128b^3}$$

$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{\left(5a^4\right)\text{Subst}\left(\int\frac{1}{\sqrt{a-bx^2}}\,dx,x,\right.}{64b^3}$$

$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{\left(5a^4\right)\text{Subst}\left(\int\frac{1}{1+bx^2}\,dx,x,\right.}{64b^3}$$

$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}$$

Mathematica [A] time = 0.136363, size = 98, normalized size = 0.77

$$\frac{\sqrt{a-bx}\left(\sqrt{b}\sqrt{x}\left(-10a^2bx-15a^3-8ab^2x^2+48b^3x^3\right)+\frac{15a^{7/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}\right)}{192b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^3 - 10*a^2*b*x - 8*a*b^2*x^2 + 48*b^
3*x^3) + (15*a^(7/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))
/(192*b^(7/2))
```

Maple [A] time = 0.007, size = 127, normalized size = 1.

$$-\frac{1}{4b}x^{\frac{5}{2}}(-bx+a)^{\frac{3}{2}}-\frac{5a}{24b^2}x^{\frac{3}{2}}(-bx+a)^{\frac{3}{2}}-\frac{5a^2}{32b^3}(-bx+a)^{\frac{3}{2}}\sqrt{x}+\frac{5a^3}{64b^3}\sqrt{x}\sqrt{-bx+a}+\frac{5a^4}{128}\sqrt{x(-bx+a)}\arctan\left(\sqrt{b}\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(1/2),x)
```

```
[Out] -1/4/b*x^(5/2)*(-b*x+a)^(3/2)-5/24/b^2*a*x^(3/2)*(-b*x+a)^(3/2)-5/32/b^3*a^
2*x^(1/2)*(-b*x+a)^(3/2)+5/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b^3+5/128/b^(7/2)*
a^4*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/(-
b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62109, size = 424, normalized size = 3.34

$$\left[-\frac{15\,a^4\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(48\,b^4x^3-8\,ab^3x^2-10\,a^2b^2x-15\,a^3b\right)\sqrt{-bx+a}\sqrt{x}}{384\,b^4},-\frac{15\,a^4\sqrt{b}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) - 2*(48*b^4*x^3 - 8*a*b^3*x^2 - 10*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a)*s
qrt(x))/b^4, -1/192*(15*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))
) - (48*b^4*x^3 - 8*a*b^3*x^2 - 10*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a)*sqr
t(x))/b^4]
```

Sympy [A] time = 21.3753, size = 325, normalized size = 2.56

$$\begin{cases} \frac{5ia^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{-1+\frac{bx}{a}}}-\frac{7i\sqrt{a}x^{\frac{7}{2}}}{24\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}}+\frac{ibx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ -\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{1-\frac{bx}{a}}}+\frac{5a^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1-\frac{bx}{a}}}+\frac{7\sqrt{a}x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}}+\frac{5a^4\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}}-\frac{bx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(7/2)*sqrt(x)/(64*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(5/2)*x
**(3/2)/(192*b**2*sqrt(-1 + b*x/a)) - I*a**(3/2)*x**(5/2)/(96*b*sqrt(-1 + b
*x/a)) - 7*I*sqrt(a)*x**(7/2)/(24*sqrt(-1 + b*x/a)) - 5*I*a**4*acosh(sqrt(b
)*sqrt(x)/sqrt(a))/(64*b**(7/2)) + I*b*x**(9/2)/(4*sqrt(a)*sqrt(-1 + b*x/a)
), Abs(b*x)/Abs(a) > 1), (-5*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 - b*x/a)) + 5
*a**(5/2)*x**(3/2)/(192*b**2*sqrt(1 - b*x/a)) + a**(3/2)*x**(5/2)/(96*b*sqr
t(1 - b*x/a)) + 7*sqrt(a)*x**(7/2)/(24*sqrt(1 - b*x/a)) + 5*a**4*asin(sqrt(
b)*sqrt(x)/sqrt(a))/(64*b**(7/2)) - b*x**(9/2)/(4*sqrt(a)*sqrt(1 - b*x/a)),
 True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.498 $\int x^{3/2}\sqrt{a-bx}\,dx$

Optimal. Leaf size=102

$$-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}$$

```
[Out] -(a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b^2) - (a*x^(3/2)*Sqrt[a - b*x])/(12*b) + (
x^(5/2)*Sqrt[a - b*x])/3 + (a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8
*b^(5/2))
```

Rubi [A] time = 0.0296454, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[a - b*x],x]
```

```
[Out] -(a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b^2) - (a*x^(3/2)*Sqrt[a - b*x])/(12*b) + (
x^(5/2)*Sqrt[a - b*x])/3 + (a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8
*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^{3/2}\sqrt{a-bx}\,dx=\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{1}{6}a\int\frac{x^{3/2}}{\sqrt{a-bx}}dx$$
$$=-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^2\int\frac{\sqrt{x}}{\sqrt{a-bx}}dx}{8b}$$
$$=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\int\frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{16b^2}$$
$$=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\,\text{Subst}\left(\int\frac{1}{\sqrt{a-bx^2}}dx,x,\sqrt{x}\right)}{8b^2}$$
$$=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\,\text{Subst}\left(\int\frac{1}{1+bx^2}dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^2}$$
$$=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}$$

Mathematica [A] time = 0.114324, size = 87, normalized size = 0.85

$$\frac{\sqrt{a-bx}\left(\sqrt{b}\sqrt{x}\left(-3a^2-2abx+8b^2x^2\right)+\frac{3a^{5/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}\right)}{24b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-3*a^2 - 2*a*b*x + 8*b^2*x^2) + (3*a^(5/2)
*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(24*b^(5/2))
```

Maple [A] time = 0.005, size = 108, normalized size = 1.1

$$-\frac{1}{3b}x^{\frac{3}{2}}(-bx+a)^{\frac{3}{2}}-\frac{a}{4b^2}(-bx+a)^{\frac{3}{2}}\sqrt{x}+\frac{a^2}{8b^2}\sqrt{x}\sqrt{-bx+a}+\frac{a^3}{16}\sqrt{x(-bx+a)}\arctan\left(\sqrt{b}\left(x-\frac{a}{2b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(1/2),x)
```

```
[Out] -1/3/b*x^(3/2)*(-b*x+a)^(3/2)-1/4/b^2*a*x^(1/2)*(-b*x+a)^(3/2)+1/8*a^2*x^(1
/2)*(-b*x+a)^(1/2)/b^2+1/16/b^(5/2)*a^3*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x
^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/(-b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60524, size = 367, normalized size = 3.6

$$\left[-\frac{3\,a^3\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(8\,b^3x^2-2\,ab^2x-3\,a^2b\right)\sqrt{-bx+a}\sqrt{x}}{48\,b^3},-\frac{3\,a^3\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
- 2*(8*b^3*x^2 - 2*a*b^2*x - 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/24*(3
*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (8*b^3*x^2 - 2*a*b^
2*x - 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^3]
```

Sympy [A] time = 7.27923, size = 262, normalized size = 2.57

$$\begin{cases} \frac{ia^{\frac{5}{2}}\sqrt{x}}{8b^2\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{-1+\frac{bx}{a}}}-\frac{5i\sqrt{a}x^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{ia^3\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{ibx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ -\frac{a^{\frac{5}{2}}\sqrt{x}}{8b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{1-\frac{bx}{a}}}+\frac{5\sqrt{a}x^{\frac{5}{2}}}{12\sqrt{1-\frac{bx}{a}}}+\frac{a^3\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{bx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((I*a**(5/2)*sqrt(x)/(8*b**2*sqrt(-1 + b*x/a)) - I*a**(3/2)*x**(3/
2)/(24*b*sqrt(-1 + b*x/a)) - 5*I*sqrt(a)*x**(5/2)/(12*sqrt(-1 + b*x/a)) - I
*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(5/2)) + I*b*x**(7/2)/(3*sqrt(a)
*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (-a**(5/2)*sqrt(x)/(8*b**2*sqrt(1
 - b*x/a)) + a**(3/2)*x**(3/2)/(24*b*sqrt(1 - b*x/a)) + 5*sqrt(a)*x**(5/2)/
(12*sqrt(1 - b*x/a)) + a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(5/2)) - b*
x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.499 $\int \sqrt{x}\sqrt{a-bx}\,dx$

Optimal. Leaf size=77

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a-bx}-\frac{a\sqrt{x}\sqrt{a-bx}}{4b}$$

```
[Out] -(a*Sqrt[x]*Sqrt[a - b*x])/(4*b) + (x^(3/2)*Sqrt[a - b*x])/2 + (a^2*ArcTan[
(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*b^(3/2))
```

Rubi [A] time = 0.0236496, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a-bx}-\frac{a\sqrt{x}\sqrt{a-bx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[a - b*x],x]
```

```
[Out] -(a*Sqrt[x]*Sqrt[a - b*x])/(4*b) + (x^(3/2)*Sqrt[a - b*x])/2 + (a^2*ArcTan[
(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\sqrt{a-bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{8b}\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{4b}\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{4b}\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0940121, size = 75, normalized size = 0.97

$$
\frac{\sqrt{a-bx}\left(\frac{a^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}+\sqrt{b}\sqrt{x}(2bx-a)\right)}{4b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-a + 2*b*x) + (a^(3/2)*ArcSin[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(4*b^(3/2))
```

Maple [A] time = 0.003, size = 86, normalized size = 1.1

$$
\frac{1}{2}x^{\frac{3}{2}}\sqrt{-bx+a}-\frac{a}{4b}\sqrt{x}\sqrt{-bx+a}+\frac{a^2}{8}\sqrt{x(-bx+a)}\arctan\left(\sqrt{b}\left(x-\frac{a}{2b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-b*x+a)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(-b*x+a)^(1/2)-1/4*a*x^(1/2)*(-b*x+a)^(1/2)/b+1/8*a^2/b^(3/2)*(
x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/(-b*x^2
+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6777, size = 311, normalized size = 4.04

$$\left[-\frac{a^2\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(2\,b^2x-ab\right)\sqrt{-bx+a}\sqrt{x}}{8\,b^2},-\frac{a^2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\left(2\,b^2x-ab\right)\sqrt{-b\right.}{4\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2
*(2*b^2*x - a*b)*sqrt(-b*x + a)*sqrt(x))/b^2, -1/4*(a^2*sqrt(b)*arctan(sqrt
(-b*x + a)/(sqrt(b)*sqrt(x))) - (2*b^2*x - a*b)*sqrt(-b*x + a)*sqrt(x))/b^2
]
```

Sympy [A] time = 3.73317, size = 209, normalized size = 2.71

$$\begin{cases} \frac{ia^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{-1+\frac{bx}{a}}}-\frac{3i\sqrt{a}x^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}}-\frac{ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+\frac{ibx^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ -\frac{a^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{1-\frac{bx}{a}}}+\frac{3\sqrt{a}x^{\frac{3}{2}}}{4\sqrt{1-\frac{bx}{a}}}+\frac{a^2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}-\frac{bx^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((I*a**(3/2)*sqrt(x)/(4*b*sqrt(-1 + b*x/a)) - 3*I*sqrt(a)*x**(3/2)
/(4*sqrt(-1 + b*x/a)) - I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2))
+ I*b*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (-a**(3/
2)*sqrt(x)/(4*b*sqrt(1 - b*x/a)) + 3*sqrt(a)*x**(3/2)/(4*sqrt(1 - b*x/a)) +
 a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2)) - b*x**(5/2)/(2*sqrt(a)*sq
rt(1 - b*x/a)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.500 $\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=46

$$\sqrt{x}\sqrt{a-bx}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

```
[Out] Sqrt[x]*Sqrt[a - b*x] + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]
```

Rubi [A] time = 0.0169224, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$\sqrt{x}\sqrt{a-bx}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[a - b*x] + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{a-bx} + \frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx$$
$$= \sqrt{x}\sqrt{a-bx} + a\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\sqrt{a-bx} + a\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)$$
$$= \sqrt{x}\sqrt{a-bx} + \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0902888, size = 65, normalized size = 1.41

$$\frac{\frac{a^{3/2}\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} + \sqrt{x}(a-bx)}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*(a - b*x) + (a^(3/2)*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sqrt[x])/Sq
rt[a]])/Sqrt[b])/Sqrt[a - b*x]
```

Maple [A] time = 0.004, size = 66, normalized size = 1.4

$$\sqrt{x}\sqrt{-bx+a} + \frac{a}{2}\sqrt{x(-bx+a)}\arctan\left(\sqrt{b}\left(x-\frac{a}{2b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1
/2)*arctan(b^(1/2)*(x-1/2/b*a)/(-b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58569, size = 257, normalized size = 5.59

$$\left[-\frac{a\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,\sqrt{-bx+a}b\sqrt{x}}{2\,b},-\frac{a\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\sqrt{-bx+a}b\sqrt{x}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2*s
qrt(-b*x + a)*b*sqrt(x))/b, -(a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt
(x))) - sqrt(-b*x + a)*b*sqrt(x))/b]
```

Sympy [A] time = 2.00933, size = 121, normalized size = 2.63

$$\begin{cases} -\frac{i\sqrt{a}\sqrt{x}}{\sqrt{-1+\frac{bx}{a}}}-\frac{ia\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{ibx^{\frac{3}{2}}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ \sqrt{a}\sqrt{x}\sqrt{1-\frac{bx}{a}}+\frac{a\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(a)*sqrt(x)/sqrt(-1 + b*x/a) - I*a*acosh(sqrt(b)*sqrt(x)/
sqrt(a))/sqrt(b) + I*b*x**(3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a)
 > 1), (sqrt(a)*sqrt(x)*sqrt(1 - b*x/a) + a*asin(sqrt(b)*sqrt(x)/sqrt(a))/s
qrt(b), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.501 $\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2\sqrt{a-bx}}{\sqrt{x}}-2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

```
[Out] (-2*Sqrt[a - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*
x]]
```

Rubi [A] time = 0.0163847, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {47, 63, 217, 203}

$$-\frac{2\sqrt{a-bx}}{\sqrt{x}}-2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(3/2),x]
```


```
[Out] (-2*Sqrt[a - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*
x]]
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx = -\frac{2\sqrt{a-bx}}{\sqrt{x}} - b\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx$$
$$= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - (2b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - (2b)\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)$$
$$= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - 2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

Mathematica [A] time = 0.0620808, size = 69, normalized size = 1.47

$$-\frac{2\left(\sqrt{a}\sqrt{b}\sqrt{x}\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+a-bx\right)}{\sqrt{x}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(3/2),x]
```

```
[Out] (-2*(a - b*x + Sqrt[a]*Sqrt[b]*Sqrt[x]*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sq
rt[x])/Sqrt[a]]))/(Sqrt[x]*Sqrt[a - b*x])
```

Maple [A] time = 0.02, size = 66, normalized size = 1.4

$$-2\,\frac{\sqrt{-bx+a}}{\sqrt{x}} - \sqrt{b}\arctan\left(\sqrt{b}\left(x-\frac{a}{2\,b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)\sqrt{x\,(-bx+a)}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(3/2),x)
```

```
[Out] -2*(-b*x+a)^(1/2)/x^(1/2)-b^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/(-b*x^2+a*x)^(
1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66598, size = 246, normalized size = 5.23

$$\left[\frac{\sqrt{-b}x\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,\sqrt{-bx+a}\sqrt{x}}{x},\frac{2\left(\sqrt{b}x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\sqrt{-bx+a}\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2*sqrt(-
b*x + a)*sqrt(x))/x, 2*(sqrt(b)*x*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))
- sqrt(-b*x + a)*sqrt(x))/x]
```

Sympy [A] time = 1.82019, size = 150, normalized size = 3.19

$$\begin{cases} \frac{2i\sqrt{a}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}}+2i\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-\frac{2ib\sqrt{x}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ -\frac{2\sqrt{a}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}-2\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\frac{2b\sqrt{x}}{\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*sqrt(a)/(sqrt(x)*sqrt(-1 + b*x/a)) + 2*I*sqrt(b)*acosh(sqrt(
b)*sqrt(x)/sqrt(a)) - 2*I*b*sqrt(x)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x)/Ab
s(a) > 1), (-2*sqrt(a)/(sqrt(x)*sqrt(1 - b*x/a)) - 2*sqrt(b)*asin(sqrt(b)*s
qrt(x)/sqrt(a)) + 2*b*sqrt(x)/(sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.502 $\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

```
[Out] (-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

Rubi [A] time = 0.0017392, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(5/2),x]

[Out] (-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx = -\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Mathematica [A] time = 0.0063745, size = 22, normalized size = 1.

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(5/2),x]

[Out] (-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

Maple [A] time = 0.003, size = 17, normalized size = 0.8

$$-\frac{2}{3\,a}\left(-bx+a\right)^{\frac{3}{2}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(5/2),x)
```

```
[Out] -2/3*(-b*x+a)^(3/2)/a/x^(3/2)
```

Maxima [A] time = 1.02236, size = 22, normalized size = 1.

$$-\frac{2\,(-bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(-b*x + a)^(3/2)/(a*x^(3/2))
```

Fricas [A] time = 1.59402, size = 57, normalized size = 2.59

$$\frac{2\,(bx-a)\sqrt{-bx+a}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x - a)*sqrt(-b*x + a)/(a*x^(3/2))
```

Sympy [B] time = 2.13464, size = 92, normalized size = 4.18

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3x} + \frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3a} & \text{for}\ \frac{|a|}{|b||x|} > 1 \\ -\frac{2i\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{3x} + \frac{2ib^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(5/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 2*b**(3/2)*sqrt(a/(b*x) - 1
)/(3*a), Abs(a)/(Abs(b)*Abs(x)) > 1), (-2*I*sqrt(b)*sqrt(-a/(b*x) + 1)/(3*x
) + 2*I*b**(3/2)*sqrt(-a/(b*x) + 1)/(3*a), True))
```

Giac [B] time = 1.24095, size = 57, normalized size = 2.59

$$\frac{2\,(bx-a)\sqrt{-bx+ab^4}}{3\,((bx-a)b+ab)^{\frac{3}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*x - a)*sqrt(-b*x + a)*b^4/(((b*x - a)*b + a*b)^(3/2)*a*abs(b))
```

3.503 $\int \frac{\sqrt{a-bx}}{x^{7/2}} dx$

Optimal. Leaf size=46

$$-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}$$

```
[Out] (-2*(a - b*x)^(3/2))/(5*a*x^(5/2)) - (4*b*(a - b*x)^(3/2))/(15*a^2*x^(3/2))
```

Rubi [A] time = 0.0051817, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(7/2),x]

[Out] (-2*(a - b*x)^(3/2))/(5*a*x^(5/2)) - (4*b*(a - b*x)^(3/2))/(15*a^2*x^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-bx}}{x^{7/2}} dx &= -\frac{2(a-bx)^{3/2}}{5ax^{5/2}}+\frac{(2b)\int \frac{\sqrt{a-bx}}{x^{5/2}} dx}{5a}\\ &= -\frac{2(a-bx)^{3/2}}{5ax^{5/2}}-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0086985, size = 30, normalized size = 0.65

$$-\frac{2(a-bx)^{3/2}(3a+2bx)}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(7/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2)*(3*a + 2*b*x))/(15*a^2*x^(5/2))
```

Maple [A] time = 0.003, size = 25, normalized size = 0.5

$$-\frac{4\,bx+6\,a}{15\,a^2}\left(-bx+a\right)^{\frac{3}{2}}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(7/2),x)
```

```
[Out] -2/15*(-b*x+a)^(3/2)*(2*b*x+3*a)/x^(5/2)/a^2
```

Maxima [A] time = 1.05495, size = 45, normalized size = 0.98

$$-\frac{2\left(\frac{5\,(-bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{3\,(-bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] -2/15*(5*(-b*x + a)^(3/2)*b/x^(3/2) + 3*(-b*x + a)^(5/2)/x^(5/2))/a^2
```

Fricas [A] time = 1.63756, size = 85, normalized size = 1.85

$$\frac{2\left(2\,b^2x^2+abx-3\,a^2\right)\sqrt{-bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b^2*x^2 + a*b*x - 3*a^2)*sqrt(-b*x + a)/(a^2*x^(5/2))
```

Sympy [A] time = 20.0139, size = 245, normalized size = 5.33

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{5x^2}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{15ax}+\frac{4b^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}{15a^2} & \text{for}\ \frac{|a|}{|b||x|}>1 \\ \frac{6ia^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{x\left(-15a^3bx+15a^2b^2x^2\right)}-\frac{8ia^2b^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2}-\frac{2iab^{\frac{7}{2}}x\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2}+\frac{4ib^{\frac{9}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(7/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(5*x**2) + 2*b**(3/2)*sqrt(a/(b*x)
- 1)/(15*a*x) + 4*b**(5/2)*sqrt(a/(b*x) - 1)/(15*a**2), Abs(a)/(Abs(b)*Abs(
x)) > 1), (6*I*a**3*b**(3/2)*sqrt(-a/(b*x) + 1)/(x*(-15*a**3*b*x + 15*a**2*
b**2*x**2)) - 8*I*a**2*b**(5/2)*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*
b**2*x**2) - 2*I*a*b**(7/2)*x*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b*
*2*x**2) + 4*I*b**(9/2)*x**2*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**
2*x**2), True))
```

Giac [A] time = 1.23375, size = 82, normalized size = 1.78

$$-\frac{(bx-a)\sqrt{-bx+a}b\left(\frac{2(bx-a)}{a^3b^4}+\frac{5}{a^2b^4}\right)}{480\left((bx-a)b+ab\right)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] -1/480*(b*x - a)*sqrt(-b*x + a)*b*(2*(b*x - a)/(a^3*b^4) + 5/(a^2*b^4))/(((
b*x - a)*b + a*b)^(5/2)*abs(b))
```

3.504 $\int \frac{\sqrt{a-bx}}{x^{9/2}} dx$

Optimal. Leaf size=71

$$-\frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a-bx)^{3/2}}{7ax^{7/2}}$$

```
[Out] (-2*(a - b*x)^(3/2))/(7*a*x^(7/2)) - (8*b*(a - b*x)^(3/2))/(35*a^2*x^(5/2))
 - (16*b^2*(a - b*x)^(3/2))/(105*a^3*x^(3/2))
```

Rubi [A] time = 0.0104142, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a-bx)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(9/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(7*a*x^(7/2)) - (8*b*(a - b*x)^(3/2))/(35*a^2*x^(5/2))
 - (16*b^2*(a - b*x)^(3/2))/(105*a^3*x^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a-bx}}{x^{9/2}} dx &= -\frac{2(a-bx)^{3/2}}{7ax^{7/2}}+\frac{(4b)\int \frac{\sqrt{a-bx}}{x^{7/2}} dx}{7a}\\
&= -\frac{2(a-bx)^{3/2}}{7ax^{7/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}+\frac{\left(8b^2\right)\int \frac{\sqrt{a-bx}}{x^{5/2}} dx}{35a^2}\\
&= -\frac{2(a-bx)^{3/2}}{7ax^{7/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}-\frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0113349, size = 41, normalized size = 0.58

$$-\frac{2(a-bx)^{3/2}\left(15a^2+12abx+8b^2x^2\right)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(9/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2)*(15*a^2 + 12*a*b*x + 8*b^2*x^2))/(105*a^3*x^(7/2))
```

Maple [A] time = 0.003, size = 36, normalized size = 0.5

$$-\frac{16\,b^2x^2+24\,abx+30\,a^2}{105\,a^3}(-bx+a)^{\frac{3}{2}}x^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(9/2),x)
```

```
[Out] -2/105*(-b*x+a)^(3/2)*(8*b^2*x^2+12*a*b*x+15*a^2)/x^(7/2)/a^3
```

Maxima [A] time = 1.03152, size = 66, normalized size = 0.93

$$-\frac{2\left(\frac{35(-bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}+\frac{42(-bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15(-bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(35*(-b*x + a)^(3/2)*b^2/x^(3/2) + 42*(-b*x + a)^(5/2)*b/x^(5/2) + 1
5*(-b*x + a)^(7/2)/x^(7/2))/a^3
```

Fricas [A] time = 1.51523, size = 112, normalized size = 1.58

$$\frac{2\left(8\,b^3x^3+4\,ab^2x^2+3\,a^2bx-15\,a^3\right)\sqrt{-bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(8*b^3*x^3 + 4*a*b^2*x^2 + 3*a^2*b*x - 15*a^3)*sqrt(-b*x + a)/(a^3*x^
(7/2))
```

Sympy [B] time = 114.542, size = 711, normalized size = 10.01

$$\begin{cases} -\frac{30a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5}+\frac{66a^4b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5}-\frac{34a^3b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5}+\frac{6a^2b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}-1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5}- \\ -\frac{30ia^5b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5}+\frac{66ia^4b^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5}-\frac{34ia^3b^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5}+\frac{6ia^2b^{\frac{15}{2}}x^3\sqrt{-\frac{a}{bx}+1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5}- \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(9/2),x)
```

```
[Out] Piecewise((-30*a**5*b**(9/2)*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a*
*4*b**5*x**4 + 105*a**3*b**6*x**5) + 66*a**4*b**(11/2)*x*sqrt(a/(b*x) - 1)/
(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*a**3*b*
*(13/2)*x**2*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 1
05*a**3*b**6*x**5) + 6*a**2*b**(15/2)*x**3*sqrt(a/(b*x) - 1)/(105*a**5*b**4
*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*a*b**(17/2)*x**4*sqrt
(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5
) + 16*b**(19/2)*x**5*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5
*x**4 + 105*a**3*b**6*x**5), Abs(a)/(Abs(b)*Abs(x)) > 1), (-30*I*a**5*b**(9
/2)*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*
b**6*x**5) + 66*I*a**4*b**(11/2)*x*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 -
 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*I*a**3*b**(13/2)*x**2*sqrt(-
a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5)
 + 6*I*a**2*b**(15/2)*x**3*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**
4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*I*a*b**(17/2)*x**4*sqrt(-a/(b*x) + 1
)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 16*I*b**
(19/2)*x**5*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 1
05*a**3*b**6*x**5), True))
```

Giac [A] time = 1.22038, size = 107, normalized size = 1.51

$$\frac{(bx-a)\sqrt{-bx+a}\left(4\,(bx-a)\left(\frac{2\,(bx-a)}{a^4b^5}+\frac{7}{a^3b^5}\right)+\frac{35}{a^2b^5}\right)b}{40320\,((bx-a)b+ab)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 1/40320*(b*x - a)*sqrt(-b*x + a)*(4*(b*x - a)*(2*(b*x - a)/(a^4*b^5) + 7/(a
^3*b^5)) + 35/(a^2*b^5))*b/(((b*x - a)*b + a*b)^(7/2)*abs(b))
```

3.505 $\int x^{5/2}\sqrt{2+bx}\,dx$

Optimal. Leaf size=108

$$-\frac{5x^{3/2}\sqrt{bx+2}}{24b^2}+\frac{5\sqrt{x}\sqrt{bx+2}}{8b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{4}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{12b}$$

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(8*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(24*b^2) + (x
^(5/2)*Sqrt[2 + b*x])/(12*b) + (x^(7/2)*Sqrt[2 + b*x])/4 - (5*ArcSinh[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rubi [A] time = 0.0315586, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$-\frac{5x^{3/2}\sqrt{bx+2}}{24b^2}+\frac{5\sqrt{x}\sqrt{bx+2}}{8b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{4}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[2 + b*x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(8*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(24*b^2) + (x
^(5/2)*Sqrt[2 + b*x])/(12*b) + (x^(7/2)*Sqrt[2 + b*x])/4 - (5*ArcSinh[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{5/2}\sqrt{2+bx}\,dx = \frac{1}{4}x^{7/2}\sqrt{2+bx}+\frac{1}{4}\int \frac{x^{5/2}}{\sqrt{2+bx}}dx$$
$$= \frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}dx}{12b}$$
$$= -\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}+\frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}dx}{8b^2}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\int \frac{1}{\sqrt{x}\sqrt{2+bx}}dx}{8b^3}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}dx,x,\sqrt{x}\right)}{4b^3}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

Mathematica [A] time = 0.0520828, size = 70, normalized size = 0.65

$$\frac{\sqrt{x}\sqrt{bx+2}\left(6b^3x^3+2b^2x^2-5bx+15\right)}{24b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 6*b^3*x^3))/(24*b^3) - (5*
ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Maple [A] time = 0.007, size = 108, normalized size = 1.

$$\frac{1}{4b}x^{\frac{5}{2}}(bx+2)^{\frac{3}{2}}-\frac{5}{12b^2}x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}+\frac{5}{8b^3}(bx+2)^{\frac{3}{2}}\sqrt{x}-\frac{5}{8b^3}\sqrt{x}\sqrt{bx+2}-\frac{5}{8}\sqrt{x(bx+2)}\ln\left((bx+1)\frac{1}{\sqrt{b}}+\sqrt{bx^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+2)^(1/2),x)
```

```
[Out] 1/4/b*x^(5/2)*(b*x+2)^(3/2)-5/12/b^2*x^(3/2)*(b*x+2)^(3/2)+5/8/b^3*x^(1/2)*
(b*x+2)^(3/2)-5/8*x^(1/2)*(b*x+2)^(1/2)/b^3-5/8/b^(7/2)*(x*(b*x+2))^(1/2)/(
b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69992, size = 363, normalized size = 3.36

$$\left[\frac{\left(6\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\sqrt{x}+15\,\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{24\,b^4},\ \frac{\left(6\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+\right.}{24\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 15*
sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/24*((6*b^4*x^3
 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(s
qrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^4]
```

Sympy [A] time = 18.0572, size = 117, normalized size = 1.08

$$\frac{bx^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{7x^{\frac{7}{2}}}{12\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{24b\sqrt{bx+2}}+\frac{5x^{\frac{3}{2}}}{24b^2\sqrt{bx+2}}+\frac{5\sqrt{x}}{4b^3\sqrt{bx+2}}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(9/2)/(4*sqrt(b*x + 2)) + 7*x**(7/2)/(12*sqrt(b*x + 2)) - x**(5/2)/(24
*b*sqrt(b*x + 2)) + 5*x**(3/2)/(24*b**2*sqrt(b*x + 2)) + 5*sqrt(x)/(4*b**3*
sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(7/2))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.506 $\int x^{3/2}\sqrt{2+bx}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{x}\sqrt{bx+2}}{2b^2}+\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{1}{3}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{6b}$$

```
[Out] -(Sqrt[x]*Sqrt[2 + b*x])/(2*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(6*b) + (x^(5/2)
*Sqrt[2 + b*x])/3 + ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(5/2)
```

Rubi [A] time = 0.0200607, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$-\frac{\sqrt{x}\sqrt{bx+2}}{2b^2}+\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{1}{3}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[2 + b*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 + b*x])/(2*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(6*b) + (x^(5/2)
*Sqrt[2 + b*x])/3 + ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{3/2}\sqrt{2+bx}\,dx=\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{1}{3}\int\frac{x^{3/2}}{\sqrt{2+bx}}dx$$
$$=\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}-\frac{\int\frac{\sqrt{x}}{\sqrt{2+bx}}dx}{2b}$$
$$=-\frac{\sqrt{x}\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{\int\frac{1}{\sqrt{x}\sqrt{2+bx}}dx}{2b^2}$$
$$=-\frac{\sqrt{x}\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{\text{Subst}\left(\int\frac{1}{\sqrt{2+bx^2}}dx,x,\sqrt{x}\right)}{b^2}$$
$$=-\frac{\sqrt{x}\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.0428565, size = 58, normalized size = 0.69

$$\frac{\sqrt{x}\sqrt{bx+2}\left(2b^2x^2+bx-3\right)}{6b^2}+\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(-3 + b*x + 2*b^2*x^2))/(6*b^2) + ArcSinh[(Sqrt[b]*S
qrt[x])/Sqrt[2]]/b^(5/2)
```

Maple [A] time = 0.003, size = 93, normalized size = 1.1

$$\frac{1}{3b}x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}-\frac{1}{2b^2}(bx+2)^{\frac{3}{2}}\sqrt{x}+\frac{1}{2b^2}\sqrt{x}\sqrt{bx+2}+\frac{1}{2}\sqrt{x(bx+2)}\ln\left((bx+1)\frac{1}{\sqrt{b}}+\sqrt{bx^2+2x}\right)b^{-\frac{5}{2}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(1/2),x)
```

```
[Out] 1/3/b*x^(3/2)*(b*x+2)^(3/2)-1/2/b^2*x^(1/2)*(b*x+2)^(3/2)+1/2*x^(1/2)*(b*x+
2)^(1/2)/b^2+1/2/b^(5/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)
/b^(1/2)+(b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59764, size = 317, normalized size = 3.77

$$\left[\frac{\left(2\,b^3x^2+b^2x-3\,b\right)\sqrt{bx+2}\sqrt{x}+3\,\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{6\,b^3},\ \frac{\left(2\,b^3x^2+b^2x-3\,b\right)\sqrt{bx+2}\sqrt{x}-6\,\sqrt{-b}\,\mathrm{arcta}}{6\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x +
 sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/6*((2*b^3*x^2 + b^2*x - 3*b)*sq
rt(b*x + 2)*sqrt(x) - 6*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x)))
)/b^3]
```

Sympy [A] time = 6.06878, size = 90, normalized size = 1.07

$$\frac{bx^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{5x^{\frac{5}{2}}}{6\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{6b\sqrt{bx+2}}-\frac{\sqrt{x}}{b^2\sqrt{bx+2}}+\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(7/2)/(3*sqrt(b*x + 2)) + 5*x**(5/2)/(6*sqrt(b*x + 2)) - x**(3/2)/(6*b
*sqrt(b*x + 2)) - sqrt(x)/(b**2*sqrt(b*x + 2)) + asinh(sqrt(2)*sqrt(b)*sqrt
(x)/2)/b**(5/2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.507 $\int \sqrt{x}\sqrt{2+bx}\,dx$

Optimal. Leaf size=64

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\sqrt{bx+2}}{2b}$$

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/(2*b) + (x^(3/2)*Sqrt[2 + b*x])/2 - ArcSinh[(Sqrt[b
]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rubi [A] time = 0.014512, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/(2*b) + (x^(3/2)*Sqrt[2 + b*x])/2 - ArcSinh[(Sqrt[b
]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{x}\sqrt{2+bx}\,dx = \frac{1}{2}x^{3/2}\sqrt{2+bx}+\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx$$
$$= \frac{\sqrt{x}\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}-\frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b}$$
$$= \frac{\sqrt{x}\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b}$$
$$= \frac{\sqrt{x}\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.0315072, size = 51, normalized size = 0.8

$$\frac{\sqrt{x}(bx+1)\sqrt{bx+2}}{2b}-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*(1 + b*x)*Sqrt[2 + b*x])/(2*b) - ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]
]/b^(3/2)
```

Maple [A] time = 0.003, size = 75, normalized size = 1.2

$$\frac{1}{2}x^{\frac{3}{2}}\sqrt{bx+2}+\frac{1}{2\,b}\sqrt{x}\sqrt{bx+2}-\frac{1}{2}\sqrt{x\,(bx+2)}\ln\left((bx+1)\frac{1}{\sqrt{b}}+\sqrt{bx^2+2\,x}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x+2)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(b*x+2)^(1/2)+1/2*x^(1/2)*(b*x+2)^(1/2)/b-1/2/b^(3/2)*(x*(b*x+2
))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71831, size = 277, normalized size = 4.33

$$\left[\frac{\left(b^{2}x+b\right)\sqrt{bx+2}\sqrt{x}+\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{2\,b^{2}},\ \frac{\left(b^{2}x+b\right)\sqrt{bx+2}\sqrt{x}+2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)}{2\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x + b)*sqrt(b*x + 2)*sqrt(x) + sqrt(b)*log(b*x - sqrt(b*x + 2)*s
qrt(b)*sqrt(x) + 1))/b^2, 1/2*((b^2*x + b)*sqrt(b*x + 2)*sqrt(x) + 2*sqrt(-
b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^2]
```

Sympy [A] time = 3.14949, size = 71, normalized size = 1.11

$$\frac{bx^{\frac{5}{2}}}{2\sqrt{bx+2}}+\frac{3x^{\frac{3}{2}}}{2\sqrt{bx+2}}+\frac{\sqrt{x}}{b\sqrt{bx+2}}-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(5/2)/(2*sqrt(b*x + 2)) + 3*x**(3/2)/(2*sqrt(b*x + 2)) + sqrt(x)/(b*sq
rt(b*x + 2)) - asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.508 $\int \frac{\sqrt{2+bx}}{\sqrt{x}} \, dx$

Optimal. Leaf size=40

$$\sqrt{x}\sqrt{bx+2}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] Sqrt[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rubi [A] time = 0.007761, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$\sqrt{x}\sqrt{bx+2}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2+bx}}{\sqrt{x}} \, dx &= \sqrt{x}\sqrt{2+bx}+\int \frac{1}{\sqrt{x}\sqrt{2+bx}} \, dx \\
&= \sqrt{x}\sqrt{2+bx}+2\operatorname{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}} \, dx, x, \sqrt{x}\right) \\
&= \sqrt{x}\sqrt{2+bx}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.0130447, size = 40, normalized size = 1.

$$\sqrt{x}\sqrt{bx+2}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Maple [A] time = 0.004, size = 58, normalized size = 1.5

$$\sqrt{x}\sqrt{bx+2}+\sqrt{x(bx+2)}\ln\left((bx+1)\frac{1}{\sqrt{b}}+\sqrt{bx^2+2x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(b*x+2)^(1/2)+(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^
(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73182, size = 234, normalized size = 5.85

$$\left[\frac{\sqrt{bx+2}b\sqrt{x}+\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{b},\frac{\sqrt{bx+2}b\sqrt{x}-2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b*x + 2)*b*sqrt(x) + sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x)
 + 1))/b, (sqrt(b*x + 2)*b*sqrt(x) - 2*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-
b)/(b*sqrt(x))))/b]
```

Sympy [A] time = 1.78603, size = 37, normalized size = 0.92

$$\sqrt{x}\sqrt{bx+2}+\frac{2\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(1/2),x)
```

```
[Out] sqrt(x)*sqrt(b*x + 2) + 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.509 $\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx$

Optimal. Leaf size=41

$$2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

```
[Out] (-2*Sqrt[2 + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rubi [A] time = 0.009401, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {47, 54, 215}

$$2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{2+bx}}{\sqrt{x}}+b\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx\\ &= -\frac{2\sqrt{2+bx}}{\sqrt{x}}+(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= -\frac{2\sqrt{2+bx}}{\sqrt{x}}+2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0138621, size = 41, normalized size = 1.

$$2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Maple [A] time = 0.019, size = 59, normalized size = 1.4

$$-2\frac{\sqrt{bx+2}}{\sqrt{x}}+\sqrt{b}\ln\left((bx+1)\frac{1}{\sqrt{b}}+\sqrt{bx^2+2x}\right)\sqrt{x(bx+2)}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(3/2),x)
```

```
[Out] -2*(b*x+2)^(1/2)/x^(1/2)+b^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))*(x*(
b*x+2))^(1/2)/x^(1/2)/(b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50655, size = 238, normalized size = 5.8

$$\left[\frac{\sqrt{b}x\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)-2\sqrt{bx+2}\sqrt{x}}{x},-\frac{2\left(\sqrt{-b}x\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+2}\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 2*sqrt(b*x + 2)*
sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) + sqr
t(b*x + 2)*sqrt(x))/x]
```

Sympy [A] time = 1.58143, size = 48, normalized size = 1.17

$$-2\sqrt{b}\sqrt{1+\frac{2}{bx}}-\sqrt{b}\log\left(\frac{1}{bx}\right)+2\sqrt{b}\log\left(\sqrt{1+\frac{2}{bx}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(3/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(1 + 2/(b*x)) - sqrt(b)*log(1/(b*x)) + 2*sqrt(b)*log(sqrt(1
+ 2/(b*x)) + 1)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.510 $\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

```
[Out] -(2 + b*x)^(3/2)/(3*x^(3/2))
```

Rubi [A] time = 0.0014173, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(5/2),x]
```

```
[Out] -(2 + b*x)^(3/2)/(3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx = -\frac{(2+bx)^{3/2}}{3x^{3/2}}$$

Mathematica [A] time = 0.0132774, size = 18, normalized size = 1.

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(5/2),x]
```

```
[Out] -(2 + b*x)^(3/2)/(3*x^(3/2))
```

Maple [A] time = 0.002, size = 13, normalized size = 0.7

$$-\frac{1}{3}(bx+2)^{\frac{3}{2}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(5/2),x)
```

```
[Out] -1/3*(b*x+2)^(3/2)/x^(3/2)
```

Maxima [A] time = 1.03068, size = 16, normalized size = 0.89

$$-\frac{(bx+2)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A] time = 1.50226, size = 41, normalized size = 2.28

$$-\frac{(bx+2)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(b*x + 2)^(3/2)/x^(3/2)
```

Sympy [B] time = 2.11646, size = 37, normalized size = 2.06

$$-\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-\frac{2\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(5/2),x)
```

```
[Out] -b**(3/2)*sqrt(1 + 2/(b*x))/3 - 2*sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

Giac [B] time = 1.23251, size = 39, normalized size = 2.17

$$-\frac{(bx+2)^{\frac{3}{2}}b^4}{3\left((bx+2)b-2b\right)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(b*x + 2)^(3/2)*b^4/(((b*x + 2)*b - 2*b)^(3/2)*abs(b))
```

3.511 $\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx$

Optimal. Leaf size=38

$$\frac{b(bx+2)^{3/2}}{15x^{3/2}} - \frac{(bx+2)^{3/2}}{5x^{5/2}}$$

```
[Out] -(2 + b*x)^(3/2)/(5*x^(5/2)) + (b*(2 + b*x)^(3/2))/(15*x^(3/2))
```

Rubi [A] time = 0.0039924, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{b(bx+2)^{3/2}}{15x^{3/2}} - \frac{(bx+2)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(7/2),x]
```

```
[Out] -(2 + b*x)^(3/2)/(5*x^(5/2)) + (b*(2 + b*x)^(3/2))/(15*x^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx &= -\frac{(2+bx)^{3/2}}{5x^{5/2}} - \frac{1}{5}b\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx\\ &= -\frac{(2+bx)^{3/2}}{5x^{5/2}} + \frac{b(2+bx)^{3/2}}{15x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0074926, size = 23, normalized size = 0.61

$$\frac{(bx-3)(bx+2)^{3/2}}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(7/2),x]
```

```
[Out] ((-3 + b*x)*(2 + b*x)^(3/2))/(15*x^(5/2))
```

Maple [A] time = 0.004, size = 18, normalized size = 0.5

$$\frac{bx-3}{15}(bx+2)^{\frac{3}{2}}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(7/2),x)
```

```
[Out] 1/15*(b*x+2)^(3/2)*(b*x-3)/x^(5/2)
```

Maxima [A] time = 1.05509, size = 35, normalized size = 0.92

$$\frac{(bx+2)^{\frac{3}{2}}b}{6x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{10x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x + 2)^(3/2)*b/x^(3/2) - 1/10*(b*x + 2)^(5/2)/x^(5/2)
```

Fricas [A] time = 1.58403, size = 65, normalized size = 1.71

$$\frac{\left(b^2x^2-bx-6\right)\sqrt{bx+2}}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(b^2*x^2 - b*x - 6)*sqrt(b*x + 2)/x^(5/2)
```

Sympy [A] time = 17.2822, size = 56, normalized size = 1.47

$$\frac{b^{\frac{5}{2}}\sqrt{1+\frac{2}{bx}}}{15}-\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{15x}-\frac{2\sqrt{b}\sqrt{1+\frac{2}{bx}}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(7/2),x)
```

```
[Out] b**(5/2)*sqrt(1 + 2/(b*x))/15 - b**(3/2)*sqrt(1 + 2/(b*x))/(15*x) - 2*sqrt(
b)*sqrt(1 + 2/(b*x))/(5*x**2)
```

Giac [A] time = 1.21857, size = 57, normalized size = 1.5

$$\frac{\left((bx+2)b^5-5\,b^5\right)(bx+2)^{\frac{3}{2}}b}{15\,((bx+2)b-2\,b)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((b*x + 2)*b^5 - 5*b^5)*(b*x + 2)^(3/2)*b/(((b*x + 2)*b - 2*b)^(5/2)*a
bs(b))
```

3.512 $\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{2b^2(bx+2)^{3/2}}{105x^{3/2}}+\frac{2b(bx+2)^{3/2}}{35x^{5/2}}-\frac{(bx+2)^{3/2}}{7x^{7/2}}$$

```
[Out] -(2 + b*x)^(3/2)/(7*x^(7/2)) + (2*b*(2 + b*x)^(3/2))/(35*x^(5/2)) - (2*b^2*
(2 + b*x)^(3/2))/(105*x^(3/2))
```

Rubi [A] time = 0.0082292, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2(bx+2)^{3/2}}{105x^{3/2}}+\frac{2b(bx+2)^{3/2}}{35x^{5/2}}-\frac{(bx+2)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(9/2),x]
```

```
[Out] -(2 + b*x)^(3/2)/(7*x^(7/2)) + (2*b*(2 + b*x)^(3/2))/(35*x^(5/2)) - (2*b^2*
(2 + b*x)^(3/2))/(105*x^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx &= -\frac{(2+bx)^{3/2}}{7x^{7/2}}-\frac{1}{7}(2b)\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx\\ &= -\frac{(2+bx)^{3/2}}{7x^{7/2}}+\frac{2b(2+bx)^{3/2}}{35x^{5/2}}+\frac{1}{35}\left(2b^2\right)\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx\\ &= -\frac{(2+bx)^{3/2}}{7x^{7/2}}+\frac{2b(2+bx)^{3/2}}{35x^{5/2}}-\frac{2b^2(2+bx)^{3/2}}{105x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0121579, size = 32, normalized size = 0.54

$$-\frac{(bx+2)^{3/2}\left(2b^2x^2-6bx+15\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(9/2),x]
```

```
[Out] -((2 + b*x)^(3/2)*(15 - 6*b*x + 2*b^2*x^2))/(105*x^(7/2))
```

Maple [A] time = 0.003, size = 27, normalized size = 0.5

$$-\frac{2\,b^2x^2-6\,bx+15}{105}\,(bx+2)^{\frac{3}{2}}\,x^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(9/2),x)
```

```
[Out] -1/105*(b*x+2)^(3/2)*(2*b^2*x^2-6*b*x+15)/x^(7/2)
```

Maxima [A] time = 0.995352, size = 55, normalized size = 0.93

$$-\frac{(bx+2)^{\frac{3}{2}}b^2}{12\,x^{\frac{3}{2}}}+\frac{(bx+2)^{\frac{5}{2}}b}{10\,x^{\frac{5}{2}}}-\frac{(bx+2)^{\frac{7}{2}}}{28\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -1/12*(b*x + 2)^(3/2)*b^2/x^(3/2) + 1/10*(b*x + 2)^(5/2)*b/x^(5/2) - 1/28*(
b*x + 2)^(7/2)/x^(7/2)
```

Fricas [A] time = 1.55872, size = 90, normalized size = 1.53

$$-\frac{\left(2\,b^3x^3-2\,b^2x^2+3\,bx+30\right)\sqrt{bx+2}}{105\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(2*b^3*x^3 - 2*b^2*x^2 + 3*b*x + 30)*sqrt(b*x + 2)/x^(7/2)
```

Sympy [B] time = 71.3997, size = 270, normalized size = 4.58

$$-\frac{2b^{\frac{19}{2}}x^5\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{6b^{\frac{17}{2}}x^4\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{3b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{34b^{\frac{13}{2}}x^2\sqrt{\quad}}{105b^6x^5+420b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(9/2),x)
```

```
[Out] -2*b**(19/2)*x**5*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b*
*4*x**3) - 6*b**(17/2)*x**4*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**
4 + 420*b**4*x**3) - 3*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 42
0*b**5*x**4 + 420*b**4*x**3) - 34*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(105*b**
6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 132*b**(11/2)*x*sqrt(1 + 2/(b*x))
/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 120*b**(9/2)*sqrt(1 + 2/
(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3)
```

Giac [A] time = 1.22253, size = 74, normalized size = 1.25

$$-\frac{\left(35\,b^{7}+2\left((bx+2)b^{7}-7\,b^{7}\right)(bx+2)\right)(bx+2)^{\frac{3}{2}}b}{105\,((bx+2)b-2\,b)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(35*b^7 + 2*((b*x + 2)*b^7 - 7*b^7)*(b*x + 2))*(b*x + 2)^(3/2)*b/(((
b*x + 2)*b - 2*b)^(7/2)*abs(b))
```

3.513 $\int x^{5/2}\sqrt{2-bx}\,dx$

Optimal. Leaf size=112

$$-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{4}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{12b}$$

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(8*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(24*b^2) - (
x^(5/2)*Sqrt[2 - b*x])/(12*b) + (x^(7/2)*Sqrt[2 - b*x])/4 + (5*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rubi [A] time = 0.0286051, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{4}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[2 - b*x],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(8*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(24*b^2) - (
x^(5/2)*Sqrt[2 - b*x])/(12*b) + (x^(7/2)*Sqrt[2 - b*x])/4 + (5*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}\sqrt{2-bx}\,dx &= \frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{1}{4}\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx \\
&= -\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{12b} \\
&= -\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b^2} \\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{8b^3} \\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{4b^3} \\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0450378, size = 71, normalized size = 0.63

$$
\frac{\sqrt{x}\sqrt{2-bx}\left(6b^3x^3-2b^2x^2-5bx-15\right)}{24b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x - 2*b^2*x^2 + 6*b^3*x^3))/(24*b^3) + (5
*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Maple [A] time = 0.007, size = 116, normalized size = 1.

$$
-\frac{1}{4b}x^{\frac{5}{2}}(-bx+2)^{\frac{3}{2}}-\frac{5}{12b^2}x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}}-\frac{5}{8b^3}(-bx+2)^{\frac{3}{2}}\sqrt{x}+\frac{5}{8b^3}\sqrt{x}\sqrt{-bx+2}+\frac{5}{8}\sqrt{(-bx+2)x}\arctan\left(\sqrt{b}\left(x-b^{-1}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(1/2),x)
```

```
[Out] -1/4/b*x^(5/2)*(-b*x+2)^(3/2)-5/12/b^2*x^(3/2)*(-b*x+2)^(3/2)-5/8/b^3*x^(1/
2)*(-b*x+2)^(3/2)+5/8*x^(1/2)*(-b*x+2)^(1/2)/b^3+5/8/b^(7/2)*((-b*x+2)*x)^(
1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51176, size = 367, normalized size = 3.28

$$\left[\frac{\left(6\,b^4x^3-2\,b^3x^2-5\,b^2x-15\,b\right)\sqrt{-bx+2}\sqrt{x}-15\,\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{24\,b^4},\frac{\left(6\,b^4x^3-2\,b^3x^2-5\,b^2x-1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 15
*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^4, 1/24*((6*b^
4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arc
tan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^4]
```

Sympy [A] time = 17.6091, size = 252, normalized size = 2.25

$$\begin{cases} \frac{ibx^{\frac{9}{2}}}{4\sqrt{bx-2}} - \frac{7ix^{\frac{7}{2}}}{12\sqrt{bx-2}} - \frac{ix^{\frac{5}{2}}}{24b\sqrt{bx-2}} - \frac{5ix^{\frac{3}{2}}}{24b^2\sqrt{bx-2}} + \frac{5i\sqrt{x}}{4b^3\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{for } \frac{|bx|}{2} > 1 \\ -\frac{bx^{\frac{9}{2}}}{4\sqrt{-bx+2}} + \frac{7x^{\frac{7}{2}}}{12\sqrt{-bx+2}} + \frac{x^{\frac{5}{2}}}{24b\sqrt{-bx+2}} + \frac{5x^{\frac{3}{2}}}{24b^2\sqrt{-bx+2}} - \frac{5\sqrt{x}}{4b^3\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(9/2)/(4*sqrt(b*x - 2)) - 7*I*x**(7/2)/(12*sqrt(b*x - 2))
 - I*x**(5/2)/(24*b*sqrt(b*x - 2)) - 5*I*x**(3/2)/(24*b**2*sqrt(b*x - 2)) +
 5*I*sqrt(x)/(4*b**3*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/
(4*b**(7/2)), Abs(b*x)/2 > 1), (-b*x**(9/2)/(4*sqrt(-b*x + 2)) + 7*x**(7/2)
/(12*sqrt(-b*x + 2)) + x**(5/2)/(24*b*sqrt(-b*x + 2)) + 5*x**(3/2)/(24*b**2
*sqrt(-b*x + 2)) - 5*sqrt(x)/(4*b**3*sqrt(-b*x + 2)) + 5*asin(sqrt(2)*sqrt(
b)*sqrt(x)/2)/(4*b**(7/2)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.514 $\int x^{3/2}\sqrt{2-bx}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{1}{3}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{6b}$$

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x])/(2*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(6*b) + (x^(5/2)
*Sqrt[2 - b*x])/3 + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(5/2)
```

Rubi [A] time = 0.022504, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{1}{3}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[2 - b*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x])/(2*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(6*b) + (x^(5/2)
*Sqrt[2 - b*x])/3 + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}\sqrt{2-bx}\,dx &= \frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{1}{3}\int \frac{x^{3/2}}{\sqrt{2-bx}}dx \\
&= -\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\int \frac{\sqrt{x}}{\sqrt{2-bx}}dx}{2b} \\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}dx}{2b^2} \\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}dx,x,\sqrt{x}\right)}{b^2} \\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0587462, size = 60, normalized size = 0.69

$$
\frac{\sqrt{x}\sqrt{2-bx}\left(2b^2x^2-bx-3\right)}{6b^2}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-3 - b*x + 2*b^2*x^2))/(6*b^2) + ArcSin[(Sqrt[b]*Sq
rt[x])/Sqrt[2]]/b^(5/2)
```

Maple [A] time = 0.004, size = 100, normalized size = 1.2

$$
-\frac{1}{3b}x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}}-\frac{1}{2b^2}(-bx+2)^{\frac{3}{2}}\sqrt{x}+\frac{1}{2b^2}\sqrt{x}\sqrt{-bx+2}+\frac{1}{2}\sqrt{(-bx+2)x}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2x}}\right)b^{-}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(1/2),x)
```

```
[Out] -1/3/b*x^(3/2)*(-b*x+2)^(3/2)-1/2/b^2*x^(1/2)*(-b*x+2)^(3/2)+1/2*x^(1/2)*(-
b*x+2)^(1/2)/b^2+1/2/b^(5/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arct
an(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53088, size = 321, normalized size = 3.69

$$\left[\frac{\left(2\,b^{3}x^{2}-b^{2}x-3\,b\right)\sqrt{-bx+2}\sqrt{x}-3\,\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{6\,b^{3}},\frac{\left(2\,b^{3}x^{2}-b^{2}x-3\,b\right)\sqrt{-bx+2}\sqrt{x}-6\,\sqrt{b}\,\mathrm{ar}}{6\,b^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 - b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x) - 3*sqrt(-b)*log(-b*
x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, 1/6*((2*b^3*x^2 - b^2*x - 3*
b)*sqrt(-b*x + 2)*sqrt(x) - 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x
))))/b^3]
```

Sympy [A] time = 6.35489, size = 196, normalized size = 2.25

$$\begin{cases} \frac{ibx^{\frac{7}{2}}}{3\sqrt{bx-2}} - \frac{5ix^{\frac{5}{2}}}{6\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{6b\sqrt{bx-2}} + \frac{i\sqrt{x}}{b^{2}\sqrt{bx-2}} - \frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for}\: \frac{|bx|}{2} > 1 \\ -\frac{bx^{\frac{7}{2}}}{3\sqrt{-bx+2}} + \frac{5x^{\frac{5}{2}}}{6\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{6b\sqrt{-bx+2}} - \frac{\sqrt{x}}{b^{2}\sqrt{-bx+2}} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(7/2)/(3*sqrt(b*x - 2)) - 5*I*x**(5/2)/(6*sqrt(b*x - 2))
- I*x**(3/2)/(6*b*sqrt(b*x - 2)) + I*sqrt(x)/(b**2*sqrt(b*x - 2)) - I*acosh
(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2), Abs(b*x)/2 > 1), (-b*x**(7/2)/(3*sqrt
(-b*x + 2)) + 5*x**(5/2)/(6*sqrt(-b*x + 2)) + x**(3/2)/(6*b*sqrt(-b*x + 2))
 - sqrt(x)/(b**2*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.515 $\int \sqrt{x}\sqrt{2-bx}\,dx$

Optimal. Leaf size=65

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x])/(2*b) + (x^(3/2)*Sqrt[2 - b*x])/2 + ArcSin[(Sqrt[b
]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rubi [A] time = 0.0149726, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[2 - b*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x])/(2*b) + (x^(3/2)*Sqrt[2 - b*x])/2 + ArcSin[(Sqrt[b
]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{x}\sqrt{2-bx}\,dx = \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.037953, size = 51, normalized size = 0.78

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}} + \frac{\sqrt{x}\sqrt{2-bx}(bx-1)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-1 + b*x))/(2*b) + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]
]/b^(3/2)
```

Maple [A] time = 0.002, size = 81, normalized size = 1.3

$$\frac{1}{2}x^{\frac{3}{2}}\sqrt{-bx+2} - \frac{1}{2b}\sqrt{x}\sqrt{-bx+2} + \frac{1}{2}\sqrt{(-bx+2)x}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2x}}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{-bx+2}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-b*x+2)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(-b*x+2)^(1/2)-1/2*x^(1/2)*(-b*x+2)^(1/2)/b+1/2/b^(3/2)*((-b*x+
2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60081, size = 281, normalized size = 4.32

$$\left[\frac{\left(b^{2}x-b\right)\sqrt{-bx+2}\sqrt{x}-\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{2\,b^{2}},\ \frac{\left(b^{2}x-b\right)\sqrt{-bx+2}\sqrt{x}-2\,\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{2\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x - b)*sqrt(-b*x + 2)*sqrt(x) - sqrt(-b)*log(-b*x + sqrt(-b*x +
2)*sqrt(-b)*sqrt(x) + 1))/b^2, 1/2*((b^2*x - b)*sqrt(-b*x + 2)*sqrt(x) - 2*
sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^2]
```

Sympy [A] time = 3.31303, size = 156, normalized size = 2.4

$$\begin{cases} \frac{ibx^{\frac{5}{2}}}{2\sqrt{bx-2}}-\frac{3ix^{\frac{3}{2}}}{2\sqrt{bx-2}}+\frac{i\sqrt{x}}{b\sqrt{bx-2}}-\frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ -\frac{bx^{\frac{5}{2}}}{2\sqrt{-bx+2}}+\frac{3x^{\frac{3}{2}}}{2\sqrt{-bx+2}}-\frac{\sqrt{x}}{b\sqrt{-bx+2}}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(5/2)/(2*sqrt(b*x - 2)) - 3*I*x**(3/2)/(2*sqrt(b*x - 2))
+ I*sqrt(x)/(b*sqrt(b*x - 2)) - I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
, Abs(b*x)/2 > 1), (-b*x**(5/2)/(2*sqrt(-b*x + 2)) + 3*x**(3/2)/(2*sqrt(-b*
x + 2)) - sqrt(x)/(b*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(
3/2), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.516 $\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=41

$$\sqrt{x}\sqrt{2-bx}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] Sqrt[x]*Sqrt[2 - b*x] + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rubi [A] time = 0.0076871, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\sqrt{x}\sqrt{2-bx}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[2 - b*x] + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx &= \sqrt{x}\sqrt{2-bx}+\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx\\ &= \sqrt{x}\sqrt{2-bx}+2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{x}\sqrt{2-bx}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0216177, size = 41, normalized size = 1.

$$\sqrt{x}\sqrt{2-bx}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[2 - b*x] + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Maple [B] time = 0.004, size = 63, normalized size = 1.5

$$\sqrt{x}\sqrt{-bx+2}+\sqrt{(-bx+2)\,x}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2\,x}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+2}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(-b*x+2)^(1/2)+((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*ar
ctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62178, size = 238, normalized size = 5.8

$$\left[\frac{\sqrt{-bx+2}b\sqrt{x}-\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{b},\frac{\sqrt{-bx+2}b\sqrt{x}-2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b*x + 2)*b*sqrt(x) - sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sq
rt(x) + 1))/b, (sqrt(-b*x + 2)*b*sqrt(x) - 2*sqrt(b)*arctan(sqrt(-b*x + 2)/
(sqrt(b)*sqrt(x))))/b]
```

Sympy [A] time = 1.93144, size = 121, normalized size = 2.95

$$\begin{cases} \frac{ibx^{\frac{3}{2}}}{\sqrt{bx-2}} - \frac{2i\sqrt{x}}{\sqrt{bx-2}} - \frac{2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx|}{2} > 1 \\ -\frac{bx^{\frac{3}{2}}}{\sqrt{-bx+2}} + \frac{2\sqrt{x}}{\sqrt{-bx+2}} + \frac{2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(3/2)/sqrt(b*x - 2) - 2*I*sqrt(x)/sqrt(b*x - 2) - 2*I*aco
sh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x)/2 > 1), (-b*x**(3/2)/sqrt(-
b*x + 2) + 2*sqrt(x)/sqrt(-b*x + 2) + 2*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqr
t(b), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.517 $\int \frac{\sqrt{2-bx}}{x^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] (-2*Sqrt[2 - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rubi [A] time = 0.0091793, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {47, 54, 216}

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2-bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{2-bx}}{\sqrt{x}}-b\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx\\
&= -\frac{2\sqrt{2-bx}}{\sqrt{x}}-(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.014115, size = 42, normalized size = 1.

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Maple [B] time = 0.024, size = 90, normalized size = 2.1

$$2\,\frac{(bx-2)\sqrt{(-bx+2)\,x}}{\sqrt{-x\,(bx-2)}\sqrt{x}\sqrt{-bx+2}}-\sqrt{b}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2\,x}}\right)\sqrt{(-bx+2)\,x}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(3/2),x)
```

```
[Out] 2*(b*x-2)/(-x*(b*x-2))^(1/2)*((-b*x+2)*x)^(1/2)/x^(1/2)/(-b*x+2)^(1/2)-b^(1
/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))*((-b*x+2)*x)^(1/2)/x^(1/2)/(
-b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54535, size = 240, normalized size = 5.71

$$\left[\frac{\sqrt{-b}x\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)-2\sqrt{-bx+2}\sqrt{x}}{x},\ \frac{2\left(\sqrt{b}x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-\sqrt{-bx+2}\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - 2*sqrt(-b*x
 + 2)*sqrt(x))/x, 2*(sqrt(b)*x*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - sq
rt(-b*x + 2)*sqrt(x))/x]
```

Sympy [C] time = 1.74417, size = 136, normalized size = 3.24

$$\begin{cases} -2\sqrt{b}\sqrt{-1+\frac{2}{bx}} - i\sqrt{b}\log\left(\frac{1}{bx}\right) + 2i\sqrt{b}\log\left(\frac{1}{\sqrt{b}\sqrt{x}}\right) - 2\sqrt{b}\,\text{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right) & \text{for } \frac{2}{|bx|} > 1 \\ -2i\sqrt{b}\sqrt{1-\frac{2}{bx}} - i\sqrt{b}\log\left(\frac{1}{bx}\right) + 2i\sqrt{b}\log\left(\sqrt{1-\frac{2}{bx}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(-1 + 2/(b*x)) - I*sqrt(b)*log(1/(b*x)) + 2*I*sqr
t(b)*log(1/(sqrt(b)*sqrt(x))) - 2*sqrt(b)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2),
2/Abs(b*x) > 1), (-2*I*sqrt(b)*sqrt(1 - 2/(b*x)) - I*sqrt(b)*log(1/(b*x)) +
 2*I*sqrt(b)*log(sqrt(1 - 2/(b*x)) + 1), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.518 $\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

```
[Out] -(2 - b*x)^(3/2)/(3*x^(3/2))
```

Rubi [A] time = 0.0014399, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(5/2),x]
```

```
[Out] -(2 - b*x)^(3/2)/(3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx = -\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Mathematica [A] time = 0.0053656, size = 19, normalized size = 1.

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(5/2),x]
```

```
[Out] -(2 - b*x)^(3/2)/(3*x^(3/2))
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$-\frac{1}{3}\,(-bx+2)^{\frac{3}{2}}\,x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(5/2),x)
```

```
[Out] -1/3*(-b*x+2)^(3/2)/x^(3/2)
```

Maxima [A] time = 1.02419, size = 18, normalized size = 0.95

$$-\frac{(-bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A] time = 1.6011, size = 51, normalized size = 2.68

$$\frac{(bx-2)\sqrt{-bx+2}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x - 2)*sqrt(-b*x + 2)/x^(3/2)
```

Sympy [B] time = 2.32574, size = 82, normalized size = 4.32

$$\begin{cases} \frac{b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3} - \frac{2\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for}\ \frac{2}{|bx|} > 1 \\ \frac{ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3} - \frac{2i\sqrt{b}\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(5/2),x)
```

```
[Out] Piecewise((b**(3/2)*sqrt(-1 + 2/(b*x))/3 - 2*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*
x), 2/Abs(b*x) > 1), (I*b**(3/2)*sqrt(1 - 2/(b*x))/3 - 2*I*sqrt(b)*sqrt(1 -
 2/(b*x))/(3*x), True))
```

Giac [B] time = 1.28805, size = 47, normalized size = 2.47

$$\frac{(bx-2)\sqrt{-bx+2}b^4}{3\left((bx-2)b+2\,b\right)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*x - 2)*sqrt(-b*x + 2)*b^4/(((b*x - 2)*b + 2*b)^(3/2)*abs(b))
```

3.519 $\int \frac{\sqrt{2-bx}}{x^{7/2}}\,dx$

Optimal. Leaf size=40

$$-\frac{b(2-bx)^{3/2}}{15x^{3/2}}-\frac{(2-bx)^{3/2}}{5x^{5/2}}$$

```
[Out] -(2 - b*x)^(3/2)/(5*x^(5/2)) - (b*(2 - b*x)^(3/2))/(15*x^(3/2))
```

Rubi [A] time = 0.0041434, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{b(2-bx)^{3/2}}{15x^{3/2}}-\frac{(2-bx)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(7/2),x]
```

```
[Out] -(2 - b*x)^(3/2)/(5*x^(5/2)) - (b*(2 - b*x)^(3/2))/(15*x^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{x^{7/2}}\,dx &= -\frac{(2-bx)^{3/2}}{5x^{5/2}}+\frac{1}{5}b\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx\\ &= -\frac{(2-bx)^{3/2}}{5x^{5/2}}-\frac{b(2-bx)^{3/2}}{15x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.009253, size = 24, normalized size = 0.6

$$-\frac{(2-bx)^{3/2}(bx+3)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(7/2),x]
```

```
[Out] -((2 - b*x)^(3/2)*(3 + b*x))/(15*x^(5/2))
```

Maple [A] time = 0.003, size = 19, normalized size = 0.5

$$-\frac{bx+3}{15}\left(-bx+2\right)^{\frac{3}{2}}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(7/2),x)
```

```
[Out] -1/15*(b*x+3)*(-b*x+2)^(3/2)/x^(5/2)
```

Maxima [A] time = 1.08431, size = 38, normalized size = 0.95

$$-\frac{\left(-bx+2\right)^{\frac{3}{2}}b}{6\,x^{\frac{3}{2}}}-\frac{\left(-bx+2\right)^{\frac{5}{2}}}{10\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-b*x + 2)^(3/2)*b/x^(3/2) - 1/10*(-b*x + 2)^(5/2)/x^(5/2)
```

Fricas [A] time = 1.56244, size = 66, normalized size = 1.65

$$\frac{\left(b^{2}x^{2}+bx-6\right)\sqrt{-bx+2}}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(b^2*x^2 + b*x - 6)*sqrt(-b*x + 2)/x^(5/2)
```

Sympy [A] time = 18.4888, size = 194, normalized size = 4.85

$$\begin{cases} \frac{b^{\frac{5}{2}}\sqrt{-1+\frac{2}{bx}}}{15}+\frac{b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{15x}-\frac{2\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{5x^{2}} & \text{for}\ \frac{2}{|bx|}>1 \\ \frac{ib^{\frac{9}{2}}x^{2}\sqrt{1-\frac{2}{bx}}}{15b^{2}x^{2}-30bx}-\frac{ib^{\frac{7}{2}}x\sqrt{1-\frac{2}{bx}}}{15b^{2}x^{2}-30bx}-\frac{8ib^{\frac{5}{2}}\sqrt{1-\frac{2}{bx}}}{15b^{2}x^{2}-30bx}+\frac{12ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{x\left(15b^{2}x^{2}-30bx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(7/2),x)
```

```
[Out] Piecewise((b**(5/2)*sqrt(-1 + 2/(b*x))/15 + b**(3/2)*sqrt(-1 + 2/(b*x))/(15
*x) - 2*sqrt(b)*sqrt(-1 + 2/(b*x))/(5*x**2), 2/Abs(b*x) > 1), (I*b**(9/2)*x
**2*sqrt(1 - 2/(b*x))/(15*b**2*x**2 - 30*b*x) - I*b**(7/2)*x*sqrt(1 - 2/(b*
x))/(15*b**2*x**2 - 30*b*x) - 8*I*b**(5/2)*sqrt(1 - 2/(b*x))/(15*b**2*x**2
- 30*b*x) + 12*I*b**(3/2)*sqrt(1 - 2/(b*x))/(x*(15*b**2*x**2 - 30*b*x)), Tr
ue))
```

Giac [A] time = 1.17405, size = 65, normalized size = 1.62

$$\frac{\left((bx-2)b^5+5\,b^5\right)(bx-2)\sqrt{-bx+2}b}{15\left((bx-2)b+2\,b\right)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((b*x - 2)*b^5 + 5*b^5)*(b*x - 2)*sqrt(-b*x + 2)*b/((((b*x - 2)*b + 2*b
)^(5/2)*abs(b))
```

3.520 $\int \frac{\sqrt{2-bx}}{x^{9/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}-\frac{(2-bx)^{3/2}}{7x^{7/2}}$$

```
[Out] -(2 - b*x)^(3/2)/(7*x^(7/2)) - (2*b*(2 - b*x)^(3/2))/(35*x^(5/2)) - (2*b^2*
(2 - b*x)^(3/2))/(105*x^(3/2))
```

Rubi [A] time = 0.0080486, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}-\frac{(2-bx)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(9/2),x]
```

```
[Out] -(2 - b*x)^(3/2)/(7*x^(7/2)) - (2*b*(2 - b*x)^(3/2))/(35*x^(5/2)) - (2*b^2*
(2 - b*x)^(3/2))/(105*x^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2-bx}}{x^{9/2}}\,dx &= -\frac{(2-bx)^{3/2}}{7x^{7/2}}+\frac{1}{7}(2b)\int \frac{\sqrt{2-bx}}{x^{7/2}}\,dx\\
&= -\frac{(2-bx)^{3/2}}{7x^{7/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}+\frac{1}{35}\left(2b^2\right)\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx\\
&= -\frac{(2-bx)^{3/2}}{7x^{7/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}-\frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0119495, size = 33, normalized size = 0.53

$$-\frac{(2-bx)^{3/2}\left(2b^2x^2+6bx+15\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(9/2),x]
```

```
[Out] -((2 - b*x)^(3/2)*(15 + 6*b*x + 2*b^2*x^2))/(105*x^(7/2))
```

Maple [A] time = 0.003, size = 28, normalized size = 0.5

$$-\frac{2\,b^2x^2+6\,bx+15}{105}\,(-bx+2)^{\frac{3}{2}}\,x^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(9/2),x)
```

```
[Out] -1/105*(2*b^2*x^2+6*b*x+15)*(-b*x+2)^(3/2)/x^(7/2)
```

Maxima [A] time = 1.04392, size = 59, normalized size = 0.95

$$-\frac{(-bx+2)^{\frac{3}{2}}b^2}{12\,x^{\frac{3}{2}}}-\frac{(-bx+2)^{\frac{5}{2}}b}{10\,x^{\frac{5}{2}}}-\frac{(-bx+2)^{\frac{7}{2}}}{28\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-b*x + 2)^(3/2)*b^2/x^(3/2) - 1/10*(-b*x + 2)^(5/2)*b/x^(5/2) - 1/28
*(-b*x + 2)^(7/2)/x^(7/2)
```

Fricas [A] time = 1.56232, size = 90, normalized size = 1.45

$$\frac{\left(2\,b^3x^3+2\,b^2x^2+3\,bx-30\right)\sqrt{-bx+2}}{105\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*b^3*x^3 + 2*b^2*x^2 + 3*b*x - 30)*sqrt(-b*x + 2)/x^(7/2)
```

Sympy [B] time = 67.6331, size = 554, normalized size = 8.94

$$\begin{cases} \frac{2b^{\frac{19}{2}}x^5\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}-\frac{6b^{\frac{17}{2}}x^4\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}+\frac{3b^{\frac{15}{2}}x^3\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}-\frac{34b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}+\frac{132b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} \\ \frac{2ib^{\frac{19}{2}}x^5\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}-\frac{6ib^{\frac{17}{2}}x^4\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}+\frac{3ib^{\frac{15}{2}}x^3\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}-\frac{34ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}+\frac{132ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(9/2),x)
```

```
[Out] Piecewise((2*b**(19/2)*x**5*sqrt(-1 + 2/(b*x))/(105*b**6*x**5 - 420*b**5*x*
*4 + 420*b**4*x**3) - 6*b**(17/2)*x**4*sqrt(-1 + 2/(b*x))/(105*b**6*x**5 -
420*b**5*x**4 + 420*b**4*x**3) + 3*b**(15/2)*x**3*sqrt(-1 + 2/(b*x))/(105*b
**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) - 34*b**(13/2)*x**2*sqrt(-1 + 2/(
b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) + 132*b**(11/2)*x*sqr
t(-1 + 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) - 120*b**(9
/2)*sqrt(-1 + 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3), 2/A
bs(b*x) > 1), (2*I*b**(19/2)*x**5*sqrt(1 - 2/(b*x))/(105*b**6*x**5 - 420*b*
*5*x**4 + 420*b**4*x**3) - 6*I*b**(17/2)*x**4*sqrt(1 - 2/(b*x))/(105*b**6*x
**5 - 420*b**5*x**4 + 420*b**4*x**3) + 3*I*b**(15/2)*x**3*sqrt(1 - 2/(b*x))
/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) - 34*I*b**(13/2)*x**2*sqrt
(1 - 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) + 132*I*b**(1
1/2)*x*sqrt(1 - 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) -
120*I*b**(9/2)*sqrt(1 - 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*
x**3), True))
```

Giac [A] time = 1.27835, size = 82, normalized size = 1.32

$$\frac{\left(35\,b^7+2\left((bx-2)b^7+7\,b^7\right)(bx-2)\right)(bx-2)\sqrt{-bx+2}b}{105\,((bx-2)b+2\,b)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(35*b^7 + 2*((b*x - 2)*b^7 + 7*b^7)*(b*x - 2))*(b*x - 2)*sqrt(-b*x +
2)*b/(((b*x - 2)*b + 2*b)^(7/2)*abs(b))
```

3.521 $\int x^{5/2}(a+bx)^{3/2}\,dx$

Optimal. Leaf size=143

$$-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{7/2}}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

```
[Out] (3*a^4*Sqrt[x]*Sqrt[a + b*x])/(128*b^3) - (a^3*x^(3/2)*Sqrt[a + b*x])/(64*b
^2) + (a^2*x^(5/2)*Sqrt[a + b*x])/(80*b) + (3*a*x^(7/2)*Sqrt[a + b*x])/40 +
 (x^(7/2)*(a + b*x)^(3/2))/5 - (3*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*
x]])/(128*b^(7/2))
```

Rubi [A] time = 0.051225, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{7/2}}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^(3/2),x]
```

```
[Out] (3*a^4*Sqrt[x]*Sqrt[a + b*x])/(128*b^3) - (a^3*x^(3/2)*Sqrt[a + b*x])/(64*b
^2) + (a^2*x^(5/2)*Sqrt[a + b*x])/(80*b) + (3*a*x^(7/2)*Sqrt[a + b*x])/40 +
 (x^(7/2)*(a + b*x)^(3/2))/5 - (3*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*
x]])/(128*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^{5/2}(a+bx)^{3/2}\,dx = \frac{1}{5}x^{7/2}(a+bx)^{3/2}+\frac{1}{10}(3a)\int x^{5/2}\sqrt{a+bx}\,dx$$
$$= \frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}+\frac{1}{80}\left(3a^2\right)\int\frac{x^{5/2}}{\sqrt{a+bx}}\,dx$$
$$= \frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}-\frac{a^3\int\frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{32b}$$
$$= -\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}+\frac{\left(3a^4\right)\int\frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{128b^2}$$
$$= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}-\frac{\left(3a^5\right)\int}{2}$$
$$= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}-\frac{\left(3a^5\right)\mathrm{S}}{}$$
$$= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}-\frac{\left(3a^5\right)\mathrm{S}}{}$$
$$= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}-\frac{3a^5\tan}{1}$$

Mathematica [A] time = 0.224587, size = 107, normalized size = 0.75

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(8a^2b^2x^2-10a^3bx+15a^4+176ab^3x^3+128b^4x^4\right)-\frac{15a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{640b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^4 - 10*a^3*b*x + 8*a^2*b^2*x^2 + 176*
a*b^3*x^3 + 128*b^4*x^4) - (15*a^(9/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/
Sqrt[1 + (b*x)/a]))/(640*b^(7/2))
```

Maple [A] time = 0.005, size = 138, normalized size = 1.

$$\frac{1}{5b}x^{\frac{5}{2}}(bx+a)^{\frac{5}{2}}-\frac{a}{8b^2}x^{\frac{3}{2}}(bx+a)^{\frac{5}{2}}+\frac{a^2}{16b^3}(bx+a)^{\frac{5}{2}}\sqrt{x}-\frac{a^3}{64b^3}(bx+a)^{\frac{3}{2}}\sqrt{x}-\frac{3a^4}{128b^3}\sqrt{x}\sqrt{bx+a}-\frac{3a^5}{256}\sqrt{x(bx+a)}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(3/2),x)
```

```
[Out] 1/5/b*x^(5/2)*(b*x+a)^(5/2)-1/8/b^2*a*x^(3/2)*(b*x+a)^(5/2)+1/16/b^3*a^2*x^
(1/2)*(b*x+a)^(5/2)-1/64/b^3*a^3*(b*x+a)^(3/2)*x^(1/2)-3/128*a^4*x^(1/2)*(b
*x+a)^(1/2)/b^3-3/256/b^(7/2)*a^5*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*l
n((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59036, size = 470, normalized size = 3.29

$$\left[\frac{15\,a^{5}\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(128\,b^{5}x^{4}+176\,ab^{4}x^{3}+8\,a^{2}b^{3}x^{2}-10\,a^{3}b^{2}x+15\,a^{4}b\right)\sqrt{bx+a}\sqrt{x}}{1280\,b^{4}},\ \frac{15\,a^{5}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/1280*(15*a^5*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b^2*x + 15*a^4*b)*s
qrt(b*x + a)*sqrt(x))/b^4, 1/640*(15*a^5*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt
(-b)/(b*sqrt(x))) + (128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b
^2*x + 15*a^4*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [A] time = 52.0442, size = 178, normalized size = 1.24

$$\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^{3}\sqrt{1+\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1+\frac{bx}{a}}}+\frac{23a^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1+\frac{bx}{a}}}+\frac{19\sqrt{a}bx^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}}-\frac{3a^{5}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{b^{2}x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(3/2),x)
```

```
[Out] 3*a**(9/2)*sqrt(x)/(128*b**3*sqrt(1 + b*x/a)) + a**(7/2)*x**(3/2)/(128*b**2
*sqrt(1 + b*x/a)) - a**(5/2)*x**(5/2)/(320*b*sqrt(1 + b*x/a)) + 23*a**(3/2)
*x**(7/2)/(80*sqrt(1 + b*x/a)) + 19*sqrt(a)*b*x**(9/2)/(40*sqrt(1 + b*x/a))
 - 3*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(7/2)) + b**2*x**(11/2)/(5
*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.522 $\int x^{3/2}(a+bx)^{3/2}\,dx$

Optimal. Leaf size=119

$$-\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{5/2}}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

```
[Out] (-3*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b^2) + (a^2*x^(3/2)*Sqrt[a + b*x])/(32*b
) + (a*x^(5/2)*Sqrt[a + b*x])/8 + (x^(5/2)*(a + b*x)^(3/2))/4 + (3*a^4*ArcT
anh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(5/2))
```

Rubi [A] time = 0.0379329, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{5/2}}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^(3/2),x]
```

```
[Out] (-3*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b^2) + (a^2*x^(3/2)*Sqrt[a + b*x])/(32*b
) + (a*x^(5/2)*Sqrt[a + b*x])/8 + (x^(5/2)*(a + b*x)^(3/2))/4 + (3*a^4*ArcT
anh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^{3/2}(a+bx)^{3/2}\,dx = \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{1}{8}(3a)\int x^{3/2}\sqrt{a+bx}\,dx$$
$$= \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{1}{16}a^2\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx$$
$$= \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} - \frac{(3a^3)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{64b}$$
$$= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{(3a^4)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{128b^2}$$
$$= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{(3a^4)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+b}}\right.}{64b^2}$$
$$= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{(3a^4)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\right.}{64b^2}$$
$$= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{5/2}}$$

Mathematica [A] time = 0.123042, size = 96, normalized size = 0.81

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(2a^2bx-3a^3+24ab^2x^2+16b^3x^3\right)+\frac{3a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{64b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(-3*a^3 + 2*a^2*b*x + 24*a*b^2*x^2 + 16*b^3
*x^3) + (3*a^(7/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/
(64*b^(5/2))
```

Maple [A] time = 0.004, size = 120, normalized size = 1.

$$\frac{1}{4b}x^{\frac{3}{2}}(bx+a)^{\frac{5}{2}} - \frac{a}{8b^2}(bx+a)^{\frac{5}{2}}\sqrt{x} + \frac{a^2}{32b^2}(bx+a)^{\frac{3}{2}}\sqrt{x} + \frac{3a^3}{64b^2}\sqrt{x}\sqrt{bx+a} + \frac{3a^4}{128}\sqrt{x(bx+a)}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(3/2),x)
```

```
[Out] 1/4/b*x^(3/2)*(b*x+a)^(5/2)-1/8/b^2*a*x^(1/2)*(b*x+a)^(5/2)+1/32/b^2*a^2*(b
*x+a)^(3/2)*x^(1/2)+3/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b^2+3/128/b^(5/2)*a^4*(x
*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60805, size = 412, normalized size = 3.46

$$\left[\frac{3\,a^4\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b\right)\sqrt{bx+a}\sqrt{x}}{128\,b^3}, -\frac{3\,a^4\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}}{b\sqrt{}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(3*a^4*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(16*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/
b^3, -1/64*(3*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (16
*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/b^3
]
```

Sympy [A] time = 11.2612, size = 153, normalized size = 1.29

$$-\frac{3a^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1+\frac{bx}{a}}}+\frac{13a^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1+\frac{bx}{a}}}+\frac{5\sqrt{a}bx^{\frac{7}{2}}}{8\sqrt{1+\frac{bx}{a}}}+\frac{3a^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}}+\frac{b^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(3/2),x)
```

```
[Out] -3*a**(7/2)*sqrt(x)/(64*b**2*sqrt(1 + b*x/a)) - a**(5/2)*x**(3/2)/(64*b*sqr
t(1 + b*x/a)) + 13*a**(3/2)*x**(5/2)/(32*sqrt(1 + b*x/a)) + 5*sqrt(a)*b*x**
(7/2)/(8*sqrt(1 + b*x/a)) + 3*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(5
/2)) + b**2*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.523 $\int \sqrt{x}(a+bx)^{3/2}\,dx$

Optimal. Leaf size=95

$$-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}+\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}$$

```
[Out] (a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b) + (a*x^(3/2)*Sqrt[a + b*x])/4 + (x^(3/2)*
(a + b*x)^(3/2))/3 - (a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(8*b^(3
/2))
```

Rubi [A] time = 0.0290658, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}+\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^(3/2),x]
```

```
[Out] (a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b) + (a*x^(3/2)*Sqrt[a + b*x])/4 + (x^(3/2)*
(a + b*x)^(3/2))/3 - (a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(8*b^(3
/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}(a+bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(a+bx)^{3/2}+\frac{1}{2}a\int \sqrt{x}\sqrt{a+bx}\,dx\\
&= \frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}+\frac{1}{8}a^2\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx\\
&= \frac{a^2\sqrt{x}\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{16b}\\
&= \frac{a^2\sqrt{x}\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{8b}\\
&= \frac{a^2\sqrt{x}\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{8b}\\
&= \frac{a^2\sqrt{x}\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.114142, size = 85, normalized size = 0.89

$$
\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(3a^2+14abx+8b^2x^2\right)-\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{24b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(3*a^2 + 14*a*b*x + 8*b^2*x^2) - (3*a^(5/2)
*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(24*b^(3/2))
```

Maple [A] time = 0.003, size = 96, normalized size = 1.

$$
\frac{1}{3}x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}+\frac{a}{4}x^{\frac{3}{2}}\sqrt{bx+a}+\frac{a^2}{8b}\sqrt{x}\sqrt{bx+a}-\frac{a^3}{16}\sqrt{x(bx+a)}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*x^(1/2),x)
```

```
[Out] 1/3*x^(3/2)*(b*x+a)^(3/2)+1/4*a*x^(3/2)*(b*x+a)^(1/2)+1/8*a^2*x^(1/2)*(b*x+
a)^(1/2)/b-1/16*a^3/b^(3/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/2
*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.6472, size = 363, normalized size = 3.82

$$\left[\frac{3\,a^3\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(8\,b^3x^2+14\,ab^2x+3\,a^2b\right)\sqrt{bx+a}\sqrt{x}}{48\,b^2},\frac{3\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+(8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 14*a*b^2*x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 + 14*a*b^2*
x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [A] time = 6.18672, size = 124, normalized size = 1.31

$$\frac{a^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1+\frac{bx}{a}}}+\frac{17a^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1+\frac{bx}{a}}}+\frac{11\sqrt{a}bx^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}}-\frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{b^2x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*x**(1/2),x)
```

```
[Out] a**(5/2)*sqrt(x)/(8*b*sqrt(1 + b*x/a)) + 17*a**(3/2)*x**(3/2)/(24*sqrt(1 +
b*x/a)) + 11*sqrt(a)*b*x**(5/2)/(12*sqrt(1 + b*x/a)) - a**3*asinh(sqrt(b)*s
qrt(x)/sqrt(a))/(8*b**(3/2)) + b**2*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.524 $\int \frac{(a+bx)^{3/2}}{\sqrt{x}} dx$

Optimal. Leaf size=71

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}} + \frac{3}{4}a\sqrt{x}\sqrt{a+bx} + \frac{1}{2}\sqrt{x}(a+bx)^{3/2}$$

```
[Out] (3*a*Sqrt[x]*Sqrt[a + b*x])/4 + (Sqrt[x]*(a + b*x)^(3/2))/2 + (3*a^2*ArcTan
h[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*Sqrt[b])
```

Rubi [A] time = 0.0215816, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}} + \frac{3}{4}a\sqrt{x}\sqrt{a+bx} + \frac{1}{2}\sqrt{x}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*a*Sqrt[x]*Sqrt[a + b*x])/4 + (Sqrt[x]*(a + b*x)^(3/2))/2 + (3*a^2*ArcTan
h[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*Sqrt[b])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}(a+bx)^{3/2}+\frac{1}{4}(3a)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx\\
&= \frac{3}{4}a\sqrt{x}\sqrt{a+bx}+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx\\
&= \frac{3}{4}a\sqrt{x}\sqrt{a+bx}+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{3}{4}a\sqrt{x}\sqrt{a+bx}+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\
&= \frac{3}{4}a\sqrt{x}\sqrt{a+bx}+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.0972369, size = 69, normalized size = 0.97

$$
\frac{1}{4}\sqrt{a+bx}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx}{a}+1}}+\sqrt{x}(5a+2bx)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[x]*(5*a + 2*b*x) + (3*a^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x)/a])))/4
```

Maple [A] time = 0.003, size = 78, normalized size = 1.1

$$
\frac{1}{2}(bx+a)^{\frac{3}{2}}\sqrt{x}+\frac{3a}{4}\sqrt{x}\sqrt{bx+a}+\frac{3a^2}{8}\sqrt{x(bx+a)}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{b}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*(b*x+a)^(3/2)*x^(1/2)+3/4*a*x^(1/2)*(b*x+a)^(1/2)+3/8*a^2*(x*(b*x+a))^(
1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49584, size = 311, normalized size = 4.38

$$\left[\frac{3\,a^2\sqrt{b}\log\left(2\,bx + 2\,\sqrt{bx+a}\sqrt{b}\sqrt{x} + a\right) + 2\left(2\,b^2x + 5\,ab\right)\sqrt{bx+a}\sqrt{x}}{8\,b}, -\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right) - \left(2\,b^2x + 5\,ab\right)\sqrt{b}}{4\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b
]
```

Sympy [A] time = 3.62097, size = 75, normalized size = 1.06

$$\frac{5a^{\frac{3}{2}}\sqrt{x}\sqrt{1+\frac{bx}{a}}}{4} + \frac{\sqrt{a}bx^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{2} + \frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] 5*a**(3/2)*sqrt(x)*sqrt(1 + b*x/a)/4 + sqrt(a)*b*x**(3/2)*sqrt(1 + b*x/a)/2
 + 3*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(b))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.525 $\int \frac{(a+bx)^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=63

$$-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{a+bx}+3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

```
[Out] 3*b*Sqrt[x]*Sqrt[a + b*x] - (2*(a + b*x)^(3/2))/Sqrt[x] + 3*a*Sqrt[b]*ArcTa
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]]
```

Rubi [A] time = 0.021407, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{a+bx}+3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] 3*b*Sqrt[x]*Sqrt[a + b*x] - (2*(a + b*x)^(3/2))/Sqrt[x] + 3*a*Sqrt[b]*ArcTa
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{\sqrt{x}}+(3b)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx\\
&= 3b\sqrt{x}\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+\frac{1}{2}(3ab)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx\\
&= 3b\sqrt{x}\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+(3ab)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= 3b\sqrt{x}\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+(3ab)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\
&= 3b\sqrt{x}\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0102121, size = 46, normalized size = 0.73

$$-\frac{2a\sqrt{a+bx}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a + b*x]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x)/a)])/(Sqrt[x
]*Sqrt[1 + (b*x)/a])
```

Maple [A] time = 0.013, size = 71, normalized size = 1.1

$$-(-bx+2\,a)\sqrt{bx+a}\frac{1}{\sqrt{x}}+\frac{3\,a}{2}\sqrt{b}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\sqrt{x\,(bx+a)}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(3/2),x)
```

```
[Out] -(b*x+a)^(1/2)*(-b*x+2*a)/x^(1/2)+3/2*a*b^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x
^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65396, size = 289, normalized size = 4.59

$$\left[\frac{3\,a\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,\sqrt{bx+a}(bx-2\,a)\sqrt{x}}{2\,x}, -\frac{3\,a\sqrt{-b}x\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}(bx-2\,a)\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sq
rt(b*x + a)*(b*x - 2*a)*sqrt(x))/x, -(3*a*sqrt(-b)*x*arctan(sqrt(b*x + a)*s
qrt(-b)/(b*sqrt(x))) - sqrt(b*x + a)*(b*x - 2*a)*sqrt(x))/x]
```

Sympy [A] time = 3.33552, size = 92, normalized size = 1.46

$$-\frac{2a^{\frac{3}{2}}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}b\sqrt{x}}{\sqrt{1+\frac{bx}{a}}}+3a\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\frac{b^{2}x^{\frac{3}{2}}}{\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(3/2),x)
```

```
[Out] -2*a**(3/2)/(sqrt(x)*sqrt(1 + b*x/a)) - sqrt(a)*b*sqrt(x)/sqrt(1 + b*x/a) +
 3*a*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqrt(a)) + b**2*x**(3/2)/(sqrt(a)*sqrt(1
 + b*x/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.526 $\int \frac{(a+bx)^{3/2}}{x^{5/2}} dx$

Optimal. Leaf size=64

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{a+bx}}{\sqrt{x}}$$

```
[Out] (-2*b*Sqrt[a + b*x])/Sqrt[x] - (2*(a + b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]]
```

Rubi [A] time = 0.0214081, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 63, 217, 206}

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{a+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*b*Sqrt[a + b*x])/Sqrt[x] - (2*(a + b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{x^{5/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{3x^{3/2}}+b\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx\\ &= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+b^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx\\ &= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+\left(2b^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+\left(2b^2\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\ &= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\end{aligned}$$

Mathematica [C] time = 0.0100274, size = 48, normalized size = 0.75

$$-\frac{2a\sqrt{a+bx}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a + b*x]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x)/a)])/(3*x^(
3/2)*Sqrt[1 + (b*x)/a])
```

Maple [A] time = 0.015, size = 67, normalized size = 1.1

$$-\frac{8\,bx+2\,a}{3}\sqrt{bx+a}x^{-\frac{3}{2}}+b^{\frac{3}{2}}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\sqrt{x\,(bx+a)}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(5/2),x)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(4*b*x+a)/x^(3/2)+b^(3/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+
a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72707, size = 302, normalized size = 4.72

$$\left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(4\,bx+a)\sqrt{bx+a}\sqrt{x}}{3\,x^2},-\frac{2\left(3\,\sqrt{-b}bx^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+(4\,bx+a)\sqrt{bx+a}\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*(4
*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*
x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2]
```

Sympy [A] time = 4.60244, size = 71, normalized size = 1.11

$$-\frac{2a\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3}-b^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)+2b^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(5/2),x)
```

```
[Out] -2*a*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 8*b**(3/2)*sqrt(a/(b*x) + 1)/3 - b**
(3/2)*log(a/(b*x)) + 2*b**(3/2)*log(sqrt(a/(b*x) + 1) + 1)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.527 $\int x^{5/2}(a-bx)^{3/2}\,dx$

Optimal. Leaf size=149

$$-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}+\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{7/2}}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a - b*x])/(128*b^3) - (a^3*x^(3/2)*Sqrt[a - b*x])/(64*
b^2) - (a^2*x^(5/2)*Sqrt[a - b*x])/(80*b) + (3*a*x^(7/2)*Sqrt[a - b*x])/40
+ (x^(7/2)*(a - b*x)^(3/2))/5 + (3*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*
x]])/(128*b^(7/2))
```

Rubi [A] time = 0.0532699, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}+\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{7/2}}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a - b*x])/(128*b^3) - (a^3*x^(3/2)*Sqrt[a - b*x])/(64*
b^2) - (a^2*x^(5/2)*Sqrt[a - b*x])/(80*b) + (3*a*x^(7/2)*Sqrt[a - b*x])/40
+ (x^(7/2)*(a - b*x)^(3/2))/5 + (3*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*
x]])/(128*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^{5/2}(a-bx)^{3/2}\,dx = \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{1}{10}(3a)\int x^{5/2}\sqrt{a-bx}\,dx$$
$$= \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{1}{80}\left(3a^2\right)\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx$$
$$= -\frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{a^3\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{32b}$$
$$= -\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{\left(3a^4\right)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{128b^2}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{\left(3a^5\right)\ldots}{\ldots}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{\left(3a^5\right)\ldots}{\ldots}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{\left(3a^5\right)\ldots}{\ldots}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{3a^5\tan\ldots}{1\ldots}$$

Mathematica [A] time = 0.160506, size = 110, normalized size = 0.74

$$\frac{\sqrt{a-bx}\left(\frac{15a^{9/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}} - \sqrt{b}\sqrt{x}\left(8a^2b^2x^2 + 10a^3bx + 15a^4 - 176ab^3x^3 + 128b^4x^4\right)\right)}{640b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-(Sqrt[b]*Sqrt[x]*(15*a^4 + 10*a^3*b*x + 8*a^2*b^2*x^2 - 17
6*a*b^3*x^3 + 128*b^4*x^4)) + (15*a^(9/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]]
)/Sqrt[1 - (b*x)/a]))/(640*b^(7/2))
```

Maple [A] time = 0.005, size = 146, normalized size = 1.

$$-\frac{1}{5b}x^{\frac{5}{2}}(-bx+a)^{\frac{5}{2}} - \frac{a}{8b^2}x^{\frac{3}{2}}(-bx+a)^{\frac{5}{2}} - \frac{a^2}{16b^3}(-bx+a)^{\frac{5}{2}}\sqrt{x} + \frac{a^3}{64b^3}(-bx+a)^{\frac{3}{2}}\sqrt{x} + \frac{3a^4}{128b^3}\sqrt{x}\sqrt{-bx+a} + \frac{3a^5}{256}\sqrt{x(\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(3/2),x)
```

```
[Out] -1/5/b*x^(5/2)*(-b*x+a)^(5/2)-1/8/b^2*a*x^(3/2)*(-b*x+a)^(5/2)-1/16/b^3*a^2
*x^(1/2)*(-b*x+a)^(5/2)+1/64/b^3*a^3*(-b*x+a)^(3/2)*x^(1/2)+3/128*a^4*x^(1/
2)*(-b*x+a)^(1/2)/b^3+3/256/b^(7/2)*a^5*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x
^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/(-b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9582, size = 477, normalized size = 3.2

$$\left[-\frac{15\,a^5\sqrt{-b}\log\left(-2\,bx + 2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x} + a\right) + 2\left(128\,b^5x^4 - 176\,ab^4x^3 + 8\,a^2b^3x^2 + 10\,a^3b^2x + 15\,a^4b\right)\sqrt{-bx+a}\right.}{1280\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*a^5*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) + 2*(128*b^5*x^4 - 176*a*b^4*x^3 + 8*a^2*b^3*x^2 + 10*a^3*b^2*x + 15*a^4
*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/640*(15*a^5*sqrt(b)*arctan(sqrt(-b*x +
a)/(sqrt(b)*sqrt(x))) + (128*b^5*x^4 - 176*a*b^4*x^3 + 8*a^2*b^3*x^2 + 10*a
^3*b^2*x + 15*a^4*b)*sqrt(-b*x + a)*sqrt(x))/b^4]
```

Sympy [A] time = 56.4197, size = 377, normalized size = 2.53

$$\begin{cases} \frac{3ia^{\frac{9}{2}}\sqrt{x}}{128b^3\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^2\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{-1+\frac{bx}{a}}} - \frac{23ia^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{-1+\frac{bx}{a}}} + \frac{19i\sqrt{a}bx^{\frac{9}{2}}}{40\sqrt{-1+\frac{bx}{a}}} - \frac{3ia^5\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}} - \frac{ib^2x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|} > 1 \\ -\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^3\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^2\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1-\frac{bx}{a}}} + \frac{23a^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}} - \frac{19\sqrt{a}bx^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}} + \frac{3a^5\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}} + \frac{b^2x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((3*I*a**(9/2)*sqrt(x)/(128*b**3*sqrt(-1 + b*x/a)) - I*a**(7/2)*x*
*(3/2)/(128*b**2*sqrt(-1 + b*x/a)) - I*a**(5/2)*x**(5/2)/(320*b*sqrt(-1 + b
*x/a)) - 23*I*a**(3/2)*x**(7/2)/(80*sqrt(-1 + b*x/a)) + 19*I*sqrt(a)*b*x**(
9/2)/(40*sqrt(-1 + b*x/a)) - 3*I*a**5*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b
**(7/2)) - I*b**2*x**(11/2)/(5*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) >
 1), (-3*a**(9/2)*sqrt(x)/(128*b**3*sqrt(1 - b*x/a)) + a**(7/2)*x**(3/2)/(1
28*b**2*sqrt(1 - b*x/a)) + a**(5/2)*x**(5/2)/(320*b*sqrt(1 - b*x/a)) + 23*a
**(3/2)*x**(7/2)/(80*sqrt(1 - b*x/a)) - 19*sqrt(a)*b*x**(9/2)/(40*sqrt(1 -
b*x/a)) + 3*a**5*asin(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(7/2)) + b**2*x**(11
/2)/(5*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.528 $\int x^{3/2}(a-bx)^{3/2}\,dx$

Optimal. Leaf size=124

$$-\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}+\frac{3a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{5/2}}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

```
[Out] (-3*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b^2) - (a^2*x^(3/2)*Sqrt[a - b*x])/(32*b
) + (a*x^(5/2)*Sqrt[a - b*x])/8 + (x^(5/2)*(a - b*x)^(3/2))/4 + (3*a^4*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(5/2))
```

Rubi [A] time = 0.0414726, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$-\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}+\frac{3a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{5/2}}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-3*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b^2) - (a^2*x^(3/2)*Sqrt[a - b*x])/(32*b
) + (a*x^(5/2)*Sqrt[a - b*x])/8 + (x^(5/2)*(a - b*x)^(3/2))/4 + (3*a^4*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^{3/2}(a-bx)^{3/2}\,dx = \frac{1}{4}x^{5/2}(a-bx)^{3/2} + \frac{1}{8}(3a)\int x^{3/2}\sqrt{a-bx}\,dx$$
$$= \frac{1}{8}ax^{5/2}\sqrt{a-bx} + \frac{1}{4}x^{5/2}(a-bx)^{3/2} + \frac{1}{16}a^2\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx$$
$$= -\frac{a^2x^{3/2}\sqrt{a-bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a-bx} + \frac{1}{4}x^{5/2}(a-bx)^{3/2} + \frac{(3a^3)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{64b}$$
$$= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{3/2}\sqrt{a-bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a-bx} + \frac{1}{4}x^{5/2}(a-bx)^{3/2} + \frac{(3a^4)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{128b^2}$$
$$= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{3/2}\sqrt{a-bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a-bx} + \frac{1}{4}x^{5/2}(a-bx)^{3/2} + \frac{(3a^4)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx\right.}{64b^2}$$
$$= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{3/2}\sqrt{a-bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a-bx} + \frac{1}{4}x^{5/2}(a-bx)^{3/2} + \frac{(3a^4)\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,\right.}{64b^2}$$
$$= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{3/2}\sqrt{a-bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a-bx} + \frac{1}{4}x^{5/2}(a-bx)^{3/2} + \frac{3a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{5/2}}$$

Mathematica [A] time = 0.139047, size = 99, normalized size = 0.8

$$\frac{\sqrt{a-bx}\left(\frac{3a^{7/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}} - \sqrt{b}\sqrt{x}\left(2a^2bx+3a^3-24ab^2x^2+16b^3x^3\right)\right)}{64b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-(Sqrt[b]*Sqrt[x]*(3*a^3 + 2*a^2*b*x - 24*a*b^2*x^2 + 16*b^
3*x^3)) + (3*a^(7/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))
/(64*b^(5/2))
```

Maple [A] time = 0.003, size = 127, normalized size = 1.

$$-\frac{1}{4b}x^{\frac{3}{2}}(-bx+a)^{\frac{5}{2}} - \frac{a}{8b^2}(-bx+a)^{\frac{5}{2}}\sqrt{x} + \frac{a^2}{32b^2}(-bx+a)^{\frac{3}{2}}\sqrt{x} + \frac{3a^3}{64b^2}\sqrt{x}\sqrt{-bx+a} + \frac{3a^4}{128}\sqrt{x(-bx+a)}\arctan\left(\sqrt{b}\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(3/2),x)
```

```
[Out] -1/4/b*x^(3/2)*(-b*x+a)^(5/2)-1/8/b^2*a*x^(1/2)*(-b*x+a)^(5/2)+1/32/b^2*a^2
*(-b*x+a)^(3/2)*x^(1/2)+3/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b^2+3/128/b^(5/2)*a
^4*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/(-b
*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93496, size = 417, normalized size = 3.36

$$\left[-\frac{3\,a^4\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(16\,b^4x^3-24\,ab^3x^2+2\,a^2b^2x+3\,a^3b\right)\sqrt{-bx+a}\sqrt{x}}{128\,b^3},-\frac{3\,a^4\sqrt{b}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(3*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(16*b^4*x^3 - 24*a*b^3*x^2 + 2*a^2*b^2*x + 3*a^3*b)*sqrt(-b*x + a)*sqr
t(x))/b^3, -1/64*(3*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) +
(16*b^4*x^3 - 24*a*b^3*x^2 + 2*a^2*b^2*x + 3*a^3*b)*sqrt(-b*x + a)*sqrt(x))
/b^3]
```

Sympy [A] time = 11.931, size = 325, normalized size = 2.62

$$\begin{cases} \frac{3ia^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{-1+\frac{bx}{a}}}-\frac{13ia^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{-1+\frac{bx}{a}}}+\frac{5i\sqrt{a}bx^{\frac{7}{2}}}{8\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}}-\frac{ib^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ -\frac{3a^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1-\frac{bx}{a}}}+\frac{13a^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1-\frac{bx}{a}}}-\frac{5\sqrt{a}bx^{\frac{7}{2}}}{8\sqrt{1-\frac{bx}{a}}}+\frac{3a^4\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}}+\frac{b^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((3*I*a**(7/2)*sqrt(x)/(64*b**2*sqrt(-1 + b*x/a)) - I*a**(5/2)*x**
(3/2)/(64*b*sqrt(-1 + b*x/a)) - 13*I*a**(3/2)*x**(5/2)/(32*sqrt(-1 + b*x/a)
) + 5*I*sqrt(a)*b*x**(7/2)/(8*sqrt(-1 + b*x/a)) - 3*I*a**4*acosh(sqrt(b)*sq
rt(x)/sqrt(a))/(64*b**(5/2)) - I*b**2*x**(9/2)/(4*sqrt(a)*sqrt(-1 + b*x/a))
, Abs(b*x)/Abs(a) > 1), (-3*a**(7/2)*sqrt(x)/(64*b**2*sqrt(1 - b*x/a)) + a*
*(5/2)*x**(3/2)/(64*b*sqrt(1 - b*x/a)) + 13*a**(3/2)*x**(5/2)/(32*sqrt(1 -
b*x/a)) - 5*sqrt(a)*b*x**(7/2)/(8*sqrt(1 - b*x/a)) + 3*a**4*asin(sqrt(b)*sq
rt(x)/sqrt(a))/(64*b**(5/2)) + b**2*x**(9/2)/(4*sqrt(a)*sqrt(1 - b*x/a)), T
rue))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="giac")
```

[Out] Timed out

3.529 $\int \sqrt{x}(a-bx)^{3/2}\,dx$

Optimal. Leaf size=99

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}$$

```
[Out] -(a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b) + (a*x^(3/2)*Sqrt[a - b*x])/4 + (x^(3/2)
*(a - b*x)^(3/2))/3 + (a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8*b^(3
/2))
```

Rubi [A] time = 0.0309863, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a - b*x)^(3/2),x]
```


```
[Out] -(a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b) + (a*x^(3/2)*Sqrt[a - b*x])/4 + (x^(3/2)
*(a - b*x)^(3/2))/3 + (a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8*b^(3
/2))
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}(a-bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{1}{2}a\int \sqrt{x}\sqrt{a-bx}\,dx\\
&= \frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{1}{8}a^2\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx\\
&= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{16b}\\
&= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{8b}\\
&= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{8b}\\
&= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.114232, size = 87, normalized size = 0.88

$$\frac{\sqrt{a-bx}\left(\sqrt{b}\sqrt{x}\left(-3a^2+14abx-8b^2x^2\right)+\frac{3a^{5/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}\right)}{24b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-3*a^2 + 14*a*b*x - 8*b^2*x^2) + (3*a^(5/2
)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(24*b^(3/2))
```

Maple [A] time = 0.004, size = 102, normalized size = 1.

$$\frac{1}{3}x^{\frac{3}{2}}(-bx+a)^{\frac{3}{2}}+\frac{a}{4}x^{\frac{3}{2}}\sqrt{-bx+a}-\frac{a^2}{8b}\sqrt{x}\sqrt{-bx+a}+\frac{a^3}{16}\sqrt{x(-bx+a)}\arctan\left(\sqrt{b}\left(x-\frac{a}{2b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)*x^(1/2),x)
```

```
[Out] 1/3*x^(3/2)*(-b*x+a)^(3/2)+1/4*a*x^(3/2)*(-b*x+a)^(1/2)-1/8*a^2*x^(1/2)*(-b
*x+a)^(1/2)/b+1/16*a^3/b^(3/2)*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*ar
ctan(b^(1/2)*(x-1/2/b*a)/(-b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97079, size = 370, normalized size = 3.74

$$\left[-\frac{3\,a^3\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(8\,b^3x^2-14\,ab^2x+3\,a^2b\right)\sqrt{-bx+a}\sqrt{x}}{48\,b^2},-\frac{3\,a^3\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
+ 2*(8*b^3*x^2 - 14*a*b^2*x + 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^2, -1/24*(
3*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (8*b^3*x^2 - 14*a*
b^2*x + 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^2]
```

Sympy [A] time = 6.47694, size = 265, normalized size = 2.68

$$\begin{cases} \frac{ia^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{-1+\frac{bx}{a}}} - \frac{17ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{-1+\frac{bx}{a}}} + \frac{11i\sqrt{a}bx^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}} - \frac{ia^3\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}} - \frac{ib^2x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ -\frac{a^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1-\frac{bx}{a}}} + \frac{17a^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1-\frac{bx}{a}}} - \frac{11\sqrt{a}bx^{\frac{5}{2}}}{12\sqrt{1-\frac{bx}{a}}} + \frac{a^3\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}} + \frac{b^2x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)*x**(1/2),x)
```

```
[Out] Piecewise((I*a**(5/2)*sqrt(x)/(8*b*sqrt(-1 + b*x/a)) - 17*I*a**(3/2)*x**(3/
2)/(24*sqrt(-1 + b*x/a)) + 11*I*sqrt(a)*b*x**(5/2)/(12*sqrt(-1 + b*x/a)) -
I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(3/2)) - I*b**2*x**(7/2)/(3*sqr
t(a)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (-a**(5/2)*sqrt(x)/(8*b*sqrt(
1 - b*x/a)) + 17*a**(3/2)*x**(3/2)/(24*sqrt(1 - b*x/a)) - 11*sqrt(a)*b*x**(
5/2)/(12*sqrt(1 - b*x/a)) + a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(3/2))
 + b**2*x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.530 $\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=74

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\sqrt{a-bx}+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}$$

```
[Out] (3*a*Sqrt[x]*Sqrt[a - b*x])/4 + (Sqrt[x]*(a - b*x)^(3/2))/2 + (3*a^2*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*Sqrt[b])
```

Rubi [A] time = 0.021048, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\sqrt{a-bx}+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*a*Sqrt[x]*Sqrt[a - b*x])/4 + (Sqrt[x]*(a - b*x)^(3/2))/2 + (3*a^2*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*Sqrt[b])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}(a-bx)^{3/2}+\frac{1}{4}(3a)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx \\
&= \frac{3}{4}a\sqrt{x}\sqrt{a-bx}+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx \\
&= \frac{3}{4}a\sqrt{x}\sqrt{a-bx}+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right) \\
&= \frac{3}{4}a\sqrt{x}\sqrt{a-bx}+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right) \\
&= \frac{3}{4}a\sqrt{x}\sqrt{a-bx}+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.113653, size = 71, normalized size = 0.96

$$
\frac{1}{4}\sqrt{a-bx}\left(\frac{3a^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{1-\frac{bx}{a}}}+\sqrt{x}(5a-2bx)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[x]*(5*a - 2*b*x) + (3*a^(3/2)*ArcSin[(Sqrt[b]*Sqrt[x])
/Sqrt[a]])/(Sqrt[b]*Sqrt[1 - (b*x)/a])))/4
```

Maple [A] time = 0.006, size = 83, normalized size = 1.1

$$
\frac{1}{2}\left(-bx+a\right)^{\frac{3}{2}}\sqrt{x}+\frac{3\,a}{4}\sqrt{x}\sqrt{-bx+a}+\frac{3\,a^2}{8}\sqrt{x\left(-bx+a\right)}\arctan\left(\sqrt{b}\left(x-\frac{a}{2\,b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+a}}\frac{1}{\sqrt{b}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*x^(1/2)*(-b*x+a)^(1/2)+3/8*a^2*(x*(-b*x+a)
)^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/(-b*x^2+a
*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92642, size = 316, normalized size = 4.27

$$\left[-\frac{3\,a^2\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(2\,b^2x-5\,ab\right)\sqrt{-bx+a}\sqrt{x}}{8\,b},-\frac{3\,a^2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)+\left(2\,b^2x-5\,a\right.}{4\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*(2*b^2*x - 5*a*b)*sqrt(-b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(b)*arctan(
sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x - 5*a*b)*sqrt(-b*x + a)*sqrt(x
))/b]
```

Sympy [A] time = 3.56229, size = 192, normalized size = 2.59

$$\begin{cases} -\frac{5ia^{\frac{3}{2}}\sqrt{x}}{4\sqrt{-1+\frac{bx}{a}}}+\frac{7i\sqrt{a}bx^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}}-\frac{ib^2x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ \frac{5a^{\frac{3}{2}}\sqrt{x}\sqrt{1-\frac{bx}{a}}}{4}-\frac{\sqrt{a}bx^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{2}+\frac{3a^2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((-5*I*a**(3/2)*sqrt(x)/(4*sqrt(-1 + b*x/a)) + 7*I*sqrt(a)*b*x**(3
/2)/(4*sqrt(-1 + b*x/a)) - 3*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(
b)) - I*b**2*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (
5*a**(3/2)*sqrt(x)*sqrt(1 - b*x/a)/4 - sqrt(a)*b*x**(3/2)*sqrt(1 - b*x/a)/2
 + 3*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(b)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.531 $\int \frac{(a-bx)^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=66

$$-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\sqrt{a-bx}-3a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

```
[Out] -3*b*Sqrt[x]*Sqrt[a - b*x] - (2*(a - b*x)^(3/2))/Sqrt[x] - 3*a*Sqrt[b]*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
```

Rubi [A] time = 0.0203585, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\sqrt{a-bx}-3a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] -3*b*Sqrt[x]*Sqrt[a - b*x] - (2*(a - b*x)^(3/2))/Sqrt[x] - 3*a*Sqrt[b]*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{\sqrt{x}}-(3b)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\ &= -3b\sqrt{x}\sqrt{a-bx}-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-\frac{1}{2}(3ab)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx\\ &= -3b\sqrt{x}\sqrt{a-bx}-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-(3ab)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\ &= -3b\sqrt{x}\sqrt{a-bx}-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-(3ab)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\ &= -3b\sqrt{x}\sqrt{a-bx}-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-3a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)\end{aligned}$$

Mathematica [C] time = 0.0114121, size = 47, normalized size = 0.71

$$-\frac{2a\sqrt{a-bx}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{bx}{a}\right)}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a - b*x]*Hypergeometric2F1[-3/2, -1/2, 1/2, (b*x)/a])/(Sqrt[x]*S
qrt[1 - (b*x)/a])
```

Maple [A] time = 0.016, size = 74, normalized size = 1.1

$$-(bx+2\,a)\sqrt{-bx+a}\frac{1}{\sqrt{x}}-\frac{3\,a}{2}\sqrt{b}\arctan\left(\sqrt{b}\left(x-\frac{a}{2\,b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)\sqrt{x\,(-bx+a)}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(3/2),x)
```

```
[Out] -(-b*x+a)^(1/2)*(b*x+2*a)/x^(1/2)-3/2*a*b^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/
(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90025, size = 292, normalized size = 4.42

$$\left[\frac{3\,a\sqrt{-b}x\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,(bx+2\,a)\sqrt{-bx+a}\sqrt{x}}{2\,x},\frac{3\,a\sqrt{b}x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-(bx+2\,a)\sqrt{-bx}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) -
2*(b*x + 2*a)*sqrt(-b*x + a)*sqrt(x))/x, (3*a*sqrt(b)*x*arctan(sqrt(-b*x +
a)/(sqrt(b)*sqrt(x))) - (b*x + 2*a)*sqrt(-b*x + a)*sqrt(x))/x]
```

Sympy [A] time = 3.21243, size = 199, normalized size = 3.02

$$\begin{cases} \frac{2ia^{\frac{3}{2}}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}}-\frac{i\sqrt{a}b\sqrt{x}}{\sqrt{-1+\frac{bx}{a}}}+3ia\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-\frac{ib^2x^{\frac{3}{2}}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \frac{|bx|}{|a|}>1 \\ -\frac{2a^{\frac{3}{2}}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}+\frac{\sqrt{a}b\sqrt{x}}{\sqrt{1-\frac{bx}{a}}}-3a\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\frac{b^2x^{\frac{3}{2}}}{\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*a**(3/2)/(sqrt(x)*sqrt(-1 + b*x/a)) - I*sqrt(a)*b*sqrt(x)/sq
rt(-1 + b*x/a) + 3*I*a*sqrt(b)*acosh(sqrt(b)*sqrt(x)/sqrt(a)) - I*b**2*x**(
3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (-2*a**(3/2)/(sqrt(x
)*sqrt(1 - b*x/a)) + sqrt(a)*b*sqrt(x)/sqrt(1 - b*x/a) - 3*a*sqrt(b)*asin(s
qrt(b)*sqrt(x)/sqrt(a)) + b**2*x**(3/2)/(sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.532 $\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=67

$$2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{a-bx}}{\sqrt{x}}$$

```
[Out] (2*b*Sqrt[a - b*x])/Sqrt[x] - (2*(a - b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*A
rcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
```

Rubi [A] time = 0.0215647, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {47, 63, 217, 203}

$$2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{a-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[a - b*x])/Sqrt[x] - (2*(a - b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*A
rcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{3x^{3/2}} - b\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx \\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx \\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right) \\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right) \\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0101356, size = 49, normalized size = 0.73

$$
-\frac{2a\sqrt{a-bx}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{bx}{a}\right)}{3x^{3/2}\sqrt{1-\frac{bx}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a - b*x]*Hypergeometric2F1[-3/2, -3/2, -1/2, (b*x)/a])/(3*x^(3/2
)*Sqrt[1 - (b*x)/a])
```

Maple [A] time = 0.016, size = 71, normalized size = 1.1

$$
-\frac{-8\,bx+2\,a}{3}\sqrt{-bx+a}\,x^{-\frac{3}{2}}+b^{\frac{3}{2}}\arctan\left(\sqrt{b}\left(x-\frac{a}{2\,b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)\sqrt{x\,(-bx+a)}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(5/2),x)
```

```
[Out] -2/3*(-b*x+a)^(1/2)*(-4*b*x+a)/x^(3/2)+b^(3/2)*arctan(b^(1/2)*(x-1/2/b*a)/(
-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73592, size = 306, normalized size = 4.57

$$\left[\frac{3\sqrt{-b}bx^2\log\left(-2\,bx-2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\,(4\,bx-a)\sqrt{-bx+a}\sqrt{x}}{3\,x^2},-\frac{2\left(3\,b^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-(4\,bx-a)\sqrt{-b\right.}{3\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-b)*b*x^2*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
+ 2*(4*b*x - a)*sqrt(-b*x + a)*sqrt(x))/x^2, -2/3*(3*b^(3/2)*x^2*arctan(sqr
t(-b*x + a)/(sqrt(b)*sqrt(x))) - (4*b*x - a)*sqrt(-b*x + a)*sqrt(x))/x^2]
```

Sympy [C] time = 4.43924, size = 190, normalized size = 2.84

$$\begin{cases} -\frac{2a\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3x}+\frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3}-2ib^{\frac{3}{2}}\log\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+ib^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)+2b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) & \text{for } \frac{|a|}{|b||x|}>1 \\ -\frac{2ia\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{3x}+\frac{8ib^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3}+ib^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)-2ib^{\frac{3}{2}}\log\left(\sqrt{-\frac{a}{bx}+1}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(5/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 8*b**(3/2)*sqrt(a/(b*x) -
 1)/3 - 2*I*b**(3/2)*log(sqrt(a)/(sqrt(b)*sqrt(x))) + I*b**(3/2)*log(a/(b*x
)) + 2*b**(3/2)*asin(sqrt(b)*sqrt(x)/sqrt(a)), Abs(a)/(Abs(b)*Abs(x)) > 1),
 (-2*I*a*sqrt(b)*sqrt(-a/(b*x) + 1)/(3*x) + 8*I*b**(3/2)*sqrt(-a/(b*x) + 1)
/3 + I*b**(3/2)*log(a/(b*x)) - 2*I*b**(3/2)*log(sqrt(-a/(b*x) + 1) + 1), Tr
ue))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.533 $\int x^{5/2}(2+bx)^{3/2}\,dx$

Optimal. Leaf size=126

$$-\frac{x^{3/2}\sqrt{bx+2}}{8b^2}+\frac{3\sqrt{x}\sqrt{bx+2}}{8b^3}-\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{5}x^{7/2}(bx+2)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{20b}$$

```
[Out] (3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^3) - (x^(3/2)*Sqrt[2 + b*x])/(8*b^2) + (x^(5
/2)*Sqrt[2 + b*x])/(20*b) + (3*x^(7/2)*Sqrt[2 + b*x])/20 + (x^(7/2)*(2 + b*
x)^(3/2))/5 - (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rubi [A] time = 0.034079, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$-\frac{x^{3/2}\sqrt{bx+2}}{8b^2}+\frac{3\sqrt{x}\sqrt{bx+2}}{8b^3}-\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{5}x^{7/2}(bx+2)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^3) - (x^(3/2)*Sqrt[2 + b*x])/(8*b^2) + (x^(5
/2)*Sqrt[2 + b*x])/(20*b) + (3*x^(7/2)*Sqrt[2 + b*x])/20 + (x^(7/2)*(2 + b*
x)^(3/2))/5 - (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{5/2}(2+bx)^{3/2}\,dx = \frac{1}{5}x^{7/2}(2+bx)^{3/2}+\frac{3}{5}\int x^{5/2}\sqrt{2+bx}\,dx$$
$$=\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}+\frac{3}{20}\int\frac{x^{5/2}}{\sqrt{2+bx}}\,dx$$
$$=\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}-\frac{\int\frac{x^{3/2}}{\sqrt{2+bx}}dx}{4b}$$
$$=-\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}+\frac{3\int\frac{\sqrt{x}}{\sqrt{2+bx}}dx}{8b^2}$$
$$=\frac{3\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}-\frac{3\int\frac{1}{\sqrt{x}\sqrt{2+bx}}dx}{8b^3}$$
$$=\frac{3\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}-\frac{3\,\text{Subst}\left(\int\frac{}{\sqrt{2}}\right.}{4b}$$
$$=\frac{3\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}-\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right.}{4b^{7/2}}$$

Mathematica [A] time = 0.0479891, size = 78, normalized size = 0.62

$$\frac{\sqrt{x}\sqrt{bx+2}\left(8b^4x^4+22b^3x^3+2b^2x^2-5bx+15\right)}{40b^3}-\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 22*b^3*x^3 + 8*b^4*x^4))/(
40*b^3) - (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Maple [A] time = 0.005, size = 123, normalized size = 1.

$$\frac{1}{5b}x^{\frac{5}{2}}(bx+2)^{\frac{5}{2}}-\frac{1}{4b^2}x^{\frac{3}{2}}(bx+2)^{\frac{5}{2}}+\frac{1}{4b^3}(bx+2)^{\frac{5}{2}}\sqrt{x}-\frac{1}{8b^3}(bx+2)^{\frac{3}{2}}\sqrt{x}-\frac{3}{8b^3}\sqrt{x}\sqrt{bx+2}-\frac{3}{8}\sqrt{x(bx+2)}\ln\left((bx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+2)^(3/2),x)
```

```
[Out] 1/5/b*x^(5/2)*(b*x+2)^(5/2)-1/4/b^2*x^(3/2)*(b*x+2)^(5/2)+1/4/b^3*x^(1/2)*(
b*x+2)^(5/2)-1/8/b^3*x^(1/2)*(b*x+2)^(3/2)-3/8*x^(1/2)*(b*x+2)^(1/2)/b^3-3/
8/b^(7/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2
+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8483, size = 398, normalized size = 3.16

$$\left[\frac{\left(8\,b^{5}x^{4}+22\,b^{4}x^{3}+2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\sqrt{x}+15\,\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{40\,b^{4}}, \frac{\left(8\,b^{5}x^{4}+22\,b^{4}x^{3}+2\,b^{3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 + 22*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*
sqrt(x) + 15*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/4
0*((8*b^5*x^4 + 22*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt
(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^4]
```

Sympy [A] time = 34.5666, size = 136, normalized size = 1.08

$$\frac{b^{2}x^{\frac{11}{2}}}{5\sqrt{bx+2}}+\frac{19bx^{\frac{9}{2}}}{20\sqrt{bx+2}}+\frac{23x^{\frac{7}{2}}}{20\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{40b\sqrt{bx+2}}+\frac{x^{\frac{3}{2}}}{8b^{2}\sqrt{bx+2}}+\frac{3\sqrt{x}}{4b^{3}\sqrt{bx+2}}-\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(3/2),x)
```

```
[Out] b**2*x**(11/2)/(5*sqrt(b*x + 2)) + 19*b*x**(9/2)/(20*sqrt(b*x + 2)) + 23*x*
*(7/2)/(20*sqrt(b*x + 2)) - x**(5/2)/(40*b*sqrt(b*x + 2)) + x**(3/2)/(8*b**
2*sqrt(b*x + 2)) + 3*sqrt(x)/(4*b**3*sqrt(b*x + 2)) - 3*asinh(sqrt(2)*sqrt(
b)*sqrt(x)/2)/(4*b**(7/2))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.534 $\int x^{3/2}(2+bx)^{3/2}\,dx$

Optimal. Leaf size=105

$$-\frac{3\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(8*b) + (x^(5/
2)*Sqrt[2 + b*x])/4 + (x^(5/2)*(2 + b*x)^(3/2))/4 + (3*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(5/2))
```

Rubi [A] time = 0.0255435, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$-\frac{3\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(8*b) + (x^(5/
2)*Sqrt[2 + b*x])/4 + (x^(5/2)*(2 + b*x)^(3/2))/4 + (3*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}(2+bx)^{3/2}\,dx &= \frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{3}{4}\int x^{3/2}\sqrt{2+bx}\,dx\\
&= \frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx\\
&= \frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}-\frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b}\\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{3\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{8b^2}\\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{}\right.}{4b^2}\\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0347363, size = 70, normalized size = 0.67

$$
\frac{\sqrt{b}\sqrt{x}\sqrt{bx+2}\left(2b^3x^3+6b^2x^2+bx-3\right)+6\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[2 + b*x]*(-3 + b*x + 6*b^2*x^2 + 2*b^3*x^3) + 6*ArcSi
nh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(8*b^(5/2))
```

Maple [A] time = 0.005, size = 108, normalized size = 1.

$$
\frac{1}{4b}x^{\frac{3}{2}}(bx+2)^{\frac{5}{2}}-\frac{1}{4b^2}(bx+2)^{\frac{5}{2}}\sqrt{x}+\frac{1}{8b^2}(bx+2)^{\frac{3}{2}}\sqrt{x}+\frac{3}{8b^2}\sqrt{x}\sqrt{bx+2}+\frac{3}{8}\sqrt{x(bx+2)}\ln\left((bx+1)\frac{1}{\sqrt{b}}+\sqrt{bx^2+}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(3/2),x)
```

```
[Out] 1/4/b*x^(3/2)*(b*x+2)^(5/2)-1/4/b^2*x^(1/2)*(b*x+2)^(5/2)+1/8/b^2*x^(1/2)*(
b*x+2)^(3/2)+3/8*x^(1/2)*(b*x+2)^(1/2)/b^2+3/8/b^(5/2)*(x*(b*x+2))^(1/2)/(b
*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91894, size = 350, normalized size = 3.33

$$\left[\frac{\left(2\,b^{4}x^{3}+6\,b^{3}x^{2}+b^{2}x-3\,b\right)\sqrt{bx+2}\sqrt{x}+3\,\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{8\,b^{3}},\ \frac{\left(2\,b^{4}x^{3}+6\,b^{3}x^{2}+b^{2}x-3\,b\right)\sqrt{bx+2}\sqrt{x}\cdots}{8\,b^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((2*b^4*x^3 + 6*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(
b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/8*((2*b^4*x^3 + 6*b
^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) - 6*sqrt(-b)*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

Sympy [A] time = 10.39, size = 117, normalized size = 1.11

$$\frac{b^{2}x^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{5bx^{\frac{7}{2}}}{4\sqrt{bx+2}}+\frac{13x^{\frac{5}{2}}}{8\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{8b\sqrt{bx+2}}-\frac{3\sqrt{x}}{4b^{2}\sqrt{bx+2}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(3/2),x)
```

```
[Out] b**2*x**(9/2)/(4*sqrt(b*x + 2)) + 5*b*x**(7/2)/(4*sqrt(b*x + 2)) + 13*x**(5
/2)/(8*sqrt(b*x + 2)) - x**(3/2)/(8*b*sqrt(b*x + 2)) - 3*sqrt(x)/(4*b**2*sq
rt(b*x + 2)) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.535 $\int \sqrt{x}(2+bx)^{3/2}\,dx$

Optimal. Leaf size=82

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\sqrt{bx+2}}{2b}$$

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/(2*b) + (x^(3/2)*Sqrt[2 + b*x])/2 + (x^(3/2)*(2 + b
*x)^(3/2))/3 - ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rubi [A] time = 0.0163102, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/(2*b) + (x^(3/2)*Sqrt[2 + b*x])/2 + (x^(3/2)*(2 + b
*x)^(3/2))/3 - ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}(2+bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(2+bx)^{3/2} + \int \sqrt{x}\sqrt{2+bx}\,dx \\
&= \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{3}x^{3/2}(2+bx)^{3/2} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx \\
&= \frac{\sqrt{x}\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{3}x^{3/2}(2+bx)^{3/2} - \frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b} \\
&= \frac{\sqrt{x}\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{3}x^{3/2}(2+bx)^{3/2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b} \\
&= \frac{\sqrt{x}\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{3}x^{3/2}(2+bx)^{3/2} - \frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0323364, size = 60, normalized size = 0.73

$$\frac{\sqrt{x}\sqrt{bx+2}\left(2b^2x^2+7bx+3\right)}{6b} - \frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(3 + 7*b*x + 2*b^2*x^2))/(6*b) - ArcSinh[(Sqrt[b]*Sq
rt[x])/Sqrt[2]]/b^(3/2)
```

Maple [A] time = 0.003, size = 87, normalized size = 1.1

$$\frac{1}{3}x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}} + \frac{1}{2}x^{\frac{3}{2}}\sqrt{bx+2} + \frac{1}{2b}\sqrt{x}\sqrt{bx+2} - \frac{1}{2}\sqrt{x(bx+2)}\ln\left((bx+1)\frac{1}{\sqrt{b}} + \sqrt{bx^2+2x}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)*x^(1/2),x)
```

```
[Out] 1/3*x^(3/2)*(b*x+2)^(3/2)+1/2*x^(3/2)*(b*x+2)^(1/2)+1/2*x^(1/2)*(b*x+2)^(1/
2)/b-1/2/b^(3/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)
+(b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79548, size = 323, normalized size = 3.94

$$\left[\frac{\left(2\,b^3x^2+7\,b^2x+3\,b\right)\sqrt{bx+2}\sqrt{x}+3\,\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{6\,b^2},\frac{\left(2\,b^3x^2+7\,b^2x+3\,b\right)\sqrt{bx+2}\sqrt{x}+6\,\sqrt{-b}\,\mathrm{ar}}{6\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 + 7*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x
 - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^2, 1/6*((2*b^3*x^2 + 7*b^2*x + 3*b
)*sqrt(b*x + 2)*sqrt(x) + 6*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(
x))))/b^2]
```

Sympy [A] time = 5.32213, size = 92, normalized size = 1.12

$$\frac{b^2x^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{11bx^{\frac{5}{2}}}{6\sqrt{bx+2}}+\frac{17x^{\frac{3}{2}}}{6\sqrt{bx+2}}+\frac{\sqrt{x}}{b\sqrt{bx+2}}-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)*x**(1/2),x)
```

```
[Out] b**2*x**(7/2)/(3*sqrt(b*x + 2)) + 11*b*x**(5/2)/(6*sqrt(b*x + 2)) + 17*x**(
3/2)/(6*sqrt(b*x + 2)) + sqrt(x)/(b*sqrt(b*x + 2)) - asinh(sqrt(2)*sqrt(b)*
sqrt(x)/2)/b**(3/2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.536 $\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$\frac{1}{2}\sqrt{x}(bx+2)^{3/2}+\frac{3}{2}\sqrt{x}\sqrt{bx+2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] (3*Sqrt[x]*Sqrt[2 + b*x])/2 + (Sqrt[x]*(2 + b*x)^(3/2))/2 + (3*ArcSinh[(Sqr
t[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rubi [A] time = 0.0109375, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$\frac{1}{2}\sqrt{x}(bx+2)^{3/2}+\frac{3}{2}\sqrt{x}\sqrt{bx+2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 + b*x])/2 + (Sqrt[x]*(2 + b*x)^(3/2))/2 + (3*ArcSinh[(Sqr
t[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}(2+bx)^{3/2}+\frac{3}{2}\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx \\
&= \frac{3}{2}\sqrt{x}\sqrt{2+bx}+\frac{1}{2}\sqrt{x}(2+bx)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx \\
&= \frac{3}{2}\sqrt{x}\sqrt{2+bx}+\frac{1}{2}\sqrt{x}(2+bx)^{3/2}+3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right) \\
&= \frac{3}{2}\sqrt{x}\sqrt{2+bx}+\frac{1}{2}\sqrt{x}(2+bx)^{3/2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.0240026, size = 48, normalized size = 0.79

$$
\frac{1}{2}\sqrt{x}\sqrt{bx+2}(bx+5)+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(5 + b*x))/2 + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
)/Sqrt[b]
```

Maple [A] time = 0.004, size = 72, normalized size = 1.2

$$
\frac{1}{2}(bx+2)^{\frac{3}{2}}\sqrt{x}+\frac{3}{2}\sqrt{x}\sqrt{bx+2}+\frac{3}{2}\sqrt{x(bx+2)}\ln\left((bx+1)\frac{1}{\sqrt{b}}+\sqrt{bx^2+2x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*(b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(b*x+2)^(1/2)+3/2*(x*(b*x+2))^(1/2)/(
b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73564, size = 279, normalized size = 4.57

$$\left[\frac{\left(b^2x+5\,b\right)\sqrt{bx+2}\sqrt{x}+3\,\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{2\,b},\frac{\left(b^2x+5\,b\right)\sqrt{bx+2}\sqrt{x}-6\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x + 5*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x + sqrt(b*x +
2)*sqrt(b)*sqrt(x) + 1))/b, 1/2*((b^2*x + 5*b)*sqrt(b*x + 2)*sqrt(x) - 6*sq
rt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b]
```

Sympy [A] time = 3.14048, size = 76, normalized size = 1.25

$$\frac{b^2x^{\frac{5}{2}}}{2\sqrt{bx+2}}+\frac{7bx^{\frac{3}{2}}}{2\sqrt{bx+2}}+\frac{5\sqrt{x}}{\sqrt{bx+2}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] b**2*x**(5/2)/(2*sqrt(b*x + 2)) + 7*b*x**(3/2)/(2*sqrt(b*x + 2)) + 5*sqrt(x
)/sqrt(b*x + 2) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.537 $\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{2(bx+2)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{bx+2}+6\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] 3*b*Sqrt[x]*Sqrt[2 + b*x] - (2*(2 + b*x)^(3/2))/Sqrt[x] + 6*Sqrt[b]*ArcSinh
[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rubi [A] time = 0.0117686, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$-\frac{2(bx+2)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{bx+2}+6\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] 3*b*Sqrt[x]*Sqrt[2 + b*x] - (2*(2 + b*x)^(3/2))/Sqrt[x] + 6*Sqrt[b]*ArcSinh
[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx = -\frac{2(2+bx)^{3/2}}{\sqrt{x}} + (3b)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx$$
$$= 3b\sqrt{x}\sqrt{2+bx} - \frac{2(2+bx)^{3/2}}{\sqrt{x}} + (3b)\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx$$
$$= 3b\sqrt{x}\sqrt{2+bx} - \frac{2(2+bx)^{3/2}}{\sqrt{x}} + (6b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= 3b\sqrt{x}\sqrt{2+bx} - \frac{2(2+bx)^{3/2}}{\sqrt{x}} + 6\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Mathematica [C] time = 0.0043338, size = 28, normalized size = 0.48

$$-\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx}{2}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] (-4*Sqrt[2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -(b*x)/2])/Sqrt[x]
```

Maple [A] time = 0.012, size = 72, normalized size = 1.2

$$(b^2x^2 - 2\,bx - 8)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+2}} + 3\,\frac{\sqrt{b}\sqrt{x\,(bx+2)}}{\sqrt{x}\sqrt{bx+2}}\ln\left(\frac{bx+1}{\sqrt{b}} + \sqrt{bx^2+2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(3/2),x)
```

```
[Out] (b^2*x^2-2*b*x-8)/x^(1/2)/(b*x+2)^(1/2)+3*b^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2
+2*x)^(1/2))*(x*(b*x+2))^(1/2)/x^(1/2)/(b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88847, size = 265, normalized size = 4.57

$$\left[\frac{3\sqrt{b}x\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+\sqrt{bx+2}(bx-4)\sqrt{x}}{x}, -\frac{6\sqrt{-b}x\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+2}(bx-4)\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + sqrt(b*x + 2)*
(b*x - 4)*sqrt(x))/x, -(6*sqrt(-b)*x*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(
x))) - sqrt(b*x + 2)*(b*x - 4)*sqrt(x))/x]
```

Sympy [A] time = 2.84553, size = 73, normalized size = 1.26

$$6\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)+\frac{b^2x^{\frac{3}{2}}}{\sqrt{bx+2}}-\frac{2b\sqrt{x}}{\sqrt{bx+2}}-\frac{8}{\sqrt{x}\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(3/2),x)
```

```
[Out] 6*sqrt(b)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**2*x**(3/2)/sqrt(b*x + 2) -
2*b*sqrt(x)/sqrt(b*x + 2) - 8/(sqrt(x)*sqrt(b*x + 2))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.538 $\int \frac{(2+bx)^{3/2}}{x^{5/2}} dx$

Optimal. Leaf size=60

$$2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(bx+2)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{bx+2}}{\sqrt{x}}$$

```
[Out] (-2*b*Sqrt[2 + b*x])/Sqrt[x] - (2*(2 + b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*
ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rubi [A] time = 0.013916, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {47, 54, 215}

$$2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(bx+2)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*b*Sqrt[2 + b*x])/Sqrt[x] - (2*(2 + b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*
ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(2+bx)^{3/2}}{x^{5/2}}\,dx = -\frac{2(2+bx)^{3/2}}{3x^{3/2}} + b\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx$$
$$= -\frac{2b\sqrt{2+bx}}{\sqrt{x}} - \frac{2(2+bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx$$
$$= -\frac{2b\sqrt{2+bx}}{\sqrt{x}} - \frac{2(2+bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2b\sqrt{2+bx}}{\sqrt{x}} - \frac{2(2+bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Mathematica [C] time = 0.0052794, size = 30, normalized size = 0.5

$$-\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx}{2}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, -(b*x)/2])/(3*x^(3/2))
```

Maple [A] time = 0.016, size = 73, normalized size = 1.2

$$-\frac{8\,b^2x^2+20\,bx+8}{3}x^{-\frac{3}{2}}\frac{1}{\sqrt{bx+2}}+b^{\frac{3}{2}}\ln\left((bx+1)\frac{1}{\sqrt{b}}+\sqrt{bx^2+2\,x}\right)\sqrt{x\,(bx+2)}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(5/2),x)
```

```
[Out] -4/3*(2*b^2*x^2+5*b*x+2)/x^(3/2)/(b*x+2)^(1/2)+b^(3/2)*ln((b*x+1)/b^(1/2)+(
b*x^2+2*x)^(1/2))*(x*(b*x+2))^(1/2)/x^(1/2)/(b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89723, size = 300, normalized size = 5.

$$\left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)-4\,(2\,bx+1)\sqrt{bx+2}\sqrt{x}}{3\,x^2}, -\frac{2\left(3\,\sqrt{-b}bx^2\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)+2\,(2\,bx+1)\sqrt{bx+\right.}{3\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b^(3/2)*x^2*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 4*(2*b*x
 + 1)*sqrt(b*x + 2)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))) + 2*(2*b*x + 1)*sqrt(b*x + 2)*sqrt(x))/x^2]
```

Sympy [A] time = 4.73204, size = 70, normalized size = 1.17

$$-\frac{8b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-b^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)+2b^{\frac{3}{2}}\log\left(\sqrt{1+\frac{2}{bx}}+1\right)-\frac{4\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(5/2),x)
```

```
[Out] -8*b**(3/2)*sqrt(1 + 2/(b*x))/3 - b**(3/2)*log(1/(b*x)) + 2*b**(3/2)*log(sq
rt(1 + 2/(b*x)) + 1) - 4*sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.539 $\int x^{5/2}(2-bx)^{3/2}\,dx$

Optimal. Leaf size=131

$$-\frac{x^{3/2}\sqrt{2-bx}}{8b^2}-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{20b}$$

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^3) - (x^(3/2)*Sqrt[2 - b*x])/(8*b^2) - (x^(
5/2)*Sqrt[2 - b*x])/(20*b) + (3*x^(7/2)*Sqrt[2 - b*x])/20 + (x^(7/2)*(2 - b
*x)^(3/2))/5 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rubi [A] time = 0.0399673, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{x^{3/2}\sqrt{2-bx}}{8b^2}-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^3) - (x^(3/2)*Sqrt[2 - b*x])/(8*b^2) - (x^(
5/2)*Sqrt[2 - b*x])/(20*b) + (3*x^(7/2)*Sqrt[2 - b*x])/20 + (x^(7/2)*(2 - b
*x)^(3/2))/5 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^{5/2}(2-bx)^{3/2}\,dx = \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3}{5}\int x^{5/2}\sqrt{2-bx}\,dx$$
$$= \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3}{20}\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{4b}$$
$$= -\frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{8b^3}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3\,\text{Subst}\left(\int \frac{}{\sqrt{2}}\right.}{4}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

Mathematica [A] time = 0.0616042, size = 79, normalized size = 0.6

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}} - \frac{\sqrt{x}\sqrt{2-bx}\left(8b^4x^4-22b^3x^3+2b^2x^2+5bx+15\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 - b*x)^(3/2),x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x]*(15 + 5*b*x + 2*b^2*x^2 - 22*b^3*x^3 + 8*b^4*x^4))/
(40*b^3) + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Maple [A] time = 0.005, size = 132, normalized size = 1.

$$-\frac{1}{5b}x^{\frac{5}{2}}(-bx+2)^{\frac{5}{2}} - \frac{1}{4b^2}x^{\frac{3}{2}}(-bx+2)^{\frac{5}{2}} - \frac{1}{4b^3}(-bx+2)^{\frac{5}{2}}\sqrt{x} + \frac{1}{8b^3}(-bx+2)^{\frac{3}{2}}\sqrt{x} + \frac{3}{8b^3}\sqrt{x}\sqrt{-bx+2} + \frac{3}{8}\sqrt{(-bx+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(3/2),x)
```

```
[Out] -1/5/b*x^(5/2)*(-b*x+2)^(5/2)-1/4/b^2*x^(3/2)*(-b*x+2)^(5/2)-1/4/b^3*x^(1/2
)*(-b*x+2)^(5/2)+1/8/b^3*x^(1/2)*(-b*x+2)^(3/2)+3/8*x^(1/2)*(-b*x+2)^(1/2)/
b^3+3/8/b^(7/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x
-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.88604, size = 405, normalized size = 3.09

$$\left[-\frac{\left(8\,b^5x^4-22\,b^4x^3+2\,b^3x^2+5\,b^2x+15\,b\right)\sqrt{-bx+2}\sqrt{x}+15\,\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{40\,b^4},-\frac{\left(8\,b^5x^4-22\,b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="fricas")

[Out] [-1/40*((8*b^5*x^4 - 22*b^4*x^3 + 2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x + 2)*sqrt(x) + 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^4, -1/40*((8*b^5*x^4 - 22*b^4*x^3 + 2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x + 2)*sqrt(x) + 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^4]

Sympy [A] time = 35.1992, size = 291, normalized size = 2.22

$$\begin{cases} -\frac{ib^2x^{\frac{11}{2}}}{5\sqrt{bx-2}}+\frac{19ibx^{\frac{9}{2}}}{20\sqrt{bx-2}}-\frac{23ix^{\frac{7}{2}}}{20\sqrt{bx-2}}-\frac{ix^{\frac{5}{2}}}{40b\sqrt{bx-2}}-\frac{ix^{\frac{3}{2}}}{8b^2\sqrt{bx-2}}+\frac{3i\sqrt{x}}{4b^3\sqrt{bx-2}}-\frac{3i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ \frac{b^2x^{\frac{11}{2}}}{5\sqrt{-bx+2}}-\frac{19bx^{\frac{9}{2}}}{20\sqrt{-bx+2}}+\frac{23x^{\frac{7}{2}}}{20\sqrt{-bx+2}}+\frac{x^{\frac{5}{2}}}{40b\sqrt{-bx+2}}+\frac{x^{\frac{3}{2}}}{8b^2\sqrt{-bx+2}}-\frac{3\sqrt{x}}{4b^3\sqrt{-bx+2}}+\frac{3\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(5/2)*(-b*x+2)**(3/2),x)

[Out] Piecewise((-I*b**2*x**(11/2)/(5*sqrt(b*x - 2)) + 19*I*b*x**(9/2)/(20*sqrt(b*x - 2)) - 23*I*x**(7/2)/(20*sqrt(b*x - 2)) - I*x**(5/2)/(40*b*sqrt(b*x - 2)) - I*x**(3/2)/(8*b**2*sqrt(b*x - 2)) + 3*I*sqrt(x)/(4*b**3*sqrt(b*x - 2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(7/2)), Abs(b*x)/2 > 1), (b**2*x**(11/2)/(5*sqrt(-b*x + 2)) - 19*b*x**(9/2)/(20*sqrt(-b*x + 2)) + 23*x**(7/2)/(20*sqrt(-b*x + 2)) + x**(5/2)/(40*b*sqrt(-b*x + 2)) + x**(3/2)/(8*b**2*sqrt(-b*x + 2)) - 3*sqrt(x)/(4*b**3*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(7/2)), True))

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="giac")

[Out] Timed out

3.540 $\int x^{3/2}(2-bx)^{3/2}\,dx$

Optimal. Leaf size=109

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(8*b) + (x^(5/
2)*Sqrt[2 - b*x])/4 + (x^(5/2)*(2 - b*x)^(3/2))/4 + (3*ArcSin[(Sqrt[b]*Sqrt
[x])/Sqrt[2]])/(4*b^(5/2))
```

Rubi [A] time = 0.0279175, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(8*b) + (x^(5/
2)*Sqrt[2 - b*x])/4 + (x^(5/2)*(2 - b*x)^(3/2))/4 + (3*ArcSin[(Sqrt[b]*Sqrt
[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^{3/2}(2-bx)^{3/2}\,dx = \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3}{4}\int x^{3/2}\sqrt{2-bx}\,dx$$
$$= \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{8b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{4b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

Mathematica [A] time = 0.0539062, size = 70, normalized size = 0.64

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}} - \frac{\sqrt{x}\sqrt{2-bx}\left(2b^3x^3 - 6b^2x^2 + bx + 3\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 - b*x)^(3/2),x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x]*(3 + b*x - 6*b^2*x^2 + 2*b^3*x^3))/(8*b^2) + (3*Arc
Sin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Maple [A] time = 0.004, size = 116, normalized size = 1.1

$$-\frac{1}{4b}x^{\frac{3}{2}}(-bx+2)^{\frac{5}{2}} - \frac{1}{4b^2}(-bx+2)^{\frac{5}{2}}\sqrt{x} + \frac{1}{8b^2}(-bx+2)^{\frac{3}{2}}\sqrt{x} + \frac{3}{8b^2}\sqrt{x}\sqrt{-bx+2} + \frac{3}{8}\sqrt{(-bx+2)x}\arctan\left(\sqrt{b}\left(x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(3/2),x)
```

```
[Out] -1/4/b*x^(3/2)*(-b*x+2)^(5/2)-1/4/b^2*x^(1/2)*(-b*x+2)^(5/2)+1/8/b^2*x^(1/2
)*(-b*x+2)^(3/2)+3/8*x^(1/2)*(-b*x+2)^(1/2)/b^2+3/8/b^(5/2)*((-b*x+2)*x)^(1
/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83026, size = 356, normalized size = 3.27

$$\left[-\frac{\left(2\,b^4x^3-6\,b^3x^2+b^2x+3\,b\right)\sqrt{-bx+2}\sqrt{x}+3\,\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{8\,b^3}, -\frac{\left(2\,b^4x^3-6\,b^3x^2+b^2x+3\,b\right)\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*b^4*x^3 - 6*b^3*x^2 + b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqr
t(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, -1/8*((2*b^4*x^
3 - 6*b^3*x^2 + b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 6*sqrt(b)*arctan(sqrt
(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

Sympy [A] time = 10.743, size = 252, normalized size = 2.31

$$\begin{cases} -\frac{ib^2x^{\frac{9}{2}}}{4\sqrt{bx-2}} + \frac{5ibx^{\frac{7}{2}}}{4\sqrt{bx-2}} - \frac{13ix^{\frac{5}{2}}}{8\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{8b\sqrt{bx-2}} + \frac{3i\sqrt{x}}{4b^2\sqrt{bx-2}} - \frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{for}\ \frac{|bx|}{2} > 1 \\ \frac{b^2x^{\frac{9}{2}}}{4\sqrt{-bx+2}} - \frac{5bx^{\frac{7}{2}}}{4\sqrt{-bx+2}} + \frac{13x^{\frac{5}{2}}}{8\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{8b\sqrt{-bx+2}} - \frac{3\sqrt{x}}{4b^2\sqrt{-bx+2}} + \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(9/2)/(4*sqrt(b*x - 2)) + 5*I*b*x**(7/2)/(4*sqrt(b*x
- 2)) - 13*I*x**(5/2)/(8*sqrt(b*x - 2)) - I*x**(3/2)/(8*b*sqrt(b*x - 2)) +
3*I*sqrt(x)/(4*b**2*sqrt(b*x - 2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(
4*b**(5/2)), Abs(b*x)/2 > 1), (b**2*x**(9/2)/(4*sqrt(-b*x + 2)) - 5*b*x**(7
/2)/(4*sqrt(-b*x + 2)) + 13*x**(5/2)/(8*sqrt(-b*x + 2)) + x**(3/2)/(8*b*sqr
t(-b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqrt(b)*s
qrt(x)/2)/(4*b**(5/2)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.541 $\int \sqrt{x}(2-bx)^{3/2}\,dx$

Optimal. Leaf size=84

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x])/(2*b) + (x^(3/2)*Sqrt[2 - b*x])/2 + (x^(3/2)*(2 -
b*x)^(3/2))/3 + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rubi [A] time = 0.0157307, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 - b*x)^(3/2),x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x])/(2*b) + (x^(3/2)*Sqrt[2 - b*x])/2 + (x^(3/2)*(2 -
b*x)^(3/2))/3 + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{x}(2-bx)^{3/2}\,dx = \frac{1}{3}x^{3/2}(2-bx)^{3/2} + \int \sqrt{x}\sqrt{2-bx}\,dx$$
$$= \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{1}{3}x^{3/2}(2-bx)^{3/2} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{1}{3}x^{3/2}(2-bx)^{3/2} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{1}{3}x^{3/2}(2-bx)^{3/2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{1}{3}x^{3/2}(2-bx)^{3/2} + \frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.040948, size = 60, normalized size = 0.71

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}} - \frac{\sqrt{x}\sqrt{2-bx}\left(2b^2x^2-7bx+3\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 - b*x)^(3/2),x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x]*(3 - 7*b*x + 2*b^2*x^2))/(6*b) + ArcSin[(Sqrt[b]*Sq
rt[x])/Sqrt[2]]/b^(3/2)
```

Maple [A] time = 0.004, size = 94, normalized size = 1.1

$$\frac{1}{3}x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}} + \frac{1}{2}x^{\frac{3}{2}}\sqrt{-bx+2} - \frac{1}{2b}\sqrt{x}\sqrt{-bx+2} + \frac{1}{2}\sqrt{(-bx+2)x}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2x}}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{-bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)*x^(1/2),x)
```

```
[Out] 1/3*x^(3/2)*(-b*x+2)^(3/2)+1/2*x^(3/2)*(-b*x+2)^(1/2)-1/2*x^(1/2)*(-b*x+2)^
(1/2)/b+1/2/b^(3/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan(b^(1/2
)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86114, size = 329, normalized size = 3.92

$$\left[-\frac{\left(2\,b^{3}x^{2}-7\,b^{2}x+3\,b\right)\sqrt{-bx+2}\sqrt{x}+3\,\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{6\,b^{2}},-\frac{\left(2\,b^{3}x^{2}-7\,b^{2}x+3\,b\right)\sqrt{-bx+2}\sqrt{x}}{6\,b^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*b^3*x^2 - 7*b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(
-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^2, -1/6*((2*b^3*x^2 - 7*b^2*
x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*
sqrt(x))))/b^2]
```

Sympy [A] time = 5.46052, size = 199, normalized size = 2.37

$$\begin{cases} -\frac{ib^{2}x^{\frac{7}{2}}}{3\sqrt{bx-2}}+\frac{11ibx^{\frac{5}{2}}}{6\sqrt{bx-2}}-\frac{17ix^{\frac{3}{2}}}{6\sqrt{bx-2}}+\frac{i\sqrt{x}}{b\sqrt{bx-2}}-\frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ \frac{b^{2}x^{\frac{7}{2}}}{3\sqrt{-bx+2}}-\frac{11bx^{\frac{5}{2}}}{6\sqrt{-bx+2}}+\frac{17x^{\frac{3}{2}}}{6\sqrt{-bx+2}}-\frac{\sqrt{x}}{b\sqrt{-bx+2}}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)*x**(1/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(7/2)/(3*sqrt(b*x - 2)) + 11*I*b*x**(5/2)/(6*sqrt(b*x
 - 2)) - 17*I*x**(3/2)/(6*sqrt(b*x - 2)) + I*sqrt(x)/(b*sqrt(b*x - 2)) - I*
acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2), Abs(b*x)/2 > 1), (b**2*x**(7/2)/
(3*sqrt(-b*x + 2)) - 11*b*x**(5/2)/(6*sqrt(-b*x + 2)) + 17*x**(3/2)/(6*sqrt
(-b*x + 2)) - sqrt(x)/(b*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/
b**(3/2), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.542 $\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=63

$$\frac{1}{2}\sqrt{x}(2-bx)^{3/2}+\frac{3}{2}\sqrt{x}\sqrt{2-bx}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] (3*Sqrt[x]*Sqrt[2 - b*x])/2 + (Sqrt[x]*(2 - b*x)^(3/2))/2 + (3*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rubi [A] time = 0.011925, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{1}{2}\sqrt{x}(2-bx)^{3/2}+\frac{3}{2}\sqrt{x}\sqrt{2-bx}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 - b*x])/2 + (Sqrt[x]*(2 - b*x)^(3/2))/2 + (3*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx = \frac{1}{2}\sqrt{x}(2-bx)^{3/2} + \frac{3}{2}\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx$$
$$= \frac{3}{2}\sqrt{x}\sqrt{2-bx} + \frac{1}{2}\sqrt{x}(2-bx)^{3/2} + \frac{3}{2}\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx$$
$$= \frac{3}{2}\sqrt{x}\sqrt{2-bx} + \frac{1}{2}\sqrt{x}(2-bx)^{3/2} + 3\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{3}{2}\sqrt{x}\sqrt{2-bx} + \frac{1}{2}\sqrt{x}(2-bx)^{3/2} + \frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0275864, size = 49, normalized size = 0.78

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}} - \frac{1}{2}\sqrt{x}\sqrt{2-bx}(bx-5)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x]*(-5 + b*x))/2 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]
])/Sqrt[b]
```

Maple [A] time = 0.005, size = 78, normalized size = 1.2

$$\frac{1}{2}(-bx+2)^{\frac{3}{2}}\sqrt{x} + \frac{3}{2}\sqrt{x}\sqrt{-bx+2} + \frac{3}{2}\sqrt{(-bx+2)\,x}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2\,x}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+2}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*(-b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(-b*x+2)^(1/2)+3/2*((-b*x+2)*x)^(1/2
)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83868, size = 286, normalized size = 4.54

$$\left[-\frac{\left(b^2x-5\,b\right)\sqrt{-bx+2}\sqrt{x}+3\,\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{2\,b},-\frac{\left(b^2x-5\,b\right)\sqrt{-bx+2}\sqrt{x}+6\,\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*x - 5*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(-b*x + sqrt(-b
*x + 2)*sqrt(-b)*sqrt(x) + 1))/b, -1/2*((b^2*x - 5*b)*sqrt(-b*x + 2)*sqrt(x
) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b]
```

Sympy [A] time = 3.24902, size = 167, normalized size = 2.65

$$\begin{cases} -\frac{ib^2x^{\frac{5}{2}}}{2\sqrt{bx-2}}+\frac{7ibx^{\frac{3}{2}}}{2\sqrt{bx-2}}-\frac{5i\sqrt{x}}{\sqrt{bx-2}}-\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for } \frac{|bx|}{2}>1 \\ \frac{b^2x^{\frac{5}{2}}}{2\sqrt{-bx+2}}-\frac{7bx^{\frac{3}{2}}}{2\sqrt{-bx+2}}+\frac{5\sqrt{x}}{\sqrt{-bx+2}}+\frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(5/2)/(2*sqrt(b*x - 2)) + 7*I*b*x**(3/2)/(2*sqrt(b*x
- 2)) - 5*I*sqrt(x)/sqrt(b*x - 2) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sq
rt(b), Abs(b*x)/2 > 1), (b**2*x**(5/2)/(2*sqrt(-b*x + 2)) - 7*b*x**(3/2)/(2
*sqrt(-b*x + 2)) + 5*sqrt(x)/sqrt(-b*x + 2) + 3*asin(sqrt(2)*sqrt(b)*sqrt(x
)/2)/sqrt(b), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.543 $\int \frac{(2-bx)^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=60

$$-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\sqrt{2-bx}-6\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -3*b*Sqrt[x]*Sqrt[2 - b*x] - (2*(2 - b*x)^(3/2))/Sqrt[x] - 6*Sqrt[b]*ArcSin
[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rubi [A] time = 0.0115209, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {47, 50, 54, 216}

$$-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\sqrt{2-bx}-6\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/x^(3/2),x]
```


```
[Out] -3*b*Sqrt[x]*Sqrt[2 - b*x] - (2*(2 - b*x)^(3/2))/Sqrt[x] - 6*Sqrt[b]*ArcSin
[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(2-bx)^{3/2}}{\sqrt{x}} - (3b)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx \\ &= -3b\sqrt{x}\sqrt{2-bx} - \frac{2(2-bx)^{3/2}}{\sqrt{x}} - (3b)\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx \\ &= -3b\sqrt{x}\sqrt{2-bx} - \frac{2(2-bx)^{3/2}}{\sqrt{x}} - (6b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right) \\ &= -3b\sqrt{x}\sqrt{2-bx} - \frac{2(2-bx)^{3/2}}{\sqrt{x}} - 6\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [C] time = 0.0051878, size = 28, normalized size = 0.47

$$-\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{bx}{2}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] (-4*Sqrt[2]*Hypergeometric2F1[-3/2, -1/2, 1/2, (b*x)/2])/Sqrt[x]
```

Maple [B] time = 0.018, size = 97, normalized size = 1.6

$$(b^2x^2+2bx-8)\sqrt{(-bx+2)x}\frac{1}{\sqrt{-x(bx-2)}}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+2}}-3\frac{\sqrt{b}\sqrt{(-bx+2)x}}{\sqrt{x}\sqrt{-bx+2}}\arctan\left(\frac{\sqrt{b}}{\sqrt{-bx^2+2x}}\left(x-b^{-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(3/2),x)
```

```
[Out] (b^2*x^2+2*b*x-8)/(-x*(b*x-2))^(1/2)*((-b*x+2)*x)^(1/2)/x^(1/2)/(-b*x+2)^(1
/2)-3*b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))*((-b*x+2)*x)^(1/2)
/x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85379, size = 267, normalized size = 4.45

$$\left[\frac{3\sqrt{-b}x\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)-(bx+4)\sqrt{-bx+2}\sqrt{x}}{x},\frac{6\sqrt{b}x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-(bx+4)\sqrt{-bx+2}\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - (b*x + 4)*
sqrt(-b*x + 2)*sqrt(x))/x, (6*sqrt(b)*x*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt
(x))) - (b*x + 4)*sqrt(-b*x + 2)*sqrt(x))/x]
```

Sympy [A] time = 2.92056, size = 160, normalized size = 2.67

$$\begin{cases} 6i\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)-\frac{ib^2x^{\frac{3}{2}}}{\sqrt{bx-2}}-\frac{2ib\sqrt{x}}{\sqrt{bx-2}}+\frac{8i}{\sqrt{x}\sqrt{bx-2}} & \text{for } \frac{|bx|}{2}>1 \\ -6\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)+\frac{b^2x^{\frac{3}{2}}}{\sqrt{-bx+2}}+\frac{2b\sqrt{x}}{\sqrt{-bx+2}}-\frac{8}{\sqrt{x}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(3/2),x)
```

```
[Out] Piecewise((6*I*sqrt(b)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2) - I*b**2*x**(3/2)/s
qrt(b*x - 2) - 2*I*b*sqrt(x)/sqrt(b*x - 2) + 8*I/(sqrt(x)*sqrt(b*x - 2)), A
bs(b*x)/2 > 1), (-6*sqrt(b)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**2*x**(3/2)
/sqrt(-b*x + 2) + 2*b*sqrt(x)/sqrt(-b*x + 2) - 8/(sqrt(x)*sqrt(-b*x + 2)),
True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.544 $\int \frac{(2-bx)^{3/2}}{x^{5/2}} dx$

Optimal. Leaf size=62

$$2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{2-bx}}{\sqrt{x}}$$

```
[Out] (2*b*Sqrt[2 - b*x])/Sqrt[x] - (2*(2 - b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*A
rcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rubi [A] time = 0.0132896, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {47, 54, 216}

$$2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[2 - b*x])/Sqrt[x] - (2*(2 - b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*A
rcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-bx)^{3/2}}{x^{5/2}}\,dx &= -\frac{2(2-bx)^{3/2}}{3x^{3/2}} - b\int \frac{\sqrt{2-bx}}{x^{3/2}}\,dx\\ &= \frac{2b\sqrt{2-bx}}{\sqrt{x}} - \frac{2(2-bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx\\ &= \frac{2b\sqrt{2-bx}}{\sqrt{x}} - \frac{2(2-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2b\sqrt{2-bx}}{\sqrt{x}} - \frac{2(2-bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [C] time = 0.0053345, size = 30, normalized size = 0.48

$$-\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{bx}{2}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (b*x)/2])/(3*x^(3/2))
```

Maple [B] time = 0.018, size = 98, normalized size = 1.6

$$-\frac{8\,b^2x^2-20\,bx+8}{3}\sqrt{(-bx+2)\,x}x^{-\frac{3}{2}}\frac{1}{\sqrt{-x\,(bx-2)}}\frac{1}{\sqrt{-bx+2}}+b^{\frac{3}{2}}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2\,x}}\right)\sqrt{(-bx+2)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(5/2),x)
```

```
[Out] -4/3*(2*b^2*x^2-5*b*x+2)/x^(3/2)/(-x*(b*x-2))^(1/2)*((-b*x+2)*x)^(1/2)/(-b*
x+2)^(1/2)+b^(3/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))*((-b*x+2)*x)^
(1/2)/x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92141, size = 304, normalized size = 4.9

$$\left[\frac{3\sqrt{-b}bx^2\log\left(-bx-\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)+4\,(2\,bx-1)\sqrt{-bx+2}\sqrt{x}}{3\,x^2}, -\frac{2\left(3\,b^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-2\,(2\,bx-1)\sqrt{-}\right.}{3\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-b)*b*x^2*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + 4*
(2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x))/x^2, -2/3*(3*b^(3/2)*x^2*arctan(sqrt(-b
*x + 2)/(sqrt(b)*sqrt(x))) - 2*(2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x))/x^2]
```

Sympy [C] time = 4.92877, size = 182, normalized size = 2.94

$$\begin{cases} \frac{8b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3} + ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right) - 2ib^{\frac{3}{2}}\log\left(\frac{1}{\sqrt{b}\sqrt{x}}\right) + 2b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right) - \frac{4\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for } \frac{2}{|bx|} > 1 \\ \frac{8ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3} + ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right) - 2ib^{\frac{3}{2}}\log\left(\sqrt{1-\frac{2}{bx}}+1\right) - \frac{4i\sqrt{b}\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(5/2),x)
```

```
[Out] Piecewise((8*b**(3/2)*sqrt(-1 + 2/(b*x))/3 + I*b**(3/2)*log(1/(b*x)) - 2*I*
b**(3/2)*log(1/(sqrt(b)*sqrt(x))) + 2*b**(3/2)*asin(sqrt(2)*sqrt(b)*sqrt(x)
/2) - 4*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*x), 2/Abs(b*x) > 1), (8*I*b**(3/2)*sq
rt(1 - 2/(b*x))/3 + I*b**(3/2)*log(1/(b*x)) - 2*I*b**(3/2)*log(sqrt(1 - 2/(
b*x)) + 1) - 4*I*sqrt(b)*sqrt(1 - 2/(b*x))/(3*x), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.545 $\int x^{5/2}(a+bx)^{5/2}\,dx$

Optimal. Leaf size=164

$$-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{512b^{7/2}}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}\cdot$$

```
[Out] (5*a^5*Sqrt[x]*Sqrt[a + b*x])/(512*b^3) - (5*a^4*x^(3/2)*Sqrt[a + b*x])/(76
8*b^2) + (a^3*x^(5/2)*Sqrt[a + b*x])/(192*b) + (a^2*x^(7/2)*Sqrt[a + b*x])/
32 + (a*x^(7/2)*(a + b*x)^(3/2))/12 + (x^(7/2)*(a + b*x)^(5/2))/6 - (5*a^6*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(512*b^(7/2))
```

Rubi [A] time = 0.0632622, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{512b^{7/2}}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^(5/2),x]
```

```
[Out] (5*a^5*Sqrt[x]*Sqrt[a + b*x])/(512*b^3) - (5*a^4*x^(3/2)*Sqrt[a + b*x])/(76
8*b^2) + (a^3*x^(5/2)*Sqrt[a + b*x])/(192*b) + (a^2*x^(7/2)*Sqrt[a + b*x])/
32 + (a*x^(7/2)*(a + b*x)^(3/2))/12 + (x^(7/2)*(a + b*x)^(5/2))/6 - (5*a^6*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(512*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}(a+bx)^{5/2}\,dx &= \frac{1}{6}x^{7/2}(a+bx)^{5/2}+\frac{1}{12}(5a)\int x^{5/2}(a+bx)^{3/2}\,dx\\
&= \frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x^{7/2}(a+bx)^{5/2}+\frac{1}{8}a^2\int x^{5/2}\sqrt{a+bx}\,dx\\
&= \frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x^{7/2}(a+bx)^{5/2}+\frac{1}{64}a^3\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx\\
&= \frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x^{7/2}(a+bx)^{5/2}-\frac{\left(5a^4\right)\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{384b}\\
&= -\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x^{7/2}(a+bx)^{5/2}+\\
&= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x\\
&= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x\\
&= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x\\
&= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x
\end{aligned}
$$

Mathematica [A] time = 0.233578, size = 118, normalized size = 0.72

$$
\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(8a^3b^2x^2+432a^2b^3x^3-10a^4bx+15a^5+640ab^4x^4+256b^5x^5\right)-\frac{15a^{11/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{1536b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^5 - 10*a^4*b*x + 8*a^3*b^2*x^2 + 432*
a^2*b^3*x^3 + 640*a*b^4*x^4 + 256*b^5*x^5) - (15*a^(11/2)*ArcSinh[(Sqrt[b]*
Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(1536*b^(7/2))
```

Maple [A] time = 0.003, size = 156, normalized size = 1.

$$
\frac{1}{6b}x^{\frac{5}{2}}(bx+a)^{\frac{7}{2}}-\frac{a}{12b^2}x^{\frac{3}{2}}(bx+a)^{\frac{7}{2}}+\frac{a^2}{32b^3}\sqrt{x}(bx+a)^{\frac{7}{2}}-\frac{a^3}{192b^3}(bx+a)^{\frac{5}{2}}\sqrt{x}-\frac{5a^4}{768b^3}(bx+a)^{\frac{3}{2}}\sqrt{x}-\frac{5a^5}{512b^3}\sqrt{x}\sqrt{bx}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(5/2),x)
```

```
[Out] 1/6/b*x^(5/2)*(b*x+a)^(7/2)-1/12/b^2*a*x^(3/2)*(b*x+a)^(7/2)+1/32/b^3*a^2*x
^(1/2)*(b*x+a)^(7/2)-1/192/b^3*a^3*(b*x+a)^(5/2)*x^(1/2)-5/768/b^3*a^4*(b*x
+a)^(3/2)*x^(1/2)-5/512*a^5*x^(1/2)*(b*x+a)^(1/2)/b^3-5/1024/b^(7/2)*a^6*(x
```

```
*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9423, size = 520, normalized size = 3.17

$$\left[\frac{15\,a^{6}\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(256\,b^{6}x^{5}+640\,ab^{5}x^{4}+432\,a^{2}b^{4}x^{3}+8\,a^{3}b^{3}x^{2}-10\,a^{4}b^{2}x+15\,a^{5}b\right)\sqrt{b}}{3072\,b^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3072*(15*a^6*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(256*b^6*x^5 + 640*a*b^5*x^4 + 432*a^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b
^2*x + 15*a^5*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/1536*(15*a^6*sqrt(-b)*arctan
(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (256*b^6*x^5 + 640*a*b^5*x^4 + 432*a
^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b^2*x + 15*a^5*b)*sqrt(b*x + a)*sqrt(x)
)/b^4]
```

Sympy [A] time = 102.469, size = 207, normalized size = 1.26

$$\frac{5a^{\frac{11}{2}}\sqrt{x}}{512b^{3}\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{1+\frac{bx}{a}}}+\frac{55a^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{1+\frac{bx}{a}}}+\frac{67a^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{1+\frac{bx}{a}}}+\frac{7\sqrt{a}b^{2}x^{\frac{11}{2}}}{12\sqrt{1+\frac{bx}{a}}}-\frac{5a^{6}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(5/2),x)
```

```
[Out] 5*a**(11/2)*sqrt(x)/(512*b**3*sqrt(1 + b*x/a)) + 5*a**(9/2)*x**(3/2)/(1536*
b**2*sqrt(1 + b*x/a)) - a**(7/2)*x**(5/2)/(768*b*sqrt(1 + b*x/a)) + 55*a**(
5/2)*x**(7/2)/(192*sqrt(1 + b*x/a)) + 67*a**(3/2)*b*x**(9/2)/(96*sqrt(1 + b
*x/a)) + 7*sqrt(a)*b**2*x**(11/2)/(12*sqrt(1 + b*x/a)) - 5*a**6*asinh(sqrt(
b)*sqrt(x)/sqrt(a))/(512*b**(7/2)) + b**3*x**(13/2)/(6*sqrt(a)*sqrt(1 + b*x
/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.546 $\int x^{3/2}(a+bx)^{5/2}\,dx$

Optimal. Leaf size=140

$$-\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{5/2}}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a + b*x])/(128*b^2) + (a^3*x^(3/2)*Sqrt[a + b*x])/(64*
b) + (a^2*x^(5/2)*Sqrt[a + b*x])/16 + (a*x^(5/2)*(a + b*x)^(3/2))/8 + (x^(5
/2)*(a + b*x)^(5/2))/5 + (3*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(
128*b^(5/2))
```

Rubi [A] time = 0.0476703, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{5/2}}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^(5/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a + b*x])/(128*b^2) + (a^3*x^(3/2)*Sqrt[a + b*x])/(64*
b) + (a^2*x^(5/2)*Sqrt[a + b*x])/16 + (a*x^(5/2)*(a + b*x)^(3/2))/8 + (x^(5
/2)*(a + b*x)^(5/2))/5 + (3*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(
128*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^{3/2}(a+bx)^{5/2}\,dx = \frac{1}{5}x^{5/2}(a+bx)^{5/2} + \frac{1}{2}a\int x^{3/2}(a+bx)^{3/2}\,dx$$
$$= \frac{1}{8}ax^{5/2}(a+bx)^{3/2} + \frac{1}{5}x^{5/2}(a+bx)^{5/2} + \frac{1}{16}\left(3a^2\right)\int x^{3/2}\sqrt{a+bx}\,dx$$
$$= \frac{1}{16}a^2x^{5/2}\sqrt{a+bx} + \frac{1}{8}ax^{5/2}(a+bx)^{3/2} + \frac{1}{5}x^{5/2}(a+bx)^{5/2} + \frac{1}{32}a^3\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx$$
$$= \frac{a^3x^{3/2}\sqrt{a+bx}}{64b} + \frac{1}{16}a^2x^{5/2}\sqrt{a+bx} + \frac{1}{8}ax^{5/2}(a+bx)^{3/2} + \frac{1}{5}x^{5/2}(a+bx)^{5/2} - \frac{\left(3a^4\right)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{128b}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2} + \frac{a^3x^{3/2}\sqrt{a+bx}}{64b} + \frac{1}{16}a^2x^{5/2}\sqrt{a+bx} + \frac{1}{8}ax^{5/2}(a+bx)^{3/2} + \frac{1}{5}x^{5/2}(a+bx)^{5/2} + \frac{(3}{}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2} + \frac{a^3x^{3/2}\sqrt{a+bx}}{64b} + \frac{1}{16}a^2x^{5/2}\sqrt{a+bx} + \frac{1}{8}ax^{5/2}(a+bx)^{3/2} + \frac{1}{5}x^{5/2}(a+bx)^{5/2} + \frac{(3}{}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2} + \frac{a^3x^{3/2}\sqrt{a+bx}}{64b} + \frac{1}{16}a^2x^{5/2}\sqrt{a+bx} + \frac{1}{8}ax^{5/2}(a+bx)^{3/2} + \frac{1}{5}x^{5/2}(a+bx)^{5/2} + \frac{(3}{}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2} + \frac{a^3x^{3/2}\sqrt{a+bx}}{64b} + \frac{1}{16}a^2x^{5/2}\sqrt{a+bx} + \frac{1}{8}ax^{5/2}(a+bx)^{3/2} + \frac{1}{5}x^{5/2}(a+bx)^{5/2} + \frac{3a}{}$$

Mathematica [A] time = 0.131813, size = 107, normalized size = 0.76

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(248a^2b^2x^2+10a^3bx-15a^4+336ab^3x^3+128b^4x^4\right)+\frac{15a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^4 + 10*a^3*b*x + 248*a^2*b^2*x^2 + 3
36*a*b^3*x^3 + 128*b^4*x^4) + (15*a^(9/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]
])/Sqrt[1 + (b*x)/a]))/(640*b^(5/2))
```

Maple [A] time = 0.004, size = 138, normalized size = 1.

$$\frac{1}{5b}x^{\frac{3}{2}}(bx+a)^{\frac{7}{2}} - \frac{3a}{40b^2}\sqrt{x}(bx+a)^{\frac{7}{2}} + \frac{a^2}{80b^2}(bx+a)^{\frac{5}{2}}\sqrt{x} + \frac{a^3}{64b^2}(bx+a)^{\frac{3}{2}}\sqrt{x} + \frac{3a^4}{128b^2}\sqrt{x}\sqrt{bx+a} + \frac{3a^5}{256}\sqrt{x(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(5/2),x)
```

```
[Out] 1/5/b*x^(3/2)*(b*x+a)^(7/2)-3/40/b^2*a*x^(1/2)*(b*x+a)^(7/2)+1/80/b^2*a^2*(
b*x+a)^(5/2)*x^(1/2)+1/64/b^2*a^3*(b*x+a)^(3/2)*x^(1/2)+3/128*a^4*x^(1/2)*(
b*x+a)^(1/2)/b^2+3/256/b^(5/2)*a^5*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*
ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94763, size = 477, normalized size = 3.41

$$\left[\frac{15\,a^5\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(128\,b^5x^4+336\,ab^4x^3+248\,a^2b^3x^2+10\,a^3b^2x-15\,a^4b\right)\sqrt{bx+a}\sqrt{x}}{1280\,b^3},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/1280*(15*a^5*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*a^3*b^2*x - 15*a^4*b)
*sqrt(b*x + a)*sqrt(x))/b^3, -1/640*(15*a^5*sqrt(-b)*arctan(sqrt(b*x + a)*s
qrt(-b)/(b*sqrt(x))) - (128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*
a^3*b^2*x - 15*a^4*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

Sympy [A] time = 50.7545, size = 180, normalized size = 1.29

$$-\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{1+\frac{bx}{a}}}+\frac{129a^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{1+\frac{bx}{a}}}+\frac{73a^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{1+\frac{bx}{a}}}+\frac{29\sqrt{a}b^2x^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}}+\frac{3a^5\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{b^3x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(5/2),x)
```

```
[Out] -3*a**(9/2)*sqrt(x)/(128*b**2*sqrt(1 + b*x/a)) - a**(7/2)*x**(3/2)/(128*b*s
qrt(1 + b*x/a)) + 129*a**(5/2)*x**(5/2)/(320*sqrt(1 + b*x/a)) + 73*a**(3/2)
*b*x**(7/2)/(80*sqrt(1 + b*x/a)) + 29*sqrt(a)*b**2*x**(9/2)/(40*sqrt(1 + b*
x/a)) + 3*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(5/2)) + b**3*x**(11/
2)/(5*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.547 $\int \sqrt{x}(a+bx)^{5/2}\,dx$

Optimal. Leaf size=116

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{3/2}}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

```
[Out] (5*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b) + (5*a^2*x^(3/2)*Sqrt[a + b*x])/32 + (
5*a*x^(3/2)*(a + b*x)^(3/2))/24 + (x^(3/2)*(a + b*x)^(5/2))/4 - (5*a^4*ArcT
anh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(3/2))
```

Rubi [A] time = 0.0382128, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{3/2}}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^(5/2),x]
```

```
[Out] (5*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b) + (5*a^2*x^(3/2)*Sqrt[a + b*x])/32 + (
5*a*x^(3/2)*(a + b*x)^(3/2))/24 + (x^(3/2)*(a + b*x)^(5/2))/4 - (5*a^4*ArcT
anh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(a+bx)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(a+bx)^{5/2}+\frac{1}{8}(5a)\int \sqrt{x}(a+bx)^{3/2}\,dx\\ &= \frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}+\frac{1}{16}\left(5a^2\right)\int \sqrt{x}\sqrt{a+bx}\,dx\\ &= \frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}+\frac{1}{64}\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx\\ &= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-\frac{\left(5a^4\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}d}{128b}\\ &= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-\frac{\left(5a^4\right)\text{Subst}\left(\int \frac{}{}\right.}{64}\\ &= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-\frac{\left(5a^4\right)\text{Subst}\left(\int \frac{}{1}\right.}{64}\\ &= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{}}{\sqrt{a+b}}\right.}{64b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.179235, size = 96, normalized size = 0.83

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(118a^2bx+15a^3+136ab^2x^2+48b^3x^3\right)-\frac{15a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{192b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^3 + 118*a^2*b*x + 136*a*b^2*x^2 + 48*
b^3*x^3) - (15*a^(7/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a
]))/(192*b^(3/2))
```

Maple [A] time = 0.005, size = 111, normalized size = 1.

$$\frac{1}{4}x^{\frac{3}{2}}(bx+a)^{\frac{5}{2}}+\frac{5a}{24}x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}+\frac{5a^2}{32}x^{\frac{3}{2}}\sqrt{bx+a}+\frac{5a^3}{64b}\sqrt{x}\sqrt{bx+a}-\frac{5a^4}{128}\sqrt{x(bx+a)}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*x^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(b*x+a)^(5/2)+5/24*a*x^(3/2)*(b*x+a)^(3/2)+5/32*a^2*x^(3/2)*(b*
x+a)^(1/2)+5/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b-5/128*a^4/b^(3/2)*(x*(b*x+a))^(
1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86037, size = 425, normalized size = 3.66

$$\left[\frac{15\,a^4\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(48\,b^4x^3+136\,ab^3x^2+118\,a^2b^2x+15\,a^3b\right)\sqrt{bx+a}\sqrt{x}}{384\,b^2}, \frac{15\,a^4\sqrt{-b}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt
(x))/b^2, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))
 + (48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sq
rt(x))/b^2]
```

Sympy [A] time = 19.5142, size = 155, normalized size = 1.34

$$\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1+\frac{bx}{a}}}+\frac{133a^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1+\frac{bx}{a}}}+\frac{127a^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1+\frac{bx}{a}}}+\frac{23\sqrt{a}b^2x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}}-\frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}}+\frac{b^3x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*x**(1/2),x)
```

```
[Out] 5*a**(7/2)*sqrt(x)/(64*b*sqrt(1 + b*x/a)) + 133*a**(5/2)*x**(3/2)/(192*sqrt
(1 + b*x/a)) + 127*a**(3/2)*b*x**(5/2)/(96*sqrt(1 + b*x/a)) + 23*sqrt(a)*b*
*2*x**(7/2)/(24*sqrt(1 + b*x/a)) - 5*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(6
4*b**(3/2)) + b**3*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.548 $\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=92

$$\frac{5}{8}a^2\sqrt{x}\sqrt{a+bx}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}+\frac{5}{12}a\sqrt{x}(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}$$

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a + b*x])/8 + (5*a*Sqrt[x]*(a + b*x)^(3/2))/12 + (Sqrt[
x]*(a + b*x)^(5/2))/3 + (5*a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(8
*Sqrt[b])
```

Rubi [A] time = 0.0277851, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{5}{8}a^2\sqrt{x}\sqrt{a+bx}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}+\frac{5}{12}a\sqrt{x}(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a + b*x])/8 + (5*a*Sqrt[x]*(a + b*x)^(3/2))/12 + (Sqrt[
x]*(a + b*x)^(5/2))/3 + (5*a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(8
*Sqrt[b])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}(a+bx)^{5/2} + \frac{1}{6}(5a)\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx$$
$$= \frac{5}{12}a\sqrt{x}(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}(a+bx)^{5/2} + \frac{1}{8}\left(5a^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$$
$$= \frac{5}{8}a^2\sqrt{x}\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}(a+bx)^{5/2} + \frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx$$
$$= \frac{5}{8}a^2\sqrt{x}\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}(a+bx)^{5/2} + \frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{5}{8}a^2\sqrt{x}\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}(a+bx)^{5/2} + \frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)$$
$$= \frac{5}{8}a^2\sqrt{x}\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}(a+bx)^{5/2} + \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}$$

Mathematica [A] time = 0.110568, size = 80, normalized size = 0.87

$$\frac{1}{24}\sqrt{a+bx}\left(\sqrt{x}\left(33a^2+26abx+8b^2x^2\right)+\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx}{a}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[x]*(33*a^2 + 26*a*b*x + 8*b^2*x^2) + (15*a^(5/2)*ArcSi
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x)/a])))/24
```

Maple [A] time = 0.004, size = 93, normalized size = 1.

$$\frac{1}{3}(bx+a)^{\frac{5}{2}}\sqrt{x}+\frac{5a}{12}(bx+a)^{\frac{3}{2}}\sqrt{x}+\frac{5a^2}{8}\sqrt{x}\sqrt{bx+a}+\frac{5a^3}{16}\sqrt{x(bx+a)}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*(b*x+a)^(5/2)*x^(1/2)+5/12*a*(b*x+a)^(3/2)*x^(1/2)+5/8*a^2*x^(1/2)*(b*x
+a)^(1/2)+5/16*a^3*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b
^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85233, size = 365, normalized size = 3.97

$$\left[\frac{15\,a^3\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(8\,b^3x^2+26\,ab^2x+33\,a^2b\right)\sqrt{bx+a}\sqrt{x}}{48\,b}, -\frac{15\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 + 26*a*b^2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b, -1/24*(15*a^3
*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 26*a*b^
2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b]
```

Sympy [A] time = 11.7799, size = 102, normalized size = 1.11

$$\frac{11a^{\frac{5}{2}}\sqrt{x}\sqrt{1+\frac{bx}{a}}}{8}+\frac{13a^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{12}+\frac{\sqrt{a}b^2x^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{3}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] 11*a**(5/2)*sqrt(x)*sqrt(1 + b*x/a)/8 + 13*a**(3/2)*b*x**(3/2)*sqrt(1 + b*x
/a)/12 + sqrt(a)*b**2*x**(5/2)*sqrt(1 + b*x/a)/3 + 5*a**3*asinh(sqrt(b)*sqr
t(x)/sqrt(a))/(8*sqrt(b))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.549 $\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{15}{4}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}(a+bx)^{3/2}+\frac{15}{4}ab\sqrt{x}\sqrt{a+bx}$$

```
[Out] (15*a*b*Sqrt[x]*Sqrt[a + b*x])/4 + (5*b*Sqrt[x]*(a + b*x)^(3/2))/2 - (2*(a
+ b*x)^(5/2))/Sqrt[x] + (15*a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
b*x]])/4
```

Rubi [A] time = 0.0279171, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$\frac{15}{4}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}(a+bx)^{3/2}+\frac{15}{4}ab\sqrt{x}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (15*a*b*Sqrt[x]*Sqrt[a + b*x])/4 + (5*b*Sqrt[x]*(a + b*x)^(3/2))/2 - (2*(a
+ b*x)^(5/2))/Sqrt[x] + (15*a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
b*x]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(a+bx)^{5/2}}{\sqrt{x}}+(5b)\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx\\
&= \frac{5}{2}b\sqrt{x}(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{1}{4}(15ab)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx\\
&= \frac{15}{4}ab\sqrt{x}\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{1}{8}\left(15a^2b\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx\\
&= \frac{15}{4}ab\sqrt{x}\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{1}{4}\left(15a^2b\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{15}{4}ab\sqrt{x}\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{1}{4}\left(15a^2b\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\
&= \frac{15}{4}ab\sqrt{x}\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{15}{4}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0119474, size = 48, normalized size = 0.54

$$-\frac{2a^2\sqrt{a+bx}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-2*a^2*Sqrt[a + b*x]*Hypergeometric2F1[-5/2, -1/2, 1/2, -((b*x)/a)])/(Sqrt
[x]*Sqrt[1 + (b*x)/a])
```

Maple [A] time = 0.014, size = 84, normalized size = 0.9

$$-\frac{-2\,b^2x^2-9\,abx+8\,a^2}{4}\sqrt{bx+a}\frac{1}{\sqrt{x}}+\frac{15\,a^2}{8}\sqrt{b}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\sqrt{x\,(bx+a)}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^(3/2),x)
```

```
[Out] -1/4*(b*x+a)^(1/2)*(-2*b^2*x^2-9*a*b*x+8*a^2)/x^(1/2)+15/8*a^2*b^(1/2)*ln((
1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97287, size = 351, normalized size = 3.94

$$\left[\frac{15\,a^{2}\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(2\,b^{2}x^{2}+9\,abx-8\,a^{2}\right)\sqrt{bx+a}\sqrt{x}}{8\,x},-\frac{15\,a^{2}\sqrt{-b}x\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*a^2*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(2*b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt(b*x + a)*sqrt(x))/x, -1/4*(15*a^2*sqrt(
-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x^2 + 9*a*b*x - 8
*a^2)*sqrt(b*x + a)*sqrt(x))/x]
```

Sympy [A] time = 12.4347, size = 126, normalized size = 1.42

$$-\frac{2a^{\frac{5}{2}}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{1+\frac{bx}{a}}}+\frac{11\sqrt{a}b^{2}x^{\frac{3}{2}}}{4\sqrt{1+\frac{bx}{a}}}+\frac{15a^{2}\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4}+\frac{b^{3}x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(3/2),x)
```

```
[Out] -2*a**(5/2)/(sqrt(x)*sqrt(1 + b*x/a)) + a**(3/2)*b*sqrt(x)/(4*sqrt(1 + b*x/
a)) + 11*sqrt(a)*b**2*x**(3/2)/(4*sqrt(1 + b*x/a)) + 15*a**2*sqrt(b)*asinh(
sqrt(b)*sqrt(x)/sqrt(a))/4 + b**3*x**(5/2)/(2*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.550 $\int \frac{(a+bx)^{5/2}}{x^{5/2}} dx$

Optimal. Leaf size=86

$$5b^2\sqrt{x}\sqrt{a+bx}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{5/2}}{3x^{3/2}}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}$$

```
[Out] 5*b^2*Sqrt[x]*Sqrt[a + b*x] - (10*b*(a + b*x)^(3/2))/(3*Sqrt[x]) - (2*(a +
b*x)^(5/2))/(3*x^(3/2)) + 5*a*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*
x]]
```

Rubi [A] time = 0.0262959, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$5b^2\sqrt{x}\sqrt{a+bx}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{5/2}}{3x^{3/2}}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[a + b*x] - (10*b*(a + b*x)^(3/2))/(3*Sqrt[x]) - (2*(a +
b*x)^(5/2))/(3*x^(3/2)) + 5*a*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*
x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\frac{1}{3}(5b)\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx\\
&= -\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx\\
&= 5b^2\sqrt{x}\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx\\
&= 5b^2\sqrt{x}\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= 5b^2\sqrt{x}\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\
&= 5b^2\sqrt{x}\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0102402, size = 50, normalized size = 0.58

$$-\frac{2a^2\sqrt{a+bx}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (-2*a^2*Sqrt[a + b*x]*Hypergeometric2F1[-5/2, -3/2, -1/2, -((b*x)/a)])/(3*x
^(3/2)*Sqrt[1 + (b*x)/a])
```

Maple [A] time = 0.017, size = 82, normalized size = 1.

$$-\frac{-3\,b^2x^2+14\,abx+2\,a^2}{3}\sqrt{bx+a}x^{-\frac{3}{2}}+\frac{5\,a}{2}b^{\frac{3}{2}}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\sqrt{x\,(bx+a)}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^(5/2),x)
```

```
[Out] -1/3*(b*x+a)^(1/2)*(-3*b^2*x^2+14*a*b*x+2*a^2)/x^(3/2)+5/2*b^(3/2)*a*ln((1/
2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83555, size = 362, normalized size = 4.21

$$\left[\frac{15\,ab^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(3\,b^2x^2-14\,abx-2\,a^2\right)\sqrt{bx+a}\sqrt{x}}{6\,x^2},-\frac{15\,a\sqrt{-b}bx^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(3*b^2*x^2 - 14*a*b*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2, -1/3*(15*a*sqrt
(-b)*b*x^2*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 14*a*b
*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2]
```

Sympy [A] time = 11.8364, size = 99, normalized size = 1.15

$$-\frac{2a^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{14ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3}-\frac{5ab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}+5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx}+1}+1\right)+b^{\frac{5}{2}}x\sqrt{\frac{a}{bx}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(5/2),x)
```

```
[Out] -2*a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 14*a*b**(3/2)*sqrt(a/(b*x) + 1)/3
 - 5*a*b**(3/2)*log(a/(b*x))/2 + 5*a*b**(3/2)*log(sqrt(a/(b*x) + 1) + 1) +
b**(5/2)*x*sqrt(a/(b*x) + 1)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.551 $\int x^{5/2}(a-bx)^{5/2}\,dx$

Optimal. Leaf size=171

$$-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}+\frac{5a^6\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{512b^{7/2}}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^7$$

```
[Out] (-5*a^5*Sqrt[x]*Sqrt[a - b*x])/(512*b^3) - (5*a^4*x^(3/2)*Sqrt[a - b*x])/(7
68*b^2) - (a^3*x^(5/2)*Sqrt[a - b*x])/(192*b) + (a^2*x^(7/2)*Sqrt[a - b*x])
/32 + (a*x^(7/2)*(a - b*x)^(3/2))/12 + (x^(7/2)*(a - b*x)^(5/2))/6 + (5*a^6
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(512*b^(7/2))
```

Rubi [A] time = 0.0608304, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}+\frac{5a^6\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{512b^{7/2}}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^7$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a - b*x)^(5/2),x]
```

```
[Out] (-5*a^5*Sqrt[x]*Sqrt[a - b*x])/(512*b^3) - (5*a^4*x^(3/2)*Sqrt[a - b*x])/(7
68*b^2) - (a^3*x^(5/2)*Sqrt[a - b*x])/(192*b) + (a^2*x^(7/2)*Sqrt[a - b*x])
/32 + (a*x^(7/2)*(a - b*x)^(3/2))/12 + (x^(7/2)*(a - b*x)^(5/2))/6 + (5*a^6
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(512*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^{5/2}(a-bx)^{5/2}\,dx = \frac{1}{6}x^{7/2}(a-bx)^{5/2}+\frac{1}{12}(5a)\int x^{5/2}(a-bx)^{3/2}\,dx$$
$$= \frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}+\frac{1}{8}a^2\int x^{5/2}\sqrt{a-bx}\,dx$$
$$= \frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}+\frac{1}{64}a^3\int\frac{x^{5/2}}{\sqrt{a-bx}}\,dx$$
$$= -\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}+\frac{\left(5a^4\right)\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{384b}$$
$$= -\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}$$
$$= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+$$
$$= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+$$
$$= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+$$
$$= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+$$

Mathematica [A] time = 0.186593, size = 120, normalized size = 0.7

$$\frac{\sqrt{a-bx}\left(\sqrt{b}\sqrt{x}\left(-8a^3b^2x^2+432a^2b^3x^3-10a^4bx-15a^5-640ab^4x^4+256b^5x^5\right)+\frac{15a^{11/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}\right)}{1536b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^5 - 10*a^4*b*x - 8*a^3*b^2*x^2 + 432
*a^2*b^3*x^3 - 640*a*b^4*x^4 + 256*b^5*x^5) + (15*a^(11/2)*ArcSin[(Sqrt[b]*
Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(1536*b^(7/2))
```

Maple [A] time = 0.003, size = 165, normalized size = 1.

$$-\frac{1}{6b}x^{\frac{5}{2}}(-bx+a)^{\frac{7}{2}}-\frac{a}{12b^2}x^{\frac{3}{2}}(-bx+a)^{\frac{7}{2}}-\frac{a^2}{32b^3}\sqrt{x}(-bx+a)^{\frac{7}{2}}+\frac{a^3}{192b^3}(-bx+a)^{\frac{5}{2}}\sqrt{x}+\frac{5a^4}{768b^3}(-bx+a)^{\frac{3}{2}}\sqrt{x}+\frac{5}{51}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(5/2),x)
```

```
[Out] -1/6/b*x^(5/2)*(-b*x+a)^(7/2)-1/12/b^2*a*x^(3/2)*(-b*x+a)^(7/2)-1/32/b^3*a^
2*x^(1/2)*(-b*x+a)^(7/2)+1/192/b^3*a^3*(-b*x+a)^(5/2)*x^(1/2)+5/768/b^3*a^4
*(-b*x+a)^(3/2)*x^(1/2)+5/512*a^5*x^(1/2)*(-b*x+a)^(1/2)/b^3+5/1024/b^(7/2)
```

```
*a^6*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/(
-b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89305, size = 527, normalized size = 3.08

$$\left[-\frac{15\,a^6\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(256\,b^6x^5-640\,ab^5x^4+432\,a^2b^4x^3-8\,a^3b^3x^2-10\,a^4b^2x-15\,a^5b\right)}{3072\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3072*(15*a^6*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) - 2*(256*b^6*x^5 - 640*a*b^5*x^4 + 432*a^2*b^4*x^3 - 8*a^3*b^3*x^2 - 10*
a^4*b^2*x - 15*a^5*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/1536*(15*a^6*sqrt(b)*
arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (256*b^6*x^5 - 640*a*b^5*x^4 + 4
32*a^2*b^4*x^3 - 8*a^3*b^3*x^2 - 10*a^4*b^2*x - 15*a^5*b)*sqrt(-b*x + a)*sq
rt(x))/b^4]
```

Sympy [A] time = 105.6, size = 437, normalized size = 2.56

$$\begin{cases} \frac{5ia^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{-1+\frac{bx}{a}}} - \frac{55ia^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{-1+\frac{bx}{a}}} + \frac{67ia^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{-1+\frac{bx}{a}}} - \frac{7i\sqrt{a}b^2x^{\frac{11}{2}}}{12\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^6\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}} + \frac{ib^3x^{\frac{13}{2}}}{6\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\: \frac{|bx|}{|a|} > \\ -\frac{5a^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{1-\frac{bx}{a}}} + \frac{5a^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{1-\frac{bx}{a}}} + \frac{55a^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{1-\frac{bx}{a}}} - \frac{67a^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{1-\frac{bx}{a}}} + \frac{7\sqrt{a}b^2x^{\frac{11}{2}}}{12\sqrt{1-\frac{bx}{a}}} + \frac{5a^6\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}} - \frac{b^3x^{\frac{13}{2}}}{6\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((5*I*a**(11/2)*sqrt(x)/(512*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(9/2)
*x**(3/2)/(1536*b**2*sqrt(-1 + b*x/a)) - I*a**(7/2)*x**(5/2)/(768*b*sqrt(-1
 + b*x/a)) - 55*I*a**(5/2)*x**(7/2)/(192*sqrt(-1 + b*x/a)) + 67*I*a**(3/2)*
b*x**(9/2)/(96*sqrt(-1 + b*x/a)) - 7*I*sqrt(a)*b**2*x**(11/2)/(12*sqrt(-1 +
 b*x/a)) - 5*I*a**6*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(512*b**(7/2)) + I*b**3*
x**(13/2)/(6*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (-5*a**(11/2)
*sqrt(x)/(512*b**3*sqrt(1 - b*x/a)) + 5*a**(9/2)*x**(3/2)/(1536*b**2*sqrt(1
 - b*x/a)) + a**(7/2)*x**(5/2)/(768*b*sqrt(1 - b*x/a)) + 55*a**(5/2)*x**(7/
2)/(192*sqrt(1 - b*x/a)) - 67*a**(3/2)*b*x**(9/2)/(96*sqrt(1 - b*x/a)) + 7*
```

```
sqrt(a)*b**2*x**(11/2)/(12*sqrt(1 - b*x/a)) + 5*a**6*asin(sqrt(b)*sqrt(x)/s
qrt(a))/(512*b**(7/2)) - b**3*x**(13/2)/(6*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.552 $\int x^{3/2}(a-bx)^{5/2}\,dx$

Optimal. Leaf size=146

$$-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}+\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{5/2}}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a - b*x])/(128*b^2) - (a^3*x^(3/2)*Sqrt[a - b*x])/(64*
b) + (a^2*x^(5/2)*Sqrt[a - b*x])/16 + (a*x^(5/2)*(a - b*x)^(3/2))/8 + (x^(5
/2)*(a - b*x)^(5/2))/5 + (3*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(1
28*b^(5/2))
```

Rubi [A] time = 0.0505688, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}+\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{5/2}}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a - b*x)^(5/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a - b*x])/(128*b^2) - (a^3*x^(3/2)*Sqrt[a - b*x])/(64*
b) + (a^2*x^(5/2)*Sqrt[a - b*x])/16 + (a*x^(5/2)*(a - b*x)^(3/2))/8 + (x^(5
/2)*(a - b*x)^(5/2))/5 + (3*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(1
28*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}(a-bx)^{5/2}\,dx &= \frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{1}{2}a\int x^{3/2}(a-bx)^{3/2}\,dx\\
&= \frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{1}{16}\left(3a^2\right)\int x^{3/2}\sqrt{a-bx}\,dx\\
&= \frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{1}{32}a^3\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx\\
&= -\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{\left(3a^4\right)\int\frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{128b}\\
&= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\\
&= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\\
&= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\\
&= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+
\end{aligned}$$

Mathematica [A] time = 0.142884, size = 109, normalized size = 0.75

$$\frac{\sqrt{a-bx}\left(\sqrt{b}\sqrt{x}\left(248a^2b^2x^2-10a^3bx-15a^4-336ab^3x^3+128b^4x^4\right)+\frac{15a^{9/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}\right)}{640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^4 - 10*a^3*b*x + 248*a^2*b^2*x^2 - 3
36*a*b^3*x^3 + 128*b^4*x^4) + (15*a^(9/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]]
)/Sqrt[1 - (b*x)/a]))/(640*b^(5/2))
```

Maple [A] time = 0.005, size = 146, normalized size = 1.

$$-\frac{1}{5b}x^{\frac{3}{2}}(-bx+a)^{\frac{7}{2}}-\frac{3a}{40b^2}\sqrt{x}(-bx+a)^{\frac{7}{2}}+\frac{a^2}{80b^2}(-bx+a)^{\frac{5}{2}}\sqrt{x}+\frac{a^3}{64b^2}(-bx+a)^{\frac{3}{2}}\sqrt{x}+\frac{3a^4}{128b^2}\sqrt{x}\sqrt{-bx+a}+\frac{3a^5}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(5/2),x)
```

```
[Out] -1/5/b*x^(3/2)*(-b*x+a)^(7/2)-3/40/b^2*a*x^(1/2)*(-b*x+a)^(7/2)+1/80/b^2*a^
2*(-b*x+a)^(5/2)*x^(1/2)+1/64/b^2*a^3*(-b*x+a)^(3/2)*x^(1/2)+3/128*a^4*x^(1
/2)*(-b*x+a)^(1/2)/b^2+3/256/b^(5/2)*a^5*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/
x^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/(-b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90761, size = 482, normalized size = 3.3

$$\left[-\frac{15\,a^5\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(128\,b^5x^4-336\,ab^4x^3+248\,a^2b^3x^2-10\,a^3b^2x-15\,a^4b\right)\sqrt{-bx+a}}{1280\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*a^5*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) - 2*(128*b^5*x^4 - 336*a*b^4*x^3 + 248*a^2*b^3*x^2 - 10*a^3*b^2*x - 15*a
^4*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/640*(15*a^5*sqrt(b)*arctan(sqrt(-b*x
+ a)/(sqrt(b)*sqrt(x))) - (128*b^5*x^4 - 336*a*b^4*x^3 + 248*a^2*b^3*x^2 -
10*a^3*b^2*x - 15*a^4*b)*sqrt(-b*x + a)*sqrt(x))/b^3]
```

Sympy [A] time = 52.7216, size = 381, normalized size = 2.61

$$\begin{cases} \frac{3ia^{\frac{9}{2}}\sqrt{x}}{128b^2\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{-1+\frac{bx}{a}}}-\frac{129ia^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{-1+\frac{bx}{a}}}+\frac{73ia^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{-1+\frac{bx}{a}}}-\frac{29i\sqrt{a}b^2x^{\frac{9}{2}}}{40\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^5\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{ib^3x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ -\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{1-\frac{bx}{a}}}+\frac{129a^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{1-\frac{bx}{a}}}-\frac{73a^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}}+\frac{29\sqrt{a}b^2x^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}}+\frac{3a^5\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}-\frac{b^3x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((3*I*a**(9/2)*sqrt(x)/(128*b**2*sqrt(-1 + b*x/a)) - I*a**(7/2)*x*
*(3/2)/(128*b*sqrt(-1 + b*x/a)) - 129*I*a**(5/2)*x**(5/2)/(320*sqrt(-1 + b*
x/a)) + 73*I*a**(3/2)*b*x**(7/2)/(80*sqrt(-1 + b*x/a)) - 29*I*sqrt(a)*b**2*
x**(9/2)/(40*sqrt(-1 + b*x/a)) - 3*I*a**5*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(1
28*b**(5/2)) + I*b**3*x**(11/2)/(5*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(
a) > 1), (-3*a**(9/2)*sqrt(x)/(128*b**2*sqrt(1 - b*x/a)) + a**(7/2)*x**(3/2
)/(128*b*sqrt(1 - b*x/a)) + 129*a**(5/2)*x**(5/2)/(320*sqrt(1 - b*x/a)) - 7
3*a**(3/2)*b*x**(7/2)/(80*sqrt(1 - b*x/a)) + 29*sqrt(a)*b**2*x**(9/2)/(40*s
qrt(1 - b*x/a)) + 3*a**5*asin(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(5/2)) - b**
3*x**(11/2)/(5*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.553 $\int \sqrt{x}(a-bx)^{5/2}\,dx$

Optimal. Leaf size=121

$$\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{3/2}}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

```
[Out] (-5*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b) + (5*a^2*x^(3/2)*Sqrt[a - b*x])/32 +
(5*a*x^(3/2)*(a - b*x)^(3/2))/24 + (x^(3/2)*(a - b*x)^(5/2))/4 + (5*a^4*Arc
Tan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(3/2))
```

Rubi [A] time = 0.0377049, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{3/2}}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a - b*x)^(5/2),x]
```

```
[Out] (-5*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b) + (5*a^2*x^(3/2)*Sqrt[a - b*x])/32 +
(5*a*x^(3/2)*(a - b*x)^(3/2))/24 + (x^(3/2)*(a - b*x)^(5/2))/4 + (5*a^4*Arc
Tan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(a-bx)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{1}{8}(5a)\int \sqrt{x}(a-bx)^{3/2}\,dx\\ &= \frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{1}{16}\left(5a^2\right)\int \sqrt{x}\sqrt{a-bx}\,dx\\ &= \frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{1}{64}\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx\\ &= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{\left(5a^4\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}}{128b}\\ &= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{\left(5a^4\right)\text{Subst}\left(\int\right.}{64}\\ &= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{\left(5a^4\right)\text{Subst}\left(\int\right.}{6}\\ &= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.120693, size = 98, normalized size = 0.81

$$\frac{\sqrt{a-bx}\left(\sqrt{b}\sqrt{x}\left(118a^2bx-15a^3-136ab^2x^2+48b^3x^3\right)+\frac{15a^{7/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}\right)}{192b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^3 + 118*a^2*b*x - 136*a*b^2*x^2 + 48
*b^3*x^3) + (15*a^(7/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a
]))/(192*b^(3/2))
```

Maple [A] time = 0.005, size = 118, normalized size = 1.

$$\frac{1}{4}x^{\frac{3}{2}}(-bx+a)^{\frac{5}{2}}+\frac{5a}{24}x^{\frac{3}{2}}(-bx+a)^{\frac{3}{2}}+\frac{5a^2}{32}x^{\frac{3}{2}}\sqrt{-bx+a}-\frac{5a^3}{64b}\sqrt{x}\sqrt{-bx+a}+\frac{5a^4}{128}\sqrt{x(-bx+a)}\arctan\left(\sqrt{b}\left(x-\frac{a}{2b}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)*x^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(-b*x+a)^(5/2)+5/24*a*x^(3/2)*(-b*x+a)^(3/2)+5/32*a^2*x^(3/2)*(
-b*x+a)^(1/2)-5/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b+5/128*a^4/b^(3/2)*(x*(-b*x+
a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/(-b*x^2+a*x)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78927, size = 432, normalized size = 3.57

$$\left[-\frac{15\,a^{4}\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(48\,b^{4}x^{3}-136\,ab^{3}x^{2}+118\,a^{2}b^{2}x-15\,a^{3}b\right)\sqrt{-bx+a}\sqrt{x}}{384\,b^{2}},-\frac{15\,a^{4}\sqrt{b}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) - 2*(48*b^4*x^3 - 136*a*b^3*x^2 + 118*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a
)*sqrt(x))/b^2, -1/192*(15*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(
x))) - (48*b^4*x^3 - 136*a*b^3*x^2 + 118*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x +
a)*sqrt(x))/b^2]
```

Sympy [A] time = 19.0609, size = 328, normalized size = 2.71

$$\begin{cases} \frac{5ia^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{-1+\frac{bx}{a}}}-\frac{133ia^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{-1+\frac{bx}{a}}}+\frac{127ia^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{-1+\frac{bx}{a}}}-\frac{23i\sqrt{a}b^{2}x^{\frac{7}{2}}}{24\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{4}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}}+\frac{ib^{3}x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ -\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1-\frac{bx}{a}}}+\frac{133a^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1-\frac{bx}{a}}}-\frac{127a^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1-\frac{bx}{a}}}+\frac{23\sqrt{a}b^{2}x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}}+\frac{5a^{4}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}}-\frac{b^{3}x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)*x**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(7/2)*sqrt(x)/(64*b*sqrt(-1 + b*x/a)) - 133*I*a**(5/2)*x*
*(3/2)/(192*sqrt(-1 + b*x/a)) + 127*I*a**(3/2)*b*x**(5/2)/(96*sqrt(-1 + b*x
/a)) - 23*I*sqrt(a)*b**2*x**(7/2)/(24*sqrt(-1 + b*x/a)) - 5*I*a**4*acosh(sq
rt(b)*sqrt(x)/sqrt(a))/(64*b**(3/2)) + I*b**3*x**(9/2)/(4*sqrt(a)*sqrt(-1 +
 b*x/a)), Abs(b*x)/Abs(a) > 1), (-5*a**(7/2)*sqrt(x)/(64*b*sqrt(1 - b*x/a))
 + 133*a**(5/2)*x**(3/2)/(192*sqrt(1 - b*x/a)) - 127*a**(3/2)*b*x**(5/2)/(9
6*sqrt(1 - b*x/a)) + 23*sqrt(a)*b**2*x**(7/2)/(24*sqrt(1 - b*x/a)) + 5*a**4
*asin(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(3/2)) - b**3*x**(9/2)/(4*sqrt(a)*sqr
t(1 - b*x/a)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.554 $\int \frac{(a-bx)^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=96

$$\frac{5}{8}a^2\sqrt{x}\sqrt{a-bx}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}+\frac{5}{12}a\sqrt{x}(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}$$

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a - b*x])/8 + (5*a*Sqrt[x]*(a - b*x)^(3/2))/12 + (Sqrt[
x]*(a - b*x)^(5/2))/3 + (5*a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8*
Sqrt[b])
```

Rubi [A] time = 0.0304088, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$\frac{5}{8}a^2\sqrt{x}\sqrt{a-bx}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}+\frac{5}{12}a\sqrt{x}(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a - b*x])/8 + (5*a*Sqrt[x]*(a - b*x)^(3/2))/12 + (Sqrt[
x]*(a - b*x)^(5/2))/3 + (5*a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8*
Sqrt[b])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(a-bx)^{5/2}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}(a-bx)^{5/2}+\frac{1}{6}(5a)\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx$$
$$=\frac{5}{12}a\sqrt{x}(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}+\frac{1}{8}\left(5a^2\right)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx$$
$$=\frac{5}{8}a^2\sqrt{x}\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}+\frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx$$
$$=\frac{5}{8}a^2\sqrt{x}\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}+\frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)$$
$$=\frac{5}{8}a^2\sqrt{x}\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}+\frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)$$
$$=\frac{5}{8}a^2\sqrt{x}\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}$$

Mathematica [A] time = 0.117461, size = 82, normalized size = 0.85

$$\frac{1}{24}\sqrt{a-bx}\left(\sqrt{x}\left(33a^2-26abx+8b^2x^2\right)+\frac{15a^{5/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{1-\frac{bx}{a}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[x]*(33*a^2 - 26*a*b*x + 8*b^2*x^2) + (15*a^(5/2)*ArcSi
n[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 - (b*x)/a])))/24
```

Maple [A] time = 0.004, size = 99, normalized size = 1.

$$\frac{1}{3}(-bx+a)^{\frac{5}{2}}\sqrt{x}+\frac{5a}{12}(-bx+a)^{\frac{3}{2}}\sqrt{x}+\frac{5a^2}{8}\sqrt{x}\sqrt{-bx+a}+\frac{5a^3}{16}\sqrt{x(-bx+a)}\arctan\left(\sqrt{b}\left(x-\frac{a}{2b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*(-b*x+a)^(5/2)*x^(1/2)+5/12*a*(-b*x+a)^(3/2)*x^(1/2)+5/8*a^2*x^(1/2)*(-
b*x+a)^(1/2)+5/16*a^3*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arc
tan(b^(1/2)*(x-1/2/b*a)/(-b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86577, size = 370, normalized size = 3.85

$$\left[-\frac{15\,a^3\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(8\,b^3x^2-26\,ab^2x+33\,a^2b\right)\sqrt{-bx+a}\sqrt{x}}{48\,b},-\frac{15\,a^3\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 - 2*(8*b^3*x^2 - 26*a*b^2*x + 33*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b, -1/24*(
15*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (8*b^3*x^2 - 26*a
*b^2*x + 33*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b]
```

Sympy [A] time = 11.2105, size = 248, normalized size = 2.58

$$\begin{cases} -\frac{11ia^{\frac{5}{2}}\sqrt{x}}{8\sqrt{-1+\frac{bx}{a}}}+\frac{59ia^{\frac{3}{2}}bx^{\frac{3}{2}}}{24\sqrt{-1+\frac{bx}{a}}}-\frac{17i\sqrt{a}b^2x^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^3\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{ib^3x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ \frac{11a^{\frac{5}{2}}\sqrt{x}\sqrt{1-\frac{bx}{a}}}{8}-\frac{13a^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{12}+\frac{\sqrt{a}b^2x^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{3}+\frac{5a^3\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((-11*I*a**(5/2)*sqrt(x)/(8*sqrt(-1 + b*x/a)) + 59*I*a**(3/2)*b*x*
*(3/2)/(24*sqrt(-1 + b*x/a)) - 17*I*sqrt(a)*b**2*x**(5/2)/(12*sqrt(-1 + b*x
/a)) - 5*I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqrt(b)) + I*b**3*x**(7/2
)/(3*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (11*a**(5/2)*sqrt(x)*
sqrt(1 - b*x/a)/8 - 13*a**(3/2)*b*x**(3/2)*sqrt(1 - b*x/a)/12 + sqrt(a)*b**
2*x**(5/2)*sqrt(1 - b*x/a)/3 + 5*a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqrt
(b)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.555 $\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{15}{4}a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}(a-bx)^{3/2}-\frac{15}{4}ab\sqrt{x}\sqrt{a-bx}$$

```
[Out] (-15*a*b*Sqrt[x]*Sqrt[a - b*x])/4 - (5*b*Sqrt[x]*(a - b*x)^(3/2))/2 - (2*(a
 - b*x)^(5/2))/Sqrt[x] - (15*a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a -
b*x]])/4
```

Rubi [A] time = 0.0281738, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$-\frac{15}{4}a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}(a-bx)^{3/2}-\frac{15}{4}ab\sqrt{x}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-15*a*b*Sqrt[x]*Sqrt[a - b*x])/4 - (5*b*Sqrt[x]*(a - b*x)^(3/2))/2 - (2*(a
 - b*x)^(5/2))/Sqrt[x] - (15*a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a -
b*x]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(a-bx)^{5/2}}{\sqrt{x}}-(5b)\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx\\
&= -\frac{5}{2}b\sqrt{x}(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}(15ab)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= -\frac{15}{4}ab\sqrt{x}\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{8}\left(15a^2b\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx\\
&= -\frac{15}{4}ab\sqrt{x}\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}\left(15a^2b\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{15}{4}ab\sqrt{x}\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}\left(15a^2b\right)\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\
&= -\frac{15}{4}ab\sqrt{x}\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{15}{4}a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0120249, size = 49, normalized size = 0.53

$$-\frac{2a^2\sqrt{a-bx}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{bx}{a}\right)}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-2*a^2*Sqrt[a - b*x]*Hypergeometric2F1[-5/2, -1/2, 1/2, (b*x)/a])/(Sqrt[x]
*Sqrt[1 - (b*x)/a])
```

Maple [A] time = 0.013, size = 88, normalized size = 1.

$$-\frac{-2\,b^2x^2+9\,abx+8\,a^2}{4}\sqrt{-bx+a}\frac{1}{\sqrt{x}}-\frac{15\,a^2}{8}\sqrt{b}\arctan\left(\sqrt{b}\left(x-\frac{a}{2\,b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)\sqrt{x\,(-bx+a)}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(3/2),x)
```

```
[Out] -1/4*(-b*x+a)^(1/2)*(-2*b^2*x^2+9*a*b*x+8*a^2)/x^(1/2)-15/8*a^2*b^(1/2)*arc
tan(b^(1/2)*(x-1/2/b*a)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*
x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86165, size = 354, normalized size = 3.81

$$\left[\frac{15\,a^2\sqrt{-b}x\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(2\,b^2x^2-9\,abx-8\,a^2\right)\sqrt{-bx+a}\sqrt{x}}{8\,x},\frac{15\,a^2\sqrt{b}x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*a^2*sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(2*b^2*x^2 - 9*a*b*x - 8*a^2)*sqrt(-b*x + a)*sqrt(x))/x, 1/4*(15*a^2*s
qrt(b)*x*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x^2 - 9*a*b*x -
8*a^2)*sqrt(-b*x + a)*sqrt(x))/x]
```

Sympy [A] time = 12.6059, size = 269, normalized size = 2.89

$$\begin{cases} \frac{2ia^{\frac{5}{2}}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}}+\frac{ia^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{-1+\frac{bx}{a}}}-\frac{11i\sqrt{a}b^2x^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}}+\frac{15ia^2\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4}+\frac{ib^3x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ -\frac{2a^{\frac{5}{2}}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{1-\frac{bx}{a}}}+\frac{11\sqrt{a}b^2x^{\frac{3}{2}}}{4\sqrt{1-\frac{bx}{a}}}-\frac{15a^2\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4}-\frac{b^3x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*a**(5/2)/(sqrt(x)*sqrt(-1 + b*x/a)) + I*a**(3/2)*b*sqrt(x)/(
4*sqrt(-1 + b*x/a)) - 11*I*sqrt(a)*b**2*x**(3/2)/(4*sqrt(-1 + b*x/a)) + 15*
I*a**2*sqrt(b)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/4 + I*b**3*x**(5/2)/(2*sqrt(a
)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (-2*a**(5/2)/(sqrt(x)*sqrt(1 - b
*x/a)) - a**(3/2)*b*sqrt(x)/(4*sqrt(1 - b*x/a)) + 11*sqrt(a)*b**2*x**(3/2)/
(4*sqrt(1 - b*x/a)) - 15*a**2*sqrt(b)*asin(sqrt(b)*sqrt(x)/sqrt(a))/4 - b**
3*x**(5/2)/(2*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="giac")

[Out] Timed out
```

3.556 $\int \frac{(a-bx)^{5/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=90

$$5b^2\sqrt{x}\sqrt{a-bx}+5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}$$

```
[Out] 5*b^2*Sqrt[x]*Sqrt[a - b*x] + (10*b*(a - b*x)^(3/2))/(3*Sqrt[x]) - (2*(a -
b*x)^(5/2))/(3*x^(3/2)) + 5*a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x
]]
```

Rubi [A] time = 0.0296783, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$5b^2\sqrt{x}\sqrt{a-bx}+5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[a - b*x] + (10*b*(a - b*x)^(3/2))/(3*Sqrt[x]) - (2*(a -
b*x)^(5/2))/(3*x^(3/2)) + 5*a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x
]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(a-bx)^{5/2}}{3x^{3/2}}-\frac{1}{3}(5b)\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx\\
&= \frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= 5b^2\sqrt{x}\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx\\
&= 5b^2\sqrt{x}\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= 5b^2\sqrt{x}\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\
&= 5b^2\sqrt{x}\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0107629, size = 51, normalized size = 0.57

$$-\frac{2a^2\sqrt{a-bx}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{bx}{a}\right)}{3x^{3/2}\sqrt{1-\frac{bx}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (-2*a^2*Sqrt[a - b*x]*Hypergeometric2F1[-5/2, -3/2, -1/2, (b*x)/a])/(3*x^(3
/2)*Sqrt[1 - (b*x)/a])
```

Maple [A] time = 0.017, size = 86, normalized size = 1.

$$-\frac{-3\,b^2x^2-14\,abx+2\,a^2}{3}\sqrt{-bx+ax}^{-\frac{3}{2}}+\frac{5\,a}{2}b^{\frac{3}{2}}\arctan\left(\sqrt{b}\left(x-\frac{a}{2\,b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)\sqrt{x\,(-bx+a)}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(5/2),x)
```

```
[Out] -1/3*(-b*x+a)^(1/2)*(-3*b^2*x^2-14*a*b*x+2*a^2)/x^(3/2)+5/2*b^(3/2)*a*arcta
n(b^(1/2)*(x-1/2/b*a)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+
a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82039, size = 366, normalized size = 4.07

$$\left[\frac{15\,a\sqrt{-b}bx^2\log\left(-2\,bx-2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(3\,b^2x^2+14\,abx-2\,a^2\right)\sqrt{-bx+a}\sqrt{x}}{6\,x^2}, -\frac{15\,ab^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx}}{\sqrt{b}\sqrt{}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*sqrt(-b)*b*x^2*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) + 2*(3*b^2*x^2 + 14*a*b*x - 2*a^2)*sqrt(-b*x + a)*sqrt(x))/x^2, -1/3*(15
*a*b^(3/2)*x^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (3*b^2*x^2 + 14*a
*b*x - 2*a^2)*sqrt(-b*x + a)*sqrt(x))/x^2]
```

Sympy [C] time = 12.2156, size = 248, normalized size = 2.76

$$\begin{cases} -\frac{2a^2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3x}+\frac{14ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3}-5iab^{\frac{3}{2}}\log\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{5iab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}+5ab^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+b^{\frac{5}{2}}x\sqrt{\frac{a}{bx}-1} & \text{for } \frac{|a|}{|b||x|}>1 \\ -\frac{2ia^2\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{3x}+\frac{14iab^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3}+\frac{5iab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}-5iab^{\frac{3}{2}}\log\left(\sqrt{-\frac{a}{bx}+1}+1\right)+ib^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 14*a*b**(3/2)*sqrt(a/(
b*x) - 1)/3 - 5*I*a*b**(3/2)*log(sqrt(a)/(sqrt(b)*sqrt(x))) + 5*I*a*b**(3/2
)*log(a/(b*x))/2 + 5*a*b**(3/2)*asin(sqrt(b)*sqrt(x)/sqrt(a)) + b**(5/2)*x*
sqrt(a/(b*x) - 1), Abs(a)/(Abs(b)*Abs(x)) > 1), (-2*I*a**2*sqrt(b)*sqrt(-a/
(b*x) + 1)/(3*x) + 14*I*a*b**(3/2)*sqrt(-a/(b*x) + 1)/3 + 5*I*a*b**(3/2)*lo
g(a/(b*x))/2 - 5*I*a*b**(3/2)*log(sqrt(-a/(b*x) + 1) + 1) + I*b**(5/2)*x*sq
rt(-a/(b*x) + 1), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.557 $\int x^{5/2}(2+bx)^{5/2}\,dx$

Optimal. Leaf size=144

$$-\frac{5x^{3/2}\sqrt{bx+2}}{48b^2}+\frac{5\sqrt{x}\sqrt{bx+2}}{16b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}+\frac{1}{6}x^{7/2}(bx+2)^{5/2}+\frac{1}{6}x^{7/2}(bx+2)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+}}{24b}$$

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(16*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(48*b^2) + (
x^(5/2)*Sqrt[2 + b*x])/(24*b) + (x^(7/2)*Sqrt[2 + b*x])/8 + (x^(7/2)*(2 + b
*x)^(3/2))/6 + (x^(7/2)*(2 + b*x)^(5/2))/6 - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/(8*b^(7/2))
```

Rubi [A] time = 0.0450716, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$-\frac{5x^{3/2}\sqrt{bx+2}}{48b^2}+\frac{5\sqrt{x}\sqrt{bx+2}}{16b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}+\frac{1}{6}x^{7/2}(bx+2)^{5/2}+\frac{1}{6}x^{7/2}(bx+2)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(16*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(48*b^2) + (
x^(5/2)*Sqrt[2 + b*x])/(24*b) + (x^(7/2)*Sqrt[2 + b*x])/8 + (x^(7/2)*(2 + b
*x)^(3/2))/6 + (x^(7/2)*(2 + b*x)^(5/2))/6 - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/(8*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}(2+bx)^{5/2}\,dx &= \frac{1}{6}x^{7/2}(2+bx)^{5/2}+\frac{5}{6}\int x^{5/2}(2+bx)^{3/2}\,dx\\
&= \frac{1}{6}x^{7/2}(2+bx)^{3/2}+\frac{1}{6}x^{7/2}(2+bx)^{5/2}+\frac{1}{2}\int x^{5/2}\sqrt{2+bx}\,dx\\
&= \frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx)^{3/2}+\frac{1}{6}x^{7/2}(2+bx)^{5/2}+\frac{1}{8}\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx\\
&= \frac{x^{5/2}\sqrt{2+bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx)^{3/2}+\frac{1}{6}x^{7/2}(2+bx)^{5/2}-\frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{24b}\\
&= -\frac{5x^{3/2}\sqrt{2+bx}}{48b^2}+\frac{x^{5/2}\sqrt{2+bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx)^{3/2}+\frac{1}{6}x^{7/2}(2+bx)^{5/2}+\frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}}{16b^2}\\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{48b^2}+\frac{x^{5/2}\sqrt{2+bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx)^{3/2}+\frac{1}{6}x^{7/2}(2+bx)^{5/}\\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{48b^2}+\frac{x^{5/2}\sqrt{2+bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx)^{3/2}+\frac{1}{6}x^{7/2}(2+bx)^{5/}\\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{48b^2}+\frac{x^{5/2}\sqrt{2+bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx)^{3/2}+\frac{1}{6}x^{7/2}(2+bx)^{5/}
\end{aligned}
$$

Mathematica [A] time = 0.0562539, size = 86, normalized size = 0.6

$$
\frac{\sqrt{x}\sqrt{bx+2}\left(8b^5x^5+40b^4x^4+54b^3x^3+2b^2x^2-5bx+15\right)}{48b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 54*b^3*x^3 + 40*b^4*x^4 +
8*b^5*x^5))/(48*b^3) - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(8*b^(7/2))
```

Maple [A] time = 0.004, size = 138, normalized size = 1.

$$
\frac{1}{6b}x^{\frac{5}{2}}(bx+2)^{\frac{7}{2}}-\frac{1}{6b^2}x^{\frac{3}{2}}(bx+2)^{\frac{7}{2}}+\frac{1}{8b^3}\sqrt{x}(bx+2)^{\frac{7}{2}}-\frac{1}{24b^3}(bx+2)^{\frac{5}{2}}\sqrt{x}-\frac{5}{48b^3}(bx+2)^{\frac{3}{2}}\sqrt{x}-\frac{5}{16b^3}\sqrt{x}\sqrt{bx+2}-
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+2)^(5/2),x)
```

```
[Out] 1/6/b*x^(5/2)*(b*x+2)^(7/2)-1/6/b^2*x^(3/2)*(b*x+2)^(7/2)+1/8/b^3*x^(1/2)*(
b*x+2)^(7/2)-1/24/b^3*x^(1/2)*(b*x+2)^(5/2)-5/48/b^3*x^(1/2)*(b*x+2)^(3/2)-
5/16*x^(1/2)*(b*x+2)^(1/2)/b^3-5/16/b^(7/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)
/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90134, size = 433, normalized size = 3.01

$$\left[\frac{\left(8\,b^{6}x^{5}+40\,b^{5}x^{4}+54\,b^{4}x^{3}+2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\sqrt{x}+15\,\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{48\,b^{4}},\frac{\left(8\,b^{6}x^{5}+40\,b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*((8*b^6*x^5 + 40*b^5*x^4 + 54*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*s
qrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) +
 1))/b^4, 1/48*((8*b^6*x^5 + 40*b^5*x^4 + 54*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x
+ 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(
b*sqrt(x))))/b^4]
```

Sympy [A] time = 67.264, size = 158, normalized size = 1.1

$$\frac{b^{3}x^{\frac{13}{2}}}{6\sqrt{bx+2}}+\frac{7b^{2}x^{\frac{11}{2}}}{6\sqrt{bx+2}}+\frac{67bx^{\frac{9}{2}}}{24\sqrt{bx+2}}+\frac{55x^{\frac{7}{2}}}{24\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{48b\sqrt{bx+2}}+\frac{5x^{\frac{3}{2}}}{48b^{2}\sqrt{bx+2}}+\frac{5\sqrt{x}}{8b^{3}\sqrt{bx+2}}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(5/2),x)
```

```
[Out] b**3*x**(13/2)/(6*sqrt(b*x + 2)) + 7*b**2*x**(11/2)/(6*sqrt(b*x + 2)) + 67*
b*x**(9/2)/(24*sqrt(b*x + 2)) + 55*x**(7/2)/(24*sqrt(b*x + 2)) - x**(5/2)/(
48*b*sqrt(b*x + 2)) + 5*x**(3/2)/(48*b**2*sqrt(b*x + 2)) + 5*sqrt(x)/(8*b**
3*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(8*b**(7/2))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.558 $\int x^{3/2}(2+bx)^{5/2}\,dx$

Optimal. Leaf size=123

$$-\frac{3\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(8*b) + (x^(5/
2)*Sqrt[2 + b*x])/4 + (x^(5/2)*(2 + b*x)^(3/2))/4 + (x^(5/2)*(2 + b*x)^(5/2
))/5 + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Rubi [A] time = 0.0282731, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$-\frac{3\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(8*b) + (x^(5/
2)*Sqrt[2 + b*x])/4 + (x^(5/2)*(2 + b*x)^(3/2))/4 + (x^(5/2)*(2 + b*x)^(5/2
))/5 + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{3/2}(2+bx)^{5/2}\,dx = \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \int x^{3/2}(2+bx)^{3/2}\,dx$$
$$= \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \frac{3}{4}\int x^{3/2}\sqrt{2+bx}\,dx$$
$$= \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx$$
$$= \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b}$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \frac{3\int \frac{}{\sqrt{x}\sqrt{}}}{8b}$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \frac{3\,\mathrm{Subs}}{}$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \frac{3\sinh^{-}}{4b}$$

Mathematica [A] time = 0.0506403, size = 78, normalized size = 0.63

$$\frac{\sqrt{x}\sqrt{bx+2}\left(8b^4x^4+42b^3x^3+62b^2x^2+5bx-15\right)}{40b^2} + \frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(-15 + 5*b*x + 62*b^2*x^2 + 42*b^3*x^3 + 8*b^4*x^4))
/(40*b^2) + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Maple [A] time = 0.004, size = 123, normalized size = 1.

$$\frac{1}{5b}x^{\frac{3}{2}}(bx+2)^{\frac{7}{2}} - \frac{3}{20b^2}\sqrt{x}(bx+2)^{\frac{7}{2}} + \frac{1}{20b^2}(bx+2)^{\frac{5}{2}}\sqrt{x} + \frac{1}{8b^2}(bx+2)^{\frac{3}{2}}\sqrt{x} + \frac{3}{8b^2}\sqrt{x}\sqrt{bx+2} + \frac{3}{8}\sqrt{x(bx+2)}\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(5/2),x)
```

```
[Out] 1/5/b*x^(3/2)*(b*x+2)^(7/2)-3/20/b^2*x^(1/2)*(b*x+2)^(7/2)+1/20/b^2*x^(1/2)
*(b*x+2)^(5/2)+1/8/b^2*x^(1/2)*(b*x+2)^(3/2)+3/8*x^(1/2)*(b*x+2)^(1/2)/b^2+
3/8/b^(5/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x
^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93715, size = 401, normalized size = 3.26

$$\left[\frac{\left(8\,b^{5}x^{4}+42\,b^{4}x^{3}+62\,b^{3}x^{2}+5\,b^{2}x-15\,b\right)\sqrt{bx+2}\sqrt{x}+15\,\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{40\,b^{3}}, \frac{\left(8\,b^{5}x^{4}+42\,b^{4}x^{3}+62\,b^{3}x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 + 42*b^4*x^3 + 62*b^3*x^2 + 5*b^2*x - 15*b)*sqrt(b*x + 2)
*sqrt(x) + 15*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/
40*((8*b^5*x^4 + 42*b^4*x^3 + 62*b^3*x^2 + 5*b^2*x - 15*b)*sqrt(b*x + 2)*sq
rt(x) - 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

Sympy [A] time = 34.2153, size = 138, normalized size = 1.12

$$\frac{b^{3}x^{\frac{11}{2}}}{5\sqrt{bx+2}}+\frac{29b^{2}x^{\frac{9}{2}}}{20\sqrt{bx+2}}+\frac{73bx^{\frac{7}{2}}}{20\sqrt{bx+2}}+\frac{129x^{\frac{5}{2}}}{40\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{8b\sqrt{bx+2}}-\frac{3\sqrt{x}}{4b^{2}\sqrt{bx+2}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(5/2),x)
```

```
[Out] b**3*x**(11/2)/(5*sqrt(b*x + 2)) + 29*b**2*x**(9/2)/(20*sqrt(b*x + 2)) + 73
*b*x**(7/2)/(20*sqrt(b*x + 2)) + 129*x**(5/2)/(40*sqrt(b*x + 2)) - x**(3/2)
/(8*b*sqrt(b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(b*x + 2)) + 3*asinh(sqrt(2)*s
qrt(b)*sqrt(x)/2)/(4*b**(5/2))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.559 $\int \sqrt{x}(2+bx)^{5/2}\,dx$

Optimal. Leaf size=102

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(bx+2)^{5/2}+\frac{5}{12}x^{3/2}(bx+2)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{bx+2}+\frac{5\sqrt{x}\sqrt{bx+2}}{8b}$$

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(8*b) + (5*x^(3/2)*Sqrt[2 + b*x])/8 + (5*x^(3/2)*
(2 + b*x)^(3/2))/12 + (x^(3/2)*(2 + b*x)^(5/2))/4 - (5*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(3/2))
```

Rubi [A] time = 0.0221546, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(bx+2)^{5/2}+\frac{5}{12}x^{3/2}(bx+2)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{bx+2}+\frac{5\sqrt{x}\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 + b*x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(8*b) + (5*x^(3/2)*Sqrt[2 + b*x])/8 + (5*x^(3/2)*
(2 + b*x)^(3/2))/12 + (x^(3/2)*(2 + b*x)^(5/2))/4 - (5*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(2+bx)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(2+bx)^{5/2}+\frac{5}{4}\int \sqrt{x}(2+bx)^{3/2}\,dx\\ &= \frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}+\frac{5}{4}\int \sqrt{x}\sqrt{2+bx}\,dx\\ &= \frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}+\frac{5}{8}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx\\ &= \frac{5\sqrt{x}\sqrt{2+bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}-\frac{5\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{8b}\\ &= \frac{5\sqrt{x}\sqrt{2+bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}-\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{\right.}{4b}\\ &= \frac{5\sqrt{x}\sqrt{2+bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0403282, size = 70, normalized size = 0.69

$$\frac{\sqrt{x}\sqrt{bx+2}\left(6b^3x^3+34b^2x^2+59bx+15\right)}{24b}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 + 59*b*x + 34*b^2*x^2 + 6*b^3*x^3))/(24*b) - (5*
ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(3/2))
```

Maple [A] time = 0.003, size = 99, normalized size = 1.

$$\frac{1}{4}x^{\frac{3}{2}}(bx+2)^{\frac{5}{2}}+\frac{5}{12}x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}+\frac{5}{8}x^{\frac{3}{2}}\sqrt{bx+2}+\frac{5}{8b}\sqrt{x}\sqrt{bx+2}-\frac{5}{8}\sqrt{x(bx+2)}\ln\left((bx+1)\frac{1}{\sqrt{b}}+\sqrt{bx^2+2x}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)*x^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(b*x+2)^(5/2)+5/12*x^(3/2)*(b*x+2)^(3/2)+5/8*x^(3/2)*(b*x+2)^(1
/2)+5/8*x^(1/2)*(b*x+2)^(1/2)/b-5/8/b^(3/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)
/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81017, size = 369, normalized size = 3.62

$$\left[\frac{\left(6\,b^4x^3+34\,b^3x^2+59\,b^2x+15\,b\right)\sqrt{bx+2}\sqrt{x}+15\,\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{24\,b^2}, \frac{\left(6\,b^4x^3+34\,b^3x^2+59\,b^2x+15\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 + 34*b^3*x^2 + 59*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 1
5*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^2, 1/24*((6*b^4*x
^3 + 34*b^3*x^2 + 59*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arct
an(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^2]
```

Sympy [A] time = 16.3658, size = 119, normalized size = 1.17

$$\frac{b^3x^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{23b^2x^{\frac{7}{2}}}{12\sqrt{bx+2}}+\frac{127bx^{\frac{5}{2}}}{24\sqrt{bx+2}}+\frac{133x^{\frac{3}{2}}}{24\sqrt{bx+2}}+\frac{5\sqrt{x}}{4b\sqrt{bx+2}}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)*x**(1/2),x)
```

```
[Out] b**3*x**(9/2)/(4*sqrt(b*x + 2)) + 23*b**2*x**(7/2)/(12*sqrt(b*x + 2)) + 127
*b*x**(5/2)/(24*sqrt(b*x + 2)) + 133*x**(3/2)/(24*sqrt(b*x + 2)) + 5*sqrt(x
)/(4*b*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(3/2))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.560 $\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=79

$$\frac{1}{3}\sqrt{x}(bx+2)^{5/2}+\frac{5}{6}\sqrt{x}(bx+2)^{3/2}+\frac{5}{2}\sqrt{x}\sqrt{bx+2}+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/2 + (5*Sqrt[x]*(2 + b*x)^(3/2))/6 + (Sqrt[x]*(2 +
 b*x)^(5/2))/3 + (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rubi [A] time = 0.0159358, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$\frac{1}{3}\sqrt{x}(bx+2)^{5/2}+\frac{5}{6}\sqrt{x}(bx+2)^{3/2}+\frac{5}{2}\sqrt{x}\sqrt{bx+2}+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/2 + (5*Sqrt[x]*(2 + b*x)^(3/2))/6 + (Sqrt[x]*(2 +
 b*x)^(5/2))/3 + (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}(2+bx)^{5/2} + \frac{5}{3}\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx$$
$$= \frac{5}{6}\sqrt{x}(2+bx)^{3/2} + \frac{1}{3}\sqrt{x}(2+bx)^{5/2} + \frac{5}{2}\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx$$
$$= \frac{5}{2}\sqrt{x}\sqrt{2+bx} + \frac{5}{6}\sqrt{x}(2+bx)^{3/2} + \frac{1}{3}\sqrt{x}(2+bx)^{5/2} + \frac{5}{2}\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx$$
$$= \frac{5}{2}\sqrt{x}\sqrt{2+bx} + \frac{5}{6}\sqrt{x}(2+bx)^{3/2} + \frac{1}{3}\sqrt{x}(2+bx)^{5/2} + 5\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{5}{2}\sqrt{x}\sqrt{2+bx} + \frac{5}{6}\sqrt{x}(2+bx)^{3/2} + \frac{1}{3}\sqrt{x}(2+bx)^{5/2} + \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0303438, size = 57, normalized size = 0.72

$$\frac{1}{6}\sqrt{x}\sqrt{bx+2}\left(2b^2x^2+13bx+33\right) + \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(33 + 13*b*x + 2*b^2*x^2))/6 + (5*ArcSinh[(Sqrt[b]*S
qrt[x])/Sqrt[2]])/Sqrt[b]
```

Maple [A] time = 0.003, size = 84, normalized size = 1.1

$$\frac{1}{3}(bx+2)^{\frac{5}{2}}\sqrt{x} + \frac{5}{6}(bx+2)^{\frac{3}{2}}\sqrt{x} + \frac{5}{2}\sqrt{x}\sqrt{bx+2} + \frac{5}{2}\sqrt{x(bx+2)}\ln\left((bx+1)\frac{1}{\sqrt{b}} + \sqrt{bx^2+2x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*(b*x+2)^(5/2)*x^(1/2)+5/6*(b*x+2)^(3/2)*x^(1/2)+5/2*x^(1/2)*(b*x+2)^(1/
2)+5/2*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*
x)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75174, size = 325, normalized size = 4.11

$$\left[\frac{\left(2 b^3 x^2+13 b^2 x+33 b\right) \sqrt{b x+2} \sqrt{x}+15 \sqrt{b} \log \left(b x+\sqrt{b x+2} \sqrt{b} \sqrt{x}+1\right)}{6 b}, \frac{\left(2 b^3 x^2+13 b^2 x+33 b\right) \sqrt{b x+2} \sqrt{x}-30 \sqrt{-\right.}{6 b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 + 13*b^2*x + 33*b)*sqrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(
b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b, 1/6*((2*b^3*x^2 + 13*b^2*x + 3
3*b)*sqrt(b*x + 2)*sqrt(x) - 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*s
qrt(x))))/b]
```

Sympy [A] time = 10.1798, size = 97, normalized size = 1.23

$$\frac{b^3 x^{\frac{7}{2}}}{3 \sqrt{b x+2}}+\frac{17 b^2 x^{\frac{5}{2}}}{6 \sqrt{b x+2}}+\frac{59 b x^{\frac{3}{2}}}{6 \sqrt{b x+2}}+\frac{11 \sqrt{x}}{\sqrt{b x+2}}+\frac{5 \operatorname{asinh}\left(\frac{\sqrt{2} \sqrt{b} \sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] b**3*x**(7/2)/(3*sqrt(b*x + 2)) + 17*b**2*x**(5/2)/(6*sqrt(b*x + 2)) + 59*b
*x**(3/2)/(6*sqrt(b*x + 2)) + 11*sqrt(x)/sqrt(b*x + 2) + 5*asinh(sqrt(2)*sq
rt(b)*sqrt(x)/2)/sqrt(b)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.561 $\int \frac{(2+bx)^{5/2}}{x^{3/2}} \, dx$

Optimal. Leaf size=79

$$-\frac{2(bx+2)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}(bx+2)^{3/2}+\frac{15}{2}b\sqrt{x}\sqrt{bx+2}+15\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] (15*b*Sqrt[x]*Sqrt[2 + b*x])/2 + (5*b*Sqrt[x]*(2 + b*x)^(3/2))/2 - (2*(2 +
b*x)^(5/2))/Sqrt[x] + 15*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rubi [A] time = 0.0162733, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$-\frac{2(bx+2)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}(bx+2)^{3/2}+\frac{15}{2}b\sqrt{x}\sqrt{bx+2}+15\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/x^(3/2),x]
```


```
[Out] (15*b*Sqrt[x]*Sqrt[2 + b*x])/2 + (5*b*Sqrt[x]*(2 + b*x)^(3/2))/2 - (2*(2 +
b*x)^(5/2))/Sqrt[x] + 15*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(2+bx)^{5/2}}{\sqrt{x}} + (5b)\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx \\
&= \frac{5}{2}b\sqrt{x}(2+bx)^{3/2} - \frac{2(2+bx)^{5/2}}{\sqrt{x}} + \frac{1}{2}(15b)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx \\
&= \frac{15}{2}b\sqrt{x}\sqrt{2+bx} + \frac{5}{2}b\sqrt{x}(2+bx)^{3/2} - \frac{2(2+bx)^{5/2}}{\sqrt{x}} + \frac{1}{2}(15b)\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx \\
&= \frac{15}{2}b\sqrt{x}\sqrt{2+bx} + \frac{5}{2}b\sqrt{x}(2+bx)^{3/2} - \frac{2(2+bx)^{5/2}}{\sqrt{x}} + (15b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right) \\
&= \frac{15}{2}b\sqrt{x}\sqrt{2+bx} + \frac{5}{2}b\sqrt{x}(2+bx)^{3/2} - \frac{2(2+bx)^{5/2}}{\sqrt{x}} + 15\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0050738, size = 28, normalized size = 0.35

$$
-\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx}{2}\right)}{\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-8*Sqrt[2]*Hypergeometric2F1[-5/2, -1/2, 1/2, -(b*x)/2])/Sqrt[x]
```

Maple [A] time = 0.015, size = 81, normalized size = 1.

$$
\frac{b^3x^3+11\,b^2x^2+2\,bx-32}{2}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+2}}+\frac{15}{2}\sqrt{b}\ln\left((bx+1)\frac{1}{\sqrt{b}}+\sqrt{bx^2+2\,x}\right)\sqrt{x\,(bx+2)}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(3/2),x)
```

```
[Out] 1/2*(b^3*x^3+11*b^2*x^2+2*b*x-32)/x^(1/2)/(b*x+2)^(1/2)+15/2*b^(1/2)*ln((b*
x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))*(x*(b*x+2))^(1/2)/x^(1/2)/(b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86295, size = 313, normalized size = 3.96

$$\left[\frac{15\sqrt{b}x\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+\left(b^2x^2+9\,bx-16\right)\sqrt{bx+2}\sqrt{x}}{2\,x}, -\frac{30\sqrt{-b}x\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)-\left(b^2x^2+9\,bx-\right.}{2\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (b^2*x^2
+ 9*b*x - 16)*sqrt(b*x + 2)*sqrt(x))/x, -1/2*(30*sqrt(-b)*x*arctan(sqrt(b*x
 + 2)*sqrt(-b)/(b*sqrt(x))) - (b^2*x^2 + 9*b*x - 16)*sqrt(b*x + 2)*sqrt(x))
/x]
```

Sympy [A] time = 11.0624, size = 94, normalized size = 1.19

$$15\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)+\frac{b^3x^{\frac{5}{2}}}{2\sqrt{bx+2}}+\frac{11b^2x^{\frac{3}{2}}}{2\sqrt{bx+2}}+\frac{b\sqrt{x}}{\sqrt{bx+2}}-\frac{16}{\sqrt{x}\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(3/2),x)
```

```
[Out] 15*sqrt(b)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**3*x**(5/2)/(2*sqrt(b*x + 2
)) + 11*b**2*x**(3/2)/(2*sqrt(b*x + 2)) + b*sqrt(x)/sqrt(b*x + 2) - 16/(sqr
t(x)*sqrt(b*x + 2))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.562 $\int \frac{(2+bx)^{5/2}}{x^{5/2}} dx$

Optimal. Leaf size=81

$$5b^2\sqrt{x}\sqrt{bx+2}+10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(bx+2)^{5/2}}{3x^{3/2}}-\frac{10b(bx+2)^{3/2}}{3\sqrt{x}}$$

```
[Out] 5*b^2*Sqrt[x]*Sqrt[2 + b*x] - (10*b*(2 + b*x)^(3/2))/(3*Sqrt[x]) - (2*(2 +
b*x)^(5/2))/(3*x^(3/2)) + 10*b^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rubi [A] time = 0.0171751, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$5b^2\sqrt{x}\sqrt{bx+2}+10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(bx+2)^{5/2}}{3x^{3/2}}-\frac{10b(bx+2)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[2 + b*x] - (10*b*(2 + b*x)^(3/2))/(3*Sqrt[x]) - (2*(2 +
b*x)^(5/2))/(3*x^(3/2)) + 10*b^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx = -\frac{2(2+bx)^{5/2}}{3x^{3/2}} + \frac{1}{3}(5b)\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx$$
$$= -\frac{10b(2+bx)^{3/2}}{3\sqrt{x}} - \frac{2(2+bx)^{5/2}}{3x^{3/2}} + \left(5b^2\right)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx$$
$$= 5b^2\sqrt{x}\sqrt{2+bx} - \frac{10b(2+bx)^{3/2}}{3\sqrt{x}} - \frac{2(2+bx)^{5/2}}{3x^{3/2}} + \left(5b^2\right)\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx$$
$$= 5b^2\sqrt{x}\sqrt{2+bx} - \frac{10b(2+bx)^{3/2}}{3\sqrt{x}} - \frac{2(2+bx)^{5/2}}{3x^{3/2}} + \left(10b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= 5b^2\sqrt{x}\sqrt{2+bx} - \frac{10b(2+bx)^{3/2}}{3\sqrt{x}} - \frac{2(2+bx)^{5/2}}{3x^{3/2}} + 10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Mathematica [C] time = 0.0070435, size = 30, normalized size = 0.37

$$-\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx}{2}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (-8*Sqrt[2]*Hypergeometric2F1[-5/2, -3/2, -1/2, -(b*x)/2])/(3*x^(3/2))
```

Maple [A] time = 0.014, size = 82, normalized size = 1.

$$\frac{3\,b^3x^3-22\,b^2x^2-64\,bx-16}{3}x^{-\frac{3}{2}}\frac{1}{\sqrt{bx+2}}+5\,\frac{b^{3/2}\sqrt{x\,(bx+2)}}{\sqrt{x}\sqrt{bx+2}}\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{bx^2+2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(5/2),x)
```

```
[Out] 1/3*(3*b^3*x^3-22*b^2*x^2-64*b*x-16)/x^(3/2)/(b*x+2)^(1/2)+5*b^(3/2)*ln((b*
x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))*(x*(b*x+2))^(1/2)/x^(1/2)/(b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80796, size = 332, normalized size = 4.1

$$\left[\frac{15\,b^{\frac{3}{2}}x^2\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+\left(3\,b^2x^2-28\,bx-8\right)\sqrt{bx+2}\sqrt{x}}{3\,x^2},-\frac{30\,\sqrt{-b}bx^2\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)-\left(3\,b^2x^2-28\right.}{3\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*b^(3/2)*x^2*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (3*b^2*
x^2 - 28*b*x - 8)*sqrt(b*x + 2)*sqrt(x))/x^2, -1/3*(30*sqrt(-b)*b*x^2*arcta
n(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 28*b*x - 8)*sqrt(b*x +
 2)*sqrt(x))/x^2]
```

Sympy [A] time = 10.9004, size = 88, normalized size = 1.09

$$b^{\frac{5}{2}}x\sqrt{1+\frac{2}{bx}}-\frac{28b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-5b^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)+10b^{\frac{3}{2}}\log\left(\sqrt{1+\frac{2}{bx}}+1\right)-\frac{8\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(5/2),x)
```

```
[Out] b**(5/2)*x*sqrt(1 + 2/(b*x)) - 28*b**(3/2)*sqrt(1 + 2/(b*x))/3 - 5*b**(3/2)
*log(1/(b*x)) + 10*b**(3/2)*log(sqrt(1 + 2/(b*x)) + 1) - 8*sqrt(b)*sqrt(1 +
 2/(b*x))/(3*x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.563 $\int x^{5/2}(2-bx)^{5/2}\,dx$

Optimal. Leaf size=150

$$-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{24b}$$

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(16*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(48*b^2) -
(x^(5/2)*Sqrt[2 - b*x])/(24*b) + (x^(7/2)*Sqrt[2 - b*x])/8 + (x^(7/2)*(2 -
b*x)^(3/2))/6 + (x^(7/2)*(2 - b*x)^(5/2))/6 + (5*ArcSin[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/(8*b^(7/2))
```

Rubi [A] time = 0.0452038, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(16*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(48*b^2) -
(x^(5/2)*Sqrt[2 - b*x])/(24*b) + (x^(7/2)*Sqrt[2 - b*x])/8 + (x^(7/2)*(2 -
b*x)^(3/2))/6 + (x^(7/2)*(2 - b*x)^(5/2))/6 + (5*ArcSin[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/(8*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^{5/2}(2-bx)^{5/2}\,dx &= \frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{5}{6}\int x^{5/2}(2-bx)^{3/2}\,dx\\
&= \frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{2}\int x^{5/2}\sqrt{2-bx}\,dx\\
&= \frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{8}\int\frac{x^{5/2}}{\sqrt{2-bx}}\,dx\\
&= -\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{5\int\frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{24b}\\
&= -\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{5\int\frac{\sqrt{x}}{\sqrt{2-bx}}\,d}{16b^2}\\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5}\\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5}\\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5}
\end{aligned}$$

Mathematica [A] time = 0.0647716, size = 87, normalized size = 0.58

$$\frac{\sqrt{x}\sqrt{2-bx}\left(8b^5x^5-40b^4x^4+54b^3x^3-2b^2x^2-5bx-15\right)}{48b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x - 2*b^2*x^2 + 54*b^3*x^3 - 40*b^4*x^4 +
 8*b^5*x^5))/(48*b^3) + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(8*b^(7/2))
```

Maple [A] time = 0.004, size = 148, normalized size = 1.

$$-\frac{1}{6\,b}x^{\frac{5}{2}}\left(-bx+2\right)^{\frac{7}{2}}-\frac{1}{6\,b^2}x^{\frac{3}{2}}\left(-bx+2\right)^{\frac{7}{2}}-\frac{1}{8\,b^3}\sqrt{x}\left(-bx+2\right)^{\frac{7}{2}}+\frac{1}{24\,b^3}\left(-bx+2\right)^{\frac{5}{2}}\sqrt{x}+\frac{5}{48\,b^3}\left(-bx+2\right)^{\frac{3}{2}}\sqrt{x}+\frac{5}{16\,b^3}\sqrt{x}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(5/2),x)
```

```
[Out] -1/6/b*x^(5/2)*(-b*x+2)^(7/2)-1/6/b^2*x^(3/2)*(-b*x+2)^(7/2)-1/8/b^3*x^(1/2
)*(-b*x+2)^(7/2)+1/24/b^3*x^(1/2)*(-b*x+2)^(5/2)+5/48/b^3*x^(1/2)*(-b*x+2)^
(3/2)+5/16*x^(1/2)*(-b*x+2)^(1/2)/b^3+5/16/b^(7/2)*((-b*x+2)*x)^(1/2)/(-b*x
+2)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91903, size = 437, normalized size = 2.91

$$\left[\frac{\left(8\,b^6x^5 - 40\,b^5x^4 + 54\,b^4x^3 - 2\,b^3x^2 - 5\,b^2x - 15\,b\right)\sqrt{-bx+2}\sqrt{x} - 15\,\sqrt{-b}\log\left(-bx + \sqrt{-bx+2}\sqrt{-b}\sqrt{x} + 1\right)}{48\,b^4}, \frac{\left(8\,b^6x^5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*((8*b^6*x^5 - 40*b^5*x^4 + 54*b^4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*s
qrt(-b*x + 2)*sqrt(x) - 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt
(x) + 1))/b^4, 1/48*((8*b^6*x^5 - 40*b^5*x^4 + 54*b^4*x^3 - 2*b^3*x^2 - 5*b
^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqr
t(b)*sqrt(x))))/b^4]
```

Sympy [A] time = 67.4091, size = 337, normalized size = 2.25

$$\begin{cases} \frac{ib^3x^{\frac{13}{2}}}{6\sqrt{bx-2}} - \frac{7ib^2x^{\frac{11}{2}}}{6\sqrt{bx-2}} + \frac{67ibx^{\frac{9}{2}}}{24\sqrt{bx-2}} - \frac{55ix^{\frac{7}{2}}}{24\sqrt{bx-2}} - \frac{ix^{\frac{5}{2}}}{48b\sqrt{bx-2}} - \frac{5ix^{\frac{3}{2}}}{48b^2\sqrt{bx-2}} + \frac{5i\sqrt{x}}{8b^3\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}} & \text{for}\: \frac{|bx|}{2} > 1 \\ -\frac{b^3x^{\frac{13}{2}}}{6\sqrt{-bx+2}} + \frac{7b^2x^{\frac{11}{2}}}{6\sqrt{-bx+2}} - \frac{67bx^{\frac{9}{2}}}{24\sqrt{-bx+2}} + \frac{55x^{\frac{7}{2}}}{24\sqrt{-bx+2}} + \frac{x^{\frac{5}{2}}}{48b\sqrt{-bx+2}} + \frac{5x^{\frac{3}{2}}}{48b^2\sqrt{-bx+2}} - \frac{5\sqrt{x}}{8b^3\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((I*b**3*x**(13/2)/(6*sqrt(b*x - 2)) - 7*I*b**2*x**(11/2)/(6*sqrt(
b*x - 2)) + 67*I*b*x**(9/2)/(24*sqrt(b*x - 2)) - 55*I*x**(7/2)/(24*sqrt(b*x
 - 2)) - I*x**(5/2)/(48*b*sqrt(b*x - 2)) - 5*I*x**(3/2)/(48*b**2*sqrt(b*x -
 2)) + 5*I*sqrt(x)/(8*b**3*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(
x)/2)/(8*b**(7/2)), Abs(b*x)/2 > 1), (-b**3*x**(13/2)/(6*sqrt(-b*x + 2)) +
7*b**2*x**(11/2)/(6*sqrt(-b*x + 2)) - 67*b*x**(9/2)/(24*sqrt(-b*x + 2)) + 5
5*x**(7/2)/(24*sqrt(-b*x + 2)) + x**(5/2)/(48*b*sqrt(-b*x + 2)) + 5*x**(3/2
)/(48*b**2*sqrt(-b*x + 2)) - 5*sqrt(x)/(8*b**3*sqrt(-b*x + 2)) + 5*asin(sqr
t(2)*sqrt(b)*sqrt(x)/2)/(8*b**(7/2)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.564 $\int x^{3/2}(2-bx)^{5/2}\,dx$

Optimal. Leaf size=128

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(8*b) + (x^(5/
2)*Sqrt[2 - b*x])/4 + (x^(5/2)*(2 - b*x)^(3/2))/4 + (x^(5/2)*(2 - b*x)^(5/2
))/5 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Rubi [A] time = 0.0285975, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(8*b) + (x^(5/
2)*Sqrt[2 - b*x])/4 + (x^(5/2)*(2 - b*x)^(3/2))/4 + (x^(5/2)*(2 - b*x)^(5/2
))/5 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^{3/2}(2-bx)^{5/2}\,dx = \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \int x^{3/2}(2-bx)^{3/2}\,dx$$
$$= \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \frac{3}{4}\int x^{3/2}\sqrt{2-bx}\,dx$$
$$= \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \frac{3\int \frac{1}{\sqrt{x}\sqrt{2-bx}}}{8b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \frac{3\,\text{Subst}\Big(\int}{}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \frac{3\sin^{-1}\Big(\frac{\sqrt{b}}{\sqrt{2}}}{4b^{5/2}}$$

Mathematica [A] time = 0.0464512, size = 79, normalized size = 0.62

$$\frac{\sqrt{x}\sqrt{2-bx}\left(8b^4x^4-42b^3x^3+62b^2x^2-5bx-15\right)}{40b^2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x + 62*b^2*x^2 - 42*b^3*x^3 + 8*b^4*x^4))
/(40*b^2) + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Maple [A] time = 0.005, size = 132, normalized size = 1.

$$-\frac{1}{5b}x^{\frac{3}{2}}(-bx+2)^{\frac{7}{2}}-\frac{3}{20b^2}\sqrt{x}(-bx+2)^{\frac{7}{2}}+\frac{1}{20b^2}(-bx+2)^{\frac{5}{2}}\sqrt{x}+\frac{1}{8b^2}(-bx+2)^{\frac{3}{2}}\sqrt{x}+\frac{3}{8b^2}\sqrt{x}\sqrt{-bx+2}+\frac{3}{8}\sqrt{(-bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(5/2),x)
```

```
[Out] -1/5/b*x^(3/2)*(-b*x+2)^(7/2)-3/20/b^2*x^(1/2)*(-b*x+2)^(7/2)+1/20/b^2*x^(1
/2)*(-b*x+2)^(5/2)+1/8/b^2*x^(1/2)*(-b*x+2)^(3/2)+3/8*x^(1/2)*(-b*x+2)^(1/2
)/b^2+3/8/b^(5/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan(b^(1/2)*
(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89604, size = 405, normalized size = 3.16

$$\left[\frac{\left(8\,b^{5}x^{4}-42\,b^{4}x^{3}+62\,b^{3}x^{2}-5\,b^{2}x-15\,b\right)\sqrt{-bx+2}\sqrt{x}-15\,\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{40\,b^{3}}, \frac{\left(8\,b^{5}x^{4}-42\,b^{4}x^{3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 - 42*b^4*x^3 + 62*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2
)*sqrt(x) - 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^
3, 1/40*((8*b^5*x^4 - 42*b^4*x^3 + 62*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x +
 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

Sympy [A] time = 33.3455, size = 294, normalized size = 2.3

$$\begin{cases} \frac{ib^{3}x^{\frac{11}{2}}}{5\sqrt{bx-2}}-\frac{29ib^{2}x^{\frac{9}{2}}}{20\sqrt{bx-2}}+\frac{73ibx^{\frac{7}{2}}}{20\sqrt{bx-2}}-\frac{129ix^{\frac{5}{2}}}{40\sqrt{bx-2}}-\frac{ix^{\frac{3}{2}}}{8b\sqrt{bx-2}}+\frac{3i\sqrt{x}}{4b^{2}\sqrt{bx-2}}-\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ -\frac{b^{3}x^{\frac{11}{2}}}{5\sqrt{-bx+2}}+\frac{29b^{2}x^{\frac{9}{2}}}{20\sqrt{-bx+2}}-\frac{73bx^{\frac{7}{2}}}{20\sqrt{-bx+2}}+\frac{129x^{\frac{5}{2}}}{40\sqrt{-bx+2}}+\frac{x^{\frac{3}{2}}}{8b\sqrt{-bx+2}}-\frac{3\sqrt{x}}{4b^{2}\sqrt{-bx+2}}+\frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((I*b**3*x**(11/2)/(5*sqrt(b*x - 2)) - 29*I*b**2*x**(9/2)/(20*sqrt
(b*x - 2)) + 73*I*b*x**(7/2)/(20*sqrt(b*x - 2)) - 129*I*x**(5/2)/(40*sqrt(b
*x - 2)) - I*x**(3/2)/(8*b*sqrt(b*x - 2)) + 3*I*sqrt(x)/(4*b**2*sqrt(b*x -
2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2)), Abs(b*x)/2 > 1), (
-b**3*x**(11/2)/(5*sqrt(-b*x + 2)) + 29*b**2*x**(9/2)/(20*sqrt(-b*x + 2)) -
 73*b*x**(7/2)/(20*sqrt(-b*x + 2)) + 129*x**(5/2)/(40*sqrt(-b*x + 2)) + x**
(3/2)/(8*b*sqrt(-b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(-b*x + 2)) + 3*asin(sqr
t(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.565 $\int \sqrt{x}(2-bx)^{5/2}\,dx$

Optimal. Leaf size=106

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{2-bx}-\frac{5\sqrt{x}\sqrt{2-bx}}{8b}$$

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(8*b) + (5*x^(3/2)*Sqrt[2 - b*x])/8 + (5*x^(3/2)
*(2 - b*x)^(3/2))/12 + (x^(3/2)*(2 - b*x)^(5/2))/4 + (5*ArcSin[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(3/2))
```

Rubi [A] time = 0.022558, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{2-bx}-\frac{5\sqrt{x}\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 - b*x)^(5/2),x]
```


```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(8*b) + (5*x^(3/2)*Sqrt[2 - b*x])/8 + (5*x^(3/2)
*(2 - b*x)^(3/2))/12 + (x^(3/2)*(2 - b*x)^(5/2))/4 + (5*ArcSin[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(3/2))
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(2-bx)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5}{4}\int \sqrt{x}(2-bx)^{3/2}\,dx \\ &= \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5}{4}\int \sqrt{x}\sqrt{2-bx}\,dx \\ &= \frac{5}{8}x^{3/2}\sqrt{2-bx} + \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5}{8}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx \\ &= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2-bx} + \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{8b} \\ &= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2-bx} + \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x\right.}{4b} \\ &= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2-bx} + \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0437954, size = 71, normalized size = 0.67

$$\frac{\sqrt{x}\sqrt{2-bx}\left(6b^3x^3-34b^2x^2+59bx-15\right)}{24b}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 + 59*b*x - 34*b^2*x^2 + 6*b^3*x^3))/(24*b) + (5
*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(3/2))
```

Maple [A] time = 0.004, size = 107, normalized size = 1.

$$\frac{1}{4}x^{\frac{3}{2}}(-bx+2)^{\frac{5}{2}}+\frac{5}{12}x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}}+\frac{5}{8}x^{\frac{3}{2}}\sqrt{-bx+2}-\frac{5}{8b}\sqrt{x}\sqrt{-bx+2}+\frac{5}{8}\sqrt{(-bx+2)x}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)*x^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(-b*x+2)^(5/2)+5/12*x^(3/2)*(-b*x+2)^(3/2)+5/8*x^(3/2)*(-b*x+2)
^(1/2)-5/8*x^(1/2)*(-b*x+2)^(1/2)/b+5/8/b^(3/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)
^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88979, size = 373, normalized size = 3.52

$$\left[\frac{\left(6\,b^4x^3-34\,b^3x^2+59\,b^2x-15\,b\right)\sqrt{-bx+2}\sqrt{x}-15\,\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{24\,b^2},\ \frac{\left(6\,b^4x^3-34\,b^3x^2+59\,b^2x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 - 34*b^3*x^2 + 59*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) -
15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^2, 1/24*((6*
b^4*x^3 - 34*b^3*x^2 + 59*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)
*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^2]
```

Sympy [A] time = 15.8152, size = 255, normalized size = 2.41

$$\begin{cases} \frac{ib^3x^{\frac{9}{2}}}{4\sqrt{bx-2}}-\frac{23ib^2x^{\frac{7}{2}}}{12\sqrt{bx-2}}+\frac{127ibx^{\frac{5}{2}}}{24\sqrt{bx-2}}-\frac{133ix^{\frac{3}{2}}}{24\sqrt{bx-2}}+\frac{5i\sqrt{x}}{4b\sqrt{bx-2}}-\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ -\frac{b^3x^{\frac{9}{2}}}{4\sqrt{-bx+2}}+\frac{23b^2x^{\frac{7}{2}}}{12\sqrt{-bx+2}}-\frac{127bx^{\frac{5}{2}}}{24\sqrt{-bx+2}}+\frac{133x^{\frac{3}{2}}}{24\sqrt{-bx+2}}-\frac{5\sqrt{x}}{4b\sqrt{-bx+2}}+\frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)*x**(1/2),x)
```

```
[Out] Piecewise((I*b**3*x**(9/2)/(4*sqrt(b*x - 2)) - 23*I*b**2*x**(7/2)/(12*sqrt(
b*x - 2)) + 127*I*b*x**(5/2)/(24*sqrt(b*x - 2)) - 133*I*x**(3/2)/(24*sqrt(b
*x - 2)) + 5*I*sqrt(x)/(4*b*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt
(x)/2)/(4*b**(3/2)), Abs(b*x)/2 > 1), (-b**3*x**(9/2)/(4*sqrt(-b*x + 2)) +
23*b**2*x**(7/2)/(12*sqrt(-b*x + 2)) - 127*b*x**(5/2)/(24*sqrt(-b*x + 2)) +
 133*x**(3/2)/(24*sqrt(-b*x + 2)) - 5*sqrt(x)/(4*b*sqrt(-b*x + 2)) + 5*asin
(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(3/2)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.566 $\int \frac{(2-bx)^{5/2}}{\sqrt{x}} dx$

Optimal. Leaf size=82

$$\frac{1}{3}\sqrt{x}(2-bx)^{5/2}+\frac{5}{6}\sqrt{x}(2-bx)^{3/2}+\frac{5}{2}\sqrt{x}\sqrt{2-bx}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] (5*Sqrt[x]*Sqrt[2 - b*x])/2 + (5*Sqrt[x]*(2 - b*x)^(3/2))/6 + (Sqrt[x]*(2 -
 b*x)^(5/2))/3 + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rubi [A] time = 0.0165851, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{1}{3}\sqrt{x}(2-bx)^{5/2}+\frac{5}{6}\sqrt{x}(2-bx)^{3/2}+\frac{5}{2}\sqrt{x}\sqrt{2-bx}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 - b*x])/2 + (5*Sqrt[x]*(2 - b*x)^(3/2))/6 + (Sqrt[x]*(2 -
 b*x)^(5/2))/3 + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(2-bx)^{5/2}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}(2-bx)^{5/2} + \frac{5}{3}\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx$$
$$= \frac{5}{6}\sqrt{x}(2-bx)^{3/2} + \frac{1}{3}\sqrt{x}(2-bx)^{5/2} + \frac{5}{2}\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx$$
$$= \frac{5}{2}\sqrt{x}\sqrt{2-bx} + \frac{5}{6}\sqrt{x}(2-bx)^{3/2} + \frac{1}{3}\sqrt{x}(2-bx)^{5/2} + \frac{5}{2}\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx$$
$$= \frac{5}{2}\sqrt{x}\sqrt{2-bx} + \frac{5}{6}\sqrt{x}(2-bx)^{3/2} + \frac{1}{3}\sqrt{x}(2-bx)^{5/2} + 5\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{5}{2}\sqrt{x}\sqrt{2-bx} + \frac{5}{6}\sqrt{x}(2-bx)^{3/2} + \frac{1}{3}\sqrt{x}(2-bx)^{5/2} + \frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0367934, size = 58, normalized size = 0.71

$$\frac{1}{6}\sqrt{x}\sqrt{2-bx}\left(2b^2x^2 - 13bx + 33\right) + \frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(33 - 13*b*x + 2*b^2*x^2))/6 + (5*ArcSin[(Sqrt[b]*Sq
rt[x])/Sqrt[2]])/Sqrt[b]
```

Maple [A] time = 0.003, size = 91, normalized size = 1.1

$$\frac{1}{3}(-bx+2)^{\frac{5}{2}}\sqrt{x} + \frac{5}{6}(-bx+2)^{\frac{3}{2}}\sqrt{x} + \frac{5}{2}\sqrt{x}\sqrt{-bx+2} + \frac{5}{2}\sqrt{(-bx+2)\,x}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2\,x}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*(-b*x+2)^(5/2)*x^(1/2)+5/6*(-b*x+2)^(3/2)*x^(1/2)+5/2*x^(1/2)*(-b*x+2)^
(1/2)+5/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*
(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84821, size = 329, normalized size = 4.01

$$\left[\frac{\left(2\,b^{3}x^{2}-13\,b^{2}x+33\,b\right)\sqrt{-bx+2}\sqrt{x}-15\,\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{6\,b},\ \frac{\left(2\,b^{3}x^{2}-13\,b^{2}x+33\,b\right)\sqrt{-bx+2}\cdots}{6\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 - 13*b^2*x + 33*b)*sqrt(-b*x + 2)*sqrt(x) - 15*sqrt(-b)*lo
g(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b, 1/6*((2*b^3*x^2 - 13*b^2*
x + 33*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b
)*sqrt(x))))/b]
```

Sympy [A] time = 9.95409, size = 209, normalized size = 2.55

$$\begin{cases} \frac{ib^{3}x^{\frac{7}{2}}}{3\sqrt{bx-2}} - \frac{17ib^{2}x^{\frac{5}{2}}}{6\sqrt{bx-2}} + \frac{59ibx^{\frac{3}{2}}}{6\sqrt{bx-2}} - \frac{11i\sqrt{x}}{\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for}\: \frac{\left|bx\right|}{2} > 1 \\ -\frac{b^{3}x^{\frac{7}{2}}}{3\sqrt{-bx+2}} + \frac{17b^{2}x^{\frac{5}{2}}}{6\sqrt{-bx+2}} - \frac{59bx^{\frac{3}{2}}}{6\sqrt{-bx+2}} + \frac{11\sqrt{x}}{\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*b**3*x**(7/2)/(3*sqrt(b*x - 2)) - 17*I*b**2*x**(5/2)/(6*sqrt(b
*x - 2)) + 59*I*b*x**(3/2)/(6*sqrt(b*x - 2)) - 11*I*sqrt(x)/sqrt(b*x - 2) -
 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x)/2 > 1), (-b**3*x**(
7/2)/(3*sqrt(-b*x + 2)) + 17*b**2*x**(5/2)/(6*sqrt(-b*x + 2)) - 59*b*x**(3/
2)/(6*sqrt(-b*x + 2)) + 11*sqrt(x)/sqrt(-b*x + 2) + 5*asin(sqrt(2)*sqrt(b)*
sqrt(x)/2)/sqrt(b), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.567 $\int \frac{(2-bx)^{5/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}(2-bx)^{3/2}-\frac{15}{2}b\sqrt{x}\sqrt{2-bx}-15\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] (-15*b*Sqrt[x]*Sqrt[2 - b*x])/2 - (5*b*Sqrt[x]*(2 - b*x)^(3/2))/2 - (2*(2 -
 b*x)^(5/2))/Sqrt[x] - 15*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rubi [A] time = 0.0169737, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {47, 50, 54, 216}

$$-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}(2-bx)^{3/2}-\frac{15}{2}b\sqrt{x}\sqrt{2-bx}-15\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-15*b*Sqrt[x]*Sqrt[2 - b*x])/2 - (5*b*Sqrt[x]*(2 - b*x)^(3/2))/2 - (2*(2 -
 b*x)^(5/2))/Sqrt[x] - 15*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2-bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(2-bx)^{5/2}}{\sqrt{x}} - (5b)\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx \\
&= -\frac{5}{2}b\sqrt{x}(2-bx)^{3/2} - \frac{2(2-bx)^{5/2}}{\sqrt{x}} - \frac{1}{2}(15b)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx \\
&= -\frac{15}{2}b\sqrt{x}\sqrt{2-bx} - \frac{5}{2}b\sqrt{x}(2-bx)^{3/2} - \frac{2(2-bx)^{5/2}}{\sqrt{x}} - \frac{1}{2}(15b)\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx \\
&= -\frac{15}{2}b\sqrt{x}\sqrt{2-bx} - \frac{5}{2}b\sqrt{x}(2-bx)^{3/2} - \frac{2(2-bx)^{5/2}}{\sqrt{x}} - (15b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{15}{2}b\sqrt{x}\sqrt{2-bx} - \frac{5}{2}b\sqrt{x}(2-bx)^{3/2} - \frac{2(2-bx)^{5/2}}{\sqrt{x}} - 15\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0053384, size = 28, normalized size = 0.34

$$-\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{bx}{2}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-8*Sqrt[2]*Hypergeometric2F1[-5/2, -1/2, 1/2, (b*x)/2])/Sqrt[x]
```

Maple [A] time = 0.018, size = 106, normalized size = 1.3

$$-\frac{b^3x^3-11\,b^2x^2+2\,bx+32}{2}\sqrt{(-bx+2)\,x}\frac{1}{\sqrt{-x\,(bx-2)}}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+2}}-\frac{15}{2}\sqrt{b}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2\,x}}\right)\sqrt{(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(3/2),x)
```

```
[Out] -1/2*(b^3*x^3-11*b^2*x^2+2*b*x+32)/(-x*(b*x-2))^(1/2)*((-b*x+2)*x)^(1/2)/x^
(1/2)/(-b*x+2)^(1/2)-15/2*b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2)
)*((-b*x+2)*x)^(1/2)/x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95462, size = 316, normalized size = 3.85

$$\left[\frac{15\,\sqrt{-b}x\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)+\left(b^2x^2-9\,bx-16\right)\sqrt{-bx+2}\sqrt{x}}{2\,x},\ \frac{30\,\sqrt{b}x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)+\left(b^2x^2-9\,bx-\right.}{2\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + (b^2*
x^2 - 9*b*x - 16)*sqrt(-b*x + 2)*sqrt(x))/x, 1/2*(30*sqrt(b)*x*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^2*x^2 - 9*b*x - 16)*sqrt(-b*x + 2)*sqrt(x
))/x]
```

Sympy [A] time = 11.0837, size = 202, normalized size = 2.46

$$\begin{cases} 15i\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)+\frac{ib^3x^{\frac{5}{2}}}{2\sqrt{bx-2}}-\frac{11ib^2x^{\frac{3}{2}}}{2\sqrt{bx-2}}+\frac{ib\sqrt{x}}{\sqrt{bx-2}}+\frac{16i}{\sqrt{x}\sqrt{bx-2}} & \text{for}\ \frac{|bx|}{2}>1 \\ -15\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)-\frac{b^3x^{\frac{5}{2}}}{2\sqrt{-bx+2}}+\frac{11b^2x^{\frac{3}{2}}}{2\sqrt{-bx+2}}-\frac{b\sqrt{x}}{\sqrt{-bx+2}}-\frac{16}{\sqrt{x}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(3/2),x)
```

```
[Out] Piecewise((15*I*sqrt(b)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2) + I*b**3*x**(5/2)/
(2*sqrt(b*x - 2)) - 11*I*b**2*x**(3/2)/(2*sqrt(b*x - 2)) + I*b*sqrt(x)/sqrt
(b*x - 2) + 16*I/(sqrt(x)*sqrt(b*x - 2)), Abs(b*x)/2 > 1), (-15*sqrt(b)*asi
n(sqrt(2)*sqrt(b)*sqrt(x)/2) - b**3*x**(5/2)/(2*sqrt(-b*x + 2)) + 11*b**2*x
**(3/2)/(2*sqrt(-b*x + 2)) - b*sqrt(x)/sqrt(-b*x + 2) - 16/(sqrt(x)*sqrt(-b
*x + 2)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.568 $\int \frac{(2-bx)^{5/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=84

$$5b^2\sqrt{x}\sqrt{2-bx}+10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}$$

```
[Out] 5*b^2*Sqrt[x]*Sqrt[2 - b*x] + (10*b*(2 - b*x)^(3/2))/(3*Sqrt[x]) - (2*(2 -
b*x)^(5/2))/(3*x^(3/2)) + 10*b^(3/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rubi [A] time = 0.0172905, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {47, 50, 54, 216}

$$5b^2\sqrt{x}\sqrt{2-bx}+10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/x^(5/2),x]
```


```
[Out] 5*b^2*Sqrt[x]*Sqrt[2 - b*x] + (10*b*(2 - b*x)^(3/2))/(3*Sqrt[x]) - (2*(2 -
b*x)^(5/2))/(3*x^(3/2)) + 10*b^(3/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(2-bx)^{5/2}}{3x^{3/2}}-\frac{1}{3}(5b)\int \frac{(2-bx)^{3/2}}{x^{3/2}}\,dx\\ &= \frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\ &= 5b^2\sqrt{x}\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx\\ &= 5b^2\sqrt{x}\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(10b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)\\ &= 5b^2\sqrt{x}\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [C] time = 0.0069384, size = 30, normalized size = 0.36

$$-\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{bx}{2}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (-8*Sqrt[2]*Hypergeometric2F1[-5/2, -3/2, -1/2, (b*x)/2])/(3*x^(3/2))
```

Maple [A] time = 0.017, size = 107, normalized size = 1.3

$$-\frac{3\,b^3x^3+22\,b^2x^2-64\,bx+16}{3}\sqrt{(-bx+2)\,x}x^{-\frac{3}{2}}\frac{1}{\sqrt{-x\,(bx-2)}}\frac{1}{\sqrt{-bx+2}}+5\,\frac{b^{3/2}\sqrt{(-bx+2)\,x}}{\sqrt{x}\sqrt{-bx+2}}\arctan\left(\frac{\sqrt{b}}{\sqrt{-bx^2+2\,x}}\left(x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(5/2),x)
```

```
[Out] -1/3*(3*b^3*x^3+22*b^2*x^2-64*b*x+16)/x^(3/2)/(-x*(b*x-2))^(1/2)*((-b*x+2)*
x)^(1/2)/(-b*x+2)^(1/2)+5*b^(3/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2)
)*((-b*x+2)*x)^(1/2)/x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92463, size = 336, normalized size = 4.

$$\left[\frac{15\sqrt{-b}bx^2\log\left(-bx-\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)+\left(3b^2x^2+28bx-8\right)\sqrt{-bx+2}\sqrt{x}}{3x^2}, -\frac{30b^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-\left(3b^2\right.}{3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*sqrt(-b)*b*x^2*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + (
3*b^2*x^2 + 28*b*x - 8)*sqrt(-b*x + 2)*sqrt(x))/x^2, -1/3*(30*b^(3/2)*x^2*a
rctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - (3*b^2*x^2 + 28*b*x - 8)*sqrt(-b*
x + 2)*sqrt(x))/x^2]
```

Sympy [C] time = 10.6367, size = 221, normalized size = 2.63

$$\begin{cases} b^{\frac{5}{2}}x\sqrt{-1+\frac{2}{bx}}+\frac{28b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3}+5ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)-10ib^{\frac{3}{2}}\log\left(\frac{1}{\sqrt{b}\sqrt{x}}\right)+10b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)-\frac{8\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for}\ \frac{2}{|bx|}>1 \\ ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}+\frac{28ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3}+5ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)-10ib^{\frac{3}{2}}\log\left(\sqrt{1-\frac{2}{bx}}+1\right)-\frac{8i\sqrt{b}\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(5/2),x)
```

```
[Out] Piecewise((b**(5/2)*x*sqrt(-1 + 2/(b*x)) + 28*b**(3/2)*sqrt(-1 + 2/(b*x))/3
 + 5*I*b**(3/2)*log(1/(b*x)) - 10*I*b**(3/2)*log(1/(sqrt(b)*sqrt(x))) + 10*
b**(3/2)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2) - 8*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*
x), 2/Abs(b*x) > 1), (I*b**(5/2)*x*sqrt(1 - 2/(b*x)) + 28*I*b**(3/2)*sqrt(1
 - 2/(b*x))/3 + 5*I*b**(3/2)*log(1/(b*x)) - 10*I*b**(3/2)*log(sqrt(1 - 2/(b
*x)) + 1) - 8*I*sqrt(b)*sqrt(1 - 2/(b*x))/(3*x), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.569 $\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=101

$$\frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}-\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2}+\frac{x^{5/2}\sqrt{a+bx}}{3b}$$

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b^3) - (5*a*x^(3/2)*Sqrt[a + b*x])/(12*b^2
) + (x^(5/2)*Sqrt[a + b*x])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a
 + b*x]])/(8*b^(7/2))
```

Rubi [A] time = 0.0304051, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}-\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2}+\frac{x^{5/2}\sqrt{a+bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b^3) - (5*a*x^(3/2)*Sqrt[a + b*x])/(12*b^2
) + (x^(5/2)*Sqrt[a + b*x])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a
 + b*x]])/(8*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx = \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{(5a)\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{6b}$$
$$= -\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} + \frac{\left(5a^2\right)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{8b^2}$$
$$= \frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{\left(5a^3\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{16b^3}$$
$$= \frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{8b^3}$$
$$= \frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^3}$$
$$= \frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}$$

Mathematica [A] time = 0.170838, size = 85, normalized size = 0.84

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(15a^2-10abx+8b^2x^2\right)-\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{24b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^2 - 10*a*b*x + 8*b^2*x^2) - (15*a^(5/
2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(24*b^(7/2))
```

Maple [A] time = 0.004, size = 102, normalized size = 1.

$$\frac{1}{3b}x^{\frac{5}{2}}\sqrt{bx+a}-\frac{5a}{12b^2}x^{\frac{3}{2}}\sqrt{bx+a}+\frac{5a^2}{8b^3}\sqrt{x}\sqrt{bx+a}-\frac{5a^3}{16}\sqrt{x(bx+a)}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)b^{-\frac{7}{2}}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/3*x^(5/2)*(b*x+a)^(1/2)/b-5/12*a*x^(3/2)*(b*x+a)^(1/2)/b^2+5/8*a^2*x^(1/2
)*(b*x+a)^(1/2)/b^3-5/16/b^(7/2)*a^3*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2
)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94789, size = 369, normalized size = 3.65

$$\left[\frac{15\,a^{3}\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(8\,b^{3}x^{2}-10\,ab^{2}x+15\,a^{2}b\right)\sqrt{bx+a}\sqrt{x}}{48\,b^{4}},\frac{15\,a^{3}\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(8\right.}{24}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/24*(15*a^
3*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 - 10*a*b
^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [A] time = 13.0539, size = 128, normalized size = 1.27

$$\frac{5a^{\frac{5}{2}}\sqrt{x}}{8b^{3}\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}x^{\frac{5}{2}}}{12b\sqrt{1+\frac{bx}{a}}}-\frac{5a^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*sqrt(x)/(8*b**3*sqrt(1 + b*x/a)) + 5*a**(3/2)*x**(3/2)/(24*b**2*
sqrt(1 + b*x/a)) - sqrt(a)*x**(5/2)/(12*b*sqrt(1 + b*x/a)) - 5*a**3*asinh(s
qrt(b)*sqrt(x)/sqrt(a))/(8*b**(7/2)) + x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.570 $\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=77

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2}+\frac{x^{3/2}\sqrt{a+bx}}{2b}$$

```
[Out] (-3*a*Sqrt[x]*Sqrt[a + b*x])/(4*b^2) + (x^(3/2)*Sqrt[a + b*x])/(2*b) + (3*a
^2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*b^(5/2))
```

Rubi [A] time = 0.0224716, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2}+\frac{x^{3/2}\sqrt{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (-3*a*Sqrt[x]*Sqrt[a + b*x])/(4*b^2) + (x^(3/2)*Sqrt[a + b*x])/(2*b) + (3*a
^2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx = \frac{x^{3/2}\sqrt{a+bx}}{2b} - \frac{(3a)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{4b}$$
$$= -\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{(3a^2)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{8b^2}$$
$$= -\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{4b^2}$$
$$= -\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^2}$$
$$= -\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}$$

Mathematica [A] time = 0.0591006, size = 85, normalized size = 1.1

$$\frac{\sqrt{b}\sqrt{x}\left(-3a^2-abx+2b^2x^2\right)+3a^{5/2}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*(-3*a^2 - a*b*x + 2*b^2*x^2) + 3*a^(5/2)*Sqrt[1 + (b*x)/a]
*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[a + b*x])
```

Maple [A] time = 0.003, size = 84, normalized size = 1.1

$$\frac{1}{2b}x^{\frac{3}{2}}\sqrt{bx+a} - \frac{3a}{4b^2}\sqrt{x}\sqrt{bx+a} + \frac{3a^2}{8}\sqrt{x(bx+a)}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)b^{-\frac{5}{2}}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(b*x+a)^(1/2)/b-3/4*a*x^(1/2)*(b*x+a)^(1/2)/b^2+3/8/b^(5/2)*a^2
*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94413, size = 316, normalized size = 4.1

$$\left[\frac{3\,a^2\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(2\,b^2x-3\,ab\right)\sqrt{bx+a}\sqrt{x}}{8\,b^3},-\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\left(2\,b^2x-3\,ab\right)\sqrt{bx+a}\sqrt{x}}{4\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(-b)*arctan(sqr
t(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))
/b^3]
```

Sympy [A] time = 4.54513, size = 100, normalized size = 1.3

$$-\frac{3a^{\frac{3}{2}}\sqrt{x}}{4b^2\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}x^{\frac{3}{2}}}{4b\sqrt{1+\frac{bx}{a}}}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*sqrt(x)/(4*b**2*sqrt(1 + b*x/a)) - sqrt(a)*x**(3/2)/(4*b*sqrt(1
 + b*x/a)) + 3*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(5/2)) + x**(5/2)/
(2*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.571 $\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=48

$$\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

```
[Out] (Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^
(3/2)
```

Rubi [A] time = 0.0162688, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^
(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{2b}$$
$$= \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b}$$
$$= \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.0379529, size = 68, normalized size = 1.42

$$\frac{\sqrt{b}\sqrt{x}(a+bx) - a^{3/2}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*(a + b*x) - a^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/(b^(3/2)*Sqrt[a + b*x])
```

Maple [A] time = 0.004, size = 65, normalized size = 1.4

$$\frac{1}{b}\sqrt{x}\sqrt{bx+a} - \frac{a}{2}\sqrt{x(bx+a)}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}} + \sqrt{bx^2+ax}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b-1/2*a/b^(3/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/
2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84526, size = 255, normalized size = 5.31

$$\left[\frac{a\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,\sqrt{bx+a}b\sqrt{x}}{2\,b^2}, \frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}b\sqrt{x}}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b^2, (a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x
))) + sqrt(b*x + a)*b*sqrt(x))/b^2]
```

Sympy [A] time = 2.25168, size = 44, normalized size = 0.92

$$\frac{\sqrt{a}\sqrt{x}\sqrt{1+\frac{bx}{a}}}{b}-\frac{a\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x)*sqrt(1 + b*x/a)/b - a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/
2)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.572 $\int \frac{1}{\sqrt{x}\sqrt{a+bx}} dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]
```

Rubi [A] time = 0.0127668, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a + b*x]),x]

[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{a+bx}} dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.012968, size = 50, normalized size = 1.79

$$\frac{2\sqrt{a}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a + b*x]),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*S
qrt[a + b*x])
```

Maple [B] time = 0.002, size = 48, normalized size = 1.7

$$\sqrt{x(bx+a)}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] (x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^
(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79051, size = 162, normalized size = 5.79

$$\left[\frac{\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a)/sqrt(b), -2*sqrt(-b)*arct
an(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))/b]
```

Sympy [A] time = 1.1561, size = 22, normalized size = 0.79

$$\frac{2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.573 $\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=19

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

```
[Out] (-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

Rubi [A] time = 0.001619, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a + b*x]),x]

[Out] (-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Mathematica [A] time = 0.004231, size = 19, normalized size = 1.

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a + b*x]),x]

[Out] (-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$-2\,\frac{\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2*(b*x+a)^(1/2)/a/x^(1/2)
```

Maxima [A] time = 1.05798, size = 20, normalized size = 1.05

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

Fricas [A] time = 1.9059, size = 41, normalized size = 2.16

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

Sympy [A] time = 1.01257, size = 19, normalized size = 1.

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/a
```

Giac [B] time = 1.06644, size = 45, normalized size = 2.37

$$-\frac{2\sqrt{bx+a}b^2}{\sqrt{(bx+a)b-ab}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a*abs(b))
```

3.574 $\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=44

$$\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}$$

```
[Out] (-2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])
```

Rubi [A] time = 0.0047552, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx &= -\frac{2\sqrt{a+bx}}{3ax^{3/2}}-\frac{(2b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a}\\ &= -\frac{2\sqrt{a+bx}}{3ax^{3/2}}+\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0074476, size = 27, normalized size = 0.61

$$-\frac{2(a-2bx)\sqrt{a+bx}}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*(a - 2*b*x)*Sqrt[a + b*x])/(3*a^2*x^(3/2))
```

Maple [A] time = 0.003, size = 22, normalized size = 0.5

$$-\frac{-4\,bx+2\,a}{3\,a^2}\sqrt{bx+a}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(-2*b*x+a)/x^(3/2)/a^2
```

Maxima [A] time = 1.16657, size = 42, normalized size = 0.95

$$\frac{2\left(\frac{3\sqrt{bx+a}b}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(b*x + a)*b/sqrt(x) - (b*x + a)^(3/2)/x^(3/2))/a^2
```

Fricas [A] time = 1.71568, size = 61, normalized size = 1.39

$$\frac{2\,(2\,bx-a)\sqrt{bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*x - a)*sqrt(b*x + a)/(a^2*x^(3/2))
```

Sympy [A] time = 3.21835, size = 42, normalized size = 0.95

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3ax}+\frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*a*x) + 4*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a**2
)
```

Giac [A] time = 1.07579, size = 68, normalized size = 1.55

$$-\frac{\sqrt{bx+a}b\left(\frac{2(bx+a)}{a^2b^3}-\frac{3}{ab^3}\right)}{24\left((bx+a)b-ab\right)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(b*x + a)*b*(2*(b*x + a)/(a^2*b^3) - 3/(a*b^3))/(((b*x + a)*b - a
*b)^(3/2)*abs(b))
```

3.575 $\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=68

$$-\frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}$$

```
[Out] (-2*Sqrt[a + b*x])/(5*a*x^(5/2)) + (8*b*Sqrt[a + b*x])/(15*a^2*x^(3/2)) - (
16*b^2*Sqrt[a + b*x])/(15*a^3*Sqrt[x])
```

Rubi [A] time = 0.0102229, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(5*a*x^(5/2)) + (8*b*Sqrt[a + b*x])/(15*a^2*x^(3/2)) - (
16*b^2*Sqrt[a + b*x])/(15*a^3*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx &= -\frac{2\sqrt{a+bx}}{5ax^{5/2}}-\frac{(4b)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{5a}\\
&= -\frac{2\sqrt{a+bx}}{5ax^{5/2}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}+\frac{\left(8b^2\right)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{15a^2}\\
&= -\frac{2\sqrt{a+bx}}{5ax^{5/2}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.008939, size = 40, normalized size = 0.59

$$-\frac{2\sqrt{a+bx}\left(3a^2-4abx+8b^2x^2\right)}{15a^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(5/2))
```

Maple [A] time = 0.004, size = 35, normalized size = 0.5

$$-\frac{16\,b^2x^2-8\,abx+6\,a^2}{15\,a^3}\sqrt{bx+a}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)^(1/2)*(8*b^2*x^2-4*a*b*x+3*a^2)/x^(5/2)/a^3
```

Maxima [A] time = 1.14235, size = 62, normalized size = 0.91

$$-\frac{2\left(\frac{15\sqrt{bx+a}b^2}{\sqrt{x}}-\frac{10\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{3\,(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*(15*sqrt(b*x + a)*b^2/sqrt(x) - 10*(b*x + a)^(3/2)*b/x^(3/2) + 3*(b*x
 + a)^(5/2)/x^(5/2))/a^3
```

Fricas [A] time = 1.81237, size = 88, normalized size = 1.29

$$-\frac{2\left(8\,b^2x^2-4\,abx+3\,a^2\right)\sqrt{bx+a}}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x + a)/(a^3*x^(5/2))
```

Sympy [B] time = 30.2744, size = 287, normalized size = 4.22

$$-\frac{6a^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{4a^3b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{6a^2b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{24al}{15a^5b^4x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a)**(1/2),x)
```

```
[Out] -6*a**4*b**(9/2)*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 +
 15*a**3*b**6*x**4) - 4*a**3*b**(11/2)*x*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x*
*2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 6*a**2*b**(13/2)*x**2*sqrt(a/
(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 24
*a*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3
+ 15*a**3*b**6*x**4) - 16*b**(17/2)*x**4*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x*
*2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4)
```

Giac [A] time = 1.08107, size = 89, normalized size = 1.31

$$\frac{\sqrt{bx+a}\left(4\,(bx+a)\left(\frac{2\,(bx+a)}{a^3b^4}-\frac{5}{a^2b^4}\right)+\frac{15}{ab^4}\right)b}{480\,((bx+a)b-ab)^{\frac{5}{2}}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/480*sqrt(b*x + a)*(4*(b*x + a)*(2*(b*x + a)/(a^3*b^4) - 5/(a^2*b^4)) + 15
/(a*b^4))*b/(((b*x + a)*b - a*b)^(5/2)*abs(b))
```

3.576 $\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=92

$$-\frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}}+\frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}+\frac{12b\sqrt{a+bx}}{35a^2x^{5/2}}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}$$

```
[Out] (-2*Sqrt[a + b*x])/(7*a*x^(7/2)) + (12*b*Sqrt[a + b*x])/(35*a^2*x^(5/2)) -
(16*b^2*Sqrt[a + b*x])/(35*a^3*x^(3/2)) + (32*b^3*Sqrt[a + b*x])/(35*a^4*Sq
rt[x])
```

Rubi [A] time = 0.0163696, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}}+\frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}+\frac{12b\sqrt{a+bx}}{35a^2x^{5/2}}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[a + b*x]),x]
```


```
[Out] (-2*Sqrt[a + b*x])/(7*a*x^(7/2)) + (12*b*Sqrt[a + b*x])/(35*a^2*x^(5/2)) -
(16*b^2*Sqrt[a + b*x])/(35*a^3*x^(3/2)) + (32*b^3*Sqrt[a + b*x])/(35*a^4*Sq
rt[x])
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx &= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} - \frac{(6b)\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx}{7a}\\ &= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} + \frac{12b\sqrt{a+bx}}{35a^2x^{5/2}} + \frac{(24b^2)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{35a^2}\\ &= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} + \frac{12b\sqrt{a+bx}}{35a^2x^{5/2}} - \frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}} - \frac{(16b^3)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{35a^3}\\ &= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} + \frac{12b\sqrt{a+bx}}{35a^2x^{5/2}} - \frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}} + \frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.011458, size = 51, normalized size = 0.55

$$-\frac{2\sqrt{a+bx}\left(-6a^2bx+5a^3+8ab^2x^2-16b^3x^3\right)}{35a^4x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(5*a^3 - 6*a^2*b*x + 8*a*b^2*x^2 - 16*b^3*x^3))/(35*a^4*x
^(7/2))
```

Maple [A] time = 0.004, size = 46, normalized size = 0.5

$$-\frac{-32\,b^3x^3+16\,ab^2x^2-12\,a^2bx+10\,a^3}{35\,a^4}\sqrt{bx+a}x^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)^(1/2)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/x^(7/2)/a^4
```

Maxima [A] time = 1.3045, size = 82, normalized size = 0.89

$$\frac{2\left(\frac{35\sqrt{bx+a}b^3}{\sqrt{x}} - \frac{35\,(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}} + \frac{21\,(bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}} - \frac{5\,(bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{35\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(35*sqrt(b*x + a)*b^3/sqrt(x) - 35*(b*x + a)^(3/2)*b^2/x^(3/2) + 21*(b
*x + a)^(5/2)*b/x^(5/2) - 5*(b*x + a)^(7/2)/x^(7/2))/a^4
```

Fricas [A] time = 1.77953, size = 109, normalized size = 1.18

$$\frac{2\left(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3\right)\sqrt{bx+a}}{35\,a^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^4*x^(7
/2))
```

Sympy [B] time = 174.652, size = 488, normalized size = 5.3

$$-\frac{10a^6b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}-\frac{18a^5b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}-\frac{}{35a^7b^9x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x+a)**(1/2),x)
```

```
[Out] -10*a**6*b**(19/2)*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x*
*4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) - 18*a**5*b**(21/2)*x*sqrt(a
/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5
+ 35*a**4*b**12*x**6) - 10*a**4*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(35*a**7*b
**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6)
+ 10*a**3*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b*
*10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) + 60*a**2*b**(27/2)*x*
*4*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b*
*11*x**5 + 35*a**4*b**12*x**6) + 80*a*b**(29/2)*x**5*sqrt(a/(b*x) + 1)/(35*
a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*
x**6) + 32*b**(31/2)*x**6*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b
**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6)
```

Giac [A] time = 1.0962, size = 111, normalized size = 1.21

$$-\frac{\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{2\,(bx+a)}{a^4b^5}-\frac{7}{a^3b^5}\right)+\frac{35}{a^2b^5}\right)-\frac{35}{ab^5}\right)\sqrt{bx+a}b}{13440\,((bx+a)b-ab)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/13440*(2*(b*x + a)*(4*(b*x + a)*(2*(b*x + a)/(a^4*b^5) - 7/(a^3*b^5)) +
35/(a^2*b^5)) - 35/(a*b^5))*sqrt(b*x + a)*b/(((b*x + a)*b - a*b)^(7/2)*abs(
b))
```

3.577 $\int \frac{x^{5/2}}{(a+bx)^{3/2}} dx$

Optimal. Leaf size=96

$$\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}-\frac{15a\sqrt{x}\sqrt{a+bx}}{4b^3}-\frac{2x^{5/2}}{b\sqrt{a+bx}}$$

```
[Out] (-2*x^(5/2))/(b*Sqrt[a + b*x]) - (15*a*Sqrt[x]*Sqrt[a + b*x])/(4*b^3) + (5*
x^(3/2)*Sqrt[a + b*x])/(2*b^2) + (15*a^2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
 b*x]])/(4*b^(7/2))
```

Rubi [A] time = 0.0293033, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}-\frac{15a\sqrt{x}\sqrt{a+bx}}{4b^3}-\frac{2x^{5/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^(5/2))/(b*Sqrt[a + b*x]) - (15*a*Sqrt[x]*Sqrt[a + b*x])/(4*b^3) + (5*
x^(3/2)*Sqrt[a + b*x])/(2*b^2) + (15*a^2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
 b*x]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2x^{5/2}}{b\sqrt{a+bx}}+\frac{5\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{b}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}-\frac{(15a)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{4b^2}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{\left(15a^2\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{8b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0097587, size = 50, normalized size = 0.52

$$\frac{2x^{7/2}\sqrt{\frac{bx}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};-\frac{bx}{a}\right)}{7a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 + (b*x)/a]*Hypergeometric2F1[3/2, 7/2, 9/2, -((b*x)/a)])/
(7*a*Sqrt[a + b*x])
```

Maple [A] time = 0.023, size = 119, normalized size = 1.2

$$-\frac{-2\,bx+7\,a}{4\,b^3}\sqrt{x}\sqrt{bx+a}+\left(\frac{15\,a^2}{8}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)b^{-\frac{7}{2}}-2\,\frac{a^2}{b^4}\sqrt{b\left(x+\frac{a}{b}\right)^2-a\left(x+\frac{a}{b}\right)}\left(x+\frac{a}{b}\right)^{-1}\right)\sqrt{x\,(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(3/2),x)
```

```
[Out] -1/4*(-2*b*x+7*a)*x^(1/2)*(b*x+a)^(1/2)/b^3+(15/8/b^(7/2)*a^2*ln((1/2*a+b*x
)/b^(1/2)+(b*x^2+a*x)^(1/2))-2/b^4*a^2/(x+1/b*a)*(b*(x+1/b*a)^2-a*(x+1/b*a)
)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89192, size = 429, normalized size = 4.47

$$\left[\frac{15\left(a^2bx+a^3\right)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(2\,b^3x^2-5\,ab^2x-15\,a^2b\right)\sqrt{bx+a}\sqrt{x}}{8\left(b^5x+ab^4\right)}, -\frac{15\left(a^2bx+a^3\right)\sqrt{-b}\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(a^2*b*x + a^3)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x
) + a) + 2*(2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x
 + a*b^4), -1/4*(15*(a^2*b*x + a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/
(b*sqrt(x))) - (2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x + a*b^4)]
```

Sympy [A] time = 15.9217, size = 105, normalized size = 1.09

$$-\frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{1+\frac{bx}{a}}}-\frac{5\sqrt{a}x^{\frac{3}{2}}}{4b^2\sqrt{1+\frac{bx}{a}}}+\frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] -15*a**(3/2)*sqrt(x)/(4*b**3*sqrt(1 + b*x/a)) - 5*sqrt(a)*x**(3/2)/(4*b**2*
sqrt(1 + b*x/a)) + 15*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2)) + x*
*(5/2)/(2*sqrt(a)*b*sqrt(1 + b*x/a))
```

Giac [A] time = 60.6907, size = 177, normalized size = 1.84

$$\frac{\left(2\sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(\frac{2\,(bx+a)}{b^3}-\frac{9\,a}{b^3}\right)-\frac{32\,a^3}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)b^{\frac{3}{2}}}-\frac{15\,a^2\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{b^{\frac{5}{2}}}\right)|b|}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)/b^3 - 9*a/b^3) -
32*a^3/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*b^(3/2)
) - 15*a^2*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(5/2)
)*abs(b)/b^2
```

3.578 $\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{b\sqrt{a+bx}}$$

```
[Out] (-2*x^(3/2))/(b*Sqrt[a + b*x]) + (3*Sqrt[x]*Sqrt[a + b*x])/b^2 - (3*a*ArcTa
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(5/2)
```

Rubi [A] time = 0.0215275, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^(3/2))/(b*Sqrt[a + b*x]) + (3*Sqrt[x]*Sqrt[a + b*x])/b^2 - (3*a*ArcTa
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{b}\\ &= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{(3a)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{2b^2}\\ &= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\ &= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{(3a)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^2}\\ &= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.0095008, size = 50, normalized size = 0.74

$$\frac{2x^{5/2}\sqrt{\frac{bx}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-\frac{bx}{a}\right)}{5a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2)*Sqrt[1 + (b*x)/a]*Hypergeometric2F1[3/2, 5/2, 7/2, -((b*x)/a)])/
(5*a*Sqrt[a + b*x])
```

Maple [B] time = 0.02, size = 106, normalized size = 1.6

$$\frac{1}{b^2}\sqrt{x}\sqrt{bx+a}+\left(-\frac{3a}{2}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)b^{-\frac{5}{2}}+2\frac{a}{b^3}\sqrt{b\left(x+\frac{a}{b}\right)^2-a\left(x+\frac{a}{b}\right)}\left(x+\frac{a}{b}\right)^{-1}\right)\sqrt{x(bx+a)}\frac{1}{\sqrt{x}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b^2+(-3/2/b^(5/2)*a*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x
)^(1/2))+2/b^3*a/(x+1/b*a)*(b*(x+1/b*a)^2-a*(x+1/b*a))^(1/2))*(x*(b*x+a))^(
1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94129, size = 363, normalized size = 5.34

$$\left[\frac{3\left(abx+a^2\right)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(b^2x+3\,ab\right)\sqrt{bx+a}\sqrt{x}}{2\left(b^4x+ab^3\right)}, \frac{3\left(abx+a^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+}{b^4x+ab^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*b*x + a^2)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) +
 a) + 2*(b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3), (3*(a*b*x +
 a^2)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 3*a*b)
*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3)]
```

Sympy [A] time = 4.40012, size = 71, normalized size = 1.04

$$\frac{3\sqrt{a}\sqrt{x}}{b^2\sqrt{1+\frac{bx}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}} + \frac{x^{\frac{3}{2}}}{\sqrt{a}b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(3/2),x)
```

```
[Out] 3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 + b*x/a)) - 3*a*asinh(sqrt(b)*sqrt(x)/sqrt(a
))/b**(5/2) + x**(3/2)/(sqrt(a)*b*sqrt(1 + b*x/a))
```

Giac [B] time = 59.6162, size = 155, normalized size = 2.28

$$\frac{\left(\frac{8\,a^2\sqrt{b}}{\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab} + \frac{3\,a\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}} + \frac{2\,\sqrt{(bx+a)b-ab}\sqrt{bx+a}}{b}\right)|b|}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(8*a^2*sqrt(b)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a
*b) + 3*a*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b)
+ 2*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)/b)*abs(b)/b^3
```

3.579 $\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}$$

```
[Out] (-2*Sqrt[x])/(b*Sqrt[a + b*x]) + (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]
])/b^(3/2)
```

Rubi [A] time = 0.0161718, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[a + b*x]) + (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]
])/b^(3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx = -\frac{2\sqrt{x}}{b\sqrt{a+bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{a+bx}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.0684489, size = 64, normalized size = 1.33

$$\frac{2\left(\sqrt{a}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-\sqrt{b}\sqrt{x}\right)}{b^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-(Sqrt[b]*Sqrt[x]) + Sqrt[a]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]]))/(b^(3/2)*Sqrt[a + b*x])
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \sqrt{x}\,(bx+a)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(3/2),x)
```

```
[Out] int(x^(1/2)/(b*x+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87175, size = 308, normalized size = 6.42

$$\left[\frac{(bx+a)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,\sqrt{bx+a}b\sqrt{x}}{b^3x+ab^2}, -\frac{2\left((bx+a)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}b\sqrt{x}\right)}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + a)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sq
rt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2), -2*((b*x + a)*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2)]
```

Sympy [A] time = 2.10617, size = 46, normalized size = 0.96

$$\frac{2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{a}b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(3/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) - 2*sqrt(x)/(sqrt(a)*b*sqrt(1 + b
*x/a))
```

Giac [B] time = 59.6491, size = 115, normalized size = 2.4

$$-\frac{\left(\frac{4\,a\sqrt{b}}{\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab}+\frac{\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(4*a*sqrt(b)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b) +
 log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b))*abs(b)/b
^2
```

3.580 $\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

```
[Out] (2*Sqrt[x])/(a*Sqrt[a + b*x])
```

Rubi [A] time = 0.001838, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^(3/2)),x]

[Out] (2*Sqrt[x])/(a*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Mathematica [A] time = 0.0044999, size = 19, normalized size = 1.

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^(3/2)),x]

[Out] (2*Sqrt[x])/(a*Sqrt[a + b*x])
```

Maple [A] time = 0.002, size = 16, normalized size = 0.8

$$2\,\frac{\sqrt{x}}{a\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] 2*x^(1/2)/a/(b*x+a)^(1/2)
```

Maxima [A] time = 1.09507, size = 20, normalized size = 1.05

$$\frac{2\sqrt{x}}{\sqrt{bx + a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)/(sqrt(b*x + a)*a)
```

Fricas [A] time = 1.81284, size = 53, normalized size = 2.79

$$\frac{2\sqrt{bx + a}\sqrt{x}}{abx + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)*sqrt(x)/(a*b*x + a^2)
```

Sympy [A] time = 1.2184, size = 17, normalized size = 0.89

$$\frac{2}{a\sqrt{b}\sqrt{\frac{a}{bx} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] 2/(a*sqrt(b)*sqrt(a/(b*x) + 1))
```

Giac [B] time = 1.05505, size = 61, normalized size = 3.21

$$\frac{4\,b^{\frac{3}{2}}}{\left(\left(\sqrt{bx + a}\sqrt{b} - \sqrt{(bx + a)b - ab}\right)^2 + ab\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 4*b^(3/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*abs(
b))
```

3.581 $\int \frac{1}{x^{3/2}(a+bx)^{3/2}} dx$

Optimal. Leaf size=39

$$\frac{2}{a\sqrt{x}\sqrt{a+bx}} - \frac{4\sqrt{a+bx}}{a^2\sqrt{x}}$$

```
[Out] 2/(a*Sqrt[x]*Sqrt[a + b*x]) - (4*Sqrt[a + b*x])/(a^2*Sqrt[x])
```

Rubi [A] time = 0.0047896, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{2}{a\sqrt{x}\sqrt{a+bx}} - \frac{4\sqrt{a+bx}}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[x]*Sqrt[a + b*x]) - (4*Sqrt[a + b*x])/(a^2*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a+bx)^{3/2}} dx &= \frac{2}{a\sqrt{x}\sqrt{a+bx}} + \frac{2\int \frac{1}{x^{3/2}\sqrt{a+bx}} dx}{a} \\ &= \frac{2}{a\sqrt{x}\sqrt{a+bx}} - \frac{4\sqrt{a+bx}}{a^2\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0095566, size = 25, normalized size = 0.64

$$-\frac{2(a+2bx)}{a^2\sqrt{x}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*(a + 2*b*x))/(a^2*Sqrt[x]*Sqrt[a + b*x])
```

Maple [A] time = 0.003, size = 22, normalized size = 0.6

$$-2\frac{2bx+a}{a^{2}\sqrt{x}\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] -2*(2*b*x+a)/x^(1/2)/(b*x+a)^(1/2)/a^2
```

Maxima [A] time = 1.12064, size = 43, normalized size = 1.1

$$-\frac{2b\sqrt{x}}{\sqrt{bx+a}a^{2}}-\frac{2\sqrt{bx+a}}{a^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b*sqrt(x)/(sqrt(b*x + a)*a^2) - 2*sqrt(b*x + a)/(a^2*sqrt(x))
```

Fricas [A] time = 1.95174, size = 78, normalized size = 2.

$$-\frac{2(2bx+a)\sqrt{bx+a}\sqrt{x}}{a^{2}bx^{2}+a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*x + a)*sqrt(b*x + a)*sqrt(x)/(a^2*b*x^2 + a^3*x)
```

Sympy [A] time = 2.79672, size = 41, normalized size = 1.05

$$-\frac{2}{a\sqrt{bx}\sqrt{\frac{a}{bx}+1}}-\frac{4\sqrt{b}}{a^{2}\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(3/2),x)
```

```
[Out] -2/(a*sqrt(b)*x*sqrt(a/(b*x) + 1)) - 4*sqrt(b)/(a**2*sqrt(a/(b*x) + 1))
```

Giac [B] time = 1.07059, size = 111, normalized size = 2.85

$$-\frac{4\,b^{\frac{5}{2}}}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)a|b|}-\frac{2\,\sqrt{bx+a}b^2}{\sqrt{(bx+a)b-ab}a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -4*b^(5/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*a*a
bs(b)) - 2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a^2*abs(b))
```

3.582 $\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}+\frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}+\frac{2}{ax^{3/2}\sqrt{a+bx}}$$

```
[Out] 2/(a*x^(3/2)*Sqrt[a + b*x]) - (8*Sqrt[a + b*x])/(3*a^2*x^(3/2)) + (16*b*Sqr
t[a + b*x])/(3*a^3*Sqrt[x])
```

Rubi [A] time = 0.0098531, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}+\frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}+\frac{2}{ax^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(3/2)*Sqrt[a + b*x]) - (8*Sqrt[a + b*x])/(3*a^2*x^(3/2)) + (16*b*Sqr
t[a + b*x])/(3*a^3*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx &= \frac{2}{ax^{3/2}\sqrt{a+bx}}+\frac{4\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{a}\\ &= \frac{2}{ax^{3/2}\sqrt{a+bx}}-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}-\frac{(8b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^2}\\ &= \frac{2}{ax^{3/2}\sqrt{a+bx}}-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}+\frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0094522, size = 38, normalized size = 0.6

$$-\frac{2\left(a^2-4abx-8b^2x^2\right)}{3a^3x^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*(a^2 - 4*a*b*x - 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a + b*x])
```

Maple [A] time = 0.005, size = 33, normalized size = 0.5

$$-\frac{-16\,b^2x^2-8\,abx+2\,a^2}{3\,a^3}x^{-\frac{3}{2}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(3/2),x)
```

```
[Out] -2/3*(-8*b^2*x^2-4*a*b*x+a^2)/x^(3/2)/(b*x+a)^(1/2)/a^3
```

Maxima [A] time = 1.16904, size = 68, normalized size = 1.08

$$\frac{2\,b^2\sqrt{x}}{\sqrt{bx+a}a^3}+\frac{2\left(\frac{6\,\sqrt{bx+a}b}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(b*x + a)*a^3) + 2/3*(6*sqrt(b*x + a)*b/sqrt(x) - (b*x +
 a)^(3/2)/x^(3/2))/a^3
```

Fricas [A] time = 2.11559, size = 104, normalized size = 1.65

$$\frac{2\left(8\,b^2x^2+4\,abx-a^2\right)\sqrt{bx+a}\sqrt{x}}{3\left(a^3bx^3+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*b^2*x^2 + 4*a*b*x - a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b*x^3 + a^4*x^2)
```

Sympy [B] time = 16.9072, size = 219, normalized size = 3.48

$$-\frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{6a^2b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{24ab^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{16b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] -2*a**3*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a*
*3*b**6*x**3) + 6*a**2*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**
4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*a*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(3*
a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 16*b**(15/2)*x**3*sqrt
(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3)
```

Giac [A] time = 1.09024, size = 126, normalized size = 2.

$$-\frac{\sqrt{bx+a}\left(\frac{5\,(bx+a)|b|}{b^2}-\frac{6\,a|b|}{b^2}\right)}{24\left((bx+a)b-ab\right)^{\frac{3}{2}}}+\frac{4\,b^{\frac{7}{2}}}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(b*x + a)*(5*(b*x + a)*abs(b)/b^2 - 6*a*abs(b)/b^2)/((b*x + a)*b
- a*b)^(3/2) + 4*b^(7/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b)
)^2 + a*b)*a^2*abs(b))
```

3.583 $\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{2}{ax^{5/2}\sqrt{a+bx}}$$

```
[Out] 2/(a*x^(5/2)*Sqrt[a + b*x]) - (12*Sqrt[a + b*x])/(5*a^2*x^(5/2)) + (16*b*Sq
rt[a + b*x])/(5*a^3*x^(3/2)) - (32*b^2*Sqrt[a + b*x])/(5*a^4*Sqrt[x])
```

Rubi [A] time = 0.016875, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{2}{ax^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x)^(3/2)),x]
```



```
[Out] 2/(a*x^(5/2)*Sqrt[a + b*x]) - (12*Sqrt[a + b*x])/(5*a^2*x^(5/2)) + (16*b*Sq
rt[a + b*x])/(5*a^3*x^(3/2)) - (32*b^2*Sqrt[a + b*x])/(5*a^4*Sqrt[x])
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx &= \frac{2}{ax^{5/2}\sqrt{a+bx}}+\frac{6\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx}{a}\\
&= \frac{2}{ax^{5/2}\sqrt{a+bx}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}-\frac{(24b)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{5a^2}\\
&= \frac{2}{ax^{5/2}\sqrt{a+bx}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}+\frac{\left(16b^2\right)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{5a^3}\\
&= \frac{2}{ax^{5/2}\sqrt{a+bx}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}-\frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.010603, size = 49, normalized size = 0.56

$$-\frac{2\left(-2a^2bx+a^3+8ab^2x^2+16b^3x^3\right)}{5a^4x^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*(a^3 - 2*a^2*b*x + 8*a*b^2*x^2 + 16*b^3*x^3))/(5*a^4*x^(5/2)*Sqrt[a + b
*x])
```

Maple [A] time = 0.006, size = 44, normalized size = 0.5

$$-\frac{32\,b^3x^3+16\,ab^2x^2-4\,a^2bx+2\,a^3}{5\,a^4}x^{-\frac{5}{2}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a)^(3/2),x)
```

```
[Out] -2/5*(16*b^3*x^3+8*a*b^2*x^2-2*a^2*b*x+a^3)/x^(5/2)/(b*x+a)^(1/2)/a^4
```

Maxima [A] time = 1.03385, size = 86, normalized size = 0.99

$$-\frac{2\,b^3\sqrt{x}}{\sqrt{bx+a}a^4}-\frac{2\left(\frac{15\,\sqrt{bx+a}b^2}{\sqrt{x}}-\frac{5\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{5\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b^3*sqrt(x)/(sqrt(b*x + a)*a^4) - 2/5*(15*sqrt(b*x + a)*b^2/sqrt(x) - 5*
(b*x + a)^(3/2)*b/x^(3/2) + (b*x + a)^(5/2)/x^(5/2))/a^4
```

Fricas [A] time = 2.04874, size = 128, normalized size = 1.47

$$-\frac{2\left(16\,b^3x^3+8\,ab^2x^2-2\,a^2bx+a^3\right)\sqrt{bx+a}\sqrt{x}}{5\left(a^4bx^4+a^5x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*b^3*x^3 + 8*a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b*x^4 + a^5*x^3)
```

Sympy [B] time = 106.382, size = 348, normalized size = 4.

$$-\frac{2a^5b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{10a^3b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{60a^2b^{\frac{25}{2}}x^3}{5a^7b^9x^2+15a^6b^{10}x^3+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a)**(3/2),x)
```

```
[Out] -2*a**5*b**(19/2)*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3
+ 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 10*a**3*b**(23/2)*x**2*sqrt(a/(
b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a
**4*b**12*x**5) - 60*a**2*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**
2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 80*a*b**
(27/2)*x**4*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a
**5*b**11*x**4 + 5*a**4*b**12*x**5) - 32*b**(29/2)*x**5*sqrt(a/(b*x) + 1)/(
5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x
**5)
```

Giac [A] time = 1.10179, size = 147, normalized size = 1.69

$$-\frac{4\,b^{\frac{9}{2}}}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)a^3|b|}+\frac{\left(\frac{15\,a^4}{b}+\left(\frac{11\,(bx+a)a^2}{b}-\frac{25\,a^3}{b}\right)(bx+a)\right)\sqrt{bx+a}}{40\,((bx+a)b-ab)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -4*b^(9/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*a^3
*abs(b)) + 1/40*(15*a^4/b + (11*(b*x + a)*a^2/b - 25*a^3/b)*(b*x + a))*sqrt
(b*x + a)/((b*x + a)*b - a*b)^(5/2)
```

3.584 $\int \frac{x^{5/2}}{(a+bx)^{5/2}} dx$

Optimal. Leaf size=91

$$-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}$$

```
[Out] (-2*x^(5/2))/(3*b*(a + b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[a + b*x]) + (
5*Sqrt[x]*Sqrt[a + b*x])/b^3 - (5*a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]
])/b^(7/2)
```

Rubi [A] time = 0.0281517, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^(5/2))/(3*b*(a + b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[a + b*x]) + (
5*Sqrt[x]*Sqrt[a + b*x])/b^3 - (5*a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]
])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^{5/2}}\,dx &= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}+\frac{5\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx}{3b}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{b^2}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{(5a)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{2b^3}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{(5a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{(5a)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^3}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0104971, size = 50, normalized size = 0.55

$$\frac{2x^{7/2}\sqrt{\frac{bx}{a}+1}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};-\frac{bx}{a}\right)}{7a^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 + (b*x)/a]*Hypergeometric2F1[5/2, 7/2, 9/2, -((b*x)/a)])/
(7*a^2*Sqrt[a + b*x])
```

Maple [B] time = 0.031, size = 147, normalized size = 1.6

$$\frac{1}{b^3}\sqrt{x}\sqrt{bx+a}+\left(-\frac{5a}{2}\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)b^{-\frac{7}{2}}-\frac{2a^2}{3b^5}\sqrt{b\left(x+\frac{a}{b}\right)^2-a\left(x+\frac{a}{b}\right)}\left(x+\frac{a}{b}\right)^{-2}+\frac{14a}{3b^4}\sqrt{b\left(x+\frac{a}{b}\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(5/2),x)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b^3+(-5/2/b^(7/2)*a*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x
)^(1/2))-2/3/b^5*a^2/(x+1/b*a)^2*(b*(x+1/b*a)^2-a*(x+1/b*a))^(1/2)+14/3/b^4
*a/(x+1/b*a)*(b*(x+1/b*a)^2-a*(x+1/b*a))^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(
b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11829, size = 512, normalized size = 5.63

$$\left[\frac{15\left(ab^2x^2+2\,a^2bx+a^3\right)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(3\,b^3x^2+20\,ab^2x+15\,a^2b\right)\sqrt{bx+a}\sqrt{x}}{6\left(b^6x^2+2\,ab^5x+a^2b^4\right)},\frac{15\left(ab^2x^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*
sqrt(b)*sqrt(x) + a) + 2*(3*b^3*x^2 + 20*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*
sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/3*(15*(a*b^2*x^2 + 2*a^2*b*x +
a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 + 20*
a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [B] time = 15.8031, size = 396, normalized size = 4.35

$$-\frac{15a^{\frac{81}{2}}b^{22}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{15a^{\frac{79}{2}}b^{23}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{15a^{40}b^{\frac{45}{2}}x^{2}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] -15*a**(81/2)*b**22*x**(51/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a)
)/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*
x**(53/2)*sqrt(1 + b*x/a)) - 15*a**(79/2)*b**23*x**(53/2)*sqrt(1 + b*x/a)*a
sinh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x
/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a)) + 15*a**40*b**(45/2)
*x**26/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(5
3/2)*x**(53/2)*sqrt(1 + b*x/a)) + 20*a**39*b**(47/2)*x**27/(3*a**(79/2)*b**
(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 +
 b*x/a)) + 3*a**38*b**(49/2)*x**28/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1
+ b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a))
```

Giac [B] time = 59.5621, size = 266, normalized size = 2.92

$$\frac{\left(\frac{15\,a\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}\right)}{b^{\frac{5}{2}}}+\frac{6\sqrt{(bx+a)b-ab}\sqrt{bx+a}}{b^{3}}+\frac{8\left(9\,a^{2}\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{4}\sqrt{b}+12\,a^{3}\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}b^{\frac{3}{2}}+7\,a^{4}b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)^{3}b^{2}}\right)|b|}{6\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(15*a*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(5/2)
+ 6*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)/b^3 + 8*(9*a^2*(sqrt(b*x + a)*sqr
t(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + 12*a^3*(sqrt(b*x + a)*sqrt(b) -
 sqrt((b*x + a)*b - a*b))^2*b^(3/2) + 7*a^4*b^(5/2))/(((sqrt(b*x + a)*sqrt(
b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*b^2))*abs(b)/b^2
```

3.585 $\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}$$

```
[Out] (-2*x^(3/2))/(3*b*(a + b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[a + b*x]) + (2*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(5/2)
```

Rubi [A] time = 0.021607, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 63, 217, 206}

$$-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*b*(a + b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[a + b*x]) + (2*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx = -\frac{2x^{3/2}}{3b(a+bx)^{3/2}} + \frac{\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx}{b}$$
$$= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a+bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{b^2}$$
$$= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^2}$$
$$= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a+bx}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.130184, size = 80, normalized size = 1.16

$$\frac{6\sqrt{a}(a+bx)\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - 2\sqrt{b}\sqrt{x}(3a+4bx)}{3b^{5/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[b]*Sqrt[x]*(3*a + 4*b*x) + 6*Sqrt[a]*(a + b*x)*Sqrt[1 + (b*x)/a]*A
rcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(3*b^(5/2)*(a + b*x)^(3/2))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}} (bx+a)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] int(x^(3/2)/(b*x+a)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08751, size = 451, normalized size = 6.54

$$\left[\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\left(4\,b^2x+3\,ab\right)\sqrt{bx+a}\sqrt{x}}{3\left(b^5x^2+2\,ab^4x+a^2b^3\right)},-\frac{2\left(3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(
b)*sqrt(x) + a) - 2*(4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x^2 + 2*a
*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-b)*arctan(sqrt(b
*x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x^2 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [B] time = 6.02493, size = 328, normalized size = 4.75

$$\frac{6a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{6a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{6a^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(5/2),x)
```

```
[Out] 6*a**(39/2)*b**11*x**(27/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a))/
(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x*
*(29/2)*sqrt(1 + b*x/a)) + 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 + b*x/a)*asin
h(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a)
 + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/a)) - 6*a**19*b**(23/2)*x**
14/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)
*x**(29/2)*sqrt(1 + b*x/a)) - 8*a**18*b**(25/2)*x**15/(3*a**(39/2)*b**(27/2
)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/
a))
```

Giac [B] time = 58.8763, size = 223, normalized size = 3.23

$$-\frac{\left(\frac{3\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}}+\frac{8\left(3\,a\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4\sqrt{b}+3\,a^2\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{3}{2}}+2\,a^3b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3}\right)|b|}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b) +
8*(3*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + 3*a^2*
(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(3/2) + 2*a^3*b^(5/2)
)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3)*abs(b)/b^3
```

3.586 $\int \frac{\sqrt{x}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=21

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

[Out] (2*x^(3/2))/(3*a*(a + b*x)^(3/2))

Rubi [A] time = 0.001626, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a + b*x)^(5/2),x]

[Out] (2*x^(3/2))/(3*a*(a + b*x)^(3/2))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx)^{5/2}}\,dx = \frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Mathematica [A] time = 0.005987, size = 21, normalized size = 1.

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(a + b*x)^(5/2),x]

[Out] (2*x^(3/2))/(3*a*(a + b*x)^(3/2))

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$\frac{2}{3\,a}x^{\frac{3}{2}}\,(bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(5/2),x)
```

```
[Out] 2/3*x^(3/2)/a/(b*x+a)^(3/2)
```

Maxima [A] time = 1.04428, size = 20, normalized size = 0.95

$$\frac{2\,x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/((b*x + a)^(3/2)*a)
```

Fricas [B] time = 2.07273, size = 77, normalized size = 3.67

$$\frac{2\sqrt{bx+a}x^{\frac{3}{2}}}{3\left(ab^2x^2+2\,a^2bx+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x + a)*x^(3/2)/(a*b^2*x^2 + 2*a^2*b*x + a^3)
```

Sympy [B] time = 2.42667, size = 42, normalized size = 2.

$$\frac{2x^{\frac{3}{2}}}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(5/2),x)
```

```
[Out] 2*x**(3/2)/(3*a**(5/2)*sqrt(1 + b*x/a) + 3*a**(3/2)*b*x*sqrt(1 + b*x/a))
```

Giac [B] time = 1.15023, size = 116, normalized size = 5.52

$$\frac{4\left(3\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4\sqrt{b}+a^2b^{\frac{5}{2}}\right)|b|}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + a^2*b^
(5/2))*abs(b)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^
3*b^2)
```

3.587 $\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}+\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}$$

```
[Out] (2*Sqrt[x])/(3*a*(a + b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a + b*x])
```

Rubi [A] time = 0.0046125, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}+\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^(5/2)),x]

[Out] (2*Sqrt[x])/(3*a*(a + b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx &= \frac{2\sqrt{x}}{3a(a+bx)^{3/2}}+\frac{2\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx}{3a}\\ &= \frac{2\sqrt{x}}{3a(a+bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}\end{aligned}$$

Mathematica [A] time = 0.0117225, size = 29, normalized size = 0.67

$$\frac{2\sqrt{x}(3a+2bx)}{3a^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x]*(3*a + 2*b*x))/(3*a^2*(a + b*x)^(3/2))
```

Maple [A] time = 0.004, size = 24, normalized size = 0.6

$$\frac{4\,bx+6\,a}{3\,a^{2}}\sqrt{x}\,(bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 2/3*x^(1/2)*(2*b*x+3*a)/(b*x+a)^(3/2)/a^2
```

Maxima [A] time = 1.10958, size = 36, normalized size = 0.84

$$-\frac{2\left(b-\frac{3\,(bx+a)}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b - 3*(b*x + a)/x)*x^(3/2)/((b*x + a)^(3/2)*a^2)
```

Fricas [A] time = 2.16803, size = 99, normalized size = 2.3

$$\frac{2\,(2\,bx+3\,a)\sqrt{bx+a}\sqrt{x}}{3\left(a^{2}b^{2}x^{2}+2\,a^{3}bx+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*x + 3*a)*sqrt(b*x + a)*sqrt(x)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)
```

Sympy [B] time = 3.69002, size = 92, normalized size = 2.14

$$\frac{6a}{3a^{3}\sqrt{b}\sqrt{\frac{a}{bx}+1}+3a^{2}b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}+\frac{4bx}{3a^{3}\sqrt{b}\sqrt{\frac{a}{bx}+1}+3a^{2}b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] 6*a/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x) + 1)
) + 4*b*x/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x
) + 1))
```

Giac [B] time = 1.15429, size = 109, normalized size = 2.53

$$\frac{8\left(3\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)b^{\frac{5}{2}}}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*b^(5/2)/(
((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*abs(b))
```

3.588 $\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\sqrt{a+bx}}+\frac{2}{3a\sqrt{x}(a+bx)^{3/2}}$$

```
[Out] 2/(3*a*Sqrt[x]*(a + b*x)^(3/2)) + 8/(3*a^2*Sqrt[x]*Sqrt[a + b*x]) - (16*Sqr
t[a + b*x])/(3*a^3*Sqrt[x])
```

Rubi [A] time = 0.0091767, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\sqrt{a+bx}}+\frac{2}{3a\sqrt{x}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*Sqrt[x]*(a + b*x)^(3/2)) + 8/(3*a^2*Sqrt[x]*Sqrt[a + b*x]) - (16*Sqr
t[a + b*x])/(3*a^3*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx &= \frac{2}{3a\sqrt{x}(a+bx)^{3/2}}+\frac{4\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx}{3a}\\
&= \frac{2}{3a\sqrt{x}(a+bx)^{3/2}}+\frac{8}{3a^2\sqrt{x}\sqrt{a+bx}}+\frac{8\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^2}\\
&= \frac{2}{3a\sqrt{x}(a+bx)^{3/2}}+\frac{8}{3a^2\sqrt{x}\sqrt{a+bx}}-\frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.014205, size = 40, normalized size = 0.62

$$-\frac{2\left(3a^2+12abx+8b^2x^2\right)}{3a^3\sqrt{x}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^(5/2)),x]
```

```
[Out] (-2*(3*a^2 + 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a + b*x)^(3/2))
```

Maple [A] time = 0.004, size = 35, normalized size = 0.6

$$-\frac{16\,b^2x^2+24\,abx+6\,a^2}{3\,a^3}(bx+a)^{-\frac{3}{2}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] -2/3*(8*b^2*x^2+12*a*b*x+3*a^2)/x^(1/2)/(b*x+a)^(3/2)/a^3
```

Maxima [A] time = 1.1227, size = 62, normalized size = 0.97

$$\frac{2\left(b^2-\frac{6\,(bx+a)b}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^3}-\frac{2\,\sqrt{bx+a}}{a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2 - 6*(b*x + a)*b/x)*x^(3/2)/((b*x + a)^(3/2)*a^3) - 2*sqrt(b*x + a)
/(a^3*sqrt(x))
```

Fricas [A] time = 2.10661, size = 128, normalized size = 2.

$$-\frac{2\left(8\,b^2x^2+12\,abx+3\,a^2\right)\sqrt{bx+a}\sqrt{x}}{3\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 + 12*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b^2*x^3 + 2*
a^4*b*x^2 + a^5*x)
```

Sympy [B] time = 17.4037, size = 153, normalized size = 2.39

$$-\frac{6a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}-\frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}-\frac{16b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**2*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x + 3*a**3*b*
*6*x**2) - 24*a*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x
+ 3*a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*
a**4*b**5*x + 3*a**3*b**6*x**2)
```

Giac [B] time = 1.11258, size = 215, normalized size = 3.36

$$-\frac{2\sqrt{bx+a}b^2}{\sqrt{(bx+a)b-ab}a^3|b|}-\frac{4\left(3\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4b^{\frac{5}{2}}+12a\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{7}{2}}+5a^2b^{\frac{9}{2}}\right)}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a^3*abs(b)) - 4/3*(3*(sqrt(b*
x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*b^(5/2) + 12*a*(sqrt(b*x + a)*s
qrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(7/2) + 5*a^2*b^(9/2))/(((sqrt(b*x +
a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*a^2*abs(b))
```

3.589 $\int \frac{1}{x^{5/2}(a+bx)^{5/2}} dx$

Optimal. Leaf size=84

$$-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}+\frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}$$

```
[Out] 2/(3*a*x^(3/2)*(a + b*x)^(3/2)) + 4/(a^2*x^(3/2)*Sqrt[a + b*x]) - (16*Sqrt[
a + b*x])/(3*a^3*x^(3/2)) + (32*b*Sqrt[a + b*x])/(3*a^4*Sqrt[x])
```

Rubi [A] time = 0.0153343, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}+\frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x^(3/2)*(a + b*x)^(3/2)) + 4/(a^2*x^(3/2)*Sqrt[a + b*x]) - (16*Sqrt[
a + b*x])/(3*a^3*x^(3/2)) + (32*b*Sqrt[a + b*x])/(3*a^4*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(a+bx)^{5/2}} dx &= \frac{2}{3ax^{3/2}(a+bx)^{3/2}}+\frac{2\int \frac{1}{x^{5/2}(a+bx)^{3/2}} dx}{a}\\
&= \frac{2}{3ax^{3/2}(a+bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}+\frac{8\int \frac{1}{x^{5/2}\sqrt{a+bx}} dx}{a^2}\\
&= \frac{2}{3ax^{3/2}(a+bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}-\frac{(16b)\int \frac{1}{x^{3/2}\sqrt{a+bx}} dx}{3a^3}\\
&= \frac{2}{3ax^{3/2}(a+bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}+\frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0234266, size = 49, normalized size = 0.58

$$-\frac{2\left(-6a^2bx+a^3-24ab^2x^2-16b^3x^3\right)}{3a^4x^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^(5/2)),x]
```

```
[Out] (-2*(a^3 - 6*a^2*b*x - 24*a*b^2*x^2 - 16*b^3*x^3))/(3*a^4*x^(3/2)*(a + b*x)
^(3/2))
```

Maple [A] time = 0.003, size = 44, normalized size = 0.5

$$-\frac{-32\,b^3x^3-48\,ab^2x^2-12\,a^2bx+2\,a^3}{3\,a^4}x^{-\frac{3}{2}}\,(bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(5/2),x)
```

```
[Out] -2/3*(-16*b^3*x^3-24*a*b^2*x^2-6*a^2*b*x+a^3)/x^(3/2)/(b*x+a)^(3/2)/a^4
```

Maxima [A] time = 1.12725, size = 86, normalized size = 1.02

$$\frac{2\left(\frac{9\,\sqrt{bx+a}b}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^4}-\frac{2\left(b^3-\frac{9\,(bx+a)b^2}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(9*sqrt(b*x + a)*b/sqrt(x) - (b*x + a)^(3/2)/x^(3/2))/a^4 - 2/3*(b^3 -
9*(b*x + a)*b^2/x)*x^(3/2)/((b*x + a)^(3/2)*a^4)
```

Fricas [A] time = 2.05912, size = 150, normalized size = 1.79

$$\frac{2\left(16\,b^3x^3+24\,ab^2x^2+6\,a^2bx-a^3\right)\sqrt{bx+a}\sqrt{x}}{3\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*x^3 + 24*a*b^2*x^2 + 6*a^2*b*x - a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)
```

Sympy [B] time = 40.0572, size = 337, normalized size = 4.01

$$-\frac{2a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{10a^3b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{60a^2b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] -2*a**4*b**(19/2)*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*
a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*a**3*b**(21/2)*x*sqrt(a/(b*x) + 1
)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**
4) + 60*a**2*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10
*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80*a*b**(25/2)*x**3*sqrt(a
/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4
*b**12*x**4) + 32*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*
b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4)
```

Giac [B] time = 1.15531, size = 234, normalized size = 2.79

$$-\frac{\sqrt{bx+a}\left(\frac{8\,(bx+a)a|b|}{b^2}-\frac{9\,a^2|b|}{b^2}\right)}{24\left((bx+a)b-ab\right)^{\frac{3}{2}}}+\frac{8\left(3\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4b^{\frac{7}{2}}+9\,a\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{9}{2}}+4\right.}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3a^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(b*x + a)*(8*(b*x + a)*a*abs(b)/b^2 - 9*a^2*abs(b)/b^2)/((b*x + a
)*b - a*b)^(3/2) + 8/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))
^4*b^(7/2) + 9*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(9/2
) + 4*a^2*b^(11/2))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 +
 a*b)^3*a^3*abs(b))
```

3.590 $\int \frac{x^{5/2}}{\sqrt{a-bx}} dx$

Optimal. Leaf size=105

$$-\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}$$

```
[Out] (-5*a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b^3) - (5*a*x^(3/2)*Sqrt[a - b*x])/(12*b^
2) - (x^(5/2)*Sqrt[a - b*x])/(3*b) + (5*a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a
 - b*x]])/(8*b^(7/2))
```

Rubi [A] time = 0.0303057, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$-\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a - b*x],x]
```

```
[Out] (-5*a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b^3) - (5*a*x^(3/2)*Sqrt[a - b*x])/(12*b^
2) - (x^(5/2)*Sqrt[a - b*x])/(3*b) + (5*a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a
 - b*x]])/(8*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx &= -\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{(5a)\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{6b}\\
&= -\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{\left(5a^2\right)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{8b^2}\\
&= -\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{\left(5a^3\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{16b^3}\\
&= -\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{8b^3}\\
&= -\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^3}\\
&= -\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.173854, size = 88, normalized size = 0.84

$$
\frac{\sqrt{a-bx}\left(\frac{15a^{5/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}-\sqrt{b}\sqrt{x}\left(15a^2+10abx+8b^2x^2\right)\right)}{24b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(-(Sqrt[b]*Sqrt[x]*(15*a^2 + 10*a*b*x + 8*b^2*x^2)) + (15*a^
(5/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(24*b^(7/2))
```

Maple [A] time = 0.003, size = 108, normalized size = 1.

$$
-\frac{1}{3b}x^{\frac{5}{2}}\sqrt{-bx+a}-\frac{5a}{12b^2}x^{\frac{3}{2}}\sqrt{-bx+a}-\frac{5a^2}{8b^3}\sqrt{x}\sqrt{-bx+a}+\frac{5a^3}{16}\sqrt{x(-bx+a)}\arctan\left(\sqrt{b}\left(x-\frac{a}{2b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)b^{-\frac{7}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -1/3*x^(5/2)*(-b*x+a)^(1/2)/b-5/12*a*x^(3/2)*(-b*x+a)^(1/2)/b^2-5/8*a^2*x^(
1/2)*(-b*x+a)^(1/2)/b^3+5/16/b^(7/2)*a^3*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a
)^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/(-b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.15587, size = 375, normalized size = 3.57

$$\left[-\frac{15\,a^3\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(8\,b^3x^2+10\,ab^2x+15\,a^2b\right)\sqrt{-bx+a}\sqrt{x}}{48\,b^4},-\frac{15\,a^3\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(8*b^3*x^2 + 10*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/24
*(15*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (8*b^3*x^2 + 10
*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^4]
```

Sympy [A] time = 15.0901, size = 272, normalized size = 2.59

$$\begin{cases} \frac{5ia^{\frac{5}{2}}\sqrt{x}}{8b^3\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^2\sqrt{-1+\frac{bx}{a}}} - \frac{i\sqrt{a}x^{\frac{5}{2}}}{12b\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^3\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}} - \frac{ix^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|} > 1 \\ -\frac{5a^{\frac{5}{2}}\sqrt{x}}{8b^3\sqrt{1-\frac{bx}{a}}} + \frac{5a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^2\sqrt{1-\frac{bx}{a}}} + \frac{\sqrt{a}x^{\frac{5}{2}}}{12b\sqrt{1-\frac{bx}{a}}} + \frac{5a^3\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}} + \frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(5/2)*sqrt(x)/(8*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(3/2)*x*
*(3/2)/(24*b**2*sqrt(-1 + b*x/a)) - I*sqrt(a)*x**(5/2)/(12*b*sqrt(-1 + b*x/
a)) - 5*I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(7/2)) - I*x**(7/2)/(3*
sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (-5*a**(5/2)*sqrt(x)/(8*b*
*3*sqrt(1 - b*x/a)) + 5*a**(3/2)*x**(3/2)/(24*b**2*sqrt(1 - b*x/a)) + sqrt(
a)*x**(5/2)/(12*b*sqrt(1 - b*x/a)) + 5*a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(
8*b**(7/2)) + x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.591 $\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=80

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}$$

```
[Out] (-3*a*Sqrt[x]*Sqrt[a - b*x])/(4*b^2) - (x^(3/2)*Sqrt[a - b*x])/(2*b) + (3*a
^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*b^(5/2))
```

Rubi [A] time = 0.0228779, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a - b*x],x]
```

```
[Out] (-3*a*Sqrt[x]*Sqrt[a - b*x])/(4*b^2) - (x^(3/2)*Sqrt[a - b*x])/(2*b) + (3*a
^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx = -\frac{x^{3/2}\sqrt{a-bx}}{2b} + \frac{(3a)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{4b}$$
$$= -\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2} - \frac{x^{3/2}\sqrt{a-bx}}{2b} + \frac{(3a^2)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{8b^2}$$
$$= -\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2} - \frac{x^{3/2}\sqrt{a-bx}}{2b} + \frac{(3a^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{4b^2}$$
$$= -\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2} - \frac{x^{3/2}\sqrt{a-bx}}{2b} + \frac{(3a^2)\operatorname{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^2}$$
$$= -\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2} - \frac{x^{3/2}\sqrt{a-bx}}{2b} + \frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}$$

Mathematica [A] time = 0.050758, size = 86, normalized size = 1.08

$$\frac{\sqrt{b}\sqrt{x}\left(-3a^2+abx+2b^2x^2\right)+3a^{5/2}\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{5/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*(-3*a^2 + a*b*x + 2*b^2*x^2) + 3*a^(5/2)*Sqrt[1 - (b*x)/a]
*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[a - b*x])
```

Maple [A] time = 0.004, size = 89, normalized size = 1.1

$$-\frac{1}{2b}x^{\frac{3}{2}}\sqrt{-bx+a}-\frac{3a}{4b^2}\sqrt{x}\sqrt{-bx+a}+\frac{3a^2}{8}\sqrt{x(-bx+a)}\arctan\left(\sqrt{b}\left(x-\frac{a}{2b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)b^{-\frac{5}{2}}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -1/2*x^(3/2)*(-b*x+a)^(1/2)/b-3/4*a*x^(1/2)*(-b*x+a)^(1/2)/b^2+3/8/b^(5/2)*
a^2*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/(-
b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08532, size = 321, normalized size = 4.01

$$\left[-\frac{3\,a^2\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(2\,b^2x+3\,ab\right)\sqrt{-bx+a}\sqrt{x}}{8\,b^3},\,-\frac{3\,a^2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)+\left(2\,b^2x+\right.}{4\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*(2*b^2*x + 3*a*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(b)*arcta
n(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x + 3*a*b)*sqrt(-b*x + a)*sqrt
(x))/b^3]
```

Sympy [A] time = 5.28609, size = 216, normalized size = 2.7

$$\begin{cases} \frac{3ia^{\frac{3}{2}}\sqrt{x}}{4b^2\sqrt{-1+\frac{bx}{a}}} - \frac{i\sqrt{a}x^{\frac{3}{2}}}{4b\sqrt{-1+\frac{bx}{a}}} - \frac{3ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} - \frac{ix^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|} > 1 \\ -\frac{3a^{\frac{3}{2}}\sqrt{x}}{4b^2\sqrt{1-\frac{bx}{a}}} + \frac{\sqrt{a}x^{\frac{3}{2}}}{4b\sqrt{1-\frac{bx}{a}}} + \frac{3a^2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} + \frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((3*I*a**(3/2)*sqrt(x)/(4*b**2*sqrt(-1 + b*x/a)) - I*sqrt(a)*x**(3
/2)/(4*b*sqrt(-1 + b*x/a)) - 3*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**
(5/2)) - I*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (-3
*a**(3/2)*sqrt(x)/(4*b**2*sqrt(1 - b*x/a)) + sqrt(a)*x**(3/2)/(4*b*sqrt(1 -
 b*x/a)) + 3*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(5/2)) + x**(5/2)/(2*
sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.592 $\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=50

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}$$

```
[Out] -((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/
b^(3/2)
```

Rubi [A] time = 0.0171857, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {50, 63, 217, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/
b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx = -\frac{\sqrt{x}\sqrt{a-bx}}{b} + \frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{2b}$$
$$= -\frac{\sqrt{x}\sqrt{a-bx}}{b} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{\sqrt{x}\sqrt{a-bx}}{b} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b}$$
$$= -\frac{\sqrt{x}\sqrt{a-bx}}{b} + \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.043236, size = 71, normalized size = 1.42

$$\frac{a^{3/2}\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\sqrt{b}\sqrt{x}(bx-a)}{b^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*(-a + b*x) + a^(3/2)*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/(b^(3/2)*Sqrt[a - b*x])
```

Maple [A] time = 0.004, size = 70, normalized size = 1.4

$$-\frac{1}{b}\sqrt{x}\sqrt{-bx+a}+\frac{a}{2}\sqrt{x\,(-bx+a)}\arctan\left(\sqrt{b}\left(x-\frac{a}{2\,b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -x^(1/2)*(-b*x+a)^(1/2)/b+1/2*a/b^(3/2)*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)
^(1/2)*arctan(b^(1/2)*(x-1/2/b*a)/(-b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.15506, size = 262, normalized size = 5.24

$$\left[-\frac{a\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\,\sqrt{-bx+a}b\sqrt{x}}{2\,b^2},\,-\frac{a\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)+\sqrt{-bx+a}b\sqrt{x}}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) + 2*s
qrt(-b*x + a)*b*sqrt(x))/b^2, -(a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sq
rt(x))) + sqrt(-b*x + a)*b*sqrt(x))/b^2]
```

Sympy [A] time = 2.59369, size = 122, normalized size = 2.44

$$\begin{cases} -\frac{i\sqrt{a}\sqrt{x}\sqrt{-1+\frac{bx}{a}}}{b}-\frac{ia\,\mathrm{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} & \text{for } \frac{|bx|}{|a|}>1 \\ -\frac{\sqrt{a}\sqrt{x}}{b\sqrt{1-\frac{bx}{a}}}+\frac{a\,\mathrm{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(a)*sqrt(x)*sqrt(-1 + b*x/a)/b - I*a*acosh(sqrt(b)*sqrt(x
)/sqrt(a))/b**(3/2), Abs(b*x)/Abs(a) > 1), (-sqrt(a)*sqrt(x)/(b*sqrt(1 - b*
x/a)) + a*asin(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) + x**(3/2)/(sqrt(a)*sqrt(1
 - b*x/a)), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.593 $\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]
```

Rubi [A] time = 0.0125623, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {63, 217, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a - b*x]),x]

[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.014671, size = 52, normalized size = 1.79

$$\frac{2\sqrt{a}\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a - b*x]),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sq
rt[a - b*x])
```

Maple [B] time = 0.004, size = 51, normalized size = 1.8

$$\sqrt{x(-bx+a)}\arctan\left(\sqrt{b}\left(x-\frac{a}{2b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] (x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2/b*a
)/(-b*x^2+a*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13147, size = 165, normalized size = 5.69

$$\left[-\frac{\sqrt{-b}\log\left(-2bx+2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)}{b}, -\frac{2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)/b, -2*arctan
(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

Sympy [A] time = 1.35913, size = 56, normalized size = 1.93

$$\begin{cases} -\frac{2i \operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for } \frac{|bx|}{|a|} > 1 \\ \frac{2 \operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b), Abs(b*x)/Abs(a) > 1
), (2*asin(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.594 $\int \frac{1}{x^{3/2}\sqrt{a-bx}} dx$

Optimal. Leaf size=20

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

```
[Out] (-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

Rubi [A] time = 0.0017315, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}} dx = -\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Mathematica [A] time = 0.0106507, size = 20, normalized size = 1.

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

Maple [A] time = 0.004, size = 17, normalized size = 0.9

$$-2\frac{\sqrt{-bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -2*(-b*x+a)^(1/2)/a/x^(1/2)
```

Maxima [A] time = 0.996761, size = 22, normalized size = 1.1

$$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-b*x + a)/(a*sqrt(x))
```

Fricas [A] time = 2.08596, size = 42, normalized size = 2.1

$$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-b*x + a)/(a*sqrt(x))
```

Sympy [A] time = 1.23314, size = 49, normalized size = 2.45

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{a} & \text{for}\ \frac{|a|}{|b||x|} > 1 \\ -\frac{2i\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/a, Abs(a)/(Abs(b)*Abs(x)) > 1), (-2
*I*sqrt(b)*sqrt(-a/(b*x) + 1)/a, True))
```

Giac [B] time = 1.08071, size = 47, normalized size = 2.35

$$-\frac{2\sqrt{-bx+a}b^2}{\sqrt{(bx-a)b+ab}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a*abs(b))
```

3.595 $\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx$

Optimal. Leaf size=46

$$-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}$$

```
[Out] (-2*Sqrt[a - b*x])/(3*a*x^(3/2)) - (4*b*Sqrt[a - b*x])/(3*a^2*Sqrt[x])
```

Rubi [A] time = 0.005001, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(3*a*x^(3/2)) - (4*b*Sqrt[a - b*x])/(3*a^2*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx &= -\frac{2\sqrt{a-bx}}{3ax^{3/2}}+\frac{(2b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a}\\ &= -\frac{2\sqrt{a-bx}}{3ax^{3/2}}-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0091297, size = 28, normalized size = 0.61

$$-\frac{2\sqrt{a-bx}(a+2bx)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x]*(a + 2*b*x))/(3*a^2*x^(3/2))
```

Maple [A] time = 0.004, size = 23, normalized size = 0.5

$$-\frac{4\,bx+2\,a}{3\,a^2}\sqrt{-bx+a}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -2/3*(-b*x+a)^(1/2)*(2*b*x+a)/x^(3/2)/a^2
```

Maxima [A] time = 0.988458, size = 43, normalized size = 0.93

$$-\frac{2\left(\frac{3\sqrt{-bx+a}b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(-b*x + a)*b/sqrt(x) + (-b*x + a)^(3/2)/x^(3/2))/a^2
```

Fricas [A] time = 1.92803, size = 63, normalized size = 1.37

$$-\frac{2\,(2\,bx+a)\sqrt{-bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*b*x + a)*sqrt(-b*x + a)/(a^2*x^(3/2))
```

Sympy [A] time = 3.81764, size = 180, normalized size = 3.91

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3ax}-\frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3a^2} & \text{for } \frac{|a|}{|b||x|}>1 \\ \frac{2ia^2b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2}+\frac{2iab^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2}-\frac{4ib^{\frac{7}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*a*x) - 4*b**(3/2)*sqrt(a/(b*x) -
 1)/(3*a**2), Abs(a)/(Abs(b)*Abs(x)) > 1), (2*I*a**2*b**(3/2)*sqrt(-a/(b*x)
 + 1)/(-3*a**3*b*x + 3*a**2*b**2*x**2) + 2*I*a*b**(5/2)*x*sqrt(-a/(b*x) + 1
)/(-3*a**3*b*x + 3*a**2*b**2*x**2) - 4*I*b**(7/2)*x**2*sqrt(-a/(b*x) + 1)/(
-3*a**3*b*x + 3*a**2*b**2*x**2), True))
```

Giac [A] time = 1.09614, size = 73, normalized size = 1.59

$$-\frac{\sqrt{-bx+a}\,b\left(\frac{2\,(bx-a)}{a^2b^3}+\frac{3}{ab^3}\right)}{24\,((bx-a)b+ab)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-b*x + a)*b*(2*(b*x - a)/(a^2*b^3) + 3/(a*b^3))/(((b*x - a)*b +
a*b)^(3/2)*abs(b))
```

3.596 $\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{2x^{5/2}}{b\sqrt{a-bx}}$$

```
[Out] (2*x^(5/2))/(b*Sqrt[a - b*x]) + (15*a*Sqrt[x]*Sqrt[a - b*x])/(4*b^3) + (5*x
^(3/2)*Sqrt[a - b*x])/(2*b^2) - (15*a^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b
*x]])/(4*b^(7/2))
```

Rubi [A] time = 0.0300671, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$-\frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{2x^{5/2}}{b\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2))/(b*Sqrt[a - b*x]) + (15*a*Sqrt[x]*Sqrt[a - b*x])/(4*b^3) + (5*x
^(3/2)*Sqrt[a - b*x])/(2*b^2) - (15*a^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b
*x]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx &= \frac{2x^{5/2}}{b\sqrt{a-bx}}-\frac{5\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{b}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{(15a)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{4b^2}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{\left(15a^2\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{8b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0125512, size = 51, normalized size = 0.51

$$\frac{2x^{7/2}\sqrt{1-\frac{bx}{a}}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{bx}{a}\right)}{7a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 - (b*x)/a]*Hypergeometric2F1[3/2, 7/2, 9/2, (b*x)/a])/(7*
a*Sqrt[a - b*x])
```

Maple [A] time = 0.023, size = 127, normalized size = 1.3

$$\frac{2\,bx+7\,a}{4\,b^3}\sqrt{x}\sqrt{-bx+a}+\left(-\frac{15\,a^2}{8}\arctan\left(\sqrt{b}\left(x-\frac{a}{2\,b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)b^{-\frac{7}{2}}-2\,\frac{a^2}{b^4}\sqrt{-b\left(x-\frac{a}{b}\right)^2-a\left(x-\frac{a}{b}\right)}\left(x-\frac{a}{b}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(3/2),x)
```

```
[Out] 1/4*(2*b*x+7*a)/b^3*x^(1/2)*(-b*x+a)^(1/2)+(-15/8*a^2/b^(7/2)*arctan(b^(1/2
)*(x-1/2/b*a)/(-b*x^2+a*x)^(1/2))-2*a^2/b^4/(x-1/b*a)*(-b*(x-1/b*a)^2-a*(x-
1/b*a))^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94584, size = 433, normalized size = 4.33

$$\left[-\frac{15\left(a^{2}bx-a^{3}\right)\sqrt{-b}\log\left(-2\,bx-2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(2\,b^{3}x^{2}+5\,ab^{2}x-15\,a^{2}b\right)\sqrt{-bx+a}\sqrt{x}}{8\left(b^{5}x-ab^{4}\right)},\ \frac{15\left(a^{2}bx-a^{3}\right)\sqrt{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(a^2*b*x - a^3)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*s
qrt(x) + a) - 2*(2*b^3*x^2 + 5*a*b^2*x - 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/
(b^5*x - a*b^4), 1/4*(15*(a^2*b*x - a^3)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqr
t(b)*sqrt(x))) + (2*b^3*x^2 + 5*a*b^2*x - 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))
/(b^5*x - a*b^4)]
```

Sympy [A] time = 15.6244, size = 226, normalized size = 2.26

$$\begin{cases} -\frac{15ia^{\frac{3}{2}}\sqrt{x}}{4b^{3}\sqrt{-1+\frac{bx}{a}}}+\frac{5i\sqrt{a}x^{\frac{3}{2}}}{4b^{2}\sqrt{-1+\frac{bx}{a}}}+\frac{15ia^{2}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}}+\frac{ix^{\frac{5}{2}}}{2\sqrt{a}b\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|}>1 \\ \frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^{3}\sqrt{1-\frac{bx}{a}}}-\frac{5\sqrt{a}x^{\frac{3}{2}}}{4b^{2}\sqrt{1-\frac{bx}{a}}}-\frac{15a^{2}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}}-\frac{x^{\frac{5}{2}}}{2\sqrt{a}b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-15*I*a**(3/2)*sqrt(x)/(4*b**3*sqrt(-1 + b*x/a)) + 5*I*sqrt(a)*x
**(3/2)/(4*b**2*sqrt(-1 + b*x/a)) + 15*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a)
)/(4*b**(7/2)) + I*x**(5/2)/(2*sqrt(a)*b*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a)
 > 1), (15*a**(3/2)*sqrt(x)/(4*b**3*sqrt(1 - b*x/a)) - 5*sqrt(a)*x**(3/2)/(
4*b**2*sqrt(1 - b*x/a)) - 15*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2)
) - x**(5/2)/(2*sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

Giac [B] time = 59.2308, size = 208, normalized size = 2.08

$$\frac{\left(2\sqrt{(bx-a)b+ab}\sqrt{-bx+a}\left(\frac{2(bx-a)}{b^3}+\frac{9a}{b^3}\right)-\frac{32a^3\sqrt{-b}}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)b^2}+\frac{15a^2\sqrt{-b}\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{b^3}\right)|b|}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)/b^3 + 9*a/b^3) -
 32*a^3*sqrt(-b)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 -
a*b)*b^2) + 15*a^2*sqrt(-b)*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b
 + a*b))^2)/b^3)*abs(b)/b^2
```

3.597 $\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{3\sqrt{x}\sqrt{a-bx}}{b^2}-\frac{3a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{b\sqrt{a-bx}}$$

```
[Out] (2*x^(3/2))/(b*Sqrt[a - b*x]) + (3*Sqrt[x]*Sqrt[a - b*x])/b^2 - (3*a*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(5/2)
```

Rubi [A] time = 0.0220824, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$\frac{3\sqrt{x}\sqrt{a-bx}}{b^2}-\frac{3a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{b\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a - b*x)^(3/2),x]
```


```
[Out] (2*x^(3/2))/(b*Sqrt[a - b*x]) + (3*Sqrt[x]*Sqrt[a - b*x])/b^2 - (3*a*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(5/2)
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx &= \frac{2x^{3/2}}{b\sqrt{a-bx}}-\frac{3\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{b}\\ &= \frac{2x^{3/2}}{b\sqrt{a-bx}}+\frac{3\sqrt{x}\sqrt{a-bx}}{b^2}-\frac{(3a)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{2b^2}\\ &= \frac{2x^{3/2}}{b\sqrt{a-bx}}+\frac{3\sqrt{x}\sqrt{a-bx}}{b^2}-\frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\ &= \frac{2x^{3/2}}{b\sqrt{a-bx}}+\frac{3\sqrt{x}\sqrt{a-bx}}{b^2}-\frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^2}\\ &= \frac{2x^{3/2}}{b\sqrt{a-bx}}+\frac{3\sqrt{x}\sqrt{a-bx}}{b^2}-\frac{3a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.0107771, size = 51, normalized size = 0.72

$$\frac{2x^{5/2}\sqrt{1-\frac{bx}{a}}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{bx}{a}\right)}{5a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2)*Sqrt[1 - (b*x)/a]*Hypergeometric2F1[3/2, 5/2, 7/2, (b*x)/a])/(5*
a*Sqrt[a - b*x])
```

Maple [B] time = 0.02, size = 114, normalized size = 1.6

$$\frac{1}{b^2}\sqrt{x}\sqrt{-bx+a}+\left(-\frac{3a}{2}\arctan\left(\sqrt{b}\left(x-\frac{a}{2b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)b^{-\frac{5}{2}}-2\frac{a}{b^3}\sqrt{-b\left(x-\frac{a}{b}\right)^2-a\left(x-\frac{a}{b}\right)}\left(x-\frac{a}{b}\right)^{-1}\right)\sqrt{x(-bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(3/2),x)
```

```
[Out] x^(1/2)*(-b*x+a)^(1/2)/b^2+(-3/2/b^(5/2)*a*arctan(b^(1/2)*(x-1/2/b*a)/(-b*x
^2+a*x)^(1/2))-2/b^3*a/(x-1/b*a)*(-b*(x-1/b*a)^2-a*(x-1/b*a))^(1/2))*(x*(-b
*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79657, size = 369, normalized size = 5.2

$$\left[-\frac{3\left(abx-a^2\right)\sqrt{-b}\log\left(-2\,bx-2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(b^2x-3\,ab\right)\sqrt{-bx+a}\sqrt{x}}{2\left(b^4x-ab^3\right)}, \frac{3\left(abx-a^2\right)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)+ \dots}{b^4x-ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(a*b*x - a^2)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt
(x) + a) - 2*(b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x))/(b^4*x - a*b^3), (3*(a
*b*x - a^2)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (b^2*x - 3*a
*b)*sqrt(-b*x + a)*sqrt(x))/(b^4*x - a*b^3)]
```

Sympy [A] time = 4.24528, size = 156, normalized size = 2.2

$$\begin{cases} -\frac{3i\sqrt{a}\sqrt{x}}{b^2\sqrt{-1+\frac{bx}{a}}}+\frac{3ia\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}+\frac{ix^{\frac{3}{2}}}{\sqrt{a}b\sqrt{-1+\frac{bx}{a}}} & \text{for } \frac{|bx|}{|a|}>1 \\ \frac{3\sqrt{a}\sqrt{x}}{b^2\sqrt{1-\frac{bx}{a}}}-\frac{3a\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}-\frac{x^{\frac{3}{2}}}{\sqrt{a}b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-3*I*sqrt(a)*sqrt(x)/(b**2*sqrt(-1 + b*x/a)) + 3*I*a*acosh(sqrt(
b)*sqrt(x)/sqrt(a))/b**(5/2) + I*x**(3/2)/(sqrt(a)*b*sqrt(-1 + b*x/a)), Abs
(b*x)/Abs(a) > 1), (3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 - b*x/a)) - 3*a*asin(sqr
t(b)*sqrt(x)/sqrt(a))/b**(5/2) - x**(3/2)/(sqrt(a)*b*sqrt(1 - b*x/a)), True
))
```

Giac [B] time = 59.3328, size = 176, normalized size = 2.48

$$-\frac{\left(\frac{8\,a^2\sqrt{-b}}{\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab}+\frac{3\,a\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}}-\frac{2\,\sqrt{(bx-a)b+ab}\sqrt{-bx+a}}{b}\right)|b|}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(8*a^2*sqrt(-b)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2
 - a*b) + 3*a*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/sq
rt(-b) - 2*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)/b)*abs(b)/b^3
```

3.598 $\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

```
[Out] (2*Sqrt[x])/(b*Sqrt[a - b*x]) - (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
/b^(3/2)
```

Rubi [A] time = 0.0162758, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {47, 63, 217, 203}

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[a - b*x]) - (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
/b^(3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.0737895, size = 66, normalized size = 1.32

$$\frac{2\sqrt{b}\sqrt{x} - 2\sqrt{a}\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b]*Sqrt[x] - 2*Sqrt[a]*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sqrt[x])/S
qrt[a]])/(b^(3/2)*Sqrt[a - b*x])
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \sqrt{x}\,(-bx+a)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(3/2),x)
```

```
[Out] int(x^(1/2)/(-b*x+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7616, size = 312, normalized size = 6.24

$$\left[-\frac{(bx-a)\sqrt{-b}\log\left(-2\,bx-2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\,\sqrt{-bx+a}b\sqrt{x}}{b^3x-ab^2}, \frac{2\left((bx-a)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\sqrt{-bx+a}b\sqrt{x}\right)}{b^3x-ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((b*x - a)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*sqrt(-b*x + a)*b*sqrt(x))/(b^3*x - a*b^2), 2*((b*x - a)*sqrt(b)*arctan(s
qrt(-b*x + a)/(sqrt(b)*sqrt(x))) - sqrt(-b*x + a)*b*sqrt(x))/(b^3*x - a*b^2
)]
```

Sympy [A] time = 2.11018, size = 104, normalized size = 2.08

$$\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{2i\sqrt{x}}{\sqrt{a}b\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \frac{|bx|}{|a|} > 1 \\ -\frac{2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} + \frac{2\sqrt{x}}{\sqrt{a}b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) - 2*I*sqrt(x)/(sqrt(
a)*b*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (-2*asin(sqrt(b)*sqrt(x)/sqrt
(a))/b**(3/2) + 2*sqrt(x)/(sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

Giac [B] time = 59.564, size = 138, normalized size = 2.76

$$-\frac{\left(\frac{4\,a\sqrt{-b}}{\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab}-\frac{\sqrt{-b}\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{b}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(4*a*sqrt(-b)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b
) - sqrt(-b)*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/b)*
abs(b)/b^2
```

3.599 $\int \frac{1}{\sqrt{x}(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=20

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

```
[Out] (2*Sqrt[x])/(a*Sqrt[a - b*x])
```

Rubi [A] time = 0.0017072, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a - b*x)^(3/2)),x]

[Out] (2*Sqrt[x])/(a*Sqrt[a - b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(a-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Mathematica [A] time = 0.0045051, size = 20, normalized size = 1.

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a - b*x)^(3/2)),x]

[Out] (2*Sqrt[x])/(a*Sqrt[a - b*x])
```

Maple [A] time = 0.005, size = 17, normalized size = 0.9

$$2\,\frac{\sqrt{x}}{a\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] 2*x^(1/2)/a/(-b*x+a)^(1/2)
```

Maxima [A] time = 1.02052, size = 22, normalized size = 1.1

$$\frac{2\sqrt{x}}{\sqrt{-bx+a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)/(sqrt(-b*x + a)*a)
```

Fricas [A] time = 1.72294, size = 55, normalized size = 2.75

$$-\frac{2\sqrt{-bx+a}\sqrt{x}}{abx-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-b*x + a)*sqrt(x)/(a*b*x - a^2)
```

Sympy [A] time = 1.14351, size = 48, normalized size = 2.4

$$\begin{cases} \frac{2}{a\sqrt{b}\sqrt{\frac{a}{bx}-1}} & \text{for}\ \frac{|a|}{|b||x|} > 1 \\ -\frac{2i}{a\sqrt{b}\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((2/(a*sqrt(b)*sqrt(a/(b*x) - 1)), Abs(a)/(Abs(b)*Abs(x)) > 1), (-
2*I/(a*sqrt(b)*sqrt(-a/(b*x) + 1)), True))
```

Giac [B] time = 1.07288, size = 72, normalized size = 3.6

$$-\frac{4\sqrt{-b}b}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-b)*b/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b
)*abs(b))
```

3.600 $\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2}{a\sqrt{x}\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}$$

```
[Out] 2/(a*Sqrt[x]*Sqrt[a - b*x]) - (4*Sqrt[a - b*x])/(a^2*Sqrt[x])
```

Rubi [A] time = 0.0049461, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$\frac{2}{a\sqrt{x}\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a - b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[x]*Sqrt[a - b*x]) - (4*Sqrt[a - b*x])/(a^2*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx &= \frac{2}{a\sqrt{x}\sqrt{a-bx}}+\frac{2\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{a}\\ &= \frac{2}{a\sqrt{x}\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0082571, size = 26, normalized size = 0.63

$$-\frac{2(a-2bx)}{a^2\sqrt{x}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a - b*x)^(3/2)),x]
```

```
[Out] (-2*(a - 2*b*x))/(a^2*Sqrt[x]*Sqrt[a - b*x])
```

Maple [A] time = 0.003, size = 23, normalized size = 0.6

$$-2\,\frac{-2\,bx+a}{a^{2}\sqrt{x}\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(3/2),x)
```

```
[Out] -2*(-2*b*x+a)/x^(1/2)/(-b*x+a)^(1/2)/a^2
```

Maxima [A] time = 1.05453, size = 46, normalized size = 1.12

$$\frac{2\,b\sqrt{x}}{\sqrt{-bx+a}a^{2}}-\frac{2\,\sqrt{-bx+a}}{a^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b*sqrt(x)/(sqrt(-b*x + a)*a^2) - 2*sqrt(-b*x + a)/(a^2*sqrt(x))
```

Fricas [A] time = 1.74941, size = 80, normalized size = 1.95

$$-\frac{2\,(2\,bx-a)\sqrt{-bx+a}\sqrt{x}}{a^{2}bx^{2}-a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*x - a)*sqrt(-b*x + a)*sqrt(x)/(a^2*b*x^2 - a^3*x)
```

Sympy [A] time = 2.53384, size = 116, normalized size = 2.83

$$\begin{cases} -\frac{2}{a\sqrt{b}x\sqrt{\frac{a}{bx}-1}}+\frac{4\sqrt{b}}{a^{2}\sqrt{\frac{a}{bx}-1}} & \text{for}\ \frac{|a|}{|b||x|}>1\\ \frac{2iab^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{-a^{3}b+a^{2}b^{2}x}-\frac{4ib^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}{-a^{3}b+a^{2}b^{2}x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-2/(a*sqrt(b)*x*sqrt(a/(b*x) - 1)) + 4*sqrt(b)/(a**2*sqrt(a/(b*x
) - 1)), Abs(a)/(Abs(b)*Abs(x)) > 1), (2*I*a*b**(3/2)*sqrt(-a/(b*x) + 1)/(-
a**3*b + a**2*b**2*x) - 4*I*b**(5/2)*x*sqrt(-a/(b*x) + 1)/(-a**3*b + a**2*b
**2*x), True))
```

Giac [B] time = 1.07418, size = 127, normalized size = 3.1

$$-\frac{4\sqrt{-b}b^2}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)a|b|}-\frac{2\sqrt{-bx+a}b^2}{\sqrt{(bx-a)b+ab}a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-b)*b^2/((((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a
*b)*a*abs(b)) - 2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a^2*abs(b))
```

3.601 $\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}-\frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}+\frac{2}{ax^{3/2}\sqrt{a-bx}}$$

```
[Out] 2/(a*x^(3/2)*Sqrt[a - b*x]) - (8*Sqrt[a - b*x])/(3*a^2*x^(3/2)) - (16*b*Sqr
t[a - b*x])/(3*a^3*Sqrt[x])
```

Rubi [A] time = 0.0104674, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}-\frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}+\frac{2}{ax^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a - b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(3/2)*Sqrt[a - b*x]) - (8*Sqrt[a - b*x])/(3*a^2*x^(3/2)) - (16*b*Sqr
t[a - b*x])/(3*a^3*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx &= \frac{2}{ax^{3/2}\sqrt{a-bx}}+\frac{4\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx}{a}\\
&= \frac{2}{ax^{3/2}\sqrt{a-bx}}-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}+\frac{(8b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^2}\\
&= \frac{2}{ax^{3/2}\sqrt{a-bx}}-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}-\frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0112065, size = 39, normalized size = 0.59

$$-\frac{2\left(a^2+4abx-8b^2x^2\right)}{3a^3x^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a - b*x)^(3/2)),x]
```

```
[Out] (-2*(a^2 + 4*a*b*x - 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a - b*x])
```

Maple [A] time = 0.005, size = 34, normalized size = 0.5

$$-\frac{-16\,b^2x^2+8\,abx+2\,a^2}{3\,a^3}x^{-\frac{3}{2}}\frac{1}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(3/2),x)
```

```
[Out] -2/3*(-8*b^2*x^2+4*a*b*x+a^2)/x^(3/2)/(-b*x+a)^(1/2)/a^3
```

Maxima [A] time = 1.03725, size = 70, normalized size = 1.06

$$\frac{2\,b^2\sqrt{x}}{\sqrt{-bx+a}a^3}-\frac{2\left(\frac{6\,\sqrt{-bx+a}b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(-b*x + a)*a^3) - 2/3*(6*sqrt(-b*x + a)*b/sqrt(x) + (-b*
x + a)^(3/2)/x^(3/2))/a^3
```

Fricas [A] time = 1.84799, size = 107, normalized size = 1.62

$$-\frac{2\left(8\,b^2x^2-4\,abx-a^2\right)\sqrt{-bx+a}\sqrt{x}}{3\left(a^3bx^3-a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 - 4*a*b*x - a^2)*sqrt(-b*x + a)*sqrt(x)/(a^3*b*x^3 - a^4*x^
2)
```

Sympy [B] time = 16.2018, size = 456, normalized size = 6.91

$$\begin{cases} -\frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} - \frac{6a^2b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} + \frac{24ab^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} - \frac{16b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} & \text{for } \frac{|a|}{|b||x|} > 1 \\ -\frac{2ia^3b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} - \frac{6ia^2b^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} + \frac{24iab^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} - \frac{16ib^{\frac{15}{2}}x^3\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-2*a**3*b**(9/2)*sqrt(a/(b*x) - 1)/(3*a**5*b**4*x - 6*a**4*b**5*
x**2 + 3*a**3*b**6*x**3) - 6*a**2*b**(11/2)*x*sqrt(a/(b*x) - 1)/(3*a**5*b**
4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*a*b**(13/2)*x**2*sqrt(a/(b*
x) - 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) - 16*b**(15/2
)*x**3*sqrt(a/(b*x) - 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x*
*3), Abs(a)/(Abs(b)*Abs(x)) > 1), (-2*I*a**3*b**(9/2)*sqrt(-a/(b*x) + 1)/(3
*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) - 6*I*a**2*b**(11/2)*x*
sqrt(-a/(b*x) + 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) +
24*I*a*b**(13/2)*x**2*sqrt(-a/(b*x) + 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2
+ 3*a**3*b**6*x**3) - 16*I*b**(15/2)*x**3*sqrt(-a/(b*x) + 1)/(3*a**5*b**4*x
 - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3), True))
```

Giac [B] time = 1.09395, size = 144, normalized size = 2.18

$$-\frac{\sqrt{-bx+a}\left(\frac{5\,(bx-a)|b|}{b^2}+\frac{6\,a|b|}{b^2}\right)}{24\,((bx-a)b+ab)^{\frac{3}{2}}}-\frac{4\,\sqrt{-b}b^3}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-b*x + a)*(5*(b*x - a)*abs(b)/b^2 + 6*a*abs(b)/b^2)/((b*x - a)*b
 + a*b)^(3/2) - 4*sqrt(-b)*b^3/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*
b + a*b))^2 - a*b)*a^2*abs(b))
```

3.602 $\int \frac{x^{5/2}}{(a-bx)^{5/2}} dx$

Optimal. Leaf size=95

$$-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{5a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}+\frac{2x^{5/2}}{3b(a-bx)^{3/2}}$$

```
[Out] (2*x^(5/2))/(3*b*(a - b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[a - b*x]) - (5
*Sqrt[x]*Sqrt[a - b*x])/b^3 + (5*a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
/b^(7/2)
```

Rubi [A] time = 0.0285071, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{5a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}+\frac{2x^{5/2}}{3b(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(5/2))/(3*b*(a - b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[a - b*x]) - (5
*Sqrt[x]*Sqrt[a - b*x])/b^3 + (5*a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx &= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{5\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx}{3b}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{b^2}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{(5a)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{2b^3}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{(5a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{(5a)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^3}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{5a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0115855, size = 51, normalized size = 0.54

$$\frac{2x^{7/2}\sqrt{1-\frac{bx}{a}}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{bx}{a}\right)}{7a^2\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 - (b*x)/a]*Hypergeometric2F1[5/2, 7/2, 9/2, (b*x)/a])/(7*
a^2*Sqrt[a - b*x])
```

Maple [B] time = 0.032, size = 160, normalized size = 1.7

$$-\frac{1}{b^3}\sqrt{x}\sqrt{-bx+a}+\left(\frac{5a}{2}\arctan\left(\sqrt{b}\left(x-\frac{a}{2b}\right)\frac{1}{\sqrt{-bx^2+ax}}\right)\right)b^{-\frac{7}{2}}+\frac{2a^2}{3b^5}\sqrt{-b\left(x-\frac{a}{b}\right)^2-a\left(x-\frac{a}{b}\right)}\left(x-\frac{a}{b}\right)^{-2}+\frac{14a}{3b^4}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(5/2),x)
```

```
[Out] -x^(1/2)*(-b*x+a)^(1/2)/b^3+(5/2/b^(7/2)*a*arctan(b^(1/2)*(x-1/2/b*a)/(-b*x
^2+a*x)^(1/2))+2/3/b^5*a^2/(x-1/b*a)^2*(-b*(x-1/b*a)^2-a*(x-1/b*a))^(1/2)+1
4/3/b^4*a/(x-1/b*a)*(-b*(x-1/b*a)^2-a*(x-1/b*a))^(1/2))*(x*(-b*x+a))^(1/2)/
x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84299, size = 518, normalized size = 5.45

$$\left[-\frac{15\left(ab^2x^2-2\,a^2bx+a^3\right)\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(3\,b^3x^2-20\,ab^2x+15\,a^2b\right)\sqrt{-bx+a}\sqrt{x}}{6\left(b^6x^2-2\,ab^5x+a^2b^4\right)},-\frac{15(\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x +
 a)*sqrt(-b)*sqrt(x) + a) + 2*(3*b^3*x^2 - 20*a*b^2*x + 15*a^2*b)*sqrt(-b*x
 + a)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4), -1/3*(15*(a*b^2*x^2 - 2*a^2
*b*x + a^3)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (3*b^3*x^2 -
 20*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*
b^4)]
```

Sympy [B] time = 15.4067, size = 972, normalized size = 10.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-30*I*a**(81/2)*b**22*x**(51/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*s
qrt(x)/sqrt(a))/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(7
7/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 15*pi*a**(81/2)*b**22*x**(51/2
)*sqrt(-1 + b*x/a)/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a*
*(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 30*I*a**(79/2)*b**23*x**(53
/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(6*a**(79/2)*b**(51/2)*
x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/
a)) - 15*pi*a**(79/2)*b**23*x**(53/2)*sqrt(-1 + b*x/a)/(6*a**(79/2)*b**(51/
2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b
*x/a)) + 30*I*a**40*b**(45/2)*x**26/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-
1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) - 40*I*a**39
```

```
*b**(47/2)*x**27/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(
77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 6*I*a**38*b**(49/2)*x**28/(6*
a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(
53/2)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (15*a**(81/2)*b**22*x**(51/2
)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**(51/2)*x**(
51/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) -
15*a**(79/2)*b**23*x**(53/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/
(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x*
*(53/2)*sqrt(1 - b*x/a)) - 15*a**40*b**(45/2)*x**26/(3*a**(79/2)*b**(51/2)*
x**(51/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)
) + 20*a**39*b**(47/2)*x**27/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/
a) - 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) - 3*a**38*b**(49/2)*x
**28/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/
2)*x**(53/2)*sqrt(1 - b*x/a)), True))
```

Giac [B] time = 59.3158, size = 298, normalized size = 3.14

$$\frac{\left(\frac{15\,a\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}\right)}{\sqrt{-b}b^{2}}-\frac{6\sqrt{(bx-a)b+ab}\sqrt{-bx+a}}{b^{3}}-\frac{8\left(9\,a^{2}\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{4}-12\,a^{3}\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}b+7\,a^{4}b}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}-ab\right)^{3}\sqrt{-b}b}\right.}{6\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(15*a*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/(sqrt(
-b)*b^2) - 6*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)/b^3 - 8*(9*a^2*(sqrt(-b
*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4 - 12*a^3*(sqrt(-b*x + a)*sqrt
(-b) - sqrt((b*x - a)*b + a*b))^2*b + 7*a^4*b^2)/(((sqrt(-b*x + a)*sqrt(-b)
 - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*sqrt(-b)*b))*abs(b)/b^2
```

3.603 $\int \frac{x^{3/2}}{(a-bx)^{5/2}} dx$

Optimal. Leaf size=72

$$-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{3b(a-bx)^{3/2}}$$

```
[Out] (2*x^(3/2))/(3*b*(a - b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[a - b*x]) + (2*Ar
cTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(5/2)
```

Rubi [A] time = 0.0220513, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {47, 63, 217, 203}

$$-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{3b(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*b*(a - b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[a - b*x]) + (2*Ar
cTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx = \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx}{b}$$
$$= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a-bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{b^2}$$
$$= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a-bx}} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a-bx}} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^2}$$
$$= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a-bx}} + \frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.169204, size = 82, normalized size = 1.14

$$\frac{2\left(\sqrt{b}\sqrt{x}(4bx-3a)+3\sqrt{a}(a-bx)\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\right)}{3b^{5/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*(Sqrt[b]*Sqrt[x]*(-3*a + 4*b*x) + 3*Sqrt[a]*(a - b*x)*Sqrt[1 - (b*x)/a]*
ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(3*b^(5/2)*(a - b*x)^(3/2))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}} (-bx+a)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(5/2),x)
```

```
[Out] int(x^(3/2)/(-b*x+a)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90345, size = 456, normalized size = 6.33

$$\left[-\frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(4\,b^2x-3\,ab\right)\sqrt{-bx+a}\sqrt{x}}{3\left(b^5x^2-2\,ab^4x+a^2b^3\right)}, -\frac{2\left(3\left(b^2x^2-2\,abx+a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*s
qrt(-b)*sqrt(x) + a) - 2*(4*b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x))/(b^5*x^2
 - 2*a*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(b)*arctan(s
qrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (4*b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x)
)/(b^5*x^2 - 2*a*b^4*x + a^2*b^3)]
```

Sympy [B] time = 6.04487, size = 835, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-6*I*a**(39/2)*b**11*x**(27/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sq
rt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37
/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) + 3*pi*a**(39/2)*b**11*x**(27/2)*
sqrt(-1 + b*x/a)/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(
37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) + 6*I*a**(37/2)*b**12*x**(29/2)
*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**
(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a))
 - 3*pi*a**(37/2)*b**12*x**(29/2)*sqrt(-1 + b*x/a)/(3*a**(39/2)*b**(27/2)*x
**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a
)) + 6*I*a**19*b**(23/2)*x**14/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b
*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) - 8*I*a**18*b**(2
5/2)*x**15/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*
b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (6*a**(39/2)*b
**11*x**(27/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b
**(27/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1
 - b*x/a)) - 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(
x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*
b**(29/2)*x**(29/2)*sqrt(1 - b*x/a)) - 6*a**19*b**(23/2)*x**14/(3*a**(39/2)
*b**(27/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt
(1 - b*x/a)) + 8*a**18*b**(25/2)*x**15/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqr
t(1 - b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 - b*x/a)), True))
```

Giac [B] time = 59.183, size = 266, normalized size = 3.69

$$-\frac{\left(\frac{3\,\sqrt{-b}\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{b}-\frac{8\left(3\,a\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}-3\,a^2\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\sqrt{-b}b+2\,a^3\sqrt{-b}b^2\right)}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3}\right)|b|}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(-b)*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)
/b - 8*(3*a*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b)
- 3*a^2*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2*sqrt(-b)*b +
2*a^3*sqrt(-b)*b^2)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2
- a*b)^3)*abs(b)/b^3
```

3.604 $\int \frac{\sqrt{x}}{(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

[Out] (2*x^(3/2))/(3*a*(a - b*x)^(3/2))

Rubi [A] time = 0.0016898, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a - b*x)^(5/2),x]

[Out] (2*x^(3/2))/(3*a*(a - b*x)^(3/2))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}}\,dx = \frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Mathematica [A] time = 0.0055816, size = 22, normalized size = 1.

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(a - b*x)^(5/2),x]

[Out] (2*x^(3/2))/(3*a*(a - b*x)^(3/2))

Maple [A] time = 0.003, size = 17, normalized size = 0.8

$$\frac{2}{3a}x^{\frac{3}{2}}(-bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(5/2),x)
```

```
[Out] 2/3*x^(3/2)/a/(-b*x+a)^(3/2)
```

Maxima [A] time = 1.05743, size = 22, normalized size = 1.

$$\frac{2\,x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/((-b*x + a)^(3/2)*a)
```

Fricas [B] time = 1.77506, size = 78, normalized size = 3.55

$$\frac{2\,\sqrt{-bx+a}x^{\frac{3}{2}}}{3\left(ab^2x^2-2\,a^2bx+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-b*x + a)*x^(3/2)/(a*b^2*x^2 - 2*a^2*b*x + a^3)
```

Sympy [B] time = 2.25948, size = 97, normalized size = 4.41

$$\begin{cases} \frac{2ix^{\frac{3}{2}}}{-3a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{-1+\frac{bx}{a}}} & \text{for } \frac{|bx|}{|a|}>1 \\ -\frac{2x^{\frac{3}{2}}}{-3a^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((2*I*x**(3/2)/(-3*a**(5/2)*sqrt(-1 + b*x/a) + 3*a**(3/2)*b*x*sqrt
(-1 + b*x/a)), Abs(b*x)/Abs(a) > 1), (-2*x**(3/2)/(-3*a**(5/2)*sqrt(1 - b*x
/a) + 3*a**(3/2)*b*x*sqrt(1 - b*x/a)), True))
```

Giac [B] time = 1.1171, size = 138, normalized size = 6.27

$$\frac{4\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}+a^2\sqrt{-b}b^2\right)|b|}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b) + a^2
*sqrt(-b)*b^2)*abs(b)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))
^2 - a*b)^3*b^2)
```

3.605 $\int \frac{1}{\sqrt{x}(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=45

$$\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}+\frac{2\sqrt{x}}{3a(a-bx)^{3/2}}$$

```
[Out] (2*Sqrt[x])/(3*a*(a - b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a - b*x])
```

Rubi [A] time = 0.0050084, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}+\frac{2\sqrt{x}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a - b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x])/(3*a*(a - b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a - b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(a-bx)^{5/2}}\,dx &= \frac{2\sqrt{x}}{3a(a-bx)^{3/2}}+\frac{2\int \frac{1}{\sqrt{x}(a-bx)^{3/2}}\,dx}{3a}\\ &= \frac{2\sqrt{x}}{3a(a-bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}\end{aligned}$$

Mathematica [A] time = 0.009055, size = 30, normalized size = 0.67

$$\frac{2\sqrt{x}(3a-2bx)}{3a^2(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a - b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x]*(3*a - 2*b*x))/(3*a^2*(a - b*x)^(3/2))
```

Maple [A] time = 0.003, size = 25, normalized size = 0.6

$$\frac{-4\,bx+6\,a}{3\,a^2}\sqrt{x}\,(-bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 2/3*x^(1/2)*(-2*b*x+3*a)/(-b*x+a)^(3/2)/a^2
```

Maxima [A] time = 1.02868, size = 41, normalized size = 0.91

$$\frac{2\left(b-\frac{3\,(bx-a)}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b - 3*(b*x - a)/x)*x^(3/2)/((-b*x + a)^(3/2)*a^2)
```

Fricas [A] time = 1.87127, size = 101, normalized size = 2.24

$$-\frac{2\,(2\,bx-3\,a)\sqrt{-bx+a}\sqrt{x}}{3\left(a^2b^2x^2-2\,a^3bx+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*b*x - 3*a)*sqrt(-b*x + a)*sqrt(x)/(a^2*b^2*x^2 - 2*a^3*b*x + a^4)
```

Sympy [B] time = 3.56506, size = 201, normalized size = 4.47

$$\begin{cases} -\frac{6a}{-3a^3\sqrt{b}\sqrt{\frac{a}{bx}-1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}-1}}+\frac{4bx}{-3a^3\sqrt{b}\sqrt{\frac{a}{bx}-1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}-1}} & \text{for}\ \frac{|a|}{|b||x|}>1 \\ \frac{6iab}{-3a^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}+3a^2b^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}-\frac{4ib^2x}{-3a^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}+3a^2b^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((-6*a/(-3*a**3*sqrt(b)*sqrt(a/(b*x) - 1) + 3*a**2*b**(3/2)*x*sqrt
(a/(b*x) - 1)) + 4*b*x/(-3*a**3*sqrt(b)*sqrt(a/(b*x) - 1) + 3*a**2*b**(3/2)
*x*sqrt(a/(b*x) - 1)), Abs(a)/(Abs(b)*Abs(x)) > 1), (6*I*a*b/(-3*a**3*b**(3
/2)*sqrt(-a/(b*x) + 1) + 3*a**2*b**(5/2)*x*sqrt(-a/(b*x) + 1)) - 4*I*b**2*x
/(-3*a**3*b**(3/2)*sqrt(-a/(b*x) + 1) + 3*a**2*b**(5/2)*x*sqrt(-a/(b*x) + 1
)), True))
```

Giac [B] time = 1.0963, size = 130, normalized size = 2.89

$$\frac{8\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}-ab\right)\sqrt{-b}b^{2}}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}-ab\right)^{3}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)*sqrt(-b
)*b^2/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*abs(
b))
```

3.606 $\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\sqrt{a-bx}}+\frac{2}{3a\sqrt{x}(a-bx)^{3/2}}$$

```
[Out] 2/(3*a*Sqrt[x]*(a - b*x)^(3/2)) + 8/(3*a^2*Sqrt[x]*Sqrt[a - b*x]) - (16*Sqr
t[a - b*x])/(3*a^3*Sqrt[x])
```

Rubi [A] time = 0.009829, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\sqrt{a-bx}}+\frac{2}{3a\sqrt{x}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a - b*x)^(5/2)),x]
```



```
[Out] 2/(3*a*Sqrt[x]*(a - b*x)^(3/2)) + 8/(3*a^2*Sqrt[x]*Sqrt[a - b*x]) - (16*Sqr
t[a - b*x])/(3*a^3*Sqrt[x])
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx &= \frac{2}{3a\sqrt{x}(a-bx)^{3/2}}+\frac{4\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx}{3a}\\ &= \frac{2}{3a\sqrt{x}(a-bx)^{3/2}}+\frac{8}{3a^2\sqrt{x}\sqrt{a-bx}}+\frac{8\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^2}\\ &= \frac{2}{3a\sqrt{x}(a-bx)^{3/2}}+\frac{8}{3a^2\sqrt{x}\sqrt{a-bx}}-\frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0116426, size = 41, normalized size = 0.61

$$-\frac{2\left(3a^2-12abx+8b^2x^2\right)}{3a^3\sqrt{x}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a - b*x)^(5/2)),x]
```

```
[Out] (-2*(3*a^2 - 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a - b*x)^(3/2))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.5

$$-\frac{16\,b^2x^2-24\,abx+6\,a^2}{3\,a^3}(-bx+a)^{-\frac{3}{2}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(5/2),x)
```

```
[Out] -2/3*(8*b^2*x^2-12*a*b*x+3*a^2)/x^(1/2)/(-b*x+a)^(3/2)/a^3
```

Maxima [A] time = 1.11762, size = 68, normalized size = 1.01

$$\frac{2\left(b^2-\frac{6\,(bx-a)b}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a^3}-\frac{2\,\sqrt{-bx+a}}{a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2 - 6*(b*x - a)*b/x)*x^(3/2)/((-b*x + a)^(3/2)*a^3) - 2*sqrt(-b*x +
a)/(a^3*sqrt(x))
```

Fricas [A] time = 1.85453, size = 130, normalized size = 1.94

$$-\frac{2\left(8\,b^2x^2-12\,abx+3\,a^2\right)\sqrt{-bx+a}\sqrt{x}}{3\left(a^3b^2x^3-2\,a^4bx^2+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 - 12*a*b*x + 3*a^2)*sqrt(-b*x + a)*sqrt(x)/(a^3*b^2*x^3 - 2
*a^4*b*x^2 + a^5*x)
```

Sympy [B] time = 16.0072, size = 318, normalized size = 4.75

$$\begin{cases} -\frac{6a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} + \frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} - \frac{16b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} & \text{for } \frac{|a|}{|b||x|} > 1 \\ -\frac{6ia^2b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} + \frac{24iab^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} - \frac{16ib^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-6*a**2*b**(9/2)*sqrt(a/(b*x) - 1)/(3*a**5*b**4 - 6*a**4*b**5*x
+ 3*a**3*b**6*x**2) + 24*a*b**(11/2)*x*sqrt(a/(b*x) - 1)/(3*a**5*b**4 - 6*a
**4*b**5*x + 3*a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) - 1)/(3*a**
5*b**4 - 6*a**4*b**5*x + 3*a**3*b**6*x**2), Abs(a)/(Abs(b)*Abs(x)) > 1), (-
6*I*a**2*b**(9/2)*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a**4*b**5*x + 3*a**3*
b**6*x**2) + 24*I*a*b**(11/2)*x*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a**4*b*
*5*x + 3*a**3*b**6*x**2) - 16*I*b**(13/2)*x**2*sqrt(-a/(b*x) + 1)/(3*a**5*b
**4 - 6*a**4*b**5*x + 3*a**3*b**6*x**2), True))
```

Giac [B] time = 1.15958, size = 255, normalized size = 3.81

$$-\frac{2\sqrt{-bx+a}b^2}{\sqrt{(bx-a)b+ab}a^3|b|} - \frac{4\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}b^2 - 12\,a\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\sqrt{-b}b\right.}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a^3*abs(b)) - 4/3*(3*(sqrt(-
b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b)*b^2 - 12*a*(sqrt(-b
*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2*sqrt(-b)*b^3 + 5*a^2*sqrt(-b)
*b^4)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*a^2*
abs(b))
```

3.607 $\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}-\frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}+\frac{2}{3ax^{3/2}(a-bx)^{3/2}}$$

```
[Out] 2/(3*a*x^(3/2)*(a - b*x)^(3/2)) + 4/(a^2*x^(3/2)*Sqrt[a - b*x]) - (16*Sqrt[
a - b*x])/(3*a^3*x^(3/2)) - (32*b*Sqrt[a - b*x])/(3*a^4*Sqrt[x])
```

Rubi [A] time = 0.0163789, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}-\frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}+\frac{2}{3ax^{3/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x^(3/2)*(a - b*x)^(3/2)) + 4/(a^2*x^(3/2)*Sqrt[a - b*x]) - (16*Sqrt[
a - b*x])/(3*a^3*x^(3/2)) - (32*b*Sqrt[a - b*x])/(3*a^4*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx &= \frac{2}{3ax^{3/2}(a-bx)^{3/2}}+\frac{2\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx}{a}\\
&= \frac{2}{3ax^{3/2}(a-bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}+\frac{8\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx}{a^2}\\
&= \frac{2}{3ax^{3/2}(a-bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}+\frac{(16b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^3}\\
&= \frac{2}{3ax^{3/2}(a-bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}-\frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0135244, size = 50, normalized size = 0.57

$$-\frac{2\left(6a^2bx+a^3-24ab^2x^2+16b^3x^3\right)}{3a^4x^{3/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

```
[Out] (-2*(a^3 + 6*a^2*b*x - 24*a*b^2*x^2 + 16*b^3*x^3))/(3*a^4*x^(3/2)*(a - b*x)
^(3/2))
```

Maple [A] time = 0.003, size = 45, normalized size = 0.5

$$-\frac{32\,b^3x^3-48\,ab^2x^2+12\,a^2bx+2\,a^3}{3\,a^4}x^{-\frac{3}{2}}\left(-bx+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(5/2),x)
```

```
[Out] -2/3*(16*b^3*x^3-24*a*b^2*x^2+6*a^2*b*x+a^3)/x^(3/2)/(-b*x+a)^(3/2)/a^4
```

Maxima [A] time = 1.02535, size = 92, normalized size = 1.05

$$-\frac{2\left(\frac{9\sqrt{-bx+a}b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^4}+\frac{2\left(b^3-\frac{9\,(bx-a)b^2}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(9*sqrt(-b*x + a)*b/sqrt(x) + (-b*x + a)^(3/2)/x^(3/2))/a^4 + 2/3*(b^3
 - 9*(b*x - a)*b^2/x)*x^(3/2)/((-b*x + a)^(3/2)*a^4)
```

Fricas [A] time = 1.8497, size = 153, normalized size = 1.74

$$-\frac{2\left(16\,b^3x^3-24\,ab^2x^2+6\,a^2bx+a^3\right)\sqrt{-bx+a}\sqrt{x}}{3\left(a^4b^2x^4-2\,a^5bx^3+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(16*b^3*x^3 - 24*a*b^2*x^2 + 6*a^2*b*x + a^3)*sqrt(-b*x + a)*sqrt(x)/(
a^4*b^2*x^4 - 2*a^5*b*x^3 + a^6*x^2)
```

Sympy [B] time = 36.5423, size = 692, normalized size = 7.86

$$\begin{cases} \frac{2a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{10a^3b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{60a^2b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{80ab^{\frac{25}{2}}x^3 \cdots}{-3a^7b^9x+9a^6b^{10}x^2-9 \cdots} \\ \frac{2ia^4b^{\frac{19}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{10ia^3b^{\frac{21}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{60ia^2b^{\frac{23}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{80iab^{\frac{25}{2}}x^3 \cdots}{-3a^7b^9x+9a^6b^{10}x^2-9 \cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((2*a**4*b**(19/2)*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**1
0*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*a**3*b**(21/2)*x*sqrt(
a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a*
*4*b**12*x**4) - 60*a**2*b**(23/2)*x**2*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x +
 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80*a*b**(25/2
)*x**3*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11
*x**3 + 3*a**4*b**12*x**4) - 32*b**(27/2)*x**4*sqrt(a/(b*x) - 1)/(-3*a**7*b
**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4), Abs(a)/
(Abs(b)*Abs(x)) > 1), (2*I*a**4*b**(19/2)*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*
x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*I*a**3*
b**(21/2)*x*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5
*b**11*x**3 + 3*a**4*b**12*x**4) - 60*I*a**2*b**(23/2)*x**2*sqrt(-a/(b*x) +
 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*
x**4) + 80*I*a*b**(25/2)*x**3*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6*b
**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) - 32*I*b**(27/2)*x**4*sq
rt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 +
3*a**4*b**12*x**4), True))
```

Giac [B] time = 1.15586, size = 277, normalized size = 3.15

$$-\frac{\sqrt{-bx+a}\left(\frac{8\,(bx-a)a|b|}{b^2}+\frac{9\,a^2|b|}{b^2}\right)}{24\,((bx-a)b+ab)^{\frac{3}{2}}} - \frac{8\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}b^3 - 9\,a\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ \cdots}\right.\right.}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3 a^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-b*x + a)*(8*(b*x - a)*a*abs(b)/b^2 + 9*a^2*abs(b)/b^2)/((b*x -
a)*b + a*b)^(3/2) - 8/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*
b))^4*sqrt(-b)*b^3 - 9*a*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b)
)^2*sqrt(-b)*b^4 + 4*a^2*sqrt(-b)*b^5)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b
*x - a)*b + a*b))^2 - a*b)^3*a^3*abs(b))
```

3.608 $\int \frac{x^{5/2}}{\sqrt{2+bx}} dx$

Optimal. Leaf size=88

$$-\frac{5x^{3/2}\sqrt{bx+2}}{6b^2}+\frac{5\sqrt{x}\sqrt{bx+2}}{2b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{x^{5/2}\sqrt{bx+2}}{3b}$$

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(2*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(6*b^2) + (x^
(5/2)*Sqrt[2 + b*x])/(3*b) - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Rubi [A] time = 0.021807, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$-\frac{5x^{3/2}\sqrt{bx+2}}{6b^2}+\frac{5\sqrt{x}\sqrt{bx+2}}{2b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{x^{5/2}\sqrt{bx+2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[2 + b*x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(2*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(6*b^2) + (x^
(5/2)*Sqrt[2 + b*x])/(3*b) - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx = \frac{x^{5/2}\sqrt{2+bx}}{3b} - \frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{3b}$$
$$= -\frac{5x^{3/2}\sqrt{2+bx}}{6b^2} + \frac{x^{5/2}\sqrt{2+bx}}{3b} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b^2}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{6b^2} + \frac{x^{5/2}\sqrt{2+bx}}{3b} - \frac{5\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b^3}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{6b^2} + \frac{x^{5/2}\sqrt{2+bx}}{3b} - \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^3}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{6b^2} + \frac{x^{5/2}\sqrt{2+bx}}{3b} - \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

Mathematica [A] time = 0.0408326, size = 60, normalized size = 0.68

$$\frac{\sqrt{x}\sqrt{bx+2}\left(2b^2x^2-5bx+15\right)}{6b^3} - \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2))/(6*b^3) - (5*ArcSinh[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Maple [A] time = 0.005, size = 93, normalized size = 1.1

$$\frac{1}{3b}x^{\frac{5}{2}}\sqrt{bx+2} - \frac{5}{6b^2}x^{\frac{3}{2}}\sqrt{bx+2} + \frac{5}{2b^3}\sqrt{x}\sqrt{bx+2} - \frac{5}{2}\sqrt{x(bx+2)}\ln\left((bx+1)\frac{1}{\sqrt{b}} + \sqrt{bx^2+2x}\right)b^{-\frac{7}{2}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(1/2),x)
```

```
[Out] 1/3*x^(5/2)*(b*x+2)^(1/2)/b-5/6*x^(3/2)*(b*x+2)^(1/2)/b^2+5/2*x^(1/2)*(b*x+
2)^(1/2)/b^3-5/2/b^(7/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)
/b^(1/2)+(b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82923, size = 328, normalized size = 3.73

$$\left[\frac{\left(2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\sqrt{x}+15\,\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{6\,b^{4}},\frac{\left(2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\sqrt{x}+30\,\sqrt{-b}\,\mathrm{a}}{6\,b^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(b
*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/6*((2*b^3*x^2 - 5*b^2*x + 1
5*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*s
qrt(x))))/b^4]
```

Sympy [A] time = 12.5306, size = 95, normalized size = 1.08

$$\frac{x^{\frac{7}{2}}}{3\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{6b\sqrt{bx+2}}+\frac{5x^{\frac{3}{2}}}{6b^{2}\sqrt{bx+2}}+\frac{5\sqrt{x}}{b^{3}\sqrt{bx+2}}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(7/2)/(3*sqrt(b*x + 2)) - x**(5/2)/(6*b*sqrt(b*x + 2)) + 5*x**(3/2)/(6*b
**2*sqrt(b*x + 2)) + 5*sqrt(x)/(b**3*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(
b)*sqrt(x)/2)/b**(7/2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.609 $\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx$

Optimal. Leaf size=67

$$-\frac{3\sqrt{x}\sqrt{bx+2}}{2b^2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{x^{3/2}\sqrt{bx+2}}{2b}$$

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(2*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(2*b) + (3*Arc
Sinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rubi [A] time = 0.0135392, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$-\frac{3\sqrt{x}\sqrt{bx+2}}{2b^2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{x^{3/2}\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[2 + b*x],x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(2*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(2*b) + (3*Arc
Sinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx = \frac{x^{3/2}\sqrt{2+bx}}{2b} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b}$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{2b} + \frac{3\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{2b} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{2b} + \frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.0283173, size = 51, normalized size = 0.76

$$\frac{\sqrt{x}\sqrt{bx+2}(bx-3)}{2b^2} + \frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*(-3 + b*x)*Sqrt[2 + b*x])/(2*b^2) + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/b^(5/2)
```

Maple [A] time = 0.005, size = 78, normalized size = 1.2

$$\frac{1}{2b}x^{\frac{3}{2}}\sqrt{bx+2} - \frac{3}{2b^2}\sqrt{x}\sqrt{bx+2} + \frac{3}{2}\sqrt{x(bx+2)}\ln\left((bx+1)\frac{1}{\sqrt{b}} + \sqrt{bx^2+2x}\right)b^{-\frac{5}{2}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(b*x+2)^(1/2)/b-3/2*x^(1/2)*(b*x+2)^(1/2)/b^2+3/2/b^(5/2)*(x*(b
*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92437, size = 285, normalized size = 4.25

$$\left[\frac{\left(b^2x-3\,b\right)\sqrt{bx+2}\sqrt{x}+3\,\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{2\,b^3},\frac{\left(b^2x-3\,b\right)\sqrt{bx+2}\sqrt{x}-6\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)}{2\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x + sqrt(b*x +
2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/2*((b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) - 6*
sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

Sympy [A] time = 4.22213, size = 75, normalized size = 1.12

$$\frac{x^{\frac{5}{2}}}{2\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{2b\sqrt{bx+2}}-\frac{3\sqrt{x}}{b^2\sqrt{bx+2}}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(5/2)/(2*sqrt(b*x + 2)) - x**(3/2)/(2*b*sqrt(b*x + 2)) - 3*sqrt(x)/(b**2
*sqrt(b*x + 2)) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.610 $\int \frac{\sqrt{x}}{\sqrt{2+bx}} dx$

Optimal. Leaf size=43

$$\frac{\sqrt{x}\sqrt{bx+2}}{b} - \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)
```

Rubi [A] time = 0.0087786, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$\frac{\sqrt{x}\sqrt{bx+2}}{b} - \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{2+bx}} dx &= \frac{\sqrt{x}\sqrt{2+bx}}{b} - \frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}} dx}{b} \\
&= \frac{\sqrt{x}\sqrt{2+bx}}{b} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}} dx, x, \sqrt{x}\right)}{b} \\
&= \frac{\sqrt{x}\sqrt{2+bx}}{b} - \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0148595, size = 43, normalized size = 1.

$$\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)
```

Maple [A] time = 0.004, size = 62, normalized size = 1.4

$$\frac{1}{b}\sqrt{x}\sqrt{bx+2}-\sqrt{x(bx+2)}\ln\left((bx+1)\frac{1}{\sqrt{b}}+\sqrt{bx^2+2x}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] x^(1/2)*(b*x+2)^(1/2)/b-1/b^(3/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*l
n((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83451, size = 239, normalized size = 5.56

$$\left[\frac{\sqrt{bx+2}b\sqrt{x}+\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{b^2},\frac{\sqrt{bx+2}b\sqrt{x}+2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b*x + 2)*b*sqrt(x) + sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x)
 + 1))/b^2, (sqrt(b*x + 2)*b*sqrt(x) + 2*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt
(-b)/(b*sqrt(x))))/b^2]
```

Sympy [A] time = 2.25833, size = 54, normalized size = 1.26

$$\frac{x^{\frac{3}{2}}}{\sqrt{bx+2}}+\frac{2\sqrt{x}}{b\sqrt{bx+2}}-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(3/2)/sqrt(b*x + 2) + 2*sqrt(x)/(b*sqrt(b*x + 2)) - 2*asinh(sqrt(2)*sqrt
(b)*sqrt(x)/2)/b**(3/2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.611 $\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=24

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rubi [A] time = 0.0055053, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {54, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0038526, size = 24, normalized size = 1.

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Maple [B] time = 0.002, size = 46, normalized size = 1.9

$$\sqrt{x(bx+2)}\ln\left((bx+1)\frac{1}{\sqrt{b}}+\sqrt{bx^2+2x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] (x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2
))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83676, size = 157, normalized size = 6.54

$$\left[\frac{\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1)/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x + 2)*sqrt(-b)/(b*sqrt(x)))/b]
```

Sympy [A] time = 1.16283, size = 24, normalized size = 1.

$$\frac{2\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.612 $\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

```
[Out] -(Sqrt[2 + b*x]/Sqrt[x])
```

Rubi [A] time = 0.0012056, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -(Sqrt[2 + b*x]/Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{2+bx}}{\sqrt{x}}$$

Mathematica [A] time = 0.0034954, size = 16, normalized size = 1.

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -(Sqrt[2 + b*x]/Sqrt[x])
```

Maple [A] time = 0.003, size = 13, normalized size = 0.8

$$-\sqrt{bx+2}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(1/2),x)
```

```
[Out] -(b*x+2)^(1/2)/x^(1/2)
```

Maxima [A] time = 1.02366, size = 16, normalized size = 1.

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*x + 2)/sqrt(x)
```

Fricas [A] time = 1.8098, size = 32, normalized size = 2.

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x + 2)/sqrt(x)
```

Sympy [A] time = 1.12139, size = 15, normalized size = 0.94

$$-\sqrt{b}\sqrt{1+\frac{2}{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(1 + 2/(b*x))
```

Giac [B] time = 1.07529, size = 39, normalized size = 2.44

$$-\frac{\sqrt{bx+2}b^2}{\sqrt{(bx+2)b-2b}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b))
```

3.613 $\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=38

$$\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}$$

```
[Out] -Sqrt[2 + b*x]/(3*x^(3/2)) + (b*Sqrt[2 + b*x])/(3*Sqrt[x])
```

Rubi [A] time = 0.0035665, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -Sqrt[2 + b*x]/(3*x^(3/2)) + (b*Sqrt[2 + b*x])/(3*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx &= -\frac{\sqrt{2+bx}}{3x^{3/2}}-\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= -\frac{\sqrt{2+bx}}{3x^{3/2}}+\frac{b\sqrt{2+bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0053017, size = 23, normalized size = 0.61

$$\frac{(bx-1)\sqrt{bx+2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[2 + b*x]),x]
```

```
[Out] ((-1 + b*x)*Sqrt[2 + b*x])/(3*x^(3/2))
```

Maple [A] time = 0.003, size = 18, normalized size = 0.5

$$\frac{bx-1}{3}\sqrt{bx+2}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(1/2),x)
```

```
[Out] 1/3*(b*x+2)^(1/2)*(b*x-1)/x^(3/2)
```

Maxima [A] time = 1.19424, size = 35, normalized size = 0.92

$$\frac{\sqrt{bx+2}b}{2\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}}{6x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x + 2)*b/sqrt(x) - 1/6*(b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A] time = 1.86946, size = 50, normalized size = 1.32

$$\frac{\sqrt{bx+2}(bx-1)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x + 2)*(b*x - 1)/x^(3/2)
```

Sympy [A] time = 4.14656, size = 34, normalized size = 0.89

$$\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-\frac{\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(1/2),x)
```

```
[Out] b**(3/2)*sqrt(1 + 2/(b*x))/3 - sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

Giac [A] time = 1.07499, size = 57, normalized size = 1.5

$$\frac{\left((bx+2)b^3-3\,b^3\right)\sqrt{bx+2}b}{3\,((bx+2)b-2\,b)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x + 2)*b^3 - 3*b^3)*sqrt(b*x + 2)*b/(((b*x + 2)*b - 2*b)^(3/2)*abs(
b))
```

3.614 $\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=59

$$-\frac{2b^2\sqrt{bx+2}}{15\sqrt{x}}+\frac{2b\sqrt{bx+2}}{15x^{3/2}}-\frac{\sqrt{bx+2}}{5x^{5/2}}$$

```
[Out] -Sqrt[2 + b*x]/(5*x^(5/2)) + (2*b*Sqrt[2 + b*x])/(15*x^(3/2)) - (2*b^2*Sqrt
[2 + b*x])/(15*Sqrt[x])
```

Rubi [A] time = 0.0074262, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2\sqrt{bx+2}}{15\sqrt{x}}+\frac{2b\sqrt{bx+2}}{15x^{3/2}}-\frac{\sqrt{bx+2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -Sqrt[2 + b*x]/(5*x^(5/2)) + (2*b*Sqrt[2 + b*x])/(15*x^(3/2)) - (2*b^2*Sqrt
[2 + b*x])/(15*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx &= -\frac{\sqrt{2+bx}}{5x^{5/2}}-\frac{1}{5}(2b)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx\\ &= -\frac{\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{15x^{3/2}}+\frac{1}{15}\left(2b^2\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= -\frac{\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{15x^{3/2}}-\frac{2b^2\sqrt{2+bx}}{15\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.008169, size = 32, normalized size = 0.54

$$-\frac{\sqrt{bx+2}\left(2b^2x^2-2bx+3\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -(Sqrt[2 + b*x]*(3 - 2*b*x + 2*b^2*x^2))/(15*x^(5/2))
```

Maple [A] time = 0.003, size = 27, normalized size = 0.5

$$-\frac{2\,b^2x^2-2\,bx+3}{15}\sqrt{bx+2}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+2)^(1/2),x)
```

```
[Out] -1/15*(b*x+2)^(1/2)*(2*b^2*x^2-2*b*x+3)/x^(5/2)
```

Maxima [A] time = 1.02854, size = 55, normalized size = 0.93

$$-\frac{\sqrt{bx+2}b^2}{4\sqrt{x}}+\frac{(bx+2)^{\frac{3}{2}}b}{6\,x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{20\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(b*x + 2)*b^2/sqrt(x) + 1/6*(b*x + 2)^(3/2)*b/x^(3/2) - 1/20*(b*x
+ 2)^(5/2)/x^(5/2)
```

Fricas [A] time = 1.77065, size = 72, normalized size = 1.22

$$-\frac{\left(2\,b^2x^2-2\,bx+3\right)\sqrt{bx+2}}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*b^2*x^2 - 2*b*x + 3)*sqrt(b*x + 2)/x^(5/2)
```

Sympy [B] time = 28.0066, size = 224, normalized size = 3.8

$$-\frac{2b^{\frac{17}{2}}x^4\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{6b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{3b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{4b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\ 15$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+2)**(1/2),x)
```

```
[Out] -2*b**(17/2)*x**4*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x**3 + 60*b**4*
x**2) - 6*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x**3 + 6
0*b**4*x**2) - 3*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x
**3 + 60*b**4*x**2) - 4*b**(11/2)*x*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b*
*5*x**3 + 60*b**4*x**2) - 12*b**(9/2)*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*
b**5*x**3 + 60*b**4*x**2)
```

Giac [A] time = 1.09795, size = 74, normalized size = 1.25

$$-\frac{\left(15\,b^5+2\left((bx+2)b^5-5\,b^5\right)(bx+2)\right)\sqrt{bx+2}b}{15\left((bx+2)b-2\,b\right)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(15*b^5 + 2*((b*x + 2)*b^5 - 5*b^5)*(b*x + 2))*sqrt(b*x + 2)*b/(((b*x
 + 2)*b - 2*b)^(5/2)*abs(b))
```

3.615 $\int \frac{1}{x^{9/2}\sqrt{2+bx}} dx$

Optimal. Leaf size=80

$$-\frac{2b^2\sqrt{bx+2}}{35x^{3/2}}+\frac{2b^3\sqrt{bx+2}}{35\sqrt{x}}+\frac{3b\sqrt{bx+2}}{35x^{5/2}}-\frac{\sqrt{bx+2}}{7x^{7/2}}$$

```
[Out] -Sqrt[2 + b*x]/(7*x^(7/2)) + (3*b*Sqrt[2 + b*x])/(35*x^(5/2)) - (2*b^2*Sqrt
[2 + b*x])/(35*x^(3/2)) + (2*b^3*Sqrt[2 + b*x])/(35*Sqrt[x])
```

Rubi [A] time = 0.0123568, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2\sqrt{bx+2}}{35x^{3/2}}+\frac{2b^3\sqrt{bx+2}}{35\sqrt{x}}+\frac{3b\sqrt{bx+2}}{35x^{5/2}}-\frac{\sqrt{bx+2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -Sqrt[2 + b*x]/(7*x^(7/2)) + (3*b*Sqrt[2 + b*x])/(35*x^(5/2)) - (2*b^2*Sqrt
[2 + b*x])/(35*x^(3/2)) + (2*b^3*Sqrt[2 + b*x])/(35*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx &= -\frac{\sqrt{2+bx}}{7x^{7/2}}-\frac{1}{7}(3b)\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx\\
&= -\frac{\sqrt{2+bx}}{7x^{7/2}}+\frac{3b\sqrt{2+bx}}{35x^{5/2}}+\frac{1}{35}\left(6b^2\right)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx\\
&= -\frac{\sqrt{2+bx}}{7x^{7/2}}+\frac{3b\sqrt{2+bx}}{35x^{5/2}}-\frac{2b^2\sqrt{2+bx}}{35x^{3/2}}-\frac{1}{35}\left(2b^3\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\
&= -\frac{\sqrt{2+bx}}{7x^{7/2}}+\frac{3b\sqrt{2+bx}}{35x^{5/2}}-\frac{2b^2\sqrt{2+bx}}{35x^{3/2}}+\frac{2b^3\sqrt{2+bx}}{35\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0090322, size = 40, normalized size = 0.5

$$\frac{\sqrt{bx+2}\left(2b^3x^3-2b^2x^2+3bx-5\right)}{35x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[2 + b*x]),x]
```

```
[Out] (Sqrt[2 + b*x]*(-5 + 3*b*x - 2*b^2*x^2 + 2*b^3*x^3))/(35*x^(7/2))
```

Maple [A] time = 0.005, size = 35, normalized size = 0.4

$$\frac{2\,b^3x^3-2\,b^2x^2+3\,bx-5}{35}\sqrt{bx+2}x^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x+2)^(1/2),x)
```

```
[Out] 1/35*(b*x+2)^(1/2)*(2*b^3*x^3-2*b^2*x^2+3*b*x-5)/x^(7/2)
```

Maxima [A] time = 1.08572, size = 76, normalized size = 0.95

$$\frac{\sqrt{bx+2}b^3}{8\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}b^2}{8\,x^{\frac{3}{2}}}+\frac{3\,(bx+2)^{\frac{5}{2}}b}{40\,x^{\frac{5}{2}}}-\frac{(bx+2)^{\frac{7}{2}}}{56\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(b*x + 2)*b^3/sqrt(x) - 1/8*(b*x + 2)^(3/2)*b^2/x^(3/2) + 3/40*(b*x
 + 2)^(5/2)*b/x^(5/2) - 1/56*(b*x + 2)^(7/2)/x^(7/2)
```

Fricas [A] time = 1.75463, size = 86, normalized size = 1.08

$$\frac{\left(2\,b^3x^3-2\,b^2x^2+3\,bx-5\right)\sqrt{bx+2}}{35\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(2*b^3*x^3 - 2*b^2*x^2 + 3*b*x - 5)*sqrt(b*x + 2)/x^(7/2)
```

Sympy [B] time = 104.571, size = 374, normalized size = 4.68

$$\frac{2b^{\frac{31}{2}}x^6\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{10b^{\frac{29}{2}}x^5\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{15b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4 \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*b**(31/2)*x**6*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b*
*10*x**4 + 280*b**9*x**3) + 10*b**(29/2)*x**5*sqrt(1 + 2/(b*x))/(35*b**12*x
**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b**9*x**3) + 15*b**(27/2)*x**4*
sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b*
*9*x**3) + 5*b**(25/2)*x**3*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x*
*5 + 420*b**10*x**4 + 280*b**9*x**3) - 10*b**(23/2)*x**2*sqrt(1 + 2/(b*x))/
(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b**9*x**3) - 36*b**(
21/2)*x*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4
+ 280*b**9*x**3) - 40*b**(19/2)*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**1
1*x**5 + 420*b**10*x**4 + 280*b**9*x**3)
```

Giac [A] time = 1.10613, size = 92, normalized size = 1.15

$$-\frac{\left(35\,b^7-\left(35\,b^7+2\left((bx+2)b^7-7\,b^7\right)(bx+2)\right)(bx+2)\right)\sqrt{bx+2}b}{35\left((bx+2)b-2\,b\right)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/35*(35*b^7 - (35*b^7 + 2*((b*x + 2)*b^7 - 7*b^7)*(b*x + 2))*(b*x + 2))*s
qrt(b*x + 2)*b/(((b*x + 2)*b - 2*b)^(7/2)*abs(b))
```

3.616 $\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{5x^{3/2}\sqrt{bx+2}}{2b^2}-\frac{15\sqrt{x}\sqrt{bx+2}}{2b^3}+\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{2x^{5/2}}{b\sqrt{bx+2}}$$

```
[Out] (-2*x^(5/2))/(b*Sqrt[2 + b*x]) - (15*Sqrt[x]*Sqrt[2 + b*x])/(2*b^3) + (5*x^
(3/2)*Sqrt[2 + b*x])/(2*b^2) + (15*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7
/2)
```

Rubi [A] time = 0.0204427, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$\frac{5x^{3/2}\sqrt{bx+2}}{2b^2}-\frac{15\sqrt{x}\sqrt{bx+2}}{2b^3}+\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{2x^{5/2}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*x^(5/2))/(b*Sqrt[2 + b*x]) - (15*Sqrt[x]*Sqrt[2 + b*x])/(2*b^3) + (5*x^
(3/2)*Sqrt[2 + b*x])/(2*b^2) + (15*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7
/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx = -\frac{2x^{5/2}}{b\sqrt{2+bx}}+\frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{b}$$

$$= -\frac{2x^{5/2}}{b\sqrt{2+bx}}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}-\frac{15\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b^2}$$

$$= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b^3}$$

$$= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}$$

$$= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

Mathematica [C] time = 0.0058571, size = 30, normalized size = 0.35

$$\frac{x^{7/2}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};-\frac{bx}{2}\right)}{7\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (x^(7/2)*Hypergeometric2F1[3/2, 7/2, 9/2, -(b*x)/2])/(7*Sqrt[2])
```

Maple [A] time = 0.02, size = 106, normalized size = 1.2

$$\frac{bx-7}{2\,b^3}\sqrt{x}\sqrt{bx+2}+\left(\frac{15}{2}\ln\left((bx+1)\frac{1}{\sqrt{b}}+\sqrt{bx^2+2\,x}\right)b^{-\frac{7}{2}}-8\,\frac{1}{b^4}\sqrt{b\left(x+2\,b^{-1}\right)^2-2\,x-4\,b^{-1}}\left(x+2\,b^{-1}\right)^{-1}\right)\sqrt{x\,(bx+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(3/2),x)
```

```
[Out] 1/2*(b*x-7)*x^(1/2)*(b*x+2)^(1/2)/b^3+(15/2/b^(7/2)*ln((b*x+1)/b^(1/2)+(b*x
^2+2*x)^(1/2))-8/b^4/(x+2/b)*(b*(x+2/b)^2-2*x-4/b)^(1/2))*(x*(b*x+2))^(1/2)
/x^(1/2)/(b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96873, size = 383, normalized size = 4.45

$$\left[\frac{15\,(bx+2)\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+\left(b^3x^2-5\,b^2x-30\,b\right)\sqrt{bx+2}\sqrt{x}}{2\left(b^5x+2\,b^4\right)},-\frac{30\,(bx+2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)}{2\left(b^5x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*(b*x + 2)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (
b^3*x^2 - 5*b^2*x - 30*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x + 2*b^4), -1/2*(30*
(b*x + 2)*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) - (b^3*x^2 -
5*b^2*x - 30*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x + 2*b^4)]
```

Sympy [A] time = 13.4935, size = 80, normalized size = 0.93

$$\frac{x^{\frac{5}{2}}}{2b\sqrt{bx+2}}-\frac{5x^{\frac{3}{2}}}{2b^2\sqrt{bx+2}}-\frac{15\sqrt{x}}{b^3\sqrt{bx+2}}+\frac{15\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(3/2),x)
```

```
[Out] x**(5/2)/(2*b*sqrt(b*x + 2)) - 5*x**(3/2)/(2*b**2*sqrt(b*x + 2)) - 15*sqrt(
x)/(b**3*sqrt(b*x + 2)) + 15*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(7/2)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.617 $\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{3\sqrt{x}\sqrt{bx+2}}{b^2}-\frac{6\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{b\sqrt{bx+2}}$$

```
[Out] (-2*x^(3/2))/(b*Sqrt[2 + b*x]) + (3*Sqrt[x]*Sqrt[2 + b*x])/b^2 - (6*ArcSinh
[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rubi [A] time = 0.0138479, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$\frac{3\sqrt{x}\sqrt{bx+2}}{b^2}-\frac{6\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*x^(3/2))/(b*Sqrt[2 + b*x]) + (3*Sqrt[x]*Sqrt[2 + b*x])/b^2 - (6*ArcSinh
[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx = -\frac{2x^{3/2}}{b\sqrt{2+bx}} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{b}$$
$$= -\frac{2x^{3/2}}{b\sqrt{2+bx}} + \frac{3\sqrt{x}\sqrt{2+bx}}{b^2} - \frac{3\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b^2}$$
$$= -\frac{2x^{3/2}}{b\sqrt{2+bx}} + \frac{3\sqrt{x}\sqrt{2+bx}}{b^2} - \frac{6\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{2x^{3/2}}{b\sqrt{2+bx}} + \frac{3\sqrt{x}\sqrt{2+bx}}{b^2} - \frac{6\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [C] time = 0.005861, size = 30, normalized size = 0.48

$$\frac{x^{5/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-\frac{bx}{2}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (x^(5/2)*Hypergeometric2F1[3/2, 5/2, 7/2, -(b*x)/2])/(5*Sqrt[2])
```

Maple [B] time = 0.019, size = 100, normalized size = 1.6

$$\frac{1}{b^2}\sqrt{x}\sqrt{bx+2}+\left(-3\frac{1}{b^{5/2}}\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{bx^2+2x}\right)+4\frac{1}{b^3}\sqrt{b\left(x+2b^{-1}\right)^2-2x-4b^{-1}}\left(x+2b^{-1}\right)^{-1}\right)\sqrt{x(bx+2)}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(3/2),x)
```

```
[Out] x^(1/2)*(b*x+2)^(1/2)/b^2+(-3/b^(5/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
+4/b^3/(x+2/b)*(b*(x+2/b)^2-2*x-4/b)^(1/2))*(x*(b*x+2))^(1/2)/x^(1/2)/(b*x+
2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74284, size = 333, normalized size = 5.29

$$\left[\frac{3\,(bx+2)\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+\left(b^2x+6\,b\right)\sqrt{bx+2}\sqrt{x}}{b^4x+2\,b^3},\ \frac{6\,(bx+2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)+\left(b^2x+6\,b\right)\sqrt{b}}{b^4x+2\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*(b*x + 2)*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (b^2*x
 + 6*b)*sqrt(b*x + 2)*sqrt(x))/(b^4*x + 2*b^3), (6*(b*x + 2)*sqrt(-b)*arcta
n(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 6*b)*sqrt(b*x + 2)*sqrt(x)
)/(b^4*x + 2*b^3)]
```

Sympy [A] time = 3.78501, size = 58, normalized size = 0.92

$$\frac{x^{\frac{3}{2}}}{b\sqrt{bx+2}}+\frac{6\sqrt{x}}{b^2\sqrt{bx+2}}-\frac{6\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(3/2),x)
```

```
[Out] x**(3/2)/(b*sqrt(b*x + 2)) + 6*sqrt(x)/(b**2*sqrt(b*x + 2)) - 6*asinh(sqrt(
2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.618 $\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=44

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

```
[Out] (-2*Sqrt[x])/(b*Sqrt[2 + b*x]) + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(
3/2)
```

Rubi [A] time = 0.0093868, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {47, 54, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[2 + b*x]) + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(
3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx = -\frac{2\sqrt{x}}{b\sqrt{2+bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{2+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{2+bx}} + \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.0289237, size = 44, normalized size = 1.

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}} - \frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[2 + b*x]) + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(
3/2)
```

Maple [A] time = 0.034, size = 48, normalized size = 1.1

$$2\,\frac{1}{b^{3/2}\sqrt{\pi}}\left(-1/2\,\frac{\sqrt{\pi}\sqrt{x}\sqrt{2}\sqrt{b}}{\sqrt{1/2\,bx+1}} + \sqrt{\pi}\text{Arcsinh}\left(1/2\,\sqrt{b}\sqrt{x}\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(3/2),x)
```

```
[Out] 2/b^(3/2)/Pi^(1/2)*(-1/2*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)/(1/2*b*x+1)^(1/2)
+Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86869, size = 302, normalized size = 6.86

$$\left[\frac{(bx+2)\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right) - 2\sqrt{bx+2}b\sqrt{x}}{b^3x+2\,b^2}, -\frac{2\left((bx+2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right) + \sqrt{bx+2}b\sqrt{x}\right)}{b^3x+2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + 2)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 2*sqrt(b
*x + 2)*b*sqrt(x))/(b^3*x + 2*b^2), -2*((b*x + 2)*sqrt(-b)*arctan(sqrt(b*x
+ 2)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + 2)*b*sqrt(x))/(b^3*x + 2*b^2)]
```

Sympy [A] time = 1.75628, size = 41, normalized size = 0.93

$$-\frac{2\sqrt{x}}{b\sqrt{bx+2}}+\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(3/2),x)
```

```
[Out] -2*sqrt(x)/(b*sqrt(b*x + 2)) + 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
```

Giac [B] time = 24.9303, size = 111, normalized size = 2.52

$$-\frac{\left(\frac{\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}\right)}{\sqrt{b}}+\frac{8\,\sqrt{b}}{\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}+2\,b}\right)|b|}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -(log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) + 8*sqrt
(b)/((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b))*abs(b)/b^2
```

3.619 $\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

```
[Out] Sqrt[x]/Sqrt[2 + b*x]
```

Rubi [A] time = 0.0013378, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 + b*x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx = \frac{\sqrt{x}}{\sqrt{2+bx}}$$

Mathematica [A] time = 0.0032298, size = 15, normalized size = 1.

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 + b*x]
```

Maple [A] time = 0.003, size = 12, normalized size = 0.8

$$\sqrt{x}\frac{1}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] x^(1/2)/(b*x+2)^(1/2)
```

Maxima [A] time = 1.08694, size = 15, normalized size = 1.

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/sqrt(b*x + 2)
```

Fricas [A] time = 1.48699, size = 31, normalized size = 2.07

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x)/sqrt(b*x + 2)
```

Sympy [A] time = 1.11237, size = 15, normalized size = 1.

$$\frac{1}{\sqrt{b}\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] 1/(sqrt(b)*sqrt(1 + 2/(b*x)))
```

Giac [B] time = 1.07288, size = 59, normalized size = 3.93

$$\frac{4\,b^{\frac{3}{2}}}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}+2\,b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 4*b^(3/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(
b))
```

3.620 $\int \frac{1}{x^{3/2}(2+bx)^{3/2}} dx$

Optimal. Leaf size=32

$$\frac{1}{\sqrt{x}\sqrt{bx+2}} - \frac{\sqrt{bx+2}}{\sqrt{x}}$$

```
[Out] 1/(Sqrt[x]*Sqrt[2 + b*x]) - Sqrt[2 + b*x]/Sqrt[x]
```

Rubi [A] time = 0.0030264, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{1}{\sqrt{x}\sqrt{bx+2}} - \frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 + b*x)^(3/2)),x]

[Out] 1/(Sqrt[x]*Sqrt[2 + b*x]) - Sqrt[2 + b*x]/Sqrt[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2+bx)^{3/2}} dx &= \frac{1}{\sqrt{x}\sqrt{2+bx}} + \int \frac{1}{x^{3/2}\sqrt{2+bx}} dx \\ &= \frac{1}{\sqrt{x}\sqrt{2+bx}} - \frac{\sqrt{2+bx}}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0061833, size = 21, normalized size = 0.66

$$\frac{-bx-1}{\sqrt{x}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] (-1 - b*x)/(Sqrt[x]*Sqrt[2 + b*x])
```

Maple [A] time = 0.002, size = 18, normalized size = 0.6

$$-(bx+1)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(3/2),x)
```

```
[Out] -(b*x+1)/x^(1/2)/(b*x+2)^(1/2)
```

Maxima [A] time = 1.04309, size = 35, normalized size = 1.09

$$-\frac{b\sqrt{x}}{2\sqrt{bx+2}}-\frac{\sqrt{bx+2}}{2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*sqrt(x)/sqrt(b*x + 2) - 1/2*sqrt(b*x + 2)/sqrt(x)
```

Fricas [A] time = 1.56168, size = 65, normalized size = 2.03

$$-\frac{\sqrt{bx+2}(bx+1)\sqrt{x}}{bx^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x + 2)*(b*x + 1)*sqrt(x)/(b*x^2 + 2*x)
```

Sympy [A] time = 2.52736, size = 34, normalized size = 1.06

$$-\frac{\sqrt{b}}{\sqrt{1+\frac{2}{bx}}}-\frac{1}{\sqrt{b}x\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(3/2),x)
```

```
[Out] -sqrt(b)/sqrt(1 + 2/(b*x)) - 1/(sqrt(b)*x*sqrt(1 + 2/(b*x)))
```

Giac [B] time = 1.08926, size = 100, normalized size = 3.12

$$-\frac{\sqrt{bx+2}b^2}{2\sqrt{(bx+2)b-2b}|b|}-\frac{2b^{\frac{5}{2}}}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b)) - 2*b^(5/2)/(((sqrt
(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(b))
```

3.621 $\int \frac{1}{x^{5/2}(2+bx)^{3/2}} dx$

Optimal. Leaf size=53

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

```
[Out] 1/(x^(3/2)*Sqrt[2 + b*x]) - (2*Sqrt[2 + b*x])/(3*x^(3/2)) + (2*b*Sqrt[2 + b
*x])/(3*Sqrt[x])
```

Rubi [A] time = 0.0073547, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(x^(3/2)*Sqrt[2 + b*x]) - (2*Sqrt[2 + b*x])/(3*x^(3/2)) + (2*b*Sqrt[2 + b
*x])/(3*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx &= \frac{1}{x^{3/2}\sqrt{2+bx}}+2\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}-\frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}+\frac{2b\sqrt{2+bx}}{3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.006948, size = 32, normalized size = 0.6

$$\frac{2b^2x^2+2bx-1}{3x^{3/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] (-1 + 2*b*x + 2*b^2*x^2)/(3*x^(3/2)*Sqrt[2 + b*x])
```

Maple [A] time = 0.003, size = 27, normalized size = 0.5

$$\frac{2\,b^2x^2+2\,bx-1}{3}x^{-\frac{3}{2}}\frac{1}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(3/2),x)
```

```
[Out] 1/3*(2*b^2*x^2+2*b*x-1)/x^(3/2)/(b*x+2)^(1/2)
```

Maxima [A] time = 1.07154, size = 55, normalized size = 1.04

$$\frac{b^2\sqrt{x}}{4\sqrt{bx+2}}+\frac{\sqrt{bx+2}b}{2\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}}{12\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*sqrt(x)/sqrt(b*x + 2) + 1/2*sqrt(b*x + 2)*b/sqrt(x) - 1/12*(b*x + 2
)^(3/2)/x^(3/2)
```

Fricas [A] time = 1.48534, size = 90, normalized size = 1.7

$$\frac{\left(2\,b^2x^2+2\,bx-1\right)\sqrt{bx+2}\sqrt{x}}{3\left(bx^3+2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^2*x^2 + 2*b*x - 1)*sqrt(b*x + 2)*sqrt(x)/(b*x^3 + 2*x^2)
```

Sympy [B] time = 13.9313, size = 170, normalized size = 3.21

$$\frac{2b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}+\frac{6b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}+\frac{3b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}-\frac{2b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(3/2),x)
```

```
[Out] 2*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4*x)
 + 6*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4
*x) + 3*b**(11/2)*x*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4
*x) - 2*b**(9/2)*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4*x)
```

Giac [B] time = 1.08034, size = 116, normalized size = 2.19

$$\frac{b^{\frac{7}{2}}}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}+2\,b\right)|b|}+\frac{\left(5\,(bx+2)b^{2}|b|-12\,b^{2}|b|\right)\sqrt{bx+2}}{12\,((bx+2)b-2\,b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] b^(7/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(b)
) + 1/12*(5*(b*x + 2)*b^2*abs(b) - 12*b^2*abs(b))*sqrt(b*x + 2)/((b*x + 2)*
b - 2*b)^(3/2)
```

3.622 $\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2b^2\sqrt{bx+2}}{5\sqrt{x}}+\frac{2b\sqrt{bx+2}}{5x^{3/2}}-\frac{3\sqrt{bx+2}}{5x^{5/2}}+\frac{1}{x^{5/2}\sqrt{bx+2}}$$

```
[Out] 1/(x^(5/2)*Sqrt[2 + b*x]) - (3*Sqrt[2 + b*x])/(5*x^(5/2)) + (2*b*Sqrt[2 + b
*x])/(5*x^(3/2)) - (2*b^2*Sqrt[2 + b*x])/(5*Sqrt[x])
```

Rubi [A] time = 0.013534, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2\sqrt{bx+2}}{5\sqrt{x}}+\frac{2b\sqrt{bx+2}}{5x^{3/2}}-\frac{3\sqrt{bx+2}}{5x^{5/2}}+\frac{1}{x^{5/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(x^(5/2)*Sqrt[2 + b*x]) - (3*Sqrt[2 + b*x])/(5*x^(5/2)) + (2*b*Sqrt[2 + b
*x])/(5*x^(3/2)) - (2*b^2*Sqrt[2 + b*x])/(5*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx &= \frac{1}{x^{5/2}\sqrt{2+bx}}+3\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{x^{5/2}\sqrt{2+bx}}-\frac{3\sqrt{2+bx}}{5x^{5/2}}-\frac{1}{5}(6b)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{x^{5/2}\sqrt{2+bx}}-\frac{3\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{5x^{3/2}}+\frac{1}{5}\left(2b^2\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{x^{5/2}\sqrt{2+bx}}-\frac{3\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{5x^{3/2}}-\frac{2b^2\sqrt{2+bx}}{5\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.009784, size = 39, normalized size = 0.53

$$\frac{-2b^3x^3-2b^2x^2+bx-1}{5x^{5/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] (-1 + b*x - 2*b^2*x^2 - 2*b^3*x^3)/(5*x^(5/2)*Sqrt[2 + b*x])
```

Maple [A] time = 0.004, size = 35, normalized size = 0.5

$$-\frac{2\,b^3x^3+2\,b^2x^2-bx+1}{5}x^{-\frac{5}{2}}\frac{1}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+2)^(3/2),x)
```

```
[Out] -1/5*(2*b^3*x^3+2*b^2*x^2-b*x+1)/x^(5/2)/(b*x+2)^(1/2)
```

Maxima [A] time = 1.02739, size = 76, normalized size = 1.03

$$-\frac{b^3\sqrt{x}}{8\sqrt{bx+2}}-\frac{3\sqrt{bx+2}b^2}{8\sqrt{x}}+\frac{(bx+2)^{\frac{3}{2}}b}{8\,x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{40\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*b^3*sqrt(x)/sqrt(b*x + 2) - 3/8*sqrt(b*x + 2)*b^2/sqrt(x) + 1/8*(b*x +
 2)^(3/2)*b/x^(3/2) - 1/40*(b*x + 2)^(5/2)/x^(5/2)
```

Fricas [A] time = 1.58721, size = 105, normalized size = 1.42

$$-\frac{\left(2\,b^3x^3+2\,b^2x^2-bx+1\right)\sqrt{bx+2}\sqrt{x}}{5\left(bx^4+2\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*b^3*x^3 + 2*b^2*x^2 - b*x + 1)*sqrt(b*x + 2)*sqrt(x)/(b*x^4 + 2*x^3
)
```

Sympy [B] time = 55.9985, size = 269, normalized size = 3.64

$$-\frac{2b^{\frac{29}{2}}x^5\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{10b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{15b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+2)**(3/2),x)
```

```
[Out] -2*b**(29/2)*x**5*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**1
0*x**3 + 40*b**9*x**2) - 10*b**(27/2)*x**4*sqrt(1 + 2/(b*x))/(5*b**12*x**5
+ 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2) - 15*b**(25/2)*x**3*sqrt(1
+ 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2) -
5*b**(23/2)*x**2*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10
*x**3 + 40*b**9*x**2) - 4*b**(19/2)*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b*
*11*x**4 + 60*b**10*x**3 + 40*b**9*x**2)
```

Giac [B] time = 1.09133, size = 144, normalized size = 1.95

$$-\frac{b^{\frac{9}{2}}}{2\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)|b|}-\frac{\left(\frac{60\,b^6}{|b|}+\left(\frac{11\,(bx+2)b^6}{|b|}-\frac{50\,b^6}{|b|}\right)(bx+2)\right)\sqrt{bx+2}}{40\,((bx+2)b-2\,b)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*b^(9/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*a
bs(b)) - 1/40*(60*b^6/abs(b) + (11*(b*x + 2)*b^6/abs(b) - 50*b^6/abs(b))*(b
*x + 2))*sqrt(b*x + 2)/((b*x + 2)*b - 2*b)^(5/2)
```

3.623 $\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{10x^{3/2}}{3b^2\sqrt{bx+2}}+\frac{5\sqrt{x}\sqrt{bx+2}}{b^3}-\frac{10\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{2x^{5/2}}{3b(bx+2)^{3/2}}$$

```
[Out] (-2*x^(5/2))/(3*b*(2 + b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[2 + b*x]) + (
5*Sqrt[x]*Sqrt[2 + b*x])/b^3 - (10*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7
/2)
```

Rubi [A] time = 0.0201415, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$-\frac{10x^{3/2}}{3b^2\sqrt{bx+2}}+\frac{5\sqrt{x}\sqrt{bx+2}}{b^3}-\frac{10\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{2x^{5/2}}{3b(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-2*x^(5/2))/(3*b*(2 + b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[2 + b*x]) + (
5*Sqrt[x]*Sqrt[2 + b*x])/b^3 - (10*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7
/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx = -\frac{2x^{5/2}}{3b(2+bx)^{3/2}} + \frac{5\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx}{3b}$$
$$= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2+bx}} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{b^2}$$
$$= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2+bx}} + \frac{5\sqrt{x}\sqrt{2+bx}}{b^3} - \frac{5\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b^3}$$
$$= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2+bx}} + \frac{5\sqrt{x}\sqrt{2+bx}}{b^3} - \frac{10\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^3}$$
$$= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2+bx}} + \frac{5\sqrt{x}\sqrt{2+bx}}{b^3} - \frac{10\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

Mathematica [C] time = 0.006098, size = 30, normalized size = 0.35

$$\frac{x^{7/2}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};-\frac{bx}{2}\right)}{14\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (x^(7/2)*Hypergeometric2F1[5/2, 7/2, 9/2, -(b*x)/2])/(14*Sqrt[2])
```

Maple [B] time = 0.028, size = 136, normalized size = 1.6

$$\frac{1}{b^3}\sqrt{x}\sqrt{bx+2} + \left(-5\frac{1}{b^{7/2}}\ln\left(\frac{bx+1}{\sqrt{b}} + \sqrt{bx^2+2x}\right) - \frac{8}{3b^5}\sqrt{b\left(x+2b^{-1}\right)^2 - 2x - 4b^{-1}}\left(x+2b^{-1}\right)^{-2} + \frac{28}{3b^4}\sqrt{b\left(x+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(5/2),x)
```

```
[Out] x^(1/2)*(b*x+2)^(1/2)/b^3+(-5/b^(7/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
-8/3/b^5/(x+2/b)^2*(b*(x+2/b)^2-2*x-4/b)^(1/2)+28/3/b^4/(x+2/b)*(b*(x+2/b)^
2-2*x-4/b)^(1/2))*(x*(b*x+2))^(1/2)/x^(1/2)/(b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55062, size = 455, normalized size = 5.29

$$\left[\frac{15\left(b^{2}x^{2}+4\,bx+4\right)\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+\left(3\,b^{3}x^{2}+40\,b^{2}x+60\,b\right)\sqrt{bx+2}\sqrt{x}}{3\left(b^{6}x^{2}+4\,b^{5}x+4\,b^{4}\right)},\frac{30\left(b^{2}x^{2}+4\,bx+4\right)\sqrt{-b}\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*(b^2*x^2 + 4*b*x + 4)*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt
(x) + 1) + (3*b^3*x^2 + 40*b^2*x + 60*b)*sqrt(b*x + 2)*sqrt(x))/(b^6*x^2 +
4*b^5*x + 4*b^4), 1/3*(30*(b^2*x^2 + 4*b*x + 4)*sqrt(-b)*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 + 40*b^2*x + 60*b)*sqrt(b*x + 2)*sqrt
(x))/(b^6*x^2 + 4*b^5*x + 4*b^4)]
```

Sympy [B] time = 12.9109, size = 308, normalized size = 3.58

$$\frac{3b^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}+\frac{40b^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}+\frac{60b^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}-\frac{30b^{1}}{3b^{\frac{27}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(5/2),x)
```

```
[Out] 3*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25
/2)*sqrt(b*x + 2)) + 40*b**(21/2)*x**14/(3*b**(27/2)*x**(27/2)*sqrt(b*x + 2
) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) + 60*b**(19/2)*x**13/(3*b**(27/2)*
x**(27/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) - 30*b**10*x
**(27/2)*sqrt(b*x + 2)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27
/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) - 60*b**9*x**(25/2
)*sqrt(b*x + 2)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27/2)*sqr
t(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.624 $\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2\sqrt{x}}{b^2\sqrt{bx+2}}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{3b(bx+2)^{3/2}}$$

```
[Out] (-2*x^(3/2))/(3*b*(2 + b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[2 + b*x]) + (2*A
rcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rubi [A] time = 0.0144139, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {47, 54, 215}

$$-\frac{2\sqrt{x}}{b^2\sqrt{bx+2}}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{3b(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*b*(2 + b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[2 + b*x]) + (2*A
rcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx = -\frac{2x^{3/2}}{3b(2+bx)^{3/2}} + \frac{\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx}{b}$$
$$= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2+bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b^2}$$
$$= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2+bx}} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2+bx}} + \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.0793311, size = 52, normalized size = 0.8

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}} - \frac{4\sqrt{x}(2bx+3)}{3b^2(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-4*Sqrt[x]*(3 + 2*b*x))/(3*b^2*(2 + b*x)^(3/2)) + (2*ArcSinh[(Sqrt[b]*Sqrt
[x])/Sqrt[2]])/b^(5/2)
```

Maple [A] time = 0.029, size = 55, normalized size = 0.9

$$\frac{4}{3\sqrt{\pi}}\left(-\frac{\sqrt{\pi}\sqrt{2}\,(10\,bx+15)}{20}\sqrt{b}\sqrt{x}\left(\frac{bx}{2}+1\right)^{-\frac{3}{2}} + \frac{3\sqrt{\pi}}{2}\mathrm{Arcsinh}\left(\frac{\sqrt{2}}{2}\sqrt{b}\sqrt{x}\right)\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(5/2),x)
```

```
[Out] 4/3/b^(5/2)/Pi^(1/2)*(-1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(10*b*x+15)/(1
/2*b*x+1)^(3/2)+3/2*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65766, size = 421, normalized size = 6.48

$$\left[\frac{3\left(b^2x^2+4bx+4\right)\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)-4\left(2b^2x+3b\right)\sqrt{bx+2}\sqrt{x}}{3\left(b^5x^2+4b^4x+4b^3\right)},-\frac{2\left(3\left(b^2x^2+4bx+4\right)\sqrt{-b}\arctan\right.}{3\left(b^5x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 4*b*x + 4)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(
x) + 1) - 4*(2*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x^2 + 4*b^4*x + 4*b
^3), -2/3*(3*(b^2*x^2 + 4*b*x + 4)*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(
b*sqrt(x))) + 2*(2*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x^2 + 4*b^4*x +
 4*b^3)]
```

Sympy [B] time = 5.86438, size = 257, normalized size = 3.95

$$-\frac{8b^{\frac{11}{2}}x^8}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}-\frac{12b^{\frac{9}{2}}x^7}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}+\frac{6b^5x^{\frac{15}{2}}\sqrt{bx+2}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(5/2),x)
```

```
[Out] -8*b**(11/2)*x**8/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(13/2)*x**(13
/2)*sqrt(b*x + 2)) - 12*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2)
+ 6*b**(13/2)*x**(13/2)*sqrt(b*x + 2)) + 6*b**5*x**(15/2)*sqrt(b*x + 2)*asi
nh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(
13/2)*x**(13/2)*sqrt(b*x + 2)) + 12*b**4*x**(13/2)*sqrt(b*x + 2)*asinh(sqrt
(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(13/2)*x
**(13/2)*sqrt(b*x + 2))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.625 $\int \frac{\sqrt{x}}{(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=18

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

```
[Out] x^(3/2)/(3*(2 + b*x)^(3/2))
```

Rubi [A] time = 0.0012821, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 + b*x)^(5/2),x]
```

```
[Out] x^(3/2)/(3*(2 + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2+bx)^{5/2}}\,dx = \frac{x^{3/2}}{3(2+bx)^{3/2}}$$

Mathematica [A] time = 0.0042477, size = 18, normalized size = 1.

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 + b*x)^(5/2),x]
```

```
[Out] x^(3/2)/(3*(2 + b*x)^(3/2))
```

Maple [A] time = 0.004, size = 13, normalized size = 0.7

$$\frac{1}{3}x^{\frac{3}{2}}\left(bx+2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(5/2),x)
```

```
[Out] 1/3*x^(3/2)/(b*x+2)^(3/2)
```

Maxima [A] time = 1.01698, size = 16, normalized size = 0.89

$$\frac{x^{\frac{3}{2}}}{3\,(bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^(3/2)/(b*x + 2)^(3/2)
```

Fricas [B] time = 1.52731, size = 66, normalized size = 3.67

$$\frac{\sqrt{bx+2}x^{\frac{3}{2}}}{3\left(b^2x^2+4\,bx+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x + 2)*x^(3/2)/(b^2*x^2 + 4*b*x + 4)
```

Sympy [A] time = 2.27685, size = 27, normalized size = 1.5

$$\frac{x^{\frac{3}{2}}}{3bx\sqrt{bx+2}+6\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(5/2),x)
```

```
[Out] x**(3/2)/(3*b*x*sqrt(b*x + 2) + 6*sqrt(b*x + 2))
```

Giac [B] time = 1.10089, size = 111, normalized size = 6.17

$$\frac{4\left(3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^4\sqrt{b}+4\,b^{\frac{5}{2}}\right)|b|}{3\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*sqrt(b) + 4*b^(5
/2))*abs(b)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*
b^2)
```

3.626 $\int \frac{1}{\sqrt{x}(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{x}}{3\sqrt{bx+2}}+\frac{\sqrt{x}}{3(bx+2)^{3/2}}$$

```
[Out] Sqrt[x]/(3*(2 + b*x)^(3/2)) + Sqrt[x]/(3*Sqrt[2 + b*x])
```

Rubi [A] time = 0.0034164, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{\sqrt{x}}{3\sqrt{bx+2}}+\frac{\sqrt{x}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
```

```
[Out] Sqrt[x]/(3*(2 + b*x)^(3/2)) + Sqrt[x]/(3*Sqrt[2 + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(2+bx)^{5/2}}\,dx &= \frac{\sqrt{x}}{3(2+bx)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx\\ &= \frac{\sqrt{x}}{3(2+bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2+bx}}\end{aligned}$$

Mathematica [A] time = 0.0061133, size = 23, normalized size = 0.62

$$\frac{\sqrt{x}(bx+3)}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
```

```
[Out] (Sqrt[x]*(3 + b*x))/(3*(2 + b*x)^(3/2))
```

Maple [A] time = 0.003, size = 18, normalized size = 0.5

$$\frac{bx+3}{3}\sqrt{x}\,(bx+2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*x^(1/2)*(b*x+3)/(b*x+2)^(3/2)
```

Maxima [A] time = 1.00239, size = 32, normalized size = 0.86

$$-\frac{\left(b-\frac{3\,(bx+2)}{x}\right)x^{\frac{3}{2}}}{6\,(bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(b - 3*(b*x + 2)/x)*x^(3/2)/(b*x + 2)^(3/2)
```

Fricas [A] time = 1.4751, size = 80, normalized size = 2.16

$$\frac{(bx+3)\sqrt{bx+2}\sqrt{x}}{3\left(b^2x^2+4\,bx+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x + 3)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^2 + 4*b*x + 4)
```

Sympy [B] time = 3.98838, size = 75, normalized size = 2.03

$$\frac{bx}{3b^{\frac{3}{2}}x\sqrt{1+\frac{2}{bx}}+6\sqrt{b}\sqrt{1+\frac{2}{bx}}}+\frac{3}{3b^{\frac{3}{2}}x\sqrt{1+\frac{2}{bx}}+6\sqrt{b}\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] b*x/(3*b**(3/2)*x*sqrt(1 + 2/(b*x)) + 6*sqrt(b)*sqrt(1 + 2/(b*x))) + 3/(3*b
**(3/2)*x*sqrt(1 + 2/(b*x)) + 6*sqrt(b)*sqrt(1 + 2/(b*x)))
```

Giac [B] time = 1.09351, size = 107, normalized size = 2.89

$$\frac{8\left(3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2b\right)b^{\frac{5}{2}}}{3\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*b^(5/2)/(
((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs(b))
```

3.627 $\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=55

$$-\frac{2\sqrt{bx+2}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\sqrt{bx+2}}+\frac{1}{3\sqrt{x}(bx+2)^{3/2}}$$

```
[Out] 1/(3*Sqrt[x]*(2 + b*x)^(3/2)) + 2/(3*Sqrt[x]*Sqrt[2 + b*x]) - (2*Sqrt[2 + b
*x])/(3*Sqrt[x])
```

Rubi [A] time = 0.0059313, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2\sqrt{bx+2}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\sqrt{bx+2}}+\frac{1}{3\sqrt{x}(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] 1/(3*Sqrt[x]*(2 + b*x)^(3/2)) + 2/(3*Sqrt[x]*Sqrt[2 + b*x]) - (2*Sqrt[2 + b
*x])/(3*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx &= \frac{1}{3\sqrt{x}(2+bx)^{3/2}}+\frac{2}{3}\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx\\ &= \frac{1}{3\sqrt{x}(2+bx)^{3/2}}+\frac{2}{3\sqrt{x}\sqrt{2+bx}}+\frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= \frac{1}{3\sqrt{x}(2+bx)^{3/2}}+\frac{2}{3\sqrt{x}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0095435, size = 32, normalized size = 0.58

$$\frac{-2b^2x^2-6bx-3}{3\sqrt{x}(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] (-3 - 6*b*x - 2*b^2*x^2)/(3*Sqrt[x]*(2 + b*x)^(3/2))
```

Maple [A] time = 0.003, size = 27, normalized size = 0.5

$$-\frac{2\,b^2x^2+6\,bx+3}{3}(bx+2)^{-\frac{3}{2}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(5/2),x)
```

```
[Out] -1/3*(2*b^2*x^2+6*b*x+3)/x^(1/2)/(b*x+2)^(3/2)
```

Maxima [A] time = 0.974572, size = 54, normalized size = 0.98

$$\frac{\left(b^2-\frac{6\,(bx+2)b}{x}\right)x^{\frac{3}{2}}}{12\,(bx+2)^{\frac{3}{2}}}-\frac{\sqrt{bx+2}}{4\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2 - 6*(b*x + 2)*b/x)*x^(3/2)/(b*x + 2)^(3/2) - 1/4*sqrt(b*x + 2)/sq
rt(x)
```

Fricas [A] time = 1.62944, size = 105, normalized size = 1.91

$$-\frac{\left(2\,b^2x^2+6\,bx+3\right)\sqrt{bx+2}\sqrt{x}}{3\left(b^2x^3+4\,bx^2+4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^2 + 6*b*x + 3)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^3 + 4*b*x^2 + 4*x
)
```

Sympy [B] time = 14.1047, size = 117, normalized size = 2.13

$$-\frac{2b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}-\frac{6b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}-\frac{3b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(5/2),x)
```

```
[Out] -2*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4) - 6
*b**(11/2)*x*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4) - 3*b**(
9/2)*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4)
```

Giac [B] time = 1.13735, size = 196, normalized size = 3.56

$$-\frac{\sqrt{bx+2}b^2}{4\sqrt{(bx+2)b-2b}|b|}-\frac{3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^4b^{\frac{5}{2}}+24\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2b^{\frac{7}{2}}+20b^{\frac{9}{2}}}{3\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b)) - 1/3*(3*(sqrt(b*x
+ 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*b^(5/2) + 24*(sqrt(b*x + 2)*sqrt(
b) - sqrt((b*x + 2)*b - 2*b))^2*b^(7/2) + 20*b^(9/2))/(((sqrt(b*x + 2)*sqrt
(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs(b))
```

3.628 $\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{1}{3x^{3/2}(bx+2)^{3/2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

```
[Out] 1/(3*x^(3/2)*(2 + b*x)^(3/2)) + 1/(x^(3/2)*Sqrt[2 + b*x]) - (2*Sqrt[2 + b*x
])/(3*x^(3/2)) + (2*b*Sqrt[2 + b*x])/(3*Sqrt[x])
```

Rubi [A] time = 0.0089598, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{1}{3x^{3/2}(bx+2)^{3/2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] 1/(3*x^(3/2)*(2 + b*x)^(3/2)) + 1/(x^(3/2)*Sqrt[2 + b*x]) - (2*Sqrt[2 + b*x
])/(3*x^(3/2)) + (2*b*Sqrt[2 + b*x])/(3*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx &= \frac{1}{3x^{3/2}(2+bx)^{3/2}}+\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx\\
&= \frac{1}{3x^{3/2}(2+bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2+bx}}+2\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{3x^{3/2}(2+bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}-\frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{3x^{3/2}(2+bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}+\frac{2b\sqrt{2+bx}}{3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0109285, size = 40, normalized size = 0.56

$$\frac{2b^3x^3+6b^2x^2+3bx-1}{3x^{3/2}(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] (-1 + 3*b*x + 6*b^2*x^2 + 2*b^3*x^3)/(3*x^(3/2)*(2 + b*x)^(3/2))
```

Maple [A] time = 0.003, size = 35, normalized size = 0.5

$$\frac{2\,b^3x^3+6\,b^2x^2+3\,bx-1}{3}x^{-\frac{3}{2}}\,(bx+2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(5/2),x)
```

```
[Out] 1/3*(2*b^3*x^3+6*b^2*x^2+3*b*x-1)/x^(3/2)/(b*x+2)^(3/2)
```

Maxima [A] time = 0.977377, size = 74, normalized size = 1.04

$$\frac{3\sqrt{bx+2}b}{8\sqrt{x}}-\frac{\left(b^3-\frac{9\,(bx+2)b^2}{x}\right)x^{\frac{3}{2}}}{24\,(bx+2)^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{3}{2}}}{24\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(b*x + 2)*b/sqrt(x) - 1/24*(b^3 - 9*(b*x + 2)*b^2/x)*x^(3/2)/(b*x +
 2)^(3/2) - 1/24*(b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A] time = 1.51536, size = 123, normalized size = 1.73

$$\frac{\left(2\,b^3x^3+6\,b^2x^2+3\,bx-1\right)\sqrt{bx+2}\sqrt{x}}{3\left(b^2x^4+4\,bx^3+4\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^3*x^3 + 6*b^2*x^2 + 3*b*x - 1)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^4 + 4*
b*x^3 + 4*x^2)
```

Sympy [B] time = 31.4723, size = 257, normalized size = 3.62

$$\frac{2b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}+\frac{10b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}+\frac{15b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(5/2),x)
```

```
[Out] 2*b**(27/2)*x**4*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10
*x**2 + 24*b**9*x) + 10*b**(25/2)*x**3*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18
*b**11*x**3 + 36*b**10*x**2 + 24*b**9*x) + 15*b**(23/2)*x**2*sqrt(1 + 2/(b*
x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10*x**2 + 24*b**9*x) + 5*b**(21/2
)*x*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10*x**2 + 24*b*
*9*x) - 2*b**(19/2)*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b*
*10*x**2 + 24*b**9*x)
```

Giac [B] time = 1.13935, size = 213, normalized size = 3.

$$\frac{\left(4\,(bx+2)b^2|b|-9\,b^2|b|\right)\sqrt{bx+2}}{12\,((bx+2)b-2\,b)^{\frac{3}{2}}}+\frac{3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^4b^{\frac{7}{2}}+18\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2b^{\frac{9}{2}}+}{3\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*(4*(b*x + 2)*b^2*abs(b) - 9*b^2*abs(b))*sqrt(b*x + 2)/((b*x + 2)*b - 2
*b)^(3/2) + 1/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*b^(7
/2) + 18*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2*b^(9/2) + 16*b
^(11/2))/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs
(b))
```

3.629 $\int \frac{x^{5/2}}{\sqrt{2-bx}} \, dx$

Optimal. Leaf size=91

$$-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{x^{5/2}\sqrt{2-bx}}{3b}$$

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(2*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(6*b^2) - (x
^(5/2)*Sqrt[2 - b*x])/(3*b) + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Rubi [A] time = 0.0205853, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{x^{5/2}\sqrt{2-bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[2 - b*x],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(2*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(6*b^2) - (x
^(5/2)*Sqrt[2 - b*x])/(3*b) + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx = -\frac{x^{5/2}\sqrt{2-bx}}{3b} + \frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{3b}$$
$$= -\frac{5x^{3/2}\sqrt{2-bx}}{6b^2} - \frac{x^{5/2}\sqrt{2-bx}}{3b} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b^2}$$
$$= -\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{6b^2} - \frac{x^{5/2}\sqrt{2-bx}}{3b} + \frac{5\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b^3}$$
$$= -\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{6b^2} - \frac{x^{5/2}\sqrt{2-bx}}{3b} + \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^3}$$
$$= -\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{6b^2} - \frac{x^{5/2}\sqrt{2-bx}}{3b} + \frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

Mathematica [A] time = 0.0411747, size = 61, normalized size = 0.67

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}} - \frac{\sqrt{x}\sqrt{2-bx}\left(2b^2x^2+5bx+15\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[2 - b*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x]*(15 + 5*b*x + 2*b^2*x^2))/(6*b^3) + (5*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Maple [A] time = 0.005, size = 100, normalized size = 1.1

$$-\frac{1}{3b}x^{\frac{5}{2}}\sqrt{-bx+2} - \frac{5}{6b^2}x^{\frac{3}{2}}\sqrt{-bx+2} - \frac{5}{2b^3}\sqrt{x}\sqrt{-bx+2} + \frac{5}{2}\sqrt{(-bx+2)x}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2x}}\right)b^{-\frac{7}{2}}\frac{}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -1/3*x^(5/2)*(-b*x+2)^(1/2)/b-5/6*x^(3/2)*(-b*x+2)^(1/2)/b^2-5/2*x^(1/2)*(-
b*x+2)^(1/2)/b^3+5/2/b^(7/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arct
an(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58267, size = 335, normalized size = 3.68

$$\left[-\frac{\left(2\,b^{3}x^{2}+5\,b^{2}x+15\,b\right)\sqrt{-bx+2}\sqrt{x}+15\,\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{6\,b^{4}},-\frac{\left(2\,b^{3}x^{2}+5\,b^{2}x+15\,b\right)\sqrt{-bx+2}\sqrt{x}}{6\,b^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x + 2)*sqrt(x) + 15*sqrt(-b)*lo
g(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^4, -1/6*((2*b^3*x^2 + 5*b^
2*x + 15*b)*sqrt(-b*x + 2)*sqrt(x) + 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt
(b)*sqrt(x))))/b^4]
```

Sympy [A] time = 12.717, size = 206, normalized size = 2.26

$$\begin{cases} -\frac{ix^{\frac{7}{2}}}{3\sqrt{bx-2}}-\frac{ix^{\frac{5}{2}}}{6b\sqrt{bx-2}}-\frac{5ix^{\frac{3}{2}}}{6b^{2}\sqrt{bx-2}}+\frac{5i\sqrt{x}}{b^{3}\sqrt{bx-2}}-\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{for}\: \frac{|bx|}{2}>1 \\ \frac{x^{\frac{7}{2}}}{3\sqrt{-bx+2}}+\frac{x^{\frac{5}{2}}}{6b\sqrt{-bx+2}}+\frac{5x^{\frac{3}{2}}}{6b^{2}\sqrt{-bx+2}}-\frac{5\sqrt{x}}{b^{3}\sqrt{-bx+2}}+\frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(7/2)/(3*sqrt(b*x - 2)) - I*x**(5/2)/(6*b*sqrt(b*x - 2)) -
 5*I*x**(3/2)/(6*b**2*sqrt(b*x - 2)) + 5*I*sqrt(x)/(b**3*sqrt(b*x - 2)) - 5
*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(7/2), Abs(b*x)/2 > 1), (x**(7/2)/(3
*sqrt(-b*x + 2)) + x**(5/2)/(6*b*sqrt(-b*x + 2)) + 5*x**(3/2)/(6*b**2*sqrt(
-b*x + 2)) - 5*sqrt(x)/(b**3*sqrt(-b*x + 2)) + 5*asin(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(7/2), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.630 $\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx$

Optimal. Leaf size=69

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{x^{3/2}\sqrt{2-bx}}{2b}$$

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(2*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(2*b) + (3*Arc
Sin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rubi [A] time = 0.01492, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{x^{3/2}\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[2 - b*x],x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(2*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(2*b) + (3*Arc
Sin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx = -\frac{x^{3/2}\sqrt{2-bx}}{2b} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{2b} + \frac{3\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{2b} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{2b} + \frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.0295956, size = 52, normalized size = 0.75

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}} - \frac{\sqrt{x}\sqrt{2-bx}(bx+3)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[2 - b*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x]*(3 + b*x))/(2*b^2) + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sq
rt[2]])/b^(5/2)
```

Maple [A] time = 0.005, size = 84, normalized size = 1.2

$$-\frac{1}{2\,b}x^{\frac{3}{2}}\sqrt{-bx+2} - \frac{3}{2\,b^2}\sqrt{x}\sqrt{-bx+2} + \frac{3}{2}\sqrt{(-bx+2)\,x}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2\,x}}\right)b^{-\frac{5}{2}}\frac{1}{\sqrt{-bx+2}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -1/2*x^(3/2)*(-b*x+2)^(1/2)/b-3/2*x^(1/2)*(-b*x+2)^(1/2)/b^2+3/2/b^(5/2)*((
-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56412, size = 292, normalized size = 4.23

$$\left[-\frac{\left(b^2x+3\,b\right)\sqrt{-bx+2}\sqrt{x}+3\,\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{2\,b^3},-\frac{\left(b^2x+3\,b\right)\sqrt{-bx+2}\sqrt{x}+6\,\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{2\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(-b*x + sqrt(-b
*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, -1/2*((b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt
(x) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

Sympy [A] time = 4.37159, size = 163, normalized size = 2.36

$$\begin{cases} -\frac{ix^{\frac{5}{2}}}{2\sqrt{bx-2}}-\frac{ix^{\frac{3}{2}}}{2b\sqrt{bx-2}}+\frac{3i\sqrt{x}}{b^2\sqrt{bx-2}}-\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ \frac{x^{\frac{5}{2}}}{2\sqrt{-bx+2}}+\frac{x^{\frac{3}{2}}}{2b\sqrt{-bx+2}}-\frac{3\sqrt{x}}{b^2\sqrt{-bx+2}}+\frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(5/2)/(2*sqrt(b*x - 2)) - I*x**(3/2)/(2*b*sqrt(b*x - 2)) +
 3*I*sqrt(x)/(b**2*sqrt(b*x - 2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b*
*(5/2), Abs(b*x)/2 > 1), (x**(5/2)/(2*sqrt(-b*x + 2)) + x**(3/2)/(2*b*sqrt(
-b*x + 2)) - 3*sqrt(x)/(b**2*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(5/2), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.631 $\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx$

Optimal. Leaf size=45

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{2-bx}}{b}$$

```
[Out] -((Sqrt[x]*Sqrt[2 - b*x])/b) + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2
)
```

Rubi [A] time = 0.009505, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{2-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[2 - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[2 - b*x])/b) + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx = -\frac{\sqrt{x}\sqrt{2-bx}}{b} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{b} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{b} + \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.0155848, size = 45, normalized size = 1.

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}} - \frac{\sqrt{x}\sqrt{2-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[2 - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[2 - b*x])/b) + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2
)
```

Maple [A] time = 0.004, size = 67, normalized size = 1.5

$$-\frac{1}{b}\sqrt{x}\sqrt{-bx+2} + \sqrt{(-bx+2)\,x}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2\,x}}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{-bx+2}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -x^(1/2)*(-b*x+2)^(1/2)/b+1/b^(3/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/
2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5645, size = 246, normalized size = 5.47

$$\left[-\frac{\sqrt{-bx+2}b\sqrt{x} + \sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{b^2}, -\frac{\sqrt{-bx+2}b\sqrt{x} + 2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-b*x + 2)*b*sqrt(x) + sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*s
qrt(x) + 1))/b^2, -(sqrt(-b*x + 2)*b*sqrt(x) + 2*sqrt(b)*arctan(sqrt(-b*x +
 2)/(sqrt(b)*sqrt(x))))/b^2]
```

Sympy [A] time = 2.2682, size = 121, normalized size = 2.69

$$\begin{cases} -\frac{ix^{\frac{3}{2}}}{\sqrt{bx-2}} + \frac{2i\sqrt{x}}{b\sqrt{bx-2}} - \frac{2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for}\ \frac{|bx|}{2} > 1 \\ \frac{x^{\frac{3}{2}}}{\sqrt{-bx+2}} - \frac{2\sqrt{x}}{b\sqrt{-bx+2}} + \frac{2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(3/2)/sqrt(b*x - 2) + 2*I*sqrt(x)/(b*sqrt(b*x - 2)) - 2*I*
acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2), Abs(b*x)/2 > 1), (x**(3/2)/sqrt(
-b*x + 2) - 2*sqrt(x)/(b*sqrt(-b*x + 2)) + 2*asin(sqrt(2)*sqrt(b)*sqrt(x)/2
)/b**(3/2), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.632 $\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=24

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rubi [A] time = 0.0067891, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {54, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[2 - b*x]),x]
```

```
[Out] (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0044428, size = 24, normalized size = 1.

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[2 - b*x]),x]
```

```
[Out] (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Maple [B] time = 0.003, size = 50, normalized size = 2.1

$$\sqrt{(-bx+2)\,x}\,\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2\,x}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+2}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] ((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-
b*x^2+2*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48754, size = 159, normalized size = 6.62

$$\left[-\frac{\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{b},\,-\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1)/b, -2*arctan(sqr
t(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

Sympy [A] time = 1.22131, size = 58, normalized size = 2.42

$$\begin{cases}-\frac{2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx|}{2}>1\\ \frac{2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x)/2 > 1),
(2*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.633 $\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=17

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

```
[Out] -(Sqrt[2 - b*x]/Sqrt[x])
```

Rubi [A] time = 0.001366, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[2 - b*x]),x]

[Out] -(Sqrt[2 - b*x]/Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx = -\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Mathematica [A] time = 0.0038564, size = 17, normalized size = 1.

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[2 - b*x]),x]

[Out] -(Sqrt[2 - b*x]/Sqrt[x])
```

Maple [A] time = 0.003, size = 14, normalized size = 0.8

$$-\sqrt{-bx+2}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -(-b*x+2)^(1/2)/x^(1/2)
```

Maxima [A] time = 1.00026, size = 18, normalized size = 1.06

$$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-b*x + 2)/sqrt(x)
```

Fricas [A] time = 1.54185, size = 34, normalized size = 2.

$$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x + 2)/sqrt(x)
```

Sympy [A] time = 1.17606, size = 39, normalized size = 2.29

$$\begin{cases} -\sqrt{b}\sqrt{-1+\frac{2}{bx}} & \text{for}\ \frac{2}{|bx|}>1 \\ -i\sqrt{b}\sqrt{1-\frac{2}{bx}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(b)*sqrt(-1 + 2/(b*x)), 2/Abs(b*x) > 1), (-I*sqrt(b)*sqrt(1
 - 2/(b*x)), True))
```

Giac [B] time = 1.06825, size = 41, normalized size = 2.41

$$-\frac{\sqrt{-bx+2}b^2}{\sqrt{(bx-2)b+2b}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-b*x + 2)*b^2/(sqrt((b*x - 2)*b + 2*b)*abs(b))
```

3.634 $\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}$$

```
[Out] -Sqrt[2 - b*x]/(3*x^(3/2)) - (b*Sqrt[2 - b*x])/(3*Sqrt[x])
```

Rubi [A] time = 0.0039226, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -Sqrt[2 - b*x]/(3*x^(3/2)) - (b*Sqrt[2 - b*x])/(3*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx &= -\frac{\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\ &= -\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0112824, size = 24, normalized size = 0.6

$$-\frac{\sqrt{2-bx}(bx+1)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -(Sqrt[2 - b*x]*(1 + b*x))/(3*x^(3/2))
```

Maple [A] time = 0.002, size = 19, normalized size = 0.5

$$-\frac{bx+1}{3}\sqrt{-bx+2}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -1/3*(b*x+1)/x^(3/2)*(-b*x+2)^(1/2)
```

Maxima [A] time = 1.01113, size = 38, normalized size = 0.95

$$-\frac{\sqrt{-bx+2}b}{2\sqrt{x}}-\frac{(-bx+2)^{\frac{3}{2}}}{6x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-b*x + 2)*b/sqrt(x) - 1/6*(-b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A] time = 1.60481, size = 53, normalized size = 1.32

$$-\frac{(bx+1)\sqrt{-bx+2}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(b*x + 1)*sqrt(-b*x + 2)/x^(3/2)
```

Sympy [A] time = 4.10904, size = 139, normalized size = 3.48

$$\begin{cases} -\frac{b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3}-\frac{\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for } \frac{2}{|bx|}>1 \\ -\frac{ib^{\frac{7}{2}}x^2\sqrt{1-\frac{2}{bx}}}{3b^2x^2-6bx}+\frac{ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}}{3b^2x^2-6bx}+\frac{2ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3b^2x^2-6bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-b**(3/2)*sqrt(-1 + 2/(b*x))/3 - sqrt(b)*sqrt(-1 + 2/(b*x))/(3*x
), 2/Abs(b*x) > 1), (-I*b**(7/2)*x**2*sqrt(1 - 2/(b*x))/(3*b**2*x**2 - 6*b*
x) + I*b**(5/2)*x*sqrt(1 - 2/(b*x))/(3*b**2*x**2 - 6*b*x) + 2*I*b**(3/2)*sq
rt(1 - 2/(b*x))/(3*b**2*x**2 - 6*b*x), True))
```

Giac [A] time = 1.06432, size = 58, normalized size = 1.45

$$-\frac{\left((bx-2)b^3+3\,b^3\right)\sqrt{-bx+2}b}{3\,((bx-2)b+2\,b)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*((b*x - 2)*b^3 + 3*b^3)*sqrt(-b*x + 2)*b/(((b*x - 2)*b + 2*b)^(3/2)*ab
s(b))
```

3.635 $\int \frac{x^{5/2}}{(2-bx)^{3/2}} dx$

Optimal. Leaf size=89

$$\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}+\frac{15\sqrt{x}\sqrt{2-bx}}{2b^3}-\frac{15\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{2x^{5/2}}{b\sqrt{2-bx}}$$

```
[Out] (2*x^(5/2))/(b*Sqrt[2 - b*x]) + (15*Sqrt[x]*Sqrt[2 - b*x])/(2*b^3) + (5*x^(
3/2)*Sqrt[2 - b*x])/(2*b^2) - (15*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2
)
```

Rubi [A] time = 0.0207852, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {47, 50, 54, 216}

$$\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}+\frac{15\sqrt{x}\sqrt{2-bx}}{2b^3}-\frac{15\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{2x^{5/2}}{b\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2))/(b*Sqrt[2 - b*x]) + (15*Sqrt[x]*Sqrt[2 - b*x])/(2*b^3) + (5*x^(
3/2)*Sqrt[2 - b*x])/(2*b^2) - (15*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2
)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(2-bx)^{3/2}}\,dx &= \frac{2x^{5/2}}{b\sqrt{2-bx}}-\frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{b}\\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}}+\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}-\frac{15\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b^2}\\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}}+\frac{15\sqrt{x}\sqrt{2-bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}-\frac{15\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}}+\frac{15\sqrt{x}\sqrt{2-bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}-\frac{15\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}}+\frac{15\sqrt{x}\sqrt{2-bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}-\frac{15\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0061238, size = 30, normalized size = 0.34

$$\frac{x^{7/2}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{bx}{2}\right)}{7\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (x^(7/2)*Hypergeometric2F1[3/2, 7/2, 9/2, (b*x)/2])/(7*Sqrt[2])
```

Maple [B] time = 0.026, size = 138, normalized size = 1.6

$$-\frac{(bx+7)(bx-2)}{2b^3}\sqrt{x}\sqrt{(-bx+2)x}\frac{1}{\sqrt{-x(bx-2)}}\frac{1}{\sqrt{-bx+2}}-\left(\frac{15}{2}\arctan\left(\sqrt{b}\left(x-b^{-1}\right)\frac{1}{\sqrt{-bx^2+2x}}\right)b^{-\frac{7}{2}}+8\frac{1}{b^4}\sqrt{-b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(3/2),x)
```

```
[Out] -1/2*(b*x+7)*x^(1/2)*(b*x-2)/b^3/(-x*(b*x-2))^(1/2)*((-b*x+2)*x)^(1/2)/(-b*
x+2)^(1/2)-(15/2/b^(7/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))+8/b^4/(
x-2/b)*(-b*(x-2/b)^2-2*x+4/b)^(1/2))*((-b*x+2)*x)^(1/2)/x^(1/2)/(-b*x+2)^(1
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.53743, size = 387, normalized size = 4.35

$$\left[-\frac{15\,(bx-2)\sqrt{-b}\log\left(-bx-\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)-\left(b^3x^2+5\,b^2x-30\,b\right)\sqrt{-bx+2}\sqrt{x}}{2\left(b^5x-2\,b^4\right)},\ \frac{30\,(bx-2)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{2\left(b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(15*(b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1
) - (b^3*x^2 + 5*b^2*x - 30*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x - 2*b^4), 1/2
*(30*(b*x - 2)*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^3*x^2
+ 5*b^2*x - 30*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x - 2*b^4)]
```

Sympy [A] time = 12.8081, size = 173, normalized size = 1.94

$$\begin{cases} \frac{ix^{\frac{5}{2}}}{2b\sqrt{bx-2}}+\frac{5ix^{\frac{3}{2}}}{2b^2\sqrt{bx-2}}-\frac{15i\sqrt{x}}{b^3\sqrt{bx-2}}+\frac{15i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{for } \frac{|bx|}{2}>1 \\ -\frac{x^{\frac{5}{2}}}{2b\sqrt{-bx+2}}-\frac{5x^{\frac{3}{2}}}{2b^2\sqrt{-bx+2}}+\frac{15\sqrt{x}}{b^3\sqrt{-bx+2}}-\frac{15\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((I*x**(5/2)/(2*b*sqrt(b*x - 2)) + 5*I*x**(3/2)/(2*b**2*sqrt(b*x -
 2)) - 15*I*sqrt(x)/(b**3*sqrt(b*x - 2)) + 15*I*acosh(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(7/2), Abs(b*x)/2 > 1), (-x**(5/2)/(2*b*sqrt(-b*x + 2)) - 5*x**(3/
2)/(2*b**2*sqrt(-b*x + 2)) + 15*sqrt(x)/(b**3*sqrt(-b*x + 2)) - 15*asin(sqr
t(2)*sqrt(b)*sqrt(x)/2)/b**(7/2), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.636 $\int \frac{x^{3/2}}{(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{3\sqrt{x}\sqrt{2-bx}}{b^2}-\frac{6\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{b\sqrt{2-bx}}$$

```
[Out] (2*x^(3/2))/(b*Sqrt[2 - b*x]) + (3*Sqrt[x]*Sqrt[2 - b*x])/b^2 - (6*ArcSin[(
Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rubi [A] time = 0.0144168, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {47, 50, 54, 216}

$$\frac{3\sqrt{x}\sqrt{2-bx}}{b^2}-\frac{6\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{b\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (2*x^(3/2))/(b*Sqrt[2 - b*x]) + (3*Sqrt[x]*Sqrt[2 - b*x])/b^2 - (6*ArcSin[(
Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^{3/2}}{(2-bx)^{3/2}}\,dx = \frac{2x^{3/2}}{b\sqrt{2-bx}} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{b}$$
$$= \frac{2x^{3/2}}{b\sqrt{2-bx}} + \frac{3\sqrt{x}\sqrt{2-bx}}{b^2} - \frac{3\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b^2}$$
$$= \frac{2x^{3/2}}{b\sqrt{2-bx}} + \frac{3\sqrt{x}\sqrt{2-bx}}{b^2} - \frac{6\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= \frac{2x^{3/2}}{b\sqrt{2-bx}} + \frac{3\sqrt{x}\sqrt{2-bx}}{b^2} - \frac{6\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [C] time = 0.0049503, size = 30, normalized size = 0.46

$$\frac{x^{5/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{bx}{2}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (x^(5/2)*Hypergeometric2F1[3/2, 5/2, 7/2, (b*x)/2])/(5*Sqrt[2])
```

Maple [B] time = 0.023, size = 133, normalized size = 2.1

$$-\frac{bx-2}{b^2}\sqrt{x}\sqrt{(-bx+2)\,x}\frac{1}{\sqrt{-x\,(bx-2)}}\frac{1}{\sqrt{-bx+2}} - \left(3\,\frac{1}{b^{5/2}}\arctan\left(\frac{\sqrt{b}}{\sqrt{-bx^2+2\,x}}\left(x-b^{-1}\right)\right) + 4\,\frac{1}{b^3}\sqrt{-b\left(x-2\,b^{-1}\right)^2-2\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(3/2),x)
```

```
[Out] -1/b^2*x^(1/2)*(b*x-2)/(-x*(b*x-2))^(1/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)
-(3/b^(5/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))+4/b^3/(x-2/b)*(-b*(x
-2/b)^2-2*x+4/b)^(1/2))*((-b*x+2)*x)^(1/2)/x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60111, size = 339, normalized size = 5.22

$$\left[-\frac{3\,(bx-2)\sqrt{-b}\log\left(-bx-\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)-\left(b^2x-6\,b\right)\sqrt{-bx+2}\sqrt{x}}{b^4x-2\,b^3},\ \frac{6\,(bx-2)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)+\left(b^2x-\right.}{b^4x-2\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(3*(b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - (
b^2*x - 6*b)*sqrt(-b*x + 2)*sqrt(x))/(b^4*x - 2*b^3), (6*(b*x - 2)*sqrt(b)*
arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^2*x - 6*b)*sqrt(-b*x + 2)*sqr
t(x))/(b^4*x - 2*b^3)]
```

Sympy [A] time = 3.58133, size = 128, normalized size = 1.97

$$\begin{cases} \frac{ix^{\frac{3}{2}}}{b\sqrt{bx-2}} - \frac{6i\sqrt{x}}{b^2\sqrt{bx-2}} + \frac{6i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for}\ \frac{|bx|}{2} > 1 \\ -\frac{x^{\frac{3}{2}}}{b\sqrt{-bx+2}} + \frac{6\sqrt{x}}{b^2\sqrt{-bx+2}} - \frac{6\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((I*x**(3/2)/(b*sqrt(b*x - 2)) - 6*I*sqrt(x)/(b**2*sqrt(b*x - 2))
+ 6*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2), Abs(b*x)/2 > 1), (-x**(3/2
)/(b*sqrt(-b*x + 2)) + 6*sqrt(x)/(b**2*sqrt(-b*x + 2)) - 6*asin(sqrt(2)*sqr
t(b)*sqrt(x)/2)/b**(5/2), True))
```

Giac [B] time = 17.237, size = 162, normalized size = 2.49

$$\frac{\left(\frac{3\sqrt{-b}\log\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\right)}{b}+\frac{\sqrt{(bx-2)b+2\,b}\sqrt{-bx+2}}{b}-\frac{16\sqrt{-b}}{\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b}\right)|b|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] (3*sqrt(-b)*log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/b +
sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)/b - 16*sqrt(-b)/((sqrt(-b*x + 2)*sqr
t(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b))*abs(b)/b^3
```

3.637 $\int \frac{\sqrt{x}}{(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}}-\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] (2*Sqrt[x])/(b*Sqrt[2 - b*x]) - (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/
2)
```

Rubi [A] time = 0.0094423, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {47, 54, 216}

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}}-\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[2 - b*x]) - (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/
2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.0391526, size = 45, normalized size = 1.

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[2 - b*x]) - (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/
2)
```

Maple [A] time = 0.028, size = 67, normalized size = 1.5

$$-2\,\frac{1}{\sqrt{-b}\sqrt{\pi}b}\left(1/2\,\frac{\sqrt{\pi}\sqrt{x}\sqrt{2}\,(-b)^{3/2}}{b\sqrt{-1/2\,bx+1}} - \frac{\sqrt{\pi}\,(-b)^{3/2}\arcsin\left(1/2\,\sqrt{b}\sqrt{x}\sqrt{2}\right)}{b^{3/2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(3/2),x)
```

```
[Out] -2/(-b)^(1/2)/Pi^(1/2)/b*(1/2*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(3/2)/b/(-1/2*b
*x+1)^(1/2)-Pi^(1/2)*(-b)^(3/2)/b^(3/2)*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56244, size = 306, normalized size = 6.8

$$\left[-\frac{(bx-2)\sqrt{-b}\log\left(-bx-\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)+2\sqrt{-bx+2}b\sqrt{x}}{b^3x-2b^2}, \frac{2\left((bx-2)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-\sqrt{-bx+2}b\sqrt{x}\right)}{b^3x-2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + 2*s
qrt(-b*x + 2)*b*sqrt(x))/(b^3*x - 2*b^2), 2*((b*x - 2)*sqrt(b)*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) - sqrt(-b*x + 2)*b*sqrt(x))/(b^3*x - 2*b^2)]
```

Sympy [A] time = 1.83719, size = 92, normalized size = 2.04

$$\begin{cases} -\frac{2i\sqrt{x}}{b\sqrt{bx-2}} + \frac{2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for}\: \frac{|bx|}{2} > 1 \\ \frac{2\sqrt{x}}{b\sqrt{-bx+2}} - \frac{2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-2*I*sqrt(x)/(b*sqrt(b*x - 2)) + 2*I*acosh(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(3/2), Abs(b*x)/2 > 1), (2*sqrt(x)/(b*sqrt(-b*x + 2)) - 2*asin(sqr
t(2)*sqrt(b)*sqrt(x)/2)/b**(3/2), True))
```

Giac [B] time = 17.9262, size = 124, normalized size = 2.76

$$-\frac{\left(\frac{\log\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2\right)}{\sqrt{-b}} + \frac{8\sqrt{-b}}{\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2-2b}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -(log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/sqrt(-b) + 8*s
qrt(-b)/((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b))*abs(
b)/b^2
```

3.638 $\int \frac{1}{\sqrt{x}(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

```
[Out] Sqrt[x]/Sqrt[2 - b*x]
```

Rubi [A] time = 0.0013493, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 - b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 - b*x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(2-bx)^{3/2}}\,dx = \frac{\sqrt{x}}{\sqrt{2-bx}}$$

Mathematica [A] time = 0.0037654, size = 16, normalized size = 1.

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 - b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 - b*x]
```

Maple [A] time = 0.003, size = 13, normalized size = 0.8

$$\sqrt{x}\frac{1}{\sqrt{-bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [A] time = 1.01769, size = 16, normalized size = 1.

$$\frac{\sqrt{x}}{\sqrt{-bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/sqrt(-b*x + 2)
```

Fricas [A] time = 1.5107, size = 47, normalized size = 2.94

$$-\frac{\sqrt{-bx+2}\sqrt{x}}{bx-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x + 2)*sqrt(x)/(b*x - 2)
```

Sympy [A] time = 1.12369, size = 39, normalized size = 2.44

$$\begin{cases} \frac{1}{\sqrt{b}\sqrt{-1+\frac{2}{bx}}} & \text{for}\: \frac{2}{|bx|} > 1 \\ -\frac{i}{\sqrt{b}\sqrt{1-\frac{2}{bx}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((1/(sqrt(b)*sqrt(-1 + 2/(b*x))), 2/Abs(b*x) > 1), (-I/(sqrt(b)*sq
rt(1 - 2/(b*x))), True))
```

Giac [B] time = 1.08857, size = 68, normalized size = 4.25

$$-\frac{4\sqrt{-b}b}{\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2-2b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-b)*b/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b
)*abs(b))
```

3.639 $\int \frac{1}{x^{3/2}(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

```
[Out] 1/(Sqrt[x]*Sqrt[2 - b*x]) - Sqrt[2 - b*x]/Sqrt[x]
```

Rubi [A] time = 0.0028815, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$\frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] 1/(Sqrt[x]*Sqrt[2 - b*x]) - Sqrt[2 - b*x]/Sqrt[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2-bx)^{3/2}}\,dx &= \frac{1}{\sqrt{x}\sqrt{2-bx}}+\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\ &= \frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0068941, size = 21, normalized size = 0.62

$$\frac{bx-1}{\sqrt{x}\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] (-1 + b*x)/(Sqrt[x]*Sqrt[2 - b*x])
```

Maple [A] time = 0.003, size = 18, normalized size = 0.5

$$(bx-1)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(3/2),x)
```

```
[Out] (b*x-1)/x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [A] time = 1.03326, size = 38, normalized size = 1.12

$$\frac{b\sqrt{x}}{2\sqrt{-bx+2}}-\frac{\sqrt{-bx+2}}{2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*sqrt(x)/sqrt(-b*x + 2) - 1/2*sqrt(-b*x + 2)/sqrt(x)
```

Fricas [A] time = 1.40884, size = 66, normalized size = 1.94

$$-\frac{(bx-1)\sqrt{-bx+2}\sqrt{x}}{bx^2-2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(b*x - 1)*sqrt(-b*x + 2)*sqrt(x)/(b*x^2 - 2*x)
```

Sympy [A] time = 2.51213, size = 90, normalized size = 2.65

$$\begin{cases} \frac{\sqrt{b}}{\sqrt{-1+\frac{2}{bx}}}-\frac{1}{\sqrt{bx}\sqrt{-1+\frac{2}{bx}}} & \text{for } \frac{2}{|bx|}>1 \\ -\frac{ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}}{b^2x-2b}+\frac{ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{b^2x-2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((sqrt(b)/sqrt(-1 + 2/(b*x)) - 1/(sqrt(b)*x*sqrt(-1 + 2/(b*x))), 2
/Abs(b*x) > 1), (-I*b**(5/2)*x*sqrt(1 - 2/(b*x))/(b**2*x - 2*b) + I*b**(3/2
)*sqrt(1 - 2/(b*x))/(b**2*x - 2*b), True))
```

Giac [B] time = 1.09955, size = 112, normalized size = 3.29

$$-\frac{\sqrt{-bx+2}b^2}{2\sqrt{(bx-2)b+2b}|b|}-\frac{2\sqrt{-b}b^2}{\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2-2b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-b*x + 2)*b^2/(sqrt((b*x - 2)*b + 2*b)*abs(b)) - 2*sqrt(-b)*b^2/(
((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)*abs(b))
```

3.640 $\int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

```
[Out] 1/(x^(3/2)*Sqrt[2 - b*x]) - (2*Sqrt[2 - b*x])/(3*x^(3/2)) - (2*b*Sqrt[2 - b
*x])/(3*Sqrt[x])
```

Rubi [A] time = 0.0076754, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] 1/(x^(3/2)*Sqrt[2 - b*x]) - (2*Sqrt[2 - b*x])/(3*x^(3/2)) - (2*b*Sqrt[2 - b
*x])/(3*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx &= \frac{1}{x^{3/2}\sqrt{2-bx}}+2\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx\\
&= \frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\
&= \frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3x^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0074608, size = 33, normalized size = 0.59

$$\frac{2b^2x^2-2bx-1}{3x^{3/2}\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] (-1 - 2*b*x + 2*b^2*x^2)/(3*x^(3/2)*Sqrt[2 - b*x])
```

Maple [A] time = 0.004, size = 28, normalized size = 0.5

$$\frac{2\,b^2x^2-2\,bx-1}{3}x^{-\frac{3}{2}}\frac{1}{\sqrt{-bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(3/2),x)
```

```
[Out] 1/3*(2*b^2*x^2-2*b*x-1)/x^(3/2)/(-b*x+2)^(1/2)
```

Maxima [A] time = 1.00596, size = 59, normalized size = 1.05

$$\frac{b^2\sqrt{x}}{4\,\sqrt{-bx+2}}-\frac{\sqrt{-bx+2}b}{2\,\sqrt{x}}-\frac{(-bx+2)^{\frac{3}{2}}}{12\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*sqrt(x)/sqrt(-b*x + 2) - 1/2*sqrt(-b*x + 2)*b/sqrt(x) - 1/12*(-b*x
+ 2)^(3/2)/x^(3/2)
```

Fricas [A] time = 1.4673, size = 93, normalized size = 1.66

$$-\frac{\left(2\,b^2x^2-2\,bx-1\right)\sqrt{-bx+2}\sqrt{x}}{3\left(bx^3-2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^2 - 2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x)/(b*x^3 - 2*x^2)
```

Sympy [B] time = 13.4028, size = 354, normalized size = 6.32

$$\begin{cases} -\frac{2b^{\frac{15}{2}}x^3\sqrt{-1+\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x}+\frac{6b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x}-\frac{3b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x}-\frac{2b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x} & \text{for}\ \frac{2}{|bx|}>1 \\ -\frac{2ib^{\frac{15}{2}}x^3\sqrt{1-\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x}+\frac{6ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x}-\frac{3ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x}-\frac{2ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-2*b**(15/2)*x**3*sqrt(-1 + 2/(b*x))/(3*b**6*x**3 - 12*b**5*x**2
 + 12*b**4*x) + 6*b**(13/2)*x**2*sqrt(-1 + 2/(b*x))/(3*b**6*x**3 - 12*b**5*
x**2 + 12*b**4*x) - 3*b**(11/2)*x*sqrt(-1 + 2/(b*x))/(3*b**6*x**3 - 12*b**5
*x**2 + 12*b**4*x) - 2*b**(9/2)*sqrt(-1 + 2/(b*x))/(3*b**6*x**3 - 12*b**5*x
**2 + 12*b**4*x), 2/Abs(b*x) > 1), (-2*I*b**(15/2)*x**3*sqrt(1 - 2/(b*x))/(
3*b**6*x**3 - 12*b**5*x**2 + 12*b**4*x) + 6*I*b**(13/2)*x**2*sqrt(1 - 2/(b*
x))/(3*b**6*x**3 - 12*b**5*x**2 + 12*b**4*x) - 3*I*b**(11/2)*x*sqrt(1 - 2/(
b*x))/(3*b**6*x**3 - 12*b**5*x**2 + 12*b**4*x) - 2*I*b**(9/2)*sqrt(1 - 2/(b
*x))/(3*b**6*x**3 - 12*b**5*x**2 + 12*b**4*x), True))
```

Giac [B] time = 1.06899, size = 130, normalized size = 2.32

$$-\frac{\sqrt{-b}b^{3}}{\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^{2}-2\,b\right)|b|}-\frac{\left(5\,(bx-2)b^{2}|b|+12\,b^{2}|b|\right)\sqrt{-bx+2}}{12\,((bx-2)b+2\,b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(-b)*b^3/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b
)*abs(b)) - 1/12*(5*(b*x - 2)*b^2*abs(b) + 12*b^2*abs(b))*sqrt(-b*x + 2)/((
b*x - 2)*b + 2*b)^(3/2)
```

3.641 $\int \frac{x^{5/2}}{(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}-\frac{5\sqrt{x}\sqrt{2-bx}}{b^3}+\frac{10\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{2x^{5/2}}{3b(2-bx)^{3/2}}$$

```
[Out] (2*x^(5/2))/(3*b*(2 - b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[2 - b*x]) - (5
*Sqrt[x]*Sqrt[2 - b*x])/b^3 + (10*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2
)
```

Rubi [A] time = 0.0216252, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {47, 50, 54, 216}

$$-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}-\frac{5\sqrt{x}\sqrt{2-bx}}{b^3}+\frac{10\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{2x^{5/2}}{3b(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (2*x^(5/2))/(3*b*(2 - b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[2 - b*x]) - (5
*Sqrt[x]*Sqrt[2 - b*x])/b^3 + (10*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2
)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^{5/2}}{(2-bx)^{5/2}}\,dx = \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{5\int \frac{x^{3/2}}{(2-bx)^{3/2}}\,dx}{3b}$$
$$= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{b^2}$$
$$= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} - \frac{5\sqrt{x}\sqrt{2-bx}}{b^3} + \frac{5\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b^3}$$
$$= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} - \frac{5\sqrt{x}\sqrt{2-bx}}{b^3} + \frac{10\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^3}$$
$$= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} - \frac{5\sqrt{x}\sqrt{2-bx}}{b^3} + \frac{10\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

Mathematica [C] time = 0.0062723, size = 30, normalized size = 0.34

$$\frac{x^{7/2}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{bx}{2}\right)}{14\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (x^(7/2)*Hypergeometric2F1[5/2, 7/2, 9/2, (b*x)/2])/(14*Sqrt[2])
```

Maple [B] time = 0.029, size = 168, normalized size = 1.9

$$\frac{bx-2}{b^3}\sqrt{x}\sqrt{(-bx+2)\,x}\frac{1}{\sqrt{-x\,(bx-2)}}\frac{1}{\sqrt{-bx+2}} + \left(5\,\frac{1}{b^{7/2}}\arctan\left(\frac{\sqrt{b}}{\sqrt{-bx^2+2\,x}}\left(x-b^{-1}\right)\right) + \frac{28}{3\,b^4}\sqrt{-b\left(x-2\,b^{-1}\right)^2-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(5/2),x)
```

```
[Out] 1/b^3*x^(1/2)*(b*x-2)/(-x*(b*x-2))^(1/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)+
(5/b^(7/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))+28/3/b^4/(x-2/b)*(-b*
(x-2/b)^2-2*x+4/b)^(1/2)+8/3/b^5/(x-2/b)^2*(-b*(x-2/b)^2-2*x+4/b)^(1/2))*((
-b*x+2)*x)^(1/2)/x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68149, size = 462, normalized size = 5.19

$$\left[-\frac{15\left(b^{2}x^{2}-4\,bx+4\right)\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)+\left(3\,b^{3}x^{2}-40\,b^{2}x+60\,b\right)\sqrt{-bx+2}\sqrt{x}}{3\left(b^{6}x^{2}-4\,b^{5}x+4\,b^{4}\right)},-\frac{30\left(b^{2}x^{2}-4\,bx+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(15*(b^2*x^2 - 4*b*x + 4)*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)
*sqrt(x) + 1) + (3*b^3*x^2 - 40*b^2*x + 60*b)*sqrt(-b*x + 2)*sqrt(x))/(b^6*
x^2 - 4*b^5*x + 4*b^4), -1/3*(30*(b^2*x^2 - 4*b*x + 4)*sqrt(b)*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) + (3*b^3*x^2 - 40*b^2*x + 60*b)*sqrt(-b*x + 2)
*sqrt(x))/(b^6*x^2 - 4*b^5*x + 4*b^4)]
```

Sympy [B] time = 12.4095, size = 753, normalized size = 8.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((-3*I*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b*
*(25/2)*x**(25/2)*sqrt(b*x - 2)) + 40*I*b**(21/2)*x**14/(3*b**(27/2)*x**(27
/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) - 60*I*b**(19/2)*x
**13/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x
- 2)) - 30*I*b**10*x**(27/2)*sqrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)
/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)
) + 15*pi*b**10*x**(27/2)*sqrt(b*x - 2)/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2
) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) + 60*I*b**9*x**(25/2)*sqrt(b*x - 2
)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6
*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) - 30*pi*b**9*x**(25/2)*sqrt(b*x - 2)/(3
*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)),
Abs(b*x)/2 > 1), (3*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) -
 6*b**(25/2)*x**(25/2)*sqrt(-b*x + 2)) - 40*b**(21/2)*x**14/(3*b**(27/2)*x*
*(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*x + 2)) + 60*b**(19/
2)*x**13/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt
(-b*x + 2)) + 30*b**10*x**(27/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(b)*sqrt(x
)/2)/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*
x + 2)) - 60*b**9*x**(25/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/
(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*x + 2
)), True))
```

Giac [B] time = 18.1029, size = 270, normalized size = 3.03

$$\frac{\left(\frac{15\log\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^{2}\right)}{\sqrt{-b}b^{2}}-\frac{3\sqrt{(bx-2)b+2\,b}\sqrt{-bx+2}}{b^{3}}-\frac{16\left(9\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^{4}-24\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^{2}b+28\,b^{2}\right)}{\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^{2}-2\,b\right)^{3}\sqrt{-b}b}\right)|b|}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(15*log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/(sqrt(-b
)*b^2) - 3*sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)/b^3 - 16*(9*(sqrt(-b*x +
2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4 - 24*(sqrt(-b*x + 2)*sqrt(-b) - sq
rt((b*x - 2)*b + 2*b))^2*b + 28*b^2)/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x
 - 2)*b + 2*b))^2 - 2*b)^3*sqrt(-b)*b))*abs(b)/b^2
```

3.642 $\int \frac{x^{3/2}}{(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{3b(2-bx)^{3/2}}$$

```
[Out] (2*x^(3/2))/(3*b*(2 - b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[2 - b*x]) + (2*Ar
cSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rubi [A] time = 0.0145247, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {47, 54, 216}

$$-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{3b(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*b*(2 - b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[2 - b*x]) + (2*Ar
cSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^{3/2}}{(2-bx)^{5/2}}\,dx = \frac{2x^{3/2}}{3b(2-bx)^{3/2}} - \frac{\int \frac{\sqrt{x}}{(2-bx)^{3/2}}\,dx}{b}$$
$$= \frac{2x^{3/2}}{3b(2-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2-bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b^2}$$
$$= \frac{2x^{3/2}}{3b(2-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2-bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= \frac{2x^{3/2}}{3b(2-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2-bx}} + \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.0547212, size = 53, normalized size = 0.79

$$\frac{4\sqrt{x}(2bx-3)}{3b^2(2-bx)^{3/2}} + \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (4*Sqrt[x]*(-3 + 2*b*x))/(3*b^2*(2 - b*x)^(3/2)) + (2*ArcSin[(Sqrt[b]*Sqrt[
x])/Sqrt[2]])/b^(5/2)
```

Maple [A] time = 0.032, size = 73, normalized size = 1.1

$$-\frac{4}{3\sqrt{\pi}b}\left(-\frac{\sqrt{\pi}\sqrt{2}\,(-10\,bx+15)}{20\,b^2}\sqrt{x}\,(-b)^{\frac{5}{2}}\left(-\frac{bx}{2}+1\right)^{-\frac{3}{2}} + \frac{3\sqrt{\pi}}{2}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{2}}{2}\sqrt{b}\sqrt{x}\right)b^{-\frac{5}{2}}\right)(-b)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(5/2),x)
```

```
[Out] -4/3/(-b)^(3/2)/Pi^(1/2)/b*(-1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(5/2)*(-10*
b*x+15)/b^2/(-1/2*b*x+1)^(3/2)+3/2*Pi^(1/2)*(-b)^(5/2)/b^(5/2)*arcsin(1/2*b
^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66662, size = 427, normalized size = 6.37

$$\left[-\frac{3\left(b^2x^2-4\,bx+4\right)\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)-4\left(2\,b^2x-3\,b\right)\sqrt{-bx+2}\sqrt{x}}{3\left(b^5x^2-4\,b^4x+4\,b^3\right)},-\frac{2\left(3\left(b^2x^2-4\,bx+4\right)\sqrt{b}\,\mathrm{arc}\right.}{3\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 4*b*x + 4)*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*
sqrt(x) + 1) - 4*(2*b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x^2 - 4*b^4*x
 + 4*b^3), -2/3*(3*(b^2*x^2 - 4*b*x + 4)*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqr
t(b)*sqrt(x))) - 2*(2*b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x^2 - 4*b^4
*x + 4*b^3)]
```

Sympy [B] time = 5.82932, size = 649, normalized size = 9.69

$$\begin{cases}\frac{8ib^{\frac{11}{2}}x^8}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}-\frac{12ib^{\frac{9}{2}}x^7}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}-\frac{6ib^5x^{\frac{15}{2}}\sqrt{bx-2}\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}+\frac{3\pi b^5x^{\frac{15}{2}}\sqrt{bx-2}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}+\frac{12ib^4x\cdots}{3b\cdots} \\ -\frac{8b^{\frac{11}{2}}x^8}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}}+\frac{12b^{\frac{9}{2}}x^7}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}}+\frac{6b^5x^{\frac{15}{2}}\sqrt{-bx+2}\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}}-\frac{12b^4x^{\frac{13}{2}}\sqrt{-bx+2}\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right.}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx\cdots}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((8*I*b**(11/2)*x**8/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(
13/2)*x**(13/2)*sqrt(b*x - 2)) - 12*I*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*
sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)) - 6*I*b**5*x**(15/2)*s
qrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b
*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)) + 3*pi*b**5*x**(15/2)*sqrt(b
*x - 2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b
*x - 2)) + 12*I*b**4*x**(13/2)*sqrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/
2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x -
2)) - 6*pi*b**4*x**(13/2)*sqrt(b*x - 2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2
) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)), Abs(b*x)/2 > 1), (-8*b**(11/2)*x*
*8/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(13/2)*sqrt(-b*x
+ 2)) + 12*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2
)*x**(13/2)*sqrt(-b*x + 2)) + 6*b**5*x**(15/2)*sqrt(-b*x + 2)*asin(sqrt(2)*
sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(
13/2)*sqrt(-b*x + 2)) - 12*b**4*x**(13/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(
b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(13/2)
*sqrt(-b*x + 2)), True))
```

Giac [B] time = 18.0308, size = 244, normalized size = 3.64

$$-\frac{\left(\frac{3\sqrt{-b}\log\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\right)}{b}-\frac{16\left(3\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^4\sqrt{-b}-6\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\sqrt{-b}b+8\sqrt{-b}b^2\right)}{\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3}\right)|b|}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(-b)*log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)
/b - 16*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*sqrt(-b) -
 6*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2*sqrt(-b)*b + 8*sqr
t(-b)*b^2)/((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)^3)
*abs(b)/b^3
```

3.643 $\int \frac{\sqrt{x}}{(2-bx)^{5/2}} dx$

Optimal. Leaf size=19

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

```
[Out] x^(3/2)/(3*(2 - b*x)^(3/2))
```

Rubi [A] time = 0.001343, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 - b*x)^(5/2),x]

[Out] x^(3/2)/(3*(2 - b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2-bx)^{5/2}} dx = \frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Mathematica [A] time = 0.0044959, size = 19, normalized size = 1.

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 - b*x)^(5/2),x]

[Out] x^(3/2)/(3*(2 - b*x)^(3/2))
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$\frac{1}{3}x^{\frac{3}{2}}(-bx+2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(5/2),x)
```

```
[Out] 1/3*x^(3/2)/(-b*x+2)^(3/2)
```

Maxima [A] time = 1.07818, size = 18, normalized size = 0.95

$$\frac{x^{\frac{3}{2}}}{3\,(-bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^(3/2)/(-b*x + 2)^(3/2)
```

Fricas [B] time = 1.6524, size = 68, normalized size = 3.58

$$\frac{\sqrt{-bx+2}\,x^{\frac{3}{2}}}{3\left(b^2x^2-4\,bx+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-b*x + 2)*x^(3/2)/(b^2*x^2 - 4*b*x + 4)
```

Sympy [B] time = 2.26752, size = 65, normalized size = 3.42

$$\begin{cases} \frac{ix^{\frac{3}{2}}}{3bx\sqrt{bx-2}-6\sqrt{bx-2}} & \text{for } \frac{|bx|}{2} > 1 \\ -\frac{x^{\frac{3}{2}}}{3bx\sqrt{-bx+2}-6\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((I*x**(3/2)/(3*b*x*sqrt(b*x - 2) - 6*sqrt(b*x - 2)), Abs(b*x)/2 >
 1), (-x**(3/2)/(3*b*x*sqrt(-b*x + 2) - 6*sqrt(-b*x + 2)), True))
```

Giac [B] time = 1.0942, size = 128, normalized size = 6.74

$$\frac{4\left(3\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^4\sqrt{-b}+4\,\sqrt{-b}b^2\right)|b|}{3\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*sqrt(-b) + 4*s
qrt(-b)*b^2)*abs(b)/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2
 - 2*b)^3*b^2)
```

3.644 $\int \frac{1}{\sqrt{x}(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{x}}{3\sqrt{2-bx}}+\frac{\sqrt{x}}{3(2-bx)^{3/2}}$$

```
[Out] Sqrt[x]/(3*(2 - b*x)^(3/2)) + Sqrt[x]/(3*Sqrt[2 - b*x])
```

Rubi [A] time = 0.003746, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$\frac{\sqrt{x}}{3\sqrt{2-bx}}+\frac{\sqrt{x}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 - b*x)^(5/2)),x]
```

```
[Out] Sqrt[x]/(3*(2 - b*x)^(3/2)) + Sqrt[x]/(3*Sqrt[2 - b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(2-bx)^{5/2}}\,dx &= \frac{\sqrt{x}}{3(2-bx)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{x}(2-bx)^{3/2}}\,dx\\ &= \frac{\sqrt{x}}{3(2-bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2-bx}}\end{aligned}$$

Mathematica [A] time = 0.0074441, size = 24, normalized size = 0.62

$$-\frac{\sqrt{x}(bx-3)}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 - b*x)^(5/2)),x]

[Out] -(Sqrt[x]*(-3 + b*x))/(3*(2 - b*x)^(3/2))
```

Maple [A] time = 0.004, size = 19, normalized size = 0.5

$$-\frac{bx-3}{3}\sqrt{x}\,(-bx+2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(5/2)/x^(1/2),x)

[Out] -1/3*x^(1/2)*(b*x-3)/(-b*x+2)^(3/2)
```

Maxima [A] time = 1.05575, size = 34, normalized size = 0.87

$$\frac{\left(b-\frac{3\,(bx-2)}{x}\right)x^{\frac{3}{2}}}{6\,(-bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")

[Out] 1/6*(b - 3*(b*x - 2)/x)*x^(3/2)/(-b*x + 2)^(3/2)
```

Fricas [A] time = 1.62095, size = 82, normalized size = 2.1

$$-\frac{(bx-3)\sqrt{-bx+2}\sqrt{x}}{3\left(b^2x^2-4\,bx+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")

[Out] -1/3*(b*x - 3)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^2 - 4*b*x + 4)
```

Sympy [B] time = 4.08189, size = 163, normalized size = 4.18

$$\begin{cases} \frac{bx}{3b^{\frac{3}{2}}x\sqrt{-1+\frac{2}{bx}}-6\sqrt{b}\sqrt{-1+\frac{2}{bx}}} - \frac{3}{3b^{\frac{3}{2}}x\sqrt{-1+\frac{2}{bx}}-6\sqrt{b}\sqrt{-1+\frac{2}{bx}}} & \text{for}\ \frac{2}{|bx|} > 1 \\ -\frac{ib^2x}{3b^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}-6b^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}} + \frac{3ib}{3b^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}-6b^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((b*x/(3*b**(3/2)*x*sqrt(-1 + 2/(b*x)) - 6*sqrt(b)*sqrt(-1 + 2/(b*
x))) - 3/(3*b**(3/2)*x*sqrt(-1 + 2/(b*x)) - 6*sqrt(b)*sqrt(-1 + 2/(b*x))),
2/Abs(b*x) > 1), (-I*b**2*x/(3*b**(5/2)*x*sqrt(1 - 2/(b*x)) - 6*b**(3/2)*sq
rt(1 - 2/(b*x))) + 3*I*b/(3*b**(5/2)*x*sqrt(1 - 2/(b*x)) - 6*b**(3/2)*sqrt(
1 - 2/(b*x))), True))
```

Giac [B] time = 1.08853, size = 122, normalized size = 3.13

$$\frac{8\left(3\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)\sqrt{-b}b^2}{3\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)*sqrt(-b
)*b^2/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)^3*abs(
b))
```

3.645 $\int \frac{1}{x^{3/2}(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{2\sqrt{2-bx}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\sqrt{2-bx}}+\frac{1}{3\sqrt{x}(2-bx)^{3/2}}$$

```
[Out] 1/(3*Sqrt[x]*(2 - b*x)^(3/2)) + 2/(3*Sqrt[x]*Sqrt[2 - b*x]) - (2*Sqrt[2 - b
*x])/(3*Sqrt[x])
```

Rubi [A] time = 0.0063941, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{2\sqrt{2-bx}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\sqrt{2-bx}}+\frac{1}{3\sqrt{x}(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 - b*x)^(5/2)),x]
```


```
[Out] 1/(3*Sqrt[x]*(2 - b*x)^(3/2)) + 2/(3*Sqrt[x]*Sqrt[2 - b*x]) - (2*Sqrt[2 - b
*x])/(3*Sqrt[x])
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}(2-bx)^{5/2}}\,dx &= \frac{1}{3\sqrt{x}(2-bx)^{3/2}}+\frac{2}{3}\int \frac{1}{x^{3/2}(2-bx)^{3/2}}\,dx\\
&= \frac{1}{3\sqrt{x}(2-bx)^{3/2}}+\frac{2}{3\sqrt{x}\sqrt{2-bx}}+\frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\
&= \frac{1}{3\sqrt{x}(2-bx)^{3/2}}+\frac{2}{3\sqrt{x}\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0107665, size = 33, normalized size = 0.57

$$-\frac{2b^2x^2-6bx+3}{3\sqrt{x}(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] -(3 - 6*b*x + 2*b^2*x^2)/(3*Sqrt[x]*(2 - b*x)^(3/2))
```

Maple [A] time = 0.003, size = 28, normalized size = 0.5

$$-\frac{2\,b^2x^2-6\,bx+3}{3}\left(-bx+2\right)^{-\frac{3}{2}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(5/2),x)
```

```
[Out] -1/3*(2*b^2*x^2-6*b*x+3)/x^(1/2)/(-b*x+2)^(3/2)
```

Maxima [A] time = 1.02109, size = 57, normalized size = 0.98

$$\frac{\left(b^2-\frac{6\,(bx-2)b}{x}\right)x^{\frac{3}{2}}}{12\,(-bx+2)^{\frac{3}{2}}}-\frac{\sqrt{-bx+2}}{4\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2 - 6*(b*x - 2)*b/x)*x^(3/2)/(-b*x + 2)^(3/2) - 1/4*sqrt(-b*x + 2)/
sqrt(x)
```

Fricas [A] time = 1.58853, size = 107, normalized size = 1.84

$$-\frac{\left(2\,b^2x^2-6\,bx+3\right)\sqrt{-bx+2}\sqrt{x}}{3\left(b^2x^3-4\,bx^2+4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^2 - 6*b*x + 3)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^3 - 4*b*x^2 + 4*
x)
```

Sympy [B] time = 13.9576, size = 243, normalized size = 4.19

$$\begin{cases} -\frac{2b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} + \frac{6b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} - \frac{3b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} & \text{for } \frac{2}{|bx|} > 1 \\ -\frac{2ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} + \frac{6ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} - \frac{3ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((-2*b**(13/2)*x**2*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x +
12*b**4) + 6*b**(11/2)*x*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x + 12*b
**4) - 3*b**(9/2)*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x + 12*b**4), 2
/Abs(b*x) > 1), (-2*I*b**(13/2)*x**2*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*b*
*5*x + 12*b**4) + 6*I*b**(11/2)*x*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*b**5*
x + 12*b**4) - 3*I*b**(9/2)*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*b**5*x + 12
*b**4), True))
```

Giac [B] time = 1.11142, size = 230, normalized size = 3.97

$$-\frac{\sqrt{-bx+2}b^2}{4\sqrt{(bx-2)b+2b}|b|} - \frac{3\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^4\sqrt{-b}b^2 - 24\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2\sqrt{-b}b^3 +}{3\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2-2b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-b*x + 2)*b^2/(sqrt((b*x - 2)*b + 2*b)*abs(b)) - 1/3*(3*(sqrt(-b*
x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*sqrt(-b)*b^2 - 24*(sqrt(-b*x +
 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2*sqrt(-b)*b^3 + 20*sqrt(-b)*b^4)/(
((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)^3*abs(b))
```

3.646 $\int \frac{1}{x^{5/2}(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}+\frac{1}{3x^{3/2}(2-bx)^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

```
[Out] 1/(3*x^(3/2)*(2 - b*x)^(3/2)) + 1/(x^(3/2)*Sqrt[2 - b*x]) - (2*Sqrt[2 - b*x
])/(3*x^(3/2)) - (2*b*Sqrt[2 - b*x])/(3*Sqrt[x])
```

Rubi [A] time = 0.0095324, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}+\frac{1}{3x^{3/2}(2-bx)^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] 1/(3*x^(3/2)*(2 - b*x)^(3/2)) + 1/(x^(3/2)*Sqrt[2 - b*x]) - (2*Sqrt[2 - b*x
])/(3*x^(3/2)) - (2*b*Sqrt[2 - b*x])/(3*Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(2-bx)^{5/2}}\,dx &= \frac{1}{3x^{3/2}(2-bx)^{3/2}}+\int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx\\
&= \frac{1}{3x^{3/2}(2-bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}+2\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx\\
&= \frac{1}{3x^{3/2}(2-bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\
&= \frac{1}{3x^{3/2}(2-bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3x^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0127586, size = 41, normalized size = 0.55

$$-\frac{2b^3x^3-6b^2x^2+3bx+1}{3x^{3/2}(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] -(1 + 3*b*x - 6*b^2*x^2 + 2*b^3*x^3)/(3*x^(3/2)*(2 - b*x)^(3/2))
```

Maple [A] time = 0.003, size = 36, normalized size = 0.5

$$-\frac{2\,b^3x^3-6\,b^2x^2+3\,bx+1}{3}x^{-\frac{3}{2}}\,(-bx+2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(5/2),x)
```

```
[Out] -1/3*(2*b^3*x^3-6*b^2*x^2+3*b*x+1)/x^(3/2)/(-b*x+2)^(3/2)
```

Maxima [A] time = 0.990595, size = 78, normalized size = 1.04

$$-\frac{3\,\sqrt{-bx+2}b}{8\,\sqrt{x}}+\frac{\left(b^3-\frac{9\,(bx-2)b^2}{x}\right)x^{\frac{3}{2}}}{24\,(-bx+2)^{\frac{3}{2}}}-\frac{(-bx+2)^{\frac{3}{2}}}{24\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/8*sqrt(-b*x + 2)*b/sqrt(x) + 1/24*(b^3 - 9*(b*x - 2)*b^2/x)*x^(3/2)/(-b*
x + 2)^(3/2) - 1/24*(-b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A] time = 1.63185, size = 126, normalized size = 1.68

$$-\frac{\left(2\,b^3x^3-6\,b^2x^2+3\,bx+1\right)\sqrt{-bx+2}\sqrt{x}}{3\left(b^2x^4-4\,bx^3+4\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^3*x^3 - 6*b^2*x^2 + 3*b*x + 1)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^4 -
4*b*x^3 + 4*x^2)
```

Sympy [B] time = 30.8685, size = 529, normalized size = 7.05

$$\begin{cases} -\frac{2b^{\frac{27}{2}}x^4\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^4-18b^{11}x^3+36b^{10}x^2-24b^9x} + \frac{10b^{\frac{25}{2}}x^3\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^4-18b^{11}x^3+36b^{10}x^2-24b^9x} - \frac{15b^{\frac{23}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^4-18b^{11}x^3+36b^{10}x^2-24b^9x} + \frac{5b^{\frac{21}{2}}x\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^4-18b^{11}x^3+36b^{10}x^2-24b^9x} + \ldots \\ -\frac{2ib^{\frac{27}{2}}x^4\sqrt{1-\frac{2}{bx}}}{3b^{12}x^4-18b^{11}x^3+36b^{10}x^2-24b^9x} + \frac{10ib^{\frac{25}{2}}x^3\sqrt{1-\frac{2}{bx}}}{3b^{12}x^4-18b^{11}x^3+36b^{10}x^2-24b^9x} - \frac{15ib^{\frac{23}{2}}x^2\sqrt{1-\frac{2}{bx}}}{3b^{12}x^4-18b^{11}x^3+36b^{10}x^2-24b^9x} + \frac{5ib^{\frac{21}{2}}x\sqrt{1-\frac{2}{bx}}}{3b^{12}x^4-18b^{11}x^3+36b^{10}x^2-24b^9x} + \ldots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((-2*b**(27/2)*x**4*sqrt(-1 + 2/(b*x))/(3*b**12*x**4 - 18*b**11*x*
*3 + 36*b**10*x**2 - 24*b**9*x) + 10*b**(25/2)*x**3*sqrt(-1 + 2/(b*x))/(3*b
**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b**9*x) - 15*b**(23/2)*x**2*
sqrt(-1 + 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b**9*
x) + 5*b**(21/2)*x*sqrt(-1 + 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b*
*10*x**2 - 24*b**9*x) + 2*b**(19/2)*sqrt(-1 + 2/(b*x))/(3*b**12*x**4 - 18*b
**11*x**3 + 36*b**10*x**2 - 24*b**9*x), 2/Abs(b*x) > 1), (-2*I*b**(27/2)*x*
*4*sqrt(1 - 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b**
9*x) + 10*I*b**(25/2)*x**3*sqrt(1 - 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3
+ 36*b**10*x**2 - 24*b**9*x) - 15*I*b**(23/2)*x**2*sqrt(1 - 2/(b*x))/(3*b**
12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b**9*x) + 5*I*b**(21/2)*x*sqrt
(1 - 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b**9*x) +
2*I*b**(19/2)*sqrt(1 - 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b**10*x*
*2 - 24*b**9*x), True))
```

Giac [B] time = 1.16395, size = 247, normalized size = 3.29

$$-\frac{\left(4\,(bx-2)b^2|b|+9\,b^2|b|\right)\sqrt{-bx+2}}{12\,((bx-2)b+2\,b)^{\frac{3}{2}}} - \frac{3\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^4\sqrt{-b}b^3 - 18\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)\ldots}\right.}{3\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(4*(b*x - 2)*b^2*abs(b) + 9*b^2*abs(b))*sqrt(-b*x + 2)/((b*x - 2)*b +
 2*b)^(3/2) - 1/3*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*
sqrt(-b)*b^3 - 18*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2*sqr
t(-b)*b^4 + 16*sqrt(-b)*b^5)/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b
+ 2*b))^2 - 2*b)^3*abs(b))
```

3.647 $\int \frac{\sqrt{x}}{\sqrt{1-x}} dx$

Optimal. Leaf size=27

$$-\sqrt{1-x}\sqrt{x} - \frac{1}{2}\sin^{-1}(1-2x)$$

[Out] -(Sqrt[1 - x]*Sqrt[x]) - ArcSin[1 - 2*x]/2

Rubi [A] time = 0.005159, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 53, 619, 216}

$$-\sqrt{1-x}\sqrt{x} - \frac{1}{2}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/Sqrt[1 - x],x]

[Out] -(Sqrt[1 - x]*Sqrt[x]) - ArcSin[1 - 2*x]/2

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx &= -\sqrt{1-x}\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx\\ &= -\sqrt{1-x}\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{x-x^2}}\,dx\\ &= -\sqrt{1-x}\sqrt{x}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)\\ &= -\sqrt{1-x}\sqrt{x}-\frac{1}{2}\sin^{-1}(1-2x)\end{aligned}$$

Mathematica [A] time = 0.0080975, size = 25, normalized size = 0.93

$$-\sqrt{-(x-1)x}-\sin^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[1 - x],x]
```

```
[Out] -Sqrt[-((-1 + x)*x)] - ArcSin[Sqrt[1 - x]]
```

Maple [A] time = 0.005, size = 41, normalized size = 1.5

$$-\sqrt{1-x}\sqrt{x}+\frac{\arcsin(2x-1)}{2}\sqrt{x(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1-x)^(1/2),x)
```

```
[Out] -(1-x)^(1/2)*x^(1/2)+1/2*(x*(1-x))^(1/2)/x^(1/2)/(1-x)^(1/2)*arcsin(2*x-1)
```

Maxima [A] time = 1.59106, size = 50, normalized size = 1.85

$$\frac{\sqrt{-x+1}}{\sqrt{x}\left(\frac{x-1}{x}-1\right)}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-x + 1)/(sqrt(x)*((x - 1)/x - 1)) - arctan(sqrt(-x + 1)/sqrt(x))
```

Fricas [A] time = 1.45219, size = 73, normalized size = 2.7

$$-\sqrt{x}\sqrt{-x+1}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(x)*sqrt(-x + 1) - arctan(sqrt(-x + 1)/sqrt(x))
```

Sympy [A] time = 1.75505, size = 54, normalized size = 2.

$$\begin{cases} -i\sqrt{x}\sqrt{x-1} - i\operatorname{acosh}\left(\sqrt{x}\right) & \text{for } |x| > 1 \\ \frac{x^{\frac{3}{2}}}{\sqrt{1-x}} - \frac{\sqrt{x}}{\sqrt{1-x}} + \operatorname{asin}\left(\sqrt{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(x)*sqrt(x - 1) - I*acosh(sqrt(x)), Abs(x) > 1), (x**(3/2
)/sqrt(1 - x) - sqrt(x)/sqrt(1 - x) + asin(sqrt(x)), True))
```

Giac [A] time = 1.0529, size = 23, normalized size = 0.85

$$-\sqrt{x}\sqrt{-x+1} + \arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(x)*sqrt(-x + 1) + arcsin(sqrt(x))
```

3.648 $\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(1-2x)$$

```
[Out] -ArcSin[1 - 2*x]
```

Rubi [A] time = 0.0030581, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {53, 619, 216}

$$-\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*Sqrt[x]),x]

[Out] -ArcSin[1 - 2*x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx &= \int \frac{1}{\sqrt{x-x^2}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right)\\
&= -\sin^{-1}(1-2x)
\end{aligned}$$

Mathematica [A] time = 0.0079737, size = 12, normalized size = 1.5

$$-2\sin^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*Sqrt[x]),x]
```

```
[Out] -2*ArcSin[Sqrt[1 - x]]
```

Maple [B] time = 0.003, size = 27, normalized size = 3.4

$$\arcsin(2x-1)\sqrt{x(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/2)/x^(1/2),x)
```

```
[Out] (x*(1-x))^(1/2)/x^(1/2)/(1-x)^(1/2)*arcsin(2*x-1)
```

Maxima [B] time = 1.55005, size = 19, normalized size = 2.38

$$-2\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-x + 1)/sqrt(x))
```

Fricas [B] time = 1.53014, size = 45, normalized size = 5.62

$$-2\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-x + 1)/sqrt(x))
```

Sympy [A] time = 1.00491, size = 20, normalized size = 2.5

$$\begin{cases} -2i\operatorname{acosh}\left(\sqrt{x}\right) & \text{for}\ |x|>1 \\ 2\operatorname{asin}\left(\sqrt{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(x)), Abs(x) > 1), (2*asin(sqrt(x)), True))
```

Giac [A] time = 1.06984, size = 8, normalized size = 1.

$$2 \arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(sqrt(x))
```

3.649 $\int \frac{1}{\sqrt{x}\sqrt{1-bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcSin[Sqrt[b]*Sqrt[x]])/Sqrt[b]
```

Rubi [A] time = 0.004793, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {54, 216}

$$\frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 - b*x]),x]

[Out] (2*ArcSin[Sqrt[b]*Sqrt[x]])/Sqrt[b]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{1-bx}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0066255, size = 19, normalized size = 1.

$$\frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 - b*x]),x]
```

```
[Out] (2*ArcSin[Sqrt[b]*Sqrt[x]])/Sqrt[b]
```

Maple [B] time = 0.007, size = 48, normalized size = 2.5

$$\sqrt{x(-bx+1)}\arctan\left(\sqrt{b}\left(x-\frac{1}{2b}\right)\frac{1}{\sqrt{-bx^2+x}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-bx+1}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+1)^(1/2),x)
```

```
[Out] (x*(-b*x+1))^(1/2)/x^(1/2)/(-b*x+1)^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2/b)/
(-b*x^2+x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53397, size = 165, normalized size = 8.68

$$\left[-\frac{\sqrt{-b}\log\left(-2bx+2\sqrt{-bx+1}\sqrt{-b}\sqrt{x}+1\right)}{b}, -\frac{2\arctan\left(\frac{\sqrt{-bx+1}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + 1)*sqrt(-b)*sqrt(x) + 1)/b, -2*arctan
(sqrt(-b*x + 1)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

Sympy [A] time = 1.15522, size = 42, normalized size = 2.21

$$\begin{cases} -\frac{2i\operatorname{acosh}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}} & \text{for}\ |bx| > 1 \\ \frac{2\operatorname{asin}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+1)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(b)*sqrt(x))/sqrt(b), Abs(b*x) > 1), (2*asin(sqrt
(b)*sqrt(x))/sqrt(b), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.650 $\int x^{5/3}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}$$

```
[Out] (3*a*x^(8/3))/8 + (3*b*x^(11/3))/11
```

Rubi [A] time = 0.0035423, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(8/3))/8 + (3*b*x^(11/3))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)\,dx &= \int \left(ax^{5/3}+bx^{8/3}\right)dx\\ &= \frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}\end{aligned}$$

Mathematica [A] time = 0.00556, size = 17, normalized size = 0.81

$$\frac{3}{88}x^{8/3}(11a+8bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x),x]
```

```
[Out] (3*x^(8/3)*(11*a + 8*b*x))/88
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$\frac{24\,bx+33\,a}{88}x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a),x)
```

```
[Out] 3/88*x^(8/3)*(8*b*x+11*a)
```

Maxima [A] time = 1.07222, size = 18, normalized size = 0.86

$$\frac{3}{11}\,bx^{\frac{11}{3}}+\frac{3}{8}\,ax^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 3/11*b*x^(11/3) + 3/8*a*x^(8/3)
```

Fricas [A] time = 1.38988, size = 47, normalized size = 2.24

$$\frac{3}{88}\left(8\,bx^{3}+11\,ax^{2}\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 3/88*(8*b*x^3 + 11*a*x^2)*x^(2/3)
```

Sympy [A] time = 2.21241, size = 19, normalized size = 0.9

$$\frac{3ax^{\frac{8}{3}}}{8}+\frac{3bx^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(8/3)/8 + 3*b*x**(11/3)/11
```

Giac [A] time = 1.04928, size = 18, normalized size = 0.86

$$\frac{3}{11}\,bx^{\frac{11}{3}}+\frac{3}{8}\,ax^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="giac")
```

```
[Out] 3/11*b*x^(11/3) + 3/8*a*x^(8/3)
```

3.651 $\int x^{4/3}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}$$

```
[Out] (3*a*x^(7/3))/7 + (3*b*x^(10/3))/10
```

Rubi [A] time = 0.0035655, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(7/3))/7 + (3*b*x^(10/3))/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)\,dx &= \int \left(ax^{4/3}+bx^{7/3}\right)\,dx\\ &= \frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}\end{aligned}$$

Mathematica [A] time = 0.0044912, size = 17, normalized size = 0.81

$$\frac{3}{70}x^{7/3}(10a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x),x]
```

```
[Out] (3*x^(7/3)*(10*a + 7*b*x))/70
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$\frac{21\,bx+30\,a}{70}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a),x)
```

```
[Out] 3/70*x^(7/3)*(7*b*x+10*a)
```

Maxima [A] time = 1.10987, size = 18, normalized size = 0.86

$$\frac{3}{10}\,bx^{\frac{10}{3}} + \frac{3}{7}\,ax^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 3/10*b*x^(10/3) + 3/7*a*x^(7/3)
```

Fricas [A] time = 1.53166, size = 47, normalized size = 2.24

$$\frac{3}{70}\left(7\,bx^3 + 10\,ax^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 3/70*(7*b*x^3 + 10*a*x^2)*x^(1/3)
```

Sympy [A] time = 1.46949, size = 19, normalized size = 0.9

$$\frac{3ax^{\frac{7}{3}}}{7} + \frac{3bx^{\frac{10}{3}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(7/3)/7 + 3*b*x**(10/3)/10
```

Giac [A] time = 1.05425, size = 18, normalized size = 0.86

$$\frac{3}{10}\,bx^{\frac{10}{3}} + \frac{3}{7}\,ax^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="giac")
```

```
[Out] 3/10*b*x^(10/3) + 3/7*a*x^(7/3)
```

3.652 $\int x^{2/3}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}$$

```
[Out] (3*a*x^(5/3))/5 + (3*b*x^(8/3))/8
```

Rubi [A] time = 0.003643, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(5/3))/5 + (3*b*x^(8/3))/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)\,dx&=\int\left(ax^{2/3}+bx^{5/3}\right)\,dx\\&=\frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}\end{aligned}$$

Mathematica [A] time = 0.0046576, size = 17, normalized size = 0.81

$$\frac{3}{40}x^{5/3}(8a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x),x]
```

```
[Out] (3*x^(5/3)*(8*a + 5*b*x))/40
```

Maple [A] time = 0.001, size = 14, normalized size = 0.7

$$\frac{15\,bx+24\,a}{40}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a),x)
```

```
[Out] 3/40*x^(5/3)*(5*b*x+8*a)
```

Maxima [A] time = 1.0489, size = 18, normalized size = 0.86

$$\frac{3}{8}\,bx^{\frac{8}{3}} + \frac{3}{5}\,ax^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 3/8*b*x^(8/3) + 3/5*a*x^(5/3)
```

Fricas [A] time = 1.54449, size = 43, normalized size = 2.05

$$\frac{3}{40}\left(5\,bx^{2} + 8\,ax\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b*x^2 + 8*a*x)*x^(2/3)
```

Sympy [A] time = 0.485269, size = 19, normalized size = 0.9

$$\frac{3ax^{\frac{5}{3}}}{5} + \frac{3bx^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(5/3)/5 + 3*b*x**(8/3)/8
```

Giac [A] time = 1.05589, size = 18, normalized size = 0.86

$$\frac{3}{8}\,bx^{\frac{8}{3}} + \frac{3}{5}\,ax^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="giac")
```

```
[Out] 3/8*b*x^(8/3) + 3/5*a*x^(5/3)
```

3.653 $\int \sqrt[3]{x}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}$$

```
[Out] (3*a*x^(4/3))/4 + (3*b*x^(7/3))/7
```

Rubi [A] time = 0.0035914, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(4/3))/4 + (3*b*x^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}(a+bx)\,dx &= \int \left(a\sqrt[3]{x}+bx^{4/3}\right)\,dx\\ &= \frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}\end{aligned}$$

Mathematica [A] time = 0.0044191, size = 17, normalized size = 0.81

$$\frac{3}{28}x^{4/3}(7a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x),x]
```

```
[Out] (3*x^(4/3)*(7*a + 4*b*x))/28
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$\frac{12\,bx+21\,a}{28}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a),x)
```

```
[Out] 3/28*x^(4/3)*(4*b*x+7*a)
```

Maxima [A] time = 1.05329, size = 18, normalized size = 0.86

$$\frac{3}{7}\,bx^{\frac{7}{3}} + \frac{3}{4}\,ax^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 3/7*b*x^(7/3) + 3/4*a*x^(4/3)
```

Fricas [A] time = 1.50137, size = 43, normalized size = 2.05

$$\frac{3}{28}\left(4\,bx^2 + 7\,ax\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 3/28*(4*b*x^2 + 7*a*x)*x^(1/3)
```

Sympy [A] time = 1.24474, size = 19, normalized size = 0.9

$$\frac{3ax^{\frac{4}{3}}}{4} + \frac{3bx^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(4/3)/4 + 3*b*x**(7/3)/7
```

Giac [A] time = 1.05017, size = 18, normalized size = 0.86

$$\frac{3}{7}\,bx^{\frac{7}{3}} + \frac{3}{4}\,ax^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="giac")
```

```
[Out] 3/7*b*x^(7/3) + 3/4*a*x^(4/3)
```

3.654 $\int \frac{a+bx}{\sqrt[3]{x}} \, dx$

Optimal. Leaf size=21

$$\frac{3}{2}ax^{2/3} + \frac{3}{5}bx^{5/3}$$

```
[Out] (3*a*x^(2/3))/2 + (3*b*x^(5/3))/5
```

Rubi [A] time = 0.0038604, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{2}ax^{2/3} + \frac{3}{5}bx^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(1/3),x]
```

```
[Out] (3*a*x^(2/3))/2 + (3*b*x^(5/3))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{\sqrt[3]{x}} \, dx &= \int \left(\frac{a}{\sqrt[3]{x}} + bx^{2/3}\right) dx \\ &= \frac{3}{2}ax^{2/3} + \frac{3}{5}bx^{5/3} \end{aligned}$$

Mathematica [A] time = 0.0045825, size = 17, normalized size = 0.81

$$\frac{3}{10}x^{2/3}(5a + 2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(1/3),x]
```

```
[Out] (3*x^(2/3)*(5*a + 2*b*x))/10
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$\frac{6\,bx + 15\,a}{10} x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(1/3),x)
```

```
[Out] 3/10*x^(2/3)*(2*b*x+5*a)
```

Maxima [A] time = 1.06304, size = 18, normalized size = 0.86

$$\frac{3}{5}\,bx^{\frac{5}{3}} + \frac{3}{2}\,ax^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/5*b*x^(5/3) + 3/2*a*x^(2/3)
```

Fricas [A] time = 1.53034, size = 38, normalized size = 1.81

$$\frac{3}{10}\,(2\,bx + 5\,a)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(2*b*x + 5*a)*x^(2/3)
```

Sympy [A] time = 1.32003, size = 19, normalized size = 0.9

$$\frac{3ax^{\frac{2}{3}}}{2} + \frac{3bx^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(1/3),x)
```

```
[Out] 3*a*x**(2/3)/2 + 3*b*x**(5/3)/5
```

Giac [A] time = 1.07289, size = 18, normalized size = 0.86

$$\frac{3}{5}\,bx^{\frac{5}{3}} + \frac{3}{2}\,ax^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="giac")
```

```
[Out] 3/5*b*x^(5/3) + 3/2*a*x^(2/3)
```

3.655 $\int \frac{a+bx}{x^{2/3}} dx$

Optimal. Leaf size=19

$$3a\sqrt[3]{x} + \frac{3}{4}bx^{4/3}$$

```
[Out] 3*a*x^(1/3) + (3*b*x^(4/3))/4
```

Rubi [A] time = 0.0034785, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$3a\sqrt[3]{x} + \frac{3}{4}bx^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(2/3),x]
```

```
[Out] 3*a*x^(1/3) + (3*b*x^(4/3))/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{x^{2/3}} dx &= \int \left(\frac{a}{x^{2/3}} + b\sqrt[3]{x}\right) dx \\ &= 3a\sqrt[3]{x} + \frac{3}{4}bx^{4/3} \end{aligned}$$

Mathematica [A] time = 0.0045585, size = 16, normalized size = 0.84

$$\frac{3}{4}\sqrt[3]{x}(4a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(2/3),x]
```

```
[Out] (3*x^(1/3)*(4*a + b*x))/4
```

Maple [A] time = 0.002, size = 13, normalized size = 0.7

$$\frac{3\,bx + 12\,a}{4}\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(2/3),x)
```

```
[Out] 3/4*x^(1/3)*(b*x+4*a)
```

Maxima [A] time = 1.01697, size = 18, normalized size = 0.95

$$\frac{3}{4}\, b x^{\frac{4}{3}} + 3\, a x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/4*b*x^(4/3) + 3*a*x^(1/3)
```

Fricas [A] time = 1.53769, size = 34, normalized size = 1.79

$$\frac{3}{4}\, (b x + 4\, a) x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x + 4*a)*x^(1/3)
```

Sympy [A] time = 1.14289, size = 17, normalized size = 0.89

$$3a\sqrt[3]{x} + \frac{3bx^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(2/3),x)
```

```
[Out] 3*a*x**(1/3) + 3*b*x**(4/3)/4
```

Giac [A] time = 1.05519, size = 18, normalized size = 0.95

$$\frac{3}{4}\, b x^{\frac{4}{3}} + 3\, a x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="giac")
```

```
[Out] 3/4*b*x^(4/3) + 3*a*x^(1/3)
```

3.656 $\int \frac{a+bx}{x^{4/3}} \, dx$

Optimal. Leaf size=19

$$\frac{3}{2}bx^{2/3} - \frac{3a}{\sqrt[3]{x}}$$

```
[Out] (-3*a)/x^(1/3) + (3*b*x^(2/3))/2
```

Rubi [A] time = 0.0036323, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{2}bx^{2/3} - \frac{3a}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(4/3),x]
```

```
[Out] (-3*a)/x^(1/3) + (3*b*x^(2/3))/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a + bx}{x^{4/3}} \, dx &= \int \left(\frac{a}{x^{4/3}} + \frac{b}{\sqrt[3]{x}}\right) dx\\ &= -\frac{3a}{\sqrt[3]{x}} + \frac{3}{2}bx^{2/3}\end{aligned}$$

Mathematica [A] time = 0.0051168, size = 16, normalized size = 0.84

$$\frac{3(bx - 2a)}{2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(4/3),x]
```

```
[Out] (3*(-2*a + b*x))/(2*x^(1/3))
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$-\frac{-3\,bx + 6\,a}{2}\frac{1}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(4/3),x)
```

```
[Out] -3/2*(-b*x+2*a)/x^(1/3)
```

Maxima [A] time = 1.07173, size = 18, normalized size = 0.95

$$\frac{3}{2}bx^{\frac{2}{3}} - \frac{3a}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="maxima")
```

```
[Out] 3/2*b*x^(2/3) - 3*a/x^(1/3)
```

Fricas [A] time = 1.53729, size = 34, normalized size = 1.79

$$\frac{3(bx-2a)}{2x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x - 2*a)/x^(1/3)
```

Sympy [A] time = 0.437249, size = 17, normalized size = 0.89

$$-\frac{3a}{\sqrt[3]{x}} + \frac{3bx^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(4/3),x)
```

```
[Out] -3*a/x**(1/3) + 3*b*x**(2/3)/2
```

Giac [A] time = 1.06447, size = 18, normalized size = 0.95

$$\frac{3}{2}bx^{\frac{2}{3}} - \frac{3a}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="giac")
```

```
[Out] 3/2*b*x^(2/3) - 3*a/x^(1/3)
```

3.657 $\int \frac{a+bx}{x^{5/3}} dx$

Optimal. Leaf size=19

$$3b\sqrt[3]{x} - \frac{3a}{2x^{2/3}}$$

[Out] (-3*a)/(2*x^(2/3)) + 3*b*x^(1/3)

Rubi [A] time = 0.0035201, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$3b\sqrt[3]{x} - \frac{3a}{2x^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/x^(5/3),x]

[Out] (-3*a)/(2*x^(2/3)) + 3*b*x^(1/3)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{x^{5/3}} dx &= \int \left(\frac{a}{x^{5/3}} + \frac{b}{x^{2/3}}\right) dx \\ &= -\frac{3a}{2x^{2/3}} + 3b\sqrt[3]{x} \end{aligned}$$

Mathematica [A] time = 0.0052261, size = 19, normalized size = 1.

$$3b\sqrt[3]{x} - \frac{3a}{2x^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/x^(5/3),x]

[Out] (-3*a)/(2*x^(2/3)) + 3*b*x^(1/3)

Maple [A] time = 0.002, size = 12, normalized size = 0.6

$$-\frac{-6\,bx + 3\,a}{2} x^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(5/3),x)
```

```
[Out] -3/2*(-2*b*x+a)/x^(2/3)
```

Maxima [A] time = 1.00163, size = 18, normalized size = 0.95

$$3\,bx^{\frac{1}{3}} - \frac{3\,a}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="maxima")
```

```
[Out] 3*b*x^(1/3) - 3/2*a/x^(2/3)
```

Fricas [A] time = 1.4177, size = 34, normalized size = 1.79

$$\frac{3\,(2\,bx - a)}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="fricas")
```

```
[Out] 3/2*(2*b*x - a)/x^(2/3)
```

Sympy [A] time = 0.486049, size = 17, normalized size = 0.89

$$-\frac{3a}{2x^{\frac{2}{3}}} + 3b\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(5/3),x)
```

```
[Out] -3*a/(2*x**(2/3)) + 3*b*x**(1/3)
```

Giac [A] time = 1.05946, size = 18, normalized size = 0.95

$$3\,bx^{\frac{1}{3}} - \frac{3\,a}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="giac")
```

```
[Out] 3*b*x^(1/3) - 3/2*a/x^(2/3)
```

3.658 $\int x^{5/3}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}$$

```
[Out] (3*a^2*x^(8/3))/8 + (6*a*b*x^(11/3))/11 + (3*b^2*x^(14/3))/14
```

Rubi [A] time = 0.0067997, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(8/3))/8 + (6*a*b*x^(11/3))/11 + (3*b^2*x^(14/3))/14
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)^2\,dx&=\int\left(a^2x^{5/3}+2abx^{8/3}+b^2x^{11/3}\right)dx\\&=\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}\end{aligned}$$

Mathematica [A] time = 0.0083215, size = 28, normalized size = 0.78

$$\frac{3}{616}x^{8/3}\left(77a^2+112abx+44b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(8/3)*(77*a^2 + 112*a*b*x + 44*b^2*x^2))/616
```

Maple [A] time = 0.003, size = 25, normalized size = 0.7

$$\frac{132\,b^2x^2+336\,abx+231\,a^2}{616}x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a)^2,x)
```

```
[Out] 3/616*x^(8/3)*(44*b^2*x^2+112*a*b*x+77*a^2)
```

Maxima [A] time = 1.00203, size = 32, normalized size = 0.89

$$\frac{3}{14}\,b^2x^{\frac{14}{3}} + \frac{6}{11}\,abx^{\frac{11}{3}} + \frac{3}{8}\,a^2x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/14*b^2*x^(14/3) + 6/11*a*b*x^(11/3) + 3/8*a^2*x^(8/3)
```

Fricas [A] time = 1.46021, size = 74, normalized size = 2.06

$$\frac{3}{616}\left(44\,b^2x^4 + 112\,abx^3 + 77\,a^2x^2\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 3/616*(44*b^2*x^4 + 112*a*b*x^3 + 77*a^2*x^2)*x^(2/3)
```

Sympy [A] time = 4.0259, size = 34, normalized size = 0.94

$$\frac{3a^2x^{\frac{8}{3}}}{8} + \frac{6abx^{\frac{11}{3}}}{11} + \frac{3b^2x^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a)**2,x)
```

```
[Out] 3*a**2*x**(8/3)/8 + 6*a*b*x**(11/3)/11 + 3*b**2*x**(14/3)/14
```

Giac [A] time = 1.07298, size = 32, normalized size = 0.89

$$\frac{3}{14}\,b^2x^{\frac{14}{3}} + \frac{6}{11}\,abx^{\frac{11}{3}} + \frac{3}{8}\,a^2x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/14*b^2*x^(14/3) + 6/11*a*b*x^(11/3) + 3/8*a^2*x^(8/3)
```

3.659 $\int x^{4/3}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}$$

```
[Out] (3*a^2*x^(7/3))/7 + (3*a*b*x^(10/3))/5 + (3*b^2*x^(13/3))/13
```

Rubi [A] time = 0.0070806, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(7/3))/7 + (3*a*b*x^(10/3))/5 + (3*b^2*x^(13/3))/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)^2\,dx &= \int \left(a^2x^{4/3}+2abx^{7/3}+b^2x^{10/3}\right)dx\\ &= \frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}\end{aligned}$$

Mathematica [A] time = 0.0076244, size = 28, normalized size = 0.78

$$\frac{3}{455}x^{7/3}\left(65a^2+91abx+35b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(7/3)*(65*a^2 + 91*a*b*x + 35*b^2*x^2))/455
```

Maple [A] time = 0.005, size = 25, normalized size = 0.7

$$\frac{105\,b^2x^2+273\,abx+195\,a^2}{455}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a)^2,x)
```

```
[Out] 3/455*x^(7/3)*(35*b^2*x^2+91*a*b*x+65*a^2)
```

Maxima [A] time = 1.03683, size = 32, normalized size = 0.89

$$\frac{3}{13}\,b^2x^{\frac{13}{3}} + \frac{3}{5}\,abx^{\frac{10}{3}} + \frac{3}{7}\,a^2x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/13*b^2*x^(13/3) + 3/5*a*b*x^(10/3) + 3/7*a^2*x^(7/3)
```

Fricas [A] time = 1.46162, size = 73, normalized size = 2.03

$$\frac{3}{455}\left(35\,b^2x^4 + 91\,abx^3 + 65\,a^2x^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 3/455*(35*b^2*x^4 + 91*a*b*x^3 + 65*a^2*x^2)*x^(1/3)
```

Sympy [A] time = 3.16779, size = 34, normalized size = 0.94

$$\frac{3a^2x^{\frac{7}{3}}}{7} + \frac{3abx^{\frac{10}{3}}}{5} + \frac{3b^2x^{\frac{13}{3}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a)**2,x)
```

```
[Out] 3*a**2*x**(7/3)/7 + 3*a*b*x**(10/3)/5 + 3*b**2*x**(13/3)/13
```

Giac [A] time = 1.05798, size = 32, normalized size = 0.89

$$\frac{3}{13}\,b^2x^{\frac{13}{3}} + \frac{3}{5}\,abx^{\frac{10}{3}} + \frac{3}{7}\,a^2x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/13*b^2*x^(13/3) + 3/5*a*b*x^(10/3) + 3/7*a^2*x^(7/3)
```

3.660 $\int x^{2/3}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}$$

```
[Out] (3*a^2*x^(5/3))/5 + (3*a*b*x^(8/3))/4 + (3*b^2*x^(11/3))/11
```

Rubi [A] time = 0.0068258, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(5/3))/5 + (3*a*b*x^(8/3))/4 + (3*b^2*x^(11/3))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)^2\,dx &= \int \left(a^2x^{2/3}+2abx^{5/3}+b^2x^{8/3}\right)dx\\ &= \frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}\end{aligned}$$

Mathematica [A] time = 0.0074095, size = 28, normalized size = 0.78

$$\frac{3}{220}x^{5/3}\left(44a^2+55abx+20b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(5/3)*(44*a^2 + 55*a*b*x + 20*b^2*x^2))/220
```

Maple [A] time = 0.004, size = 25, normalized size = 0.7

$$\frac{60\,b^2x^2+165\,abx+132\,a^2}{220}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a)^2,x)
```

```
[Out] 3/220*x^(5/3)*(20*b^2*x^2+55*a*b*x+44*a^2)
```

Maxima [A] time = 1.04631, size = 32, normalized size = 0.89

$$\frac{3}{11}\,b^2x^{\frac{11}{3}}+\frac{3}{4}\,abx^{\frac{8}{3}}+\frac{3}{5}\,a^2x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/11*b^2*x^(11/3) + 3/4*a*b*x^(8/3) + 3/5*a^2*x^(5/3)
```

Fricas [A] time = 1.51761, size = 70, normalized size = 1.94

$$\frac{3}{220}\left(20\,b^2x^3+55\,abx^2+44\,a^2x\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 3/220*(20*b^2*x^3 + 55*a*b*x^2 + 44*a^2*x)*x^(2/3)
```

Sympy [A] time = 1.29595, size = 34, normalized size = 0.94

$$\frac{3a^2x^{\frac{5}{3}}}{5}+\frac{3abx^{\frac{8}{3}}}{4}+\frac{3b^2x^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a)**2,x)
```

```
[Out] 3*a**2*x**(5/3)/5 + 3*a*b*x**(8/3)/4 + 3*b**2*x**(11/3)/11
```

Giac [A] time = 1.06283, size = 32, normalized size = 0.89

$$\frac{3}{11}\,b^2x^{\frac{11}{3}}+\frac{3}{4}\,abx^{\frac{8}{3}}+\frac{3}{5}\,a^2x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/11*b^2*x^(11/3) + 3/4*a*b*x^(8/3) + 3/5*a^2*x^(5/3)
```

3.661 $\int \sqrt[3]{x}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}$$

```
[Out] (3*a^2*x^(4/3))/4 + (6*a*b*x^(7/3))/7 + (3*b^2*x^(10/3))/10
```

Rubi [A] time = 0.0069011, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(4/3))/4 + (6*a*b*x^(7/3))/7 + (3*b^2*x^(10/3))/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}(a+bx)^2\,dx &= \int \left(a^2\sqrt[3]{x}+2abx^{4/3}+b^2x^{7/3}\right)dx\\ &= \frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}\end{aligned}$$

Mathematica [A] time = 0.007176, size = 28, normalized size = 0.78

$$\frac{3}{140}x^{4/3}\left(35a^2+40abx+14b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(4/3)*(35*a^2 + 40*a*b*x + 14*b^2*x^2))/140
```

Maple [A] time = 0.004, size = 25, normalized size = 0.7

$$\frac{42\,b^2x^2+120\,abx+105\,a^2}{140}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a)^2,x)
```

```
[Out] 3/140*x^(4/3)*(14*b^2*x^2+40*a*b*x+35*a^2)
```

Maxima [A] time = 0.995491, size = 32, normalized size = 0.89

$$\frac{3}{10}\,b^2x^{\frac{10}{3}}+\frac{6}{7}\,abx^{\frac{7}{3}}+\frac{3}{4}\,a^2x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/10*b^2*x^(10/3) + 6/7*a*b*x^(7/3) + 3/4*a^2*x^(4/3)
```

Fricas [A] time = 1.54584, size = 70, normalized size = 1.94

$$\frac{3}{140}\left(14\,b^2x^3+40\,abx^2+35\,a^2x\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 3/140*(14*b^2*x^3 + 40*a*b*x^2 + 35*a^2*x)*x^(1/3)
```

Sympy [C] time = 2.69543, size = 2635, normalized size = 73.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a)**2,x)
```

```
[Out] Piecewise((27*a**(34/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)/(-140*a**
8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*
a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3
*exp(2*I*pi/3)) + 27*a**(34/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7
*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I
*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 72*a**(31/3)*b*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2
*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(
a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3))
- 81*a**(31/3)*b*(a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**
(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/
3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 60*a**(28/3)*b**2*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp
(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)
*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)
) + 81*a**(28/3)*b**2*(a/b + x)**2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*
```

```
a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp
(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(25/3)*
b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2*I*pi/3)/(-140*a**8*b**(
4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b
**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2
*I*pi/3)) - 27*a**(25/3)*b**3*(a/b + x)**3/(-140*a**8*b**(4/3)*exp(2*I*pi/3
) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x
)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 135*a
**(22/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(2*I*pi/3)/(-140*
a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 4
20*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)
**3*exp(2*I*pi/3)) - 132*a**(19/3)*b**5*(-1 + b*(a/b + x)/a)**(1/3)*(a/b +
x)**5*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(
a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 14
0*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 42*a**(16/3)*b**6*(-1 + b*(a
/b + x)/a)**(1/3)*(a/b + x)**6*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2*I*pi
/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b +
 x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)), Abs(
b*(a/b + x))/Abs(a) > 1), (-27*a**(34/3)*(1 - b*(a/b + x)/a)**(1/3)/(-140*a
**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 42
0*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)*
*3*exp(2*I*pi/3)) + 27*a**(34/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a*
*7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2
*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 72*a**(31/3)*b*
(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 42
0*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*e
xp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 81*a**(31/3
)*b*(a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b +
x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5
*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(28/3)*b**2*(1 - b*(a/b + x)
/a)**(1/3)*(a/b + x)**2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/
3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3)
+ 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 81*a**(28/3)*b**2*(a/b +
 x)**2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(
2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13
/3)*(a/b + x)**3*exp(2*I*pi/3)) + 60*a**(25/3)*b**3*(1 - b*(a/b + x)/a)**(1
/3)*(a/b + x)**3/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b
 + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a
**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 27*a**(25/3)*b**3*(a/b + x)**3/
(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/
3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/
b + x)**3*exp(2*I*pi/3)) - 135*a**(22/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a
/b + x)**4/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*
exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b*
*(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 132*a**(19/3)*b**5*(1 - b*(a/b + x)/a
)**(1/3)*(a/b + x)**5/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)
*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) +
140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 42*a**(16/3)*b**6*(1 - b*(
a/b + x)/a)**(1/3)*(a/b + x)**6/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**
7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*
I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)), True))
```

Giac [A] time = 1.0448, size = 32, normalized size = 0.89

$$\frac{3}{10}\,b^{2}x^{\frac{10}{3}}+\frac{6}{7}\,abx^{\frac{7}{3}}+\frac{3}{4}\,a^{2}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/10*b^2*x^(10/3) + 6/7*a*b*x^(7/3) + 3/4*a^2*x^(4/3)
```

3.662 $\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=36

$$\frac{3}{2}a^2x^{2/3}+\frac{6}{5}abx^{5/3}+\frac{3}{8}b^2x^{8/3}$$

```
[Out] (3*a^2*x^(2/3))/2 + (6*a*b*x^(5/3))/5 + (3*b^2*x^(8/3))/8
```

Rubi [A] time = 0.0076617, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{2}a^2x^{2/3}+\frac{6}{5}abx^{5/3}+\frac{3}{8}b^2x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(1/3),x]
```

```
[Out] (3*a^2*x^(2/3))/2 + (6*a*b*x^(5/3))/5 + (3*b^2*x^(8/3))/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx &= \int \left(\frac{a^2}{\sqrt[3]{x}}+2abx^{2/3}+b^2x^{5/3}\right)dx\\ &= \frac{3}{2}a^2x^{2/3}+\frac{6}{5}abx^{5/3}+\frac{3}{8}b^2x^{8/3}\end{aligned}$$

Mathematica [A] time = 0.0073862, size = 28, normalized size = 0.78

$$\frac{3}{40}x^{2/3}\left(20a^2+16abx+5b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(1/3),x]
```

```
[Out] (3*x^(2/3)*(20*a^2 + 16*a*b*x + 5*b^2*x^2))/40
```

Maple [A] time = 0.003, size = 25, normalized size = 0.7

$$\frac{15\,b^2x^2+48\,abx+60\,a^2}{40}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(1/3),x)
```

```
[Out] 3/40*x^(2/3)*(5*b^2*x^2+16*a*b*x+20*a^2)
```

Maxima [A] time = 1.02346, size = 32, normalized size = 0.89

$$\frac{3}{8}\,b^{2}x^{\frac{8}{3}}+\frac{6}{5}\,abx^{\frac{5}{3}}+\frac{3}{2}\,a^{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*b^2*x^(8/3) + 6/5*a*b*x^(5/3) + 3/2*a^2*x^(2/3)
```

Fricas [A] time = 1.50496, size = 62, normalized size = 1.72

$$\frac{3}{40}\left(5\,b^{2}x^{2}+16\,abx+20\,a^{2}\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^2*x^2 + 16*a*b*x + 20*a^2)*x^(2/3)
```

Sympy [C] time = 2.16043, size = 1766, normalized size = 49.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(1/3),x)
```

```
[Out] Piecewise((-27*a**(32/3)*(-1 + b*(a/b + x)/a)**(2/3)/(-40*a**8*b**(2/3) + 1
20*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(1
1/3)*(a/b + x)**3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a*
*7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*
(a/b + x)**3) + 63*a**(29/3)*b*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)/(-40*a
**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2
 + 40*a**5*b**(11/3)*(a/b + x)**3) - 81*a**(29/3)*b*(a/b + x)*exp(2*I*pi/3)
/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b
+ x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 42*a**(26/3)*b**2*(-1 + b*(a/b
+ x)/a)**(2/3)*(a/b + x)**2/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x
) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 81*a
**(26/3)*b**2*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(
5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b +
x)**3) + 18*a**(23/3)*b**3*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(-40*a*
*8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2
+ 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(23/3)*b**3*(a/b + x)**3*exp(2*I*
```

```
pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*
(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(20/3)*b**4*(-1 + b*
(a/b + x)/a)**(2/3)*(a/b + x)**4/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/
b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) +
 15*a**(17/3)*b**5*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**5/(-40*a**8*b**(2
/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**
5*b**(11/3)*(a/b + x)**3), Abs(b*(a/b + x))/Abs(a) > 1), (-27*a**(32/3)*(1
- b*(a/b + x)/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3
)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)*
*3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/
b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) +
 63*a**(29/3)*b*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)*exp(2*I*pi/3)/(-40*a**
8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 +
 40*a**5*b**(11/3)*(a/b + x)**3) - 81*a**(29/3)*b*(a/b + x)*exp(2*I*pi/3)/(
-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b +
x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 42*a**(26/3)*b**2*(1 - b*(a/b + x
)/a)**(2/3)*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/
3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)
**3) + 81*a**(26/3)*b**2*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 12
0*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11
/3)*(a/b + x)**3) + 18*a**(23/3)*b**3*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)*
*3*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**
6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(23/3)*b*
*3*(a/b + x)**3*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b +
 x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27
*a**(20/3)*b**4*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(2*I*pi/3)/(-40*
a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**
2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 15*a**(17/3)*b**5*(1 - b*(a/b + x)/a)
**(2/3)*(a/b + x)**5*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(
a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3)
, True))
```

Giac [A] time = 1.0473, size = 32, normalized size = 0.89

$$\frac{3}{8}b^2x^{\frac{8}{3}} + \frac{6}{5}abx^{\frac{5}{3}} + \frac{3}{2}a^2x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="giac")
```

```
[Out] 3/8*b^2*x^(8/3) + 6/5*a*b*x^(5/3) + 3/2*a^2*x^(2/3)
```

3.663 $\int \frac{(a+bx)^2}{x^{2/3}} dx$

Optimal. Leaf size=34

$$3a^2\sqrt[3]{x} + \frac{3}{2}abx^{4/3} + \frac{3}{7}b^2x^{7/3}$$

```
[Out] 3*a^2*x^(1/3) + (3*a*b*x^(4/3))/2 + (3*b^2*x^(7/3))/7
```

Rubi [A] time = 0.0066598, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$3a^2\sqrt[3]{x} + \frac{3}{2}abx^{4/3} + \frac{3}{7}b^2x^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(2/3),x]
```

```
[Out] 3*a^2*x^(1/3) + (3*a*b*x^(4/3))/2 + (3*b^2*x^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^2}{x^{2/3}} dx &= \int \left(\frac{a^2}{x^{2/3}} + 2ab\sqrt[3]{x} + b^2x^{4/3}\right) dx \\ &= 3a^2\sqrt[3]{x} + \frac{3}{2}abx^{4/3} + \frac{3}{7}b^2x^{7/3} \end{aligned}$$

Mathematica [A] time = 0.007176, size = 28, normalized size = 0.82

$$\frac{3}{14}\sqrt[3]{x}\left(14a^2 + 7abx + 2b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(2/3),x]
```

```
[Out] (3*x^(1/3)*(14*a^2 + 7*a*b*x + 2*b^2*x^2))/14
```

Maple [A] time = 0.003, size = 25, normalized size = 0.7

$$\frac{6\,b^2x^2 + 21\,abx + 42\,a^2}{14}\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(2/3),x)
```

```
[Out] 3/14*x^(1/3)*(2*b^2*x^2+7*a*b*x+14*a^2)
```

Maxima [A] time = 1.50062, size = 32, normalized size = 0.94

$$\frac{3}{7}b^2x^{\frac{7}{3}} + \frac{3}{2}abx^{\frac{4}{3}} + 3a^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/7*b^2*x^(7/3) + 3/2*a*b*x^(4/3) + 3*a^2*x^(1/3)
```

Fricas [A] time = 1.50627, size = 61, normalized size = 1.79

$$\frac{3}{14}\left(2b^2x^2 + 7abx + 14a^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3/14*(2*b^2*x^2 + 7*a*b*x + 14*a^2)*x^(1/3)
```

Sympy [C] time = 2.39756, size = 1742, normalized size = 51.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(2/3),x)
```

```
[Out] Piecewise((-27*a**(31/3)*(-1 + b*(a/b + x)/a)**(1/3)/(-14*a**8*b**(1/3) + 4
2*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/
3)*(a/b + x)**3) + 27*a**(31/3)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b*
*(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b +
 x)**3) + 72*a**(28/3)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)/(-14*a**8*b*
*(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a*
*5*b**(10/3)*(a/b + x)**3) - 81*a**(28/3)*b*(a/b + x)*exp(I*pi/3)/(-14*a**8
*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14
*a**5*b**(10/3)*(a/b + x)**3) - 60*a**(25/3)*b**2*(-1 + b*(a/b + x)/a)**(1/
3)*(a/b + x)**2/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b
**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 81*a**(25/3)*b**2*
(a/b + x)**2*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) -
42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 18*a**(22
/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(-14*a**8*b**(1/3) + 42*a
**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*
(a/b + x)**3) - 27*a**(22/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/
```

```
3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b
**(10/3)*(a/b + x)**3) - 9*a**(19/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b
+ x)**4/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*
(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 6*a**(16/3)*b**5*(-1 + b*(
a/b + x)/a)**(1/3)*(a/b + x)**5/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b
+ x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3), Abs
(b*(a/b + x))/Abs(a) > 1), (-27*a**(31/3)*(1 - b*(a/b + x)/a)**(1/3)*exp(I*
pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a
/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 27*a**(31/3)*exp(I*pi/3)/(-1
4*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**
2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 72*a**(28/3)*b*(1 - b*(a/b + x)/a)**(
1/3)*(a/b + x)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x)
- 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 81*a**(
28/3)*b*(a/b + x)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b +
x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 60*a
**(25/3)*b**2*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(I*pi/3)/(-14*a**8
*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14
*a**5*b**(10/3)*(a/b + x)**3) + 81*a**(25/3)*b**2*(a/b + x)**2*exp(I*pi/3)/
(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x
)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 18*a**(22/3)*b**3*(1 - b*(a/b + x)
/a)**(1/3)*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(
a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3)
- 27*a**(22/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b
**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b
+ x)**3) - 9*a**(19/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(I*p
i/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/
b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 6*a**(16/3)*b**5*(1 - b*(a/b
+ x)/a)**(1/3)*(a/b + x)**5*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/
3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)*
*3), True))
```

Giac [A] time = 1.0645, size = 32, normalized size = 0.94

$$\frac{3}{7}\,b^{2}x^{\frac{7}{3}}+\frac{3}{2}\,abx^{\frac{4}{3}}+3\,a^{2}x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="giac")
```

```
[Out] 3/7*b^2*x^(7/3) + 3/2*a*b*x^(4/3) + 3*a^2*x^(1/3)
```

3.664 $\int \frac{(a+bx)^2}{x^{4/3}} dx$

Optimal. Leaf size=32

$$-\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}$$

```
[Out] (-3*a^2)/x^(1/3) + 3*a*b*x^(2/3) + (3*b^2*x^(5/3))/5
```

Rubi [A] time = 0.0067156, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(4/3),x]
```

```
[Out] (-3*a^2)/x^(1/3) + 3*a*b*x^(2/3) + (3*b^2*x^(5/3))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{4/3}}\,dx &= \int \left(\frac{a^2}{x^{4/3}}+\frac{2ab}{\sqrt[3]{x}}+b^2x^{2/3}\right)dx\\ &= -\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}\end{aligned}$$

Mathematica [A] time = 0.0075086, size = 27, normalized size = 0.84

$$\frac{3\left(-5a^2+5abx+b^2x^2\right)}{5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(4/3),x]
```

```
[Out] (3*(-5*a^2 + 5*a*b*x + b^2*x^2))/(5*x^(1/3))
```

Maple [A] time = 0.003, size = 25, normalized size = 0.8

$$-\frac{-3 b^2 x^2 - 15 abx + 15 a^2}{5} \frac{1}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(4/3),x)
```

```
[Out] -3/5*(-b^2*x^2-5*a*b*x+5*a^2)/x^(1/3)
```

Maxima [A] time = 1.33996, size = 32, normalized size = 1.

$$\frac{3}{5} b^2 x^{\frac{5}{3}} + 3 abx^{\frac{2}{3}} - \frac{3 a^2}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="maxima")
```

```
[Out] 3/5*b^2*x^(5/3) + 3*a*b*x^(2/3) - 3*a^2/x^(1/3)
```

Fricas [A] time = 1.4694, size = 55, normalized size = 1.72

$$\frac{3 \left(b^2 x^2 + 5 abx - 5 a^2\right)}{5 x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="fricas")
```

```
[Out] 3/5*(b^2*x^2 + 5*a*b*x - 5*a^2)/x^(1/3)
```

Sympy [C] time = 2.22666, size = 1828, normalized size = 57.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(4/3),x)
```

```
[Out] Piecewise((-27*a**(29/3)*b**(1/3)*(-1 + b*(a/b + x)/a)**(2/3)*exp(I*pi/3)/(
-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
 x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 27*a**(29/3)*b
**(1/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b*
*2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 63*a*
*(26/3)*b**(4/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)*exp(I*pi/3)/(-5*a**8
*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*
exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 81*a**(26/3)*b**(4/3)
*(a/b + x)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6
```

```
*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 42
*a**(23/3)*b**(7/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**2*exp(I*pi/3)/(-
5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 81*a**(23/3)*b*
*(7/3)*(a/b + x)**2/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3)
- 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi
/3)) + 3*a**(20/3)*b**(10/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(I
*pi/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**
2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 27*a**
(20/3)*b**(10/3)*(a/b + x)**3/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*ex
p(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**
3*exp(I*pi/3)) + 3*a**(17/3)*b**(13/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x
)**4*exp(I*pi/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 1
5*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)
), Abs(b*(a/b + x))/Abs(a) > 1), (27*a**(29/3)*b**(1/3)*(1 - b*(a/b + x)/a)
**(2/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b*
*2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 27*a*
*(29/3)*b**(1/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 1
5*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)
) - 63*a**(26/3)*b**(4/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)/(-5*a**8*exp
(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(
I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 81*a**(26/3)*b**(4/3)*(a/
b + x)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**
2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 42*a**
(23/3)*b**(7/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(-5*a**8*exp(I*pi/3
) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3)
 + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 81*a**(23/3)*b**(7/3)*(a/b + x)*
*2/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a
/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 3*a**(20/3
)*b**(10/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(-5*a**8*exp(I*pi/3) +
15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5
*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 27*a**(20/3)*b**(10/3)*(a/b + x)**3/
(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b
+ x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 3*a**(17/3)*b
**(13/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**4/(-5*a**8*exp(I*pi/3) + 15*
a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a*
*5*b**3*(a/b + x)**3*exp(I*pi/3)), True))
```

Giac [A] time = 1.05394, size = 32, normalized size = 1.

$$\frac{3}{5}\,b^2x^{\frac{5}{3}} + 3\,abx^{\frac{2}{3}} - \frac{3\,a^2}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="giac")
```

```
[Out] 3/5*b^2*x^(5/3) + 3*a*b*x^(2/3) - 3*a^2/x^(1/3)
```

3.665 $\int \frac{(a+bx)^2}{x^{5/3}} dx$

Optimal. Leaf size=34

$$-\frac{3a^2}{2x^{2/3}}+6ab\sqrt[3]{x}+\frac{3}{4}b^2x^{4/3}$$

```
[Out] (-3*a^2)/(2*x^(2/3)) + 6*a*b*x^(1/3) + (3*b^2*x^(4/3))/4
```

Rubi [A] time = 0.0071943, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{3a^2}{2x^{2/3}}+6ab\sqrt[3]{x}+\frac{3}{4}b^2x^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(5/3),x]
```

```
[Out] (-3*a^2)/(2*x^(2/3)) + 6*a*b*x^(1/3) + (3*b^2*x^(4/3))/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{5/3}}\,dx &= \int \left(\frac{a^2}{x^{5/3}}+\frac{2ab}{x^{2/3}}+b^2\sqrt[3]{x}\right)dx\\ &= -\frac{3a^2}{2x^{2/3}}+6ab\sqrt[3]{x}+\frac{3}{4}b^2x^{4/3}\end{aligned}$$

Mathematica [A] time = 0.0074894, size = 27, normalized size = 0.79

$$\frac{3\left(-2a^2+8abx+b^2x^2\right)}{4x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(5/3),x]
```

```
[Out] (3*(-2*a^2 + 8*a*b*x + b^2*x^2))/(4*x^(2/3))
```

Maple [A] time = 0.004, size = 25, normalized size = 0.7

$$-\frac{-3\,b^2x^2-24\,abx+6\,a^2}{4}x^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(5/3),x)
```

```
[Out] -3/4*(-b^2*x^2-8*a*b*x+2*a^2)/x^(2/3)
```

Maxima [A] time = 1.5608, size = 32, normalized size = 0.94

$$\frac{3}{4} b^2 x^{\frac{4}{3}} + 6 abx^{\frac{1}{3}} - \frac{3 a^2}{2 x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="maxima")
```

```
[Out] 3/4*b^2*x^(4/3) + 6*a*b*x^(1/3) - 3/2*a^2/x^(2/3)
```

Fricas [A] time = 1.45979, size = 55, normalized size = 1.62

$$\frac{3 \left(b^2 x^2 + 8 abx - 2 a^2\right)}{4 x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b^2*x^2 + 8*a*b*x - 2*a^2)/x^(2/3)
```

Sympy [C] time = 2.18521, size = 1958, normalized size = 57.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(5/3),x)
```

```
[Out] Piecewise((-27*a**(28/3)*b**(2/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)
/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*
(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 27*a
**(28/3)*b**(2/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3
) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(
2*I*pi/3)) + 72*a**(25/3)*b**(5/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*ex
p(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12
*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi
/3)) + 81*a**(25/3)*b**(5/3)*(a/b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(
a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b
**3*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(22/3)*b**(8/3)*(-1 + b*(a/b + x)/a
)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b
 + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3
*(a/b + x)**3*exp(2*I*pi/3)) - 81*a**(22/3)*b**(8/3)*(a/b + x)**2/(-4*a**8*
exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)*
```

```
*2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) + 12*a**(19/3)*b
**(11/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2*I*pi/3)/(-4*a**8*ex
p(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2
*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) + 27*a**(19/3)*b**
(11/3)*(a/b + x)**3/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi
/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*ex
p(2*I*pi/3)) + 3*a**(16/3)*b**(14/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*
*4*exp(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3)
 - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2
*I*pi/3)), Abs(b*(a/b + x))/Abs(a) > 1), (27*a**(28/3)*b**(2/3)*(1 - b*(a/b
 + x)/a)**(1/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3)
- 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*
I*pi/3)) - 27*a**(28/3)*b**(2/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b +
x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a
/b + x)**3*exp(2*I*pi/3)) - 72*a**(25/3)*b**(5/3)*(1 - b*(a/b + x)/a)**(1/3
)*(a/b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12
*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi
/3)) + 81*a**(25/3)*b**(5/3)*(a/b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(
a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b
**3*(a/b + x)**3*exp(2*I*pi/3)) + 60*a**(22/3)*b**(8/3)*(1 - b*(a/b + x)/a)
**(1/3)*(a/b + x)**2/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*p
i/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*e
xp(2*I*pi/3)) - 81*a**(22/3)*b**(8/3)*(a/b + x)**2/(-4*a**8*exp(2*I*pi/3) +
 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3
) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 12*a**(19/3)*b**(11/3)*(1 - b
*(a/b + x)/a)**(1/3)*(a/b + x)**3/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b +
 x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(
a/b + x)**3*exp(2*I*pi/3)) + 27*a**(19/3)*b**(11/3)*(a/b + x)**3/(-4*a**8*e
xp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**
2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 3*a**(16/3)*b**
(14/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(-4*a**8*exp(2*I*pi/3) + 12*
a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) +
4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)), True))
```

Giac [A] time = 1.05725, size = 32, normalized size = 0.94

$$\frac{3}{4}\,b^2x^{\frac{4}{3}} + 6\,abx^{\frac{1}{3}} - \frac{3\,a^2}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="giac")
```

```
[Out] 3/4*b^2*x^(4/3) + 6*a*b*x^(1/3) - 3/2*a^2/x^(2/3)
```

3.666 $\int x^{5/3}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{9}{11}a^2bx^{11/3}+\frac{3}{8}a^3x^{8/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}$$

```
[Out] (3*a^3*x^(8/3))/8 + (9*a^2*b*x^(11/3))/11 + (9*a*b^2*x^(14/3))/14 + (3*b^3*
x^(17/3))/17
```

Rubi [A] time = 0.0104423, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{11}a^2bx^{11/3}+\frac{3}{8}a^3x^{8/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(8/3))/8 + (9*a^2*b*x^(11/3))/11 + (9*a*b^2*x^(14/3))/14 + (3*b^3*
x^(17/3))/17
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)^3\,dx &= \int \left(a^3x^{5/3}+3a^2bx^{8/3}+3ab^2x^{11/3}+b^3x^{14/3}\right)dx\\ &= \frac{3}{8}a^3x^{8/3}+\frac{9}{11}a^2bx^{11/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}\end{aligned}$$

Mathematica [A] time = 0.0117228, size = 39, normalized size = 0.76

$$\frac{3x^{8/3}\left(2856a^2bx+1309a^3+2244ab^2x^2+616b^3x^3\right)}{10472}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(8/3)*(1309*a^3 + 2856*a^2*b*x + 2244*a*b^2*x^2 + 616*b^3*x^3))/10472
```

Maple [A] time = 0.004, size = 36, normalized size = 0.7

$$\frac{1848\,b^3x^3+6732\,ab^2x^2+8568\,a^2bx+3927\,a^3}{10472}x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a)^3,x)
```

```
[Out] 3/10472*x^(8/3)*(616*b^3*x^3+2244*a*b^2*x^2+2856*a^2*b*x+1309*a^3)
```

Maxima [A] time = 1.42024, size = 47, normalized size = 0.92

$$\frac{3}{17}\,b^3x^{\frac{17}{3}}+\frac{9}{14}\,ab^2x^{\frac{14}{3}}+\frac{9}{11}\,a^2bx^{\frac{11}{3}}+\frac{3}{8}\,a^3x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/17*b^3*x^(17/3) + 9/14*a*b^2*x^(14/3) + 9/11*a^2*b*x^(11/3) + 3/8*a^3*x^(
8/3)
```

Fricas [A] time = 1.54968, size = 108, normalized size = 2.12

$$\frac{3}{10472}\left(616\,b^3x^5+2244\,ab^2x^4+2856\,a^2bx^3+1309\,a^3x^2\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/10472*(616*b^3*x^5 + 2244*a*b^2*x^4 + 2856*a^2*b*x^3 + 1309*a^3*x^2)*x^(2
/3)
```

Sympy [A] time = 7.31497, size = 49, normalized size = 0.96

$$\frac{3a^3x^{\frac{8}{3}}}{8}+\frac{9a^2bx^{\frac{11}{3}}}{11}+\frac{9ab^2x^{\frac{14}{3}}}{14}+\frac{3b^3x^{\frac{17}{3}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(8/3)/8 + 9*a**2*b*x**(11/3)/11 + 9*a*b**2*x**(14/3)/14 + 3*b**3*
x**(17/3)/17
```

Giac [A] time = 1.07897, size = 47, normalized size = 0.92

$$\frac{3}{17}\,b^3x^{\frac{17}{3}}+\frac{9}{14}\,ab^2x^{\frac{14}{3}}+\frac{9}{11}\,a^2bx^{\frac{11}{3}}+\frac{3}{8}\,a^3x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/17*b^3*x^(17/3) + 9/14*a*b^2*x^(14/3) + 9/11*a^2*b*x^(11/3) + 3/8*a^3*x^(
8/3)
```

3.667 $\int x^{4/3}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{9}{10}a^2bx^{10/3}+\frac{3}{7}a^3x^{7/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}$$

```
[Out] (3*a^3*x^(7/3))/7 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*x^(13/3))/13 + (3*b^3*
x^(16/3))/16
```

Rubi [A] time = 0.0109155, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{10}a^2bx^{10/3}+\frac{3}{7}a^3x^{7/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(7/3))/7 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*x^(13/3))/13 + (3*b^3*
x^(16/3))/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)^3\,dx &= \int \left(a^3x^{4/3}+3a^2bx^{7/3}+3ab^2x^{10/3}+b^3x^{13/3}\right)dx\\ &= \frac{3}{7}a^3x^{7/3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}\end{aligned}$$

Mathematica [A] time = 0.0112979, size = 39, normalized size = 0.76

$$\frac{3x^{7/3}\left(2184a^2bx+1040a^3+1680ab^2x^2+455b^3x^3\right)}{7280}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(7/3)*(1040*a^3 + 2184*a^2*b*x + 1680*a*b^2*x^2 + 455*b^3*x^3))/7280
```

Maple [A] time = 0.004, size = 36, normalized size = 0.7

$$\frac{1365\,b^3x^3+5040\,ab^2x^2+6552\,a^2bx+3120\,a^3}{7280}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a)^3,x)
```

```
[Out] 3/7280*x^(7/3)*(455*b^3*x^3+1680*a*b^2*x^2+2184*a^2*b*x+1040*a^3)
```

Maxima [A] time = 1.4617, size = 47, normalized size = 0.92

$$\frac{3}{16}\,b^3x^{\frac{16}{3}} + \frac{9}{13}\,ab^2x^{\frac{13}{3}} + \frac{9}{10}\,a^2bx^{\frac{10}{3}} + \frac{3}{7}\,a^3x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/16*b^3*x^(16/3) + 9/13*a*b^2*x^(13/3) + 9/10*a^2*b*x^(10/3) + 3/7*a^3*x^(
7/3)
```

Fricas [A] time = 1.47577, size = 107, normalized size = 2.1

$$\frac{3}{7280}\left(455\,b^3x^5 + 1680\,ab^2x^4 + 2184\,a^2bx^3 + 1040\,a^3x^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/7280*(455*b^3*x^5 + 1680*a*b^2*x^4 + 2184*a^2*b*x^3 + 1040*a^3*x^2)*x^(1/
3)
```

Sympy [A] time = 5.23708, size = 49, normalized size = 0.96

$$\frac{3a^3x^{\frac{7}{3}}}{7} + \frac{9a^2bx^{\frac{10}{3}}}{10} + \frac{9ab^2x^{\frac{13}{3}}}{13} + \frac{3b^3x^{\frac{16}{3}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(7/3)/7 + 9*a**2*b*x**(10/3)/10 + 9*a*b**2*x**(13/3)/13 + 3*b**3*
x**(16/3)/16
```

Giac [A] time = 1.05051, size = 47, normalized size = 0.92

$$\frac{3}{16}\,b^3x^{\frac{16}{3}} + \frac{9}{13}\,ab^2x^{\frac{13}{3}} + \frac{9}{10}\,a^2bx^{\frac{10}{3}} + \frac{3}{7}\,a^3x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/16*b^3*x^(16/3) + 9/13*a*b^2*x^(13/3) + 9/10*a^2*b*x^(10/3) + 3/7*a^3*x^(
7/3)
```

3.668 $\int x^{2/3}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{9}{8}a^2bx^{8/3}+\frac{3}{5}a^3x^{5/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}$$

```
[Out] (3*a^3*x^(5/3))/5 + (9*a^2*b*x^(8/3))/8 + (9*a*b^2*x^(11/3))/11 + (3*b^3*x^
(14/3))/14
```

Rubi [A] time = 0.010782, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{8}a^2bx^{8/3}+\frac{3}{5}a^3x^{5/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(5/3))/5 + (9*a^2*b*x^(8/3))/8 + (9*a*b^2*x^(11/3))/11 + (3*b^3*x^
(14/3))/14
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)^3\,dx &= \int \left(a^3x^{2/3}+3a^2bx^{5/3}+3ab^2x^{8/3}+b^3x^{11/3}\right)dx\\ &= \frac{3}{5}a^3x^{5/3}+\frac{9}{8}a^2bx^{8/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}\end{aligned}$$

Mathematica [A] time = 0.0103705, size = 39, normalized size = 0.76

$$\frac{3x^{5/3}\left(1155a^2bx+616a^3+840ab^2x^2+220b^3x^3\right)}{3080}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(5/3)*(616*a^3 + 1155*a^2*b*x + 840*a*b^2*x^2 + 220*b^3*x^3))/3080
```

Maple [A] time = 0.004, size = 36, normalized size = 0.7

$$\frac{660\,b^3x^3+2520\,ab^2x^2+3465\,a^2bx+1848\,a^3}{3080}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a)^3,x)
```

```
[Out] 3/3080*x^(5/3)*(220*b^3*x^3+840*a*b^2*x^2+1155*a^2*b*x+616*a^3)
```

Maxima [A] time = 1.18694, size = 47, normalized size = 0.92

$$\frac{3}{14}\,b^{3}x^{\frac{14}{3}} + \frac{9}{11}\,ab^{2}x^{\frac{11}{3}} + \frac{9}{8}\,a^{2}bx^{\frac{8}{3}} + \frac{3}{5}\,a^{3}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/14*b^3*x^(14/3) + 9/11*a*b^2*x^(11/3) + 9/8*a^2*b*x^(8/3) + 3/5*a^3*x^(5/
3)
```

Fricas [A] time = 1.45842, size = 101, normalized size = 1.98

$$\frac{3}{3080}\left(220\,b^{3}x^{4} + 840\,ab^{2}x^{3} + 1155\,a^{2}bx^{2} + 616\,a^{3}x\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/3080*(220*b^3*x^4 + 840*a*b^2*x^3 + 1155*a^2*b*x^2 + 616*a^3*x)*x^(2/3)
```

Sympy [A] time = 2.50533, size = 49, normalized size = 0.96

$$\frac{3a^{3}x^{\frac{5}{3}}}{5} + \frac{9a^{2}bx^{\frac{8}{3}}}{8} + \frac{9ab^{2}x^{\frac{11}{3}}}{11} + \frac{3b^{3}x^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(5/3)/5 + 9*a**2*b*x**(8/3)/8 + 9*a*b**2*x**(11/3)/11 + 3*b**3*x*
*(14/3)/14
```

Giac [A] time = 1.05635, size = 47, normalized size = 0.92

$$\frac{3}{14}\,b^{3}x^{\frac{14}{3}} + \frac{9}{11}\,ab^{2}x^{\frac{11}{3}} + \frac{9}{8}\,a^{2}bx^{\frac{8}{3}} + \frac{3}{5}\,a^{3}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/14*b^3*x^(14/3) + 9/11*a*b^2*x^(11/3) + 9/8*a^2*b*x^(8/3) + 3/5*a^3*x^(5/
3)
```

3.669 $\int \sqrt[3]{x}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{9}{7}a^2bx^{7/3}+\frac{3}{4}a^3x^{4/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}$$

```
[Out] (3*a^3*x^(4/3))/4 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^(10/3))/10 + (3*b^3*x^
(13/3))/13
```

Rubi [A] time = 0.0111296, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{7}a^2bx^{7/3}+\frac{3}{4}a^3x^{4/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(4/3))/4 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^(10/3))/10 + (3*b^3*x^
(13/3))/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}(a+bx)^3\,dx&=\int \left(a^3\sqrt[3]{x}+3a^2bx^{4/3}+3ab^2x^{7/3}+b^3x^{10/3}\right)dx\\&=\frac{3}{4}a^3x^{4/3}+\frac{9}{7}a^2bx^{7/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}\end{aligned}$$

Mathematica [A] time = 0.0100032, size = 39, normalized size = 0.76

$$\frac{3x^{4/3}\left(780a^2bx+455a^3+546ab^2x^2+140b^3x^3\right)}{1820}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(4/3)*(455*a^3 + 780*a^2*b*x + 546*a*b^2*x^2 + 140*b^3*x^3))/1820
```

Maple [A] time = 0.003, size = 36, normalized size = 0.7

$$\frac{420\,b^3x^3+1638\,ab^2x^2+2340\,a^2bx+1365\,a^3}{1820}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a)^3,x)
```

```
[Out] 3/1820*x^(4/3)*(140*b^3*x^3+546*a*b^2*x^2+780*a^2*b*x+455*a^3)
```

Maxima [A] time = 1.18618, size = 47, normalized size = 0.92

$$\frac{3}{13}\,b^3x^{\frac{13}{3}}+\frac{9}{10}\,ab^2x^{\frac{10}{3}}+\frac{9}{7}\,a^2bx^{\frac{7}{3}}+\frac{3}{4}\,a^3x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/13*b^3*x^(13/3) + 9/10*a*b^2*x^(10/3) + 9/7*a^2*b*x^(7/3) + 3/4*a^3*x^(4/
3)
```

Fricas [A] time = 1.44294, size = 100, normalized size = 1.96

$$\frac{3}{1820}\left(140\,b^3x^4+546\,ab^2x^3+780\,a^2bx^2+455\,a^3x\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/1820*(140*b^3*x^4 + 546*a*b^2*x^3 + 780*a^2*b*x^2 + 455*a^3*x)*x^(1/3)
```

Sympy [C] time = 4.11791, size = 5013, normalized size = 98.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a)**3,x)
```

```
[Out] Piecewise((-243*a**(73/3)*(-1 + b*(a/b + x)/a)**(1/3)/(1820*a**20*b**(4/3)
- 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 364
00*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 1092
0*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a
**(73/3)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x)
+ 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 +
 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 +
1820*a**14*b**(22/3)*(a/b + x)**6) + 1377*a**(70/3)*b*(-1 + b*(a/b + x)/a)*
*(1/3)*(a/b + x)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27
300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 273
00*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820
*a**14*b**(22/3)*(a/b + x)**6) - 1458*a**(70/3)*b*(a/b + x)*exp(I*pi/3)/(18
20*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*
(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(
a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/
```

```
b + x)**6) - 3213*a**(67/3)*b**2*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(
1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3
)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)
*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(
a/b + x)**6) + 3645*a**(67/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(1820*a**20*b**
(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2
 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4
- 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) +
 3927*a**(64/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(1820*a**20*b
**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)*
*2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**
4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6)
 - 4860*a**(64/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(1820*a**20*b**(4/3) - 1092
0*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**
17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**1
5*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 2163*a**(61
/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(1820*a**20*b**(4/3) - 10
920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a
**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a*
*15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 3645*a**(
61/3)*b**4*(a/b + x)**4*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(
7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)
*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*
(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 1827*a**(58/3)*b**5*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**5/(1820*a**20*b**(4/3) - 10920*a**19*b*
*(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/
3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3
)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 1458*a**(58/3)*b**5*(
a/b + x)**5*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b +
x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**
3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5
 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 6573*a**(55/3)*b**6*(-1 + b*(a/b +
x)/a)**(1/3)*(a/b + x)**6/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b
+ x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)
**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)*
*5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a**(55/3)*b**6*(a/b + x)**6*e
xp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a*
*18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**
16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14
*b**(22/3)*(a/b + x)**6) - 8787*a**(52/3)*b**7*(-1 + b*(a/b + x)/a)**(1/3)*
(a/b + x)**7/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*
a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a
**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**
14*b**(22/3)*(a/b + x)**6) + 6498*a**(49/3)*b**8*(-1 + b*(a/b + x)/a)**(1/3
)*(a/b + x)**8/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 2730
0*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300
*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a
**14*b**(22/3)*(a/b + x)**6) - 2562*a**(46/3)*b**9*(-1 + b*(a/b + x)/a)**(1
/3)*(a/b + x)**9/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27
300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 273
00*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820
*a**14*b**(22/3)*(a/b + x)**6) + 420*a**(43/3)*b**10*(-1 + b*(a/b + x)/a)**
(1/3)*(a/b + x)**10/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) +
 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 +
27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1
820*a**14*b**(22/3)*(a/b + x)**6), Abs(b*(a/b + x))/Abs(a) > 1), (-243*a**(
73/3)*(1 - b*(a/b + x)/a)**(1/3)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a
**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*
b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b
```

```
**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a**(73/3)*
exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a
**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a*
*16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**1
4*b**(22/3)*(a/b + x)**6) + 1377*a**(70/3)*b*(1 - b*(a/b + x)/a)**(1/3)*(a/
b + x)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) +
27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 2
7300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 18
20*a**14*b**(22/3)*(a/b + x)**6) - 1458*a**(70/3)*b*(a/b + x)*exp(I*pi/3)/(
1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3
)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)
*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(
a/b + x)**6) - 3213*a**(67/3)*b**2*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*
exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a
**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a*
*16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**1
4*b**(22/3)*(a/b + x)**6) + 3645*a**(67/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(1
820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)
*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*
(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a
/b + x)**6) + 3927*a**(64/3)*b**3*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**3*e
xp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a*
*18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**
16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14
*b**(22/3)*(a/b + x)**6) - 4860*a**(64/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(18
20*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*
(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(
a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/
b + x)**6) - 2163*a**(61/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4*ex
p(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**
18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**1
6*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*
b**(22/3)*(a/b + x)**6) + 3645*a**(61/3)*b**4*(a/b + x)**4*exp(I*pi/3)/(182
0*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(
a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a
/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b
 + x)**6) - 1827*a**(58/3)*b**5*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**5*exp
(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**1
8*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16
*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b
**(22/3)*(a/b + x)**6) - 1458*a**(58/3)*b**5*(a/b + x)**5*exp(I*pi/3)/(1820
*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a
/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/
b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b
+ x)**6) + 6573*a**(55/3)*b**6*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**6*exp(
I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18
*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*
b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b*
*(22/3)*(a/b + x)**6) + 243*a**(55/3)*b**6*(a/b + x)**6*exp(I*pi/3)/(1820*a
**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b
 + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b
+ x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b +
x)**6) - 8787*a**(52/3)*b**7*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**7*exp(I*
pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b
**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b*
*(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(
22/3)*(a/b + x)**6) + 6498*a**(49/3)*b**8*(1 - b*(a/b + x)/a)**(1/3)*(a/b +
 x)**8*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) +
27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 2
```

```
7300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 18
20*a**14*b**(22/3)*(a/b + x)**6) - 2562*a**(46/3)*b**9*(1 - b*(a/b + x)/a)*
*(1/3)*(a/b + x)**9*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)
*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/
b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b
 + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 420*a**(43/3)*b**10*(1 - b*
(a/b + x)/a)**(1/3)*(a/b + x)**10*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*
a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17
*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*
b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6), True))
```

Giac [A] time = 1.05415, size = 47, normalized size = 0.92

$$\frac{3}{13}\,b^3x^{\frac{13}{3}} + \frac{9}{10}\,ab^2x^{\frac{10}{3}} + \frac{9}{7}\,a^2bx^{\frac{7}{3}} + \frac{3}{4}\,a^3x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/13*b^3*x^(13/3) + 9/10*a*b^2*x^(10/3) + 9/7*a^2*b*x^(7/3) + 3/4*a^3*x^(4/
3)
```

3.670 $\int \frac{(a+bx)^3}{\sqrt[3]{x}} \, dx$

Optimal. Leaf size=51

$$\frac{9}{5}a^2bx^{5/3} + \frac{3}{2}a^3x^{2/3} + \frac{9}{8}ab^2x^{8/3} + \frac{3}{11}b^3x^{11/3}$$

```
[Out] (3*a^3*x^(2/3))/2 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^(8/3))/8 + (3*b^3*x^(1
1/3))/11
```

Rubi [A] time = 0.0104117, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{5}a^2bx^{5/3} + \frac{3}{2}a^3x^{2/3} + \frac{9}{8}ab^2x^{8/3} + \frac{3}{11}b^3x^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(1/3),x]
```

```
[Out] (3*a^3*x^(2/3))/2 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^(8/3))/8 + (3*b^3*x^(1
1/3))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^3}{\sqrt[3]{x}} \, dx &= \int \left(\frac{a^3}{\sqrt[3]{x}} + 3a^2bx^{2/3} + 3ab^2x^{5/3} + b^3x^{8/3} \right) dx \\ &= \frac{3}{2}a^3x^{2/3} + \frac{9}{5}a^2bx^{5/3} + \frac{9}{8}ab^2x^{8/3} + \frac{3}{11}b^3x^{11/3} \end{aligned}$$

Mathematica [A] time = 0.0102657, size = 39, normalized size = 0.76

$$\frac{3}{440}x^{2/3}\left(264a^2bx + 220a^3 + 165ab^2x^2 + 40b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(1/3),x]
```

```
[Out] (3*x^(2/3)*(220*a^3 + 264*a^2*b*x + 165*a*b^2*x^2 + 40*b^3*x^3))/440
```

Maple [A] time = 0.004, size = 36, normalized size = 0.7

$$\frac{120\,b^{3}x^{3}+495\,ab^{2}x^{2}+792\,a^{2}bx+660\,a^{3}}{440}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(1/3),x)
```

```
[Out] 3/440*x^(2/3)*(40*b^3*x^3+165*a*b^2*x^2+264*a^2*b*x+220*a^3)
```

Maxima [A] time = 1.1043, size = 47, normalized size = 0.92

$$\frac{3}{11}\,b^{3}x^{\frac{11}{3}}+\frac{9}{8}\,ab^{2}x^{\frac{8}{3}}+\frac{9}{5}\,a^{2}bx^{\frac{5}{3}}+\frac{3}{2}\,a^{3}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*b^3*x^(11/3) + 9/8*a*b^2*x^(8/3) + 9/5*a^2*b*x^(5/3) + 3/2*a^3*x^(2/3)
```

Fricas [A] time = 1.50903, size = 92, normalized size = 1.8

$$\frac{3}{440}\left(40\,b^{3}x^{3}+165\,ab^{2}x^{2}+264\,a^{2}bx+220\,a^{3}\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(40*b^3*x^3 + 165*a*b^2*x^2 + 264*a^2*b*x + 220*a^3)*x^(2/3)
```

Sympy [C] time = 3.63954, size = 6248, normalized size = 122.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(1/3),x)
```

```
[Out] Piecewise((243*a**(71/3)*(-1 + b*(a/b + x)/a)**(2/3)*exp(I*pi/3)/(440*a**20
*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a*
*18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*e
xp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**
(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi
/3)) + 243*a**(71/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*
(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800
*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)*
*4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*
b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1296*a**(68/3)*b*(-1 + b*(a/b + x)/a)
**(2/3)*(a/b + x)*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*
```

b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/
3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(
a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 4
40*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1458*a**(68/3)*b*(a/b + x)/(
440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3)
+ 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b
+ x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*
a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6
*exp(I*pi/3)) + 2808*a**(65/3)*b**2*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**
2*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b +
x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*
b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(
I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/
3)*(a/b + x)**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*(a/b + x)**2/(440*a**20*
b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**
18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*ex
p(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(
17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/
3)) - 3120*a**(62/3)*b**3*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(I*pi
/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*p
i/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*
(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) -
2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b +
x)**6*exp(I*pi/3)) - 4860*a**(62/3)*b**3*(a/b + x)**3/(440*a**20*b**(2/3)*e
xp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3
)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3)
+ 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b
 + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1710
*a**(59/3)*b**4*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(I*pi/3)/(440*a
20*b(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 660
0*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)*
*3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15
*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(
I*pi/3)) + 3645*a**(59/3)*b**4*(a/b + x)**4/(440*a**20*b**(2/3)*exp(I*pi/3)
 - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x
)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**
16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*e
xp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 72*a**(56/3)*b
5*(-1 + b*(a/b + x)/a)(2/3)*(a/b + x)**5*exp(I*pi/3)/(440*a**20*b**(2/3
)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(
8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/
3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(
a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1
458*a**(56/3)*b**5*(a/b + x)**5/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**1
9*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*p
i/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)
*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) +
 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1104*a**(53/3)*b**6*(-1 +
b*(a/b + x)/a)**(2/3)*(a/b + x)**6*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi
/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b
+ x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*
a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**
5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 243*a**(53/
3)*b**6*(a/b + x)**6/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*
(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800
*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)*
*4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*
b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1152*a**(50/3)*b**7*(-1 + b*(a/b + x)
/a)**(2/3)*(a/b + x)**7*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*

```
a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp
(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(1
4/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/
3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 585*a**(47/3)*b**8*(-1
 + b*(a/b + x)/a)**(2/3)*(a/b + x)**8*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I
*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a
/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 66
00*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x
)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 120*a**(
44/3)*b**9*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**9*exp(I*pi/3)/(440*a**20*
b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**
18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*ex
p(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(
17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/
3)), Abs(b*(a/b + x))/Abs(a) > 1), (-243*a**(71/3)*(1 - b*(a/b + x)/a)**(2/
3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi
/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(
a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2
640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x
)**6*exp(I*pi/3)) + 243*a**(71/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a*
*19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I
*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/
3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3)
 + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1296*a**(68/3)*b*(1 - b*
(a/b + x)/a)**(2/3)*(a/b + x)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*
b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/
3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(
a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 4
40*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1458*a**(68/3)*b*(a/b + x)/(
440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3)
+ 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b
+ x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*
a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6
*exp(I*pi/3)) - 2808*a**(65/3)*b**2*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**2
/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3
) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/
b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 264
0*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)*
*6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*(a/b + x)**2/(440*a**20*b**(2/3)*exp(
I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(
a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6
600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b +
x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 3120*a*
*(62/3)*b**3*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(440*a**20*b**(2/3)*ex
p(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)
*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) +
 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b
+ x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 4860*
a**(62/3)*b**3*(a/b + x)**3/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b*
*(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3)
 - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/
b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440
*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1710*a**(59/3)*b**4*(1 - b*(a/
b + x)/a)**(2/3)*(a/b + x)**4/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*
b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/
3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(
a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 4
40*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 3645*a**(59/3)*b**4*(a/b + x
)**4/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*
```

```
pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)
*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) -
 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b +
 x)**6*exp(I*pi/3)) - 72*a**(56/3)*b**5*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x
)**5/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*
pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)
*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) -
 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b +
 x)**6*exp(I*pi/3)) - 1458*a**(56/3)*b**5*(a/b + x)**5/(440*a**20*b**(2/3)*
exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/
3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3)
 + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/
b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 110
4*a**(53/3)*b**6*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**6/(440*a**20*b**(2/3
)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(
8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/
3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(
a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 2
43*a**(53/3)*b**6*(a/b + x)**6/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19
*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi
/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*
(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) +
440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1152*a**(50/3)*b**7*(1 - b*
(a/b + x)/a)**(2/3)*(a/b + x)**7/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**
19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*
pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3
)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3)
+ 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 585*a**(47/3)*b**8*(1 - b
*(a/b + x)/a)**(2/3)*(a/b + x)**8/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a*
*19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I
*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/
3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3)
 + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 120*a**(44/3)*b**9*(1 -
b*(a/b + x)/a)**(2/3)*(a/b + x)**9/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a
**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(
I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14
/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3
) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)), True))
```

Giac [A] time = 1.06085, size = 47, normalized size = 0.92

$$\frac{3}{11}\,b^3x^{\frac{11}{3}} + \frac{9}{8}\,ab^2x^{\frac{8}{3}} + \frac{9}{5}\,a^2bx^{\frac{5}{3}} + \frac{3}{2}\,a^3x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="giac")
```

```
[Out] 3/11*b^3*x^(11/3) + 9/8*a*b^2*x^(8/3) + 9/5*a^2*b*x^(5/3) + 3/2*a^3*x^(2/3)
```

3.671 $\int \frac{(a+bx)^3}{x^{2/3}}\,dx$

Optimal. Leaf size=49

$$\frac{9}{4}a^2bx^{4/3}+3a^3\sqrt[3]{x}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}$$

```
[Out] 3*a^3*x^(1/3) + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(7/3))/7 + (3*b^3*x^(10/3)
)/10
```

Rubi [A] time = 0.010978, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{4}a^2bx^{4/3}+3a^3\sqrt[3]{x}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(2/3),x]
```


```
[Out] 3*a^3*x^(1/3) + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(7/3))/7 + (3*b^3*x^(10/3)
)/10
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{2/3}}\,dx &= \int \left(\frac{a^3}{x^{2/3}}+3a^2b\sqrt[3]{x}+3ab^2x^{4/3}+b^3x^{7/3}\right)dx\\ &= 3a^3\sqrt[3]{x}+\frac{9}{4}a^2bx^{4/3}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}\end{aligned}$$

Mathematica [A] time = 0.009869, size = 39, normalized size = 0.8

$$\frac{3}{140}\sqrt[3]{x}\left(105a^2bx+140a^3+60ab^2x^2+14b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(2/3),x]
```

```
[Out] (3*x^(1/3)*(140*a^3 + 105*a^2*b*x + 60*a*b^2*x^2 + 14*b^3*x^3))/140
```

Maple [A] time = 0.003, size = 36, normalized size = 0.7

$$\frac{42\,b^{3}x^{3}+180\,ab^{2}x^{2}+315\,a^{2}bx+420\,a^{3}}{140}\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(2/3),x)
```

```
[Out] 3/140*x^(1/3)*(14*b^3*x^3+60*a*b^2*x^2+105*a^2*b*x+140*a^3)
```

Maxima [A] time = 1.03714, size = 47, normalized size = 0.96

$$\frac{3}{10}\,b^{3}x^{\frac{10}{3}}+\frac{9}{7}\,ab^{2}x^{\frac{7}{3}}+\frac{9}{4}\,a^{2}bx^{\frac{4}{3}}+3\,a^{3}x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/10*b^3*x^(10/3) + 9/7*a*b^2*x^(7/3) + 9/4*a^2*b*x^(4/3) + 3*a^3*x^(1/3)
```

Fricas [A] time = 1.55949, size = 90, normalized size = 1.84

$$\frac{3}{140}\left(14\,b^{3}x^{3}+60\,ab^{2}x^{2}+105\,a^{2}bx+140\,a^{3}\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3/140*(14*b^3*x^3 + 60*a*b^2*x^2 + 105*a^2*b*x + 140*a^3)*x^(1/3)
```

Sympy [C] time = 3.84548, size = 6669, normalized size = 136.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(2/3),x)
```

```
[Out] Piecewise((243*a**(70/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)/(140*a**
20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 21
00*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b +
x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840
*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)
**6*exp(2*I*pi/3)) + 243*a**(70/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*
a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*e
xp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16
*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*ex
p(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 1377*a**(67
/3)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(2*I*pi/3)/(140*a**20*b**(1/
```

```
3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*
b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp
(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b*
*(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2
*I*pi/3)) - 1458*a**(67/3)*b*(a/b + x)/(140*a**20*b**(1/3)*exp(2*I*pi/3) -
840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)*
*2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a
**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**
5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3213*a*
*(64/3)*b**2*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(140*a*
*20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2
100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b +
 x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 84
0*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x
)**6*exp(2*I*pi/3)) + 3645*a**(64/3)*b**2*(a/b + x)**2/(140*a**20*b**(1/3)*
exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**
(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(1
6/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*
pi/3)) - 3927*a**(61/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2
*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*e
xp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*
b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*ex
p(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b*
*(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 4860*a**(61/3)*b**3*(a/b + x)**3/(140
*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3)
+ 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/
b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) -
 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b
+ x)**6*exp(2*I*pi/3)) + 2583*a**(58/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a
/b + x)**4*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(
4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/
3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)
*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3
) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3645*a**(58/3)*b**4*(
a/b + x)**4/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x
)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**
17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4
*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14
*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 693*a**(55/3)*b**5*(-1 + b*(a/b +
x)/a)**(1/3)*(a/b + x)**5*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) -
 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)
**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*
a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)*
*5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 1458*a
**(55/3)*b**5*(a/b + x)**5/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b*
*(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*p
i/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/
3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi
/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 273*a**(52/3)*b**6*
(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**6*exp(2*I*pi/3)/(140*a**20*b**(1/3)*
exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**
(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(1
6/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*
pi/3)) + 243*a**(52/3)*b**6*(a/b + x)**6/(140*a**20*b**(1/3)*exp(2*I*pi/3)
- 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x
)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100
*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)
```

```
**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 387*a
**(49/3)*b**7*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**7*exp(2*I*pi/3)/(140*a
**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) +
2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b
+ x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 8
40*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b +
x)**6*exp(2*I*pi/3)) - 198*a**(46/3)*b**8*(-1 + b*(a/b + x)/a)**(1/3)*(a/b
+ x)**8*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3
)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3)
- 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a
/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) +
 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 42*a**(43/3)*b**9*(-1 +
b*(a/b + x)/a)**(1/3)*(a/b + x)**9*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*
I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*
(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3
) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(
a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3))
, Abs(b*(a/b + x))/Abs(a) > 1), (-243*a**(70/3)*(1 - b*(a/b + x)/a)**(1/3)/
(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi
/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)
*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/
3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(
a/b + x)**6*exp(2*I*pi/3)) + 243*a**(70/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3
) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
 x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 21
00*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b +
x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 137
7*a**(67/3)*b*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)/(140*a**20*b**(1/3)*exp(
2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3
)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi
/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)
*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3
)) - 1458*a**(67/3)*b*(a/b + x)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**
19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(
2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b*
*(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2
*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 3213*a**(64/3)
*b**2*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(140*a**20*b**(1/3)*exp(2*I*p
i/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/
b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) +
 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b
 + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) +
3645*a**(64/3)*b**2*(a/b + x)**2/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a*
*19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp
(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b
**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(
2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3927*a**(61/3
)*b**3*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(140*a**20*b**(1/3)*exp(2*I*
pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a
/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3)
+ 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/
b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) -
 4860*a**(61/3)*b**3*(a/b + x)**3/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a
**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*ex
p(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*
b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp
(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 2583*a**(58/
3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(140*a**20*b**(1/3)*exp(2*I
*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(
```

```
a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3)
 + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a
/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3))
+ 3645*a**(58/3)*b**4*(a/b + x)**4/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*
a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*e
xp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16
*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*ex
p(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 693*a**(55/
3)*b**5*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**5/(140*a**20*b**(1/3)*exp(2*I
*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(
a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3)
 + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a
/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3))
- 1458*a**(55/3)*b**5*(a/b + x)**5/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*
a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*e
xp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16
*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*ex
p(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 273*a**(52/
3)*b**6*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**6/(140*a**20*b**(1/3)*exp(2*I
*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(
a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3)
 + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a
/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3))
+ 243*a**(52/3)*b**6*(a/b + x)**6/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a
**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*ex
p(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*
b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp
(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 387*a**(49/3
)*b**7*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**7/(140*a**20*b**(1/3)*exp(2*I*
pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a
/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3)
+ 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/
b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) +
 198*a**(46/3)*b**8*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**8/(140*a**20*b**(
1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**1
8*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*e
xp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*
b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp
(2*I*pi/3)) - 42*a**(43/3)*b**9*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**9/(14
0*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3)
 + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a
/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3)
- 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b
 + x)**6*exp(2*I*pi/3)), True))
```

Giac [A] time = 1.05315, size = 47, normalized size = 0.96

$$\frac{3}{10}\,b^3x^{\frac{10}{3}}+\frac{9}{7}\,ab^2x^{\frac{7}{3}}+\frac{9}{4}\,a^2bx^{\frac{4}{3}}+3\,a^3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="giac")
```

```
[Out] 3/10*b^3*x^(10/3) + 9/7*a*b^2*x^(7/3) + 9/4*a^2*b*x^(4/3) + 3*a^3*x^(1/3)
```

3.672 $\int \frac{(a+bx)^3}{x^{4/3}} dx$

Optimal. Leaf size=49

$$\frac{9}{2}a^2bx^{2/3} - \frac{3a^3}{\sqrt[3]{x}} + \frac{9}{5}ab^2x^{5/3} + \frac{3}{8}b^3x^{8/3}$$

```
[Out] (-3*a^3)/x^(1/3) + (9*a^2*b*x^(2/3))/2 + (9*a*b^2*x^(5/3))/5 + (3*b^3*x^(8/
3))/8
```

Rubi [A] time = 0.0109002, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{2}a^2bx^{2/3} - \frac{3a^3}{\sqrt[3]{x}} + \frac{9}{5}ab^2x^{5/3} + \frac{3}{8}b^3x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(4/3),x]
```

```
[Out] (-3*a^3)/x^(1/3) + (9*a^2*b*x^(2/3))/2 + (9*a*b^2*x^(5/3))/5 + (3*b^3*x^(8/
3))/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{4/3}} dx &= \int \left(\frac{a^3}{x^{4/3}} + \frac{3a^2b}{\sqrt[3]{x}} + 3ab^2x^{2/3} + b^3x^{5/3}\right) dx \\ &= -\frac{3a^3}{\sqrt[3]{x}} + \frac{9}{2}a^2bx^{2/3} + \frac{9}{5}ab^2x^{5/3} + \frac{3}{8}b^3x^{8/3}\end{aligned}$$

Mathematica [A] time = 0.0101344, size = 39, normalized size = 0.8

$$\frac{3\left(60a^2bx - 40a^3 + 24ab^2x^2 + 5b^3x^3\right)}{40\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(4/3),x]
```

```
[Out] (3*(-40*a^3 + 60*a^2*b*x + 24*a*b^2*x^2 + 5*b^3*x^3))/(40*x^(1/3))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.7

$$-\frac{-15\,b^3x^3-72\,ab^2x^2-180\,a^2bx+120\,a^3}{40}\frac{1}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(4/3),x)
```

```
[Out] -3/40*(-5*b^3*x^3-24*a*b^2*x^2-60*a^2*b*x+40*a^3)/x^(1/3)
```

Maxima [A] time = 1.03584, size = 47, normalized size = 0.96

$$\frac{3}{8}\,b^3x^{\frac{8}{3}}+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{2}\,a^2bx^{\frac{2}{3}}-\frac{3\,a^3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="maxima")
```

```
[Out] 3/8*b^3*x^(8/3) + 9/5*a*b^2*x^(5/3) + 9/2*a^2*b*x^(2/3) - 3*a^3/x^(1/3)
```

Fricas [A] time = 1.52177, size = 85, normalized size = 1.73

$$\frac{3\left(5\,b^3x^3+24\,ab^2x^2+60\,a^2bx-40\,a^3\right)}{40\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^3*x^3 + 24*a*b^2*x^2 + 60*a^2*b*x - 40*a^3)/x^(1/3)
```

Sympy [C] time = 3.62714, size = 4005, normalized size = 81.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(4/3),x)
```

```
[Out] Piecewise((243*a**(68/3)*b**(1/3)*(-1 + b*(a/b + x)/a)**(2/3)/(40*a**20 - 2
40*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b +
x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**1
4*b**6*(a/b + x)**6) - 243*a**(68/3)*b**(1/3)*exp(2*I*pi/3)/(40*a**20 - 240
*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)
**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*
b**6*(a/b + x)**6) - 1296*a**(65/3)*b**(4/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 8
00*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(
```

a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(65/3)*b**(4/3)*(a/b +
x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b +
x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**
15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 2808*a**(62/3)*b**(7/3
)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(40*a**20 - 240*a**19*b*(a/b + x
) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b
4*(a/b + x)4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6
) - 3645*a**(62/3)*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3)/(40*a**20 - 240*a**1
9*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 +
600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*
(a/b + x)**6) - 3120*a**(59/3)*b**(10/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b +
x)**3/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 80
0*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a
/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 4860*a**(59/3)*b**(10/3)*(a/b +
x)**3*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b
+ x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*
a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1830*a**(56/3)*b**(
13/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**4/(40*a**20 - 240*a**19*b*(a/b
+ x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**
16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x
)**6) - 3645*a**(56/3)*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3)/(40*a**20 - 240
*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)
3 + 600*a16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*
b**6*(a/b + x)**6) - 528*a**(53/3)*b**(16/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)**5/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(53/3)*b**(16/3)*(a/
b + x)**5*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*
(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 -
240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 96*a**(50/3)*b*
(19/3)(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**6/(40*a**20 - 240*a**19*b*(a
/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a
16*b4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b +
x)**6) - 243*a**(50/3)*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)/(40*a**20 - 24
0*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x
)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14
*b**6*(a/b + x)**6) - 48*a**(47/3)*b**(22/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)**7/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 15*a**(44/3)*b**(25/3)*(-1 +
b*(a/b + x)/a)**(2/3)*(a/b + x)**8/(40*a**20 - 240*a**19*b*(a/b + x) + 600
*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/
b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6), Abs(
b*(a/b + x))/Abs(a) > 1), (243*a**(68/3)*b**(1/3)*(1 - b*(a/b + x)/a)**(2/3
)*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x
)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**1
5*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 243*a**(68/3)*b**(1/3)*
exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)*
*2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*
b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 1296*a**(65/3)*b**(4/3)*(
1 - b*(a/b + x)/a)**(2/3)*(a/b + x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(
a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*
a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b
+ x)**6) + 1458*a**(65/3)*b**(4/3)*(a/b + x)*exp(2*I*pi/3)/(40*a**20 - 240*
a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)*
*3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b
6*(a/b + x)6) + 2808*a**(62/3)*b**(7/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b
+ x)**2*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(
a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 2

```
40*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 3645*a**(62/3)*b
**(7/3)*(a/b + x)**2*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*
a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b
 + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 3120
*a**(59/3)*b**(10/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(2*I*pi/3)/
(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17
*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)
**5 + 40*a**14*b**6*(a/b + x)**6) + 4860*a**(59/3)*b**(10/3)*(a/b + x)**3*e
xp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**
2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b
**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1830*a**(56/3)*b**(13/3)*(
1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(2*I*pi/3)/(40*a**20 - 240*a**19*
b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 6
00*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a
/b + x)**6) - 3645*a**(56/3)*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3)/(40*a**20
 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/
b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*
a**14*b**6*(a/b + x)**6) - 528*a**(53/3)*b**(16/3)*(1 - b*(a/b + x)/a)**(2/
3)*(a/b + x)**5*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18
*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)
**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(
53/3)*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x
) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b
**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6
) + 96*a**(50/3)*b**(19/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**6*exp(2*I*
pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800
*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/
b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 243*a**(50/3)*b**(19/3)*(a/b + x)
**6*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b +
 x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a*
*15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 48*a**(47/3)*b**(22/3
)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**7*exp(2*I*pi/3)/(40*a**20 - 240*a**
19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3
+ 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6
*(a/b + x)**6) + 15*a**(44/3)*b**(25/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x
)**8*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b
+ x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a
**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6), True))
```

Giac [A] time = 1.05932, size = 47, normalized size = 0.96

$$\frac{3}{8}b^3x^{\frac{8}{3}} + \frac{9}{5}ab^2x^{\frac{5}{3}} + \frac{9}{2}a^2bx^{\frac{2}{3}} - \frac{3a^3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="giac")
```

```
[Out] 3/8*b^3*x^(8/3) + 9/5*a*b^2*x^(5/3) + 9/2*a^2*b*x^(2/3) - 3*a^3/x^(1/3)
```

3.673 $\int \frac{(a+bx)^3}{x^{5/3}} dx$

Optimal. Leaf size=49

$$9a^2b\sqrt[3]{x}-\frac{3a^3}{2x^{2/3}}+\frac{9}{4}ab^2x^{4/3}+\frac{3}{7}b^3x^{7/3}$$

```
[Out] (-3*a^3)/(2*x^(2/3)) + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(4/3))/4 + (3*b^3*x^(7/3))/7
```

Rubi [A] time = 0.0107705, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$9a^2b\sqrt[3]{x}-\frac{3a^3}{2x^{2/3}}+\frac{9}{4}ab^2x^{4/3}+\frac{3}{7}b^3x^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(5/3),x]
```

```
[Out] (-3*a^3)/(2*x^(2/3)) + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(4/3))/4 + (3*b^3*x^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{5/3}} dx &= \int \left(\frac{a^3}{x^{5/3}}+\frac{3a^2b}{x^{2/3}}+3ab^2\sqrt[3]{x}+b^3x^{4/3}\right) dx\\ &= -\frac{3a^3}{2x^{2/3}}+9a^2b\sqrt[3]{x}+\frac{9}{4}ab^2x^{4/3}+\frac{3}{7}b^3x^{7/3}\end{aligned}$$

Mathematica [A] time = 0.0105027, size = 39, normalized size = 0.8

$$\frac{3\left(84a^2bx-14a^3+21ab^2x^2+4b^3x^3\right)}{28x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(5/3),x]
```

```
[Out] (3*(-14*a^3 + 84*a^2*b*x + 21*a*b^2*x^2 + 4*b^3*x^3))/(28*x^(2/3))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.7

$$-\frac{-12\,b^3x^3-63\,ab^2x^2-252\,a^2bx+42\,a^3}{28}x^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(5/3),x)
```

```
[Out] -3/28*(-4*b^3*x^3-21*a*b^2*x^2-84*a^2*b*x+14*a^3)/x^(2/3)
```

Maxima [A] time = 1.00691, size = 47, normalized size = 0.96

$$\frac{3}{7}\,b^3x^{\frac{7}{3}}+\frac{9}{4}\,ab^2x^{\frac{4}{3}}+9\,a^2bx^{\frac{1}{3}}-\frac{3\,a^3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="maxima")
```

```
[Out] 3/7*b^3*x^(7/3) + 9/4*a*b^2*x^(4/3) + 9*a^2*b*x^(1/3) - 3/2*a^3/x^(2/3)
```

Fricas [A] time = 1.48839, size = 85, normalized size = 1.73

$$\frac{3\left(4\,b^3x^3+21\,ab^2x^2+84\,a^2bx-14\,a^3\right)}{28\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="fricas")
```

```
[Out] 3/28*(4*b^3*x^3 + 21*a*b^2*x^2 + 84*a^2*b*x - 14*a^3)/x^(2/3)
```

Sympy [C] time = 3.65347, size = 3966, normalized size = 80.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(5/3),x)
```

```
[Out] Piecewise((243*a**(67/3)*b**(2/3)*(-1 + b*(a/b + x)/a)**(1/3)/(28*a**20 - 1
68*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b +
x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**1
4*b**6*(a/b + x)**6) - 243*a**(67/3)*b**(2/3)*exp(I*pi/3)/(28*a**20 - 168*a
**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**
3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b*
*6*(a/b + x)**6) - 1377*a**(64/3)*b**(5/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b
 + x)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560
*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/
b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(64/3)*b**(5/3)*(a/b + x)
```

```
*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**
2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b
**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 3213*a**(61/3)*b**(8/3)*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(28*a**20 - 168*a**19*b*(a/b + x) +
420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*
(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) -
3645*a**(61/3)*b**(8/3)*(a/b + x)**2*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a
/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a
**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b +
 x)**6) - 3927*a**(58/3)*b**(11/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3
/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**1
7*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x
)**5 + 28*a**14*b**6*(a/b + x)**6) + 4860*a**(58/3)*b**(11/3)*(a/b + x)**3*
exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2
 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b*
*5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 2625*a**(55/3)*b**(14/3)*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(28*a**20 - 168*a**19*b*(a/b + x) +
420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*
(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) -
3645*a**(55/3)*b**(14/3)*(a/b + x)**4*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(
a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*
a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b
+ x)**6) - 903*a**(52/3)*b**(17/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**5
/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**1
7*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x
)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(52/3)*b**(17/3)*(a/b + x)**5*
exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2
 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b*
*5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 147*a**(49/3)*b**(20/3)*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**6/(28*a**20 - 168*a**19*b*(a/b + x) + 4
20*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(
a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 2
43*a**(49/3)*b**(20/3)*(a/b + x)**6*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/
b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a*
*16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b +
x)**6) - 33*a**(46/3)*b**(23/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**7/(2
8*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b
**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**
5 + 28*a**14*b**6*(a/b + x)**6) + 12*a**(43/3)*b**(26/3)*(-1 + b*(a/b + x)/
a)**(1/3)*(a/b + x)**8/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(
a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 1
68*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6), Abs(b*(a/b + x))/
Abs(a) > 1), (243*a**(67/3)*b**(2/3)*(1 - b*(a/b + x)/a)**(1/3)*exp(I*pi/3)
/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**1
7*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x
)**5 + 28*a**14*b**6*(a/b + x)**6) - 243*a**(67/3)*b**(2/3)*exp(I*pi/3)/(28
*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b*
*3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5
 + 28*a**14*b**6*(a/b + x)**6) - 1377*a**(64/3)*b**(5/3)*(1 - b*(a/b + x)/a
)**(1/3)*(a/b + x)*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**1
8*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x
)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**
(64/3)*b**(5/3)*(a/b + x)*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 4
20*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(
a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 3
213*a**(61/3)*b**(8/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(I*pi/3)/
(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17
*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)
**5 + 28*a**14*b**6*(a/b + x)**6) - 3645*a**(61/3)*b**(8/3)*(a/b + x)**2*ex
```

```
p(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 -
 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5
*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 3927*a**(58/3)*b**(11/3)*(1 -
 b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/
b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a*
*16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b +
x)**6) + 4860*a**(58/3)*b**(11/3)*(a/b + x)**3*exp(I*pi/3)/(28*a**20 - 168*
a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)*
*3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b
**6*(a/b + x)**6) + 2625*a**(55/3)*b**(14/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/
b + x)**4*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a
/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 16
8*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 3645*a**(55/3)*b*
*(14/3)*(a/b + x)**4*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a*
*18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b +
 x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 903*a*
*(52/3)*b**(17/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**5*exp(I*pi/3)/(28*a
**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3
*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 +
 28*a**14*b**6*(a/b + x)**6) + 1458*a**(52/3)*b**(17/3)*(a/b + x)**5*exp(I*
pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560
*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/
b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 147*a**(49/3)*b**(20/3)*(1 - b*(a
/b + x)/a)**(1/3)*(a/b + x)**6*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x
) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b
**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6
) - 243*a**(49/3)*b**(20/3)*(a/b + x)**6*exp(I*pi/3)/(28*a**20 - 168*a**19*
b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 4
20*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a
/b + x)**6) - 33*a**(46/3)*b**(23/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**
7*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)*
*2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*
b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 12*a**(43/3)*b**(26/3)*(1
 - b*(a/b + x)/a)**(1/3)*(a/b + x)**8*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(
a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*
a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b
+ x)**6), True))
```

Giac [A] time = 1.06923, size = 47, normalized size = 0.96

$$\frac{3}{7}b^3x^{\frac{7}{3}} + \frac{9}{4}ab^2x^{\frac{4}{3}} + 9a^2bx^{\frac{1}{3}} - \frac{3a^3}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="giac")
```

```
[Out] 3/7*b^3*x^(7/3) + 9/4*a*b^2*x^(4/3) + 9*a^2*b*x^(1/3) - 3/2*a^3/x^(2/3)
```

3.674 $\int \frac{x^{5/3}}{a+bx}\,dx$

Optimal. Leaf size=125

$$-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{a^{5/3}\log(a+bx)}{2b^{8/3}}-\frac{\sqrt{3}a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{8/3}}-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}$$

```
[Out] (-3*a*x^(2/3))/(2*b^2) + (3*x^(5/3))/(5*b) - (Sqrt[3]*a^(5/3)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(8/3) - (3*a^(5/3)*Log[a^(1/3
) + b^(1/3)*x^(1/3)])/(2*b^(8/3)) + (a^(5/3)*Log[a + b*x])/(2*b^(8/3))
```

Rubi [A] time = 0.0700581, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 56, 617, 204, 31}

$$-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{a^{5/3}\log(a+bx)}{2b^{8/3}}-\frac{\sqrt{3}a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{8/3}}-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x),x]
```

```
[Out] (-3*a*x^(2/3))/(2*b^2) + (3*x^(5/3))/(5*b) - (Sqrt[3]*a^(5/3)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(8/3) - (3*a^(5/3)*Log[a^(1/3
) + b^(1/3)*x^(1/3)])/(2*b^(8/3)) + (a^(5/3)*Log[a + b*x])/(2*b^(8/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{5/3}}{a+bx}dx = \frac{3x^{5/3}}{5b} - \frac{a\int \frac{x^{2/3}}{a+bx}dx}{b}$$
$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} + \frac{a^2\int \frac{1}{\sqrt[3]{x}(a+bx)}dx}{b^2}$$
$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} + \frac{a^{5/3}\log(a+bx)}{2b^{8/3}} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}x}{\sqrt[3]{b}} + x^2}dx, x, \sqrt[3]{x}\right)}{2b^3} - \frac{(3a^{5/3})\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\right.}{2b^{8/3}}$$
$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} - \frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}} + \frac{a^{5/3}\log(a+bx)}{2b^{8/3}} + \frac{(3a^{5/3})\,\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1 - \frac{2\sqrt[3]{b}}{\sqrt[3]{a}}\right.}{b^{8/3}}$$
$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} - \frac{\sqrt{3}a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{8/3}} - \frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}} + \frac{a^{5/3}\log(a+bx)}{2b^{8/3}}$$

Mathematica [C] time = 0.0093248, size = 38, normalized size = 0.3

$$\frac{3x^{2/3}\left(5a\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx}{a}\right) - 5a + 2bx\right)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x),x]
```

```
[Out] (3*x^(2/3)*(-5*a + 2*b*x + 5*a*Hypergeometric2F1[2/3, 1, 5/3, -((b*x)/a)]))
/(10*b^2)
```

Maple [A] time = 0.007, size = 122, normalized size = 1.

$$\frac{3}{5b}x^{\frac{5}{3}} - \frac{3a}{2b^2}x^{\frac{2}{3}} - \frac{a^2}{b^3}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{a^2}{2b^3}\ln\left(x^{\frac{2}{3}} - \sqrt[3]{\frac{a}{b}}\sqrt[3]{x} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{a^2\sqrt{3}}{b^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a),x)
```

```
[Out] 3/5*x^(5/3)/b-3/2*a*x^(2/3)/b^2-a^2/b^3/(1/b*a)^(1/3)*ln(x^(1/3)+(1/b*a)^(1
/3))+1/2*a^2/b^3/(1/b*a)^(1/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/
3))+a^2/b^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x^(1/
```

```
3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50858, size = 381, normalized size = 3.05

$$\frac{10\sqrt{3}a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3a}\right)-5a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+10a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(b\left(-\frac{a^2}{b^2}\right)\right.}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/10*(10*sqrt(3)*a*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a^2/b
^2)^(1/3) + sqrt(3)*a)/a) - 5*a*(-a^2/b^2)^(1/3)*log(-b*x^(1/3)*(-a^2/b^2)^
(2/3) + a*x^(2/3) - a*(-a^2/b^2)^(1/3)) + 10*a*(-a^2/b^2)^(1/3)*log(b*(-a^2
/b^2)^(2/3) + a*x^(1/3)) + 3*(2*b*x - 5*a)*x^(2/3))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09942, size = 186, normalized size = 1.49

$$-\frac{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b^2}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^4}+\frac{\left(-ab^2\right)^{\frac{2}{3}}a\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^4}+\frac{3(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -a*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 - sqrt(3)*(-a*b^2)^(2/
3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/2*
(-a*b^2)^(2/3)*a*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 3
/10*(2*b^4*x^(5/3) - 5*a*b^3*x^(2/3))/b^5
```

3.675 $\int \frac{x^{4/3}}{a+bx} dx$

Optimal. Leaf size=123

$$\frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}}-\frac{a^{4/3}\log(a+bx)}{2b^{7/3}}-\frac{\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{7/3}}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}$$

```
[Out] (-3*a*x^(1/3))/b^2 + (3*x^(4/3))/(4*b) - (Sqrt[3]*a^(4/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(7/3) + (3*a^(4/3)*Log[a^(1/3) +
b^(1/3)*x^(1/3)])/(2*b^(7/3)) - (a^(4/3)*Log[a + b*x])/(2*b^(7/3))
```

Rubi [A] time = 0.0563835, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 58, 617, 204, 31}

$$\frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}}-\frac{a^{4/3}\log(a+bx)}{2b^{7/3}}-\frac{\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{7/3}}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x),x]
```

```
[Out] (-3*a*x^(1/3))/b^2 + (3*x^(4/3))/(4*b) - (Sqrt[3]*a^(4/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(7/3) + (3*a^(4/3)*Log[a^(1/3) +
b^(1/3)*x^(1/3)])/(2*b^(7/3)) - (a^(4/3)*Log[a + b*x])/(2*b^(7/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{4/3}}{a+bx}\,dx = \frac{3x^{4/3}}{4b} - \frac{a\int \frac{\sqrt[3]{x}}{a+bx}\,dx}{b}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} + \frac{a^2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{b^2}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} - \frac{a^{4/3}\log(a+bx)}{2b^{7/3}} + \frac{\left(3a^{5/3}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^{8/3}} + \frac{\left(3a^{4/3}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\cdots}\right.}{2b^{7/3}}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} + \frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}} - \frac{a^{4/3}\log(a+bx)}{2b^{7/3}} + \frac{\left(3a^{4/3}\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\cdots}{\sqrt[3]{a}}\right.}{b^{7/3}}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} - \frac{\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{7/3}} + \frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}} - \frac{a^{4/3}\log(a+bx)}{2b^{7/3}}$$

Mathematica [A] time = 0.0578726, size = 140, normalized size = 1.14

$$\frac{-2a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)+4a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)-4\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-12a\sqrt[3]{b}\sqrt[3]{x}+3b^{4/3}x^{4/3}}{4b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x),x]
```

```
[Out] (-12*a*b^(1/3)*x^(1/3) + 3*b^(4/3)*x^(4/3) - 4*Sqrt[3]*a^(4/3)*ArcTan[(1 -
(2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[a^(1/3) + b^(1/3)*x^(
1/3)] - 2*a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])
/(4*b^(7/3))
```

Maple [A] time = 0.005, size = 121, normalized size = 1.

$$\frac{3}{4\,b}x^{\frac{4}{3}} - 3\,\frac{a\sqrt[3]{x}}{b^2} + \frac{a^2}{b^3}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{a^2}{2\,b^3}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{a^2\sqrt{3}}{b^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a),x)
```

```
[Out] 3/4*x^(4/3)/b-3*a*x^(1/3)/b^2+a^2/b^3/(1/b*a)^(2/3)*ln(x^(1/3)+(1/b*a)^(1/3
))-1/2*a^2/b^3/(1/b*a)^(2/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/3)
)+a^2/b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x^(1/3)
-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56, size = 312, normalized size = 2.54

$$\frac{4\sqrt{3}a\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-2a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+4a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+3(bx-4a)x^{\frac{1}{3}}}{4b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(3)*a*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(a/b)^(2/3) -
sqrt(3)*a)/a) - 2*a*(a/b)^(1/3)*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(
2/3)) + 4*a*(a/b)^(1/3)*log(x^(1/3) + (a/b)^(1/3)) + 3*(b*x - 4*a)*x^(1/3))
/b^2
```

Sympy [A] time = 43.7102, size = 240, normalized size = 1.95

$$\begin{cases} \tilde{\infty}x^{\frac{4}{3}} & \text{fo} \\ \frac{3x^{\frac{7}{3}}}{7a} & \text{fo} \\ \frac{3x^{\frac{4}{3}}}{4b} & \text{fo} \\ -\frac{\sqrt[3]{-1}a^{\frac{4}{3}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{b^{3}\left(\frac{1}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt[3]{-1}a^{\frac{4}{3}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{2b^{3}\left(\frac{1}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt[3]{-1}\sqrt{3}a^{\frac{4}{3}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{b^{3}\left(\frac{1}{b}\right)^{\frac{2}{3}}}-\frac{3a\sqrt[3]{x}}{b^{2}}+\frac{3x^{\frac{4}{3}}}{4b} & \text{o} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(4/3), Eq(a, 0) & Eq(b, 0)), (3*x**(7/3)/(7*a), Eq(b, 0))
, (3*x**(4/3)/(4*b), Eq(a, 0)), (-(-1)**(1/3)*a**(4/3)*log(-(-1)**(1/3)*a**
(1/3)*(1/b)**(1/3) + x**(1/3))/(b**3*(1/b)**(2/3)) + (-1)**(1/3)*a**(4/3)*l
og(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1
/b)**(1/3) + 4*x**(2/3))/(2*b**3*(1/b)**(2/3)) + (-1)**(1/3)*sqrt(3)*a**(4/
```

```
3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)
))/(b**3*(1/b)**(2/3)) - 3*a*x**(1/3)/b**2 + 3*x**(4/3)/(4*b), True))
```

Giac [A] time = 1.0752, size = 184, normalized size = 1.5

$$-\frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b^2}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3}+\frac{\left(-ab^2\right)^{\frac{1}{3}}a\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -a*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 + sqrt(3)*(-a*b^2)^(1/
3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/2*
(-a*b^2)^(1/3)*a*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 3
/4*(b^3*x^(4/3) - 4*a*b^2*x^(1/3))/b^4
```

3.676 $\int \frac{x^{2/3}}{a+bx} dx$

Optimal. Leaf size=111

$$\frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{5/3}}-\frac{a^{2/3}\log(a+bx)}{2b^{5/3}}+\frac{\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{5/3}}+\frac{3x^{2/3}}{2b}$$

```
[Out] (3*x^(2/3))/(2*b) + (Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(
Sqrt[3]*a^(1/3))])/b^(5/3) + (3*a^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*
b^(5/3)) - (a^(2/3)*Log[a + b*x])/(2*b^(5/3))
```

Rubi [A] time = 0.0396263, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 56, 617, 204, 31}

$$\frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{5/3}}-\frac{a^{2/3}\log(a+bx)}{2b^{5/3}}+\frac{\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{5/3}}+\frac{3x^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x),x]
```

```
[Out] (3*x^(2/3))/(2*b) + (Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(
Sqrt[3]*a^(1/3))])/b^(5/3) + (3*a^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*
b^(5/3)) - (a^(2/3)*Log[a + b*x])/(2*b^(5/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{2/3}}{a+bx}\,dx = \frac{3x^{2/3}}{2b} - \frac{a\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{b}$$

$$= \frac{3x^{2/3}}{2b} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^2} + \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{2b^{5/3}}$$

$$= \frac{3x^{2/3}}{2b} + \frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{5/3}} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}} - \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{b^{5/3}}$$

$$= \frac{3x^{2/3}}{2b} + \frac{\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} + \frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{5/3}} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}}$$

Mathematica [C] time = 0.0066705, size = 29, normalized size = 0.26

$$-\frac{3x^{2/3}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx}{a}\right)-1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x),x]
```

```
[Out] (-3*x^(2/3)*(-1 + Hypergeometric2F1[2/3, 1, 5/3, -((b*x)/a)]))/(2*b)
```

Maple [A] time = 0.005, size = 107, normalized size = 1.

$$\frac{3}{2\,b}x^{\frac{2}{3}} + \frac{a}{b^2}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{a}{2\,b^2}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{a\sqrt{3}}{b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(b*x+a),x)
```

```
[Out] 3/2*x^(2/3)/b+1/b^2*a/(1/b*a)^(1/3)*ln(x^(1/3)+(1/b*a)^(1/3))-1/2/b^2*a/(1/
b*a)^(1/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/3))-1/b^2*a*3^(1/2)/
(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78864, size = 336, normalized size = 3.03

$$-\frac{2\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)+\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-2\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)-3x^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(a^2/b^2)^(
1/3) - sqrt(3)*a)/a) + (a^2/b^2)^(1/3)*log(-b*x^(1/3)*(a^2/b^2)^(2/3) + a*x
^(2/3) + a*(a^2/b^2)^(1/3)) - 2*(a^2/b^2)^(1/3)*log(b*(a^2/b^2)^(2/3) + a*x
^(1/3)) - 3*x^(2/3))/b
```

Sympy [A] time = 12.8211, size = 228, normalized size = 2.05

$$\begin{cases} \tilde{\infty}x^{\frac{2}{3}} & \text{for } a = \\ \frac{3x^{\frac{5}{3}}}{5a} & \text{for } b = \\ \frac{3x^{\frac{2}{3}}}{2b} & \text{for } a = \\ \frac{(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{b^7\left(\frac{1}{b}\right)^{\frac{16}{3}}} - \frac{(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{2b^7\left(\frac{1}{b}\right)^{\frac{16}{3}}} + \frac{(-1)^{\frac{2}{3}}\sqrt{3}a^{\frac{2}{3}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{b^7\left(\frac{1}{b}\right)^{\frac{16}{3}}} + \frac{3x^{\frac{2}{3}}}{2b} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(5/3)/(5*a), Eq(b, 0))
, (3*x**(2/3)/(2*b), Eq(a, 0)), ((-1)**(2/3)*a**(2/3)*log(-(-1)**(1/3)*a**(
1/3)*(1/b)**(1/3) + x**(1/3))/(b**7*(1/b)**(16/3)) - (-1)**(2/3)*a**(2/3)*l
og(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1
/b)**(1/3) + 4*x**(2/3))/(2*b**7*(1/b)**(16/3)) + (-1)**(2/3)*sqrt(3)*a**(2
/3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3
)))/(b**7*(1/b)**(16/3)) + 3*x**(2/3)/(2*b), True))
```

Giac [A] time = 1.0966, size = 159, normalized size = 1.43

$$\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b}+\frac{3x^{\frac{2}{3}}}{2b}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] (-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b + 3/2*x^(2/3)/b + sqrt(3)*(
-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b
^3 - 1/2*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/
b^3
```

3.677 $\int \frac{\sqrt[3]{x}}{a+bx}\,dx$

Optimal. Leaf size=109

$$-\frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}}+\frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}+\frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}}+\frac{3\sqrt[3]{x}}{b}$$

```
[Out] (3*x^(1/3))/b + (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt
[3]*a^(1/3))])/b^(4/3) - (3*a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*b^(4
/3)) + (a^(1/3)*Log[a + b*x])/(2*b^(4/3))
```

Rubi [A] time = 0.0389769, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 58, 617, 204, 31}

$$-\frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}}+\frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}+\frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}}+\frac{3\sqrt[3]{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x),x]
```

```
[Out] (3*x^(1/3))/b + (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt
[3]*a^(1/3))])/b^(4/3) - (3*a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*b^(4
/3)) + (a^(1/3)*Log[a + b*x])/(2*b^(4/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{a+bx}\,dx = \frac{3\sqrt[3]{x}}{b} - \frac{a\int \frac{1}{x^{2/3}(a+bx)}\,dx}{b}$$

$$= \frac{3\sqrt[3]{x}}{b} + \frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}} - \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^{5/3}} - \frac{\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{2b^{4/3}}$$

$$= \frac{3\sqrt[3]{x}}{b} - \frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}} - \frac{\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{b^{4/3}}$$

$$= \frac{3\sqrt[3]{x}}{b} + \frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}} - \frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}$$

Mathematica [A] time = 0.0275063, size = 126, normalized size = 1.16

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)+2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6\sqrt[3]{b}\sqrt[3]{x}}{2b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x),x]
```

```
[Out] (6*b^(1/3)*x^(1/3) + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1
/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] + a^(1/3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*b^(4/3))
```

Maple [A] time = 0.004, size = 108, normalized size = 1.

$$3\frac{\sqrt[3]{x}}{b}-\frac{a}{b^2}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{a}{2b^2}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{a\sqrt{3}}{b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a),x)
```

```
[Out] 3*x^(1/3)/b-1/b^2*a/(1/b*a)^(2/3)*ln(x^(1/3)+(1/b*a)^(1/3))+1/2/b^2*a/(1/b*
a)^(2/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/3))-1/b^2*a/(1/b*a)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86441, size = 292, normalized size = 2.68

$$\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+6x^{\frac{1}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a/b)^(2/3) -
sqrt(3)*a)/a) - (-a/b)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2
/3)) + 2*(-a/b)^(1/3)*log(x^(1/3) - (-a/b)^(1/3)) + 6*x^(1/3))/b
```

Sympy [A] time = 7.68076, size = 224, normalized size = 2.06

$$\begin{cases} \tilde{\infty}\sqrt[3]{x} & \text{for } a = 0 \\ \frac{3x^{\frac{4}{3}}}{4a} & \text{for } b = 0 \\ \frac{3\sqrt[3]{x}}{b} & \text{for } a = 0 \\ \frac{\sqrt[3]{-1}\sqrt[3]{a}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{b^2\left(\frac{1}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt[3]{-1}\sqrt[3]{a}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{2b^2\left(\frac{1}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt[3]{-1}\sqrt{3}\sqrt[3]{a}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{b^2\left(\frac{1}{b}\right)^{\frac{2}{3}}} + \frac{3\sqrt[3]{x}}{b} & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(4/3)/(4*a), Eq(b, 0))
, (3*x**(1/3)/b, Eq(a, 0)), ((-1)**(1/3)*a**(1/3)*log(-(-1)**(1/3)*a**(1/3)
*(1/b)**(1/3) + x**(1/3))/(b**2*(1/b)**(2/3)) - (-1)**(1/3)*a**(1/3)*log(4*
(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**
(1/3) + 4*x**(2/3))/(2*b**2*(1/b)**(2/3)) - (-1)**(1/3)*sqrt(3)*a**(1/3)*at
an(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(b
**2*(1/b)**(2/3)) + 3*x**(1/3)/b, True))
```

Giac [A] time = 1.08902, size = 161, normalized size = 1.48

$$\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}}+\frac{3x^{\frac{1}{3}}}{b}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] (-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b - sqrt(3)*(-a*b^2)^(1/3)*ar
ctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 + 3*x^(1/3)/b
 - 1/2*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^
2
```

3.678 $\int \frac{1}{\sqrt[3]{x}(a+bx)} \, dx$

Optimal. Leaf size=100

$$-\frac{3 \log \left(\sqrt[3]{a}+\sqrt[3]{b} \sqrt[3]{x}\right)}{2 \sqrt[3]{a} b^{2/3}}+\frac{\log (a+b x)}{2 \sqrt[3]{a} b^{2/3}}-\frac{\sqrt{3} \tan ^{-1}\left(\frac{\sqrt[3]{a}-2 \sqrt[3]{b} \sqrt[3]{x}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt[3]{a} b^{2/3}}$$

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(1/3
)*b^(2/3))) - (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3)) + Log[
a + b*x]/(2*a^(1/3)*b^(2/3))
```

Rubi [A] time = 0.0284094, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {56, 617, 204, 31}

$$-\frac{3 \log \left(\sqrt[3]{a}+\sqrt[3]{b} \sqrt[3]{x}\right)}{2 \sqrt[3]{a} b^{2/3}}+\frac{\log (a+b x)}{2 \sqrt[3]{a} b^{2/3}}-\frac{\sqrt{3} \tan ^{-1}\left(\frac{\sqrt[3]{a}-2 \sqrt[3]{b} \sqrt[3]{x}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt[3]{a} b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(1/3
)*b^(2/3))) - (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3)) + Log[
a + b*x]/(2*a^(1/3)*b^(2/3))
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx = \frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}$$

$$= -\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}} + \frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}} + \frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}$$

Mathematica [C] time = 0.0054872, size = 27, normalized size = 0.27

$$\frac{3x^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx}{a}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)),x]
```

```
[Out] (3*x^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, -((b*x)/a)])/(2*a)
```

Maple [A] time = 0.004, size = 96, normalized size = 1.

$$-\frac{1}{b}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{1}{2b}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{\sqrt{3}}{b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a),x)
```

```
[Out] -1/b/(1/b*a)^(1/3)*ln(x^(1/3)+(1/b*a)^(1/3))+1/2/b/(1/b*a)^(1/3)*ln(x^(2/3)
-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/3))+3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^
(1/2)*(2/(1/b*a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87932, size = 801, normalized size = 8.01

$$\left[\frac{\sqrt{3}ab\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+\sqrt{3}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab\right.\right.}{2\,ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x - a*b + sqrt(3)*(a*b*
x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/
a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x + a)) + (-a*b^2)^(2/3)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x^(1/
3) - (-a*b^2)^(1/3)))/(a*b^2), 1/2*(2*sqrt(3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*a
rctan(1/3*sqrt(3)*(2*b*x^(1/3) + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b)
 + (-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/
3)) - 2*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)))/(a*b^2)]
```

Sympy [A] time = 9.68224, size = 218, normalized size = 2.18

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3x^{\frac{2}{3}}}{2a} & \text{for}\: b = 0 \\ -\frac{3}{b\sqrt[3]{x}} & \text{for}\: a = 0 \\ -\frac{(-1)^{\frac{2}{3}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{\sqrt[3]{a}b^2\left(\frac{1}{b}\right)^{\frac{4}{3}}}+\frac{(-1)^{\frac{2}{3}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{2\sqrt[3]{a}b^2\left(\frac{1}{b}\right)^{\frac{4}{3}}}-\frac{(-1)^{\frac{2}{3}}\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{\sqrt[3]{a}b^2\left(\frac{1}{b}\right)^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(2/3)/(2*a), Eq(b, 0))
, (-3/(b*x**(1/3)), Eq(a, 0)), (-(-1)**(2/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b
)**(1/3) + x**(1/3))/(a**(1/3)*b**2*(1/b)**(4/3)) + (-1)**(2/3)*log(4*(-1)*
*(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3)
 + 4*x**(2/3))/(2*a**(1/3)*b**2*(1/b)**(4/3)) - (-1)**(2/3)*sqrt(3)*atan(sq
rt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(a**(1/
3)*b**2*(1/b)**(4/3)), True))
```

Giac [A] time = 1.09265, size = 159, normalized size = 1.59

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a - sqrt(3)*(-a*b^2)^(2/3)*a
rctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/2*(-
a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

3.679 $\int \frac{1}{x^{2/3}(a+bx)}\,dx$

Optimal. Leaf size=100

$$\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(2/3
)*b^(1/3))) + (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3)) - Log[
a + b*x]/(2*a^(2/3)*b^(1/3))
```

Rubi [A] time = 0.02787, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {58, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)),x]
```


```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(2/3
)*b^(1/3))) + (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3)) - Log[
a + b*x]/(2*a^(2/3)*b^(1/3))
```


Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x^{2/3}(a+bx)}\,dx = -\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}$$

$$= \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}} + \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.0229514, size = 103, normalized size = 1.03

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/
3) + b^(1/3)*x^(1/3)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(
2/3)])/(2*a^(2/3)*b^(1/3))
```

Maple [A] time = 0.003, size = 95, normalized size = 1.

$$\frac{1}{b}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{1}{2b}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}}{b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a),x)
```

```
[Out] 1/b/(1/b*a)^(2/3)*ln(x^(1/3)+(1/b*a)^(1/3))-1/2/b/(1/b*a)^(2/3)*ln(x^(2/3)-
(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/3))+1/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(1/b*a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9316, size = 807, normalized size = 8.07

$$\left[\frac{\sqrt{3}ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx-a^2+\sqrt{3}\left(2\,abx^{\frac{2}{3}}-\left(a^2b\right)^{\frac{1}{3}}a+\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+}{2\,a^2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x - a^2 + sqrt(3)*(2*a*
b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b)
 - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x + a)) - (a^2*b)^(2/3)*log(a*b*x^(2/3) +
(a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*log(a*b*x^(1/3)
+ (a^2*b)^(2/3)))/(a^2*b), 1/2*(2*sqrt(3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(
-1/3*sqrt(3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)
/b)/a^2) - (a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*
x^(1/3)) + 2*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [A] time = 16.8926, size = 218, normalized size = 2.18

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for}\ a = 0 \wedge b = 0 \\ \frac{3\sqrt[3]{x}}{a} & \text{for}\ b = 0 \\ -\frac{3}{2bx^{\frac{2}{3}}} & \text{for}\ a = 0 \\ -\frac{\sqrt[3]{-1}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{a^{\frac{2}{3}}b^3\left(\frac{1}{b}\right)^{\frac{8}{3}}}+\frac{\sqrt[3]{-1}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{2a^{\frac{2}{3}}b^3\left(\frac{1}{b}\right)^{\frac{8}{3}}}+\frac{\sqrt[3]{-1}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{a^{\frac{2}{3}}b^3\left(\frac{1}{b}\right)^{\frac{8}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(1/3)/a, Eq(b, 0)), (-
3/(2*b*x**(2/3)), Eq(a, 0)), (-(-1)**(1/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b)*
*(1/3) + x**(1/3))/(a**(2/3)*b**3*(1/b)**(8/3)) + (-1)**(1/3)*log(4*(-1)**(
2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) +
 4*x**(2/3))/(2*a**(2/3)*b**3*(1/b)**(8/3)) + (-1)**(1/3)*sqrt(3)*atan(sqrt
(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(a**(2/3)
*b**3*(1/b)**(8/3)), True))
```

Giac [A] time = 1.07621, size = 158, normalized size = 1.58

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a + sqrt(3)*(-a*b^2)^(1/3)*a
rctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/2*(-a*
b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

3.680 $\int \frac{1}{x^{4/3}(a+bx)}\,dx$

Optimal. Leaf size=109

$$\frac{3\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}}-\frac{\sqrt[3]{b}\log(a+bx)}{2a^{4/3}}+\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{3}{a\sqrt[3]{x}}$$

```
[Out] -3/(a*x^(1/3)) + (Sqrt[3]*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqr
t[3]*a^(1/3))])/a^(4/3) + (3*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(
4/3)) - (b^(1/3)*Log[a + b*x])/(2*a^(4/3))
```

Rubi [A] time = 0.0375527, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$\frac{3\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}}-\frac{\sqrt[3]{b}\log(a+bx)}{2a^{4/3}}+\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{3}{a\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)),x]
```

```
[Out] -3/(a*x^(1/3)) + (Sqrt[3]*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqr
t[3]*a^(1/3))])/a^(4/3) + (3*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(
4/3)) - (b^(1/3)*Log[a + b*x])/(2*a^(4/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{4/3}(a+bx)}\,dx &= -\frac{3}{a\sqrt[3]{x}} - \frac{b\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{a}\\
&= -\frac{3}{a\sqrt[3]{x}} - \frac{\sqrt[3]{b}\log(a+bx)}{2a^{4/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2a} + \frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{2a^{4/3}}\\
&= -\frac{3}{a\sqrt[3]{x}} + \frac{3\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\sqrt[3]{b}\log(a+bx)}{2a^{4/3}} - \frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{4/3}}\\
&= -\frac{3}{a\sqrt[3]{x}} + \frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} + \frac{3\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\sqrt[3]{b}\log(a+bx)}{2a^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.0046366, size = 25, normalized size = 0.23

$$-\frac{3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};-\frac{bx}{a}\right)}{a\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/3, 1, 2/3, -((b*x)/a)])/(a*x^(1/3))
```

Maple [A] time = 0.006, size = 104, normalized size = 1.

$$\frac{1}{a}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{1}{2a}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{\sqrt{3}}{a}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-3\frac{1}{a\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3)/(b*x+a),x)
```

```
[Out] 1/a/(1/b*a)^(1/3)*ln(x^(1/3)+(1/b*a)^(1/3))-1/2/a/(1/b*a)^(1/3)*ln(x^(2/3)-
(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/3))-1/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3
^(1/2)*(2/(1/b*a)^(1/3)*x^(1/3)-1))-3/a/x^(1/3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72538, size = 304, normalized size = 2.79

$$-\frac{2\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)+6}{2ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x^(1/3)*(b/a)^(1/3) - 1/3*
sqrt(3)) + x*(b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/3) + b*x^(2/3) + a*(b/a)^(
1/3)) - 2*x*(b/a)^(1/3)*log(a*(b/a)^(2/3) + b*x^(1/3)) + 6*x^(2/3))/(a*x)
```

Sympy [A] time = 46.0869, size = 223, normalized size = 2.05

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} & \text{for}\: a = 0 \wedge b \\ -\frac{3}{a\sqrt[3]{x}} & \text{for}\: b = 0 \\ -\frac{3}{4bx^{\frac{4}{3}}} & \text{for}\: a = 0 \\ -\frac{3}{a\sqrt[3]{x}} + \frac{(-1)^{\frac{2}{3}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{a^{\frac{4}{3}}b\left(\frac{1}{b}\right)^{\frac{4}{3}}} - \frac{(-1)^{\frac{2}{3}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{2a^{\frac{4}{3}}b\left(\frac{1}{b}\right)^{\frac{4}{3}}} + \frac{(-1)^{\frac{2}{3}}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{a^{\frac{4}{3}}b\left(\frac{1}{b}\right)^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (-3/(a*x**(1/3)), Eq(b, 0)),
 (-3/(4*b*x**(4/3)), Eq(a, 0)), (-3/(a*x**(1/3)) + (-1)**(2/3)*log(-(-1)**(
1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(a**(4/3)*b*(1/b)**(4/3)) - (-1)**(2
/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/
3)*(1/b)**(1/3) + 4*x**(2/3))/(2*a**(4/3)*b*(1/b)**(4/3)) + (-1)**(2/3)*sqr
t(3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/
3)))/(a**(4/3)*b*(1/b)**(4/3)), True))
```

Giac [A] time = 1.07443, size = 169, normalized size = 1.55

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b}-\frac{3}{ax^{\frac{1}{3}}}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + sqrt(3)*(-a*b^2)^(2/3
)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - 3/(
a*x^(1/3)) - 1/2*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)
^(2/3))/(a^2*b)
```

3.681 $\int \frac{1}{x^{5/3}(a+bx)}\,dx$

Optimal. Leaf size=111

$$-\frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}}+\frac{b^{2/3}\log(a+bx)}{2a^{5/3}}+\frac{\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}-\frac{3}{2ax^{2/3}}$$

```
[Out] -3/(2*a*x^(2/3)) + (Sqrt[3]*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(S
qrt[3]*a^(1/3))])/a^(5/3) - (3*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a
^(5/3)) + (b^(2/3)*Log[a + b*x])/(2*a^(5/3))
```

Rubi [A] time = 0.0391327, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$-\frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}}+\frac{b^{2/3}\log(a+bx)}{2a^{5/3}}+\frac{\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}-\frac{3}{2ax^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)),x]
```

```
[Out] -3/(2*a*x^(2/3)) + (Sqrt[3]*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(S
qrt[3]*a^(1/3))])/a^(5/3) - (3*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a
^(5/3)) + (b^(2/3)*Log[a + b*x])/(2*a^(5/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x^{5/3}(a+bx)}\,dx = -\frac{3}{2ax^{2/3}} - \frac{b\int \frac{1}{x^{2/3}(a+bx)}\,dx}{a}$$

$$= -\frac{3}{2ax^{2/3}} + \frac{b^{2/3}\log(a+bx)}{2a^{5/3}} - \frac{\left(3\sqrt[3]{b}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\left(3b^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,dx\right.}{2a^{5/3}}$$

$$= -\frac{3}{2ax^{2/3}} - \frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}} + \frac{b^{2/3}\log(a+bx)}{2a^{5/3}} - \frac{\left(3b^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{5/3}}$$

$$= -\frac{3}{2ax^{2/3}} + \frac{\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} - \frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}} + \frac{b^{2/3}\log(a+bx)}{2a^{5/3}}$$

Mathematica [C] time = 0.0047665, size = 27, normalized size = 0.24

$$-\frac{3\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};-\frac{bx}{a}\right)}{2ax^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)),x]
```

```
[Out] (-3*Hypergeometric2F1[-2/3, 1, 1/3, -((b*x)/a)])/(2*a*x^(2/3))
```

Maple [A] time = 0.005, size = 105, normalized size = 1.

$$-\frac{1}{a}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{1}{2a}\ln\left(x^{\frac{2}{3}} - \sqrt[3]{\frac{a}{b}}\sqrt[3]{x} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{\sqrt{3}}{a}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{3}{2a}x^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a),x)
```

```
[Out] -1/a/(1/b*a)^(2/3)*ln(x^(1/3)+(1/b*a)^(1/3))+1/2/a/(1/b*a)^(2/3)*ln(x^(2/3)
-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/3))-1/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(1/b*a)^(1/3)*x^(1/3)-1))-3/2/a/x^(2/3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52118, size = 365, normalized size = 3.29

$$\frac{2\sqrt{3}x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^{\frac{1}{3}}-a\left(-\right.\right.}{2ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x^(1/3)*(-b^2/a^2
)^(2/3) - sqrt(3)*b)/b) - x*(-b^2/a^2)^(1/3)*log(b^2*x^(2/3) + a*b*x^(1/3)*
(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x*(-b^2/a^2)^(1/3)*log(b*x^(1/
3) - a*(-b^2/a^2)^(1/3)) - 3*x^(1/3))/(a*x)
```

Sympy [A] time = 59.7891, size = 231, normalized size = 2.08

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} & \text{for } a = 0 \wedge b = \\ -\frac{3}{2ax^{\frac{2}{3}}} & \text{for } b = 0 \\ -\frac{3}{5bx^{\frac{5}{3}}} & \text{for } a = 0 \\ -\frac{3}{2ax^{\frac{2}{3}}}+\frac{\sqrt[3]{-1}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{a^{\frac{5}{3}}b^4\left(\frac{1}{b}\right)^{\frac{14}{3}}}-\frac{\sqrt[3]{-1}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{2a^{\frac{5}{3}}b^4\left(\frac{1}{b}\right)^{\frac{14}{3}}}-\frac{\sqrt[3]{-1}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{a^{\frac{5}{3}}b^4\left(\frac{1}{b}\right)^{\frac{14}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (-3/(2*a*x**(2/3)), Eq(b, 0)
), (-3/(5*b*x**(5/3)), Eq(a, 0)), (-3/(2*a*x**(2/3)) + (-1)**(1/3)*log(-(-1
)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(a**(5/3)*b**4*(1/b)**(14/3)) -
(-1)**(1/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3
)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(2*a**(5/3)*b**4*(1/b)**(14/3)) - (-1
)**(1/3)*sqrt(3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3
)*(1/b)**(1/3)))/(a**(5/3)*b**4*(1/b)**(14/3)), True))
```

Giac [A] time = 1.08904, size = 162, normalized size = 1.46

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^{2}}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a^{2}}-\frac{3}{2ax^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 - sqrt(3)*(-a*b^2)^(1/3
)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - 1/2*(-a
*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/2/(a
*x^(2/3))
```

3.682 $\int \frac{x^{5/3}}{(a+bx)^2} dx$

Optimal. Leaf size=129

$$\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}+\frac{5a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}-\frac{x^{5/3}}{b(a+bx)}+\frac{5x^{2/3}}{2b^2}$$

```
[Out] (5*x^(2/3))/(2*b^2) - x^(5/3)/(b*(a + b*x)) + (5*a^(2/3)*ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (5*a^(2/3)*Log[a
^(1/3) + b^(1/3)*x^(1/3)])/(2*b^(8/3)) - (5*a^(2/3)*Log[a + b*x])/(6*b^(8/3
))
```

Rubi [A] time = 0.0470636, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 50, 56, 617, 204, 31}

$$\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}+\frac{5a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}-\frac{x^{5/3}}{b(a+bx)}+\frac{5x^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x)^2,x]
```

```
[Out] (5*x^(2/3))/(2*b^2) - x^(5/3)/(b*(a + b*x)) + (5*a^(2/3)*ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (5*a^(2/3)*Log[a
^(1/3) + b^(1/3)*x^(1/3)])/(2*b^(8/3)) - (5*a^(2/3)*Log[a + b*x])/(6*b^(8/3
))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/3}}{(a+bx)^2}\,dx &= -\frac{x^{5/3}}{b(a+bx)}+\frac{5\int \frac{x^{2/3}}{a+bx}\,dx}{3b}\\
&= \frac{5x^{2/3}}{2b^2}-\frac{x^{5/3}}{b(a+bx)}-\frac{(5a)\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{3b^2}\\
&= \frac{5x^{2/3}}{2b^2}-\frac{x^{5/3}}{b(a+bx)}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}-\frac{(5a)\,\mathrm{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{2b^3}+\frac{\left(5a^{2/3}\right)\mathrm{Subst}\left(\int \right.}{2b^{8/3}}\\
&= \frac{5x^{2/3}}{2b^2}-\frac{x^{5/3}}{b(a+bx)}+\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}-\frac{\left(5a^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\right.}{b^{8/3}}\\
&= \frac{5x^{2/3}}{2b^2}-\frac{x^{5/3}}{b(a+bx)}+\frac{5a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^{8/3}}+\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}
\end{aligned}$$

Mathematica [C] time = 0.0048991, size = 27, normalized size = 0.21

$$\frac{3x^{8/3}\,{}_2F_1\left(2,\frac{8}{3};\frac{11}{3};-\frac{bx}{a}\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x)^2,x]
```

```
[Out] (3*x^(8/3)*Hypergeometric2F1[2, 8/3, 11/3, -((b*x)/a)])/(8*a^2)
```

Maple [A] time = 0.011, size = 123, normalized size = 1.

$$\frac{3}{2b^2}x^{\frac{2}{3}}+\frac{a}{b^2(bx+a)}x^{\frac{2}{3}}+\frac{5a}{3b^3}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{5a}{6b^3}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{5a\sqrt{3}}{3b^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a)^2,x)
```

```
[Out] 3/2*x^(2/3)/b^2+1/b^2*a*x^(2/3)/(b*x+a)+5/3/b^3*a/(1/b*a)^(1/3)*ln(x^(1/3)+
(1/b*a)^(1/3))-5/6/b^3*a/(1/b*a)^(1/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/
b*a)^(2/3))-5/3/b^3*a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(
1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55877, size = 420, normalized size = 3.26

$$-\frac{10\sqrt{3}(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)+5(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-10(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}}{6\left(b^3x+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(10*sqrt(3)*(b*x + a)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*
(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(b*x + a)*(a^2/b^2)^(1/3)*log(-b*x^(1/3
)*(a^2/b^2)^(2/3) + a*x^(2/3) + a*(a^2/b^2)^(1/3)) - 10*(b*x + a)*(a^2/b^2)
^(1/3)*log(b*(a^2/b^2)^(2/3) + a*x^(1/3)) - 3*(3*b*x + 5*a)*x^(2/3))/(b^3*x
 + a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07725, size = 182, normalized size = 1.41

$$\frac{5\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b^{2}}+\frac{ax^{\frac{2}{3}}}{(bx+a)b^{2}}+\frac{3\,x^{\frac{2}{3}}}{2\,b^{2}}+\frac{5\,\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{4}}-\frac{5\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right.}{6\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 + a*x^(2/3)/((b*x + a
)*b^2) + 3/2*x^(2/3)/b^2 + 5/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2
*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 5/6*(-a*b^2)^(2/3)*log(x^(2/3)
 + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4
```

3.683 $\int \frac{x^{4/3}}{(a+bx)^2} dx$

Optimal. Leaf size=125

$$-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}+\frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}+\frac{4\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}}-\frac{x^{4/3}}{b(a+bx)}+\frac{4\sqrt[3]{x}}{b^2}$$

```
[Out] (4*x^(1/3))/b^2 - x^(4/3)/(b*(a + b*x)) + (4*a^(1/3)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (2*a^(1/3)*Log[a^(1/
3) + b^(1/3)*x^(1/3)])/b^(7/3) + (2*a^(1/3)*Log[a + b*x])/(3*b^(7/3))
```

Rubi [A] time = 0.0465502, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 50, 58, 617, 204, 31}

$$-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}+\frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}+\frac{4\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}}-\frac{x^{4/3}}{b(a+bx)}+\frac{4\sqrt[3]{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x)^2,x]
```

```
[Out] (4*x^(1/3))/b^2 - x^(4/3)/(b*(a + b*x)) + (4*a^(1/3)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (2*a^(1/3)*Log[a^(1/
3) + b^(1/3)*x^(1/3)])/b^(7/3) + (2*a^(1/3)*Log[a + b*x])/(3*b^(7/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{4/3}}{(a+bx)^2}\,dx = -\frac{x^{4/3}}{b(a+bx)}+\frac{4\int \frac{\sqrt[3]{x}}{a+bx}\,dx}{3b}$$
$$= \frac{4\sqrt[3]{x}}{b^2}-\frac{x^{4/3}}{b(a+bx)}-\frac{(4a)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3b^2}$$
$$= \frac{4\sqrt[3]{x}}{b^2}-\frac{x^{4/3}}{b(a+bx)}+\frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}-\frac{\left(2a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{b^{8/3}}-\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \cdots\right.}{b^{7/\cdots}}$$
$$= \frac{4\sqrt[3]{x}}{b^2}-\frac{x^{4/3}}{b(a+bx)}-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}+\frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}-\frac{\left(4\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\cdots}{\cdots}\right.}{b^{7/3}}$$
$$= \frac{4\sqrt[3]{x}}{b^2}-\frac{x^{4/3}}{b(a+bx)}+\frac{4\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}}-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}+\frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}$$

Mathematica [C] time = 0.0043447, size = 27, normalized size = 0.22

$$\frac{3x^{7/3}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\frac{bx}{a}\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x)^2,x]
```

```
[Out] (3*x^(7/3)*Hypergeometric2F1[2, 7/3, 10/3, -((b*x)/a)])/(7*a^2)
```

Maple [A] time = 0.01, size = 123, normalized size = 1.

$$3\,\frac{\sqrt[3]{x}}{b^2}+\frac{a}{b^2\,(bx+a)}\sqrt[3]{x}-\frac{4\,a}{3\,b^3}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2\,a}{3\,b^3}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{4\,a\sqrt{3}}{3\,b^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a)^2,x)
```

```
[Out] 3*x^(1/3)/b^2+1/b^2*a*x^(1/3)/(b*x+a)-4/3/b^3*a/(1/b*a)^(2/3)*ln(x^(1/3)+(1
/b*a)^(1/3))+2/3/b^3*a/(1/b*a)^(2/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/b*
a)^(2/3))-4/3/b^3*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/
3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62347, size = 373, normalized size = 2.98

$$\frac{4\sqrt{3}(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-2(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+4(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x\right.}{3\left(b^3x+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(4*sqrt(3)*(b*x + a)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a/b
)^(2/3) - sqrt(3)*a)/a) - 2*(b*x + a)*(-a/b)^(1/3)*log(x^(2/3) + x^(1/3)*(-
a/b)^(1/3) + (-a/b)^(2/3)) + 4*(b*x + a)*(-a/b)^(1/3)*log(x^(1/3) - (-a/b)^
(1/3)) + 3*(3*b*x + 4*a)*x^(1/3))/(b^3*x + a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07739, size = 182, normalized size = 1.46

$$\frac{4\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3b^2}-\frac{4\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3}+\frac{ax^{\frac{1}{3}}}{(bx+a)b^2}+\frac{3x^{\frac{1}{3}}}{b^2}-\frac{2\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right.\right.}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 - 4/3*sqrt(3)*(-a*b^2
)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + a
*x^(1/3)/((b*x + a)*b^2) + 3*x^(1/3)/b^2 - 2/3*(-a*b^2)^(1/3)*log(x^(2/3) +
 x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3
```

3.684 $\int \frac{x^{2/3}}{(a+bx)^2}\,dx$

Optimal. Leaf size=115

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}b^{5/3}}+\frac{\log(a+bx)}{3\sqrt[3]{a}b^{5/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}}-\frac{x^{2/3}}{b(a+bx)}$$

```
[Out] -(x^(2/3)/(b*(a + b*x))) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(
1/3)*b^(5/3)) + Log[a + b*x]/(3*a^(1/3)*b^(5/3))
```

Rubi [A] time = 0.0377792, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 56, 617, 204, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}b^{5/3}}+\frac{\log(a+bx)}{3\sqrt[3]{a}b^{5/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}}-\frac{x^{2/3}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x)^2,x]
```

```
[Out] -(x^(2/3)/(b*(a + b*x))) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(
1/3)*b^(5/3)) + Log[a + b*x]/(3*a^(1/3)*b^(5/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{2/3}}{(a+bx)^2}\,dx &= -\frac{x^{2/3}}{b(a+bx)} + \frac{2\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{3b} \\
&= -\frac{x^{2/3}}{b(a+bx)} + \frac{\log(a+bx)}{3\sqrt[3]{a}b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{b^2} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{a}b^{5/3}} \\
&= -\frac{x^{2/3}}{b(a+bx)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}b^{5/3}} + \frac{\log(a+bx)}{3\sqrt[3]{a}b^{5/3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{5/3}} \\
&= -\frac{x^{2/3}}{b(a+bx)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}b^{5/3}} + \frac{\log(a+bx)}{3\sqrt[3]{a}b^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.0039834, size = 27, normalized size = 0.23

$$\frac{3x^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};-\frac{bx}{a}\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x)^2,x]
```

```
[Out] (3*x^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, -((b*x)/a)])/(5*a^2)
```

Maple [A] time = 0.009, size = 112, normalized size = 1.

$$-\frac{1}{b\,(bx+a)}x^{\frac{2}{3}} - \frac{2}{3\,b^2}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{1}{3\,b^2}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{2\sqrt{3}}{3\,b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right) \frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(b*x+a)^2,x)
```

```
[Out] -x^(2/3)/b/(b*x+a)-2/3/b^2/(1/b*a)^(1/3)*ln(x^(1/3)+(1/b*a)^(1/3))+1/3/b^2/
(1/b*a)^(1/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/3))+2/3/b^2*3^(1/
2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.39975, size = 987, normalized size = 8.58

$$\left[-\frac{3\,ab^2x^{\frac{2}{3}} - 3\sqrt{\frac{1}{3}}\left(ab^2x + a^2b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x - ab + 3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}} + \left(-ab^2\right)^{\frac{1}{3}}a + 2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}} - 3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx + a}\right) - \left(-ab^2\right)^{\frac{2}{3}}(bx + a}{3\left(ab^4x + a^2b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/3*(3*a*b^2*x^(2/3) - 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt((-a*b^2)^(1/3)/
a)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a
*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*
x + a)) - (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/
3) + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(b*x + a)*log(b*x^(1/3) - (-a*b^2)^
(1/3)))/(a*b^4*x + a^2*b^3), -1/3*(3*a*b^2*x^(2/3) - 6*sqrt(1/3)*(a*b^2*x +
 a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^2)^(1
/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(b*x + a)*l
og(b*x^(1/3) - (-a*b^2)^(1/3)))/(a*b^4*x + a^2*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11597, size = 184, normalized size = 1.6

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab} - \frac{x^{\frac{2}{3}}}{(bx + a)b} - \frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3} + \frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right.\right.}{3\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b) - x^(2/3)/((b*x +
a)*b) - 2/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(
1/3))/(-a/b)^(1/3))/(a*b^3) + 1/3*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b^3)
```

3.685 $\int \frac{\sqrt[3]{x}}{(a+bx)^2} \, dx$

Optimal. Leaf size=117

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}-\frac{\sqrt[3]{x}}{b(a+bx)}$$

```
[Out] -(x^(1/3)/(b*(a + b*x))) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^
(1/3))]/(Sqrt[3]*a^(2/3)*b^(4/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(2*a^(2/
3)*b^(4/3)) - Log[a + b*x]/(6*a^(2/3)*b^(4/3))
```

Rubi [A] time = 0.0379469, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 58, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}-\frac{\sqrt[3]{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x)^2,x]
```

```
[Out] -(x^(1/3)/(b*(a + b*x))) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^
(1/3))]/(Sqrt[3]*a^(2/3)*b^(4/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(2*a^(2/
3)*b^(4/3)) - Log[a + b*x]/(6*a^(2/3)*b^(4/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx = -\frac{\sqrt[3]{x}}{b(a+bx)} + \frac{\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3b}$$

$$= -\frac{\sqrt[3]{x}}{b(a+bx)} - \frac{\log(a+bx)}{6a^{2/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,dx, x, \sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}$$

$$= -\frac{\sqrt[3]{x}}{b(a+bx)} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}} - \frac{\log(a+bx)}{6a^{2/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}b^{4/3}}$$

$$= -\frac{\sqrt[3]{x}}{b(a+bx)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}} - \frac{\log(a+bx)}{6a^{2/3}b^{4/3}}$$

Mathematica [C] time = 0.0045437, size = 27, normalized size = 0.23

$$\frac{3x^{4/3}\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};-\frac{bx}{a}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x)^2,x]
```

```
[Out] (3*x^(4/3)*Hypergeometric2F1[4/3, 2, 7/3, -((b*x)/a)])/(4*a^2)
```

Maple [A] time = 0.008, size = 112, normalized size = 1.

$$-\frac{1}{b(bx+a)}\sqrt[3]{x} + \frac{1}{3b^2}\ln\left(\sqrt[3]{x} + \sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{1}{6b^2}\ln\left(x^{\frac{2}{3}} - \sqrt[3]{\frac{a}{b}}\sqrt[3]{x} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}}{3b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a)^2,x)
```

```
[Out] -x^(1/3)/b/(b*x+a)+1/3/b^2/(1/b*a)^(2/3)*ln(x^(1/3)+(1/b*a)^(1/3))-1/6/b^2/
(1/b*a)^(2/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/3))+1/3/b^2/(1/b*
a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.30111, size = 999, normalized size = 8.54

$$\left[-\frac{6\,a^2bx^{\frac{1}{3}} - 3\sqrt{\frac{1}{3}}\left(ab^2x + a^2b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^{\frac{2}{3}}-\left(a^2b\right)^{\frac{1}{3}}a+\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right) + \left(a^2b\right)^{\frac{2}{3}}(bx+a)\,\mathrm{lo}}{6\left(a^2b^3x + a^3b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*a^2*b*x^(1/3) - 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt(-(a^2*b)^(1/3)/
b)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2
*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
 + a)) + (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b
)^(2/3)*x^(1/3)) - 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3
)))/(a^2*b^3*x + a^3*b^2), -1/6*(6*a^2*b*x^(1/3) - 6*sqrt(1/3)*(a*b^2*x + a
^2*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^
(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(2/3)*(b*x + a)*log(a*b
*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) - 2*(a^2*b)^(2/3)*(b*x
+ a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^2*b^3*x + a^3*b^2)]
```

Sympy [A] time = 136.351, size = 638, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(4/3)/(4*a**2), Eq(b,
0)), (-3/(2*b**2*x**(2/3)), Eq(a, 0)), (-2*(-1)**(1/3)*a**(7/3)*b*(1/b)**(4
/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(6*a**3*b + 6*a**2*b
**2*x) + (-1)**(1/3)*a**(7/3)*b*(1/b)**(4/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/
b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(6*a
**3*b + 6*a**2*b**2*x) + 2*(-1)**(1/3)*sqrt(3)*a**(7/3)*b*(1/b)**(4/3)*atan
(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(6*a
**3*b + 6*a**2*b**2*x) - 2*(-1)**(1/3)*a**(7/3)*b*(1/b)**(4/3)*log(2)/(6*a*
*3*b + 6*a**2*b**2*x) - 2*(-1)**(1/3)*a**(4/3)*b**2*x*(1/b)**(4/3)*log(-(-1
```

```
)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(6*a**3*b + 6*a**2*b**2*x) + (-1
)**(1/3)*a**(4/3)*b**2*x*(1/b)**(4/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/
3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(6*a**3*b +
 6*a**2*b**2*x) + 2*(-1)**(1/3)*sqrt(3)*a**(4/3)*b**2*x*(1/b)**(4/3)*atan(s
qrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(6*a**
3*b + 6*a**2*b**2*x) - 2*(-1)**(1/3)*a**(4/3)*b**2*x*(1/b)**(4/3)*log(2)/(6
*a**3*b + 6*a**2*b**2*x) - 6*a**2*x**(1/3)/(6*a**3*b + 6*a**2*b**2*x), True
))
```

Giac [A] time = 1.08737, size = 184, normalized size = 1.57

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^{2}}-\frac{x^{\frac{1}{3}}}{(bx+a)b}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right.\right.}{6\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*(-a*
b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b
^2) - x^(1/3)/((b*x + a)*b) + 1/6*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

3.686 $\int \frac{1}{\sqrt[3]{x}(a+bx)^2} \, dx$

Optimal. Leaf size=116

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}+\frac{\log(a+bx)}{6a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}}+\frac{x^{2/3}}{a(a+bx)}$$

```
[Out] x^(2/3)/(a*(a + b*x)) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/
3))]/(Sqrt[3]*a^(4/3)*b^(2/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(2*a^(4/3)*
b^(2/3)) + Log[a + b*x]/(6*a^(4/3)*b^(2/3))
```

Rubi [A] time = 0.0414263, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}+\frac{\log(a+bx)}{6a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}}+\frac{x^{2/3}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)^2),x]
```

```
[Out] x^(2/3)/(a*(a + b*x)) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/
3))]/(Sqrt[3]*a^(4/3)*b^(2/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(2*a^(4/3)*
b^(2/3)) + Log[a + b*x]/(6*a^(4/3)*b^(2/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx = \frac{x^{2/3}}{a(a+bx)} + \frac{\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{3a}$$

$$= \frac{x^{2/3}}{a(a+bx)} + \frac{\log(a+bx)}{6a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2ab} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,dx, x, \sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}$$

$$= \frac{x^{2/3}}{a(a+bx)} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}} + \frac{\log(a+bx)}{6a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{4/3}b^{2/3}}$$

$$= \frac{x^{2/3}}{a(a+bx)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}} + \frac{\log(a+bx)}{6a^{4/3}b^{2/3}}$$

Mathematica [C] time = 0.0047714, size = 27, normalized size = 0.23

$$\frac{3x^{2/3}\,{}_2F_1\left(\frac{2}{3}, 2; \frac{5}{3}; -\frac{bx}{a}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)^2),x]
```

```
[Out] (3*x^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, -((b*x)/a)])/(2*a^2)
```

Maple [A] time = 0.006, size = 120, normalized size = 1.

$$\frac{1}{a(bx+a)}x^{\frac{2}{3}} - \frac{1}{3ab}\ln\left(\sqrt[3]{x} + \sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{1}{6ab}\ln\left(x^{\frac{2}{3}} - \sqrt[3]{\frac{a}{b}}\sqrt[3]{x} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{\sqrt{3}}{3ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a)^2,x)
```

```
[Out] x^(2/3)/a/(b*x+a)-1/3/a/b/(1/b*a)^(1/3)*ln(x^(1/3)+(1/b*a)^(1/3))+1/6/a/b/(
1/b*a)^(1/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/3))+1/3/a*3^(1/2)/
b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.27565, size = 990, normalized size = 8.53

$$\left[\frac{6\,ab^2x^{\frac{2}{3}} + 3\sqrt{\frac{1}{3}}\left(ab^2x + a^2b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right) + \left(-ab^2\right)^{\frac{2}{3}}(bx + a}{6\left(a^2b^3x + a^3b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(6*a*b^2*x^(2/3) + 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*
b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
 + a)) + (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3
) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*(b*x + a)*log(b*x^(1/3) - (-a*b^2)^(
1/3)))/(a^2*b^3*x + a^3*b^2), 1/6*(6*a*b^2*x^(2/3) + 6*sqrt(1/3)*(a*b^2*x +
 a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^2)^(1
/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*(b*x + a)*l
og(b*x^(1/3) - (-a*b^2)^(1/3)))/(a^2*b^3*x + a^3*b^2)]
```

Sympy [A] time = 159.72, size = 819, normalized size = 7.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (3*x**(2/3)/(2*a**2), Eq(b,
0)), (-3/(4*b**2*x**(4/3)), Eq(a, 0)), (-6*(-1)**(1/3)*a**(4/3)*b**2*x**(2/
3)*(1/b)**(4/3)/(-6*(-1)**(1/3)*a**(10/3)*b**2*(1/b)**(4/3) - 6*(-1)**(1/3)
*a**(7/3)*b**3*x*(1/b)**(4/3)) - 2*a**2*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1
/3) + x**(1/3))/(-6*(-1)**(1/3)*a**(10/3)*b**2*(1/b)**(4/3) - 6*(-1)**(1/3)
*a**(7/3)*b**3*x*(1/b)**(4/3)) + a**2*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/
3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(-6*(-1)**(
1/3)*a**(10/3)*b**2*(1/b)**(4/3) - 6*(-1)**(1/3)*a**(7/3)*b**3*x*(1/b)**(4/
3)) - 2*sqrt(3)*a**2*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**
```

```
(1/3)*(1/b)**(1/3)))/(-6*(-1)**(1/3)*a**(10/3)*b**2*(1/b)**(4/3) - 6*(-1)**
(1/3)*a**(7/3)*b**3*x*(1/b)**(4/3)) - 2*a**2*log(2)/(-6*(-1)**(1/3)*a**(10/
3)*b**2*(1/b)**(4/3) - 6*(-1)**(1/3)*a**(7/3)*b**3*x*(1/b)**(4/3)) - 2*a*b*
x*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(-6*(-1)**(1/3)*a**(10
/3)*b**2*(1/b)**(4/3) - 6*(-1)**(1/3)*a**(7/3)*b**3*x*(1/b)**(4/3)) + a*b*x
*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*
(1/b)**(1/3) + 4*x**(2/3))/(-6*(-1)**(1/3)*a**(10/3)*b**2*(1/b)**(4/3) - 6*
(-1)**(1/3)*a**(7/3)*b**3*x*(1/b)**(4/3)) - 2*sqrt(3)*a*b*x*atan(sqrt(3)/3
- 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(-6*(-1)**(1/3)
*a**(10/3)*b**2*(1/b)**(4/3) - 6*(-1)**(1/3)*a**(7/3)*b**3*x*(1/b)**(4/3))
- 2*a*b*x*log(2)/(-6*(-1)**(1/3)*a**(10/3)*b**2*(1/b)**(4/3) - 6*(-1)**(1/3
)*a**(7/3)*b**3*x*(1/b)**(4/3)), True))
```

Giac [A] time = 1.08371, size = 178, normalized size = 1.53

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2}+\frac{x^{\frac{2}{3}}}{(bx+a)a}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right.\right.}{6\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + x^(2/3)/((b*x + a)
*a) - 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^2*b^2) + 1/6*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2)
```

3.687 $\int \frac{1}{x^{2/3}(a+bx)^2} dx$

Optimal. Leaf size=113

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

```
[Out] x^(1/3)/(a*(a + b*x)) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(5/3)*b^(1/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(5/3
)*b^(1/3)) - Log[a + b*x]/(3*a^(5/3)*b^(1/3))
```

Rubi [A] time = 0.0404265, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)^2),x]
```

```
[Out] x^(1/3)/(a*(a + b*x)) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(5/3)*b^(1/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(5/3
)*b^(1/3)) - Log[a + b*x]/(3*a^(5/3)*b^(1/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{2/3}(a+bx)^2}\,dx &= \frac{\sqrt[3]{x}}{a(a+bx)}+\frac{2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3a}\\
&= \frac{\sqrt[3]{x}}{a(a+bx)}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{a^{4/3}b^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx,x,\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}\\
&= \frac{\sqrt[3]{x}}{a(a+bx)}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}+\frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{5/3}\sqrt[3]{b}}\\
&= \frac{\sqrt[3]{x}}{a(a+bx)}-\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [C] time = 0.0046421, size = 25, normalized size = 0.22

$$\frac{3\sqrt[3]{x}\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};-\frac{bx}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)^2),x]
```

```
[Out] (3*x^(1/3)*Hypergeometric2F1[1/3, 2, 4/3, -((b*x)/a)])/a^2
```

Maple [A] time = 0.007, size = 120, normalized size = 1.1

$$\frac{1}{a\,(bx+a)}\sqrt[3]{x}+\frac{2}{3\,ab}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{1}{3\,ab}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2\sqrt{3}}{3\,ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a)^2,x)
```

```
[Out] x^(1/3)/a/(b*x+a)+2/3/a/b/(1/b*a)^(2/3)*ln(x^(1/3)+(1/b*a)^(1/3))-1/3/a/b/(
1/b*a)^(2/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/3))+2/3/a/b/(1/b*a
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.33606, size = 991, normalized size = 8.77

$$\left[\frac{3\,a^2bx^{\frac{1}{3}} + 3\,\sqrt{\frac{1}{3}}\left(ab^2x + a^2b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx - a^2 + 3\,\sqrt{\frac{1}{3}}\left(2\,abx^{\frac{2}{3}} - \left(a^2b\right)^{\frac{1}{3}}a + \left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}} - 3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx + a}\right) - \left(a^2b\right)^{\frac{2}{3}}(bx + a)\log \cdots}{3\left(a^3b^2x + a^4b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^2*b*x^(1/3) + 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*
b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
+ a)) - (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)
^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)
))/(a^3*b^2*x + a^4*b), 1/3*(3*a^2*b*x^(1/3) + 6*sqrt(1/3)*(a*b^2*x + a^2*b
)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^(2/3
)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(
2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*(b*x + a)
*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^3*b^2*x + a^4*b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08328, size = 178, normalized size = 1.58

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} + \frac{2\,\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b} + \frac{x^{\frac{1}{3}}}{(bx + a)a} + \frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\cdots\right.\right.}{3\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + 2/3*sqrt(3)*(-a*b^
2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b
) + x^(1/3)/((b*x + a)*a) + 1/3*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)
^(1/3) + (-a/b)^(2/3))/(a^2*b)
```

3.688 $\int \frac{1}{x^{4/3}(a+bx)^2}\,dx$

Optimal. Leaf size=124

$$\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{7/3}}-\frac{2\sqrt[3]{b}\log(a+bx)}{3a^{7/3}}+\frac{4\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}}-\frac{4}{a^2\sqrt[3]{x}}+\frac{1}{a\sqrt[3]{x}(a+bx)}$$

```
[Out] -4/(a^2*x^(1/3)) + 1/(a*x^(1/3)*(a + b*x)) + (4*b^(1/3)*ArcTan[(a^(1/3) - 2
*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (2*b^(1/3)*Log[a^
(1/3) + b^(1/3)*x^(1/3)])/a^(7/3) - (2*b^(1/3)*Log[a + b*x])/(3*a^(7/3))
```

Rubi [A] time = 0.0491247, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{7/3}}-\frac{2\sqrt[3]{b}\log(a+bx)}{3a^{7/3}}+\frac{4\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}}-\frac{4}{a^2\sqrt[3]{x}}+\frac{1}{a\sqrt[3]{x}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)^2),x]
```


```
[Out] -4/(a^2*x^(1/3)) + 1/(a*x^(1/3)*(a + b*x)) + (4*b^(1/3)*ArcTan[(a^(1/3) - 2
*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (2*b^(1/3)*Log[a^
(1/3) + b^(1/3)*x^(1/3)])/a^(7/3) - (2*b^(1/3)*Log[a + b*x])/(3*a^(7/3))
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x^{4/3}(a+bx)^2}\,dx = \frac{1}{a\sqrt[3]{x}(a+bx)} + \frac{4\int \frac{1}{x^{4/3}(a+bx)}\,dx}{3a}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}(a+bx)} - \frac{(4b)\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{3a^2}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}(a+bx)} - \frac{2\sqrt[3]{b}\log(a+bx)}{3a^{7/3}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{a^2} + \frac{\left(2\sqrt[3]{b}\right)\mathrm{Subst}}{}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}(a+bx)} + \frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{7/3}} - \frac{2\sqrt[3]{b}\log(a+bx)}{3a^{7/3}} - \frac{\left(4\sqrt[3]{b}\right)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{a^{7/3}}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}(a+bx)} + \frac{4\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{7/3}} + \frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{7/3}} - \frac{2\sqrt[3]{b}\log(a+bx)}{3a^{7/3}}$$

Mathematica [C] time = 0.0056071, size = 25, normalized size = 0.2

$$-\frac{3\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};-\frac{bx}{a}\right)}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)^2),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/3, 2, 2/3, -((b*x)/a)])/(a^2*x^(1/3))
```

Maple [A] time = 0.012, size = 121, normalized size = 1.

$$-3\frac{1}{a^2\sqrt[3]{x}} - \frac{b}{a^2(bx+a)}x^{\frac{2}{3}} + \frac{4}{3a^2}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{2}{3a^2}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{4\sqrt{3}}{3a^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3)/(b*x+a)^2,x)
```

```
[Out] -3/a^2/x^(1/3)-b/a^2*x^(2/3)/(b*x+a)+4/3/a^2/(1/b*a)^(1/3)*ln(x^(1/3)+(1/b*
a)^(1/3))-2/3/a^2/(1/b*a)^(1/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2
```

```
/3))-4/3/a^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x^(1
/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35309, size = 393, normalized size = 3.17

$$-\frac{4\sqrt{3}\left(bx^2+ax\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+2\left(bx^2+ax\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-4\left(bx^2+ax\right.}{3\left(a^2bx^2+a^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*sqrt(3)*(b*x^2 + a*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x^(1/3)*(b/a)^
(1/3) - 1/3*sqrt(3)) + 2*(b*x^2 + a*x)*(b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/
3) + b*x^(2/3) + a*(b/a)^(1/3)) - 4*(b*x^2 + a*x)*(b/a)^(1/3)*log(a*(b/a)^(
2/3) + b*x^(1/3)) + 3*(4*b*x + 3*a)*x^(2/3))/(a^2*b*x^2 + a^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0914, size = 196, normalized size = 1.58

$$\frac{4\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3}+\frac{4\,\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b}-\frac{4\,bx+3\,a}{\left(bx^{\frac{4}{3}}+ax^{\frac{1}{3}}\right)a^2}-\frac{2\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{3\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4/3*b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 + 4/3*sqrt(3)*(-a*b
^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*
b) - (4*b*x + 3*a)/((b*x^(4/3) + a*x^(1/3))*a^2) - 2/3*(-a*b^2)^(2/3)*log(x
^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b)
```

3.689 $\int \frac{1}{x^{5/3}(a+bx)^2}\,dx$

Optimal. Leaf size=128

$$-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}}+\frac{5b^{2/3}\log(a+bx)}{6a^{8/3}}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}}-\frac{5}{2a^2x^{2/3}}+\frac{1}{ax^{2/3}(a+bx)}$$

```
[Out] -5/(2*a^2*x^(2/3)) + 1/(a*x^(2/3)*(a + b*x)) + (5*b^(2/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) - (5*b^(2/3)*Log[
a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(8/3)) + (5*b^(2/3)*Log[a + b*x])/(6*a^(8/
3))
```

Rubi [A] time = 0.0501187, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}}+\frac{5b^{2/3}\log(a+bx)}{6a^{8/3}}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}}-\frac{5}{2a^2x^{2/3}}+\frac{1}{ax^{2/3}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)^2),x]
```

```
[Out] -5/(2*a^2*x^(2/3)) + 1/(a*x^(2/3)*(a + b*x)) + (5*b^(2/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) - (5*b^(2/3)*Log[
a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(8/3)) + (5*b^(2/3)*Log[a + b*x])/(6*a^(8/
3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x^{5/3}(a+bx)^2}\,dx = \frac{1}{ax^{2/3}(a+bx)} + \frac{5\int \frac{1}{x^{5/3}(a+bx)}\,dx}{3a}$$

$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} - \frac{(5b)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3a^2}$$

$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} + \frac{5b^{2/3}\log(a+bx)}{6a^{8/3}} - \frac{\left(5\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2a^{7/3}} - \frac{\left(5b^{2/3}\right.}{}$$

$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}} + \frac{5b^{2/3}\log(a+bx)}{6a^{8/3}} - \frac{\left(5b^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-}\right.}{a^{8/}}$$

$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} + \frac{5b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{8/3}} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}} + \frac{5b^{2/3}\log(a+bx}{6a^{8/3}}$$

Mathematica [C] time = 0.0050339, size = 27, normalized size = 0.21

$$-\frac{3\,{}_2F_1\left(-\frac{2}{3},2;\frac{1}{3};-\frac{bx}{a}\right)}{2a^2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)^2),x]
```

```
[Out] (-3*Hypergeometric2F1[-2/3, 2, 1/3, -((b*x)/a)])/(2*a^2*x^(2/3))
```

Maple [A] time = 0.012, size = 121, normalized size = 1.

$$-\frac{3}{2\,a^2}x^{-\frac{2}{3}} - \frac{b}{a^2\,(bx+a)}\sqrt[3]{x} - \frac{5}{3\,a^2}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{5}{6\,a^2}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{5\sqrt{3}}{3\,a^2}\arctan\left(\frac{\sqrt{3}}{3}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a)^2,x)
```

```
[Out] -3/2/a^2/x^(2/3)-b/a^2*x^(1/3)/(b*x+a)-5/3/a^2/(1/b*a)^(2/3)*ln(x^(1/3)+(1/
b*a)^(1/3))+5/6/a^2/(1/b*a)^(2/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^
```

```
(2/3))-5/3/a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x^
(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.45074, size = 456, normalized size = 3.56

$$\frac{10\sqrt{3}\left(bx^2+ax\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-5\left(bx^2+ax\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+10\left(\right.}{6\left(a^2bx^2+a^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(10*sqrt(3)*(b*x^2 + a*x)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x^(1
/3)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(b*x^2 + a*x)*(-b^2/a^2)^(1/3)*log
(b^2*x^(2/3) + a*b*x^(1/3)*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 10*(b
*x^2 + a*x)*(-b^2/a^2)^(1/3)*log(b*x^(1/3) - a*(-b^2/a^2)^(1/3)) - 3*(5*b*x
 + 3*a)*x^(1/3))/(a^2*b*x^2 + a^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08042, size = 185, normalized size = 1.45

$$\frac{5b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3}-\frac{5\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3}-\frac{bx^{\frac{1}{3}}}{(bx+a)a^2}-\frac{5\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(\right.\right.}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5/3*b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 - 5/3*sqrt(3)*(-a*b
^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3 -
 b*x^(1/3)/((b*x + a)*a^2) - 5/6*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b
)^(1/3) + (-a/b)^(2/3))/a^3 - 3/2/(a^2*x^(2/3))
```

3.690 $\int \frac{x^{5/3}}{(a+bx)^3} dx$

Optimal. Leaf size=140

$$-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}b^{8/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{8/3}}-\frac{x^{5/3}}{2b(a+bx)^2}$$

```
[Out] -x^(5/3)/(2*b*(a + b*x)^2) - (5*x^(2/3))/(6*b^2*(a + b*x)) - (5*ArcTan[(a^(
1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(8/3)) -
 (5*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(6*a^(1/3)*b^(8/3)) + (5*Log[a + b*x])/
(18*a^(1/3)*b^(8/3))
```

Rubi [A] time = 0.0486111, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 56, 617, 204, 31}

$$-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}b^{8/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{8/3}}-\frac{x^{5/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x)^3,x]
```

```
[Out] -x^(5/3)/(2*b*(a + b*x)^2) - (5*x^(2/3))/(6*b^2*(a + b*x)) - (5*ArcTan[(a^(
1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(8/3)) -
 (5*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(6*a^(1/3)*b^(8/3)) + (5*Log[a + b*x])/
(18*a^(1/3)*b^(8/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{5/3}}{(a+bx)^3}\,dx = -\frac{x^{5/3}}{2b(a+bx)^2} + \frac{5\int \frac{x^{2/3}}{(a+bx)^2}\,dx}{6b}$$

$$= -\frac{x^{5/3}}{2b(a+bx)^2} - \frac{5x^{2/3}}{6b^2(a+bx)} + \frac{5\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{9b^2}$$

$$= -\frac{x^{5/3}}{2b(a+bx)^2} - \frac{5x^{2/3}}{6b^2(a+bx)} + \frac{5\log(a+bx)}{18\sqrt[3]{a}b^{8/3}} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{6b^3} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}\cdots}\right.}{6\sqrt[3]{ab}\cdots}$$

$$= -\frac{x^{5/3}}{2b(a+bx)^2} - \frac{5x^{2/3}}{6b^2(a+bx)} - \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}b^{8/3}} + \frac{5\log(a+bx)}{18\sqrt[3]{a}b^{8/3}} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\cdots}{\cdots}\right.}{3\sqrt[3]{a}b^{8/3}}$$

$$= -\frac{x^{5/3}}{2b(a+bx)^2} - \frac{5x^{2/3}}{6b^2(a+bx)} - \frac{5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{8/3}} - \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}b^{8/3}} + \frac{5\log(a+bx)}{18\sqrt[3]{a}b^{8/3}}$$

Mathematica [C] time = 0.0043703, size = 27, normalized size = 0.19

$$\frac{3x^{8/3}\,{}_2F_1\left(\frac{8}{3},3;\frac{11}{3};-\frac{bx}{a}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x)^3,x]
```

```
[Out] (3*x^(8/3)*Hypergeometric2F1[8/3, 3, 11/3, -((b*x)/a)])/(8*a^3)
```

Maple [A] time = 0.011, size = 124, normalized size = 0.9

$$3\frac{1}{(bx+a)^2}\left(-4/9\frac{x^{5/3}}{b} - \frac{5\,ax^{2/3}}{18\,b^2}\right) - \frac{5}{9\,b^3}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{5}{18\,b^3}\ln\left(x^{\frac{2}{3}} - \sqrt[3]{\frac{a}{b}}\sqrt[3]{x} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{5\sqrt{3}}{9\,b^3}\arctan\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a)^3,x)
```

```
[Out] 3*(-4/9*x^(5/3)/b-5/18*a*x^(2/3)/b^2)/(b*x+a)^2-5/9/b^3/(1/b*a)^(1/3)*ln(x^
(1/3)+(1/b*a)^(1/3))+5/18/b^3/(1/b*a)^(1/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3
)+(1/b*a)^(2/3))+5/9/b^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a
)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.4179, size = 1219, normalized size = 8.71

$$\left[\frac{15\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+5\left(b^2x^2+2abx\right.}{18\left(ab^6x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*
b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
 + a)) + 5*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b
^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 10*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^
2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) - 3*(8*a*b^3*x + 5*a^2*b^2)*x^(2/3
))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4), 1/18*(30*sqrt(1/3)*(a*b^3*x^2 + 2*a
^2*b^2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 5*(b^2*x^2 + 2*a*b*x + a^2)*(-a
*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) -
10*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3))
 - 3*(8*a*b^3*x + 5*a^2*b^2)*x^(2/3))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a)**3,x)
```

[Out] Timed out

Giac [A] time = 1.08823, size = 197, normalized size = 1.41

$$-\frac{5\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2}-\frac{5\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4}-\frac{8\,bx^{\frac{5}{3}}+5\,ax^{\frac{2}{3}}}{6\,(bx+a)^2b^2}+\frac{5\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\right.\right.}{18\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -5/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b^2) - 5/9*sqrt(3)*(-
a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
*b^4) - 1/6*(8*b*x^(5/3) + 5*a*x^(2/3))/((b*x + a)^2*b^2) + 5/18*(-a*b^2)^(
2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4)
```

3.691 $\int \frac{x^{4/3}}{(a+bx)^3}\,dx$

Optimal. Leaf size=140

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{x^{4/3}}{2b(a+bx)^2}$$

```
[Out] -x^(4/3)/(2*b*(a + b*x)^2) - (2*x^(1/3))/(3*b^2*(a + b*x)) - (2*ArcTan[(a^(
1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(7/3)) +
 Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(2/3)*b^(7/3)) - Log[a + b*x]/(9*a^(2/
3)*b^(7/3))
```

Rubi [A] time = 0.053599, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 58, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{x^{4/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x)^3,x]
```

```
[Out] -x^(4/3)/(2*b*(a + b*x)^2) - (2*x^(1/3))/(3*b^2*(a + b*x)) - (2*ArcTan[(a^(
1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(7/3)) +
 Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(2/3)*b^(7/3)) - Log[a + b*x]/(9*a^(2/
3)*b^(7/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{4/3}}{(a+bx)^3}\,dx &= -\frac{x^{4/3}}{2b(a+bx)^2}+\frac{2\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx}{3b}\\ &= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}+\frac{2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9b^2}\\ &= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{3\sqrt[3]{a}b^{8/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx\right.}{3a^{2/3}b^{7/3}}\\ &= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}+\frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\sqrt[3]{}}{\sqrt[3]{a}}\right.}{3a^{2/3}b^{7/3}}\\ &= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}\end{aligned}$$

Mathematica [C] time = 0.0041966, size = 27, normalized size = 0.19

$$\frac{3x^{7/3}\,{}_2F_1\left(\frac{7}{3},3;\frac{10}{3};-\frac{bx}{a}\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x)^3,x]
```

```
[Out] (3*x^(7/3)*Hypergeometric2F1[7/3, 3, 10/3, -((b*x)/a)])/(7*a^3)
```

Maple [A] time = 0.01, size = 124, normalized size = 0.9

$$3\frac{1}{(bx+a)^2}\left(-\frac{7x^{4/3}}{18b}-2/9\frac{a\sqrt[3]{x}}{b^2}\right)+\frac{2}{9b^3}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{1}{9b^3}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2\sqrt{3}}{9b^3}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a)^3,x)
```

```
[Out] 3*(-7/18*x^(4/3)/b-2/9*a*x^(1/3)/b^2)/(b*x+a)^2+2/9/b^3/(1/b*a)^(2/3)*ln(x^
(1/3)+(1/b*a)^(1/3))-1/9/b^3/(1/b*a)^(2/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)
```

```
+(1/b*a)^(2/3))+2/9/b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)
^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.47622, size = 1227, normalized size = 8.76

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx-a^2+3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}}-(a^2b)^{\frac{1}{3}}a+(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}-3(a^2b)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-2\left(b^2x^2+2abx+\cdots\right.}{18\left(a^2b^5x^2+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b
)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x +
 a)) - 2*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^
(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 4*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3
)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) - 3*(7*a^2*b^2*x + 4*a^3*b)*x^(1/3))/(a^
2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3), 1/18*(12*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b
^2*x + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*
(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(b^2*x^2 + 2*a*b*x +
a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3
)) + 4*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2
/3)) - 3*(7*a^2*b^2*x + 4*a^3*b)*x^(1/3))/(a^2*b^5*x^2 + 2*a^3*b^4*x + a^4*
b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a)**3,x)
```

[Out] Timed out

Giac [A] time = 1.08043, size = 197, normalized size = 1.41

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^{2}}+\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{3}}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^{3}}-\frac{7\,}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b^2) + 2/9*sqrt(3)*(-
a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
*b^3) + 1/9*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3
))/(a*b^3) - 1/6*(7*b*x^(4/3) + 4*a*x^(1/3))/((b*x + a)^2*b^2)
```

3.692 $\int \frac{x^{2/3}}{(a+bx)^3} \, dx$

Optimal. Leaf size=143

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}}+\frac{\log(a+bx)}{18a^{4/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}}+\frac{x^{2/3}}{3ab(a+bx)}-\frac{x^{2/3}}{2b(a+bx)^2}$$

```
[Out] -x^(2/3)/(2*b*(a + b*x)^2) + x^(2/3)/(3*a*b*(a + b*x)) - ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - Log[a^(
1/3) + b^(1/3)*x^(1/3)]/(6*a^(4/3)*b^(5/3)) + Log[a + b*x]/(18*a^(4/3)*b^(5
/3))
```

Rubi [A] time = 0.0499361, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 51, 56, 617, 204, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}}+\frac{\log(a+bx)}{18a^{4/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}}+\frac{x^{2/3}}{3ab(a+bx)}-\frac{x^{2/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x)^3,x]
```

```
[Out] -x^(2/3)/(2*b*(a + b*x)^2) + x^(2/3)/(3*a*b*(a + b*x)) - ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - Log[a^(
1/3) + b^(1/3)*x^(1/3)]/(6*a^(4/3)*b^(5/3)) + Log[a + b*x]/(18*a^(4/3)*b^(5
/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{2/3}}{(a+bx)^3}\,dx &= -\frac{x^{2/3}}{2b(a+bx)^2}+\frac{\int \frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx}{3b}\\
&= -\frac{x^{2/3}}{2b(a+bx)^2}+\frac{x^{2/3}}{3ab(a+bx)}+\frac{\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{9ab}\\
&= -\frac{x^{2/3}}{2b(a+bx)^2}+\frac{x^{2/3}}{3ab(a+bx)}+\frac{\log(a+bx)}{18a^{4/3}b^{5/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{6ab^2}-\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx\right.}{6a^{4/3}b^{5/3}}\\
&= -\frac{x^{2/3}}{2b(a+bx)^2}+\frac{x^{2/3}}{3ab(a+bx)}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}}+\frac{\log(a+bx)}{18a^{4/3}b^{5/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right.}{3a^{4/3}b^{5/3}}\\
&= -\frac{x^{2/3}}{2b(a+bx)^2}+\frac{x^{2/3}}{3ab(a+bx)}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}}+\frac{\log(a+bx)}{18a^{4/3}b^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.0048033, size = 27, normalized size = 0.19

$$\frac{3x^{5/3}\,{}_2F_1\left(\frac{5}{3},3;\frac{8}{3};-\frac{bx}{a}\right)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x)^3,x]
```

```
[Out] (3*x^(5/3)*Hypergeometric2F1[5/3, 3, 8/3, -((b*x)/a)])/(5*a^3)
```

Maple [A] time = 0.01, size = 132, normalized size = 0.9

$$3\frac{1}{(bx+a)^2}\left(1/9\frac{x^{5/3}}{a}-1/18\frac{x^{2/3}}{b}\right)-\frac{1}{9b^2a}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{1}{18b^2a}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{\sqrt{3}}{9b^2a}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(b*x+a)^3,x)
```

```
[Out] 3*(1/9/a*x^(5/3)-1/18*x^(2/3)/b)/(b*x+a)^2-1/9/b^2/a/(1/b*a)^(1/3)*ln(x^(1/
3)+(1/b*a)^(1/3))+1/18/b^2/a/(1/b*a)^(1/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)
+(1/b*a)^(2/3))+1/9/b^2/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*
a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.66037, size = 1208, normalized size = 8.45

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+\left(b^2x^2+2abx+\right.}{18\left(a^2b^5x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b
^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
+ a)) + (b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)
^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(
2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(2*a*b^3*x - a^2*b^2)*x^(2/3))/(a^
2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3), 1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^
2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^
2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(
2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(b^2*
x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(2*
a*b^3*x - a^2*b^2)*x^(2/3))/(a^2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09467, size = 201, normalized size = 1.41

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^3}+\frac{2\,bx^{\frac{5}{3}}-ax^{\frac{2}{3}}}{6\,(bx+a)^2ab}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\right.}{18\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a^2*b) - 1/9*sqrt(3)*(-
a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
^2*b^3) + 1/6*(2*b*x^(5/3) - a*x^(2/3))/((b*x + a)^2*a*b) + 1/18*(-a*b^2)^(
2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3)
```

3.693 $\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx$

Optimal. Leaf size=143

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}}-\frac{\log(a+bx)}{18a^{5/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{\sqrt[3]{x}}{6ab(a+bx)}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

```
[Out] -x^(1/3)/(2*b*(a + b*x)^2) + x^(1/3)/(6*a*b*(a + b*x)) - ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + Log[a^(
1/3) + b^(1/3)*x^(1/3)]/(6*a^(5/3)*b^(4/3)) - Log[a + b*x]/(18*a^(5/3)*b^(4
/3))
```

Rubi [A] time = 0.0519344, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 51, 58, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}}-\frac{\log(a+bx)}{18a^{5/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{\sqrt[3]{x}}{6ab(a+bx)}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x)^3,x]
```

```
[Out] -x^(1/3)/(2*b*(a + b*x)^2) + x^(1/3)/(6*a*b*(a + b*x)) - ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + Log[a^(
1/3) + b^(1/3)*x^(1/3)]/(6*a^(5/3)*b^(4/3)) - Log[a + b*x]/(18*a^(5/3)*b^(4
/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx &= -\frac{\sqrt[3]{x}}{2b(a+bx)^2}+\frac{\int \frac{1}{x^{2/3}(a+bx)^2}\,dx}{6b}\\ &= -\frac{\sqrt[3]{x}}{2b(a+bx)^2}+\frac{\sqrt[3]{x}}{6ab(a+bx)}+\frac{\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9ab}\\ &= -\frac{\sqrt[3]{x}}{2b(a+bx)^2}+\frac{\sqrt[3]{x}}{6ab(a+bx)}-\frac{\log(a+bx)}{18a^{5/3}b^{4/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx\right.}{6a^{5/3}b^{4/3}}\\ &= -\frac{\sqrt[3]{x}}{2b(a+bx)^2}+\frac{\sqrt[3]{x}}{6ab(a+bx)}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}}-\frac{\log(a+bx)}{18a^{5/3}b^{4/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right.}{3a^{5/3}b^{4/3}}\\ &= -\frac{\sqrt[3]{x}}{2b(a+bx)^2}+\frac{\sqrt[3]{x}}{6ab(a+bx)}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}}-\frac{\log(a+bx)}{18a^{5/3}b^{4/3}}\end{aligned}$$

Mathematica [C] time = 0.0051409, size = 27, normalized size = 0.19

$$\frac{3x^{4/3}\,{}_2F_1\left(\frac{4}{3},3;\frac{7}{3};-\frac{bx}{a}\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x)^3,x]
```

```
[Out] (3*x^(4/3)*Hypergeometric2F1[4/3, 3, 7/3, -((b*x)/a)])/(4*a^3)
```

Maple [A] time = 0.01, size = 132, normalized size = 0.9

$$3\,\frac{1}{(bx+a)^2}\left(1/18\,\frac{x^{4/3}}{a}-1/9\,\frac{\sqrt[3]{x}}{b}\right)+\frac{1}{9\,b^2a}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{1}{18\,b^2a}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}}{9\,b^2a}\,\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a)^3,x)
```

```
[Out] 3*(1/18/a*x^(4/3)-1/9*x^(1/3)/b)/(b*x+a)^2+1/9/b^2/a/(1/b*a)^(2/3)*ln(x^(1/
3)+(1/b*a)^(1/3))-1/18/b^2/a/(1/b*a)^(2/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)
+(1/b*a)^(2/3))+1/9/b^2/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*
a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.72668, size = 1215, normalized size = 8.5

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx-a^2+3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}}-\left(a^2b\right)^{\frac{1}{3}}a+\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-\left(b^2x^2+2abx+a^{2}\cdots\right.}{18\left(a^3b^4x^2+2\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b
)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x +
 a)) - (b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1
/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*
log(a*b*x^(1/3) + (a^2*b)^(2/3)) + 3*(a^2*b^2*x - 2*a^3*b)*x^(1/3))/(a^3*b^
4*x^2 + 2*a^4*b^3*x + a^5*b^2), 1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x
+ a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*
b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^2 + 2*a*b*x + a^2)*(a
^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*
(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) +
3*(a^2*b^2*x - 2*a^3*b)*x^(1/3))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0748, size = 200, normalized size = 1.4

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{2}b}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{2}b^{2}}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{2}b^{2}}+\frac{bx^{\frac{4}{3}}-}{6\,(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a^2*b) + 1/9*sqrt(3)*(-
a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
^2*b^2) + 1/18*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(
2/3))/(a^2*b^2) + 1/6*(b*x^(4/3) - 2*a*x^(1/3))/((b*x + a)^2*a*b)
```

3.694 $\int \frac{1}{\sqrt[3]{x}(a+bx)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}}+\frac{\log(a+bx)}{9a^{7/3}b^{2/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}}+\frac{2x^{2/3}}{3a^2(a+bx)}+\frac{x^{2/3}}{2a(a+bx)^2}$$

```
[Out] x^(2/3)/(2*a*(a + b*x)^2) + (2*x^(2/3))/(3*a^2*(a + b*x)) - (2*ArcTan[(a^(1
/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(2/3)) -
Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(7/3)*b^(2/3)) + Log[a + b*x]/(9*a^(7/3
)*b^(2/3))
```

Rubi [A] time = 0.0528097, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}}+\frac{\log(a+bx)}{9a^{7/3}b^{2/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}}+\frac{2x^{2/3}}{3a^2(a+bx)}+\frac{x^{2/3}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)^3),x]
```

```
[Out] x^(2/3)/(2*a*(a + b*x)^2) + (2*x^(2/3))/(3*a^2*(a + b*x)) - (2*ArcTan[(a^(1
/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(2/3)) -
Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(7/3)*b^(2/3)) + Log[a + b*x]/(9*a^(7/3
)*b^(2/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^3}\,dx = \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2\int \frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx}{3a}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} + \frac{2\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{9a^2}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{3a^2b} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\, d\right.}{3a^{7/3}b^{2/3}}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}}{\sqrt[3]{}}\right.}{3a^{7/3}b^{2/3}}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}}$$

Mathematica [C] time = 0.004572, size = 27, normalized size = 0.19

$$\frac{3x^{2/3}\,{}_2F_1\left(\frac{2}{3}, 3; \frac{5}{3}; -\frac{bx}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)^3),x]
```

```
[Out] (3*x^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, -((b*x)/a)])/(2*a^3)
```

Maple [A] time = 0.006, size = 136, normalized size = 1.

$$\frac{1}{2\,a\,(bx+a)^2}x^{\frac{2}{3}} + \frac{2}{3\,a^2\,(bx+a)}x^{\frac{2}{3}} - \frac{2}{9\,a^2b}\ln\left(\sqrt[3]{x} + \sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{1}{9\,a^2b}\ln\left(x^{\frac{2}{3}} - \sqrt[3]{\frac{a}{b}}\sqrt[3]{x} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{2\sqrt{3}}{9\,a^2b}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a)^3,x)
```

```
[Out] 1/2*x^(2/3)/a/(b*x+a)^2+2/3*x^(2/3)/a^2/(b*x+a)-2/9/a^2/b/(1/b*a)^(1/3)*ln(
x^(1/3)+(1/b*a)^(1/3))+1/9/a^2/b/(1/b*a)^(1/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^(
```

```
1/3)+(1/b*a)^(2/3))+2/9/a^2*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(
1/b*a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.67306, size = 1220, normalized size = 8.71

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+2\left(b^2x^2+2abx\right.}{18\left(a^3b^4x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b
^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
+ a)) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^
2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 4*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)
^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(4*a*b^3*x + 7*a^2*b^2)*x^(2/3))
/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2), 1/18*(12*sqrt(1/3)*(a*b^3*x^2 + 2*a
^2*b^2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a
*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) -
4*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3))
+ 3*(4*a*b^3*x + 7*a^2*b^2)*x^(2/3))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08428, size = 193, normalized size = 1.38

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{3}}-\frac{2\,\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}b^{2}}+\frac{4\,bx^{\frac{5}{3}}+7\,ax^{\frac{2}{3}}}{6\,(bx+a)^{2}a^{2}}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)\right.}{9\,a^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 - 2/9*sqrt(3)*(-a*b^
2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b
^2) + 1/6*(4*b*x^(5/3) + 7*a*x^(2/3))/((b*x + a)^2*a^2) + 1/9*(-a*b^2)^(2/3
)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2)
```

3.695 $\int \frac{1}{x^{2/3}(a+bx)^3} dx$

Optimal. Leaf size=140

$$\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

```
[Out] x^(1/3)/(2*a*(a + b*x)^2) + (5*x^(1/3))/(6*a^2*(a + b*x)) - (5*ArcTan[(a^(1
/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(1/3)) +
(5*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(6*a^(8/3)*b^(1/3)) - (5*Log[a + b*x])/(
18*a^(8/3)*b^(1/3))
```

Rubi [A] time = 0.0496618, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)^3),x]
```

```
[Out] x^(1/3)/(2*a*(a + b*x)^2) + (5*x^(1/3))/(6*a^2*(a + b*x)) - (5*ArcTan[(a^(1
/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(1/3)) +
(5*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(6*a^(8/3)*b^(1/3)) - (5*Log[a + b*x])/(
18*a^(8/3)*b^(1/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{2/3}(a+bx)^3}\,dx &= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\int \frac{1}{x^{2/3}(a+bx)^2}\,dx}{6a} \\
&= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{5\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9a^2} \\
&= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}+\frac{5\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{6a^{7/3}b^{2/3}}+\frac{5\,\text{Subst}\left(\int \right.}{6a^{8}} \\
&= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}+\frac{5\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\right.}{3a^{8/3}\sqrt[3]{b}} \\
&= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}-\frac{5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [C] time = 0.0039178, size = 25, normalized size = 0.18

$$\frac{3\sqrt[3]{x}\,{}_2F_1\left(\frac{1}{3},3;\frac{4}{3};-\frac{bx}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)^3),x]
```

```
[Out] (3*x^(1/3)*Hypergeometric2F1[1/3, 3, 4/3, -((b*x)/a)])/a^3
```

Maple [A] time = 0.006, size = 136, normalized size = 1.

$$\frac{1}{2a(bx+a)^2}\sqrt[3]{x}+\frac{5}{6a^2(bx+a)}\sqrt[3]{x}+\frac{5}{9a^2b}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{5}{18a^2b}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{5\sqrt{3}}{9a^2b}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a)^3,x)
```

```
[Out] 1/2*x^(1/3)/a/(b*x+a)^2+5/6*x^(1/3)/a^2/(b*x+a)+5/9/a^2/b/(1/b*a)^(2/3)*ln(
x^(1/3)+(1/b*a)^(1/3))-5/18/a^2/b/(1/b*a)^(2/3)*ln(x^(2/3)-(1/b*a)^(1/3)*x^
```

```
(1/3)+(1/b*a)^(2/3))+5/9/a^2/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(1/b*a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.6851, size = 1226, normalized size = 8.76

$$\left[\frac{15\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx-a^2+3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}}-\left(a^2b\right)^{\frac{1}{3}}a+\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-5\left(b^2x^2+2abx+\cdots\right.}{18\left(a^4b^3x^2+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="fricas")

[Out] [1/18*(15*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*
b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
+ a)) - 5*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)
^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 10*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2
/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) + 3*(5*a^2*b^2*x + 8*a^3*b)*x^(1/3))/(
a^4*b^3*x^2 + 2*a^5*b^2*x + a^6*b), 1/18*(30*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b
^2*x + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*
(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^2 + 2*a*b*x +
a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3
)) + 10*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(
2/3)) + 3*(5*a^2*b^2*x + 8*a^3*b)*x^(1/3))/(a^4*b^3*x^2 + 2*a^5*b^2*x + a^6
*b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07824, size = 193, normalized size = 1.38

$$-\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{3}}+\frac{5\,\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}b}+\frac{5\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{3}b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -5/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 + 5/9*sqrt(3)*(-a*b^
2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b
) + 5/18*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/
(a^3*b) + 1/6*(5*b*x^(4/3) + 8*a*x^(1/3))/((b*x + a)^2*a^2)
```

3.696 $\int \frac{1}{x^{4/3}(a+bx)^3} \, dx$

Optimal. Leaf size=152

$$\frac{7}{6a^2\sqrt[3]{x}(a+bx)}+\frac{7\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{10/3}}-\frac{7\sqrt[3]{b}\log(a+bx)}{9a^{10/3}}+\frac{14\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}-\frac{14}{3a^3\sqrt[3]{x}}+\frac{1}{2a\sqrt[3]{x}(a+bx)^2}$$

```
[Out] -14/(3*a^3*x^(1/3)) + 1/(2*a*x^(1/3)*(a + b*x)^2) + 7/(6*a^2*x^(1/3)*(a + b
*x)) + (14*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(10/3)) + (7*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(3*a^(10
/3)) - (7*b^(1/3)*Log[a + b*x])/(9*a^(10/3))
```

Rubi [A] time = 0.0596014, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$\frac{7}{6a^2\sqrt[3]{x}(a+bx)}+\frac{7\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{10/3}}-\frac{7\sqrt[3]{b}\log(a+bx)}{9a^{10/3}}+\frac{14\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}-\frac{14}{3a^3\sqrt[3]{x}}+\frac{1}{2a\sqrt[3]{x}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)^3),x]
```


```
[Out] -14/(3*a^3*x^(1/3)) + 1/(2*a*x^(1/3)*(a + b*x)^2) + 7/(6*a^2*x^(1/3)*(a + b
*x)) + (14*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(10/3)) + (7*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(3*a^(10
/3)) - (7*b^(1/3)*Log[a + b*x])/(9*a^(10/3))
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x^{4/3}(a+bx)^3}\,dx = \frac{1}{2a\sqrt[3]{x}(a+bx)^2} + \frac{7\int \frac{1}{x^{4/3}(a+bx)^2}\,dx}{6a}$$

$$= \frac{1}{2a\sqrt[3]{x}(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}(a+bx)} + \frac{14\int \frac{1}{x^{4/3}(a+bx)}\,dx}{9a^2}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}(a+bx)} - \frac{(14b)\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{9a^3}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}(a+bx)} - \frac{7\sqrt[3]{b}\log(a+bx)}{9a^{10/3}} - \frac{7\,\mathrm{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx, x, \right.}{3a^3}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}(a+bx)} + \frac{7\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{10/3}} - \frac{7\sqrt[3]{b}\log(a+bx)}{9a^{10/3}} - \frac{(14}{}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}(a+bx)} + \frac{14\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{10/3}} + \frac{7\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{10/3}}$$

Mathematica [C] time = 0.0052801, size = 25, normalized size = 0.16

$$-\frac{3\,{}_2F_1\left(-\frac{1}{3},3;\frac{2}{3};-\frac{bx}{a}\right)}{a^3\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)^3),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/3, 3, 2/3, -((b*x)/a)])/(a^3*x^(1/3))
```

Maple [A] time = 0.013, size = 139, normalized size = 0.9

$$-3\,\frac{1}{a^3\sqrt[3]{x}} - \frac{5\,b^2}{3\,a^3\,(bx+a)^2}x^{\frac{5}{3}} - \frac{13\,b}{6\,a^2\,(bx+a)^2}x^{\frac{2}{3}} + \frac{14}{9\,a^3}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{7}{9\,a^3}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{14}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3)/(b*x+a)^3,x)
```

```
[Out] -3/a^3/x^(1/3)-5/3*b^2/a^3/(b*x+a)^2*x^(5/3)-13/6*b/a^2/(b*x+a)^2*x^(2/3)+1
4/9/a^3/(1/b*a)^(1/3)*ln(x^(1/3)+(1/b*a)^(1/3))-7/9/a^3/(1/b*a)^(1/3)*ln(x^
(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/3))-14/9/a^3*3^(1/2)/(1/b*a)^(1/3)*a
rctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7421, size = 510, normalized size = 3.36

$$-\frac{28\sqrt{3}\left(b^{2}x^{3}+2\,abx^{2}+a^{2}x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+14\left(b^{2}x^{3}+2\,abx^{2}+a^{2}x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}\right.}{18\left(a^{3}b^{2}x^{3}+2\,a^{4}bx^{2}+a^{5}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/18*(28*sqrt(3)*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(b/a)^(1/3)*arctan(2/3*sqrt
(3)*x^(1/3)*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(
b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/3) + b*x^(2/3) + a*(b/a)^(1/3)) - 28*(b^
2*x^3 + 2*a*b*x^2 + a^2*x)*(b/a)^(1/3)*log(a*(b/a)^(2/3) + b*x^(1/3)) + 3*(
28*b^2*x^2 + 49*a*b*x + 18*a^2)*x^(2/3))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08608, size = 209, normalized size = 1.38

$$\frac{14\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{4}}+\frac{14\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{4}b}-\frac{3}{a^{3}x^{\frac{1}{3}}}-\frac{7\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right.\right.}{9\,a^{4}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 14/9*b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^4 + 14/9*sqrt(3)*(-a
*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^
4*b) - 3/(a^3*x^(1/3)) - 7/9*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a^4*b) - 1/6*(10*b^2*x^(5/3) + 13*a*b*x^(2/3))/((b*x +
 a)^2*a^3)
```

3.697 $\int \frac{1}{x^{5/3}(a+bx)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}}+\frac{10b^{2/3}\log(a+bx)}{9a^{11/3}}+\frac{20b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}}+\frac{4}{3a^2x^{2/3}(a+bx)}-\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx}$$

```
[Out] -10/(3*a^3*x^(2/3)) + 1/(2*a*x^(2/3)*(a + b*x)^2) + 4/(3*a^2*x^(2/3)*(a + b
*x)) + (20*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(11/3)) - (10*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(3*a^(1
1/3)) + (10*b^(2/3)*Log[a + b*x])/(9*a^(11/3))
```

Rubi [A] time = 0.0606935, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}}+\frac{10b^{2/3}\log(a+bx)}{9a^{11/3}}+\frac{20b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}}+\frac{4}{3a^2x^{2/3}(a+bx)}-\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)^3),x]
```

```
[Out] -10/(3*a^3*x^(2/3)) + 1/(2*a*x^(2/3)*(a + b*x)^2) + 4/(3*a^2*x^(2/3)*(a + b
*x)) + (20*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(11/3)) - (10*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(3*a^(1
1/3)) + (10*b^(2/3)*Log[a + b*x])/(9*a^(11/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/3}(a+bx)^3}\,dx &= \frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4\int \frac{1}{x^{5/3}(a+bx)^2}\,dx}{3a}\\
&= \frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4}{3a^2x^{2/3}(a+bx)}+\frac{20\int \frac{1}{x^{5/3}(a+bx)}\,dx}{9a^2}\\
&= -\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4}{3a^2x^{2/3}(a+bx)}-\frac{(20b)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9a^3}\\
&= -\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4}{3a^2x^{2/3}(a+bx)}+\frac{10b^{2/3}\log(a+bx)}{9a^{11/3}}-\frac{\left(10\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}\cdots}\right.}{3a^{10/3}}\\
&= -\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4}{3a^2x^{2/3}(a+bx)}-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}}+\frac{10b^{2/3}\log(a+bx)}{9a^{11/3}}\cdots\\
&= -\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4}{3a^2x^{2/3}(a+bx)}+\frac{20b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{11/3}}-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{3a^{11/3}}
\end{aligned}
$$

Mathematica [C] time = 0.0057024, size = 27, normalized size = 0.18

$$
-\frac{3\,{}_2F_1\left(-\frac{2}{3},3;\frac{1}{3};-\frac{bx}{a}\right)}{2a^3x^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)^3),x]
```

```
[Out] (-3*Hypergeometric2F1[-2/3, 3, 1/3, -((b*x)/a)])/(2*a^3*x^(2/3))
```

Maple [A] time = 0.014, size = 139, normalized size = 0.9

$$
-\frac{3}{2\,a^3}x^{-\frac{2}{3}}-\frac{11\,b^2}{6\,a^3\,(bx+a)^2}x^{\frac{4}{3}}-\frac{7\,b}{3\,a^2\,(bx+a)^2}\sqrt[3]{x}-\frac{20}{9\,a^3}\ln\left(\sqrt[3]{x}+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{10}{9\,a^3}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{\frac{a}{b}}\sqrt[3]{x}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a)^3,x)
```

```
[Out] -3/2/a^3/x^(2/3)-11/6/a^3*b^2/(b*x+a)^2*x^(4/3)-7/3/a^2*b/(b*x+a)^2*x^(1/3)
-20/9/a^3/(1/b*a)^(2/3)*ln(x^(1/3)+(1/b*a)^(1/3))+10/9/a^3/(1/b*a)^(2/3)*ln
(x^(2/3)-(1/b*a)^(1/3)*x^(1/3)+(1/b*a)^(2/3))-20/9/a^3/(1/b*a)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.56814, size = 570, normalized size = 3.75

$$\frac{40\sqrt{3}\left(b^2x^3+2\,abx^2+a^2x\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-20\left(b^2x^3+2\,abx^2+a^2x\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right.\right.}{18\left(a^3b^2x^3+2\,a^4bx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(40*sqrt(3)*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(-b^2/a^2)^(1/3)*arctan(1/3*
(2*sqrt(3)*a*x^(1/3)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 20*(b^2*x^3 + 2*a*b
*x^2 + a^2*x)*(-b^2/a^2)^(1/3)*log(b^2*x^(2/3) + a*b*x^(1/3)*(-b^2/a^2)^(1/
3) + a^2*(-b^2/a^2)^(2/3)) + 40*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(-b^2/a^2)^(1
/3)*log(b*x^(1/3) - a*(-b^2/a^2)^(1/3)) - 3*(20*b^2*x^2 + 32*a*b*x + 9*a^2)
*x^(1/3))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07723, size = 203, normalized size = 1.34

$$\frac{20\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4}-\frac{20\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4}-\frac{10\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^4}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 20/9*b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^4 - 20/9*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4
 - 10/9*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/a
^4 - 1/6*(20*b^2*x^2 + 32*a*b*x + 9*a^2)/((b*x^(4/3) + a*x^(1/3))^2*a^3)
```

3.698 $\int \frac{\sqrt[4]{1-x}}{1+x} \, dx$

Optimal. Leaf size=58

$$4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*ArcTan[(1 - x)^(1/4)/2^(1/4)] - 2*2^(1/4)*ArcTa
nh[(1 - x)^(1/4)/2^(1/4)]
```

Rubi [A] time = 0.0209206, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {50, 63, 212, 206, 203}

$$4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/4)/(1 + x),x]
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*ArcTan[(1 - x)^(1/4)/2^(1/4)] - 2*2^(1/4)*ArcTa
nh[(1 - x)^(1/4)/2^(1/4)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{1-x}}{1+x}\,dx &= 4\sqrt[4]{1-x}+2\int \frac{1}{(1-x)^{3/4}(1+x)}\,dx\\ &= 4\sqrt[4]{1-x}-8\,\text{Subst}\left(\int \frac{1}{2-x^4}\,dx,x,\sqrt[4]{1-x}\right)\\ &= 4\sqrt[4]{1-x}-\left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx,x,\sqrt[4]{1-x}\right)-\left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\,dx,x,\sqrt[4]{1-x}\right)\\ &= 4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0158726, size = 58, normalized size = 1.

$$4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/4)/(1 + x),x]
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*ArcTan[(1 - x)^(1/4)/2^(1/4)] - 2*2^(1/4)*ArcTa
nh[(1 - x)^(1/4)/2^(1/4)]
```

Maple [A] time = 0.007, size = 62, normalized size = 1.1

$$4\sqrt[4]{1-x}-2\sqrt[4]{2}\arctan\left(1/2\sqrt[4]{1-x}2^{3/4}\right)-\sqrt[4]{2}\ln\left(\left(\sqrt[4]{1-x}+\sqrt[4]{2}\right)\left(\sqrt[4]{1-x}-\sqrt[4]{2}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/4)/(1+x),x)
```

```
[Out] 4*(1-x)^(1/4)-2*2^(1/4)*arctan(1/2*(1-x)^(1/4)*2^(3/4))-2^(1/4)*ln(((1-x)^(
1/4)+2^(1/4))/((1-x)^(1/4)-2^(1/4)))
```

Maxima [A] time = 1.54982, size = 82, normalized size = 1.41

$$-2\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}(-x+1)^{\frac{1}{4}}\right)+2^{\frac{1}{4}}\log\left(-\frac{2^{\frac{1}{4}}-(-x+1)^{\frac{1}{4}}}{2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}}\right)+4(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="maxima")
```

```
[Out] -2*2^(1/4)*arctan(1/2*2^(3/4)*(-x + 1)^(1/4)) + 2^(1/4)*log(-(2^(1/4) - (-x
 + 1)^(1/4))/(2^(1/4) + (-x + 1)^(1/4))) + 4*(-x + 1)^(1/4)
```

Fricas [A] time = 1.66822, size = 255, normalized size = 4.4

$$4\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{\sqrt{2}+\sqrt{-x+1}}-\frac{1}{2}\cdot 2^{\frac{3}{4}}(-x+1)^{\frac{1}{4}}\right)-2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right)+2^{\frac{1}{4}}\log\left(-2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right)+4(-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="fricas")
```

```
[Out] 4*2^(1/4)*arctan(1/2*2^(3/4)*sqrt(sqrt(2) + sqrt(-x + 1)) - 1/2*2^(3/4)*(-x
 + 1)^(1/4)) - 2^(1/4)*log(2^(1/4) + (-x + 1)^(1/4)) + 2^(1/4)*log(-2^(1/4)
 + (-x + 1)^(1/4)) + 4*(-x + 1)^(1/4)
```

Sympy [C] time = 2.20202, size = 243, normalized size = 4.19

$$\frac{5\sqrt[4]{-1}\sqrt[4]{x-1}\Gamma\left(\frac{5}{4}\right)}{\Gamma\left(\frac{9}{4}\right)}+\frac{5\sqrt[4]{-2}e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}e^{\frac{i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{5(-1)^{\frac{3}{4}}\sqrt[4]{2}e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}e^{\frac{3i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{5\sqrt[4]{-2}e^{-\frac{i\pi}{4}}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/4)/(1+x),x)
```

```
[Out] 5*(-1)**(1/4)*(x - 1)**(1/4)*gamma(5/4)/gamma(9/4) + 5*(-2)**(1/4)*exp(-I*p
i/4)*log(-2**(3/4)*(x - 1)**(1/4)*exp_polar(I*pi/4)/2 + 1)*gamma(5/4)/(4*ga
mma(9/4)) - 5*(-1)**(3/4)*2**(1/4)*exp(-I*pi/4)*log(-2**(3/4)*(x - 1)**(1/4
)*exp_polar(3*I*pi/4)/2 + 1)*gamma(5/4)/(4*gamma(9/4)) - 5*(-2)**(1/4)*exp(
-I*pi/4)*log(-2**(3/4)*(x - 1)**(1/4)*exp_polar(5*I*pi/4)/2 + 1)*gamma(5/4)
/(4*gamma(9/4)) + 5*(-1)**(3/4)*2**(1/4)*exp(-I*pi/4)*log(-2**(3/4)*(x - 1)
**(1/4)*exp_polar(7*I*pi/4)/2 + 1)*gamma(5/4)/(4*gamma(9/4))
```

Giac [A] time = 1.09497, size = 86, normalized size = 1.48

$$-2\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}(-x+1)^{\frac{1}{4}}\right)-2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right)+2^{\frac{1}{4}}\log\left(\left|-2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right|\right)+4(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="giac")
```

```
[Out] -2*2^(1/4)*arctan(1/2*2^(3/4)*(-x + 1)^(1/4)) - 2^(1/4)*log(2^(1/4) + (-x +
 1)^(1/4)) + 2^(1/4)*log(abs(-2^(1/4) + (-x + 1)^(1/4))) + 4*(-x + 1)^(1/4)
```

3.699 $\int x^m (a+bx)^{10}\,dx$

Optimal. Leaf size=187

$$\frac{45a^8b^2x^{m+3}}{m+3}+\frac{120a^7b^3x^{m+4}}{m+4}+\frac{210a^6b^4x^{m+5}}{m+5}+\frac{252a^5b^5x^{m+6}}{m+6}+\frac{210a^4b^6x^{m+7}}{m+7}+\frac{120a^3b^7x^{m+8}}{m+8}+\frac{45a^2b^8x^{m+9}}{m+9}+\frac{10a^9bx}{m+}$$

```
[Out] (a^10*x^(1 + m))/(1 + m) + (10*a^9*b*x^(2 + m))/(2 + m) + (45*a^8*b^2*x^(3
+ m))/(3 + m) + (120*a^7*b^3*x^(4 + m))/(4 + m) + (210*a^6*b^4*x^(5 + m))/(
5 + m) + (252*a^5*b^5*x^(6 + m))/(6 + m) + (210*a^4*b^6*x^(7 + m))/(7 + m)
+ (120*a^3*b^7*x^(8 + m))/(8 + m) + (45*a^2*b^8*x^(9 + m))/(9 + m) + (10*a*
b^9*x^(10 + m))/(10 + m) + (b^10*x^(11 + m))/(11 + m)
```

Rubi [A] time = 0.0846196, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{45a^8b^2x^{m+3}}{m+3}+\frac{120a^7b^3x^{m+4}}{m+4}+\frac{210a^6b^4x^{m+5}}{m+5}+\frac{252a^5b^5x^{m+6}}{m+6}+\frac{210a^4b^6x^{m+7}}{m+7}+\frac{120a^3b^7x^{m+8}}{m+8}+\frac{45a^2b^8x^{m+9}}{m+9}+\frac{10a^9bx}{m+}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^10,x]
```

```
[Out] (a^10*x^(1 + m))/(1 + m) + (10*a^9*b*x^(2 + m))/(2 + m) + (45*a^8*b^2*x^(3
+ m))/(3 + m) + (120*a^7*b^3*x^(4 + m))/(4 + m) + (210*a^6*b^4*x^(5 + m))/(
5 + m) + (252*a^5*b^5*x^(6 + m))/(6 + m) + (210*a^4*b^6*x^(7 + m))/(7 + m)
+ (120*a^3*b^7*x^(8 + m))/(8 + m) + (45*a^2*b^8*x^(9 + m))/(9 + m) + (10*a*
b^9*x^(10 + m))/(10 + m) + (b^10*x^(11 + m))/(11 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^{10}\,dx &= \int \left(a^{10}x^m+10a^9bx^{1+m}+45a^8b^2x^{2+m}+120a^7b^3x^{3+m}+210a^6b^4x^{4+m}+252a^5b^5x^{5+m}+210a^4b^6x\right.\\ &= \frac{a^{10}x^{1+m}}{1+m}+\frac{10a^9bx^{2+m}}{2+m}+\frac{45a^8b^2x^{3+m}}{3+m}+\frac{120a^7b^3x^{4+m}}{4+m}+\frac{210a^6b^4x^{5+m}}{5+m}+\frac{252a^5b^5x^{6+m}}{6+m}+\frac{210a^4b}{7+}\end{aligned}$$

Mathematica [A] time = 0.105001, size = 166, normalized size = 0.89

$$x^{m+1}\left(\frac{45a^8b^2x^2}{m+3}+\frac{120a^7b^3x^3}{m+4}+\frac{210a^6b^4x^4}{m+5}+\frac{252a^5b^5x^5}{m+6}+\frac{210a^4b^6x^6}{m+7}+\frac{120a^3b^7x^7}{m+8}+\frac{45a^2b^8x^8}{m+9}+\frac{10a^9bx}{m+2}+\frac{a^{10}}{m+1}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^10,x]
```

```
[Out] x^(1 + m)*(a^10/(1 + m) + (10*a^9*b*x)/(2 + m) + (45*a^8*b^2*x^2)/(3 + m) +
 (120*a^7*b^3*x^3)/(4 + m) + (210*a^6*b^4*x^4)/(5 + m) + (252*a^5*b^5*x^5)/
(6 + m) + (210*a^4*b^6*x^6)/(7 + m) + (120*a^3*b^7*x^7)/(8 + m) + (45*a^2*b
^8*x^8)/(9 + m) + (10*a*b^9*x^9)/(10 + m) + (b^10*x^10)/(11 + m))
```

Maple [B] time = 0.006, size = 1535, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^10,x)
```

```
[Out] x^(1+m)*(b^10*m^10*x^10+10*a*b^9*m^10*x^9+55*b^10*m^9*x^10+45*a^2*b^8*m^10*
x^8+560*a*b^9*m^9*x^9+1320*b^10*m^8*x^10+120*a^3*b^7*m^10*x^7+2565*a^2*b^8*
m^9*x^8+13650*a*b^9*m^8*x^9+18150*b^10*m^7*x^10+210*a^4*b^6*m^10*x^6+6960*a
^3*b^7*m^9*x^7+63540*a^2*b^8*m^8*x^8+190200*a*b^9*m^7*x^9+157773*b^10*m^6*x
^10+252*a^5*b^5*m^10*x^5+12390*a^4*b^6*m^9*x^6+175320*a^3*b^7*m^8*x^7+89829
0*a^2*b^8*m^7*x^8+1672230*a*b^9*m^6*x^9+902055*b^10*m^5*x^10+210*a^6*b^4*m^
10*x^4+15120*a^5*b^5*m^9*x^5+317520*a^4*b^6*m^8*x^6+2517840*a^3*b^7*m^7*x^7
+7999425*a^2*b^8*m^6*x^8+9653280*a*b^9*m^5*x^9+3416930*b^10*m^4*x^10+120*a^
7*b^3*m^10*x^3+12810*a^6*b^4*m^9*x^4+394380*a^5*b^5*m^8*x^5+4638060*a^4*b^6
*m^7*x^6+22748040*a^3*b^7*m^6*x^7+46695285*a^2*b^8*m^5*x^8+36862550*a*b^9*m
^4*x^9+8409500*b^10*m^3*x^10+45*a^8*b^2*m^10*x^2+7440*a^7*b^3*m^9*x^3+34020
0*a^6*b^4*m^8*x^4+5866560*a^5*b^5*m^7*x^5+42592410*a^4*b^6*m^6*x^6+13452264
0*a^3*b^7*m^5*x^7+180021510*a^2*b^8*m^4*x^8+91331800*a*b^9*m^3*x^9+12753576
*b^10*m^2*x^10+10*a^9*b*m^10*x+2835*a^8*b^2*m^9*x^2+201240*a^7*b^3*m^8*x^3+
5159700*a^6*b^4*m^7*x^4+54871236*a^5*b^5*m^6*x^5+255740310*a^4*b^6*m^5*x^6+
524563080*a^3*b^7*m^4*x^7+449614260*a^2*b^8*m^3*x^8+139262760*a*b^9*m^2*x^9
+10628640*b^10*m*x^10+a^10*m^10+640*a^9*b*m^9*x+78120*a^8*b^2*m^8*x^2+31154
40*a^7*b^3*m^7*x^3+49260330*a^6*b^4*m^6*x^4+335437200*a^5*b^5*m^5*x^5+10111
20180*a^4*b^6*m^4*x^6+1322982960*a^3*b^7*m^3*x^7+690085080*a^2*b^8*m^2*x^8+
116552160*a*b^9*m*x^9+3628800*b^10*x^10+65*a^10*m^9+17970*a^9*b*m^8*x+12357
90*a^8*b^2*m^7*x^2+30429000*a^7*b^3*m^6*x^3+307585530*a^6*b^4*m^5*x^4+13489
39620*a^5*b^5*m^4*x^5+2581262040*a^4*b^6*m^3*x^6+2047105440*a^3*b^7*m^2*x^7
+580543200*a^2*b^8*m*x^8+39916800*a*b^9*x^9+1860*a^10*m^8+290760*a^9*b*m^7*
x+12376665*a^8*b^2*m^6*x^2+194790960*a^7*b^3*m^5*x^3+1263374700*a^6*b^4*m^4
*x^4+3497286240*a^5*b^5*m^3*x^5+4035361680*a^4*b^6*m^2*x^6+1733313600*a^3*b
^7*m*x^7+199584000*a^2*b^8*x^8+30810*a^10*m^7+2992710*a^9*b*m^6*x+81560115*
a^8*b^2*m^5*x^2+821580360*a^7*b^3*m^4*x^3+3342229800*a^6*b^4*m^3*x^4+554131
7712*a^5*b^5*m^2*x^5+3445243200*a^4*b^6*m*x^6+598752000*a^3*b^7*x^7+326613*
a^10*m^6+20390160*a^9*b*m^5*x+355598730*a^8*b^2*m^4*x^2+2233166160*a^7*b^3*
m^3*x^3+5393046960*a^6*b^4*m^2*x^4+4783423680*a^5*b^5*m*x^5+1197504000*a^4*
b^6*x^6+2310945*a^10*m^5+92615030*a^9*b*m^4*x+1003011660*a^8*b^2*m^3*x^2+36
98304480*a^7*b^3*m^2*x^3+4727540160*a^6*b^4*m*x^4+1676505600*a^5*b^5*x^5+11
028590*a^10*m^4+274727240*a^9*b*m^3*x+1727578440*a^8*b^2*m^2*x^2+3316939200
*a^7*b^3*m*x^3+1676505600*a^6*b^4*x^4+34967140*a^10*m^3+503126280*a^9*b*m^2
*x+1608573600*a^8*b^2*m*x^2+1197504000*a^7*b^3*x^3+70290936*a^10*m^2+502927
200*a^9*b*m*x+598752000*a^8*b^2*x^2+80627040*a^10*m+199584000*a^9*b*x+39916
800*a^10)/(11+m)/(10+m)/(9+m)/(8+m)/(7+m)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+
m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.7809, size = 3479, normalized size = 18.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] ((b^10*m^10 + 55*b^10*m^9 + 1320*b^10*m^8 + 18150*b^10*m^7 + 157773*b^10*m^
6 + 902055*b^10*m^5 + 3416930*b^10*m^4 + 8409500*b^10*m^3 + 12753576*b^10*m
^2 + 10628640*b^10*m + 3628800*b^10)*x^11 + 10*(a*b^9*m^10 + 56*a*b^9*m^9 +
 1365*a*b^9*m^8 + 19020*a*b^9*m^7 + 167223*a*b^9*m^6 + 965328*a*b^9*m^5 + 3
686255*a*b^9*m^4 + 9133180*a*b^9*m^3 + 13926276*a*b^9*m^2 + 11655216*a*b^9*
m + 3991680*a*b^9)*x^10 + 45*(a^2*b^8*m^10 + 57*a^2*b^8*m^9 + 1412*a^2*b^8*
m^8 + 19962*a^2*b^8*m^7 + 177765*a^2*b^8*m^6 + 1037673*a^2*b^8*m^5 + 400047
8*a^2*b^8*m^4 + 9991428*a^2*b^8*m^3 + 15335224*a^2*b^8*m^2 + 12900960*a^2*b
^8*m + 4435200*a^2*b^8)*x^9 + 120*(a^3*b^7*m^10 + 58*a^3*b^7*m^9 + 1461*a^3
*b^7*m^8 + 20982*a^3*b^7*m^7 + 189567*a^3*b^7*m^6 + 1121022*a^3*b^7*m^5 + 4
371359*a^3*b^7*m^4 + 11024858*a^3*b^7*m^3 + 17059212*a^3*b^7*m^2 + 14444280
*a^3*b^7*m + 4989600*a^3*b^7)*x^8 + 210*(a^4*b^6*m^10 + 59*a^4*b^6*m^9 + 15
12*a^4*b^6*m^8 + 22086*a^4*b^6*m^7 + 202821*a^4*b^6*m^6 + 1217811*a^4*b^6*m
^5 + 4814858*a^4*b^6*m^4 + 12291724*a^4*b^6*m^3 + 19216008*a^4*b^6*m^2 + 16
405920*a^4*b^6*m + 5702400*a^4*b^6)*x^7 + 252*(a^5*b^5*m^10 + 60*a^5*b^5*m^
9 + 1565*a^5*b^5*m^8 + 23280*a^5*b^5*m^7 + 217743*a^5*b^5*m^6 + 1331100*a^5
*b^5*m^5 + 5352935*a^5*b^5*m^4 + 13878120*a^5*b^5*m^3 + 21989356*a^5*b^5*m^
2 + 18981840*a^5*b^5*m + 6652800*a^5*b^5)*x^6 + 210*(a^6*b^4*m^10 + 61*a^6*
b^4*m^9 + 1620*a^6*b^4*m^8 + 24570*a^6*b^4*m^7 + 234573*a^6*b^4*m^6 + 14646
93*a^6*b^4*m^5 + 6016070*a^6*b^4*m^4 + 15915380*a^6*b^4*m^3 + 25681176*a^6*
b^4*m^2 + 22512096*a^6*b^4*m + 7983360*a^6*b^4)*x^5 + 120*(a^7*b^3*m^10 + 6
2*a^7*b^3*m^9 + 1677*a^7*b^3*m^8 + 25962*a^7*b^3*m^7 + 253575*a^7*b^3*m^6 +
 1623258*a^7*b^3*m^5 + 6846503*a^7*b^3*m^4 + 18609718*a^7*b^3*m^3 + 3081920
4*a^7*b^3*m^2 + 27641160*a^7*b^3*m + 9979200*a^7*b^3)*x^4 + 45*(a^8*b^2*m^1
0 + 63*a^8*b^2*m^9 + 1736*a^8*b^2*m^8 + 27462*a^8*b^2*m^7 + 275037*a^8*b^2*
m^6 + 1812447*a^8*b^2*m^5 + 7902194*a^8*b^2*m^4 + 22289148*a^8*b^2*m^3 + 38
390632*a^8*b^2*m^2 + 35746080*a^8*b^2*m + 13305600*a^8*b^2)*x^3 + 10*(a^9*b
*m^10 + 64*a^9*b*m^9 + 1797*a^9*b*m^8 + 29076*a^9*b*m^7 + 299271*a^9*b*m^6
+ 2039016*a^9*b*m^5 + 9261503*a^9*b*m^4 + 27472724*a^9*b*m^3 + 50312628*a^9
*b*m^2 + 50292720*a^9*b*m + 19958400*a^9*b)*x^2 + (a^10*m^10 + 65*a^10*m^9
+ 1860*a^10*m^8 + 30810*a^10*m^7 + 326613*a^10*m^6 + 2310945*a^10*m^5 + 110
28590*a^10*m^4 + 34967140*a^10*m^3 + 70290936*a^10*m^2 + 80627040*a^10*m +
39916800*a^10)*x)*x^m/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 +
 2637558*m^6 + 13339535*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2
+ 120543840*m + 39916800)
```

Sympy [A] time = 6.45937, size = 9996, normalized size = 53.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**10,x)
```

```
[Out] Piecewise((-a**10/(10*x**10) - 10*a**9*b/(9*x**9) - 45*a**8*b**2/(8*x**8) -
 120*a**7*b**3/(7*x**7) - 35*a**6*b**4/x**6 - 252*a**5*b**5/(5*x**5) - 105*
a**4*b**6/(2*x**4) - 40*a**3*b**7/x**3 - 45*a**2*b**8/(2*x**2) - 10*a*b**9/
x + b**10*log(x), Eq(m, -11)), (-a**10/(9*x**9) - 5*a**9*b/(4*x**8) - 45*a*
*8*b**2/(7*x**7) - 20*a**7*b**3/x**6 - 42*a**6*b**4/x**5 - 63*a**5*b**5/x**
4 - 70*a**4*b**6/x**3 - 60*a**3*b**7/x**2 - 45*a**2*b**8/x + 10*a*b**9*log(
x) + b**10*x, Eq(m, -10)), (-a**10/(8*x**8) - 10*a**9*b/(7*x**7) - 15*a**8*
b**2/(2*x**6) - 24*a**7*b**3/x**5 - 105*a**6*b**4/(2*x**4) - 84*a**5*b**5/x
**3 - 105*a**4*b**6/x**2 - 120*a**3*b**7/x + 45*a**2*b**8*log(x) + 10*a*b**
9*x + b**10*x**2/2, Eq(m, -9)), (-a**10/(7*x**7) - 5*a**9*b/(3*x**6) - 9*a*
*8*b**2/x**5 - 30*a**7*b**3/x**4 - 70*a**6*b**4/x**3 - 126*a**5*b**5/x**2 -
 210*a**4*b**6/x + 120*a**3*b**7*log(x) + 45*a**2*b**8*x + 5*a*b**9*x**2 +
b**10*x**3/3, Eq(m, -8)), (-a**10/(6*x**6) - 2*a**9*b/x**5 - 45*a**8*b**2/(
4*x**4) - 40*a**7*b**3/x**3 - 105*a**6*b**4/x**2 - 252*a**5*b**5/x + 210*a*
*4*b**6*log(x) + 120*a**3*b**7*x + 45*a**2*b**8*x**2/2 + 10*a*b**9*x**3/3 +
 b**10*x**4/4, Eq(m, -7)), (-a**10/(5*x**5) - 5*a**9*b/(2*x**4) - 15*a**8*b
**2/x**3 - 60*a**7*b**3/x**2 - 210*a**6*b**4/x + 252*a**5*b**5*log(x) + 210
*a**4*b**6*x + 60*a**3*b**7*x**2 + 15*a**2*b**8*x**3 + 5*a*b**9*x**4/2 + b*
*10*x**5/5, Eq(m, -6)), (-a**10/(4*x**4) - 10*a**9*b/(3*x**3) - 45*a**8*b**
2/(2*x**2) - 120*a**7*b**3/x + 210*a**6*b**4*log(x) + 252*a**5*b**5*x + 105
*a**4*b**6*x**2 + 40*a**3*b**7*x**3 + 45*a**2*b**8*x**4/4 + 2*a*b**9*x**5 +
 b**10*x**6/6, Eq(m, -5)), (-a**10/(3*x**3) - 5*a**9*b/x**2 - 45*a**8*b**2/
x + 120*a**7*b**3*log(x) + 210*a**6*b**4*x + 126*a**5*b**5*x**2 + 70*a**4*b
**6*x**3 + 30*a**3*b**7*x**4 + 9*a**2*b**8*x**5 + 5*a*b**9*x**6/3 + b**10*x
**7/7, Eq(m, -4)), (-a**10/(2*x**2) - 10*a**9*b/x + 45*a**8*b**2*log(x) + 1
20*a**7*b**3*x + 105*a**6*b**4*x**2 + 84*a**5*b**5*x**3 + 105*a**4*b**6*x**
4/2 + 24*a**3*b**7*x**5 + 15*a**2*b**8*x**6/2 + 10*a*b**9*x**7/7 + b**10*x*
*8/8, Eq(m, -3)), (-a**10/x + 10*a**9*b*log(x) + 45*a**8*b**2*x + 60*a**7*b
**3*x**2 + 70*a**6*b**4*x**3 + 63*a**5*b**5*x**4 + 42*a**4*b**6*x**5 + 20*a
**3*b**7*x**6 + 45*a**2*b**8*x**7/7 + 5*a*b**9*x**8/4 + b**10*x**9/9, Eq(m,
 -2)), (a**10*log(x) + 10*a**9*b*x + 45*a**8*b**2*x**2/2 + 40*a**7*b**3*x**
3 + 105*a**6*b**4*x**4/2 + 252*a**5*b**5*x**5/5 + 35*a**4*b**6*x**6 + 120*a
**3*b**7*x**7/7 + 45*a**2*b**8*x**8/8 + 10*a*b**9*x**9/9 + b**10*x**10/10,
Eq(m, -1)), (a**10*m**10*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 65*a**10*m**9*x*x**m/(m**11
 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 1860*a**10*m**8*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m
**3 + 150917976*m**2 + 120543840*m + 39916800) + 30810*a**10*m**7*x*x**m/(m
**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 326613*a**10*m**6*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 2310945*a**10*m**5*x
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 11028590*a**10*m**4*x*x**m/(m**11 + 66*m**10 + 1925*m
**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 34967140*a
**10*m**3*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 70290936*a**10*m**2*x*x**m/(m**11 + 66*m**
10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
 80627040*a**10*m*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 35742
```

3*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15
0917976*m**2 + 120543840*m + 39916800) + 39916800*a**10*x*x**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 10*a**9*b*m**10*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
 + 150917976*m**2 + 120543840*m + 39916800) + 640*a**9*b*m**9*x**2*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
 39916800) + 17970*a**9*b*m**8*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32
670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105
258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 290760*a**9*b*m**7
*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
 120543840*m + 39916800) + 2992710*a**9*b*m**6*x**2*x**m/(m**11 + 66*m**10
+ 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459
95730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 20
390160*a**9*b*m**5*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 3
57423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
+ 150917976*m**2 + 120543840*m + 39916800) + 92615030*a**9*b*m**4*x**2*x**m
/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840
*m + 39916800) + 274727240*a**9*b*m**3*x**2*x**m/(m**11 + 66*m**10 + 1925*m
9 + 32670*m8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 503126280*
a**9*b*m**2*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m
7 + 2637558*m6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091
7976*m**2 + 120543840*m + 39916800) + 502927200*a**9*b*m*x**2*x**m/(m**11 +
 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 199584000*a**9*b*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m*
*8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076
*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 45*a**8*b**2*m**10*x**3*
x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**
6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12054
3840*m + 39916800) + 2835*a**8*b**2*m**9*x**3*x**m/(m**11 + 66*m**10 + 1925
*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*
m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 78120*a*
*8*b**2*m**8*x**3*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*
m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1509
17976*m**2 + 120543840*m + 39916800) + 1235790*a**8*b**2*m**7*x**3*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
 39916800) + 12376665*a**8*b**2*m**6*x**3*x**m/(m**11 + 66*m**10 + 1925*m**
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 81560115*a**
8*b**2*m**5*x**3*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m
7 + 2637558*m6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091
7976*m**2 + 120543840*m + 39916800) + 355598730*a**8*b**2*m**4*x**3*x**m/(m
11 + 66*m10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 1003011660*a**8*b**2*m**3*x**3*x**m/(m**11 + 66*m**10 + 1925*
m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m
4 + 105258076*m3 + 150917976*m**2 + 120543840*m + 39916800) + 172757844
0*a**8*b**2*m**2*x**3*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357
423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
150917976*m**2 + 120543840*m + 39916800) + 1608573600*a**8*b**2*m*x**3*x**m
/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840

*m + 39916800) + 598752000*a**8*b**2*x**3*x**m/(m**11 + 66*m**10 + 1925*m**
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 120*a**7*b**
3*m**10*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 7440*a**7*b**3*m**9*x**4*x**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 201240*a**7*b**3*m**8*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580
76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3115440*a**7*b**3*m**7
*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
 120543840*m + 39916800) + 30429000*a**7*b**3*m**6*x**4*x**m/(m**11 + 66*m*
*10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800)
+ 194790960*a**7*b**3*m**5*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580
76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 821580360*a**7*b**3*m*
*4*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263
7558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
 + 120543840*m + 39916800) + 2233166160*a**7*b**3*m**3*x**4*x**m/(m**11 + 6
6*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m*
*5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 399168
00) + 3698304480*a**7*b**3*m**2*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3316939200*a**7*b
3*m*x4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m
2 + 120543840*m + 39916800) + 1197504000*a7*b**3*x**4*x**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 210*a**6*b**4*m**10*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**
8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*
m**3 + 150917976*m**2 + 120543840*m + 39916800) + 12810*a**6*b**4*m**9*x**5
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 340200*a**6*b**4*m**8*x**5*x**m/(m**11 + 66*m**10 + 1
925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459957
30*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 51597
00*a**6*b**4*m**7*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 35
7423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
 150917976*m**2 + 120543840*m + 39916800) + 49260330*a**6*b**4*m**6*x**5*x*
*m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6
+ 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205438
40*m + 39916800) + 307585530*a**6*b**4*m**5*x**5*x**m/(m**11 + 66*m**10 + 1
925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459957
30*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 12633
74700*a**6*b**4*m**4*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**
3 + 150917976*m**2 + 120543840*m + 39916800) + 3342229800*a**6*b**4*m**3*x*
*5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 5393046960*a**6*b**4*m**2*x**5*x**m/(m**11 + 66*m**
10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
 4727540160*a**6*b**4*m*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**
8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*
m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1676505600*a**6*b**4*x**5

*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 252*a**5*b**5*m**10*x**6*x**m/(m**11 + 66*m**10 + 192
5*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730
*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 15120*a
5*b5*m**9*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423
*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150
917976*m**2 + 120543840*m + 39916800) + 394380*a**5*b**5*m**8*x**6*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
39916800) + 5866560*a**5*b**5*m**7*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
+ 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 54871236*a**5
*b**5*m**6*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m*
*7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917
976*m**2 + 120543840*m + 39916800) + 335437200*a**5*b**5*m**5*x**6*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
39916800) + 1348939620*a**5*b**5*m**4*x**6*x**m/(m**11 + 66*m**10 + 1925*m
9 + 32670*m8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3497286240
*a**5*b**5*m**3*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 3574
23*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1
50917976*m**2 + 120543840*m + 39916800) + 5541317712*a**5*b**5*m**2*x**6*x*
*m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6
+ 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205438
40*m + 39916800) + 4783423680*a**5*b**5*m*x**6*x**m/(m**11 + 66*m**10 + 192
5*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730
*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1676505
600*a**5*b**5*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423
*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150
917976*m**2 + 120543840*m + 39916800) + 210*a**4*b**6*m**10*x**7*x**m/(m**1
1 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133395
35*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3
9916800) + 12390*a**4*b**6*m**9*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 317520*a**4*b**6*
m**8*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2
637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m*
*2 + 120543840*m + 39916800) + 4638060*a**4*b**6*m**7*x**7*x**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 42592410*a**4*b**6*m**6*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 255740310*a**4*b**6*
m**5*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2
637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m*
*2 + 120543840*m + 39916800) + 1011120180*a**4*b**6*m**4*x**7*x**m/(m**11 +
66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 2581262040*a**4*b**6*m**3*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 +
32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 4035361680*a**4
*b**6*m**2*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m*
*7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917
976*m**2 + 120543840*m + 39916800) + 3445243200*a**4*b**6*m*x**7*x**m/(m**1
1 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133395
35*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3
9916800) + 1197504000*a**4*b**6*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3

2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 120*a**3*b**7*m**
10*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263
7558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
 + 120543840*m + 39916800) + 6960*a**3*b**7*m**9*x**8*x**m/(m**11 + 66*m**1
0 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4
5995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
175320*a**3*b**7*m**8*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 2517840*a**3*b**7*m**7*x**8
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 22748040*a**3*b**7*m**6*x**8*x**m/(m**11 + 66*m**10 +
 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4599
5730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 134
522640*a**3*b**7*m**5*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 524563080*a**3*b**7*m**4*x*
*8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 1322982960*a**3*b**7*m**3*x**8*x**m/(m**11 + 66*m**
10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
 2047105440*a**3*b**7*m**2*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580
76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1733313600*a**3*b**7*m
*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
 120543840*m + 39916800) + 598752000*a**3*b**7*x**8*x**m/(m**11 + 66*m**10
+ 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459
95730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 45
*a**2*b**8*m**10*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357
423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
150917976*m**2 + 120543840*m + 39916800) + 2565*a**2*b**8*m**9*x**9*x**m/(m
11 + 66*m10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 63540*a**2*b**8*m**8*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 898290*a**2*b*
*8*m**7*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 7999425*a**2*b**8*m**6*x**9*x**m/(m**11 +
 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 46695285*a**2*b**8*m**5*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 180021510*a**2*b*
*8*m**4*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 449614260*a**2*b**8*m**3*x**9*x**m/(m**11
 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 690085080*a**2*b**8*m**2*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 580543200*a**2
*b**8*m*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 199584000*a**2*b**8*x**9*x**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**

```
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 10*a*b**9*m**10*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 560*a*b**9*m**9*x**10*x**m/
(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1
3339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*
m + 39916800) + 13650*a*b**9*m**8*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 190200*a*b**9*
m**7*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m
**2 + 120543840*m + 39916800) + 1672230*a*b**9*m**6*x**10*x**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 9653280*a*b**9*m**5*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m*
*8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076
*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 36862550*a*b**9*m**4*x**
10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 91331800*a*b**9*m**3*x**10*x**m/(m**11 + 66*m**10 +
 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4599
5730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 139
262760*a*b**9*m**2*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
 + 150917976*m**2 + 120543840*m + 39916800) + 116552160*a*b**9*m*x**10*x**m
/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840
*m + 39916800) + 39916800*a*b**9*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 +
 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + b**10*m**10*x**
11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 55*b**10*m**9*x**11*x**m/(m**11 + 66*m**10 + 1925*m
**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1320*b**10
*m**8*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 18150*b**10*m**7*x**11*x**m/(m**11 + 66*m*
*10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800)
+ 157773*b**10*m**6*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**
3 + 150917976*m**2 + 120543840*m + 39916800) + 902055*b**10*m**5*x**11*x**m
/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840
*m + 39916800) + 3416930*b**10*m**4*x**11*x**m/(m**11 + 66*m**10 + 1925*m**
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 8409500*b**1
0*m**3*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 12753576*b**10*m**2*x**11*x**m/(m**11 + 6
6*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m*
*5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 399168
00) + 10628640*b**10*m*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**
8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*
m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3628800*b**10*x**11*x**m/
(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1
3339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*
m + 39916800), True))
```

Giac [B] time = 1.10491, size = 2599, normalized size = 13.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] (b^10*m^10*x^11*x^m + 10*a*b^9*m^10*x^10*x^m + 55*b^10*m^9*x^11*x^m + 45*a^
2*b^8*m^10*x^9*x^m + 560*a*b^9*m^9*x^10*x^m + 1320*b^10*m^8*x^11*x^m + 120*
a^3*b^7*m^10*x^8*x^m + 2565*a^2*b^8*m^9*x^9*x^m + 13650*a*b^9*m^8*x^10*x^m
+ 18150*b^10*m^7*x^11*x^m + 210*a^4*b^6*m^10*x^7*x^m + 6960*a^3*b^7*m^9*x^8
*x^m + 63540*a^2*b^8*m^8*x^9*x^m + 190200*a*b^9*m^7*x^10*x^m + 157773*b^10*
m^6*x^11*x^m + 252*a^5*b^5*m^10*x^6*x^m + 12390*a^4*b^6*m^9*x^7*x^m + 17532
0*a^3*b^7*m^8*x^8*x^m + 898290*a^2*b^8*m^7*x^9*x^m + 1672230*a*b^9*m^6*x^10
*x^m + 902055*b^10*m^5*x^11*x^m + 210*a^6*b^4*m^10*x^5*x^m + 15120*a^5*b^5*
m^9*x^6*x^m + 317520*a^4*b^6*m^8*x^7*x^m + 2517840*a^3*b^7*m^7*x^8*x^m + 79
99425*a^2*b^8*m^6*x^9*x^m + 9653280*a*b^9*m^5*x^10*x^m + 3416930*b^10*m^4*x
^11*x^m + 120*a^7*b^3*m^10*x^4*x^m + 12810*a^6*b^4*m^9*x^5*x^m + 394380*a^5
*b^5*m^8*x^6*x^m + 4638060*a^4*b^6*m^7*x^7*x^m + 22748040*a^3*b^7*m^6*x^8*x
^m + 46695285*a^2*b^8*m^5*x^9*x^m + 36862550*a*b^9*m^4*x^10*x^m + 8409500*b
^10*m^3*x^11*x^m + 45*a^8*b^2*m^10*x^3*x^m + 7440*a^7*b^3*m^9*x^4*x^m + 340
200*a^6*b^4*m^8*x^5*x^m + 5866560*a^5*b^5*m^7*x^6*x^m + 42592410*a^4*b^6*m^
6*x^7*x^m + 134522640*a^3*b^7*m^5*x^8*x^m + 180021510*a^2*b^8*m^4*x^9*x^m +
 91331800*a*b^9*m^3*x^10*x^m + 12753576*b^10*m^2*x^11*x^m + 10*a^9*b*m^10*x
^2*x^m + 2835*a^8*b^2*m^9*x^3*x^m + 201240*a^7*b^3*m^8*x^4*x^m + 5159700*a^
6*b^4*m^7*x^5*x^m + 54871236*a^5*b^5*m^6*x^6*x^m + 255740310*a^4*b^6*m^5*x^
7*x^m + 524563080*a^3*b^7*m^4*x^8*x^m + 449614260*a^2*b^8*m^3*x^9*x^m + 139
262760*a*b^9*m^2*x^10*x^m + 10628640*b^10*m*x^11*x^m + a^10*m^10*x*x^m + 64
0*a^9*b*m^9*x^2*x^m + 78120*a^8*b^2*m^8*x^3*x^m + 3115440*a^7*b^3*m^7*x^4*x
^m + 49260330*a^6*b^4*m^6*x^5*x^m + 335437200*a^5*b^5*m^5*x^6*x^m + 1011120
180*a^4*b^6*m^4*x^7*x^m + 1322982960*a^3*b^7*m^3*x^8*x^m + 690085080*a^2*b^
8*m^2*x^9*x^m + 116552160*a*b^9*m*x^10*x^m + 3628800*b^10*x^11*x^m + 65*a^1
0*m^9*x*x^m + 17970*a^9*b*m^8*x^2*x^m + 1235790*a^8*b^2*m^7*x^3*x^m + 30429
000*a^7*b^3*m^6*x^4*x^m + 307585530*a^6*b^4*m^5*x^5*x^m + 1348939620*a^5*b^
5*m^4*x^6*x^m + 2581262040*a^4*b^6*m^3*x^7*x^m + 2047105440*a^3*b^7*m^2*x^8
*x^m + 580543200*a^2*b^8*m*x^9*x^m + 39916800*a*b^9*x^10*x^m + 1860*a^10*m^
8*x*x^m + 290760*a^9*b*m^7*x^2*x^m + 12376665*a^8*b^2*m^6*x^3*x^m + 1947909
60*a^7*b^3*m^5*x^4*x^m + 1263374700*a^6*b^4*m^4*x^5*x^m + 3497286240*a^5*b^
5*m^3*x^6*x^m + 4035361680*a^4*b^6*m^2*x^7*x^m + 1733313600*a^3*b^7*m*x^8*x
^m + 199584000*a^2*b^8*x^9*x^m + 30810*a^10*m^7*x*x^m + 2992710*a^9*b*m^6*x
^2*x^m + 81560115*a^8*b^2*m^5*x^3*x^m + 821580360*a^7*b^3*m^4*x^4*x^m + 334
2229800*a^6*b^4*m^3*x^5*x^m + 5541317712*a^5*b^5*m^2*x^6*x^m + 3445243200*a
^4*b^6*m*x^7*x^m + 598752000*a^3*b^7*x^8*x^m + 326613*a^10*m^6*x*x^m + 2039
0160*a^9*b*m^5*x^2*x^m + 355598730*a^8*b^2*m^4*x^3*x^m + 2233166160*a^7*b^3
*m^3*x^4*x^m + 5393046960*a^6*b^4*m^2*x^5*x^m + 4783423680*a^5*b^5*m*x^6*x^
m + 1197504000*a^4*b^6*x^7*x^m + 2310945*a^10*m^5*x*x^m + 92615030*a^9*b*m^
4*x^2*x^m + 1003011660*a^8*b^2*m^3*x^3*x^m + 3698304480*a^7*b^3*m^2*x^4*x^m
 + 4727540160*a^6*b^4*m*x^5*x^m + 1676505600*a^5*b^5*x^6*x^m + 11028590*a^1
0*m^4*x*x^m + 274727240*a^9*b*m^3*x^2*x^m + 1727578440*a^8*b^2*m^2*x^3*x^m
+ 3316939200*a^7*b^3*m*x^4*x^m + 1676505600*a^6*b^4*x^5*x^m + 34967140*a^10
*m^3*x*x^m + 503126280*a^9*b*m^2*x^2*x^m + 1608573600*a^8*b^2*m*x^3*x^m + 1
197504000*a^7*b^3*x^4*x^m + 70290936*a^10*m^2*x*x^m + 502927200*a^9*b*m*x^2
*x^m + 598752000*a^8*b^2*x^3*x^m + 80627040*a^10*m*x*x^m + 199584000*a^9*b*
x^2*x^m + 39916800*a^10*x*x^m)/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357
423*m^7 + 2637558*m^6 + 13339535*m^5 + 45995730*m^4 + 105258076*m^3 + 15091
```

```
7976*m^2 + 120543840*m + 39916800)
```

3.700 $\int x^m(a+bx)^7\,dx$

Optimal. Leaf size=133

$$\frac{21a^5b^2x^{m+3}}{m+3}+\frac{35a^4b^3x^{m+4}}{m+4}+\frac{35a^3b^4x^{m+5}}{m+5}+\frac{21a^2b^5x^{m+6}}{m+6}+\frac{7a^6bx^{m+2}}{m+2}+\frac{a^7x^{m+1}}{m+1}+\frac{7ab^6x^{m+7}}{m+7}+\frac{b^7x^{m+8}}{m+8}$$

```
[Out] (a^7*x^(1 + m))/(1 + m) + (7*a^6*b*x^(2 + m))/(2 + m) + (21*a^5*b^2*x^(3 +
m))/(3 + m) + (35*a^4*b^3*x^(4 + m))/(4 + m) + (35*a^3*b^4*x^(5 + m))/(5 +
m) + (21*a^2*b^5*x^(6 + m))/(6 + m) + (7*a*b^6*x^(7 + m))/(7 + m) + (b^7*x^
(8 + m))/(8 + m)
```

Rubi [A] time = 0.0500478, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21a^5b^2x^{m+3}}{m+3}+\frac{35a^4b^3x^{m+4}}{m+4}+\frac{35a^3b^4x^{m+5}}{m+5}+\frac{21a^2b^5x^{m+6}}{m+6}+\frac{7a^6bx^{m+2}}{m+2}+\frac{a^7x^{m+1}}{m+1}+\frac{7ab^6x^{m+7}}{m+7}+\frac{b^7x^{m+8}}{m+8}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^7,x]
```

```
[Out] (a^7*x^(1 + m))/(1 + m) + (7*a^6*b*x^(2 + m))/(2 + m) + (21*a^5*b^2*x^(3 +
m))/(3 + m) + (35*a^4*b^3*x^(4 + m))/(4 + m) + (35*a^3*b^4*x^(5 + m))/(5 +
m) + (21*a^2*b^5*x^(6 + m))/(6 + m) + (7*a*b^6*x^(7 + m))/(7 + m) + (b^7*x^
(8 + m))/(8 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^7\,dx &= \int \left(a^7x^m+7a^6bx^{1+m}+21a^5b^2x^{2+m}+35a^4b^3x^{3+m}+35a^3b^4x^{4+m}+21a^2b^5x^{5+m}+7ab^6x^{6+m}+b^7x^{7+m}\right)dx\\ &= \frac{a^7x^{1+m}}{1+m}+\frac{7a^6bx^{2+m}}{2+m}+\frac{21a^5b^2x^{3+m}}{3+m}+\frac{35a^4b^3x^{4+m}}{4+m}+\frac{35a^3b^4x^{5+m}}{5+m}+\frac{21a^2b^5x^{6+m}}{6+m}+\frac{7ab^6x^{7+m}}{7+m}+\frac{b^7x^{8+m}}{8+m}\end{aligned}$$

Mathematica [A] time = 0.0667215, size = 118, normalized size = 0.89

$$x^{m+1}\left(\frac{21a^5b^2x^2}{m+3}+\frac{35a^4b^3x^3}{m+4}+\frac{35a^3b^4x^4}{m+5}+\frac{21a^2b^5x^5}{m+6}+\frac{7a^6bx}{m+2}+\frac{a^7}{m+1}+\frac{7ab^6x^6}{m+7}+\frac{b^7x^7}{m+8}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^7,x]
```

```
[Out] x^(1 + m)*(a^7/(1 + m) + (7*a^6*b*x)/(2 + m) + (21*a^5*b^2*x^2)/(3 + m) + (
35*a^4*b^3*x^3)/(4 + m) + (35*a^3*b^4*x^4)/(5 + m) + (21*a^2*b^5*x^5)/(6 +
```

```
m) + (7*a*b^6*x^6)/(7 + m) + (b^7*x^7)/(8 + m))
```

Maple [B] time = 0.005, size = 782, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^7,x)
```

```
[Out] x^(1+m)*(b^7*m^7*x^7+7*a*b^6*m^7*x^6+28*b^7*m^6*x^7+21*a^2*b^5*m^7*x^5+203*
a*b^6*m^6*x^6+322*b^7*m^5*x^7+35*a^3*b^4*m^7*x^4+630*a^2*b^5*m^6*x^5+2401*a
*b^6*m^5*x^6+1960*b^7*m^4*x^7+35*a^4*b^3*m^7*x^3+1085*a^3*b^4*m^6*x^4+7686*
a^2*b^5*m^5*x^5+14945*a*b^6*m^4*x^6+6769*b^7*m^3*x^7+21*a^5*b^2*m^7*x^2+112
0*a^4*b^3*m^6*x^3+13685*a^3*b^4*m^5*x^4+49140*a^2*b^5*m^4*x^5+52528*a*b^6*m
^3*x^6+13132*b^7*m^2*x^7+7*a^6*b*m^7*x+693*a^5*b^2*m^6*x^2+14630*a^4*b^3*m^
5*x^3+90335*a^3*b^4*m^4*x^4+176589*a^2*b^5*m^3*x^5+103292*a*b^6*m^2*x^6+130
68*b^7*m*x^7+a^7*m^7+238*a^6*b*m^6*x+9387*a^5*b^2*m^5*x^2+100240*a^4*b^3*m^
4*x^3+334040*a^3*b^4*m^3*x^4+353430*a^2*b^5*m^2*x^5+103824*a*b^6*m*x^6+5040
*b^7*x^7+35*a^7*m^6+3346*a^6*b*m^5*x+67095*a^5*b^2*m^4*x^2+384755*a^4*b^3*m
^3*x^3+684740*a^3*b^4*m^2*x^4+360024*a^2*b^5*m*x^5+40320*a*b^6*x^6+511*a^7*
m^5+25060*a^6*b*m^4*x+270144*a^5*b^2*m^3*x^2+815920*a^4*b^3*m^2*x^3+710640*
a^3*b^4*m*x^4+141120*a^2*b^5*x^5+4025*a^7*m^4+107023*a^6*b*m^3*x+602532*a^5
*b^2*m^2*x^2+870660*a^4*b^3*m*x^3+282240*a^3*b^4*x^4+18424*a^7*m^3+256942*a
^6*b*m^2*x+673008*a^5*b^2*m*x^2+352800*a^4*b^3*x^3+48860*a^7*m^2+312984*a^6
*b*m*x+282240*a^5*b^2*x^2+69264*a^7*m+141120*a^6*b*x+40320*a^7)/(8+m)/(7+m)
/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60821, size = 1608, normalized size = 12.09

$$\frac{\left(\left(b^7m^7 + 28\,b^7m^6 + 322\,b^7m^5 + 1960\,b^7m^4 + 6769\,b^7m^3 + 13132\,b^7m^2 + 13068\,b^7m + 5040\,b^7\right)x^8 + 7\left(ab^6m^7 + 29\,ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] ((b^7*m^7 + 28*b^7*m^6 + 322*b^7*m^5 + 1960*b^7*m^4 + 6769*b^7*m^3 + 13132*
b^7*m^2 + 13068*b^7*m + 5040*b^7)*x^8 + 7*(a*b^6*m^7 + 29*a*b^6*m^6 + 343*a
*b^6*m^5 + 2135*a*b^6*m^4 + 7504*a*b^6*m^3 + 14756*a*b^6*m^2 + 14832*a*b^6*
m + 5760*a*b^6)*x^7 + 21*(a^2*b^5*m^7 + 30*a^2*b^5*m^6 + 366*a^2*b^5*m^5 +
2340*a^2*b^5*m^4 + 8409*a^2*b^5*m^3 + 16830*a^2*b^5*m^2 + 17144*a^2*b^5*m +
```

```
 6720*a^2*b^5)*x^6 + 35*(a^3*b^4*m^7 + 31*a^3*b^4*m^6 + 391*a^3*b^4*m^5 + 2
581*a^3*b^4*m^4 + 9544*a^3*b^4*m^3 + 19564*a^3*b^4*m^2 + 20304*a^3*b^4*m +
8064*a^3*b^4)*x^5 + 35*(a^4*b^3*m^7 + 32*a^4*b^3*m^6 + 418*a^4*b^3*m^5 + 28
64*a^4*b^3*m^4 + 10993*a^4*b^3*m^3 + 23312*a^4*b^3*m^2 + 24876*a^4*b^3*m +
10080*a^4*b^3)*x^4 + 21*(a^5*b^2*m^7 + 33*a^5*b^2*m^6 + 447*a^5*b^2*m^5 + 3
195*a^5*b^2*m^4 + 12864*a^5*b^2*m^3 + 28692*a^5*b^2*m^2 + 32048*a^5*b^2*m +
 13440*a^5*b^2)*x^3 + 7*(a^6*b*m^7 + 34*a^6*b*m^6 + 478*a^6*b*m^5 + 3580*a^
6*b*m^4 + 15289*a^6*b*m^3 + 36706*a^6*b*m^2 + 44712*a^6*b*m + 20160*a^6*b)*
x^2 + (a^7*m^7 + 35*a^7*m^6 + 511*a^7*m^5 + 4025*a^7*m^4 + 18424*a^7*m^3 +
48860*a^7*m^2 + 69264*a^7*m + 40320*a^7)*x)*x^m/(m^8 + 36*m^7 + 546*m^6 + 4
536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)
```

Sympy [A] time = 2.90442, size = 4257, normalized size = 32.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**7,x)
```

```
[Out] Piecewise((-a**7/(7*x**7) - 7*a**6*b/(6*x**6) - 21*a**5*b**2/(5*x**5) - 35*
a**4*b**3/(4*x**4) - 35*a**3*b**4/(3*x**3) - 21*a**2*b**5/(2*x**2) - 7*a*b*
*6/x + b**7*log(x), Eq(m, -8)), (-a**7/(6*x**6) - 7*a**6*b/(5*x**5) - 21*a*
*5*b**2/(4*x**4) - 35*a**4*b**3/(3*x**3) - 35*a**3*b**4/(2*x**2) - 21*a**2*
b**5/x + 7*a*b**6*log(x) + b**7*x, Eq(m, -7)), (-a**7/(5*x**5) - 7*a**6*b/(
4*x**4) - 7*a**5*b**2/x**3 - 35*a**4*b**3/(2*x**2) - 35*a**3*b**4/x + 21*a*
*2*b**5*log(x) + 7*a*b**6*x + b**7*x**2/2, Eq(m, -6)), (-a**7/(4*x**4) - 7*
a**6*b/(3*x**3) - 21*a**5*b**2/(2*x**2) - 35*a**4*b**3/x + 35*a**3*b**4*log
(x) + 21*a**2*b**5*x + 7*a*b**6*x**2/2 + b**7*x**3/3, Eq(m, -5)), (-a**7/(3
*x**3) - 7*a**6*b/(2*x**2) - 21*a**5*b**2/x + 35*a**4*b**3*log(x) + 35*a**3
*b**4*x + 21*a**2*b**5*x**2/2 + 7*a*b**6*x**3/3 + b**7*x**4/4, Eq(m, -4)),
(-a**7/(2*x**2) - 7*a**6*b/x + 21*a**5*b**2*log(x) + 35*a**4*b**3*x + 35*a*
*3*b**4*x**2/2 + 7*a**2*b**5*x**3 + 7*a*b**6*x**4/4 + b**7*x**5/5, Eq(m, -3
)), (-a**7/x + 7*a**6*b*log(x) + 21*a**5*b**2*x + 35*a**4*b**3*x**2/2 + 35*
a**3*b**4*x**3/3 + 21*a**2*b**5*x**4/4 + 7*a*b**6*x**5/5 + b**7*x**6/6, Eq(
m, -2)), (a**7*log(x) + 7*a**6*b*x + 21*a**5*b**2*x**2/2 + 35*a**4*b**3*x**
3/3 + 35*a**3*b**4*x**4/4 + 21*a**2*b**5*x**5/5 + 7*a*b**6*x**6/6 + b**7*x*
*7/7, Eq(m, -1)), (a**7*m**7*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5
+ 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**7*m**6*
x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 1
18124*m**2 + 109584*m + 40320) + 511*a**7*m**5*x*x**m/(m**8 + 36*m**7 + 546
*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 4032
0) + 4025*a**7*m**4*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m
**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 18424*a**7*m**3*x*x**m
/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*
m**2 + 109584*m + 40320) + 48860*a**7*m**2*x*x**m/(m**8 + 36*m**7 + 546*m**
6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) +
 69264*a**7*m*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 +
67284*m**3 + 118124*m**2 + 109584*m + 40320) + 40320*a**7*x*x**m/(m**8 + 36
*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 1095
84*m + 40320) + 7*a**6*b*m**7*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m
**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 238*a**6*
b*m**6*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 6728
4*m**3 + 118124*m**2 + 109584*m + 40320) + 3346*a**6*b*m**5*x**2*x**m/(m**8
 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 +
 109584*m + 40320) + 25060*a**6*b*m**4*x**2*x**m/(m**8 + 36*m**7 + 546*m**6
 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) +
```

```
107023*a**6*b*m**3*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449
*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 256942*a**6*b*m**2*x
**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
 118124*m**2 + 109584*m + 40320) + 312984*a**6*b*m*x**2*x**m/(m**8 + 36*m**
7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m
 + 40320) + 141120*a**6*b*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5
+ 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 21*a**5*b**2*
m**7*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*
m**3 + 118124*m**2 + 109584*m + 40320) + 693*a**5*b**2*m**6*x**3*x**m/(m**8
 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 +
 109584*m + 40320) + 9387*a**5*b**2*m**5*x**3*x**m/(m**8 + 36*m**7 + 546*m*
*6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320)
+ 67095*a**5*b**2*m**4*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 2
2449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 270144*a**5*b**2
*m**3*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284
*m**3 + 118124*m**2 + 109584*m + 40320) + 602532*a**5*b**2*m**2*x**3*x**m/(
m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m*
*2 + 109584*m + 40320) + 673008*a**5*b**2*m*x**3*x**m/(m**8 + 36*m**7 + 546
*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 4032
0) + 282240*a**5*b**2*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22
449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**4*b**3*m**7
*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3
 + 118124*m**2 + 109584*m + 40320) + 1120*a**4*b**3*m**6*x**4*x**m/(m**8 +
36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 10
9584*m + 40320) + 14630*a**4*b**3*m**5*x**4*x**m/(m**8 + 36*m**7 + 546*m**6
 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) +
100240*a**4*b**3*m**4*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22
449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 384755*a**4*b**3*
m**3*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*
m**3 + 118124*m**2 + 109584*m + 40320) + 815920*a**4*b**3*m**2*x**4*x**m/(m
**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**
2 + 109584*m + 40320) + 870660*a**4*b**3*m*x**4*x**m/(m**8 + 36*m**7 + 546*
m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320
) + 352800*a**4*b**3*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 224
49*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**3*b**4*m**7*
x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3
+ 118124*m**2 + 109584*m + 40320) + 1085*a**3*b**4*m**6*x**5*x**m/(m**8 + 3
6*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109
584*m + 40320) + 13685*a**3*b**4*m**5*x**5*x**m/(m**8 + 36*m**7 + 546*m**6
+ 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 9
0335*a**3*b**4*m**4*x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 2244
9*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 334040*a**3*b**4*m*
*3*x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m*
*3 + 118124*m**2 + 109584*m + 40320) + 684740*a**3*b**4*m**2*x**5*x**m/(m**
8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2
+ 109584*m + 40320) + 710640*a**3*b**4*m*x**5*x**m/(m**8 + 36*m**7 + 546*m*
*6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320)
+ 282240*a**3*b**4*x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449
*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 21*a**2*b**5*m**7*x*
*6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
118124*m**2 + 109584*m + 40320) + 630*a**2*b**5*m**6*x**6*x**m/(m**8 + 36*m
**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584
*m + 40320) + 7686*a**2*b**5*m**5*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 45
36*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 49140
*a**2*b**5*m**4*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m*
*4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 176589*a**2*b**5*m**3*x
**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
 118124*m**2 + 109584*m + 40320) + 353430*a**2*b**5*m**2*x**6*x**m/(m**8 +
36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 10
```

```
9584*m + 40320) + 360024*a**2*b**5*m*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 +
 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 14
1120*a**2*b**5*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**
4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 7*a*b**6*m**7*x**7*x**m/
(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m
**2 + 109584*m + 40320) + 203*a*b**6*m**6*x**7*x**m/(m**8 + 36*m**7 + 546*m
**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320)
 + 2401*a*b**6*m**5*x**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 2244
9*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 14945*a*b**6*m**4*x
**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
 118124*m**2 + 109584*m + 40320) + 52528*a*b**6*m**3*x**7*x**m/(m**8 + 36*m
**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584
*m + 40320) + 103292*a*b**6*m**2*x**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 453
6*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 103824
*a*b**6*m*x**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 6
7284*m**3 + 118124*m**2 + 109584*m + 40320) + 40320*a*b**6*x**7*x**m/(m**8
+ 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 +
109584*m + 40320) + b**7*m**7*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m
**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 28*b**7*m
**6*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m
**3 + 118124*m**2 + 109584*m + 40320) + 322*b**7*m**5*x**8*x**m/(m**8 + 36*
m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 10958
4*m + 40320) + 1960*b**7*m**4*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m
**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 6769*b**7
*m**3*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284
*m**3 + 118124*m**2 + 109584*m + 40320) + 13132*b**7*m**2*x**8*x**m/(m**8 +
 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 1
09584*m + 40320) + 13068*b**7*m*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536
*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 5040*b*
*7*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m*
*3 + 118124*m**2 + 109584*m + 40320), True))
```

Giac [B] time = 1.08466, size = 1339, normalized size = 10.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] (b^7*m^7*x^8*x^m + 7*a*b^6*m^7*x^7*x^m + 28*b^7*m^6*x^8*x^m + 21*a^2*b^5*m^
7*x^6*x^m + 203*a*b^6*m^6*x^7*x^m + 322*b^7*m^5*x^8*x^m + 35*a^3*b^4*m^7*x^
5*x^m + 630*a^2*b^5*m^6*x^6*x^m + 2401*a*b^6*m^5*x^7*x^m + 1960*b^7*m^4*x^8
*x^m + 35*a^4*b^3*m^7*x^4*x^m + 1085*a^3*b^4*m^6*x^5*x^m + 7686*a^2*b^5*m^5
*x^6*x^m + 14945*a*b^6*m^4*x^7*x^m + 6769*b^7*m^3*x^8*x^m + 21*a^5*b^2*m^7*
x^3*x^m + 1120*a^4*b^3*m^6*x^4*x^m + 13685*a^3*b^4*m^5*x^5*x^m + 49140*a^2*
b^5*m^4*x^6*x^m + 52528*a*b^6*m^3*x^7*x^m + 13132*b^7*m^2*x^8*x^m + 7*a^6*b
*m^7*x^2*x^m + 693*a^5*b^2*m^6*x^3*x^m + 14630*a^4*b^3*m^5*x^4*x^m + 90335*
a^3*b^4*m^4*x^5*x^m + 176589*a^2*b^5*m^3*x^6*x^m + 103292*a*b^6*m^2*x^7*x^m
 + 13068*b^7*m*x^8*x^m + a^7*m^7*x*x^m + 238*a^6*b*m^6*x^2*x^m + 9387*a^5*b
^2*m^5*x^3*x^m + 100240*a^4*b^3*m^4*x^4*x^m + 334040*a^3*b^4*m^3*x^5*x^m +
353430*a^2*b^5*m^2*x^6*x^m + 103824*a*b^6*m*x^7*x^m + 5040*b^7*x^8*x^m + 35
*a^7*m^6*x*x^m + 3346*a^6*b*m^5*x^2*x^m + 67095*a^5*b^2*m^4*x^3*x^m + 38475
5*a^4*b^3*m^3*x^4*x^m + 684740*a^3*b^4*m^2*x^5*x^m + 360024*a^2*b^5*m*x^6*x
^m + 40320*a*b^6*x^7*x^m + 511*a^7*m^5*x*x^m + 25060*a^6*b*m^4*x^2*x^m + 27
0144*a^5*b^2*m^3*x^3*x^m + 815920*a^4*b^3*m^2*x^4*x^m + 710640*a^3*b^4*m*x^
5*x^m + 141120*a^2*b^5*x^6*x^m + 4025*a^7*m^4*x*x^m + 107023*a^6*b*m^3*x^2*
```

```
x^m + 602532*a^5*b^2*m^2*x^3*x^m + 870660*a^4*b^3*m*x^4*x^m + 282240*a^3*b^
4*x^5*x^m + 18424*a^7*m^3*x*x^m + 256942*a^6*b*m^2*x^2*x^m + 673008*a^5*b^2
*m*x^3*x^m + 352800*a^4*b^3*x^4*x^m + 48860*a^7*m^2*x*x^m + 312984*a^6*b*m*
x^2*x^m + 282240*a^5*b^2*x^3*x^m + 69264*a^7*m*x*x^m + 141120*a^6*b*x^2*x^m
 + 40320*a^7*x*x^m)/(m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*
m^3 + 118124*m^2 + 109584*m + 40320)
```

3.701 $\int x^m(a+bx)^3\,dx$

Optimal. Leaf size=61

$$\frac{3a^2bx^{m+2}}{m+2}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(2 + m))/(2 + m) + (3*a*b^2*x^(3 + m))
/(3 + m) + (b^3*x^(4 + m))/(4 + m)
```

Rubi [A] time = 0.0182403, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2bx^{m+2}}{m+2}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(2 + m))/(2 + m) + (3*a*b^2*x^(3 + m))
/(3 + m) + (b^3*x^(4 + m))/(4 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^3\,dx&=\int\left(a^3x^m+3a^2bx^{1+m}+3ab^2x^{2+m}+b^3x^{3+m}\right)dx\\&=\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m}\end{aligned}$$

Mathematica [A] time = 0.0300431, size = 54, normalized size = 0.89

$$x^{m+1}\left(\frac{3a^2bx}{m+2}+\frac{a^3}{m+1}+\frac{3ab^2x^2}{m+3}+\frac{b^3x^3}{m+4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x)/(2 + m) + (3*a*b^2*x^2)/(3 + m) + (b^3
*x^3)/(4 + m))
```

Maple [B] time = 0., size = 170, normalized size = 2.8

$$\frac{x^{1+m}\left(b^3m^3x^3+3\,ab^2m^3x^2+6\,b^3m^2x^3+3\,a^2bm^3x+21\,ab^2m^2x^2+11\,b^3mx^3+a^3m^3+24\,a^2bm^2x+42\,ab^2mx^2+6\,b^3\right.}{(4+m)\,(3+m)\,(2+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^3,x)
```

```
[Out] x^(1+m)*(b^3*m^3*x^3+3*a*b^2*m^3*x^2+6*b^3*m^2*x^3+3*a^2*b*m^3*x+21*a*b^2*m
^2*x^2+11*b^3*m*x^3+a^3*m^3+24*a^2*b*m^2*x+42*a*b^2*m*x^2+6*b^3*x^3+9*a^3*m
^2+57*a^2*b*m*x+24*a*b^2*x^2+26*a^3*m+36*a^2*b*x+24*a^3)/(4+m)/(3+m)/(2+m)/
(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.58413, size = 336, normalized size = 5.51

$$\frac{\left(\left(b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3\right)x^4+3\left(ab^2m^3+7\,ab^2m^2+14\,ab^2m+8\,ab^2\right)x^3+3\left(a^2bm^3+8\,a^2bm^2+19\,a^2bm+1\right.\right.}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
+ 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*
a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*x^m/(m^4 + 10*m^3
 + 35*m^2 + 50*m + 24)
```

Sympy [A] time = 0.769065, size = 663, normalized size = 10.87

$$\begin{cases} -\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x) \\ -\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log(x)+b^3x \\ -\frac{a^3}{x}+3a^2b\log(x)+3ab^2x+\frac{b^3x^2}{2} \\ a^3\log(x)+3a^2bx+\frac{3ab^2x^2}{2}+\frac{b^3x^3}{3} \\ \frac{a^3m^3xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{9a^3m^2xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{26a^3mxx^m}{m^4+10m^3+35m^2+50m+24}+\frac{24a^3xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{3a^2bm^3x^2x^m}{m^4+10m^3+35m^2+50m+24} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x), E
q(m, -4)), (-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**3*x, Eq(m, -
3)), (-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2, Eq(m, -2)), (a*
*3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3, Eq(m, -1)), (a**3*m
**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 9*a**3*m**2*x*x**m/(m**
4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*a**3*m*x*x**m/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24) + 24*a**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
 3*a**2*b*m**3*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*a**2*b
*m**2*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 57*a**2*b*m*x**2*x
**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 36*a**2*b*x**2*x**m/(m**4 + 10
*m**3 + 35*m**2 + 50*m + 24) + 3*a*b**2*m**3*x**3*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 21*a*b**2*m**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 5
0*m + 24) + 42*a*b**2*m*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
24*a*b**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + b**3*m**3*x**4
*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**3*m**2*x**4*x**m/(m**4
+ 10*m**3 + 35*m**2 + 50*m + 24) + 11*b**3*m*x**4*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24
), True))
```

Giac [B] time = 1.07291, size = 302, normalized size = 4.95

$$\frac{b^3m^3x^4x^m + 3\,ab^2m^3x^3x^m + 6\,b^3m^2x^4x^m + 3\,a^2bm^3x^2x^m + 21\,ab^2m^2x^3x^m + 11\,b^3mx^4x^m + a^3m^3xx^m + 24\,a^2bm^2x^2x^m + 4}{m^4 + 10\,m^3 + 35\,m^2 + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^4*x^m + 3*a*b^2*m^3*x^3*x^m + 6*b^3*m^2*x^4*x^m + 3*a^2*b*m^3*x^
2*x^m + 21*a*b^2*m^2*x^3*x^m + 11*b^3*m*x^4*x^m + a^3*m^3*x*x^m + 24*a^2*b*
m^2*x^2*x^m + 42*a*b^2*m*x^3*x^m + 6*b^3*x^4*x^m + 9*a^3*m^2*x*x^m + 57*a^2
*b*m*x^2*x^m + 24*a*b^2*x^3*x^m + 26*a^3*m*x*x^m + 36*a^2*b*x^2*x^m + 24*a^
3*x*x^m)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

3.702 $\int x^m (a+bx)^2 \, dx$

Optimal. Leaf size=43

$$\frac{a^2 x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2 x^{m+3}}{m+3}$$

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(2 + m))/(2 + m) + (b^2*x^(3 + m))/(3 +
m)
```

Rubi [A] time = 0.012549, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2 x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2 x^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(2 + m))/(2 + m) + (b^2*x^(3 + m))/(3 +
m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m (a+bx)^2 \, dx &= \int \left(a^2 x^m + 2abx^{1+m} + b^2 x^{2+m}\right) dx\\ &= \frac{a^2 x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2 x^{3+m}}{3+m}\end{aligned}$$

Mathematica [A] time = 0.0279385, size = 38, normalized size = 0.88

$$x^{m+1}\left(\frac{a^2}{m+1}+\frac{2abx}{m+2}+\frac{b^2x^2}{m+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))
```

Maple [A] time = 0.002, size = 87, normalized size = 2.

$$\frac{x^{1+m}\left(b^{2}m^{2}x^{2}+2\,ab\,m^{2}x+3\,b^{2}m\,x^{2}+a^{2}m^{2}+8\,abmx+2\,b^{2}x^{2}+5\,a^{2}m+6\,abx+6\,a^{2}\right)}{\left(3+m\right)\left(2+m\right)\left(1+m\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^2,x)
```

```
[Out] x^(1+m)*(b^2*m^2*x^2+2*a*b*m^2*x+3*b^2*m*x^2+a^2*m^2+8*a*b*m*x+2*b^2*x^2+5*
a^2*m+6*a*b*x+6*a^2)/(3+m)/(2+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56175, size = 178, normalized size = 4.14

$$\frac{\left(\left(b^{2}m^{2}+3\,b^{2}m+2\,b^{2}\right)x^{3}+2\left(abm^{2}+4\,abm+3\,ab\right)x^{2}+\left(a^{2}m^{2}+5\,a^{2}m+6\,a^{2}\right)x\right)x^{m}}{m^{3}+6\,m^{2}+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2
*m^2 + 5*a^2*m + 6*a^2)*x)*x^m/(m^3 + 6*m^2 + 11*m + 6)
```

Sympy [A] time = 0.475533, size = 299, normalized size = 6.95

$$\begin{cases} -\frac{a^{2}}{2x^{2}}-\frac{2ab}{x}+b^{2}\log\left(x\right) \\ -\frac{a^{2}}{x}+2ab\log\left(x\right)+b^{2}x \\ a^{2}\log\left(x\right)+2abx+\frac{b^{2}x^{2}}{2} \\ \frac{a^{2}m^{2}xx^{m}}{m^{3}+6m^{2}+11m+6}+\frac{5a^{2}mxx^{m}}{m^{3}+6m^{2}+11m+6}+\frac{6a^{2}xx^{m}}{m^{3}+6m^{2}+11m+6}+\frac{2abm^{2}x^{2}x^{m}}{m^{3}+6m^{2}+11m+6}+\frac{8abmx^{2}x^{m}}{m^{3}+6m^{2}+11m+6}+\frac{6abx^{2}x^{m}}{m^{3}+6m^{2}+11m+6}+\frac{b^{2}m^{2}x^{3}x^{m}}{m^{3}+6m^{2}+11m+6}+\frac{3}{m^{3}+6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x), Eq(m, -3)), (-a**2/x + 2
*a*b*log(x) + b**2*x, Eq(m, -2)), (a**2*log(x) + 2*a*b*x + b**2*x**2/2, Eq(
m, -1)), (a**2*m**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 5*a**2*m*x*x**m/(m*
*3 + 6*m**2 + 11*m + 6) + 6*a**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 2*a*b*
```

```
m**2*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 8*a*b*m*x**2*x**m/(m**3 + 6*m**
2 + 11*m + 6) + 6*a*b*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + b**2*m**2*x**3
*x**m/(m**3 + 6*m**2 + 11*m + 6) + 3*b**2*m*x**3*x**m/(m**3 + 6*m**2 + 11*m
 + 6) + 2*b**2*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

Giac [B] time = 1.05492, size = 158, normalized size = 3.67

$$\frac{b^2m^2x^3x^m + 2\,abm^2x^2x^m + 3\,b^2mx^3x^m + a^2m^2xx^m + 8\,abmx^2x^m + 2\,b^2x^3x^m + 5\,a^2mxx^m + 6\,abx^2x^m + 6\,a^2xx^m}{m^3 + 6\,m^2 + 11\,m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^3*x^m + 2*a*b*m^2*x^2*x^m + 3*b^2*m*x^3*x^m + a^2*m^2*x*x^m + 8*
a*b*m*x^2*x^m + 2*b^2*x^3*x^m + 5*a^2*m*x*x^m + 6*a*b*x^2*x^m + 6*a^2*x*x^m
)/(m^3 + 6*m^2 + 11*m + 6)
```

3.703 $\int x^m(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(2 + m))/(2 + m)
```

Rubi [A] time = 0.0064292, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(2 + m))/(2 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)\,dx &= \int \left(ax^m+bx^{1+m}\right)\,dx\\ &= \frac{ax^{1+m}}{1+m}+\frac{bx^{2+m}}{2+m}\end{aligned}$$

Mathematica [A] time = 0.012999, size = 22, normalized size = 0.88

$$x^{m+1}\left(\frac{a}{m+1}+\frac{bx}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x),x]
```

```
[Out] x^(1 + m)*(a/(1 + m) + (b*x)/(2 + m))
```

Maple [A] time = 0., size = 31, normalized size = 1.2

$$\frac{x^{1+m}\,(bmx+am+bx+2\,a)}{(2+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a),x)
```

```
[Out] x^(1+m)*(b*m*x+a*m+b*x+2*a)/(2+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57877, size = 72, normalized size = 2.88

$$\frac{\left((bm+b)x^2+(am+2\,a)x\right)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [A] time = 0.262392, size = 87, normalized size = 3.48

$$\begin{cases} -\frac{a}{x}+b\log(x) & \text{for } m=-2 \\ a\log(x)+bx & \text{for } m=-1 \\ \frac{amxx^m}{m^2+3m+2}+\frac{2axx^m}{m^2+3m+2}+\frac{bmx^2x^m}{m^2+3m+2}+\frac{bx^2x^m}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a),x)
```

```
[Out] Piecewise((-a/x + b*log(x), Eq(m, -2)), (a*log(x) + b*x, Eq(m, -1)), (a*m*x
*x**m/(m**2 + 3*m + 2) + 2*a*x*x**m/(m**2 + 3*m + 2) + b*m*x**2*x**m/(m**2
+ 3*m + 2) + b*x**2*x**m/(m**2 + 3*m + 2), True))
```

Giac [A] time = 1.06295, size = 58, normalized size = 2.32

$$\frac{bmx^2x^m+amxx^m+bx^2x^m+2\,axx^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="giac")
```

```
[Out] (b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

3.704 $\int \frac{x^m}{a+bx} dx$

Optimal. Leaf size=29

$$\frac{x^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{bx}{a}\right)}{a(m+1)}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a*(1 + m))
```

Rubi [A] time = 0.0056343, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {64}

$$\frac{x^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{bx}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a*(1 + m))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{x^m}{a+bx}\,dx = \frac{x^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{bx}{a}\right)}{a(1+m)}$$

Mathematica [A] time = 0.0060936, size = 29, normalized size = 1.

$$\frac{x^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{bx}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a*(1 + m))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{x^m}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a),x)
```

```
[Out] int(x^m/(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b*x + a), x)
```

Sympy [C] time = 0.629635, size = 61, normalized size = 2.1

$$\frac{mxx^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma\left(m+1\right)}{a\Gamma\left(m+2\right)}+\frac{xx^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma\left(m+1\right)}{a\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a),x)
```

```
[Out] m*x*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a*gamma(m
+ 2)) + x*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a*ga
mma(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x + a), x)
```

3.705 $\int \frac{x^m}{(a+bx)^2} dx$

Optimal. Leaf size=29

$$\frac{x^{m+1} {}_2F_1\left(2, m+1; m+2; -\frac{bx}{a}\right)}{a^2(m+1)}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/a)])/(a^2*(1 + m))
```

Rubi [A] time = 0.0052433, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {64}

$$\frac{x^{m+1} {}_2F_1\left(2, m+1; m+2; -\frac{bx}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/a)])/(a^2*(1 + m))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{x^m}{(a+bx)^2} dx = \frac{x^{1+m} {}_2F_1\left(2, 1+m; 2+m; -\frac{bx}{a}\right)}{a^2(1+m)}$$

Mathematica [A] time = 0.0050386, size = 29, normalized size = 1.

$$\frac{x^{m+1} {}_2F_1\left(2, m+1; m+2; -\frac{bx}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/a)])/(a^2*(1 + m))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^2,x)
```

```
[Out] int(x^m/(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{b^2x^2+2\,abx+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] time = 0.841628, size = 262, normalized size = 9.03

$$-\frac{am^2xx^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)}-\frac{amxx^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)}+\frac{amxx^m\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)}+\frac{}{a^3\Gamma(m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**2,x)
```

```
[Out] -a*m**2*x*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a**3
*gamma(m + 2) + a**2*b*x*gamma(m + 2)) - a*m*x*x**m*lerchphi(b*x*exp_polar(
I*pi)/a, 1, m + 1)*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2))
 + a*m*x*x**m*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2)) + a*
x*x**m*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2)) - b*m**2*x*
*2*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a**3*gamma(
m + 2) + a**2*b*x*gamma(m + 2)) - b*m*x**2*x**m*lerchphi(b*x*exp_polar(I*pi
)/a, 1, m + 1)*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x + a)^2, x)
```

3.706 $\int \frac{x^m}{(a+bx)^3} dx$

Optimal. Leaf size=29

$$\frac{x^{m+1} {}_2F_1\left(3,m+1;m+2;-\frac{bx}{a}\right)}{a^3(m+1)}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/a)])/(a^3*(1 + m))
```

Rubi [A] time = 0.0054327, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {64}

$$\frac{x^{m+1} {}_2F_1\left(3,m+1;m+2;-\frac{bx}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/a)])/(a^3*(1 + m))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{x^m}{(a+bx)^3} dx = \frac{x^{1+m} {}_2F_1\left(3,1+m;2+m;-\frac{bx}{a}\right)}{a^3(1+m)}$$

Mathematica [A] time = 0.0054128, size = 29, normalized size = 1.

$$\frac{x^{m+1} {}_2F_1\left(3,m+1;m+2;-\frac{bx}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/a)])/(a^3*(1 + m))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^3,x)
```

```
[Out] int(x^m/(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [C] time = 1.16805, size = 717, normalized size = 24.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**3,x)
```

```
[Out] a**2*m**3*x*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*
a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)
) - a**2*m**2*x*x**m*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m
 + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) - a**2*m*x*x**m*lerchphi(b*x*exp_pol
ar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(
m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) + a**2*m*x*x**m*gamma(m + 1)/(2*a**
5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) +
 2*a**2*x*x**m*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2)
+ 2*a**3*b**2*x**2*gamma(m + 2)) + 2*a*b*m**3*x**2*x**m*lerchphi(b*x*exp_po
lar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma
(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) - a*b*m**2*x**2*x**m*gamma(m + 1)/
(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m +
 2)) - 2*a*b*m*x**2*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m
+ 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamm
a(m + 2)) + a*b*x**2*x**m*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*ga
```

```
mma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) + b**2*m**3*x**3*x**m*lerchphi(
b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4
*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) - b**2*m*x**3*x**m*lerch
phi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*
a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x + a)^3, x)
```

3.707 $\int x^m (a+bx)^{5/2} \, dx$

Optimal. Leaf size=48

$$\frac{2x^m(a+bx)^{7/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{7}{2},-m;\frac{9}{2};\frac{bx}{a}+1\right)}{7b}$$

```
[Out] (2*x^m*(a + b*x)^(7/2)*Hypergeometric2F1[7/2, -m, 9/2, 1 + (b*x)/a])/(7*b*(
-((b*x)/a))^m)
```

Rubi [A] time = 0.0112384, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {67, 65}

$$\frac{2x^m(a+bx)^{7/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{7}{2},-m;\frac{9}{2};\frac{bx}{a}+1\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^(5/2),x]
```

```
[Out] (2*x^m*(a + b*x)^(7/2)*Hypergeometric2F1[7/2, -m, 9/2, 1 + (b*x)/a])/(7*b*(
-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^{5/2}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int\left(-\frac{bx}{a}\right)^m(a+bx)^{5/2}\,dx\\ &= \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{7/2}\,{}_2F_1\left(\frac{7}{2},-m;\frac{9}{2};1+\frac{bx}{a}\right)}{7b}\end{aligned}$$

Mathematica [A] time = 0.0629255, size = 48, normalized size = 1.

$$\frac{2x^m(a+bx)^{7/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{7}{2},-m;\frac{9}{2};\frac{bx}{a}+1\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^(5/2),x]
```

```
[Out] (2*x^m*(a + b*x)^(7/2)*Hypergeometric2F1[7/2, -m, 9/2, 1 + (b*x)/a])/(7*b*(
-((b*x)/a))^m)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^{m}(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^(5/2),x)
```

```
[Out] int(x^m*(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{2}}x^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)*x^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{bx+a}x^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*x^m, x)
```

Sympy [C] time = 53.2687, size = 37, normalized size = 0.77

$$\frac{a^{\frac{5}{2}}xx^{m}\Gamma(m+1)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{2},m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**(5/2),x)
```

```
[Out] a**(5/2)*x*x**m*gamma(m + 1)*hyper((-5/2, m + 1), (m + 2,), b*x*exp_polar(I
*pi)/a)/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^{\frac{5}{2}} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/2)*x^m, x)
```

3.708 $\int x^m (a+bx)^{3/2} \, dx$

Optimal. Leaf size=48

$$\frac{2x^m(a+bx)^{5/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{5}{2},-m;\frac{7}{2};\frac{bx}{a}+1\right)}{5b}$$

```
[Out] (2*x^m*(a + b*x)^(5/2)*Hypergeometric2F1[5/2, -m, 7/2, 1 + (b*x)/a])/(5*b*(
-((b*x)/a))^m)
```

Rubi [A] time = 0.0115617, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {67, 65}

$$\frac{2x^m(a+bx)^{5/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{5}{2},-m;\frac{7}{2};\frac{bx}{a}+1\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^(3/2),x]
```

```
[Out] (2*x^m*(a + b*x)^(5/2)*Hypergeometric2F1[5/2, -m, 7/2, 1 + (b*x)/a])/(5*b*(
-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int x^m(a+bx)^{3/2}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int\left(-\frac{bx}{a}\right)^m(a+bx)^{3/2}\,dx\\
&= \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{5/2}\,{}_2F_1\left(\frac{5}{2},-m;\frac{7}{2};1+\frac{bx}{a}\right)}{5b}
\end{aligned}$$

Mathematica [A] time = 0.0399808, size = 48, normalized size = 1.

$$\frac{2x^m(a+bx)^{5/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{5}{2},-m;\frac{7}{2};\frac{bx}{a}+1\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^(3/2),x]
```

```
[Out] (2*x^m*(a + b*x)^(5/2)*Hypergeometric2F1[5/2, -m, 7/2, 1 + (b*x)/a])/(5*b*(
-((b*x)/a))^m)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^m (bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^(3/2),x)
```

```
[Out] int(x^m*(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*x^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^{\frac{3}{2}} x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*x^m, x)
```

Sympy [C] time = 5.22225, size = 37, normalized size = 0.77

$$\frac{a^{\frac{3}{2}} x x^m \Gamma(m+1)\, {}_2F_1\left(\left.\begin{matrix} -\frac{3}{2}, m+1 \\ m+2 \end{matrix}\right| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**(3/2),x)
```

```
[Out] a**(3/2)*x*x**m*gamma(m + 1)*hyper((-3/2, m + 1), (m + 2,), b*x*exp_polar(I
*pi)/a)/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)*x^m, x)
```

3.709 $\int x^m \sqrt{a+bx}\,dx$

Optimal. Leaf size=48

$$\frac{2x^m(a+bx)^{3/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};\frac{bx}{a}+1\right)}{3b}$$

```
[Out] (2*x^m*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, -m, 5/2, 1 + (b*x)/a])/(3*b*(
-((b*x)/a))^m)
```

Rubi [A] time = 0.0109918, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {67, 65}

$$\frac{2x^m(a+bx)^{3/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};\frac{bx}{a}+1\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, -m, 5/2, 1 + (b*x)/a])/(3*b*(
-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int x^m\sqrt{a+bx}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int\left(-\frac{bx}{a}\right)^m\sqrt{a+bx}\,dx\\ &= \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};1+\frac{bx}{a}\right)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0298459, size = 48, normalized size = 1.

$$\frac{2x^m(a+bx)^{3/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};\frac{bx}{a}+1\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, -m, 5/2, 1 + (b*x)/a])/(3*b*(
-((b*x)/a))^m)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^m \sqrt{bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^(1/2),x)
```

```
[Out] int(x^m*(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*x^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx+a}x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*x^m, x)
```

Sympy [C] time = 1.44426, size = 37, normalized size = 0.77

$$\frac{\sqrt{a}xx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2}, m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*x**m*gamma(m + 1)*hyper((-1/2, m + 1), (m + 2,), b*x*exp_polar(I*
pi)/a)/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)*x^m, x)
```

3.710 $\int \frac{x^m}{\sqrt{a+bx}} dx$

Optimal. Leaf size=46

$$\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

```
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```

Rubi [A] time = 0.010974, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {67, 65}

$$\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx}} dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{\sqrt{a+bx}} dx\\ &= \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0229852, size = 46, normalized size = 1.

$$\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^{m} \frac{1}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^(1/2),x)
```

```
[Out] int(x^m/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m}}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(b*x + a), x)
```

Sympy [C] time = 1.23456, size = 36, normalized size = 0.78

$$\frac{xx^{m}\Gamma\left(m+1\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, m+1 \\ m+2 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**(1/2),x)
```

```
[Out] x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(s
qrt(a)*gamma(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x + a), x)
```

3.711 $\int \frac{x^m}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};\frac{bx}{a}+1\right)}{b\sqrt{a+bx}}$$

```
[Out] (-2*x^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/(b*(-((b*x)/a))^m*Sq
rt[a + b*x])
```

Rubi [A] time = 0.010546, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {67, 65}

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};\frac{bx}{a}+1\right)}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/(b*(-((b*x)/a))^m*Sq
rt[a + b*x])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{(a+bx)^{3/2}}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{(a+bx)^{3/2}}\,dx\\ &= -\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};1+\frac{bx}{a}\right)}{b\sqrt{a+bx}}\end{aligned}$$

Mathematica [A] time = 0.0349451, size = 46, normalized size = 1.

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};\frac{bx}{a}+1\right)}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/(b*(-((b*x)/a))^m*Sq
rt[a + b*x])
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int x^{m} (bx + a)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^(3/2),x)
```

```
[Out] int(x^m/(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{(bx + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx + a}x^{m}}{b^{2}x^{2} + 2\,abx + a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*x^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] time = 1.95313, size = 36, normalized size = 0.78

$$\frac{xx^{m}\Gamma(m+1)\,{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2}, m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{2}}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**(3/2),x)
```

```
[Out] x*x**m*gamma(m + 1)*hyper((3/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(a
**(3/2)*gamma(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x + a)^(3/2), x)
```

3.712 $\int \frac{x^m}{(a+bx)^{5/2}} \, dx$

Optimal. Leaf size=48

$$-\frac{2x^m \left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(-\frac{3}{2},-m;-\frac{1}{2};\frac{bx}{a}+1\right)}{3b(a+bx)^{3/2}}$$

```
[Out] (-2*x^m*Hypergeometric2F1[-3/2, -m, -1/2, 1 + (b*x)/a])/(3*b*(-((b*x)/a))^m
*(a + b*x)^(3/2))
```

Rubi [A] time = 0.0110132, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {67, 65}

$$-\frac{2x^m \left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(-\frac{3}{2},-m;-\frac{1}{2};\frac{bx}{a}+1\right)}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^m*Hypergeometric2F1[-3/2, -m, -1/2, 1 + (b*x)/a])/(3*b*(-((b*x)/a))^m
*(a + b*x)^(3/2))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^m}{(a+bx)^{5/2}} \, dx &= \left(x^m \left(-\frac{bx}{a}\right)^{-m}\right) \int \frac{\left(-\frac{bx}{a}\right)^m}{(a+bx)^{5/2}} \, dx \\ &= -\frac{2x^m \left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(-\frac{3}{2},-m;-\frac{1}{2};1+\frac{bx}{a}\right)}{3b(a+bx)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0382125, size = 48, normalized size = 1.

$$-\frac{2x^m \left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(-\frac{3}{2},-m;-\frac{1}{2};\frac{bx}{a}+1\right)}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^m*Hypergeometric2F1[-3/2, -m, -1/2, 1 + (b*x)/a])/(3*b*(-((b*x)/a))^m
*(a + b*x)^(3/2))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int x^m (bx+a)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^(5/2),x)
```

```
[Out] int(x^m/(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}x^m}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*x^m/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [C] time = 5.98714, size = 36, normalized size = 0.75

$$\frac{xx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{5}{2}, m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{2}}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**(5/2),x)
```

```
[Out] x*x**m*gamma(m + 1)*hyper((5/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(a
**(5/2)*gamma(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x + a)^(5/2), x)
```

3.713 $\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=51

$$\frac{2a^2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m-2;\frac{3}{2};\frac{bx}{a}+1\right)}{b^3}$$

```
[Out] (2*a^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -2 - m, 3/2, 1 + (b*x)/a])/
(b^3*(-((b*x)/a))^m)
```

Rubi [A] time = 0.0118605, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {67, 65}

$$\frac{2a^2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m-2;\frac{3}{2};\frac{bx}{a}+1\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -2 - m, 3/2, 1 + (b*x)/a])/
(b^3*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx &= \frac{\left(a^2x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{2+m}}{\sqrt{a+bx}}\,dx}{b^2}\\ &= \frac{2a^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},-2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0358965, size = 51, normalized size = 1.

$$\frac{2a^2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m-2;\frac{3}{2};\frac{bx}{a}+1\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -2 - m, 3/2, 1 + (b*x)/a])/
(b^3*(-((b*x)/a))^m)
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int x^{2+m}\frac{1}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)/(b*x+a)^(1/2),x)
```

```
[Out] int(x^(2+m)/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m+2}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 2)/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m+2}}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m + 2)/sqrt(b*x + a), x)
```

Sympy [C] time = 4.70403, size = 37, normalized size = 0.73

$$\frac{x^3x^m\Gamma(m+3)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m+3\\ m+4\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\Gamma(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)/(b*x+a)**(1/2),x)
```

```
[Out] x**3*x**m*gamma(m + 3)*hyper((1/2, m + 3), (m + 4,), b*x*exp_polar(I*pi)/a)
/(sqrt(a)*gamma(m + 4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m+2}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)/sqrt(b*x + a), x)
```

3.714 $\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=49

$$-\frac{2ax^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m-1;\frac{3}{2};\frac{bx}{a}+1\right)}{b^2}$$

```
[Out] (-2*a*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -1 - m, 3/2, 1 + (b*x)/a])/(
b^2*(-((b*x)/a))^m)
```

Rubi [A] time = 0.0121783, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {67, 65}

$$-\frac{2ax^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m-1;\frac{3}{2};\frac{bx}{a}+1\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -1 - m, 3/2, 1 + (b*x)/a])/(
b^2*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx &= -\frac{\left(ax^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{1+m}}{\sqrt{a+bx}}\,dx}{b}\\ &= -\frac{2ax^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},-1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0315547, size = 49, normalized size = 1.

$$-\frac{2ax^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m-1;\frac{3}{2};\frac{bx}{a}+1\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -1 - m, 3/2, 1 + (b*x)/a])/(
b^2*(-((b*x)/a))^m)
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int x^{1+m} \frac{1}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)/(b*x+a)^(1/2),x)
```

```
[Out] int(x^(1+m)/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m+1}}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 1)/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m+1}}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m + 1)/sqrt(b*x + a), x)
```

Sympy [C] time = 2.71689, size = 37, normalized size = 0.76

$$\frac{x^2 x^m \Gamma(m+2)\, {}_2F_1\left(\begin{matrix} \frac{1}{2}, m+2 \\ m+3 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)/(b*x+a)**(1/2),x)
```

```
[Out] x**2*x**m*gamma(m + 2)*hyper((1/2, m + 2), (m + 3,), b*x*exp_polar(I*pi)/a)
/(sqrt(a)*gamma(m + 3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m+1}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)/sqrt(b*x + a), x)
```

3.715 $\int \frac{x^m}{\sqrt{a+bx}} dx$

Optimal. Leaf size=46

$$\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

```
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```

Rubi [A] time = 0.010804, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {67, 65}

$$\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x],x]
```


```
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```


Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx}} dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{\sqrt{a+bx}} dx \\ &= \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0081201, size = 46, normalized size = 1.

$$\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m}\frac{1}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^(1/2),x)
```

```
[Out] int(x^m/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m}}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(b*x + a), x)
```

Sympy [C] time = 1.25598, size = 36, normalized size = 0.78

$$\frac{xx^{m}\Gamma\left(m+1\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2}, m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**(1/2),x)
```

```
[Out] x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(s
qrt(a)*gamma(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x + a), x)
```

3.716 $\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=48

$$-\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a}$$

```
[Out] (-2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*x)/a])/(a*(
-((b*x)/a))^m)
```

Rubi [A] time = 0.0116183, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {67, 65}

$$-\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*x)/a])/(a*(
-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx &= -\frac{\left(bx^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{-1+m}}{\sqrt{a+bx}}\,dx}{a}\\ &= -\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0291351, size = 48, normalized size = 1.

$$-\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*x)/a])/(a*(
-((b*x)/a))^m)
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int x^{-1+m}\frac{1}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)/(b*x+a)^(1/2),x)
```

```
[Out] int(x^(-1+m)/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-1}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 1)/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m-1}}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m - 1)/sqrt(b*x + a), x)
```

Sympy [C] time = 5.56003, size = 31, normalized size = 0.65

$$\frac{x^m\Gamma(m)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m\\ m+1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\Gamma(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)/(b*x+a)**(1/2),x)
```

```
[Out] x**m*gamma(m)*hyper((1/2, m), (m + 1,), b*x*exp_polar(I*pi)/a)/(sqrt(a)*gam
ma(m + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-1}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)/sqrt(b*x + a), x)
```

3.717 $\int \frac{x^{-2+m}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=49

$$\frac{2bx^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{1}{2},2-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a^2}$$

```
[Out] (2*b*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 2 - m, 3/2, 1 + (b*x)/a])/(a^
2*(-((b*x)/a))^m)
```

Rubi [A] time = 0.011609, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {67, 65}

$$\frac{2bx^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{1}{2},2-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)/Sqrt[a + b*x],x]
```

```
[Out] (2*b*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 2 - m, 3/2, 1 + (b*x)/a])/(a^
2*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-2+m}}{\sqrt{a+bx}}\,dx &= \frac{\left(b^2x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{-2+m}}{\sqrt{a+bx}}\,dx}{a^2}\\ &= \frac{2bx^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0307771, size = 49, normalized size = 1.

$$\frac{2bx^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{1}{2},2-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)/Sqrt[a + b*x],x]
```

```
[Out] (2*b*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 2 - m, 3/2, 1 + (b*x)/a])/(a^
2*(-((b*x)/a))^m)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^{-2+m} \frac{1}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)/(b*x+a)^(1/2),x)
```

```
[Out] int(x^(-2+m)/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-2}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 2)/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m-2}}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m - 2)/sqrt(b*x + a), x)
```

Sympy [C] time = 34.9807, size = 32, normalized size = 0.65

$$\frac{x^m\Gamma(m-1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m-1\\ m\end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}x\Gamma(m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)/(b*x+a)**(1/2),x)
```

```
[Out] x**m*gamma(m - 1)*hyper((1/2, m - 1), (m,), b*x*exp_polar(I*pi)/a)/(sqrt(a)
*x*gamma(m))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-2}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)/sqrt(b*x + a), x)
```

3.718 $\int \frac{x^{-3+m}}{\sqrt{a+bx}} dx$

Optimal. Leaf size=51

$$-\frac{2b^2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},3-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a^3}$$

```
[Out] (-2*b^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 3 - m, 3/2, 1 + (b*x)/a])/
(a^3*(-((b*x)/a))^m)
```

Rubi [A] time = 0.0125032, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {67, 65}

$$-\frac{2b^2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},3-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*b^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 3 - m, 3/2, 1 + (b*x)/a])/
(a^3*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-3+m}}{\sqrt{a+bx}} dx &= -\frac{\left(b^3x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{-3+m}}{\sqrt{a+bx}} dx}{a^3}\\ &= -\frac{2b^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},3-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0322882, size = 51, normalized size = 1.

$$-\frac{2b^2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},3-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*b^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 3 - m, 3/2, 1 + (b*x)/a])/
(a^3*(-((b*x)/a))^m)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x^{-3+m} \frac{1}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)/(b*x+a)^(1/2),x)
```

```
[Out] int(x^(-3+m)/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-3}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 3)/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m-3}}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m - 3)/sqrt(b*x + a), x)
```

Sympy [C] time = 137.449, size = 37, normalized size = 0.73

$$\frac{x^m \Gamma(m-2)\, {}_2F_1\left(\begin{matrix} \frac{1}{2}, m-2 \\ m-1 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}x^2\Gamma(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)/(b*x+a)**(1/2),x)
```

```
[Out] x**m*gamma(m - 2)*hyper((1/2, m - 2), (m - 1,), b*x*exp_polar(I*pi)/a)/(sqr
t(a)*x**2*gamma(m - 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-3}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)/sqrt(b*x + a), x)
```

3.719 $\int \frac{x^m}{\sqrt{2+3x}}\,dx$

Optimal. Leaf size=31

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;-\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1 + m)
)
```

Rubi [A] time = 0.0055908, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {64}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;-\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[2 + 3*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1 + m)
)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{x^m}{\sqrt{2+3x}}\,dx=\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}(1+m)}$$

Mathematica [A] time = 0.005287, size = 31, normalized size = 1.

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;-\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[2 + 3*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1 + m)
)
```

Maple [A] time = 0.023, size = 29, normalized size = 0.9

$$\frac{x^{1+m}\sqrt{2}}{2+2m}{}_2\mathrm{F}_1(\frac{1}{2},1+m;\,2+m;\,-\frac{3x}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(2+3*x)^(1/2),x)
```

```
[Out] 1/2*x^(1+m)*hypergeom([1/2,1+m],[2+m],-3/2*x)/(1+m)*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(3*x + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{\sqrt{3x+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(3*x + 2), x)
```

Sympy [C] time = 1.00617, size = 37, normalized size = 1.19

$$\frac{\sqrt{2}xx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m+1\\m+2\end{matrix}\middle|\frac{3xe^{i\pi}}{2}\right)}{2\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(2+3*x)**(1/2),x)
```

```
[Out] sqrt(2)*x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), 3*x*exp_polar(I*p
i)/2)/(2*gamma(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(3*x + 2), x)
```

3.720 $\int \frac{x^m}{\sqrt{2-3x}}\,dx$

Optimal. Leaf size=31

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1 + m))
```

Rubi [A] time = 0.0040992, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {64}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[2 - 3*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1 + m))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{x^m}{\sqrt{2-3x}}\,dx = \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}(1+m)}$$

Mathematica [A] time = 0.0060529, size = 31, normalized size = 1.

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[2 - 3*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1 + m))
```

Maple [A] time = 0.026, size = 29, normalized size = 0.9

$$\frac{x^{1+m}\sqrt{2}}{2+2m}{}_2\mathrm{F}_1(\frac{1}{2},1+m;\,2+m;\,\frac{3x}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(2-3*x)^(1/2),x)
```

```
[Out] 1/2*x^(1+m)*hypergeom([1/2,1+m],[2+m],3/2*x)/(1+m)*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2-3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(-3*x + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{x^m\sqrt{-3x+2}}{3x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2-3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-x^m*sqrt(-3*x + 2)/(3*x - 2), x)
```

Sympy [C] time = 1.0146, size = 46, normalized size = 1.48

$$-\frac{2\cdot 2^m\sqrt{3}\cdot 3^{-m}i\sqrt{x-\frac{2}{3}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|\frac{3\left(x-\frac{2}{3}\right)e^{i\pi}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(2-3*x)**(1/2),x)
```

```
[Out] -2*2**m*sqrt(3)*3**(-m)*I*sqrt(x - 2/3)*hyper((1/2, -m), (3/2,), 3*(x - 2/3
)*exp_polar(I*pi)/2)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2-3*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(-3*x + 2), x)
```

3.721 $\int \frac{x^m}{\sqrt{-2+3x}}\,dx$

Optimal. Leaf size=36

$$\left(\frac{3}{2}\right)^{-m-1}\sqrt{3x-2}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

```
[Out] (3/2)^(-1 - m)*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - (3*x)/2]
```

Rubi [A] time = 0.0083152, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {65}

$$\left(\frac{3}{2}\right)^{-m-1}\sqrt{3x-2}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[-2 + 3*x],x]
```

```
[Out] (3/2)^(-1 - m)*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - (3*x)/2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{x^m}{\sqrt{-2+3x}}\,dx = \left(\frac{3}{2}\right)^{-1-m}\sqrt{-2+3x}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Mathematica [A] time = 0.006302, size = 36, normalized size = 1.

$$\left(\frac{3}{2}\right)^{-m-1}\sqrt{3x-2}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[-2 + 3*x],x]
```

```
[Out] (3/2)^(-1 - m)*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - (3*x)/2]
```

Maple [C] time = 0.033, size = 43, normalized size = 1.2

$$\frac{\sqrt{2}x^{1+m}}{2+2m}\sqrt{-\mathrm{signum}\left(x-\frac{2}{3}\right)}\,{}_2\mathrm{F}_1(\frac{1}{2},1+m;\,2+m;\,\frac{3x}{2})\frac{1}{\sqrt{\mathrm{signum}\left(x-\frac{2}{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-2+3*x)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)/signum(x-2/3)^(1/2)*(-signum(x-2/3))^(1/2)/(1+m)*x^(1+m)*hyperg
eom([1/2,1+m],[2+m],3/2*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{3x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(3*x - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\sqrt{3x-2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(3*x - 2), x)
```

Sympy [C] time = 1.01632, size = 36, normalized size = 1.

$$-\frac{\sqrt{2}ixx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m+1\\ m+2\end{matrix}\middle|\frac{3x}{2}\right)}{2\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-2+3*x)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), 3*x/2)/(2*gamm
a(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{3x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2+3*x)^(1/2),x, algorithm="giac")

[Out] integrate(x^m/sqrt(3*x - 2), x)
```

3.722 $\int \frac{x^m}{\sqrt{-2-3x}}\,dx$

Optimal. Leaf size=50

$$-2^{m+1}3^{-m-1}\sqrt{-3x-2}(-x)^{-m}x^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{3x}{2}+1\right)$$

```
[Out] -((2^(1 + m)*3^(-1 - m)*Sqrt[-2 - 3*x]*x^m*Hypergeometric2F1[1/2, -m, 3/2,
1 + (3*x)/2])/(-x)^m)
```

Rubi [A] time = 0.0109464, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {67, 12, 65}

$$-2^{m+1}3^{-m-1}\sqrt{-3x-2}(-x)^{-m}x^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{3x}{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[-2 - 3*x],x]
```

```
[Out] -((2^(1 + m)*3^(-1 - m)*Sqrt[-2 - 3*x]*x^m*Hypergeometric2F1[1/2, -m, 3/2,
1 + (3*x)/2])/(-x)^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{-2-3x}}\,dx &= \left(\left(\frac{2}{3}\right)^m(-x)^{-m}x^m\right)\int \frac{\left(\frac{3}{2}\right)^m(-x)^m}{\sqrt{-2-3x}}\,dx\\ &= \left((-x)^{-m}x^m\right)\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx\\ &= -2^{1+m}3^{-1-m}\sqrt{-2-3x}(-x)^{-m}x^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0228393, size = 48, normalized size = 0.96

$$-\frac{2}{3}\sqrt{-3x-2}\left(\frac{1}{2}(-3x-2)+1\right)^{-m}x^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{3x}{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[-2 - 3*x],x]
```

```
[Out] (-2*Sqrt[-2 - 3*x]*x^m*Hypergeometric2F1[1/2, -m, 3/2, 1 + (3*x)/2])/(3*(1
+ (-2 - 3*x)/2)^m)
```

Maple [C] time = 0.016, size = 30, normalized size = 0.6

$$\frac{-\frac{i}{2}x^{1+m}\sqrt{2}}{1+m}{}_2F_1(\frac{1}{2},1+m;\,2+m;\,-\frac{3x}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-2-3*x)^(1/2),x)
```

```
[Out] -1/2*I*x^(1+m)*hypergeom([1/2,1+m],[2+m],-3/2*x)/(1+m)*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{-3x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2-3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(-3*x - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{x^m\sqrt{-3x-2}}{3x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2-3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-x^m*sqrt(-3*x - 2)/(3*x + 2), x)
```

Sympy [C] time = 1.01834, size = 41, normalized size = 0.82

$$-\frac{\sqrt{2}ixx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m+1\\m+2\end{matrix}\middle|\frac{3xe^{i\pi}}{2}\right)}{2\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-2-3*x)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), 3*x*exp_polar(
I*pi)/2)/(2*gamma(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{-3x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2-3*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(-3*x - 2), x)
```

3.723 $\int \frac{(-x)^m}{\sqrt{a+bx}} dx$

Optimal. Leaf size=48

$$\frac{2(-x)^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

```
[Out] (2*(-x)^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-
((b*x)/a))^m)
```

Rubi [A] time = 0.0108051, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {67, 65}

$$\frac{2(-x)^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[a + b*x],x]
```

```
[Out] (2*(-x)^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-
((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx &= \left((-x)^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{\sqrt{a+bx}}\,dx\\ &= \frac{2(-x)^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0167387, size = 48, normalized size = 1.

$$\frac{2(-x)^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[a + b*x],x]
```

```
[Out] (2*(-x)^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b*x)/a))^m)
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int (-x)^m \frac{1}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(b*x+a)^(1/2),x)
```

```
[Out] int((-x)^m/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x)^m}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x)^m/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-x)^m}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-x)^m/sqrt(b*x + a), x)
```

Sympy [C] time = 1.27127, size = 42, normalized size = 0.88

$$\frac{xx^m e^{i\pi m}\Gamma(m+1)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, m+1\\ m+2\end{matrix}\right|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(b*x+a)**(1/2),x)
```

```
[Out] x*x**m*exp(I*pi*m)*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), b*x*exp_polar
(I*pi)/a)/(sqrt(a)*gamma(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x)^m}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-x)^m/sqrt(b*x + a), x)
```

3.724 $\int \frac{(-x)^m}{\sqrt{2+3x}}\,dx$

Optimal. Leaf size=34

$$-\frac{(-x)^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;-\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

```
[Out] -(((-x)^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1
 + m)))
```

Rubi [A] time = 0.0060438, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {64}

$$-\frac{(-x)^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;-\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[2 + 3*x],x]
```

```
[Out] -(((-x)^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1
 + m)))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{(-x)^m}{\sqrt{2+3x}}\,dx=-\frac{(-x)^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}(1+m)}$$

Mathematica [A] time = 0.0050011, size = 32, normalized size = 0.94

$$\frac{x(-x)^m\,{}_2F_1\left(\frac{1}{2},m+1;m+2;-\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[2 + 3*x],x]
```

```
[Out] ((-x)^m*x*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1 + m))
```

Maple [A] time = 0.017, size = 30, normalized size = 0.9

$$\frac{\sqrt{2}\,(-x)^{m}\,x}{2+2m}\,{}_2\mathrm{F}_1\!\left(\frac{1}{2},1+m;\,2+m;\,-\frac{3x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(2+3*x)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(-x)^m/(1+m)*x*hypergeom([1/2,1+m],[2+m],-3/2*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x)^{m}}{\sqrt{3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x)^m/sqrt(3*x + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-x)^{m}}{\sqrt{3x+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-x)^m/sqrt(3*x + 2), x)
```

Sympy [C] time = 0.989137, size = 44, normalized size = 1.29

$$\frac{2\cdot 2^{m}\sqrt{3}\cdot 3^{-m}\sqrt{x+\frac{2}{3}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|\frac{3\left(x+\frac{2}{3}\right)e^{2i\pi}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(2+3*x)**(1/2),x)
```

```
[Out] 2*2**m*sqrt(3)*3**(-m)*sqrt(x + 2/3)*hyper((1/2, -m), (3/2,), 3*(x + 2/3)*e
xp_polar(2*I*pi)/2)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x)^{m}}{\sqrt{3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-x)^m/sqrt(3*x + 2), x)
```

3.725 $\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx$

Optimal. Leaf size=34

$$-\frac{(-x)^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

```
[Out] -(((-x)^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1
+ m)))
```

Rubi [A] time = 0.0048189, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {64}

$$-\frac{(-x)^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[2 - 3*x],x]
```

```
[Out] -(((-x)^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1
+ m)))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx = -\frac{(-x)^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}(1+m)}$$

Mathematica [A] time = 0.0050443, size = 32, normalized size = 0.94

$$\frac{x(-x)^m\,{}_2F_1\left(\frac{1}{2},m+1;m+2;\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[2 - 3*x],x]
```

```
[Out] ((-x)^m*x*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1 + m))
```

Maple [A] time = 0.023, size = 30, normalized size = 0.9

$$\frac{\sqrt{2}\,(-x)^{m}\,x}{2+2\,m}\,{}_2\mathrm{F}_1\!\left(\frac{1}{2},1+m;\,2+m;\,\frac{3\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(2-3*x)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(-x)^m/(1+m)*x*hypergeom([1/2,1+m],[2+m],3/2*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x)^{m}}{\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2-3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x)^m/sqrt(-3*x + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(-x)^{m}\,\sqrt{-3\,x+2}}{3\,x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2-3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(-x)^m*sqrt(-3*x + 2)/(3*x - 2), x)
```

Sympy [C] time = 1.04734, size = 53, normalized size = 1.56

$$-\frac{2\cdot 2^{m}\sqrt{3}\cdot 3^{-m}i\sqrt{x-\frac{2}{3}}e^{i\pi m}{}_2F_1\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|\frac{3\left(x-\frac{2}{3}\right)e^{i\pi}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(2-3*x)**(1/2),x)
```

```
[Out] -2*2**m*sqrt(3)*3**(-m)*I*sqrt(x - 2/3)*exp(I*pi*m)*hyper((1/2, -m), (3/2,)
, 3*(x - 2/3)*exp_polar(I*pi)/2)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x)^{m}}{\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2-3*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-x)^m/sqrt(-3*x + 2), x)
```

3.726 $\int \frac{(-x)^m}{\sqrt{-2+3x}} dx$

Optimal. Leaf size=49

$$2^{m+1}3^{-m-1}\sqrt{3x-2}(-x)^m x^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

```
[Out] (2^(1 + m)*3^(-1 - m)*(-x)^m*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2,
 1 - (3*x)/2])/x^m
```

Rubi [A] time = 0.0110271, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {67, 12, 65}

$$2^{m+1}3^{-m-1}\sqrt{3x-2}(-x)^m x^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[-2 + 3*x],x]
```

```
[Out] (2^(1 + m)*3^(-1 - m)*(-x)^m*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2,
 1 - (3*x)/2])/x^m
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(-x)^m}{\sqrt{-2+3x}}\,dx &= \left(\left(\frac{2}{3}\right)^m (-x)^m x^{-m}\right)\int \frac{\left(\frac{3}{2}\right)^m x^m}{\sqrt{-2+3x}}\,dx\\
&= \left((-x)^m x^{-m}\right)\int \frac{x^m}{\sqrt{-2+3x}}\,dx\\
&= 2^{1+m}3^{-1-m}(-x)^m x^{-m}\sqrt{-2+3x}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0074754, size = 49, normalized size = 1.

$$2^{m+1}3^{-m-1}\sqrt{3x-2}(-x)^m x^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[-2 + 3*x],x]
```

```
[Out] (2^(1 + m)*3^(-1 - m)*(-x)^m*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2,
 1 - (3*x)/2])/x^m
```

Maple [C] time = 0.033, size = 44, normalized size = 0.9

$$\frac{\sqrt{2}\,(-x)^m\,x}{2+2\,m}\sqrt{-\mathrm{signum}\left(x-\frac{2}{3}\right)}{}_2\mathrm{F}_1(\frac{1}{2},1+m;\,2+m;\,\frac{3\,x}{2})\frac{1}{\sqrt{\mathrm{signum}\left(x-\frac{2}{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(-2+3*x)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(-x)^m/signum(x-2/3)^(1/2)*(-signum(x-2/3))^(1/2)/(1+m)*x*hyper
geom([1/2,1+m],[2+m],3/2*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x)^m}{\sqrt{3\,x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x)^m/sqrt(3*x - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(-x)^m}{\sqrt{3\,x-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-x)^m/sqrt(3*x - 2), x)
```

Sympy [C] time = 1.03686, size = 42, normalized size = 0.86

$$-\frac{\sqrt{2}ixx^{m}e^{i\pi m}\Gamma\left(m+1\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},m+1\\m+2\end{matrix}\middle|\frac{3x}{2}\right)}{2\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(-2+3*x)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*x**m*exp(I*pi*m)*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), 3*
x/2)/(2*gamma(m + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x)^m}{\sqrt{3x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-x)^m/sqrt(3*x - 2), x)
```

3.727 $\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx$

Optimal. Leaf size=37

$$-\left(\frac{3}{2}\right)^{-m-1}\sqrt{-3x-2}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{3x}{2}+1\right)$$

```
[Out] -((3/2)^(-1 - m)*Sqrt[-2 - 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (3*x)/2
])
```

Rubi [A] time = 0.0046895, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {65}

$$-\left(\frac{3}{2}\right)^{-m-1}\sqrt{-3x-2}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{3x}{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[-2 - 3*x],x]
```

```
[Out] -((3/2)^(-1 - m)*Sqrt[-2 - 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (3*x)/2
])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx=-\left(\frac{3}{2}\right)^{-1-m}\sqrt{-2-3x}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)$$

Mathematica [A] time = 0.0143071, size = 57, normalized size = 1.54

$$-\frac{2}{3}\sqrt{-3x-2}\left(\frac{1}{2}(-3x-2)+1\right)^{-m}x^{-m}\left(-x^2\right)^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{3x}{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[-2 - 3*x],x]
```

```
[Out] (-2*Sqrt[-2 - 3*x]*(-x^2)^m*Hypergeometric2F1[1/2, -m, 3/2, 1 + (3*x)/2])/(
3*(1 + (-2 - 3*x)/2)^m*x^m)
```

Maple [C] time = 0.017, size = 31, normalized size = 0.8

$$\frac{-\frac{i}{2}\sqrt{2}\,(-x)^{m}\,x}{1+m}\,{}_2\mathrm{F}_1\left(\frac{1}{2},1+m;\,2+m;\,-\frac{3x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(-2-3*x)^(1/2),x)
```

```
[Out] -1/2*I*2^(1/2)*(-x)^m/(1+m)*x*hypergeom([1/2,1+m],[2+m],-3/2*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x)^m}{\sqrt{-3x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2-3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x)^m/sqrt(-3*x - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(-x)^m\sqrt{-3x-2}}{3x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2-3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(-x)^m*sqrt(-3*x - 2)/(3*x + 2), x)
```

Sympy [C] time = 1.05786, size = 48, normalized size = 1.3

$$-\frac{2\cdot 2^{m}\sqrt{3}\cdot 3^{-m}i\sqrt{x+\frac{2}{3}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|\frac{3\left(x+\frac{2}{3}\right)e^{2i\pi}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(-2-3*x)**(1/2),x)
```

```
[Out] -2*2**m*sqrt(3)*3**(-m)*I*sqrt(x + 2/3)*hyper((1/2, -m), (3/2,), 3*(x + 2/3
)*exp_polar(2*I*pi)/2)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x)^m}{\sqrt{-3x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2-3*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-x)^m/sqrt(-3*x - 2), x)
```

3.728 $\int \frac{x^n}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=26

$$-2\sqrt{1-x}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)$$

```
[Out] -2*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x]
```

Rubi [A] time = 0.0042752, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {65}

$$-2\sqrt{1-x}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^n/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{x^n}{\sqrt{1-x}}\,dx = -2\sqrt{1-x}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)$$

Mathematica [A] time = 0.0045975, size = 26, normalized size = 1.

$$-2\sqrt{1-x}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x]
```

Maple [A] time = 0.03, size = 23, normalized size = 0.9

$$\frac{x^{1+n}}{1+n}{}_2\mathrm{F}_1(\frac{1}{2},1+n;\,2+n;\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/(1-x)^(1/2),x)
```

```
[Out] 1/(1+n)*x^(1+n)*hypergeom([1/2,1+n],[2+n],x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^n}{\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^n/sqrt(-x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^n\sqrt{-x+1}}{x-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-x^n*sqrt(-x + 1)/(x - 1), x)
```

Sympy [C] time = 0.929018, size = 26, normalized size = 1.

$$-2i\sqrt{x-1}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-n\\ \frac{3}{2}\end{matrix}\middle|(x-1)\,e^{i\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n/(1-x)**(1/2),x)
```

```
[Out] -2*I*sqrt(x - 1)*hyper((1/2, -n), (3/2,), (x - 1)*exp_polar(I*pi))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^n}{\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^n/sqrt(-x + 1), x)
```

3.729 $\int \frac{x^n}{\sqrt{a-ax}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\sqrt{a-ax}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)}{a}$$

```
[Out] (-2*Sqrt[a - a*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x])/a
```

Rubi [A] time = 0.0044462, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {65}

$$-\frac{2\sqrt{a-ax}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^n/Sqrt[a - a*x],x]
```

```
[Out] (-2*Sqrt[a - a*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x])/a
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{x^n}{\sqrt{a-ax}}\,dx = -\frac{2\sqrt{a-ax}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)}{a}$$

Mathematica [A] time = 0.0073741, size = 30, normalized size = 1.

$$-\frac{2\sqrt{a-ax}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n/Sqrt[a - a*x],x]
```

```
[Out] (-2*Sqrt[a - a*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x])/a
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x^n \frac{1}{\sqrt{-ax+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/(-a*x+a)^(1/2),x)
```

```
[Out] int(x^n/(-a*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^n}{\sqrt{-ax+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(-a*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^n/sqrt(-a*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-ax+a}\,x^n}{ax-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(-a*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*x + a)*x^n/(a*x - a), x)
```

Sympy [C] time = 0.995525, size = 31, normalized size = 1.03

$$-\frac{2i\sqrt{x-1}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-n\\ \frac{3}{2}\end{matrix}\middle|(x-1)\,e^{i\pi}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n/(-a*x+a)**(1/2),x)
```

```
[Out] -2*I*sqrt(x - 1)*hyper((1/2, -n), (3/2,), (x - 1)*exp_polar(I*pi))/sqrt(a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^n}{\sqrt{-ax+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(-a*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^n/sqrt(-a*x + a), x)
```

3.730 $\int x^m (a+bx)^n \, dx$

Optimal. Leaf size=47

$$\frac{x^{m+1}(a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{bx}{a}\right)}{m+1}$$

```
[Out] (x^(1 + m)*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])/((1
 + m)*(1 + (b*x)/a)^n)
```

Rubi [A] time = 0.0111344, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {66, 64}

$$\frac{x^{m+1}(a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{bx}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^n,x]
```

```
[Out] (x^(1 + m)*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])/((1
 + m)*(1 + (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int x^m (a+bx)^n \, dx &= \left((a+bx)^n \left(1+\frac{bx}{a}\right)^{-n}\right) \int x^m \left(1+\frac{bx}{a}\right)^n \, dx\\ &= \frac{x^{1+m}(a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.0096855, size = 47, normalized size = 1.

$$\frac{x^{m+1}(a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{bx}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^n,x]
```

```
[Out] (x^(1 + m)*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])/((1
 + m)*(1 + (b*x)/a)^n)
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int x^{m}\left(bx+a\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^n,x)
```

```
[Out] int(x^m*(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{n} x^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx+a)^{n} x^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n*x^m, x)
```

Sympy [C] time = 2.69871, size = 34, normalized size = 0.72

$$\frac{a^{n} x x^{m} \Gamma\left(m+1\right) {}_{2}F_{1}\left(\left.\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\right| \frac{bxe^{i\pi}}{a}\right)}{\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**n,x)
```

```
[Out] a**n*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a
)/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^m, x)
```

3.731 $\int (cx)^m(a+bx)^n\,dx$

Optimal. Leaf size=52

$$\frac{(cx)^{m+1}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(m+1,-n;m+2;-\frac{bx}{a}\right)}{c(m+1)}$$

```
[Out] ((c*x)^(1 + m)*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])
/(c*(1 + m)*(1 + (b*x)/a)^n)
```

Rubi [A] time = 0.0135314, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {66, 64}

$$\frac{(cx)^{m+1}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(m+1,-n;m+2;-\frac{bx}{a}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x)^n,x]
```

```
[Out] ((c*x)^(1 + m)*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])
/(c*(1 + m)*(1 + (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int (cx)^m(a+bx)^n\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (cx)^m\left(1+\frac{bx}{a}\right)^n\,dx\\ &= \frac{(cx)^{1+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{c(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0065342, size = 48, normalized size = 0.92

$$\frac{x(cx)^m(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(m+1,-n;m+2;-\frac{bx}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x)^n,x]
```

```
[Out] (x*(c*x)^m*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])/((1
 + m)*(1 + (b*x)/a)^n)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int (cx)^m (bx+a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x+a)^n,x)
```

```
[Out] int((c*x)^m*(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(c*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^n (cx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n*(c*x)^m, x)
```

Sympy [C] time = 2.18966, size = 37, normalized size = 0.71

$$\frac{a^n c^m x x^m \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x+a)**n,x)
```

```
[Out] a**n*c**m*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), b*x*exp_polar(I*
pi)/a)/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n (cx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(c*x)^m, x)
```

3.732 $\int x^3(a+bx)^n\,dx$

Optimal. Leaf size=83

$$-\frac{a^3(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2(a+bx)^{n+2}}{b^4(n+2)}-\frac{3a(a+bx)^{n+3}}{b^4(n+3)}+\frac{(a+bx)^{n+4}}{b^4(n+4)}$$

```
[Out] -((a^3*(a + b*x)^(1 + n))/(b^4*(1 + n))) + (3*a^2*(a + b*x)^(2 + n))/(b^4*(
2 + n)) - (3*a*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (a + b*x)^(4 + n)/(b^4*(4
 + n))
```

Rubi [A] time = 0.031316, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^3(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2(a+bx)^{n+2}}{b^4(n+2)}-\frac{3a(a+bx)^{n+3}}{b^4(n+3)}+\frac{(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^n,x]
```

```
[Out] -((a^3*(a + b*x)^(1 + n))/(b^4*(1 + n))) + (3*a^2*(a + b*x)^(2 + n))/(b^4*(
2 + n)) - (3*a*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (a + b*x)^(4 + n)/(b^4*(4
 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^n\,dx&=\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx\\&=-\frac{a^3(a+bx)^{1+n}}{b^4(1+n)}+\frac{3a^2(a+bx)^{2+n}}{b^4(2+n)}-\frac{3a(a+bx)^{3+n}}{b^4(3+n)}+\frac{(a+bx)^{4+n}}{b^4(4+n)}\end{aligned}$$

Mathematica [A] time = 0.0496236, size = 67, normalized size = 0.81

$$\frac{(a+bx)^{n+1}\left(\frac{3a^2(a+bx)}{n+2}-\frac{a^3}{n+1}-\frac{3a(a+bx)^2}{n+3}+\frac{(a+bx)^3}{n+4}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-(a^3/(1 + n)) + (3*a^2*(a + b*x))/(2 + n) - (3*a*(a +
b*x)^2)/(3 + n) + (a + b*x)^3/(4 + n)))/b^4
```

Maple [A] time = 0.005, size = 126, normalized size = 1.5

$$-\frac{(bx+a)^{1+n}\left(-b^3n^3x^3-6\,b^3n^2x^3+3\,ab^2n^2x^2-11\,b^3nx^3+9\,ab^2nx^2-6\,b^3x^3-6\,a^2bnx+6\,ab^2x^2-6\,a^2bx+6\,a^3\right)}{b^4\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3+9*a
*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/b^4/(n^4+10*n
^3+35*n^2+50*n+24)
```

Maxima [A] time = 1.04599, size = 136, normalized size = 1.64

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4)
```

Fricas [A] time = 1.54057, size = 292, normalized size = 3.52

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)(bx+a)^n}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*(b*
x + a)^n/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Sympy [A] time = 1.80595, size = 1319, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a
**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 2*a**3/(6*a**3*b**4 + 18*a**2*
```

```
b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log(a
/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 9*a
*b**2*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
6*b**3*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6
*b**7*x**3) - 9*b**3*x**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
6*b**7*x**3), Eq(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) - 3*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b
*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*lo
g(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 6*a*b**2*x**2/(2*a**2
*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2), Eq(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a
**3/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) -
 3*a*b**2*x**2/(2*a*b**4 + 2*b**5*x) + b**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(
n, -2)), (-a**3*log(a/b + x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3
*b), Eq(n, -1)), (-6*a**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**
4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n**2*x**2*(a + b
*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*
a**2*b**2*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50
*b**4*n + 24*b**4) + a*b**3*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**
3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*n**2*x**3*(a + b*x)**n/(
b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*n
*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 2
4*b**4) + b**4*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n
**2 + 50*b**4*n + 24*b**4) + 6*b**4*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*
b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*n*x**4*(a + b*x)*
*n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4
*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 2
4*b**4), True))
```

Giac [B] time = 1.05998, size = 305, normalized size = 3.67

$$\frac{(bx+a)^n b^4 n^3 x^4 + (bx+a)^n a b^3 n^3 x^3 + 6\,(bx+a)^n b^4 n^2 x^4 + 3\,(bx+a)^n a b^3 n^2 x^3 + 11\,(bx+a)^n b^4 n x^4 - 3\,(bx+a)^n a^2 b^2 n}{b^4 n^4 + 10\,b^4 n^3 + 35\,b^4 n^2 + 50}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*n^3*x^4 + (b*x + a)^n*a*b^3*n^3*x^3 + 6*(b*x + a)^n*b^4*n^
2*x^4 + 3*(b*x + a)^n*a*b^3*n^2*x^3 + 11*(b*x + a)^n*b^4*n*x^4 - 3*(b*x + a
)^n*a^2*b^2*n^2*x^2 + 2*(b*x + a)^n*a*b^3*n*x^3 + 6*(b*x + a)^n*b^4*x^4 - 3
*(b*x + a)^n*a^2*b^2*n*x^2 + 6*(b*x + a)^n*a^3*b*n*x - 6*(b*x + a)^n*a^4)/(
b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

3.733 $\int x^2(a+bx)^n\,dx$

Optimal. Leaf size=60

$$\frac{a^2(a+bx)^{n+1}}{b^3(n+1)}-\frac{2a(a+bx)^{n+2}}{b^3(n+2)}+\frac{(a+bx)^{n+3}}{b^3(n+3)}$$

```
[Out] (a^2*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*(a + b*x)^(2 + n))/(b^3*(2 + n
)) + (a + b*x)^(3 + n)/(b^3*(3 + n))
```

Rubi [A] time = 0.0189535, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{n+1}}{b^3(n+1)}-\frac{2a(a+bx)^{n+2}}{b^3(n+2)}+\frac{(a+bx)^{n+3}}{b^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n,x]
```

```
[Out] (a^2*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*(a + b*x)^(2 + n))/(b^3*(2 + n
)) + (a + b*x)^(3 + n)/(b^3*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\,dx &= \int\left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx\\ &= \frac{a^2(a+bx)^{1+n}}{b^3(1+n)}-\frac{2a(a+bx)^{2+n}}{b^3(2+n)}+\frac{(a+bx)^{3+n}}{b^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.0252241, size = 57, normalized size = 0.95

$$\frac{(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(b^
3*(1 + n)*(2 + n)*(3 + n))
```

Maple [A] time = 0.004, size = 73, normalized size = 1.2

$$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3\,b^2nx^2-2\,abnx+2\,b^2x^2-2\,abx+2\,a^2\right)}{b^3\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n,x)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)/b
^3/(n^3+6*n^2+11*n+6)
```

Maxima [A] time = 1.10318, size = 92, normalized size = 1.53

$$\frac{\left(\left(n^2+3\,n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [A] time = 1.62979, size = 188, normalized size = 3.13

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)(bx+a)^n}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*(b*x + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

Sympy [A] time = 1.05871, size = 597, normalized size = 9.95

$$\begin{cases} \frac{a^nx^3}{3} \\ \frac{2a^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{a^2}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{2b^2x^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}-\frac{2b^2x^2}{2a^2b^3+4ab^4x+2b^5x^2} \\ -\frac{2a^2\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}-\frac{2a^2}{ab^3+b^4x}-\frac{2abx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}+\frac{b^2x^2}{ab^3+b^4x} \\ \frac{a^2\log\left(\frac{a}{b}+x\right)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b} \\ \frac{2a^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}-\frac{2a^2bnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{ab^2n^2x^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{ab^2nx^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{b^3n^2x^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{3b^3\cdots}{b^3n^3+6b\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a*
b**4*x + 2*b**5*x**2) + a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a
*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*lo
g(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - 2*b**2*x**2/(2*a**2*b
**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)/(a*b**3
+ b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b**4*
x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)/b**3 - a*x
/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n*
*2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n*
*2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**
3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*n**3 + 6*b*
*3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n/(b**3*n**3 + 6*
b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)**n/(b**3*n**3 + 6
*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)**n/(b**3*n**3 + 6*
b**3*n**2 + 11*b**3*n + 6*b**3), True))
```

Giac [B] time = 1.06967, size = 189, normalized size = 3.15

$$\frac{(bx+a)^n b^3 n^2 x^3 + (bx+a)^n a b^2 n^2 x^2 + 3\,(bx+a)^n b^3 n x^3 + (bx+a)^n a b^2 n x^2 + 2\,(bx+a)^n b^3 x^3 - 2\,(bx+a)^n a^2 b n x + 2\,(bx+}{b^3 n^3 + 6\,b^3 n^2 + 11\,b^3 n + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^3*n^2*x^3 + (b*x + a)^n*a*b^2*n^2*x^2 + 3*(b*x + a)^n*b^3*n*
x^3 + (b*x + a)^n*a*b^2*n*x^2 + 2*(b*x + a)^n*b^3*x^3 - 2*(b*x + a)^n*a^2*b
*n*x + 2*(b*x + a)^n*a^3)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

3.734 $\int x(a+bx)^n\,dx$

Optimal. Leaf size=39

$$\frac{(a+bx)^{n+2}}{b^2(n+2)}-\frac{a(a+bx)^{n+1}}{b^2(n+1)}$$

```
[Out] -((a*(a + b*x)^(1 + n))/(b^2*(1 + n))) + (a + b*x)^(2 + n)/(b^2*(2 + n))
```

Rubi [A] time = 0.0115666, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^{n+2}}{b^2(n+2)}-\frac{a(a+bx)^{n+1}}{b^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n,x]
```

```
[Out] -((a*(a + b*x)^(1 + n))/(b^2*(1 + n))) + (a + b*x)^(2 + n)/(b^2*(2 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n\,dx &= \int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx\\ &= -\frac{a(a+bx)^{1+n}}{b^2(1+n)}+\frac{(a+bx)^{2+n}}{b^2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.0161724, size = 33, normalized size = 0.85

$$\frac{(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n))
```

Maple [A] time = 0.003, size = 36, normalized size = 0.9

$$-\frac{(bx+a)^{1+n}(-xnb-bx+a)}{b^2(n^2+3n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(1+n)*(-b*n*x-b*x+a)/b^2/(n^2+3*n+2)
```

Maxima [A] time = 1.09751, size = 57, normalized size = 1.46

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2)
```

Fricas [A] time = 1.66871, size = 104, normalized size = 2.67

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)(bx+a)^n}{b^2n^2+3\,b^2n+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*(b*x + a)^n/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

Sympy [A] time = 0.566216, size = 201, normalized size = 5.15

$$\begin{cases} \frac{a^nx^2}{2} & \text{for}\: b=0 \\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x}+\frac{a}{ab^2+b^3x}+\frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for}\: n=-2 \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2}+\frac{x}{b} & \text{for}\: n=-1 \\ -\frac{a^2(a+bx)^n}{b^2n^2+3b^2n+2b^2}+\frac{abnx(a+bx)^n}{b^2n^2+3b^2n+2b^2}+\frac{b^2nx^2(a+bx)^n}{b^2n^2+3b^2n+2b^2}+\frac{b^2x^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(a
*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(n, -2)), (-a*log(a
/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n +
 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*n*x*
*2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(a + b*x)**n/(b
**2*n**2 + 3*b**2*n + 2*b**2), True))
```

Giac [A] time = 1.06191, size = 103, normalized size = 2.64

$$\frac{(bx+a)^n b^2 n x^2 + (bx+a)^n abnx + (bx+a)^n b^2 x^2 - (bx+a)^n a^2}{b^2 n^2 + 3\,b^2 n + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^2*n*x^2 + (b*x + a)^n*a*b*n*x + (b*x + a)^n*b^2*x^2 - (b*x +
 a)^n*a^2)/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

3.735 $\int (a+bx)^n\,dx$

Optimal. Leaf size=18

$$\frac{(a+bx)^{n+1}}{b(n+1)}$$

```
[Out] (a + b*x)^(1 + n)/(b*(1 + n))
```

Rubi [A] time = 0.003058, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n,x]
```

```
[Out] (a + b*x)^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^n\,dx=\frac{(a+bx)^{1+n}}{b(1+n)}$$

Mathematica [A] time = 0.0085913, size = 17, normalized size = 0.94

$$\frac{(a+bx)^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n,x]
```

```
[Out] (a + b*x)^(1 + n)/(b + b*n)
```

Maple [A] time = 0.001, size = 19, normalized size = 1.1

$$\frac{(bx+a)^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n,x)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53451, size = 45, normalized size = 2.5

$$\frac{(bx+a)(bx+a)^n}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(b*x + a)^n/(b*n + b)
```

Sympy [A] time = 0.060148, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(a+bx)^{n+1}}{n+1} & \text{for}\ n \neq -1 \\ \log(a+bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n,x)
```

```
[Out] Piecewise(((a + b*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(a + b*x), True))/b
```

Giac [A] time = 1.07766, size = 24, normalized size = 1.33

$$\frac{(bx+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="giac")
```

```
[Out] (b*x + a)^(n + 1)/(b*(n + 1))
```

3.736 $\int \frac{(a+bx)^n}{x} dx$

Optimal. Leaf size=35

$$-\frac{(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[Out] -(((a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1
 + n)))
```

Rubi [A] time = 0.0062213, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {65}

$$-\frac{(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x,x]
```

```
[Out] -(((a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1
 + n)))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n}{x} dx = -\frac{(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

Mathematica [A] time = 0.0107766, size = 35, normalized size = 1.

$$-\frac{(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x,x]
```

```
[Out] -(((a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1
 + n)))
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x,x)
```

```
[Out] int((b*x+a)^n/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/x, x)
```

Sympy [B] time = 1.42799, size = 83, normalized size = 2.37

$$-\frac{bb^n n\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n \Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma(n+2)}-\frac{bb^n\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n \Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x,x)
```

```
[Out] -b*b**n*n*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n
+ 1)/(a*gamma(n + 2)) - b*b**n*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/
a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/x, x)
```

3.737 $\int \frac{(a+bx)^n}{x^2} dx$

Optimal. Leaf size=35

$$\frac{b(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)}$$

```
[Out] (b*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^2*
(1 + n))
```

Rubi [A] time = 0.0068788, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {65}

$$\frac{b(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x^2,x]
```

```
[Out] (b*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^2*
(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n}{x^2}\,dx = \frac{b(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)}$$

Mathematica [A] time = 0.0057215, size = 35, normalized size = 1.

$$\frac{b(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x^2,x]
```

```
[Out] (b*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^2*
(1 + n))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^2,x)
```

```
[Out] int((b*x+a)^n/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^n}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/x^2, x)
```

Sympy [B] time = 1.90122, size = 354, normalized size = 10.11

$$\frac{ab^2b^nn^2\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{-a^3\Gamma(n+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(n+2)}+\frac{ab^2b^nn\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{-a^3\Gamma(n+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(n+2)}-\frac{ab^2b^nn}{-a^3\Gamma(n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**2,x)
```

```
[Out] a*b**2*b**n*n**2*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*g
amma(n + 1)/(-a**3*gamma(n + 2) + a**2*b*(a/b + x)*gamma(n + 2)) + a*b**2*b
**n*n*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)
/(-a**3*gamma(n + 2) + a**2*b*(a/b + x)*gamma(n + 2)) - a*b**2*b**n*n*(a/b
+ x)*(a/b + x)**n*gamma(n + 1)/(-a**3*gamma(n + 2) + a**2*b*(a/b + x)*gamma
(n + 2)) - a*b**2*b**n*(a/b + x)*(a/b + x)**n*gamma(n + 1)/(-a**3*gamma(n +
 2) + a**2*b*(a/b + x)*gamma(n + 2)) - b**3*b**n*n**2*(a/b + x)**2*(a/b + x
)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(-a**3*gamma(n + 2) + a
```

```
**2*b*(a/b + x)*gamma(n + 2)) - b**3*b**n*n*(a/b + x)**2*(a/b + x)**n*lerch
phi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(-a**3*gamma(n + 2) + a**2*b*(a/b
 + x)*gamma(n + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/x^2, x)
```

3.738 $\int \frac{(a+bx)^n}{x^3} dx$

Optimal. Leaf size=38

$$-\frac{b^2(a+bx)^{n+1} {}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)}$$

```
[Out] -((b^2*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^3*(1 + n)))
```

Rubi [A] time = 0.0076062, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {65}

$$-\frac{b^2(a+bx)^{n+1} {}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x^3,x]
```

```
[Out] -((b^2*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^3*(1 + n)))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n}{x^3} dx = -\frac{b^2(a+bx)^{1+n} {}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)}$$

Mathematica [A] time = 0.0059817, size = 38, normalized size = 1.

$$-\frac{b^2(a+bx)^{n+1} {}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x^3,x]
```

```
[Out] -((b^2*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^3*(1 + n)))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^3,x)
```

```
[Out] int((b*x+a)^n/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(bx+a)^n}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/x^3, x)
```

Sympy [B] time = 2.50652, size = 918, normalized size = 24.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**3,x)
```

```
[Out] -a**2*b**3*b**n*n**3*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n +
1)*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*
a**3*b**2*(a/b + x)**2*gamma(n + 2)) + a**2*b**3*b**n*n**2*(a/b + x)*(a/b +
 x)**n*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2)
+ 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)) + a**2*b**3*b**n*n*(a/b + x)*(a/b
+ x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(2*a**5*gamma(n + 2)
 - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2))
 - a**2*b**3*b**n*n*(a/b + x)*(a/b + x)**n*gamma(n + 1)/(2*a**5*gamma(n + 2
) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)
) - 2*a**2*b**3*b**n*(a/b + x)*(a/b + x)**n*gamma(n + 1)/(2*a**5*gamma(n +
2) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2
```

```
)) + 2*a*b**4*b**n*n**3*(a/b + x)**2*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1
, n + 1)*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2
) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)) - a*b**4*b**n*n**2*(a/b + x)**2*
(a/b + x)**n*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n
 + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)) - 2*a*b**4*b**n*n*(a/b + x)*
*2*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(2*a**5*gamm
a(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma
(n + 2)) + a*b**4*b**n*(a/b + x)**2*(a/b + x)**n*gamma(n + 1)/(2*a**5*gamma
(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(
n + 2)) - b**5*b**n*n**3*(a/b + x)**3*(a/b + x)**n*lerchphi(b*(a/b + x)/a,
1, n + 1)*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n +
2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)) + b**5*b**n*n*(a/b + x)**3*(a/b
 + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(2*a**5*gamma(n + 2
) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/x^3, x)
```

3.739 $\int x^{-4+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=110

$$-\frac{2b^2x^{n-1}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}+\frac{2bx^{n-2}(a+bx)^{1-n}}{a^2(2-n)(3-n)}-\frac{x^{n-3}(a+bx)^{1-n}}{a(3-n)}$$

```
[Out] -((x^(-3 + n)*(a + b*x)^(1 - n))/(a*(3 - n))) + (2*b*x^(-2 + n)*(a + b*x)^(
1 - n))/(a^2*(2 - n)*(3 - n)) - (2*b^2*x^(-1 + n)*(a + b*x)^(1 - n))/(a^3*(
1 - n)*(2 - n)*(3 - n))
```

Rubi [A] time = 0.0356771, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2x^{n-1}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}+\frac{2bx^{n-2}(a+bx)^{1-n}}{a^2(2-n)(3-n)}-\frac{x^{n-3}(a+bx)^{1-n}}{a(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 + n)/(a + b*x)^n,x]
```

```
[Out] -((x^(-3 + n)*(a + b*x)^(1 - n))/(a*(3 - n))) + (2*b*x^(-2 + n)*(a + b*x)^(
1 - n))/(a^2*(2 - n)*(3 - n)) - (2*b^2*x^(-1 + n)*(a + b*x)^(1 - n))/(a^3*(
1 - n)*(2 - n)*(3 - n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^{-4+n}(a+bx)^{-n}\,dx &= -\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)}-\frac{(2b)\int x^{-3+n}(a+bx)^{-n}\,dx}{a(3-n)}\\ &= -\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)}+\frac{2bx^{-2+n}(a+bx)^{1-n}}{a^2(2-n)(3-n)}+\frac{\left(2b^2\right)\int x^{-2+n}(a+bx)^{-n}\,dx}{a^2(2-n)(3-n)}\\ &= -\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)}+\frac{2bx^{-2+n}(a+bx)^{1-n}}{a^2(2-n)(3-n)}-\frac{2b^2x^{-1+n}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}\end{aligned}$$

Mathematica [A] time = 0.025607, size = 64, normalized size = 0.58

$$\frac{x^{n-3}(a+bx)^{1-n}\left(a^2\left(n^2-3n+2\right)+2ab(n-1)x+2b^2x^2\right)}{a^3(n-3)(n-2)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 + n)/(a + b*x)^n,x]
```

```
[Out] (x^(-3 + n)*(a + b*x)^(1 - n)*(a^2*(2 - 3*n + n^2) + 2*a*b*(-1 + n)*x + 2*b
^2*x^2))/(a^3*(-3 + n)*(-2 + n)*(-1 + n))
```

Maple [A] time = 0.007, size = 77, normalized size = 0.7

$$\frac{(bx+a)\,x^{-3+n}\left(a^2n^2+2\,abnx+2\,b^2x^2-3\,a^2n-2\,abx+2\,a^2\right)}{(bx+a)^n\,(-3+n)\,(-2+n)\,(-1+n)\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4+n)/((b*x+a)^n),x)
```

```
[Out] (b*x+a)*x^(-3+n)*(a^2*n^2+2*a*b*n*x+2*b^2*x^2-3*a^2*n-2*a*b*x+2*a^2)/((b*x+
a)^n)/(-3+n)/(-2+n)/(-1+n)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{n-4}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 4)/(b*x + a)^n, x)
```

Fricas [A] time = 1.62168, size = 208, normalized size = 1.89

$$\frac{\left(2\,ab^2nx^3+2\,b^3x^4+\left(a^2bn^2-a^2bn\right)x^2+\left(a^3n^2-3\,a^3n+2\,a^3\right)x\right)x^{n-4}}{\left(a^3n^3-6\,a^3n^2+11\,a^3n-6\,a^3\right)(bx+a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] (2*a*b^2*n*x^3 + 2*b^3*x^4 + (a^2*b*n^2 - a^2*b*n)*x^2 + (a^3*n^2 - 3*a^3*n
 + 2*a^3)*x)*x^(n - 4)/((a^3*n^3 - 6*a^3*n^2 + 11*a^3*n - 6*a^3)*(b*x + a)^
n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4+n)/((b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{n-4}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 4)/(b*x + a)^n, x)
```

3.740 $\int x^{-3+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=64

$$\frac{bx^{n-1}(a+bx)^{1-n}}{a^2(1-n)(2-n)}-\frac{x^{n-2}(a+bx)^{1-n}}{a(2-n)}$$

```
[Out] -((x^(-2 + n)*(a + b*x)^(1 - n))/(a*(2 - n))) + (b*x^(-1 + n)*(a + b*x)^(1
- n))/(a^2*(1 - n)*(2 - n))
```

Rubi [A] time = 0.0091874, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{bx^{n-1}(a+bx)^{1-n}}{a^2(1-n)(2-n)}-\frac{x^{n-2}(a+bx)^{1-n}}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + n)/(a + b*x)^n,x]
```

```
[Out] -((x^(-2 + n)*(a + b*x)^(1 - n))/(a*(2 - n))) + (b*x^(-1 + n)*(a + b*x)^(1
- n))/(a^2*(1 - n)*(2 - n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^{-3+n}(a+bx)^{-n}\,dx&=-\frac{x^{-2+n}(a+bx)^{1-n}}{a(2-n)}-\frac{b\int x^{-2+n}(a+bx)^{-n}\,dx}{a(2-n)}\\&=-\frac{x^{-2+n}(a+bx)^{1-n}}{a(2-n)}+\frac{bx^{-1+n}(a+bx)^{1-n}}{a^2(1-n)(2-n)}\end{aligned}$$

Mathematica [A] time = 0.0155736, size = 39, normalized size = 0.61

$$\frac{x^{n-2}(a+bx)^{1-n}(a(n-1)+bx)}{a^2(n-2)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + n)/(a + b*x)^n,x]
```

```
[Out] (x^(-2 + n)*(a + b*x)^(1 - n)*(a*(-1 + n) + b*x))/(a^2*(-2 + n)*(-1 + n))
```

Maple [A] time = 0.004, size = 44, normalized size = 0.7

$$\frac{x^{-2+n}\,(an+bx-a)\,(bx+a)}{(bx+a)^{n}\,(-2+n)\,(-1+n)\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+n)/((b*x+a)^n),x)
```

```
[Out] x^(-2+n)*(a*n+b*x-a)*(b*x+a)/((b*x+a)^n)/(-2+n)/(-1+n)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{n-3}}{(bx+a)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 3)/(b*x + a)^n, x)
```

Fricas [A] time = 1.64152, size = 126, normalized size = 1.97

$$\frac{\left(abnx^{2}+b^{2}x^{3}+\left(a^{2}n-a^{2}\right)x\right)x^{n-3}}{\left(a^{2}n^{2}-3\,a^{2}n+2\,a^{2}\right)(bx+a)^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] (a*b*n*x^2 + b^2*x^3 + (a^2*n - a^2)*x)*x^(n - 3)/((a^2*n^2 - 3*a^2*n + 2*a
^2)*(b*x + a)^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+n)/((b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{n-3}}{(bx+a)^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 3)/(b*x + a)^n, x)
```

3.741 $\int x^{-2+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=28

$$-\frac{x^{n-1}(a+bx)^{1-n}}{a(1-n)}$$

```
[Out] -((x^(-1 + n)*(a + b*x)^(1 - n))/(a*(1 - n)))
```

Rubi [A] time = 0.0027961, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{x^{n-1}(a+bx)^{1-n}}{a(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + n)/(a + b*x)^n,x]
```

```
[Out] -((x^(-1 + n)*(a + b*x)^(1 - n))/(a*(1 - n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int x^{-2+n}(a+bx)^{-n}\,dx=-\frac{x^{-1+n}(a+bx)^{1-n}}{a(1-n)}$$

Mathematica [A] time = 0.0063209, size = 25, normalized size = 0.89

$$\frac{x^{n-1}(a+bx)^{1-n}}{a(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + n)/(a + b*x)^n,x]
```

```
[Out] (x^(-1 + n)*(a + b*x)^(1 - n))/(a*(-1 + n))
```

Maple [A] time = 0.003, size = 29, normalized size = 1.

$$\frac{x^{-1+n}\,(bx+a)}{a\,(-1+n)\,(bx+a)^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+n)/((b*x+a)^n),x)
```

```
[Out] x^(-1+n)*(b*x+a)/a/(-1+n)/((b*x+a)^n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{n-2}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 2)/(b*x + a)^n, x)
```

Fricas [A] time = 1.6588, size = 66, normalized size = 2.36

$$\frac{\left(bx^2+ax\right)x^{n-2}}{(an-a)(bx+a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*x^(n - 2)/((a*n - a)*(b*x + a)^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+n)/((b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{n-2}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 2)/(b*x + a)^n, x)
```

3.742 $\int x^{-1+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=39

$$\frac{x^n(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^n{}_2F_1\left(n,n;n+1;-\frac{bx}{a}\right)}{n}$$

```
[Out] (x^n*(1 + (b*x)/a)^n*Hypergeometric2F1[n, n, 1 + n, -((b*x)/a)])/(n*(a + b*
x)^n)
```

Rubi [A] time = 0.0097928, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {66, 64}

$$\frac{x^n(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^n{}_2F_1\left(n,n;n+1;-\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a + b*x)^n,x]
```

```
[Out] (x^n*(1 + (b*x)/a)^n*Hypergeometric2F1[n, n, 1 + n, -((b*x)/a)])/(n*(a + b*
x)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] &&  !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}(a+bx)^{-n}\,dx &= \left((a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n\right)\int x^{-1+n}\left(1+\frac{bx}{a}\right)^{-n}dx\\ &= \frac{x^n(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n{}_2F_1\left(n,n;1+n;-\frac{bx}{a}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.0071447, size = 39, normalized size = 1.

$$\frac{x^n(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^n{}_2F_1\left(n,n;n+1;-\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a + b*x)^n,x]
```

```
[Out] (x^n*(1 + (b*x)/a)^n*Hypergeometric2F1[n, n, 1 + n, -((b*x)/a)])/(n*(a + b*
x)^n)
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{x^{-1+n}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)/((b*x+a)^n),x)
```

```
[Out] int(x^(-1+n)/((b*x+a)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{n-1}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 1)/(b*x + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{n-1}}{(bx+a)^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral(x^(n - 1)/(b*x + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/((b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{n-1}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 1)/(b*x + a)^n, x)
```

3.743 $\int x^n(a+bx)^{-n}\,dx$

Optimal. Leaf size=45

$$\frac{x^{n+1}(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^n {}_2F_1\left(n,n+1;n+2;-\frac{bx}{a}\right)}{n+1}$$

```
[Out] (x^(1 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 1 + n, 2 + n, -((b*x)/a)])/
((1 + n)*(a + b*x)^n)
```

Rubi [A] time = 0.0102049, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {66, 64}

$$\frac{x^{n+1}(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^n {}_2F_1\left(n,n+1;n+2;-\frac{bx}{a}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n/(a + b*x)^n,x]
```

```
[Out] (x^(1 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 1 + n, 2 + n, -((b*x)/a)])/
((1 + n)*(a + b*x)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int x^n(a+bx)^{-n}\,dx &= \left((a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n\right)\int x^n\left(1+\frac{bx}{a}\right)^{-n}\,dx\\ &= \frac{x^{1+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{bx}{a}\right)}{1+n}\end{aligned}$$

Mathematica [A] time = 0.0070472, size = 45, normalized size = 1.

$$\frac{x^{n+1}(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^n {}_2F_1\left(n,n+1;n+2;-\frac{bx}{a}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n/(a + b*x)^n,x]
```

```
[Out] (x^(1 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 1 + n, 2 + n, -((b*x)/a)])/
((1 + n)*(a + b*x)^n)
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{x^n}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/((b*x+a)^n),x)
```

```
[Out] int(x^n/((b*x+a)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^n}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^n/(b*x + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^n}{(bx+a)^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral(x^n/(b*x + a)^n, x)
```

Sympy [C] time = 41.13, size = 32, normalized size = 0.71

$$\frac{a^{-n}xx^n\Gamma(n+1)\,{}_2F_1\left(\begin{matrix} n, n+1 \\ n+2 \end{matrix}\middle|\, \frac{bxe^{i\pi}}{a}\right)}{\Gamma(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n/((b*x+a)**n),x)
```

```
[Out] a**(-n)*x*x**n*gamma(n + 1)*hyper((n, n + 1), (n + 2,), b*x*exp_polar(I*pi)
/a)/gamma(n + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^n}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^n/(b*x + a)^n, x)
```

3.744 $\int x^{1+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=45

$$\frac{x^{n+2}(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^n {}_2F_1\left(n,n+2;n+3;-\frac{bx}{a}\right)}{n+2}$$

```
[Out] (x^(2 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 2 + n, 3 + n, -((b*x)/a)])/
((2 + n)*(a + b*x)^n)
```

Rubi [A] time = 0.0105547, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {66, 64}

$$\frac{x^{n+2}(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^n {}_2F_1\left(n,n+2;n+3;-\frac{bx}{a}\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + n)/(a + b*x)^n,x]
```

```
[Out] (x^(2 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 2 + n, 3 + n, -((b*x)/a)])/
((2 + n)*(a + b*x)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int x^{1+n}(a+bx)^{-n}\,dx &= \left((a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n\right)\int x^{1+n}\left(1+\frac{bx}{a}\right)^{-n}dx\\ &= \frac{x^{2+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,2+n;3+n;-\frac{bx}{a}\right)}{2+n}\end{aligned}$$

Mathematica [A] time = 0.0080227, size = 45, normalized size = 1.

$$\frac{x^{n+2}(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^n {}_2F_1\left(n,n+2;n+3;-\frac{bx}{a}\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + n)/(a + b*x)^n,x]
```

```
[Out] (x^(2 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 2 + n, 3 + n, -((b*x)/a)])/
((2 + n)*(a + b*x)^n)
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{x^{1+n}}{(bx+a)^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+n)/((b*x+a)^n),x)
```

```
[Out] int(x^(1+n)/((b*x+a)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{n+1}}{(bx+a)^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n + 1)/(b*x + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{n+1}}{(bx+a)^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+n)/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral(x^(n + 1)/(b*x + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+n)/((b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{n+1}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+n)/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n + 1)/(b*x + a)^n, x)
```

3.745 $\int x^{3/2}(a+bx)^n\,dx$

Optimal. Leaf size=45

$$\frac{2}{5}x^{5/2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{bx}{a}\right)$$

```
[Out] (2*x^(5/2)*(a + b*x)^n*Hypergeometric2F1[5/2, -n, 7/2, -((b*x)/a)])/(5*(1 +
 (b*x)/a)^n)
```

Rubi [A] time = 0.0087233, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {66, 64}

$$\frac{2}{5}x^{5/2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^n,x]
```

```
[Out] (2*x^(5/2)*(a + b*x)^n*Hypergeometric2F1[5/2, -n, 7/2, -((b*x)/a)])/(5*(1 +
 (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] &&  !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^n\,dx&=\left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int x^{3/2}\left(1+\frac{bx}{a}\right)^n dx\\&=\frac{2}{5}x^{5/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{bx}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0070115, size = 45, normalized size = 1.

$$\frac{2}{5}x^{5/2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^n,x]
```

```
[Out] (2*x^(5/2)*(a + b*x)^n*Hypergeometric2F1[5/2, -n, 7/2, -((b*x)/a)])/(5*(1 +
 (b*x)/a)^n)
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}} (bx+a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^n,x)
```

```
[Out] int(x^(3/2)*(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n x^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^n x^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n*x^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**n,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^(3/2), x)
```

3.746 $\int \sqrt{x}(a+bx)^n\,dx$

Optimal. Leaf size=45

$$\frac{2}{3}x^{3/2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{bx}{a}\right)$$

```
[Out] (2*x^(3/2)*(a + b*x)^n*Hypergeometric2F1[3/2, -n, 5/2, -((b*x)/a)])/(3*(1 +
 (b*x)/a)^n)
```

Rubi [A] time = 0.0087533, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {66, 64}

$$\frac{2}{3}x^{3/2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^n,x]
```

```
[Out] (2*x^(3/2)*(a + b*x)^n*Hypergeometric2F1[3/2, -n, 5/2, -((b*x)/a)])/(3*(1 +
 (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(a+bx)^n\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int \sqrt{x}\left(1+\frac{bx}{a}\right)^n\,dx\\ &= \frac{2}{3}x^{3/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{bx}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0074904, size = 45, normalized size = 1.

$$\frac{2}{3}x^{3/2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^n,x]
```

```
[Out] (2*x^(3/2)*(a + b*x)^n*Hypergeometric2F1[3/2, -n, 5/2, -((b*x)/a)])/(3*(1 +
 (b*x)/a)^n)
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \sqrt{x}\,(bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x+a)^n,x)
```

```
[Out] int(x^(1/2)*(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx+a)^n\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n*sqrt(x), x)
```

Sympy [C] time = 13.8212, size = 27, normalized size = 0.6

$$\frac{2a^n x^{\frac{3}{2}}\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-n\\ \frac{5}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x+a)**n,x)
```

```
[Out] 2*a**n*x**(3/2)*hyper((3/2, -n), (5/2,), b*x*exp_polar(I*pi)/a)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^n \sqrt{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*sqrt(x), x)
```

3.747 $\int \frac{(a+bx)^n}{\sqrt{x}}\,dx$

Optimal. Leaf size=43

$$2\sqrt{x}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{bx}{a}\right)$$

```
[Out] (2*Sqrt[x]*(a + b*x)^n*Hypergeometric2F1[1/2, -n, 3/2, -((b*x)/a)])/(1 + (b
*x)/a)^n
```

Rubi [A] time = 0.0086755, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {66, 64}

$$2\sqrt{x}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/Sqrt[x],x]
```


```
[Out] (2*Sqrt[x]*(a + b*x)^n*Hypergeometric2F1[1/2, -n, 3/2, -((b*x)/a)])/(1 + (b
*x)/a)^n
```


Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{\sqrt{x}}\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int \frac{\left(1+\frac{bx}{a}\right)^n}{\sqrt{x}}\,dx\\ &= 2\sqrt{x}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{bx}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0060289, size = 43, normalized size = 1.

$$2\sqrt{x}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(a + b*x)^n*Hypergeometric2F1[1/2, -n, 3/2, -((b*x)/a)])/(1 + (b
*x)/a)^n
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int (bx+a)^n \frac{1}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^(1/2),x)
```

```
[Out] int((b*x+a)^n/x^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^n}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/sqrt(x), x)
```

Sympy [C] time = 7.76647, size = 26, normalized size = 0.6

$$2a^n\sqrt{x}\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, -n \\ \frac{3}{2}\end{matrix}\middle|\,\frac{bxe^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**(1/2),x)
```

```
[Out] 2*a**n*sqrt(x)*hyper((1/2, -n), (3/2,), b*x*exp_polar(I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/sqrt(x), x)
```

3.748 $\int \frac{(a+bx)^n}{x^{3/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{2(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}}$$

```
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((b*x)/a)])/(Sqrt[x]*(1 +
 (b*x)/a)^n)
```

Rubi [A] time = 0.0086445, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {66, 64}

$$-\frac{2(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x^(3/2),x]
```

```
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((b*x)/a)])/(Sqrt[x]*(1 +
 (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{x^{3/2}}\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int \frac{\left(1+\frac{bx}{a}\right)^n}{x^{3/2}}\,dx\\ &= -\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.0072898, size = 43, normalized size = 1.

$$-\frac{2(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x^(3/2),x]
```

```
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((b*x)/a)])/(Sqrt[x]*(1 +
 (b*x)/a)^n)
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (bx+a)^n x^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^(3/2),x)
```

```
[Out] int((b*x+a)^n/x^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^n}{x^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/x^(3/2), x)
```

Sympy [C] time = 54.5904, size = 29, normalized size = 0.67

$$-\frac{2a^n {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -n \\ \frac{1}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**(3/2),x)
```

```
[Out] -2*a**n*hyper((-1/2, -n), (1/2,), b*x*exp_polar(I*pi)/a)/sqrt(x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/x^(3/2), x)
```

3.749 $\int \frac{(a+bx)^n}{x^{5/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}}$$

```
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((b*x)/a)])/(3*x^(3/2)*(
1 + (b*x)/a)^n)
```

Rubi [A] time = 0.0090092, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {66, 64}

$$-\frac{2(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((b*x)/a)])/(3*x^(3/2)*(
1 + (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{x^{5/2}}\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int \frac{\left(1+\frac{bx}{a}\right)^n}{x^{5/2}}\,dx\\ &= -\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0067415, size = 45, normalized size = 1.

$$-\frac{2(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((b*x)/a)])/(3*x^(3/2)*(
1 + (b*x)/a)^n)
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int (bx + a)^n x^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^(5/2),x)
```

```
[Out] int((b*x+a)^n/x^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^n}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/x^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx + a)^n}{x^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/x^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/x^(5/2), x)
```

3.750 $\int (bx)^m (2 + dx)^n \, dx$

Optimal. Leaf size=35

$$\frac{2^n (bx)^{m+1} \, {}_2F_1\left(m+1,-n;m+2;-\frac{dx}{2}\right)}{b(m+1)}$$

```
[Out] (2^n*(b*x)^(1 + m)*Hypergeometric2F1[1 + m, -n, 2 + m, -(d*x)/2])/(b*(1 + m
))
```

Rubi [A] time = 0.0108964, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {64}

$$\frac{2^n (bx)^{m+1} \, {}_2F_1\left(m+1,-n;m+2;-\frac{dx}{2}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^m*(2 + d*x)^n,x]
```

```
[Out] (2^n*(b*x)^(1 + m)*Hypergeometric2F1[1 + m, -n, 2 + m, -(d*x)/2])/(b*(1 + m
))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int (bx)^m (2+dx)^n \, dx = \frac{2^n (bx)^{1+m} \, {}_2F_1\left(1+m,-n;2+m;-\frac{dx}{2}\right)}{b(1+m)}$$

Mathematica [A] time = 0.0062759, size = 31, normalized size = 0.89

$$\frac{2^n x (bx)^m \, {}_2F_1\left(m+1,-n;m+2;-\frac{dx}{2}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^m*(2 + d*x)^n,x]
```

```
[Out] (2^n*x*(b*x)^m*Hypergeometric2F1[1 + m, -n, 2 + m, -(d*x)/2])/(1 + m)
```

Maple [A] time = 0.069, size = 32, normalized size = 0.9

$$\frac{2^{n} (bx)^{m} x}{1+m} {}_2\mathrm{F}_1(-n,1+m;\,2+m;\,-\frac{dx}{2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*(d*x+2)^n,x)
```

```
[Out] 2^n*(b*x)^m/(1+m)*x*hypergeom([-n,1+m],[2+m],-1/2*d*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx)^{m} (dx+2)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x)^m*(d*x + 2)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx)^{m} (dx+2)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+2)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x)^m*(d*x + 2)^n, x)
```

Sympy [C] time = 2.12556, size = 37, normalized size = 1.06

$$\frac{2^{n} b^{m} x x^{m} \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\middle| \frac{dxe^{i\pi}}{2}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*(d*x+2)**n,x)
```

```
[Out] 2**n*b**m*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), d*x*exp_polar(I*
pi)/2)/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx)^{m} (dx+2)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x)^m*(d*x + 2)^n, x)
```

3.751 $\int (bx)^m (c-bcx)^n \, dx$

Optimal. Leaf size=40

$$-\frac{(c-bcx)^{n+1} {}_2F_1(-m,n+1;n+2;1-bx)}{bc(n+1)}$$

```
[Out] -(((c - b*c*x)^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, 1 - b*x])/(b*c*(
1 + n)))
```

Rubi [A] time = 0.0091687, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {65}

$$-\frac{(c-bcx)^{n+1} {}_2F_1(-m,n+1;n+2;1-bx)}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^m*(c - b*c*x)^n,x]
```

```
[Out] -(((c - b*c*x)^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, 1 - b*x])/(b*c*(
1 + n)))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (bx)^m (c-bcx)^n \, dx = -\frac{(c-bcx)^{1+n} {}_2F_1(-m,1+n;2+n;1-bx)}{bc(1+n)}$$

Mathematica [A] time = 0.0102818, size = 44, normalized size = 1.1

$$\frac{x(bx)^m(1-bx)^{-n}(c-bcx)^n {}_2F_1(m+1,-n;m+2;bx)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^m*(c - b*c*x)^n,x]
```

```
[Out] (x*(b*x)^m*(c - b*c*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, b*x])/((1 + m)
*(1 - b*x)^n)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (bx)^m (-bcx+c)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*(-b*c*x+c)^n,x)
```

```
[Out] int((b*x)^m*(-b*c*x+c)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-bcx + c)^n (bx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(-b*c*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((-b*c*x + c)^n*(b*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-bcx + c)^n (bx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(-b*c*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral((-b*c*x + c)^n*(b*x)^m, x)
```

Sympy [C] time = 2.1403, size = 37, normalized size = 0.92

$$\frac{b^m c^n x x^m \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix} \middle| bxe^{2i\pi}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*(-b*c*x+c)**n,x)
```

```
[Out] b**m*c**n*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), b*x*exp_polar(2*
I*pi))/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-bcx + c)^n (bx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(-b*c*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((-b*c*x + c)^n*(b*x)^m, x)
```

3.752 $\int (bx)^m (c+dx)^n \, dx$

Optimal. Leaf size=52

$$\frac{(bx)^{m+1}(c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{dx}{c}\right)}{b(m+1)}$$

```
[Out] ((b*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*x)/c)])
/(b*(1 + m)*(1 + (d*x)/c)^n)
```

Rubi [A] time = 0.0128916, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {66, 64}

$$\frac{(bx)^{m+1}(c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{dx}{c}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^m*(c + d*x)^n,x]
```

```
[Out] ((b*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*x)/c)])
/(b*(1 + m)*(1 + (d*x)/c)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int (bx)^m (c+dx)^n \, dx &= \left((c+dx)^n \left(1+\frac{dx}{c}\right)^{-n}\right) \int (bx)^m \left(1+\frac{dx}{c}\right)^n \, dx \\
&= \frac{(bx)^{1+m}(c+dx)^n \left(1+\frac{dx}{c}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;-\frac{dx}{c}\right)}{b(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.0120037, size = 48, normalized size = 0.92

$$\frac{x(bx)^m(c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{dx}{c}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^m*(c + d*x)^n,x]
```

```
[Out] (x*(b*x)^m*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*x)/c)])/((1
 + m)*(1 + (d*x)/c)^n)
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int (bx)^m (dx + c)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*(d*x+c)^n,x)
```

```
[Out] int((b*x)^m*(d*x+c)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m (dx + c)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x)^m*(d*x + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx)^m (dx + c)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x)^m*(d*x + c)^n, x)
```

Sympy [C] time = 2.15417, size = 37, normalized size = 0.71

$$\frac{b^m c^n x x^m \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\middle| \frac{dxe^{i\pi}}{c}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*(d*x+c)**n,x)
```

```
[Out] b**m*c**n*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), d*x*exp_polar(I*
pi)/c)/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m (dx + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x)^m*(d*x + c)^n, x)
```

3.753 $\int x^{-1+n}(a+bx)^{-1-n}\,dx$

Optimal. Leaf size=19

$$\frac{x^n(a+bx)^{-n}}{an}$$

```
[Out] x^n/(a*n*(a + b*x)^n)
```

Rubi [A] time = 0.00262, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{x^n(a+bx)^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(a + b*x)^(-1 - n),x]
```

```
[Out] x^n/(a*n*(a + b*x)^n)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int x^{-1+n}(a+bx)^{-1-n}\,dx=\frac{x^n(a+bx)^{-n}}{an}$$

Mathematica [A] time = 0.0047965, size = 19, normalized size = 1.

$$\frac{x^n(a+bx)^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(a + b*x)^(-1 - n),x]
```

```
[Out] x^n/(a*n*(a + b*x)^n)
```

Maple [A] time = 0.001, size = 20, normalized size = 1.1

$$\frac{x^n\,(bx+a)^{-n}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b*x+a)^(-1-n),x)
```

```
[Out] x^n*(b*x+a)^(-n)/a/n
```

Maxima [A] time = 1.13209, size = 30, normalized size = 1.58

$$\frac{e^{(-n\log(bx+a)+n\log(x))}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="maxima")
```

```
[Out] e^(-n*log(b*x + a) + n*log(x))/(a*n)
```

Fricas [A] time = 1.6104, size = 68, normalized size = 3.58

$$\frac{\left(bx^2+ax\right)(bx+a)^{-n-1}x^{n-1}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^(-n - 1)*x^(n - 1)/(a*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b*x+a)**(-1-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{-n-1}x^{n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 1)*x^(n - 1), x)
```

3.754 $\int x^{-3-n}(a+bx)^n\,dx$

Optimal. Leaf size=58

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

```
[Out] -((x^(-2 - n)*(a + b*x)^(1 + n))/(a*(2 + n))) + (b*x^(-1 - n)*(a + b*x)^(1
+ n))/(a^2*(1 + n)*(2 + n))
```

Rubi [A] time = 0.0134934, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - n)*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + b*x)^(1 + n))/(a*(2 + n))) + (b*x^(-1 - n)*(a + b*x)^(1
+ n))/(a^2*(1 + n)*(2 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^{-3-n}(a+bx)^n\,dx&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-n}(a+bx)^n\,dx}{a(2+n)}\\&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A] time = 0.0140357, size = 40, normalized size = 0.69

$$-\frac{x^{-n-2}(an+a-bx)(a+bx)^{n+1}}{a^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - n)*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + a*n - b*x)*(a + b*x)^(1 + n))/(a^2*(1 + n)*(2 + n)))
```

Maple [A] time = 0.005, size = 41, normalized size = 0.7

$$-\frac{(bx+a)^{1+n}\,x^{-2-n}\,(an-bx+a)}{(2+n)\,(1+n)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-n)*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(1+n)*x^(-2-n)*(a*n-b*x+a)/(2+n)/(1+n)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n x^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

Fricas [A] time = 1.69656, size = 126, normalized size = 2.17

$$-\frac{\left(abnx^2-b^2x^3+\left(a^2n+a^2\right)x\right)(bx+a)^n x^{-n-3}}{a^2n^2+3\,a^2n+2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x + a)^n*x^(-n - 3)/(a^2*n^2 +
3*a^2*n + 2*a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-n)*(b*x+a)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^n x^{-n-3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

3.755 $\int x^{2n-3(1+n)}(a+bx)^n\,dx$

Optimal. Leaf size=58

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

```
[Out] -((x^(-2 - n)*(a + b*x)^(1 + n))/(a*(2 + n))) + (b*x^(-1 - n)*(a + b*x)^(1
+ n))/(a^2*(1 + n)*(2 + n))
```

Rubi [A] time = 0.0111185, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*n - 3*(1 + n))*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + b*x)^(1 + n))/(a*(2 + n))) + (b*x^(-1 - n)*(a + b*x)^(1
+ n))/(a^2*(1 + n)*(2 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^{2n-3(1+n)}(a+bx)^n\,dx&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-n}(a+bx)^n\,dx}{a(2+n)}\\&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A] time = 0.0021056, size = 40, normalized size = 0.69

$$-\frac{x^{-n-2}(an+a-bx)(a+bx)^{n+1}}{a^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*n - 3*(1 + n))*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + a*n - b*x)*(a + b*x)^(1 + n))/(a^2*(1 + n)*(2 + n)))
```

Maple [A] time = 0., size = 41, normalized size = 0.7

$$-\frac{(bx+a)^{1+n} x^{-2-n} (an-bx+a)}{(2+n)(1+n) a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-n)*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(1+n)*x^(-2-n)*(a*n-b*x+a)/(2+n)/(1+n)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n x^{-n-3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

Fricas [A] time = 1.56186, size = 126, normalized size = 2.17

$$-\frac{\left(abnx^2 - b^2x^3 + \left(a^2n + a^2\right)x\right)(bx+a)^n x^{-n-3}}{a^2n^2 + 3\,a^2n + 2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x + a)^n*x^(-n - 3)/(a^2*n^2 +
3*a^2*n + 2*a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-n)*(b*x+a)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n x^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

3.756 $\int x^3\sqrt{cx^2}(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{1}{5}ax^4\sqrt{cx^2}+\frac{1}{6}bx^5\sqrt{cx^2}$$

```
[Out] (a*x^4*Sqrt[c*x^2])/5 + (b*x^5*Sqrt[c*x^2])/6
```

Rubi [A] time = 0.0114102, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{5}ax^4\sqrt{cx^2}+\frac{1}{6}bx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x^4*Sqrt[c*x^2])/5 + (b*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{cx^2}(a+bx)\,dx&=\frac{\sqrt{cx^2}\int x^4(a+bx)\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(ax^4+bx^5\right)\,dx}{x}\\&=\frac{1}{5}ax^4\sqrt{cx^2}+\frac{1}{6}bx^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0045488, size = 24, normalized size = 0.69

$$\frac{1}{30}x^4\sqrt{cx^2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (x^4*Sqrt[c*x^2]*(6*a + 5*b*x))/30
```

Maple [A] time = 0.004, size = 21, normalized size = 0.6

$$\frac{x^4 (5 bx + 6 a)}{30} \sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)*(c*x^2)^(1/2),x)
```

```
[Out] 1/30*x^4*(5*b*x+6*a)*(c*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52525, size = 51, normalized size = 1.46

$$\frac{1}{30} \left(5 bx^5 + 6 ax^4\right) \sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(5*b*x^5 + 6*a*x^4)*sqrt(c*x^2)
```

Sympy [A] time = 0.394821, size = 36, normalized size = 1.03

$$\frac{a\sqrt{c}x^4\sqrt{x^2}}{5} + \frac{b\sqrt{c}x^5\sqrt{x^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)*(c*x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x**4*sqrt(x**2)/5 + b*sqrt(c)*x**5*sqrt(x**2)/6
```

Giac [A] time = 1.04993, size = 30, normalized size = 0.86

$$\frac{1}{30} \left(5 bx^6 \mathrm{sgn}\left(x\right) + 6 ax^5 \mathrm{sgn}\left(x\right)\right) \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/30*(5*b*x^6*sgn(x) + 6*a*x^5*sgn(x))*sqrt(c)
```

3.757 $\int x^2\sqrt{cx^2}(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{1}{4}ax^3\sqrt{cx^2}+\frac{1}{5}bx^4\sqrt{cx^2}$$

```
[Out] (a*x^3*Sqrt[c*x^2])/4 + (b*x^4*Sqrt[c*x^2])/5
```

Rubi [A] time = 0.0099123, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{4}ax^3\sqrt{cx^2}+\frac{1}{5}bx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x),x]

[Out] (a*x^3*Sqrt[c*x^2])/4 + (b*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{cx^2}(a+bx)\,dx&=\frac{\sqrt{cx^2}\int x^3(a+bx)\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(ax^3+bx^4\right)dx}{x}\\&=\frac{1}{4}ax^3\sqrt{cx^2}+\frac{1}{5}bx^4\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.003507, size = 24, normalized size = 0.69

$$\frac{1}{20}x^3\sqrt{cx^2}(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (x^3*Sqrt[c*x^2]*(5*a + 4*b*x))/20
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$\frac{x^3 (4 bx + 5 a)}{20} \sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)*(c*x^2)^(1/2),x)
```

```
[Out] 1/20*x^3*(4*b*x+5*a)*(c*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56547, size = 51, normalized size = 1.46

$$\frac{1}{20} \left(4 bx^4 + 5 ax^3\right) \sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*x^4 + 5*a*x^3)*sqrt(c*x^2)
```

Sympy [A] time = 0.305324, size = 36, normalized size = 1.03

$$\frac{a\sqrt{c}x^3\sqrt{x^2}}{4} + \frac{b\sqrt{c}x^4\sqrt{x^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)*(c*x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x**3*sqrt(x**2)/4 + b*sqrt(c)*x**4*sqrt(x**2)/5
```

Giac [A] time = 1.06644, size = 30, normalized size = 0.86

$$\frac{1}{20} \left(4 bx^5 \mathrm{sgn}\left(x\right) + 5 ax^4 \mathrm{sgn}\left(x\right)\right) \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*(4*b*x^5*sgn(x) + 5*a*x^4*sgn(x))*sqrt(c)
```

3.758 $\int x\sqrt{cx^2}(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}$$

```
[Out] (a*x^2*Sqrt[c*x^2])/3 + (b*x^3*Sqrt[c*x^2])/4
```

Rubi [A] time = 0.0086389, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$\frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x^2*Sqrt[c*x^2])/3 + (b*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{cx^2}(a+bx)\,dx &= \frac{\sqrt{cx^2}\int x^2(a+bx)\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int\left(ax^2+bx^3\right)dx}{x}\\ &= \frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.003476, size = 24, normalized size = 0.69

$$\frac{1}{12}x^2\sqrt{cx^2}(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

Maple [A] time = 0.002, size = 21, normalized size = 0.6

$$\frac{x^2(3bx+4a)}{12}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)*(c*x^2)^(1/2),x)
```

```
[Out] 1/12*x^2*(3*b*x+4*a)*(c*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46539, size = 51, normalized size = 1.46

$$\frac{1}{12}\left(3bx^3+4ax^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*x^3 + 4*a*x^2)*sqrt(c*x^2)
```

Sympy [A] time = 0.23566, size = 36, normalized size = 1.03

$$\frac{a\sqrt{c}x^2\sqrt{x^2}}{3}+\frac{b\sqrt{c}x^3\sqrt{x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x**2*sqrt(x**2)/3 + b*sqrt(c)*x**3*sqrt(x**2)/4
```

Giac [A] time = 1.07218, size = 30, normalized size = 0.86

$$\frac{1}{12}\left(3bx^4\operatorname{sgn}(x)+4ax^3\operatorname{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*x^4*sgn(x) + 4*a*x^3*sgn(x))*sqrt(c)
```

3.759 $\int \sqrt{cx^2}(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}$$

```
[Out] (a*x*Sqrt[c*x^2])/2 + (b*x^2*Sqrt[c*x^2])/3
```

Rubi [A] time = 0.0078348, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$\frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x*Sqrt[c*x^2])/2 + (b*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx^2}(a+bx)\,dx &= \frac{\sqrt{cx^2}\int x(a+bx)\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(ax+bx^2\right)\,dx}{x}\\ &= \frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0029974, size = 22, normalized size = 0.67

$$\frac{1}{6}x\sqrt{cx^2}(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

Maple [A] time = 0.004, size = 19, normalized size = 0.6

$$\frac{x(2bx+3a)}{6}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2),x)
```

```
[Out] 1/6*x*(2*b*x+3*a)*(c*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57154, size = 47, normalized size = 1.42

$$\frac{1}{6}\left(2bx^2+3ax\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*x^2 + 3*a*x)*sqrt(c*x^2)
```

Sympy [A] time = 0.193054, size = 34, normalized size = 1.03

$$\frac{a\sqrt{c}x\sqrt{x^2}}{2}+\frac{b\sqrt{c}x^2\sqrt{x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x*sqrt(x**2)/2 + b*sqrt(c)*x**2*sqrt(x**2)/3
```

Giac [A] time = 1.04885, size = 30, normalized size = 0.91

$$\frac{1}{6}\left(2bx^3\operatorname{sgn}(x)+3ax^2\operatorname{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*x^3*sgn(x) + 3*a*x^2*sgn(x))*sqrt(c)
```

3.760 $\int \frac{\sqrt{cx^2}(a+bx)}{x}\,dx$

Optimal. Leaf size=27

$$a\sqrt{cx^2}+\frac{1}{2}bx\sqrt{cx^2}$$

```
[Out] a*Sqrt[c*x^2] + (b*x*Sqrt[c*x^2])/2
```

Rubi [A] time = 0.0040651, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {15}

$$a\sqrt{cx^2}+\frac{1}{2}bx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

```
[Out] a*Sqrt[c*x^2] + (b*x*Sqrt[c*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)}{x}\,dx &= \frac{\sqrt{cx^2}\int (a+bx)\,dx}{x}\\ &= a\sqrt{cx^2}+\frac{1}{2}bx\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0045681, size = 24, normalized size = 0.89

$$\frac{cx^2(2a+bx)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

```
[Out] (c*x^2*(2*a + b*x))/(2*Sqrt[c*x^2])
```

Maple [A] time = 0.002, size = 17, normalized size = 0.6

$$\frac{bx+2\,a}{2}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x,x)
```

```
[Out] 1/2*(b*x+2*a)*(c*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4784, size = 39, normalized size = 1.44

$$\frac{1}{2}\sqrt{cx^2}(bx+2a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)
```

Sympy [A] time = 0.194566, size = 29, normalized size = 1.07

$$a\sqrt{c}\sqrt{x^2}+\frac{b\sqrt{c}x\sqrt{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x,x)
```

```
[Out] a*sqrt(c)*sqrt(x**2) + b*sqrt(c)*x*sqrt(x**2)/2
```

Giac [A] time = 1.0545, size = 23, normalized size = 0.85

$$\frac{1}{2}\left(bx^2+2\,ax\right)\sqrt{c}\mathrm{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*sqrt(c)*sgn(x)
```

3.761 $\int \frac{\sqrt{cx^2}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=28

$$\frac{a\sqrt{cx^2}\log(x)}{x}+b\sqrt{cx^2}$$

```
[Out] b*Sqrt[c*x^2] + (a*Sqrt[c*x^2]*Log[x])/x
```

Rubi [A] time = 0.0049333, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{a\sqrt{cx^2}\log(x)}{x}+b\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
```

```
[Out] b*Sqrt[c*x^2] + (a*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)}{x^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{a+bx}{x}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(b+\frac{a}{x}\right)dx}{x}\\ &= b\sqrt{cx^2}+\frac{a\sqrt{cx^2}\log(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.0042371, size = 20, normalized size = 0.71

$$\frac{cx(a\log(x)+bx)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
```

```
[Out] (c*x*(b*x + a*Log[x]))/Sqrt[c*x^2]
```

Maple [A] time = 0.004, size = 20, normalized size = 0.7

$$\frac{bx + a\ln(x)}{x}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^2,x)
```

```
[Out] (c*x^2)^(1/2)/x*(b*x+a*ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.52884, size = 43, normalized size = 1.54

$$\frac{\sqrt{cx^2}(bx + a\log(x))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a*log(x))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}\,(a + bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)/x**2, x)
```

Giac [A] time = 1.06821, size = 23, normalized size = 0.82

$$(bx\operatorname{sgn}(x) + a\log(|x|)\operatorname{sgn}(x))\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b*x*sgn(x) + a*log(abs(x))*sgn(x))*sqrt(c)
```

3.762 $\int \frac{\sqrt{cx^2}(a+bx)}{x^3} dx$

Optimal. Leaf size=32

$$\frac{b\sqrt{cx^2}\log(x)}{x} - \frac{a\sqrt{cx^2}}{x^2}$$

```
[Out] -((a*Sqrt[c*x^2])/x^2) + (b*Sqrt[c*x^2]*Log[x])/x
```

Rubi [A] time = 0.0067455, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{b\sqrt{cx^2}\log(x)}{x} - \frac{a\sqrt{cx^2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^3,x]

[Out] -((a*Sqrt[c*x^2])/x^2) + (b*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)}{x^3} dx &= \frac{\sqrt{cx^2}\int \frac{a+bx}{x^2} dx}{x} \\
&= \frac{\sqrt{cx^2}\int \left(\frac{a}{x^2}+\frac{b}{x}\right) dx}{x} \\
&= -\frac{a\sqrt{cx^2}}{x^2}+\frac{b\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A] time = 0.0050649, size = 20, normalized size = 0.62

$$\frac{c(bx\log(x)-a)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^3,x]
```

```
[Out] (c*(-a + b*x*Log[x]))/Sqrt[c*x^2]
```

Maple [A] time = 0.005, size = 21, normalized size = 0.7

$$\frac{b\ln\left(x\right)x-a}{x^{2}}\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^3,x)
```

```
[Out] (c*x^2)^(1/2)*(b*ln(x)*x-a)/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.52064, size = 46, normalized size = 1.44

$$\frac{\sqrt{cx^{2}}(bx\log\left(x\right)-a)}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx^{2}}\left(a+bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)/x**3, x)
```

Giac [A] time = 1.05994, size = 27, normalized size = 0.84

$$\left(b\log\left(|x|\right)\operatorname{sgn}\left(x\right)-\frac{a\operatorname{sgn}\left(x\right)}{x}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (b*log(abs(x))*sgn(x) - a*sgn(x)/x)*sqrt(c)
```

3.763 $\int \frac{\sqrt{cx^2}(a+bx)}{x^4} dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3}$$

```
[Out] -(Sqrt[c*x^2]*(a + b*x)^2)/(2*a*x^3)
```

Rubi [A] time = 0.0040421, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 37}

$$-\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
```

```
[Out] -(Sqrt[c*x^2]*(a + b*x)^2)/(2*a*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)}{x^4} dx &= \frac{\sqrt{cx^2}\int \frac{a+bx}{x^3} dx}{x}\\ &= -\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3}\end{aligned}$$

Mathematica [A] time = 0.0042912, size = 22, normalized size = 0.85

$$-\frac{\sqrt{cx^2}(a+2bx)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
```

```
[Out] -(Sqrt[c*x^2]*(a + 2*b*x))/(2*x^3)
```

Maple [A] time = 0.004, size = 19, normalized size = 0.7

$$-\frac{2\,bx+a}{2\,x^3}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*(2*b*x+a)*(c*x^2)^(1/2)/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.45878, size = 46, normalized size = 1.77

$$-\frac{\sqrt{cx^2}(2\,bx+a)}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/x^3
```

Sympy [A] time = 0.482106, size = 36, normalized size = 1.38

$$-\frac{a\sqrt{c}\sqrt{x^2}}{2x^3}-\frac{b\sqrt{c}\sqrt{x^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**4,x)
```

```
[Out] -a*sqrt(c)*sqrt(x**2)/(2*x**3) - b*sqrt(c)*sqrt(x**2)/x**2
```

Giac [A] time = 1.06677, size = 26, normalized size = 1.

$$-\frac{(2\,bx\mathrm{sgn}\,(x)+a\mathrm{sgn}\,(x))\sqrt{c}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x*sgn(x) + a*sgn(x))*sqrt(c)/x^2
```

3.764 $\int x^3 \left(cx^2\right)^{3/2} (a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}$$

```
[Out] (a*c*x^6*Sqrt[c*x^2])/7 + (b*c*x^7*Sqrt[c*x^2])/8
```

Rubi [A] time = 0.0133218, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^6*Sqrt[c*x^2])/7 + (b*c*x^7*Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3 \left(cx^2\right)^{3/2} (a+bx)\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^6(a+bx)\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^6+bx^7\right)dx}{x}\\ &= \frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0066866, size = 24, normalized size = 0.65

$$\frac{1}{56}x^4\left(cx^2\right)^{3/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (x^4*(c*x^2)^(3/2)*(8*a + 7*b*x))/56
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$\frac{x^4 (7bx + 8a)}{56} (cx^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] 1/56*x^4*(7*b*x+8*a)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49586, size = 57, normalized size = 1.54

$$\frac{1}{56} (7bcx^7 + 8acx^6) \sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/56*(7*b*c*x^7 + 8*a*c*x^6)*sqrt(c*x^2)
```

Sympy [A] time = 1.09628, size = 36, normalized size = 0.97

$$\frac{ac^{\frac{3}{2}} x^4 (x^2)^{\frac{3}{2}}}{7} + \frac{bc^{\frac{3}{2}} x^5 (x^2)^{\frac{3}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*c**(3/2)*x**4*(x**2)**(3/2)/7 + b*c**(3/2)*x**5*(x**2)**(3/2)/8
```

Giac [A] time = 1.06378, size = 30, normalized size = 0.81

$$\frac{1}{56} (7bx^8 \operatorname{sgn}(x) + 8ax^7 \operatorname{sgn}(x)) c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/56*(7*b*x^8*sgn(x) + 8*a*x^7*sgn(x))*c^(3/2)
```

3.765 $\int x^2 \left(cx^2\right)^{3/2} (a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}$$

```
[Out] (a*c*x^5*Sqrt[c*x^2])/6 + (b*c*x^6*Sqrt[c*x^2])/7
```

Rubi [A] time = 0.0124938, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^5*Sqrt[c*x^2])/6 + (b*c*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2 \left(cx^2\right)^{3/2} (a+bx)\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^5(a+bx)\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^5+bx^6\right)\,dx}{x}\\ &= \frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0061477, size = 24, normalized size = 0.65

$$\frac{1}{42}x^3\left(cx^2\right)^{3/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (x^3*(c*x^2)^(3/2)*(7*a + 6*b*x))/42
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$\frac{x^3 (6 bx + 7 a)}{42} \left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] 1/42*x^3*(6*b*x+7*a)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54471, size = 57, normalized size = 1.54

$$\frac{1}{42} \left(6 bcx^6 + 7 acx^5\right) \sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/42*(6*b*c*x^6 + 7*a*c*x^5)*sqrt(c*x^2)
```

Sympy [A] time = 0.832824, size = 36, normalized size = 0.97

$$\frac{ac^{\frac{3}{2}} x^3 \left(x^2\right)^{\frac{3}{2}}}{6} + \frac{bc^{\frac{3}{2}} x^4 \left(x^2\right)^{\frac{3}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*c**(3/2)*x**3*(x**2)**(3/2)/6 + b*c**(3/2)*x**4*(x**2)**(3/2)/7
```

Giac [A] time = 1.05123, size = 30, normalized size = 0.81

$$\frac{1}{42} \left(6 bx^7 \mathrm{sgn}\left(x\right) + 7 ax^6 \mathrm{sgn}\left(x\right)\right) c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/42*(6*b*x^7*sgn(x) + 7*a*x^6*sgn(x))*c^(3/2)
```

3.766 $\int x\left(cx^2\right)^{3/2}(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}$$

```
[Out] (a*c*x^4*Sqrt[c*x^2])/5 + (b*c*x^5*Sqrt[c*x^2])/6
```

Rubi [A] time = 0.0108016, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^4*Sqrt[c*x^2])/5 + (b*c*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x\left(cx^2\right)^{3/2}(a+bx)\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^4+bx^5\right)\,dx}{x}\\
&= \frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.005597, size = 24, normalized size = 0.65

$$\frac{1}{30}x^2\left(cx^2\right)^{3/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

Maple [A] time = 0.002, size = 21, normalized size = 0.6

$$\frac{x^2\left(5\,bx+6\,a\right)}{30}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] 1/30*x^2*(5*b*x+6*a)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51571, size = 57, normalized size = 1.54

$$\frac{1}{30}\left(5\,bcx^5+6\,acx^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(5*b*c*x^5 + 6*a*c*x^4)*sqrt(c*x^2)
```

Sympy [A] time = 0.651173, size = 36, normalized size = 0.97

$$\frac{ac^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}{5}+\frac{bc^{\frac{3}{2}}x^3\left(x^2\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*c**(3/2)*x**2*(x**2)**(3/2)/5 + b*c**(3/2)*x**3*(x**2)**(3/2)/6
```

Giac [A] time = 1.06546, size = 30, normalized size = 0.81

$$\frac{1}{30}\left(5\,bx^6\mathrm{sgn}\left(x\right)+6\,ax^5\mathrm{sgn}\left(x\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/30*(5*b*x^6*sgn(x) + 6*a*x^5*sgn(x))*c^(3/2)
```

3.767 $\int \left(cx^2\right)^{3/2} (a+bx) \, dx$

Optimal. Leaf size=37

$$\frac{1}{4} a c x^3 \sqrt{c x^2}+\frac{1}{5} b c x^4 \sqrt{c x^2}$$

```
[Out] (a*c*x^3*Sqrt[c*x^2])/4 + (b*c*x^4*Sqrt[c*x^2])/5
```

Rubi [A] time = 0.0097555, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$\frac{1}{4} a c x^3 \sqrt{c x^2}+\frac{1}{5} b c x^4 \sqrt{c x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^3*Sqrt[c*x^2])/4 + (b*c*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(cx^2\right)^{3/2} (a+bx) \, dx &= \frac{\left(c\sqrt{cx^2}\right) \int x^3 (a+bx) \, dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right) \int \left(ax^3+bx^4\right) \, dx}{x}\\
&= \frac{1}{4} a c x^3 \sqrt{c x^2}+\frac{1}{5} b c x^4 \sqrt{c x^2}
\end{aligned}$$

Mathematica [A] time = 0.0052157, size = 22, normalized size = 0.59

$$\frac{1}{20} x \left(cx^2\right)^{3/2} (5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

Maple [A] time = 0.002, size = 19, normalized size = 0.5

$$\frac{x(4bx+5a)}{20}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] 1/20*x*(4*b*x+5*a)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54806, size = 57, normalized size = 1.54

$$\frac{1}{20}\left(4bcx^4+5acx^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*c*x^4 + 5*a*c*x^3)*sqrt(c*x^2)
```

Sympy [A] time = 0.484755, size = 34, normalized size = 0.92

$$\frac{ac^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}{4}+\frac{bc^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*c**(3/2)*x*(x**2)**(3/2)/4 + b*c**(3/2)*x**2*(x**2)**(3/2)/5
```

Giac [A] time = 1.0628, size = 30, normalized size = 0.81

$$\frac{1}{20}\left(4bx^5\operatorname{sgn}(x)+5ax^4\operatorname{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/20*(4*b*x^5*sgn(x) + 5*a*x^4*sgn(x))*c^(3/2)
```

3.768 $\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx$

Optimal. Leaf size=37

$$\frac{1}{3}acx^2\sqrt{cx^2}+\frac{1}{4}bcx^3\sqrt{cx^2}$$

```
[Out] (a*c*x^2*Sqrt[c*x^2])/3 + (b*c*x^3*Sqrt[c*x^2])/4
```

Rubi [A] time = 0.0088063, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{3}acx^2\sqrt{cx^2}+\frac{1}{4}bcx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x,x]
```

```
[Out] (a*c*x^2*Sqrt[c*x^2])/3 + (b*c*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^2+bx^3\right)dx}{x}\\ &= \frac{1}{3}acx^2\sqrt{cx^2}+\frac{1}{4}bcx^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0021944, size = 25, normalized size = 0.68

$$\frac{1}{12}cx^2\sqrt{cx^2}(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x,x]
```

```
[Out] (c*x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

Maple [A] time = 0.003, size = 18, normalized size = 0.5

$$\frac{3\,bx+4\,a}{12}\left(cx^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x,x)
```

```
[Out] 1/12*(3*b*x+4*a)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39588, size = 57, normalized size = 1.54

$$\frac{1}{12}\left(3\,bcx^{3}+4\,acx^{2}\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c*x^3 + 4*a*c*x^2)*sqrt(c*x^2)
```

Sympy [A] time = 0.520868, size = 31, normalized size = 0.84

$$\frac{ac^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}{3}+\frac{bc^{\frac{3}{2}}x\left(x^{2}\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x,x)
```

```
[Out] a*c**(3/2)*(x**2)**(3/2)/3 + b*c**(3/2)*x*(x**2)**(3/2)/4
```

Giac [A] time = 1.05761, size = 30, normalized size = 0.81

$$\frac{1}{12}\left(3\,bx^4\operatorname{sgn}(x) + 4\,ax^3\operatorname{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="giac")
```

```
[Out] 1/12*(3*b*x^4*sgn(x) + 4*a*x^3*sgn(x))*c^(3/2)
```

3.769 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^2} dx$

Optimal. Leaf size=35

$$\frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}$$

```
[Out] (a*c*x*Sqrt[c*x^2])/2 + (b*c*x^2*Sqrt[c*x^2])/3
```

Rubi [A] time = 0.0084287, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
```

```
[Out] (a*c*x*Sqrt[c*x^2])/2 + (b*c*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)}{x^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)\,dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax+bx^2\right)\,dx}{x} \\
&= \frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.0020262, size = 23, normalized size = 0.66

$$\frac{1}{6}cx\sqrt{cx^2}(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
```

```
[Out] (c*x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

Maple [A] time = 0.002, size = 21, normalized size = 0.6

$$\frac{2bx+3a}{6x}\left(cx^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^2,x)
```

```
[Out] 1/6/x*(2*b*x+3*a)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52435, size = 53, normalized size = 1.51

$$\frac{1}{6}\left(2\,bcx^{2}+3\,acx\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c*x^2 + 3*a*c*x)*sqrt(c*x^2)
```

Sympy [A] time = 0.51171, size = 31, normalized size = 0.89

$$\frac{ac^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}{2x}+\frac{bc^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**2,x)
```

```
[Out] a*c**(3/2)*(x**2)**(3/2)/(2*x) + b*c**(3/2)*(x**2)**(3/2)/3
```

Giac [A] time = 1.07138, size = 30, normalized size = 0.86

$$\frac{1}{6}\left(2\,bx^3\mathrm{sgn}\left(x\right) + 3\,ax^2\mathrm{sgn}\left(x\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*b*x^3*sgn(x) + 3*a*x^2*sgn(x))*c^(3/2)
```

3.770 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^3} dx$

Optimal. Leaf size=29

$$ac\sqrt{cx^2} + \frac{1}{2}bcx\sqrt{cx^2}$$

```
[Out] a*c*Sqrt[c*x^2] + (b*c*x*Sqrt[c*x^2])/2
```

Rubi [A] time = 0.0043358, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {15}

$$ac\sqrt{cx^2} + \frac{1}{2}bcx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

```
[Out] a*c*Sqrt[c*x^2] + (b*c*x*Sqrt[c*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)}{x^3} dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)\,dx}{x}\\ &= ac\sqrt{cx^2} + \frac{1}{2}bcx\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0028447, size = 21, normalized size = 0.72

$$\frac{1}{2}c\sqrt{cx^2}(2a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

```
[Out] (c*Sqrt[c*x^2]*(2*a + b*x))/2
```

Maple [A] time = 0.002, size = 20, normalized size = 0.7

$$\frac{bx+2a}{2x^2}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^3,x)
```

```
[Out] 1/2/x^2*(b*x+2*a)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.49662, size = 45, normalized size = 1.55

$$\frac{1}{2}\,(bcx + 2\,ac)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x + 2*a*c)*sqrt(c*x^2)
```

Sympy [A] time = 0.695981, size = 32, normalized size = 1.1

$$\frac{ac^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{x^2} + \frac{bc^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**3,x)
```

```
[Out] a*c**(3/2)*(x**2)**(3/2)/x**2 + b*c**(3/2)*(x**2)**(3/2)/(2*x)
```

Giac [A] time = 1.06579, size = 23, normalized size = 0.79

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)c^{\frac{3}{2}}\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*c^(3/2)*sgn(x)
```

3.771 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^4} dx$

Optimal. Leaf size=30

$$\frac{ac\sqrt{cx^2}\log(x)}{x} + bc\sqrt{cx^2}$$

```
[Out] b*c*Sqrt[c*x^2] + (a*c*Sqrt[c*x^2]*Log[x])/x
```

Rubi [A] time = 0.0052881, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{ac\sqrt{cx^2}\log(x)}{x} + bc\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
```

```
[Out] b*c*Sqrt[c*x^2] + (a*c*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)}{x^4} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{a+bx}{x} dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(b+\frac{a}{x}\right) dx}{x} \\
&= bc\sqrt{cx^2} + \frac{ac\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A] time = 0.0043884, size = 21, normalized size = 0.7

$$\frac{(cx^2)^{3/2}(a\log(x)+bx)}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x + a*Log[x]))/x^3
```

Maple [A] time = 0.004, size = 20, normalized size = 0.7

$$\frac{bx + a\ln(x)}{x^3}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^4,x)
```

```
[Out] (c*x^2)^(3/2)/x^3*(b*x+a*ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.8643, size = 49, normalized size = 1.63

$$\frac{(bcx + ac\log(x))\sqrt{cx^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] (b*c*x + a*c*log(x))*sqrt(c*x^2)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}(a + bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**4,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)/x**4, x)
```

Giac [A] time = 1.06045, size = 23, normalized size = 0.77

$$(bx\operatorname{sgn}(x) + a\log(|x|)\operatorname{sgn}(x))c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] (b*x*sgn(x) + a*log(abs(x))*sgn(x))*c^(3/2)
```

3.772 $\int x^3 \left(cx^2\right)^{5/2} (a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}$$

```
[Out] (a*c^2*x^8*Sqrt[c*x^2])/9 + (b*c^2*x^9*Sqrt[c*x^2])/10
```

Rubi [A] time = 0.0159981, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^8*Sqrt[c*x^2])/9 + (b*c^2*x^9*Sqrt[c*x^2])/10
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(cx^2\right)^{5/2}(a+bx)\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^8(a+bx)\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int\left(ax^8+bx^9\right)\,dx}{x}\\&=\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0066605, size = 24, normalized size = 0.59

$$\frac{1}{90}x^4\left(cx^2\right)^{5/2}(10a+9bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (x^4*(c*x^2)^(5/2)*(10*a + 9*b*x))/90
```

Maple [A] time = 0.003, size = 21, normalized size = 0.5

$$\frac{x^4 (9 bx + 10 a)}{90} \left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(5/2)*(b*x+a),x)
```

```
[Out] 1/90*x^4*(9*b*x+10*a)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82478, size = 63, normalized size = 1.54

$$\frac{1}{90} \left(9 bc^2 x^9 + 10 ac^2 x^8\right) \sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/90*(9*b*c^2*x^9 + 10*a*c^2*x^8)*sqrt(c*x^2)
```

Sympy [A] time = 2.8722, size = 36, normalized size = 0.88

$$\frac{ac^{\frac{5}{2}} x^4 \left(x^2\right)^{\frac{5}{2}}}{9} + \frac{bc^{\frac{5}{2}} x^5 \left(x^2\right)^{\frac{5}{2}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*c**(5/2)*x**4*(x**2)**(5/2)/9 + b*c**(5/2)*x**5*(x**2)**(5/2)/10
```

Giac [A] time = 1.05748, size = 38, normalized size = 0.93

$$\frac{1}{90} \left(9 bc^2 x^{10} \mathrm{sgn}\left(x\right) + 10 ac^2 x^9 \mathrm{sgn}\left(x\right)\right) \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/90*(9*b*c^2*x^10*sgn(x) + 10*a*c^2*x^9*sgn(x))*sqrt(c)
```

3.773 $\int x^2 \left(cx^2\right)^{5/2} (a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{1}{8}ac^2x^7\sqrt{cx^2}+\frac{1}{9}bc^2x^8\sqrt{cx^2}$$

```
[Out] (a*c^2*x^7*Sqrt[c*x^2])/8 + (b*c^2*x^8*Sqrt[c*x^2])/9
```

Rubi [A] time = 0.0150992, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{8}ac^2x^7\sqrt{cx^2}+\frac{1}{9}bc^2x^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^7*Sqrt[c*x^2])/8 + (b*c^2*x^8*Sqrt[c*x^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(cx^2\right)^{5/2} (a+bx)\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^7(a+bx)\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^7+bx^8\right)\,dx}{x}\\
&= \frac{1}{8}ac^2x^7\sqrt{cx^2}+\frac{1}{9}bc^2x^8\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.0065112, size = 24, normalized size = 0.59

$$\frac{1}{72}x^3\left(cx^2\right)^{5/2}(9a+8bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (x^3*(c*x^2)^(5/2)*(9*a + 8*b*x))/72
```

Maple [A] time = 0.003, size = 21, normalized size = 0.5

$$\frac{x^3 (8 bx + 9 a)}{72} \left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(5/2)*(b*x+a),x)
```

```
[Out] 1/72*x^3*(8*b*x+9*a)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66498, size = 62, normalized size = 1.51

$$\frac{1}{72} \left(8 bc^2 x^8 + 9 ac^2 x^7\right) \sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/72*(8*b*c^2*x^8 + 9*a*c^2*x^7)*sqrt(c*x^2)
```

Sympy [A] time = 2.03589, size = 36, normalized size = 0.88

$$\frac{ac^{\frac{5}{2}} x^3 \left(x^2\right)^{\frac{5}{2}}}{8} + \frac{bc^{\frac{5}{2}} x^4 \left(x^2\right)^{\frac{5}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*c**(5/2)*x**3*(x**2)**(5/2)/8 + b*c**(5/2)*x**4*(x**2)**(5/2)/9
```

Giac [A] time = 1.0729, size = 38, normalized size = 0.93

$$\frac{1}{72} \left(8 bc^2 x^9 \mathrm{sgn}\left(x\right) + 9 ac^2 x^8 \mathrm{sgn}\left(x\right)\right) \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/72*(8*b*c^2*x^9*sgn(x) + 9*a*c^2*x^8*sgn(x))*sqrt(c)
```

3.774 $\int x\left(cx^2\right)^{5/2}(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}$$

```
[Out] (a*c^2*x^6*Sqrt[c*x^2])/7 + (b*c^2*x^7*Sqrt[c*x^2])/8
```

Rubi [A] time = 0.0132595, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^6*Sqrt[c*x^2])/7 + (b*c^2*x^7*Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{5/2}(a+bx)\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^6(a+bx)\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int\left(ax^6+bx^7\right)\,dx}{x}\\&=\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0063578, size = 24, normalized size = 0.59

$$\frac{1}{56}x^2\left(cx^2\right)^{5/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (x^2*(c*x^2)^(5/2)*(8*a + 7*b*x))/56
```

Maple [A] time = 0.002, size = 21, normalized size = 0.5

$$\frac{x^2 (7 bx + 8 a)}{56} (cx^2)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(b*x+a),x)
```

```
[Out] 1/56*x^2*(7*b*x+8*a)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75293, size = 62, normalized size = 1.51

$$\frac{1}{56} \left(7 bc^2 x^7 + 8 ac^2 x^6\right) \sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/56*(7*b*c^2*x^7 + 8*a*c^2*x^6)*sqrt(c*x^2)
```

Sympy [A] time = 1.73241, size = 36, normalized size = 0.88

$$\frac{ac^{\frac{5}{2}} x^2 \left(x^2\right)^{\frac{5}{2}}}{7} + \frac{bc^{\frac{5}{2}} x^3 \left(x^2\right)^{\frac{5}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*c**(5/2)*x**2*(x**2)**(5/2)/7 + b*c**(5/2)*x**3*(x**2)**(5/2)/8
```

Giac [A] time = 1.04845, size = 38, normalized size = 0.93

$$\frac{1}{56} \left(7 bc^2 x^8 \mathrm{sgn}\left(x\right) + 8 ac^2 x^7 \mathrm{sgn}\left(x\right)\right) \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/56*(7*b*c^2*x^8*sgn(x) + 8*a*c^2*x^7*sgn(x))*sqrt(c)
```

3.775 $\int \left(cx^2\right)^{5/2} (a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{1}{6}ac^2x^5\sqrt{cx^2}+\frac{1}{7}bc^2x^6\sqrt{cx^2}$$

```
[Out] (a*c^2*x^5*Sqrt[c*x^2])/6 + (b*c^2*x^6*Sqrt[c*x^2])/7
```

Rubi [A] time = 0.0128006, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$\frac{1}{6}ac^2x^5\sqrt{cx^2}+\frac{1}{7}bc^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^5*Sqrt[c*x^2])/6 + (b*c^2*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(cx^2\right)^{5/2}(a+bx)\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^5+bx^6\right)\,dx}{x}\\ &= \frac{1}{6}ac^2x^5\sqrt{cx^2}+\frac{1}{7}bc^2x^6\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0060522, size = 22, normalized size = 0.54

$$\frac{1}{42}x\left(cx^2\right)^{5/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (x*(c*x^2)^(5/2)*(7*a + 6*b*x))/42
```

Maple [A] time = 0.002, size = 19, normalized size = 0.5

$$\frac{x(6bx+7a)}{42}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a),x)
```

```
[Out] 1/42*x*(6*b*x+7*a)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73336, size = 62, normalized size = 1.51

$$\frac{1}{42}\left(6\,bc^2x^6+7\,ac^2x^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/42*(6*b*c^2*x^6 + 7*a*c^2*x^5)*sqrt(c*x^2)
```

Sympy [A] time = 1.3229, size = 34, normalized size = 0.83

$$\frac{ac^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}{6}+\frac{bc^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*c**(5/2)*x*(x**2)**(5/2)/6 + b*c**(5/2)*x**2*(x**2)**(5/2)/7
```

Giac [A] time = 1.06079, size = 38, normalized size = 0.93

$$\frac{1}{42}\left(6\,bc^2x^7\mathrm{sgn}\left(x\right)+7\,ac^2x^6\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/42*(6*b*c^2*x^7*sgn(x) + 7*a*c^2*x^6*sgn(x))*sqrt(c)
```

3.776 $\int \frac{(cx^2)^{5/2}(a+bx)}{x} dx$

Optimal. Leaf size=41

$$\frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}$$

```
[Out] (a*c^2*x^4*Sqrt[c*x^2])/5 + (b*c^2*x^5*Sqrt[c*x^2])/6
```

Rubi [A] time = 0.0120243, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x,x]
```

```
[Out] (a*c^2*x^4*Sqrt[c*x^2])/5 + (b*c^2*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)}{x}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^4+bx^5\right)dx}{x}\\ &= \frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.002977, size = 25, normalized size = 0.61

$$\frac{1}{30}cx^2\left(cx^2\right)^{3/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x,x]
```

```
[Out] (c*x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

Maple [A] time = 0.002, size = 18, normalized size = 0.4

$$\frac{5bx + 6a}{30} \left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x,x)
```

```
[Out] 1/30*(5*b*x+6*a)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80116, size = 62, normalized size = 1.51

$$\frac{1}{30} \left(5 bc^2x^5 + 6 ac^2x^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] 1/30*(5*b*c^2*x^5 + 6*a*c^2*x^4)*sqrt(c*x^2)
```

Sympy [A] time = 1.33393, size = 31, normalized size = 0.76

$$\frac{ac^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}}{5} + \frac{bc^{\frac{5}{2}} x \left(x^2\right)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x,x)
```

```
[Out] a*c**(5/2)*(x**2)**(5/2)/5 + b*c**(5/2)*x*(x**2)**(5/2)/6
```

Giac [A] time = 1.06855, size = 38, normalized size = 0.93

$$\frac{1}{30}\left(5\,bc^{2}x^{6}\mathrm{sgn}\left(x\right)+6\,ac^{2}x^{5}\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="giac")
```

```
[Out] 1/30*(5*b*c^2*x^6*sgn(x) + 6*a*c^2*x^5*sgn(x))*sqrt(c)
```

3.777 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^2} dx$

Optimal. Leaf size=41

$$\frac{1}{4}ac^2x^3\sqrt{cx^2} + \frac{1}{5}bc^2x^4\sqrt{cx^2}$$

```
[Out] (a*c^2*x^3*Sqrt[c*x^2])/4 + (b*c^2*x^4*Sqrt[c*x^2])/5
```

Rubi [A] time = 0.011029, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{4}ac^2x^3\sqrt{cx^2} + \frac{1}{5}bc^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
```

```
[Out] (a*c^2*x^3*Sqrt[c*x^2])/4 + (b*c^2*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)}{x^2} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)\,dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^3+bx^4\right)\,dx}{x} \\
&= \frac{1}{4}ac^2x^3\sqrt{cx^2} + \frac{1}{5}bc^2x^4\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.002578, size = 23, normalized size = 0.56

$$\frac{1}{20}cx\left(cx^2\right)^{3/2}(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
```

```
[Out] (c*x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

Maple [A] time = 0.002, size = 21, normalized size = 0.5

$$\frac{4\,bx+5\,a}{20\,x}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^2,x)
```

```
[Out] 1/20/x*(4*b*x+5*a)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8442, size = 62, normalized size = 1.51

$$\frac{1}{20}\left(4\,bc^2x^4+5\,ac^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*c^2*x^4 + 5*a*c^2*x^3)*sqrt(c*x^2)
```

Sympy [A] time = 1.35926, size = 31, normalized size = 0.76

$$\frac{ac^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{4x}+\frac{bc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**2,x)
```

```
[Out] a*c**(5/2)*(x**2)**(5/2)/(4*x) + b*c**(5/2)*(x**2)**(5/2)/5
```

Giac [A] time = 1.04246, size = 38, normalized size = 0.93

$$\frac{1}{20}\left(4\,bc^2x^5\mathrm{sgn}\left(x\right)+5\,ac^2x^4\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/20*(4*b*c^2*x^5*sgn(x) + 5*a*c^2*x^4*sgn(x))*sqrt(c)
```

3.778 $\int \frac{\left(cx^2\right)^{5/2}(a+bx)}{x^3}\,dx$

Optimal. Leaf size=41

$$\frac{1}{3}ac^2x^2\sqrt{cx^2}+\frac{1}{4}bc^2x^3\sqrt{cx^2}$$

```
[Out] (a*c^2*x^2*Sqrt[c*x^2])/3 + (b*c^2*x^3*Sqrt[c*x^2])/4
```

Rubi [A] time = 0.0100404, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{3}ac^2x^2\sqrt{cx^2}+\frac{1}{4}bc^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^3,x]
```

```
[Out] (a*c^2*x^2*Sqrt[c*x^2])/3 + (b*c^2*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cx^2\right)^{5/2}(a+bx)}{x^3}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^2+bx^3\right)\,dx}{x}\\ &= \frac{1}{3}ac^2x^2\sqrt{cx^2}+\frac{1}{4}bc^2x^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0026706, size = 27, normalized size = 0.66

$$\frac{1}{12}c^2x^2\sqrt{cx^2}(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^3,x]
```

```
[Out] (c^2*x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

Maple [A] time = 0.003, size = 21, normalized size = 0.5

$$\frac{3\,bx+4\,a}{12\,x^{2}}\left(cx^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^3,x)
```

```
[Out] 1/12/x^2*(3*b*x+4*a)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88457, size = 62, normalized size = 1.51

$$\frac{1}{12}\left(3\,bc^{2}x^{3}+4\,ac^{2}x^{2}\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c^2*x^3 + 4*a*c^2*x^2)*sqrt(c*x^2)
```

Sympy [A] time = 1.52712, size = 34, normalized size = 0.83

$$\frac{ac^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{3x^{2}}+\frac{bc^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**3,x)
```

```
[Out] a*c**(5/2)*(x**2)**(5/2)/(3*x**2) + b*c**(5/2)*(x**2)**(5/2)/(4*x)
```

Giac [A] time = 1.05465, size = 38, normalized size = 0.93

$$\frac{1}{12} \left(3 \, b c^2 x^4 \mathrm{sgn}\left(x\right) + 4 \, a c^2 x^3 \mathrm{sgn}\left(x\right)\right) \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/12*(3*b*c^2*x^4*sgn(x) + 4*a*c^2*x^3*sgn(x))*sqrt(c)
```

3.779 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^4} dx$

Optimal. Leaf size=39

$$\frac{1}{2}ac^2x\sqrt{cx^2}+\frac{1}{3}bc^2x^2\sqrt{cx^2}$$

```
[Out] (a*c^2*x*Sqrt[c*x^2])/2 + (b*c^2*x^2*Sqrt[c*x^2])/3
```

Rubi [A] time = 0.0084051, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{2}ac^2x\sqrt{cx^2}+\frac{1}{3}bc^2x^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
```

```
[Out] (a*c^2*x*Sqrt[c*x^2])/2 + (b*c^2*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)}{x^4}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax+bx^2\right)\,dx}{x}\\ &= \frac{1}{2}ac^2x\sqrt{cx^2}+\frac{1}{3}bc^2x^2\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0022244, size = 25, normalized size = 0.64

$$\frac{1}{6}c^2x\sqrt{cx^2}(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
```

```
[Out] (c^2*x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

Maple [A] time = 0.003, size = 21, normalized size = 0.5

$$\frac{2\,bx+3\,a}{6\,x^{3}}\left(cx^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^4,x)
```

```
[Out] 1/6/x^3*(2*b*x+3*a)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.82724, size = 58, normalized size = 1.49

$$\frac{1}{6}\left(2\,bc^{2}x^{2}+3\,ac^{2}x\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c^2*x^2 + 3*a*c^2*x)*sqrt(c*x^2)
```

Sympy [A] time = 1.52252, size = 36, normalized size = 0.92

$$\frac{ac^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{2x^{3}}+\frac{bc^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{3x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**4,x)
```

```
[Out] a*c**(5/2)*(x**2)**(5/2)/(2*x**3) + b*c**(5/2)*(x**2)**(5/2)/(3*x**2)
```

Giac [A] time = 1.07041, size = 38, normalized size = 0.97

$$\frac{1}{6}\left(2\,bc^2x^3\mathrm{sgn}\left(x\right)+3\,ac^2x^2\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c^2*x^3*sgn(x) + 3*a*c^2*x^2*sgn(x))*sqrt(c)
```

3.780 $\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$\frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}$$

```
[Out] (a*x^4)/(3*Sqrt[c*x^2]) + (b*x^5)/(4*Sqrt[c*x^2])
```

Rubi [A] time = 0.008578, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^4)/(3*Sqrt[c*x^2]) + (b*x^5)/(4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx &= \frac{x\int x^2(a+bx)\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(ax^2+bx^3\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0043804, size = 24, normalized size = 0.69

$$\frac{x^4(4a+3bx)}{12\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (x^4*(4*a + 3*b*x))/(12*Sqrt[c*x^2])
```

Maple [A] time = 0.002, size = 21, normalized size = 0.6

$$\frac{x^4 (3 bx + 4 a)}{12} \frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/12*x^4*(3*b*x+4*a)/(c*x^2)^(1/2)
```

Maxima [A] time = 1.04352, size = 45, normalized size = 1.29

$$\frac{\sqrt{cx^2} bx^3}{4 c} + \frac{\sqrt{cx^2} ax^2}{3 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^2)*b*x^3/c + 1/3*sqrt(c*x^2)*a*x^2/c
```

Fricas [A] time = 1.80944, size = 54, normalized size = 1.54

$$\frac{\left(3 bx^3 + 4 ax^2\right) \sqrt{cx^2}}{12 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*x^3 + 4*a*x^2)*sqrt(c*x^2)/c
```

Sympy [A] time = 0.561217, size = 36, normalized size = 1.03

$$\frac{ax^4}{3\sqrt{c}\sqrt{x^2}} + \frac{bx^5}{4\sqrt{c}\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] a*x**4/(3*sqrt(c)*sqrt(x**2)) + b*x**5/(4*sqrt(c)*sqrt(x**2))
```

Giac [A] time = 1.06321, size = 35, normalized size = 1.

$$\frac{1}{12}\sqrt{cx^2}\left(\frac{3\,bx}{c}+\frac{4\,a}{c}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(c*x^2)*(3*b*x/c + 4*a/c)*x^2
```

3.781 $\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$\frac{ax^3}{2\sqrt{cx^2}}+\frac{bx^4}{3\sqrt{cx^2}}$$

```
[Out] (a*x^3)/(2*Sqrt[c*x^2]) + (b*x^4)/(3*Sqrt[c*x^2])
```

Rubi [A] time = 0.0081311, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{ax^3}{2\sqrt{cx^2}}+\frac{bx^4}{3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^3)/(2*Sqrt[c*x^2]) + (b*x^4)/(3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx &= \frac{x\int x(a+bx)\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(ax+bx^2\right)dx}{\sqrt{cx^2}}\\ &= \frac{ax^3}{2\sqrt{cx^2}}+\frac{bx^4}{3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0041718, size = 24, normalized size = 0.69

$$\frac{x^3(3a+2bx)}{6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (x^3*(3*a + 2*b*x))/(6*Sqrt[c*x^2])
```

Maple [A] time = 0.001, size = 21, normalized size = 0.6

$$\frac{x^3\left(2bx+3a\right)}{6}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/6*x^3*(2*b*x+3*a)/(c*x^2)^(1/2)
```

Maxima [A] time = 1.0442, size = 35, normalized size = 1.

$$\frac{\sqrt{cx^2}bx^2}{3c}+\frac{ax^2}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2)*b*x^2/c + 1/2*a*x^2/sqrt(c)
```

Fricas [A] time = 1.69145, size = 50, normalized size = 1.43

$$\frac{\left(2bx^2+3ax\right)\sqrt{cx^2}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*x^2 + 3*a*x)*sqrt(c*x^2)/c
```

Sympy [A] time = 0.526608, size = 36, normalized size = 1.03

$$\frac{ax^3}{2\sqrt{c}\sqrt{x^2}}+\frac{bx^4}{3\sqrt{c}\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] a*x**3/(2*sqrt(c)*sqrt(x**2)) + b*x**4/(3*sqrt(c)*sqrt(x**2))
```

Giac [A] time = 1.06585, size = 32, normalized size = 0.91

$$\frac{1}{6}\sqrt{cx^2}\left(\frac{2\,bx}{c}+\frac{3\,a}{c}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2)*(2*b*x/c + 3*a/c)*x
```

3.782 $\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=32

$$\frac{ax^2}{\sqrt{cx^2}}+\frac{bx^3}{2\sqrt{cx^2}}$$

```
[Out] (a*x^2)/Sqrt[c*x^2] + (b*x^3)/(2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0042455, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {15}

$$\frac{ax^2}{\sqrt{cx^2}}+\frac{bx^3}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^2)/Sqrt[c*x^2] + (b*x^3)/(2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx &= \frac{x\int (a+bx)\,dx}{\sqrt{cx^2}}\\ &= \frac{ax^2}{\sqrt{cx^2}}+\frac{bx^3}{2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0014175, size = 23, normalized size = 0.72

$$\frac{x^2(2a+bx)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (x^2*(2*a + b*x))/(2*Sqrt[c*x^2])
```

Maple [A] time = 0.002, size = 20, normalized size = 0.6

$$\frac{x^2\,(bx+2\,a)}{2}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/2*x^2*(b*x+2*a)/(c*x^2)^(1/2)
```

Maxima [A] time = 1.00629, size = 30, normalized size = 0.94

$$\frac{bx^2}{2\sqrt{c}} + \frac{\sqrt{cx^2}a}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2/sqrt(c) + sqrt(c*x^2)*a/c
```

Fricas [A] time = 1.66548, size = 42, normalized size = 1.31

$$\frac{\sqrt{cx^2}(bx + 2a)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)/c
```

Sympy [A] time = 0.437677, size = 34, normalized size = 1.06

$$\frac{ax^2}{\sqrt{c}\sqrt{x^2}} + \frac{bx^3}{2\sqrt{c}\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] a*x**2/(sqrt(c)*sqrt(x**2)) + b*x**3/(2*sqrt(c)*sqrt(x**2))
```

Giac [A] time = 1.07768, size = 30, normalized size = 0.94

$$\frac{1}{2}\sqrt{cx^2}\left(\frac{bx}{c} + \frac{2a}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2)*(b*x/c + 2*a/c)
```

3.783 $\int \frac{a+bx}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=29

$$\frac{ax\log(x)}{\sqrt{cx^2}}+\frac{bx^2}{\sqrt{cx^2}}$$

```
[Out] (b*x^2)/Sqrt[c*x^2] + (a*x*Log[x])/Sqrt[c*x^2]
```

Rubi [A] time = 0.0051277, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$\frac{ax\log(x)}{\sqrt{cx^2}}+\frac{bx^2}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[c*x^2],x]
```

```
[Out] (b*x^2)/Sqrt[c*x^2] + (a*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(b+\frac{a}{x}\right)dx}{\sqrt{cx^2}}\\ &= \frac{bx^2}{\sqrt{cx^2}}+\frac{ax\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0016435, size = 19, normalized size = 0.66

$$\frac{x(a\log(x)+bx)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[c*x^2],x]

[Out] (x*(b*x + a*Log[x]))/Sqrt[c*x^2]
```

Maple [A] time = 0.002, size = 18, normalized size = 0.6

$$x(bx + a\ln(x))\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(1/2),x)

[Out] 1/(c*x^2)^(1/2)*x*(b*x+a*ln(x))
```

Maxima [A] time = 1.04629, size = 27, normalized size = 0.93

$$\frac{a\log(x)}{\sqrt{c}} + \frac{\sqrt{cx^2}b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")

[Out] a*log(x)/sqrt(c) + sqrt(c*x^2)*b/c
```

Fricas [A] time = 1.79276, size = 49, normalized size = 1.69

$$\frac{\sqrt{cx^2}(bx + a\log(x))}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")

[Out] sqrt(c*x^2)*(b*x + a*log(x))/(c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(1/2),x)

[Out] Integral((a + b*x)/sqrt(c*x**2), x)
```

Giac [A] time = 1.07634, size = 47, normalized size = 1.62

$$-\frac{a\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{\sqrt{c}}+\frac{\sqrt{cx^2}b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -a*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/sqrt(c) + sqrt(c*x^2)*b/c
```

3.784 $\int \frac{a+bx}{x\sqrt{cx^2}}\,dx$

Optimal. Leaf size=27

$$\frac{bx\log(x)}{\sqrt{cx^2}}-\frac{a}{\sqrt{cx^2}}$$

```
[Out] -(a/Sqrt[c*x^2]) + (b*x*Log[x])/Sqrt[c*x^2]
```

Rubi [A] time = 0.0060676, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{bx\log(x)}{\sqrt{cx^2}}-\frac{a}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x*Sqrt[c*x^2]),x]
```

```
[Out] -(a/Sqrt[c*x^2]) + (b*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{a}{\sqrt{cx^2}}+\frac{bx\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0060521, size = 23, normalized size = 0.85

$$\frac{cx^2(bx\log(x)-a)}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*Sqrt[c*x^2]),x]
```

```
[Out] (c*x^2*(-a + b*x*Log[x]))/(c*x^2)^(3/2)
```

Maple [A] time = 0.003, size = 18, normalized size = 0.7

$$(b\ln(x)\,x-a)\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(1/2),x)
```

```
[Out] (b*ln(x)*x-a)/(c*x^2)^(1/2)
```

Maxima [A] time = 1.07141, size = 23, normalized size = 0.85

$$\frac{b\log(x)}{\sqrt{c}}-\frac{a}{\sqrt{c}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*log(x)/sqrt(c) - a/(sqrt(c)*x)
```

Fricas [A] time = 1.82753, size = 51, normalized size = 1.89

$$\frac{\sqrt{cx^2}(bx\log(x)-a)}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+bx}{x\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)/(x*sqrt(c*x**2)), x)
```

Giac [B] time = 1.07331, size = 63, normalized size = 2.33

$$-\frac{b\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)-\frac{2a\sqrt{c}}{\sqrt{c}x-\sqrt{cx^2}}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - 2*a*sqrt(c)/(sqrt(c)*x - sqrt(c*x^
2)))/sqrt(c)
```

3.785 $\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{(a+bx)^2}{2ax\sqrt{cx^2}}$$

```
[Out] -(a + b*x)^2/(2*a*x*Sqrt[c*x^2])
```

Rubi [A] time = 0.0038607, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2ax\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] -(a + b*x)^2/(2*a*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^3}\,dx}{\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2ax\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0059155, size = 23, normalized size = 0.88

$$\frac{cx(-a-2bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] (c*x*(-a - 2*b*x))/(2*(c*x^2)^(3/2))
```

Maple [A] time = 0.002, size = 19, normalized size = 0.7

$$-\frac{2\,bx+a}{2\,x}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(1/2),x)
```

```
[Out] -1/2*(2*b*x+a)/x/(c*x^2)^(1/2)
```

Maxima [A] time = 1.05018, size = 26, normalized size = 1.

$$-\frac{b}{\sqrt{c}x}-\frac{a}{2\,\sqrt{c}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b/(sqrt(c)*x) - 1/2*a/(sqrt(c)*x^2)
```

Fricas [A] time = 1.69186, size = 51, normalized size = 1.96

$$-\frac{\sqrt{cx^2}(2\,bx+a)}{2\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c*x^3)
```

Sympy [A] time = 0.505369, size = 31, normalized size = 1.19

$$-\frac{a}{2\sqrt{c}x\sqrt{x^2}}-\frac{b}{\sqrt{c}\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(1/2),x)
```

```
[Out] -a/(2*sqrt(c)*x*sqrt(x**2)) - b/(sqrt(c)*sqrt(x**2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.786 $\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}$$

```
[Out] -a/(3*x^2*Sqrt[c*x^2]) - b/(2*x*Sqrt[c*x^2])
```

Rubi [A] time = 0.0067594, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] -a/(3*x^2*Sqrt[c*x^2]) - b/(2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^4}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0061041, size = 22, normalized size = 0.63

$$\frac{c(-2a-3bx)}{6\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] (c*(-2*a - 3*b*x))/(6*(c*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$-\frac{3\,bx+2\,a}{6\,x^2}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(1/2),x)
```

```
[Out] -1/6*(3*b*x+2*a)/x^2/(c*x^2)^(1/2)
```

Maxima [A] time = 1.02184, size = 26, normalized size = 0.74

$$-\frac{b}{2\,\sqrt{c}x^2}-\frac{a}{3\,\sqrt{c}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b/(sqrt(c)*x^2) - 1/3*a/(sqrt(c)*x^3)
```

Fricas [A] time = 1.54425, size = 54, normalized size = 1.54

$$-\frac{\sqrt{cx^2}(3\,bx+2\,a)}{6\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c*x^4)
```

Sympy [A] time = 0.592965, size = 36, normalized size = 1.03

$$-\frac{a}{3\sqrt{c}x^2\sqrt{x^2}}-\frac{b}{2\sqrt{c}x\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(1/2),x)
```

```
[Out] -a/(3*sqrt(c)*x**2*sqrt(x**2)) - b/(2*sqrt(c)*x*sqrt(x**2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{\sqrt{cx^2}x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/(sqrt(c*x^2)*x^3), x)
```

3.787 $\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}$$

```
[Out] -a/(4*x^3*Sqrt[c*x^2]) - b/(3*x^2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0067296, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] -a/(4*x^3*Sqrt[c*x^2]) - b/(3*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0049902, size = 24, normalized size = 0.69

$$\frac{-3a-4bx}{12x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] (-3*a - 4*b*x)/(12*x^3*Sqrt[c*x^2])
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$-\frac{4\,bx+3\,a}{12\,x^{3}}\frac{1}{\sqrt{cx^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(1/2),x)
```

```
[Out] -1/12*(4*b*x+3*a)/x^3/(c*x^2)^(1/2)
```

Maxima [A] time = 1.03886, size = 26, normalized size = 0.74

$$-\frac{b}{3\,\sqrt{c}x^{3}}-\frac{a}{4\,\sqrt{c}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*b/(sqrt(c)*x^3) - 1/4*a/(sqrt(c)*x^4)
```

Fricas [A] time = 1.58097, size = 55, normalized size = 1.57

$$-\frac{\sqrt{cx^{2}}(4\,bx+3\,a)}{12\,cx^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c*x^5)
```

Sympy [A] time = 0.729381, size = 37, normalized size = 1.06

$$-\frac{a}{4\sqrt{c}x^{3}\sqrt{x^{2}}}-\frac{b}{3\sqrt{c}x^{2}\sqrt{x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(1/2),x)
```

```
[Out] -a/(4*sqrt(c)*x**3*sqrt(x**2)) - b/(3*sqrt(c)*x**2*sqrt(x**2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.788 $\int \frac{x^3(a+bx)}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$\frac{ax^2}{c\sqrt{cx^2}}+\frac{bx^3}{2c\sqrt{cx^2}}$$

```
[Out] (a*x^2)/(c*Sqrt[c*x^2]) + (b*x^3)/(2*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0056526, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {15}

$$\frac{ax^2}{c\sqrt{cx^2}}+\frac{bx^3}{2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (a*x^2)/(c*Sqrt[c*x^2]) + (b*x^3)/(2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int (a+bx)\,dx}{c\sqrt{cx^2}}\\ &= \frac{ax^2}{c\sqrt{cx^2}}+\frac{bx^3}{2c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0040282, size = 23, normalized size = 0.61

$$\frac{x^4(2a+bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (x^4*(2*a + b*x))/(2*(c*x^2)^(3/2))
```

Maple [A] time = 0.002, size = 20, normalized size = 0.5

$$\frac{x^{4}\left(bx+2a\right)}{2}\left(cx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] 1/2*x^4*(b*x+2*a)/(c*x^2)^(3/2)
```

Maxima [A] time = 1.0237, size = 43, normalized size = 1.13

$$\frac{bx^{3}}{2\sqrt{cx^{2}}c}+\frac{ax^{2}}{\sqrt{cx^{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^3/(sqrt(c*x^2)*c) + a*x^2/(sqrt(c*x^2)*c)
```

Fricas [A] time = 1.54675, size = 45, normalized size = 1.18

$$\frac{\sqrt{cx^{2}}(bx+2a)}{2c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)/c^2
```

Sympy [A] time = 0.597934, size = 34, normalized size = 0.89

$$\frac{ax^{4}}{c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}+\frac{bx^{5}}{2c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] a*x**4/(c**(3/2)*(x**2)**(3/2)) + b*x**5/(2*c**(3/2)*(x**2)**(3/2))
```

Giac [A] time = 1.07361, size = 34, normalized size = 0.89

$$\frac{\sqrt{cx^{2}}\left(\frac{bx}{c}+\frac{2a}{c}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2)*(b*x/c + 2*a/c)/c
```

3.789 $\int \frac{x^2(a+bx)}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{ax\log(x)}{c\sqrt{cx^2}}+\frac{bx^2}{c\sqrt{cx^2}}$$

```
[Out] (b*x^2)/(c*Sqrt[c*x^2]) + (a*x*Log[x])/(c*Sqrt[c*x^2])
```

Rubi [A] time = 0.005395, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{ax\log(x)}{c\sqrt{cx^2}}+\frac{bx^2}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (b*x^2)/(c*Sqrt[c*x^2]) + (a*x*Log[x])/(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(b+\frac{a}{x}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{bx^2}{c\sqrt{cx^2}}+\frac{ax\log(x)}{c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0039511, size = 21, normalized size = 0.6

$$\frac{x^3(a\log(x)+bx)}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(b*x + a*Log[x]))/(c*x^2)^(3/2)
```

Maple [A] time = 0.002, size = 20, normalized size = 0.6

$$x^{3}\left(bx+a\ln\left(x\right)\right)\left(cx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] 1/(c*x^2)^(3/2)*x^3*(b*x+a*ln(x))
```

Maxima [A] time = 1.07308, size = 31, normalized size = 0.89

$$\frac{bx^{2}}{\sqrt{cx^{2}}c}+\frac{a\log\left(x\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*x^2/(sqrt(c*x^2)*c) + a*log(x)/c^(3/2)
```

Fricas [A] time = 1.60319, size = 51, normalized size = 1.46

$$\frac{\sqrt{cx^{2}}(bx+a\log\left(x\right))}{c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a*log(x))/(c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{2}\left(a+bx\right)}{\left(cx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x)/(c*x**2)**(3/2), x)
```

Giac [A] time = 1.06013, size = 54, normalized size = 1.54

$$-\frac{\frac{a\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{\sqrt{c}}-\frac{\sqrt{cx^2}b}{c}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(a*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/sqrt(c) - sqrt(c*x^2)*b/c)/c
```

3.790 $\int \frac{x(a+bx)}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=33

$$\frac{bx\log(x)}{c\sqrt{cx^2}}-\frac{a}{c\sqrt{cx^2}}$$

```
[Out] -(a/(c*Sqrt[c*x^2])) + (b*x*Log[x])/(c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0067799, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$\frac{bx\log(x)}{c\sqrt{cx^2}}-\frac{a}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] -(a/(c*Sqrt[c*x^2])) + (b*x*Log[x])/(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{c\sqrt{cx^2}}+\frac{bx\log(x)}{c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0029206, size = 22, normalized size = 0.67

$$\frac{x^2(bx\log(x)-a)}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (x^2*(-a + b*x*Log[x]))/(c*x^2)^(3/2)
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$x^2 (b \ln (x) x - a) \left(c x^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] x^2*(b*ln(x)*x-a)/(c*x^2)^(3/2)
```

Maxima [A] time = 1.06939, size = 28, normalized size = 0.85

$$\frac{b \log (x)}{c^{\frac{3}{2}}} - \frac{a}{\sqrt{c x^2} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*log(x)/c^(3/2) - a/(sqrt(c*x^2)*c)
```

Fricas [A] time = 1.61938, size = 54, normalized size = 1.64

$$\frac{\sqrt{c x^2}(b x \log (x) - a)}{c^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x (a + b x)}{\left(c x^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x)/(c*x**2)**(3/2), x)
```

Giac [A] time = 1.0683, size = 63, normalized size = 1.91

$$-\frac{b\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)-\frac{2a\sqrt{c}}{\sqrt{c}x-\sqrt{cx^2}}}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - 2*a*sqrt(c)/(sqrt(c)*x - sqrt(c*x^
2)))/c^(3/2)
```

3.791 $\int \frac{a+bx}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^2}{2acx\sqrt{cx^2}}$$

```
[Out] -(a + b*x)^2/(2*a*c*x*Sqrt[c*x^2])
```

Rubi [A] time = 0.0043576, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2acx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c*x^2)^(3/2),x]
```

```
[Out] -(a + b*x)^2/(2*a*c*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^3}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2acx\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0027527, size = 22, normalized size = 0.76

$$\frac{x(-a-2bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(-a - 2*b*x))/(2*(c*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 17, normalized size = 0.6

$$-\frac{x\,(2\,bx+a)}{2}\left(cx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] -1/2*x*(2*b*x+a)/(c*x^2)^(3/2)
```

Maxima [A] time = 1.06174, size = 31, normalized size = 1.07

$$-\frac{b}{\sqrt{cx^{2}}c}-\frac{a}{2\,c^{\frac{3}{2}}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -b/(sqrt(c*x^2)*c) - 1/2*a/(c^(3/2)*x^2)
```

Fricas [A] time = 1.62874, size = 54, normalized size = 1.86

$$-\frac{\sqrt{cx^{2}}(2\,bx+a)}{2\,c^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c^2*x^3)
```

Sympy [A] time = 0.505787, size = 34, normalized size = 1.17

$$-\frac{ax}{2c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}-\frac{bx^{2}}{c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] -a*x/(2*c**(3/2)*(x**2)**(3/2)) - b*x**2/(c**(3/2)*(x**2)**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.792 $\int \frac{a+bx}{x\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}$$

```
[Out] -a/(3*c*x^2*Sqrt[c*x^2]) - b/(2*c*x*Sqrt[c*x^2])
```

Rubi [A] time = 0.0077756, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x*(c*x^2)^(3/2)),x]
```

```
[Out] -a/(3*c*x^2*Sqrt[c*x^2]) - b/(2*c*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^4}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0087747, size = 25, normalized size = 0.61

$$\frac{cx^2(-2a-3bx)}{6\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (c*x^2*(-2*a - 3*b*x))/(6*(c*x^2)^(5/2))
```

Maple [A] time = 0.003, size = 18, normalized size = 0.4

$$-\frac{3\,bx+2\,a}{6}\left(cx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(3/2),x)
```

```
[Out] -1/6*(3*b*x+2*a)/(c*x^2)^(3/2)
```

Maxima [A] time = 1.12331, size = 26, normalized size = 0.63

$$-\frac{b}{2\,c^{\frac{3}{2}}x^{2}}-\frac{a}{3\,c^{\frac{3}{2}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b/(c^(3/2)*x^2) - 1/3*a/(c^(3/2)*x^3)
```

Fricas [A] time = 1.45478, size = 57, normalized size = 1.39

$$-\frac{\sqrt{cx^{2}}(3\,bx+2\,a)}{6\,c^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c^2*x^4)
```

Sympy [A] time = 0.637379, size = 32, normalized size = 0.78

$$-\frac{a}{3c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}-\frac{bx}{2c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(3/2),x)
```

```
[Out] -a/(3*c**(3/2)*(x**2)**(3/2)) - b*x/(2*c**(3/2)*(x**2)**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{\left(cx^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((c*x^2)^(3/2)*x), x)
```

3.793 $\int \frac{a+bx}{x^2(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}$$

```
[Out] -a/(4*c*x^3*Sqrt[c*x^2]) - b/(3*c*x^2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0074609, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -a/(4*c*x^3*Sqrt[c*x^2]) - b/(3*c*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0082576, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}(3a+4bx)}{12c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]

[Out] -(Sqrt[c*x^2]*(3*a + 4*b*x))/(12*c^2*x^5)
```

Maple [A] time = 0.003, size = 21, normalized size = 0.5

$$-\frac{4\,bx+3\,a}{12\,x}\left(cx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(3/2),x)

[Out] -1/12*(4*b*x+3*a)/x/(c*x^2)^(3/2)
```

Maxima [A] time = 1.04431, size = 26, normalized size = 0.63

$$-\frac{b}{3\,c^{\frac{3}{2}}x^{3}}-\frac{a}{4\,c^{\frac{3}{2}}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="maxima")

[Out] -1/3*b/(c^(3/2)*x^3) - 1/4*a/(c^(3/2)*x^4)
```

Fricas [A] time = 1.58013, size = 58, normalized size = 1.41

$$-\frac{\sqrt{cx^{2}}(4\,bx+3\,a)}{12\,c^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="fricas")

[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c^2*x^5)
```

Sympy [A] time = 0.712332, size = 32, normalized size = 0.78

$$-\frac{a}{4c^{\frac{3}{2}}x\left(x^{2}\right)^{\frac{3}{2}}}-\frac{b}{3c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(3/2),x)

[Out] -a/(4*c**(3/2)*x*(x**2)**(3/2)) - b/(3*c**(3/2)*(x**2)**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.794 $\int \frac{a+bx}{x^3 (cx^2)^{3/2}} dx$

Optimal. Leaf size=41

$$-\frac{a}{5cx^4\sqrt{cx^2}} - \frac{b}{4cx^3\sqrt{cx^2}}$$

```
[Out] -a/(5*c*x^4*Sqrt[c*x^2]) - b/(4*c*x^3*Sqrt[c*x^2])
```

Rubi [A] time = 0.0075206, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{5cx^4\sqrt{cx^2}} - \frac{b}{4cx^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] -a/(5*c*x^4*Sqrt[c*x^2]) - b/(4*c*x^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^3 \left(cx^2\right)^{3/2}} dx &= \frac{x \int \frac{a+bx}{x^6} dx}{c\sqrt{cx^2}} \\
&= \frac{x \int \left(\frac{a}{x^6} + \frac{b}{x^5}\right) dx}{c\sqrt{cx^2}} \\
&= -\frac{a}{5cx^4\sqrt{cx^2}} - \frac{b}{4cx^3\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0079341, size = 22, normalized size = 0.54

$$\frac{c(-4a-5bx)}{20\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] (c*(-4*a - 5*b*x))/(20*(c*x^2)^(5/2))
```

Maple [A] time = 0.003, size = 21, normalized size = 0.5

$$-\frac{5\,bx+4\,a}{20\,x^{2}}\left(cx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(3/2),x)
```

```
[Out] -1/20*(5*b*x+4*a)/x^2/(c*x^2)^(3/2)
```

Maxima [A] time = 1.03121, size = 26, normalized size = 0.63

$$-\frac{b}{4\,c^{\frac{3}{2}}x^{4}}-\frac{a}{5\,c^{\frac{3}{2}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*b/(c^(3/2)*x^4) - 1/5*a/(c^(3/2)*x^5)
```

Fricas [A] time = 1.4947, size = 58, normalized size = 1.41

$$-\frac{\sqrt{cx^{2}}(5\,bx+4\,a)}{20\,c^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(c*x^2)*(5*b*x + 4*a)/(c^2*x^6)
```

Sympy [A] time = 0.859538, size = 36, normalized size = 0.88

$$-\frac{a}{5c^{\frac{3}{2}}x^{2}\left(x^{2}\right)^{\frac{3}{2}}}-\frac{b}{4c^{\frac{3}{2}}x\left(x^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(3/2),x)
```

```
[Out] -a/(5*c**(3/2)*x**2*(x**2)**(3/2)) - b/(4*c**(3/2)*x*(x**2)**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{\left(cx^2\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((c*x^2)^(3/2)*x^3), x)
```

3.795 $\int \frac{a+bx}{x^4(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}$$

```
[Out] -a/(6*c*x^5*Sqrt[c*x^2]) - b/(5*c*x^4*Sqrt[c*x^2])
```

Rubi [A] time = 0.0074406, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]

[Out] -a/(6*c*x^5*Sqrt[c*x^2]) - b/(5*c*x^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^4\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^7}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^7}+\frac{b}{x^6}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0072439, size = 24, normalized size = 0.59

$$\frac{-5a-6bx}{30x^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] (-5*a - 6*b*x)/(30*x^3*(c*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 21, normalized size = 0.5

$$-\frac{6\,bx+5\,a}{30\,x^{3}}\left(cx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(3/2),x)
```

```
[Out] -1/30*(6*b*x+5*a)/x^3/(c*x^2)^(3/2)
```

Maxima [A] time = 1.06289, size = 26, normalized size = 0.63

$$-\frac{b}{5\,c^{\frac{3}{2}}x^{5}}-\frac{a}{6\,c^{\frac{3}{2}}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*b/(c^(3/2)*x^5) - 1/6*a/(c^(3/2)*x^6)
```

Fricas [A] time = 1.49702, size = 58, normalized size = 1.41

$$-\frac{\sqrt{cx^{2}}(6\,bx+5\,a)}{30\,c^{2}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/30*sqrt(c*x^2)*(6*b*x + 5*a)/(c^2*x^7)
```

Sympy [A] time = 1.00212, size = 37, normalized size = 0.9

$$-\frac{a}{6c^{\frac{3}{2}}x^{3}\left(x^{2}\right)^{\frac{3}{2}}}-\frac{b}{5c^{\frac{3}{2}}x^{2}\left(x^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(3/2),x)
```

```
[Out] -a/(6*c**(3/2)*x**3*(x**2)**(3/2)) - b/(5*c**(3/2)*x**2*(x**2)**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.796 $\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=33

$$\frac{bx\log(x)}{c^2\sqrt{cx^2}}-\frac{a}{c^2\sqrt{cx^2}}$$

```
[Out] -(a/(c^2*Sqrt[c*x^2])) + (b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0075182, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{bx\log(x)}{c^2\sqrt{cx^2}}-\frac{a}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -(a/(c^2*Sqrt[c*x^2])) + (b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{c^2\sqrt{cx^2}}+\frac{bx\log(x)}{c^2\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0055764, size = 22, normalized size = 0.67

$$\frac{bx\log(x)-a}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] (-a + b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$x^4 (b\ln(x)\, x - a) \left(cx^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] x^4*(b*ln(x)*x-a)/(c*x^2)^(5/2)
```

Maxima [A] time = 1.10431, size = 32, normalized size = 0.97

$$-\frac{ax^2}{\left(cx^2\right)^{\frac{3}{2}} c} + \frac{b\log(x)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -a*x^2/((c*x^2)^(3/2)*c) + b*log(x)/c^(5/2)
```

Fricas [A] time = 1.50131, size = 54, normalized size = 1.64

$$\frac{\sqrt{cx^2}(bx\log(x) - a)}{c^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (a + bx)}{\left(cx^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*x)/(c*x**2)**(5/2), x)
```

Giac [A] time = 1.10259, size = 63, normalized size = 1.91

$$-\frac{b\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)-\frac{2a\sqrt{c}}{\sqrt{c}x-\sqrt{cx^2}}}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] -(b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - 2*a*sqrt(c)/(sqrt(c)*x - sqrt(c*x^
2)))/c^(5/2)
```

3.797 $\int \frac{x^2(a+bx)}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}$$

```
[Out] -(a + b*x)^2/(2*a*c^2*x*Sqrt[c*x^2])
```

Rubi [A] time = 0.0048614, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -(a + b*x)^2/(2*a*c^2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^3}\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0063874, size = 24, normalized size = 0.83

$$\frac{x^3(-a-2bx)}{2(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] (x^3*(-a - 2*b*x))/(2*(c*x^2)^(5/2))
```

Maple [A] time = 0.003, size = 19, normalized size = 0.7

$$-\frac{x^3(2bx+a)}{2}\left(cx^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] -1/2*x^3*(2*b*x+a)/(c*x^2)^(5/2)
```

Maxima [A] time = 1.01444, size = 35, normalized size = 1.21

$$-\frac{bx^2}{\left(cx^2\right)^{\frac{3}{2}}c}-\frac{a}{2c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -b*x^2/((c*x^2)^(3/2)*c) - 1/2*a/(c^(5/2)*x^2)
```

Fricas [A] time = 1.46749, size = 54, normalized size = 1.86

$$-\frac{\sqrt{cx^2}(2bx+a)}{2c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c^3*x^3)
```

Sympy [A] time = 0.854968, size = 36, normalized size = 1.24

$$-\frac{ax^3}{2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{bx^4}{c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] -a*x**3/(2*c**(5/2)*(x**2)**(5/2)) - b*x**4/(c**(5/2)*(x**2)**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.798 $\int \frac{x(a+bx)}{(cx^2)^{5/2}} \, dx$

Optimal. Leaf size=41

$$-\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}$$

```
[Out] -a/(3*c^2*x^2*Sqrt[c*x^2]) - b/(2*c^2*x*Sqrt[c*x^2])
```

Rubi [A] time = 0.0082408, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$-\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -a/(3*c^2*x^2*Sqrt[c*x^2]) - b/(2*c^2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx)}{(cx^2)^{5/2}} \, dx &= \frac{x\int \frac{a+bx}{x^4} \, dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right) \, dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0037681, size = 24, normalized size = 0.59

$$\frac{x^2(-2a-3bx)}{6(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] (x^2*(-2*a - 3*b*x))/(6*(c*x^2)^(5/2))
```

Maple [A] time = 0.001, size = 21, normalized size = 0.5

$$-\frac{x^2 (3bx + 2a)}{6} \left(cx^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] -1/6*x^2*(3*b*x+2*a)/(c*x^2)^(5/2)
```

Maxima [A] time = 1.069, size = 31, normalized size = 0.76

$$-\frac{a}{3 \left(cx^2\right)^{\frac{3}{2}} c} - \frac{b}{2 c^{\frac{5}{2}} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a/((c*x^2)^(3/2)*c) - 1/2*b/(c^(5/2)*x^2)
```

Fricas [A] time = 1.59097, size = 57, normalized size = 1.39

$$-\frac{\sqrt{cx^2}(3bx + 2a)}{6 c^3 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c^3*x^4)
```

Sympy [A] time = 0.852636, size = 37, normalized size = 0.9

$$-\frac{ax^2}{3c^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}} - \frac{bx^3}{2c^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] -a*x**2/(3*c**(5/2)*(x**2)**(5/2)) - b*x**3/(2*c**(5/2)*(x**2)**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)x}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*x/(c*x^2)^(5/2), x)
```

3.799 $\int \frac{a+bx}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}$$

```
[Out] -a/(4*c^2*x^3*Sqrt[c*x^2]) - b/(3*c^2*x^2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0072754, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$-\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c*x^2)^(5/2),x]
```

```
[Out] -a/(4*c^2*x^3*Sqrt[c*x^2]) - b/(3*c^2*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0027211, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}(3a+4bx)}{12c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c*x^2)^(5/2),x]
```

```
[Out] -(Sqrt[c*x^2]*(3*a + 4*b*x))/(12*c^3*x^5)
```

Maple [A] time = 0.002, size = 19, normalized size = 0.5

$$-\frac{x\,(4\,bx+3\,a)}{12}\left(cx^{2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] -1/12*x*(4*b*x+3*a)/(c*x^2)^(5/2)
```

Maxima [A] time = 1.06456, size = 31, normalized size = 0.76

$$-\frac{b}{3\,\left(cx^{2}\right)^{\frac{3}{2}}c}-\frac{a}{4\,c^{\frac{5}{2}}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*b/((c*x^2)^(3/2)*c) - 1/4*a/(c^(5/2)*x^4)
```

Fricas [A] time = 1.54431, size = 58, normalized size = 1.41

$$-\frac{\sqrt{cx^{2}}(4\,bx+3\,a)}{12\,c^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c^3*x^5)
```

Sympy [A] time = 0.879129, size = 36, normalized size = 0.88

$$-\frac{ax}{4c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}-\frac{bx^{2}}{3c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] -a*x/(4*c**(5/2)*(x**2)**(5/2)) - b*x**2/(3*c**(5/2)*(x**2)**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.800 $\int \frac{a+bx}{x(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}$$

```
[Out] -a/(5*c^2*x^4*Sqrt[c*x^2]) - b/(4*c^2*x^3*Sqrt[c*x^2])
```

Rubi [A] time = 0.0081763, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -a/(5*c^2*x^4*Sqrt[c*x^2]) - b/(4*c^2*x^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^6}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^6}+\frac{b}{x^5}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0079968, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}(4a+5bx)}{20c^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[c*x^2]*(4*a + 5*b*x))/(20*c^3*x^6)
```

Maple [A] time = 0.004, size = 18, normalized size = 0.4

$$-\frac{5\,bx+4\,a}{20}\left(cx^{2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(5/2),x)
```

```
[Out] -1/20*(5*b*x+4*a)/(c*x^2)^(5/2)
```

Maxima [A] time = 1.12213, size = 26, normalized size = 0.63

$$-\frac{b}{4\,c^{\frac{5}{2}}x^{4}}-\frac{a}{5\,c^{\frac{5}{2}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*b/(c^(5/2)*x^4) - 1/5*a/(c^(5/2)*x^5)
```

Fricas [A] time = 1.51195, size = 58, normalized size = 1.41

$$-\frac{\sqrt{cx^{2}}(5\,bx+4\,a)}{20\,c^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(c*x^2)*(5*b*x + 4*a)/(c^3*x^6)
```

Sympy [A] time = 0.994106, size = 32, normalized size = 0.78

$$-\frac{a}{5c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}-\frac{bx}{4c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(5/2),x)
```

```
[Out] -a/(5*c**(5/2)*(x**2)**(5/2)) - b*x/(4*c**(5/2)*(x**2)**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{\left(cx^2\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((c*x^2)^(5/2)*x), x)
```

3.801 $\int \frac{a+bx}{x^2(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}$$

```
[Out] -a/(6*c^2*x^5*Sqrt[c*x^2]) - b/(5*c^2*x^4*Sqrt[c*x^2])
```

Rubi [A] time = 0.0076584, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -a/(6*c^2*x^5*Sqrt[c*x^2]) - b/(5*c^2*x^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^7}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^7}+\frac{b}{x^6}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0071941, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}(5a+6bx)}{30c^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[c*x^2]*(5*a + 6*b*x))/(30*c^3*x^7)
```

Maple [A] time = 0.002, size = 21, normalized size = 0.5

$$-\frac{6\,bx+5\,a}{30\,x}\left(cx^{2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(5/2),x)
```

```
[Out] -1/30*(6*b*x+5*a)/x/(c*x^2)^(5/2)
```

Maxima [A] time = 1.03392, size = 26, normalized size = 0.63

$$-\frac{b}{5\,c^{\frac{5}{2}}x^{5}}-\frac{a}{6\,c^{\frac{5}{2}}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*b/(c^(5/2)*x^5) - 1/6*a/(c^(5/2)*x^6)
```

Fricas [A] time = 1.5722, size = 58, normalized size = 1.41

$$-\frac{\sqrt{cx^{2}}(6\,bx+5\,a)}{30\,c^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*sqrt(c*x^2)*(6*b*x + 5*a)/(c^3*x^7)
```

Sympy [A] time = 1.23448, size = 32, normalized size = 0.78

$$-\frac{a}{6c^{\frac{5}{2}}x\left(x^{2}\right)^{\frac{5}{2}}}-\frac{b}{5c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(5/2),x)
```

```
[Out] -a/(6*c**(5/2)*x*(x**2)**(5/2)) - b/(5*c**(5/2)*(x**2)**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.802 $\int \frac{a+bx}{x^3(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}$$

```
[Out] -a/(7*c^2*x^6*Sqrt[c*x^2]) - b/(6*c^2*x^5*Sqrt[c*x^2])
```

Rubi [A] time = 0.0088245, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] -a/(7*c^2*x^6*Sqrt[c*x^2]) - b/(6*c^2*x^5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^3\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^8}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^8}+\frac{b}{x^7}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0081205, size = 22, normalized size = 0.54

$$\frac{c(-6a-7bx)}{42\left(cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] (c*(-6*a - 7*b*x))/(42*(c*x^2)^(7/2))
```

Maple [A] time = 0.003, size = 21, normalized size = 0.5

$$-\frac{7\,bx+6\,a}{42\,x^{2}}\left(cx^{2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(5/2),x)
```

```
[Out] -1/42*(7*b*x+6*a)/x^2/(c*x^2)^(5/2)
```

Maxima [A] time = 1.08664, size = 26, normalized size = 0.63

$$-\frac{b}{6\,c^{\frac{5}{2}}x^{6}}-\frac{a}{7\,c^{\frac{5}{2}}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b/(c^(5/2)*x^6) - 1/7*a/(c^(5/2)*x^7)
```

Fricas [A] time = 1.60592, size = 58, normalized size = 1.41

$$-\frac{\sqrt{cx^{2}}(7\,bx+6\,a)}{42\,c^{3}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/42*sqrt(c*x^2)*(7*b*x + 6*a)/(c^3*x^8)
```

Sympy [A] time = 1.49165, size = 36, normalized size = 0.88

$$-\frac{a}{7c^{\frac{5}{2}}x^{2}\left(x^{2}\right)^{\frac{5}{2}}}-\frac{b}{6c^{\frac{5}{2}}x\left(x^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(5/2),x)
```

```
[Out] -a/(7*c**(5/2)*x**2*(x**2)**(5/2)) - b/(6*c**(5/2)*x*(x**2)**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{\left(cx^2\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((c*x^2)^(5/2)*x^3), x)
```

3.803 $\int \frac{a+bx}{x^4(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}$$

```
[Out] -a/(8*c^2*x^7*Sqrt[c*x^2]) - b/(7*c^2*x^6*Sqrt[c*x^2])
```

Rubi [A] time = 0.0083773, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] -a/(8*c^2*x^7*Sqrt[c*x^2]) - b/(7*c^2*x^6*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^4\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^9}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^9}+\frac{b}{x^8}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0066191, size = 24, normalized size = 0.59

$$\frac{-7a-8bx}{56x^3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] (-7*a - 8*b*x)/(56*x^3*(c*x^2)^(5/2))
```

Maple [A] time = 0.003, size = 21, normalized size = 0.5

$$-\frac{8\,bx+7\,a}{56\,x^3}\left(cx^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(5/2),x)
```

```
[Out] -1/56*(8*b*x+7*a)/x^3/(c*x^2)^(5/2)
```

Maxima [A] time = 1.0884, size = 26, normalized size = 0.63

$$-\frac{b}{7\,c^{\frac{5}{2}}x^7}-\frac{a}{8\,c^{\frac{5}{2}}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/7*b/(c^(5/2)*x^7) - 1/8*a/(c^(5/2)*x^8)
```

Fricas [A] time = 1.48948, size = 58, normalized size = 1.41

$$-\frac{\sqrt{cx^2}(8\,bx+7\,a)}{56\,c^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/56*sqrt(c*x^2)*(8*b*x + 7*a)/(c^3*x^9)
```

Sympy [A] time = 1.7776, size = 37, normalized size = 0.9

$$-\frac{a}{8c^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}}-\frac{b}{7c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(5/2),x)
```

```
[Out] -a/(8*c**(5/2)*x**3*(x**2)**(5/2)) - b/(7*c**(5/2)*x**2*(x**2)**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.804 $\int x^3\sqrt{cx^2}(a+bx)^2\,dx$

Optimal. Leaf size=57

$$\frac{1}{5}a^2x^4\sqrt{cx^2}+\frac{1}{3}abx^5\sqrt{cx^2}+\frac{1}{7}b^2x^6\sqrt{cx^2}$$

```
[Out] (a^2*x^4*Sqrt[c*x^2])/5 + (a*b*x^5*Sqrt[c*x^2])/3 + (b^2*x^6*Sqrt[c*x^2])/7
```

Rubi [A] time = 0.0154403, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{1}{5}a^2x^4\sqrt{cx^2}+\frac{1}{3}abx^5\sqrt{cx^2}+\frac{1}{7}b^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^4*Sqrt[c*x^2])/5 + (a*b*x^5*Sqrt[c*x^2])/3 + (b^2*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{cx^2}(a+bx)^2\,dx &= \frac{\sqrt{cx^2}\int x^4(a+bx)^2\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int\left(a^2x^4+2abx^5+b^2x^6\right)dx}{x}\\ &= \frac{1}{5}a^2x^4\sqrt{cx^2}+\frac{1}{3}abx^5\sqrt{cx^2}+\frac{1}{7}b^2x^6\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0055549, size = 35, normalized size = 0.61

$$\frac{1}{105}x^4\sqrt{cx^2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x^4*Sqrt[c*x^2]*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

Maple [A] time = 0.002, size = 32, normalized size = 0.6

$$\frac{x^4 \left(15 b^2 x^2 + 35 abx + 21 a^2\right)}{105} \sqrt{c x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2*(c*x^2)^(1/2),x)
```

```
[Out] 1/105*x^4*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51526, size = 78, normalized size = 1.37

$$\frac{1}{105} \left(15 b^2 x^6 + 35 abx^5 + 21 a^2 x^4\right) \sqrt{c x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*x^6 + 35*a*b*x^5 + 21*a^2*x^4)*sqrt(c*x^2)
```

Sympy [A] time = 0.525169, size = 60, normalized size = 1.05

$$\frac{a^2 \sqrt{c} x^4 \sqrt{x^2}}{5} + \frac{ab \sqrt{c} x^5 \sqrt{x^2}}{3} + \frac{b^2 \sqrt{c} x^6 \sqrt{x^2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x**4*sqrt(x**2)/5 + a*b*sqrt(c)*x**5*sqrt(x**2)/3 + b**2*sqrt(
c)*x**6*sqrt(x**2)/7
```

Giac [A] time = 1.0584, size = 47, normalized size = 0.82

$$\frac{1}{105} \left(15 \, b^2 x^7 \operatorname{sgn}(x) + 35 \, abx^6 \operatorname{sgn}(x) + 21 \, a^2 x^5 \operatorname{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*x^7*sgn(x) + 35*a*b*x^6*sgn(x) + 21*a^2*x^5*sgn(x))*sqrt(c)
```

3.805 $\int x^2\sqrt{cx^2}(a+bx)^2\,dx$

Optimal. Leaf size=57

$$\frac{1}{4}a^2x^3\sqrt{cx^2}+\frac{2}{5}abx^4\sqrt{cx^2}+\frac{1}{6}b^2x^5\sqrt{cx^2}$$

```
[Out] (a^2*x^3*Sqrt[c*x^2])/4 + (2*a*b*x^4*Sqrt[c*x^2])/5 + (b^2*x^5*Sqrt[c*x^2])
/6
```

Rubi [A] time = 0.0146784, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{1}{4}a^2x^3\sqrt{cx^2}+\frac{2}{5}abx^4\sqrt{cx^2}+\frac{1}{6}b^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3*Sqrt[c*x^2])/4 + (2*a*b*x^4*Sqrt[c*x^2])/5 + (b^2*x^5*Sqrt[c*x^2])
/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{cx^2}(a+bx)^2\,dx&=\frac{\sqrt{cx^2}\int x^3(a+bx)^2\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(a^2x^3+2abx^4+b^2x^5\right)dx}{x}\\&=\frac{1}{4}a^2x^3\sqrt{cx^2}+\frac{2}{5}abx^4\sqrt{cx^2}+\frac{1}{6}b^2x^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.005374, size = 35, normalized size = 0.61

$$\frac{1}{60}x^3\sqrt{cx^2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x^3*Sqrt[c*x^2]*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

Maple [A] time = 0.003, size = 32, normalized size = 0.6

$$\frac{x^3\left(10\,b^2x^2+24\,abx+15\,a^2\right)}{60}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2*(c*x^2)^(1/2),x)
```

```
[Out] 1/60*x^3*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60178, size = 77, normalized size = 1.35

$$\frac{1}{60}\left(10\,b^2x^5+24\,abx^4+15\,a^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*x^5 + 24*a*b*x^4 + 15*a^2*x^3)*sqrt(c*x^2)
```

Sympy [A] time = 0.406481, size = 61, normalized size = 1.07

$$\frac{a^2\sqrt{c}x^3\sqrt{x^2}}{4}+\frac{2ab\sqrt{c}x^4\sqrt{x^2}}{5}+\frac{b^2\sqrt{c}x^5\sqrt{x^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x**3*sqrt(x**2)/4 + 2*a*b*sqrt(c)*x**4*sqrt(x**2)/5 + b**2*sqr
t(c)*x**5*sqrt(x**2)/6
```

Giac [A] time = 1.06048, size = 47, normalized size = 0.82

$$\frac{1}{60}\left(10\,b^2x^6\mathrm{sgn}\left(x\right)+24\,abx^5\mathrm{sgn}\left(x\right)+15\,a^2x^4\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/60*(10*b^2*x^6*sgn(x) + 24*a*b*x^5*sgn(x) + 15*a^2*x^4*sgn(x))*sqrt(c)
```

3.806 $\int x\sqrt{cx^2}(a+bx)^2\,dx$

Optimal. Leaf size=57

$$\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}$$

```
[Out] (a^2*x^2*Sqrt[c*x^2])/3 + (a*b*x^3*Sqrt[c*x^2])/2 + (b^2*x^4*Sqrt[c*x^2])/5
```

Rubi [A] time = 0.0134847, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2*Sqrt[c*x^2])/3 + (a*b*x^3*Sqrt[c*x^2])/2 + (b^2*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{cx^2}(a+bx)^2\,dx&=\frac{\sqrt{cx^2}\int x^2(a+bx)^2\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(a^2x^2+2abx^3+b^2x^4\right)dx}{x}\\&=\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0052016, size = 35, normalized size = 0.61

$$\frac{1}{30}x^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

Maple [A] time = 0.003, size = 32, normalized size = 0.6

$$\frac{x^2 \left(6 b^2 x^2 + 15 a b x + 10 a^2\right)}{30} \sqrt{c x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2*(c*x^2)^(1/2),x)
```

```
[Out] 1/30*x^2*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50877, size = 76, normalized size = 1.33

$$\frac{1}{30} \left(6 b^2 x^4 + 15 a b x^3 + 10 a^2 x^2\right) \sqrt{c x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^2*x^4 + 15*a*b*x^3 + 10*a^2*x^2)*sqrt(c*x^2)
```

Sympy [A] time = 0.347242, size = 60, normalized size = 1.05

$$\frac{a^2 \sqrt{c} x^2 \sqrt{x^2}}{3} + \frac{a b \sqrt{c} x^3 \sqrt{x^2}}{2} + \frac{b^2 \sqrt{c} x^4 \sqrt{x^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x**2*sqrt(x**2)/3 + a*b*sqrt(c)*x**3*sqrt(x**2)/2 + b**2*sqrt(
c)*x**4*sqrt(x**2)/5
```

Giac [A] time = 1.05721, size = 47, normalized size = 0.82

$$\frac{1}{30}\left(6\,b^2x^5\mathrm{sgn}\left(x\right)+15\,abx^4\mathrm{sgn}\left(x\right)+10\,a^2x^3\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*x^5*sgn(x) + 15*a*b*x^4*sgn(x) + 10*a^2*x^3*sgn(x))*sqrt(c)
```

3.807 $\int \sqrt{cx^2}(a+bx)^2\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}a^2x\sqrt{cx^2}+\frac{2}{3}abx^2\sqrt{cx^2}+\frac{1}{4}b^2x^3\sqrt{cx^2}$$

```
[Out] (a^2*x*Sqrt[c*x^2])/2 + (2*a*b*x^2*Sqrt[c*x^2])/3 + (b^2*x^3*Sqrt[c*x^2])/4
```

Rubi [A] time = 0.0136003, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{1}{2}a^2x\sqrt{cx^2}+\frac{2}{3}abx^2\sqrt{cx^2}+\frac{1}{4}b^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x*Sqrt[c*x^2])/2 + (2*a*b*x^2*Sqrt[c*x^2])/3 + (b^2*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{cx^2}(a+bx)^2\,dx &= \frac{\sqrt{cx^2}\int x(a+bx)^2\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(a^2x+2abx^2+b^2x^3\right)dx}{x}\\
&= \frac{1}{2}a^2x\sqrt{cx^2}+\frac{2}{3}abx^2\sqrt{cx^2}+\frac{1}{4}b^2x^3\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.0063803, size = 33, normalized size = 0.6

$$\frac{1}{12}x\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

Maple [A] time = 0.001, size = 30, normalized size = 0.6

$$\frac{x\left(3\,b^{2}x^{2}+8\,abx+6\,a^{2}\right)}{12}\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2),x)
```

```
[Out] 1/12*x*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54565, size = 70, normalized size = 1.27

$$\frac{1}{12}\left(3\,b^{2}x^{3}+8\,abx^{2}+6\,a^{2}x\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*x^3 + 8*a*b*x^2 + 6*a^2*x)*sqrt(c*x^2)
```

Sympy [A] time = 0.258771, size = 60, normalized size = 1.09

$$\frac{a^{2}\sqrt{c}x\sqrt{x^{2}}}{2}+\frac{2ab\sqrt{c}x^{2}\sqrt{x^{2}}}{3}+\frac{b^{2}\sqrt{c}x^{3}\sqrt{x^{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x*sqrt(x**2)/2 + 2*a*b*sqrt(c)*x**2*sqrt(x**2)/3 + b**2*sqrt(c
)*x**3*sqrt(x**2)/4
```

Giac [A] time = 1.07227, size = 47, normalized size = 0.85

$$\frac{1}{12}\left(3\,b^2x^4\mathrm{sgn}\left(x\right)+8\,abx^3\mathrm{sgn}\left(x\right)+6\,a^2x^2\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*x^4*sgn(x) + 8*a*b*x^3*sgn(x) + 6*a^2*x^2*sgn(x))*sqrt(c)
```

3.808 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x} dx$

Optimal. Leaf size=26

$$\frac{\sqrt{cx^2}(a+bx)^3}{3bx}$$

[Out] (Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)

Rubi [A] time = 0.0036536, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 32}

$$\frac{\sqrt{cx^2}(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]

[Out] (Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{cx^2}(a+bx)^2}{x} dx &= \frac{\sqrt{cx^2} \int (a+bx)^2 dx}{x} \\ &= \frac{\sqrt{cx^2}(a+bx)^3}{3bx} \end{aligned}$$

Mathematica [A] time = 0.005424, size = 25, normalized size = 0.96

$$\frac{cx(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]

[Out] (c*x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])

Maple [A] time = 0.003, size = 28, normalized size = 1.1

$$\frac{b^2x^2 + 3\,abx + 3\,a^2}{3}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x,x)
```

```
[Out] 1/3*(b^2*x^2+3*a*b*x+3*a^2)*(c*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55154, size = 61, normalized size = 2.35

$$\frac{1}{3}\left(b^2x^2 + 3\,abx + 3\,a^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)
```

Sympy [B] time = 0.258034, size = 51, normalized size = 1.96

$$a^2\sqrt{c}\sqrt{x^2} + ab\sqrt{c}x\sqrt{x^2} + \frac{b^2\sqrt{c}x^2\sqrt{x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x,x)
```

```
[Out] a**2*sqrt(c)*sqrt(x**2) + a*b*sqrt(c)*x*sqrt(x**2) + b**2*sqrt(c)*x**2*sqrt
(x**2)/3
```

Giac [A] time = 1.04565, size = 39, normalized size = 1.5

$$\frac{1}{3}\left(\frac{(bx+a)^3\mathrm{sgn}\,(x)}{b} - \frac{a^3\mathrm{sgn}\,(x)}{b}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/3*((b*x + a)^3*sgn(x)/b - a^3*sgn(x)/b)*sqrt(c)
```

3.809 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2} \, dx$

Optimal. Leaf size=49

$$\frac{a^2\sqrt{cx^2}\log(x)}{x}+2ab\sqrt{cx^2}+\frac{1}{2}b^2x\sqrt{cx^2}$$

```
[Out] 2*a*b*Sqrt[c*x^2] + (b^2*x*Sqrt[c*x^2])/2 + (a^2*Sqrt[c*x^2]*Log[x])/x
```

Rubi [A] time = 0.009421, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2\sqrt{cx^2}\log(x)}{x}+2ab\sqrt{cx^2}+\frac{1}{2}b^2x\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
```

```
[Out] 2*a*b*Sqrt[c*x^2] + (b^2*x*Sqrt[c*x^2])/2 + (a^2*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{x}\\
&= 2ab\sqrt{cx^2}+\frac{1}{2}b^2x\sqrt{cx^2}+\frac{a^2\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A] time = 0.0097211, size = 33, normalized size = 0.67

$$\frac{cx\left(2a^2\log(x)+bx(4a+bx)\right)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
```

```
[Out] (c*x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

Maple [A] time = 0.006, size = 33, normalized size = 0.7

$$\frac{b^2x^2 + 2\,a^2\ln\left(x\right) + 4\,abx}{2\,x}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^2,x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58105, size = 73, normalized size = 1.49

$$\frac{\left(b^2x^2 + 4\,abx + 2\,a^2\log\left(x\right)\right)\sqrt{cx^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}\left(a + bx\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**2, x)
```

Giac [A] time = 1.0843, size = 43, normalized size = 0.88

$$\frac{1}{2}\left(b^2x^2\mathrm{sgn}\,(x) + 4\,abx\mathrm{sgn}\,(x) + 2\,a^2\log\left(|x|\right)\mathrm{sgn}\,(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2*sgn(x) + 4*a*b*x*sgn(x) + 2*a^2*log(abs(x))*sgn(x))*sqrt(c)
```

3.810 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx$

Optimal. Leaf size=49

$$-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\log(x)}{x}+b^2\sqrt{cx^2}$$

```
[Out] b^2*Sqrt[c*x^2] - (a^2*Sqrt[c*x^2])/x^2 + (2*a*b*Sqrt[c*x^2]*Log[x])/x
```

Rubi [A] time = 0.0113116, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\log(x)}{x}+b^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
```

```
[Out] b^2*Sqrt[c*x^2] - (a^2*Sqrt[c*x^2])/x^2 + (2*a*b*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x^2}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx}{x}\\ &= b^2\sqrt{cx^2}-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\log(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.0119556, size = 31, normalized size = 0.63

$$\frac{c\left(-a^2+2abx\log(x)+b^2x^2\right)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
```

```
[Out] (c*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/Sqrt[c*x^2]
```

Maple [A] time = 0.009, size = 32, normalized size = 0.7

$$\frac{2\,ab\ln(x)\,x+b^2x^2-a^2}{x^2}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^3,x)
```

```
[Out] (c*x^2)^(1/2)*(2*a*b*ln(x)*x+b^2*x^2-a^2)/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.73218, size = 68, normalized size = 1.39

$$\frac{\left(b^2x^2+2\,abx\log(x)-a^2\right)\sqrt{cx^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}\,(a+bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**3, x)
```

Giac [A] time = 1.06002, size = 42, normalized size = 0.86

$$\left(b^2x\text{sgn}\left(x\right)+2\,ab\log\left(|x|\right)\text{sgn}\left(x\right)-\frac{a^2\text{sgn}\left(x\right)}{x}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (b^2*x*sgn(x) + 2*a*b*log(abs(x))*sgn(x) - a^2*sgn(x)/x)*sqrt(c)
```

3.811 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4} dx$

Optimal. Leaf size=54

$$-\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{x}$$

```
[Out] -(a^2*Sqrt[c*x^2])/(2*x^3) - (2*a*b*Sqrt[c*x^2])/x^2 + (b^2*Sqrt[c*x^2]*Log
[x])/x
```

Rubi [A] time = 0.0115592, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
```

```
[Out] -(a^2*Sqrt[c*x^2])/(2*x^3) - (2*a*b*Sqrt[c*x^2])/x^2 + (b^2*Sqrt[c*x^2]*Log
[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4}\,dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x^3}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{x}\\
&= -\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A] time = 0.0092095, size = 36, normalized size = 0.67

$$\frac{\sqrt{cx^2}\left(2b^2x^2\log(x)-a(a+4bx)\right)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
```

```
[Out] (Sqrt[c*x^2]*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*x^3)
```

Maple [A] time = 0.008, size = 34, normalized size = 0.6

$$\frac{2\,b^{2}\ln\left(x\right)x^{2}-4\,abx-a^{2}}{2\,x^{3}}\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^4,x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.60008, size = 76, normalized size = 1.41

$$\frac{\left(2\,b^{2}x^{2}\log\left(x\right)-4\,abx-a^{2}\right)\sqrt{cx^{2}}}{2\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx^{2}}\left(a+bx\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**4, x)
```

Giac [A] time = 1.06497, size = 47, normalized size = 0.87

$$\frac{1}{2}\left(2\,b^2\log\left(|x|\right)\operatorname{sgn}\left(x\right)-\frac{4\,abx\operatorname{sgn}\left(x\right)+a^2\operatorname{sgn}\left(x\right)}{x^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*log(abs(x))*sgn(x) - (4*a*b*x*sgn(x) + a^2*sgn(x))/x^2)*sqrt(c)
```

3.812 $\int x^3 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=60

$$\frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}$$

```
[Out] (a^2*c*x^6*Sqrt[c*x^2])/7 + (a*b*c*x^7*Sqrt[c*x^2])/4 + (b^2*c*x^8*Sqrt[c*x
^2])/9
```

Rubi [A] time = 0.0187081, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^6*Sqrt[c*x^2])/7 + (a*b*c*x^7*Sqrt[c*x^2])/4 + (b^2*c*x^8*Sqrt[c*x
^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^6(a+bx)^2 \, dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^6+2abx^7+b^2x^8\right) dx}{x}\\ &= \frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0084166, size = 35, normalized size = 0.58

$$\frac{1}{252}x^4\left(cx^2\right)^{3/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x^4*(c*x^2)^(3/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

Maple [A] time = 0.005, size = 32, normalized size = 0.5

$$\frac{x^4\left(28\,b^2x^2+63\,abx+36\,a^2\right)}{252}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/252*x^4*(28*b^2*x^2+63*a*b*x+36*a^2)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48707, size = 86, normalized size = 1.43

$$\frac{1}{252}\left(28\,b^2cx^8+63\,abcx^7+36\,a^2cx^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^2*c*x^8 + 63*a*b*c*x^7 + 36*a^2*c*x^6)*sqrt(c*x^2)
```

Sympy [A] time = 1.38573, size = 60, normalized size = 1.

$$\frac{a^2c^{\frac{3}{2}}x^4\left(x^2\right)^{\frac{3}{2}}}{7}+\frac{abc^{\frac{3}{2}}x^5\left(x^2\right)^{\frac{3}{2}}}{4}+\frac{b^2c^{\frac{3}{2}}x^6\left(x^2\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(3/2)*x**4*(x**2)**(3/2)/7 + a*b*c**(3/2)*x**5*(x**2)**(3/2)/4 + b*
*2*c**(3/2)*x**6*(x**2)**(3/2)/9
```

Giac [A] time = 1.05908, size = 47, normalized size = 0.78

$$\frac{1}{252}\left(28\,b^2x^9\mathrm{sgn}\left(x\right)+63\,abx^8\mathrm{sgn}\left(x\right)+36\,a^2x^7\mathrm{sgn}\left(x\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/252*(28*b^2*x^9*sgn(x) + 63*a*b*x^8*sgn(x) + 36*a^2*x^7*sgn(x))*c^(3/2)
```

3.813 $\int x^2 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=60

$$\frac{1}{6}a^2cx^5\sqrt{cx^2}+\frac{2}{7}abcx^6\sqrt{cx^2}+\frac{1}{8}b^2cx^7\sqrt{cx^2}$$

```
[Out] (a^2*c*x^5*Sqrt[c*x^2])/6 + (2*a*b*c*x^6*Sqrt[c*x^2])/7 + (b^2*c*x^7*Sqrt[c
*x^2])/8
```

Rubi [A] time = 0.017045, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{1}{6}a^2cx^5\sqrt{cx^2}+\frac{2}{7}abcx^6\sqrt{cx^2}+\frac{1}{8}b^2cx^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^5*Sqrt[c*x^2])/6 + (2*a*b*c*x^6*Sqrt[c*x^2])/7 + (b^2*c*x^7*Sqrt[c
*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^5(a+bx)^2 \, dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^5+2abx^6+b^2x^7\right) dx}{x}\\
&= \frac{1}{6}a^2cx^5\sqrt{cx^2}+\frac{2}{7}abcx^6\sqrt{cx^2}+\frac{1}{8}b^2cx^7\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.0088444, size = 35, normalized size = 0.58

$$\frac{1}{168}x^3\left(cx^2\right)^{3/2}\left(28a^2+48abx+21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x^3*(c*x^2)^(3/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

Maple [A] time = 0.005, size = 32, normalized size = 0.5

$$\frac{x^3 \left(21 b^2 x^2 + 48 abx + 28 a^2\right)}{168} \left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/168*x^3*(21*b^2*x^2+48*a*b*x+28*a^2)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45785, size = 86, normalized size = 1.43

$$\frac{1}{168} \left(21 b^2 cx^7 + 48 abcx^6 + 28 a^2 cx^5\right) \sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(21*b^2*c*x^7 + 48*a*b*c*x^6 + 28*a^2*c*x^5)*sqrt(c*x^2)
```

Sympy [A] time = 1.13822, size = 61, normalized size = 1.02

$$\frac{a^2 c^{\frac{3}{2}} x^3 \left(x^2\right)^{\frac{3}{2}}}{6} + \frac{2abc^{\frac{3}{2}} x^4 \left(x^2\right)^{\frac{3}{2}}}{7} + \frac{b^2 c^{\frac{3}{2}} x^5 \left(x^2\right)^{\frac{3}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(3/2)*x**3*(x**2)**(3/2)/6 + 2*a*b*c**(3/2)*x**4*(x**2)**(3/2)/7 +
b**2*c**(3/2)*x**5*(x**2)**(3/2)/8
```

Giac [A] time = 1.05139, size = 47, normalized size = 0.78

$$\frac{1}{168}\left(21\,b^2x^8\mathrm{sgn}\left(x\right)+48\,abx^7\mathrm{sgn}\left(x\right)+28\,a^2x^6\mathrm{sgn}\left(x\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/168*(21*b^2*x^8*sgn(x) + 48*a*b*x^7*sgn(x) + 28*a^2*x^6*sgn(x))*c^(3/2)
```

3.814 $\int x\left(cx^2\right)^{3/2}(a+bx)^2\,dx$

Optimal. Leaf size=60

$$\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}$$

```
[Out] (a^2*c*x^4*Sqrt[c*x^2])/5 + (a*b*c*x^5*Sqrt[c*x^2])/3 + (b^2*c*x^6*Sqrt[c*x
^2])/7
```

Rubi [A] time = 0.0161078, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^4*Sqrt[c*x^2])/5 + (a*b*c*x^5*Sqrt[c*x^2])/3 + (b^2*c*x^6*Sqrt[c*x
^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{3/2}(a+bx)^2\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)^2\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(a^2x^4+2abx^5+b^2x^6\right)dx}{x}\\&=\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0075748, size = 35, normalized size = 0.58

$$\frac{1}{105}x^2\left(cx^2\right)^{3/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x^2*(c*x^2)^(3/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

Maple [A] time = 0.004, size = 32, normalized size = 0.5

$$\frac{x^2 \left(15 b^2 x^2 + 35 abx + 21 a^2\right)}{105} \left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/105*x^2*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42549, size = 86, normalized size = 1.43

$$\frac{1}{105} \left(15 b^2 c x^6 + 35 abc x^5 + 21 a^2 c x^4\right) \sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*c*x^6 + 35*a*b*c*x^5 + 21*a^2*c*x^4)*sqrt(c*x^2)
```

Sympy [A] time = 0.877928, size = 60, normalized size = 1.

$$\frac{a^2 c^{\frac{3}{2}} x^2 \left(x^2\right)^{\frac{3}{2}}}{5} + \frac{abc^{\frac{3}{2}} x^3 \left(x^2\right)^{\frac{3}{2}}}{3} + \frac{b^2 c^{\frac{3}{2}} x^4 \left(x^2\right)^{\frac{3}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(3/2)*x**2*(x**2)**(3/2)/5 + a*b*c**(3/2)*x**3*(x**2)**(3/2)/3 + b*
*2*c**(3/2)*x**4*(x**2)**(3/2)/7
```

Giac [A] time = 1.0684, size = 47, normalized size = 0.78

$$\frac{1}{105}\left(15\,b^2x^7\mathrm{sgn}\left(x\right)+35\,abx^6\mathrm{sgn}\left(x\right)+21\,a^2x^5\mathrm{sgn}\left(x\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*x^7*sgn(x) + 35*a*b*x^6*sgn(x) + 21*a^2*x^5*sgn(x))*c^(3/2)
```

3.815 $\int \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=60

$$\frac{1}{4}a^2cx^3\sqrt{cx^2}+\frac{2}{5}abcx^4\sqrt{cx^2}+\frac{1}{6}b^2cx^5\sqrt{cx^2}$$

```
[Out] (a^2*c*x^3*Sqrt[c*x^2])/4 + (2*a*b*c*x^4*Sqrt[c*x^2])/5 + (b^2*c*x^5*Sqrt[c
*x^2])/6
```

Rubi [A] time = 0.0146034, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{1}{4}a^2cx^3\sqrt{cx^2}+\frac{2}{5}abcx^4\sqrt{cx^2}+\frac{1}{6}b^2cx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^3*Sqrt[c*x^2])/4 + (2*a*b*c*x^4*Sqrt[c*x^2])/5 + (b^2*c*x^5*Sqrt[c
*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(cx^2\right)^{3/2} (a+bx)^2 \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(a+bx)^2 \, dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^3+2abx^4+b^2x^5\right) dx}{x}\\ &= \frac{1}{4}a^2cx^3\sqrt{cx^2}+\frac{2}{5}abcx^4\sqrt{cx^2}+\frac{1}{6}b^2cx^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0065863, size = 33, normalized size = 0.55

$$\frac{1}{60}x\left(cx^2\right)^{3/2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

Maple [A] time = 0.003, size = 30, normalized size = 0.5

$$\frac{x\left(10\,b^2x^2+24\,abx+15\,a^2\right)}{60}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/60*x*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54387, size = 85, normalized size = 1.42

$$\frac{1}{60}\left(10\,b^2cx^5+24\,abcx^4+15\,a^2cx^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*c*x^5 + 24*a*b*c*x^4 + 15*a^2*c*x^3)*sqrt(c*x^2)
```

Sympy [A] time = 0.73236, size = 60, normalized size = 1.

$$\frac{a^2c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}{4}+\frac{2abc^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}{5}+\frac{b^2c^{\frac{3}{2}}x^3\left(x^2\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(3/2)*x*(x**2)**(3/2)/4 + 2*a*b*c**(3/2)*x**2*(x**2)**(3/2)/5 + b**
2*c**(3/2)*x**3*(x**2)**(3/2)/6
```

Giac [A] time = 1.0478, size = 47, normalized size = 0.78

$$\frac{1}{60}\left(10\,b^2x^6\mathrm{sgn}\left(x\right)+24\,abx^5\mathrm{sgn}\left(x\right)+15\,a^2x^4\mathrm{sgn}\left(x\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^2*x^6*sgn(x) + 24*a*b*x^5*sgn(x) + 15*a^2*x^4*sgn(x))*c^(3/2)
```

3.816 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x} dx$

Optimal. Leaf size=60

$$\frac{1}{3}a^2cx^2\sqrt{cx^2} + \frac{1}{2}abcx^3\sqrt{cx^2} + \frac{1}{5}b^2cx^4\sqrt{cx^2}$$

```
[Out] (a^2*c*x^2*Sqrt[c*x^2])/3 + (a*b*c*x^3*Sqrt[c*x^2])/2 + (b^2*c*x^4*Sqrt[c*x
^2])/5
```

Rubi [A] time = 0.0146076, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{1}{3}a^2cx^2\sqrt{cx^2} + \frac{1}{2}abcx^3\sqrt{cx^2} + \frac{1}{5}b^2cx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
```

```
[Out] (a^2*c*x^2*Sqrt[c*x^2])/3 + (a*b*c*x^3*Sqrt[c*x^2])/2 + (b^2*c*x^4*Sqrt[c*x
^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)^2}{x}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)^2\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^2+2abx^3+b^2x^4\right)dx}{x}\\
&= \frac{1}{3}a^2cx^2\sqrt{cx^2} + \frac{1}{2}abcx^3\sqrt{cx^2} + \frac{1}{5}b^2cx^4\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.0039654, size = 36, normalized size = 0.6

$$\frac{1}{30}cx^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
```

```
[Out] (c*x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

Maple [A] time = 0.003, size = 29, normalized size = 0.5

$$\frac{6\,b^2x^2+15\,abx+10\,a^2}{30}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x,x)
```

```
[Out] 1/30*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51166, size = 84, normalized size = 1.4

$$\frac{1}{30}\left(6\,b^2cx^4+15\,abcx^3+10\,a^2cx^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^2*c*x^4 + 15*a*b*c*x^3 + 10*a^2*c*x^2)*sqrt(c*x^2)
```

Sympy [A] time = 0.687988, size = 54, normalized size = 0.9

$$\frac{a^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{3}+\frac{abc^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}{2}+\frac{b^2c^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x,x)
```

```
[Out] a**2*c**(3/2)*(x**2)**(3/2)/3 + a*b*c**(3/2)*x*(x**2)**(3/2)/2 + b**2*c**(3
/2)*x**2*(x**2)**(3/2)/5
```

Giac [A] time = 1.08133, size = 47, normalized size = 0.78

$$\frac{1}{30}\left(6\,b^2x^5\mathrm{sgn}\left(x\right)+15\,abx^4\mathrm{sgn}\left(x\right)+10\,a^2x^3\mathrm{sgn}\left(x\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*x^5*sgn(x) + 15*a*b*x^4*sgn(x) + 10*a^2*x^3*sgn(x))*c^(3/2)
```

3.817 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2} dx$

Optimal. Leaf size=58

$$\frac{1}{2}a^2cx\sqrt{cx^2}+\frac{2}{3}abcx^2\sqrt{cx^2}+\frac{1}{4}b^2cx^3\sqrt{cx^2}$$

```
[Out] (a^2*c*x*Sqrt[c*x^2])/2 + (2*a*b*c*x^2*Sqrt[c*x^2])/3 + (b^2*c*x^3*Sqrt[c*x
^2])/4
```

Rubi [A] time = 0.0131542, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{1}{2}a^2cx\sqrt{cx^2}+\frac{2}{3}abcx^2\sqrt{cx^2}+\frac{1}{4}b^2cx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (a^2*c*x*Sqrt[c*x^2])/2 + (2*a*b*c*x^2*Sqrt[c*x^2])/3 + (b^2*c*x^3*Sqrt[c*x
^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)^2\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x+2abx^2+b^2x^3\right)dx}{x}\\ &= \frac{1}{2}a^2cx\sqrt{cx^2}+\frac{2}{3}abcx^2\sqrt{cx^2}+\frac{1}{4}b^2cx^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0036646, size = 34, normalized size = 0.59

$$\frac{1}{12}cx\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (c*x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

Maple [A] time = 0.002, size = 32, normalized size = 0.6

$$\frac{3\,b^2x^2+8\,abx+6\,a^2}{12\,x}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^2,x)
```

```
[Out] 1/12/x*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41586, size = 78, normalized size = 1.34

$$\frac{1}{12}\left(3\,b^2cx^3+8\,abcx^2+6\,a^2cx\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c*x^3 + 8*a*b*c*x^2 + 6*a^2*c*x)*sqrt(c*x^2)
```

Sympy [A] time = 0.709454, size = 54, normalized size = 0.93

$$\frac{a^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{2x}+\frac{2abc^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{3}+\frac{b^2c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**2,x)
```

```
[Out] a**2*c**(3/2)*(x**2)**(3/2)/(2*x) + 2*a*b*c**(3/2)*(x**2)**(3/2)/3 + b**2*c
**(3/2)*x*(x**2)**(3/2)/4
```

Giac [A] time = 1.05584, size = 47, normalized size = 0.81

$$\frac{1}{12}\left(3\,b^2x^4\mathrm{sgn}\left(x\right)+8\,abx^3\mathrm{sgn}\left(x\right)+6\,a^2x^2\mathrm{sgn}\left(x\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*x^4*sgn(x) + 8*a*b*x^3*sgn(x) + 6*a^2*x^2*sgn(x))*c^(3/2)
```

3.818 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3} dx$

Optimal. Leaf size=27

$$\frac{c\sqrt{cx^2}(a+bx)^3}{3bx}$$

```
[Out] (c*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rubi [A] time = 0.0044034, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 32}

$$\frac{c\sqrt{cx^2}(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] (c*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3} dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)^2 dx}{x}\\ &= \frac{c\sqrt{cx^2}(a+bx)^3}{3bx}\end{aligned}$$

Mathematica [A] time = 0.0052405, size = 26, normalized size = 0.96

$$\frac{(cx^2)^{3/2}(a+bx)^3}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^3)/(3*b*x^3)
```

Maple [A] time = 0.002, size = 31, normalized size = 1.2

$$\frac{b^2x^2 + 3\,abx + 3\,a^2}{3\,x^2}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^3,x)
```

```
[Out] 1/3/x^2*(b^2*x^2+3*a*b*x+3*a^2)*(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43776, size = 69, normalized size = 2.56

$$\frac{1}{3}\left(b^2cx^2 + 3\,abcx + 3\,a^2c\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*c*x^2 + 3*a*b*c*x + 3*a^2*c)*sqrt(c*x^2)
```

Sympy [B] time = 0.854937, size = 51, normalized size = 1.89

$$\frac{a^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{x^2} + \frac{abc^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{x} + \frac{b^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**3,x)
```

```
[Out] a**2*c**(3/2)*(x**2)**(3/2)/x**2 + a*b*c**(3/2)*(x**2)**(3/2)/x + b**2*c**(
3/2)*(x**2)**(3/2)/3
```

Giac [A] time = 1.06025, size = 39, normalized size = 1.44

$$\frac{1}{3}\left(\frac{(bx+a)^3\mathrm{sgn}\left(x\right)}{b} - \frac{a^3\mathrm{sgn}\left(x\right)}{b}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/3*((b*x + a)^3*sgn(x)/b - a^3*sgn(x)/b)*c^(3/2)
```

3.819 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4} dx$

Optimal. Leaf size=52

$$\frac{a^2c\sqrt{cx^2}\log(x)}{x} + 2abc\sqrt{cx^2} + \frac{1}{2}b^2cx\sqrt{cx^2}$$

```
[Out] 2*a*b*c*Sqrt[c*x^2] + (b^2*c*x*Sqrt[c*x^2])/2 + (a^2*c*Sqrt[c*x^2]*Log[x])/
x
```

Rubi [A] time = 0.0097471, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2c\sqrt{cx^2}\log(x)}{x} + 2abc\sqrt{cx^2} + \frac{1}{2}b^2cx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] 2*a*b*c*Sqrt[c*x^2] + (b^2*c*x*Sqrt[c*x^2])/2 + (a^2*c*Sqrt[c*x^2]*Log[x])/
x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^2}{x} dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(2ab + \frac{a^2}{x} + b^2x\right) dx}{x} \\
&= 2abc\sqrt{cx^2} + \frac{1}{2}b^2cx\sqrt{cx^2} + \frac{a^2c\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A] time = 0.0081543, size = 34, normalized size = 0.65

$$\frac{(cx^2)^{3/2}\left(2a^2\log(x) + bx(4a + bx)\right)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*x^3)
```

Maple [A] time = 0.004, size = 33, normalized size = 0.6

$$\frac{b^2x^2 + 2a^2\ln(x) + 4abx}{2x^3}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^4,x)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.50387, size = 81, normalized size = 1.56

$$\frac{\left(b^2cx^2 + 4abcx + 2a^2c\log(x)\right)\sqrt{cx^2}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c*x^2 + 4*a*b*c*x + 2*a^2*c*log(x))*sqrt(c*x^2)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}(a+bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**4,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**2/x**4, x)
```

Giac [A] time = 1.05001, size = 43, normalized size = 0.83

$$\frac{1}{2}\left(b^2x^2\text{sgn}\left(x\right)+4\,abx\text{sgn}\left(x\right)+2\,a^2\log\left(|x|\right)\text{sgn}\left(x\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2*sgn(x) + 4*a*b*x*sgn(x) + 2*a^2*log(abs(x))*sgn(x))*c^(3/2)
```

3.820 $\int x \left(cx^2\right)^{5/2} (a+bx)^2 \, dx$

Optimal. Leaf size=66

$$\frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}$$

```
[Out] (a^2*c^2*x^6*Sqrt[c*x^2])/7 + (a*b*c^2*x^7*Sqrt[c*x^2])/4 + (b^2*c^2*x^8*Sq
rt[c*x^2])/9
```

Rubi [A] time = 0.0190345, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*x^6*Sqrt[c*x^2])/7 + (a*b*c^2*x^7*Sqrt[c*x^2])/4 + (b^2*c^2*x^8*Sq
rt[c*x^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x \left(cx^2\right)^{5/2} (a+bx)^2 \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^6(a+bx)^2 \, dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^6+2abx^7+b^2x^8\right) dx}{x}\\
&= \frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.008282, size = 35, normalized size = 0.53

$$\frac{1}{252}x^2\left(cx^2\right)^{5/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (x^2*(c*x^2)^(5/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

Maple [A] time = 0.003, size = 32, normalized size = 0.5

$$\frac{x^2\left(28\,b^2x^2+63\,abx+36\,a^2\right)}{252}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(b*x+a)^2,x)
```

```
[Out] 1/252*x^2*(28*b^2*x^2+63*a*b*x+36*a^2)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53779, size = 95, normalized size = 1.44

$$\frac{1}{252}\left(28\,b^2c^2x^8+63\,abc^2x^7+36\,a^2c^2x^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^2*c^2*x^8 + 63*a*b*c^2*x^7 + 36*a^2*c^2*x^6)*sqrt(c*x^2)
```

Sympy [A] time = 2.08741, size = 60, normalized size = 0.91

$$\frac{a^2c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}{7}+\frac{abc^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}}{4}+\frac{b^2c^{\frac{5}{2}}x^4\left(x^2\right)^{\frac{5}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(5/2)*x**2*(x**2)**(5/2)/7 + a*b*c**(5/2)*x**3*(x**2)**(5/2)/4 + b*
*2*c**(5/2)*x**4*(x**2)**(5/2)/9
```

Giac [A] time = 1.05773, size = 59, normalized size = 0.89

$$\frac{1}{252}\left(28\,b^2c^2x^9\mathrm{sgn}\left(x\right)+63\,abc^2x^8\mathrm{sgn}\left(x\right)+36\,a^2c^2x^7\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/252*(28*b^2*c^2*x^9*sgn(x) + 63*a*b*c^2*x^8*sgn(x) + 36*a^2*c^2*x^7*sgn(x
))*sqrt(c)
```

3.821 $\int \left(cx^2\right)^{5/2} (a+bx)^2 \, dx$

Optimal. Leaf size=66

$$\frac{1}{6}a^2c^2x^5\sqrt{cx^2}+\frac{2}{7}abc^2x^6\sqrt{cx^2}+\frac{1}{8}b^2c^2x^7\sqrt{cx^2}$$

```
[Out] (a^2*c^2*x^5*Sqrt[c*x^2])/6 + (2*a*b*c^2*x^6*Sqrt[c*x^2])/7 + (b^2*c^2*x^7*
Sqrt[c*x^2])/8
```

Rubi [A] time = 0.0169404, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{1}{6}a^2c^2x^5\sqrt{cx^2}+\frac{2}{7}abc^2x^6\sqrt{cx^2}+\frac{1}{8}b^2c^2x^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*x^5*Sqrt[c*x^2])/6 + (2*a*b*c^2*x^6*Sqrt[c*x^2])/7 + (b^2*c^2*x^7*
Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(cx^2\right)^{5/2} (a+bx)^2 \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(a+bx)^2 \, dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^5+2abx^6+b^2x^7\right) dx}{x}\\
&= \frac{1}{6}a^2c^2x^5\sqrt{cx^2}+\frac{2}{7}abc^2x^6\sqrt{cx^2}+\frac{1}{8}b^2c^2x^7\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.0078313, size = 33, normalized size = 0.5

$$\frac{1}{168}x\left(cx^2\right)^{5/2}\left(28a^2+48abx+21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (x*(c*x^2)^(5/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

Maple [A] time = 0.003, size = 30, normalized size = 0.5

$$\frac{x\left(21\,b^{2}x^{2}+48\,abx+28\,a^{2}\right)}{168}\left(cx^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2,x)
```

```
[Out] 1/168*x*(21*b^2*x^2+48*a*b*x+28*a^2)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50974, size = 95, normalized size = 1.44

$$\frac{1}{168}\left(21\,b^{2}c^{2}x^{7}+48\,abc^{2}x^{6}+28\,a^{2}c^{2}x^{5}\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(21*b^2*c^2*x^7 + 48*a*b*c^2*x^6 + 28*a^2*c^2*x^5)*sqrt(c*x^2)
```

Sympy [A] time = 1.679, size = 60, normalized size = 0.91

$$\frac{a^{2}c^{\frac{5}{2}}x\left(x^{2}\right)^{\frac{5}{2}}}{6}+\frac{2abc^{\frac{5}{2}}x^{2}\left(x^{2}\right)^{\frac{5}{2}}}{7}+\frac{b^{2}c^{\frac{5}{2}}x^{3}\left(x^{2}\right)^{\frac{5}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(5/2)*x*(x**2)**(5/2)/6 + 2*a*b*c**(5/2)*x**2*(x**2)**(5/2)/7 + b**
2*c**(5/2)*x**3*(x**2)**(5/2)/8
```

Giac [A] time = 1.04871, size = 59, normalized size = 0.89

$$\frac{1}{168}\left(21\,b^2c^2x^8\mathrm{sgn}\,(x)+48\,abc^2x^7\mathrm{sgn}\,(x)+28\,a^2c^2x^6\mathrm{sgn}\,(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/168*(21*b^2*c^2*x^8*sgn(x) + 48*a*b*c^2*x^7*sgn(x) + 28*a^2*c^2*x^6*sgn(x
))*sqrt(c)
```

3.822 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x}\,dx$

Optimal. Leaf size=66

$$\frac{1}{5}a^2c^2x^4\sqrt{cx^2}+\frac{1}{3}abc^2x^5\sqrt{cx^2}+\frac{1}{7}b^2c^2x^6\sqrt{cx^2}$$

```
[Out] (a^2*c^2*x^4*Sqrt[c*x^2])/5 + (a*b*c^2*x^5*Sqrt[c*x^2])/3 + (b^2*c^2*x^6*Sq
rt[c*x^2])/7
```

Rubi [A] time = 0.0163015, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{1}{5}a^2c^2x^4\sqrt{cx^2}+\frac{1}{3}abc^2x^5\sqrt{cx^2}+\frac{1}{7}b^2c^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
```

```
[Out] (a^2*c^2*x^4*Sqrt[c*x^2])/5 + (a*b*c^2*x^5*Sqrt[c*x^2])/3 + (b^2*c^2*x^6*Sq
rt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)^2\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^4+2abx^5+b^2x^6\right)dx}{x}\\ &= \frac{1}{5}a^2c^2x^4\sqrt{cx^2}+\frac{1}{3}abc^2x^5\sqrt{cx^2}+\frac{1}{7}b^2c^2x^6\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0040024, size = 36, normalized size = 0.55

$$\frac{1}{105}cx^2\left(cx^2\right)^{3/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
```

```
[Out] (c*x^2*(c*x^2)^(3/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

Maple [A] time = 0.003, size = 29, normalized size = 0.4

$$\frac{15\,b^2x^2+35\,abx+21\,a^2}{105}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x,x)
```

```
[Out] 1/105*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56017, size = 95, normalized size = 1.44

$$\frac{1}{105}\left(15\,b^2c^2x^6+35\,abc^2x^5+21\,a^2c^2x^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*c^2*x^6 + 35*a*b*c^2*x^5 + 21*a^2*c^2*x^4)*sqrt(c*x^2)
```

Sympy [A] time = 1.69717, size = 54, normalized size = 0.82

$$\frac{a^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{5}+\frac{abc^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}{3}+\frac{b^2c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/5 + a*b*c**(5/2)*x*(x**2)**(5/2)/3 + b**2*c**(5
/2)*x**2*(x**2)**(5/2)/7
```

Giac [A] time = 1.09911, size = 59, normalized size = 0.89

$$\frac{1}{105}\left(15\,b^2c^2x^7\mathrm{sgn}\left(x\right)+35\,abc^2x^6\mathrm{sgn}\left(x\right)+21\,a^2c^2x^5\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*c^2*x^7*sgn(x) + 35*a*b*c^2*x^6*sgn(x) + 21*a^2*c^2*x^5*sgn(x
))*sqrt(c)
```

3.823 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=66

$$\frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}$$

```
[Out] (a^2*c^2*x^3*Sqrt[c*x^2])/4 + (2*a*b*c^2*x^4*Sqrt[c*x^2])/5 + (b^2*c^2*x^5*
Sqrt[c*x^2])/6
```

Rubi [A] time = 0.0151678, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (a^2*c^2*x^3*Sqrt[c*x^2])/4 + (2*a*b*c^2*x^4*Sqrt[c*x^2])/5 + (b^2*c^2*x^5*
Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)^2\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^3+2abx^4+b^2x^5\right)dx}{x}\\ &= \frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0046783, size = 34, normalized size = 0.52

$$\frac{1}{60}cx\left(cx^2\right)^{3/2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (c*x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

Maple [A] time = 0.005, size = 32, normalized size = 0.5

$$\frac{10\,b^2x^2+24\,abx+15\,a^2}{60\,x}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^2,x)
```

```
[Out] 1/60/x*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55942, size = 93, normalized size = 1.41

$$\frac{1}{60}\left(10\,b^2c^2x^5+24\,abc^2x^4+15\,a^2c^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*c^2*x^5 + 24*a*b*c^2*x^4 + 15*a^2*c^2*x^3)*sqrt(c*x^2)
```

Sympy [A] time = 1.75327, size = 54, normalized size = 0.82

$$\frac{a^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{4x}+\frac{2abc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{5}+\frac{b^2c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**2,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/(4*x) + 2*a*b*c**(5/2)*(x**2)**(5/2)/5 + b**2*c
**(5/2)*x*(x**2)**(5/2)/6
```

Giac [A] time = 1.05756, size = 59, normalized size = 0.89

$$\frac{1}{60}\left(10\,b^{2}c^{2}x^{6}\mathrm{sgn}\left(x\right)+24\,abc^{2}x^{5}\mathrm{sgn}\left(x\right)+15\,a^{2}c^{2}x^{4}\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^2*c^2*x^6*sgn(x) + 24*a*b*c^2*x^5*sgn(x) + 15*a^2*c^2*x^4*sgn(x)
)*sqrt(c)
```

3.824 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3}\,dx$

Optimal. Leaf size=66

$$\frac{1}{3}a^2c^2x^2\sqrt{cx^2}+\frac{1}{2}abc^2x^3\sqrt{cx^2}+\frac{1}{5}b^2c^2x^4\sqrt{cx^2}$$

```
[Out] (a^2*c^2*x^2*Sqrt[c*x^2])/3 + (a*b*c^2*x^3*Sqrt[c*x^2])/2 + (b^2*c^2*x^4*Sq
rt[c*x^2])/5
```

Rubi [A] time = 0.0142389, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{1}{3}a^2c^2x^2\sqrt{cx^2}+\frac{1}{2}abc^2x^3\sqrt{cx^2}+\frac{1}{5}b^2c^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] (a^2*c^2*x^2*Sqrt[c*x^2])/3 + (a*b*c^2*x^3*Sqrt[c*x^2])/2 + (b^2*c^2*x^4*Sq
rt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)^2\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^2+2abx^3+b^2x^4\right)dx}{x}\\ &= \frac{1}{3}a^2c^2x^2\sqrt{cx^2}+\frac{1}{2}abc^2x^3\sqrt{cx^2}+\frac{1}{5}b^2c^2x^4\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.003676, size = 38, normalized size = 0.58

$$\frac{1}{30}c^2x^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] (c^2*x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

Maple [A] time = 0.004, size = 32, normalized size = 0.5

$$\frac{6\,b^2x^2+15\,abx+10\,a^2}{30\,x^2}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^3,x)
```

```
[Out] 1/30/x^2*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46576, size = 92, normalized size = 1.39

$$\frac{1}{30}\left(6\,b^2c^2x^4+15\,abc^2x^3+10\,a^2c^2x^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^2*c^2*x^4 + 15*a*b*c^2*x^3 + 10*a^2*c^2*x^2)*sqrt(c*x^2)
```

Sympy [A] time = 1.85787, size = 54, normalized size = 0.82

$$\frac{a^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{3x^2}+\frac{abc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{2x}+\frac{b^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**3,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/(3*x**2) + a*b*c**(5/2)*(x**2)**(5/2)/(2*x) + b
**2*c**(5/2)*(x**2)**(5/2)/5
```

Giac [A] time = 1.05104, size = 59, normalized size = 0.89

$$\frac{1}{30}\left(6\,b^2c^2x^5\mathrm{sgn}\left(x\right)+15\,abc^2x^4\mathrm{sgn}\left(x\right)+10\,a^2c^2x^3\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*c^2*x^5*sgn(x) + 15*a*b*c^2*x^4*sgn(x) + 10*a^2*c^2*x^3*sgn(x))
*sqrt(c)
```

3.825 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4} dx$

Optimal. Leaf size=64

$$\frac{1}{2}a^2c^2x\sqrt{cx^2}+\frac{2}{3}abc^2x^2\sqrt{cx^2}+\frac{1}{4}b^2c^2x^3\sqrt{cx^2}$$

```
[Out] (a^2*c^2*x*Sqrt[c*x^2])/2 + (2*a*b*c^2*x^2*Sqrt[c*x^2])/3 + (b^2*c^2*x^3*Sq
rt[c*x^2])/4
```

Rubi [A] time = 0.0146246, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{1}{2}a^2c^2x\sqrt{cx^2}+\frac{2}{3}abc^2x^2\sqrt{cx^2}+\frac{1}{4}b^2c^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] (a^2*c^2*x*Sqrt[c*x^2])/2 + (2*a*b*c^2*x^2*Sqrt[c*x^2])/3 + (b^2*c^2*x^3*Sq
rt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)^2\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x+2abx^2+b^2x^3\right)dx}{x}\\ &= \frac{1}{2}a^2c^2x\sqrt{cx^2}+\frac{2}{3}abc^2x^2\sqrt{cx^2}+\frac{1}{4}b^2c^2x^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.0028009, size = 36, normalized size = 0.56

$$\frac{1}{12}c^2x\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] (c^2*x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

Maple [A] time = 0.002, size = 32, normalized size = 0.5

$$\frac{3\,b^2x^2+8\,abx+6\,a^2}{12\,x^3}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^4,x)
```

```
[Out] 1/12/x^3*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44262, size = 86, normalized size = 1.34

$$\frac{1}{12}\left(3\,b^2c^2x^3+8\,abc^2x^2+6\,a^2c^2x\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^2*x^3 + 8*a*b*c^2*x^2 + 6*a^2*c^2*x)*sqrt(c*x^2)
```

Sympy [A] time = 1.8784, size = 60, normalized size = 0.94

$$\frac{a^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{2x^3}+\frac{2abc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{3x^2}+\frac{b^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**4,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/(2*x**3) + 2*a*b*c**(5/2)*(x**2)**(5/2)/(3*x**2
) + b**2*c**(5/2)*(x**2)**(5/2)/(4*x)
```

Giac [A] time = 1.04537, size = 59, normalized size = 0.92

$$\frac{1}{12}\left(3\,b^2c^2x^4\mathrm{sgn}\left(x\right)+8\,abc^2x^3\mathrm{sgn}\left(x\right)+6\,a^2c^2x^2\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*c^2*x^4*sgn(x) + 8*a*b*c^2*x^3*sgn(x) + 6*a^2*c^2*x^2*sgn(x))*s
qrt(c)
```

3.826 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5} \, dx$

Optimal. Leaf size=29

$$\frac{c^2\sqrt{cx^2}(a+bx)^3}{3bx}$$

```
[Out] (c^2*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rubi [A] time = 0.0042798, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 32}

$$\frac{c^2\sqrt{cx^2}(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

```
[Out] (c^2*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5} \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int (a+bx)^2 \, dx}{x}\\ &= \frac{c^2\sqrt{cx^2}(a+bx)^3}{3bx}\end{aligned}$$

Mathematica [A] time = 0.0058366, size = 26, normalized size = 0.9

$$\frac{(cx^2)^{5/2}(a+bx)^3}{3bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

```
[Out] ((c*x^2)^(5/2)*(a + b*x)^3)/(3*b*x^5)
```

Maple [A] time = 0.002, size = 31, normalized size = 1.1

$$\frac{b^2x^2 + 3\,abx + 3\,a^2}{3\,x^4}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^5,x)
```

```
[Out] 1/3/x^4*(b^2*x^2+3*a*b*x+3*a^2)*(c*x^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.49357, size = 77, normalized size = 2.66

$$\frac{1}{3}\left(b^2c^2x^2 + 3\,abc^2x + 3\,a^2c^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*c^2*x^2 + 3*a*b*c^2*x + 3*a^2*c^2)*sqrt(c*x^2)
```

Sympy [B] time = 1.90511, size = 56, normalized size = 1.93

$$\frac{a^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{x^4} + \frac{abc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{x^3} + \frac{b^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**5,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/x**4 + a*b*c**(5/2)*(x**2)**(5/2)/x**3 + b**2*c
**(5/2)*(x**2)**(5/2)/(3*x**2)
```

Giac [A] time = 1.07392, size = 55, normalized size = 1.9

$$\frac{1}{3}\left(b^2c^2x^3\mathrm{sgn}\left(x\right) + 3\,abc^2x^2\mathrm{sgn}\left(x\right) + 3\,a^2c^2x\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*c^2*x^3*sgn(x) + 3*a*b*c^2*x^2*sgn(x) + 3*a^2*c^2*x*sgn(x))*sqrt(c
)
```

3.827 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6} dx$

Optimal. Leaf size=58

$$\frac{a^2c^2\sqrt{cx^2}\log(x)}{x} + 2abc^2\sqrt{cx^2} + \frac{1}{2}b^2c^2x\sqrt{cx^2}$$

```
[Out] 2*a*b*c^2*Sqrt[c*x^2] + (b^2*c^2*x*Sqrt[c*x^2])/2 + (a^2*c^2*Sqrt[c*x^2]*Lo
g[x])/x
```

Rubi [A] time = 0.0114799, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2c^2\sqrt{cx^2}\log(x)}{x} + 2abc^2\sqrt{cx^2} + \frac{1}{2}b^2c^2x\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
```

```
[Out] 2*a*b*c^2*Sqrt[c*x^2] + (b^2*c^2*x*Sqrt[c*x^2])/2 + (a^2*c^2*Sqrt[c*x^2]*Lo
g[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{(a+bx)^2}{x} dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(2ab + \frac{a^2}{x} + b^2x\right) dx}{x} \\
&= 2abc^2\sqrt{cx^2} + \frac{1}{2}b^2c^2x\sqrt{cx^2} + \frac{a^2c^2\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A] time = 0.0095722, size = 35, normalized size = 0.6

$$\frac{c^3x\left(2a^2\log(x) + bx(4a+bx)\right)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
```

```
[Out] (c^3*x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

Maple [A] time = 0.005, size = 33, normalized size = 0.6

$$\frac{b^2x^2 + 2a^2\ln(x) + 4abx}{2x^5}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^6,x)
```

```
[Out] 1/2*(c*x^2)^(5/2)*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.48463, size = 89, normalized size = 1.53

$$\frac{\left(b^2c^2x^2 + 4abc^2x + 2a^2c^2\log(x)\right)\sqrt{cx^2}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2 + 4*a*b*c^2*x + 2*a^2*c^2*log(x))*sqrt(c*x^2)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2\right)^{\frac{5}{2}}(a+bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**6,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**2/x**6, x)
```

Giac [A] time = 1.04798, size = 55, normalized size = 0.95

$$\frac{1}{2}\left(b^2c^2x^2\mathrm{sgn}\left(x\right) + 4\,abc^2x\mathrm{sgn}\left(x\right) + 2\,a^2c^2\log\left(|x|\right)\mathrm{sgn}\left(x\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2*x^2*sgn(x) + 4*a*b*c^2*x*sgn(x) + 2*a^2*c^2*log(abs(x))*sgn(x)
)*sqrt(c)
```

3.828 $\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=57

$$\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}$$

```
[Out] (a^2*x^4)/(3*Sqrt[c*x^2]) + (a*b*x^5)/(2*Sqrt[c*x^2]) + (b^2*x^6)/(5*Sqrt[c
*x^2])
```

Rubi [A] time = 0.0132603, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x^4)/(3*Sqrt[c*x^2]) + (a*b*x^5)/(2*Sqrt[c*x^2]) + (b^2*x^6)/(5*Sqrt[c
*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx&=\frac{x\int x^2(a+bx)^2\,dx}{\sqrt{cx^2}}\\&=\frac{x\int \left(a^2x^2+2abx^3+b^2x^4\right)dx}{\sqrt{cx^2}}\\&=\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0060105, size = 35, normalized size = 0.61

$$\frac{x^4\left(10a^2+15abx+6b^2x^2\right)}{30\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x^4*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/(30*Sqrt[c*x^2])
```

Maple [A] time = 0.003, size = 32, normalized size = 0.6

$$\frac{x^4 \left(6 b^2 x^2 + 15 abx + 10 a^2\right)}{30} \frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/30*x^4*(6*b^2*x^2+15*a*b*x+10*a^2)/(c*x^2)^(1/2)
```

Maxima [A] time = 1.03058, size = 73, normalized size = 1.28

$$\frac{\sqrt{cx^2} b^2 x^4}{5 c} + \frac{\sqrt{cx^2} abx^3}{2 c} + \frac{\sqrt{cx^2} a^2 x^2}{3 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c*x^2)*b^2*x^4/c + 1/2*sqrt(c*x^2)*a*b*x^3/c + 1/3*sqrt(c*x^2)*a^2
*x^2/c
```

Fricas [A] time = 1.44338, size = 78, normalized size = 1.37

$$\frac{\left(6 b^2 x^4 + 15 abx^3 + 10 a^2 x^2\right) \sqrt{cx^2}}{30 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^2*x^4 + 15*a*b*x^3 + 10*a^2*x^2)*sqrt(c*x^2)/c
```

Sympy [A] time = 0.695983, size = 60, normalized size = 1.05

$$\frac{a^2 x^4}{3 \sqrt{c} \sqrt{x^2}} + \frac{abx^5}{2 \sqrt{c} \sqrt{x^2}} + \frac{b^2 x^6}{5 \sqrt{c} \sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**4/(3*sqrt(c)*sqrt(x**2)) + a*b*x**5/(2*sqrt(c)*sqrt(x**2)) + b**2*x
**6/(5*sqrt(c)*sqrt(x**2))
```

Giac [A] time = 1.06773, size = 55, normalized size = 0.96

$$\frac{1}{30}\sqrt{cx^2}\left(3\left(\frac{2b^2x}{c}+\frac{5ab}{c}\right)x+\frac{10a^2}{c}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(c*x^2)*(3*(2*b^2*x/c + 5*a*b/c)*x + 10*a^2/c)*x^2
```

3.829 $\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=57

$$\frac{a^2x^3}{2\sqrt{cx^2}}+\frac{2abx^4}{3\sqrt{cx^2}}+\frac{b^2x^5}{4\sqrt{cx^2}}$$

```
[Out] (a^2*x^3)/(2*Sqrt[c*x^2]) + (2*a*b*x^4)/(3*Sqrt[c*x^2]) + (b^2*x^5)/(4*Sqrt
[c*x^2])
```

Rubi [A] time = 0.0121125, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2x^3}{2\sqrt{cx^2}}+\frac{2abx^4}{3\sqrt{cx^2}}+\frac{b^2x^5}{4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x^3)/(2*Sqrt[c*x^2]) + (2*a*b*x^4)/(3*Sqrt[c*x^2]) + (b^2*x^5)/(4*Sqrt
[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int x(a+bx)^2\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(a^2x+2abx^2+b^2x^3\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{a^2x^3}{2\sqrt{cx^2}}+\frac{2abx^4}{3\sqrt{cx^2}}+\frac{b^2x^5}{4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0048516, size = 35, normalized size = 0.61

$$\frac{x^3\left(6a^2+8abx+3b^2x^2\right)}{12\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x^3*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/(12*Sqrt[c*x^2])
```

Maple [A] time = 0.003, size = 32, normalized size = 0.6

$$\frac{x^3\left(3\,b^2x^2+8\,abx+6\,a^2\right)}{12}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/12*x^3*(3*b^2*x^2+8*a*b*x+6*a^2)/(c*x^2)^(1/2)
```

Maxima [A] time = 1.04988, size = 63, normalized size = 1.11

$$\frac{\sqrt{cx^2}b^2x^3}{4\,c}+\frac{2\,\sqrt{cx^2}abx^2}{3\,c}+\frac{a^2x^2}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^2)*b^2*x^3/c + 2/3*sqrt(c*x^2)*a*b*x^2/c + 1/2*a^2*x^2/sqrt(c)
```

Fricas [A] time = 1.52525, size = 73, normalized size = 1.28

$$\frac{\left(3\,b^2x^3+8\,abx^2+6\,a^2x\right)\sqrt{cx^2}}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*x^3 + 8*a*b*x^2 + 6*a^2*x)*sqrt(c*x^2)/c
```

Sympy [A] time = 0.59411, size = 61, normalized size = 1.07

$$\frac{a^2x^3}{2\sqrt{c}\sqrt{x^2}}+\frac{2abx^4}{3\sqrt{c}\sqrt{x^2}}+\frac{b^2x^5}{4\sqrt{c}\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**3/(2*sqrt(c)*sqrt(x**2)) + 2*a*b*x**4/(3*sqrt(c)*sqrt(x**2)) + b**2
*x**5/(4*sqrt(c)*sqrt(x**2))
```

Giac [A] time = 1.08845, size = 51, normalized size = 0.89

$$\frac{1}{12}\sqrt{cx^2}\left(\left(\frac{3b^2x}{c}+\frac{8ab}{c}\right)x+\frac{6a^2}{c}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(c*x^2)*((3*b^2*x/c + 8*a*b/c)*x + 6*a^2/c)*x
```

3.830 $\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=24

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

[Out] (x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])

Rubi [A] time = 0.0033883, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 32}

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*x)^2)/Sqrt[c*x^2],x]

[Out] (x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int (a+bx)^2\,dx}{\sqrt{cx^2}}\\ &= \frac{x(a+bx)^3}{3b\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.001783, size = 24, normalized size = 1.

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a + b*x)^2)/Sqrt[c*x^2],x]

[Out] (x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])

Maple [A] time = 0.001, size = 31, normalized size = 1.3

$$\frac{x^2 \left(b^2 x^2 + 3\,abx + 3\,a^2\right)}{3} \frac{1}{\sqrt{c x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/3*x^2*(b^2*x^2+3*a*b*x+3*a^2)/(c*x^2)^(1/2)
```

Maxima [B] time = 1.04516, size = 57, normalized size = 2.38

$$\frac{\sqrt{c x^2} b^2 x^2}{3\,c} + \frac{abx^2}{\sqrt{c}} + \frac{\sqrt{c x^2} a^2}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2)*b^2*x^2/c + a*b*x^2/sqrt(c) + sqrt(c*x^2)*a^2/c
```

Fricas [A] time = 1.45534, size = 63, normalized size = 2.62

$$\frac{\left(b^2 x^2 + 3\,abx + 3\,a^2\right) \sqrt{c x^2}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)/c
```

Sympy [B] time = 0.507174, size = 56, normalized size = 2.33

$$\frac{a^2 x^2}{\sqrt{c}\sqrt{x^2}} + \frac{abx^3}{\sqrt{c}\sqrt{x^2}} + \frac{b^2 x^4}{3\sqrt{c}\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**2/(sqrt(c)*sqrt(x**2)) + a*b*x**3/(sqrt(c)*sqrt(x**2)) + b**2*x**4/
(3*sqrt(c)*sqrt(x**2))
```

Giac [A] time = 1.06458, size = 49, normalized size = 2.04

$$\frac{1}{3}\sqrt{cx^2}\left(\left(\frac{b^2x}{c}+\frac{3\,ab}{c}\right)x+\frac{3\,a^2}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(c*x^2)*((b^2*x/c + 3*a*b/c)*x + 3*a^2/c)
```

3.831 $\int \frac{(a+bx)^2}{\sqrt{cx^2}} dx$

Optimal. Leaf size=52

$$\frac{a^2 x \log(x)}{\sqrt{cx^2}}+\frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}$$

```
[Out] (2*a*b*x^2)/Sqrt[c*x^2] + (b^2*x^3)/(2*Sqrt[c*x^2]) + (a^2*x*Log[x])/Sqrt[c
*x^2]
```

Rubi [A] time = 0.0100162, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a^2 x \log(x)}{\sqrt{cx^2}}+\frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[c*x^2],x]
```

```
[Out] (2*a*b*x^2)/Sqrt[c*x^2] + (b^2*x^3)/(2*Sqrt[c*x^2]) + (a^2*x*Log[x])/Sqrt[c
*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{\sqrt{cx^2}}\\ &= \frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}+\frac{a^2x\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0032546, size = 32, normalized size = 0.62

$$\frac{x\left(2a^2\log(x)+bx(4a+bx)\right)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[c*x^2],x]
```

```
[Out] (x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

Maple [A] time = 0.004, size = 31, normalized size = 0.6

$$\frac{x\left(b^2x^2+2a^2\ln(x)+4abx\right)}{2}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/(c*x^2)^(1/2)
```

Maxima [A] time = 1.03481, size = 47, normalized size = 0.9

$$\frac{b^2x^2}{2\sqrt{c}}+\frac{a^2\log(x)}{\sqrt{c}}+\frac{2\sqrt{cx^2}ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/sqrt(c) + a^2*log(x)/sqrt(c) + 2*sqrt(c*x^2)*a*b/c
```

Fricas [A] time = 1.56805, size = 78, normalized size = 1.5

$$\frac{\left(b^2x^2+4abx+2a^2\log(x)\right)\sqrt{cx^2}}{2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/(c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+bx)^2}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2/sqrt(c*x**2), x)
```

Giac [A] time = 1.07069, size = 68, normalized size = 1.31

$$-\frac{a^2 \log\left(\left|-\sqrt{c}x + \sqrt{cx^2}\right|\right)}{\sqrt{c}} + \frac{1}{2}\sqrt{cx^2}\left(\frac{b^2x}{c} + \frac{4\,ab}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -a^2*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/sqrt(c) + 1/2*sqrt(c*x^2)*(b^2*x/c
+ 4*a*b/c)
```

3.832 $\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{a^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}$$

```
[Out] -(a^2/Sqrt[c*x^2]) + (b^2*x^2)/Sqrt[c*x^2] + (2*a*b*x*Log[x])/Sqrt[c*x^2]
```

Rubi [A] time = 0.0105107, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
```

```
[Out] -(a^2/Sqrt[c*x^2]) + (b^2*x^2)/Sqrt[c*x^2] + (2*a*b*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx}{\sqrt{cx^2}}\\
&= -\frac{a^2}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0093845, size = 34, normalized size = 0.72

$$\frac{cx^2\left(-a^2+2abx\log(x)+b^2x^2\right)}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
```

```
[Out] (c*x^2*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/(c*x^2)^(3/2)
```

Maple [A] time = 0.003, size = 29, normalized size = 0.6

$$(2\,ab\ln(x)\,x+b^2x^2-a^2)\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(1/2),x)
```

```
[Out] (2*a*b*ln(x)*x+b^2*x^2-a^2)/(c*x^2)^(1/2)
```

Maxima [A] time = 1.01363, size = 47, normalized size = 1.

$$\frac{2\,ab\log(x)}{\sqrt{c}}+\frac{\sqrt{cx^2}b^2}{c}-\frac{a^2}{\sqrt{c}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*a*b*log(x)/sqrt(c) + sqrt(c*x^2)*b^2/c - a^2/(sqrt(c)*x)
```

Fricas [A] time = 1.5568, size = 73, normalized size = 1.55

$$\frac{\left(b^2x^2+2\,abx\log(x)-a^2\right)\sqrt{cx^2}}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2/(x*sqrt(c*x**2)), x)
```

Giac [A] time = 1.0755, size = 88, normalized size = 1.87

$$\frac{\sqrt{cx^2}b^2}{c} - \frac{2\left(ab\log\left(\left|-\sqrt{c}x + \sqrt{cx^2}\right|\right) - \frac{a^2\sqrt{c}}{\sqrt{c}x - \sqrt{cx^2}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2)*b^2/c - 2*(a*b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - a^2*sqrt(c)
/(sqrt(c)*x - sqrt(c*x^2)))/sqrt(c)
```

3.833 $\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{a^2}{2x\sqrt{cx^2}}-\frac{2ab}{\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}$$

```
[Out] (-2*a*b)/Sqrt[c*x^2] - a^2/(2*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/Sqrt[c*x^2]
```

Rubi [A] time = 0.0113112, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2}{2x\sqrt{cx^2}}-\frac{2ab}{\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] (-2*a*b)/Sqrt[c*x^2] - a^2/(2*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int\left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{2ab}{\sqrt{cx^2}}-\frac{a^2}{2x\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0080317, size = 35, normalized size = 0.71

$$\frac{cx\left(2b^2x^2\log(x)-a(a+4bx)\right)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] (c*x*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 34, normalized size = 0.7

$$\frac{2\,b^2\ln(x)\,x^2-4\,abx-a^2}{2\,x}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/2/x*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/(c*x^2)^(1/2)
```

Maxima [A] time = 1.08382, size = 42, normalized size = 0.86

$$\frac{b^2\log(x)}{\sqrt{c}}-\frac{2\,ab}{\sqrt{c}x}-\frac{a^2}{2\,\sqrt{c}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*log(x)/sqrt(c) - 2*a*b/(sqrt(c)*x) - 1/2*a^2/(sqrt(c)*x^2)
```

Fricas [A] time = 1.46495, size = 81, normalized size = 1.65

$$\frac{\left(2\,b^2x^2\log(x)-4\,abx-a^2\right)\sqrt{cx^2}}{2\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2/(x**2*sqrt(c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.834 $\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}$$

```
[Out] -(a + b*x)^3/(3*a*x^2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0036094, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] -(a + b*x)^3/(3*a*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^4}\,dx}{\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0096495, size = 33, normalized size = 1.27

$$\frac{c\left(-a^2-3abx-3b^2x^2\right)}{3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] (c*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 30, normalized size = 1.2

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^2}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(1/2),x)
```

```
[Out] -1/3*(3*b^2*x^2+3*a*b*x+a^2)/x^2/(c*x^2)^(1/2)
```

Maxima [A] time = 1.07142, size = 45, normalized size = 1.73

$$-\frac{b^2}{\sqrt{c}x}-\frac{ab}{\sqrt{c}x^2}-\frac{a^2}{3\,\sqrt{c}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b^2/(sqrt(c)*x) - a*b/(sqrt(c)*x^2) - 1/3*a^2/(sqrt(c)*x^3)
```

Fricas [A] time = 1.49316, size = 73, normalized size = 2.81

$$-\frac{\left(3\,b^2x^2+3\,abx+a^2\right)\sqrt{cx^2}}{3\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c*x^4)
```

Sympy [B] time = 0.608357, size = 53, normalized size = 2.04

$$-\frac{a^2}{3\sqrt{c}x^2\sqrt{x^2}}-\frac{ab}{\sqrt{c}x\sqrt{x^2}}-\frac{b^2}{\sqrt{c}\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(1/2),x)
```

```
[Out] -a**2/(3*sqrt(c)*x**2*sqrt(x**2)) - a*b/(sqrt(c)*x*sqrt(x**2)) - b**2/(sqrt
(c)*sqrt(x**2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^2}{\sqrt{c x^2}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2/(sqrt(c*x^2)*x^3), x)
```

3.835 $\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}$$

```
[Out] -a^2/(4*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*x^2*Sqrt[c*x^2]) - b^2/(2*x*Sqrt[c*x^
2])
```

Rubi [A] time = 0.0128204, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] -a^2/(4*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*x^2*Sqrt[c*x^2]) - b^2/(2*x*Sqrt[c*x^
2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^5}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0080514, size = 35, normalized size = 0.61

$$\frac{-3a^2-8abx-6b^2x^2}{12x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] (-3*a^2 - 8*a*b*x - 6*b^2*x^2)/(12*x^3*Sqrt[c*x^2])
```

Maple [A] time = 0.003, size = 32, normalized size = 0.6

$$-\frac{6b^2x^2+8abx+3a^2}{12x^3}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(1/2),x)
```

```
[Out] -1/12*(6*b^2*x^2+8*a*b*x+3*a^2)/x^3/(c*x^2)^(1/2)
```

Maxima [A] time = 1.08714, size = 45, normalized size = 0.79

$$-\frac{b^2}{2\sqrt{c}x^2}-\frac{2ab}{3\sqrt{c}x^3}-\frac{a^2}{4\sqrt{c}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b^2/(sqrt(c)*x^2) - 2/3*a*b/(sqrt(c)*x^3) - 1/4*a^2/(sqrt(c)*x^4)
```

Fricas [A] time = 1.60211, size = 77, normalized size = 1.35

$$-\frac{\left(6b^2x^2+8abx+3a^2\right)\sqrt{cx^2}}{12cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c*x^5)
```

Sympy [A] time = 0.75472, size = 61, normalized size = 1.07

$$-\frac{a^2}{4\sqrt{c}x^3\sqrt{x^2}}-\frac{2ab}{3\sqrt{c}x^2\sqrt{x^2}}-\frac{b^2}{2\sqrt{c}x\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(1/2),x)
```

```
[Out] -a**2/(4*sqrt(c)*x**3*sqrt(x**2)) - 2*a*b/(3*sqrt(c)*x**2*sqrt(x**2)) - b**
2/(2*sqrt(c)*x*sqrt(x**2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.836 $\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x(a+bx)^3}{3bc\sqrt{cx^2}}$$

```
[Out] (x*(a + b*x)^3)/(3*b*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0042818, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 32}

$$\frac{x(a+bx)^3}{3bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*x)^3)/(3*b*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}}\,dx &= \frac{x\int (a+bx)^2\,dx}{c\sqrt{cx^2}}\\ &= \frac{x(a+bx)^3}{3bc\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0041706, size = 26, normalized size = 0.96

$$\frac{x^3(a+bx)^3}{3b\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^3)/(3*b*(c*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 31, normalized size = 1.2

$$\frac{x^4\left(b^2x^2+3\,abx+3\,a^2\right)}{3}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(3/2),x)
```

```
[Out] 1/3*x^4*(b^2*x^2+3*a*b*x+3*a^2)/(c*x^2)^(3/2)
```

Maxima [B] time = 1.06627, size = 70, normalized size = 2.59

$$\frac{b^2x^4}{3\sqrt{cx^2}c}+\frac{abx^3}{\sqrt{cx^2}c}+\frac{a^2x^2}{\sqrt{cx^2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^4/(sqrt(c*x^2)*c) + a*b*x^3/(sqrt(c*x^2)*c) + a^2*x^2/(sqrt(c*x^2
)*c)
```

Fricas [A] time = 1.49582, size = 66, normalized size = 2.44

$$\frac{\left(b^2x^2+3\,abx+3\,a^2\right)\sqrt{cx^2}}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)/c^2
```

Sympy [B] time = 0.758949, size = 56, normalized size = 2.07

$$\frac{a^2x^4}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}+\frac{abx^5}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}+\frac{b^2x^6}{3c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] a**2*x**4/(c**(3/2)*(x**2)**(3/2)) + a*b*x**5/(c**(3/2)*(x**2)**(3/2)) + b*
*2*x**6/(3*c**(3/2)*(x**2)**(3/2))
```

Giac [A] time = 1.07006, size = 53, normalized size = 1.96

$$\frac{\sqrt{cx^2}\left(\left(\frac{b^2x}{c}+\frac{3\,ab}{c}\right)x+\frac{3\,a^2}{c}\right)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(c*x^2)*((b^2*x/c + 3*a*b/c)*x + 3*a^2/c)/c
```

3.837 $\int \frac{x^2(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=61

$$\frac{a^2x\log(x)}{c\sqrt{cx^2}}+\frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}$$

```
[Out] (2*a*b*x^2)/(c*Sqrt[c*x^2]) + (b^2*x^3)/(2*c*Sqrt[c*x^2]) + (a^2*x*Log[x])/
(c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0109616, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2x\log(x)}{c\sqrt{cx^2}}+\frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (2*a*b*x^2)/(c*Sqrt[c*x^2]) + (b^2*x^3)/(2*c*Sqrt[c*x^2]) + (a^2*x*Log[x])/
(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{c\sqrt{cx^2}}\\ &= \frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}+\frac{a^2x\log(x)}{c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0072139, size = 34, normalized size = 0.56

$$\frac{x^3\left(2a^2\log(x)+bx(4a+bx)\right)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*(c*x^2)^(3/2))
```

Maple [A] time = 0.005, size = 33, normalized size = 0.5

$$\frac{x^{3}\left(b^{2}x^{2}+2\,a^{2}\ln\left(x\right)+4\,abx\right)}{2}\left(cx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(3/2),x)
```

```
[Out] 1/2*x^3*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/(c*x^2)^(3/2)
```

Maxima [A] time = 1.04843, size = 61, normalized size = 1.

$$\frac{b^{2}x^{3}}{2\,\sqrt{cx^{2}}c}+\frac{2\,abx^{2}}{\sqrt{cx^{2}}c}+\frac{a^{2}\log\left(x\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^3/(sqrt(c*x^2)*c) + 2*a*b*x^2/(sqrt(c*x^2)*c) + a^2*log(x)/c^(3/2
)
```

Fricas [A] time = 1.54526, size = 81, normalized size = 1.33

$$\frac{\left(b^{2}x^{2}+4\,abx+2\,a^{2}\log\left(x\right)\right)\sqrt{cx^{2}}}{2\,c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/(c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{2}\left(a+bx\right)^{2}}{\left(cx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x)**2/(c*x**2)**(3/2), x)
```

Giac [A] time = 1.07647, size = 74, normalized size = 1.21

$$-\frac{\frac{2\,a^2\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{\sqrt{c}}-\sqrt{cx^2}\left(\frac{b^2x}{c}+\frac{4\,ab}{c}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/sqrt(c) - sqrt(c*x^2)*(b^2*x
/c + 4*a*b/c))/c
```

3.838 $\int \frac{x(a+bx)^2}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}$$

```
[Out] -(a^2/(c*Sqrt[c*x^2])) + (b^2*x^2)/(c*Sqrt[c*x^2]) + (2*a*b*x*Log[x])/(c*Sq
rt[c*x^2])
```

Rubi [A] time = 0.0120212, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] -(a^2/(c*Sqrt[c*x^2])) + (b^2*x^2)/(c*Sqrt[c*x^2]) + (2*a*b*x*Log[x])/(c*Sq
rt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^2}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a^2}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0055392, size = 33, normalized size = 0.59

$$\frac{x^2\left(-a^2+2abx\log(x)+b^2x^2\right)}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x^2*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/(c*x^2)^(3/2)
```

Maple [A] time = 0.003, size = 32, normalized size = 0.6

$$x^2\left(2\,ab\ln(x)\,x+b^2x^2-a^2\right)\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(3/2),x)
```

```
[Out] x^2*(2*a*b*ln(x)*x+b^2*x^2-a^2)/(c*x^2)^(3/2)
```

Maxima [A] time = 1.06537, size = 57, normalized size = 1.02

$$\frac{b^2x^2}{\sqrt{cx^2}c}+\frac{2\,ab\log(x)}{c^{\frac{3}{2}}}-\frac{a^2}{\sqrt{cx^2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*x^2/(sqrt(c*x^2)*c) + 2*a*b*log(x)/c^(3/2) - a^2/(sqrt(c*x^2)*c)
```

Fricas [A] time = 1.53961, size = 76, normalized size = 1.36

$$\frac{\left(b^2x^2+2\,abx\log(x)-a^2\right)\sqrt{cx^2}}{c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\,(a+bx)^2}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x)**2/(c*x**2)**(3/2), x)
```

Giac [A] time = 1.07066, size = 93, normalized size = 1.66

$$\frac{\frac{\sqrt{cx^2}b^2}{c} - \frac{2\left(ab\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)-\frac{a^2\sqrt{c}}{\sqrt{c}x-\sqrt{cx^2}}\right)}{\sqrt{c}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(c*x^2)*b^2/c - 2*(a*b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - a^2*sqrt(c
)/(sqrt(c)*x - sqrt(c*x^2)))/sqrt(c))/c
```

3.839 $\int \frac{(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{a^2}{2cx\sqrt{cx^2}}-\frac{2ab}{c\sqrt{cx^2}}+\frac{b^2x\log(x)}{c\sqrt{cx^2}}$$

```
[Out] (-2*a*b)/(c*Sqrt[c*x^2]) - a^2/(2*c*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(c*Sqrt
[c*x^2])
```

Rubi [A] time = 0.0122137, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$-\frac{a^2}{2cx\sqrt{cx^2}}-\frac{2ab}{c\sqrt{cx^2}}+\frac{b^2x\log(x)}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c*x^2)^(3/2),x]
```

```
[Out] (-2*a*b)/(c*Sqrt[c*x^2]) - a^2/(2*c*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(c*Sqrt
[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{2ab}{c\sqrt{cx^2}}-\frac{a^2}{2cx\sqrt{cx^2}}+\frac{b^2x\log(x)}{c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0041649, size = 34, normalized size = 0.59

$$\frac{x\left(2b^2x^2\log(x)-a(a+4bx)\right)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c*x^2)^(3/2),x]
```

```
[Out] (x*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 32, normalized size = 0.6

$$\frac{x\left(2\,b^2\ln\left(x\right)x^2-4\,abx-a^2\right)}{2}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(3/2),x)
```

```
[Out] 1/2*x*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/(c*x^2)^(3/2)
```

Maxima [A] time = 1.05825, size = 47, normalized size = 0.81

$$\frac{b^2\log\left(x\right)}{c^{\frac{3}{2}}}-\frac{2\,ab}{\sqrt{cx^2}c}-\frac{a^2}{2\,c^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*log(x)/c^(3/2) - 2*a*b/(sqrt(c*x^2)*c) - 1/2*a^2/(c^(3/2)*x^2)
```

Fricas [A] time = 1.58561, size = 84, normalized size = 1.45

$$\frac{\left(2\,b^2x^2\log\left(x\right)-4\,abx-a^2\right)\sqrt{cx^2}}{2\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c^2*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx\right)^2}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Integral((a + b*x)**2/(c*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.840 $\int \frac{(a+bx)^2}{x\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}$$

```
[Out] -(a + b*x)^3/(3*a*c*x^2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0043833, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
```

```
[Out] -(a + b*x)^3/(3*a*c*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^4}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0117217, size = 36, normalized size = 1.24

$$\frac{cx^2\left(-a^2-3abx-3b^2x^2\right)}{3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (c*x^2*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(5/2))
```

Maple [A] time = 0.003, size = 27, normalized size = 0.9

$$-\frac{3b^2x^2 + 3abx + a^2}{3}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(3/2),x)
```

```
[Out] -1/3*(3*b^2*x^2+3*a*b*x+a^2)/(c*x^2)^(3/2)
```

Maxima [A] time = 1.05268, size = 50, normalized size = 1.72

$$-\frac{b^2}{\sqrt{cx^2}c} - \frac{ab}{c^{\frac{3}{2}}x^2} - \frac{a^2}{3c^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -b^2/(sqrt(c*x^2)*c) - a*b/(c^(3/2)*x^2) - 1/3*a^2/(c^(3/2)*x^3)
```

Fricas [A] time = 1.48277, size = 76, normalized size = 2.62

$$-\frac{\left(3b^2x^2 + 3abx + a^2\right)\sqrt{cx^2}}{3c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c^2*x^4)
```

Sympy [B] time = 0.610677, size = 53, normalized size = 1.83

$$-\frac{a^2}{3c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} - \frac{abx}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} - \frac{b^2x^2}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(3*c**(3/2)*(x**2)**(3/2)) - a*b*x/(c**(3/2)*(x**2)**(3/2)) - b**2*x*
*2/(c**(3/2)*(x**2)**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^2}{\left(cx^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2/((c*x^2)^(3/2)*x), x)
```

3.841 $\int \frac{(a+bx)^2}{x^2 (cx^2)^{3/2}} \, dx$

Optimal. Leaf size=66

$$-\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}$$

```
[Out] -a^2/(4*c*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*c*x^2*Sqrt[c*x^2]) - b^2/(2*c*x*Sqr
t[c*x^2])
```

Rubi [A] time = 0.0128758, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -a^2/(4*c*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*c*x^2*Sqrt[c*x^2]) - b^2/(2*c*x*Sqr
t[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{x^2\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^5}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx}{c\sqrt{cx^2}}\\
&= -\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0095725, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\left(3a^2+8abx+6b^2x^2\right)}{12c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -(Sqrt[c*x^2]*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(12*c^2*x^5)
```

Maple [A] time = 0.004, size = 32, normalized size = 0.5

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(3/2),x)
```

```
[Out] -1/12*(6*b^2*x^2+8*a*b*x+3*a^2)/x/(c*x^2)^(3/2)
```

Maxima [A] time = 1.04647, size = 45, normalized size = 0.68

$$-\frac{b^2}{2\,c^{\frac{3}{2}}x^2}-\frac{2\,ab}{3\,c^{\frac{3}{2}}x^3}-\frac{a^2}{4\,c^{\frac{3}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b^2/(c^(3/2)*x^2) - 2/3*a*b/(c^(3/2)*x^3) - 1/4*a^2/(c^(3/2)*x^4)
```

Fricas [A] time = 1.51039, size = 80, normalized size = 1.21

$$-\frac{\left(6\,b^2x^2+8\,abx+3\,a^2\right)\sqrt{cx^2}}{12\,c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c^2*x^5)
```

Sympy [A] time = 0.733367, size = 56, normalized size = 0.85

$$-\frac{a^2}{4c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}-\frac{2ab}{3c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}-\frac{b^2x}{2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(4*c**(3/2)*x*(x**2)**(3/2)) - 2*a*b/(3*c**(3/2)*(x**2)**(3/2)) - b**
2*x/(2*c**(3/2)*(x**2)**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.842 $\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{5cx^4\sqrt{cx^2}}-\frac{ab}{2cx^3\sqrt{cx^2}}-\frac{b^2}{3cx^2\sqrt{cx^2}}$$

```
[Out] -a^2/(5*c*x^4*Sqrt[c*x^2]) - (a*b)/(2*c*x^3*Sqrt[c*x^2]) - b^2/(3*c*x^2*Sqr
t[c*x^2])
```

Rubi [A] time = 0.0126852, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2}{5cx^4\sqrt{cx^2}}-\frac{ab}{2cx^3\sqrt{cx^2}}-\frac{b^2}{3cx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] -a^2/(5*c*x^4*Sqrt[c*x^2]) - (a*b)/(2*c*x^3*Sqrt[c*x^2]) - b^2/(3*c*x^2*Sqr
t[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^6}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right)dx}{c\sqrt{cx^2}}\\
&= -\frac{a^2}{5cx^4\sqrt{cx^2}}-\frac{ab}{2cx^3\sqrt{cx^2}}-\frac{b^2}{3cx^2\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0120577, size = 33, normalized size = 0.5

$$\frac{c\left(-6a^2-15abx-10b^2x^2\right)}{30\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] (c*(-6*a^2 - 15*a*b*x - 10*b^2*x^2))/(30*(c*x^2)^(5/2))
```

Maple [A] time = 0.003, size = 32, normalized size = 0.5

$$-\frac{10\,b^2x^2+15\,abx+6\,a^2}{30\,x^2}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(3/2),x)
```

```
[Out] -1/30*(10*b^2*x^2+15*a*b*x+6*a^2)/x^2/(c*x^2)^(3/2)
```

Maxima [A] time = 1.14835, size = 45, normalized size = 0.68

$$-\frac{b^2}{3\,c^{\frac{3}{2}}x^3}-\frac{ab}{2\,c^{\frac{3}{2}}x^4}-\frac{a^2}{5\,c^{\frac{3}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*b^2/(c^(3/2)*x^3) - 1/2*a*b/(c^(3/2)*x^4) - 1/5*a^2/(c^(3/2)*x^5)
```

Fricas [A] time = 1.46748, size = 82, normalized size = 1.24

$$-\frac{\left(10\,b^2x^2+15\,abx+6\,a^2\right)\sqrt{cx^2}}{30\,c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)*sqrt(c*x^2)/(c^2*x^6)
```

Sympy [A] time = 0.90496, size = 56, normalized size = 0.85

$$-\frac{a^2}{5c^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}-\frac{ab}{2c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}-\frac{b^2}{3c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(5*c**(3/2)*x**2*(x**2)**(3/2)) - a*b/(2*c**(3/2)*x*(x**2)**(3/2)) -
b**2/(3*c**(3/2)*(x**2)**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^2}{(cx^2)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2/((c*x^2)^(3/2)*x^3), x)
```

3.843 $\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{6cx^5\sqrt{cx^2}}-\frac{2ab}{5cx^4\sqrt{cx^2}}-\frac{b^2}{4cx^3\sqrt{cx^2}}$$

```
[Out] -a^2/(6*c*x^5*Sqrt[c*x^2]) - (2*a*b)/(5*c*x^4*Sqrt[c*x^2]) - b^2/(4*c*x^3*S
qrt[c*x^2])
```

Rubi [A] time = 0.0139281, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2}{6cx^5\sqrt{cx^2}}-\frac{2ab}{5cx^4\sqrt{cx^2}}-\frac{b^2}{4cx^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] -a^2/(6*c*x^5*Sqrt[c*x^2]) - (2*a*b)/(5*c*x^4*Sqrt[c*x^2]) - b^2/(4*c*x^3*S
qrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^7}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a^2}{6cx^5\sqrt{cx^2}}-\frac{2ab}{5cx^4\sqrt{cx^2}}-\frac{b^2}{4cx^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0079686, size = 35, normalized size = 0.53

$$\frac{-10a^2-24abx-15b^2x^2}{60x^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] (-10*a^2 - 24*a*b*x - 15*b^2*x^2)/(60*x^3*(c*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 32, normalized size = 0.5

$$-\frac{15\,b^2x^2+24\,abx+10\,a^2}{60\,x^3}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(3/2),x)
```

```
[Out] -1/60*(15*b^2*x^2+24*a*b*x+10*a^2)/x^3/(c*x^2)^(3/2)
```

Maxima [A] time = 1.01388, size = 45, normalized size = 0.68

$$-\frac{b^2}{4\,c^{\frac{3}{2}}x^4}-\frac{2\,ab}{5\,c^{\frac{3}{2}}x^5}-\frac{a^2}{6\,c^{\frac{3}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*b^2/(c^(3/2)*x^4) - 2/5*a*b/(c^(3/2)*x^5) - 1/6*a^2/(c^(3/2)*x^6)
```

Fricas [A] time = 1.46298, size = 84, normalized size = 1.27

$$-\frac{\left(15\,b^2x^2+24\,abx+10\,a^2\right)\sqrt{cx^2}}{60\,c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)*sqrt(c*x^2)/(c^2*x^7)
```

Sympy [A] time = 1.06204, size = 61, normalized size = 0.92

$$-\frac{a^2}{6c^{\frac{3}{2}}x^3\left(x^2\right)^{\frac{3}{2}}}-\frac{2ab}{5c^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}-\frac{b^2}{4c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(6*c**(3/2)*x**3*(x**2)**(3/2)) - 2*a*b/(5*c**(3/2)*x**2*(x**2)**(3/2
)) - b**2/(4*c**(3/2)*x*(x**2)**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.844 $\int \frac{x^3(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}$$

```
[Out] -(a^2/(c^2*Sqrt[c*x^2])) + (b^2*x^2)/(c^2*Sqrt[c*x^2]) + (2*a*b*x*Log[x])/(
c^2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0126618, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -(a^2/(c^2*Sqrt[c*x^2])) + (b^2*x^2)/(c^2*Sqrt[c*x^2]) + (2*a*b*x*Log[x])/(
c^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a^2}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0092408, size = 33, normalized size = 0.59

$$\frac{-a^2+2abx\log(x)+b^2x^2}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (-a^2 + b^2*x^2 + 2*a*b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

Maple [A] time = 0.003, size = 32, normalized size = 0.6

$$x^4 \left(2 ab \ln (x) x + b^2 x^2 - a^2\right) \left(c x^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(5/2),x)
```

```
[Out] x^4*(2*a*b*ln(x)*x+b^2*x^2-a^2)/(c*x^2)^(5/2)
```

Maxima [A] time = 1.09722, size = 61, normalized size = 1.09

$$\frac{b^2 x^4}{\left(c x^2\right)^{\frac{3}{2}} c} - \frac{a^2 x^2}{\left(c x^2\right)^{\frac{3}{2}} c} + \frac{2 ab \log (x)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] b^2*x^4/((c*x^2)^(3/2)*c) - a^2*x^2/((c*x^2)^(3/2)*c) + 2*a*b*log(x)/c^(5/2
)
```

Fricas [A] time = 1.4926, size = 76, normalized size = 1.36

$$\frac{\left(b^2 x^2 + 2 abx \log (x) - a^2\right) \sqrt{c x^2}}{c^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (a + bx)^2}{\left(c x^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*x)**2/(c*x**2)**(5/2), x)
```

Giac [A] time = 1.07175, size = 88, normalized size = 1.57

$$\frac{\sqrt{cx^2}b^2}{c^3} - \frac{2\left(ab\log\left(\left|-\sqrt{c}x + \sqrt{cx^2}\right|\right) - \frac{a^2\sqrt{c}}{\sqrt{c}x-\sqrt{cx^2}}\right)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2)*b^2/c^3 - 2*(a*b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - a^2*sqrt(
c)/(sqrt(c)*x - sqrt(c*x^2)))/c^(5/2)
```

3.845 $\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}} \, dx$

Optimal. Leaf size=58

$$-\frac{a^2}{2c^2x\sqrt{cx^2}}-\frac{2ab}{c^2\sqrt{cx^2}}+\frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

```
[Out] (-2*a*b)/(c^2*Sqrt[c*x^2]) - a^2/(2*c^2*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(c^
2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0122728, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2}{2c^2x\sqrt{cx^2}}-\frac{2ab}{c^2\sqrt{cx^2}}+\frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (-2*a*b)/(c^2*Sqrt[c*x^2]) - a^2/(2*c^2*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(c^
2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}} \, dx &= \frac{x\int \frac{(a+bx)^2}{x^3} \, dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right) dx}{c^2\sqrt{cx^2}}\\
&= -\frac{2ab}{c^2\sqrt{cx^2}}-\frac{a^2}{2c^2x\sqrt{cx^2}}+\frac{b^2x\log(x)}{c^2\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0090575, size = 36, normalized size = 0.62

$$\frac{x^3\left(2b^2x^2\log(x)-a(a+4bx)\right)}{2\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (x^3*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(5/2))
```

Maple [A] time = 0.004, size = 34, normalized size = 0.6

$$\frac{x^{3}\left(2\,b^{2}\ln\left(x\right)x^{2}-4\,abx-a^{2}\right)}{2}\left(cx^{2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(5/2),x)
```

```
[Out] 1/2*x^3*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/(c*x^2)^(5/2)
```

Maxima [A] time = 1.10453, size = 51, normalized size = 0.88

$$-\frac{2\,abx^{2}}{\left(cx^{2}\right)^{\frac{3}{2}}c}+\frac{b^{2}\log\left(x\right)}{c^{\frac{5}{2}}}-\frac{a^{2}}{2\,c^{\frac{5}{2}}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -2*a*b*x^2/((c*x^2)^(3/2)*c) + b^2*log(x)/c^(5/2) - 1/2*a^2/(c^(5/2)*x^2)
```

Fricas [A] time = 1.5738, size = 84, normalized size = 1.45

$$\frac{\left(2\,b^{2}x^{2}\log\left(x\right)-4\,abx-a^{2}\right)\sqrt{cx^{2}}}{2\,c^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c^3*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{2}\left(a+bx\right)^{2}}{\left(cx^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*x)**2/(c*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.846 $\int \frac{x(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}$$

```
[Out] -(a + b*x)^3/(3*a*c^2*x^2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0041975, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -(a + b*x)^3/(3*a*c^2*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^4}\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0061763, size = 35, normalized size = 1.21

$$\frac{x^2\left(-a^2-3abx-3b^2x^2\right)}{3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (x^2*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(5/2))
```

Maple [A] time = 0.004, size = 30, normalized size = 1.

$$-\frac{x^2 \left(3 b^2 x^2 + 3 abx + a^2\right)}{3} \left(c x^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(5/2),x)
```

```
[Out] -1/3*x^2*(3*b^2*x^2+3*a*b*x+a^2)/(c*x^2)^(5/2)
```

Maxima [A] time = 1.0646, size = 59, normalized size = 2.03

$$-\frac{b^2 x^2}{\left(c x^2\right)^{\frac{3}{2}} c} - \frac{a^2}{3 \left(c x^2\right)^{\frac{3}{2}} c} - \frac{ab}{c^{\frac{5}{2}} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -b^2*x^2/((c*x^2)^(3/2)*c) - 1/3*a^2/((c*x^2)^(3/2)*c) - a*b/(c^(5/2)*x^2)
```

Fricas [A] time = 1.48859, size = 76, normalized size = 2.62

$$-\frac{\left(3 b^2 x^2 + 3 abx + a^2\right) \sqrt{c x^2}}{3 c^3 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c^3*x^4)
```

Sympy [B] time = 0.867778, size = 58, normalized size = 2.

$$-\frac{a^2 x^2}{3 c^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}} - \frac{abx^3}{c^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}} - \frac{b^2 x^4}{c^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] -a**2*x**2/(3*c**(5/2)*(x**2)**(5/2)) - a*b*x**3/(c**(5/2)*(x**2)**(5/2)) -
 b**2*x**4/(c**(5/2)*(x**2)**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^2x}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*x/(c*x^2)^(5/2), x)
```

3.847 $\int \frac{(a+bx)^2}{(cx^2)^{5/2}} dx$

Optimal. Leaf size=66

$$-\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}$$

```
[Out] -a^2/(4*c^2*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*c^2*x^2*Sqrt[c*x^2]) - b^2/(2*c^2
*x*Sqrt[c*x^2])
```

Rubi [A] time = 0.0125624, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$-\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c*x^2)^(5/2),x]
```

```
[Out] -a^2/(4*c^2*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*c^2*x^2*Sqrt[c*x^2]) - b^2/(2*c^2
*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{(cx^2)^{5/2}} dx &= \frac{x\int \frac{(a+bx)^2}{x^5} dx}{c^2\sqrt{cx^2}} \\
&= \frac{x\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right) dx}{c^2\sqrt{cx^2}} \\
&= -\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0037259, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\left(3a^2+8abx+6b^2x^2\right)}{12c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c*x^2)^(5/2),x]
```

```
[Out] -(Sqrt[c*x^2]*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(12*c^3*x^5)
```

Maple [A] time = 0.004, size = 30, normalized size = 0.5

$$-\frac{x\left(6b^{2}x^{2}+8abx+3a^{2}\right)}{12}\left(cx^{2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(5/2),x)
```

```
[Out] -1/12*x*(6*b^2*x^2+8*a*b*x+3*a^2)/(c*x^2)^(5/2)
```

Maxima [A] time = 1.06597, size = 50, normalized size = 0.76

$$-\frac{2\,ab}{3\left(cx^{2}\right)^{\frac{3}{2}}c}-\frac{b^{2}}{2\,c^{\frac{5}{2}}x^{2}}-\frac{a^{2}}{4\,c^{\frac{5}{2}}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*a*b/((c*x^2)^(3/2)*c) - 1/2*b^2/(c^(5/2)*x^2) - 1/4*a^2/(c^(5/2)*x^4)
```

Fricas [A] time = 1.53816, size = 80, normalized size = 1.21

$$-\frac{\left(6\,b^{2}x^{2}+8\,abx+3\,a^{2}\right)\sqrt{cx^{2}}}{12\,c^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c^3*x^5)
```

Sympy [A] time = 0.901204, size = 61, normalized size = 0.92

$$-\frac{a^{2}x}{4c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}-\frac{2abx^{2}}{3c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}-\frac{b^{2}x^{3}}{2c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] -a**2*x/(4*c**(5/2)*(x**2)**(5/2)) - 2*a*b*x**2/(3*c**(5/2)*(x**2)**(5/2))
- b**2*x**3/(2*c**(5/2)*(x**2)**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.848 $\int \frac{(a+bx)^2}{x(cx^2)^{5/2}} dx$

Optimal. Leaf size=66

$$-\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

```
[Out] -a^2/(5*c^2*x^4*Sqrt[c*x^2]) - (a*b)/(2*c^2*x^3*Sqrt[c*x^2]) - b^2/(3*c^2*x
^2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0124531, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -a^2/(5*c^2*x^4*Sqrt[c*x^2]) - (a*b)/(2*c^2*x^3*Sqrt[c*x^2]) - b^2/(3*c^2*x
^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^6}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0122521, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\left(6a^2+15abx+10b^2x^2\right)}{30c^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[c*x^2]*(6*a^2 + 15*a*b*x + 10*b^2*x^2))/(30*c^3*x^6)
```

Maple [A] time = 0.005, size = 29, normalized size = 0.4

$$-\frac{10 b^2 x^2 + 15 abx + 6 a^2}{30} \left(c x^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(5/2),x)
```

```
[Out] -1/30*(10*b^2*x^2+15*a*b*x+6*a^2)/(c*x^2)^(5/2)
```

Maxima [A] time = 1.08766, size = 50, normalized size = 0.76

$$-\frac{b^2}{3 \left(c x^2\right)^{\frac{3}{2}} c} - \frac{ab}{2 c^{\frac{5}{2}} x^4} - \frac{a^2}{5 c^{\frac{5}{2}} x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*b^2/((c*x^2)^(3/2)*c) - 1/2*a*b/(c^(5/2)*x^4) - 1/5*a^2/(c^(5/2)*x^5)
```

Fricas [A] time = 1.47218, size = 82, normalized size = 1.24

$$-\frac{\left(10 b^2 x^2 + 15 abx + 6 a^2\right) \sqrt{c x^2}}{30 c^3 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)*sqrt(c*x^2)/(c^3*x^6)
```

Sympy [A] time = 1.03065, size = 56, normalized size = 0.85

$$-\frac{a^2}{5c^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}} - \frac{abx}{2c^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}} - \frac{b^2 x^2}{3c^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(5*c**(5/2)*(x**2)**(5/2)) - a*b*x/(2*c**(5/2)*(x**2)**(5/2)) - b**2*
x**2/(3*c**(5/2)*(x**2)**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^2}{(cx^2)^{\frac{5}{2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2/((c*x^2)^(5/2)*x), x)
```

3.849 $\int \frac{(a+bx)^2}{x^2\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{6c^2x^5\sqrt{cx^2}}-\frac{2ab}{5c^2x^4\sqrt{cx^2}}-\frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

```
[Out] -a^2/(6*c^2*x^5*Sqrt[c*x^2]) - (2*a*b)/(5*c^2*x^4*Sqrt[c*x^2]) - b^2/(4*c^2
*x^3*Sqrt[c*x^2])
```

Rubi [A] time = 0.013062, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2}{6c^2x^5\sqrt{cx^2}}-\frac{2ab}{5c^2x^4\sqrt{cx^2}}-\frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -a^2/(6*c^2*x^5*Sqrt[c*x^2]) - (2*a*b)/(5*c^2*x^4*Sqrt[c*x^2]) - b^2/(4*c^2
*x^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^2\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^7}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a^2}{6c^2x^5\sqrt{cx^2}}-\frac{2ab}{5c^2x^4\sqrt{cx^2}}-\frac{b^2}{4c^2x^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0090606, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\left(10a^2+24abx+15b^2x^2\right)}{60c^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[c*x^2]*(10*a^2 + 24*a*b*x + 15*b^2*x^2))/(60*c^3*x^7)
```

Maple [A] time = 0.004, size = 32, normalized size = 0.5

$$-\frac{15 b^2x^2 + 24 abx + 10 a^2}{60 x} \left(cx^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(5/2),x)
```

```
[Out] -1/60*(15*b^2*x^2+24*a*b*x+10*a^2)/x/(c*x^2)^(5/2)
```

Maxima [A] time = 1.08683, size = 45, normalized size = 0.68

$$-\frac{b^2}{4 c^{\frac{5}{2}} x^4} - \frac{2 ab}{5 c^{\frac{5}{2}} x^5} - \frac{a^2}{6 c^{\frac{5}{2}} x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*b^2/(c^(5/2)*x^4) - 2/5*a*b/(c^(5/2)*x^5) - 1/6*a^2/(c^(5/2)*x^6)
```

Fricas [A] time = 1.50181, size = 84, normalized size = 1.27

$$-\frac{\left(15 b^2x^2 + 24 abx + 10 a^2\right)\sqrt{cx^2}}{60 c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)*sqrt(c*x^2)/(c^3*x^7)
```

Sympy [A] time = 1.30085, size = 56, normalized size = 0.85

$$-\frac{a^2}{6c^{\frac{5}{2}} x \left(x^2\right)^{\frac{5}{2}}} - \frac{2ab}{5c^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}} - \frac{b^2x}{4c^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(6*c**(5/2)*x*(x**2)**(5/2)) - 2*a*b/(5*c**(5/2)*(x**2)**(5/2)) - b**
2*x/(4*c**(5/2)*(x**2)**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.850 $\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

```
[Out] -a^2/(7*c^2*x^6*Sqrt[c*x^2]) - (a*b)/(3*c^2*x^5*Sqrt[c*x^2]) - b^2/(5*c^2*x
^4*Sqrt[c*x^2])
```

Rubi [A] time = 0.0133345, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
```



```
[Out] -a^2/(7*c^2*x^6*Sqrt[c*x^2]) - (a*b)/(3*c^2*x^5*Sqrt[c*x^2]) - b^2/(5*c^2*x
^4*Sqrt[c*x^2])
```


Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^8}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{x^8}+\frac{2ab}{x^7}+\frac{b^2}{x^6}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0134686, size = 33, normalized size = 0.5

$$\frac{c\left(-15a^2-35abx-21b^2x^2\right)}{105\left(cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] (c*(-15*a^2 - 35*a*b*x - 21*b^2*x^2))/(105*(c*x^2)^(7/2))
```

Maple [A] time = 0.004, size = 32, normalized size = 0.5

$$-\frac{21\,b^2x^2+35\,abx+15\,a^2}{105\,x^2}\left(cx^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(5/2),x)
```

```
[Out] -1/105*(21*b^2*x^2+35*a*b*x+15*a^2)/x^2/(c*x^2)^(5/2)
```

Maxima [A] time = 1.06856, size = 45, normalized size = 0.68

$$-\frac{b^2}{5\,c^{\frac{5}{2}}x^5}-\frac{ab}{3\,c^{\frac{5}{2}}x^6}-\frac{a^2}{7\,c^{\frac{5}{2}}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*b^2/(c^(5/2)*x^5) - 1/3*a*b/(c^(5/2)*x^6) - 1/7*a^2/(c^(5/2)*x^7)
```

Fricas [A] time = 1.51058, size = 85, normalized size = 1.29

$$-\frac{\left(21\,b^2x^2+35\,abx+15\,a^2\right)\sqrt{cx^2}}{105\,c^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)*sqrt(c*x^2)/(c^3*x^8)
```

Sympy [A] time = 1.53685, size = 56, normalized size = 0.85

$$-\frac{a^2}{7c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}-\frac{ab}{3c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}-\frac{b^2}{5c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(7*c**(5/2)*x**2*(x**2)**(5/2)) - a*b/(3*c**(5/2)*x*(x**2)**(5/2)) -
b**2/(5*c**(5/2)*(x**2)**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^2}{\left(cx^2\right)^{\frac{5}{2}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2/((c*x^2)^(5/2)*x^3), x)
```

3.851 $\int \frac{(a+bx)^2}{x^4 \left(cx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=66

$$-\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

```
[Out] -a^2/(8*c^2*x^7*Sqrt[c*x^2]) - (2*a*b)/(7*c^2*x^6*Sqrt[c*x^2]) - b^2/(6*c^2
*x^5*Sqrt[c*x^2])
```

Rubi [A] time = 0.0134873, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] -a^2/(8*c^2*x^7*Sqrt[c*x^2]) - (2*a*b)/(7*c^2*x^6*Sqrt[c*x^2]) - b^2/(6*c^2
*x^5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{x^4 \left(cx^2\right)^{5/2}} \, dx &= \frac{x \int \frac{(a+bx)^2}{x^9} \, dx}{c^2\sqrt{cx^2}}\\
&= \frac{x \int \left(\frac{a^2}{x^9}+\frac{2ab}{x^8}+\frac{b^2}{x^7}\right) dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0093735, size = 35, normalized size = 0.53

$$\frac{-21a^2-48abx-28b^2x^2}{168x^3 \left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] (-21*a^2 - 48*a*b*x - 28*b^2*x^2)/(168*x^3*(c*x^2)^(5/2))
```

Maple [A] time = 0.005, size = 32, normalized size = 0.5

$$-\frac{28\,b^2x^2+48\,abx+21\,a^2}{168\,x^3}\left(cx^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(5/2),x)
```

```
[Out] -1/168*(28*b^2*x^2+48*a*b*x+21*a^2)/x^3/(c*x^2)^(5/2)
```

Maxima [A] time = 1.01, size = 45, normalized size = 0.68

$$-\frac{b^2}{6\,c^{\frac{5}{2}}x^6}-\frac{2\,ab}{7\,c^{\frac{5}{2}}x^7}-\frac{a^2}{8\,c^{\frac{5}{2}}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b^2/(c^(5/2)*x^6) - 2/7*a*b/(c^(5/2)*x^7) - 1/8*a^2/(c^(5/2)*x^8)
```

Fricas [A] time = 1.5617, size = 85, normalized size = 1.29

$$-\frac{\left(28\,b^2x^2+48\,abx+21\,a^2\right)\sqrt{cx^2}}{168\,c^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/168*(28*b^2*x^2 + 48*a*b*x + 21*a^2)*sqrt(c*x^2)/(c^3*x^9)
```

Sympy [A] time = 1.83628, size = 61, normalized size = 0.92

$$-\frac{a^2}{8c^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}}-\frac{2ab}{7c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}-\frac{b^2}{6c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(8*c**(5/2)*x**3*(x**2)**(5/2)) - 2*a*b/(7*c**(5/2)*x**2*(x**2)**(5/2
)) - b**2/(6*c**(5/2)*x*(x**2)**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.852 $\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx$

Optimal. Leaf size=102

$$-\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}+\frac{a^4\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}$$

```
[Out] -((a^3*Sqrt[c*x^2])/b^4) + (a^2*x*Sqrt[c*x^2])/(2*b^3) - (a*x^2*Sqrt[c*x^2]
)/(3*b^2) + (x^3*Sqrt[c*x^2])/(4*b) + (a^4*Sqrt[c*x^2]*Log[a + b*x])/(b^5*x
)
```

Rubi [A] time = 0.035834, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}+\frac{a^4\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] -((a^3*Sqrt[c*x^2])/b^4) + (a^2*x*Sqrt[c*x^2])/(2*b^3) - (a*x^2*Sqrt[c*x^2]
)/(3*b^2) + (x^3*Sqrt[c*x^2])/(4*b) + (a^4*Sqrt[c*x^2]*Log[a + b*x])/(b^5*x
)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^4}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}\\ &= -\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}+\frac{a^4\sqrt{cx^2}\log(a+bx)}{b^5x}\end{aligned}$$

Mathematica [A] time = 0.0182006, size = 63, normalized size = 0.62

$$\frac{cx\left(bx\left(6a^2bx-12a^3-4ab^2x^2+3b^3x^3\right)+12a^4\log(a+bx)\right)}{12b^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (c*x*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*a^4*Log[a +
b*x]))/(12*b^5*Sqrt[c*x^2])
```

Maple [A] time = 0.006, size = 63, normalized size = 0.6

$$\frac{3\,b^4x^4-4\,x^3ab^3+6\,x^2a^2b^2+12\,a^4\ln\left(bx+a\right)-12\,bxa^3}{12\,b^5x}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] 1/12*(c*x^2)^(1/2)*(3*b^4*x^4-4*x^3*a*b^3+6*x^2*a^2*b^2+12*a^4*ln(b*x+a)-12
*b*x*a^3)/x/b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49567, size = 139, normalized size = 1.36

$$\frac{\left(3\,b^4x^4-4\,ab^3x^3+6\,a^2b^2x^2-12\,a^3bx+12\,a^4\log\left(bx+a\right)\right)\sqrt{cx^2}}{12\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))*sqrt(c*x^2)/(b^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x**3*sqrt(c*x**2)/(a + b*x), x)
```

Giac [A] time = 1.05774, size = 109, normalized size = 1.07

$$\frac{1}{12}\sqrt{c}\left(\frac{12\,a^4\log\left(|bx+a|\right)\operatorname{sgn}\left(x\right)}{b^5}-\frac{12\,a^4\log\left(|a|\right)\operatorname{sgn}\left(x\right)}{b^5}+\frac{3\,b^3x^4\operatorname{sgn}\left(x\right)-4\,ab^2x^3\operatorname{sgn}\left(x\right)+6\,a^2bx^2\operatorname{sgn}\left(x\right)-12\,a^3x\operatorname{sgn}}{b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(c)*(12*a^4*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*log(abs(a))*sgn(
x)/b^5 + (3*b^3*x^4*sgn(x) - 4*a*b^2*x^3*sgn(x) + 6*a^2*b*x^2*sgn(x) - 12*a
^3*x*sgn(x))/b^4)
```

3.853 $\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx$

Optimal. Leaf size=80

$$\frac{a^2\sqrt{cx^2}}{b^3}-\frac{a^3\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}$$

```
[Out] (a^2*Sqrt[c*x^2])/b^3 - (a*x*Sqrt[c*x^2])/(2*b^2) + (x^2*Sqrt[c*x^2])/(3*b)
 - (a^3*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rubi [A] time = 0.0249259, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2\sqrt{cx^2}}{b^3}-\frac{a^3\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (a^2*Sqrt[c*x^2])/b^3 - (a*x*Sqrt[c*x^2])/(2*b^2) + (x^2*Sqrt[c*x^2])/(3*b)
 - (a^3*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^3}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{x}\\ &= \frac{a^2\sqrt{cx^2}}{b^3}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}-\frac{a^3\sqrt{cx^2}\log(a+bx)}{b^4x}\end{aligned}$$

Mathematica [A] time = 0.0152084, size = 52, normalized size = 0.65

$$\frac{cx\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (c*x*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*Sqrt[
c*x^2])
```

Maple [A] time = 0.007, size = 52, normalized size = 0.7

$$-\frac{-2\,b^3x^3+3\,ab^2x^2+6\,a^3\ln\left(bx+a\right)-6\,a^2bx}{6\,xb^4}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] -1/6*(c*x^2)^(1/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/x/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49981, size = 113, normalized size = 1.41

$$\frac{\left(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x**2*sqrt(c*x**2)/(a + b*x), x)
```

Giac [A] time = 1.08084, size = 93, normalized size = 1.16

$$-\frac{1}{6}\sqrt{c}\left(\frac{6a^3\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b^4}-\frac{6a^3\log\left(|a|\right)\operatorname{sgn}(x)}{b^4}-\frac{2b^2x^3\operatorname{sgn}(x)-3abx^2\operatorname{sgn}(x)+6a^2x\operatorname{sgn}(x)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(c)*(6*a^3*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*log(abs(a))*sgn(x)
/b^4 - (2*b^2*x^3*sgn(x) - 3*a*b*x^2*sgn(x) + 6*a^2*x*sgn(x))/b^3)
```

3.854 $\int \frac{x\sqrt{cx^2}}{a+bx}\,dx$

Optimal. Leaf size=58

$$\frac{a^2\sqrt{cx^2}\log(a+bx)}{b^3x}-\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}$$

```
[Out] -((a*Sqrt[c*x^2])/b^2) + (x*Sqrt[c*x^2])/(2*b) + (a^2*Sqrt[c*x^2]*Log[a + b
*x])/(b^3*x)
```

Rubi [A] time = 0.0182118, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{a^2\sqrt{cx^2}\log(a+bx)}{b^3x}-\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] -((a*Sqrt[c*x^2])/b^2) + (x*Sqrt[c*x^2])/(2*b) + (a^2*Sqrt[c*x^2]*Log[a + b
*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{cx^2}}{a+bx}\,dx&=\frac{\sqrt{cx^2}\int \frac{x^2}{a+bx}\,dx}{x}\\&=\frac{\sqrt{cx^2}\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{x}\\&=-\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}+\frac{a^2\sqrt{cx^2}\log(a+bx)}{b^3x}\end{aligned}$$

Mathematica [A] time = 0.0113913, size = 40, normalized size = 0.69

$$\frac{cx\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (c*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

Maple [A] time = 0.006, size = 40, normalized size = 0.7

$$\frac{b^2x^2 + 2\,a^2 \ln\left(bx + a\right) - 2\,abx}{2\,b^3x}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/x/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60804, size = 89, normalized size = 1.53

$$\frac{\left(b^2x^2 - 2\,abx + 2\,a^2 \log\left(bx + a\right)\right)\sqrt{cx^2}}{2\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{cx^2}}{a + bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x*sqrt(c*x**2)/(a + b*x), x)
```

Giac [A] time = 1.07681, size = 73, normalized size = 1.26

$$\frac{1}{2}\sqrt{c}\left(\frac{2\,a^2\log\left(|bx+a|\right)\operatorname{sgn}\left(x\right)}{b^3}-\frac{2\,a^2\log\left(|a|\right)\operatorname{sgn}\left(x\right)}{b^3}+\frac{bx^2\operatorname{sgn}\left(x\right)-2\,ax\operatorname{sgn}\left(x\right)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c)*(2*a^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*log(abs(a))*sgn(x)/
b^3 + (b*x^2*sgn(x) - 2*a*x*sgn(x))/b^2)
```

3.855 $\int \frac{\sqrt{cx^2}}{a+bx} \, dx$

Optimal. Leaf size=38

$$\frac{\sqrt{cx^2}}{b}-\frac{a\sqrt{cx^2}\log(a+bx)}{b^2x}$$

```
[Out] Sqrt[c*x^2]/b - (a*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rubi [A] time = 0.0124095, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{\sqrt{cx^2}}{b}-\frac{a\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(a + b*x),x]
```

```
[Out] Sqrt[c*x^2]/b - (a*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{x}\\ &= \frac{\sqrt{cx^2}}{b}-\frac{a\sqrt{cx^2}\log(a+bx)}{b^2x}\end{aligned}$$

Mathematica [A] time = 0.0071633, size = 28, normalized size = 0.74

$$\frac{cx(bx-a\log(a+bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(a + b*x),x]
```

```
[Out] (c*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

Maple [A] time = 0.004, size = 29, normalized size = 0.8

$$-\frac{a\ln(bx+a)-bx}{b^2x}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] -(c*x^2)^(1/2)*(a*ln(b*x+a)-b*x)/b^2/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55409, size = 59, normalized size = 1.55

$$\frac{\sqrt{cx^2}(bx-a\log(bx+a))}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(a + b*x), x)
```

Giac [A] time = 1.05594, size = 50, normalized size = 1.32

$$\sqrt{c}\left(\frac{x\text{sgn}(x)}{b} - \frac{a\log(|bx+a|)\text{sgn}(x)}{b^2} + \frac{a\log(|a|)\text{sgn}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] sqrt(c)*(x*sgn(x)/b - a*log(abs(b*x + a))*sgn(x)/b^2 + a*log(abs(a))*sgn(x)
/b^2)
```

3.856 $\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx$

Optimal. Leaf size=22

$$\frac{\sqrt{cx^2}\log(a+bx)}{bx}$$

```
[Out] (Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rubi [A] time = 0.0032198, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 31}

$$\frac{\sqrt{cx^2}\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x*(a + b*x)),x]
```

```
[Out] (Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\log(a+bx)}{bx}\end{aligned}$$

Mathematica [A] time = 0.0036721, size = 21, normalized size = 0.95

$$\frac{cx\log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x*(a + b*x)),x]
```

```
[Out] (c*x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

Maple [A] time = 0.002, size = 21, normalized size = 1.

$$\frac{\ln\left(bx+a\right)\sqrt{c x^{2}}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x/(b*x+a),x)
```

```
[Out] ln(b*x+a)*(c*x^2)^(1/2)/b/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46506, size = 43, normalized size = 1.95

$$\frac{\sqrt{c x^{2}}\log\left(bx+a\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*log(b*x + a)/(b*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c x^{2}}}{x\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x*(a + b*x)), x)
```

Giac [A] time = 1.05504, size = 38, normalized size = 1.73

$$\sqrt{c}\left(\frac{\log\left(|bx+a|\right)\operatorname{sgn}\left(x\right)}{b}-\frac{\log\left(|a|\right)\operatorname{sgn}\left(x\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] sqrt(c)*(log(abs(b*x + a))*sgn(x)/b - log(abs(a))*sgn(x)/b)
```

3.857 $\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{cx^2}\log(x)}{ax}-\frac{\sqrt{cx^2}\log(a+bx)}{ax}$$

```
[Out] (Sqrt[c*x^2]*Log[x])/(a*x) - (Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rubi [A] time = 0.007027, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {15, 36, 29, 31}

$$\frac{\sqrt{cx^2}\log(x)}{ax}-\frac{\sqrt{cx^2}\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
```

```
[Out] (Sqrt[c*x^2]*Log[x])/(a*x) - (Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{x(a+bx)}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \frac{1}{x}\,dx}{ax}-\frac{\left(b\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{ax}\\ &= \frac{\sqrt{cx^2}\log(x)}{ax}-\frac{\sqrt{cx^2}\log(a+bx)}{ax}\end{aligned}$$

Mathematica [A] time = 0.007199, size = 26, normalized size = 0.62

$$\frac{cx(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
```

```
[Out] (c*x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

Maple [A] time = 0.008, size = 26, normalized size = 0.6

$$\frac{\ln(x)-\ln(bx+a)}{ax}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^2/(b*x+a),x)
```

```
[Out] (c*x^2)^(1/2)*(ln(x)-ln(b*x+a))/a/x
```

Maxima [A] time = 1.07241, size = 32, normalized size = 0.76

$$-\frac{\sqrt{c}\log(bx+a)}{a}+\frac{\sqrt{c}\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(c)*log(b*x + a)/a + sqrt(c)*log(x)/a
```

Fricas [A] time = 1.63121, size = 139, normalized size = 3.31

$$\left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{ax}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b*x +
 a)*sqrt(-c)/(a*c*x))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}}{x^2 (a + bx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**2/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**2*(a + b*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.858 $\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx$

Optimal. Leaf size=61

$$-\frac{b\sqrt{cx^2}\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{\sqrt{cx^2}}{ax^2}$$

```
[Out] -(Sqrt[c*x^2]/(a*x^2)) - (b*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*Sqrt[c*x^2]*Lo
g[a + b*x])/(a^2*x)
```

Rubi [A] time = 0.017243, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$-\frac{b\sqrt{cx^2}\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
```

```
[Out] -(Sqrt[c*x^2]/(a*x^2)) - (b*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*Sqrt[c*x^2]*Lo
g[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{x^2(a+bx)}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{x}\\
&= -\frac{\sqrt{cx^2}}{ax^2}-\frac{b\sqrt{cx^2}\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\log(a+bx)}{a^2x}
\end{aligned}$$

Mathematica [A] time = 0.0115274, size = 32, normalized size = 0.52

$$-\frac{c(-bx\log(a+bx)+a+bx\log(x))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
```

```
[Out] -((c*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

Maple [A] time = 0.01, size = 33, normalized size = 0.5

$$-\frac{b\ln(x)\,x - b\ln(bx+a)\,x + a}{a^2x^2}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^3/(b*x+a),x)
```

```
[Out] -(c*x^2)^(1/2)*(b*ln(x)*x-b*ln(b*x+a)*x+a)/a^2/x^2
```

Maxima [A] time = 1.07367, size = 50, normalized size = 0.82

$$\frac{b\sqrt{c}\log(bx+a)}{a^2} - \frac{b\sqrt{c}\log(x)}{a^2} - \frac{\sqrt{c}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*sqrt(c)*log(b*x + a)/a^2 - b*sqrt(c)*log(x)/a^2 - sqrt(c)/(a*x)
```

Fricas [A] time = 1.57438, size = 68, normalized size = 1.11

$$\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right) - a\right)}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}}{x^3\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**3/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**3*(a + b*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.859 $\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx$

Optimal. Leaf size=84

$$\frac{b^2\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\log(a+bx)}{a^3x}+\frac{b\sqrt{cx^2}}{a^2x^2}-\frac{\sqrt{cx^2}}{2ax^3}$$

```
[Out] -Sqrt[c*x^2]/(2*a*x^3) + (b*Sqrt[c*x^2])/(a^2*x^2) + (b^2*Sqrt[c*x^2]*Log[x
])/(a^3*x) - (b^2*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rubi [A] time = 0.0226037, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$\frac{b^2\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\log(a+bx)}{a^3x}+\frac{b\sqrt{cx^2}}{a^2x^2}-\frac{\sqrt{cx^2}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
```

```
[Out] -Sqrt[c*x^2]/(2*a*x^3) + (b*Sqrt[c*x^2])/(a^2*x^2) + (b^2*Sqrt[c*x^2]*Log[x
])/(a^3*x) - (b^2*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{x^3(a+bx)}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{x}\\ &= -\frac{\sqrt{cx^2}}{2ax^3}+\frac{b\sqrt{cx^2}}{a^2x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\log(a+bx)}{a^3x}\end{aligned}$$

Mathematica [A] time = 0.0146526, size = 53, normalized size = 0.63

$$\frac{\sqrt{cx^2}\left(-2b^2x^2\log(a+bx)-a(a-2bx)+2b^2x^2\log(x)\right)}{2a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
```

```
[Out] (Sqrt[c*x^2]*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x])
)/(2*a^3*x^3)
```

Maple [A] time = 0.01, size = 51, normalized size = 0.6

$$\frac{2 b^2 \ln (x) x^2-2 b^2 \ln (bx+a) x^2+2 abx-a^2}{2 a^3x^3}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^4/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/a^3/x^3
```

Maxima [A] time = 1.09557, size = 70, normalized size = 0.83

$$-\frac{b^2\sqrt{c}\log (bx+a)}{a^3}+\frac{b^2\sqrt{c}\log (x)}{a^3}+\frac{2 b\sqrt{c}x-a\sqrt{c}}{2 a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^2*sqrt(c)*log(b*x + a)/a^3 + b^2*sqrt(c)*log(x)/a^3 + 1/2*(2*b*sqrt(c)*x
 - a*sqrt(c))/(a^2*x^2)
```

Fricas [A] time = 1.62491, size = 97, normalized size = 1.15

$$\frac{\left(2 b^2x^2 \log \left(\frac{x}{bx+a}\right)+2 abx-a^2\right)\sqrt{cx^2}}{2 a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}}{x^4 (a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**4/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**4*(a + b*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.860 $\int \frac{x(cx^2)^{3/2}}{a+bx} dx$

Optimal. Leaf size=107

$$-\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}+\frac{a^4c\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}$$

```
[Out] -((a^3*c*Sqrt[c*x^2])/b^4) + (a^2*c*x*Sqrt[c*x^2])/(2*b^3) - (a*c*x^2*Sqrt[
c*x^2])/(3*b^2) + (c*x^3*Sqrt[c*x^2])/(4*b) + (a^4*c*Sqrt[c*x^2]*Log[a + b*
x])/(b^5*x)
```

Rubi [A] time = 0.0321806, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}+\frac{a^4c\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

```
[Out] -((a^3*c*Sqrt[c*x^2])/b^4) + (a^2*c*x*Sqrt[c*x^2])/(2*b^3) - (a*c*x^2*Sqrt[
c*x^2])/(3*b^2) + (c*x^3*Sqrt[c*x^2])/(4*b) + (a^4*c*Sqrt[c*x^2]*Log[a + b*
x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(cx^2\right)^{3/2}}{a+bx}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^4}{a+bx}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}\\ &= -\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}+\frac{a^4c\sqrt{cx^2}\log(a+bx)}{b^5x}\end{aligned}$$

Mathematica [A] time = 0.013049, size = 64, normalized size = 0.6

$$\frac{\left(cx^2\right)^{3/2}\left(bx\left(6a^2bx-12a^3-4ab^2x^2+3b^3x^3\right)+12a^4\log(a+bx)\right)}{12b^5x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*a^
4*Log[a + b*x]))/(12*b^5*x^3)
```

Maple [A] time = 0.004, size = 63, normalized size = 0.6

$$\frac{3\,b^4x^4-4\,x^3ab^3+6\,x^2a^2b^2+12\,a^4\ln\left(bx+a\right)-12\,bxa^3}{12\,b^5x^3}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/12*(c*x^2)^(3/2)*(3*b^4*x^4-4*x^3*a*b^3+6*x^2*a^2*b^2+12*a^4*ln(b*x+a)-12
*b*x*a^3)/x^3/b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60599, size = 153, normalized size = 1.43

$$\frac{\left(3\,b^4cx^4-4\,ab^3cx^3+6\,a^2b^2cx^2-12\,a^3bcx+12\,a^4c\log\left(bx+a\right)\right)\sqrt{cx^2}}{12\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*c*x^4 - 4*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 - 12*a^3*b*c*x + 12*a^4
*c*log(b*x + a))*sqrt(c*x^2)/(b^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(cx^2\right)^{\frac{3}{2}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)/(b*x+a),x)
```

[Out] Integral(x*(c*x**2)**(3/2)/(a + b*x), x)

Giac [A] time = 1.05754, size = 109, normalized size = 1.02

$$\frac{1}{12}c^{\frac{3}{2}}\left(\frac{12\,a^4\log\left(|bx+a|\right)\mathrm{sgn}\left(x\right)}{b^5}-\frac{12\,a^4\log\left(|a|\right)\mathrm{sgn}\left(x\right)}{b^5}+\frac{3\,b^3x^4\mathrm{sgn}\left(x\right)-4\,ab^2x^3\mathrm{sgn}\left(x\right)+6\,a^2bx^2\mathrm{sgn}\left(x\right)-12\,a^3x\mathrm{sgn}\left(\right.}{b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")

[Out] 1/12*c^(3/2)*(12*a^4*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*log(abs(a))*sgn(x)/b^5 + (3*b^3*x^4*sgn(x) - 4*a*b^2*x^3*sgn(x) + 6*a^2*b*x^2*sgn(x) - 12*a^3*x*sgn(x))/b^4)

3.861 $\int \frac{(cx^2)^{3/2}}{a+bx} dx$

Optimal. Leaf size=84

$$\frac{a^2c\sqrt{cx^2}}{b^3}-\frac{a^3c\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}$$

```
[Out] (a^2*c*Sqrt[c*x^2])/b^3 - (a*c*x*Sqrt[c*x^2])/(2*b^2) + (c*x^2*Sqrt[c*x^2])
/(3*b) - (a^3*c*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rubi [A] time = 0.0246509, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a^2c\sqrt{cx^2}}{b^3}-\frac{a^3c\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(a + b*x),x]
```

```
[Out] (a^2*c*Sqrt[c*x^2])/b^3 - (a*c*x*Sqrt[c*x^2])/(2*b^2) + (c*x^2*Sqrt[c*x^2])
/(3*b) - (a^3*c*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{a+bx} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^3}{a+bx} dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right) dx}{x}\\
&= \frac{a^2c\sqrt{cx^2}}{b^3}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}-\frac{a^3c\sqrt{cx^2}\log(a+bx)}{b^4x}
\end{aligned}$$

Mathematica [A] time = 0.0114378, size = 53, normalized size = 0.63

$$\frac{(cx^2)^{3/2}\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(a + b*x),x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6
*b^4*x^3)
```

Maple [A] time = 0.005, size = 52, normalized size = 0.6

$$-\frac{-2\,b^3x^3+3\,ab^2x^2+6\,a^3\ln\left(bx+a\right)-6\,a^2bx}{6\,x^3b^4}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -1/6*(c*x^2)^(3/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/x^3/b
^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59563, size = 124, normalized size = 1.48

$$\frac{\left(2\,b^3cx^3-3\,ab^2cx^2+6\,a^2bcx-6\,a^3c\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*c*x^3 - 3*a*b^2*c*x^2 + 6*a^2*b*c*x - 6*a^3*c*log(b*x + a))*sqrt
(c*x^2)/(b^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(a + b*x), x)
```

Giac [A] time = 1.05621, size = 93, normalized size = 1.11

$$-\frac{1}{6}c^{\frac{3}{2}}\left(\frac{6a^3\log\left(|bx+a|\right)\operatorname{sgn}\left(x\right)}{b^4}-\frac{6a^3\log\left(|a|\right)\operatorname{sgn}\left(x\right)}{b^4}-\frac{2b^2x^3\operatorname{sgn}\left(x\right)-3abx^2\operatorname{sgn}\left(x\right)+6a^2x\operatorname{sgn}\left(x\right)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*c^(3/2)*(6*a^3*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*log(abs(a))*sgn(x)
/b^4 - (2*b^2*x^3*sgn(x) - 3*a*b*x^2*sgn(x) + 6*a^2*x*sgn(x))/b^3)
```

3.862 $\int \frac{(cx^2)^{3/2}}{x(a+bx)} dx$

Optimal. Leaf size=61

$$\frac{a^2c\sqrt{cx^2}\log(a+bx)}{b^3x}-\frac{ac\sqrt{cx^2}}{b^2}+\frac{cx\sqrt{cx^2}}{2b}$$

```
[Out] -((a*c*Sqrt[c*x^2])/b^2) + (c*x*Sqrt[c*x^2])/(2*b) + (a^2*c*Sqrt[c*x^2]*Log
[a + b*x])/(b^3*x)
```

Rubi [A] time = 0.0185233, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2c\sqrt{cx^2}\log(a+bx)}{b^3x}-\frac{ac\sqrt{cx^2}}{b^2}+\frac{cx\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x*(a + b*x)),x]
```

```
[Out] -((a*c*Sqrt[c*x^2])/b^2) + (c*x*Sqrt[c*x^2])/(2*b) + (a^2*c*Sqrt[c*x^2]*Log
[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x(a+bx)}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^2}{a+bx}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{x}\\ &= -\frac{ac\sqrt{cx^2}}{b^2}+\frac{cx\sqrt{cx^2}}{2b}+\frac{a^2c\sqrt{cx^2}\log(a+bx)}{b^3x}\end{aligned}$$

Mathematica [A] time = 0.0048889, size = 42, normalized size = 0.69

$$\frac{c^2x\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x*(a + b*x)),x]
```

```
[Out] (c^2*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

Maple [A] time = 0.006, size = 40, normalized size = 0.7

$$\frac{b^2x^2 + 2a^2\ln(bx+a) - 2abx}{2b^3x^3}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/b^3/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51577, size = 97, normalized size = 1.59

$$\frac{\left(b^2cx^2 - 2abcx + 2a^2c\log(bx+a)\right)\sqrt{cx^2}}{2b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c*x^2 - 2*a*b*c*x + 2*a^2*c*log(b*x + a))*sqrt(c*x^2)/(b^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x*(a + b*x)), x)
```

Giac [A] time = 1.06519, size = 73, normalized size = 1.2

$$\frac{1}{2}\,c^{\frac{3}{2}}\left(\frac{2\,a^2\log\left(|bx+a|\right)\operatorname{sgn}\left(x\right)}{b^3}-\frac{2\,a^2\log\left(|a|\right)\operatorname{sgn}\left(x\right)}{b^3}+\frac{bx^2\operatorname{sgn}\left(x\right)-2\,ax\operatorname{sgn}\left(x\right)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*c^(3/2)*(2*a^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*log(abs(a))*sgn(x)/
b^3 + (b*x^2*sgn(x) - 2*a*x*sgn(x))/b^2)
```

3.863 $\int \frac{(cx^2)^{3/2}}{x^2(a+bx)} \, dx$

Optimal. Leaf size=40

$$\frac{c\sqrt{cx^2}}{b} - \frac{ac\sqrt{cx^2}\log(a+bx)}{b^2x}$$

```
[Out] (c*Sqrt[c*x^2])/b - (a*c*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rubi [A] time = 0.0116974, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{c\sqrt{cx^2}}{b} - \frac{ac\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c*Sqrt[c*x^2])/b - (a*c*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x^2(a+bx)} \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x}{a+bx} \, dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx}{x} \\
&= \frac{c\sqrt{cx^2}}{b} - \frac{ac\sqrt{cx^2}\log(a+bx)}{b^2x}
\end{aligned}$$

Mathematica [A] time = 0.0033153, size = 30, normalized size = 0.75

$$\frac{c^2x(bx-a\log(a+bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c^2*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

Maple [A] time = 0.003, size = 29, normalized size = 0.7

$$-\frac{a\ln(bx+a)-bx}{x^3b^2}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^2/(b*x+a),x)
```

```
[Out] -(c*x^2)^(3/2)*(a*ln(b*x+a)-b*x)/x^3/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63481, size = 65, normalized size = 1.62

$$\frac{(bcx-ac\log(bx+a))\sqrt{cx^2}}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c*x - a*c*log(b*x + a))*sqrt(c*x^2)/(b^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}}{x^2\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**2/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**2*(a + b*x)), x)
```

Giac [A] time = 1.06222, size = 50, normalized size = 1.25

$$c^{\frac{3}{2}}\left(\frac{x\text{sgn}\left(x\right)}{b}-\frac{a\log\left(\left|bx+a\right|\right)\text{sgn}\left(x\right)}{b^2}+\frac{a\log\left(\left|a\right|\right)\text{sgn}\left(x\right)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] c^(3/2)*(x*sgn(x)/b - a*log(abs(b*x + a))*sgn(x)/b^2 + a*log(abs(a))*sgn(x)
/b^2)
```

3.864 $\int \frac{(cx^2)^{3/2}}{x^3(a+bx)} \, dx$

Optimal. Leaf size=23

$$\frac{c\sqrt{cx^2}\log(a+bx)}{bx}$$

```
[Out] (c*Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rubi [A] time = 0.0038999, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 31}

$$\frac{c\sqrt{cx^2}\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
```

```
[Out] (c*Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^3(a+bx)} \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{a+bx} \, dx}{x}\\ &= \frac{c\sqrt{cx^2}\log(a+bx)}{bx}\end{aligned}$$

Mathematica [A] time = 0.0032357, size = 22, normalized size = 0.96

$$\frac{(cx^2)^{3/2}\log(a+bx)}{bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(3/2)*Log[a + b*x])/(b*x^3)
```

Maple [A] time = 0.002, size = 21, normalized size = 0.9

$$\frac{\ln (bx + a)}{bx^3} \left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^3/(b*x+a),x)
```

```
[Out] (c*x^2)^(3/2)/x^3*ln(b*x+a)/b
```

Maxima [A] time = 1.05648, size = 18, normalized size = 0.78

$$\frac{c^{\frac{3}{2}} \log (bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] c^(3/2)*log(b*x + a)/b
```

Fricas [A] time = 1.57137, size = 46, normalized size = 2.

$$\frac{\sqrt{cx^2} c \log (bx + a)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*c*log(b*x + a)/(b*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^3 (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**3/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**3*(a + b*x)), x)
```

Giac [A] time = 1.05631, size = 38, normalized size = 1.65

$$c^{\frac{3}{2}}\left(\frac{\log\left(|bx+a|\right)\operatorname{sgn}\left(x\right)}{b}-\frac{\log\left(|a|\right)\operatorname{sgn}\left(x\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] c^(3/2)*(log(abs(b*x + a))*sgn(x)/b - log(abs(a))*sgn(x)/b)
```

3.865 $\int \frac{(cx^2)^{3/2}}{x^4(a+bx)} \, dx$

Optimal. Leaf size=44

$$\frac{c\sqrt{cx^2}\log(x)}{ax} - \frac{c\sqrt{cx^2}\log(a+bx)}{ax}$$

```
[Out] (c*Sqrt[c*x^2]*Log[x])/(a*x) - (c*Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rubi [A] time = 0.0068632, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {15, 36, 29, 31}

$$\frac{c\sqrt{cx^2}\log(x)}{ax} - \frac{c\sqrt{cx^2}\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]
```

```
[Out] (c*Sqrt[c*x^2]*Log[x])/(a*x) - (c*Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x^4(a+bx)} \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)} \, dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x} \, dx}{ax} - \frac{\left(bc\sqrt{cx^2}\right)\int \frac{1}{a+bx} \, dx}{ax} \\
&= \frac{c\sqrt{cx^2}\log(x)}{ax} - \frac{c\sqrt{cx^2}\log(a+bx)}{ax}
\end{aligned}$$

Mathematica [A] time = 0.0076032, size = 27, normalized size = 0.61

$$\frac{(cx^2)^{3/2}(\log(x)-\log(a+bx))}{ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(3/2)*(Log[x] - Log[a + b*x]))/(a*x^3)
```

Maple [A] time = 0.004, size = 26, normalized size = 0.6

$$\frac{\ln(x)-\ln(bx+a)}{ax^3}(cx^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^4/(b*x+a),x)
```

```
[Out] (c*x^2)^(3/2)*(ln(x)-ln(b*x+a))/x^3/a
```

Maxima [A] time = 1.03084, size = 32, normalized size = 0.73

$$-\frac{c^{\frac{3}{2}}\log(bx+a)}{a}+\frac{c^{\frac{3}{2}}\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] -c^(3/2)*log(b*x + a)/a + c^(3/2)*log(x)/a
```

Fricas [A] time = 1.59278, size = 144, normalized size = 3.27

$$\left[\frac{\sqrt{cx^2}c\log\left(\frac{x}{bx+a}\right)}{ax}, \frac{2\sqrt{-c}c\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*c*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*c*arctan(sqrt(c*x^2)*(2*b
*x + a)*sqrt(-c)/(a*c*x))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^4\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**4/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**4*(a + b*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.866 $\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx$

Optimal. Leaf size=64

$$-\frac{bc\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{c\sqrt{cx^2}}{ax^2}$$

```
[Out] -((c*Sqrt[c*x^2])/(a*x^2)) - (b*c*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*c*Sqrt[c
*x^2]*Log[a + b*x])/(a^2*x)
```

Rubi [A] time = 0.016311, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$-\frac{bc\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{c\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(a*x^2)) - (b*c*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*c*Sqrt[c
*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)}\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{x}\\
&= -\frac{c\sqrt{cx^2}}{ax^2}-\frac{bc\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc\sqrt{cx^2}\log(a+bx)}{a^2x}
\end{aligned}$$

Mathematica [A] time = 0.0103781, size = 34, normalized size = 0.53

$$-\frac{c^2(-bx\log(a+bx)+a+bx\log(x))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]
```

```
[Out] -((c^2*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

Maple [A] time = 0.005, size = 33, normalized size = 0.5

$$-\frac{b\ln(x)x - b\ln(bx+a)x + a}{a^2x^4}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^5/(b*x+a),x)
```

```
[Out] -(c*x^2)^(3/2)*(b*ln(x)*x-b*ln(b*x+a)*x+a)/x^4/a^2
```

Maxima [A] time = 1.05236, size = 50, normalized size = 0.78

$$\frac{bc^{\frac{3}{2}}\log(bx+a)}{a^2} - \frac{bc^{\frac{3}{2}}\log(x)}{a^2} - \frac{c^{\frac{3}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(3/2)*log(b*x + a)/a^2 - b*c^(3/2)*log(x)/a^2 - c^(3/2)/(a*x)
```

Fricas [A] time = 1.55071, size = 73, normalized size = 1.14

$$\frac{\left(bcx\log\left(\frac{bx+a}{x}\right) - ac\right)\sqrt{cx^2}}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c*x*log((b*x + a)/x) - a*c)*sqrt(c*x^2)/(a^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^5(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**5/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**5*(a + b*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.867 $\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx$

Optimal. Leaf size=88

$$\frac{b^2c\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\log(a+bx)}{a^3x}+\frac{bc\sqrt{cx^2}}{a^2x^2}-\frac{c\sqrt{cx^2}}{2ax^3}$$

```
[Out] -(c*Sqrt[c*x^2])/(2*a*x^3) + (b*c*Sqrt[c*x^2])/(a^2*x^2) + (b^2*c*Sqrt[c*x^
2]*Log[x])/(a^3*x) - (b^2*c*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rubi [A] time = 0.0236948, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$\frac{b^2c\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\log(a+bx)}{a^3x}+\frac{bc\sqrt{cx^2}}{a^2x^2}-\frac{c\sqrt{cx^2}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
```

```
[Out] -(c*Sqrt[c*x^2])/(2*a*x^3) + (b*c*Sqrt[c*x^2])/(a^2*x^2) + (b^2*c*Sqrt[c*x^
2]*Log[x])/(a^3*x) - (b^2*c*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^3(a+bx)}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{x}\\ &= -\frac{c\sqrt{cx^2}}{2ax^3}+\frac{bc\sqrt{cx^2}}{a^2x^2}+\frac{b^2c\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\log(a+bx)}{a^3x}\end{aligned}$$

Mathematica [A] time = 0.0142415, size = 53, normalized size = 0.6

$$\frac{(cx^2)^{3/2}\left(-2b^2x^2\log(a+bx)-a(a-2bx)+2b^2x^2\log(x)\right)}{2a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(3/2)*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x
]))/(2*a^3*x^5)
```

Maple [A] time = 0.003, size = 51, normalized size = 0.6

$$\frac{2\,b^{2}\ln\left(x\right)x^{2}-2\,b^{2}\ln\left(bx+a\right)x^{2}+2\,abx-a^{2}}{2\,a^{3}x^{5}}\left(cx^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^6/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/x^5/a^3
```

Maxima [A] time = 1.11167, size = 70, normalized size = 0.8

$$-\frac{b^{2}c^{\frac{3}{2}}\log\left(bx+a\right)}{a^{3}}+\frac{b^{2}c^{\frac{3}{2}}\log\left(x\right)}{a^{3}}+\frac{2\,bc^{\frac{3}{2}}x-ac^{\frac{3}{2}}}{2\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^2*c^(3/2)*log(b*x + a)/a^3 + b^2*c^(3/2)*log(x)/a^3 + 1/2*(2*b*c^(3/2)*x
 - a*c^(3/2))/(a^2*x^2)
```

Fricas [A] time = 1.61361, size = 105, normalized size = 1.19

$$\frac{\left(2\,b^{2}cx^{2}\log\left(\frac{x}{bx+a}\right)+2\,abcx-a^{2}c\right)\sqrt{cx^{2}}}{2\,a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*c*x^2*log(x/(b*x + a)) + 2*a*b*c*x - a^2*c)*sqrt(c*x^2)/(a^3*x^3
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^{2}\right)^{\frac{3}{2}}}{x^{6}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**6/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**6*(a + b*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.868 $\int \frac{(cx^2)^{3/2}}{x^7(a+bx)} dx$

Optimal. Leaf size=112

$$-\frac{b^2c\sqrt{cx^2}}{a^3x^2}-\frac{b^3c\sqrt{cx^2}\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\log(a+bx)}{a^4x}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{c\sqrt{cx^2}}{3ax^4}$$

```
[Out] -(c*Sqrt[c*x^2])/(3*a*x^4) + (b*c*Sqrt[c*x^2])/(2*a^2*x^3) - (b^2*c*Sqrt[c*
x^2])/(a^3*x^2) - (b^3*c*Sqrt[c*x^2]*Log[x])/(a^4*x) + (b^3*c*Sqrt[c*x^2]*L
og[a + b*x])/(a^4*x)
```

Rubi [A] time = 0.0310682, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$-\frac{b^2c\sqrt{cx^2}}{a^3x^2}-\frac{b^3c\sqrt{cx^2}\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\log(a+bx)}{a^4x}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{c\sqrt{cx^2}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
```

```
[Out] -(c*Sqrt[c*x^2])/(3*a*x^4) + (b*c*Sqrt[c*x^2])/(2*a^2*x^3) - (b^2*c*Sqrt[c*
x^2])/(a^3*x^2) - (b^3*c*Sqrt[c*x^2]*Log[x])/(a^4*x) + (b^3*c*Sqrt[c*x^2]*L
og[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x^7(a+bx)} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^4(a+bx)} dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right) dx}{x}\\
&= -\frac{c\sqrt{cx^2}}{3ax^4}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{b^2c\sqrt{cx^2}}{a^3x^2}-\frac{b^3c\sqrt{cx^2}\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\log(a+bx)}{a^4x}
\end{aligned}$$

Mathematica [A] time = 0.0251771, size = 65, normalized size = 0.58

$$-\frac{(cx^2)^{3/2}\left(a\left(2a^2-3abx+6b^2x^2\right)-6b^3x^3\log(a+bx)+6b^3x^3\log(x)\right)}{6a^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
```

```
[Out] -((c*x^2)^(3/2)*(a*(2*a^2 - 3*a*b*x + 6*b^2*x^2) + 6*b^3*x^3*Log[x] - 6*b^3
*x^3*Log[a + b*x]))/(6*a^4*x^6)
```

Maple [A] time = 0.01, size = 62, normalized size = 0.6

$$-\frac{6\,b^3\ln(x)\,x^3-6\,b^3\ln(bx+a)\,x^3+6\,ab^2x^2-3\,a^2bx+2\,a^3}{6\,x^6a^4}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^7/(b*x+a),x)
```

```
[Out] -1/6*(c*x^2)^(3/2)*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*a*b^2*x^2-3*a^2*b
*x+2*a^3)/x^6/a^4
```

Maxima [A] time = 1.04894, size = 89, normalized size = 0.79

$$\frac{b^3c^{\frac{3}{2}}\log(bx+a)}{a^4}-\frac{b^3c^{\frac{3}{2}}\log(x)}{a^4}-\frac{6\,b^2c^{\frac{3}{2}}x^2-3\,abc^{\frac{3}{2}}x+2\,a^2c^{\frac{3}{2}}}{6\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^3*c^(3/2)*log(b*x + a)/a^4 - b^3*c^(3/2)*log(x)/a^4 - 1/6*(6*b^2*c^(3/2)*
x^2 - 3*a*b*c^(3/2)*x + 2*a^2*c^(3/2))/(a^3*x^3)
```

Fricas [A] time = 1.66753, size = 132, normalized size = 1.18

$$\frac{\left(6\,b^3cx^3\log\left(\frac{bx+a}{x}\right)-6\,ab^2cx^2+3\,a^2bcx-2\,a^3c\right)\sqrt{cx^2}}{6\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*c*x^3*log((b*x + a)/x) - 6*a*b^2*c*x^2 + 3*a^2*b*c*x - 2*a^3*c)*
sqrt(c*x^2)/(a^4*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^7 (a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**7/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**7*(a + b*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.869 $\int \frac{(cx^2)^{5/2}}{a+bx}\,dx$

Optimal. Leaf size=142

$$\frac{a^4c^2\sqrt{cx^2}}{b^5}-\frac{a^3c^2x\sqrt{cx^2}}{2b^4}+\frac{a^2c^2x^2\sqrt{cx^2}}{3b^3}-\frac{a^5c^2\sqrt{cx^2}\log(a+bx)}{b^6x}-\frac{ac^2x^3\sqrt{cx^2}}{4b^2}+\frac{c^2x^4\sqrt{cx^2}}{5b}$$

```
[Out] (a^4*c^2*Sqrt[c*x^2])/b^5 - (a^3*c^2*x*Sqrt[c*x^2])/(2*b^4) + (a^2*c^2*x^2*
Sqrt[c*x^2])/(3*b^3) - (a*c^2*x^3*Sqrt[c*x^2])/(4*b^2) + (c^2*x^4*Sqrt[c*x^
2])/(5*b) - (a^5*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^6*x)
```

Rubi [A] time = 0.044857, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a^4c^2\sqrt{cx^2}}{b^5}-\frac{a^3c^2x\sqrt{cx^2}}{2b^4}+\frac{a^2c^2x^2\sqrt{cx^2}}{3b^3}-\frac{a^5c^2\sqrt{cx^2}\log(a+bx)}{b^6x}-\frac{ac^2x^3\sqrt{cx^2}}{4b^2}+\frac{c^2x^4\sqrt{cx^2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(a + b*x),x]
```

```
[Out] (a^4*c^2*Sqrt[c*x^2])/b^5 - (a^3*c^2*x*Sqrt[c*x^2])/(2*b^4) + (a^2*c^2*x^2*
Sqrt[c*x^2])/(3*b^3) - (a*c^2*x^3*Sqrt[c*x^2])/(4*b^2) + (c^2*x^4*Sqrt[c*x^
2])/(5*b) - (a^5*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^6*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}}{a+bx}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^5}{a+bx}\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}-\frac{a^5}{b^5(a+bx)}\right)dx}{x}\\
&= \frac{a^4c^2\sqrt{cx^2}}{b^5}-\frac{a^3c^2x\sqrt{cx^2}}{2b^4}+\frac{a^2c^2x^2\sqrt{cx^2}}{3b^3}-\frac{ac^2x^3\sqrt{cx^2}}{4b^2}+\frac{c^2x^4\sqrt{cx^2}}{5b}-\frac{a^5c^2\sqrt{cx^2}\log(a+bx)}{b^6x}
\end{aligned}$$

Mathematica [A] time = 0.0215217, size = 76, normalized size = 0.54

$$\frac{c^3x\left(bx\left(20a^2b^2x^2-30a^3bx+60a^4-15ab^3x^3+12b^4x^4\right)-60a^5\log(a+bx)\right)}{60b^6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(a + b*x),x]
```

```
[Out] (c^3*x*(b*x*(60*a^4 - 30*a^3*b*x + 20*a^2*b^2*x^2 - 15*a*b^3*x^3 + 12*b^4*x
^4) - 60*a^5*Log[a + b*x]))/(60*b^6*Sqrt[c*x^2])
```

Maple [A] time = 0.005, size = 74, normalized size = 0.5

$$-\frac{-12\,b^5x^5+15\,ab^4x^4-20\,a^2b^3x^3+30\,a^3b^2x^2+60\,a^5\ln(bx+a)-60\,a^4bx}{60\,x^5b^6}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(b*x+a),x)
```

```
[Out] -1/60*(c*x^2)^(5/2)*(-12*b^5*x^5+15*a*b^4*x^4-20*a^2*b^3*x^3+30*a^3*b^2*x^2
+60*a^5*ln(b*x+a)-60*a^4*b*x)/x^5/b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59848, size = 198, normalized size = 1.39

$$\frac{\left(12\,b^5c^2x^5-15\,ab^4c^2x^4+20\,a^2b^3c^2x^3-30\,a^3b^2c^2x^2+60\,a^4bc^2x-60\,a^5c^2\log(bx+a)\right)\sqrt{cx^2}}{60\,b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*c^2*x^5 - 15*a*b^4*c^2*x^4 + 20*a^2*b^3*c^2*x^3 - 30*a^3*b^2*c
^2*x^2 + 60*a^4*b*c^2*x - 60*a^5*c^2*log(b*x + a))*sqrt(c*x^2)/(b^6*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2\right)^{\frac{5}{2}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/(b*x+a),x)
```

[Out] Integral((c*x**2)**(5/2)/(a + b*x), x)

Giac [A] time = 1.04936, size = 157, normalized size = 1.11

$$-\frac{1}{60}\left(\frac{60\,a^5c^2\log\left(|bx+a|\right)\text{sgn}\left(x\right)}{b^6}-\frac{60\,a^5c^2\log\left(|a|\right)\text{sgn}\left(x\right)}{b^6}-\frac{12\,b^4c^2x^5\text{sgn}\left(x\right)-15\,ab^3c^2x^4\text{sgn}\left(x\right)+20\,a^2b^2c^2x^3\text{sgn}}{b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/60*(60*a^5*c^2*log(abs(b*x + a))*sgn(x)/b^6 - 60*a^5*c^2*log(abs(a))*sgn
(x)/b^6 - (12*b^4*c^2*x^5*sgn(x) - 15*a*b^3*c^2*x^4*sgn(x) + 20*a^2*b^2*c^2
*x^3*sgn(x) - 30*a^3*b*c^2*x^2*sgn(x) + 60*a^4*c^2*x*sgn(x))/b^5)*sqrt(c)
```

3.870 $\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx$

Optimal. Leaf size=117

$$-\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}+\frac{a^4c^2\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}$$

```
[Out] -((a^3*c^2*Sqrt[c*x^2])/b^4) + (a^2*c^2*x*Sqrt[c*x^2])/(2*b^3) - (a*c^2*x^2
*Sqrt[c*x^2])/(3*b^2) + (c^2*x^3*Sqrt[c*x^2])/(4*b) + (a^4*c^2*Sqrt[c*x^2]*
Log[a + b*x])/(b^5*x)
```

Rubi [A] time = 0.0510223, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}+\frac{a^4c^2\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x*(a + b*x)),x]
```


```
[Out] -((a^3*c^2*Sqrt[c*x^2])/b^4) + (a^2*c^2*x*Sqrt[c*x^2])/(2*b^3) - (a*c^2*x^2
*Sqrt[c*x^2])/(3*b^2) + (c^2*x^3*Sqrt[c*x^2])/(4*b) + (a^4*c^2*Sqrt[c*x^2]*
Log[a + b*x])/(b^5*x)
```


Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^4}{a+bx}\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}\\
&= -\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}+\frac{a^4c^2\sqrt{cx^2}\log(a+bx)}{b^5x}
\end{aligned}$$

Mathematica [A] time = 0.0058649, size = 65, normalized size = 0.56

$$\frac{c\left(cx^2\right)^{3/2}\left(bx\left(6a^2bx-12a^3-4ab^2x^2+3b^3x^3\right)+12a^4\log(a+bx)\right)}{12b^5x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x*(a + b*x)),x]
```

```
[Out] (c*(c*x^2)^(3/2)*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*
a^4*Log[a + b*x]))/(12*b^5*x^3)
```

Maple [A] time = 0.004, size = 63, normalized size = 0.5

$$\frac{3\,b^4x^4-4\,x^3ab^3+6\,x^2a^2b^2+12\,a^4\ln\left(bx+a\right)-12\,bxa^3}{12\,b^5x^5}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x/(b*x+a),x)
```

```
[Out] 1/12*(c*x^2)^(5/2)*(3*b^4*x^4-4*x^3*a*b^3+6*x^2*a^2*b^2+12*a^4*ln(b*x+a)-12
*b*x*a^3)/b^5/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61623, size = 166, normalized size = 1.42

$$\frac{\left(3\,b^4c^2x^4-4\,ab^3c^2x^3+6\,a^2b^2c^2x^2-12\,a^3bc^2x+12\,a^4c^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{12\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*c^2*x^4 - 4*a*b^3*c^2*x^3 + 6*a^2*b^2*c^2*x^2 - 12*a^3*b*c^2*x
+ 12*a^4*c^2*log(b*x + a))*sqrt(c*x^2)/(b^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2\right)^{\frac{5}{2}}}{x\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x*(a + b*x)), x)
```

Giac [A] time = 1.1095, size = 134, normalized size = 1.15

$$\frac{1}{12}\left(\frac{12\,a^4c^2\log\left(|bx+a|\right)\operatorname{sgn}\left(x\right)}{b^5}-\frac{12\,a^4c^2\log\left(|a|\right)\operatorname{sgn}\left(x\right)}{b^5}+\frac{3\,b^3c^2x^4\operatorname{sgn}\left(x\right)-4\,ab^2c^2x^3\operatorname{sgn}\left(x\right)+6\,a^2bc^2x^2\operatorname{sgn}\left(x\right)-12\,a}{b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(12*a^4*c^2*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*c^2*log(abs(a))*sgn(
x)/b^5 + (3*b^3*c^2*x^4*sgn(x) - 4*a*b^2*c^2*x^3*sgn(x) + 6*a^2*b*c^2*x^2*s
gn(x) - 12*a^3*c^2*x*sgn(x))/b^4)*sqrt(c)
```

3.871 $\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx$

Optimal. Leaf size=92

$$\frac{a^2c^2\sqrt{cx^2}}{b^3}-\frac{a^3c^2\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{ac^2x\sqrt{cx^2}}{2b^2}+\frac{c^2x^2\sqrt{cx^2}}{3b}$$

```
[Out] (a^2*c^2*Sqrt[c*x^2])/b^3 - (a*c^2*x*Sqrt[c*x^2])/(2*b^2) + (c^2*x^2*Sqrt[c
*x^2])/(3*b) - (a^3*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rubi [A] time = 0.0303088, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2c^2\sqrt{cx^2}}{b^3}-\frac{a^3c^2\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{ac^2x\sqrt{cx^2}}{2b^2}+\frac{c^2x^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
```

```
[Out] (a^2*c^2*Sqrt[c*x^2])/b^3 - (a*c^2*x*Sqrt[c*x^2])/(2*b^2) + (c^2*x^2*Sqrt[c
*x^2])/(3*b) - (a^3*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^3}{a+bx}\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{x}\\
&= \frac{a^2c^2\sqrt{cx^2}}{b^3}-\frac{ac^2x\sqrt{cx^2}}{2b^2}+\frac{c^2x^2\sqrt{cx^2}}{3b}-\frac{a^3c^2\sqrt{cx^2}\log(a+bx)}{b^4x}
\end{aligned}$$

Mathematica [A] time = 0.0050529, size = 54, normalized size = 0.59

$$\frac{c\left(cx^2\right)^{3/2}\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c*(c*x^2)^(3/2)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/
(6*b^4*x^3)
```

Maple [A] time = 0.004, size = 52, normalized size = 0.6

$$-\frac{-2\,b^3x^3+3\,ab^2x^2+6\,a^3\ln\left(bx+a\right)-6\,a^2bx}{6\,b^4x^5}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^2/(b*x+a),x)
```

```
[Out] -1/6*(c*x^2)^(5/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/b^4/x
^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58303, size = 135, normalized size = 1.47

$$\frac{\left(2\,b^3c^2x^3-3\,ab^2c^2x^2+6\,a^2bc^2x-6\,a^3c^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*c^2*x^3 - 3*a*b^2*c^2*x^2 + 6*a^2*b*c^2*x - 6*a^3*c^2*log(b*x +
a))*sqrt(c*x^2)/(b^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2\right)^{\frac{5}{2}}}{x^2\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**2/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**2*(a + b*x)), x)
```

Giac [A] time = 1.05407, size = 113, normalized size = 1.23

$$-\frac{1}{6}\left(\frac{6\,a^3c^2\log\left(|bx+a|\right)\operatorname{sgn}\left(x\right)}{b^4}-\frac{6\,a^3c^2\log\left(|a|\right)\operatorname{sgn}\left(x\right)}{b^4}-\frac{2\,b^2c^2x^3\operatorname{sgn}\left(x\right)-3\,abc^2x^2\operatorname{sgn}\left(x\right)+6\,a^2c^2x\operatorname{sgn}\left(x\right)}{b^3}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*(6*a^3*c^2*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*c^2*log(abs(a))*sgn(x)
/b^4 - (2*b^2*c^2*x^3*sgn(x) - 3*a*b*c^2*x^2*sgn(x) + 6*a^2*c^2*x*sgn(x))/b
^3)*sqrt(c)
```

3.872 $\int \frac{(cx^2)^{5/2}}{x^3(a+bx)} dx$

Optimal. Leaf size=67

$$\frac{a^2c^2\sqrt{cx^2}\log(a+bx)}{b^3x}-\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}$$

```
[Out] -((a*c^2*Sqrt[c*x^2])/b^2) + (c^2*x*Sqrt[c*x^2])/(2*b) + (a^2*c^2*Sqrt[c*x^
2]*Log[a + b*x])/(b^3*x)
```

Rubi [A] time = 0.0197543, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2c^2\sqrt{cx^2}\log(a+bx)}{b^3x}-\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
```

```
[Out] -((a*c^2*Sqrt[c*x^2])/b^2) + (c^2*x*Sqrt[c*x^2])/(2*b) + (a^2*c^2*Sqrt[c*x^
2]*Log[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^2}{a+bx}\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{x}\\ &= -\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}+\frac{a^2c^2\sqrt{cx^2}\log(a+bx)}{b^3x}\end{aligned}$$

Mathematica [A] time = 0.0050563, size = 42, normalized size = 0.63

$$\frac{c^3x\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
```

```
[Out] (c^3*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

Maple [A] time = 0.005, size = 40, normalized size = 0.6

$$\frac{b^2x^2 + 2a^2\ln(bx + a) - 2abx}{2x^5b^3}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^3/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(5/2)*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/x^5/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54965, size = 105, normalized size = 1.57

$$\frac{\left(b^2c^2x^2 - 2abc^2x + 2a^2c^2\log(bx + a)\right)\sqrt{cx^2}}{2b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2 - 2*a*b*c^2*x + 2*a^2*c^2*log(b*x + a))*sqrt(c*x^2)/(b^3*x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2\right)^{\frac{5}{2}}}{x^3(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**3/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**3*(a + b*x)), x)
```

Giac [A] time = 1.07709, size = 89, normalized size = 1.33

$$\frac{1}{2}\left(\frac{2\,a^2c^2\log\left(|bx+a|\right)\operatorname{sgn}\left(x\right)}{b^3}-\frac{2\,a^2c^2\log\left(|a|\right)\operatorname{sgn}\left(x\right)}{b^3}+\frac{bc^2x^2\operatorname{sgn}\left(x\right)-2\,ac^2x\operatorname{sgn}\left(x\right)}{b^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*c^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*c^2*log(abs(a))*sgn(x)/
b^3 + (b*c^2*x^2*sgn(x) - 2*a*c^2*x*sgn(x))/b^2)*sqrt(c)
```

3.873 $\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx$

Optimal. Leaf size=44

$$\frac{c^2\sqrt{cx^2}}{b} - \frac{ac^2\sqrt{cx^2}\log(a+bx)}{b^2x}$$

```
[Out] (c^2*Sqrt[c*x^2])/b - (a*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rubi [A] time = 0.0122541, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{c^2\sqrt{cx^2}}{b} - \frac{ac^2\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
```

```
[Out] (c^2*Sqrt[c*x^2])/b - (a*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x}{a+bx}\,dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{x} \\
&= \frac{c^2\sqrt{cx^2}}{b} - \frac{ac^2\sqrt{cx^2}\log(a+bx)}{b^2x}
\end{aligned}$$

Mathematica [A] time = 0.0040608, size = 30, normalized size = 0.68

$$\frac{c^3x(bx-a\log(a+bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
```

```
[Out] (c^3*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

Maple [A] time = 0.002, size = 29, normalized size = 0.7

$$-\frac{a\ln\left(bx+a\right)-bx}{b^{2}x^{5}}\left(cx^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^4/(b*x+a),x)
```

```
[Out] -(c*x^2)^(5/2)*(a*ln(b*x+a)-b*x)/x^5/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59132, size = 70, normalized size = 1.59

$$\frac{\left(bc^{2}x-ac^{2}\log\left(bx+a\right)\right)\sqrt{cx^{2}}}{b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c^2*x - a*c^2*log(b*x + a))*sqrt(c*x^2)/(b^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^{2}\right)^{\frac{5}{2}}}{x^{4}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**4/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**4*(a + b*x)), x)
```

Giac [A] time = 1.05969, size = 62, normalized size = 1.41

$$\left(\frac{c^2 x \operatorname{sgn}(x)}{b} - \frac{ac^2 \log(|bx+a|)\operatorname{sgn}(x)}{b^2} + \frac{ac^2 \log(|a|)\operatorname{sgn}(x)}{b^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] (c^2*x*sgn(x)/b - a*c^2*log(abs(b*x + a))*sgn(x)/b^2 + a*c^2*log(abs(a))*sg
n(x)/b^2)*sqrt(c)
```

3.874 $\int \frac{(cx^2)^{5/2}}{x^5(a+bx)}\,dx$

Optimal. Leaf size=25

$$\frac{c^2\sqrt{cx^2}\log(a+bx)}{bx}$$

```
[Out] (c^2*Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rubi [A] time = 0.0043513, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 31}

$$\frac{c^2\sqrt{cx^2}\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]
```

```
[Out] (c^2*Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}}{x^5(a+bx)}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{x}\\ &= \frac{c^2\sqrt{cx^2}\log(a+bx)}{bx}\end{aligned}$$

Mathematica [A] time = 0.00404, size = 22, normalized size = 0.88

$$\frac{(cx^2)^{5/2}\log(a+bx)}{bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(5/2)*Log[a + b*x])/(b*x^5)
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$\frac{\ln(bx+a)}{bx^5}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^5/(b*x+a),x)
```

```
[Out] (c*x^2)^(5/2)/x^5*ln(b*x+a)/b
```

Maxima [A] time = 1.07215, size = 18, normalized size = 0.72

$$\frac{c^{\frac{5}{2}}\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] c^(5/2)*log(b*x + a)/b
```

Fricas [A] time = 1.8374, size = 49, normalized size = 1.96

$$\frac{\sqrt{cx^2}c^2\log(bx+a)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*c^2*log(b*x + a)/(b*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2\right)^{\frac{5}{2}}}{x^5(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**5/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**5*(a + b*x)), x)
```

Giac [A] time = 1.06493, size = 46, normalized size = 1.84

$$\left(\frac{c^2\log\left(|bx+a|\right)\operatorname{sgn}\left(x\right)}{b}-\frac{c^2\log\left(|a|\right)\operatorname{sgn}\left(x\right)}{b}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] (c^2*log(abs(b*x + a))*sgn(x)/b - c^2*log(abs(a))*sgn(x)/b)*sqrt(c)
```

3.875 $\int \frac{(cx^2)^{5/2}}{x^6(a+bx)} dx$

Optimal. Leaf size=48

$$\frac{c^2\sqrt{cx^2}\log(x)}{ax} - \frac{c^2\sqrt{cx^2}\log(a+bx)}{ax}$$

```
[Out] (c^2*Sqrt[c*x^2]*Log[x])/(a*x) - (c^2*Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rubi [A] time = 0.0080137, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {15, 36, 29, 31}

$$\frac{c^2\sqrt{cx^2}\log(x)}{ax} - \frac{c^2\sqrt{cx^2}\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]
```

```
[Out] (c^2*Sqrt[c*x^2]*Log[x])/(a*x) - (c^2*Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}}{x^6(a+bx)} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)} dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x} dx}{ax} - \frac{\left(bc^2\sqrt{cx^2}\right)\int \frac{1}{a+bx} dx}{ax} \\
&= \frac{c^2\sqrt{cx^2}\log(x)}{ax} - \frac{c^2\sqrt{cx^2}\log(a+bx)}{ax}
\end{aligned}$$

Mathematica [A] time = 0.0091546, size = 28, normalized size = 0.58

$$\frac{c^3x(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]
```

```
[Out] (c^3*x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

Maple [A] time = 0.003, size = 26, normalized size = 0.5

$$\frac{\ln(x)-\ln(bx+a)}{ax^5}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^6/(b*x+a),x)
```

```
[Out] (c*x^2)^(5/2)*(ln(x)-ln(b*x+a))/a/x^5
```

Maxima [A] time = 1.04982, size = 32, normalized size = 0.67

$$-\frac{c^{\frac{5}{2}}\log(bx+a)}{a}+\frac{c^{\frac{5}{2}}\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="maxima")
```

```
[Out] -c^(5/2)*log(b*x + a)/a + c^(5/2)*log(x)/a
```

Fricas [A] time = 1.72324, size = 150, normalized size = 3.12

$$\left[\frac{\sqrt{cx^2}c^2\log\left(\frac{x}{bx+a}\right)}{ax}, \frac{2\sqrt{-c}c^2\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*c^2*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*c^2*arctan(sqrt(c*x^2)*
(2*b*x + a)*sqrt(-c)/(a*c*x))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2)^{\frac{5}{2}}}{x^6 (a + bx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**6/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**6*(a + b*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.876 $\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx$

Optimal. Leaf size=70

$$-\frac{bc^2\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{c^2\sqrt{cx^2}}{ax^2}$$

```
[Out] -((c^2*Sqrt[c*x^2])/(a*x^2)) - (b*c^2*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*c^2*
Sqrt[c*x^2]*Log[a + b*x])/(a^2*x)
```

Rubi [A] time = 0.0169617, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$-\frac{bc^2\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{c^2\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
```

```
[Out] -((c^2*Sqrt[c*x^2])/(a*x^2)) - (b*c^2*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*c^2*
Sqrt[c*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)}\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{x}\\&=-\frac{c^2\sqrt{cx^2}}{ax^2}-\frac{bc^2\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\log(a+bx)}{a^2x}\end{aligned}$$

Mathematica [A] time = 0.0116956, size = 34, normalized size = 0.49

$$-\frac{c^3(-bx\log(a+bx)+a+bx\log(x))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
```

```
[Out] -((c^3*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

Maple [A] time = 0.003, size = 33, normalized size = 0.5

$$-\frac{b\ln(x)\,x-b\ln(bx+a)\,x+a}{x^6a^2}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^7/(b*x+a),x)
```

```
[Out] -(c*x^2)^(5/2)*(b*ln(x)*x-b*ln(b*x+a)*x+a)/x^6/a^2
```

Maxima [A] time = 1.04171, size = 50, normalized size = 0.71

$$\frac{bc^{\frac{5}{2}}\log(bx+a)}{a^2}-\frac{bc^{\frac{5}{2}}\log(x)}{a^2}-\frac{c^{\frac{5}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(5/2)*log(b*x + a)/a^2 - b*c^(5/2)*log(x)/a^2 - c^(5/2)/(a*x)
```

Fricas [A] time = 1.83502, size = 78, normalized size = 1.11

$$\frac{\left(bc^2x\log\left(\frac{bx+a}{x}\right)-ac^2\right)\sqrt{cx^2}}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c^2*x*log((b*x + a)/x) - a*c^2)*sqrt(c*x^2)/(a^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2\right)^{\frac{5}{2}}}{x^7\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**7/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**7*(a + b*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.877 $\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=83

$$\frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}$$

```
[Out] (a^2*x^2)/(b^3*Sqrt[c*x^2]) - (a*x^3)/(2*b^2*Sqrt[c*x^2]) + x^4/(3*b*Sqrt[c
*x^2]) - (a^3*x*Log[a + b*x])/(b^4*Sqrt[c*x^2])
```

Rubi [A] time = 0.0240749, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (a^2*x^2)/(b^3*Sqrt[c*x^2]) - (a*x^3)/(2*b^2*Sqrt[c*x^2]) + x^4/(3*b*Sqrt[c
*x^2]) - (a^3*x*Log[a + b*x])/(b^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{x^3}{a+bx}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= \frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0103318, size = 51, normalized size = 0.61

$$\frac{x\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*Sqrt[c*
x^2])
```

Maple [A] time = 0.004, size = 50, normalized size = 0.6

$$-\frac{x\left(-2\,b^{3}x^{3}+3\,ab^{2}x^{2}+6\,a^{3}\ln\left(bx+a\right)-6\,a^{2}bx\right)}{6\,b^{4}}\frac{1}{\sqrt{cx^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] -1/6*x*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/(c*x^2)^(1/2)/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82468, size = 116, normalized size = 1.4

$$\frac{\left(2\,b^{3}x^{3}-3\,ab^{2}x^{2}+6\,a^{2}bx-6\,a^{3}\log\left(bx+a\right)\right)\sqrt{cx^{2}}}{6\,b^{4}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{4}}{\sqrt{cx^{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [A] time = 1.10552, size = 108, normalized size = 1.3

$$\frac{1}{6}\sqrt{cx^2}\left(x\left(\frac{2x}{bc}-\frac{3a}{b^2c}\right)+\frac{6a^2}{b^3c}\right)+\frac{a^3\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2a\sqrt{c}\right|\right)}{b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2)*(x*(2*x/(b*c) - 3*a/(b^2*c)) + 6*a^2/(b^3*c)) + a^3*log(abs
(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(b^4*sqrt(c))
```

3.878 $\int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=61

$$\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}-\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}$$

```
[Out] -((a*x^2)/(b^2*Sqrt[c*x^2])) + x^3/(2*b*Sqrt[c*x^2]) + (a^2*x*Log[a + b*x])
/(b^3*Sqrt[c*x^2])
```

Rubi [A] time = 0.0170711, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}-\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -((a*x^2)/(b^2*Sqrt[c*x^2])) + x^3/(2*b*Sqrt[c*x^2]) + (a^2*x*Log[a + b*x])
/(b^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{x^2}{a+bx}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0122701, size = 39, normalized size = 0.64

$$\frac{x\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

Maple [A] time = 0.004, size = 38, normalized size = 0.6

$$\frac{x\left(b^2x^2+2\,a^2\ln\left(bx+a\right)-2\,abx\right)}{2\,b^3}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/(c*x^2)^(1/2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80387, size = 92, normalized size = 1.51

$$\frac{\left(b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,b^3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [A] time = 1.06914, size = 89, normalized size = 1.46

$$\frac{1}{2}\sqrt{cx^2}\left(\frac{x}{bc}-\frac{2a}{b^2c}\right)-\frac{a^2\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2a\sqrt{c}\right|\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2)*(x/(b*c) - 2*a/(b^2*c)) - a^2*log(abs(-(sqrt(c)*x - sqrt(c*
x^2))*b - 2*a*sqrt(c)))/(b^3*sqrt(c))
```

3.879 $\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=39

$$\frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}$$

```
[Out] x^2/(b*Sqrt[c*x^2]) - (a*x*Log[a + b*x])/(b^2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0121325, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] x^2/(b*Sqrt[c*x^2]) - (a*x*Log[a + b*x])/(b^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{x}{a+bx}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= \frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0079885, size = 27, normalized size = 0.69

$$\frac{x(bx-a\log(a+bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)),x]

[Out] (x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

Maple [A] time = 0.003, size = 27, normalized size = 0.7

$$-\frac{x\left(a\ln\left(bx+a\right)-bx\right)}{b^{2}}\frac{1}{\sqrt{cx^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)/(c*x^2)^(1/2),x)

[Out] -x*(a*ln(b*x+a)-b*x)/(c*x^2)^(1/2)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79, size = 62, normalized size = 1.59

$$\frac{\sqrt{cx^{2}}(bx-a\log\left(bx+a\right))}{b^{2}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")

[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{2}}{\sqrt{cx^{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)/(c*x**2)**(1/2),x)

[Out] Integral(x**2/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [A] time = 1.08518, size = 68, normalized size = 1.74

$$\frac{a \log\left(\left|-\left(\sqrt{c}x - \sqrt{cx^2}\right)b - 2\,a\sqrt{c}\right|\right)}{b^2\sqrt{c}} + \frac{\sqrt{cx^2}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(b^2*sqrt(c)) + sqrt
(c*x^2)/(b*c)
```

3.880 $\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=20

$$\frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

```
[Out] (x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

Rubi [A] time = 0.0037139, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 31}

$$\frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{1}{a+bx}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\log(a+bx)}{b\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0021436, size = 20, normalized size = 1.

$$\frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

Maple [A] time = 0.003, size = 19, normalized size = 1.

$$\frac{x\ln(bx+a)}{b}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] x*ln(b*x+a)/b/(c*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81381, size = 46, normalized size = 2.3

$$\frac{\sqrt{cx^2}\log(bx+a)}{bcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*log(b*x + a)/(b*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [A] time = 1.08111, size = 47, normalized size = 2.35

$$-\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2\,a\sqrt{c}\right|\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(b*sqrt(c))
```

3.881 $\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{x\log(x)}{a\sqrt{cx^2}}-\frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

```
[Out] (x*Log[x])/(a*Sqrt[c*x^2]) - (x*Log[a + b*x])/(a*Sqrt[c*x^2])
```

Rubi [A] time = 0.0071107, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {15, 36, 29, 31}

$$\frac{x\log(x)}{a\sqrt{cx^2}}-\frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*Log[x])/(a*Sqrt[c*x^2]) - (x*Log[a + b*x])/(a*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x(a+bx)}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \frac{1}{x}\,dx}{a\sqrt{cx^2}}-\frac{(bx)\int \frac{1}{a+bx}\,dx}{a\sqrt{cx^2}}\\ &= \frac{x\log(x)}{a\sqrt{cx^2}}-\frac{x\log(a+bx)}{a\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0035858, size = 25, normalized size = 0.66

$$\frac{x(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

Maple [A] time = 0.002, size = 24, normalized size = 0.6

$$\frac{x(\ln(x)-\ln(bx+a))}{a}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] x*(ln(x)-ln(b*x+a))/(c*x^2)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82093, size = 147, normalized size = 3.87

$$\left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{acx}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*c*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b*x
 + a)*sqrt(-c)/(a*c*x))/(a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [A] time = 1.10163, size = 80, normalized size = 2.11

$$\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2\,a\sqrt{c}\right|\right)}{a\sqrt{c}}-\frac{\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(a*sqrt(c)) - log(abs(
-sqrt(c)*x + sqrt(c*x^2)))/(a*sqrt(c))
```

3.882 $\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=54

$$-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}-\frac{1}{a\sqrt{cx^2}}$$

```
[Out] -(1/(a*Sqrt[c*x^2])) - (b*x*Log[x])/(a^2*Sqrt[c*x^2]) + (b*x*Log[a + b*x])/
(a^2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0155541, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}-\frac{1}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -(1/(a*Sqrt[c*x^2])) - (b*x*Log[x])/(a^2*Sqrt[c*x^2]) + (b*x*Log[a + b*x])/
(a^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^2(a+bx)}\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{\sqrt{cx^2}}\\
&= -\frac{1}{a\sqrt{cx^2}}-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0108824, size = 36, normalized size = 0.67

$$\frac{cx^2(bx\log(a+bx)-a-bx\log(x))}{a^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (c*x^2*(-a - b*x*Log[x] + b*x*Log[a + b*x]))/(a^2*(c*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 30, normalized size = 0.6

$$-\frac{b\ln(x)x - b\ln(bx+a)x + a}{a^2}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] -(b*ln(x)*x-b*ln(b*x+a)*x+a)/(c*x^2)^(1/2)/a^2
```

Maxima [A] time = 1.02622, size = 50, normalized size = 0.93

$$\frac{b\log(bx+a)}{a^2\sqrt{c}} - \frac{b\log(x)}{a^2\sqrt{c}} - \frac{1}{a\sqrt{c}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/(a^2*sqrt(c)) - b*log(x)/(a^2*sqrt(c)) - 1/(a*sqrt(c)*x)
```

Fricas [A] time = 1.66856, size = 70, normalized size = 1.3

$$\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right) - a\right)}{a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*c*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*x**2)*(a + b*x)), x)
```

Giac [A] time = 1.08413, size = 123, normalized size = 2.28

$$-\sqrt{c}\left(\frac{b\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2\,a\sqrt{c}\right|\right)}{a^2c}-\frac{b\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{a^2c}-\frac{2}{\left(\sqrt{c}x-\sqrt{cx^2}\right)a\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(c)*(b*log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(a^2*c) -
b*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/(a^2*c) - 2/((sqrt(c)*x - sqrt(c*x^2))
*a*sqrt(c)))
```

3.883 $\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=77

$$\frac{b^2x\log(x)}{a^3\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}+\frac{b}{a^2\sqrt{cx^2}}-\frac{1}{2ax\sqrt{cx^2}}$$

```
[Out] b/(a^2*Sqrt[c*x^2]) - 1/(2*a*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(a^3*Sqrt[c*x^
2]) - (b^2*x*Log[a + b*x])/(a^3*Sqrt[c*x^2])
```

Rubi [A] time = 0.0285021, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$\frac{b^2x\log(x)}{a^3\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}+\frac{b}{a^2\sqrt{cx^2}}-\frac{1}{2ax\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] b/(a^2*Sqrt[c*x^2]) - 1/(2*a*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(a^3*Sqrt[c*x^
2]) - (b^2*x*Log[a + b*x])/(a^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^3(a+bx)}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= \frac{b}{a^2\sqrt{cx^2}}-\frac{1}{2ax\sqrt{cx^2}}+\frac{b^2x\log(x)}{a^3\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0108957, size = 52, normalized size = 0.68

$$\frac{cx\left(-2b^2x^2\log(a+bx)-a(a-2bx)+2b^2x^2\log(x)\right)}{2a^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (c*x*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x]))/(2*a^3
*(c*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 51, normalized size = 0.7

$$\frac{2 b^2 \ln(x) x^2 - 2 b^2 \ln(bx+a) x^2 + 2abx - a^2}{2 x a^3} \frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/2/x*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/(c*x^2)^(1/2)/a^3
```

Maxima [A] time = 1.02626, size = 74, normalized size = 0.96

$$-\frac{b^2 \log(bx+a)}{a^3 \sqrt{c}} + \frac{b^2 \log(x)}{a^3 \sqrt{c}} + \frac{2 b \sqrt{c} x - a \sqrt{c}}{2 a^2 c x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/(a^3*sqrt(c)) + b^2*log(x)/(a^3*sqrt(c)) + 1/2*(2*b*sqrt(
c)*x - a*sqrt(c))/(a^2*c*x^2)
```

Fricas [A] time = 1.58499, size = 100, normalized size = 1.3

$$\frac{\left(2 b^2 x^2 \log\left(\frac{x}{bx+a}\right) + 2abx - a^2\right)\sqrt{cx^2}}{2 a^3 c x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*c*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{cx^2} (a+bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c*x**2)*(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.884 $\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=100

$$-\frac{b^2}{a^3\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}$$

```
[Out] -(b^2/(a^3*Sqrt[c*x^2])) - 1/(3*a*x^2*Sqrt[c*x^2]) + b/(2*a^2*x*Sqrt[c*x^2]
) - (b^3*x*Log[x])/(a^4*Sqrt[c*x^2]) + (b^3*x*Log[a + b*x])/(a^4*Sqrt[c*x^2
])
```

Rubi [A] time = 0.0263064, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$-\frac{b^2}{a^3\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -(b^2/(a^3*Sqrt[c*x^2])) - 1/(3*a*x^2*Sqrt[c*x^2]) + b/(2*a^2*x*Sqrt[c*x^2]
) - (b^3*x*Log[x])/(a^4*Sqrt[c*x^2]) + (b^3*x*Log[a + b*x])/(a^4*Sqrt[c*x^2
])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^4(a+bx)}\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx}{\sqrt{cx^2}}\\
&= -\frac{b^2}{a^3\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0117815, size = 63, normalized size = 0.63

$$\frac{c\left(a\left(-2a^2+3abx-6b^2x^2\right)+6b^3x^3\log(a+bx)-6b^3x^3\log(x)\right)}{6a^4\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (c*(a*(-2*a^2 + 3*a*b*x - 6*b^2*x^2) - 6*b^3*x^3*Log[x] + 6*b^3*x^3*Log[a +
 b*x]))/(6*a^4*(c*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 62, normalized size = 0.6

$$-\frac{6\,b^3\ln(x)\,x^3-6\,b^3\ln(bx+a)\,x^3+6\,ab^2x^2-3\,a^2bx+2\,a^3}{6\,x^2a^4}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] -1/6/x^2*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*a*b^2*x^2-3*a^2*b*x+2*a^3)/
(c*x^2)^(1/2)/a^4
```

Maxima [A] time = 1.03203, size = 93, normalized size = 0.93

$$\frac{b^3\log(bx+a)}{a^4\sqrt{c}}-\frac{b^3\log(x)}{a^4\sqrt{c}}-\frac{6\,b^2\sqrt{c}x^2-3\,ab\sqrt{c}x+2\,a^2\sqrt{c}}{6\,a^3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/(a^4*sqrt(c)) - b^3*log(x)/(a^4*sqrt(c)) - 1/6*(6*b^2*sqrt
(c)*x^2 - 3*a*b*sqrt(c)*x + 2*a^2*sqrt(c))/(a^3*c*x^3)
```

Fricas [A] time = 1.57203, size = 124, normalized size = 1.24

$$\frac{\left(6\,b^3x^3\log\left(\frac{bx+a}{x}\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3\right)\sqrt{cx^2}}{6\,a^4cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log((b*x + a)/x) - 6*a*b^2*x^2 + 3*a^2*b*x - 2*a^3)*sqrt(c*x
^2)/(a^4*c*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(c*x**2)*(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2}(bx+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2)*(b*x + a)*x^3), x)
```

3.885 $\int \frac{x^6}{\left(cx^2\right)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=95

$$\frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}$$

```
[Out] (a^2*x^2)/(b^3*c*Sqrt[c*x^2]) - (a*x^3)/(2*b^2*c*Sqrt[c*x^2]) + x^4/(3*b*c*
Sqrt[c*x^2]) - (a^3*x*Log[a + b*x])/(b^4*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0271292, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (a^2*x^2)/(b^3*c*Sqrt[c*x^2]) - (a*x^3)/(2*b^2*c*Sqrt[c*x^2]) + x^4/(3*b*c*
Sqrt[c*x^2]) - (a^3*x*Log[a + b*x])/(b^4*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(cx^2\right)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{x^3}{a+bx}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0110408, size = 53, normalized size = 0.56

$$\frac{x^3\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*(c*x^
2)^(3/2))
```

Maple [A] time = 0.004, size = 52, normalized size = 0.6

$$-\frac{x^3\left(-2\,b^3x^3+3\,ab^2x^2+6\,a^3\ln\left(bx+a\right)-6\,a^2bx\right)}{6\,b^4}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -1/6*x^3*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/(c*x^2)^(3/2)/b
^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50659, size = 119, normalized size = 1.25

$$\frac{\left(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^6}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**6/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A] time = 1.08555, size = 115, normalized size = 1.21

$$\frac{\sqrt{cx^2}\left(x\left(\frac{2x}{bc} - \frac{3a}{b^2c}\right) + \frac{6a^2}{b^3c}\right) + \frac{6a^3\log\left(\left|-\left(\sqrt{c}x - \sqrt{cx^2}\right)b - 2a\sqrt{c}\right|\right)}{b^4\sqrt{c}}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(c*x^2)*(x*(2*x/(b*c) - 3*a/(b^2*c)) + 6*a^2/(b^3*c)) + 6*a^3*log(
abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(b^4*sqrt(c)))/c
```

3.886 $\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=70

$$\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}-\frac{ax^2}{b^2c\sqrt{cx^2}}+\frac{x^3}{2bc\sqrt{cx^2}}$$

```
[Out] -((a*x^2)/(b^2*c*Sqrt[c*x^2])) + x^3/(2*b*c*Sqrt[c*x^2]) + (a^2*x*Log[a + b
*x])/(b^3*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0200026, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}-\frac{ax^2}{b^2c\sqrt{cx^2}}+\frac{x^3}{2bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -((a*x^2)/(b^2*c*Sqrt[c*x^2])) + x^3/(2*b*c*Sqrt[c*x^2]) + (a^2*x*Log[a + b
*x])/(b^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{x^2}{a+bx}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{ax^2}{b^2c\sqrt{cx^2}}+\frac{x^3}{2bc\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0078899, size = 41, normalized size = 0.59

$$\frac{x^3\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*(c*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 40, normalized size = 0.6

$$\frac{x^3\left(b^2x^2+2a^2\ln\left(bx+a\right)-2abx\right)}{2b^3}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/2*x^3*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/(c*x^2)^(3/2)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56188, size = 95, normalized size = 1.36

$$\frac{\left(b^2x^2-2abx+2a^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{2b^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\left(cx^2\right)^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**5/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A] time = 1.07358, size = 95, normalized size = 1.36

$$\frac{\sqrt{cx^2}\left(\frac{x}{bc}-\frac{2a}{b^2c}\right)-\frac{2a^2\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2a\sqrt{c}\right|\right)}{b^3\sqrt{c}}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(c*x^2)*(x/(b*c) - 2*a/(b^2*c)) - 2*a^2*log(abs(-(sqrt(c)*x - sqrt
(c*x^2))*b - 2*a*sqrt(c)))/(b^3*sqrt(c)))/c
```

3.887 $\int \frac{x^4}{\left(cx^2\right)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=45

$$\frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}$$

```
[Out] x^2/(b*c*Sqrt[c*x^2]) - (a*x*Log[a + b*x])/(b^2*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0131412, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] x^2/(b*c*Sqrt[c*x^2]) - (a*x*Log[a + b*x])/(b^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(cx^2\right)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{x}{a+bx}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0065016, size = 29, normalized size = 0.64

$$\frac{x^3(bx-a\log(a+bx))}{b^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*(b*x - a*Log[a + b*x]))/(b^2*(c*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 29, normalized size = 0.6

$$-\frac{x^3 (a \ln (bx + a) - bx)}{b^2} \left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -x^3*(a*ln(b*x+a)-b*x)/(c*x^2)^(3/2)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70125, size = 65, normalized size = 1.44

$$\frac{\sqrt{cx^2}(bx - a \log (bx + a))}{b^2 c^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(cx^2\right)^{\frac{3}{2}} (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**4/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A] time = 1.08184, size = 73, normalized size = 1.62

$$\frac{\frac{a\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2\,a\sqrt{c}\right|\right)}{b^2\sqrt{c}}+\frac{\sqrt{cx^2}}{bc}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] (a*log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(b^2*sqrt(c)) + sqr
t(c*x^2)/(b*c))/c
```

3.888 $\int \frac{x^3}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=23

$$\frac{x\log(a+bx)}{bc\sqrt{cx^2}}$$

```
[Out] (x*Log[a + b*x])/(b*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0039192, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 31}

$$\frac{x\log(a+bx)}{bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x*Log[a + b*x])/(b*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{a+bx}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\log(a+bx)}{bc\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0037252, size = 22, normalized size = 0.96

$$\frac{x^3\log(a+bx)}{b(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*Log[a + b*x])/(b*(c*x^2)^(3/2))
```

Maple [A] time = 0.001, size = 21, normalized size = 0.9

$$\frac{x^3 \ln (bx+a)}{b} \left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/(c*x^2)^(3/2)*x^3*ln(b*x+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44449, size = 49, normalized size = 2.13

$$\frac{\sqrt{cx^2} \log (bx+a)}{bc^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*log(b*x + a)/(b*c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(cx^2\right)^{\frac{3}{2}} (a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**3/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A] time = 1.07928, size = 47, normalized size = 2.04

$$-\frac{\log\left(\left|-\left(\sqrt{c}x - \sqrt{cx^2}\right)b - 2\,a\sqrt{c}\right|\right)}{bc^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(b*c^(3/2))
```

3.889 $\int \frac{x^2}{\left(cx^2\right)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=44

$$\frac{x\log(x)}{ac\sqrt{cx^2}}-\frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

```
[Out] (x*Log[x])/(a*c*Sqrt[c*x^2]) - (x*Log[a + b*x])/(a*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0080417, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {15, 36, 29, 31}

$$\frac{x\log(x)}{ac\sqrt{cx^2}}-\frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x*Log[x])/(a*c*Sqrt[c*x^2]) - (x*Log[a + b*x])/(a*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(cx^2\right)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x(a+bx)}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \frac{1}{x}\,dx}{ac\sqrt{cx^2}}-\frac{(bx)\int \frac{1}{a+bx}\,dx}{ac\sqrt{cx^2}}\\
&= \frac{x\log(x)}{ac\sqrt{cx^2}}-\frac{x\log(a+bx)}{ac\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0071789, size = 27, normalized size = 0.61

$$\frac{x^3(\log(x) - \log(a + bx))}{a\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*(Log[x] - Log[a + b*x]))/(a*(c*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 26, normalized size = 0.6

$$\frac{x^3\left(\ln(x) - \ln(bx + a)\right)}{a}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] x^3*(ln(x)-ln(b*x+a))/(c*x^2)^(3/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54608, size = 153, normalized size = 3.48

$$\left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{ac^2x}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*c^2*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b
*x + a)*sqrt(-c)/(a*c*x))/(a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**2/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A] time = 1.08139, size = 85, normalized size = 1.93

$$\frac{\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2\,a\sqrt{c}\right|\right)}{a\sqrt{c}}-\frac{\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{a\sqrt{c}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] (log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(a*sqrt(c)) - log(abs
(-sqrt(c)*x + sqrt(c*x^2)))/(a*sqrt(c)))/c
```

3.890 $\int \frac{x}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=63

$$-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}-\frac{1}{ac\sqrt{cx^2}}$$

```
[Out] -(1/(a*c*Sqrt[c*x^2])) - (b*x*Log[x])/(a^2*c*Sqrt[c*x^2]) + (b*x*Log[a + b*
x])/(a^2*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0169767, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 44}

$$-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}-\frac{1}{ac\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -(1/(a*c*Sqrt[c*x^2])) - (b*x*Log[x])/(a^2*c*Sqrt[c*x^2]) + (b*x*Log[a + b*
x])/(a^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^2(a+bx)}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{1}{ac\sqrt{cx^2}}-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0059688, size = 35, normalized size = 0.56

$$\frac{x^2(bx\log(a+bx)-a-bx\log(x))}{a^2(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^2*(-a - b*x*Log[x] + b*x*Log[a + b*x]))/(a^2*(c*x^2)^(3/2))
```

Maple [A] time = 0.002, size = 33, normalized size = 0.5

$$-\frac{x^2 \left(b \ln(x) x - b \ln(bx+a) x + a\right)}{a^2} \left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -x^2*(b*ln(x)*x-b*ln(b*x+a)*x+a)/(c*x^2)^(3/2)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63943, size = 73, normalized size = 1.16

$$\frac{\sqrt{cx^2}\left(bx \log\left(\frac{bx+a}{x}\right) - a\right)}{a^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*c^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(cx^2\right)^{\frac{3}{2}} (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A] time = 1.08624, size = 123, normalized size = 1.95

$$-\frac{\frac{b\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2\,a\sqrt{c}\right|\right)}{a^2c}-\frac{b\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{a^2c}-\frac{2}{\left(\sqrt{c}x-\sqrt{cx^2}\right)a\sqrt{c}}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(a^2*c) - b*log(ab
s(-sqrt(c)*x + sqrt(c*x^2)))/(a^2*c) - 2/((sqrt(c)*x - sqrt(c*x^2))*a*sqrt(
c)))/sqrt(c)
```

3.891 $\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=89

$$\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}+\frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}$$

```
[Out] b/(a^2*c*Sqrt[c*x^2]) - 1/(2*a*c*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(a^3*c*Sqr
t[c*x^2]) - (b^2*x*Log[a + b*x])/(a^3*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0222743, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 44}

$$\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}+\frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] b/(a^2*c*Sqrt[c*x^2]) - 1/(2*a*c*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(a^3*c*Sqr
t[c*x^2]) - (b^2*x*Log[a + b*x])/(a^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^3(a+bx)}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}+\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.006718, size = 51, normalized size = 0.57

$$\frac{x\left(-2b^2x^2\log(a+bx)-a(a-2bx)+2b^2x^2\log(x)\right)}{2a^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x]))/(2*a^3*(
c*x^2)^(3/2))
```

Maple [A] time = 0.005, size = 49, normalized size = 0.6

$$\frac{x\left(2\,b^{2}\ln\left(x\right)x^{2}-2\,b^{2}\ln\left(bx+a\right)x^{2}+2\,abx-a^{2}\right)}{2\,a^{3}}\left(cx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/2*x*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/(c*x^2)^(3/2)/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49573, size = 103, normalized size = 1.16

$$\frac{\left(2\,b^{2}x^{2}\log\left(\frac{x}{bx+a}\right)+2\,abx-a^{2}\right)\sqrt{cx^{2}}}{2\,a^{3}c^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*c^2*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(cx^{2}\right)^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(1/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.892 $\int \frac{1}{x(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=115

$$-\frac{b^2}{a^3c\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}$$

```
[Out] -(b^2/(a^3*c*Sqrt[c*x^2])) - 1/(3*a*c*x^2*Sqrt[c*x^2]) + b/(2*a^2*c*x*Sqrt[
c*x^2]) - (b^3*x*Log[x])/(a^4*c*Sqrt[c*x^2]) + (b^3*x*Log[a + b*x])/(a^4*c*
Sqrt[c*x^2])
```

Rubi [A] time = 0.0283939, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$-\frac{b^2}{a^3c\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -(b^2/(a^3*c*Sqrt[c*x^2])) - 1/(3*a*c*x^2*Sqrt[c*x^2]) + b/(2*a^2*c*x*Sqrt[
c*x^2]) - (b^3*x*Log[x])/(a^4*c*Sqrt[c*x^2]) + (b^3*x*Log[a + b*x])/(a^4*c*
Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(cx^2\right)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^4(a+bx)}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx}{c\sqrt{cx^2}}\\
&= -\frac{b^2}{a^3c\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.015498, size = 66, normalized size = 0.57

$$\frac{cx^2\left(a\left(-2a^2+3abx-6b^2x^2\right)+6b^3x^3\log(a+bx)-6b^3x^3\log(x)\right)}{6a^4\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (c*x^2*(a*(-2*a^2 + 3*a*b*x - 6*b^2*x^2) - 6*b^3*x^3*Log[x] + 6*b^3*x^3*Log
[a + b*x]))/(6*a^4*(c*x^2)^(5/2))
```

Maple [A] time = 0.004, size = 59, normalized size = 0.5

$$-\frac{6\,b^3\ln(x)\,x^3-6\,b^3\ln(bx+a)\,x^3+6\,ab^2x^2-3\,a^2bx+2\,a^3}{6\,a^4}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -1/6*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*a*b^2*x^2-3*a^2*b*x+2*a^3)/(c*x
^2)^(3/2)/a^4
```

Maxima [A] time = 1.04303, size = 93, normalized size = 0.81

$$\frac{b^3\log(bx+a)}{a^4c^{\frac{3}{2}}}-\frac{b^3\log(x)}{a^4c^{\frac{3}{2}}}-\frac{6\,b^2\sqrt{c}x^2-3\,ab\sqrt{c}x+2\,a^2\sqrt{c}}{6\,a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/(a^4*c^(3/2)) - b^3*log(x)/(a^4*c^(3/2)) - 1/6*(6*b^2*sqrt
(c)*x^2 - 3*a*b*sqrt(c)*x + 2*a^2*sqrt(c))/(a^3*c^2*x^3)
```

Fricas [A] time = 1.60288, size = 127, normalized size = 1.1

$$\frac{\left(6\,b^3x^3\log\left(\frac{bx+a}{x}\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3\right)\sqrt{cx^2}}{6\,a^4c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log((b*x + a)/x) - 6*a*b^2*x^2 + 3*a^2*b*x - 2*a^3)*sqrt(c*x
^2)/(a^4*c^2*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(cx^2\right)^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(1/(x*(c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2\right)^{\frac{3}{2}}(bx+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2)^(3/2)*(b*x + a)*x), x)
```

3.893 $\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=106

$$-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}+\frac{3a^2\sqrt{cx^2}}{b^4}-\frac{4a^3\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}$$

```
[Out] (3*a^2*Sqrt[c*x^2])/b^4 - (a*x*Sqrt[c*x^2])/b^3 + (x^2*Sqrt[c*x^2])/(3*b^2)
 - (a^4*Sqrt[c*x^2])/(b^5*x*(a + b*x)) - (4*a^3*Sqrt[c*x^2]*Log[a + b*x])/(
b^5*x)
```

Rubi [A] time = 0.039058, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}+\frac{3a^2\sqrt{cx^2}}{b^4}-\frac{4a^3\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```


```
[Out] (3*a^2*Sqrt[c*x^2])/b^4 - (a*x*Sqrt[c*x^2])/b^3 + (x^2*Sqrt[c*x^2])/(3*b^2)
 - (a^4*Sqrt[c*x^2])/(b^5*x*(a + b*x)) - (4*a^3*Sqrt[c*x^2]*Log[a + b*x])/(
b^5*x)
```


Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^4}{(a+bx)^2}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx}{x}\\
&= \frac{3a^2\sqrt{cx^2}}{b^4}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3\sqrt{cx^2}\log(a+bx)}{b^5x}
\end{aligned}$$

Mathematica [A] time = 0.0275, size = 81, normalized size = 0.76

$$\frac{cx\left(6a^2b^2x^2+9a^3bx-12a^3(a+bx)\log(a+bx)-3a^4-2ab^3x^3+b^4x^4\right)}{3b^5\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (c*x*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4 - 12*a^3*(
a + b*x)*Log[a + b*x]))/(3*b^5*Sqrt[c*x^2]*(a + b*x))
```

Maple [A] time = 0.009, size = 88, normalized size = 0.8

$$-\frac{-b^4x^4+2x^3ab^3+12\ln(bx+a)xa^3b-6x^2a^2b^2+12a^4\ln(bx+a)-9bxa^3+3a^4}{3b^5x(bx+a)}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] -1/3*(c*x^2)^(1/2)*(-b^4*x^4+2*x^3*a*b^3+12*ln(b*x+a)*x*a^3*b-6*x^2*a^2*b^2
+12*a^4*ln(b*x+a)-9*b*x*a^3+3*a^4)/x/b^5/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65858, size = 177, normalized size = 1.67

$$\frac{\left(b^4x^4-2ab^3x^3+6a^2b^2x^2+9a^3bx-3a^4-12\left(a^3bx+a^4\right)\log(bx+a)\right)\sqrt{cx^2}}{3\left(b^6x^2+ab^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))*sqrt(c*x^2)/(b^6*x^2 + a*b^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**3*sqrt(c*x**2)/(a + b*x)**2, x)
```

Giac [A] time = 1.05304, size = 130, normalized size = 1.23

$$-\frac{1}{3}\sqrt{c}\left(\frac{12\,a^3\log\left(|bx+a|\right)\operatorname{sgn}\left(x\right)}{b^5}+\frac{3\,a^4\operatorname{sgn}\left(x\right)}{(bx+a)b^5}-\frac{3\left(4\,a^3\log\left(|a|\right)+a^3\right)\operatorname{sgn}\left(x\right)}{b^5}-\frac{b^4x^3\operatorname{sgn}\left(x\right)-3\,ab^3x^2\operatorname{sgn}\left(x\right)+9\,a^2b^{\ldots}}{b^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(c)*(12*a^3*log(abs(b*x + a))*sgn(x)/b^5 + 3*a^4*sgn(x)/((b*x + a)
*b^5) - 3*(4*a^3*log(abs(a)) + a^3)*sgn(x)/b^5 - (b^4*x^3*sgn(x) - 3*a*b^3*
x^2*sgn(x) + 9*a^2*b^2*x*sgn(x))/b^6)
```

3.894 $\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=85

$$\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}$$

```
[Out] (-2*a*Sqrt[c*x^2])/b^3 + (x*Sqrt[c*x^2])/(2*b^2) + (a^3*Sqrt[c*x^2])/(b^4*x
*(a + b*x)) + (3*a^2*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rubi [A] time = 0.032175, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (-2*a*Sqrt[c*x^2])/b^3 + (x*Sqrt[c*x^2])/(2*b^2) + (a^3*Sqrt[c*x^2])/(b^4*x
*(a + b*x)) + (3*a^2*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^3}{(a+bx)^2}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx}{x}\\
&= -\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}+\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\log(a+bx)}{b^4x}
\end{aligned}$$

Mathematica [A] time = 0.020186, size = 70, normalized size = 0.82

$$\frac{cx\left(-4a^2bx+6a^2(a+bx)\log(a+bx)+2a^3-3ab^2x^2+b^3x^3\right)}{2b^4\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (c*x*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)*Log[a + b
*x]))/(2*b^4*Sqrt[c*x^2]*(a + b*x))
```

Maple [A] time = 0.009, size = 76, normalized size = 0.9

$$\frac{b^3x^3 + 6\ln(bx+a)\,xa^2b - 3\,ab^2x^2 + 6\,a^3\ln(bx+a) - 4\,a^2bx + 2\,a^3}{2\,xb^4\,(bx+a)}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(b^3*x^3+6*ln(b*x+a)*x*a^2*b-3*a*b^2*x^2+6*a^3*ln(b*x+a)-
4*a^2*b*x+2*a^3)/x/b^4/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53493, size = 154, normalized size = 1.81

$$\frac{\left(b^3x^3 - 3\,ab^2x^2 - 4\,a^2bx + 2\,a^3 + 6\left(a^2bx + a^3\right)\log(bx+a)\right)\sqrt{cx^2}}{2\left(b^5x^2 + ab^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))*sqrt(c*x^2)/(b^5*x^2 + a*b^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2*sqrt(c*x**2)/(a + b*x)**2, x)
```

Giac [A] time = 1.04986, size = 108, normalized size = 1.27

$$\frac{1}{2}\sqrt{c}\left(\frac{6a^2\log\left(|bx+a|\right)\operatorname{sgn}\left(x\right)}{b^4}+\frac{2a^3\operatorname{sgn}\left(x\right)}{(bx+a)b^4}-\frac{2\left(3a^2\log\left(|a|\right)+a^2\right)\operatorname{sgn}\left(x\right)}{b^4}+\frac{b^2x^2\operatorname{sgn}\left(x\right)-4abx\operatorname{sgn}\left(x\right)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c)*(6*a^2*log(abs(b*x + a))*sgn(x)/b^4 + 2*a^3*sgn(x)/((b*x + a)*b
^4) - 2*(3*a^2*log(abs(a)) + a^2)*sgn(x)/b^4 + (b^2*x^2*sgn(x) - 4*a*b*x*sg
n(x))/b^4)
```

3.895 $\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=65

$$-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\log(a+bx)}{b^3x}+\frac{\sqrt{cx^2}}{b^2}$$

```
[Out] Sqrt[c*x^2]/b^2 - (a^2*Sqrt[c*x^2])/(b^3*x*(a + b*x)) - (2*a*Sqrt[c*x^2]*Lo
g[a + b*x])/(b^3*x)
```

Rubi [A] time = 0.021723, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\log(a+bx)}{b^3x}+\frac{\sqrt{cx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] Sqrt[c*x^2]/b^2 - (a^2*Sqrt[c*x^2])/(b^3*x*(a + b*x)) - (2*a*Sqrt[c*x^2]*Lo
g[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^2}{(a+bx)^2}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{x}\\
&= \frac{\sqrt{cx^2}}{b^2}-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\log(a+bx)}{b^3x}
\end{aligned}$$

Mathematica [A] time = 0.019721, size = 53, normalized size = 0.82

$$\frac{cx\left(-a^2+abx-2a(a+bx)\log(a+bx)+b^2x^2\right)}{b^3\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (c*x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x^2
]*(a + b*x))
```

Maple [A] time = 0.009, size = 62, normalized size = 1.

$$-\frac{2\ln(bx+a)xab-b^2x^2+2a^2\ln(bx+a)-abx+a^2}{b^3x(bx+a)}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(1/2)*(2*ln(b*x+a)*x*a*b-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/x/b^3/
(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57574, size = 119, normalized size = 1.83

$$\frac{\left(b^2x^2+abx-a^2-2\left(abx+a^2\right)\log(bx+a)\right)\sqrt{cx^2}}{b^4x^2+ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*x^2
 + a*b^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x*sqrt(c*x**2)/(a + b*x)**2, x)
```

Giac [A] time = 1.07773, size = 78, normalized size = 1.2

$$\sqrt{c}\left(\frac{x\operatorname{sgn}(x)}{b^2}-\frac{2\,a\log(|bx+a|)\operatorname{sgn}(x)}{b^3}+\frac{(2\,a\log(|a|)+a)\operatorname{sgn}(x)}{b^3}-\frac{a^2\operatorname{sgn}(x)}{(bx+a)b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] sqrt(c)*(x*sgn(x)/b^2 - 2*a*log(abs(b*x + a))*sgn(x)/b^3 + (2*a*log(abs(a))
 + a)*sgn(x)/b^3 - a^2*sgn(x)/((b*x + a)*b^3))
```

3.896 $\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=47

$$\frac{a\sqrt{cx^2}}{b^2x(a+bx)}+\frac{\sqrt{cx^2}\log(a+bx)}{b^2x}$$

```
[Out] (a*Sqrt[c*x^2])/(b^2*x*(a + b*x)) + (Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rubi [A] time = 0.0162818, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a\sqrt{cx^2}}{b^2x(a+bx)}+\frac{\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(a + b*x)^2,x]
```

```
[Out] (a*Sqrt[c*x^2])/(b^2*x*(a + b*x)) + (Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{x}{(a+bx)^2}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx}{x}\\ &= \frac{a\sqrt{cx^2}}{b^2x(a+bx)}+\frac{\sqrt{cx^2}\log(a+bx)}{b^2x}\end{aligned}$$

Mathematica [A] time = 0.0116019, size = 36, normalized size = 0.77

$$\frac{cx((a+bx)\log(a+bx)+a)}{b^2\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(a + b*x)^2,x]
```

```
[Out] (c*x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

Maple [A] time = 0.008, size = 41, normalized size = 0.9

$$\frac{b\ln(bx+a)x + a\ln(bx+a) + a}{b^2x(bx+a)}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] (c*x^2)^(1/2)*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/x/b^2/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51192, size = 84, normalized size = 1.79

$$\frac{\sqrt{cx^2}((bx+a)\log(bx+a)+a)}{b^3x^2+ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*x^2 + a*b^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(a + b*x)**2, x)
```

Giac [A] time = 1.07896, size = 62, normalized size = 1.32

$$-\sqrt{c}\left(\frac{(\log(|a|)+1)\operatorname{sgn}(x)}{b^2}-\frac{\log(|bx+a|)\operatorname{sgn}(x)}{b^2}-\frac{a\operatorname{sgn}(x)}{(bx+a)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -sqrt(c)*((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))*sgn(x)/b^2 - a*s
gn(x)/((b*x + a)*b^2))
```

3.897 $\int \frac{\sqrt{cx^2}}{x(a+bx)^2} \, dx$

Optimal. Leaf size=24

$$-\frac{\sqrt{cx^2}}{bx(a+bx)}$$

```
[Out] -(Sqrt[c*x^2]/(b*x*(a + b*x)))
```

Rubi [A] time = 0.0036378, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 32}

$$-\frac{\sqrt{cx^2}}{bx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
```

```
[Out] -(Sqrt[c*x^2]/(b*x*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{cx^2}}{x(a+bx)^2} \, dx &= \frac{\sqrt{cx^2} \int \frac{1}{(a+bx)^2} \, dx}{x} \\ &= -\frac{\sqrt{cx^2}}{bx(a+bx)} \end{aligned}$$

Mathematica [A] time = 0.0061281, size = 23, normalized size = 0.96

$$-\frac{cx}{b\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
```

```
[Out] -((c*x)/(b*Sqrt[c*x^2]*(a + b*x)))
```

Maple [A] time = 0.003, size = 23, normalized size = 1.

$$-\frac{1}{bx\,(bx+a)}\sqrt{c x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(1/2)/b/x/(b*x+a)
```

Maxima [A] time = 1.03162, size = 22, normalized size = 0.92

$$-\frac{\sqrt{c}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(c)/(b^2*x + a*b)
```

Fricas [A] time = 1.26492, size = 43, normalized size = 1.79

$$-\frac{\sqrt{c x^2}}{b^2x^2+abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^2)/(b^2*x^2 + a*b*x)
```

Sympy [A] time = 0.801927, size = 39, normalized size = 1.62

$$\begin{cases} -\frac{\sqrt{c}\sqrt{x^2}}{abx+b^2x^2} & \text{for } b\neq 0 \\ \frac{\sqrt{c}\sqrt{x^2}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x/(b*x+a)**2,x)
```

```
[Out] Piecewise((-sqrt(c)*sqrt(x**2)/(a*b*x + b**2*x**2), Ne(b, 0)), (sqrt(c)*sqr
t(x**2)/a**2, True))
```

Giac [A] time = 1.06232, size = 39, normalized size = 1.62

$$-\sqrt{c}\left(\frac{\mathrm{sgn}\,(x)}{(bx+a)b}-\frac{\mathrm{sgn}\,(x)}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -sqrt(c)*(sgn(x)/((b*x + a)*b) - sgn(x)/(a*b))
```

3.898 $\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{cx^2}\log(a+bx)}{a^2x}+\frac{\sqrt{cx^2}\log(x)}{a^2x}+\frac{\sqrt{cx^2}}{ax(a+bx)}$$

```
[Out] Sqrt[c*x^2]/(a*x*(a + b*x)) + (Sqrt[c*x^2]*Log[x])/(a^2*x) - (Sqrt[c*x^2]*L
og[a + b*x])/(a^2*x)
```

Rubi [A] time = 0.0179407, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$-\frac{\sqrt{cx^2}\log(a+bx)}{a^2x}+\frac{\sqrt{cx^2}\log(x)}{a^2x}+\frac{\sqrt{cx^2}}{ax(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]/(a*x*(a + b*x)) + (Sqrt[c*x^2]*Log[x])/(a^2*x) - (Sqrt[c*x^2]*L
og[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{x(a+bx)^2}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx}{x}\\ &= \frac{\sqrt{cx^2}}{ax(a+bx)}+\frac{\sqrt{cx^2}\log(x)}{a^2x}-\frac{\sqrt{cx^2}\log(a+bx)}{a^2x}\end{aligned}$$

Mathematica [A] time = 0.014482, size = 45, normalized size = 0.69

$$\frac{cx(\log(x)(a+bx)-(a+bx)\log(a+bx)+a)}{a^2\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
```

```
[Out] (c*x*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*Sqrt[c*x^2]*(a +
 b*x))
```

Maple [A] time = 0.01, size = 52, normalized size = 0.8

$$\frac{b\ln(x)x - b\ln(bx+a)x + a\ln(x) - a\ln(bx+a) + a}{a^2x(bx+a)}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^2/(b*x+a)^2,x)
```

```
[Out] (c*x^2)^(1/2)*(b*ln(x)*x-b*ln(b*x+a)*x+a*ln(x)-a*ln(b*x+a)+a)/x/a^2/(b*x+a)
```

Maxima [A] time = 1.03193, size = 51, normalized size = 0.78

$$\frac{\sqrt{c}}{abx+a^2} - \frac{\sqrt{c}\log(bx+a)}{a^2} + \frac{\sqrt{c}\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c)/(a*b*x + a^2) - sqrt(c)*log(b*x + a)/a^2 + sqrt(c)*log(x)/a^2
```

Fricas [A] time = 1.43205, size = 89, normalized size = 1.37

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*x^2 + a^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**2/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**2*(a + b*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.899 $\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2} dx$

Optimal. Leaf size=87

$$-\frac{b\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2b\sqrt{cx^2}\log(x)}{a^3x}+\frac{2b\sqrt{cx^2}\log(a+bx)}{a^3x}-\frac{\sqrt{cx^2}}{a^2x^2}$$

```
[Out] -(Sqrt[c*x^2]/(a^2*x^2)) - (b*Sqrt[c*x^2])/(a^2*x*(a + b*x)) - (2*b*Sqrt[c*
x^2]*Log[x])/(a^3*x) + (2*b*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rubi [A] time = 0.0270424, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$-\frac{b\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2b\sqrt{cx^2}\log(x)}{a^3x}+\frac{2b\sqrt{cx^2}\log(a+bx)}{a^3x}-\frac{\sqrt{cx^2}}{a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
```

```
[Out] -(Sqrt[c*x^2]/(a^2*x^2)) - (b*Sqrt[c*x^2])/(a^2*x*(a + b*x)) - (2*b*Sqrt[c*
x^2]*Log[x])/(a^3*x) + (2*b*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{x^2(a+bx)^2}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx}{x}\\ &= -\frac{\sqrt{cx^2}}{a^2x^2}-\frac{b\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2b\sqrt{cx^2}\log(x)}{a^3x}+\frac{2b\sqrt{cx^2}\log(a+bx)}{a^3x}\end{aligned}$$

Mathematica [A] time = 0.0268839, size = 57, normalized size = 0.66

$$-\frac{c(a(a+2bx)+2bx\log(x)(a+bx)-2bx(a+bx)\log(a+bx))}{a^3\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
```

```
[Out] -((c*(a*(a + 2*b*x) + 2*b*x*(a + b*x)*Log[x] - 2*b*x*(a + b*x)*Log[a + b*x]
))/(a^3*Sqrt[c*x^2]*(a + b*x)))
```

Maple [A] time = 0.003, size = 74, normalized size = 0.9

$$-\frac{2\,b^{2}\ln\left(x\right)x^{2}-2\,b^{2}\ln\left(bx+a\right)x^{2}+2\,ab\ln\left(x\right)x-2\,\ln\left(bx+a\right)xab+2\,abx+a^{2}}{x^{2}a^{3}\left(bx+a\right)}\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^3/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(1/2)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*ln(x)*x-2*ln(b*x+
a)*x*a*b+2*a*b*x+a^2)/x^2/a^3/(b*x+a)
```

Maxima [A] time = 1.02047, size = 78, normalized size = 0.9

$$-\frac{2\,b\sqrt{c}x+a\sqrt{c}}{a^{2}bx^{2}+a^{3}x}+\frac{2\,b\sqrt{c}\log\left(bx+a\right)}{a^{3}}-\frac{2\,b\sqrt{c}\log\left(x\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*sqrt(c)*x + a*sqrt(c))/(a^2*b*x^2 + a^3*x) + 2*b*sqrt(c)*log(b*x + a)
/a^3 - 2*b*sqrt(c)*log(x)/a^3
```

Fricas [A] time = 1.32187, size = 123, normalized size = 1.41

$$-\frac{\left(2\,abx+a^{2}-2\left(b^{2}x^{2}+abx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^{2}}}{a^{3}bx^{3}+a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
x^3 + a^4*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx^{2}}}{x^{3}\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**3/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**3*(a + b*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.900 $\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2} dx$

Optimal. Leaf size=112

$$\frac{b^2\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2\sqrt{cx^2}\log(x)}{a^4x}-\frac{3b^2\sqrt{cx^2}\log(a+bx)}{a^4x}+\frac{2b\sqrt{cx^2}}{a^3x^2}-\frac{\sqrt{cx^2}}{2a^2x^3}$$

```
[Out] -Sqrt[c*x^2]/(2*a^2*x^3) + (2*b*Sqrt[c*x^2])/(a^3*x^2) + (b^2*Sqrt[c*x^2])/
(a^3*x*(a + b*x)) + (3*b^2*Sqrt[c*x^2]*Log[x])/(a^4*x) - (3*b^2*Sqrt[c*x^2]
*Log[a + b*x])/(a^4*x)
```

Rubi [A] time = 0.0356999, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$\frac{b^2\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2\sqrt{cx^2}\log(x)}{a^4x}-\frac{3b^2\sqrt{cx^2}\log(a+bx)}{a^4x}+\frac{2b\sqrt{cx^2}}{a^3x^2}-\frac{\sqrt{cx^2}}{2a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

```
[Out] -Sqrt[c*x^2]/(2*a^2*x^3) + (2*b*Sqrt[c*x^2])/(a^3*x^2) + (b^2*Sqrt[c*x^2])/
(a^3*x*(a + b*x)) + (3*b^2*Sqrt[c*x^2]*Log[x])/(a^4*x) - (3*b^2*Sqrt[c*x^2]
*Log[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2} dx &= \frac{\sqrt{cx^2}\int \frac{1}{x^3(a+bx)^2} dx}{x} \\
&= \frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right) dx}{x} \\
&= -\frac{\sqrt{cx^2}}{2a^2x^3}+\frac{2b\sqrt{cx^2}}{a^3x^2}+\frac{b^2\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2\sqrt{cx^2}\log(x)}{a^4x}-\frac{3b^2\sqrt{cx^2}\log(a+bx)}{a^4x}
\end{aligned}$$

Mathematica [A] time = 0.0271562, size = 82, normalized size = 0.73

$$\frac{\sqrt{cx^2}\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2\log(x)(a+bx)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

```
[Out] (Sqrt[c*x^2]*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] -
 6*b^2*x^2*(a + b*x)*Log[a + b*x]))/(2*a^4*x^3*(a + b*x))
```

Maple [A] time = 0.012, size = 95, normalized size = 0.9

$$\frac{6 b^3 \ln(x) x^3 - 6 b^3 \ln(bx+a) x^3 + 6 \ln(x) x^2 a b^2 - 6 \ln(bx+a) x^2 a b^2 + 6 a b^2 x^2 + 3 a^2 b x - a^3}{2 x^3 a^4 (bx+a)} \sqrt{c x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^4/(b*x+a)^2,x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*ln(x)*x^2*a*b^2-6*
ln(b*x+a)*x^2*a*b^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/x^3/a^4/(b*x+a)
```

Maxima [A] time = 0.994223, size = 107, normalized size = 0.96

$$\frac{6 b^2 \sqrt{c} x^2 + 3 a b \sqrt{c} x - a^2 \sqrt{c}}{2 \left(a^3 b x^3 + a^4 x^2\right)} - \frac{3 b^2 \sqrt{c} \log(bx+a)}{a^4} + \frac{3 b^2 \sqrt{c} \log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*sqrt(c)*x^2 + 3*a*b*sqrt(c)*x - a^2*sqrt(c))/(a^3*b*x^3 + a^4*x^
2) - 3*b^2*sqrt(c)*log(b*x + a)/a^4 + 3*b^2*sqrt(c)*log(x)/a^4
```

Fricas [A] time = 1.34271, size = 154, normalized size = 1.38

$$\frac{\left(6 a b^2 x^2 + 3 a^2 b x - a^3 + 6 \left(b^3 x^3 + a b^2 x^2\right) \log\left(\frac{x}{bx+a}\right)\right) \sqrt{c x^2}}{2 \left(a^4 b x^4 + a^5 x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*x^4 + a^5*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c x^2}}{x^4 (a+bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**4/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**4*(a + b*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.901 $\int \frac{x(cx^2)^{3/2}}{(a+bx)^2} dx$

Optimal. Leaf size=111

$$-\frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)}+\frac{3a^2c\sqrt{cx^2}}{b^4}-\frac{4a^3c\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{acx\sqrt{cx^2}}{b^3}+\frac{cx^2\sqrt{cx^2}}{3b^2}$$

```
[Out] (3*a^2*c*Sqrt[c*x^2])/b^4 - (a*c*x*Sqrt[c*x^2])/b^3 + (c*x^2*Sqrt[c*x^2])/(
3*b^2) - (a^4*c*Sqrt[c*x^2])/(b^5*x*(a + b*x)) - (4*a^3*c*Sqrt[c*x^2]*Log[a
 + b*x])/(b^5*x)
```

Rubi [A] time = 0.0373899, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)}+\frac{3a^2c\sqrt{cx^2}}{b^4}-\frac{4a^3c\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{acx\sqrt{cx^2}}{b^3}+\frac{cx^2\sqrt{cx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
```

```
[Out] (3*a^2*c*Sqrt[c*x^2])/b^4 - (a*c*x*Sqrt[c*x^2])/b^3 + (c*x^2*Sqrt[c*x^2])/(
3*b^2) - (a^4*c*Sqrt[c*x^2])/(b^5*x*(a + b*x)) - (4*a^3*c*Sqrt[c*x^2]*Log[a
 + b*x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(cx^2\right)^{3/2}}{(a+bx)^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^4}{(a+bx)^2}\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx}{x}\\
&= \frac{3a^2c\sqrt{cx^2}}{b^4}-\frac{acx\sqrt{cx^2}}{b^3}+\frac{cx^2\sqrt{cx^2}}{3b^2}-\frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3c\sqrt{cx^2}\log(a+bx)}{b^5x}
\end{aligned}$$

Mathematica [A] time = 0.0196553, size = 82, normalized size = 0.74

$$\frac{(cx^2)^{3/2}\left(6a^2b^2x^2+9a^3bx-12a^3(a+bx)\log(a+bx)-3a^4-2ab^3x^3+b^4x^4\right)}{3b^5x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
```

```
[Out] ((c*x^2)^(3/2)*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4
- 12*a^3*(a + b*x)*Log[a + b*x]))/(3*b^5*x^3*(a + b*x))
```

Maple [A] time = 0.004, size = 88, normalized size = 0.8

$$-\frac{-b^4x^4+2x^3ab^3+12\ln(bx+a)xa^3b-6x^2a^2b^2+12a^4\ln(bx+a)-9bxa^3+3a^4}{3b^5x^3(bx+a)}(cx^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] -1/3*(c*x^2)^(3/2)*(-b^4*x^4+2*x^3*a*b^3+12*ln(b*x+a)*x*a^3*b-6*x^2*a^2*b^2
+12*a^4*ln(b*x+a)-9*b*x*a^3+3*a^4)/x^3/b^5/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37743, size = 196, normalized size = 1.77

$$\frac{\left(b^4cx^4-2ab^3cx^3+6a^2b^2cx^2+9a^3bcx-3a^4c-12\left(a^3bcx+a^4c\right)\log(bx+a)\right)\sqrt{cx^2}}{3\left(b^6x^2+ab^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*c*x^4 - 2*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 + 9*a^3*b*c*x - 3*a^4*c -
12*(a^3*b*c*x + a^4*c)*log(b*x + a))*sqrt(c*x^2)/(b^6*x^2 + a*b^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x(cx^2)^{\frac{3}{2}}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)/(a + b*x)**2, x)
```

Giac [A] time = 1.0597, size = 130, normalized size = 1.17

$$-\frac{1}{3}c^{\frac{3}{2}}\left(\frac{12a^3\log\left(|bx+a|\right)\operatorname{sgn}\left(x\right)}{b^5}+\frac{3a^4\operatorname{sgn}\left(x\right)}{(bx+a)b^5}-\frac{3\left(4a^3\log\left(|a|\right)+a^3\right)\operatorname{sgn}\left(x\right)}{b^5}-\frac{b^4x^3\operatorname{sgn}\left(x\right)-3ab^3x^2\operatorname{sgn}\left(x\right)+9a^2b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*c^(3/2)*(12*a^3*log(abs(b*x + a))*sgn(x)/b^5 + 3*a^4*sgn(x)/((b*x + a)
*b^5) - 3*(4*a^3*log(abs(a)) + a^3)*sgn(x)/b^5 - (b^4*x^3*sgn(x) - 3*a*b^3*
x^2*sgn(x) + 9*a^2*b^2*x*sgn(x))/b^6)
```

3.902 $\int \frac{(cx^2)^{3/2}}{(a+bx)^2} dx$

Optimal. Leaf size=89

$$\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}$$

```
[Out] (-2*a*c*Sqrt[c*x^2])/b^3 + (c*x*Sqrt[c*x^2])/(2*b^2) + (a^3*c*Sqrt[c*x^2])/
(b^4*x*(a + b*x)) + (3*a^2*c*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rubi [A] time = 0.0285655, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (-2*a*c*Sqrt[c*x^2])/b^3 + (c*x*Sqrt[c*x^2])/(2*b^2) + (a^3*c*Sqrt[c*x^2])/
(b^4*x*(a + b*x)) + (3*a^2*c*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{(a+bx)^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^3}{(a+bx)^2}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx}{x}\\ &= -\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}+\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\log(a+bx)}{b^4x}\end{aligned}$$

Mathematica [A] time = 0.0176026, size = 71, normalized size = 0.8

$$\frac{(cx^2)^{3/2}\left(-4a^2bx+6a^2(a+bx)\log(a+bx)+2a^3-3ab^2x^2+b^3x^3\right)}{2b^4x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] ((c*x^2)^(3/2)*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)
*Log[a + b*x]))/(2*b^4*x^3*(a + b*x))
```

Maple [A] time = 0.005, size = 76, normalized size = 0.9

$$\frac{b^3x^3 + 6\ln(bx+a)\,xa^2b - 3\,ab^2x^2 + 6\,a^3\ln(bx+a) - 4\,a^2bx + 2\,a^3}{2\,x^3b^4\,(bx+a)}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(b^3*x^3+6*ln(b*x+a)*x*a^2*b-3*a*b^2*x^2+6*a^3*ln(b*x+a)-
4*a^2*b*x+2*a^3)/x^3/b^4/(b*x+a)
```

Maploma [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27045, size = 170, normalized size = 1.91

$$\frac{\left(b^3cx^3 - 3\,ab^2cx^2 - 4\,a^2bcx + 2\,a^3c + 6\left(a^2bcx + a^3c\right)\log(bx+a)\right)\sqrt{cx^2}}{2\left(b^5x^2 + ab^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*c*x^3 - 3*a*b^2*c*x^2 - 4*a^2*b*c*x + 2*a^3*c + 6*(a^2*b*c*x + a^3
*c)*log(b*x + a))*sqrt(c*x^2)/(b^5*x^2 + a*b^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(a + b*x)**2, x)
```

Giac [A] time = 1.0887, size = 108, normalized size = 1.21

$$\frac{1}{2}c^{\frac{3}{2}}\left(\frac{6a^2\log(|bx+a|)\operatorname{sgn}(x)}{b^4}+\frac{2a^3\operatorname{sgn}(x)}{(bx+a)b^4}-\frac{2\left(3a^2\log(|a|)+a^2\right)\operatorname{sgn}(x)}{b^4}+\frac{b^2x^2\operatorname{sgn}(x)-4abx\operatorname{sgn}(x)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^(3/2)*(6*a^2*log(abs(b*x + a))*sgn(x)/b^4 + 2*a^3*sgn(x)/((b*x + a)*b
^4) - 2*(3*a^2*log(abs(a)) + a^2)*sgn(x)/b^4 + (b^2*x^2*sgn(x) - 4*a*b*x*sg
n(x))/b^4)
```

3.903 $\int \frac{(cx^2)^{3/2}}{x(a+bx)^2} \, dx$

Optimal. Leaf size=68

$$-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\log(a+bx)}{b^3x}+\frac{c\sqrt{cx^2}}{b^2}$$

```
[Out] (c*Sqrt[c*x^2])/b^2 - (a^2*c*Sqrt[c*x^2])/(b^3*x*(a + b*x)) - (2*a*c*Sqrt[c
*x^2]*Log[a + b*x])/(b^3*x)
```

Rubi [A] time = 0.020345, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\log(a+bx)}{b^3x}+\frac{c\sqrt{cx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
```

```
[Out] (c*Sqrt[c*x^2])/b^2 - (a^2*c*Sqrt[c*x^2])/(b^3*x*(a + b*x)) - (2*a*c*Sqrt[c
*x^2]*Log[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x(a+bx)^2} \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^2}{(a+bx)^2} \, dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right) dx}{x}\\ &= \frac{c\sqrt{cx^2}}{b^2}-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\log(a+bx)}{b^3x}\end{aligned}$$

Mathematica [A] time = 0.0063461, size = 55, normalized size = 0.81

$$\frac{c^2x\left(-a^2+abx-2a(a+bx)\log(a+bx)+b^2x^2\right)}{b^3\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
```

```
[Out] (c^2*x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x
^2]*(a + b*x))
```

Maple [A] time = 0.003, size = 62, normalized size = 0.9

$$-\frac{2\ln\left(bx+a\right)xab-b^{2}x^{2}+2\,a^{2}\ln\left(bx+a\right)-abx+a^{2}}{b^{3}x^{3}\left(bx+a\right)}\left(cx^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(3/2)*(2*ln(b*x+a)*x*a*b-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/x^3/b^
3/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.21537, size = 132, normalized size = 1.94

$$\frac{\left(b^{2}cx^{2}+abcx-a^{2}c-2\left(abcx+a^{2}c\right)\log\left(bx+a\right)\right)\sqrt{cx^{2}}}{b^{4}x^{2}+ab^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x^2 + a*b*c*x - a^2*c - 2*(a*b*c*x + a^2*c)*log(b*x + a))*sqrt(c*x^2
)/(b^4*x^2 + a*b^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^{2}\right)^{\frac{3}{2}}}{x\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x*(a + b*x)**2), x)
```

Giac [A] time = 1.06311, size = 78, normalized size = 1.15

$$c^{\frac{3}{2}}\left(\frac{x\mathrm{sgn}\left(x\right)}{b^2} - \frac{2\,a\log\left(|bx+a|\right)\mathrm{sgn}\left(x\right)}{b^3} + \frac{\left(2\,a\log\left(|a|\right)+a\right)\mathrm{sgn}\left(x\right)}{b^3} - \frac{a^2\mathrm{sgn}\left(x\right)}{(bx+a)b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] c^(3/2)*(x*sgn(x)/b^2 - 2*a*log(abs(b*x + a))*sgn(x)/b^3 + (2*a*log(abs(a))
 + a)*sgn(x)/b^3 - a^2*sgn(x)/((b*x + a)*b^3))
```

3.904 $\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2} dx$

Optimal. Leaf size=49

$$\frac{ac\sqrt{cx^2}}{b^2x(a+bx)}+\frac{c\sqrt{cx^2}\log(a+bx)}{b^2x}$$

```
[Out] (a*c*Sqrt[c*x^2])/(b^2*x*(a + b*x)) + (c*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rubi [A] time = 0.0145969, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{ac\sqrt{cx^2}}{b^2x(a+bx)}+\frac{c\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
```

```
[Out] (a*c*Sqrt[c*x^2])/(b^2*x*(a + b*x)) + (c*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x}{(a+bx)^2} dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right) dx}{x}\\ &= \frac{ac\sqrt{cx^2}}{b^2x(a+bx)}+\frac{c\sqrt{cx^2}\log(a+bx)}{b^2x}\end{aligned}$$

Mathematica [A] time = 0.0088697, size = 38, normalized size = 0.78

$$\frac{c^2x((a+bx)\log(a+bx)+a)}{b^2\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
```

```
[Out] (c^2*x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

Maple [A] time = 0.003, size = 41, normalized size = 0.8

$$\frac{b\ln(bx+a)x+a\ln(bx+a)+a}{x^3b^2(bx+a)}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^2/(b*x+a)^2,x)
```

```
[Out] (c*x^2)^(3/2)*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/x^3/b^2/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.21269, size = 92, normalized size = 1.88

$$\frac{\sqrt{cx^2}(ac+(bcx+ac)\log(bx+a))}{b^3x^2+ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*c + (b*c*x + a*c)*log(b*x + a))/(b^3*x^2 + a*b^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}}{x^2(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**2/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**2*(a + b*x)**2), x)
```

Giac [A] time = 1.06821, size = 62, normalized size = 1.27

$$-c^{\frac{3}{2}}\left(\frac{(\log(|a|)+1)\text{sgn}(x)}{b^2}-\frac{\log(|bx+a|)\,\text{sgn}(x)}{b^2}-\frac{a\text{sgn}(x)}{(bx+a)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -c^(3/2)*((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))*sgn(x)/b^2 - a*s
gn(x)/((b*x + a)*b^2))
```

3.905 $\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2}\,dx$

Optimal. Leaf size=25

$$-\frac{c\sqrt{cx^2}}{bx(a+bx)}$$

```
[Out] -((c*Sqrt[c*x^2])/(b*x*(a + b*x)))
```

Rubi [A] time = 0.004015, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 32}

$$-\frac{c\sqrt{cx^2}}{bx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(b*x*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{(a+bx)^2}\,dx}{x}\\ &= -\frac{c\sqrt{cx^2}}{bx(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0055979, size = 24, normalized size = 0.96

$$-\frac{(cx^2)^{3/2}}{bx^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
```

```
[Out] -((c*x^2)^(3/2)/(b*x^3*(a + b*x)))
```

Maple [A] time = 0.002, size = 23, normalized size = 0.9

$$-\frac{1}{(bx+a)\,bx^{3}}\left(cx^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^3/(b*x+a)^2,x)
```

```
[Out] -1/(b*x+a)/b*(c*x^2)^(3/2)/x^3
```

Maxima [A] time = 1.04378, size = 22, normalized size = 0.88

$$-\frac{c^{\frac{3}{2}}}{b^{2}x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -c^(3/2)/(b^2*x + a*b)
```

Fricas [A] time = 1.25167, size = 46, normalized size = 1.84

$$-\frac{\sqrt{cx^{2}}c}{b^{2}x^{2}+abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^2)*c/(b^2*x^2 + a*b*x)
```

Sympy [A] time = 2.19917, size = 44, normalized size = 1.76

$$\begin{cases} -\frac{c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}{abx^{3}+b^{2}x^{4}} & \text{for}\: b\neq 0 \\ \frac{c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}{a^{2}x^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**3/(b*x+a)**2,x)
```

```
[Out] Piecewise((-c**(3/2)*(x**2)**(3/2)/(a*b*x**3 + b**2*x**4), Ne(b, 0)), (c**(
3/2)*(x**2)**(3/2)/(a**2*x**2), True))
```

<u>Giac</u> [A] time = 1.0644, size = 39, normalized size = 1.56

$$-c^{\frac{3}{2}}\left(\frac{\mathrm{sgn}\left(x\right)}{(bx+a)b}-\frac{\mathrm{sgn}\left(x\right)}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -c^(3/2)*(sgn(x)/((b*x + a)*b) - sgn(x)/(a*b))
```

3.906 $\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2} dx$

Optimal. Leaf size=68

$$-\frac{c\sqrt{cx^2}\log(a+bx)}{a^2x}+\frac{c\sqrt{cx^2}\log(x)}{a^2x}+\frac{c\sqrt{cx^2}}{ax(a+bx)}$$

```
[Out] (c*Sqrt[c*x^2])/(a*x*(a + b*x)) + (c*Sqrt[c*x^2]*Log[x])/(a^2*x) - (c*Sqrt[
c*x^2]*Log[a + b*x])/(a^2*x)
```

Rubi [A] time = 0.0177966, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$-\frac{c\sqrt{cx^2}\log(a+bx)}{a^2x}+\frac{c\sqrt{cx^2}\log(x)}{a^2x}+\frac{c\sqrt{cx^2}}{ax(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
```

```
[Out] (c*Sqrt[c*x^2])/(a*x*(a + b*x)) + (c*Sqrt[c*x^2]*Log[x])/(a^2*x) - (c*Sqrt[
c*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)^2} dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) dx}{x} \\
&= \frac{c\sqrt{cx^2}}{ax(a+bx)}+\frac{c\sqrt{cx^2}\log(x)}{a^2x}-\frac{c\sqrt{cx^2}\log(a+bx)}{a^2x}
\end{aligned}$$

Mathematica [A] time = 0.0155185, size = 46, normalized size = 0.68

$$\frac{(cx^2)^{3/2}(\log(x)(a+bx)-(a+bx)\log(a+bx)+a)}{a^2x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
```

```
[Out] ((c*x^2)^(3/2)*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*x^3*(a
 + b*x))
```

Maple [A] time = 0.003, size = 52, normalized size = 0.8

$$\frac{b\ln(x)x - b\ln(bx+a)x + a\ln(x) - a\ln(bx+a) + a}{a^2x^3(bx+a)}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^4/(b*x+a)^2,x)
```

```
[Out] (c*x^2)^(3/2)*(b*ln(x)*x-b*ln(b*x+a)*x+a*ln(x)-a*ln(b*x+a)+a)/x^3/a^2/(b*x+
a)
```

Maxima [A] time = 1.04309, size = 51, normalized size = 0.75

$$\frac{c^{\frac{3}{2}}}{abx+a^2} - \frac{c^{\frac{3}{2}}\log(bx+a)}{a^2} + \frac{c^{\frac{3}{2}}\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] c^(3/2)/(a*b*x + a^2) - c^(3/2)*log(b*x + a)/a^2 + c^(3/2)*log(x)/a^2
```

Fricas [A] time = 1.31839, size = 97, normalized size = 1.43

$$\frac{\sqrt{cx^2}\left(ac + (bcx+ac)\log\left(\frac{x}{bx+a}\right)\right)}{a^2bx^2 + a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*c + (b*c*x + a*c)*log(x/(b*x + a)))/(a^2*b*x^2 + a^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^4(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**4/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**4*(a + b*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.907 $\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{bc\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2bc\sqrt{cx^2}\log(x)}{a^3x}+\frac{2bc\sqrt{cx^2}\log(a+bx)}{a^3x}-\frac{c\sqrt{cx^2}}{a^2x^2}$$

```
[Out] -((c*Sqrt[c*x^2])/(a^2*x^2)) - (b*c*Sqrt[c*x^2])/(a^2*x*(a + b*x)) - (2*b*c
*Sqrt[c*x^2]*Log[x])/(a^3*x) + (2*b*c*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rubi [A] time = 0.0239688, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$-\frac{bc\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2bc\sqrt{cx^2}\log(x)}{a^3x}+\frac{2bc\sqrt{cx^2}\log(a+bx)}{a^3x}-\frac{c\sqrt{cx^2}}{a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(a^2*x^2)) - (b*c*Sqrt[c*x^2])/(a^2*x*(a + b*x)) - (2*b*c
*Sqrt[c*x^2]*Log[x])/(a^3*x) + (2*b*c*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)^2}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx}{x}\\ &= -\frac{c\sqrt{cx^2}}{a^2x^2}-\frac{bc\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2bc\sqrt{cx^2}\log(x)}{a^3x}+\frac{2bc\sqrt{cx^2}\log(a+bx)}{a^3x}\end{aligned}$$

Mathematica [A] time = 0.0214687, size = 59, normalized size = 0.65

$$-\frac{c^2(a(a+2bx)+2bx\log(x)(a+bx)-2bx(a+bx)\log(a+bx))}{a^3\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
```

```
[Out] -((c^2*(a*(a + 2*b*x) + 2*b*x*(a + b*x)*Log[x] - 2*b*x*(a + b*x)*Log[a + b*
x]))/(a^3*Sqrt[c*x^2]*(a + b*x)))
```

Maple [A] time = 0.003, size = 74, normalized size = 0.8

$$-\frac{2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2ab\ln(x)x-2\ln(bx+a)xab+2abx+a^2}{a^3x^4(bx+a)}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^5/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(3/2)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*ln(x)*x-2*ln(b*x+
a)*x*a*b+2*a*b*x+a^2)/x^4/a^3/(b*x+a)
```

Maxima [A] time = 1.02072, size = 78, normalized size = 0.86

$$\frac{2bc^{\frac{3}{2}}\log(bx+a)}{a^3}-\frac{2bc^{\frac{3}{2}}\log(x)}{a^3}-\frac{2bc^{\frac{3}{2}}x+ac^{\frac{3}{2}}}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*b*c^(3/2)*log(b*x + a)/a^3 - 2*b*c^(3/2)*log(x)/a^3 - (2*b*c^(3/2)*x + a*
c^(3/2))/(a^2*b*x^2 + a^3*x)
```

Fricas [A] time = 1.30933, size = 134, normalized size = 1.47

$$-\frac{\left(2abcx+a^2c-2\left(b^2cx^2+abcx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bx^3+a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*c*x + a^2*c - 2*(b^2*c*x^2 + a*b*c*x)*log((b*x + a)/x))*sqrt(c*x^2)
/(a^3*b*x^3 + a^4*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}}{x^5\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**5/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**5*(a + b*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.908 $\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2} dx$

Optimal. Leaf size=117

$$\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2c\sqrt{cx^2}\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\log(a+bx)}{a^4x}+\frac{2bc\sqrt{cx^2}}{a^3x^2}-\frac{c\sqrt{cx^2}}{2a^2x^3}$$

```
[Out] -(c*Sqrt[c*x^2])/(2*a^2*x^3) + (2*b*c*Sqrt[c*x^2])/(a^3*x^2) + (b^2*c*Sqrt[
c*x^2])/(a^3*x*(a + b*x)) + (3*b^2*c*Sqrt[c*x^2]*Log[x])/(a^4*x) - (3*b^2*c
*Sqrt[c*x^2]*Log[a + b*x])/(a^4*x)
```

Rubi [A] time = 0.0317061, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2c\sqrt{cx^2}\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\log(a+bx)}{a^4x}+\frac{2bc\sqrt{cx^2}}{a^3x^2}-\frac{c\sqrt{cx^2}}{2a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
```

```
[Out] -(c*Sqrt[c*x^2])/(2*a^2*x^3) + (2*b*c*Sqrt[c*x^2])/(a^3*x^2) + (b^2*c*Sqrt[
c*x^2])/(a^3*x*(a + b*x)) + (3*b^2*c*Sqrt[c*x^2]*Log[x])/(a^4*x) - (3*b^2*c
*Sqrt[c*x^2]*Log[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^3(a+bx)^2} dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right) dx}{x}\\ &= -\frac{c\sqrt{cx^2}}{2a^2x^3}+\frac{2bc\sqrt{cx^2}}{a^3x^2}+\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2c\sqrt{cx^2}\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\log(a+bx)}{a^4x}\end{aligned}$$

Mathematica [A] time = 0.0210971, size = 82, normalized size = 0.7

$$\frac{(cx^2)^{3/2}\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2\log(x)(a+bx)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4x^5(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
```

```
[Out] ((c*x^2)^(3/2)*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x]
 - 6*b^2*x^2*(a + b*x)*Log[a + b*x]))/(2*a^4*x^5*(a + b*x))
```

Maple [A] time = 0.004, size = 95, normalized size = 0.8

$$\frac{6\,b^3\ln(x)\,x^3-6\,b^3\ln(bx+a)\,x^3+6\,\ln(x)\,x^2ab^2-6\,\ln(bx+a)\,x^2ab^2+6\,ab^2x^2+3\,a^2bx-a^3}{2\,x^5a^4\,(bx+a)}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^6/(b*x+a)^2,x)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*ln(x)*x^2*a*b^2-6*
ln(b*x+a)*x^2*a*b^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/x^5/a^4/(b*x+a)
```

Maxima [A] time = 1.05454, size = 107, normalized size = 0.91

$$-\frac{3\,b^2c^{\frac{3}{2}}\log(bx+a)}{a^4}+\frac{3\,b^2c^{\frac{3}{2}}\log(x)}{a^4}+\frac{6\,b^2c^{\frac{3}{2}}x^2+3\,abc^{\frac{3}{2}}x-a^2c^{\frac{3}{2}}}{2\left(a^3bx^3+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -3*b^2*c^(3/2)*log(b*x + a)/a^4 + 3*b^2*c^(3/2)*log(x)/a^4 + 1/2*(6*b^2*c^(
3/2)*x^2 + 3*a*b*c^(3/2)*x - a^2*c^(3/2))/(a^3*b*x^3 + a^4*x^2)
```

Fricas [A] time = 1.37117, size = 167, normalized size = 1.43

$$\frac{\left(6\,ab^2cx^2+3\,a^2bcx-a^3c+6\left(b^3cx^3+ab^2cx^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bx^4+a^5x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*c*x^2 + 3*a^2*b*c*x - a^3*c + 6*(b^3*c*x^3 + a*b^2*c*x^2)*log(
x/(b*x + a)))*sqrt(c*x^2)/(a^4*b*x^4 + a^5*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^6\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**6/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**6*(a + b*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.909 $\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2} dx$

Optimal. Leaf size=107

$$-\frac{a^4 x}{b^5 \sqrt{cx^2}(a+bx)}+\frac{3 a^2 x^2}{b^4 \sqrt{cx^2}}-\frac{4 a^3 x \log (a+bx)}{b^5 \sqrt{cx^2}}-\frac{a x^3}{b^3 \sqrt{cx^2}}+\frac{x^4}{3 b^2 \sqrt{cx^2}}$$

```
[Out] (3*a^2*x^2)/(b^4*Sqrt[c*x^2]) - (a*x^3)/(b^3*Sqrt[c*x^2]) + x^4/(3*b^2*Sqrt
[c*x^2]) - (a^4*x)/(b^5*Sqrt[c*x^2]*(a + b*x)) - (4*a^3*x*Log[a + b*x])/(b^
5*Sqrt[c*x^2])
```

Rubi [A] time = 0.032868, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^4 x}{b^5 \sqrt{cx^2}(a+bx)}+\frac{3 a^2 x^2}{b^4 \sqrt{cx^2}}-\frac{4 a^3 x \log (a+bx)}{b^5 \sqrt{cx^2}}-\frac{a x^3}{b^3 \sqrt{cx^2}}+\frac{x^4}{3 b^2 \sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (3*a^2*x^2)/(b^4*Sqrt[c*x^2]) - (a*x^3)/(b^3*Sqrt[c*x^2]) + x^4/(3*b^2*Sqrt
[c*x^2]) - (a^4*x)/(b^5*Sqrt[c*x^2]*(a + b*x)) - (4*a^3*x*Log[a + b*x])/(b^
5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2} dx &= \frac{x \int \frac{x^4}{(a+bx)^2} dx}{\sqrt{cx^2}}\\
&= \frac{x \int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right) dx}{\sqrt{cx^2}}\\
&= \frac{3a^2x^2}{b^4\sqrt{cx^2}}-\frac{ax^3}{b^3\sqrt{cx^2}}+\frac{x^4}{3b^2\sqrt{cx^2}}-\frac{a^4x}{b^5\sqrt{cx^2}(a+bx)}-\frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0149218, size = 80, normalized size = 0.75

$$\frac{x\left(6a^2b^2x^2+9a^3bx-12a^3(a+bx)\log(a+bx)-3a^4-2ab^3x^3+b^4x^4\right)}{3b^5\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4 - 12*a^3*(a
+ b*x)*Log[a + b*x]))/(3*b^5*Sqrt[c*x^2]*(a + b*x))
```

Maple [A] time = 0.004, size = 86, normalized size = 0.8

$$-\frac{x\left(-b^4x^4+2x^3ab^3+12\ln(bx+a)xa^3b-6x^2a^2b^2+12a^4\ln(bx+a)-9bxa^3+3a^4\right)}{3b^5(bx+a)}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -1/3*x*(-b^4*x^4+2*x^3*a*b^3+12*ln(b*x+a)*x*a^3*b-6*x^2*a^2*b^2+12*a^4*ln(b
*x+a)-9*b*x*a^3+3*a^4)/(c*x^2)^(1/2)/b^5/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.25569, size = 182, normalized size = 1.7

$$\frac{\left(b^4x^4-2ab^3x^3+6a^2b^2x^2+9a^3bx-3a^4-12\left(a^3bx+a^4\right)\log(bx+a)\right)\sqrt{cx^2}}{3\left(b^6cx^2+ab^5cx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))*sqrt(c*x^2)/(b^6*c*x^2 + a*b^5*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\sqrt{cx^2}(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{cx^2}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/(sqrt(c*x^2)*(b*x + a)^2), x)
```

3.910 $\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2} \, dx$

Optimal. Leaf size=86

$$\frac{a^3x}{b^4\sqrt{cx^2}(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}-\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}$$

```
[Out] (-2*a*x^2)/(b^3*Sqrt[c*x^2]) + x^3/(2*b^2*Sqrt[c*x^2]) + (a^3*x)/(b^4*Sqrt[
c*x^2]*(a + b*x)) + (3*a^2*x*Log[a + b*x])/(b^4*Sqrt[c*x^2])
```

Rubi [A] time = 0.02566, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^3x}{b^4\sqrt{cx^2}(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}-\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (-2*a*x^2)/(b^3*Sqrt[c*x^2]) + x^3/(2*b^2*Sqrt[c*x^2]) + (a^3*x)/(b^4*Sqrt[
c*x^2]*(a + b*x)) + (3*a^2*x*Log[a + b*x])/(b^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2} \, dx &= \frac{x\int \frac{x^3}{(a+bx)^2} \, dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right) dx}{\sqrt{cx^2}}\\
&= -\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}+\frac{a^3x}{b^4\sqrt{cx^2}(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0129685, size = 69, normalized size = 0.8

$$\frac{x\left(-4a^2bx+6a^2(a+bx)\log(a+bx)+2a^3-3ab^2x^2+b^3x^3\right)}{2b^4\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)*Log[a + b*x
]))/(2*b^4*Sqrt[c*x^2]*(a + b*x))
```

Maple [A] time = 0.004, size = 74, normalized size = 0.9

$$\frac{x\left(b^3x^3+6\,\ln\left(bx+a\right)xa^2b-3\,ab^2x^2+6\,a^3\ln\left(bx+a\right)-4\,a^2bx+2\,a^3\right)}{2\,b^4\left(bx+a\right)}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(b^3*x^3+6*ln(b*x+a)*x*a^2*b-3*a*b^2*x^2+6*a^3*ln(b*x+a)-4*a^2*b*x+2*
a^3)/(c*x^2)^(1/2)/b^4/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.16849, size = 159, normalized size = 1.85

$$\frac{\left(b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\left(a^2bx+a^3\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\left(b^5cx^2+ab^4cx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))*sqrt(c*x^2)/(b^5*c*x^2 + a*b^4*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\sqrt{cx^2}\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{cx^2}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(c*x^2)*(b*x + a)^2), x)
```

3.911 $\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal. Leaf size=64

$$-\frac{a^2x}{b^3\sqrt{cx^2}(a+bx)}-\frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}+\frac{x^2}{b^2\sqrt{cx^2}}$$

```
[Out] x^2/(b^2*Sqrt[c*x^2]) - (a^2*x)/(b^3*Sqrt[c*x^2]*(a + b*x)) - (2*a*x*Log[a
+ b*x])/(b^3*Sqrt[c*x^2])
```

Rubi [A] time = 0.0196238, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2x}{b^3\sqrt{cx^2}(a+bx)}-\frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}+\frac{x^2}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] x^2/(b^2*Sqrt[c*x^2]) - (a^2*x)/(b^3*Sqrt[c*x^2]*(a + b*x)) - (2*a*x*Log[a
+ b*x])/(b^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx &= \frac{x\int \frac{x^2}{(a+bx)^2}\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{\sqrt{cx^2}}\\
&= \frac{x^2}{b^2\sqrt{cx^2}}-\frac{a^2x}{b^3\sqrt{cx^2}(a+bx)}-\frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0119881, size = 52, normalized size = 0.81

$$\frac{x\left(-a^2+abx-2a(a+bx)\log(a+bx)+b^2x^2\right)}{b^3\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x^2]*
(a + b*x))
```

Maple [A] time = 0.004, size = 60, normalized size = 0.9

$$-\frac{x\left(2\ln\left(bx+a\right)xab-b^{2}x^{2}+2a^{2}\ln\left(bx+a\right)-abx+a^{2}\right)}{b^{3}\left(bx+a\right)}\frac{1}{\sqrt{cx^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -x*(2*ln(b*x+a)*x*a*b-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/(c*x^2)^(1/2)/b^3/
(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34299, size = 124, normalized size = 1.94

$$\frac{\left(b^{2}x^{2}+abx-a^{2}-2\left(abx+a^{2}\right)\log\left(bx+a\right)\right)\sqrt{cx^{2}}}{b^{4}cx^{2}+ab^{3}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*c*x
^2 + a*b^3*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{3}}{\sqrt{cx^{2}}\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^2}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(c*x^2)*(b*x + a)^2), x)
```

3.912 $\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2} dx$

Optimal. Leaf size=43

$$\frac{ax}{b^2\sqrt{cx^2}(a+bx)}+\frac{x\log(a+bx)}{b^2\sqrt{cx^2}}$$

```
[Out] (a*x)/(b^2*Sqrt[c*x^2]*(a + b*x)) + (x*Log[a + b*x])/(b^2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0131761, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{ax}{b^2\sqrt{cx^2}(a+bx)}+\frac{x\log(a+bx)}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (a*x)/(b^2*Sqrt[c*x^2]*(a + b*x)) + (x*Log[a + b*x])/(b^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2} dx &= \frac{x\int \frac{x}{(a+bx)^2} dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right) dx}{\sqrt{cx^2}}\\
&= \frac{ax}{b^2\sqrt{cx^2}(a+bx)}+\frac{x\log(a+bx)}{b^2\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0090779, size = 35, normalized size = 0.81

$$\frac{x((a+bx)\log(a+bx)+a)}{b^2\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

Maple [A] time = 0.003, size = 39, normalized size = 0.9

$$\frac{x\left(b\ln\left(bx+a\right)x+a\ln\left(bx+a\right)+a\right)}{b^{2}\left(bx+a\right)}\frac{1}{\sqrt{cx^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] x*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/(c*x^2)^(1/2)/b^2/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29448, size = 89, normalized size = 2.07

$$\frac{\sqrt{cx^{2}}\left((bx+a)\log\left(bx+a\right)+a\right)}{b^{3}cx^{2}+ab^{2}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*c*x^2 + a*b^2*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{2}}{\sqrt{cx^{2}}\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^2}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(c*x^2)*(b*x + a)^2), x)
```

3.913 $\int \frac{x}{\sqrt{cx^2}(a+bx)^2} dx$

Optimal. Leaf size=22

$$-\frac{x}{b\sqrt{cx^2}(a+bx)}$$

```
[Out] -(x/(b*Sqrt[c*x^2]*(a + b*x)))
```

Rubi [A] time = 0.0033827, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 32}

$$-\frac{x}{b\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(x/(b*Sqrt[c*x^2]*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{cx^2}(a+bx)^2} dx &= \frac{x \int \frac{1}{(a+bx)^2} dx}{\sqrt{cx^2}}\\ &= -\frac{x}{b\sqrt{cx^2}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0028723, size = 22, normalized size = 1.

$$-\frac{x}{b\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(x/(b*Sqrt[c*x^2]*(a + b*x)))
```

Maple [A] time = 0.001, size = 21, normalized size = 1.

$$-\frac{x}{b(bx+a)}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -x/b/(b*x+a)/(c*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33952, size = 49, normalized size = 2.23

$$-\frac{\sqrt{cx^2}}{b^2cx^2+abcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^2)/(b^2*c*x^2 + a*b*c*x)
```

Sympy [A] time = 1.11411, size = 85, normalized size = 3.86

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{c}\sqrt{x^2}} & \text{for}\ a=0 \wedge b=0 \\ -\frac{1}{b^2\sqrt{c}\sqrt{x^2}} & \text{for}\ a=0 \\ \frac{\tilde{\infty}x^2}{\sqrt{c}\sqrt{x^2}} & \text{for}\ b=-\frac{a}{x} \\ \frac{x^2}{a^2\sqrt{c}\sqrt{x^2}+ab\sqrt{c}x\sqrt{x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((zoo/(sqrt(c)*sqrt(x**2)), Eq(a, 0) & Eq(b, 0)), (-1/(b**2*sqrt(c
)*sqrt(x**2)), Eq(a, 0)), (zoo*x**2/(sqrt(c)*sqrt(x**2)), Eq(b, -a/x)), (x*
*2/(a**2*sqrt(c)*sqrt(x**2) + a*b*sqrt(c)*x*sqrt(x**2)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^2}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(c*x^2)*(b*x + a)^2), x)
```

3.914 $\int \frac{1}{\sqrt{cx^2}(a+bx)^2} dx$

Optimal. Leaf size=59

$$-\frac{x\log(a+bx)}{a^2\sqrt{cx^2}}+\frac{x\log(x)}{a^2\sqrt{cx^2}}+\frac{x}{a\sqrt{cx^2}(a+bx)}$$

```
[Out] x/(a*Sqrt[c*x^2]*(a + b*x)) + (x*Log[x])/(a^2*Sqrt[c*x^2]) - (x*Log[a + b*x
])/(a^2*Sqrt[c*x^2])
```

Rubi [A] time = 0.0164084, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 44}

$$-\frac{x\log(a+bx)}{a^2\sqrt{cx^2}}+\frac{x\log(x)}{a^2\sqrt{cx^2}}+\frac{x}{a\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] x/(a*Sqrt[c*x^2]*(a + b*x)) + (x*Log[x])/(a^2*Sqrt[c*x^2]) - (x*Log[a + b*x
])/(a^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{cx^2}(a+bx)^2} dx &= \frac{x\int \frac{1}{x(a+bx)^2} dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) dx}{\sqrt{cx^2}}\\ &= \frac{x}{a\sqrt{cx^2}(a+bx)}+\frac{x\log(x)}{a^2\sqrt{cx^2}}-\frac{x\log(a+bx)}{a^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0053642, size = 44, normalized size = 0.75

$$\frac{x(\log(x)(a+bx)-(a+bx)\log(a+bx)+a)}{a^2\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*Sqrt[c*x^2]*(a + b
*x))
```

Maple [A] time = 0.004, size = 50, normalized size = 0.9

$$\frac{x \left(b \ln \left(x \right) x - b \ln \left(bx + a \right) x + a \ln \left(x \right) - a \ln \left(bx + a \right) + a \right)}{a^{2} \left(bx + a \right)} \frac{1}{\sqrt{c x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] x*(b*ln(x)*x-b*ln(b*x+a)*x+a*ln(x)-a*ln(b*x+a)+a)/(c*x^2)^(1/2)/a^2/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38414, size = 95, normalized size = 1.61

$$\frac{\sqrt{c x^{2}} \left(\left(bx + a\right) \log \left(\frac{x}{bx + a} \right) + a \right)}{a^{2} b c x^{2} + a^{3} c x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*c*x^2 + a^3*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c x^{2}} \left(a + b x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [A] time = 1.12313, size = 116, normalized size = 1.97

$$-\frac{\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^2\sqrt{c}\,\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{1}{(bx+a)a\sqrt{c}\,\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-a/(b*x + a) + 1))/(a^2*sqrt(c)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2
)) - 1/((b*x + a)*a*sqrt(c)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2))
```

3.915 $\int \frac{1}{x\sqrt{cx^2}(a+bx)^2} dx$

Optimal. Leaf size=78

$$-\frac{bx}{a^2\sqrt{cx^2}(a+bx)}-\frac{2bx\log(x)}{a^3\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}-\frac{1}{a^2\sqrt{cx^2}}$$

```
[Out] -(1/(a^2*Sqrt[c*x^2])) - (b*x)/(a^2*Sqrt[c*x^2]*(a + b*x)) - (2*b*x*Log[x])
/(a^3*Sqrt[c*x^2]) + (2*b*x*Log[a + b*x])/(a^3*Sqrt[c*x^2])
```

Rubi [A] time = 0.0224452, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$-\frac{bx}{a^2\sqrt{cx^2}(a+bx)}-\frac{2bx\log(x)}{a^3\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}-\frac{1}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*Sqrt[c*x^2])) - (b*x)/(a^2*Sqrt[c*x^2]*(a + b*x)) - (2*b*x*Log[x])
/(a^3*Sqrt[c*x^2]) + (2*b*x*Log[a + b*x])/(a^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx &= \frac{x\int \frac{1}{x^2(a+bx)^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{1}{a^2\sqrt{cx^2}}-\frac{bx}{a^2\sqrt{cx^2}(a+bx)}-\frac{2bx\log(x)}{a^3\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0247229, size = 60, normalized size = 0.77

$$\frac{cx^2(-a(a+2bx)-2bx\log(x)(a+bx)+2bx(a+bx)\log(a+bx))}{a^3\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (c*x^2*(-(a*(a + 2*b*x)) - 2*b*x*(a + b*x)*Log[x] + 2*b*x*(a + b*x)*Log[a +
 b*x]))/(a^3*(c*x^2)^(3/2)*(a + b*x))
```

Maple [A] time = 0.003, size = 71, normalized size = 0.9

$$-\frac{2\,b^2\ln(x)\,x^2-2\,b^2\ln(bx+a)\,x^2+2\,ab\ln(x)\,x-2\,\ln(bx+a)\,xab+2\,abx+a^2}{a^3\,(bx+a)}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*ln(x)*x-2*ln(b*x+a)*x*a*b+2*a*b
*x+a^2)/(c*x^2)^(1/2)/a^3/(b*x+a)
```

Maxima [A] time = 1.06927, size = 77, normalized size = 0.99

$$-\frac{2\,bx+a}{a^2b\sqrt{c}x^2+a^3\sqrt{c}x}+\frac{2\,b\log(bx+a)}{a^3\sqrt{c}}-\frac{2\,b\log(x)}{a^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*sqrt(c)*x^2 + a^3*sqrt(c)*x) + 2*b*log(b*x + a)/(a^3*sq
rt(c)) - 2*b*log(x)/(a^3*sqrt(c))
```

Fricas [A] time = 1.3451, size = 128, normalized size = 1.64

$$-\frac{\left(2\,abx+a^2-2\left(b^2x^2+abx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bcx^3+a^4cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
c*x^3 + a^4*c*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.916 $\int \frac{1}{x^2\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal. Leaf size=103

$$\frac{b^2x}{a^3\sqrt{cx^2}(a+bx)}+\frac{3b^2x\log(x)}{a^4\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}+\frac{2b}{a^3\sqrt{cx^2}}-\frac{1}{2a^2x\sqrt{cx^2}}$$

```
[Out] (2*b)/(a^3*Sqrt[c*x^2]) - 1/(2*a^2*x*Sqrt[c*x^2]) + (b^2*x)/(a^3*Sqrt[c*x^2
]*(a + b*x)) + (3*b^2*x*Log[x])/(a^4*Sqrt[c*x^2]) - (3*b^2*x*Log[a + b*x])/
(a^4*Sqrt[c*x^2])
```

Rubi [A] time = 0.0316542, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$\frac{b^2x}{a^3\sqrt{cx^2}(a+bx)}+\frac{3b^2x\log(x)}{a^4\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}+\frac{2b}{a^3\sqrt{cx^2}}-\frac{1}{2a^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (2*b)/(a^3*Sqrt[c*x^2]) - 1/(2*a^2*x*Sqrt[c*x^2]) + (b^2*x)/(a^3*Sqrt[c*x^2
]*(a + b*x)) + (3*b^2*x*Log[x])/(a^4*Sqrt[c*x^2]) - (3*b^2*x*Log[a + b*x])/
(a^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{cx^2}(a+bx)^2}\,dx &= \frac{x\int \frac{1}{x^3(a+bx)^2}\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx}{\sqrt{cx^2}}\\
&= \frac{2b}{a^3\sqrt{cx^2}}-\frac{1}{2a^2x\sqrt{cx^2}}+\frac{b^2x}{a^3\sqrt{cx^2}(a+bx)}+\frac{3b^2x\log(x)}{a^4\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0170383, size = 81, normalized size = 0.79

$$\frac{cx\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2\log(x)(a+bx)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (c*x*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] - 6*b^2*x
^2*(a + b*x)*Log[a + b*x]))/(2*a^4*(c*x^2)^(3/2)*(a + b*x))
```

Maple [A] time = 0.005, size = 95, normalized size = 0.9

$$\frac{6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6\ln(x)x^2ab^2-6\ln(bx+a)x^2ab^2+6ab^2x^2+3a^2bx-a^3}{2xa^4(bx+a)}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/2/x*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*ln(x)*x^2*a*b^2-6*ln(b*x+a)*x^
2*a*b^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/(c*x^2)^(1/2)/a^4/(b*x+a)
```

Maxima [A] time = 1.05652, size = 103, normalized size = 1.

$$\frac{6b^2x^2+3abx-a^2}{2\left(a^3b\sqrt{c}x^3+a^4\sqrt{c}x^2\right)}-\frac{3b^2\log(bx+a)}{a^4\sqrt{c}}+\frac{3b^2\log(x)}{a^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*sqrt(c)*x^3 + a^4*sqrt(c)*x^2) - 3*b
^2*log(b*x + a)/(a^4*sqrt(c)) + 3*b^2*log(x)/(a^4*sqrt(c))
```

Fricas [A] time = 1.3217, size = 159, normalized size = 1.54

$$\frac{\left(6ab^2x^2+3a^2bx-a^3+6\left(b^3x^3+ab^2x^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bcx^4+a^5cx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*c*x^4 + a^5*c*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.917 $\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=73

$$-\frac{a^2x}{b^3c\sqrt{cx^2}(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}+\frac{x^2}{b^2c\sqrt{cx^2}}$$

```
[Out] x^2/(b^2*c*Sqrt[c*x^2]) - (a^2*x)/(b^3*c*Sqrt[c*x^2]*(a + b*x)) - (2*a*x*Lo
g[a + b*x])/(b^3*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.02105, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^2x}{b^3c\sqrt{cx^2}(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}+\frac{x^2}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] x^2/(b^2*c*Sqrt[c*x^2]) - (a^2*x)/(b^3*c*Sqrt[c*x^2]*(a + b*x)) - (2*a*x*Lo
g[a + b*x])/(b^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{x^2}{(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{x^2}{b^2c\sqrt{cx^2}}-\frac{a^2x}{b^3c\sqrt{cx^2}(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0142128, size = 54, normalized size = 0.74

$$\frac{x^3\left(-a^2+abx-2a(a+bx)\log(a+bx)+b^2x^2\right)}{b^3\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^3*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*(c*x^2)^(3
/2)*(a + b*x))
```

Maple [A] time = 0.004, size = 62, normalized size = 0.9

$$-\frac{x^{3}\left(2\ln\left(bx+a\right)xab-b^{2}x^{2}+2a^{2}\ln\left(bx+a\right)-abx+a^{2}\right)}{b^{3}\left(bx+a\right)}\left(cx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] -x^3*(2*ln(b*x+a)*x*a*b-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/(c*x^2)^(3/2)/b^
3/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.21525, size = 130, normalized size = 1.78

$$\frac{\left(b^{2}x^{2}+abx-a^{2}-2\left(abx+a^{2}\right)\log\left(bx+a\right)\right)\sqrt{cx^{2}}}{b^{4}c^{2}x^{2}+ab^{3}c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*c^2
*x^2 + a*b^3*c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{5}}{\left(cx^{2}\right)^{\frac{3}{2}}\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**5/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(cx^2\right)^{\frac{3}{2}}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5/((c*x^2)^(3/2)*(b*x + a)^2), x)
```

3.918 $\int \frac{x^4}{\left(cx^2\right)^{3/2}(a+bx)^2} dx$

Optimal. Leaf size=49

$$\frac{ax}{b^2c\sqrt{cx^2}(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}$$

```
[Out] (a*x)/(b^2*c*Sqrt[c*x^2]*(a + b*x)) + (x*Log[a + b*x])/(b^2*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0138987, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{ax}{b^2c\sqrt{cx^2}(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (a*x)/(b^2*c*Sqrt[c*x^2]*(a + b*x)) + (x*Log[a + b*x])/(b^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(cx^2\right)^{3/2}(a+bx)^2} dx &= \frac{x\int \frac{x}{(a+bx)^2} dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right) dx}{c\sqrt{cx^2}}\\
&= \frac{ax}{b^2c\sqrt{cx^2}(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0110324, size = 37, normalized size = 0.76

$$\frac{x^3((a+bx)\log(a+bx)+a)}{b^2\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^3*(a + (a + b*x)*Log[a + b*x]))/(b^2*(c*x^2)^(3/2)*(a + b*x))
```

Maple [A] time = 0.003, size = 41, normalized size = 0.8

$$\frac{x^3 (b \ln (bx+a) x + a \ln (bx+a) + a)}{b^2 (bx+a)} \left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] x^3*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/(c*x^2)^(3/2)/b^2/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38408, size = 95, normalized size = 1.94

$$\frac{\sqrt{cx^2}((bx+a)\log (bx+a) + a)}{b^3c^2x^2 + ab^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*c^2*x^2 + a*b^2*c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(cx^2\right)^{\frac{3}{2}} (a+bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**4/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(cx^2\right)^{\frac{3}{2}}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/((c*x^2)^(3/2)*(b*x + a)^2), x)
```

3.919 $\int \frac{x^3}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=25

$$-\frac{x}{bc\sqrt{cx^2}(a+bx)}$$

```
[Out] -(x/(b*c*Sqrt[c*x^2]*(a + b*x)))
```

Rubi [A] time = 0.0038696, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 32}

$$-\frac{x}{bc\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(x/(b*c*Sqrt[c*x^2]*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{1}{(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{x}{bc\sqrt{cx^2}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0059425, size = 24, normalized size = 0.96

$$-\frac{x^3}{b\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(x^3/(b*(c*x^2)^(3/2)*(a + b*x)))
```

Maple [A] time = 0.001, size = 23, normalized size = 0.9

$$-\frac{x^3}{(bx+a)\,b}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] -1/(b*x+a)/b*x^3/(c*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33424, size = 54, normalized size = 2.16

$$-\frac{\sqrt{cx^2}}{b^2c^2x^2+abc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^2)/(b^2*c^2*x^2 + a*b*c^2*x)
```

Sympy [A] time = 1.77616, size = 90, normalized size = 3.6

$$\begin{cases} \frac{\tilde{\infty}x^2}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} & \text{for } a=0 \wedge b=0 \\ -\frac{x^2}{b^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} & \text{for } a=0 \\ \frac{\tilde{\infty}x^4}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} & \text{for } b=-\frac{a}{x} \\ \frac{x^4}{a^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}+abc^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**2/(c**(3/2)*(x**2)**(3/2)), Eq(a, 0) & Eq(b, 0)), (-x**2/
(b**2*c**(3/2)*(x**2)**(3/2)), Eq(a, 0)), (zoo*x**4/(c**(3/2)*(x**2)**(3/2)
), Eq(b, -a/x)), (x**4/(a**2*c**(3/2)*(x**2)**(3/2) + a*b*c**(3/2)*x*(x**2)
**(3/2)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(cx^2\right)^{\frac{3}{2}}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/((c*x^2)^(3/2)*(b*x + a)^2), x)
```

3.920 $\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=68

$$-\frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}+\frac{x\log(x)}{a^2c\sqrt{cx^2}}+\frac{x}{ac\sqrt{cx^2}(a+bx)}$$

```
[Out] x/(a*c*Sqrt[c*x^2]*(a + b*x)) + (x*Log[x])/(a^2*c*Sqrt[c*x^2]) - (x*Log[a +
 b*x])/(a^2*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0167383, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 44}

$$-\frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}+\frac{x\log(x)}{a^2c\sqrt{cx^2}}+\frac{x}{ac\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] x/(a*c*Sqrt[c*x^2]*(a + b*x)) + (x*Log[x])/(a^2*c*Sqrt[c*x^2]) - (x*Log[a +
 b*x])/(a^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{1}{x(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{x}{ac\sqrt{cx^2}(a+bx)}+\frac{x\log(x)}{a^2c\sqrt{cx^2}}-\frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0134991, size = 46, normalized size = 0.68

$$\frac{x^3(\log(x)(a+bx)-(a+bx)\log(a+bx)+a)}{a^2\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^3*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*(c*x^2)^(3/2)*(a
 + b*x))
```

Maple [A] time = 0.005, size = 52, normalized size = 0.8

$$\frac{x^{3}\left(b\ln\left(x\right)x-b\ln\left(bx+a\right)x+a\ln\left(x\right)-a\ln\left(bx+a\right)+a\right)}{a^{2}\left(bx+a\right)}\left(cx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] x^3*(b*ln(x)*x-b*ln(b*x+a)*x+a*ln(x)-a*ln(b*x+a)+a)/(c*x^2)^(3/2)/a^2/(b*x+
a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38343, size = 100, normalized size = 1.47

$$\frac{\sqrt{cx^{2}}\left(\left(bx+a\right)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^{2}bc^{2}x^{2}+a^{3}c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*c^2*x^2 + a^3*c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(cx^{2}\right)^{\frac{3}{2}}\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.921 $\int \frac{x}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{bx}{a^2c\sqrt{cx^2}(a+bx)}-\frac{2bx\log(x)}{a^3c\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}-\frac{1}{a^2c\sqrt{cx^2}}$$

```
[Out] -(1/(a^2*c*Sqrt[c*x^2])) - (b*x)/(a^2*c*Sqrt[c*x^2]*(a + b*x)) - (2*b*x*Log
[x])/(a^3*c*Sqrt[c*x^2]) + (2*b*x*Log[a + b*x])/(a^3*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0236142, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 44}

$$-\frac{bx}{a^2c\sqrt{cx^2}(a+bx)}-\frac{2bx\log(x)}{a^3c\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}-\frac{1}{a^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*c*Sqrt[c*x^2])) - (b*x)/(a^2*c*Sqrt[c*x^2]*(a + b*x)) - (2*b*x*Log
[x])/(a^3*c*Sqrt[c*x^2]) + (2*b*x*Log[a + b*x])/(a^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{1}{x^2(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{1}{a^2c\sqrt{cx^2}}-\frac{bx}{a^2c\sqrt{cx^2}(a+bx)}-\frac{2bx\log(x)}{a^3c\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0105107, size = 59, normalized size = 0.66

$$\frac{x^2(-a(a+2bx)-2bx\log(x)(a+bx)+2bx(a+bx)\log(a+bx))}{a^3\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^2*(-(a*(a + 2*b*x)) - 2*b*x*(a + b*x)*Log[x] + 2*b*x*(a + b*x)*Log[a + b
*x]))/(a^3*(c*x^2)^(3/2)*(a + b*x))
```

Maple [A] time = 0.004, size = 74, normalized size = 0.8

$$-\frac{x^{2}\left(2\,b^{2}\ln\left(x\right)x^{2}-2\,b^{2}\ln\left(bx+a\right)x^{2}+2\,ab\ln\left(x\right)x-2\,\ln\left(bx+a\right)xab+2\,abx+a^{2}\right)}{a^{3}\left(bx+a\right)}\left(cx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] -x^2*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*ln(x)*x-2*ln(b*x+a)*x*a*b+2
*a*b*x+a^2)/(c*x^2)^(3/2)/a^3/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.16488, size = 134, normalized size = 1.49

$$-\frac{\left(2\,abx+a^{2}-2\left(b^{2}x^{2}+abx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^{2}}}{a^{3}bc^{2}x^{3}+a^{4}c^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
c^2*x^3 + a^4*c^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(cx^{2}\right)^{\frac{3}{2}}\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.922 $\int \frac{1}{(cx^2)^{3/2}(a+bx)^2} dx$

Optimal. Leaf size=118

$$\frac{b^2x}{a^3c\sqrt{cx^2}(a+bx)}+\frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}+\frac{2b}{a^3c\sqrt{cx^2}}-\frac{1}{2a^2cx\sqrt{cx^2}}$$

```
[Out] (2*b)/(a^3*c*Sqrt[c*x^2]) - 1/(2*a^2*c*x*Sqrt[c*x^2]) + (b^2*x)/(a^3*c*Sqrt
[c*x^2]*(a + b*x)) + (3*b^2*x*Log[x])/(a^4*c*Sqrt[c*x^2]) - (3*b^2*x*Log[a
+ b*x])/(a^4*c*Sqrt[c*x^2])
```

Rubi [A] time = 0.0323105, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 44}

$$\frac{b^2x}{a^3c\sqrt{cx^2}(a+bx)}+\frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}+\frac{2b}{a^3c\sqrt{cx^2}}-\frac{1}{2a^2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (2*b)/(a^3*c*Sqrt[c*x^2]) - 1/(2*a^2*c*x*Sqrt[c*x^2]) + (b^2*x)/(a^3*c*Sqrt
[c*x^2]*(a + b*x)) + (3*b^2*x*Log[x])/(a^4*c*Sqrt[c*x^2]) - (3*b^2*x*Log[a
+ b*x])/(a^4*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{1}{x^3(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx}{c\sqrt{cx^2}}\\
&= \frac{2b}{a^3c\sqrt{cx^2}}-\frac{1}{2a^2cx\sqrt{cx^2}}+\frac{b^2x}{a^3c\sqrt{cx^2}(a+bx)}+\frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0086627, size = 80, normalized size = 0.68

$$\frac{x\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2\log(x)(a+bx)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] - 6*b^2*x^2
*(a + b*x)*Log[a + b*x]))/(2*a^4*(c*x^2)^(3/2)*(a + b*x))
```

Maple [A] time = 0.006, size = 93, normalized size = 0.8

$$\frac{x\left(6\,b^3\ln(x)\,x^3-6\,b^3\ln(bx+a)\,x^3+6\,\ln(x)\,x^2ab^2-6\,\ln(bx+a)\,x^2ab^2+6\,ab^2x^2+3\,a^2bx-a^3\right)}{2\,a^4\,(bx+a)}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] 1/2*x*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*ln(x)*x^2*a*b^2-6*ln(b*x+a)*x^
2*a*b^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/(c*x^2)^(3/2)/a^4/(b*x+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.24315, size = 165, normalized size = 1.4

$$\frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\left(b^3x^3+ab^2x^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bc^2x^4+a^5c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*c^2*x^4 + a^5*c^2*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2\right)^{\frac{3}{2}} (a+bx)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(1/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.923 $\int x^2\sqrt{cx^2}(a+bx)^n\,dx$

Optimal. Leaf size=131

$$-\frac{a^3\sqrt{cx^2}(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2\sqrt{cx^2}(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3a\sqrt{cx^2}(a+bx)^{n+3}}{b^4(n+3)x}+\frac{\sqrt{cx^2}(a+bx)^{n+4}}{b^4(n+4)x}$$

```
[Out] -((a^3*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*Sqrt[c*x^2]
*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*Sqrt[c*x^2]*(a + b*x)^(3 + n))/(
b^4*(3 + n)*x) + (Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 + n)*x)
```

Rubi [A] time = 0.0374866, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^3\sqrt{cx^2}(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2\sqrt{cx^2}(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3a\sqrt{cx^2}(a+bx)^{n+3}}{b^4(n+3)x}+\frac{\sqrt{cx^2}(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] -((a^3*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*Sqrt[c*x^2]
*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*Sqrt[c*x^2]*(a + b*x)^(3 + n))/(
b^4*(3 + n)*x) + (Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{cx^2}(a+bx)^n\,dx&=\frac{\sqrt{cx^2}\int x^3(a+bx)^n\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{x}\\&=-\frac{a^3\sqrt{cx^2}(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2\sqrt{cx^2}(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3a\sqrt{cx^2}(a+bx)^{3+n}}{b^4(3+n)x}+\frac{\sqrt{cx^2}(a+bx)^{4+n}}{b^4(4+n)x}\end{aligned}$$

Mathematica [A] time = 0.0660456, size = 97, normalized size = 0.74

$$\frac{cx(a+bx)^{n+1}\left(6a^2b(n+1)x-6a^3-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2
)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*Sqrt[c*x^2])
```

Maple [A] time = 0.004, size = 136, normalized size = 1.

$$-\frac{(bx+a)^{1+n}\left(-b^3n^3x^3-6\,b^3n^2x^3+3\,ab^2n^2x^2-11\,b^3nx^3+9\,ab^2nx^2-6\,b^3x^3-6\,a^2bnx+6\,ab^2x^2-6\,a^2bx+6\,a^3\right)\sqrt{cx^2}}{xb^4\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(c*x^2)^(1/2),x)
```

```
[Out] -(c*x^2)^(1/2)*(b*x+a)^(1+n)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A] time = 1.06055, size = 157, normalized size = 1.2

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4\sqrt{c}x^4+\left(n^3+3\,n^2+2\,n\right)ab^3\sqrt{c}x^3-3\left(n^2+n\right)a^2b^2\sqrt{c}x^2+6\,a^3b\sqrt{c}nx-6\,a^4\sqrt{c}\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*sqrt(c)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*sqrt(
c)*x^3 - 3*(n^2 + n)*a^2*b^2*sqrt(c)*x^2 + 6*a^3*b*sqrt(c)*n*x - 6*a^4*sqrt
(c))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

Fricas [A] time = 1.31143, size = 313, normalized size = 2.39

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\sqrt{cx^2}(bx+a}{\left(b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^
4)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.06487, size = 405, normalized size = 3.09

$$\left(\frac{6\,a^4a^n\mathrm{sgn}\,(x)}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}+\frac{(bx+a)^n b^4n^3x^4\mathrm{sgn}\,(x)+(bx+a)^n ab^3n^3x^3\mathrm{sgn}\,(x)+6\,(bx+a)^n b^4n^2x^4\mathrm{s}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] (6*a^4*a^n*sgn(x)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4) +
 ((b*x + a)^n*b^4*n^3*x^4*sgn(x) + (b*x + a)^n*a*b^3*n^3*x^3*sgn(x) + 6*(b*
x + a)^n*b^4*n^2*x^4*sgn(x) + 3*(b*x + a)^n*a*b^3*n^2*x^3*sgn(x) + 11*(b*x
+ a)^n*b^4*n*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b^2*n^2*x^2*sgn(x) + 2*(b*x + a
)^n*a*b^3*n*x^3*sgn(x) + 6*(b*x + a)^n*b^4*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b
^2*n*x^2*sgn(x) + 6*(b*x + a)^n*a^3*b*n*x*sgn(x) - 6*(b*x + a)^n*a^4*sgn(x)
)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4))*sqrt(c)
```

3.924 $\int x\sqrt{cx^2}(a+bx)^n\,dx$

Optimal. Leaf size=96

$$\frac{a^2\sqrt{cx^2}(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2a\sqrt{cx^2}(a+bx)^{n+2}}{b^3(n+2)x}+\frac{\sqrt{cx^2}(a+bx)^{n+3}}{b^3(n+3)x}$$

```
[Out] (a^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*Sqrt[c*x^2]*(a +
 b*x)^(2 + n))/(b^3*(2 + n)*x) + (Sqrt[c*x^2]*(a + b*x)^(3 + n))/(b^3*(3 +
n)*x)
```

Rubi [A] time = 0.0287581, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{a^2\sqrt{cx^2}(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2a\sqrt{cx^2}(a+bx)^{n+2}}{b^3(n+2)x}+\frac{\sqrt{cx^2}(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (a^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*Sqrt[c*x^2]*(a +
 b*x)^(2 + n))/(b^3*(2 + n)*x) + (Sqrt[c*x^2]*(a + b*x)^(3 + n))/(b^3*(3 +
n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{cx^2}(a+bx)^n\,dx&=\frac{\sqrt{cx^2}\int x^2(a+bx)^n\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}\\&=\frac{a^2\sqrt{cx^2}(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2a\sqrt{cx^2}(a+bx)^{2+n}}{b^3(2+n)x}+\frac{\sqrt{cx^2}(a+bx)^{3+n}}{b^3(3+n)x}\end{aligned}$$

Mathematica [A] time = 0.0460786, size = 68, normalized size = 0.71

$$\frac{cx(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))
/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Maple [A] time = 0.004, size = 83, normalized size = 0.9

$$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3\,b^2nx^2-2\,abnx+2\,b^2x^2-2\,abx+2\,a^2\right)\sqrt{cx^2}}{xb^3\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(c*x^2)^(1/2),x)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(1/2)/x/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A] time = 1.02689, size = 108, normalized size = 1.12

$$\frac{\left(\left(n^2+3\,n+2\right)b^3\sqrt{c}x^3+\left(n^2+n\right)ab^2\sqrt{c}x^2-2\,a^2b\sqrt{c}nx+2\,a^3\sqrt{c}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [A] time = 1.44238, size = 209, normalized size = 2.18

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\sqrt{cx^2}(bx+a)^n}{\left(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.08288, size = 270, normalized size = 2.81

$$-\left(\frac{2\,a^3a^n\mathrm{sgn}\left(x\right)}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}-\frac{\left(bx+a\right)^nb^3n^2x^3\mathrm{sgn}\left(x\right)+\left(bx+a\right)^nab^2n^2x^2\mathrm{sgn}\left(x\right)+3\left(bx+a\right)^nb^3nx^3\mathrm{sgn}\left(x\right)+\left(bx+\right.}{b^3n^3+6\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a^3*a^n*sgn(x)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3) - ((b*x + a)^n*
b^3*n^2*x^3*sgn(x) + (b*x + a)^n*a*b^2*n^2*x^2*sgn(x) + 3*(b*x + a)^n*b^3*n
*x^3*sgn(x) + (b*x + a)^n*a*b^2*n*x^2*sgn(x) + 2*(b*x + a)^n*b^3*x^3*sgn(x)
 - 2*(b*x + a)^n*a^2*b*n*x*sgn(x) + 2*(b*x + a)^n*a^3*sgn(x))/(b^3*n^3 + 6*
b^3*n^2 + 11*b^3*n + 6*b^3))*sqrt(c)
```

3.925 $\int \sqrt{cx^2}(a+bx)^n\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{cx^2}(a+bx)^{n+2}}{b^2(n+2)x}-\frac{a\sqrt{cx^2}(a+bx)^{n+1}}{b^2(n+1)x}$$

```
[Out] -((a*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (Sqrt[c*x^2]*(a + b*
x)^(2 + n))/(b^2*(2 + n)*x)
```

Rubi [A] time = 0.0169286, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{\sqrt{cx^2}(a+bx)^{n+2}}{b^2(n+2)x}-\frac{a\sqrt{cx^2}(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] -((a*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (Sqrt[c*x^2]*(a + b*
x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx^2}(a+bx)^n\,dx &= \frac{\sqrt{cx^2}\int x(a+bx)^n\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{x}\\ &= -\frac{a\sqrt{cx^2}(a+bx)^{1+n}}{b^2(1+n)x}+\frac{\sqrt{cx^2}(a+bx)^{2+n}}{b^2(2+n)x}\end{aligned}$$

Mathematica [A] time = 0.0304787, size = 44, normalized size = 0.7

$$\frac{cx(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^2]
)
```

Maple [A] time = 0.002, size = 46, normalized size = 0.7

$$-\frac{(bx+a)^{1+n}(-bxn-bx+a)}{xb^2(n^2+3n+2)}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2),x)
```

```
[Out] -(c*x^2)^(1/2)*(b*x+a)^(1+n)*(-b*n*x-b*x+a)/x/b^2/(n^2+3*n+2)
```

Maxima [A] time = 1.03805, size = 69, normalized size = 1.1

$$\frac{\left(b^2\sqrt{c}(n+1)x^2+ab\sqrt{c}nx-a^2\sqrt{c}\right)(bx+a)^n}{(n^2+3n+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b^2*sqrt(c)*(n + 1)*x^2 + a*b*sqrt(c)*n*x - a^2*sqrt(c))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

Fricas [A] time = 1.21974, size = 126, normalized size = 2.

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\sqrt{cx^2}(bx+a)^n}{(b^2n^2+3b^2n+2b^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*n^2 + 3*b
^2*n + 2*b^2)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.06232, size = 161, normalized size = 2.56

$$\left(\frac{a^2a^n\mathrm{sgn}\left(x\right)}{b^2n^2+3\,b^2n+2\,b^2}+\frac{(bx+a)^nb^2nx^2\mathrm{sgn}\left(x\right)+(bx+a)^nabnx\mathrm{sgn}\left(x\right)+(bx+a)^nb^2x^2\mathrm{sgn}\left(x\right)-(bx+a)^na^2\mathrm{sgn}\left(x\right)}{b^2n^2+3\,b^2n+2\,b^2}\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] (a^2*a^n*sgn(x)/(b^2*n^2 + 3*b^2*n + 2*b^2) + ((b*x + a)^n*b^2*n*x^2*sgn(x)
 + (b*x + a)^n*a*b*n*x*sgn(x) + (b*x + a)^n*b^2*x^2*sgn(x) - (b*x + a)^n*a^
2*sgn(x))/(b^2*n^2 + 3*b^2*n + 2*b^2))*sqrt(c)
```

3.926 $\int \frac{\sqrt{cx^2}(a+bx)^n}{x} dx$

Optimal. Leaf size=30

$$\frac{\sqrt{cx^2}(a+bx)^{n+1}}{b(n+1)x}$$

[Out] (Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)

Rubi [A] time = 0.0056715, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 32}

$$\frac{\sqrt{cx^2}(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x,x]

[Out] (Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)^n}{x} dx &= \frac{\sqrt{cx^2}\int (a+bx)^n dx}{x}\\ &= \frac{\sqrt{cx^2}(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.0125384, size = 29, normalized size = 0.97

$$\frac{cx(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x,x]

[Out] (c*x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])

Maple [A] time = 0.001, size = 29, normalized size = 1.

$$\frac{(bx+a)^{1+n}}{b(1+n)x}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x,x)
```

```
[Out] (b*x+a)^(1+n)*(c*x^2)^(1/2)/b/(1+n)/x
```

Maxima [A] time = 1.05858, size = 38, normalized size = 1.27

$$\frac{\left(b\sqrt{c}x+a\sqrt{c}\right)(bx+a)^n}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*(n + 1))
```

Fricas [A] time = 1.33024, size = 66, normalized size = 2.2

$$\frac{\sqrt{cx^2}(bx+a)(bx+a)^n}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*n + b)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.06366, size = 57, normalized size = 1.9

$$-\sqrt{c}\left(\frac{a^{n+1}\operatorname{sgn}(x)}{bn+b}-\frac{(bx+a)^{n+1}\operatorname{sgn}(x)}{b(n+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -sqrt(c)*(a^(n + 1)*sgn(x)/(b*n + b) - (b*x + a)^(n + 1)*sgn(x)/(b*(n + 1))
)
```

3.927 $\int \frac{\sqrt{cx^2}(a+bx)^n}{x^2} \, dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{cx^2}(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x}$$

```
[Out] -((Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*
x)/a])/(a*(1 + n)*x))
```

Rubi [A] time = 0.0110567, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$-\frac{\sqrt{cx^2}(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x^2,x]
```

```
[Out] -((Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*
x)/a])/(a*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)^n}{x^2} \, dx &= \frac{\sqrt{cx^2} \int \frac{(a+bx)^n}{x} \, dx}{x} \\
&= -\frac{\sqrt{cx^2}(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}
\end{aligned}$$

Mathematica [A] time = 0.0104535, size = 46, normalized size = 0.98

$$-\frac{cx(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x^2,x]
```

```
[Out] -((c*x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(
a*(1 + n)*Sqrt[c*x^2]))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^2}\sqrt{cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x^2,x)
```

```
[Out] int((b*x+a)^n*(c*x^2)^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}(bx+a)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**n/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}(bx+a)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^2, x)
```

3.928 $\int \frac{\sqrt{cx^2}(a+bx)^n}{x^3}\,dx$

Optimal. Leaf size=47

$$\frac{b\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x}$$

```
[Out] (b*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*
x)/a])/(a^2*(1 + n)*x)
```

Rubi [A] time = 0.0106019, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$\frac{b\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x^3,x]
```

```
[Out] (b*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*
x)/a])/(a^2*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)^n}{x^3}\,dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^n}{x^2}\,dx}{x}\\ &= \frac{b\sqrt{cx^2}(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.0141412, size = 47, normalized size = 1.

$$\frac{b\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x^3,x]
```

```
[Out] (b*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*
x)/a])/(a^2*(1 + n)*x)
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^3}\sqrt{cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x^3,x)
```

```
[Out] int((b*x+a)^n*(c*x^2)^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}(bx+a)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**n/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}(bx+a)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^3, x)
```

3.929 $\int \frac{\sqrt{cx^2}(a+bx)^n}{x^4} dx$

Optimal. Leaf size=50

$$-\frac{b^2\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)x}$$

```
[Out] -((b^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a^3*(1 + n)*x))
```

Rubi [A] time = 0.0115907, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$-\frac{b^2\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x^4,x]
```

```
[Out] -((b^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a^3*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)^n}{x^4} dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^n}{x^3} dx}{x}\\
&= -\frac{b^2\sqrt{cx^2}(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x}
\end{aligned}$$

Mathematica [A] time = 0.0155292, size = 50, normalized size = 1.

$$-\frac{b^2\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x^4,x]
```

```
[Out] -((b^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a^3*(1 + n)*x))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^4}\sqrt{cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x^4,x)
```

```
[Out] int((b*x+a)^n*(c*x^2)^(1/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}(bx+a)^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**n/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2}(bx+a)^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^4, x)
```

3.930 $\int x\left(cx^2\right)^{3/2}(a+bx)^n\,dx$

Optimal. Leaf size=169

$$\frac{a^4c\sqrt{cx^2}(a+bx)^{n+1}}{b^5(n+1)x}-\frac{4a^3c\sqrt{cx^2}(a+bx)^{n+2}}{b^5(n+2)x}+\frac{6a^2c\sqrt{cx^2}(a+bx)^{n+3}}{b^5(n+3)x}-\frac{4ac\sqrt{cx^2}(a+bx)^{n+4}}{b^5(n+4)x}+\frac{c\sqrt{cx^2}(a+bx)^{n+5}}{b^5(n+5)x}$$

```
[Out] (a^4*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^5*(1 + n)*x) - (4*a^3*c*Sqrt[c*x^2
]*(a + b*x)^(2 + n))/(b^5*(2 + n)*x) + (6*a^2*c*Sqrt[c*x^2]*(a + b*x)^(3 +
n))/(b^5*(3 + n)*x) - (4*a*c*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^5*(4 + n)*x)
 + (c*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^5*(5 + n)*x)
```

Rubi [A] time = 0.0553475, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{a^4c\sqrt{cx^2}(a+bx)^{n+1}}{b^5(n+1)x}-\frac{4a^3c\sqrt{cx^2}(a+bx)^{n+2}}{b^5(n+2)x}+\frac{6a^2c\sqrt{cx^2}(a+bx)^{n+3}}{b^5(n+3)x}-\frac{4ac\sqrt{cx^2}(a+bx)^{n+4}}{b^5(n+4)x}+\frac{c\sqrt{cx^2}(a+bx)^{n+5}}{b^5(n+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] (a^4*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^5*(1 + n)*x) - (4*a^3*c*Sqrt[c*x^2
]*(a + b*x)^(2 + n))/(b^5*(2 + n)*x) + (6*a^2*c*Sqrt[c*x^2]*(a + b*x)^(3 +
n))/(b^5*(3 + n)*x) - (4*a*c*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^5*(4 + n)*x)
 + (c*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^5*(5 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{3/2}(a+bx)^n\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)^n\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int\left(\frac{a^4(a+bx)^n}{b^4}-\frac{4a^3(a+bx)^{1+n}}{b^4}+\frac{6a^2(a+bx)^{2+n}}{b^4}-\frac{4a(a+bx)^{3+n}}{b^4}+\frac{(a+bx)^{4+n}}{b^4}\right)dx}{x}\\ &= \frac{a^4c\sqrt{cx^2}(a+bx)^{1+n}}{b^5(1+n)x}-\frac{4a^3c\sqrt{cx^2}(a+bx)^{2+n}}{b^5(2+n)x}+\frac{6a^2c\sqrt{cx^2}(a+bx)^{3+n}}{b^5(3+n)x}-\frac{4ac\sqrt{cx^2}(a+bx)^{4+n}}{b^5(4+n)x}+\end{aligned}$$

Mathematica [A] time = 0.0744413, size = 132, normalized size = 0.78

$$\frac{\left(cx^2\right)^{3/2}(a+bx)^{n+1}\left(12a^2b^2\left(n^2+3n+2\right)x^2-24a^3b(n+1)x+24a^4-4ab^3\left(n^3+6n^2+11n+6\right)x^3+b^4\left(n^4+10n^3\right.\right.}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n)*(24*a^4 - 24*a^3*b*(1 + n)*x + 12*a^2*b^2*
(2 + 3*n + n^2)*x^2 - 4*a*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 + b^4*(24 + 50*n
 + 35*n^2 + 10*n^3 + n^4)*x^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n
)*x^3)
```

Maple [A] time = 0.005, size = 199, normalized size = 1.2

$$\frac{(bx+a)^{1+n}\left(b^4n^4x^4+10\,b^4n^3x^4-4\,ab^3n^3x^3+35\,b^4n^2x^4-24\,ab^3n^2x^3+50\,b^4nx^4+12\,a^2b^2n^2x^2-44\,ab^3nx^3+24\,b^4x\right.}{x^3b^5\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a)^n,x)
```

```
[Out] (b*x+a)^(1+n)*(b^4*n^4*x^4+10*b^4*n^3*x^4-4*a*b^3*n^3*x^3+35*b^4*n^2*x^4-24
*a*b^3*n^2*x^3+50*b^4*n*x^4+12*a^2*b^2*n^2*x^2-44*a*b^3*n*x^3+24*b^4*x^4+36
*a^2*b^2*n*x^2-24*a*b^3*x^3-24*a^3*b*n*x+24*a^2*b^2*x^2-24*a^3*b*x+24*a^4)*
(c*x^2)^(3/2)/x^3/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

Maxima [A] time = 1.07084, size = 212, normalized size = 1.25

$$\frac{\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5c^{\frac{3}{2}}x^5+\left(n^4+6\,n^3+11\,n^2+6\,n\right)ab^4c^{\frac{3}{2}}x^4-4\left(n^3+3\,n^2+2\,n\right)a^2b^3c^{\frac{3}{2}}x^3+12\left(n^2+\right.\right.}{\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^(3/2)*x^5 + (n^4 + 6*n^3 + 11*n^
2 + 6*n)*a*b^4*c^(3/2)*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^(3/2)*x^3 + 12
*(n^2 + n)*a^3*b^2*c^(3/2)*x^2 - 24*a^4*b*c^(3/2)*n*x + 24*a^5*c^(3/2))*(b*
x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

Fricas [A] time = 1.57399, size = 494, normalized size = 2.92

$$-\frac{\left(24\,a^4bcnx-24\,a^5c-\left(b^5cn^4+10\,b^5cn^3+35\,b^5cn^2+50\,b^5cn+24\,b^5c\right)x^5-\left(ab^4cn^4+6\,ab^4cn^3+11\,ab^4cn^2+6\,ab^4c\right.\right.}{\left(b^5n^5+15\,b^5n^4+85\,b^5n^3+225\,b^5n^2+274\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(24*a^4*b*c*n*x - 24*a^5*c - (b^5*c*n^4 + 10*b^5*c*n^3 + 35*b^5*c*n^2 + 50
*b^5*c*n + 24*b^5*c)*x^5 - (a*b^4*c*n^4 + 6*a*b^4*c*n^3 + 11*a*b^4*c*n^2 +
6*a*b^4*c*n)*x^4 + 4*(a^2*b^3*c*n^3 + 3*a^2*b^3*c*n^2 + 2*a^2*b^3*c*n)*x^3
- 12*(a^3*b^2*c*n^2 + a^3*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^5*n^5 +
 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(cx^2\right)^{\frac{3}{2}} (a+bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a)**n,x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)*(a + b*x)**n, x)
```

Giac [B] time = 1.07302, size = 575, normalized size = 3.4

$$-\left(\frac{24\,a^5 a^n \mathrm{sgn}\left(x\right)}{b^5 n^5 + 15\,b^5 n^4 + 85\,b^5 n^3 + 225\,b^5 n^2 + 274\,b^5 n + 120\,b^5} - \frac{(bx+a)^n b^5 n^4 x^5 \mathrm{sgn}\left(x\right) + (bx+a)^n a b^4 n^4 x^4 \mathrm{sgn}\left(x\right) + 10\,(bx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] -(24*a^5*a^n*sgn(x)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*
b^5*n + 120*b^5) - ((b*x + a)^n*b^5*n^4*x^5*sgn(x) + (b*x + a)^n*a*b^4*n^4*
x^4*sgn(x) + 10*(b*x + a)^n*b^5*n^3*x^5*sgn(x) + 6*(b*x + a)^n*a*b^4*n^3*x^
4*sgn(x) + 35*(b*x + a)^n*b^5*n^2*x^5*sgn(x) - 4*(b*x + a)^n*a^2*b^3*n^3*x^
3*sgn(x) + 11*(b*x + a)^n*a*b^4*n^2*x^4*sgn(x) + 50*(b*x + a)^n*b^5*n*x^5*s
gn(x) - 12*(b*x + a)^n*a^2*b^3*n^2*x^3*sgn(x) + 6*(b*x + a)^n*a*b^4*n*x^4*s
gn(x) + 24*(b*x + a)^n*b^5*x^5*sgn(x) + 12*(b*x + a)^n*a^3*b^2*n^2*x^2*sgn(
x) - 8*(b*x + a)^n*a^2*b^3*n*x^3*sgn(x) + 12*(b*x + a)^n*a^3*b^2*n*x^2*sgn(
x) - 24*(b*x + a)^n*a^4*b*n*x*sgn(x) + 24*(b*x + a)^n*a^5*sgn(x))/(b^5*n^5
+ 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5))*c^(3/2)
```

3.931 $\int \left(cx^2\right)^{3/2} (a+bx)^n \, dx$

Optimal. Leaf size=135

$$-\frac{a^3c\sqrt{cx^2}(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c\sqrt{cx^2}(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac\sqrt{cx^2}(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c\sqrt{cx^2}(a+bx)^{n+4}}{b^4(n+4)x}$$

```
[Out] -((a^3*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*c*Sqrt[c*
x^2]*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*c*Sqrt[c*x^2]*(a + b*x)^(3 +
 n))/(b^4*(3 + n)*x) + (c*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 + n)*x)
```

Rubi [A] time = 0.0386992, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$-\frac{a^3c\sqrt{cx^2}(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c\sqrt{cx^2}(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac\sqrt{cx^2}(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c\sqrt{cx^2}(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x)^n,x]
```


```
[Out] -((a^3*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*c*Sqrt[c*
x^2]*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*c*Sqrt[c*x^2]*(a + b*x)^(3 +
 n))/(b^4*(3 + n)*x) + (c*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 + n)*x)
```


Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(cx^2\right)^{3/2} (a+bx)^n \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(a+bx)^n \, dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right) dx}{x}\\ &= -\frac{a^3c\sqrt{cx^2}(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2c\sqrt{cx^2}(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3ac\sqrt{cx^2}(a+bx)^{3+n}}{b^4(3+n)x}+\frac{c\sqrt{cx^2}(a+bx)^{4+n}}{b^4(4+n)x}\end{aligned}$$

Mathematica [A] time = 0.0562247, size = 98, normalized size = 0.73

$$\frac{\left(cx^2\right)^{3/2}(a+bx)^{n+1}\left(6a^2b(n+1)x-6a^3-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 +
 3*n + n^2)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(
3 + n)*(4 + n)*x^3)
```

Maple [A] time = 0.005, size = 136, normalized size = 1.

$$-\frac{(bx+a)^{1+n}\left(-b^3n^3x^3-6\,b^3n^2x^3+3\,ab^2n^2x^2-11\,b^3nx^3+9\,ab^2nx^2-6\,b^3x^3-6\,a^2bnx+6\,ab^2x^2-6\,a^2bx+6\,a^3\right)}{x^3b^4\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(3/2)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x^3/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A] time = 1.01576, size = 157, normalized size = 1.16

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4c^{\frac{3}{2}}x^4+\left(n^3+3\,n^2+2\,n\right)ab^3c^{\frac{3}{2}}x^3-3\left(n^2+n\right)a^2b^2c^{\frac{3}{2}}x^2+6\,a^3bc^{\frac{3}{2}}nx-6\,a^4c^{\frac{3}{2}}\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*c^(3/2)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^(3/
2)*x^3 - 3*(n^2 + n)*a^2*b^2*c^(3/2)*x^2 + 6*a^3*b*c^(3/2)*n*x - 6*a^4*c^(3
/2))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

Fricas [A] time = 1.53218, size = 343, normalized size = 2.54

$$\frac{\left(6\,a^3bcnx-6\,a^4c+\left(b^4cn^3+6\,b^4cn^2+11\,b^4cn+6\,b^4c\right)x^4+\left(ab^3cn^3+3\,ab^3cn^2+2\,ab^3cn\right)x^3-3\left(a^2b^2cn^2+a^2b^2cn\right)x^2\right)\sqrt{cx^2}(bx+a)^n}{\left(b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*c*n*x - 6*a^4*c + (b^4*c*n^3 + 6*b^4*c*n^2 + 11*b^4*c*n + 6*b^4*c)
*x^4 + (a*b^3*c*n^3 + 3*a*b^3*c*n^2 + 2*a*b^3*c*n)*x^3 - 3*(a^2*b^2*c*n^2 +
 a^2*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^4*n^4 + 10*b^4*n^3 + 35*b^4*
n^2 + 50*b^4*n + 24*b^4)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^{\frac{3}{2}} (a+bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n, x)
```

Giac [B] time = 1.07286, size = 405, normalized size = 3.

$$\left(\frac{6\,a^4a^n\mathrm{sgn}\,(x)}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}+\frac{(bx+a)^nb^4n^3x^4\mathrm{sgn}\,(x)+(bx+a)^nab^3n^3x^3\mathrm{sgn}\,(x)+6\,(bx+a)^nb^4n^2x^4\mathrm{s}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] (6*a^4*a^n*sgn(x)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4) +
 ((b*x + a)^n*b^4*n^3*x^4*sgn(x) + (b*x + a)^n*a*b^3*n^3*x^3*sgn(x) + 6*(b*
x + a)^n*b^4*n^2*x^4*sgn(x) + 3*(b*x + a)^n*a*b^3*n^2*x^3*sgn(x) + 11*(b*x
+ a)^n*b^4*n*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b^2*n^2*x^2*sgn(x) + 2*(b*x + a
)^n*a*b^3*n*x^3*sgn(x) + 6*(b*x + a)^n*b^4*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b
^2*n*x^2*sgn(x) + 6*(b*x + a)^n*a^3*b*n*x*sgn(x) - 6*(b*x + a)^n*a^4*sgn(x)
)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4))*c^(3/2)
```

3.932 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x}\,dx$

Optimal. Leaf size=99

$$\frac{a^2c\sqrt{cx^2}(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac\sqrt{cx^2}(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c\sqrt{cx^2}(a+bx)^{n+3}}{b^3(n+3)x}$$

```
[Out] (a^2*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*c*Sqrt[c*x^2]*
(a + b*x)^(2 + n))/(b^3*(2 + n)*x) + (c*Sqrt[c*x^2]*(a + b*x)^(3 + n))/(b^3
*(3 + n)*x)
```

Rubi [A] time = 0.0284546, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2c\sqrt{cx^2}(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac\sqrt{cx^2}(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c\sqrt{cx^2}(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x,x]
```

```
[Out] (a^2*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*c*Sqrt[c*x^2]*
(a + b*x)^(2 + n))/(b^3*(2 + n)*x) + (c*Sqrt[c*x^2]*(a + b*x)^(3 + n))/(b^3
*(3 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)^n}{x}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)^n\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}\\
&= \frac{a^2c\sqrt{cx^2}(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2ac\sqrt{cx^2}(a+bx)^{2+n}}{b^3(2+n)x}+\frac{c\sqrt{cx^2}(a+bx)^{3+n}}{b^3(3+n)x}
\end{aligned}$$

Mathematica [A] time = 0.0322038, size = 70, normalized size = 0.71

$$\frac{c^2x(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x,x]
```

```
[Out] (c^2*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2
))/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Maple [A] time = 0.004, size = 83, normalized size = 0.8

$$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3\,b^2nx^2-2\,abnx+2\,b^2x^2-2\,abx+2\,a^2\right)}{x^3b^3\left(n^3+6\,n^2+11\,n+6\right)}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x,x)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(3/2)/x^3/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A] time = 1.0552, size = 108, normalized size = 1.09

$$\frac{\left(\left(n^2+3\,n+2\right)b^3c^{\frac{3}{2}}x^3+\left(n^2+n\right)ab^2c^{\frac{3}{2}}x^2-2\,a^2bc^{\frac{3}{2}}nx+2\,a^3c^{\frac{3}{2}}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*c^(3/2)*x^3 + (n^2 + n)*a*b^2*c^(3/2)*x^2 - 2*a^2*b*c^
(3/2)*n*x + 2*a^3*c^(3/2))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [A] time = 1.72208, size = 228, normalized size = 2.3

$$-\frac{\left(2\,a^2bcnx-2\,a^3c-\left(b^3cn^2+3\,b^3cn+2\,b^3c\right)x^3-\left(ab^2cn^2+ab^2cn\right)x^2\right)\sqrt{cx^2}(bx+a)^n}{\left(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*c*n*x - 2*a^3*c - (b^3*c*n^2 + 3*b^3*c*n + 2*b^3*c)*x^3 - (a*b^2*
c*n^2 + a*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6*b^3*n^2 + 11*
b^3*n + 6*b^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2)^{\frac{3}{2}} (a + bx)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2)^{\frac{3}{2}} (bx + a)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x, x)
```

3.933 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^2}\,dx$

Optimal. Leaf size=65

$$\frac{c\sqrt{cx^2}(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac\sqrt{cx^2}(a+bx)^{n+1}}{b^2(n+1)x}$$

```
[Out] -((a*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (c*Sqrt[c*x^2]*(a
+ b*x)^(2 + n))/(b^2*(2 + n)*x)
```

Rubi [A] time = 0.0179092, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{c\sqrt{cx^2}(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac\sqrt{cx^2}(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] -((a*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (c*Sqrt[c*x^2]*(a
+ b*x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)^n\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{x}\\ &= -\frac{ac\sqrt{cx^2}(a+bx)^{1+n}}{b^2(1+n)x}+\frac{c\sqrt{cx^2}(a+bx)^{2+n}}{b^2(2+n)x}\end{aligned}$$

Mathematica [A] time = 0.007752, size = 46, normalized size = 0.71

$$\frac{c^2x(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] (c^2*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

Maple [A] time = 0.001, size = 46, normalized size = 0.7

$$-\frac{(bx+a)^{1+n}(-bxn-bx+a)}{x^3b^2(n^2+3n+2)}(cx^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^2,x)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(3/2)*(-b*n*x-b*x+a)/x^3/b^2/(n^2+3*n+2)
```

Maxima [A] time = 1.02159, size = 69, normalized size = 1.06

$$\frac{\left(b^2c^{\frac{3}{2}}(n+1)x^2+abc^{\frac{3}{2}}nx-a^2c^{\frac{3}{2}}\right)(bx+a)^n}{(n^2+3n+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="maxima")
```

```
[Out] (b^2*c^(3/2)*(n + 1)*x^2 + a*b*c^(3/2)*n*x - a^2*c^(3/2))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

Fricas [A] time = 1.60575, size = 136, normalized size = 2.09

$$\frac{\left(abcnx-a^2c+\left(b^2cn+b^2c\right)x^2\right)\sqrt{cx^2}(bx+a)^n}{\left(b^2n^2+3b^2n+2b^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="fricas")
```

```
[Out] (a*b*c*n*x - a^2*c + (b^2*c*n + b^2*c)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*n
^2 + 3*b^2*n + 2*b^2)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.08445, size = 161, normalized size = 2.48

$$\left(\frac{a^2a^n\text{sgn}(x)}{b^2n^2+3b^2n+2b^2}+\frac{(bx+a)^nb^2nx^2\text{sgn}(x)+(bx+a)^nabnx\text{sgn}(x)+(bx+a)^nb^2x^2\text{sgn}(x)-(bx+a)^na^2\text{sgn}(x)}{b^2n^2+3b^2n+2b^2}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="giac")
```

```
[Out] (a^2*a^n*sgn(x)/(b^2*n^2 + 3*b^2*n + 2*b^2) + ((b*x + a)^n*b^2*n*x^2*sgn(x)
 + (b*x + a)^n*a*b*n*x*sgn(x) + (b*x + a)^n*b^2*x^2*sgn(x) - (b*x + a)^n*a^
2*sgn(x))/(b^2*n^2 + 3*b^2*n + 2*b^2))*c^(3/2)
```

3.934 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^3}\,dx$

Optimal. Leaf size=31

$$\frac{c\sqrt{cx^2}(a+bx)^{n+1}}{b(n+1)x}$$

```
[Out] (c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rubi [A] time = 0.0059132, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 32}

$$\frac{c\sqrt{cx^2}(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^3}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)^n\,dx}{x}\\ &= \frac{c\sqrt{cx^2}(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.0138324, size = 30, normalized size = 0.97

$$\frac{(cx^2)^{3/2}(a+bx)^{n+1}}{b(n+1)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n))/(b*(1 + n)*x^3)
```

Maple [A] time = 0.001, size = 29, normalized size = 0.9

$$\frac{(bx+a)^{1+n}}{b\,(1+n)\,x^3}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^3,x)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)*(c*x^2)^(3/2)/x^3
```

Maxima [A] time = 0.991608, size = 38, normalized size = 1.23

$$\frac{\left(bc^{\frac{3}{2}}x+ac^{\frac{3}{2}}\right)(bx+a)^n}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="maxima")
```

```
[Out] (b*c^(3/2)*x + a*c^(3/2))*(b*x + a)^n/(b*(n + 1))
```

Fricas [A] time = 1.61113, size = 72, normalized size = 2.32

$$\frac{(bcx+ac)\sqrt{cx^2}(bx+a)^n}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="fricas")
```

```
[Out] (b*c*x + a*c)*sqrt(c*x^2)*(b*x + a)^n/((b*n + b)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.05204, size = 57, normalized size = 1.84

$$-c^{\frac{3}{2}}\left(\frac{a^{n+1}\mathrm{sgn}\,(x)}{bn+b}-\frac{(bx+a)^{n+1}\mathrm{sgn}\,(x)}{b(n+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="giac")
```

```
[Out] -c^(3/2)*(a^(n + 1)*sgn(x)/(b*n + b) - (b*x + a)^(n + 1)*sgn(x)/(b*(n + 1))
)
```

3.935 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^4}\,dx$

Optimal. Leaf size=48

$$-\frac{c\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x}$$

```
[Out] -((c*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (
b*x)/a])/(a*(1 + n)*x))
```

Rubi [A] time = 0.0113252, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$-\frac{c\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] -((c*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (
b*x)/a])/(a*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^4}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x}\,dx}{x}\\ &= -\frac{c\sqrt{cx^2}(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.0097353, size = 47, normalized size = 0.98

$$-\frac{(cx^2)^{3/2}(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] -(((c*x^2)^(3/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (
b*x)/a])/(a*(1 + n)*x^3))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^4} \left(cx^2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^4,x)
```

```
[Out] int((c*x^2)^(3/2)*(b*x+a)^n/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}} (bx+a)^n}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n c}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}} (a+bx)^n}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**4,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2)^{\frac{3}{2}}(bx+a)^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^4, x)
```

3.936 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^5} dx$

Optimal. Leaf size=48

$$\frac{bc\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x}$$

```
[Out] (b*c*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (
b*x)/a])/(a^2*(1 + n)*x)
```

Rubi [A] time = 0.0111331, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$\frac{bc\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (b*c*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (
b*x)/a])/(a^2*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^5}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x^2}\,dx}{x}\\ &= \frac{bc\sqrt{cx^2}(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.0108275, size = 47, normalized size = 0.98

$$\frac{b\left(cx^2\right)^{3/2}(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (b*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (
b*x)/a])/(a^2*(1 + n)*x^3)
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^5}\left(cx^2\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^5,x)
```

```
[Out] int((c*x^2)^(3/2)*(b*x+a)^n/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}(bx+a)^n}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^5,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n c}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}(a+bx)^n}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**5,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}(bx+a)^n}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^5, x)
```

3.937 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^6}\,dx$

Optimal. Leaf size=51

$$-\frac{b^2c\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)x}$$

```
[Out] -((b^2*c*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1
 + (b*x)/a])/(a^3*(1 + n)*x))
```

Rubi [A] time = 0.0130143, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$-\frac{b^2c\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^6,x]
```

```
[Out] -((b^2*c*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1
 + (b*x)/a])/(a^3*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cx^2\right)^{3/2}(a+bx)^n}{x^6}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x^3}\,dx}{x}\\ &= -\frac{b^2c\sqrt{cx^2}(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.0114088, size = 50, normalized size = 0.98

$$-\frac{b^2\left(cx^2\right)^{3/2}(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^6,x]
```

```
[Out] -((b^2*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1
 + (b*x)/a])/(a^3*(1 + n)*x^3))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^6}\left(cx^2\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^6,x)
```

```
[Out] int((c*x^2)^(3/2)*(b*x+a)^n/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}(bx+a)^n}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n c}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}(a+bx)^n}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**6,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}(bx+a)^n}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^6,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^6, x)
```

3.938 $\int \left(cx^2\right)^{5/2} (a+bx)^n \, dx$

Optimal. Leaf size=217

$$-\frac{a^5c^2\sqrt{cx^2}(a+bx)^{n+1}}{b^6(n+1)x}+\frac{5a^4c^2\sqrt{cx^2}(a+bx)^{n+2}}{b^6(n+2)x}-\frac{10a^3c^2\sqrt{cx^2}(a+bx)^{n+3}}{b^6(n+3)x}+\frac{10a^2c^2\sqrt{cx^2}(a+bx)^{n+4}}{b^6(n+4)x}-\frac{5ac^2\sqrt{cx^2}(a+bx)}{b^6(n+5)x}$$

```
[Out] -((a^5*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^6*(1 + n)*x)) + (5*a^4*c^2*Sqr
t[c*x^2]*(a + b*x)^(2 + n))/(b^6*(2 + n)*x) - (10*a^3*c^2*Sqrt[c*x^2]*(a +
b*x)^(3 + n))/(b^6*(3 + n)*x) + (10*a^2*c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/
(b^6*(4 + n)*x) - (5*a*c^2*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^6*(5 + n)*x) +
 (c^2*Sqrt[c*x^2]*(a + b*x)^(6 + n))/(b^6*(6 + n)*x)
```

Rubi [A] time = 0.073794, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$-\frac{a^5c^2\sqrt{cx^2}(a+bx)^{n+1}}{b^6(n+1)x}+\frac{5a^4c^2\sqrt{cx^2}(a+bx)^{n+2}}{b^6(n+2)x}-\frac{10a^3c^2\sqrt{cx^2}(a+bx)^{n+3}}{b^6(n+3)x}+\frac{10a^2c^2\sqrt{cx^2}(a+bx)^{n+4}}{b^6(n+4)x}-\frac{5ac^2\sqrt{cx^2}(a+bx)}{b^6(n+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] -((a^5*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^6*(1 + n)*x)) + (5*a^4*c^2*Sqr
t[c*x^2]*(a + b*x)^(2 + n))/(b^6*(2 + n)*x) - (10*a^3*c^2*Sqrt[c*x^2]*(a +
b*x)^(3 + n))/(b^6*(3 + n)*x) + (10*a^2*c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/
(b^6*(4 + n)*x) - (5*a*c^2*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^6*(5 + n)*x) +
 (c^2*Sqrt[c*x^2]*(a + b*x)^(6 + n))/(b^6*(6 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(cx^2\right)^{5/2}(a+bx)^n\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(a+bx)^n\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a^5(a+bx)^n}{b^5}+\frac{5a^4(a+bx)^{1+n}}{b^5}-\frac{10a^3(a+bx)^{2+n}}{b^5}+\frac{10a^2(a+bx)^{3+n}}{b^5}-\frac{5a(a+bx)^{4+n}}{b^5}+\frac{(a+bx)^{5+n}}{b^5}\right)dx}{x}\\ &= -\frac{a^5c^2\sqrt{cx^2}(a+bx)^{1+n}}{b^6(1+n)x}+\frac{5a^4c^2\sqrt{cx^2}(a+bx)^{2+n}}{b^6(2+n)x}-\frac{10a^3c^2\sqrt{cx^2}(a+bx)^{3+n}}{b^6(3+n)x}+\frac{10a^2c^2\sqrt{cx^2}(a+b}{b^6(4+n)x}\end{aligned}$$

Mathematica [A] time = 0.1091, size = 172, normalized size = 0.79

$$\frac{c^3x(a+bx)^{n+1}\left(-60a^3b^2\left(n^2+3n+2\right)x^2+20a^2b^3\left(n^3+6n^2+11n+6\right)x^3+120a^4b(n+1)x-120a^5-5ab^4\left(n^4+10\right.\right.}{b^6(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(-120*a^5 + 120*a^4*b*(1 + n)*x - 60*a^3*b^2*(2 +
3*n + n^2)*x^2 + 20*a^2*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 - 5*a*b^4*(24 + 50
*n + 35*n^2 + 10*n^3 + n^4)*x^4 + b^5*(120 + 274*n + 225*n^2 + 85*n^3 + 15*
n^4 + n^5)*x^5))/(b^6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*Sqrt[
c*x^2])
```

Maple [A] time = 0.006, size = 280, normalized size = 1.3

$$-\frac{(bx+a)^{1+n}\left(-b^5n^5x^5-15\,b^5n^4x^5+5\,ab^4n^4x^4-85\,b^5n^3x^5+50\,ab^4n^3x^4-225\,b^5n^2x^5-20\,a^2b^3n^3x^3+175\,ab^4n^2x^4\right.}{x^5b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(5/2)*(-b^5*n^5*x^5-15*b^5*n^4*x^5+5*a*b^4*n^4*x^4-8
5*b^5*n^3*x^5+50*a*b^4*n^3*x^4-225*b^5*n^2*x^5-20*a^2*b^3*n^3*x^3+175*a*b^4
*n^2*x^4-274*b^5*n*x^5-120*a^2*b^3*n^2*x^3+250*a*b^4*n*x^4-120*b^5*x^5+60*a
^3*b^2*n^2*x^2-220*a^2*b^3*n*x^3+120*a*b^4*x^4+180*a^3*b^2*n*x^2-120*a^2*b^
3*x^3-120*a^4*b*n*x+120*a^3*b^2*x^2-120*a^4*b*x+120*a^5)/x^5/b^6/(n^6+21*n^
5+175*n^4+735*n^3+1624*n^2+1764*n+720)
```

Maxima [A] time = 1.12252, size = 274, normalized size = 1.26

$$\frac{\left(\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)b^6c^{\frac{5}{2}}x^6+\left(n^5+10\,n^4+35\,n^3+50\,n^2+24\,n\right)ab^5c^{\frac{5}{2}}x^5-5\left(n^4+6\,n^3+1\right.\right.}{\left(n^6+21\,n^5+175\,n^4+735\,n^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^6*c^(5/2)*x^6 + (n^5 + 1
0*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*c^(5/2)*x^5 - 5*(n^4 + 6*n^3 + 11*n^2
 + 6*n)*a^2*b^4*c^(5/2)*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*c^(5/2)*x^3 -
60*(n^2 + n)*a^4*b^2*c^(5/2)*x^2 + 120*a^5*b*c^(5/2)*n*x - 120*a^6*c^(5/2))
*(b*x + a)^n/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*
b^6)
```

Fricas [A] time = 1.71717, size = 733, normalized size = 3.38

$$\frac{\left(120\,a^5bc^2nx - 120\,a^6c^2 + \left(b^6c^2n^5 + 15\,b^6c^2n^4 + 85\,b^6c^2n^3 + 225\,b^6c^2n^2 + 274\,b^6c^2n + 120\,b^6c^2\right)x^6 + \left(ab^5c^2n^5 + 10\,ab^5c\right.\right.}{(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (120*a^5*b*c^2*n*x - 120*a^6*c^2 + (b^6*c^2*n^5 + 15*b^6*c^2*n^4 + 85*b^6*c
^2*n^3 + 225*b^6*c^2*n^2 + 274*b^6*c^2*n + 120*b^6*c^2)*x^6 + (a*b^5*c^2*n^
5 + 10*a*b^5*c^2*n^4 + 35*a*b^5*c^2*n^3 + 50*a*b^5*c^2*n^2 + 24*a*b^5*c^2*n
)*x^5 - 5*(a^2*b^4*c^2*n^4 + 6*a^2*b^4*c^2*n^3 + 11*a^2*b^4*c^2*n^2 + 6*a^2
*b^4*c^2*n)*x^4 + 20*(a^3*b^3*c^2*n^3 + 3*a^3*b^3*c^2*n^2 + 2*a^3*b^3*c^2*n
)*x^3 - 60*(a^4*b^2*c^2*n^2 + a^4*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/(
(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6
*n + 720*b^6)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^{\frac{5}{2}} (a + bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n, x)
```

Giac [B] time = 1.08614, size = 864, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] (120*a^6*a^n*c^2*sgn(x)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 +
 1624*b^6*n^2 + 1764*b^6*n + 720*b^6) + ((b*x + a)^n*b^6*c^2*n^5*x^6*sgn(x)
 + (b*x + a)^n*a*b^5*c^2*n^5*x^5*sgn(x) + 15*(b*x + a)^n*b^6*c^2*n^4*x^6*sg
n(x) + 10*(b*x + a)^n*a*b^5*c^2*n^4*x^5*sgn(x) + 85*(b*x + a)^n*b^6*c^2*n^3
*x^6*sgn(x) - 5*(b*x + a)^n*a^2*b^4*c^2*n^4*x^4*sgn(x) + 35*(b*x + a)^n*a*b
^5*c^2*n^3*x^5*sgn(x) + 225*(b*x + a)^n*b^6*c^2*n^2*x^6*sgn(x) - 30*(b*x +
a)^n*a^2*b^4*c^2*n^3*x^4*sgn(x) + 50*(b*x + a)^n*a*b^5*c^2*n^2*x^5*sgn(x) +
 274*(b*x + a)^n*b^6*c^2*n*x^6*sgn(x) + 20*(b*x + a)^n*a^3*b^3*c^2*n^3*x^3*
sgn(x) - 55*(b*x + a)^n*a^2*b^4*c^2*n^2*x^4*sgn(x) + 24*(b*x + a)^n*a*b^5*c
^2*n*x^5*sgn(x) + 120*(b*x + a)^n*b^6*c^2*x^6*sgn(x) + 60*(b*x + a)^n*a^3*b
^3*c^2*n^2*x^3*sgn(x) - 30*(b*x + a)^n*a^2*b^4*c^2*n*x^4*sgn(x) - 60*(b*x +
 a)^n*a^4*b^2*c^2*n^2*x^2*sgn(x) + 40*(b*x + a)^n*a^3*b^3*c^2*n*x^3*sgn(x)
- 60*(b*x + a)^n*a^4*b^2*c^2*n*x^2*sgn(x) + 120*(b*x + a)^n*a^5*b*c^2*n*x*s
gn(x) - 120*(b*x + a)^n*a^6*c^2*sgn(x))/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4
 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6))*sqrt(c)
```

3.939 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x} dx$

Optimal. Leaf size=179

$$\frac{a^4c^2\sqrt{cx^2}(a+bx)^{n+1}}{b^5(n+1)x} - \frac{4a^3c^2\sqrt{cx^2}(a+bx)^{n+2}}{b^5(n+2)x} + \frac{6a^2c^2\sqrt{cx^2}(a+bx)^{n+3}}{b^5(n+3)x} - \frac{4ac^2\sqrt{cx^2}(a+bx)^{n+4}}{b^5(n+4)x} + \frac{c^2\sqrt{cx^2}(a+bx)^{n+5}}{b^5(n+5)x}$$

```
[Out] (a^4*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^5*(1 + n)*x) - (4*a^3*c^2*Sqrt[c
*x^2]*(a + b*x)^(2 + n))/(b^5*(2 + n)*x) + (6*a^2*c^2*Sqrt[c*x^2]*(a + b*x)
^(3 + n))/(b^5*(3 + n)*x) - (4*a*c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^5*(4
 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^5*(5 + n)*x)
```

Rubi [A] time = 0.051275, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^4c^2\sqrt{cx^2}(a+bx)^{n+1}}{b^5(n+1)x} - \frac{4a^3c^2\sqrt{cx^2}(a+bx)^{n+2}}{b^5(n+2)x} + \frac{6a^2c^2\sqrt{cx^2}(a+bx)^{n+3}}{b^5(n+3)x} - \frac{4ac^2\sqrt{cx^2}(a+bx)^{n+4}}{b^5(n+4)x} + \frac{c^2\sqrt{cx^2}(a+bx)^{n+5}}{b^5(n+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x,x]
```

```
[Out] (a^4*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^5*(1 + n)*x) - (4*a^3*c^2*Sqrt[c
*x^2]*(a + b*x)^(2 + n))/(b^5*(2 + n)*x) + (6*a^2*c^2*Sqrt[c*x^2]*(a + b*x)
^(3 + n))/(b^5*(3 + n)*x) - (4*a*c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^5*(4
 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^5*(5 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^n}{x} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)^n dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^4(a+bx)^n}{b^4} - \frac{4a^3(a+bx)^{1+n}}{b^4} + \frac{6a^2(a+bx)^{2+n}}{b^4} - \frac{4a(a+bx)^{3+n}}{b^4} + \frac{(a+bx)^{4+n}}{b^4}\right) dx}{x} \\
&= \frac{a^4c^2\sqrt{cx^2}(a+bx)^{1+n}}{b^5(1+n)x} - \frac{4a^3c^2\sqrt{cx^2}(a+bx)^{2+n}}{b^5(2+n)x} + \frac{6a^2c^2\sqrt{cx^2}(a+bx)^{3+n}}{b^5(3+n)x} - \frac{4ac^2\sqrt{cx^2}(a+bx}{b^5(4+n)x}
\end{aligned}$$

Mathematica [A] time = 0.0218738, size = 133, normalized size = 0.74

$$\frac{c\left(cx^2\right)^{3/2}(a+bx)^{n+1}\left(12a^2b^2\left(n^2+3n+2\right)x^2-24a^3b(n+1)x+24a^4-4ab^3\left(n^3+6n^2+11n+6\right)x^3+b^4\left(n^4+10n^3+\right.\right.}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x,x]
```

```
[Out] (c*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*(24*a^4 - 24*a^3*b*(1 + n)*x + 12*a^2*b^
2*(2 + 3*n + n^2)*x^2 - 4*a*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 + b^4*(24 + 50
*n + 35*n^2 + 10*n^3 + n^4)*x^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 +
 n)*x^3)
```

Maple [A] time = 0.006, size = 199, normalized size = 1.1

$$\frac{(bx+a)^{1+n}\left(b^4n^4x^4+10\,b^4n^3x^4-4\,ab^3n^3x^3+35\,b^4n^2x^4-24\,ab^3n^2x^3+50\,b^4nx^4+12\,a^2b^2n^2x^2-44\,ab^3nx^3+24\,b^4x^4+\right.}{x^5b^5\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x,x)
```

```
[Out] (b*x+a)^(1+n)*(b^4*n^4*x^4+10*b^4*n^3*x^4-4*a*b^3*n^3*x^3+35*b^4*n^2*x^4-24
*a*b^3*n^2*x^3+50*b^4*n*x^4+12*a^2*b^2*n^2*x^2-44*a*b^3*n*x^3+24*b^4*x^4+36
*a^2*b^2*n*x^2-24*a*b^3*x^3-24*a^3*b*n*x+24*a^2*b^2*x^2-24*a^3*b*x+24*a^4)*
(c*x^2)^(5/2)/x^5/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

Maxima [A] time = 1.00942, size = 212, normalized size = 1.18

$$\frac{\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5c^{\frac{5}{2}}x^5+\left(n^4+6\,n^3+11\,n^2+6\,n\right)ab^4c^{\frac{5}{2}}x^4-4\left(n^3+3\,n^2+2\,n\right)a^2b^3c^{\frac{5}{2}}x^3+12\left(n^2+n\right)\right.}{\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="maxima")
```

```
[Out] ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^(5/2)*x^5 + (n^4 + 6*n^3 + 11*n^
2 + 6*n)*a*b^4*c^(5/2)*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^(5/2)*x^3 + 12
*(n^2 + n)*a^3*b^2*c^(5/2)*x^2 - 24*a^4*b*c^(5/2)*n*x + 24*a^5*c^(5/2))*(b*
x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

Fricas [A] time = 1.596, size = 537, normalized size = 3.

$$-\frac{\left(24\,a^4bc^2nx-24\,a^5c^2-\left(b^5c^2n^4+10\,b^5c^2n^3+35\,b^5c^2n^2+50\,b^5c^2n+24\,b^5c^2\right)x^5-\left(ab^4c^2n^4+6\,ab^4c^2n^3+11\,ab^4c^2n^2+\right.\right.}{\left(b^5n^5+15\,b^5n^4+85\,b^5n^3+225\,b^5n^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="fricas")
```

```
[Out] -(24*a^4*b*c^2*n*x - 24*a^5*c^2 - (b^5*c^2*n^4 + 10*b^5*c^2*n^3 + 35*b^5*c^
2*n^2 + 50*b^5*c^2*n + 24*b^5*c^2)*x^5 - (a*b^4*c^2*n^4 + 6*a*b^4*c^2*n^3 +
 11*a*b^4*c^2*n^2 + 6*a*b^4*c^2*n)*x^4 + 4*(a^2*b^3*c^2*n^3 + 3*a^2*b^3*c^2
*n^2 + 2*a^2*b^3*c^2*n)*x^3 - 12*(a^3*b^2*c^2*n^2 + a^3*b^2*c^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 27
4*b^5*n + 120*b^5)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}(a+bx)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x, x)
```

3.940 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^2} dx$

Optimal. Leaf size=143

$$-\frac{a^3c^2\sqrt{cx^2}(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c^2\sqrt{cx^2}(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac^2\sqrt{cx^2}(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c^2\sqrt{cx^2}(a+bx)^{n+4}}{b^4(n+4)x}$$

```
[Out] -((a^3*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*c^2*Sqr
t[c*x^2]*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*c^2*Sqrt[c*x^2]*(a + b*x
)^(3 + n))/(b^4*(3 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 +
n)*x)
```

Rubi [A] time = 0.0424376, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^3c^2\sqrt{cx^2}(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c^2\sqrt{cx^2}(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac^2\sqrt{cx^2}(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c^2\sqrt{cx^2}(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] -((a^3*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*c^2*Sqr
t[c*x^2]*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*c^2*Sqrt[c*x^2]*(a + b*x
)^(3 + n))/(b^4*(3 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 +
n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^2}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)^n\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{x}\\
&= -\frac{a^3c^2\sqrt{cx^2}(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2c^2\sqrt{cx^2}(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3ac^2\sqrt{cx^2}(a+bx)^{3+n}}{b^4(3+n)x}+\frac{c^2\sqrt{cx^2}(a+bx)^{4+n}}{b^4(4+n)x}
\end{aligned}$$

Mathematica [A] time = 0.0187142, size = 99, normalized size = 0.69

$$\frac{c\left(cx^2\right)^{3/2}(a+bx)^{n+1}\left(6a^2b(n+1)x-6a^3-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] (c*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2
 + 3*n + n^2)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)
*(3 + n)*(4 + n)*x^3)
```

Maple [A] time = 0.004, size = 136, normalized size = 1.

$$-\frac{(bx+a)^{1+n}\left(-b^3n^3x^3-6\,b^3n^2x^3+3\,ab^2n^2x^2-11\,b^3nx^3+9\,ab^2nx^2-6\,b^3x^3-6\,a^2bnx+6\,ab^2x^2-6\,a^2bx+6\,a^3\right)}{x^5b^4\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^2,x)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(5/2)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x^5/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A] time = 1.04549, size = 157, normalized size = 1.1

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4c^{\frac{5}{2}}x^4+\left(n^3+3\,n^2+2\,n\right)ab^3c^{\frac{5}{2}}x^3-3\left(n^2+n\right)a^2b^2c^{\frac{5}{2}}x^2+6\,a^3bc^{\frac{5}{2}}nx-6\,a^4c^{\frac{5}{2}}\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*c^(5/2)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^(5/
2)*x^3 - 3*(n^2 + n)*a^2*b^2*c^(5/2)*x^2 + 6*a^3*b*c^(5/2)*n*x - 6*a^4*c^(5
/2))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

Fricas [A] time = 1.626, size = 373, normalized size = 2.61

$$\frac{\left(6\,a^3bc^2nx-6\,a^4c^2+\left(b^4c^2n^3+6\,b^4c^2n^2+11\,b^4c^2n+6\,b^4c^2\right)x^4+\left(ab^3c^2n^3+3\,ab^3c^2n^2+2\,ab^3c^2n\right)x^3-3\left(a^2b^2c^2n^2+\right.\right.}{\left(b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*c^2*n*x - 6*a^4*c^2 + (b^4*c^2*n^3 + 6*b^4*c^2*n^2 + 11*b^4*c^2*n
+ 6*b^4*c^2)*x^4 + (a*b^3*c^2*n^3 + 3*a*b^3*c^2*n^2 + 2*a*b^3*c^2*n)*x^3 -
```

```
3*(a^2*b^2*c^2*n^2 + a^2*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^4*n^4
+ 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2)^{\frac{5}{2}}(a+bx)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**2,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2)^{\frac{5}{2}}(bx+a)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^2, x)
```

3.941 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^3}\,dx$

Optimal. Leaf size=105

$$\frac{a^2c^2\sqrt{cx^2}(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac^2\sqrt{cx^2}(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c^2\sqrt{cx^2}(a+bx)^{n+3}}{b^3(n+3)x}$$

```
[Out] (a^2*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*c^2*Sqrt[c*x
^2]*(a + b*x)^(2 + n))/(b^3*(2 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(3 + n)
)/(b^3*(3 + n)*x)
```

Rubi [A] time = 0.029299, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2c^2\sqrt{cx^2}(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac^2\sqrt{cx^2}(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c^2\sqrt{cx^2}(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (a^2*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*c^2*Sqrt[c*x
^2]*(a + b*x)^(2 + n))/(b^3*(2 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(3 + n)
)/(b^3*(3 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^3}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)^n\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}\\
&= \frac{a^2c^2\sqrt{cx^2}(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2ac^2\sqrt{cx^2}(a+bx)^{2+n}}{b^3(2+n)x}+\frac{c^2\sqrt{cx^2}(a+bx)^{3+n}}{b^3(3+n)x}
\end{aligned}$$

Mathematica [A] time = 0.0455615, size = 70, normalized size = 0.67

$$\frac{c^3x(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2
))/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Maple [A] time = 0.005, size = 83, normalized size = 0.8

$$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3\,b^2nx^2-2\,abnx+2\,b^2x^2-2\,abx+2\,a^2\right)}{x^5b^3\left(n^3+6\,n^2+11\,n+6\right)}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^3,x)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(5/2)/x^5/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A] time = 1.01093, size = 108, normalized size = 1.03

$$\frac{\left(\left(n^2+3\,n+2\right)b^3c^{\frac{5}{2}}x^3+\left(n^2+n\right)ab^2c^{\frac{5}{2}}x^2-2\,a^2bc^{\frac{5}{2}}nx+2\,a^3c^{\frac{5}{2}}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*c^(5/2)*x^3 + (n^2 + n)*a*b^2*c^(5/2)*x^2 - 2*a^2*b*c^
(5/2)*n*x + 2*a^3*c^(5/2))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [A] time = 1.69188, size = 247, normalized size = 2.35

$$-\frac{\left(2\,a^2bc^2nx-2\,a^3c^2-\left(b^3c^2n^2+3\,b^3c^2n+2\,b^3c^2\right)x^3-\left(ab^2c^2n^2+ab^2c^2n\right)x^2\right)\sqrt{cx^2}(bx+a)^n}{\left(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*c^2*n*x - 2*a^3*c^2 - (b^3*c^2*n^2 + 3*b^3*c^2*n + 2*b^3*c^2)*x^3
 - (a*b^2*c^2*n^2 + a*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6
*b^3*n^2 + 11*b^3*n + 6*b^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}(a+bx)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**3,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}(bx+a)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^3, x)
```

3.942 $\int \frac{\left(cx^2\right)^{5/2}(a+bx)^n}{x^4}\,dx$

Optimal. Leaf size=69

$$\frac{c^2\sqrt{cx^2}(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac^2\sqrt{cx^2}(a+bx)^{n+1}}{b^2(n+1)x}$$

```
[Out] -((a*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (c^2*Sqrt[c*x^2]
*(a + b*x)^(2 + n))/(b^2*(2 + n)*x)
```

Rubi [A] time = 0.0187664, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{c^2\sqrt{cx^2}(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac^2\sqrt{cx^2}(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] -((a*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (c^2*Sqrt[c*x^2]
*(a + b*x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(cx^2\right)^{5/2}(a+bx)^n}{x^4}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)^n\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{x}\\
&= -\frac{ac^2\sqrt{cx^2}(a+bx)^{1+n}}{b^2(1+n)x}+\frac{c^2\sqrt{cx^2}(a+bx)^{2+n}}{b^2(2+n)x}
\end{aligned}$$

Mathematica [A] time = 0.0080047, size = 46, normalized size = 0.67

$$\frac{c^3x(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

Maple [A] time = 0.002, size = 46, normalized size = 0.7

$$-\frac{(bx+a)^{1+n}(-bxn-bx+a)}{x^5b^2(n^2+3n+2)}(cx^2)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^4,x)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(5/2)*(-b*n*x-b*x+a)/x^5/b^2/(n^2+3*n+2)
```

Maxima [A] time = 1.0375, size = 69, normalized size = 1.

$$\frac{\left(b^2c^{\frac{5}{2}}(n+1)x^2+abc^{\frac{5}{2}}nx-a^2c^{\frac{5}{2}}\right)(bx+a)^n}{(n^2+3n+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="maxima")
```

```
[Out] (b^2*c^(5/2)*(n + 1)*x^2 + a*b*c^(5/2)*n*x - a^2*c^(5/2))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

Fricas [A] time = 1.52213, size = 147, normalized size = 2.13

$$\frac{\left(abc^2nx-a^2c^2+\left(b^2c^2n+b^2c^2\right)x^2\right)\sqrt{cx^2}(bx+a)^n}{\left(b^2n^2+3b^2n+2b^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="fricas")
```

```
[Out] (a*b*c^2*n*x - a^2*c^2 + (b^2*c^2*n + b^2*c^2)*x^2)*sqrt(c*x^2)*(b*x + a)^n
/((b^2*n^2 + 3*b^2*n + 2*b^2)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**4,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}} (bx+a)^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^4, x)
```

3.943 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^5}\,dx$

Optimal. Leaf size=33

$$\frac{c^2\sqrt{cx^2}(a+bx)^{n+1}}{b(n+1)x}$$

```
[Out] (c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rubi [A] time = 0.0064947, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 32}

$$\frac{c^2\sqrt{cx^2}(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^5}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int (a+bx)^n\,dx}{x}\\ &= \frac{c^2\sqrt{cx^2}(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.016853, size = 31, normalized size = 0.94

$$\frac{c^3x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Maple [A] time = 0.002, size = 29, normalized size = 0.9

$$\frac{(bx+a)^{1+n}}{b(1+n)x^5}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^5,x)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)*(c*x^2)^(5/2)/x^5
```

Maxima [A] time = 1.02733, size = 38, normalized size = 1.15

$$\frac{\left(bc^{\frac{5}{2}}x+ac^{\frac{5}{2}}\right)(bx+a)^n}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="maxima")
```

```
[Out] (b*c^(5/2)*x + a*c^(5/2))*(b*x + a)^n/(b*(n + 1))
```

Fricas [A] time = 1.57848, size = 77, normalized size = 2.33

$$\frac{\left(bc^2x+ac^2\right)\sqrt{cx^2}(bx+a)^n}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="fricas")
```

```
[Out] (b*c^2*x + a*c^2)*sqrt(c*x^2)*(b*x + a)^n/((b*n + b)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**5,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}} (bx+a)^n}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^5, x)
```

3.944 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^6} dx$

Optimal. Leaf size=50

$$-\frac{c^2\sqrt{cx^2}(a+bx)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x}$$

```
[Out] -((c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a*(1 + n)*x))
```

Rubi [A] time = 0.0117057, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$-\frac{c^2\sqrt{cx^2}(a+bx)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^6,x]
```

```
[Out] -((c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^6} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x} dx}{x}\\ &= -\frac{c^2\sqrt{cx^2}(a+bx)^{1+n}{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.0110851, size = 47, normalized size = 0.94

$$-\frac{(cx^2)^{5/2}(a+bx)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^6,x]
```

```
[Out] -(((c*x^2)^(5/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (
b*x)/a])/(a*(1 + n)*x^5))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n}}{x^{6}}\left(cx^{2}\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^6,x)
```

```
[Out] int((c*x^2)^(5/2)*(b*x+a)^n/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2}\right)^{\frac{5}{2}}(bx+a)^{n}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^{2}}(bx+a)^{n}c^{2}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2}\right)^{\frac{5}{2}}(a+bx)^{n}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**6,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2)^{\frac{5}{2}} (bx + a)^n}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^6,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^6, x)
```

3.945 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^7} dx$

Optimal. Leaf size=50

$$\frac{bc^2\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x}$$

```
[Out] (b*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a^2*(1 + n)*x)
```

Rubi [A] time = 0.0117204, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$\frac{bc^2\sqrt{cx^2}(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^7,x]
```

```
[Out] (b*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a^2*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cx^2\right)^{5/2}(a+bx)^n}{x^7}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x^2}\,dx}{x}\\ &= \frac{bc^2\sqrt{cx^2}(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.0116575, size = 47, normalized size = 0.94

$$\frac{b\left(cx^2\right)^{5/2}(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^7,x]
```

```
[Out] (b*(c*x^2)^(5/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (
b*x)/a])/(a^2*(1 + n)*x^5)
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^7}\left(cx^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^7,x)
```

```
[Out] int((c*x^2)^(5/2)*(b*x+a)^n/x^7,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}(bx+a)^n}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^7,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n c^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^7,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c^2/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2)^{\frac{5}{2}} (bx + a)^n}{x^7} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^7, x)
```

3.946 $\int \frac{x^4(a+bx)^n}{\sqrt{cx^2}} dx$

Optimal. Leaf size=123

$$-\frac{a^3x(a+bx)^{n+1}}{b^4(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4(n+4)\sqrt{cx^2}}$$

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a + b*x)
^(2 + n))/(b^4*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^4*(3 + n
)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*(4 + n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0371531, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a + b*x)
^(2 + n))/(b^4*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^4*(3 + n
)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*(4 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(a+bx)^n}{\sqrt{cx^2}} dx &= \frac{x\int x^3(a+bx)^n\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{\sqrt{cx^2}}\\
&= -\frac{a^3x(a+bx)^{1+n}}{b^4(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4(4+n)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0366258, size = 96, normalized size = 0.78

$$\frac{x(a+bx)^{n+1}\left(6a^2b(n+1)x-6a^3-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2)*
x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 +
n)*Sqrt[c*x^2])
```

Maple [A] time = 0.003, size = 134, normalized size = 1.1

$$-\frac{(bx+a)^{1+n}x\left(-b^3n^3x^3-6b^3n^2x^3+3ab^2n^2x^2-11b^3nx^3+9ab^2nx^2-6b^3x^3-6a^2bnx+6ab^2x^2-6a^2bx+6a^3\right)}{b^4\left(n^4+10n^3+35n^2+50n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] -(b*x+a)^(1+n)*x*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3+9
*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(1/
2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A] time = 1.02278, size = 140, normalized size = 1.14

$$\frac{\left(\left(n^3+6n^2+11n+6\right)b^4x^4+\left(n^3+3n^2+2n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6a^3bnx-6a^4\right)(bx+a)^n}{\left(n^4+10n^3+35n^2+50n+24\right)b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*sqrt(c))
```

Fricas [A] time = 1.58553, size = 327, normalized size = 2.66

$$\frac{\left(6a^3bnx+\left(b^4n^3+6b^4n^2+11b^4n+6b^4\right)x^4-6a^4+\left(ab^3n^3+3ab^3n^2+2ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\sqrt{cx^2}(bx}{\left(b^4cn^4+10b^4cn^3+35b^4cn^2+50b^4cn+24b^4c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
```

```
t(c*x^2)*(b*x + a)^n/((b^4*c*n^4 + 10*b^4*c*n^3 + 35*b^4*c*n^2 + 50*b^4*c*n
 + 24*b^4*c)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^4}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^4/sqrt(c*x^2), x)
```

3.947 $\int \frac{x^3(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{a^2x(a+bx)^{n+1}}{b^3(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3(n+3)\sqrt{cx^2}}$$

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^(2 +
 n))/(b^3*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*(3 + n)*Sqrt[c*
x^2])
```

Rubi [A] time = 0.0258797, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2x(a+bx)^{n+1}}{b^3(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^(2 +
 n))/(b^3*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*(3 + n)*Sqrt[c*
x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^n}{\sqrt{cx^2}}\,dx &= \frac{x\int x^2(a+bx)^n\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{\sqrt{cx^2}}\\ &= \frac{a^2x(a+bx)^{1+n}}{b^3(1+n)\sqrt{cx^2}}-\frac{2ax(a+bx)^{2+n}}{b^3(2+n)\sqrt{cx^2}}+\frac{x(a+bx)^{3+n}}{b^3(3+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0277289, size = 67, normalized size = 0.74

$$\frac{x(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(
b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Maple [A] time = 0.005, size = 81, normalized size = 0.9

$$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3\,b^2nx^2-2\,abnx+2\,b^2x^2-2\,abx+2\,a^2\right)x}{b^3\left(n^3+6\,n^2+11\,n+6\right)}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
/(c*x^2)^(1/2)/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A] time = 1.03458, size = 112, normalized size = 1.24

$$\frac{\left(\left(n^2+3\,n+2\right)b^3\sqrt{c}x^3+\left(n^2+n\right)ab^2\sqrt{c}x^2-2\,a^2b\sqrt{c}nx+2\,a^3\sqrt{c}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c)
```

Fricas [A] time = 1.62616, size = 220, normalized size = 2.44

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\sqrt{cx^2}(bx+a)^n}{\left(b^3cn^3+6\,b^3cn^2+11\,b^3cn+6\,b^3c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c*n^3 + 6*b^3*c*n^2 + 11*b^3*c*n +
6*b^3*c)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^3}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^3/sqrt(c*x^2), x)
```

3.948 $\int \frac{x^2(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=59

$$\frac{x(a+bx)^{n+2}}{b^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2(n+1)\sqrt{cx^2}}$$

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2 + n)
)/(b^2*(2 + n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0157513, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{x(a+bx)^{n+2}}{b^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2 + n)
)/(b^2*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)^n}{\sqrt{cx^2}}\,dx &= \frac{x\int x(a+bx)^n\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{ax(a+bx)^{1+n}}{b^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2(2+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0206006, size = 43, normalized size = 0.73

$$\frac{x(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^2])
```

Maple [A] time = 0.002, size = 44, normalized size = 0.8

$$-\frac{(bx+a)^{1+n}\,x\,(-bxn-bx+a)}{b^2\left(n^2+3\,n+2\right)}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] -(b*x+a)^(1+n)*x*(-b*n*x-b*x+a)/(c*x^2)^(1/2)/b^2/(n^2+3*n+2)
```

Maxima [A] time = 1.02931, size = 61, normalized size = 1.03

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*sqrt(c))
```

Fricas [A] time = 1.70888, size = 134, normalized size = 2.27

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\sqrt{cx^2}(bx+a)^n}{\left(b^2cn^2+3\,b^2cn+2\,b^2c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c*n^2 + 3
*b^2*c*n + 2*b^2*c)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^2}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^2/sqrt(c*x^2), x)
```

3.949 $\int \frac{x(a+bx)^n}{\sqrt{cx^2}} dx$

Optimal. Leaf size=28

$$\frac{x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

```
[Out] (x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0048816, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x(a+bx)^n}{\sqrt{cx^2}} dx &= \frac{x \int (a+bx)^n dx}{\sqrt{cx^2}} \\ &= \frac{x(a+bx)^{1+n}}{b(1+n)\sqrt{cx^2}} \end{aligned}$$

Mathematica [A] time = 0.0106076, size = 28, normalized size = 1.

$$\frac{x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Maple [A] time = 0., size = 27, normalized size = 1.

$$\frac{x\left(bx+a\right)^{1+n}}{b\left(1+n\right)}\frac{1}{\sqrt{c x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] x*(b*x+a)^(1+n)/b/(1+n)/(c*x^2)^(1/2)
```

Maxima [A] time = 1.01808, size = 42, normalized size = 1.5

$$\frac{\left(b\sqrt{c}x+a\sqrt{c}\right)(bx+a)^{n}}{bc(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c*(n + 1))
```

Fricas [A] time = 1.6154, size = 72, normalized size = 2.57

$$\frac{\sqrt{cx^{2}}(bx+a)(bx+a)^{n}}{(bcn+bc)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c*n + b*c)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(bx+a)^{n}x}{\sqrt{cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x/sqrt(c*x^2), x)
```

3.950 $\int \frac{(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=45

$$-\frac{x(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)\sqrt{cx^2}}$$

```
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
(1 + n)*Sqrt[c*x^2]))
```

Rubi [A] time = 0.0097081, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 65}

$$-\frac{x(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/Sqrt[c*x^2],x]
```

```
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x}\,dx}{\sqrt{cx^2}}\\ &= -\frac{x(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0023348, size = 45, normalized size = 1.

$$-\frac{x(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/Sqrt[c*x^2],x]
```

```
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
(1 + n)*Sqrt[c*x^2]))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int (bx+a)^n \frac{1}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] int((b*x+a)^n/(c*x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/sqrt(c*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n}{cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^n}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**n/sqrt(c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/sqrt(c*x^2), x)
```

3.951 $\int \frac{(a+bx)^n}{x\sqrt{cx^2}} dx$

Optimal. Leaf size=45

$$\frac{bx(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)\sqrt{cx^2}}$$

```
[Out] (b*x*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^
2*(1 + n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0100599, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$\frac{bx(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(x*Sqrt[c*x^2]),x]
```

```
[Out] (b*x*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^
2*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{x\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{bx(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0114595, size = 48, normalized size = 1.07

$$\frac{bcx^3(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(x*Sqrt[c*x^2]),x]
```

```
[Out] (b*c*x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^2*(1 + n)*(c*x^2)^(3/2))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x} \frac{1}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x/(c*x^2)^(1/2),x)
```

```
[Out] int((b*x+a)^n/x/(c*x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{\sqrt{cx^2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/(sqrt(c*x^2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n}{cx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^n}{x\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**n/(x*sqrt(c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{\sqrt{cx^2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/(sqrt(c*x^2)*x), x)
```

3.952 $\int \frac{(a+bx)^n}{x^2\sqrt{cx^2}}\,dx$

Optimal. Leaf size=48

$$-\frac{b^2x(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)\sqrt{cx^2}}$$

```
[Out] -((b^2*x*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^3*(1 + n)*Sqrt[c*x^2]))
```

Rubi [A] time = 0.011783, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$-\frac{b^2x(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] -((b^2*x*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^3*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{x^2\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^3}\,dx}{\sqrt{cx^2}}\\ &= -\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0106825, size = 51, normalized size = 1.06

$$-\frac{b^2cx^3(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] -((b^2*c*x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)
/a])/(a^3*(1 + n)*(c*x^2)^(3/2)))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x^2}\frac{1}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^2/(c*x^2)^(1/2),x)
```

```
[Out] int((b*x+a)^n/x^2/(c*x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{\sqrt{cx^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/(sqrt(c*x^2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n}{cx^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^n}{x^2\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**n/(x**2*sqrt(c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{\sqrt{cx^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/(sqrt(c*x^2)*x^2), x)
```

3.953 $\int \frac{x^6(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c(n+4)\sqrt{cx^2}}$$

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*c*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a + b*
x)^(2 + n))/(b^4*c*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^4*c*
(3 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*c*(4 + n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0436374, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*c*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a + b*
x)^(2 + n))/(b^4*c*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^4*c*
(3 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*c*(4 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int x^3(a+bx)^n\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a^3x(a+bx)^{1+n}}{b^4c(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c(4+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.040864, size = 98, normalized size = 0.73

$$\frac{x^3(a+bx)^{n+1}\left(6a^2b(n+1)x-6a^3-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2
)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*(c*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 136, normalized size = 1.

$$-\frac{(bx+a)^{1+n}\,x^3\left(-b^3n^3x^3-6\,b^3n^2x^3+3\,ab^2n^2x^2-11\,b^3nx^3+9\,ab^2nx^2-6\,b^3x^3-6\,a^2bnx+6\,ab^2x^2-6\,a^2bx+6\,a^3\right)}{b^4\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] -(b*x+a)^(1+n)*x^3*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3
+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(
3/2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A] time = 1.03533, size = 140, normalized size = 1.04

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*c^(3/2))
```

Fricas [A] time = 1.34892, size = 340, normalized size = 2.52

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\sqrt{cx^2}(bx+a}{\left(b^4c^2n^4+10\,b^4c^2n^3+35\,b^4c^2n^2+50\,b^4c^2n+24\,b^4c^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*c^2*n^4 + 10*b^4*c^2*n^3 + 35*b^4*c^2*n^2 + 50*b
^4*c^2*n + 24*b^4*c^2)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^6}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^6/(c*x^2)^(3/2), x)
```

3.954 $\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=99

$$\frac{a^2x(a+bx)^{n+1}}{b^3c(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c(n+3)\sqrt{cx^2}}$$

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*c*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^(2
 + n))/(b^3*c*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*c*(3 + n)*S
qrt[c*x^2])
```

Rubi [A] time = 0.0311947, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2x(a+bx)^{n+1}}{b^3c(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*c*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^(2
 + n))/(b^3*c*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*c*(3 + n)*S
qrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int x^2(a+bx)^n\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{a^2x(a+bx)^{1+n}}{b^3c(1+n)\sqrt{cx^2}}-\frac{2ax(a+bx)^{2+n}}{b^3c(2+n)\sqrt{cx^2}}+\frac{x(a+bx)^{3+n}}{b^3c(3+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.03314, size = 69, normalized size = 0.7

$$\frac{x^3(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))
/(b^3*(1 + n)*(2 + n)*(3 + n)*(c*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 83, normalized size = 0.8

$$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3\,b^2nx^2-2\,abnx+2\,b^2x^2-2\,abx+2\,a^2\right)x^3}{b^3\left(n^3+6\,n^2+11\,n+6\right)}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
^3/(c*x^2)^(3/2)/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A] time = 1.02913, size = 112, normalized size = 1.13

$$\frac{\left(\left(n^2+3\,n+2\right)b^3\sqrt{c}x^3+\left(n^2+n\right)ab^2\sqrt{c}x^2-2\,a^2b\sqrt{c}nx+2\,a^3\sqrt{c}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c^2)
```

Fricas [A] time = 1.37366, size = 231, normalized size = 2.33

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\sqrt{cx^2}(bx+a)^n}{\left(b^3c^2n^3+6\,b^3c^2n^2+11\,b^3c^2n+6\,b^3c^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c^2*n^3 + 6*b^3*c^2*n^2 + 11*b^3*c^
2*n + 6*b^3*c^2)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^5}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^5/(c*x^2)^(3/2), x)
```

3.955 $\int \frac{x^4(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{x(a+bx)^{n+2}}{b^2c(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c(n+1)\sqrt{cx^2}}$$

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*c*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2 +
n))/(b^2*c*(2 + n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.018915, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{x(a+bx)^{n+2}}{b^2c(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*c*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2 +
n))/(b^2*c*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int x(a+bx)^n\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{c\sqrt{cx^2}}\\
&= -\frac{ax(a+bx)^{1+n}}{b^2c(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2c(2+n)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0222306, size = 45, normalized size = 0.69

$$\frac{x^3(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*(c*x^2)^(3/
2))
```

Maple [A] time = 0.001, size = 46, normalized size = 0.7

$$-\frac{(bx+a)^{1+n}x^3(-bxn-bx+a)}{b^2(n^2+3n+2)}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] -(b*x+a)^(1+n)*x^3*(-b*n*x-b*x+a)/(c*x^2)^(3/2)/b^2/(n^2+3*n+2)
```

Maxima [A] time = 1.01701, size = 61, normalized size = 0.94

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n}{\left(n^2+3n+2\right)b^2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*c^(3/2))
```

Fricas [A] time = 1.35539, size = 142, normalized size = 2.18

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\sqrt{cx^2}(bx+a)^n}{\left(b^2c^2n^2+3b^2c^2n+2b^2c^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c^2*n^2 +
 3*b^2*c^2*n + 2*b^2*c^2)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^4}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^4/(c*x^2)^(3/2), x)
```

3.956 $\int \frac{x^3(a+bx)^n}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=31

$$\frac{x(a+bx)^{n+1}}{bc(n+1)\sqrt{cx^2}}$$

```
[Out] (x*(a + b*x)^(1 + n))/(b*c*(1 + n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0067218, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{bc(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*c*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^n}{(cx^2)^{3/2}}\,dx &= \frac{x\int (a+bx)^n\,dx}{c\sqrt{cx^2}}\\ &= \frac{x(a+bx)^{1+n}}{bc(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0141891, size = 30, normalized size = 0.97

$$\frac{x^3(a+bx)^{n+1}}{b(n+1)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n))/(b*(1 + n)*(c*x^2)^(3/2))
```

Maple [A] time = 0.001, size = 29, normalized size = 0.9

$$\frac{(bx+a)^{1+n}x^3}{b(1+n)}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)*x^3/(c*x^2)^(3/2)
```

Maxima [A] time = 1.02099, size = 42, normalized size = 1.35

$$\frac{\left(b\sqrt{c}x+a\sqrt{c}\right)(bx+a)^n}{bc^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c^2*(n + 1))
```

Fricas [A] time = 1.28886, size = 77, normalized size = 2.48

$$\frac{\sqrt{cx^2}(bx+a)(bx+a)^n}{\left(bc^2n+bc^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c^2*n + b*c^2)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^3}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^3/(c*x^2)^(3/2), x)
```

3.957 $\int \frac{x^2(a+bx)^n}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=48

$$-\frac{x(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{ac(n+1)\sqrt{cx^2}}$$

```
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
c*(1 + n)*Sqrt[c*x^2]))
```

Rubi [A] time = 0.0124052, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$-\frac{x(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{ac(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
c*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)^n}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{x(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{ac(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0081135, size = 47, normalized size = 0.98

$$-\frac{x^3(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(
a*(1 + n)*(c*x^2)^(3/2)))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int x^{2} \left(b x + a\right)^{n} \left(c \,x^{2}\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] int(x^2*(b*x+a)^n/(c*x^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(b x + a\right)}^{n} x^{2}}{\left(c x^{2}\right)^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^2/(c*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c x^{2}} {\left(b x + a\right)}^{n}}{c^{2} x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2} \left(a + b x\right)^{n}}{\left(c x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x)**n/(c*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^2}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^2/(c*x^2)^(3/2), x)
```

3.958 $\int \frac{x(a+bx)^n}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{bx(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2c(n+1)\sqrt{cx^2}}$$

```
[Out] (b*x*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^
2*c*(1 + n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0120192, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 65}

$$\frac{bx(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (b*x*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^
2*c*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^n}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{bx(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2c(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0025044, size = 47, normalized size = 0.98

$$\frac{bx^3(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (b*x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^2*(1 + n)*(c*x^2)^(3/2))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int x\left(bx+a\right)^{n}\left(c\,x^{2}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] int(x*(b*x+a)^n/(c*x^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n}x}{\left(cx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x/(c*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^{2}}(bx+a)^{n}}{c^{2}x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+bx\right)^{n}}{\left(cx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x)**n/(c*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x/(c*x^2)^(3/2), x)
```

3.959 $\int \frac{(a+bx)^n}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=51

$$-\frac{b^2x(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3c(n+1)\sqrt{cx^2}}$$

```
[Out] -((b^2*x*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^3*c*(1 + n)*Sqrt[c*x^2]))
```

Rubi [A] time = 0.0136584, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 65}

$$-\frac{b^2x(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(c*x^2)^(3/2),x]
```

```
[Out] -((b^2*x*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^3*c*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{(cx^2)^{3/2}} dx &= \frac{x\int \frac{(a+bx)^n}{x^3} dx}{c\sqrt{cx^2}}\\ &= -\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3c(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0025603, size = 50, normalized size = 0.98

$$-\frac{b^2x^3(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(c*x^2)^(3/2),x]
```

```
[Out] -((b^2*x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a
])/(a^3*(1 + n)*(c*x^2)^(3/2)))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int (bx+a)^n \left(cx^2\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] int((b*x+a)^n/(c*x^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{\left(cx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/(c*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n}{c^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^n}{\left(cx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Integral((a + b*x)**n/(c*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/(c*x^2)^(3/2), x)
```

3.960 $\int \frac{(a+bx)^n}{x\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=50

$$\frac{b^3x(a+bx)^{n+1}\,{}_2F_1\left(4,n+1;n+2;\frac{bx}{a}+1\right)}{a^4c(n+1)\sqrt{cx^2}}$$

```
[Out] (b^3*x*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^4*c*(1 + n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0141199, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$\frac{b^3x(a+bx)^{n+1}\,{}_2F_1\left(4,n+1;n+2;\frac{bx}{a}+1\right)}{a^4c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (b^3*x*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^4*c*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n}{x\left(cx^2\right)^{3/2}}\,dx = \frac{x\int \frac{(a+bx)^n}{x^4}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{b^3x(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4c(1+n)\sqrt{cx^2}}$$

Mathematica [A] time = 0.0134882, size = 50, normalized size = 1.

$$\frac{b^3cx^5(a+bx)^{n+1}\,{}_2F_1\left(4,n+1;n+2;\frac{bx}{a}+1\right)}{a^4(n+1)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (b^3*c*x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a
])/(a^4*(1 + n)*(c*x^2)^(5/2))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{x} \left(cx^2\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x/(c*x^2)^(3/2),x)
```

```
[Out] int((b*x+a)^n/x/(c*x^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{\left(cx^2\right)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/((c*x^2)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n}{c^2x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^2*x^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^n}{x\left(cx^2\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x/(c*x**2)**(3/2),x)
```

```
[Out] Integral((a + b*x)**n/(x*(c*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{(cx^2)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/((c*x^2)^(3/2)*x), x)
```

3.961 $\int \frac{x^8(a+bx)^n}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c^2(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c^2(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c^2(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c^2(n+4)\sqrt{cx^2}}$$

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*c^2*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a +
b*x)^(2 + n))/(b^4*c^2*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^
4*c^2*(3 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*c^2*(4 + n)*Sqrt[c*
x^2])
```

Rubi [A] time = 0.0460681, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c^2(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c^2(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c^2(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c^2(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*c^2*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a +
b*x)^(2 + n))/(b^4*c^2*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^
4*c^2*(3 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*c^2*(4 + n)*Sqrt[c*
x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8(a+bx)^n}{(cx^2)^{5/2}}\,dx &= \frac{x\int x^3(a+bx)^n\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a^3x(a+bx)^{1+n}}{b^4c^2(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c^2(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c^2(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c^2(4+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0331838, size = 99, normalized size = 0.73

$$\frac{x(a+bx)^{n+1}\left(6a^2b(n+1)x-6a^3-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4c^2(n+1)(n+2)(n+3)(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2)*
x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*c^2*(1 + n)*(2 + n)*(3 + n)*(
4 + n)*Sqrt[c*x^2])
```

Maple [A] time = 0.005, size = 136, normalized size = 1.

$$-\frac{(bx+a)^{1+n}x^5\left(-b^3n^3x^3-6\,b^3n^2x^3+3\,ab^2n^2x^2-11\,b^3nx^3+9\,ab^2nx^2-6\,b^3x^3-6\,a^2bnx+6\,ab^2x^2-6\,a^2bx+6\,a^3\right)}{b^4\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] -(b*x+a)^(1+n)*x^5*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3
+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(
5/2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A] time = 1.03958, size = 140, normalized size = 1.04

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*c^(5/2))
```

Fricas [A] time = 1.4507, size = 340, normalized size = 2.52

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\sqrt{cx^2}(bx+a}{\left(b^4c^3n^4+10\,b^4c^3n^3+35\,b^4c^3n^2+50\,b^4c^3n+24\,b^4c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
```

```
t(c*x^2)*(b*x + a)^n/((b^4*c^3*n^4 + 10*b^4*c^3*n^3 + 35*b^4*c^3*n^2 + 50*b
^4*c^3*n + 24*b^4*c^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^8}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^8/(c*x^2)^(5/2), x)
```

3.962 $\int \frac{x^7(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=99

$$\frac{a^2x(a+bx)^{n+1}}{b^3c^2(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c^2(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c^2(n+3)\sqrt{cx^2}}$$

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*c^2*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^
(2 + n))/(b^3*c^2*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*c^2*(3
+ n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0307015, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{a^2x(a+bx)^{n+1}}{b^3c^2(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c^2(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c^2(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*c^2*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^
(2 + n))/(b^3*c^2*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*c^2*(3
+ n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int x^2(a+bx)^n\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{c^2\sqrt{cx^2}}\\
&= \frac{a^2x(a+bx)^{1+n}}{b^3c^2(1+n)\sqrt{cx^2}}-\frac{2ax(a+bx)^{2+n}}{b^3c^2(2+n)\sqrt{cx^2}}+\frac{x(a+bx)^{3+n}}{b^3c^2(3+n)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0300613, size = 70, normalized size = 0.71

$$\frac{x(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3c^2(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(
b^3*c^2*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Maple [A] time = 0.004, size = 83, normalized size = 0.8

$$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3\,b^2nx^2-2\,abnx+2\,b^2x^2-2\,abx+2\,a^2\right)x^5}{b^3\left(n^3+6\,n^2+11\,n+6\right)}\left(cx^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
^5/(c*x^2)^(5/2)/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A] time = 1.05935, size = 112, normalized size = 1.13

$$\frac{\left(\left(n^2+3\,n+2\right)b^3\sqrt{c}x^3+\left(n^2+n\right)ab^2\sqrt{c}x^2-2\,a^2b\sqrt{c}nx+2\,a^3\sqrt{c}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c^3)
```

Fricas [A] time = 1.35503, size = 231, normalized size = 2.33

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\sqrt{cx^2}(bx+a)^n}{\left(b^3c^3n^3+6\,b^3c^3n^2+11\,b^3c^3n+6\,b^3c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c^3*n^3 + 6*b^3*c^3*n^2 + 11*b^3*c^
3*n + 6*b^3*c^3)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^7}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^7/(c*x^2)^(5/2), x)
```

3.963 $\int \frac{x^6(a+bx)^n}{\left(cx^2\right)^{5/2}} dx$

Optimal. Leaf size=65

$$\frac{x(a+bx)^{n+2}}{b^2c^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c^2(n+1)\sqrt{cx^2}}$$

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*c^2*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2
+ n))/(b^2*c^2*(2 + n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0202965, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 43}

$$\frac{x(a+bx)^{n+2}}{b^2c^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*c^2*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2
+ n))/(b^2*c^2*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int x(a+bx)^n\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{ax(a+bx)^{1+n}}{b^2c^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2c^2(2+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0200956, size = 46, normalized size = 0.71

$$\frac{x(a+bx)^{n+1}(b(n+1)x-a)}{b^2c^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*c^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

Maple [A] time = 0.002, size = 46, normalized size = 0.7

$$-\frac{(bx+a)^{1+n}\,x^5\,(-bnx-bx+a)}{b^2\left(n^2+3\,n+2\right)}\left(cx^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] -(b*x+a)^(1+n)*x^5*(-b*n*x-b*x+a)/(c*x^2)^(5/2)/b^2/(n^2+3*n+2)
```

Maxima [A] time = 1.05973, size = 61, normalized size = 0.94

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*c^(5/2))
```

Fricas [A] time = 1.46468, size = 142, normalized size = 2.18

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\sqrt{cx^2}(bx+a)^n}{\left(b^2c^3n^2+3\,b^2c^3n+2\,b^2c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c^3*n^2 +
 3*b^2*c^3*n + 2*b^2*c^3)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^6}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^6/(c*x^2)^(5/2), x)
```

3.964 $\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=31

$$\frac{x(a+bx)^{n+1}}{bc^2(n+1)\sqrt{cx^2}}$$

```
[Out] (x*(a + b*x)^(1 + n))/(b*c^2*(1 + n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0066318, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{bc^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*c^2*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int (a+bx)^n\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x(a+bx)^{1+n}}{bc^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0115729, size = 31, normalized size = 1.

$$\frac{x(a+bx)^{n+1}}{bc^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*c^2*(1 + n)*Sqrt[c*x^2])
```

Maple [A] time = 0.002, size = 29, normalized size = 0.9

$$\frac{(bx+a)^{1+n}x^5}{b(1+n)}\left(cx^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)*x^5/(c*x^2)^(5/2)
```

Maxima [A] time = 1.03933, size = 42, normalized size = 1.35

$$\frac{\left(b\sqrt{c}x+a\sqrt{c}\right)(bx+a)^n}{bc^3(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c^3*(n + 1))
```

Fricas [A] time = 1.25077, size = 77, normalized size = 2.48

$$\frac{\sqrt{cx^2}(bx+a)(bx+a)^n}{\left(bc^3n+bc^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c^3*n + b*c^3)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^5}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^5/(c*x^2)^(5/2), x)
```

3.965 $\int \frac{x^4(a+bx)^n}{(cx^2)^{5/2}} \, dx$

Optimal. Leaf size=48

$$-\frac{x(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{ac^2(n+1)\sqrt{cx^2}}$$

```
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
c^2*(1 + n)*Sqrt[c*x^2]))
```

Rubi [A] time = 0.0125406, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$-\frac{x(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{ac^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
c^2*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4(a+bx)^n}{(cx^2)^{5/2}} \, dx &= \frac{x \int \frac{(a+bx)^n}{x} \, dx}{c^2\sqrt{cx^2}}\\ &= -\frac{x(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{ac^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0084175, size = 47, normalized size = 0.98

$$-\frac{x^5(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(
a*(1 + n)*(c*x^2)^(5/2)))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int x^{4} (bx + a)^{n} \left(c x^{2}\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] int(x^4*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^{n} x^{4}}{\left(c x^{2}\right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^4/(c*x^2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c x^{2}} (bx + a)^{n}}{c^{3} x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4} (a + bx)^{n}}{\left(c x^{2}\right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^4}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^4/(c*x^2)^(5/2), x)
```

3.966 $\int \frac{x^3(a+bx)^n}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=48

$$\frac{bx(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2c^2(n+1)\sqrt{cx^2}}$$

```
[Out] (b*x*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^
2*c^2*(1 + n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0126808, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$\frac{bx(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (b*x*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^
2*c^2*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{x^3(a+bx)^n}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^n}{x^2}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{bx(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2c^2(1+n)\sqrt{cx^2}}$$

Mathematica [A] time = 0.0088574, size = 47, normalized size = 0.98

$$\frac{bx^5(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (b*x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^2*(1 + n)*(c*x^2)^(5/2))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int x^{3} (bx + a)^{n} \left(c x^{2}\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] int(x^3*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^{n} x^{3}}{\left(c x^{2}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^3/(c*x^2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c x^{2}}(bx + a)^{n}}{c^{3} x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^3*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3} (a + bx)^{n}}{\left(c x^{2}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^3}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^3/(c*x^2)^(5/2), x)
```

3.967 $\int \frac{x^2(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{b^2x(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3c^2(n+1)\sqrt{cx^2}}$$

```
[Out] -((b^2*x*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^3*c^2*(1 + n)*Sqrt[c*x^2]))
```

Rubi [A] time = 0.0146581, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 65}

$$-\frac{b^2x(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((b^2*x*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^3*c^2*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^3}\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3c^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0100845, size = 50, normalized size = 0.98

$$-\frac{b^2x^5(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((b^2*x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a
])/(a^3*(1 + n)*(c*x^2)^(5/2)))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x^{2} (bx+a)^{n} \left(cx^{2}\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] int(x^2*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n} x^{2}}{\left(cx^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^2/(c*x^2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^{2}}(bx+a)^{n}}{c^{3}x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^3*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2} (a+bx)^{n}}{\left(cx^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x^2}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^2/(c*x^2)^(5/2), x)
```

3.968 $\int \frac{x(a+bx)^n}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=50

$$\frac{b^3x(a+bx)^{n+1}\,{}_2F_1\left(4,n+1;n+2;\frac{bx}{a}+1\right)}{a^4c^2(n+1)\sqrt{cx^2}}$$

```
[Out] (b^3*x*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^4*c^2*(1 + n)*Sqrt[c*x^2])
```

Rubi [A] time = 0.013246, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 65}

$$\frac{b^3x(a+bx)^{n+1}\,{}_2F_1\left(4,n+1;n+2;\frac{bx}{a}+1\right)}{a^4c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (b^3*x*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^4*c^2*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^n}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^4}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{b^3x(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4c^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0028012, size = 49, normalized size = 0.98

$$\frac{b^3x^5(a+bx)^{n+1}\,{}_2F_1\left(4,n+1;n+2;\frac{bx}{a}+1\right)}{a^4(n+1)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (b^3*x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^4*(1 + n)*(c*x^2)^(5/2))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int x (bx+a)^n \left(c x^2\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] int(x*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x}{\left(c x^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x/(c*x^2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c x^2}(bx+a)^n}{c^3 x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^3*x^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x (a+bx)^n}{\left(c x^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n x}{(cx^2)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x/(c*x^2)^(5/2), x)
```

3.969 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{ac^2\sqrt{cx^2}(dx)^{m+6}}{d^6(m+6)x}+\frac{bc^2\sqrt{cx^2}(dx)^{m+7}}{d^7(m+7)x}$$

```
[Out] (a*c^2*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 + m)*x) + (b*c^2*(d*x)^(7 + m)*Sq
rt[c*x^2])/(d^7*(7 + m)*x)
```

Rubi [A] time = 0.0304476, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {15, 16, 43}

$$\frac{ac^2\sqrt{cx^2}(dx)^{m+6}}{d^6(m+6)x}+\frac{bc^2\sqrt{cx^2}(dx)^{m+7}}{d^7(m+7)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 + m)*x) + (b*c^2*(d*x)^(7 + m)*Sq
rt[c*x^2])/(d^7*(7 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(dx)^m(a+bx)\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int (dx)^{5+m}(a+bx)\,dx}{d^5x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a(dx)^{5+m}+\frac{b(dx)^{6+m}}{d}\right)dx}{d^5x}\\
&= \frac{ac^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}+\frac{bc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x}
\end{aligned}$$

Mathematica [A] time = 0.0273481, size = 38, normalized size = 0.58

$$\frac{x\left(cx^2\right)^{5/2}(dx)^m(a(m+7)+b(m+6)x)}{(m+6)(m+7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^(5/2)*(a*(7 + m) + b*(6 + m)*x))/((6 + m)*(7 + m))
```

Maple [A] time = 0.004, size = 40, normalized size = 0.6

$$\frac{(bmx+am+6\,bx+7\,a)\,x\,(dx)^m}{(7+m)\,(6+m)}\left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x)
```

```
[Out] x*(b*m*x+a*m+6*b*x+7*a)*(d*x)^m*(c*x^2)^(5/2)/(7+m)/(6+m)
```

Maxima [A] time = 1.06589, size = 53, normalized size = 0.82

$$\frac{bc^{\frac{5}{2}}d^m x^7 x^m}{m+7}+\frac{ac^{\frac{5}{2}}d^m x^6 x^m}{m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(5/2)*d^m*x^7*x^m/(m + 7) + a*c^(5/2)*d^m*x^6*x^m/(m + 6)
```

Fricas [A] time = 1.3552, size = 123, normalized size = 1.89

$$\frac{\left(\left(bc^2m+6\,bc^2\right)x^6+\left(ac^2m+7\,ac^2\right)x^5\right)\sqrt{cx^2}\,(dx)^m}{m^2+13\,m+42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*c^2*m + 6*b*c^2)*x^6 + (a*c^2*m + 7*a*c^2)*x^5)*sqrt(c*x^2)*(d*x)^m/(m^
2 + 13*m + 42)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.970 $\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx$

Optimal. Leaf size=61

$$\frac{ac\sqrt{cx^2}(dx)^{m+4}}{d^4(m+4)x}+\frac{bc\sqrt{cx^2}(dx)^{m+5}}{d^5(m+5)x}$$

```
[Out] (a*c*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x) + (b*c*(d*x)^(5 + m)*Sqrt[c
*x^2])/(d^5*(5 + m)*x)
```

Rubi [A] time = 0.0294383, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {15, 16, 43}

$$\frac{ac\sqrt{cx^2}(dx)^{m+4}}{d^4(m+4)x}+\frac{bc\sqrt{cx^2}(dx)^{m+5}}{d^5(m+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x) + (b*c*(d*x)^(5 + m)*Sqrt[c
*x^2])/(d^5*(5 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(dx)^m(a+bx)\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int (dx)^{3+m}(a+bx)\,dx}{d^3x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a(dx)^{3+m}+\frac{b(dx)^{4+m}}{d}\right)dx}{d^3x}\\ &= \frac{ac(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}+\frac{bc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x}\end{aligned}$$

Mathematica [A] time = 0.0304621, size = 38, normalized size = 0.62

$$\frac{x\left(cx^2\right)^{3/2}(dx)^m(a(m+5)+b(m+4)x)}{(m+4)(m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^(3/2)*(a*(5 + m) + b*(4 + m)*x))/((4 + m)*(5 + m))
```

Maple [A] time = 0.002, size = 40, normalized size = 0.7

$$\frac{(bmx+am+4bx+5a)x(dx)^m}{(5+m)(4+m)}\left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] x*(b*m*x+a*m+4*b*x+5*a)*(d*x)^m*(c*x^2)^(3/2)/(5+m)/(4+m)
```

Maxima [A] time = 1.05781, size = 53, normalized size = 0.87

$$\frac{bc^{\frac{3}{2}}d^mx^5x^m}{m+5}+\frac{ac^{\frac{3}{2}}d^mx^4x^m}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(3/2)*d^m*x^5*x^m/(m + 5) + a*c^(3/2)*d^m*x^4*x^m/(m + 4)
```

Fricas [A] time = 1.37498, size = 111, normalized size = 1.82

$$\frac{\left((bcm+4bc)x^4+(acm+5ac)x^3\right)\sqrt{cx^2}(dx)^m}{m^2+9m+20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*c*m + 4*b*c)*x^4 + (a*c*m + 5*a*c)*x^3)*sqrt(c*x^2)*(d*x)^m/(m^2 + 9*m
+ 20)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.971 $\int (dx)^m \sqrt{cx^2}(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{a\sqrt{cx^2}(dx)^{m+2}}{d^2(m+2)x}+\frac{b\sqrt{cx^2}(dx)^{m+3}}{d^3(m+3)x}$$

```
[Out] (a*(d*x)^(2 + m)*Sqrt[c*x^2])/(d^2*(2 + m)*x) + (b*(d*x)^(3 + m)*Sqrt[c*x^2
])/(d^3*(3 + m)*x)
```

Rubi [A] time = 0.0267448, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {15, 16, 43}

$$\frac{a\sqrt{cx^2}(dx)^{m+2}}{d^2(m+2)x}+\frac{b\sqrt{cx^2}(dx)^{m+3}}{d^3(m+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*(d*x)^(2 + m)*Sqrt[c*x^2])/(d^2*(2 + m)*x) + (b*(d*x)^(3 + m)*Sqrt[c*x^2
])/(d^3*(3 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m\sqrt{cx^2}(a+bx)\,dx &= \frac{\sqrt{cx^2}\int x(dx)^m(a+bx)\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int (dx)^{1+m}(a+bx)\,dx}{dx}\\
&= \frac{\sqrt{cx^2}\int \left(a(dx)^{1+m}+\frac{b(dx)^{2+m}}{d}\right)dx}{dx}\\
&= \frac{a(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x}+\frac{b(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x}
\end{aligned}$$

Mathematica [A] time = 0.0203458, size = 38, normalized size = 0.64

$$\frac{x\sqrt{cx^2}(dx)^m(a(m+3)+b(m+2)x)}{(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (x*(d*x)^m*Sqrt[c*x^2]*(a*(3 + m) + b*(2 + m)*x))/((2 + m)*(3 + m))
```

Maple [A] time = 0.002, size = 40, normalized size = 0.7

$$\frac{(bmx+am+2\,bx+3\,a)\,x\,(dx)^m}{(3+m)\,(2+m)}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x)
```

```
[Out] x*(b*m*x+a*m+2*b*x+3*a)*(d*x)^m*(c*x^2)^(1/2)/(3+m)/(2+m)
```

Maxima [A] time = 1.04417, size = 53, normalized size = 0.9

$$\frac{b\sqrt{c}d^m x^3 x^m}{m+3}+\frac{a\sqrt{c}d^m x^2 x^m}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*sqrt(c)*d^m*x^3*x^m/(m + 3) + a*sqrt(c)*d^m*x^2*x^m/(m + 2)
```

Fricas [A] time = 1.29818, size = 96, normalized size = 1.63

$$\frac{\left((bm+2\,b)x^2+(am+3\,a)x\right)\sqrt{cx^2}\,(dx)^m}{m^2+5\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*m + 2*b)*x^2 + (a*m + 3*a)*x)*sqrt(c*x^2)*(d*x)^m/(m^2 + 5*m + 6)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.972 $\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{ax(dx)^m}{m\sqrt{cx^2}}+\frac{bx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}$$

```
[Out] (a*x*(d*x)^m)/(m*Sqrt[c*x^2]) + (b*x*(d*x)^(1 + m))/(d*(1 + m)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0193592, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {15, 16, 43}

$$\frac{ax(dx)^m}{m\sqrt{cx^2}}+\frac{bx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x*(d*x)^m)/(m*Sqrt[c*x^2]) + (b*x*(d*x)^(1 + m))/(d*(1 + m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)}{x}\,dx}{\sqrt{cx^2}}\\ &= \frac{(dx)\int (dx)^{-1+m}(a+bx)\,dx}{\sqrt{cx^2}}\\ &= \frac{(dx)\int \left(a(dx)^{-1+m}+\frac{b(dx)^m}{d}\right)dx}{\sqrt{cx^2}}\\ &= \frac{ax(dx)^m}{m\sqrt{cx^2}}+\frac{bx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0156799, size = 33, normalized size = 0.69

$$\frac{x(dx)^m(am+a+bmx)}{m(m+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (x*(d*x)^m*(a + a*m + b*m*x))/(m*(1 + m)*Sqrt[c*x^2])
```

Maple [A] time = 0.002, size = 32, normalized size = 0.7

$$\frac{(bmx+am+a)\,x\,(dx)^m}{(1+m)\,m}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] x*(b*m*x+a*m+a)*(d*x)^m/(1+m)/m/(c*x^2)^(1/2)
```

Maxima [A] time = 1.05922, size = 43, normalized size = 0.9

$$\frac{bd^mxx^m}{\sqrt{c}(m+1)}+\frac{ad^mx^m}{\sqrt{c}m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*d^m*x*x^m/(sqrt(c)*(m + 1)) + a*d^m*x^m/(sqrt(c)*m)
```

Fricas [A] time = 1.35777, size = 77, normalized size = 1.6

$$\frac{(bmx+am+a)\sqrt{cx^2}\,(dx)^m}{\left(cm^2+cm\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b*m*x + a*m + a)*sqrt(c*x^2)*(d*x)^m/((c*m^2 + c*m)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)(dx)^m}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(d*x)^m/sqrt(c*x^2), x)
```

3.973 $\int \frac{(dx)^m(a+bx)}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{ad^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{bdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}$$

```
[Out] -((a*d^2*x*(d*x)^(-2 + m))/(c*(2 - m)*Sqrt[c*x^2])) - (b*d*x*(d*x)^(-1 + m)
)/(c*(1 - m)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0358064, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {15, 16, 43}

$$-\frac{ad^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{bdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] -((a*d^2*x*(d*x)^(-2 + m))/(c*(2 - m)*Sqrt[c*x^2])) - (b*d*x*(d*x)^(-1 + m)
)/(c*(1 - m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)}{x^3}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{(d^3x)\int (dx)^{-3+m}(a+bx)\,dx}{c\sqrt{cx^2}}$$
$$= \frac{(d^3x)\int \left(a(dx)^{-3+m} + \frac{b(dx)^{-2+m}}{d}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{ad^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}} - \frac{bdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}}$$

Mathematica [A] time = 0.0261452, size = 38, normalized size = 0.58

$$\frac{x(dx)^m(a(m-1)+b(m-2)x)}{(m-2)(m-1)(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a*(-1 + m) + b*(-2 + m)*x))/((-2 + m)*(-1 + m)*(c*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 40, normalized size = 0.6

$$\frac{(bmx+am-2\,bx-a)\,x\,(dx)^m}{(-1+m)\,(-2+m)}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] x*(b*m*x+a*m-2*b*x-a)*(d*x)^m/(-1+m)/(-2+m)/(c*x^2)^(3/2)
```

Maxima [A] time = 1.06085, size = 53, normalized size = 0.82

$$\frac{bd^mx^m}{c^{\frac{3}{2}}(m-1)x} + \frac{ad^mx^m}{c^{\frac{3}{2}}(m-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*d^m*x^m/(c^(3/2)*(m - 1)*x) + a*d^m*x^m/(c^(3/2)*(m - 2)*x^2)
```

Fricas [A] time = 1.3613, size = 109, normalized size = 1.68

$$\frac{\sqrt{cx^2}(am+(bm-2\,b)x-a)\,(dx)^m}{\left(c^2m^2-3\,c^2m+2\,c^2\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*m + (b*m - 2*b)*x - a)*(d*x)^m/((c^2*m^2 - 3*c^2*m + 2*c^2)*
x^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)\,(dx)^m}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(d*x)^m/(c*x^2)^(3/2), x)
```

3.974 $\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{ad^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{bd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}$$

```
[Out] -((a*d^4*x*(d*x)^(-4 + m))/(c^2*(4 - m)*Sqrt[c*x^2])) - (b*d^3*x*(d*x)^(-3
+ m))/(c^2*(3 - m)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0365748, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {15, 16, 43}

$$-\frac{ad^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{bd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -((a*d^4*x*(d*x)^(-4 + m))/(c^2*(4 - m)*Sqrt[c*x^2])) - (b*d^3*x*(d*x)^(-3
+ m))/(c^2*(3 - m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)}{x^5}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{(d^5x)\int (dx)^{-5+m}(a+bx)\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{(d^5x)\int \left(a(dx)^{-5+m}+\frac{b(dx)^{-4+m}}{d}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{ad^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}} - \frac{bd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}$$

Mathematica [A] time = 0.0547224, size = 38, normalized size = 0.57

$$\frac{x(dx)^m(a(m-3)+b(m-4)x)}{(m-4)(m-3)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a*(-3 + m) + b*(-4 + m)*x))/((-4 + m)*(-3 + m)*(c*x^2)^(5/2))
```

Maple [A] time = 0.002, size = 40, normalized size = 0.6

$$\frac{(bmx+am-4\,bx-3\,a)\,x\,(dx)^m}{(-3+m)\,(-4+m)}\left(cx^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] x*(b*m*x+a*m-4*b*x-3*a)*(d*x)^m/(-3+m)/(-4+m)/(c*x^2)^(5/2)
```

Maxima [A] time = 1.04902, size = 53, normalized size = 0.79

$$\frac{bd^mx^m}{c^{\frac{5}{2}}(m-3)x^3} + \frac{ad^mx^m}{c^{\frac{5}{2}}(m-4)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] b*d^m*x^m/(c^(5/2)*(m - 3)*x^3) + a*d^m*x^m/(c^(5/2)*(m - 4)*x^4)
```

Fricas [A] time = 1.32064, size = 113, normalized size = 1.69

$$\frac{\sqrt{cx^2}(am+(bm-4\,b)x-3\,a)\,(dx)^m}{\left(c^3m^2-7\,c^3m+12\,c^3\right)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*m + (b*m - 4*b)*x - 3*a)*(d*x)^m/((c^3*m^2 - 7*c^3*m + 12*c^
3)*x^5)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)\,(dx)^m}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(d*x)^m/(c*x^2)^(5/2), x)
```

3.975 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^2 \, dx$

Optimal. Leaf size=103

$$\frac{a^2c^2\sqrt{cx^2}(dx)^{m+6}}{d^6(m+6)x}+\frac{2abc^2\sqrt{cx^2}(dx)^{m+7}}{d^7(m+7)x}+\frac{b^2c^2\sqrt{cx^2}(dx)^{m+8}}{d^8(m+8)x}$$

```
[Out] (a^2*c^2*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 + m)*x) + (2*a*b*c^2*(d*x)^(7 +
 m)*Sqrt[c*x^2])/(d^7*(7 + m)*x) + (b^2*c^2*(d*x)^(8 + m)*Sqrt[c*x^2])/(d^8
*(8 + m)*x)
```

Rubi [A] time = 0.0489113, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$\frac{a^2c^2\sqrt{cx^2}(dx)^{m+6}}{d^6(m+6)x}+\frac{2abc^2\sqrt{cx^2}(dx)^{m+7}}{d^7(m+7)x}+\frac{b^2c^2\sqrt{cx^2}(dx)^{m+8}}{d^8(m+8)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 + m)*x) + (2*a*b*c^2*(d*x)^(7 +
 m)*Sqrt[c*x^2])/(d^7*(7 + m)*x) + (b^2*c^2*(d*x)^(8 + m)*Sqrt[c*x^2])/(d^8
*(8 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^2 \, dx = \frac{\left(c^2\sqrt{cx^2}\right) \int x^5 (dx)^m (a+bx)^2 \, dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right) \int (dx)^{5+m} (a+bx)^2 \, dx}{d^5 x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right) \int \left(a^2 (dx)^{5+m} + \frac{2ab(dx)^{6+m}}{d} + \frac{b^2 (dx)^{7+m}}{d^2}\right) dx}{d^5 x}$$
$$= \frac{a^2c^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x} + \frac{2abc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x} + \frac{b^2c^2(dx)^{8+m}\sqrt{cx^2}}{d^8(8+m)x}$$

Mathematica [A] time = 0.0703456, size = 48, normalized size = 0.47

$$x \left(cx^2\right)^{5/2} (dx)^m \left(\frac{a^2}{m+6} + \frac{2abx}{m+7} + \frac{b^2x^2}{m+8}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] x*(d*x)^m*(c*x^2)^(5/2)*(a^2/(6 + m) + (2*a*b*x)/(7 + m) + (b^2*x^2)/(8 + m
))
```

Maple [A] time = 0.005, size = 95, normalized size = 0.9

$$\frac{\left(b^2m^2x^2 + 2\,abm^2x + 13\,b^2mx^2 + a^2m^2 + 28\,abmx + 42\,b^2x^2 + 15\,a^2m + 96\,abx + 56\,a^2\right) x\,(dx)^m}{(8+m)\,(7+m)\,(6+m)} \left(cx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+13*b^2*m*x^2+a^2*m^2+28*a*b*m*x+42*b^2*x^2+15*a^
2*m+96*a*b*x+56*a^2)*(d*x)^m*(c*x^2)^(5/2)/(8+m)/(7+m)/(6+m)
```

Maxima [A] time = 1.07237, size = 86, normalized size = 0.83

$$\frac{b^2c^{\frac{5}{2}}d^mx^8x^m}{m+8} + \frac{2\,abc^{\frac{5}{2}}d^mx^7x^m}{m+7} + \frac{a^2c^{\frac{5}{2}}d^mx^6x^m}{m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^(5/2)*d^m*x^8*x^m/(m + 8) + 2*a*b*c^(5/2)*d^m*x^7*x^m/(m + 7) + a^2*c
^(5/2)*d^m*x^6*x^m/(m + 6)
```

Fricas [A] time = 1.37286, size = 265, normalized size = 2.57

$$\frac{\left(\left(b^2c^2m^2 + 13\, b^2c^2m + 42\, b^2c^2\right)x^7 + 2\left(abc^2m^2 + 14\, abc^2m + 48\, abc^2\right)x^6 + \left(a^2c^2m^2 + 15\, a^2c^2m + 56\, a^2c^2\right)x^5\right)\sqrt{cx^2}\,(dx}{m^3 + 21\, m^2 + 146\, m + 336}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*c^2*m^2 + 13*b^2*c^2*m + 42*b^2*c^2)*x^7 + 2*(a*b*c^2*m^2 + 14*a*b*c^
2*m + 48*a*b*c^2)*x^6 + (a^2*c^2*m^2 + 15*a^2*c^2*m + 56*a^2*c^2)*x^5)*sqrt
(c*x^2)*(d*x)^m/(m^3 + 21*m^2 + 146*m + 336)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.976 $\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=97

$$\frac{a^2c\sqrt{cx^2}(dx)^{m+4}}{d^4(m+4)x}+\frac{2abc\sqrt{cx^2}(dx)^{m+5}}{d^5(m+5)x}+\frac{b^2c\sqrt{cx^2}(dx)^{m+6}}{d^6(m+6)x}$$

```
[Out] (a^2*c*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x) + (2*a*b*c*(d*x)^(5 + m)*
Sqrt[c*x^2])/(d^5*(5 + m)*x) + (b^2*c*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 +
m)*x)
```

Rubi [A] time = 0.0446737, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$\frac{a^2c\sqrt{cx^2}(dx)^{m+4}}{d^4(m+4)x}+\frac{2abc\sqrt{cx^2}(dx)^{m+5}}{d^5(m+5)x}+\frac{b^2c\sqrt{cx^2}(dx)^{m+6}}{d^6(m+6)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x) + (2*a*b*c*(d*x)^(5 + m)*
Sqrt[c*x^2])/(d^5*(5 + m)*x) + (b^2*c*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 +
m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x^3 (dx)^m (a+bx)^2\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int (dx)^{3+m} (a+bx)^2\,dx}{d^3 x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2 (dx)^{3+m} + \frac{2ab(dx)^{4+m}}{d} + \frac{b^2 (dx)^{5+m}}{d^2}\right)dx}{d^3 x}$$
$$= \frac{a^2 c (dx)^{4+m}\sqrt{cx^2}}{d^4 (4+m)x} + \frac{2abc(dx)^{5+m}\sqrt{cx^2}}{d^5 (5+m)x} + \frac{b^2 c (dx)^{6+m}\sqrt{cx^2}}{d^6 (6+m)x}$$

Mathematica [A] time = 0.0562466, size = 48, normalized size = 0.49

$$x\left(cx^2\right)^{3/2} (dx)^m \left(\frac{a^2}{m+4} + \frac{2abx}{m+5} + \frac{b^2x^2}{m+6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] x*(d*x)^m*(c*x^2)^(3/2)*(a^2/(4 + m) + (2*a*b*x)/(5 + m) + (b^2*x^2)/(6 + m
))
```

Maple [A] time = 0.004, size = 95, normalized size = 1.

$$\frac{\left(b^2m^2x^2 + 2\,abm^2x + 9\,b^2mx^2 + a^2m^2 + 20\,abmx + 20\,b^2x^2 + 11\,a^2m + 48\,abx + 30\,a^2\right)x\,(dx)^m}{(6+m)\,(5+m)\,(4+m)} \left(cx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+9*b^2*m*x^2+a^2*m^2+20*a*b*m*x+20*b^2*x^2+11*a^2
*m+48*a*b*x+30*a^2)*(d*x)^m*(c*x^2)^(3/2)/(6+m)/(5+m)/(4+m)
```

Maxima [A] time = 1.06926, size = 86, normalized size = 0.89

$$\frac{b^2c^{\frac{3}{2}}d^m x^6 x^m}{m+6} + \frac{2\,abc^{\frac{3}{2}}d^m x^5 x^m}{m+5} + \frac{a^2c^{\frac{3}{2}}d^m x^4 x^m}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^(3/2)*d^m*x^6*x^m/(m + 6) + 2*a*b*c^(3/2)*d^m*x^5*x^m/(m + 5) + a^2*c
^(3/2)*d^m*x^4*x^m/(m + 4)
```

Fricas [A] time = 1.23126, size = 238, normalized size = 2.45

$$\frac{\left(\left(b^2cm^2+9\,b^2cm+20\,b^2c\right)x^5+2\left(abcm^2+10\,abcm+24\,abc\right)x^4+\left(a^2cm^2+11\,a^2cm+30\,a^2c\right)x^3\right)\sqrt{cx^2}\,(dx)^m}{m^3+15\,m^2+74\,m+120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*c*m^2 + 9*b^2*c*m + 20*b^2*c)*x^5 + 2*(a*b*c*m^2 + 10*a*b*c*m + 24*a*
b*c)*x^4 + (a^2*c*m^2 + 11*a^2*c*m + 30*a^2*c)*x^3)*sqrt(c*x^2)*(d*x)^m/(m^
3 + 15*m^2 + 74*m + 120)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.977 $\int (dx)^m \sqrt{cx^2}(a+bx)^2\,dx$

Optimal. Leaf size=94

$$\frac{a^2\sqrt{cx^2}(dx)^{m+2}}{d^2(m+2)x}+\frac{2ab\sqrt{cx^2}(dx)^{m+3}}{d^3(m+3)x}+\frac{b^2\sqrt{cx^2}(dx)^{m+4}}{d^4(m+4)x}$$

```
[Out] (a^2*(d*x)^(2 + m)*Sqrt[c*x^2])/(d^2*(2 + m)*x) + (2*a*b*(d*x)^(3 + m)*Sqrt
[c*x^2])/(d^3*(3 + m)*x) + (b^2*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x)
```

Rubi [A] time = 0.0410875, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$\frac{a^2\sqrt{cx^2}(dx)^{m+2}}{d^2(m+2)x}+\frac{2ab\sqrt{cx^2}(dx)^{m+3}}{d^3(m+3)x}+\frac{b^2\sqrt{cx^2}(dx)^{m+4}}{d^4(m+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*(d*x)^(2 + m)*Sqrt[c*x^2])/(d^2*(2 + m)*x) + (2*a*b*(d*x)^(3 + m)*Sqrt
[c*x^2])/(d^3*(3 + m)*x) + (b^2*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m\sqrt{cx^2}(a+bx)^2\,dx&=\frac{\sqrt{cx^2}\int x(dx)^m(a+bx)^2\,dx}{x}\\&=\frac{\sqrt{cx^2}\int (dx)^{1+m}(a+bx)^2\,dx}{dx}\\&=\frac{\sqrt{cx^2}\int\left(a^2(dx)^{1+m}+\frac{2ab(dx)^{2+m}}{d}+\frac{b^2(dx)^{3+m}}{d^2}\right)dx}{dx}\\&=\frac{a^2(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x}+\frac{2ab(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x}+\frac{b^2(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}\end{aligned}$$

Mathematica [A] time = 0.049885, size = 72, normalized size = 0.77

$$\frac{x\sqrt{cx^2}(dx)^m\left(a^2\left(m^2+7m+12\right)+2ab\left(m^2+6m+8\right)x+b^2\left(m^2+5m+6\right)x^2\right)}{(m+2)(m+3)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x*(d*x)^m*Sqrt[c*x^2]*(a^2*(12 + 7*m + m^2) + 2*a*b*(8 + 6*m + m^2)*x + b^
2*(6 + 5*m + m^2)*x^2))/((2 + m)*(3 + m)*(4 + m))
```

Maple [A] time = 0.005, size = 95, normalized size = 1.

$$\frac{\left(b^2m^2x^2+2\,abm^2x+5\,b^2mx^2+a^2m^2+12\,abmx+6\,b^2x^2+7\,a^2m+16\,abx+12\,a^2\right)x\,(dx)^m}{(4+m)\,(3+m)\,(2+m)}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+5*b^2*m*x^2+a^2*m^2+12*a*b*m*x+6*b^2*x^2+7*a^2*m
+16*a*b*x+12*a^2)*(d*x)^m*(c*x^2)^(1/2)/(4+m)/(3+m)/(2+m)
```

Maxima [A] time = 1.08932, size = 86, normalized size = 0.91

$$\frac{b^2\sqrt{c}d^mx^4x^m}{m+4}+\frac{2\,ab\sqrt{c}d^mx^3x^m}{m+3}+\frac{a^2\sqrt{c}d^mx^2x^m}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*sqrt(c)*d^m*x^4*x^m/(m + 4) + 2*a*b*sqrt(c)*d^m*x^3*x^m/(m + 3) + a^2*s
qrt(c)*d^m*x^2*x^m/(m + 2)
```

Fricas [A] time = 1.35785, size = 203, normalized size = 2.16

$$\frac{\left(\left(b^2m^2+5\,b^2m+6\,b^2\right)x^3+2\left(abm^2+6\,abm+8\,ab\right)x^2+\left(a^2m^2+7\,a^2m+12\,a^2\right)x\right)\sqrt{cx^2}\,(dx)^m}{m^3+9\,m^2+26\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 5*b^2*m + 6*b^2)*x^3 + 2*(a*b*m^2 + 6*a*b*m + 8*a*b)*x^2 + (a^2
*m^2 + 7*a^2*m + 12*a^2)*x)*sqrt(c*x^2)*(d*x)^m/(m^3 + 9*m^2 + 26*m + 24)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a)**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.978 $\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=81

$$\frac{a^2x(dx)^m}{m\sqrt{cx^2}}+\frac{2abx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}+\frac{b^2x(dx)^{m+2}}{d^2(m+2)\sqrt{cx^2}}$$

```
[Out] (a^2*x*(d*x)^m)/(m*Sqrt[c*x^2]) + (2*a*b*x*(d*x)^(1 + m))/(d*(1 + m)*Sqrt[c
*x^2]) + (b^2*x*(d*x)^(2 + m))/(d^2*(2 + m)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0355124, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$\frac{a^2x(dx)^m}{m\sqrt{cx^2}}+\frac{2abx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}+\frac{b^2x(dx)^{m+2}}{d^2(m+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x*(d*x)^m)/(m*Sqrt[c*x^2]) + (2*a*b*x*(d*x)^(1 + m))/(d*(1 + m)*Sqrt[c
*x^2]) + (b^2*x*(d*x)^(2 + m))/(d^2*(2 + m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)^2}{x}\,dx}{\sqrt{cx^2}}\\
&= \frac{(dx)\int (dx)^{-1+m}(a+bx)^2\,dx}{\sqrt{cx^2}}\\
&= \frac{(dx)\int \left(a^2(dx)^{-1+m}+\frac{2ab(dx)^m}{d}+\frac{b^2(dx)^{1+m}}{d^2}\right)dx}{\sqrt{cx^2}}\\
&= \frac{a^2x(dx)^m}{m\sqrt{cx^2}}+\frac{2abx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}}+\frac{b^2x(dx)^{2+m}}{d^2(2+m)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.032731, size = 62, normalized size = 0.77

$$\frac{x(dx)^m\left(a^2\left(m^2+3m+2\right)+2abm(m+2)x+b^2m(m+1)x^2\right)}{m(m+1)(m+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x*(d*x)^m*(a^2*(2 + 3*m + m^2) + 2*a*b*m*(2 + m)*x + b^2*m*(1 + m)*x^2))/(
m*(1 + m)*(2 + m)*Sqrt[c*x^2])
```

Maple [A] time = 0.004, size = 79, normalized size = 1.

$$\frac{\left(b^2x^2m^2+2\,abxm^2+b^2x^2m+a^2m^2+4\,abxm+3\,a^2m+2\,a^2\right)x\,(dx)^m}{(2+m)\,(1+m)\,m}\frac{1}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+b^2*m*x^2+a^2*m^2+4*a*b*m*x+3*a^2*m+2*a^2)*(d*x)
^m/(2+m)/(1+m)/m/(c*x^2)^(1/2)
```

Maxima [A] time = 1.08843, size = 77, normalized size = 0.95

$$\frac{b^2d^mx^2x^m}{\sqrt{c}(m+2)}+\frac{2\,abd^mxx^m}{\sqrt{c}(m+1)}+\frac{a^2d^mx^m}{\sqrt{c}m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^2*x^m/(sqrt(c)*(m + 2)) + 2*a*b*d^m*x*x^m/(sqrt(c)*(m + 1)) + a^2
*d^m*x^m/(sqrt(c)*m)
```

Fricas [A] time = 1.42444, size = 174, normalized size = 2.15

$$\frac{\left(a^2m^2+3\,a^2m+\left(b^2m^2+b^2m\right)x^2+2\,a^2+2\left(abm^2+2\,abm\right)x\right)\sqrt{cx^2}\,(dx)^m}{\left(cm^3+3\,cm^2+2\,cm\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 + 3*a^2*m + (b^2*m^2 + b^2*m)*x^2 + 2*a^2 + 2*(a*b*m^2 + 2*a*b*m)*
x)*sqrt(c*x^2)*(d*x)^m/((c*m^3 + 3*c*m^2 + 2*c*m)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^2 (dx)^m}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*(d*x)^m/sqrt(c*x^2), x)
```

3.979 $\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{a^2d^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{2abdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}+\frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

```
[Out] -((a^2*d^2*x*(d*x)^(-2 + m))/(c*(2 - m)*Sqrt[c*x^2])) - (2*a*b*d*x*(d*x)^(-
1 + m))/(c*(1 - m)*Sqrt[c*x^2]) + (b^2*x*(d*x)^m)/(c*m*Sqrt[c*x^2])
```

Rubi [A] time = 0.0450581, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$-\frac{a^2d^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{2abdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}+\frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] -((a^2*d^2*x*(d*x)^(-2 + m))/(c*(2 - m)*Sqrt[c*x^2])) - (2*a*b*d*x*(d*x)^(-
1 + m))/(c*(1 - m)*Sqrt[c*x^2]) + (b^2*x*(d*x)^m)/(c*m*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)^2}{x^3}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{\left(d^3x\right)\int (dx)^{-3+m}(a+bx)^2\,dx}{c\sqrt{cx^2}}$$
$$= \frac{\left(d^3x\right)\int \left(a^2(dx)^{-3+m} + \frac{2ab(dx)^{-2+m}}{d} + \frac{b^2(dx)^{-1+m}}{d^2}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2d^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}} - \frac{2abdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}} + \frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

Mathematica [A] time = 0.0803281, size = 62, normalized size = 0.67

$$\frac{x(dx)^m\left(a^2(m-1)m + 2ab(m-2)mx + b^2\left(m^2-3m+2\right)x^2\right)}{(m-2)(m-1)m\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a^2*(-1 + m)*m + 2*a*b*(-2 + m)*m*x + b^2*(2 - 3*m + m^2)*x^2))
/((-2 + m)*(-1 + m)*m*(c*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 83, normalized size = 0.9

$$\frac{\left(b^2m^2x^2 + 2\,abxm^2 - 3\,b^2mx^2 + a^2m^2 - 4\,abxm + 2\,b^2x^2 - a^2m\right)x\,(dx)^m}{m\,(-1+m)\,(-2+m)}\left(cx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x-3*b^2*m*x^2+a^2*m^2-4*a*b*m*x+2*b^2*x^2-a^2*m)*(
d*x)^m/m/(-1+m)/(-2+m)/(c*x^2)^(3/2)
```

Maxima [A] time = 1.09372, size = 80, normalized size = 0.86

$$\frac{b^2d^mx^m}{c^{\frac{3}{2}}m} + \frac{2\,abd^mx^m}{c^{\frac{3}{2}}(m-1)x} + \frac{a^2d^mx^m}{c^{\frac{3}{2}}(m-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^m/(c^(3/2)*m) + 2*a*b*d^m*x^m/(c^(3/2)*(m - 1)*x) + a^2*d^m*x^m/(
c^(3/2)*(m - 2)*x^2)
```

Fricas [A] time = 1.30778, size = 185, normalized size = 1.99

$$\frac{\left(a^2m^2 - a^2m + \left(b^2m^2 - 3\,b^2m + 2\,b^2\right)x^2 + 2\,\left(abm^2 - 2\,abm\right)x\right)\sqrt{cx^2}\,(dx)^m}{\left(c^2m^3 - 3\,c^2m^2 + 2\,c^2m\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 - a^2*m + (b^2*m^2 - 3*b^2*m + 2*b^2)*x^2 + 2*(a*b*m^2 - 2*a*b*m)*
x)*sqrt(c*x^2)*(d*x)^m/((c^2*m^3 - 3*c^2*m^2 + 2*c^2*m)*x^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^2\,(dx)^m}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*(d*x)^m/(c*x^2)^(3/2), x)
```

3.980 $\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{5/2}} dx$

Optimal. Leaf size=105

$$-\frac{a^2d^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{m-2}}{c^2(2-m)\sqrt{cx^2}}$$

```
[Out] -((a^2*d^4*x*(d*x)^(-4 + m))/(c^2*(4 - m)*Sqrt[c*x^2])) - (2*a*b*d^3*x*(d*x
)^(-3 + m))/(c^2*(3 - m)*Sqrt[c*x^2]) - (b^2*d^2*x*(d*x)^(-2 + m))/(c^2*(2
- m)*Sqrt[c*x^2])
```

Rubi [A] time = 0.0542726, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$-\frac{a^2d^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{m-2}}{c^2(2-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -((a^2*d^4*x*(d*x)^(-4 + m))/(c^2*(4 - m)*Sqrt[c*x^2])) - (2*a*b*d^3*x*(d*x
)^(-3 + m))/(c^2*(3 - m)*Sqrt[c*x^2]) - (b^2*d^2*x*(d*x)^(-2 + m))/(c^2*(2
- m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)^2}{x^5}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{\left(d^5x\right)\int (dx)^{-5+m}(a+bx)^2\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{\left(d^5x\right)\int \left(a^2(dx)^{-5+m}+\frac{2ab(dx)^{-4+m}}{d}+\frac{b^2(dx)^{-3+m}}{d^2}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2d^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{-2+m}}{c^2(2-m)\sqrt{cx^2}}$$

Mathematica [A] time = 0.0553227, size = 72, normalized size = 0.69

$$\frac{x(dx)^m\left(a^2\left(m^2-5m+6\right)+2ab\left(m^2-6m+8\right)x+b^2\left(m^2-7m+12\right)x^2\right)}{(m-4)(m-3)(m-2)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a^2*(6 - 5*m + m^2) + 2*a*b*(8 - 6*m + m^2)*x + b^2*(12 - 7*m +
 m^2)*x^2))/((-4 + m)*(-3 + m)*(-2 + m)*(c*x^2)^(5/2))
```

Maple [A] time = 0.005, size = 95, normalized size = 0.9

$$\frac{\left(b^2m^2x^2+2\,abm^2x-7\,b^2mx^2+a^2m^2-12\,abmx+12\,b^2x^2-5\,a^2m+16\,abx+6\,a^2\right)x\,(dx)^m}{(-2+m)\,(-3+m)\,(-4+m)}\left(cx^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x-7*b^2*m*x^2+a^2*m^2-12*a*b*m*x+12*b^2*x^2-5*a^2*
m+16*a*b*x+6*a^2)*(d*x)^m/(-2+m)/(-3+m)/(-4+m)/(c*x^2)^(5/2)
```

Maxima [A] time = 1.09451, size = 86, normalized size = 0.82

$$\frac{b^2d^mx^m}{c^{\frac{5}{2}}(m-2)x^2}+\frac{2\,abd^mx^m}{c^{\frac{5}{2}}(m-3)x^3}+\frac{a^2d^mx^m}{c^{\frac{5}{2}}(m-4)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^m/(c^(5/2)*(m - 2)*x^2) + 2*a*b*d^m*x^m/(c^(5/2)*(m - 3)*x^3) + a
^2*d^m*x^m/(c^(5/2)*(m - 4)*x^4)
```

Fricas [A] time = 1.34616, size = 224, normalized size = 2.13

$$\frac{\left(a^2m^2 - 5\,a^2m + \left(b^2m^2 - 7\,b^2m + 12\,b^2\right)x^2 + 6\,a^2 + 2\left(abm^2 - 6\,abm + 8\,ab\right)x\right)\sqrt{cx^2}\,(dx)^m}{\left(c^3m^3 - 9\,c^3m^2 + 26\,c^3m - 24\,c^3\right)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 - 5*a^2*m + (b^2*m^2 - 7*b^2*m + 12*b^2)*x^2 + 6*a^2 + 2*(a*b*m^2
- 6*a*b*m + 8*a*b)*x)*sqrt(c*x^2)*(d*x)^m/((c^3*m^3 - 9*c^3*m^2 + 26*c^3*m
- 24*c^3)*x^5)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^2\,(dx)^m}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*(d*x)^m/(c*x^2)^(5/2), x)
```

3.981 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^n \, dx$

Optimal. Leaf size=67

$$\frac{c^2\sqrt{cx^2}(dx)^{m+6}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m+6,-n;m+7;-\frac{bx}{a}\right)}{d^6(m+6)x}$$

```
[Out] (c^2*(d*x)^(6 + m)*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[6 + m, -n, 7 +
 m, -((b*x)/a)])/(d^6*(6 + m)*x*(1 + (b*x)/a)^n)
```

Rubi [A] time = 0.028676, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {15, 16, 66, 64}

$$\frac{c^2\sqrt{cx^2}(dx)^{m+6}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m+6,-n;m+7;-\frac{bx}{a}\right)}{d^6(m+6)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n,x]
```


```
[Out] (c^2*(d*x)^(6 + m)*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[6 + m, -n, 7 +
 m, -((b*x)/a)])/(d^6*(6 + m)*x*(1 + (b*x)/a)^n)
```


Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] &&  !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^n \, dx = \frac{\left(c^2\sqrt{cx^2}\right) \int x^5 (dx)^m (a+bx)^n \, dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right) \int (dx)^{5+m} (a+bx)^n \, dx}{d^5 x}$$
$$= \frac{\left(c^2\sqrt{cx^2}(a+bx)^n \left(1+\frac{bx}{a}\right)^{-n}\right) \int (dx)^{5+m} \left(1+\frac{bx}{a}\right)^n \, dx}{d^5 x}$$
$$= \frac{c^2 (dx)^{6+m} \sqrt{cx^2}(a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(6+m,-n;7+m;-\frac{bx}{a}\right)}{d^6(6+m)x}$$

Mathematica [A] time = 0.0203262, size = 57, normalized size = 0.85

$$\frac{x \left(cx^2\right)^{5/2} (dx)^m (a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(m+6,-n;m+7;-\frac{bx}{a}\right)}{m+6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n,x]

[Out] (x*(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n*Hypergeometric2F1[6 + m, -n, 7 + m, -(
(b*x)/a)])/((6 + m)*(1 + (b*x)/a)^n)
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^2\right)^{\frac{5}{2}} (bx+a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x)

[Out] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^{\frac{5}{2}} (bx+a)^n (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="maxima")

[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2}(bx+a)^n\,(dx)^m\,c^2x^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m*c^2*x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^{\frac{5}{2}}(bx+a)^n\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n*(d*x)^m, x)
```

3.982 $\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^n \, dx$

Optimal. Leaf size=65

$$\frac{c\sqrt{cx^2}(dx)^{m+4}(a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(m+4,-n;m+5;-\frac{bx}{a}\right)}{d^4(m+4)x}$$

```
[Out] (c*(d*x)^(4 + m)*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[4 + m, -n, 5 + m
, -((b*x)/a)])/(d^4*(4 + m)*x*(1 + (b*x)/a)^n)
```

Rubi [A] time = 0.0285395, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {15, 16, 66, 64}

$$\frac{c\sqrt{cx^2}(dx)^{m+4}(a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(m+4,-n;m+5;-\frac{bx}{a}\right)}{d^4(m+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] (c*(d*x)^(4 + m)*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[4 + m, -n, 5 + m
, -((b*x)/a)])/(d^4*(4 + m)*x*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^n\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x^3(dx)^m(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int (dx)^{3+m}(a+bx)^n\,dx}{d^3x}$$
$$= \frac{\left(c\sqrt{cx^2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{3+m}\left(1+\frac{bx}{a}\right)^n dx}{d^3x}$$
$$= \frac{c(dx)^{4+m}\sqrt{cx^2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(4+m,-n;5+m;-\frac{bx}{a}\right)}{d^4(4+m)x}$$

Mathematica [A] time = 0.0229955, size = 57, normalized size = 0.88

$$\frac{x\left(cx^2\right)^{3/2}(dx)^m(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m+4,-n;m+5;-\frac{bx}{a}\right)}{m+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n*Hypergeometric2F1[4 + m, -n, 5 + m, -(
(b*x)/a)])/((4 + m)*(1 + (b*x)/a)^n)
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^2\right)^{\frac{3}{2}} (bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x)
```

```
[Out] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^{\frac{3}{2}} (bx+a)^n (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2}(bx+a)^n\,(dx)^m\,cx^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m*c*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^{\frac{3}{2}}(bx+a)^n\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n*(d*x)^m, x)
```

3.983 $\int (dx)^m \sqrt{cx^2}(a+bx)^n\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{cx^2}(dx)^{m+2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m+2,-n;m+3;-\frac{bx}{a}\right)}{d^2(m+2)x}$$

```
[Out] ((d*x)^(2 + m)*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[2 + m, -n, 3 + m,
-((b*x)/a)])/(d^2*(2 + m)*x*(1 + (b*x)/a)^n)
```

Rubi [A] time = 0.0281022, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {15, 16, 66, 64}

$$\frac{\sqrt{cx^2}(dx)^{m+2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m+2,-n;m+3;-\frac{bx}{a}\right)}{d^2(m+2)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] ((d*x)^(2 + m)*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[2 + m, -n, 3 + m,
-((b*x)/a)])/(d^2*(2 + m)*x*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int (dx)^m\sqrt{cx^2}(a+bx)^n\,dx = \frac{\sqrt{cx^2}\int x(dx)^m(a+bx)^n\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int (dx)^{1+m}(a+bx)^n\,dx}{dx}$$
$$= \frac{\left(\sqrt{cx^2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{1+m}\left(1+\frac{bx}{a}\right)^n\,dx}{dx}$$
$$= \frac{(dx)^{2+m}\sqrt{cx^2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(2+m,-n;3+m;-\frac{bx}{a}\right)}{d^2(2+m)x}$$

Mathematica [A] time = 0.0140299, size = 57, normalized size = 0.89

$$\frac{x\sqrt{cx^2}(dx)^m(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m+2,-n;m+3;-\frac{bx}{a}\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (x*(d*x)^m*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[2 + m, -n, 3 + m, -((b
*x)/a)])/((2 + m)*(1 + (b*x)/a)^n)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (dx)^m\sqrt{cx^2}(bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x)
```

```
[Out] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2}(bx+a)^n(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2}(bx+a)^n(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2}\,(dx)^m\,(a+bx)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a)**n,x)
```

```
[Out] Integral(sqrt(c*x**2)*(d*x)**m*(a + b*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2}(bx+a)^n\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m, x)
```

3.984 $\int \frac{(dx)^m(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{x(dx)^m(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(m,-n;m+1;-\frac{bx}{a}\right)}{m\sqrt{cx^2}}$$

```
[Out] (x*(d*x)^m*(a + b*x)^n*Hypergeometric2F1[m, -n, 1 + m, -((b*x)/a)])/(m*Sqrt
[c*x^2]*(1 + (b*x)/a)^n)
```

Rubi [A] time = 0.0213641, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {15, 16, 66, 64}

$$\frac{x(dx)^m(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(m,-n;m+1;-\frac{bx}{a}\right)}{m\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(d*x)^m*(a + b*x)^n*Hypergeometric2F1[m, -n, 1 + m, -((b*x)/a)])/(m*Sqrt
[c*x^2]*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(dx)^m(a+bx)^n}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)^n}{x}\,dx}{\sqrt{cx^2}} \\
&= \frac{(dx)\int (dx)^{-1+m}(a+bx)^n\,dx}{\sqrt{cx^2}} \\
&= \frac{\left(dx(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{-1+m}\left(1+\frac{bx}{a}\right)^n\,dx}{\sqrt{cx^2}} \\
&= \frac{x(dx)^m(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(m,-n;1+m;-\frac{bx}{a}\right)}{m\sqrt{cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0100182, size = 53, normalized size = 1.

$$
\frac{x(dx)^m(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m,-n;m+1;-\frac{bx}{a}\right)}{m\sqrt{cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(d*x)^m*(a + b*x)^n*Hypergeometric2F1[m, -n, 1 + m, -((b*x)/a)])/(m*Sqrt
[c*x^2]*(1 + (b*x)/a)^n)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$
\int (dx)^m (bx+a)^n \frac{1}{\sqrt{cx^2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] int((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{(bx+a)^n (dx)^m}{\sqrt{cx^2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x)^m/sqrt(c*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n (dx)^m}{cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m/(c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m (a+bx)^n}{\sqrt{cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x)**n/sqrt(c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n (dx)^m}{\sqrt{cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x)^m/sqrt(c*x^2), x)
```

3.985 $\int \frac{(dx)^m(a+bx)^n}{\left(cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=68

$$-\frac{d^2x(dx)^{m-2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m-2,-n;m-1;-\frac{bx}{a}\right)}{c(2-m)\sqrt{cx^2}}$$

```
[Out] -((d^2*x*(d*x)^(-2 + m)*(a + b*x)^n*Hypergeometric2F1[-2 + m, -n, -1 + m, -
((b*x)/a)])/(c*(2 - m)*Sqrt[c*x^2]*(1 + (b*x)/a)^n))
```

Rubi [A] time = 0.0315716, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {15, 16, 66, 64}

$$-\frac{d^2x(dx)^{m-2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m-2,-n;m-1;-\frac{bx}{a}\right)}{c(2-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((d^2*x*(d*x)^(-2 + m)*(a + b*x)^n*Hypergeometric2F1[-2 + m, -n, -1 + m, -
((b*x)/a)])/(c*(2 - m)*Sqrt[c*x^2]*(1 + (b*x)/a)^n))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] &&  !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)^n}{(cx^2)^{3/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)^n}{x^3}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{(d^3x)\int (dx)^{-3+m}(a+bx)^n\,dx}{c\sqrt{cx^2}}$$
$$= \frac{\left(d^3x(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{-3+m}\left(1+\frac{bx}{a}\right)^n\,dx}{c\sqrt{cx^2}}$$
$$= -\frac{d^2x(dx)^{-2+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-2+m,-n;-1+m;-\frac{bx}{a}\right)}{c(2-m)\sqrt{cx^2}}$$

Mathematica [A] time = 0.015123, size = 57, normalized size = 0.84

$$\frac{x(dx)^m(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m-2,-n;m-1;-\frac{bx}{a}\right)}{(m-2)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x)^n*Hypergeometric2F1[-2 + m, -n, -1 + m, -((b*x)/a)])/(
(-2 + m)*(c*x^2)^(3/2)*(1 + (b*x)/a)^n)
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (dx)^m (bx+a)^n \left(cx^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] int((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n (dx)^m}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x)^m/(c*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n\,(dx)^m}{c^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m/(c^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^m\,(a+bx)^n}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x)**n/(c*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(bx+a)^n\,(dx)^m}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x)^m/(c*x^2)^(3/2), x)
```

3.986 $\int \frac{(dx)^m(a+bx)^n}{\left(cx^2\right)^{5/2}} dx$

Optimal. Leaf size=68

$$-\frac{d^4x(dx)^{m-4}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m-4,-n;m-3;-\frac{bx}{a}\right)}{c^2(4-m)\sqrt{cx^2}}$$

```
[Out] -((d^4*x*(d*x)^(-4 + m)*(a + b*x)^n*Hypergeometric2F1[-4 + m, -n, -3 + m, -
((b*x)/a)])/(c^2*(4 - m)*Sqrt[c*x^2]*(1 + (b*x)/a)^n))
```

Rubi [A] time = 0.0309549, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {15, 16, 66, 64}

$$-\frac{d^4x(dx)^{m-4}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m-4,-n;m-3;-\frac{bx}{a}\right)}{c^2(4-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^n)/(c*x^2)^(5/2),x]
```


```
[Out] -((d^4*x*(d*x)^(-4 + m)*(a + b*x)^n*Hypergeometric2F1[-4 + m, -n, -3 + m, -
((b*x)/a)])/(c^2*(4 - m)*Sqrt[c*x^2]*(1 + (b*x)/a)^n))
```


Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)^n}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)^n}{x^5}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{(d^5x)\int (dx)^{-5+m}(a+bx)^n\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{\left(d^5x(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{-5+m}\left(1+\frac{bx}{a}\right)^n\,dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{d^4x(dx)^{-4+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-4+m,-n;-3+m;-\frac{bx}{a}\right)}{c^2(4-m)\sqrt{cx^2}}$$

Mathematica [A] time = 0.0160141, size = 57, normalized size = 0.84

$$\frac{x(dx)^m(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m-4,-n;m-3;-\frac{bx}{a}\right)}{(m-4)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x)^n*Hypergeometric2F1[-4 + m, -n, -3 + m, -((b*x)/a)])/(
(-4 + m)*(c*x^2)^(5/2)*(1 + (b*x)/a)^n)
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int (dx)^m (bx+a)^n \left(cx^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] int((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n (dx)^m}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x)^m/(c*x^2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2}(bx+a)^n\,(dx)^m}{c^3x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m/(c^3*x^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n\,(dx)^m}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x)^m/(c*x^2)^(5/2), x)
```

3.987 $\int x^3 \left(cx^2\right)^p (a+bx)^{-5-2p}\,dx$

Optimal. Leaf size=33

$$\frac{x^4 \left(cx^2\right)^p (a+bx)^{-2(p+2)}}{2a(p+2)}$$

```
[Out] (x^4*(c*x^2)^p)/(2*a*(2 + p)*(a + b*x)^(2*(2 + p)))
```

Rubi [A] time = 0.0095883, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$\frac{x^4 \left(cx^2\right)^p (a+bx)^{-2(p+2)}}{2a(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p),x]
```

```
[Out] (x^4*(c*x^2)^p)/(2*a*(2 + p)*(a + b*x)^(2*(2 + p)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(cx^2\right)^p (a+bx)^{-5-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{3+2p}(a+bx)^{-5-2p}\,dx\\ &= \frac{x^4\left(cx^2\right)^p (a+bx)^{-2(2+p)}}{2a(2+p)}\end{aligned}$$

Mathematica [A] time = 0.018029, size = 32, normalized size = 0.97

$$\frac{x^4 \left(cx^2\right)^p (a+bx)^{-2p-4}}{a(2p+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p),x]
```

```
[Out] (x^4*(c*x^2)^p*(a + b*x)^(-4 - 2*p))/(a*(4 + 2*p))
```

Maple [A] time = 0.003, size = 32, normalized size = 1.

$$\frac{x^4 (bx + a)^{-4-2p} \left(cx^2\right)^p}{2a\left(2 + p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x)
```

```
[Out] 1/2*x^4*(b*x+a)^(-4-2*p)/a/(2+p)*(c*x^2)^p
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^p (bx + a)^{-2p-5} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 5)*x^3, x)
```

Fricas [A] time = 1.74501, size = 86, normalized size = 2.61

$$\frac{\left(bx^5 + ax^4\right) \left(cx^2\right)^p (bx + a)^{-2p-5}}{2\left(ap + 2a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^5 + a*x^4)*(c*x^2)^p*(b*x + a)^(-2*p - 5)/(a*p + 2*a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**p*(b*x+a)**(-5-2*p),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.09765, size = 100, normalized size = 3.03

$$\frac{\left(cx^2\right)^p bx^5 e^{\left(-2\,p\log(bx+a)-5\,\log(bx+a)\right)} + \left(cx^2\right)^p ax^4 e^{\left(-2\,p\log(bx+a)-5\,\log(bx+a)\right)}}{2\left(ap + 2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="giac")
```

```
[Out] 1/2*((c*x^2)^p*b*x^5*e^(-2*p*log(b*x + a) - 5*log(b*x + a)) + (c*x^2)^p*a*x
^4*e^(-2*p*log(b*x + a) - 5*log(b*x + a)))/(a*p + 2*a)
```

3.988 $\int x^2 \left(cx^2\right)^p (a+bx)^{-4-2p}\,dx$

Optimal. Leaf size=32

$$\frac{x^3 \left(cx^2\right)^p (a+bx)^{-2p-3}}{a(2p+3)}$$

```
[Out] (x^3*(c*x^2)^p*(a + b*x)^(-3 - 2*p))/(a*(3 + 2*p))
```

Rubi [A] time = 0.0086787, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$\frac{x^3 \left(cx^2\right)^p (a+bx)^{-2p-3}}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p),x]
```

```
[Out] (x^3*(c*x^2)^p*(a + b*x)^(-3 - 2*p))/(a*(3 + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(cx^2\right)^p (a+bx)^{-4-2p}\,dx &= \left(x^{-2p} \left(cx^2\right)^p\right) \int x^{2+2p}(a+bx)^{-4-2p}\,dx\\ &= \frac{x^3 \left(cx^2\right)^p (a+bx)^{-3-2p}}{a(3+2p)}\end{aligned}$$

Mathematica [A] time = 0.0198317, size = 34, normalized size = 1.06

$$\frac{x^3 \left(cx^2\right)^p (a+bx)^{1-2(p+2)}}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p),x]
```

```
[Out] (x^3*(c*x^2)^p*(a + b*x)^(1 - 2*(2 + p)))/(a*(3 + 2*p))
```

Maple [A] time = 0.003, size = 33, normalized size = 1.

$$\frac{x^3 \left(c x^2\right)^p (bx+a)^{-3-2p}}{a \left(3+2p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x)
```

```
[Out] x^3*(c*x^2)^p*(b*x+a)^(-3-2*p)/a/(3+2*p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^2\right)^p (bx+a)^{-2p-4} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 4)*x^2, x)
```

Fricas [A] time = 1.58956, size = 84, normalized size = 2.62

$$\frac{\left(b x^4 + a x^3\right) \left(c x^2\right)^p (bx+a)^{-2p-4}}{2ap + 3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="fricas")
```

```
[Out] (b*x^4 + a*x^3)*(c*x^2)^p*(b*x + a)^(-2*p - 4)/(2*a*p + 3*a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**p*(b*x+a)**(-4-2*p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^p (bx+a)^{-2p-4}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 4)*x^2, x)
```

3.989 $\int x \left(cx^2\right)^p (a+bx)^{-3-2p} \, dx$

Optimal. Leaf size=33

$$\frac{x^2 \left(cx^2\right)^p (a+bx)^{-2(p+1)}}{2a(p+1)}$$

```
[Out] (x^2*(c*x^2)^p)/(2*a*(1 + p)*(a + b*x)^(2*(1 + p)))
```

Rubi [A] time = 0.0097638, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 37}

$$\frac{x^2 \left(cx^2\right)^p (a+bx)^{-2(p+1)}}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^p*(a + b*x)^(-3 - 2*p),x]
```

```
[Out] (x^2*(c*x^2)^p)/(2*a*(1 + p)*(a + b*x)^(2*(1 + p)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x \left(cx^2\right)^p (a+bx)^{-3-2p} \, dx &= \left(x^{-2p} \left(cx^2\right)^p\right) \int x^{1+2p} (a+bx)^{-3-2p} \, dx \\ &= \frac{x^2 \left(cx^2\right)^p (a+bx)^{-2(1+p)}}{2a(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0157256, size = 32, normalized size = 0.97

$$\frac{x^2 \left(cx^2\right)^p (a+bx)^{-2p-2}}{a(2p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^p*(a + b*x)^(-3 - 2*p),x]
```

```
[Out] (x^2*(c*x^2)^p*(a + b*x)^(-2 - 2*p))/(a*(2 + 2*p))
```

Maple [A] time = 0.003, size = 32, normalized size = 1.

$$\frac{x^2 (bx+a)^{-2-2p} \left(cx^2\right)^p}{2a\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x)
```

```
[Out] 1/2*x^2*(b*x+a)^(-2-2*p)/a/(1+p)*(c*x^2)^p
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^p (bx+a)^{-2p-3} x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 3)*x, x)
```

Fricas [A] time = 1.63409, size = 84, normalized size = 2.55

$$\frac{\left(bx^3+ax^2\right)\left(cx^2\right)^p (bx+a)^{-2p-3}}{2\left(ap+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^3 + a*x^2)*(c*x^2)^p*(b*x + a)^(-2*p - 3)/(a*p + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**p*(b*x+a)**(-3-2*p),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.09472, size = 97, normalized size = 2.94

$$\frac{\left(cx^2\right)^p bx^3 e^{\left(-2\,p\log(bx+a)-3\,\log(bx+a)\right)} + \left(cx^2\right)^p ax^2 e^{\left(-2\,p\log(bx+a)-3\,\log(bx+a)\right)}}{2\left(ap+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="giac")
```

```
[Out] 1/2*((c*x^2)^p*b*x^3*e^(-2*p*log(b*x + a) - 3*log(b*x + a)) + (c*x^2)^p*a*x
^2*e^(-2*p*log(b*x + a) - 3*log(b*x + a)))/(a*p + a)
```

3.990 $\int \left(cx^2\right)^p (a+bx)^{-2-2p}\,dx$

Optimal. Leaf size=30

$$\frac{x\left(cx^2\right)^p (a+bx)^{-2p-1}}{a(2p+1)}$$

[Out] (x*(c*x^2)^p*(a + b*x)^(-1 - 2*p))/(a*(1 + 2*p))

Rubi [A] time = 0.0079995, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 37}

$$\frac{x\left(cx^2\right)^p (a+bx)^{-2p-1}}{a(2p+1)}$$

Antiderivative was successfully verified.

[In] Int[(c*x^2)^p*(a + b*x)^(-2 - 2*p),x]

[Out] (x*(c*x^2)^p*(a + b*x)^(-1 - 2*p))/(a*(1 + 2*p))

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \left(cx^2\right)^p (a+bx)^{-2-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{2p}(a+bx)^{-2-2p}\,dx\\ &= \frac{x\left(cx^2\right)^p (a+bx)^{-1-2p}}{a(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.0161289, size = 28, normalized size = 0.93

$$\frac{x\left(cx^2\right)^p (a+bx)^{-2p-1}}{2ap+a}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x^2)^p*(a + b*x)^(-2 - 2*p),x]

```
[Out] (x*(c*x^2)^p*(a + b*x)^(-1 - 2*p))/(a + 2*a*p)
```

Maple [A] time = 0.003, size = 31, normalized size = 1.

$$\frac{x\left(cx^2\right)^p (bx+a)^{-1-2p}}{a\left(1+2p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(-2-2*p),x)
```

```
[Out] x*(c*x^2)^p*(b*x+a)^(-1-2*p)/a/(1+2*p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^p (bx+a)^{-2p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 2), x)
```

Fricas [A] time = 1.56111, size = 78, normalized size = 2.6

$$\frac{\left(bx^2+ax\right)\left(cx^2\right)^p (bx+a)^{-2p-2}}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(c*x^2)^p*(b*x + a)^(-2*p - 2)/(2*a*p + a)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(-2-2*p),x)
```

```
[Out] Exception raised: TypeError
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^p (bx+a)^{-2p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 2), x)
```

3.991 $\int \frac{(cx^2)^p (a+bx)^{-1-2p}}{x} dx$

Optimal. Leaf size=26

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

```
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

Rubi [A] time = 0.0061454, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x,x]
```

```
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^p (a+bx)^{-1-2p}}{x} dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{-1+2p}(a+bx)^{-1-2p} dx \\ &= \frac{(cx^2)^p (a+bx)^{-2p}}{2ap}\end{aligned}$$

Mathematica [A] time = 0.0071077, size = 26, normalized size = 1.

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x,x]
```

```
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

Maple [A] time = 0.003, size = 25, normalized size = 1.

$$\frac{(bx+a)^{-2p}\left(cx^2\right)^p}{2ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x)
```

```
[Out] 1/2*(b*x+a)^(-2*p)/a/p*(c*x^2)^p
```

Maxima [A] time = 1.06811, size = 36, normalized size = 1.38

$$\frac{c^p e^{\left(-2\,p\log(bx+a)+2\,p\log(x)\right)}}{2\,ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*c^p*e^(-2*p*log(b*x + a) + 2*p*log(x))/(a*p)
```

Fricas [A] time = 1.72138, size = 70, normalized size = 2.69

$$\frac{(bx+a)\left(cx^2\right)^p(bx+a)^{-2\,p-1}}{2\,ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p - 1)/(a*p)
```

Sympy [A] time = 70.4082, size = 264, normalized size = 10.15

$$\begin{cases} -\frac{b^{-2p}c^p x^{-2p}\left(x^2\right)^p}{bx} & \text{for } a=0 \\ \frac{0^{-2p-1}c^p\left(x^2\right)^p}{2p} & \text{for } a=-bx \\ \frac{c^p\left(0^{\frac{1}{p}}\right)^{-2p-1}\left(x^2\right)^p}{2p} & \text{for } a=0^{\frac{1}{p}} - \\ \frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x\right)}{a} & \text{for } p=0 \\ \frac{a^2c^p\left(x^2\right)^p}{2a^3p(a+bx)^{2p}+4a^2bpx(a+bx)^{2p}+2ab^2px^2(a+bx)^{2p}}+\frac{abc^px\left(x^2\right)^p}{2a^3p(a+bx)^{2p}+4a^2bpx(a+bx)^{2p}+2ab^2px^2(a+bx)^{2p}}+\frac{bc^px\left(x^2\right)^p}{2a^2p(a+bx)^{2p}+2abpx(a+bx)^{2p}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(-1-2*p)/x,x)
```

```
[Out] Piecewise((-b**(-2*p)*c**p*x**(-2*p)*(x**2)**p/(b*x), Eq(a, 0)), (0**(-2*p
- 1)*c**p*(x**2)**p/(2*p), Eq(a, -b*x)), (c**p*(0**(1/p))**(-2*p - 1)*(x**2
)**p/(2*p), Eq(a, 0**(1/p) - b*x)), (log(x)/a - log(a/b + x)/a, Eq(p, 0)),
(a**2*c**p*(x**2)**p/(2*a**3*p*(a + b*x)**(2*p) + 4*a**2*b*p*x*(a + b*x)**(
2*p) + 2*a*b**2*p*x**2*(a + b*x)**(2*p)) + a*b*c**p*x*(x**2)**p/(2*a**3*p*(
a + b*x)**(2*p) + 4*a**2*b*p*x*(a + b*x)**(2*p) + 2*a*b**2*p*x**2*(a + b*x)
**(2*p)) + b*c**p*x*(x**2)**p/(2*a**2*p*(a + b*x)**(2*p) + 2*a*b*p*x*(a + b
*x)**(2*p)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^p (bx+a)^{-2p-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 1)/x, x)
```

3.992 $\int \frac{(cx^2)^p (a+bx)^{-2p}}{x^2} dx$

Optimal. Leaf size=33

$$-\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}$$

```
[Out] -(((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*(1 - 2*p)*x))
```

Rubi [A] time = 0.0108223, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {15, 37}

$$-\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^p/(x^2*(a + b*x)^(2*p)),x]
```

```
[Out] -(((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*(1 - 2*p)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(cx^2)^p (a+bx)^{-2p}}{x^2} dx = \left(x^{-2p} (cx^2)^p\right) \int x^{-2+2p}(a+bx)^{-2p} dx$$
$$= -\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}$$

Mathematica [A] time = 0.0098735, size = 32, normalized size = 0.97

$$\frac{(cx^2)^p (a+bx)^{1-2p}}{a(2p-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^p/(x^2*(a + b*x)^(2*p)),x]
```

```
[Out] ((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*(-1 + 2*p)*x)
```

Maple [A] time = 0.003, size = 38, normalized size = 1.2

$$\frac{(bx+a)\left(cx^{2}\right)^{p}}{\left(2p-1\right)ax\left(bx+a\right)^{2p}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/x^2/((b*x+a)^(2*p)),x)
```

```
[Out] 1/x*(b*x+a)/a/(2*p-1)*(c*x^2)^p/((b*x+a)^(2*p))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2}\right)^{p}}{\left(bx+a\right)^{2p}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p/((b*x + a)^(2*p)*x^2), x)
```

Fricas [A] time = 1.63047, size = 72, normalized size = 2.18

$$\frac{(bx+a)\left(cx^{2}\right)^{p}}{\left(2\,ap-a\right)\left(bx+a\right)^{2p}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="fricas")
```

```
[Out] (b*x + a)*(c*x^2)^p/((2*a*p - a)*(b*x + a)^(2*p)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p/x**2/((b*x+a)**(2*p)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^p}{\left(bx+a\right)^{2p}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p/((b*x + a)^(2*p)*x^2), x)
```

3.993 $\int \frac{(cx^2)^p (a+bx)^{1-2p}}{x^3} dx$

Optimal. Leaf size=35

$$-\frac{(cx^2)^p (a+bx)^{2-2p}}{2a(1-p)x^2}$$

```
[Out] -((c*x^2)^p*(a + b*x)^(2 - 2*p))/(2*a*(1 - p)*x^2)
```

Rubi [A] time = 0.01086, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$-\frac{(cx^2)^p (a+bx)^{2-2p}}{2a(1-p)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x]
```

```
[Out] -((c*x^2)^p*(a + b*x)^(2 - 2*p))/(2*a*(1 - p)*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^p (a+bx)^{1-2p}}{x^3} dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{-3+2p}(a+bx)^{1-2p} dx\\ &= -\frac{(cx^2)^p (a+bx)^{2-2p}}{2a(1-p)x^2}\end{aligned}$$

Mathematica [A] time = 0.0106003, size = 32, normalized size = 0.91

$$\frac{(cx^2)^p (a+bx)^{2-2p}}{a(2p-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x]
```

```
[Out] ((c*x^2)^p*(a + b*x)^(2 - 2*p))/(a*(-2 + 2*p)*x^2)
```

Maple [A] time = 0.002, size = 32, normalized size = 0.9

$$\frac{(bx+a)^{2-2p}\left(cx^2\right)^p}{2\,x^2a\left(p-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x)
```

```
[Out] 1/2/x^2*(b*x+a)^(2-2*p)/a/(p-1)*(c*x^2)^p
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^p (bx+a)^{-2p+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 1)/x^3, x)
```

Fricas [A] time = 1.78616, size = 84, normalized size = 2.4

$$\frac{(bx+a)\left(cx^2\right)^p (bx+a)^{-2p+1}}{2\left(ap-a\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p + 1)/((a*p - a)*x^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(1-2*p)/x**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2)^p (bx+a)^{-2p+1}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 1)/x^3, x)
```

3.994 $\int \frac{(cx^2)^p (a+bx)^{2-2p}}{x^4} dx$

Optimal. Leaf size=33

$$-\frac{(cx^2)^p (a+bx)^{3-2p}}{a(3-2p)x^3}$$

[Out] -(((c*x^2)^p*(a + b*x)^(3 - 2*p))/(a*(3 - 2*p)*x^3))

Rubi [A] time = 0.0104211, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$-\frac{(cx^2)^p (a+bx)^{3-2p}}{a(3-2p)x^3}$$

Antiderivative was successfully verified.

[In] Int[((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x]

[Out] -(((c*x^2)^p*(a + b*x)^(3 - 2*p))/(a*(3 - 2*p)*x^3))

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^p (a+bx)^{2-2p}}{x^4} dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{-4+2p}(a+bx)^{2-2p} dx\\ &= -\frac{(cx^2)^p (a+bx)^{3-2p}}{a(3-2p)x^3}\end{aligned}$$

Mathematica [A] time = 0.0098643, size = 32, normalized size = 0.97

$$\frac{(cx^2)^p (a+bx)^{3-2p}}{a(2p-3)x^3}$$

Antiderivative was successfully verified.

[In] Integrate[((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x]

```
[Out] ((c*x^2)^p*(a + b*x)^(3 - 2*p))/(a*(-3 + 2*p)*x^3)
```

Maple [A] time = 0.002, size = 33, normalized size = 1.

$$\frac{(bx+a)^{3-2p}\left(cx^2\right)^p}{x^3a\left(2p-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x)
```

```
[Out] 1/x^3*(b*x+a)^(3-2*p)/a/(2*p-3)*(c*x^2)^p
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^p(bx+a)^{-2p+2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 2)/x^4, x)
```

Fricas [A] time = 1.6147, size = 84, normalized size = 2.55

$$\frac{(bx+a)\left(cx^2\right)^p(bx+a)^{-2p+2}}{\left(2\,ap-3\,a\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p + 2)/((2*a*p - 3*a)*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(2-2*p)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2\right)^p (bx+a)^{-2p+2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 2)/x^4, x)
```

3.995 $\int x^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx$

Optimal. Leaf size=38

$$\frac{x^{m+1}\left(cx^2\right)^p (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

```
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Rubi [A] time = 0.0102633, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {15, 37}

$$\frac{x^{m+1}\left(cx^2\right)^p (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
```

```
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{m+2p}(a+bx)^{-2-m-2p}\,dx\\ &= \frac{x^{1+m}\left(cx^2\right)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)}\end{aligned}$$

Mathematica [A] time = 0.0289237, size = 38, normalized size = 1.

$$\frac{x^{m+1}\left(cx^2\right)^p (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
```

```
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Maple [A] time = 0.003, size = 39, normalized size = 1.

$$\frac{x^{1+m} \left(c x^{2}\right)^{p} \left(b x +a \right)^{-1-m-2 p}}{a \left(1+m +2 p \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x)
```

```
[Out] x^(1+m)*(c*x^2)^p*(b*x+a)^(-1-m-2*p)/a/(1+m+2*p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^{2}\right)^{p} (b x + a)^{-m-2 p-2} x^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*x^m, x)
```

Fricas [A] time = 1.62989, size = 119, normalized size = 3.13

$$\frac{\left(b x^{2} + a x\right) (b x + a)^{-m-2 p-2} x^{m} e^{\left(p \log(c)+2 p \log(x)\right)}}{a m + 2 a p + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^(-m - 2*p - 2)*x^m*e^(p*log(c) + 2*p*log(x))/(a*m +
 2*a*p + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**2)**p*(b*x+a)**(-2-m-2*p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^p (bx+a)^{-m-2p-2} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*x^m, x)
```

3.996 $\int (dx)^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx$

Optimal. Leaf size=39

$$\frac{x\left(cx^2\right)^p (dx)^m (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

```
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Rubi [A] time = 0.0103912, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 20, 37}

$$\frac{x\left(cx^2\right)^p (dx)^m (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{2p}(dx)^m(a+bx)^{-2-m-2p}\,dx\\
&= \left(x^{-m-2p}(dx)^m\left(cx^2\right)^p\right)\int x^{m+2p}(a+bx)^{-2-m-2p}\,dx\\
&= \frac{x(dx)^m\left(cx^2\right)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)}
\end{aligned}$$

Mathematica [A] time = 0.015029, size = 39, normalized size = 1.

$$\frac{x\left(cx^2\right)^p(dx)^m(a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Maple [A] time = 0.003, size = 40, normalized size = 1.

$$\frac{x\left(dx\right)^m\left(cx^2\right)^p\left(bx+a\right)^{-1-m-2p}}{a\left(1+m+2p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x)
```

```
[Out] x*(d*x)^m*(c*x^2)^p*(b*x+a)^(-1-m-2*p)/a/(1+m+2*p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^2\right)^p\left(bx+a\right)^{-m-2p-2}\left(dx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*(d*x)^m, x)
```

Fricas [A] time = 1.59401, size = 132, normalized size = 3.38

$$\frac{\left(bx^2+ax\right)\left(bx+a\right)^{-m-2p-2}\left(dx\right)^m e^{\left(2p\log(dx)+p\log\left(\frac{c}{d^2}\right)\right)}}{am+2ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^(-m - 2*p - 2)*(d*x)^m*e^(2*p*log(d*x) + p*log(c/d^
2))/(a*m + 2*a*p + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**p*(b*x+a)**(-2-m-2*p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^p (bx+a)^{-m-2p-2} (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*(d*x)^m, x)
```

3.997 $\int x^m \left(cx^2\right)^p (a+bx)^n\,dx$

Optimal. Leaf size=63

$$\frac{x^{m+1}\left(cx^2\right)^p (a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(-n,m+2p+1;m+2p+2;-\frac{bx}{a}\right)}{m+2p+1}$$

```
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 + m +
 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Rubi [A] time = 0.0188879, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {15, 66, 64}

$$\frac{x^{m+1}\left(cx^2\right)^p (a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(-n,m+2p+1;m+2p+2;-\frac{bx}{a}\right)}{m+2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*x^2)^p*(a + b*x)^n,x]
```

```
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 + m +
 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int x^m \left(cx^2\right)^p (a+bx)^n\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{m+2p}(a+bx)^n\,dx\\
&= \left(x^{-2p}\left(cx^2\right)^p (a+bx)^n \left(1+\frac{bx}{a}\right)^{-n}\right)\int x^{m+2p}\left(1+\frac{bx}{a}\right)^n\,dx\\
&= \frac{x^{1+m}\left(cx^2\right)^p (a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{1+m+2p}
\end{aligned}$$

Mathematica [A] time = 0.0125635, size = 63, normalized size = 1.

$$\frac{x^{m+1}\left(cx^2\right)^p(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(-n,m+2p+1;m+2p+2;-\frac{bx}{a}\right)}{m+2p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*x^2)^p*(a + b*x)^n,x]
```

```
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 + m +
 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int x^m\left(cx^2\right)^p(bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^2)^p*(b*x+a)^n,x)
```

```
[Out] int(x^m*(c*x^2)^p*(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^p(bx+a)^n x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^n*x^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^2\right)^p(bx+a)^n x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((c*x^2)^p*(b*x + a)^n*x^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m\left(cx^2\right)^p(a+bx)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**2)**p*(b*x+a)**n,x)
```

```
[Out] Integral(x**m*(c*x**2)**p*(a + b*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^p (bx+a)^n x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^n*x^m, x)
```

3.998 $\int (dx)^m \left(cx^2\right)^p (a+bx)^n\,dx$

Optimal. Leaf size=68

$$\frac{\left(cx^2\right)^p (dx)^{m+1}(a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(-n,m+2p+1;m+2p+2;-\frac{bx}{a}\right)}{d(m+2p+1)}$$

```
[Out] ((d*x)^(1 + m)*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 +
 m + 2*p, -((b*x)/a)])/(d*(1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Rubi [A] time = 0.0219284, antiderivative size = 64, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {15, 20, 66, 64}

$$\frac{x\left(cx^2\right)^p (dx)^m(a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(-n,m+2p+1;m+2p+2;-\frac{bx}{a}\right)}{m+2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^p*(a + b*x)^n,x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 + m +
 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^p (a+bx)^n\,dx = \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{2p}(dx)^m(a+bx)^n\,dx$$
$$= \left(x^{-m-2p}(dx)^m\left(cx^2\right)^p\right)\int x^{m+2p}(a+bx)^n\,dx$$
$$= \left(x^{-m-2p}(dx)^m\left(cx^2\right)^p(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int x^{m+2p}\left(1+\frac{bx}{a}\right)^n dx$$
$$= \frac{x(dx)^m\left(cx^2\right)^p(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{1+m+2p}$$

Mathematica [A] time = 0.0084658, size = 64, normalized size = 0.94

$$\frac{x\left(cx^2\right)^p(dx)^m(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(-n,m+2p+1;m+2p+2;-\frac{bx}{a}\right)}{m+2p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^p*(a + b*x)^n,x]

[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 + m +
 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int (dx)^m\left(cx^2\right)^p(bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^p*(b*x+a)^n,x)

[Out] int((d*x)^m*(c*x^2)^p*(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^p(bx+a)^n(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="maxima")

[Out] integrate((c*x^2)^p*(b*x + a)^n*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2\right)^p(bx+a)^n(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((c*x^2)^p*(b*x + a)^n*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^p (dx)^m (a+bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**p*(b*x+a)**n,x)
```

```
[Out] Integral((c*x**2)**p*(d*x)**m*(a + b*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2\right)^p (bx+a)^n (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^n*(d*x)^m, x)
```

3.999 $\int \frac{(a+bx)^5}{\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=17

$$\frac{b^2(a+bx)^3}{3d^3}$$

```
[Out] (b^2*(a + b*x)^3)/(3*d^3)
```

Rubi [A] time = 0.0037963, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$\frac{b^2(a+bx)^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^3)/(3*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int (a+bx)^2 dx}{d^3} \\ &= \frac{b^2(a+bx)^3}{3d^3} \end{aligned}$$

Mathematica [A] time = 0.0019294, size = 17, normalized size = 1.

$$\frac{b^2(a+bx)^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^3)/(3*d^3)
```

Maple [A] time = 0.001, size = 16, normalized size = 0.9

$$\frac{b^2 (bx + a)^3}{3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(a*d/b+d*x)^3,x)
```

```
[Out] 1/3*b^2*(b*x+a)^3/d^3
```

Maxima [B] time = 1.028, size = 42, normalized size = 2.47

$$\frac{b^5 x^3 + 3\, a b^4 x^2 + 3\, a^2 b^3 x}{3\, d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

Fricas [B] time = 1.49167, size = 63, normalized size = 3.71

$$\frac{b^5 x^3 + 3\, a b^4 x^2 + 3\, a^2 b^3 x}{3\, d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

Sympy [B] time = 0.101171, size = 34, normalized size = 2.

$$\frac{a^2 b^3 x}{d^3} + \frac{a b^4 x^2}{d^3} + \frac{b^5 x^3}{3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(a*d/b+d*x)**3,x)
```

```
[Out] a**2*b**3*x/d**3 + a*b**4*x**2/d**3 + b**5*x**3/(3*d**3)
```

Giac [B] time = 1.06376, size = 42, normalized size = 2.47

$$\frac{b^5 x^3 + 3\, a b^4 x^2 + 3\, a^2 b^3 x}{3\, d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

3.1000 $\int \frac{(a+bx)^4}{\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=23

$$\frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3}$$

```
[Out] (a*b^3*x)/d^3 + (b^4*x^2)/(2*d^3)
```

Rubi [A] time = 0.0057754, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {21}

$$\frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/((a*d)/b + d*x)^3,x]
```

```
[Out] (a*b^3*x)/d^3 + (b^4*x^2)/(2*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^4}{\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int (a+bx)\, dx}{d^3} \\ &= \frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3} \end{aligned}$$

Mathematica [A] time = 0.0009898, size = 19, normalized size = 0.83

$$\frac{b^3\left(ax+\frac{bx^2}{2}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^3*(a*x + (b*x^2)/2))/d^3
```

Maple [A] time = 0., size = 18, normalized size = 0.8

$$\frac{b^3\left(ax+\frac{bx^2}{2}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(a*d/b+d*x)^3,x)
```

```
[Out] b^3/d^3*(a*x+1/2*b*x^2)
```

Maxima [A] time = 1.01743, size = 27, normalized size = 1.17

$$\frac{b^4x^2+2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

Fricas [A] time = 1.56128, size = 42, normalized size = 1.83

$$\frac{b^4x^2+2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

Sympy [A] time = 0.096131, size = 20, normalized size = 0.87

$$\frac{ab^3x}{d^3}+\frac{b^4x^2}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(a*d/b+d*x)**3,x)
```

```
[Out] a*b**3*x/d**3 + b**4*x**2/(2*d**3)
```

Giac [A] time = 1.05981, size = 27, normalized size = 1.17

$$\frac{b^4x^2+2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

3.1001 $\int \frac{(a+bx)^3}{\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=8

$$\frac{b^3x}{d^3}$$

[Out] (b^3*x)/d^3

Rubi [A] time = 0.001698, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 8}

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^3/((a*d)/b + d*x)^3,x]

[Out] (b^3*x)/d^3

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^3}{\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int 1 \, dx}{d^3} \\ &= \frac{b^3x}{d^3} \end{aligned}$$

Mathematica [A] time = 0.0003215, size = 8, normalized size = 1.

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^3/((a*d)/b + d*x)^3,x]

```
[Out] (b^3*x)/d^3
```

Maple [A] time = 0.002, size = 9, normalized size = 1.1

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(a*d/b+d*x)^3,x)
```

```
[Out] b^3*x/d^3
```

Maxima [A] time = 1.00235, size = 11, normalized size = 1.38

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3
```

Fricas [A] time = 1.50043, size = 15, normalized size = 1.88

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] b^3*x/d^3
```

Sympy [A] time = 0.086447, size = 7, normalized size = 0.88

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(a*d/b+d*x)**3,x)
```

```
[Out] b**3*x/d**3
```

Giac [A] time = 1.06505, size = 11, normalized size = 1.38

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3
```

3.1002 $\int \frac{(a+bx)^2}{\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=13

$$\frac{b^2\log(a+bx)}{d^3}$$

```
[Out] (b^2*Log[a + b*x])/d^3
```

Rubi [A] time = 0.0033044, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 31}

$$\frac{b^2\log(a+bx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*Log[a + b*x])/d^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{a+bx}\,dx}{d^3}\\ &= \frac{b^2\log(a+bx)}{d^3}\end{aligned}$$

Mathematica [A] time = 0.0015988, size = 13, normalized size = 1.

$$\frac{b^2\log(a+bx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*Log[a + b*x])/d^3
```

Maple [A] time = 0.001, size = 14, normalized size = 1.1

$$\frac{b^2 \ln (bx + a)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(a*d/b+d*x)^3,x)
```

```
[Out] b^2*ln(b*x+a)/d^3
```

Maxima [A] time = 1.02462, size = 18, normalized size = 1.38

$$\frac{b^2 \log (bx + a)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/d^3
```

Fricas [A] time = 1.48896, size = 30, normalized size = 2.31

$$\frac{b^2 \log (bx + a)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] b^2*log(b*x + a)/d^3
```

Sympy [A] time = 0.091023, size = 19, normalized size = 1.46

$$\frac{b^2 \log \left(ad^3 + bd^3 x\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(a*d/b+d*x)**3,x)
```

```
[Out] b**2*log(a*d**3 + b*d**3*x)/d**3
```

Giac [A] time = 1.06317, size = 19, normalized size = 1.46

$$\frac{b^2 \log(|bx + a|)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] b^2*log(abs(b*x + a))/d^3
```

3.1003 $\int \frac{a+bx}{\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=15

$$-\frac{b^2}{d^3(a+bx)}$$

[Out] -(b^2/(d^3*(a + b*x)))

Rubi [A] time = 0.0033476, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$-\frac{b^2}{d^3(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/((a*d)/b + d*x)^3,x]

[Out] -(b^2/(d^3*(a + b*x)))

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^2}\,dx}{d^3}\\ &= -\frac{b^2}{d^3(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0039192, size = 15, normalized size = 1.

$$-\frac{b^2}{d^3(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/((a*d)/b + d*x)^3,x]

```
[Out] -(b^2/(d^3*(a + b*x)))
```

Maple [A] time = 0.001, size = 16, normalized size = 1.1

$$-\frac{b^2}{d^3 (bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(a*d/b+d*x)^3,x)
```

```
[Out] -b^2/d^3/(b*x+a)
```

Maxima [A] time = 0.979857, size = 26, normalized size = 1.73

$$-\frac{b^2}{bd^3x + ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -b^2/(b*d^3*x + a*d^3)
```

Fricas [A] time = 1.5042, size = 32, normalized size = 2.13

$$-\frac{b^2}{bd^3x + ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] -b^2/(b*d^3*x + a*d^3)
```

Sympy [A] time = 0.309068, size = 19, normalized size = 1.27

$$-\frac{b^3}{abd^3 + b^2d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(a*b*d**3 + b**2*d**3*x)
```

Giac [A] time = 1.06447, size = 20, normalized size = 1.33

$$-\frac{b^2}{(bx+a)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] -b^2/((b*x + a)*d^3)
```

3.1004 $\int \frac{1}{(a+bx)\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=17

$$-\frac{b^2}{3d^3(a+bx)^3}$$

[Out] -b^2/(3*d^3*(a + b*x)^3)

Rubi [A] time = 0.0033228, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$-\frac{b^2}{3d^3(a+bx)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)*((a*d)/b + d*x)^3),x]

[Out] -b^2/(3*d^3*(a + b*x)^3)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int \frac{1}{(a+bx)^4} dx}{d^3}\\ &= -\frac{b^2}{3d^3(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.0048979, size = 17, normalized size = 1.

$$-\frac{b^2}{3d^3(a+bx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x)*((a*d)/b + d*x)^3),x]

```
[Out] -b^2/(3*d^3*(a + b*x)^3)
```

Maple [A] time = 0., size = 16, normalized size = 0.9

$$-\frac{b^2}{3\,d^3\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(a*d/b+d*x)^3,x)
```

```
[Out] -1/3*b^2/d^3/(b*x+a)^3
```

Maxima [B] time = 0.99459, size = 63, normalized size = 3.71

$$-\frac{b^2}{3\left(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*b^2/(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
```

Fricas [B] time = 1.64416, size = 92, normalized size = 5.41

$$-\frac{b^2}{3\left(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/3*b^2/(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
```

Sympy [B] time = 0.404026, size = 53, normalized size = 3.12

$$-\frac{b^3}{3a^3bd^3+9a^2b^2d^3x+9ab^3d^3x^2+3b^4d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(3*a**3*b*d**3 + 9*a**2*b**2*d**3*x + 9*a*b**3*d**3*x**2 + 3*b**4*d**
3*x**3)
```

Giac [A] time = 1.07017, size = 20, normalized size = 1.18

$$-\frac{b^2}{3\,(bx+a)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] -1/3*b^2/((b*x + a)^3*d^3)
```

3.1005 $\int \frac{1}{(a+bx)^2\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{b^2}{4d^3(a+bx)^4}$$

```
[Out] -b^2/(4*d^3*(a + b*x)^4)
```

Rubi [A] time = 0.0032718, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$-\frac{b^2}{4d^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*((a*d)/b + d*x)^3),x]

[Out] -b^2/(4*d^3*(a + b*x)^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^5}\,dx}{d^3}\\ &= -\frac{b^2}{4d^3(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.0054998, size = 17, normalized size = 1.

$$-\frac{b^2}{4d^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*((a*d)/b + d*x)^3),x]
```

```
[Out] -b^2/(4*d^3*(a + b*x)^4)
```

Maple [A] time = 0., size = 16, normalized size = 0.9

$$-\frac{b^2}{4\,d^3\,(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(a*d/b+d*x)^3,x)
```

```
[Out] -1/4*b^2/d^3/(b*x+a)^4
```

Maxima [B] time = 1.0036, size = 82, normalized size = 4.82

$$-\frac{b^2}{4\left(b^4d^3x^4+4\,ab^3d^3x^3+6\,a^2b^2d^3x^2+4\,a^3bd^3x+a^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^2/(b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^3*b*d^3*x
 + a^4*d^3)
```

Fricas [B] time = 1.63316, size = 119, normalized size = 7.

$$-\frac{b^2}{4\left(b^4d^3x^4+4\,ab^3d^3x^3+6\,a^2b^2d^3x^2+4\,a^3bd^3x+a^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*b^2/(b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^3*b*d^3*x
 + a^4*d^3)
```

Sympy [B] time = 0.474918, size = 68, normalized size = 4.

$$-\frac{b^3}{4a^4bd^3+16a^3b^2d^3x+24a^2b^3d^3x^2+16ab^4d^3x^3+4b^5d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(4*a**4*b*d**3 + 16*a**3*b**2*d**3*x + 24*a**2*b**3*d**3*x**2 + 16*a*
b**4*d**3*x**3 + 4*b**5*d**3*x**4)
```

Giac [A] time = 1.0528, size = 20, normalized size = 1.18

$$-\frac{b^2}{4\,(bx+a)^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] -1/4*b^2/((b*x + a)^4*d^3)
```

3.1006 $\int \frac{1}{(a+bx)^3\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{b^2}{5d^3(a+bx)^5}$$

```
[Out] -b^2/(5*d^3*(a + b*x)^5)
```

Rubi [A] time = 0.003293, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$-\frac{b^2}{5d^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*((a*d)/b + d*x)^3),x]
```

```
[Out] -b^2/(5*d^3*(a + b*x)^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^6}\,dx}{d^3}\\ &= -\frac{b^2}{5d^3(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.0064176, size = 17, normalized size = 1.

$$-\frac{b^2}{5d^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*((a*d)/b + d*x)^3),x]
```

```
[Out] -b^2/(5*d^3*(a + b*x)^5)
```

Maple [A] time = 0.002, size = 16, normalized size = 0.9

$$-\frac{b^2}{5\,d^3\,(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(a*d/b+d*x)^3,x)
```

```
[Out] -1/5*b^2/d^3/(b*x+a)^5
```

Maxima [B] time = 1.02923, size = 101, normalized size = 5.94

$$-\frac{b^2}{5\left(b^5d^3x^5+5\,ab^4d^3x^4+10\,a^2b^3d^3x^3+10\,a^3b^2d^3x^2+5\,a^4bd^3x+a^5d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/5*b^2/(b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^2*b^3*d^3*x^3 + 10*a^3*b^2*d
^3*x^2 + 5*a^4*b*d^3*x + a^5*d^3)
```

Fricas [B] time = 1.69989, size = 149, normalized size = 8.76

$$-\frac{b^2}{5\left(b^5d^3x^5+5\,ab^4d^3x^4+10\,a^2b^3d^3x^3+10\,a^3b^2d^3x^2+5\,a^4bd^3x+a^5d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/5*b^2/(b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^2*b^3*d^3*x^3 + 10*a^3*b^2*d
^3*x^2 + 5*a^4*b*d^3*x + a^5*d^3)
```

Sympy [B] time = 0.546246, size = 83, normalized size = 4.88

$$-\frac{b^3}{5a^5bd^3+25a^4b^2d^3x+50a^3b^3d^3x^2+50a^2b^4d^3x^3+25ab^5d^3x^4+5b^6d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(5*a**5*b*d**3 + 25*a**4*b**2*d**3*x + 50*a**3*b**3*d**3*x**2 + 50*a*
*2*b**4*d**3*x**3 + 25*a*b**5*d**3*x**4 + 5*b**6*d**3*x**5)
```

Giac [A] time = 1.05078, size = 20, normalized size = 1.18

$$-\frac{b^2}{5\,(bx+a)^5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] -1/5*b^2/((b*x + a)^5*d^3)
```

3.1007 $\int \frac{\left(\frac{bc}{d}+bx\right)^5}{(c+dx)^3} \, dx$

Optimal. Leaf size=17

$$\frac{b^5(c+dx)^3}{3d^6}$$

```
[Out] (b^5*(c + d*x)^3)/(3*d^6)
```

Rubi [A] time = 0.0036141, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (b^5*(c + d*x)^3)/(3*d^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^5}{(c+dx)^3} \, dx &= \frac{b^5 \int (c+dx)^2 \, dx}{d^5}\\ &= \frac{b^5(c+dx)^3}{3d^6}\end{aligned}$$

Mathematica [A] time = 0.0024827, size = 17, normalized size = 1.

$$\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (b^5*(c + d*x)^3)/(3*d^6)
```

Maple [A] time = 0.003, size = 16, normalized size = 0.9

$$\frac{b^5 (dx + c)^3}{3 d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^5/(d*x+c)^3,x)
```

```
[Out] 1/3*b^5*(d*x+c)^3/d^6
```

Maxima [B] time = 1.03419, size = 47, normalized size = 2.76

$$\frac{b^5 d^2 x^3 + 3 b^5 c d x^2 + 3 b^5 c^2 x}{3 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

Fricas [B] time = 1.73966, size = 72, normalized size = 4.24

$$\frac{b^5 d^2 x^3 + 3 b^5 c d x^2 + 3 b^5 c^2 x}{3 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

Sympy [B] time = 0.107849, size = 34, normalized size = 2.

$$\frac{b^5 c^2 x}{d^5} + \frac{b^5 c x^2}{d^4} + \frac{b^5 x^3}{3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**5/(d*x+c)**3,x)
```

```
[Out] b**5*c**2*x/d**5 + b**5*c*x**2/d**4 + b**5*x**3/(3*d**3)
```

Giac [B] time = 1.06061, size = 47, normalized size = 2.76

$$\frac{b^5 d^2 x^3 + 3 b^5 c d x^2 + 3 b^5 c^2 x}{3 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

3.1008 $\int \frac{\left(\frac{bc}{d}+bx\right)^4}{(c+dx)^3}\,dx$

Optimal. Leaf size=23

$$\frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}$$

```
[Out] (b^4*c*x)/d^4 + (b^4*x^2)/(2*d^3)
```

Rubi [A] time = 0.0049852, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {21}

$$\frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (b^4*c*x)/d^4 + (b^4*x^2)/(2*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^4}{(c+dx)^3}\,dx &= \frac{b^4\int (c+dx)\,dx}{d^4}\\ &= \frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}\end{aligned}$$

Mathematica [A] time = 0.001033, size = 19, normalized size = 0.83

$$\frac{b^4\left(cx+\frac{dx^2}{2}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (b^4*(c*x + (d*x^2)/2))/d^4
```

Maple [A] time = 0.001, size = 18, normalized size = 0.8

$$\frac{b^4}{d^4}\left(cx+\frac{dx^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^4/(d*x+c)^3,x)
```

```
[Out] b^4/d^4*(c*x+1/2*d*x^2)
```

Maxima [A] time = 1.02181, size = 28, normalized size = 1.22

$$\frac{b^4dx^2+2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

Fricas [A] time = 1.78849, size = 45, normalized size = 1.96

$$\frac{b^4dx^2+2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

Sympy [A] time = 0.100628, size = 20, normalized size = 0.87

$$\frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**4/(d*x+c)**3,x)
```

```
[Out] b**4*c*x/d**4 + b**4*x**2/(2*d**3)
```

Giac [A] time = 1.05597, size = 28, normalized size = 1.22

$$\frac{b^4dx^2+2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

3.1009 $\int \frac{\left(\frac{bc}{d}+bx\right)^3}{(c+dx)^3}\,dx$

Optimal. Leaf size=8

$$\frac{b^3x}{d^3}$$

```
[Out] (b^3*x)/d^3
```

Rubi [A] time = 0.0014703, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 8}

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^3}{(c+dx)^3}\,dx &= \frac{b^3\int 1\,dx}{d^3}\\ &= \frac{b^3x}{d^3}\end{aligned}$$

Mathematica [A] time = 0.0003523, size = 8, normalized size = 1.

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3
```

Maple [A] time = 0., size = 9, normalized size = 1.1

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^3/(d*x+c)^3,x)
```

```
[Out] b^3*x/d^3
```

Maxima [A] time = 1.00999, size = 11, normalized size = 1.38

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3
```

Fricas [A] time = 1.64186, size = 15, normalized size = 1.88

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] b^3*x/d^3
```

Sympy [A] time = 0.088005, size = 7, normalized size = 0.88

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**3/(d*x+c)**3,x)
```

```
[Out] b**3*x/d**3
```

Giac [A] time = 1.08361, size = 11, normalized size = 1.38

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3
```

3.1010 $\int \frac{\left(\frac{bc}{d}+bx\right)^2}{(c+dx)^3} dx$

Optimal. Leaf size=13

$$\frac{b^2 \log(c+dx)}{d^3}$$

```
[Out] (b^2*Log[c + d*x])/d^3
```

Rubi [A] time = 0.0030361, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 31}

$$\frac{b^2 \log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Log[c + d*x])/d^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\left(\frac{bc}{d}+bx\right)^2}{(c+dx)^3} dx &= \frac{b^2 \int \frac{1}{c+dx} dx}{d^2} \\ &= \frac{b^2 \log(c+dx)}{d^3} \end{aligned}$$

Mathematica [A] time = 0.0019214, size = 13, normalized size = 1.

$$\frac{b^2 \log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Log[c + d*x])/d^3
```

Maple [A] time = 0.001, size = 14, normalized size = 1.1

$$\frac{b^2 \ln (dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^2/(d*x+c)^3,x)

[Out] b^2*ln(d*x+c)/d^3
```

Maxima [A] time = 1.00713, size = 18, normalized size = 1.38

$$\frac{b^2 \log (dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="maxima")

[Out] b^2*log(d*x + c)/d^3
```

Fricas [A] time = 1.55221, size = 30, normalized size = 2.31

$$\frac{b^2 \log (dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="fricas")

[Out] b^2*log(d*x + c)/d^3
```

Sympy [A] time = 0.088541, size = 17, normalized size = 1.31

$$\frac{b^2 \log \left(cd^2 + d^3 x\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**2/(d*x+c)**3,x)

[Out] b**2*log(c*d**2 + d**3*x)/d**3
```

Giac [A] time = 1.07414, size = 19, normalized size = 1.46

$$\frac{b^2 \log(|dx + c|)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^2*log(abs(d*x + c))/d^3
```

3.1011 $\int \frac{\frac{bc}{d}+bx}{(c+dx)^3} dx$

Optimal. Leaf size=13

$$-\frac{b}{d^2(c+dx)}$$

```
[Out] -(b/(d^2*(c + d*x)))
```

Rubi [A] time = 0.0032366, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$-\frac{b}{d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)/(c + d*x)^3,x]

[Out] -(b/(d^2*(c + d*x)))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\frac{bc}{d}+bx}{(c+dx)^3} dx &= \frac{b \int \frac{1}{(c+dx)^2} dx}{d} \\ &= -\frac{b}{d^2(c+dx)} \end{aligned}$$

Mathematica [A] time = 0.0040674, size = 13, normalized size = 1.

$$-\frac{b}{d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)/(c + d*x)^3,x]
```

```
[Out] -(b/(d^2*(c + d*x)))
```

Maple [A] time = 0.001, size = 14, normalized size = 1.1

$$-\frac{b}{d^2(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)/(d*x+c)^3,x)
```

```
[Out] -b/d^2/(d*x+c)
```

Maxima [A] time = 0.987419, size = 22, normalized size = 1.69

$$-\frac{b}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -b/(d^3*x + c*d^2)
```

Fricas [A] time = 1.45861, size = 27, normalized size = 2.08

$$-\frac{b}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -b/(d^3*x + c*d^2)
```

Sympy [A] time = 0.284583, size = 12, normalized size = 0.92

$$-\frac{b}{cd^2+d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)**3,x)
```

```
[Out] -b/(c*d**2 + d**3*x)
```

Giac [A] time = 1.05929, size = 18, normalized size = 1.38

$$-\frac{b}{(dx+c)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -b/((d*x + c)*d^2)
```

3.1012 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)(c+dx)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{3b(c+dx)^3}$$

```
[Out] -1/(3*b*(c + d*x)^3)
```

Rubi [A] time = 0.0029554, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$-\frac{1}{3b(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*c)/d + b*x)*(c + d*x)^3),x]
```

```
[Out] -1/(3*b*(c + d*x)^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx\right)(c+dx)^3}\,dx &= \frac{d\int \frac{1}{(c+dx)^4}\,dx}{b}\\ &= -\frac{1}{3b(c+dx)^3}\end{aligned}$$

Mathematica [A] time = 0.0052916, size = 14, normalized size = 1.

$$-\frac{1}{3b(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)*(c + d*x)^3),x]
```

```
[Out] -1/(3*b*(c + d*x)^3)
```

Maple [A] time = 0.001, size = 13, normalized size = 0.9

$$-\frac{1}{3\,b\,(dx+c)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)/(d*x+c)^3,x)
```

```
[Out] -1/3/b/(d*x+c)^3
```

Maxima [B] time = 1.00733, size = 49, normalized size = 3.5

$$-\frac{1}{3\left(bd^{3}x^{3}+3\,bcd^{2}x^{2}+3\,bc^{2}dx+bc^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/3/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)
```

Fricas [B] time = 1.44479, size = 76, normalized size = 5.43

$$-\frac{1}{3\left(bd^{3}x^{3}+3\,bcd^{2}x^{2}+3\,bc^{2}dx+bc^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)
```

Sympy [B] time = 0.399423, size = 44, normalized size = 3.14

$$-\frac{d}{3bc^{3}d+9bc^{2}d^{2}x+9bcd^{3}x^{2}+3bd^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)**3,x)
```

```
[Out] -d/(3*b*c**3*d + 9*b*c**2*d**2*x + 9*b*c*d**3*x**2 + 3*b*d**4*x**3)
```

Giac [A] time = 1.0682, size = 16, normalized size = 1.14

$$-\frac{1}{3\,(dx+c)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/3/((d*x + c)^3*b)
```

3.1013 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)^2 (c+dx)^3} \, dx$

Optimal. Leaf size=15

$$-\frac{d}{4b^2(c+dx)^4}$$

```
[Out] -d/(4*b^2*(c + d*x)^4)
```

Rubi [A] time = 0.0032222, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$-\frac{d}{4b^2(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*c)/d + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -d/(4*b^2*(c + d*x)^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx\right)^2 (c+dx)^3} \, dx &= \frac{d^2 \int \frac{1}{(c+dx)^5} \, dx}{b^2}\\ &= -\frac{d}{4b^2(c+dx)^4}\end{aligned}$$

Mathematica [A] time = 0.0053454, size = 15, normalized size = 1.

$$-\frac{d}{4b^2(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -d/(4*b^2*(c + d*x)^4)
```

Maple [A] time = 0.001, size = 14, normalized size = 0.9

$$-\frac{d}{4 b^2 (dx + c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)^2/(d*x+c)^3,x)
```

```
[Out] -1/4*d/b^2/(d*x+c)^4
```

Maxima [B] time = 1.00136, size = 80, normalized size = 5.33

$$-\frac{d}{4 \left(b^2 d^4 x^4 + 4 b^2 c d^3 x^3 + 6 b^2 c^2 d^2 x^2 + 4 b^2 c^3 dx + b^2 c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*d/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
 b^2*c^4)
```

Fricas [B] time = 1.44963, size = 116, normalized size = 7.73

$$-\frac{d}{4 \left(b^2 d^4 x^4 + 4 b^2 c d^3 x^3 + 6 b^2 c^2 d^2 x^2 + 4 b^2 c^3 dx + b^2 c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*d/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
 b^2*c^4)
```

Sympy [B] time = 0.48291, size = 68, normalized size = 4.53

$$-\frac{d^2}{4b^2c^4d + 16b^2c^3d^2x + 24b^2c^2d^3x^2 + 16b^2cd^4x^3 + 4b^2d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)**2/(d*x+c)**3,x)
```

```
[Out] -d**2/(4*b**2*c**4*d + 16*b**2*c**3*d**2*x + 24*b**2*c**2*d**3*x**2 + 16*b*
*2*c*d**4*x**3 + 4*b**2*d**5*x**4)
```

Giac [A] time = 1.06731, size = 27, normalized size = 1.8

$$-\frac{b^2}{4\left(bx+\frac{bc}{d}\right)^4 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*b^2/((b*x + b*c/d)^4*d^3)
```

3.1014 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)^3 (c+dx)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{d^2}{5b^3(c+dx)^5}$$

```
[Out] -d^2/(5*b^3*(c + d*x)^5)
```

Rubi [A] time = 0.0034984, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$-\frac{d^2}{5b^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*c)/d + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -d^2/(5*b^3*(c + d*x)^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx\right)^3 (c+dx)^3}\,dx &= \frac{d^3 \int \frac{1}{(c+dx)^6}\,dx}{b^3}\\ &= -\frac{d^2}{5b^3(c+dx)^5}\end{aligned}$$

Mathematica [A] time = 0.0057995, size = 17, normalized size = 1.

$$-\frac{d^2}{5b^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -d^2/(5*b^3*(c + d*x)^5)
```

Maple [A] time = 0., size = 16, normalized size = 0.9

$$-\frac{d^2}{5\,b^3\,(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)^3/(d*x+c)^3,x)
```

```
[Out] -1/5*d^2/b^3/(d*x+c)^5
```

Maxima [B] time = 1.05315, size = 101, normalized size = 5.94

$$-\frac{d^2}{5\left(b^3d^5x^5+5\,b^3cd^4x^4+10\,b^3c^2d^3x^3+10\,b^3c^3d^2x^2+5\,b^3c^4dx+b^3c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/5*d^2/(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d
^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)
```

Fricas [B] time = 1.39587, size = 149, normalized size = 8.76

$$-\frac{d^2}{5\left(b^3d^5x^5+5\,b^3cd^4x^4+10\,b^3c^2d^3x^3+10\,b^3c^3d^2x^2+5\,b^3c^4dx+b^3c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/5*d^2/(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d
^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)
```

Sympy [B] time = 0.571301, size = 83, normalized size = 4.88

$$-\frac{d^3}{5b^3c^5d+25b^3c^4d^2x+50b^3c^3d^3x^2+50b^3c^2d^4x^3+25b^3cd^5x^4+5b^3d^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)**3/(d*x+c)**3,x)
```

```
[Out] -d**3/(5*b**3*c**5*d + 25*b**3*c**4*d**2*x + 50*b**3*c**3*d**3*x**2 + 50*b*
*3*c**2*d**4*x**3 + 25*b**3*c*d**5*x**4 + 5*b**3*d**6*x**5)
```

Giac [A] time = 1.05843, size = 20, normalized size = 1.18

$$-\frac{d^2}{5(dx+c)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/5*d^2/((d*x + c)^5*b^3)
```

3.1015 $\int (a+bx)^5(ac+bcx)^n\,dx$

Optimal. Leaf size=24

$$\frac{(ac+bcx)^{n+6}}{bc^6(n+6)}$$

```
[Out] (a*c + b*c*x)^(6 + n)/(b*c^6*(6 + n))
```

Rubi [A] time = 0.0088863, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{(ac+bcx)^{n+6}}{bc^6(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^n,x]

[Out] (a*c + b*c*x)^(6 + n)/(b*c^6*(6 + n))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^n\,dx &= \frac{\int (ac+bcx)^{5+n}\,dx}{c^5}\\ &= \frac{(ac+bcx)^{6+n}}{bc^6(6+n)}\end{aligned}$$

Mathematica [A] time = 0.0196693, size = 25, normalized size = 1.04

$$\frac{(a+bx)^6(c(a+bx))^n}{b(n+6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^n,x]

[Out] ((a + b*x)^6*(c*(a + b*x))^n)/(b*(6 + n))
```

Maple [A] time = 0.002, size = 27, normalized size = 1.1

$$\frac{(bx+a)^6(bcx+ac)^n}{b(6+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^n,x)
```

```
[Out] (b*x+a)^6/b/(6+n)*(b*c*x+a*c)^n
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.6815, size = 165, normalized size = 6.88

$$\frac{\left(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6\right)(bcx+ac)^n}{bn+6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] (b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 +
 6*a^5*b*x + a^6)*(b*c*x + a*c)^n/(b*n + 6*b)
```

Sympy [A] time = 1.76593, size = 212, normalized size = 8.83

$$\begin{cases} \frac{x}{ac^6} & \text{for } b = 0 \\ a^5x(ac)^n & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b}+x\right)}{bc^6} & \text{for } n = - \\ \frac{a^6(ac+bcx)^n}{bn+6b} + \frac{6a^5bx(ac+bcx)^n}{bn+6b} + \frac{15a^4b^2x^2(ac+bcx)^n}{bn+6b} + \frac{20a^3b^3x^3(ac+bcx)^n}{bn+6b} + \frac{15a^2b^4x^4(ac+bcx)^n}{bn+6b} + \frac{6ab^5x^5(ac+bcx)^n}{bn+6b} + \frac{b^6x^6(ac+bcx)^n}{bn+6b} & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((x/(a*c**6), Eq(b, 0) & Eq(n, -6)), (a**5*x*(a*c)**n, Eq(b, 0)),
(log(a/b + x)/(b*c**6), Eq(n, -6)), (a**6*(a*c + b*c*x)**n/(b*n + 6*b) + 6*
a**5*b*x*(a*c + b*c*x)**n/(b*n + 6*b) + 15*a**4*b**2*x**2*(a*c + b*c*x)**n/
(b*n + 6*b) + 20*a**3*b**3*x**3*(a*c + b*c*x)**n/(b*n + 6*b) + 15*a**2*b**4
```

```
*x**4*(a*c + b*c*x)**n/(b*n + 6*b) + 6*a*b**5*x**5*(a*c + b*c*x)**n/(b*n +
6*b) + b**6*x**6*(a*c + b*c*x)**n/(b*n + 6*b), True))
```

Giac [B] time = 1.07213, size = 190, normalized size = 7.92

$$\frac{(bcx+ac)^n b^6x^6 + 6\,(bcx+ac)^n ab^5x^5 + 15\,(bcx+ac)^n a^2b^4x^4 + 20\,(bcx+ac)^n a^3b^3x^3 + 15\,(bcx+ac)^n a^4b^2x^2 + 6\,(bcx+}{bn+6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="giac")
```

```
[Out] ((b*c*x + a*c)^n*b^6*x^6 + 6*(b*c*x + a*c)^n*a*b^5*x^5 + 15*(b*c*x + a*c)^n
*a^2*b^4*x^4 + 20*(b*c*x + a*c)^n*a^3*b^3*x^3 + 15*(b*c*x + a*c)^n*a^4*b^2*
x^2 + 6*(b*c*x + a*c)^n*a^5*b*x + (b*c*x + a*c)^n*a^6)/(b*n + 6*b)
```

3.1016 $\int (a+bx)^5(ac+bcx)^3\,dx$

Optimal. Leaf size=17

$$\frac{c^3(a+bx)^9}{9b}$$

```
[Out] (c^3*(a + b*x)^9)/(9*b)
```

Rubi [A] time = 0.0038523, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{c^3(a+bx)^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^3,x]
```

```
[Out] (c^3*(a + b*x)^9)/(9*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^3\,dx &= c^3\int (a+bx)^8\,dx\\ &= \frac{c^3(a+bx)^9}{9b}\end{aligned}$$

Mathematica [A] time = 0.0020879, size = 17, normalized size = 1.

$$\frac{c^3(a+bx)^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^3,x]
```

```
[Out] (c^3*(a + b*x)^9)/(9*b)
```

Maple [B] time = 0.001, size = 114, normalized size = 6.7

$$\frac{b^8c^3x^9}{9} + ab^7c^3x^8 + 4\,a^2b^6c^3x^7 + \frac{28\,a^3b^5c^3x^6}{3} + 14\,a^4b^4c^3x^5 + 14\,a^5b^3c^3x^4 + \frac{28\,a^6c^3b^2x^3}{3} + 4\,a^7c^3bx^2 + a^8c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^3,x)
```

```
[Out] 1/9*b^8*c^3*x^9+a*b^7*c^3*x^8+4*a^2*b^6*c^3*x^7+28/3*a^3*b^5*c^3*x^6+14*a^4
*b^4*c^3*x^5+14*a^5*b^3*c^3*x^4+28/3*a^6*c^3*b^2*x^3+4*a^7*c^3*b*x^2+a^8*c^
3*x
```

Maxima [B] time = 1.03238, size = 153, normalized size = 9.

$$\frac{1}{9}\,b^8c^3x^9 + ab^7c^3x^8 + 4\,a^2b^6c^3x^7 + \frac{28}{3}\,a^3b^5c^3x^6 + 14\,a^4b^4c^3x^5 + 14\,a^5b^3c^3x^4 + \frac{28}{3}\,a^6b^2c^3x^3 + 4\,a^7bc^3x^2 + a^8c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^8*c^3*x^9 + a*b^7*c^3*x^8 + 4*a^2*b^6*c^3*x^7 + 28/3*a^3*b^5*c^3*x^6
+ 14*a^4*b^4*c^3*x^5 + 14*a^5*b^3*c^3*x^4 + 28/3*a^6*b^2*c^3*x^3 + 4*a^7*b*
c^3*x^2 + a^8*c^3*x
```

Fricas [B] time = 1.32902, size = 231, normalized size = 13.59

$$\frac{1}{9}x^9c^3b^8 + x^8c^3b^7a + 4x^7c^3b^6a^2 + \frac{28}{3}x^6c^3b^5a^3 + 14x^5c^3b^4a^4 + 14x^4c^3b^3a^5 + \frac{28}{3}x^3c^3b^2a^6 + 4x^2c^3ba^7 + xc^3a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^3*b^8 + x^8*c^3*b^7*a + 4*x^7*c^3*b^6*a^2 + 28/3*x^6*c^3*b^5*a^3
+ 14*x^5*c^3*b^4*a^4 + 14*x^4*c^3*b^3*a^5 + 28/3*x^3*c^3*b^2*a^6 + 4*x^2*c^
3*b*a^7 + x*c^3*a^8
```

Sympy [B] time = 0.088348, size = 124, normalized size = 7.29

$$a^8c^3x + 4a^7bc^3x^2 + \frac{28a^6b^2c^3x^3}{3} + 14a^5b^3c^3x^4 + 14a^4b^4c^3x^5 + \frac{28a^3b^5c^3x^6}{3} + 4a^2b^6c^3x^7 + ab^7c^3x^8 + \frac{b^8c^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**3,x)
```

```
[Out] a**8*c**3*x + 4*a**7*b*c**3*x**2 + 28*a**6*b**2*c**3*x**3/3 + 14*a**5*b**3*
c**3*x**4 + 14*a**4*b**4*c**3*x**5 + 28*a**3*b**5*c**3*x**6/3 + 4*a**2*b**6
*c**3*x**7 + a*b**7*c**3*x**8 + b**8*c**3*x**9/9
```

Giac [B] time = 1.07406, size = 153, normalized size = 9.

$$\frac{1}{9}b^8c^3x^9 + ab^7c^3x^8 + 4a^2b^6c^3x^7 + \frac{28}{3}a^3b^5c^3x^6 + 14a^4b^4c^3x^5 + 14a^5b^3c^3x^4 + \frac{28}{3}a^6b^2c^3x^3 + 4a^7bc^3x^2 + a^8c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] 1/9*b^8*c^3*x^9 + a*b^7*c^3*x^8 + 4*a^2*b^6*c^3*x^7 + 28/3*a^3*b^5*c^3*x^6
+ 14*a^4*b^4*c^3*x^5 + 14*a^5*b^3*c^3*x^4 + 28/3*a^6*b^2*c^3*x^3 + 4*a^7*b*
c^3*x^2 + a^8*c^3*x
```

3.1017 $\int (a+bx)^5(ac+bcx)^2\,dx$

Optimal. Leaf size=17

$$\frac{c^2(a+bx)^8}{8b}$$

```
[Out] (c^2*(a + b*x)^8)/(8*b)
```

Rubi [A] time = 0.0037584, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{c^2(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^2,x]
```

```
[Out] (c^2*(a + b*x)^8)/(8*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^2\,dx &= c^2\int (a+bx)^7\,dx\\ &= \frac{c^2(a+bx)^8}{8b}\end{aligned}$$

Mathematica [A] time = 0.0020709, size = 17, normalized size = 1.

$$\frac{c^2(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^2,x]
```

```
[Out] (c^2*(a + b*x)^8)/(8*b)
```

Maple [B] time = 0.002, size = 100, normalized size = 5.9

$$\frac{b^7c^2x^8}{8} + ab^6c^2x^7 + \frac{7\,a^2b^5c^2x^6}{2} + 7\,a^3b^4c^2x^5 + \frac{35\,a^4b^3c^2x^4}{4} + 7\,a^5b^2c^2x^3 + \frac{7\,a^6c^2bx^2}{2} + a^7c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^2,x)
```

```
[Out] 1/8*b^7*c^2*x^8+a*b^6*c^2*x^7+7/2*a^2*b^5*c^2*x^6+7*a^3*b^4*c^2*x^5+35/4*a^
4*b^3*c^2*x^4+7*a^5*b^2*c^2*x^3+7/2*a^6*c^2*b*x^2+a^7*c^2*x
```

Maxima [B] time = 1.00701, size = 134, normalized size = 7.88

$$\frac{1}{8}\,b^7c^2x^8 + ab^6c^2x^7 + \frac{7}{2}\,a^2b^5c^2x^6 + 7\,a^3b^4c^2x^5 + \frac{35}{4}\,a^4b^3c^2x^4 + 7\,a^5b^2c^2x^3 + \frac{7}{2}\,a^6bc^2x^2 + a^7c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^7*c^2*x^8 + a*b^6*c^2*x^7 + 7/2*a^2*b^5*c^2*x^6 + 7*a^3*b^4*c^2*x^5 +
 35/4*a^4*b^3*c^2*x^4 + 7*a^5*b^2*c^2*x^3 + 7/2*a^6*b*c^2*x^2 + a^7*c^2*x
```

Fricas [B] time = 1.37589, size = 203, normalized size = 11.94

$$\frac{1}{8}x^8c^2b^7 + x^7c^2b^6a + \frac{7}{2}x^6c^2b^5a^2 + 7x^5c^2b^4a^3 + \frac{35}{4}x^4c^2b^3a^4 + 7x^3c^2b^2a^5 + \frac{7}{2}x^2c^2ba^6 + xc^2a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*c^2*b^7 + x^7*c^2*b^6*a + 7/2*x^6*c^2*b^5*a^2 + 7*x^5*c^2*b^4*a^3 +
 35/4*x^4*c^2*b^3*a^4 + 7*x^3*c^2*b^2*a^5 + 7/2*x^2*c^2*b*a^6 + x*c^2*a^7
```

Sympy [B] time = 0.085406, size = 110, normalized size = 6.47

$$a^7c^2x + \frac{7a^6bc^2x^2}{2} + 7a^5b^2c^2x^3 + \frac{35a^4b^3c^2x^4}{4} + 7a^3b^4c^2x^5 + \frac{7a^2b^5c^2x^6}{2} + ab^6c^2x^7 + \frac{b^7c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**2,x)
```

```
[Out] a**7*c**2*x + 7*a**6*b*c**2*x**2/2 + 7*a**5*b**2*c**2*x**3 + 35*a**4*b**3*c
**2*x**4/4 + 7*a**3*b**4*c**2*x**5 + 7*a**2*b**5*c**2*x**6/2 + a*b**6*c**2*
x**7 + b**7*c**2*x**8/8
```

Giac [B] time = 1.07167, size = 134, normalized size = 7.88

$$\frac{1}{8}b^7c^2x^8 + ab^6c^2x^7 + \frac{7}{2}a^2b^5c^2x^6 + 7a^3b^4c^2x^5 + \frac{35}{4}a^4b^3c^2x^4 + 7a^5b^2c^2x^3 + \frac{7}{2}a^6bc^2x^2 + a^7c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^7*c^2*x^8 + a*b^6*c^2*x^7 + 7/2*a^2*b^5*c^2*x^6 + 7*a^3*b^4*c^2*x^5 +
 35/4*a^4*b^3*c^2*x^4 + 7*a^5*b^2*c^2*x^3 + 7/2*a^6*b*c^2*x^2 + a^7*c^2*x
```

3.1018 $\int (a+bx)^5(ac+bcx)\,dx$

Optimal. Leaf size=15

$$\frac{c(a+bx)^7}{7b}$$

```
[Out] (c*(a + b*x)^7)/(7*b)
```

Rubi [A] time = 0.0029926, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {21, 32}

$$\frac{c(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x),x]
```

```
[Out] (c*(a + b*x)^7)/(7*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)\,dx &= c\int (a+bx)^6\,dx\\ &= \frac{c(a+bx)^7}{7b}\end{aligned}$$

Mathematica [A] time = 0.0017526, size = 15, normalized size = 1.

$$\frac{c(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x),x]
```

```
[Out] (c*(a + b*x)^7)/(7*b)
```

Maple [B] time = 0.001, size = 72, normalized size = 4.8

$$\frac{b^6 c x^7}{7} + a b^5 c x^6 + 3 a^2 b^4 c x^5 + 5 a^3 b^3 c x^4 + 5 a^4 b^2 c x^3 + 3 a^5 b c x^2 + a^6 c x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c),x)
```

```
[Out] 1/7*b^6*c*x^7+a*b^5*c*x^6+3*a^2*b^4*c*x^5+5*a^3*b^3*c*x^4+5*a^4*b^2*c*x^3+3
*a^5*b*c*x^2+a^6*c*x
```

Maxima [B] time = 0.993697, size = 96, normalized size = 6.4

$$\frac{1}{7} b^6 c x^7 + a b^5 c x^6 + 3 a^2 b^4 c x^5 + 5 a^3 b^3 c x^4 + 5 a^4 b^2 c x^3 + 3 a^5 b c x^2 + a^6 c x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/7*b^6*c*x^7 + a*b^5*c*x^6 + 3*a^2*b^4*c*x^5 + 5*a^3*b^3*c*x^4 + 5*a^4*b^2
*c*x^3 + 3*a^5*b*c*x^2 + a^6*c*x
```

Fricas [B] time = 1.3624, size = 147, normalized size = 9.8

$$\frac{1}{7} x^7 c b^6 + x^6 c b^5 a + 3 x^5 c b^4 a^2 + 5 x^4 c b^3 a^3 + 5 x^3 c b^2 a^4 + 3 x^2 c b a^5 + x c a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c*b^6 + x^6*c*b^5*a + 3*x^5*c*b^4*a^2 + 5*x^4*c*b^3*a^3 + 5*x^3*c*b
^2*a^4 + 3*x^2*c*b*a^5 + x*c*a^6
```

Sympy [B] time = 0.075771, size = 78, normalized size = 5.2

$$a^6 c x + 3 a^5 b c x^2 + 5 a^4 b^2 c x^3 + 5 a^3 b^3 c x^4 + 3 a^2 b^4 c x^5 + a b^5 c x^6 + \frac{b^6 c x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c),x)
```

```
[Out] a**6*c*x + 3*a**5*b*c*x**2 + 5*a**4*b**2*c*x**3 + 5*a**3*b**3*c*x**4 + 3*a*
*2*b**4*c*x**5 + a*b**5*c*x**6 + b**6*c*x**7/7
```

Giac [B] time = 1.05803, size = 96, normalized size = 6.4

$$\frac{1}{7} b^6 c x^7 + a b^5 c x^6 + 3 a^2 b^4 c x^5 + 5 a^3 b^3 c x^4 + 5 a^4 b^2 c x^3 + 3 a^5 b c x^2 + a^6 c x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] 1/7*b^6*c*x^7 + a*b^5*c*x^6 + 3*a^2*b^4*c*x^5 + 5*a^3*b^3*c*x^4 + 5*a^4*b^2
*c*x^3 + 3*a^5*b*c*x^2 + a^6*c*x
```

3.1019 $\int \frac{(a+bx)^5}{ac+bcx} dx$

Optimal. Leaf size=17

$$\frac{(a+bx)^5}{5bc}$$

```
[Out] (a + b*x)^5/(5*b*c)
```

Rubi [A] time = 0.0035367, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{(a+bx)^5}{5bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x),x]
```

```
[Out] (a + b*x)^5/(5*b*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{ac+bcx} dx &= \frac{\int (a+bx)^4 dx}{c}\\ &= \frac{(a+bx)^5}{5bc}\end{aligned}$$

Mathematica [A] time = 0.0014878, size = 17, normalized size = 1.

$$\frac{(a+bx)^5}{5bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x),x]
```

```
[Out] (a + b*x)^5/(5*b*c)
```

Maple [A] time = 0.001, size = 16, normalized size = 0.9

$$\frac{(bx+a)^5}{5\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c),x)
```

```
[Out] 1/5*(b*x+a)^5/b/c
```

Maxima [B] time = 1.01414, size = 65, normalized size = 3.82

$$\frac{b^4x^5+5\,ab^3x^4+10\,a^2b^2x^3+10\,a^3bx^2+5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

Fricas [B] time = 1.41319, size = 99, normalized size = 5.82

$$\frac{b^4x^5+5\,ab^3x^4+10\,a^2b^2x^3+10\,a^3bx^2+5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

Sympy [B] time = 0.089336, size = 51, normalized size = 3.

$$\frac{a^4x}{c}+\frac{2a^3bx^2}{c}+\frac{2a^2b^2x^3}{c}+\frac{ab^3x^4}{c}+\frac{b^4x^5}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c),x)
```

```
[Out] a**4*x/c + 2*a**3*b*x**2/c + 2*a**2*b**2*x**3/c + a*b**3*x**4/c + b**4*x**5
/(5*c)
```

Giac [B] time = 1.08084, size = 65, normalized size = 3.82

$$\frac{b^4x^5+5\,ab^3x^4+10\,a^2b^2x^3+10\,a^3bx^2+5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

3.1020 $\int \frac{(a+bx)^5}{(ac+bcx)^2} dx$

Optimal. Leaf size=17

$$\frac{(a+bx)^4}{4bc^2}$$

```
[Out] (a + b*x)^4/(4*b*c^2)
```

Rubi [A] time = 0.0040407, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{(a+bx)^4}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^2,x]
```

```
[Out] (a + b*x)^4/(4*b*c^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^2} dx &= \frac{\int (a+bx)^3 \, dx}{c^2}\\ &= \frac{(a+bx)^4}{4bc^2}\end{aligned}$$

Mathematica [A] time = 0.0013063, size = 17, normalized size = 1.

$$\frac{(a+bx)^4}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^2,x]
```

```
[Out] (a + b*x)^4/(4*b*c^2)
```

Maple [A] time = 0., size = 16, normalized size = 0.9

$$\frac{(bx+a)^4}{4\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^2,x)
```

```
[Out] 1/4*(b*x+a)^4/b/c^2
```

Maxima [B] time = 0.987722, size = 50, normalized size = 2.94

$$\frac{b^3x^4+4\,ab^2x^3+6\,a^2bx^2+4\,a^3x}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)/c^2
```

Fricas [B] time = 1.42017, size = 77, normalized size = 4.53

$$\frac{b^3x^4+4\,ab^2x^3+6\,a^2bx^2+4\,a^3x}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)/c^2
```

Sympy [B] time = 0.094837, size = 46, normalized size = 2.71

$$\frac{a^3x}{c^2}+\frac{3a^2bx^2}{2c^2}+\frac{ab^2x^3}{c^2}+\frac{b^3x^4}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**2,x)
```

```
[Out] a**3*x/c**2 + 3*a**2*b*x**2/(2*c**2) + a*b**2*x**3/c**2 + b**3*x**4/(4*c**2
)
```

Giac [A] time = 1.07711, size = 24, normalized size = 1.41

$$\frac{(bcx+ac)^4}{4\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*c*x + a*c)^4/(b*c^6)
```

3.1021 $\int \frac{(a+bx)^5}{(ac+bcx)^3}\,dx$

Optimal. Leaf size=17

$$\frac{(a+bx)^3}{3bc^3}$$

[Out] (a + b*x)^3/(3*b*c^3)

Rubi [A] time = 0.0040241, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{(a+bx)^3}{3bc^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^3,x]

[Out] (a + b*x)^3/(3*b*c^3)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^3}\,dx &= \frac{\int (a+bx)^2\,dx}{c^3}\\ &= \frac{(a+bx)^3}{3bc^3}\end{aligned}$$

Mathematica [A] time = 0.0009627, size = 17, normalized size = 1.

$$\frac{(a+bx)^3}{3bc^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^3,x]

[Out] (a + b*x)^3/(3*b*c^3)

Maple [A] time = 0.001, size = 16, normalized size = 0.9

$$\frac{(bx+a)^3}{3\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^3,x)
```

```
[Out] 1/3*(b*x+a)^3/b/c^3
```

Maxima [A] time = 1.01137, size = 35, normalized size = 2.06

$$\frac{b^2x^3+3\,abx^2+3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

Fricas [A] time = 1.45946, size = 55, normalized size = 3.24

$$\frac{b^2x^3+3\,abx^2+3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

Sympy [B] time = 0.094063, size = 29, normalized size = 1.71

$$\frac{a^2x}{c^3}+\frac{abx^2}{c^3}+\frac{b^2x^3}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**3,x)
```

```
[Out] a**2*x/c**3 + a*b*x**2/c**3 + b**2*x**3/(3*c**3)
```

Giac [A] time = 1.05813, size = 35, normalized size = 2.06

$$\frac{b^2x^3+3\,abx^2+3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

3.1022 $\int \frac{(a+bx)^5}{(ac+bcx)^4}\,dx$

Optimal. Leaf size=18

$$\frac{ax}{c^4}+\frac{bx^2}{2c^4}$$

```
[Out] (a*x)/c^4 + (b*x^2)/(2*c^4)
```

Rubi [A] time = 0.004596, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {21}

$$\frac{ax}{c^4}+\frac{bx^2}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^4,x]

[Out] (a*x)/c^4 + (b*x^2)/(2*c^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^4}\,dx &= \frac{\int (a+bx)\,dx}{c^4}\\ &= \frac{ax}{c^4}+\frac{bx^2}{2c^4}\end{aligned}$$

Mathematica [A] time = 0.0006691, size = 16, normalized size = 0.89

$$\frac{ax+\frac{bx^2}{2}}{c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^4,x]

[Out] (a*x + (b*x^2)/2)/c^4
```

Maple [A] time = 0., size = 15, normalized size = 0.8

$$\frac{1}{c^4}\left(ax+\frac{bx^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^4,x)
```

```
[Out] 1/c^4*(a*x+1/2*b*x^2)
```

Maxima [A] time = 1.02607, size = 20, normalized size = 1.11

$$\frac{bx^2 + 2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

Fricas [A] time = 1.56144, size = 34, normalized size = 1.89

$$\frac{bx^2 + 2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

Sympy [A] time = 0.144051, size = 15, normalized size = 0.83

$$\frac{ax}{c^4} + \frac{bx^2}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**4,x)
```

```
[Out] a*x/c**4 + b*x**2/(2*c**4)
```

Giac [A] time = 1.0618, size = 20, normalized size = 1.11

$$\frac{bx^2 + 2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

3.1023 $\int \frac{(a+bx)^5}{(ac+bcx)^5} dx$

Optimal. Leaf size=5

$$\frac{x}{c^5}$$

[Out] x/c^5

Rubi [A] time = 0.0013297, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 8}

$$\frac{x}{c^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^5,x]

[Out] x/c^5

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^5} dx &= \frac{\int 1 \, dx}{c^5}\\ &= \frac{x}{c^5}\end{aligned}$$

Mathematica [A] time = 0.0003026, size = 5, normalized size = 1.

$$\frac{x}{c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^5,x]

[Out] x/c^5

Maple [A] time = 0.002, size = 6, normalized size = 1.2

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^5,x)
```

```
[Out] x/c^5
```

Maxima [A] time = 1.03508, size = 7, normalized size = 1.4

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] x/c^5
```

Fricas [A] time = 1.45602, size = 9, normalized size = 1.8

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="fricas")
```

```
[Out] x/c^5
```

Sympy [A] time = 0.098591, size = 3, normalized size = 0.6

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**5,x)
```

```
[Out] x/c**5
```

Giac [B] time = 1.07033, size = 20, normalized size = 4.

$$\frac{bcx + ac}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="giac")
```

```
[Out] (b*c*x + a*c)/(b*c^6)
```

3.1024 $\int \frac{(a+bx)^5}{(ac+bcx)^6} dx$

Optimal. Leaf size=13

$$\frac{\log(a+bx)}{bc^6}$$

```
[Out] Log[a + b*x]/(b*c^6)
```

Rubi [A] time = 0.0037264, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 31}

$$\frac{\log(a+bx)}{bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^6,x]
```

```
[Out] Log[a + b*x]/(b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^6} dx &= \frac{\int \frac{1}{a+bx} dx}{c^6} \\ &= \frac{\log(a+bx)}{bc^6} \end{aligned}$$

Mathematica [A] time = 0.0018879, size = 13, normalized size = 1.

$$\frac{\log(a+bx)}{bc^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^6,x]
```

```
[Out] Log[a + b*x]/(b*c^6)
```

Maple [A] time = 0.001, size = 14, normalized size = 1.1

$$\frac{\ln\left(bx+a\right)}{b c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^6,x)
```

```
[Out] ln(b*x+a)/b/c^6
```

Maxima [A] time = 1.02807, size = 18, normalized size = 1.38

$$\frac{\log\left(bx+a\right)}{b c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="maxima")
```

```
[Out] log(b*x + a)/(b*c^6)
```

Fricas [A] time = 1.52776, size = 30, normalized size = 2.31

$$\frac{\log\left(bx+a\right)}{b c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] log(b*x + a)/(b*c^6)
```

Sympy [A] time = 0.107435, size = 17, normalized size = 1.31

$$\frac{\log\left(ac^{6}+bc^{6}x\right)}{b c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**6,x)
```

```
[Out] log(a*c**6 + b*c**6*x)/(b*c**6)
```

Giac [A] time = 1.05024, size = 19, normalized size = 1.46

$$\frac{\log\left(|bx+a|\right)}{b c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/(b*c^6)
```

3.1025 $\int \frac{(a+bx)^5}{(ac+bcx)^7} dx$

Optimal. Leaf size=15

$$-\frac{1}{bc^7(a+bx)}$$

[Out] -(1/(b*c^7*(a + b*x)))

Rubi [A] time = 0.0041574, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$-\frac{1}{bc^7(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^7,x]

[Out] -(1/(b*c^7*(a + b*x)))

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^7} dx &= \frac{\int \frac{1}{(a+bx)^2} dx}{c^7}\\ &= -\frac{1}{bc^7(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0021952, size = 15, normalized size = 1.

$$-\frac{1}{bc^7(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^7,x]

[Out] -(1/(b*c^7*(a + b*x)))

Maple [A] time = 0.002, size = 16, normalized size = 1.1

$$-\frac{1}{bc^7\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^7,x)
```

```
[Out] -1/b/c^7/(b*x+a)
```

Maxima [A] time = 1.03331, size = 26, normalized size = 1.73

$$-\frac{1}{b^2c^7x+abc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="maxima")
```

```
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

Fricas [A] time = 1.50465, size = 35, normalized size = 2.33

$$-\frac{1}{b^2c^7x+abc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="fricas")
```

```
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

Sympy [A] time = 0.42156, size = 17, normalized size = 1.13

$$-\frac{1}{abc^7+b^2c^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**7,x)
```

```
[Out] -1/(a*b*c**7 + b**2*c**7*x)
```

Giac [A] time = 1.05078, size = 20, normalized size = 1.33

$$-\frac{1}{(bx+a)bc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="giac")

[Out] -1/((b*x + a)*b*c^7)
```

3.1026 $\int \frac{(a+bx)^5}{(ac+bcx)^8} dx$

Optimal. Leaf size=17

$$-\frac{1}{2bc^8(a+bx)^2}$$

```
[Out] -1/(2*b*c^8*(a + b*x)^2)
```

Rubi [A] time = 0.0038258, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$-\frac{1}{2bc^8(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^8,x]
```

```
[Out] -1/(2*b*c^8*(a + b*x)^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^8} dx &= \frac{\int \frac{1}{(a+bx)^3} dx}{c^8} \\ &= -\frac{1}{2bc^8(a+bx)^2} \end{aligned}$$

Mathematica [A] time = 0.0041017, size = 17, normalized size = 1.

$$-\frac{1}{2bc^8(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^8,x]
```

```
[Out] -1/(2*b*c^8*(a + b*x)^2)
```

Maple [A] time = 0.001, size = 16, normalized size = 0.9

$$-\frac{1}{2\,b c^8 \left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^8,x)
```

```
[Out] -1/2/b/c^8/(b*x+a)^2
```

Maxima [B] time = 1.0553, size = 45, normalized size = 2.65

$$-\frac{1}{2\left(b^3c^8x^2+2\,ab^2c^8x+a^2bc^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="maxima")
```

```
[Out] -1/2/(b^3*c^8*x^2 + 2*a*b^2*c^8*x + a^2*b*c^8)
```

Fricas [B] time = 1.45614, size = 65, normalized size = 3.82

$$-\frac{1}{2\left(b^3c^8x^2+2\,ab^2c^8x+a^2bc^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*c^8*x^2 + 2*a*b^2*c^8*x + a^2*b*c^8)
```

Sympy [B] time = 0.385585, size = 36, normalized size = 2.12

$$-\frac{1}{2a^2bc^8+4ab^2c^8x+2b^3c^8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**8,x)
```

```
[Out] -1/(2*a**2*b*c**8 + 4*a*b**2*c**8*x + 2*b**3*c**8*x**2)
```

Giac [A] time = 1.04775, size = 20, normalized size = 1.18

$$-\frac{1}{2\left(bx+a\right)^2 bc^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="giac")
```

```
[Out] -1/2/((b*x + a)^2*b*c^8)
```

3.1027 $\int \frac{1}{\sqrt{-2-3x}\sqrt{2+3x}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{3x+2}\log(3x+2)}{3\sqrt{-3x-2}}$$

```
[Out] (Sqrt[2 + 3*x]*Log[2 + 3*x])/(3*Sqrt[-2 - 3*x])
```

Rubi [A] time = 0.0030387, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {23, 31}

$$\frac{\sqrt{3x+2}\log(3x+2)}{3\sqrt{-3x-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] (Sqrt[2 + 3*x]*Log[2 + 3*x])/(3*Sqrt[-2 - 3*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2-3x}\sqrt{2+3x}}\,dx &= \frac{\sqrt{2+3x}\int \frac{1}{2+3x}\,dx}{\sqrt{-2-3x}}\\ &= \frac{\sqrt{2+3x}\log(2+3x)}{3\sqrt{-2-3x}}\end{aligned}$$

Mathematica [A] time = 0.0059376, size = 28, normalized size = 1.

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

Maple [A] time = 0.002, size = 23, normalized size = 0.8

$$\frac{\ln(2+3x)}{3}\sqrt{2+3x}\frac{1}{\sqrt{-2-3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x)
```

```
[Out] 1/3*ln(2+3*x)*(2+3*x)^(1/2)/(-2-3*x)^(1/2)
```

Maxima [C] time = 1.55498, size = 8, normalized size = 0.29

$$\frac{1}{3}i\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*I*log(x + 2/3)
```

Fricas [A] time = 1.48654, size = 4, normalized size = 0.14

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [C] time = 1.40529, size = 53, normalized size = 1.89

$$\begin{cases} -\frac{i\log\left(x+\frac{2}{3}\right)}{3} & \text{for}\ \left|x+\frac{2}{3}\right|<1 \\ \frac{i\log\left(\frac{1}{x+\frac{2}{3}}\right)}{3} & \text{for}\ \frac{1}{\left|x+\frac{2}{3}\right|}<1 \\ \frac{iG_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x+\frac{2}{3}\right)}{3}-\frac{iG_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x+\frac{2}{3}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((-I*log(x + 2/3)/3, Abs(x + 2/3) < 1), (I*log(1/(x + 2/3))/3, 1/A
bs(x + 2/3) < 1), (I*meijerg(((), (1, 1)), ((0, 0), ()), x + 2/3)/3 - I*mei
jerg(((1, 1), ()), ((), (0, 0)), x + 2/3)/3, True))
```

Giac [C] time = 1.06409, size = 15, normalized size = 0.54

$$-\frac{1}{3}i\log\left(|3x+2|\right)\operatorname{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*I*log(abs(3*x + 2))*sgn(x)
```

3.1028 $\int (a+bx)(ac-bcx)^3\,dx$

Optimal. Leaf size=38

$$\frac{c^3(a-bx)^5}{5b}-\frac{ac^3(a-bx)^4}{2b}$$

```
[Out] -(a*c^3*(a - b*x)^4)/(2*b) + (c^3*(a - b*x)^5)/(5*b)
```

Rubi [A] time = 0.0116105, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{c^3(a-bx)^5}{5b}-\frac{ac^3(a-bx)^4}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^3,x]
```

```
[Out] -(a*c^3*(a - b*x)^4)/(2*b) + (c^3*(a - b*x)^5)/(5*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^3\,dx &= \int \left(2a(ac-bcx)^3-\frac{(ac-bcx)^4}{c}\right)dx\\ &= -\frac{ac^3(a-bx)^4}{2b}+\frac{c^3(a-bx)^5}{5b}\end{aligned}$$

Mathematica [A] time = 0.0029372, size = 40, normalized size = 1.05

$$c^3\left(-a^3bx^2+a^4x+\frac{1}{2}ab^3x^4-\frac{1}{5}b^4x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^3,x]
```

```
[Out] c^3*(a^4*x - a^3*b*x^2 + (a*b^3*x^4)/2 - (b^4*x^5)/5)
```

Maple [A] time = 0., size = 45, normalized size = 1.2

$$-\frac{b^4c^3x^5}{5}+\frac{ab^3c^3x^4}{2}-a^3c^3bx^2+a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^3,x)
```

```
[Out] -1/5*b^4*c^3*x^5+1/2*a*b^3*c^3*x^4-a^3*c^3*b*x^2+a^4*c^3*x
```

Maxima [A] time = 1.01166, size = 59, normalized size = 1.55

$$-\frac{1}{5}b^4c^3x^5 + \frac{1}{2}ab^3c^3x^4 - a^3bc^3x^2 + a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

Fricas [A] time = 1.34728, size = 89, normalized size = 2.34

$$-\frac{1}{5}x^5c^3b^4 + \frac{1}{2}x^4c^3b^3a - x^2c^3ba^3 + xc^3a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/5*x^5*c^3*b^4 + 1/2*x^4*c^3*b^3*a - x^2*c^3*b*a^3 + x*c^3*a^4
```

Sympy [A] time = 0.070738, size = 44, normalized size = 1.16

$$a^4c^3x - a^3bc^3x^2 + \frac{ab^3c^3x^4}{2} - \frac{b^4c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**3,x)
```

```
[Out] a**4*c**3*x - a**3*b*c**3*x**2 + a*b**3*c**3*x**4/2 - b**4*c**3*x**5/5
```

Giac [A] time = 1.04091, size = 59, normalized size = 1.55

$$-\frac{1}{5}b^4c^3x^5 + \frac{1}{2}ab^3c^3x^4 - a^3bc^3x^2 + a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] -1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

3.1029 $\int (a+bx)(ac-bcx)^2\,dx$

Optimal. Leaf size=38

$$\frac{c^2(a-bx)^4}{4b}-\frac{2ac^2(a-bx)^3}{3b}$$

```
[Out] (-2*a*c^2*(a - b*x)^3)/(3*b) + (c^2*(a - b*x)^4)/(4*b)
```

Rubi [A] time = 0.0170622, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{c^2(a-bx)^4}{4b}-\frac{2ac^2(a-bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^2,x]

[Out] (-2*a*c^2*(a - b*x)^3)/(3*b) + (c^2*(a - b*x)^4)/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^2\,dx &= \int \left(2a(ac-bcx)^2-\frac{(ac-bcx)^3}{c}\right)dx\\ &= -\frac{2ac^2(a-bx)^3}{3b}+\frac{c^2(a-bx)^4}{4b}\end{aligned}$$

Mathematica [A] time = 0.0023617, size = 42, normalized size = 1.11

$$c^2\left(-\frac{1}{2}a^2bx^2+a^3x-\frac{1}{3}ab^2x^3+\frac{b^3x^4}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^2,x]

[Out] c^2*(a^3*x - (a^2*b*x^2)/2 - (a*b^2*x^3)/3 + (b^3*x^4)/4)
```

Maple [A] time = 0.001, size = 45, normalized size = 1.2

$$\frac{b^3c^2x^4}{4}-\frac{ab^2c^2x^3}{3}-\frac{a^2c^2bx^2}{2}+a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^2,x)
```

```
[Out] 1/4*b^3*c^2*x^4-1/3*a*b^2*c^2*x^3-1/2*a^2*c^2*b*x^2+a^3*c^2*x
```

Maxima [A] time = 1.00264, size = 59, normalized size = 1.55

$$\frac{1}{4}\,b^3c^2x^4-\frac{1}{3}\,ab^2c^2x^3-\frac{1}{2}\,a^2bc^2x^2+a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

Fricas [A] time = 1.31821, size = 93, normalized size = 2.45

$$\frac{1}{4}x^4c^2b^3-\frac{1}{3}x^3c^2b^2a-\frac{1}{2}x^2c^2ba^2+xc^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*c^2*b^3 - 1/3*x^3*c^2*b^2*a - 1/2*x^2*c^2*b*a^2 + x*c^2*a^3
```

Sympy [A] time = 0.065695, size = 46, normalized size = 1.21

$$a^3c^2x-\frac{a^2bc^2x^2}{2}-\frac{ab^2c^2x^3}{3}+\frac{b^3c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**2,x)
```

```
[Out] a**3*c**2*x - a**2*b*c**2*x**2/2 - a*b**2*c**2*x**3/3 + b**3*c**2*x**4/4
```

Giac [A] time = 1.05925, size = 59, normalized size = 1.55

$$\frac{1}{4}\,b^3c^2x^4-\frac{1}{3}\,ab^2c^2x^3-\frac{1}{2}\,a^2bc^2x^2+a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] 1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

3.1030 $\int (a+bx)(ac-bcx)\,dx$

Optimal. Leaf size=18

$$a^2cx-\frac{1}{3}b^2cx^3$$

```
[Out] a^2*c*x - (b^2*c*x^3)/3
```

Rubi [A] time = 0.0042267, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {41}

$$a^2cx-\frac{1}{3}b^2cx^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x),x]
```

```
[Out] a^2*c*x - (b^2*c*x^3)/3
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)\,dx &= \int \left(a^2c-b^2cx^2\right)\,dx\\ &= a^2cx-\frac{1}{3}b^2cx^3\end{aligned}$$

Mathematica [A] time = 0.0016718, size = 18, normalized size = 1.

$$c\left(a^2x-\frac{b^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x),x]
```

```
[Out] c*(a^2*x - (b^2*x^3)/3)
```

Maple [A] time = 0., size = 17, normalized size = 0.9

$$a^2cx-\frac{b^2cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c),x)

[Out] a^2*c*x-1/3*b^2*c*x^3
```

Maxima [A] time = 1.02333, size = 22, normalized size = 1.22

$$-\frac{1}{3}\,b^2cx^3 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="maxima")

[Out] -1/3*b^2*c*x^3 + a^2*c*x
```

Fricas [A] time = 1.24425, size = 35, normalized size = 1.94

$$-\frac{1}{3}x^3cb^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="fricas")

[Out] -1/3*x^3*c*b^2 + x*c*a^2
```

Sympy [A] time = 0.057572, size = 15, normalized size = 0.83

$$a^2cx - \frac{b^2cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x)

[Out] a**2*c*x - b**2*c*x**3/3
```

Giac [A] time = 1.05493, size = 22, normalized size = 1.22

$$-\frac{1}{3}\,b^2cx^3 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="giac")

[Out] -1/3*b^2*c*x^3 + a^2*c*x
```

3.1031 $\int (a+bx)\,dx$

Optimal. Leaf size=12

$$ax+\frac{bx^2}{2}$$

```
[Out] a*x + (b*x^2)/2
```

Rubi [A] time = 0.0020981, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x,x]
```

```
[Out] a*x + (b*x^2)/2
```

Rubi steps

$$\int (a+bx)\,dx=ax+\frac{bx^2}{2}$$

Mathematica [A] time = 0.0000315, size = 12, normalized size = 1.

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x,x]
```

```
[Out] a*x + (b*x^2)/2
```

Maple [A] time = 0.001, size = 11, normalized size = 0.9

$$ax+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a,x)
```

```
[Out] a*x+1/2*b*x^2
```

Maxima [A] time = 0.978564, size = 14, normalized size = 1.17

$$\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

Fricas [A] time = 1.33125, size = 23, normalized size = 1.92

$$\frac{1}{2}x^2b+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*b + x*a
```

Sympy [A] time = 0.050369, size = 8, normalized size = 0.67

$$ax+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x)
```

```
[Out] a*x + b*x**2/2
```

Giac [A] time = 1.03693, size = 14, normalized size = 1.17

$$\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

3.1032 $\int \frac{a+bx}{ac-bcx}\,dx$

Optimal. Leaf size=23

$$-\frac{2a\log(a-bx)}{bc}-\frac{x}{c}$$

```
[Out] -(x/c) - (2*a*Log[a - b*x])/(b*c)
```

Rubi [A] time = 0.012133, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2a\log(a-bx)}{bc}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x),x]
```

```
[Out] -(x/c) - (2*a*Log[a - b*x])/(b*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{ac-bcx}\,dx &= \int \left(-\frac{1}{c}+\frac{2a}{c(a-bx)}\right)dx\\ &= -\frac{x}{c}-\frac{2a\log(a-bx)}{bc}\end{aligned}$$

Mathematica [A] time = 0.0048164, size = 23, normalized size = 1.

$$-\frac{2a\log(a-bx)}{bc}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x),x]
```

```
[Out] -(x/c) - (2*a*Log[a - b*x])/(b*c)
```

Maple [A] time = 0.002, size = 25, normalized size = 1.1

$$-\frac{x}{c}-2\,\frac{a\ln(bx-a)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c),x)
```

```
[Out] -x/c-2/c/b*a*ln(b*x-a)
```

Maxima [A] time = 1.02062, size = 32, normalized size = 1.39

$$-\frac{x}{c} - \frac{2\,a\log\left(bx - a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -x/c - 2*a*log(b*x - a)/(b*c)
```

Fricas [A] time = 1.47799, size = 45, normalized size = 1.96

$$-\frac{bx + 2\,a\log\left(bx - a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] -(b*x + 2*a*log(b*x - a))/(b*c)
```

Sympy [A] time = 0.285939, size = 17, normalized size = 0.74

$$-\frac{2a\log\left(-a + bx\right)}{bc} - \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x)
```

```
[Out] -2*a*log(-a + b*x)/(b*c) - x/c
```

Giac [A] time = 1.05307, size = 34, normalized size = 1.48

$$-\frac{x}{c} - \frac{2\,a\log\left(|bx - a|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="giac")
```

```
[Out] -x/c - 2*a*log(abs(b*x - a))/(b*c)
```

3.1033 $\int \frac{a+bx}{(ac-bcx)^2} dx$

Optimal. Leaf size=32

$$\frac{2a}{bc^2(a-bx)}+\frac{\log(a-bx)}{bc^2}$$

```
[Out] (2*a)/(b*c^2*(a - b*x)) + Log[a - b*x]/(b*c^2)
```

Rubi [A] time = 0.017297, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2a}{bc^2(a-bx)}+\frac{\log(a-bx)}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^2,x]
```

```
[Out] (2*a)/(b*c^2*(a - b*x)) + Log[a - b*x]/(b*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^2} dx &= \int \left(\frac{2a}{c^2(a-bx)^2}-\frac{1}{c^2(a-bx)}\right) dx\\ &= \frac{2a}{bc^2(a-bx)}+\frac{\log(a-bx)}{bc^2}\end{aligned}$$

Mathematica [A] time = 0.0166424, size = 28, normalized size = 0.88

$$\frac{\log(c(a-bx))+\frac{2a}{a-bx}}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^2,x]
```

```
[Out] ((2*a)/(a - b*x) + Log[c*(a - b*x)])/(b*c^2)
```

Maple [A] time = 0.005, size = 35, normalized size = 1.1

$$-2\frac{a}{c^2b(bx-a)}+\frac{\ln(bx-a)}{c^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^2,x)
```

```
[Out] -2/c^2/b*a/(b*x-a)+1/c^2/b*ln(b*x-a)
```

Maxima [A] time = 1.02122, size = 50, normalized size = 1.56

$$-\frac{2\,a}{b^2c^2x - abc^2} + \frac{\log\left(bx - a\right)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -2*a/(b^2*c^2*x - a*b*c^2) + log(b*x - a)/(b*c^2)
```

Fricas [A] time = 1.41035, size = 73, normalized size = 2.28

$$\frac{\left(bx - a\right)\log\left(bx - a\right) - 2\,a}{b^2c^2x - abc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] ((b*x - a)*log(b*x - a) - 2*a)/(b^2*c^2*x - a*b*c^2)
```

Sympy [A] time = 0.338311, size = 29, normalized size = 0.91

$$-\frac{2a}{-abc^2 + b^2c^2x} + \frac{\log\left(-a + bx\right)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**2,x)
```

```
[Out] -2*a/(-a*b*c**2 + b**2*c**2*x) + log(-a + b*x)/(b*c**2)
```

Giac [B] time = 1.07027, size = 109, normalized size = 3.41

$$-\frac{\frac{a}{(bcx-ac)b} + \frac{\log\left(\frac{|bcx-ac|}{(bcx-ac)^2|b||c|}\right)}{bc}}{c} - \frac{a}{(bcx-ac)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -(a/((b*c*x - a*c)*b) + log(abs(b*c*x - a*c)/((b*c*x - a*c)^2*abs(b)*abs(c)
))/(b*c))/c - a/((b*c*x - a*c)*b*c)
```

3.1034 $\int \frac{a+bx}{(ac-bcx)^3}\,dx$

Optimal. Leaf size=13

$$\frac{x}{c^3(a-bx)^2}$$

```
[Out] x/(c^3*(a - b*x)^2)
```

Rubi [A] time = 0.0021911, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {34}

$$\frac{x}{c^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^3,x]

[Out] x/(c^3*(a - b*x)^2)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\int \frac{a+bx}{(ac-bcx)^3}\,dx=\frac{x}{c^3(a-bx)^2}$$

Mathematica [A] time = 0.0078797, size = 13, normalized size = 1.

$$\frac{x}{c^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^3,x]

[Out] x/(c^3*(a - b*x)^2)
```

Maple [B] time = 0.003, size = 33, normalized size = 2.5

$$\frac{1}{c^3}\left(\frac{1}{b\,(bx-a)}+\frac{a}{b\,(bx-a)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^3,x)
```

```
[Out] 1/c^3*(1/b/(b*x-a)+1/b*a/(b*x-a)^2)
```

Maxima [B] time = 0.989695, size = 41, normalized size = 3.15

$$\frac{x}{b^2c^3x^2 - 2\,abc^3x + a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] x/(b^2*c^3*x^2 - 2*a*b*c^3*x + a^2*c^3)
```

Fricas [B] time = 1.48136, size = 55, normalized size = 4.23

$$\frac{x}{b^2c^3x^2 - 2\,abc^3x + a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] x/(b^2*c^3*x^2 - 2*a*b*c^3*x + a^2*c^3)
```

Sympy [B] time = 0.368923, size = 27, normalized size = 2.08

$$\frac{x}{a^2c^3 - 2abc^3x + b^2c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**3,x)
```

```
[Out] x/(a**2*c**3 - 2*a*b*c**3*x + b**2*c**3*x**2)
```

Giac [A] time = 1.05118, size = 19, normalized size = 1.46

$$\frac{x}{(bx-a)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] x/((b*x - a)^2*c^3)
```

3.1035 $\int \frac{a+bx}{(ac-bcx)^4}\,dx$

Optimal. Leaf size=38

$$\frac{2a}{3bc^4(a-bx)^3}-\frac{1}{2bc^4(a-bx)^2}$$

```
[Out] (2*a)/(3*b*c^4*(a - b*x)^3) - 1/(2*b*c^4*(a - b*x)^2)
```

Rubi [A] time = 0.0187421, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2a}{3bc^4(a-bx)^3}-\frac{1}{2bc^4(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^4,x]
```

```
[Out] (2*a)/(3*b*c^4*(a - b*x)^3) - 1/(2*b*c^4*(a - b*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^4}\,dx &= \int \left(\frac{2a}{c^4(a-bx)^4}-\frac{1}{c^4(a-bx)^3}\right)dx\\ &= \frac{2a}{3bc^4(a-bx)^3}-\frac{1}{2bc^4(a-bx)^2}\end{aligned}$$

Mathematica [A] time = 0.0113416, size = 25, normalized size = 0.66

$$-\frac{a+3bx}{6bc^4(bx-a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^4,x]
```

```
[Out] -(a + 3*b*x)/(6*b*c^4*(-a + b*x)^3)
```

Maple [A] time = 0.005, size = 35, normalized size = 0.9

$$\frac{1}{c^4}\left(-\frac{2\,a}{3\,b\,(bx-a)^3}-\frac{1}{2\,b\,(bx-a)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^4,x)
```

```
[Out] 1/c^4*(-2/3/b*a/(b*x-a)^3-1/2/b/(b*x-a)^2)
```

Maxima [A] time = 1.00579, size = 73, normalized size = 1.92

$$-\frac{3\,bx+a}{6\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*x + a)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a^3*b*c
^4)
```

Fricas [A] time = 1.48177, size = 108, normalized size = 2.84

$$-\frac{3\,bx+a}{6\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x + a)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a^3*b*c
^4)
```

Sympy [A] time = 0.486193, size = 56, normalized size = 1.47

$$-\frac{a+3bx}{-6a^3bc^4+18a^2b^2c^4x-18ab^3c^4x^2+6b^4c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**4,x)
```

```
[Out] -(a + 3*b*x)/(-6*a**3*b*c**4 + 18*a**2*b**2*c**4*x - 18*a*b**3*c**4*x**2 +
6*b**4*c**4*x**3)
```

Giac [A] time = 1.05926, size = 31, normalized size = 0.82

$$-\frac{3\,bx+a}{6\,(bx-a)^3bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*x + a)/((b*x - a)^3*b*c^4)
```

3.1036 $\int \frac{a+bx}{(ac-bcx)^5}\,dx$

Optimal. Leaf size=38

$$\frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}$$

```
[Out] a/(2*b*c^5*(a - b*x)^4) - 1/(3*b*c^5*(a - b*x)^3)
```

Rubi [A] time = 0.018712, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^5,x]
```

```
[Out] a/(2*b*c^5*(a - b*x)^4) - 1/(3*b*c^5*(a - b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^5}\,dx &= \int \left(\frac{2a}{c^5(a-bx)^5}-\frac{1}{c^5(a-bx)^4}\right)dx\\ &= \frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}\end{aligned}$$

Mathematica [A] time = 0.0105647, size = 24, normalized size = 0.63

$$\frac{a+2bx}{6bc^5(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^5,x]
```

```
[Out] (a + 2*b*x)/(6*b*c^5*(a - b*x)^4)
```

Maple [A] time = 0.005, size = 35, normalized size = 0.9

$$\frac{1}{c^5}\left(\frac{a}{2\,b\,(bx-a)^4}+\frac{1}{3\,b\,(bx-a)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^5,x)
```

```
[Out] 1/c^5*(1/2/b*a/(b*x-a)^4+1/3/b/(b*x-a)^3)
```

Maxima [A] time = 1.01628, size = 90, normalized size = 2.37

$$\frac{2\,bx+a}{6\left(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*x + a)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*c^5*x^2 - 4*a^3*
b^2*c^5*x + a^4*b*c^5)
```

Fricas [A] time = 1.45573, size = 134, normalized size = 3.53

$$\frac{2\,bx+a}{6\left(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*x + a)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*c^5*x^2 - 4*a^3*
b^2*c^5*x + a^4*b*c^5)
```

Sympy [B] time = 0.52499, size = 70, normalized size = 1.84

$$\frac{a+2bx}{6a^4bc^5-24a^3b^2c^5x+36a^2b^3c^5x^2-24ab^4c^5x^3+6b^5c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**5,x)
```

```
[Out] (a + 2*b*x)/(6*a**4*b*c**5 - 24*a**3*b**2*c**5*x + 36*a**2*b**3*c**5*x**2 -
 24*a*b**4*c**5*x**3 + 6*b**5*c**5*x**4)
```

Giac [A] time = 1.09206, size = 54, normalized size = 1.42

$$\frac{a}{2\left(bcx-ac\right)^4bc}+\frac{1}{3\left(bcx-ac\right)^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="giac")

[Out] 1/2*a/((b*c*x - a*c)^4*b*c) + 1/3/((b*c*x - a*c)^3*b*c^2)
```

3.1037 $\int \frac{a+bx}{(ac-bcx)^6}\,dx$

Optimal. Leaf size=38

$$\frac{2a}{5bc^6(a-bx)^5}-\frac{1}{4bc^6(a-bx)^4}$$

```
[Out] (2*a)/(5*b*c^6*(a - b*x)^5) - 1/(4*b*c^6*(a - b*x)^4)
```

Rubi [A] time = 0.0189225, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2a}{5bc^6(a-bx)^5}-\frac{1}{4bc^6(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^6,x]
```

```
[Out] (2*a)/(5*b*c^6*(a - b*x)^5) - 1/(4*b*c^6*(a - b*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^6}\,dx &= \int \left(\frac{2a}{c^6(a-bx)^6}-\frac{1}{c^6(a-bx)^5}\right)dx\\ &= \frac{2a}{5bc^6(a-bx)^5}-\frac{1}{4bc^6(a-bx)^4}\end{aligned}$$

Mathematica [A] time = 0.0121554, size = 27, normalized size = 0.71

$$-\frac{3a+5bx}{20bc^6(bx-a)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^6,x]
```

```
[Out] -(3*a + 5*b*x)/(20*b*c^6*(-a + b*x)^5)
```

Maple [A] time = 0.004, size = 35, normalized size = 0.9

$$\frac{1}{c^6}\left(-\frac{1}{4\,b\,(bx-a)^4}-\frac{2\,a}{5\,b\,(bx-a)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^6,x)
```

```
[Out] 1/c^6*(-1/4/b/(b*x-a)^4-2/5/b*a/(b*x-a)^5)
```

Maxima [B] time = 1.00619, size = 113, normalized size = 2.97

$$-\frac{5\,bx+3\,a}{20\,\left(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(5*b*x + 3*a)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2*b^4*c^6*x^3 - 1
0*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Fricas [B] time = 1.44014, size = 169, normalized size = 4.45

$$-\frac{5\,bx+3\,a}{20\,\left(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] -1/20*(5*b*x + 3*a)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2*b^4*c^6*x^3 - 1
0*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Sympy [B] time = 0.617279, size = 88, normalized size = 2.32

$$-\frac{3a+5bx}{-20a^5bc^6+100a^4b^2c^6x-200a^3b^3c^6x^2+200a^2b^4c^6x^3-100ab^5c^6x^4+20b^6c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**6,x)
```

```
[Out] -(3*a + 5*b*x)/(-20*a**5*b*c**6 + 100*a**4*b**2*c**6*x - 200*a**3*b**3*c**6
*x**2 + 200*a**2*b**4*c**6*x**3 - 100*a*b**5*c**6*x**4 + 20*b**6*c**6*x**5)
```

Giac [A] time = 1.05226, size = 34, normalized size = 0.89

$$-\frac{5\,bx+3\,a}{20\,(bx-a)^5bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="giac")

[Out] -1/20*(5*b*x + 3*a)/((b*x - a)^5*b*c^6)
```

3.1038 $\int (a+bx)^2(ac-bcx)^3\,dx$

Optimal. Leaf size=57

$$-\frac{a^2c^3(a-bx)^4}{b}-\frac{c^3(a-bx)^6}{6b}+\frac{4ac^3(a-bx)^5}{5b}$$

```
[Out] -((a^2*c^3*(a - b*x)^4)/b) + (4*a*c^3*(a - b*x)^5)/(5*b) - (c^3*(a - b*x)^6
)/(6*b)
```

Rubi [A] time = 0.0303072, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{a^2c^3(a-bx)^4}{b}-\frac{c^3(a-bx)^6}{6b}+\frac{4ac^3(a-bx)^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

```
[Out] -((a^2*c^3*(a - b*x)^4)/b) + (4*a*c^3*(a - b*x)^5)/(5*b) - (c^3*(a - b*x)^6
)/(6*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^3\,dx &= \int \left(4a^2(ac-bcx)^3-\frac{4a(ac-bcx)^4}{c}+\frac{(ac-bcx)^5}{c^2}\right)dx\\ &= -\frac{a^2c^3(a-bx)^4}{b}+\frac{4ac^3(a-bx)^5}{5b}-\frac{c^3(a-bx)^6}{6b}\end{aligned}$$

Mathematica [A] time = 0.0029511, size = 68, normalized size = 1.19

$$c^3\left(\frac{1}{2}a^2b^3x^4-\frac{2}{3}a^3b^2x^3-\frac{1}{2}a^4bx^2+a^5x+\frac{1}{5}ab^4x^5-\frac{1}{6}b^5x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

```
[Out] c^3*(a^5*x - (a^4*b*x^2)/2 - (2*a^3*b^2*x^3)/3 + (a^2*b^3*x^4)/2 + (a*b^4*x
^5)/5 - (b^5*x^6)/6)
```

Maple [A] time = 0., size = 73, normalized size = 1.3

$$-\frac{b^5c^3x^6}{6}+\frac{ab^4c^3x^5}{5}+\frac{a^2b^3c^3x^4}{2}-\frac{2\,a^3c^3b^2x^3}{3}-\frac{a^4c^3bx^2}{2}+a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^3,x)
```

```
[Out] -1/6*b^5*c^3*x^6+1/5*a*b^4*c^3*x^5+1/2*a^2*b^3*c^3*x^4-2/3*a^3*c^3*b^2*x^3-
1/2*a^4*c^3*b*x^2+a^5*c^3*x
```

Maxima [A] time = 0.99351, size = 97, normalized size = 1.7

$$-\frac{1}{6}\,b^5c^3x^6+\frac{1}{5}\,ab^4c^3x^5+\frac{1}{2}\,a^2b^3c^3x^4-\frac{2}{3}\,a^3b^2c^3x^3-\frac{1}{2}\,a^4bc^3x^2+a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c^
3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

Fricas [A] time = 1.34014, size = 154, normalized size = 2.7

$$-\frac{1}{6}x^6c^3b^5+\frac{1}{5}x^5c^3b^4a+\frac{1}{2}x^4c^3b^3a^2-\frac{2}{3}x^3c^3b^2a^3-\frac{1}{2}x^2c^3ba^4+xc^3a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/6*x^6*c^3*b^5 + 1/5*x^5*c^3*b^4*a + 1/2*x^4*c^3*b^3*a^2 - 2/3*x^3*c^3*b^
2*a^3 - 1/2*x^2*c^3*b*a^4 + x*c^3*a^5
```

Sympy [A] time = 0.076897, size = 78, normalized size = 1.37

$$a^5c^3x-\frac{a^4bc^3x^2}{2}-\frac{2a^3b^2c^3x^3}{3}+\frac{a^2b^3c^3x^4}{2}+\frac{ab^4c^3x^5}{5}-\frac{b^5c^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**3,x)
```

```
[Out] a**5*c**3*x - a**4*b*c**3*x**2/2 - 2*a**3*b**2*c**3*x**3/3 + a**2*b**3*c**3
*x**4/2 + a*b**4*c**3*x**5/5 - b**5*c**3*x**6/6
```

Giac [A] time = 1.05994, size = 97, normalized size = 1.7

$$-\frac{1}{6}\,b^5c^3x^6+\frac{1}{5}\,ab^4c^3x^5+\frac{1}{2}\,a^2b^3c^3x^4-\frac{2}{3}\,a^3b^2c^3x^3-\frac{1}{2}\,a^4bc^3x^2+a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] -1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c^
3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

3.1039 $\int (a+bx)^2(ac-bcx)^2\,dx$

Optimal. Leaf size=38

$$-\frac{2}{3}a^2b^2c^2x^3+a^4c^2x+\frac{1}{5}b^4c^2x^5$$

```
[Out] a^4*c^2*x - (2*a^2*b^2*c^2*x^3)/3 + (b^4*c^2*x^5)/5
```

Rubi [A] time = 0.0171346, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {41, 194}

$$-\frac{2}{3}a^2b^2c^2x^3+a^4c^2x+\frac{1}{5}b^4c^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^2,x]
```

```
[Out] a^4*c^2*x - (2*a^2*b^2*c^2*x^3)/3 + (b^4*c^2*x^5)/5
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^2\,dx &= \int \left(a^2c-b^2cx^2\right)^2\,dx\\ &= \int \left(a^4c^2-2a^2b^2c^2x^2+b^4c^2x^4\right)\,dx\\ &= a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5\end{aligned}$$

Mathematica [A] time = 0.0016806, size = 38, normalized size = 1.

$$-\frac{2}{3}a^2b^2c^2x^3+a^4c^2x+\frac{1}{5}b^4c^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^2,x]
```

```
[Out] a^4*c^2*x - (2*a^2*b^2*c^2*x^3)/3 + (b^4*c^2*x^5)/5
```

Maple [A] time = 0.001, size = 35, normalized size = 0.9

$$a^4c^2x - \frac{2\,a^2b^2c^2x^3}{3} + \frac{b^4c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^2,x)
```

```
[Out] a^4*c^2*x-2/3*a^2*b^2*c^2*x^3+1/5*b^4*c^2*x^5
```

Maxima [A] time = 1.02374, size = 46, normalized size = 1.21

$$\frac{1}{5}\,b^4c^2x^5 - \frac{2}{3}\,a^2b^2c^2x^3 + a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

Fricas [A] time = 1.28935, size = 69, normalized size = 1.82

$$\frac{1}{5}x^5c^2b^4 - \frac{2}{3}x^3c^2b^2a^2 + xc^2a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c^2*b^4 - 2/3*x^3*c^2*b^2*a^2 + x*c^2*a^4
```

Sympy [A] time = 0.067727, size = 36, normalized size = 0.95

$$a^4c^2x - \frac{2a^2b^2c^2x^3}{3} + \frac{b^4c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**2,x)
```

```
[Out] a**4*c**2*x - 2*a**2*b**2*c**2*x**3/3 + b**4*c**2*x**5/5
```

Giac [A] time = 1.0603, size = 46, normalized size = 1.21

$$\frac{1}{5}\,b^4c^2x^5 - \frac{2}{3}\,a^2b^2c^2x^3 + a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

3.1040 $\int (a+bx)^2(ac-bcx)\,dx$

Optimal. Leaf size=32

$$\frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}$$

```
[Out] (2*a*c*(a + b*x)^3)/(3*b) - (c*(a + b*x)^4)/(4*b)
```

Rubi [A] time = 0.0149207, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x),x]
```

```
[Out] (2*a*c*(a + b*x)^3)/(3*b) - (c*(a + b*x)^4)/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)\,dx &= \int \left(2ac(a+bx)^2-c(a+bx)^3\right)dx\\ &= \frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}\end{aligned}$$

Mathematica [A] time = 0.001673, size = 40, normalized size = 1.25

$$c\left(\frac{1}{2}a^2bx^2+a^3x-\frac{1}{3}ab^2x^3-\frac{1}{4}b^3x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x),x]
```

```
[Out] c*(a^3*x + (a^2*b*x^2)/2 - (a*b^2*x^3)/3 - (b^3*x^4)/4)
```

Maple [A] time = 0., size = 37, normalized size = 1.2

$$-\frac{b^3cx^4}{4}-\frac{ab^2cx^3}{3}+\frac{a^2bcx^2}{2}+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c),x)
```

```
[Out] -1/4*b^3*c*x^4-1/3*a*b^2*c*x^3+1/2*a^2*b*c*x^2+a^3*c*x
```

Maxima [A] time = 1.01329, size = 49, normalized size = 1.53

$$-\frac{1}{4}\,b^3cx^4 - \frac{1}{3}\,ab^2cx^3 + \frac{1}{2}\,a^2bcx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x
```

Fricas [A] time = 1.35025, size = 84, normalized size = 2.62

$$-\frac{1}{4}x^4cb^3 - \frac{1}{3}x^3cb^2a + \frac{1}{2}x^2cba^2 + xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] -1/4*x^4*c*b^3 - 1/3*x^3*c*b^2*a + 1/2*x^2*c*b*a^2 + x*c*a^3
```

Sympy [A] time = 0.066783, size = 39, normalized size = 1.22

$$a^3cx + \frac{a^2bcx^2}{2} - \frac{ab^2cx^3}{3} - \frac{b^3cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c),x)
```

```
[Out] a**3*c*x + a**2*b*c*x**2/2 - a*b**2*c*x**3/3 - b**3*c*x**4/4
```

Giac [A] time = 1.05419, size = 49, normalized size = 1.53

$$-\frac{1}{4}\,b^3cx^4 - \frac{1}{3}\,ab^2cx^3 + \frac{1}{2}\,a^2bcx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="giac")
```

```
[Out] -1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x
```

3.1041 $\int (a+bx)^2 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^3}{3b}$$

[Out] (a + b*x)^3/(3*b)

Rubi [A] time = 0.0014589, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^2,x]

[Out] (a + b*x)^3/(3*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^2 \, dx = \frac{(a+bx)^3}{3b}$$

Mathematica [A] time = 0.001089, size = 14, normalized size = 1.

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^2,x]

[Out] (a + b*x)^3/(3*b)

Maple [A] time = 0., size = 13, normalized size = 0.9

$$\frac{(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^2,x)

```
[Out] 1/3*(b*x+a)^3/b
```

Maxima [A] time = 1.02022, size = 27, normalized size = 1.93

$$\frac{1}{3}b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Fricas [A] time = 1.26147, size = 42, normalized size = 3.

$$\frac{1}{3}x^3b^2 + x^2ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*b^2 + x^2*b*a + x*a^2
```

Sympy [B] time = 0.057453, size = 19, normalized size = 1.36

$$a^2x + abx^2 + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2,x)
```

```
[Out] a**2*x + a*b*x**2 + b**2*x**3/3
```

Giac [A] time = 1.13932, size = 16, normalized size = 1.14

$$\frac{(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a)^3/b
```

3.1042 $\int \frac{(a+bx)^2}{ac-bcx} \, dx$

Optimal. Leaf size=43

$$-\frac{4a^2\log(a-bx)}{bc}-\frac{(a+bx)^2}{2bc}-\frac{2ax}{c}$$

```
[Out] (-2*a*x)/c - (a + b*x)^2/(2*b*c) - (4*a^2*Log[a - b*x])/(b*c)
```

Rubi [A] time = 0.0137355, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{4a^2\log(a-bx)}{bc}-\frac{(a+bx)^2}{2bc}-\frac{2ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x),x]
```

```
[Out] (-2*a*x)/c - (a + b*x)^2/(2*b*c) - (4*a^2*Log[a - b*x])/(b*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{ac-bcx}\,dx &= \int \left(-\frac{2a}{c}-\frac{a+bx}{c}+\frac{4a^2}{ac-bcx}\right)dx\\ &= -\frac{2ax}{c}-\frac{(a+bx)^2}{2bc}-\frac{4a^2\log(a-bx)}{bc}\end{aligned}$$

Mathematica [A] time = 0.0069963, size = 37, normalized size = 0.86

$$-\frac{4a^2\log(a-bx)}{bc}-\frac{3ax}{c}-\frac{bx^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x),x]
```

```
[Out] (-3*a*x)/c - (b*x^2)/(2*c) - (4*a^2*Log[a - b*x])/(b*c)
```

Maple [A] time = 0.002, size = 37, normalized size = 0.9

$$-\frac{bx^2}{2c}-3\frac{ax}{c}-4\frac{a^2\ln(bx-a)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c),x)
```

```
[Out] -1/2/c*b*x^2-3*a*x/c-4/c*a^2/b*ln(b*x-a)
```

Maxima [A] time = 1.01021, size = 47, normalized size = 1.09

$$-\frac{4\,a^2\log\left(bx-a\right)}{bc}-\frac{bx^2+6\,ax}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -4*a^2*log(b*x - a)/(b*c) - 1/2*(b*x^2 + 6*a*x)/c
```

Fricas [A] time = 1.51219, size = 72, normalized size = 1.67

$$-\frac{b^2x^2+6\,abx+8\,a^2\log\left(bx-a\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 + 6*a*b*x + 8*a^2*log(b*x - a))/(b*c)
```

Sympy [A] time = 0.297853, size = 31, normalized size = 0.72

$$-\frac{4a^2\log\left(-a+bx\right)}{bc}-\frac{3ax}{c}-\frac{bx^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c),x)
```

```
[Out] -4*a**2*log(-a + b*x)/(b*c) - 3*a*x/c - b*x**2/(2*c)
```

Giac [A] time = 1.04645, size = 62, normalized size = 1.44

$$-\frac{4\,a^2\log\left(|bx-a|\right)}{bc}-\frac{b^3cx^2+6\,ab^2cx}{2\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="giac")
```

```
[Out] -4*a^2*log(abs(b*x - a))/(b*c) - 1/2*(b^3*c*x^2 + 6*a*b^2*c*x)/(b^2*c^2)
```

3.1043 $\int \frac{(a+bx)^2}{(ac-bcx)^2} dx$

Optimal. Leaf size=41

$$\frac{4a^2}{bc^2(a-bx)} + \frac{4a\log(a-bx)}{bc^2} + \frac{x}{c^2}$$

```
[Out] x/c^2 + (4*a^2)/(b*c^2*(a - b*x)) + (4*a*Log[a - b*x])/(b*c^2)
```

Rubi [A] time = 0.0221189, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{4a^2}{bc^2(a-bx)} + \frac{4a\log(a-bx)}{bc^2} + \frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^2,x]
```

```
[Out] x/c^2 + (4*a^2)/(b*c^2*(a - b*x)) + (4*a*Log[a - b*x])/(b*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^2} dx &= \int \left(\frac{1}{c^2} + \frac{4a^2}{c^2(a-bx)^2} - \frac{4a}{c^2(a-bx)}\right) dx \\ &= \frac{x}{c^2} + \frac{4a^2}{bc^2(a-bx)} + \frac{4a\log(a-bx)}{bc^2}\end{aligned}$$

Mathematica [A] time = 0.0295893, size = 35, normalized size = 0.85

$$\frac{\frac{4a^2}{b(a-bx)} + \frac{4a\log(a-bx)}{b} + x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^2,x]
```

```
[Out] (x + (4*a^2)/(b*(a - b*x)) + (4*a*Log[a - b*x])/b)/c^2
```

Maple [A] time = 0.005, size = 44, normalized size = 1.1

$$\frac{x}{c^2} - 4\,\frac{a^2}{c^2 b\,(bx - a)} + 4\,\frac{a\ln(bx - a)}{c^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^2,x)
```

```
[Out] x/c^2-4/c^2*a^2/b/(b*x-a)+4/c^2/b*a*ln(b*x-a)
```

Maxima [A] time = 1.00933, size = 62, normalized size = 1.51

$$-\frac{4\,a^2}{b^2 c^2 x - abc^2} + \frac{x}{c^2} + \frac{4\,a\log(bx - a)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -4*a^2/(b^2*c^2*x - a*b*c^2) + x/c^2 + 4*a*log(b*x - a)/(b*c^2)
```

Fricas [A] time = 1.51304, size = 108, normalized size = 2.63

$$\frac{b^2x^2 - abx - 4\,a^2 + 4\left(abx - a^2\right)\log(bx - a)}{b^2c^2x - abc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 - a*b*x - 4*a^2 + 4*(a*b*x - a^2)*log(b*x - a))/(b^2*c^2*x - a*b*c
^2)
```

Sympy [A] time = 0.358625, size = 39, normalized size = 0.95

$$-\frac{4a^2}{-abc^2 + b^2c^2x} + \frac{4a\log(-a + bx)}{bc^2} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**2,x)
```

```
[Out] -4*a**2/(-a*b*c**2 + b**2*c**2*x) + 4*a*log(-a + b*x)/(b*c**2) + x/c**2
```

Giac [A] time = 1.08186, size = 107, normalized size = 2.61

$$-\frac{4\,a^2}{(bcx - ac)bc} - \frac{4\,a\log\left(\frac{|bcx-ac|}{(bcx-ac)^2|b||c|}\right)}{bc^2} + \frac{bcx - ac}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -4*a^2/((b*c*x - a*c)*b*c) - 4*a*log(abs(b*c*x - a*c)/((b*c*x - a*c)^2*abs(
b)*abs(c)))/(b*c^2) + (b*c*x - a*c)/(b*c^3)
```

3.1044 $\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx$

Optimal. Leaf size=52

$$\frac{2a^2}{bc^3(a-bx)^2}-\frac{4a}{bc^3(a-bx)}-\frac{\log(a-bx)}{bc^3}$$

```
[Out] (2*a^2)/(b*c^3*(a - b*x)^2) - (4*a)/(b*c^3*(a - b*x)) - Log[a - b*x]/(b*c^3
)
```

Rubi [A] time = 0.0280616, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{2a^2}{bc^3(a-bx)^2}-\frac{4a}{bc^3(a-bx)}-\frac{\log(a-bx)}{bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^3,x]
```

```
[Out] (2*a^2)/(b*c^3*(a - b*x)^2) - (4*a)/(b*c^3*(a - b*x)) - Log[a - b*x]/(b*c^3
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx &= \int \left(\frac{4a^2}{c^3(a-bx)^3}-\frac{4a}{c^3(a-bx)^2}+\frac{1}{c^3(a-bx)}\right)dx\\ &= \frac{2a^2}{bc^3(a-bx)^2}-\frac{4a}{bc^3(a-bx)}-\frac{\log(a-bx)}{bc^3}\end{aligned}$$

Mathematica [A] time = 0.0244032, size = 33, normalized size = 0.63

$$-\frac{\frac{2a(a-2bx)}{(a-bx)^2}+\log(a-bx)}{bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^3,x]
```

```
[Out] -(((2*a*(a - 2*b*x))/(a - b*x)^2 + Log[a - b*x])/(b*c^3))
```

Maple [A] time = 0.005, size = 56, normalized size = 1.1

$$4\frac{a}{c^3b(bx-a)}+2\frac{a^2}{c^3b(bx-a)^2}-\frac{\ln(bx-a)}{c^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^3,x)
```

```
[Out] 4/c^3/b*a/(b*x-a)+2/c^3*a^2/b/(b*x-a)^2-1/c^3/b*ln(b*x-a)
```

Maxima [A] time = 1.03558, size = 82, normalized size = 1.58

$$\frac{2\left(2\,abx-a^2\right)}{b^3c^3x^2-2\,ab^2c^3x+a^2bc^3}-\frac{\log(bx-a)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] 2*(2*a*b*x - a^2)/(b^3*c^3*x^2 - 2*a*b^2*c^3*x + a^2*b*c^3) - log(b*x - a)/
(b*c^3)
```

Fricas [A] time = 1.57548, size = 138, normalized size = 2.65

$$\frac{4\,abx-2\,a^2-\left(b^2x^2-2\,abx+a^2\right)\log(bx-a)}{b^3c^3x^2-2\,ab^2c^3x+a^2bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] (4*a*b*x - 2*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x - a))/(b^3*c^3*x^2 - 2
*a*b^2*c^3*x + a^2*b*c^3)
```

Sympy [A] time = 0.447033, size = 53, normalized size = 1.02

$$\frac{-2a^2+4abx}{a^2bc^3-2ab^2c^3x+b^3c^3x^2}-\frac{\log(-a+bx)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**3,x)
```

```
[Out] (-2*a**2 + 4*a*b*x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2) - log(
-a + b*x)/(b*c**3)
```

Giac [A] time = 1.05647, size = 62, normalized size = 1.19

$$-\frac{\log(|bx-a|)}{bc^3}+\frac{2\left(2\,abx-a^2\right)}{(bx-a)^2bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*x - a))/(b*c^3) + 2*(2*a*b*x - a^2)/((b*x - a)^2*b*c^3)
```

3.1045 $\int \frac{(a+bx)^2}{(ac-bcx)^4} dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

```
[Out] (a + b*x)^3/(6*a*b*c^4*(a - b*x)^3)
```

Rubi [A] time = 0.0051261, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

```
[Out] (a + b*x)^3/(6*a*b*c^4*(a - b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^2}{(ac-bcx)^4} dx = \frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

Mathematica [A] time = 0.0182353, size = 31, normalized size = 1.11

$$-\frac{a^2+3b^2x^2}{3bc^4(bx-a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

```
[Out] -(a^2 + 3*b^2*x^2)/(3*b*c^4*(-a + b*x)^3)
```

Maple [A] time = 0.004, size = 52, normalized size = 1.9

$$\frac{1}{c^4}\left(-\frac{4\,a^2}{3\,b\,(bx-a)^3}-\frac{1}{b\,(bx-a)}-2\,\frac{a}{b\,(bx-a)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^4,x)
```

```
[Out] 1/c^4*(-4/3*a^2/b/(b*x-a)^3-1/b/(b*x-a)-2/b*a/(b*x-a)^2)
```

Maxima [B] time = 1.0375, size = 81, normalized size = 2.89

$$-\frac{3\,b^2x^2+a^2}{3\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a
^3*b*c^4)
```

Fricas [B] time = 1.55696, size = 116, normalized size = 4.14

$$-\frac{3\,b^2x^2+a^2}{3\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a
^3*b*c^4)
```

Sympy [B] time = 0.507735, size = 61, normalized size = 2.18

$$-\frac{a^2+3b^2x^2}{-3a^3bc^4+9a^2b^2c^4x-9ab^3c^4x^2+3b^4c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**4,x)
```

```
[Out] -(a**2 + 3*b**2*x**2)/(-3*a**3*b*c**4 + 9*a**2*b**2*c**4*x - 9*a*b**3*c**4*
x**2 + 3*b**4*c**4*x**3)
```

Giac [A] time = 1.06528, size = 39, normalized size = 1.39

$$-\frac{3\,b^2x^2+a^2}{3\,(bx-a)^3bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/((b*x - a)^3*b*c^4)
```

3.1046 $\int \frac{(a+bx)^2}{(ac-bcx)^5} dx$

Optimal. Leaf size=56

$$\frac{a^2}{bc^5(a-bx)^4} - \frac{4a}{3bc^5(a-bx)^3} + \frac{1}{2bc^5(a-bx)^2}$$

```
[Out] a^2/(b*c^5*(a - b*x)^4) - (4*a)/(3*b*c^5*(a - b*x)^3) + 1/(2*b*c^5*(a - b*x
)^2)
```

Rubi [A] time = 0.0237935, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{a^2}{bc^5(a-bx)^4} - \frac{4a}{3bc^5(a-bx)^3} + \frac{1}{2bc^5(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^5,x]
```

```
[Out] a^2/(b*c^5*(a - b*x)^4) - (4*a)/(3*b*c^5*(a - b*x)^3) + 1/(2*b*c^5*(a - b*x
)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^5} dx &= \int \left(\frac{4a^2}{c^5(a-bx)^5} - \frac{4a}{c^5(a-bx)^4} + \frac{1}{c^5(a-bx)^3}\right) dx\\ &= \frac{a^2}{bc^5(a-bx)^4} - \frac{4a}{3bc^5(a-bx)^3} + \frac{1}{2bc^5(a-bx)^2}\end{aligned}$$

Mathematica [A] time = 0.0119657, size = 35, normalized size = 0.62

$$\frac{a^2+2abx+3b^2x^2}{6bc^5(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^5,x]
```

```
[Out] (a^2 + 2*a*b*x + 3*b^2*x^2)/(6*b*c^5*(a - b*x)^4)
```

Maple [A] time = 0.004, size = 51, normalized size = 0.9

$$\frac{1}{c^5}\left(\frac{a^2}{b\,(bx-a)^4}+\frac{4\,a}{3\,b\,(bx-a)^3}+\frac{1}{2\,b\,(bx-a)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^5,x)
```

```
[Out] 1/c^5*(a^2/b/(b*x-a)^4+4/3/b*a/(b*x-a)^3+1/2/b/(b*x-a)^2)
```

Maxima [A] time = 1.06078, size = 105, normalized size = 1.88

$$\frac{3\,b^2x^2+2\,abx+a^2}{6\left(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(3*b^2*x^2 + 2*a*b*x + a^2)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*
c^5*x^2 - 4*a^3*b^2*c^5*x + a^4*b*c^5)
```

Fricas [A] time = 1.48695, size = 155, normalized size = 2.77

$$\frac{3\,b^2x^2+2\,abx+a^2}{6\left(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*x^2 + 2*a*b*x + a^2)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*
c^5*x^2 - 4*a^3*b^2*c^5*x + a^4*b*c^5)
```

Sympy [A] time = 0.584813, size = 82, normalized size = 1.46

$$\frac{a^2+2abx+3b^2x^2}{6a^4bc^5-24a^3b^2c^5x+36a^2b^3c^5x^2-24ab^4c^5x^3+6b^5c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**5,x)
```

```
[Out] (a**2 + 2*a*b*x + 3*b**2*x**2)/(6*a**4*b*c**5 - 24*a**3*b**2*c**5*x + 36*a*
*2*b**3*c**5*x**2 - 24*a*b**4*c**5*x**3 + 6*b**5*c**5*x**4)
```

Giac [A] time = 1.05837, size = 86, normalized size = 1.54

$$\frac{\frac{6a^2}{(bcx-ac)^4b} + \frac{8a}{(bcx-ac)^3bc} + \frac{3}{(bcx-ac)^2bc^2}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="giac")
```

```
[Out] 1/6*(6*a^2/((b*c*x - a*c)^4*b) + 8*a/((b*c*x - a*c)^3*b*c) + 3/((b*c*x - a*
c)^2*b*c^2))/c
```

3.1047 $\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx$

Optimal. Leaf size=57

$$\frac{4a^2}{5bc^6(a-bx)^5}-\frac{a}{bc^6(a-bx)^4}+\frac{1}{3bc^6(a-bx)^3}$$

```
[Out] (4*a^2)/(5*b*c^6*(a - b*x)^5) - a/(b*c^6*(a - b*x)^4) + 1/(3*b*c^6*(a - b*x
)^3)
```

Rubi [A] time = 0.0248059, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{4a^2}{5bc^6(a-bx)^5}-\frac{a}{bc^6(a-bx)^4}+\frac{1}{3bc^6(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

```
[Out] (4*a^2)/(5*b*c^6*(a - b*x)^5) - a/(b*c^6*(a - b*x)^4) + 1/(3*b*c^6*(a - b*x
)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx &= \int \left(\frac{4a^2}{c^6(a-bx)^6}-\frac{4a}{c^6(a-bx)^5}+\frac{1}{c^6(a-bx)^4}\right)dx\\ &= \frac{4a^2}{5bc^6(a-bx)^5}-\frac{a}{bc^6(a-bx)^4}+\frac{1}{3bc^6(a-bx)^3}\end{aligned}$$

Mathematica [A] time = 0.0182162, size = 38, normalized size = 0.67

$$-\frac{2a^2+5abx+5b^2x^2}{15bc^6(bx-a)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

```
[Out] -(2*a^2 + 5*a*b*x + 5*b^2*x^2)/(15*b*c^6*(-a + b*x)^5)
```

Maple [A] time = 0.005, size = 52, normalized size = 0.9

$$\frac{1}{c^6}\left(-\frac{a}{b\,(bx-a)^4}-\frac{1}{3\,b\,(bx-a)^3}-\frac{4\,a^2}{5\,b\,(bx-a)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^6,x)
```

```
[Out] 1/c^6*(-1/b*a/(b*x-a)^4-1/3/b/(b*x-a)^3-4/5*a^2/b/(b*x-a)^5)
```

Maxima [A] time = 1.0339, size = 128, normalized size = 2.25

$$-\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\left(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="maxima")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2
*b^4*c^6*x^3 - 10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Fricas [A] time = 1.50276, size = 190, normalized size = 3.33

$$-\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\left(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2
*b^4*c^6*x^3 - 10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Sympy [B] time = 0.651973, size = 100, normalized size = 1.75

$$-\frac{2a^2+5abx+5b^2x^2}{-15a^5bc^6+75a^4b^2c^6x-150a^3b^3c^6x^2+150a^2b^4c^6x^3-75ab^5c^6x^4+15b^6c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**6,x)
```

```
[Out] -(2*a**2 + 5*a*b*x + 5*b**2*x**2)/(-15*a**5*b*c**6 + 75*a**4*b**2*c**6*x -
150*a**3*b**3*c**6*x**2 + 150*a**2*b**4*c**6*x**3 - 75*a*b**5*c**6*x**4 + 1
5*b**6*c**6*x**5)
```

<u>Giac</u> [A] time = 1.06136, size = 49, normalized size = 0.86

$$-\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\,(bx-a)^5bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="giac")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/((b*x - a)^5*b*c^6)
```

3.1048 $\int \frac{(a+bx)^2}{(ac-bcx)^7}\,dx$

Optimal. Leaf size=59

$$\frac{2a^2}{3bc^7(a-bx)^6}-\frac{4a}{5bc^7(a-bx)^5}+\frac{1}{4bc^7(a-bx)^4}$$

```
[Out] (2*a^2)/(3*b*c^7*(a - b*x)^6) - (4*a)/(5*b*c^7*(a - b*x)^5) + 1/(4*b*c^7*(a
 - b*x)^4)
```

Rubi [A] time = 0.0290148, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{2a^2}{3bc^7(a-bx)^6}-\frac{4a}{5bc^7(a-bx)^5}+\frac{1}{4bc^7(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

```
[Out] (2*a^2)/(3*b*c^7*(a - b*x)^6) - (4*a)/(5*b*c^7*(a - b*x)^5) + 1/(4*b*c^7*(a
 - b*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^7}\,dx &= \int \left(\frac{4a^2}{c^7(a-bx)^7}-\frac{4a}{c^7(a-bx)^6}+\frac{1}{c^7(a-bx)^5}\right)dx\\ &= \frac{2a^2}{3bc^7(a-bx)^6}-\frac{4a}{5bc^7(a-bx)^5}+\frac{1}{4bc^7(a-bx)^4}\end{aligned}$$

Mathematica [A] time = 0.0148413, size = 37, normalized size = 0.63

$$\frac{7a^2+18abx+15b^2x^2}{60bc^7(a-bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

```
[Out] (7*a^2 + 18*a*b*x + 15*b^2*x^2)/(60*b*c^7*(a - b*x)^6)
```

Maple [A] time = 0.004, size = 52, normalized size = 0.9

$$\frac{1}{c^7}\left(\frac{1}{4\,b\,(bx-a)^4}+\frac{4\,a}{5\,b\,(bx-a)^5}+\frac{2\,a^2}{3\,b\,(bx-a)^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^7,x)
```

```
[Out] 1/c^7*(1/4/b/(b*x-a)^4+4/5/b*a/(b*x-a)^5+2/3*a^2/b/(b*x-a)^6)
```

Maxima [A] time = 1.05498, size = 146, normalized size = 2.47

$$\frac{15\,b^2x^2+18\,abx+7\,a^2}{60\left(b^7c^7x^6-6\,ab^6c^7x^5+15\,a^2b^5c^7x^4-20\,a^3b^4c^7x^3+15\,a^4b^3c^7x^2-6\,a^5b^2c^7x+a^6bc^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/(b^7*c^7*x^6 - 6*a*b^6*c^7*x^5 + 15*a^
2*b^5*c^7*x^4 - 20*a^3*b^4*c^7*x^3 + 15*a^4*b^3*c^7*x^2 - 6*a^5*b^2*c^7*x +
 a^6*b*c^7)
```

Fricas [A] time = 1.45674, size = 220, normalized size = 3.73

$$\frac{15\,b^2x^2+18\,abx+7\,a^2}{60\left(b^7c^7x^6-6\,ab^6c^7x^5+15\,a^2b^5c^7x^4-20\,a^3b^4c^7x^3+15\,a^4b^3c^7x^2-6\,a^5b^2c^7x+a^6bc^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/(b^7*c^7*x^6 - 6*a*b^6*c^7*x^5 + 15*a^
2*b^5*c^7*x^4 - 20*a^3*b^4*c^7*x^3 + 15*a^4*b^3*c^7*x^2 - 6*a^5*b^2*c^7*x +
 a^6*b*c^7)
```

Sympy [B] time = 0.759735, size = 114, normalized size = 1.93

$$\frac{7a^2+18abx+15b^2x^2}{60a^6bc^7-360a^5b^2c^7x+900a^4b^3c^7x^2-1200a^3b^4c^7x^3+900a^2b^5c^7x^4-360ab^6c^7x^5+60b^7c^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**7,x)
```

```
[Out] (7*a**2 + 18*a*b*x + 15*b**2*x**2)/(60*a**6*b*c**7 - 360*a**5*b**2*c**7*x +
 900*a**4*b**3*c**7*x**2 - 1200*a**3*b**4*c**7*x**3 + 900*a**2*b**5*c**7*x*
*4 - 360*a*b**6*c**7*x**5 + 60*b**7*c**7*x**6)
```

Giac [A] time = 1.06035, size = 49, normalized size = 0.83

$$\frac{15 b^2 x^2 + 18 abx + 7 a^2}{60 (bx - a)^6 bc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="giac")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/((b*x - a)^6*b*c^7)
```

3.1049 $\int \frac{(ac-bcx)^3}{a+bx}\,dx$

Optimal. Leaf size=61

$$\frac{8a^3c^3\log(a+bx)}{b}-4a^2c^3x+\frac{c^3(a-bx)^3}{3b}+\frac{ac^3(a-bx)^2}{b}$$

```
[Out] -4*a^2*c^3*x + (a*c^3*(a - b*x)^2)/b + (c^3*(a - b*x)^3)/(3*b) + (8*a^3*c^3
*Log[a + b*x])/b
```

Rubi [A] time = 0.0212865, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{8a^3c^3\log(a+bx)}{b}-4a^2c^3x+\frac{c^3(a-bx)^3}{3b}+\frac{ac^3(a-bx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^3/(a + b*x),x]
```

```
[Out] -4*a^2*c^3*x + (a*c^3*(a - b*x)^2)/b + (c^3*(a - b*x)^3)/(3*b) + (8*a^3*c^3
*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^3}{a+bx}\,dx &= \int \left(-4a^2c^3+\frac{8a^3c^3}{a+bx}-2ac^2(ac-bcx)-c(ac-bcx)^2\right)dx\\ &= -4a^2c^3x+\frac{ac^3(a-bx)^2}{b}+\frac{c^3(a-bx)^3}{3b}+\frac{8a^3c^3\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0056005, size = 42, normalized size = 0.69

$$c^3\left(\frac{8a^3\log(a+bx)}{b}-7a^2x+2abx^2-\frac{b^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^3/(a + b*x),x]
```

```
[Out] c^3*(-7*a^2*x + 2*a*b*x^2 - (b^2*x^3)/3 + (8*a^3*Log[a + b*x])/b)
```

Maple [A] time = 0.002, size = 49, normalized size = 0.8

$$-\frac{c^3x^3b^2}{3}+2\,c^3bx^2a-7\,a^2c^3x+8\,\frac{a^3c^3\ln\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^3/(b*x+a),x)
```

```
[Out] -1/3*c^3*x^3*b^2+2*c^3*b*x^2*a-7*a^2*c^3*x+8*a^3*c^3*ln(b*x+a)/b
```

Maxima [A] time = 1.05212, size = 65, normalized size = 1.07

$$-\frac{1}{3}\,b^2c^3x^3+2\,abc^3x^2-7\,a^2c^3x+\frac{8\,a^3c^3\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/3*b^2*c^3*x^3 + 2*a*b*c^3*x^2 - 7*a^2*c^3*x + 8*a^3*c^3*log(b*x + a)/b
```

Fricas [A] time = 1.56054, size = 112, normalized size = 1.84

$$-\frac{b^3c^3x^3-6\,ab^2c^3x^2+21\,a^2bc^3x-24\,a^3c^3\log\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*c^3*x^3 - 6*a*b^2*c^3*x^2 + 21*a^2*b*c^3*x - 24*a^3*c^3*log(b*x +
 a))/b
```

Sympy [A] time = 0.317511, size = 49, normalized size = 0.8

$$\frac{8a^3c^3\log\left(a+bx\right)}{b}-7a^2c^3x+2abc^3x^2-\frac{b^2c^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**3/(b*x+a),x)
```

```
[Out] 8*a**3*c**3*log(a + b*x)/b - 7*a**2*c**3*x + 2*a*b*c**3*x**2 - b**2*c**3*x*
*3/3
```

Giac [A] time = 1.04772, size = 80, normalized size = 1.31

$$\frac{8\,a^3c^3\log\left(|bx+a|\right)}{b}-\frac{b^5c^3x^3-6\,ab^4c^3x^2+21\,a^2b^3c^3x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 8*a^3*c^3*log(abs(b*x + a))/b - 1/3*(b^5*c^3*x^3 - 6*a*b^4*c^3*x^2 + 21*a^2
*b^3*c^3*x)/b^3
```

3.1050 $\int \frac{(ac-bcx)^2}{a+bx}\,dx$

Optimal. Leaf size=43

$$\frac{4a^2c^2\log(a+bx)}{b}+\frac{c^2(a-bx)^2}{2b}-2ac^2x$$

```
[Out] -2*a*c^2*x + (c^2*(a - b*x)^2)/(2*b) + (4*a^2*c^2*Log[a + b*x])/b
```

Rubi [A] time = 0.0143916, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{4a^2c^2\log(a+bx)}{b}+\frac{c^2(a-bx)^2}{2b}-2ac^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^2/(a + b*x),x]
```

```
[Out] -2*a*c^2*x + (c^2*(a - b*x)^2)/(2*b) + (4*a^2*c^2*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^2}{a+bx}\,dx &= \int \left(-2ac^2+\frac{4a^2c^2}{a+bx}-c(ac-bcx)\right)dx\\ &= -2ac^2x+\frac{c^2(a-bx)^2}{2b}+\frac{4a^2c^2\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0061941, size = 31, normalized size = 0.72

$$c^2\left(\frac{4a^2\log(a+bx)}{b}-3ax+\frac{bx^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^2/(a + b*x),x]
```

```
[Out] c^2*(-3*a*x + (b*x^2)/2 + (4*a^2*Log[a + b*x])/b)
```

Maple [A] time = 0.001, size = 35, normalized size = 0.8

$$\frac{c^2bx^2}{2}-3\,ac^2x+4\,\frac{a^2c^2\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^2/(b*x+a),x)
```

```
[Out] 1/2*c^2*b*x^2-3*a*c^2*x+4*a^2*c^2*ln(b*x+a)/b
```

Maxima [A] time = 1.02392, size = 46, normalized size = 1.07

$$\frac{1}{2}\,bc^{2}x^{2} - 3\,ac^{2}x + \frac{4\,a^{2}c^{2}\log\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*b*c^2*x^2 - 3*a*c^2*x + 4*a^2*c^2*log(b*x + a)/b
```

Fricas [A] time = 1.4938, size = 81, normalized size = 1.88

$$\frac{b^{2}c^{2}x^{2} - 6\,abc^{2}x + 8\,a^{2}c^{2}\log\left(bx + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2 - 6*a*b*c^2*x + 8*a^2*c^2*log(b*x + a))/b
```

Sympy [A] time = 0.296108, size = 34, normalized size = 0.79

$$\frac{4a^{2}c^{2}\log\left(a + bx\right)}{b} - 3ac^{2}x + \frac{bc^{2}x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**2/(b*x+a),x)
```

```
[Out] 4*a**2*c**2*log(a + b*x)/b - 3*a*c**2*x + b*c**2*x**2/2
```

Giac [A] time = 1.0569, size = 61, normalized size = 1.42

$$\frac{4\,a^{2}c^{2}\log\left(|bx + a|\right)}{b} + \frac{b^{3}c^{2}x^{2} - 6\,ab^{2}c^{2}x}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="giac")
```

```
[Out] 4*a^2*c^2*log(abs(b*x + a))/b + 1/2*(b^3*c^2*x^2 - 6*a*b^2*c^2*x)/b^2
```

3.1051 $\int \frac{ac-bcx}{a+bx}\,dx$

Optimal. Leaf size=18

$$\frac{2ac\log(a+bx)}{b}-cx$$

```
[Out] -(c*x) + (2*a*c*Log[a + b*x])/b
```

Rubi [A] time = 0.0103672, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2ac\log(a+bx)}{b}-cx$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)/(a + b*x),x]
```

```
[Out] -(c*x) + (2*a*c*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ac-bcx}{a+bx}\,dx &= \int \left(-c+\frac{2ac}{a+bx}\right)dx\\ &= -cx+\frac{2ac\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0039915, size = 18, normalized size = 1.

$$c\left(\frac{2a\log(a+bx)}{b}-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)/(a + b*x),x]
```

```
[Out] c*(-x + (2*a*Log[a + b*x])/b)
```

Maple [A] time = 0.001, size = 19, normalized size = 1.1

$$-cx+2\,\frac{ac\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)/(b*x+a),x)
```

```
[Out] -c*x+2*a*c*ln(b*x+a)/b
```

Maxima [A] time = 1.0294, size = 24, normalized size = 1.33

$$-cx + \frac{2\,ac\log\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -c*x + 2*a*c*log(b*x + a)/b
```

Fricas [A] time = 1.41083, size = 45, normalized size = 2.5

$$-\frac{bcx - 2\,ac\log\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*c*x - 2*a*c*log(b*x + a))/b
```

Sympy [A] time = 0.276684, size = 15, normalized size = 0.83

$$\frac{2ac\log\left(a + bx\right)}{b} - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x)
```

```
[Out] 2*a*c*log(a + b*x)/b - c*x
```

Giac [A] time = 1.04863, size = 26, normalized size = 1.44

$$-cx + \frac{2\,ac\log\left(|bx + a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="giac")
```

```
[Out] -c*x + 2*a*c*log(abs(b*x + a))/b
```

3.1052 $\int \frac{1}{a+bx} dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

```
[Out] Log[a + b*x]/b
```

Rubi [A] time = 0.0015238, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+bx} dx = \frac{\log(a+bx)}{b}$$

Mathematica [A] time = 0.0007695, size = 10, normalized size = 1.

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Maple [A] time = 0.001, size = 11, normalized size = 1.1

$$\frac{\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a),x)
```

```
[Out] ln(b*x+a)/b
```

Maxima [A] time = 1.06101, size = 14, normalized size = 1.4

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/b
```

Fricas [A] time = 1.42756, size = 22, normalized size = 2.2

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

Sympy [A] time = 0.058831, size = 7, normalized size = 0.7

$$\frac{\log(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
```

```
[Out] log(a + b*x)/b
```

Giac [A] time = 1.06475, size = 15, normalized size = 1.5

$$\frac{\log(|bx+a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

3.1053 $\int \frac{1}{(a+bx)(ac-bcx)}\,dx$

Optimal. Leaf size=17

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

```
[Out] ArcTanh[(b*x)/a]/(a*b*c)
```

Rubi [A] time = 0.0085704, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {35, 208}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b*c)
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(ac-bcx)}\,dx &= \int \frac{1}{a^2c-b^2cx^2}\,dx\\ &= \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}\end{aligned}$$

Mathematica [A] time = 0.007866, size = 17, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b*c)
```

Maple [B] time = 0.004, size = 38, normalized size = 2.2

$$\frac{\ln (bx+a)}{2bca}-\frac{\ln (bx-a)}{2bca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c),x)
```

```
[Out] 1/2/c/b/a*ln(b*x+a)-1/2/c/b/a*ln(b*x-a)
```

Maxima [B] time = 1.03954, size = 50, normalized size = 2.94

$$\frac{\log (bx+a)}{2abc}-\frac{\log (bx-a)}{2abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)/(a*b*c) - 1/2*log(b*x - a)/(a*b*c)
```

Fricas [A] time = 1.42231, size = 58, normalized size = 3.41

$$\frac{\log (bx+a)-\log (bx-a)}{2abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/2*(log(b*x + a) - log(b*x - a))/(a*b*c)
```

Sympy [B] time = 0.158794, size = 22, normalized size = 1.29

$$-\frac{\frac{\log \left(-\frac{a}{b}+x\right)}{2}-\frac{\log \left(\frac{a}{b}+x\right)}{2}}{abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x)
```

```
[Out] -(log(-a/b + x)/2 - log(a/b + x)/2)/(a*b*c)
```

Giac [B] time = 1.06458, size = 53, normalized size = 3.12

$$\frac{\log (|bx+a|)}{2abc}-\frac{\log (|bx-a|)}{2abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x + a))/(a*b*c) - 1/2*log(abs(b*x - a))/(a*b*c)
```

3.1054 $\int \frac{1}{(a+bx)(ac-bcx)^2} dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2}+\frac{1}{2abc^2(a-bx)}$$

```
[Out] 1/(2*a*b*c^2*(a - b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c^2)
```

Rubi [A] time = 0.0292954, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {44, 208}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2}+\frac{1}{2abc^2(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)^2),x]
```

```
[Out] 1/(2*a*b*c^2*(a - b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)(ac-bcx)^2} dx &= \int \left(\frac{1}{2ac^2(a-bx)^2}+\frac{1}{2ac^2\left(a^2-b^2x^2\right)}\right) dx\\
&= \frac{1}{2abc^2(a-bx)}+\frac{\int \frac{1}{a^2-b^2x^2} dx}{2ac^2}\\
&= \frac{1}{2abc^2(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2}
\end{aligned}$$

Mathematica [A] time = 0.0152711, size = 53, normalized size = 1.26

$$\frac{(bx-a)\log(a-bx)+(a-bx)\log(a+bx)+2a}{4a^2bc^2(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)^2),x]
```

```
[Out] (2*a + (-a + b*x)*Log[a - b*x] + (a - b*x)*Log[a + b*x])/(4*a^2*b*c^2*(a -
b*x))
```

Maple [A] time = 0.007, size = 58, normalized size = 1.4

$$\frac{\ln(bx+a)}{4c^2a^2b} - \frac{\ln(bx-a)}{4c^2a^2b} - \frac{1}{2c^2ba(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c)^2,x)
```

```
[Out] 1/4/c^2/a^2/b*ln(b*x+a)-1/4/c^2/a^2/b*ln(b*x-a)-1/2/c^2/b/a/(b*x-a)
```

Maxima [A] time = 1.06501, size = 81, normalized size = 1.93

$$-\frac{1}{2\left(ab^2c^2x - a^2bc^2\right)} + \frac{\log(bx+a)}{4a^2bc^2} - \frac{\log(bx-a)}{4a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(a*b^2*c^2*x - a^2*b*c^2) + 1/4*log(b*x + a)/(a^2*b*c^2) - 1/4*log(b*x
 - a)/(a^2*b*c^2)
```

Fricas [A] time = 1.53512, size = 120, normalized size = 2.86

$$\frac{(bx-a)\log(bx+a) - (bx-a)\log(bx-a) - 2a}{4\left(a^2b^2c^2x - a^3bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((b*x - a)*log(b*x + a) - (b*x - a)*log(b*x - a) - 2*a)/(a^2*b^2*c^2*x
- a^3*b*c^2)
```

Sympy [A] time = 0.468911, size = 48, normalized size = 1.14

$$-\frac{1}{-2a^2bc^2 + 2ab^2c^2x} + \frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4} + \frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)**2,x)
```

```
[Out] -1/(-2*a**2*b*c**2 + 2*a*b**2*c**2*x) + (-log(-a/b + x)/4 + log(a/b + x)/4)
/(a**2*b*c**2)
```

Giac [A] time = 1.07984, size = 72, normalized size = 1.71

$$-\frac{1}{2\,(bcx-ac)abc}+\frac{\log\left(\left|-\frac{2\,ac}{bcx-ac}-1\right|\right)}{4\,a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -1/2/((b*c*x - a*c)*a*b*c) + 1/4*log(abs(-2*a*c/(b*c*x - a*c) - 1))/(a^2*b*
c^2)
```

3.1055 $\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx$

Optimal. Leaf size=63

$$\frac{1}{4a^2bc^3(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}+\frac{1}{4abc^3(a-bx)^2}$$

```
[Out] 1/(4*a*b*c^3*(a - b*x)^2) + 1/(4*a^2*b*c^3*(a - b*x)) + ArcTanh[(b*x)/a]/(4
*a^3*b*c^3)
```

Rubi [A] time = 0.0385831, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {44, 208}

$$\frac{1}{4a^2bc^3(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}+\frac{1}{4abc^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

```
[Out] 1/(4*a*b*c^3*(a - b*x)^2) + 1/(4*a^2*b*c^3*(a - b*x)) + ArcTanh[(b*x)/a]/(4
*a^3*b*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx &= \int \left(\frac{1}{2ac^3(a-bx)^3}+\frac{1}{4a^2c^3(a-bx)^2}+\frac{1}{4a^2c^3\left(a^2-b^2x^2\right)}\right)dx\\ &= \frac{1}{4abc^3(a-bx)^2}+\frac{1}{4a^2bc^3(a-bx)}+\frac{\int \frac{1}{a^2-b^2x^2}\,dx}{4a^2c^3}\\ &= \frac{1}{4abc^3(a-bx)^2}+\frac{1}{4a^2bc^3(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}\end{aligned}$$

Mathematica [A] time = 0.0205471, size = 65, normalized size = 1.03

$$\frac{2a(2a-bx)+(a-bx)^2(-\log(a-bx))+(a-bx)^2\log(a+bx)}{8a^3bc^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

```
[Out] (2*a*(2*a - b*x) - (a - b*x)^2*Log[a - b*x] + (a - b*x)^2*Log[a + b*x])/(8*
a^3*b*c^3*(a - b*x)^2)
```

Maple [A] time = 0.005, size = 78, normalized size = 1.2

$$\frac{\ln(bx+a)}{8c^3a^3b} - \frac{\ln(bx-a)}{8c^3a^3b} - \frac{1}{4c^3a^2b(bx-a)} + \frac{1}{4c^3ba(bx-a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c)^3,x)
```

```
[Out] 1/8/c^3/a^3/b*ln(b*x+a)-1/8/c^3/a^3/b*ln(b*x-a)-1/4/c^3/a^2/b/(b*x-a)+1/4/c
^3/b/a/(b*x-a)^2
```

Maxima [A] time = 1.02586, size = 111, normalized size = 1.76

$$-\frac{bx-2a}{4\left(a^2b^3c^3x^2 - 2a^3b^2c^3x + a^4bc^3\right)} + \frac{\log(bx+a)}{8a^3bc^3} - \frac{\log(bx-a)}{8a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x - 2*a)/(a^2*b^3*c^3*x^2 - 2*a^3*b^2*c^3*x + a^4*b*c^3) + 1/8*log(
b*x + a)/(a^3*b*c^3) - 1/8*log(b*x - a)/(a^3*b*c^3)
```

Fricas [A] time = 1.51312, size = 208, normalized size = 3.3

$$-\frac{2abx - 4a^2 - \left(b^2x^2 - 2abx + a^2\right)\log(bx+a) + \left(b^2x^2 - 2abx + a^2\right)\log(bx-a)}{8\left(a^3b^3c^3x^2 - 2a^4b^2c^3x + a^5bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*b*x - 4*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x + a) + (b^2*x^2 -
 2*a*b*x + a^2)*log(b*x - a))/(a^3*b^3*c^3*x^2 - 2*a^4*b^2*c^3*x + a^5*b*c^
3)
```

Sympy [A] time = 0.521837, size = 71, normalized size = 1.13

$$-\frac{-2a+bx}{4a^4bc^3 - 8a^3b^2c^3x + 4a^2b^3c^3x^2} - \frac{\frac{\log\left(-\frac{a}{b}+x\right)}{8} - \frac{\log\left(\frac{a}{b}+x\right)}{8}}{a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)**3,x)
```

```
[Out] -(-2*a + b*x)/(4*a**4*b*c**3 - 8*a**3*b**2*c**3*x + 4*a**2*b**3*c**3*x**2)
- (log(-a/b + x)/8 - log(a/b + x)/8)/(a**3*b*c**3)
```

Giac [A] time = 1.07019, size = 93, normalized size = 1.48

$$\frac{\log\left(|bx+a|\right)}{8\,a^3bc^3}-\frac{\log\left(|bx-a|\right)}{8\,a^3bc^3}-\frac{abx-2\,a^2}{4\,(bx-a)^2a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] 1/8*log(abs(b*x + a))/(a^3*b*c^3) - 1/8*log(abs(b*x - a))/(a^3*b*c^3) - 1/4
*(a*b*x - 2*a^2)/((b*x - a)^2*a^3*b*c^3)
```

3.1056 $\int \frac{(ac-bcx)^3}{(a+bx)^2} dx$

Optimal. Leaf size=54

$$-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}+5ac^3x-\frac{1}{2}bc^3x^2$$

```
[Out] 5*a*c^3*x - (b*c^3*x^2)/2 - (8*a^3*c^3)/(b*(a + b*x)) - (12*a^2*c^3*Log[a +
 b*x])/b
```

Rubi [A] time = 0.0313632, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}+5ac^3x-\frac{1}{2}bc^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^3/(a + b*x)^2,x]
```

```
[Out] 5*a*c^3*x - (b*c^3*x^2)/2 - (8*a^3*c^3)/(b*(a + b*x)) - (12*a^2*c^3*Log[a +
 b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx &= \int \left(5ac^3-bc^3x+\frac{8a^3c^3}{(a+bx)^2}-\frac{12a^2c^3}{a+bx}\right)dx\\ &= 5ac^3x-\frac{1}{2}bc^3x^2-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0188758, size = 46, normalized size = 0.85

$$c^3\left(-\frac{8a^3}{b(a+bx)}-\frac{12a^2\log(a+bx)}{b}+5ax-\frac{bx^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^3/(a + b*x)^2,x]
```

```
[Out] c^3*(5*a*x - (b*x^2)/2 - (8*a^3)/(b*(a + b*x)) - (12*a^2*Log[a + b*x])/b)
```

Maple [A] time = 0.005, size = 53, normalized size = 1.

$$5\,ac^3x - \frac{bc^3x^2}{2} - 8\,\frac{a^3c^3}{b\,(bx+a)} - 12\,\frac{a^2c^3\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^3/(b*x+a)^2,x)
```

```
[Out] 5*a*c^3*x-1/2*b*c^3*x^2-8*a^3*c^3/b/(b*x+a)-12*a^2*c^3*ln(b*x+a)/b
```

Maxima [A] time = 1.01918, size = 72, normalized size = 1.33

$$-\frac{1}{2}\,bc^3x^2 - \frac{8\,a^3c^3}{b^2x+ab} + 5\,ac^3x - \frac{12\,a^2c^3\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*c^3*x^2 - 8*a^3*c^3/(b^2*x + a*b) + 5*a*c^3*x - 12*a^2*c^3*log(b*x +
 a)/b
```

Fricas [A] time = 1.51771, size = 167, normalized size = 3.09

$$-\frac{b^3c^3x^3 - 9\,ab^2c^3x^2 - 10\,a^2bc^3x + 16\,a^3c^3 + 24\left(a^2bc^3x + a^3c^3\right)\log(bx+a)}{2\left(b^2x+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*c^3*x^3 - 9*a*b^2*c^3*x^2 - 10*a^2*b*c^3*x + 16*a^3*c^3 + 24*(a^2
*b*c^3*x + a^3*c^3)*log(b*x + a))/(b^2*x + a*b)
```

Sympy [A] time = 0.388527, size = 51, normalized size = 0.94

$$-\frac{8a^3c^3}{ab+b^2x} - \frac{12a^2c^3\log(a+bx)}{b} + 5ac^3x - \frac{bc^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**3/(b*x+a)**2,x)
```

```
[Out] -8*a**3*c**3/(a*b + b**2*x) - 12*a**2*c**3*log(a + b*x)/b + 5*a*c**3*x - b*
c**3*x**2/2
```

Giac [A] time = 1.05334, size = 108, normalized size = 2.

$$\frac{12\,a^2c^3\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}-\frac{8\,a^3c^3}{(bx+a)b}+\frac{\left(\frac{12\,ac^3}{bx+a}-c^3\right)(bx+a)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 12*a^2*c^3*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - 8*a^3*c^3/((b*x + a)*
b) + 1/2*(12*a*c^3/(b*x + a) - c^3)*(b*x + a)^2/b
```

3.1057 $\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx$

Optimal. Leaf size=39

$$-\frac{4a^2c^2}{b(a+bx)}-\frac{4ac^2\log(a+bx)}{b}+c^2x$$

```
[Out] c^2*x - (4*a^2*c^2)/(b*(a + b*x)) - (4*a*c^2*Log[a + b*x])/b
```

Rubi [A] time = 0.0209701, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{4a^2c^2}{b(a+bx)}-\frac{4ac^2\log(a+bx)}{b}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^2/(a + b*x)^2,x]
```

```
[Out] c^2*x - (4*a^2*c^2)/(b*(a + b*x)) - (4*a*c^2*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx &= \int \left(c^2+\frac{4a^2c^2}{(a+bx)^2}-\frac{4ac^2}{a+bx}\right)dx\\ &= c^2x-\frac{4a^2c^2}{b(a+bx)}-\frac{4ac^2\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0179474, size = 33, normalized size = 0.85

$$c^2\left(-\frac{4a^2}{b(a+bx)}-\frac{4a\log(a+bx)}{b}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^2/(a + b*x)^2,x]
```

```
[Out] c^2*(x - (4*a^2)/(b*(a + b*x)) - (4*a*Log[a + b*x])/b)
```

Maple [A] time = 0.005, size = 40, normalized size = 1.

$$c^2x-4\,\frac{a^2c^2}{b\,(bx+a)}-4\,\frac{ac^2\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^2/(b*x+a)^2,x)
```

```
[Out] c^2*x-4*a^2*c^2/b/(b*x+a)-4*a*c^2*ln(b*x+a)/b
```

Maxima [A] time = 1.04708, size = 54, normalized size = 1.38

$$-\frac{4\,a^2c^2}{b^2x+ab}+c^2x-\frac{4\,ac^2\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -4*a^2*c^2/(b^2*x + a*b) + c^2*x - 4*a*c^2*log(b*x + a)/b
```

Fricas [A] time = 1.5316, size = 124, normalized size = 3.18

$$\frac{b^2c^2x^2+abc^2x-4\,a^2c^2-4\left(abc^2x+a^2c^2\right)\log\left(bx+a\right)}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*c^2*x^2 + a*b*c^2*x - 4*a^2*c^2 - 4*(a*b*c^2*x + a^2*c^2)*log(b*x + a)
)/(b^2*x + a*b)
```

Sympy [A] time = 0.344966, size = 36, normalized size = 0.92

$$-\frac{4a^2c^2}{ab+b^2x}-\frac{4ac^2\log\left(a+bx\right)}{b}+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**2/(b*x+a)**2,x)
```

```
[Out] -4*a**2*c**2/(a*b + b**2*x) - 4*a*c**2*log(a + b*x)/b + c**2*x
```

Giac [A] time = 1.07997, size = 80, normalized size = 2.05

$$\frac{4\,ac^2\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}+\frac{(bx+a)c^2}{b}-\frac{4\,a^2c^2}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4*a*c^2*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b + (b*x + a)*c^2/b - 4*a^2*
c^2/((b*x + a)*b)
```

3.1058 $\int \frac{ac-bcx}{(a+bx)^2}\,dx$

Optimal. Leaf size=27

$$-\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}$$

```
[Out] (-2*a*c)/(b*(a + b*x)) - (c*Log[a + b*x])/b
```

Rubi [A] time = 0.0132458, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)/(a + b*x)^2,x]
```

```
[Out] (-2*a*c)/(b*(a + b*x)) - (c*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ac-bcx}{(a+bx)^2}\,dx &= \int \left(\frac{2ac}{(a+bx)^2}-\frac{c}{a+bx}\right)dx\\ &= -\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.009045, size = 23, normalized size = 0.85

$$-\frac{c\left(\frac{2a}{a+bx}+\log(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)/(a + b*x)^2,x]
```

```
[Out] -((c*((2*a)/(a + b*x) + Log[a + b*x]))/b)
```

Maple [A] time = 0.004, size = 28, normalized size = 1.

$$-2\,\frac{ac}{b\,(bx+a)}-\frac{c\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)/(b*x+a)^2,x)
```

```
[Out] -2*a*c/b/(b*x+a)-c*ln(b*x+a)/b
```

Maxima [A] time = 1.02968, size = 38, normalized size = 1.41

$$-\frac{2\,ac}{b^2x+ab}-\frac{c\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*a*c/(b^2*x + a*b) - c*log(b*x + a)/b
```

Fricas [A] time = 1.53493, size = 72, normalized size = 2.67

$$-\frac{2\,ac+\left(bcx+ac\right)\log\left(bx+a\right)}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*c + (b*c*x + a*c)*log(b*x + a))/(b^2*x + a*b)
```

Sympy [A] time = 0.318086, size = 24, normalized size = 0.89

$$-\frac{2ac}{ab+b^2x}-\frac{c\log\left(a+bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)**2,x)
```

```
[Out] -2*a*c/(a*b + b**2*x) - c*log(a + b*x)/b
```

Giac [B] time = 1.05256, size = 73, normalized size = 2.7

$$c\left(\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}-\frac{a}{(bx+a)b}\right)-\frac{ac}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] c*(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b)) - a*c/((b*x
+ a)*b)
```

3.1059 $\int \frac{1}{(a+bx)^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

[Out] -(1/(b*(a + b*x)))

Rubi [A] time = 0.0014791, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-2),x]

[Out] -(1/(b*(a + b*x)))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

Mathematica [A] time = 0.0018908, size = 12, normalized size = 1.

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-2),x]

[Out] -(1/(b*(a + b*x)))

Maple [A] time = 0., size = 13, normalized size = 1.1

$$-\frac{1}{b(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2,x)
```

```
[Out] -1/b/(b*x+a)
```

Maxima [A] time = 1.01185, size = 16, normalized size = 1.33

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

Fricas [A] time = 1.4018, size = 24, normalized size = 2.

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

Sympy [A] time = 0.304446, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2,x)
```

```
[Out] -1/(a*b + b**2*x)
```

Giac [A] time = 1.04688, size = 16, normalized size = 1.33

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b)
```

3.1060 $\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx$

Optimal. Leaf size=41

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}-\frac{1}{2abc(a+bx)}$$

```
[Out] -1/(2*a*b*c*(a + b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c)
```

Rubi [A] time = 0.0279136, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {44, 208}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}-\frac{1}{2abc(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

```
[Out] -1/(2*a*b*c*(a + b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx &= \int \left(\frac{1}{2ac(a+bx)^2}+\frac{1}{2ac\left(a^2-b^2x^2\right)}\right)dx\\
&= -\frac{1}{2abc(a+bx)}+\frac{\int \frac{1}{a^2-b^2x^2}\,dx}{2ac}\\
&= -\frac{1}{2abc(a+bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}
\end{aligned}$$

Mathematica [A] time = 0.0136945, size = 50, normalized size = 1.22

$$\frac{-(a+bx)\log(a-bx)+(a+bx)\log(a+bx)-2a}{4a^2bc(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

```
[Out] (-2*a - (a + b*x)*Log[a - b*x] + (a + b*x)*Log[a + b*x])/(4*a^2*b*c*(a + b*
x))
```

Maple [A] time = 0.009, size = 56, normalized size = 1.4

$$\frac{\ln(bx+a)}{4ca^2b} - \frac{1}{2abc(bx+a)} - \frac{\ln(bx-a)}{4ca^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c),x)
```

```
[Out] 1/4/c/a^2/b*ln(b*x+a)-1/2/a/b/c/(b*x+a)-1/4/c/a^2/b*ln(b*x-a)
```

Maxima [A] time = 1.01729, size = 74, normalized size = 1.8

$$-\frac{1}{2\left(ab^2cx+a^2bc\right)} + \frac{\log(bx+a)}{4a^2bc} - \frac{\log(bx-a)}{4a^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -1/2/(a*b^2*c*x + a^2*b*c) + 1/4*log(b*x + a)/(a^2*b*c) - 1/4*log(b*x - a)/
(a^2*b*c)
```

Fricas [A] time = 1.53883, size = 115, normalized size = 2.8

$$\frac{(bx+a)\log(bx+a) - (bx+a)\log(bx-a) - 2a}{4\left(a^2b^2cx+a^3bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x + a)*log(b*x + a) - (b*x + a)*log(b*x - a) - 2*a)/(a^2*b^2*c*x +
a^3*b*c)
```

Sympy [A] time = 0.421321, size = 44, normalized size = 1.07

$$-\frac{1}{2a^2bc+2ab^2cx} - \frac{\frac{\log\left(-\frac{a}{b}+x\right)}{4} - \frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c),x)
```

```
[Out] -1/(2*a**2*b*c + 2*a*b**2*c*x) - (log(-a/b + x)/4 - log(a/b + x)/4)/(a**2*b
*c)
```

Giac [A] time = 1.06203, size = 59, normalized size = 1.44

$$-\frac{\log\left(\left|-\frac{2a}{bx+a}+1\right|\right)}{4a^2bc}-\frac{1}{2(bx+a)abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(-2*a/(b*x + a) + 1))/(a^2*b*c) - 1/2/((b*x + a)*a*b*c)
```

3.1061 $\int \frac{1}{(a+bx)^2(ac-bcx)^2} \, dx$

Optimal. Leaf size=46

$$\frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}$$

```
[Out] x/(2*a^2*c^2*(a^2 - b^2*x^2)) + ArcTanh[(b*x)/a]/(2*a^3*b*c^2)
```

Rubi [A] time = 0.0166867, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {41, 199, 208}

$$\frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

```
[Out] x/(2*a^2*c^2*(a^2 - b^2*x^2)) + ArcTanh[(b*x)/a]/(2*a^3*b*c^2)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^2(ac-bcx)^2} \, dx &= \int \frac{1}{\left(a^2c-b^2cx^2\right)^2} \, dx \\
&= \frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}+\frac{\int \frac{1}{a^2c-b^2cx^2} \, dx}{2a^2c} \\
&= \frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}
\end{aligned}$$

Mathematica [A] time = 0.0232392, size = 74, normalized size = 1.61

$$\frac{\left(b^2x^2-a^2\right)\log(a-bx)+\left(a^2-b^2x^2\right)\log(a+bx)+2abx}{4a^3bc^2(a-bx)(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

```
[Out] (2*a*b*x + (-a^2 + b^2*x^2)*Log[a - b*x] + (a^2 - b^2*x^2)*Log[a + b*x])/(4
*a^3*b*c^2*(a - b*x)*(a + b*x))
```

Maple [A] time = 0.01, size = 76, normalized size = 1.7

$$\frac{\ln(bx+a)}{4c^2a^3b}-\frac{1}{4c^2a^2b(bx+a)}-\frac{\ln(bx-a)}{4c^2a^3b}-\frac{1}{4c^2a^2b(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c)^2,x)
```

```
[Out] 1/4/c^2/a^3/b*ln(b*x+a)-1/4/c^2/a^2/b/(b*x+a)-1/4/c^2/a^3/b*ln(b*x-a)-1/4/c
^2/a^2/b/(b*x-a)
```

Maxima [A] time = 1.03254, size = 86, normalized size = 1.87

$$-\frac{x}{2\left(a^2b^2c^2x^2-a^4c^2\right)}+\frac{\log(bx+a)}{4a^3bc^2}-\frac{\log(bx-a)}{4a^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(a^2*b^2*c^2*x^2 - a^4*c^2) + 1/4*log(b*x + a)/(a^3*b*c^2) - 1/4*log
(b*x - a)/(a^3*b*c^2)
```

Fricas [A] time = 1.64226, size = 146, normalized size = 3.17

$$-\frac{2abx-\left(b^2x^2-a^2\right)\log(bx+a)+\left(b^2x^2-a^2\right)\log(bx-a)}{4\left(a^3b^3c^2x^2-a^5bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b*x - (b^2*x^2 - a^2)*log(b*x + a) + (b^2*x^2 - a^2)*log(b*x - a)
)/(a^3*b^3*c^2*x^2 - a^5*b*c^2)
```

Sympy [A] time = 0.407274, size = 49, normalized size = 1.07

$$-\frac{x}{-2a^4c^2+2a^2b^2c^2x^2}+\frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4}+\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c)**2,x)
```

```
[Out] -x/(-2*a**4*c**2 + 2*a**2*b**2*c**2*x**2) + (-log(-a/b + x)/4 + log(a/b + x
)/4)/(a**3*b*c**2)
```

Giac [A] time = 1.058, size = 112, normalized size = 2.43

$$-\frac{1}{4\,(bcx-ac)a^2bc}+\frac{\log\left(\left|-\frac{2\,ac}{bcx-ac}-1\right|\right)}{4\,a^3bc^2}+\frac{1}{8\,a^3b\left(\frac{2\,ac}{bcx-ac}+1\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -1/4/((b*c*x - a*c)*a^2*b*c) + 1/4*log(abs(-2*a*c/(b*c*x - a*c) - 1))/(a^3*
b*c^2) + 1/8/(a^3*b*(2*a*c/(b*c*x - a*c) + 1)*c^2)
```

3.1062 $\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx$

Optimal. Leaf size=83

$$\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{1}{8a^2bc^3(a-bx)^2}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}$$

```
[Out] 1/(8*a^2*b*c^3*(a - b*x)^2) + 1/(4*a^3*b*c^3*(a - b*x)) - 1/(8*a^3*b*c^3*(a
 + b*x)) + (3*ArcTanh[(b*x)/a])/(8*a^4*b*c^3)
```

Rubi [A] time = 0.0495295, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {44, 208}

$$\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{1}{8a^2bc^3(a-bx)^2}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

```
[Out] 1/(8*a^2*b*c^3*(a - b*x)^2) + 1/(4*a^3*b*c^3*(a - b*x)) - 1/(8*a^3*b*c^3*(a
 + b*x)) + (3*ArcTanh[(b*x)/a])/(8*a^4*b*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx &= \int \left(\frac{1}{4a^2c^3(a-bx)^3}+\frac{1}{4a^3c^3(a-bx)^2}+\frac{1}{8a^3c^3(a+bx)^2}+\frac{3}{8a^3c^3\left(a^2-b^2x^2\right)}\right)dx\\
&= \frac{1}{8a^2bc^3(a-bx)^2}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{3\int \frac{1}{a^2-b^2x^2}\,dx}{8a^3c^3}\\
&= \frac{1}{8a^2bc^3(a-bx)^2}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}
\end{aligned}$$

Mathematica [A] time = 0.0355216, size = 68, normalized size = 0.82

$$\frac{\frac{2a\left(2a^2+3abx-3b^2x^2\right)}{(a-bx)^2(a+bx)}-3\log(a-bx)+3\log(a+bx)}{16a^4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

```
[Out] ((2*a*(2*a^2 + 3*a*b*x - 3*b^2*x^2))/((a - b*x)^2*(a + b*x)) - 3*Log[a - b*
x] + 3*Log[a + b*x])/(16*a^4*b*c^3)
```

Maple [A] time = 0.01, size = 96, normalized size = 1.2

$$\frac{3\ln(bx+a)}{16c^3a^4b}-\frac{1}{8a^3bc^3(bx+a)}-\frac{3\ln(bx-a)}{16c^3a^4b}-\frac{1}{4a^3bc^3(bx-a)}+\frac{1}{8c^3a^2b(bx-a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c)^3,x)
```

```
[Out] 3/16/c^3/a^4/b*ln(b*x+a)-1/8/a^3/b/c^3/(b*x+a)-3/16/c^3/a^4/b*ln(b*x-a)-1/4
/c^3/a^3/b/(b*x-a)+1/8/c^3/a^2/b/(b*x-a)^2
```

Maxima [A] time = 1.07872, size = 146, normalized size = 1.76

$$-\frac{3b^2x^2-3abx-2a^2}{8\left(a^3b^4c^3x^3-a^4b^3c^3x^2-a^5b^2c^3x+a^6bc^3\right)}+\frac{3\log(bx+a)}{16a^4bc^3}-\frac{3\log(bx-a)}{16a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b^2*x^2 - 3*a*b*x - 2*a^2)/(a^3*b^4*c^3*x^3 - a^4*b^3*c^3*x^2 - a^5
*b^2*c^3*x + a^6*b*c^3) + 3/16*log(b*x + a)/(a^4*b*c^3) - 3/16*log(b*x - a)
/(a^4*b*c^3)
```

Fricas [A] time = 1.66517, size = 290, normalized size = 3.49

$$-\frac{6ab^2x^2-6a^2bx-4a^3-3\left(b^3x^3-ab^2x^2-a^2bx+a^3\right)\log(bx+a)+3\left(b^3x^3-ab^2x^2-a^2bx+a^3\right)\log(bx-a)}{16\left(a^4b^4c^3x^3-a^5b^3c^3x^2-a^6b^2c^3x+a^7bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*a*b^2*x^2 - 6*a^2*b*x - 4*a^3 - 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x +
 a^3)*log(b*x + a) + 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x + a^3)*log(b*x - a))/
(a^4*b^4*c^3*x^3 - a^5*b^3*c^3*x^2 - a^6*b^2*c^3*x + a^7*b*c^3)
```

Sympy [A] time = 0.655645, size = 104, normalized size = 1.25

$$-\frac{-2a^2-3abx+3b^2x^2}{8a^6bc^3-8a^5b^2c^3x-8a^4b^3c^3x^2+8a^3b^4c^3x^3}-\frac{\frac{3\log\left(-\frac{a}{b}+x\right)}{16}-\frac{3\log\left(\frac{a}{b}+x\right)}{16}}{a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c)**3,x)
```

```
[Out] -(-2*a**2 - 3*a*b*x + 3*b**2*x**2)/(8*a**6*b*c**3 - 8*a**5*b**2*c**3*x - 8*
a**4*b**3*c**3*x**2 + 8*a**3*b**4*c**3*x**3) - (3*log(-a/b + x)/16 - 3*log(
a/b + x)/16)/(a**4*b*c**3)
```

Giac [A] time = 1.06237, size = 109, normalized size = 1.31

$$-\frac{3\,\log\left(\left|-\frac{2\,a}{bx+a}+1\right|\right)}{16\,a^4bc^3}-\frac{1}{8\,(bx+a)a^3bc^3}+\frac{\frac{12\,a}{bx+a}-5}{32\,a^4bc^3\left(\frac{2\,a}{bx+a}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] -3/16*log(abs(-2*a/(b*x + a) + 1))/(a^4*b*c^3) - 1/8/((b*x + a)*a^3*b*c^3)
+ 1/32*(12*a/(b*x + a) - 5)/(a^4*b*c^3*(2*a/(b*x + a) - 1)^2)
```

3.1063 $\int (1-x)^{9/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=108

$$\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}+\frac{3}{10}(x+1)^{3/2}(1-x)^{7/2}+\frac{21}{40}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{8}(x+1)^{3/2}(1-x)^{3/2}+\frac{21}{16}x\sqrt{x+1}\sqrt{1-x}+\frac{21}{16}$$

```
[Out] (21*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (7*(1 - x)^(3/2)*(1 + x)^(3/2))/8 + (21
*(1 - x)^(5/2)*(1 + x)^(3/2))/40 + (3*(1 - x)^(7/2)*(1 + x)^(3/2))/10 + ((1
 - x)^(9/2)*(1 + x)^(3/2))/6 + (21*ArcSin[x])/16
```

Rubi [A] time = 0.02088, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}+\frac{3}{10}(x+1)^{3/2}(1-x)^{7/2}+\frac{21}{40}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{8}(x+1)^{3/2}(1-x)^{3/2}+\frac{21}{16}x\sqrt{x+1}\sqrt{1-x}+\frac{21}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*Sqrt[1 + x],x]
```

```
[Out] (21*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (7*(1 - x)^(3/2)*(1 + x)^(3/2))/8 + (21
*(1 - x)^(5/2)*(1 + x)^(3/2))/40 + (3*(1 - x)^(7/2)*(1 + x)^(3/2))/10 + ((1
 - x)^(9/2)*(1 + x)^(3/2))/6 + (21*ArcSin[x])/16
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (1-x)^{9/2}\sqrt{1+x}\,dx = \frac{1}{6}(1-x)^{9/2}(1+x)^{3/2} + \frac{3}{2}\int (1-x)^{7/2}\sqrt{1+x}\,dx$$
$$= \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2} + \frac{1}{6}(1-x)^{9/2}(1+x)^{3/2} + \frac{21}{10}\int (1-x)^{5/2}\sqrt{1+x}\,dx$$
$$= \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2} + \frac{1}{6}(1-x)^{9/2}(1+x)^{3/2} + \frac{21}{8}\int (1-x)^{3/2}\sqrt{1+x}\,dx$$
$$= \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2} + \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2} + \frac{1}{6}(1-x)^{9/2}(1+x)^{3/2} + \frac{21}{8}$$
$$= \frac{21}{16}\sqrt{1-x}x\sqrt{1+x} + \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2} + \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2} + \frac{1}{6}(1$$
$$= \frac{21}{16}\sqrt{1-x}x\sqrt{1+x} + \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2} + \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2} + \frac{1}{6}(1$$
$$= \frac{21}{16}\sqrt{1-x}x\sqrt{1+x} + \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2} + \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2} + \frac{1}{6}(1$$

Mathematica [A] time = 0.0510854, size = 60, normalized size = 0.56

$$\frac{1}{240}\left(\sqrt{1-x^2}\left(40x^5-192x^4+350x^3-256x^2-75x+448\right)-630\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x^2]*(448 - 75*x - 256*x^2 + 350*x^3 - 192*x^4 + 40*x^5) - 630*Ar
cSin[Sqrt[1 - x]/Sqrt[2]])/240
```

Maple [A] time = 0.005, size = 113, normalized size = 1.1

$$\frac{1}{6}(1-x)^{\frac{9}{2}}(1+x)^{\frac{3}{2}} + \frac{3}{10}(1-x)^{\frac{7}{2}}(1+x)^{\frac{3}{2}} + \frac{21}{40}(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}} + \frac{7}{8}(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}} + \frac{21}{16}\sqrt{1-x}(1+x)^{\frac{3}{2}} - \frac{21}{16}\sqrt{1-x}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(9/2)*(1+x)^(1/2),x)
```

```
[Out] 1/6*(1-x)^(9/2)*(1+x)^(3/2)+3/10*(1-x)^(7/2)*(1+x)^(3/2)+21/40*(1-x)^(5/2)*
(1+x)^(3/2)+7/8*(1-x)^(3/2)*(1+x)^(3/2)+21/16*(1-x)^(1/2)*(1+x)^(3/2)-21/16
*(1-x)^(1/2)*(1+x)^(1/2)+21/16*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*
arcsin(x)
```

Maxima [A] time = 1.57081, size = 92, normalized size = 0.85

$$-\frac{1}{6}\left(-x^2+1\right)^{\frac{3}{2}}x^3 + \frac{4}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2 - \frac{13}{8}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{28}{15}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{21}{16}\sqrt{-x^2+1}x + \frac{21}{16}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(3/2)*x^3 + 4/5*(-x^2 + 1)^(3/2)*x^2 - 13/8*(-x^2 + 1)^(3/2
)*x + 28/15*(-x^2 + 1)^(3/2) + 21/16*sqrt(-x^2 + 1)*x + 21/16*arcsin(x)
```

Fricas [A] time = 1.6009, size = 178, normalized size = 1.65

$$\frac{1}{240}\left(40\,x^{5}-192\,x^{4}+350\,x^{3}-256\,x^{2}-75\,x+448\right)\sqrt{x+1}\sqrt{-x+1}-\frac{21}{8}\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/240*(40*x^5 - 192*x^4 + 350*x^3 - 256*x^2 - 75*x + 448)*sqrt(x + 1)*sqrt(
-x + 1) - 21/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12889, size = 201, normalized size = 1.86

$$-\frac{4}{15}\,((3\,(x+1)(x-3)+17)(x+1)-10)(x+1)^{\frac{3}{2}}\sqrt{-x+1}-\frac{4}{3}\,(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1}+\frac{1}{48}\,((2\,((4\,(x+1)(x-4)+39)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -4/15*((3*(x + 1)*(x - 3) + 17)*(x + 1) - 10)*(x + 1)^(3/2)*sqrt(-x + 1) -
4/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) + 1/48*((2*((4*(x + 1)*(x - 4) + 39)
*(x + 1) - 37)*(x + 1) + 31)*(x + 1) - 3)*sqrt(x + 1)*sqrt(-x + 1) + 3/4*((
2*(x + 1)*(x - 2) + 5)*(x + 1) - 1)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x +
 1)*x*sqrt(-x + 1) + 21/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1064 $\int (1-x)^{7/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=88

$$\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}+\frac{7}{20}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{8}x\sqrt{x+1}\sqrt{1-x}+\frac{7}{8}\sin^{-1}(x)$$

```
[Out] (7*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + (7*(1 - x)^(3/2)*(1 + x)^(3/2))/12 + (7*(
1 - x)^(5/2)*(1 + x)^(3/2))/20 + ((1 - x)^(7/2)*(1 + x)^(3/2))/5 + (7*ArcSi
n[x])/8
```

Rubi [A] time = 0.0153782, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}+\frac{7}{20}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{8}x\sqrt{x+1}\sqrt{1-x}+\frac{7}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

```
[Out] (7*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + (7*(1 - x)^(3/2)*(1 + x)^(3/2))/12 + (7*(
1 - x)^(5/2)*(1 + x)^(3/2))/20 + ((1 - x)^(7/2)*(1 + x)^(3/2))/5 + (7*ArcSi
n[x])/8
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{7/2}\sqrt{1+x}\,dx &= \frac{1}{5}(1-x)^{7/2}(1+x)^{3/2} + \frac{7}{5}\int (1-x)^{5/2}\sqrt{1+x}\,dx \\ &= \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2} + \frac{1}{5}(1-x)^{7/2}(1+x)^{3/2} + \frac{7}{4}\int (1-x)^{3/2}\sqrt{1+x}\,dx \\ &= \frac{7}{12}(1-x)^{3/2}(1+x)^{3/2} + \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2} + \frac{1}{5}(1-x)^{7/2}(1+x)^{3/2} + \frac{7}{4}\int \sqrt{1-x}\sqrt{1+x}\,dx \\ &= \frac{7}{8}\sqrt{1-x}x\sqrt{1+x} + \frac{7}{12}(1-x)^{3/2}(1+x)^{3/2} + \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2} + \frac{1}{5}(1-x)^{7/2}(1+x)^{3/2} + \frac{7}{8} \\ &= \frac{7}{8}\sqrt{1-x}x\sqrt{1+x} + \frac{7}{12}(1-x)^{3/2}(1+x)^{3/2} + \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2} + \frac{1}{5}(1-x)^{7/2}(1+x)^{3/2} + \frac{7}{8} \\ &= \frac{7}{8}\sqrt{1-x}x\sqrt{1+x} + \frac{7}{12}(1-x)^{3/2}(1+x)^{3/2} + \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2} + \frac{1}{5}(1-x)^{7/2}(1+x)^{3/2} + \frac{7}{8}\end{aligned}$$

Mathematica [A] time = 0.0528489, size = 56, normalized size = 0.64

$$\frac{1}{120}\sqrt{1-x^2}\left(-24x^4+90x^3-112x^2+15x+136\right)-\frac{7}{4}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x^2]*(136 + 15*x - 112*x^2 + 90*x^3 - 24*x^4))/120 - (7*ArcSin[Sq
rt[1 - x]/Sqrt[2]])/4
```

Maple [A] time = 0.003, size = 99, normalized size = 1.1

$$\frac{1}{5}(1-x)^{\frac{7}{2}}(1+x)^{\frac{3}{2}}+\frac{7}{20}(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}+\frac{7}{12}(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}+\frac{7}{8}\sqrt{1-x}(1+x)^{\frac{3}{2}}-\frac{7}{8}\sqrt{1-x}\sqrt{1+x}+\frac{7\arcsin(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)*(1+x)^(1/2),x)
```

```
[Out] 1/5*(1-x)^(7/2)*(1+x)^(3/2)+7/20*(1-x)^(5/2)*(1+x)^(3/2)+7/12*(1-x)^(3/2)*(
1+x)^(3/2)+7/8*(1-x)^(1/2)*(1+x)^(3/2)-7/8*(1-x)^(1/2)*(1+x)^(1/2)+7/8*((1+
x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.5388, size = 73, normalized size = 0.83

$$\frac{1}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2-\frac{3}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{17}{15}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{7}{8}\sqrt{-x^2+1}x+\frac{7}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(3/2)*x^2 - 3/4*(-x^2 + 1)^(3/2)*x + 17/15*(-x^2 + 1)^(3/2)
+ 7/8*sqrt(-x^2 + 1)*x + 7/8*arcsin(x)
```

Fricas [A] time = 1.54134, size = 163, normalized size = 1.85

$$-\frac{1}{120}\left(24x^4-90x^3+112x^2-15x-136\right)\sqrt{x+1}\sqrt{-x+1}-\frac{7}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(24*x^4 - 90*x^3 + 112*x^2 - 15*x - 136)*sqrt(x + 1)*sqrt(-x + 1) -
7/4*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [A] time = 74.6343, size = 253, normalized size = 2.88

$$\begin{cases} -\frac{7i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}}+\frac{39i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}}-\frac{449i(x+1)^{\frac{7}{2}}}{60\sqrt{x-1}}+\frac{1657i(x+1)^{\frac{5}{2}}}{120\sqrt{x-1}}-\frac{263i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}}+\frac{7i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1 \\ \frac{7\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}}-\frac{39(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}}+\frac{449(x+1)^{\frac{7}{2}}}{60\sqrt{1-x}}-\frac{1657(x+1)^{\frac{5}{2}}}{120\sqrt{1-x}}+\frac{263(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}}-\frac{7\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-7*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) + 39*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) - 449*I*(x + 1)**(7/2)/(60*s
qrt(x - 1)) + 1657*I*(x + 1)**(5/2)/(120*sqrt(x - 1)) - 263*I*(x + 1)**(3/2
)/(24*sqrt(x - 1)) + 7*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1)/2 > 1), (7
*asin(sqrt(2)*sqrt(x + 1)/2)/4 + (x + 1)**(11/2)/(5*sqrt(1 - x)) - 39*(x +
1)**(9/2)/(20*sqrt(1 - x)) + 449*(x + 1)**(7/2)/(60*sqrt(1 - x)) - 1657*(x
+ 1)**(5/2)/(120*sqrt(1 - x)) + 263*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 7*sqr
t(x + 1)/(4*sqrt(1 - x)), True))
```

Giac [A] time = 1.10213, size = 143, normalized size = 1.62

$$-\frac{1}{15}\left((3(x+1)(x-3)+17)(x+1)-10\right)(x+1)^{\frac{3}{2}}\sqrt{-x+1}-(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1}+\frac{3}{8}\left((2(x+1)(x-2)+5)(x+1)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*((3*(x + 1)*(x - 3) + 17)*(x + 1) - 10)*(x + 1)^(3/2)*sqrt(-x + 1) -
(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) + 3/8*((2*(x + 1)*(x - 2) + 5)*(x + 1) -
 1)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + 7/4*arcsin(
1/2*sqrt(2)*sqrt(x + 1))
```

3.1065 $\int (1-x)^{5/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=68

$$\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}+\frac{5}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{5}{8}x\sqrt{x+1}\sqrt{1-x}+\frac{5}{8}\sin^{-1}(x)$$

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + (5*(1 - x)^(3/2)*(1 + x)^(3/2))/12 + ((1
- x)^(5/2)*(1 + x)^(3/2))/4 + (5*ArcSin[x])/8
```

Rubi [A] time = 0.010737, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}+\frac{5}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{5}{8}x\sqrt{x+1}\sqrt{1-x}+\frac{5}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + (5*(1 - x)^(3/2)*(1 + x)^(3/2))/12 + ((1
- x)^(5/2)*(1 + x)^(3/2))/4 + (5*ArcSin[x])/8
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{5/2}\sqrt{1+x}\,dx &= \frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{4}\int (1-x)^{3/2}\sqrt{1+x}\,dx\\ &= \frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{4}\int \sqrt{1-x}\sqrt{1+x}\,dx\\ &= \frac{5}{8}\sqrt{1-x}x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx\\ &= \frac{5}{8}\sqrt{1-x}x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{5}{8}\sqrt{1-x}x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0403838, size = 50, normalized size = 0.74

$$\frac{1}{24}\left(\sqrt{1-x^2}\left(6x^3-16x^2+9x+16\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x^2]*(16 + 9*x - 16*x^2 + 6*x^3) - 30*ArcSin[Sqrt[1 - x]/Sqrt[2]]
)/24
```

Maple [A] time = 0.005, size = 85, normalized size = 1.3

$$\frac{1}{4}(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}+\frac{5}{12}(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}+\frac{5}{8}\sqrt{1-x}(1+x)^{\frac{3}{2}}-\frac{5}{8}\sqrt{1-x}\sqrt{1+x}+\frac{5\arcsin(x)}{8}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}\sqrt{\cdot}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)*(1+x)^(1/2),x)
```

```
[Out] 1/4*(1-x)^(5/2)*(1+x)^(3/2)+5/12*(1-x)^(3/2)*(1+x)^(3/2)+5/8*(1-x)^(1/2)*(1
+x)^(3/2)-5/8*(1-x)^(1/2)*(1+x)^(1/2)+5/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(
1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.54095, size = 54, normalized size = 0.79

$$-\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{-x^2+1}x+\frac{5}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)*x + 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/8
*arcsin(x)
```

Fricas [A] time = 1.58123, size = 143, normalized size = 2.1

$$\frac{1}{24}\left(6x^3-16x^2+9x+16\right)\sqrt{x+1}\sqrt{-x+1}-\frac{5}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 - 16*x^2 + 9*x + 16)*sqrt(x + 1)*sqrt(-x + 1) - 5/4*arctan((sqr
t(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [A] time = 15.673, size = 218, normalized size = 3.21

$$\begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}}-\frac{23i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}}+\frac{127i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}}-\frac{133i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}}+\frac{5i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}}+\frac{23(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}}-\frac{127(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}}+\frac{133(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}}-\frac{5\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) - 23*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) + 127*I*(x + 1)**(5/2)/(24*sq
rt(x - 1)) - 133*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(4*sqr
t(x - 1)), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(
9/2)/(4*sqrt(1 - x)) + 23*(x + 1)**(7/2)/(12*sqrt(1 - x)) - 127*(x + 1)**(5
/2)/(24*sqrt(1 - x)) + 133*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 5*sqrt(x + 1)/
(4*sqrt(1 - x)), True))
```

Giac [A] time = 1.0887, size = 103, normalized size = 1.51

$$-\frac{2}{3}(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1}+\frac{1}{8}((2(x+1)(x-2)+5)(x+1)-1)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\sqrt{x+1}x\sqrt{-x+1}+\frac{5}{4}\arcsin\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) + 1/8*((2*(x + 1)*(x - 2) + 5)*(x +
 1) - 1)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + 5/4*ar
csin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1066 $\int (1-x)^{3/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=48

$$\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 + ((1 - x)^(3/2)*(1 + x)^(3/2))/3 + ArcSin[x]
/2
```

Rubi [A] time = 0.0056664, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 + ((1 - x)^(3/2)*(1 + x)^(3/2))/3 + ArcSin[x]
/2
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (1-x)^{3/2}\sqrt{1+x}\,dx = \frac{1}{3}(1-x)^{3/2}(1+x)^{3/2} + \int \sqrt{1-x}\sqrt{1+x}\,dx$$
$$= \frac{1}{2}\sqrt{1-x}x\sqrt{1+x} + \frac{1}{3}(1-x)^{3/2}(1+x)^{3/2} + \frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$$
$$= \frac{1}{2}\sqrt{1-x}x\sqrt{1+x} + \frac{1}{3}(1-x)^{3/2}(1+x)^{3/2} + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{1}{2}\sqrt{1-x}x\sqrt{1+x} + \frac{1}{3}(1-x)^{3/2}(1+x)^{3/2} + \frac{1}{2}\sin^{-1}(x)$$

Mathematica [A] time = 0.0375313, size = 44, normalized size = 0.92

$$\frac{1}{6}\left(-2x^2+3x+2\right)\sqrt{1-x^2} - \sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*Sqrt[1 + x],x]
```

```
[Out] ((2 + 3*x - 2*x^2)*Sqrt[1 - x^2])/6 - ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

Maple [B] time = 0.003, size = 71, normalized size = 1.5

$$\frac{1}{3}(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}} + \frac{1}{2}\sqrt{1-x}(1+x)^{\frac{3}{2}} - \frac{1}{2}\sqrt{1-x}\sqrt{1+x} + \frac{\arcsin(x)}{2}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)*(1+x)^(1/2),x)
```

```
[Out] 1/3*(1-x)^(3/2)*(1+x)^(3/2)+1/2*(1-x)^(1/2)*(1+x)^(3/2)-1/2*(1-x)^(1/2)*(1+
x)^(1/2)+1/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.55182, size = 38, normalized size = 0.79

$$\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A] time = 1.53546, size = 124, normalized size = 2.58

$$-\frac{1}{6}\left(2x^2-3x-2\right)\sqrt{x+1}\sqrt{-x+1} - \arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*x^2 - 3*x - 2)*sqrt(x + 1)*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(
-x + 1) - 1)/x)
```

Sympy [B] time = 4.92475, size = 168, normalized size = 3.5

$$\begin{cases} -i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} + \frac{11i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} - \frac{17i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} - \frac{11(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} + \frac{17(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x - 1
)) + 11*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 17*I*(x + 1)**(3/2)/(6*sqrt(x -
1)) + I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (asin(sqrt(2)*sqrt(x +
1)/2) + (x + 1)**(7/2)/(3*sqrt(1 - x)) - 11*(x + 1)**(5/2)/(6*sqrt(1 - x))
+ 17*(x + 1)**(3/2)/(6*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A] time = 1.0681, size = 59, normalized size = 1.23

$$-\frac{1}{3}(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1} + \frac{1}{2}\sqrt{x+1}x\sqrt{-x+1} + \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) +
arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1067 $\int \sqrt{1-x}\sqrt{1+x}\,dx$

Optimal. Leaf size=28

$$\frac{1}{2}\sqrt{1-x}\sqrt{x+1}x+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 + ArcSin[x]/2
```

Rubi [A] time = 0.0035754, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {38, 41, 216}

$$\frac{1}{2}\sqrt{1-x}\sqrt{x+1}x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 + ArcSin[x]/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x}\sqrt{1+x}\,dx &= \frac{1}{2}\sqrt{1-x}x\sqrt{1+x}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}x\sqrt{1+x}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}x\sqrt{1+x}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0064911, size = 20, normalized size = 0.71

$$\frac{1}{2}\left(\sqrt{1-x^2}x+\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*Sqrt[1 + x],x]
```

```
[Out] (x*Sqrt[1 - x^2] + ArcSin[x])/2
```

Maple [B] time = 0.003, size = 57, normalized size = 2.

$$-\frac{1}{2}(1-x)^{\frac{3}{2}}\sqrt{1+x}+\frac{1}{2}\sqrt{1-x}\sqrt{1+x}+\frac{\arcsin(x)}{2}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)*(1+x)^(1/2),x)
```

```
[Out] -1/2*(1-x)^(3/2)*(1+x)^(1/2)+1/2*(1-x)^(1/2)*(1+x)^(1/2)+1/2*((1+x)*(1-x))^
(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.54687, size = 23, normalized size = 0.82

$$\frac{1}{2}\sqrt{-x^2+1}x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A] time = 1.48498, size = 101, normalized size = 3.61

$$\frac{1}{2}\sqrt{x+1}x\sqrt{-x+1}-\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x + 1)*x*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [B] time = 2.67175, size = 133, normalized size = 4.75

$$\begin{cases} -i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}-\frac{3i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}+\frac{3(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x - 1
)) - 3*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + I*sqrt(x + 1)/sqrt(x - 1), Abs(x
+ 1)/2 > 1), (asin(sqrt(2)*sqrt(x + 1)/2) - (x + 1)**(5/2)/(2*sqrt(1 - x))
+ 3*(x + 1)**(3/2)/(2*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A] time = 1.09921, size = 36, normalized size = 1.29

$$\frac{1}{2}\sqrt{x+1}x\sqrt{-x+1}+\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1068 $\int \frac{\sqrt{1+x}}{\sqrt{1-x}} dx$

Optimal. Leaf size=21

$$\sin^{-1}(x) - \sqrt{1-x}\sqrt{x+1}$$

```
[Out] -(Sqrt[1 - x]*Sqrt[1 + x]) + ArcSin[x]
```

Rubi [A] time = 0.00379, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\sin^{-1}(x) - \sqrt{1-x}\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/Sqrt[1 - x],x]

[Out] -(Sqrt[1 - x]*Sqrt[1 + x]) + ArcSin[x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x}}{\sqrt{1-x}} dx &= -\sqrt{1-x}\sqrt{1+x} + \int \frac{1}{\sqrt{1-x}\sqrt{1+x}} dx \\
&= -\sqrt{1-x}\sqrt{1+x} + \int \frac{1}{\sqrt{1-x^2}} dx \\
&= -\sqrt{1-x}\sqrt{1+x} + \sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0066771, size = 32, normalized size = 1.52

$$-\sqrt{1-x^2} - 2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/Sqrt[1 - x],x]
```

```
[Out] -Sqrt[1 - x^2] - 2*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

Maple [B] time = 0.003, size = 42, normalized size = 2.

$$-\sqrt{1-x}\,\sqrt{1+x}+\arcsin \left(x \right) \sqrt{\left(1+x \right) \left(1-x \right)}\, \frac{1}{\sqrt{1-x}}\, \frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(1/2),x)
```

```
[Out] -(1-x)^(1/2)*(1+x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin
(x)
```

Maxima [A] time = 1.525, size = 19, normalized size = 0.9

$$-\sqrt{-x^{2}+1}+\arcsin \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [B] time = 1.63437, size = 97, normalized size = 4.62

$$-\sqrt{x+1}\sqrt{-x+1}-2\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(x + 1)*sqrt(-x + 1) - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [B] time = 1.84416, size = 100, normalized size = 4.76

$$\begin{cases} -2 i \operatorname{acosh}{\left(\frac{\sqrt{2} \sqrt{x+1}}{2} \right)} - \frac{i (x+1)^{\frac{3}{2}}}{\sqrt{x-1}} + \frac{2 i \sqrt{x+1}}{\sqrt{x-1}} & \text{for}\: \frac{\left|{x+1}\right|}{2} > 1 \\2 \operatorname{asin}{\left(\frac{\sqrt{2} \sqrt{x+1}}{2} \right)} + \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} - \frac{2 \sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(3/2)/sqrt(x - 1)
 + 2*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (2*asin(sqrt(2)*sqrt(x +
 1)/2) + (x + 1)**(3/2)/sqrt(1 - x) - 2*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A] time = 1.06617, size = 38, normalized size = 1.81

$$-\sqrt{x+1}\sqrt{-x+1}+2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(x + 1)*sqrt(-x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1069 $\int \frac{\sqrt{1+x}}{(1-x)^{3/2}} \, dx$

Optimal. Leaf size=23

$$\frac{2\sqrt{x+1}}{\sqrt{1-x}} - \sin^{-1}(x)$$

```
[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x]
```

Rubi [A] time = 0.0037246, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$\frac{2\sqrt{x+1}}{\sqrt{1-x}} - \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(3/2),x]

[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x}}{(1-x)^{3/2}} \, dx &= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x}\sqrt{1+x}} \, dx \\
&= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x^2}} \, dx \\
&= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0130225, size = 36, normalized size = 1.57

$$2\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}+\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(3/2),x]
```

```
[Out] 2*(Sqrt[1 + x]/Sqrt[1 - x] + ArcSin[Sqrt[1 - x]/Sqrt[2]])
```

Maple [B] time = 0.022, size = 64, normalized size = 2.8

$$2\frac{\sqrt{1+x}\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\sqrt{1-x}}-\arcsin(x)\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(3/2),x)
```

```
[Out] 2*(1+x)^(1/2)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)-((1+x)*
(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.56908, size = 28, normalized size = 1.22

$$-\frac{2\sqrt{-x^2+1}}{x-1}-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

Fricas [B] time = 1.50013, size = 131, normalized size = 5.7

$$\frac{2\left((x-1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+x-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + x - sqrt(x + 1)*sqrt(
-x + 1) - 1)/(x - 1)
```

Sympy [A] time = 1.57789, size = 71, normalized size = 3.09

$$\begin{cases} 2i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ -2 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - 2*I*sqrt(x + 1)/sqrt(x - 1),
Abs(x + 1)/2 > 1), (-2*asin(sqrt(2)*sqrt(x + 1)/2) + 2*sqrt(x + 1)/sqrt(1 -
 x), True))
```

Giac [A] time = 1.07497, size = 45, normalized size = 1.96

$$-\frac{2\sqrt{x+1}\sqrt{-x+1}}{x-1} - 2 \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1070 $\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx$

Optimal. Leaf size=20

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

```
[Out] (1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

Rubi [A] time = 0.0015894, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(5/2),x]
```

```
[Out] (1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx = \frac{(1+x)^{3/2}}{3(1-x)^{3/2}}$$

Mathematica [A] time = 0.0055352, size = 20, normalized size = 1.

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(5/2),x]
```

```
[Out] (1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$\frac{1}{3}(1+x)^{\frac{3}{2}}(1-x)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(5/2),x)
```

```
[Out] 1/3*(1+x)^(3/2)/(1-x)^(3/2)
```

Maxima [B] time = 1.02511, size = 51, normalized size = 2.55

$$\frac{2\sqrt{-x^2+1}}{3\left(x^2-2x+1\right)}+\frac{\sqrt{-x^2+1}}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [B] time = 1.53824, size = 89, normalized size = 4.45

$$\frac{x^2+(x+1)^{\frac{3}{2}}\sqrt{-x+1}-2x+1}{3\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^2 + (x + 1)^(3/2)*sqrt(-x + 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

Sympy [A] time = 2.36126, size = 61, normalized size = 3.05

$$\begin{cases}\frac{i(x+1)^{\frac{3}{2}}}{3\sqrt{x-1}(x+1)-6\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1\\ -\frac{(x+1)^{\frac{3}{2}}}{3\sqrt{1-x}(x+1)-6\sqrt{1-x}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(5/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(3/2)/(3*sqrt(x - 1)*(x + 1) - 6*sqrt(x - 1)), Abs(x
+ 1)/2 > 1), (-(x + 1)**(3/2)/(3*sqrt(1 - x)*(x + 1) - 6*sqrt(1 - x)), True
))
```

Giac [A] time = 1.08294, size = 26, normalized size = 1.3

$$\frac{(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{3(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(x + 1)^(3/2)*sqrt(-x + 1)/(x - 1)^2
```

3.1071 $\int \frac{\sqrt{1+x}}{(1-x)^{7/2}} dx$

Optimal. Leaf size=41

$$\frac{(x+1)^{3/2}}{15(1-x)^{3/2}}+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}$$

```
[Out] (1 + x)^(3/2)/(5*(1 - x)^(5/2)) + (1 + x)^(3/2)/(15*(1 - x)^(3/2))
```

Rubi [A] time = 0.0042463, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{(x+1)^{3/2}}{15(1-x)^{3/2}}+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(7/2),x]

[Out] (1 + x)^(3/2)/(5*(1 - x)^(5/2)) + (1 + x)^(3/2)/(15*(1 - x)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}} dx &= \frac{(1+x)^{3/2}}{5(1-x)^{5/2}}+\frac{1}{5}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}} dx\\ &= \frac{(1+x)^{3/2}}{5(1-x)^{5/2}}+\frac{(1+x)^{3/2}}{15(1-x)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0081319, size = 23, normalized size = 0.56

$$-\frac{(x-4)(x+1)^{3/2}}{15(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(7/2),x]
```

```
[Out] -((-4 + x)*(1 + x)^(3/2))/(15*(1 - x)^(5/2))
```

Maple [A] time = 0.002, size = 18, normalized size = 0.4

$$-\frac{x-4}{15}(1+x)^{\frac{3}{2}}(1-x)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(7/2),x)
```

```
[Out] -1/15*(1+x)^(3/2)*(x-4)/(1-x)^(5/2)
```

Maxima [B] time = 1.03062, size = 86, normalized size = 2.1

$$-\frac{2\sqrt{-x^2+1}}{5\left(x^3-3x^2+3x-1\right)}-\frac{\sqrt{-x^2+1}}{15\left(x^2-2x+1\right)}+\frac{\sqrt{-x^2+1}}{15(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 1/15*sqrt(-x^2 + 1)/(x^2 - 2*
x + 1) + 1/15*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A] time = 1.58183, size = 136, normalized size = 3.32

$$\frac{4x^3-12x^2+\left(x^2-3x-4\right)\sqrt{x+1}\sqrt{-x+1}+12x-4}{15\left(x^3-3x^2+3x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(4*x^3 - 12*x^2 + (x^2 - 3*x - 4)*sqrt(x + 1)*sqrt(-x + 1) + 12*x - 4)
/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [B] time = 23.1053, size = 173, normalized size = 4.22

$$\begin{cases} \frac{i(x+1)^{\frac{5}{2}}}{15\sqrt{x-1}(x+1)^2-60\sqrt{x-1}(x+1)+60\sqrt{x-1}}-\frac{5i(x+1)^{\frac{3}{2}}}{15\sqrt{x-1}(x+1)^2-60\sqrt{x-1}(x+1)+60\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1 \\ -\frac{(x+1)^{\frac{5}{2}}}{15\sqrt{1-x}(x+1)^2-60\sqrt{1-x}(x+1)+60\sqrt{1-x}}+\frac{5(x+1)^{\frac{3}{2}}}{15\sqrt{1-x}(x+1)^2-60\sqrt{1-x}(x+1)+60\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(5/2)/(15*sqrt(x - 1)*(x + 1)**2 - 60*sqrt(x - 1)*(x
+ 1) + 60*sqrt(x - 1)) - 5*I*(x + 1)**(3/2)/(15*sqrt(x - 1)*(x + 1)**2 - 60
*sqrt(x - 1)*(x + 1) + 60*sqrt(x - 1)), Abs(x + 1)/2 > 1), (-(x + 1)**(5/2)
/(15*sqrt(1 - x)*(x + 1)**2 - 60*sqrt(1 - x)*(x + 1) + 60*sqrt(1 - x)) + 5*
(x + 1)**(3/2)/(15*sqrt(1 - x)*(x + 1)**2 - 60*sqrt(1 - x)*(x + 1) + 60*sqr
t(1 - x)), True))
```

Giac [A] time = 1.09774, size = 30, normalized size = 0.73

$$\frac{(x+1)^{\frac{3}{2}}(x-4)\sqrt{-x+1}}{15\,(x-1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x + 1)^(3/2)*(x - 4)*sqrt(-x + 1)/(x - 1)^3
```

3.1072 $\int \frac{\sqrt{1+x}}{(1-x)^{9/2}} dx$

Optimal. Leaf size=61

$$\frac{2(x+1)^{3/2}}{105(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}$$

```
[Out] (1 + x)^(3/2)/(7*(1 - x)^(7/2)) + (2*(1 + x)^(3/2))/(35*(1 - x)^(5/2)) + (2
*(1 + x)^(3/2))/(105*(1 - x)^(3/2))
```

Rubi [A] time = 0.0082057, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{3/2}}{105(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(3/2)/(7*(1 - x)^(7/2)) + (2*(1 + x)^(3/2))/(35*(1 - x)^(5/2)) + (2
*(1 + x)^(3/2))/(105*(1 - x)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x}}{(1-x)^{9/2}} \, dx &= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}}+\frac{2}{7} \int \frac{\sqrt{1+x}}{(1-x)^{7/2}} \, dx\\
&= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{35(1-x)^{5/2}}+\frac{2}{35} \int \frac{\sqrt{1+x}}{(1-x)^{5/2}} \, dx\\
&= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{35(1-x)^{5/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0114946, size = 30, normalized size = 0.49

$$\frac{(x+1)^{3/2}\left(2x^2-10x+23\right)}{105(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(9/2),x]
```

```
[Out] ((1 + x)^(3/2)*(23 - 10*x + 2*x^2))/(105*(1 - x)^(7/2))
```

Maple [A] time = 0.004, size = 25, normalized size = 0.4

$$\frac{2\,x^2-10\,x+23}{105}\,(1+x)^{\frac{3}{2}}\,(1-x)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(9/2),x)
```

```
[Out] 1/105*(1+x)^(3/2)*(2*x^2-10*x+23)/(1-x)^(7/2)
```

Maxima [B] time = 1.03258, size = 128, normalized size = 2.1

$$\frac{2\,\sqrt{-x^2+1}}{7\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}+\frac{\sqrt{-x^2+1}}{35\left(x^3-3\,x^2+3\,x-1\right)}-\frac{2\,\sqrt{-x^2+1}}{105\left(x^2-2\,x+1\right)}+\frac{2\,\sqrt{-x^2+1}}{105\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/35*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) - 2/105*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/105*sqrt(-
x^2 + 1)/(x - 1)
```

Fricas [A] time = 1.52901, size = 181, normalized size = 2.97

$$\frac{23\,x^4-92\,x^3+138\,x^2+\left(2\,x^3-8\,x^2+13\,x+23\right)\sqrt{x+1}\sqrt{-x+1}-92\,x+23}{105\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(23*x^4 - 92*x^3 + 138*x^2 + (2*x^3 - 8*x^2 + 13*x + 23)*sqrt(x + 1)*
sqrt(-x + 1) - 92*x + 23)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [B] time = 87.1587, size = 568, normalized size = 9.31

$$\begin{cases} \frac{2i(x+1)^{\frac{9}{2}}}{105\sqrt{x-1}(x+1)^4-840\sqrt{x-1}(x+1)^3+2520\sqrt{x-1}(x+1)^2-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}} - \frac{18i(x+1)^{\frac{7}{2}}}{105\sqrt{x-1}(x+1)^4-840\sqrt{x-1}(x+1)^3+2520\sqrt{x-1}(x+1)^2-3360\sqrt{x-1}(x+1)+} \\ -\frac{2(x+1)^{\frac{9}{2}}}{105\sqrt{1-x}(x+1)^4-840\sqrt{1-x}(x+1)^3+2520\sqrt{1-x}(x+1)^2-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}} + \frac{18(x+1)^{\frac{7}{2}}}{105\sqrt{1-x}(x+1)^4-840\sqrt{1-x}(x+1)^3+2520\sqrt{1-x}(x+1)^2-3360\sqrt{1-x}(x+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((2*I*(x + 1)**(9/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840*sqrt(x - 1)
*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x + 1) + 1680
*sqrt(x - 1)) - 18*I*(x + 1)**(7/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840*sqrt(
x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x + 1)
+ 1680*sqrt(x - 1)) + 63*I*(x + 1)**(5/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840
*sqrt(x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x
 + 1) + 1680*sqrt(x - 1)) - 70*I*(x + 1)**(3/2)/(105*sqrt(x - 1)*(x + 1)**4
 - 840*sqrt(x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x -
 1)*(x + 1) + 1680*sqrt(x - 1)), Abs(x + 1)/2 > 1), (-2*(x + 1)**(9/2)/(105
*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1 - x)*(x
+ 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) + 18*(x + 1)**(7/2)/
(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1 - x)
*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) - 63*(x + 1)**(5
/2)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1
- x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) + 70*(x + 1)
**(3/2)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqr
t(1 - x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)), True))
```

Giac [A] time = 1.07231, size = 39, normalized size = 0.64

$$\frac{(2(x+1)(x-6)+35)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{105(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(2*(x + 1)*(x - 6) + 35)*(x + 1)^(3/2)*sqrt(-x + 1)/(x - 1)^4
```

3.1073 $\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx$

Optimal. Leaf size=81

$$\frac{2(x+1)^{3/2}}{315(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{105(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{21(1-x)^{7/2}}+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}$$

```
[Out] (1 + x)^(3/2)/(9*(1 - x)^(9/2)) + (1 + x)^(3/2)/(21*(1 - x)^(7/2)) + (2*(1
+ x)^(3/2))/(105*(1 - x)^(5/2)) + (2*(1 + x)^(3/2))/(315*(1 - x)^(3/2))
```

Rubi [A] time = 0.0130847, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{3/2}}{315(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{105(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{21(1-x)^{7/2}}+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(11/2),x]
```

```
[Out] (1 + x)^(3/2)/(9*(1 - x)^(9/2)) + (1 + x)^(3/2)/(21*(1 - x)^(7/2)) + (2*(1
+ x)^(3/2))/(105*(1 - x)^(5/2)) + (2*(1 + x)^(3/2))/(315*(1 - x)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{1}{3}\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx\\
&= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2}{21}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx\\
&= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{5/2}}+\frac{2}{105}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx\\
&= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{5/2}}+\frac{2(1+x)^{3/2}}{315(1-x)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0135358, size = 35, normalized size = 0.43

$$\frac{(x+1)^{3/2}\left(-2x^3+12x^2-33x+58\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(11/2),x]
```

```
[Out] ((1 + x)^(3/2)*(58 - 33*x + 12*x^2 - 2*x^3))/(315*(1 - x)^(9/2))
```

Maple [A] time = 0.002, size = 30, normalized size = 0.4

$$-\frac{2\,x^3-12\,x^2+33\,x-58}{315}\,(1+x)^{\frac{3}{2}}\,(1-x)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(11/2),x)
```

```
[Out] -1/315*(1+x)^(3/2)*(2*x^3-12*x^2+33*x-58)/(1-x)^(9/2)
```

Maxima [B] time = 1.03188, size = 177, normalized size = 2.19

$$-\frac{2\sqrt{-x^2+1}}{9\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}-\frac{\sqrt{-x^2+1}}{63\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}+\frac{\sqrt{-x^2+1}}{105\left(x^3-3\,x^2+3\,x-1\right)}-\frac{2\sqrt{-x^2+1}}{315\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 1/63*sqrt(-
x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/105*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) - 2/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/315*sqrt(-x^2 + 1)/
(x - 1)
```

Fricas [A] time = 1.54972, size = 224, normalized size = 2.77

$$\frac{58\,x^5-290\,x^4+580\,x^3-580\,x^2+\left(2\,x^4-10\,x^3+21\,x^2-25\,x-58\right)\sqrt{x+1}\sqrt{-x+1}+290\,x-58}{315\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(58*x^5 - 290*x^4 + 580*x^3 - 580*x^2 + (2*x^4 - 10*x^3 + 21*x^2 - 25
*x - 58)*sqrt(x + 1)*sqrt(-x + 1) + 290*x - 58)/(x^5 - 5*x^4 + 10*x^3 - 10*
x^2 + 5*x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08433, size = 47, normalized size = 0.58

$$\frac{((2(x+1)(x-8)+63)(x+1)-105)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{315(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*((2*(x + 1)*(x - 8) + 63)*(x + 1) - 105)*(x + 1)^(3/2)*sqrt(-x + 1)/(
x - 1)^5
```

3.1074 $\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx$

Optimal. Leaf size=101

$$\frac{8(x+1)^{3/2}}{3465(1-x)^{3/2}}+\frac{8(x+1)^{3/2}}{1155(1-x)^{5/2}}+\frac{4(x+1)^{3/2}}{231(1-x)^{7/2}}+\frac{4(x+1)^{3/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{3/2}}{11(1-x)^{11/2}}$$

```
[Out] (1 + x)^(3/2)/(11*(1 - x)^(11/2)) + (4*(1 + x)^(3/2))/(99*(1 - x)^(9/2)) +
(4*(1 + x)^(3/2))/(231*(1 - x)^(7/2)) + (8*(1 + x)^(3/2))/(1155*(1 - x)^(5/
2)) + (8*(1 + x)^(3/2))/(3465*(1 - x)^(3/2))
```

Rubi [A] time = 0.0186567, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{8(x+1)^{3/2}}{3465(1-x)^{3/2}}+\frac{8(x+1)^{3/2}}{1155(1-x)^{5/2}}+\frac{4(x+1)^{3/2}}{231(1-x)^{7/2}}+\frac{4(x+1)^{3/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{3/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(3/2)/(11*(1 - x)^(11/2)) + (4*(1 + x)^(3/2))/(99*(1 - x)^(9/2)) +
(4*(1 + x)^(3/2))/(231*(1 - x)^(7/2)) + (8*(1 + x)^(3/2))/(1155*(1 - x)^(5/
2)) + (8*(1 + x)^(3/2))/(3465*(1 - x)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx &= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4}{11}\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx \\
&= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4(1+x)^{3/2}}{99(1-x)^{9/2}} + \frac{4}{33}\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx \\
&= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4(1+x)^{3/2}}{99(1-x)^{9/2}} + \frac{4(1+x)^{3/2}}{231(1-x)^{7/2}} + \frac{8}{231}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx \\
&= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4(1+x)^{3/2}}{99(1-x)^{9/2}} + \frac{4(1+x)^{3/2}}{231(1-x)^{7/2}} + \frac{8(1+x)^{3/2}}{1155(1-x)^{5/2}} + \frac{8\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx}{1155} \\
&= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4(1+x)^{3/2}}{99(1-x)^{9/2}} + \frac{4(1+x)^{3/2}}{231(1-x)^{7/2}} + \frac{8(1+x)^{3/2}}{1155(1-x)^{5/2}} + \frac{8(1+x)^{3/2}}{3465(1-x)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.017413, size = 40, normalized size = 0.4

$$
\frac{(x+1)^{3/2}\left(8x^4-56x^3+180x^2-364x+547\right)}{3465(1-x)^{11/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(3/2)*(547 - 364*x + 180*x^2 - 56*x^3 + 8*x^4))/(3465*(1 - x)^(11/
2))
```

Maple [A] time = 0.002, size = 35, normalized size = 0.4

$$
\frac{8\,x^4-56\,x^3+180\,x^2-364\,x+547}{3465}(1+x)^{\frac{3}{2}}(1-x)^{-\frac{11}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(13/2),x)
```

```
[Out] 1/3465*(1+x)^(3/2)*(8*x^4-56*x^3+180*x^2-364*x+547)/(1-x)^(11/2)
```

Maxima [B] time = 1.04257, size = 232, normalized size = 2.3

$$
\frac{2\sqrt{-x^2+1}}{11\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}+\frac{\sqrt{-x^2+1}}{99\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}-\frac{4\sqrt{-x^2+1}}{693\left(x^4-4\,x^3+6\,x^2-4\,x\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] 2/11*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 1/
99*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 4/693*sqrt(-x
^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/1155*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) - 8/3465*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 8/3465*sqrt(-x^2 + 1
```

```
)/(x - 1)
```

Fricas [A] time = 1.53397, size = 279, normalized size = 2.76

$$\frac{547\,x^6-3282\,x^5+8205\,x^4-10940\,x^3+8205\,x^2+\left(8\,x^5-48\,x^4+124\,x^3-184\,x^2+183\,x+547\right)\sqrt{x+1}\sqrt{-x+1}-3282}{3465\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(547*x^6 - 3282*x^5 + 8205*x^4 - 10940*x^3 + 8205*x^2 + (8*x^5 - 48*
x^4 + 124*x^3 - 184*x^2 + 183*x + 547)*sqrt(x + 1)*sqrt(-x + 1) - 3282*x +
547)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(13/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15099, size = 57, normalized size = 0.56

$$\frac{(4\,((2\,(x+1)(x-10)+99)(x+1)-231)(x+1)+1155)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{3465\,(x-1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="giac")
```

```
[Out] 1/3465*(4*((2*(x + 1)*(x - 10) + 99)*(x + 1) - 231)*(x + 1) + 1155)*(x + 1)
^(3/2)*sqrt(-x + 1)/(x - 1)^6
```

3.1075 $\int (1-x)^{9/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=109

$$\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}+\frac{3}{14}(x+1)^{5/2}(1-x)^{7/2}+\frac{3}{10}(x+1)^{5/2}(1-x)^{5/2}+\frac{3}{8}x(x+1)^{3/2}(1-x)^{3/2}+\frac{9}{16}x\sqrt{x+1}\sqrt{1-x}+\frac{9}{16}$$

```
[Out] (9*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (3*(1 - x)^(3/2)*x*(1 + x)^(3/2))/8 + (3
*(1 - x)^(5/2)*(1 + x)^(5/2))/10 + (3*(1 - x)^(7/2)*(1 + x)^(5/2))/14 + ((1
 - x)^(9/2)*(1 + x)^(5/2))/7 + (9*ArcSin[x])/16
```

Rubi [A] time = 0.0197551, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}+\frac{3}{14}(x+1)^{5/2}(1-x)^{7/2}+\frac{3}{10}(x+1)^{5/2}(1-x)^{5/2}+\frac{3}{8}x(x+1)^{3/2}(1-x)^{3/2}+\frac{9}{16}x\sqrt{x+1}\sqrt{1-x}+\frac{9}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*(1 + x)^(3/2),x]
```

```
[Out] (9*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (3*(1 - x)^(3/2)*x*(1 + x)^(3/2))/8 + (3
*(1 - x)^(5/2)*(1 + x)^(5/2))/10 + (3*(1 - x)^(7/2)*(1 + x)^(5/2))/14 + ((1
 - x)^(9/2)*(1 + x)^(5/2))/7 + (9*ArcSin[x])/16
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (1-x)^{9/2}(1+x)^{3/2}\,dx = \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2} + \frac{9}{7}\int (1-x)^{7/2}(1+x)^{3/2}\,dx$$
$$= \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2} + \frac{3}{2}\int (1-x)^{5/2}(1+x)^{3/2}\,dx$$
$$= \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2} + \frac{3}{2}\int (1-x)^{3/2}(1+x)^{3/2}$$
$$= \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2} +$$
$$= \frac{9}{16}\sqrt{1-x}x\sqrt{1+x} + \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}$$
$$= \frac{9}{16}\sqrt{1-x}x\sqrt{1+x} + \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}$$
$$= \frac{9}{16}\sqrt{1-x}x\sqrt{1+x} + \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}$$

Mathematica [A] time = 0.0572892, size = 66, normalized size = 0.61

$$\frac{1}{560}\sqrt{1-x^2}\left(80x^6 - 280x^5 + 208x^4 + 350x^3 - 656x^2 + 245x + 368\right) - \frac{9}{8}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(368 + 245*x - 656*x^2 + 350*x^3 + 208*x^4 - 280*x^5 + 80*x^
6))/560 - (9*ArcSin[Sqrt[1 - x]/Sqrt[2]])/8
```

Maple [A] time = 0.004, size = 127, normalized size = 1.2

$$\frac{1}{7}(1-x)^{\frac{9}{2}}(1+x)^{\frac{5}{2}} + \frac{3}{14}(1-x)^{\frac{7}{2}}(1+x)^{\frac{5}{2}} + \frac{3}{10}(1-x)^{\frac{5}{2}}(1+x)^{\frac{5}{2}} + \frac{3}{8}(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}} + \frac{3}{8}\sqrt{1-x}(1+x)^{\frac{5}{2}} - \frac{3}{16}\sqrt{1-x}(1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(9/2)*(1+x)^(3/2),x)
```

```
[Out] 1/7*(1-x)^(9/2)*(1+x)^(5/2)+3/14*(1-x)^(7/2)*(1+x)^(5/2)+3/10*(1-x)^(5/2)*(
1+x)^(5/2)+3/8*(1-x)^(3/2)*(1+x)^(5/2)+3/8*(1-x)^(1/2)*(1+x)^(5/2)-3/16*(1-
x)^(1/2)*(1+x)^(3/2)-9/16*(1-x)^(1/2)*(1+x)^(1/2)+9/16*((1+x)*(1-x))^(1/2)/
(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.53441, size = 89, normalized size = 0.82

$$\frac{1}{7}\left(-x^2+1\right)^{\frac{5}{2}}x^2 - \frac{1}{2}\left(-x^2+1\right)^{\frac{5}{2}}x + \frac{23}{35}\left(-x^2+1\right)^{\frac{5}{2}} + \frac{3}{8}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{9}{16}\sqrt{-x^2+1}x + \frac{9}{16}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*(-x^2 + 1)^(5/2)*x^2 - 1/2*(-x^2 + 1)^(5/2)*x + 23/35*(-x^2 + 1)^(5/2)
+ 3/8*(-x^2 + 1)^(3/2)*x + 9/16*sqrt(-x^2 + 1)*x + 9/16*arcsin(x)
```

Fricas [A] time = 1.5773, size = 192, normalized size = 1.76

$$\frac{1}{560}\left(80\,x^6-280\,x^5+208\,x^4+350\,x^3-656\,x^2+245\,x+368\right)\sqrt{x+1}\sqrt{-x+1}-\frac{9}{8}\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/560*(80*x^6 - 280*x^5 + 208*x^4 + 350*x^3 - 656*x^2 + 245*x + 368)*sqrt(x
 + 1)*sqrt(-x + 1) - 9/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21059, size = 259, normalized size = 2.38

$$\frac{1}{105}\,((3\,((5\,(x+1)(x-5)+74)(x+1)-96)(x+1)+203)(x+1)-70)(x+1)^{\frac{3}{2}}\sqrt{-x+1}+\frac{2}{15}\,((3\,(x+1)(x-3)+17)(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/105*((3*((5*(x + 1)*(x - 5) + 74)*(x + 1) - 96)*(x + 1) + 203)*(x + 1) -
70)*(x + 1)^(3/2)*sqrt(-x + 1) + 2/15*((3*(x + 1)*(x - 3) + 17)*(x + 1) - 1
0)*(x + 1)^(3/2)*sqrt(-x + 1) - (x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) - 1/16*(
(2*((4*(x + 1)*(x - 4) + 39)*(x + 1) - 37)*(x + 1) + 31)*(x + 1) - 3)*sqrt(
x + 1)*sqrt(-x + 1) + 1/4*((2*(x + 1)*(x - 2) + 5)*(x + 1) - 1)*sqrt(x + 1)
*sqrt(-x + 1) + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + 9/8*arcsin(1/2*sqrt(2)*sqr
t(x + 1))
```

3.1076 $\int (1-x)^{7/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}+\frac{7}{30}(x+1)^{5/2}(1-x)^{5/2}+\frac{7}{24}x(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{16}x\sqrt{x+1}\sqrt{1-x}+\frac{7}{16}\sin^{-1}(x)$$

```
[Out] (7*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (7*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
7*(1 - x)^(5/2)*(1 + x)^(5/2))/30 + ((1 - x)^(7/2)*(1 + x)^(5/2))/6 + (7*Ar
cSin[x])/16
```

Rubi [A] time = 0.0133954, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}+\frac{7}{30}(x+1)^{5/2}(1-x)^{5/2}+\frac{7}{24}x(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{16}x\sqrt{x+1}\sqrt{1-x}+\frac{7}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*(1 + x)^(3/2),x]
```

```
[Out] (7*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (7*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
7*(1 - x)^(5/2)*(1 + x)^(5/2))/30 + ((1 - x)^(7/2)*(1 + x)^(5/2))/6 + (7*Ar
cSin[x])/16
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (1-x)^{7/2}(1+x)^{3/2}\,dx = \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2} + \frac{7}{6}\int (1-x)^{5/2}(1+x)^{3/2}\,dx$$
$$= \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2} + \frac{7}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx$$
$$= \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2} + \frac{7}{8}\int \sqrt{1-x}\sqrt{1+x}$$
$$= \frac{7}{16}\sqrt{1-x}x\sqrt{1+x} + \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2}$$
$$= \frac{7}{16}\sqrt{1-x}x\sqrt{1+x} + \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2}$$
$$= \frac{7}{16}\sqrt{1-x}x\sqrt{1+x} + \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2}$$

Mathematica [A] time = 0.0542176, size = 61, normalized size = 0.69

$$\frac{1}{240}\sqrt{1-x^2}\left(-40x^5+96x^4+10x^3-192x^2+135x+96\right)-\frac{7}{8}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(96 + 135*x - 192*x^2 + 10*x^3 + 96*x^4 - 40*x^5))/240 - (7*
ArcSin[Sqrt[1 - x]/Sqrt[2]])/8
```

Maple [A] time = 0.005, size = 113, normalized size = 1.3

$$\frac{1}{6}(1-x)^{\frac{7}{2}}(1+x)^{\frac{5}{2}}+\frac{7}{30}(1-x)^{\frac{5}{2}}(1+x)^{\frac{5}{2}}+\frac{7}{24}(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}+\frac{7}{24}\sqrt{1-x}(1+x)^{\frac{5}{2}}-\frac{7}{48}\sqrt{1-x}(1+x)^{\frac{3}{2}}-\frac{7}{16}\sqrt{1-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)*(1+x)^(3/2),x)
```

```
[Out] 1/6*(1-x)^(7/2)*(1+x)^(5/2)+7/30*(1-x)^(5/2)*(1+x)^(5/2)+7/24*(1-x)^(3/2)*(
1+x)^(5/2)+7/24*(1-x)^(1/2)*(1+x)^(5/2)-7/48*(1-x)^(1/2)*(1+x)^(3/2)-7/16*(
1-x)^(1/2)*(1+x)^(1/2)+7/16*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arc
sin(x)
```

Maxima [A] time = 1.53828, size = 70, normalized size = 0.79

$$-\frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{2}{5}\left(-x^2+1\right)^{\frac{5}{2}}+\frac{7}{24}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{7}{16}\sqrt{-x^2+1}x+\frac{7}{16}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(5/2)*x + 2/5*(-x^2 + 1)^(5/2) + 7/24*(-x^2 + 1)^(3/2)*x +
7/16*sqrt(-x^2 + 1)*x + 7/16*arcsin(x)
```

Fricas [A] time = 1.55404, size = 176, normalized size = 1.98

$$-\frac{1}{240}\left(40x^5-96x^4-10x^3+192x^2-135x-96\right)\sqrt{x+1}\sqrt{-x+1}-\frac{7}{8}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(40*x^5 - 96*x^4 - 10*x^3 + 192*x^2 - 135*x - 96)*sqrt(x + 1)*sqrt(-
x + 1) - 7/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [A] time = 137.498, size = 289, normalized size = 3.25

$$\begin{cases} -\frac{7i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}-\frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}}+\frac{47i(x+1)^{\frac{11}{2}}}{30\sqrt{x-1}}-\frac{683i(x+1)^{\frac{9}{2}}}{120\sqrt{x-1}}+\frac{1151i(x+1)^{\frac{7}{2}}}{120\sqrt{x-1}}-\frac{1543i(x+1)^{\frac{5}{2}}}{240\sqrt{x-1}}-\frac{7i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}}+\frac{7i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{7\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}+\frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}}-\frac{47(x+1)^{\frac{11}{2}}}{30\sqrt{1-x}}+\frac{683(x+1)^{\frac{9}{2}}}{120\sqrt{1-x}}-\frac{1151(x+1)^{\frac{7}{2}}}{120\sqrt{1-x}}+\frac{1543(x+1)^{\frac{5}{2}}}{240\sqrt{1-x}}+\frac{7(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}}-\frac{7\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-7*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 - I*(x + 1)**(13/2)/(6*sqrt(
x - 1)) + 47*I*(x + 1)**(11/2)/(30*sqrt(x - 1)) - 683*I*(x + 1)**(9/2)/(120
*sqrt(x - 1)) + 1151*I*(x + 1)**(7/2)/(120*sqrt(x - 1)) - 1543*I*(x + 1)**(
5/2)/(240*sqrt(x - 1)) - 7*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 7*I*sqrt(x +
 1)/(8*sqrt(x - 1)), Abs(x + 1)/2 > 1), (7*asin(sqrt(2)*sqrt(x + 1)/2)/8 +
(x + 1)**(13/2)/(6*sqrt(1 - x)) - 47*(x + 1)**(11/2)/(30*sqrt(1 - x)) + 683
*(x + 1)**(9/2)/(120*sqrt(1 - x)) - 1151*(x + 1)**(7/2)/(120*sqrt(1 - x)) +
 1543*(x + 1)**(5/2)/(240*sqrt(1 - x)) + 7*(x + 1)**(3/2)/(48*sqrt(1 - x))
- 7*sqrt(x + 1)/(8*sqrt(1 - x)), True))
```

Giac [A] time = 1.0971, size = 161, normalized size = 1.81

$$\frac{2}{15}\left((3(x+1)(x-3)+17)(x+1)-10\right)(x+1)^{\frac{3}{2}}\sqrt{-x+1}-\frac{2}{3}(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1}-\frac{1}{48}\left((2\left((4(x+1)(x-4)+39)(x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*((3*(x + 1)*(x - 3) + 17)*(x + 1) - 10)*(x + 1)^(3/2)*sqrt(-x + 1) - 2
/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) - 1/48*((2*((4*(x + 1)*(x - 4) + 39)*
(x + 1) - 37)*(x + 1) + 31)*(x + 1) - 3)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqr
t(x + 1)*x*sqrt(-x + 1) + 7/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1077 $\int (1-x)^{5/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 + ((1 -
 x)^(5/2)*(1 + x)^(5/2))/5 + (3*ArcSin[x])/8
```

Rubi [A] time = 0.0081353, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 + ((1 -
 x)^(5/2)*(1 + x)^(5/2))/5 + (3*ArcSin[x])/8
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{5/2}(1+x)^{3/2}\,dx &= \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \int (1-x)^{3/2}(1+x)^{3/2}\,dx \\ &= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{4}\int \sqrt{1-x}\sqrt{1+x}\,dx \\ &= \frac{3}{8}\sqrt{1-x}x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{8}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx \\ &= \frac{3}{8}\sqrt{1-x}x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{3}{8}\sqrt{1-x}x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{8}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0444378, size = 55, normalized size = 0.8

$$\frac{1}{40}\left(\sqrt{1-x^2}\left(8x^4-10x^3-16x^2+25x+8\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(8 + 25*x - 16*x^2 - 10*x^3 + 8*x^4) - 30*ArcSin[Sqrt[1 - x]
/Sqrt[2]])/40
```

Maple [A] time = 0.005, size = 99, normalized size = 1.4

$$\frac{1}{5}(1-x)^{\frac{5}{2}}(1+x)^{\frac{5}{2}} + \frac{1}{4}(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}} + \frac{1}{4}\sqrt{1-x}\,(1+x)^{\frac{5}{2}} - \frac{1}{8}\sqrt{1-x}\,(1+x)^{\frac{3}{2}} - \frac{3}{8}\sqrt{1-x}\sqrt{1+x} + \frac{3\arcsin(x)}{8}\sqrt{(1+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)*(1+x)^(3/2),x)
```

```
[Out] 1/5*(1-x)^(5/2)*(1+x)^(5/2)+1/4*(1-x)^(3/2)*(1+x)^(5/2)+1/4*(1-x)^(1/2)*(1+
x)^(5/2)-1/8*(1-x)^(1/2)*(1+x)^(3/2)-3/8*(1-x)^(1/2)*(1+x)^(1/2)+3/8*((1+x)
*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.57968, size = 54, normalized size = 0.78

$$\frac{1}{5}\left(-x^2+1\right)^{\frac{5}{2}} + \frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{3}{8}\sqrt{-x^2+1}x + \frac{3}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(5/2) + 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8*
arcsin(x)
```

Fricas [A] time = 1.48994, size = 155, normalized size = 2.25

$$\frac{1}{40}\left(8x^4-10x^3-16x^2+25x+8\right)\sqrt{x+1}\sqrt{-x+1}-\frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 - 10*x^3 - 16*x^2 + 25*x + 8)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*ar
ctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [B] time = 37.3425, size = 250, normalized size = 3.62

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}}-\frac{29i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}}+\frac{73i(x+1)^{\frac{7}{2}}}{20\sqrt{x-1}}-\frac{129i(x+1)^{\frac{5}{2}}}{40\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}+\frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}}+\frac{29(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}}-\frac{73(x+1)^{\frac{7}{2}}}{20\sqrt{1-x}}+\frac{129(x+1)^{\frac{5}{2}}}{40\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}-\frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) - 29*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) + 73*I*(x + 1)**(7/2)/(20*sq
rt(x - 1)) - 129*I*(x + 1)**(5/2)/(40*sqrt(x - 1)) - I*(x + 1)**(3/2)/(8*sq
rt(x - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1)/2 > 1), (3*asin(sq
rt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(11/2)/(5*sqrt(1 - x)) + 29*(x + 1)**(9/2
)/(20*sqrt(1 - x)) - 73*(x + 1)**(7/2)/(20*sqrt(1 - x)) + 129*(x + 1)**(5/2
)/(40*sqrt(1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt
(1 - x)), True))
```

Giac [B] time = 1.09546, size = 143, normalized size = 2.07

$$\frac{1}{15}\left((3(x+1)(x-3)+17)(x+1)-10\right)(x+1)^{\frac{3}{2}}\sqrt{-x+1}-\frac{1}{3}(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1}-\frac{1}{8}\left((2(x+1)(x-2)+5)(x+1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*((3*(x + 1)*(x - 3) + 17)*(x + 1) - 10)*(x + 1)^(3/2)*sqrt(-x + 1) - 1
/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) - 1/8*((2*(x + 1)*(x - 2) + 5)*(x + 1
) - 1)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + 3/4*arcs
in(1/2*sqrt(2)*sqrt(x + 1))
```

3.1078 $\int (1-x)^{3/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 + (3*Ar
cSin[x])/8
```

Rubi [A] time = 0.0066708, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {38, 41, 216}

$$\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 + (3*Ar
cSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{3/2}(1+x)^{3/2}\,dx &= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{4}\int \sqrt{1-x}\sqrt{1+x}\,dx\\
&= \frac{3}{8}\sqrt{1-x}x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{8}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx\\
&= \frac{3}{8}\sqrt{1-x}x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{3}{8}\sqrt{1-x}x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{8}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0109164, size = 29, normalized size = 0.59

$$\frac{1}{8}\left(x\sqrt{1-x^2}\left(5-2x^2\right)+3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

```
[Out] (x*(5 - 2*x^2)*Sqrt[1 - x^2] + 3*ArcSin[x])/8
```

Maple [B] time = 0.004, size = 85, normalized size = 1.7

$$\frac{1}{4}(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}+\frac{1}{4}\sqrt{1-x}\,(1+x)^{\frac{5}{2}}-\frac{1}{8}\sqrt{1-x}\,(1+x)^{\frac{3}{2}}-\frac{3}{8}\sqrt{1-x}\sqrt{1+x}+\frac{3\arcsin(x)}{8}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)*(1+x)^(3/2),x)
```

```
[Out] 1/4*(1-x)^(3/2)*(1+x)^(5/2)+1/4*(1-x)^(1/2)*(1+x)^(5/2)-1/8*(1-x)^(1/2)*(1+
x)^(3/2)-3/8*(1-x)^(1/2)*(1+x)^(1/2)+3/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1
-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.52964, size = 39, normalized size = 0.8

$$\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{-x^2+1}x+\frac{3}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8*arcsin(x)
```

Fricas [A] time = 1.55559, size = 124, normalized size = 2.53

$$-\frac{1}{8}\left(2x^3-5x\right)\sqrt{x+1}\sqrt{-x+1}-\frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^3 - 5*x)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*arctan((sqrt(x + 1)*sqrt(
-x + 1) - 1)/x)
```

Sympy [B] time = 9.93612, size = 214, normalized size = 4.37

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}} + \frac{5i(x+1)^{\frac{7}{2}}}{4\sqrt{x-1}} - \frac{13i(x+1)^{\frac{5}{2}}}{8\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}} + \frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}} - \frac{5(x+1)^{\frac{7}{2}}}{4\sqrt{1-x}} + \frac{13(x+1)^{\frac{5}{2}}}{8\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}} - \frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) + 5*I*(x + 1)**(7/2)/(4*sqrt(x - 1)) - 13*I*(x + 1)**(5/2)/(8*sqrt(x
 - 1)) - I*(x + 1)**(3/2)/(8*sqrt(x - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1))
, Abs(x + 1)/2 > 1), (3*asin(sqrt(2)*sqrt(x + 1)/2)/4 + (x + 1)**(9/2)/(4*s
qrt(1 - x)) - 5*(x + 1)**(7/2)/(4*sqrt(1 - x)) + 13*(x + 1)**(5/2)/(8*sqrt(
1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1 - x)), T
rue))
```

Giac [A] time = 1.09649, size = 80, normalized size = 1.63

$$-\frac{1}{8}\left(\left(2\left(x+1\right)\left(x-2\right)+5\right)\left(x+1\right)-1\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\sqrt{x+1}x\sqrt{-x+1}+\frac{3}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*((2*(x + 1)*(x - 2) + 5)*(x + 1) - 1)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*s
qrt(x + 1)*x*sqrt(-x + 1) + 3/4*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1079 $\int \sqrt{1-x}(1+x)^{3/2}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 - ((1 - x)^(3/2)*(1 + x)^(3/2))/3 + ArcSin[x]
/2
```

Rubi [A] time = 0.0055341, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$-\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 - ((1 - x)^(3/2)*(1 + x)^(3/2))/3 + ArcSin[x]
/2
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{1-x}(1+x)^{3/2}\,dx = -\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2} + \int \sqrt{1-x}\sqrt{1+x}\,dx$$
$$= \frac{1}{2}\sqrt{1-x}x\sqrt{1+x} - \frac{1}{3}(1-x)^{3/2}(1+x)^{3/2} + \frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$$
$$= \frac{1}{2}\sqrt{1-x}x\sqrt{1+x} - \frac{1}{3}(1-x)^{3/2}(1+x)^{3/2} + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{1}{2}\sqrt{1-x}x\sqrt{1+x} - \frac{1}{3}(1-x)^{3/2}(1+x)^{3/2} + \frac{1}{2}\sin^{-1}(x)$$

Mathematica [A] time = 0.0336247, size = 44, normalized size = 0.92

$$\frac{1}{6}\sqrt{1-x^2}\left(2x^2+3x-2\right) - \sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(-2 + 3*x + 2*x^2))/6 - ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

Maple [B] time = 0.003, size = 71, normalized size = 1.5

$$\frac{1}{3}\sqrt{1-x}\,(1+x)^{\frac{5}{2}} - \frac{1}{6}\sqrt{1-x}\,(1+x)^{\frac{3}{2}} - \frac{1}{2}\sqrt{1-x}\sqrt{1+x} + \frac{\arcsin(x)}{2}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)*(1+x)^(3/2),x)
```

```
[Out] 1/3*(1-x)^(1/2)*(1+x)^(5/2)-1/6*(1-x)^(1/2)*(1+x)^(3/2)-1/2*(1-x)^(1/2)*(1+
x)^(1/2)+1/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.52172, size = 38, normalized size = 0.79

$$-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A] time = 1.52655, size = 123, normalized size = 2.56

$$\frac{1}{6}\left(2x^2+3x-2\right)\sqrt{x+1}\sqrt{-x+1} - \arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 + 3*x - 2)*sqrt(x + 1)*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(-
x + 1) - 1)/x)
```

Sympy [B] time = 5.359, size = 165, normalized size = 3.44

$$\begin{cases} -i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} - \frac{5i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} + \frac{5(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x - 1
)) - 5*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - I*(x + 1)**(3/2)/(6*sqrt(x - 1))
+ I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (asin(sqrt(2)*sqrt(x + 1)/2
) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 5*(x + 1)**(5/2)/(6*sqrt(1 - x)) + (x
+ 1)**(3/2)/(6*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A] time = 1.07302, size = 59, normalized size = 1.23

$$\frac{1}{3}(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1} + \frac{1}{2}\sqrt{x+1}x\sqrt{-x+1} + \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + a
rcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1080 $\int \frac{(1+x)^{3/2}}{\sqrt{1-x}} \, dx$

Optimal. Leaf size=47

$$-\frac{1}{2}\sqrt{1-x}(x+1)^{3/2}-\frac{3}{2}\sqrt{1-x}\sqrt{x+1}+\frac{3}{2}\sin^{-1}(x)$$

```
[Out] (-3*Sqrt[1 - x]*Sqrt[1 + x])/2 - (Sqrt[1 - x]*(1 + x)^(3/2))/2 + (3*ArcSin[
x])/2
```

Rubi [A] time = 0.006518, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$-\frac{1}{2}\sqrt{1-x}(x+1)^{3/2}-\frac{3}{2}\sqrt{1-x}\sqrt{x+1}+\frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/Sqrt[1 - x],x]
```

```
[Out] (-3*Sqrt[1 - x]*Sqrt[1 + x])/2 - (Sqrt[1 - x]*(1 + x)^(3/2))/2 + (3*ArcSin[
x])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{3/2}}{\sqrt{1-x}} \, dx &= -\frac{1}{2}\sqrt{1-x}(1+x)^{3/2}+\frac{3}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}} \, dx \\
&= -\frac{3}{2}\sqrt{1-x}\sqrt{1+x}-\frac{1}{2}\sqrt{1-x}(1+x)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}} \, dx \\
&= -\frac{3}{2}\sqrt{1-x}\sqrt{1+x}-\frac{1}{2}\sqrt{1-x}(1+x)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{1-x^2}} \, dx \\
&= -\frac{3}{2}\sqrt{1-x}\sqrt{1+x}-\frac{1}{2}\sqrt{1-x}(1+x)^{3/2}+\frac{3}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0115537, size = 37, normalized size = 0.79

$$-\frac{1}{2}\sqrt{1-x^2}(x+4)-3\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/Sqrt[1 - x],x]
```

```
[Out] -((4 + x)*Sqrt[1 - x^2])/2 - 3*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

Maple [A] time = 0.004, size = 57, normalized size = 1.2

$$-\frac{1}{2}\sqrt{1-x}\,(1+x)^{\frac{3}{2}}-\frac{3}{2}\sqrt{1-x}\sqrt{1+x}+\frac{3\arcsin(x)}{2}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(1/2),x)
```

```
[Out] -1/2*(1-x)^(1/2)*(1+x)^(3/2)-3/2*(1-x)^(1/2)*(1+x)^(1/2)+3/2*((1+x)*(1-x))^
(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.54716, size = 38, normalized size = 0.81

$$-\frac{1}{2}\sqrt{-x^2+1}x-2\sqrt{-x^2+1}+\frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

Fricas [A] time = 1.54625, size = 113, normalized size = 2.4

$$-\frac{1}{2}(x+4)\sqrt{x+1}\sqrt{-x+1}-3\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x + 4)*sqrt(x + 1)*sqrt(-x + 1) - 3*arctan((sqrt(x + 1)*sqrt(-x + 1)
- 1)/x)
```

Sympy [A] time = 3.5997, size = 136, normalized size = 2.89

$$\begin{cases} -3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} + \frac{3i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ 3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} - \frac{3\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) - I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + 3*I*sqrt(x + 1)/sqrt(x - 1), Abs(
x + 1)/2 > 1), (3*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sqrt(1 -
x)) + (x + 1)**(3/2)/(2*sqrt(1 - x)) - 3*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A] time = 1.06404, size = 42, normalized size = 0.89

$$-\frac{1}{2}(x+4)\sqrt{x+1}\sqrt{-x+1} + 3\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(x + 4)*sqrt(x + 1)*sqrt(-x + 1) + 3*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1081 $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2(x+1)^{3/2}}{\sqrt{1-x}}+3\sqrt{1-x}\sqrt{x+1}-3\sin^{-1}(x)$$

```
[Out] 3*Sqrt[1 - x]*Sqrt[1 + x] + (2*(1 + x)^(3/2))/Sqrt[1 - x] - 3*ArcSin[x]
```

Rubi [A] time = 0.0065135, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$\frac{2(x+1)^{3/2}}{\sqrt{1-x}}+3\sqrt{1-x}\sqrt{x+1}-3\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(3/2),x]
```

```
[Out] 3*Sqrt[1 - x]*Sqrt[1 + x] + (2*(1 + x)^(3/2))/Sqrt[1 - x] - 3*ArcSin[x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx = \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx$$
$$= 3\sqrt{1-x}\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$$
$$= 3\sqrt{1-x}\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= 3\sqrt{1-x}\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\sin^{-1}(x)$$

Mathematica [C] time = 0.0064706, size = 35, normalized size = 0.85

$$\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{1-x}{2}\right)}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(3/2),x]
```

```
[Out] (4*Sqrt[2]*Hypergeometric2F1[-3/2, -1/2, 1/2, (1 - x)/2])/Sqrt[1 - x]
```

Maple [B] time = 0.014, size = 72, normalized size = 1.8

$$-(x^2-4x-5)\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{-(1+x)(-1+x)}}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}} - 3\frac{\sqrt{(1+x)(1-x)}\arcsin(x)}{\sqrt{1-x}\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(3/2),x)
```

```
[Out] -(x^2-4*x-5)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)/(1+x)^(1
/2)-3*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.52968, size = 57, normalized size = 1.39

$$-\frac{(-x^2+1)^{\frac{3}{2}}}{x^2-2x+1} - \frac{6\sqrt{-x^2+1}}{x-1} - 3\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -(-x^2 + 1)^(3/2)/(x^2 - 2*x + 1) - 6*sqrt(-x^2 + 1)/(x - 1) - 3*arcsin(x)
```

Fricas [A] time = 1.47791, size = 144, normalized size = 3.51

$$\frac{\sqrt{x+1}(x-5)\sqrt{-x+1} + 6(x-1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + 5x - 5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(x + 1)*(x - 5)*sqrt(-x + 1) + 6*(x - 1)*arctan((sqrt(x + 1)*sqrt(-x +
 1) - 1)/x) + 5*x - 5)/(x - 1)
```

Sympy [A] time = 3.27815, size = 100, normalized size = 2.44

$$\begin{cases} 6i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}} - \frac{6i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ -6\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} + \frac{6\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((6*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(3/2)/sqrt(x - 1)
- 6*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (-6*asin(sqrt(2)*sqrt(x +
 1)/2) - (x + 1)**(3/2)/sqrt(1 - x) + 6*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A] time = 1.08252, size = 47, normalized size = 1.15

$$\frac{\sqrt{x+1}(x-5)\sqrt{-x+1}}{x-1} - 6\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*(x - 5)*sqrt(-x + 1)/(x - 1) - 6*arcsin(1/2*sqrt(2)*sqrt(x + 1)
)
```

3.1082 $\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}-\frac{2\sqrt{x+1}}{\sqrt{1-x}}+\sin^{-1}(x)$$

```
[Out] (-2*Sqrt[1 + x])/Sqrt[1 - x] + (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + ArcSin
[x]
```

Rubi [A] time = 0.0050945, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}-\frac{2\sqrt{x+1}}{\sqrt{1-x}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(5/2),x]
```

```
[Out] (-2*Sqrt[1 + x])/Sqrt[1 - x] + (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + ArcSin
[x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx = \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} - \int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx$$
$$= -\frac{2\sqrt{1+x}}{\sqrt{1-x}} + \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$$
$$= -\frac{2\sqrt{1+x}}{\sqrt{1-x}} + \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2\sqrt{1+x}}{\sqrt{1-x}} + \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \sin^{-1}(x)$$

Mathematica [C] time = 0.0063796, size = 37, normalized size = 0.9

$$\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1-x}{2}\right)}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(5/2),x]
```

```
[Out] (4*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - x)/2])/(3*(1 - x)^(3/2)
)
```

Maple [B] time = 0.016, size = 76, normalized size = 1.9

$$-\frac{8x^2+4x-4}{-3+3x}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{-(1+x)(-1+x)}}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}+\arcsin(x)\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(5/2),x)
```

```
[Out] -4/3*(2*x^2+x-1)/(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/
2)/(1+x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [B] time = 1.51751, size = 89, normalized size = 2.17

$$-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^3-3x^2+3x-1\right)}+\frac{2\sqrt{-x^2+1}}{3\left(x^2-2x+1\right)}+\frac{7\sqrt{-x^2+1}}{3(x-1)}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2)/(x^3 - 3*x^2 + 3*x - 1) + 2/3*sqrt(-x^2 + 1)/(x^2 - 2
*x + 1) + 7/3*sqrt(-x^2 + 1)/(x - 1) + arcsin(x)
```

Fricas [B] time = 1.46875, size = 189, normalized size = 4.61

$$-\frac{2\left(2x^2-2(2x-1)\sqrt{x+1}\sqrt{-x+1}+3\left(x^2-2x+1\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-4x+2\right)}{3\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*x^2 - 2*(2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) + 3*(x^2 - 2*x + 1)*arct
an((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 4*x + 2)/(x^2 - 2*x + 1)
```

Sympy [B] time = 5.58419, size = 500, normalized size = 12.2

$$\begin{cases} -\frac{6i\sqrt{x-1}(x+1)^{\frac{15}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{x-1}(x+1)^{\frac{15}{2}}-6\sqrt{x-1}(x+1)^{\frac{13}{2}}}+\frac{3\pi\sqrt{x-1}(x+1)^{\frac{15}{2}}}{3\sqrt{x-1}(x+1)^{\frac{15}{2}}-6\sqrt{x-1}(x+1)^{\frac{13}{2}}}+\frac{12i\sqrt{x-1}(x+1)^{\frac{13}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{x-1}(x+1)^{\frac{15}{2}}-6\sqrt{x-1}(x+1)^{\frac{13}{2}}}-\frac{6\pi\sqrt{x-1}(x+1)^{\frac{13}{2}}}{3\sqrt{x-1}(x+1)^{\frac{15}{2}}-6\sqrt{x-1}(x+1)^{\frac{13}{2}}}+\frac{}{3\sqrt{x-1}} \\ \frac{6\sqrt{1-x}(x+1)^{\frac{15}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{15}{2}}-6\sqrt{1-x}(x+1)^{\frac{13}{2}}}-\frac{12\sqrt{1-x}(x+1)^{\frac{13}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{15}{2}}-6\sqrt{1-x}(x+1)^{\frac{13}{2}}}-\frac{8(x+1)^8}{3\sqrt{1-x}(x+1)^{\frac{15}{2}}-6\sqrt{1-x}(x+1)^{\frac{13}{2}}}+\frac{12(x+1)^7}{3\sqrt{1-x}(x+1)^{\frac{15}{2}}-6\sqrt{1-x}(x+1)^{\frac{13}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(5/2),x)
```

```
[Out] Piecewise((-6*I*sqrt(x - 1)*(x + 1)**(15/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(3
*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x + 1)**(13/2)) + 3*pi*sqrt(x
 - 1)*(x + 1)**(15/2)/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x + 1
)**(13/2)) + 12*I*sqrt(x - 1)*(x + 1)**(13/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/
(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x + 1)**(13/2)) - 6*pi*sqrt
(x - 1)*(x + 1)**(13/2)/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x +
 1)**(13/2)) + 8*I*(x + 1)**8/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1
)*(x + 1)**(13/2)) - 12*I*(x + 1)**7/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqr
t(x - 1)*(x + 1)**(13/2)), Abs(x + 1)/2 > 1), (6*sqrt(1 - x)*(x + 1)**(15/2
)*asin(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x
)*(x + 1)**(13/2)) - 12*sqrt(1 - x)*(x + 1)**(13/2)*asin(sqrt(2)*sqrt(x + 1
)/2)/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/2)) - 8*(x
 + 1)**8/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/2)) +
12*(x + 1)**7/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/2
)), True))
```

Giac [A] time = 1.07073, size = 51, normalized size = 1.24

$$\frac{4(2x-1)\sqrt{x+1}\sqrt{-x+1}}{3(x-1)^2}+2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(2*x - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 + 2*arcsin(1/2*sqrt(2)*sqr
t(x + 1))
```

3.1083 $\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx$

Optimal. Leaf size=20

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

```
[Out] (1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

Rubi [A] time = 0.0016671, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(7/2),x]

[Out] (1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx = \frac{(1+x)^{5/2}}{5(1-x)^{5/2}}$$

Mathematica [A] time = 0.0071806, size = 20, normalized size = 1.

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(7/2),x]

[Out] (1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$\frac{1}{5}(1+x)^{\frac{5}{2}}(1-x)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(7/2),x)
```

```
[Out] 1/5*(1+x)^(5/2)/(1-x)^(5/2)
```

Maxima [B] time = 1.04933, size = 127, normalized size = 6.35

$$\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^4-4x^3+6x^2-4x+1}+\frac{6\sqrt{-x^2+1}}{5\left(x^3-3x^2+3x-1\right)}+\frac{\sqrt{-x^2+1}}{5\left(x^2-2x+1\right)}-\frac{\sqrt{-x^2+1}}{5(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(3/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 6/5*sqrt(-x^2 + 1)/(x^3
- 3*x^2 + 3*x - 1) + 1/5*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/5*sqrt(-x^2 + 1
)/(x - 1)
```

Fricas [B] time = 1.55843, size = 130, normalized size = 6.5

$$\frac{x^3-3x^2-\left(x^2+2x+1\right)\sqrt{x+1}\sqrt{-x+1}+3x-1}{5\left(x^3-3x^2+3x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(x^3 - 3*x^2 - (x^2 + 2*x + 1)*sqrt(x + 1)*sqrt(-x + 1) + 3*x - 1)/(x^3
 - 3*x^2 + 3*x - 1)
```

Sympy [B] time = 23.4347, size = 88, normalized size = 4.4

$$\begin{cases} -\frac{i(x+1)^{\frac{5}{2}}}{5\sqrt{x-1}(x+1)^2-20\sqrt{x-1}(x+1)+20\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1 \\ \frac{(x+1)^{\frac{5}{2}}}{5\sqrt{1-x}(x+1)^2-20\sqrt{1-x}(x+1)+20\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((-I*(x + 1)**(5/2)/(5*sqrt(x - 1)*(x + 1)**2 - 20*sqrt(x - 1)*(x
+ 1) + 20*sqrt(x - 1)), Abs(x + 1)/2 > 1), ((x + 1)**(5/2)/(5*sqrt(1 - x)*(
x + 1)**2 - 20*sqrt(1 - x)*(x + 1) + 20*sqrt(1 - x)), True))
```

Giac [A] time = 1.07877, size = 26, normalized size = 1.3

$$-\frac{(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{5\,(x-1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*(x + 1)^(5/2)*sqrt(-x + 1)/(x - 1)^3
```

3.1084 $\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}} dx$

Optimal. Leaf size=41

$$\frac{(x+1)^{5/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}$$

```
[Out] (1 + x)^(5/2)/(7*(1 - x)^(7/2)) + (1 + x)^(5/2)/(35*(1 - x)^(5/2))
```

Rubi [A] time = 0.0041896, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{(x+1)^{5/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(5/2)/(7*(1 - x)^(7/2)) + (1 + x)^(5/2)/(35*(1 - x)^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int \frac{(1+x)^{3/2}}{(1-x)^{9/2}} dx &= \frac{(1+x)^{5/2}}{7(1-x)^{7/2}}+\frac{1}{7}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx \\ &= \frac{(1+x)^{5/2}}{7(1-x)^{7/2}}+\frac{(1+x)^{5/2}}{35(1-x)^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.0107669, size = 23, normalized size = 0.56

$$-\frac{(x-6)(x+1)^{5/2}}{35(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(9/2),x]

[Out] -((-6 + x)*(1 + x)^(5/2))/(35*(1 - x)^(7/2))
```

Maple [A] time = 0.002, size = 18, normalized size = 0.4

$$-\frac{x-6}{35}(1+x)^{\frac{5}{2}}(1-x)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(9/2),x)

[Out] -1/35*(1+x)^(5/2)*(x-6)/(1-x)^(7/2)
```

Maxima [B] time = 1.02331, size = 177, normalized size = 4.32

$$-\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{2\left(x^{5}-5x^{4}+10x^{3}-10x^{2}+5x-1\right)}-\frac{3\sqrt{-x^{2}+1}}{7\left(x^{4}-4x^{3}+6x^{2}-4x+1\right)}-\frac{3\sqrt{-x^{2}+1}}{70\left(x^{3}-3x^{2}+3x-1\right)}+\frac{\sqrt{-x^{2}+1}}{35\left(x^{2}-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="maxima")

[Out] -1/2*(-x^2 + 1)^(3/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 3/7*sqrt(
-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 3/70*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) + 1/35*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/35*sqrt(-x^2 + 1)/(x
 - 1)
```

Fricas [B] time = 1.67713, size = 171, normalized size = 4.17

$$\frac{6x^{4}-24x^{3}+36x^{2}-\left(x^{3}-4x^{2}-11x-6\right)\sqrt{x+1}\sqrt{-x+1}-24x+6}{35\left(x^{4}-4x^{3}+6x^{2}-4x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="fricas")

[Out] 1/35*(6*x^4 - 24*x^3 + 36*x^2 - (x^3 - 4*x^2 - 11*x - 6)*sqrt(x + 1)*sqrt(-
x + 1) - 24*x + 6)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [B] time = 86.9206, size = 228, normalized size = 5.56

$$\begin{cases} -\frac{i(x+1)^{\frac{7}{2}}}{35\sqrt{x-1}(x+1)^{3}-210\sqrt{x-1}(x+1)^{2}+420\sqrt{x-1}(x+1)-280\sqrt{x-1}}+\frac{7i(x+1)^{\frac{5}{2}}}{35\sqrt{x-1}(x+1)^{3}-210\sqrt{x-1}(x+1)^{2}+420\sqrt{x-1}(x+1)-280\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{(x+1)^{\frac{7}{2}}}{35\sqrt{1-x}(x+1)^{3}-210\sqrt{1-x}(x+1)^{2}+420\sqrt{1-x}(x+1)-280\sqrt{1-x}}-\frac{7(x+1)^{\frac{5}{2}}}{35\sqrt{1-x}(x+1)^{3}-210\sqrt{1-x}(x+1)^{2}+420\sqrt{1-x}(x+1)-280\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((-I*(x + 1)**(7/2)/(35*sqrt(x - 1)*(x + 1)**3 - 210*sqrt(x - 1)*(
x + 1)**2 + 420*sqrt(x - 1)*(x + 1) - 280*sqrt(x - 1)) + 7*I*(x + 1)**(5/2)
/(35*sqrt(x - 1)*(x + 1)**3 - 210*sqrt(x - 1)*(x + 1)**2 + 420*sqrt(x - 1)*
(x + 1) - 280*sqrt(x - 1)), Abs(x + 1)/2 > 1), ((x + 1)**(7/2)/(35*sqrt(1 -
 x)*(x + 1)**3 - 210*sqrt(1 - x)*(x + 1)**2 + 420*sqrt(1 - x)*(x + 1) - 280
*sqrt(1 - x)) - 7*(x + 1)**(5/2)/(35*sqrt(1 - x)*(x + 1)**3 - 210*sqrt(1 -
x)*(x + 1)**2 + 420*sqrt(1 - x)*(x + 1) - 280*sqrt(1 - x)), True))
```

Giac [A] time = 1.08455, size = 30, normalized size = 0.73

$$-\frac{(x+1)^{\frac{5}{2}}(x-6)\sqrt{-x+1}}{35\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(x + 1)^(5/2)*(x - 6)*sqrt(-x + 1)/(x - 1)^4
```

3.1085 $\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx$

Optimal. Leaf size=61

$$\frac{2(x+1)^{5/2}}{315(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}$$

```
[Out] (1 + x)^(5/2)/(9*(1 - x)^(9/2)) + (2*(1 + x)^(5/2))/(63*(1 - x)^(7/2)) + (2
*(1 + x)^(5/2))/(315*(1 - x)^(5/2))
```

Rubi [A] time = 0.0080453, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{5/2}}{315(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

```
[Out] (1 + x)^(5/2)/(9*(1 - x)^(9/2)) + (2*(1 + x)^(5/2))/(63*(1 - x)^(7/2)) + (2
*(1 + x)^(5/2))/(315*(1 - x)^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx &= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2}{9}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{63(1-x)^{7/2}}+\frac{2}{63}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{63(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{315(1-x)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0117375, size = 30, normalized size = 0.49

$$\frac{(x+1)^{5/2}\left(2x^2-14x+47\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

```
[Out] ((1 + x)^(5/2)*(47 - 14*x + 2*x^2))/(315*(1 - x)^(9/2))
```

Maple [A] time = 0.004, size = 25, normalized size = 0.4

$$\frac{2\,x^2-14\,x+47}{315}(1+x)^{\frac{5}{2}}(1-x)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(11/2),x)
```

```
[Out] 1/315*(1+x)^(5/2)*(2*x^2-14*x+47)/(1-x)^(9/2)
```

Maxima [B] time = 1.00336, size = 232, normalized size = 3.8

$$\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}+\frac{2\sqrt{-x^2+1}}{9\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}+\frac{\sqrt{-x^2+1}}{63\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(3/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 2
/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 1/63*sqrt(-x^
2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 1/105*sqrt(-x^2 + 1)/(x^3 - 3*x^2
+ 3*x - 1) + 2/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/315*sqrt(-x^2 + 1)/(x
 - 1)
```

Fricas [A] time = 1.6927, size = 224, normalized size = 3.67

$$\frac{47\,x^5-235\,x^4+470\,x^3-470\,x^2-\left(2\,x^4-10\,x^3+21\,x^2+80\,x+47\right)\sqrt{x+1}\sqrt{-x+1}+235\,x-47}{315\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(47*x^5 - 235*x^4 + 470*x^3 - 470*x^2 - (2*x^4 - 10*x^3 + 21*x^2 + 80
*x + 47)*sqrt(x + 1)*sqrt(-x + 1) + 235*x - 47)/(x^5 - 5*x^4 + 10*x^3 - 10*
x^2 + 5*x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09394, size = 39, normalized size = 0.64

$$-\frac{(2(x+1)(x-8)+63)(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{315(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="giac")
```

```
[Out] -1/315*(2*(x + 1)*(x - 8) + 63)*(x + 1)^(5/2)*sqrt(-x + 1)/(x - 1)^5
```

3.1086 $\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx$

Optimal. Leaf size=81

$$\frac{2(x+1)^{5/2}}{1155(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{231(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{33(1-x)^{9/2}}+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}$$

```
[Out] (1 + x)^(5/2)/(11*(1 - x)^(11/2)) + (1 + x)^(5/2)/(33*(1 - x)^(9/2)) + (2*(
1 + x)^(5/2))/(231*(1 - x)^(7/2)) + (2*(1 + x)^(5/2))/(1155*(1 - x)^(5/2))
```

Rubi [A] time = 0.0131675, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{5/2}}{1155(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{231(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{33(1-x)^{9/2}}+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(5/2)/(11*(1 - x)^(11/2)) + (1 + x)^(5/2)/(33*(1 - x)^(9/2)) + (2*(
1 + x)^(5/2))/(231*(1 - x)^(7/2)) + (2*(1 + x)^(5/2))/(1155*(1 - x)^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{3}{11}\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx\\
&= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2}{33}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx\\
&= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{231(1-x)^{7/2}}+\frac{2}{231}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx\\
&= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{231(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{1155(1-x)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0229021, size = 35, normalized size = 0.43

$$\frac{(x+1)^{5/2}\left(-2x^3+16x^2-61x+152\right)}{1155(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(5/2)*(152 - 61*x + 16*x^2 - 2*x^3))/(1155*(1 - x)^(11/2))
```

Maple [A] time = 0.003, size = 30, normalized size = 0.4

$$-\frac{2\,x^3-16\,x^2+61\,x-152}{1155}\,(1+x)^{\frac{5}{2}}\,(1-x)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(13/2),x)
```

```
[Out] -1/1155*(1+x)^(5/2)*(2*x^3-16*x^2+61*x-152)/(1-x)^(11/2)
```

Maxima [B] time = 1.04473, size = 294, normalized size = 3.63

$$-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{4\left(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1\right)}-\frac{3\sqrt{-x^2+1}}{22\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}-\frac{}{132\left(x^5-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*
x - 1) - 3/22*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x
+ 1) - 1/132*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 1/2
31*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 1/385*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) + 2/1155*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/1155*sqrt
(-x^2 + 1)/(x - 1)
```

Fricas [A] time = 1.54324, size = 273, normalized size = 3.37

$$\frac{152\,x^6-912\,x^5+2280\,x^4-3040\,x^3+2280\,x^2-\left(2\,x^5-12\,x^4+31\,x^3-46\,x^2-243\,x-152\right)\sqrt{x+1}\sqrt{-x+1}-912\,x+}{1155\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(152*x^6 - 912*x^5 + 2280*x^4 - 3040*x^3 + 2280*x^2 - (2*x^5 - 12*x^
4 + 31*x^3 - 46*x^2 - 243*x - 152)*sqrt(x + 1)*sqrt(-x + 1) - 912*x + 152)/
```

```
(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(13/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09859, size = 47, normalized size = 0.58

$$-\frac{((2(x+1)(x-10)+99)(x+1)-231)(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{1155(x-1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="giac")
```

```
[Out] -1/1155*((2*(x + 1)*(x - 10) + 99)*(x + 1) - 231)*(x + 1)^(5/2)*sqrt(-x + 1
)/(x - 1)^6
```

3.1087 $\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}} dx$

Optimal. Leaf size=101

$$\frac{8(x+1)^{5/2}}{15015(1-x)^{5/2}}+\frac{8(x+1)^{5/2}}{3003(1-x)^{7/2}}+\frac{4(x+1)^{5/2}}{429(1-x)^{9/2}}+\frac{4(x+1)^{5/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{5/2}}{13(1-x)^{13/2}}$$

```
[Out] (1 + x)^(5/2)/(13*(1 - x)^(13/2)) + (4*(1 + x)^(5/2))/(143*(1 - x)^(11/2))
+ (4*(1 + x)^(5/2))/(429*(1 - x)^(9/2)) + (8*(1 + x)^(5/2))/(3003*(1 - x)^(
7/2)) + (8*(1 + x)^(5/2))/(15015*(1 - x)^(5/2))
```

Rubi [A] time = 0.0192798, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{8(x+1)^{5/2}}{15015(1-x)^{5/2}}+\frac{8(x+1)^{5/2}}{3003(1-x)^{7/2}}+\frac{4(x+1)^{5/2}}{429(1-x)^{9/2}}+\frac{4(x+1)^{5/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{5/2}}{13(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

```
[Out] (1 + x)^(5/2)/(13*(1 - x)^(13/2)) + (4*(1 + x)^(5/2))/(143*(1 - x)^(11/2))
+ (4*(1 + x)^(5/2))/(429*(1 - x)^(9/2)) + (8*(1 + x)^(5/2))/(3003*(1 - x)^(
7/2)) + (8*(1 + x)^(5/2))/(15015*(1 - x)^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx = \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4}{13}\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx$$
$$= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{12}{143}\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx$$
$$= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{4(1+x)^{5/2}}{429(1-x)^{9/2}} + \frac{8}{429}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx$$
$$= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{4(1+x)^{5/2}}{429(1-x)^{9/2}} + \frac{8(1+x)^{5/2}}{3003(1-x)^{7/2}} + \frac{8\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx}{3003}$$
$$= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{4(1+x)^{5/2}}{429(1-x)^{9/2}} + \frac{8(1+x)^{5/2}}{3003(1-x)^{7/2}} + \frac{8(1+x)^{5/2}}{15015(1-x)^{5/2}}$$

Mathematica [A] time = 0.0186784, size = 40, normalized size = 0.4

$$\frac{(x+1)^{5/2}\left(8x^4-72x^3+308x^2-852x+1763\right)}{15015(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

```
[Out] ((1 + x)^(5/2)*(1763 - 852*x + 308*x^2 - 72*x^3 + 8*x^4))/(15015*(1 - x)^(1
3/2))
```

Maple [A] time = 0.002, size = 35, normalized size = 0.4

$$\frac{8x^4-72x^3+308x^2-852x+1763}{15015}(1+x)^{\frac{5}{2}}(1-x)^{-\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(15/2),x)
```

```
[Out] 1/15015*(1+x)^(5/2)*(8*x^4-72*x^3+308*x^2-852*x+1763)/(1-x)^(13/2)
```

Maxima [B] time = 1.03483, size = 363, normalized size = 3.59

$$\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{5\left(x^8-8x^7+28x^6-56x^5+70x^4-56x^3+28x^2-8x+1\right)} + \frac{6\sqrt{-x^2+1}}{65\left(x^7-7x^6+21x^5-35x^4+35x^3-21x^2+7x-1\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(3/2)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*
x^2 - 8*x + 1) + 6/65*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^
3 - 21*x^2 + 7*x - 1) + 3/715*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3
 + 15*x^2 - 6*x + 1) - 1/429*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2
```

```
+ 5*x - 1) + 4/3003*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 4/5005
*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) + 8/15015*sqrt(-x^2 + 1)/(x^2 - 2*x
 + 1) - 8/15015*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A] time = 1.60946, size = 333, normalized size = 3.3

$$\frac{1763\,x^7 - 12341\,x^6 + 37023\,x^5 - 61705\,x^4 + 61705\,x^3 - 37023\,x^2 - \left(8\,x^6 - 56\,x^5 + 172\,x^4 - 308\,x^3 + 367\,x^2 + 2674\,x}{15015\left(x^7 - 7\,x^6 + 21\,x^5 - 35\,x^4 + 35\,x^3 - 21\,x^2 + 7\,x - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(1763*x^7 - 12341*x^6 + 37023*x^5 - 61705*x^4 + 61705*x^3 - 37023*x
^2 - (8*x^6 - 56*x^5 + 172*x^4 - 308*x^3 + 367*x^2 + 2674*x + 1763)*sqrt(x
+ 1)*sqrt(-x + 1) + 12341*x - 1763)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3
 - 21*x^2 + 7*x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(15/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12134, size = 57, normalized size = 0.56

$$-\frac{(4\,((2\,(x+1)(x-12)+143)(x+1)-429)(x+1)+3003)(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{15015\,(x-1)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="giac")
```

```
[Out] -1/15015*(4*((2*(x + 1)*(x - 12) + 143)*(x + 1) - 429)*(x + 1) + 3003)*(x +
 1)^(5/2)*sqrt(-x + 1)/(x - 1)^7
```

3.1088 $\int (1-x)^{11/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=130

$$\frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}+\frac{11}{72}(x+1)^{7/2}(1-x)^{9/2}+\frac{11}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{11}{48}x(x+1)^{5/2}(1-x)^{5/2}+\frac{55}{192}x(x+1)^{3/2}(1-x)^{3/2}$$

```
[Out] (55*Sqrt[1 - x]*x*Sqrt[1 + x])/128 + (55*(1 - x)^(3/2)*x*(1 + x)^(3/2))/192
 + (11*(1 - x)^(5/2)*x*(1 + x)^(5/2))/48 + (11*(1 - x)^(7/2)*(1 + x)^(7/2))
/56 + (11*(1 - x)^(9/2)*(1 + x)^(7/2))/72 + ((1 - x)^(11/2)*(1 + x)^(7/2))/
9 + (55*ArcSin[x])/128
```

Rubi [A] time = 0.0254404, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}+\frac{11}{72}(x+1)^{7/2}(1-x)^{9/2}+\frac{11}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{11}{48}x(x+1)^{5/2}(1-x)^{5/2}+\frac{55}{192}x(x+1)^{3/2}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(11/2)*(1 + x)^(5/2),x]
```

```
[Out] (55*Sqrt[1 - x]*x*Sqrt[1 + x])/128 + (55*(1 - x)^(3/2)*x*(1 + x)^(3/2))/192
 + (11*(1 - x)^(5/2)*x*(1 + x)^(5/2))/48 + (11*(1 - x)^(7/2)*(1 + x)^(7/2))
/56 + (11*(1 - x)^(9/2)*(1 + x)^(7/2))/72 + ((1 - x)^(11/2)*(1 + x)^(7/2))/
9 + (55*ArcSin[x])/128
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (1-x)^{11/2}(1+x)^{5/2}\,dx = \frac{1}{9}(1-x)^{11/2}(1+x)^{7/2} + \frac{11}{9}\int (1-x)^{9/2}(1+x)^{5/2}\,dx$$
$$= \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2} + \frac{1}{9}(1-x)^{11/2}(1+x)^{7/2} + \frac{11}{8}\int (1-x)^{7/2}(1+x)^{5/2}\,dx$$
$$= \frac{11}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2} + \frac{1}{9}(1-x)^{11/2}(1+x)^{7/2} + \frac{11}{8}\int (1-x)^{5/2}(1+$$
$$= \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2} + \frac{1}{9}(1-x)^{11/2}(1+$$
$$= \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{11}{72}(1-x)^{9/2}(1$$
$$= \frac{55}{128}\sqrt{1-x}x\sqrt{1+x} + \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x)^{7/2}(1+$$
$$= \frac{55}{128}\sqrt{1-x}x\sqrt{1+x} + \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x)^{7/2}(1+$$
$$= \frac{55}{128}\sqrt{1-x}x\sqrt{1+x} + \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x)^{7/2}(1+$$

Mathematica [A] time = 0.0706632, size = 75, normalized size = 0.58

$$\frac{\sqrt{1-x^2}\left(-896x^8+3024x^7-1024x^6-7224x^5+8448x^4+3066x^3-10240x^2+4599x+3712\right)-6930\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{8064}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(11/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(3712 + 4599*x - 10240*x^2 + 3066*x^3 + 8448*x^4 - 7224*x^5
- 1024*x^6 + 3024*x^7 - 896*x^8) - 6930*ArcSin[Sqrt[1 - x]/Sqrt[2]])/8064
```

Maple [A] time = 0.003, size = 155, normalized size = 1.2

$$\frac{1}{9}(1-x)^{\frac{11}{2}}(1+x)^{\frac{7}{2}} + \frac{11}{72}(1-x)^{\frac{9}{2}}(1+x)^{\frac{7}{2}} + \frac{11}{56}(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}} + \frac{11}{48}(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}} + \frac{11}{48}(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}} + \frac{11}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(11/2)*(1+x)^(5/2),x)
```

```
[Out] 1/9*(1-x)^(11/2)*(1+x)^(7/2)+11/72*(1-x)^(9/2)*(1+x)^(7/2)+11/56*(1-x)^(7/2
)*(1+x)^(7/2)+11/48*(1-x)^(5/2)*(1+x)^(7/2)+11/48*(1-x)^(3/2)*(1+x)^(7/2)+1
1/64*(1-x)^(1/2)*(1+x)^(7/2)-11/192*(1-x)^(1/2)*(1+x)^(5/2)-55/384*(1-x)^(1
/2)*(1+x)^(3/2)-55/128*(1-x)^(1/2)*(1+x)^(1/2)+55/128*((1+x)*(1-x))^(1/2)/(
1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.55367, size = 105, normalized size = 0.81

$$\frac{1}{9}\left(-x^2+1\right)^{\frac{7}{2}}x^2 - \frac{3}{8}\left(-x^2+1\right)^{\frac{7}{2}}x + \frac{29}{63}\left(-x^2+1\right)^{\frac{7}{2}} + \frac{11}{48}\left(-x^2+1\right)^{\frac{5}{2}}x + \frac{55}{192}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{55}{128}\sqrt{-x^2+1}x + \frac{55}{128}\,\mathrm{arcs}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(-x^2 + 1)^(7/2)*x^2 - 3/8*(-x^2 + 1)^(7/2)*x + 29/63*(-x^2 + 1)^(7/2)
+ 11/48*(-x^2 + 1)^(5/2)*x + 55/192*(-x^2 + 1)^(3/2)*x + 55/128*sqrt(-x^2 +
 1)*x + 55/128*arcsin(x)
```

Fricas [A] time = 1.5462, size = 238, normalized size = 1.83

$$-\frac{1}{8064}\left(896\,x^8-3024\,x^7+1024\,x^6+7224\,x^5-8448\,x^4-3066\,x^3+10240\,x^2-4599\,x-3712\right)\sqrt{x+1}\sqrt{-x+1}-\frac{55}{64}\,\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8064*(896*x^8 - 3024*x^7 + 1024*x^6 + 7224*x^5 - 8448*x^4 - 3066*x^3 + 1
0240*x^2 - 4599*x - 3712)*sqrt(x + 1)*sqrt(-x + 1) - 55/64*arctan((sqrt(x +
 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(11/2)*(1+x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21076, size = 409, normalized size = 3.15

$$-\frac{1}{315}\left(\left(\left(\left(5\left(\left(7\left(x+1\right)\left(x-7\right)+195\right)\left(x+1\right)-386\right)\left(x+1\right)+2369\right)\left(x+1\right)-1836\right)\left(x+1\right)+861\right)\left(x+1\right)-210\right)\left(x+1\right)^{\frac{3}{2}}\sqrt{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/315*((((5*((7*(x + 1)*(x - 7) + 195)*(x + 1) - 386)*(x + 1) + 2369)*(x +
 1) - 1836)*(x + 1) + 861)*(x + 1) - 210)*(x + 1)^(3/2)*sqrt(-x + 1) - 1/10
5*((3*((5*(x + 1)*(x - 5) + 74)*(x + 1) - 96)*(x + 1) + 203)*(x + 1) - 70)*
(x + 1)^(3/2)*sqrt(-x + 1) + 1/3*((3*(x + 1)*(x - 3) + 17)*(x + 1) - 10)*(x
 + 1)^(3/2)*sqrt(-x + 1) - (x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) + 1/128*((2*(
(4*((6*(x + 1)*(x - 6) + 125)*(x + 1) - 205)*(x + 1) + 795)*(x + 1) - 449)*
(x + 1) + 251)*(x + 1) - 15)*sqrt(x + 1)*sqrt(-x + 1) - 5/48*((2*((4*(x + 1
)*(x - 4) + 39)*(x + 1) - 37)*(x + 1) + 31)*(x + 1) - 3)*sqrt(x + 1)*sqrt(-
x + 1) + 1/8*((2*(x + 1)*(x - 2) + 5)*(x + 1) - 1)*sqrt(x + 1)*sqrt(-x + 1)
 + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + 55/64*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1089 $\int (1-x)^{9/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=110

$$\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}+\frac{9}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{3}{16}x(x+1)^{5/2}(1-x)^{5/2}+\frac{15}{64}x(x+1)^{3/2}(1-x)^{3/2}+\frac{45}{128}x\sqrt{x+1}\sqrt{1-x}$$

```
[Out] (45*Sqrt[1 - x]*x*Sqrt[1 + x])/128 + (15*(1 - x)^(3/2)*x*(1 + x)^(3/2))/64
+ (3*(1 - x)^(5/2)*x*(1 + x)^(5/2))/16 + (9*(1 - x)^(7/2)*(1 + x)^(7/2))/56
 + ((1 - x)^(9/2)*(1 + x)^(7/2))/8 + (45*ArcSin[x])/128
```

Rubi [A] time = 0.0191855, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}+\frac{9}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{3}{16}x(x+1)^{5/2}(1-x)^{5/2}+\frac{15}{64}x(x+1)^{3/2}(1-x)^{3/2}+\frac{45}{128}x\sqrt{x+1}\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*(1 + x)^(5/2),x]
```

```
[Out] (45*Sqrt[1 - x]*x*Sqrt[1 + x])/128 + (15*(1 - x)^(3/2)*x*(1 + x)^(3/2))/64
+ (3*(1 - x)^(5/2)*x*(1 + x)^(5/2))/16 + (9*(1 - x)^(7/2)*(1 + x)^(7/2))/56
 + ((1 - x)^(9/2)*(1 + x)^(7/2))/8 + (45*ArcSin[x])/128
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{9/2}(1+x)^{5/2}\,dx &= \frac{1}{8}(1-x)^{9/2}(1+x)^{7/2} + \frac{9}{8}\int (1-x)^{7/2}(1+x)^{5/2}\,dx \\ &= \frac{9}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{1}{8}(1-x)^{9/2}(1+x)^{7/2} + \frac{9}{8}\int (1-x)^{5/2}(1+x)^{5/2}\,dx \\ &= \frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2} + \frac{9}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{1}{8}(1-x)^{9/2}(1+x)^{7/2} + \frac{15}{16}\int (1-x)^{3/2}(1+x)^{3/2} \\ &= \frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2} + \frac{9}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{1}{8}(1-x)^{9/2}(1+x)^{7/2} \\ &= \frac{45}{128}\sqrt{1-x}x\sqrt{1+x} + \frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2} + \frac{9}{56}(1-x)^{7/2}(1+x)^{7/2} \\ &= \frac{45}{128}\sqrt{1-x}x\sqrt{1+x} + \frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2} + \frac{9}{56}(1-x)^{7/2}(1+x)^{7/2} \\ &= \frac{45}{128}\sqrt{1-x}x\sqrt{1+x} + \frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2} + \frac{9}{56}(1-x)^{7/2}(1+x)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0657086, size = 70, normalized size = 0.64

$$\frac{1}{896}\left(\sqrt{1-x^2}\left(112x^7-256x^6-168x^5+768x^4-210x^3-768x^2+581x+256\right)-630\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(256 + 581*x - 768*x^2 - 210*x^3 + 768*x^4 - 168*x^5 - 256*x
^6 + 112*x^7) - 630*ArcSin[Sqrt[1 - x]/Sqrt[2]])/896
```

Maple [A] time = 0.004, size = 141, normalized size = 1.3

$$\frac{1}{8}(1-x)^{\frac{9}{2}}(1+x)^{\frac{7}{2}} + \frac{9}{56}(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}} + \frac{3}{16}(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}} + \frac{3}{16}(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}} + \frac{9}{64}\sqrt{1-x}(1+x)^{\frac{7}{2}} - \frac{3}{64}\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(9/2)*(1+x)^(5/2),x)
```

```
[Out] 1/8*(1-x)^(9/2)*(1+x)^(7/2)+9/56*(1-x)^(7/2)*(1+x)^(7/2)+3/16*(1-x)^(5/2)*(
1+x)^(7/2)+3/16*(1-x)^(3/2)*(1+x)^(7/2)+9/64*(1-x)^(1/2)*(1+x)^(7/2)-3/64*(
1-x)^(1/2)*(1+x)^(5/2)-15/128*(1-x)^(1/2)*(1+x)^(3/2)-45/128*(1-x)^(1/2)*(1
+x)^(1/2)+45/128*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.52654, size = 86, normalized size = 0.78

$$-\frac{1}{8}\left(-x^2+1\right)^{\frac{7}{2}}x + \frac{2}{7}\left(-x^2+1\right)^{\frac{7}{2}} + \frac{3}{16}\left(-x^2+1\right)^{\frac{5}{2}}x + \frac{15}{64}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{45}{128}\sqrt{-x^2+1}x + \frac{45}{128}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(-x^2 + 1)^(7/2)*x + 2/7*(-x^2 + 1)^(7/2) + 3/16*(-x^2 + 1)^(5/2)*x +
15/64*(-x^2 + 1)^(3/2)*x + 45/128*sqrt(-x^2 + 1)*x + 45/128*arcsin(x)
```

Fricas [A] time = 1.62988, size = 209, normalized size = 1.9

$$\frac{1}{896}\left(112\,x^{7}-256\,x^{6}-168\,x^{5}+768\,x^{4}-210\,x^{3}-768\,x^{2}+581\,x+256\right)\sqrt{x+1}\sqrt{-x+1}-\frac{45}{64}\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/896*(112*x^7 - 256*x^6 - 168*x^5 + 768*x^4 - 210*x^3 - 768*x^2 + 581*x +
256)*sqrt(x + 1)*sqrt(-x + 1) - 45/64*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)
/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14905, size = 335, normalized size = 3.05

$$-\frac{2}{105}\left(\left(3\left(\left(5\left(x+1\right)\left(x-5\right)+74\right)\left(x+1\right)-96\right)\left(x+1\right)+203\right)\left(x+1\right)-70\right)\left(x+1\right)^{\frac{3}{2}}\sqrt{-x+1}+\frac{4}{15}\left(\left(3\left(x+1\right)\left(x-3\right)+17\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/105*((3*((5*(x + 1)*(x - 5) + 74)*(x + 1) - 96)*(x + 1) + 203)*(x + 1) -
 70)*(x + 1)^(3/2)*sqrt(-x + 1) + 4/15*((3*(x + 1)*(x - 3) + 17)*(x + 1) -
10)*(x + 1)^(3/2)*sqrt(-x + 1) - 2/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) + 1
/384*((2*((4*((6*(x + 1)*(x - 6) + 125)*(x + 1) - 205)*(x + 1) + 795)*(x +
1) - 449)*(x + 1) + 251)*(x + 1) - 15)*sqrt(x + 1)*sqrt(-x + 1) - 1/48*((2*
((4*(x + 1)*(x - 4) + 39)*(x + 1) - 37)*(x + 1) + 31)*(x + 1) - 3)*sqrt(x +
 1)*sqrt(-x + 1) - 1/8*((2*(x + 1)*(x - 2) + 5)*(x + 1) - 1)*sqrt(x + 1)*sq
rt(-x + 1) + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + 45/64*arcsin(1/2*sqrt(2)*sqrt
(x + 1))
```

3.1090 $\int (1-x)^{7/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=90

$$\frac{1}{7}(1-x)^{7/2}(x+1)^{7/2}+\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (5*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
(1 - x)^(5/2)*x*(1 + x)^(5/2))/6 + ((1 - x)^(7/2)*(1 + x)^(7/2))/7 + (5*Arc
Sin[x])/16
```

Rubi [A] time = 0.0119287, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{7}(1-x)^{7/2}(x+1)^{7/2}+\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (5*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
(1 - x)^(5/2)*x*(1 + x)^(5/2))/6 + ((1 - x)^(7/2)*(1 + x)^(7/2))/7 + (5*Arc
Sin[x])/16
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{7/2}(1+x)^{5/2}\,dx &= \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2} + \int (1-x)^{5/2}(1+x)^{5/2}\,dx \\ &= \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2} + \frac{5}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx \\ &= \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2} + \frac{5}{8}\int \sqrt{1-x}\sqrt{1+x} \\ &= \frac{5}{16}\sqrt{1-x}x\sqrt{1+x} + \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2} \\ &= \frac{5}{16}\sqrt{1-x}x\sqrt{1+x} + \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2} \\ &= \frac{5}{16}\sqrt{1-x}x\sqrt{1+x} + \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0630693, size = 66, normalized size = 0.73

$$\frac{1}{336}\sqrt{1-x^2}\left(-48x^6+56x^5+144x^4-182x^3-144x^2+231x+48\right)-\frac{5}{8}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(48 + 231*x - 144*x^2 - 182*x^3 + 144*x^4 + 56*x^5 - 48*x^6)
)/336 - (5*ArcSin[Sqrt[1 - x]/Sqrt[2]])/8
```

Maple [A] time = 0.003, size = 127, normalized size = 1.4

$$\frac{1}{7}(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}}+\frac{1}{6}(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}+\frac{1}{6}(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}+\frac{1}{8}\sqrt{1-x}(1+x)^{\frac{7}{2}}-\frac{1}{24}\sqrt{1-x}(1+x)^{\frac{5}{2}}-\frac{5}{48}\sqrt{1-x}(1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)*(1+x)^(5/2),x)
```

```
[Out] 1/7*(1-x)^(7/2)*(1+x)^(7/2)+1/6*(1-x)^(5/2)*(1+x)^(7/2)+1/6*(1-x)^(3/2)*(1+
x)^(7/2)+1/8*(1-x)^(1/2)*(1+x)^(7/2)-1/24*(1-x)^(1/2)*(1+x)^(5/2)-5/48*(1-x
)^(1/2)*(1+x)^(3/2)-5/16*(1-x)^(1/2)*(1+x)^(1/2)+5/16*((1+x)*(1-x))^(1/2)/(
1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.52674, size = 70, normalized size = 0.78

$$\frac{1}{7}\left(-x^2+1\right)^{\frac{7}{2}}+\frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{5}{24}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{5}{16}\sqrt{-x^2+1}x+\frac{5}{16}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/7*(-x^2 + 1)^(7/2) + 1/6*(-x^2 + 1)^(5/2)*x + 5/24*(-x^2 + 1)^(3/2)*x + 5
/16*sqrt(-x^2 + 1)*x + 5/16*arcsin(x)
```

Fricas [A] time = 1.56064, size = 190, normalized size = 2.11

$$-\frac{1}{336}\left(48x^6-56x^5-144x^4+182x^3+144x^2-231x-48\right)\sqrt{x+1}\sqrt{-x+1}-\frac{5}{8}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/336*(48*x^6 - 56*x^5 - 144*x^4 + 182*x^3 + 144*x^2 - 231*x - 48)*sqrt(x
+ 1)*sqrt(-x + 1) - 5/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)*(1+x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17025, size = 259, normalized size = 2.88

$$-\frac{1}{105}((3((5(x+1)(x-5)+74)(x+1)-96)(x+1)+203)(x+1)-70)(x+1)^{\frac{3}{2}}\sqrt{-x+1}+\frac{2}{15}((3(x+1)(x-3)+17)(x-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/105*((3*((5*(x + 1)*(x - 5) + 74)*(x + 1) - 96)*(x + 1) + 203)*(x + 1) -
 70)*(x + 1)^(3/2)*sqrt(-x + 1) + 2/15*((3*(x + 1)*(x - 3) + 17)*(x + 1) -
10)*(x + 1)^(3/2)*sqrt(-x + 1) - 1/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) + 1
/48*((2*((4*(x + 1)*(x - 4) + 39)*(x + 1) - 37)*(x + 1) + 31)*(x + 1) - 3)*
sqrt(x + 1)*sqrt(-x + 1) - 1/4*((2*(x + 1)*(x - 2) + 5)*(x + 1) - 1)*sqrt(x
 + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + 5/8*arcsin(1/2*sqrt(2
)*sqrt(x + 1))
```

3.1091 $\int (1-x)^{5/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (5*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
(1 - x)^(5/2)*x*(1 + x)^(5/2))/6 + (5*ArcSin[x])/16
```

Rubi [A] time = 0.0107479, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {38, 41, 216}

$$\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (5*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
(1 - x)^(5/2)*x*(1 + x)^(5/2))/6 + (5*ArcSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{5/2}(1+x)^{5/2}\,dx &= \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx\\
&= \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{8}\int \sqrt{1-x}\sqrt{1+x}\,dx\\
&= \frac{5}{16}\sqrt{1-x}x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\\
&= \frac{5}{16}\sqrt{1-x}x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{5}{16}\sqrt{1-x}x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0153676, size = 34, normalized size = 0.49

$$\frac{1}{48}\left(x\sqrt{1-x^2}\left(8x^4-26x^2+33\right)+15\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*(1 + x)^(5/2),x]
```

```
[Out] (x*Sqrt[1 - x^2]*(33 - 26*x^2 + 8*x^4) + 15*ArcSin[x])/48
```

Maple [B] time = 0.003, size = 113, normalized size = 1.6

$$\frac{1}{6}(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}+\frac{1}{6}(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}+\frac{1}{8}\sqrt{1-x}\,(1+x)^{\frac{7}{2}}-\frac{1}{24}\sqrt{1-x}\,(1+x)^{\frac{5}{2}}-\frac{5}{48}\sqrt{1-x}\,(1+x)^{\frac{3}{2}}-\frac{5}{16}\sqrt{1-x}\sqrt{1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)*(1+x)^(5/2),x)
```

```
[Out] 1/6*(1-x)^(5/2)*(1+x)^(7/2)+1/6*(1-x)^(3/2)*(1+x)^(7/2)+1/8*(1-x)^(1/2)*(1+
x)^(7/2)-1/24*(1-x)^(1/2)*(1+x)^(5/2)-5/48*(1-x)^(1/2)*(1+x)^(3/2)-5/16*(1-
x)^(1/2)*(1+x)^(1/2)+5/16*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsi
n(x)
```

Maxima [A] time = 1.4925, size = 55, normalized size = 0.79

$$\frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{5}{24}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{5}{16}\sqrt{-x^2+1}x+\frac{5}{16}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-x^2 + 1)^(5/2)*x + 5/24*(-x^2 + 1)^(3/2)*x + 5/16*sqrt(-x^2 + 1)*x +
5/16*arcsin(x)
```

Fricas [A] time = 1.50185, size = 138, normalized size = 1.97

$$\frac{1}{48}\left(8x^5-26x^3+33x\right)\sqrt{x+1}\sqrt{-x+1}-\frac{5}{8}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^5 - 26*x^3 + 33*x)*sqrt(x + 1)*sqrt(-x + 1) - 5/8*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [B] time = 79.3469, size = 286, normalized size = 4.09

$$\begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} + \frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}} - \frac{7i(x+1)^{\frac{11}{2}}}{6\sqrt{x-1}} + \frac{67i(x+1)^{\frac{9}{2}}}{24\sqrt{x-1}} - \frac{55i(x+1)^{\frac{7}{2}}}{24\sqrt{x-1}} - \frac{i(x+1)^{\frac{5}{2}}}{48\sqrt{x-1}} - \frac{5i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}} + \frac{5i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} - \frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}} + \frac{7(x+1)^{\frac{11}{2}}}{6\sqrt{1-x}} - \frac{67(x+1)^{\frac{9}{2}}}{24\sqrt{1-x}} + \frac{55(x+1)^{\frac{7}{2}}}{24\sqrt{1-x}} + \frac{(x+1)^{\frac{5}{2}}}{48\sqrt{1-x}} + \frac{5(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}} - \frac{5\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 + I*(x + 1)**(13/2)/(6*sqrt(
x - 1)) - 7*I*(x + 1)**(11/2)/(6*sqrt(x - 1)) + 67*I*(x + 1)**(9/2)/(24*sqr
t(x - 1)) - 55*I*(x + 1)**(7/2)/(24*sqrt(x - 1)) - I*(x + 1)**(5/2)/(48*sqr
t(x - 1)) - 5*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(8*sqrt(x
 - 1)), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sqrt(x + 1)/2)/8 - (x + 1)**(13/
2)/(6*sqrt(1 - x)) + 7*(x + 1)**(11/2)/(6*sqrt(1 - x)) - 67*(x + 1)**(9/2)/
(24*sqrt(1 - x)) + 55*(x + 1)**(7/2)/(24*sqrt(1 - x)) + (x + 1)**(5/2)/(48*
sqrt(1 - x)) + 5*(x + 1)**(3/2)/(48*sqrt(1 - x)) - 5*sqrt(x + 1)/(8*sqrt(1
- x)), True))
```

Giac [B] time = 1.12763, size = 138, normalized size = 1.97

$$\frac{1}{48}\left(\left(2\left(\left(4\left(x+1\right)\left(x-4\right)+39\right)\left(x+1\right)-37\right)\left(x+1\right)+31\right)\left(x+1\right)-3\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{4}\left(\left(2\left(x+1\right)\left(x-2\right)+5\right)\left(x+1\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*((4*(x + 1)*(x - 4) + 39)*(x + 1) - 37)*(x + 1) + 31)*(x + 1) - 3)
*sqrt(x + 1)*sqrt(-x + 1) - 1/4*((2*(x + 1)*(x - 2) + 5)*(x + 1) - 1)*sqrt(
x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + 5/8*arcsin(1/2*sqrt(
2)*sqrt(x + 1))
```

3.1092 $\int (1-x)^{3/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 - ((1 -
 x)^(5/2)*(1 + x)^(5/2))/5 + (3*ArcSin[x])/8
```

Rubi [A] time = 0.0080006, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$-\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 - ((1 -
 x)^(5/2)*(1 + x)^(5/2))/5 + (3*ArcSin[x])/8
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{3/2}(1+x)^{5/2}\,dx &= -\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \int (1-x)^{3/2}(1+x)^{3/2}\,dx \\
&= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} - \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{4}\int \sqrt{1-x}\sqrt{1+x}\,dx \\
&= \frac{3}{8}\sqrt{1-x}x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} - \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{8}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx \\
&= \frac{3}{8}\sqrt{1-x}x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} - \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{3}{8}\sqrt{1-x}x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} - \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{8}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0460824, size = 55, normalized size = 0.8

$$
\frac{1}{40}\left(\sqrt{1-x^2}\left(-8x^4-10x^3+16x^2+25x-8\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(-8 + 25*x + 16*x^2 - 10*x^3 - 8*x^4) - 30*ArcSin[Sqrt[1 - x
]/Sqrt[2]])/40
```

Maple [A] time = 0.005, size = 99, normalized size = 1.4

$$
\frac{1}{5}(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}} + \frac{3}{20}\sqrt{1-x}(1+x)^{\frac{7}{2}} - \frac{1}{20}\sqrt{1-x}(1+x)^{\frac{5}{2}} - \frac{1}{8}\sqrt{1-x}(1+x)^{\frac{3}{2}} - \frac{3}{8}\sqrt{1-x}\sqrt{1+x} + \frac{3\arcsin(x)}{8}\sqrt{(
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)*(1+x)^(5/2),x)
```

```
[Out] 1/5*(1-x)^(3/2)*(1+x)^(7/2)+3/20*(1-x)^(1/2)*(1+x)^(7/2)-1/20*(1-x)^(1/2)*(
1+x)^(5/2)-1/8*(1-x)^(1/2)*(1+x)^(3/2)-3/8*(1-x)^(1/2)*(1+x)^(1/2)+3/8*((1+
x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.49357, size = 54, normalized size = 0.78

$$
-\frac{1}{5}\left(-x^2+1\right)^{\frac{5}{2}} + \frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{3}{8}\sqrt{-x^2+1}x + \frac{3}{8}\arcsin(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 1)^(5/2) + 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8
*arcsin(x)
```

Fricas [A] time = 1.47241, size = 157, normalized size = 2.28

$$-\frac{1}{40}\left(8x^4+10x^3-16x^2-25x+8\right)\sqrt{x+1}\sqrt{-x+1}-\frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/40*(8*x^4 + 10*x^3 - 16*x^2 - 25*x + 8)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*a
rctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [B] time = 38.5431, size = 246, normalized size = 3.57

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}}+\frac{19i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}}-\frac{23i(x+1)^{\frac{7}{2}}}{20\sqrt{x-1}}-\frac{i(x+1)^{\frac{5}{2}}}{40\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}+\frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}}-\frac{19(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}}+\frac{23(x+1)^{\frac{7}{2}}}{20\sqrt{1-x}}+\frac{(x+1)^{\frac{5}{2}}}{40\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}-\frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) + 19*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) - 23*I*(x + 1)**(7/2)/(20*sq
rt(x - 1)) - I*(x + 1)**(5/2)/(40*sqrt(x - 1)) - I*(x + 1)**(3/2)/(8*sqrt(x
 - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1)/2 > 1), (3*asin(sqrt(2
)*sqrt(x + 1)/2)/4 + (x + 1)**(11/2)/(5*sqrt(1 - x)) - 19*(x + 1)**(9/2)/(2
0*sqrt(1 - x)) + 23*(x + 1)**(7/2)/(20*sqrt(1 - x)) + (x + 1)**(5/2)/(40*sq
rt(1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1 - x))
, True))
```

Giac [B] time = 1.09612, size = 143, normalized size = 2.07

$$-\frac{1}{15}\left((3(x+1)(x-3)+17)(x+1)-10\right)(x+1)^{\frac{3}{2}}\sqrt{-x+1}+\frac{1}{3}(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1}-\frac{1}{8}\left((2(x+1)(x-2)+5)(x+1)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/15*((3*(x + 1)*(x - 3) + 17)*(x + 1) - 10)*(x + 1)^(3/2)*sqrt(-x + 1) +
1/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) - 1/8*((2*(x + 1)*(x - 2) + 5)*(x +
1) - 1)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + 3/4*arc
sin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1093 $\int \sqrt{1-x}(1+x)^{5/2}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{4}(1-x)^{3/2}(x+1)^{5/2}-\frac{5}{12}(1-x)^{3/2}(x+1)^{3/2}+\frac{5}{8}\sqrt{1-x}x\sqrt{x+1}+\frac{5}{8}\sin^{-1}(x)$$

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/8 - (5*(1 - x)^(3/2)*(1 + x)^(3/2))/12 - ((1
- x)^(3/2)*(1 + x)^(5/2))/4 + (5*ArcSin[x])/8
```

Rubi [A] time = 0.0091997, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$-\frac{1}{4}(1-x)^{3/2}(x+1)^{5/2}-\frac{5}{12}(1-x)^{3/2}(x+1)^{3/2}+\frac{5}{8}\sqrt{1-x}x\sqrt{x+1}+\frac{5}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/8 - (5*(1 - x)^(3/2)*(1 + x)^(3/2))/12 - ((1
- x)^(3/2)*(1 + x)^(5/2))/4 + (5*ArcSin[x])/8
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{1-x}(1+x)^{5/2}\,dx &= -\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2} + \frac{5}{4}\int \sqrt{1-x}(1+x)^{3/2}\,dx \\
&= -\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2} - \frac{1}{4}(1-x)^{3/2}(1+x)^{5/2} + \frac{5}{4}\int \sqrt{1-x}\sqrt{1+x}\,dx \\
&= \frac{5}{8}\sqrt{1-x}x\sqrt{1+x} - \frac{5}{12}(1-x)^{3/2}(1+x)^{3/2} - \frac{1}{4}(1-x)^{3/2}(1+x)^{5/2} + \frac{5}{8}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx \\
&= \frac{5}{8}\sqrt{1-x}x\sqrt{1+x} - \frac{5}{12}(1-x)^{3/2}(1+x)^{3/2} - \frac{1}{4}(1-x)^{3/2}(1+x)^{5/2} + \frac{5}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{5}{8}\sqrt{1-x}x\sqrt{1+x} - \frac{5}{12}(1-x)^{3/2}(1+x)^{3/2} - \frac{1}{4}(1-x)^{3/2}(1+x)^{5/2} + \frac{5}{8}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0383331, size = 50, normalized size = 0.74

$$
\frac{1}{24}\left(\sqrt{1-x^2}\left(6x^3+16x^2+9x-16\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(-16 + 9*x + 16*x^2 + 6*x^3) - 30*ArcSin[Sqrt[1 - x]/Sqrt[2]
])/24
```

Maple [A] time = 0.003, size = 85, normalized size = 1.3

$$
\frac{1}{4}\sqrt{1-x}\,(1+x)^{\frac{7}{2}}-\frac{1}{12}\sqrt{1-x}\,(1+x)^{\frac{5}{2}}-\frac{5}{24}\sqrt{1-x}\,(1+x)^{\frac{3}{2}}-\frac{5}{8}\sqrt{1-x}\sqrt{1+x}+\frac{5\arcsin(x)}{8}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)*(1+x)^(5/2),x)
```

```
[Out] 1/4*(1-x)^(1/2)*(1+x)^(7/2)-1/12*(1-x)^(1/2)*(1+x)^(5/2)-5/24*(1-x)^(1/2)*(
1+x)^(3/2)-5/8*(1-x)^(1/2)*(1+x)^(1/2)+5/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/
(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.51081, size = 54, normalized size = 0.79

$$
-\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x-\frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{-x^2+1}x+\frac{5}{8}\arcsin(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)*x - 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/8
*arcsin(x)
```

Fricas [A] time = 1.77627, size = 143, normalized size = 2.1

$$\frac{1}{24}\left(6x^3+16x^2+9x-16\right)\sqrt{x+1}\sqrt{-x+1}-\frac{5}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 + 16*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(-x + 1) - 5/4*arctan((sqr
t(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [A] time = 17.1041, size = 214, normalized size = 3.15

$$\begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}}-\frac{7i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}}-\frac{i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}}-\frac{5i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}}+\frac{5i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}}+\frac{7(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}}+\frac{(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}}+\frac{5(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}}-\frac{5\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) - 7*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) - I*(x + 1)**(5/2)/(24*sqrt(x
- 1)) - 5*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(4*sqrt(x - 1
)), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(9/2)/(4
*sqrt(1 - x)) + 7*(x + 1)**(7/2)/(12*sqrt(1 - x)) + (x + 1)**(5/2)/(24*sqrt
(1 - x)) + 5*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 5*sqrt(x + 1)/(4*sqrt(1 - x)
), True))
```

Giac [A] time = 1.10434, size = 103, normalized size = 1.51

$$\frac{2}{3}(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1}+\frac{1}{8}\left((2(x+1)(x-2)+5)(x+1)-1\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\sqrt{x+1}x\sqrt{-x+1}+\frac{5}{4}\arcsin\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) + 1/8*((2*(x + 1)*(x - 2) + 5)*(x +
1) - 1)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + 5/4*arc
sin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1094 $\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{3}\sqrt{1-x}(x+1)^{5/2}-\frac{5}{6}\sqrt{1-x}(x+1)^{3/2}-\frac{5}{2}\sqrt{1-x}\sqrt{x+1}+\frac{5}{2}\sin^{-1}(x)$$

```
[Out] (-5*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*Sqrt[1 - x]*(1 + x)^(3/2))/6 - (Sqrt[1
 - x]*(1 + x)^(5/2))/3 + (5*ArcSin[x])/2
```

Rubi [A] time = 0.0103963, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$-\frac{1}{3}\sqrt{1-x}(x+1)^{5/2}-\frac{5}{6}\sqrt{1-x}(x+1)^{3/2}-\frac{5}{2}\sqrt{1-x}\sqrt{x+1}+\frac{5}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/Sqrt[1 - x],x]
```

```
[Out] (-5*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*Sqrt[1 - x]*(1 + x)^(3/2))/6 - (Sqrt[1
 - x]*(1 + x)^(5/2))/3 + (5*ArcSin[x])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx = -\frac{1}{3}\sqrt{1-x}(1+x)^{5/2} + \frac{5}{3}\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx$$
$$= -\frac{5}{6}\sqrt{1-x}(1+x)^{3/2} - \frac{1}{3}\sqrt{1-x}(1+x)^{5/2} + \frac{5}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx$$
$$= -\frac{5}{2}\sqrt{1-x}\sqrt{1+x} - \frac{5}{6}\sqrt{1-x}(1+x)^{3/2} - \frac{1}{3}\sqrt{1-x}(1+x)^{5/2} + \frac{5}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$$
$$= -\frac{5}{2}\sqrt{1-x}\sqrt{1+x} - \frac{5}{6}\sqrt{1-x}(1+x)^{3/2} - \frac{1}{3}\sqrt{1-x}(1+x)^{5/2} + \frac{5}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{5}{2}\sqrt{1-x}\sqrt{1+x} - \frac{5}{6}\sqrt{1-x}(1+x)^{3/2} - \frac{1}{3}\sqrt{1-x}(1+x)^{5/2} + \frac{5}{2}\sin^{-1}(x)$$

Mathematica [A] time = 0.0220195, size = 44, normalized size = 0.66

$$-\frac{1}{6}\sqrt{1-x^2}\left(2x^2+9x+22\right) - 5\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/Sqrt[1 - x],x]
```

```
[Out] -(Sqrt[1 - x^2]*(22 + 9*x + 2*x^2))/6 - 5*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

Maple [A] time = 0.004, size = 71, normalized size = 1.1

$$-\frac{1}{3}\sqrt{1-x}\,(1+x)^{\frac{5}{2}} - \frac{5}{6}\sqrt{1-x}\,(1+x)^{\frac{3}{2}} - \frac{5}{2}\sqrt{1-x}\sqrt{1+x} + \frac{5\arcsin(x)}{2}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(1/2),x)
```

```
[Out] -1/3*(1-x)^(1/2)*(1+x)^(5/2)-5/6*(1-x)^(1/2)*(1+x)^(3/2)-5/2*(1-x)^(1/2)*(1
+x)^(1/2)+5/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.55981, size = 57, normalized size = 0.85

$$-\frac{1}{3}\sqrt{-x^2+1}x^2 - \frac{3}{2}\sqrt{-x^2+1}x - \frac{11}{3}\sqrt{-x^2+1} + \frac{5}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x - 11/3*sqrt(-x^2 + 1) + 5/2*
arcsin(x)
```

Fricas [A] time = 1.63123, size = 128, normalized size = 1.91

$$-\frac{1}{6}\left(2x^2+9x+22\right)\sqrt{x+1}\sqrt{-x+1}-5\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*x^2 + 9*x + 22)*sqrt(x + 1)*sqrt(-x + 1) - 5*arctan((sqrt(x + 1)*sq
rt(-x + 1) - 1)/x)
```

Sympy [A] time = 11.5651, size = 172, normalized size = 2.57

$$\begin{cases} -5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}-\frac{i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}-\frac{5i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{5i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ 5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{5(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{5\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) - I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 5*I*(x + 1)**(3/2)/(6*sqrt(x - 1)
) + 5*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sqrt(x
+ 1)/2) + (x + 1)**(7/2)/(3*sqrt(1 - x)) + (x + 1)**(5/2)/(6*sqrt(1 - x)) +
 5*(x + 1)**(3/2)/(6*sqrt(1 - x)) - 5*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A] time = 1.0604, size = 53, normalized size = 0.79

$$-\frac{1}{6}\left((2x+7)(x+1)+15\right)\sqrt{x+1}\sqrt{-x+1}+5\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*((2*x + 7)*(x + 1) + 15)*sqrt(x + 1)*sqrt(-x + 1) + 5*arcsin(1/2*sqrt(
2)*sqrt(x + 1))
```

3.1095 $\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}} dx$

Optimal. Leaf size=65

$$\frac{2(x+1)^{5/2}}{\sqrt{1-x}}+\frac{5}{2}\sqrt{1-x}(x+1)^{3/2}+\frac{15}{2}\sqrt{1-x}\sqrt{x+1}-\frac{15}{2}\sin^{-1}(x)$$

```
[Out] (15*Sqrt[1 - x]*Sqrt[1 + x])/2 + (5*Sqrt[1 - x]*(1 + x)^(3/2))/2 + (2*(1 +
x)^(5/2))/Sqrt[1 - x] - (15*ArcSin[x])/2
```

Rubi [A] time = 0.0103244, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$\frac{2(x+1)^{5/2}}{\sqrt{1-x}}+\frac{5}{2}\sqrt{1-x}(x+1)^{3/2}+\frac{15}{2}\sqrt{1-x}\sqrt{x+1}-\frac{15}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(3/2),x]
```

```
[Out] (15*Sqrt[1 - x]*Sqrt[1 + x])/2 + (5*Sqrt[1 - x]*(1 + x)^(3/2))/2 + (2*(1 +
x)^(5/2))/Sqrt[1 - x] - (15*ArcSin[x])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx &= \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - 5\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx \\
&= \frac{5}{2}\sqrt{1-x}(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\
&= \frac{15}{2}\sqrt{1-x}\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx \\
&= \frac{15}{2}\sqrt{1-x}\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{15}{2}\sqrt{1-x}\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [C] time = 0.0065163, size = 35, normalized size = 0.54

$$
\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{1-x}{2}\right)}{\sqrt{1-x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(3/2),x]
```

```
[Out] (8*Sqrt[2]*Hypergeometric2F1[-5/2, -1/2, 1/2, (1 - x)/2])/Sqrt[1 - x]
```

Maple [A] time = 0.014, size = 77, normalized size = 1.2

$$
-\frac{x^3+8x^2-17x-24}{2}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{-(1+x)(-1+x)}}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}-\frac{15\arcsin(x)}{2}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(3/2),x)
```

```
[Out] -1/2*(x^3+8*x^2-17*x-24)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1
/2)/(1+x)^(1/2)-15/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.49699, size = 76, normalized size = 1.17

$$
-\frac{x^3}{2\sqrt{-x^2+1}}-\frac{4x^2}{\sqrt{-x^2+1}}+\frac{17x}{2\sqrt{-x^2+1}}+\frac{12}{\sqrt{-x^2+1}}-\frac{15}{2}\arcsin(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^3/sqrt(-x^2 + 1) - 4*x^2/sqrt(-x^2 + 1) + 17/2*x/sqrt(-x^2 + 1) + 12
/sqrt(-x^2 + 1) - 15/2*arcsin(x)
```

Fricas [A] time = 1.56274, size = 166, normalized size = 2.55

$$\frac{\left(x^2 + 7\,x - 24\right)\sqrt{x+1}\sqrt{-x+1} + 30\,(x-1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + 24\,x - 24}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 7*x - 24)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x - 1)*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x) + 24*x - 24)/(x - 1)
```

Sympy [A] time = 14.3842, size = 139, normalized size = 2.14

$$\begin{cases} 15i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} + \frac{5i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} - \frac{15i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ -15\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} - \frac{5(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} + \frac{15\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((15*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) + 5*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) - 15*I*sqrt(x + 1)/sqrt(x - 1), A
bs(x + 1)/2 > 1), (-15*asin(sqrt(2)*sqrt(x + 1)/2) - (x + 1)**(5/2)/(2*sqrt
(1 - x)) - 5*(x + 1)**(3/2)/(2*sqrt(1 - x)) + 15*sqrt(x + 1)/sqrt(1 - x), T
rue))
```

Giac [A] time = 1.08772, size = 57, normalized size = 0.88

$$\frac{((x+6)(x+1)-30)\sqrt{x+1}\sqrt{-x+1}}{2\,(x-1)} - 15\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((x + 6)*(x + 1) - 30)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 15*arcsin(1/2
*sqrt(2)*sqrt(x + 1))
```

3.1096 $\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}} \, dx$

Optimal. Leaf size=63

$$\frac{2(x+1)^{5/2}}{3(1-x)^{3/2}}-\frac{10(x+1)^{3/2}}{3\sqrt{1-x}}-5\sqrt{1-x}\sqrt{x+1}+5\sin^{-1}(x)$$

```
[Out] -5*Sqrt[1 - x]*Sqrt[1 + x] - (10*(1 + x)^(3/2))/(3*Sqrt[1 - x]) + (2*(1 + x
)^(5/2))/(3*(1 - x)^(3/2)) + 5*ArcSin[x]
```

Rubi [A] time = 0.0101658, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$\frac{2(x+1)^{5/2}}{3(1-x)^{3/2}}-\frac{10(x+1)^{3/2}}{3\sqrt{1-x}}-5\sqrt{1-x}\sqrt{x+1}+5\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(5/2),x]
```

```
[Out] -5*Sqrt[1 - x]*Sqrt[1 + x] - (10*(1 + x)^(3/2))/(3*Sqrt[1 - x]) + (2*(1 + x
)^(5/2))/(3*(1 - x)^(3/2)) + 5*ArcSin[x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}}\,dx = \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} - \frac{5}{3}\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx$$
$$= -\frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx$$
$$= -5\sqrt{1-x}\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$$
$$= -5\sqrt{1-x}\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -5\sqrt{1-x}\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\sin^{-1}(x)$$

Mathematica [C] time = 0.0080356, size = 37, normalized size = 0.59

$$\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{1-x}{2}\right)}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(5/2),x]
```

```
[Out] (8*Sqrt[2]*Hypergeometric2F1[-5/2, -3/2, -1/2, (1 - x)/2])/(3*(1 - x)^(3/2)
)
```

Maple [A] time = 0.017, size = 84, normalized size = 1.3

$$\frac{3x^3-31x^2-11x+23}{-3+3x}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{-(1+x)(-1+x)}}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}+5\frac{\sqrt{(1+x)(1-x)}\arcsin(x)}{\sqrt{1-x}\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(5/2),x)
```

```
[Out] 1/3*(3*x^3-31*x^2-11*x+23)/(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)
/(1-x)^(1/2)/(1+x)^(1/2)+5*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcs
in(x)
```

Maxima [B] time = 1.4886, size = 134, normalized size = 2.13

$$-\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{x^4-4x^3+6x^2-4x+1}-\frac{5\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^3-3x^2+3x-1\right)}+\frac{10\sqrt{-x^2+1}}{3\left(x^2-2x+1\right)}+\frac{35\sqrt{-x^2+1}}{3(x-1)}+5\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] -(-x^2 + 1)^(5/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 5/3*(-x^2 + 1)^(3/2)/(x
^3 - 3*x^2 + 3*x - 1) + 10/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 35/3*sqrt(-x^
```

```
2 + 1)/(x - 1) + 5*arcsin(x)
```

Fricas [A] time = 1.579, size = 205, normalized size = 3.25

$$-\frac{23\,x^2+\left(3\,x^2-34\,x+23\right)\sqrt{x+1}\sqrt{-x+1}+30\,\left(x^2-2\,x+1\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-46\,x+23}{3\,\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(23*x^2 + (3*x^2 - 34*x + 23)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x^2 - 2*x
 + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 46*x + 23)/(x^2 - 2*x + 1)
```

Sympy [B] time = 14.2402, size = 576, normalized size = 9.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(5/2),x)
```

```
[Out] Piecewise((-30*I*sqrt(x - 1)*(x + 1)**(27/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(
3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(x + 1)**(25/2)) + 15*pi*sqrt
(x - 1)*(x + 1)**(27/2)/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(x +
 1)**(25/2)) + 60*I*sqrt(x - 1)*(x + 1)**(25/2)*acosh(sqrt(2)*sqrt(x + 1)/2
)/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(x + 1)**(25/2)) - 30*pi*s
qrt(x - 1)*(x + 1)**(25/2)/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(
x + 1)**(25/2)) - 3*I*(x + 1)**15/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x
 - 1)*(x + 1)**(25/2)) + 40*I*(x + 1)**14/(3*sqrt(x - 1)*(x + 1)**(27/2) -
6*sqrt(x - 1)*(x + 1)**(25/2)) - 60*I*(x + 1)**13/(3*sqrt(x - 1)*(x + 1)**(
27/2) - 6*sqrt(x - 1)*(x + 1)**(25/2)), Abs(x + 1)/2 > 1), (30*sqrt(1 - x)*
(x + 1)**(27/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(27/2)
- 6*sqrt(1 - x)*(x + 1)**(25/2)) - 60*sqrt(1 - x)*(x + 1)**(25/2)*asin(sqrt
(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x)*(x + 1)**
(25/2)) + 3*(x + 1)**15/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x)*(x +
 1)**(25/2)) - 40*(x + 1)**14/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x
)*(x + 1)**(25/2)) + 60*(x + 1)**13/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt
(1 - x)*(x + 1)**(25/2)), True))
```

Giac [A] time = 1.09, size = 59, normalized size = 0.94

$$-\frac{((3\,x-37)(x+1)+60)\sqrt{x+1}\sqrt{-x+1}}{3\,(x-1)^2}+10\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((3*x - 37)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 + 10*arcs
in(1/2*sqrt(2)*sqrt(x + 1))
```

3.1097 $\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2(x+1)^{5/2}}{5(1-x)^{5/2}}-\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\sin^{-1}(x)$$

```
[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + (2*(1 +
 x)^(5/2))/(5*(1 - x)^(5/2)) - ArcSin[x]
```

Rubi [A] time = 0.0073775, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$\frac{2(x+1)^{5/2}}{5(1-x)^{5/2}}-\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

```
[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + (2*(1 +
 x)^(5/2))/(5*(1 - x)^(5/2)) - ArcSin[x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx &= \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx \\ &= -\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} + \int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx \\ &= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx \\ &= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \sin^{-1}(x)\end{aligned}$$

Mathematica [C] time = 0.0083435, size = 37, normalized size = 0.59

$$\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{1-x}{2}\right)}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

```
[Out] (8*Sqrt[2]*Hypergeometric2F1[-5/2, -5/2, -3/2, (1 - x)/2])/(5*(1 - x)^(5/2)
)
```

Maple [A] time = 0.017, size = 84, normalized size = 1.3

$$\frac{46\,x^3-2\,x^2-22\,x+26}{15\,(-1+x)^2}\sqrt{(1+x)\,(1-x)}\frac{1}{\sqrt{-(1+x)\,(-1+x)}}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}-\arcsin(x)\sqrt{(1+x)\,(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(7/2),x)
```

```
[Out] 2/15*(23*x^3-x^2-11*x+13)/(-1+x)^2/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2
)/(1-x)^(1/2)/(1+x)^(1/2)-((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsi
n(x)
```

Maxima [B] time = 1.55084, size = 216, normalized size = 3.43

$$-\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{5\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}+\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^4-4\,x^3+6\,x^2-4\,x+1}+\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^3-3\,x^2+3\,x-1\right)}+\frac{6\sqrt{-x^2+1}}{5\left(x^3-3\,x^2+3\,x-1\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 1)^(5/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + (-x^2 + 1
)^(3/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/3*(-x^2 + 1)^(3/2)/(x^3 - 3*x^2
```

```
+ 3*x - 1) + 6/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 7/15*sqrt(-x^2 +
1)/(x^2 - 2*x + 1) - 38/15*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

Fricas [A] time = 1.60495, size = 240, normalized size = 3.81

$$\frac{2\left(13x^3-39x^2-\left(23x^2-24x+13\right)\sqrt{x+1}\sqrt{-x+1}+15\left(x^3-3x^2+3x-1\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+39x-13\right)}{15\left(x^3-3x^2+3x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(13*x^3 - 39*x^2 - (23*x^2 - 24*x + 13)*sqrt(x + 1)*sqrt(-x + 1) + 15*
(x^3 - 3*x^2 + 3*x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 39*x - 1
3)/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [B] time = 37.5888, size = 1608, normalized size = 25.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((30*I*sqrt(x - 1)*(x + 1)**(35/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(1
5*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x
 - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 15*pi*sqrt(x - 1
)*(x + 1)**(35/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)*
*(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)
) - 180*I*sqrt(x - 1)*(x + 1)**(33/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt
(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*
(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 90*pi*sqrt(x - 1)*(x +
 1)**(33/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2
) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 36
0*I*sqrt(x - 1)*(x + 1)**(31/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(x - 1
)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*(x + 1
)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 180*pi*sqrt(x - 1)*(x + 1)**
(31/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 1
80*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 240*I*s
qrt(x - 1)*(x + 1)**(29/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(x - 1)*(x
+ 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*(x + 1)**(3
1/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 120*pi*sqrt(x - 1)*(x + 1)**(29/2
)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sq
rt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 46*I*(x + 1)
**18/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180
*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 232*I*(x
+ 1)**17/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) +
 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 400*I
*(x + 1)**16/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/
2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 2
40*I*(x + 1)**15/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**
(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2))
, Abs(x + 1)/2 > 1), (-30*sqrt(1 - x)*(x + 1)**(35/2)*asin(sqrt(2)*sqrt(x +
```

```
 1)/2)/(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 1
80*sqrt(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 180*sqr
t(1 - x)*(x + 1)**(33/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(1 - x)*(x + 1
)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 - x)*(x + 1)**(31/2
) - 120*sqrt(1 - x)*(x + 1)**(29/2)) - 360*sqrt(1 - x)*(x + 1)**(31/2)*asin
(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x
 + 1)**(33/2) + 180*sqrt(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**
(29/2)) + 240*sqrt(1 - x)*(x + 1)**(29/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(15*s
qrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 -
x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 46*(x + 1)**18/(15*
sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 -
 x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) - 232*(x + 1)**17/(1
5*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1
 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 400*(x + 1)**16/
(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt
(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) - 240*(x + 1)**1
5/(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sq
rt(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)), True))
```

Giac [A] time = 1.06599, size = 59, normalized size = 0.94

$$-\frac{2\left((23x-47)(x+1)+60\right)\sqrt{x+1}\sqrt{-x+1}}{15(x-1)^3}-2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*((23*x - 47)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^3 - 2*arc
sin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1098 $\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}} dx$

Optimal. Leaf size=20

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

```
[Out] (1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

Rubi [A] time = 0.0017708, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(9/2),x]

[Out] (1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}} dx = \frac{(1+x)^{7/2}}{7(1-x)^{7/2}}$$

Mathematica [A] time = 0.007489, size = 20, normalized size = 1.

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(9/2),x]

[Out] (1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

Maple [A] time = 0.001, size = 15, normalized size = 0.8

$$\frac{1}{7}(1+x)^{\frac{7}{2}}(1-x)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(9/2),x)
```

```
[Out] 1/7*(1+x)^(7/2)/(1-x)^(7/2)
```

Maxima [B] time = 1.02651, size = 231, normalized size = 11.55

$$\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}+\frac{5\left(-x^2+1\right)^{\frac{3}{2}}}{2\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}+\frac{15\sqrt{-x^2+1}}{7\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}+\frac{}{14(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(5/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 5/2*(
-x^2 + 1)^(3/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 15/7*sqrt(-x^2
+ 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 3/14*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3
*x - 1) - 1/7*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/7*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [B] time = 1.5855, size = 162, normalized size = 8.1

$$\frac{x^4-4\,x^3+6\,x^2+\left(x^3+3\,x^2+3\,x+1\right)\sqrt{x+1}\sqrt{-x+1}-4\,x+1}{7\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7*(x^4 - 4*x^3 + 6*x^2 + (x^3 + 3*x^2 + 3*x + 1)*sqrt(x + 1)*sqrt(-x + 1)
 - 4*x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [B] time = 86.5537, size = 116, normalized size = 5.8

$$\begin{cases} \frac{i(x+1)^{\frac{7}{2}}}{7\sqrt{x-1}(x+1)^3-42\sqrt{x-1}(x+1)^2+84\sqrt{x-1}(x+1)-56\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ -\frac{(x+1)^{\frac{7}{2}}}{7\sqrt{1-x}(x+1)^3-42\sqrt{1-x}(x+1)^2+84\sqrt{1-x}(x+1)-56\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(7/2)/(7*sqrt(x - 1)*(x + 1)**3 - 42*sqrt(x - 1)*(x +
 1)**2 + 84*sqrt(x - 1)*(x + 1) - 56*sqrt(x - 1)), Abs(x + 1)/2 > 1), (-(x
+ 1)**(7/2)/(7*sqrt(1 - x)*(x + 1)**3 - 42*sqrt(1 - x)*(x + 1)**2 + 84*sqrt
(1 - x)*(x + 1) - 56*sqrt(1 - x)), True))
```

Giac [A] time = 1.08752, size = 26, normalized size = 1.3

$$\frac{(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{7\,(x-1)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^4
```

3.1099 $\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx$

Optimal. Leaf size=41

$$\frac{(x+1)^{7/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}$$

```
[Out] (1 + x)^(7/2)/(9*(1 - x)^(9/2)) + (1 + x)^(7/2)/(63*(1 - x)^(7/2))
```

Rubi [A] time = 0.0043577, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{(x+1)^{7/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(11/2),x]

[Out] (1 + x)^(7/2)/(9*(1 - x)^(9/2)) + (1 + x)^(7/2)/(63*(1 - x)^(7/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx &= \frac{(1+x)^{7/2}}{9(1-x)^{9/2}}+\frac{1}{9}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx\\ &= \frac{(1+x)^{7/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{7/2}}{63(1-x)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0148399, size = 23, normalized size = 0.56

$$-\frac{(x-8)(x+1)^{7/2}}{63(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(11/2),x]

[Out] -((-8 + x)*(1 + x)^(7/2))/(63*(1 - x)^(9/2))
```

Maple [A] time = 0.002, size = 18, normalized size = 0.4

$$-\frac{x-8}{63}(1+x)^{\frac{7}{2}}(1-x)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(11/2),x)

[Out] -1/63*(1+x)^(7/2)*(x-8)/(1-x)^(9/2)
```

Maxima [B] time = 1.02933, size = 294, normalized size = 7.17

$$-\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{2\left(x^7-7x^6+21x^5-35x^4+35x^3-21x^2+7x-1\right)}-\frac{5\left(-x^2+1\right)^{\frac{3}{2}}}{6\left(x^6-6x^5+15x^4-20x^3+15x^2-6x+1\right)}-\frac{}{9\left(x^5-5x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="maxima")

[Out] -1/2*(-x^2 + 1)^(5/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*
x - 1) - 5/6*(-x^2 + 1)^(3/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x
 + 1) - 5/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 5/12
6*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/42*sqrt(-x^2 + 1)/(x^3
 - 3*x^2 + 3*x - 1) - 1/63*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/63*sqrt(-x^2
+ 1)/(x - 1)
```

Fricas [B] time = 1.67573, size = 209, normalized size = 5.1

$$\frac{8x^5-40x^4+80x^3-80x^2+\left(x^4-5x^3-21x^2-23x-8\right)\sqrt{x+1}\sqrt{-x+1}+40x-8}{63\left(x^5-5x^4+10x^3-10x^2+5x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="fricas")

[Out] 1/63*(8*x^5 - 40*x^4 + 80*x^3 - 80*x^2 + (x^4 - 5*x^3 - 21*x^2 - 23*x - 8)*
sqrt(x + 1)*sqrt(-x + 1) + 40*x - 8)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x -
 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09704, size = 30, normalized size = 0.73

$$\frac{(x+1)^{\frac{7}{2}}(x-8)\sqrt{-x+1}}{63\,(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="giac")
```

```
[Out] 1/63*(x + 1)^(7/2)*(x - 8)*sqrt(-x + 1)/(x - 1)^5
```

3.1100 $\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}} dx$

Optimal. Leaf size=61

$$\frac{2(x+1)^{7/2}}{693(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}$$

```
[Out] (1 + x)^(7/2)/(11*(1 - x)^(11/2)) + (2*(1 + x)^(7/2))/(99*(1 - x)^(9/2)) +
(2*(1 + x)^(7/2))/(693*(1 - x)^(7/2))
```

Rubi [A] time = 0.0079077, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{7/2}}{693(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(7/2)/(11*(1 - x)^(11/2)) + (2*(1 + x)^(7/2))/(99*(1 - x)^(9/2)) +
(2*(1 + x)^(7/2))/(693*(1 - x)^(7/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}} dx &= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}}+\frac{2}{11}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}} dx \\
&= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{99(1-x)^{9/2}}+\frac{2}{99}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}} dx \\
&= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{99(1-x)^{9/2}}+\frac{2(1+x)^{7/2}}{693(1-x)^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0185849, size = 30, normalized size = 0.49

$$\frac{(x+1)^{7/2}\left(2x^2-18x+79\right)}{693(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(7/2)*(79 - 18*x + 2*x^2))/(693*(1 - x)^(11/2))
```

Maple [A] time = 0.003, size = 25, normalized size = 0.4

$$\frac{2\,x^2-18\,x+79}{693}\,(1+x)^{\frac{7}{2}}\,(1-x)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(13/2),x)
```

```
[Out] 1/693*(1+x)^(7/2)*(2*x^2-18*x+79)/(1-x)^(11/2)
```

Maxima [B] time = 1.02213, size = 363, normalized size = 5.95

$$\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{3\left(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1\right)}+\frac{5\left(-x^2+1\right)^{\frac{3}{2}}}{12\left(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(5/2)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*
x^2 - 8*x + 1) + 5/12*(-x^2 + 1)^(3/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*
x^3 - 21*x^2 + 7*x - 1) + 5/22*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^
3 + 15*x^2 - 6*x + 1) + 5/396*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2
 + 5*x - 1) - 5/693*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/231*
sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 2/693*sqrt(-x^2 + 1)/(x^2 - 2*x +
1) + 2/693*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [B] time = 1.65722, size = 269, normalized size = 4.41

$$\frac{79\,x^6-474\,x^5+1185\,x^4-1580\,x^3+1185\,x^2+\left(2\,x^5-12\,x^4+31\,x^3+185\,x^2+219\,x+79\right)\sqrt{x+1}\sqrt{-x+1}-474\,x+79}{693\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/693*(79*x^6 - 474*x^5 + 1185*x^4 - 1580*x^3 + 1185*x^2 + (2*x^5 - 12*x^4
+ 31*x^3 + 185*x^2 + 219*x + 79)*sqrt(x + 1)*sqrt(-x + 1) - 474*x + 79)/(x^
```

```
6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(13/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10346, size = 39, normalized size = 0.64

$$\frac{(2(x+1)(x-10)+99)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{693(x-1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="giac")
```

```
[Out] 1/693*(2*(x + 1)*(x - 10) + 99)*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^6
```

3.1101 $\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}} \, dx$

Optimal. Leaf size=81

$$\frac{2(x+1)^{7/2}}{3003(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{429(1-x)^{9/2}}+\frac{3(x+1)^{7/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}$$

```
[Out] (1 + x)^(7/2)/(13*(1 - x)^(13/2)) + (3*(1 + x)^(7/2))/(143*(1 - x)^(11/2))
+ (2*(1 + x)^(7/2))/(429*(1 - x)^(9/2)) + (2*(1 + x)^(7/2))/(3003*(1 - x)^(
7/2))
```

Rubi [A] time = 0.0127514, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{7/2}}{3003(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{429(1-x)^{9/2}}+\frac{3(x+1)^{7/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(15/2),x]
```


```
[Out] (1 + x)^(7/2)/(13*(1 - x)^(13/2)) + (3*(1 + x)^(7/2))/(143*(1 - x)^(11/2))
+ (2*(1 + x)^(7/2))/(429*(1 - x)^(9/2)) + (2*(1 + x)^(7/2))/(3003*(1 - x)^(
7/2))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}} \, dx &= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3}{13}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}} \, dx\\
&= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3(1+x)^{7/2}}{143(1-x)^{11/2}}+\frac{6}{143}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}} \, dx\\
&= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3(1+x)^{7/2}}{143(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{429(1-x)^{9/2}}+\frac{2}{429}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}} \, dx\\
&= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3(1+x)^{7/2}}{143(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{429(1-x)^{9/2}}+\frac{2(1+x)^{7/2}}{3003(1-x)^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0173916, size = 35, normalized size = 0.43

$$\frac{(x+1)^{7/2}\left(-2x^3+20x^2-97x+310\right)}{3003(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(15/2),x]
```

```
[Out] ((1 + x)^(7/2)*(310 - 97*x + 20*x^2 - 2*x^3))/(3003*(1 - x)^(13/2))
```

Maple [A] time = 0.003, size = 30, normalized size = 0.4

$$-\frac{2x^3-20x^2+97x-310}{3003}(1+x)^{\frac{7}{2}}(1-x)^{-\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(15/2),x)
```

```
[Out] -1/3003*(1+x)^(7/2)*(2*x^3-20*x^2+97*x-310)/(1-x)^(13/2)
```

Maxima [B] time = 1.00994, size = 439, normalized size = 5.42

$$-\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{4\left(x^9-9x^8+36x^7-84x^6+126x^5-126x^4+84x^3-36x^2+9x-1\right)}-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{4\left(x^8-8x^7+28x^6-56x^5+70x^4-56x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(5/2)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4 +
84*x^3 - 36*x^2 + 9*x - 1) - 1/4*(-x^2 + 1)^(3/2)/(x^8 - 8*x^7 + 28*x^6 - 5
6*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1) - 3/26*sqrt(-x^2 + 1)/(x^7 - 7*
x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*x - 1) - 3/572*sqrt(-x^2 + 1)/(
x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 5/1716*sqrt(-x^2 + 1)/(
x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 5/3003*sqrt(-x^2 + 1)/(x^4 - 4*x
^3 + 6*x^2 - 4*x + 1) + 1/1001*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 2/3
003*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/3003*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [B] time = 1.49529, size = 319, normalized size = 3.94

$$\frac{310x^7-2170x^6+6510x^5-10850x^4+10850x^3-6510x^2+\left(2x^6-14x^5+43x^4-77x^3-659x^2-833x-310\right)\sqrt{}}{3003\left(x^7-7x^6+21x^5-35x^4+35x^3-21x^2+7x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="fricas")
```

```
[Out] 1/3003*(310*x^7 - 2170*x^6 + 6510*x^5 - 10850*x^4 + 10850*x^3 - 6510*x^2 +
(2*x^6 - 14*x^5 + 43*x^4 - 77*x^3 - 659*x^2 - 833*x - 310)*sqrt(x + 1)*sqrt
(-x + 1) + 2170*x - 310)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 +
 7*x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(15/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12125, size = 47, normalized size = 0.58

$$\frac{((2\,(x+1)(x-12)+143)(x+1)-429)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{3003\,(x-1)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="giac")
```

```
[Out] 1/3003*((2*(x + 1)*(x - 12) + 143)*(x + 1) - 429)*(x + 1)^(7/2)*sqrt(-x + 1
)/(x - 1)^7
```

3.1102 $\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}} \, dx$

Optimal. Leaf size=101

$$\frac{8(x+1)^{7/2}}{45045(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{6435(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{715(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{195(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{15(1-x)^{15/2}}$$

```
[Out] (1 + x)^(7/2)/(15*(1 - x)^(15/2)) + (4*(1 + x)^(7/2))/(195*(1 - x)^(13/2))
+ (4*(1 + x)^(7/2))/(715*(1 - x)^(11/2)) + (8*(1 + x)^(7/2))/(6435*(1 - x)^
(9/2)) + (8*(1 + x)^(7/2))/(45045*(1 - x)^(7/2))
```

Rubi [A] time = 0.0192351, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{8(x+1)^{7/2}}{45045(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{6435(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{715(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{195(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{15(1-x)^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(17/2),x]
```

```
[Out] (1 + x)^(7/2)/(15*(1 - x)^(15/2)) + (4*(1 + x)^(7/2))/(195*(1 - x)^(13/2))
+ (4*(1 + x)^(7/2))/(715*(1 - x)^(11/2)) + (8*(1 + x)^(7/2))/(6435*(1 - x)^
(9/2)) + (8*(1 + x)^(7/2))/(45045*(1 - x)^(7/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx &= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4}{15}\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx\\ &= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4}{65}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx\\ &= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{715(1-x)^{11/2}}+\frac{8}{715}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx\\ &= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{715(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{6435(1-x)^{9/2}}+\frac{8\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx}{6435}\\ &= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{715(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{6435(1-x)^{9/2}}+\frac{8(1+x)^{7/2}}{45045(1-x)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.020974, size = 40, normalized size = 0.4

$$\frac{(x+1)^{7/2}\left(8x^4-88x^3+468x^2-1628x+4243\right)}{45045(1-x)^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(17/2),x]
```

```
[Out] ((1 + x)^(7/2)*(4243 - 1628*x + 468*x^2 - 88*x^3 + 8*x^4))/(45045*(1 - x)^(
15/2))
```

Maple [A] time = 0.003, size = 35, normalized size = 0.4

$$\frac{8\,x^4-88\,x^3+468\,x^2-1628\,x+4243}{45045}(1+x)^{\frac{7}{2}}(1-x)^{-\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(17/2),x)
```

```
[Out] 1/45045*(1+x)^(7/2)*(8*x^4-88*x^3+468*x^2-1628*x+4243)/(1-x)^(15/2)
```

Maxima [B] time = 1.01575, size = 521, normalized size = 5.16

$$\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{5\left(x^{10}-10\,x^9+45\,x^8-120\,x^7+210\,x^6-252\,x^5+210\,x^4-120\,x^3+45\,x^2-10\,x+1\right)}+\frac{(\cdot}{6\left(x^9-9\,x^8+36\,x^7-84\,x^6+1\cdot\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(5/2)/(x^10 - 10*x^9 + 45*x^8 - 120*x^7 + 210*x^6 - 252*x^5
+ 210*x^4 - 120*x^3 + 45*x^2 - 10*x + 1) + 1/6*(-x^2 + 1)^(3/2)/(x^9 - 9*x^
8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4 + 84*x^3 - 36*x^2 + 9*x - 1) + 1/15
*sqrt(-x^2 + 1)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 -
```

```
 8*x + 1) + 1/390*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 -
21*x^2 + 7*x - 1) - 1/715*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 1
5*x^2 - 6*x + 1) + 1/1287*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5
*x - 1) - 4/9009*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/15015*s
qrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 8/45045*sqrt(-x^2 + 1)/(x^2 - 2*x +
 1) + 8/45045*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A] time = 1.65709, size = 385, normalized size = 3.81

$$\frac{4243\,x^{8}-33944\,x^{7}+118804\,x^{6}-237608\,x^{5}+297010\,x^{4}-237608\,x^{3}+118804\,x^{2}+\left(8\,x^{7}-64\,x^{6}+228\,x^{5}-480\,x^{4}\right.}{45045\left(x^{8}-8\,x^{7}+28\,x^{6}-56\,x^{5}+70\,x^{4}-56\,x^{3}+28\,x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="fricas")
```

```
[Out] 1/45045*(4243*x^8 - 33944*x^7 + 118804*x^6 - 237608*x^5 + 297010*x^4 - 2376
08*x^3 + 118804*x^2 + (8*x^7 - 64*x^6 + 228*x^5 - 480*x^4 + 675*x^3 + 8313*
x^2 + 11101*x + 4243)*sqrt(x + 1)*sqrt(-x + 1) - 33944*x + 4243)/(x^8 - 8*x
^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(17/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12352, size = 57, normalized size = 0.56

$$\frac{(4\,((2\,(x+1)(x-14)+195)(x+1)-715)(x+1)+6435)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{45045\,(x-1)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="giac")
```

```
[Out] 1/45045*(4*((2*(x + 1)*(x - 14) + 195)*(x + 1) - 715)*(x + 1) + 6435)*(x +
1)^(7/2)*sqrt(-x + 1)/(x - 1)^8
```

3.1103 $\int \frac{(1+x)^{5/2}}{(1-x)^{19/2}} dx$

Optimal. Leaf size=121

$$\frac{8(x+1)^{7/2}}{153153(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{21879(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{2431(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{663(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{51(1-x)^{15/2}}+\frac{(x+1)^{7/2}}{17(1-x)^{17/2}}$$

```
[Out] (1 + x)^(7/2)/(17*(1 - x)^(17/2)) + (1 + x)^(7/2)/(51*(1 - x)^(15/2)) + (4*
(1 + x)^(7/2))/(663*(1 - x)^(13/2)) + (4*(1 + x)^(7/2))/(2431*(1 - x)^(11/2
)) + (8*(1 + x)^(7/2))/(21879*(1 - x)^(9/2)) + (8*(1 + x)^(7/2))/(153153*(1
 - x)^(7/2))
```

Rubi [A] time = 0.0246597, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{8(x+1)^{7/2}}{153153(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{21879(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{2431(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{663(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{51(1-x)^{15/2}}+\frac{(x+1)^{7/2}}{17(1-x)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(19/2),x]
```

```
[Out] (1 + x)^(7/2)/(17*(1 - x)^(17/2)) + (1 + x)^(7/2)/(51*(1 - x)^(15/2)) + (4*
(1 + x)^(7/2))/(663*(1 - x)^(13/2)) + (4*(1 + x)^(7/2))/(2431*(1 - x)^(11/2
)) + (8*(1 + x)^(7/2))/(21879*(1 - x)^(9/2)) + (8*(1 + x)^(7/2))/(153153*(1
 - x)^(7/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{(1-x)^{19/2}}dx = \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{5}{17}\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}dx$$
$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4}{51}\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}dx$$
$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{663(1-x)^{13/2}} + \frac{4}{221}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}dx$$
$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{663(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}} + \frac{8\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}dx}{2431}$$
$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{663(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}} + \frac{8(1+x)^{7/2}}{21879(1-x)^{9/2}} + \frac{8\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}dx}{21879}$$
$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{663(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}} + \frac{8(1+x)^{7/2}}{21879(1-x)^{9/2}} + \frac{8(1+x)^{7/2}}{153153(1-x}$$

Mathematica [A] time = 0.0214536, size = 45, normalized size = 0.37

$$\frac{(x+1)^{7/2}\left(-8x^5+96x^4-556x^3+2096x^2-5871x+13252\right)}{153153(1-x)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(19/2),x]
```

```
[Out] ((1 + x)^(7/2)*(13252 - 5871*x + 2096*x^2 - 556*x^3 + 96*x^4 - 8*x^5))/(153
153*(1 - x)^(17/2))
```

Maple [A] time = 0.003, size = 40, normalized size = 0.3

$$-\frac{8\,x^5-96\,x^4+556\,x^3-2096\,x^2+5871\,x-13252}{153153}(1+x)^{\frac{7}{2}}(1-x)^{-\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(19/2),x)
```

```
[Out] -1/153153*(1+x)^(7/2)*(8*x^5-96*x^4+556*x^3-2096*x^2+5871*x-13252)/(1-x)^(1
7/2)
```

Maxima [B] time = 1.03146, size = 610, normalized size = 5.04

$$-\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{6\left(x^{11}-11\,x^{10}+55\,x^9-165\,x^8+330\,x^7-462\,x^6+462\,x^5-330\,x^4+165\,x^3-55\,x^2+11\,x-1\right)} - \frac{}{42\left(x^{10}-10\,x^9+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(5/2)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 + 330*x^7 - 462*x^
6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1) - 5/42*(-x^2 + 1)^(3/2
)/(x^10 - 10*x^9 + 45*x^8 - 120*x^7 + 210*x^6 - 252*x^5 + 210*x^4 - 120*x^3
 + 45*x^2 - 10*x + 1) - 5/119*sqrt(-x^2 + 1)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6
 + 126*x^5 - 126*x^4 + 84*x^3 - 36*x^2 + 9*x - 1) - 1/714*sqrt(-x^2 + 1)/(x
^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1) + 1/1326
*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*x - 1)
 - 1/2431*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
 + 5/21879*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 20/15
3153*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/51051*sqrt(-x^2 + 1
)/(x^3 - 3*x^2 + 3*x - 1) - 8/153153*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 8/153
153*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A] time = 1.56385, size = 448, normalized size = 3.7

$$\frac{13252\,x^9-119268\,x^8+477072\,x^7-1113168\,x^6+1669752\,x^5-1669752\,x^4+1113168\,x^3-477072\,x^2+\left(8\,x^8-72\,x^7+2\right.}{153153\left(x^9-9\,x^8+36\,x^7-84\,x^6+126\,x^5-12\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="fricas")
```

```
[Out] 1/153153*(13252*x^9 - 119268*x^8 + 477072*x^7 - 1113168*x^6 + 1669752*x^5 -
 1669752*x^4 + 1113168*x^3 - 477072*x^2 + (8*x^8 - 72*x^7 + 292*x^6 - 708*x
^5 + 1155*x^4 - 1371*x^3 - 24239*x^2 - 33885*x - 13252)*sqrt(x + 1)*sqrt(-x
 + 1) + 119268*x - 13252)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^
4 + 84*x^3 - 36*x^2 + 9*x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(19/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16272, size = 65, normalized size = 0.54

$$\frac{((4\,((2\,(x+1)(x-16)+255)(x+1)-1105)(x+1)+12155)(x+1)-21879)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{153153\,(x-1)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="giac")
```

```
[Out] 1/153153*((4*((2*(x + 1)*(x - 16) + 255)*(x + 1) - 1105)*(x + 1) + 12155)*(
x + 1) - 21879)*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^9
```

3.1104 $\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{1-ax}(ax+1)^{3/2}}{2a}-\frac{3\sqrt{1-ax}\sqrt{ax+1}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

```
[Out] (-3*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(2*a) - (Sqrt[1 - a*x]*(1 + a*x)^(3/2))/(2
*a) + (3*ArcSin[a*x])/(2*a)
```

Rubi [A] time = 0.0117152, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {50, 41, 216}

$$-\frac{\sqrt{1-ax}(ax+1)^{3/2}}{2a}-\frac{3\sqrt{1-ax}\sqrt{ax+1}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
```

```
[Out] (-3*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(2*a) - (Sqrt[1 - a*x]*(1 + a*x)^(3/2))/(2
*a) + (3*ArcSin[a*x])/(2*a)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx = -\frac{\sqrt{1-ax}(1+ax)^{3/2}}{2a} + \frac{3}{2}\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx$$
$$= -\frac{3\sqrt{1-ax}\sqrt{1+ax}}{2a} - \frac{\sqrt{1-ax}(1+ax)^{3/2}}{2a} + \frac{3}{2}\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx$$
$$= -\frac{3\sqrt{1-ax}\sqrt{1+ax}}{2a} - \frac{\sqrt{1-ax}(1+ax)^{3/2}}{2a} + \frac{3}{2}\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{3\sqrt{1-ax}\sqrt{1+ax}}{2a} - \frac{\sqrt{1-ax}(1+ax)^{3/2}}{2a} + \frac{3\sin^{-1}(ax)}{2a}$$

Mathematica [A] time = 0.0401253, size = 47, normalized size = 0.73

$$-\frac{\sqrt{1-a^2x^2}(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
```

```
[Out] -((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/(2*a)
```

Maple [A] time = 0.01, size = 98, normalized size = 1.5

$$-\frac{1}{2a}(ax+1)^{\frac{3}{2}}\sqrt{-ax+1}-\frac{3}{2a}\sqrt{-ax+1}\sqrt{ax+1}+\frac{3}{2}\sqrt{(ax+1)(-ax+1)}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{-ax+1}}\frac{1}{\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^(3/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/2*(a*x+1)^(3/2)*(-a*x+1)^(1/2)/a-3/2*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a+3/2*
((a*x+1)*(-a*x+1))^(1/2)/(a*x+1)^(1/2)/(-a*x+1)^(1/2)/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.47734, size = 69, normalized size = 1.08

$$-\frac{1}{2}\sqrt{-a^2x^2+1}x+\frac{3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}-\frac{2\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x + 3/2*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - 2*sqrt(
-a^2*x^2 + 1)/a
```

Fricas [A] time = 1.64875, size = 138, normalized size = 2.16

$$-\frac{(ax+4)\sqrt{ax+1}\sqrt{-ax+1}+6\arctan\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{ax}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((a*x + 4)*sqrt(a*x + 1)*sqrt(-a*x + 1) + 6*arctan((sqrt(a*x + 1)*sqrt
(-a*x + 1) - 1)/(a*x)))/a
```

Sympy [A] time = 24.9349, size = 75, normalized size = 1.17

$$\begin{cases} \frac{2\left(\begin{cases} -\frac{ax\sqrt{-ax+1}\sqrt{ax+1}}{4}-\sqrt{-ax+1}\sqrt{ax+1}+\frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{ax+1}}{2}\right)}{2} & \text{for}\ ax-1\geq -2\wedge ax-1<0 \end{cases}\right)}{a} & \text{for}\ a\neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(3/2)/(-a*x+1)**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((-a*x*sqrt(-a*x + 1)*sqrt(a*x + 1)/4 - sqrt(-a*x + 1
)*sqrt(a*x + 1) + 3*asin(sqrt(2)*sqrt(a*x + 1)/2)/2, (a*x - 1 >= -2) & (a*x
 - 1 < 0)))/a, Ne(a, 0)), (x, True))
```

Giac [A] time = 1.0726, size = 57, normalized size = 0.89

$$-\frac{(ax+4)\sqrt{ax+1}\sqrt{-ax+1}-6\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{ax+1}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*((a*x + 4)*sqrt(a*x + 1)*sqrt(-a*x + 1) - 6*arcsin(1/2*sqrt(2)*sqrt(a*
x + 1)))/a
```

3.1105 $\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx$

Optimal. Leaf size=62

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(2*a) - (1 - a^2*x^2)^(3/2)/(2*a*(1 - a*x)) + (3*Arc
Sin[a*x])/(2*a)
```

Rubi [A] time = 0.028291, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {795, 665, 216}

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
```

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(2*a) - (1 - a^2*x^2)^(3/2)/(2*a*(1 - a*x)) + (3*Arc
Sin[a*x])/(2*a)
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx = -\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)} + \frac{3}{2}\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx$$
$$= -\frac{3\sqrt{1-a^2x^2}}{2a} - \frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)} + \frac{3}{2}\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{3\sqrt{1-a^2x^2}}{2a} - \frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)} + \frac{3\sin^{-1}(ax)}{2a}$$

Mathematica [A] time = 0.103726, size = 91, normalized size = 1.47

$$\frac{\sqrt{1-a^2x^2}\left(6\sqrt{ax+1}\sin^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)-\sqrt{1-ax}\left(a^2x^2+5ax+4\right)\right)}{2a\sqrt{1-ax}(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-(Sqrt[1 - a*x]*(4 + 5*a*x + a^2*x^2)) + 6*Sqrt[1 + a*x
]*ArcSin[Sqrt[1 + a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*(1 + a*x))
```

Maple [B] time = 0.011, size = 118, normalized size = 1.9

$$-\frac{x}{2}\sqrt{-a^2x^2+1}-\frac{1}{2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-2\frac{1}{a}\sqrt{-\left(x-a^{-1}\right)^2a^2-2\,a\left(x-a^{-1}\right)}+2\frac{1}{\sqrt{a^2}}\arctan\left(\sqrt{a^2}x\frac{}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x)
```

```
[Out] -1/2*x*(-a^2*x^2+1)^(1/2)-1/2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))-2/a*(-(x-1/a)^2*a^2-2*a*(x-1/a))^(1/2)+2/(a^2)^(1/2)*arctan((a^2)^(
1/2)*x/(-(x-1/a)^2*a^2-2*a*(x-1/a))^(1/2))
```

Maxima [A] time = 1.48671, size = 57, normalized size = 0.92

$$-\frac{1}{2}\sqrt{-a^2x^2+1}x+\frac{3\arcsin\left(ax\right)}{2\,a}-\frac{2\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x + 3/2*arcsin(a*x)/a - 2*sqrt(-a^2*x^2 + 1)/a
```

Fricas [A] time = 1.5794, size = 111, normalized size = 1.79

$$-\frac{\sqrt{-a^2x^2+1}(ax+4)+6\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-a^2*x^2 + 1)*(a*x + 4) + 6*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x
)))/a
```

Sympy [A] time = 4.49177, size = 76, normalized size = 1.23

$$-\begin{cases}-\frac{-\sqrt{-a^2x^2+1}+\operatorname{asin}(ax)}{a} & \text{for}\: ax>-1\wedge ax<1\end{cases}-\begin{cases}-\frac{-\frac{ax\sqrt{-a^2x^2+1}}{2}-\sqrt{-a^2x^2+1}+\frac{\operatorname{asin}(ax)}{2}}{a} & \text{for}\: ax>-1\wedge ax<1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*x+1),x)
```

```
[Out] -Piecewise((-(-sqrt(-a**2*x**2 + 1) + asin(a*x))/a, (a*x > -1) & (a*x < 1))
) - Piecewise((-(-a*x*sqrt(-a**2*x**2 + 1)/2 - sqrt(-a**2*x**2 + 1) + asin(
a*x)/2)/a, (a*x > -1) & (a*x < 1)))
```

Giac [A] time = 1.07473, size = 46, normalized size = 0.74

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\left(x+\frac{4}{a}\right)+\frac{3\arcsin(ax)\operatorname{sgn}(a)}{2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(x + 4/a) + 3/2*arcsin(a*x)*sgn(a)/abs(a)
```

3.1106 $\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=87

$$\frac{1}{4}\sqrt{x+1}(1-x)^{7/2}+\frac{7}{12}\sqrt{x+1}(1-x)^{5/2}+\frac{35}{24}\sqrt{x+1}(1-x)^{3/2}+\frac{35}{8}\sqrt{x+1}\sqrt{1-x}+\frac{35}{8}\sin^{-1}(x)$$

```
[Out] (35*Sqrt[1 - x]*Sqrt[1 + x])/8 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/24 + (7*(1
- x)^(5/2)*Sqrt[1 + x])/12 + ((1 - x)^(7/2)*Sqrt[1 + x])/4 + (35*ArcSin[x])
/8
```

Rubi [A] time = 0.0176468, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\frac{1}{4}\sqrt{x+1}(1-x)^{7/2}+\frac{7}{12}\sqrt{x+1}(1-x)^{5/2}+\frac{35}{24}\sqrt{x+1}(1-x)^{3/2}+\frac{35}{8}\sqrt{x+1}\sqrt{1-x}+\frac{35}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/Sqrt[1 + x],x]
```

```
[Out] (35*Sqrt[1 - x]*Sqrt[1 + x])/8 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/24 + (7*(1
- x)^(5/2)*Sqrt[1 + x])/12 + ((1 - x)^(7/2)*Sqrt[1 + x])/4 + (35*ArcSin[x])
/8
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx &= \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{7}{4}\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx \\
&= \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{12}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{8}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= \frac{35}{8}\sqrt{1-x}\sqrt{1+x} + \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{8}\int \frac{1}{\sqrt{1-x}\sqrt{1+\ }} \\
&= \frac{35}{8}\sqrt{1-x}\sqrt{1+x} + \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{35}{8}\sqrt{1-x}\sqrt{1+x} + \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{8}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.026514, size = 61, normalized size = 0.7

$$
\frac{\sqrt{x+1}\left(6x^4-38x^3+113x^2-241x+160\right)}{24\sqrt{1-x}} - \frac{35}{4}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(160 - 241*x + 113*x^2 - 38*x^3 + 6*x^4))/(24*Sqrt[1 - x]) - (
35*ArcSin[Sqrt[1 - x]/Sqrt[2]])/4
```

Maple [A] time = 0.001, size = 85, normalized size = 1.

$$
\frac{1}{4}(1-x)^{\frac{7}{2}}\sqrt{1+x} + \frac{7}{12}(1-x)^{\frac{5}{2}}\sqrt{1+x} + \frac{35}{24}(1-x)^{\frac{3}{2}}\sqrt{1+x} + \frac{35}{8}\sqrt{1-x}\sqrt{1+x} + \frac{35\arcsin(x)}{8}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)/(1+x)^(1/2),x)
```

```
[Out] 1/4*(1-x)^(7/2)*(1+x)^(1/2)+7/12*(1-x)^(5/2)*(1+x)^(1/2)+35/24*(1-x)^(3/2)*
(1+x)^(1/2)+35/8*(1-x)^(1/2)*(1+x)^(1/2)+35/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/
2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.51879, size = 76, normalized size = 0.87

$$
-\frac{1}{4}\sqrt{-x^2+1}x^3 + \frac{4}{3}\sqrt{-x^2+1}x^2 - \frac{27}{8}\sqrt{-x^2+1}x + \frac{20}{3}\sqrt{-x^2+1} + \frac{35}{8}\arcsin(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-x^2 + 1)*x^3 + 4/3*sqrt(-x^2 + 1)*x^2 - 27/8*sqrt(-x^2 + 1)*x +
20/3*sqrt(-x^2 + 1) + 35/8*arcsin(x)
```

Fricas [A] time = 1.67869, size = 149, normalized size = 1.71

$$-\frac{1}{24}\left(6\,x^{3}-32\,x^{2}+81\,x-160\right)\sqrt{x+1}\sqrt{-x+1}-\frac{35}{4}\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*x^3 - 32*x^2 + 81*x - 160)*sqrt(x + 1)*sqrt(-x + 1) - 35/4*arctan(
(sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [A] time = 50.5507, size = 201, normalized size = 2.31

$$\begin{cases} -\frac{35i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}}+\frac{31i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}}-\frac{263i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}}+\frac{605i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}}-\frac{93i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ -\frac{\sqrt{1-x}(x+1)^{\frac{7}{2}}}{4}+\frac{25\sqrt{1-x}(x+1)^{\frac{5}{2}}}{12}-\frac{163\sqrt{1-x}(x+1)^{\frac{3}{2}}}{24}+\frac{93\sqrt{1-x}\sqrt{x+1}}{8}+\frac{35\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-35*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(9/2)/(4*sqrt(
x - 1)) + 31*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) - 263*I*(x + 1)**(5/2)/(24*s
qrt(x - 1)) + 605*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) - 93*I*sqrt(x + 1)/(4*s
qrt(x - 1)), Abs(x + 1)/2 > 1), (-sqrt(1 - x)*(x + 1)**(7/2)/4 + 25*sqrt(1
- x)*(x + 1)**(5/2)/12 - 163*sqrt(1 - x)*(x + 1)**(3/2)/24 + 93*sqrt(1 - x)
*sqrt(x + 1)/8 + 35*asin(sqrt(2)*sqrt(x + 1)/2)/4, True))
```

Giac [A] time = 1.12798, size = 136, normalized size = 1.56

$$-\frac{1}{24}\left((2\,(3\,x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\left((2\,x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}-\frac{3}{2}\,\sqrt{x+1}(x-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) +
 1/2*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 3/2*sqrt(x + 1)*(x
- 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 35/4*arcsin(1/2*sqrt(2)*sqrt
(x + 1))
```

3.1107 $\int \frac{(1-x)^{5/2}}{\sqrt{1+x}} dx$

Optimal. Leaf size=67

$$\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}+\frac{5}{6}\sqrt{x+1}(1-x)^{3/2}+\frac{5}{2}\sqrt{x+1}\sqrt{1-x}+\frac{5}{2}\sin^{-1}(x)$$

```
[Out] (5*Sqrt[1 - x]*Sqrt[1 + x])/2 + (5*(1 - x)^(3/2)*Sqrt[1 + x])/6 + ((1 - x)^
(5/2)*Sqrt[1 + x])/3 + (5*ArcSin[x])/2
```

Rubi [A] time = 0.0106446, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}+\frac{5}{6}\sqrt{x+1}(1-x)^{3/2}+\frac{5}{2}\sqrt{x+1}\sqrt{1-x}+\frac{5}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

```
[Out] (5*Sqrt[1 - x]*Sqrt[1 + x])/2 + (5*(1 - x)^(3/2)*Sqrt[1 + x])/6 + ((1 - x)^
(5/2)*Sqrt[1 + x])/3 + (5*ArcSin[x])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx &= \frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx\\ &= \frac{5}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx\\ &= \frac{5}{2}\sqrt{1-x}\sqrt{1+x}+\frac{5}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx\\ &= \frac{5}{2}\sqrt{1-x}\sqrt{1+x}+\frac{5}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{5}{2}\sqrt{1-x}\sqrt{1+x}+\frac{5}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0256524, size = 54, normalized size = 0.81

$$\frac{\sqrt{x+1}\left(-2x^3+11x^2-31x+22\right)}{6\sqrt{1-x}}-5\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(22 - 31*x + 11*x^2 - 2*x^3))/(6*Sqrt[1 - x]) - 5*ArcSin[Sqrt[
1 - x]/Sqrt[2]]
```

Maple [A] time = 0.005, size = 71, normalized size = 1.1

$$\frac{1}{3}(1-x)^{\frac{5}{2}}\sqrt{1+x}+\frac{5}{6}(1-x)^{\frac{3}{2}}\sqrt{1+x}+\frac{5}{2}\sqrt{1-x}\sqrt{1+x}+\frac{5\arcsin(x)}{2}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)/(1+x)^(1/2),x)
```

```
[Out] 1/3*(1-x)^(5/2)*(1+x)^(1/2)+5/6*(1-x)^(3/2)*(1+x)^(1/2)+5/2*(1-x)^(1/2)*(1+
x)^(1/2)+5/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.50438, size = 57, normalized size = 0.85

$$\frac{1}{3}\sqrt{-x^2+1}x^2-\frac{3}{2}\sqrt{-x^2+1}x+\frac{11}{3}\sqrt{-x^2+1}+\frac{5}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x + 11/3*sqrt(-x^2 + 1) + 5/2*a
rcsin(x)
```

Fricas [A] time = 1.54617, size = 127, normalized size = 1.9

$$\frac{1}{6}\left(2x^2 - 9x + 22\right)\sqrt{x+1}\sqrt{-x+1} - 5\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 - 9*x + 22)*sqrt(x + 1)*sqrt(-x + 1) - 5*arctan((sqrt(x + 1)*sqr
t(-x + 1) - 1)/x)
```

Sympy [A] time = 10.0033, size = 175, normalized size = 2.61

$$\begin{cases} -5i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} - \frac{17i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} + \frac{59i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} - \frac{11i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ 5\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} + \frac{17(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} - \frac{59(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} + \frac{11\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) - 17*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) + 59*I*(x + 1)**(3/2)/(6*sqrt(x
- 1)) - 11*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sq
rt(x + 1)/2) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 17*(x + 1)**(5/2)/(6*sqrt(1
 - x)) - 59*(x + 1)**(3/2)/(6*sqrt(1 - x)) + 11*sqrt(x + 1)/sqrt(1 - x), Tr
ue))
```

Giac [A] time = 1.07833, size = 93, normalized size = 1.39

$$\frac{1}{6}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} - \sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + 5\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*
sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 5*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1108 $\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=47

$$\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}+\frac{3}{2}\sqrt{x+1}\sqrt{1-x}+\frac{3}{2}\sin^{-1}(x)$$

```
[Out] (3*Sqrt[1 - x]*Sqrt[1 + x])/2 + ((1 - x)^(3/2)*Sqrt[1 + x])/2 + (3*ArcSin[x
])/2
```

Rubi [A] time = 0.0069494, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}+\frac{3}{2}\sqrt{x+1}\sqrt{1-x}+\frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)/Sqrt[1 + x],x]
```

```
[Out] (3*Sqrt[1 - x]*Sqrt[1 + x])/2 + ((1 - x)^(3/2)*Sqrt[1 + x])/2 + (3*ArcSin[x
])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx &= \frac{1}{2}(1-x)^{3/2}\sqrt{1+x}+\frac{3}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx\\
&= \frac{3}{2}\sqrt{1-x}\sqrt{1+x}+\frac{1}{2}(1-x)^{3/2}\sqrt{1+x}+\frac{3}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx\\
&= \frac{3}{2}\sqrt{1-x}\sqrt{1+x}+\frac{1}{2}(1-x)^{3/2}\sqrt{1+x}+\frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{3}{2}\sqrt{1-x}\sqrt{1+x}+\frac{1}{2}(1-x)^{3/2}\sqrt{1+x}+\frac{3}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0200142, size = 47, normalized size = 1.

$$\frac{\sqrt{x+1}\left(x^2-5x+4\right)}{2\sqrt{1-x}}-3\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(4 - 5*x + x^2))/(2*Sqrt[1 - x]) - 3*ArcSin[Sqrt[1 - x]/Sqrt[2
]]
```

Maple [A] time = 0.003, size = 57, normalized size = 1.2

$$\frac{1}{2}(1-x)^{\frac{3}{2}}\sqrt{1+x}+\frac{3}{2}\sqrt{1-x}\sqrt{1+x}+\frac{3\arcsin(x)}{2}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)/(1+x)^(1/2),x)
```

```
[Out] 1/2*(1-x)^(3/2)*(1+x)^(1/2)+3/2*(1-x)^(1/2)*(1+x)^(1/2)+3/2*((1+x)*(1-x))^(
1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.50223, size = 38, normalized size = 0.81

$$-\frac{1}{2}\sqrt{-x^2+1}x+2\sqrt{-x^2+1}+\frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x + 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

Fricas [A] time = 1.55196, size = 113, normalized size = 2.4

$$-\frac{1}{2}\sqrt{x+1}(x-4)\sqrt{-x+1}-3\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(x + 1)*(x - 4)*sqrt(-x + 1) - 3*arctan((sqrt(x + 1)*sqrt(-x + 1)
- 1)/x)
```

Sympy [A] time = 2.95077, size = 139, normalized size = 2.96

$$\begin{cases} -3i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} + \frac{7i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} - \frac{5i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ 3\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} - \frac{7(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} + \frac{5\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) + 7*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) - 5*I*sqrt(x + 1)/sqrt(x - 1), Ab
s(x + 1)/2 > 1), (3*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sqrt(1
- x)) - 7*(x + 1)**(3/2)/(2*sqrt(1 - x)) + 5*sqrt(x + 1)/sqrt(1 - x), True)
)
```

Giac [A] time = 1.07467, size = 59, normalized size = 1.26

$$-\frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + 3\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 3*arcsin
(1/2*sqrt(2)*sqrt(x + 1))
```

3.1109 $\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{1-x}\sqrt{x+1}+\sin^{-1}(x)$$

```
[Out] Sqrt[1 - x]*Sqrt[1 + x] + ArcSin[x]
```

Rubi [A] time = 0.0034088, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\sqrt{1-x}\sqrt{x+1}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/Sqrt[1 + x],x]

[Out] Sqrt[1 - x]*Sqrt[1 + x] + ArcSin[x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx &= \sqrt{1-x}\sqrt{1+x}+\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx\\ &= \sqrt{1-x}\sqrt{1+x}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{1-x}\sqrt{1+x}+\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0135955, size = 30, normalized size = 1.5

$$\sqrt{1-x^2}-2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[1 - x^2] - 2*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

Maple [B] time = 0.002, size = 41, normalized size = 2.1

$$\sqrt{1-x}\,\sqrt{1+x}+\arcsin\left(x\right)\sqrt{\left(1+x\right)\left(1-x\right)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] (1-x)^(1/2)*(1+x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(
x)
```

Maxima [A] time = 1.52339, size = 16, normalized size = 0.8

$$\sqrt{-x^2+1}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [B] time = 1.89186, size = 96, normalized size = 4.8

$$\sqrt{x+1}\sqrt{-x+1}-2\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x + 1)*sqrt(-x + 1) - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [B] time = 1.60976, size = 100, normalized size = 5.

$$\begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}}-\frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(3/2)/sqrt(x - 1)
 - 2*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (2*asin(sqrt(2)*sqrt(x +
 1)/2) - (x + 1)**(3/2)/sqrt(1 - x) + 2*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A] time = 1.05982, size = 36, normalized size = 1.8

$$\sqrt{x+1}\sqrt{-x+1} + 2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*sqrt(-x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1110 $\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

```
[Out] ArcSin[x]
```

Rubi [A] time = 0.0014997, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {41, 216}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx &= \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.0042368, size = 2, normalized size = 1.

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] ArcSin[x]
```

Maple [B] time = 0.002, size = 27, normalized size = 13.5

$$\arcsin(x)\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] ((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.53142, size = 3, normalized size = 1.5

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x)
```

Fricas [B] time = 1.81425, size = 61, normalized size = 30.5

$$-2\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [B] time = 1.09657, size = 41, normalized size = 20.5

$$\begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for}\ \frac{|x+1|}{2}>1 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2), Abs(x + 1)/2 > 1), (2*asin(sq
rt(2)*sqrt(x + 1)/2), True))
```

Giac [B] time = 1.05936, size = 18, normalized size = 9.

$$2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1111 $\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=17

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

```
[Out] Sqrt[1 + x]/Sqrt[1 - x]
```

Rubi [A] time = 0.0031938, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/Sqrt[1 - x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{\sqrt{1-x}}$$

Mathematica [A] time = 0.0031577, size = 17, normalized size = 1.

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/Sqrt[1 - x]
```

Maple [A] time = 0.002, size = 14, normalized size = 0.8

$$\sqrt{1+x}\frac{1}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(3/2)/(1+x)^(1/2),x)
```

```
[Out] (1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [A] time = 1.48572, size = 22, normalized size = 1.29

$$-\frac{\sqrt{-x^2+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A] time = 1.76067, size = 59, normalized size = 3.47

$$\frac{x-\sqrt{x+1}\sqrt{-x+1}-1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] (x - sqrt(x + 1)*sqrt(-x + 1) - 1)/(x - 1)
```

Sympy [A] time = 1.04886, size = 29, normalized size = 1.71

$$\begin{cases} \frac{1}{\sqrt{-1+\frac{2}{x+1}}} & \text{for } \frac{2}{|x+1|}>1 \\ -\frac{i}{\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((1/sqrt(-1 + 2/(x + 1)), 2/Abs(x + 1) > 1), (-I/sqrt(1 - 2/(x + 1
)), True))
```

Giac [A] time = 1.08491, size = 26, normalized size = 1.53

$$-\frac{\sqrt{x+1}\sqrt{-x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(x + 1)*sqrt(-x + 1)/(x - 1)
```

3.1112 $\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=41

$$\frac{\sqrt{x+1}}{3\sqrt{1-x}}+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}$$

```
[Out] Sqrt[1 + x]/(3*(1 - x)^(3/2)) + Sqrt[1 + x]/(3*Sqrt[1 - x])
```

Rubi [A] time = 0.0043011, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{\sqrt{x+1}}{3\sqrt{1-x}}+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(3*(1 - x)^(3/2)) + Sqrt[1 + x]/(3*Sqrt[1 - x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{3(1-x)^{3/2}}+\frac{1}{3}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{3(1-x)^{3/2}}+\frac{\sqrt{1+x}}{3\sqrt{1-x}}\end{aligned}$$

Mathematica [A] time = 0.0063659, size = 23, normalized size = 0.56

$$-\frac{(x-2)\sqrt{x+1}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
```

```
[Out] -((-2 + x)*Sqrt[1 + x])/(3*(1 - x)^(3/2))
```

Maple [A] time = 0.003, size = 18, normalized size = 0.4

$$-\frac{-2+x}{3}\sqrt{1+x}\,(1-x)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(5/2)/(1+x)^(1/2),x)
```

```
[Out] -1/3*(1+x)^(1/2)*(-2+x)/(1-x)^(3/2)
```

Maxima [A] time = 1.49189, size = 51, normalized size = 1.24

$$\frac{\sqrt{-x^2+1}}{3\left(x^2-2\,x+1\right)}-\frac{\sqrt{-x^2+1}}{3\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/3*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A] time = 1.72421, size = 100, normalized size = 2.44

$$\frac{2\,x^2-\sqrt{x+1}(x-2)\sqrt{-x+1}-4\,x+2}{3\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x^2 - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - 4*x + 2)/(x^2 - 2*x + 1)
```

Sympy [A] time = 4.17845, size = 126, normalized size = 3.07

$$\begin{cases}\frac{x+1}{3\sqrt{-1+\frac{2}{x+1}}(x+1)-6\sqrt{-1+\frac{2}{x+1}}}-\frac{3}{3\sqrt{-1+\frac{2}{x+1}}(x+1)-6\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{2}{|x+1|}>1\\ -\frac{i(x+1)}{3\sqrt{1-\frac{2}{x+1}}(x+1)-6\sqrt{1-\frac{2}{x+1}}}+\frac{3i}{3\sqrt{1-\frac{2}{x+1}}(x+1)-6\sqrt{1-\frac{2}{x+1}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise(((x + 1)/(3*sqrt(-1 + 2/(x + 1))*(x + 1) - 6*sqrt(-1 + 2/(x + 1))
) - 3/(3*sqrt(-1 + 2/(x + 1))*(x + 1) - 6*sqrt(-1 + 2/(x + 1))), 2/Abs(x +
1) > 1), (-I*(x + 1)/(3*sqrt(1 - 2/(x + 1))*(x + 1) - 6*sqrt(1 - 2/(x + 1))
) + 3*I/(3*sqrt(1 - 2/(x + 1))*(x + 1) - 6*sqrt(1 - 2/(x + 1))), True))
```

Giac [A] time = 1.06955, size = 30, normalized size = 0.73

$$-\frac{\sqrt{x+1}(x-2)\sqrt{-x+1}}{3\,(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(x + 1)*(x - 2)*sqrt(-x + 1)/(x - 1)^2
```

3.1113 $\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{x+1}}{15\sqrt{1-x}}+\frac{2\sqrt{x+1}}{15(1-x)^{3/2}}+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}$$

```
[Out] Sqrt[1 + x]/(5*(1 - x)^(5/2)) + (2*Sqrt[1 + x])/(15*(1 - x)^(3/2)) + (2*Sqr
t[1 + x])/(15*Sqrt[1 - x])
```

Rubi [A] time = 0.0078398, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2\sqrt{x+1}}{15\sqrt{1-x}}+\frac{2\sqrt{x+1}}{15(1-x)^{3/2}}+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(5*(1 - x)^(5/2)) + (2*Sqrt[1 + x])/(15*(1 - x)^(3/2)) + (2*Sqr
t[1 + x])/(15*Sqrt[1 - x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{5(1-x)^{5/2}}+\frac{2}{5}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx\\
&= \frac{\sqrt{1+x}}{5(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{15(1-x)^{3/2}}+\frac{2}{15}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx\\
&= \frac{\sqrt{1+x}}{5(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{15(1-x)^{3/2}}+\frac{2\sqrt{1+x}}{15\sqrt{1-x}}
\end{aligned}$$

Mathematica [A] time = 0.0093584, size = 30, normalized size = 0.49

$$\frac{\sqrt{x+1}\left(2x^2-6x+7\right)}{15(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(7 - 6*x + 2*x^2))/(15*(1 - x)^(5/2))
```

Maple [A] time = 0.003, size = 25, normalized size = 0.4

$$\frac{2\,x^2-6\,x+7}{15}\sqrt{1+x}\,(1-x)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(7/2)/(1+x)^(1/2),x)
```

```
[Out] 1/15*(1+x)^(1/2)*(2*x^2-6*x+7)/(1-x)^(5/2)
```

Maxima [A] time = 1.4832, size = 86, normalized size = 1.41

$$-\frac{\sqrt{-x^2+1}}{5\left(x^3-3\,x^2+3\,x-1\right)}+\frac{2\,\sqrt{-x^2+1}}{15\left(x^2-2\,x+1\right)}-\frac{2\,\sqrt{-x^2+1}}{15\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) + 2/15*sqrt(-x^2 + 1)/(x^2 - 2*
x + 1) - 2/15*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A] time = 1.86757, size = 139, normalized size = 2.28

$$\frac{7\,x^3-21\,x^2-\left(2\,x^2-6\,x+7\right)\sqrt{x+1}\sqrt{-x+1}+21\,x-7}{15\left(x^3-3\,x^2+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*x^3 - 21*x^2 - (2*x^2 - 6*x + 7)*sqrt(x + 1)*sqrt(-x + 1) + 21*x -
7)/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [B] time = 35.3623, size = 301, normalized size = 4.93

$$\begin{cases} \frac{2(x+1)^2}{15\sqrt{-1+\frac{2}{x+1}}(x+1)^2-60\sqrt{-1+\frac{2}{x+1}}(x+1)+60\sqrt{-1+\frac{2}{x+1}}} - \frac{10(x+1)}{15\sqrt{-1+\frac{2}{x+1}}(x+1)^2-60\sqrt{-1+\frac{2}{x+1}}(x+1)+60\sqrt{-1+\frac{2}{x+1}}} + \frac{15}{15\sqrt{-1+\frac{2}{x+1}}(x+1)^2-60\sqrt{-1+\frac{2}{x+1}}(x+ \\ -\frac{2i(x+1)^2}{15\sqrt{1-\frac{2}{x+1}}(x+1)^2-60\sqrt{1-\frac{2}{x+1}}(x+1)+60\sqrt{1-\frac{2}{x+1}}} + \frac{10i(x+1)}{15\sqrt{1-\frac{2}{x+1}}(x+1)^2-60\sqrt{1-\frac{2}{x+1}}(x+1)+60\sqrt{1-\frac{2}{x+1}}} - \frac{15i}{15\sqrt{1-\frac{2}{x+1}}(x+1)^2-60\sqrt{1-\frac{2}{x+1}}(x+1)+60\sqrt{1-} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*(x + 1)**2/(15*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 60*sqrt(-1 +
2/(x + 1))*(x + 1) + 60*sqrt(-1 + 2/(x + 1))) - 10*(x + 1)/(15*sqrt(-1 + 2/
(x + 1))*(x + 1)**2 - 60*sqrt(-1 + 2/(x + 1))*(x + 1) + 60*sqrt(-1 + 2/(x +
 1))) + 15/(15*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 60*sqrt(-1 + 2/(x + 1))*(x
 + 1) + 60*sqrt(-1 + 2/(x + 1))), 2/Abs(x + 1) > 1), (-2*I*(x + 1)**2/(15*s
qrt(1 - 2/(x + 1))*(x + 1)**2 - 60*sqrt(1 - 2/(x + 1))*(x + 1) + 60*sqrt(1
- 2/(x + 1))) + 10*I*(x + 1)/(15*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 60*sqrt(1
 - 2/(x + 1))*(x + 1) + 60*sqrt(1 - 2/(x + 1))) - 15*I/(15*sqrt(1 - 2/(x +
1))*(x + 1)**2 - 60*sqrt(1 - 2/(x + 1))*(x + 1) + 60*sqrt(1 - 2/(x + 1))),
True))
```

Giac [A] time = 1.08045, size = 39, normalized size = 0.64

$$-\frac{(2(x+1)(x-4)+15)\sqrt{x+1}\sqrt{-x+1}}{15(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(2*(x + 1)*(x - 4) + 15)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^3
```

3.1114 $\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=81

$$\frac{2\sqrt{x+1}}{35\sqrt{1-x}}+\frac{2\sqrt{x+1}}{35(1-x)^{3/2}}+\frac{3\sqrt{x+1}}{35(1-x)^{5/2}}+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}$$

```
[Out] Sqrt[1 + x]/(7*(1 - x)^(7/2)) + (3*Sqrt[1 + x])/(35*(1 - x)^(5/2)) + (2*Sqr
t[1 + x])/(35*(1 - x)^(3/2)) + (2*Sqrt[1 + x])/(35*Sqrt[1 - x])
```

Rubi [A] time = 0.0138147, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2\sqrt{x+1}}{35\sqrt{1-x}}+\frac{2\sqrt{x+1}}{35(1-x)^{3/2}}+\frac{3\sqrt{x+1}}{35(1-x)^{5/2}}+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(7*(1 - x)^(7/2)) + (3*Sqrt[1 + x])/(35*(1 - x)^(5/2)) + (2*Sqr
t[1 + x])/(35*(1 - x)^(3/2)) + (2*Sqrt[1 + x])/(35*Sqrt[1 - x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3}{7}\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx\\
&= \frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3\sqrt{1+x}}{35(1-x)^{5/2}}+\frac{6}{35}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx\\
&= \frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3\sqrt{1+x}}{35(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{35(1-x)^{3/2}}+\frac{2}{35}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx\\
&= \frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3\sqrt{1+x}}{35(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{35(1-x)^{3/2}}+\frac{2\sqrt{1+x}}{35\sqrt{1-x}}
\end{aligned}$$

Mathematica [A] time = 0.010828, size = 35, normalized size = 0.43

$$\frac{\sqrt{x+1}\left(-2x^3+8x^2-13x+12\right)}{35(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(12 - 13*x + 8*x^2 - 2*x^3))/(35*(1 - x)^(7/2))
```

Maple [A] time = 0.003, size = 30, normalized size = 0.4

$$-\frac{2\,x^3-8\,x^2+13\,x-12}{35}\sqrt{1+x}\,(1-x)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(9/2)/(1+x)^(1/2),x)
```

```
[Out] -1/35*(1+x)^(1/2)*(2*x^3-8*x^2+13*x-12)/(1-x)^(7/2)
```

Maxima [A] time = 1.49483, size = 128, normalized size = 1.58

$$\frac{\sqrt{-x^2+1}}{7\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}-\frac{3\sqrt{-x^2+1}}{35\left(x^3-3\,x^2+3\,x-1\right)}+\frac{2\sqrt{-x^2+1}}{35\left(x^2-2\,x+1\right)}-\frac{2\sqrt{-x^2+1}}{35\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 3/35*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) + 2/35*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/35*sqrt(-x^
2 + 1)/(x - 1)
```

Fricas [A] time = 1.83633, size = 178, normalized size = 2.2

$$\frac{12\,x^4-48\,x^3+72\,x^2-\left(2\,x^3-8\,x^2+13\,x-12\right)\sqrt{x+1}\sqrt{-x+1}-48\,x+12}{35\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(12*x^4 - 48*x^3 + 72*x^2 - (2*x^3 - 8*x^2 + 13*x - 12)*sqrt(x + 1)*sq
rt(-x + 1) - 48*x + 12)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [B] time = 126.932, size = 541, normalized size = 6.68

$$\begin{cases} \frac{2(x+1)^3}{35\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420\sqrt{-1+\frac{2}{x+1}}(x+1)-280\sqrt{-1+\frac{2}{x+1}}} - \frac{14(x+1)^2}{35\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420\sqrt{-1+\frac{2}{x+1}}(x+1)-280\sqrt{-1+\frac{2}{x+1}}} \\ -\frac{2i(x+1)^3}{35\sqrt{1-\frac{2}{x+1}}(x+1)^3-210\sqrt{1-\frac{2}{x+1}}(x+1)^2+420\sqrt{1-\frac{2}{x+1}}(x+1)-280\sqrt{1-\frac{2}{x+1}}} + \frac{14i(x+1)^2}{35\sqrt{1-\frac{2}{x+1}}(x+1)^3-210\sqrt{1-\frac{2}{x+1}}(x+1)^2+420\sqrt{1-\frac{2}{x+1}}(x+1)-280\sqrt{1-\frac{2}{x+1}}} - \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*(x + 1)**3/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-1 +
 2/(x + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1 + 2
/(x + 1))) - 14*(x + 1)**2/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-
1 + 2/(x + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1
+ 2/(x + 1))) + 35*(x + 1)/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-
1 + 2/(x + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1
+ 2/(x + 1))) - 35/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-1 + 2/(x
 + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1 + 2/(x +
 1))), 2/Abs(x + 1) > 1), (-2*I*(x + 1)**3/(35*sqrt(1 - 2/(x + 1))*(x + 1)*
*3 - 210*sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1))*(x + 1) -
 280*sqrt(1 - 2/(x + 1))) + 14*I*(x + 1)**2/(35*sqrt(1 - 2/(x + 1))*(x + 1)
**3 - 210*sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1))*(x + 1)
- 280*sqrt(1 - 2/(x + 1))) - 35*I*(x + 1)/(35*sqrt(1 - 2/(x + 1))*(x + 1)**
3 - 210*sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1))*(x + 1) -
280*sqrt(1 - 2/(x + 1))) + 35*I/(35*sqrt(1 - 2/(x + 1))*(x + 1)**3 - 210*sq
rt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1))*(x + 1) - 280*sqrt(1
 - 2/(x + 1))), True))
```

Giac [A] time = 1.08594, size = 47, normalized size = 0.58

$$-\frac{((2(x+1)(x-6)+35)(x+1)-35)\sqrt{x+1}\sqrt{-x+1}}{35(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/35*((2*(x + 1)*(x - 6) + 35)*(x + 1) - 35)*sqrt(x + 1)*sqrt(-x + 1)/(x -
 1)^4
```

3.1115 $\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=101

$$\frac{8\sqrt{x+1}}{315\sqrt{1-x}}+\frac{8\sqrt{x+1}}{315(1-x)^{3/2}}+\frac{4\sqrt{x+1}}{105(1-x)^{5/2}}+\frac{4\sqrt{x+1}}{63(1-x)^{7/2}}+\frac{\sqrt{x+1}}{9(1-x)^{9/2}}$$

```
[Out] Sqrt[1 + x]/(9*(1 - x)^(9/2)) + (4*Sqrt[1 + x])/(63*(1 - x)^(7/2)) + (4*Sqr
t[1 + x])/(105*(1 - x)^(5/2)) + (8*Sqrt[1 + x])/(315*(1 - x)^(3/2)) + (8*Sq
rt[1 + x])/(315*Sqrt[1 - x])
```

Rubi [A] time = 0.0182229, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{8\sqrt{x+1}}{315\sqrt{1-x}}+\frac{8\sqrt{x+1}}{315(1-x)^{3/2}}+\frac{4\sqrt{x+1}}{105(1-x)^{5/2}}+\frac{4\sqrt{x+1}}{63(1-x)^{7/2}}+\frac{\sqrt{x+1}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(9*(1 - x)^(9/2)) + (4*Sqrt[1 + x])/(63*(1 - x)^(7/2)) + (4*Sqr
t[1 + x])/(105*(1 - x)^(5/2)) + (8*Sqrt[1 + x])/(315*(1 - x)^(3/2)) + (8*Sq
rt[1 + x])/(315*Sqrt[1 - x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{9(1-x)^{9/2}} + \frac{4}{9}\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1+x}}{9(1-x)^{9/2}} + \frac{4\sqrt{1+x}}{63(1-x)^{7/2}} + \frac{4}{21}\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1+x}}{9(1-x)^{9/2}} + \frac{4\sqrt{1+x}}{63(1-x)^{7/2}} + \frac{4\sqrt{1+x}}{105(1-x)^{5/2}} + \frac{8}{105}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1+x}}{9(1-x)^{9/2}} + \frac{4\sqrt{1+x}}{63(1-x)^{7/2}} + \frac{4\sqrt{1+x}}{105(1-x)^{5/2}} + \frac{8\sqrt{1+x}}{315(1-x)^{3/2}} + \frac{8}{315}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1+x}}{9(1-x)^{9/2}} + \frac{4\sqrt{1+x}}{63(1-x)^{7/2}} + \frac{4\sqrt{1+x}}{105(1-x)^{5/2}} + \frac{8\sqrt{1+x}}{315(1-x)^{3/2}} + \frac{8\sqrt{1+x}}{315\sqrt{1-x}}
\end{aligned}
$$

Mathematica [A] time = 0.0112795, size = 40, normalized size = 0.4

$$\frac{\sqrt{x+1}\left(8x^4-40x^3+84x^2-100x+83\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(83 - 100*x + 84*x^2 - 40*x^3 + 8*x^4))/(315*(1 - x)^(9/2))
```

Maple [A] time = 0.003, size = 35, normalized size = 0.4

$$\frac{8\,x^4-40\,x^3+84\,x^2-100\,x+83}{315}\sqrt{1+x}\,(1-x)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(11/2)/(1+x)^(1/2),x)
```

```
[Out] 1/315*(1+x)^(1/2)*(8*x^4-40*x^3+84*x^2-100*x+83)/(1-x)^(9/2)
```

Maxima [A] time = 1.50946, size = 177, normalized size = 1.75

$$-\frac{\sqrt{-x^2+1}}{9\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}+\frac{4\sqrt{-x^2+1}}{63\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}-\frac{4\sqrt{-x^2+1}}{105\left(x^3-3\,x^2+3\,x-1\right)}+\frac{8\sqrt{-x^2+1}}{315\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 4/63*sqrt(-
x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 4/105*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) + 8/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 8/315*sqrt(-x^2 + 1)/
(x - 1)
```

Fricas [A] time = 1.84438, size = 225, normalized size = 2.23

$$\frac{83x^5 - 415x^4 + 830x^3 - 830x^2 - (8x^4 - 40x^3 + 84x^2 - 100x + 83)\sqrt{x+1}\sqrt{-x+1} + 415x - 83}{315(x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(83*x^5 - 415*x^4 + 830*x^3 - 830*x^2 - (8*x^4 - 40*x^3 + 84*x^2 - 10
0*x + 83)*sqrt(x + 1)*sqrt(-x + 1) + 415*x - 83)/(x^5 - 5*x^4 + 10*x^3 - 10
*x^2 + 5*x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08185, size = 57, normalized size = 0.56

$$-\frac{(4((2(x+1)(x-8)+63)(x+1)-105)(x+1)+315)\sqrt{x+1}\sqrt{-x+1}}{315(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/315*(4*((2*(x + 1)*(x - 8) + 63)*(x + 1) - 105)*(x + 1) + 315)*sqrt(x +
1)*sqrt(-x + 1)/(x - 1)^5
```

3.1116 $\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}} dx$

Optimal. Leaf size=85

$$-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}-\frac{7}{3}\sqrt{x+1}(1-x)^{5/2}-\frac{35}{6}\sqrt{x+1}(1-x)^{3/2}-\frac{35}{2}\sqrt{x+1}\sqrt{1-x}-\frac{35}{2}\sin^{-1}(x)$$

```
[Out] (-2*(1 - x)^(7/2))/Sqrt[1 + x] - (35*Sqrt[1 - x]*Sqrt[1 + x])/2 - (35*(1 -
x)^(3/2)*Sqrt[1 + x])/6 - (7*(1 - x)^(5/2)*Sqrt[1 + x])/3 - (35*ArcSin[x])/
2
```

Rubi [A] time = 0.0162829, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}-\frac{7}{3}\sqrt{x+1}(1-x)^{5/2}-\frac{35}{6}\sqrt{x+1}(1-x)^{3/2}-\frac{35}{2}\sqrt{x+1}\sqrt{1-x}-\frac{35}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(7/2))/Sqrt[1 + x] - (35*Sqrt[1 - x]*Sqrt[1 + x])/2 - (35*(1 -
x)^(3/2)*Sqrt[1 + x])/6 - (7*(1 - x)^(5/2)*Sqrt[1 + x])/3 - (35*ArcSin[x])/
2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx &= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - 7\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,d \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [C] time = 0.0121307, size = 37, normalized size = 0.44

$$
-\frac{(1-x)^{9/2}\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{1-x}{2}\right)}{9\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/(1 + x)^(3/2),x]
```

```
[Out] -((1 - x)^(9/2)*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - x)/2])/(9*Sqrt[2])
```

Maple [A] time = 0.015, size = 84, normalized size = 1.

$$
\frac{2x^4-15x^3+68x^2+111x-166}{6}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{-(1+x)(-1+x)}}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}-\frac{35\arcsin(x)}{2}\sqrt{(1+x)(1-x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)/(1+x)^(3/2),x)
```

```
[Out] 1/6*(2*x^4-15*x^3+68*x^2+111*x-166)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/
2)/(1-x)^(1/2)/(1+x)^(1/2)-35/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)
*arcsin(x)
```

Maxima [A] time = 1.54404, size = 95, normalized size = 1.12

$$
\frac{x^4}{3\sqrt{-x^2+1}}-\frac{5x^3}{2\sqrt{-x^2+1}}+\frac{34x^2}{3\sqrt{-x^2+1}}+\frac{37x}{2\sqrt{-x^2+1}}-\frac{83}{3\sqrt{-x^2+1}}-\frac{35}{2}\arcsin(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^4/sqrt(-x^2 + 1) - 5/2*x^3/sqrt(-x^2 + 1) + 34/3*x^2/sqrt(-x^2 + 1) +
 37/2*x/sqrt(-x^2 + 1) - 83/3/sqrt(-x^2 + 1) - 35/2*arcsin(x)
```

Fricas [A] time = 1.85903, size = 189, normalized size = 2.22

$$-\frac{\left(2\,x^{3}-13\,x^{2}+55\,x+166\right)\sqrt{x+1}\sqrt{-x+1}-210\,(x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+166\,x+166}{6\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*((2*x^3 - 13*x^2 + 55*x + 166)*sqrt(x + 1)*sqrt(-x + 1) - 210*(x + 1)*
arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 166*x + 166)/(x + 1)
```

Sympy [A] time = 61.9999, size = 207, normalized size = 2.44

$$\begin{cases} 35i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}+\frac{23i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}-\frac{125i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{13i\sqrt{x+1}}{\sqrt{x-1}}+\frac{32i}{\sqrt{x-1}\sqrt{x+1}} & \text{for}\ \frac{|x+1|}{2}>1\\ -35\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}-\frac{23(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{125(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{13\sqrt{x+1}}{\sqrt{1-x}}-\frac{32}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((35*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) + 23*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 125*I*(x + 1)**(3/2)/(6*sqrt(x
 - 1)) + 13*I*sqrt(x + 1)/sqrt(x - 1) + 32*I/(sqrt(x - 1)*sqrt(x + 1)), Abs
(x + 1)/2 > 1), (-35*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(7/2)/(3*sqrt(1
 - x)) - 23*(x + 1)**(5/2)/(6*sqrt(1 - x)) + 125*(x + 1)**(3/2)/(6*sqrt(1 -
 x)) - 13*sqrt(x + 1)/sqrt(1 - x) - 32/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Giac [A] time = 1.30713, size = 109, normalized size = 1.28

$$-\frac{1}{6}\left((2\,x-17)(x+1)+87\right)\sqrt{x+1}\sqrt{-x+1}+\frac{8\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}-\frac{8\,\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-35\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*((2*x - 17)*(x + 1) + 87)*sqrt(x + 1)*sqrt(-x + 1) + 8*(sqrt(2) - sqrt
(-x + 1))/sqrt(x + 1) - 8*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 35*arcsin(
1/2*sqrt(2)*sqrt(x + 1))
```

3.1117 $\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}} \, dx$

Optimal. Leaf size=65

$$-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}-\frac{5}{2}\sqrt{x+1}(1-x)^{3/2}-\frac{15}{2}\sqrt{x+1}\sqrt{1-x}-\frac{15}{2}\sin^{-1}(x)$$

```
[Out] (-2*(1 - x)^(5/2))/Sqrt[1 + x] - (15*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*(1 - x
)^(3/2)*Sqrt[1 + x])/2 - (15*ArcSin[x])/2
```

Rubi [A] time = 0.0108989, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}-\frac{5}{2}\sqrt{x+1}(1-x)^{3/2}-\frac{15}{2}\sqrt{x+1}\sqrt{1-x}-\frac{15}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(5/2))/Sqrt[1 + x] - (15*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*(1 - x
)^(3/2)*Sqrt[1 + x])/2 - (15*ArcSin[x])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx &= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - 5\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\ &= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\ &= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{15}{2}\sqrt{1-x}\sqrt{1+x} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx \\ &= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{15}{2}\sqrt{1-x}\sqrt{1+x} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{15}{2}\sqrt{1-x}\sqrt{1+x} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [C] time = 0.0102438, size = 37, normalized size = 0.57

$$-\frac{(1-x)^{7/2}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{1-x}{2}\right)}{7\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/(1 + x)^(3/2),x]
```

```
[Out] -((1 - x)^(7/2)*Hypergeometric2F1[3/2, 7/2, 9/2, (1 - x)/2])/(7*Sqrt[2])
```

Maple [A] time = 0.012, size = 77, normalized size = 1.2

$$-\frac{x^3-8x^2-17x+24}{2}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{-(1+x)(-1+x)}}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}-\frac{15\arcsin(x)}{2}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)/(1+x)^(3/2),x)
```

```
[Out] -1/2*(x^3-8*x^2-17*x+24)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1
/2)/(1+x)^(1/2)-15/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.47965, size = 76, normalized size = 1.17

$$-\frac{x^3}{2\sqrt{-x^2+1}}+\frac{4x^2}{\sqrt{-x^2+1}}+\frac{17x}{2\sqrt{-x^2+1}}-\frac{12}{\sqrt{-x^2+1}}-\frac{15}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^3/sqrt(-x^2 + 1) + 4*x^2/sqrt(-x^2 + 1) + 17/2*x/sqrt(-x^2 + 1) - 12
/sqrt(-x^2 + 1) - 15/2*arcsin(x)
```

Fricas [A] time = 1.79065, size = 166, normalized size = 2.55

$$\frac{\left(x^2-7x-24\right)\sqrt{x+1}\sqrt{-x+1}+30(x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-24x-24}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 7*x - 24)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x + 1)*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x) - 24*x - 24)/(x + 1)
```

Sympy [A] time = 13.6217, size = 168, normalized size = 2.58

$$\begin{cases} 15i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}-\frac{11i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}}+\frac{16i}{\sqrt{x-1}\sqrt{x+1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ -15\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}+\frac{11(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}}-\frac{16}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((15*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) - 11*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + I*sqrt(x + 1)/sqrt(x - 1) + 16
*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1)/2 > 1), (-15*asin(sqrt(2)*sqrt(x +
 1)/2) - (x + 1)**(5/2)/(2*sqrt(1 - x)) + 11*(x + 1)**(3/2)/(2*sqrt(1 - x))
 - sqrt(x + 1)/sqrt(1 - x) - 16/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Giac [A] time = 1.10505, size = 99, normalized size = 1.52

$$\frac{1}{2}\sqrt{x+1}(x-8)\sqrt{-x+1}+\frac{4\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}-\frac{4\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-15\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + 1)*(x - 8)*sqrt(-x + 1) + 4*(sqrt(2) - sqrt(-x + 1))/sqrt(x +
1) - 4*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 15*arcsin(1/2*sqrt(2)*sqrt(x
+ 1))
```

3.1118 $\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{2(1-x)^{3/2}}{\sqrt{x+1}}-3\sqrt{x+1}\sqrt{1-x}-3\sin^{-1}(x)$$

```
[Out] (-2*(1 - x)^(3/2))/Sqrt[1 + x] - 3*Sqrt[1 - x]*Sqrt[1 + x] - 3*ArcSin[x]
```

Rubi [A] time = 0.0065795, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{3/2}}{\sqrt{x+1}}-3\sqrt{x+1}\sqrt{1-x}-3\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(3/2))/Sqrt[1 + x] - 3*Sqrt[1 - x]*Sqrt[1 + x] - 3*ArcSin[x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx &= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}} - 3\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}} - 3\sqrt{1-x}\sqrt{1+x} - 3\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}} - 3\sqrt{1-x}\sqrt{1+x} - 3\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}} - 3\sqrt{1-x}\sqrt{1+x} - 3\sin^{-1}(x)
\end{aligned}
$$

Mathematica [C] time = 0.0085057, size = 37, normalized size = 0.9

$$
-\frac{(1-x)^{5/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{1-x}{2}\right)}{5\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/(1 + x)^(3/2),x]
```

```
[Out] -((1 - x)^(5/2)*Hypergeometric2F1[3/2, 5/2, 7/2, (1 - x)/2])/(5*Sqrt[2])
```

Maple [B] time = 0.013, size = 71, normalized size = 1.7

$$
(x^2+4x-5)\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{-(1+x)(-1+x)}}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}} - 3\frac{\sqrt{(1+x)(1-x)}\arcsin(x)}{\sqrt{1-x}\sqrt{1+x}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)/(1+x)^(3/2),x)
```

```
[Out] (x^2+4*x-5)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)/(1+x)^(1/
2)-3*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.57728, size = 55, normalized size = 1.34

$$
\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^2+2x+1} - \frac{6\sqrt{-x^2+1}}{x+1} - 3\arcsin(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(3/2)/(x^2 + 2*x + 1) - 6*sqrt(-x^2 + 1)/(x + 1) - 3*arcsin(x)
```

Fricas [A] time = 1.85204, size = 146, normalized size = 3.56

$$
-\frac{(x+5)\sqrt{x+1}\sqrt{-x+1} - 6(x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + 5x + 5}{x+1}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -((x + 5)*sqrt(x + 1)*sqrt(-x + 1) - 6*(x + 1)*arctan((sqrt(x + 1)*sqrt(-x
+ 1) - 1)/x) + 5*x + 5)/(x + 1)
```

Sympy [A] time = 2.81974, size = 133, normalized size = 3.24

$$\begin{cases} 6i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}} - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} + \frac{8i}{\sqrt{x-1}\sqrt{x+1}} & \text{for } \frac{|x+1|}{2} > 1 \\ -6\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} + \frac{2\sqrt{x+1}}{\sqrt{1-x}} - \frac{8}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((6*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(3/2)/sqrt(x - 1)
- 2*I*sqrt(x + 1)/sqrt(x - 1) + 8*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1)/2
 > 1), (-6*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(3/2)/sqrt(1 - x) + 2*sqr
t(x + 1)/sqrt(1 - x) - 8/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Giac [B] time = 1.0769, size = 95, normalized size = 2.32

$$-\sqrt{x+1}\sqrt{-x+1} + \frac{2\left(\sqrt{2} - \sqrt{-x+1}\right)}{\sqrt{x+1}} - \frac{2\sqrt{x+1}}{\sqrt{2} - \sqrt{-x+1}} - 6\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(x + 1)*sqrt(-x + 1) + 2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 2*sqrt
(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 6*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1119 $\int \frac{\sqrt{1-x}}{(1+x)^{3/2}} \, dx$

Optimal. Leaf size=23

$$-\frac{2\sqrt{1-x}}{\sqrt{x+1}}-\sin^{-1}(x)$$

```
[Out] (-2*Sqrt[1 - x])/Sqrt[1 + x] - ArcSin[x]
```

Rubi [A] time = 0.0033808, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$-\frac{2\sqrt{1-x}}{\sqrt{x+1}}-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 + x)^(3/2),x]

[Out] (-2*Sqrt[1 - x])/Sqrt[1 + x] - ArcSin[x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x}}{(1+x)^{3/2}} \, dx &= -\frac{2\sqrt{1-x}}{\sqrt{1+x}}-\int \frac{1}{\sqrt{1-x}\sqrt{1+x}} \, dx\\
&= -\frac{2\sqrt{1-x}}{\sqrt{1+x}}-\int \frac{1}{\sqrt{1-x^2}} \, dx\\
&= -\frac{2\sqrt{1-x}}{\sqrt{1+x}}-\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0368303, size = 34, normalized size = 1.48

$$2\left(\frac{x-1}{\sqrt{1-x^2}}+\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + x)^(3/2),x]
```

```
[Out] 2*((-1 + x)/Sqrt[1 - x^2] + ArcSin[Sqrt[1 - x]/Sqrt[2]])
```

Maple [B] time = 0.011, size = 67, normalized size = 2.9

$$2\,\frac{(-1+x)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\sqrt{1-x}\sqrt{1+x}}-\arcsin(x)\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(1+x)^(3/2),x)
```

```
[Out] 2*(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)-
((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.53654, size = 28, normalized size = 1.22

$$-\frac{2\sqrt{-x^2+1}}{x+1}-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/(x + 1) - arcsin(x)
```

Fricas [B] time = 1.76957, size = 131, normalized size = 5.7

$$\frac{2\left((x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-x-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - x - sqrt(x + 1)*sqrt(
-x + 1) - 1)/(x + 1)
```

Sympy [B] time = 1.46917, size = 104, normalized size = 4.52

$$\begin{cases} 2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} + \frac{4i}{\sqrt{x-1}\sqrt{x+1}} & \text{for } \frac{|x+1|}{2} > 1 \\ -2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{2\sqrt{x+1}}{\sqrt{1-x}} - \frac{4}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - 2*I*sqrt(x + 1)/sqrt(x - 1) +
 4*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1)/2 > 1), (-2*asin(sqrt(2)*sqrt(x
+ 1)/2) + 2*sqrt(x + 1)/sqrt(1 - x) - 4/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Giac [B] time = 1.06955, size = 74, normalized size = 3.22

$$\frac{\sqrt{2} - \sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2} - \sqrt{-x+1}} - 2 \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
 - 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1120 $\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}} \, dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

Rubi [A] time = 0.0015644, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}} \, dx = -\frac{\sqrt{1-x}}{\sqrt{1+x}}$$

Mathematica [A] time = 0.0034147, size = 18, normalized size = 1.

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$-\sqrt{1-x}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/2)/(1+x)^(3/2),x)
```

```
[Out] -(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A] time = 1.55, size = 22, normalized size = 1.22

$$-\frac{\sqrt{-x^2+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/(x + 1)
```

Fricas [A] time = 1.7723, size = 61, normalized size = 3.39

$$-\frac{x+\sqrt{x+1}\sqrt{-x+1}+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -(x + sqrt(x + 1)*sqrt(-x + 1) + 1)/(x + 1)
```

Sympy [A] time = 1.07564, size = 29, normalized size = 1.61

$$\begin{cases} -\sqrt{-1+\frac{2}{x+1}} & \text{for}\ \frac{2}{|x+1|}>1 \\ -i\sqrt{1-\frac{2}{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1)), 2/Abs(x + 1) > 1), (-I*sqrt(1 - 2/(x + 1)
), True))
```

Giac [B] time = 1.07944, size = 58, normalized size = 3.22

$$\frac{\sqrt{2}-\sqrt{-x+1}}{2\sqrt{x+1}}-\frac{\sqrt{x+1}}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/2*sqrt(x + 1)/(sqrt(2) - sqrt(
-x + 1))
```

3.1121 $\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=18

$$\frac{x}{\sqrt{1-x}\sqrt{x+1}}$$

```
[Out] x/(Sqrt[1 - x]*Sqrt[1 + x])
```

Rubi [A] time = 0.0016086, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {39}

$$\frac{x}{\sqrt{1-x}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] x/(Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx = \frac{x}{\sqrt{1-x}\sqrt{1+x}}$$

Mathematica [A] time = 0.0027818, size = 13, normalized size = 0.72

$$\frac{x}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] x/Sqrt[1 - x^2]
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$x\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(3/2)/(1+x)^(3/2),x)
```

```
[Out] x/(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A] time = 1.01873, size = 15, normalized size = 0.83

$$\frac{x}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(-x^2 + 1)
```

Fricas [A] time = 1.74749, size = 53, normalized size = 2.94

$$-\frac{\sqrt{x+1}x\sqrt{-x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(x + 1)*x*sqrt(-x + 1)/(x^2 - 1)
```

Sympy [A] time = 2.71814, size = 65, normalized size = 3.61

$$\begin{cases} \frac{1}{\sqrt{-1+\frac{2}{x+1}}} - \frac{1}{\sqrt{-1+\frac{2}{x+1}}(x+1)} & \text{for}\ \frac{2}{|x+1|} > 1 \\ -\frac{i\sqrt{1-\frac{2}{x+1}}(x+1)}{x-1} + \frac{i\sqrt{1-\frac{2}{x+1}}}{x-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((1/sqrt(-1 + 2/(x + 1)) - 1/(sqrt(-1 + 2/(x + 1))*(x + 1)), 2/Abs
(x + 1) > 1), (-I*sqrt(1 - 2/(x + 1))*(x + 1)/(x - 1) + I*sqrt(1 - 2/(x + 1
))/(x - 1), True))
```

Giac [B] time = 1.06418, size = 84, normalized size = 4.67

$$\frac{\sqrt{2}-\sqrt{-x+1}}{4\sqrt{x+1}} - \frac{\sqrt{x+1}\sqrt{-x+1}}{2(x-1)} - \frac{\sqrt{x+1}}{4\left(\sqrt{2}-\sqrt{-x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/2*sqrt(x + 1)*sqrt(-x + 1)/(x
- 1) - 1/4*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
```

3.1122 $\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{2x}{3\sqrt{1-x}\sqrt{x+1}}+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}$$

```
[Out] 1/(3*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(3*Sqrt[1 - x]*Sqrt[1 + x])
```

Rubi [A] time = 0.0045252, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 39}

$$\frac{2x}{3\sqrt{1-x}\sqrt{x+1}}+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]

[Out] 1/(3*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(3*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx &= \frac{1}{3(1-x)^{3/2}\sqrt{1+x}}+\frac{2}{3}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\ &= \frac{1}{3(1-x)^{3/2}\sqrt{1+x}}+\frac{2x}{3\sqrt{1-x}\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.007004, size = 30, normalized size = 0.71

$$\frac{2x^2-2x-1}{3(x-1)\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]
```

```
[Out] (-1 - 2*x + 2*x^2)/(3*(-1 + x)*Sqrt[1 - x^2])
```

Maple [A] time = 0.002, size = 25, normalized size = 0.6

$$-\frac{2x^2-2x-1}{3}(1-x)^{-\frac{3}{2}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(5/2)/(1+x)^(3/2),x)
```

```
[Out] -1/3*(2*x^2-2*x-1)/(1+x)^(1/2)/(1-x)^(3/2)
```

Maxima [A] time = 1.04189, size = 54, normalized size = 1.29

$$\frac{2x}{3\sqrt{-x^2+1}}-\frac{1}{3\left(\sqrt{-x^2+1}x-\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(-x^2 + 1) - 1/3/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Fricas [A] time = 1.74559, size = 122, normalized size = 2.9

$$\frac{x^3-x^2-\left(2x^2-2x-1\right)\sqrt{x+1}\sqrt{-x+1}-x+1}{3\left(x^3-x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 - x^2 - (2*x^2 - 2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) - x + 1)/(x^3 -
 x^2 - x + 1)
```

Sympy [B] time = 17.5289, size = 158, normalized size = 3.76

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-12x+3(x+1)^2}+\frac{6\sqrt{-1+\frac{2}{x+1}}(x+1)}{-12x+3(x+1)^2}-\frac{3\sqrt{-1+\frac{2}{x+1}}}{-12x+3(x+1)^2} & \text{for } \frac{2}{|x+1|}>1 \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-12x+3(x+1)^2}+\frac{6i\sqrt{1-\frac{2}{x+1}}(x+1)}{-12x+3(x+1)^2}-\frac{3i\sqrt{1-\frac{2}{x+1}}}{-12x+3(x+1)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-12*x + 3*(x + 1)**2) + 6*sq
rt(-1 + 2/(x + 1))*(x + 1)/(-12*x + 3*(x + 1)**2) - 3*sqrt(-1 + 2/(x + 1))/
(-12*x + 3*(x + 1)**2), 2/Abs(x + 1) > 1), (-2*I*sqrt(1 - 2/(x + 1))*(x + 1
)**2/(-12*x + 3*(x + 1)**2) + 6*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-12*x + 3*(x
 + 1)**2) - 3*I*sqrt(1 - 2/(x + 1))/(-12*x + 3*(x + 1)**2), True))
```

Giac [B] time = 1.07, size = 90, normalized size = 2.14

$$\frac{\sqrt{2}-\sqrt{-x+1}}{8\sqrt{x+1}}-\frac{(5x-7)\sqrt{x+1}\sqrt{-x+1}}{12(x-1)^2}-\frac{\sqrt{x+1}}{8\left(\sqrt{2}-\sqrt{-x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/12*(5*x - 7)*sqrt(x + 1)*sqrt(
-x + 1)/(x - 1)^2 - 1/8*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
```

3.1123 $\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{2x}{5\sqrt{1-x}\sqrt{x+1}}+\frac{1}{5(1-x)^{3/2}\sqrt{x+1}}+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}$$

```
[Out] 1/(5*(1 - x)^(5/2)*Sqrt[1 + x]) + 1/(5*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(
5*Sqrt[1 - x]*Sqrt[1 + x])
```

Rubi [A] time = 0.008368, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 39}

$$\frac{2x}{5\sqrt{1-x}\sqrt{x+1}}+\frac{1}{5(1-x)^{3/2}\sqrt{x+1}}+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(5*(1 - x)^(5/2)*Sqrt[1 + x]) + 1/(5*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(
5*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx &= \frac{1}{5(1-x)^{5/2}\sqrt{1+x}}+\frac{3}{5}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx\\
&= \frac{1}{5(1-x)^{5/2}\sqrt{1+x}}+\frac{1}{5(1-x)^{3/2}\sqrt{1+x}}+\frac{2}{5}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\
&= \frac{1}{5(1-x)^{5/2}\sqrt{1+x}}+\frac{1}{5(1-x)^{3/2}\sqrt{1+x}}+\frac{2x}{5\sqrt{1-x}\sqrt{1+x}}
\end{aligned}$$

Mathematica [A] time = 0.0084147, size = 33, normalized size = 0.53

$$\frac{2x^3-4x^2+x+2}{5(x-1)^2\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

```
[Out] (2 + x - 4*x^2 + 2*x^3)/(5*(-1 + x)^2*Sqrt[1 - x^2])
```

Maple [A] time = 0.003, size = 28, normalized size = 0.5

$$\frac{2x^3-4x^2+x+2}{5}(1-x)^{-\frac{5}{2}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(7/2)/(1+x)^(3/2),x)
```

```
[Out] 1/5*(2*x^3-4*x^2+x+2)/(1+x)^(1/2)/(1-x)^(5/2)
```

Maxima [A] time = 1.06284, size = 107, normalized size = 1.73

$$\frac{2x}{5\sqrt{-x^2+1}}+\frac{1}{5\left(\sqrt{-x^2+1}x^2-2\sqrt{-x^2+1}x+\sqrt{-x^2+1}\right)}-\frac{1}{5\left(\sqrt{-x^2+1}x-\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x/sqrt(-x^2 + 1) + 1/5/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2 + 1)*x + sqrt(
-x^2 + 1)) - 1/5/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Fricas [A] time = 1.94946, size = 143, normalized size = 2.31

$$\frac{2x^4-4x^3-\left(2x^3-4x^2+x+2\right)\sqrt{x+1}\sqrt{-x+1}+4x-2}{5\left(x^4-2x^3+2x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*x^4 - 4*x^3 - (2*x^3 - 4*x^2 + x + 2)*sqrt(x + 1)*sqrt(-x + 1) + 4*x
 - 2)/(x^4 - 2*x^3 + 2*x - 1)
```

Sympy [B] time = 70.1819, size = 282, normalized size = 4.55

$$\begin{cases}-\frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{60x+5(x+1)^3-30(x+1)^2+20}+\frac{10\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{60x+5(x+1)^3-30(x+1)^2+20}-\frac{15\sqrt{-1+\frac{2}{x+1}}(x+1)}{60x+5(x+1)^3-30(x+1)^2+20}+\frac{5\sqrt{-1+\frac{2}{x+1}}}{60x+5(x+1)^3-30(x+1)^2+20} & \text{for } \frac{2}{|x+1|}>1\\ -\frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{60x+5(x+1)^3-30(x+1)^2+20}+\frac{10i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{60x+5(x+1)^3-30(x+1)^2+20}-\frac{15i\sqrt{1-\frac{2}{x+1}}(x+1)}{60x+5(x+1)^3-30(x+1)^2+20}+\frac{5i\sqrt{1-\frac{2}{x+1}}}{60x+5(x+1)^3-30(x+1)^2+20} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(60*x + 5*(x + 1)**3 - 30*(x
+ 1)**2 + 20) + 10*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(60*x + 5*(x + 1)**3 - 3
0*(x + 1)**2 + 20) - 15*sqrt(-1 + 2/(x + 1))*(x + 1)/(60*x + 5*(x + 1)**3 -
 30*(x + 1)**2 + 20) + 5*sqrt(-1 + 2/(x + 1))/(60*x + 5*(x + 1)**3 - 30*(x
+ 1)**2 + 20), 2/Abs(x + 1) > 1), (-2*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(60*
x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20) + 10*I*sqrt(1 - 2/(x + 1))*(x + 1)**
2/(60*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20) - 15*I*sqrt(1 - 2/(x + 1))*(x
+ 1)/(60*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20) + 5*I*sqrt(1 - 2/(x + 1))/(
60*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20), True))
```

Giac [A] time = 1.0699, size = 99, normalized size = 1.6

$$\frac{\sqrt{2}-\sqrt{-x+1}}{16\sqrt{x+1}}-\frac{\sqrt{x+1}}{16\left(\sqrt{2}-\sqrt{-x+1}\right)}-\frac{((11\,x-39)(x+1)+60)\sqrt{x+1}\sqrt{-x+1}}{40\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/16*sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)) - 1/40*((11*x - 39)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x -
1)^3
```

3.1124 $\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}} dx$

Optimal. Leaf size=82

$$\frac{8x}{35\sqrt{1-x}\sqrt{x+1}}+\frac{4}{35(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{35(1-x)^{5/2}\sqrt{x+1}}+\frac{1}{7(1-x)^{7/2}\sqrt{x+1}}$$

```
[Out] 1/(7*(1 - x)^(7/2)*Sqrt[1 + x]) + 4/(35*(1 - x)^(5/2)*Sqrt[1 + x]) + 4/(35*
(1 - x)^(3/2)*Sqrt[1 + x]) + (8*x)/(35*Sqrt[1 - x]*Sqrt[1 + x])
```

Rubi [A] time = 0.0134444, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 39}

$$\frac{8x}{35\sqrt{1-x}\sqrt{x+1}}+\frac{4}{35(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{35(1-x)^{5/2}\sqrt{x+1}}+\frac{1}{7(1-x)^{7/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]
```


```
[Out] 1/(7*(1 - x)^(7/2)*Sqrt[1 + x]) + 4/(35*(1 - x)^(5/2)*Sqrt[1 + x]) + 4/(35*
(1 - x)^(3/2)*Sqrt[1 + x]) + (8*x)/(35*Sqrt[1 - x]*Sqrt[1 + x])
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}} dx &= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{7}\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}} dx\\
&= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{5/2}\sqrt{1+x}}+\frac{12}{35}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}} dx\\
&= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{3/2}\sqrt{1+x}}+\frac{8}{35}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}} dx\\
&= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{3/2}\sqrt{1+x}}+\frac{8x}{35\sqrt{1-x}\sqrt{1+x}}
\end{aligned}$$

Mathematica [A] time = 0.0096104, size = 40, normalized size = 0.49

$$\frac{8x^4-24x^3+20x^2+4x-13}{35(x-1)^3\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]
```

```
[Out] (-13 + 4*x + 20*x^2 - 24*x^3 + 8*x^4)/(35*(-1 + x)^3*Sqrt[1 - x^2])
```

Maple [A] time = 0.003, size = 35, normalized size = 0.4

$$-\frac{8x^4-24x^3+20x^2+4x-13}{35}(1-x)^{-\frac{7}{2}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(9/2)/(1+x)^(3/2),x)
```

```
[Out] -1/35*(8*x^4-24*x^3+20*x^2+4*x-13)/(1+x)^(1/2)/(1-x)^(7/2)
```

Maxima [B] time = 1.00894, size = 181, normalized size = 2.21

$$\frac{8x}{35\sqrt{-x^2+1}}-\frac{1}{7\left(\sqrt{-x^2+1}x^3-3\sqrt{-x^2+1}x^2+3\sqrt{-x^2+1}x-\sqrt{-x^2+1}\right)}+\frac{4}{35\left(\sqrt{-x^2+1}x^2-2\sqrt{-x^2+1}x+\sqrt{-x^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/35*x/sqrt(-x^2 + 1) - 1/7/(sqrt(-x^2 + 1)*x^3 - 3*sqrt(-x^2 + 1)*x^2 + 3*
sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1)) + 4/35/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2
 + 1)*x + sqrt(-x^2 + 1)) - 4/35/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Fricas [A] time = 1.80086, size = 213, normalized size = 2.6

$$\frac{13x^5-39x^4+26x^3+26x^2-\left(8x^4-24x^3+20x^2+4x-13\right)\sqrt{x+1}\sqrt{-x+1}-39x+13}{35\left(x^5-3x^4+2x^3+2x^2-3x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(13*x^5 - 39*x^4 + 26*x^3 + 26*x^2 - (8*x^4 - 24*x^3 + 20*x^2 + 4*x -
13)*sqrt(x + 1)*sqrt(-x + 1) - 39*x + 13)/(x^5 - 3*x^4 + 2*x^3 + 2*x^2 - 3*
x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08629, size = 107, normalized size = 1.3

$$\frac{\sqrt{2}-\sqrt{-x+1}}{32\sqrt{x+1}}-\frac{\sqrt{x+1}}{32\left(\sqrt{2}-\sqrt{-x+1}\right)}-\frac{\left(\left(\left(93\,x-523\right)\left(x+1\right)+1400\right)\left(x+1\right)-1120\right)\sqrt{x+1}\sqrt{-x+1}}{560\,\left(x-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/32*sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)) - 1/560*(((93*x - 523)*(x + 1) + 1400)*(x + 1) - 1120)*sqrt(x +
1)*sqrt(-x + 1)/(x - 1)^4
```

3.1125 $\int \frac{1}{(1-x)^{11/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=102

$$\frac{8x}{63\sqrt{1-x}\sqrt{x+1}}+\frac{4}{63(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{63(1-x)^{5/2}\sqrt{x+1}}+\frac{5}{63(1-x)^{7/2}\sqrt{x+1}}+\frac{1}{9(1-x)^{9/2}\sqrt{x+1}}$$

```
[Out] 1/(9*(1 - x)^(9/2)*Sqrt[1 + x]) + 5/(63*(1 - x)^(7/2)*Sqrt[1 + x]) + 4/(63*
(1 - x)^(5/2)*Sqrt[1 + x]) + 4/(63*(1 - x)^(3/2)*Sqrt[1 + x]) + (8*x)/(63*S
qrt[1 - x]*Sqrt[1 + x])
```

Rubi [A] time = 0.0203065, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 39}

$$\frac{8x}{63\sqrt{1-x}\sqrt{x+1}}+\frac{4}{63(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{63(1-x)^{5/2}\sqrt{x+1}}+\frac{5}{63(1-x)^{7/2}\sqrt{x+1}}+\frac{1}{9(1-x)^{9/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(9*(1 - x)^(9/2)*Sqrt[1 + x]) + 5/(63*(1 - x)^(7/2)*Sqrt[1 + x]) + 4/(63*
(1 - x)^(5/2)*Sqrt[1 + x]) + 4/(63*(1 - x)^(3/2)*Sqrt[1 + x]) + (8*x)/(63*S
qrt[1 - x]*Sqrt[1 + x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-x)^{11/2}(1+x)^{3/2}}\,dx &= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}}+\frac{5}{9}\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx\\
&= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}}+\frac{5}{63(1-x)^{7/2}\sqrt{1+x}}+\frac{20}{63}\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx\\
&= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}}+\frac{5}{63(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{21}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\\
&= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}}+\frac{5}{63(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{3/2}\sqrt{1+x}}+\frac{8}{63}\\
&= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}}+\frac{5}{63(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{3/2}\sqrt{1+x}}+\frac{}{63}
\end{aligned}$$

Mathematica [A] time = 0.0115801, size = 45, normalized size = 0.44

$$\frac{8x^5-32x^4+44x^3-16x^2-17x+20}{63(x-1)^4\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*(1 + x)^(3/2)),x]
```

```
[Out] (20 - 17*x - 16*x^2 + 44*x^3 - 32*x^4 + 8*x^5)/(63*(-1 + x)^4*Sqrt[1 - x^2]
)
```

Maple [A] time = 0.003, size = 40, normalized size = 0.4

$$\frac{8\,x^5-32\,x^4+44\,x^3-16\,x^2-17\,x+20}{63}(1-x)^{-\frac{9}{2}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(11/2)/(1+x)^(3/2),x)
```

```
[Out] 1/63*(8*x^5-32*x^4+44*x^3-16*x^2-17*x+20)/(1+x)^(1/2)/(1-x)^(9/2)
```

Maxima [B] time = 1.03223, size = 271, normalized size = 2.66

$$\frac{8\,x}{63\,\sqrt{-x^2+1}}+\frac{1}{9\left(\sqrt{-x^2+1}x^4-4\,\sqrt{-x^2+1}x^3+6\,\sqrt{-x^2+1}x^2-4\,\sqrt{-x^2+1}x+\sqrt{-x^2+1}\right)}-\frac{}{63\left(\sqrt{-x^2+1}x^3-3\,\sqrt{-x^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/63*x/sqrt(-x^2 + 1) + 1/9/(sqrt(-x^2 + 1)*x^4 - 4*sqrt(-x^2 + 1)*x^3 + 6*
sqrt(-x^2 + 1)*x^2 - 4*sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1)) - 5/63/(sqrt(-x^2
 + 1)*x^3 - 3*sqrt(-x^2 + 1)*x^2 + 3*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1)) + 4
/63/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1)) - 4/63/(sqrt
(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Fricas [A] time = 1.7967, size = 230, normalized size = 2.25

$$\frac{20\,x^6-80\,x^5+100\,x^4-100\,x^2-\left(8\,x^5-32\,x^4+44\,x^3-16\,x^2-17\,x+20\right)\sqrt{x+1}\sqrt{-x+1}+80\,x-20}{63\left(x^6-4\,x^5+5\,x^4-5\,x^2+4\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/63*(20*x^6 - 80*x^5 + 100*x^4 - 100*x^2 - (8*x^5 - 32*x^4 + 44*x^3 - 16*x
^2 - 17*x + 20)*sqrt(x + 1)*sqrt(-x + 1) + 80*x - 20)/(x^6 - 4*x^5 + 5*x^4
```

```
- 5*x^2 + 4*x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.06085, size = 115, normalized size = 1.13

$$\frac{\sqrt{2}-\sqrt{-x+1}}{64\sqrt{x+1}}-\frac{\sqrt{x+1}}{64\left(\sqrt{2}-\sqrt{-x+1}\right)}-\frac{((((193x-1481)(x+1)+5544)(x+1)-8400)(x+1)+5040)\sqrt{x+1}\sqrt{-x+1}}{2016(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/64*sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)) - 1/2016*((((193*x - 1481)*(x + 1) + 5544)*(x + 1) - 8400)*(x +
1) + 5040)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^5
```

3.1126 $\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{x+1}}+7\sqrt{x+1}(1-x)^{5/2}+\frac{35}{2}\sqrt{x+1}(1-x)^{3/2}+\frac{105}{2}\sqrt{x+1}\sqrt{1-x}+\frac{105}{2}\sin^{-1}(x)$$

```
[Out] (-2*(1 - x)^(9/2))/(3*(1 + x)^(3/2)) + (6*(1 - x)^(7/2))/Sqrt[1 + x] + (105
*Sqrt[1 - x]*Sqrt[1 + x])/2 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/2 + 7*(1 - x)^
(5/2)*Sqrt[1 + x] + (105*ArcSin[x])/2
```

Rubi [A] time = 0.0214789, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{x+1}}+7\sqrt{x+1}(1-x)^{5/2}+\frac{35}{2}\sqrt{x+1}(1-x)^{3/2}+\frac{105}{2}\sqrt{x+1}\sqrt{1-x}+\frac{105}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(9/2))/(3*(1 + x)^(3/2)) + (6*(1 - x)^(7/2))/Sqrt[1 + x] + (105
*Sqrt[1 - x]*Sqrt[1 + x])/2 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/2 + 7*(1 - x)^
(5/2)*Sqrt[1 + x] + (105*ArcSin[x])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}} dx = -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} - 3\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}} dx$$
$$= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + 21\int \frac{(1-x)^{5/2}}{\sqrt{1+x}} dx$$
$$= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + 7(1-x)^{5/2}\sqrt{1+x} + 35\int \frac{(1-x)^{3/2}}{\sqrt{1+x}} dx$$
$$= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5/2}\sqrt{1+x} + \frac{105}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}} dx$$
$$= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{105}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5/2}\sqrt{1+x} + \frac{105}{2}\int$$
$$= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{105}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5/2}\sqrt{1+x} + \frac{105}{2}\int$$
$$= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{105}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5/2}\sqrt{1+x} + \frac{105}{2}\sin$$

Mathematica [C] time = 0.0138066, size = 37, normalized size = 0.36

$$-\frac{(1-x)^{11/2}\,{}_2F_1\left(\frac{5}{2},\frac{11}{2};\frac{13}{2};\frac{1-x}{2}\right)}{22\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

```
[Out] -((1 - x)^(11/2)*Hypergeometric2F1[5/2, 11/2, 13/2, (1 - x)/2])/(22*Sqrt[2]
)
```

Maple [A] time = 0.018, size = 89, normalized size = 0.9

$$-\frac{2x^5-19x^4+119x^3+577x^2-185x-494}{6}\sqrt{(1+x)(1-x)}\,(1+x)^{-\frac{3}{2}}\frac{1}{\sqrt{-(1+x)(-1+x)}}\frac{1}{\sqrt{1-x}}+\frac{105\arcsin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(9/2)/(1+x)^(5/2),x)
```

```
[Out] -1/6*(2*x^5-19*x^4+119*x^3+577*x^2-185*x-494)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(
1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)+105/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/
(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.48302, size = 169, normalized size = 1.64

$$\frac{x^6}{3\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{7x^5}{2\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{23x^4}{\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{35}{2}x\left(\frac{3x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right)-\frac{143x}{6\sqrt{-x^2+1}}-\frac{127x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{22}{3\left(-x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^6/(-x^2 + 1)^(3/2) - 7/2*x^5/(-x^2 + 1)^(3/2) + 23*x^4/(-x^2 + 1)^(3/
2) + 35/2*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 143/6*x/sqrt(-x
^2 + 1) - 127*x^2/(-x^2 + 1)^(3/2) + 22/3*x/(-x^2 + 1)^(3/2) + 247/3/(-x^2
+ 1)^(3/2) + 105/2*arcsin(x)
```

Fricas [A] time = 1.91111, size = 238, normalized size = 2.31

$$\frac{494\,x^2 + \left(2\,x^4 - 17\,x^3 + 102\,x^2 + 679\,x + 494\right)\sqrt{x+1}\sqrt{-x+1} - 630\left(x^2 + 2\,x + 1\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + 988\,x + 494}{6\left(x^2 + 2\,x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(494*x^2 + (2*x^4 - 17*x^3 + 102*x^2 + 679*x + 494)*sqrt(x + 1)*sqrt(-x
 + 1) - 630*(x^2 + 2*x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 988*
x + 494)/(x^2 + 2*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)/(1+x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19299, size = 171, normalized size = 1.66

$$\frac{1}{6}\left((2\,x-23)(x+1)+165\right)\sqrt{x+1}\sqrt{-x+1} + \frac{2\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{3\,(x+1)^{\frac{3}{2}}} - \frac{34\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}} + \frac{2\,(x+1)^{\frac{3}{2}}\left(\frac{51\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1} - 1\right)}{3\left(\sqrt{2}-\sqrt{-x+1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x - 23)*(x + 1) + 165)*sqrt(x + 1)*sqrt(-x + 1) + 2/3*(sqrt(2) - sq
rt(-x + 1))^3/(x + 1)^(3/2) - 34*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 2/3
*(x + 1)^(3/2)*(51*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(
-x + 1))^3 + 105*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1127 $\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2(1-x)^{7/2}}{3(x+1)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{x+1}}+\frac{35}{6}\sqrt{x+1}(1-x)^{3/2}+\frac{35}{2}\sqrt{x+1}\sqrt{1-x}+\frac{35}{2}\sin^{-1}(x)$$

```
[Out] (-2*(1 - x)^(7/2))/(3*(1 + x)^(3/2)) + (14*(1 - x)^(5/2))/(3*Sqrt[1 + x]) +
 (35*Sqrt[1 - x]*Sqrt[1 + x])/2 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/6 + (35*Ar
cSin[x])/2
```

Rubi [A] time = 0.0152415, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{7/2}}{3(x+1)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{x+1}}+\frac{35}{6}\sqrt{x+1}(1-x)^{3/2}+\frac{35}{2}\sqrt{x+1}\sqrt{1-x}+\frac{35}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(7/2))/(3*(1 + x)^(3/2)) + (14*(1 - x)^(5/2))/(3*Sqrt[1 + x]) +
 (35*Sqrt[1 - x]*Sqrt[1 + x])/2 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/6 + (35*Ar
cSin[x])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}-\frac{7}{3}\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx\\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{1+x}}+\frac{35}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx\\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{1+x}}+\frac{35}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{35}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx\\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{1+x}}+\frac{35}{2}\sqrt{1-x}\sqrt{1+x}+\frac{35}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{35}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx\\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{1+x}}+\frac{35}{2}\sqrt{1-x}\sqrt{1+x}+\frac{35}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{35}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{1+x}}+\frac{35}{2}\sqrt{1-x}\sqrt{1+x}+\frac{35}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{35}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [C] time = 0.0118406, size = 37, normalized size = 0.43

$$-\frac{(1-x)^{9/2}\,{}_2F_1\left(\frac{5}{2},\frac{9}{2};\frac{11}{2};\frac{1-x}{2}\right)}{18\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

```
[Out] -((1 - x)^(9/2)*Hypergeometric2F1[5/2, 9/2, 11/2, (1 - x)/2])/(18*Sqrt[2])
```

Maple [A] time = 0.017, size = 84, normalized size = 1.

$$\frac{3x^4-33x^3-199x^2+65x+164}{6}\sqrt{(1+x)(1-x)}\,(1+x)^{-\frac{3}{2}}\frac{1}{\sqrt{-(1+x)(-1+x)}}\frac{1}{\sqrt{1-x}}+\frac{35\arcsin(x)}{2}\sqrt{(1+x)(1-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)/(1+x)^(5/2),x)
```

```
[Out] 1/6*(3*x^4-33*x^3-199*x^2+65*x+164)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(1/2)*((1+x
)*(1-x))^(1/2)/(1-x)^(1/2)+35/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)
*arcsin(x)
```

Maxima [A] time = 1.52355, size = 150, normalized size = 1.72

$$-\frac{x^5}{2\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{6x^4}{\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{35}{6}x\left(\frac{3x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right)-\frac{61x}{6\sqrt{-x^2+1}}-\frac{44x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{16x}{3\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{82}{3\left(-x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^5/(-x^2 + 1)^(3/2) + 6*x^4/(-x^2 + 1)^(3/2) + 35/6*x*(3*x^2/(-x^2 +
1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 61/6*x/sqrt(-x^2 + 1) - 44*x^2/(-x^2 + 1)^
(3/2) + 16/3*x/(-x^2 + 1)^(3/2) + 82/3/(-x^2 + 1)^(3/2) + 35/2*arcsin(x)
```

Fricas [A] time = 1.82426, size = 224, normalized size = 2.57

$$\frac{164\,x^2-\left(3\,x^3-30\,x^2-229\,x-164\right)\sqrt{x+1}\sqrt{-x+1}-210\left(x^2+2\,x+1\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+328\,x+164}{6\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(164*x^2 - (3*x^3 - 30*x^2 - 229*x - 164)*sqrt(x + 1)*sqrt(-x + 1) - 21
0*(x^2 + 2*x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 328*x + 164)/(
x^2 + 2*x + 1)
```

Sympy [C] time = 60.2526, size = 207, normalized size = 2.38

$$\begin{cases} -\frac{\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{2}+\frac{13\sqrt{-1+\frac{2}{x+1}}(x+1)}{2}+\frac{80\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{16\sqrt{-1+\frac{2}{x+1}}}{3(x+1)}+\frac{35i\log\left(\frac{1}{x+1}\right)}{2}+\frac{35i\log(x+1)}{2}+35\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for } \frac{2}{|x+1|} > 1 \\ -\frac{i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{2}+\frac{13i\sqrt{1-\frac{2}{x+1}}(x+1)}{2}+\frac{80i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{16i\sqrt{1-\frac{2}{x+1}}}{3(x+1)}+\frac{35i\log\left(\frac{1}{x+1}\right)}{2}-35i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1))*(x + 1)**2/2 + 13*sqrt(-1 + 2/(x + 1))*(x
+ 1)/2 + 80*sqrt(-1 + 2/(x + 1))/3 - 16*sqrt(-1 + 2/(x + 1))/(3*(x + 1)) +
35*I*log(1/(x + 1))/2 + 35*I*log(x + 1)/2 + 35*asin(sqrt(2)*sqrt(x + 1)/2),
 2/Abs(x + 1) > 1), (-I*sqrt(1 - 2/(x + 1))*(x + 1)**2/2 + 13*I*sqrt(1 - 2/
(x + 1))*(x + 1)/2 + 80*I*sqrt(1 - 2/(x + 1))/3 - 16*I*sqrt(1 - 2/(x + 1))/
(3*(x + 1)) + 35*I*log(1/(x + 1))/2 - 35*I*log(sqrt(1 - 2/(x + 1)) + 1), Tr
ue))
```

Giac [A] time = 1.13977, size = 161, normalized size = 1.85

$$-\frac{1}{2}\sqrt{x+1}(x-12)\sqrt{-x+1}+\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{3\,(x+1)^{\frac{3}{2}}}-\frac{13\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}+\frac{(x+1)^{\frac{3}{2}}\left(\frac{39\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{3\left(\sqrt{2}-\sqrt{-x+1}\right)^3}+35\,\arcsin\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(x + 1)*(x - 12)*sqrt(-x + 1) + 1/3*(sqrt(2) - sqrt(-x + 1))^3/(x
+ 1)^(3/2) - 13*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 1/3*(x + 1)^(3/2)*(3
9*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1))^3 + 35*a
rcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1128 $\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2(1-x)^{5/2}}{3(x+1)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{x+1}}+5\sqrt{x+1}\sqrt{1-x}+5\sin^{-1}(x)$$

```
[Out] (-2*(1 - x)^(5/2))/(3*(1 + x)^(3/2)) + (10*(1 - x)^(3/2))/(3*Sqrt[1 + x]) +
 5*Sqrt[1 - x]*Sqrt[1 + x] + 5*ArcSin[x]
```

Rubi [A] time = 0.010729, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{5/2}}{3(x+1)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{x+1}}+5\sqrt{x+1}\sqrt{1-x}+5\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/(1 + x)^(5/2),x]
```


```
[Out] (-2*(1 - x)^(5/2))/(3*(1 + x)^(3/2)) + (10*(1 - x)^(3/2))/(3*Sqrt[1 + x]) +
 5*Sqrt[1 - x]*Sqrt[1 + x] + 5*ArcSin[x]
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} - \frac{5}{3}\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx \\ &= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} + \frac{10(1-x)^{3/2}}{3\sqrt{1+x}} + 5\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\ &= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} + \frac{10(1-x)^{3/2}}{3\sqrt{1+x}} + 5\sqrt{1-x}\sqrt{1+x} + 5\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx \\ &= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} + \frac{10(1-x)^{3/2}}{3\sqrt{1+x}} + 5\sqrt{1-x}\sqrt{1+x} + 5\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} + \frac{10(1-x)^{3/2}}{3\sqrt{1+x}} + 5\sqrt{1-x}\sqrt{1+x} + 5\sin^{-1}(x)\end{aligned}$$

Mathematica [C] time = 0.0102119, size = 37, normalized size = 0.59

$$-\frac{(1-x)^{7/2}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{1-x}{2}\right)}{14\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/(1 + x)^(5/2),x]
```

```
[Out] -((1 - x)^(7/2)*Hypergeometric2F1[5/2, 7/2, 9/2, (1 - x)/2])/(14*Sqrt[2])
```

Maple [A] time = 0.019, size = 79, normalized size = 1.3

$$-\frac{3x^3+31x^2-11x-23}{3}\sqrt{(1+x)(1-x)}\,(1+x)^{-\frac{3}{2}}\frac{1}{\sqrt{-(1+x)(-1+x)}}\frac{1}{\sqrt{1-x}}+5\frac{\sqrt{(1+x)(1-x)}\arcsin(x)}{\sqrt{1-x}\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)/(1+x)^(5/2),x)
```

```
[Out] -1/3*(3*x^3+31*x^2-11*x-23)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))
^(1/2)/(1-x)^(1/2)+5*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [B] time = 1.50109, size = 132, normalized size = 2.1

$$\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{x^4+4x^3+6x^2+4x+1}-\frac{5\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^3+3x^2+3x+1\right)}-\frac{10\sqrt{-x^2+1}}{3\left(x^2+2x+1\right)}+\frac{35\sqrt{-x^2+1}}{3(x+1)}+5\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(5/2)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1) - 5/3*(-x^2 + 1)^(3/2)/(x^
3 + 3*x^2 + 3*x + 1) - 10/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 35/3*sqrt(-x^2
 + 1)/(x + 1) + 5*arcsin(x)
```

Fricas [A] time = 1.85971, size = 204, normalized size = 3.24

$$\frac{23\,x^2+\left(3\,x^2+34\,x+23\right)\sqrt{x+1}\sqrt{-x+1}-30\,\left(x^2+2\,x+1\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+46\,x+23}{3\,\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(23*x^2 + (3*x^2 + 34*x + 23)*sqrt(x + 1)*sqrt(-x + 1) - 30*(x^2 + 2*x
+ 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 46*x + 23)/(x^2 + 2*x + 1)
```

Sympy [C] time = 12.9692, size = 160, normalized size = 2.54

$$\begin{cases} \sqrt{-1+\frac{2}{x+1}}\,(x+1)+\frac{28\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{8\sqrt{-1+\frac{2}{x+1}}}{3(x+1)}+5i\log\left(\frac{1}{x+1}\right)+5i\log\left(x+1\right)+10\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for}\ \frac{2}{|x+1|}>1 \\ i\sqrt{1-\frac{2}{x+1}}\,(x+1)+\frac{28i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{8i\sqrt{1-\frac{2}{x+1}}}{3(x+1)}+5i\log\left(\frac{1}{x+1}\right)-10i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((sqrt(-1 + 2/(x + 1))*(x + 1) + 28*sqrt(-1 + 2/(x + 1))/3 - 8*sqr
t(-1 + 2/(x + 1))/(3*(x + 1)) + 5*I*log(1/(x + 1)) + 5*I*log(x + 1) + 10*as
in(sqrt(2)*sqrt(x + 1)/2), 2/Abs(x + 1) > 1), (I*sqrt(1 - 2/(x + 1))*(x + 1
) + 28*I*sqrt(1 - 2/(x + 1))/3 - 8*I*sqrt(1 - 2/(x + 1))/(3*(x + 1)) + 5*I*
log(1/(x + 1)) - 10*I*log(sqrt(1 - 2/(x + 1)) + 1), True))
```

Giac [B] time = 1.11284, size = 155, normalized size = 2.46

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{6\,(x+1)^{\frac{3}{2}}}+\sqrt{x+1}\sqrt{-x+1}-\frac{9\left(\sqrt{2}-\sqrt{-x+1}\right)}{2\,\sqrt{x+1}}+\frac{(x+1)^{\frac{3}{2}}\left(\frac{27\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{6\left(\sqrt{2}-\sqrt{-x+1}\right)^3}+10\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + sqrt(x + 1)*sqrt(-x + 1) - 9
/2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 1/6*(x + 1)^(3/2)*(27*(sqrt(2) -
sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1))^3 + 10*arcsin(1/2*sqr
t(2)*sqrt(x + 1))
```

3.1129 $\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}} \, dx$

Optimal. Leaf size=41

$$-\frac{2(1-x)^{3/2}}{3(x+1)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{x+1}}+\sin^{-1}(x)$$

```
[Out] (-2*(1 - x)^(3/2))/(3*(1 + x)^(3/2)) + (2*Sqrt[1 - x])/Sqrt[1 + x] + ArcSin
[x]
```

Rubi [A] time = 0.0055739, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$-\frac{2(1-x)^{3/2}}{3(x+1)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{x+1}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(3/2))/(3*(1 + x)^(3/2)) + (2*Sqrt[1 - x])/Sqrt[1 + x] + ArcSin
[x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}}\,dx = -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} - \int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx$$
$$= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} + \frac{2\sqrt{1-x}}{\sqrt{1+x}} + \int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$$
$$= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} + \frac{2\sqrt{1-x}}{\sqrt{1+x}} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} + \frac{2\sqrt{1-x}}{\sqrt{1+x}} + \sin^{-1}(x)$$

Mathematica [A] time = 0.0538163, size = 49, normalized size = 1.2

$$\frac{-8x^2+4x+4}{3\sqrt{1-x}(x+1)^{3/2}} - 2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/(1 + x)^(5/2),x]
```

```
[Out] (4 + 4*x - 8*x^2)/(3*Sqrt[1 - x]*(1 + x)^(3/2)) - 2*ArcSin[Sqrt[1 - x]/Sqrt
[2]]
```

Maple [B] time = 0.015, size = 73, normalized size = 1.8

$$-\frac{8x^2-4x-4}{3}\sqrt{(1+x)(1-x)}(1+x)^{-\frac{3}{2}}\frac{1}{\sqrt{-(1+x)(-1+x)}}\frac{1}{\sqrt{1-x}} + \arcsin(x)\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)/(1+x)^(5/2),x)
```

```
[Out] -4/3*(2*x^2-x-1)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x
)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [B] time = 1.5003, size = 89, normalized size = 2.17

$$-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^3+3x^2+3x+1\right)} - \frac{2\sqrt{-x^2+1}}{3\left(x^2+2x+1\right)} + \frac{7\sqrt{-x^2+1}}{3(x+1)} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2)/(x^3 + 3*x^2 + 3*x + 1) - 2/3*sqrt(-x^2 + 1)/(x^2 + 2
*x + 1) + 7/3*sqrt(-x^2 + 1)/(x + 1) + arcsin(x)
```

Fricas [B] time = 1.81969, size = 188, normalized size = 4.59

$$\frac{2\left(2x^2+2(2x+1)\sqrt{x+1}\sqrt{-x+1}-3(x^2+2x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+4x+2\right)}{3(x^2+2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*x^2 + 2*(2*x + 1)*sqrt(x + 1)*sqrt(-x + 1) - 3*(x^2 + 2*x + 1)*arcta
n((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 4*x + 2)/(x^2 + 2*x + 1)
```

Sympy [C] time = 5.09716, size = 126, normalized size = 3.07

$$\begin{cases} \frac{8\sqrt{-1+\frac{2}{x+1}}}{3} - \frac{4\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} + i\log\left(\frac{1}{x+1}\right) + i\log(x+1) + 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for } \frac{2}{|x+1|} > 1 \\ \frac{8i\sqrt{1-\frac{2}{x+1}}}{3} - \frac{4i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} + i\log\left(\frac{1}{x+1}\right) - 2i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((8*sqrt(-1 + 2/(x + 1))/3 - 4*sqrt(-1 + 2/(x + 1))/(3*(x + 1)) +
I*log(1/(x + 1)) + I*log(x + 1) + 2*asin(sqrt(2)*sqrt(x + 1)/2), 2/Abs(x +
1) > 1), (8*I*sqrt(1 - 2/(x + 1))/3 - 4*I*sqrt(1 - 2/(x + 1))/(3*(x + 1)) +
 I*log(1/(x + 1)) - 2*I*log(sqrt(1 - 2/(x + 1)) + 1), True))
```

Giac [B] time = 1.1396, size = 138, normalized size = 3.37

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{12(x+1)^{\frac{3}{2}}} - \frac{5\left(\sqrt{2}-\sqrt{-x+1}\right)}{4\sqrt{x+1}} + \frac{(x+1)^{\frac{3}{2}}\left(\frac{15\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{12\left(\sqrt{2}-\sqrt{-x+1}\right)^3} + 2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) - 5/4*(sqrt(2) - sqrt(-x + 1)
)/sqrt(x + 1) + 1/12*(x + 1)^(3/2)*(15*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) -
 1)/(sqrt(2) - sqrt(-x + 1))^3 + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

3.1130 $\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

```
[Out] -(1 - x)^(3/2)/(3*(1 + x)^(3/2))
```

Rubi [A] time = 0.0015594, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 + x)^(5/2),x]

[Out] -(1 - x)^(3/2)/(3*(1 + x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx = -\frac{(1-x)^{3/2}}{3(1+x)^{3/2}}$$

Mathematica [A] time = 0.0040988, size = 20, normalized size = 1.

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + x)^(5/2),x]

[Out] -(1 - x)^(3/2)/(3*(1 + x)^(3/2))
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$-\frac{1}{3}(1-x)^{\frac{3}{2}}(1+x)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(1+x)^(5/2),x)
```

```
[Out] -1/3*(1-x)^(3/2)/(1+x)^(3/2)
```

Maxima [B] time = 0.982584, size = 51, normalized size = 2.55

$$-\frac{2\sqrt{-x^2+1}}{3\left(x^2+2x+1\right)}+\frac{\sqrt{-x^2+1}}{3(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x + 1)
```

Fricas [B] time = 1.85103, size = 99, normalized size = 4.95

$$-\frac{x^2-\sqrt{x+1}(x-1)\sqrt{-x+1}+2x+1}{3\left(x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(x^2 - sqrt(x + 1)*(x - 1)*sqrt(-x + 1) + 2*x + 1)/(x^2 + 2*x + 1)
```

Sympy [A] time = 2.41654, size = 65, normalized size = 3.25

$$\begin{cases} \frac{\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{2\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} & \text{for } \frac{2}{|x+1|}>1 \\ \frac{i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{2i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((sqrt(-1 + 2/(x + 1))/3 - 2*sqrt(-1 + 2/(x + 1))/(3*(x + 1)), 2/A
bs(x + 1) > 1), (I*sqrt(1 - 2/(x + 1))/3 - 2*I*sqrt(1 - 2/(x + 1))/(3*(x +
1)), True))
```

Giac [B] time = 1.07845, size = 120, normalized size = 6.

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{24(x+1)^{\frac{3}{2}}}-\frac{\sqrt{2}-\sqrt{-x+1}}{8\sqrt{x+1}}+\frac{(x+1)^{\frac{3}{2}}\left(\frac{3\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{24\left(\sqrt{2}-\sqrt{-x+1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) - 1/8*(sqrt(2) - sqrt(-x + 1)
)/sqrt(x + 1) + 1/24*(x + 1)^(3/2)*(3*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) -
1)/(sqrt(2) - sqrt(-x + 1))^3
```

3.1131 $\int \frac{1}{\sqrt{1-x}(1+x)^{5/2}} dx$

Optimal. Leaf size=41

$$-\frac{\sqrt{1-x}}{3\sqrt{x+1}}-\frac{\sqrt{1-x}}{3(x+1)^{3/2}}$$

```
[Out] -Sqrt[1 - x]/(3*(1 + x)^(3/2)) - Sqrt[1 - x]/(3*Sqrt[1 + x])
```

Rubi [A] time = 0.0043043, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$-\frac{\sqrt{1-x}}{3\sqrt{x+1}}-\frac{\sqrt{1-x}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
```

```
[Out] -Sqrt[1 - x]/(3*(1 + x)^(3/2)) - Sqrt[1 - x]/(3*Sqrt[1 + x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x}(1+x)^{5/2}} dx &= -\frac{\sqrt{1-x}}{3(1+x)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}} dx\\ &= -\frac{\sqrt{1-x}}{3(1+x)^{3/2}}-\frac{\sqrt{1-x}}{3\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.0111907, size = 23, normalized size = 0.56

$$-\frac{\sqrt{1-x}(x+2)}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
```

```
[Out] -(Sqrt[1 - x]*(2 + x))/(3*(1 + x)^(3/2))
```

Maple [A] time = 0.001, size = 18, normalized size = 0.4

$$-\frac{2+x}{3}\sqrt{1-x}\,(1+x)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/2)/(1+x)^(5/2),x)
```

```
[Out] -1/3*(2+x)/(1+x)^(3/2)*(1-x)^(1/2)
```

Maxima [A] time = 1.49294, size = 51, normalized size = 1.24

$$-\frac{\sqrt{-x^2+1}}{3\left(x^2+2\,x+1\right)}-\frac{\sqrt{-x^2+1}}{3\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) - 1/3*sqrt(-x^2 + 1)/(x + 1)
```

Fricas [A] time = 1.92189, size = 101, normalized size = 2.46

$$-\frac{2\,x^2+(x+2)\sqrt{x+1}\sqrt{-x+1}+4\,x+2}{3\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^2 + (x + 2)*sqrt(x + 1)*sqrt(-x + 1) + 4*x + 2)/(x^2 + 2*x + 1)
```

Sympy [A] time = 4.23274, size = 65, normalized size = 1.59

$$\begin{cases} -\frac{\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} & \text{for}\ \frac{2}{|x+1|}>1 \\ -\frac{i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1))/3 - sqrt(-1 + 2/(x + 1))/(3*(x + 1)), 2/Ab
s(x + 1) > 1), (-I*sqrt(1 - 2/(x + 1))/3 - I*sqrt(1 - 2/(x + 1))/(3*(x + 1)
), True))
```

Giac [B] time = 1.07142, size = 120, normalized size = 2.93

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{48\,(x+1)^{\frac{3}{2}}}+\frac{3\left(\sqrt{2}-\sqrt{-x+1}\right)}{16\,\sqrt{x+1}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{9\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}+1\right)}{48\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 3/16*(sqrt(2) - sqrt(-x + 1
))/sqrt(x + 1) - 1/48*(x + 1)^(3/2)*(9*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) +
 1)/(sqrt(2) - sqrt(-x + 1))^3
```

3.1132 $\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{2\sqrt{1-x}}{3\sqrt{x+1}}-\frac{2\sqrt{1-x}}{3(x+1)^{3/2}}+\frac{1}{(x+1)^{3/2}\sqrt{1-x}}$$

```
[Out] 1/(Sqrt[1 - x]*(1 + x)^(3/2)) - (2*Sqrt[1 - x])/(3*(1 + x)^(3/2)) - (2*Sqrt
[1 - x])/(3*Sqrt[1 + x])
```

Rubi [A] time = 0.0084024, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$-\frac{2\sqrt{1-x}}{3\sqrt{x+1}}-\frac{2\sqrt{1-x}}{3(x+1)^{3/2}}+\frac{1}{(x+1)^{3/2}\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(Sqrt[1 - x]*(1 + x)^(3/2)) - (2*Sqrt[1 - x])/(3*(1 + x)^(3/2)) - (2*Sqrt
[1 - x])/(3*Sqrt[1 + x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx &= \frac{1}{\sqrt{1-x}(1+x)^{3/2}}+2\int \frac{1}{\sqrt{1-x}(1+x)^{5/2}}\,dx\\
&= \frac{1}{\sqrt{1-x}(1+x)^{3/2}}-\frac{2\sqrt{1-x}}{3(1+x)^{3/2}}+\frac{2}{3}\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}}\,dx\\
&= \frac{1}{\sqrt{1-x}(1+x)^{3/2}}-\frac{2\sqrt{1-x}}{3(1+x)^{3/2}}-\frac{2\sqrt{1-x}}{3\sqrt{1+x}}
\end{aligned}$$

Mathematica [A] time = 0.0058078, size = 30, normalized size = 0.52

$$\frac{2x^2+2x-1}{3\sqrt{1-x}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]
```

```
[Out] (-1 + 2*x + 2*x^2)/(3*Sqrt[1 - x]*(1 + x)^(3/2))
```

Maple [A] time = 0.004, size = 25, normalized size = 0.4

$$\frac{2x^2+2x-1}{3}\frac{1}{\sqrt{1-x}}(1+x)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(3/2)/(1+x)^(5/2),x)
```

```
[Out] 1/3*(2*x^2+2*x-1)/(1+x)^(3/2)/(1-x)^(1/2)
```

Maxima [A] time = 0.991325, size = 51, normalized size = 0.88

$$\frac{2x}{3\sqrt{-x^2+1}}-\frac{1}{3\left(\sqrt{-x^2+1}x+\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(-x^2 + 1) - 1/3/(sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1))
```

Fricas [A] time = 1.90355, size = 123, normalized size = 2.12

$$-\frac{x^3+x^2+\left(2x^2+2x-1\right)\sqrt{x+1}\sqrt{-x+1}-x-1}{3\left(x^3+x^2-x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(x^3 + x^2 + (2*x^2 + 2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) - x - 1)/(x^3
+ x^2 - x - 1)
```

Sympy [A] time = 17.4688, size = 165, normalized size = 2.84

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-6x+3(x+1)^2-6}+\frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)}{-6x+3(x+1)^2-6}+\frac{\sqrt{-1+\frac{2}{x+1}}}{-6x+3(x+1)^2-6} & \text{for } \frac{2}{|x+1|}>1 \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-6x+3(x+1)^2-6}+\frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)}{-6x+3(x+1)^2-6}+\frac{i\sqrt{1-\frac{2}{x+1}}}{-6x+3(x+1)^2-6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-6*x + 3*(x + 1)**2 - 6) + 2
*sqrt(-1 + 2/(x + 1))*(x + 1)/(-6*x + 3*(x + 1)**2 - 6) + sqrt(-1 + 2/(x +
1))/(-6*x + 3*(x + 1)**2 - 6), 2/Abs(x + 1) > 1), (-2*I*sqrt(1 - 2/(x + 1))
*(x + 1)**2/(-6*x + 3*(x + 1)**2 - 6) + 2*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-6
*x + 3*(x + 1)**2 - 6) + I*sqrt(1 - 2/(x + 1))/(-6*x + 3*(x + 1)**2 - 6), T
rue))
```

Giac [B] time = 1.08483, size = 146, normalized size = 2.52

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{96\,(x+1)^{\frac{3}{2}}}+\frac{7\left(\sqrt{2}-\sqrt{-x+1}\right)}{32\,\sqrt{x+1}}-\frac{\sqrt{x+1}\sqrt{-x+1}}{4\,(x-1)}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{21\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}+1\right)}{96\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/96*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 7/32*(sqrt(2) - sqrt(-x + 1
))/sqrt(x + 1) - 1/4*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 1/96*(x + 1)^(3/2)*
(21*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3
```

3.1133 $\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}} dx$

Optimal. Leaf size=43

$$\frac{2x}{3\sqrt{1-x}\sqrt{x+1}}+\frac{x}{3(1-x)^{3/2}(x+1)^{3/2}}$$

```
[Out] x/(3*(1 - x)^(3/2)*(1 + x)^(3/2)) + (2*x)/(3*Sqrt[1 - x]*Sqrt[1 + x])
```

Rubi [A] time = 0.0047007, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {40, 39}

$$\frac{2x}{3\sqrt{1-x}\sqrt{x+1}}+\frac{x}{3(1-x)^{3/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
```

```
[Out] x/(3*(1 - x)^(3/2)*(1 + x)^(3/2)) + (2*x)/(3*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}} dx &= \frac{x}{3(1-x)^{3/2}(1+x)^{3/2}}+\frac{2}{3}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}} dx\\ &= \frac{x}{3(1-x)^{3/2}(1+x)^{3/2}}+\frac{2x}{3\sqrt{1-x}\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.0064667, size = 23, normalized size = 0.53

$$-\frac{x\left(2x^2-3\right)}{3\left(1-x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
```

```
[Out] -(x*(-3 + 2*x^2))/(3*(1 - x^2)^(3/2))
```

Maple [A] time = 0.002, size = 23, normalized size = 0.5

$$-\frac{x\left(2x^2-3\right)}{3}(1-x)^{-\frac{3}{2}}(1+x)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(5/2)/(1+x)^(5/2),x)
```

```
[Out] -1/3*x*(2*x^2-3)/(1+x)^(3/2)/(1-x)^(3/2)
```

Maxima [A] time = 0.992303, size = 34, normalized size = 0.79

$$\frac{2x}{3\sqrt{-x^2+1}}+\frac{x}{3\left(-x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(-x^2 + 1) + 1/3*x/(-x^2 + 1)^(3/2)
```

Fricas [A] time = 1.58676, size = 85, normalized size = 1.98

$$-\frac{\left(2x^3-3x\right)\sqrt{x+1}\sqrt{-x+1}}{3\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^3 - 3*x)*sqrt(x + 1)*sqrt(-x + 1)/(x^4 - 2*x^2 + 1)
```

Sympy [B] time = 38.4734, size = 279, normalized size = 6.49

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{12x+3(x+1)^3-12(x+1)^2+12}+\frac{6\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{12x+3(x+1)^3-12(x+1)^2+12}-\frac{3\sqrt{-1+\frac{2}{x+1}}(x+1)}{12x+3(x+1)^3-12(x+1)^2+12}-\frac{\sqrt{-1+\frac{2}{x+1}}}{12x+3(x+1)^3-12(x+1)^2+12} & \text{for } \frac{2}{|x+1|}>1 \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{12x+3(x+1)^3-12(x+1)^2+12}+\frac{6i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{12x+3(x+1)^3-12(x+1)^2+12}-\frac{3i\sqrt{1-\frac{2}{x+1}}(x+1)}{12x+3(x+1)^3-12(x+1)^2+12}-\frac{i\sqrt{1-\frac{2}{x+1}}}{12x+3(x+1)^3-12(x+1)^2+12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(12*x + 3*(x + 1)**3 - 12*(x
+ 1)**2 + 12) + 6*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(12*x + 3*(x + 1)**3 - 12
```

```
*(x + 1)**2 + 12) - 3*sqrt(-1 + 2/(x + 1))*(x + 1)/(12*x + 3*(x + 1)**3 - 1
2*(x + 1)**2 + 12) - sqrt(-1 + 2/(x + 1))/(12*x + 3*(x + 1)**3 - 12*(x + 1)
**2 + 12), 2/Abs(x + 1) > 1), (-2*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(12*x +
3*(x + 1)**3 - 12*(x + 1)**2 + 12) + 6*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(12
*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - 3*I*sqrt(1 - 2/(x + 1))*(x + 1)/(
12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - I*sqrt(1 - 2/(x + 1))/(12*x + 3
*(x + 1)**3 - 12*(x + 1)**2 + 12), True))
```

Giac [B] time = 1.08233, size = 153, normalized size = 3.56

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{192\,(x+1)^{\frac{3}{2}}}+\frac{11\left(\sqrt{2}-\sqrt{-x+1}\right)}{64\,\sqrt{x+1}}-\frac{(4\,x-5)\sqrt{x+1}\sqrt{-x+1}}{12\,(x-1)^{2}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{33\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}+1\right)}{192\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/192*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 11/64*(sqrt(2) - sqrt(-x +
 1))/sqrt(x + 1) - 1/12*(4*x - 5)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 - 1/19
2*(x + 1)^(3/2)*(33*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) + 1)/(sqrt(2) - sqrt
(-x + 1))^3
```

3.1134 $\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx$

Optimal. Leaf size=63

$$\frac{8x}{15\sqrt{1-x}\sqrt{x+1}}+\frac{4x}{15(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}$$

```
[Out] 1/(5*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*x)/(15*(1 - x)^(3/2)*(1 + x)^(3/2))
+ (8*x)/(15*Sqrt[1 - x]*Sqrt[1 + x])
```

Rubi [A] time = 0.0081462, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {45, 40, 39}

$$\frac{8x}{15\sqrt{1-x}\sqrt{x+1}}+\frac{4x}{15(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(5*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*x)/(15*(1 - x)^(3/2)*(1 + x)^(3/2))
+ (8*x)/(15*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx &= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4}{5}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{15(1-x)^{3/2}(1+x)^{3/2}}+\frac{8}{15}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\ &= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{15(1-x)^{3/2}(1+x)^{3/2}}+\frac{8x}{15\sqrt{1-x}\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.0111156, size = 40, normalized size = 0.63

$$\frac{8x^4-8x^3-12x^2+12x+3}{15(1-x)^{5/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

```
[Out] (3 + 12*x - 12*x^2 - 8*x^3 + 8*x^4)/(15*(1 - x)^(5/2)*(1 + x)^(3/2))
```

Maple [A] time = 0.003, size = 35, normalized size = 0.6

$$\frac{8\,x^4-8\,x^3-12\,x^2+12\,x+3}{15}\,(1-x)^{-\frac{5}{2}}\,(1+x)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(7/2)/(1+x)^(5/2),x)
```

```
[Out] 1/15*(8*x^4-8*x^3-12*x^2+12*x+3)/(1+x)^(3/2)/(1-x)^(5/2)
```

Maxima [A] time = 0.999271, size = 70, normalized size = 1.11

$$\frac{8\,x}{15\,\sqrt{-x^2+1}}+\frac{4\,x}{15\,\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{1}{5\left(\left(-x^2+1\right)^{\frac{3}{2}}x-\left(-x^2+1\right)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/15*x/sqrt(-x^2 + 1) + 4/15*x/(-x^2 + 1)^(3/2) - 1/5/((-x^2 + 1)^(3/2)*x -
 (-x^2 + 1)^(3/2))
```

Fricas [A] time = 1.50386, size = 198, normalized size = 3.14

$$\frac{3\,x^5-3\,x^4-6\,x^3+6\,x^2-\left(8\,x^4-8\,x^3-12\,x^2+12\,x+3\right)\sqrt{x+1}\sqrt{-x+1}+3\,x-3}{15\left(x^5-x^4-2\,x^3+2\,x^2+x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^5 - 3*x^4 - 6*x^3 + 6*x^2 - (8*x^4 - 8*x^3 - 12*x^2 + 12*x + 3)*s
qrt(x + 1)*sqrt(-x + 1) + 3*x - 3)/(x^5 - x^4 - 2*x^3 + 2*x^2 + x - 1)
```

Sympy [B] time = 149.631, size = 423, normalized size = 6.71

$$\begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}(x+1)^4}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{40\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} - \frac{60\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{}{-120x+1} \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}(x+1)^4}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{40i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} - \frac{60i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{}{-120x+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**4/(-120*x + 15*(x + 1)**4 - 90*
(x + 1)**3 + 180*(x + 1)**2 - 120) + 40*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(-1
20*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) - 60*sqrt(-1 +
 2/(x + 1))*(x + 1)**2/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1
)**2 - 120) + 20*sqrt(-1 + 2/(x + 1))*(x + 1)/(-120*x + 15*(x + 1)**4 - 90*
(x + 1)**3 + 180*(x + 1)**2 - 120) + 5*sqrt(-1 + 2/(x + 1))/(-120*x + 15*(x
 + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120), 2/Abs(x + 1) > 1), (-8*I*
sqrt(1 - 2/(x + 1))*(x + 1)**4/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 18
0*(x + 1)**2 - 120) + 40*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(-120*x + 15*(x +
 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) - 60*I*sqrt(1 - 2/(x + 1))*(
x + 1)**2/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) +
 20*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 +
 180*(x + 1)**2 - 120) + 5*I*sqrt(1 - 2/(x + 1))/(-120*x + 15*(x + 1)**4 -
90*(x + 1)**3 + 180*(x + 1)**2 - 120), True))
```

Giac [B] time = 1.09646, size = 161, normalized size = 2.56

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{384\,(x+1)^{\frac{3}{2}}} + \frac{15\left(\sqrt{2}-\sqrt{-x+1}\right)}{128\,\sqrt{x+1}} - \frac{(x+1)^{\frac{3}{2}}\left(\frac{45\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{384\left(\sqrt{2}-\sqrt{-x+1}\right)^3} - \frac{((73\,x-247)(x+1)+360)\sqrt{x+1}\sqrt{-x+1}}{240\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 15/128*(sqrt(2) - sqrt(-x
+ 1))/sqrt(x + 1) - 1/384*(x + 1)^(3/2)*(45*(sqrt(2) - sqrt(-x + 1))^2/(x +
 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/240*((73*x - 247)*(x + 1) + 360)*sq
rt(x + 1)*sqrt(-x + 1)/(x - 1)^3
```

3.1135 $\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}} dx$

Optimal. Leaf size=83

$$\frac{8x}{21\sqrt{1-x}\sqrt{x+1}}+\frac{4x}{21(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{5/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}$$

```
[Out] 1/(7*(1 - x)^(7/2)*(1 + x)^(3/2)) + 1/(7*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*
x)/(21*(1 - x)^(3/2)*(1 + x)^(3/2)) + (8*x)/(21*Sqrt[1 - x]*Sqrt[1 + x])
```

Rubi [A] time = 0.0137007, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {45, 40, 39}

$$\frac{8x}{21\sqrt{1-x}\sqrt{x+1}}+\frac{4x}{21(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{5/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(7*(1 - x)^(7/2)*(1 + x)^(3/2)) + 1/(7*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*
x)/(21*(1 - x)^(3/2)*(1 + x)^(3/2)) + (8*x)/(21*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx = \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}} + \frac{5}{7}\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx$$
$$= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}} + \frac{1}{7(1-x)^{5/2}(1+x)^{3/2}} + \frac{4}{7}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx$$
$$= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}} + \frac{1}{7(1-x)^{5/2}(1+x)^{3/2}} + \frac{4x}{21(1-x)^{3/2}(1+x)^{3/2}} + \frac{8}{21}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}$$
$$= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}} + \frac{1}{7(1-x)^{5/2}(1+x)^{3/2}} + \frac{4x}{21(1-x)^{3/2}(1+x)^{3/2}} + \frac{8x}{21\sqrt{1-x}\sqrt{1+x}}$$

Mathematica [A] time = 0.0133994, size = 45, normalized size = 0.54

$$\frac{-8x^5+16x^4+4x^3-24x^2+9x+6}{21(1-x)^{7/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

```
[Out] (6 + 9*x - 24*x^2 + 4*x^3 + 16*x^4 - 8*x^5)/(21*(1 - x)^(7/2)*(1 + x)^(3/2)
)
```

Maple [A] time = 0.001, size = 40, normalized size = 0.5

$$-\frac{8x^5-16x^4-4x^3+24x^2-9x-6}{21}(1-x)^{-\frac{7}{2}}(1+x)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(9/2)/(1+x)^(5/2),x)
```

```
[Out] -1/21*(8*x^5-16*x^4-4*x^3+24*x^2-9*x-6)/(1+x)^(3/2)/(1-x)^(7/2)
```

Maxima [A] time = 0.994069, size = 123, normalized size = 1.48

$$\frac{8x}{21\sqrt{-x^2+1}} + \frac{4x}{21(-x^2+1)^{\frac{3}{2}}} + \frac{1}{7\left((-x^2+1)^{\frac{3}{2}}x^2 - 2(-x^2+1)^{\frac{3}{2}}x + (-x^2+1)^{\frac{3}{2}}\right)} - \frac{1}{7\left((-x^2+1)^{\frac{3}{2}}x - (-x^2+1)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/21*x/sqrt(-x^2 + 1) + 4/21*x/(-x^2 + 1)^(3/2) + 1/7/((-x^2 + 1)^(3/2)*x^2
 - 2*(-x^2 + 1)^(3/2)*x + (-x^2 + 1)^(3/2)) - 1/7/((-x^2 + 1)^(3/2)*x - (-x
^2 + 1)^(3/2))
```

Fricas [A] time = 1.55858, size = 235, normalized size = 2.83

$$\frac{6\,x^6-12\,x^5-6\,x^4+24\,x^3-6\,x^2-\left(8\,x^5-16\,x^4-4\,x^3+24\,x^2-9\,x-6\right)\sqrt{x+1}\sqrt{-x+1}-12\,x+6}{21\,\left(x^6-2\,x^5-x^4+4\,x^3-x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*(6*x^6 - 12*x^5 - 6*x^4 + 24*x^3 - 6*x^2 - (8*x^5 - 16*x^4 - 4*x^3 + 2
4*x^2 - 9*x - 6)*sqrt(x + 1)*sqrt(-x + 1) - 12*x + 6)/(x^6 - 2*x^5 - x^4 +
4*x^3 - x^2 - 2*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.08924, size = 169, normalized size = 2.04

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{768\,(x+1)^{\frac{3}{2}}}+\frac{19\left(\sqrt{2}-\sqrt{-x+1}\right)}{256\,\sqrt{x+1}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{57\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{768\left(\sqrt{2}-\sqrt{-x+1}\right)^3}-\frac{(((79\,x-432)(x+1)+1120)(x+1)-840)}{336\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/768*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 19/256*(sqrt(2) - sqrt(-x
+ 1))/sqrt(x + 1) - 1/768*(x + 1)^(3/2)*(57*(sqrt(2) - sqrt(-x + 1))^2/(x +
 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/336*(((79*x - 432)*(x + 1) + 1120)*
(x + 1) - 840)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^4
```

3.1136 $\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}} dx$

Optimal. Leaf size=103

$$\frac{16x}{63\sqrt{1-x}\sqrt{x+1}}+\frac{8x}{63(1-x)^{3/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{5/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{7/2}(x+1)^{3/2}}+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

```
[Out] 1/(9*(1 - x)^(9/2)*(1 + x)^(3/2)) + 2/(21*(1 - x)^(7/2)*(1 + x)^(3/2)) + 2/
(21*(1 - x)^(5/2)*(1 + x)^(3/2)) + (8*x)/(63*(1 - x)^(3/2)*(1 + x)^(3/2)) +
 (16*x)/(63*Sqrt[1 - x]*Sqrt[1 + x])
```

Rubi [A] time = 0.0193206, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {45, 40, 39}

$$\frac{16x}{63\sqrt{1-x}\sqrt{x+1}}+\frac{8x}{63(1-x)^{3/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{5/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{7/2}(x+1)^{3/2}}+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(9*(1 - x)^(9/2)*(1 + x)^(3/2)) + 2/(21*(1 - x)^(7/2)*(1 + x)^(3/2)) + 2/
(21*(1 - x)^(5/2)*(1 + x)^(3/2)) + (8*x)/(63*(1 - x)^(3/2)*(1 + x)^(3/2)) +
 (16*x)/(63*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}dx = \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}} + \frac{2}{3}\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}dx$$
$$= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{7/2}(1+x)^{3/2}} + \frac{10}{21}\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}dx$$
$$= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{7/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{5/2}(1+x)^{3/2}} + \frac{8}{21}\int \frac{1}{(1-x)^{5/2}(1}$$
$$= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{7/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{5/2}(1+x)^{3/2}} + \frac{8x}{63(1-x)^{3/2}(1+x}$$
$$= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{7/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{5/2}(1+x)^{3/2}} + \frac{8x}{63(1-x)^{3/2}(1+x}$$

Mathematica [A] time = 0.0127024, size = 50, normalized size = 0.49

$$\frac{16x^6-48x^5+24x^4+56x^3-66x^2+6x+19}{63(1-x)^{9/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

```
[Out] (19 + 6*x - 66*x^2 + 56*x^3 + 24*x^4 - 48*x^5 + 16*x^6)/(63*(1 - x)^(9/2)*(
1 + x)^(3/2))
```

Maple [A] time = 0.002, size = 45, normalized size = 0.4

$$\frac{16x^6-48x^5+24x^4+56x^3-66x^2+6x+19}{63}(1-x)^{-\frac{9}{2}}(1+x)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(11/2)/(1+x)^(5/2),x)
```

```
[Out] 1/63*(16*x^6-48*x^5+24*x^4+56*x^3-66*x^2+6*x+19)/(1+x)^(3/2)/(1-x)^(9/2)
```

Maxima [A] time = 1.00592, size = 197, normalized size = 1.91

$$\frac{16x}{63\sqrt{-x^2+1}} + \frac{8x}{63(-x^2+1)^{\frac{3}{2}}} - \frac{1}{9\left((-x^2+1)^{\frac{3}{2}}x^3 - 3(-x^2+1)^{\frac{3}{2}}x^2 + 3(-x^2+1)^{\frac{3}{2}}x - (-x^2+1)^{\frac{3}{2}}\right)} + \frac{}{21\left((-x^2+1)^{\frac{3}{2}}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/63*x/sqrt(-x^2 + 1) + 8/63*x/(-x^2 + 1)^(3/2) - 1/9/((-x^2 + 1)^(3/2)*x^
3 - 3*(-x^2 + 1)^(3/2)*x^2 + 3*(-x^2 + 1)^(3/2)*x - (-x^2 + 1)^(3/2)) + 2/2
1/((-x^2 + 1)^(3/2)*x^2 - 2*(-x^2 + 1)^(3/2)*x + (-x^2 + 1)^(3/2)) - 2/21/(
(-x^2 + 1)^(3/2)*x - (-x^2 + 1)^(3/2))
```

Fricas [A] time = 1.53236, size = 279, normalized size = 2.71

$$\frac{19\,x^7 - 57\,x^6 + 19\,x^5 + 95\,x^4 - 95\,x^3 - 19\,x^2 - \left(16\,x^6 - 48\,x^5 + 24\,x^4 + 56\,x^3 - 66\,x^2 + 6\,x + 19\right)\sqrt{x+1}\sqrt{-x+1} + 57\,x - }{63\left(x^7 - 3\,x^6 + x^5 + 5\,x^4 - 5\,x^3 - x^2 + 3\,x - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/63*(19*x^7 - 57*x^6 + 19*x^5 + 95*x^4 - 95*x^3 - 19*x^2 - (16*x^6 - 48*x^
5 + 24*x^4 + 56*x^3 - 66*x^2 + 6*x + 19)*sqrt(x + 1)*sqrt(-x + 1) + 57*x -
19)/(x^7 - 3*x^6 + x^5 + 5*x^4 - 5*x^3 - x^2 + 3*x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08194, size = 177, normalized size = 1.72

$$\frac{\left(\sqrt{2} - \sqrt{-x+1}\right)^3}{1536\,(x+1)^{\frac{3}{2}}} + \frac{23\left(\sqrt{2} - \sqrt{-x+1}\right)}{512\,\sqrt{x+1}} - \frac{(x+1)^{\frac{3}{2}}\left(\frac{69\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1} + 1\right)}{1536\left(\sqrt{2} - \sqrt{-x+1}\right)^3} - \frac{((((667\,x - 5021)(x+1) + 18396)(x+1) - 268}{4032\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1536*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 23/512*(sqrt(2) - sqrt(-x
 + 1))/sqrt(x + 1) - 1/1536*(x + 1)^(3/2)*(69*(sqrt(2) - sqrt(-x + 1))^2/(x
 + 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/4032*((((667*x - 5021)*(x + 1) +
18396)*(x + 1) - 26880)*(x + 1) + 15120)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^5
```

3.1137 $\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx$

Optimal. Leaf size=126

$$\frac{5}{16}a^2c^2x\sqrt{ax+a}\sqrt{c-cx}+\frac{5}{8}a^{5/2}c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)+\frac{5}{24}acx(ax+a)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(ax+a)^{5/2}(c-cx)^{5/2}$$

```
[Out] (5*a^2*c^2*x*Sqrt[a + a*x]*Sqrt[c - c*x])/16 + (5*a*c*x*(a + a*x)^(3/2)*(c
- c*x)^(3/2))/24 + (x*(a + a*x)^(5/2)*(c - c*x)^(5/2))/6 + (5*a^(5/2)*c^(5/
2)*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])])/8
```

Rubi [A] time = 0.0548007, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {38, 63, 217, 203}

$$\frac{5}{16}a^2c^2x\sqrt{ax+a}\sqrt{c-cx}+\frac{5}{8}a^{5/2}c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)+\frac{5}{24}acx(ax+a)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(ax+a)^{5/2}(c-cx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
```


```
[Out] (5*a^2*c^2*x*Sqrt[a + a*x]*Sqrt[c - c*x])/16 + (5*a*c*x*(a + a*x)^(3/2)*(c
- c*x)^(3/2))/24 + (x*(a + a*x)^(5/2)*(c - c*x)^(5/2))/6 + (5*a^(5/2)*c^(5/
2)*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])])/8
```


Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx = \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} + \frac{1}{6}(5ac)\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx$$
$$= \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} + \frac{1}{8}\left(5a^2c^2\right)\int \sqrt{a+ax}\sqrt{c-cx}\,dx$$
$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} + \frac{1}{16}\left(5a^3c\right.$$
$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} + \frac{1}{8}\left(5a^2c^3\right.$$
$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} + \frac{1}{8}\left(5a^2c^3\right.$$
$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} + \frac{5}{8}a^{5/2}c^{5/2}$$

Mathematica [A] time = 0.0980313, size = 114, normalized size = 0.9

$$\frac{c^{3/2}(a(x+1))^{5/2}\sqrt{c-cx}\left(\sqrt{c}x\sqrt{x+1}\left(8x^5-8x^4-26x^3+26x^2+33x-33\right)+30\sqrt{c-cx}\sin^{-1}\left(\frac{\sqrt{c-cx}}{\sqrt{2}\sqrt{c}}\right)\right)}{48(x-1)(x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
```

```
[Out] (c^(3/2)*(a*(1 + x))^(5/2)*Sqrt[c - c*x]*(Sqrt[c]*x*Sqrt[1 + x]*(-33 + 33*x
 + 26*x^2 - 26*x^3 - 8*x^4 + 8*x^5) + 30*Sqrt[c - c*x]*ArcSin[Sqrt[c - c*x]
/(Sqrt[2]*Sqrt[c])]))/(48*(-1 + x)*(1 + x)^(5/2))
```

Maple [B] time = 0.013, size = 193, normalized size = 1.5

$$-\frac{1}{6c}(ax+a)^{\frac{5}{2}}(-cx+c)^{\frac{7}{2}} - \frac{a}{6c}(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{7}{2}} - \frac{a^2}{8c}\sqrt{ax+a}\,(-cx+c)^{\frac{7}{2}} + \frac{a^2}{24}(-cx+c)^{\frac{5}{2}}\sqrt{ax+a} + \frac{5a^2c}{48}(-cx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(5/2)*(-c*x+c)^(5/2),x)
```

```
[Out] -1/6/c*(a*x+a)^(5/2)*(-c*x+c)^(7/2)-1/6*a/c*(a*x+a)^(3/2)*(-c*x+c)^(7/2)-1/
8*a^2/c*(a*x+a)^(1/2)*(-c*x+c)^(7/2)+1/24*a^2*(-c*x+c)^(5/2)*(a*x+a)^(1/2)+
5/48*a^2*c*(-c*x+c)^(3/2)*(a*x+a)^(1/2)+5/16*a^2*c^2*(-c*x+c)^(1/2)*(a*x+a)
^(1/2)+5/16*a^3*c^3*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2)/(
a*c)^(1/2)*arctan((a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66731, size = 478, normalized size = 3.79

$$\left[\frac{5}{32}\sqrt{-ac}a^2c^2\log\left(2\,acx^2+2\,\sqrt{-ac}\sqrt{ax+a}\sqrt{-cx+c}x-ac\right)+\frac{1}{48}\left(8\,a^2c^2x^5-26\,a^2c^2x^3+33\,a^2c^2x\right)\sqrt{ax+a}\sqrt{-cx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [5/32*sqrt(-a*c)*a^2*c^2*log(2*a*c*x^2 + 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c
*x + c)*x - a*c) + 1/48*(8*a^2*c^2*x^5 - 26*a^2*c^2*x^3 + 33*a^2*c^2*x)*sqr
t(a*x + a)*sqrt(-c*x + c), -5/16*sqrt(a*c)*a^2*c^2*arctan(sqrt(a*c)*sqrt(a*
x + a)*sqrt(-c*x + c)*x/(a*c*x^2 - a*c)) + 1/48*(8*a^2*c^2*x^5 - 26*a^2*c^2
*x^3 + 33*a^2*c^2*x)*sqrt(a*x + a)*sqrt(-c*x + c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(x+1))^{\frac{5}{2}}(-c(x-1))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(5/2)*(-c*x+c)**(5/2),x)
```

```
[Out] Integral((a*(x + 1))**(5/2)*(-c*(x - 1))**(5/2), x)
```

Giac [B] time = 1.35166, size = 478, normalized size = 3.79

$$-\frac{\left(\frac{6\,a^3c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}}-\sqrt{-(ax+a)ac+2\,a^2c}\left(\left(2\left((ax+a)\left(4\,(ax+a)\left(\frac{ax+a}{a^4}-\frac{5}{a^3}\right)+\frac{39}{a^2}\right)-\frac{37}{a}\right)(ax+a)\right.\right.\right.}{48\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/48*(6*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*
a^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*((2*((a*x + a)*(4*(a*x
 + a)*((a*x + a)/a^4 - 5/a^3) + 39/a^2) - 37/a)*(a*x + a) + 31)*(a*x + a) -
 3*a)*sqrt(a*x + a))*c^2*abs(a)/a - 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a
*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c)*sqrt(a*x + a)*a*x)*c^2*abs(a)/a + 1/4*(2*a^3*c*log(abs(-sqrt(-a
*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-(a*
x + a)*a*c + 2*a^2*c)*((a*x + a)*(2*(a*x + a)*((a*x + a)/a^2 - 3/a) + 5) -
a)*sqrt(a*x + a))*c^2*abs(a)/a
```

3.1138 $\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx$

Optimal. Leaf size=96

$$\frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)+\frac{3}{8}acx\sqrt{ax+a}\sqrt{c-cx}+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}$$

```
[Out] (3*a*c*x*Sqrt[a + a*x]*Sqrt[c - c*x])/8 + (x*(a + a*x)^(3/2)*(c - c*x)^(3/2
))/4 + (3*a^(3/2)*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c -
c*x])])/4
```

Rubi [A] time = 0.0365369, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {38, 63, 217, 203}

$$\frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)+\frac{3}{8}acx\sqrt{ax+a}\sqrt{c-cx}+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

```
[Out] (3*a*c*x*Sqrt[a + a*x]*Sqrt[c - c*x])/8 + (x*(a + a*x)^(3/2)*(c - c*x)^(3/2
))/4 + (3*a^(3/2)*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c -
c*x])])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx = \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{4}(3ac)\int \sqrt{a+ax}\sqrt{c-cx}\,dx$$
$$= \frac{3}{8}acx\sqrt{a+ax}\sqrt{c-cx} + \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{8}\left(3a^2c^2\right)\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx$$
$$= \frac{3}{8}acx\sqrt{a+ax}\sqrt{c-cx} + \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{4}\left(3ac^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx, x, \sqrt{\right.$$
$$= \frac{3}{8}acx\sqrt{a+ax}\sqrt{c-cx} + \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{4}\left(3ac^2\right)\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx, x, \frac{\sqrt{a+}}{\sqrt{c-}}\right.$$
$$= \frac{3}{8}acx\sqrt{a+ax}\sqrt{c-cx} + \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)$$

Mathematica [A] time = 0.0788277, size = 104, normalized size = 1.08

$$\frac{\sqrt{c}(a(x+1))^{3/2}\sqrt{c-cx}\left(\sqrt{c}x\sqrt{x+1}\left(-2x^3+2x^2+5x-5\right)+6\sqrt{c-cx}\sin^{-1}\left(\frac{\sqrt{c-cx}}{\sqrt{2}\sqrt{c}}\right)\right)}{8(x-1)(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

```
[Out] (Sqrt[c]*(a*(1 + x))^(3/2)*Sqrt[c - c*x]*(Sqrt[c]*x*Sqrt[1 + x]*(-5 + 5*x +
 2*x^2 - 2*x^3) + 6*Sqrt[c - c*x]*ArcSin[Sqrt[c - c*x]/(Sqrt[2]*Sqrt[c])]))
/(8*(-1 + x)*(1 + x)^(3/2))
```

Maple [B] time = 0.004, size = 143, normalized size = 1.5

$$-\frac{1}{4c}(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{5}{2}} - \frac{a}{4c}\sqrt{ax+a}(-cx+c)^{\frac{5}{2}} + \frac{a}{8}\sqrt{ax+a}(-cx+c)^{\frac{3}{2}} + \frac{3ac}{8}\sqrt{ax+a}\sqrt{-cx+c} + \frac{3a^2c^2}{8}\sqrt{(-cx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(3/2)*(-c*x+c)^(3/2),x)
```

```
[Out] -1/4/c*(a*x+a)^(3/2)*(-c*x+c)^(5/2)-1/4*a/c*(a*x+a)^(1/2)*(-c*x+c)^(5/2)+1/
8*(a*x+a)^(1/2)*(-c*x+c)^(3/2)*a+3/8*a*c*(-c*x+c)^(1/2)*(a*x+a)^(1/2)+3/8*a
^2*c^2*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2)/(a*c)^(1/2)*ar
ctan((a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63984, size = 393, normalized size = 4.09

$$\left[\frac{3}{16}\,\sqrt{-ac}ac\log\left(2\,acx^2+2\,\sqrt{-ac}\sqrt{ax+a}\sqrt{-cx+c}x-ac\right)-\frac{1}{8}\left(2\,acx^3-5\,acx\right)\sqrt{ax+a}\sqrt{-cx+c},-\frac{3}{8}\,\sqrt{ac}ac\arctan\left(\frac{\sqrt{ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [3/16*sqrt(-a*c)*a*c*log(2*a*c*x^2 + 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c*x +
 c)*x - a*c) - 1/8*(2*a*c*x^3 - 5*a*c*x)*sqrt(a*x + a)*sqrt(-c*x + c), -3/8
*sqrt(a*c)*a*c*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/(a*c*x^2 - a
*c)) - 1/8*(2*a*c*x^3 - 5*a*c*x)*sqrt(a*x + a)*sqrt(-c*x + c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\,(x+1))^{\frac{3}{2}}\,(-c\,(x-1))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(3/2)*(-c*x+c)**(3/2),x)
```

```
[Out] Integral((a*(x + 1))**(3/2)*(-c*(x - 1))**(3/2), x)
```

Giac [B] time = 1.24884, size = 277, normalized size = 2.89

$$-\frac{\left(\frac{2\,a^3c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}}-\sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}ax\right)c|a|}{2\,a^2}+\frac{\left(\frac{2\,a^3c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}}-\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a
^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*a*x)*c*ab
s(a)/a^2 + 1/8*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)
*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*(2
*(a*x + a)*((a*x + a)/a^2 - 3/a) + 5) - a)*sqrt(a*x + a))*c*abs(a)/a^2
```

3.1139 $\int \sqrt{a+ax}\sqrt{c-cx}\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}+\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)$$

```
[Out] (x*Sqrt[a + a*x]*Sqrt[c - c*x])/2 + Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
+ a*x])/(Sqrt[a]*Sqrt[c - c*x])]
```

Rubi [A] time = 0.0292047, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {38, 63, 217, 203}

$$\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}+\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*x]*Sqrt[c - c*x],x]
```

```
[Out] (x*Sqrt[a + a*x]*Sqrt[c - c*x])/2 + Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
+ a*x])/(Sqrt[a]*Sqrt[c - c*x])]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+ax}\sqrt{c-cx}\,dx = \frac{1}{2}x\sqrt{a+ax}\sqrt{c-cx}+\frac{1}{2}(ac)\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx$$
$$=\frac{1}{2}x\sqrt{a+ax}\sqrt{c-cx}+c\,\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx,x,\sqrt{a+ax}\right)$$
$$=\frac{1}{2}x\sqrt{a+ax}\sqrt{c-cx}+c\,\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx,x,\frac{\sqrt{a+ax}}{\sqrt{c-cx}}\right)$$
$$=\frac{1}{2}x\sqrt{a+ax}\sqrt{c-cx}+\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)$$

Mathematica [A] time = 0.0594009, size = 69, normalized size = 1.03

$$\frac{\sqrt{a(x+1)}\left(x\sqrt{x+1}\sqrt{c-cx}-2\sqrt{c}\sin^{-1}\left(\frac{\sqrt{c-cx}}{\sqrt{2}\sqrt{c}}\right)\right)}{2\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*x]*Sqrt[c - c*x],x]
```

```
[Out] (Sqrt[a*(1 + x)]*(x*Sqrt[1 + x]*Sqrt[c - c*x] - 2*Sqrt[c]*ArcSin[Sqrt[c - c
*x]/(Sqrt[2]*Sqrt[c])]))/(2*Sqrt[1 + x])
```

Maple [A] time = 0.004, size = 98, normalized size = 1.5

$$-\frac{1}{2c}\sqrt{ax+a}\,(-cx+c)^{\frac{3}{2}}+\frac{1}{2}\sqrt{ax+a}\sqrt{-cx+c}+\frac{ac}{2}\sqrt{(-cx+c)(ax+a)}\arctan\left(x\sqrt{ac}\frac{1}{\sqrt{-acx^2+ac}}\right)\frac{1}{\sqrt{ax+a}}\frac{1}{\sqrt{-cx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(1/2)*(-c*x+c)^(1/2),x)
```

```
[Out] -1/2/c*(a*x+a)^(1/2)*(-c*x+c)^(3/2)+1/2*(a*x+a)^(1/2)*(-c*x+c)^(1/2)+1/2*a*
c*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2)/(a*c)^(1/2)*arctan(
(a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61877, size = 325, normalized size = 4.85

$$\left[\frac{1}{2}\sqrt{ax+a}\sqrt{-cx+c}x + \frac{1}{4}\sqrt{-ac}\log\left(2\,acx^2 + 2\sqrt{-ac}\sqrt{ax+a}\sqrt{-cx+c}x - ac\right), \frac{1}{2}\sqrt{ax+a}\sqrt{-cx+c}x - \frac{1}{2}\sqrt{ac}\,\mathrm{arctan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a*x + a)*sqrt(-c*x + c)*x + 1/4*sqrt(-a*c)*log(2*a*c*x^2 + 2*sqrt
(-a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x - a*c), 1/2*sqrt(a*x + a)*sqrt(-c*x +
 c)*x - 1/2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/(a*c*
x^2 - a*c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\,(x+1)}\sqrt{-c\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(1/2)*(-c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(x + 1))*sqrt(-c*(x - 1)), x)
```

Giac [A] time = 1.13747, size = 115, normalized size = 1.72

$$-\frac{\left(\frac{2\,a^3 c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}} - \sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}\,aax\right)|a|}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a
^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*a*x)*abs(
a)/a^3
```

3.1140 $\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx$

Optimal. Leaf size=43

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

```
[Out] (2*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])])/(Sqrt[a]*Sqrt[c
])
```

Rubi [A] time = 0.0239438, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {63, 217, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])])/(Sqrt[a]*Sqrt[c
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx, x, \sqrt{a+ax}\right)}{a}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx, x, \frac{\sqrt{a+ax}}{\sqrt{c-cx}}\right)}{a}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

Mathematica [A] time = 0.0163171, size = 47, normalized size = 1.09

$$\frac{2\sqrt{x+1}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{x+1}}{\sqrt{c-cx}}\right)}{\sqrt{c}\sqrt{a(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
```

```
[Out] (2*Sqrt[1 + x]*ArcTan[(Sqrt[c]*Sqrt[1 + x])/Sqrt[c - c*x]])/(Sqrt[c]*Sqrt[a
*(1 + x)])
```

Maple [A] time = 0.005, size = 57, normalized size = 1.3

$$\sqrt{(-cx+c)(ax+a)}\arctan\left(x\sqrt{ac}\frac{1}{\sqrt{-acx^2+ac}}\right)\frac{1}{\sqrt{ax+a}}\frac{1}{\sqrt{-cx+c}}\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x)
```

```
[Out] ((-c*x+c)*(a*x+a))^(1/2)/(a*x+a)^(1/2)/(-c*x+c)^(1/2)/(a*c)^(1/2)*arctan((a
*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66179, size = 239, normalized size = 5.56

$$\left[-\frac{\sqrt{-ac}\log\left(2\,acx^2-2\,\sqrt{-ac}\sqrt{ax+a}\sqrt{-cx+c}x-ac\right)}{2\,ac},-\frac{\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{ax+a}\sqrt{-cx+c}x}{acx^2-ac}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(2*a*c*x^2 - 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*
x - a*c)/(a*c), -sqrt(a*c)*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/
(a*c*x^2 - a*c))/(a*c)]
```

Sympy [C] time = 3.1786, size = 85, normalized size = 1.98

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}\sqrt{a}\sqrt{c}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\-\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^2}\right)}{4\pi^{\frac{3}{2}}\sqrt{a}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(1/2)/(-c*x+c)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
x**(-2))/(4*pi**(3/2)*sqrt(a)*sqrt(c)) + meijerg(((-1/2, -1/4, 0, 1/4, 1/2,
 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/x**2)/(4*pi**(
3/2)*sqrt(a)*sqrt(c))
```

Giac [A] time = 1.14295, size = 66, normalized size = 1.53

$$-\frac{2\,a\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/(
sqrt(-a*c)*abs(a))
```

3.1141 $\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x}{ac\sqrt{ax+a}\sqrt{c-cx}}$$

```
[Out] x/(a*c*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rubi [A] time = 0.0031864, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {39}

$$\frac{x}{ac\sqrt{ax+a}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
```

```
[Out] x/(a*c*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx = \frac{x}{ac\sqrt{a+ax}\sqrt{c-cx}}$$

Mathematica [A] time = 0.0163892, size = 27, normalized size = 1.

$$\frac{x(x+1)}{c(a(x+1))^{3/2}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
```

```
[Out] (x*(1 + x))/(c*(a*(1 + x))^(3/2)*Sqrt[c - c*x])
```

Maple [A] time = 0.001, size = 25, normalized size = 0.9

$$-(1+x)(-1+x)\,x\,(ax+a)^{-\frac{3}{2}}(-cx+c)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x)
```

```
[Out] -(1+x)*(-1+x)*x/(a*x+a)^(3/2)/(-c*x+c)^(3/2)
```

Maxima [A] time = 1.00216, size = 28, normalized size = 1.04

$$\frac{x}{\sqrt{-acx^2 + ac}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-a*c*x^2 + a*c)*a*c)
```

Fricas [A] time = 1.57577, size = 77, normalized size = 2.85

$$-\frac{\sqrt{ax + a}\sqrt{-cx + c}x}{a^2c^2x^2 - a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(a*x + a)*sqrt(-c*x + c)*x/(a^2*c^2*x^2 - a^2*c^2)
```

Sympy [C] time = 5.44478, size = 82, normalized size = 3.04

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & \frac{1}{2}, \frac{3}{2}, 2 \\ \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 2 & 0 \end{matrix} \middle| \frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}a^{\frac{3}{2}}c^{\frac{3}{2}}} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 & \\ \frac{1}{4}, \frac{3}{4} & -\frac{1}{2}, 0, 1, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}a^{\frac{3}{2}}c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(3/2)/(-c*x+c)**(3/2),x)
```

```
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), x
**(-2))/(2*pi**(3/2)*a**(3/2)*c**(3/2)) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4,
 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), exp_polar(-2*I*pi)/x**2)/(2*pi**(3
/2)*a**(3/2)*c**(3/2))
```

Giac [B] time = 1.0892, size = 157, normalized size = 5.81

$$-\frac{2\sqrt{-ac}a}{\left(2a^2c - \left(\sqrt{-ac}\sqrt{ax + a} - \sqrt{-(ax + a)ac + 2a^2c}\right)^2\right)c|a|} - \frac{\sqrt{-(ax + a)ac + 2a^2c}\sqrt{ax + a}}{2\left((ax + a)ac - 2a^2c\right)c|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-a*c)*a/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^2)*c*abs(a)) - 1/2*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a
)/(((a*x + a)*a*c - 2*a^2*c)*c*abs(a))
```

3.1142 $\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{2x}{3a^2c^2\sqrt{ax+a}\sqrt{c-cx}}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}$$

```
[Out] x/(3*a*c*(a + a*x)^(3/2)*(c - c*x)^(3/2)) + (2*x)/(3*a^2*c^2*Sqrt[a + a*x]*
Sqrt[c - c*x])
```

Rubi [A] time = 0.0098129, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {40, 39}

$$\frac{2x}{3a^2c^2\sqrt{ax+a}\sqrt{c-cx}}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
```

```
[Out] x/(3*a*c*(a + a*x)^(3/2)*(c - c*x)^(3/2)) + (2*x)/(3*a^2*c^2*Sqrt[a + a*x]*
Sqrt[c - c*x])
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx &= \frac{x}{3ac(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{2\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx}{3ac}\\ &= \frac{x}{3ac(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{2x}{3a^2c^2\sqrt{a+ax}\sqrt{c-cx}}\end{aligned}$$

Mathematica [A] time = 0.0272902, size = 42, normalized size = 0.69

$$\frac{x(x+1)\left(2x^2-3\right)}{3c^2(x-1)(a(x+1))^{5/2}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
```

```
[Out] (x*(1 + x)*(-3 + 2*x^2))/(3*c^2*(-1 + x)*(a*(1 + x))^(5/2)*Sqrt[c - c*x])
```

Maple [A] time = 0.001, size = 32, normalized size = 0.5

$$\frac{(1+x)(-1+x)x\left(2x^2-3\right)}{3}(ax+a)^{-\frac{5}{2}}(-cx+c)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x)
```

```
[Out] 1/3*(1+x)*(-1+x)*x*(2*x^2-3)/(a*x+a)^(5/2)/(-c*x+c)^(5/2)
```

Maxima [A] time = 0.98581, size = 61, normalized size = 1.

$$\frac{x}{3\left(-acx^2+ac\right)^{\frac{3}{2}}ac}+\frac{2x}{3\sqrt{-acx^2+ac}a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x/((-a*c*x^2 + a*c)^(3/2)*a*c) + 2/3*x/(sqrt(-a*c*x^2 + a*c)*a^2*c^2)
```

Fricas [A] time = 1.53267, size = 120, normalized size = 1.97

$$-\frac{\left(2x^3-3x\right)\sqrt{ax+a}\sqrt{-cx+c}}{3\left(a^3c^3x^4-2a^3c^3x^2+a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^3 - 3*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^3*c^3*x^4 - 2*a^3*c^3*x^
2 + a^3*c^3)
```

Sympy [C] time = 53.7394, size = 82, normalized size = 1.34

$$\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{1}{2},\frac{5}{2},3\\ \frac{5}{4},\frac{7}{4},2,\frac{5}{2},3 & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{3\pi^{\frac{3}{2}}a^{\frac{5}{2}}c^{\frac{5}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},\frac{3}{4},\frac{5}{4},1 & \\ \frac{3}{4},\frac{5}{4} & -\frac{1}{2},0,2,0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^2}\right)}{3\pi^{\frac{3}{2}}a^{\frac{5}{2}}c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(5/2)/(-c*x+c)**(5/2),x)
```

```
[Out] I*meijerg(((5/4, 7/4, 1), (1/2, 5/2, 3)), ((5/4, 7/4, 2, 5/2, 3), (0,)), x*
*(-2))/(3*pi**(3/2)*a**(5/2)*c**(5/2)) + meijerg(((-1/2, 0, 1/2, 3/4, 5/4,
1), ()), ((3/4, 5/4), (-1/2, 0, 2, 0)), exp_polar(-2*I*pi)/x**2)/(3*pi**(3/
2)*a**(5/2)*c**(5/2))
```

Giac [B] time = 1.2169, size = 320, normalized size = 5.25

$$-\frac{\sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}\left(\frac{4\,(ax+a)|a|}{a^2c}-\frac{9\,|a|}{ac}\right)}{12\left((ax+a)ac-2\,a^2c\right)^2}-\frac{16\,\sqrt{-ac}a^4c^2-18\,\sqrt{-ac}\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2a^2c+}{3\left(2\,a^2c-\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*(4*(a*x + a)*abs(a)/(a^2
*c) - 9*abs(a)/(a*c))/((a*x + a)*a*c - 2*a^2*c)^2 - 1/3*(16*sqrt(-a*c)*a^4*
c^2 - 18*sqrt(-a*c)*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2
*c))^2*a^2*c + 3*sqrt(-a*c)*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^4)/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^2)^3*c^2*abs(a))
```

3.1143 $\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx$

Optimal. Leaf size=91

$$\frac{8x}{15a^3c^3\sqrt{ax+a}\sqrt{c-cx}}+\frac{4x}{15a^2c^2(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}$$

```
[Out] x/(5*a*c*(a + a*x)^(5/2)*(c - c*x)^(5/2)) + (4*x)/(15*a^2*c^2*(a + a*x)^(3/
2)*(c - c*x)^(3/2)) + (8*x)/(15*a^3*c^3*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rubi [A] time = 0.0179005, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {40, 39}

$$\frac{8x}{15a^3c^3\sqrt{ax+a}\sqrt{c-cx}}+\frac{4x}{15a^2c^2(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

```
[Out] x/(5*a*c*(a + a*x)^(5/2)*(c - c*x)^(5/2)) + (4*x)/(15*a^2*c^2*(a + a*x)^(3/
2)*(c - c*x)^(3/2)) + (8*x)/(15*a^3*c^3*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx&=\frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx}{5ac}\\&=\frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4x}{15a^2c^2(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{8\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx}{15a^2c^2}\\&=\frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4x}{15a^2c^2(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{8x}{15a^3c^3\sqrt{a+ax}\sqrt{c-cx}}\end{aligned}$$

Mathematica [A] time = 0.0362571, size = 49, normalized size = 0.54

$$\frac{x\left(8x^4-20x^2+15\right)}{15a^3c^3\left(x^2-1\right)^2\sqrt{a(x+1)}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

```
[Out] (x*(15 - 20*x^2 + 8*x^4))/(15*a^3*c^3*Sqrt[a*(1 + x)]*Sqrt[c - c*x]*(-1 + x
^2)^2)
```

Maple [A] time = 0.004, size = 37, normalized size = 0.4

$$-\frac{(1+x)(-1+x)x\left(8x^4-20x^2+15\right)}{15}(ax+a)^{-\frac{7}{2}}(-cx+c)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x)
```

```
[Out] -1/15*(1+x)*(-1+x)*x*(8*x^4-20*x^2+15)/(a*x+a)^(7/2)/(-c*x+c)^(7/2)
```

Maxima [A] time = 0.981987, size = 90, normalized size = 0.99

$$\frac{x}{5\left(-acx^2+ac\right)^{\frac{5}{2}}ac}+\frac{4x}{15\left(-acx^2+ac\right)^{\frac{3}{2}}a^2c^2}+\frac{8x}{15\sqrt{-acx^2+ac}a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-a*c*x^2 + a*c)^(5/2)*a*c) + 4/15*x/((-a*c*x^2 + a*c)^(3/2)*a^2*c^2
) + 8/15*x/(sqrt(-a*c*x^2 + a*c)*a^3*c^3)
```

Fricas [A] time = 1.60806, size = 157, normalized size = 1.73

$$-\frac{\left(8x^5-20x^3+15x\right)\sqrt{ax+a}\sqrt{-cx+c}}{15\left(a^4c^4x^6-3a^4c^4x^4+3a^4c^4x^2-a^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*x^5 - 20*x^3 + 15*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^4*c^4*x^6 - 3
*a^4*c^4*x^4 + 3*a^4*c^4*x^2 - a^4*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(7/2)/(-c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.57128, size = 450, normalized size = 4.95

$$-\frac{\sqrt{-(ax+a)ac+2a^2c}\sqrt{ax+a}\left((ax+a)\left(\frac{64(ax+a)}{c|a|}-\frac{275a}{c|a|}\right)+\frac{300a^2}{c|a|}\right)}{240\left((ax+a)ac-2a^2c\right)^3}+\frac{1024a^8c^4-2200\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)a\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/240*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(64*(a*x + a
)/(c*abs(a)) - 275*a/(c*abs(a))) + 300*a^2/(c*abs(a)))/((a*x + a)*a*c - 2*a
^2*c)^3 + 1/60*(1024*a^8*c^4 - 2200*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
+ a)*a*c + 2*a^2*c))^2*a^6*c^3 + 1660*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*
x + a)*a*c + 2*a^2*c))^4*a^4*c^2 - 450*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a
*x + a)*a*c + 2*a^2*c))^6*a^2*c + 45*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
 + a)*a*c + 2*a^2*c))^8)/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
 + a)*a*c + 2*a^2*c))^2)^5*sqrt(-a*c)*c^2*abs(a))
```

3.1144 $\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx$

Optimal. Leaf size=121

$$\frac{16x}{35a^4c^4\sqrt{ax+a}\sqrt{c-cx}}+\frac{8x}{35a^3c^3(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{6x}{35a^2c^2(ax+a)^{5/2}(c-cx)^{5/2}}+\frac{x}{7ac(ax+a)^{7/2}(c-cx)^{7/2}}$$

```
[Out] x/(7*a*c*(a + a*x)^(7/2)*(c - c*x)^(7/2)) + (6*x)/(35*a^2*c^2*(a + a*x)^(5/
2)*(c - c*x)^(5/2)) + (8*x)/(35*a^3*c^3*(a + a*x)^(3/2)*(c - c*x)^(3/2)) +
(16*x)/(35*a^4*c^4*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rubi [A] time = 0.0276369, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {40, 39}

$$\frac{16x}{35a^4c^4\sqrt{ax+a}\sqrt{c-cx}}+\frac{8x}{35a^3c^3(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{6x}{35a^2c^2(ax+a)^{5/2}(c-cx)^{5/2}}+\frac{x}{7ac(ax+a)^{7/2}(c-cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

```
[Out] x/(7*a*c*(a + a*x)^(7/2)*(c - c*x)^(7/2)) + (6*x)/(35*a^2*c^2*(a + a*x)^(5/
2)*(c - c*x)^(5/2)) + (8*x)/(35*a^3*c^3*(a + a*x)^(3/2)*(c - c*x)^(3/2)) +
(16*x)/(35*a^4*c^4*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx &= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}}+\frac{6\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx}{7ac}\\
&= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}}+\frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{24\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx}{35a^2c^2}\\
&= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}}+\frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{8x}{35a^3c^3(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{16}{}\\
&= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}}+\frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{8x}{35a^3c^3(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{}{35}
\end{aligned}$$

Mathematica [A] time = 0.0394339, size = 54, normalized size = 0.45

$$\frac{x\left(16x^6-56x^4+70x^2-35\right)}{35a^4c^4\left(x^2-1\right)^3\sqrt{a(x+1)}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

```
[Out] (x*(-35 + 70*x^2 - 56*x^4 + 16*x^6))/(35*a^4*c^4*Sqrt[a*(1 + x)]*Sqrt[c - c
*x]*(-1 + x^2)^3)
```

Maple [A] time = 0.003, size = 42, normalized size = 0.4

$$\frac{(1+x)(-1+x)x\left(16x^6-56x^4+70x^2-35\right)}{35}(ax+a)^{-\frac{9}{2}}(-cx+c)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x)
```

```
[Out] 1/35*(1+x)*(-1+x)*x*(16*x^6-56*x^4+70*x^2-35)/(a*x+a)^(9/2)/(-c*x+c)^(9/2)
```

Maxima [A] time = 0.999581, size = 120, normalized size = 0.99

$$\frac{x}{7\left(-acx^2+ac\right)^{\frac{7}{2}}ac}+\frac{6x}{35\left(-acx^2+ac\right)^{\frac{5}{2}}a^2c^2}+\frac{8x}{35\left(-acx^2+ac\right)^{\frac{3}{2}}a^3c^3}+\frac{16x}{35\sqrt{-acx^2+ac}a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/7*x/((-a*c*x^2 + a*c)^(7/2)*a*c) + 6/35*x/((-a*c*x^2 + a*c)^(5/2)*a^2*c^2
) + 8/35*x/((-a*c*x^2 + a*c)^(3/2)*a^3*c^3) + 16/35*x/(sqrt(-a*c*x^2 + a*c)
*a^4*c^4)
```

Fricas [A] time = 1.59296, size = 192, normalized size = 1.59

$$-\frac{\left(16x^7-56x^5+70x^3-35x\right)\sqrt{ax+a}\sqrt{-cx+c}}{35\left(a^5c^5x^8-4a^5c^5x^6+6a^5c^5x^4-4a^5c^5x^2+a^5c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*x^7 - 56*x^5 + 70*x^3 - 35*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^5*c
^5*x^8 - 4*a^5*c^5*x^6 + 6*a^5*c^5*x^4 - 4*a^5*c^5*x^2 + a^5*c^5)
```

<u>**Sympy**</u> **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(9/2)/(-c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[B]** time = 1.81705, size = 590, normalized size = 4.88

$$-\frac{\sqrt{-(ax+a)ac+2\,a^2c}\left((ax+a)\left((ax+a)\left(\frac{256\,(ax+a)|a|}{a^2c}-\frac{1617\,|a|}{ac}\right)+\frac{3430\,|a|}{c}\right)-\frac{2450\,a|a|}{c}\right)\sqrt{ax+a}}{1120\left((ax+a)ac-2\,a^2c\right)^4}+\frac{16384\,a^{12}c^6-51744\left(\sqrt{-a\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] -1/1120*sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*((a*x + a)*(256*(a*x + a)
*abs(a)/(a^2*c) - 1617*abs(a)/(a*c)) + 3430*abs(a)/c) - 2450*a*abs(a)/c)*sq
rt(a*x + a)/((a*x + a)*a*c - 2*a^2*c)^4 + 1/280*(16384*a^12*c^6 - 51744*(sq
rt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^2*a^10*c^5 + 66416
*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^4*a^8*c^4 - 43
120*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^6*a^6*c^3 +
 14280*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^8*a^4*c^
2 - 2450*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^10*a^2
*c + 175*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^12)/((
2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^2)^7*
sqrt(-a*c)*a*c^3*abs(a))
```

3.1145 $\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx$

Optimal. Leaf size=135

$$\frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{5a^6c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{8b}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

```
[Out] (5*a^4*c^2*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/16 + (5*a^2*c*x*(a + b*x)^(3/
2)*(a*c - b*c*x)^(3/2))/24 + (x*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2))/6 + (5
*a^6*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a - b*x)]])/(8*b)
```

Rubi [A] time = 0.0522339, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {38, 63, 217, 203}

$$\frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{5a^6c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{8b}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

```
[Out] (5*a^4*c^2*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/16 + (5*a^2*c*x*(a + b*x)^(3/
2)*(a*c - b*c*x)^(3/2))/24 + (x*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2))/6 + (5
*a^6*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a - b*x)]])/(8*b)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx = \frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2} + \frac{1}{6}\left(5a^2c\right)\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx$$
$$= \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2} + \frac{1}{8}\left(5a^4c^2\right)\int \sqrt{a+bx}\sqrt{ac-}$$
$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$
$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$
$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$
$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

Mathematica [A] time = 0.147235, size = 120, normalized size = 0.89

$$\frac{c^3\left(34a^2b^5x^5 - 59a^4b^3x^3 + 33a^6bx - 30a^{13/2}\sqrt{a-bx}\sqrt{\frac{bx}{a}+1}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right) - 8b^7x^7\right)}{48b\sqrt{a+bx}\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

```
[Out] (c^3*(33*a^6*b*x - 59*a^4*b^3*x^3 + 34*a^2*b^5*x^5 - 8*b^7*x^7 - 30*a^(13/2
)*Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]))
/(48*b*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

Maple [B] time = 0.01, size = 243, normalized size = 1.8

$$-\frac{1}{6\,bc}(bx+a)^{\frac{5}{2}}(-bcx+ac)^{\frac{7}{2}} - \frac{a}{6\,bc}(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{7}{2}} - \frac{a^2}{8\,bc}\sqrt{bx+a}(-bcx+ac)^{\frac{7}{2}} + \frac{a^3}{24\,b}(-bcx+ac)^{\frac{5}{2}}\sqrt{bx+a} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x)
```

```
[Out] -1/6/b/c*(b*x+a)^(5/2)*(-b*c*x+a*c)^(7/2)-1/6*a/b/c*(b*x+a)^(3/2)*(-b*c*x+a
*c)^(7/2)-1/8*a^2/b/c*(b*x+a)^(1/2)*(-b*c*x+a*c)^(7/2)+1/24*a^3/b*(-b*c*x+a
*c)^(5/2)*(b*x+a)^(1/2)+5/48*a^4*c/b*(-b*c*x+a*c)^(3/2)*(b*x+a)^(1/2)+5/16*
a^5*c^2/b*(-b*c*x+a*c)^(1/2)*(b*x+a)^(1/2)+5/16*a^6*c^3*((b*x+a)*(-b*c*x+a*
c))^(1/2)/(-b*c*x+a*c)^(1/2)/(b*x+a)^(1/2)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/
2)*x/(-b^2*c*x^2+a^2*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70367, size = 524, normalized size = 3.88

$$\left[\frac{15\,a^6\sqrt{-c}c^2\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\left(8\,b^5c^2x^5-26\,a^2b^3c^2x^3+33\,a^4bc^2x\right)\sqrt{-bcx+ac}\sqrt{}}{96\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^6*sqrt(-c)*c^2*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x
+ a)*b*sqrt(-c)*x - a^2*c) + 2*(8*b^5*c^2*x^5 - 26*a^2*b^3*c^2*x^3 + 33*a^4
*b*c^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b, -1/48*(15*a^6*c^(5/2)*arctan
(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (8*b^5
*c^2*x^5 - 26*a^2*b^3*c^2*x^3 + 33*a^4*b*c^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x
 + a))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\left(-a+bx\right)\right)^{\frac{5}{2}}\left(a+bx\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(-b*c*x+a*c)**(5/2),x)
```

```
[Out] Integral((-c*(-a + b*x))**(5/2)*(a + b*x)**(5/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1146 $\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx$

Optimal. Leaf size=102

$$\frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{4b}+\frac{3}{8}a^2cx\sqrt{a+bx}\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}$$

```
[Out] (3*a^2*c*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 + (x*(a + b*x)^(3/2)*(a*c - b
*c*x)^(3/2))/4 + (3*a^4*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a -
b*x)]])/(4*b)
```

Rubi [A] time = 0.0361188, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {38, 63, 217, 203}

$$\frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{4b}+\frac{3}{8}a^2cx\sqrt{a+bx}\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
```

```
[Out] (3*a^2*c*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 + (x*(a + b*x)^(3/2)*(a*c - b
*c*x)^(3/2))/4 + (3*a^4*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a -
b*x)]])/(4*b)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx = \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{4}\left(3a^2c\right)\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{8}\left(3a^4c^2\right)\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bc}$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{\left(3a^4c^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2ac-cx^2}}\,dx,\right.}{4b}$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{\left(3a^4c^2\right)\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx,x,\right.}{4b}$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{4b}$$

Mathematica [A] time = 0.122282, size = 109, normalized size = 1.07

$$\frac{c^2\left(-7a^2b^3x^3+5a^4bx-6a^{9/2}\sqrt{a-bx}\sqrt{\frac{bx}{a}+1}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)+2b^5x^5\right)}{8b\sqrt{a+bx}\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
```

```
[Out] (c^2*(5*a^4*b*x - 7*a^2*b^3*x^3 + 2*b^5*x^5 - 6*a^(9/2)*Sqrt[a - b*x]*Sqrt[
1 + (b*x)/a]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]))/(8*b*Sqrt[c*(a - b*x
)]*Sqrt[a + b*x])
```

Maple [B] time = 0.004, size = 185, normalized size = 1.8

$$-\frac{1}{4\,bc}(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{5}{2}}-\frac{a}{4\,bc}\sqrt{bx+a}\,(-bcx+ac)^{\frac{5}{2}}+\frac{a^2}{8\,b}(-bcx+ac)^{\frac{3}{2}}\sqrt{bx+a}+\frac{3\,a^3c}{8\,b}\sqrt{bx+a}\sqrt{-bcx+ac}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x)
```

```
[Out] -1/4/b/c*(b*x+a)^(3/2)*(-b*c*x+a*c)^(5/2)-1/4*a/b/c*(b*x+a)^(1/2)*(-b*c*x+a
*c)^(5/2)+1/8*a^2/b*(-b*c*x+a*c)^(3/2)*(b*x+a)^(1/2)+3/8*a^3*c/b*(-b*c*x+a*
c)^(1/2)*(b*x+a)^(1/2)+3/8*a^4*c^2*((b*x+a)*(-b*c*x+a*c))^(1/2)/(-b*c*x+a*c
)^(1/2)/(b*x+a)^(1/2)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*
c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62313, size = 447, normalized size = 4.38

$$\left[\frac{3\,a^4\sqrt{-c}c\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)-2\left(2\,b^3cx^3-5\,a^2bcx\right)\sqrt{-bcx+ac}\sqrt{bx+a}}{16\,b},-\frac{3\,a^4c^{\frac{3}{2}}\arctan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^4*sqrt(-c)*c*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a
)*b*sqrt(-c)*x - a^2*c) - 2*(2*b^3*c*x^3 - 5*a^2*b*c*x)*sqrt(-b*c*x + a*c)*
sqrt(b*x + a))/b, -1/8*(3*a^4*c^(3/2)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x +
a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (2*b^3*c*x^3 - 5*a^2*b*c*x)*sqrt(-b*c
*x + a*c)*sqrt(b*x + a))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(-c(-a+bx))^{\frac{3}{2}}(a+bx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(-b*c*x+a*c)**(3/2),x)
```

```
[Out] Integral((-c*(-a + b*x))**(3/2)*(a + b*x)**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1147 $\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx$

Optimal. Leaf size=68

$$\frac{a^2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}$$

```
[Out] (x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/2 + (a^2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
 + b*x])/Sqrt[c*(a - b*x)]])/b
```

Rubi [A] time = 0.0265255, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {38, 63, 217, 203}

$$\frac{a^2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
```

```
[Out] (x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/2 + (a^2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
 + b*x])/Sqrt[c*(a - b*x)]])/b
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx = \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{1}{2}\left(a^2c\right)\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{\left(a^2c\right)\text{Subst}\left(\int \frac{1}{\sqrt{2ac-cx^2}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{\left(a^2c\right)\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}$$
$$= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{a^2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}$$

Mathematica [A] time = 0.118363, size = 95, normalized size = 1.4

$$\frac{c\left(a^2bx - 2a^{5/2}\sqrt{a-bx}\sqrt{\frac{bx}{a}+1}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right) - b^3x^3\right)}{2b\sqrt{a+bx}\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
```

```
[Out] (c*(a^2*b*x - b^3*x^3 - 2*a^(5/2)*Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*ArcSin[Sq
rt[a - b*x]/(Sqrt[2]*Sqrt[a])]))/(2*b*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

Maple [B] time = 0.004, size = 127, normalized size = 1.9

$$-\frac{1}{2\,bc}\sqrt{bx+a}\,(-bcx+ac)^{\frac{3}{2}} + \frac{a}{2\,b}\sqrt{bx+a}\sqrt{-bcx+ac} + \frac{a^2c}{2}\sqrt{(bx+a)\,(-bcx+ac)}\,\arctan\left(x\sqrt{b^2c}\frac{1}{\sqrt{-b^2cx^2+a^2c}}\right)\frac{1}{\sqrt{bx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x)
```

```
[Out] -1/2/b/c*(b*x+a)^(1/2)*(-b*c*x+a*c)^(3/2)+1/2*a/b*(-b*c*x+a*c)^(1/2)*(b*x+a
)^(1/2)+1/2*a^2*c*((b*x+a)*(-b*c*x+a*c))^(1/2)/(-b*c*x+a*c)^(1/2)/(b*x+a)^(
1/2)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66942, size = 373, normalized size = 5.49

$$\left[\frac{a^2\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\,\sqrt{-bcx+ac}\sqrt{bx+a}bx}{4\,b},-\frac{a^2\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)-\sqrt{-bcx+ac}\sqrt{bx+a}bx}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a^2*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*s
qrt(-c)*x - a^2*c) + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*x)/b, -1/2*(a^2*s
qrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2
*c)) - sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*x)/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-c\,(-a+bx)}\sqrt{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1148 $\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}} dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}$$

```
[Out] (2*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a - b*x)]])/(b*Sqrt[c])
```

Rubi [A] time = 0.0199325, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {63, 217, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a - b*x)]])/(b*Sqrt[c])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2ac-cx^2}}\,dx, x, \sqrt{a+bx}\right)}{b}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.01539, size = 48, normalized size = 1.26

$$-\frac{2\sqrt{a-bx}\tan^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{a+bx}}\right)}{b\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x]*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x]])/(b*Sqrt[c*(a - b*x)]
)
```

Maple [B] time = 0.005, size = 71, normalized size = 1.9

$$\sqrt{(bx+a)(-bcx+ac)}\arctan\left(x\sqrt{b^2c}\frac{1}{\sqrt{-b^2cx^2+a^2c}}\right)\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{-bcx+ac}}\frac{1}{\sqrt{b^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] ((b*x+a)*(-b*c*x+a*c))^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(b^2*c)^(1/2)
*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57401, size = 252, normalized size = 6.63

$$\left[-\frac{\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)}{2\,bc},-\frac{\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)}{b\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(
-c)*x - a^2*c)/(b*c), -arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/
(b^2*c*x^2 - a^2*c))/(b*sqrt(c))]
```

Sympy [C] time = 3.42973, size = 90, normalized size = 2.37

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix} \middle| \frac{a^2}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix} \middle| \frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
a**2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(c)) + meijerg(((-1/2, -1/4, 0, 1/4, 1
/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(-2*I*pi)/(b**2*
x**2))/(4*pi**(3/2)*b*sqrt(c))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1149 $\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{x}{a^2c\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[Out] x/(a^2*c*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rubi [A] time = 0.0038488, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {39}

$$\frac{x}{a^2c\sqrt{a+bx}\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
```

```
[Out] x/(a^2*c*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx = \frac{x}{a^2c\sqrt{a+bx}\sqrt{ac-bcx}}$$

Mathematica [A] time = 0.0129921, size = 29, normalized size = 0.97

$$\frac{x}{a^2c\sqrt{a+bx}\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
```

```
[Out] x/(a^2*c*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

Maple [A] time = 0.002, size = 30, normalized size = 1.

$$\frac{x\,(-bx+a)}{a^2}\frac{1}{\sqrt{bx+a}}(-bcx+ac)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x)
```

```
[Out] 1/(b*x+a)^(1/2)*(-b*x+a)/a^2*x/(-b*c*x+a*c)^(3/2)
```

Maxima [A] time = 1.00213, size = 34, normalized size = 1.13

$$\frac{x}{\sqrt{-b^2cx^2 + a^2c}a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-b^2*c*x^2 + a^2*c)*a^2*c)
```

Fricas [A] time = 1.56822, size = 88, normalized size = 2.93

$$-\frac{\sqrt{-bcx + ac}\sqrt{bx + a}x}{a^2b^2c^2x^2 - a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*c*x + a*c)*sqrt(b*x + a)*x/(a^2*b^2*c^2*x^2 - a^4*c^2)
```

Sympy [C] time = 5.54556, size = 94, normalized size = 3.13

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & \frac{1}{2}, \frac{3}{2}, 2 \\ \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 2 & 0 \end{matrix} \middle| \frac{a^2}{b^2x^2}\right)}{2\pi^{\frac{3}{2}}a^2bc^{\frac{3}{2}}} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 & \\ \frac{1}{4}, \frac{3}{4} & -\frac{1}{2}, 0, 1, 0 \end{matrix} \middle| \frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{2\pi^{\frac{3}{2}}a^2bc^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(-b*c*x+a*c)**(3/2),x)
```

```
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), a
**2/(b**2*x**2))/(2*pi**(3/2)*a**2*b*c**(3/2)) + meijerg(((-1/2, 0, 1/4, 1/
2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), a**2*exp_polar(-2*I*pi)/(b*
*2*x**2))/(2*pi**(3/2)*a**2*b*c**(3/2))
```

Giac [B] time = 1.11973, size = 155, normalized size = 5.17

$$\frac{2\sqrt{-c}c}{\left(2\,ac^2 - \left(\sqrt{-bcx + ac}\sqrt{-c} - \sqrt{2\,ac^2 + (bcx - ac)c}\right)^2\right)ab|c|} - \frac{\sqrt{-bcx + ac}}{2\sqrt{2\,ac^2 + (bcx - ac)c}a^2b|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(-c)*c/((2*a*c^2 - (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c
*x - a*c)*c))^2)*a*b*abs(c)) - 1/2*sqrt(-b*c*x + a*c)/(sqrt(2*a*c^2 + (b*c*
x - a*c)*c)*a^2*b*abs(c))
```

3.1150 $\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{2x}{3a^4c^2\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}$$

```
[Out] x/(3*a^2*c*(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)) + (2*x)/(3*a^4*c^2*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x])
```

Rubi [A] time = 0.011025, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {40, 39}

$$\frac{2x}{3a^4c^2\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

```
[Out] x/(3*a^2*c*(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)) + (2*x)/(3*a^4*c^2*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x])
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx &= \frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{2\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx}{3a^2c}\\ &= \frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{2x}{3a^4c^2\sqrt{a+bx}\sqrt{ac-bcx}}\end{aligned}$$

Mathematica [A] time = 0.0237397, size = 46, normalized size = 0.69

$$\frac{3a^2x-2b^2x^3}{3a^4c(a+bx)^{3/2}(c(a-bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

```
[Out] (3*a^2*x - 2*b^2*x^3)/(3*a^4*c*(c*(a - b*x))^(3/2)*(a + b*x)^(3/2))
```

Maple [A] time = 0.001, size = 45, normalized size = 0.7

$$\frac{(-bx+a)\,x\left(-2\,b^2x^2+3\,a^2\right)}{3\,a^4}(bx+a)^{-\frac{3}{2}}(-bcx+ac)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x)
```

```
[Out] 1/3*(-b*x+a)*x*(-2*b^2*x^2+3*a^2)/(b*x+a)^(3/2)/a^4/(-b*c*x+a*c)^(5/2)
```

Maxima [A] time = 0.98127, size = 72, normalized size = 1.07

$$\frac{x}{3\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}a^2c}+\frac{2\,x}{3\,\sqrt{-b^2cx^2+a^2c}a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x/((-b^2*c*x^2 + a^2*c)^(3/2)*a^2*c) + 2/3*x/(sqrt(-b^2*c*x^2 + a^2*c)*
a^4*c^2)
```

Fricas [A] time = 1.64699, size = 147, normalized size = 2.19

$$-\frac{\left(2\,b^2x^3-3\,a^2x\right)\sqrt{-bcx+ac}\sqrt{bx+a}}{3\left(a^4b^4c^3x^4-2\,a^6b^2c^3x^2+a^8c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^3 - 3*a^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^4*b^4*c^3*x^
4 - 2*a^6*b^2*c^3*x^2 + a^8*c^3)
```

Sympy [C] time = 47.5316, size = 94, normalized size = 1.4

$$\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{1}{2},\frac{5}{2},3\\ \frac{5}{4},\frac{7}{4},2,\frac{5}{2},3 & 0\end{matrix}\middle|\frac{a^2}{b^2x^2}\right)}{3\pi^{\frac{3}{2}}a^4bc^{\frac{5}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},\frac{3}{4},\frac{5}{4},1 & \\ \frac{3}{4},\frac{5}{4} & -\frac{1}{2},0,2,0\end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{3\pi^{\frac{3}{2}}a^4bc^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(-b*c*x+a*c)**(5/2),x)
```

```
[Out] I*meijerg(((5/4, 7/4, 1), (1/2, 5/2, 3)), ((5/4, 7/4, 2, 5/2, 3), (0,)), a*
*2/(b**2*x**2))/(3*pi**(3/2)*a**4*b*c**(5/2)) + meijerg(((-1/2, 0, 1/2, 3/4
, 5/4, 1), ()), ((3/4, 5/4), (-1/2, 0, 2, 0)), a**2*exp_polar(-2*I*pi)/(b**
2*x**2))/(3*pi**(3/2)*a**4*b*c**(5/2))
```

Giac [B] time = 1.25254, size = 339, normalized size = 5.06

$$-\frac{\sqrt{-bcx+ac}\left(\frac{9\,|c|}{a^3bc}+\frac{4\,(bcx-ac)|c|}{a^4bc^2}\right)}{12\left(2\,ac^2+(bcx-ac)c\right)^{\frac{3}{2}}}+\frac{16\,a^2\sqrt{-c}c^4-18\,a\left(\sqrt{-bcx+ac}\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^2\sqrt{-c}c^2+3\left(\sqrt{-bcx+ac}\sqrt{}\right.}{3\left(2\,ac^2-\left(\sqrt{-bcx+ac}\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^2\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(-b*c*x + a*c)*(9*abs(c)/(a^3*b*c) + 4*(b*c*x - a*c)*abs(c)/(a^4*
b*c^2))/(2*a*c^2 + (b*c*x - a*c)*c)^(3/2) + 1/3*(16*a^2*sqrt(-c)*c^4 - 18*a
*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)
*c^2 + 3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*
sqrt(-c))/((2*a*c^2 - (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x
- a*c)*c))^2)^3*a^3*b*abs(c))
```

3.1151 $\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx$

Optimal. Leaf size=100

$$\frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}$$

```
[Out] x/(5*a^2*c*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)) + (4*x)/(15*a^4*c^2*(a + b*
x)^(3/2)*(a*c - b*c*x)^(3/2)) + (8*x)/(15*a^6*c^3*Sqrt[a + b*x]*Sqrt[a*c -
b*c*x])
```

Rubi [A] time = 0.0199559, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {40, 39}

$$\frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

```
[Out] x/(5*a^2*c*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)) + (4*x)/(15*a^4*c^2*(a + b*
x)^(3/2)*(a*c - b*c*x)^(3/2)) + (8*x)/(15*a^6*c^3*Sqrt[a + b*x]*Sqrt[a*c -
b*c*x])
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx &= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx}{5a^2c}\\ &= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{8\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx}{15a^4c^2}\\ &= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{ac-bcx}}\end{aligned}$$

Mathematica [A] time = 0.0302893, size = 57, normalized size = 0.57

$$\frac{-20a^2b^2x^3+15a^4x+8b^4x^5}{15a^6c(a+bx)^{5/2}(c(a-bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

```
[Out] (15*a^4*x - 20*a^2*b^2*x^3 + 8*b^4*x^5)/(15*a^6*c*(c*(a - b*x))^(5/2)*(a +
b*x)^(5/2))
```

Maple [A] time = 0.004, size = 56, normalized size = 0.6

$$\frac{(-bx+a)\,x\left(8\,b^4x^4-20\,x^2a^2b^2+15\,a^4\right)}{15\,a^6}(bx+a)^{-\frac{5}{2}}(-bcx+ac)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x)
```

```
[Out] 1/15*(-b*x+a)*x*(8*b^4*x^4-20*a^2*b^2*x^2+15*a^4)/(b*x+a)^(5/2)/a^6/(-b*c*x
+a*c)^(7/2)
```

Maxima [A] time = 0.996784, size = 107, normalized size = 1.07

$$\frac{x}{5\left(-b^2cx^2+a^2c\right)^{\frac{5}{2}}a^2c}+\frac{4\,x}{15\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}a^4c^2}+\frac{8\,x}{15\,\sqrt{-b^2cx^2+a^2c}a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-b^2*c*x^2 + a^2*c)^(5/2)*a^2*c) + 4/15*x/((-b^2*c*x^2 + a^2*c)^(3/
2)*a^4*c^2) + 8/15*x/(sqrt(-b^2*c*x^2 + a^2*c)*a^6*c^3)
```

Fricas [A] time = 1.75966, size = 203, normalized size = 2.03

$$-\frac{\left(8\,b^4x^5-20\,a^2b^2x^3+15\,a^4x\right)\sqrt{-bcx+ac}\sqrt{bx+a}}{15\left(a^6b^6c^4x^6-3\,a^8b^4c^4x^4+3\,a^{10}b^2c^4x^2-a^{12}c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*b^4*x^5 - 20*a^2*b^2*x^3 + 15*a^4*x)*sqrt(-b*c*x + a*c)*sqrt(b*x +
 a)/(a^6*b^6*c^4*x^6 - 3*a^8*b^4*c^4*x^4 + 3*a^10*b^2*c^4*x^2 - a^12*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(-b*c*x+a*c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42209, size = 494, normalized size = 4.94

$$-\frac{\sqrt{-bcx+ac}\left((bcx-ac)\left(\frac{275\,c}{a^5b|c|}+\frac{64\,(bcx-ac)}{a^6b|c|}\right)+\frac{300\,c^2}{a^4b|c|}\right)}{240\left(2\,ac^2+(bcx-ac)c\right)^{\frac{5}{2}}}-\frac{1024\,a^4c^8-2200\,a^3\left(\sqrt{-bcx+ac}\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/240*sqrt(-b*c*x + a*c)*((b*c*x - a*c)*(275*c/(a^5*b*abs(c)) + 64*(b*c*x
- a*c)/(a^6*b*abs(c))) + 300*c^2/(a^4*b*abs(c)))/(2*a*c^2 + (b*c*x - a*c)*c
)^(5/2) - 1/60*(1024*a^4*c^8 - 2200*a^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt
(2*a*c^2 + (b*c*x - a*c)*c))^2*c^6 + 1660*a^2*(sqrt(-b*c*x + a*c)*sqrt(-c)
- sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*c^4 - 450*a*(sqrt(-b*c*x + a*c)*sqrt(-
c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*c^2 + 45*(sqrt(-b*c*x + a*c)*sqrt(-
c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^8)/((2*a*c^2 - (sqrt(-b*c*x + a*c)*sq
rt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2)^5*a^5*b*sqrt(-c)*abs(c))
```

3.1152 $\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx$

Optimal. Leaf size=133

$$\frac{16x}{35a^8c^4\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}$$

```
[Out] x/(7*a^2*c*(a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)) + (6*x)/(35*a^4*c^2*(a + b*
x)^(5/2)*(a*c - b*c*x)^(5/2)) + (8*x)/(35*a^6*c^3*(a + b*x)^(3/2)*(a*c - b*
c*x)^(3/2)) + (16*x)/(35*a^8*c^4*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rubi [A] time = 0.033947, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {40, 39}

$$\frac{16x}{35a^8c^4\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]
```

```
[Out] x/(7*a^2*c*(a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)) + (6*x)/(35*a^4*c^2*(a + b*
x)^(5/2)*(a*c - b*c*x)^(5/2)) + (8*x)/(35*a^6*c^3*(a + b*x)^(3/2)*(a*c - b*
c*x)^(3/2)) + (16*x)/(35*a^8*c^4*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx &= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}+\frac{6\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx}{7a^2c}\\
&= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{24\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx}{35a^4c^2}\\
&= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}(ac-bc}\\
&= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}(ac-bc}
\end{aligned}$$

Mathematica [A] time = 0.0412714, size = 76, normalized size = 0.57

$$\frac{x\left(-70a^4b^2x^2+56a^2b^4x^4+35a^6-16b^6x^6\right)\sqrt{c(a-bx)}}{35a^8c^5(a-bx)^4(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]
```

```
[Out] (x*Sqrt[c*(a - b*x)]*(35*a^6 - 70*a^4*b^2*x^2 + 56*a^2*b^4*x^4 - 16*b^6*x^6
))/(35*a^8*c^5*(a - b*x)^4*(a + b*x)^(7/2))
```

Maple [A] time = 0.004, size = 67, normalized size = 0.5

$$\frac{(-bx+a)\,x\left(-16\,b^6x^6+56\,b^4x^4a^2-70\,b^2x^2a^4+35\,a^6\right)}{35\,a^8}(bx+a)^{-\frac{7}{2}}(-bcx+ac)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x)
```

```
[Out] 1/35*(-b*x+a)*x*(-16*b^6*x^6+56*a^2*b^4*x^4-70*a^4*b^2*x^2+35*a^6)/(b*x+a)^
(7/2)/a^8/(-b*c*x+a*c)^(9/2)
```

Maxima [A] time = 0.971443, size = 142, normalized size = 1.07

$$\frac{x}{7\left(-b^2cx^2+a^2c\right)^{\frac{7}{2}}a^2c}+\frac{6\,x}{35\left(-b^2cx^2+a^2c\right)^{\frac{5}{2}}a^4c^2}+\frac{8\,x}{35\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}a^6c^3}+\frac{16\,x}{35\sqrt{-b^2cx^2+a^2c}a^8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/7*x/((-b^2*c*x^2 + a^2*c)^(7/2)*a^2*c) + 6/35*x/((-b^2*c*x^2 + a^2*c)^(5/
2)*a^4*c^2) + 8/35*x/((-b^2*c*x^2 + a^2*c)^(3/2)*a^6*c^3) + 16/35*x/(sqrt(-
b^2*c*x^2 + a^2*c)*a^8*c^4)
```

Fricas [A] time = 1.87974, size = 257, normalized size = 1.93

$$-\frac{\left(16\,b^6x^7-56\,a^2b^4x^5+70\,a^4b^2x^3-35\,a^6x\right)\sqrt{-bcx+ac}\sqrt{bx+a}}{35\left(a^8b^8c^5x^8-4\,a^{10}b^6c^5x^6+6\,a^{12}b^4c^5x^4-4\,a^{14}b^2c^5x^2+a^{16}c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*b^6*x^7 - 56*a^2*b^4*x^5 + 70*a^4*b^2*x^3 - 35*a^6*x)*sqrt(-b*c*x
 + a*c)*sqrt(b*x + a)/(a^8*b^8*c^5*x^8 - 4*a^10*b^6*c^5*x^6 + 6*a^12*b^4*c^
5*x^4 - 4*a^14*b^2*c^5*x^2 + a^16*c^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(-b*c*x+a*c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.05113, size = 657, normalized size = 4.94

$$-\frac{\sqrt{-bcx+ac}\left((bcx-ac)\left((bcx-ac)\left(\frac{1617\,|c|}{a^7bc}+\frac{256\,(bcx-ac)|c|}{a^8bc^2}\right)+\frac{3430\,|c|}{a^6b}\right)+\frac{2450\,c|c|}{a^5b}\right)}{1120\left(2\,ac^2+(bcx-ac)c\right)^{\frac{7}{2}}}-\frac{16384\,a^6c^{12}-51744\,a^5\left(\sqrt{-bcx+ac}\sqrt{-c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="giac")
```

```
[Out] -1/1120*sqrt(-b*c*x + a*c)*((b*c*x - a*c)*((b*c*x - a*c)*(1617*abs(c)/(a^7*
b*c) + 256*(b*c*x - a*c)*abs(c)/(a^8*b*c^2)) + 3430*abs(c)/(a^6*b)) + 2450*
c*abs(c)/(a^5*b))/(2*a*c^2 + (b*c*x - a*c)*c)^(7/2) - 1/280*(16384*a^6*c^12
 - 51744*a^5*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c)
)^2*c^10 + 66416*a^4*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x -
 a*c)*c))^4*c^8 - 43120*a^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (
b*c*x - a*c)*c))^6*c^6 + 14280*a^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*
c^2 + (b*c*x - a*c)*c))^8*c^4 - 2450*a*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(
2*a*c^2 + (b*c*x - a*c)*c))^10*c^2 + 175*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqr
t(2*a*c^2 + (b*c*x - a*c)*c))^12)/((2*a*c^2 - (sqrt(-b*c*x + a*c)*sqrt(-c)
- sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2)^7*a^7*b*sqrt(-c)*c*abs(c))
```

3.1153 $\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx$

Optimal. Leaf size=100

$$6\sqrt{6}(1-2x)^{5/2}x(2x+1)^{5/2}+15\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{45}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}x\sqrt{2x+1}+\frac{45}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

```
[Out] (45*Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x])/2 + 15*Sqrt[3/2]*(1 - 2*x)^(3/
2)*x*(1 + 2*x)^(3/2) + 6*Sqrt[6]*(1 - 2*x)^(5/2)*x*(1 + 2*x)^(5/2) + (45*Sq
rt[3/2]*ArcSin[2*x])/4
```

Rubi [A] time = 0.0166831, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {38, 41, 216}

$$6\sqrt{6}(1-2x)^{5/2}x(2x+1)^{5/2}+15\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{45}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}x\sqrt{2x+1}+\frac{45}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

```
[Out] (45*Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x])/2 + 15*Sqrt[3/2]*(1 - 2*x)^(3/
2)*x*(1 + 2*x)^(3/2) + 6*Sqrt[6]*(1 - 2*x)^(5/2)*x*(1 + 2*x)^(5/2) + (45*Sq
rt[3/2]*ArcSin[2*x])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx = 6\sqrt{6}(1-2x)^{5/2}x(1+2x)^{5/2} + 5\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx$$
$$= 15\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}(1-2x)^{5/2}x(1+2x)^{5/2} + \frac{45}{2}\int \sqrt{3-6x}\sqrt{2+4x}\,dx$$
$$= \frac{45}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}x\sqrt{1+2x} + 15\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}(1-2x)^{5/2}x(1+2x)^{5/2} + \frac{135}{2}$$
$$= \frac{45}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}x\sqrt{1+2x} + 15\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}(1-2x)^{5/2}x(1+2x)^{5/2} + \frac{135}{2}$$
$$= \frac{45}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}x\sqrt{1+2x} + 15\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}(1-2x)^{5/2}x(1+2x)^{5/2} + \frac{45}{4}$$

Mathematica [A] time = 0.0326457, size = 44, normalized size = 0.44

$$\frac{3}{4}\sqrt{\frac{3}{2}}\left(2x\sqrt{1-4x^2}\left(128x^4-104x^2+33\right)+15\sin^{-1}(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

```
[Out] (3*Sqrt[3/2]*(2*x*Sqrt[1 - 4*x^2]*(33 - 104*x^2 + 128*x^4) + 15*ArcSin[2*x]
))/4
```

Maple [A] time = 0.006, size = 134, normalized size = 1.3

$$\frac{1}{24}(3-6x)^{\frac{5}{2}}(2+4x)^{\frac{7}{2}} + \frac{1}{8}(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{7}{2}} + \frac{9}{32}\sqrt{3-6x}(2+4x)^{\frac{7}{2}} - \frac{3}{16}(2+4x)^{\frac{5}{2}}\sqrt{3-6x} - \frac{15}{16}(2+4x)^{\frac{3}{2}}\sqrt{3-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^(5/2)*(2+4*x)^(5/2),x)
```

```
[Out] 1/24*(3-6*x)^(5/2)*(2+4*x)^(7/2)+1/8*(3-6*x)^(3/2)*(2+4*x)^(7/2)+9/32*(3-6*
x)^(1/2)*(2+4*x)^(7/2)-3/16*(2+4*x)^(5/2)*(3-6*x)^(1/2)-15/16*(2+4*x)^(3/2)
*(3-6*x)^(1/2)-45/8*(3-6*x)^(1/2)*(2+4*x)^(1/2)+45/8*((2+4*x)*(3-6*x))^(1/2
)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*6^(1/2)*arcsin(2*x)
```

Maxima [A] time = 1.46279, size = 62, normalized size = 0.62

$$\frac{1}{6}\left(-24x^2+6\right)^{\frac{5}{2}}x + \frac{5}{4}\left(-24x^2+6\right)^{\frac{3}{2}}x + \frac{45}{4}\sqrt{-24x^2+6x} + \frac{45}{8}\sqrt{6}\arcsin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-24*x^2 + 6)^(5/2)*x + 5/4*(-24*x^2 + 6)^(3/2)*x + 45/4*sqrt(-24*x^2 +
 6)*x + 45/8*sqrt(6)*arcsin(2*x)
```

Fricas [A] time = 1.65272, size = 194, normalized size = 1.94

$$\frac{3}{4}\left(128\,x^5 - 104\,x^3 + 33\,x\right)\sqrt{4\,x+2}\sqrt{-6\,x+3} - \frac{45}{8}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}\sqrt{4\,x+2}\sqrt{-6\,x+3}}{12\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 3/4*(128*x^5 - 104*x^3 + 33*x)*sqrt(4*x + 2)*sqrt(-6*x + 3) - 45/8*sqrt(3)*
sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(5/2)*(4*x+2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10393, size = 174, normalized size = 1.74

$$\frac{3}{8}\sqrt{3}\sqrt{2}\left(((2\,((8\,(2\,x+1)(x-2)+39)(2\,x+1)-37)(2\,x+1)+31)(2\,x+1)-3)\sqrt{2\,x+1}\sqrt{-2\,x+1}-12\,((4\,(2\,x+1)(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(3)*sqrt(2)*(((2*((8*(2*x + 1)*(x - 2) + 39)*(2*x + 1) - 37)*(2*x +
 1) + 31)*(2*x + 1) - 3)*sqrt(2*x + 1)*sqrt(-2*x + 1) - 12*((4*(2*x + 1)*(x
 - 1) + 5)*(2*x + 1) - 1)*sqrt(2*x + 1)*sqrt(-2*x + 1) + 48*sqrt(2*x + 1)*x
*sqrt(-2*x + 1) + 30*arcsin(1/2*sqrt(2)*sqrt(2*x + 1)))
```

3.1154 $\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx$

Optimal. Leaf size=74

$$3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}x\sqrt{2x+1}+\frac{9}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

```
[Out] (9*Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x])/2 + 3*Sqrt[3/2]*(1 - 2*x)^(3/2)
*x*(1 + 2*x)^(3/2) + (9*Sqrt[3/2]*ArcSin[2*x])/4
```

Rubi [A] time = 0.0100821, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {38, 41, 216}

$$3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}x\sqrt{2x+1}+\frac{9}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

```
[Out] (9*Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x])/2 + 3*Sqrt[3/2]*(1 - 2*x)^(3/2)
*x*(1 + 2*x)^(3/2) + (9*Sqrt[3/2]*ArcSin[2*x])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx &= 3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2}+\frac{9}{2}\int \sqrt{3-6x}\sqrt{2+4x}\,dx\\
&= \frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}x\sqrt{1+2x}+3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2}+\frac{27}{2}\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx\\
&= \frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}x\sqrt{1+2x}+3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2}+\frac{27}{2}\int \frac{1}{\sqrt{6-24x^2}}\,dx\\
&= \frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}x\sqrt{1+2x}+3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2}+\frac{9}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)
\end{aligned}$$

Mathematica [A] time = 0.0335005, size = 39, normalized size = 0.53

$$\frac{3}{4}\sqrt{\frac{3}{2}}\left(2x\sqrt{1-4x^2}\left(5-8x^2\right)+3\sin^{-1}(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

```
[Out] (3*Sqrt[3/2]*(2*x*(5 - 8*x^2)*Sqrt[1 - 4*x^2] + 3*ArcSin[2*x]))/4
```

Maple [B] time = 0.003, size = 102, normalized size = 1.4

$$\frac{1}{16}(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{5}{2}}+\frac{3}{16}(2+4x)^{\frac{5}{2}}\sqrt{3-6x}-\frac{3}{16}(2+4x)^{\frac{3}{2}}\sqrt{3-6x}-\frac{9}{8}\sqrt{3-6x}\sqrt{2+4x}+\frac{9\arcsin(2x)\sqrt{6}}{8}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^(3/2)*(2+4*x)^(3/2),x)
```

```
[Out] 1/16*(3-6*x)^(3/2)*(2+4*x)^(5/2)+3/16*(2+4*x)^(5/2)*(3-6*x)^(1/2)-3/16*(2+4
*x)^(3/2)*(3-6*x)^(1/2)-9/8*(3-6*x)^(1/2)*(2+4*x)^(1/2)+9/8*((2+4*x)*(3-6*x
))^(1/2)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*6^(1/2)*arcsin(2*x)
```

Maxima [A] time = 1.50259, size = 46, normalized size = 0.62

$$\frac{1}{4}\left(-24x^2+6\right)^{\frac{3}{2}}x+\frac{9}{4}\sqrt{-24x^2+6}x+\frac{9}{8}\sqrt{6}\arcsin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-24*x^2 + 6)^(3/2)*x + 9/4*sqrt(-24*x^2 + 6)*x + 9/8*sqrt(6)*arcsin(2*
x)
```

Fricas [A] time = 1.55817, size = 177, normalized size = 2.39

$$-\frac{3}{4}\left(8x^3-5x\right)\sqrt{4x+2}\sqrt{-6x+3}-\frac{9}{8}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}\sqrt{4x+2}\sqrt{-6x+3}}{12x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -3/4*(8*x^3 - 5*x)*sqrt(4*x + 2)*sqrt(-6*x + 3) - 9/8*sqrt(3)*sqrt(2)*arcta
n(1/12*sqrt(3)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(3/2)*(4*x+2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09449, size = 103, normalized size = 1.39

$$-\frac{3}{8}\sqrt{3}\sqrt{2}\left(((4(2x+1)(x-1)+5)(2x+1)-1)\sqrt{2x+1}\sqrt{-2x+1}-8\sqrt{2x+1}x\sqrt{-2x+1}-6\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{2x+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -3/8*sqrt(3)*sqrt(2)*(((4*(2*x + 1)*(x - 1) + 5)*(2*x + 1) - 1)*sqrt(2*x +
1)*sqrt(-2*x + 1) - 8*sqrt(2*x + 1)*x*sqrt(-2*x + 1) - 6*arcsin(1/2*sqrt(2)
*sqrt(2*x + 1)))
```

3.1155 $\int \sqrt{3-6x}\sqrt{2+4x}\,dx$

Optimal. Leaf size=43

$$\sqrt{\frac{3}{2}}\sqrt{1-2x}\sqrt{2x+1}x+\frac{1}{2}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

```
[Out] Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x] + (Sqrt[3/2]*ArcSin[2*x])/2
```

Rubi [A] time = 0.0057703, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {38, 41, 216}

$$\sqrt{\frac{3}{2}}\sqrt{1-2x}\sqrt{2x+1}x+\frac{1}{2}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]

[Out] Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x] + (Sqrt[3/2]*ArcSin[2*x])/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-6x}\sqrt{2+4x}\,dx &= \sqrt{\frac{3}{2}}\sqrt{1-2x}x\sqrt{1+2x}+3\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx\\ &= \sqrt{\frac{3}{2}}\sqrt{1-2x}x\sqrt{1+2x}+3\int \frac{1}{\sqrt{6-24x^2}}\,dx\\ &= \sqrt{\frac{3}{2}}\sqrt{1-2x}x\sqrt{1+2x}+\frac{1}{2}\sqrt{\frac{3}{2}}\sin^{-1}(2x)\end{aligned}$$

Mathematica [A] time = 0.009989, size = 30, normalized size = 0.7

$$\frac{1}{2}\sqrt{\frac{3}{2}}\left(2\sqrt{1-4x^2}x+\sin^{-1}(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]
```

```
[Out] (Sqrt[3/2]*(2*x*Sqrt[1 - 4*x^2] + ArcSin[2*x]))/2
```

Maple [B] time = 0.003, size = 70, normalized size = 1.6

$$-\frac{1}{12}\sqrt{2+4x}\,(3-6x)^{\frac{3}{2}}+\frac{1}{4}\sqrt{3-6x}\sqrt{2+4x}+\frac{\arcsin(2x)\sqrt{6}}{4}\sqrt{(2+4x)(3-6x)}\frac{1}{\sqrt{3-6x}}\frac{1}{\sqrt{2+4x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^(1/2)*(2+4*x)^(1/2),x)
```

```
[Out] -1/12*(2+4*x)^(1/2)*(3-6*x)^(3/2)+1/4*(3-6*x)^(1/2)*(2+4*x)^(1/2)+1/4*((2+4
*x)*(3-6*x))^(1/2)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*6^(1/2)*arcsin(2*x)
```

Maxima [A] time = 1.48093, size = 30, normalized size = 0.7

$$\frac{1}{2}\sqrt{-24x^2+6}x+\frac{1}{4}\sqrt{6}\arcsin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-24*x^2 + 6)*x + 1/4*sqrt(6)*arcsin(2*x)
```

Fricas [A] time = 1.46261, size = 159, normalized size = 3.7

$$\frac{1}{2}\sqrt{4x+2}x\sqrt{-6x+3}-\frac{1}{4}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}\sqrt{4x+2}\sqrt{-6x+3}}{12x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*x + 2)*x*sqrt(-6*x + 3) - 1/4*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3
)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

Sympy [B] time = 5.52426, size = 187, normalized size = 4.35

$$\begin{cases} -\frac{\sqrt{6}i\operatorname{acosh}\left(\sqrt{x+\frac{1}{2}}\right)}{2}+\frac{\sqrt{6}i\left(x+\frac{1}{2}\right)^{\frac{5}{2}}}{\sqrt{x-\frac{1}{2}}}-\frac{3\sqrt{6}i\left(x+\frac{1}{2}\right)^{\frac{3}{2}}}{2\sqrt{x-\frac{1}{2}}}+\frac{\sqrt{6}i\sqrt{x+\frac{1}{2}}}{2\sqrt{x-\frac{1}{2}}} & \text{for}\ \left|x+\frac{1}{2}\right|>1 \\ \frac{\sqrt{6}\operatorname{asin}\left(\sqrt{x+\frac{1}{2}}\right)}{2}-\frac{\sqrt{6}\left(x+\frac{1}{2}\right)^{\frac{5}{2}}}{\sqrt{\frac{1}{2}-x}}+\frac{3\sqrt{6}\left(x+\frac{1}{2}\right)^{\frac{3}{2}}}{2\sqrt{\frac{1}{2}-x}}-\frac{\sqrt{6}\sqrt{x+\frac{1}{2}}}{2\sqrt{\frac{1}{2}-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(1/2)*(4*x+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(6)*I*acosh(sqrt(x + 1/2))/2 + sqrt(6)*I*(x + 1/2)**(5/2)/s
qrt(x - 1/2) - 3*sqrt(6)*I*(x + 1/2)**(3/2)/(2*sqrt(x - 1/2)) + sqrt(6)*I*s
qrt(x + 1/2)/(2*sqrt(x - 1/2)), Abs(x + 1/2) > 1), (sqrt(6)*asin(sqrt(x + 1
/2))/2 - sqrt(6)*(x + 1/2)**(5/2)/sqrt(1/2 - x) + 3*sqrt(6)*(x + 1/2)**(3/2
)/(2*sqrt(1/2 - x)) - sqrt(6)*sqrt(x + 1/2)/(2*sqrt(1/2 - x)), True))
```

Giac [A] time = 1.06266, size = 51, normalized size = 1.19

$$\frac{1}{2}\sqrt{3}\sqrt{2}\left(\sqrt{2x+1}x\sqrt{-2x+1}+\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{2x+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*sqrt(2)*(sqrt(2*x + 1)*x*sqrt(-2*x + 1) + arcsin(1/2*sqrt(2)*sq
rt(2*x + 1)))
```

3.1156 $\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx$

Optimal. Leaf size=13

$$\frac{\sin^{-1}(2x)}{2\sqrt{6}}$$

[Out] ArcSin[2*x]/(2*Sqrt[6])

Rubi [A] time = 0.0025095, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {41, 216}

$$\frac{\sin^{-1}(2x)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]

[Out] ArcSin[2*x]/(2*Sqrt[6])

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx &= \int \frac{1}{\sqrt{6-24x^2}}\,dx\\ &= \frac{\sin^{-1}(2x)}{2\sqrt{6}}\end{aligned}$$

Mathematica [A] time = 0.0151583, size = 13, normalized size = 1.

$$\frac{\sin^{-1}(2x)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]

[Out] ArcSin[2*x]/(2*Sqrt[6])

Maple [B] time = 0.003, size = 37, normalized size = 2.9

$$\frac{\arcsin\left(2x\right)\sqrt{6}}{12}\sqrt{\left(2+4x\right)\left(3-6x\right)}\frac{1}{\sqrt{3-6x}}\frac{1}{\sqrt{2+4x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-6*x)^(1/2)/(2+4*x)^(1/2),x)
```

```
[Out] 1/12*((2+4*x)*(3-6*x))^(1/2)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*6^(1/2)*arcsin(2*x
)
```

Maxima [A] time = 1.43274, size = 12, normalized size = 0.92

$$\frac{1}{12}\sqrt{6}\arcsin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(6)*arcsin(2*x)
```

Fricas [B] time = 1.46493, size = 90, normalized size = 6.92

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{4x+2}\sqrt{-6x+3}}{12x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*arctan(1/12*sqrt(6)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

Sympy [A] time = 5.28419, size = 41, normalized size = 3.15

$$\begin{cases} -\frac{\sqrt{6}i\operatorname{acosh}\left(\sqrt{x+\frac{1}{2}}\right)}{6} & \text{for}\: \left|{x+\frac{1}{2}}\right| > 1 \\ \frac{\sqrt{6}\operatorname{asin}\left(\sqrt{x+\frac{1}{2}}\right)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(1/2)/(4*x+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(6)*I*acosh(sqrt(x + 1/2))/6, Abs(x + 1/2) > 1), (sqrt(6)*a
sin(sqrt(x + 1/2))/6, True))
```

<u>Giac</u> [A] time = 1.07293, size = 20, normalized size = 1.54

$$\frac{1}{6}\sqrt{6}\arcsin\left(\frac{1}{2}\sqrt{4x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*arcsin(1/2*sqrt(4*x + 2))
```

3.1157 $\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\frac{x}{6\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}$$

```
[Out] x/(6*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x])
```

Rubi [A] time = 0.0019949, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {39}

$$\frac{x}{6\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]
```

```
[Out] x/(6*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx = \frac{x}{6\sqrt{6}\sqrt{1-2x}\sqrt{1+2x}}$$

Mathematica [A] time = 0.0156117, size = 16, normalized size = 0.57

$$\frac{x}{6\sqrt{6-24x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]
```

```
[Out] x/(6*Sqrt[6 - 24*x^2])
```

Maple [A] time = 0.003, size = 28, normalized size = 1.

$$-(2x-1)(1+2x)x(3-6x)^{-\frac{3}{2}}(2+4x)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-6*x)^(3/2)/(2+4*x)^(3/2),x)

[Out] -(2*x-1)*(1+2*x)*x/(3-6*x)^(3/2)/(2+4*x)^(3/2)
```

Maxima [A] time = 0.963899, size = 16, normalized size = 0.57

$$\frac{x}{6\sqrt{-24x^2+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="maxima")

[Out] 1/6*x/sqrt(-24*x^2 + 6)
```

Fricas [A] time = 1.55657, size = 68, normalized size = 2.43

$$-\frac{\sqrt{4x+2}x\sqrt{-6x+3}}{36\left(4x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="fricas")

[Out] -1/36*sqrt(4*x + 2)*x*sqrt(-6*x + 3)/(4*x^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(3/2)/(4*x+2)**(3/2),x)

[Out] Timed out
```

Giac [B] time = 1.05424, size = 96, normalized size = 3.43

$$-\frac{\sqrt{6}\left(\sqrt{-4x+2}-2\right)}{288\sqrt{4x+2}}-\frac{\sqrt{6}\sqrt{4x+2}\sqrt{-4x+2}}{288(2x-1)}+\frac{\sqrt{6}\sqrt{4x+2}}{288\left(\sqrt{-4x+2}-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="giac")

[Out] -1/288*sqrt(6)*(sqrt(-4*x + 2) - 2)/sqrt(4*x + 2) - 1/288*sqrt(6)*sqrt(4*x
+ 2)*sqrt(-4*x + 2)/(2*x - 1) + 1/288*sqrt(6)*sqrt(4*x + 2)/(sqrt(-4*x + 2)
 - 2)
```

3.1158 $\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx$

Optimal. Leaf size=57

$$\frac{x}{54\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}+\frac{x}{108\sqrt{6}(1-2x)^{3/2}(2x+1)^{3/2}}$$

```
[Out] x/(108*Sqrt[6]*(1 - 2*x)^(3/2)*(1 + 2*x)^(3/2)) + x/(54*Sqrt[6]*Sqrt[1 - 2*
x]*Sqrt[1 + 2*x])
```

Rubi [A] time = 0.0062288, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {40, 39}

$$\frac{x}{54\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}+\frac{x}{108\sqrt{6}(1-2x)^{3/2}(2x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

```
[Out] x/(108*Sqrt[6]*(1 - 2*x)^(3/2)*(1 + 2*x)^(3/2)) + x/(54*Sqrt[6]*Sqrt[1 - 2*
x]*Sqrt[1 + 2*x])
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx &= \frac{x}{108\sqrt{6}(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{1}{9}\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx\\ &= \frac{x}{108\sqrt{6}(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{x}{54\sqrt{6}\sqrt{1-2x}\sqrt{1+2x}}\end{aligned}$$

Mathematica [A] time = 0.0243268, size = 37, normalized size = 0.65

$$\frac{x\left(8x^2-3\right)}{108\sqrt{6-12x}(2x-1)(2x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

```
[Out] (x*(-3 + 8*x^2))/(108*Sqrt[6 - 12*x]*(-1 + 2*x)*(1 + 2*x)^(3/2))
```

Maple [A] time = 0.004, size = 35, normalized size = 0.6

$$\frac{(2x-1)(1+2x)x(8x^2-3)}{3}(3-6x)^{-\frac{5}{2}}(2+4x)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-6*x)^(5/2)/(2+4*x)^(5/2),x)
```

```
[Out] 1/3*(2*x-1)*(1+2*x)*x*(8*x^2-3)/(3-6*x)^(5/2)/(2+4*x)^(5/2)
```

Maxima [A] time = 0.968932, size = 34, normalized size = 0.6

$$\frac{x}{54\sqrt{-24x^2+6}}+\frac{x}{18(-24x^2+6)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/54*x/sqrt(-24*x^2 + 6) + 1/18*x/(-24*x^2 + 6)^(3/2)
```

Fricas [A] time = 1.55853, size = 97, normalized size = 1.7

$$-\frac{(8x^3-3x)\sqrt{4x+2}\sqrt{-6x+3}}{648(16x^4-8x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/648*(8*x^3 - 3*x)*sqrt(4*x + 2)*sqrt(-6*x + 3)/(16*x^4 - 8*x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(5/2)/(4*x+2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.08276, size = 174, normalized size = 3.05

$$-\frac{\sqrt{6}\left(\sqrt{-4x+2}-2\right)^{3}}{82944\,(4x+2)^{\frac{3}{2}}}-\frac{11\sqrt{6}\left(\sqrt{-4x+2}-2\right)}{27648\sqrt{4x+2}}-\frac{\left(4\sqrt{6}(2x+1)-9\sqrt{6}\right)\sqrt{4x+2}\sqrt{-4x+2}}{10368\,(2x-1)^{2}}+\frac{\sqrt{6}(4x+2)^{\frac{3}{2}}\left(\frac{33\left(\sqrt{-4x+2}-2\right)^{2}}{2x+1}+2\right)}{165888\left(\sqrt{-4x+2}-2\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/82944*sqrt(6)*(sqrt(-4*x + 2) - 2)^3/(4*x + 2)^(3/2) - 11/27648*sqrt(6)*
(sqrt(-4*x + 2) - 2)/sqrt(4*x + 2) - 1/10368*(4*sqrt(6)*(2*x + 1) - 9*sqrt(
6))*sqrt(4*x + 2)*sqrt(-4*x + 2)/(2*x - 1)^2 + 1/165888*sqrt(6)*(4*x + 2)^(
3/2)*(33*(sqrt(-4*x + 2) - 2)^2/(2*x + 1) + 2)/(sqrt(-4*x + 2) - 2)^3
```

3.1159 $\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx$

Optimal. Leaf size=85

$$\frac{x}{405\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}+\frac{x}{810\sqrt{6}(1-2x)^{3/2}(2x+1)^{3/2}}+\frac{x}{1080\sqrt{6}(1-2x)^{5/2}(2x+1)^{5/2}}$$

```
[Out] x/(1080*Sqrt[6]*(1 - 2*x)^(5/2)*(1 + 2*x)^(5/2)) + x/(810*Sqrt[6]*(1 - 2*x)
^(3/2)*(1 + 2*x)^(3/2)) + x/(405*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x])
```

Rubi [A] time = 0.0110413, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {40, 39}

$$\frac{x}{405\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}+\frac{x}{810\sqrt{6}(1-2x)^{3/2}(2x+1)^{3/2}}+\frac{x}{1080\sqrt{6}(1-2x)^{5/2}(2x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

```
[Out] x/(1080*Sqrt[6]*(1 - 2*x)^(5/2)*(1 + 2*x)^(5/2)) + x/(810*Sqrt[6]*(1 - 2*x)
^(3/2)*(1 + 2*x)^(3/2)) + x/(405*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x])
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx &= \frac{x}{1080\sqrt{6}(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{2}{15}\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx\\ &= \frac{x}{1080\sqrt{6}(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{x}{810\sqrt{6}(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{2}{135}\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx\\ &= \frac{x}{1080\sqrt{6}(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{x}{810\sqrt{6}(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{x}{405\sqrt{6}\sqrt{1-2x}\sqrt{1+2x}}\end{aligned}$$

Mathematica [A] time = 0.0304035, size = 42, normalized size = 0.49

$$\frac{x\left(128x^4-80x^2+15\right)}{3240\sqrt{6-12x}(1-2x)^2(2x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

```
[Out] (x*(15 - 80*x^2 + 128*x^4))/(3240*Sqrt[6 - 12*x]*(1 - 2*x)^2*(1 + 2*x)^(5/2
))
```

Maple [A] time = 0.002, size = 40, normalized size = 0.5

$$-\frac{(2x-1)(1+2x)x\left(128x^4-80x^2+15\right)}{15}(3-6x)^{-\frac{7}{2}}(2+4x)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-6*x)^(7/2)/(2+4*x)^(7/2),x)
```

```
[Out] -1/15*(2*x-1)*(1+2*x)*x*(128*x^4-80*x^2+15)/(3-6*x)^(7/2)/(2+4*x)^(7/2)
```

Maxima [A] time = 1.01568, size = 50, normalized size = 0.59

$$\frac{x}{405\sqrt{-24x^2+6}}+\frac{x}{135\left(-24x^2+6\right)^{\frac{3}{2}}}+\frac{x}{30\left(-24x^2+6\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/405*x/sqrt(-24*x^2 + 6) + 1/135*x/(-24*x^2 + 6)^(3/2) + 1/30*x/(-24*x^2 +
 6)^(5/2)
```

Fricas [A] time = 1.54768, size = 130, normalized size = 1.53

$$-\frac{\left(128x^5-80x^3+15x\right)\sqrt{4x+2}\sqrt{-6x+3}}{19440\left(64x^6-48x^4+12x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/19440*(128*x^5 - 80*x^3 + 15*x)*sqrt(4*x + 2)*sqrt(-6*x + 3)/(64*x^6 - 4
8*x^4 + 12*x^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(7/2)/(4*x+2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1112, size = 248, normalized size = 2.92

$$-\frac{\sqrt{6}\left(\sqrt{-4x+2}-2\right)^{5}}{13271040\,(4x+2)^{\frac{5}{2}}}-\frac{17\sqrt{6}\left(\sqrt{-4x+2}-2\right)^{3}}{7962624\,(4x+2)^{\frac{3}{2}}}-\frac{71\sqrt{6}\left(\sqrt{-4x+2}-2\right)}{1327104\sqrt{4x+2}}-\frac{\left(\left(64\sqrt{6}(2x+1)-275\sqrt{6}\right)(2x+1)+300\sqrt{}\right.}{1244160\,(2x-1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/13271040*sqrt(6)*(sqrt(-4*x + 2) - 2)^5/(4*x + 2)^(5/2) - 17/7962624*sqr
t(6)*(sqrt(-4*x + 2) - 2)^3/(4*x + 2)^(3/2) - 71/1327104*sqrt(6)*(sqrt(-4*x
 + 2) - 2)/sqrt(4*x + 2) - 1/1244160*((64*sqrt(6)*(2*x + 1) - 275*sqrt(6))*
(2*x + 1) + 300*sqrt(6))*sqrt(4*x + 2)*sqrt(-4*x + 2)/(2*x - 1)^3 + 1/79626
240*sqrt(6)*(1065*(sqrt(-4*x + 2) - 2)^4/(2*x + 1)^2 + 85*(sqrt(-4*x + 2) -
 2)^2/(2*x + 1) + 6)*(4*x + 2)^(5/2)/(sqrt(-4*x + 2) - 2)^5
```

3.1160 $\int (3-x)^{3/2}(-2+x)^{3/2}\,dx$

Optimal. Leaf size=91

$$-\frac{1}{4}(x-2)^{3/2}(3-x)^{5/2}-\frac{1}{8}\sqrt{x-2}(3-x)^{5/2}+\frac{1}{32}\sqrt{x-2}(3-x)^{3/2}+\frac{3}{64}\sqrt{x-2}\sqrt{3-x}-\frac{3}{128}\sin^{-1}(5-2x)$$

```
[Out] (3*Sqrt[3 - x]*Sqrt[-2 + x])/64 + ((3 - x)^(3/2)*Sqrt[-2 + x])/32 - ((3 - x
)^(5/2)*Sqrt[-2 + x])/8 - ((3 - x)^(5/2)*(-2 + x)^(3/2))/4 - (3*ArcSin[5 -
2*x])/128
```

Rubi [A] time = 0.0220188, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {50, 53, 619, 216}

$$-\frac{1}{4}(x-2)^{3/2}(3-x)^{5/2}-\frac{1}{8}\sqrt{x-2}(3-x)^{5/2}+\frac{1}{32}\sqrt{x-2}(3-x)^{3/2}+\frac{3}{64}\sqrt{x-2}\sqrt{3-x}-\frac{3}{128}\sin^{-1}(5-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x)^(3/2)*(-2 + x)^(3/2),x]
```


```
[Out] (3*Sqrt[3 - x]*Sqrt[-2 + x])/64 + ((3 - x)^(3/2)*Sqrt[-2 + x])/32 - ((3 - x
)^(5/2)*Sqrt[-2 + x])/8 - ((3 - x)^(5/2)*(-2 + x)^(3/2))/4 - (3*ArcSin[5 -
2*x])/128
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (3-x)^{3/2}(-2+x)^{3/2}\,dx = -\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2} + \frac{3}{8}\int (3-x)^{3/2}\sqrt{-2+x}\,dx$$
$$= -\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x} - \frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2} + \frac{1}{16}\int \frac{(3-x)^{3/2}}{\sqrt{-2+x}}\,dx$$
$$= \frac{1}{32}(3-x)^{3/2}\sqrt{-2+x} - \frac{1}{8}(3-x)^{5/2}\sqrt{-2+x} - \frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2} + \frac{3}{64}\int \frac{\sqrt{3-x}}{\sqrt{-2+x}}\,dx$$
$$= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x} + \frac{1}{32}(3-x)^{3/2}\sqrt{-2+x} - \frac{1}{8}(3-x)^{5/2}\sqrt{-2+x} - \frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2} +$$
$$= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x} + \frac{1}{32}(3-x)^{3/2}\sqrt{-2+x} - \frac{1}{8}(3-x)^{5/2}\sqrt{-2+x} - \frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2} +$$
$$= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x} + \frac{1}{32}(3-x)^{3/2}\sqrt{-2+x} - \frac{1}{8}(3-x)^{5/2}\sqrt{-2+x} - \frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2} -$$
$$= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x} + \frac{1}{32}(3-x)^{3/2}\sqrt{-2+x} - \frac{1}{8}(3-x)^{5/2}\sqrt{-2+x} - \frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2} -$$

Mathematica [A] time = 0.0471677, size = 80, normalized size = 0.88

$$\frac{\sqrt{-x^2+5x-6}\left(\sqrt{x-2}\left(-16x^4+168x^3-650x^2+1095x-675\right)+3\sqrt{3-x}\sin^{-1}\left(\sqrt{3-x}\right)\right)}{64(x-3)\sqrt{x-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x)^(3/2)*(-2 + x)^(3/2),x]
```

```
[Out] (Sqrt[-6 + 5*x - x^2]*(Sqrt[-2 + x]*(-675 + 1095*x - 650*x^2 + 168*x^3 - 16
*x^4) + 3*Sqrt[3 - x]*ArcSin[Sqrt[3 - x]]))/(64*(-3 + x)*Sqrt[-2 + x])
```

Maple [A] time = 0.005, size = 89, normalized size = 1.

$$\frac{1}{4}(3-x)^{\frac{3}{2}}(-2+x)^{\frac{5}{2}} + \frac{1}{8}\sqrt{3-x}(-2+x)^{\frac{5}{2}} - \frac{1}{32}\sqrt{3-x}(-2+x)^{\frac{3}{2}} - \frac{3}{64}\sqrt{3-x}\sqrt{-2+x} + \frac{3\arcsin(2x-5)}{128}\sqrt{(-2+x)(3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-x)^(3/2)*(-2+x)^(3/2),x)
```

```
[Out] 1/4*(3-x)^(3/2)*(-2+x)^(5/2)+1/8*(3-x)^(1/2)*(-2+x)^(5/2)-1/32*(3-x)^(1/2)*
(-2+x)^(3/2)-3/64*(3-x)^(1/2)*(-2+x)^(1/2)+3/128*((-2+x)*(3-x))^(1/2)/(-2+x
)^(1/2)/(3-x)^(1/2)*arcsin(2*x-5)
```

Maxima [A] time = 1.47638, size = 90, normalized size = 0.99

$$\frac{1}{4}\left(-x^2+5x-6\right)^{\frac{3}{2}}x - \frac{5}{8}\left(-x^2+5x-6\right)^{\frac{3}{2}} + \frac{3}{32}\sqrt{-x^2+5x-6}x - \frac{15}{64}\sqrt{-x^2+5x-6} + \frac{3}{128}\arcsin(2x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + 5*x - 6)^(3/2)*x - 5/8*(-x^2 + 5*x - 6)^(3/2) + 3/32*sqrt(-x^2
+ 5*x - 6)*x - 15/64*sqrt(-x^2 + 5*x - 6) + 3/128*arcsin(2*x - 5)
```

Fricas [A] time = 1.57946, size = 184, normalized size = 2.02

$$-\frac{1}{64}\left(16\,x^3-120\,x^2+290\,x-225\right)\sqrt{x-2}\sqrt{-x+3}-\frac{3}{128}\,\arctan\left(\frac{(2\,x-5)\sqrt{x-2}\sqrt{-x+3}}{2\left(x^2-5\,x+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(16*x^3 - 120*x^2 + 290*x - 225)*sqrt(x - 2)*sqrt(-x + 3) - 3/128*arc
tan(1/2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

Sympy [A] time = 10.2779, size = 199, normalized size = 2.19

$$\begin{cases} -\frac{3i\,\mathrm{acosh}\left(\sqrt{x-2}\right)}{64}-\frac{i(x-2)^{\frac{9}{2}}}{4\sqrt{x-3}}+\frac{5i(x-2)^{\frac{7}{2}}}{8\sqrt{x-3}}-\frac{13i(x-2)^{\frac{5}{2}}}{32\sqrt{x-3}}-\frac{i(x-2)^{\frac{3}{2}}}{64\sqrt{x-3}}+\frac{3i\sqrt{x-2}}{64\sqrt{x-3}} & \text{for } |x-2|>1 \\ \frac{3\,\mathrm{asin}\left(\sqrt{x-2}\right)}{64}+\frac{(x-2)^{\frac{9}{2}}}{4\sqrt{3-x}}-\frac{5(x-2)^{\frac{7}{2}}}{8\sqrt{3-x}}+\frac{13(x-2)^{\frac{5}{2}}}{32\sqrt{3-x}}+\frac{(x-2)^{\frac{3}{2}}}{64\sqrt{3-x}}-\frac{3\sqrt{x-2}}{64\sqrt{3-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)**(3/2)*(-2+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(x - 2))/64 - I*(x - 2)**(9/2)/(4*sqrt(x - 3)) +
5*I*(x - 2)**(7/2)/(8*sqrt(x - 3)) - 13*I*(x - 2)**(5/2)/(32*sqrt(x - 3)) -
 I*(x - 2)**(3/2)/(64*sqrt(x - 3)) + 3*I*sqrt(x - 2)/(64*sqrt(x - 3)), Abs(
x - 2) > 1), (3*asin(sqrt(x - 2))/64 + (x - 2)**(9/2)/(4*sqrt(3 - x)) - 5*(
x - 2)**(7/2)/(8*sqrt(3 - x)) + 13*(x - 2)**(5/2)/(32*sqrt(3 - x)) + (x - 2
)**(3/2)/(64*sqrt(3 - x)) - 3*sqrt(x - 2)/(64*sqrt(3 - x)), True))
```

Giac [A] time = 1.09911, size = 117, normalized size = 1.29

$$-\frac{1}{192}\left(2\left(4\left(6\,x+19\right)(x-2)+155\right)(x-2)-303\right)\sqrt{x-2}\sqrt{-x+3}+\frac{5}{24}\left(2\left(4\,x+3\right)(x-2)-15\right)\sqrt{x-2}\sqrt{-x+3}-\frac{3}{2}\,(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/192*(2*(4*(6*x + 19)*(x - 2) + 155)*(x - 2) - 303)*sqrt(x - 2)*sqrt(-x +
 3) + 5/24*(2*(4*x + 3)*(x - 2) - 15)*sqrt(x - 2)*sqrt(-x + 3) - 3/2*(2*x -
 5)*sqrt(x - 2)*sqrt(-x + 3) + 3/64*arcsin(sqrt(x - 2))
```

3.1161 $\int \sqrt{3-x}\sqrt{-2+x}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{x-2}(3-x)^{3/2}+\frac{1}{4}\sqrt{x-2}\sqrt{3-x}-\frac{1}{8}\sin^{-1}(5-2x)$$

```
[Out] (Sqrt[3 - x]*Sqrt[-2 + x])/4 - ((3 - x)^(3/2)*Sqrt[-2 + x])/2 - ArcSin[5 -
2*x]/8
```

Rubi [A] time = 0.0097638, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {50, 53, 619, 216}

$$-\frac{1}{2}\sqrt{x-2}(3-x)^{3/2}+\frac{1}{4}\sqrt{x-2}\sqrt{3-x}-\frac{1}{8}\sin^{-1}(5-2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x]*Sqrt[-2 + x],x]
```

```
[Out] (Sqrt[3 - x]*Sqrt[-2 + x])/4 - ((3 - x)^(3/2)*Sqrt[-2 + x])/2 - ArcSin[5 -
2*x]/8
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{3-x}\sqrt{-2+x}\,dx = -\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{4}\int \frac{\sqrt{3-x}}{\sqrt{-2+x}}\,dx$$
$$= \frac{1}{4}\sqrt{3-x}\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{8}\int \frac{1}{\sqrt{3-x}\sqrt{-2+x}}\,dx$$
$$= \frac{1}{4}\sqrt{3-x}\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{8}\int \frac{1}{\sqrt{-6+5x-x^2}}\,dx$$
$$= \frac{1}{4}\sqrt{3-x}\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,5-2x\right)$$
$$= \frac{1}{4}\sqrt{3-x}\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}\sin^{-1}(5-2x)$$

Mathematica [A] time = 0.0226581, size = 69, normalized size = 1.35

$$\frac{\sqrt{-x^2+5x-6}\left(\sqrt{x-2}\left(2x^2-11x+15\right)+\sqrt{3-x}\sin^{-1}\left(\sqrt{3-x}\right)\right)}{4(x-3)\sqrt{x-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x]*Sqrt[-2 + x],x]
```

```
[Out] (Sqrt[-6 + 5*x - x^2]*(Sqrt[-2 + x]*(15 - 11*x + 2*x^2) + Sqrt[3 - x]*ArcSi
n[Sqrt[3 - x]]))/(4*(-3 + x)*Sqrt[-2 + x])
```

Maple [A] time = 0.004, size = 61, normalized size = 1.2

$$-\frac{1}{2}(3-x)^{\frac{3}{2}}\sqrt{-2+x}+\frac{1}{4}\sqrt{3-x}\sqrt{-2+x}+\frac{\arcsin(2x-5)}{8}\sqrt{(-2+x)(3-x)}\frac{1}{\sqrt{3-x}}\frac{1}{\sqrt{-2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-x)^(1/2)*(-2+x)^(1/2),x)
```

```
[Out] -1/2*(3-x)^(3/2)*(-2+x)^(1/2)+1/4*(3-x)^(1/2)*(-2+x)^(1/2)+1/8*((-2+x)*(3-x
))^(1/2)/(-2+x)^(1/2)/(3-x)^(1/2)*arcsin(2*x-5)
```

Maxima [A] time = 1.423, size = 51, normalized size = 1.

$$\frac{1}{2}\sqrt{-x^2+5x-6}x-\frac{5}{4}\sqrt{-x^2+5x-6}+\frac{1}{8}\arcsin(2x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 5*x - 6)*x - 5/4*sqrt(-x^2 + 5*x - 6) + 1/8*arcsin(2*x - 5)
```

Fricas [A] time = 1.49894, size = 147, normalized size = 2.88

$$\frac{1}{4}(2x-5)\sqrt{x-2}\sqrt{-x+3}-\frac{1}{8}\arctan\left(\frac{(2x-5)\sqrt{x-2}\sqrt{-x+3}}{2\left(x^2-5x+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3) - 1/8*arctan(1/2*(2*x - 5)*sqrt(x -
2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

Sympy [A] time = 3.0069, size = 124, normalized size = 2.43

$$\begin{cases} -\frac{i\,\mathrm{acosh}\left(\sqrt{x-2}\right)}{4}+\frac{i(x-2)^{\frac{5}{2}}}{2\sqrt{x-3}}-\frac{3i(x-2)^{\frac{3}{2}}}{4\sqrt{x-3}}+\frac{i\sqrt{x-2}}{4\sqrt{x-3}} & \text{for } |x-2|>1 \\ \frac{\mathrm{asin}\left(\sqrt{x-2}\right)}{4}-\frac{(x-2)^{\frac{5}{2}}}{2\sqrt{3-x}}+\frac{3(x-2)^{\frac{3}{2}}}{4\sqrt{3-x}}-\frac{\sqrt{x-2}}{4\sqrt{3-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)**(1/2)*(-2+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(x - 2))/4 + I*(x - 2)**(5/2)/(2*sqrt(x - 3)) - 3*I
*(x - 2)**(3/2)/(4*sqrt(x - 3)) + I*sqrt(x - 2)/(4*sqrt(x - 3)), Abs(x - 2)
 > 1), (asin(sqrt(x - 2))/4 - (x - 2)**(5/2)/(2*sqrt(3 - x)) + 3*(x - 2)**(
3/2)/(4*sqrt(3 - x)) - sqrt(x - 2)/(4*sqrt(3 - x)), True))
```

Giac [A] time = 1.09726, size = 38, normalized size = 0.75

$$\frac{1}{4}(2x-5)\sqrt{x-2}\sqrt{-x+3}+\frac{1}{4}\arcsin\left(\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3) + 1/4*arcsin(sqrt(x - 2))
```

3.1162 $\int \frac{1}{\sqrt{3-x}\sqrt{-2+x}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(5-2x)$$

```
[Out] -ArcSin[5 - 2*x]
```

Rubi [A] time = 0.0041647, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {53, 619, 216}

$$-\sin^{-1}(5-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x]*Sqrt[-2 + x]),x]

[Out] -ArcSin[5 - 2*x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3-x}\sqrt{-2+x}}\,dx &= \int \frac{1}{\sqrt{-6+5x-x^2}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 5-2x\right)\\
&= -\sin^{-1}(5-2x)
\end{aligned}$$

Mathematica [A] time = 0.0100673, size = 12, normalized size = 1.5

$$-2\sin^{-1}\left(\sqrt{3-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x]*Sqrt[-2 + x]),x]

[Out] -2*ArcSin[Sqrt[3 - x]]
```

Maple [B] time = 0.003, size = 31, normalized size = 3.9

$$\arcsin(2x-5)\sqrt{(-2+x)(3-x)}\frac{1}{\sqrt{3-x}}\frac{1}{\sqrt{-2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)^(1/2)/(-2+x)^(1/2),x)

[Out] ((-2+x)*(3-x))^(1/2)/(-2+x)^(1/2)/(3-x)^(1/2)*arcsin(2*x-5)
```

Maxima [A] time = 1.48708, size = 8, normalized size = 1.

$$\arcsin(2x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="maxima")

[Out] arcsin(2*x - 5)
```

Fricas [B] time = 1.58787, size = 88, normalized size = 11.

$$-\arctan\left(\frac{(2x-5)\sqrt{x-2}\sqrt{-x+3}}{2\left(x^2-5x+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="fricas")

[Out] -arctan(1/2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

Sympy [A] time = 1.61958, size = 26, normalized size = 3.25

$$\begin{cases} -2i\operatorname{acosh}\left(\sqrt{x-2}\right) & \text{for}\ |x-2|>1 \\ 2\operatorname{asin}\left(\sqrt{x-2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(1/2)/(-2+x)**(1/2),x)

[Out] Piecewise((-2*I*acosh(sqrt(x - 2)), Abs(x - 2) > 1), (2*asin(sqrt(x - 2)),
True))
```

Giac [A] time = 1.08056, size = 11, normalized size = 1.38

$$2\arcsin\left(\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(sqrt(x - 2))
```

3.1163 $\int \frac{1}{(3-x)^{3/2}(-2+x)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{\sqrt{3-x}\sqrt{x-2}}-\frac{4\sqrt{3-x}}{\sqrt{x-2}}$$

```
[Out] 2/(Sqrt[3 - x]*Sqrt[-2 + x]) - (4*Sqrt[3 - x])/Sqrt[-2 + x]
```

Rubi [A] time = 0.0044166, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2}{\sqrt{3-x}\sqrt{x-2}}-\frac{4\sqrt{3-x}}{\sqrt{x-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(3/2)*(-2 + x)^(3/2)),x]

[Out] 2/(Sqrt[3 - x]*Sqrt[-2 + x]) - (4*Sqrt[3 - x])/Sqrt[-2 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-x)^{3/2}(-2+x)^{3/2}}\,dx &= \frac{2}{\sqrt{3-x}\sqrt{-2+x}}+2\int \frac{1}{\sqrt{3-x}(-2+x)^{3/2}}\,dx\\ &= \frac{2}{\sqrt{3-x}\sqrt{-2+x}}-\frac{4\sqrt{3-x}}{\sqrt{-2+x}}\end{aligned}$$

Mathematica [A] time = 0.0069226, size = 21, normalized size = 0.57

$$\frac{2(2x-5)}{\sqrt{-x^2+5x-6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(3/2)*(-2 + x)^(3/2)),x]
```

```
[Out] (2*(-5 + 2*x))/Sqrt[-6 + 5*x - x^2]
```

Maple [A] time = 0.003, size = 20, normalized size = 0.5

$$2\,\frac{2\,x-5}{\sqrt{3-x}\sqrt{-2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)^(3/2)/(-2+x)^(3/2),x)
```

```
[Out] 2*(2*x-5)/(-2+x)^(1/2)/(3-x)^(1/2)
```

Maxima [A] time = 0.978491, size = 41, normalized size = 1.11

$$\frac{4\,x}{\sqrt{-x^2+5\,x-6}}-\frac{10}{\sqrt{-x^2+5\,x-6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 5*x - 6) - 10/sqrt(-x^2 + 5*x - 6)
```

Fricas [A] time = 1.52193, size = 74, normalized size = 2.

$$-\frac{2\,(2\,x-5)\sqrt{x-2}\sqrt{-x+3}}{x^2-5\,x+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6)
```

Sympy [A] time = 3.25889, size = 100, normalized size = 2.7

$$\begin{cases} -\frac{4i\sqrt{x-3}(x-2)}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}}+\frac{2i\sqrt{x-3}}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} & \text{for } |x-2|>1 \\ -\frac{4\sqrt{3-x}(x-2)}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}}+\frac{2\sqrt{3-x}}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(3/2)/(-2+x)**(3/2),x)
```

```
[Out] Piecewise((-4*I*sqrt(x - 3)*(x - 2)/((x - 2)**(3/2) - sqrt(x - 2)) + 2*I*sq
rt(x - 3)/((x - 2)**(3/2) - sqrt(x - 2)), Abs(x - 2) > 1), (-4*sqrt(3 - x)*
(x - 2)/((x - 2)**(3/2) - sqrt(x - 2)) + 2*sqrt(3 - x)/((x - 2)**(3/2) - sq
rt(x - 2)), True))
```

Giac [A] time = 1.0689, size = 72, normalized size = 1.95

$$-\frac{\sqrt{-x+3}-1}{\sqrt{x-2}}-\frac{2\sqrt{x-2}\sqrt{-x+3}}{x-3}+\frac{\sqrt{x-2}}{\sqrt{-x+3}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(-x + 3) - 1)/sqrt(x - 2) - 2*sqrt(x - 2)*sqrt(-x + 3)/(x - 3) + sqrt
(x - 2)/(sqrt(-x + 3) - 1)
```

3.1164 $\int \frac{1}{(3-x)^{5/2}(-2+x)^{5/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{32\sqrt{3-x}}{3\sqrt{x-2}}-\frac{16\sqrt{3-x}}{3(x-2)^{3/2}}+\frac{4}{(x-2)^{3/2}\sqrt{3-x}}+\frac{2}{3(x-2)^{3/2}(3-x)^{3/2}}$$

```
[Out] 2/(3*(3 - x)^(3/2)*(-2 + x)^(3/2)) + 4/(Sqrt[3 - x]*(-2 + x)^(3/2)) - (16*S
qrt[3 - x])/(3*(-2 + x)^(3/2)) - (32*Sqrt[3 - x])/(3*Sqrt[-2 + x])
```

Rubi [A] time = 0.0127822, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$-\frac{32\sqrt{3-x}}{3\sqrt{x-2}}-\frac{16\sqrt{3-x}}{3(x-2)^{3/2}}+\frac{4}{(x-2)^{3/2}\sqrt{3-x}}+\frac{2}{3(x-2)^{3/2}(3-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(5/2)*(-2 + x)^(5/2)),x]
```

```
[Out] 2/(3*(3 - x)^(3/2)*(-2 + x)^(3/2)) + 4/(Sqrt[3 - x]*(-2 + x)^(3/2)) - (16*S
qrt[3 - x])/(3*(-2 + x)^(3/2)) - (32*Sqrt[3 - x])/(3*Sqrt[-2 + x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x)^{5/2}(-2+x)^{5/2}}\,dx &= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}}+2\int \frac{1}{(3-x)^{3/2}(-2+x)^{5/2}}\,dx\\
&= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}}+\frac{4}{\sqrt{3-x}(-2+x)^{3/2}}+8\int \frac{1}{\sqrt{3-x}(-2+x)^{5/2}}\,dx\\
&= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}}+\frac{4}{\sqrt{3-x}(-2+x)^{3/2}}-\frac{16\sqrt{3-x}}{3(-2+x)^{3/2}}+\frac{16}{3}\int \frac{1}{\sqrt{3-x}(-2+x)^{3/2}}\,dx\\
&= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}}+\frac{4}{\sqrt{3-x}(-2+x)^{3/2}}-\frac{16\sqrt{3-x}}{3(-2+x)^{3/2}}-\frac{32\sqrt{3-x}}{3\sqrt{-2+x}}
\end{aligned}$$

Mathematica [A] time = 0.0130121, size = 33, normalized size = 0.42

$$\frac{-32x^3+240x^2-588x+470}{3\left(-x^2+5x-6\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(5/2)*(-2 + x)^(5/2)),x]
```

```
[Out] (470 - 588*x + 240*x^2 - 32*x^3)/(3*(-6 + 5*x - x^2)^(3/2))
```

Maple [A] time = 0.003, size = 30, normalized size = 0.4

$$-\frac{32\,x^{3}-240\,x^{2}+588\,x-470}{3}\,(3-x)^{-\frac{3}{2}}\,(-2+x)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)^(5/2)/(-2+x)^(5/2),x)
```

```
[Out] -2/3*(16*x^3-120*x^2+294*x-235)/(-2+x)^(3/2)/(3-x)^(3/2)
```

Maxima [A] time = 0.990339, size = 80, normalized size = 1.01

$$\frac{32\,x}{3\,\sqrt{-x^{2}+5\,x-6}}-\frac{80}{3\,\sqrt{-x^{2}+5\,x-6}}+\frac{4\,x}{3\,\left(-x^{2}+5\,x-6\right)^{\frac{3}{2}}}-\frac{10}{3\,\left(-x^{2}+5\,x-6\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x/sqrt(-x^2 + 5*x - 6) - 80/3/sqrt(-x^2 + 5*x - 6) + 4/3*x/(-x^2 + 5*x
 - 6)^(3/2) - 10/3/(-x^2 + 5*x - 6)^(3/2)
```

Fricas [A] time = 1.58898, size = 135, normalized size = 1.71

$$-\frac{2\,\left(16\,x^{3}-120\,x^{2}+294\,x-235\right)\sqrt{x-2}\sqrt{-x+3}}{3\,\left(x^{4}-10\,x^{3}+37\,x^{2}-60\,x+36\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(16*x^3 - 120*x^2 + 294*x - 235)*sqrt(x - 2)*sqrt(-x + 3)/(x^4 - 10*x^
3 + 37*x^2 - 60*x + 36)
```

Sympy [B] time = 39.3784, size = 282, normalized size = 3.57

$$\begin{cases} -\frac{32\sqrt{-1+\frac{1}{x-2}}(x-2)^3}{3x+3(x-2)^3-6(x-2)^2-6}+\frac{48\sqrt{-1+\frac{1}{x-2}}(x-2)^2}{3x+3(x-2)^3-6(x-2)^2-6}-\frac{12\sqrt{-1+\frac{1}{x-2}}(x-2)}{3x+3(x-2)^3-6(x-2)^2-6}-\frac{2\sqrt{-1+\frac{1}{x-2}}}{3x+3(x-2)^3-6(x-2)^2-6} & \text{for } \frac{1}{|x-2|}>1 \\ -\frac{32i\sqrt{1-\frac{1}{x-2}}(x-2)^3}{3x+3(x-2)^3-6(x-2)^2-6}+\frac{48i\sqrt{1-\frac{1}{x-2}}(x-2)^2}{3x+3(x-2)^3-6(x-2)^2-6}-\frac{12i\sqrt{1-\frac{1}{x-2}}(x-2)}{3x+3(x-2)^3-6(x-2)^2-6}-\frac{2i\sqrt{1-\frac{1}{x-2}}}{3x+3(x-2)^3-6(x-2)^2-6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(5/2)/(-2+x)**(5/2),x)
```

```
[Out] Piecewise((-32*sqrt(-1 + 1/(x - 2))*(x - 2)**3/(3*x + 3*(x - 2)**3 - 6*(x -
 2)**2 - 6) + 48*sqrt(-1 + 1/(x - 2))*(x - 2)**2/(3*x + 3*(x - 2)**3 - 6*(x
 - 2)**2 - 6) - 12*sqrt(-1 + 1/(x - 2))*(x - 2)/(3*x + 3*(x - 2)**3 - 6*(x
- 2)**2 - 6) - 2*sqrt(-1 + 1/(x - 2))/(3*x + 3*(x - 2)**3 - 6*(x - 2)**2 -
6), 1/Abs(x - 2) > 1), (-32*I*sqrt(1 - 1/(x - 2))*(x - 2)**3/(3*x + 3*(x -
2)**3 - 6*(x - 2)**2 - 6) + 48*I*sqrt(1 - 1/(x - 2))*(x - 2)**2/(3*x + 3*(x
 - 2)**3 - 6*(x - 2)**2 - 6) - 12*I*sqrt(1 - 1/(x - 2))*(x - 2)/(3*x + 3*(x
 - 2)**3 - 6*(x - 2)**2 - 6) - 2*I*sqrt(1 - 1/(x - 2))/(3*x + 3*(x - 2)**3
- 6*(x - 2)**2 - 6), True))
```

Giac [A] time = 1.08445, size = 131, normalized size = 1.66

$$-\frac{\left(\sqrt{-x+3}-1\right)^3}{12\,(x-2)^{\frac{3}{2}}}-\frac{11\left(\sqrt{-x+3}-1\right)}{4\sqrt{x-2}}-\frac{2\,(8\,x-25)\sqrt{x-2}\sqrt{-x+3}}{3\,(x-3)^2}+\frac{(x-2)^{\frac{3}{2}}\left(\frac{33\left(\sqrt{-x+3}-1\right)^2}{x-2}+1\right)}{12\left(\sqrt{-x+3}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(-x + 3) - 1)^3/(x - 2)^(3/2) - 11/4*(sqrt(-x + 3) - 1)/sqrt(x -
 2) - 2/3*(8*x - 25)*sqrt(x - 2)*sqrt(-x + 3)/(x - 3)^2 + 1/12*(x - 2)^(3/2
)*(33*(sqrt(-x + 3) - 1)^2/(x - 2) + 1)/(sqrt(-x + 3) - 1)^3
```

3.1165 $\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx$

Optimal. Leaf size=21

$$\frac{x}{9\sqrt{3-x}\sqrt{x+3}}$$

[Out] x/(9*Sqrt[3 - x]*Sqrt[3 + x])

Rubi [A] time = 0.0016875, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {39}

$$\frac{x}{9\sqrt{3-x}\sqrt{x+3}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]

[Out] x/(9*Sqrt[3 - x]*Sqrt[3 + x])

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx = \frac{x}{9\sqrt{3-x}\sqrt{3+x}}$$

Mathematica [A] time = 0.0043093, size = 16, normalized size = 0.76

$$\frac{x}{9\sqrt{9-x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]

[Out] x/(9*Sqrt[9 - x^2])

Maple [A] time = 0.002, size = 16, normalized size = 0.8

$$\frac{x}{9}\frac{1}{\sqrt{3-x}}\frac{1}{\sqrt{3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)^(3/2)/(3+x)^(3/2),x)
```

```
[Out] 1/9*x/(3-x)^(1/2)/(3+x)^(1/2)
```

Maxima [A] time = 0.984483, size = 16, normalized size = 0.76

$$\frac{x}{9\sqrt{-x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*x/sqrt(-x^2 + 9)
```

Fricas [A] time = 1.48049, size = 58, normalized size = 2.76

$$-\frac{\sqrt{x+3}x\sqrt{-x+3}}{9\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(x + 3)*x*sqrt(-x + 3)/(x^2 - 9)
```

Sympy [A] time = 2.84558, size = 73, normalized size = 3.48

$$\begin{cases} \frac{1}{9\sqrt{-1+\frac{6}{x+3}}} - \frac{1}{3\sqrt{-1+\frac{6}{x+3}}(x+3)} & \text{for}\ \frac{6}{|x+3|} > 1 \\ -\frac{i\sqrt{1-\frac{6}{x+3}}(x+3)}{9x-27} + \frac{3i\sqrt{1-\frac{6}{x+3}}}{9x-27} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(3/2)/(3+x)**(3/2),x)
```

```
[Out] Piecewise((1/(9*sqrt(-1 + 6/(x + 3))) - 1/(3*sqrt(-1 + 6/(x + 3))*(x + 3)),
 6/Abs(x + 3) > 1), (-I*sqrt(1 - 6/(x + 3))*(x + 3)/(9*x - 27) + 3*I*sqrt(1
 - 6/(x + 3))/(9*x - 27), True))
```

Giac [B] time = 1.07454, size = 84, normalized size = 4.

$$\frac{\sqrt{6}-\sqrt{-x+3}}{36\sqrt{x+3}} - \frac{\sqrt{x+3}\sqrt{-x+3}}{18(x-3)} - \frac{\sqrt{x+3}}{36\left(\sqrt{6}-\sqrt{-x+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/36*(sqrt(6) - sqrt(-x + 3))/sqrt(x + 3) - 1/18*sqrt(x + 3)*sqrt(-x + 3)/(
x - 3) - 1/36*sqrt(x + 3)/(sqrt(6) - sqrt(-x + 3))
```

3.1166 $\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\frac{x}{9\sqrt{3-bx}\sqrt{bx+3}}$$

```
[Out] x/(9*Sqrt[3 - b*x]*Sqrt[3 + b*x])
```

Rubi [A] time = 0.0025522, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {39}

$$\frac{x}{9\sqrt{3-bx}\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(9*Sqrt[3 - b*x]*Sqrt[3 + b*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx = \frac{x}{9\sqrt{3-bx}\sqrt{3+bx}}$$

Mathematica [A] time = 0.007636, size = 19, normalized size = 0.79

$$\frac{x}{9\sqrt{9-b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(9*Sqrt[9 - b^2*x^2])
```

Maple [A] time = 0.003, size = 19, normalized size = 0.8

$$\frac{x}{9}\frac{1}{\sqrt{-bx+3}}\frac{1}{\sqrt{bx+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x)
```

```
[Out] 1/9*x/(-b*x+3)^(1/2)/(b*x+3)^(1/2)
```

Maxima [A] time = 0.995133, size = 20, normalized size = 0.83

$$\frac{x}{9\sqrt{-b^2x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*x/sqrt(-b^2*x^2 + 9)
```

Fricas [A] time = 1.55026, size = 69, normalized size = 2.88

$$-\frac{\sqrt{bx+3}\sqrt{-bx+3}x}{9\left(b^2x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(b*x + 3)*sqrt(-b*x + 3)*x/(b^2*x^2 - 9)
```

Sympy [C] time = 5.32864, size = 73, normalized size = 3.04

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{9}{b^2x^2}\right)}{18\pi^{\frac{3}{2}}b}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{9e^{-2i\pi}}{b^2x^2}\right)}{18\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)**(3/2)/(b*x+3)**(3/2),x)
```

```
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), 9
/(b**2*x**2))/(18*pi**(3/2)*b) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4, 1), ()),
 ((1/4, 3/4), (-1/2, 0, 1, 0)), 9*exp_polar(-2*I*pi)/(b**2*x**2))/(18*pi**(
3/2)*b)
```

Giac [B] time = 1.06276, size = 111, normalized size = 4.62

$$\frac{\sqrt{6}-\sqrt{-bx+3}}{36\sqrt{bx+3}b}-\frac{\sqrt{bx+3}\sqrt{-bx+3}}{18(bx-3)b}-\frac{\sqrt{bx+3}}{36b\left(\sqrt{6}-\sqrt{-bx+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/36*(sqrt(6) - sqrt(-b*x + 3))/(sqrt(b*x + 3)*b) - 1/18*sqrt(b*x + 3)*sqrt
(-b*x + 3)/((b*x - 3)*b) - 1/36*sqrt(b*x + 3)/(b*(sqrt(6) - sqrt(-b*x + 3))
)
```

3.1167 $\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\frac{x}{18\sqrt{2}\sqrt{3-x}\sqrt{x+3}}$$

```
[Out] x/(18*Sqrt[2]*Sqrt[3 - x]*Sqrt[3 + x])
```

Rubi [A] time = 0.0016731, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {39}

$$\frac{x}{18\sqrt{2}\sqrt{3-x}\sqrt{x+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[2]*Sqrt[3 - x]*Sqrt[3 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx = \frac{x}{18\sqrt{2}\sqrt{3-x}\sqrt{3+x}}$$

Mathematica [A] time = 0.0123053, size = 21, normalized size = 0.81

$$\frac{x}{18\sqrt{6-2x}\sqrt{x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[6 - 2*x]*Sqrt[3 + x])
```

Maple [A] time = 0.001, size = 19, normalized size = 0.7

$$-\frac{(-3+x)\,x}{9}\frac{1}{\sqrt{3+x}}(6-2\,x)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6-2*x)^(3/2)/(3+x)^(3/2),x)
```

```
[Out] -1/9*(-3+x)/(3+x)^(1/2)*x/(6-2*x)^(3/2)
```

Maxima [A] time = 1.00331, size = 16, normalized size = 0.62

$$\frac{x}{18\sqrt{-2x^2+18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*x/sqrt(-2*x^2 + 18)
```

Fricas [A] time = 1.48798, size = 62, normalized size = 2.38

$$-\frac{\sqrt{x+3}x\sqrt{-2x+6}}{36(x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(x + 3)*x*sqrt(-2*x + 6)/(x^2 - 9)
```

Sympy [A] time = 123.053, size = 90, normalized size = 3.46

$$\begin{cases} \frac{\sqrt{2}}{36\sqrt{-1+\frac{6}{x+3}}} - \frac{\sqrt{2}}{12\sqrt{-1+\frac{6}{x+3}}(x+3)} & \text{for}\ \frac{6}{|x+3|} > 1 \\ -\frac{\sqrt{2}i\sqrt{1-\frac{6}{x+3}}(x+3)}{36x-108} + \frac{3\sqrt{2}i\sqrt{1-\frac{6}{x+3}}}{36x-108} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)**(3/2)/(3+x)**(3/2),x)
```

```
[Out] Piecewise((sqrt(2)/(36*sqrt(-1 + 6/(x + 3))) - sqrt(2)/(12*sqrt(-1 + 6/(x +
 3))*(x + 3)), 6/Abs(x + 3) > 1), (-sqrt(2)*I*sqrt(1 - 6/(x + 3))*(x + 3)/(
36*x - 108) + 3*sqrt(2)*I*sqrt(1 - 6/(x + 3))/(36*x - 108), True))
```

Giac [B] time = 1.07278, size = 96, normalized size = 3.69

$$\frac{\sqrt{2}\left(\sqrt{6}-\sqrt{-x+3}\right)}{144\sqrt{x+3}} - \frac{\sqrt{2}\sqrt{x+3}\sqrt{-x+3}}{72(x-3)} - \frac{\sqrt{2}\sqrt{x+3}}{144\left(\sqrt{6}-\sqrt{-x+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/144*sqrt(2)*(sqrt(6) - sqrt(-x + 3))/sqrt(x + 3) - 1/72*sqrt(2)*sqrt(x +
3)*sqrt(-x + 3)/(x - 3) - 1/144*sqrt(2)*sqrt(x + 3)/(sqrt(6) - sqrt(-x + 3)
)
```

3.1168 $\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}} \, dx$

Optimal. Leaf size=29

$$\frac{x}{18\sqrt{2}\sqrt{3-bx}\sqrt{bx+3}}$$

```
[Out] x/(18*Sqrt[2]*Sqrt[3 - b*x]*Sqrt[3 + b*x])
```

Rubi [A] time = 0.0025479, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {39}

$$\frac{x}{18\sqrt{2}\sqrt{3-bx}\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[2]*Sqrt[3 - b*x]*Sqrt[3 + b*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}} \, dx = \frac{x}{18\sqrt{2}\sqrt{3-bx}\sqrt{3+bx}}$$

Mathematica [A] time = 0.0177079, size = 19, normalized size = 0.66

$$\frac{x}{18\sqrt{18-2b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[18 - 2*b^2*x^2])
```

Maple [A] time = 0.002, size = 24, normalized size = 0.8

$$-\frac{(bx-3)x}{9}\frac{1}{\sqrt{bx+3}}(-2bx+6)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x)
```

```
[Out] -1/9*(b*x-3)/(b*x+3)^(1/2)*x/(-2*b*x+6)^(3/2)
```

Maxima [A] time = 0.969727, size = 20, normalized size = 0.69

$$\frac{x}{18\sqrt{-2\,b^2x^2+18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*x/sqrt(-2*b^2*x^2 + 18)
```

Fricas [A] time = 1.59389, size = 73, normalized size = 2.52

$$-\frac{\sqrt{bx+3}\sqrt{-2\,bx+6}x}{36\left(b^2x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(b*x + 3)*sqrt(-2*b*x + 6)*x/(b^2*x^2 - 9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)**(3/2)/(b*x+3)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.07303, size = 123, normalized size = 4.24

$$\frac{\sqrt{2}\left(\sqrt{6}-\sqrt{-bx+3}\right)}{144\sqrt{bx+3}b}-\frac{\sqrt{2}\sqrt{bx+3}\sqrt{-bx+3}}{72\,(bx-3)b}-\frac{\sqrt{2}\sqrt{bx+3}}{144\,b\left(\sqrt{6}-\sqrt{-bx+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/144*sqrt(2)*(sqrt(6) - sqrt(-b*x + 3))/(sqrt(b*x + 3)*b) - 1/72*sqrt(2)*s
qrt(b*x + 3)*sqrt(-b*x + 3)/((b*x - 3)*b) - 1/144*sqrt(2)*sqrt(b*x + 3)/(b*
(sqrt(6) - sqrt(-b*x + 3)))
```

3.1169 $\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx$

Optimal. Leaf size=39

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bdx-ad}}\right)}{b\sqrt{d}}$$

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(a*d) + b*d*x]])/(b*Sqrt[d])
```

Rubi [A] time = 0.024387, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bdx-ad}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(a*d) + b*d*x]])/(b*Sqrt[d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{-2ad+dx^2}}\,dx, x, \sqrt{a+bx}\right)}{b}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.0213767, size = 39, normalized size = 1.

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bdx-ad}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(a*d) + b*d*x]])/(b*Sqrt[d])
```

Maple [B] time = 0.007, size = 76, normalized size = 2.

$$\sqrt{(bx+a)(bdx-ad)}\ln\left(b^2dx\frac{1}{\sqrt{b^2d}}+\sqrt{b^2dx^2-a^2d}\right)\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{bdx-ad}}\frac{1}{\sqrt{b^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x)
```

```
[Out] ((b*x+a)*(b*d*x-a*d))^(1/2)/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2)*ln(b^2*d*x/(b^2
*d)^(1/2)+(b^2*d*x^2-a^2*d)^(1/2))/(b^2*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58296, size = 248, normalized size = 6.36

$$\left[\frac{\log\left(2\,b^2dx^2+2\,\sqrt{bdx-ad}\sqrt{bx+a}b\sqrt{d}x-a^2d\right)}{2\,b\sqrt{d}},-\frac{\sqrt{-d}\arctan\left(\frac{\sqrt{bdx-ad}\sqrt{bx+a}b\sqrt{-d}x}{b^2dx^2-a^2d}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*b^2*d*x^2 + 2*sqrt(b*d*x - a*d)*sqrt(b*x + a)*b*sqrt(d)*x - a^2*
d)/(b*sqrt(d)), -sqrt(-d)*arctan(sqrt(b*d*x - a*d)*sqrt(b*x + a)*b*sqrt(-d)
*x/(b^2*d*x^2 - a^2*d))/(b*d)]
```

Sympy [C] time = 3.35799, size = 88, normalized size = 2.26

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| \frac{a^2}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{d}} - \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| \frac{a^2e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(b*d*x-a*d)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), a**
2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(d)) - I*meijerg(((-1/2, -1/4, 0, 1/4, 1/
2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(2*I*pi)/(b**2*x*
*2))/(4*pi**(3/2)*b*sqrt(d))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1170 $\int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx$

Optimal. Leaf size=241

$$-\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}e}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{e\cdots}}\cdots\right)}{\sqrt[4]{3}e}$$

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)*e)
 - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)
*e) - Log[(Sqrt[6 - 3*e*x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt
[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]]/(Sqrt[2]*3^(1/4)*e) + Log[(Sqrt[6 - 3*e*x
] + Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2
 + e*x]]/(Sqrt[2]*3^(1/4)*e)
```

Rubi [A] time = 0.254286, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}e}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{e\cdots}}\cdots\right)}{\sqrt[4]{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)*e)
 - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)
*e) - Log[(Sqrt[6 - 3*e*x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt
[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]]/(Sqrt[2]*3^(1/4)*e) + Log[(Sqrt[6 - 3*e*x
] + Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2
 + e*x]]/(Sqrt[2]*3^(1/4)*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx = -\frac{4\,\text{Subst}\left(\int \frac{x^2}{\left(4-\frac{x^4}{3}\right)^{3/4}}\,dx, x, \sqrt[4]{6-3ex}\right)}{3e}$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e} - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{3}+x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{3}-\sqrt{2}\sqrt[4]{3}x+x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{3}+\sqrt{2}\sqrt[4]{3}x+x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} + \frac{\log\left(\frac{\sqrt{2-ex}+\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} - \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\right.}{\sqrt[4]{3}}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}e} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}e} - \frac{\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} + \frac{\text{lo}}{}$$

Mathematica [C] time = 0.0210897, size = 42, normalized size = 0.17

$$-\frac{\sqrt{2}(6-3ex)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{12}(6-3ex)\right)}{9e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
```

```
[Out] -(Sqrt[2]*(6 - 3*e*x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (6 - 3*e*x)/12
])/(9*e)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{-3\,ex+6}}\,(ex+2)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
```

```
[Out] int(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex+2)^{\frac{3}{4}}(-3\,ex+6)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x + 2)^(3/4)*(-3*e*x + 6)^(1/4)), x)
```

Fricas [B] time = 1.75919, size = 1565, normalized size = 6.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*(1/3)^(1/4)*(e^(-4))^(1/4)*arctan(-(sqrt(2)*(1/3)^(3/4)*(e*x + 2)
^(1/4)*(-3*e*x + 6)^(3/4)*e^3*(e^(-4))^(3/4) - sqrt(3)*sqrt(2)*(1/3)^(3/4)*
(e^4*x - 2*e^3)*sqrt((sqrt(2)*(1/3)^(1/4)*(e*x + 2)^(1/4)*(-3*e*x + 6)^(3/4
)*e*(e^(-4))^(1/4) + 3*sqrt(1/3)*(e^3*x - 2*e^2)*sqrt(e^(-4)) - sqrt(e*x +
2)*sqrt(-3*e*x + 6))/(e*x - 2))*(e^(-4))^(3/4) + e*x - 2)/(e*x - 2)) + 2*sq
rt(2)*(1/3)^(1/4)*(e^(-4))^(1/4)*arctan(-(sqrt(2)*(1/3)^(3/4)*(e*x + 2)^(1/
4)*(-3*e*x + 6)^(3/4)*e^3*(e^(-4))^(3/4) - sqrt(3)*sqrt(2)*(1/3)^(3/4)*(e^4
*x - 2*e^3)*sqrt(-(sqrt(2)*(1/3)^(1/4)*(e*x + 2)^(1/4)*(-3*e*x + 6)^(3/4)*e
*(e^(-4))^(1/4) - 3*sqrt(1/3)*(e^3*x - 2*e^2)*sqrt(e^(-4)) + sqrt(e*x + 2)*
sqrt(-3*e*x + 6))/(e*x - 2))*(e^(-4))^(3/4) - e*x + 2)/(e*x - 2)) - 1/2*sqr
t(2)*(1/3)^(1/4)*(e^(-4))^(1/4)*log(3*(sqrt(2)*(1/3)^(1/4)*(e*x + 2)^(1/4)*
(-3*e*x + 6)^(3/4)*e*(e^(-4))^(1/4) + 3*sqrt(1/3)*(e^3*x - 2*e^2)*sqrt(e^(-
4)) - sqrt(e*x + 2)*sqrt(-3*e*x + 6))/(e*x - 2)) + 1/2*sqrt(2)*(1/3)^(1/4)*
(e^(-4))^(1/4)*log(-3*(sqrt(2)*(1/3)^(1/4)*(e*x + 2)^(1/4)*(-3*e*x + 6)^(3/
4)*e*(e^(-4))^(1/4) - 3*sqrt(1/3)*(e^3*x - 2*e^2)*sqrt(e^(-4)) + sqrt(e*x +
 2)*sqrt(-3*e*x + 6))/(e*x - 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{3^{\frac{3}{4}} \int \frac{1}{\sqrt[4]{-ex+2}(ex+2)^{\frac{3}{4}}}\, dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)**(1/4)/(e*x+2)**(3/4),x)
```

```
[Out] 3**(3/4)*Integral(1/((-e*x + 2)**(1/4)*(e*x + 2)**(3/4)), x)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex+2)^{\frac{3}{4}}(-3\,ex+6)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + 2)^(3/4)*(-3*e*x + 6)^(1/4)), x)
```

3.1171 $\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}} dx$

Optimal. Leaf size=144

$$-\frac{14a^2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{14a^2x}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}$$

```
[Out] (14*a^2*x)/(5*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) - ((14*I)/15)*(a - I*a*x
)^(3/4)*(a + I*a*x)^(3/4) - (((2*I)/5)*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4))
/a - (14*a^2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(5*(a - I*a*x)^(1/4
)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0343024, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 42, 229, 227, 196}

$$-\frac{14a^2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{14a^2x}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(1/4),x]
```


```
[Out] (14*a^2*x)/(5*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) - ((14*I)/15)*(a - I*a*x
)^(3/4)*(a + I*a*x)^(3/4) - (((2*I)/5)*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4))
/a - (14*a^2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(5*(a - I*a*x)^(1/4
)*(a + I*a*x)^(1/4))
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}+\frac{1}{5}(7a)\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx\\
&= -\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}+\frac{1}{5}\left(7a^2\right)\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx\\
&= -\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}+\frac{\left(7a^2\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= -\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}+\frac{\left(7a^2\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= \frac{14a^2x}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}-\frac{\left(7a^2\sqrt[4]{1+x^2}\right)\int \frac{}{(1+}}{5\sqrt[4]{a-iax}\sqrt[4]{a+}}\\
&= \frac{14a^2x}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}-\frac{14a^2\sqrt[4]{1+x^2}E\left(\frac{1}{2}\text{t}\right.}{5\sqrt[4]{a-iax}\sqrt[4]{a}}
\end{aligned}$$

Mathematica [C] time = 0.0366124, size = 70, normalized size = 0.49

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{11/4}\,{}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] (((2*I)/11)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[1/
4, 11/4, 15/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.059, size = 104, normalized size = 0.7

$$-\frac{(20\,i+6\,x)\,(x+i)\,(x-i)\,a^2}{15}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}\frac{1}{\sqrt[4]{a\,(1+ix)}}+\frac{7\,a^2x}{5}{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^2)\sqrt[4]{-a^2\,(-1+ix)\,(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a\,(-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] -2/15*(10*I+3*x)*(x+I)*(x-I)*a^2/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)+7/5/
(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)*a^2*(-a^2*(-1+I*x)*(1+I*x))^(
```

```
1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-iax+a)^{\frac{7}{4}}}{(iax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}\left(3\,x^2+10i\,x-21\right)-15\,x\mathrm{integral}\left(\frac{14\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}{5\,(x^4+x^2)},x\right)}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 + 10*I*x - 21) - 15*x*
integral(14/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(x^4 + x^2), x))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1172 $\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=106

$$-\frac{2a\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2ax}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}$$

```
[Out] (2*a*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) - (((2*I)/3)*(a - I*a*x)^(3/4
)*(a + I*a*x)^(3/4))/a - (2*a*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/((
a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0223588, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 42, 229, 227, 196}

$$-\frac{2a\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2ax}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(1/4),x]
```


```
[Out] (2*a*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) - (((2*I)/3)*(a - I*a*x)^(3/4
)*(a + I*a*x)^(3/4))/a - (2*a*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/((
a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}+a\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx\\
&= -\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}+\frac{\left(a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= -\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}+\frac{\left(a\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= \frac{2ax}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}-\frac{\left(a\sqrt[4]{1+x^2}\right)\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= \frac{2ax}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}-\frac{2a\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}
\end{aligned}$$

Mathematica [C] time = 0.0227682, size = 70, normalized size = 0.66

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{7/4}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] (((2*I)/7)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[1/4,
 7/4, 11/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.039, size = 94, normalized size = 0.9

$$-\frac{2i}{3}(x+i)(x-i)a\frac{1}{\sqrt[4]{-a(-1+ix)}}\frac{1}{\sqrt[4]{a(1+ix)}}+ax{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^2\right)\sqrt[4]{-a^2(-1+ix)(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a(-1+ix)}}\frac{1}{\sqrt[4]{a(1+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] -2/3*I*(x+I)*(x-I)*a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)+1/(a^2)^(1/4)*x*
hypergeom([1/4,1/2],[3/2],-x^2)*a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x
))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-iax+a)^{\frac{3}{4}}}{(iax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,ax\mathrm{integral}\left(\frac{2\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{3}{4}}}{ax^4+ax^2},x\right)-2\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{3}{4}}(ix-3)}{3\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*x*integral(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a*x^4 + a*x^2),
 x) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(I*x - 3))/(a*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a\,(ix-1))^{\frac{3}{4}}}{\sqrt[4]{a\,(ix+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral((-a*(I*x - 1))**(3/4)/(a*(I*x + 1))**(1/4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1173 $\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=71

$$\frac{2x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

```
[Out] (2*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) - (2*(1 + x^2)^(1/4)*EllipticE[
ArcTan[x]/2, 2])/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0119873, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {42, 229, 227, 196}

$$\frac{2x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)),x]
```


```
[Out] (2*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) - (2*(1 + x^2)^(1/4)*EllipticE[
ArcTan[x]/2, 2])/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```


Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}dx = \frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$
$$= \frac{\sqrt[4]{1+x^2}\int \frac{1}{\sqrt[4]{1+x^2}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$
$$= \frac{2x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} - \frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$
$$= \frac{2x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} - \frac{2\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Mathematica [C] time = 0.0220942, size = 70, normalized size = 0.99

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((2*I)/3)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[1/4,
 3/4, 7/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a-iax}}\frac{1}{\sqrt[4]{a+iax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 x \text{integral}\left(\frac{2 (i a x+a)^{\frac{3}{4}} (-i a x+a)^{\frac{3}{4}}}{a^2 x^4+a^2 x^2}, x\right) + 2 (i a x + a)^{\frac{3}{4}} (-i a x + a)^{\frac{3}{4}}}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] (a^2*x*integral(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^2*x^4 + a^2*x^2),
 x) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4))/(a^2*x)
```

Sympy [A] time = 3.69559, size = 102, normalized size = 1.44

$$-\frac{i G_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{8}, \frac{5}{8}, 1 & \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \\ -\frac{1}{4}, \frac{1}{8}, \frac{1}{4}, \frac{5}{8}, \frac{3}{4} & 0 \end{matrix} \middle| \frac{e^{-3i\pi}}{x^2}\right) e^{\frac{i\pi}{4}}}{4\pi\sqrt{a}\Gamma\left(\frac{1}{4}\right)} + \frac{i G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{3}{8}, 0, \frac{1}{8}, \frac{1}{2}, 1 & \\ -\frac{3}{8}, \frac{1}{8} & -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \middle| \frac{e^{-i\pi}}{x^2}\right)}{4\pi\sqrt{a}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] -I*meijerg(((1/8, 5/8, 1), (1/4, 1/2, 3/4)), ((-1/4, 1/8, 1/4, 5/8, 3/4), (
0,)), exp_polar(-3*I*pi)/x**2)*exp(I*pi/4)/(4*pi*sqrt(a)*gamma(1/4)) + I*me
ijerg(((-1/2, -3/8, 0, 1/8, 1/2, 1), ()), ((-3/8, 1/8), (-1/2, -1/4, 0, 0))
, exp_polar(-I*pi)/x**2)/(4*pi*sqrt(a)*gamma(1/4))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1174 $\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=78

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

```
[Out] (-2*I)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Ellipti
cE[ArcTan[x]/2, 2])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0159803, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {48, 42, 197, 196}

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (-2*I)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Ellipti
cE[ArcTan[x]/2, 2])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 48

```
Int[1/(((a_) + (b_.)*(x_))^(5/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-2/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4)), x] + Dist[c, Int[1/((a + b*x)^(
5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0
] && NegQ[a^2*b^2]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}} + a\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx$$
$$= -\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}} + \frac{\left(a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\left(a^2+a^2x^2\right)^{5/4}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$
$$= -\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}} + \frac{\sqrt[4]{1+x^2}\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$
$$= -\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}} + \frac{2\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Mathematica [C] time = 0.0223079, size = 68, normalized size = 0.87

$$-\frac{2i2^{3/4}\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-2*I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, 1/2 - (I/
2)*x])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.05, size = 94, normalized size = 1.2

$$2\frac{x-i}{a\sqrt[4]{-a(-1+ix)}\sqrt[4]{a(1+ix)}} - \frac{x}{a}{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^2\right)\sqrt[4]{-a^2(-1+ix)(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a(-1+ix)}}\frac{1}{\sqrt[4]{a(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] 2*(x-I)/a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/(a^2)^(1/4)*x*hypergeom([
1/4,1/2],[3/2],-x^2)/a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a
*(1+I*x))^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(5/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(a^3x^2+i\,a^3x\right)\mathrm{integral}\left(-\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}{a^3x^4+a^3x^2},x\right)-2i\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}{a^3x^2+i\,a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] ((a^3*x^2 + I*a^3*x)*integral(-2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^3*
x^4 + a^3*x^2), x) - 2*I*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4))/(a^3*x^2 + I
*a^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt[4]{a\,(ix+1)}\,(-a\,(ix-1))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral(1/((a*(I*x + 1))**(1/4)*(-a*(I*x - 1))**(5/4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1175 $\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=82

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

```
[Out] ((-4*I)/5)/(a*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Ell
ipticE[ArcTan[x]/2, 2])/(5*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0165606, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {46, 42, 197, 196}

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-4*I)/5)/(a*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Ell
ipticE[ArcTan[x]/2, 2])/(5*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 46

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx = -\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{1}{5}\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx$$

$$= -\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{\left(a^2+a^2x^2\right)^{5/4}}\,dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$= -\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{\sqrt[4]{1+x^2}\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$= -\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{2\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Mathematica [C] time = 0.0258731, size = 70, normalized size = 0.85

$$-\frac{2i2^{3/4}\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/5)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, 1/2
- (I/2)*x])/(a*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.053, size = 105, normalized size = 1.3

$$\frac{2x^2+4+2ix}{(5x+5i)a^2}\frac{1}{\sqrt[4]{-a(-1+ix)}}\frac{1}{\sqrt[4]{a(1+ix)}} - \frac{x}{5a^2}{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^2)\sqrt[4]{-a^2(-1+ix)(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a(-1+ix)}}\frac{1}{\sqrt[4]{a(1+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] 2/5*(x^2+2+I*x)/(x+I)/a^2/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/5/(a^2)^(
1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^2*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-
a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(9/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}(2\,x+4i)+\left(5\,a^4x^2+10i\,a^4x-5\,a^4\right)\mathrm{integral}\left(-\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}{5\left(a^4x^2+a^4\right)},x\right)}{5\,a^4x^2+10i\,a^4x-5\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] ((I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(2*x + 4*I) + (5*a^4*x^2 + 10*I*a^4*x
 - 5*a^4)*integral(-1/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^4*x^2 + a^4
), x))/(5*a^4*x^2 + 10*I*a^4*x - 5*a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1176 $\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=115

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{15a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}-\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

```
[Out] ((-4*I)/15)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) - (((2*I)/9)*(a + I*a
*x)^(3/4))/(a^2*(a - I*a*x)^(9/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]
/2, 2])/(15*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.02639, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 46, 42, 197, 196}

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{15a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}-\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(1/4)),x]
```


```
[Out] ((-4*I)/15)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) - (((2*I)/9)*(a + I*a
*x)^(3/4))/(a^2*(a - I*a*x)^(9/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]
/2, 2])/(15*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 46

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}+\frac{\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx}{3a}\\
&= -\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}+\frac{\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx}{15a}\\
&= -\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}+\frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{\left(a^2+a^2x^2\right)^{5/4}}\,dx}{15a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= -\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}+\frac{\sqrt[4]{1+x^2}\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx}{15a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= -\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}+\frac{2\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{15a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}
\end{aligned}$$

Mathematica [C] time = 0.0277512, size = 70, normalized size = 0.61

$$-\frac{2i2^{3/4}\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/9)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, 1/2
- (I/2)*x])/(a*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.059, size = 113, normalized size = 1.

$$\frac{12\,ix^2+6\,x^3-4\,x+22\,i}{45\,(x+i)^2\,a^3}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}\frac{1}{\sqrt[4]{a\,(1+ix)}}-\frac{x}{15\,a^3}{}_2F_1(\frac{1}{4},\frac{1}{2};\,\frac{3}{2};\,-x^2)\sqrt[4]{-a^2\,(-1+ix)\,(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] 2/45*(6*I*x^2+3*x^3-2*x+11*I)/(x+I)^2/a^3/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(
1/4)-1/15/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^3*(-a^2*(-1+I*x)*
(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(13/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{3}{4}}\left(3\,x^2+9i\,x-11\right)+\left(45\,a^5x^3+135i\,a^5x^2-135\,a^5x-45i\,a^5\right)\mathrm{integral}\left(-\frac{(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{3}{4}}}{15\,(a^5x^2+a^5)},x\right)}{45\,a^5x^3+135i\,a^5x^2-135\,a^5x-45i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] (2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 + 9*I*x - 11) + (45*a^5*x^3
+ 135*I*a^5*x^2 - 135*a^5*x - 45*I*a^5)*integral(-1/15*(I*a*x + a)^(3/4)*(-
I*a*x + a)^(3/4)/(a^5*x^2 + a^5), x))/(45*a^5*x^3 + 135*I*a^5*x^2 - 135*a^5
*x - 45*I*a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1177 $\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=148

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{39a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}-\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

```
[Out] ((-4*I)/39)/(a^3*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) - (((2*I)/13)*(a + I*
a*x)^(3/4))/(a^2*(a - I*a*x)^(13/4)) - (((10*I)/117)*(a + I*a*x)^(3/4))/(a^
3*(a - I*a*x)^(9/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(39*a^
4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0384858, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 46, 42, 197, 196}

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{39a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}-\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(1/4)),x]
```


```
[Out] ((-4*I)/39)/(a^3*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) - (((2*I)/13)*(a + I*
a*x)^(3/4))/(a^2*(a - I*a*x)^(13/4)) - (((10*I)/117)*(a + I*a*x)^(3/4))/(a^
3*(a - I*a*x)^(9/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(39*a^
4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 46

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
```

```
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}} + \frac{5\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx}{13a}$$

$$= -\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}} - \frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}} + \frac{5\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx}{39a^2}$$

$$= -\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} - \frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}} - \frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}} + \frac{\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}}{39a^2}$$

$$= -\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} - \frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}} - \frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}} + \frac{\sqrt[4]{a^2+a^2x^2}\int \frac{}{(a^2+}}{39a^2\sqrt[4]{a-iax}\sqrt[4]{}}$$

$$= -\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} - \frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}} - \frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}} + \frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/}}}{39a^4\sqrt[4]{a-iax}\sqrt[4]{a-}}$$

$$= -\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} - \frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}} - \frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}} + \frac{2\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan\right.}{39a^4\sqrt[4]{a-iax}\sqrt[4]{}}$$

Mathematica [C] time = 0.0281626, size = 70, normalized size = 0.47

$$-\frac{2i2^{3/4}\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{13}{4},\frac{1}{4};-\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{13a(a-iax)^{13/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/13)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-13/4, 1/4, -9/4, 1/
2 - (I/2)*x])/(a*(a - I*a*x)^(13/4)*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.059, size = 114, normalized size = 0.8

$$\frac{18\,ix^3+6\,x^4-40-16\,x^2}{117\,(x+i)^3\,a^4}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}\frac{1}{\sqrt[4]{a\,(1+ix)}} - \frac{x}{39\,a^4}{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^2)\sqrt[4]{-a^2\,(-1+ix)\,(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a\,(-1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] 2/117*(9*I*x^3+3*x^4-20-8*x^2)/(x+I)^3/a^4/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^
(1/4)-1/39/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^4*(-a^2*(-1+I*x)
```

```
*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{17}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(17/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(6\,x^3+24i\,x^2-40\,x-40i\right)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}+\left(117\,a^6x^4+468i\,a^6x^3-702\,a^6x^2-468i\,a^6x+117\,a^6\right)\mathrm{integral}\left(-\frac{(i\,a}{}\right.}{117\,a^6x^4+468i\,a^6x^3-702\,a^6x^2-468i\,a^6x+117\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] ((6*x^3 + 24*I*x^2 - 40*x - 40*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4) + (1
17*a^6*x^4 + 468*I*a^6*x^3 - 702*a^6*x^2 - 468*I*a^6*x + 117*a^6)*integral(
-1/39*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^6*x^2 + a^6), x))/(117*a^6*x^
4 + 468*I*a^6*x^3 - 702*a^6*x^2 - 468*I*a^6*x + 117*a^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(17/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1178 $\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=256

$$-\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{i\,\mathrm{ta}}{}$$

```
[Out] ((-I)*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))/a - (I*ArcTan[1 - (Sqrt[2]*(a -
I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (I*ArcTan[1 + (Sqrt[2]*(a - I*a
*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] - ((I/2)*Log[1 + Sqrt[a - I*a*x]/Sqr
t[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + ((
I/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/
(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rubi [A] time = 0.172916, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{i\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] ((-I)*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))/a - (I*ArcTan[1 - (Sqrt[2]*(a -
I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (I*ArcTan[1 + (Sqrt[2]*(a - I*a
*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] - ((I/2)*Log[1 + Sqrt[a - I*a*x]/Sqr
t[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + ((
I/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/
(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx = -\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} + \frac{1}{2}a\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$

$$= -\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} + 2i\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)$$

$$= -\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} + 2i\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= -\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} + i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) + i\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= -\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} + \frac{1}{2}i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) + \frac{1}{2}i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\right.$$

$$= -\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}} + \frac{i\,\text{Subst}(\ldots}{}$$

$$= -\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} - \frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} + \frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{a\ldots}}\right.}{2\sqrt{2}}$$

Mathematica [C] time = 0.0208058, size = 70, normalized size = 0.27

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{5/4}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] (((2*I)/5)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[1/4,
 5/4, 9/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \sqrt[4]{a-iax}\frac{1}{\sqrt[4]{a+iax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] int((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(1/4), x)
```

Fricas [A] time = 1.67944, size = 540, normalized size = 2.11

$$\frac{\sqrt{i}a\log\left(\frac{\sqrt{i}(ax-ia)+(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{x-i}\right)-\sqrt{i}a\log\left(-\frac{\sqrt{i}(ax-ia)-(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{x-i}\right)+\sqrt{-i}a\log\left(\frac{\sqrt{-i}(ax-ia)+(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{x-i}\right)-\sqrt{-}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(I)*a*log((sqrt(I)*(a*x - I*a) + (I*a*x + a)^(3/4)*(-I*a*x + a)^(1
/4))/(x - I)) - sqrt(I)*a*log(-(sqrt(I)*(a*x - I*a) - (I*a*x + a)^(3/4)*(-I
*a*x + a)^(1/4))/(x - I)) + sqrt(-I)*a*log((sqrt(-I)*(a*x - I*a) + (I*a*x +
 a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - sqrt(-I)*a*log(-(sqrt(-I)*(a*x - I
*a) - (I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 2*I*(I*a*x + a)^(3/4
)*(-I*a*x + a)^(1/4))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{-a\,(ix-1)}}{\sqrt[4]{a\,(ix+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral((-a*(I*x - 1))**(1/4)/(a*(I*x + 1))**(1/4), x)
```

Giac [A] time = 1.25306, size = 252, normalized size = 0.98

$$\frac{1}{2}i\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2(-iax+a)^{\frac{1}{4}}}{(iax+a)^{\frac{1}{4}}}\right)\right)+\frac{1}{2}i\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2(-iax+a)^{\frac{1}{4}}}{(iax+a)^{\frac{1}{4}}}\right)\right)+\frac{1}{4}i\sqrt{2}\log\left(\frac{\sqrt{2}(-iax+a)^{\frac{1}{4}}}{(iax+a)^{\frac{1}{4}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/2*I*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(-I*a*x + a)^(1/4)/(I*a*x + a
)^(1/4))) + 1/2*I*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(-I*a*x + a)^(1/
4)/(I*a*x + a)^(1/4))) + 1/4*I*sqrt(2)*log(sqrt(2)*(-I*a*x + a)^(1/4)/(I*a*
x + a)^(1/4) + sqrt(-I*a*x + a)/sqrt(I*a*x + a) + 1) - 1/4*I*sqrt(2)*log(-s
qrt(2)*(-I*a*x + a)^(1/4)/(I*a*x + a)^(1/4) + sqrt(-I*a*x + a)/sqrt(I*a*x +
 a) + 1) + I*(-I*a*x + a)^(1/4)*(-I*a*x - a)/((I*a*x + a)^(1/4)*a)
```

3.1179 $\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=233

$$-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

```
[Out] ((-I)*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
+ (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a -
 (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(
a + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rubi [A] time = 0.130697, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
```


```
[Out] ((-I)*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
+ (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a -
 (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(
a + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```


Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx = \frac{(4i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$

$$= \frac{(4i)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{(2i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\,\text{Subst}\left(\int \frac{\sqrt{2}+\ldots}{-1-\sqrt{2}\ldots}\right.}{\sqrt{\ldots}}$$

$$= -\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} + \frac{\left(i\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,\ldots\right.}{a}$$

$$= -\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} + \ldots$$

Mathematica [C] time = 0.0218541, size = 68, normalized size = 0.29

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}\sqrt[4]{a-iax}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((2*I)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4
, 5/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (a-iax)^{-\frac{3}{4}}\frac{1}{\sqrt[4]{a+iax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [A] time = 1.65726, size = 601, normalized size = 2.58

$$\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}+2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,x-2i}\right)-\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}-2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,x-2i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(4*I/a^2) + 2*(I*a*x + a)^(3/4)*
(-I*a*x + a)^(1/4))/(2*x - 2*I)) - 1/2*sqrt(4*I/a^2)*log(-((a^2*x - I*a^2)*
sqrt(4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) + 1/2*
sqrt(-4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(3/4)*(-
I*a*x + a)^(1/4))/(2*x - 2*I)) - 1/2*sqrt(-4*I/a^2)*log(-((a^2*x - I*a^2)*s
qrt(-4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a\,(ix+1)}\,(-a\,(ix-1))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral(1/((a*(I*x + 1))**(1/4)*(-a*(I*x - 1))**(3/4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1180 $\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=33

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

```
[Out] (((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

Rubi [A] time = 0.0032194, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {37}

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Mathematica [A] time = 0.013589, size = 33, normalized size = 1.

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

Maple [A] time = 0.03, size = 31, normalized size = 0.9

$$\frac{2x-2i}{3a}(-a(-1+ix))^{-\frac{3}{4}}\frac{1}{\sqrt[4]{a(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] 2/3/a/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(x-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(7/4)), x)
```

Fricas [A] time = 1.5323, size = 78, normalized size = 2.36

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\left(a^3x+i\,a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/3*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(a^3*x + I*a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a\,(ix+1)}\,(-a\,(ix-1))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral(1/((a*(I*x + 1))**(1/4)*(-a*(I*x - 1))**(7/4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1181 $\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=67

$$-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}$$

```
[Out] (((-2*I)/7)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(7/4)) - (((4*I)/21)*(a + I
*a*x)^(3/4))/(a^3*(a - I*a*x)^(3/4))
```

Rubi [A] time = 0.0094478, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {45, 37}

$$-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/7)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(7/4)) - (((4*I)/21)*(a + I
*a*x)^(3/4))/(a^3*(a - I*a*x)^(3/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}+\frac{2\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx}{7a}\\ &= -\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.0210978, size = 45, normalized size = 0.67

$$\frac{2(5-2ix)(a+iax)^{3/4}}{21a^3(x+i)(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*(5 - (2*I)*x)*(a + I*a*x)^(3/4))/(21*a^3*(I + x)*(a - I*a*x)^(3/4))
```

Maple [A] time = 0.035, size = 44, normalized size = 0.7

$$\frac{4x^2+10+6ix}{21a^2(x+i)}(-a(-1+ix))^{-\frac{3}{4}}\frac{1}{\sqrt[4]{a(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] 2/21/a^2/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(2*x^2+5+3*I*x)/(x+I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(11/4)), x)
```

Fricas [A] time = 1.63675, size = 116, normalized size = 1.73

$$\frac{(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}(4x+10i)}{21a^4x^2+42ia^4x-21a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] (I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(4*x + 10*I)/(21*a^4*x^2 + 42*I*a^4*x
- 21*a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1182 $\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=100

$$-\frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}$$

```
[Out] (((-2*I)/11)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(11/4)) - (((8*I)/77)*(a +
 I*a*x)^(3/4))/(a^3*(a - I*a*x)^(7/4)) - (((16*I)/231)*(a + I*a*x)^(3/4))/(
a^4*(a - I*a*x)^(3/4))
```

Rubi [A] time = 0.0182027, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {45, 37}

$$-\frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/11)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(11/4)) - (((8*I)/77)*(a +
 I*a*x)^(3/4))/(a^3*(a - I*a*x)^(7/4)) - (((16*I)/231)*(a + I*a*x)^(3/4))/(
a^4*(a - I*a*x)^(3/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}+\frac{4\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx}{11a}\\
&= -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}+\frac{8\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx}{77a^2}\\
&= -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}-\frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.0245436, size = 52, normalized size = 0.52

$$\frac{2\left(-8ix^2+28x+41i\right)(a+iax)^{3/4}}{231a^4(x+i)^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*(a + I*a*x)^(3/4)*(41*I + 28*x - (8*I)*x^2))/(231*a^4*(I + x)^2*(a - I*a
*x)^(3/4))
```

Maple [A] time = 0.038, size = 50, normalized size = 0.5

$$\frac{40\,ix^2+16\,x^3-26\,x+82\,i}{231\,a^3\,(x+i)^2}(-a\,(-1+ix))^{-\frac{3}{4}}\frac{1}{\sqrt[4]{a\,(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] 2/231/a^3/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(20*I*x^2+8*x^3-13*x+41*I)/
(x+I)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{15}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(15/4)), x)
```

Fricas [A] time = 1.55214, size = 157, normalized size = 1.57

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}\left(8\,x^2+28i\,x-41\right)}{231\,a^5x^3+693i\,a^5x^2-693\,a^5x-231i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 + 28*I*x - 41)/(231*a^5*x^3 +
 693*I*a^5*x^2 - 693*a^5*x - 231*I*a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(15/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1183 $\int \frac{1}{(a-iax)^{19/4} \sqrt[4]{a+iax}} dx$

Optimal. Leaf size=133

$$-\frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}-\frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}$$

```
[Out] (((-2*I)/15)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(15/4)) - (((4*I)/55)*(a +
 I*a*x)^(3/4))/(a^3*(a - I*a*x)^(11/4)) - (((16*I)/385)*(a + I*a*x)^(3/4))/
(a^4*(a - I*a*x)^(7/4)) - (((32*I)/1155)*(a + I*a*x)^(3/4))/(a^5*(a - I*a*x
)^(3/4))
```

Rubi [A] time = 0.0283329, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {45, 37}

$$-\frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}-\frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```


```
[Out] (((-2*I)/15)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(15/4)) - (((4*I)/55)*(a +
 I*a*x)^(3/4))/(a^3*(a - I*a*x)^(11/4)) - (((16*I)/385)*(a + I*a*x)^(3/4))/
(a^4*(a - I*a*x)^(7/4)) - (((32*I)/1155)*(a + I*a*x)^(3/4))/(a^5*(a - I*a*x
)^(3/4))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} + \frac{2\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx}{5a}$$
$$= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} - \frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}} + \frac{8\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx}{55a^2}$$
$$= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} - \frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}} - \frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}} + \frac{16\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx}{385a^3}$$
$$= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} - \frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}} - \frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}} - \frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}$$

Mathematica [A] time = 0.0284822, size = 57, normalized size = 0.43

$$\frac{2\left(-16ix^3+72x^2+138ix-159\right)(a+iax)^{3/4}}{1155a^5(x+i)^3(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*(a + I*a*x)^(3/4)*(-159 + (138*I)*x + 72*x^2 - (16*I)*x^3))/(1155*a^5*(I
 + x)^3*(a - I*a*x)^(3/4))
```

Maple [A] time = 0.039, size = 55, normalized size = 0.4

$$\frac{112\,ix^3+32\,x^4-42\,ix-318-132\,x^2}{1155\,a^4\,(x+i)^3}(-a\,(-1+ix))^{-\frac{3}{4}}\frac{1}{\sqrt[4]{a\,(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] 2/1155/a^4/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(56*I*x^3+16*x^4-21*I*x-15
9-66*x^2)/(x+I)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{19}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(19/4)), x)
```

Fricas [A] time = 1.52049, size = 200, normalized size = 1.5

$$\frac{\left(32\,x^{3}+144i\,x^{2}-276\,x-318i\right)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{1155\,a^{6}x^{4}+4620i\,a^{6}x^{3}-6930\,a^{6}x^{2}-4620i\,a^{6}x+1155\,a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] (32*x^3 + 144*I*x^2 - 276*x - 318*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(
1155*a^6*x^4 + 4620*I*a^6*x^3 - 6930*a^6*x^2 - 4620*I*a^6*x + 1155*a^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(19/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1184 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=256

$$-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i}{}$$

```
[Out] ((-I)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(
a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + ((3*I)*ArcTan[1 + (Sqrt[2]*
(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (((3*I)/2)*Log[1 + Sqrt[a
- I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/
Sqrt[2] - (((3*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a
- I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rubi [A] time = 0.158251, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-I)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(
a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + ((3*I)*ArcTan[1 + (Sqrt[2]*
(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (((3*I)/2)*Log[1 + Sqrt[a
- I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/
Sqrt[2] - (((3*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a
- I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx = -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{1}{2}(3a)\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}}\,dx$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+6i\,\mathrm{Subst}\left(\int \frac{x^2}{\left(2a-x^4\right)^{3/4}}\,dx,x,\sqrt[4]{a-iax}\right)$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+6i\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}-3i\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+3i\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3}{2}i\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+\frac{3}{2}i\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\right.$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}-\frac{3i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}+\frac{(3i)\,\mathrm{Sub}}{}$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\right.}{2\sqrt{2}}$$

Mathematica [C] time = 0.0248626, size = 70, normalized size = 0.27

$$\frac{2i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{7/4}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] (((2*I)/7)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[3/4,
 7/4, 11/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (a-iax)^{\frac{3}{4}}(a+iax)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(3/4), x)
```

Fricas [A] time = 1.58805, size = 594, normalized size = 2.32

$$\frac{\sqrt{9i}a\log\left(\frac{\sqrt{9i}(ax+ia)+3(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3x+3i}\right)-\sqrt{9i}a\log\left(-\frac{\sqrt{9i}(ax+ia)-3(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3x+3i}\right)+\sqrt{-9i}a\log\left(\frac{\sqrt{-9i}(ax+ia)+3(iax+a)^{\frac{1}{4}}(-i}{3x+3i}\right.}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(9*I)*a*log((sqrt(9*I)*(a*x + I*a) + 3*(I*a*x + a)^(1/4)*(-I*a*x +
 a)^(3/4))/(3*x + 3*I)) - sqrt(9*I)*a*log(-(sqrt(9*I)*(a*x + I*a) - 3*(I*a*
x + a)^(1/4)*(-I*a*x + a)^(3/4))/(3*x + 3*I)) + sqrt(-9*I)*a*log((sqrt(-9*I
)*(a*x + I*a) + 3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(3*x + 3*I)) - sqrt
(-9*I)*a*log(-(sqrt(-9*I)*(a*x + I*a) - 3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3
/4))/(3*x + 3*I)) - 2*I*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a(ix-1))^{\frac{3}{4}}}{(a(ix+1))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral((-a*(I*x - 1))**(3/4)/(a*(I*x + 1))**(3/4), x)
```

Giac [A] time = 1.18519, size = 242, normalized size = 0.95

$$\frac{3}{2}i\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2(-iax+a)^{\frac{1}{4}}}{(iax+a)^{\frac{1}{4}}}\right)\right)+\frac{3}{2}i\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2(-iax+a)^{\frac{1}{4}}}{(iax+a)^{\frac{1}{4}}}\right)\right)-\frac{3}{4}i\sqrt{2}\log\left(\frac{\sqrt{2}(-iax+}{(iax+a)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] 3/2*I*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(-I*a*x + a)^(1/4)/(I*a*x + a
)^(1/4))) + 3/2*I*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(-I*a*x + a)^(1/
4)/(I*a*x + a)^(1/4))) - 3/4*I*sqrt(2)*log(sqrt(2)*(-I*a*x + a)^(1/4)/(I*a*
x + a)^(1/4) + sqrt(-I*a*x + a)/sqrt(I*a*x + a) + 1) + 3/4*I*sqrt(2)*log(-s
qrt(2)*(-I*a*x + a)^(1/4)/(I*a*x + a)^(1/4) + sqrt(-I*a*x + a)/sqrt(I*a*x +
 a) + 1) - I*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)/a
```

3.1185 $\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=233

$$\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

```
[Out] ((-I)*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
+ (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a +
 (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(
a + I*a*x)^(1/4)])/(Sqrt[2]*a) - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rubi [A] time = 0.135098, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-I)*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
+ (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a +
 (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(
a + I*a*x)^(1/4)])/(Sqrt[2]*a) - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}}\,dx = \frac{(4i)\,\text{Subst}\left(\int \frac{x^2}{\left(2a-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$

$$= \frac{(4i)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= -\frac{(2i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\int \frac{1}{-1 \cdots}\right.}{\cdots}$$

$$= \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} + \frac{\left(i\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx \cdots\right.}{a}$$

$$= -\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a}$$

Mathematica [C] time = 0.0220721, size = 70, normalized size = 0.3

$$\frac{2i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((2*I)/3)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[3/4,
 3/4, 7/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a-iax}}(a+iax)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [A] time = 1.62714, size = 601, normalized size = 2.58

$$\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x+i\,a^2\right)\sqrt{\frac{4i}{a^2}}+2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{2\,x+2i}\right)-\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x+i\,a^2\right)\sqrt{\frac{4i}{a^2}}-2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{2\,x+2i}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I/a^2)*log(((a^2*x + I*a^2)*sqrt(4*I/a^2) + 2*(I*a*x + a)^(1/4)*
(-I*a*x + a)^(3/4))/(2*x + 2*I)) - 1/2*sqrt(4*I/a^2)*log(-((a^2*x + I*a^2)*
sqrt(4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x + 2*I)) + 1/2*
sqrt(-4*I/a^2)*log(((a^2*x + I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(1/4)*(-
I*a*x + a)^(3/4))/(2*x + 2*I)) - 1/2*sqrt(-4*I/a^2)*log(-((a^2*x + I*a^2)*s
qrt(-4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x + 2*I))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\,(ix+1))^{\frac{3}{4}}\,\sqrt[4]{-a\,(ix-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((a*(I*x + 1))**(3/4)*(-a*(I*x - 1))**(1/4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1186 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=31

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

[Out] ((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))

Rubi [A] time = 0.0031736, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {37}

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]

[Out] ((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Mathematica [A] time = 0.0126921, size = 31, normalized size = 1.

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]

[Out] ((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))

Maple [A] time = 0.029, size = 31, normalized size = 1.

$$2\,\frac{x-i}{a\,(a\,(1+ix))^{3/4}\,\sqrt[4]{-a\,(-1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] 2/a/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(x-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(5/4)), x)
```

Fricas [A] time = 1.54243, size = 76, normalized size = 2.45

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{a^3x+i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)/(a^3*x + I*a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\,(ix+1))^{\frac{3}{4}}\,(-a\,(ix-1))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((a*(I*x + 1))**(3/4)*(-a*(I*x - 1))**(5/4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1187 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=67

$$-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}$$

```
[Out] (((-2*I)/5)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(5/4)) - (((4*I)/5)*(a + I*
a*x)^(1/4))/(a^3*(a - I*a*x)^(1/4))
```

Rubi [A] time = 0.0098718, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {45, 37}

$$-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
```


```
[Out] (((-2*I)/5)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(5/4)) - (((4*I)/5)*(a + I*
a*x)^(1/4))/(a^3*(a - I*a*x)^(1/4))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx &= -\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}+\frac{2\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx}{5a}\\ &= -\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}\end{aligned}$$

Mathematica [A] time = 0.0179804, size = 45, normalized size = 0.67

$$\frac{2(3-2ix)\sqrt[4]{a+iax}}{5a^3(x+i)\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (2*(3 - (2*I)*x)*(a + I*a*x)^(1/4))/(5*a^3*(I + x)*(a - I*a*x)^(1/4))
```

Maple [A] time = 0.034, size = 44, normalized size = 0.7

$$\frac{4x^2+6+2ix}{5a^2(x+i)}(a(1+ix))^{-\frac{3}{4}}\frac{1}{\sqrt[4]{-a(-1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] 2/5/a^2/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(2*x^2+3+I*x)/(x+I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(9/4)), x)
```

Fricas [A] time = 1.58109, size = 112, normalized size = 1.67

$$\frac{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(4x+6i)}{5a^4x^2+10i\,a^4x-5a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] (I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(4*x + 6*I)/(5*a^4*x^2 + 10*I*a^4*x -
5*a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1188 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}-\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}$$

```
[Out] (((-2*I)/9)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(9/4)) - (((8*I)/45)*(a + I
*a*x)^(1/4))/(a^3*(a - I*a*x)^(5/4)) - (((16*I)/45)*(a + I*a*x)^(1/4))/(a^4
*(a - I*a*x)^(1/4))
```

Rubi [A] time = 0.0179842, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {45, 37}

$$-\frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}-\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```


```
[Out] (((-2*I)/9)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(9/4)) - (((8*I)/45)*(a + I
*a*x)^(1/4))/(a^3*(a - I*a*x)^(5/4)) - (((16*I)/45)*(a + I*a*x)^(1/4))/(a^4
*(a - I*a*x)^(1/4))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx &= -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}+\frac{4\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx}{9a}\\
&= -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}+\frac{8\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx}{45a^2}\\
&= -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}-\frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}
\end{aligned}$$

Mathematica [A] time = 0.0235545, size = 52, normalized size = 0.52

$$\frac{2(-8ix^2+20x+17i)\sqrt[4]{a+iax}}{45a^4(x+i)^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (2*(a + I*a*x)^(1/4)*(17*I + 20*x - (8*I)*x^2))/(45*a^4*(I + x)^2*(a - I*a*
x)^(1/4))
```

Maple [A] time = 0.038, size = 50, normalized size = 0.5

$$\frac{24\,ix^2+16\,x^3+6\,x+34\,i}{45\,a^3\,(x+i)^2}(a\,(1+ix))^{-\frac{3}{4}}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] 2/45/a^3/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(12*I*x^2+8*x^3+3*x+17*I)/(x
+I)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(13/4)), x)
```

Fricas [A] time = 1.45051, size = 154, normalized size = 1.54

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}\left(8\,x^2+20i\,x-17\right)}{45\,a^5x^3+135i\,a^5x^2-135\,a^5x-45i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(8*x^2 + 20*I*x - 17)/(45*a^5*x^3 +
135*I*a^5*x^2 - 135*a^5*x - 45*I*a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1189 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=112

$$\frac{10a^2\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}(x),2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{10}{3}i\sqrt[4]{a-iax}\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}$$

```
[Out] ((-10*I)/3)*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4) - (((2*I)/3)*(a - I*a*x)^(5
/4)*(a + I*a*x)^(1/4))/a + (10*a^2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2
])/(3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rubi [A] time = 0.0241203, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {50, 42, 233, 231}

$$\frac{10a^2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{10}{3}i\sqrt[4]{a-iax}\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-10*I)/3)*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4) - (((2*I)/3)*(a - I*a*x)^(5
/4)*(a + I*a*x)^(1/4))/a + (10*a^2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2
])/(3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] && !IntegerQ[2*m]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx = -\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}+\frac{1}{3}(5a)\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx$$
$$= -\frac{10}{3}i\sqrt[4]{a-iax}\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}+\frac{1}{3}\left(5a^2\right)\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx$$
$$= -\frac{10}{3}i\sqrt[4]{a-iax}\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}+\frac{\left(5a^2\left(a^2+a^2x^2\right)^{3/4}\right)\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= -\frac{10}{3}i\sqrt[4]{a-iax}\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}+\frac{\left(5a^2\left(1+x^2\right)^{3/4}\right)\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= -\frac{10}{3}i\sqrt[4]{a-iax}\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}+\frac{10a^2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

Mathematica [C] time = 0.0261663, size = 70, normalized size = 0.62

$$\frac{2i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{9/4}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] (((2*I)/9)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[3/4,
 9/4, 13/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int (a-iax)^{\frac{5}{4}}(a+iax)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{5}{4}}}{(i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x+12i)+\text{integral}\left(\frac{5\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\,(x^2+1)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(2*x + 12*I) + integral(5/3*(I*a*
x + a)^(1/4)*(-I*a*x + a)^(1/4)/(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(-a\,(ix-1))^{\frac{5}{4}}}{(a\,(ix+1))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral((-a*(I*x - 1))**(5/4)/(a*(I*x + 1))**(3/4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1190 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=76

$$\frac{2a\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}(x),2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}$$

```
[Out] ((-2*I)*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))/a + (2*a*(1 + x^2)^(3/4)*Ellip
ticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rubi [A] time = 0.0145622, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {50, 42, 233, 231}

$$\frac{2a\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-2*I)*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))/a + (2*a*(1 + x^2)^(3/4)*Ellip
ticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a} + a\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx$$
$$= -\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a} + \frac{\left(a\left(a^2+a^2x^2\right)^{3/4}\right)\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= -\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a} + \frac{\left(a\left(1+x^2\right)^{3/4}\right)\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= -\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a} + \frac{2a\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

Mathematica [C] time = 0.0206476, size = 70, normalized size = 0.92

$$\frac{2i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{5/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] (((2*I)/5)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[3/4,
 5/4, 9/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \sqrt[4]{a-iax}\,(a+iax)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\mathrm{integral}\left(\frac{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{ax^2+a},x\right)-2i\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] (a*integral((I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a*x^2 + a), x) - 2*I*(I*a
*x + a)^(1/4)*(-I*a*x + a)^(1/4))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt[4]{-a\,(ix-1)}}{(a\,(ix+1))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral((-a*(I*x - 1))**(1/4)/(a*(I*x + 1))**(3/4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1191 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}} \, dx$

Optimal. Leaf size=43

$$\frac{2\left(x^2+1\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\tan^{-1}(x),2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] (2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a*x
)^(3/4))
```

Rubi [A] time = 0.0081492, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {42, 233, 231}

$$\frac{2\left(x^2+1\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a*x
)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] && !IntegerQ[2*m]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}} \, dx &= \frac{\left(a^2+a^2x^2\right)^{3/4} \int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}} \, dx}{(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= \frac{\left(1+x^2\right)^{3/4} \int \frac{1}{\left(1+x^2\right)^{3/4}} \, dx}{(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= \frac{2\left(1+x^2\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0198353, size = 68, normalized size = 1.58

$$\frac{2i\sqrt[4]{2}(1+ix)^{3/4}\sqrt[4]{a-iax}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((2*I)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 3/4
, 5/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int (a-iax)^{-\frac{3}{4}}(a+iax)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{a^2x^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a^2*x^2 + a^2), x)
```

Sympy [A] time = 10.2738, size = 100, normalized size = 2.33

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{8},\frac{7}{8},1 & \frac{1}{2},\frac{3}{4},\frac{5}{4}\\ \frac{1}{4},\frac{3}{8},\frac{3}{4},\frac{7}{8},\frac{5}{4} & 0\end{matrix}\middle|\frac{e^{-3i\pi}}{x^2}\right)e^{\frac{3i\pi}{4}}}{4\pi a^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{8},0,\frac{3}{8},\frac{1}{2},1 & \\ -\frac{1}{8},\frac{3}{8} & -\frac{1}{2},0,\frac{1}{4},0\end{matrix}\middle|\frac{e^{-i\pi}}{x^2}\right)}{4\pi a^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] -I*meijerg(((3/8, 7/8, 1), (1/2, 3/4, 5/4)), ((1/4, 3/8, 3/4, 7/8, 5/4), (0
,)), exp_polar(-3*I*pi)/x**2)*exp(3*I*pi/4)/(4*pi*a**(3/2)*gamma(3/4)) + I*
meijerg(((-1/2, -1/8, 0, 3/8, 1/2, 1), ()), ((-1/8, 3/8), (-1/2, 0, 1/4, 0)
), exp_polar(-I*pi)/x**2)/(4*pi*a**(3/2)*gamma(3/4))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1192 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=82

$$\frac{2\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}(x),2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}$$

```
[Out] (((-2*I)/3)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)
*EllipticF[ArcTan[x]/2, 2])/(3*a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rubi [A] time = 0.0147231, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {51, 42, 233, 231}

$$\frac{2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)
*EllipticF[ArcTan[x]/2, 2])/(3*a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx &= -\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx}{3a}\\ &= -\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= -\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{\left(1+x^2\right)^{3/4}\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= -\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.0206751, size = 70, normalized size = 0.85

$$-\frac{2i\sqrt[4]{2}(1+ix)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/3)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, 1/2 -
 (I/2)*x])/(a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (a-iax)^{-\frac{7}{4}}(a+iax)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(7/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(a^{3}x+i\,a^{3}\right)\mathrm{integral}\left(\frac{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\left(a^{3}x^{2}+a^{3}\right)},x\right)+2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\left(a^{3}x+i\,a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^3*x + I*a^3)*integral(1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a
^3*x^2 + a^3), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^3*x + I*a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\,(ix+1))^{\frac{3}{4}}\,(-a\,(ix-1))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((a*(I*x + 1))**(3/4)*(-a*(I*x - 1))**(7/4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1193 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=115

$$\frac{2\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}(x),2\right)}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}$$

```
[Out] (((-2*I)/7)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(7/4)) - (((2*I)/7)*(a + I*
a*x)^(1/4))/(a^3*(a - I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x
]/2, 2])/(7*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rubi [A] time = 0.0250462, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {51, 42, 233, 231}

$$\frac{2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)),x]
```


```
[Out] (((-2*I)/7)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(7/4)) - (((2*I)/7)*(a + I*
a*x)^(1/4))/(a^3*(a - I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x
]/2, 2])/(7*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}} + \frac{3\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx}{7a}$$
$$= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}} - \frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}} + \frac{\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx}{7a^2}$$
$$= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}} - \frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}} + \frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}} - \frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}} + \frac{\left(1+x^2\right)^{3/4}\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}} - \frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}} + \frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

Mathematica [C] time = 0.0244519, size = 70, normalized size = 0.61

$$-\frac{2i\sqrt[4]{2}(1+ix)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a(a-iax)^{7/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/7)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, 1/2
- (I/2)*x])/(a*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int (a-iax)^{-\frac{11}{4}}(a+iax)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(11/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{(7\,a^4x^2+14i\,a^4x-7\,a^4)\text{integral}\left(\frac{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{7\,(a^4x^2+a^4)},x\right)+(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x+4i)}{7\,a^4x^2+14i\,a^4x-7\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] ((7*a^4*x^2 + 14*I*a^4*x - 7*a^4)*integral(1/7*(I*a*x + a)^(1/4)*(-I*a*x +
a)^(1/4)/(a^4*x^2 + a^4), x) + (I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(2*x +
4*I))/(7*a^4*x^2 + 14*I*a^4*x - 7*a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1194 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=291

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i\sqrt[4]{a+iax}(a-iax)^{3/4}}{3a}-\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\tan^{-1}}{}$$

```
[Out] (((4*I)/3)*(a - I*a*x)^(7/4))/(a*(a + I*a*x)^(3/4)) + (((7*I)/3)*(a - I*a*x
)^(3/4)*(a + I*a*x)^(1/4))/a + ((7*I)*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4)
)/(a + I*a*x)^(1/4)])/Sqrt[2] - ((7*I)*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/Sqrt[2] - (((7*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a
+ I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (((7*I
)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/
(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rubi [A] time = 0.180831, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {47, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i\sqrt[4]{a+iax}(a-iax)^{3/4}}{3a}-\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```


```
[Out] (((4*I)/3)*(a - I*a*x)^(7/4))/(a*(a + I*a*x)^(3/4)) + (((7*I)/3)*(a - I*a*x
)^(3/4)*(a + I*a*x)^(1/4))/a + ((7*I)*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4)
)/(a + I*a*x)^(1/4)])/Sqrt[2] - ((7*I)*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/Sqrt[2] - (((7*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a
+ I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (((7*I
)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/
(a + I*a*x)^(1/4)])/Sqrt[2]
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx = \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} - \frac{7}{3}\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - \frac{1}{2}(7a)\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}}\,dx$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - 14i\,\text{Subst}\left(\int \frac{x^2}{\left(2a-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right)$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - 14i\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} + 7i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) - 7i\,\text{Subst}\left(\int \frac{1}{1}\right.$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - \frac{7}{2}i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) - \frac{7}{2}i\,\text{Subs}$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - \frac{7i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}} + \frac{7i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\right.}{2\sqrt{2}}$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} + \frac{7i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} - \frac{7i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} - \cdots$$

Mathematica [C] time = 0.0273962, size = 70, normalized size = 0.24

$$\frac{i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{11/4}\,{}_2F_1\left(\frac{7}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/11)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[7/4, 1
1/4, 15/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (a-iax)^{\frac{7}{4}}(a+iax)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{7}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(7/4), x)
```

Fricas [A] time = 1.70361, size = 701, normalized size = 2.41

$$-\frac{3\sqrt{49i}(ax-ia)\log\left(\frac{\sqrt{49i}(ax+ia)+7(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7x+7i}\right)-3\sqrt{49i}(ax-ia)\log\left(-\frac{\sqrt{49i}(ax+ia)-7(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7x+7i}\right)+3\sqrt{-49i}(ax}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(49*I)*(a*x - I*a)*log((sqrt(49*I)*(a*x + I*a) + 7*(I*a*x + a)^
(1/4)*(-I*a*x + a)^(3/4))/(7*x + 7*I)) - 3*sqrt(49*I)*(a*x - I*a)*log(-(sqr
t(49*I)*(a*x + I*a) - 7*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(7*x + 7*I))
+ 3*sqrt(-49*I)*(a*x - I*a)*log((sqrt(-49*I)*(a*x + I*a) + 7*(I*a*x + a)^(1
/4)*(-I*a*x + a)^(3/4))/(7*x + 7*I)) - 3*sqrt(-49*I)*(a*x - I*a)*log(-(sqrt
(-49*I)*(a*x + I*a) - 7*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(7*x + 7*I))
+ 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(-3*I*x - 11))/(a*x - I*a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1195 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=266

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}} - \frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}} + 1\right)}{\sqrt{2}a} + \frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}} + \frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}} + 1\right)}{\sqrt{2}a} + \frac{i\sqrt{2}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - i\sqrt{2}\tan$$

```
[Out] (((4*I)/3)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(3/4)) + (I*Sqrt[2]*ArcTan[1 -
 (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Sqrt[2]*ArcTan[1 +
(Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Log[1 + Sqrt[a - I*a
*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt
[2]*a) + (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^
(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rubi [A] time = 0.139989, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {47, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}} - \frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}} + 1\right)}{\sqrt{2}a} + \frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}} + \frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}} + 1\right)}{\sqrt{2}a} + \frac{i\sqrt{2}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - i\sqrt{2}\tan$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(3/4)) + (I*Sqrt[2]*ArcTan[1 -
 (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Sqrt[2]*ArcTan[1 +
(Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Log[1 + Sqrt[a - I*a
*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt
[2]*a) + (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^
(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx = \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}}\,dx$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{(4i)\,\mathrm{Subst}\left(\int \frac{x^2}{(2a-x^4)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{(4i)\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} + \frac{(2i)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{(2i)\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{i\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\,\mathrm{Subs}}{}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} - \frac{\left(i\sqrt{2}\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} + \frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{a}}\right.}{\sqrt{2}a}$$

Mathematica [C] time = 0.0223752, size = 70, normalized size = 0.26

$$\frac{i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{7/4}\,{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/7)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[7/4, 7/4
, 11/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (a-iax)^{\frac{3}{4}}(a+iax)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(7/4), x)
```

Fricas [A] time = 1.67501, size = 763, normalized size = 2.87

$$-\frac{3\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x+i\,a^2\right)\sqrt{\frac{4i}{a^2}}+2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{2\,x+2i}\right)-3\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x+i\,a^2\right)\sqrt{\frac{4i}{a^2}}-2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{2\,x+2i}\right)+3\left(a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(a^2*x - I*a^2)*sqrt(4*I/a^2)*log(((a^2*x + I*a^2)*sqrt(4*I/a^2) +
2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x + 2*I)) - 3*(a^2*x - I*a^2)*sq
rt(4*I/a^2)*log(-((a^2*x + I*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a
*x + a)^(3/4))/(2*x + 2*I)) + 3*(a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(((a^2*x
+ I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x + 2*
I)) - 3*(a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(-((a^2*x + I*a^2)*sqrt(-4*I/a^2)
 - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x + 2*I)) - 8*(I*a*x + a)^(1/
4)*(-I*a*x + a)^(3/4))/(a^2*x - I*a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a\,(ix-1))^{\frac{3}{4}}}{(a\,(ix+1))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral((-a*(I*x - 1))**(3/4)/(a*(I*x + 1))**(7/4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1196 $\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=33

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

```
[Out] (((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

Rubi [A] time = 0.0034473, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {37}

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]

[Out] (((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx = \frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Mathematica [A] time = 0.0106085, size = 33, normalized size = 1.

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]

[Out] (((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

Maple [A] time = 0.03, size = 31, normalized size = 0.9

$$\frac{2x+2i}{3a}(a(1+ix))^{-\frac{3}{4}}\frac{1}{\sqrt[4]{-a(-1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] 2/3/a/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(x+I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [A] time = 1.55673, size = 78, normalized size = 2.36

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{3\left(a^3x-i\,a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 2/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)/(a^3*x - I*a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\,(ix+1))^{\frac{7}{4}}\,\sqrt[4]{-a\,(ix-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral(1/((a*(I*x + 1))**(7/4)*(-a*(I*x - 1))**(1/4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1197 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}} dx$

Optimal. Leaf size=65

$$\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}-\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{3/4}}$$

```
[Out] (-2*I)/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((4*I)/3)*(a - I*a*x)^(
3/4))/(a^3*(a + I*a*x)^(3/4))
```

Rubi [A] time = 0.0090441, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {45, 37}

$$\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}-\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (-2*I)/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((4*I)/3)*(a - I*a*x)^(
3/4))/(a^3*(a + I*a*x)^(3/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx &= -\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{3/4}}+\frac{2\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx}{a}\\ &= -\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{3/4}}+\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.0162802, size = 38, normalized size = 0.58

$$\frac{4x-2i}{3a^2\sqrt[4]{a-iax}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (-2*I + 4*x)/(3*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))
```

Maple [A] time = 0.039, size = 33, normalized size = 0.5

$$\frac{-2\,i+4\,x}{3\,a^{2}}\,(a\,(1+ix))^{-\frac{3}{4}}\,\frac{1}{\sqrt[4]{-a\,(-1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] 2/3/a^2/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(-I+2*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(5/4)), x)
```

Fricas [A] time = 1.56689, size = 95, normalized size = 1.46

$$\frac{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(4\,x-2i)}{3\left(a^{4}x^{2}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(4*x - 2*I)/(a^4*x^2 + a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1198 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=100

$$\frac{16i(a-iax)^{3/4}}{15a^4(a+iax)^{3/4}}-\frac{8i}{5a^3(a+iax)^{3/4}\sqrt[4]{a-iax}}-\frac{2i}{5a^2(a+iax)^{3/4}(a-iax)^{5/4}}$$

```
[Out] ((-2*I)/5)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)) - ((8*I)/5)/(a^3*(a -
I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((16*I)/15)*(a - I*a*x)^(3/4))/(a^4*(a +
 I*a*x)^(3/4))
```

Rubi [A] time = 0.0173897, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {45, 37}

$$\frac{16i(a-iax)^{3/4}}{15a^4(a+iax)^{3/4}}-\frac{8i}{5a^3(a+iax)^{3/4}\sqrt[4]{a-iax}}-\frac{2i}{5a^2(a+iax)^{3/4}(a-iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-2*I)/5)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)) - ((8*I)/5)/(a^3*(a -
I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((16*I)/15)*(a - I*a*x)^(3/4))/(a^4*(a +
 I*a*x)^(3/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx &= -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}}+\frac{4\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx}{5a}\\
&= -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}}-\frac{8i}{5a^3\sqrt[4]{a-iax}(a+iax)^{3/4}}+\frac{8\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx}{5a^2}\\
&= -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}}-\frac{8i}{5a^3\sqrt[4]{a-iax}(a+iax)^{3/4}}+\frac{16i(a-iax)^{3/4}}{15a^4(a+iax)^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.0213935, size = 50, normalized size = 0.5

$$\frac{2\left(8x^2+4ix+7\right)}{15a^3(x+i)\sqrt[4]{a-iax}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (2*(7 + (4*I)*x + 8*x^2))/(15*a^3*(I + x)*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/
4))
```

Maple [A] time = 0.044, size = 44, normalized size = 0.4

$$\frac{16\,x^2+8\,ix+14}{15\,a^3\,(x+i)}\,(a\,(1+ix))^{-\frac{3}{4}}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] 2/15/a^3/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(8*x^2+4*I*x+7)/(x+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.57352, size = 149, normalized size = 1.49

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}\left(8\,x^2+4i\,x+7\right)}{15\,a^5x^3+15i\,a^5x^2+15\,a^5x+15i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(8*x^2 + 4*I*x + 7)/(15*a^5*x^3 + 15
*I*a^5*x^2 + 15*a^5*x + 15*I*a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1199 $\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=139

$$-\frac{10a^2\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}(x),2\right)}{(a+iax)^{3/4}(a-iax)^{3/4}}+\frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}+\frac{2i\sqrt[4]{a+iax}(a-iax)^{5/4}}{a}+10i\sqrt[4]{a+iax}\sqrt[4]{a-iax}$$

```
[Out] (((4*I)/3)*(a - I*a*x)^(9/4))/(a*(a + I*a*x)^(3/4)) + (10*I)*(a - I*a*x)^(1
/4)*(a + I*a*x)^(1/4) + ((2*I)*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))/a - (10
*a^2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a
*x)^(3/4))
```

Rubi [A] time = 0.033715, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {47, 50, 42, 233, 231}

$$-\frac{10a^2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{(a+iax)^{3/4}(a-iax)^{3/4}}+\frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}+\frac{2i\sqrt[4]{a+iax}(a-iax)^{5/4}}{a}+10i\sqrt[4]{a+iax}\sqrt[4]{a-iax}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(9/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(9/4))/(a*(a + I*a*x)^(3/4)) + (10*I)*(a - I*a*x)^(1
/4)*(a + I*a*x)^(1/4) + ((2*I)*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))/a - (10
*a^2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a
*x)^(3/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx &= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}-3\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx\\
&= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}+\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a}-(5a)\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx\\
&= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}+10i\sqrt[4]{a-iax}\sqrt[4]{a+iax}+\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a}-\left(5a^2\right)\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx\\
&= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}+10i\sqrt[4]{a-iax}\sqrt[4]{a+iax}+\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a}-\frac{\left(5a^2\left(a^2+a^2x^2\right)^{3/4}\right)\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}+10i\sqrt[4]{a-iax}\sqrt[4]{a+iax}+\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a}-\frac{\left(5a^2\left(1+x^2\right)^{3/4}\right)\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}+10i\sqrt[4]{a-iax}\sqrt[4]{a+iax}+\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a}-\frac{10a^2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0293677, size = 70, normalized size = 0.5

$$\frac{i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{13/4}\,{}_2F_1\left(\frac{7}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}-\frac{ix}{2}\right)}{13a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(9/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/13)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(13/4)*Hypergeometric2F1[7/4, 1
3/4, 17/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (a-iax)^{\frac{9}{4}}(a+iax)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-iax+a)^{\frac{9}{4}}}{(iax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(9/4)/(I*a*x + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{(3x-3i)\mathrm{integral}\left(-\frac{5(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{1}{4}}}{x^2+1},x\right)+2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{1}{4}}\left(x^2+11ix+20\right)}{3x-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] ((3*x - 3*I)*integral(-5*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(x^2 + 1), x)
 + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(x^2 + 11*I*x + 20))/(3*x - 3*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(9/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1200 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}} dx$

Optimal. Leaf size=113

$$-\frac{10a\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}(x),2\right)}{3(a+iax)^{3/4}(a-iax)^{3/4}}+\frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a+iax}\sqrt[4]{a-iax}}{3a}$$

```
[Out] (((4*I)/3)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(3/4)) + (((10*I)/3)*(a - I*a*
x)^(1/4)*(a + I*a*x)^(1/4))/a - (10*a*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2
, 2])/(3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rubi [A] time = 0.0242478, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {47, 50, 42, 233, 231}

$$-\frac{10a\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a+iax)^{3/4}(a-iax)^{3/4}}+\frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a+iax}\sqrt[4]{a-iax}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(7/4),x]
```


```
[Out] (((4*I)/3)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(3/4)) + (((10*I)/3)*(a - I*a*
x)^(1/4)*(a + I*a*x)^(1/4))/a - (10*a*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2
, 2])/(3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}\,dx &= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}-\frac{5}{3}\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx\\
&= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{3a}-\frac{1}{3}(5a)\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx\\
&= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{3a}-\frac{\left(5a\left(a^2+a^2x^2\right)^{3/4}\right)\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{3a}-\frac{\left(5a\left(1+x^2\right)^{3/4}\right)\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{3a}-\frac{10a\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0252346, size = 70, normalized size = 0.62

$$\frac{i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{9/4}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/9)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[7/4, 9/4
, 13/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int (a-iax)^{\frac{5}{4}}(a+iax)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{5}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,(ax-i\,a)\text{integral}\left(-\frac{5\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\,(ax^2+a)},x\right)-2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}(-3i\,x-7)}{3\,(ax-i\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a*x - I*a)*integral(-5/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a*x^
2 + a), x) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(-3*I*x - 7))/(a*x - I*
a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1201 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=79

$$\frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{2\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}(x),2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] (((4*I)/3)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(3/4)) - (2*(1 + x^2)^(3/4)*El
lipticF[ArcTan[x]/2, 2])/(3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rubi [A] time = 0.0139069, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {47, 42, 233, 231}

$$\frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(3/4)) - (2*(1 + x^2)^(3/4)*El
lipticF[ArcTan[x]/2, 2])/(3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx = \frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}} - \frac{1}{3}\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx$$

$$= \frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}} - \frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

$$= \frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}} - \frac{\left(1+x^2\right)^{3/4}\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

$$= \frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}} - \frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

Mathematica [C] time = 0.0212477, size = 70, normalized size = 0.89

$$\frac{i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/5)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[5/4, 7/4
, 9/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \sqrt[4]{a-iax}\,(a+iax)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(a^2x-i\,a^2\right)\text{integral}\left(-\frac{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\left(a^2x^2+a^2\right)},x\right)+4\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\left(a^2x-i\,a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2*x - I*a^2)*integral(-1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(
a^2*x^2 + a^2), x) + 4*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^2*x - I*a^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt[4]{-a\,(ix-1)}}{(a\,(ix+1))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral((-a*(I*x - 1))**(1/4)/(a*(I*x + 1))**(7/4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1202 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=82

$$\frac{2\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}(x),2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}$$

```
[Out] (((2*I)/3)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)*
EllipticF[ArcTan[x]/2, 2])/(3*a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rubi [A] time = 0.0154676, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {51, 42, 233, 231}

$$\frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (((2*I)/3)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)*
EllipticF[ArcTan[x]/2, 2])/(3*a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}}\,dx = \frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}} + \frac{\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx}{3a}$$
$$= \frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}} + \frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= \frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}} + \frac{\left(1+x^2\right)^{3/4}\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= \frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}} + \frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

Mathematica [C] time = 0.0198246, size = 68, normalized size = 0.83

$$\frac{i\sqrt[4]{2}(1+ix)^{3/4}\sqrt[4]{a-iax}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (I*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 7/4, 5/
4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int (a-iax)^{-\frac{3}{4}}(a+iax)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(a^3x - i\,a^3\right)\mathrm{integral}\left(\frac{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\left(a^3x^2+a^3\right)}, x\right) + 2\,(i\,ax + a)^{\frac{1}{4}}(-i\,ax + a)^{\frac{1}{4}}}{3\left(a^3x - i\,a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^3*x - I*a^3)*integral(1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a
^3*x^2 + a^3), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^3*x - I*a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\,(ix+1))^{\frac{7}{4}}\,(-a\,(ix-1))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral(1/((a*(I*x + 1))**(7/4)*(-a*(I*x - 1))**(3/4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1203 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=81

$$\frac{2\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}(x),2\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] (2*x)/(3*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)*Elli
pticF[ArcTan[x]/2, 2])/(3*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rubi [A] time = 0.0148509, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {42, 199, 233, 231}

$$\frac{2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (2*x)/(3*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)*Elli
pticF[ArcTan[x]/2, 2])/(3*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx = \frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{7/4}}dx}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

$$= \frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}dx}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

$$= \frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{\left(1+x^2\right)^{3/4}\int \frac{1}{\left(1+x^2\right)^{3/4}}dx}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

$$= \frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

Mathematica [C] time = 0.0219649, size = 70, normalized size = 0.86

$$-\frac{i\sqrt[4]{2}(1+ix)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4};\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-I/3)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-3/4, 7/4, 1/4, 1/2 - (I/
2)*x])/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int (a-iax)^{-\frac{7}{4}}(a+iax)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(7/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(a^4x^2+a^4\right)\text{integral}\left(\frac{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\left(a^4x^2+a^4\right)},x\right)+2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}x}{3\left(a^4x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^4*x^2 + a^4)*integral(1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a
^4*x^2 + a^4), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*x)/(a^4*x^2 + a^
4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1204 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=114

$$\frac{10\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}(x),2\right)}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}$$

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)) + (10*x)/(21*a^3*(a -
I*a*x)^(3/4)*(a + I*a*x)^(3/4)) + (10*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2
, 2])/(21*a^3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rubi [A] time = 0.0246717, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 42, 199, 233, 231}

$$\frac{10\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)) + (10*x)/(21*a^3*(a -
I*a*x)^(3/4)*(a + I*a*x)^(3/4)) + (10*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2
, 2])/(21*a^3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx = -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{5\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx}{7a}$$
$$= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{\left(5\left(a^2+a^2x^2\right)^{3/4}\right)\int \frac{1}{\left(a^2+a^2x^2\right)^{7/4}}\,dx}{7a(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{\left(5\left(a^2+a^2x^2\right)^{3/4}\right)\int \frac{}{(a^2+}}{21a^3(a-iax)^{3/4}(a+i}$$
$$= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{\left(5\left(1+x^2\right)^{3/4}\right)\int \frac{1}{\left(1+x^2\right)^{3/4}}}{21a^3(a-iax)^{3/4}(a+iax)}$$
$$= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{10\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\right.}{21a^3(a-iax)^{3/4}(a+ia}$$

Mathematica [C] time = 0.0239229, size = 70, normalized size = 0.61

$$-\frac{i\sqrt[4]{2}(1+ix)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{7}{4};-\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-I/7)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-7/4, 7/4, -3/4, 1/2 - (I
/2)*x])/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int (a-iax)^{-\frac{11}{4}}(a+iax)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(21\,a^5x^3+21i\,a^5x^2+21\,a^5x+21i\,a^5\right)\mathrm{integral}\left(\frac{5\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{21\left(a^5x^2+a^5\right)},x\right)+2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}\left(5\,x^2+5i\,x+3\right)}{21\,a^5x^3+21i\,a^5x^2+21\,a^5x+21i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] ((21*a^5*x^3 + 21*I*a^5*x^2 + 21*a^5*x + 21*I*a^5)*integral(5/21*(I*a*x + a
)^(1/4)*(-I*a*x + a)^(1/4)/(a^5*x^2 + a^5), x) + 2*(I*a*x + a)^(1/4)*(-I*a*
x + a)^(1/4)*(5*x^2 + 5*I*x + 3))/(21*a^5*x^3 + 21*I*a^5*x^2 + 21*a^5*x + 2
1*I*a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1205 $\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=147

$$\frac{10\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}(x),2\right)}{33a^4(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{10x}{33a^4(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}}-\frac{2i}{11a^2(a-iax)^{11/}}$$

```
[Out] ((-2*I)/11)/(a^2*(a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)) - ((2*I)/11)/(a^3*(a
 - I*a*x)^(7/4)*(a + I*a*x)^(3/4)) + (10*x)/(33*a^4*(a - I*a*x)^(3/4)*(a +
I*a*x)^(3/4)) + (10*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/(33*a^4*(a -
 I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rubi [A] time = 0.0352481, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 42, 199, 233, 231}

$$\frac{10\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{33a^4(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{10x}{33a^4(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}}-\frac{2i}{11a^2(a-iax)^{11/4}(a+ia}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(7/4)),x]
```


```
[Out] ((-2*I)/11)/(a^2*(a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)) - ((2*I)/11)/(a^3*(a
 - I*a*x)^(7/4)*(a + I*a*x)^(3/4)) + (10*x)/(33*a^4*(a - I*a*x)^(3/4)*(a +
I*a*x)^(3/4)) + (10*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/(33*a^4*(a -
 I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}}\,dx = -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} + \frac{7\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx}{11a}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{5\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx}{11a^2}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{\left(5\left(a^2+a^2x^2\right)^{3/4}\right)\int \frac{1}{(a^2+a^2}}{11a^2(a-iax)^{3/4}(a+ia}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{10x}{33a^4(a-iax)^{3/4}(a+iax)^{3/}}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{10x}{33a^4(a-iax)^{3/4}(a+iax)^{3/}}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{10x}{33a^4(a-iax)^{3/4}(a+iax)^{3/}}$$

Mathematica [C] time = 0.0282298, size = 70, normalized size = 0.48

$$-\frac{i\sqrt[4]{2}(1+ix)^{3/4}\,{}_2F_1\left(-\frac{11}{4},\frac{7}{4};-\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-I/11)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-11/4, 7/4, -7/4, 1/2 -
(I/2)*x])/(a^2*(a - I*a*x)^(11/4)*(a + I*a*x)^(3/4))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int (a-iax)^{-\frac{15}{4}}(a+iax)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(33\,a^6x^4+66i\,a^6x^3+66i\,a^6x-33\,a^6\right)\mathrm{integral}\left(\frac{5\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}{33\,\left(a^6x^2+a^6\right)},x\right)+\left(10\,x^3+20i\,x^2-4\,x+12i\right)(i\,ax+a)^{\frac{1}{4}}(-i\,ax+}{33\,a^6x^4+66i\,a^6x^3+66i\,a^6x-33\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] ((33*a^6*x^4 + 66*I*a^6*x^3 + 66*I*a^6*x - 33*a^6)*integral(5/33*(I*a*x + a
)^(1/4)*(-I*a*x + a)^(1/4)/(a^6*x^2 + a^6), x) + (10*x^3 + 20*I*x^2 - 4*x +
 12*I)*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(33*a^6*x^4 + 66*I*a^6*x^3 + 6
6*I*a^6*x - 33*a^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(15/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1206 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=137

$$\frac{14a\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a+iax}\sqrt[4]{a-iax}}+\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}+\frac{14i(a+iax)^{3/4}(a-iax)^{3/4}}{3a}-\frac{14ax}{\sqrt[4]{a+iax}\sqrt[4]{a-iax}}$$

```
[Out] (-14*a*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + ((4*I)*(a - I*a*x)^(7/4))
/(a*(a + I*a*x)^(1/4)) + (((14*I)/3)*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))/a
 + (14*a*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/((a - I*a*x)^(1/4)*(a +
 I*a*x)^(1/4))
```

Rubi [A] time = 0.0320576, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {47, 50, 42, 229, 227, 196}

$$\frac{14a\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a+iax}\sqrt[4]{a-iax}}+\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}+\frac{14i(a+iax)^{3/4}(a-iax)^{3/4}}{3a}-\frac{14ax}{\sqrt[4]{a+iax}\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] (-14*a*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + ((4*I)*(a - I*a*x)^(7/4))
/(a*(a + I*a*x)^(1/4)) + (((14*I)/3)*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))/a
 + (14*a*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/((a - I*a*x)^(1/4)*(a +
 I*a*x)^(1/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx &= \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}-7\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx\\
&= \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}+\frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}-(7a)\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx\\
&= \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}+\frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}-\frac{\left(7a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}+\frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}-\frac{\left(7a\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= -\frac{14ax}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}+\frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}+\frac{\left(7a\sqrt[4]{1+x^2}\right)\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= -\frac{14ax}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}+\frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}+\frac{14a\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}
\end{aligned}
$$

Mathematica [C] time = 0.0283203, size = 70, normalized size = 0.51

$$
\frac{i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{11/4}\,{}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11a^2\sqrt[4]{a+iax}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((I/11)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[5/4, 1
1/4, 15/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.051, size = 96, normalized size = 0.7

$$
\frac{2i}{3}\left(x^2+13-12ix\right)a\frac{1}{\sqrt[4]{-a(-1+ix)}}\frac{1}{\sqrt[4]{a(1+ix)}}-7\frac{x_2F_1(1/4,1/2;\,3/2;\,-x^2)a\sqrt[4]{-a^2(-1+ix)(1+ix)}}{\sqrt[4]{a^2}\sqrt[4]{-a(-1+ix)}\sqrt[4]{a(1+ix)}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] 2/3*I*(x^2+13-12*I*x)*a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-7/(a^2)^(1/4)
*x*hypergeom([1/4,1/2],[3/2],-x^2)*a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+
I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{7}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}\left(2i\,x^2-16\,x+42i\right)+3\left(ax^2-i\,ax\right)\mathrm{integral}\left(-\frac{14\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}{ax^4+ax^2},x\right)}{3\left(ax^2-i\,ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/3*((I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(2*I*x^2 - 16*x + 42*I) + 3*(a*x^
2 - I*a*x)*integral(-14*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a*x^4 + a*x^2
), x))/(a*x^2 - I*a*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.1207 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=102

$$\frac{6\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{6x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}$$

```
[Out] (-6*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + ((4*I)*(a - I*a*x)^(3/4))/(a
*(a + I*a*x)^(1/4)) + (6*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/((a - I
*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0199544, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {47, 42, 229, 227, 196}

$$\frac{6\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{6x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(5/4),x]
```


```
[Out] (-6*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + ((4*I)*(a - I*a*x)^(3/4))/(a
*(a + I*a*x)^(1/4)) + (6*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/((a - I
*a*x)^(1/4)*(a + I*a*x)^(1/4))
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx &= \frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}} - 3\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx \\
&= \frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}} - \frac{\left(3\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} \\
&= \frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}} - \frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} \\
&= -\frac{6x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} + \frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}} + \frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} \\
&= -\frac{6x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} + \frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}} + \frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}
\end{aligned}$$

Mathematica [C] time = 0.0220003, size = 70, normalized size = 0.69

$$\frac{i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{7/4}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((I/7)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[5/4, 7/4
, 11/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.042, size = 88, normalized size = 0.9

$$4\,\frac{x+i}{\sqrt[4]{-a\,(-1+ix)}\sqrt[4]{a\,(1+ix)}} - 3\,\frac{x{}_2F_1(1/4,1/2;\,3/2;\,-x^2)\sqrt[4]{-a^2\,(-1+ix)\,(1+ix)}}{\sqrt[4]{a^2}\sqrt[4]{-a\,(-1+ix)}\sqrt[4]{a\,(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] 4*(x+I)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-3/(a^2)^(1/4)*x*hypergeom([1/
4,1/2],[3/2],-x^2)*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+
I*x))^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}(2\,x-6i)-\left(a^2x^2-i\,a^2x\right)\mathrm{integral}\left(-\frac{6\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}{a^2x^4+a^2x^2},x\right)}{a^2x^2-i\,a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] -((I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(2*x - 6*I) - (a^2*x^2 - I*a^2*x)*in
tegral(-6*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^2*x^4 + a^2*x^2), x))/(a^
2*x^2 - I*a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a\,(ix-1))^{\frac{3}{4}}}{(a\,(ix+1))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral((-a*(I*x - 1))**(3/4)/(a*(I*x + 1))**(5/4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1208 $\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=78

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

```
[Out] (2*I)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Elliptic
E[ArcTan[x]/2, 2])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0158888, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {48, 42, 197, 196}

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] (2*I)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Elliptic
E[ArcTan[x]/2, 2])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 48

```
Int[1/(((a_) + (b_.)*(x_))^(5/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-2/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4)), x] + Dist[c, Int[1/((a + b*x)^(
5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0
] && NegQ[a^2*b^2]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{5/4}}dx = \frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}} + a\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}dx$$
$$= \frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}} + \frac{\left(a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$
$$= \frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}} + \frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}dx}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$
$$= \frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}} + \frac{2\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Mathematica [C] time = 0.0218235, size = 70, normalized size = 0.9

$$\frac{i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((I/3)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[3/4, 5/4
, 7/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.033, size = 94, normalized size = 1.2

$$2\frac{x+i}{a\sqrt[4]{-a(-1+ix)}\sqrt[4]{a(1+ix)}} - \frac{x}{a}{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^2)\sqrt[4]{-a^2(-1+ix)(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a(-1+ix)}}\frac{1}{\sqrt[4]{a(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] 2*(x+I)/a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/(a^2)^(1/4)*x*hypergeom([
1/4,1/2],[3/2],-x^2)/a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a
*(1+I*x))^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(a^3x^2 - i\,a^3x\right)\mathrm{integral}\left(-\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}{a^3x^4+a^3x^2}, x\right) + 2i\,(i\,ax + a)^{\frac{3}{4}}(-i\,ax + a)^{\frac{3}{4}}}{a^3x^2 - i\,a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] ((a^3*x^2 - I*a^3*x)*integral(-2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^3*
x^4 + a^3*x^2), x) + 2*I*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4))/(a^3*x^2 - I
*a^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\,(ix+1))^{\frac{5}{4}}\,\sqrt[4]{-a\,(ix-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((a*(I*x + 1))**(5/4)*(-a*(I*x - 1))**(1/4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1209 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

```
[Out] (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(a^2*(a - I*a*x)^(1/4)*(a + I
*a*x)^(1/4))
```

Rubi [A] time = 0.0085974, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {42, 197, 196}

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(a^2*(a - I*a*x)^(1/4)*(a + I
*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx = \frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{\left(a^2+a^2x^2\right)^{5/4}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$
$$= \frac{\sqrt[4]{1+x^2}\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx}{a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$
$$= \frac{2\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Mathematica [C] time = 0.0229696, size = 68, normalized size = 1.48

$$-\frac{i2^{3/4}\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, 1/2 - (I/2)
*x])/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.038, size = 91, normalized size = 2.

$$2\,\frac{x}{a^2\sqrt[4]{-a\left(-1+ix\right)}\sqrt[4]{a\left(1+ix\right)}}-\frac{x}{a^2}{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^2\right)\sqrt[4]{-a^2\left(-1+ix\right)\left(1+ix\right)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a\left(-1+ix\right)}}\frac{1}{\sqrt[4]{a\left(1+ix\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] 2*x/a^2/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/(a^2)^(1/4)*x*hypergeom([1/
4,1/2],[3/2],-x^2)/a^2*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a
*(1+I*x))^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(5/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}x+\left(a^4x^2+a^4\right)\text{integral}\left(-\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}{a^4x^2+a^4},x\right)}{a^4x^2+a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] (2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*x + (a^4*x^2 + a^4)*integral(-(I*a*
x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^4*x^2 + a^4), x))/(a^4*x^2 + a^4)
```

Sympy [A] time = 54.0124, size = 97, normalized size = 2.11

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{8},\frac{9}{8},1 & \frac{1}{2},\frac{5}{4},\frac{7}{4}\\ \frac{5}{8},\frac{3}{4},\frac{9}{8},\frac{5}{4},\frac{7}{4} & 0\end{matrix}\middle|\frac{e^{-3i\pi}}{x^2}\right)e^{-\frac{3i\pi}{4}}}{4\pi a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{8},\frac{1}{2},\frac{5}{8},1 & \\ \frac{1}{8},\frac{5}{8} & -\frac{1}{2},0,\frac{3}{4},0\end{matrix}\middle|\frac{e^{-i\pi}}{x^2}\right)}{4\pi a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] -I*meijerg(((5/8, 9/8, 1), (1/2, 5/4, 7/4)), ((5/8, 3/4, 9/8, 5/4, 7/4), (0
,)), exp_polar(-3*I*pi)/x**2)*exp(-3*I*pi/4)/(4*pi*a**(5/2)*gamma(5/4)) + I
*meijerg(((-1/2, 0, 1/8, 1/2, 5/8, 1), ()), ((1/8, 5/8), (-1/2, 0, 3/4, 0))
, exp_polar(-I*pi)/x**2)/(4*pi*a**(5/2)*gamma(5/4))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1210 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=82

$$\frac{6\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

```
[Out] ((-2*I)/5)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (6*(1 + x^2)^(1/4)*E
llipticE[ArcTan[x]/2, 2])/(5*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0160064, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {51, 42, 197, 196}

$$\frac{6\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/5)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (6*(1 + x^2)^(1/4)*E
llipticE[ArcTan[x]/2, 2])/(5*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx = -\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{3\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx}{5a}$$
$$= -\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{\left(3\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\left(a^2+a^2x^2\right)^{5/4}}\,dx}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$
$$= -\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$
$$= -\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{6\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Mathematica [C] time = 0.024226, size = 70, normalized size = 0.85

$$-\frac{i2^{3/4}\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{4};-\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-I/5)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, 1/2 - (I
/2)*x])/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.059, size = 107, normalized size = 1.3

$$\frac{6\,ix+6\,x^2+2}{(5\,x+5\,i)\,a^3}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}\frac{1}{\sqrt[4]{a\,(1+ix)}}-\frac{3\,x}{5\,a^3}{}_2F_1(\frac{1}{4},\frac{1}{2};\,\frac{3}{2};\,-x^2)\sqrt[4]{-a^2\,(-1+ix)\,(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}\frac{1}{\sqrt[4]{a\,(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] 2/5*(3*I*x+3*x^2+1)/(x+I)/a^3/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-3/5/(a^
2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^3*(-a^2*(-1+I*x)*(1+I*x))^(1/4
)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}\left(3\,x^2+3i\,x+1\right)+\left(5\,a^5x^3+5i\,a^5x^2+5\,a^5x+5i\,a^5\right)\mathrm{integral}\left(-\frac{3\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}{5\left(a^5x^2+a^5\right)},x\right)}{5\,a^5x^3+5i\,a^5x^2+5\,a^5x+5i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] (2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 + 3*I*x + 1) + (5*a^5*x^3 +
5*I*a^5*x^2 + 5*a^5*x + 5*I*a^5)*integral(-3/5*(I*a*x + a)^(3/4)*(-I*a*x +
a)^(3/4)/(a^5*x^2 + a^5), x))/(5*a^5*x^3 + 5*I*a^5*x^2 + 5*a^5*x + 5*I*a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1211 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=115

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

```
[Out] ((-2*I)/9)/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)) - ((2*I)/9)/(a^3*(a -
I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2,
 2])/(3*a^4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0264505, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {51, 42, 197, 196}

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/9)/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)) - ((2*I)/9)/(a^3*(a -
I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2,
 2])/(3*a^4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx &= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}+\frac{5\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx}{9a}\\ &= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx}{3a^2}\\ &= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{\left(a^2+a^2x^2\right)^{5/4}}\,dx}{3a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\ &= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\sqrt[4]{1+x^2}\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx}{3a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\ &= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] time = 0.0259067, size = 70, normalized size = 0.61

$$-\frac{i2^{3/4}\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{9}{4},\frac{5}{4};-\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-I/9)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-9/4, 5/4, -5/4, 1/2 - (I
/2)*x])/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.066, size = 113, normalized size = 1.

$$\frac{12\,ix^2+6\,x^3-4\,x+4\,i}{9\,(x+i)^2\,a^4}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}\frac{1}{\sqrt[4]{a\,(1+ix)}}-\frac{x}{3\,a^4}{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^2)\sqrt[4]{-a^2\,(-1+ix)\,(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] 2/9*(6*I*x^2+3*x^3-2*x+2*I)/(x+I)^2/a^4/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/
4)-1/3/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^4*(-a^2*(-1+I*x)*(1+
I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(6\,x^3+12i\,x^2-4\,x+4i\right)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}+\left(9\,a^6x^4+18i\,a^6x^3+18i\,a^6x-9\,a^6\right)\mathrm{integral}\left(-\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}{3\left(a^6x^2+a^6\right)},x\right)}{9\,a^6x^4+18i\,a^6x^3+18i\,a^6x-9\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] ((6*x^3 + 12*I*x^2 - 4*x + 4*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4) + (9*a
^6*x^4 + 18*I*a^6*x^3 + 18*I*a^6*x - 9*a^6)*integral(-1/3*(I*a*x + a)^(3/4)
*(-I*a*x + a)^(3/4)/(a^6*x^2 + a^6), x))/(9*a^6*x^4 + 18*I*a^6*x^3 + 18*I*a
^6*x - 9*a^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1212 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=287

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}+\frac{5i(a+iax)^{3/4}\sqrt[4]{a-iax}}{a}+\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{5i\tan^{-1}\left(\right.}{}$$

```
[Out] ((4*I)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(1/4)) + ((5*I)*(a - I*a*x)^(1/4)*
(a + I*a*x)^(3/4))/a + ((5*I)*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I
*a*x)^(1/4)])/Sqrt[2] - ((5*I)*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a +
I*a*x)^(1/4)])/Sqrt[2] + (((5*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] - (((5*I)/2)*Log
[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a
*x)^(1/4)])/Sqrt[2]
```

Rubi [A] time = 0.177951, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {47, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}+\frac{5i(a+iax)^{3/4}\sqrt[4]{a-iax}}{a}+\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{5i\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((4*I)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(1/4)) + ((5*I)*(a - I*a*x)^(1/4)*
(a + I*a*x)^(3/4))/a + ((5*I)*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I
*a*x)^(1/4)])/Sqrt[2] - ((5*I)*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a +
I*a*x)^(1/4)])/Sqrt[2] + (((5*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] - (((5*I)/2)*Log
[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a
*x)^(1/4)])/Sqrt[2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}dx = \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} - 5\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}dx$$
$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} - \frac{1}{2}(5a)\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}dx$$
$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} - 10i\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}dx, x, \sqrt[4]{a-iax}\right)$$
$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} - 10i\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$
$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} - 5i\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) - 5i\,\mathrm{Subst}\left(\int \frac{1+}{1+}\right.$$
$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} - \frac{5}{2}i\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) - \frac{5}{2}i\,\mathrm{Subst}$$
$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} + \frac{5i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}} - \frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}} - \frac{5i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}} + \frac{\sqrt{2}}{}\right.}{2\sqrt{2}}$$
$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} + \frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} - \frac{5i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} + \frac{5i}{}$$

Mathematica [C] time = 0.0270854, size = 70, normalized size = 0.24

$$\frac{i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{9/4}\,{}_2F_1\left(\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((I/9)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[5/4, 9/4
, 13/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (a-iax)^{\frac{5}{4}}(a+iax)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] int((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{5}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(5/4), x)
```

Fricas [A] time = 1.81101, size = 686, normalized size = 2.39

$$-\frac{\sqrt{25i}(ax-i\,a)\log\left(\frac{\sqrt{25i}(ax-i\,a)+5\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{5\,x-5i}\right)-\sqrt{25i}(ax-i\,a)\log\left(-\frac{\sqrt{25i}(ax-i\,a)-5\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{5\,x-5i}\right)+\sqrt{-25i}(ax-i\,a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(25*I)*(a*x - I*a)*log((sqrt(25*I)*(a*x - I*a) + 5*(I*a*x + a)^(3
/4)*(-I*a*x + a)^(1/4))/(5*x - 5*I)) - sqrt(25*I)*(a*x - I*a)*log(-(sqrt(25
*I)*(a*x - I*a) - 5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(5*x - 5*I)) + sq
rt(-25*I)*(a*x - I*a)*log((sqrt(-25*I)*(a*x - I*a) + 5*(I*a*x + a)^(3/4)*(-
I*a*x + a)^(1/4))/(5*x - 5*I)) - sqrt(-25*I)*(a*x - I*a)*log(-(sqrt(-25*I)*
(a*x - I*a) - 5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(5*x - 5*I)) + 2*(I*a
*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(-I*x - 9))/(a*x - I*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a\,(ix-1))^{\frac{5}{4}}}{(a\,(ix+1))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral((-a*(I*x - 1))**(5/4)/(a*(I*x + 1))**(5/4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1213 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=264

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\tan^{-1}\left(\right.}{}$$

```
[Out] ((4*I)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(1/4)) + (I*Sqrt[2]*ArcTan[1 - (Sq
rt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Sqrt[2]*ArcTan[1 + (Sqr
t[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a + (I*Log[1 + Sqrt[a - I*a*x]/
Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*
a) - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rubi [A] time = 0.138632, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {47, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```


```
[Out] ((4*I)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(1/4)) + (I*Sqrt[2]*ArcTan[1 - (Sq
rt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Sqrt[2]*ArcTan[1 + (Sqr
t[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a + (I*Log[1 + Sqrt[a - I*a*x]/
Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*
a) - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx = \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$
$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(4i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$
$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(4i)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$
$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(2i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{(2i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$
$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\right.}{}$$
$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} - \frac{\left(i\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-1\cdots}\right.}{}$$
$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-i\cdots}}{\sqrt[4]{a+ia\cdots}}\right.}{\sqrt{2}a}$$

Mathematica [C] time = 0.0220569, size = 70, normalized size = 0.27

$$\frac{i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((I/5)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[5/4, 5/4
, 9/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \sqrt[4]{a-iax}\,(a+iax)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] int((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(5/4), x)
```

Fricas [A] time = 1.9733, size = 752, normalized size = 2.85

$$-\frac{\left(a^2x - i\,a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}+2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,x-2i}\right) - \left(a^2x - i\,a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}-2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,x-2i}\right) + \left(a^2x - i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(4*I/a^2) + 2*
(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (a^2*x - I*a^2)*sqrt(4
*I/a^2)*log(-((a^2*x - I*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x +
 a)^(1/4))/(2*x - 2*I)) + (a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(((a^2*x - I*a^
2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) -
(a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(-((a^2*x - I*a^2)*sqrt(-4*I/a^2) - 2*(I*
a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - 8*(I*a*x + a)^(3/4)*(-I*a
*x + a)^(1/4))/(a^2*x - I*a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{-a\,(ix-1)}}{(a\,(ix+1))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral((-a*(I*x - 1))**(1/4)/(a*(I*x + 1))**(5/4), x)
```

Giac [A] time = 1.13795, size = 244, normalized size = 0.92

$$-\frac{2i\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2\,(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{1}{4}}}\right)\right) + 2i\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2\,(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{1}{4}}}\right)\right) + i\,\sqrt{2}\log\left(\frac{\sqrt{2}(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{1}{4}}}+\frac{\sqrt{-i\,ax+a}}{\sqrt{i\,ax+a}}+1\right.}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] -1/2*(2*I*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(-I*a*x + a)^(1/4)/(I*a*x
 + a)^(1/4))) + 2*I*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(-I*a*x + a)^(
1/4)/(I*a*x + a)^(1/4))) + I*sqrt(2)*log(sqrt(2)*(-I*a*x + a)^(1/4)/(I*a*x
```

```
+ a)^(1/4) + sqrt(-I*a*x + a)/sqrt(I*a*x + a) + 1) - I*sqrt(2)*log(-sqrt(2)
*(-I*a*x + a)^(1/4)/(I*a*x + a)^(1/4) + sqrt(-I*a*x + a)/sqrt(I*a*x + a) +
1) - 8*I*(-I*a*x + a)^(1/4)/(I*a*x + a)^(1/4))/a
```

3.1214 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=31

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

```
[Out] ((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.003278, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {37}

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx=\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Mathematica [A] time = 0.0107523, size = 31, normalized size = 1.

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))
```

Maple [A] time = 0.028, size = 31, normalized size = 1.

$$2\,\frac{x+i}{a\left(-a\left(-1+ix\right)\right)^{3/4}\sqrt[4]{a\left(1+ix\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] 2/a/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(x+I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax + a)^{\frac{5}{4}}(-i\,ax + a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [A] time = 1.77303, size = 76, normalized size = 2.45

$$\frac{2\,(i\,ax + a)^{\frac{3}{4}}(-i\,ax + a)^{\frac{1}{4}}}{a^3x - i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(a^3*x - I*a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\,(ix+1))^{\frac{5}{4}}\,(-a\,(ix-1))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((a*(I*x + 1))**(5/4)*(-a*(I*x - 1))**(3/4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1215 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=67

$$\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

```
[Out] ((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((4*I)/3)*(a - I*a*
x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0094655, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {45, 37}

$$\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((4*I)/3)*(a - I*a*
x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx &= -\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}+\frac{2\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{3a}\\ &= -\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}+\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [A] time = 0.0169206, size = 38, normalized size = 0.57

$$\frac{4x+2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] (2*I + 4*x)/(3*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4))
```

Maple [A] time = 0.033, size = 33, normalized size = 0.5

$$\frac{2\,i+4\,x}{3\,a^{2}}\,(-a\,(-1+ix))^{-\frac{3}{4}}\,\frac{1}{\sqrt[4]{a\,(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] 2/3/a^2/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(I+2*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(7/4)), x)
```

Fricas [A] time = 1.73902, size = 95, normalized size = 1.42

$$\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(4\,x+2i)}{3\left(a^{4}x^{2}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/3*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(4*x + 2*I)/(a^4*x^2 + a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\,(ix+1))^{\frac{5}{4}}\,(-a\,(ix-1))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((a*(I*x + 1))**(5/4)*(-a*(I*x - 1))**(7/4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1216 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=100

$$\frac{16i\sqrt[4]{a-iax}}{21a^4\sqrt[4]{a+iax}}-\frac{8i}{21a^3\sqrt[4]{a+iax}(a-iax)^{3/4}}-\frac{2i}{7a^2\sqrt[4]{a+iax}(a-iax)^{7/4}}$$

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)) - ((8*I)/21)/(a^3*(a -
 I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((16*I)/21)*(a - I*a*x)^(1/4))/(a^4*(a
+ I*a*x)^(1/4))
```

Rubi [A] time = 0.0181517, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {45, 37}

$$\frac{16i\sqrt[4]{a-iax}}{21a^4\sqrt[4]{a+iax}}-\frac{8i}{21a^3\sqrt[4]{a+iax}(a-iax)^{3/4}}-\frac{2i}{7a^2\sqrt[4]{a+iax}(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)) - ((8*I)/21)/(a^3*(a -
 I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((16*I)/21)*(a - I*a*x)^(1/4))/(a^4*(a
+ I*a*x)^(1/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx &= -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}}+\frac{4\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx}{7a}\\ &= -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}}-\frac{8i}{21a^3(a-iax)^{3/4}\sqrt[4]{a+iax}}+\frac{8\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{21a^2}\\ &= -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}}-\frac{8i}{21a^3(a-iax)^{3/4}\sqrt[4]{a+iax}}+\frac{16i\sqrt[4]{a-iax}}{21a^4\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [A] time = 0.020365, size = 50, normalized size = 0.5

$$\frac{16x^2+24ix-2}{21a^3(x+i)(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] (-2 + (24*I)*x + 16*x^2)/(21*a^3*(I + x)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4
))
```

Maple [A] time = 0.041, size = 44, normalized size = 0.4

$$\frac{16\,x^2+24\,ix-2}{21\,a^3\,(x+i)}(-a\,(-1+ix))^{-\frac{3}{4}}\frac{1}{\sqrt[4]{a\,(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] 2/21/a^3/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(8*x^2+12*I*x-1)/(x+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.91745, size = 150, normalized size = 1.5

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}\left(8\,x^2+12i\,x-1\right)}{21\,a^5x^3+21i\,a^5x^2+21\,a^5x+21i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 + 12*I*x - 1)/(21*a^5*x^3 + 2
1*I*a^5*x^2 + 21*a^5*x + 21*I*a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1217 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=141

$$-\frac{42\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5\sqrt[4]{a+iax}\sqrt[4]{a-iax}}+\frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}}+\frac{42x}{5\sqrt[4]{a+iax}\sqrt[4]{a-iax}}$$

```
[Out] (((4*I)/5)*(a - I*a*x)^(7/4))/(a*(a + I*a*x)^(5/4)) + (42*x)/(5*(a - I*a*x)
^(1/4)*(a + I*a*x)^(1/4)) - (((28*I)/5)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(
1/4)) - (42*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(5*(a - I*a*x)^(1/4)
*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0303233, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {47, 42, 229, 227, 196}

$$-\frac{42\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5\sqrt[4]{a+iax}\sqrt[4]{a-iax}}+\frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}}+\frac{42x}{5\sqrt[4]{a+iax}\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((4*I)/5)*(a - I*a*x)^(7/4))/(a*(a + I*a*x)^(5/4)) + (42*x)/(5*(a - I*a*x)
^(1/4)*(a + I*a*x)^(1/4)) - (((28*I)/5)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(
1/4)) - (42*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(5*(a - I*a*x)^(1/4)
*(a + I*a*x)^(1/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx &= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}} - \frac{7}{5}\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx \\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}} - \frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}} + \frac{21}{5}\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx \\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}} - \frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}} + \frac{\left(21\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}} \\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}} - \frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}} + \frac{\left(21\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}} \\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}} + \frac{42x}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}} - \frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}} - \frac{\left(21\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}} \\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}} + \frac{42x}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}} - \frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}} - \frac{42\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}
\end{aligned}
$$

Mathematica [C] time = 0.0378105, size = 70, normalized size = 0.5

$$
\frac{i\sqrt[4]{1+ix}(a-iax)^{11/4}\,{}_2F_1\left(\frac{9}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11\sqrt[4]{2}a^3\sqrt[4]{a+iax}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] ((I/11)*(1 + I*x)^(1/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[9/4, 11/4, 15/
4, 1/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.05, size = 101, normalized size = 0.7

$$
-\frac{32x^2+24+8ix}{5x-5i}\frac{1}{\sqrt[4]{-a(-1+ix)}}\frac{1}{\sqrt[4]{a(1+ix)}}+\frac{21x}{5}{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^2\right)\sqrt[4]{-a^2(-1+ix)(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a(-1+ix)}}\frac{1}{\sqrt[4]{a(}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] -8/5*(4*x^2+3+I*x)/(x-I)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)+21/5/(a^2)^(
1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-
```

```
1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-iax+a)^{\frac{7}{4}}}{(iax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{3}{4}}\left(5\,x^2-30i\,x-21\right)+5\left(a^2x^3-2i\,a^2x^2-a^2x\right)\mathrm{integral}\left(\frac{42\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{3}{4}}}{5\left(a^2x^4+a^2x^2\right)},x\right)}{5\left(a^2x^3-2i\,a^2x^2-a^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(5*x^2 - 30*I*x - 21) + 5*(a^2*
x^3 - 2*I*a^2*x^2 - a^2*x)*integral(42/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/
4)/(a^2*x^4 + a^2*x^2), x))/(a^2*x^3 - 2*I*a^2*x^2 - a^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1218 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=115

$$-\frac{6\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{6i}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}$$

```
[Out] (((4*I)/5)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(5/4)) - ((6*I)/5)/(a*(a - I*a
*x)^(1/4)*(a + I*a*x)^(1/4)) - (6*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2]
)/(5*a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0252828, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {47, 48, 42, 197, 196}

$$-\frac{6\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{6i}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(9/4),x]
```


```
[Out] (((4*I)/5)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(5/4)) - ((6*I)/5)/(a*(a - I*a
*x)^(1/4)*(a + I*a*x)^(1/4)) - (6*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2]
)/(5*a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 48

```
Int[1/(((a_) + (b_.)*(x_))^(5/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-2/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4)), x] + Dist[c, Int[1/((a + b*x)^(
5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0
] && NegQ[a^2*b^2]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx &= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}-\frac{3}{5}\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{5/4}}\,dx\\
&= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}-\frac{6i}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{1}{5}(3a)\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx\\
&= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}-\frac{6i}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{\left(3a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\left(a^2+a^2x^2\right)^{5/4}}\,dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}-\frac{6i}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}-\frac{6i}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{6\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}
\end{aligned}
$$

Mathematica [C] time = 0.0227565, size = 70, normalized size = 0.61

$$
\frac{i\sqrt[4]{1+ix}(a-iax)^{7/4}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7\sqrt[4]{2}a^3\sqrt[4]{a+iax}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] ((I/7)*(1 + I*x)^(1/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[7/4, 9/4, 11/4,
1/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.034, size = 107, normalized size = 0.9

$$
-\frac{6x^2+2+4ix}{(5x-5i)a}\frac{1}{\sqrt[4]{-a(-1+ix)}}\frac{1}{\sqrt[4]{a(1+ix)}}+\frac{3x}{5a}{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^2)\sqrt[4]{-a^2(-1+ix)(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a(-1+ix)}}\frac{1}{\sqrt[4]{a(1+ix)}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] -2/5*(3*x^2+1+2*I*x)/(x-I)/a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)+3/5/(a^2
)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(
-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-iax+a)^{\frac{3}{4}}}{(iax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{3}{4}}(5ix+3)-\left(5\,a^3x^3-10i\,a^3x^2-5\,a^3x\right)\mathrm{integral}\left(\frac{6\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{3}{4}}}{5\left(a^3x^4+a^3x^2\right)},x\right)}{5\,a^3x^3-10i\,a^3x^2-5\,a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] -(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(5*I*x + 3) - (5*a^3*x^3 - 10*I*a^
3*x^2 - 5*a^3*x)*integral(6/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^3*x^4
 + a^3*x^2), x))/(5*a^3*x^3 - 10*I*a^3*x^2 - 5*a^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1219 $\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=82

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}$$

```
[Out] ((4*I)/5)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)) + (2*(1 + x^2)^(1/4)*Elli
pticE[ArcTan[x]/2, 2])/(5*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0156894, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {46, 42, 197, 196}

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((4*I)/5)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)) + (2*(1 + x^2)^(1/4)*Elli
pticE[ArcTan[x]/2, 2])/(5*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 46

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{9/4}}\,dx &= \frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}+\frac{1}{5}\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx\\ &= \frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}+\frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{\left(a^2+a^2x^2\right)^{5/4}}\,dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\ &= \frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}+\frac{\sqrt[4]{1+x^2}\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\ &= \frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}+\frac{2\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] time = 0.02265, size = 70, normalized size = 0.85

$$\frac{i\sqrt[4]{1+ix}(a-iax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3\sqrt[4]{2}a^3\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((I/3)*(1 + I*x)^(1/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[3/4, 9/4, 7/4, 1
/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.035, size = 105, normalized size = 1.3

$$\frac{2x^2+4-2ix}{(5x-5i)a^2}\frac{1}{\sqrt[4]{-a(-1+ix)}}\frac{1}{\sqrt[4]{a(1+ix)}}-\frac{x}{5a^2}{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^2\right)\sqrt[4]{-a^2(-1+ix)(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a(-1+ix)}}\frac{1}{\sqrt[4]{a(1+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] 2/5*(x^2+2-I*x)/(x-I)/a^2/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/5/(a^2)^(
1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^2*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-
a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}(2\,x-4i)+\left(5\,a^4x^2-10i\,a^4x-5\,a^4\right)\mathrm{integral}\left(-\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}{5\left(a^4x^2+a^4\right)},x\right)}{5\,a^4x^2-10i\,a^4x-5\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] ((I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(2*x - 4*I) + (5*a^4*x^2 - 10*I*a^4*x
 - 5*a^4)*integral(-1/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^4*x^2 + a^4
), x))/(5*a^4*x^2 - 10*I*a^4*x - 5*a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1220 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=82

$$\frac{6\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2i}{5a^2\sqrt[4]{a-iax}(a+iax)^{5/4}}$$

```
[Out] ((2*I)/5)/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)) + (6*(1 + x^2)^(1/4)*El
lipticE[ArcTan[x]/2, 2])/(5*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0246731, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 46, 42, 197, 196}

$$\frac{6\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2i}{5a^2\sqrt[4]{a-iax}(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((2*I)/5)/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)) + (6*(1 + x^2)^(1/4)*El
lipticE[ArcTan[x]/2, 2])/(5*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 46

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx &= -\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{5/4}}+\frac{3\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{9/4}}\,dx}{a}\\ &= \frac{2i}{5a^2\sqrt[4]{a-iax}(a+iax)^{5/4}}+\frac{3\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx}{5a}\\ &= \frac{2i}{5a^2\sqrt[4]{a-iax}(a+iax)^{5/4}}+\frac{\left(3\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\left(a^2+a^2x^2\right)^{5/4}}\,dx}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\ &= \frac{2i}{5a^2\sqrt[4]{a-iax}(a+iax)^{5/4}}+\frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\ &= \frac{2i}{5a^2\sqrt[4]{a-iax}(a+iax)^{5/4}}+\frac{6\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] time = 0.0256838, size = 68, normalized size = 0.83

$$-\frac{i\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{1}{4},\frac{9}{4};\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{\sqrt[4]{2}a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-I)*(1 + I*x)^(1/4)*Hypergeometric2F1[-1/4, 9/4, 3/4, 1/2 - (I/2)*x])/(2^
(1/4)*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.056, size = 107, normalized size = 1.3

$$\frac{-6\,ix+6\,x^2+2}{(5\,x-5\,i)\,a^3}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}\frac{1}{\sqrt[4]{a\,(1+ix)}}-\frac{3\,x}{5\,a^3}{}_2F_1(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -x^2)\sqrt[4]{-a^2\,(-1+ix)\,(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}\frac{1}{\sqrt[4]{a\,(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] 2/5*(-3*I*x+3*x^2+1)/(x-I)/a^3/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-3/5/(a
^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^3*(-a^2*(-1+I*x)*(1+I*x))^(1/
4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}\left(3\,x^2-3i\,x+1\right)+\left(5\,a^5x^3-5i\,a^5x^2+5\,a^5x-5i\,a^5\right)\mathrm{integral}\left(-\frac{3\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}{5\left(a^5x^2+a^5\right)},x\right)}{5\,a^5x^3-5i\,a^5x^2+5\,a^5x-5i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] (2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 - 3*I*x + 1) + (5*a^5*x^3 -
5*I*a^5*x^2 + 5*a^5*x - 5*I*a^5)*integral(-3/5*(I*a*x + a)^(3/4)*(-I*a*x +
a)^(3/4)/(a^5*x^2 + a^5), x))/(5*a^5*x^3 - 5*I*a^5*x^2 + 5*a^5*x - 5*I*a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1221 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=88

$$\frac{2x}{5a^4\left(x^2+1\right)\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{6\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

```
[Out] (2*x)/(5*a^4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)*(1 + x^2)) + (6*(1 + x^2)^
(1/4)*EllipticE[ArcTan[x]/2, 2])/(5*a^4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)
)
```

Rubi [A] time = 0.0158891, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {42, 199, 197, 196}

$$\frac{2x}{5a^4\left(x^2+1\right)\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{6\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*x)/(5*a^4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)*(1 + x^2)) + (6*(1 + x^2)^
(1/4)*EllipticE[ArcTan[x]/2, 2])/(5*a^4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)
)
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx = \frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{(a^2+a^2x^2)^{9/4}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$= \frac{2x}{5a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}\left(1+x^2\right)} + \frac{\left(3\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$= \frac{2x}{5a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}\left(1+x^2\right)} + \frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$= \frac{2x}{5a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}\left(1+x^2\right)} + \frac{6\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Mathematica [C] time = 0.0275565, size = 70, normalized size = 0.8

$$-\frac{i\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4};-\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5\sqrt[4]{2}a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-I/5)*(1 + I*x)^(1/4)*Hypergeometric2F1[-5/4, 9/4, -1/4, 1/2 - (I/2)*x])/
(2^(1/4)*a^3*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (a-iax)^{-\frac{9}{4}}(a+iax)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(9/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(3x^3+4x\right)(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{3}{4}}+5\left(a^6x^4+2a^6x^2+a^6\right)\mathrm{integral}\left(-\frac{3(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{3}{4}}}{5\left(a^6x^2+a^6\right)},x\right)}{5\left(a^6x^4+2a^6x^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/5*(2*(3*x^3 + 4*x)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4) + 5*(a^6*x^4 + 2*
a^6*x^2 + a^6)*integral(-3/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^6*x^2
+ a^6), x))/(a^6*x^4 + 2*a^6*x^2 + a^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1222 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=121

$$\frac{14x}{45a^5\left(x^2+1\right)\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{14\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{15a^5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}$$

```
[Out] ((-2*I)/9)/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)) + (14*x)/(45*a^5*(a -
I*a*x)^(1/4)*(a + I*a*x)^(1/4)*(1 + x^2)) + (14*(1 + x^2)^(1/4)*EllipticE[A
rcTan[x]/2, 2])/(15*a^5*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0257717, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 42, 199, 197, 196}

$$\frac{14x}{45a^5\left(x^2+1\right)\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{14\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{15a^5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/9)/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)) + (14*x)/(45*a^5*(a -
I*a*x)^(1/4)*(a + I*a*x)^(1/4)*(1 + x^2)) + (14*(1 + x^2)^(1/4)*EllipticE[A
rcTan[x]/2, 2])/(15*a^5*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
```

```
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx = -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{7\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx}{9a}$$

$$= -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{\left(7\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\left(a^2+a^2x^2\right)^{9/4}}\,dx}{9a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$= -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{14x}{45a^5\sqrt[4]{a-iax}\sqrt[4]{a+iax}\left(1+x^2\right)} + \frac{\left(7\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2 ...}}{15a^3\sqrt[4]{a-iax}\sqrt[4]{a+ ...}}$$

$$= -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{14x}{45a^5\sqrt[4]{a-iax}\sqrt[4]{a+iax}\left(1+x^2\right)} + \frac{\left(7\sqrt[4]{1+x^2}\right)\int \frac{1}{\left(1+x^2\right)^{5/4}} ...}{15a^5\sqrt[4]{a-iax}\sqrt[4]{a+iax ...}}$$

$$= -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{14x}{45a^5\sqrt[4]{a-iax}\sqrt[4]{a+iax}\left(1+x^2\right)} + \frac{14\sqrt[4]{1+x^2}E\left(\frac{1}{2}\tan^{-1}(x ...\right.}{15a^5\sqrt[4]{a-iax}\sqrt[4]{a+i ...}}$$

Mathematica [C] time = 0.0279646, size = 70, normalized size = 0.58

$$-\frac{i\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{9}{4},\frac{9}{4};-\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9\sqrt[4]{2}a^3(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-I/9)*(1 + I*x)^(1/4)*Hypergeometric2F1[-9/4, 9/4, -5/4, 1/2 - (I/2)*x])/
(2^(1/4)*a^3*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.077, size = 124, normalized size = 1.

$$\frac{42\,ix^3+42\,x^4+56\,ix+56\,x^2+10}{(45\,x-45\,i)\,(x+i)^2\,a^5}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}\frac{1}{\sqrt[4]{a\,(1+ix)}}-\frac{7\,x}{15\,a^5}{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^2\right)\sqrt[4]{-a^2\,(-1+ix)\,(1+ix)}\frac{1}{\sqrt[4]{a^2}}\frac{1}{\sqrt[4]{-a\,(...}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] 2/45*(21*I*x^3+21*x^4+28*I*x+28*x^2+5)/(x-I)/(x+I)^2/a^5/(-a*(-1+I*x))^(1/4
)/(a*(1+I*x))^(1/4)-7/15/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^5*
(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(42\,x^{4}+42i\,x^{3}+56\,x^{2}+56i\,x+10\right)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}+\left(45\,a^{7}x^{5}+45i\,a^{7}x^{4}+90\,a^{7}x^{3}+90i\,a^{7}x^{2}+45\,a^{7}x+45i\right.}{45\,a^{7}x^{5}+45i\,a^{7}x^{4}+90\,a^{7}x^{3}+90i\,a^{7}x^{2}+45\,a^{7}x+45i\,a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] ((42*x^4 + 42*I*x^3 + 56*x^2 + 56*I*x + 10)*(I*a*x + a)^(3/4)*(-I*a*x + a)^
(3/4) + (45*a^7*x^5 + 45*I*a^7*x^4 + 90*a^7*x^3 + 90*I*a^7*x^2 + 45*a^7*x +
 45*I*a^7)*integral(-7/15*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^7*x^2 + a
^7), x))/(45*a^7*x^5 + 45*I*a^7*x^4 + 90*a^7*x^3 + 90*I*a^7*x^2 + 45*a^7*x
+ 45*I*a^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1223 $\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=154

$$\frac{14x}{65a^6\left(x^2+1\right)\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{42\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{65a^6\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}}-\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}$$

```
[Out] ((-2*I)/13)/(a^2*(a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)) - ((2*I)/13)/(a^3*(a
 - I*a*x)^(9/4)*(a + I*a*x)^(5/4)) + (14*x)/(65*a^6*(a - I*a*x)^(1/4)*(a +
I*a*x)^(1/4)*(1 + x^2)) + (42*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(6
5*a^6*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0399608, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {51, 42, 199, 197, 196}

$$\frac{14x}{65a^6\left(x^2+1\right)\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{42\sqrt[4]{x^2+1}E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{65a^6\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}}-\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(9/4)),x]
```


```
[Out] ((-2*I)/13)/(a^2*(a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)) - ((2*I)/13)/(a^3*(a
 - I*a*x)^(9/4)*(a + I*a*x)^(5/4)) + (14*x)/(65*a^6*(a - I*a*x)^(1/4)*(a +
I*a*x)^(1/4)*(1 + x^2)) + (42*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(6
5*a^6*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx = -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}} + \frac{9\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx}{13a}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}} - \frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{7\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx}{13a^2}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}} - \frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{\left(7\sqrt[4]{a^2+a^2x^2}\right)\int \frac{}{(a^2+a^{}}}{13a^2\sqrt[4]{a-iax}\sqrt[4]{a+}}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}} - \frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{14x}{65a^6\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}} - \frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{14x}{65a^6\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}} - \frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{14x}{65a^6\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Mathematica [C] time = 0.0336205, size = 70, normalized size = 0.45

$$-\frac{i\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{13}{4},\frac{9}{4};-\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{13\sqrt[4]{2}a^3(a-iax)^{13/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-I/13)*(1 + I*x)^(1/4)*Hypergeometric2F1[-13/4, 9/4, -9/4, 1/2 - (I/2)*x]
)/(2^(1/4)*a^3*(a - I*a*x)^(13/4)*(a + I*a*x)^(1/4))
```

Maple [C] time = 0.086, size = 130, normalized size = 0.8

$$\frac{84\,ix^4+42\,x^5+112\,ix^2-46\,x+14\,x^3+20\,i}{(65\,x-65\,i)\,(x+i)^3\,a^6}\frac{1}{\sqrt[4]{-a\,(-1+ix)}}\frac{1}{\sqrt[4]{a\,(1+ix)}}-\frac{21\,x}{65\,a^6}{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^2)\sqrt[4]{-a^2\,(-1+ix)\,(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] 2/65*(42*I*x^4+21*x^5+56*I*x^2-23*x+7*x^3+10*I)/(x-I)/(x+I)^3/a^6/(-a*(-1+I
*x))^(1/4)/(a*(1+I*x))^(1/4)-21/65/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],
-x^2)/a^6*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/
4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(42\,x^5+84i\,x^4+14\,x^3+112i\,x^2-46\,x+20i\right)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}+\left(65\,a^8x^6+130i\,a^8x^5+65\,a^8x^4+260i\,a^8x^3-65\,a\right.}{65\,a^8x^6+130i\,a^8x^5+65\,a^8x^4+260i\,a^8x^3-65\,a^8x^2+130i\,a^8x-65}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] ((42*x^5 + 84*I*x^4 + 14*x^3 + 112*I*x^2 - 46*x + 20*I)*(I*a*x + a)^(3/4)*(
-I*a*x + a)^(3/4) + (65*a^8*x^6 + 130*I*a^8*x^5 + 65*a^8*x^4 + 260*I*a^8*x^
3 - 65*a^8*x^2 + 130*I*a^8*x - 65*a^8)*integral(-21/65*(I*a*x + a)^(3/4)*(-
I*a*x + a)^(3/4)/(a^8*x^2 + a^8), x))/(65*a^8*x^6 + 130*I*a^8*x^5 + 65*a^8*
x^4 + 260*I*a^8*x^3 - 65*a^8*x^2 + 130*I*a^8*x - 65*a^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(17/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1224 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=297

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

```
[Out] ((((4*I)/5)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(5/4)) - ((4*I)*(a - I*a*x)^(1
/4))/(a*(a + I*a*x)^(1/4)) - (I*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/
4))/(a + I*a*x)^(1/4)])/a + (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/a - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sq
rt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*Log[1 + Sqrt[
a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]
)/(Sqrt[2]*a)
```

Rubi [A] time = 0.140118, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {47, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] ((((4*I)/5)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(5/4)) - ((4*I)*(a - I*a*x)^(1
/4))/(a*(a + I*a*x)^(1/4)) - (I*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/
4))/(a + I*a*x)^(1/4)])/a + (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/a - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sq
rt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*Log[1 + Sqrt[
a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]
)/(Sqrt[2]*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx &= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(4i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(4i)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(2i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} + \ldots \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\log\left(1+\ldots\right.}{\ldots}
\end{aligned}
$$

Mathematica [C] time = 0.0240654, size = 70, normalized size = 0.24

$$
\frac{i\sqrt[4]{1+ix}(a-iax)^{9/4}\,{}_2F_1\left(\frac{9}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9\sqrt[4]{2}a^3\sqrt[4]{a+iax}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] ((I/9)*(1 + I*x)^(1/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[9/4, 9/4, 13/4,
1/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$
\int (a-iax)^{\frac{5}{4}}(a+iax)^{-\frac{9}{4}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] int((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{5}{4}}}{(i\,ax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(9/4), x)
```

Fricas [A] time = 2.19028, size = 879, normalized size = 2.96

$$\frac{\left(5\,a^2x^2-10i\,a^2x-5\,a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}+2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,x-2i}\right)-\left(5\,a^2x^2-10i\,a^2x-5\,a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}-2\,(i\,ax-}{2\,x-2i}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] ((5*a^2*x^2 - 10*I*a^2*x - 5*a^2)*sqrt(4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(4
*I/a^2) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (5*a^2*x^2
 - 10*I*a^2*x - 5*a^2)*sqrt(4*I/a^2)*log(-((a^2*x - I*a^2)*sqrt(4*I/a^2) -
2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) + (5*a^2*x^2 - 10*I*a^
2*x - 5*a^2)*sqrt(-4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x
+ a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (5*a^2*x^2 - 10*I*a^2*x - 5*a
^2)*sqrt(-4*I/a^2)*log(-((a^2*x - I*a^2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)^(3/
4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*
(48*x - 32*I))/(10*a^2*x^2 - 20*I*a^2*x - 10*a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1225 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=33

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

```
[Out] (((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

Rubi [A] time = 0.0032964, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {37}

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]

[Out] (((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx = \frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Mathematica [A] time = 0.011016, size = 33, normalized size = 1.

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]

[Out] (((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

Maple [B] time = 0.034, size = 50, normalized size = 1.5

$$\frac{4\,ix+2\,x^2-2}{5\,a^2\,(-1+ix)\,(x-i)}\sqrt[4]{-a\,(-1+ix)}\frac{1}{\sqrt[4]{a\,(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] 2/5/a^2*(-a*(-1+I*x))^(1/4)/(-1+I*x)/(a*(1+I*x))^(1/4)*(2*I*x+x^2-1)/(x-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(9/4), x)
```

Fricas [B] time = 1.985, size = 113, normalized size = 3.42

$$-\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x+2i)}{5\,a^3x^2-10i\,a^3x-5\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] -(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(2*x + 2*I)/(5*a^3*x^2 - 10*I*a^3*x -
 5*a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{-a\,(ix-1)}}{(a\,(ix+1))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral((-a*(I*x - 1))**(1/4)/(a*(I*x + 1))**(9/4), x)
```

Giac [A] time = 1.11823, size = 46, normalized size = 1.39

$$-\frac{(-i\,ax+a)^{\frac{1}{4}}\left(-\frac{4i\,a}{i\,ax+a}+2i\right)}{5\,(i\,ax+a)^{\frac{1}{4}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="giac")

[Out] -1/5*(-I*a*x + a)^(1/4)*(-4*I*a/(I*a*x + a) + 2*I)/((I*a*x + a)^(1/4)*a^2)
```

3.1226 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=67

$$\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}$$

```
[Out] (((2*I)/5)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(5/4)) + (((4*I)/5)*(a - I*a
*x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0098011, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {45, 37}

$$\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(5/4)) + (((4*I)/5)*(a - I*a
*x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx &= \frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}+\frac{2\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{5a}\\ &= \frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}+\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [A] time = 0.0189533, size = 45, normalized size = 0.67

$$\frac{2(3+2ix)\sqrt[4]{a-iax}}{5a^3(x-i)\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*(3 + (2*I)*x)*(a - I*a*x)^(1/4))/(5*a^3*(-I + x)*(a + I*a*x)^(1/4))
```

Maple [A] time = 0.036, size = 44, normalized size = 0.7

$$\frac{4x^2+6-2ix}{5a^2(x-i)}(-a(-1+ix))^{-\frac{3}{4}}\frac{1}{\sqrt[4]{a(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] 2/5/a^2/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(2*x^2+3-I*x)/(x-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(iax+a)^{\frac{9}{4}}(-iax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [A] time = 2.16991, size = 112, normalized size = 1.67

$$\frac{(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}(4x-6i)}{5a^4x^2-10ia^4x-5a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] (I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(4*x - 6*I)/(5*a^4*x^2 - 10*I*a^4*x -
5*a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1227 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=100

$$\frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}-\frac{2i}{3a^2(a+iax)^{5/4}(a-iax)^{3/4}}$$

```
[Out] ((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (((8*I)/15)*(a - I*a
*x)^(1/4))/(a^3*(a + I*a*x)^(5/4)) + (((16*I)/15)*(a - I*a*x)^(1/4))/(a^4*(
a + I*a*x)^(1/4))
```

Rubi [A] time = 0.018242, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {45, 37}

$$\frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}-\frac{2i}{3a^2(a+iax)^{5/4}(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (((8*I)/15)*(a - I*a
*x)^(1/4))/(a^3*(a + I*a*x)^(5/4)) + (((16*I)/15)*(a - I*a*x)^(1/4))/(a^4*(
a + I*a*x)^(1/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx &= -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{4\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx}{3a}\\
&= -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}+\frac{8\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{15a^2}\\
&= -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}+\frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}
\end{aligned}$$

Mathematica [A] time = 0.0213835, size = 50, normalized size = 0.5

$$\frac{2\left(8x^2-4ix+7\right)}{15a^3(x-i)(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*(7 - (4*I)*x + 8*x^2))/(15*a^3*(-I + x)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1
/4))
```

Maple [A] time = 0.041, size = 44, normalized size = 0.4

$$\frac{16x^2-8ix+14}{15a^3(x-i)}(-a(-1+ix))^{-\frac{3}{4}}\frac{1}{\sqrt[4]{a(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] 2/15/a^3/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(8*x^2-4*I*x+7)/(x-I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.21088, size = 149, normalized size = 1.49

$$\frac{2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}\left(8x^2-4ix+7\right)}{15a^5x^3-15ia^5x^2+15a^5x-15ia^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 - 4*I*x + 7)/(15*a^5*x^3 - 15
*I*a^5*x^2 + 15*a^5*x - 15*I*a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1228 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=133

$$\frac{32i\sqrt[4]{a-iax}}{35a^5\sqrt[4]{a+iax}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}-\frac{4i}{7a^3(a+iax)^{5/4}(a-iax)^{3/4}}-\frac{2i}{7a^2(a+iax)^{5/4}(a-iax)^{7/4}}$$

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)) - ((4*I)/7)/(a^3*(a -
I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (((16*I)/35)*(a - I*a*x)^(1/4))/(a^4*(a +
 I*a*x)^(5/4)) + (((32*I)/35)*(a - I*a*x)^(1/4))/(a^5*(a + I*a*x)^(1/4))
```

Rubi [A] time = 0.0294059, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {45, 37}

$$\frac{32i\sqrt[4]{a-iax}}{35a^5\sqrt[4]{a+iax}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}-\frac{4i}{7a^3(a+iax)^{5/4}(a-iax)^{3/4}}-\frac{2i}{7a^2(a+iax)^{5/4}(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)) - ((4*I)/7)/(a^3*(a -
I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (((16*I)/35)*(a - I*a*x)^(1/4))/(a^4*(a +
 I*a*x)^(5/4)) + (((32*I)/35)*(a - I*a*x)^(1/4))/(a^5*(a + I*a*x)^(1/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx &= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}+\frac{6\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx}{7a}\\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}-\frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{8\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx}{7a^2}\\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}-\frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}+\frac{16\int \frac{1}{(a-ia}{}\\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}-\frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}+\frac{32i\sqrt[4]{a-}}{35a^5\sqrt[4]{a+}}
\end{aligned}$$

Mathematica [A] time = 0.0296752, size = 57, normalized size = 0.43

$$\frac{2\left(16x^3+8ix^2+22x+9i\right)}{35a^4\left(x^2+1\right)(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*(9*I + 22*x + (8*I)*x^2 + 16*x^3))/(35*a^4*(a - I*a*x)^(3/4)*(a + I*a*x)
^(1/4)*(1 + x^2))
```

Maple [A] time = 0.048, size = 56, normalized size = 0.4

$$\frac{32\,x^3+16\,ix^2+44\,x+18\,i}{35\,a^4\,(x-i)\,(x+i)}(-a\,(-1+ix))^{-\frac{3}{4}}\frac{1}{\sqrt[4]{a\,(1+ix)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] 2/35/a^4/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(16*x^3+8*I*x^2+22*x+9*I)/(x
-I)/(x+I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(11/4)), x)
```

Fricas [A] time = 2.05048, size = 142, normalized size = 1.07

$$\frac{\left(32\,x^3+16i\,x^2+44\,x+18i\right)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{35\left(a^6x^4+2\,a^6x^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/35*(32*x^3 + 16*I*x^2 + 44*x + 18*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)
/(a^6*x^4 + 2*a^6*x^2 + a^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1229 $\int (a+bx)^2(ac-bcx)^n\,dx$

Optimal. Leaf size=83

$$-\frac{4a^2(ac-bcx)^{n+1}}{bc(n+1)}+\frac{4a(ac-bcx)^{n+2}}{bc^2(n+2)}-\frac{(ac-bcx)^{n+3}}{bc^3(n+3)}$$

```
[Out] (-4*a^2*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (4*a*(a*c - b*c*x)^(2 + n))/
(b*c^2*(2 + n)) - (a*c - b*c*x)^(3 + n)/(b*c^3*(3 + n))
```

Rubi [A] time = 0.0285228, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{4a^2(ac-bcx)^{n+1}}{bc(n+1)}+\frac{4a(ac-bcx)^{n+2}}{bc^2(n+2)}-\frac{(ac-bcx)^{n+3}}{bc^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

```
[Out] (-4*a^2*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (4*a*(a*c - b*c*x)^(2 + n))/
(b*c^2*(2 + n)) - (a*c - b*c*x)^(3 + n)/(b*c^3*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^n\,dx&=\int\left(4a^2(ac-bcx)^n-\frac{4a(ac-bcx)^{1+n}}{c}+\frac{(ac-bcx)^{2+n}}{c^2}\right)dx\\&=-\frac{4a^2(ac-bcx)^{1+n}}{bc(1+n)}+\frac{4a(ac-bcx)^{2+n}}{bc^2(2+n)}-\frac{(ac-bcx)^{3+n}}{bc^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.0382107, size = 77, normalized size = 0.93

$$\frac{(bx-a)\left(a^2\left(n^2+7n+14\right)+2ab\left(n^2+5n+4\right)x+b^2\left(n^2+3n+2\right)x^2\right)(c(a-bx))^n}{b(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

```
[Out] ((c*(a - b*x))^n*(-a + b*x)*(a^2*(14 + 7*n + n^2) + 2*a*b*(4 + 5*n + n^2)*x
 + b^2*(2 + 3*n + n^2)*x^2))/(b*(1 + n)*(2 + n)*(3 + n))
```

Maple [A] time = 0.006, size = 103, normalized size = 1.2

$$-\frac{\left(b^{2} n^{2} x^{2}+2 a b n^{2} x+3 b^{2} n x^{2}+a^{2} n^{2}+10 a b n x+2 b^{2} x^{2}+7 a^{2} n+8 a b x+14 a^{2}\right)(-b x+a)(-b c x+a c)^{n}}{b\left(n^{3}+6 n^{2}+11 n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^n,x)
```

```
[Out] -(-b*x+a)*(b^2*n^2*x^2+2*a*b*n^2*x+3*b^2*n*x^2+a^2*n^2+10*a*b*n*x+2*b^2*x^2
+7*a^2*n+8*a*b*x+14*a^2)*(-b*c*x+a*c)^n/b/(n^3+6*n^2+11*n+6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.24628, size = 261, normalized size = 3.14

$$-\frac{\left(a^{3} n^{2}+7 a^{3} n-\left(b^{3} n^{2}+3 b^{3} n+2 b^{3}\right) x^{3}+14 a^{3}-\left(a b^{2} n^{2}+7 a b^{2} n+6 a b^{2}\right) x^{2}+\left(a^{2} b n^{2}+3 a^{2} b n-6 a^{2} b\right) x\right)(-b c x+a c)^{n}}{b n^{3}+6 b n^{2}+11 b n+6 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] -(a^3*n^2 + 7*a^3*n - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 + 14*a^3 - (a*b^2*n^2
 + 7*a*b^2*n + 6*a*b^2)*x^2 + (a^2*b*n^2 + 3*a^2*b*n - 6*a^2*b)*x)*(-b*c*x
+ a*c)^n/(b*n^3 + 6*b*n^2 + 11*b*n + 6*b)
```

Sympy [A] time = 1.07409, size = 785, normalized size = 9.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((a**2*x*(a*c)**n, Eq(b, 0)), (-a**2*log(-a/b + x)/(a**2*b*c**3 -
2*a*b**2*c**3*x + b**3*c**3*x**2) + 2*a*b*x*log(-a/b + x)/(a**2*b*c**3 - 2*
a*b**2*c**3*x + b**3*c**3*x**2) - b**2*x**2*log(-a/b + x)/(a**2*b*c**3 - 2*
a*b**2*c**3*x + b**3*c**3*x**2) + 2*b**2*x**2/(a**2*b*c**3 - 2*a*b**2*c**3*
x + b**3*c**3*x**2), Eq(n, -3)), (-4*a**2*log(-a/b + x)/(-a*b*c**2 + b**2*c
**2*x) - 5*a**2/(-a*b*c**2 + b**2*c**2*x) + 4*a*b*x*log(-a/b + x)/(-a*b*c**
2 + b**2*c**2*x) + b**2*x**2/(-a*b*c**2 + b**2*c**2*x), Eq(n, -2)), (-4*a**
2*log(-a/b + x)/(b*c) - 3*a*x/c - b*x**2/(2*c), Eq(n, -1)), (-a**3*n**2*(a*
```

```
c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) - 7*a**3*n*(a*c - b*c*x)**
n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) - 14*a**3*(a*c - b*c*x)**n/(b*n**3 + 6
*b*n**2 + 11*b*n + 6*b) - a**2*b*n**2*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2
 + 11*b*n + 6*b) - 3*a**2*b*n*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*
n + 6*b) + 6*a**2*b*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) +
 a*b**2*n**2*x**2*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 7*a
*b**2*n*x**2*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 6*a*b**2
*x**2*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + b**3*n**2*x**3*
(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 3*b**3*n*x**3*(a*c -
b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 2*b**3*x**3*(a*c - b*c*x)**n
/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b), True))
```

Giac [B] time = 1.07298, size = 346, normalized size = 4.17

$$(-bcx+ac)^n b^3 n^2 x^3 + (-bcx+ac)^n a b^2 n^2 x^2 + 3\,(-bcx+ac)^n b^3 n x^3 - (-bcx+ac)^n a^2 b n^2 x + 7\,(-bcx+ac)^n a b^2 n x^2 + 2\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="giac")
```

```
[Out] ((-b*c*x + a*c)^n*b^3*n^2*x^3 + (-b*c*x + a*c)^n*a*b^2*n^2*x^2 + 3*(-b*c*x
+ a*c)^n*b^3*n*x^3 - (-b*c*x + a*c)^n*a^2*b*n^2*x + 7*(-b*c*x + a*c)^n*a*b^
2*n*x^2 + 2*(-b*c*x + a*c)^n*b^3*x^3 - (-b*c*x + a*c)^n*a^3*n^2 - 3*(-b*c*x
 + a*c)^n*a^2*b*n*x + 6*(-b*c*x + a*c)^n*a*b^2*x^2 - 7*(-b*c*x + a*c)^n*a^3
*n + 6*(-b*c*x + a*c)^n*a^2*b*x - 14*(-b*c*x + a*c)^n*a^3)/(b*n^3 + 6*b*n^2
 + 11*b*n + 6*b)
```

3.1230 $\int (a+bx)(ac-bcx)^n \,dx$

Optimal. Leaf size=53

$$\frac{(ac-bcx)^{n+2}}{bc^2(n+2)}-\frac{2a(ac-bcx)^{n+1}}{bc(n+1)}$$

```
[Out] (-2*a*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (a*c - b*c*x)^(2 + n)/(b*c^2*(
2 + n))
```

Rubi [A] time = 0.0170643, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{(ac-bcx)^{n+2}}{bc^2(n+2)}-\frac{2a(ac-bcx)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^n,x]
```

```
[Out] (-2*a*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (a*c - b*c*x)^(2 + n)/(b*c^2*(
2 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^n \,dx &= \int \left(2a(ac-bcx)^n-\frac{(ac-bcx)^{1+n}}{c}\right) dx\\ &= -\frac{2a(ac-bcx)^{1+n}}{bc(1+n)}+\frac{(ac-bcx)^{2+n}}{bc^2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.0210716, size = 43, normalized size = 0.81

$$\frac{(bx-a)(a(n+3)+b(n+1)x)(c(a-bx))^n}{b(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^n,x]
```

```
[Out] ((c*(a - b*x))^n*(-a + b*x)*(a*(3 + n) + b*(1 + n)*x))/(b*(1 + n)*(2 + n))
```

Maple [A] time = 0.002, size = 47, normalized size = 0.9

$$-\frac{(-bcx+ac)^{n}(bnx+an+bx+3a)(-bx+a)}{b(n^{2}+3n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^n,x)
```

```
[Out] -(-b*c*x+a*c)^n*(b*n*x+a*n+b*x+3*a)*(-b*x+a)/b/(n^2+3*n+2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13825, size = 117, normalized size = 2.21

$$-\frac{\left(a^{2}n-2abx-\left(b^{2}n+b^{2}\right)x^{2}+3a^{2}\right)(-bcx+ac)^{n}}{bn^{2}+3bn+2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] -(a^2*n - 2*a*b*x - (b^2*n + b^2)*x^2 + 3*a^2)*(-b*c*x + a*c)^n/(b*n^2 + 3*
b*n + 2*b)
```

Sympy [A] time = 0.615571, size = 245, normalized size = 4.62

$$\begin{cases} ax(ac)^{n} & \text{for } b=0 \\ -\frac{a\log\left(-\frac{a}{b}+x\right)}{-abc^{2}+b^{2}c^{2}x}-\frac{2a}{-abc^{2}+b^{2}c^{2}x}+\frac{bx\log\left(-\frac{a}{b}+x\right)}{-abc^{2}+b^{2}c^{2}x} & \text{for } n=-2 \\ -\frac{2a\log\left(-\frac{a}{b}+x\right)}{bc}-\frac{x}{c} & \text{for } n=-1 \\ -\frac{a^{2}n(ac-bcx)^{n}}{bn^{2}+3bn+2b}-\frac{3a^{2}(ac-bcx)^{n}}{bn^{2}+3bn+2b}+\frac{2abx(ac-bcx)^{n}}{bn^{2}+3bn+2b}+\frac{b^{2}nx^{2}(ac-bcx)^{n}}{bn^{2}+3bn+2b}+\frac{b^{2}x^{2}(ac-bcx)^{n}}{bn^{2}+3bn+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((a*x*(a*c)**n, Eq(b, 0)), (-a*log(-a/b + x)/(-a*b*c**2 + b**2*c**
2*x) - 2*a/(-a*b*c**2 + b**2*c**2*x) + b*x*log(-a/b + x)/(-a*b*c**2 + b**2*
c**2*x), Eq(n, -2)), (-2*a*log(-a/b + x)/(b*c) - x/c, Eq(n, -1)), (-a**2*n*
(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) - 3*a**2*(a*c - b*c*x)**n/(b*n**2 +
 3*b*n + 2*b) + 2*a*b*x*(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) + b**2*n*x*
```

```
*2*(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) + b**2*x**2*(a*c - b*c*x)**n/(b*
n**2 + 3*b*n + 2*b), True))
```

Giac [A] time = 1.05295, size = 139, normalized size = 2.62

$$\frac{(-bcx+ac)^n b^2nx^2 + (-bcx+ac)^n b^2x^2 - (-bcx+ac)^n a^2n + 2(-bcx+ac)^n abx - 3(-bcx+ac)^n a^2}{bn^2 + 3bn + 2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="giac")
```

```
[Out] ((-b*c*x + a*c)^n*b^2*n*x^2 + (-b*c*x + a*c)^n*b^2*x^2 - (-b*c*x + a*c)^n*a
^2*n + 2*(-b*c*x + a*c)^n*a*b*x - 3*(-b*c*x + a*c)^n*a^2)/(b*n^2 + 3*b*n +
2*b)
```

3.1231 $\int \frac{(ac-bcx)^n}{a+bx} dx$

Optimal. Leaf size=52

$$-\frac{(ac-bcx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a-bx}{2a}\right)}{2abc(n+1)}$$

```
[Out] -((a*c - b*c*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a - b*x)/(2*a)]
)/(2*a*b*c*(1 + n))
```

Rubi [A] time = 0.0145496, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {68}

$$-\frac{(ac-bcx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a-bx}{2a}\right)}{2abc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^n/(a + b*x),x]
```

```
[Out] -((a*c - b*c*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a - b*x)/(2*a)]
)/(2*a*b*c*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(ac-bcx)^n}{a+bx}\,dx = -\frac{(ac-bcx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{a-bx}{2a}\right)}{2abc(1+n)}$$

Mathematica [A] time = 0.0128123, size = 52, normalized size = 1.

$$-\frac{(a-bx)(c(a-bx))^n\,{}_2F_1\left(1,n+1;n+2;\frac{a-bx}{2a}\right)}{2ab(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^n/(a + b*x),x]
```

```
[Out] -((a - b*x)*(c*(a - b*x))^n*Hypergeometric2F1[1, 1 + n, 2 + n, (a - b*x)/(2
*a)])/(2*a*b*(1 + n))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{(-bcx + ac)^n}{bx + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^n/(b*x+a),x)
```

```
[Out] int((-b*c*x+a*c)^n/(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bcx + ac)^n}{bx + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((-b*c*x + a*c)^n/(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-bcx + ac)^n}{bx + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((-b*c*x + a*c)^n/(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c(-a + bx))^n}{a + bx} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**n/(b*x+a),x)
```

```
[Out] Integral((-c*(-a + b*x))**n/(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bcx + ac)^n}{bx + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a),x, algorithm="giac")

[Out] integrate((-b*c*x + a*c)^n/(b*x + a), x)
```

3.1232 $\int \frac{(ac-bcx)^n}{(a+bx)^2}\,dx$

Optimal. Leaf size=52

$$-\frac{(ac-bcx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{a-bx}{2a}\right)}{4a^2bc(n+1)}$$

```
[Out] -((a*c - b*c*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (a - b*x)/(2*a)]
)/(4*a^2*b*c*(1 + n))
```

Rubi [A] time = 0.0114906, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {68}

$$-\frac{(ac-bcx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{a-bx}{2a}\right)}{4a^2bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^n/(a + b*x)^2,x]
```

```
[Out] -((a*c - b*c*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (a - b*x)/(2*a)]
)/(4*a^2*b*c*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(ac-bcx)^n}{(a+bx)^2}\,dx = -\frac{(ac-bcx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;\frac{a-bx}{2a}\right)}{4a^2bc(1+n)}$$

Mathematica [A] time = 0.0109877, size = 52, normalized size = 1.

$$-\frac{(a-bx)(c(a-bx))^n\,{}_2F_1\left(2,n+1;n+2;\frac{a-bx}{2a}\right)}{4a^2b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^n/(a + b*x)^2,x]
```

```
[Out] -((a - b*x)*(c*(a - b*x))^n*Hypergeometric2F1[2, 1 + n, 2 + n, (a - b*x)/(2
*a)])/(4*a^2*b*(1 + n))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{(-bcx+ac)^n}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^n/(b*x+a)^2,x)
```

```
[Out] int((-b*c*x+a*c)^n/(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bcx+ac)^n}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*c*x + a*c)^n/(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(-bcx+ac)^n}{b^2x^2+2\,abx+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((-b*c*x + a*c)^n/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-c(-a+bx))^n}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**n/(b*x+a)**2,x)
```

```
[Out] Integral((-c*(-a + b*x))**n/(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bcx+ac)^n}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*c*x + a*c)^n/(b*x + a)^2, x)
```

3.1233 $\int (a+ax)^m(c-cx)^m\,dx$

Optimal. Leaf size=41

$$x\left(1-x^2\right)^{-m}(ax+a)^m(c-cx)^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};x^2\right)$$

```
[Out] (x*(a + a*x)^m*(c - c*x)^m*Hypergeometric2F1[1/2, -m, 3/2, x^2])/(1 - x^2)^
m
```

Rubi [A] time = 0.0116055, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {42, 246, 245}

$$x\left(1-x^2\right)^{-m}(ax+a)^m(c-cx)^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x)^m*(c - c*x)^m,x]
```

```
[Out] (x*(a + a*x)^m*(c - c*x)^m*Hypergeometric2F1[1/2, -m, 3/2, x^2])/(1 - x^2)^
m
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] && !IntegerQ[2*m]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+ax)^m(c-cx)^m\,dx&=\left((a+ax)^m(c-cx)^m\left(ac-acx^2\right)^{-m}\right)\int\left(ac-acx^2\right)^m\,dx\\&=\left((a+ax)^m(c-cx)^m\left(1-x^2\right)^{-m}\right)\int\left(1-x^2\right)^m\,dx\\&=x(a+ax)^m(c-cx)^m\left(1-x^2\right)^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0219677, size = 53, normalized size = 1.29

$$\frac{2^m(x-1)(x+1)^{-m}(a(x+1))^m(c-cx)^m\,{}_2F_1\left(-m,m+1;m+2;\frac{1}{2}-\frac{x}{2}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x)^m*(c - c*x)^m,x]
```

```
[Out] (2^m*(-1 + x)*(a*(1 + x))^m*(c - c*x)^m*Hypergeometric2F1[-m, 1 + m, 2 + m,
 1/2 - x/2])/((1 + m)*(1 + x)^m)
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int (ax+a)^m (-cx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^m*(-c*x+c)^m,x)
```

```
[Out] int((a*x+a)^m*(-c*x+c)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ax+a)^m(-cx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^m*(-c*x+c)^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + a)^m*(-c*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ax+a)^m(-cx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^m*(-c*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + a)^m*(-c*x + c)^m, x)
```

Sympy [C] time = 4.37577, size = 124, normalized size = 3.02

$$\frac{a^m c^m G_{6,6}^{5,3}\left(\begin{matrix} -\frac{m}{2},\frac{1}{2}-\frac{m}{2},1 & \frac{1}{2},-m,\frac{1}{2}-m \\ -m-\frac{1}{2},-m,-\frac{m}{2},\frac{1}{2}-m,\frac{1}{2}-\frac{m}{2} & 0 \end{matrix}\middle|\,\frac{e^{-2i\pi}}{x^2}\right)e^{-i\pi m}}{4\pi\Gamma(-m)} - \frac{a^m c^m G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{2},-\frac{m}{2}-\frac{1}{2},-\frac{m}{2},1 & \\ -\frac{m}{2}-\frac{1}{2},-\frac{m}{2} & -\frac{1}{2},0,-m-\frac{1}{2} \end{matrix}\right.}{4\pi\Gamma(-m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**m*(-c*x+c)**m,x)
```

```
[Out] a**m*c**m*meijerg(((-m/2, 1/2 - m/2, 1), (1/2, -m, 1/2 - m)), ((-m - 1/2, -
m, -m/2, 1/2 - m, 1/2 - m/2), (0,)), exp_polar(-2*I*pi)/x**2)*exp(-I*pi*m)/
(4*pi*gamma(-m)) - a**m*c**m*meijerg(((-1/2, 0, 1/2, -m/2 - 1/2, -m/2, 1),
()), ((-m/2 - 1/2, -m/2), (-1/2, 0, -m - 1/2, 0)), x**(-2))/(4*pi*gamma(-m)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ax+a)^m(-cx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^m*(-c*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((a*x + a)^m*(-c*x + c)^m, x)
```

3.1234 $\int (a+bx)^m(ac-bcx)^m\,dx$

Optimal. Leaf size=57

$$x(a+bx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}(ac-bcx)^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{b^2x^2}{a^2}\right)$$

```
[Out] (x*(a + b*x)^m*(a*c - b*c*x)^m*Hypergeometric2F1[1/2, -m, 3/2, (b^2*x^2)/a^
2])/(1 - (b^2*x^2)/a^2)^m
```

Rubi [A] time = 0.0179656, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {42, 246, 245}

$$x(a+bx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}(ac-bcx)^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{b^2x^2}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a*c - b*c*x)^m,x]
```

```
[Out] (x*(a + b*x)^m*(a*c - b*c*x)^m*Hypergeometric2F1[1/2, -m, 3/2, (b^2*x^2)/a^
2])/(1 - (b^2*x^2)/a^2)^m
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[((
a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b*d*x^2)^FracPart[m], In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] && !IntegerQ[2*m]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(ac-bcx)^m\,dx &= \left((a+bx)^m(ac-bcx)^m\left(a^2c-b^2cx^2\right)^{-m}\right)\int \left(a^2c-b^2cx^2\right)^m\,dx\\ &= \left((a+bx)^m(ac-bcx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}\right)\int \left(1-\frac{b^2x^2}{a^2}\right)^m\,dx\\ &= x(a+bx)^m(ac-bcx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{b^2x^2}{a^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0239115, size = 72, normalized size = 1.26

$$-\frac{2^m(a-bx)(a+bx)^m\left(\frac{a+bx}{a}\right)^{-m}(c(a-bx))^m\,{}_2F_1\left(-m,m+1;m+2;\frac{a-bx}{2a}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a*c - b*c*x)^m,x]
```

```
[Out] -((2^m*(a - b*x)*(c*(a - b*x))^m*(a + b*x)^m*Hypergeometric2F1[-m, 1 + m, 2
 + m, (a - b*x)/(2*a)])/(b*(1 + m)*((a + b*x)/a)^m))
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int (bx+a)^m(-bcx+ac)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(-b*c*x+a*c)^m,x)
```

```
[Out] int((b*x+a)^m*(-b*c*x+a*c)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-bcx+ac)^m(bx+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b*c*x+a*c)^m,x, algorithm="maxima")
```

```
[Out] integrate((-b*c*x + a*c)^m*(b*x + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-bcx+ac)^m(bx+a)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b*c*x+a*c)^m,x, algorithm="fricas")
```

```
[Out] integral((-b*c*x + a*c)^m*(b*x + a)^m, x)
```

Sympy [C] time = 5.12157, size = 146, normalized size = 2.56

$$\frac{aa^{2m}c^mG_{6,6}^{5,3}\left(\begin{matrix}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},1 & \frac{1}{2},-m,\frac{1}{2}-m\\ -m-\frac{1}{2},-m,-\frac{m}{2},\frac{1}{2}-m,\frac{1}{2}-\frac{m}{2} & 0\end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^2}\right)e^{-i\pi m}}{4\pi b\Gamma(-m)}-\frac{aa^{2m}c^mG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},-\frac{m}{2}-\frac{1}{2},-\frac{m}{2},1\\ -\frac{m}{2}-\frac{1}{2},-\frac{m}{2} & -\frac{1}{2},\end{matrix}\right.}{4\pi b\Gamma(-m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(-b*c*x+a*c)**m,x)
```

```
[Out] a*a**(2*m)*c**m*meijerg(((-m/2, 1/2 - m/2, 1), (1/2, -m, 1/2 - m)), ((-m -
1/2, -m, -m/2, 1/2 - m, 1/2 - m/2), (0,)), a**2*exp_polar(-2*I*pi)/(b**2*x*
*2))*exp(-I*pi*m)/(4*pi*b*gamma(-m)) - a*a**(2*m)*c**m*meijerg(((-1/2, 0, 1
/2, -m/2 - 1/2, -m/2, 1), ()), ((-m/2 - 1/2, -m/2), (-1/2, 0, -m - 1/2, 0))
, a**2/(b**2*x**2))/(4*pi*b*gamma(-m))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-bcx + ac)^m (bx + a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b*c*x+a*c)^m,x, algorithm="giac")
```

```
[Out] integrate((-b*c*x + a*c)^m*(b*x + a)^m, x)
```

3.1235 $\int (3-6x)^m (2+4x)^m \, dx$

Optimal. Leaf size=20

$$6^m x \, _2F_1\left(\frac{1}{2},-m;\frac{3}{2};4x^2\right)$$

```
[Out] 6^m*x*Hypergeometric2F1[1/2, -m, 3/2, 4*x^2]
```

Rubi [A] time = 0.0057414, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {41, 245}

$$6^m x \, _2F_1\left(\frac{1}{2},-m;\frac{3}{2};4x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 6*x)^m*(2 + 4*x)^m,x]
```

```
[Out] 6^m*x*Hypergeometric2F1[1/2, -m, 3/2, 4*x^2]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (3-6x)^m (2+4x)^m \, dx &= \int \left(6-24x^2\right)^m \, dx \\ &= 6^m x \, _2F_1\left(\frac{1}{2},-m;\frac{3}{2};4x^2\right) \end{aligned}$$

Mathematica [A] time = 0.0049051, size = 20, normalized size = 1.

$$6^m x \, _2F_1\left(\frac{1}{2},-m;\frac{3}{2};4x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 6*x)^m*(2 + 4*x)^m,x]
```

```
[Out] 6^m*x*Hypergeometric2F1[1/2, -m, 3/2, 4*x^2]
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int (3-6x)^m (2+4x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^m*(2+4*x)^m,x)
```

```
[Out] int((3-6*x)^m*(2+4*x)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (4x+2)^m (-6x+3)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^m*(4*x+2)^m,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 2)^m*(-6*x + 3)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((4x+2)^m(-6x+3)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^m*(4*x+2)^m,x, algorithm="fricas")
```

```
[Out] integral((4*x + 2)^m*(-6*x + 3)^m, x)
```

Sympy [C] time = 4.61746, size = 42, normalized size = 2.1

$$\frac{24^m \left(x+\frac{1}{2}\right)\left(x+\frac{1}{2}\right)^m \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -m, m+1 \\ m+2 \end{matrix} \middle| \left(x+\frac{1}{2}\right) e^{2i\pi}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**m*(4*x+2)**m,x)
```

```
[Out] 24**m*(x + 1/2)*(x + 1/2)**m*gamma(m + 1)*hyper((-m, m + 1), (m + 2,), (x + 
 1/2)*exp_polar(2*I*pi))/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (4x+2)^m (-6x+3)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^m*(4*x+2)^m,x, algorithm="giac")
```

```
[Out] integrate((4*x + 2)^m*(-6*x + 3)^m, x)
```

3.1236 $\int (a+bx)^4(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{(a+bx)^5(bc-ad)}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

```
[Out] ((b*c - a*d)*(a + b*x)^5)/(5*b^2) + (d*(a + b*x)^6)/(6*b^2)
```

Rubi [A] time = 0.0147325, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(a+bx)^5(bc-ad)}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x),x]

[Out] ((b*c - a*d)*(a + b*x)^5)/(5*b^2) + (d*(a + b*x)^6)/(6*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^4}{b}+\frac{d(a+bx)^5}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^5}{5b^2}+\frac{d(a+bx)^6}{6b^2}\end{aligned}$$

Mathematica [B] time = 0.0180957, size = 84, normalized size = 2.21

$$\frac{1}{30}x\left(15a^2b^2x^2(4c+3dx)+20a^3bx(3c+2dx)+15a^4(2c+dx)+6ab^3x^3(5c+4dx)+b^4x^4(6c+5dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x),x]

[Out] (x*(15*a^4*(2*c + d*x) + 20*a^3*b*x*(3*c + 2*d*x) + 15*a^2*b^2*x^2*(4*c + 3
*d*x) + 6*a*b^3*x^3*(5*c + 4*d*x) + b^4*x^4*(6*c + 5*d*x)))/30
```

Maple [B] time = 0.001, size = 97, normalized size = 2.6

$$\frac{b^4dx^6}{6}+\frac{\left(4\,ab^3d+b^4c\right)x^5}{5}+\frac{\left(6\,b^2a^2d+4\,ab^3c\right)x^4}{4}+\frac{\left(4\,a^3bd+6\,b^2a^2c\right)x^3}{3}+\frac{\left(a^4d+4\,a^3bc\right)x^2}{2}+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c),x)
```

```
[Out] 1/6*b^4*d*x^6+1/5*(4*a*b^3*d+b^4*c)*x^5+1/4*(6*a^2*b^2*d+4*a*b^3*c)*x^4+1/3
*(4*a^3*b*d+6*a^2*b^2*c)*x^3+1/2*(a^4*d+4*a^3*b*c)*x^2+a^4*c*x
```

Maxima [B] time = 0.962154, size = 130, normalized size = 3.42

$$\frac{1}{6}\,b^4dx^6 + a^4cx + \frac{1}{5}\left(b^4c + 4\,ab^3d\right)x^5 + \frac{1}{2}\left(2\,ab^3c + 3\,a^2b^2d\right)x^4 + \frac{2}{3}\left(3\,a^2b^2c + 2\,a^3bd\right)x^3 + \frac{1}{2}\left(4\,a^3bc + a^4d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*b^4*d*x^6 + a^4*c*x + 1/5*(b^4*c + 4*a*b^3*d)*x^5 + 1/2*(2*a*b^3*c + 3*
a^2*b^2*d)*x^4 + 2/3*(3*a^2*b^2*c + 2*a^3*b*d)*x^3 + 1/2*(4*a^3*b*c + a^4*d
)*x^2
```

Fricas [B] time = 1.71768, size = 217, normalized size = 5.71

$$\frac{1}{6}x^6db^4 + \frac{1}{5}x^5cb^4 + \frac{4}{5}x^5db^3a + x^4cb^3a + \frac{3}{2}x^4db^2a^2 + 2x^3cb^2a^2 + \frac{4}{3}x^3dba^3 + 2x^2cba^3 + \frac{1}{2}x^2da^4 + xca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*d*b^4 + 1/5*x^5*c*b^4 + 4/5*x^5*d*b^3*a + x^4*c*b^3*a + 3/2*x^4*d*b
^2*a^2 + 2*x^3*c*b^2*a^2 + 4/3*x^3*d*b*a^3 + 2*x^2*c*b*a^3 + 1/2*x^2*d*a^4
+ x*c*a^4
```

Sympy [B] time = 0.075686, size = 100, normalized size = 2.63

$$a^4cx + \frac{b^4dx^6}{6} + x^5\left(\frac{4ab^3d}{5} + \frac{b^4c}{5}\right) + x^4\left(\frac{3a^2b^2d}{2} + ab^3c\right) + x^3\left(\frac{4a^3bd}{3} + 2a^2b^2c\right) + x^2\left(\frac{a^4d}{2} + 2a^3bc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c),x)
```

```
[Out] a**4*c*x + b**4*d*x**6/6 + x**5*(4*a*b**3*d/5 + b**4*c/5) + x**4*(3*a**2*b*
*2*d/2 + a*b**3*c) + x**3*(4*a**3*b*d/3 + 2*a**2*b**2*c) + x**2*(a**4*d/2 +
 2*a**3*b*c)
```

Giac [B] time = 1.06349, size = 131, normalized size = 3.45

$$\frac{1}{6}\,b^4dx^6 + \frac{1}{5}\,b^4cx^5 + \frac{4}{5}\,ab^3dx^5 + ab^3cx^4 + \frac{3}{2}\,a^2b^2dx^4 + 2\,a^2b^2cx^3 + \frac{4}{3}\,a^3bdx^3 + 2\,a^3bcx^2 + \frac{1}{2}\,a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*b^4*d*x^6 + 1/5*b^4*c*x^5 + 4/5*a*b^3*d*x^5 + a*b^3*c*x^4 + 3/2*a^2*b^2
*d*x^4 + 2*a^2*b^2*c*x^3 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + 1/2*a^4*d*x^2
+ a^4*c*x
```

3.1237 $\int (a+bx)^3(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{(a+bx)^4(bc-ad)}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

```
[Out] ((b*c - a*d)*(a + b*x)^4)/(4*b^2) + (d*(a + b*x)^5)/(5*b^2)
```

Rubi [A] time = 0.01236, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(a+bx)^4(bc-ad)}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^4)/(4*b^2) + (d*(a + b*x)^5)/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^3}{b}+\frac{d(a+bx)^4}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^4}{4b^2}+\frac{d(a+bx)^5}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.0100097, size = 67, normalized size = 1.76

$$\frac{1}{2}a^2x^2(ad+3bc)+a^3cx+\frac{1}{4}b^2x^4(3ad+bc)+abx^3(ad+bc)+\frac{1}{5}b^3dx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^2)/2 + a*b*(b*c + a*d)*x^3 + (b^2*(b*c + 3*a
*d)*x^4)/4 + (b^3*d*x^5)/5
```

Maple [B] time = 0., size = 73, normalized size = 1.9

$$\frac{b^3dx^5}{5}+\frac{\left(3\,ab^2d+b^3c\right)x^4}{4}+\frac{\left(3\,a^2bd+3\,ab^2c\right)x^3}{3}+\frac{\left(a^3d+3\,a^2bc\right)x^2}{2}+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c),x)
```

```
[Out] 1/5*b^3*d*x^5+1/4*(3*a*b^2*d+b^3*c)*x^4+1/3*(3*a^2*b*d+3*a*b^2*c)*x^3+1/2*(
a^3*d+3*a^2*b*c)*x^2+a^3*c*x
```

Maxima [B] time = 0.977541, size = 93, normalized size = 2.45

$$\frac{1}{5}\,b^3dx^5 + a^3cx + \frac{1}{4}\left(b^3c + 3\,ab^2d\right)x^4 + \left(ab^2c + a^2bd\right)x^3 + \frac{1}{2}\left(3\,a^2bc + a^3d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/5*b^3*d*x^5 + a^3*c*x + 1/4*(b^3*c + 3*a*b^2*d)*x^4 + (a*b^2*c + a^2*b*d)
*x^3 + 1/2*(3*a^2*b*c + a^3*d)*x^2
```

Fricas [B] time = 1.71222, size = 163, normalized size = 4.29

$$\frac{1}{5}x^5db^3 + \frac{1}{4}x^4cb^3 + \frac{3}{4}x^4db^2a + x^3cb^2a + x^3dba^2 + \frac{3}{2}x^2cba^2 + \frac{1}{2}x^2da^3 + xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d*b^3 + 1/4*x^4*c*b^3 + 3/4*x^4*d*b^2*a + x^3*c*b^2*a + x^3*d*b*a^2
 + 3/2*x^2*c*b*a^2 + 1/2*x^2*d*a^3 + x*c*a^3
```

Sympy [B] time = 0.070427, size = 73, normalized size = 1.92

$$a^3cx + \frac{b^3dx^5}{5} + x^4\left(\frac{3ab^2d}{4} + \frac{b^3c}{4}\right) + x^3\left(a^2bd + ab^2c\right) + x^2\left(\frac{a^3d}{2} + \frac{3a^2bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c),x)
```

```
[Out] a**3*c*x + b**3*d*x**5/5 + x**4*(3*a*b**2*d/4 + b**3*c/4) + x**3*(a**2*b*d
+ a*b**2*c) + x**2*(a**3*d/2 + 3*a**2*b*c/2)
```

Giac [B] time = 1.06763, size = 97, normalized size = 2.55

$$\frac{1}{5}\,b^3dx^5 + \frac{1}{4}\,b^3cx^4 + \frac{3}{4}\,ab^2dx^4 + ab^2cx^3 + a^2bdx^3 + \frac{3}{2}\,a^2bcx^2 + \frac{1}{2}\,a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/5*b^3*d*x^5 + 1/4*b^3*c*x^4 + 3/4*a*b^2*d*x^4 + a*b^2*c*x^3 + a^2*b*d*x^3
 + 3/2*a^2*b*c*x^2 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.1238 $\int (a+bx)^2(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{(a+bx)^3(bc-ad)}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

```
[Out] ((b*c - a*d)*(a + b*x)^3)/(3*b^2) + (d*(a + b*x)^4)/(4*b^2)
```

Rubi [A] time = 0.0268233, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(a+bx)^3(bc-ad)}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^3)/(3*b^2) + (d*(a + b*x)^4)/(4*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^2}{b}+\frac{d(a+bx)^3}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^3}{3b^2}+\frac{d(a+bx)^4}{4b^2}\end{aligned}$$

Mathematica [A] time = 0.0078615, size = 46, normalized size = 1.21

$$\frac{1}{12}x\left(6a^2(2c+dx)+4abx(3c+2dx)+b^2x^2(4c+3dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x),x]
```

```
[Out] (x*(6*a^2*(2*c + d*x) + 4*a*b*x*(3*c + 2*d*x) + b^2*x^2*(4*c + 3*d*x)))/12
```

Maple [A] time = 0., size = 49, normalized size = 1.3

$$\frac{b^2dx^4}{4}+\frac{\left(2\,abd+b^2c\right)x^3}{3}+\frac{\left(a^2d+2\,abc\right)x^2}{2}+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c),x)
```

```
[Out] 1/4*b^2*d*x^4+1/3*(2*a*b*d+b^2*c)*x^3+1/2*(a^2*d+2*a*b*c)*x^2+a^2*c*x
```

Maxima [A] time = 0.95809, size = 65, normalized size = 1.71

$$\frac{1}{4}b^2dx^4 + a^2cx + \frac{1}{3}\left(b^2c + 2\,abd\right)x^3 + \frac{1}{2}\left(2\,abc + a^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d*x^4 + a^2*c*x + 1/3*(b^2*c + 2*a*b*d)*x^3 + 1/2*(2*a*b*c + a^2*d)
*x^2
```

Fricas [A] time = 1.75886, size = 115, normalized size = 3.03

$$\frac{1}{4}x^4db^2 + \frac{1}{3}x^3cb^2 + \frac{2}{3}x^3dba + x^2cba + \frac{1}{2}x^2da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*d*b^2 + 1/3*x^3*c*b^2 + 2/3*x^3*d*b*a + x^2*c*b*a + 1/2*x^2*d*a^2 +
 x*c*a^2
```

Sympy [A] time = 0.078587, size = 49, normalized size = 1.29

$$a^2cx + \frac{b^2dx^4}{4} + x^3\left(\frac{2abd}{3} + \frac{b^2c}{3}\right) + x^2\left(\frac{a^2d}{2} + abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**4/4 + x**3*(2*a*b*d/3 + b**2*c/3) + x**2*(a**2*d/2 + a
*b*c)
```

Giac [A] time = 1.06686, size = 66, normalized size = 1.74

$$\frac{1}{4}b^2dx^4 + \frac{1}{3}b^2cx^3 + \frac{2}{3}abdx^3 + abcx^2 + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*b^2*d*x^4 + 1/3*b^2*c*x^3 + 2/3*a*b*d*x^3 + a*b*c*x^2 + 1/2*a^2*d*x^2 +
 a^2*c*x
```

3.1239 $\int (a+bx)(c+dx)\,dx$

Optimal. Leaf size=28

$$\frac{1}{2}x^2(ad+bc)+acx+\frac{1}{3}bdx^3$$

```
[Out] a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

Rubi [A] time = 0.015441, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{1}{2}x^2(ad+bc)+acx+\frac{1}{3}bdx^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)\,dx &= \int \left(ac+(bc+ad)x+bdx^2\right)\,dx\\ &= acx+\frac{1}{2}(bc+ad)x^2+\frac{1}{3}bdx^3\end{aligned}$$

Mathematica [A] time = 0.0039147, size = 28, normalized size = 1.

$$\frac{1}{2}x^2(ad+bc)+acx+\frac{1}{3}bdx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

Maple [A] time = 0.001, size = 25, normalized size = 0.9

$$acx+\frac{(ad+bc)\,x^2}{2}+\frac{bdx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c),x)
```

```
[Out] a*c*x+1/2*(a*d+b*c)*x^2+1/3*b*d*x^3
```

Maxima [A] time = 0.970086, size = 32, normalized size = 1.14

$$\frac{1}{3}\,bdx^3 + acx + \frac{1}{2}\,(bc + ad)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/3*b*d*x^3 + a*c*x + 1/2*(b*c + a*d)*x^2
```

Fricas [A] time = 1.73416, size = 66, normalized size = 2.36

$$\frac{1}{3}x^3db + \frac{1}{2}x^2cb + \frac{1}{2}x^2da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*d*b + 1/2*x^2*c*b + 1/2*x^2*d*a + x*c*a
```

Sympy [A] time = 0.055744, size = 26, normalized size = 0.93

$$acx + \frac{bdx^3}{3} + x^2\left(\frac{ad}{2} + \frac{bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x)
```

```
[Out] a*c*x + b*d*x**3/3 + x**2*(a*d/2 + b*c/2)
```

Giac [A] time = 1.0495, size = 35, normalized size = 1.25

$$\frac{1}{3}\,bdx^3 + \frac{1}{2}\,bcx^2 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/3*b*d*x^3 + 1/2*b*c*x^2 + 1/2*a*d*x^2 + a*c*x
```

3.1240 $\int (c+dx)\,dx$

Optimal. Leaf size=12

$$cx+\frac{dx^2}{2}$$

```
[Out] c*x + (d*x^2)/2
```

Rubi [A] time = 0.0021542, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[c + d*x,x]
```

```
[Out] c*x + (d*x^2)/2
```

Rubi steps

$$\int (c+dx)\,dx=cx+\frac{dx^2}{2}$$

Mathematica [A] time = 0.000037, size = 12, normalized size = 1.

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[c + d*x,x]
```

```
[Out] c*x + (d*x^2)/2
```

Maple [A] time = 0.001, size = 11, normalized size = 0.9

$$cx+\frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x+c,x)
```

```
[Out] c*x+1/2*d*x^2
```

Maxima [A] time = 0.94133, size = 14, normalized size = 1.17

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + c*x
```

Fricas [A] time = 1.74649, size = 23, normalized size = 1.92

$$\frac{1}{2}x^2d + xc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*d + x*c
```

Sympy [A] time = 0.050014, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x)
```

```
[Out] c*x + d*x**2/2
```

Giac [A] time = 1.05364, size = 14, normalized size = 1.17

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + c*x
```

3.1241 $\int \frac{c+dx}{a+bx} dx$

Optimal. Leaf size=25

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

```
[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

Rubi [A] time = 0.0170204, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{a+bx}\,dx &= \int \left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx\\ &= \frac{dx}{b}+\frac{(bc-ad)\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0079084, size = 25, normalized size = 1.

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

Maple [A] time = 0.002, size = 32, normalized size = 1.3

$$\frac{dx}{b}-\frac{a\ln(bx+a)\,d}{b^2}+\frac{c\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a),x)
```

```
[Out] d*x/b-1/b^2*ln(b*x+a)*a*d+c*ln(b*x+a)/b
```

Maxima [A] time = 0.983127, size = 34, normalized size = 1.36

$$\frac{dx}{b} + \frac{(bc - ad)\log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] d*x/b + (b*c - a*d)*log(b*x + a)/b^2
```

Fricas [A] time = 1.93326, size = 54, normalized size = 2.16

$$\frac{bdx + (bc - ad)\log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*d*x + (b*c - a*d)*log(b*x + a))/b^2
```

Sympy [A] time = 0.29266, size = 20, normalized size = 0.8

$$\frac{dx}{b} - \frac{(ad - bc)\log(a + bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x)
```

```
[Out] d*x/b - (a*d - b*c)*log(a + b*x)/b**2
```

Giac [A] time = 1.07774, size = 35, normalized size = 1.4

$$\frac{dx}{b} + \frac{(bc - ad)\log(|bx + a|)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] d*x/b + (b*c - a*d)*log(abs(b*x + a))/b^2
```

3.1242 $\int \frac{c+dx}{(a+bx)^2}\,dx$

Optimal. Leaf size=32

$$\frac{d\log(a+bx)}{b^2}-\frac{bc-ad}{b^2(a+bx)}$$

```
[Out] -((b*c - a*d)/(b^2*(a + b*x))) + (d*Log[a + b*x])/b^2
```

Rubi [A] time = 0.0193538, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{d\log(a+bx)}{b^2}-\frac{bc-ad}{b^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^2,x]
```

```
[Out] -((b*c - a*d)/(b^2*(a + b*x))) + (d*Log[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^2}\,dx &= \int \left(\frac{bc-ad}{b(a+bx)^2}+\frac{d}{b(a+bx)}\right)dx\\ &= -\frac{bc-ad}{b^2(a+bx)}+\frac{d\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0107254, size = 31, normalized size = 0.97

$$\frac{ad-bc}{b^2(a+bx)}+\frac{d\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^2,x]
```

```
[Out] (-(b*c) + a*d)/(b^2*(a + b*x)) + (d*Log[a + b*x])/b^2
```

Maple [A] time = 0.006, size = 39, normalized size = 1.2

$$\frac{ad}{b^2\,(bx+a)}-\frac{c}{b\,(bx+a)}+\frac{d\ln(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^2,x)
```

```
[Out] 1/b^2/(b*x+a)*a*d-1/b/(b*x+a)*c+d*ln(b*x+a)/b^2
```

Maxima [A] time = 1.05097, size = 47, normalized size = 1.47

$$-\frac{bc-ad}{b^3x+ab^2}+\frac{d\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)/(b^3*x + a*b^2) + d*log(b*x + a)/b^2
```

Fricas [A] time = 1.91972, size = 80, normalized size = 2.5

$$-\frac{bc-ad-(bdx+ad)\log(bx+a)}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b*c - a*d - (b*d*x + a*d)*log(b*x + a))/(b^3*x + a*b^2)
```

Sympy [A] time = 0.341905, size = 27, normalized size = 0.84

$$\frac{ad-bc}{ab^2+b^3x}+\frac{d\log(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**2,x)
```

```
[Out] (a*d - b*c)/(a*b**2 + b**3*x) + d*log(a + b*x)/b**2
```

Giac [A] time = 1.06566, size = 77, normalized size = 2.41

$$-\frac{d\left(\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}-\frac{a}{(bx+a)b}\right)}{b}-\frac{c}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -d*(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b))/b - c/((b*x
 + a)*b)
```

3.1243 $\int \frac{c+dx}{(a+bx)^3} dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^2}{2(a+bx)^2(bc-ad)}$$

```
[Out] -(c + d*x)^2/(2*(b*c - a*d)*(a + b*x)^2)
```

Rubi [A] time = 0.0043352, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {37}

$$-\frac{(c+dx)^2}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^3,x]
```

```
[Out] -(c + d*x)^2/(2*(b*c - a*d)*(a + b*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{c+dx}{(a+bx)^3} dx = -\frac{(c+dx)^2}{2(bc-ad)(a+bx)^2}$$

Mathematica [A] time = 0.0093039, size = 26, normalized size = 0.93

$$-\frac{ad+b(c+2dx)}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^3,x]
```

```
[Out] -(a*d + b*(c + 2*d*x))/(2*b^2*(a + b*x)^2)
```

Maple [A] time = 0.006, size = 35, normalized size = 1.3

$$-\frac{d}{b^2(bx+a)} - \frac{-ad+bc}{2b^2(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^3,x)
```

```
[Out] -d/b^2/(b*x+a)-1/2*(-a*d+b*c)/b^2/(b*x+a)^2
```

Maxima [A] time = 0.965442, size = 51, normalized size = 1.82

$$-\frac{2\,bdx+bc+ad}{2\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Fricas [A] time = 1.96827, size = 81, normalized size = 2.89

$$-\frac{2\,bdx+bc+ad}{2\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [A] time = 0.428885, size = 39, normalized size = 1.39

$$-\frac{ad+bc+2bdx}{2a^{2}b^{2}+4ab^{3}x+2b^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**3,x)
```

```
[Out] -(a*d + b*c + 2*b*d*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

Giac [A] time = 1.06438, size = 32, normalized size = 1.14

$$-\frac{2\,bdx+bc+ad}{2\,(bx+a)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/((b*x + a)^2*b^2)
```

3.1244 $\int \frac{c+dx}{(a+bx)^4} dx$

Optimal. Leaf size=38

$$-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}$$

```
[Out] -(b*c - a*d)/(3*b^2*(a + b*x)^3) - d/(2*b^2*(a + b*x)^2)
```

Rubi [A] time = 0.0199422, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^4,x]
```

```
[Out] -(b*c - a*d)/(3*b^2*(a + b*x)^3) - d/(2*b^2*(a + b*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^4} dx &= \int \left(\frac{bc-ad}{b(a+bx)^4}+\frac{d}{b(a+bx)^3}\right) dx\\ &= -\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}\end{aligned}$$

Mathematica [A] time = 0.0087701, size = 27, normalized size = 0.71

$$-\frac{ad+2bc+3bdx}{6b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^4,x]
```

```
[Out] -(2*b*c + a*d + 3*b*d*x)/(6*b^2*(a + b*x)^3)
```

Maple [A] time = 0.004, size = 35, normalized size = 0.9

$$-\frac{d}{2\,b^2\,(bx+a)^2}-\frac{-ad+bc}{3\,b^2\,(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^4,x)
```

```
[Out] -1/2*d/b^2/(b*x+a)^2-1/3*(-a*d+b*c)/b^2/(b*x+a)^3
```

Maxima [A] time = 0.956878, size = 68, normalized size = 1.79

$$-\frac{3\,bdx+2\,bc+ad}{6\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)
```

Fricas [A] time = 1.94994, size = 105, normalized size = 2.76

$$-\frac{3\,bdx+2\,bc+ad}{6\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)
```

Sympy [A] time = 0.501469, size = 53, normalized size = 1.39

$$-\frac{ad+2bc+3bdx}{6a^3b^2+18a^2b^3x+18ab^4x^2+6b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**4,x)
```

```
[Out] -(a*d + 2*b*c + 3*b*d*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6
*b**5*x**3)
```

Giac [A] time = 1.07809, size = 34, normalized size = 0.89

$$-\frac{3\,bdx+2\,bc+ad}{6\,(bx+a)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/((b*x + a)^3*b^2)
```

3.1245 $\int \frac{c+dx}{(a+bx)^5} dx$

Optimal. Leaf size=38

$$-\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}$$

```
[Out] -(b*c - a*d)/(4*b^2*(a + b*x)^4) - d/(3*b^2*(a + b*x)^3)
```

Rubi [A] time = 0.019943, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^5,x]
```

```
[Out] -(b*c - a*d)/(4*b^2*(a + b*x)^4) - d/(3*b^2*(a + b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^5} dx &= \int \left(\frac{bc-ad}{b(a+bx)^5}+\frac{d}{b(a+bx)^4}\right) dx\\ &= -\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.0097732, size = 27, normalized size = 0.71

$$-\frac{ad+3bc+4bdx}{12b^2(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^5,x]
```

```
[Out] -(3*b*c + a*d + 4*b*d*x)/(12*b^2*(a + b*x)^4)
```

Maple [A] time = 0.005, size = 35, normalized size = 0.9

$$-\frac{d}{3\,b^2\,(bx+a)^3}-\frac{-ad+bc}{4\,b^2\,(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^5,x)
```

```
[Out] -1/3*d/b^2/(b*x+a)^3-1/4*(-a*d+b*c)/b^2/(b*x+a)^4
```

Maxima [A] time = 0.948785, size = 82, normalized size = 2.16

$$-\frac{4\,bdx + 3\,bc + ad}{12\,\left(b^{6}x^{4} + 4\,ab^{5}x^{3} + 6\,a^{2}b^{4}x^{2} + 4\,a^{3}b^{3}x + a^{4}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^
3*b^3*x + a^4*b^2)
```

Fricas [A] time = 2.01209, size = 128, normalized size = 3.37

$$-\frac{4\,bdx + 3\,bc + ad}{12\,\left(b^{6}x^{4} + 4\,ab^{5}x^{3} + 6\,a^{2}b^{4}x^{2} + 4\,a^{3}b^{3}x + a^{4}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^
3*b^3*x + a^4*b^2)
```

Sympy [B] time = 0.593464, size = 65, normalized size = 1.71

$$-\frac{ad + 3bc + 4bdx}{12a^{4}b^{2} + 48a^{3}b^{3}x + 72a^{2}b^{4}x^{2} + 48ab^{5}x^{3} + 12b^{6}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**5,x)
```

```
[Out] -(a*d + 3*b*c + 4*b*d*x)/(12*a**4*b**2 + 48*a**3*b**3*x + 72*a**2*b**4*x**2
 + 48*a*b**5*x**3 + 12*b**6*x**4)
```

Giac [A] time = 1.05095, size = 55, normalized size = 1.45

$$-\frac{c}{4\,(bx + a)^{4}b} - \frac{d}{3\,(bx + a)^{3}b^{2}} + \frac{ad}{4\,(bx + a)^{4}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*c/((b*x + a)^4*b) - 1/3*d/((b*x + a)^3*b^2) + 1/4*a*d/((b*x + a)^4*b^2
)
```

3.1246 $\int (a+bx)^4(c+dx)^2\,dx$

Optimal. Leaf size=65

$$\frac{d(a+bx)^6(bc-ad)}{3b^3}+\frac{(a+bx)^5(bc-ad)^2}{5b^3}+\frac{d^2(a+bx)^7}{7b^3}$$

```
[Out] ((b*c - a*d)^2*(a + b*x)^5)/(5*b^3) + (d*(b*c - a*d)*(a + b*x)^6)/(3*b^3) +
 (d^2*(a + b*x)^7)/(7*b^3)
```

Rubi [A] time = 0.0877685, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d(a+bx)^6(bc-ad)}{3b^3}+\frac{(a+bx)^5(bc-ad)^2}{5b^3}+\frac{d^2(a+bx)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^5)/(5*b^3) + (d*(b*c - a*d)*(a + b*x)^6)/(3*b^3) +
 (d^2*(a + b*x)^7)/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^4}{b^2}+\frac{2d(bc-ad)(a+bx)^5}{b^2}+\frac{d^2(a+bx)^6}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^5}{5b^3}+\frac{d(bc-ad)(a+bx)^6}{3b^3}+\frac{d^2(a+bx)^7}{7b^3}\end{aligned}$$

Mathematica [B] time = 0.0257322, size = 148, normalized size = 2.28

$$\frac{1}{5}b^2x^5\left(6a^2d^2+8abcd+b^2c^2\right)+abx^4\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{3}a^2x^3\left(a^2d^2+8abcd+6b^2c^2\right)+a^3cx^2(ad+2bc)+a^4c^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^2,x]
```

```
[Out] a^4*c^2*x + a^3*c*(2*b*c + a*d)*x^2 + (a^2*(6*b^2*c^2 + 8*a*b*c*d + a^2*d^2
)*x^3)/3 + a*b*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^4 + (b^2*(b^2*c^2 + 8*a*b*
c*d + 6*a^2*d^2)*x^5)/5 + (b^3*d*(b*c + 2*a*d)*x^6)/3 + (b^4*d^2*x^7)/7
```

Maple [B] time = 0.002, size = 163, normalized size = 2.5

$$\frac{b^4d^2x^7}{7}+\frac{(4\,ab^3d^2+2\,b^4cd)\,x^6}{6}+\frac{(6\,b^2a^2d^2+8\,ab^3cd+b^4c^2)\,x^5}{5}+\frac{(4\,a^3bd^2+12\,b^2a^2cd+4\,ab^3c^2)\,x^4}{4}+\frac{(a^4d^2+8\,a^3bcd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^2,x)
```

```
[Out] 1/7*b^4*d^2*x^7+1/6*(4*a*b^3*d^2+2*b^4*c*d)*x^6+1/5*(6*a^2*b^2*d^2+8*a*b^3*
c*d+b^4*c^2)*x^5+1/4*(4*a^3*b*d^2+12*a^2*b^2*c*d+4*a*b^3*c^2)*x^4+1/3*(a^4*
d^2+8*a^3*b*c*d+6*a^2*b^2*c^2)*x^3+1/2*(2*a^4*c*d+4*a^3*b*c^2)*x^2+a^4*c^2*
x
```

Maxima [B] time = 0.96925, size = 211, normalized size = 3.25

$$\frac{1}{7}\,b^4d^2x^7+a^4c^2x+\frac{1}{3}\,(b^4cd+2\,ab^3d^2)x^6+\frac{1}{5}\,(b^4c^2+8\,ab^3cd+6\,a^2b^2d^2)x^5+(ab^3c^2+3\,a^2b^2cd+a^3bd^2)x^4+\frac{1}{3}\,(6\,a^2b^2c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^4*d^2*x^7 + a^4*c^2*x + 1/3*(b^4*c*d + 2*a*b^3*d^2)*x^6 + 1/5*(b^4*c^
2 + 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x^5 + (a*b^3*c^2 + 3*a^2*b^2*c*d + a^3*b*d
^2)*x^4 + 1/3*(6*a^2*b^2*c^2 + 8*a^3*b*c*d + a^4*d^2)*x^3 + (2*a^3*b*c^2 +
a^4*c*d)*x^2
```

Fricas [B] time = 1.72042, size = 363, normalized size = 5.58

$$\frac{1}{7}x^7d^2b^4+\frac{1}{3}x^6dcb^4+\frac{2}{3}x^6d^2b^3a+\frac{1}{5}x^5c^2b^4+\frac{8}{5}x^5dcb^3a+\frac{6}{5}x^5d^2b^2a^2+x^4c^2b^3a+3x^4dcb^2a^2+x^4d^2ba^3+2x^3c^2b^2a^2+\frac{8}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d^2*b^4 + 1/3*x^6*d*c*b^4 + 2/3*x^6*d^2*b^3*a + 1/5*x^5*c^2*b^4 + 8
/5*x^5*d*c*b^3*a + 6/5*x^5*d^2*b^2*a^2 + x^4*c^2*b^3*a + 3*x^4*d*c*b^2*a^2
+ x^4*d^2*b*a^3 + 2*x^3*c^2*b^2*a^2 + 8/3*x^3*d*c*b*a^3 + 1/3*x^3*d^2*a^4 +
 2*x^2*c^2*b*a^3 + x^2*d*c*a^4 + x*c^2*a^4
```

Sympy [B] time = 0.089372, size = 168, normalized size = 2.58

$$a^4c^2x+\frac{b^4d^2x^7}{7}+x^6\left(\frac{2ab^3d^2}{3}+\frac{b^4cd}{3}\right)+x^5\left(\frac{6a^2b^2d^2}{5}+\frac{8ab^3cd}{5}+\frac{b^4c^2}{5}\right)+x^4\left(a^3bd^2+3a^2b^2cd+ab^3c^2\right)+x^3\left(\frac{a^4d^2}{3}+\frac{8a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**2,x)
```

```
[Out] a**4*c**2*x + b**4*d**2*x**7/7 + x**6*(2*a*b**3*d**2/3 + b**4*c*d/3) + x**5
*(6*a**2*b**2*d**2/5 + 8*a*b**3*c*d/5 + b**4*c**2/5) + x**4*(a**3*b*d**2 +
3*a**2*b**2*c*d + a*b**3*c**2) + x**3*(a**4*d**2/3 + 8*a**3*b*c*d/3 + 2*a**
2*b**2*c**2) + x**2*(a**4*c*d + 2*a**3*b*c**2)
```

Giac [B] time = 1.05064, size = 230, normalized size = 3.54

$$\frac{1}{7}\,b^4d^2x^7+\frac{1}{3}\,b^4cdx^6+\frac{2}{3}\,ab^3d^2x^6+\frac{1}{5}\,b^4c^2x^5+\frac{8}{5}\,ab^3cdx^5+\frac{6}{5}\,a^2b^2d^2x^5+ab^3c^2x^4+3\,a^2b^2cdx^4+a^3bd^2x^4+2\,a^2b^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^4*d^2*x^7 + 1/3*b^4*c*d*x^6 + 2/3*a*b^3*d^2*x^6 + 1/5*b^4*c^2*x^5 + 8
/5*a*b^3*c*d*x^5 + 6/5*a^2*b^2*d^2*x^5 + a*b^3*c^2*x^4 + 3*a^2*b^2*c*d*x^4
+ a^3*b*d^2*x^4 + 2*a^2*b^2*c^2*x^3 + 8/3*a^3*b*c*d*x^3 + 1/3*a^4*d^2*x^3 +
 2*a^3*b*c^2*x^2 + a^4*c*d*x^2 + a^4*c^2*x
```

3.1247 $\int (a+bx)^3(c+dx)^2\,dx$

Optimal. Leaf size=65

$$\frac{2d(a+bx)^5(bc-ad)}{5b^3}+\frac{(a+bx)^4(bc-ad)^2}{4b^3}+\frac{d^2(a+bx)^6}{6b^3}$$

```
[Out] ((b*c - a*d)^2*(a + b*x)^4)/(4*b^3) + (2*d*(b*c - a*d)*(a + b*x)^5)/(5*b^3)
 + (d^2*(a + b*x)^6)/(6*b^3)
```

Rubi [A] time = 0.0636893, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2d(a+bx)^5(bc-ad)}{5b^3}+\frac{(a+bx)^4(bc-ad)^2}{4b^3}+\frac{d^2(a+bx)^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^4)/(4*b^3) + (2*d*(b*c - a*d)*(a + b*x)^5)/(5*b^3)
 + (d^2*(a + b*x)^6)/(6*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^3}{b^2}+\frac{2d(bc-ad)(a+bx)^4}{b^2}+\frac{d^2(a+bx)^5}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^4}{4b^3}+\frac{2d(bc-ad)(a+bx)^5}{5b^3}+\frac{d^2(a+bx)^6}{6b^3}\end{aligned}$$

Mathematica [A] time = 0.0146819, size = 122, normalized size = 1.88

$$\frac{1}{4}bx^4\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}ax^3\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{2}a^2cx^2(2ad+3bc)+a^3c^2x+\frac{1}{5}b^2dx^5(3ad+2bc)+\frac{1}{6}b^3d^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^2)/2 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^3)/3 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4 + (b^2*d*(2*b*c
+ 3*a*d)*x^5)/5 + (b^3*d^2*x^6)/6
```

Maple [B] time = 0.001, size = 125, normalized size = 1.9

$$\frac{b^3d^2x^6}{6}+\frac{\left(3\,ab^2d^2+2\,b^3cd\right)x^5}{5}+\frac{\left(3\,a^2bd^2+6\,ab^2cd+b^3c^2\right)x^4}{4}+\frac{\left(a^3d^2+6\,a^2bcd+3\,ab^2c^2\right)x^3}{3}+\frac{\left(2\,a^3cd+3\,a^2bc^2\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^2,x)
```

```
[Out] 1/6*b^3*d^2*x^6+1/5*(3*a*b^2*d^2+2*b^3*c*d)*x^5+1/4*(3*a^2*b*d^2+6*a*b^2*c*
d+b^3*c^2)*x^4+1/3*(a^3*d^2+6*a^2*b*c*d+3*a*b^2*c^2)*x^3+1/2*(2*a^3*c*d+3*a
^2*b*c^2)*x^2+a^3*c^2*x
```

Maxima [B] time = 0.947646, size = 167, normalized size = 2.57

$$\frac{1}{6}\,b^3d^2x^6+a^3c^2x+\frac{1}{5}\left(2\,b^3cd+3\,ab^2d^2\right)x^5+\frac{1}{4}\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^4+\frac{1}{3}\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^3+\frac{1}{2}\left(3\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*d^2*x^6 + a^3*c^2*x + 1/5*(2*b^3*c*d + 3*a*b^2*d^2)*x^5 + 1/4*(b^3*
c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^4 + 1/3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3
*d^2)*x^3 + 1/2*(3*a^2*b*c^2 + 2*a^3*c*d)*x^2
```

Fricas [B] time = 1.73361, size = 285, normalized size = 4.38

$$\frac{1}{6}x^6d^2b^3+\frac{2}{5}x^5dcb^3+\frac{3}{5}x^5d^2b^2a+\frac{1}{4}x^4c^2b^3+\frac{3}{2}x^4dcb^2a+\frac{3}{4}x^4d^2ba^2+x^3c^2b^2a+2x^3dcba^2+\frac{1}{3}x^3d^2a^3+\frac{3}{2}x^2c^2ba^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*d^2*b^3 + 2/5*x^5*d*c*b^3 + 3/5*x^5*d^2*b^2*a + 1/4*x^4*c^2*b^3 + 3
/2*x^4*d*c*b^2*a + 3/4*x^4*d^2*b*a^2 + x^3*c^2*b^2*a + 2*x^3*d*c*b*a^2 + 1/
3*x^3*d^2*a^3 + 3/2*x^2*c^2*b*a^2 + x^2*d*c*a^3 + x*c^2*a^3
```

Sympy [B] time = 0.081453, size = 133, normalized size = 2.05

$$a^3c^2x+\frac{b^3d^2x^6}{6}+x^5\left(\frac{3ab^2d^2}{5}+\frac{2b^3cd}{5}\right)+x^4\left(\frac{3a^2bd^2}{4}+\frac{3ab^2cd}{2}+\frac{b^3c^2}{4}\right)+x^3\left(\frac{a^3d^2}{3}+2a^2bcd+ab^2c^2\right)+x^2\left(a^3cd+\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**2,x)
```

```
[Out] a**3*c**2*x + b**3*d**2*x**6/6 + x**5*(3*a*b**2*d**2/5 + 2*b**3*c*d/5) + x*
*4*(3*a**2*b*d**2/4 + 3*a*b**2*c*d/2 + b**3*c**2/4) + x**3*(a**3*d**2/3 + 2
*a**2*b*c*d + a*b**2*c**2) + x**2*(a**3*c*d + 3*a**2*b*c**2/2)
```

Giac [B] time = 1.05346, size = 176, normalized size = 2.71

$$\frac{1}{6}b^3d^2x^6 + \frac{2}{5}b^3cdx^5 + \frac{3}{5}ab^2d^2x^5 + \frac{1}{4}b^3c^2x^4 + \frac{3}{2}ab^2cdx^4 + \frac{3}{4}a^2bd^2x^4 + ab^2c^2x^3 + 2\,a^2bcdx^3 + \frac{1}{3}a^3d^2x^3 + \frac{3}{2}a^2bc^2x^2 + a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*b^3*d^2*x^6 + 2/5*b^3*c*d*x^5 + 3/5*a*b^2*d^2*x^5 + 1/4*b^3*c^2*x^4 + 3
/2*a*b^2*c*d*x^4 + 3/4*a^2*b*d^2*x^4 + a*b^2*c^2*x^3 + 2*a^2*b*c*d*x^3 + 1/
3*a^3*d^2*x^3 + 3/2*a^2*b*c^2*x^2 + a^3*c*d*x^2 + a^3*c^2*x
```

3.1248 $\int (a+bx)^2(c+dx)^2\,dx$

Optimal. Leaf size=65

$$\frac{d(a+bx)^4(bc-ad)}{2b^3}+\frac{(a+bx)^3(bc-ad)^2}{3b^3}+\frac{d^2(a+bx)^5}{5b^3}$$

```
[Out] ((b*c - a*d)^2*(a + b*x)^3)/(3*b^3) + (d*(b*c - a*d)*(a + b*x)^4)/(2*b^3) +
 (d^2*(a + b*x)^5)/(5*b^3)
```

Rubi [A] time = 0.0454584, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d(a+bx)^4(bc-ad)}{2b^3}+\frac{(a+bx)^3(bc-ad)^2}{3b^3}+\frac{d^2(a+bx)^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^3)/(3*b^3) + (d*(b*c - a*d)*(a + b*x)^4)/(2*b^3) +
 (d^2*(a + b*x)^5)/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^2}{b^2}+\frac{2d(bc-ad)(a+bx)^3}{b^2}+\frac{d^2(a+bx)^4}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^3}{3b^3}+\frac{d(bc-ad)(a+bx)^4}{2b^3}+\frac{d^2(a+bx)^5}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.0100173, size = 79, normalized size = 1.22

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{2}bdx^4(ad+bc)+acx^2(ad+bc)+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^2,x]
```

```
[Out] a^2*c^2*x + a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3)/3 +
 (b*d*(b*c + a*d)*x^4)/2 + (b^2*d^2*x^5)/5
```

Maple [A] time = 0., size = 87, normalized size = 1.3

$$\frac{b^{2}d^{2}x^{5}}{5}+\frac{\left(2\,abd^{2}+2\,b^{2}cd\right)x^{4}}{4}+\frac{\left(a^{2}d^{2}+4\,abcd+b^{2}c^{2}\right)x^{3}}{3}+\frac{\left(2\,a^{2}cd+2\,abc^{2}\right)x^{2}}{2}+a^{2}c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^2,x)
```

```
[Out] 1/5*b^2*d^2*x^5+1/4*(2*a*b*d^2+2*b^2*c*d)*x^4+1/3*(a^2*d^2+4*a*b*c*d+b^2*c^
2)*x^3+1/2*(2*a^2*c*d+2*a*b*c^2)*x^2+a^2*c^2*x
```

Maxima [A] time = 0.952025, size = 109, normalized size = 1.68

$$\frac{1}{5}\,b^{2}d^{2}x^{5}+a^{2}c^{2}x+\frac{1}{2}\left(b^{2}cd+abd^{2}\right)x^{4}+\frac{1}{3}\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)x^{3}+\left(abc^{2}+a^{2}cd\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*d^2*x^5 + a^2*c^2*x + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/3*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^3 + (a*b*c^2 + a^2*c*d)*x^2
```

Fricas [A] time = 1.70661, size = 198, normalized size = 3.05

$$\frac{1}{5}x^{5}d^{2}b^{2}+\frac{1}{2}x^{4}dcb^{2}+\frac{1}{2}x^{4}d^{2}ba+\frac{1}{3}x^{3}c^{2}b^{2}+\frac{4}{3}x^{3}dcba+\frac{1}{3}x^{3}d^{2}a^{2}+x^{2}c^{2}ba+x^{2}dca^{2}+xc^{2}a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d^2*b^2 + 1/2*x^4*d*c*b^2 + 1/2*x^4*d^2*b*a + 1/3*x^3*c^2*b^2 + 4/3
*x^3*d*c*b*a + 1/3*x^3*d^2*a^2 + x^2*c^2*b*a + x^2*d*c*a^2 + x*c^2*a^2
```

Sympy [A] time = 0.074827, size = 87, normalized size = 1.34

$$a^{2}c^{2}x+\frac{b^{2}d^{2}x^{5}}{5}+x^{4}\left(\frac{abd^{2}}{2}+\frac{b^{2}cd}{2}\right)+x^{3}\left(\frac{a^{2}d^{2}}{3}+\frac{4abcd}{3}+\frac{b^{2}c^{2}}{3}\right)+x^{2}\left(a^{2}cd+abc^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**5/5 + x**4*(a*b*d**2/2 + b**2*c*d/2) + x**3*(a**
2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3) + x**2*(a**2*c*d + a*b*c**2)
```

Giac [A] time = 1.05125, size = 120, normalized size = 1.85

$$\frac{1}{5}\,b^{2}d^{2}x^{5}+\frac{1}{2}\,b^{2}cdx^{4}+\frac{1}{2}\,abd^{2}x^{4}+\frac{1}{3}\,b^{2}c^{2}x^{3}+\frac{4}{3}\,abcdx^{3}+\frac{1}{3}\,a^{2}d^{2}x^{3}+abc^{2}x^{2}+a^{2}cdx^{2}+a^{2}c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*d^2*x^5 + 1/2*b^2*c*d*x^4 + 1/2*a*b*d^2*x^4 + 1/3*b^2*c^2*x^3 + 4/3
*a*b*c*d*x^3 + 1/3*a^2*d^2*x^3 + a*b*c^2*x^2 + a^2*c*d*x^2 + a^2*c^2*x
```

3.1249 $\int (a+bx)(c+dx)^2\,dx$

Optimal. Leaf size=38

$$\frac{b(c+dx)^4}{4d^2}-\frac{(c+dx)^3(bc-ad)}{3d^2}$$

```
[Out] -((b*c - a*d)*(c + d*x)^3)/(3*d^2) + (b*(c + d*x)^4)/(4*d^2)
```

Rubi [A] time = 0.0273636, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^4}{4d^2}-\frac{(c+dx)^3(bc-ad)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^2,x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^3)/(3*d^2) + (b*(c + d*x)^4)/(4*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^2\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^2}{d}+\frac{b(c+dx)^3}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^3}{3d^2}+\frac{b(c+dx)^4}{4d^2}\end{aligned}$$

Mathematica [A] time = 0.0097692, size = 47, normalized size = 1.24

$$\frac{1}{12}x\left(4dx^2(ad+2bc)+6cx(2ad+bc)+12ac^2+3bd^2x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^2,x]
```

```
[Out] (x*(12*a*c^2 + 6*c*(b*c + 2*a*d)*x + 4*d*(2*b*c + a*d)*x^2 + 3*b*d^2*x^3))/
12
```

Maple [A] time = 0., size = 49, normalized size = 1.3

$$\frac{bd^2x^4}{4}+\frac{\left(ad^2+2\,bcd\right)x^3}{3}+\frac{\left(2\,acd+bc^2\right)x^2}{2}+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^2,x)
```

```
[Out] 1/4*b*d^2*x^4+1/3*(a*d^2+2*b*c*d)*x^3+1/2*(2*a*c*d+b*c^2)*x^2+a*c^2*x
```

Maxima [A] time = 0.948592, size = 65, normalized size = 1.71

$$\frac{1}{4}\,bd^{2}x^{4} + ac^{2}x + \frac{1}{3}\left(2\,bcd + ad^{2}\right)x^{3} + \frac{1}{2}\left(bc^{2} + 2\,acd\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*d^2*x^4 + a*c^2*x + 1/3*(2*b*c*d + a*d^2)*x^3 + 1/2*(b*c^2 + 2*a*c*d)
*x^2
```

Fricas [A] time = 1.78969, size = 115, normalized size = 3.03

$$\frac{1}{4}x^{4}d^{2}b + \frac{2}{3}x^{3}dcb + \frac{1}{3}x^{3}d^{2}a + \frac{1}{2}x^{2}c^{2}b + x^{2}dca + xc^{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*d^2*b + 2/3*x^3*d*c*b + 1/3*x^3*d^2*a + 1/2*x^2*c^2*b + x^2*d*c*a +
 x*c^2*a
```

Sympy [A] time = 0.067188, size = 49, normalized size = 1.29

$$ac^{2}x + \frac{bd^{2}x^{4}}{4} + x^{3}\left(\frac{ad^{2}}{3} + \frac{2bcd}{3}\right) + x^{2}\left(acd + \frac{bc^{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**4/4 + x**3*(a*d**2/3 + 2*b*c*d/3) + x**2*(a*c*d + b*c*
*2/2)
```

Giac [A] time = 1.05866, size = 66, normalized size = 1.74

$$\frac{1}{4}\,bd^{2}x^{4} + \frac{2}{3}\,bcdx^{3} + \frac{1}{3}\,ad^{2}x^{3} + \frac{1}{2}\,bc^{2}x^{2} + acdx^{2} + ac^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*b*d^2*x^4 + 2/3*b*c*d*x^3 + 1/3*a*d^2*x^3 + 1/2*b*c^2*x^2 + a*c*d*x^2 +
 a*c^2*x
```

3.1250 $\int (c+dx)^2\,dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^3}{3d}$$

```
[Out] (c + d*x)^3/(3*d)
```

Rubi [A] time = 0.0015332, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2,x]
```

```
[Out] (c + d*x)^3/(3*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^2\,dx=\frac{(c+dx)^3}{3d}$$

Mathematica [A] time = 0.0013158, size = 14, normalized size = 1.

$$\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2,x]
```

```
[Out] (c + d*x)^3/(3*d)
```

Maple [A] time = 0., size = 13, normalized size = 0.9

$$\frac{(dx+c)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2,x)
```

```
[Out] 1/3*(d*x+c)^3/d
```

Maxima [A] time = 0.950799, size = 27, normalized size = 1.93

$$\frac{1}{3}d^2x^3 + cdx^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x
```

Fricas [A] time = 1.76396, size = 42, normalized size = 3.

$$\frac{1}{3}x^3d^2 + x^2dc + xc^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*d^2 + x^2*d*c + x*c^2
```

Sympy [B] time = 0.056934, size = 19, normalized size = 1.36

$$c^2x + cdx^2 + \frac{d^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2,x)
```

```
[Out] c**2*x + c*d*x**2 + d**2*x**3/3
```

Giac [A] time = 1.07417, size = 16, normalized size = 1.14

$$\frac{(dx+c)^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(d*x + c)^3/d
```

3.1251 $\int \frac{(c+dx)^2}{a+bx} dx$

Optimal. Leaf size=49

$$\frac{dx(bc-ad)}{b^2}+\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{(c+dx)^2}{2b}$$

```
[Out] (d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b*c - a*d)^2*Log[a + b*x])/b^
3
```

Rubi [A] time = 0.0186455, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{dx(bc-ad)}{b^2}+\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{(c+dx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b*c - a*d)^2*Log[a + b*x])/b^
3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{a+bx}\,dx &= \int \left(\frac{d(bc-ad)}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(c+dx)}{b}\right)dx\\ &= \frac{d(bc-ad)x}{b^2}+\frac{(c+dx)^2}{2b}+\frac{(bc-ad)^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0160432, size = 43, normalized size = 0.88

$$\frac{bdx(-2ad+4bc+bdx)+2(bc-ad)^2\log(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x),x]
```

```
[Out] (b*d*x*(4*b*c - 2*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[a + b*x])/(2*b^3)
```

Maple [A] time = 0.003, size = 74, normalized size = 1.5

$$\frac{d^2x^2}{2b} - \frac{ad^2x}{b^2} + 2\frac{dxc}{b} + \frac{a^2\ln(bx+a)d^2}{b^3} - 2\frac{a\ln(bx+a)cd}{b^2} + \frac{\ln(bx+a)c^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a),x)
```

```
[Out] 1/2*d^2/b*x^2-d^2/b^2*a*x+2*d/b*x*c+1/b^3*ln(b*x+a)*a^2*d^2-2/b^2*ln(b*x+a)
*a*c*d+1/b*ln(b*x+a)*c^2
```

Maxima [A] time = 0.976674, size = 82, normalized size = 1.67

$$\frac{bd^2x^2 + 2\left(2\,bcd - ad^2\right)x}{2\,b^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*d^2*x^2 + 2*(2*b*c*d - a*d^2)*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(b*x + a)/b^3
```

Fricas [A] time = 2.06895, size = 135, normalized size = 2.76

$$\frac{b^2d^2x^2 + 2\left(2\,b^2cd - abd^2\right)x + 2\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log(bx+a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*(2*b^2*c*d - a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(b*x + a))/b^3
```

Sympy [A] time = 0.378542, size = 44, normalized size = 0.9

$$\frac{d^2x^2}{2b} - \frac{x\left(ad^2 - 2bcd\right)}{b^2} + \frac{(ad-bc)^2\log(a+bx)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a),x)
```

```
[Out] d**2*x**2/(2*b) - x*(a*d**2 - 2*b*c*d)/b**2 + (a*d - b*c)**2*log(a + b*x)/b
**3
```

Giac [A] time = 1.06733, size = 81, normalized size = 1.65

$$\frac{bd^2x^2 + 4\,bcdx - 2\,ad^2x}{2\,b^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(|bx + a|\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2 + 4*b*c*d*x - 2*a*d^2*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(abs(b*x + a))/b^3
```

3.1252 $\int \frac{(c+dx)^2}{(a+bx)^2} dx$

Optimal. Leaf size=51

$$-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}+\frac{d^2x}{b^2}$$

```
[Out] (d^2*x)/b^2 - (b*c - a*d)^2/(b^3*(a + b*x)) + (2*d*(b*c - a*d)*Log[a + b*x]
)/b^3
```

Rubi [A] time = 0.0346731, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^2,x]
```

```
[Out] (d^2*x)/b^2 - (b*c - a*d)^2/(b^3*(a + b*x)) + (2*d*(b*c - a*d)*Log[a + b*x]
)/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^2} dx &= \int \left(\frac{d^2}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)^2}+\frac{2d(bc-ad)}{b^2(a+bx)}\right) dx\\ &= \frac{d^2x}{b^2}-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0362308, size = 47, normalized size = 0.92

$$\frac{-\frac{(bc-ad)^2}{a+bx}+2d(bc-ad)\log(a+bx)+bd^2x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^2,x]
```

```
[Out] (b*d^2*x - (b*c - a*d)^2/(a + b*x) + 2*d*(b*c - a*d)*Log[a + b*x])/b^3
```

Maple [A] time = 0.005, size = 86, normalized size = 1.7

$$\frac{d^2x}{b^2}-\frac{a^2d^2}{b^3\left(bx+a\right)}+2\frac{acd}{b^2\left(bx+a\right)}-\frac{c^2}{b\left(bx+a\right)}-2\frac{d^2\ln\left(bx+a\right)a}{b^3}+2\frac{d\ln\left(bx+a\right)c}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^2,x)
```

```
[Out] d^2*x/b^2-1/b^3/(b*x+a)*a^2*d^2+2/b^2/(b*x+a)*a*c*d-1/b/(b*x+a)*c^2-2/b^3*d
^2*ln(b*x+a)*a+2/b^2*d*ln(b*x+a)*c
```

Maxima [A] time = 0.960575, size = 90, normalized size = 1.76

$$\frac{d^2x}{b^2}-\frac{b^2c^2-2\,abcd+a^2d^2}{b^4x+ab^3}+\frac{2\left(bcd-ad^2\right)\log\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] d^2*x/b^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b^4*x + a*b^3) + 2*(b*c*d - a*
d^2)*log(b*x + a)/b^3
```

Fricas [A] time = 1.93322, size = 184, normalized size = 3.61

$$\frac{b^2d^2x^2+abd^2x-b^2c^2+2\,abcd-a^2d^2+2\left(abcd-a^2d^2+\left(b^2cd-abd^2\right)x\right)\log\left(bx+a\right)}{b^4x+ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + a*b*d^2*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 + 2*(a*b*c*d - a^2
*d^2 + (b^2*c*d - a*b*d^2)*x)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A] time = 0.562278, size = 60, normalized size = 1.18

$$-\frac{a^2d^2-2abcd+b^2c^2}{ab^3+b^4x}+\frac{d^2x}{b^2}-\frac{2d\left(ad-bc\right)\log\left(a+bx\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**2,x)
```

```
[Out] -(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(a*b**3 + b**4*x) + d**2*x/b**2 - 2*d*
(a*d - b*c)*log(a + b*x)/b**3
```

Giac [A] time = 1.06726, size = 132, normalized size = 2.59

$$\frac{(bx+a)d^2}{b^3} - \frac{2\left(bcd - ad^2\right)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3} - \frac{\frac{b^3c^2}{bx+a} - \frac{2\,ab^2cd}{bx+a} + \frac{a^2bd^2}{bx+a}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)*d^2/b^3 - 2*(b*c*d - a*d^2)*log(abs(b*x + a)/((b*x + a)^2*abs(b))
)/b^3 - (b^3*c^2/(b*x + a) - 2*a*b^2*c*d/(b*x + a) + a^2*b*d^2/(b*x + a))/b
^4
```

3.1253 $\int \frac{(c+dx)^2}{(a+bx)^3} dx$

Optimal. Leaf size=59

$$-\frac{2d(bc-ad)}{b^3(a+bx)}-\frac{(bc-ad)^2}{2b^3(a+bx)^2}+\frac{d^2\log(a+bx)}{b^3}$$

```
[Out] -(b*c - a*d)^2/(2*b^3*(a + b*x)^2) - (2*d*(b*c - a*d))/(b^3*(a + b*x)) + (d
^2*Log[a + b*x])/b^3
```

Rubi [A] time = 0.0350432, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2d(bc-ad)}{b^3(a+bx)}-\frac{(bc-ad)^2}{2b^3(a+bx)^2}+\frac{d^2\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^3,x]
```

```
[Out] -(b*c - a*d)^2/(2*b^3*(a + b*x)^2) - (2*d*(b*c - a*d))/(b^3*(a + b*x)) + (d
^2*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^3}dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^3}+\frac{2d(bc-ad)}{b^2(a+bx)^2}+\frac{d^2}{b^2(a+bx)}\right)dx\\ &= -\frac{(bc-ad)^2}{2b^3(a+bx)^2}-\frac{2d(bc-ad)}{b^3(a+bx)}+\frac{d^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0234555, size = 49, normalized size = 0.83

$$\frac{2d^2\log(a+bx)-\frac{(bc-ad)(3ad+b(c+4dx))}{(a+bx)^2}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^3,x]
```

```
[Out] (-(((b*c - a*d)*(3*a*d + b*(c + 4*d*x)))/(a + b*x)^2) + 2*d^2*Log[a + b*x])
/(2*b^3)
```

Maple [A] time = 0.005, size = 92, normalized size = 1.6

$$2\frac{ad^2}{b^3(bx+a)}-2\frac{cd}{b^2(bx+a)}-\frac{a^2d^2}{2b^3(bx+a)^2}+\frac{acd}{b^2(bx+a)^2}-\frac{c^2}{2b(bx+a)^2}+\frac{d^2\ln(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^3,x)
```

```
[Out] 2/b^3*d^2/(b*x+a)*a-2/b^2*d/(b*x+a)*c-1/2/b^3/(b*x+a)^2*a^2*d^2+1/b^2/(b*x+
a)^2*a*c*d-1/2/b/(b*x+a)^2*c^2+d^2*ln(b*x+a)/b^3
```

Maxima [A] time = 0.964279, size = 107, normalized size = 1.81

$$-\frac{b^2c^2+2\,abcd-3\,a^2d^2+4\left(b^2cd-abd^2\right)x}{2\left(b^5x^2+2\,ab^4x+a^2b^3\right)}+\frac{d^2\log(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3) + d^2*log(b*x + a)/b^3
```

Fricas [A] time = 1.88601, size = 207, normalized size = 3.51

$$-\frac{b^2c^2+2\,abcd-3\,a^2d^2+4\left(b^2cd-abd^2\right)x-2\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\log(bx+a)}{2\left(b^5x^2+2\,ab^4x+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d^
2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3
)
```

Sympy [A] time = 0.630731, size = 80, normalized size = 1.36

$$\frac{3a^2d^2-2abcd-b^2c^2+x\left(4abd^2-4b^2cd\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{d^2\log(a+bx)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**3,x)
```

```
[Out] (3*a**2*d**2 - 2*a*b*c*d - b**2*c**2 + x*(4*a*b*d**2 - 4*b**2*c*d))/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) + d**2*log(a + b*x)/b**3
```

Giac [A] time = 1.0557, size = 92, normalized size = 1.56

$$\frac{d^2\log\left(|bx+a|\right)}{b^3}-\frac{4\left(bcd-ad^2\right)x+\frac{b^2c^2+2\,abcd-3\,a^2d^2}{b}}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] d^2*log(abs(b*x + a))/b^3 - 1/2*(4*(b*c*d - a*d^2)*x + (b^2*c^2 + 2*a*b*c*d
 - 3*a^2*d^2)/b)/((b*x + a)^2*b^2)
```

3.1254 $\int \frac{(c+dx)^2}{(a+bx)^4} dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^3}{3(a+bx)^3(bc-ad)}$$

```
[Out] -(c + d*x)^3/(3*(b*c - a*d)*(a + b*x)^3)
```

Rubi [A] time = 0.0042436, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(c+dx)^3}{3(a+bx)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^4,x]
```

```
[Out] -(c + d*x)^3/(3*(b*c - a*d)*(a + b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+bx)^4} dx = -\frac{(c+dx)^3}{3(bc-ad)(a+bx)^3}$$

Mathematica [A] time = 0.0215738, size = 53, normalized size = 1.89

$$-\frac{a^2d^2+abd(c+3dx)+b^2\left(c^2+3cdx+3d^2x^2\right)}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^4,x]
```

```
[Out] -(a^2*d^2 + a*b*d*(c + 3*d*x) + b^2*(c^2 + 3*c*d*x + 3*d^2*x^2))/(3*b^3*(a
+ b*x)^3)
```

Maple [B] time = 0.004, size = 70, normalized size = 2.5

$$-\frac{d^2}{b^3(bx+a)}+\frac{d(ad-bc)}{b^3(bx+a)^2}-\frac{a^2d^2-2abcd+b^2c^2}{3b^3(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^4,x)
```

```
[Out] -d^2/b^3/(b*x+a)+d*(a*d-b*c)/b^3/(b*x+a)^2-1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/
b^3/(b*x+a)^3
```

Maxima [B] time = 0.963286, size = 113, normalized size = 4.04

$$-\frac{3\,b^2d^2x^2+b^2c^2+abcd+a^2d^2+3\left(b^2cd+abd^2\right)x}{3\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Fricas [B] time = 1.98506, size = 170, normalized size = 6.07

$$-\frac{3\,b^2d^2x^2+b^2c^2+abcd+a^2d^2+3\left(b^2cd+abd^2\right)x}{3\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Sympy [B] time = 0.773407, size = 88, normalized size = 3.14

$$-\frac{a^2d^2+abcd+b^2c^2+3b^2d^2x^2+x\left(3abd^2+3b^2cd\right)}{3a^3b^3+9a^2b^4x+9ab^5x^2+3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**4,x)
```

```
[Out] -(a**2*d**2 + a*b*c*d + b**2*c**2 + 3*b**2*d**2*x**2 + x*(3*a*b*d**2 + 3*b*
*2*c*d))/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**2 + 3*b**6*x**3)
```

Giac [B] time = 1.05775, size = 80, normalized size = 2.86

$$-\frac{3\,b^2d^2x^2+3\,b^2cdx+3\,abd^2x+b^2c^2+abcd+a^2d^2}{3\,(bx+a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="giac")

[Out] -1/3*(3*b^2*d^2*x^2 + 3*b^2*c*d*x + 3*a*b*d^2*x + b^2*c^2 + a*b*c*d + a^2*d
^2)/((b*x + a)^3*b^3)
```

3.1255 $\int \frac{(c+dx)^2}{(a+bx)^5} dx$

Optimal. Leaf size=65

$$-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{d^2}{2b^3(a+bx)^2}$$

```
[Out] -(b*c - a*d)^2/(4*b^3*(a + b*x)^4) - (2*d*(b*c - a*d))/(3*b^3*(a + b*x)^3)
- d^2/(2*b^3*(a + b*x)^2)
```

Rubi [A] time = 0.0331657, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{d^2}{2b^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^5,x]
```

```
[Out] -(b*c - a*d)^2/(4*b^3*(a + b*x)^4) - (2*d*(b*c - a*d))/(3*b^3*(a + b*x)^3)
- d^2/(2*b^3*(a + b*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^5}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^5}+\frac{2d(bc-ad)}{b^2(a+bx)^4}+\frac{d^2}{b^2(a+bx)^3}\right)dx\\ &= -\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{d^2}{2b^3(a+bx)^2}\end{aligned}$$

Mathematica [A] time = 0.0187041, size = 56, normalized size = 0.86

$$-\frac{a^2d^2+2abd(c+2dx)+b^2\left(3c^2+8cdx+6d^2x^2\right)}{12b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^5,x]
```

```
[Out] -(a^2*d^2 + 2*a*b*d*(c + 2*d*x) + b^2*(3*c^2 + 8*c*d*x + 6*d^2*x^2))/(12*b^
3*(a + b*x)^4)
```

Maple [A] time = 0.005, size = 71, normalized size = 1.1

$$-\frac{d^2}{2\,b^3\,(bx+a)^2}+\frac{2\,d\,(ad-bc)}{3\,b^3\,(bx+a)^3}-\frac{a^2d^2-2\,abcd+b^2c^2}{4\,b^3\,(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^5,x)
```

```
[Out] -1/2*d^2/b^3/(b*x+a)^2+2/3*d*(a*d-b*c)/b^3/(b*x+a)^3-1/4*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/b^3/(b*x+a)^4
```

Maxima [A] time = 0.962005, size = 132, normalized size = 2.03

$$-\frac{6\,b^2d^2x^2+3\,b^2c^2+2\,abcd+a^2d^2+4\left(2\,b^2cd+abd^2\right)x}{12\left(b^7x^4+4\,ab^6x^3+6\,a^2b^5x^2+4\,a^3b^4x+a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*b
*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

Fricas [A] time = 2.00984, size = 201, normalized size = 3.09

$$-\frac{6\,b^2d^2x^2+3\,b^2c^2+2\,abcd+a^2d^2+4\left(2\,b^2cd+abd^2\right)x}{12\left(b^7x^4+4\,ab^6x^3+6\,a^2b^5x^2+4\,a^3b^4x+a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*b
*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

Sympy [A] time = 0.938739, size = 104, normalized size = 1.6

$$-\frac{a^2d^2+2abcd+3b^2c^2+6b^2d^2x^2+x\left(4abd^2+8b^2cd\right)}{12a^4b^3+48a^3b^4x+72a^2b^5x^2+48ab^6x^3+12b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**5,x)
```

```
[Out] -(a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2 + 6*b**2*d**2*x**2 + x*(4*a*b*d**2 +
8*b**2*c*d))/(12*a**4*b**3 + 48*a**3*b**4*x + 72*a**2*b**5*x**2 + 48*a*b**6
*x**3 + 12*b**7*x**4)
```

Giac [A] time = 1.06059, size = 130, normalized size = 2.

$$-\frac{\frac{3c^2}{(bx+a)^4}+\frac{8cd}{(bx+a)^3b}-\frac{6acd}{(bx+a)^4b}+\frac{6d^2}{(bx+a)^2b^2}-\frac{8ad^2}{(bx+a)^3b^2}+\frac{3a^2d^2}{(bx+a)^4b^2}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/12*(3*c^2/(b*x + a)^4 + 8*c*d/((b*x + a)^3*b) - 6*a*c*d/((b*x + a)^4*b)
+ 6*d^2/((b*x + a)^2*b^2) - 8*a*d^2/((b*x + a)^3*b^2) + 3*a^2*d^2/((b*x + a
)^4*b^2))/b
```

3.1256 $\int \frac{(c+dx)^2}{(a+bx)^6} \, dx$

Optimal. Leaf size=65

$$-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d^2}{3b^3(a+bx)^3}$$

```
[Out] -(b*c - a*d)^2/(5*b^3*(a + b*x)^5) - (d*(b*c - a*d))/(2*b^3*(a + b*x)^4) -
d^2/(3*b^3*(a + b*x)^3)
```

Rubi [A] time = 0.0334299, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d^2}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^6,x]
```

```
[Out] -(b*c - a*d)^2/(5*b^3*(a + b*x)^5) - (d*(b*c - a*d))/(2*b^3*(a + b*x)^4) -
d^2/(3*b^3*(a + b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^6} \, dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^6}+\frac{2d(bc-ad)}{b^2(a+bx)^5}+\frac{d^2}{b^2(a+bx)^4}\right) dx \\ &= -\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{d^2}{3b^3(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.0246052, size = 57, normalized size = 0.88

$$-\frac{a^2d^2+abd(3c+5dx)+b^2\left(6c^2+15cdx+10d^2x^2\right)}{30b^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^6,x]
```

```
[Out] -(a^2*d^2 + a*b*d*(3*c + 5*d*x) + b^2*(6*c^2 + 15*c*d*x + 10*d^2*x^2))/(30*
b^3*(a + b*x)^5)
```

Maple [A] time = 0.005, size = 71, normalized size = 1.1

$$-\frac{d^2}{3\,b^3\,(bx+a)^3}-\frac{a^2d^2-2\,abcd+b^2c^2}{5\,b^3\,(bx+a)^5}+\frac{d\,(ad-bc)}{2\,b^3\,(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^6,x)
```

```
[Out] -1/3*d^2/b^3/(b*x+a)^3-1/5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^5+1/2*d*
(a*d-b*c)/b^3/(b*x+a)^4
```

Maxima [A] time = 1.10207, size = 147, normalized size = 2.26

$$-\frac{10\,b^2d^2x^2+6\,b^2c^2+3\,abcd+a^2d^2+5\left(3\,b^2cd+abd^2\right)x}{30\left(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a*
b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

Fricas [A] time = 1.99109, size = 227, normalized size = 3.49

$$-\frac{10\,b^2d^2x^2+6\,b^2c^2+3\,abcd+a^2d^2+5\left(3\,b^2cd+abd^2\right)x}{30\left(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a*
b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

Sympy [B] time = 1.1061, size = 116, normalized size = 1.78

$$-\frac{a^2d^2+3abcd+6b^2c^2+10b^2d^2x^2+x\left(5abd^2+15b^2cd\right)}{30a^5b^3+150a^4b^4x+300a^3b^5x^2+300a^2b^6x^3+150ab^7x^4+30b^8x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**6,x)
```

```
[Out] -(a**2*d**2 + 3*a*b*c*d + 6*b**2*c**2 + 10*b**2*d**2*x**2 + x*(5*a*b*d**2 +
 15*b**2*c*d))/(30*a**5*b**3 + 150*a**4*b**4*x + 300*a**3*b**5*x**2 + 300*a
```

```
**2*b**6*x**3 + 150*a*b**7*x**4 + 30*b**8*x**5)
```

Giac [A] time = 1.08119, size = 82, normalized size = 1.26

$$-\frac{10\,b^2d^2x^2+15\,b^2cdx+5\,abd^2x+6\,b^2c^2+3\,abcd+a^2d^2}{30\,(bx+a)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 15*b^2*c*d*x + 5*a*b*d^2*x + 6*b^2*c^2 + 3*a*b*c*d
+ a^2*d^2)/((b*x + a)^5*b^3)
```

3.1257 $\int \frac{(c+dx)^2}{(a+bx)^7} dx$

Optimal. Leaf size=65

$$-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{d^2}{4b^3(a+bx)^4}$$

```
[Out] -(b*c - a*d)^2/(6*b^3*(a + b*x)^6) - (2*d*(b*c - a*d))/(5*b^3*(a + b*x)^5)
- d^2/(4*b^3*(a + b*x)^4)
```

Rubi [A] time = 0.0329904, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{d^2}{4b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^7,x]
```

```
[Out] -(b*c - a*d)^2/(6*b^3*(a + b*x)^6) - (2*d*(b*c - a*d))/(5*b^3*(a + b*x)^5)
- d^2/(4*b^3*(a + b*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^7}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^7}+\frac{2d(bc-ad)}{b^2(a+bx)^6}+\frac{d^2}{b^2(a+bx)^5}\right)dx\\ &= -\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{d^2}{4b^3(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.0202034, size = 58, normalized size = 0.89

$$-\frac{a^2d^2+2abd(2c+3dx)+b^2\left(10c^2+24cdx+15d^2x^2\right)}{60b^3(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^7,x]
```

```
[Out] -(a^2*d^2 + 2*a*b*d*(2*c + 3*d*x) + b^2*(10*c^2 + 24*c*d*x + 15*d^2*x^2))/(
60*b^3*(a + b*x)^6)
```

Maple [A] time = 0.006, size = 71, normalized size = 1.1

$$-\frac{a^{2}d^{2}-2\,abcd+b^{2}c^{2}}{6\,b^{3}\left(bx+a\right)^{6}}+\frac{2\,d\left(ad-bc\right)}{5\,b^{3}\left(bx+a\right)^{5}}-\frac{d^{2}}{4\,b^{3}\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^7,x)
```

```
[Out] -1/6*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^6+2/5*d*(a*d-b*c)/b^3/(b*x+a)^
5-1/4*d^2/b^3/(b*x+a)^4
```

Maxima [B] time = 1.15195, size = 162, normalized size = 2.49

$$-\frac{15\,b^{2}d^{2}x^{2}+10\,b^{2}c^{2}+4\,abcd+a^{2}d^{2}+6\left(4\,b^{2}cd+abd^{2}\right)x}{60\left(b^{9}x^{6}+6\,ab^{8}x^{5}+15\,a^{2}b^{7}x^{4}+20\,a^{3}b^{6}x^{3}+15\,a^{4}b^{5}x^{2}+6\,a^{5}b^{4}x+a^{6}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d + a
*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*a^
4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Fricas [B] time = 1.91312, size = 251, normalized size = 3.86

$$-\frac{15\,b^{2}d^{2}x^{2}+10\,b^{2}c^{2}+4\,abcd+a^{2}d^{2}+6\left(4\,b^{2}cd+abd^{2}\right)x}{60\left(b^{9}x^{6}+6\,ab^{8}x^{5}+15\,a^{2}b^{7}x^{4}+20\,a^{3}b^{6}x^{3}+15\,a^{4}b^{5}x^{2}+6\,a^{5}b^{4}x+a^{6}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d + a
*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*a^
4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Sympy [B] time = 1.34307, size = 128, normalized size = 1.97

$$-\frac{a^{2}d^{2}+4abcd+10b^{2}c^{2}+15b^{2}d^{2}x^{2}+x\left(6abd^{2}+24b^{2}cd\right)}{60a^{6}b^{3}+360a^{5}b^{4}x+900a^{4}b^{5}x^{2}+1200a^{3}b^{6}x^{3}+900a^{2}b^{7}x^{4}+360ab^{8}x^{5}+60b^{9}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**7,x)
```

```
[Out] -(a**2*d**2 + 4*a*b*c*d + 10*b**2*c**2 + 15*b**2*d**2*x**2 + x*(6*a*b*d**2
+ 24*b**2*c*d))/(60*a**6*b**3 + 360*a**5*b**4*x + 900*a**4*b**5*x**2 + 1200
```

```
*a**3*b**6*x**3 + 900*a**2*b**7*x**4 + 360*a*b**8*x**5 + 60*b**9*x**6)
```

Giac [A] time = 1.07213, size = 82, normalized size = 1.26

$$-\frac{15\,b^2d^2x^2+24\,b^2cdx+6\,abd^2x+10\,b^2c^2+4\,abcd+a^2d^2}{60\,(bx+a)^6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 24*b^2*c*d*x + 6*a*b*d^2*x + 10*b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)/((b*x + a)^6*b^3)
```

3.1258 $\int (a+bx)^5(c+dx)^3\,dx$

Optimal. Leaf size=92

$$\frac{3d^2(a+bx)^8(bc-ad)}{8b^4}+\frac{3d(a+bx)^7(bc-ad)^2}{7b^4}+\frac{(a+bx)^6(bc-ad)^3}{6b^4}+\frac{d^3(a+bx)^9}{9b^4}$$

```
[Out] ((b*c - a*d)^3*(a + b*x)^6)/(6*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^7)/(7*b^
4) + (3*d^2*(b*c - a*d)*(a + b*x)^8)/(8*b^4) + (d^3*(a + b*x)^9)/(9*b^4)
```

Rubi [A] time = 0.157189, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3d^2(a+bx)^8(bc-ad)}{8b^4}+\frac{3d(a+bx)^7(bc-ad)^2}{7b^4}+\frac{(a+bx)^6(bc-ad)^3}{6b^4}+\frac{d^3(a+bx)^9}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^3,x]
```


```
[Out] ((b*c - a*d)^3*(a + b*x)^6)/(6*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^7)/(7*b^
4) + (3*d^2*(b*c - a*d)*(a + b*x)^8)/(8*b^4) + (d^3*(a + b*x)^9)/(9*b^4)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^5}{b^3}+\frac{3d(bc-ad)^2(a+bx)^6}{b^3}+\frac{3d^2(bc-ad)(a+bx)^7}{b^3}+\frac{d^3(a+bx)^8}{b^3}\right)dx\\ &= \frac{(bc-ad)^3(a+bx)^6}{6b^4}+\frac{3d(bc-ad)^2(a+bx)^7}{7b^4}+\frac{3d^2(bc-ad)(a+bx)^8}{8b^4}+\frac{d^3(a+bx)^9}{9b^4}\end{aligned}$$

Mathematica [B] time = 0.0739038, size = 235, normalized size = 2.55

$$\frac{1}{504}x\left(84a^3b^2x^2\left(45c^2dx+20c^3+36cd^2x^2+10d^3x^3\right)+36a^2b^3x^3\left(84c^2dx+35c^3+70cd^2x^2+20d^3x^3\right)+126a^4bx\left(20c^2d\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^3,x]
```

```
[Out] (x*(126*a^5*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 126*a^4*b*x*(10*c
^3 + 20*c^2*d*x + 15*c*d^2*x^2 + 4*d^3*x^3) + 84*a^3*b^2*x^2*(20*c^3 + 45*c
^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3) + 36*a^2*b^3*x^3*(35*c^3 + 84*c^2*d*x +
 70*c*d^2*x^2 + 20*d^3*x^3) + 9*a*b^4*x^4*(56*c^3 + 140*c^2*d*x + 120*c*d^2
*x^2 + 35*d^3*x^3) + b^5*x^5*(84*c^3 + 216*c^2*d*x + 189*c*d^2*x^2 + 56*d^3
```

```
*x^3)))/504
```

Maple [B] time = 0.001, size = 281, normalized size = 3.1

$$\frac{b^5d^3x^9}{9}+\frac{(5\,ab^4d^3+3\,b^5cd^2)\,x^8}{8}+\frac{(10\,a^2b^3d^3+15\,ab^4cd^2+3\,b^5c^2d)\,x^7}{7}+\frac{(10\,a^3b^2d^3+30\,a^2b^3cd^2+15\,ab^4c^2d+b^5c^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^3,x)
```

```
[Out] 1/9*b^5*d^3*x^9+1/8*(5*a*b^4*d^3+3*b^5*c*d^2)*x^8+1/7*(10*a^2*b^3*d^3+15*a*
b^4*c*d^2+3*b^5*c^2*d)*x^7+1/6*(10*a^3*b^2*d^3+30*a^2*b^3*c*d^2+15*a*b^4*c^
2*d+b^5*c^3)*x^6+1/5*(5*a^4*b*d^3+30*a^3*b^2*c*d^2+30*a^2*b^3*c^2*d+5*a*b^4
*c^3)*x^5+1/4*(a^5*d^3+15*a^4*b*c*d^2+30*a^3*b^2*c^2*d+10*a^2*b^3*c^3)*x^4+
1/3*(3*a^5*c*d^2+15*a^4*b*c^2*d+10*a^3*b^2*c^3)*x^3+1/2*(3*a^5*c^2*d+5*a^4*
b*c^3)*x^2+a^5*c^3*x
```

Maxima [B] time = 0.947845, size = 374, normalized size = 4.07

$$\frac{1}{9}\,b^5d^3x^9+a^5c^3x+\frac{1}{8}\left(3\,b^5cd^2+5\,ab^4d^3\right)x^8+\frac{1}{7}\left(3\,b^5c^2d+15\,ab^4cd^2+10\,a^2b^3d^3\right)x^7+\frac{1}{6}\left(b^5c^3+15\,ab^4c^2d+30\,a^2b^3c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*d^3*x^9 + a^5*c^3*x + 1/8*(3*b^5*c*d^2 + 5*a*b^4*d^3)*x^8 + 1/7*(3*
b^5*c^2*d + 15*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^7 + 1/6*(b^5*c^3 + 15*a*b^4*
c^2*d + 30*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^6 + (a*b^4*c^3 + 6*a^2*b^3*c^2
*d + 6*a^3*b^2*c*d^2 + a^4*b*d^3)*x^5 + 1/4*(10*a^2*b^3*c^3 + 30*a^3*b^2*c^
2*d + 15*a^4*b*c*d^2 + a^5*d^3)*x^4 + 1/3*(10*a^3*b^2*c^3 + 15*a^4*b*c^2*d
+ 3*a^5*c*d^2)*x^3 + 1/2*(5*a^4*b*c^3 + 3*a^5*c^2*d)*x^2
```

Fricas [B] time = 1.72465, size = 651, normalized size = 7.08

$$\frac{1}{9}x^9d^3b^5+\frac{3}{8}x^8d^2cb^5+\frac{5}{8}x^8d^3b^4a+\frac{3}{7}x^7dc^2b^5+\frac{15}{7}x^7d^2cb^4a+\frac{10}{7}x^7d^3b^3a^2+\frac{1}{6}x^6c^3b^5+\frac{5}{2}x^6dc^2b^4a+5x^6d^2cb^3a^2+\frac{5}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^3*b^5 + 3/8*x^8*d^2*c*b^5 + 5/8*x^8*d^3*b^4*a + 3/7*x^7*d*c^2*b^5
 + 15/7*x^7*d^2*c*b^4*a + 10/7*x^7*d^3*b^3*a^2 + 1/6*x^6*c^3*b^5 + 5/2*x^6*
d*c^2*b^4*a + 5*x^6*d^2*c*b^3*a^2 + 5/3*x^6*d^3*b^2*a^3 + x^5*c^3*b^4*a + 6
*x^5*d*c^2*b^3*a^2 + 6*x^5*d^2*c*b^2*a^3 + x^5*d^3*b*a^4 + 5/2*x^4*c^3*b^3*
a^2 + 15/2*x^4*d*c^2*b^2*a^3 + 15/4*x^4*d^2*c*b*a^4 + 1/4*x^4*d^3*a^5 + 10/
3*x^3*c^3*b^2*a^3 + 5*x^3*d*c^2*b*a^4 + x^3*d^2*c*a^5 + 5/2*x^2*c^3*b*a^4 +
 3/2*x^2*d*c^2*a^5 + x*c^3*a^5
```

Sympy [B] time = 0.101289, size = 308, normalized size = 3.35

$$a^5c^3x + \frac{b^5d^3x^9}{9} + x^8\left(\frac{5ab^4d^3}{8} + \frac{3b^5cd^2}{8}\right) + x^7\left(\frac{10a^2b^3d^3}{7} + \frac{15ab^4cd^2}{7} + \frac{3b^5c^2d}{7}\right) + x^6\left(\frac{5a^3b^2d^3}{3} + 5a^2b^3cd^2 + \frac{5ab^4c^2d}{2} + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**3,x)
```

```
[Out] a**5*c**3*x + b**5*d**3*x**9/9 + x**8*(5*a*b**4*d**3/8 + 3*b**5*c*d**2/8) +
 x**7*(10*a**2*b**3*d**3/7 + 15*a*b**4*c*d**2/7 + 3*b**5*c**2*d/7) + x**6*(
5*a**3*b**2*d**3/3 + 5*a**2*b**3*c*d**2 + 5*a*b**4*c**2*d/2 + b**5*c**3/6)
+ x**5*(a**4*b*d**3 + 6*a**3*b**2*c*d**2 + 6*a**2*b**3*c**2*d + a*b**4*c**3
) + x**4*(a**5*d**3/4 + 15*a**4*b*c*d**2/4 + 15*a**3*b**2*c**2*d/2 + 5*a**2
*b**3*c**3/2) + x**3*(a**5*c*d**2 + 5*a**4*b*c**2*d + 10*a**3*b**2*c**3/3)
+ x**2*(3*a**5*c**2*d/2 + 5*a**4*b*c**3/2)
```

Giac [B] time = 1.05207, size = 409, normalized size = 4.45

$$\frac{1}{9}b^5d^3x^9 + \frac{3}{8}b^5cd^2x^8 + \frac{5}{8}ab^4d^3x^8 + \frac{3}{7}b^5c^2dx^7 + \frac{15}{7}ab^4cd^2x^7 + \frac{10}{7}a^2b^3d^3x^7 + \frac{1}{6}b^5c^3x^6 + \frac{5}{2}ab^4c^2dx^6 + 5\,a^2b^3cd^2x^6 + \frac{5}{3}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/9*b^5*d^3*x^9 + 3/8*b^5*c*d^2*x^8 + 5/8*a*b^4*d^3*x^8 + 3/7*b^5*c^2*d*x^7
 + 15/7*a*b^4*c*d^2*x^7 + 10/7*a^2*b^3*d^3*x^7 + 1/6*b^5*c^3*x^6 + 5/2*a*b^
4*c^2*d*x^6 + 5*a^2*b^3*c*d^2*x^6 + 5/3*a^3*b^2*d^3*x^6 + a*b^4*c^3*x^5 + 6
*a^2*b^3*c^2*d*x^5 + 6*a^3*b^2*c*d^2*x^5 + a^4*b*d^3*x^5 + 5/2*a^2*b^3*c^3*
x^4 + 15/2*a^3*b^2*c^2*d*x^4 + 15/4*a^4*b*c*d^2*x^4 + 1/4*a^5*d^3*x^4 + 10/
3*a^3*b^2*c^3*x^3 + 5*a^4*b*c^2*d*x^3 + a^5*c*d^2*x^3 + 5/2*a^4*b*c^3*x^2 +
 3/2*a^5*c^2*d*x^2 + a^5*c^3*x
```

3.1259 $\int (a+bx)^4(c+dx)^3\,dx$

Optimal. Leaf size=92

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

```
[Out] ((b*c - a*d)^3*(a + b*x)^5)/(5*b^4) + (d*(b*c - a*d)^2*(a + b*x)^6)/(2*b^4)
 + (3*d^2*(b*c - a*d)*(a + b*x)^7)/(7*b^4) + (d^3*(a + b*x)^8)/(8*b^4)
```

Rubi [A] time = 0.114227, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^5)/(5*b^4) + (d*(b*c - a*d)^2*(a + b*x)^6)/(2*b^4)
 + (3*d^2*(b*c - a*d)*(a + b*x)^7)/(7*b^4) + (d^3*(a + b*x)^8)/(8*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^4}{b^3}+\frac{3d(bc-ad)^2(a+bx)^5}{b^3}+\frac{3d^2(bc-ad)(a+bx)^6}{b^3}+\frac{d^3(a+bx)^7}{b^3}\right)dx\\ &= \frac{(bc-ad)^3(a+bx)^5}{5b^4}+\frac{d(bc-ad)^2(a+bx)^6}{2b^4}+\frac{3d^2(bc-ad)(a+bx)^7}{7b^4}+\frac{d^3(a+bx)^8}{8b^4}\end{aligned}$$

Mathematica [B] time = 0.0265199, size = 217, normalized size = 2.36

$$\frac{1}{2}b^2dx^6\left(2a^2d^2+4abcd+b^2c^2\right)+\frac{1}{5}bx^5\left(18a^2bcd^2+4a^3d^3+12ab^2c^2d+b^3c^3\right)+\frac{1}{4}ax^4\left(12a^2bcd^2+a^3d^3+18ab^2c^2d+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^3,x]
```

```
[Out] a^4*c^3*x + (a^3*c^2*(4*b*c + 3*a*d)*x^2)/2 + a^2*c*(2*b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^3 + (a*(4*b^3*c^3 + 18*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^3)
*x^4)/4 + (b*(b^3*c^3 + 12*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 4*a^3*d^3)*x^5)/5
 + (b^2*d*(b^2*c^2 + 4*a*b*c*d + 2*a^2*d^2)*x^6)/2 + (b^3*d^2*(3*b*c + 4*a*
d)*x^7)/7 + (b^4*d^3*x^8)/8
```

Maple [B] time = 0.001, size = 229, normalized size = 2.5

$$\frac{b^4d^3x^8}{8}+\frac{(4\,ab^3d^3+3\,b^4cd^2)\,x^7}{7}+\frac{(6\,b^2a^2d^3+12\,ab^3cd^2+3\,b^4c^2d)\,x^6}{6}+\frac{(4\,a^3bd^3+18\,b^2a^2cd^2+12\,ab^3c^2d+b^4c^3)\,x^5}{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^3,x)
```

```
[Out] 1/8*b^4*d^3*x^8+1/7*(4*a*b^3*d^3+3*b^4*c*d^2)*x^7+1/6*(6*a^2*b^2*d^3+12*a*b
^3*c*d^2+3*b^4*c^2*d)*x^6+1/5*(4*a^3*b*d^3+18*a^2*b^2*c*d^2+12*a*b^3*c^2*d+
b^4*c^3)*x^5+1/4*(a^4*d^3+12*a^3*b*c*d^2+18*a^2*b^2*c^2*d+4*a*b^3*c^3)*x^4+
1/3*(3*a^4*c*d^2+12*a^3*b*c^2*d+6*a^2*b^2*c^3)*x^3+1/2*(3*a^4*c^2*d+4*a^3*b
*c^3)*x^2+a^4*c^3*x
```

Maxima [B] time = 0.967014, size = 304, normalized size = 3.3

$$\frac{1}{8}\,b^4d^3x^8+a^4c^3x+\frac{1}{7}\left(3\,b^4cd^2+4\,ab^3d^3\right)x^7+\frac{1}{2}\left(b^4c^2d+4\,ab^3cd^2+2\,a^2b^2d^3\right)x^6+\frac{1}{5}\left(b^4c^3+12\,ab^3c^2d+18\,a^2b^2cd^2+4\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b^
4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^2*
d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*c^2
*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a^4*c
*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Fricas [B] time = 1.73457, size = 520, normalized size = 5.65

$$\frac{1}{8}x^8d^3b^4+\frac{3}{7}x^7d^2cb^4+\frac{4}{7}x^7d^3b^3a+\frac{1}{2}x^6dc^2b^4+2x^6d^2cb^3a+x^6d^3b^2a^2+\frac{1}{5}x^5c^3b^4+\frac{12}{5}x^5dc^2b^3a+\frac{18}{5}x^5d^2cb^2a^2+\frac{4}{5}x^5d^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d^3*b^4 + 3/7*x^7*d^2*c*b^4 + 4/7*x^7*d^3*b^3*a + 1/2*x^6*d*c^2*b^4
 + 2*x^6*d^2*c*b^3*a + x^6*d^3*b^2*a^2 + 1/5*x^5*c^3*b^4 + 12/5*x^5*d*c^2*b
^3*a + 18/5*x^5*d^2*c*b^2*a^2 + 4/5*x^5*d^3*b*a^3 + x^4*c^3*b^3*a + 9/2*x^4
*d*c^2*b^2*a^2 + 3*x^4*d^2*c*b*a^3 + 1/4*x^4*d^3*a^4 + 2*x^3*c^3*b^2*a^2 +
4*x^3*d*c^2*b*a^3 + x^3*d^2*c*a^4 + 2*x^2*c^3*b*a^3 + 3/2*x^2*d*c^2*a^4 + x
*c^3*a^4
```

Sympy [B] time = 0.093947, size = 243, normalized size = 2.64

$$a^4c^3x+\frac{b^4d^3x^8}{8}+x^7\left(\frac{4ab^3d^3}{7}+\frac{3b^4cd^2}{7}\right)+x^6\left(a^2b^2d^3+2ab^3cd^2+\frac{b^4c^2d}{2}\right)+x^5\left(\frac{4a^3bd^3}{5}+\frac{18a^2b^2cd^2}{5}+\frac{12ab^3c^2d}{5}+\frac{b^4c^3}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**3,x)
```

```
[Out] a**4*c**3*x + b**4*d**3*x**8/8 + x**7*(4*a*b**3*d**3/7 + 3*b**4*c*d**2/7) +
 x**6*(a**2*b**2*d**3 + 2*a*b**3*c*d**2 + b**4*c**2*d/2) + x**5*(4*a**3*b*d
**3/5 + 18*a**2*b**2*c*d**2/5 + 12*a*b**3*c**2*d/5 + b**4*c**3/5) + x**4*(a
**4*d**3/4 + 3*a**3*b*c*d**2 + 9*a**2*b**2*c**2*d/2 + a*b**3*c**3) + x**3*(
a**4*c*d**2 + 4*a**3*b*c**2*d + 2*a**2*b**2*c**3) + x**2*(3*a**4*c**2*d/2 +
 2*a**3*b*c**3)
```

Giac [B] time = 1.06627, size = 331, normalized size = 3.6

$$\frac{1}{8}b^4d^3x^8 + \frac{3}{7}b^4cd^2x^7 + \frac{4}{7}ab^3d^3x^7 + \frac{1}{2}b^4c^2dx^6 + 2\,ab^3cd^2x^6 + a^2b^2d^3x^6 + \frac{1}{5}b^4c^3x^5 + \frac{12}{5}ab^3c^2dx^5 + \frac{18}{5}a^2b^2cd^2x^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 4/7*a*b^3*d^3*x^7 + 1/2*b^4*c^2*d*x^6
 + 2*a*b^3*c*d^2*x^6 + a^2*b^2*d^3*x^6 + 1/5*b^4*c^3*x^5 + 12/5*a*b^3*c^2*d
*x^5 + 18/5*a^2*b^2*c*d^2*x^5 + 4/5*a^3*b*d^3*x^5 + a*b^3*c^3*x^4 + 9/2*a^2
*b^2*c^2*d*x^4 + 3*a^3*b*c*d^2*x^4 + 1/4*a^4*d^3*x^4 + 2*a^2*b^2*c^3*x^3 +
4*a^3*b*c^2*d*x^3 + a^4*c*d^2*x^3 + 2*a^3*b*c^3*x^2 + 3/2*a^4*c^2*d*x^2 + a
^4*c^3*x
```

3.1260 $\int (a+bx)^3(c+dx)^3\,dx$

Optimal. Leaf size=92

$$\frac{d^2(a+bx)^6(bc-ad)}{2b^4}+\frac{3d(a+bx)^5(bc-ad)^2}{5b^4}+\frac{(a+bx)^4(bc-ad)^3}{4b^4}+\frac{d^3(a+bx)^7}{7b^4}$$

```
[Out] ((b*c - a*d)^3*(a + b*x)^4)/(4*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^5)/(5*b^
4) + (d^2*(b*c - a*d)*(a + b*x)^6)/(2*b^4) + (d^3*(a + b*x)^7)/(7*b^4)
```

Rubi [A] time = 0.0826116, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^2(a+bx)^6(bc-ad)}{2b^4}+\frac{3d(a+bx)^5(bc-ad)^2}{5b^4}+\frac{(a+bx)^4(bc-ad)^3}{4b^4}+\frac{d^3(a+bx)^7}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^4)/(4*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^5)/(5*b^
4) + (d^2*(b*c - a*d)*(a + b*x)^6)/(2*b^4) + (d^3*(a + b*x)^7)/(7*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^3\,dx&=\int\left(\frac{(bc-ad)^3(a+bx)^3}{b^3}+\frac{3d(bc-ad)^2(a+bx)^4}{b^3}+\frac{3d^2(bc-ad)(a+bx)^5}{b^3}+\frac{d^3(a+bx)^6}{b^3}\right)dx\\&=\frac{(bc-ad)^3(a+bx)^4}{4b^4}+\frac{3d(bc-ad)^2(a+bx)^5}{5b^4}+\frac{d^2(bc-ad)(a+bx)^6}{2b^4}+\frac{d^3(a+bx)^7}{7b^4}\end{aligned}$$

Mathematica [A] time = 0.0212754, size = 161, normalized size = 1.75

$$\frac{3}{5}bdx^5\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{4}x^4\left(9a^2bcd^2+a^3d^3+9ab^2c^2d+b^3c^3\right)+acx^3\left(a^2d^2+3abcd+b^2c^2\right)+\frac{3}{2}a^2c^2x^2(ad+bc)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^3,x]
```

```
[Out] a^3*c^3*x + (3*a^2*c^2*(b*c + a*d)*x^2)/2 + a*c*(b^2*c^2 + 3*a*b*c*d + a^2*
d^2)*x^3 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^4)/4 + (3
*b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5)/5 + (b^2*d^2*(b*c + a*d)*x^6)/2 +
 (b^3*d^3*x^7)/7
```

Maple [B] time = 0., size = 177, normalized size = 1.9

$$\frac{b^{3}d^{3}x^{7}}{7}+\frac{\left(3\,ab^{2}d^{3}+3\,b^{3}cd^{2}\right)x^{6}}{6}+\frac{\left(3\,a^{2}bd^{3}+9\,ab^{2}cd^{2}+3\,b^{3}c^{2}d\right)x^{5}}{5}+\frac{\left(a^{3}d^{3}+9\,a^{2}bcd^{2}+9\,ab^{2}c^{2}d+b^{3}c^{3}\right)x^{4}}{4}+\frac{(3\,a^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^3,x)
```

```
[Out] 1/7*b^3*d^3*x^7+1/6*(3*a*b^2*d^3+3*b^3*c*d^2)*x^6+1/5*(3*a^2*b*d^3+9*a*b^2*
c*d^2+3*b^3*c^2*d)*x^5+1/4*(a^3*d^3+9*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3)*x^
4+1/3*(3*a^3*c*d^2+9*a^2*b*c^2*d+3*a*b^2*c^3)*x^3+1/2*(3*a^3*c^2*d+3*a^2*b*
c^3)*x^2+a^3*c^3*x
```

Maxima [A] time = 0.961389, size = 225, normalized size = 2.45

$$\frac{1}{7}\,b^{3}d^{3}x^{7}+a^{3}c^{3}x+\frac{1}{2}\left(b^{3}cd^{2}+ab^{2}d^{3}\right)x^{6}+\frac{3}{5}\left(b^{3}c^{2}d+3\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x^{5}+\frac{1}{4}\left(b^{3}c^{3}+9\,ab^{2}c^{2}d+9\,a^{2}bcd^{2}+a^{3}d^{3}\right)x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*d^3*x^7 + a^3*c^3*x + 1/2*(b^3*c*d^2 + a*b^2*d^3)*x^6 + 3/5*(b^3*c^
2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^5 + 1/4*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2
*b*c*d^2 + a^3*d^3)*x^4 + (a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x^3 + 3/2
*(a^2*b*c^3 + a^3*c^2*d)*x^2
```

Fricas [B] time = 1.803, size = 409, normalized size = 4.45

$$\frac{1}{7}x^{7}d^{3}b^{3}+\frac{1}{2}x^{6}d^{2}cb^{3}+\frac{1}{2}x^{6}d^{3}b^{2}a+\frac{3}{5}x^{5}dc^{2}b^{3}+\frac{9}{5}x^{5}d^{2}cb^{2}a+\frac{3}{5}x^{5}d^{3}ba^{2}+\frac{1}{4}x^{4}c^{3}b^{3}+\frac{9}{4}x^{4}dc^{2}b^{2}a+\frac{9}{4}x^{4}d^{2}cba^{2}+\frac{1}{4}x^{4}d^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d^3*b^3 + 1/2*x^6*d^2*c*b^3 + 1/2*x^6*d^3*b^2*a + 3/5*x^5*d*c^2*b^3
 + 9/5*x^5*d^2*c*b^2*a + 3/5*x^5*d^3*b*a^2 + 1/4*x^4*c^3*b^3 + 9/4*x^4*d*c^
2*b^2*a + 9/4*x^4*d^2*c*b*a^2 + 1/4*x^4*d^3*a^3 + x^3*c^3*b^2*a + 3*x^3*d*c
^2*b*a^2 + x^3*d^2*c*a^3 + 3/2*x^2*c^3*b*a^2 + 3/2*x^2*d*c^2*a^3 + x*c^3*a^
3
```

Sympy [B] time = 0.086416, size = 190, normalized size = 2.07

$$a^{3}c^{3}x+\frac{b^{3}d^{3}x^{7}}{7}+x^{6}\left(\frac{ab^{2}d^{3}}{2}+\frac{b^{3}cd^{2}}{2}\right)+x^{5}\left(\frac{3a^{2}bd^{3}}{5}+\frac{9ab^{2}cd^{2}}{5}+\frac{3b^{3}c^{2}d}{5}\right)+x^{4}\left(\frac{a^{3}d^{3}}{4}+\frac{9a^{2}bcd^{2}}{4}+\frac{9ab^{2}c^{2}d}{4}+\frac{b^{3}c^{3}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**3,x)
```

```
[Out] a**3*c**3*x + b**3*d**3*x**7/7 + x**6*(a*b**2*d**3/2 + b**3*c*d**2/2) + x**
5*(3*a**2*b*d**3/5 + 9*a*b**2*c*d**2/5 + 3*b**3*c**2*d/5) + x**4*(a**3*d**3
/4 + 9*a**2*b*c*d**2/4 + 9*a*b**2*c**2*d/4 + b**3*c**3/4) + x**3*(a**3*c*d*
*2 + 3*a**2*b*c**2*d + a*b**2*c**3) + x**2*(3*a**3*c**2*d/2 + 3*a**2*b*c**3
/2)
```

Giac [B] time = 1.0964, size = 254, normalized size = 2.76

$$\frac{1}{7}b^3d^3x^7 + \frac{1}{2}b^3cd^2x^6 + \frac{1}{2}ab^2d^3x^6 + \frac{3}{5}b^3c^2dx^5 + \frac{9}{5}ab^2cd^2x^5 + \frac{3}{5}a^2bd^3x^5 + \frac{1}{4}b^3c^3x^4 + \frac{9}{4}ab^2c^2dx^4 + \frac{9}{4}a^2bcd^2x^4 + \frac{1}{4}a^3d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*d^3*x^7 + 1/2*b^3*c*d^2*x^6 + 1/2*a*b^2*d^3*x^6 + 3/5*b^3*c^2*d*x^5
 + 9/5*a*b^2*c*d^2*x^5 + 3/5*a^2*b*d^3*x^5 + 1/4*b^3*c^3*x^4 + 9/4*a*b^2*c^
2*d*x^4 + 9/4*a^2*b*c*d^2*x^4 + 1/4*a^3*d^3*x^4 + a*b^2*c^3*x^3 + 3*a^2*b*c
^2*d*x^3 + a^3*c*d^2*x^3 + 3/2*a^2*b*c^3*x^2 + 3/2*a^3*c^2*d*x^2 + a^3*c^3*
x
```

3.1261 $\int (a+bx)^2(c+dx)^3\,dx$

Optimal. Leaf size=65

$$-\frac{2b(c+dx)^5(bc-ad)}{5d^3}+\frac{(c+dx)^4(bc-ad)^2}{4d^3}+\frac{b^2(c+dx)^6}{6d^3}$$

```
[Out] ((b*c - a*d)^2*(c + d*x)^4)/(4*d^3) - (2*b*(b*c - a*d)*(c + d*x)^5)/(5*d^3)
 + (b^2*(c + d*x)^6)/(6*d^3)
```

Rubi [A] time = 0.0644285, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b(c+dx)^5(bc-ad)}{5d^3}+\frac{(c+dx)^4(bc-ad)^2}{4d^3}+\frac{b^2(c+dx)^6}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^4)/(4*d^3) - (2*b*(b*c - a*d)*(c + d*x)^5)/(5*d^3)
 + (b^2*(c + d*x)^6)/(6*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^3\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^3}{d^2}-\frac{2b(bc-ad)(c+dx)^4}{d^2}+\frac{b^2(c+dx)^5}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^4}{4d^3}-\frac{2b(bc-ad)(c+dx)^5}{5d^3}+\frac{b^2(c+dx)^6}{6d^3}\end{aligned}$$

Mathematica [A] time = 0.0140066, size = 122, normalized size = 1.88

$$\frac{1}{4}dx^4\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{6}b$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d^2*(3*b*c
+ 2*a*d)*x^5)/5 + (b^2*d^3*x^6)/6
```

Maple [B] time = 0., size = 125, normalized size = 1.9

$$\frac{b^2d^3x^6}{6}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^5}{5}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^4}{4}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^3}{3}+\frac{\left(3\,a^2c^2d+2\,abc^3\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^3,x)
```

```
[Out] 1/6*b^2*d^3*x^6+1/5*(2*a*b*d^3+3*b^2*c*d^2)*x^5+1/4*(a^2*d^3+6*a*b*c*d^2+3*
b^2*c^2*d)*x^4+1/3*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^3+1/2*(3*a^2*c^2*d+2
*a*b*c^3)*x^2+a^2*c^3*x
```

Maxima [B] time = 1.0054, size = 167, normalized size = 2.57

$$\frac{1}{6}\,b^2d^3x^6+a^2c^3x+\frac{1}{5}\left(3\,b^2cd^2+2\,abd^3\right)x^5+\frac{1}{4}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^4+\frac{1}{3}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^3+\frac{1}{2}\left(2\,abc^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d^3*x^6 + a^2*c^3*x + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/4*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^4 + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c
*d^2)*x^3 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

Fricas [B] time = 1.7463, size = 285, normalized size = 4.38

$$\frac{1}{6}x^6d^3b^2+\frac{3}{5}x^5d^2cb^2+\frac{2}{5}x^5d^3ba+\frac{3}{4}x^4dc^2b^2+\frac{3}{2}x^4d^2cba+\frac{1}{4}x^4d^3a^2+\frac{1}{3}x^3c^3b^2+2x^3dc^2ba+x^3d^2ca^2+x^2c^3ba+\frac{3}{2}x^2dc^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*d^3*b^2 + 3/5*x^5*d^2*c*b^2 + 2/5*x^5*d^3*b*a + 3/4*x^4*d*c^2*b^2 +
 3/2*x^4*d^2*c*b*a + 1/4*x^4*d^3*a^2 + 1/3*x^3*c^3*b^2 + 2*x^3*d*c^2*b*a +
x^3*d^2*c*a^2 + x^2*c^3*b*a + 3/2*x^2*d*c^2*a^2 + x*c^3*a^2
```

Sympy [B] time = 0.082265, size = 133, normalized size = 2.05

$$a^2c^3x+\frac{b^2d^3x^6}{6}+x^5\left(\frac{2abd^3}{5}+\frac{3b^2cd^2}{5}\right)+x^4\left(\frac{a^2d^3}{4}+\frac{3abcd^2}{2}+\frac{3b^2c^2d}{4}\right)+x^3\left(a^2cd^2+2abc^2d+\frac{b^2c^3}{3}\right)+x^2\left(\frac{3a^2c^2d}{2}+al\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**6/6 + x**5*(2*a*b*d**3/5 + 3*b**2*c*d**2/5) + x*
*4*(a**2*d**3/4 + 3*a*b*c*d**2/2 + 3*b**2*c**2*d/4) + x**3*(a**2*c*d**2 + 2
*a*b*c**2*d + b**2*c**3/3) + x**2*(3*a**2*c**2*d/2 + a*b*c**3)
```

Giac [B] time = 1.05247, size = 176, normalized size = 2.71

$$\frac{1}{6}b^2d^3x^6 + \frac{3}{5}b^2cd^2x^5 + \frac{2}{5}abd^3x^5 + \frac{3}{4}b^2c^2dx^4 + \frac{3}{2}abcd^2x^4 + \frac{1}{4}a^2d^3x^4 + \frac{1}{3}b^2c^3x^3 + 2\,abc^2dx^3 + a^2cd^2x^3 + abc^3x^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/6*b^2*d^3*x^6 + 3/5*b^2*c*d^2*x^5 + 2/5*a*b*d^3*x^5 + 3/4*b^2*c^2*d*x^4 +
 3/2*a*b*c*d^2*x^4 + 1/4*a^2*d^3*x^4 + 1/3*b^2*c^3*x^3 + 2*a*b*c^2*d*x^3 +
a^2*c*d^2*x^3 + a*b*c^3*x^2 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x
```

3.1262 $\int (a+bx)(c+dx)^3\,dx$

Optimal. Leaf size=38

$$\frac{b(c+dx)^5}{5d^2}-\frac{(c+dx)^4(bc-ad)}{4d^2}$$

```
[Out] -((b*c - a*d)*(c + d*x)^4)/(4*d^2) + (b*(c + d*x)^5)/(5*d^2)
```

Rubi [A] time = 0.0150808, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^5}{5d^2}-\frac{(c+dx)^4(bc-ad)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^3,x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^4)/(4*d^2) + (b*(c + d*x)^5)/(5*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^3\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^3}{d}+\frac{b(c+dx)^4}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^4}{4d^2}+\frac{b(c+dx)^5}{5d^2}\end{aligned}$$

Mathematica [A] time = 0.0083032, size = 67, normalized size = 1.76

$$\frac{1}{2}c^2x^2(3ad+bc)+\frac{1}{4}d^2x^4(ad+3bc)+cdx^3(ad+bc)+ac^3x+\frac{1}{5}bd^3x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^2)/2 + c*d*(b*c + a*d)*x^3 + (d^2*(3*b*c + a
*d)*x^4)/4 + (b*d^3*x^5)/5
```

Maple [B] time = 0., size = 73, normalized size = 1.9

$$\frac{bd^3x^5}{5}+\frac{\left(ad^3+3\,bcd^2\right)x^4}{4}+\frac{\left(3\,acd^2+3\,bc^2d\right)x^3}{3}+\frac{\left(3\,ac^2d+bc^3\right)x^2}{2}+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^3,x)
```

```
[Out] 1/5*b*d^3*x^5+1/4*(a*d^3+3*b*c*d^2)*x^4+1/3*(3*a*c*d^2+3*b*c^2*d)*x^3+1/2*(
3*a*c^2*d+b*c^3)*x^2+a*c^3*x
```

Maxima [B] time = 0.973537, size = 93, normalized size = 2.45

$$\frac{1}{5}\,bd^3x^5 + ac^3x + \frac{1}{4}\left(3\,bcd^2 + ad^3\right)x^4 + \left(bc^2d + acd^2\right)x^3 + \frac{1}{2}\left(bc^3 + 3\,ac^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/5*b*d^3*x^5 + a*c^3*x + 1/4*(3*b*c*d^2 + a*d^3)*x^4 + (b*c^2*d + a*c*d^2)
*x^3 + 1/2*(b*c^3 + 3*a*c^2*d)*x^2
```

Fricas [B] time = 1.77785, size = 163, normalized size = 4.29

$$\frac{1}{5}x^5d^3b + \frac{3}{4}x^4d^2cb + \frac{1}{4}x^4d^3a + x^3dc^2b + x^3d^2ca + \frac{1}{2}x^2c^3b + \frac{3}{2}x^2dc^2a + xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d^3*b + 3/4*x^4*d^2*c*b + 1/4*x^4*d^3*a + x^3*d*c^2*b + x^3*d^2*c*a
 + 1/2*x^2*c^3*b + 3/2*x^2*d*c^2*a + x*c^3*a
```

Sympy [B] time = 0.070579, size = 73, normalized size = 1.92

$$ac^3x + \frac{bd^3x^5}{5} + x^4\left(\frac{ad^3}{4} + \frac{3bcd^2}{4}\right) + x^3\left(acd^2 + bc^2d\right) + x^2\left(\frac{3ac^2d}{2} + \frac{bc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**5/5 + x**4*(a*d**3/4 + 3*b*c*d**2/4) + x**3*(a*c*d**2
+ b*c**2*d) + x**2*(3*a*c**2*d/2 + b*c**3/2)
```

Giac [B] time = 1.04949, size = 97, normalized size = 2.55

$$\frac{1}{5}\,bd^3x^5 + \frac{3}{4}\,bcd^2x^4 + \frac{1}{4}\,ad^3x^4 + bc^2dx^3 + acd^2x^3 + \frac{1}{2}\,bc^3x^2 + \frac{3}{2}\,ac^2dx^2 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/5*b*d^3*x^5 + 3/4*b*c*d^2*x^4 + 1/4*a*d^3*x^4 + b*c^2*d*x^3 + a*c*d^2*x^3
 + 1/2*b*c^3*x^2 + 3/2*a*c^2*d*x^2 + a*c^3*x
```

3.1263 $\int (c+dx)^3 \, dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^4}{4d}$$

```
[Out] (c + d*x)^4/(4*d)
```

Rubi [A] time = 0.0015485, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3,x]
```

```
[Out] (c + d*x)^4/(4*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^3 \, dx = \frac{(c+dx)^4}{4d}$$

Mathematica [A] time = 0.0014879, size = 14, normalized size = 1.

$$\frac{(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3,x]
```

```
[Out] (c + d*x)^4/(4*d)
```

Maple [A] time = 0., size = 13, normalized size = 0.9

$$\frac{(dx+c)^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3,x)
```

```
[Out] 1/4*(d*x+c)^4/d
```

Maxima [B] time = 0.966924, size = 42, normalized size = 3.

$$\frac{1}{4}d^3x^4 + cd^2x^3 + \frac{3}{2}c^2dx^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x
```

Fricas [B] time = 1.76399, size = 66, normalized size = 4.71

$$\frac{1}{4}x^4d^3 + x^3d^2c + \frac{3}{2}x^2dc^2 + xc^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*d^3 + x^3*d^2*c + 3/2*x^2*d*c^2 + x*c^3
```

Sympy [B] time = 0.059863, size = 32, normalized size = 2.29

$$c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3,x)
```

```
[Out] c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
```

Giac [A] time = 1.05997, size = 16, normalized size = 1.14

$$\frac{(dx + c)^4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(d*x + c)^4/d
```

3.1264 $\int \frac{(c+dx)^3}{a+bx} \, dx$

Optimal. Leaf size=73

$$\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{(c+dx)^3}{3b}$$

```
[Out] (d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x)^2)/(2*b^2) + (c + d*x)^3/(
3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4
```

Rubi [A] time = 0.0276175, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{(c+dx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x)^2)/(2*b^2) + (c + d*x)^3/(
3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{a+bx} \, dx &= \int \left(\frac{d(bc-ad)^2}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)(c+dx)}{b^2}+\frac{d(c+dx)^2}{b}\right) dx\\ &= \frac{d(bc-ad)^2x}{b^3}+\frac{(bc-ad)(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{3b}+\frac{(bc-ad)^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0302777, size = 74, normalized size = 1.01

$$\frac{bdx\left(6a^2d^2-3abd(6c+dx)+b^2\left(18c^2+9cdx+2d^2x^2\right)\right)+6(bc-ad)^3\log(a+bx)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x),x]
```

```
[Out] (b*d*x*(6*a^2*d^2 - 3*a*b*d*(6*c + d*x) + b^2*(18*c^2 + 9*c*d*x + 2*d^2*x^2
)) + 6*(b*c - a*d)^3*Log[a + b*x])/(6*b^4)
```

Maple [A] time = 0.003, size = 133, normalized size = 1.8

$$\frac{d^3x^3}{3b}-\frac{d^3x^2a}{2b^2}+\frac{3d^2x^2c}{2b}+\frac{a^2d^3x}{b^3}-3\frac{acd^2x}{b^2}+3\frac{dc^2x}{b}-\frac{a^3\ln(bx+a)d^3}{b^4}+3\frac{a^2\ln(bx+a)cd^2}{b^3}-3\frac{a\ln(bx+a)c^2d}{b^2}+\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a),x)
```

```
[Out] 1/3*d^3/b*x^3-1/2*d^3/b^2*x^2*a+3/2*d^2/b*x^2*c+d^3/b^3*a^2*x-3*d^2/b^2*a*c
*x+3*d/b*c^2*x-1/b^4*ln(b*x+a)*a^3*d^3+3/b^3*ln(b*x+a)*a^2*c*d^2-3/b^2*ln(b
*x+a)*a*c^2*d+1/b*ln(b*x+a)*c^3
```

Maxima [A] time = 0.992634, size = 154, normalized size = 2.11

$$\frac{2\,b^2d^3x^3+3\left(3\,b^2cd^2-abd^3\right)x^2+6\left(3\,b^2c^2d-3\,abcd^2+a^2d^3\right)x}{6\,b^3}+\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*d^3*x^3 + 3*(3*b^2*c*d^2 - a*b*d^3)*x^2 + 6*(3*b^2*c^2*d - 3*a*b
*c*d^2 + a^2*d^3)*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(b*x + a)/b^4
```

Fricas [A] time = 1.93933, size = 238, normalized size = 3.26

$$\frac{2\,b^3d^3x^3+3\left(3\,b^3cd^2-ab^2d^3\right)x^2+6\left(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3\right)x+6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(bx+a\right)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*d^3*x^3 + 3*(3*b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(3*b^3*c^2*d - 3*a
*b^2*c*d^2 + a^2*b*d^3)*x + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(b*x + a))/b^4
```

Sympy [A] time = 0.457561, size = 82, normalized size = 1.12

$$\frac{d^3x^3}{3b}-\frac{x^2\left(ad^3-3bcd^2\right)}{2b^2}+\frac{x\left(a^2d^3-3abcd^2+3b^2c^2d\right)}{b^3}-\frac{(ad-bc)^3\log\left(a+bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a),x)
```

```
[Out] d**3*x**3/(3*b) - x**2*(a*d**3 - 3*b*c*d**2)/(2*b**2) + x*(a**2*d**3 - 3*a*
b*c*d**2 + 3*b**2*c**2*d)/b**3 - (a*d - b*c)**3*log(a + b*x)/b**4
```

Giac [A] time = 1.04778, size = 155, normalized size = 2.12

$$\frac{2\,b^2d^3x^3+9\,b^2cd^2x^2-3\,abd^3x^2+18\,b^2c^2dx-18\,abcd^2x+6\,a^2d^3x}{6\,b^3}+\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(|bx+a|\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^2*d^3*x^3 + 9*b^2*c*d^2*x^2 - 3*a*b*d^3*x^2 + 18*b^2*c^2*d*x - 18*
a*b*c*d^2*x + 6*a^2*d^3*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(abs(b*x + a))/b^4
```

3.1265 $\int \frac{(c+dx)^3}{(a+bx)^2} dx$

Optimal. Leaf size=75

$$\frac{d^2x(3bc-2ad)}{b^3}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}+\frac{d^3x^2}{2b^2}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^2)/(2*b^2) - (b*c - a*d)^3/(b^4*(a + b
*x)) + (3*d*(b*c - a*d)^2*Log[a + b*x])/b^4
```

Rubi [A] time = 0.0550575, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^2x(3bc-2ad)}{b^3}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}+\frac{d^3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^2)/(2*b^2) - (b*c - a*d)^3/(b^4*(a + b
*x)) + (3*d*(b*c - a*d)^2*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^2}\,dx &= \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x}{b^2}+\frac{(bc-ad)^3}{b^3(a+bx)^2}+\frac{3d(bc-ad)^2}{b^3(a+bx)}\right)dx\\ &= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^2}{2b^2}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0526268, size = 72, normalized size = 0.96

$$\frac{2bd^2x(3bc-2ad)-\frac{2(bc-ad)^3}{a+bx}+6d(bc-ad)^2\log(a+bx)+b^2d^3x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^2,x]
```

```
[Out] (2*b*d^2*(3*b*c - 2*a*d)*x + b^2*d^3*x^2 - (2*(b*c - a*d)^3)/(a + b*x) + 6*
d*(b*c - a*d)^2*Log[a + b*x])/(2*b^4)
```

Maple [B] time = 0.006, size = 149, normalized size = 2.

$$\frac{d^3x^2}{2\,b^2}-2\,\frac{ad^3x}{b^3}+3\,\frac{d^2xc}{b^2}+\frac{a^3d^3}{b^4\,(bx+a)}-3\,\frac{a^2cd^2}{b^3\,(bx+a)}+3\,\frac{ac^2d}{b^2\,(bx+a)}-\frac{c^3}{b\,(bx+a)}+3\,\frac{d^3\ln(bx+a)\,a^2}{b^4}-6\,\frac{d^2\ln(bx+}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^2,x)
```

```
[Out] 1/2*d^3*x^2/b^2-2*d^3/b^3*a*x+3*d^2/b^2*x*c+1/b^4/(b*x+a)*a^3*d^3-3/b^3/(b*
x+a)*a^2*c*d^2+3/b^2/(b*x+a)*a*c^2*d-1/b/(b*x+a)*c^3+3/b^4*d^3*ln(b*x+a)*a^
2-6/b^3*d^2*ln(b*x+a)*a*c+3/b^2*d*ln(b*x+a)*c^2
```

Maxima [A] time = 0.964306, size = 159, normalized size = 2.12

$$-\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{b^5x+ab^4}+\frac{bd^3x^2+2\left(3\,bcd^2-2\,ad^3\right)x}{2\,b^3}+\frac{3\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\log(bx+a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(b^5*x + a*b^4) + 1/2*
(b*d^3*x^2 + 2*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*log(b*x + a)/b^4
```

Fricas [B] time = 2.24533, size = 354, normalized size = 4.72

$$\frac{b^3d^3x^3-2\,b^3c^3+6\,ab^2c^2d-6\,a^2bcd^2+2\,a^3d^3+3\left(2\,b^3cd^2-ab^2d^3\right)x^2+2\left(3\,ab^2cd^2-2\,a^2bd^3\right)x+6\left(ab^2c^2d-2\,a^2bc\right.}{2\left(b^5x+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
3*(2*b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(3*a*b^2*c*d^2 - 2*a^2*b*d^3)*x + 6*(a*
b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x)*log(b*x + a))/(b^5*x + a*b^4)
```

Sympy [A] time = 0.700389, size = 100, normalized size = 1.33

$$\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{ab^4+b^5x}+\frac{d^3x^2}{2b^2}-\frac{x\left(2ad^3-3bcd^2\right)}{b^3}+\frac{3d\,(ad-bc)^2\log(a+bx)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**2,x)
```

```
[Out] (a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(a*b**4 + b**5*
x) + d**3*x**2/(2*b**2) - x*(2*a*d**3 - 3*b*c*d**2)/b**3 + 3*d*(a*d - b*c)*
```

```
*2*log(a + b*x)/b**4
```

Giac [B] time = 1.05533, size = 225, normalized size = 3.

$$\frac{\left(d^3+\frac{6\left(b^2cd^2-abd^3\right)}{(bx+a)b}\right)(bx+a)^2}{2\,b^4}-\frac{3\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4}-\frac{\frac{b^5c^3}{bx+a}-\frac{3\,ab^4c^2d}{bx+a}+\frac{3\,a^2b^3cd^2}{bx+a}-\frac{a^3b^2d^3}{bx+a}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(d^3 + 6*(b^2*c*d^2 - a*b*d^3)/((b*x + a)*b))*(b*x + a)^2/b^4 - 3*(b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^4 -
 (b^5*c^3/(b*x + a) - 3*a*b^4*c^2*d/(b*x + a) + 3*a^2*b^3*c*d^2/(b*x + a) -
 a^3*b^2*d^3/(b*x + a))/b^6
```

3.1266 $\int \frac{(c+dx)^3}{(a+bx)^3}\,dx$

Optimal. Leaf size=78

$$\frac{3d^2(bc-ad)\log(a+bx)}{b^4}-\frac{3d(bc-ad)^2}{b^4(a+bx)}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}+\frac{d^3x}{b^3}$$

```
[Out] (d^3*x)/b^3 - (b*c - a*d)^3/(2*b^4*(a + b*x)^2) - (3*d*(b*c - a*d)^2)/(b^4*
(a + b*x)) + (3*d^2*(b*c - a*d)*Log[a + b*x])/b^4
```

Rubi [A] time = 0.0523061, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3d^2(bc-ad)\log(a+bx)}{b^4}-\frac{3d(bc-ad)^2}{b^4(a+bx)}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}+\frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^3,x]
```

```
[Out] (d^3*x)/b^3 - (b*c - a*d)^3/(2*b^4*(a + b*x)^2) - (3*d*(b*c - a*d)^2)/(b^4*
(a + b*x)) + (3*d^2*(b*c - a*d)*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^3}\,dx &= \int \left(\frac{d^3}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)^3}+\frac{3d(bc-ad)^2}{b^3(a+bx)^2}+\frac{3d^2(bc-ad)}{b^3(a+bx)}\right)dx\\ &= \frac{d^3x}{b^3}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}-\frac{3d(bc-ad)^2}{b^4(a+bx)}+\frac{3d^2(bc-ad)\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0432346, size = 114, normalized size = 1.46

$$\frac{a^2bd^2(9c-4dx)-5a^3d^3+ab^2d\left(-3c^2+12cdx+4d^2x^2\right)-6d^2(a+bx)^2(ad-bc)\log(a+bx)+b^3\left(-\left(6c^2dx+c^3-2d^3\right.\right.}{2b^4(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^3,x]
```

```
[Out] (-5*a^3*d^3 + a^2*b*d^2*(9*c - 4*d*x) + a*b^2*d*(-3*c^2 + 12*c*d*x + 4*d^2*
x^2) - b^3*(c^3 + 6*c^2*d*x - 2*d^3*x^3) - 6*d^2*(-(b*c) + a*d)*(a + b*x)^2
*Log[a + b*x])/(2*b^4*(a + b*x)^2)
```

Maple [B] time = 0.006, size = 160, normalized size = 2.1

$$\frac{d^3x}{b^3}-3\frac{a^2d^3}{b^4(bx+a)}+6\frac{acd^2}{b^3(bx+a)}-3\frac{dc^2}{b^2(bx+a)}+\frac{a^3d^3}{2b^4(bx+a)^2}-\frac{3a^2cd^2}{2b^3(bx+a)^2}+\frac{3ac^2d}{2b^2(bx+a)^2}-\frac{c^3}{2b(bx+a)^2}-3\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^3,x)
```

```
[Out] d^3*x/b^3-3/b^4*d^3/(b*x+a)*a^2+6/b^3*d^2/(b*x+a)*a*c-3/b^2*d/(b*x+a)*c^2+1
/2/b^4/(b*x+a)^2*a^3*d^3-3/2/b^3/(b*x+a)^2*a^2*c*d^2+3/2/b^2/(b*x+a)^2*a*c^
2*d-1/2/b/(b*x+a)^2*c^3-3/b^4*d^3*ln(b*x+a)*a+3/b^3*d^2*ln(b*x+a)*c
```

Maxima [A] time = 0.970695, size = 169, normalized size = 2.17

$$\frac{d^3x}{b^3}-\frac{b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3+6\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x}{2\left(b^6x^2+2\,ab^5x+a^2b^4\right)}+\frac{3\left(bcd^2-ad^3\right)\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] d^3*x/b^3 - 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) +
3*(b*c*d^2 - a*d^3)*log(b*x + a)/b^4
```

Fricas [B] time = 2.20461, size = 375, normalized size = 4.81

$$\frac{2\,b^3d^3x^3+4\,ab^2d^3x^2-b^3c^3-3\,ab^2c^2d+9\,a^2bcd^2-5\,a^3d^3-2\left(3\,b^3c^2d-6\,ab^2cd^2+2\,a^2bd^3\right)x+6\left(a^2bcd^2-a^3d^3+\left(b^3cd\right.\right.}{2\left(b^6x^2+2\,ab^5x+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 4*a*b^2*d^3*x^2 - b^3*c^3 - 3*a*b^2*c^2*d + 9*a^2*b*c*
d^2 - 5*a^3*d^3 - 2*(3*b^3*c^2*d - 6*a*b^2*c*d^2 + 2*a^2*b*d^3)*x + 6*(a^2*
b*c*d^2 - a^3*d^3 + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(a*b^2*c*d^2 - a^2*b*d^
3)*x)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [A] time = 1.04473, size = 128, normalized size = 1.64

$$-\frac{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3+x\left(6a^2bd^3-12ab^2cd^2+6b^3c^2d\right)}{2a^2b^4+4ab^5x+2b^6x^2}+\frac{d^3x}{b^3}-\frac{3d^2\left(ad-bc\right)\log\left(a+bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**3,x)
```

```
[Out] -(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3*c**3 + x*(6*a**2*b
*d**3 - 12*a*b**2*c*d**2 + 6*b**3*c**2*d))/(2*a**2*b**4 + 4*a*b**5*x + 2*b*
*6*x**2) + d**3*x/b**3 - 3*d**2*(a*d - b*c)*log(a + b*x)/b**4
```

Giac [A] time = 1.07987, size = 151, normalized size = 1.94

$$\frac{d^3x}{b^3} + \frac{3\left(bcd^2 - ad^3\right)\log\left(|bx + a|\right)}{b^4} - \frac{b^3c^3 + 3\,ab^2c^2d - 9\,a^2bcd^2 + 5\,a^3d^3 + 6\left(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3\right)x}{2\,(bx + a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] d^3*x/b^3 + 3*(b*c*d^2 - a*d^3)*log(abs(b*x + a))/b^4 - 1/2*(b^3*c^3 + 3*a*
b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/((b*x + a)^2*b^4)
```

3.1267 $\int \frac{(c+dx)^3}{(a+bx)^4} dx$

Optimal. Leaf size=86

$$-\frac{3d^2(bc-ad)}{b^4(a+bx)}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{(bc-ad)^3}{3b^4(a+bx)^3}+\frac{d^3\log(a+bx)}{b^4}$$

```
[Out] -(b*c - a*d)^3/(3*b^4*(a + b*x)^3) - (3*d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^2
) - (3*d^2*(b*c - a*d))/(b^4*(a + b*x)) + (d^3*Log[a + b*x])/b^4
```

Rubi [A] time = 0.0501509, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3d^2(bc-ad)}{b^4(a+bx)}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{(bc-ad)^3}{3b^4(a+bx)^3}+\frac{d^3\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^4,x]
```

```
[Out] -(b*c - a*d)^3/(3*b^4*(a + b*x)^3) - (3*d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^2
) - (3*d^2*(b*c - a*d))/(b^4*(a + b*x)) + (d^3*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^4}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^4}+\frac{3d(bc-ad)^2}{b^3(a+bx)^3}+\frac{3d^2(bc-ad)}{b^3(a+bx)^2}+\frac{d^3}{b^3(a+bx)}\right)dx\\ &= -\frac{(bc-ad)^3}{3b^4(a+bx)^3}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{3d^2(bc-ad)}{b^4(a+bx)}+\frac{d^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.0420197, size = 80, normalized size = 0.93

$$\frac{6d^3\log(a+bx)-\frac{(bc-ad)\left(11a^2d^2+abd(5c+27dx)+b^2\left(2c^2+9cdx+18d^2x^2\right)\right)}{(a+bx)^3}}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^4,x]
```

```
[Out] (-(((b*c - a*d)*(11*a^2*d^2 + a*b*d*(5*c + 27*d*x) + b^2*(2*c^2 + 9*c*d*x +
 18*d^2*x^2)))/(a + b*x)^3) + 6*d^3*Log[a + b*x])/(6*b^4)
```

Maple [B] time = 0.006, size = 166, normalized size = 1.9

$$3\frac{ad^3}{b^4(bx+a)}-3\frac{cd^2}{b^3(bx+a)}-\frac{3a^2d^3}{2b^4(bx+a)^2}+3\frac{acd^2}{b^3(bx+a)^2}-\frac{3c^2d}{2b^2(bx+a)^2}+\frac{a^3d^3}{3b^4(bx+a)^3}-\frac{a^2cd^2}{b^3(bx+a)^3}+\frac{ac}{b^2(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^4,x)
```

```
[Out] 3/b^4*d^3/(b*x+a)*a-3/b^3*d^2/(b*x+a)*c-3/2*d^3/b^4/(b*x+a)^2*a^2+3*d^2/b^3
/(b*x+a)^2*a*c-3/2*d/b^2/(b*x+a)^2*c^2+1/3/b^4/(b*x+a)^3*a^3*d^3-1/b^3/(b*x
+a)^3*a^2*c*d^2+1/b^2/(b*x+a)^3*a*c^2*d-1/3/b/(b*x+a)^3*c^3+d^3*ln(b*x+a)/b
^4
```

Maxima [A] time = 0.976149, size = 192, normalized size = 2.23

$$-\frac{2b^3c^3+3ab^2c^2d+6a^2bcd^2-11a^3d^3+18\left(b^3cd^2-ab^2d^3\right)x^2+9\left(b^3c^2d+2ab^2cd^2-3a^2bd^3\right)x}{6\left(b^7x^3+3ab^6x^2+3a^2b^5x+a^3b^4\right)}+\frac{d^3\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d^
2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x)/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4) + d^3*log(b*x + a)/b^4
```

Fricas [B] time = 2.24528, size = 360, normalized size = 4.19

$$-\frac{2b^3c^3+3ab^2c^2d+6a^2bcd^2-11a^3d^3+18\left(b^3cd^2-ab^2d^3\right)x^2+9\left(b^3c^2d+2ab^2cd^2-3a^2bd^3\right)x-6\left(b^3d^3x^3+3ab^2d\right.}{6\left(b^7x^3+3ab^6x^2+3a^2b^5x+a^3b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d^
2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3
*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

Sympy [A] time = 1.38809, size = 148, normalized size = 1.72

$$\frac{11a^3d^3-6a^2bcd^2-3ab^2c^2d-2b^3c^3+x^2\left(18ab^2d^3-18b^3cd^2\right)+x\left(27a^2bd^3-18ab^2cd^2-9b^3c^2d\right)}{6a^3b^4+18a^2b^5x+18ab^6x^2+6b^7x^3}+\frac{d^3\log\left(a+bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**4,x)
```

```
[Out] (11*a**3*d**3 - 6*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 2*b**3*c**3 + x**2*(18*
a*b**2*d**3 - 18*b**3*c*d**2) + x*(27*a**2*b*d**3 - 18*a*b**2*c*d**2 - 9*b*
*3*c**2*d))/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
 d**3*log(a + b*x)/b**4
```

Giac [A] time = 1.05631, size = 159, normalized size = 1.85

$$\frac{d^3 \log \left(|bx + a|\right)}{b^4} - \frac{18 \left(b^2 c d^2 - a b d^3\right) x^2 + 9 \left(b^2 c^2 d + 2\, a b c d^2 - 3\, a^2 d^3\right) x + \frac{2\, b^3 c^3 + 3\, a b^2 c^2 d + 6\, a^2 b c d^2 - 11\, a^3 d^3}{b}}{6\, (bx + a)^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] d^3*log(abs(b*x + a))/b^4 - 1/6*(18*(b^2*c*d^2 - a*b*d^3)*x^2 + 9*(b^2*c^2*
d + 2*a*b*c*d^2 - 3*a^2*d^3)*x + (2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2
 - 11*a^3*d^3)/b)/((b*x + a)^3*b^3)
```

3.1268 $\int \frac{(c+dx)^3}{(a+bx)^5} dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^4}{4(a+bx)^4(bc-ad)}$$

```
[Out] -(c + d*x)^4/(4*(b*c - a*d)*(a + b*x)^4)
```

Rubi [A] time = 0.0029669, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(c+dx)^4}{4(a+bx)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^5,x]
```

```
[Out] -(c + d*x)^4/(4*(b*c - a*d)*(a + b*x)^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+bx)^5} dx = -\frac{(c+dx)^4}{4(bc-ad)(a+bx)^4}$$

Mathematica [B] time = 0.0317511, size = 91, normalized size = 3.25

$$-\frac{a^2bd^2(c+4dx)+a^3d^3+ab^2d\left(c^2+4cdx+6d^2x^2\right)+b^3\left(4c^2dx+c^3+6cd^2x^2+4d^3x^3\right)}{4b^4(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^5,x]
```

```
[Out] -(a^3*d^3 + a^2*b*d^2*(c + 4*d*x) + a*b^2*d*(c^2 + 4*c*d*x + 6*d^2*x^2) + b
^3*(c^3 + 4*c^2*d*x + 6*c*d^2*x^2 + 4*d^3*x^3))/(4*b^4*(a + b*x)^4)
```

Maple [B] time = 0.005, size = 122, normalized size = 4.4

$$-\frac{d^3}{b^4(bx+a)}+\frac{3d^2(ad-bc)}{2b^4(bx+a)^2}-\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{b^4(bx+a)^3}-\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{4b^4(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^5,x)
```

```
[Out] -d^3/b^4/(b*x+a)+3/2*d^2*(a*d-b*c)/b^4/(b*x+a)^2-d*(a^2*d^2-2*a*b*c*d+b^2*c
^2)/b^4/(b*x+a)^3-1/4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b
*x+a)^4
```

Maxima [B] time = 0.979592, size = 193, normalized size = 6.89

$$-\frac{4\,b^3d^3x^3+b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3+6\left(b^3cd^2+ab^2d^3\right)x^2+4\left(b^3c^2d+ab^2cd^2+a^2bd^3\right)x}{4\left(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b^
3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^8*
x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Fricas [B] time = 2.28969, size = 284, normalized size = 10.14

$$-\frac{4\,b^3d^3x^3+b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3+6\left(b^3cd^2+ab^2d^3\right)x^2+4\left(b^3c^2d+ab^2cd^2+a^2bd^3\right)x}{4\left(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b^
3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^8*
x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Sympy [B] time = 1.76393, size = 153, normalized size = 5.46

$$-\frac{a^3d^3+a^2bcd^2+ab^2c^2d+b^3c^3+4b^3d^3x^3+x^2\left(6ab^2d^3+6b^3cd^2\right)+x\left(4a^2bd^3+4ab^2cd^2+4b^3c^2d\right)}{4a^4b^4+16a^3b^5x+24a^2b^6x^2+16ab^7x^3+4b^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**5,x)
```

```
[Out] -(a**3*d**3 + a**2*b*c*d**2 + a*b**2*c**2*d + b**3*c**3 + 4*b**3*d**3*x**3
+ x**2*(6*a*b**2*d**3 + 6*b**3*c*d**2) + x*(4*a**2*b*d**3 + 4*a*b**2*c*d**2
 + 4*b**3*c**2*d))/(4*a**4*b**4 + 16*a**3*b**5*x + 24*a**2*b**6*x**2 + 16*a
*b**7*x**3 + 4*b**8*x**4)
```

Giac [B] time = 1.05459, size = 215, normalized size = 7.68

$$-\frac{\frac{b^2c^3}{(bx+a)^4}+\frac{4\,bc^2d}{(bx+a)^3}-\frac{3\,abc^2d}{(bx+a)^4}+\frac{6\,cd^2}{(bx+a)^2}-\frac{8\,acd^2}{(bx+a)^3}+\frac{3\,a^2cd^2}{(bx+a)^4}+\frac{4\,d^3}{(bx+a)b}-\frac{6\,ad^3}{(bx+a)^2b}+\frac{4\,a^2d^3}{(bx+a)^3b}-\frac{a^3d^3}{(bx+a)^4b}}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*(b^2*c^3/(b*x + a)^4 + 4*b*c^2*d/(b*x + a)^3 - 3*a*b*c^2*d/(b*x + a)^4
 + 6*c*d^2/(b*x + a)^2 - 8*a*c*d^2/(b*x + a)^3 + 3*a^2*c*d^2/(b*x + a)^4 +
4*d^3/((b*x + a)*b) - 6*a*d^3/((b*x + a)^2*b) + 4*a^2*d^3/((b*x + a)^3*b) -
 a^3*d^3/((b*x + a)^4*b))/b^3
```

3.1269 $\int \frac{(c+dx)^3}{(a+bx)^6} dx$

Optimal. Leaf size=58

$$\frac{d(c+dx)^4}{20(a+bx)^4(bc-ad)^2} - \frac{(c+dx)^4}{5(a+bx)^5(bc-ad)}$$

```
[Out] -(c + d*x)^4/(5*(b*c - a*d)*(a + b*x)^5) + (d*(c + d*x)^4)/(20*(b*c - a*d)^
2*(a + b*x)^4)
```

Rubi [A] time = 0.0103058, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d(c+dx)^4}{20(a+bx)^4(bc-ad)^2} - \frac{(c+dx)^4}{5(a+bx)^5(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^6,x]
```

```
[Out] -(c + d*x)^4/(5*(b*c - a*d)*(a + b*x)^5) + (d*(c + d*x)^4)/(20*(b*c - a*d)^
2*(a + b*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^3}{(a+bx)^6} dx &= -\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5} - \frac{d \int \frac{(c+dx)^3}{(a+bx)^5} dx}{5(bc-ad)} \\
&= -\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5} + \frac{d(c+dx)^4}{20(bc-ad)^2(a+bx)^4}
\end{aligned}$$

Mathematica [A] time = 0.0358266, size = 97, normalized size = 1.67

$$-\frac{a^2bd^2(2c+5dx)+a^3d^3+ab^2d\left(3c^2+10cdx+10d^2x^2\right)+b^3\left(15c^2dx+4c^3+20cd^2x^2+10d^3x^3\right)}{20b^4(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^6,x]
```

```
[Out] -(a^3*d^3 + a^2*b*d^2*(2*c + 5*d*x) + a*b^2*d*(3*c^2 + 10*c*d*x + 10*d^2*x^
2) + b^3*(4*c^3 + 15*c^2*d*x + 20*c*d^2*x^2 + 10*d^3*x^3))/(20*b^4*(a + b*x
)^5)
```

Maple [B] time = 0.005, size = 121, normalized size = 2.1

$$-\frac{d^3}{2\,b^4\,(bx+a)^2}+\frac{d^2\,(ad-bc)}{b^4\,(bx+a)^3}-\frac{-a^3d^3+3\,a^2bcd^2-3\,ab^2c^2d+b^3c^3}{5\,b^4\,(bx+a)^5}-\frac{3\,d\left(a^2d^2-2\,abcd+b^2c^2\right)}{4\,b^4\,(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^6,x)
```

```
[Out] -1/2*d^3/b^4/(b*x+a)^2+d^2*(a*d-b*c)/b^4/(b*x+a)^3-1/5*(-a^3*d^3+3*a^2*b*c*
d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^5-3/4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/
b^4/(b*x+a)^4
```

Maxima [B] time = 0.963151, size = 216, normalized size = 3.72

$$-\frac{10\,b^3d^3x^3+4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3+10\left(2\,b^3cd^2+ab^2d^3\right)x^2+5\left(3\,b^3c^2d+2\,ab^2cd^2+a^2bd^3\right)x}{20\left(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3
 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

Fricas [B] time = 2.33128, size = 328, normalized size = 5.66

$$-\frac{10\,b^3d^3x^3+4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3+10\left(2\,b^3cd^2+ab^2d^3\right)x^2+5\left(3\,b^3c^2d+2\,ab^2cd^2+a^2bd^3\right)x}{20\left(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3
 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

Sympy [B] time = 2.29929, size = 170, normalized size = 2.93

$$-\frac{a^3d^3+2a^2bcd^2+3ab^2c^2d+4b^3c^3+10b^3d^3x^3+x^2\left(10ab^2d^3+20b^3cd^2\right)+x\left(5a^2bd^3+10ab^2cd^2+15b^3c^2d\right)}{20a^5b^4+100a^4b^5x+200a^3b^6x^2+200a^2b^7x^3+100ab^8x^4+20b^9x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**6,x)
```

```
[Out] -(a**3*d**3 + 2*a**2*b*c*d**2 + 3*a*b**2*c**2*d + 4*b**3*c**3 + 10*b**3*d**
3*x**3 + x**2*(10*a*b**2*d**3 + 20*b**3*c*d**2) + x*(5*a**2*b*d**3 + 10*a*b
**2*c*d**2 + 15*b**3*c**2*d))/(20*a**5*b**4 + 100*a**4*b**5*x + 200*a**3*b*
*6*x**2 + 200*a**2*b**7*x**3 + 100*a*b**8*x**4 + 20*b**9*x**5)
```

Giac [B] time = 1.05608, size = 154, normalized size = 2.66

$$-\frac{10\,b^3d^3x^3+20\,b^3cd^2x^2+10\,ab^2d^3x^2+15\,b^3c^2dx+10\,ab^2cd^2x+5\,a^2bd^3x+4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3}{20\,(bx+a)^5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 20*b^3*c*d^2*x^2 + 10*a*b^2*d^3*x^2 + 15*b^3*c^2*d*
x + 10*a*b^2*c*d^2*x + 5*a^2*b*d^3*x + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*
c*d^2 + a^3*d^3)/((b*x + a)^5*b^4)
```

3.1270 $\int \frac{(c+dx)^3}{(a+bx)^7}\,dx$

Optimal. Leaf size=92

$$-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{d^3}{3b^4(a+bx)^3}$$

```
[Out] -(b*c - a*d)^3/(6*b^4*(a + b*x)^6) - (3*d*(b*c - a*d)^2)/(5*b^4*(a + b*x)^5
) - (3*d^2*(b*c - a*d))/(4*b^4*(a + b*x)^4) - d^3/(3*b^4*(a + b*x)^3)
```

Rubi [A] time = 0.0504132, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{d^3}{3b^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^7,x]
```

```
[Out] -(b*c - a*d)^3/(6*b^4*(a + b*x)^6) - (3*d*(b*c - a*d)^2)/(5*b^4*(a + b*x)^5
) - (3*d^2*(b*c - a*d))/(4*b^4*(a + b*x)^4) - d^3/(3*b^4*(a + b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^7}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^7}+\frac{3d(bc-ad)^2}{b^3(a+bx)^6}+\frac{3d^2(bc-ad)}{b^3(a+bx)^5}+\frac{d^3}{b^3(a+bx)^4}\right)dx\\ &= -\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{d^3}{3b^4(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.0350424, size = 97, normalized size = 1.05

$$-\frac{3a^2bd^2(c+2dx)+a^3d^3+3ab^2d\left(2c^2+6cdx+5d^2x^2\right)+b^3\left(36c^2dx+10c^3+45cd^2x^2+20d^3x^3\right)}{60b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^7,x]
```

```
[Out] -(a^3*d^3 + 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*d*(2*c^2 + 6*c*d*x + 5*d^2*x^
2) + b^3*(10*c^3 + 36*c^2*d*x + 45*c*d^2*x^2 + 20*d^3*x^3))/(60*b^4*(a + b*
x)^6)
```

Maple [A] time = 0.006, size = 122, normalized size = 1.3

$$-\frac{-a^{3}d^{3}+3\,a^{2}bc\,d^{2}-3\,a\,b^{2}c^{2}d+b^{3}c^{3}}{6\,b^{4}\left(bx+a\right)^{6}}-\frac{d^{3}}{3\,b^{4}\left(bx+a\right)^{3}}-\frac{3\,d\left(a^{2}d^{2}-2\,abcd+b^{2}c^{2}\right)}{5\,b^{4}\left(bx+a\right)^{5}}+\frac{3\,d^{2}\left(ad-bc\right)}{4\,b^{4}\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^7,x)
```

```
[Out] -1/6*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^6-1/3*d^3/b
^4/(b*x+a)^3-3/5*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^5+3/4*d^2*(a*d-b
*c)/b^4/(b*x+a)^4
```

Maxima [B] time = 0.970962, size = 231, normalized size = 2.51

$$-\frac{20\,b^{3}d^{3}x^{3}+10\,b^{3}c^{3}+6\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}+a^{3}d^{3}+15\left(3\,b^{3}cd^{2}+ab^{2}d^{3}\right)x^{2}+6\left(6\,b^{3}c^{2}d+3\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x}{60\left(b^{10}x^{6}+6\,ab^{9}x^{5}+15\,a^{2}b^{8}x^{4}+20\,a^{3}b^{7}x^{3}+15\,a^{4}b^{6}x^{2}+6\,a^{5}b^{5}x+a^{6}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^
3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a^2
*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a
^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

Fricas [B] time = 2.23976, size = 354, normalized size = 3.85

$$-\frac{20\,b^{3}d^{3}x^{3}+10\,b^{3}c^{3}+6\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}+a^{3}d^{3}+15\left(3\,b^{3}cd^{2}+ab^{2}d^{3}\right)x^{2}+6\left(6\,b^{3}c^{2}d+3\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x}{60\left(b^{10}x^{6}+6\,ab^{9}x^{5}+15\,a^{2}b^{8}x^{4}+20\,a^{3}b^{7}x^{3}+15\,a^{4}b^{6}x^{2}+6\,a^{5}b^{5}x+a^{6}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^
3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a^2
*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a
^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

Sympy [B] time = 3.05433, size = 182, normalized size = 1.98

$$-\frac{a^{3}d^{3}+3a^{2}bcd^{2}+6ab^{2}c^{2}d+10b^{3}c^{3}+20b^{3}d^{3}x^{3}+x^{2}\left(15ab^{2}d^{3}+45b^{3}cd^{2}\right)+x\left(6a^{2}bd^{3}+18ab^{2}cd^{2}+36b^{3}c^{2}d\right)}{60a^{6}b^{4}+360a^{5}b^{5}x+900a^{4}b^{6}x^{2}+1200a^{3}b^{7}x^{3}+900a^{2}b^{8}x^{4}+360ab^{9}x^{5}+60b^{10}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**7,x)
```

```
[Out] -(a**3*d**3 + 3*a**2*b*c*d**2 + 6*a*b**2*c**2*d + 10*b**3*c**3 + 20*b**3*d*
*3*x**3 + x**2*(15*a*b**2*d**3 + 45*b**3*c*d**2) + x*(6*a**2*b*d**3 + 18*a*
b**2*c*d**2 + 36*b**3*c**2*d))/(60*a**6*b**4 + 360*a**5*b**5*x + 900*a**4*b
**6*x**2 + 1200*a**3*b**7*x**3 + 900*a**2*b**8*x**4 + 360*a*b**9*x**5 + 60*
b**10*x**6)
```

Giac [A] time = 1.05761, size = 154, normalized size = 1.67

$$-\frac{20\,b^3d^3x^3 + 45\,b^3cd^2x^2 + 15\,ab^2d^3x^2 + 36\,b^3c^2dx + 18\,ab^2cd^2x + 6\,a^2bd^3x + 10\,b^3c^3 + 6\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3}{60\,(bx+a)^6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 45*b^3*c*d^2*x^2 + 15*a*b^2*d^3*x^2 + 36*b^3*c^2*d*
x + 18*a*b^2*c*d^2*x + 6*a^2*b*d^3*x + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b
*c*d^2 + a^3*d^3)/((b*x + a)^6*b^4)
```

3.1271 $\int \frac{(c+dx)^3}{(a+bx)^8} dx$

Optimal. Leaf size=92

$$-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d^3}{4b^4(a+bx)^4}$$

```
[Out] -(b*c - a*d)^3/(7*b^4*(a + b*x)^7) - (d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^6)
- (3*d^2*(b*c - a*d))/(5*b^4*(a + b*x)^5) - d^3/(4*b^4*(a + b*x)^4)
```

Rubi [A] time = 0.049082, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d^3}{4b^4(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^8,x]
```

```
[Out] -(b*c - a*d)^3/(7*b^4*(a + b*x)^7) - (d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^6)
- (3*d^2*(b*c - a*d))/(5*b^4*(a + b*x)^5) - d^3/(4*b^4*(a + b*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^8}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^8}+\frac{3d(bc-ad)^2}{b^3(a+bx)^7}+\frac{3d^2(bc-ad)}{b^3(a+bx)^6}+\frac{d^3}{b^3(a+bx)^5}\right)dx\\ &= -\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d^3}{4b^4(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.0309266, size = 97, normalized size = 1.05

$$-\frac{a^2bd^2(4c+7dx)+a^3d^3+ab^2d\left(10c^2+28cdx+21d^2x^2\right)+b^3\left(70c^2dx+20c^3+84cd^2x^2+35d^3x^3\right)}{140b^4(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^8,x]
```

```
[Out] -(a^3*d^3 + a^2*b*d^2*(4*c + 7*d*x) + a*b^2*d*(10*c^2 + 28*c*d*x + 21*d^2*x
^2) + b^3*(20*c^3 + 70*c^2*d*x + 84*c*d^2*x^2 + 35*d^3*x^3))/(140*b^4*(a +
b*x)^7)
```

Maple [A] time = 0.004, size = 122, normalized size = 1.3

$$-\frac{d\left(a^2d^2-2\,abcd+b^2c^2\right)}{2\,b^4\left(bx+a\right)^6}+\frac{3\,d^2\left(ad-bc\right)}{5\,b^4\left(bx+a\right)^5}-\frac{d^3}{4\,b^4\left(bx+a\right)^4}-\frac{-a^3d^3+3\,a^2bcd^2-3\,ab^2c^2d+b^3c^3}{7\,b^4\left(bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^8,x)
```

```
[Out] -1/2*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^6+3/5*d^2*(a*d-b*c)/b^4/(b*x
+a)^5-1/4*d^3/b^4/(b*x+a)^4-1/7*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c
^3)/b^4/(b*x+a)^7
```

Maxima [B] time = 1.01489, size = 246, normalized size = 2.67

$$-\frac{35\,b^3d^3x^3+20\,b^3c^3+10\,ab^2c^2d+4\,a^2bcd^2+a^3d^3+21\left(4\,b^3cd^2+ab^2d^3\right)x^2+7\left(10\,b^3c^2d+4\,ab^2cd^2+a^2bd^3\right)x}{140\left(b^{11}x^7+7\,ab^{10}x^6+21\,a^2b^9x^5+35\,a^3b^8x^4+35\,a^4b^7x^3+21\,a^5b^6x^2+7\,a^6b^5x+a^7b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3*
d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4 +
35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

Fricas [B] time = 2.11933, size = 382, normalized size = 4.15

$$-\frac{35\,b^3d^3x^3+20\,b^3c^3+10\,ab^2c^2d+4\,a^2bcd^2+a^3d^3+21\left(4\,b^3cd^2+ab^2d^3\right)x^2+7\left(10\,b^3c^2d+4\,ab^2cd^2+a^2bd^3\right)x}{140\left(b^{11}x^7+7\,ab^{10}x^6+21\,a^2b^9x^5+35\,a^3b^8x^4+35\,a^4b^7x^3+21\,a^5b^6x^2+7\,a^6b^5x+a^7b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3*
d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4 +
35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

Sympy [B] time = 3.79294, size = 194, normalized size = 2.11

$$-\frac{a^3d^3+4a^2bcd^2+10ab^2c^2d+20b^3c^3+35b^3d^3x^3+x^2\left(21ab^2d^3+84b^3cd^2\right)+x\left(7a^2bd^3+28ab^2cd^2+70b^3c^2d\right)}{140a^7b^4+980a^6b^5x+2940a^5b^6x^2+4900a^4b^7x^3+4900a^3b^8x^4+2940a^2b^9x^5+980ab^{10}x^6+140b^{11}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**8,x)
```

```
[Out] -(a**3*d**3 + 4*a**2*b*c*d**2 + 10*a*b**2*c**2*d + 20*b**3*c**3 + 35*b**3*d
**3*x**3 + x**2*(21*a*b**2*d**3 + 84*b**3*c*d**2) + x*(7*a**2*b*d**3 + 28*a
*b**2*c*d**2 + 70*b**3*c**2*d))/(140*a**7*b**4 + 980*a**6*b**5*x + 2940*a**
5*b**6*x**2 + 4900*a**4*b**7*x**3 + 4900*a**3*b**8*x**4 + 2940*a**2*b**9*x*
*5 + 980*a*b**10*x**6 + 140*b**11*x**7)
```

Giac [A] time = 1.06903, size = 154, normalized size = 1.67

$$-\frac{35\,b^3d^3x^3 + 84\,b^3cd^2x^2 + 21\,ab^2d^3x^2 + 70\,b^3c^2dx + 28\,ab^2cd^2x + 7\,a^2bd^3x + 20\,b^3c^3 + 10\,ab^2c^2d + 4\,a^2bcd^2 + a^3d^3}{140\,(bx+a)^7b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 84*b^3*c*d^2*x^2 + 21*a*b^2*d^3*x^2 + 70*b^3*c^2*d
*x + 28*a*b^2*c*d^2*x + 7*a^2*b*d^3*x + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2
*b*c*d^2 + a^3*d^3)/((b*x + a)^7*b^4)
```

3.1272 $\int \frac{(c+dx)^3}{(a+bx)^9} \, dx$

Optimal. Leaf size=92

$$-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{d^3}{5b^4(a+bx)^5}$$

```
[Out] -(b*c - a*d)^3/(8*b^4*(a + b*x)^8) - (3*d*(b*c - a*d)^2)/(7*b^4*(a + b*x)^7
) - (d^2*(b*c - a*d))/(2*b^4*(a + b*x)^6) - d^3/(5*b^4*(a + b*x)^5)
```

Rubi [A] time = 0.0463985, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{d^3}{5b^4(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^9,x]
```

```
[Out] -(b*c - a*d)^3/(8*b^4*(a + b*x)^8) - (3*d*(b*c - a*d)^2)/(7*b^4*(a + b*x)^7
) - (d^2*(b*c - a*d))/(2*b^4*(a + b*x)^6) - d^3/(5*b^4*(a + b*x)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^9}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^9}+\frac{3d(bc-ad)^2}{b^3(a+bx)^8}+\frac{3d^2(bc-ad)}{b^3(a+bx)^7}+\frac{d^3}{b^3(a+bx)^6}\right)dx\\ &= -\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{d^3}{5b^4(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.0352527, size = 97, normalized size = 1.05

$$-\frac{a^2bd^2(5c+8dx)+a^3d^3+ab^2d\left(15c^2+40cdx+28d^2x^2\right)+b^3\left(120c^2dx+35c^3+140cd^2x^2+56d^3x^3\right)}{280b^4(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^9,x]
```

```
[Out] -(a^3*d^3 + a^2*b*d^2*(5*c + 8*d*x) + a*b^2*d*(15*c^2 + 40*c*d*x + 28*d^2*x
^2) + b^3*(35*c^3 + 120*c^2*d*x + 140*c*d^2*x^2 + 56*d^3*x^3))/(280*b^4*(a
+ b*x)^8)
```

Maple [A] time = 0.006, size = 122, normalized size = 1.3

$$-\frac{-a^3d^3+3\,a^2bcd^2-3\,ab^2c^2d+b^3c^3}{8\,b^4\,(bx+a)^8}+\frac{d^2\,(ad-bc)}{2\,b^4\,(bx+a)^6}-\frac{d^3}{5\,b^4\,(bx+a)^5}-\frac{3\,d\left(a^2d^2-2\,abcd+b^2c^2\right)}{7\,b^4\,(bx+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^9,x)
```

```
[Out] -1/8*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^8+1/2*d^2*(
a*d-b*c)/b^4/(b*x+a)^6-1/5*d^3/b^4/(b*x+a)^5-3/7*d*(a^2*d^2-2*a*b*c*d+b^2*c
^2)/b^4/(b*x+a)^7
```

Maxima [B] time = 1.03103, size = 261, normalized size = 2.84

$$-\frac{56\,b^3d^3x^3+35\,b^3c^3+15\,ab^2c^2d+5\,a^2bcd^2+a^3d^3+28\left(5\,b^3cd^2+ab^2d^3\right)x^2+8\left(15\,b^3c^2d+5\,ab^2cd^2+a^2bd^3\right)x}{280\left(b^{12}x^8+8\,ab^{11}x^7+28\,a^2b^{10}x^6+56\,a^3b^9x^5+70\,a^4b^8x^4+56\,a^5b^7x^3+28\,a^6b^6x^2+8\,a^7b^5x+a^8b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*
d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^5 +
 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b^4)
```

Fricas [B] time = 2.25509, size = 406, normalized size = 4.41

$$-\frac{56\,b^3d^3x^3+35\,b^3c^3+15\,ab^2c^2d+5\,a^2bcd^2+a^3d^3+28\left(5\,b^3cd^2+ab^2d^3\right)x^2+8\left(15\,b^3c^2d+5\,ab^2cd^2+a^2bd^3\right)x}{280\left(b^{12}x^8+8\,ab^{11}x^7+28\,a^2b^{10}x^6+56\,a^3b^9x^5+70\,a^4b^8x^4+56\,a^5b^7x^3+28\,a^6b^6x^2+8\,a^7b^5x+a^8b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*
d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^5 +
 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b^4)
```

Sympy [B] time = 4.83922, size = 206, normalized size = 2.24

$$-\frac{a^3d^3+5a^2bcd^2+15ab^2c^2d+35b^3c^3+56b^3d^3x^3+x^2\left(28ab^2d^3+140b^3cd^2\right)+x\left(8a^2bd^3+40ab^2cd^2+120b^3c^2d\right)}{280a^8b^4+2240a^7b^5x+7840a^6b^6x^2+15680a^5b^7x^3+19600a^4b^8x^4+15680a^3b^9x^5+7840a^2b^{10}x^6+2240ab^{11}x^7+280b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**9,x)
```

```
[Out] -(a**3*d**3 + 5*a**2*b*c*d**2 + 15*a*b**2*c**2*d + 35*b**3*c**3 + 56*b**3*d
**3*x**3 + x**2*(28*a*b**2*d**3 + 140*b**3*c*d**2) + x*(8*a**2*b*d**3 + 40*
a*b**2*c*d**2 + 120*b**3*c**2*d))/(280*a**8*b**4 + 2240*a**7*b**5*x + 7840*
a**6*b**6*x**2 + 15680*a**5*b**7*x**3 + 19600*a**4*b**8*x**4 + 15680*a**3*b
**9*x**5 + 7840*a**2*b**10*x**6 + 2240*a*b**11*x**7 + 280*b**12*x**8)
```

Giac [A] time = 1.06038, size = 154, normalized size = 1.67

$$-\frac{56\,b^3d^3x^3+140\,b^3cd^2x^2+28\,ab^2d^3x^2+120\,b^3c^2dx+40\,ab^2cd^2x+8\,a^2bd^3x+35\,b^3c^3+15\,ab^2c^2d+5\,a^2bcd^2+a^3d^3}{280\,(bx+a)^8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 140*b^3*c*d^2*x^2 + 28*a*b^2*d^3*x^2 + 120*b^3*c^2
*d*x + 40*a*b^2*c*d^2*x + 8*a^2*b*d^3*x + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a
^2*b*c*d^2 + a^3*d^3)/((b*x + a)^8*b^4)
```

3.1273 $\int (a+bx)^9(c+dx)^7\,dx$

Optimal. Leaf size=200

$$\frac{7d^6(a+bx)^{16}(bc-ad)}{16b^8}+\frac{7d^5(a+bx)^{15}(bc-ad)^2}{5b^8}+\frac{5d^4(a+bx)^{14}(bc-ad)^3}{2b^8}+\frac{35d^3(a+bx)^{13}(bc-ad)^4}{13b^8}+\frac{7d^2(a+bx)^{12}(}{4b^8}$$

```
[Out] ((b*c - a*d)^7*(a + b*x)^10)/(10*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^11)/(1
1*b^8) + (7*d^2*(b*c - a*d)^5*(a + b*x)^12)/(4*b^8) + (35*d^3*(b*c - a*d)^4
*(a + b*x)^13)/(13*b^8) + (5*d^4*(b*c - a*d)^3*(a + b*x)^14)/(2*b^8) + (7*d
^5*(b*c - a*d)^2*(a + b*x)^15)/(5*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^16)/(
16*b^8) + (d^7*(a + b*x)^17)/(17*b^8)
```

Rubi [A] time = 0.676466, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^{16}(bc-ad)}{16b^8}+\frac{7d^5(a+bx)^{15}(bc-ad)^2}{5b^8}+\frac{5d^4(a+bx)^{14}(bc-ad)^3}{2b^8}+\frac{35d^3(a+bx)^{13}(bc-ad)^4}{13b^8}+\frac{7d^2(a+bx)^{12}(}{4b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9*(c + d*x)^7,x]
```



```
[Out] ((b*c - a*d)^7*(a + b*x)^10)/(10*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^11)/(1
1*b^8) + (7*d^2*(b*c - a*d)^5*(a + b*x)^12)/(4*b^8) + (35*d^3*(b*c - a*d)^4
*(a + b*x)^13)/(13*b^8) + (5*d^4*(b*c - a*d)^3*(a + b*x)^14)/(2*b^8) + (7*d
^5*(b*c - a*d)^2*(a + b*x)^15)/(5*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^16)/(
16*b^8) + (d^7*(a + b*x)^17)/(17*b^8)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^9(c+dx)^7\,dx &= \int \left(\frac{(bc-ad)^7(a+bx)^9}{b^7}+\frac{7d(bc-ad)^6(a+bx)^{10}}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^{11}}{b^7}+\frac{35d^3(bc-ad)^4}{b^7}\right.\\ &= \frac{(bc-ad)^7(a+bx)^{10}}{10b^8}+\frac{7d(bc-ad)^6(a+bx)^{11}}{11b^8}+\frac{7d^2(bc-ad)^5(a+bx)^{12}}{4b^8}+\frac{35d^3(bc-ad)^4(a+}{13b^8}\end{aligned}$$

Mathematica [B] time = 0.139842, size = 993, normalized size = 4.96

$$\frac{1}{17}b^9d^7x^{17}+\frac{1}{16}b^8d^6(7bc+9ad)x^{16}+\frac{1}{5}b^7d^5\left(7b^2c^2+21abdc+12a^2d^2\right)x^{15}+\frac{1}{2}b^6d^4\left(5b^3c^3+27ab^2dc^2+36a^2bd^2c+12a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9*(c + d*x)^7,x]
```

```
[Out] a^9*c^7*x + (a^8*c^6*(9*b*c + 7*a*d)*x^2)/2 + a^7*c^5*(12*b^2*c^2 + 21*a*b*
c*d + 7*a^2*d^2)*x^3 + (7*a^6*c^4*(12*b^3*c^3 + 36*a*b^2*c^2*d + 27*a^2*b*c
*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^5*c^3*(18*b^4*c^4 + 84*a*b^3*c^3*d + 108*a^
2*b^2*c^2*d^2 + 45*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^4*c^2*(6*b^5*c^5
+ 42*a*b^4*c^4*d + 84*a^2*b^3*c^3*d^2 + 60*a^3*b^2*c^2*d^3 + 15*a^4*b*c*d^4
 + a^5*d^5)*x^6)/2 + a^3*c*(12*b^6*c^6 + 126*a*b^5*c^5*d + 378*a^2*b^4*c^4*
d^2 + 420*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^4 + 27*a^5*b*c*d^5 + a^6*d^6)
*x^7 + (a^2*(36*b^7*c^7 + 588*a*b^6*c^6*d + 2646*a^2*b^5*c^5*d^2 + 4410*a^3
*b^4*c^4*d^3 + 2940*a^4*b^3*c^3*d^4 + 756*a^5*b^2*c^2*d^5 + 63*a^6*b*c*d^6
+ a^7*d^7)*x^8)/8 + a*b*(b^7*c^7 + 28*a*b^6*c^6*d + 196*a^2*b^5*c^5*d^2 + 4
90*a^3*b^4*c^4*d^3 + 490*a^4*b^3*c^3*d^4 + 196*a^5*b^2*c^2*d^5 + 28*a^6*b*c
*d^6 + a^7*d^7)*x^9 + (b^2*(b^7*c^7 + 63*a*b^6*c^6*d + 756*a^2*b^5*c^5*d^2
+ 2940*a^3*b^4*c^4*d^3 + 4410*a^4*b^3*c^3*d^4 + 2646*a^5*b^2*c^2*d^5 + 588*
a^6*b*c*d^6 + 36*a^7*d^7)*x^10)/10 + (7*b^3*d*(b^6*c^6 + 27*a*b^5*c^5*d + 1
80*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d^3 + 378*a^4*b^2*c^2*d^4 + 126*a^5*b*
c*d^5 + 12*a^6*d^6)*x^11)/11 + (7*b^4*d^2*(b^5*c^5 + 15*a*b^4*c^4*d + 60*a^
2*b^3*c^3*d^2 + 84*a^3*b^2*c^2*d^3 + 42*a^4*b*c*d^4 + 6*a^5*d^5)*x^12)/4 +
(7*b^5*d^3*(5*b^4*c^4 + 45*a*b^3*c^3*d + 108*a^2*b^2*c^2*d^2 + 84*a^3*b*c*d
^3 + 18*a^4*d^4)*x^13)/13 + (b^6*d^4*(5*b^3*c^3 + 27*a*b^2*c^2*d + 36*a^2*b
*c*d^2 + 12*a^3*d^3)*x^14)/2 + (b^7*d^5*(7*b^2*c^2 + 21*a*b*c*d + 12*a^2*d^
2)*x^15)/5 + (b^8*d^6*(7*b*c + 9*a*d)*x^16)/16 + (b^9*d^7*x^17)/17
```

Maple [B] time = 0.003, size = 1033, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9*(d*x+c)^7,x)
```

```
[Out] 1/17*b^9*d^7*x^17+1/16*(9*a*b^8*d^7+7*b^9*c*d^6)*x^16+1/15*(36*a^2*b^7*d^7+
63*a*b^8*c*d^6+21*b^9*c^2*d^5)*x^15+1/14*(84*a^3*b^6*d^7+252*a^2*b^7*c*d^6+
189*a*b^8*c^2*d^5+35*b^9*c^3*d^4)*x^14+1/13*(126*a^4*b^5*d^7+588*a^3*b^6*c*
d^6+756*a^2*b^7*c^2*d^5+315*a*b^8*c^3*d^4+35*b^9*c^4*d^3)*x^13+1/12*(126*a^
5*b^4*d^7+882*a^4*b^5*c*d^6+1764*a^3*b^6*c^2*d^5+1260*a^2*b^7*c^3*d^4+315*a
*b^8*c^4*d^3+21*b^9*c^5*d^2)*x^12+1/11*(84*a^6*b^3*d^7+882*a^5*b^4*c*d^6+26
46*a^4*b^5*c^2*d^5+2940*a^3*b^6*c^3*d^4+1260*a^2*b^7*c^4*d^3+189*a*b^8*c^5*
d^2+7*b^9*c^6*d)*x^11+1/10*(36*a^7*b^2*d^7+588*a^6*b^3*c*d^6+2646*a^5*b^4*c
^2*d^5+4410*a^4*b^5*c^3*d^4+2940*a^3*b^6*c^4*d^3+756*a^2*b^7*c^5*d^2+63*a*b
^8*c^6*d+b^9*c^7)*x^10+1/9*(9*a^8*b*d^7+252*a^7*b^2*c*d^6+1764*a^6*b^3*c^2*
d^5+4410*a^5*b^4*c^3*d^4+4410*a^4*b^5*c^4*d^3+1764*a^3*b^6*c^5*d^2+252*a^2*
b^7*c^6*d+9*a*b^8*c^7)*x^9+1/8*(a^9*d^7+63*a^8*b*c*d^6+756*a^7*b^2*c^2*d^5+
2940*a^6*b^3*c^3*d^4+4410*a^5*b^4*c^4*d^3+2646*a^4*b^5*c^5*d^2+588*a^3*b^6*
c^6*d+36*a^2*b^7*c^7)*x^8+1/7*(7*a^9*c*d^6+189*a^8*b*c^2*d^5+1260*a^7*b^2*c
^3*d^4+2940*a^6*b^3*c^4*d^3+2646*a^5*b^4*c^5*d^2+882*a^4*b^5*c^6*d+84*a^3*b
^6*c^7)*x^7+1/6*(21*a^9*c^2*d^5+315*a^8*b*c^3*d^4+1260*a^7*b^2*c^4*d^3+1764
*a^6*b^3*c^5*d^2+882*a^5*b^4*c^6*d+126*a^4*b^5*c^7)*x^6+1/5*(35*a^9*c^3*d^4
+315*a^8*b*c^4*d^3+756*a^7*b^2*c^5*d^2+588*a^6*b^3*c^6*d+126*a^5*b^4*c^7)*x
^5+1/4*(35*a^9*c^4*d^3+189*a^8*b*c^5*d^2+252*a^7*b^2*c^6*d+84*a^6*b^3*c^7)*
x^4+1/3*(21*a^9*c^5*d^2+63*a^8*b*c^6*d+36*a^7*b^2*c^7)*x^3+1/2*(7*a^9*c^6*d
+9*a^8*b*c^7)*x^2+a^9*c^7*x
```

Maxima [B] time = 0.980516, size = 1381, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/17*b^9*d^7*x^17 + a^9*c^7*x + 1/16*(7*b^9*c*d^6 + 9*a*b^8*d^7)*x^16 + 1/5
*(7*b^9*c^2*d^5 + 21*a*b^8*c*d^6 + 12*a^2*b^7*d^7)*x^15 + 1/2*(5*b^9*c^3*d^
4 + 27*a*b^8*c^2*d^5 + 36*a^2*b^7*c*d^6 + 12*a^3*b^6*d^7)*x^14 + 7/13*(5*b^
9*c^4*d^3 + 45*a*b^8*c^3*d^4 + 108*a^2*b^7*c^2*d^5 + 84*a^3*b^6*c*d^6 + 18*
a^4*b^5*d^7)*x^13 + 7/4*(b^9*c^5*d^2 + 15*a*b^8*c^4*d^3 + 60*a^2*b^7*c^3*d^
4 + 84*a^3*b^6*c^2*d^5 + 42*a^4*b^5*c*d^6 + 6*a^5*b^4*d^7)*x^12 + 7/11*(b^9
*c^6*d + 27*a*b^8*c^5*d^2 + 180*a^2*b^7*c^4*d^3 + 420*a^3*b^6*c^3*d^4 + 378
*a^4*b^5*c^2*d^5 + 126*a^5*b^4*c*d^6 + 12*a^6*b^3*d^7)*x^11 + 1/10*(b^9*c^7
 + 63*a*b^8*c^6*d + 756*a^2*b^7*c^5*d^2 + 2940*a^3*b^6*c^4*d^3 + 4410*a^4*b
^5*c^3*d^4 + 2646*a^5*b^4*c^2*d^5 + 588*a^6*b^3*c*d^6 + 36*a^7*b^2*d^7)*x^1
0 + (a*b^8*c^7 + 28*a^2*b^7*c^6*d + 196*a^3*b^6*c^5*d^2 + 490*a^4*b^5*c^4*d
^3 + 490*a^5*b^4*c^3*d^4 + 196*a^6*b^3*c^2*d^5 + 28*a^7*b^2*c*d^6 + a^8*b*d
^7)*x^9 + 1/8*(36*a^2*b^7*c^7 + 588*a^3*b^6*c^6*d + 2646*a^4*b^5*c^5*d^2 +
4410*a^5*b^4*c^4*d^3 + 2940*a^6*b^3*c^3*d^4 + 756*a^7*b^2*c^2*d^5 + 63*a^8*
b*c*d^6 + a^9*d^7)*x^8 + (12*a^3*b^6*c^7 + 126*a^4*b^5*c^6*d + 378*a^5*b^4*
c^5*d^2 + 420*a^6*b^3*c^4*d^3 + 180*a^7*b^2*c^3*d^4 + 27*a^8*b*c^2*d^5 + a^
9*c*d^6)*x^7 + 7/2*(6*a^4*b^5*c^7 + 42*a^5*b^4*c^6*d + 84*a^6*b^3*c^5*d^2 +
 60*a^7*b^2*c^4*d^3 + 15*a^8*b*c^3*d^4 + a^9*c^2*d^5)*x^6 + 7/5*(18*a^5*b^4
*c^7 + 84*a^6*b^3*c^6*d + 108*a^7*b^2*c^5*d^2 + 45*a^8*b*c^4*d^3 + 5*a^9*c^
3*d^4)*x^5 + 7/4*(12*a^6*b^3*c^7 + 36*a^7*b^2*c^6*d + 27*a^8*b*c^5*d^2 + 5*
a^9*c^4*d^3)*x^4 + (12*a^7*b^2*c^7 + 21*a^8*b*c^6*d + 7*a^9*c^5*d^2)*x^3 +
1/2*(9*a^8*b*c^7 + 7*a^9*c^6*d)*x^2
```

Fricas [B] time = 1.92556, size = 2626, normalized size = 13.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^7*b^9 + 7/16*x^16*d^6*c*b^9 + 9/16*x^16*d^7*b^8*a + 7/5*x^15*d^
5*c^2*b^9 + 21/5*x^15*d^6*c*b^8*a + 12/5*x^15*d^7*b^7*a^2 + 5/2*x^14*d^4*c^
3*b^9 + 27/2*x^14*d^5*c^2*b^8*a + 18*x^14*d^6*c*b^7*a^2 + 6*x^14*d^7*b^6*a^
3 + 35/13*x^13*d^3*c^4*b^9 + 315/13*x^13*d^4*c^3*b^8*a + 756/13*x^13*d^5*c^
2*b^7*a^2 + 588/13*x^13*d^6*c*b^6*a^3 + 126/13*x^13*d^7*b^5*a^4 + 7/4*x^12*
d^2*c^5*b^9 + 105/4*x^12*d^3*c^4*b^8*a + 105*x^12*d^4*c^3*b^7*a^2 + 147*x^1
2*d^5*c^2*b^6*a^3 + 147/2*x^12*d^6*c*b^5*a^4 + 21/2*x^12*d^7*b^4*a^5 + 7/11
*x^11*d*c^6*b^9 + 189/11*x^11*d^2*c^5*b^8*a + 1260/11*x^11*d^3*c^4*b^7*a^2
+ 2940/11*x^11*d^4*c^3*b^6*a^3 + 2646/11*x^11*d^5*c^2*b^5*a^4 + 882/11*x^11
*d^6*c*b^4*a^5 + 84/11*x^11*d^7*b^3*a^6 + 1/10*x^10*c^7*b^9 + 63/10*x^10*d*
c^6*b^8*a + 378/5*x^10*d^2*c^5*b^7*a^2 + 294*x^10*d^3*c^4*b^6*a^3 + 441*x^1
0*d^4*c^3*b^5*a^4 + 1323/5*x^10*d^5*c^2*b^4*a^5 + 294/5*x^10*d^6*c*b^3*a^6
+ 18/5*x^10*d^7*b^2*a^7 + x^9*c^7*b^8*a + 28*x^9*d*c^6*b^7*a^2 + 196*x^9*d^
2*c^5*b^6*a^3 + 490*x^9*d^3*c^4*b^5*a^4 + 490*x^9*d^4*c^3*b^4*a^5 + 196*x^9
*d^5*c^2*b^3*a^6 + 28*x^9*d^6*c*b^2*a^7 + x^9*d^7*b*a^8 + 9/2*x^8*c^7*b^7*a
^2 + 147/2*x^8*d*c^6*b^6*a^3 + 1323/4*x^8*d^2*c^5*b^5*a^4 + 2205/4*x^8*d^3*
c^4*b^4*a^5 + 735/2*x^8*d^4*c^3*b^3*a^6 + 189/2*x^8*d^5*c^2*b^2*a^7 + 63/8*
x^8*d^6*c*b*a^8 + 1/8*x^8*d^7*a^9 + 12*x^7*c^7*b^6*a^3 + 126*x^7*d*c^6*b^5*
a^4 + 378*x^7*d^2*c^5*b^4*a^5 + 420*x^7*d^3*c^4*b^3*a^6 + 180*x^7*d^4*c^3*b
^2*a^7 + 27*x^7*d^5*c^2*b*a^8 + x^7*d^6*c*a^9 + 21*x^6*c^7*b^5*a^4 + 147*x^
6*d*c^6*b^4*a^5 + 294*x^6*d^2*c^5*b^3*a^6 + 210*x^6*d^3*c^4*b^2*a^7 + 105/2
*x^6*d^4*c^3*b*a^8 + 7/2*x^6*d^5*c^2*a^9 + 126/5*x^5*c^7*b^4*a^5 + 588/5*x^
```

```
5*d*c^6*b^3*a^6 + 756/5*x^5*d^2*c^5*b^2*a^7 + 63*x^5*d^3*c^4*b*a^8 + 7*x^5*
d^4*c^3*a^9 + 21*x^4*c^7*b^3*a^6 + 63*x^4*d*c^6*b^2*a^7 + 189/4*x^4*d^2*c^5
*b*a^8 + 35/4*x^4*d^3*c^4*a^9 + 12*x^3*c^7*b^2*a^7 + 21*x^3*d*c^6*b*a^8 + 7
*x^3*d^2*c^5*a^9 + 9/2*x^2*c^7*b*a^8 + 7/2*x^2*d*c^6*a^9 + x*c^7*a^9
```

Sympy [B] time = 0.204455, size = 1163, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9*(d*x+c)**7,x)
```

```
[Out] a**9*c**7*x + b**9*d**7*x**17/17 + x**16*(9*a*b**8*d**7/16 + 7*b**9*c*d**6/
16) + x**15*(12*a**2*b**7*d**7/5 + 21*a*b**8*c*d**6/5 + 7*b**9*c**2*d**5/5)
 + x**14*(6*a**3*b**6*d**7 + 18*a**2*b**7*c*d**6 + 27*a*b**8*c**2*d**5/2 +
5*b**9*c**3*d**4/2) + x**13*(126*a**4*b**5*d**7/13 + 588*a**3*b**6*c*d**6/1
3 + 756*a**2*b**7*c**2*d**5/13 + 315*a*b**8*c**3*d**4/13 + 35*b**9*c**4*d**
3/13) + x**12*(21*a**5*b**4*d**7/2 + 147*a**4*b**5*c*d**6/2 + 147*a**3*b**6
*c**2*d**5 + 105*a**2*b**7*c**3*d**4 + 105*a*b**8*c**4*d**3/4 + 7*b**9*c**5
*d**2/4) + x**11*(84*a**6*b**3*d**7/11 + 882*a**5*b**4*c*d**6/11 + 2646*a**
4*b**5*c**2*d**5/11 + 2940*a**3*b**6*c**3*d**4/11 + 1260*a**2*b**7*c**4*d**
3/11 + 189*a*b**8*c**5*d**2/11 + 7*b**9*c**6*d/11) + x**10*(18*a**7*b**2*d*
*7/5 + 294*a**6*b**3*c*d**6/5 + 1323*a**5*b**4*c**2*d**5/5 + 441*a**4*b**5*
c**3*d**4 + 294*a**3*b**6*c**4*d**3 + 378*a**2*b**7*c**5*d**2/5 + 63*a*b**8
*c**6*d/10 + b**9*c**7/10) + x**9*(a**8*b*d**7 + 28*a**7*b**2*c*d**6 + 196*
a**6*b**3*c**2*d**5 + 490*a**5*b**4*c**3*d**4 + 490*a**4*b**5*c**4*d**3 + 1
96*a**3*b**6*c**5*d**2 + 28*a**2*b**7*c**6*d + a*b**8*c**7) + x**8*(a**9*d*
*7/8 + 63*a**8*b*c*d**6/8 + 189*a**7*b**2*c**2*d**5/2 + 735*a**6*b**3*c**3*
d**4/2 + 2205*a**5*b**4*c**4*d**3/4 + 1323*a**4*b**5*c**5*d**2/4 + 147*a**3
*b**6*c**6*d/2 + 9*a**2*b**7*c**7/2) + x**7*(a**9*c*d**6 + 27*a**8*b*c**2*d
**5 + 180*a**7*b**2*c**3*d**4 + 420*a**6*b**3*c**4*d**3 + 378*a**5*b**4*c**
5*d**2 + 126*a**4*b**5*c**6*d + 12*a**3*b**6*c**7) + x**6*(7*a**9*c**2*d**5
/2 + 105*a**8*b*c**3*d**4/2 + 210*a**7*b**2*c**4*d**3 + 294*a**6*b**3*c**5*
d**2 + 147*a**5*b**4*c**6*d + 21*a**4*b**5*c**7) + x**5*(7*a**9*c**3*d**4 +
 63*a**8*b*c**4*d**3 + 756*a**7*b**2*c**5*d**2/5 + 588*a**6*b**3*c**6*d/5 +
 126*a**5*b**4*c**7/5) + x**4*(35*a**9*c**4*d**3/4 + 189*a**8*b*c**5*d**2/4
 + 63*a**7*b**2*c**6*d + 21*a**6*b**3*c**7) + x**3*(7*a**9*c**5*d**2 + 21*a
**8*b*c**6*d + 12*a**7*b**2*c**7) + x**2*(7*a**9*c**6*d/2 + 9*a**8*b*c**7/2
)
```

Giac [B] time = 1.07534, size = 1586, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/17*b^9*d^7*x^17 + 7/16*b^9*c*d^6*x^16 + 9/16*a*b^8*d^7*x^16 + 7/5*b^9*c^2
*d^5*x^15 + 21/5*a*b^8*c*d^6*x^15 + 12/5*a^2*b^7*d^7*x^15 + 5/2*b^9*c^3*d^4
*x^14 + 27/2*a*b^8*c^2*d^5*x^14 + 18*a^2*b^7*c*d^6*x^14 + 6*a^3*b^6*d^7*x^1
4 + 35/13*b^9*c^4*d^3*x^13 + 315/13*a*b^8*c^3*d^4*x^13 + 756/13*a^2*b^7*c^2
*d^5*x^13 + 588/13*a^3*b^6*c*d^6*x^13 + 126/13*a^4*b^5*d^7*x^13 + 7/4*b^9*c
```

^5*d^2*x^12 + 105/4*a*b^8*c^4*d^3*x^12 + 105*a^2*b^7*c^3*d^4*x^12 + 147*a^3
*b^6*c^2*d^5*x^12 + 147/2*a^4*b^5*c*d^6*x^12 + 21/2*a^5*b^4*d^7*x^12 + 7/11
*b^9*c^6*d*x^11 + 189/11*a*b^8*c^5*d^2*x^11 + 1260/11*a^2*b^7*c^4*d^3*x^11
+ 2940/11*a^3*b^6*c^3*d^4*x^11 + 2646/11*a^4*b^5*c^2*d^5*x^11 + 882/11*a^5*
b^4*c*d^6*x^11 + 84/11*a^6*b^3*d^7*x^11 + 1/10*b^9*c^7*x^10 + 63/10*a*b^8*c
^6*d*x^10 + 378/5*a^2*b^7*c^5*d^2*x^10 + 294*a^3*b^6*c^4*d^3*x^10 + 441*a^4
*b^5*c^3*d^4*x^10 + 1323/5*a^5*b^4*c^2*d^5*x^10 + 294/5*a^6*b^3*c*d^6*x^10
+ 18/5*a^7*b^2*d^7*x^10 + a*b^8*c^7*x^9 + 28*a^2*b^7*c^6*d*x^9 + 196*a^3*b^
6*c^5*d^2*x^9 + 490*a^4*b^5*c^4*d^3*x^9 + 490*a^5*b^4*c^3*d^4*x^9 + 196*a^6
*b^3*c^2*d^5*x^9 + 28*a^7*b^2*c*d^6*x^9 + a^8*b*d^7*x^9 + 9/2*a^2*b^7*c^7*x
^8 + 147/2*a^3*b^6*c^6*d*x^8 + 1323/4*a^4*b^5*c^5*d^2*x^8 + 2205/4*a^5*b^4*
c^4*d^3*x^8 + 735/2*a^6*b^3*c^3*d^4*x^8 + 189/2*a^7*b^2*c^2*d^5*x^8 + 63/8*
a^8*b*c*d^6*x^8 + 1/8*a^9*d^7*x^8 + 12*a^3*b^6*c^7*x^7 + 126*a^4*b^5*c^6*d*
x^7 + 378*a^5*b^4*c^5*d^2*x^7 + 420*a^6*b^3*c^4*d^3*x^7 + 180*a^7*b^2*c^3*d
^4*x^7 + 27*a^8*b*c^2*d^5*x^7 + a^9*c*d^6*x^7 + 21*a^4*b^5*c^7*x^6 + 147*a^
5*b^4*c^6*d*x^6 + 294*a^6*b^3*c^5*d^2*x^6 + 210*a^7*b^2*c^4*d^3*x^6 + 105/2
*a^8*b*c^3*d^4*x^6 + 7/2*a^9*c^2*d^5*x^6 + 126/5*a^5*b^4*c^7*x^5 + 588/5*a^
6*b^3*c^6*d*x^5 + 756/5*a^7*b^2*c^5*d^2*x^5 + 63*a^8*b*c^4*d^3*x^5 + 7*a^9*
c^3*d^4*x^5 + 21*a^6*b^3*c^7*x^4 + 63*a^7*b^2*c^6*d*x^4 + 189/4*a^8*b*c^5*d
^2*x^4 + 35/4*a^9*c^4*d^3*x^4 + 12*a^7*b^2*c^7*x^3 + 21*a^8*b*c^6*d*x^3 + 7
*a^9*c^5*d^2*x^3 + 9/2*a^8*b*c^7*x^2 + 7/2*a^9*c^6*d*x^2 + a^9*c^7*x

3.1274 $\int (a+bx)^8(c+dx)^7\,dx$

Optimal. Leaf size=200

$$\frac{7d^6(a+bx)^{15}(bc-ad)}{15b^8}+\frac{3d^5(a+bx)^{14}(bc-ad)^2}{2b^8}+\frac{35d^4(a+bx)^{13}(bc-ad)^3}{13b^8}+\frac{35d^3(a+bx)^{12}(bc-ad)^4}{12b^8}+\frac{21d^2(a+$$

```
[Out] ((b*c - a*d)^7*(a + b*x)^9)/(9*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^10)/(10*
b^8) + (21*d^2*(b*c - a*d)^5*(a + b*x)^11)/(11*b^8) + (35*d^3*(b*c - a*d)^4
*(a + b*x)^12)/(12*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^13)/(13*b^8) + (3
*d^5*(b*c - a*d)^2*(a + b*x)^14)/(2*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^15)
/(15*b^8) + (d^7*(a + b*x)^16)/(16*b^8)
```

Rubi [A] time = 0.573663, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^{15}(bc-ad)}{15b^8}+\frac{3d^5(a+bx)^{14}(bc-ad)^2}{2b^8}+\frac{35d^4(a+bx)^{13}(bc-ad)^3}{13b^8}+\frac{35d^3(a+bx)^{12}(bc-ad)^4}{12b^8}+\frac{21d^2(a+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8*(c + d*x)^7,x]
```


```
[Out] ((b*c - a*d)^7*(a + b*x)^9)/(9*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^10)/(10*
b^8) + (21*d^2*(b*c - a*d)^5*(a + b*x)^11)/(11*b^8) + (35*d^3*(b*c - a*d)^4
*(a + b*x)^12)/(12*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^13)/(13*b^8) + (3
*d^5*(b*c - a*d)^2*(a + b*x)^14)/(2*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^15)
/(15*b^8) + (d^7*(a + b*x)^16)/(16*b^8)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^8(c+dx)^7\,dx=\int \left(\frac{(bc-ad)^7(a+bx)^8}{b^7}+\frac{7d(bc-ad)^6(a+bx)^9}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^{10}}{b^7}+\frac{35d^3(bc-ac}{b}\right.$$
$$=\frac{(bc-ad)^7(a+bx)^9}{9b^8}+\frac{7d(bc-ad)^6(a+bx)^{10}}{10b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{11}}{11b^8}+\frac{35d^3(bc-ad)^4(}{12b^8}$$

Mathematica [B] time = 0.103973, size = 897, normalized size = 4.48

$$\frac{1}{16}b^8d^7x^{16}+\frac{1}{15}b^7d^6(7bc+8ad)x^{15}+\frac{1}{2}b^6d^5\left(3b^2c^2+8abdc+4a^2d^2\right)x^{14}+\frac{7}{13}b^5d^4\left(5b^3c^3+24ab^2dc^2+28a^2bd^2c+8\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8*(c + d*x)^7,x]
```

```
[Out] a^8*c^7*x + (a^7*c^6*(8*b*c + 7*a*d)*x^2)/2 + (7*a^6*c^5*(4*b^2*c^2 + 8*a*b
*c*d + 3*a^2*d^2)*x^3)/3 + (7*a^5*c^4*(8*b^3*c^3 + 28*a*b^2*c^2*d + 24*a^2*
b*c*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^4*c^3*(10*b^4*c^4 + 56*a*b^3*c^3*d + 84*
a^2*b^2*c^2*d^2 + 40*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^3*c^2*(8*b^5*c^
5 + 70*a*b^4*c^4*d + 168*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 40*a^4*b*c
*d^4 + 3*a^5*d^5)*x^6)/6 + a^2*c*(4*b^6*c^6 + 56*a*b^5*c^5*d + 210*a^2*b^4*
c^4*d^2 + 280*a^3*b^3*c^3*d^3 + 140*a^4*b^2*c^2*d^4 + 24*a^5*b*c*d^5 + a^6*
d^6)*x^7 + (a*(8*b^7*c^7 + 196*a*b^6*c^6*d + 1176*a^2*b^5*c^5*d^2 + 2450*a^
3*b^4*c^4*d^3 + 1960*a^4*b^3*c^3*d^4 + 588*a^5*b^2*c^2*d^5 + 56*a^6*b*c*d^6
 + a^7*d^7)*x^8)/8 + (b*(b^7*c^7 + 56*a*b^6*c^6*d + 588*a^2*b^5*c^5*d^2 + 1
960*a^3*b^4*c^4*d^3 + 2450*a^4*b^3*c^3*d^4 + 1176*a^5*b^2*c^2*d^5 + 196*a^6
*b*c*d^6 + 8*a^7*d^7)*x^9)/9 + (7*b^2*d*(b^6*c^6 + 24*a*b^5*c^5*d + 140*a^2
*b^4*c^4*d^2 + 280*a^3*b^3*c^3*d^3 + 210*a^4*b^2*c^2*d^4 + 56*a^5*b*c*d^5 +
 4*a^6*d^6)*x^10)/10 + (7*b^3*d^2*(3*b^5*c^5 + 40*a*b^4*c^4*d + 140*a^2*b^3
*c^3*d^2 + 168*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 8*a^5*d^5)*x^11)/11 + (7*
b^4*d^3*(5*b^4*c^4 + 40*a*b^3*c^3*d + 84*a^2*b^2*c^2*d^2 + 56*a^3*b*c*d^3 +
 10*a^4*d^4)*x^12)/12 + (7*b^5*d^4*(5*b^3*c^3 + 24*a*b^2*c^2*d + 28*a^2*b*c
*d^2 + 8*a^3*d^3)*x^13)/13 + (b^6*d^5*(3*b^2*c^2 + 8*a*b*c*d + 4*a^2*d^2)*x
^14)/2 + (b^7*d^6*(7*b*c + 8*a*d)*x^15)/15 + (b^8*d^7*x^16)/16
```

Maple [B] time = 0.001, size = 925, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8*(d*x+c)^7,x)
```

```
[Out] 1/16*b^8*d^7*x^16+1/15*(8*a*b^7*d^7+7*b^8*c*d^6)*x^15+1/14*(28*a^2*b^6*d^7+
56*a*b^7*c*d^6+21*b^8*c^2*d^5)*x^14+1/13*(56*a^3*b^5*d^7+196*a^2*b^6*c*d^6+
168*a*b^7*c^2*d^5+35*b^8*c^3*d^4)*x^13+1/12*(70*a^4*b^4*d^7+392*a^3*b^5*c*d
^6+588*a^2*b^6*c^2*d^5+280*a*b^7*c^3*d^4+35*b^8*c^4*d^3)*x^12+1/11*(56*a^5*
b^3*d^7+490*a^4*b^4*c*d^6+1176*a^3*b^5*c^2*d^5+980*a^2*b^6*c^3*d^4+280*a*b^
7*c^4*d^3+21*b^8*c^5*d^2)*x^11+1/10*(28*a^6*b^2*d^7+392*a^5*b^3*c*d^6+1470*
a^4*b^4*c^2*d^5+1960*a^3*b^5*c^3*d^4+980*a^2*b^6*c^4*d^3+168*a*b^7*c^5*d^2+
7*b^8*c^6*d)*x^10+1/9*(8*a^7*b*d^7+196*a^6*b^2*c*d^6+1176*a^5*b^3*c^2*d^5+2
450*a^4*b^4*c^3*d^4+1960*a^3*b^5*c^4*d^3+588*a^2*b^6*c^5*d^2+56*a*b^7*c^6*d
+b^8*c^7)*x^9+1/8*(a^8*d^7+56*a^7*b*c*d^6+588*a^6*b^2*c^2*d^5+1960*a^5*b^3*
c^3*d^4+2450*a^4*b^4*c^4*d^3+1176*a^3*b^5*c^5*d^2+196*a^2*b^6*c^6*d+8*a*b^7
*c^7)*x^8+1/7*(7*a^8*c*d^6+168*a^7*b*c^2*d^5+980*a^6*b^2*c^3*d^4+1960*a^5*b
^3*c^4*d^3+1470*a^4*b^4*c^5*d^2+392*a^3*b^5*c^6*d+28*a^2*b^6*c^7)*x^7+1/6*(
21*a^8*c^2*d^5+280*a^7*b*c^3*d^4+980*a^6*b^2*c^4*d^3+1176*a^5*b^3*c^5*d^2+4
90*a^4*b^4*c^6*d+56*a^3*b^5*c^7)*x^6+1/5*(35*a^8*c^3*d^4+280*a^7*b*c^4*d^3+
588*a^6*b^2*c^5*d^2+392*a^5*b^3*c^6*d+70*a^4*b^4*c^7)*x^5+1/4*(35*a^8*c^4*d
^3+168*a^7*b*c^5*d^2+196*a^6*b^2*c^6*d+56*a^5*b^3*c^7)*x^4+1/3*(21*a^8*c^5*
d^2+56*a^7*b*c^6*d+28*a^6*b^2*c^7)*x^3+1/2*(7*a^8*c^6*d+8*a^7*b*c^7)*x^2+a^
8*c^7*x
```

Maxima [B] time = 1.0119, size = 1243, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/16*b^8*d^7*x^16 + a^8*c^7*x + 1/15*(7*b^8*c*d^6 + 8*a*b^7*d^7)*x^15 + 1/2
*(3*b^8*c^2*d^5 + 8*a*b^7*c*d^6 + 4*a^2*b^6*d^7)*x^14 + 7/13*(5*b^8*c^3*d^4
 + 24*a*b^7*c^2*d^5 + 28*a^2*b^6*c*d^6 + 8*a^3*b^5*d^7)*x^13 + 7/12*(5*b^8*
c^4*d^3 + 40*a*b^7*c^3*d^4 + 84*a^2*b^6*c^2*d^5 + 56*a^3*b^5*c*d^6 + 10*a^4
*b^4*d^7)*x^12 + 7/11*(3*b^8*c^5*d^2 + 40*a*b^7*c^4*d^3 + 140*a^2*b^6*c^3*d
^4 + 168*a^3*b^5*c^2*d^5 + 70*a^4*b^4*c*d^6 + 8*a^5*b^3*d^7)*x^11 + 7/10*(b
^8*c^6*d + 24*a*b^7*c^5*d^2 + 140*a^2*b^6*c^4*d^3 + 280*a^3*b^5*c^3*d^4 + 2
10*a^4*b^4*c^2*d^5 + 56*a^5*b^3*c*d^6 + 4*a^6*b^2*d^7)*x^10 + 1/9*(b^8*c^7
+ 56*a*b^7*c^6*d + 588*a^2*b^6*c^5*d^2 + 1960*a^3*b^5*c^4*d^3 + 2450*a^4*b^
4*c^3*d^4 + 1176*a^5*b^3*c^2*d^5 + 196*a^6*b^2*c*d^6 + 8*a^7*b*d^7)*x^9 + 1
/8*(8*a*b^7*c^7 + 196*a^2*b^6*c^6*d + 1176*a^3*b^5*c^5*d^2 + 2450*a^4*b^4*c
^4*d^3 + 1960*a^5*b^3*c^3*d^4 + 588*a^6*b^2*c^2*d^5 + 56*a^7*b*c*d^6 + a^8*
d^7)*x^8 + (4*a^2*b^6*c^7 + 56*a^3*b^5*c^6*d + 210*a^4*b^4*c^5*d^2 + 280*a^
5*b^3*c^4*d^3 + 140*a^6*b^2*c^3*d^4 + 24*a^7*b*c^2*d^5 + a^8*c*d^6)*x^7 + 7
/6*(8*a^3*b^5*c^7 + 70*a^4*b^4*c^6*d + 168*a^5*b^3*c^5*d^2 + 140*a^6*b^2*c^
4*d^3 + 40*a^7*b*c^3*d^4 + 3*a^8*c^2*d^5)*x^6 + 7/5*(10*a^4*b^4*c^7 + 56*a^
5*b^3*c^6*d + 84*a^6*b^2*c^5*d^2 + 40*a^7*b*c^4*d^3 + 5*a^8*c^3*d^4)*x^5 +
7/4*(8*a^5*b^3*c^7 + 28*a^6*b^2*c^6*d + 24*a^7*b*c^5*d^2 + 5*a^8*c^4*d^3)*x
^4 + 7/3*(4*a^6*b^2*c^7 + 8*a^7*b*c^6*d + 3*a^8*c^5*d^2)*x^3 + 1/2*(8*a^7*b
*c^7 + 7*a^8*c^6*d)*x^2
```

Fricas [B] time = 1.79439, size = 2337, normalized size = 11.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^7*b^8 + 7/15*x^15*d^6*c*b^8 + 8/15*x^15*d^7*b^7*a + 3/2*x^14*d^
5*c^2*b^8 + 4*x^14*d^6*c*b^7*a + 2*x^14*d^7*b^6*a^2 + 35/13*x^13*d^4*c^3*b^
8 + 168/13*x^13*d^5*c^2*b^7*a + 196/13*x^13*d^6*c*b^6*a^2 + 56/13*x^13*d^7*
b^5*a^3 + 35/12*x^12*d^3*c^4*b^8 + 70/3*x^12*d^4*c^3*b^7*a + 49*x^12*d^5*c^
2*b^6*a^2 + 98/3*x^12*d^6*c*b^5*a^3 + 35/6*x^12*d^7*b^4*a^4 + 21/11*x^11*d^
2*c^5*b^8 + 280/11*x^11*d^3*c^4*b^7*a + 980/11*x^11*d^4*c^3*b^6*a^2 + 1176/
11*x^11*d^5*c^2*b^5*a^3 + 490/11*x^11*d^6*c*b^4*a^4 + 56/11*x^11*d^7*b^3*a^
5 + 7/10*x^10*d*c^6*b^8 + 84/5*x^10*d^2*c^5*b^7*a + 98*x^10*d^3*c^4*b^6*a^2
 + 196*x^10*d^4*c^3*b^5*a^3 + 147*x^10*d^5*c^2*b^4*a^4 + 196/5*x^10*d^6*c*b
^3*a^5 + 14/5*x^10*d^7*b^2*a^6 + 1/9*x^9*c^7*b^8 + 56/9*x^9*d*c^6*b^7*a + 1
96/3*x^9*d^2*c^5*b^6*a^2 + 1960/9*x^9*d^3*c^4*b^5*a^3 + 2450/9*x^9*d^4*c^3*
b^4*a^4 + 392/3*x^9*d^5*c^2*b^3*a^5 + 196/9*x^9*d^6*c*b^2*a^6 + 8/9*x^9*d^7
*b*a^7 + x^8*c^7*b^7*a + 49/2*x^8*d*c^6*b^6*a^2 + 147*x^8*d^2*c^5*b^5*a^3 +
 1225/4*x^8*d^3*c^4*b^4*a^4 + 245*x^8*d^4*c^3*b^3*a^5 + 147/2*x^8*d^5*c^2*b
^2*a^6 + 7*x^8*d^6*c*b*a^7 + 1/8*x^8*d^7*a^8 + 4*x^7*c^7*b^6*a^2 + 56*x^7*d
*c^6*b^5*a^3 + 210*x^7*d^2*c^5*b^4*a^4 + 280*x^7*d^3*c^4*b^3*a^5 + 140*x^7*
d^4*c^3*b^2*a^6 + 24*x^7*d^5*c^2*b*a^7 + x^7*d^6*c*a^8 + 28/3*x^6*c^7*b^5*a
^3 + 245/3*x^6*d*c^6*b^4*a^4 + 196*x^6*d^2*c^5*b^3*a^5 + 490/3*x^6*d^3*c^4*
b^2*a^6 + 140/3*x^6*d^4*c^3*b*a^7 + 7/2*x^6*d^5*c^2*a^8 + 14*x^5*c^7*b^4*a^
4 + 392/5*x^5*d*c^6*b^3*a^5 + 588/5*x^5*d^2*c^5*b^2*a^6 + 56*x^5*d^3*c^4*b*
a^7 + 7*x^5*d^4*c^3*a^8 + 14*x^4*c^7*b^3*a^5 + 49*x^4*d*c^6*b^2*a^6 + 42*x^
4*d^2*c^5*b*a^7 + 35/4*x^4*d^3*c^4*a^8 + 28/3*x^3*c^7*b^2*a^6 + 56/3*x^3*d*
c^6*b*a^7 + 7*x^3*d^2*c^5*a^8 + 4*x^2*c^7*b*a^7 + 7/2*x^2*d*c^6*a^8 + x*c^7
*a^8
```

Sympy [B] time = 0.189895, size = 1046, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8*(d*x+c)**7,x)
```

```
[Out] a**8*c**7*x + b**8*d**7*x**16/16 + x**15*(8*a*b**7*d**7/15 + 7*b**8*c*d**6/
15) + x**14*(2*a**2*b**6*d**7 + 4*a*b**7*c*d**6 + 3*b**8*c**2*d**5/2) + x**
13*(56*a**3*b**5*d**7/13 + 196*a**2*b**6*c*d**6/13 + 168*a*b**7*c**2*d**5/1
3 + 35*b**8*c**3*d**4/13) + x**12*(35*a**4*b**4*d**7/6 + 98*a**3*b**5*c*d**
6/3 + 49*a**2*b**6*c**2*d**5 + 70*a*b**7*c**3*d**4/3 + 35*b**8*c**4*d**3/12
) + x**11*(56*a**5*b**3*d**7/11 + 490*a**4*b**4*c*d**6/11 + 1176*a**3*b**5*
c**2*d**5/11 + 980*a**2*b**6*c**3*d**4/11 + 280*a*b**7*c**4*d**3/11 + 21*b*
*8*c**5*d**2/11) + x**10*(14*a**6*b**2*d**7/5 + 196*a**5*b**3*c*d**6/5 + 14
7*a**4*b**4*c**2*d**5 + 196*a**3*b**5*c**3*d**4 + 98*a**2*b**6*c**4*d**3 +
84*a*b**7*c**5*d**2/5 + 7*b**8*c**6*d/10) + x**9*(8*a**7*b*d**7/9 + 196*a**
6*b**2*c*d**6/9 + 392*a**5*b**3*c**2*d**5/3 + 2450*a**4*b**4*c**3*d**4/9 +
1960*a**3*b**5*c**4*d**3/9 + 196*a**2*b**6*c**5*d**2/3 + 56*a*b**7*c**6*d/9
 + b**8*c**7/9) + x**8*(a**8*d**7/8 + 7*a**7*b*c*d**6 + 147*a**6*b**2*c**2*
d**5/2 + 245*a**5*b**3*c**3*d**4 + 1225*a**4*b**4*c**4*d**3/4 + 147*a**3*b*
*5*c**5*d**2 + 49*a**2*b**6*c**6*d/2 + a*b**7*c**7) + x**7*(a**8*c*d**6 + 2
4*a**7*b*c**2*d**5 + 140*a**6*b**2*c**3*d**4 + 280*a**5*b**3*c**4*d**3 + 21
0*a**4*b**4*c**5*d**2 + 56*a**3*b**5*c**6*d + 4*a**2*b**6*c**7) + x**6*(7*a
**8*c**2*d**5/2 + 140*a**7*b*c**3*d**4/3 + 490*a**6*b**2*c**4*d**3/3 + 196*
a**5*b**3*c**5*d**2 + 245*a**4*b**4*c**6*d/3 + 28*a**3*b**5*c**7/3) + x**5*
(7*a**8*c**3*d**4 + 56*a**7*b*c**4*d**3 + 588*a**6*b**2*c**5*d**2/5 + 392*a
**5*b**3*c**6*d/5 + 14*a**4*b**4*c**7) + x**4*(35*a**8*c**4*d**3/4 + 42*a**
7*b*c**5*d**2 + 49*a**6*b**2*c**6*d + 14*a**5*b**3*c**7) + x**3*(7*a**8*c**
5*d**2 + 56*a**7*b*c**6*d/3 + 28*a**6*b**2*c**7/3) + x**2*(7*a**8*c**6*d/2
+ 4*a**7*b*c**7)
```

Giac [B] time = 1.05724, size = 1418, normalized size = 7.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/16*b^8*d^7*x^16 + 7/15*b^8*c*d^6*x^15 + 8/15*a*b^7*d^7*x^15 + 3/2*b^8*c^2
*d^5*x^14 + 4*a*b^7*c*d^6*x^14 + 2*a^2*b^6*d^7*x^14 + 35/13*b^8*c^3*d^4*x^1
3 + 168/13*a*b^7*c^2*d^5*x^13 + 196/13*a^2*b^6*c*d^6*x^13 + 56/13*a^3*b^5*d
^7*x^13 + 35/12*b^8*c^4*d^3*x^12 + 70/3*a*b^7*c^3*d^4*x^12 + 49*a^2*b^6*c^2
*d^5*x^12 + 98/3*a^3*b^5*c*d^6*x^12 + 35/6*a^4*b^4*d^7*x^12 + 21/11*b^8*c^5
*d^2*x^11 + 280/11*a*b^7*c^4*d^3*x^11 + 980/11*a^2*b^6*c^3*d^4*x^11 + 1176/
11*a^3*b^5*c^2*d^5*x^11 + 490/11*a^4*b^4*c*d^6*x^11 + 56/11*a^5*b^3*d^7*x^1
1 + 7/10*b^8*c^6*d*x^10 + 84/5*a*b^7*c^5*d^2*x^10 + 98*a^2*b^6*c^4*d^3*x^10
 + 196*a^3*b^5*c^3*d^4*x^10 + 147*a^4*b^4*c^2*d^5*x^10 + 196/5*a^5*b^3*c*d^
6*x^10 + 14/5*a^6*b^2*d^7*x^10 + 1/9*b^8*c^7*x^9 + 56/9*a*b^7*c^6*d*x^9 + 1
96/3*a^2*b^6*c^5*d^2*x^9 + 1960/9*a^3*b^5*c^4*d^3*x^9 + 2450/9*a^4*b^4*c^3*
d^4*x^9 + 392/3*a^5*b^3*c^2*d^5*x^9 + 196/9*a^6*b^2*c*d^6*x^9 + 8/9*a^7*b*d
^7*x^9 + a*b^7*c^7*x^8 + 49/2*a^2*b^6*c^6*d*x^8 + 147*a^3*b^5*c^5*d^2*x^8 +
 1225/4*a^4*b^4*c^4*d^3*x^8 + 245*a^5*b^3*c^3*d^4*x^8 + 147/2*a^6*b^2*c^2*d
^5*x^8 + 7*a^7*b*c*d^6*x^8 + 1/8*a^8*d^7*x^8 + 4*a^2*b^6*c^7*x^7 + 56*a^3*b
```

^5*c^6*d*x^7 + 210*a^4*b^4*c^5*d^2*x^7 + 280*a^5*b^3*c^4*d^3*x^7 + 140*a^6*b^2*c^3*d^4*x^7 + 24*a^7*b*c^2*d^5*x^7 + a^8*c*d^6*x^7 + 28/3*a^3*b^5*c^7*x^6 + 245/3*a^4*b^4*c^6*d*x^6 + 196*a^5*b^3*c^5*d^2*x^6 + 490/3*a^6*b^2*c^4*d^3*x^6 + 140/3*a^7*b*c^3*d^4*x^6 + 7/2*a^8*c^2*d^5*x^6 + 14*a^4*b^4*c^7*x^5 + 392/5*a^5*b^3*c^6*d*x^5 + 588/5*a^6*b^2*c^5*d^2*x^5 + 56*a^7*b*c^4*d^3*x^5 + 7*a^8*c^3*d^4*x^5 + 14*a^5*b^3*c^7*x^4 + 49*a^6*b^2*c^6*d*x^4 + 42*a^7*b*c^5*d^2*x^4 + 35/4*a^8*c^4*d^3*x^4 + 28/3*a^6*b^2*c^7*x^3 + 56/3*a^7*b*c^6*d*x^3 + 7*a^8*c^5*d^2*x^3 + 4*a^7*b*c^7*x^2 + 7/2*a^8*c^6*d*x^2 + a^8*c^7*x

3.1275 $\int (a+bx)^7(c+dx)^7\,dx$

Optimal. Leaf size=200

$$\frac{d^6(a+bx)^{14}(bc-ad)}{2b^8}+\frac{21d^5(a+bx)^{13}(bc-ad)^2}{13b^8}+\frac{35d^4(a+bx)^{12}(bc-ad)^3}{12b^8}+\frac{35d^3(a+bx)^{11}(bc-ad)^4}{11b^8}+\frac{21d^2(a+bx)}{10b}$$

```
[Out] ((b*c - a*d)^7*(a + b*x)^8)/(8*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^9)/(9*b^
8) + (21*d^2*(b*c - a*d)^5*(a + b*x)^10)/(10*b^8) + (35*d^3*(b*c - a*d)^4*(
a + b*x)^11)/(11*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^12)/(12*b^8) + (21*
d^5*(b*c - a*d)^2*(a + b*x)^13)/(13*b^8) + (d^6*(b*c - a*d)*(a + b*x)^14)/(
2*b^8) + (d^7*(a + b*x)^15)/(15*b^8)
```

Rubi [A] time = 0.453697, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^6(a+bx)^{14}(bc-ad)}{2b^8}+\frac{21d^5(a+bx)^{13}(bc-ad)^2}{13b^8}+\frac{35d^4(a+bx)^{12}(bc-ad)^3}{12b^8}+\frac{35d^3(a+bx)^{11}(bc-ad)^4}{11b^8}+\frac{21d^2(a+bx)}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^7*(a + b*x)^8)/(8*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^9)/(9*b^
8) + (21*d^2*(b*c - a*d)^5*(a + b*x)^10)/(10*b^8) + (35*d^3*(b*c - a*d)^4*(
a + b*x)^11)/(11*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^12)/(12*b^8) + (21*
d^5*(b*c - a*d)^2*(a + b*x)^13)/(13*b^8) + (d^6*(b*c - a*d)*(a + b*x)^14)/(
2*b^8) + (d^7*(a + b*x)^15)/(15*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^7(c+dx)^7\,dx &= \int \left(\frac{(bc-ad)^7(a+bx)^7}{b^7}+\frac{7d(bc-ad)^6(a+bx)^8}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^9}{b^7}+\frac{35d^3(bc-ad)^4(a}{b^7}\right.\\ &= \frac{(bc-ad)^7(a+bx)^8}{8b^8}+\frac{7d(bc-ad)^6(a+bx)^9}{9b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{10}}{10b^8}+\frac{35d^3(bc-ad)^4(a+}{11b^8}\end{aligned}$$

Mathematica [B] time = 0.0799013, size = 785, normalized size = 3.92

$$\frac{7}{13}b^5d^5x^{13}\left(3a^2d^2+7abcd+3b^2c^2\right)+\frac{7}{12}b^4d^4x^{12}\left(21a^2bcd^2+5a^3d^3+21ab^2c^2d+5b^3c^3\right)+\frac{7}{11}b^3d^3x^{11}\left(63a^2b^2c^2d^2+35a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7*(c + d*x)^7,x]
```

```
[Out] a^7*c^7*x + (7*a^6*c^6*(b*c + a*d)*x^2)/2 + (7*a^5*c^5*(3*b^2*c^2 + 7*a*b*c
*d + 3*a^2*d^2)*x^3)/3 + (7*a^4*c^4*(5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*
c*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^3*c^3*(5*b^4*c^4 + 35*a*b^3*c^3*d + 63*a^2
*b^2*c^2*d^2 + 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^2*c^2*(3*b^5*c^5 +
 35*a*b^4*c^4*d + 105*a^2*b^3*c^3*d^2 + 105*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^
4 + 3*a^5*d^5)*x^6)/6 + a*c*(b^6*c^6 + 21*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2
 + 175*a^3*b^3*c^3*d^3 + 105*a^4*b^2*c^2*d^4 + 21*a^5*b*c*d^5 + a^6*d^6)*x^
7 + ((b^7*c^7 + 49*a*b^6*c^6*d + 441*a^2*b^5*c^5*d^2 + 1225*a^3*b^4*c^4*d^3
 + 1225*a^4*b^3*c^3*d^4 + 441*a^5*b^2*c^2*d^5 + 49*a^6*b*c*d^6 + a^7*d^7)*x
^8)/8 + (7*b*d*(b^6*c^6 + 21*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2 + 175*a^3*b^
3*c^3*d^3 + 105*a^4*b^2*c^2*d^4 + 21*a^5*b*c*d^5 + a^6*d^6)*x^9)/9 + (7*b^2
*d^2*(3*b^5*c^5 + 35*a*b^4*c^4*d + 105*a^2*b^3*c^3*d^2 + 105*a^3*b^2*c^2*d^
3 + 35*a^4*b*c*d^4 + 3*a^5*d^5)*x^10)/10 + (7*b^3*d^3*(5*b^4*c^4 + 35*a*b^3
*c^3*d + 63*a^2*b^2*c^2*d^2 + 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^11)/11 + (7*b^4
*d^4*(5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*c*d^2 + 5*a^3*d^3)*x^12)/12 + (
7*b^5*d^5*(3*b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*x^13)/13 + (b^6*d^6*(b*c + a*
d)*x^14)/2 + (b^7*d^7*x^15)/15
```

Maple [B] time = 0.002, size = 817, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7*(d*x+c)^7,x)
```

```
[Out] 1/15*b^7*d^7*x^15+1/14*(7*a*b^6*d^7+7*b^7*c*d^6)*x^14+1/13*(21*a^2*b^5*d^7+
49*a*b^6*c*d^6+21*b^7*c^2*d^5)*x^13+1/12*(35*a^3*b^4*d^7+147*a^2*b^5*c*d^6+
147*a*b^6*c^2*d^5+35*b^7*c^3*d^4)*x^12+1/11*(35*a^4*b^3*d^7+245*a^3*b^4*c*d
^6+441*a^2*b^5*c^2*d^5+245*a*b^6*c^3*d^4+35*b^7*c^4*d^3)*x^11+1/10*(21*a^5*
b^2*d^7+245*a^4*b^3*c*d^6+735*a^3*b^4*c^2*d^5+735*a^2*b^5*c^3*d^4+245*a*b^6
*c^4*d^3+21*b^7*c^5*d^2)*x^10+1/9*(7*a^6*b*d^7+147*a^5*b^2*c*d^6+735*a^4*b^
3*c^2*d^5+1225*a^3*b^4*c^3*d^4+735*a^2*b^5*c^4*d^3+147*a*b^6*c^5*d^2+7*b^7*
c^6*d)*x^9+1/8*(a^7*d^7+49*a^6*b*c*d^6+441*a^5*b^2*c^2*d^5+1225*a^4*b^3*c^3
*d^4+1225*a^3*b^4*c^4*d^3+441*a^2*b^5*c^5*d^2+49*a*b^6*c^6*d+b^7*c^7)*x^8+1
/7*(7*a^7*c*d^6+147*a^6*b*c^2*d^5+735*a^5*b^2*c^3*d^4+1225*a^4*b^3*c^4*d^3+
735*a^3*b^4*c^5*d^2+147*a^2*b^5*c^6*d+7*a*b^6*c^7)*x^7+1/6*(21*a^7*c^2*d^5+
245*a^6*b*c^3*d^4+735*a^5*b^2*c^4*d^3+735*a^4*b^3*c^5*d^2+245*a^3*b^4*c^6*d
+21*a^2*b^5*c^7)*x^6+1/5*(35*a^7*c^3*d^4+245*a^6*b*c^4*d^3+441*a^5*b^2*c^5*
d^2+245*a^4*b^3*c^6*d+35*a^3*b^4*c^7)*x^5+1/4*(35*a^7*c^4*d^3+147*a^6*b*c^5
*d^2+147*a^5*b^2*c^6*d+35*a^4*b^3*c^7)*x^4+1/3*(21*a^7*c^5*d^2+49*a^6*b*c^6
*d+21*a^5*b^2*c^7)*x^3+1/2*(7*a^7*c^6*d+7*a^6*b*c^7)*x^2+a^7*c^7*x
```

Maxima [B] time = 0.994162, size = 1089, normalized size = 5.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/15*b^7*d^7*x^15 + a^7*c^7*x + 1/2*(b^7*c*d^6 + a*b^6*d^7)*x^14 + 7/13*(3*
b^7*c^2*d^5 + 7*a*b^6*c*d^6 + 3*a^2*b^5*d^7)*x^13 + 7/12*(5*b^7*c^3*d^4 + 2
1*a*b^6*c^2*d^5 + 21*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^12 + 7/11*(5*b^7*c^4*
```

```
d^3 + 35*a*b^6*c^3*d^4 + 63*a^2*b^5*c^2*d^5 + 35*a^3*b^4*c*d^6 + 5*a^4*b^3*
d^7)*x^11 + 7/10*(3*b^7*c^5*d^2 + 35*a*b^6*c^4*d^3 + 105*a^2*b^5*c^3*d^4 +
105*a^3*b^4*c^2*d^5 + 35*a^4*b^3*c*d^6 + 3*a^5*b^2*d^7)*x^10 + 7/9*(b^7*c^6
*d + 21*a*b^6*c^5*d^2 + 105*a^2*b^5*c^4*d^3 + 175*a^3*b^4*c^3*d^4 + 105*a^4
*b^3*c^2*d^5 + 21*a^5*b^2*c*d^6 + a^6*b*d^7)*x^9 + 1/8*(b^7*c^7 + 49*a*b^6*
c^6*d + 441*a^2*b^5*c^5*d^2 + 1225*a^3*b^4*c^4*d^3 + 1225*a^4*b^3*c^3*d^4 +
 441*a^5*b^2*c^2*d^5 + 49*a^6*b*c*d^6 + a^7*d^7)*x^8 + (a*b^6*c^7 + 21*a^2*
b^5*c^6*d + 105*a^3*b^4*c^5*d^2 + 175*a^4*b^3*c^4*d^3 + 105*a^5*b^2*c^3*d^4
 + 21*a^6*b*c^2*d^5 + a^7*c*d^6)*x^7 + 7/6*(3*a^2*b^5*c^7 + 35*a^3*b^4*c^6*
d + 105*a^4*b^3*c^5*d^2 + 105*a^5*b^2*c^4*d^3 + 35*a^6*b*c^3*d^4 + 3*a^7*c^
2*d^5)*x^6 + 7/5*(5*a^3*b^4*c^7 + 35*a^4*b^3*c^6*d + 63*a^5*b^2*c^5*d^2 + 3
5*a^6*b*c^4*d^3 + 5*a^7*c^3*d^4)*x^5 + 7/4*(5*a^4*b^3*c^7 + 21*a^5*b^2*c^6*
d + 21*a^6*b*c^5*d^2 + 5*a^7*c^4*d^3)*x^4 + 7/3*(3*a^5*b^2*c^7 + 7*a^6*b*c^
6*d + 3*a^7*c^5*d^2)*x^3 + 7/2*(a^6*b*c^7 + a^7*c^6*d)*x^2
```

Fricas [B] time = 1.95135, size = 2079, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*d^7*b^7 + 1/2*x^14*d^6*c*b^7 + 1/2*x^14*d^7*b^6*a + 21/13*x^13*d^
5*c^2*b^7 + 49/13*x^13*d^6*c*b^6*a + 21/13*x^13*d^7*b^5*a^2 + 35/12*x^12*d^
4*c^3*b^7 + 49/4*x^12*d^5*c^2*b^6*a + 49/4*x^12*d^6*c*b^5*a^2 + 35/12*x^12*
d^7*b^4*a^3 + 35/11*x^11*d^3*c^4*b^7 + 245/11*x^11*d^4*c^3*b^6*a + 441/11*x
^11*d^5*c^2*b^5*a^2 + 245/11*x^11*d^6*c*b^4*a^3 + 35/11*x^11*d^7*b^3*a^4 +
21/10*x^10*d^2*c^5*b^7 + 49/2*x^10*d^3*c^4*b^6*a + 147/2*x^10*d^4*c^3*b^5*a
^2 + 147/2*x^10*d^5*c^2*b^4*a^3 + 49/2*x^10*d^6*c*b^3*a^4 + 21/10*x^10*d^7*
b^2*a^5 + 7/9*x^9*d*c^6*b^7 + 49/3*x^9*d^2*c^5*b^6*a + 245/3*x^9*d^3*c^4*b^
5*a^2 + 1225/9*x^9*d^4*c^3*b^4*a^3 + 245/3*x^9*d^5*c^2*b^3*a^4 + 49/3*x^9*d
^6*c*b^2*a^5 + 7/9*x^9*d^7*b*a^6 + 1/8*x^8*c^7*b^7 + 49/8*x^8*d*c^6*b^6*a +
 441/8*x^8*d^2*c^5*b^5*a^2 + 1225/8*x^8*d^3*c^4*b^4*a^3 + 1225/8*x^8*d^4*c^
3*b^3*a^4 + 441/8*x^8*d^5*c^2*b^2*a^5 + 49/8*x^8*d^6*c*b*a^6 + 1/8*x^8*d^7*
a^7 + x^7*c^7*b^6*a + 21*x^7*d*c^6*b^5*a^2 + 105*x^7*d^2*c^5*b^4*a^3 + 175*
x^7*d^3*c^4*b^3*a^4 + 105*x^7*d^4*c^3*b^2*a^5 + 21*x^7*d^5*c^2*b*a^6 + x^7*
d^6*c*a^7 + 7/2*x^6*c^7*b^5*a^2 + 245/6*x^6*d*c^6*b^4*a^3 + 245/2*x^6*d^2*c
^5*b^3*a^4 + 245/2*x^6*d^3*c^4*b^2*a^5 + 245/6*x^6*d^4*c^3*b*a^6 + 7/2*x^6*
d^5*c^2*a^7 + 7*x^5*c^7*b^4*a^3 + 49*x^5*d*c^6*b^3*a^4 + 441/5*x^5*d^2*c^5*
b^2*a^5 + 49*x^5*d^3*c^4*b*a^6 + 7*x^5*d^4*c^3*a^7 + 35/4*x^4*c^7*b^3*a^4 +
 147/4*x^4*d*c^6*b^2*a^5 + 147/4*x^4*d^2*c^5*b*a^6 + 35/4*x^4*d^3*c^4*a^7 +
 7*x^3*c^7*b^2*a^5 + 49/3*x^3*d*c^6*b*a^6 + 7*x^3*d^2*c^5*a^7 + 7/2*x^2*c^7
*b*a^6 + 7/2*x^2*d*c^6*a^7 + x*c^7*a^7
```

Sympy [B] time = 0.195297, size = 935, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7*(d*x+c)**7,x)
```

```
[Out] a**7*c**7*x + b**7*d**7*x**15/15 + x**14*(a*b**6*d**7/2 + b**7*c*d**6/2) +
x**13*(21*a**2*b**5*d**7/13 + 49*a*b**6*c*d**6/13 + 21*b**7*c**2*d**5/13) +
```

```
 x**12*(35*a**3*b**4*d**7/12 + 49*a**2*b**5*c*d**6/4 + 49*a*b**6*c**2*d**5/
4 + 35*b**7*c**3*d**4/12) + x**11*(35*a**4*b**3*d**7/11 + 245*a**3*b**4*c*d
**6/11 + 441*a**2*b**5*c**2*d**5/11 + 245*a*b**6*c**3*d**4/11 + 35*b**7*c**
4*d**3/11) + x**10*(21*a**5*b**2*d**7/10 + 49*a**4*b**3*c*d**6/2 + 147*a**3
*b**4*c**2*d**5/2 + 147*a**2*b**5*c**3*d**4/2 + 49*a*b**6*c**4*d**3/2 + 21*
b**7*c**5*d**2/10) + x**9*(7*a**6*b*d**7/9 + 49*a**5*b**2*c*d**6/3 + 245*a*
*4*b**3*c**2*d**5/3 + 1225*a**3*b**4*c**3*d**4/9 + 245*a**2*b**5*c**4*d**3/
3 + 49*a*b**6*c**5*d**2/3 + 7*b**7*c**6*d/9) + x**8*(a**7*d**7/8 + 49*a**6*
b*c*d**6/8 + 441*a**5*b**2*c**2*d**5/8 + 1225*a**4*b**3*c**3*d**4/8 + 1225*
a**3*b**4*c**4*d**3/8 + 441*a**2*b**5*c**5*d**2/8 + 49*a*b**6*c**6*d/8 + b*
*7*c**7/8) + x**7*(a**7*c*d**6 + 21*a**6*b*c**2*d**5 + 105*a**5*b**2*c**3*d
**4 + 175*a**4*b**3*c**4*d**3 + 105*a**3*b**4*c**5*d**2 + 21*a**2*b**5*c**6
*d + a*b**6*c**7) + x**6*(7*a**7*c**2*d**5/2 + 245*a**6*b*c**3*d**4/6 + 245
*a**5*b**2*c**4*d**3/2 + 245*a**4*b**3*c**5*d**2/2 + 245*a**3*b**4*c**6*d/6
 + 7*a**2*b**5*c**7/2) + x**5*(7*a**7*c**3*d**4 + 49*a**6*b*c**4*d**3 + 441
*a**5*b**2*c**5*d**2/5 + 49*a**4*b**3*c**6*d + 7*a**3*b**4*c**7) + x**4*(35
*a**7*c**4*d**3/4 + 147*a**6*b*c**5*d**2/4 + 147*a**5*b**2*c**6*d/4 + 35*a*
*4*b**3*c**7/4) + x**3*(7*a**7*c**5*d**2 + 49*a**6*b*c**6*d/3 + 7*a**5*b**2
*c**7) + x**2*(7*a**7*c**6*d/2 + 7*a**6*b*c**7/2)
```

Giac [B] time = 1.05949, size = 1247, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/15*b^7*d^7*x^15 + 1/2*b^7*c*d^6*x^14 + 1/2*a*b^6*d^7*x^14 + 21/13*b^7*c^2
*d^5*x^13 + 49/13*a*b^6*c*d^6*x^13 + 21/13*a^2*b^5*d^7*x^13 + 35/12*b^7*c^3
*d^4*x^12 + 49/4*a*b^6*c^2*d^5*x^12 + 49/4*a^2*b^5*c*d^6*x^12 + 35/12*a^3*b
^4*d^7*x^12 + 35/11*b^7*c^4*d^3*x^11 + 245/11*a*b^6*c^3*d^4*x^11 + 441/11*a
^2*b^5*c^2*d^5*x^11 + 245/11*a^3*b^4*c*d^6*x^11 + 35/11*a^4*b^3*d^7*x^11 +
21/10*b^7*c^5*d^2*x^10 + 49/2*a*b^6*c^4*d^3*x^10 + 147/2*a^2*b^5*c^3*d^4*x^
10 + 147/2*a^3*b^4*c^2*d^5*x^10 + 49/2*a^4*b^3*c*d^6*x^10 + 21/10*a^5*b^2*d
^7*x^10 + 7/9*b^7*c^6*d*x^9 + 49/3*a*b^6*c^5*d^2*x^9 + 245/3*a^2*b^5*c^4*d^
3*x^9 + 1225/9*a^3*b^4*c^3*d^4*x^9 + 245/3*a^4*b^3*c^2*d^5*x^9 + 49/3*a^5*b
^2*c*d^6*x^9 + 7/9*a^6*b*d^7*x^9 + 1/8*b^7*c^7*x^8 + 49/8*a*b^6*c^6*d*x^8 +
 441/8*a^2*b^5*c^5*d^2*x^8 + 1225/8*a^3*b^4*c^4*d^3*x^8 + 1225/8*a^4*b^3*c^
3*d^4*x^8 + 441/8*a^5*b^2*c^2*d^5*x^8 + 49/8*a^6*b*c*d^6*x^8 + 1/8*a^7*d^7*
x^8 + a*b^6*c^7*x^7 + 21*a^2*b^5*c^6*d*x^7 + 105*a^3*b^4*c^5*d^2*x^7 + 175*
a^4*b^3*c^4*d^3*x^7 + 105*a^5*b^2*c^3*d^4*x^7 + 21*a^6*b*c^2*d^5*x^7 + a^7*
c*d^6*x^7 + 7/2*a^2*b^5*c^7*x^6 + 245/6*a^3*b^4*c^6*d*x^6 + 245/2*a^4*b^3*c
^5*d^2*x^6 + 245/2*a^5*b^2*c^4*d^3*x^6 + 245/6*a^6*b*c^3*d^4*x^6 + 7/2*a^7*
c^2*d^5*x^6 + 7*a^3*b^4*c^7*x^5 + 49*a^4*b^3*c^6*d*x^5 + 441/5*a^5*b^2*c^5*
d^2*x^5 + 49*a^6*b*c^4*d^3*x^5 + 7*a^7*c^3*d^4*x^5 + 35/4*a^4*b^3*c^7*x^4 +
 147/4*a^5*b^2*c^6*d*x^4 + 147/4*a^6*b*c^5*d^2*x^4 + 35/4*a^7*c^4*d^3*x^4 +
 7*a^5*b^2*c^7*x^3 + 49/3*a^6*b*c^6*d*x^3 + 7*a^7*c^5*d^2*x^3 + 7/2*a^6*b*c
^7*x^2 + 7/2*a^7*c^6*d*x^2 + a^7*c^7*x
```

3.1276 $\int (a+bx)^6(c+dx)^7\,dx$

Optimal. Leaf size=173

$$-\frac{6b^5(c+dx)^{13}(bc-ad)}{13d^7}+\frac{5b^4(c+dx)^{12}(bc-ad)^2}{4d^7}-\frac{20b^3(c+dx)^{11}(bc-ad)^3}{11d^7}+\frac{3b^2(c+dx)^{10}(bc-ad)^4}{2d^7}-\frac{2b(c+dx)^9(b}{3d^7}$$

```
[Out] ((b*c - a*d)^6*(c + d*x)^8)/(8*d^7) - (2*b*(b*c - a*d)^5*(c + d*x)^9)/(3*d^
7) + (3*b^2*(b*c - a*d)^4*(c + d*x)^10)/(2*d^7) - (20*b^3*(b*c - a*d)^3*(c
+ d*x)^11)/(11*d^7) + (5*b^4*(b*c - a*d)^2*(c + d*x)^12)/(4*d^7) - (6*b^5*(
b*c - a*d)*(c + d*x)^13)/(13*d^7) + (b^6*(c + d*x)^14)/(14*d^7)
```

Rubi [A] time = 0.434592, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{6b^5(c+dx)^{13}(bc-ad)}{13d^7}+\frac{5b^4(c+dx)^{12}(bc-ad)^2}{4d^7}-\frac{20b^3(c+dx)^{11}(bc-ad)^3}{11d^7}+\frac{3b^2(c+dx)^{10}(bc-ad)^4}{2d^7}-\frac{2b(c+dx)^9(b}{3d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6*(c + d*x)^7,x]
```


```
[Out] ((b*c - a*d)^6*(c + d*x)^8)/(8*d^7) - (2*b*(b*c - a*d)^5*(c + d*x)^9)/(3*d^
7) + (3*b^2*(b*c - a*d)^4*(c + d*x)^10)/(2*d^7) - (20*b^3*(b*c - a*d)^3*(c
+ d*x)^11)/(11*d^7) + (5*b^4*(b*c - a*d)^2*(c + d*x)^12)/(4*d^7) - (6*b^5*(
b*c - a*d)*(c + d*x)^13)/(13*d^7) + (b^6*(c + d*x)^14)/(14*d^7)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^6(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^6(c+dx)^7}{d^6}-\frac{6b(bc-ad)^5(c+dx)^8}{d^6}+\frac{15b^2(bc-ad)^4(c+dx)^9}{d^6}-\frac{20b^3(bc-ad)^3(}{d^6}\right.\\ &= \frac{(bc-ad)^6(c+dx)^8}{8d^7}-\frac{2b(bc-ad)^5(c+dx)^9}{3d^7}+\frac{3b^2(bc-ad)^4(c+dx)^{10}}{2d^7}-\frac{20b^3(bc-ad)^3(c+dx}{11d^7}\end{aligned}$$

Mathematica [B] time = 0.0792028, size = 684, normalized size = 3.95

$$\frac{1}{4}b^4d^5x^{12}\left(5a^2d^2+14abcd+7b^2c^2\right)+\frac{1}{11}b^3d^4x^{11}\left(105a^2bcd^2+20a^3d^3+126ab^2c^2d+35b^3c^3\right)+\frac{1}{2}b^2d^3x^{10}\left(63a^2b^2c^2d^2+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6*(c + d*x)^7,x]
```

```
[Out] a^6*c^7*x + (a^5*c^6*(6*b*c + 7*a*d)*x^2)/2 + a^4*c^5*(5*b^2*c^2 + 14*a*b*c
*d + 7*a^2*d^2)*x^3 + (a^3*c^4*(20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*
```

```
d^2 + 35*a^3*d^3)*x^4)/4 + a^2*c^3*(3*b^4*c^4 + 28*a*b^3*c^3*d + 63*a^2*b^2
*c^2*d^2 + 42*a^3*b*c*d^3 + 7*a^4*d^4)*x^5 + (a*c^2*(2*b^5*c^5 + 35*a*b^4*c
^4*d + 140*a^2*b^3*c^3*d^2 + 175*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 7*a^5*d
^5)*x^6)/2 + (c*(b^6*c^6 + 42*a*b^5*c^5*d + 315*a^2*b^4*c^4*d^2 + 700*a^3*b
^3*c^3*d^3 + 525*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 + 7*a^6*d^6)*x^7)/7 + (d
*(7*b^6*c^6 + 126*a*b^5*c^5*d + 525*a^2*b^4*c^4*d^2 + 700*a^3*b^3*c^3*d^3 +
 315*a^4*b^2*c^2*d^4 + 42*a^5*b*c*d^5 + a^6*d^6)*x^8)/8 + (b*d^2*(7*b^5*c^5
 + 70*a*b^4*c^4*d + 175*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 35*a^4*b*c*
d^4 + 2*a^5*d^5)*x^9)/3 + (b^2*d^3*(7*b^4*c^4 + 42*a*b^3*c^3*d + 63*a^2*b^2
*c^2*d^2 + 28*a^3*b*c*d^3 + 3*a^4*d^4)*x^10)/2 + (b^3*d^4*(35*b^3*c^3 + 126
*a*b^2*c^2*d + 105*a^2*b*c*d^2 + 20*a^3*d^3)*x^11)/11 + (b^4*d^5*(7*b^2*c^2
 + 14*a*b*c*d + 5*a^2*d^2)*x^12)/4 + (b^5*d^6*(7*b*c + 6*a*d)*x^13)/13 + (b
^6*d^7*x^14)/14
```

Maple [B] time = 0.001, size = 709, normalized size = 4.1

$$\frac{b^6d^7x^{14}}{14}+\frac{\left(6\,ab^5d^7+7\,b^6cd^6\right)x^{13}}{13}+\frac{\left(15\,a^2b^4d^7+42\,ab^5cd^6+21\,b^6c^2d^5\right)x^{12}}{12}+\frac{\left(20\,a^3b^3d^7+105\,a^2b^4cd^6+126\,ab^5c^2d\right.}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6*(d*x+c)^7,x)
```

```
[Out] 1/14*b^6*d^7*x^14+1/13*(6*a*b^5*d^7+7*b^6*c*d^6)*x^13+1/12*(15*a^2*b^4*d^7+
42*a*b^5*c*d^6+21*b^6*c^2*d^5)*x^12+1/11*(20*a^3*b^3*d^7+105*a^2*b^4*c*d^6+
126*a*b^5*c^2*d^5+35*b^6*c^3*d^4)*x^11+1/10*(15*a^4*b^2*d^7+140*a^3*b^3*c*d
^6+315*a^2*b^4*c^2*d^5+210*a*b^5*c^3*d^4+35*b^6*c^4*d^3)*x^10+1/9*(6*a^5*b*
d^7+105*a^4*b^2*c*d^6+420*a^3*b^3*c^2*d^5+525*a^2*b^4*c^3*d^4+210*a*b^5*c^4
*d^3+21*b^6*c^5*d^2)*x^9+1/8*(a^6*d^7+42*a^5*b*c*d^6+315*a^4*b^2*c^2*d^5+70
0*a^3*b^3*c^3*d^4+525*a^2*b^4*c^4*d^3+126*a*b^5*c^5*d^2+7*b^6*c^6*d)*x^8+1/
7*(7*a^6*c*d^6+126*a^5*b*c^2*d^5+525*a^4*b^2*c^3*d^4+700*a^3*b^3*c^4*d^3+31
5*a^2*b^4*c^5*d^2+42*a*b^5*c^6*d+b^6*c^7)*x^7+1/6*(21*a^6*c^2*d^5+210*a^5*b
*c^3*d^4+525*a^4*b^2*c^4*d^3+420*a^3*b^3*c^5*d^2+105*a^2*b^4*c^6*d+6*a*b^5*
c^7)*x^6+1/5*(35*a^6*c^3*d^4+210*a^5*b*c^4*d^3+315*a^4*b^2*c^5*d^2+140*a^3*
b^3*c^6*d+15*a^2*b^4*c^7)*x^5+1/4*(35*a^6*c^4*d^3+126*a^5*b*c^5*d^2+105*a^4
*b^2*c^6*d+20*a^3*b^3*c^7)*x^4+1/3*(21*a^6*c^5*d^2+42*a^5*b*c^6*d+15*a^4*b^
2*c^7)*x^3+1/2*(7*a^6*c^6*d+6*a^5*b*c^7)*x^2+a^6*c^7*x
```

Maxima [B] time = 0.980019, size = 953, normalized size = 5.51

$$\frac{1}{14}\,b^6d^7x^{14}+a^6c^7x+\frac{1}{13}\left(7\,b^6cd^6+6\,ab^5d^7\right)x^{13}+\frac{1}{4}\left(7\,b^6c^2d^5+14\,ab^5cd^6+5\,a^2b^4d^7\right)x^{12}+\frac{1}{11}\left(35\,b^6c^3d^4+126\,ab^5c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/14*b^6*d^7*x^14 + a^6*c^7*x + 1/13*(7*b^6*c*d^6 + 6*a*b^5*d^7)*x^13 + 1/4
*(7*b^6*c^2*d^5 + 14*a*b^5*c*d^6 + 5*a^2*b^4*d^7)*x^12 + 1/11*(35*b^6*c^3*d
^4 + 126*a*b^5*c^2*d^5 + 105*a^2*b^4*c*d^6 + 20*a^3*b^3*d^7)*x^11 + 1/2*(7*
b^6*c^4*d^3 + 42*a*b^5*c^3*d^4 + 63*a^2*b^4*c^2*d^5 + 28*a^3*b^3*c*d^6 + 3*
a^4*b^2*d^7)*x^10 + 1/3*(7*b^6*c^5*d^2 + 70*a*b^5*c^4*d^3 + 175*a^2*b^4*c^3
*d^4 + 140*a^3*b^3*c^2*d^5 + 35*a^4*b^2*c*d^6 + 2*a^5*b*d^7)*x^9 + 1/8*(7*b
^6*c^6*d + 126*a*b^5*c^5*d^2 + 525*a^2*b^4*c^4*d^3 + 700*a^3*b^3*c^3*d^4 +
315*a^4*b^2*c^2*d^5 + 42*a^5*b*c*d^6 + a^6*d^7)*x^8 + 1/7*(b^6*c^7 + 42*a*b
```

```
^5*c^6*d + 315*a^2*b^4*c^5*d^2 + 700*a^3*b^3*c^4*d^3 + 525*a^4*b^2*c^3*d^4
+ 126*a^5*b*c^2*d^5 + 7*a^6*c*d^6)*x^7 + 1/2*(2*a*b^5*c^7 + 35*a^2*b^4*c^6*
d + 140*a^3*b^3*c^5*d^2 + 175*a^4*b^2*c^4*d^3 + 70*a^5*b*c^3*d^4 + 7*a^6*c^
2*d^5)*x^6 + (3*a^2*b^4*c^7 + 28*a^3*b^3*c^6*d + 63*a^4*b^2*c^5*d^2 + 42*a^
5*b*c^4*d^3 + 7*a^6*c^3*d^4)*x^5 + 1/4*(20*a^3*b^3*c^7 + 105*a^4*b^2*c^6*d
+ 126*a^5*b*c^5*d^2 + 35*a^6*c^4*d^3)*x^4 + (5*a^4*b^2*c^7 + 14*a^5*b*c^6*d
 + 7*a^6*c^5*d^2)*x^3 + 1/2*(6*a^5*b*c^7 + 7*a^6*c^6*d)*x^2
```

Fricas [B] time = 2.09939, size = 1736, normalized size = 10.03

$$\frac{1}{14}x^{14}d^7b^6 + \frac{7}{13}x^{13}d^6cb^6 + \frac{6}{13}x^{13}d^7b^5a + \frac{7}{4}x^{12}d^5c^2b^6 + \frac{7}{2}x^{12}d^6cb^5a + \frac{5}{4}x^{12}d^7b^4a^2 + \frac{35}{11}x^{11}d^4c^3b^6 + \frac{126}{11}x^{11}d^5c^2b^5a + \frac{105}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*d^7*b^6 + 7/13*x^13*d^6*c*b^6 + 6/13*x^13*d^7*b^5*a + 7/4*x^12*d^
5*c^2*b^6 + 7/2*x^12*d^6*c*b^5*a + 5/4*x^12*d^7*b^4*a^2 + 35/11*x^11*d^4*c^
3*b^6 + 126/11*x^11*d^5*c^2*b^5*a + 105/11*x^11*d^6*c*b^4*a^2 + 20/11*x^11*
d^7*b^3*a^3 + 7/2*x^10*d^3*c^4*b^6 + 21*x^10*d^4*c^3*b^5*a + 63/2*x^10*d^5*
c^2*b^4*a^2 + 14*x^10*d^6*c*b^3*a^3 + 3/2*x^10*d^7*b^2*a^4 + 7/3*x^9*d^2*c^
5*b^6 + 70/3*x^9*d^3*c^4*b^5*a + 175/3*x^9*d^4*c^3*b^4*a^2 + 140/3*x^9*d^5*
c^2*b^3*a^3 + 35/3*x^9*d^6*c*b^2*a^4 + 2/3*x^9*d^7*b*a^5 + 7/8*x^8*d*c^6*b^
6 + 63/4*x^8*d^2*c^5*b^5*a + 525/8*x^8*d^3*c^4*b^4*a^2 + 175/2*x^8*d^4*c^3*
b^3*a^3 + 315/8*x^8*d^5*c^2*b^2*a^4 + 21/4*x^8*d^6*c*b*a^5 + 1/8*x^8*d^7*a^
6 + 1/7*x^7*c^7*b^6 + 6*x^7*d*c^6*b^5*a + 45*x^7*d^2*c^5*b^4*a^2 + 100*x^7*
d^3*c^4*b^3*a^3 + 75*x^7*d^4*c^3*b^2*a^4 + 18*x^7*d^5*c^2*b*a^5 + x^7*d^6*c
*a^6 + x^6*c^7*b^5*a + 35/2*x^6*d*c^6*b^4*a^2 + 70*x^6*d^2*c^5*b^3*a^3 + 17
5/2*x^6*d^3*c^4*b^2*a^4 + 35*x^6*d^4*c^3*b*a^5 + 7/2*x^6*d^5*c^2*a^6 + 3*x^
5*c^7*b^4*a^2 + 28*x^5*d*c^6*b^3*a^3 + 63*x^5*d^2*c^5*b^2*a^4 + 42*x^5*d^3*
c^4*b*a^5 + 7*x^5*d^4*c^3*a^6 + 5*x^4*c^7*b^3*a^3 + 105/4*x^4*d*c^6*b^2*a^4
 + 63/2*x^4*d^2*c^5*b*a^5 + 35/4*x^4*d^3*c^4*a^6 + 5*x^3*c^7*b^2*a^4 + 14*x
^3*d*c^6*b*a^5 + 7*x^3*d^2*c^5*a^6 + 3*x^2*c^7*b*a^5 + 7/2*x^2*d*c^6*a^6 +
x*c^7*a^6
```

Sympy [B] time = 0.15926, size = 796, normalized size = 4.6

$$a^6c^7x + \frac{b^6d^7x^{14}}{14} + x^{13}\left(\frac{6ab^5d^7}{13} + \frac{7b^6cd^6}{13}\right) + x^{12}\left(\frac{5a^2b^4d^7}{4} + \frac{7ab^5cd^6}{2} + \frac{7b^6c^2d^5}{4}\right) + x^{11}\left(\frac{20a^3b^3d^7}{11} + \frac{105a^2b^4cd^6}{11} + \frac{126a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6*(d*x+c)**7,x)
```

```
[Out] a**6*c**7*x + b**6*d**7*x**14/14 + x**13*(6*a*b**5*d**7/13 + 7*b**6*c*d**6/
13) + x**12*(5*a**2*b**4*d**7/4 + 7*a*b**5*c*d**6/2 + 7*b**6*c**2*d**5/4) +
 x**11*(20*a**3*b**3*d**7/11 + 105*a**2*b**4*c*d**6/11 + 126*a*b**5*c**2*d*
*5/11 + 35*b**6*c**3*d**4/11) + x**10*(3*a**4*b**2*d**7/2 + 14*a**3*b**3*c*
d**6 + 63*a**2*b**4*c**2*d**5/2 + 21*a*b**5*c**3*d**4 + 7*b**6*c**4*d**3/2)
 + x**9*(2*a**5*b*d**7/3 + 35*a**4*b**2*c*d**6/3 + 140*a**3*b**3*c**2*d**5/
3 + 175*a**2*b**4*c**3*d**4/3 + 70*a*b**5*c**4*d**3/3 + 7*b**6*c**5*d**2/3)
 + x**8*(a**6*d**7/8 + 21*a**5*b*c*d**6/4 + 315*a**4*b**2*c**2*d**5/8 + 175
*a**3*b**3*c**3*d**4/2 + 525*a**2*b**4*c**4*d**3/8 + 63*a*b**5*c**5*d**2/4
+ 7*b**6*c**6*d/8) + x**7*(a**6*c*d**6 + 18*a**5*b*c**2*d**5 + 75*a**4*b**2
```

```
*c**3*d**4 + 100*a**3*b**3*c**4*d**3 + 45*a**2*b**4*c**5*d**2 + 6*a*b**5*c*
*6*d + b**6*c**7/7) + x**6*(7*a**6*c**2*d**5/2 + 35*a**5*b*c**3*d**4 + 175*
a**4*b**2*c**4*d**3/2 + 70*a**3*b**3*c**5*d**2 + 35*a**2*b**4*c**6*d/2 + a*
b**5*c**7) + x**5*(7*a**6*c**3*d**4 + 42*a**5*b*c**4*d**3 + 63*a**4*b**2*c*
*5*d**2 + 28*a**3*b**3*c**6*d + 3*a**2*b**4*c**7) + x**4*(35*a**6*c**4*d**3
/4 + 63*a**5*b*c**5*d**2/2 + 105*a**4*b**2*c**6*d/4 + 5*a**3*b**3*c**7) + x
**3*(7*a**6*c**5*d**2 + 14*a**5*b*c**6*d + 5*a**4*b**2*c**7) + x**2*(7*a**6
*c**6*d/2 + 3*a**5*b*c**7)
```

Giac [B] time = 1.05637, size = 1077, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/14*b^6*d^7*x^14 + 7/13*b^6*c*d^6*x^13 + 6/13*a*b^5*d^7*x^13 + 7/4*b^6*c^2
*d^5*x^12 + 7/2*a*b^5*c*d^6*x^12 + 5/4*a^2*b^4*d^7*x^12 + 35/11*b^6*c^3*d^4
*x^11 + 126/11*a*b^5*c^2*d^5*x^11 + 105/11*a^2*b^4*c*d^6*x^11 + 20/11*a^3*b
^3*d^7*x^11 + 7/2*b^6*c^4*d^3*x^10 + 21*a*b^5*c^3*d^4*x^10 + 63/2*a^2*b^4*c
^2*d^5*x^10 + 14*a^3*b^3*c*d^6*x^10 + 3/2*a^4*b^2*d^7*x^10 + 7/3*b^6*c^5*d^
2*x^9 + 70/3*a*b^5*c^4*d^3*x^9 + 175/3*a^2*b^4*c^3*d^4*x^9 + 140/3*a^3*b^3*
c^2*d^5*x^9 + 35/3*a^4*b^2*c*d^6*x^9 + 2/3*a^5*b*d^7*x^9 + 7/8*b^6*c^6*d*x^
8 + 63/4*a*b^5*c^5*d^2*x^8 + 525/8*a^2*b^4*c^4*d^3*x^8 + 175/2*a^3*b^3*c^3*
d^4*x^8 + 315/8*a^4*b^2*c^2*d^5*x^8 + 21/4*a^5*b*c*d^6*x^8 + 1/8*a^6*d^7*x^
8 + 1/7*b^6*c^7*x^7 + 6*a*b^5*c^6*d*x^7 + 45*a^2*b^4*c^5*d^2*x^7 + 100*a^3*
b^3*c^4*d^3*x^7 + 75*a^4*b^2*c^3*d^4*x^7 + 18*a^5*b*c^2*d^5*x^7 + a^6*c*d^6
*x^7 + a*b^5*c^7*x^6 + 35/2*a^2*b^4*c^6*d*x^6 + 70*a^3*b^3*c^5*d^2*x^6 + 17
5/2*a^4*b^2*c^4*d^3*x^6 + 35*a^5*b*c^3*d^4*x^6 + 7/2*a^6*c^2*d^5*x^6 + 3*a^
2*b^4*c^7*x^5 + 28*a^3*b^3*c^6*d*x^5 + 63*a^4*b^2*c^5*d^2*x^5 + 42*a^5*b*c^
4*d^3*x^5 + 7*a^6*c^3*d^4*x^5 + 5*a^3*b^3*c^7*x^4 + 105/4*a^4*b^2*c^6*d*x^4
 + 63/2*a^5*b*c^5*d^2*x^4 + 35/4*a^6*c^4*d^3*x^4 + 5*a^4*b^2*c^7*x^3 + 14*a
^5*b*c^6*d*x^3 + 7*a^6*c^5*d^2*x^3 + 3*a^5*b*c^7*x^2 + 7/2*a^6*c^6*d*x^2 +
a^6*c^7*x
```

3.1277 $\int (a+bx)^5(c+dx)^7\,dx$

Optimal. Leaf size=144

$$-\frac{5b^4(c+dx)^{12}(bc-ad)}{12d^6}+\frac{10b^3(c+dx)^{11}(bc-ad)^2}{11d^6}-\frac{b^2(c+dx)^{10}(bc-ad)^3}{d^6}+\frac{5b(c+dx)^9(bc-ad)^4}{9d^6}-\frac{(c+dx)^8(bc-ad}{8d^6}$$

```
[Out] -((b*c - a*d)^5*(c + d*x)^8)/(8*d^6) + (5*b*(b*c - a*d)^4*(c + d*x)^9)/(9*d
^6) - (b^2*(b*c - a*d)^3*(c + d*x)^10)/d^6 + (10*b^3*(b*c - a*d)^2*(c + d*x
)^11)/(11*d^6) - (5*b^4*(b*c - a*d)*(c + d*x)^12)/(12*d^6) + (b^5*(c + d*x)
^13)/(13*d^6)
```

Rubi [A] time = 0.359666, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{5b^4(c+dx)^{12}(bc-ad)}{12d^6}+\frac{10b^3(c+dx)^{11}(bc-ad)^2}{11d^6}-\frac{b^2(c+dx)^{10}(bc-ad)^3}{d^6}+\frac{5b(c+dx)^9(bc-ad)^4}{9d^6}-\frac{(c+dx)^8(bc-ad}{8d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^7,x]
```

```
[Out] -((b*c - a*d)^5*(c + d*x)^8)/(8*d^6) + (5*b*(b*c - a*d)^4*(c + d*x)^9)/(9*d
^6) - (b^2*(b*c - a*d)^3*(c + d*x)^10)/d^6 + (10*b^3*(b*c - a*d)^2*(c + d*x
)^11)/(11*d^6) - (5*b^4*(b*c - a*d)*(c + d*x)^12)/(12*d^6) + (b^5*(c + d*x)
^13)/(13*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^5(c+dx)^7}{d^5}+\frac{5b(bc-ad)^4(c+dx)^8}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^9}{d^5}+\frac{10b^3(bc-ad)^2(}{d^5}\right.\\ &= -\frac{(bc-ad)^5(c+dx)^8}{8d^6}+\frac{5b(bc-ad)^4(c+dx)^9}{9d^6}-\frac{b^2(bc-ad)^3(c+dx)^{10}}{d^6}+\frac{10b^3(bc-ad)^2(c+dx}{11d^6}\end{aligned}$$

Mathematica [B] time = 0.0676412, size = 574, normalized size = 3.99

$$\frac{1}{11}b^3d^5x^{11}\left(10a^2d^2+35abcd+21b^2c^2\right)+\frac{1}{2}b^2d^4x^{10}\left(14a^2bcd^2+2a^3d^3+21ab^2c^2d+7b^3c^3\right)+\frac{5}{9}bd^3x^9\left(42a^2b^2c^2d^2+14a^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^7,x]
```

```
[Out] a^5*c^7*x + (a^4*c^6*(5*b*c + 7*a*d)*x^2)/2 + (a^3*c^5*(10*b^2*c^2 + 35*a*b
*c*d + 21*a^2*d^2)*x^3)/3 + (5*a^2*c^4*(2*b^3*c^3 + 14*a*b^2*c^2*d + 21*a^2
```

```
*b*c*d^2 + 7*a^3*d^3)*x^4)/4 + a*c^3*(b^4*c^4 + 14*a*b^3*c^3*d + 42*a^2*b^2
*c^2*d^2 + 35*a^3*b*c*d^3 + 7*a^4*d^4)*x^5 + (c^2*(b^5*c^5 + 35*a*b^4*c^4*d
 + 210*a^2*b^3*c^3*d^2 + 350*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d^4 + 21*a^5*d^5
)*x^6)/6 + c*d*(b^5*c^5 + 15*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 + 50*a^3*b^2*
c^2*d^3 + 15*a^4*b*c*d^4 + a^5*d^5)*x^7 + (d^2*(21*b^5*c^5 + 175*a*b^4*c^4*
d + 350*a^2*b^3*c^3*d^2 + 210*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 + a^5*d^5)*x
^8)/8 + (5*b*d^3*(7*b^4*c^4 + 35*a*b^3*c^3*d + 42*a^2*b^2*c^2*d^2 + 14*a^3*
b*c*d^3 + a^4*d^4)*x^9)/9 + (b^2*d^4*(7*b^3*c^3 + 21*a*b^2*c^2*d + 14*a^2*b
*c*d^2 + 2*a^3*d^3)*x^10)/2 + (b^3*d^5*(21*b^2*c^2 + 35*a*b*c*d + 10*a^2*d^
2)*x^11)/11 + (b^4*d^6*(7*b*c + 5*a*d)*x^12)/12 + (b^5*d^7*x^13)/13
```

Maple [B] time = 0.001, size = 601, normalized size = 4.2

$$\frac{b^5d^7x^{13}}{13}+\frac{\left(5\,ab^4d^7+7\,b^5cd^6\right)x^{12}}{12}+\frac{\left(10\,a^2b^3d^7+35\,ab^4cd^6+21\,b^5c^2d^5\right)x^{11}}{11}+\frac{\left(10\,a^3b^2d^7+70\,a^2b^3cd^6+105\,ab^4c^2d^5\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^7,x)
```

```
[Out] 1/13*b^5*d^7*x^13+1/12*(5*a*b^4*d^7+7*b^5*c*d^6)*x^12+1/11*(10*a^2*b^3*d^7+
35*a*b^4*c*d^6+21*b^5*c^2*d^5)*x^11+1/10*(10*a^3*b^2*d^7+70*a^2*b^3*c*d^6+1
05*a*b^4*c^2*d^5+35*b^5*c^3*d^4)*x^10+1/9*(5*a^4*b*d^7+70*a^3*b^2*c*d^6+210
*a^2*b^3*c^2*d^5+175*a*b^4*c^3*d^4+35*b^5*c^4*d^3)*x^9+1/8*(a^5*d^7+35*a^4*
b*c*d^6+210*a^3*b^2*c^2*d^5+350*a^2*b^3*c^3*d^4+175*a*b^4*c^4*d^3+21*b^5*c^
5*d^2)*x^8+1/7*(7*a^5*c*d^6+105*a^4*b*c^2*d^5+350*a^3*b^2*c^3*d^4+350*a^2*b
^3*c^4*d^3+105*a*b^4*c^5*d^2+7*b^5*c^6*d)*x^7+1/6*(21*a^5*c^2*d^5+175*a^4*b
*c^3*d^4+350*a^3*b^2*c^4*d^3+210*a^2*b^3*c^5*d^2+35*a*b^4*c^6*d+b^5*c^7)*x^
6+1/5*(35*a^5*c^3*d^4+175*a^4*b*c^4*d^3+210*a^3*b^2*c^5*d^2+70*a^2*b^3*c^6*
d+5*a*b^4*c^7)*x^5+1/4*(35*a^5*c^4*d^3+105*a^4*b*c^5*d^2+70*a^3*b^2*c^6*d+1
0*a^2*b^3*c^7)*x^4+1/3*(21*a^5*c^5*d^2+35*a^4*b*c^6*d+10*a^3*b^2*c^7)*x^3+1
/2*(7*a^5*c^6*d+5*a^4*b*c^7)*x^2+a^5*c^7*x
```

Maxima [B] time = 0.978672, size = 802, normalized size = 5.57

$$\frac{1}{13}\,b^5d^7x^{13}+a^5c^7x+\frac{1}{12}\left(7\,b^5cd^6+5\,ab^4d^7\right)x^{12}+\frac{1}{11}\left(21\,b^5c^2d^5+35\,ab^4cd^6+10\,a^2b^3d^7\right)x^{11}+\frac{1}{2}\left(7\,b^5c^3d^4+21\,ab^4c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/13*b^5*d^7*x^13 + a^5*c^7*x + 1/12*(7*b^5*c*d^6 + 5*a*b^4*d^7)*x^12 + 1/1
1*(21*b^5*c^2*d^5 + 35*a*b^4*c*d^6 + 10*a^2*b^3*d^7)*x^11 + 1/2*(7*b^5*c^3*
d^4 + 21*a*b^4*c^2*d^5 + 14*a^2*b^3*c*d^6 + 2*a^3*b^2*d^7)*x^10 + 5/9*(7*b^
5*c^4*d^3 + 35*a*b^4*c^3*d^4 + 42*a^2*b^3*c^2*d^5 + 14*a^3*b^2*c*d^6 + a^4*
b*d^7)*x^9 + 1/8*(21*b^5*c^5*d^2 + 175*a*b^4*c^4*d^3 + 350*a^2*b^3*c^3*d^4
+ 210*a^3*b^2*c^2*d^5 + 35*a^4*b*c*d^6 + a^5*d^7)*x^8 + (b^5*c^6*d + 15*a*b
^4*c^5*d^2 + 50*a^2*b^3*c^4*d^3 + 50*a^3*b^2*c^3*d^4 + 15*a^4*b*c^2*d^5 + a
^5*c*d^6)*x^7 + 1/6*(b^5*c^7 + 35*a*b^4*c^6*d + 210*a^2*b^3*c^5*d^2 + 350*a
^3*b^2*c^4*d^3 + 175*a^4*b*c^3*d^4 + 21*a^5*c^2*d^5)*x^6 + (a*b^4*c^7 + 14*
a^2*b^3*c^6*d + 42*a^3*b^2*c^5*d^2 + 35*a^4*b*c^4*d^3 + 7*a^5*c^3*d^4)*x^5
+ 5/4*(2*a^2*b^3*c^7 + 14*a^3*b^2*c^6*d + 21*a^4*b*c^5*d^2 + 7*a^5*c^4*d^3)
*x^4 + 1/3*(10*a^3*b^2*c^7 + 35*a^4*b*c^6*d + 21*a^5*c^5*d^2)*x^3 + 1/2*(5*
```

```
a^4*b*c^7 + 7*a^5*c^6*d)*x^2
```

Fricas [B] time = 2.07709, size = 1467, normalized size = 10.19

$$\frac{1}{13}x^{13}d^7b^5 + \frac{7}{12}x^{12}d^6cb^5 + \frac{5}{12}x^{12}d^7b^4a + \frac{21}{11}x^{11}d^5c^2b^5 + \frac{35}{11}x^{11}d^6cb^4a + \frac{10}{11}x^{11}d^7b^3a^2 + \frac{7}{2}x^{10}d^4c^3b^5 + \frac{21}{2}x^{10}d^5c^2b^4a + 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^7*b^5 + 7/12*x^12*d^6*c*b^5 + 5/12*x^12*d^7*b^4*a + 21/11*x^11*
d^5*c^2*b^5 + 35/11*x^11*d^6*c*b^4*a + 10/11*x^11*d^7*b^3*a^2 + 7/2*x^10*d^
4*c^3*b^5 + 21/2*x^10*d^5*c^2*b^4*a + 7*x^10*d^6*c*b^3*a^2 + x^10*d^7*b^2*a
^3 + 35/9*x^9*d^3*c^4*b^5 + 175/9*x^9*d^4*c^3*b^4*a + 70/3*x^9*d^5*c^2*b^3*
a^2 + 70/9*x^9*d^6*c*b^2*a^3 + 5/9*x^9*d^7*b*a^4 + 21/8*x^8*d^2*c^5*b^5 + 1
75/8*x^8*d^3*c^4*b^4*a + 175/4*x^8*d^4*c^3*b^3*a^2 + 105/4*x^8*d^5*c^2*b^2*
a^3 + 35/8*x^8*d^6*c*b*a^4 + 1/8*x^8*d^7*a^5 + x^7*d*c^6*b^5 + 15*x^7*d^2*c
^5*b^4*a + 50*x^7*d^3*c^4*b^3*a^2 + 50*x^7*d^4*c^3*b^2*a^3 + 15*x^7*d^5*c^2
*b*a^4 + x^7*d^6*c*a^5 + 1/6*x^6*c^7*b^5 + 35/6*x^6*d*c^6*b^4*a + 35*x^6*d^
2*c^5*b^3*a^2 + 175/3*x^6*d^3*c^4*b^2*a^3 + 175/6*x^6*d^4*c^3*b*a^4 + 7/2*x
^6*d^5*c^2*a^5 + x^5*c^7*b^4*a + 14*x^5*d*c^6*b^3*a^2 + 42*x^5*d^2*c^5*b^2*
a^3 + 35*x^5*d^3*c^4*b*a^4 + 7*x^5*d^4*c^3*a^5 + 5/2*x^4*c^7*b^3*a^2 + 35/2
*x^4*d*c^6*b^2*a^3 + 105/4*x^4*d^2*c^5*b*a^4 + 35/4*x^4*d^3*c^4*a^5 + 10/3*
x^3*c^7*b^2*a^3 + 35/3*x^3*d*c^6*b*a^4 + 7*x^3*d^2*c^5*a^5 + 5/2*x^2*c^7*b*
a^4 + 7/2*x^2*d*c^6*a^5 + x*c^7*a^5
```

Sympy [B] time = 0.141385, size = 673, normalized size = 4.67

$$a^5c^7x + \frac{b^5d^7x^{13}}{13} + x^{12}\left(\frac{5ab^4d^7}{12} + \frac{7b^5cd^6}{12}\right) + x^{11}\left(\frac{10a^2b^3d^7}{11} + \frac{35ab^4cd^6}{11} + \frac{21b^5c^2d^5}{11}\right) + x^{10}\left(a^3b^2d^7 + 7a^2b^3cd^6 + \frac{21ab^4c^2}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**7,x)
```

```
[Out] a**5*c**7*x + b**5*d**7*x**13/13 + x**12*(5*a*b**4*d**7/12 + 7*b**5*c*d**6/
12) + x**11*(10*a**2*b**3*d**7/11 + 35*a*b**4*c*d**6/11 + 21*b**5*c**2*d**5
/11) + x**10*(a**3*b**2*d**7 + 7*a**2*b**3*c*d**6 + 21*a*b**4*c**2*d**5/2 +
 7*b**5*c**3*d**4/2) + x**9*(5*a**4*b*d**7/9 + 70*a**3*b**2*c*d**6/9 + 70*a
**2*b**3*c**2*d**5/3 + 175*a*b**4*c**3*d**4/9 + 35*b**5*c**4*d**3/9) + x**8
*(a**5*d**7/8 + 35*a**4*b*c*d**6/8 + 105*a**3*b**2*c**2*d**5/4 + 175*a**2*b
**3*c**3*d**4/4 + 175*a*b**4*c**4*d**3/8 + 21*b**5*c**5*d**2/8) + x**7*(a**
5*c*d**6 + 15*a**4*b*c**2*d**5 + 50*a**3*b**2*c**3*d**4 + 50*a**2*b**3*c**4
*d**3 + 15*a*b**4*c**5*d**2 + b**5*c**6*d) + x**6*(7*a**5*c**2*d**5/2 + 175
*a**4*b*c**3*d**4/6 + 175*a**3*b**2*c**4*d**3/3 + 35*a**2*b**3*c**5*d**2 +
35*a*b**4*c**6*d/6 + b**5*c**7/6) + x**5*(7*a**5*c**3*d**4 + 35*a**4*b*c**4
*d**3 + 42*a**3*b**2*c**5*d**2 + 14*a**2*b**3*c**6*d + a*b**4*c**7) + x**4*
(35*a**5*c**4*d**3/4 + 105*a**4*b*c**5*d**2/4 + 35*a**3*b**2*c**6*d/2 + 5*a
**2*b**3*c**7/2) + x**3*(7*a**5*c**5*d**2 + 35*a**4*b*c**6*d/3 + 10*a**3*b*
*2*c**7/3) + x**2*(7*a**5*c**6*d/2 + 5*a**4*b*c**7/2)
```

Giac [B] time = 1.06003, size = 905, normalized size = 6.28

$$\frac{1}{13}b^5d^7x^{13} + \frac{7}{12}b^5cd^6x^{12} + \frac{5}{12}ab^4d^7x^{12} + \frac{21}{11}b^5c^2d^5x^{11} + \frac{35}{11}ab^4cd^6x^{11} + \frac{10}{11}a^2b^3d^7x^{11} + \frac{7}{2}b^5c^3d^4x^{10} + \frac{21}{2}ab^4c^2d^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/13*b^5*d^7*x^13 + 7/12*b^5*c*d^6*x^12 + 5/12*a*b^4*d^7*x^12 + 21/11*b^5*c
^2*d^5*x^11 + 35/11*a*b^4*c*d^6*x^11 + 10/11*a^2*b^3*d^7*x^11 + 7/2*b^5*c^3
*d^4*x^10 + 21/2*a*b^4*c^2*d^5*x^10 + 7*a^2*b^3*c*d^6*x^10 + a^3*b^2*d^7*x^
10 + 35/9*b^5*c^4*d^3*x^9 + 175/9*a*b^4*c^3*d^4*x^9 + 70/3*a^2*b^3*c^2*d^5*
x^9 + 70/9*a^3*b^2*c*d^6*x^9 + 5/9*a^4*b*d^7*x^9 + 21/8*b^5*c^5*d^2*x^8 + 1
75/8*a*b^4*c^4*d^3*x^8 + 175/4*a^2*b^3*c^3*d^4*x^8 + 105/4*a^3*b^2*c^2*d^5*
x^8 + 35/8*a^4*b*c*d^6*x^8 + 1/8*a^5*d^7*x^8 + b^5*c^6*d*x^7 + 15*a*b^4*c^5
*d^2*x^7 + 50*a^2*b^3*c^4*d^3*x^7 + 50*a^3*b^2*c^3*d^4*x^7 + 15*a^4*b*c^2*d
^5*x^7 + a^5*c*d^6*x^7 + 1/6*b^5*c^7*x^6 + 35/6*a*b^4*c^6*d*x^6 + 35*a^2*b^
3*c^5*d^2*x^6 + 175/3*a^3*b^2*c^4*d^3*x^6 + 175/6*a^4*b*c^3*d^4*x^6 + 7/2*a
^5*c^2*d^5*x^6 + a*b^4*c^7*x^5 + 14*a^2*b^3*c^6*d*x^5 + 42*a^3*b^2*c^5*d^2*
x^5 + 35*a^4*b*c^4*d^3*x^5 + 7*a^5*c^3*d^4*x^5 + 5/2*a^2*b^3*c^7*x^4 + 35/2
*a^3*b^2*c^6*d*x^4 + 105/4*a^4*b*c^5*d^2*x^4 + 35/4*a^5*c^4*d^3*x^4 + 10/3*
a^3*b^2*c^7*x^3 + 35/3*a^4*b*c^6*d*x^3 + 7*a^5*c^5*d^2*x^3 + 5/2*a^4*b*c^7*
x^2 + 7/2*a^5*c^6*d*x^2 + a^5*c^7*x
```

3.1278 $\int (a+bx)^4(c+dx)^7\,dx$

Optimal. Leaf size=119

$$-\frac{4b^3(c+dx)^{11}(bc-ad)}{11d^5}+\frac{3b^2(c+dx)^{10}(bc-ad)^2}{5d^5}-\frac{4b(c+dx)^9(bc-ad)^3}{9d^5}+\frac{(c+dx)^8(bc-ad)^4}{8d^5}+\frac{b^4(c+dx)^{12}}{12d^5}$$

```
[Out] ((b*c - a*d)^4*(c + d*x)^8)/(8*d^5) - (4*b*(b*c - a*d)^3*(c + d*x)^9)/(9*d^
5) + (3*b^2*(b*c - a*d)^2*(c + d*x)^10)/(5*d^5) - (4*b^3*(b*c - a*d)*(c + d
*x)^11)/(11*d^5) + (b^4*(c + d*x)^12)/(12*d^5)
```

Rubi [A] time = 0.279351, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{4b^3(c+dx)^{11}(bc-ad)}{11d^5}+\frac{3b^2(c+dx)^{10}(bc-ad)^2}{5d^5}-\frac{4b(c+dx)^9(bc-ad)^3}{9d^5}+\frac{(c+dx)^8(bc-ad)^4}{8d^5}+\frac{b^4(c+dx)^{12}}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^4*(c + d*x)^8)/(8*d^5) - (4*b*(b*c - a*d)^3*(c + d*x)^9)/(9*d^
5) + (3*b^2*(b*c - a*d)^2*(c + d*x)^10)/(5*d^5) - (4*b^3*(b*c - a*d)*(c + d
*x)^11)/(11*d^5) + (b^4*(c + d*x)^12)/(12*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^4(c+dx)^7\,dx=\int \left(\frac{(-bc+ad)^4(c+dx)^7}{d^4}-\frac{4b(bc-ad)^3(c+dx)^8}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^9}{d^4}-\frac{4b^3(bc-ad)(c+\right.$$

$$=\frac{(bc-ad)^4(c+dx)^8}{8d^5}-\frac{4b(bc-ad)^3(c+dx)^9}{9d^5}+\frac{3b^2(bc-ad)^2(c+dx)^{10}}{5d^5}-\frac{4b^3(bc-ad)(c+dx)^1}{11d^5}$$

Mathematica [B] time = 0.049573, size = 473, normalized size = 3.97

$$\frac{1}{10}b^2d^5x^{10}\left(6a^2d^2+28abcd+21b^2c^2\right)+\frac{1}{9}bd^4x^9\left(42a^2bcd^2+4a^3d^3+84ab^2c^2d+35b^3c^3\right)+\frac{1}{8}d^3x^8\left(126a^2b^2c^2d^2+28a^3b\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^7,x]
```

```
[Out] a^4*c^7*x + (a^3*c^6*(4*b*c + 7*a*d)*x^2)/2 + (a^2*c^5*(6*b^2*c^2 + 28*a*b*
c*d + 21*a^2*d^2)*x^3)/3 + (a*c^4*(4*b^3*c^3 + 42*a*b^2*c^2*d + 84*a^2*b*c*
d^2 + 35*a^3*d^3)*x^4)/4 + (c^3*(b^4*c^4 + 28*a*b^3*c^3*d + 126*a^2*b^2*c^2
*d^2 + 140*a^3*b*c*d^3 + 35*a^4*d^4)*x^5)/5 + (7*c^2*d*(b^4*c^4 + 12*a*b^3*
```

```
c^3*d + 30*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 + 3*a^4*d^4)*x^6)/6 + c*d^2*(3*
b^4*c^4 + 20*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)*x
^7 + (d^3*(35*b^4*c^4 + 140*a*b^3*c^3*d + 126*a^2*b^2*c^2*d^2 + 28*a^3*b*c*
d^3 + a^4*d^4)*x^8)/8 + (b*d^4*(35*b^3*c^3 + 84*a*b^2*c^2*d + 42*a^2*b*c*d^
2 + 4*a^3*d^3)*x^9)/9 + (b^2*d^5*(21*b^2*c^2 + 28*a*b*c*d + 6*a^2*d^2)*x^10
)/10 + (b^3*d^6*(7*b*c + 4*a*d)*x^11)/11 + (b^4*d^7*x^12)/12
```

Maple [B] time = 0.002, size = 493, normalized size = 4.1

$$\frac{b^4d^7x^{12}}{12}+\frac{\left(4\,ab^3d^7+7\,b^4cd^6\right)x^{11}}{11}+\frac{\left(6\,b^2a^2d^7+28\,ab^3cd^6+21\,b^4c^2d^5\right)x^{10}}{10}+\frac{\left(4\,a^3bd^7+42\,b^2a^2cd^6+84\,ab^3c^2d^5+3\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^7,x)
```

```
[Out] 1/12*b^4*d^7*x^12+1/11*(4*a*b^3*d^7+7*b^4*c*d^6)*x^11+1/10*(6*a^2*b^2*d^7+2
8*a*b^3*c*d^6+21*b^4*c^2*d^5)*x^10+1/9*(4*a^3*b*d^7+42*a^2*b^2*c*d^6+84*a*b
^3*c^2*d^5+35*b^4*c^3*d^4)*x^9+1/8*(a^4*d^7+28*a^3*b*c*d^6+126*a^2*b^2*c^2*
d^5+140*a*b^3*c^3*d^4+35*b^4*c^4*d^3)*x^8+1/7*(7*a^4*c*d^6+84*a^3*b*c^2*d^5
+210*a^2*b^2*c^3*d^4+140*a*b^3*c^4*d^3+21*b^4*c^5*d^2)*x^7+1/6*(21*a^4*c^2*
d^5+140*a^3*b*c^3*d^4+210*a^2*b^2*c^4*d^3+84*a*b^3*c^5*d^2+7*b^4*c^6*d)*x^6
+1/5*(35*a^4*c^3*d^4+140*a^3*b*c^4*d^3+126*a^2*b^2*c^5*d^2+28*a*b^3*c^6*d+b
^4*c^7)*x^5+1/4*(35*a^4*c^4*d^3+84*a^3*b*c^5*d^2+42*a^2*b^2*c^6*d+4*a*b^3*c
^7)*x^4+1/3*(21*a^4*c^5*d^2+28*a^3*b*c^6*d+6*a^2*b^2*c^7)*x^3+1/2*(7*a^4*c^
6*d+4*a^3*b*c^7)*x^2+a^4*c^7*x
```

Maxima [B] time = 0.983395, size = 660, normalized size = 5.55

$$\frac{1}{12}\,b^4d^7x^{12}+a^4c^7x+\frac{1}{11}\left(7\,b^4cd^6+4\,ab^3d^7\right)x^{11}+\frac{1}{10}\left(21\,b^4c^2d^5+28\,ab^3cd^6+6\,a^2b^2d^7\right)x^{10}+\frac{1}{9}\left(35\,b^4c^3d^4+84\,ab^3c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*d^7*x^12 + a^4*c^7*x + 1/11*(7*b^4*c*d^6 + 4*a*b^3*d^7)*x^11 + 1/1
0*(21*b^4*c^2*d^5 + 28*a*b^3*c*d^6 + 6*a^2*b^2*d^7)*x^10 + 1/9*(35*b^4*c^3*
d^4 + 84*a*b^3*c^2*d^5 + 42*a^2*b^2*c*d^6 + 4*a^3*b*d^7)*x^9 + 1/8*(35*b^4*
c^4*d^3 + 140*a*b^3*c^3*d^4 + 126*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^
7)*x^8 + (3*b^4*c^5*d^2 + 20*a*b^3*c^4*d^3 + 30*a^2*b^2*c^3*d^4 + 12*a^3*b*
c^2*d^5 + a^4*c*d^6)*x^7 + 7/6*(b^4*c^6*d + 12*a*b^3*c^5*d^2 + 30*a^2*b^2*c
^4*d^3 + 20*a^3*b*c^3*d^4 + 3*a^4*c^2*d^5)*x^6 + 1/5*(b^4*c^7 + 28*a*b^3*c^
6*d + 126*a^2*b^2*c^5*d^2 + 140*a^3*b*c^4*d^3 + 35*a^4*c^3*d^4)*x^5 + 1/4*(
4*a*b^3*c^7 + 42*a^2*b^2*c^6*d + 84*a^3*b*c^5*d^2 + 35*a^4*c^4*d^3)*x^4 + 1
/3*(6*a^2*b^2*c^7 + 28*a^3*b*c^6*d + 21*a^4*c^5*d^2)*x^3 + 1/2*(4*a^3*b*c^7
 + 7*a^4*c^6*d)*x^2
```

Fricas [B] time = 1.87938, size = 1184, normalized size = 9.95

$$\frac{1}{12}x^{12}d^7b^4+\frac{7}{11}x^{11}d^6cb^4+\frac{4}{11}x^{11}d^7b^3a+\frac{21}{10}x^{10}d^5c^2b^4+\frac{14}{5}x^{10}d^6cb^3a+\frac{3}{5}x^{10}d^7b^2a^2+\frac{35}{9}x^9d^4c^3b^4+\frac{28}{3}x^9d^5c^2b^3a+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*d^7*b^4 + 7/11*x^11*d^6*c*b^4 + 4/11*x^11*d^7*b^3*a + 21/10*x^10*
d^5*c^2*b^4 + 14/5*x^10*d^6*c*b^3*a + 3/5*x^10*d^7*b^2*a^2 + 35/9*x^9*d^4*c
^3*b^4 + 28/3*x^9*d^5*c^2*b^3*a + 14/3*x^9*d^6*c*b^2*a^2 + 4/9*x^9*d^7*b*a^
3 + 35/8*x^8*d^3*c^4*b^4 + 35/2*x^8*d^4*c^3*b^3*a + 63/4*x^8*d^5*c^2*b^2*a^
2 + 7/2*x^8*d^6*c*b*a^3 + 1/8*x^8*d^7*a^4 + 3*x^7*d^2*c^5*b^4 + 20*x^7*d^3*
c^4*b^3*a + 30*x^7*d^4*c^3*b^2*a^2 + 12*x^7*d^5*c^2*b*a^3 + x^7*d^6*c*a^4 +
 7/6*x^6*d*c^6*b^4 + 14*x^6*d^2*c^5*b^3*a + 35*x^6*d^3*c^4*b^2*a^2 + 70/3*x
^6*d^4*c^3*b*a^3 + 7/2*x^6*d^5*c^2*a^4 + 1/5*x^5*c^7*b^4 + 28/5*x^5*d*c^6*b
^3*a + 126/5*x^5*d^2*c^5*b^2*a^2 + 28*x^5*d^3*c^4*b*a^3 + 7*x^5*d^4*c^3*a^4
 + x^4*c^7*b^3*a + 21/2*x^4*d*c^6*b^2*a^2 + 21*x^4*d^2*c^5*b*a^3 + 35/4*x^4
*d^3*c^4*a^4 + 2*x^3*c^7*b^2*a^2 + 28/3*x^3*d*c^6*b*a^3 + 7*x^3*d^2*c^5*a^4
 + 2*x^2*c^7*b*a^3 + 7/2*x^2*d*c^6*a^4 + x*c^7*a^4
```

Sympy [B] time = 0.129943, size = 549, normalized size = 4.61

$$a^4c^7x + \frac{b^4d^7x^{12}}{12} + x^{11}\left(\frac{4ab^3d^7}{11} + \frac{7b^4cd^6}{11}\right) + x^{10}\left(\frac{3a^2b^2d^7}{5} + \frac{14ab^3cd^6}{5} + \frac{21b^4c^2d^5}{10}\right) + x^9\left(\frac{4a^3bd^7}{9} + \frac{14a^2b^2cd^6}{3} + \frac{28ab^3c}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**7,x)
```

```
[Out] a**4*c**7*x + b**4*d**7*x**12/12 + x**11*(4*a*b**3*d**7/11 + 7*b**4*c*d**6/
11) + x**10*(3*a**2*b**2*d**7/5 + 14*a*b**3*c*d**6/5 + 21*b**4*c**2*d**5/10
) + x**9*(4*a**3*b*d**7/9 + 14*a**2*b**2*c*d**6/3 + 28*a*b**3*c**2*d**5/3 +
 35*b**4*c**3*d**4/9) + x**8*(a**4*d**7/8 + 7*a**3*b*c*d**6/2 + 63*a**2*b**
2*c**2*d**5/4 + 35*a*b**3*c**3*d**4/2 + 35*b**4*c**4*d**3/8) + x**7*(a**4*c
*d**6 + 12*a**3*b*c**2*d**5 + 30*a**2*b**2*c**3*d**4 + 20*a*b**3*c**4*d**3
+ 3*b**4*c**5*d**2) + x**6*(7*a**4*c**2*d**5/2 + 70*a**3*b*c**3*d**4/3 + 35
*a**2*b**2*c**4*d**3 + 14*a*b**3*c**5*d**2 + 7*b**4*c**6*d/6) + x**5*(7*a**
4*c**3*d**4 + 28*a**3*b*c**4*d**3 + 126*a**2*b**2*c**5*d**2/5 + 28*a*b**3*c
**6*d/5 + b**4*c**7/5) + x**4*(35*a**4*c**4*d**3/4 + 21*a**3*b*c**5*d**2 +
21*a**2*b**2*c**6*d/2 + a*b**3*c**7) + x**3*(7*a**4*c**5*d**2 + 28*a**3*b*c
**6*d/3 + 2*a**2*b**2*c**7) + x**2*(7*a**4*c**6*d/2 + 2*a**3*b*c**7)
```

Giac [B] time = 1.07816, size = 737, normalized size = 6.19

$$\frac{1}{12}b^4d^7x^{12} + \frac{7}{11}b^4cd^6x^{11} + \frac{4}{11}ab^3d^7x^{11} + \frac{21}{10}b^4c^2d^5x^{10} + \frac{14}{5}ab^3cd^6x^{10} + \frac{3}{5}a^2b^2d^7x^{10} + \frac{35}{9}b^4c^3d^4x^9 + \frac{28}{3}ab^3c^2d^5x^9 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/12*b^4*d^7*x^12 + 7/11*b^4*c*d^6*x^11 + 4/11*a*b^3*d^7*x^11 + 21/10*b^4*c
^2*d^5*x^10 + 14/5*a*b^3*c*d^6*x^10 + 3/5*a^2*b^2*d^7*x^10 + 35/9*b^4*c^3*d
^4*x^9 + 28/3*a*b^3*c^2*d^5*x^9 + 14/3*a^2*b^2*c*d^6*x^9 + 4/9*a^3*b*d^7*x^
9 + 35/8*b^4*c^4*d^3*x^8 + 35/2*a*b^3*c^3*d^4*x^8 + 63/4*a^2*b^2*c^2*d^5*x^
8 + 7/2*a^3*b*c*d^6*x^8 + 1/8*a^4*d^7*x^8 + 3*b^4*c^5*d^2*x^7 + 20*a*b^3*c^
4*d^3*x^7 + 30*a^2*b^2*c^3*d^4*x^7 + 12*a^3*b*c^2*d^5*x^7 + a^4*c*d^6*x^7 +
 7/6*b^4*c^6*d*x^6 + 14*a*b^3*c^5*d^2*x^6 + 35*a^2*b^2*c^4*d^3*x^6 + 70/3*a
^3*b*c^3*d^4*x^6 + 7/2*a^4*c^2*d^5*x^6 + 1/5*b^4*c^7*x^5 + 28/5*a*b^3*c^6*d
```

```
*x^5 + 126/5*a^2*b^2*c^5*d^2*x^5 + 28*a^3*b*c^4*d^3*x^5 + 7*a^4*c^3*d^4*x^5
 + a*b^3*c^7*x^4 + 21/2*a^2*b^2*c^6*d*x^4 + 21*a^3*b*c^5*d^2*x^4 + 35/4*a^4
*c^4*d^3*x^4 + 2*a^2*b^2*c^7*x^3 + 28/3*a^3*b*c^6*d*x^3 + 7*a^4*c^5*d^2*x^3
 + 2*a^3*b*c^7*x^2 + 7/2*a^4*c^6*d*x^2 + a^4*c^7*x
```

3.1279 $\int (a+bx)^3(c+dx)^7\,dx$

Optimal. Leaf size=92

$$-\frac{3b^2(c+dx)^{10}(bc-ad)}{10d^4}+\frac{b(c+dx)^9(bc-ad)^2}{3d^4}-\frac{(c+dx)^8(bc-ad)^3}{8d^4}+\frac{b^3(c+dx)^{11}}{11d^4}$$

```
[Out] -((b*c - a*d)^3*(c + d*x)^8)/(8*d^4) + (b*(b*c - a*d)^2*(c + d*x)^9)/(3*d^4
) - (3*b^2*(b*c - a*d)*(c + d*x)^10)/(10*d^4) + (b^3*(c + d*x)^11)/(11*d^4)
```

Rubi [A] time = 0.218233, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^2(c+dx)^{10}(bc-ad)}{10d^4}+\frac{b(c+dx)^9(bc-ad)^2}{3d^4}-\frac{(c+dx)^8(bc-ad)^3}{8d^4}+\frac{b^3(c+dx)^{11}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^7,x]
```

```
[Out] -((b*c - a*d)^3*(c + d*x)^8)/(8*d^4) + (b*(b*c - a*d)^2*(c + d*x)^9)/(3*d^4
) - (3*b^2*(b*c - a*d)*(c + d*x)^10)/(10*d^4) + (b^3*(c + d*x)^11)/(11*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^7}{d^3}+\frac{3b(bc-ad)^2(c+dx)^8}{d^3}-\frac{3b^2(bc-ad)(c+dx)^9}{d^3}+\frac{b^3(c+dx)^{10}}{d^3}\right)dx\\ &= -\frac{(bc-ad)^3(c+dx)^8}{8d^4}+\frac{b(bc-ad)^2(c+dx)^9}{3d^4}-\frac{3b^2(bc-ad)(c+dx)^{10}}{10d^4}+\frac{b^3(c+dx)^{11}}{11d^4}\end{aligned}$$

Mathematica [B] time = 0.0409356, size = 360, normalized size = 3.91

$$\frac{1}{3}bd^5x^9\left(a^2d^2+7abcd+7b^2c^2\right)+\frac{1}{8}d^4x^8\left(21a^2bcd^2+a^3d^3+63ab^2c^2d+35b^3c^3\right)+cd^3x^7\left(9a^2bcd^2+a^3d^3+15ab^2c^2d+5\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^7,x]
```

```
[Out] a^3*c^7*x + (a^2*c^6*(3*b*c + 7*a*d)*x^2)/2 + a*c^5*(b^2*c^2 + 7*a*b*c*d +
7*a^2*d^2)*x^3 + (c^4*(b^3*c^3 + 21*a*b^2*c^2*d + 63*a^2*b*c*d^2 + 35*a^3*d
^3)*x^4)/4 + (7*c^3*d*(b^3*c^3 + 9*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 5*a^3*d^3
)*x^5)/5 + (7*c^2*d^2*(b^3*c^3 + 5*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*d^3)*x
^6)/2 + c*d^3*(5*b^3*c^3 + 15*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^7 +
(d^4*(35*b^3*c^3 + 63*a*b^2*c^2*d + 21*a^2*b*c*d^2 + a^3*d^3)*x^8)/8 + (b*d
```

```
^5*(7*b^2*c^2 + 7*a*b*c*d + a^2*d^2)*x^9)/3 + (b^2*d^6*(7*b*c + 3*a*d)*x^10
)/10 + (b^3*d^7*x^11)/11
```

Maple [B] time = 0.002, size = 385, normalized size = 4.2

$$\frac{b^3d^7x^{11}}{11}+\frac{\left(3\,ab^2d^7+7\,b^3cd^6\right)x^{10}}{10}+\frac{\left(3\,a^2bd^7+21\,ab^2cd^6+21\,b^3c^2d^5\right)x^9}{9}+\frac{\left(a^3d^7+21\,a^2bcd^6+63\,ab^2c^2d^5+35\,b^3c^3\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^7,x)
```

```
[Out] 1/11*b^3*d^7*x^11+1/10*(3*a*b^2*d^7+7*b^3*c*d^6)*x^10+1/9*(3*a^2*b*d^7+21*a
*b^2*c*d^6+21*b^3*c^2*d^5)*x^9+1/8*(a^3*d^7+21*a^2*b*c*d^6+63*a*b^2*c^2*d^5
+35*b^3*c^3*d^4)*x^8+1/7*(7*a^3*c*d^6+63*a^2*b*c^2*d^5+105*a*b^2*c^3*d^4+35
*b^3*c^4*d^3)*x^7+1/6*(21*a^3*c^2*d^5+105*a^2*b*c^3*d^4+105*a*b^2*c^4*d^3+2
1*b^3*c^5*d^2)*x^6+1/5*(35*a^3*c^3*d^4+105*a^2*b*c^4*d^3+63*a*b^2*c^5*d^2+7
*b^3*c^6*d)*x^5+1/4*(35*a^3*c^4*d^3+63*a^2*b*c^5*d^2+21*a*b^2*c^6*d+b^3*c^7
)*x^4+1/3*(21*a^3*c^5*d^2+21*a^2*b*c^6*d+3*a*b^2*c^7)*x^3+1/2*(7*a^3*c^6*d+
3*a^2*b*c^7)*x^2+a^3*c^7*x
```

Maxima [B] time = 0.964849, size = 508, normalized size = 5.52

$$\frac{1}{11}\,b^3d^7x^{11}+a^3c^7x+\frac{1}{10}\left(7\,b^3cd^6+3\,ab^2d^7\right)x^{10}+\frac{1}{3}\left(7\,b^3c^2d^5+7\,ab^2cd^6+a^2bd^7\right)x^9+\frac{1}{8}\left(35\,b^3c^3d^4+63\,ab^2c^2d^5+21\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d^7*x^11 + a^3*c^7*x + 1/10*(7*b^3*c*d^6 + 3*a*b^2*d^7)*x^10 + 1/3
*(7*b^3*c^2*d^5 + 7*a*b^2*c*d^6 + a^2*b*d^7)*x^9 + 1/8*(35*b^3*c^3*d^4 + 63
*a*b^2*c^2*d^5 + 21*a^2*b*c*d^6 + a^3*d^7)*x^8 + (5*b^3*c^4*d^3 + 15*a*b^2*
c^3*d^4 + 9*a^2*b*c^2*d^5 + a^3*c*d^6)*x^7 + 7/2*(b^3*c^5*d^2 + 5*a*b^2*c^4
*d^3 + 5*a^2*b*c^3*d^4 + a^3*c^2*d^5)*x^6 + 7/5*(b^3*c^6*d + 9*a*b^2*c^5*d^
2 + 15*a^2*b*c^4*d^3 + 5*a^3*c^3*d^4)*x^5 + 1/4*(b^3*c^7 + 21*a*b^2*c^6*d +
 63*a^2*b*c^5*d^2 + 35*a^3*c^4*d^3)*x^4 + (a*b^2*c^7 + 7*a^2*b*c^6*d + 7*a^
3*c^5*d^2)*x^3 + 1/2*(3*a^2*b*c^7 + 7*a^3*c^6*d)*x^2
```

Fricas [B] time = 2.01767, size = 913, normalized size = 9.92

$$\frac{1}{11}x^{11}d^7b^3+\frac{7}{10}x^{10}d^6cb^3+\frac{3}{10}x^{10}d^7b^2a+\frac{7}{3}x^9d^5c^2b^3+\frac{7}{3}x^9d^6cb^2a+\frac{1}{3}x^9d^7ba^2+\frac{35}{8}x^8d^4c^3b^3+\frac{63}{8}x^8d^5c^2b^2a+\frac{21}{8}x^8d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^7*b^3 + 7/10*x^10*d^6*c*b^3 + 3/10*x^10*d^7*b^2*a + 7/3*x^9*d^5
*c^2*b^3 + 7/3*x^9*d^6*c*b^2*a + 1/3*x^9*d^7*b*a^2 + 35/8*x^8*d^4*c^3*b^3 +
 63/8*x^8*d^5*c^2*b^2*a + 21/8*x^8*d^6*c*b*a^2 + 1/8*x^8*d^7*a^3 + 5*x^7*d^
3*c^4*b^3 + 15*x^7*d^4*c^3*b^2*a + 9*x^7*d^5*c^2*b*a^2 + x^7*d^6*c*a^3 + 7/
2*x^6*d^2*c^5*b^3 + 35/2*x^6*d^3*c^4*b^2*a + 35/2*x^6*d^4*c^3*b*a^2 + 7/2*x
```

```
^6*d^5*c^2*a^3 + 7/5*x^5*d*c^6*b^3 + 63/5*x^5*d^2*c^5*b^2*a + 21*x^5*d^3*c^
4*b*a^2 + 7*x^5*d^4*c^3*a^3 + 1/4*x^4*c^7*b^3 + 21/4*x^4*d*c^6*b^2*a + 63/4
*x^4*d^2*c^5*b*a^2 + 35/4*x^4*d^3*c^4*a^3 + x^3*c^7*b^2*a + 7*x^3*d*c^6*b*a
^2 + 7*x^3*d^2*c^5*a^3 + 3/2*x^2*c^7*b*a^2 + 7/2*x^2*d*c^6*a^3 + x*c^7*a^3
```

Sympy [B] time = 0.138434, size = 427, normalized size = 4.64

$$a^3c^7x + \frac{b^3d^7x^{11}}{11} + x^{10}\left(\frac{3ab^2d^7}{10} + \frac{7b^3cd^6}{10}\right) + x^9\left(\frac{a^2bd^7}{3} + \frac{7ab^2cd^6}{3} + \frac{7b^3c^2d^5}{3}\right) + x^8\left(\frac{a^3d^7}{8} + \frac{21a^2bcd^6}{8} + \frac{63ab^2c^2d^5}{8} + \frac{35}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**7,x)
```

```
[Out] a**3*c**7*x + b**3*d**7*x**11/11 + x**10*(3*a*b**2*d**7/10 + 7*b**3*c*d**6/
10) + x**9*(a**2*b*d**7/3 + 7*a*b**2*c*d**6/3 + 7*b**3*c**2*d**5/3) + x**8*
(a**3*d**7/8 + 21*a**2*b*c*d**6/8 + 63*a*b**2*c**2*d**5/8 + 35*b**3*c**3*d*
*4/8) + x**7*(a**3*c*d**6 + 9*a**2*b*c**2*d**5 + 15*a*b**2*c**3*d**4 + 5*b*
*3*c**4*d**3) + x**6*(7*a**3*c**2*d**5/2 + 35*a**2*b*c**3*d**4/2 + 35*a*b**
2*c**4*d**3/2 + 7*b**3*c**5*d**2/2) + x**5*(7*a**3*c**3*d**4 + 21*a**2*b*c*
*4*d**3 + 63*a*b**2*c**5*d**2/5 + 7*b**3*c**6*d/5) + x**4*(35*a**3*c**4*d**
3/4 + 63*a**2*b*c**5*d**2/4 + 21*a*b**2*c**6*d/4 + b**3*c**7/4) + x**3*(7*a
**3*c**5*d**2 + 7*a**2*b*c**6*d + a*b**2*c**7) + x**2*(7*a**3*c**6*d/2 + 3*
a**2*b*c**7/2)
```

Giac [B] time = 1.06532, size = 567, normalized size = 6.16

$$\frac{1}{11}b^3d^7x^{11} + \frac{7}{10}b^3cd^6x^{10} + \frac{3}{10}ab^2d^7x^{10} + \frac{7}{3}b^3c^2d^5x^9 + \frac{7}{3}ab^2cd^6x^9 + \frac{1}{3}a^2bd^7x^9 + \frac{35}{8}b^3c^3d^4x^8 + \frac{63}{8}ab^2c^2d^5x^8 + \frac{21}{8}a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/11*b^3*d^7*x^11 + 7/10*b^3*c*d^6*x^10 + 3/10*a*b^2*d^7*x^10 + 7/3*b^3*c^2
*d^5*x^9 + 7/3*a*b^2*c*d^6*x^9 + 1/3*a^2*b*d^7*x^9 + 35/8*b^3*c^3*d^4*x^8 +
 63/8*a*b^2*c^2*d^5*x^8 + 21/8*a^2*b*c*d^6*x^8 + 1/8*a^3*d^7*x^8 + 5*b^3*c^
4*d^3*x^7 + 15*a*b^2*c^3*d^4*x^7 + 9*a^2*b*c^2*d^5*x^7 + a^3*c*d^6*x^7 + 7/
2*b^3*c^5*d^2*x^6 + 35/2*a*b^2*c^4*d^3*x^6 + 35/2*a^2*b*c^3*d^4*x^6 + 7/2*a
^3*c^2*d^5*x^6 + 7/5*b^3*c^6*d*x^5 + 63/5*a*b^2*c^5*d^2*x^5 + 21*a^2*b*c^4*
d^3*x^5 + 7*a^3*c^3*d^4*x^5 + 1/4*b^3*c^7*x^4 + 21/4*a*b^2*c^6*d*x^4 + 63/4
*a^2*b*c^5*d^2*x^4 + 35/4*a^3*c^4*d^3*x^4 + a*b^2*c^7*x^3 + 7*a^2*b*c^6*d*x
^3 + 7*a^3*c^5*d^2*x^3 + 3/2*a^2*b*c^7*x^2 + 7/2*a^3*c^6*d*x^2 + a^3*c^7*x
```

3.1280 $\int (a+bx)^2(c+dx)^7\,dx$

Optimal. Leaf size=65

$$-\frac{2b(c+dx)^9(bc-ad)}{9d^3}+\frac{(c+dx)^8(bc-ad)^2}{8d^3}+\frac{b^2(c+dx)^{10}}{10d^3}$$

```
[Out] ((b*c - a*d)^2*(c + d*x)^8)/(8*d^3) - (2*b*(b*c - a*d)*(c + d*x)^9)/(9*d^3)
 + (b^2*(c + d*x)^10)/(10*d^3)
```

Rubi [A] time = 0.159342, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b(c+dx)^9(bc-ad)}{9d^3}+\frac{(c+dx)^8(bc-ad)^2}{8d^3}+\frac{b^2(c+dx)^{10}}{10d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^8)/(8*d^3) - (2*b*(b*c - a*d)*(c + d*x)^9)/(9*d^3)
 + (b^2*(c + d*x)^10)/(10*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^7}{d^2}-\frac{2b(bc-ad)(c+dx)^8}{d^2}+\frac{b^2(c+dx)^9}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^8}{8d^3}-\frac{2b(bc-ad)(c+dx)^9}{9d^3}+\frac{b^2(c+dx)^{10}}{10d^3}\end{aligned}$$

Mathematica [B] time = 0.0270711, size = 261, normalized size = 4.02

$$\frac{1}{8}d^5x^8\left(a^2d^2+14abcd+21b^2c^2\right)+cd^4x^7\left(a^2d^2+6abcd+5b^2c^2\right)+\frac{7}{6}c^2d^3x^6\left(3a^2d^2+10abcd+5b^2c^2\right)+\frac{7}{5}c^3d^2x^5\left(5a^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^7,x]
```

```
[Out] a^2*c^7*x + (a*c^6*(2*b*c + 7*a*d)*x^2)/2 + (c^5*(b^2*c^2 + 14*a*b*c*d + 21
*a^2*d^2)*x^3)/3 + (7*c^4*d*(b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*x^4)/4 + (7*c
^3*d^2*(3*b^2*c^2 + 10*a*b*c*d + 5*a^2*d^2)*x^5)/5 + (7*c^2*d^3*(5*b^2*c^2
+ 10*a*b*c*d + 3*a^2*d^2)*x^6)/6 + c*d^4*(5*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*
x^7 + (d^5*(21*b^2*c^2 + 14*a*b*c*d + a^2*d^2)*x^8)/8 + (b*d^6*(7*b*c + 2*a
```

```
*d)*x^9)/9 + (b^2*d^7*x^10)/10
```

Maple [B] time = 0.002, size = 277, normalized size = 4.3

$$\frac{b^2d^7x^{10}}{10}+\frac{\left(2\,abd^7+7\,b^2cd^6\right)x^9}{9}+\frac{\left(a^2d^7+14\,abcd^6+21\,b^2c^2d^5\right)x^8}{8}+\frac{\left(7\,a^2cd^6+42\,abc^2d^5+35\,b^2c^3d^4\right)x^7}{7}+\frac{\left(21\,a^2c^2d^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^7,x)
```

```
[Out] 1/10*b^2*d^7*x^10+1/9*(2*a*b*d^7+7*b^2*c*d^6)*x^9+1/8*(a^2*d^7+14*a*b*c*d^6
+21*b^2*c^2*d^5)*x^8+1/7*(7*a^2*c*d^6+42*a*b*c^2*d^5+35*b^2*c^3*d^4)*x^7+1/
6*(21*a^2*c^2*d^5+70*a*b*c^3*d^4+35*b^2*c^4*d^3)*x^6+1/5*(35*a^2*c^3*d^4+70
*a*b*c^4*d^3+21*b^2*c^5*d^2)*x^5+1/4*(35*a^2*c^4*d^3+42*a*b*c^5*d^2+7*b^2*c
^6*d)*x^4+1/3*(21*a^2*c^5*d^2+14*a*b*c^6*d+b^2*c^7)*x^3+1/2*(7*a^2*c^6*d+2*
a*b*c^7)*x^2+a^2*c^7*x
```

Maxima [B] time = 0.986425, size = 369, normalized size = 5.68

$$\frac{1}{10}\,b^2d^7x^{10}+a^2c^7x+\frac{1}{9}\left(7\,b^2cd^6+2\,abd^7\right)x^9+\frac{1}{8}\left(21\,b^2c^2d^5+14\,abcd^6+a^2d^7\right)x^8+\left(5\,b^2c^3d^4+6\,abc^2d^5+a^2cd^6\right)x^7+\frac{7}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^7*x^10 + a^2*c^7*x + 1/9*(7*b^2*c*d^6 + 2*a*b*d^7)*x^9 + 1/8*(21
*b^2*c^2*d^5 + 14*a*b*c*d^6 + a^2*d^7)*x^8 + (5*b^2*c^3*d^4 + 6*a*b*c^2*d^5
 + a^2*c*d^6)*x^7 + 7/6*(5*b^2*c^4*d^3 + 10*a*b*c^3*d^4 + 3*a^2*c^2*d^5)*x^
6 + 7/5*(3*b^2*c^5*d^2 + 10*a*b*c^4*d^3 + 5*a^2*c^3*d^4)*x^5 + 7/4*(b^2*c^6
*d + 6*a*b*c^5*d^2 + 5*a^2*c^4*d^3)*x^4 + 1/3*(b^2*c^7 + 14*a*b*c^6*d + 21*
a^2*c^5*d^2)*x^3 + 1/2*(2*a*b*c^7 + 7*a^2*c^6*d)*x^2
```

Fricas [B] time = 1.94814, size = 644, normalized size = 9.91

$$\frac{1}{10}x^{10}d^7b^2+\frac{7}{9}x^9d^6cb^2+\frac{2}{9}x^9d^7ba+\frac{21}{8}x^8d^5c^2b^2+\frac{7}{4}x^8d^6cba+\frac{1}{8}x^8d^7a^2+5x^7d^4c^3b^2+6x^7d^5c^2ba+x^7d^6ca^2+\frac{35}{6}x^6d^3c^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d^7*b^2 + 7/9*x^9*d^6*c*b^2 + 2/9*x^9*d^7*b*a + 21/8*x^8*d^5*c^2*
b^2 + 7/4*x^8*d^6*c*b*a + 1/8*x^8*d^7*a^2 + 5*x^7*d^4*c^3*b^2 + 6*x^7*d^5*c
^2*b*a + x^7*d^6*c*a^2 + 35/6*x^6*d^3*c^4*b^2 + 35/3*x^6*d^4*c^3*b*a + 7/2*
x^6*d^5*c^2*a^2 + 21/5*x^5*d^2*c^5*b^2 + 14*x^5*d^3*c^4*b*a + 7*x^5*d^4*c^3
*a^2 + 7/4*x^4*d*c^6*b^2 + 21/2*x^4*d^2*c^5*b*a + 35/4*x^4*d^3*c^4*a^2 + 1/
3*x^3*c^7*b^2 + 14/3*x^3*d*c^6*b*a + 7*x^3*d^2*c^5*a^2 + x^2*c^7*b*a + 7/2*
x^2*d*c^6*a^2 + x*c^7*a^2
```

Sympy [B] time = 0.105788, size = 303, normalized size = 4.66

$$a^2c^7x + \frac{b^2d^7x^{10}}{10} + x^9\left(\frac{2abd^7}{9} + \frac{7b^2cd^6}{9}\right) + x^8\left(\frac{a^2d^7}{8} + \frac{7abcd^6}{4} + \frac{21b^2c^2d^5}{8}\right) + x^7\left(a^2cd^6 + 6abc^2d^5 + 5b^2c^3d^4\right) + x^6\left(\frac{7a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**7,x)
```

```
[Out] a**2*c**7*x + b**2*d**7*x**10/10 + x**9*(2*a*b*d**7/9 + 7*b**2*c*d**6/9) +
x**8*(a**2*d**7/8 + 7*a*b*c*d**6/4 + 21*b**2*c**2*d**5/8) + x**7*(a**2*c*d*
*6 + 6*a*b*c**2*d**5 + 5*b**2*c**3*d**4) + x**6*(7*a**2*c**2*d**5/2 + 35*a*
b*c**3*d**4/3 + 35*b**2*c**4*d**3/6) + x**5*(7*a**2*c**3*d**4 + 14*a*b*c**4
*d**3 + 21*b**2*c**5*d**2/5) + x**4*(35*a**2*c**4*d**3/4 + 21*a*b*c**5*d**2
/2 + 7*b**2*c**6*d/4) + x**3*(7*a**2*c**5*d**2 + 14*a*b*c**6*d/3 + b**2*c**
7/3) + x**2*(7*a**2*c**6*d/2 + a*b*c**7)
```

Giac [B] time = 1.04615, size = 397, normalized size = 6.11

$$\frac{1}{10}b^2d^7x^{10} + \frac{7}{9}b^2cd^6x^9 + \frac{2}{9}abd^7x^9 + \frac{21}{8}b^2c^2d^5x^8 + \frac{7}{4}abcd^6x^8 + \frac{1}{8}a^2d^7x^8 + 5b^2c^3d^4x^7 + 6abc^2d^5x^7 + a^2cd^6x^7 + \frac{35}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^7*x^10 + 7/9*b^2*c*d^6*x^9 + 2/9*a*b*d^7*x^9 + 21/8*b^2*c^2*d^5*
x^8 + 7/4*a*b*c*d^6*x^8 + 1/8*a^2*d^7*x^8 + 5*b^2*c^3*d^4*x^7 + 6*a*b*c^2*d
^5*x^7 + a^2*c*d^6*x^7 + 35/6*b^2*c^4*d^3*x^6 + 35/3*a*b*c^3*d^4*x^6 + 7/2*
a^2*c^2*d^5*x^6 + 21/5*b^2*c^5*d^2*x^5 + 14*a*b*c^4*d^3*x^5 + 7*a^2*c^3*d^4
*x^5 + 7/4*b^2*c^6*d*x^4 + 21/2*a*b*c^5*d^2*x^4 + 35/4*a^2*c^4*d^3*x^4 + 1/
3*b^2*c^7*x^3 + 14/3*a*b*c^6*d*x^3 + 7*a^2*c^5*d^2*x^3 + a*b*c^7*x^2 + 7/2*
a^2*c^6*d*x^2 + a^2*c^7*x
```

3.1281 $\int (a+bx)(c+dx)^7\,dx$

Optimal. Leaf size=38

$$\frac{b(c+dx)^9}{9d^2}-\frac{(c+dx)^8(bc-ad)}{8d^2}$$

```
[Out] -((b*c - a*d)*(c + d*x)^8)/(8*d^2) + (b*(c + d*x)^9)/(9*d^2)
```

Rubi [A] time = 0.0164614, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^9}{9d^2}-\frac{(c+dx)^8(bc-ad)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^7,x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^8)/(8*d^2) + (b*(c + d*x)^9)/(9*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^7}{d}+\frac{b(c+dx)^8}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^8}{8d^2}+\frac{b(c+dx)^9}{9d^2}\end{aligned}$$

Mathematica [B] time = 0.0170292, size = 151, normalized size = 3.97

$$\frac{7}{6}c^2d^4x^6(3ad+5bc)+7c^3d^3x^5(ad+bc)+\frac{7}{4}c^4d^2x^4(5ad+3bc)+\frac{7}{3}c^5dx^3(3ad+bc)+\frac{1}{2}c^6x^2(7ad+bc)+\frac{1}{8}d^6x^8(ad+7bc)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^7,x]
```

```
[Out] a*c^7*x + (c^6*(b*c + 7*a*d)*x^2)/2 + (7*c^5*d*(b*c + 3*a*d)*x^3)/3 + (7*c^
4*d^2*(3*b*c + 5*a*d)*x^4)/4 + 7*c^3*d^3*(b*c + a*d)*x^5 + (7*c^2*d^4*(5*b*
c + 3*a*d)*x^6)/6 + c*d^5*(3*b*c + a*d)*x^7 + (d^6*(7*b*c + a*d)*x^8)/8 + (
b*d^7*x^9)/9
```

Maple [B] time = 0.001, size = 169, normalized size = 4.5

$$\frac{bd^7x^9}{9}+\frac{\left(ad^7+7\,bcd^6\right)x^8}{8}+\frac{\left(7\,acd^6+21\,bc^2d^5\right)x^7}{7}+\frac{\left(21\,ac^2d^5+35\,bc^3d^4\right)x^6}{6}+\frac{\left(35\,ac^3d^4+35\,bc^4d^3\right)x^5}{5}+\frac{(35\,ac^4d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^7,x)
```

```
[Out] 1/9*b*d^7*x^9+1/8*(a*d^7+7*b*c*d^6)*x^8+1/7*(7*a*c*d^6+21*b*c^2*d^5)*x^7+1/
6*(21*a*c^2*d^5+35*b*c^3*d^4)*x^6+1/5*(35*a*c^3*d^4+35*b*c^4*d^3)*x^5+1/4*(
35*a*c^4*d^3+21*b*c^5*d^2)*x^4+1/3*(21*a*c^5*d^2+7*b*c^6*d)*x^3+1/2*(7*a*c^
6*d+b*c^7)*x^2+a*c^7*x
```

Maxima [B] time = 0.945336, size = 220, normalized size = 5.79

$$\frac{1}{9}\,bd^7x^9+ac^7x+\frac{1}{8}\left(7\,bcd^6+ad^7\right)x^8+\left(3\,bc^2d^5+acd^6\right)x^7+\frac{7}{6}\left(5\,bc^3d^4+3\,ac^2d^5\right)x^6+7\left(bc^4d^3+ac^3d^4\right)x^5+\frac{7}{4}\left(3\,bc^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/9*b*d^7*x^9 + a*c^7*x + 1/8*(7*b*c*d^6 + a*d^7)*x^8 + (3*b*c^2*d^5 + a*c*
d^6)*x^7 + 7/6*(5*b*c^3*d^4 + 3*a*c^2*d^5)*x^6 + 7*(b*c^4*d^3 + a*c^3*d^4)*
x^5 + 7/4*(3*b*c^5*d^2 + 5*a*c^4*d^3)*x^4 + 7/3*(b*c^6*d + 3*a*c^5*d^2)*x^3
 + 1/2*(b*c^7 + 7*a*c^6*d)*x^2
```

Fricas [B] time = 1.85742, size = 378, normalized size = 9.95

$$\frac{1}{9}x^9d^7b+\frac{7}{8}x^8d^6cb+\frac{1}{8}x^8d^7a+3x^7d^5c^2b+x^7d^6ca+\frac{35}{6}x^6d^4c^3b+\frac{7}{2}x^6d^5c^2a+7x^5d^3c^4b+7x^5d^4c^3a+\frac{21}{4}x^4d^2c^5b+\frac{35}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^7*b + 7/8*x^8*d^6*c*b + 1/8*x^8*d^7*a + 3*x^7*d^5*c^2*b + x^7*d^6
*c*a + 35/6*x^6*d^4*c^3*b + 7/2*x^6*d^5*c^2*a + 7*x^5*d^3*c^4*b + 7*x^5*d^4
*c^3*a + 21/4*x^4*d^2*c^5*b + 35/4*x^4*d^3*c^4*a + 7/3*x^3*d*c^6*b + 7*x^3*
d^2*c^5*a + 1/2*x^2*c^7*b + 7/2*x^2*d*c^6*a + x*c^7*a
```

Sympy [B] time = 0.089849, size = 178, normalized size = 4.68

$$ac^7x+\frac{bd^7x^9}{9}+x^8\left(\frac{ad^7}{8}+\frac{7bcd^6}{8}\right)+x^7\left(acd^6+3bc^2d^5\right)+x^6\left(\frac{7ac^2d^5}{2}+\frac{35bc^3d^4}{6}\right)+x^5\left(7ac^3d^4+7bc^4d^3\right)+x^4\left(\frac{35ac^4}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**7,x)
```

```
[Out] a*c**7*x + b*d**7*x**9/9 + x**8*(a*d**7/8 + 7*b*c*d**6/8) + x**7*(a*c*d**6
+ 3*b*c**2*d**5) + x**6*(7*a*c**2*d**5/2 + 35*b*c**3*d**4/6) + x**5*(7*a*c*
*3*d**4 + 7*b*c**4*d**3) + x**4*(35*a*c**4*d**3/4 + 21*b*c**5*d**2/4) + x**
3*(7*a*c**5*d**2 + 7*b*c**6*d/3) + x**2*(7*a*c**6*d/2 + b*c**7/2)
```

Giac [B] time = 1.07522, size = 228, normalized size = 6.

$$\frac{1}{9}bd^7x^9 + \frac{7}{8}bcd^6x^8 + \frac{1}{8}ad^7x^8 + 3\,bc^2d^5x^7 + acd^6x^7 + \frac{35}{6}bc^3d^4x^6 + \frac{7}{2}ac^2d^5x^6 + 7\,bc^4d^3x^5 + 7\,ac^3d^4x^5 + \frac{21}{4}bc^5d^2x^4 + \frac{3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/9*b*d^7*x^9 + 7/8*b*c*d^6*x^8 + 1/8*a*d^7*x^8 + 3*b*c^2*d^5*x^7 + a*c*d^6
*x^7 + 35/6*b*c^3*d^4*x^6 + 7/2*a*c^2*d^5*x^6 + 7*b*c^4*d^3*x^5 + 7*a*c^3*d
^4*x^5 + 21/4*b*c^5*d^2*x^4 + 35/4*a*c^4*d^3*x^4 + 7/3*b*c^6*d*x^3 + 7*a*c^
5*d^2*x^3 + 1/2*b*c^7*x^2 + 7/2*a*c^6*d*x^2 + a*c^7*x
```

3.1282 $\int (c+dx)^7\,dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^8}{8d}$$

```
[Out] (c + d*x)^8/(8*d)
```

Rubi [A] time = 0.0015101, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7,x]
```

```
[Out] (c + d*x)^8/(8*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^7\,dx = \frac{(c+dx)^8}{8d}$$

Mathematica [A] time = 0.0015981, size = 14, normalized size = 1.

$$\frac{(c+dx)^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7,x]
```

```
[Out] (c + d*x)^8/(8*d)
```

Maple [A] time = 0., size = 13, normalized size = 0.9

$$\frac{(dx+c)^8}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7,x)
```

```
[Out] 1/8*(d*x+c)^8/d
```

Maxima [A] time = 0.943771, size = 16, normalized size = 1.14

$$\frac{(dx+c)^8}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x + c)^8/d
```

Fricas [B] time = 1.97808, size = 159, normalized size = 11.36

$$\frac{1}{8}x^8d^7 + x^7d^6c + \frac{7}{2}x^6d^5c^2 + 7x^5d^4c^3 + \frac{35}{4}x^4d^3c^4 + 7x^3d^2c^5 + \frac{7}{2}x^2dc^6 + xc^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d^7 + x^7*d^6*c + 7/2*x^6*d^5*c^2 + 7*x^5*d^4*c^3 + 35/4*x^4*d^3*c^
4 + 7*x^3*d^2*c^5 + 7/2*x^2*d*c^6 + x*c^7
```

Sympy [B] time = 0.070815, size = 83, normalized size = 5.93

$$c^7x + \frac{7c^6dx^2}{2} + 7c^5d^2x^3 + \frac{35c^4d^3x^4}{4} + 7c^3d^4x^5 + \frac{7c^2d^5x^6}{2} + cd^6x^7 + \frac{d^7x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7,x)
```

```
[Out] c**7*x + 7*c**6*d*x**2/2 + 7*c**5*d**2*x**3 + 35*c**4*d**3*x**4/4 + 7*c**3*
d**4*x**5 + 7*c**2*d**5*x**6/2 + c*d**6*x**7 + d**7*x**8/8
```

Giac [A] time = 1.04431, size = 16, normalized size = 1.14

$$\frac{(dx+c)^8}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x + c)^8/d
```

3.1283 $\int \frac{(c+dx)^7}{a+bx}\,dx$

Optimal. Leaf size=169

$$\frac{dx(bc-ad)^6}{b^7}+\frac{(c+dx)^2(bc-ad)^5}{2b^6}+\frac{(c+dx)^3(bc-ad)^4}{3b^5}+\frac{(c+dx)^4(bc-ad)^3}{4b^4}+\frac{(c+dx)^5(bc-ad)^2}{5b^3}+\frac{(c+dx)^6(bc-}{6b^2}$$

```
[Out] (d*(b*c - a*d)^6*x)/b^7 + ((b*c - a*d)^5*(c + d*x)^2)/(2*b^6) + ((b*c - a*d
)^4*(c + d*x)^3)/(3*b^5) + ((b*c - a*d)^3*(c + d*x)^4)/(4*b^4) + ((b*c - a*
d)^2*(c + d*x)^5)/(5*b^3) + ((b*c - a*d)*(c + d*x)^6)/(6*b^2) + (c + d*x)^7
/(7*b) + ((b*c - a*d)^7*Log[a + b*x])/b^8
```

Rubi [A] time = 0.0725765, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{dx(bc-ad)^6}{b^7}+\frac{(c+dx)^2(bc-ad)^5}{2b^6}+\frac{(c+dx)^3(bc-ad)^4}{3b^5}+\frac{(c+dx)^4(bc-ad)^3}{4b^4}+\frac{(c+dx)^5(bc-ad)^2}{5b^3}+\frac{(c+dx)^6(bc-}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^6*x)/b^7 + ((b*c - a*d)^5*(c + d*x)^2)/(2*b^6) + ((b*c - a*d
)^4*(c + d*x)^3)/(3*b^5) + ((b*c - a*d)^3*(c + d*x)^4)/(4*b^4) + ((b*c - a*
d)^2*(c + d*x)^5)/(5*b^3) + ((b*c - a*d)*(c + d*x)^6)/(6*b^2) + (c + d*x)^7
/(7*b) + ((b*c - a*d)^7*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{a+bx}\,dx &= \int \left(\frac{d(bc-ad)^6}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)}+\frac{d(bc-ad)^5(c+dx)}{b^6}+\frac{d(bc-ad)^4(c+dx)^2}{b^5}+\frac{d(bc-ad)^3(c+dx)^3}{b^4}\right.\\ &= \frac{d(bc-ad)^6x}{b^7}+\frac{(bc-ad)^5(c+dx)^2}{2b^6}+\frac{(bc-ad)^4(c+dx)^3}{3b^5}+\frac{(bc-ad)^3(c+dx)^4}{4b^4}+\frac{(bc-ad)^2(c+dx}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.142045, size = 304, normalized size = 1.8

$$\frac{dx\left(21a^2b^4d^2\left(140c^2d^2x^2+350c^3dx+700c^4+35cd^3x^3+4d^4x^4\right)-35a^3b^3d^3\left(126c^2dx+420c^3+28cd^2x^2+3d^3x^3\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x),x]
```

```
[Out] (d*x*(420*a^6*d^6 - 210*a^5*b*d^5*(14*c + d*x) + 70*a^4*b^2*d^4*(126*c^2 +
21*c*d*x + 2*d^2*x^2) - 35*a^3*b^3*d^3*(420*c^3 + 126*c^2*d*x + 28*c*d^2*x^
```

```
2 + 3*d^3*x^3) + 21*a^2*b^4*d^2*(700*c^4 + 350*c^3*d*x + 140*c^2*d^2*x^2 +
35*c*d^3*x^3 + 4*d^4*x^4) - 7*a*b^5*d*(1260*c^5 + 1050*c^4*d*x + 700*c^3*d^
2*x^2 + 315*c^2*d^3*x^3 + 84*c*d^4*x^4 + 10*d^5*x^5) + b^6*(2940*c^6 + 4410
*c^5*d*x + 4900*c^4*d^2*x^2 + 3675*c^3*d^3*x^3 + 1764*c^2*d^4*x^4 + 490*c*d
^5*x^5 + 60*d^6*x^6)))/(420*b^7) + ((b*c - a*d)^7*Log[a + b*x])/b^8
```

Maple [B] time = 0.006, size = 539, normalized size = 3.2

$$\frac{d^7x^3a^4}{3\,b^5}+\frac{35\,d^3x^3c^4}{3\,b}-\frac{d^7x^2a^5}{2\,b^6}+\frac{21\,d^2x^2c^5}{2\,b}-\frac{d^7x^6a}{6\,b^2}+\frac{7\,d^6x^6c}{6\,b}+\frac{d^7x^5a^2}{5\,b^3}+\frac{21\,d^5x^5c^2}{5\,b}-\frac{\ln(bx+a)\,a^7d^7}{b^8}+7\,\frac{dc^6x}{b}+\frac{a^6d^7x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a),x)
```

```
[Out] 1/3*d^7/b^5*x^3*a^4+35/3*d^3/b*x^3*c^4-1/2*d^7/b^6*x^2*a^5+21/2*d^2/b*x^2*c
^5-1/6*d^7/b^2*x^6*a+7/6*d^6/b*x^6*c+1/5*d^7/b^3*x^5*a^2+21/5*d^5/b*x^5*c^2
-1/b^8*ln(b*x+a)*a^7*d^7+7*d/b*c^6*x+d^7/b^7*a^6*x-1/4*d^7/b^4*x^4*a^3+35/4
*d^4/b*x^4*c^3-7/3*d^6/b^4*x^3*a^3*c+7*d^5/b^3*x^3*a^2*c^2-35/3*d^4/b^2*x^3
*a*c^3+7/4*d^6/b^3*x^4*a^2*c-21/4*d^5/b^2*x^4*a*c^2-7/5*d^6/b^2*x^5*a*c-21/
b^6*ln(b*x+a)*a^5*c^2*d^5+35/b^5*ln(b*x+a)*a^4*c^3*d^4-35/b^4*ln(b*x+a)*a^3
*c^4*d^3+21/b^3*ln(b*x+a)*a^2*c^5*d^2+7/b^7*ln(b*x+a)*a^6*c*d^6-21/2*d^5/b^
4*x^2*a^3*c^2-7/b^2*ln(b*x+a)*a*c^6*d-7*d^6/b^6*a^5*c*x-21*d^2/b^2*a*c^5*x+
7/2*d^6/b^5*x^2*a^4*c+35/2*d^4/b^3*x^2*a^2*c^3-35/2*d^3/b^2*x^2*a*c^4+35*d^
3/b^3*a^2*c^4*x+21*d^5/b^5*a^4*c^2*x-35*d^4/b^4*a^3*c^3*x+1/7*d^7/b*x^7+1/b
*ln(b*x+a)*c^7
```

Maxima [B] time = 0.96687, size = 621, normalized size = 3.67

$$\frac{60\,b^6d^7x^7+70\left(7\,b^6cd^6-ab^5d^7\right)x^6+84\left(21\,b^6c^2d^5-7\,ab^5cd^6+a^2b^4d^7\right)x^5+105\left(35\,b^6c^3d^4-21\,ab^5c^2d^5+7\,a^2b^4cd^6-a^3\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/420*(60*b^6*d^7*x^7 + 70*(7*b^6*c*d^6 - a*b^5*d^7)*x^6 + 84*(21*b^6*c^2*d
^5 - 7*a*b^5*c*d^6 + a^2*b^4*d^7)*x^5 + 105*(35*b^6*c^3*d^4 - 21*a*b^5*c^2*
d^5 + 7*a^2*b^4*c*d^6 - a^3*b^3*d^7)*x^4 + 140*(35*b^6*c^4*d^3 - 35*a*b^5*c
^3*d^4 + 21*a^2*b^4*c^2*d^5 - 7*a^3*b^3*c*d^6 + a^4*b^2*d^7)*x^3 + 210*(21*
b^6*c^5*d^2 - 35*a*b^5*c^4*d^3 + 35*a^2*b^4*c^3*d^4 - 21*a^3*b^3*c^2*d^5 +
7*a^4*b^2*c*d^6 - a^5*b*d^7)*x^2 + 420*(7*b^6*c^6*d - 21*a*b^5*c^5*d^2 + 35
*a^2*b^4*c^4*d^3 - 35*a^3*b^3*c^3*d^4 + 21*a^4*b^2*c^2*d^5 - 7*a^5*b*c*d^6
+ a^6*d^7)*x)/b^7 + (b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*
b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7
*d^7)*log(b*x + a)/b^8
```

Fricas [B] time = 2.32019, size = 952, normalized size = 5.63

$$\frac{60\,b^7d^7x^7+70\left(7\,b^7cd^6-ab^6d^7\right)x^6+84\left(21\,b^7c^2d^5-7\,ab^6cd^6+a^2b^5d^7\right)x^5+105\left(35\,b^7c^3d^4-21\,ab^6c^2d^5+7\,a^2b^5cd^6-a^3\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/420*(60*b^7*d^7*x^7 + 70*(7*b^7*c*d^6 - a*b^6*d^7)*x^6 + 84*(21*b^7*c^2*d
^5 - 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 105*(35*b^7*c^3*d^4 - 21*a*b^6*c^2*
d^5 + 7*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 140*(35*b^7*c^4*d^3 - 35*a*b^6*c
^3*d^4 + 21*a^2*b^5*c^2*d^5 - 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 210*(21*
b^7*c^5*d^2 - 35*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 - 21*a^3*b^4*c^2*d^5 +
7*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 420*(7*b^7*c^6*d - 21*a*b^6*c^5*d^2 +
35*a^2*b^5*c^4*d^3 - 35*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 - 7*a^5*b^2*c*
d^6 + a^6*b*d^7)*x + 420*(b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35
*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6
- a^7*d^7)*log(b*x + a))/b^8
```

Sympy [B] time = 0.968406, size = 384, normalized size = 2.27

$$\frac{d^7x^7}{7b} - \frac{x^6\left(ad^7 - 7bcd^6\right)}{6b^2} + \frac{x^5\left(a^2d^7 - 7abcd^6 + 21b^2c^2d^5\right)}{5b^3} - \frac{x^4\left(a^3d^7 - 7a^2bcd^6 + 21ab^2c^2d^5 - 35b^3c^3d^4\right)}{4b^4} + \frac{x^3\left(a^4d^7 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a),x)
```

```
[Out] d**7*x**7/(7*b) - x**6*(a*d**7 - 7*b*c*d**6)/(6*b**2) + x**5*(a**2*d**7 - 7
*a*b*c*d**6 + 21*b**2*c**2*d**5)/(5*b**3) - x**4*(a**3*d**7 - 7*a**2*b*c*d*
*6 + 21*a*b**2*c**2*d**5 - 35*b**3*c**3*d**4)/(4*b**4) + x**3*(a**4*d**7 -
7*a**3*b*c*d**6 + 21*a**2*b**2*c**2*d**5 - 35*a*b**3*c**3*d**4 + 35*b**4*c*
*4*d**3)/(3*b**5) - x**2*(a**5*d**7 - 7*a**4*b*c*d**6 + 21*a**3*b**2*c**2*d
**5 - 35*a**2*b**3*c**3*d**4 + 35*a*b**4*c**4*d**3 - 21*b**5*c**5*d**2)/(2*
b**6) + x*(a**6*d**7 - 7*a**5*b*c*d**6 + 21*a**4*b**2*c**2*d**5 - 35*a**3*b
**3*c**3*d**4 + 35*a**2*b**4*c**4*d**3 - 21*a*b**5*c**5*d**2 + 7*b**6*c**6*
d)/b**7 - (a*d - b*c)**7*log(a + b*x)/b**8
```

Giac [B] time = 1.04438, size = 671, normalized size = 3.97

$$\frac{60\,b^6d^7x^7 + 490\,b^6cd^6x^6 - 70\,ab^5d^7x^6 + 1764\,b^6c^2d^5x^5 - 588\,ab^5cd^6x^5 + 84\,a^2b^4d^7x^5 + 3675\,b^6c^3d^4x^4 - 2205\,ab^5c^2d^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/420*(60*b^6*d^7*x^7 + 490*b^6*c*d^6*x^6 - 70*a*b^5*d^7*x^6 + 1764*b^6*c^2
*d^5*x^5 - 588*a*b^5*c*d^6*x^5 + 84*a^2*b^4*d^7*x^5 + 3675*b^6*c^3*d^4*x^4
- 2205*a*b^5*c^2*d^5*x^4 + 735*a^2*b^4*c*d^6*x^4 - 105*a^3*b^3*d^7*x^4 + 49
00*b^6*c^4*d^3*x^3 - 4900*a*b^5*c^3*d^4*x^3 + 2940*a^2*b^4*c^2*d^5*x^3 - 98
0*a^3*b^3*c*d^6*x^3 + 140*a^4*b^2*d^7*x^3 + 4410*b^6*c^5*d^2*x^2 - 7350*a*b
^5*c^4*d^3*x^2 + 7350*a^2*b^4*c^3*d^4*x^2 - 4410*a^3*b^3*c^2*d^5*x^2 + 1470
*a^4*b^2*c*d^6*x^2 - 210*a^5*b*d^7*x^2 + 2940*b^6*c^6*d*x - 8820*a*b^5*c^5*
d^2*x + 14700*a^2*b^4*c^4*d^3*x - 14700*a^3*b^3*c^3*d^4*x + 8820*a^4*b^2*c^
2*d^5*x - 2940*a^5*b*c*d^6*x + 420*a^6*d^7*x)/b^7 + (b^7*c^7 - 7*a*b^6*c^6*
d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b
^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)*log(abs(b*x + a))/b^8
```

3.1284 $\int \frac{(c+dx)^7}{(a+bx)^2} dx$

Optimal. Leaf size=187

$$\frac{7d^6(a+bx)^5(bc-ad)}{5b^8}+\frac{21d^5(a+bx)^4(bc-ad)^2}{4b^8}+\frac{35d^4(a+bx)^3(bc-ad)^3}{3b^8}+\frac{35d^3(a+bx)^2(bc-ad)^4}{2b^8}+\frac{21d^2x(bc-ad)^5}{b^7}$$

```
[Out] (21*d^2*(b*c - a*d)^5*x)/b^7 - (b*c - a*d)^7/(b^8*(a + b*x)) + (35*d^3*(b*c
 - a*d)^4*(a + b*x)^2)/(2*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^3)/(3*b^8)
 + (21*d^5*(b*c - a*d)^2*(a + b*x)^4)/(4*b^8) + (7*d^6*(b*c - a*d)*(a + b*x
)^5)/(5*b^8) + (d^7*(a + b*x)^6)/(6*b^8) + (7*d*(b*c - a*d)^6*Log[a + b*x])
/b^8
```

Rubi [A] time = 0.234759, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^5(bc-ad)}{5b^8}+\frac{21d^5(a+bx)^4(bc-ad)^2}{4b^8}+\frac{35d^4(a+bx)^3(bc-ad)^3}{3b^8}+\frac{35d^3(a+bx)^2(bc-ad)^4}{2b^8}+\frac{21d^2x(bc-ad)^5}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^2,x]
```


```
[Out] (21*d^2*(b*c - a*d)^5*x)/b^7 - (b*c - a*d)^7/(b^8*(a + b*x)) + (35*d^3*(b*c
 - a*d)^4*(a + b*x)^2)/(2*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^3)/(3*b^8)
 + (21*d^5*(b*c - a*d)^2*(a + b*x)^4)/(4*b^8) + (7*d^6*(b*c - a*d)*(a + b*x
)^5)/(5*b^8) + (d^7*(a + b*x)^6)/(6*b^8) + (7*d*(b*c - a*d)^6*Log[a + b*x])
/b^8
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^2}dx &= \int \left(\frac{21d^2(bc-ad)^5}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^2}+\frac{7d(bc-ad)^6}{b^7(a+bx)}+\frac{35d^3(bc-ad)^4(a+bx)}{b^7}+\frac{35d^4(bc-ad)^3(a+bx)^2}{b^7}\right.\\ &= \frac{21d^2(bc-ad)^5x}{b^7}-\frac{(bc-ad)^7}{b^8(a+bx)}+\frac{35d^3(bc-ad)^4(a+bx)^2}{2b^8}+\frac{35d^4(bc-ad)^3(a+bx)^3}{3b^8}+\frac{21d^5(bc-ad)^2(a}{4b^8}\end{aligned}$$

Mathematica [B] time = 0.119815, size = 388, normalized size = 2.07

$$\frac{21a^2b^5d^2\left(200c^3d^2x^2+50c^2d^3x^3-200c^4dx-60c^5+10cd^4x^4+d^5x^5\right)-35a^3b^4d^3\left(90c^2d^2x^2-180c^3dx-60c^4+12cd^3x^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^2,x]
```

```
[Out] (60*a^7*d^7 - 60*a^6*b*d^6*(7*c + 6*d*x) + 210*a^5*b^2*d^5*(6*c^2 + 10*c*d*
x - d^2*x^2) + 70*a^4*b^3*d^4*(-30*c^3 - 72*c^2*d*x + 18*c*d^2*x^2 + d^3*x^
3) - 35*a^3*b^4*d^3*(-60*c^4 - 180*c^3*d*x + 90*c^2*d^2*x^2 + 12*c*d^3*x^3
+ d^4*x^4) + 21*a^2*b^5*d^2*(-60*c^5 - 200*c^4*d*x + 200*c^3*d^2*x^2 + 50*c
^2*d^3*x^3 + 10*c*d^4*x^4 + d^5*x^5) - 7*a*b^6*d*(-60*c^6 - 180*c^5*d*x + 4
50*c^4*d^2*x^2 + 200*c^3*d^3*x^3 + 75*c^2*d^4*x^4 + 18*c*d^5*x^5 + 2*d^6*x^
6) + b^7*(-60*c^7 + 1260*c^5*d^2*x^2 + 1050*c^4*d^3*x^3 + 700*c^3*d^4*x^4 +
 315*c^2*d^5*x^5 + 84*c*d^6*x^6 + 10*d^7*x^7) + 420*d*(b*c - a*d)^6*(a + b*
x)*Log[a + b*x])/(60*b^8*(a + b*x))
```

Maple [B] time = 0.011, size = 571, normalized size = 3.1

$$-\frac{c^7}{b\,(bx+a)}+\frac{d^7x^6}{6\,b^2}-\frac{2\,d^7x^5a}{5\,b^3}+\frac{7\,d^6x^5c}{5\,b^2}+\frac{3\,d^7x^4a^2}{4\,b^4}+\frac{21\,d^5x^4c^2}{4\,b^2}-\frac{4\,d^7x^3a^3}{3\,b^5}+\frac{35\,d^4x^3c^3}{3\,b^2}+\frac{5\,d^7x^2a^4}{2\,b^6}+\frac{35\,d^3x^2c^4}{2\,b^2}-6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^2,x)
```

```
[Out] -1/b/(b*x+a)*c^7+1/6*d^7/b^2*x^6-2/5*d^7/b^3*x^5*a+7/5*d^6/b^2*x^5*c+3/4*d^
7/b^4*x^4*a^2+21/4*d^5/b^2*x^4*c^2-4/3*d^7/b^5*x^3*a^3+35/3*d^4/b^2*x^3*c^3
+5/2*d^7/b^6*x^2*a^4+35/2*d^3/b^2*x^2*c^4-6*d^7/b^7*a^5*x+21*d^2/b^2*c^5*x+
1/b^8/(b*x+a)*a^7*d^7+7/b^8*d^7*ln(b*x+a)*a^6+7/b^2*d*ln(b*x+a)*c^6+63/2*d^
5/b^4*x^2*a^2*c^2-35*d^4/b^3*x^2*a*c^3-35/b^5/(b*x+a)*a^4*c^3*d^4+35/b^4/(b
*x+a)*a^3*c^4*d^3-21/b^3/(b*x+a)*a^2*c^5*d^2+7/b^2/(b*x+a)*a*c^6*d-14*d^5/b
^3*x^3*a*c^2-14*d^6/b^5*x^2*a^3*c-42/b^3*d^2*ln(b*x+a)*a*c^5+35*d^6/b^6*a^4
*c*x-84*d^5/b^5*a^3*c^2*x+105*d^4/b^4*a^2*c^3*x-70*d^3/b^3*a*c^4*x-7/b^7/(b
*x+a)*a^6*c*d^6+21/b^6/(b*x+a)*a^5*c^2*d^5+105/b^4*d^3*ln(b*x+a)*a^2*c^4-42
/b^7*d^6*ln(b*x+a)*a^5*c+105/b^6*d^5*ln(b*x+a)*a^4*c^2-140/b^5*d^4*ln(b*x+a
)*a^3*c^3-7/2*d^6/b^3*x^4*a*c+7*d^6/b^4*x^3*a^2*c
```

Maxima [B] time = 0.977581, size = 630, normalized size = 3.37

$$-\frac{b^7c^7-7\,ab^6c^6d+21\,a^2b^5c^5d^2-35\,a^3b^4c^4d^3+35\,a^4b^3c^3d^4-21\,a^5b^2c^2d^5+7\,a^6bcd^6-a^7d^7}{b^9x+ab^8}+\frac{10\,b^5d^7x^6+12\left(7\,b^5cd^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^
4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(b^9*x + a*b^
8) + 1/60*(10*b^5*d^7*x^6 + 12*(7*b^5*c*d^6 - 2*a*b^4*d^7)*x^5 + 15*(21*b^5
*c^2*d^5 - 14*a*b^4*c*d^6 + 3*a^2*b^3*d^7)*x^4 + 20*(35*b^5*c^3*d^4 - 42*a*
b^4*c^2*d^5 + 21*a^2*b^3*c*d^6 - 4*a^3*b^2*d^7)*x^3 + 30*(35*b^5*c^4*d^3 -
70*a*b^4*c^3*d^4 + 63*a^2*b^3*c^2*d^5 - 28*a^3*b^2*c*d^6 + 5*a^4*b*d^7)*x^2
 + 60*(21*b^5*c^5*d^2 - 70*a*b^4*c^4*d^3 + 105*a^2*b^3*c^3*d^4 - 84*a^3*b^2
*c^2*d^5 + 35*a^4*b*c*d^6 - 6*a^5*d^7)*x)/b^7 + 7*(b^6*c^6*d - 6*a*b^5*c^5*
d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*
b*c*d^6 + a^6*d^7)*log(b*x + a)/b^8
```

Fricas [B] time = 2.29378, size = 1310, normalized size = 7.01

$$\frac{10\,b^7d^7x^7 - 60\,b^7c^7 + 420\,ab^6c^6d - 1260\,a^2b^5c^5d^2 + 2100\,a^3b^4c^4d^3 - 2100\,a^4b^3c^3d^4 + 1260\,a^5b^2c^2d^5 - 420\,a^6bcd^6 + 60\,a^7 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^7*d^7*x^7 - 60*b^7*c^7 + 420*a*b^6*c^6*d - 1260*a^2*b^5*c^5*d^2
+ 2100*a^3*b^4*c^4*d^3 - 2100*a^4*b^3*c^3*d^4 + 1260*a^5*b^2*c^2*d^5 - 420*
a^6*b*c*d^6 + 60*a^7*d^7 + 14*(6*b^7*c*d^6 - a*b^6*d^7)*x^6 + 21*(15*b^7*c^
2*d^5 - 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 35*(20*b^7*c^3*d^4 - 15*a*b^6*c^
2*d^5 + 6*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 70*(15*b^7*c^4*d^3 - 20*a*b^6*
c^3*d^4 + 15*a^2*b^5*c^2*d^5 - 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 210*(6*
b^7*c^5*d^2 - 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 - 15*a^3*b^4*c^2*d^5 +
6*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 60*(21*a*b^6*c^5*d^2 - 70*a^2*b^5*c^4*
d^3 + 105*a^3*b^4*c^3*d^4 - 84*a^4*b^3*c^2*d^5 + 35*a^5*b^2*c*d^6 - 6*a^6*b
*d^7)*x + 420*(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^
4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d -
 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2
*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)*log(b*x + a))/(b^9*x + a*b^8)
```

Sympy [B] time = 1.73995, size = 410, normalized size = 2.19

$$\frac{a^7d^7 - 7a^6bcd^6 + 21a^5b^2c^2d^5 - 35a^4b^3c^3d^4 + 35a^3b^4c^4d^3 - 21a^2b^5c^5d^2 + 7ab^6c^6d - b^7c^7}{ab^8 + b^9x} + \frac{d^7x^6}{6b^2} - \frac{x^5\left(2ad^7 - 7bcd^6\right)}{5b^3} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**2,x)
```

```
[Out] (a**7*d**7 - 7*a**6*b*c*d**6 + 21*a**5*b**2*c**2*d**5 - 35*a**4*b**3*c**3*d
**4 + 35*a**3*b**4*c**4*d**3 - 21*a**2*b**5*c**5*d**2 + 7*a*b**6*c**6*d - b
**7*c**7)/(a*b**8 + b**9*x) + d**7*x**6/(6*b**2) - x**5*(2*a*d**7 - 7*b*c*d
**6)/(5*b**3) + x**4*(3*a**2*d**7 - 14*a*b*c*d**6 + 21*b**2*c**2*d**5)/(4*b
**4) - x**3*(4*a**3*d**7 - 21*a**2*b*c*d**6 + 42*a*b**2*c**2*d**5 - 35*b**3
*c**3*d**4)/(3*b**5) + x**2*(5*a**4*d**7 - 28*a**3*b*c*d**6 + 63*a**2*b**2*
c**2*d**5 - 70*a*b**3*c**3*d**4 + 35*b**4*c**4*d**3)/(2*b**6) - x*(6*a**5*d
**7 - 35*a**4*b*c*d**6 + 84*a**3*b**2*c**2*d**5 - 105*a**2*b**3*c**3*d**4 +
 70*a*b**4*c**4*d**3 - 21*b**5*c**5*d**2)/b**7 + 7*d*(a*d - b*c)**6*log(a +
 b*x)/b**8
```

Giac [B] time = 1.07483, size = 765, normalized size = 4.09

$$\frac{\left(10\,d^7 + \frac{84\left(b^2cd^6 - abd^7\right)}{(bx+a)b} + \frac{315\left(b^4c^2d^5 - 2\,ab^3cd^6 + a^2b^2d^7\right)}{(bx+a)^2b^2} + \frac{700\left(b^6c^3d^4 - 3\,ab^5c^2d^5 + 3\,a^2b^4cd^6 - a^3b^3d^7\right)}{(bx+a)^3b^3} + \frac{1050\left(b^8c^4d^3 - 4\,ab^7c^3d^4 + 6\,a^2b^6c^2d^5 - 4\,a^3b^5cd \cdots\right)}{(bx+a)^4b^4} \cdots\right.}{60\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*d^7 + 84*(b^2*c*d^6 - a*b*d^7)/((b*x + a)*b) + 315*(b^4*c^2*d^5 -
2*a*b^3*c*d^6 + a^2*b^2*d^7)/((b*x + a)^2*b^2) + 700*(b^6*c^3*d^4 - 3*a*b^5
*c^2*d^5 + 3*a^2*b^4*c*d^6 - a^3*b^3*d^7)/((b*x + a)^3*b^3) + 1050*(b^8*c^4
*d^3 - 4*a*b^7*c^3*d^4 + 6*a^2*b^6*c^2*d^5 - 4*a^3*b^5*c*d^6 + a^4*b^4*d^7)
/((b*x + a)^4*b^4) + 1260*(b^10*c^5*d^2 - 5*a*b^9*c^4*d^3 + 10*a^2*b^8*c^3*
d^4 - 10*a^3*b^7*c^2*d^5 + 5*a^4*b^6*c*d^6 - a^5*b^5*d^7)/((b*x + a)^5*b^5)
)*(b*x + a)^6/b^8 - 7*(b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 2
0*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*log(abs(b
*x + a)/((b*x + a)^2*abs(b)))/b^8 - (b^13*c^7/(b*x + a) - 7*a*b^12*c^6*d/(b
*x + a) + 21*a^2*b^11*c^5*d^2/(b*x + a) - 35*a^3*b^10*c^4*d^3/(b*x + a) + 3
5*a^4*b^9*c^3*d^4/(b*x + a) - 21*a^5*b^8*c^2*d^5/(b*x + a) + 7*a^6*b^7*c*d^
6/(b*x + a) - a^7*b^6*d^7/(b*x + a))/b^14
```

3.1285 $\int \frac{(c+dx)^7}{(a+bx)^3} dx$

Optimal. Leaf size=185

$$\frac{7d^6(a+bx)^4(bc-ad)}{4b^8}+\frac{7d^5(a+bx)^3(bc-ad)^2}{b^8}+\frac{35d^4(a+bx)^2(bc-ad)^3}{2b^8}+\frac{35d^3x(bc-ad)^4}{b^7}+\frac{21d^2(bc-ad)^5\log(a+b$$

```
[Out] (35*d^3*(b*c - a*d)^4*x)/b^7 - (b*c - a*d)^7/(2*b^8*(a + b*x)^2) - (7*d*(b*
c - a*d)^6)/(b^8*(a + b*x)) + (35*d^4*(b*c - a*d)^3*(a + b*x)^2)/(2*b^8) +
(7*d^5*(b*c - a*d)^2*(a + b*x)^3)/b^8 + (7*d^6*(b*c - a*d)*(a + b*x)^4)/(4*
b^8) + (d^7*(a + b*x)^5)/(5*b^8) + (21*d^2*(b*c - a*d)^5*Log[a + b*x])/b^8
```

Rubi [A] time = 0.21712, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^4(bc-ad)}{4b^8}+\frac{7d^5(a+bx)^3(bc-ad)^2}{b^8}+\frac{35d^4(a+bx)^2(bc-ad)^3}{2b^8}+\frac{35d^3x(bc-ad)^4}{b^7}+\frac{21d^2(bc-ad)^5\log(a+b$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^3,x]
```


```
[Out] (35*d^3*(b*c - a*d)^4*x)/b^7 - (b*c - a*d)^7/(2*b^8*(a + b*x)^2) - (7*d*(b*
c - a*d)^6)/(b^8*(a + b*x)) + (35*d^4*(b*c - a*d)^3*(a + b*x)^2)/(2*b^8) +
(7*d^5*(b*c - a*d)^2*(a + b*x)^3)/b^8 + (7*d^6*(b*c - a*d)*(a + b*x)^4)/(4*
b^8) + (d^7*(a + b*x)^5)/(5*b^8) + (21*d^2*(b*c - a*d)^5*Log[a + b*x])/b^8
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^3}dx = \int \left(\frac{35d^3(bc-ad)^4}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^3}+\frac{7d(bc-ad)^6}{b^7(a+bx)^2}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)}+\frac{35d^4(bc-ad)^3(a+bx)}{b^7}+\frac{21d^5(b$$

$$=\frac{35d^3(bc-ad)^4x}{b^7}-\frac{(bc-ad)^7}{2b^8(a+bx)^2}-\frac{7d(bc-ad)^6}{b^8(a+bx)}+\frac{35d^4(bc-ad)^3(a+bx)^2}{2b^8}+\frac{7d^5(bc-ad)^2(a+bx)^3}{b^8}+$$

Mathematica [B] time = 0.129932, size = 389, normalized size = 2.1

$$7a^2b^5d^2\left(-550c^3d^2x^2+200c^2d^3x^3-200c^4dx+90c^5+25cd^4x^4+2d^5x^5\right)-35a^3b^4d^3\left(-126c^2d^2x^2-20c^3dx+50c^4+20c$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^3,x]
```

```
[Out] (-130*a^7*d^7 + 10*a^6*b*d^6*(77*c + 16*d*x) + 10*a^5*b^2*d^5*(-189*c^2 - 5
6*c*d*x + 50*d^2*x^2) + 70*a^4*b^3*d^4*(35*c^3 + 6*c^2*d*x - 34*c*d^2*x^2 +
 2*d^3*x^3) - 35*a^3*b^4*d^3*(50*c^4 - 20*c^3*d*x - 126*c^2*d^2*x^2 + 20*c*
d^3*x^3 + d^4*x^4) + 7*a^2*b^5*d^2*(90*c^5 - 200*c^4*d*x - 550*c^3*d^2*x^2
+ 200*c^2*d^3*x^3 + 25*c*d^4*x^4 + 2*d^5*x^5) - 7*a*b^6*d*(10*c^6 - 120*c^5
*d*x - 200*c^4*d^2*x^2 + 200*c^3*d^3*x^3 + 50*c^2*d^4*x^4 + 10*c*d^5*x^5 +
d^6*x^6) + b^7*(-10*c^7 - 140*c^6*d*x + 700*c^4*d^3*x^3 + 350*c^3*d^4*x^4 +
 140*c^2*d^5*x^5 + 35*c*d^6*x^6 + 4*d^7*x^7) - 420*d^2*(-(b*c) + a*d)^5*(a
+ b*x)^2*Log[a + b*x])/(20*b^8*(a + b*x)^2)
```

Maple [B] time = 0.011, size = 599, normalized size = 3.2

$$-\frac{c^7}{2\,b\,(bx+a)^2}+\frac{d^7x^5}{5\,b^3}-7\,\frac{a^6d^7}{b^8\,(bx+a)}-\frac{3\,d^7x^4a}{4\,b^4}+\frac{a^7d^7}{2\,b^8\,(bx+a)^2}-21\,\frac{d^7\ln(bx+a)\,a^5}{b^8}+21\,\frac{d^2\ln(bx+a)\,c^5}{b^3}-7\,\frac{d}{b^2\,(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^3,x)
```

```
[Out] -1/2/b/(b*x+a)^2*c^7+1/5*d^7/b^3*x^5-7/b^8*d^7/(b*x+a)*a^6-3/4*d^7/b^4*x^4*
a+1/2/b^8/(b*x+a)^2*a^7*d^7-21/b^8*d^7*ln(b*x+a)*a^5+21/b^3*d^2*ln(b*x+a)*c
^5-7/b^2*d/(b*x+a)*c^6+35*d^3/b^3*c^4*x+35/2*d^4/b^3*x^2*c^3+15*d^7/b^7*a^4
*x-5*d^7/b^6*x^2*a^3+7/4*d^6/b^3*x^4*c+2*d^7/b^5*x^3*a^2+7*d^5/b^3*x^3*c^2+
21*d^6/b^5*x^2*a^2*c+210/b^5*d^4*ln(b*x+a)*a^2*c^3-105/b^4*d^3*ln(b*x+a)*a*
c^4+42/b^7*d^6/(b*x+a)*a^5*c-105/b^6*d^5/(b*x+a)*a^4*c^2+140/b^5*d^4/(b*x+a
)*a^3*c^3-105/b^4*d^3/(b*x+a)*a^2*c^4+42/b^3*d^2/(b*x+a)*a*c^5-7/2/b^7/(b*x
+a)^2*a^6*c*d^6+21/2/b^6/(b*x+a)^2*a^5*c^2*d^5-35/2/b^5/(b*x+a)^2*a^4*c^3*d
^4+35/2/b^4/(b*x+a)^2*a^3*c^4*d^3-21/2/b^3/(b*x+a)^2*a^2*c^5*d^2+7/2/b^2/(b
*x+a)^2*a*c^6*d-63/2*d^5/b^4*x^2*a*c^2-70*d^6/b^6*a^3*c*x+126*d^5/b^5*a^2*c
^2*x-105*d^4/b^4*a*c^3*x-7*d^6/b^4*x^3*a*c-210/b^6*d^5*ln(b*x+a)*a^3*c^2+10
5/b^7*d^6*ln(b*x+a)*a^4*c
```

Maxima [B] time = 1.02106, size = 639, normalized size = 3.45

$$-\frac{b^7c^7+7\,ab^6c^6d-63\,a^2b^5c^5d^2+175\,a^3b^4c^4d^3-245\,a^4b^3c^3d^4+189\,a^5b^2c^2d^5-77\,a^6bcd^6+13\,a^7d^7+14\left(b^7c^6d-6\,ab\right.}{2\left(b^{10}x^2+2\,ab^9x+a^2b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^7*c^7 + 7*a*b^6*c^6*d - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 -
245*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 77*a^6*b*c*d^6 + 13*a^7*d^7 + 1
4*(b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 +
15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^10*x^2 + 2*a*b^9*x
+ a^2*b^8) + 1/20*(4*b^4*d^7*x^5 + 5*(7*b^4*c*d^6 - 3*a*b^3*d^7)*x^4 + 20*(
7*b^4*c^2*d^5 - 7*a*b^3*c*d^6 + 2*a^2*b^2*d^7)*x^3 + 10*(35*b^4*c^3*d^4 - 6
3*a*b^3*c^2*d^5 + 42*a^2*b^2*c*d^6 - 10*a^3*b*d^7)*x^2 + 20*(35*b^4*c^4*d^3
 - 105*a*b^3*c^3*d^4 + 126*a^2*b^2*c^2*d^5 - 70*a^3*b*c*d^6 + 15*a^4*d^7)*x
)/b^7 + 21*(b^5*c^5*d^2 - 5*a*b^4*c^4*d^3 + 10*a^2*b^3*c^3*d^4 - 10*a^3*b^2
*c^2*d^5 + 5*a^4*b*c*d^6 - a^5*d^7)*log(b*x + a)/b^8
```

Fricas [B] time = 2.38854, size = 1445, normalized size = 7.81

$$\frac{4\,b^{7}d^{7}x^{7} - 10\,b^{7}c^{7} - 70\,ab^{6}c^{6}d + 630\,a^{2}b^{5}c^{5}d^{2} - 1750\,a^{3}b^{4}c^{4}d^{3} + 2450\,a^{4}b^{3}c^{3}d^{4} - 1890\,a^{5}b^{2}c^{2}d^{5} + 770\,a^{6}bcd^{6} - 130\,a^{7}d^{7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*d^7*x^7 - 10*b^7*c^7 - 70*a*b^6*c^6*d + 630*a^2*b^5*c^5*d^2 - 1
750*a^3*b^4*c^4*d^3 + 2450*a^4*b^3*c^3*d^4 - 1890*a^5*b^2*c^2*d^5 + 770*a^6
*b*c*d^6 - 130*a^7*d^7 + 7*(5*b^7*c*d^6 - a*b^6*d^7)*x^6 + 14*(10*b^7*c^2*d
^5 - 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 35*(10*b^7*c^3*d^4 - 10*a*b^6*c^2*d
^5 + 5*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 140*(5*b^7*c^4*d^3 - 10*a*b^6*c^3
*d^4 + 10*a^2*b^5*c^2*d^5 - 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 10*(140*a*
b^6*c^4*d^3 - 385*a^2*b^5*c^3*d^4 + 441*a^3*b^4*c^2*d^5 - 238*a^4*b^3*c*d^6
 + 50*a^5*b^2*d^7)*x^2 - 20*(7*b^7*c^6*d - 42*a*b^6*c^5*d^2 + 70*a^2*b^5*c^
4*d^3 - 35*a^3*b^4*c^3*d^4 - 21*a^4*b^3*c^2*d^5 + 28*a^5*b^2*c*d^6 - 8*a^6*
b*d^7)*x + 420*(a^2*b^5*c^5*d^2 - 5*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 -
10*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 - a^7*d^7 + (b^7*c^5*d^2 - 5*a*b^6*c^4*d
^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^
7)*x^2 + 2*(a*b^6*c^5*d^2 - 5*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 - 10*a^4
*b^3*c^2*d^5 + 5*a^5*b^2*c*d^6 - a^6*b*d^7)*x)*log(b*x + a))/(b^10*x^2 + 2*
a*b^9*x + a^2*b^8)
```

Sympy [B] time = 3.63683, size = 437, normalized size = 2.36

$$-\frac{13a^{7}d^{7} - 77a^{6}bcd^{6} + 189a^{5}b^{2}c^{2}d^{5} - 245a^{4}b^{3}c^{3}d^{4} + 175a^{3}b^{4}c^{4}d^{3} - 63a^{2}b^{5}c^{5}d^{2} + 7ab^{6}c^{6}d + b^{7}c^{7} + x\left(14a^{6}bd^{7} - 84a^{5}b^{2}c\right.}{2a^{2}b^{8} + 4ab^{9}x + 2b^{10}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**3,x)
```

```
[Out] -(13*a**7*d**7 - 77*a**6*b*c*d**6 + 189*a**5*b**2*c**2*d**5 - 245*a**4*b**3
*c**3*d**4 + 175*a**3*b**4*c**4*d**3 - 63*a**2*b**5*c**5*d**2 + 7*a*b**6*c*
*6*d + b**7*c**7 + x*(14*a**6*b*d**7 - 84*a**5*b**2*c*d**6 + 210*a**4*b**3*
c**2*d**5 - 280*a**3*b**4*c**3*d**4 + 210*a**2*b**5*c**4*d**3 - 84*a*b**6*c
**5*d**2 + 14*b**7*c**6*d))/(2*a**2*b**8 + 4*a*b**9*x + 2*b**10*x**2) + d**
7*x**5/(5*b**3) - x**4*(3*a*d**7 - 7*b*c*d**6)/(4*b**4) + x**3*(2*a**2*d**7
 - 7*a*b*c*d**6 + 7*b**2*c**2*d**5)/b**5 - x**2*(10*a**3*d**7 - 42*a**2*b*c
*d**6 + 63*a*b**2*c**2*d**5 - 35*b**3*c**3*d**4)/(2*b**6) + x*(15*a**4*d**7
 - 70*a**3*b*c*d**6 + 126*a**2*b**2*c**2*d**5 - 105*a*b**3*c**3*d**4 + 35*b
**4*c**4*d**3)/b**7 - 21*d**2*(a*d - b*c)**5*log(a + b*x)/b**8
```

Giac [B] time = 1.06849, size = 644, normalized size = 3.48

$$\frac{21\left(b^{5}c^{5}d^{2} - 5\,ab^{4}c^{4}d^{3} + 10\,a^{2}b^{3}c^{3}d^{4} - 10\,a^{3}b^{2}c^{2}d^{5} + 5\,a^{4}bcd^{6} - a^{5}d^{7}\right)\log\left(|bx + a|\right)}{b^{8}} - \frac{b^{7}c^{7} + 7\,ab^{6}c^{6}d - 63\,a^{2}b^{5}c^{5}d^{2} + 17}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 21*(b^5*c^5*d^2 - 5*a*b^4*c^4*d^3 + 10*a^2*b^3*c^3*d^4 - 10*a^3*b^2*c^2*d^5
 + 5*a^4*b*c*d^6 - a^5*d^7)*log(abs(b*x + a))/b^8 - 1/2*(b^7*c^7 + 7*a*b^6*
c^6*d - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 - 245*a^4*b^3*c^3*d^4 + 18
9*a^5*b^2*c^2*d^5 - 77*a^6*b*c*d^6 + 13*a^7*d^7 + 14*(b^7*c^6*d - 6*a*b^6*c
^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a
^5*b^2*c*d^6 + a^6*b*d^7)*x)/((b*x + a)^2*b^8) + 1/20*(4*b^12*d^7*x^5 + 35*
b^12*c*d^6*x^4 - 15*a*b^11*d^7*x^4 + 140*b^12*c^2*d^5*x^3 - 140*a*b^11*c*d^
6*x^3 + 40*a^2*b^10*d^7*x^3 + 350*b^12*c^3*d^4*x^2 - 630*a*b^11*c^2*d^5*x^2
 + 420*a^2*b^10*c*d^6*x^2 - 100*a^3*b^9*d^7*x^2 + 700*b^12*c^4*d^3*x - 2100
*a*b^11*c^3*d^4*x + 2520*a^2*b^10*c^2*d^5*x - 1400*a^3*b^9*c*d^6*x + 300*a^
4*b^8*d^7*x)/b^15
```

3.1286 $\int \frac{(c+dx)^7}{(a+bx)^4} dx$

Optimal. Leaf size=187

$$\frac{7d^6(a+bx)^3(bc-ad)}{3b^8}+\frac{21d^5(a+bx)^2(bc-ad)^2}{2b^8}+\frac{35d^4x(bc-ad)^3}{b^7}-\frac{21d^2(bc-ad)^5}{b^8(a+bx)}+\frac{35d^3(bc-ad)^4\log(a+bx)}{b^8}-\frac{7d}{2b}$$

```
[Out] (35*d^4*(b*c - a*d)^3*x)/b^7 - (b*c - a*d)^7/(3*b^8*(a + b*x)^3) - (7*d*(b*
c - a*d)^6)/(2*b^8*(a + b*x)^2) - (21*d^2*(b*c - a*d)^5)/(b^8*(a + b*x)) +
(21*d^5*(b*c - a*d)^2*(a + b*x)^2)/(2*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^3
)/(3*b^8) + (d^7*(a + b*x)^4)/(4*b^8) + (35*d^3*(b*c - a*d)^4*Log[a + b*x])
/b^8
```

Rubi [A] time = 0.21263, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^3(bc-ad)}{3b^8}+\frac{21d^5(a+bx)^2(bc-ad)^2}{2b^8}+\frac{35d^4x(bc-ad)^3}{b^7}-\frac{21d^2(bc-ad)^5}{b^8(a+bx)}+\frac{35d^3(bc-ad)^4\log(a+bx)}{b^8}-\frac{7d}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^4,x]
```

```
[Out] (35*d^4*(b*c - a*d)^3*x)/b^7 - (b*c - a*d)^7/(3*b^8*(a + b*x)^3) - (7*d*(b*
c - a*d)^6)/(2*b^8*(a + b*x)^2) - (21*d^2*(b*c - a*d)^5)/(b^8*(a + b*x)) +
(21*d^5*(b*c - a*d)^2*(a + b*x)^2)/(2*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^3
)/(3*b^8) + (d^7*(a + b*x)^4)/(4*b^8) + (35*d^3*(b*c - a*d)^4*Log[a + b*x])
/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^4}dx &= \int \left(\frac{35d^4(bc-ad)^3}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^4}+\frac{7d(bc-ad)^6}{b^7(a+bx)^3}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^2}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)}+\frac{21d^5(bc-ad)^2}{b^7}\right.\\ &= \frac{35d^4(bc-ad)^3x}{b^7}-\frac{(bc-ad)^7}{3b^8(a+bx)^3}-\frac{7d(bc-ad)^6}{2b^8(a+bx)^2}-\frac{21d^2(bc-ad)^5}{b^8(a+bx)}+\frac{21d^5(bc-ad)^2(a+bx)^2}{2b^8}+\frac{7d^6(bc}{}\end{aligned}$$

Mathematica [A] time = 0.10457, size = 199, normalized size = 1.06

$$\frac{6b^2d^5x^2\left(10a^2d^2-28abcd+21b^2c^2\right)+12bd^4x\left(70a^2bcd^2-20a^3d^3-84ab^2c^2d+35b^3c^3\right)+4b^3d^6x^3(7bc-4ad)+\frac{252d^2(ad}{a+bx}}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^4,x]
```

```
[Out] (12*b*d^4*(35*b^3*c^3 - 84*a*b^2*c^2*d + 70*a^2*b*c*d^2 - 20*a^3*d^3)*x + 6
*b^2*d^5*(21*b^2*c^2 - 28*a*b*c*d + 10*a^2*d^2)*x^2 + 4*b^3*d^6*(7*b*c - 4*
a*d)*x^3 + 3*b^4*d^7*x^4 - (4*(b*c - a*d)^7)/(a + b*x)^3 - (42*d*(b*c - a*d
)^6)/(a + b*x)^2 + (252*d^2*(-(b*c) + a*d)^5)/(a + b*x) + 420*d^3*(b*c - a*
d)^4*Log[a + b*x])/(12*b^8)
```

Maple [B] time = 0.011, size = 622, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^4,x)
```

```
[Out] -1/3/b/(b*x+a)^3*c^7+1/4*d^7/b^4*x^4-4/3*d^7/b^5*x^3*a+7/3*d^6/b^4*x^3*c+5*
d^7/b^6*x^2*a^2+21/2*d^5/b^4*x^2*c^2-20*d^7/b^7*a^3*x+35*d^4/b^4*c^3*x+21/b
^8*d^7/(b*x+a)*a^5-21/b^3*d^2/(b*x+a)*c^5-7/2/b^8*d^7/(b*x+a)^2*a^6-7/2/b^2
*d/(b*x+a)^2*c^6+1/3/b^8/(b*x+a)^3*a^7*d^7+35/b^8*d^7*ln(b*x+a)*a^4+35/b^4*
d^3*ln(b*x+a)*c^4+21/b^3*d^2/(b*x+a)^2*a*c^5-7/3/b^7/(b*x+a)^3*a^6*c*d^6-14
0/b^5*d^4*ln(b*x+a)*a*c^3-7/b^3/(b*x+a)^3*a^2*c^5*d^2+7/3/b^2/(b*x+a)^3*a*c
^6*d-35/3/b^5/(b*x+a)^3*a^4*c^3*d^4+35/3/b^4/(b*x+a)^3*a^3*c^4*d^3+7/b^6/(b
*x+a)^3*a^5*c^2*d^5-14*d^6/b^5*x^2*a*c+70*d^6/b^6*a^2*c*x-84*d^5/b^5*a*c^2*
x-105/b^7*d^6/(b*x+a)*a^4*c+210/b^6*d^5/(b*x+a)*a^3*c^2-210/b^5*d^4/(b*x+a)
*a^2*c^3+105/b^4*d^3/(b*x+a)*a*c^4+21/b^7*d^6/(b*x+a)^2*a^5*c-105/2/b^6*d^5
/(b*x+a)^2*a^4*c^2+70/b^5*d^4/(b*x+a)^2*a^3*c^3-105/2/b^4*d^3/(b*x+a)^2*a^2
*c^4-140/b^7*d^6*ln(b*x+a)*a^3*c+210/b^6*d^5*ln(b*x+a)*a^2*c^2
```

Maxima [B] time = 1.02643, size = 653, normalized size = 3.49

$$-\frac{2\,b^7c^7 + 7\,ab^6c^6d + 42\,a^2b^5c^5d^2 - 385\,a^3b^4c^4d^3 + 910\,a^4b^3c^3d^4 - 987\,a^5b^2c^2d^5 + 518\,a^6bcd^6 - 107\,a^7d^7 + 126\,\left(b^7c^5d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^7*c^7 + 7*a*b^6*c^6*d + 42*a^2*b^5*c^5*d^2 - 385*a^3*b^4*c^4*d^3
+ 910*a^4*b^3*c^3*d^4 - 987*a^5*b^2*c^2*d^5 + 518*a^6*b*c*d^6 - 107*a^7*d^7
 + 126*(b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2
*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 21*(b^7*c^6*d + 6*a*b^6*c^5*d^2
 - 45*a^2*b^5*c^4*d^3 + 100*a^3*b^4*c^3*d^4 - 105*a^4*b^3*c^2*d^5 + 54*a^5*
b^2*c*d^6 - 11*a^6*b*d^7)*x)/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2*b^9*x + a^3*b
^8) + 1/12*(3*b^3*d^7*x^4 + 4*(7*b^3*c*d^6 - 4*a*b^2*d^7)*x^3 + 6*(21*b^3*c
^2*d^5 - 28*a*b^2*c*d^6 + 10*a^2*b*d^7)*x^2 + 12*(35*b^3*c^3*d^4 - 84*a*b^2
*c^2*d^5 + 70*a^2*b*c*d^6 - 20*a^3*d^7)*x)/b^7 + 35*(b^4*c^4*d^3 - 4*a*b^3*
c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7)*log(b*x + a)/b^8
```

Fricas [B] time = 2.33662, size = 1520, normalized size = 8.13

$$\frac{3\,b^7d^7x^7 - 4\,b^7c^7 - 14\,ab^6c^6d - 84\,a^2b^5c^5d^2 + 770\,a^3b^4c^4d^3 - 1820\,a^4b^3c^3d^4 + 1974\,a^5b^2c^2d^5 - 1036\,a^6bcd^6 + 214\,a^7d^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*d^7*x^7 - 4*b^7*c^7 - 14*a*b^6*c^6*d - 84*a^2*b^5*c^5*d^2 + 770
*a^3*b^4*c^4*d^3 - 1820*a^4*b^3*c^3*d^4 + 1974*a^5*b^2*c^2*d^5 - 1036*a^6*b
*c*d^6 + 214*a^7*d^7 + 7*(4*b^7*c*d^6 - a*b^6*d^7)*x^6 + 21*(6*b^7*c^2*d^5
- 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 105*(4*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 +
 4*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 2*(630*a*b^6*c^3*d^4 - 1323*a^2*b^5*c
^2*d^5 + 1022*a^3*b^4*c*d^6 - 278*a^4*b^3*d^7)*x^3 - 6*(42*b^7*c^5*d^2 - 21
0*a*b^6*c^4*d^3 + 210*a^2*b^5*c^3*d^4 + 63*a^3*b^4*c^2*d^5 - 182*a^4*b^3*c*
d^6 + 68*a^5*b^2*d^7)*x^2 - 6*(7*b^7*c^6*d + 42*a*b^6*c^5*d^2 - 315*a^2*b^5
*c^4*d^3 + 630*a^3*b^4*c^3*d^4 - 567*a^4*b^3*c^2*d^5 + 238*a^5*b^2*c*d^6 -
37*a^6*b*d^7)*x + 420*(a^3*b^4*c^4*d^3 - 4*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*
d^5 - 4*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^4*d^3 - 4*a*b^6*c^3*d^4 + 6*a^2*b^5*
c^2*d^5 - 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 3*(a*b^6*c^4*d^3 - 4*a^2*b^5
*c^3*d^4 + 6*a^3*b^4*c^2*d^5 - 4*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 3*(a^2*
b^5*c^4*d^3 - 4*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 - 4*a^5*b^2*c*d^6 + a^6
*b*d^7)*x)*log(b*x + a))/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2*b^9*x + a^3*b^8)
```

Sympy [B] time = 8.26844, size = 468, normalized size = 2.5

$$\frac{107a^7d^7 - 518a^6bcd^6 + 987a^5b^2c^2d^5 - 910a^4b^3c^3d^4 + 385a^3b^4c^4d^3 - 42a^2b^5c^5d^2 - 7ab^6c^6d - 2b^7c^7 + x^2\left(126a^5b^2d^7 - 63\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**4,x)
```

```
[Out] (107*a**7*d**7 - 518*a**6*b*c*d**6 + 987*a**5*b**2*c**2*d**5 - 910*a**4*b**
3*c**3*d**4 + 385*a**3*b**4*c**4*d**3 - 42*a**2*b**5*c**5*d**2 - 7*a*b**6*c
**6*d - 2*b**7*c**7 + x**2*(126*a**5*b**2*d**7 - 630*a**4*b**3*c*d**6 + 126
0*a**3*b**4*c**2*d**5 - 1260*a**2*b**5*c**3*d**4 + 630*a*b**6*c**4*d**3 - 1
26*b**7*c**5*d**2) + x*(231*a**6*b*d**7 - 1134*a**5*b**2*c*d**6 + 2205*a**4
*b**3*c**2*d**5 - 2100*a**3*b**4*c**3*d**4 + 945*a**2*b**5*c**4*d**3 - 126*
a*b**6*c**5*d**2 - 21*b**7*c**6*d))/(6*a**3*b**8 + 18*a**2*b**9*x + 18*a*b*
*10*x**2 + 6*b**11*x**3) + d**7*x**4/(4*b**4) - x**3*(4*a*d**7 - 7*b*c*d**6
)/(3*b**5) + x**2*(10*a**2*d**7 - 28*a*b*c*d**6 + 21*b**2*c**2*d**5)/(2*b**
6) - x*(20*a**3*d**7 - 70*a**2*b*c*d**6 + 84*a*b**2*c**2*d**5 - 35*b**3*c**
3*d**4)/b**7 + 35*d**3*(a*d - b*c)**4*log(a + b*x)/b**8
```

Giac [B] time = 1.0549, size = 635, normalized size = 3.4

$$\frac{35\left(b^4c^4d^3 - 4\,ab^3c^3d^4 + 6\,a^2b^2c^2d^5 - 4\,a^3bcd^6 + a^4d^7\right)\log\left(|bx + a|\right)}{b^8} - \frac{2\,b^7c^7 + 7\,ab^6c^6d + 42\,a^2b^5c^5d^2 - 385\,a^3b^4c^4d^3 + }{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 35*(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4
*d^7)*log(abs(b*x + a))/b^8 - 1/6*(2*b^7*c^7 + 7*a*b^6*c^6*d + 42*a^2*b^5*c
^5*d^2 - 385*a^3*b^4*c^4*d^3 + 910*a^4*b^3*c^3*d^4 - 987*a^5*b^2*c^2*d^5 +
518*a^6*b*c*d^6 - 107*a^7*d^7 + 126*(b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2
```

```
*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 21
*(b^7*c^6*d + 6*a*b^6*c^5*d^2 - 45*a^2*b^5*c^4*d^3 + 100*a^3*b^4*c^3*d^4 -
105*a^4*b^3*c^2*d^5 + 54*a^5*b^2*c*d^6 - 11*a^6*b*d^7)*x)/((b*x + a)^3*b^8)
 + 1/12*(3*b^12*d^7*x^4 + 28*b^12*c*d^6*x^3 - 16*a*b^11*d^7*x^3 + 126*b^12*
c^2*d^5*x^2 - 168*a*b^11*c*d^6*x^2 + 60*a^2*b^10*d^7*x^2 + 420*b^12*c^3*d^4
*x - 1008*a*b^11*c^2*d^5*x + 840*a^2*b^10*c*d^6*x - 240*a^3*b^9*d^7*x)/b^16
```

3.1287 $\int \frac{(c+dx)^7}{(a+bx)^5}\,dx$

Optimal. Leaf size=187

$$\frac{7d^6(a+bx)^2(bc-ad)}{2b^8}+\frac{21d^5x(bc-ad)^2}{b^7}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}+\frac{35d^4(bc-ad)^3\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}$$

```
[Out] (21*d^5*(b*c - a*d)^2*x)/b^7 - (b*c - a*d)^7/(4*b^8*(a + b*x)^4) - (7*d*(b*
c - a*d)^6)/(3*b^8*(a + b*x)^3) - (21*d^2*(b*c - a*d)^5)/(2*b^8*(a + b*x)^2
) - (35*d^3*(b*c - a*d)^4)/(b^8*(a + b*x)) + (7*d^6*(b*c - a*d)*(a + b*x)^2
)/(2*b^8) + (d^7*(a + b*x)^3)/(3*b^8) + (35*d^4*(b*c - a*d)^3*Log[a + b*x])
/b^8
```

Rubi [A] time = 0.197603, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^2(bc-ad)}{2b^8}+\frac{21d^5x(bc-ad)^2}{b^7}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}+\frac{35d^4(bc-ad)^3\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^5,x]
```

```
[Out] (21*d^5*(b*c - a*d)^2*x)/b^7 - (b*c - a*d)^7/(4*b^8*(a + b*x)^4) - (7*d*(b*
c - a*d)^6)/(3*b^8*(a + b*x)^3) - (21*d^2*(b*c - a*d)^5)/(2*b^8*(a + b*x)^2
) - (35*d^3*(b*c - a*d)^4)/(b^8*(a + b*x)) + (7*d^6*(b*c - a*d)*(a + b*x)^2
)/(2*b^8) + (d^7*(a + b*x)^3)/(3*b^8) + (35*d^4*(b*c - a*d)^3*Log[a + b*x])
/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^5}\,dx &= \int \left(\frac{21d^5(bc-ad)^2}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^5}+\frac{7d(bc-ad)^6}{b^7(a+bx)^4}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^3}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^2}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)}\right. \\ &= \frac{21d^5(bc-ad)^2x}{b^7}-\frac{(bc-ad)^7}{4b^8(a+bx)^4}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}+\frac{7d^6(bc-ad)(a+\cdots}{2b^8}\end{aligned}$$

Mathematica [A] time = 0.113926, size = 173, normalized size = 0.93

$$\frac{12bd^5x\left(15a^2d^2-35abcd+21b^2c^2\right)+6b^2d^6x^2(7bc-5ad)-\frac{420d^3(bc-ad)^4}{a+bx}+\frac{126d^2(ad-bc)^5}{(a+bx)^2}+420d^4(bc-ad)^3\log(a+bx)-\cdots}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^5,x]
```

```
[Out] (12*b*d^5*(21*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2)*x + 6*b^2*d^6*(7*b*c - 5*a
*d)*x^2 + 4*b^3*d^7*x^3 - (3*(b*c - a*d)^7)/(a + b*x)^4 - (28*d*(b*c - a*d)
^6)/(a + b*x)^3 + (126*d^2*(-(b*c) + a*d)^5)/(a + b*x)^2 - (420*d^3*(b*c -
a*d)^4)/(a + b*x) + 420*d^4*(b*c - a*d)^3*Log[a + b*x])/(12*b^8)
```

Maple [B] time = 0.011, size = 641, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^5,x)
```

```
[Out] -1/4/b/(b*x+a)^4*c^7+1/3*d^7/b^5*x^3-35*d^6/b^6*a*c*x-210/b^6*d^5/(b*x+a)*a
^2*c^2-5/2*d^7/b^6*x^2*a+7/2*d^6/b^5*x^2*c+15*d^7/b^7*a^2*x+21*d^5/b^5*c^2*
x-35/b^8*d^7/(b*x+a)*a^4-35/b^4*d^3/(b*x+a)*c^4+21/2/b^8*d^7/(b*x+a)^2*a^5-
21/2/b^3*d^2/(b*x+a)^2*c^5-7/3/b^8*d^7/(b*x+a)^3*a^6-7/3/b^2*d/(b*x+a)^3*c^
6-35/b^8*d^7*ln(b*x+a)*a^3+35/b^5*d^4*ln(b*x+a)*c^3+1/4/b^8/(b*x+a)^4*a^7*d
^7+14/b^3*d^2/(b*x+a)^3*a*c^5+35/4/b^4/(b*x+a)^4*a^3*c^4*d^3-21/4/b^3/(b*x+
a)^4*a^2*c^5*d^2+7/4/b^2/(b*x+a)^4*a*c^6*d+105/b^7*d^6*ln(b*x+a)*a^2*c-105/
b^6*d^5*ln(b*x+a)*a*c^2-7/4/b^7/(b*x+a)^4*a^6*c*d^6+21/4/b^6/(b*x+a)^4*a^5*
c^2*d^5-35/4/b^5/(b*x+a)^4*a^4*c^3*d^4+140/b^5*d^4/(b*x+a)*a*c^3-105/2/b^7*
d^6/(b*x+a)^2*a^4*c+105/b^6*d^5/(b*x+a)^2*a^3*c^2-105/b^5*d^4/(b*x+a)^2*a^2
*c^3+140/b^7*d^6/(b*x+a)*a^3*c+105/2/b^4*d^3/(b*x+a)^2*a*c^4+14/b^7*d^6/(b*
x+a)^3*a^5*c-35/b^6*d^5/(b*x+a)^3*a^4*c^2+140/3/b^5*d^4/(b*x+a)^3*a^3*c^3-3
5/b^4*d^3/(b*x+a)^3*a^2*c^4
```

Maxima [B] time = 1.11407, size = 667, normalized size = 3.57

$$-\frac{3\,b^7c^7 + 7\,ab^6c^6d + 21\,a^2b^5c^5d^2 + 105\,a^3b^4c^4d^3 - 875\,a^4b^3c^3d^4 + 1617\,a^5b^2c^2d^5 - 1197\,a^6bcd^6 + 319\,a^7d^7 + 420\,\left(b^7c^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^7*c^7 + 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3
 - 875*a^4*b^3*c^3*d^4 + 1617*a^5*b^2*c^2*d^5 - 1197*a^6*b*c*d^6 + 319*a^7*
d^7 + 420*(b^7*c^4*d^3 - 4*a*b^6*c^3*d^4 + 6*a^2*b^5*c^2*d^5 - 4*a^3*b^4*c*
d^6 + a^4*b^3*d^7)*x^3 + 126*(b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 - 30*a^2*b^5*c^
3*d^4 + 50*a^3*b^4*c^2*d^5 - 35*a^4*b^3*c*d^6 + 9*a^5*b^2*d^7)*x^2 + 28*(b^
7*c^6*d + 3*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 110*a^3*b^4*c^3*d^4 + 195*
a^4*b^3*c^2*d^5 - 141*a^5*b^2*c*d^6 + 37*a^6*b*d^7)*x)/(b^12*x^4 + 4*a*b^11
*x^3 + 6*a^2*b^10*x^2 + 4*a^3*b^9*x + a^4*b^8) + 1/6*(2*b^2*d^7*x^3 + 3*(7*
b^2*c*d^6 - 5*a*b*d^7)*x^2 + 6*(21*b^2*c^2*d^5 - 35*a*b*c*d^6 + 15*a^2*d^7)
*x)/b^7 + 35*(b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*log(
b*x + a)/b^8
```

Fricas [B] time = 2.32775, size = 1551, normalized size = 8.29

$$\frac{4\,b^{7}d^{7}x^{7} - 3\,b^{7}c^{7} - 7\,ab^{6}c^{6}d - 21\,a^{2}b^{5}c^{5}d^{2} - 105\,a^{3}b^{4}c^{4}d^{3} + 875\,a^{4}b^{3}c^{3}d^{4} - 1617\,a^{5}b^{2}c^{2}d^{5} + 1197\,a^{6}bcd^{6} - 319\,a^{7}d^{7} + 14\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^7*d^7*x^7 - 3*b^7*c^7 - 7*a*b^6*c^6*d - 21*a^2*b^5*c^5*d^2 - 105*
a^3*b^4*c^4*d^3 + 875*a^4*b^3*c^3*d^4 - 1617*a^5*b^2*c^2*d^5 + 1197*a^6*b*c
*d^6 - 319*a^7*d^7 + 14*(3*b^7*c*d^6 - a*b^6*d^7)*x^6 + 84*(3*b^7*c^2*d^5 -
 3*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 4*(252*a*b^6*c^2*d^5 - 357*a^2*b^5*c*d^
6 + 139*a^3*b^4*d^7)*x^4 - 4*(105*b^7*c^4*d^3 - 420*a*b^6*c^3*d^4 + 252*a^2
*b^5*c^2*d^5 + 168*a^3*b^4*c*d^6 - 136*a^4*b^3*d^7)*x^3 - 6*(21*b^7*c^5*d^2
 + 105*a*b^6*c^4*d^3 - 630*a^2*b^5*c^3*d^4 + 882*a^3*b^4*c^2*d^5 - 462*a^4*
b^3*c*d^6 + 74*a^5*b^2*d^7)*x^2 - 4*(7*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 105*a
^2*b^5*c^4*d^3 - 770*a^3*b^4*c^3*d^4 + 1302*a^4*b^3*c^2*d^5 - 882*a^5*b^2*c
*d^6 + 214*a^6*b*d^7)*x + 420*(a^4*b^3*c^3*d^4 - 3*a^5*b^2*c^2*d^5 + 3*a^6*
b*c*d^6 - a^7*d^7 + (b^7*c^3*d^4 - 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*
b^4*d^7)*x^4 + 4*(a*b^6*c^3*d^4 - 3*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 - a^4
*b^3*d^7)*x^3 + 6*(a^2*b^5*c^3*d^4 - 3*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*d^6 -
a^5*b^2*d^7)*x^2 + 4*(a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6
 - a^6*b*d^7)*x)*log(b*x + a))/(b^12*x^4 + 4*a*b^11*x^3 + 6*a^2*b^10*x^2 +
4*a^3*b^9*x + a^4*b^8)
```

Sympy [B] time = 26.0549, size = 495, normalized size = 2.65

$$-\frac{319a^{7}d^{7} - 1197a^{6}bcd^{6} + 1617a^{5}b^{2}c^{2}d^{5} - 875a^{4}b^{3}c^{3}d^{4} + 105a^{3}b^{4}c^{4}d^{3} + 21a^{2}b^{5}c^{5}d^{2} + 7ab^{6}c^{6}d + 3b^{7}c^{7} + x^{3}\left(420a^{4}b^{3}d^{7} -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**5,x)
```

```
[Out] -(319*a**7*d**7 - 1197*a**6*b*c*d**6 + 1617*a**5*b**2*c**2*d**5 - 875*a**4*
b**3*c**3*d**4 + 105*a**3*b**4*c**4*d**3 + 21*a**2*b**5*c**5*d**2 + 7*a*b**
6*c**6*d + 3*b**7*c**7 + x**3*(420*a**4*b**3*d**7 - 1680*a**3*b**4*c*d**6 +
 2520*a**2*b**5*c**2*d**5 - 1680*a*b**6*c**3*d**4 + 420*b**7*c**4*d**3) + x
**2*(1134*a**5*b**2*d**7 - 4410*a**4*b**3*c*d**6 + 6300*a**3*b**4*c**2*d**5
 - 3780*a**2*b**5*c**3*d**4 + 630*a*b**6*c**4*d**3 + 126*b**7*c**5*d**2) +
x*(1036*a**6*b*d**7 - 3948*a**5*b**2*c*d**6 + 5460*a**4*b**3*c**2*d**5 - 30
80*a**3*b**4*c**3*d**4 + 420*a**2*b**5*c**4*d**3 + 84*a*b**6*c**5*d**2 + 28
*b**7*c**6*d))/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b
**11*x**3 + 12*b**12*x**4) + d**7*x**3/(3*b**5) - x**2*(5*a*d**7 - 7*b*c*d*
*6)/(2*b**6) + x*(15*a**2*d**7 - 35*a*b*c*d**6 + 21*b**2*c**2*d**5)/b**7 -
35*d**4*(a*d - b*c)**3*log(a + b*x)/b**8
```

Giac [B] time = 1.08144, size = 891, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/6*(2*d^7 + 21*(b^2*c*d^6 - a*b*d^7)/((b*x + a)*b) + 126*(b^4*c^2*d^5 - 2*
a*b^3*c*d^6 + a^2*b^2*d^7)/((b*x + a)^2*b^2))*(b*x + a)^3/b^8 - 35*(b^3*c^3
*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*log(abs(b*x + a)/((b*x +
a)^2*abs(b)))/b^8 - 1/12*(3*b^43*c^7/(b*x + a)^4 + 28*b^42*c^6*d/(b*x + a)^
3 - 21*a*b^42*c^6*d/(b*x + a)^4 + 126*b^41*c^5*d^2/(b*x + a)^2 - 168*a*b^41
*c^5*d^2/(b*x + a)^3 + 63*a^2*b^41*c^5*d^2/(b*x + a)^4 + 420*b^40*c^4*d^3/(
b*x + a) - 630*a*b^40*c^4*d^3/(b*x + a)^2 + 420*a^2*b^40*c^4*d^3/(b*x + a)^
3 - 105*a^3*b^40*c^4*d^3/(b*x + a)^4 - 1680*a*b^39*c^3*d^4/(b*x + a) + 1260
*a^2*b^39*c^3*d^4/(b*x + a)^2 - 560*a^3*b^39*c^3*d^4/(b*x + a)^3 + 105*a^4*
b^39*c^3*d^4/(b*x + a)^4 + 2520*a^2*b^38*c^2*d^5/(b*x + a) - 1260*a^3*b^38*
c^2*d^5/(b*x + a)^2 + 420*a^4*b^38*c^2*d^5/(b*x + a)^3 - 63*a^5*b^38*c^2*d^
5/(b*x + a)^4 - 1680*a^3*b^37*c*d^6/(b*x + a) + 630*a^4*b^37*c*d^6/(b*x + a
)^2 - 168*a^5*b^37*c*d^6/(b*x + a)^3 + 21*a^6*b^37*c*d^6/(b*x + a)^4 + 420*
a^4*b^36*d^7/(b*x + a) - 126*a^5*b^36*d^7/(b*x + a)^2 + 28*a^6*b^36*d^7/(b*
x + a)^3 - 3*a^7*b^36*d^7/(b*x + a)^4)/b^44
```

3.1288 $\int \frac{(c+dx)^7}{(a+bx)^6} dx$

Optimal. Leaf size=181

$$\frac{d^6x(7bc-6ad)}{b^7}-\frac{35d^4(bc-ad)^3}{b^8(a+bx)}-\frac{35d^3(bc-ad)^4}{2b^8(a+bx)^2}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}+\frac{21d^5(bc-ad)^2\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{(bc-}{5b^8(a}$$

```
[Out] (d^6*(7*b*c - 6*a*d)*x)/b^7 + (d^7*x^2)/(2*b^6) - (b*c - a*d)^7/(5*b^8*(a +
 b*x)^5) - (7*d*(b*c - a*d)^6)/(4*b^8*(a + b*x)^4) - (7*d^2*(b*c - a*d)^5)/
(b^8*(a + b*x)^3) - (35*d^3*(b*c - a*d)^4)/(2*b^8*(a + b*x)^2) - (35*d^4*(b
*c - a*d)^3)/(b^8*(a + b*x)) + (21*d^5*(b*c - a*d)^2*Log[a + b*x])/b^8
```

Rubi [A] time = 0.191197, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^6x(7bc-6ad)}{b^7}-\frac{35d^4(bc-ad)^3}{b^8(a+bx)}-\frac{35d^3(bc-ad)^4}{2b^8(a+bx)^2}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}+\frac{21d^5(bc-ad)^2\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{(bc-}{5b^8(a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^6,x]
```

```
[Out] (d^6*(7*b*c - 6*a*d)*x)/b^7 + (d^7*x^2)/(2*b^6) - (b*c - a*d)^7/(5*b^8*(a +
 b*x)^5) - (7*d*(b*c - a*d)^6)/(4*b^8*(a + b*x)^4) - (7*d^2*(b*c - a*d)^5)/
(b^8*(a + b*x)^3) - (35*d^3*(b*c - a*d)^4)/(2*b^8*(a + b*x)^2) - (35*d^4*(b
*c - a*d)^3)/(b^8*(a + b*x)) + (21*d^5*(b*c - a*d)^2*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^6}dx &= \int \left(\frac{d^6(7bc-6ad)}{b^7}+\frac{d^7x}{b^6}+\frac{(bc-ad)^7}{b^7(a+bx)^6}+\frac{7d(bc-ad)^6}{b^7(a+bx)^5}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^4}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^3}+\frac{35d^4(bc}{b^7(a+}\right.\\ &= \frac{d^6(7bc-6ad)x}{b^7}+\frac{d^7x^2}{2b^6}-\frac{(bc-ad)^7}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}-\frac{35d^3(bc-ad)^4}{2b^8(a+bx)^2}-\frac{35d^4(bc-}{b^8(a+b}\end{aligned}$$

Mathematica [B] time = 0.148633, size = 389, normalized size = 2.15

$$\frac{-a^2b^5d^2\left(1400c^3d^2x^2-6300c^2d^3x^3+175c^4dx+14c^5+700cd^4x^4+500d^5x^5\right)-5a^3b^4d^3\left(-1540c^2d^2x^2+140c^3dx+7c^4+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^6,x]
```

```
[Out] (459*a^7*d^7 + 3*a^6*b*d^6*(-406*c + 625*d*x) + a^5*b^2*d^5*(959*c^2 - 5250
*c*d*x + 2700*d^2*x^2) + 5*a^4*b^3*d^4*(-28*c^3 + 875*c^2*d*x - 1680*c*d^2*
x^2 + 260*d^3*x^3) - 5*a^3*b^4*d^3*(7*c^4 + 140*c^3*d*x - 1540*c^2*d^2*x^2
+ 1120*c*d^3*x^3 + 80*d^4*x^4) - a^2*b^5*d^2*(14*c^5 + 175*c^4*d*x + 1400*c
^3*d^2*x^2 - 6300*c^2*d^3*x^3 + 700*c*d^4*x^4 + 500*d^5*x^5) - 7*a*b^6*d*(c
^6 + 10*c^5*d*x + 50*c^4*d^2*x^2 + 200*c^3*d^3*x^3 - 300*c^2*d^4*x^4 - 100*
c*d^5*x^5 + 10*d^6*x^6) - b^7*(4*c^7 + 35*c^6*d*x + 140*c^5*d^2*x^2 + 350*c
^4*d^3*x^3 + 700*c^3*d^4*x^4 - 140*c*d^6*x^6 - 10*d^7*x^7) + 420*d^5*(b*c -
 a*d)^2*(a + b*x)^5*Log[a + b*x])/(20*b^8*(a + b*x)^5)
```

Maple [B] time = 0.013, size = 656, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^6,x)
```

```
[Out] -1/5/b/(b*x+a)^5*c^7+1/2*d^7*x^2/b^6+35/b^5*d^4/(b*x+a)^4*a^3*c^3-105/4/b^4
*d^3/(b*x+a)^4*a^2*c^4+21/2/b^3*d^2/(b*x+a)^4*a*c^5-105/b^7*d^6/(b*x+a)*a^2
*c+105/b^6*d^5/(b*x+a)*a*c^2+70/b^7*d^6/(b*x+a)^2*a^3*c-105/b^6*d^5/(b*x+a)
^2*a^2*c^2+70/b^5*d^4/(b*x+a)^2*a*c^3-35/b^7*d^6/(b*x+a)^3*a^4*c+70/b^6*d^5
/(b*x+a)^3*a^3*c^2-70/b^5*d^4/(b*x+a)^3*a^2*c^3+35/b^4*d^3/(b*x+a)^3*a*c^4-
7/5/b^7/(b*x+a)^5*a^6*c*d^6+7/b^4/(b*x+a)^5*a^3*c^4*d^3-21/5/b^3/(b*x+a)^5*
a^2*c^5*d^2+7/5/b^2/(b*x+a)^5*a*c^6*d-42/b^7*d^6*ln(b*x+a)*a*c+21/2/b^7*d^6
/(b*x+a)^4*a^5*c-105/4/b^6*d^5/(b*x+a)^4*a^4*c^2+21/5/b^6/(b*x+a)^5*a^5*c^2
*d^5-7/b^5/(b*x+a)^5*a^4*c^3*d^4+7*d^6/b^6*x*c+35/b^8*d^7/(b*x+a)*a^3-35/b^
5*d^4/(b*x+a)*c^3-35/2/b^8*d^7/(b*x+a)^2*a^4-35/2/b^4*d^3/(b*x+a)^2*c^4+7/b
^8*d^7/(b*x+a)^3*a^5-7/b^3*d^2/(b*x+a)^3*c^5+1/5/b^8/(b*x+a)^5*a^7*d^7+21/b
^8*d^7*ln(b*x+a)*a^2+21/b^6*d^5*ln(b*x+a)*c^2-7/4/b^8*d^7/(b*x+a)^4*a^6-7/4
/b^2*d/(b*x+a)^4*c^6-6*d^7/b^7*a*x
```

Maxima [B] time = 1.12461, size = 680, normalized size = 3.76

$$-\frac{4\,b^7c^7 + 7\,ab^6c^6d + 14\,a^2b^5c^5d^2 + 35\,a^3b^4c^4d^3 + 140\,a^4b^3c^3d^4 - 959\,a^5b^2c^2d^5 + 1218\,a^6bcd^6 - 459\,a^7d^7 + 700\,\left(b^7c^3d^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(4*b^7*c^7 + 7*a*b^6*c^6*d + 14*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3
+ 140*a^4*b^3*c^3*d^4 - 959*a^5*b^2*c^2*d^5 + 1218*a^6*b*c*d^6 - 459*a^7*d^
7 + 700*(b^7*c^3*d^4 - 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4
 + 350*(b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 - 18*a^2*b^5*c^2*d^5 + 20*a^3*b^4*c*d
^6 - 7*a^4*b^3*d^7)*x^3 + 70*(2*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 20*a^2*b^5*
c^3*d^4 - 110*a^3*b^4*c^2*d^5 + 130*a^4*b^3*c*d^6 - 47*a^5*b^2*d^7)*x^2 + 3
5*(b^7*c^6*d + 2*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 - 1
25*a^4*b^3*c^2*d^5 + 154*a^5*b^2*c*d^6 - 57*a^6*b*d^7)*x)/(b^13*x^5 + 5*a*b
^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8) + 1/2*
(b*d^7*x^2 + 2*(7*b*c*d^6 - 6*a*d^7)*x)/b^7 + 21*(b^2*c^2*d^5 - 2*a*b*c*d^6
 + a^2*d^7)*log(b*x + a)/b^8
```

Fricas [B] time = 2.24066, size = 1517, normalized size = 8.38

$$\frac{10\,b^7d^7x^7 - 4\,b^7c^7 - 7\,ab^6c^6d - 14\,a^2b^5c^5d^2 - 35\,a^3b^4c^4d^3 - 140\,a^4b^3c^3d^4 + 959\,a^5b^2c^2d^5 - 1218\,a^6bcd^6 + 459\,a^7d^7 + 70\,(\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^7*d^7*x^7 - 4*b^7*c^7 - 7*a*b^6*c^6*d - 14*a^2*b^5*c^5*d^2 - 35*
a^3*b^4*c^4*d^3 - 140*a^4*b^3*c^3*d^4 + 959*a^5*b^2*c^2*d^5 - 1218*a^6*b*c*
d^6 + 459*a^7*d^7 + 70*(2*b^7*c*d^6 - a*b^6*d^7)*x^6 + 100*(7*a*b^6*c*d^6 -
 5*a^2*b^5*d^7)*x^5 - 100*(7*b^7*c^3*d^4 - 21*a*b^6*c^2*d^5 + 7*a^2*b^5*c*d
^6 + 4*a^3*b^4*d^7)*x^4 - 50*(7*b^7*c^4*d^3 + 28*a*b^6*c^3*d^4 - 126*a^2*b^
5*c^2*d^5 + 112*a^3*b^4*c*d^6 - 26*a^4*b^3*d^7)*x^3 - 10*(14*b^7*c^5*d^2 +
35*a*b^6*c^4*d^3 + 140*a^2*b^5*c^3*d^4 - 770*a^3*b^4*c^2*d^5 + 840*a^4*b^3*
c*d^6 - 270*a^5*b^2*d^7)*x^2 - 5*(7*b^7*c^6*d + 14*a*b^6*c^5*d^2 + 35*a^2*b
^5*c^4*d^3 + 140*a^3*b^4*c^3*d^4 - 875*a^4*b^3*c^2*d^5 + 1050*a^5*b^2*c*d^6
 - 375*a^6*b*d^7)*x + 420*(a^5*b^2*c^2*d^5 - 2*a^6*b*c*d^6 + a^7*d^7 + (b^7
*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 5*(a*b^6*c^2*d^5 - 2*a^2*b^5*
c*d^6 + a^3*b^4*d^7)*x^4 + 10*(a^2*b^5*c^2*d^5 - 2*a^3*b^4*c*d^6 + a^4*b^3*
d^7)*x^3 + 10*(a^3*b^4*c^2*d^5 - 2*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 5*(a^
4*b^3*c^2*d^5 - 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)*log(b*x + a))/(b^13*x^5 + 5
*a*b^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8)
```

Sympy [B] time = 91.8753, size = 522, normalized size = 2.88

$$\frac{459a^7d^7 - 1218a^6bcd^6 + 959a^5b^2c^2d^5 - 140a^4b^3c^3d^4 - 35a^3b^4c^4d^3 - 14a^2b^5c^5d^2 - 7ab^6c^6d - 4b^7c^7 + x^4\left(700a^3b^4d^7 - 210\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**6,x)
```

```
[Out] (459*a**7*d**7 - 1218*a**6*b*c*d**6 + 959*a**5*b**2*c**2*d**5 - 140*a**4*b*
*3*c**3*d**4 - 35*a**3*b**4*c**4*d**3 - 14*a**2*b**5*c**5*d**2 - 7*a*b**6*c
**6*d - 4*b**7*c**7 + x**4*(700*a**3*b**4*d**7 - 2100*a**2*b**5*c*d**6 + 21
00*a*b**6*c**2*d**5 - 700*b**7*c**3*d**4) + x**3*(2450*a**4*b**3*d**7 - 700
0*a**3*b**4*c*d**6 + 6300*a**2*b**5*c**2*d**5 - 1400*a*b**6*c**3*d**4 - 350
*b**7*c**4*d**3) + x**2*(3290*a**5*b**2*d**7 - 9100*a**4*b**3*c*d**6 + 7700
*a**3*b**4*c**2*d**5 - 1400*a**2*b**5*c**3*d**4 - 350*a*b**6*c**4*d**3 - 14
0*b**7*c**5*d**2) + x*(1995*a**6*b*d**7 - 5390*a**5*b**2*c*d**6 + 4375*a**4
*b**3*c**2*d**5 - 700*a**3*b**4*c**3*d**4 - 175*a**2*b**5*c**4*d**3 - 70*a*
b**6*c**5*d**2 - 35*b**7*c**6*d))/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**
3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + d*
*7*x**2/(2*b**6) - x*(6*a*d**7 - 7*b*c*d**6)/b**7 + 21*d**5*(a*d - b*c)**2*
log(a + b*x)/b**8
```

Giac [B] time = 1.06106, size = 625, normalized size = 3.45

$$\frac{21\left(b^2c^2d^5 - 2\,abcd^6 + a^2d^7\right)\log\left(|bx + a|\right)}{b^8} + \frac{b^6d^7x^2 + 14\,b^6cd^6x - 12\,ab^5d^7x}{2\,b^{12}} - \frac{4\,b^7c^7 + 7\,ab^6c^6d + 14\,a^2b^5c^5d^2 + 35\,a^3b^4\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 21*(b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7)*log(abs(b*x + a))/b^8 + 1/2*(b^6*d
^7*x^2 + 14*b^6*c*d^6*x - 12*a*b^5*d^7*x)/b^12 - 1/20*(4*b^7*c^7 + 7*a*b^6*
c^6*d + 14*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 - 959
*a^5*b^2*c^2*d^5 + 1218*a^6*b*c*d^6 - 459*a^7*d^7 + 700*(b^7*c^3*d^4 - 3*a*
b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 350*(b^7*c^4*d^3 + 4*a*b
^6*c^3*d^4 - 18*a^2*b^5*c^2*d^5 + 20*a^3*b^4*c*d^6 - 7*a^4*b^3*d^7)*x^3 + 7
0*(2*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 - 110*a^3*b^4*c^2*d
^5 + 130*a^4*b^3*c*d^6 - 47*a^5*b^2*d^7)*x^2 + 35*(b^7*c^6*d + 2*a*b^6*c^5*
d^2 + 5*a^2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 - 125*a^4*b^3*c^2*d^5 + 154*a^
5*b^2*c*d^6 - 57*a^6*b*d^7)*x)/((b*x + a)^5*b^8)
```

3.1289 $\int \frac{(c+dx)^7}{(a+bx)^7} dx$

Optimal. Leaf size=186

$$-\frac{21d^5(bc-ad)^2}{b^8(a+bx)}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}+\frac{7d^6(bc-ad)\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{(bc}{6b^8(a}$$

```
[Out] (d^7*x)/b^7 - (b*c - a*d)^7/(6*b^8*(a + b*x)^6) - (7*d*(b*c - a*d)^6)/(5*b^
8*(a + b*x)^5) - (21*d^2*(b*c - a*d)^5)/(4*b^8*(a + b*x)^4) - (35*d^3*(b*c
- a*d)^4)/(3*b^8*(a + b*x)^3) - (35*d^4*(b*c - a*d)^3)/(2*b^8*(a + b*x)^2)
- (21*d^5*(b*c - a*d)^2)/(b^8*(a + b*x)) + (7*d^6*(b*c - a*d)*Log[a + b*x])
/b^8
```

Rubi [A] time = 0.172135, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{21d^5(bc-ad)^2}{b^8(a+bx)}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}+\frac{7d^6(bc-ad)\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{(bc}{6b^8(a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^7,x]
```

```
[Out] (d^7*x)/b^7 - (b*c - a*d)^7/(6*b^8*(a + b*x)^6) - (7*d*(b*c - a*d)^6)/(5*b^
8*(a + b*x)^5) - (21*d^2*(b*c - a*d)^5)/(4*b^8*(a + b*x)^4) - (35*d^3*(b*c
- a*d)^4)/(3*b^8*(a + b*x)^3) - (35*d^4*(b*c - a*d)^3)/(2*b^8*(a + b*x)^2)
- (21*d^5*(b*c - a*d)^2)/(b^8*(a + b*x)) + (7*d^6*(b*c - a*d)*Log[a + b*x])
/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^7}dx &= \int \left(\frac{d^7}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^7}+\frac{7d(bc-ad)^6}{b^7(a+bx)^6}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^5}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^4}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^3}+\frac{21d^5(bc-}{b^7(a+}\right.\\ &= \frac{d^7x}{b^7}-\frac{(bc-ad)^7}{6b^8(a+bx)^6}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}-\frac{21d^5(bc-a}{b^8(a+bx}\end{aligned}$$

Mathematica [B] time = 0.206488, size = 390, normalized size = 2.1

$$-\frac{3a^2b^5d^2\left(350c^3d^2x^2+1400c^2d^3x^3+70c^4dx+7c^5-3150cd^4x^4+120d^5x^5\right)+5a^3b^4d^3\left(630c^2d^2x^2+84c^3dx+7c^4-308\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^7,x]
```

```
[Out] -(669*a^7*d^7 + 3*a^6*b*d^6*(-343*c + 1198*d*x) + 3*a^5*b^2*d^5*(70*c^2 - 1
918*c*d*x + 2575*d^2*x^2) + 5*a^4*b^3*d^4*(14*c^3 + 252*c^2*d*x - 2625*c*d^
2*x^2 + 1640*d^3*x^3) + 5*a^3*b^4*d^3*(7*c^4 + 84*c^3*d*x + 630*c^2*d^2*x^2
 - 3080*c*d^3*x^3 + 810*d^4*x^4) + 3*a^2*b^5*d^2*(7*c^5 + 70*c^4*d*x + 350*
c^3*d^2*x^2 + 1400*c^2*d^3*x^3 - 3150*c*d^4*x^4 + 120*d^5*x^5) + a*b^6*d*(1
4*c^6 + 126*c^5*d*x + 525*c^4*d^2*x^2 + 1400*c^3*d^3*x^3 + 3150*c^2*d^4*x^4
 - 2520*c*d^5*x^5 - 360*d^6*x^6) + b^7*(10*c^7 + 84*c^6*d*x + 315*c^5*d^2*x
^2 + 700*c^4*d^3*x^3 + 1050*c^3*d^4*x^4 + 1260*c^2*d^5*x^5 - 60*d^7*x^7) +
420*d^6*(-(b*c) + a*d)*(a + b*x)^6*Log[a + b*x])/(60*b^8*(a + b*x)^6)
```

Maple [B] time = 0.012, size = 666, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^7,x)
```

```
[Out] 105/2/b^6*d^5/(b*x+a)^2*a*c^2+140/3/b^7*d^6/(b*x+a)^3*a^3*c-70/b^6*d^5/(b*x
+a)^3*a^2*c^2+140/3/b^5*d^4/(b*x+a)^3*a*c^3+28/b^5*d^4/(b*x+a)^5*a^3*c^3-21
/b^4*d^3/(b*x+a)^5*a^2*c^4+42/5/b^3*d^2/(b*x+a)^5*a*c^5-105/4/b^7*d^6/(b*x+
a)^4*a^4*c+105/2/b^6*d^5/(b*x+a)^4*a^3*c^2-105/2/b^5*d^4/(b*x+a)^4*a^2*c^3+
105/4/b^4*d^3/(b*x+a)^4*a*c^4+42/b^7*d^6/(b*x+a)*a*c-7/6/b^7/(b*x+a)^6*a^6*
c*d^6+7/2/b^6/(b*x+a)^6*a^5*c^2*d^5-35/6/b^5/(b*x+a)^6*a^4*c^3*d^4+35/6/b^4
/(b*x+a)^6*a^3*c^4*d^3-7/2/b^3/(b*x+a)^6*a^2*c^5*d^2+7/6/b^2/(b*x+a)^6*a*c^
6*d-105/2/b^7*d^6/(b*x+a)^2*a^2*c-1/6/b/(b*x+a)^6*c^7-7/5/b^8*d^7/(b*x+a)^5
*a^6-7/5/b^2*d/(b*x+a)^5*c^6-7/b^8*d^7*ln(b*x+a)*a+7/b^7*d^6*ln(b*x+a)*c+21
/4/b^8*d^7/(b*x+a)^4*a^5-21/4/b^3*d^2/(b*x+a)^4*c^5-21/b^8*d^7/(b*x+a)*a^2-
21/b^6*d^5/(b*x+a)*c^2+1/6/b^8/(b*x+a)^6*a^7*d^7+35/2/b^8*d^7/(b*x+a)^2*a^3
-35/2/b^5*d^4/(b*x+a)^2*c^3-35/3/b^8*d^7/(b*x+a)^3*a^4-35/3/b^4*d^3/(b*x+a)
^3*c^4-21/b^6*d^5/(b*x+a)^5*a^4*c^2+42/5/b^7*d^6/(b*x+a)^5*a^5*c+d^7*x/b^7
```

Maple [B] time = 1.11724, size = 697, normalized size = 3.75

$$\frac{d^7x}{b^7} - \frac{10\,b^7c^7 + 14\,ab^6c^6d + 21\,a^2b^5c^5d^2 + 35\,a^3b^4c^4d^3 + 70\,a^4b^3c^3d^4 + 210\,a^5b^2c^2d^5 - 1029\,a^6bcd^6 + 669\,a^7d^7 + 1260}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] d^7*x/b^7 - 1/60*(10*b^7*c^7 + 14*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 35*a^3
*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 - 1029*a^6*b*c*d^6
+ 669*a^7*d^7 + 1260*(b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1050
*(b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 - 9*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^4 + 70
0*(b^7*c^4*d^3 + 2*a*b^6*c^3*d^4 + 6*a^2*b^5*c^2*d^5 - 22*a^3*b^4*c*d^6 + 1
3*a^4*b^3*d^7)*x^3 + 105*(3*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*
d^4 + 30*a^3*b^4*c^2*d^5 - 125*a^4*b^3*c*d^6 + 77*a^5*b^2*d^7)*x^2 + 42*(2*
b^7*c^6*d + 3*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 + 30*a
^4*b^3*c^2*d^5 - 137*a^5*b^2*c*d^6 + 87*a^6*b*d^7)*x)/(b^14*x^6 + 6*a*b^13*
x^5 + 15*a^2*b^12*x^4 + 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a
```

```
^6*b^8) + 7*(b*c*d^6 - a*d^7)*log(b*x + a)/b^8
```

Fricas [B] time = 2.03882, size = 1436, normalized size = 7.72

$$\frac{60\,b^7d^7x^7 + 360\,ab^6d^7x^6 - 10\,b^7c^7 - 14\,ab^6c^6d - 21\,a^2b^5c^5d^2 - 35\,a^3b^4c^4d^3 - 70\,a^4b^3c^3d^4 - 210\,a^5b^2c^2d^5 + 1029\,a^6bcd^6 - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*d^7*x^7 + 360*a*b^6*d^7*x^6 - 10*b^7*c^7 - 14*a*b^6*c^6*d - 21
*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 - 70*a^4*b^3*c^3*d^4 - 210*a^5*b^2*c^
2*d^5 + 1029*a^6*b*c*d^6 - 669*a^7*d^7 - 180*(7*b^7*c^2*d^5 - 14*a*b^6*c*d^
6 + 2*a^2*b^5*d^7)*x^5 - 150*(7*b^7*c^3*d^4 + 21*a*b^6*c^2*d^5 - 63*a^2*b^5
*c*d^6 + 27*a^3*b^4*d^7)*x^4 - 100*(7*b^7*c^4*d^3 + 14*a*b^6*c^3*d^4 + 42*a
^2*b^5*c^2*d^5 - 154*a^3*b^4*c*d^6 + 82*a^4*b^3*d^7)*x^3 - 15*(21*b^7*c^5*d
^2 + 35*a*b^6*c^4*d^3 + 70*a^2*b^5*c^3*d^4 + 210*a^3*b^4*c^2*d^5 - 875*a^4*
b^3*c*d^6 + 515*a^5*b^2*d^7)*x^2 - 6*(14*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 35*
a^2*b^5*c^4*d^3 + 70*a^3*b^4*c^3*d^4 + 210*a^4*b^3*c^2*d^5 - 959*a^5*b^2*c*
d^6 + 599*a^6*b*d^7)*x + 420*(a^6*b*c*d^6 - a^7*d^7 + (b^7*c*d^6 - a*b^6*d^
7)*x^6 + 6*(a*b^6*c*d^6 - a^2*b^5*d^7)*x^5 + 15*(a^2*b^5*c*d^6 - a^3*b^4*d^
7)*x^4 + 20*(a^3*b^4*c*d^6 - a^4*b^3*d^7)*x^3 + 15*(a^4*b^3*c*d^6 - a^5*b^2
*d^7)*x^2 + 6*(a^5*b^2*c*d^6 - a^6*b*d^7)*x)*log(b*x + a))/(b^14*x^6 + 6*a*
b^13*x^5 + 15*a^2*b^12*x^4 + 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*
x + a^6*b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06446, size = 620, normalized size = 3.33

$$\frac{d^7x}{b^7} + \frac{7\left(bcd^6 - ad^7\right)\log\left(|bx + a|\right)}{b^8} - \frac{10\,b^7c^7 + 14\,ab^6c^6d + 21\,a^2b^5c^5d^2 + 35\,a^3b^4c^4d^3 + 70\,a^4b^3c^3d^4 + 210\,a^5b^2c^2d^5 - 102\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] d^7*x/b^7 + 7*(b*c*d^6 - a*d^7)*log(abs(b*x + a))/b^8 - 1/60*(10*b^7*c^7 +
14*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d
^4 + 210*a^5*b^2*c^2*d^5 - 1029*a^6*b*c*d^6 + 669*a^7*d^7 + 1260*(b^7*c^2*d
^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1050*(b^7*c^3*d^4 + 3*a*b^6*c^2*d^5
 - 9*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^4 + 700*(b^7*c^4*d^3 + 2*a*b^6*c^3*d^
```

```
4 + 6*a^2*b^5*c^2*d^5 - 22*a^3*b^4*c*d^6 + 13*a^4*b^3*d^7)*x^3 + 105*(3*b^7
*c^5*d^2 + 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 - 125*
a^4*b^3*c*d^6 + 77*a^5*b^2*d^7)*x^2 + 42*(2*b^7*c^6*d + 3*a*b^6*c^5*d^2 + 5
*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 - 137*a^5*b^2*c*
d^6 + 87*a^6*b*d^7)*x)/((b*x + a)^6*b^8)
```

3.1290 $\int \frac{(c+dx)^7}{(a+bx)^8}\,dx$

Optimal. Leaf size=194

$$-\frac{7d^6(bc-ad)}{b^8(a+bx)}-\frac{21d^5(bc-ad)^2}{2b^8(a+bx)^2}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{(bc-ad)^7}{7b^8(a+bx)^7}+\frac{d}{}$$

```
[Out] -(b*c - a*d)^7/(7*b^8*(a + b*x)^7) - (7*d*(b*c - a*d)^6)/(6*b^8*(a + b*x)^6
) - (21*d^2*(b*c - a*d)^5)/(5*b^8*(a + b*x)^5) - (35*d^3*(b*c - a*d)^4)/(4*
b^8*(a + b*x)^4) - (35*d^4*(b*c - a*d)^3)/(3*b^8*(a + b*x)^3) - (21*d^5*(b*
c - a*d)^2)/(2*b^8*(a + b*x)^2) - (7*d^6*(b*c - a*d))/(b^8*(a + b*x)) + (d^
7*Log[a + b*x])/b^8
```

Rubi [A] time = 0.155908, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{7d^6(bc-ad)}{b^8(a+bx)}-\frac{21d^5(bc-ad)^2}{2b^8(a+bx)^2}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{(bc-ad)^7}{7b^8(a+bx)^7}+\frac{d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^8,x]
```

```
[Out] -(b*c - a*d)^7/(7*b^8*(a + b*x)^7) - (7*d*(b*c - a*d)^6)/(6*b^8*(a + b*x)^6
) - (21*d^2*(b*c - a*d)^5)/(5*b^8*(a + b*x)^5) - (35*d^3*(b*c - a*d)^4)/(4*
b^8*(a + b*x)^4) - (35*d^4*(b*c - a*d)^3)/(3*b^8*(a + b*x)^3) - (21*d^5*(b*
c - a*d)^2)/(2*b^8*(a + b*x)^2) - (7*d^6*(b*c - a*d))/(b^8*(a + b*x)) + (d^
7*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^8}\,dx &= \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^8}+\frac{7d(bc-ad)^6}{b^7(a+bx)^7}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^6}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^5}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^4}+\frac{21d^5(bc-ad)^2}{b^7(a+bx)^3}\right.\\ &= -\frac{(bc-ad)^7}{7b^8(a+bx)^7}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\frac{21d^5(bc-ad)^2}{2b^8(a+bx)^2}-\end{aligned}$$

Mathematica [A] time = 0.160317, size = 308, normalized size = 1.59

$$\frac{d^7\log(a+bx)}{b^8}-\frac{(bc-ad)\left(a^2b^4d^2\left(6909c^2d^2x^2+1813c^3dx+214c^4+15925cd^3x^3+26950d^4x^4\right)+a^3b^3d^3\left(2793c^2dx+3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^8,x]
```

```
[Out] -((b*c - a*d)*(1089*a^6*d^6 + 3*a^5*b*d^5*(223*c + 2401*d*x) + 3*a^4*b^2*d^
4*(153*c^2 + 1421*c*d*x + 6713*d^2*x^2) + a^3*b^3*d^3*(319*c^3 + 2793*c^2*d
*x + 11319*c*d^2*x^2 + 30625*d^3*x^3) + a^2*b^4*d^2*(214*c^4 + 1813*c^3*d*x
 + 6909*c^2*d^2*x^2 + 15925*c*d^3*x^3 + 26950*d^4*x^4) + a*b^5*d*(130*c^5 +
 1078*c^4*d*x + 3969*c^3*d^2*x^2 + 8575*c^2*d^3*x^3 + 12250*c*d^4*x^4 + 132
30*d^5*x^5) + b^6*(60*c^6 + 490*c^5*d*x + 1764*c^4*d^2*x^2 + 3675*c^3*d^3*x
^3 + 4900*c^2*d^4*x^4 + 4410*c*d^5*x^5 + 2940*d^6*x^6)))/(420*b^8*(a + b*x)
^7) + (d^7*Log[a + b*x])/b^8
```

Maple [B] time = 0.01, size = 672, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^8,x)
```

```
[Out] 21*d^3/b^4/(b*x+a)^5*a*c^4+35*d^6/b^7/(b*x+a)^4*a^3*c-1/7/b/(b*x+a)^7*c^7-3
5/3*d^4/b^5/(b*x+a)^3*c^3+7/b^8*d^7/(b*x+a)*a-7/b^7*d^6/(b*x+a)*c-7/6*d^7/b
^8/(b*x+a)^6*a^6+21/5*d^7/b^8/(b*x+a)^5*a^5-21/5*d^2/b^3/(b*x+a)^5*c^5-35/4
*d^7/b^8/(b*x+a)^4*a^4-35/4*d^3/b^4/(b*x+a)^4*c^4+1/7/b^8/(b*x+a)^7*a^7*d^7
-7/6*d/b^2/(b*x+a)^6*c^6-21/2*d^7/b^8/(b*x+a)^2*a^2-21/2*d^5/b^6/(b*x+a)^2*
c^2+35/3*d^7/b^8/(b*x+a)^3*a^3+1/b^2/(b*x+a)^7*a*c^6*d+7*d^6/b^7/(b*x+a)^6*
a^5*c-3/b^3/(b*x+a)^7*a^2*c^5*d^2-105/2*d^5/b^6/(b*x+a)^4*a^2*c^2+35*d^4/b^
5/(b*x+a)^4*a*c^3-1/b^7/(b*x+a)^7*a^6*c*d^6+3/b^6/(b*x+a)^7*a^5*c^2*d^5-5/b
^5/(b*x+a)^7*c^3*d^4*a^4+5/b^4/(b*x+a)^7*a^3*c^4*d^3+35*d^5/b^6/(b*x+a)^3*a
*c^2-21*d^6/b^7/(b*x+a)^5*a^4*c+42*d^5/b^6/(b*x+a)^5*a^3*c^2-42*d^4/b^5/(b*
x+a)^5*a^2*c^3-35*d^6/b^7/(b*x+a)^3*a^2*c-35/2*d^5/b^6/(b*x+a)^6*a^4*c^2+70
/3*d^4/b^5/(b*x+a)^6*a^3*c^3-35/2*d^3/b^4/(b*x+a)^6*a^2*c^4+7*d^2/b^3/(b*x+
a)^6*a*c^5+21*d^6/b^7/(b*x+a)^2*a*c+d^7*ln(b*x+a)/b^8
```

Maxima [B] time = 1.07051, size = 721, normalized size = 3.72

$$-\frac{60\, b^7 c^7 + 70\, a b^6 c^6 d + 84\, a^2 b^5 c^5 d^2 + 105\, a^3 b^4 c^4 d^3 + 140\, a^4 b^3 c^3 d^4 + 210\, a^5 b^2 c^2 d^5 + 420\, a^6 b c d^6 - 1089\, a^7 d^7 + 2940\, \left(b \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/420*(60*b^7*c^7 + 70*a*b^6*c^6*d + 84*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*
d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^
7*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(b^7*c^2*d^5 + 2*a*b^6*c*d^
6 - 3*a^2*b^5*d^7)*x^5 + 2450*(2*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 6*a^2*b^5*
c*d^6 - 11*a^3*b^4*d^7)*x^4 + 1225*(3*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 6*a^2
*b^5*c^2*d^5 + 12*a^3*b^4*c*d^6 - 25*a^4*b^3*d^7)*x^3 + 147*(12*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 + 60*a^4*b^3*
c*d^6 - 137*a^5*b^2*d^7)*x^2 + 49*(10*b^7*c^6*d + 12*a*b^6*c^5*d^2 + 15*a^2
*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 + 60*a^5*b^2*c*d^6 -
 147*a^6*b*d^7)*x)/(b^15*x^7 + 7*a*b^14*x^6 + 21*a^2*b^13*x^5 + 35*a^3*b^12
*x^4 + 35*a^4*b^11*x^3 + 21*a^5*b^10*x^2 + 7*a^6*b^9*x + a^7*b^8) + d^7*log
```

```
(b*x + a)/b^8
```

Fricas [B] time = 2.15655, size = 1320, normalized size = 6.8

$$-\frac{60\,b^7c^7 + 70\,ab^6c^6d + 84\,a^2b^5c^5d^2 + 105\,a^3b^4c^4d^3 + 140\,a^4b^3c^3d^4 + 210\,a^5b^2c^2d^5 + 420\,a^6bcd^6 - 1089\,a^7d^7 + 2940\,\left(b^7cd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/420*(60*b^7*c^7 + 70*a*b^6*c^6*d + 84*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*
d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^
7*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(b^7*c^2*d^5 + 2*a*b^6*c*d^
6 - 3*a^2*b^5*d^7)*x^5 + 2450*(2*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 6*a^2*b^5*
c*d^6 - 11*a^3*b^4*d^7)*x^4 + 1225*(3*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 6*a^2
*b^5*c^2*d^5 + 12*a^3*b^4*c*d^6 - 25*a^4*b^3*d^7)*x^3 + 147*(12*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 + 60*a^4*b^3*
c*d^6 - 137*a^5*b^2*d^7)*x^2 + 49*(10*b^7*c^6*d + 12*a*b^6*c^5*d^2 + 15*a^2
*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 + 60*a^5*b^2*c*d^6 -
 147*a^6*b*d^7)*x - 420*(b^7*d^7*x^7 + 7*a*b^6*d^7*x^6 + 21*a^2*b^5*d^7*x^5
 + 35*a^3*b^4*d^7*x^4 + 35*a^4*b^3*d^7*x^3 + 21*a^5*b^2*d^7*x^2 + 7*a^6*b*d
^7*x + a^7*d^7)*log(b*x + a))/(b^15*x^7 + 7*a*b^14*x^6 + 21*a^2*b^13*x^5 +
35*a^3*b^12*x^4 + 35*a^4*b^11*x^3 + 21*a^5*b^10*x^2 + 7*a^6*b^9*x + a^7*b^8
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06038, size = 629, normalized size = 3.24

$$\frac{d^7\log\left(|bx + a|\right)}{b^8} - \frac{2940\left(b^6cd^6 - ab^5d^7\right)x^6 + 4410\left(b^6c^2d^5 + 2\,ab^5cd^6 - 3\,a^2b^4d^7\right)x^5 + 2450\left(2\,b^6c^3d^4 + 3\,ab^5c^2d^5 + 6\,a^2b^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] d^7*log(abs(b*x + a))/b^8 - 1/420*(2940*(b^6*c*d^6 - a*b^5*d^7)*x^6 + 4410*
(b^6*c^2*d^5 + 2*a*b^5*c*d^6 - 3*a^2*b^4*d^7)*x^5 + 2450*(2*b^6*c^3*d^4 + 3
*a*b^5*c^2*d^5 + 6*a^2*b^4*c*d^6 - 11*a^3*b^3*d^7)*x^4 + 1225*(3*b^6*c^4*d^
3 + 4*a*b^5*c^3*d^4 + 6*a^2*b^4*c^2*d^5 + 12*a^3*b^3*c*d^6 - 25*a^4*b^2*d^7
)*x^3 + 147*(12*b^6*c^5*d^2 + 15*a*b^5*c^4*d^3 + 20*a^2*b^4*c^3*d^4 + 30*a^
3*b^3*c^2*d^5 + 60*a^4*b^2*c*d^6 - 137*a^5*b*d^7)*x^2 + 49*(10*b^6*c^6*d +
```

```
12*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 + 20*a^3*b^3*c^3*d^4 + 30*a^4*b^2*c^2
*d^5 + 60*a^5*b*c*d^6 - 147*a^6*d^7)*x + (60*b^7*c^7 + 70*a*b^6*c^6*d + 84*
a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c
^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^7*d^7)/b)/((b*x + a)^7*b^7)
```

3.1291 $\int \frac{(c+dx)^7}{(a+bx)^9}\,dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^8}{8(a+bx)^8(bc-ad)}$$

```
[Out] -(c + d*x)^8/(8*(b*c - a*d)*(a + b*x)^8)
```

Rubi [A] time = 0.0034644, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(c+dx)^8}{8(a+bx)^8(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^9,x]
```

```
[Out] -(c + d*x)^8/(8*(b*c - a*d)*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^9}\,dx = -\frac{(c+dx)^8}{8(bc-ad)(a+bx)^8}$$

Mathematica [B] time = 0.119213, size = 353, normalized size = 12.61

$$-\frac{a^2b^5d^2\left(28c^3d^2x^2+56c^2d^3x^3+8c^4dx+c^5+70cd^4x^4+56d^5x^5\right)+a^3b^4d^3\left(28c^2d^2x^2+8c^3dx+c^4+56cd^3x^3+70d^4x^4\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^9,x]
```

```
[Out] -(a^7*d^7 + a^6*b*d^6*(c + 8*d*x) + a^5*b^2*d^5*(c^2 + 8*c*d*x + 28*d^2*x^2
) + a^4*b^3*d^4*(c^3 + 8*c^2*d*x + 28*c*d^2*x^2 + 56*d^3*x^3) + a^3*b^4*d^3
*(c^4 + 8*c^3*d*x + 28*c^2*d^2*x^2 + 56*c*d^3*x^3 + 70*d^4*x^4) + a^2*b^5*d
^2*(c^5 + 8*c^4*d*x + 28*c^3*d^2*x^2 + 56*c^2*d^3*x^3 + 70*c*d^4*x^4 + 56*d
^5*x^5) + a*b^6*d*(c^6 + 8*c^5*d*x + 28*c^4*d^2*x^2 + 56*c^3*d^3*x^3 + 70*c
^2*d^4*x^4 + 56*c*d^5*x^5 + 28*d^6*x^6) + b^7*(c^7 + 8*c^6*d*x + 28*c^5*d^2
*x^2 + 56*c^4*d^3*x^3 + 70*c^3*d^4*x^4 + 56*c^2*d^5*x^5 + 28*c*d^6*x^6 + 8*
d^7*x^7))/(8*b^8*(a + b*x)^8)
```

Maple [B] time = 0.007, size = 464, normalized size = 16.6

$$-\frac{d^7}{b^8\,(bx+a)}-\frac{-a^7d^7+7\,a^6cd^6b-21\,a^5b^2c^2d^5+35\,c^3d^4a^4b^3-35\,a^3b^4c^4d^3+21\,a^2c^5d^2b^5-7\,ac^6db^6+b^7c^7}{8\,b^8\,(bx+a)^8}+\frac{7\,d^2\,(a^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^9,x)
```

```
[Out] -d^7/b^8/(b*x+a)-1/8*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*
c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b
*x+a)^8+7/2*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^
2+5*a*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^6+7/2*d^6*(a*d-b*c)/b^8/(b*x+a)^2-7*d^
5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^3-7*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*
a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^8/(b*x+a)^5+35/4*d^4*(a^3*d^3-3*a^
2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^4-d*(a^6*d^6-6*a^5*b*c*d^5+15*
a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6
)/b^8/(b*x+a)^7
```

Maxima [B] time = 1.05627, size = 687, normalized size = 24.54

$$-\frac{8\,b^7d^7x^7+b^7c^7+ab^6c^6d+a^2b^5c^5d^2+a^3b^4c^4d^3+a^4b^3c^3d^4+a^5b^2c^2d^5+a^6bcd^6+a^7d^7+28\,\left(b^7cd^6+ab^6d^7\right)x^6+56}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4
*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7 + 28*(b^7*
c*d^6 + a*b^6*d^7)*x^6 + 56*(b^7*c^2*d^5 + a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 +
 70*(b^7*c^3*d^4 + a*b^6*c^2*d^5 + a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 56*(b
^7*c^4*d^3 + a*b^6*c^3*d^4 + a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + a^4*b^3*d^7)
*x^3 + 28*(b^7*c^5*d^2 + a*b^6*c^4*d^3 + a^2*b^5*c^3*d^4 + a^3*b^4*c^2*d^5
+ a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 8*(b^7*c^6*d + a*b^6*c^5*d^2 + a^2*b^5
*c^4*d^3 + a^3*b^4*c^3*d^4 + a^4*b^3*c^2*d^5 + a^5*b^2*c*d^6 + a^6*b*d^7)*x
)/(b^16*x^8 + 8*a*b^15*x^7 + 28*a^2*b^14*x^6 + 56*a^3*b^13*x^5 + 70*a^4*b^1
2*x^4 + 56*a^5*b^11*x^3 + 28*a^6*b^10*x^2 + 8*a^7*b^9*x + a^8*b^8)
```

Fricas [B] time = 2.10975, size = 1003, normalized size = 35.82

$$-\frac{8\,b^7d^7x^7+b^7c^7+ab^6c^6d+a^2b^5c^5d^2+a^3b^4c^4d^3+a^4b^3c^3d^4+a^5b^2c^2d^5+a^6bcd^6+a^7d^7+28\,\left(b^7cd^6+ab^6d^7\right)x^6+56}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4
*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7 + 28*(b^7*
c*d^6 + a*b^6*d^7)*x^6 + 56*(b^7*c^2*d^5 + a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 +
```

```
 70*(b^7*c^3*d^4 + a*b^6*c^2*d^5 + a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 56*(b
^7*c^4*d^3 + a*b^6*c^3*d^4 + a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + a^4*b^3*d^7)
*x^3 + 28*(b^7*c^5*d^2 + a*b^6*c^4*d^3 + a^2*b^5*c^3*d^4 + a^3*b^4*c^2*d^5
+ a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 8*(b^7*c^6*d + a*b^6*c^5*d^2 + a^2*b^5
*c^4*d^3 + a^3*b^4*c^3*d^4 + a^4*b^3*c^2*d^5 + a^5*b^2*c*d^6 + a^6*b*d^7)*x
)/(b^16*x^8 + 8*a*b^15*x^7 + 28*a^2*b^14*x^6 + 56*a^3*b^13*x^5 + 70*a^4*b^1
2*x^4 + 56*a^5*b^11*x^3 + 28*a^6*b^10*x^2 + 8*a^7*b^9*x + a^8*b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**9,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06372, size = 660, normalized size = 23.57

$$-\frac{8\,b^7d^7x^7+28\,b^7cd^6x^6+28\,ab^6d^7x^6+56\,b^7c^2d^5x^5+56\,ab^6cd^6x^5+56\,a^2b^5d^7x^5+70\,b^7c^3d^4x^4+70\,ab^6c^2d^5x^4+70\,a^2b^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + 28*b^7*c*d^6*x^6 + 28*a*b^6*d^7*x^6 + 56*b^7*c^2*d^5*
x^5 + 56*a*b^6*c*d^6*x^5 + 56*a^2*b^5*d^7*x^5 + 70*b^7*c^3*d^4*x^4 + 70*a*b
^6*c^2*d^5*x^4 + 70*a^2*b^5*c*d^6*x^4 + 70*a^3*b^4*d^7*x^4 + 56*b^7*c^4*d^3
*x^3 + 56*a*b^6*c^3*d^4*x^3 + 56*a^2*b^5*c^2*d^5*x^3 + 56*a^3*b^4*c*d^6*x^3
 + 56*a^4*b^3*d^7*x^3 + 28*b^7*c^5*d^2*x^2 + 28*a*b^6*c^4*d^3*x^2 + 28*a^2*
b^5*c^3*d^4*x^2 + 28*a^3*b^4*c^2*d^5*x^2 + 28*a^4*b^3*c*d^6*x^2 + 28*a^5*b^
2*d^7*x^2 + 8*b^7*c^6*d*x + 8*a*b^6*c^5*d^2*x + 8*a^2*b^5*c^4*d^3*x + 8*a^3
*b^4*c^3*d^4*x + 8*a^4*b^3*c^2*d^5*x + 8*a^5*b^2*c*d^6*x + 8*a^6*b*d^7*x +
b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4
 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^8*b^8)
```

3.1292 $\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=58

$$\frac{d(c+dx)^8}{72(a+bx)^8(bc-ad)^2}-\frac{(c+dx)^8}{9(a+bx)^9(bc-ad)}$$

```
[Out] -(c + d*x)^8/(9*(b*c - a*d)*(a + b*x)^9) + (d*(c + d*x)^8)/(72*(b*c - a*d)^
2*(a + b*x)^8)
```

Rubi [A] time = 0.009459, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d(c+dx)^8}{72(a+bx)^8(bc-ad)^2}-\frac{(c+dx)^8}{9(a+bx)^9(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^10,x]
```

```
[Out] -(c + d*x)^8/(9*(b*c - a*d)*(a + b*x)^9) + (d*(c + d*x)^8)/(72*(b*c - a*d)^
2*(a + b*x)^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx &= -\frac{(c+dx)^8}{9(bc-ad)(a+bx)^9}-\frac{d\int \frac{(c+dx)^7}{(a+bx)^9}\,dx}{9(bc-ad)}\\ &= -\frac{(c+dx)^8}{9(bc-ad)(a+bx)^9}+\frac{d(c+dx)^8}{72(bc-ad)^2(a+bx)^8}\end{aligned}$$

Mathematica [B] time = 0.118556, size = 367, normalized size = 6.33

$$-\frac{3a^2b^5d^2\left(48c^3d^2x^2+84c^2d^3x^3+15c^4dx+2c^5+84cd^4x^4+42d^5x^5\right)+a^3b^4d^3\left(108c^2d^2x^2+36c^3dx+5c^4+168cd^3x^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^10,x]
```

```
[Out] -(a^7*d^7 + a^6*b*d^6*(2*c + 9*d*x) + 3*a^5*b^2*d^5*(c^2 + 6*c*d*x + 12*d^2
*x^2) + a^4*b^3*d^4*(4*c^3 + 27*c^2*d*x + 72*c*d^2*x^2 + 84*d^3*x^3) + a^3*
b^4*d^3*(5*c^4 + 36*c^3*d*x + 108*c^2*d^2*x^2 + 168*c*d^3*x^3 + 126*d^4*x^4
) + 3*a^2*b^5*d^2*(2*c^5 + 15*c^4*d*x + 48*c^3*d^2*x^2 + 84*c^2*d^3*x^3 + 8
4*c*d^4*x^4 + 42*d^5*x^5) + a*b^6*d*(7*c^6 + 54*c^5*d*x + 180*c^4*d^2*x^2 +
 336*c^3*d^3*x^3 + 378*c^2*d^4*x^4 + 252*c*d^5*x^5 + 84*d^6*x^6) + b^7*(8*c
^7 + 63*c^6*d*x + 216*c^5*d^2*x^2 + 420*c^4*d^3*x^3 + 504*c^3*d^4*x^4 + 378
*c^2*d^5*x^5 + 168*c*d^6*x^6 + 36*d^7*x^7))/(72*b^8*(a + b*x)^9)
```

Maple [B] time = 0.008, size = 464, normalized size = 8.

$$-\frac{7\,d\left(a^{6}d^{6}-6\,a^{5}bcd^{5}+15\,a^{4}b^{2}c^{2}d^{4}-20\,a^{3}b^{3}c^{3}d^{3}+15\,a^{2}b^{4}c^{4}d^{2}-6\,ab^{5}c^{5}d+b^{6}c^{6}\right)}{8\,b^{8}\left(bx+a\right)^{8}}-\frac{35\,d^{3}\left(a^{4}d^{4}-4\,a^{3}bcd^{3}+6\,a^{2}b^{2}c^{2}d^{2}\right.}{6\,b^{8}\left(bx+a\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^10,x)
```

```
[Out] -7/8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*
b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^8-35/6*d^3*(a^4*d^4-4*a^3*b*
c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^8/(b*x+a)^6-1/2*d^7/b^8/(b
*x+a)^2+7/3*d^6*(a*d-b*c)/b^8/(b*x+a)^3+7*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^
2*c^2*d-b^3*c^3)/b^8/(b*x+a)^5-21/4*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*
x+a)^4-1/9*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35
*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^9+3*
d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^
4*d-b^5*c^5)/b^8/(b*x+a)^7
```

Maxima [B] time = 1.0716, size = 740, normalized size = 12.76

$$-\frac{36\,b^{7}d^{7}x^{7}+8\,b^{7}c^{7}+7\,ab^{6}c^{6}d+6\,a^{2}b^{5}c^{5}d^{2}+5\,a^{3}b^{4}c^{4}d^{3}+4\,a^{4}b^{3}c^{3}d^{4}+3\,a^{5}b^{2}c^{2}d^{5}+2\,a^{6}bcd^{6}+a^{7}d^{7}+84\left(2\,b^{7}cd^{6}+ab^{6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7 + 84*(2*b^7*c*d^6 + a*b^6*d^7)*x^6 + 126*(3*b^7*c^2*d^5 + 2*a*b^6*c*d
^6 + a^2*b^5*d^7)*x^5 + 126*(4*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 2*a^2*b^5*c*
d^6 + a^3*b^4*d^7)*x^4 + 84*(5*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 3*a^2*b^5*c^
2*d^5 + 2*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 36*(6*b^7*c^5*d^2 + 5*a*b^6*c^
4*d^3 + 4*a^2*b^5*c^3*d^4 + 3*a^3*b^4*c^2*d^5 + 2*a^4*b^3*c*d^6 + a^5*b^2*d
^7)*x^2 + 9*(7*b^7*c^6*d + 6*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 4*a^3*b^4*
c^3*d^4 + 3*a^4*b^3*c^2*d^5 + 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^17*x^9 + 9
*a*b^16*x^8 + 36*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^
5*b^12*x^4 + 84*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

Fricas [B] time = 2.03853, size = 1111, normalized size = 19.16

$$-\frac{36\,b^7d^7x^7 + 8\,b^7c^7 + 7\,ab^6c^6d + 6\,a^2b^5c^5d^2 + 5\,a^3b^4c^4d^3 + 4\,a^4b^3c^3d^4 + 3\,a^5b^2c^2d^5 + 2\,a^6bcd^6 + a^7d^7 + 84\left(2\,b^7cd^6 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7 + 84*(2*b^7*c*d^6 + a*b^6*d^7)*x^6 + 126*(3*b^7*c^2*d^5 + 2*a*b^6*c*d
^6 + a^2*b^5*d^7)*x^5 + 126*(4*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 2*a^2*b^5*c*
d^6 + a^3*b^4*d^7)*x^4 + 84*(5*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 3*a^2*b^5*c^
2*d^5 + 2*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 36*(6*b^7*c^5*d^2 + 5*a*b^6*c^
4*d^3 + 4*a^2*b^5*c^3*d^4 + 3*a^3*b^4*c^2*d^5 + 2*a^4*b^3*c*d^6 + a^5*b^2*d
^7)*x^2 + 9*(7*b^7*c^6*d + 6*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 4*a^3*b^4*
c^3*d^4 + 3*a^4*b^3*c^2*d^5 + 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^17*x^9 + 9
*a*b^16*x^8 + 36*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^
5*b^12*x^4 + 84*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**10,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06109, size = 670, normalized size = 11.55

$$-\frac{36\,b^7d^7x^7 + 168\,b^7cd^6x^6 + 84\,ab^6d^7x^6 + 378\,b^7c^2d^5x^5 + 252\,ab^6cd^6x^5 + 126\,a^2b^5d^7x^5 + 504\,b^7c^3d^4x^4 + 378\,ab^6c^2d^5x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 168*b^7*c*d^6*x^6 + 84*a*b^6*d^7*x^6 + 378*b^7*c^2*
d^5*x^5 + 252*a*b^6*c*d^6*x^5 + 126*a^2*b^5*d^7*x^5 + 504*b^7*c^3*d^4*x^4 +
 378*a*b^6*c^2*d^5*x^4 + 252*a^2*b^5*c*d^6*x^4 + 126*a^3*b^4*d^7*x^4 + 420*
b^7*c^4*d^3*x^3 + 336*a*b^6*c^3*d^4*x^3 + 252*a^2*b^5*c^2*d^5*x^3 + 168*a^3
*b^4*c*d^6*x^3 + 84*a^4*b^3*d^7*x^3 + 216*b^7*c^5*d^2*x^2 + 180*a*b^6*c^4*d
^3*x^2 + 144*a^2*b^5*c^3*d^4*x^2 + 108*a^3*b^4*c^2*d^5*x^2 + 72*a^4*b^3*c*d
^6*x^2 + 36*a^5*b^2*d^7*x^2 + 63*b^7*c^6*d*x + 54*a*b^6*c^5*d^2*x + 45*a^2*
b^5*c^4*d^3*x + 36*a^3*b^4*c^3*d^4*x + 27*a^4*b^3*c^2*d^5*x + 18*a^5*b^2*c*
d^6*x + 9*a^6*b*d^7*x + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7)/((b*x + a)^9*b^8)
```

3.1293 $\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx$

Optimal. Leaf size=89

$$-\frac{d^2(c+dx)^8}{360(a+bx)^8(bc-ad)^3}+\frac{d(c+dx)^8}{45(a+bx)^9(bc-ad)^2}-\frac{(c+dx)^8}{10(a+bx)^{10}(bc-ad)}$$

```
[Out] -(c + d*x)^8/(10*(b*c - a*d)*(a + b*x)^10) + (d*(c + d*x)^8)/(45*(b*c - a*d
)^2*(a + b*x)^9) - (d^2*(c + d*x)^8)/(360*(b*c - a*d)^3*(a + b*x)^8)
```

Rubi [A] time = 0.0218066, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{d^2(c+dx)^8}{360(a+bx)^8(bc-ad)^3}+\frac{d(c+dx)^8}{45(a+bx)^9(bc-ad)^2}-\frac{(c+dx)^8}{10(a+bx)^{10}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^11,x]
```

```
[Out] -(c + d*x)^8/(10*(b*c - a*d)*(a + b*x)^10) + (d*(c + d*x)^8)/(45*(b*c - a*d
)^2*(a + b*x)^9) - (d^2*(c + d*x)^8)/(360*(b*c - a*d)^3*(a + b*x)^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx &= -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}}-\frac{d\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx}{5(bc-ad)}\\
&= -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}}+\frac{d(c+dx)^8}{45(bc-ad)^2(a+bx)^9}+\frac{d^2\int \frac{(c+dx)^7}{(a+bx)^9}\,dx}{45(bc-ad)^2}\\
&= -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}}+\frac{d(c+dx)^8}{45(bc-ad)^2(a+bx)^9}-\frac{d^2(c+dx)^8}{360(bc-ad)^3(a+bx)^8}
\end{aligned}$$

Mathematica [B] time = 0.118284, size = 371, normalized size = 4.17

$$-\frac{3a^2b^5d^2\left(150c^3d^2x^2+240c^2d^3x^3+50c^4dx+7c^5+210cd^4x^4+84d^5x^5\right)+5a^3b^4d^3\left(54c^2d^2x^2+20c^3dx+3c^4+72cd^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^11,x]
```

```
[Out] -(a^7*d^7 + a^6*b*d^6*(3*c + 10*d*x) + 3*a^5*b^2*d^5*(2*c^2 + 10*c*d*x + 15
*d^2*x^2) + 5*a^4*b^3*d^4*(2*c^3 + 12*c^2*d*x + 27*c*d^2*x^2 + 24*d^3*x^3)
+ 5*a^3*b^4*d^3*(3*c^4 + 20*c^3*d*x + 54*c^2*d^2*x^2 + 72*c*d^3*x^3 + 42*d^
4*x^4) + 3*a^2*b^5*d^2*(7*c^5 + 50*c^4*d*x + 150*c^3*d^2*x^2 + 240*c^2*d^3*
x^3 + 210*c*d^4*x^4 + 84*d^5*x^5) + a*b^6*d*(28*c^6 + 210*c^5*d*x + 675*c^4
*d^2*x^2 + 1200*c^3*d^3*x^3 + 1260*c^2*d^4*x^4 + 756*c*d^5*x^5 + 210*d^6*x^
6) + b^7*(36*c^7 + 280*c^6*d*x + 945*c^5*d^2*x^2 + 1800*c^4*d^3*x^3 + 2100*
c^3*d^4*x^4 + 1512*c^2*d^5*x^5 + 630*c*d^6*x^6 + 120*d^7*x^7))/(360*b^8*(a
+ b*x)^10)
```

Maple [B] time = 0.006, size = 464, normalized size = 5.2

$$-\frac{-a^7d^7+7\,a^6cd^6b-21\,a^5b^2c^2d^5+35\,c^3d^4a^4b^3-35\,a^3b^4c^4d^3+21\,a^2c^5d^2b^5-7\,ac^6db^6+b^7c^7}{10\,b^8\,(bx+a)^{10}}+\frac{21\,d^2\left(a^5d^5-5\,a^4bcd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^11,x)
```

```
[Out] -1/10*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*
b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^10+21/8*d
^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4
*d-b^5*c^5)/b^8/(b*x+a)^8+35/6*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3
*c^3)/b^8/(b*x+a)^6-1/3*d^7/b^8/(b*x+a)^3-21/5*d^5*(a^2*d^2-2*a*b*c*d+b^2*c
^2)/b^8/(b*x+a)^5+7/4*d^6*(a*d-b*c)/b^8/(b*x+a)^4-7/9*d*(a^6*d^6-6*a^5*b*c*
d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+
b^6*c^6)/b^8/(b*x+a)^9-5*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b
^3*c^3*d+b^4*c^4)/b^8/(b*x+a)^7
```

Maxima [B] time = 1.08508, size = 755, normalized size = 8.48

$$-\frac{120\,b^7d^7x^7+36\,b^7c^7+28\,ab^6c^6d+21\,a^2b^5c^5d^2+15\,a^3b^4c^4d^3+10\,a^4b^3c^3d^4+6\,a^5b^2c^2d^5+3\,a^6bcd^6+a^7d^7+210\left(3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="maxima")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2
+ 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + a^7*d^7 + 210*(3*b^7*c*d^6 + a*b^6*d^7)*x^6 + 252*(6*b^7*c^2*d^5 + 3*a
*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 210*(10*b^7*c^3*d^4 + 6*a*b^6*c^2*d^5 + 3*a
^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 120*(15*b^7*c^4*d^3 + 10*a*b^6*c^3*d^4 +
6*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 45*(21*b^7*c^5*d^2
```

```
+ 15*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 6*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*
d^6 + a^5*b^2*d^7)*x^2 + 10*(28*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 15*a^2*b^5*c
^4*d^3 + 10*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6 + a^6*b*d
^7)*x)/(b^18*x^10 + 10*a*b^17*x^9 + 45*a^2*b^16*x^8 + 120*a^3*b^15*x^7 + 21
0*a^4*b^14*x^6 + 252*a^5*b^13*x^5 + 210*a^6*b^12*x^4 + 120*a^7*b^11*x^3 + 4
5*a^8*b^10*x^2 + 10*a^9*b^9*x + a^10*b^8)
```

Fricas [B] time = 2.03056, size = 1172, normalized size = 13.17

$$-\frac{120\,b^7d^7x^7 + 36\,b^7c^7 + 28\,ab^6c^6d + 21\,a^2b^5c^5d^2 + 15\,a^3b^4c^4d^3 + 10\,a^4b^3c^3d^4 + 6\,a^5b^2c^2d^5 + 3\,a^6bcd^6 + a^7d^7 + 210\left(3\,b^7c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="fricas")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2
+ 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + a^7*d^7 + 210*(3*b^7*c*d^6 + a*b^6*d^7)*x^6 + 252*(6*b^7*c^2*d^5 + 3*a
*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 210*(10*b^7*c^3*d^4 + 6*a*b^6*c^2*d^5 + 3*a
^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 120*(15*b^7*c^4*d^3 + 10*a*b^6*c^3*d^4 +
6*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 45*(21*b^7*c^5*d^2
+ 15*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 6*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*
d^6 + a^5*b^2*d^7)*x^2 + 10*(28*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 15*a^2*b^5*c
^4*d^3 + 10*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6 + a^6*b*d
^7)*x)/(b^18*x^10 + 10*a*b^17*x^9 + 45*a^2*b^16*x^8 + 120*a^3*b^15*x^7 + 21
0*a^4*b^14*x^6 + 252*a^5*b^13*x^5 + 210*a^6*b^12*x^4 + 120*a^7*b^11*x^3 + 4
5*a^8*b^10*x^2 + 10*a^9*b^9*x + a^10*b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**11,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.05768, size = 670, normalized size = 7.53

$$-\frac{120\,b^7d^7x^7 + 630\,b^7cd^6x^6 + 210\,ab^6d^7x^6 + 1512\,b^7c^2d^5x^5 + 756\,ab^6cd^6x^5 + 252\,a^2b^5d^7x^5 + 2100\,b^7c^3d^4x^4 + 1260\,ab^6c^2d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="giac")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 630*b^7*c*d^6*x^6 + 210*a*b^6*d^7*x^6 + 1512*b^7*
c^2*d^5*x^5 + 756*a*b^6*c*d^6*x^5 + 252*a^2*b^5*d^7*x^5 + 2100*b^7*c^3*d^4*
x^4 + 1260*a*b^6*c^2*d^5*x^4 + 630*a^2*b^5*c*d^6*x^4 + 210*a^3*b^4*d^7*x^4
```

+ 1800*b^7*c^4*d^3*x^3 + 1200*a*b^6*c^3*d^4*x^3 + 720*a^2*b^5*c^2*d^5*x^3 + 360*a^3*b^4*c*d^6*x^3 + 120*a^4*b^3*d^7*x^3 + 945*b^7*c^5*d^2*x^2 + 675*a*b^6*c^4*d^3*x^2 + 450*a^2*b^5*c^3*d^4*x^2 + 270*a^3*b^4*c^2*d^5*x^2 + 135*a^4*b^3*c*d^6*x^2 + 45*a^5*b^2*d^7*x^2 + 280*b^7*c^6*d*x + 210*a*b^6*c^5*d^2*x + 150*a^2*b^5*c^4*d^3*x + 100*a^3*b^4*c^3*d^4*x + 60*a^4*b^3*c^2*d^5*x + 30*a^5*b^2*c*d^6*x + 10*a^6*b*d^7*x + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^10*b^8)

3.1294 $\int \frac{(c+dx)^7}{(a+bx)^{12}}\,dx$

Optimal. Leaf size=120

$$\frac{d^3(c+dx)^8}{1320(a+bx)^8(bc-ad)^4}-\frac{d^2(c+dx)^8}{165(a+bx)^9(bc-ad)^3}+\frac{3d(c+dx)^8}{110(a+bx)^{10}(bc-ad)^2}-\frac{(c+dx)^8}{11(a+bx)^{11}(bc-ad)}$$

```
[Out] -(c + d*x)^8/(11*(b*c - a*d)*(a + b*x)^11) + (3*d*(c + d*x)^8)/(110*(b*c -
a*d)^2*(a + b*x)^10) - (d^2*(c + d*x)^8)/(165*(b*c - a*d)^3*(a + b*x)^9) +
(d^3*(c + d*x)^8)/(1320*(b*c - a*d)^4*(a + b*x)^8)
```

Rubi [A] time = 0.0334245, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d^3(c+dx)^8}{1320(a+bx)^8(bc-ad)^4}-\frac{d^2(c+dx)^8}{165(a+bx)^9(bc-ad)^3}+\frac{3d(c+dx)^8}{110(a+bx)^{10}(bc-ad)^2}-\frac{(c+dx)^8}{11(a+bx)^{11}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^12,x]
```

```
[Out] -(c + d*x)^8/(11*(b*c - a*d)*(a + b*x)^11) + (3*d*(c + d*x)^8)/(110*(b*c -
a*d)^2*(a + b*x)^10) - (d^2*(c + d*x)^8)/(165*(b*c - a*d)^3*(a + b*x)^9) +
(d^3*(c + d*x)^8)/(1320*(b*c - a*d)^4*(a + b*x)^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^7}{(a+bx)^{12}}\,dx &= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}}-\frac{(3d)\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx}{11(bc-ad)}\\
&= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}}+\frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}}+\frac{\left(3d^2\right)\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx}{55(bc-ad)^2}\\
&= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}}+\frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}}-\frac{d^2(c+dx)^8}{165(bc-ad)^3(a+bx)^9}-\frac{d^3\int \frac{(c+dx)^7}{(a+bx)^9}\,dx}{165(bc-ad)^3}\\
&= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}}+\frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}}-\frac{d^2(c+dx)^8}{165(bc-ad)^3(a+bx)^9}+\frac{d^3(c+dx)^8}{1320(bc-ad)^4(a+bx)^8}
\end{aligned}$$

Mathematica [B] time = 0.115657, size = 369, normalized size = 3.08

$$-\frac{a^2b^5d^2\left(1100c^3d^2x^2+1650c^2d^3x^3+385c^4dx+56c^5+1320cd^4x^4+462d^5x^5\right)+5a^3b^4d^3\left(110c^2d^2x^2+44c^3dx+7c^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^12,x]
```

```
[Out] -(a^7*d^7 + a^6*b*d^6*(4*c + 11*d*x) + a^5*b^2*d^5*(10*c^2 + 44*c*d*x + 55*
d^2*x^2) + 5*a^4*b^3*d^4*(4*c^3 + 22*c^2*d*x + 44*c*d^2*x^2 + 33*d^3*x^3) +
 5*a^3*b^4*d^3*(7*c^4 + 44*c^3*d*x + 110*c^2*d^2*x^2 + 132*c*d^3*x^3 + 66*d
^4*x^4) + a^2*b^5*d^2*(56*c^5 + 385*c^4*d*x + 1100*c^3*d^2*x^2 + 1650*c^2*d
^3*x^3 + 1320*c*d^4*x^4 + 462*d^5*x^5) + a*b^6*d*(84*c^6 + 616*c^5*d*x + 19
25*c^4*d^2*x^2 + 3300*c^3*d^3*x^3 + 3300*c^2*d^4*x^4 + 1848*c*d^5*x^5 + 462
*d^6*x^6) + b^7*(120*c^7 + 924*c^6*d*x + 3080*c^5*d^2*x^2 + 5775*c^4*d^3*x^
3 + 6600*c^3*d^4*x^4 + 4620*c^2*d^5*x^5 + 1848*c*d^6*x^6 + 330*d^7*x^7))/(1
320*b^8*(a + b*x)^11)
```

Maple [B] time = 0.008, size = 464, normalized size = 3.9

$$-\frac{7\,d\left(a^6d^6-6\,a^5bcd^5+15\,a^4b^2c^2d^4-20\,a^3b^3c^3d^3+15\,a^2b^4c^4d^2-6\,ab^5c^5d+b^6c^6\right)}{10\,b^8\,(bx+a)^{10}}-\frac{35\,d^3\left(a^4d^4-4\,a^3bcd^3+6\,a^2b^2c\right.}{8\,b^8\,(bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^12,x)
```

```
[Out] -7/10*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2
*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^10-35/8*d^3*(a^4*d^4-4*a^3*
b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^8/(b*x+a)^8-7/2*d^5*(a^2
*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^6-1/11*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b
^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6
*c^6*d+b^7*c^7)/b^8/(b*x+a)^11+7/5*d^6*(a*d-b*c)/b^8/(b*x+a)^5-1/4*d^7/b^8/
(b*x+a)^4+7/3*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*
d^2+5*a*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^9+5*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b
^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^7
```

Maxima [B] time = 1.07124, size = 770, normalized size = 6.42

$$-\frac{330\,b^7d^7x^7+120\,b^7c^7+84\,ab^6c^6d+56\,a^2b^5c^5d^2+35\,a^3b^4c^4d^3+20\,a^4b^3c^3d^4+10\,a^5b^2c^2d^5+4\,a^6bcd^6+a^7d^7+462}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="maxima")
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 120*b^7*c^7 + 84*a*b^6*c^6*d + 56*a^2*b^5*c^5*d^
2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 4*a^6*b*
c*d^6 + a^7*d^7 + 462*(4*b^7*c*d^6 + a*b^6*d^7)*x^6 + 462*(10*b^7*c^2*d^5 +
 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 330*(20*b^7*c^3*d^4 + 10*a*b^6*c^2*d^5
```

```
+ 4*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 165*(35*b^7*c^4*d^3 + 20*a*b^6*c^3*d
^4 + 10*a^2*b^5*c^2*d^5 + 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 55*(56*b^7*c
^5*d^2 + 35*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 10*a^3*b^4*c^2*d^5 + 4*a^4
*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 11*(84*b^7*c^6*d + 56*a*b^6*c^5*d^2 + 35*a^
2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 10*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 +
 a^6*b*d^7)*x)/(b^19*x^11 + 11*a*b^18*x^10 + 55*a^2*b^17*x^9 + 165*a^3*b^16
*x^8 + 330*a^4*b^15*x^7 + 462*a^5*b^14*x^6 + 462*a^6*b^13*x^5 + 330*a^7*b^1
2*x^4 + 165*a^8*b^11*x^3 + 55*a^9*b^10*x^2 + 11*a^10*b^9*x + a^11*b^8)
```

Fricas [B] time = 1.75301, size = 1211, normalized size = 10.09

$$-\frac{330\,b^7d^7x^7+120\,b^7c^7+84\,ab^6c^6d+56\,a^2b^5c^5d^2+35\,a^3b^4c^4d^3+20\,a^4b^3c^3d^4+10\,a^5b^2c^2d^5+4\,a^6bcd^6+a^7d^7+462\,\big(4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="fricas")
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 120*b^7*c^7 + 84*a*b^6*c^6*d + 56*a^2*b^5*c^5*d^
2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 4*a^6*b*
c*d^6 + a^7*d^7 + 462*(4*b^7*c*d^6 + a*b^6*d^7)*x^6 + 462*(10*b^7*c^2*d^5 +
 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 330*(20*b^7*c^3*d^4 + 10*a*b^6*c^2*d^5
+ 4*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 165*(35*b^7*c^4*d^3 + 20*a*b^6*c^3*d
^4 + 10*a^2*b^5*c^2*d^5 + 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 55*(56*b^7*c
^5*d^2 + 35*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 10*a^3*b^4*c^2*d^5 + 4*a^4
*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 11*(84*b^7*c^6*d + 56*a*b^6*c^5*d^2 + 35*a^
2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 10*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 +
 a^6*b*d^7)*x)/(b^19*x^11 + 11*a*b^18*x^10 + 55*a^2*b^17*x^9 + 165*a^3*b^16
*x^8 + 330*a^4*b^15*x^7 + 462*a^5*b^14*x^6 + 462*a^6*b^13*x^5 + 330*a^7*b^1
2*x^4 + 165*a^8*b^11*x^3 + 55*a^9*b^10*x^2 + 11*a^10*b^9*x + a^11*b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**12,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.07188, size = 670, normalized size = 5.58

$$-\frac{330\,b^7d^7x^7+1848\,b^7cd^6x^6+462\,ab^6d^7x^6+4620\,b^7c^2d^5x^5+1848\,ab^6cd^6x^5+462\,a^2b^5d^7x^5+6600\,b^7c^3d^4x^4+3300\,ab^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="giac")
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 1848*b^7*c*d^6*x^6 + 462*a*b^6*d^7*x^6 + 4620*b^
7*c^2*d^5*x^5 + 1848*a*b^6*c*d^6*x^5 + 462*a^2*b^5*d^7*x^5 + 6600*b^7*c^3*d
```

```
^4*x^4 + 3300*a*b^6*c^2*d^5*x^4 + 1320*a^2*b^5*c*d^6*x^4 + 330*a^3*b^4*d^7*
x^4 + 5775*b^7*c^4*d^3*x^3 + 3300*a*b^6*c^3*d^4*x^3 + 1650*a^2*b^5*c^2*d^5*
x^3 + 660*a^3*b^4*c*d^6*x^3 + 165*a^4*b^3*d^7*x^3 + 3080*b^7*c^5*d^2*x^2 +
1925*a*b^6*c^4*d^3*x^2 + 1100*a^2*b^5*c^3*d^4*x^2 + 550*a^3*b^4*c^2*d^5*x^2
 + 220*a^4*b^3*c*d^6*x^2 + 55*a^5*b^2*d^7*x^2 + 924*b^7*c^6*d*x + 616*a*b^6
*c^5*d^2*x + 385*a^2*b^5*c^4*d^3*x + 220*a^3*b^4*c^3*d^4*x + 110*a^4*b^3*c^
2*d^5*x + 44*a^5*b^2*c*d^6*x + 11*a^6*b*d^7*x + 120*b^7*c^7 + 84*a*b^6*c^6*
d + 56*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b
^2*c^2*d^5 + 4*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^11*b^8)
```

3.1295 $\int \frac{(c+dx)^7}{(a+bx)^{13}} dx$

Optimal. Leaf size=151

$$-\frac{d^4(c+dx)^8}{3960(a+bx)^8(bc-ad)^5}+\frac{d^3(c+dx)^8}{495(a+bx)^9(bc-ad)^4}-\frac{d^2(c+dx)^8}{110(a+bx)^{10}(bc-ad)^3}+\frac{d(c+dx)^8}{33(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)}{12(a+bx)^{12}(b}$$

```
[Out] -(c + d*x)^8/(12*(b*c - a*d)*(a + b*x)^12) + (d*(c + d*x)^8)/(33*(b*c - a*d
)^2*(a + b*x)^11) - (d^2*(c + d*x)^8)/(110*(b*c - a*d)^3*(a + b*x)^10) + (d
^3*(c + d*x)^8)/(495*(b*c - a*d)^4*(a + b*x)^9) - (d^4*(c + d*x)^8)/(3960*(
b*c - a*d)^5*(a + b*x)^8)
```

Rubi [A] time = 0.0466408, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{d^4(c+dx)^8}{3960(a+bx)^8(bc-ad)^5}+\frac{d^3(c+dx)^8}{495(a+bx)^9(bc-ad)^4}-\frac{d^2(c+dx)^8}{110(a+bx)^{10}(bc-ad)^3}+\frac{d(c+dx)^8}{33(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)}{12(a+bx)^{12}(b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^13,x]
```


```
[Out] -(c + d*x)^8/(12*(b*c - a*d)*(a + b*x)^12) + (d*(c + d*x)^8)/(33*(b*c - a*d
)^2*(a + b*x)^11) - (d^2*(c + d*x)^8)/(110*(b*c - a*d)^3*(a + b*x)^10) + (d
^3*(c + d*x)^8)/(495*(b*c - a*d)^4*(a + b*x)^9) - (d^4*(c + d*x)^8)/(3960*(
b*c - a*d)^5*(a + b*x)^8)
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{13}}dx = -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} - \frac{d\int \frac{(c+dx)^7}{(a+bx)^{12}}dx}{3(bc-ad)}$$
$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} + \frac{d^2\int \frac{(c+dx)^7}{(a+bx)^{11}}dx}{11(bc-ad)^2}$$
$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} - \frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}} - \frac{d^3\int \frac{(c+dx)^7}{(a+bx)^{10}}dx}{55(bc-ad)^3}$$
$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} - \frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}} + \frac{d^3(c+dx)^8}{495(bc-ad)^4(a+b}$$
$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} - \frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}} + \frac{d^3(c+dx)^8}{495(bc-ad)^4(a+b}$$

Mathematica [B] time = 0.120301, size = 371, normalized size = 2.46

$$-\frac{3a^2b^5d^2\left(770c^3d^2x^2+1100c^2d^3x^3+280c^4dx+42c^5+825cd^4x^4+264d^5x^5\right)+5a^3b^4d^3\left(198c^2d^2x^2+84c^3dx+14c^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^13,x]
```

```
[Out] -(a^7*d^7 + a^6*b*d^6*(5*c + 12*d*x) + 3*a^5*b^2*d^5*(5*c^2 + 20*c*d*x + 22
*d^2*x^2) + 5*a^4*b^3*d^4*(7*c^3 + 36*c^2*d*x + 66*c*d^2*x^2 + 44*d^3*x^3)
+ 5*a^3*b^4*d^3*(14*c^4 + 84*c^3*d*x + 198*c^2*d^2*x^2 + 220*c*d^3*x^3 + 99
*d^4*x^4) + 3*a^2*b^5*d^2*(42*c^5 + 280*c^4*d*x + 770*c^3*d^2*x^2 + 1100*c^
2*d^3*x^3 + 825*c*d^4*x^4 + 264*d^5*x^5) + a*b^6*d*(210*c^6 + 1512*c^5*d*x
+ 4620*c^4*d^2*x^2 + 7700*c^3*d^3*x^3 + 7425*c^2*d^4*x^4 + 3960*c*d^5*x^5 +
 924*d^6*x^6) + b^7*(330*c^7 + 2520*c^6*d*x + 8316*c^5*d^2*x^2 + 15400*c^4*
d^3*x^3 + 17325*c^3*d^4*x^4 + 11880*c^2*d^5*x^5 + 4620*c*d^6*x^6 + 792*d^7*
x^7))/(3960*b^8*(a + b*x)^12)
```

Maple [B] time = 0.007, size = 464, normalized size = 3.1

$$\frac{21\,d^2\left(a^5d^5-5\,a^4bcd^4+10\,a^3b^2c^2d^3-10\,a^2b^3c^3d^2+5\,ab^4c^4d-b^5c^5\right)}{10\,b^8\,(bx+a)^{10}}+\frac{35\,d^4\left(a^3d^3-3\,a^2bcd^2+3\,ab^2c^2d-b^3c^3\right)}{8\,b^8\,(bx+a)^8}+\frac{7}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^13,x)
```

```
[Out] 21/10*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*
b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^10+35/8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c
^2*d-b^3*c^3)/b^8/(b*x+a)^8+7/6*d^6*(a*d-b*c)/b^8/(b*x+a)^6-7/11*d*(a^6*d^6
-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a
*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^11-1/5*d^7/b^8/(b*x+a)^5-1/12*(-a^7*d^7+7*a
^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*
b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^12-35/9*d^3*(a^4*d^4-4*a^3*b
*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^8/(b*x+a)^9-3*d^5*(a^2*d^
2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^7
```

Maxima [B] time = 1.11723, size = 784, normalized size = 5.19

$$-\frac{792\,b^7d^7x^7+330\,b^7c^7+210\,ab^6c^6d+126\,a^2b^5c^5d^2+70\,a^3b^4c^4d^3+35\,a^4b^3c^3d^4+15\,a^5b^2c^2d^5+5\,a^6bcd^6+a^7d^7+924}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="maxima")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 330*b^7*c^7 + 210*a*b^6*c^6*d + 126*a^2*b^5*c^5*
d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 5*a^6*
b*c*d^6 + a^7*d^7 + 924*(5*b^7*c*d^6 + a*b^6*d^7)*x^6 + 792*(15*b^7*c^2*d^5
 + 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 495*(35*b^7*c^3*d^4 + 15*a*b^6*c^2*d^
5 + 5*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 220*(70*b^7*c^4*d^3 + 35*a*b^6*c^3
*d^4 + 15*a^2*b^5*c^2*d^5 + 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 66*(126*b^
7*c^5*d^2 + 70*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 + 15*a^3*b^4*c^2*d^5 + 5*
a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 12*(210*b^7*c^6*d + 126*a*b^6*c^5*d^2 +
70*a^2*b^5*c^4*d^3 + 35*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 + 5*a^5*b^2*c*
d^6 + a^6*b*d^7)*x)/(b^20*x^12 + 12*a*b^19*x^11 + 66*a^2*b^18*x^10 + 220*a^
3*b^17*x^9 + 495*a^4*b^16*x^8 + 792*a^5*b^15*x^7 + 924*a^6*b^14*x^6 + 792*a
^7*b^13*x^5 + 495*a^8*b^12*x^4 + 220*a^9*b^11*x^3 + 66*a^10*b^10*x^2 + 12*a
^11*b^9*x + a^12*b^8)
```

Fricas [B] time = 1.84802, size = 1246, normalized size = 8.25

$$-\frac{792\,b^7d^7x^7+330\,b^7c^7+210\,ab^6c^6d+126\,a^2b^5c^5d^2+70\,a^3b^4c^4d^3+35\,a^4b^3c^3d^4+15\,a^5b^2c^2d^5+5\,a^6bcd^6+a^7d^7+924}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="fricas")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 330*b^7*c^7 + 210*a*b^6*c^6*d + 126*a^2*b^5*c^5*
d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 5*a^6*
b*c*d^6 + a^7*d^7 + 924*(5*b^7*c*d^6 + a*b^6*d^7)*x^6 + 792*(15*b^7*c^2*d^5
 + 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 495*(35*b^7*c^3*d^4 + 15*a*b^6*c^2*d^
5 + 5*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 220*(70*b^7*c^4*d^3 + 35*a*b^6*c^3
*d^4 + 15*a^2*b^5*c^2*d^5 + 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 66*(126*b^
7*c^5*d^2 + 70*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 + 15*a^3*b^4*c^2*d^5 + 5*
a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 12*(210*b^7*c^6*d + 126*a*b^6*c^5*d^2 +
70*a^2*b^5*c^4*d^3 + 35*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 + 5*a^5*b^2*c*
d^6 + a^6*b*d^7)*x)/(b^20*x^12 + 12*a*b^19*x^11 + 66*a^2*b^18*x^10 + 220*a^
3*b^17*x^9 + 495*a^4*b^16*x^8 + 792*a^5*b^15*x^7 + 924*a^6*b^14*x^6 + 792*a
^7*b^13*x^5 + 495*a^8*b^12*x^4 + 220*a^9*b^11*x^3 + 66*a^10*b^10*x^2 + 12*a
^11*b^9*x + a^12*b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**13,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.08083, size = 670, normalized size = 4.44

$$-\frac{792\,b^7d^7x^7+4620\,b^7cd^6x^6+924\,ab^6d^7x^6+11880\,b^7c^2d^5x^5+3960\,ab^6cd^6x^5+792\,a^2b^5d^7x^5+17325\,b^7c^3d^4x^4+742}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="giac")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 4620*b^7*c*d^6*x^6 + 924*a*b^6*d^7*x^6 + 11880*b
^7*c^2*d^5*x^5 + 3960*a*b^6*c*d^6*x^5 + 792*a^2*b^5*d^7*x^5 + 17325*b^7*c^3
*d^4*x^4 + 7425*a*b^6*c^2*d^5*x^4 + 2475*a^2*b^5*c*d^6*x^4 + 495*a^3*b^4*d^
7*x^4 + 15400*b^7*c^4*d^3*x^3 + 7700*a*b^6*c^3*d^4*x^3 + 3300*a^2*b^5*c^2*d
^5*x^3 + 1100*a^3*b^4*c*d^6*x^3 + 220*a^4*b^3*d^7*x^3 + 8316*b^7*c^5*d^2*x^
2 + 4620*a*b^6*c^4*d^3*x^2 + 2310*a^2*b^5*c^3*d^4*x^2 + 990*a^3*b^4*c^2*d^5
*x^2 + 330*a^4*b^3*c*d^6*x^2 + 66*a^5*b^2*d^7*x^2 + 2520*b^7*c^6*d*x + 1512
*a*b^6*c^5*d^2*x + 840*a^2*b^5*c^4*d^3*x + 420*a^3*b^4*c^3*d^4*x + 180*a^4*
b^3*c^2*d^5*x + 60*a^5*b^2*c*d^6*x + 12*a^6*b*d^7*x + 330*b^7*c^7 + 210*a*b
^6*c^6*d + 126*a^2*b^5*c^5*d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 +
15*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^12*b^8)
```

3.1296 $\int \frac{(c+dx)^7}{(a+bx)^{14}}\,dx$

Optimal. Leaf size=198

$$-\frac{d^6(bc-ad)}{b^8(a+bx)^7}-\frac{21d^5(bc-ad)^2}{8b^8(a+bx)^8}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-$$

```
[Out] -(b*c - a*d)^7/(13*b^8*(a + b*x)^13) - (7*d*(b*c - a*d)^6)/(12*b^8*(a + b*x
)^12) - (21*d^2*(b*c - a*d)^5)/(11*b^8*(a + b*x)^11) - (7*d^3*(b*c - a*d)^4
)/(2*b^8*(a + b*x)^10) - (35*d^4*(b*c - a*d)^3)/(9*b^8*(a + b*x)^9) - (21*d
^5*(b*c - a*d)^2)/(8*b^8*(a + b*x)^8) - (d^6*(b*c - a*d))/(b^8*(a + b*x)^7)
 - d^7/(6*b^8*(a + b*x)^6)
```

Rubi [A] time = 0.153446, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{d^6(bc-ad)}{b^8(a+bx)^7}-\frac{21d^5(bc-ad)^2}{8b^8(a+bx)^8}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^14,x]
```


```
[Out] -(b*c - a*d)^7/(13*b^8*(a + b*x)^13) - (7*d*(b*c - a*d)^6)/(12*b^8*(a + b*x
)^12) - (21*d^2*(b*c - a*d)^5)/(11*b^8*(a + b*x)^11) - (7*d^3*(b*c - a*d)^4
)/(2*b^8*(a + b*x)^10) - (35*d^4*(b*c - a*d)^3)/(9*b^8*(a + b*x)^9) - (21*d
^5*(b*c - a*d)^2)/(8*b^8*(a + b*x)^8) - (d^6*(b*c - a*d))/(b^8*(a + b*x)^7)
 - d^7/(6*b^8*(a + b*x)^6)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{14}}\,dx = \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{14}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{13}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{12}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{11}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{10}}+\frac{21d^5(bc-ad}{b^7(a+bx)^9}\right.$$
$$=-\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\frac{21d^5(bc-ad}{8b^8(a+bx)}$$

Mathematica [A] time = 0.121782, size = 369, normalized size = 1.86

$$-\frac{3a^2b^5d^2\left(1456c^3d^2x^2+2002c^2d^3x^3+546c^4dx+84c^5+1430cd^4x^4+429d^5x^5\right)+a^3b^4d^3\left(1638c^2d^2x^2+728c^3dx+126c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^14,x]
```

```
[Out] -(a^7*d^7 + a^6*b*d^6*(6*c + 13*d*x) + 3*a^5*b^2*d^5*(7*c^2 + 26*c*d*x + 26
*d^2*x^2) + a^4*b^3*d^4*(56*c^3 + 273*c^2*d*x + 468*c*d^2*x^2 + 286*d^3*x^3
) + a^3*b^4*d^3*(126*c^4 + 728*c^3*d*x + 1638*c^2*d^2*x^2 + 1716*c*d^3*x^3
+ 715*d^4*x^4) + 3*a^2*b^5*d^2*(84*c^5 + 546*c^4*d*x + 1456*c^3*d^2*x^2 + 2
002*c^2*d^3*x^3 + 1430*c*d^4*x^4 + 429*d^5*x^5) + a*b^6*d*(462*c^6 + 3276*c
^5*d*x + 9828*c^4*d^2*x^2 + 16016*c^3*d^3*x^3 + 15015*c^2*d^4*x^4 + 7722*c*
d^5*x^5 + 1716*d^6*x^6) + b^7*(792*c^7 + 6006*c^6*d*x + 19656*c^5*d^2*x^2 +
 36036*c^4*d^3*x^3 + 40040*c^3*d^4*x^4 + 27027*c^2*d^5*x^5 + 10296*c*d^6*x^
6 + 1716*d^7*x^7))/(10296*b^8*(a + b*x)^13)
```

Maple [B] time = 0.008, size = 463, normalized size = 2.3

$$-\frac{7\,d^3\left(a^4d^4-4\,a^3bcd^3+6\,a^2b^2c^2d^2-4\,ab^3c^3d+b^4c^4\right)}{2\,b^8\,(bx+a)^{10}}-\frac{21\,d^5\left(a^2d^2-2\,abcd+b^2c^2\right)}{8\,b^8\,(bx+a)^8}-\frac{d^7}{6\,b^8\,(bx+a)^6}+\frac{21\,d^2\left(a^5d^5-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^14,x)
```

```
[Out] -7/2*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^
8/(b*x+a)^10-21/8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^8-1/6*d^7/b^8
/(b*x+a)^6+21/11*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c
^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^11-1/13*(-a^7*d^7+7*a^6*b*c*d^6-2
1*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-
7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^13-7/12*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*
b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^
8/(b*x+a)^12+35/9*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*
x+a)^9+d^6*(a*d-b*c)/b^8/(b*x+a)^7
```

Maxima [B] time = 1.11617, size = 799, normalized size = 4.04

$$-\frac{1716\,b^7d^7x^7+792\,b^7c^7+462\,ab^6c^6d+252\,a^2b^5c^5d^2+126\,a^3b^4c^4d^3+56\,a^4b^3c^3d^4+21\,a^5b^2c^2d^5+6\,a^6bcd^6+a^7d^7+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 792*b^7*c^7 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^
5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a
^6*b*c*d^6 + a^7*d^7 + 1716*(6*b^7*c*d^6 + a*b^6*d^7)*x^6 + 1287*(21*b^7*c^
2*d^5 + 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 715*(56*b^7*c^3*d^4 + 21*a*b^6*c
^2*d^5 + 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 286*(126*b^7*c^4*d^3 + 56*a*b
^6*c^3*d^4 + 21*a^2*b^5*c^2*d^5 + 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 78*(
252*b^7*c^5*d^2 + 126*a*b^6*c^4*d^3 + 56*a^2*b^5*c^3*d^4 + 21*a^3*b^4*c^2*d
^5 + 6*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 13*(462*b^7*c^6*d + 252*a*b^6*c^5
*d^2 + 126*a^2*b^5*c^4*d^3 + 56*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 + 6*a^
5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^21*x^13 + 13*a*b^20*x^12 + 78*a^2*b^19*x^11
+ 286*a^3*b^18*x^10 + 715*a^4*b^17*x^9 + 1287*a^5*b^16*x^8 + 1716*a^6*b^15*
x^7 + 1716*a^7*b^14*x^6 + 1287*a^8*b^13*x^5 + 715*a^9*b^12*x^4 + 286*a^10*b
^11*x^3 + 78*a^11*b^10*x^2 + 13*a^12*b^9*x + a^13*b^8)
```

Fricas [B] time = 1.76347, size = 1291, normalized size = 6.52

$$-\frac{1716\, b^{7} d^{7} x^{7} + 792\, b^{7} c^{7} + 462\, a b^{6} c^{6} d + 252\, a^{2} b^{5} c^{5} d^{2} + 126\, a^{3} b^{4} c^{4} d^{3} + 56\, a^{4} b^{3} c^{3} d^{4} + 21\, a^{5} b^{2} c^{2} d^{5} + 6\, a^{6} b c d^{6} + a^{7} d^{7} + 17}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 792*b^7*c^7 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^
5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a
^6*b*c*d^6 + a^7*d^7 + 1716*(6*b^7*c*d^6 + a*b^6*d^7)*x^6 + 1287*(21*b^7*c^
2*d^5 + 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 715*(56*b^7*c^3*d^4 + 21*a*b^6*c
^2*d^5 + 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 286*(126*b^7*c^4*d^3 + 56*a*b
^6*c^3*d^4 + 21*a^2*b^5*c^2*d^5 + 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 78*(
252*b^7*c^5*d^2 + 126*a*b^6*c^4*d^3 + 56*a^2*b^5*c^3*d^4 + 21*a^3*b^4*c^2*d
^5 + 6*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 13*(462*b^7*c^6*d + 252*a*b^6*c^5
*d^2 + 126*a^2*b^5*c^4*d^3 + 56*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 + 6*a^
5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^21*x^13 + 13*a*b^20*x^12 + 78*a^2*b^19*x^11
+ 286*a^3*b^18*x^10 + 715*a^4*b^17*x^9 + 1287*a^5*b^16*x^8 + 1716*a^6*b^15*
x^7 + 1716*a^7*b^14*x^6 + 1287*a^8*b^13*x^5 + 715*a^9*b^12*x^4 + 286*a^10*b
^11*x^3 + 78*a^11*b^10*x^2 + 13*a^12*b^9*x + a^13*b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**14,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06428, size = 670, normalized size = 3.38

$$-\frac{1716\, b^{7} d^{7} x^{7} + 10296\, b^{7} c d^{6} x^{6} + 1716\, a b^{6} d^{7} x^{6} + 27027\, b^{7} c^{2} d^{5} x^{5} + 7722\, a b^{6} c d^{6} x^{5} + 1287\, a^{2} b^{5} d^{7} x^{5} + 40040\, b^{7} c^{3} d^{4} x^{4} + 15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="giac")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 10296*b^7*c*d^6*x^6 + 1716*a*b^6*d^7*x^6 + 270
27*b^7*c^2*d^5*x^5 + 7722*a*b^6*c*d^6*x^5 + 1287*a^2*b^5*d^7*x^5 + 40040*b^
7*c^3*d^4*x^4 + 15015*a*b^6*c^2*d^5*x^4 + 4290*a^2*b^5*c*d^6*x^4 + 715*a^3*
b^4*d^7*x^4 + 36036*b^7*c^4*d^3*x^3 + 16016*a*b^6*c^3*d^4*x^3 + 6006*a^2*b^
5*c^2*d^5*x^3 + 1716*a^3*b^4*c*d^6*x^3 + 286*a^4*b^3*d^7*x^3 + 19656*b^7*c^
5*d^2*x^2 + 9828*a*b^6*c^4*d^3*x^2 + 4368*a^2*b^5*c^3*d^4*x^2 + 1638*a^3*b^
4*c^2*d^5*x^2 + 468*a^4*b^3*c*d^6*x^2 + 78*a^5*b^2*d^7*x^2 + 6006*b^7*c^6*d
*x + 3276*a*b^6*c^5*d^2*x + 1638*a^2*b^5*c^4*d^3*x + 728*a^3*b^4*c^3*d^4*x
+ 273*a^4*b^3*c^2*d^5*x + 78*a^5*b^2*c*d^6*x + 13*a^6*b*d^7*x + 792*b^7*c^7
 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3
*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^13*b^8)
```

3.1297 $\int \frac{(c+dx)^7}{(a+bx)^{15}}\,dx$

Optimal. Leaf size=200

$$-\frac{7d^6(bc-ad)}{8b^8(a+bx)^8}-\frac{7d^5(bc-ad)^2}{3b^8(a+bx)^9}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}$$

```
[Out] -(b*c - a*d)^7/(14*b^8*(a + b*x)^14) - (7*d*(b*c - a*d)^6)/(13*b^8*(a + b*x
)^13) - (7*d^2*(b*c - a*d)^5)/(4*b^8*(a + b*x)^12) - (35*d^3*(b*c - a*d)^4)
/(11*b^8*(a + b*x)^11) - (7*d^4*(b*c - a*d)^3)/(2*b^8*(a + b*x)^10) - (7*d^
5*(b*c - a*d)^2)/(3*b^8*(a + b*x)^9) - (7*d^6*(b*c - a*d))/(8*b^8*(a + b*x)
^8) - d^7/(7*b^8*(a + b*x)^7)
```

Rubi [A] time = 0.142819, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{7d^6(bc-ad)}{8b^8(a+bx)^8}-\frac{7d^5(bc-ad)^2}{3b^8(a+bx)^9}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^15,x]
```

```
[Out] -(b*c - a*d)^7/(14*b^8*(a + b*x)^14) - (7*d*(b*c - a*d)^6)/(13*b^8*(a + b*x
)^13) - (7*d^2*(b*c - a*d)^5)/(4*b^8*(a + b*x)^12) - (35*d^3*(b*c - a*d)^4)
/(11*b^8*(a + b*x)^11) - (7*d^4*(b*c - a*d)^3)/(2*b^8*(a + b*x)^10) - (7*d^
5*(b*c - a*d)^2)/(3*b^8*(a + b*x)^9) - (7*d^6*(b*c - a*d))/(8*b^8*(a + b*x)
^8) - d^7/(7*b^8*(a + b*x)^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^{15}}\,dx &= \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{15}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{14}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{13}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{12}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{11}}+\frac{21d^5(bc-}{b^7(a+b}\right.\\ &= -\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\frac{7d^5(bc-a}{3b^8(a+b}\end{aligned}$$

Mathematica [A] time = 0.120119, size = 371, normalized size = 1.86

$$-\frac{7a^2b^5d^2\left(1092c^3d^2x^2+1456c^2d^3x^3+420c^4dx+66c^5+1001cd^4x^4+286d^5x^5\right)+7a^3b^4d^3\left(364c^2d^2x^2+168c^3dx+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^15,x]
```

```
[Out] -(a^7*d^7 + 7*a^6*b*d^6*(c + 2*d*x) + 7*a^5*b^2*d^5*(4*c^2 + 14*c*d*x + 13*
d^2*x^2) + 7*a^4*b^3*d^4*(12*c^3 + 56*c^2*d*x + 91*c*d^2*x^2 + 52*d^3*x^3)
+ 7*a^3*b^4*d^3*(30*c^4 + 168*c^3*d*x + 364*c^2*d^2*x^2 + 364*c*d^3*x^3 + 1
43*d^4*x^4) + 7*a^2*b^5*d^2*(66*c^5 + 420*c^4*d*x + 1092*c^3*d^2*x^2 + 1456
*c^2*d^3*x^3 + 1001*c*d^4*x^4 + 286*d^5*x^5) + 7*a*b^6*d*(132*c^6 + 924*c^5
*d*x + 2730*c^4*d^2*x^2 + 4368*c^3*d^3*x^3 + 4004*c^2*d^4*x^4 + 2002*c*d^5*
x^5 + 429*d^6*x^6) + b^7*(1716*c^7 + 12936*c^6*d*x + 42042*c^5*d^2*x^2 + 76
440*c^4*d^3*x^3 + 84084*c^3*d^4*x^4 + 56056*c^2*d^5*x^5 + 21021*c*d^6*x^6 +
 3432*d^7*x^7))/(24024*b^8*(a + b*x)^14)
```

Maple [B] time = 0.008, size = 464, normalized size = 2.3

$$\frac{7\,d^4\left(a^3d^3-3\,a^2bcd^2+3\,ab^2c^2d-b^3c^3\right)}{2\,b^8\,(bx+a)^{10}}+\frac{7\,d^6\,(ad-bc)}{8\,b^8\,(bx+a)^8}-\frac{-a^7d^7+7\,a^6cd^6b-21\,a^5b^2c^2d^5+35\,c^3d^4a^4b^3-35\,a^3b^4c^4d^3+}{14\,b^8\,(bx+a)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^15,x)
```

```
[Out] 7/2*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^10+7/8*d^
6*(a*d-b*c)/b^8/(b*x+a)^8-1/14*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+3
5*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c
^7)/b^8/(b*x+a)^14-35/11*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b
^3*c^3*d+b^4*c^4)/b^8/(b*x+a)^11-7/13*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c
^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*
x+a)^13+7/4*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^
2+5*a*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^12-7/3*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)
/b^8/(b*x+a)^9-1/7*d^7/b^8/(b*x+a)^7
```

Maxima [B] time = 1.13914, size = 814, normalized size = 4.07

$$-\frac{3432\,b^7d^7x^7+1716\,b^7c^7+924\,ab^6c^6d+462\,a^2b^5c^5d^2+210\,a^3b^4c^4d^3+84\,a^4b^3c^3d^4+28\,a^5b^2c^2d^5+7\,a^6bcd^6+a^7d^7+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="maxima")
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 1716*b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c
^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*
a^6*b*c*d^6 + a^7*d^7 + 3003*(7*b^7*c*d^6 + a*b^6*d^7)*x^6 + 2002*(28*b^7*c
^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1001*(84*b^7*c^3*d^4 + 28*a*b^6
*c^2*d^5 + 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 364*(210*b^7*c^4*d^3 + 84*a
*b^6*c^3*d^4 + 28*a^2*b^5*c^2*d^5 + 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 91
*(462*b^7*c^5*d^2 + 210*a*b^6*c^4*d^3 + 84*a^2*b^5*c^3*d^4 + 28*a^3*b^4*c^2
*d^5 + 7*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 14*(924*b^7*c^6*d + 462*a*b^6*c
^5*d^2 + 210*a^2*b^5*c^4*d^3 + 84*a^3*b^4*c^3*d^4 + 28*a^4*b^3*c^2*d^5 + 7*
a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^22*x^14 + 14*a*b^21*x^13 + 91*a^2*b^20*x^1
2 + 364*a^3*b^19*x^11 + 1001*a^4*b^18*x^10 + 2002*a^5*b^17*x^9 + 3003*a^6*b
^16*x^8 + 3432*a^7*b^15*x^7 + 3003*a^8*b^14*x^6 + 2002*a^9*b^13*x^5 + 1001*
a^10*b^12*x^4 + 364*a^11*b^11*x^3 + 91*a^12*b^10*x^2 + 14*a^13*b^9*x + a^14
```

*b^8)

Fricas [B] time = 1.85291, size = 1326, normalized size = 6.63

$$-\frac{3432\,b^{7}d^{7}x^{7}+1716\,b^{7}c^{7}+924\,ab^{6}c^{6}d+462\,a^{2}b^{5}c^{5}d^{2}+210\,a^{3}b^{4}c^{4}d^{3}+84\,a^{4}b^{3}c^{3}d^{4}+28\,a^{5}b^{2}c^{2}d^{5}+7\,a^{6}bcd^{6}+a^{7}d^{7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="fricas")
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 1716*b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c
^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*
a^6*b*c*d^6 + a^7*d^7 + 3003*(7*b^7*c*d^6 + a*b^6*d^7)*x^6 + 2002*(28*b^7*c
^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1001*(84*b^7*c^3*d^4 + 28*a*b^6
*c^2*d^5 + 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 364*(210*b^7*c^4*d^3 + 84*a
*b^6*c^3*d^4 + 28*a^2*b^5*c^2*d^5 + 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 91
*(462*b^7*c^5*d^2 + 210*a*b^6*c^4*d^3 + 84*a^2*b^5*c^3*d^4 + 28*a^3*b^4*c^2
*d^5 + 7*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 14*(924*b^7*c^6*d + 462*a*b^6*c
^5*d^2 + 210*a^2*b^5*c^4*d^3 + 84*a^3*b^4*c^3*d^4 + 28*a^4*b^3*c^2*d^5 + 7*
a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^22*x^14 + 14*a*b^21*x^13 + 91*a^2*b^20*x^1
2 + 364*a^3*b^19*x^11 + 1001*a^4*b^18*x^10 + 2002*a^5*b^17*x^9 + 3003*a^6*b
^16*x^8 + 3432*a^7*b^15*x^7 + 3003*a^8*b^14*x^6 + 2002*a^9*b^13*x^5 + 1001*
a^10*b^12*x^4 + 364*a^11*b^11*x^3 + 91*a^12*b^10*x^2 + 14*a^13*b^9*x + a^14
*b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**15,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06709, size = 670, normalized size = 3.35

$$-\frac{3432\,b^{7}d^{7}x^{7}+21021\,b^{7}cd^{6}x^{6}+3003\,ab^{6}d^{7}x^{6}+56056\,b^{7}c^{2}d^{5}x^{5}+14014\,ab^{6}cd^{6}x^{5}+2002\,a^{2}b^{5}d^{7}x^{5}+84084\,b^{7}c^{3}d^{4}x^{4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="giac")
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 21021*b^7*c*d^6*x^6 + 3003*a*b^6*d^7*x^6 + 560
56*b^7*c^2*d^5*x^5 + 14014*a*b^6*c*d^6*x^5 + 2002*a^2*b^5*d^7*x^5 + 84084*b
^7*c^3*d^4*x^4 + 28028*a*b^6*c^2*d^5*x^4 + 7007*a^2*b^5*c*d^6*x^4 + 1001*a^
3*b^4*d^7*x^4 + 76440*b^7*c^4*d^3*x^3 + 30576*a*b^6*c^3*d^4*x^3 + 10192*a^2
*b^5*c^2*d^5*x^3 + 2548*a^3*b^4*c*d^6*x^3 + 364*a^4*b^3*d^7*x^3 + 42042*b^7
*c^5*d^2*x^2 + 19110*a*b^6*c^4*d^3*x^2 + 7644*a^2*b^5*c^3*d^4*x^2 + 2548*a^
```

```
3*b^4*c^2*d^5*x^2 + 637*a^4*b^3*c*d^6*x^2 + 91*a^5*b^2*d^7*x^2 + 12936*b^7*
c^6*d*x + 6468*a*b^6*c^5*d^2*x + 2940*a^2*b^5*c^4*d^3*x + 1176*a^3*b^4*c^3*
d^4*x + 392*a^4*b^3*c^2*d^5*x + 98*a^5*b^2*c*d^6*x + 14*a^6*b*d^7*x + 1716*
b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*
a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^
14*b^8)
```

3.1298 $\int \frac{(c+dx)^7}{(a+bx)^{16}}\,dx$

Optimal. Leaf size=200

$$-\frac{7d^6(bc-ad)}{9b^8(a+bx)^9}-\frac{21d^5(bc-ad)^2}{10b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}$$

```
[Out] -(b*c - a*d)^7/(15*b^8*(a + b*x)^15) - (d*(b*c - a*d)^6)/(2*b^8*(a + b*x)^1
4) - (21*d^2*(b*c - a*d)^5)/(13*b^8*(a + b*x)^13) - (35*d^3*(b*c - a*d)^4)/
(12*b^8*(a + b*x)^12) - (35*d^4*(b*c - a*d)^3)/(11*b^8*(a + b*x)^11) - (21*
d^5*(b*c - a*d)^2)/(10*b^8*(a + b*x)^10) - (7*d^6*(b*c - a*d))/(9*b^8*(a +
b*x)^9) - d^7/(8*b^8*(a + b*x)^8)
```

Rubi [A] time = 0.139636, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{7d^6(bc-ad)}{9b^8(a+bx)^9}-\frac{21d^5(bc-ad)^2}{10b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^16,x]
```

```
[Out] -(b*c - a*d)^7/(15*b^8*(a + b*x)^15) - (d*(b*c - a*d)^6)/(2*b^8*(a + b*x)^1
4) - (21*d^2*(b*c - a*d)^5)/(13*b^8*(a + b*x)^13) - (35*d^3*(b*c - a*d)^4)/
(12*b^8*(a + b*x)^12) - (35*d^4*(b*c - a*d)^3)/(11*b^8*(a + b*x)^11) - (21*
d^5*(b*c - a*d)^2)/(10*b^8*(a + b*x)^10) - (7*d^6*(b*c - a*d))/(9*b^8*(a +
b*x)^9) - d^7/(8*b^8*(a + b*x)^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{16}}\,dx=\int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{16}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{15}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{14}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{13}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{12}}+\frac{21d^5(bc-}{b^7(a+b}\right.$$
$$=-\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-\frac{21d^5(bc-}{10b^8(a+}$$

Mathematica [A] time = 0.120596, size = 371, normalized size = 1.86

$$-\frac{3a^2b^5d^2\left(4200c^3d^2x^2+5460c^2d^3x^3+1650c^4dx+264c^5+3640cd^4x^4+1001d^5x^5\right)+5a^3b^4d^3\left(756c^2d^2x^2+360c^3dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^16,x]
```

```
[Out] -(a^7*d^7 + a^6*b*d^6*(8*c + 15*d*x) + 3*a^5*b^2*d^5*(12*c^2 + 40*c*d*x + 3
5*d^2*x^2) + 5*a^4*b^3*d^4*(24*c^3 + 108*c^2*d*x + 168*c*d^2*x^2 + 91*d^3*x
^3) + 5*a^3*b^4*d^3*(66*c^4 + 360*c^3*d*x + 756*c^2*d^2*x^2 + 728*c*d^3*x^3
 + 273*d^4*x^4) + 3*a^2*b^5*d^2*(264*c^5 + 1650*c^4*d*x + 4200*c^3*d^2*x^2
+ 5460*c^2*d^3*x^3 + 3640*c*d^4*x^4 + 1001*d^5*x^5) + a*b^6*d*(1716*c^6 + 1
1880*c^5*d*x + 34650*c^4*d^2*x^2 + 54600*c^3*d^3*x^3 + 49140*c^2*d^4*x^4 +
24024*c*d^5*x^5 + 5005*d^6*x^6) + b^7*(3432*c^7 + 25740*c^6*d*x + 83160*c^5
*d^2*x^2 + 150150*c^4*d^3*x^3 + 163800*c^3*d^4*x^4 + 108108*c^2*d^5*x^5 + 4
0040*c*d^6*x^6 + 6435*d^7*x^7))/(51480*b^8*(a + b*x)^15)
```

Maple [B] time = 0.008, size = 464, normalized size = 2.3

$$-\frac{21\,d^5\left(a^2d^2-2\,abcd+b^2c^2\right)}{10\,b^8\,(bx+a)^{10}}-\frac{d^7}{8\,b^8\,(bx+a)^8}-\frac{d\left(a^6d^6-6\,a^5bcd^5+15\,a^4b^2c^2d^4-20\,a^3b^3c^3d^3+15\,a^2b^4c^4d^2-6\,ab^5c^5d\right.}{2\,b^8\,(bx+a)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^16,x)
```

```
[Out] -21/10*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^10-1/8*d^7/b^8/(b*x+a)^8
-1/2*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*
b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^14+35/11*d^4*(a^3*d^3-3*a^2*
b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^11+21/13*d^2*(a^5*d^5-5*a^4*b*c*
d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a
)^13-1/15*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*
a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^15-35
/12*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^8
/(b*x+a)^12+7/9*d^6*(a*d-b*c)/b^8/(b*x+a)^9
```

Maxima [B] time = 1.13135, size = 829, normalized size = 4.14

$$-\frac{6435\,b^7d^7x^7+3432\,b^7c^7+1716\,ab^6c^6d+792\,a^2b^5c^5d^2+330\,a^3b^4c^4d^3+120\,a^4b^3c^3d^4+36\,a^5b^2c^2d^5+8\,a^6bcd^6+a^7d^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="maxima")
```

```
[Out] -1/51480*(6435*b^7*d^7*x^7 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*
c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 +
8*a^6*b*c*d^6 + a^7*d^7 + 5005*(8*b^7*c*d^6 + a*b^6*d^7)*x^6 + 3003*(36*b^7
*c^2*d^5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1365*(120*b^7*c^3*d^4 + 36*a*
b^6*c^2*d^5 + 8*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 455*(330*b^7*c^4*d^3 + 1
20*a*b^6*c^3*d^4 + 36*a^2*b^5*c^2*d^5 + 8*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3
+ 105*(792*b^7*c^5*d^2 + 330*a*b^6*c^4*d^3 + 120*a^2*b^5*c^3*d^4 + 36*a^3*b
^4*c^2*d^5 + 8*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 15*(1716*b^7*c^6*d + 792*
a*b^6*c^5*d^2 + 330*a^2*b^5*c^4*d^3 + 120*a^3*b^4*c^3*d^4 + 36*a^4*b^3*c^2*
d^5 + 8*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^23*x^15 + 15*a*b^22*x^14 + 105*a^2
*b^21*x^13 + 455*a^3*b^20*x^12 + 1365*a^4*b^19*x^11 + 3003*a^5*b^18*x^10 +
5005*a^6*b^17*x^9 + 6435*a^7*b^16*x^8 + 6435*a^8*b^15*x^7 + 5005*a^9*b^14*x
^6 + 3003*a^10*b^13*x^5 + 1365*a^11*b^12*x^4 + 455*a^12*b^11*x^3 + 105*a^13
```

```
*b^10*x^2 + 15*a^14*b^9*x + a^15*b^8)
```

Fricas [B] time = 1.84946, size = 1369, normalized size = 6.84

$$-\frac{6435\,b^7d^7x^7 + 3432\,b^7c^7 + 1716\,ab^6c^6d + 792\,a^2b^5c^5d^2 + 330\,a^3b^4c^4d^3 + 120\,a^4b^3c^3d^4 + 36\,a^5b^2c^2d^5 + 8\,a^6bcd^6 + a^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="fricas")
```

```
[Out] -1/51480*(6435*b^7*d^7*x^7 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*
c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 +
8*a^6*b*c*d^6 + a^7*d^7 + 5005*(8*b^7*c*d^6 + a*b^6*d^7)*x^6 + 3003*(36*b^7
*c^2*d^5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1365*(120*b^7*c^3*d^4 + 36*a*
b^6*c^2*d^5 + 8*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 455*(330*b^7*c^4*d^3 + 1
20*a*b^6*c^3*d^4 + 36*a^2*b^5*c^2*d^5 + 8*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3
+ 105*(792*b^7*c^5*d^2 + 330*a*b^6*c^4*d^3 + 120*a^2*b^5*c^3*d^4 + 36*a^3*b
^4*c^2*d^5 + 8*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 15*(1716*b^7*c^6*d + 792*
a*b^6*c^5*d^2 + 330*a^2*b^5*c^4*d^3 + 120*a^3*b^4*c^3*d^4 + 36*a^4*b^3*c^2*
d^5 + 8*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^23*x^15 + 15*a*b^22*x^14 + 105*a^2
*b^21*x^13 + 455*a^3*b^20*x^12 + 1365*a^4*b^19*x^11 + 3003*a^5*b^18*x^10 +
5005*a^6*b^17*x^9 + 6435*a^7*b^16*x^8 + 6435*a^8*b^15*x^7 + 5005*a^9*b^14*x
^6 + 3003*a^10*b^13*x^5 + 1365*a^11*b^12*x^4 + 455*a^12*b^11*x^3 + 105*a^13
*b^10*x^2 + 15*a^14*b^9*x + a^15*b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**16,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06347, size = 670, normalized size = 3.35

$$-\frac{6435\,b^7d^7x^7 + 40040\,b^7cd^6x^6 + 5005\,ab^6d^7x^6 + 108108\,b^7c^2d^5x^5 + 24024\,ab^6cd^6x^5 + 3003\,a^2b^5d^7x^5 + 163800\,b^7c^3d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="giac")
```

```
[Out] -1/51480*(6435*b^7*d^7*x^7 + 40040*b^7*c*d^6*x^6 + 5005*a*b^6*d^7*x^6 + 108
108*b^7*c^2*d^5*x^5 + 24024*a*b^6*c*d^6*x^5 + 3003*a^2*b^5*d^7*x^5 + 163800
*b^7*c^3*d^4*x^4 + 49140*a*b^6*c^2*d^5*x^4 + 10920*a^2*b^5*c*d^6*x^4 + 1365
*a^3*b^4*d^7*x^4 + 150150*b^7*c^4*d^3*x^3 + 54600*a*b^6*c^3*d^4*x^3 + 16380
*a^2*b^5*c^2*d^5*x^3 + 3640*a^3*b^4*c*d^6*x^3 + 455*a^4*b^3*d^7*x^3 + 83160
*b^7*c^5*d^2*x^2 + 34650*a*b^6*c^4*d^3*x^2 + 12600*a^2*b^5*c^3*d^4*x^2 + 37
```

```
80*a^3*b^4*c^2*d^5*x^2 + 840*a^4*b^3*c*d^6*x^2 + 105*a^5*b^2*d^7*x^2 + 2574
0*b^7*c^6*d*x + 11880*a*b^6*c^5*d^2*x + 4950*a^2*b^5*c^4*d^3*x + 1800*a^3*b
^4*c^3*d^4*x + 540*a^4*b^3*c^2*d^5*x + 120*a^5*b^2*c*d^6*x + 15*a^6*b*d^7*x
 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*c^5*d^2 + 330*a^3*b^4*c^4*
d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 + 8*a^6*b*c*d^6 + a^7*d^7)/(
(b*x + a)^15*b^8)
```

3.1299 $\int (a+bx)^{12}(c+dx)^{10}\,dx$

Optimal. Leaf size=275

$$\frac{5d^9(a+bx)^{22}(bc-ad)}{11b^{11}}+\frac{15d^8(a+bx)^{21}(bc-ad)^2}{7b^{11}}+\frac{6d^7(a+bx)^{20}(bc-ad)^3}{b^{11}}+\frac{210d^6(a+bx)^{19}(bc-ad)^4}{19b^{11}}+\frac{14d^5(a+$$

```
[Out] ((b*c - a*d)^10*(a + b*x)^13)/(13*b^11) + (5*d*(b*c - a*d)^9*(a + b*x)^14)/
(7*b^11) + (3*d^2*(b*c - a*d)^8*(a + b*x)^15)/b^11 + (15*d^3*(b*c - a*d)^7*
(a + b*x)^16)/(2*b^11) + (210*d^4*(b*c - a*d)^6*(a + b*x)^17)/(17*b^11) + (
14*d^5*(b*c - a*d)^5*(a + b*x)^18)/b^11 + (210*d^6*(b*c - a*d)^4*(a + b*x)^
19)/(19*b^11) + (6*d^7*(b*c - a*d)^3*(a + b*x)^20)/b^11 + (15*d^8*(b*c - a*
d)^2*(a + b*x)^21)/(7*b^11) + (5*d^9*(b*c - a*d)*(a + b*x)^22)/(11*b^11) +
(d^10*(a + b*x)^23)/(23*b^11)
```

Rubi [A] time = 1.46517, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{5d^9(a+bx)^{22}(bc-ad)}{11b^{11}}+\frac{15d^8(a+bx)^{21}(bc-ad)^2}{7b^{11}}+\frac{6d^7(a+bx)^{20}(bc-ad)^3}{b^{11}}+\frac{210d^6(a+bx)^{19}(bc-ad)^4}{19b^{11}}+\frac{14d^5(a+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^12*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^13)/(13*b^11) + (5*d*(b*c - a*d)^9*(a + b*x)^14)/
(7*b^11) + (3*d^2*(b*c - a*d)^8*(a + b*x)^15)/b^11 + (15*d^3*(b*c - a*d)^7*
(a + b*x)^16)/(2*b^11) + (210*d^4*(b*c - a*d)^6*(a + b*x)^17)/(17*b^11) + (
14*d^5*(b*c - a*d)^5*(a + b*x)^18)/b^11 + (210*d^6*(b*c - a*d)^4*(a + b*x)^
19)/(19*b^11) + (6*d^7*(b*c - a*d)^3*(a + b*x)^20)/b^11 + (15*d^8*(b*c - a*
d)^2*(a + b*x)^21)/(7*b^11) + (5*d^9*(b*c - a*d)*(a + b*x)^22)/(11*b^11) +
(d^10*(a + b*x)^23)/(23*b^11)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{12}(c+dx)^{10}\,dx &= \int \left(\frac{(bc-ad)^{10}(a+bx)^{12}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{13}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx)^{14}}{b^{10}}+\frac{120d^3(\right.\\ &= \frac{(bc-ad)^{10}(a+bx)^{13}}{13b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{14}}{7b^{11}}+\frac{3d^2(bc-ad)^8(a+bx)^{15}}{b^{11}}+\frac{15d^3(bc-ad}{2b}\end{aligned}$$

Mathematica [B] time = 0.261506, size = 1817, normalized size = 6.61

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^12*(c + d*x)^10,x]
```

```
[Out] a^12*c^10*x + a^11*c^9*(6*b*c + 5*a*d)*x^2 + a^10*c^8*(22*b^2*c^2 + 40*a*b*
c*d + 15*a^2*d^2)*x^3 + 5*a^9*c^7*(11*b^3*c^3 + 33*a*b^2*c^2*d + 27*a^2*b*c
*d^2 + 6*a^3*d^3)*x^4 + a^8*c^6*(99*b^4*c^4 + 440*a*b^3*c^3*d + 594*a^2*b^2
*c^2*d^2 + 288*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + 3*a^7*c^5*(44*b^5*c^5 + 275*
a*b^4*c^4*d + 550*a^2*b^3*c^3*d^2 + 440*a^3*b^2*c^2*d^3 + 140*a^4*b*c*d^4 +
 14*a^5*d^5)*x^6 + (3*a^6*c^4*(308*b^6*c^6 + 2640*a*b^5*c^5*d + 7425*a^2*b^
4*c^4*d^2 + 8800*a^3*b^3*c^3*d^3 + 4620*a^4*b^2*c^2*d^4 + 1008*a^5*b*c*d^5
+ 70*a^6*d^6)*x^7)/7 + 3*a^5*c^3*(33*b^7*c^7 + 385*a*b^6*c^6*d + 1485*a^2*b
^5*c^5*d^2 + 2475*a^3*b^4*c^4*d^3 + 1925*a^4*b^3*c^3*d^4 + 693*a^5*b^2*c^2*
d^5 + 105*a^6*b*c*d^6 + 5*a^7*d^7)*x^8 + 5*a^4*c^2*(11*b^8*c^8 + 176*a*b^7*
c^7*d + 924*a^2*b^6*c^6*d^2 + 2112*a^3*b^5*c^5*d^3 + 2310*a^4*b^4*c^4*d^4 +
 1232*a^5*b^3*c^3*d^5 + 308*a^6*b^2*c^2*d^6 + 32*a^7*b*c*d^7 + a^8*d^8)*x^9
 + a^3*c*(22*b^9*c^9 + 495*a*b^8*c^8*d + 3564*a^2*b^7*c^7*d^2 + 11088*a^3*b
^6*c^6*d^3 + 16632*a^4*b^5*c^5*d^4 + 12474*a^5*b^4*c^4*d^5 + 4620*a^6*b^3*c
^3*d^6 + 792*a^7*b^2*c^2*d^7 + 54*a^8*b*c*d^8 + a^9*d^9)*x^10 + (a^2*(66*b^
10*c^10 + 2200*a*b^9*c^9*d + 22275*a^2*b^8*c^8*d^2 + 95040*a^3*b^7*c^7*d^3
+ 194040*a^4*b^6*c^6*d^4 + 199584*a^5*b^5*c^5*d^5 + 103950*a^6*b^4*c^4*d^6
+ 26400*a^7*b^3*c^3*d^7 + 2970*a^8*b^2*c^2*d^8 + 120*a^9*b*c*d^9 + a^10*d^1
0)*x^11)/11 + a*b*(b^10*c^10 + 55*a*b^9*c^9*d + 825*a^2*b^8*c^8*d^2 + 4950*
a^3*b^7*c^7*d^3 + 13860*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 + 13860*a^6
*b^4*c^4*d^6 + 4950*a^7*b^3*c^3*d^7 + 825*a^8*b^2*c^2*d^8 + 55*a^9*b*c*d^9
+ a^10*d^10)*x^12 + (b^2*(b^10*c^10 + 120*a*b^9*c^9*d + 2970*a^2*b^8*c^8*d^
2 + 26400*a^3*b^7*c^7*d^3 + 103950*a^4*b^6*c^6*d^4 + 199584*a^5*b^5*c^5*d^5
 + 194040*a^6*b^4*c^4*d^6 + 95040*a^7*b^3*c^3*d^7 + 22275*a^8*b^2*c^2*d^8 +
 2200*a^9*b*c*d^9 + 66*a^10*d^10)*x^13)/13 + (5*b^3*d*(b^9*c^9 + 54*a*b^8*c
^8*d + 792*a^2*b^7*c^7*d^2 + 4620*a^3*b^6*c^6*d^3 + 12474*a^4*b^5*c^5*d^4 +
 16632*a^5*b^4*c^4*d^5 + 11088*a^6*b^3*c^3*d^6 + 3564*a^7*b^2*c^2*d^7 + 495
*a^8*b*c*d^8 + 22*a^9*d^9)*x^14)/7 + 3*b^4*d^2*(b^8*c^8 + 32*a*b^7*c^7*d +
308*a^2*b^6*c^6*d^2 + 1232*a^3*b^5*c^5*d^3 + 2310*a^4*b^4*c^4*d^4 + 2112*a^
5*b^3*c^3*d^5 + 924*a^6*b^2*c^2*d^6 + 176*a^7*b*c*d^7 + 11*a^8*d^8)*x^15 +
(3*b^5*d^3*(5*b^7*c^7 + 105*a*b^6*c^6*d + 693*a^2*b^5*c^5*d^2 + 1925*a^3*b^
4*c^4*d^3 + 2475*a^4*b^3*c^3*d^4 + 1485*a^5*b^2*c^2*d^5 + 385*a^6*b*c*d^6 +
 33*a^7*d^7)*x^16)/2 + (3*b^6*d^4*(70*b^6*c^6 + 1008*a*b^5*c^5*d + 4620*a^2
*b^4*c^4*d^2 + 8800*a^3*b^3*c^3*d^3 + 7425*a^4*b^2*c^2*d^4 + 2640*a^5*b*c*d
^5 + 308*a^6*d^6)*x^17)/17 + b^7*d^5*(14*b^5*c^5 + 140*a*b^4*c^4*d + 440*a^
2*b^3*c^3*d^2 + 550*a^3*b^2*c^2*d^3 + 275*a^4*b*c*d^4 + 44*a^5*d^5)*x^18 +
(5*b^8*d^6*(42*b^4*c^4 + 288*a*b^3*c^3*d + 594*a^2*b^2*c^2*d^2 + 440*a^3*b*
c*d^3 + 99*a^4*d^4)*x^19)/19 + b^9*d^7*(6*b^3*c^3 + 27*a*b^2*c^2*d + 33*a^2
*b*c*d^2 + 11*a^3*d^3)*x^20 + (b^10*d^8*(15*b^2*c^2 + 40*a*b*c*d + 22*a^2*d
^2)*x^21)/7 + (b^11*d^9*(5*b*c + 6*a*d)*x^22)/11 + (b^12*d^10*x^23)/23
```

Maple [B] time = 0.004, size = 1891, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^12*(d*x+c)^10,x)
```

```
[Out] 1/23*b^12*d^10*x^23+1/22*(12*a*b^11*d^10+10*b^12*c*d^9)*x^22+1/21*(66*a^2*b
^10*d^10+120*a*b^11*c*d^9+45*b^12*c^2*d^8)*x^21+1/20*(220*a^3*b^9*d^10+660*
a^2*b^10*c*d^9+540*a*b^11*c^2*d^8+120*b^12*c^3*d^7)*x^20+1/19*(495*a^4*b^8*
d^10+2200*a^3*b^9*c*d^9+2970*a^2*b^10*c^2*d^8+1440*a*b^11*c^3*d^7+210*b^12*
c^4*d^6)*x^19+1/18*(792*a^5*b^7*d^10+4950*a^4*b^8*c*d^9+9900*a^3*b^9*c^2*d^
8+7920*a^2*b^10*c^3*d^7+2520*a*b^11*c^4*d^6+252*b^12*c^5*d^5)*x^18+1/17*(92
```

```
4*a^6*b^6*d^10+7920*a^5*b^7*c*d^9+22275*a^4*b^8*c^2*d^8+26400*a^3*b^9*c^3*d
^7+13860*a^2*b^10*c^4*d^6+3024*a*b^11*c^5*d^5+210*b^12*c^6*d^4)*x^17+1/16*(
792*a^7*b^5*d^10+9240*a^6*b^6*c*d^9+35640*a^5*b^7*c^2*d^8+59400*a^4*b^8*c^3
*d^7+46200*a^3*b^9*c^4*d^6+16632*a^2*b^10*c^5*d^5+2520*a*b^11*c^6*d^4+120*b
^12*c^7*d^3)*x^16+1/15*(495*a^8*b^4*d^10+7920*a^7*b^5*c*d^9+41580*a^6*b^6*c
^2*d^8+95040*a^5*b^7*c^3*d^7+103950*a^4*b^8*c^4*d^6+55440*a^3*b^9*c^5*d^5+1
3860*a^2*b^10*c^6*d^4+1440*a*b^11*c^7*d^3+45*b^12*c^8*d^2)*x^15+1/14*(220*a
^9*b^3*d^10+4950*a^8*b^4*c*d^9+35640*a^7*b^5*c^2*d^8+110880*a^6*b^6*c^3*d^7
+166320*a^5*b^7*c^4*d^6+124740*a^4*b^8*c^5*d^5+46200*a^3*b^9*c^6*d^4+7920*a
^2*b^10*c^7*d^3+540*a*b^11*c^8*d^2+10*b^12*c^9*d)*x^14+1/13*(66*a^10*b^2*d^
10+2200*a^9*b^3*c*d^9+22275*a^8*b^4*c^2*d^8+95040*a^7*b^5*c^3*d^7+194040*a^
6*b^6*c^4*d^6+199584*a^5*b^7*c^5*d^5+103950*a^4*b^8*c^6*d^4+26400*a^3*b^9*c
^7*d^3+2970*a^2*b^10*c^8*d^2+120*a*b^11*c^9*d+b^12*c^10)*x^13+1/12*(12*a^11
*b*d^10+660*a^10*b^2*c*d^9+9900*a^9*b^3*c^2*d^8+59400*a^8*b^4*c^3*d^7+16632
0*a^7*b^5*c^4*d^6+232848*a^6*b^6*c^5*d^5+166320*a^5*b^7*c^6*d^4+59400*a^4*b
^8*c^7*d^3+9900*a^3*b^9*c^8*d^2+660*a^2*b^10*c^9*d+12*a*b^11*c^10)*x^12+1/1
1*(a^12*d^10+120*a^11*b*c*d^9+2970*a^10*b^2*c^2*d^8+26400*a^9*b^3*c^3*d^7+1
03950*a^8*b^4*c^4*d^6+199584*a^7*b^5*c^5*d^5+194040*a^6*b^6*c^6*d^4+95040*a
^5*b^7*c^7*d^3+22275*a^4*b^8*c^8*d^2+2200*a^3*b^9*c^9*d+66*a^2*b^10*c^10)*x
^11+1/10*(10*a^12*c*d^9+540*a^11*b*c^2*d^8+7920*a^10*b^2*c^3*d^7+46200*a^9*
b^3*c^4*d^6+124740*a^8*b^4*c^5*d^5+166320*a^7*b^5*c^6*d^4+110880*a^6*b^6*c^
7*d^3+35640*a^5*b^7*c^8*d^2+4950*a^4*b^8*c^9*d+220*a^3*b^9*c^10)*x^10+1/9*(
45*a^12*c^2*d^8+1440*a^11*b*c^3*d^7+13860*a^10*b^2*c^4*d^6+55440*a^9*b^3*c^
5*d^5+103950*a^8*b^4*c^6*d^4+95040*a^7*b^5*c^7*d^3+41580*a^6*b^6*c^8*d^2+79
20*a^5*b^7*c^9*d+495*a^4*b^8*c^10)*x^9+1/8*(120*a^12*c^3*d^7+2520*a^11*b*c^
4*d^6+16632*a^10*b^2*c^5*d^5+46200*a^9*b^3*c^6*d^4+59400*a^8*b^4*c^7*d^3+35
640*a^7*b^5*c^8*d^2+9240*a^6*b^6*c^9*d+792*a^5*b^7*c^10)*x^8+1/7*(210*a^12*
c^4*d^6+3024*a^11*b*c^5*d^5+13860*a^10*b^2*c^6*d^4+26400*a^9*b^3*c^7*d^3+22
275*a^8*b^4*c^8*d^2+7920*a^7*b^5*c^9*d+924*a^6*b^6*c^10)*x^7+1/6*(252*a^12*
c^5*d^5+2520*a^11*b*c^6*d^4+7920*a^10*b^2*c^7*d^3+9900*a^9*b^3*c^8*d^2+4950
*a^8*b^4*c^9*d+792*a^7*b^5*c^10)*x^6+1/5*(210*a^12*c^6*d^4+1440*a^11*b*c^7*
d^3+2970*a^10*b^2*c^8*d^2+2200*a^9*b^3*c^9*d+495*a^8*b^4*c^10)*x^5+1/4*(120
*a^12*c^7*d^3+540*a^11*b*c^8*d^2+660*a^10*b^2*c^9*d+220*a^9*b^3*c^10)*x^4+1
/3*(45*a^12*c^8*d^2+120*a^11*b*c^9*d+66*a^10*b^2*c^10)*x^3+1/2*(10*a^12*c^9
*d+12*a^11*b*c^10)*x^2+a^12*c^10*x
```

Maxima [B] time = 1.01257, size = 2534, normalized size = 9.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/23*b^12*d^10*x^23 + a^12*c^10*x + 1/11*(5*b^12*c*d^9 + 6*a*b^11*d^10)*x^2
2 + 1/7*(15*b^12*c^2*d^8 + 40*a*b^11*c*d^9 + 22*a^2*b^10*d^10)*x^21 + (6*b^
12*c^3*d^7 + 27*a*b^11*c^2*d^8 + 33*a^2*b^10*c*d^9 + 11*a^3*b^9*d^10)*x^20
+ 5/19*(42*b^12*c^4*d^6 + 288*a*b^11*c^3*d^7 + 594*a^2*b^10*c^2*d^8 + 440*a
^3*b^9*c*d^9 + 99*a^4*b^8*d^10)*x^19 + (14*b^12*c^5*d^5 + 140*a*b^11*c^4*d^
6 + 440*a^2*b^10*c^3*d^7 + 550*a^3*b^9*c^2*d^8 + 275*a^4*b^8*c*d^9 + 44*a^5
*b^7*d^10)*x^18 + 3/17*(70*b^12*c^6*d^4 + 1008*a*b^11*c^5*d^5 + 4620*a^2*b^
10*c^4*d^6 + 8800*a^3*b^9*c^3*d^7 + 7425*a^4*b^8*c^2*d^8 + 2640*a^5*b^7*c*d
^9 + 308*a^6*b^6*d^10)*x^17 + 3/2*(5*b^12*c^7*d^3 + 105*a*b^11*c^6*d^4 + 69
3*a^2*b^10*c^5*d^5 + 1925*a^3*b^9*c^4*d^6 + 2475*a^4*b^8*c^3*d^7 + 1485*a^5
*b^7*c^2*d^8 + 385*a^6*b^6*c*d^9 + 33*a^7*b^5*d^10)*x^16 + 3*(b^12*c^8*d^2
+ 32*a*b^11*c^7*d^3 + 308*a^2*b^10*c^6*d^4 + 1232*a^3*b^9*c^5*d^5 + 2310*a^
4*b^8*c^4*d^6 + 2112*a^5*b^7*c^3*d^7 + 924*a^6*b^6*c^2*d^8 + 176*a^7*b^5*c*
```

```
d^9 + 11*a^8*b^4*d^10)*x^15 + 5/7*(b^12*c^9*d + 54*a*b^11*c^8*d^2 + 792*a^2
*b^10*c^7*d^3 + 4620*a^3*b^9*c^6*d^4 + 12474*a^4*b^8*c^5*d^5 + 16632*a^5*b^
7*c^4*d^6 + 11088*a^6*b^6*c^3*d^7 + 3564*a^7*b^5*c^2*d^8 + 495*a^8*b^4*c*d^
9 + 22*a^9*b^3*d^10)*x^14 + 1/13*(b^12*c^10 + 120*a*b^11*c^9*d + 2970*a^2*b
^10*c^8*d^2 + 26400*a^3*b^9*c^7*d^3 + 103950*a^4*b^8*c^6*d^4 + 199584*a^5*b
^7*c^5*d^5 + 194040*a^6*b^6*c^4*d^6 + 95040*a^7*b^5*c^3*d^7 + 22275*a^8*b^4
*c^2*d^8 + 2200*a^9*b^3*c*d^9 + 66*a^10*b^2*d^10)*x^13 + (a*b^11*c^10 + 55*
a^2*b^10*c^9*d + 825*a^3*b^9*c^8*d^2 + 4950*a^4*b^8*c^7*d^3 + 13860*a^5*b^7
*c^6*d^4 + 19404*a^6*b^6*c^5*d^5 + 13860*a^7*b^5*c^4*d^6 + 4950*a^8*b^4*c^3
*d^7 + 825*a^9*b^3*c^2*d^8 + 55*a^10*b^2*c*d^9 + a^11*b*d^10)*x^12 + 1/11*(
66*a^2*b^10*c^10 + 2200*a^3*b^9*c^9*d + 22275*a^4*b^8*c^8*d^2 + 95040*a^5*b
^7*c^7*d^3 + 194040*a^6*b^6*c^6*d^4 + 199584*a^7*b^5*c^5*d^5 + 103950*a^8*b
^4*c^4*d^6 + 26400*a^9*b^3*c^3*d^7 + 2970*a^10*b^2*c^2*d^8 + 120*a^11*b*c*d
^9 + a^12*d^10)*x^11 + (22*a^3*b^9*c^10 + 495*a^4*b^8*c^9*d + 3564*a^5*b^7*
c^8*d^2 + 11088*a^6*b^6*c^7*d^3 + 16632*a^7*b^5*c^6*d^4 + 12474*a^8*b^4*c^5
*d^5 + 4620*a^9*b^3*c^4*d^6 + 792*a^10*b^2*c^3*d^7 + 54*a^11*b*c^2*d^8 + a^
12*c*d^9)*x^10 + 5*(11*a^4*b^8*c^10 + 176*a^5*b^7*c^9*d + 924*a^6*b^6*c^8*d
^2 + 2112*a^7*b^5*c^7*d^3 + 2310*a^8*b^4*c^6*d^4 + 1232*a^9*b^3*c^5*d^5 + 3
08*a^10*b^2*c^4*d^6 + 32*a^11*b*c^3*d^7 + a^12*c^2*d^8)*x^9 + 3*(33*a^5*b^7
*c^10 + 385*a^6*b^6*c^9*d + 1485*a^7*b^5*c^8*d^2 + 2475*a^8*b^4*c^7*d^3 + 1
925*a^9*b^3*c^6*d^4 + 693*a^10*b^2*c^5*d^5 + 105*a^11*b*c^4*d^6 + 5*a^12*c^
3*d^7)*x^8 + 3/7*(308*a^6*b^6*c^10 + 2640*a^7*b^5*c^9*d + 7425*a^8*b^4*c^8*
d^2 + 8800*a^9*b^3*c^7*d^3 + 4620*a^10*b^2*c^6*d^4 + 1008*a^11*b*c^5*d^5 +
70*a^12*c^4*d^6)*x^7 + 3*(44*a^7*b^5*c^10 + 275*a^8*b^4*c^9*d + 550*a^9*b^3
*c^8*d^2 + 440*a^10*b^2*c^7*d^3 + 140*a^11*b*c^6*d^4 + 14*a^12*c^5*d^5)*x^6
 + (99*a^8*b^4*c^10 + 440*a^9*b^3*c^9*d + 594*a^10*b^2*c^8*d^2 + 288*a^11*b
*c^7*d^3 + 42*a^12*c^6*d^4)*x^5 + 5*(11*a^9*b^3*c^10 + 33*a^10*b^2*c^9*d +
27*a^11*b*c^8*d^2 + 6*a^12*c^7*d^3)*x^4 + (22*a^10*b^2*c^10 + 40*a^11*b*c^9
*d + 15*a^12*c^8*d^2)*x^3 + (6*a^11*b*c^10 + 5*a^12*c^9*d)*x^2
```

Fricas [B] time = 1.60203, size = 5079, normalized size = 18.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*d^10*b^12 + 5/11*x^22*d^9*c*b^12 + 6/11*x^22*d^10*b^11*a + 15/7*x
^21*d^8*c^2*b^12 + 40/7*x^21*d^9*c*b^11*a + 22/7*x^21*d^10*b^10*a^2 + 6*x^2
0*d^7*c^3*b^12 + 27*x^20*d^8*c^2*b^11*a + 33*x^20*d^9*c*b^10*a^2 + 11*x^20*
d^10*b^9*a^3 + 210/19*x^19*d^6*c^4*b^12 + 1440/19*x^19*d^7*c^3*b^11*a + 297
0/19*x^19*d^8*c^2*b^10*a^2 + 2200/19*x^19*d^9*c*b^9*a^3 + 495/19*x^19*d^10*
b^8*a^4 + 14*x^18*d^5*c^5*b^12 + 140*x^18*d^6*c^4*b^11*a + 440*x^18*d^7*c^3
*b^10*a^2 + 550*x^18*d^8*c^2*b^9*a^3 + 275*x^18*d^9*c*b^8*a^4 + 44*x^18*d^1
0*b^7*a^5 + 210/17*x^17*d^4*c^6*b^12 + 3024/17*x^17*d^5*c^5*b^11*a + 13860/
17*x^17*d^6*c^4*b^10*a^2 + 26400/17*x^17*d^7*c^3*b^9*a^3 + 22275/17*x^17*d^
8*c^2*b^8*a^4 + 7920/17*x^17*d^9*c*b^7*a^5 + 924/17*x^17*d^10*b^6*a^6 + 15/
2*x^16*d^3*c^7*b^12 + 315/2*x^16*d^4*c^6*b^11*a + 2079/2*x^16*d^5*c^5*b^10*
a^2 + 5775/2*x^16*d^6*c^4*b^9*a^3 + 7425/2*x^16*d^7*c^3*b^8*a^4 + 4455/2*x^
16*d^8*c^2*b^7*a^5 + 1155/2*x^16*d^9*c*b^6*a^6 + 99/2*x^16*d^10*b^5*a^7 + 3
*x^15*d^2*c^8*b^12 + 96*x^15*d^3*c^7*b^11*a + 924*x^15*d^4*c^6*b^10*a^2 + 3
696*x^15*d^5*c^5*b^9*a^3 + 6930*x^15*d^6*c^4*b^8*a^4 + 6336*x^15*d^7*c^3*b^
7*a^5 + 2772*x^15*d^8*c^2*b^6*a^6 + 528*x^15*d^9*c*b^5*a^7 + 33*x^15*d^10*b
^4*a^8 + 5/7*x^14*d*c^9*b^12 + 270/7*x^14*d^2*c^8*b^11*a + 3960/7*x^14*d^3*
c^7*b^10*a^2 + 3300*x^14*d^4*c^6*b^9*a^3 + 8910*x^14*d^5*c^5*b^8*a^4 + 1188
0*x^14*d^6*c^4*b^7*a^5 + 7920*x^14*d^7*c^3*b^6*a^6 + 17820/7*x^14*d^8*c^2*b
```

```
^5*a^7 + 2475/7*x^14*d^9*c*b^4*a^8 + 110/7*x^14*d^10*b^3*a^9 + 1/13*x^13*c^
10*b^12 + 120/13*x^13*d*c^9*b^11*a + 2970/13*x^13*d^2*c^8*b^10*a^2 + 26400/
13*x^13*d^3*c^7*b^9*a^3 + 103950/13*x^13*d^4*c^6*b^8*a^4 + 199584/13*x^13*d
^5*c^5*b^7*a^5 + 194040/13*x^13*d^6*c^4*b^6*a^6 + 95040/13*x^13*d^7*c^3*b^5
*a^7 + 22275/13*x^13*d^8*c^2*b^4*a^8 + 2200/13*x^13*d^9*c*b^3*a^9 + 66/13*x
^13*d^10*b^2*a^10 + x^12*c^10*b^11*a + 55*x^12*d*c^9*b^10*a^2 + 825*x^12*d^
2*c^8*b^9*a^3 + 4950*x^12*d^3*c^7*b^8*a^4 + 13860*x^12*d^4*c^6*b^7*a^5 + 19
404*x^12*d^5*c^5*b^6*a^6 + 13860*x^12*d^6*c^4*b^5*a^7 + 4950*x^12*d^7*c^3*b
^4*a^8 + 825*x^12*d^8*c^2*b^3*a^9 + 55*x^12*d^9*c*b^2*a^10 + x^12*d^10*b*a^
11 + 6*x^11*c^10*b^10*a^2 + 200*x^11*d*c^9*b^9*a^3 + 2025*x^11*d^2*c^8*b^8*
a^4 + 8640*x^11*d^3*c^7*b^7*a^5 + 17640*x^11*d^4*c^6*b^6*a^6 + 18144*x^11*d
^5*c^5*b^5*a^7 + 9450*x^11*d^6*c^4*b^4*a^8 + 2400*x^11*d^7*c^3*b^3*a^9 + 27
0*x^11*d^8*c^2*b^2*a^10 + 120/11*x^11*d^9*c*b*a^11 + 1/11*x^11*d^10*a^12 +
22*x^10*c^10*b^9*a^3 + 495*x^10*d*c^9*b^8*a^4 + 3564*x^10*d^2*c^8*b^7*a^5 +
 11088*x^10*d^3*c^7*b^6*a^6 + 16632*x^10*d^4*c^6*b^5*a^7 + 12474*x^10*d^5*c
^5*b^4*a^8 + 4620*x^10*d^6*c^4*b^3*a^9 + 792*x^10*d^7*c^3*b^2*a^10 + 54*x^1
0*d^8*c^2*b*a^11 + x^10*d^9*c*a^12 + 55*x^9*c^10*b^8*a^4 + 880*x^9*d*c^9*b^
7*a^5 + 4620*x^9*d^2*c^8*b^6*a^6 + 10560*x^9*d^3*c^7*b^5*a^7 + 11550*x^9*d^
4*c^6*b^4*a^8 + 6160*x^9*d^5*c^5*b^3*a^9 + 1540*x^9*d^6*c^4*b^2*a^10 + 160*
x^9*d^7*c^3*b*a^11 + 5*x^9*d^8*c^2*a^12 + 99*x^8*c^10*b^7*a^5 + 1155*x^8*d*
c^9*b^6*a^6 + 4455*x^8*d^2*c^8*b^5*a^7 + 7425*x^8*d^3*c^7*b^4*a^8 + 5775*x^
8*d^4*c^6*b^3*a^9 + 2079*x^8*d^5*c^5*b^2*a^10 + 315*x^8*d^6*c^4*b*a^11 + 15
*x^8*d^7*c^3*a^12 + 132*x^7*c^10*b^6*a^6 + 7920/7*x^7*d*c^9*b^5*a^7 + 22275
/7*x^7*d^2*c^8*b^4*a^8 + 26400/7*x^7*d^3*c^7*b^3*a^9 + 1980*x^7*d^4*c^6*b^2
*a^10 + 432*x^7*d^5*c^5*b*a^11 + 30*x^7*d^6*c^4*a^12 + 132*x^6*c^10*b^5*a^7
 + 825*x^6*d*c^9*b^4*a^8 + 1650*x^6*d^2*c^8*b^3*a^9 + 1320*x^6*d^3*c^7*b^2*
a^10 + 420*x^6*d^4*c^6*b*a^11 + 42*x^6*d^5*c^5*a^12 + 99*x^5*c^10*b^4*a^8 +
 440*x^5*d*c^9*b^3*a^9 + 594*x^5*d^2*c^8*b^2*a^10 + 288*x^5*d^3*c^7*b*a^11
+ 42*x^5*d^4*c^6*a^12 + 55*x^4*c^10*b^3*a^9 + 165*x^4*d*c^9*b^2*a^10 + 135*
x^4*d^2*c^8*b*a^11 + 30*x^4*d^3*c^7*a^12 + 22*x^3*c^10*b^2*a^10 + 40*x^3*d*
c^9*b*a^11 + 15*x^3*d^2*c^8*a^12 + 6*x^2*c^10*b*a^11 + 5*x^2*d*c^9*a^12 + x
*c^10*a^12
```

Sympy [B] time = 0.296201, size = 2088, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**12*(d*x+c)**10,x)
```

```
[Out] a**12*c**10*x + b**12*d**10*x**23/23 + x**22*(6*a*b**11*d**10/11 + 5*b**12*
c*d**9/11) + x**21*(22*a**2*b**10*d**10/7 + 40*a*b**11*c*d**9/7 + 15*b**12*
c**2*d**8/7) + x**20*(11*a**3*b**9*d**10 + 33*a**2*b**10*c*d**9 + 27*a*b**1
1*c**2*d**8 + 6*b**12*c**3*d**7) + x**19*(495*a**4*b**8*d**10/19 + 2200*a**
3*b**9*c*d**9/19 + 2970*a**2*b**10*c**2*d**8/19 + 1440*a*b**11*c**3*d**7/19
 + 210*b**12*c**4*d**6/19) + x**18*(44*a**5*b**7*d**10 + 275*a**4*b**8*c*d*
*9 + 550*a**3*b**9*c**2*d**8 + 440*a**2*b**10*c**3*d**7 + 140*a*b**11*c**4*
d**6 + 14*b**12*c**5*d**5) + x**17*(924*a**6*b**6*d**10/17 + 7920*a**5*b**7
*c*d**9/17 + 22275*a**4*b**8*c**2*d**8/17 + 26400*a**3*b**9*c**3*d**7/17 +
13860*a**2*b**10*c**4*d**6/17 + 3024*a*b**11*c**5*d**5/17 + 210*b**12*c**6*
d**4/17) + x**16*(99*a**7*b**5*d**10/2 + 1155*a**6*b**6*c*d**9/2 + 4455*a**
5*b**7*c**2*d**8/2 + 7425*a**4*b**8*c**3*d**7/2 + 5775*a**3*b**9*c**4*d**6/
2 + 2079*a**2*b**10*c**5*d**5/2 + 315*a*b**11*c**6*d**4/2 + 15*b**12*c**7*d
**3/2) + x**15*(33*a**8*b**4*d**10 + 528*a**7*b**5*c*d**9 + 2772*a**6*b**6*
c**2*d**8 + 6336*a**5*b**7*c**3*d**7 + 6930*a**4*b**8*c**4*d**6 + 3696*a**3
*b**9*c**5*d**5 + 924*a**2*b**10*c**6*d**4 + 96*a*b**11*c**7*d**3 + 3*b**12
```

```
*c**8*d**2) + x**14*(110*a**9*b**3*d**10/7 + 2475*a**8*b**4*c*d**9/7 + 1782
0*a**7*b**5*c**2*d**8/7 + 7920*a**6*b**6*c**3*d**7 + 11880*a**5*b**7*c**4*d
**6 + 8910*a**4*b**8*c**5*d**5 + 3300*a**3*b**9*c**6*d**4 + 3960*a**2*b**10
*c**7*d**3/7 + 270*a*b**11*c**8*d**2/7 + 5*b**12*c**9*d/7) + x**13*(66*a**1
0*b**2*d**10/13 + 2200*a**9*b**3*c*d**9/13 + 22275*a**8*b**4*c**2*d**8/13 +
 95040*a**7*b**5*c**3*d**7/13 + 194040*a**6*b**6*c**4*d**6/13 + 199584*a**5
*b**7*c**5*d**5/13 + 103950*a**4*b**8*c**6*d**4/13 + 26400*a**3*b**9*c**7*d
**3/13 + 2970*a**2*b**10*c**8*d**2/13 + 120*a*b**11*c**9*d/13 + b**12*c**10
/13) + x**12*(a**11*b*d**10 + 55*a**10*b**2*c*d**9 + 825*a**9*b**3*c**2*d**
8 + 4950*a**8*b**4*c**3*d**7 + 13860*a**7*b**5*c**4*d**6 + 19404*a**6*b**6*
c**5*d**5 + 13860*a**5*b**7*c**6*d**4 + 4950*a**4*b**8*c**7*d**3 + 825*a**3
*b**9*c**8*d**2 + 55*a**2*b**10*c**9*d + a*b**11*c**10) + x**11*(a**12*d**1
0/11 + 120*a**11*b*c*d**9/11 + 270*a**10*b**2*c**2*d**8 + 2400*a**9*b**3*c*
*3*d**7 + 9450*a**8*b**4*c**4*d**6 + 18144*a**7*b**5*c**5*d**5 + 17640*a**6
*b**6*c**6*d**4 + 8640*a**5*b**7*c**7*d**3 + 2025*a**4*b**8*c**8*d**2 + 200
*a**3*b**9*c**9*d + 6*a**2*b**10*c**10) + x**10*(a**12*c*d**9 + 54*a**11*b*
c**2*d**8 + 792*a**10*b**2*c**3*d**7 + 4620*a**9*b**3*c**4*d**6 + 12474*a**
8*b**4*c**5*d**5 + 16632*a**7*b**5*c**6*d**4 + 11088*a**6*b**6*c**7*d**3 +
3564*a**5*b**7*c**8*d**2 + 495*a**4*b**8*c**9*d + 22*a**3*b**9*c**10) + x**
9*(5*a**12*c**2*d**8 + 160*a**11*b*c**3*d**7 + 1540*a**10*b**2*c**4*d**6 +
6160*a**9*b**3*c**5*d**5 + 11550*a**8*b**4*c**6*d**4 + 10560*a**7*b**5*c**7
*d**3 + 4620*a**6*b**6*c**8*d**2 + 880*a**5*b**7*c**9*d + 55*a**4*b**8*c**1
0) + x**8*(15*a**12*c**3*d**7 + 315*a**11*b*c**4*d**6 + 2079*a**10*b**2*c**
5*d**5 + 5775*a**9*b**3*c**6*d**4 + 7425*a**8*b**4*c**7*d**3 + 4455*a**7*b*
*5*c**8*d**2 + 1155*a**6*b**6*c**9*d + 99*a**5*b**7*c**10) + x**7*(30*a**12
*c**4*d**6 + 432*a**11*b*c**5*d**5 + 1980*a**10*b**2*c**6*d**4 + 26400*a**9
*b**3*c**7*d**3/7 + 22275*a**8*b**4*c**8*d**2/7 + 7920*a**7*b**5*c**9*d/7 +
 132*a**6*b**6*c**10) + x**6*(42*a**12*c**5*d**5 + 420*a**11*b*c**6*d**4 +
1320*a**10*b**2*c**7*d**3 + 1650*a**9*b**3*c**8*d**2 + 825*a**8*b**4*c**9*d
 + 132*a**7*b**5*c**10) + x**5*(42*a**12*c**6*d**4 + 288*a**11*b*c**7*d**3
+ 594*a**10*b**2*c**8*d**2 + 440*a**9*b**3*c**9*d + 99*a**8*b**4*c**10) + x
**4*(30*a**12*c**7*d**3 + 135*a**11*b*c**8*d**2 + 165*a**10*b**2*c**9*d + 5
5*a**9*b**3*c**10) + x**3*(15*a**12*c**8*d**2 + 40*a**11*b*c**9*d + 22*a**1
0*b**2*c**10) + x**2*(5*a**12*c**9*d + 6*a**11*b*c**10)
```

Giac [B] time = 1.07181, size = 2951, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/23*b^12*d^10*x^23 + 5/11*b^12*c*d^9*x^22 + 6/11*a*b^11*d^10*x^22 + 15/7*b
^12*c^2*d^8*x^21 + 40/7*a*b^11*c*d^9*x^21 + 22/7*a^2*b^10*d^10*x^21 + 6*b^1
2*c^3*d^7*x^20 + 27*a*b^11*c^2*d^8*x^20 + 33*a^2*b^10*c*d^9*x^20 + 11*a^3*b
^9*d^10*x^20 + 210/19*b^12*c^4*d^6*x^19 + 1440/19*a*b^11*c^3*d^7*x^19 + 297
0/19*a^2*b^10*c^2*d^8*x^19 + 2200/19*a^3*b^9*c*d^9*x^19 + 495/19*a^4*b^8*d^
10*x^19 + 14*b^12*c^5*d^5*x^18 + 140*a*b^11*c^4*d^6*x^18 + 440*a^2*b^10*c^3
*d^7*x^18 + 550*a^3*b^9*c^2*d^8*x^18 + 275*a^4*b^8*c*d^9*x^18 + 44*a^5*b^7*
d^10*x^18 + 210/17*b^12*c^6*d^4*x^17 + 3024/17*a*b^11*c^5*d^5*x^17 + 13860/
17*a^2*b^10*c^4*d^6*x^17 + 26400/17*a^3*b^9*c^3*d^7*x^17 + 22275/17*a^4*b^8
*c^2*d^8*x^17 + 7920/17*a^5*b^7*c*d^9*x^17 + 924/17*a^6*b^6*d^10*x^17 + 15/
2*b^12*c^7*d^3*x^16 + 315/2*a*b^11*c^6*d^4*x^16 + 2079/2*a^2*b^10*c^5*d^5*x
^16 + 5775/2*a^3*b^9*c^4*d^6*x^16 + 7425/2*a^4*b^8*c^3*d^7*x^16 + 4455/2*a^
5*b^7*c^2*d^8*x^16 + 1155/2*a^6*b^6*c*d^9*x^16 + 99/2*a^7*b^5*d^10*x^16 + 3
*b^12*c^8*d^2*x^15 + 96*a*b^11*c^7*d^3*x^15 + 924*a^2*b^10*c^6*d^4*x^15 + 3
```

696*a^3*b^9*c^5*d^5*x^15 + 6930*a^4*b^8*c^4*d^6*x^15 + 6336*a^5*b^7*c^3*d^7
*x^15 + 2772*a^6*b^6*c^2*d^8*x^15 + 528*a^7*b^5*c*d^9*x^15 + 33*a^8*b^4*d^1
0*x^15 + 5/7*b^12*c^9*d*x^14 + 270/7*a*b^11*c^8*d^2*x^14 + 3960/7*a^2*b^10*
c^7*d^3*x^14 + 3300*a^3*b^9*c^6*d^4*x^14 + 8910*a^4*b^8*c^5*d^5*x^14 + 1188
0*a^5*b^7*c^4*d^6*x^14 + 7920*a^6*b^6*c^3*d^7*x^14 + 17820/7*a^7*b^5*c^2*d^
8*x^14 + 2475/7*a^8*b^4*c*d^9*x^14 + 110/7*a^9*b^3*d^10*x^14 + 1/13*b^12*c^
10*x^13 + 120/13*a*b^11*c^9*d*x^13 + 2970/13*a^2*b^10*c^8*d^2*x^13 + 26400/
13*a^3*b^9*c^7*d^3*x^13 + 103950/13*a^4*b^8*c^6*d^4*x^13 + 199584/13*a^5*b^
7*c^5*d^5*x^13 + 194040/13*a^6*b^6*c^4*d^6*x^13 + 95040/13*a^7*b^5*c^3*d^7*
x^13 + 22275/13*a^8*b^4*c^2*d^8*x^13 + 2200/13*a^9*b^3*c*d^9*x^13 + 66/13*a
^10*b^2*d^10*x^13 + a*b^11*c^10*x^12 + 55*a^2*b^10*c^9*d*x^12 + 825*a^3*b^9
*c^8*d^2*x^12 + 4950*a^4*b^8*c^7*d^3*x^12 + 13860*a^5*b^7*c^6*d^4*x^12 + 19
404*a^6*b^6*c^5*d^5*x^12 + 13860*a^7*b^5*c^4*d^6*x^12 + 4950*a^8*b^4*c^3*d^
7*x^12 + 825*a^9*b^3*c^2*d^8*x^12 + 55*a^10*b^2*c*d^9*x^12 + a^11*b*d^10*x^
12 + 6*a^2*b^10*c^10*x^11 + 200*a^3*b^9*c^9*d*x^11 + 2025*a^4*b^8*c^8*d^2*x
^11 + 8640*a^5*b^7*c^7*d^3*x^11 + 17640*a^6*b^6*c^6*d^4*x^11 + 18144*a^7*b^
5*c^5*d^5*x^11 + 9450*a^8*b^4*c^4*d^6*x^11 + 2400*a^9*b^3*c^3*d^7*x^11 + 27
0*a^10*b^2*c^2*d^8*x^11 + 120/11*a^11*b*c*d^9*x^11 + 1/11*a^12*d^10*x^11 +
22*a^3*b^9*c^10*x^10 + 495*a^4*b^8*c^9*d*x^10 + 3564*a^5*b^7*c^8*d^2*x^10 +
 11088*a^6*b^6*c^7*d^3*x^10 + 16632*a^7*b^5*c^6*d^4*x^10 + 12474*a^8*b^4*c^
5*d^5*x^10 + 4620*a^9*b^3*c^4*d^6*x^10 + 792*a^10*b^2*c^3*d^7*x^10 + 54*a^1
1*b*c^2*d^8*x^10 + a^12*c*d^9*x^10 + 55*a^4*b^8*c^10*x^9 + 880*a^5*b^7*c^9*
d*x^9 + 4620*a^6*b^6*c^8*d^2*x^9 + 10560*a^7*b^5*c^7*d^3*x^9 + 11550*a^8*b^
4*c^6*d^4*x^9 + 6160*a^9*b^3*c^5*d^5*x^9 + 1540*a^10*b^2*c^4*d^6*x^9 + 160*
a^11*b*c^3*d^7*x^9 + 5*a^12*c^2*d^8*x^9 + 99*a^5*b^7*c^10*x^8 + 1155*a^6*b^
6*c^9*d*x^8 + 4455*a^7*b^5*c^8*d^2*x^8 + 7425*a^8*b^4*c^7*d^3*x^8 + 5775*a^
9*b^3*c^6*d^4*x^8 + 2079*a^10*b^2*c^5*d^5*x^8 + 315*a^11*b*c^4*d^6*x^8 + 15
*a^12*c^3*d^7*x^8 + 132*a^6*b^6*c^10*x^7 + 7920/7*a^7*b^5*c^9*d*x^7 + 22275
/7*a^8*b^4*c^8*d^2*x^7 + 26400/7*a^9*b^3*c^7*d^3*x^7 + 1980*a^10*b^2*c^6*d^
4*x^7 + 432*a^11*b*c^5*d^5*x^7 + 30*a^12*c^4*d^6*x^7 + 132*a^7*b^5*c^10*x^6
 + 825*a^8*b^4*c^9*d*x^6 + 1650*a^9*b^3*c^8*d^2*x^6 + 1320*a^10*b^2*c^7*d^3
*x^6 + 420*a^11*b*c^6*d^4*x^6 + 42*a^12*c^5*d^5*x^6 + 99*a^8*b^4*c^10*x^5 +
 440*a^9*b^3*c^9*d*x^5 + 594*a^10*b^2*c^8*d^2*x^5 + 288*a^11*b*c^7*d^3*x^5
+ 42*a^12*c^6*d^4*x^5 + 55*a^9*b^3*c^10*x^4 + 165*a^10*b^2*c^9*d*x^4 + 135*
a^11*b*c^8*d^2*x^4 + 30*a^12*c^7*d^3*x^4 + 22*a^10*b^2*c^10*x^3 + 40*a^11*b
*c^9*d*x^3 + 15*a^12*c^8*d^2*x^3 + 6*a^11*b*c^10*x^2 + 5*a^12*c^9*d*x^2 + a
^12*c^10*x

3.1300 $\int (a+bx)^{11}(c+dx)^{10}\,dx$

Optimal. Leaf size=279

$$\frac{10d^9(a+bx)^{21}(bc-ad)}{21b^{11}}+\frac{9d^8(a+bx)^{20}(bc-ad)^2}{4b^{11}}+\frac{120d^7(a+bx)^{19}(bc-ad)^3}{19b^{11}}+\frac{35d^6(a+bx)^{18}(bc-ad)^4}{3b^{11}}+\frac{252d^5(a+$$

```
[Out] ((b*c - a*d)^10*(a + b*x)^12)/(12*b^11) + (10*d*(b*c - a*d)^9*(a + b*x)^13)
/(13*b^11) + (45*d^2*(b*c - a*d)^8*(a + b*x)^14)/(14*b^11) + (8*d^3*(b*c -
a*d)^7*(a + b*x)^15)/b^11 + (105*d^4*(b*c - a*d)^6*(a + b*x)^16)/(8*b^11) +
 (252*d^5*(b*c - a*d)^5*(a + b*x)^17)/(17*b^11) + (35*d^6*(b*c - a*d)^4*(a
+ b*x)^18)/(3*b^11) + (120*d^7*(b*c - a*d)^3*(a + b*x)^19)/(19*b^11) + (9*d
^8*(b*c - a*d)^2*(a + b*x)^20)/(4*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^21)
/(21*b^11) + (d^10*(a + b*x)^22)/(22*b^11)
```

Rubi [A] time = 1.2753, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{10d^9(a+bx)^{21}(bc-ad)}{21b^{11}}+\frac{9d^8(a+bx)^{20}(bc-ad)^2}{4b^{11}}+\frac{120d^7(a+bx)^{19}(bc-ad)^3}{19b^{11}}+\frac{35d^6(a+bx)^{18}(bc-ad)^4}{3b^{11}}+\frac{252d^5(a+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^11*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^12)/(12*b^11) + (10*d*(b*c - a*d)^9*(a + b*x)^13)
/(13*b^11) + (45*d^2*(b*c - a*d)^8*(a + b*x)^14)/(14*b^11) + (8*d^3*(b*c -
a*d)^7*(a + b*x)^15)/b^11 + (105*d^4*(b*c - a*d)^6*(a + b*x)^16)/(8*b^11) +
 (252*d^5*(b*c - a*d)^5*(a + b*x)^17)/(17*b^11) + (35*d^6*(b*c - a*d)^4*(a
+ b*x)^18)/(3*b^11) + (120*d^7*(b*c - a*d)^3*(a + b*x)^19)/(19*b^11) + (9*d
^8*(b*c - a*d)^2*(a + b*x)^20)/(4*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^21)
/(21*b^11) + (d^10*(a + b*x)^22)/(22*b^11)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{11}(c+dx)^{10}\,dx &= \int \left(\frac{(bc-ad)^{10}(a+bx)^{11}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{12}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx)^{13}}{b^{10}}+\frac{120d^3(bc\right.\\ &= \frac{(bc-ad)^{10}(a+bx)^{12}}{12b^{11}}+\frac{10d(bc-ad)^9(a+bx)^{13}}{13b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{14}}{14b^{11}}+\frac{8d^3(bc-ad)^7(}{b^{11}}\end{aligned}$$

Mathematica [B] time = 0.212733, size = 1702, normalized size = 6.1

$$\frac{1}{22}b^{11}d^{10}x^{22}+\frac{1}{21}b^{10}d^9(10bc+11ad)x^{21}+\frac{1}{4}b^9d^8\left(9b^2c^2+22abdc+11a^2d^2\right)x^{20}+\frac{5}{19}b^8d^7\left(24b^3c^3+99ab^2dc^2+110a^2bd\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^11*(c + d*x)^10,x]
```

```
[Out] a^11*c^10*x + (a^10*c^9*(11*b*c + 10*a*d)*x^2)/2 + (5*a^9*c^8*(11*b^2*c^2 +
 22*a*b*c*d + 9*a^2*d^2)*x^3)/3 + (5*a^8*c^7*(33*b^3*c^3 + 110*a*b^2*c^2*d
+ 99*a^2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + 3*a^7*c^6*(22*b^4*c^4 + 110*a*b^3*c
^3*d + 165*a^2*b^2*c^2*d^2 + 88*a^3*b*c*d^3 + 14*a^4*d^4)*x^5 + (a^6*c^5*(1
54*b^5*c^5 + 1100*a*b^4*c^4*d + 2475*a^2*b^3*c^3*d^2 + 2200*a^3*b^2*c^2*d^3
 + 770*a^4*b*c*d^4 + 84*a^5*d^5)*x^6)/2 + (6*a^5*c^4*(77*b^6*c^6 + 770*a*b^
5*c^5*d + 2475*a^2*b^4*c^4*d^2 + 3300*a^3*b^3*c^3*d^3 + 1925*a^4*b^2*c^2*d^
4 + 462*a^5*b*c*d^5 + 35*a^6*d^6)*x^7)/7 + (15*a^4*c^3*(11*b^7*c^7 + 154*a*
b^6*c^6*d + 693*a^2*b^5*c^5*d^2 + 1320*a^3*b^4*c^4*d^3 + 1155*a^4*b^3*c^3*d
^4 + 462*a^5*b^2*c^2*d^5 + 77*a^6*b*c*d^6 + 4*a^7*d^7)*x^8)/4 + (5*a^3*c^2*
(11*b^8*c^8 + 220*a*b^7*c^7*d + 1386*a^2*b^6*c^6*d^2 + 3696*a^3*b^5*c^5*d^3
 + 4620*a^4*b^4*c^4*d^4 + 2772*a^5*b^3*c^3*d^5 + 770*a^6*b^2*c^2*d^6 + 88*a
^7*b*c*d^7 + 3*a^8*d^8)*x^9)/3 + (a^2*c*(11*b^9*c^9 + 330*a*b^8*c^8*d + 297
0*a^2*b^7*c^7*d^2 + 11088*a^3*b^6*c^6*d^3 + 19404*a^4*b^5*c^5*d^4 + 16632*a
^5*b^4*c^4*d^5 + 6930*a^6*b^3*c^3*d^6 + 1320*a^7*b^2*c^2*d^7 + 99*a^8*b*c*d
^8 + 2*a^9*d^9)*x^10)/2 + (a*(11*b^10*c^10 + 550*a*b^9*c^9*d + 7425*a^2*b^8
*c^8*d^2 + 39600*a^3*b^7*c^7*d^3 + 97020*a^4*b^6*c^6*d^4 + 116424*a^5*b^5*c
^5*d^5 + 69300*a^6*b^4*c^4*d^6 + 19800*a^7*b^3*c^3*d^7 + 2475*a^8*b^2*c^2*d
^8 + 110*a^9*b*c*d^9 + a^10*d^10)*x^11)/11 + (b*(b^10*c^10 + 110*a*b^9*c^9*
d + 2475*a^2*b^8*c^8*d^2 + 19800*a^3*b^7*c^7*d^3 + 69300*a^4*b^6*c^6*d^4 +
116424*a^5*b^5*c^5*d^5 + 97020*a^6*b^4*c^4*d^6 + 39600*a^7*b^3*c^3*d^7 + 74
25*a^8*b^2*c^2*d^8 + 550*a^9*b*c*d^9 + 11*a^10*d^10)*x^12)/12 + (5*b^2*d*(2
*b^9*c^9 + 99*a*b^8*c^8*d + 1320*a^2*b^7*c^7*d^2 + 6930*a^3*b^6*c^6*d^3 + 1
6632*a^4*b^5*c^5*d^4 + 19404*a^5*b^4*c^4*d^5 + 11088*a^6*b^3*c^3*d^6 + 2970
*a^7*b^2*c^2*d^7 + 330*a^8*b*c*d^8 + 11*a^9*d^9)*x^13)/13 + (15*b^3*d^2*(3*
b^8*c^8 + 88*a*b^7*c^7*d + 770*a^2*b^6*c^6*d^2 + 2772*a^3*b^5*c^5*d^3 + 462
0*a^4*b^4*c^4*d^4 + 3696*a^5*b^3*c^3*d^5 + 1386*a^6*b^2*c^2*d^6 + 220*a^7*b
*c*d^7 + 11*a^8*d^8)*x^14)/14 + 2*b^4*d^3*(4*b^7*c^7 + 77*a*b^6*c^6*d + 462
*a^2*b^5*c^5*d^2 + 1155*a^3*b^4*c^4*d^3 + 1320*a^4*b^3*c^3*d^4 + 693*a^5*b^
2*c^2*d^5 + 154*a^6*b*c*d^6 + 11*a^7*d^7)*x^15 + (3*b^5*d^4*(35*b^6*c^6 + 4
62*a*b^5*c^5*d + 1925*a^2*b^4*c^4*d^2 + 3300*a^3*b^3*c^3*d^3 + 2475*a^4*b^2
*c^2*d^4 + 770*a^5*b*c*d^5 + 77*a^6*d^6)*x^16)/8 + (3*b^6*d^5*(84*b^5*c^5 +
 770*a*b^4*c^4*d + 2200*a^2*b^3*c^3*d^2 + 2475*a^3*b^2*c^2*d^3 + 1100*a^4*b
*c*d^4 + 154*a^5*d^5)*x^17)/17 + (5*b^7*d^6*(14*b^4*c^4 + 88*a*b^3*c^3*d +
165*a^2*b^2*c^2*d^2 + 110*a^3*b*c*d^3 + 22*a^4*d^4)*x^18)/6 + (5*b^8*d^7*(2
4*b^3*c^3 + 99*a*b^2*c^2*d + 110*a^2*b*c*d^2 + 33*a^3*d^3)*x^19)/19 + (b^9*
d^8*(9*b^2*c^2 + 22*a*b*c*d + 11*a^2*d^2)*x^20)/4 + (b^10*d^9*(10*b*c + 11*
a*d)*x^21)/21 + (b^11*d^10*x^22)/22
```

Maple [B] time = 0.004, size = 1741, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^11*(d*x+c)^10,x)
```

```
[Out] 1/22*b^11*d^10*x^22+1/21*(11*a*b^10*d^10+10*b^11*c*d^9)*x^21+1/20*(55*a^2*b
^9*d^10+110*a*b^10*c*d^9+45*b^11*c^2*d^8)*x^20+1/19*(165*a^3*b^8*d^10+550*a
^2*b^9*c*d^9+495*a*b^10*c^2*d^8+120*b^11*c^3*d^7)*x^19+1/18*(330*a^4*b^7*d^
10+1650*a^3*b^8*c*d^9+2475*a^2*b^9*c^2*d^8+1320*a*b^10*c^3*d^7+210*b^11*c^4
*d^6)*x^18+1/17*(462*a^5*b^6*d^10+3300*a^4*b^7*c*d^9+7425*a^3*b^8*c^2*d^8+6
600*a^2*b^9*c^3*d^7+2310*a*b^10*c^4*d^6+252*b^11*c^5*d^5)*x^17+1/16*(462*a^
6*b^5*d^10+4620*a^5*b^6*c*d^9+14850*a^4*b^7*c^2*d^8+19800*a^3*b^8*c^3*d^7+1
1550*a^2*b^9*c^4*d^6+2772*a*b^10*c^5*d^5+210*b^11*c^6*d^4)*x^16+1/15*(330*a
```

```
^7*b^4*d^10+4620*a^6*b^5*c*d^9+20790*a^5*b^6*c^2*d^8+39600*a^4*b^7*c^3*d^7+
34650*a^3*b^8*c^4*d^6+13860*a^2*b^9*c^5*d^5+2310*a*b^10*c^6*d^4+120*b^11*c^
7*d^3)*x^15+1/14*(165*a^8*b^3*d^10+3300*a^7*b^4*c*d^9+20790*a^6*b^5*c^2*d^8
+55440*a^5*b^6*c^3*d^7+69300*a^4*b^7*c^4*d^6+41580*a^3*b^8*c^5*d^5+11550*a^
2*b^9*c^6*d^4+1320*a*b^10*c^7*d^3+45*b^11*c^8*d^2)*x^14+1/13*(55*a^9*b^2*d^
10+1650*a^8*b^3*c*d^9+14850*a^7*b^4*c^2*d^8+55440*a^6*b^5*c^3*d^7+97020*a^5
*b^6*c^4*d^6+83160*a^4*b^7*c^5*d^5+34650*a^3*b^8*c^6*d^4+6600*a^2*b^9*c^7*d
^3+495*a*b^10*c^8*d^2+10*b^11*c^9*d)*x^13+1/12*(11*a^10*b*d^10+550*a^9*b^2*
c*d^9+7425*a^8*b^3*c^2*d^8+39600*a^7*b^4*c^3*d^7+97020*a^6*b^5*c^4*d^6+1164
24*a^5*b^6*c^5*d^5+69300*a^4*b^7*c^6*d^4+19800*a^3*b^8*c^7*d^3+2475*a^2*b^9
*c^8*d^2+110*a*b^10*c^9*d+b^11*c^10)*x^12+1/11*(a^11*d^10+110*a^10*b*c*d^9+
2475*a^9*b^2*c^2*d^8+19800*a^8*b^3*c^3*d^7+69300*a^7*b^4*c^4*d^6+116424*a^6
*b^5*c^5*d^5+97020*a^5*b^6*c^6*d^4+39600*a^4*b^7*c^7*d^3+7425*a^3*b^8*c^8*d
^2+550*a^2*b^9*c^9*d+11*a*b^10*c^10)*x^11+1/10*(10*a^11*c*d^9+495*a^10*b*c^
2*d^8+6600*a^9*b^2*c^3*d^7+34650*a^8*b^3*c^4*d^6+83160*a^7*b^4*c^5*d^5+9702
0*a^6*b^5*c^6*d^4+55440*a^5*b^6*c^7*d^3+14850*a^4*b^7*c^8*d^2+1650*a^3*b^8*
c^9*d+55*a^2*b^9*c^10)*x^10+1/9*(45*a^11*c^2*d^8+1320*a^10*b*c^3*d^7+11550*
a^9*b^2*c^4*d^6+41580*a^8*b^3*c^5*d^5+69300*a^7*b^4*c^6*d^4+55440*a^6*b^5*c
^7*d^3+20790*a^5*b^6*c^8*d^2+3300*a^4*b^7*c^9*d+165*a^3*b^8*c^10)*x^9+1/8*(
120*a^11*c^3*d^7+2310*a^10*b*c^4*d^6+13860*a^9*b^2*c^5*d^5+34650*a^8*b^3*c^
6*d^4+39600*a^7*b^4*c^7*d^3+20790*a^6*b^5*c^8*d^2+4620*a^5*b^6*c^9*d+330*a^
4*b^7*c^10)*x^8+1/7*(210*a^11*c^4*d^6+2772*a^10*b*c^5*d^5+11550*a^9*b^2*c^6
*d^4+19800*a^8*b^3*c^7*d^3+14850*a^7*b^4*c^8*d^2+4620*a^6*b^5*c^9*d+462*a^5
*b^6*c^10)*x^7+1/6*(252*a^11*c^5*d^5+2310*a^10*b*c^6*d^4+6600*a^9*b^2*c^7*d
^3+7425*a^8*b^3*c^8*d^2+3300*a^7*b^4*c^9*d+462*a^6*b^5*c^10)*x^6+1/5*(210*a
^11*c^6*d^4+1320*a^10*b*c^7*d^3+2475*a^9*b^2*c^8*d^2+1650*a^8*b^3*c^9*d+330
*a^7*b^4*c^10)*x^5+1/4*(120*a^11*c^7*d^3+495*a^10*b*c^8*d^2+550*a^9*b^2*c^9
*d+165*a^8*b^3*c^10)*x^4+1/3*(45*a^11*c^8*d^2+110*a^10*b*c^9*d+55*a^9*b^2*c
^10)*x^3+1/2*(10*a^11*c^9*d+11*a^10*b*c^10)*x^2+a^11*c^10*x
```

Maxima [B] time = 1.03146, size = 2349, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/22*b^11*d^10*x^22 + a^11*c^10*x + 1/21*(10*b^11*c*d^9 + 11*a*b^10*d^10)*x
^21 + 1/4*(9*b^11*c^2*d^8 + 22*a*b^10*c*d^9 + 11*a^2*b^9*d^10)*x^20 + 5/19*
(24*b^11*c^3*d^7 + 99*a*b^10*c^2*d^8 + 110*a^2*b^9*c*d^9 + 33*a^3*b^8*d^10)
*x^19 + 5/6*(14*b^11*c^4*d^6 + 88*a*b^10*c^3*d^7 + 165*a^2*b^9*c^2*d^8 + 11
0*a^3*b^8*c*d^9 + 22*a^4*b^7*d^10)*x^18 + 3/17*(84*b^11*c^5*d^5 + 770*a*b^1
0*c^4*d^6 + 2200*a^2*b^9*c^3*d^7 + 2475*a^3*b^8*c^2*d^8 + 1100*a^4*b^7*c*d^
9 + 154*a^5*b^6*d^10)*x^17 + 3/8*(35*b^11*c^6*d^4 + 462*a*b^10*c^5*d^5 + 19
25*a^2*b^9*c^4*d^6 + 3300*a^3*b^8*c^3*d^7 + 2475*a^4*b^7*c^2*d^8 + 770*a^5*
b^6*c*d^9 + 77*a^6*b^5*d^10)*x^16 + 2*(4*b^11*c^7*d^3 + 77*a*b^10*c^6*d^4 +
 462*a^2*b^9*c^5*d^5 + 1155*a^3*b^8*c^4*d^6 + 1320*a^4*b^7*c^3*d^7 + 693*a^
5*b^6*c^2*d^8 + 154*a^6*b^5*c*d^9 + 11*a^7*b^4*d^10)*x^15 + 15/14*(3*b^11*c
^8*d^2 + 88*a*b^10*c^7*d^3 + 770*a^2*b^9*c^6*d^4 + 2772*a^3*b^8*c^5*d^5 + 4
620*a^4*b^7*c^4*d^6 + 3696*a^5*b^6*c^3*d^7 + 1386*a^6*b^5*c^2*d^8 + 220*a^7
*b^4*c*d^9 + 11*a^8*b^3*d^10)*x^14 + 5/13*(2*b^11*c^9*d + 99*a*b^10*c^8*d^2
 + 1320*a^2*b^9*c^7*d^3 + 6930*a^3*b^8*c^6*d^4 + 16632*a^4*b^7*c^5*d^5 + 19
404*a^5*b^6*c^4*d^6 + 11088*a^6*b^5*c^3*d^7 + 2970*a^7*b^4*c^2*d^8 + 330*a^
8*b^3*c*d^9 + 11*a^9*b^2*d^10)*x^13 + 1/12*(b^11*c^10 + 110*a*b^10*c^9*d +
2475*a^2*b^9*c^8*d^2 + 19800*a^3*b^8*c^7*d^3 + 69300*a^4*b^7*c^6*d^4 + 1164
24*a^5*b^6*c^5*d^5 + 97020*a^6*b^5*c^4*d^6 + 39600*a^7*b^4*c^3*d^7 + 7425*a
```

```
^8*b^3*c^2*d^8 + 550*a^9*b^2*c*d^9 + 11*a^10*b*d^10)*x^12 + 1/11*(11*a*b^10
*c^10 + 550*a^2*b^9*c^9*d + 7425*a^3*b^8*c^8*d^2 + 39600*a^4*b^7*c^7*d^3 +
97020*a^5*b^6*c^6*d^4 + 116424*a^6*b^5*c^5*d^5 + 69300*a^7*b^4*c^4*d^6 + 19
800*a^8*b^3*c^3*d^7 + 2475*a^9*b^2*c^2*d^8 + 110*a^10*b*c*d^9 + a^11*d^10)*
x^11 + 1/2*(11*a^2*b^9*c^10 + 330*a^3*b^8*c^9*d + 2970*a^4*b^7*c^8*d^2 + 11
088*a^5*b^6*c^7*d^3 + 19404*a^6*b^5*c^6*d^4 + 16632*a^7*b^4*c^5*d^5 + 6930*
a^8*b^3*c^4*d^6 + 1320*a^9*b^2*c^3*d^7 + 99*a^10*b*c^2*d^8 + 2*a^11*c*d^9)*
x^10 + 5/3*(11*a^3*b^8*c^10 + 220*a^4*b^7*c^9*d + 1386*a^5*b^6*c^8*d^2 + 36
96*a^6*b^5*c^7*d^3 + 4620*a^7*b^4*c^6*d^4 + 2772*a^8*b^3*c^5*d^5 + 770*a^9*
b^2*c^4*d^6 + 88*a^10*b*c^3*d^7 + 3*a^11*c^2*d^8)*x^9 + 15/4*(11*a^4*b^7*c^
10 + 154*a^5*b^6*c^9*d + 693*a^6*b^5*c^8*d^2 + 1320*a^7*b^4*c^7*d^3 + 1155*
a^8*b^3*c^6*d^4 + 462*a^9*b^2*c^5*d^5 + 77*a^10*b*c^4*d^6 + 4*a^11*c^3*d^7)
*x^8 + 6/7*(77*a^5*b^6*c^10 + 770*a^6*b^5*c^9*d + 2475*a^7*b^4*c^8*d^2 + 33
00*a^8*b^3*c^7*d^3 + 1925*a^9*b^2*c^6*d^4 + 462*a^10*b*c^5*d^5 + 35*a^11*c^
4*d^6)*x^7 + 1/2*(154*a^6*b^5*c^10 + 1100*a^7*b^4*c^9*d + 2475*a^8*b^3*c^8*
d^2 + 2200*a^9*b^2*c^7*d^3 + 770*a^10*b*c^6*d^4 + 84*a^11*c^5*d^5)*x^6 + 3*
(22*a^7*b^4*c^10 + 110*a^8*b^3*c^9*d + 165*a^9*b^2*c^8*d^2 + 88*a^10*b*c^7*
d^3 + 14*a^11*c^6*d^4)*x^5 + 5/4*(33*a^8*b^3*c^10 + 110*a^9*b^2*c^9*d + 99*
a^10*b*c^8*d^2 + 24*a^11*c^7*d^3)*x^4 + 5/3*(11*a^9*b^2*c^10 + 22*a^10*b*c^
9*d + 9*a^11*c^8*d^2)*x^3 + 1/2*(11*a^10*b*c^10 + 10*a^11*c^9*d)*x^2
```

Fricas [B] time = 1.73361, size = 4709, normalized size = 16.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*d^10*b^11 + 10/21*x^21*d^9*c*b^11 + 11/21*x^21*d^10*b^10*a + 9/4*
x^20*d^8*c^2*b^11 + 11/2*x^20*d^9*c*b^10*a + 11/4*x^20*d^10*b^9*a^2 + 120/1
9*x^19*d^7*c^3*b^11 + 495/19*x^19*d^8*c^2*b^10*a + 550/19*x^19*d^9*c*b^9*a^
2 + 165/19*x^19*d^10*b^8*a^3 + 35/3*x^18*d^6*c^4*b^11 + 220/3*x^18*d^7*c^3*
b^10*a + 275/2*x^18*d^8*c^2*b^9*a^2 + 275/3*x^18*d^9*c*b^8*a^3 + 55/3*x^18*
d^10*b^7*a^4 + 252/17*x^17*d^5*c^5*b^11 + 2310/17*x^17*d^6*c^4*b^10*a + 660
0/17*x^17*d^7*c^3*b^9*a^2 + 7425/17*x^17*d^8*c^2*b^8*a^3 + 3300/17*x^17*d^9
*c*b^7*a^4 + 462/17*x^17*d^10*b^6*a^5 + 105/8*x^16*d^4*c^6*b^11 + 693/4*x^1
6*d^5*c^5*b^10*a + 5775/8*x^16*d^6*c^4*b^9*a^2 + 2475/2*x^16*d^7*c^3*b^8*a^
3 + 7425/8*x^16*d^8*c^2*b^7*a^4 + 1155/4*x^16*d^9*c*b^6*a^5 + 231/8*x^16*d^
10*b^5*a^6 + 8*x^15*d^3*c^7*b^11 + 154*x^15*d^4*c^6*b^10*a + 924*x^15*d^5*c
^5*b^9*a^2 + 2310*x^15*d^6*c^4*b^8*a^3 + 2640*x^15*d^7*c^3*b^7*a^4 + 1386*x
^15*d^8*c^2*b^6*a^5 + 308*x^15*d^9*c*b^5*a^6 + 22*x^15*d^10*b^4*a^7 + 45/14
*x^14*d^2*c^8*b^11 + 660/7*x^14*d^3*c^7*b^10*a + 825*x^14*d^4*c^6*b^9*a^2 +
 2970*x^14*d^5*c^5*b^8*a^3 + 4950*x^14*d^6*c^4*b^7*a^4 + 3960*x^14*d^7*c^3*
b^6*a^5 + 1485*x^14*d^8*c^2*b^5*a^6 + 1650/7*x^14*d^9*c*b^4*a^7 + 165/14*x^
14*d^10*b^3*a^8 + 10/13*x^13*d*c^9*b^11 + 495/13*x^13*d^2*c^8*b^10*a + 6600
/13*x^13*d^3*c^7*b^9*a^2 + 34650/13*x^13*d^4*c^6*b^8*a^3 + 83160/13*x^13*d^
5*c^5*b^7*a^4 + 97020/13*x^13*d^6*c^4*b^6*a^5 + 55440/13*x^13*d^7*c^3*b^5*a
^6 + 14850/13*x^13*d^8*c^2*b^4*a^7 + 1650/13*x^13*d^9*c*b^3*a^8 + 55/13*x^1
3*d^10*b^2*a^9 + 1/12*x^12*c^10*b^11 + 55/6*x^12*d*c^9*b^10*a + 825/4*x^12*
d^2*c^8*b^9*a^2 + 1650*x^12*d^3*c^7*b^8*a^3 + 5775*x^12*d^4*c^6*b^7*a^4 + 9
702*x^12*d^5*c^5*b^6*a^5 + 8085*x^12*d^6*c^4*b^5*a^6 + 3300*x^12*d^7*c^3*b^
4*a^7 + 2475/4*x^12*d^8*c^2*b^3*a^8 + 275/6*x^12*d^9*c*b^2*a^9 + 11/12*x^12
*d^10*b*a^10 + x^11*c^10*b^10*a + 50*x^11*d*c^9*b^9*a^2 + 675*x^11*d^2*c^8*
b^8*a^3 + 3600*x^11*d^3*c^7*b^7*a^4 + 8820*x^11*d^4*c^6*b^6*a^5 + 10584*x^1
1*d^5*c^5*b^5*a^6 + 6300*x^11*d^6*c^4*b^4*a^7 + 1800*x^11*d^7*c^3*b^3*a^8 +
 225*x^11*d^8*c^2*b^2*a^9 + 10*x^11*d^9*c*b*a^10 + 1/11*x^11*d^10*a^11 + 11
```

```
/2*x^10*c^10*b^9*a^2 + 165*x^10*d*c^9*b^8*a^3 + 1485*x^10*d^2*c^8*b^7*a^4 +
 5544*x^10*d^3*c^7*b^6*a^5 + 9702*x^10*d^4*c^6*b^5*a^6 + 8316*x^10*d^5*c^5*
b^4*a^7 + 3465*x^10*d^6*c^4*b^3*a^8 + 660*x^10*d^7*c^3*b^2*a^9 + 99/2*x^10*
d^8*c^2*b*a^10 + x^10*d^9*c*a^11 + 55/3*x^9*c^10*b^8*a^3 + 1100/3*x^9*d*c^9
*b^7*a^4 + 2310*x^9*d^2*c^8*b^6*a^5 + 6160*x^9*d^3*c^7*b^5*a^6 + 7700*x^9*d
^4*c^6*b^4*a^7 + 4620*x^9*d^5*c^5*b^3*a^8 + 3850/3*x^9*d^6*c^4*b^2*a^9 + 44
0/3*x^9*d^7*c^3*b*a^10 + 5*x^9*d^8*c^2*a^11 + 165/4*x^8*c^10*b^7*a^4 + 1155
/2*x^8*d*c^9*b^6*a^5 + 10395/4*x^8*d^2*c^8*b^5*a^6 + 4950*x^8*d^3*c^7*b^4*a
^7 + 17325/4*x^8*d^4*c^6*b^3*a^8 + 3465/2*x^8*d^5*c^5*b^2*a^9 + 1155/4*x^8*
d^6*c^4*b*a^10 + 15*x^8*d^7*c^3*a^11 + 66*x^7*c^10*b^6*a^5 + 660*x^7*d*c^9*
b^5*a^6 + 14850/7*x^7*d^2*c^8*b^4*a^7 + 19800/7*x^7*d^3*c^7*b^3*a^8 + 1650*
x^7*d^4*c^6*b^2*a^9 + 396*x^7*d^5*c^5*b*a^10 + 30*x^7*d^6*c^4*a^11 + 77*x^6
*c^10*b^5*a^6 + 550*x^6*d*c^9*b^4*a^7 + 2475/2*x^6*d^2*c^8*b^3*a^8 + 1100*x
^6*d^3*c^7*b^2*a^9 + 385*x^6*d^4*c^6*b*a^10 + 42*x^6*d^5*c^5*a^11 + 66*x^5*
c^10*b^4*a^7 + 330*x^5*d*c^9*b^3*a^8 + 495*x^5*d^2*c^8*b^2*a^9 + 264*x^5*d^
3*c^7*b*a^10 + 42*x^5*d^4*c^6*a^11 + 165/4*x^4*c^10*b^3*a^8 + 275/2*x^4*d*c
^9*b^2*a^9 + 495/4*x^4*d^2*c^8*b*a^10 + 30*x^4*d^3*c^7*a^11 + 55/3*x^3*c^10
*b^2*a^9 + 110/3*x^3*d*c^9*b*a^10 + 15*x^3*d^2*c^8*a^11 + 11/2*x^2*c^10*b*a
^10 + 5*x^2*d*c^9*a^11 + x*c^10*a^11
```

Sympy [B] time = 0.280701, size = 1965, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**11*(d*x+c)**10,x)
```

```
[Out] a**11*c**10*x + b**11*d**10*x**22/22 + x**21*(11*a*b**10*d**10/21 + 10*b**1
1*c*d**9/21) + x**20*(11*a**2*b**9*d**10/4 + 11*a*b**10*c*d**9/2 + 9*b**11*
c**2*d**8/4) + x**19*(165*a**3*b**8*d**10/19 + 550*a**2*b**9*c*d**9/19 + 49
5*a*b**10*c**2*d**8/19 + 120*b**11*c**3*d**7/19) + x**18*(55*a**4*b**7*d**1
0/3 + 275*a**3*b**8*c*d**9/3 + 275*a**2*b**9*c**2*d**8/2 + 220*a*b**10*c**3
*d**7/3 + 35*b**11*c**4*d**6/3) + x**17*(462*a**5*b**6*d**10/17 + 3300*a**4
*b**7*c*d**9/17 + 7425*a**3*b**8*c**2*d**8/17 + 6600*a**2*b**9*c**3*d**7/17
 + 2310*a*b**10*c**4*d**6/17 + 252*b**11*c**5*d**5/17) + x**16*(231*a**6*b*
*5*d**10/8 + 1155*a**5*b**6*c*d**9/4 + 7425*a**4*b**7*c**2*d**8/8 + 2475*a*
*3*b**8*c**3*d**7/2 + 5775*a**2*b**9*c**4*d**6/8 + 693*a*b**10*c**5*d**5/4
+ 105*b**11*c**6*d**4/8) + x**15*(22*a**7*b**4*d**10 + 308*a**6*b**5*c*d**9
 + 1386*a**5*b**6*c**2*d**8 + 2640*a**4*b**7*c**3*d**7 + 2310*a**3*b**8*c**
4*d**6 + 924*a**2*b**9*c**5*d**5 + 154*a*b**10*c**6*d**4 + 8*b**11*c**7*d**
3) + x**14*(165*a**8*b**3*d**10/14 + 1650*a**7*b**4*c*d**9/7 + 1485*a**6*b*
*5*c**2*d**8 + 3960*a**5*b**6*c**3*d**7 + 4950*a**4*b**7*c**4*d**6 + 2970*a
**3*b**8*c**5*d**5 + 825*a**2*b**9*c**6*d**4 + 660*a*b**10*c**7*d**3/7 + 45
*b**11*c**8*d**2/14) + x**13*(55*a**9*b**2*d**10/13 + 1650*a**8*b**3*c*d**9
/13 + 14850*a**7*b**4*c**2*d**8/13 + 55440*a**6*b**5*c**3*d**7/13 + 97020*a
**5*b**6*c**4*d**6/13 + 83160*a**4*b**7*c**5*d**5/13 + 34650*a**3*b**8*c**6
*d**4/13 + 6600*a**2*b**9*c**7*d**3/13 + 495*a*b**10*c**8*d**2/13 + 10*b**1
1*c**9*d/13) + x**12*(11*a**10*b*d**10/12 + 275*a**9*b**2*c*d**9/6 + 2475*a
**8*b**3*c**2*d**8/4 + 3300*a**7*b**4*c**3*d**7 + 8085*a**6*b**5*c**4*d**6
+ 9702*a**5*b**6*c**5*d**5 + 5775*a**4*b**7*c**6*d**4 + 1650*a**3*b**8*c**7
*d**3 + 825*a**2*b**9*c**8*d**2/4 + 55*a*b**10*c**9*d/6 + b**11*c**10/12) +
 x**11*(a**11*d**10/11 + 10*a**10*b*c*d**9 + 225*a**9*b**2*c**2*d**8 + 1800
*a**8*b**3*c**3*d**7 + 6300*a**7*b**4*c**4*d**6 + 10584*a**6*b**5*c**5*d**5
 + 8820*a**5*b**6*c**6*d**4 + 3600*a**4*b**7*c**7*d**3 + 675*a**3*b**8*c**8
*d**2 + 50*a**2*b**9*c**9*d + a*b**10*c**10) + x**10*(a**11*c*d**9 + 99*a**
10*b*c**2*d**8/2 + 660*a**9*b**2*c**3*d**7 + 3465*a**8*b**3*c**4*d**6 + 831
```

```
6*a**7*b**4*c**5*d**5 + 9702*a**6*b**5*c**6*d**4 + 5544*a**5*b**6*c**7*d**3
 + 1485*a**4*b**7*c**8*d**2 + 165*a**3*b**8*c**9*d + 11*a**2*b**9*c**10/2)
+ x**9*(5*a**11*c**2*d**8 + 440*a**10*b*c**3*d**7/3 + 3850*a**9*b**2*c**4*d
**6/3 + 4620*a**8*b**3*c**5*d**5 + 7700*a**7*b**4*c**6*d**4 + 6160*a**6*b**
5*c**7*d**3 + 2310*a**5*b**6*c**8*d**2 + 1100*a**4*b**7*c**9*d/3 + 55*a**3*
b**8*c**10/3) + x**8*(15*a**11*c**3*d**7 + 1155*a**10*b*c**4*d**6/4 + 3465*
a**9*b**2*c**5*d**5/2 + 17325*a**8*b**3*c**6*d**4/4 + 4950*a**7*b**4*c**7*d
**3 + 10395*a**6*b**5*c**8*d**2/4 + 1155*a**5*b**6*c**9*d/2 + 165*a**4*b**7
*c**10/4) + x**7*(30*a**11*c**4*d**6 + 396*a**10*b*c**5*d**5 + 1650*a**9*b*
*2*c**6*d**4 + 19800*a**8*b**3*c**7*d**3/7 + 14850*a**7*b**4*c**8*d**2/7 +
660*a**6*b**5*c**9*d + 66*a**5*b**6*c**10) + x**6*(42*a**11*c**5*d**5 + 385
*a**10*b*c**6*d**4 + 1100*a**9*b**2*c**7*d**3 + 2475*a**8*b**3*c**8*d**2/2
+ 550*a**7*b**4*c**9*d + 77*a**6*b**5*c**10) + x**5*(42*a**11*c**6*d**4 + 2
64*a**10*b*c**7*d**3 + 495*a**9*b**2*c**8*d**2 + 330*a**8*b**3*c**9*d + 66*
a**7*b**4*c**10) + x**4*(30*a**11*c**7*d**3 + 495*a**10*b*c**8*d**2/4 + 275
*a**9*b**2*c**9*d/2 + 165*a**8*b**3*c**10/4) + x**3*(15*a**11*c**8*d**2 + 1
10*a**10*b*c**9*d/3 + 55*a**9*b**2*c**10/3) + x**2*(5*a**11*c**9*d + 11*a**
10*b*c**10/2)
```

Giac [B] time = 1.0877, size = 2714, normalized size = 9.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/22*b^11*d^10*x^22 + 10/21*b^11*c*d^9*x^21 + 11/21*a*b^10*d^10*x^21 + 9/4*
b^11*c^2*d^8*x^20 + 11/2*a*b^10*c*d^9*x^20 + 11/4*a^2*b^9*d^10*x^20 + 120/1
9*b^11*c^3*d^7*x^19 + 495/19*a*b^10*c^2*d^8*x^19 + 550/19*a^2*b^9*c*d^9*x^1
9 + 165/19*a^3*b^8*d^10*x^19 + 35/3*b^11*c^4*d^6*x^18 + 220/3*a*b^10*c^3*d^
7*x^18 + 275/2*a^2*b^9*c^2*d^8*x^18 + 275/3*a^3*b^8*c*d^9*x^18 + 55/3*a^4*b
^7*d^10*x^18 + 252/17*b^11*c^5*d^5*x^17 + 2310/17*a*b^10*c^4*d^6*x^17 + 660
0/17*a^2*b^9*c^3*d^7*x^17 + 7425/17*a^3*b^8*c^2*d^8*x^17 + 3300/17*a^4*b^7*
c*d^9*x^17 + 462/17*a^5*b^6*d^10*x^17 + 105/8*b^11*c^6*d^4*x^16 + 693/4*a*b
^10*c^5*d^5*x^16 + 5775/8*a^2*b^9*c^4*d^6*x^16 + 2475/2*a^3*b^8*c^3*d^7*x^1
6 + 7425/8*a^4*b^7*c^2*d^8*x^16 + 1155/4*a^5*b^6*c*d^9*x^16 + 231/8*a^6*b^5
*d^10*x^16 + 8*b^11*c^7*d^3*x^15 + 154*a*b^10*c^6*d^4*x^15 + 924*a^2*b^9*c^
5*d^5*x^15 + 2310*a^3*b^8*c^4*d^6*x^15 + 2640*a^4*b^7*c^3*d^7*x^15 + 1386*a
^5*b^6*c^2*d^8*x^15 + 308*a^6*b^5*c*d^9*x^15 + 22*a^7*b^4*d^10*x^15 + 45/14
*b^11*c^8*d^2*x^14 + 660/7*a*b^10*c^7*d^3*x^14 + 825*a^2*b^9*c^6*d^4*x^14 +
 2970*a^3*b^8*c^5*d^5*x^14 + 4950*a^4*b^7*c^4*d^6*x^14 + 3960*a^5*b^6*c^3*d
^7*x^14 + 1485*a^6*b^5*c^2*d^8*x^14 + 1650/7*a^7*b^4*c*d^9*x^14 + 165/14*a^
8*b^3*d^10*x^14 + 10/13*b^11*c^9*d*x^13 + 495/13*a*b^10*c^8*d^2*x^13 + 6600
/13*a^2*b^9*c^7*d^3*x^13 + 34650/13*a^3*b^8*c^6*d^4*x^13 + 83160/13*a^4*b^7
*c^5*d^5*x^13 + 97020/13*a^5*b^6*c^4*d^6*x^13 + 55440/13*a^6*b^5*c^3*d^7*x^
13 + 14850/13*a^7*b^4*c^2*d^8*x^13 + 1650/13*a^8*b^3*c*d^9*x^13 + 55/13*a^9
*b^2*d^10*x^13 + 1/12*b^11*c^10*x^12 + 55/6*a*b^10*c^9*d*x^12 + 825/4*a^2*b
^9*c^8*d^2*x^12 + 1650*a^3*b^8*c^7*d^3*x^12 + 5775*a^4*b^7*c^6*d^4*x^12 + 9
702*a^5*b^6*c^5*d^5*x^12 + 8085*a^6*b^5*c^4*d^6*x^12 + 3300*a^7*b^4*c^3*d^7
*x^12 + 2475/4*a^8*b^3*c^2*d^8*x^12 + 275/6*a^9*b^2*c*d^9*x^12 + 11/12*a^10
*b*d^10*x^12 + a*b^10*c^10*x^11 + 50*a^2*b^9*c^9*d*x^11 + 675*a^3*b^8*c^8*d
^2*x^11 + 3600*a^4*b^7*c^7*d^3*x^11 + 8820*a^5*b^6*c^6*d^4*x^11 + 10584*a^6
*b^5*c^5*d^5*x^11 + 6300*a^7*b^4*c^4*d^6*x^11 + 1800*a^8*b^3*c^3*d^7*x^11 +
 225*a^9*b^2*c^2*d^8*x^11 + 10*a^10*b*c*d^9*x^11 + 1/11*a^11*d^10*x^11 + 11
/2*a^2*b^9*c^10*x^10 + 165*a^3*b^8*c^9*d*x^10 + 1485*a^4*b^7*c^8*d^2*x^10 +
 5544*a^5*b^6*c^7*d^3*x^10 + 9702*a^6*b^5*c^6*d^4*x^10 + 8316*a^7*b^4*c^5*d
```

^5*x^10 + 3465*a^8*b^3*c^4*d^6*x^10 + 660*a^9*b^2*c^3*d^7*x^10 + 99/2*a^10*
b*c^2*d^8*x^10 + a^11*c*d^9*x^10 + 55/3*a^3*b^8*c^10*x^9 + 1100/3*a^4*b^7*c
^9*d*x^9 + 2310*a^5*b^6*c^8*d^2*x^9 + 6160*a^6*b^5*c^7*d^3*x^9 + 7700*a^7*b
^4*c^6*d^4*x^9 + 4620*a^8*b^3*c^5*d^5*x^9 + 3850/3*a^9*b^2*c^4*d^6*x^9 + 44
0/3*a^10*b*c^3*d^7*x^9 + 5*a^11*c^2*d^8*x^9 + 165/4*a^4*b^7*c^10*x^8 + 1155
/2*a^5*b^6*c^9*d*x^8 + 10395/4*a^6*b^5*c^8*d^2*x^8 + 4950*a^7*b^4*c^7*d^3*x
^8 + 17325/4*a^8*b^3*c^6*d^4*x^8 + 3465/2*a^9*b^2*c^5*d^5*x^8 + 1155/4*a^10
*b*c^4*d^6*x^8 + 15*a^11*c^3*d^7*x^8 + 66*a^5*b^6*c^10*x^7 + 660*a^6*b^5*c^
9*d*x^7 + 14850/7*a^7*b^4*c^8*d^2*x^7 + 19800/7*a^8*b^3*c^7*d^3*x^7 + 1650*
a^9*b^2*c^6*d^4*x^7 + 396*a^10*b*c^5*d^5*x^7 + 30*a^11*c^4*d^6*x^7 + 77*a^6
*b^5*c^10*x^6 + 550*a^7*b^4*c^9*d*x^6 + 2475/2*a^8*b^3*c^8*d^2*x^6 + 1100*a
^9*b^2*c^7*d^3*x^6 + 385*a^10*b*c^6*d^4*x^6 + 42*a^11*c^5*d^5*x^6 + 66*a^7*
b^4*c^10*x^5 + 330*a^8*b^3*c^9*d*x^5 + 495*a^9*b^2*c^8*d^2*x^5 + 264*a^10*b
*c^7*d^3*x^5 + 42*a^11*c^6*d^4*x^5 + 165/4*a^8*b^3*c^10*x^4 + 275/2*a^9*b^2
*c^9*d*x^4 + 495/4*a^10*b*c^8*d^2*x^4 + 30*a^11*c^7*d^3*x^4 + 55/3*a^9*b^2*
c^10*x^3 + 110/3*a^10*b*c^9*d*x^3 + 15*a^11*c^8*d^2*x^3 + 11/2*a^10*b*c^10*
x^2 + 5*a^11*c^9*d*x^2 + a^11*c^10*x

3.1301 $\int (a+bx)^{10}(c+dx)^{10}\,dx$

Optimal. Leaf size=279

$$\frac{d^9(a+bx)^{20}(bc-ad)}{2b^{11}}+\frac{45d^8(a+bx)^{19}(bc-ad)^2}{19b^{11}}+\frac{20d^7(a+bx)^{18}(bc-ad)^3}{3b^{11}}+\frac{210d^6(a+bx)^{17}(bc-ad)^4}{17b^{11}}+\frac{63d^5(a+$$

```
[Out] ((b*c - a*d)^10*(a + b*x)^11)/(11*b^11) + (5*d*(b*c - a*d)^9*(a + b*x)^12)/
(6*b^11) + (45*d^2*(b*c - a*d)^8*(a + b*x)^13)/(13*b^11) + (60*d^3*(b*c - a
*d)^7*(a + b*x)^14)/(7*b^11) + (14*d^4*(b*c - a*d)^6*(a + b*x)^15)/b^11 + (
63*d^5*(b*c - a*d)^5*(a + b*x)^16)/(4*b^11) + (210*d^6*(b*c - a*d)^4*(a + b
*x)^17)/(17*b^11) + (20*d^7*(b*c - a*d)^3*(a + b*x)^18)/(3*b^11) + (45*d^8*
(b*c - a*d)^2*(a + b*x)^19)/(19*b^11) + (d^9*(b*c - a*d)*(a + b*x)^20)/(2*b
^11) + (d^10*(a + b*x)^21)/(21*b^11)
```

Rubi [A] time = 1.11221, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^9(a+bx)^{20}(bc-ad)}{2b^{11}}+\frac{45d^8(a+bx)^{19}(bc-ad)^2}{19b^{11}}+\frac{20d^7(a+bx)^{18}(bc-ad)^3}{3b^{11}}+\frac{210d^6(a+bx)^{17}(bc-ad)^4}{17b^{11}}+\frac{63d^5(a+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^11)/(11*b^11) + (5*d*(b*c - a*d)^9*(a + b*x)^12)/
(6*b^11) + (45*d^2*(b*c - a*d)^8*(a + b*x)^13)/(13*b^11) + (60*d^3*(b*c - a
*d)^7*(a + b*x)^14)/(7*b^11) + (14*d^4*(b*c - a*d)^6*(a + b*x)^15)/b^11 + (
63*d^5*(b*c - a*d)^5*(a + b*x)^16)/(4*b^11) + (210*d^6*(b*c - a*d)^4*(a + b
*x)^17)/(17*b^11) + (20*d^7*(b*c - a*d)^3*(a + b*x)^18)/(3*b^11) + (45*d^8*
(b*c - a*d)^2*(a + b*x)^19)/(19*b^11) + (d^9*(b*c - a*d)*(a + b*x)^20)/(2*b
^11) + (d^10*(a + b*x)^21)/(21*b^11)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^{10}(c+dx)^{10}\,dx=\int \left(\frac{(bc-ad)^{10}(a+bx)^{10}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{11}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx)^{12}}{b^{10}}+\frac{120d^3(\right.$$

$$=\frac{(bc-ad)^{10}(a+bx)^{11}}{11b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{12}}{6b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{13}}{13b^{11}}+\frac{60d^3(bc-a}{7}$$

Mathematica [B] time = 0.171711, size = 1539, normalized size = 5.52

$$\frac{1}{21}b^{10}d^{10}x^{21}+\frac{1}{2}b^9d^9(bc+ad)x^{20}+\frac{5}{19}b^8d^8\left(9b^2c^2+20abdc+9a^2d^2\right)x^{19}+\frac{5}{3}b^7d^7\left(4b^3c^3+15ab^2dc^2+15a^2bd^2c+4a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10*(c + d*x)^10,x]
```

```
[Out] a^10*c^10*x + 5*a^9*c^9*(b*c + a*d)*x^2 + (5*a^8*c^8*(9*b^2*c^2 + 20*a*b*c*
d + 9*a^2*d^2)*x^3)/3 + (15*a^7*c^7*(4*b^3*c^3 + 15*a*b^2*c^2*d + 15*a^2*b*
c*d^2 + 4*a^3*d^3)*x^4)/2 + 3*a^6*c^6*(14*b^4*c^4 + 80*a*b^3*c^3*d + 135*a^
2*b^2*c^2*d^2 + 80*a^3*b*c*d^3 + 14*a^4*d^4)*x^5 + 2*a^5*c^5*(21*b^5*c^5 +
175*a*b^4*c^4*d + 450*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d
^4 + 21*a^5*d^5)*x^6 + (30*a^4*c^4*(7*b^6*c^6 + 84*a*b^5*c^5*d + 315*a^2*b^
4*c^4*d^2 + 480*a^3*b^3*c^3*d^3 + 315*a^4*b^2*c^2*d^4 + 84*a^5*b*c*d^5 + 7*
a^6*d^6)*x^7)/7 + (15*a^3*c^3*(2*b^7*c^7 + 35*a*b^6*c^6*d + 189*a^2*b^5*c^5
*d^2 + 420*a^3*b^4*c^4*d^3 + 420*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 + 35
*a^6*b*c*d^6 + 2*a^7*d^7)*x^8)/2 + (5*a^2*c^2*(3*b^8*c^8 + 80*a*b^7*c^7*d +
 630*a^2*b^6*c^6*d^2 + 2016*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4*d^4 + 2016*a
^5*b^3*c^3*d^5 + 630*a^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + 3*a^8*d^8)*x^9)/3 +
 a*c*(b^9*c^9 + 45*a*b^8*c^8*d + 540*a^2*b^7*c^7*d^2 + 2520*a^3*b^6*c^6*d^3
 + 5292*a^4*b^5*c^5*d^4 + 5292*a^5*b^4*c^4*d^5 + 2520*a^6*b^3*c^3*d^6 + 540
*a^7*b^2*c^2*d^7 + 45*a^8*b*c*d^8 + a^9*d^9)*x^10 + ((b^10*c^10 + 100*a*b^9
*c^9*d + 2025*a^2*b^8*c^8*d^2 + 14400*a^3*b^7*c^7*d^3 + 44100*a^4*b^6*c^6*d
^4 + 63504*a^5*b^5*c^5*d^5 + 44100*a^6*b^4*c^4*d^6 + 14400*a^7*b^3*c^3*d^7
+ 2025*a^8*b^2*c^2*d^8 + 100*a^9*b*c*d^9 + a^10*d^10)*x^11)/11 + (5*b*d*(b^
9*c^9 + 45*a*b^8*c^8*d + 540*a^2*b^7*c^7*d^2 + 2520*a^3*b^6*c^6*d^3 + 5292*
a^4*b^5*c^5*d^4 + 5292*a^5*b^4*c^4*d^5 + 2520*a^6*b^3*c^3*d^6 + 540*a^7*b^2
*c^2*d^7 + 45*a^8*b*c*d^8 + a^9*d^9)*x^12)/6 + (15*b^2*d^2*(3*b^8*c^8 + 80*
a*b^7*c^7*d + 630*a^2*b^6*c^6*d^2 + 2016*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4
*d^4 + 2016*a^5*b^3*c^3*d^5 + 630*a^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + 3*a^8*
d^8)*x^13)/13 + (30*b^3*d^3*(2*b^7*c^7 + 35*a*b^6*c^6*d + 189*a^2*b^5*c^5*d
^2 + 420*a^3*b^4*c^4*d^3 + 420*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 + 35*a
^6*b*c*d^6 + 2*a^7*d^7)*x^14)/7 + 2*b^4*d^4*(7*b^6*c^6 + 84*a*b^5*c^5*d + 3
15*a^2*b^4*c^4*d^2 + 480*a^3*b^3*c^3*d^3 + 315*a^4*b^2*c^2*d^4 + 84*a^5*b*c
*d^5 + 7*a^6*d^6)*x^15 + (3*b^5*d^5*(21*b^5*c^5 + 175*a*b^4*c^4*d + 450*a^2
*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d^4 + 21*a^5*d^5)*x^16)/4
+ (15*b^6*d^6*(14*b^4*c^4 + 80*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 + 80*a^3*b
*c*d^3 + 14*a^4*d^4)*x^17)/17 + (5*b^7*d^7*(4*b^3*c^3 + 15*a*b^2*c^2*d + 15
*a^2*b*c*d^2 + 4*a^3*d^3)*x^18)/3 + (5*b^8*d^8*(9*b^2*c^2 + 20*a*b*c*d + 9*
a^2*d^2)*x^19)/19 + (b^9*d^9*(b*c + a*d)*x^20)/2 + (b^10*d^10*x^21)/21
```

Maple [B] time = 0.001, size = 1591, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10*(d*x+c)^10,x)
```

```
[Out] 1/21*b^10*d^10*x^21+1/20*(10*a*b^9*d^10+10*b^10*c*d^9)*x^20+1/19*(45*a^2*b^
8*d^10+100*a*b^9*c*d^9+45*b^10*c^2*d^8)*x^19+1/18*(120*a^3*b^7*d^10+450*a^2
*b^8*c*d^9+450*a*b^9*c^2*d^8+120*b^10*c^3*d^7)*x^18+1/17*(210*a^4*b^6*d^10+
1200*a^3*b^7*c*d^9+2025*a^2*b^8*c^2*d^8+1200*a*b^9*c^3*d^7+210*b^10*c^4*d^6
)*x^17+1/16*(252*a^5*b^5*d^10+2100*a^4*b^6*c*d^9+5400*a^3*b^7*c^2*d^8+5400*
a^2*b^8*c^3*d^7+2100*a*b^9*c^4*d^6+252*b^10*c^5*d^5)*x^16+1/15*(210*a^6*b^4
*d^10+2520*a^5*b^5*c*d^9+9450*a^4*b^6*c^2*d^8+14400*a^3*b^7*c^3*d^7+9450*a^
2*b^8*c^4*d^6+2520*a*b^9*c^5*d^5+210*b^10*c^6*d^4)*x^15+1/14*(120*a^7*b^3*d
^10+2100*a^6*b^4*c*d^9+11340*a^5*b^5*c^2*d^8+25200*a^4*b^6*c^3*d^7+25200*a^
3*b^7*c^4*d^6+11340*a^2*b^8*c^5*d^5+2100*a*b^9*c^6*d^4+120*b^10*c^7*d^3)*x^
14+1/13*(45*a^8*b^2*d^10+1200*a^7*b^3*c*d^9+9450*a^6*b^4*c^2*d^8+30240*a^5*
b^5*c^3*d^7+44100*a^4*b^6*c^4*d^6+30240*a^3*b^7*c^5*d^5+9450*a^2*b^8*c^6*d^
4+1200*a*b^9*c^7*d^3+45*b^10*c^8*d^2)*x^13+1/12*(10*a^9*b*d^10+450*a^8*b^2*
```

```
c*d^9+5400*a^7*b^3*c^2*d^8+25200*a^6*b^4*c^3*d^7+52920*a^5*b^5*c^4*d^6+5292
0*a^4*b^6*c^5*d^5+25200*a^3*b^7*c^6*d^4+5400*a^2*b^8*c^7*d^3+450*a*b^9*c^8*
d^2+10*b^10*c^9*d)*x^12+1/11*(a^10*d^10+100*a^9*b*c*d^9+2025*a^8*b^2*c^2*d^
8+14400*a^7*b^3*c^3*d^7+44100*a^6*b^4*c^4*d^6+63504*a^5*b^5*c^5*d^5+44100*a
^4*b^6*c^6*d^4+14400*a^3*b^7*c^7*d^3+2025*a^2*b^8*c^8*d^2+100*a*b^9*c^9*d+b
^10*c^10)*x^11+1/10*(10*a^10*c*d^9+450*a^9*b*c^2*d^8+5400*a^8*b^2*c^3*d^7+2
5200*a^7*b^3*c^4*d^6+52920*a^6*b^4*c^5*d^5+52920*a^5*b^5*c^6*d^4+25200*a^4*
b^6*c^7*d^3+5400*a^3*b^7*c^8*d^2+450*a^2*b^8*c^9*d+10*a*b^9*c^10)*x^10+1/9*
(45*a^10*c^2*d^8+1200*a^9*b*c^3*d^7+9450*a^8*b^2*c^4*d^6+30240*a^7*b^3*c^5*
d^5+44100*a^6*b^4*c^6*d^4+30240*a^5*b^5*c^7*d^3+9450*a^4*b^6*c^8*d^2+1200*a
^3*b^7*c^9*d+45*a^2*b^8*c^10)*x^9+1/8*(120*a^10*c^3*d^7+2100*a^9*b*c^4*d^6+
11340*a^8*b^2*c^5*d^5+25200*a^7*b^3*c^6*d^4+25200*a^6*b^4*c^7*d^3+11340*a^5
*b^5*c^8*d^2+2100*a^4*b^6*c^9*d+120*a^3*b^7*c^10)*x^8+1/7*(210*a^10*c^4*d^6
+2520*a^9*b*c^5*d^5+9450*a^8*b^2*c^6*d^4+14400*a^7*b^3*c^7*d^3+9450*a^6*b^4
*c^8*d^2+2520*a^5*b^5*c^9*d+210*a^4*b^6*c^10)*x^7+1/6*(252*a^10*c^5*d^5+210
0*a^9*b*c^6*d^4+5400*a^8*b^2*c^7*d^3+5400*a^7*b^3*c^8*d^2+2100*a^6*b^4*c^9*
d+252*a^5*b^5*c^10)*x^6+1/5*(210*a^10*c^6*d^4+1200*a^9*b*c^7*d^3+2025*a^8*b
^2*c^8*d^2+1200*a^7*b^3*c^9*d+210*a^6*b^4*c^10)*x^5+1/4*(120*a^10*c^7*d^3+4
50*a^9*b*c^8*d^2+450*a^8*b^2*c^9*d+120*a^7*b^3*c^10)*x^4+1/3*(45*a^10*c^8*d
^2+100*a^9*b*c^9*d+45*a^8*b^2*c^10)*x^3+1/2*(10*a^10*c^9*d+10*a^9*b*c^10)*x
^2+a^10*c^10*x
```

<u>Maxima</u> [B] time = 1.01421, size = 2134, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/21*b^10*d^10*x^21 + a^10*c^10*x + 1/2*(b^10*c*d^9 + a*b^9*d^10)*x^20 + 5/
19*(9*b^10*c^2*d^8 + 20*a*b^9*c*d^9 + 9*a^2*b^8*d^10)*x^19 + 5/3*(4*b^10*c^
3*d^7 + 15*a*b^9*c^2*d^8 + 15*a^2*b^8*c*d^9 + 4*a^3*b^7*d^10)*x^18 + 15/17*
(14*b^10*c^4*d^6 + 80*a*b^9*c^3*d^7 + 135*a^2*b^8*c^2*d^8 + 80*a^3*b^7*c*d^
9 + 14*a^4*b^6*d^10)*x^17 + 3/4*(21*b^10*c^5*d^5 + 175*a*b^9*c^4*d^6 + 450*
a^2*b^8*c^3*d^7 + 450*a^3*b^7*c^2*d^8 + 175*a^4*b^6*c*d^9 + 21*a^5*b^5*d^10
)*x^16 + 2*(7*b^10*c^6*d^4 + 84*a*b^9*c^5*d^5 + 315*a^2*b^8*c^4*d^6 + 480*a
^3*b^7*c^3*d^7 + 315*a^4*b^6*c^2*d^8 + 84*a^5*b^5*c*d^9 + 7*a^6*b^4*d^10)*x
^15 + 30/7*(2*b^10*c^7*d^3 + 35*a*b^9*c^6*d^4 + 189*a^2*b^8*c^5*d^5 + 420*a
^3*b^7*c^4*d^6 + 420*a^4*b^6*c^3*d^7 + 189*a^5*b^5*c^2*d^8 + 35*a^6*b^4*c*d
^9 + 2*a^7*b^3*d^10)*x^14 + 15/13*(3*b^10*c^8*d^2 + 80*a*b^9*c^7*d^3 + 630*
a^2*b^8*c^6*d^4 + 2016*a^3*b^7*c^5*d^5 + 2940*a^4*b^6*c^4*d^6 + 2016*a^5*b^
5*c^3*d^7 + 630*a^6*b^4*c^2*d^8 + 80*a^7*b^3*c*d^9 + 3*a^8*b^2*d^10)*x^13 +
 5/6*(b^10*c^9*d + 45*a*b^9*c^8*d^2 + 540*a^2*b^8*c^7*d^3 + 2520*a^3*b^7*c^
6*d^4 + 5292*a^4*b^6*c^5*d^5 + 5292*a^5*b^5*c^4*d^6 + 2520*a^6*b^4*c^3*d^7
+ 540*a^7*b^3*c^2*d^8 + 45*a^8*b^2*c*d^9 + a^9*b*d^10)*x^12 + 1/11*(b^10*c^
10 + 100*a*b^9*c^9*d + 2025*a^2*b^8*c^8*d^2 + 14400*a^3*b^7*c^7*d^3 + 44100
*a^4*b^6*c^6*d^4 + 63504*a^5*b^5*c^5*d^5 + 44100*a^6*b^4*c^4*d^6 + 14400*a^
7*b^3*c^3*d^7 + 2025*a^8*b^2*c^2*d^8 + 100*a^9*b*c*d^9 + a^10*d^10)*x^11 +
(a*b^9*c^10 + 45*a^2*b^8*c^9*d + 540*a^3*b^7*c^8*d^2 + 2520*a^4*b^6*c^7*d^3
 + 5292*a^5*b^5*c^6*d^4 + 5292*a^6*b^4*c^5*d^5 + 2520*a^7*b^3*c^4*d^6 + 540
*a^8*b^2*c^3*d^7 + 45*a^9*b*c^2*d^8 + a^10*c*d^9)*x^10 + 5/3*(3*a^2*b^8*c^1
0 + 80*a^3*b^7*c^9*d + 630*a^4*b^6*c^8*d^2 + 2016*a^5*b^5*c^7*d^3 + 2940*a^
6*b^4*c^6*d^4 + 2016*a^7*b^3*c^5*d^5 + 630*a^8*b^2*c^4*d^6 + 80*a^9*b*c^3*d
^7 + 3*a^10*c^2*d^8)*x^9 + 15/2*(2*a^3*b^7*c^10 + 35*a^4*b^6*c^9*d + 189*a^
5*b^5*c^8*d^2 + 420*a^6*b^4*c^7*d^3 + 420*a^7*b^3*c^6*d^4 + 189*a^8*b^2*c^5
*d^5 + 35*a^9*b*c^4*d^6 + 2*a^10*c^3*d^7)*x^8 + 30/7*(7*a^4*b^6*c^10 + 84*a
```

```
^5*b^5*c^9*d + 315*a^6*b^4*c^8*d^2 + 480*a^7*b^3*c^7*d^3 + 315*a^8*b^2*c^6*
d^4 + 84*a^9*b*c^5*d^5 + 7*a^10*c^4*d^6)*x^7 + 2*(21*a^5*b^5*c^10 + 175*a^6
*b^4*c^9*d + 450*a^7*b^3*c^8*d^2 + 450*a^8*b^2*c^7*d^3 + 175*a^9*b*c^6*d^4
+ 21*a^10*c^5*d^5)*x^6 + 3*(14*a^6*b^4*c^10 + 80*a^7*b^3*c^9*d + 135*a^8*b^
2*c^8*d^2 + 80*a^9*b*c^7*d^3 + 14*a^10*c^6*d^4)*x^5 + 15/2*(4*a^7*b^3*c^10
+ 15*a^8*b^2*c^9*d + 15*a^9*b*c^8*d^2 + 4*a^10*c^7*d^3)*x^4 + 5/3*(9*a^8*b^
2*c^10 + 20*a^9*b*c^9*d + 9*a^10*c^8*d^2)*x^3 + 5*(a^9*b*c^10 + a^10*c^9*d)
*x^2
```

Fricas [B] time = 1.57117, size = 4215, normalized size = 15.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*d^10*b^10 + 1/2*x^20*d^9*c*b^10 + 1/2*x^20*d^10*b^9*a + 45/19*x^1
9*d^8*c^2*b^10 + 100/19*x^19*d^9*c*b^9*a + 45/19*x^19*d^10*b^8*a^2 + 20/3*x
^18*d^7*c^3*b^10 + 25*x^18*d^8*c^2*b^9*a + 25*x^18*d^9*c*b^8*a^2 + 20/3*x^1
8*d^10*b^7*a^3 + 210/17*x^17*d^6*c^4*b^10 + 1200/17*x^17*d^7*c^3*b^9*a + 20
25/17*x^17*d^8*c^2*b^8*a^2 + 1200/17*x^17*d^9*c*b^7*a^3 + 210/17*x^17*d^10*
b^6*a^4 + 63/4*x^16*d^5*c^5*b^10 + 525/4*x^16*d^6*c^4*b^9*a + 675/2*x^16*d^
7*c^3*b^8*a^2 + 675/2*x^16*d^8*c^2*b^7*a^3 + 525/4*x^16*d^9*c*b^6*a^4 + 63/
4*x^16*d^10*b^5*a^5 + 14*x^15*d^4*c^6*b^10 + 168*x^15*d^5*c^5*b^9*a + 630*x
^15*d^6*c^4*b^8*a^2 + 960*x^15*d^7*c^3*b^7*a^3 + 630*x^15*d^8*c^2*b^6*a^4 +
 168*x^15*d^9*c*b^5*a^5 + 14*x^15*d^10*b^4*a^6 + 60/7*x^14*d^3*c^7*b^10 + 1
50*x^14*d^4*c^6*b^9*a + 810*x^14*d^5*c^5*b^8*a^2 + 1800*x^14*d^6*c^4*b^7*a^
3 + 1800*x^14*d^7*c^3*b^6*a^4 + 810*x^14*d^8*c^2*b^5*a^5 + 150*x^14*d^9*c*b
^4*a^6 + 60/7*x^14*d^10*b^3*a^7 + 45/13*x^13*d^2*c^8*b^10 + 1200/13*x^13*d^
3*c^7*b^9*a + 9450/13*x^13*d^4*c^6*b^8*a^2 + 30240/13*x^13*d^5*c^5*b^7*a^3
+ 44100/13*x^13*d^6*c^4*b^6*a^4 + 30240/13*x^13*d^7*c^3*b^5*a^5 + 9450/13*x
^13*d^8*c^2*b^4*a^6 + 1200/13*x^13*d^9*c*b^3*a^7 + 45/13*x^13*d^10*b^2*a^8
+ 5/6*x^12*d*c^9*b^10 + 75/2*x^12*d^2*c^8*b^9*a + 450*x^12*d^3*c^7*b^8*a^2
+ 2100*x^12*d^4*c^6*b^7*a^3 + 4410*x^12*d^5*c^5*b^6*a^4 + 4410*x^12*d^6*c^4
*b^5*a^5 + 2100*x^12*d^7*c^3*b^4*a^6 + 450*x^12*d^8*c^2*b^3*a^7 + 75/2*x^12
*d^9*c*b^2*a^8 + 5/6*x^12*d^10*b*a^9 + 1/11*x^11*c^10*b^10 + 100/11*x^11*d*
c^9*b^9*a + 2025/11*x^11*d^2*c^8*b^8*a^2 + 14400/11*x^11*d^3*c^7*b^7*a^3 +
44100/11*x^11*d^4*c^6*b^6*a^4 + 63504/11*x^11*d^5*c^5*b^5*a^5 + 44100/11*x^
11*d^6*c^4*b^4*a^6 + 14400/11*x^11*d^7*c^3*b^3*a^7 + 2025/11*x^11*d^8*c^2*b
^2*a^8 + 100/11*x^11*d^9*c*b*a^9 + 1/11*x^11*d^10*a^10 + x^10*c^10*b^9*a +
45*x^10*d*c^9*b^8*a^2 + 540*x^10*d^2*c^8*b^7*a^3 + 2520*x^10*d^3*c^7*b^6*a^
4 + 5292*x^10*d^4*c^6*b^5*a^5 + 5292*x^10*d^5*c^5*b^4*a^6 + 2520*x^10*d^6*c
^4*b^3*a^7 + 540*x^10*d^7*c^3*b^2*a^8 + 45*x^10*d^8*c^2*b*a^9 + x^10*d^9*c*
a^10 + 5*x^9*c^10*b^8*a^2 + 400/3*x^9*d*c^9*b^7*a^3 + 1050*x^9*d^2*c^8*b^6*
a^4 + 3360*x^9*d^3*c^7*b^5*a^5 + 4900*x^9*d^4*c^6*b^4*a^6 + 3360*x^9*d^5*c^
5*b^3*a^7 + 1050*x^9*d^6*c^4*b^2*a^8 + 400/3*x^9*d^7*c^3*b*a^9 + 5*x^9*d^8*
c^2*a^10 + 15*x^8*c^10*b^7*a^3 + 525/2*x^8*d*c^9*b^6*a^4 + 2835/2*x^8*d^2*c
^8*b^5*a^5 + 3150*x^8*d^3*c^7*b^4*a^6 + 3150*x^8*d^4*c^6*b^3*a^7 + 2835/2*x
^8*d^5*c^5*b^2*a^8 + 525/2*x^8*d^6*c^4*b*a^9 + 15*x^8*d^7*c^3*a^10 + 30*x^7
*c^10*b^6*a^4 + 360*x^7*d*c^9*b^5*a^5 + 1350*x^7*d^2*c^8*b^4*a^6 + 14400/7*
x^7*d^3*c^7*b^3*a^7 + 1350*x^7*d^4*c^6*b^2*a^8 + 360*x^7*d^5*c^5*b*a^9 + 30
*x^7*d^6*c^4*a^10 + 42*x^6*c^10*b^5*a^5 + 350*x^6*d*c^9*b^4*a^6 + 900*x^6*d
^2*c^8*b^3*a^7 + 900*x^6*d^3*c^7*b^2*a^8 + 350*x^6*d^4*c^6*b*a^9 + 42*x^6*d
^5*c^5*a^10 + 42*x^5*c^10*b^4*a^6 + 240*x^5*d*c^9*b^3*a^7 + 405*x^5*d^2*c^8
*b^2*a^8 + 240*x^5*d^3*c^7*b*a^9 + 42*x^5*d^4*c^6*a^10 + 30*x^4*c^10*b^3*a^
7 + 225/2*x^4*d*c^9*b^2*a^8 + 225/2*x^4*d^2*c^8*b*a^9 + 30*x^4*d^3*c^7*a^10
```

```
 + 15*x^3*c^10*b^2*a^8 + 100/3*x^3*d*c^9*b*a^9 + 15*x^3*d^2*c^8*a^10 + 5*x^
2*c^10*b*a^9 + 5*x^2*d*c^9*a^10 + x*c^10*a^10
```

Sympy [B] time = 0.256867, size = 1775, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10*(d*x+c)**10,x)
```

```
[Out] a**10*c**10*x + b**10*d**10*x**21/21 + x**20*(a*b**9*d**10/2 + b**10*c*d**9
/2) + x**19*(45*a**2*b**8*d**10/19 + 100*a*b**9*c*d**9/19 + 45*b**10*c**2*d
**8/19) + x**18*(20*a**3*b**7*d**10/3 + 25*a**2*b**8*c*d**9 + 25*a*b**9*c**
2*d**8 + 20*b**10*c**3*d**7/3) + x**17*(210*a**4*b**6*d**10/17 + 1200*a**3*
b**7*c*d**9/17 + 2025*a**2*b**8*c**2*d**8/17 + 1200*a*b**9*c**3*d**7/17 + 2
10*b**10*c**4*d**6/17) + x**16*(63*a**5*b**5*d**10/4 + 525*a**4*b**6*c*d**9
/4 + 675*a**3*b**7*c**2*d**8/2 + 675*a**2*b**8*c**3*d**7/2 + 525*a*b**9*c**
4*d**6/4 + 63*b**10*c**5*d**5/4) + x**15*(14*a**6*b**4*d**10 + 168*a**5*b**
5*c*d**9 + 630*a**4*b**6*c**2*d**8 + 960*a**3*b**7*c**3*d**7 + 630*a**2*b**
8*c**4*d**6 + 168*a*b**9*c**5*d**5 + 14*b**10*c**6*d**4) + x**14*(60*a**7*b
**3*d**10/7 + 150*a**6*b**4*c*d**9 + 810*a**5*b**5*c**2*d**8 + 1800*a**4*b*
*6*c**3*d**7 + 1800*a**3*b**7*c**4*d**6 + 810*a**2*b**8*c**5*d**5 + 150*a*b
**9*c**6*d**4 + 60*b**10*c**7*d**3/7) + x**13*(45*a**8*b**2*d**10/13 + 1200
*a**7*b**3*c*d**9/13 + 9450*a**6*b**4*c**2*d**8/13 + 30240*a**5*b**5*c**3*d
**7/13 + 44100*a**4*b**6*c**4*d**6/13 + 30240*a**3*b**7*c**5*d**5/13 + 9450
*a**2*b**8*c**6*d**4/13 + 1200*a*b**9*c**7*d**3/13 + 45*b**10*c**8*d**2/13)
 + x**12*(5*a**9*b*d**10/6 + 75*a**8*b**2*c*d**9/2 + 450*a**7*b**3*c**2*d**
8 + 2100*a**6*b**4*c**3*d**7 + 4410*a**5*b**5*c**4*d**6 + 4410*a**4*b**6*c*
*5*d**5 + 2100*a**3*b**7*c**6*d**4 + 450*a**2*b**8*c**7*d**3 + 75*a*b**9*c*
*8*d**2/2 + 5*b**10*c**9*d/6) + x**11*(a**10*d**10/11 + 100*a**9*b*c*d**9/1
1 + 2025*a**8*b**2*c**2*d**8/11 + 14400*a**7*b**3*c**3*d**7/11 + 44100*a**6
*b**4*c**4*d**6/11 + 63504*a**5*b**5*c**5*d**5/11 + 44100*a**4*b**6*c**6*d*
*4/11 + 14400*a**3*b**7*c**7*d**3/11 + 2025*a**2*b**8*c**8*d**2/11 + 100*a*
b**9*c**9*d/11 + b**10*c**10/11) + x**10*(a**10*c*d**9 + 45*a**9*b*c**2*d**
8 + 540*a**8*b**2*c**3*d**7 + 2520*a**7*b**3*c**4*d**6 + 5292*a**6*b**4*c**
5*d**5 + 5292*a**5*b**5*c**6*d**4 + 2520*a**4*b**6*c**7*d**3 + 540*a**3*b**
7*c**8*d**2 + 45*a**2*b**8*c**9*d + a*b**9*c**10) + x**9*(5*a**10*c**2*d**8
 + 400*a**9*b*c**3*d**7/3 + 1050*a**8*b**2*c**4*d**6 + 3360*a**7*b**3*c**5*
d**5 + 4900*a**6*b**4*c**6*d**4 + 3360*a**5*b**5*c**7*d**3 + 1050*a**4*b**6
*c**8*d**2 + 400*a**3*b**7*c**9*d/3 + 5*a**2*b**8*c**10) + x**8*(15*a**10*c
**3*d**7 + 525*a**9*b*c**4*d**6/2 + 2835*a**8*b**2*c**5*d**5/2 + 3150*a**7*
b**3*c**6*d**4 + 3150*a**6*b**4*c**7*d**3 + 2835*a**5*b**5*c**8*d**2/2 + 52
5*a**4*b**6*c**9*d/2 + 15*a**3*b**7*c**10) + x**7*(30*a**10*c**4*d**6 + 360
*a**9*b*c**5*d**5 + 1350*a**8*b**2*c**6*d**4 + 14400*a**7*b**3*c**7*d**3/7
+ 1350*a**6*b**4*c**8*d**2 + 360*a**5*b**5*c**9*d + 30*a**4*b**6*c**10) + x
**6*(42*a**10*c**5*d**5 + 350*a**9*b*c**6*d**4 + 900*a**8*b**2*c**7*d**3 +
900*a**7*b**3*c**8*d**2 + 350*a**6*b**4*c**9*d + 42*a**5*b**5*c**10) + x**5
*(42*a**10*c**6*d**4 + 240*a**9*b*c**7*d**3 + 405*a**8*b**2*c**8*d**2 + 240
*a**7*b**3*c**9*d + 42*a**6*b**4*c**10) + x**4*(30*a**10*c**7*d**3 + 225*a*
*9*b*c**8*d**2/2 + 225*a**8*b**2*c**9*d/2 + 30*a**7*b**3*c**10) + x**3*(15*
a**10*c**8*d**2 + 100*a**9*b*c**9*d/3 + 15*a**8*b**2*c**10) + x**2*(5*a**10
*c**9*d + 5*a**9*b*c**10)
```

Giac [B] time = 1.06944, size = 2475, normalized size = 8.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/21*b^10*d^10*x^21 + 1/2*b^10*c*d^9*x^20 + 1/2*a*b^9*d^10*x^20 + 45/19*b^1
0*c^2*d^8*x^19 + 100/19*a*b^9*c*d^9*x^19 + 45/19*a^2*b^8*d^10*x^19 + 20/3*b
^10*c^3*d^7*x^18 + 25*a*b^9*c^2*d^8*x^18 + 25*a^2*b^8*c*d^9*x^18 + 20/3*a^3
*b^7*d^10*x^18 + 210/17*b^10*c^4*d^6*x^17 + 1200/17*a*b^9*c^3*d^7*x^17 + 20
25/17*a^2*b^8*c^2*d^8*x^17 + 1200/17*a^3*b^7*c*d^9*x^17 + 210/17*a^4*b^6*d^
10*x^17 + 63/4*b^10*c^5*d^5*x^16 + 525/4*a*b^9*c^4*d^6*x^16 + 675/2*a^2*b^8
*c^3*d^7*x^16 + 675/2*a^3*b^7*c^2*d^8*x^16 + 525/4*a^4*b^6*c*d^9*x^16 + 63/
4*a^5*b^5*d^10*x^16 + 14*b^10*c^6*d^4*x^15 + 168*a*b^9*c^5*d^5*x^15 + 630*a
^2*b^8*c^4*d^6*x^15 + 960*a^3*b^7*c^3*d^7*x^15 + 630*a^4*b^6*c^2*d^8*x^15 +
 168*a^5*b^5*c*d^9*x^15 + 14*a^6*b^4*d^10*x^15 + 60/7*b^10*c^7*d^3*x^14 + 1
50*a*b^9*c^6*d^4*x^14 + 810*a^2*b^8*c^5*d^5*x^14 + 1800*a^3*b^7*c^4*d^6*x^1
4 + 1800*a^4*b^6*c^3*d^7*x^14 + 810*a^5*b^5*c^2*d^8*x^14 + 150*a^6*b^4*c*d^
9*x^14 + 60/7*a^7*b^3*d^10*x^14 + 45/13*b^10*c^8*d^2*x^13 + 1200/13*a*b^9*c
^7*d^3*x^13 + 9450/13*a^2*b^8*c^6*d^4*x^13 + 30240/13*a^3*b^7*c^5*d^5*x^13
+ 44100/13*a^4*b^6*c^4*d^6*x^13 + 30240/13*a^5*b^5*c^3*d^7*x^13 + 9450/13*a
^6*b^4*c^2*d^8*x^13 + 1200/13*a^7*b^3*c*d^9*x^13 + 45/13*a^8*b^2*d^10*x^13
+ 5/6*b^10*c^9*d*x^12 + 75/2*a*b^9*c^8*d^2*x^12 + 450*a^2*b^8*c^7*d^3*x^12
+ 2100*a^3*b^7*c^6*d^4*x^12 + 4410*a^4*b^6*c^5*d^5*x^12 + 4410*a^5*b^5*c^4*
d^6*x^12 + 2100*a^6*b^4*c^3*d^7*x^12 + 450*a^7*b^3*c^2*d^8*x^12 + 75/2*a^8*
b^2*c*d^9*x^12 + 5/6*a^9*b*d^10*x^12 + 1/11*b^10*c^10*x^11 + 100/11*a*b^9*c
^9*d*x^11 + 2025/11*a^2*b^8*c^8*d^2*x^11 + 14400/11*a^3*b^7*c^7*d^3*x^11 +
44100/11*a^4*b^6*c^6*d^4*x^11 + 63504/11*a^5*b^5*c^5*d^5*x^11 + 44100/11*a^
6*b^4*c^4*d^6*x^11 + 14400/11*a^7*b^3*c^3*d^7*x^11 + 2025/11*a^8*b^2*c^2*d^
8*x^11 + 100/11*a^9*b*c*d^9*x^11 + 1/11*a^10*d^10*x^11 + a*b^9*c^10*x^10 +
45*a^2*b^8*c^9*d*x^10 + 540*a^3*b^7*c^8*d^2*x^10 + 2520*a^4*b^6*c^7*d^3*x^1
0 + 5292*a^5*b^5*c^6*d^4*x^10 + 5292*a^6*b^4*c^5*d^5*x^10 + 2520*a^7*b^3*c^
4*d^6*x^10 + 540*a^8*b^2*c^3*d^7*x^10 + 45*a^9*b*c^2*d^8*x^10 + a^10*c*d^9*
x^10 + 5*a^2*b^8*c^10*x^9 + 400/3*a^3*b^7*c^9*d*x^9 + 1050*a^4*b^6*c^8*d^2*
x^9 + 3360*a^5*b^5*c^7*d^3*x^9 + 4900*a^6*b^4*c^6*d^4*x^9 + 3360*a^7*b^3*c^
5*d^5*x^9 + 1050*a^8*b^2*c^4*d^6*x^9 + 400/3*a^9*b*c^3*d^7*x^9 + 5*a^10*c^2
*d^8*x^9 + 15*a^3*b^7*c^10*x^8 + 525/2*a^4*b^6*c^9*d*x^8 + 2835/2*a^5*b^5*c
^8*d^2*x^8 + 3150*a^6*b^4*c^7*d^3*x^8 + 3150*a^7*b^3*c^6*d^4*x^8 + 2835/2*a
^8*b^2*c^5*d^5*x^8 + 525/2*a^9*b*c^4*d^6*x^8 + 15*a^10*c^3*d^7*x^8 + 30*a^4
*b^6*c^10*x^7 + 360*a^5*b^5*c^9*d*x^7 + 1350*a^6*b^4*c^8*d^2*x^7 + 14400/7*
a^7*b^3*c^7*d^3*x^7 + 1350*a^8*b^2*c^6*d^4*x^7 + 360*a^9*b*c^5*d^5*x^7 + 30
*a^10*c^4*d^6*x^7 + 42*a^5*b^5*c^10*x^6 + 350*a^6*b^4*c^9*d*x^6 + 900*a^7*b
^3*c^8*d^2*x^6 + 900*a^8*b^2*c^7*d^3*x^6 + 350*a^9*b*c^6*d^4*x^6 + 42*a^10*
c^5*d^5*x^6 + 42*a^6*b^4*c^10*x^5 + 240*a^7*b^3*c^9*d*x^5 + 405*a^8*b^2*c^8
*d^2*x^5 + 240*a^9*b*c^7*d^3*x^5 + 42*a^10*c^6*d^4*x^5 + 30*a^7*b^3*c^10*x^
4 + 225/2*a^8*b^2*c^9*d*x^4 + 225/2*a^9*b*c^8*d^2*x^4 + 30*a^10*c^7*d^3*x^4
 + 15*a^8*b^2*c^10*x^3 + 100/3*a^9*b*c^9*d*x^3 + 15*a^10*c^8*d^2*x^3 + 5*a^
9*b*c^10*x^2 + 5*a^10*c^9*d*x^2 + a^10*c^10*x
```

3.1302 $\int (a+bx)^9(c+dx)^{10}\,dx$

Optimal. Leaf size=250

$$-\frac{9b^8(c+dx)^{19}(bc-ad)}{19d^{10}}+\frac{2b^7(c+dx)^{18}(bc-ad)^2}{d^{10}}-\frac{84b^6(c+dx)^{17}(bc-ad)^3}{17d^{10}}+\frac{63b^5(c+dx)^{16}(bc-ad)^4}{8d^{10}}-\frac{42b^4(c+$$

```
[Out] -((b*c - a*d)^9*(c + d*x)^11)/(11*d^10) + (3*b*(b*c - a*d)^8*(c + d*x)^12)/
(4*d^10) - (36*b^2*(b*c - a*d)^7*(c + d*x)^13)/(13*d^10) + (6*b^3*(b*c - a*
d)^6*(c + d*x)^14)/d^10 - (42*b^4*(b*c - a*d)^5*(c + d*x)^15)/(5*d^10) + (6
3*b^5*(b*c - a*d)^4*(c + d*x)^16)/(8*d^10) - (84*b^6*(b*c - a*d)^3*(c + d*x
)^17)/(17*d^10) + (2*b^7*(b*c - a*d)^2*(c + d*x)^18)/d^10 - (9*b^8*(b*c - a
*d)*(c + d*x)^19)/(19*d^10) + (b^9*(c + d*x)^20)/(20*d^10)
```

Rubi [A] time = 1.04202, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{9b^8(c+dx)^{19}(bc-ad)}{19d^{10}}+\frac{2b^7(c+dx)^{18}(bc-ad)^2}{d^{10}}-\frac{84b^6(c+dx)^{17}(bc-ad)^3}{17d^{10}}+\frac{63b^5(c+dx)^{16}(bc-ad)^4}{8d^{10}}-\frac{42b^4(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9*(c + d*x)^10,x]
```


```
[Out] -((b*c - a*d)^9*(c + d*x)^11)/(11*d^10) + (3*b*(b*c - a*d)^8*(c + d*x)^12)/
(4*d^10) - (36*b^2*(b*c - a*d)^7*(c + d*x)^13)/(13*d^10) + (6*b^3*(b*c - a*
d)^6*(c + d*x)^14)/d^10 - (42*b^4*(b*c - a*d)^5*(c + d*x)^15)/(5*d^10) + (6
3*b^5*(b*c - a*d)^4*(c + d*x)^16)/(8*d^10) - (84*b^6*(b*c - a*d)^3*(c + d*x
)^17)/(17*d^10) + (2*b^7*(b*c - a*d)^2*(c + d*x)^18)/d^10 - (9*b^8*(b*c - a
*d)*(c + d*x)^19)/(19*d^10) + (b^9*(c + d*x)^20)/(20*d^10)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^9(c+dx)^{10}\,dx=\int\left(\frac{(-bc+ad)^9(c+dx)^{10}}{d^9}+\frac{9b(bc-ad)^8(c+dx)^{11}}{d^9}-\frac{36b^2(bc-ad)^7(c+dx)^{12}}{d^9}+\frac{84b^3(bc$$

$$=-\frac{(bc-ad)^9(c+dx)^{11}}{11d^{10}}+\frac{3b(bc-ad)^8(c+dx)^{12}}{4d^{10}}-\frac{36b^2(bc-ad)^7(c+dx)^{13}}{13d^{10}}+\frac{6b^3(bc-ad)}{d^{10}}$$

Mathematica [B] time = 0.172641, size = 1397, normalized size = 5.59

$$\frac{1}{20}b^9d^{10}x^{20}+\frac{1}{19}b^8d^9(10bc+9ad)x^{19}+\frac{1}{2}b^7d^8\left(5b^2c^2+10abdc+4a^2d^2\right)x^{18}+\frac{3}{17}b^6d^7\left(40b^3c^3+135ab^2dc^2+120a^2bc$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9*(c + d*x)^10,x]
```

```
[Out] a^9*c^10*x + (a^8*c^9*(9*b*c + 10*a*d)*x^2)/2 + 3*a^7*c^8*(4*b^2*c^2 + 10*a
*b*c*d + 5*a^2*d^2)*x^3 + (3*a^6*c^7*(28*b^3*c^3 + 120*a*b^2*c^2*d + 135*a^
2*b*c*d^2 + 40*a^3*d^3)*x^4)/4 + (6*a^5*c^6*(21*b^4*c^4 + 140*a*b^3*c^3*d +
 270*a^2*b^2*c^2*d^2 + 180*a^3*b*c*d^3 + 35*a^4*d^4)*x^5)/5 + 3*a^4*c^5*(7*
b^5*c^5 + 70*a*b^4*c^4*d + 210*a^2*b^3*c^3*d^2 + 240*a^3*b^2*c^2*d^3 + 105*
a^4*b*c*d^4 + 14*a^5*d^5)*x^6 + 6*a^3*c^4*(2*b^6*c^6 + 30*a*b^5*c^5*d + 135
*a^2*b^4*c^4*d^2 + 240*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^4 + 54*a^5*b*c*d
^5 + 5*a^6*d^6)*x^7 + (3*a^2*c^3*(6*b^7*c^7 + 140*a*b^6*c^6*d + 945*a^2*b^5
*c^5*d^2 + 2520*a^3*b^4*c^4*d^3 + 2940*a^4*b^3*c^3*d^4 + 1512*a^5*b^2*c^2*d
^5 + 315*a^6*b*c*d^6 + 20*a^7*d^7)*x^8)/4 + a*c^2*(b^8*c^8 + 40*a*b^7*c^7*d
 + 420*a^2*b^6*c^6*d^2 + 1680*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4*d^4 + 2352
*a^5*b^3*c^3*d^5 + 840*a^6*b^2*c^2*d^6 + 120*a^7*b*c*d^7 + 5*a^8*d^8)*x^9 +
 (c*(b^9*c^9 + 90*a*b^8*c^8*d + 1620*a^2*b^7*c^7*d^2 + 10080*a^3*b^6*c^6*d^
3 + 26460*a^4*b^5*c^5*d^4 + 31752*a^5*b^4*c^4*d^5 + 17640*a^6*b^3*c^3*d^6 +
 4320*a^7*b^2*c^2*d^7 + 405*a^8*b*c*d^8 + 10*a^9*d^9)*x^10)/10 + (d*(10*b^9
*c^9 + 405*a*b^8*c^8*d + 4320*a^2*b^7*c^7*d^2 + 17640*a^3*b^6*c^6*d^3 + 317
52*a^4*b^5*c^5*d^4 + 26460*a^5*b^4*c^4*d^5 + 10080*a^6*b^3*c^3*d^6 + 1620*a
^7*b^2*c^2*d^7 + 90*a^8*b*c*d^8 + a^9*d^9)*x^11)/11 + (3*b*d^2*(5*b^8*c^8 +
 120*a*b^7*c^7*d + 840*a^2*b^6*c^6*d^2 + 2352*a^3*b^5*c^5*d^3 + 2940*a^4*b^
4*c^4*d^4 + 1680*a^5*b^3*c^3*d^5 + 420*a^6*b^2*c^2*d^6 + 40*a^7*b*c*d^7 + a
^8*d^8)*x^12)/4 + (6*b^2*d^3*(20*b^7*c^7 + 315*a*b^6*c^6*d + 1512*a^2*b^5*c
^5*d^2 + 2940*a^3*b^4*c^4*d^3 + 2520*a^4*b^3*c^3*d^4 + 945*a^5*b^2*c^2*d^5
+ 140*a^6*b*c*d^6 + 6*a^7*d^7)*x^13)/13 + 3*b^3*d^4*(5*b^6*c^6 + 54*a*b^5*c
^5*d + 180*a^2*b^4*c^4*d^2 + 240*a^3*b^3*c^3*d^3 + 135*a^4*b^2*c^2*d^4 + 30
*a^5*b*c*d^5 + 2*a^6*d^6)*x^14 + (6*b^4*d^5*(14*b^5*c^5 + 105*a*b^4*c^4*d +
 240*a^2*b^3*c^3*d^2 + 210*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 7*a^5*d^5)*x^
15)/5 + (3*b^5*d^6*(35*b^4*c^4 + 180*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 14
0*a^3*b*c*d^3 + 21*a^4*d^4)*x^16)/8 + (3*b^6*d^7*(40*b^3*c^3 + 135*a*b^2*c^
2*d + 120*a^2*b*c*d^2 + 28*a^3*d^3)*x^17)/17 + (b^7*d^8*(5*b^2*c^2 + 10*a*b
*c*d + 4*a^2*d^2)*x^18)/2 + (b^8*d^9*(10*b*c + 9*a*d)*x^19)/19 + (b^9*d^10*
x^20)/20
```

Maple [B] time = 0.003, size = 1441, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9*(d*x+c)^10,x)
```

```
[Out] 1/20*b^9*d^10*x^20+1/19*(9*a*b^8*d^10+10*b^9*c*d^9)*x^19+1/18*(36*a^2*b^7*d
^10+90*a*b^8*c*d^9+45*b^9*c^2*d^8)*x^18+1/17*(84*a^3*b^6*d^10+360*a^2*b^7*c
*d^9+405*a*b^8*c^2*d^8+120*b^9*c^3*d^7)*x^17+1/16*(126*a^4*b^5*d^10+840*a^3
*b^6*c*d^9+1620*a^2*b^7*c^2*d^8+1080*a*b^8*c^3*d^7+210*b^9*c^4*d^6)*x^16+1/
15*(126*a^5*b^4*d^10+1260*a^4*b^5*c*d^9+3780*a^3*b^6*c^2*d^8+4320*a^2*b^7*c
^3*d^7+1890*a*b^8*c^4*d^6+252*b^9*c^5*d^5)*x^15+1/14*(84*a^6*b^3*d^10+1260*
a^5*b^4*c*d^9+5670*a^4*b^5*c^2*d^8+10080*a^3*b^6*c^3*d^7+7560*a^2*b^7*c^4*d
^6+2268*a*b^8*c^5*d^5+210*b^9*c^6*d^4)*x^14+1/13*(36*a^7*b^2*d^10+840*a^6*b
^3*c*d^9+5670*a^5*b^4*c^2*d^8+15120*a^4*b^5*c^3*d^7+17640*a^3*b^6*c^4*d^6+9
072*a^2*b^7*c^5*d^5+1890*a*b^8*c^6*d^4+120*b^9*c^7*d^3)*x^13+1/12*(9*a^8*b*
d^10+360*a^7*b^2*c*d^9+3780*a^6*b^3*c^2*d^8+15120*a^5*b^4*c^3*d^7+26460*a^4
*b^5*c^4*d^6+21168*a^3*b^6*c^5*d^5+7560*a^2*b^7*c^6*d^4+1080*a*b^8*c^7*d^3+
45*b^9*c^8*d^2)*x^12+1/11*(a^9*d^10+90*a^8*b*c*d^9+1620*a^7*b^2*c^2*d^8+100
80*a^6*b^3*c^3*d^7+26460*a^5*b^4*c^4*d^6+31752*a^4*b^5*c^5*d^5+17640*a^3*b^
6*c^6*d^4+4320*a^2*b^7*c^7*d^3+405*a*b^8*c^8*d^2+10*b^9*c^9*d)*x^11+1/10*(1
```

```
0*a^9*c*d^9+405*a^8*b*c^2*d^8+4320*a^7*b^2*c^3*d^7+17640*a^6*b^3*c^4*d^6+31
752*a^5*b^4*c^5*d^5+26460*a^4*b^5*c^6*d^4+10080*a^3*b^6*c^7*d^3+1620*a^2*b^
7*c^8*d^2+90*a*b^8*c^9*d+b^9*c^10)*x^10+1/9*(45*a^9*c^2*d^8+1080*a^8*b*c^3*
d^7+7560*a^7*b^2*c^4*d^6+21168*a^6*b^3*c^5*d^5+26460*a^5*b^4*c^6*d^4+15120*
a^4*b^5*c^7*d^3+3780*a^3*b^6*c^8*d^2+360*a^2*b^7*c^9*d+9*a*b^8*c^10)*x^9+1/
8*(120*a^9*c^3*d^7+1890*a^8*b*c^4*d^6+9072*a^7*b^2*c^5*d^5+17640*a^6*b^3*c^
6*d^4+15120*a^5*b^4*c^7*d^3+5670*a^4*b^5*c^8*d^2+840*a^3*b^6*c^9*d+36*a^2*b
^7*c^10)*x^8+1/7*(210*a^9*c^4*d^6+2268*a^8*b*c^5*d^5+7560*a^7*b^2*c^6*d^4+1
0080*a^6*b^3*c^7*d^3+5670*a^5*b^4*c^8*d^2+1260*a^4*b^5*c^9*d+84*a^3*b^6*c^1
0)*x^7+1/6*(252*a^9*c^5*d^5+1890*a^8*b*c^6*d^4+4320*a^7*b^2*c^7*d^3+3780*a^
6*b^3*c^8*d^2+1260*a^5*b^4*c^9*d+126*a^4*b^5*c^10)*x^6+1/5*(210*a^9*c^6*d^4
+1080*a^8*b*c^7*d^3+1620*a^7*b^2*c^8*d^2+840*a^6*b^3*c^9*d+126*a^5*b^4*c^10
)*x^5+1/4*(120*a^9*c^7*d^3+405*a^8*b*c^8*d^2+360*a^7*b^2*c^9*d+84*a^6*b^3*c
^10)*x^4+1/3*(45*a^9*c^8*d^2+90*a^8*b*c^9*d+36*a^7*b^2*c^10)*x^3+1/2*(10*a^
9*c^9*d+9*a^8*b*c^10)*x^2+a^9*c^10*x
```

Maxima [B] time = 1.00114, size = 1940, normalized size = 7.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/20*b^9*d^10*x^20 + a^9*c^10*x + 1/19*(10*b^9*c*d^9 + 9*a*b^8*d^10)*x^19 +
 1/2*(5*b^9*c^2*d^8 + 10*a*b^8*c*d^9 + 4*a^2*b^7*d^10)*x^18 + 3/17*(40*b^9*
c^3*d^7 + 135*a*b^8*c^2*d^8 + 120*a^2*b^7*c*d^9 + 28*a^3*b^6*d^10)*x^17 + 3
/8*(35*b^9*c^4*d^6 + 180*a*b^8*c^3*d^7 + 270*a^2*b^7*c^2*d^8 + 140*a^3*b^6*
c*d^9 + 21*a^4*b^5*d^10)*x^16 + 6/5*(14*b^9*c^5*d^5 + 105*a*b^8*c^4*d^6 + 2
40*a^2*b^7*c^3*d^7 + 210*a^3*b^6*c^2*d^8 + 70*a^4*b^5*c*d^9 + 7*a^5*b^4*d^1
0)*x^15 + 3*(5*b^9*c^6*d^4 + 54*a*b^8*c^5*d^5 + 180*a^2*b^7*c^4*d^6 + 240*a
^3*b^6*c^3*d^7 + 135*a^4*b^5*c^2*d^8 + 30*a^5*b^4*c*d^9 + 2*a^6*b^3*d^10)*x
^14 + 6/13*(20*b^9*c^7*d^3 + 315*a*b^8*c^6*d^4 + 1512*a^2*b^7*c^5*d^5 + 294
0*a^3*b^6*c^4*d^6 + 2520*a^4*b^5*c^3*d^7 + 945*a^5*b^4*c^2*d^8 + 140*a^6*b^
3*c*d^9 + 6*a^7*b^2*d^10)*x^13 + 3/4*(5*b^9*c^8*d^2 + 120*a*b^8*c^7*d^3 + 8
40*a^2*b^7*c^6*d^4 + 2352*a^3*b^6*c^5*d^5 + 2940*a^4*b^5*c^4*d^6 + 1680*a^5
*b^4*c^3*d^7 + 420*a^6*b^3*c^2*d^8 + 40*a^7*b^2*c*d^9 + a^8*b*d^10)*x^12 +
1/11*(10*b^9*c^9*d + 405*a*b^8*c^8*d^2 + 4320*a^2*b^7*c^7*d^3 + 17640*a^3*b
^6*c^6*d^4 + 31752*a^4*b^5*c^5*d^5 + 26460*a^5*b^4*c^4*d^6 + 10080*a^6*b^3*
c^3*d^7 + 1620*a^7*b^2*c^2*d^8 + 90*a^8*b*c*d^9 + a^9*d^10)*x^11 + 1/10*(b^
9*c^10 + 90*a*b^8*c^9*d + 1620*a^2*b^7*c^8*d^2 + 10080*a^3*b^6*c^7*d^3 + 26
460*a^4*b^5*c^6*d^4 + 31752*a^5*b^4*c^5*d^5 + 17640*a^6*b^3*c^4*d^6 + 4320*
a^7*b^2*c^3*d^7 + 405*a^8*b*c^2*d^8 + 10*a^9*c*d^9)*x^10 + (a*b^8*c^10 + 40
*a^2*b^7*c^9*d + 420*a^3*b^6*c^8*d^2 + 1680*a^4*b^5*c^7*d^3 + 2940*a^5*b^4*
c^6*d^4 + 2352*a^6*b^3*c^5*d^5 + 840*a^7*b^2*c^4*d^6 + 120*a^8*b*c^3*d^7 +
5*a^9*c^2*d^8)*x^9 + 3/4*(6*a^2*b^7*c^10 + 140*a^3*b^6*c^9*d + 945*a^4*b^5*
c^8*d^2 + 2520*a^5*b^4*c^7*d^3 + 2940*a^6*b^3*c^6*d^4 + 1512*a^7*b^2*c^5*d^
5 + 315*a^8*b*c^4*d^6 + 20*a^9*c^3*d^7)*x^8 + 6*(2*a^3*b^6*c^10 + 30*a^4*b^
5*c^9*d + 135*a^5*b^4*c^8*d^2 + 240*a^6*b^3*c^7*d^3 + 180*a^7*b^2*c^6*d^4 +
 54*a^8*b*c^5*d^5 + 5*a^9*c^4*d^6)*x^7 + 3*(7*a^4*b^5*c^10 + 70*a^5*b^4*c^9
*d + 210*a^6*b^3*c^8*d^2 + 240*a^7*b^2*c^7*d^3 + 105*a^8*b*c^6*d^4 + 14*a^9
*c^5*d^5)*x^6 + 6/5*(21*a^5*b^4*c^10 + 140*a^6*b^3*c^9*d + 270*a^7*b^2*c^8*
d^2 + 180*a^8*b*c^7*d^3 + 35*a^9*c^6*d^4)*x^5 + 3/4*(28*a^6*b^3*c^10 + 120*
a^7*b^2*c^9*d + 135*a^8*b*c^8*d^2 + 40*a^9*c^7*d^3)*x^4 + 3*(4*a^7*b^2*c^10
 + 10*a^8*b*c^9*d + 5*a^9*c^8*d^2)*x^3 + 1/2*(9*a^8*b*c^10 + 10*a^9*c^9*d)*
x^2
```

Fricas [B] time = 1.60487, size = 3740, normalized size = 14.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*d^10*b^9 + 10/19*x^19*d^9*c*b^9 + 9/19*x^19*d^10*b^8*a + 5/2*x^18
*d^8*c^2*b^9 + 5*x^18*d^9*c*b^8*a + 2*x^18*d^10*b^7*a^2 + 120/17*x^17*d^7*c
^3*b^9 + 405/17*x^17*d^8*c^2*b^8*a + 360/17*x^17*d^9*c*b^7*a^2 + 84/17*x^17
*d^10*b^6*a^3 + 105/8*x^16*d^6*c^4*b^9 + 135/2*x^16*d^7*c^3*b^8*a + 405/4*x
^16*d^8*c^2*b^7*a^2 + 105/2*x^16*d^9*c*b^6*a^3 + 63/8*x^16*d^10*b^5*a^4 + 8
4/5*x^15*d^5*c^5*b^9 + 126*x^15*d^6*c^4*b^8*a + 288*x^15*d^7*c^3*b^7*a^2 +
252*x^15*d^8*c^2*b^6*a^3 + 84*x^15*d^9*c*b^5*a^4 + 42/5*x^15*d^10*b^4*a^5 +
 15*x^14*d^4*c^6*b^9 + 162*x^14*d^5*c^5*b^8*a + 540*x^14*d^6*c^4*b^7*a^2 +
720*x^14*d^7*c^3*b^6*a^3 + 405*x^14*d^8*c^2*b^5*a^4 + 90*x^14*d^9*c*b^4*a^5
 + 6*x^14*d^10*b^3*a^6 + 120/13*x^13*d^3*c^7*b^9 + 1890/13*x^13*d^4*c^6*b^8
*a + 9072/13*x^13*d^5*c^5*b^7*a^2 + 17640/13*x^13*d^6*c^4*b^6*a^3 + 15120/1
3*x^13*d^7*c^3*b^5*a^4 + 5670/13*x^13*d^8*c^2*b^4*a^5 + 840/13*x^13*d^9*c*b
^3*a^6 + 36/13*x^13*d^10*b^2*a^7 + 15/4*x^12*d^2*c^8*b^9 + 90*x^12*d^3*c^7*
b^8*a + 630*x^12*d^4*c^6*b^7*a^2 + 1764*x^12*d^5*c^5*b^6*a^3 + 2205*x^12*d^
6*c^4*b^5*a^4 + 1260*x^12*d^7*c^3*b^4*a^5 + 315*x^12*d^8*c^2*b^3*a^6 + 30*x
^12*d^9*c*b^2*a^7 + 3/4*x^12*d^10*b*a^8 + 10/11*x^11*d*c^9*b^9 + 405/11*x^1
1*d^2*c^8*b^8*a + 4320/11*x^11*d^3*c^7*b^7*a^2 + 17640/11*x^11*d^4*c^6*b^6*
a^3 + 31752/11*x^11*d^5*c^5*b^5*a^4 + 26460/11*x^11*d^6*c^4*b^4*a^5 + 10080
/11*x^11*d^7*c^3*b^3*a^6 + 1620/11*x^11*d^8*c^2*b^2*a^7 + 90/11*x^11*d^9*c*
b*a^8 + 1/11*x^11*d^10*a^9 + 1/10*x^10*c^10*b^9 + 9*x^10*d*c^9*b^8*a + 162*
x^10*d^2*c^8*b^7*a^2 + 1008*x^10*d^3*c^7*b^6*a^3 + 2646*x^10*d^4*c^6*b^5*a^
4 + 15876/5*x^10*d^5*c^5*b^4*a^5 + 1764*x^10*d^6*c^4*b^3*a^6 + 432*x^10*d^7
*c^3*b^2*a^7 + 81/2*x^10*d^8*c^2*b*a^8 + x^10*d^9*c*a^9 + x^9*c^10*b^8*a +
40*x^9*d*c^9*b^7*a^2 + 420*x^9*d^2*c^8*b^6*a^3 + 1680*x^9*d^3*c^7*b^5*a^4 +
 2940*x^9*d^4*c^6*b^4*a^5 + 2352*x^9*d^5*c^5*b^3*a^6 + 840*x^9*d^6*c^4*b^2*
a^7 + 120*x^9*d^7*c^3*b*a^8 + 5*x^9*d^8*c^2*a^9 + 9/2*x^8*c^10*b^7*a^2 + 10
5*x^8*d*c^9*b^6*a^3 + 2835/4*x^8*d^2*c^8*b^5*a^4 + 1890*x^8*d^3*c^7*b^4*a^5
 + 2205*x^8*d^4*c^6*b^3*a^6 + 1134*x^8*d^5*c^5*b^2*a^7 + 945/4*x^8*d^6*c^4*
b*a^8 + 15*x^8*d^7*c^3*a^9 + 12*x^7*c^10*b^6*a^3 + 180*x^7*d*c^9*b^5*a^4 +
810*x^7*d^2*c^8*b^4*a^5 + 1440*x^7*d^3*c^7*b^3*a^6 + 1080*x^7*d^4*c^6*b^2*a
^7 + 324*x^7*d^5*c^5*b*a^8 + 30*x^7*d^6*c^4*a^9 + 21*x^6*c^10*b^5*a^4 + 210
*x^6*d*c^9*b^4*a^5 + 630*x^6*d^2*c^8*b^3*a^6 + 720*x^6*d^3*c^7*b^2*a^7 + 31
5*x^6*d^4*c^6*b*a^8 + 42*x^6*d^5*c^5*a^9 + 126/5*x^5*c^10*b^4*a^5 + 168*x^5
*d*c^9*b^3*a^6 + 324*x^5*d^2*c^8*b^2*a^7 + 216*x^5*d^3*c^7*b*a^8 + 42*x^5*d
^4*c^6*a^9 + 21*x^4*c^10*b^3*a^6 + 90*x^4*d*c^9*b^2*a^7 + 405/4*x^4*d^2*c^8
*b*a^8 + 30*x^4*d^3*c^7*a^9 + 12*x^3*c^10*b^2*a^7 + 30*x^3*d*c^9*b*a^8 + 15
*x^3*d^2*c^8*a^9 + 9/2*x^2*c^10*b*a^8 + 5*x^2*d*c^9*a^9 + x*c^10*a^9
```

Sympy [B] time = 0.237887, size = 1598, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9*(d*x+c)**10,x)
```

```
[Out] a**9*c**10*x + b**9*d**10*x**20/20 + x**19*(9*a*b**8*d**10/19 + 10*b**9*c*d
**9/19) + x**18*(2*a**2*b**7*d**10 + 5*a*b**8*c*d**9 + 5*b**9*c**2*d**8/2)
```

```
+ x**17*(84*a**3*b**6*d**10/17 + 360*a**2*b**7*c*d**9/17 + 405*a*b**8*c**2*
d**8/17 + 120*b**9*c**3*d**7/17) + x**16*(63*a**4*b**5*d**10/8 + 105*a**3*b
**6*c*d**9/2 + 405*a**2*b**7*c**2*d**8/4 + 135*a*b**8*c**3*d**7/2 + 105*b**
9*c**4*d**6/8) + x**15*(42*a**5*b**4*d**10/5 + 84*a**4*b**5*c*d**9 + 252*a*
*3*b**6*c**2*d**8 + 288*a**2*b**7*c**3*d**7 + 126*a*b**8*c**4*d**6 + 84*b**
9*c**5*d**5/5) + x**14*(6*a**6*b**3*d**10 + 90*a**5*b**4*c*d**9 + 405*a**4*
b**5*c**2*d**8 + 720*a**3*b**6*c**3*d**7 + 540*a**2*b**7*c**4*d**6 + 162*a*
b**8*c**5*d**5 + 15*b**9*c**6*d**4) + x**13*(36*a**7*b**2*d**10/13 + 840*a*
*6*b**3*c*d**9/13 + 5670*a**5*b**4*c**2*d**8/13 + 15120*a**4*b**5*c**3*d**7
/13 + 17640*a**3*b**6*c**4*d**6/13 + 9072*a**2*b**7*c**5*d**5/13 + 1890*a*b
**8*c**6*d**4/13 + 120*b**9*c**7*d**3/13) + x**12*(3*a**8*b*d**10/4 + 30*a*
*7*b**2*c*d**9 + 315*a**6*b**3*c**2*d**8 + 1260*a**5*b**4*c**3*d**7 + 2205*
a**4*b**5*c**4*d**6 + 1764*a**3*b**6*c**5*d**5 + 630*a**2*b**7*c**6*d**4 +
90*a*b**8*c**7*d**3 + 15*b**9*c**8*d**2/4) + x**11*(a**9*d**10/11 + 90*a**8
*b*c*d**9/11 + 1620*a**7*b**2*c**2*d**8/11 + 10080*a**6*b**3*c**3*d**7/11 +
 26460*a**5*b**4*c**4*d**6/11 + 31752*a**4*b**5*c**5*d**5/11 + 17640*a**3*b
**6*c**6*d**4/11 + 4320*a**2*b**7*c**7*d**3/11 + 405*a*b**8*c**8*d**2/11 +
10*b**9*c**9*d/11) + x**10*(a**9*c*d**9 + 81*a**8*b*c**2*d**8/2 + 432*a**7*
b**2*c**3*d**7 + 1764*a**6*b**3*c**4*d**6 + 15876*a**5*b**4*c**5*d**5/5 + 2
646*a**4*b**5*c**6*d**4 + 1008*a**3*b**6*c**7*d**3 + 162*a**2*b**7*c**8*d**
2 + 9*a*b**8*c**9*d + b**9*c**10/10) + x**9*(5*a**9*c**2*d**8 + 120*a**8*b*
c**3*d**7 + 840*a**7*b**2*c**4*d**6 + 2352*a**6*b**3*c**5*d**5 + 2940*a**5*
b**4*c**6*d**4 + 1680*a**4*b**5*c**7*d**3 + 420*a**3*b**6*c**8*d**2 + 40*a*
*2*b**7*c**9*d + a*b**8*c**10) + x**8*(15*a**9*c**3*d**7 + 945*a**8*b*c**4*
d**6/4 + 1134*a**7*b**2*c**5*d**5 + 2205*a**6*b**3*c**6*d**4 + 1890*a**5*b*
*4*c**7*d**3 + 2835*a**4*b**5*c**8*d**2/4 + 105*a**3*b**6*c**9*d + 9*a**2*b
**7*c**10/2) + x**7*(30*a**9*c**4*d**6 + 324*a**8*b*c**5*d**5 + 1080*a**7*b
**2*c**6*d**4 + 1440*a**6*b**3*c**7*d**3 + 810*a**5*b**4*c**8*d**2 + 180*a*
*4*b**5*c**9*d + 12*a**3*b**6*c**10) + x**6*(42*a**9*c**5*d**5 + 315*a**8*b
*c**6*d**4 + 720*a**7*b**2*c**7*d**3 + 630*a**6*b**3*c**8*d**2 + 210*a**5*b
**4*c**9*d + 21*a**4*b**5*c**10) + x**5*(42*a**9*c**6*d**4 + 216*a**8*b*c**
7*d**3 + 324*a**7*b**2*c**8*d**2 + 168*a**6*b**3*c**9*d + 126*a**5*b**4*c**
10/5) + x**4*(30*a**9*c**7*d**3 + 405*a**8*b*c**8*d**2/4 + 90*a**7*b**2*c**
9*d + 21*a**6*b**3*c**10) + x**3*(15*a**9*c**8*d**2 + 30*a**8*b*c**9*d + 12
*a**7*b**2*c**10) + x**2*(5*a**9*c**9*d + 9*a**8*b*c**10/2)
```

Giac [B] time = 1.05973, size = 2236, normalized size = 8.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/20*b^9*d^10*x^20 + 10/19*b^9*c*d^9*x^19 + 9/19*a*b^8*d^10*x^19 + 5/2*b^9*
c^2*d^8*x^18 + 5*a*b^8*c*d^9*x^18 + 2*a^2*b^7*d^10*x^18 + 120/17*b^9*c^3*d^
7*x^17 + 405/17*a*b^8*c^2*d^8*x^17 + 360/17*a^2*b^7*c*d^9*x^17 + 84/17*a^3*
b^6*d^10*x^17 + 105/8*b^9*c^4*d^6*x^16 + 135/2*a*b^8*c^3*d^7*x^16 + 405/4*a
^2*b^7*c^2*d^8*x^16 + 105/2*a^3*b^6*c*d^9*x^16 + 63/8*a^4*b^5*d^10*x^16 + 8
4/5*b^9*c^5*d^5*x^15 + 126*a*b^8*c^4*d^6*x^15 + 288*a^2*b^7*c^3*d^7*x^15 +
252*a^3*b^6*c^2*d^8*x^15 + 84*a^4*b^5*c*d^9*x^15 + 42/5*a^5*b^4*d^10*x^15 +
 15*b^9*c^6*d^4*x^14 + 162*a*b^8*c^5*d^5*x^14 + 540*a^2*b^7*c^4*d^6*x^14 +
720*a^3*b^6*c^3*d^7*x^14 + 405*a^4*b^5*c^2*d^8*x^14 + 90*a^5*b^4*c*d^9*x^14
 + 6*a^6*b^3*d^10*x^14 + 120/13*b^9*c^7*d^3*x^13 + 1890/13*a*b^8*c^6*d^4*x^
13 + 9072/13*a^2*b^7*c^5*d^5*x^13 + 17640/13*a^3*b^6*c^4*d^6*x^13 + 15120/1
3*a^4*b^5*c^3*d^7*x^13 + 5670/13*a^5*b^4*c^2*d^8*x^13 + 840/13*a^6*b^3*c*d^
9*x^13 + 36/13*a^7*b^2*d^10*x^13 + 15/4*b^9*c^8*d^2*x^12 + 90*a*b^8*c^7*d^3
```

*x^12 + 630*a^2*b^7*c^6*d^4*x^12 + 1764*a^3*b^6*c^5*d^5*x^12 + 2205*a^4*b^5
*c^4*d^6*x^12 + 1260*a^5*b^4*c^3*d^7*x^12 + 315*a^6*b^3*c^2*d^8*x^12 + 30*a
^7*b^2*c*d^9*x^12 + 3/4*a^8*b*d^10*x^12 + 10/11*b^9*c^9*d*x^11 + 405/11*a*b
^8*c^8*d^2*x^11 + 4320/11*a^2*b^7*c^7*d^3*x^11 + 17640/11*a^3*b^6*c^6*d^4*x
^11 + 31752/11*a^4*b^5*c^5*d^5*x^11 + 26460/11*a^5*b^4*c^4*d^6*x^11 + 10080
/11*a^6*b^3*c^3*d^7*x^11 + 1620/11*a^7*b^2*c^2*d^8*x^11 + 90/11*a^8*b*c*d^9
*x^11 + 1/11*a^9*d^10*x^11 + 1/10*b^9*c^10*x^10 + 9*a*b^8*c^9*d*x^10 + 162*
a^2*b^7*c^8*d^2*x^10 + 1008*a^3*b^6*c^7*d^3*x^10 + 2646*a^4*b^5*c^6*d^4*x^1
0 + 15876/5*a^5*b^4*c^5*d^5*x^10 + 1764*a^6*b^3*c^4*d^6*x^10 + 432*a^7*b^2*
c^3*d^7*x^10 + 81/2*a^8*b*c^2*d^8*x^10 + a^9*c*d^9*x^10 + a*b^8*c^10*x^9 +
40*a^2*b^7*c^9*d*x^9 + 420*a^3*b^6*c^8*d^2*x^9 + 1680*a^4*b^5*c^7*d^3*x^9 +
2940*a^5*b^4*c^6*d^4*x^9 + 2352*a^6*b^3*c^5*d^5*x^9 + 840*a^7*b^2*c^4*d^6*
x^9 + 120*a^8*b*c^3*d^7*x^9 + 5*a^9*c^2*d^8*x^9 + 9/2*a^2*b^7*c^10*x^8 + 10
5*a^3*b^6*c^9*d*x^8 + 2835/4*a^4*b^5*c^8*d^2*x^8 + 1890*a^5*b^4*c^7*d^3*x^8
+ 2205*a^6*b^3*c^6*d^4*x^8 + 1134*a^7*b^2*c^5*d^5*x^8 + 945/4*a^8*b*c^4*d^
6*x^8 + 15*a^9*c^3*d^7*x^8 + 12*a^3*b^6*c^10*x^7 + 180*a^4*b^5*c^9*d*x^7 +
810*a^5*b^4*c^8*d^2*x^7 + 1440*a^6*b^3*c^7*d^3*x^7 + 1080*a^7*b^2*c^6*d^4*x
^7 + 324*a^8*b*c^5*d^5*x^7 + 30*a^9*c^4*d^6*x^7 + 21*a^4*b^5*c^10*x^6 + 210
*a^5*b^4*c^9*d*x^6 + 630*a^6*b^3*c^8*d^2*x^6 + 720*a^7*b^2*c^7*d^3*x^6 + 31
5*a^8*b*c^6*d^4*x^6 + 42*a^9*c^5*d^5*x^6 + 126/5*a^5*b^4*c^10*x^5 + 168*a^6
*b^3*c^9*d*x^5 + 324*a^7*b^2*c^8*d^2*x^5 + 216*a^8*b*c^7*d^3*x^5 + 42*a^9*c
^6*d^4*x^5 + 21*a^6*b^3*c^10*x^4 + 90*a^7*b^2*c^9*d*x^4 + 405/4*a^8*b*c^8*d
^2*x^4 + 30*a^9*c^7*d^3*x^4 + 12*a^7*b^2*c^10*x^3 + 30*a^8*b*c^9*d*x^3 + 15
*a^9*c^8*d^2*x^3 + 9/2*a^8*b*c^10*x^2 + 5*a^9*c^9*d*x^2 + a^9*c^10*x

3.1303 $\int (a+bx)^8(c+dx)^{10}\,dx$

Optimal. Leaf size=225

$$-\frac{4b^7(c+dx)^{18}(bc-ad)}{9d^9}+\frac{28b^6(c+dx)^{17}(bc-ad)^2}{17d^9}-\frac{7b^5(c+dx)^{16}(bc-ad)^3}{2d^9}+\frac{14b^4(c+dx)^{15}(bc-ad)^4}{3d^9}-\frac{4b^3(c+d}{}$$

```
[Out] ((b*c - a*d)^8*(c + d*x)^11)/(11*d^9) - (2*b*(b*c - a*d)^7*(c + d*x)^12)/(3
*d^9) + (28*b^2*(b*c - a*d)^6*(c + d*x)^13)/(13*d^9) - (4*b^3*(b*c - a*d)^5
*(c + d*x)^14)/d^9 + (14*b^4*(b*c - a*d)^4*(c + d*x)^15)/(3*d^9) - (7*b^5*(
b*c - a*d)^3*(c + d*x)^16)/(2*d^9) + (28*b^6*(b*c - a*d)^2*(c + d*x)^17)/(1
7*d^9) - (4*b^7*(b*c - a*d)*(c + d*x)^18)/(9*d^9) + (b^8*(c + d*x)^19)/(19*
d^9)
```

Rubi [A] time = 0.899381, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{4b^7(c+dx)^{18}(bc-ad)}{9d^9}+\frac{28b^6(c+dx)^{17}(bc-ad)^2}{17d^9}-\frac{7b^5(c+dx)^{16}(bc-ad)^3}{2d^9}+\frac{14b^4(c+dx)^{15}(bc-ad)^4}{3d^9}-\frac{4b^3(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8*(c + d*x)^10,x]
```


```
[Out] ((b*c - a*d)^8*(c + d*x)^11)/(11*d^9) - (2*b*(b*c - a*d)^7*(c + d*x)^12)/(3
*d^9) + (28*b^2*(b*c - a*d)^6*(c + d*x)^13)/(13*d^9) - (4*b^3*(b*c - a*d)^5
*(c + d*x)^14)/d^9 + (14*b^4*(b*c - a*d)^4*(c + d*x)^15)/(3*d^9) - (7*b^5*(
b*c - a*d)^3*(c + d*x)^16)/(2*d^9) + (28*b^6*(b*c - a*d)^2*(c + d*x)^17)/(1
7*d^9) - (4*b^7*(b*c - a*d)*(c + d*x)^18)/(9*d^9) + (b^8*(c + d*x)^19)/(19*
d^9)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^8(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^8(c+dx)^{10}}{d^8}-\frac{8b(bc-ad)^7(c+dx)^{11}}{d^8}+\frac{28b^2(bc-ad)^6(c+dx)^{12}}{d^8}-\frac{56b^3(bc}{}\right.\\ &= \frac{(bc-ad)^8(c+dx)^{11}}{11d^9}-\frac{2b(bc-ad)^7(c+dx)^{12}}{3d^9}+\frac{28b^2(bc-ad)^6(c+dx)^{13}}{13d^9}-\frac{4b^3(bc-ad)^5(}{d^9}\end{aligned}$$

Mathematica [B] time = 0.153915, size = 1241, normalized size = 5.52

$$\frac{1}{19}b^8d^{10}x^{19}+\frac{1}{9}b^7d^9(5bc+4ad)x^{18}+\frac{1}{17}b^6d^8\left(45b^2c^2+80abdc+28a^2d^2\right)x^{17}+\frac{1}{2}b^5d^7\left(15b^3c^3+45ab^2dc^2+35a^2bd^2c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8*(c + d*x)^10,x]
```

```
[Out] a^8*c^10*x + a^7*c^9*(4*b*c + 5*a*d)*x^2 + (a^6*c^8*(28*b^2*c^2 + 80*a*b*c*
d + 45*a^2*d^2)*x^3)/3 + 2*a^5*c^7*(7*b^3*c^3 + 35*a*b^2*c^2*d + 45*a^2*b*c
*d^2 + 15*a^3*d^3)*x^4 + 2*a^4*c^6*(7*b^4*c^4 + 56*a*b^3*c^3*d + 126*a^2*b^
2*c^2*d^2 + 96*a^3*b*c*d^3 + 21*a^4*d^4)*x^5 + (14*a^3*c^5*(2*b^5*c^5 + 25*
a*b^4*c^4*d + 90*a^2*b^3*c^3*d^2 + 120*a^3*b^2*c^2*d^3 + 60*a^4*b*c*d^4 + 9
*a^5*d^5)*x^6)/3 + 2*a^2*c^4*(2*b^6*c^6 + 40*a*b^5*c^5*d + 225*a^2*b^4*c^4*
d^2 + 480*a^3*b^3*c^3*d^3 + 420*a^4*b^2*c^2*d^4 + 144*a^5*b*c*d^5 + 15*a^6*
d^6)*x^7 + a*c^3*(b^7*c^7 + 35*a*b^6*c^6*d + 315*a^2*b^5*c^5*d^2 + 1050*a^3
*b^4*c^4*d^3 + 1470*a^4*b^3*c^3*d^4 + 882*a^5*b^2*c^2*d^5 + 210*a^6*b*c*d^6
 + 15*a^7*d^7)*x^8 + (c^2*(b^8*c^8 + 80*a*b^7*c^7*d + 1260*a^2*b^6*c^6*d^2
+ 6720*a^3*b^5*c^5*d^3 + 14700*a^4*b^4*c^4*d^4 + 14112*a^5*b^3*c^3*d^5 + 58
80*a^6*b^2*c^2*d^6 + 960*a^7*b*c*d^7 + 45*a^8*d^8)*x^9)/9 + c*d*(b^8*c^8 +
36*a*b^7*c^7*d + 336*a^2*b^6*c^6*d^2 + 1176*a^3*b^5*c^5*d^3 + 1764*a^4*b^4*
c^4*d^4 + 1176*a^5*b^3*c^3*d^5 + 336*a^6*b^2*c^2*d^6 + 36*a^7*b*c*d^7 + a^8
*d^8)*x^10 + (d^2*(45*b^8*c^8 + 960*a*b^7*c^7*d + 5880*a^2*b^6*c^6*d^2 + 14
112*a^3*b^5*c^5*d^3 + 14700*a^4*b^4*c^4*d^4 + 6720*a^5*b^3*c^3*d^5 + 1260*a
^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + a^8*d^8)*x^11)/11 + (2*b*d^3*(15*b^7*c^7
+ 210*a*b^6*c^6*d + 882*a^2*b^5*c^5*d^2 + 1470*a^3*b^4*c^4*d^3 + 1050*a^4*b
^3*c^3*d^4 + 315*a^5*b^2*c^2*d^5 + 35*a^6*b*c*d^6 + a^7*d^7)*x^12)/3 + (14*
b^2*d^4*(15*b^6*c^6 + 144*a*b^5*c^5*d + 420*a^2*b^4*c^4*d^2 + 480*a^3*b^3*c
^3*d^3 + 225*a^4*b^2*c^2*d^4 + 40*a^5*b*c*d^5 + 2*a^6*d^6)*x^13)/13 + 2*b^3
*d^5*(9*b^5*c^5 + 60*a*b^4*c^4*d + 120*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3
 + 25*a^4*b*c*d^4 + 2*a^5*d^5)*x^14 + (2*b^4*d^6*(21*b^4*c^4 + 96*a*b^3*c^3
*d + 126*a^2*b^2*c^2*d^2 + 56*a^3*b*c*d^3 + 7*a^4*d^4)*x^15)/3 + (b^5*d^7*(
15*b^3*c^3 + 45*a*b^2*c^2*d + 35*a^2*b*c*d^2 + 7*a^3*d^3)*x^16)/2 + (b^6*d^
8*(45*b^2*c^2 + 80*a*b*c*d + 28*a^2*d^2)*x^17)/17 + (b^7*d^9*(5*b*c + 4*a*d
)*x^18)/9 + (b^8*d^10*x^19)/19
```

Maple [B] time = 0.002, size = 1291, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8*(d*x+c)^10,x)
```

```
[Out] 1/19*b^8*d^10*x^19+1/18*(8*a*b^7*d^10+10*b^8*c*d^9)*x^18+1/17*(28*a^2*b^6*d
^10+80*a*b^7*c*d^9+45*b^8*c^2*d^8)*x^17+1/16*(56*a^3*b^5*d^10+280*a^2*b^6*c
*d^9+360*a*b^7*c^2*d^8+120*b^8*c^3*d^7)*x^16+1/15*(70*a^4*b^4*d^10+560*a^3*
b^5*c*d^9+1260*a^2*b^6*c^2*d^8+960*a*b^7*c^3*d^7+210*b^8*c^4*d^6)*x^15+1/14
*(56*a^5*b^3*d^10+700*a^4*b^4*c*d^9+2520*a^3*b^5*c^2*d^8+3360*a^2*b^6*c^3*d
^7+1680*a*b^7*c^4*d^6+252*b^8*c^5*d^5)*x^14+1/13*(28*a^6*b^2*d^10+560*a^5*b
^3*c*d^9+3150*a^4*b^4*c^2*d^8+6720*a^3*b^5*c^3*d^7+5880*a^2*b^6*c^4*d^6+201
6*a*b^7*c^5*d^5+210*b^8*c^6*d^4)*x^13+1/12*(8*a^7*b*d^10+280*a^6*b^2*c*d^9+
2520*a^5*b^3*c^2*d^8+8400*a^4*b^4*c^3*d^7+11760*a^3*b^5*c^4*d^6+7056*a^2*b^
6*c^5*d^5+1680*a*b^7*c^6*d^4+120*b^8*c^7*d^3)*x^12+1/11*(a^8*d^10+80*a^7*b*
c*d^9+1260*a^6*b^2*c^2*d^8+6720*a^5*b^3*c^3*d^7+14700*a^4*b^4*c^4*d^6+14112
*a^3*b^5*c^5*d^5+5880*a^2*b^6*c^6*d^4+960*a*b^7*c^7*d^3+45*b^8*c^8*d^2)*x^1
1+1/10*(10*a^8*c*d^9+360*a^7*b*c^2*d^8+3360*a^6*b^2*c^3*d^7+11760*a^5*b^3*c
^4*d^6+17640*a^4*b^4*c^5*d^5+11760*a^3*b^5*c^6*d^4+3360*a^2*b^6*c^7*d^3+360
*a*b^7*c^8*d^2+10*b^8*c^9*d)*x^10+1/9*(45*a^8*c^2*d^8+960*a^7*b*c^3*d^7+588
0*a^6*b^2*c^4*d^6+14112*a^5*b^3*c^5*d^5+14700*a^4*b^4*c^6*d^4+6720*a^3*b^5*
c^7*d^3+1260*a^2*b^6*c^8*d^2+80*a*b^7*c^9*d+b^8*c^10)*x^9+1/8*(120*a^8*c^3*
d^7+1680*a^7*b*c^4*d^6+7056*a^6*b^2*c^5*d^5+11760*a^5*b^3*c^6*d^4+8400*a^4*
b^4*c^7*d^3+2520*a^3*b^5*c^8*d^2+280*a^2*b^6*c^9*d+8*a*b^7*c^10)*x^8+1/7*(2
```

```
10*a^8*c^4*d^6+2016*a^7*b*c^5*d^5+5880*a^6*b^2*c^6*d^4+6720*a^5*b^3*c^7*d^3
+3150*a^4*b^4*c^8*d^2+560*a^3*b^5*c^9*d+28*a^2*b^6*c^10)*x^7+1/6*(252*a^8*c
^5*d^5+1680*a^7*b*c^6*d^4+3360*a^6*b^2*c^7*d^3+2520*a^5*b^3*c^8*d^2+700*a^4
*b^4*c^9*d+56*a^3*b^5*c^10)*x^6+1/5*(210*a^8*c^6*d^4+960*a^7*b*c^7*d^3+1260
*a^6*b^2*c^8*d^2+560*a^5*b^3*c^9*d+70*a^4*b^4*c^10)*x^5+1/4*(120*a^8*c^7*d^
3+360*a^7*b*c^8*d^2+280*a^6*b^2*c^9*d+56*a^5*b^3*c^10)*x^4+1/3*(45*a^8*c^8*
d^2+80*a^7*b*c^9*d+28*a^6*b^2*c^10)*x^3+1/2*(10*a^8*c^9*d+8*a^7*b*c^10)*x^2
+a^8*c^10*x
```

Maxima [B] time = 0.997121, size = 1732, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/19*b^8*d^10*x^19 + a^8*c^10*x + 1/9*(5*b^8*c*d^9 + 4*a*b^7*d^10)*x^18 + 1
/17*(45*b^8*c^2*d^8 + 80*a*b^7*c*d^9 + 28*a^2*b^6*d^10)*x^17 + 1/2*(15*b^8*
c^3*d^7 + 45*a*b^7*c^2*d^8 + 35*a^2*b^6*c*d^9 + 7*a^3*b^5*d^10)*x^16 + 2/3*
(21*b^8*c^4*d^6 + 96*a*b^7*c^3*d^7 + 126*a^2*b^6*c^2*d^8 + 56*a^3*b^5*c*d^9
 + 7*a^4*b^4*d^10)*x^15 + 2*(9*b^8*c^5*d^5 + 60*a*b^7*c^4*d^6 + 120*a^2*b^6
*c^3*d^7 + 90*a^3*b^5*c^2*d^8 + 25*a^4*b^4*c*d^9 + 2*a^5*b^3*d^10)*x^14 + 1
4/13*(15*b^8*c^6*d^4 + 144*a*b^7*c^5*d^5 + 420*a^2*b^6*c^4*d^6 + 480*a^3*b^
5*c^3*d^7 + 225*a^4*b^4*c^2*d^8 + 40*a^5*b^3*c*d^9 + 2*a^6*b^2*d^10)*x^13 +
 2/3*(15*b^8*c^7*d^3 + 210*a*b^7*c^6*d^4 + 882*a^2*b^6*c^5*d^5 + 1470*a^3*b
^5*c^4*d^6 + 1050*a^4*b^4*c^3*d^7 + 315*a^5*b^3*c^2*d^8 + 35*a^6*b^2*c*d^9
+ a^7*b*d^10)*x^12 + 1/11*(45*b^8*c^8*d^2 + 960*a*b^7*c^7*d^3 + 5880*a^2*b^
6*c^6*d^4 + 14112*a^3*b^5*c^5*d^5 + 14700*a^4*b^4*c^4*d^6 + 6720*a^5*b^3*c^
3*d^7 + 1260*a^6*b^2*c^2*d^8 + 80*a^7*b*c*d^9 + a^8*d^10)*x^11 + (b^8*c^9*d
 + 36*a*b^7*c^8*d^2 + 336*a^2*b^6*c^7*d^3 + 1176*a^3*b^5*c^6*d^4 + 1764*a^4
*b^4*c^5*d^5 + 1176*a^5*b^3*c^4*d^6 + 336*a^6*b^2*c^3*d^7 + 36*a^7*b*c^2*d^
8 + a^8*c*d^9)*x^10 + 1/9*(b^8*c^10 + 80*a*b^7*c^9*d + 1260*a^2*b^6*c^8*d^2
 + 6720*a^3*b^5*c^7*d^3 + 14700*a^4*b^4*c^6*d^4 + 14112*a^5*b^3*c^5*d^5 + 5
880*a^6*b^2*c^4*d^6 + 960*a^7*b*c^3*d^7 + 45*a^8*c^2*d^8)*x^9 + (a*b^7*c^10
 + 35*a^2*b^6*c^9*d + 315*a^3*b^5*c^8*d^2 + 1050*a^4*b^4*c^7*d^3 + 1470*a^5
*b^3*c^6*d^4 + 882*a^6*b^2*c^5*d^5 + 210*a^7*b*c^4*d^6 + 15*a^8*c^3*d^7)*x^
8 + 2*(2*a^2*b^6*c^10 + 40*a^3*b^5*c^9*d + 225*a^4*b^4*c^8*d^2 + 480*a^5*b^
3*c^7*d^3 + 420*a^6*b^2*c^6*d^4 + 144*a^7*b*c^5*d^5 + 15*a^8*c^4*d^6)*x^7 +
 14/3*(2*a^3*b^5*c^10 + 25*a^4*b^4*c^9*d + 90*a^5*b^3*c^8*d^2 + 120*a^6*b^2
*c^7*d^3 + 60*a^7*b*c^6*d^4 + 9*a^8*c^5*d^5)*x^6 + 2*(7*a^4*b^4*c^10 + 56*a
^5*b^3*c^9*d + 126*a^6*b^2*c^8*d^2 + 96*a^7*b*c^7*d^3 + 21*a^8*c^6*d^4)*x^5
 + 2*(7*a^5*b^3*c^10 + 35*a^6*b^2*c^9*d + 45*a^7*b*c^8*d^2 + 15*a^8*c^7*d^3
)*x^4 + 1/3*(28*a^6*b^2*c^10 + 80*a^7*b*c^9*d + 45*a^8*c^8*d^2)*x^3 + (4*a^
7*b*c^10 + 5*a^8*c^9*d)*x^2
```

Fricas [B] time = 1.6589, size = 3305, normalized size = 14.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*d^10*b^8 + 5/9*x^18*d^9*c*b^8 + 4/9*x^18*d^10*b^7*a + 45/17*x^17*
d^8*c^2*b^8 + 80/17*x^17*d^9*c*b^7*a + 28/17*x^17*d^10*b^6*a^2 + 15/2*x^16*
d^7*c^3*b^8 + 45/2*x^16*d^8*c^2*b^7*a + 35/2*x^16*d^9*c*b^6*a^2 + 7/2*x^16*
d^10*b^5*a^3 + 14*x^15*d^6*c^4*b^8 + 64*x^15*d^7*c^3*b^7*a + 84*x^15*d^8*c^
2*b^6*a^2 + 112/3*x^15*d^9*c*b^5*a^3 + 14/3*x^15*d^10*b^4*a^4 + 18*x^14*d^5
*c^5*b^8 + 120*x^14*d^6*c^4*b^7*a + 240*x^14*d^7*c^3*b^6*a^2 + 180*x^14*d^8
*c^2*b^5*a^3 + 50*x^14*d^9*c*b^4*a^4 + 4*x^14*d^10*b^3*a^5 + 210/13*x^13*d^
4*c^6*b^8 + 2016/13*x^13*d^5*c^5*b^7*a + 5880/13*x^13*d^6*c^4*b^6*a^2 + 672
0/13*x^13*d^7*c^3*b^5*a^3 + 3150/13*x^13*d^8*c^2*b^4*a^4 + 560/13*x^13*d^9*
c*b^3*a^5 + 28/13*x^13*d^10*b^2*a^6 + 10*x^12*d^3*c^7*b^8 + 140*x^12*d^4*c^
6*b^7*a + 588*x^12*d^5*c^5*b^6*a^2 + 980*x^12*d^6*c^4*b^5*a^3 + 700*x^12*d^
7*c^3*b^4*a^4 + 210*x^12*d^8*c^2*b^3*a^5 + 70/3*x^12*d^9*c*b^2*a^6 + 2/3*x^
12*d^10*b*a^7 + 45/11*x^11*d^2*c^8*b^8 + 960/11*x^11*d^3*c^7*b^7*a + 5880/1
1*x^11*d^4*c^6*b^6*a^2 + 14112/11*x^11*d^5*c^5*b^5*a^3 + 14700/11*x^11*d^6*
c^4*b^4*a^4 + 6720/11*x^11*d^7*c^3*b^3*a^5 + 1260/11*x^11*d^8*c^2*b^2*a^6 +
 80/11*x^11*d^9*c*b*a^7 + 1/11*x^11*d^10*a^8 + x^10*d*c^9*b^8 + 36*x^10*d^2
*c^8*b^7*a + 336*x^10*d^3*c^7*b^6*a^2 + 1176*x^10*d^4*c^6*b^5*a^3 + 1764*x^
10*d^5*c^5*b^4*a^4 + 1176*x^10*d^6*c^4*b^3*a^5 + 336*x^10*d^7*c^3*b^2*a^6 +
 36*x^10*d^8*c^2*b*a^7 + x^10*d^9*c*a^8 + 1/9*x^9*c^10*b^8 + 80/9*x^9*d*c^9
*b^7*a + 140*x^9*d^2*c^8*b^6*a^2 + 2240/3*x^9*d^3*c^7*b^5*a^3 + 4900/3*x^9*
d^4*c^6*b^4*a^4 + 1568*x^9*d^5*c^5*b^3*a^5 + 1960/3*x^9*d^6*c^4*b^2*a^6 + 3
20/3*x^9*d^7*c^3*b*a^7 + 5*x^9*d^8*c^2*a^8 + x^8*c^10*b^7*a + 35*x^8*d*c^9*
b^6*a^2 + 315*x^8*d^2*c^8*b^5*a^3 + 1050*x^8*d^3*c^7*b^4*a^4 + 1470*x^8*d^4
*c^6*b^3*a^5 + 882*x^8*d^5*c^5*b^2*a^6 + 210*x^8*d^6*c^4*b*a^7 + 15*x^8*d^7
*c^3*a^8 + 4*x^7*c^10*b^6*a^2 + 80*x^7*d*c^9*b^5*a^3 + 450*x^7*d^2*c^8*b^4*
a^4 + 960*x^7*d^3*c^7*b^3*a^5 + 840*x^7*d^4*c^6*b^2*a^6 + 288*x^7*d^5*c^5*b
*a^7 + 30*x^7*d^6*c^4*a^8 + 28/3*x^6*c^10*b^5*a^3 + 350/3*x^6*d*c^9*b^4*a^4
 + 420*x^6*d^2*c^8*b^3*a^5 + 560*x^6*d^3*c^7*b^2*a^6 + 280*x^6*d^4*c^6*b*a^
7 + 42*x^6*d^5*c^5*a^8 + 14*x^5*c^10*b^4*a^4 + 112*x^5*d*c^9*b^3*a^5 + 252*
x^5*d^2*c^8*b^2*a^6 + 192*x^5*d^3*c^7*b*a^7 + 42*x^5*d^4*c^6*a^8 + 14*x^4*c
^10*b^3*a^5 + 70*x^4*d*c^9*b^2*a^6 + 90*x^4*d^2*c^8*b*a^7 + 30*x^4*d^3*c^7*
a^8 + 28/3*x^3*c^10*b^2*a^6 + 80/3*x^3*d*c^9*b*a^7 + 15*x^3*d^2*c^8*a^8 + 4
*x^2*c^10*b*a^7 + 5*x^2*d*c^9*a^8 + x*c^10*a^8
```

Sympy [B] time = 0.227105, size = 1428, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8*(d*x+c)**10,x)
```

```
[Out] a**8*c**10*x + b**8*d**10*x**19/19 + x**18*(4*a*b**7*d**10/9 + 5*b**8*c*d**
9/9) + x**17*(28*a**2*b**6*d**10/17 + 80*a*b**7*c*d**9/17 + 45*b**8*c**2*d*
*8/17) + x**16*(7*a**3*b**5*d**10/2 + 35*a**2*b**6*c*d**9/2 + 45*a*b**7*c**
2*d**8/2 + 15*b**8*c**3*d**7/2) + x**15*(14*a**4*b**4*d**10/3 + 112*a**3*b*
*5*c*d**9/3 + 84*a**2*b**6*c**2*d**8 + 64*a*b**7*c**3*d**7 + 14*b**8*c**4*d
**6) + x**14*(4*a**5*b**3*d**10 + 50*a**4*b**4*c*d**9 + 180*a**3*b**5*c**2*
d**8 + 240*a**2*b**6*c**3*d**7 + 120*a*b**7*c**4*d**6 + 18*b**8*c**5*d**5)
+ x**13*(28*a**6*b**2*d**10/13 + 560*a**5*b**3*c*d**9/13 + 3150*a**4*b**4*c
**2*d**8/13 + 6720*a**3*b**5*c**3*d**7/13 + 5880*a**2*b**6*c**4*d**6/13 + 2
016*a*b**7*c**5*d**5/13 + 210*b**8*c**6*d**4/13) + x**12*(2*a**7*b*d**10/3
+ 70*a**6*b**2*c*d**9/3 + 210*a**5*b**3*c**2*d**8 + 700*a**4*b**4*c**3*d**7
 + 980*a**3*b**5*c**4*d**6 + 588*a**2*b**6*c**5*d**5 + 140*a*b**7*c**6*d**4
 + 10*b**8*c**7*d**3) + x**11*(a**8*d**10/11 + 80*a**7*b*c*d**9/11 + 1260*a
**6*b**2*c**2*d**8/11 + 6720*a**5*b**3*c**3*d**7/11 + 14700*a**4*b**4*c**4*
d**6/11 + 14112*a**3*b**5*c**5*d**5/11 + 5880*a**2*b**6*c**6*d**4/11 + 960*
```

```
a*b**7*c**7*d**3/11 + 45*b**8*c**8*d**2/11) + x**10*(a**8*c*d**9 + 36*a**7*
b*c**2*d**8 + 336*a**6*b**2*c**3*d**7 + 1176*a**5*b**3*c**4*d**6 + 1764*a**
4*b**4*c**5*d**5 + 1176*a**3*b**5*c**6*d**4 + 336*a**2*b**6*c**7*d**3 + 36*
a*b**7*c**8*d**2 + b**8*c**9*d) + x**9*(5*a**8*c**2*d**8 + 320*a**7*b*c**3*
d**7/3 + 1960*a**6*b**2*c**4*d**6/3 + 1568*a**5*b**3*c**5*d**5 + 4900*a**4*
b**4*c**6*d**4/3 + 2240*a**3*b**5*c**7*d**3/3 + 140*a**2*b**6*c**8*d**2 + 8
0*a*b**7*c**9*d/9 + b**8*c**10/9) + x**8*(15*a**8*c**3*d**7 + 210*a**7*b*c*
*4*d**6 + 882*a**6*b**2*c**5*d**5 + 1470*a**5*b**3*c**6*d**4 + 1050*a**4*b*
*4*c**7*d**3 + 315*a**3*b**5*c**8*d**2 + 35*a**2*b**6*c**9*d + a*b**7*c**10
) + x**7*(30*a**8*c**4*d**6 + 288*a**7*b*c**5*d**5 + 840*a**6*b**2*c**6*d**
4 + 960*a**5*b**3*c**7*d**3 + 450*a**4*b**4*c**8*d**2 + 80*a**3*b**5*c**9*d
 + 4*a**2*b**6*c**10) + x**6*(42*a**8*c**5*d**5 + 280*a**7*b*c**6*d**4 + 56
0*a**6*b**2*c**7*d**3 + 420*a**5*b**3*c**8*d**2 + 350*a**4*b**4*c**9*d/3 +
28*a**3*b**5*c**10/3) + x**5*(42*a**8*c**6*d**4 + 192*a**7*b*c**7*d**3 + 25
2*a**6*b**2*c**8*d**2 + 112*a**5*b**3*c**9*d + 14*a**4*b**4*c**10) + x**4*(
30*a**8*c**7*d**3 + 90*a**7*b*c**8*d**2 + 70*a**6*b**2*c**9*d + 14*a**5*b**
3*c**10) + x**3*(15*a**8*c**8*d**2 + 80*a**7*b*c**9*d/3 + 28*a**6*b**2*c**1
0/3) + x**2*(5*a**8*c**9*d + 4*a**7*b*c**10)
```

Giac [B] time = 1.07491, size = 1995, normalized size = 8.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/19*b^8*d^10*x^19 + 5/9*b^8*c*d^9*x^18 + 4/9*a*b^7*d^10*x^18 + 45/17*b^8*c
^2*d^8*x^17 + 80/17*a*b^7*c*d^9*x^17 + 28/17*a^2*b^6*d^10*x^17 + 15/2*b^8*c
^3*d^7*x^16 + 45/2*a*b^7*c^2*d^8*x^16 + 35/2*a^2*b^6*c*d^9*x^16 + 7/2*a^3*b
^5*d^10*x^16 + 14*b^8*c^4*d^6*x^15 + 64*a*b^7*c^3*d^7*x^15 + 84*a^2*b^6*c^2
*d^8*x^15 + 112/3*a^3*b^5*c*d^9*x^15 + 14/3*a^4*b^4*d^10*x^15 + 18*b^8*c^5*
d^5*x^14 + 120*a*b^7*c^4*d^6*x^14 + 240*a^2*b^6*c^3*d^7*x^14 + 180*a^3*b^5*
c^2*d^8*x^14 + 50*a^4*b^4*c*d^9*x^14 + 4*a^5*b^3*d^10*x^14 + 210/13*b^8*c^6
*d^4*x^13 + 2016/13*a*b^7*c^5*d^5*x^13 + 5880/13*a^2*b^6*c^4*d^6*x^13 + 672
0/13*a^3*b^5*c^3*d^7*x^13 + 3150/13*a^4*b^4*c^2*d^8*x^13 + 560/13*a^5*b^3*c
*d^9*x^13 + 28/13*a^6*b^2*d^10*x^13 + 10*b^8*c^7*d^3*x^12 + 140*a*b^7*c^6*d
^4*x^12 + 588*a^2*b^6*c^5*d^5*x^12 + 980*a^3*b^5*c^4*d^6*x^12 + 700*a^4*b^4
*c^3*d^7*x^12 + 210*a^5*b^3*c^2*d^8*x^12 + 70/3*a^6*b^2*c*d^9*x^12 + 2/3*a^
7*b*d^10*x^12 + 45/11*b^8*c^8*d^2*x^11 + 960/11*a*b^7*c^7*d^3*x^11 + 5880/1
1*a^2*b^6*c^6*d^4*x^11 + 14112/11*a^3*b^5*c^5*d^5*x^11 + 14700/11*a^4*b^4*c
^4*d^6*x^11 + 6720/11*a^5*b^3*c^3*d^7*x^11 + 1260/11*a^6*b^2*c^2*d^8*x^11 +
 80/11*a^7*b*c*d^9*x^11 + 1/11*a^8*d^10*x^11 + b^8*c^9*d*x^10 + 36*a*b^7*c^
8*d^2*x^10 + 336*a^2*b^6*c^7*d^3*x^10 + 1176*a^3*b^5*c^6*d^4*x^10 + 1764*a^
4*b^4*c^5*d^5*x^10 + 1176*a^5*b^3*c^4*d^6*x^10 + 336*a^6*b^2*c^3*d^7*x^10 +
 36*a^7*b*c^2*d^8*x^10 + a^8*c*d^9*x^10 + 1/9*b^8*c^10*x^9 + 80/9*a*b^7*c^9
*d*x^9 + 140*a^2*b^6*c^8*d^2*x^9 + 2240/3*a^3*b^5*c^7*d^3*x^9 + 4900/3*a^4*
b^4*c^6*d^4*x^9 + 1568*a^5*b^3*c^5*d^5*x^9 + 1960/3*a^6*b^2*c^4*d^6*x^9 + 3
20/3*a^7*b*c^3*d^7*x^9 + 5*a^8*c^2*d^8*x^9 + a*b^7*c^10*x^8 + 35*a^2*b^6*c^
9*d*x^8 + 315*a^3*b^5*c^8*d^2*x^8 + 1050*a^4*b^4*c^7*d^3*x^8 + 1470*a^5*b^3
*c^6*d^4*x^8 + 882*a^6*b^2*c^5*d^5*x^8 + 210*a^7*b*c^4*d^6*x^8 + 15*a^8*c^3
*d^7*x^8 + 4*a^2*b^6*c^10*x^7 + 80*a^3*b^5*c^9*d*x^7 + 450*a^4*b^4*c^8*d^2*
x^7 + 960*a^5*b^3*c^7*d^3*x^7 + 840*a^6*b^2*c^6*d^4*x^7 + 288*a^7*b*c^5*d^5
*x^7 + 30*a^8*c^4*d^6*x^7 + 28/3*a^3*b^5*c^10*x^6 + 350/3*a^4*b^4*c^9*d*x^6
 + 420*a^5*b^3*c^8*d^2*x^6 + 560*a^6*b^2*c^7*d^3*x^6 + 280*a^7*b*c^6*d^4*x^
6 + 42*a^8*c^5*d^5*x^6 + 14*a^4*b^4*c^10*x^5 + 112*a^5*b^3*c^9*d*x^5 + 252*
a^6*b^2*c^8*d^2*x^5 + 192*a^7*b*c^7*d^3*x^5 + 42*a^8*c^6*d^4*x^5 + 14*a^5*b
```

```
^3*c^10*x^4 + 70*a^6*b^2*c^9*d*x^4 + 90*a^7*b*c^8*d^2*x^4 + 30*a^8*c^7*d^3*
x^4 + 28/3*a^6*b^2*c^10*x^3 + 80/3*a^7*b*c^9*d*x^3 + 15*a^8*c^8*d^2*x^3 + 4
*a^7*b*c^10*x^2 + 5*a^8*c^9*d*x^2 + a^8*c^10*x
```

3.1304 $\int (a+bx)^7 (c+dx)^{10}\,dx$

Optimal. Leaf size=200

$$-\frac{7b^6(c+dx)^{17}(bc-ad)}{17d^8}+\frac{21b^5(c+dx)^{16}(bc-ad)^2}{16d^8}-\frac{7b^4(c+dx)^{15}(bc-ad)^3}{3d^8}+\frac{5b^3(c+dx)^{14}(bc-ad)^4}{2d^8}-\frac{21b^2(c+d}{}$$

```
[Out] -((b*c - a*d)^7*(c + d*x)^11)/(11*d^8) + (7*b*(b*c - a*d)^6*(c + d*x)^12)/(
12*d^8) - (21*b^2*(b*c - a*d)^5*(c + d*x)^13)/(13*d^8) + (5*b^3*(b*c - a*d)
^4*(c + d*x)^14)/(2*d^8) - (7*b^4*(b*c - a*d)^3*(c + d*x)^15)/(3*d^8) + (21
*b^5*(b*c - a*d)^2*(c + d*x)^16)/(16*d^8) - (7*b^6*(b*c - a*d)*(c + d*x)^17
)/(17*d^8) + (b^7*(c + d*x)^18)/(18*d^8)
```

Rubi [A] time = 0.76581, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{7b^6(c+dx)^{17}(bc-ad)}{17d^8}+\frac{21b^5(c+dx)^{16}(bc-ad)^2}{16d^8}-\frac{7b^4(c+dx)^{15}(bc-ad)^3}{3d^8}+\frac{5b^3(c+dx)^{14}(bc-ad)^4}{2d^8}-\frac{21b^2(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)^7*(c + d*x)^11)/(11*d^8) + (7*b*(b*c - a*d)^6*(c + d*x)^12)/(
12*d^8) - (21*b^2*(b*c - a*d)^5*(c + d*x)^13)/(13*d^8) + (5*b^3*(b*c - a*d)
^4*(c + d*x)^14)/(2*d^8) - (7*b^4*(b*c - a*d)^3*(c + d*x)^15)/(3*d^8) + (21
*b^5*(b*c - a*d)^2*(c + d*x)^16)/(16*d^8) - (7*b^6*(b*c - a*d)*(c + d*x)^17
)/(17*d^8) + (b^7*(c + d*x)^18)/(18*d^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^7(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^7(c+dx)^{10}}{d^7}+\frac{7b(bc-ad)^6(c+dx)^{11}}{d^7}-\frac{21b^2(bc-ad)^5(c+dx)^{12}}{d^7}+\frac{35b^3(bc}{}\right.$$

$$= -\frac{(bc-ad)^7(c+dx)^{11}}{11d^8}+\frac{7b(bc-ad)^6(c+dx)^{12}}{12d^8}-\frac{21b^2(bc-ad)^5(c+dx)^{13}}{13d^8}+\frac{5b^3(bc-ad)}{2d}$$

Mathematica [B] time = 0.124828, size = 1105, normalized size = 5.52

$$\frac{1}{18}b^7d^{10}x^{18}+\frac{1}{17}b^6d^9(10bc+7ad)x^{17}+\frac{1}{16}b^5d^8\left(45b^2c^2+70abdc+21a^2d^2\right)x^{16}+\frac{1}{3}b^4d^7\left(24b^3c^3+63ab^2dc^2+42a^2bc\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7*(c + d*x)^10,x]
```

```
[Out] a^7*c^10*x + (a^6*c^9*(7*b*c + 10*a*d)*x^2)/2 + (a^5*c^8*(21*b^2*c^2 + 70*a
*b*c*d + 45*a^2*d^2)*x^3)/3 + (5*a^4*c^7*(7*b^3*c^3 + 42*a*b^2*c^2*d + 63*a
^2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + 7*a^3*c^6*(b^4*c^4 + 10*a*b^3*c^3*d + 27*
a^2*b^2*c^2*d^2 + 24*a^3*b*c*d^3 + 6*a^4*d^4)*x^5 + (7*a^2*c^5*(3*b^5*c^5 +
 50*a*b^4*c^4*d + 225*a^2*b^3*c^3*d^2 + 360*a^3*b^2*c^2*d^3 + 210*a^4*b*c*d
^4 + 36*a^5*d^5)*x^6)/6 + a*c^4*(b^6*c^6 + 30*a*b^5*c^5*d + 225*a^2*b^4*c^4
*d^2 + 600*a^3*b^3*c^3*d^3 + 630*a^4*b^2*c^2*d^4 + 252*a^5*b*c*d^5 + 30*a^6
*d^6)*x^7 + (c^3*(b^7*c^7 + 70*a*b^6*c^6*d + 945*a^2*b^5*c^5*d^2 + 4200*a^3
*b^4*c^4*d^3 + 7350*a^4*b^3*c^3*d^4 + 5292*a^5*b^2*c^2*d^5 + 1470*a^6*b*c*d
^6 + 120*a^7*d^7)*x^8)/8 + (5*c^2*d*(2*b^7*c^7 + 63*a*b^6*c^6*d + 504*a^2*b
^5*c^5*d^2 + 1470*a^3*b^4*c^4*d^3 + 1764*a^4*b^3*c^3*d^4 + 882*a^5*b^2*c^2*
d^5 + 168*a^6*b*c*d^6 + 9*a^7*d^7)*x^9)/9 + (c*d^2*(9*b^7*c^7 + 168*a*b^6*c
^6*d + 882*a^2*b^5*c^5*d^2 + 1764*a^3*b^4*c^4*d^3 + 1470*a^4*b^3*c^3*d^4 +
504*a^5*b^2*c^2*d^5 + 63*a^6*b*c*d^6 + 2*a^7*d^7)*x^10)/2 + (d^3*(120*b^7*c
^7 + 1470*a*b^6*c^6*d + 5292*a^2*b^5*c^5*d^2 + 7350*a^3*b^4*c^4*d^3 + 4200*
a^4*b^3*c^3*d^4 + 945*a^5*b^2*c^2*d^5 + 70*a^6*b*c*d^6 + a^7*d^7)*x^11)/11
+ (7*b*d^4*(30*b^6*c^6 + 252*a*b^5*c^5*d + 630*a^2*b^4*c^4*d^2 + 600*a^3*b^
3*c^3*d^3 + 225*a^4*b^2*c^2*d^4 + 30*a^5*b*c*d^5 + a^6*d^6)*x^12)/12 + (7*b
^2*d^5*(36*b^5*c^5 + 210*a*b^4*c^4*d + 360*a^2*b^3*c^3*d^2 + 225*a^3*b^2*c^
2*d^3 + 50*a^4*b*c*d^4 + 3*a^5*d^5)*x^13)/13 + (5*b^3*d^6*(6*b^4*c^4 + 24*a
*b^3*c^3*d + 27*a^2*b^2*c^2*d^2 + 10*a^3*b*c*d^3 + a^4*d^4)*x^14)/2 + (b^4*
d^7*(24*b^3*c^3 + 63*a*b^2*c^2*d + 42*a^2*b*c*d^2 + 7*a^3*d^3)*x^15)/3 + (b
^5*d^8*(45*b^2*c^2 + 70*a*b*c*d + 21*a^2*d^2)*x^16)/16 + (b^6*d^9*(10*b*c +
 7*a*d)*x^17)/17 + (b^7*d^10*x^18)/18
```

Maple [B] time = 0.003, size = 1141, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7*(d*x+c)^10,x)
```

```
[Out] 1/18*b^7*d^10*x^18+1/17*(7*a*b^6*d^10+10*b^7*c*d^9)*x^17+1/16*(21*a^2*b^5*d
^10+70*a*b^6*c*d^9+45*b^7*c^2*d^8)*x^16+1/15*(35*a^3*b^4*d^10+210*a^2*b^5*c
*d^9+315*a*b^6*c^2*d^8+120*b^7*c^3*d^7)*x^15+1/14*(35*a^4*b^3*d^10+350*a^3*
b^4*c*d^9+945*a^2*b^5*c^2*d^8+840*a*b^6*c^3*d^7+210*b^7*c^4*d^6)*x^14+1/13*
(21*a^5*b^2*d^10+350*a^4*b^3*c*d^9+1575*a^3*b^4*c^2*d^8+2520*a^2*b^5*c^3*d^
7+1470*a*b^6*c^4*d^6+252*b^7*c^5*d^5)*x^13+1/12*(7*a^6*b*d^10+210*a^5*b^2*c
*d^9+1575*a^4*b^3*c^2*d^8+4200*a^3*b^4*c^3*d^7+4410*a^2*b^5*c^4*d^6+1764*a*
b^6*c^5*d^5+210*b^7*c^6*d^4)*x^12+1/11*(a^7*d^10+70*a^6*b*c*d^9+945*a^5*b^2
*c^2*d^8+4200*a^4*b^3*c^3*d^7+7350*a^3*b^4*c^4*d^6+5292*a^2*b^5*c^5*d^5+147
0*a*b^6*c^6*d^4+120*b^7*c^7*d^3)*x^11+1/10*(10*a^7*c*d^9+315*a^6*b*c^2*d^8+
2520*a^5*b^2*c^3*d^7+7350*a^4*b^3*c^4*d^6+8820*a^3*b^4*c^5*d^5+4410*a^2*b^5
*c^6*d^4+840*a*b^6*c^7*d^3+45*b^7*c^8*d^2)*x^10+1/9*(45*a^7*c^2*d^8+840*a^6
*b*c^3*d^7+4410*a^5*b^2*c^4*d^6+8820*a^4*b^3*c^5*d^5+7350*a^3*b^4*c^6*d^4+2
520*a^2*b^5*c^7*d^3+315*a*b^6*c^8*d^2+10*b^7*c^9*d)*x^9+1/8*(120*a^7*c^3*d^
7+1470*a^6*b*c^4*d^6+5292*a^5*b^2*c^5*d^5+7350*a^4*b^3*c^6*d^4+4200*a^3*b^4
*c^7*d^3+945*a^2*b^5*c^8*d^2+70*a*b^6*c^9*d+b^7*c^10)*x^8+1/7*(210*a^7*c^4*
d^6+1764*a^6*b*c^5*d^5+4410*a^5*b^2*c^6*d^4+4200*a^4*b^3*c^7*d^3+1575*a^3*b
^4*c^8*d^2+210*a^2*b^5*c^9*d+7*a*b^6*c^10)*x^7+1/6*(252*a^7*c^5*d^5+1470*a^
6*b*c^6*d^4+2520*a^5*b^2*c^7*d^3+1575*a^4*b^3*c^8*d^2+350*a^3*b^4*c^9*d+21*
a^2*b^5*c^10)*x^6+1/5*(210*a^7*c^6*d^4+840*a^6*b*c^7*d^3+945*a^5*b^2*c^8*d^
2+350*a^4*b^3*c^9*d+35*a^3*b^4*c^10)*x^5+1/4*(120*a^7*c^7*d^3+315*a^6*b*c^8
*d^2+210*a^5*b^2*c^9*d+35*a^4*b^3*c^10)*x^4+1/3*(45*a^7*c^8*d^2+70*a^6*b*c^
9*d+21*a^5*b^2*c^10)*x^3+1/2*(10*a^7*c^9*d+7*a^6*b*c^10)*x^2+a^7*c^10*x
```

Maxima [B] time = 0.980027, size = 1532, normalized size = 7.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/18*b^7*d^10*x^18 + a^7*c^10*x + 1/17*(10*b^7*c*d^9 + 7*a*b^6*d^10)*x^17 +
 1/16*(45*b^7*c^2*d^8 + 70*a*b^6*c*d^9 + 21*a^2*b^5*d^10)*x^16 + 1/3*(24*b^
7*c^3*d^7 + 63*a*b^6*c^2*d^8 + 42*a^2*b^5*c*d^9 + 7*a^3*b^4*d^10)*x^15 + 5/
2*(6*b^7*c^4*d^6 + 24*a*b^6*c^3*d^7 + 27*a^2*b^5*c^2*d^8 + 10*a^3*b^4*c*d^9
 + a^4*b^3*d^10)*x^14 + 7/13*(36*b^7*c^5*d^5 + 210*a*b^6*c^4*d^6 + 360*a^2*
b^5*c^3*d^7 + 225*a^3*b^4*c^2*d^8 + 50*a^4*b^3*c*d^9 + 3*a^5*b^2*d^10)*x^13
 + 7/12*(30*b^7*c^6*d^4 + 252*a*b^6*c^5*d^5 + 630*a^2*b^5*c^4*d^6 + 600*a^3
*b^4*c^3*d^7 + 225*a^4*b^3*c^2*d^8 + 30*a^5*b^2*c*d^9 + a^6*b*d^10)*x^12 +
1/11*(120*b^7*c^7*d^3 + 1470*a*b^6*c^6*d^4 + 5292*a^2*b^5*c^5*d^5 + 7350*a^
3*b^4*c^4*d^6 + 4200*a^4*b^3*c^3*d^7 + 945*a^5*b^2*c^2*d^8 + 70*a^6*b*c*d^9
 + a^7*d^10)*x^11 + 1/2*(9*b^7*c^8*d^2 + 168*a*b^6*c^7*d^3 + 882*a^2*b^5*c^
6*d^4 + 1764*a^3*b^4*c^5*d^5 + 1470*a^4*b^3*c^4*d^6 + 504*a^5*b^2*c^3*d^7 +
 63*a^6*b*c^2*d^8 + 2*a^7*c*d^9)*x^10 + 5/9*(2*b^7*c^9*d + 63*a*b^6*c^8*d^2
 + 504*a^2*b^5*c^7*d^3 + 1470*a^3*b^4*c^6*d^4 + 1764*a^4*b^3*c^5*d^5 + 882*
a^5*b^2*c^4*d^6 + 168*a^6*b*c^3*d^7 + 9*a^7*c^2*d^8)*x^9 + 1/8*(b^7*c^10 +
70*a*b^6*c^9*d + 945*a^2*b^5*c^8*d^2 + 4200*a^3*b^4*c^7*d^3 + 7350*a^4*b^3*
c^6*d^4 + 5292*a^5*b^2*c^5*d^5 + 1470*a^6*b*c^4*d^6 + 120*a^7*c^3*d^7)*x^8
+ (a*b^6*c^10 + 30*a^2*b^5*c^9*d + 225*a^3*b^4*c^8*d^2 + 600*a^4*b^3*c^7*d^
3 + 630*a^5*b^2*c^6*d^4 + 252*a^6*b*c^5*d^5 + 30*a^7*c^4*d^6)*x^7 + 7/6*(3*
a^2*b^5*c^10 + 50*a^3*b^4*c^9*d + 225*a^4*b^3*c^8*d^2 + 360*a^5*b^2*c^7*d^3
 + 210*a^6*b*c^6*d^4 + 36*a^7*c^5*d^5)*x^6 + 7*(a^3*b^4*c^10 + 10*a^4*b^3*c
^9*d + 27*a^5*b^2*c^8*d^2 + 24*a^6*b*c^7*d^3 + 6*a^7*c^6*d^4)*x^5 + 5/4*(7*
a^4*b^3*c^10 + 42*a^5*b^2*c^9*d + 63*a^6*b*c^8*d^2 + 24*a^7*c^7*d^3)*x^4 +
1/3*(21*a^5*b^2*c^10 + 70*a^6*b*c^9*d + 45*a^7*c^8*d^2)*x^3 + 1/2*(7*a^6*b*
c^10 + 10*a^7*c^9*d)*x^2
```

Fricas [B] time = 1.61419, size = 2931, normalized size = 14.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*d^10*b^7 + 10/17*x^17*d^9*c*b^7 + 7/17*x^17*d^10*b^6*a + 45/16*x^
16*d^8*c^2*b^7 + 35/8*x^16*d^9*c*b^6*a + 21/16*x^16*d^10*b^5*a^2 + 8*x^15*d
^7*c^3*b^7 + 21*x^15*d^8*c^2*b^6*a + 14*x^15*d^9*c*b^5*a^2 + 7/3*x^15*d^10*
b^4*a^3 + 15*x^14*d^6*c^4*b^7 + 60*x^14*d^7*c^3*b^6*a + 135/2*x^14*d^8*c^2*
b^5*a^2 + 25*x^14*d^9*c*b^4*a^3 + 5/2*x^14*d^10*b^3*a^4 + 252/13*x^13*d^5*c
^5*b^7 + 1470/13*x^13*d^6*c^4*b^6*a + 2520/13*x^13*d^7*c^3*b^5*a^2 + 1575/1
3*x^13*d^8*c^2*b^4*a^3 + 350/13*x^13*d^9*c*b^3*a^4 + 21/13*x^13*d^10*b^2*a^
5 + 35/2*x^12*d^4*c^6*b^7 + 147*x^12*d^5*c^5*b^6*a + 735/2*x^12*d^6*c^4*b^5
*a^2 + 350*x^12*d^7*c^3*b^4*a^3 + 525/4*x^12*d^8*c^2*b^3*a^4 + 35/2*x^12*d^
9*c*b^2*a^5 + 7/12*x^12*d^10*b*a^6 + 120/11*x^11*d^3*c^7*b^7 + 1470/11*x^11
*d^4*c^6*b^6*a + 5292/11*x^11*d^5*c^5*b^5*a^2 + 7350/11*x^11*d^6*c^4*b^4*a^
3 + 4200/11*x^11*d^7*c^3*b^3*a^4 + 945/11*x^11*d^8*c^2*b^2*a^5 + 70/11*x^11
*d^9*c*b*a^6 + 1/11*x^11*d^10*a^7 + 9/2*x^10*d^2*c^8*b^7 + 84*x^10*d^3*c^7*
```

```
b^6*a + 441*x^10*d^4*c^6*b^5*a^2 + 882*x^10*d^5*c^5*b^4*a^3 + 735*x^10*d^6*
c^4*b^3*a^4 + 252*x^10*d^7*c^3*b^2*a^5 + 63/2*x^10*d^8*c^2*b*a^6 + x^10*d^9
*c*a^7 + 10/9*x^9*d*c^9*b^7 + 35*x^9*d^2*c^8*b^6*a + 280*x^9*d^3*c^7*b^5*a^
2 + 2450/3*x^9*d^4*c^6*b^4*a^3 + 980*x^9*d^5*c^5*b^3*a^4 + 490*x^9*d^6*c^4*
b^2*a^5 + 280/3*x^9*d^7*c^3*b*a^6 + 5*x^9*d^8*c^2*a^7 + 1/8*x^8*c^10*b^7 +
35/4*x^8*d*c^9*b^6*a + 945/8*x^8*d^2*c^8*b^5*a^2 + 525*x^8*d^3*c^7*b^4*a^3
+ 3675/4*x^8*d^4*c^6*b^3*a^4 + 1323/2*x^8*d^5*c^5*b^2*a^5 + 735/4*x^8*d^6*c
^4*b*a^6 + 15*x^8*d^7*c^3*a^7 + x^7*c^10*b^6*a + 30*x^7*d*c^9*b^5*a^2 + 225
*x^7*d^2*c^8*b^4*a^3 + 600*x^7*d^3*c^7*b^3*a^4 + 630*x^7*d^4*c^6*b^2*a^5 +
252*x^7*d^5*c^5*b*a^6 + 30*x^7*d^6*c^4*a^7 + 7/2*x^6*c^10*b^5*a^2 + 175/3*x
^6*d*c^9*b^4*a^3 + 525/2*x^6*d^2*c^8*b^3*a^4 + 420*x^6*d^3*c^7*b^2*a^5 + 24
5*x^6*d^4*c^6*b*a^6 + 42*x^6*d^5*c^5*a^7 + 7*x^5*c^10*b^4*a^3 + 70*x^5*d*c^
9*b^3*a^4 + 189*x^5*d^2*c^8*b^2*a^5 + 168*x^5*d^3*c^7*b*a^6 + 42*x^5*d^4*c^
6*a^7 + 35/4*x^4*c^10*b^3*a^4 + 105/2*x^4*d*c^9*b^2*a^5 + 315/4*x^4*d^2*c^8
*b*a^6 + 30*x^4*d^3*c^7*a^7 + 7*x^3*c^10*b^2*a^5 + 70/3*x^3*d*c^9*b*a^6 + 1
5*x^3*d^2*c^8*a^7 + 7/2*x^2*c^10*b*a^6 + 5*x^2*d*c^9*a^7 + x*c^10*a^7
```

Sympy [B] time = 0.233145, size = 1280, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7*(d*x+c)**10,x)
```

```
[Out] a**7*c**10*x + b**7*d**10*x**18/18 + x**17*(7*a*b**6*d**10/17 + 10*b**7*c*d
**9/17) + x**16*(21*a**2*b**5*d**10/16 + 35*a*b**6*c*d**9/8 + 45*b**7*c**2*
d**8/16) + x**15*(7*a**3*b**4*d**10/3 + 14*a**2*b**5*c*d**9 + 21*a*b**6*c**
2*d**8 + 8*b**7*c**3*d**7) + x**14*(5*a**4*b**3*d**10/2 + 25*a**3*b**4*c*d*
*9 + 135*a**2*b**5*c**2*d**8/2 + 60*a*b**6*c**3*d**7 + 15*b**7*c**4*d**6) +
 x**13*(21*a**5*b**2*d**10/13 + 350*a**4*b**3*c*d**9/13 + 1575*a**3*b**4*c*
*2*d**8/13 + 2520*a**2*b**5*c**3*d**7/13 + 1470*a*b**6*c**4*d**6/13 + 252*b
**7*c**5*d**5/13) + x**12*(7*a**6*b*d**10/12 + 35*a**5*b**2*c*d**9/2 + 525*
a**4*b**3*c**2*d**8/4 + 350*a**3*b**4*c**3*d**7 + 735*a**2*b**5*c**4*d**6/2
 + 147*a*b**6*c**5*d**5 + 35*b**7*c**6*d**4/2) + x**11*(a**7*d**10/11 + 70*
a**6*b*c*d**9/11 + 945*a**5*b**2*c**2*d**8/11 + 4200*a**4*b**3*c**3*d**7/11
 + 7350*a**3*b**4*c**4*d**6/11 + 5292*a**2*b**5*c**5*d**5/11 + 1470*a*b**6*
c**6*d**4/11 + 120*b**7*c**7*d**3/11) + x**10*(a**7*c*d**9 + 63*a**6*b*c**2
*d**8/2 + 252*a**5*b**2*c**3*d**7 + 735*a**4*b**3*c**4*d**6 + 882*a**3*b**4
*c**5*d**5 + 441*a**2*b**5*c**6*d**4 + 84*a*b**6*c**7*d**3 + 9*b**7*c**8*d*
*2/2) + x**9*(5*a**7*c**2*d**8 + 280*a**6*b*c**3*d**7/3 + 490*a**5*b**2*c**
4*d**6 + 980*a**4*b**3*c**5*d**5 + 2450*a**3*b**4*c**6*d**4/3 + 280*a**2*b*
*5*c**7*d**3 + 35*a*b**6*c**8*d**2 + 10*b**7*c**9*d/9) + x**8*(15*a**7*c**3
*d**7 + 735*a**6*b*c**4*d**6/4 + 1323*a**5*b**2*c**5*d**5/2 + 3675*a**4*b**
3*c**6*d**4/4 + 525*a**3*b**4*c**7*d**3 + 945*a**2*b**5*c**8*d**2/8 + 35*a*
b**6*c**9*d/4 + b**7*c**10/8) + x**7*(30*a**7*c**4*d**6 + 252*a**6*b*c**5*d
**5 + 630*a**5*b**2*c**6*d**4 + 600*a**4*b**3*c**7*d**3 + 225*a**3*b**4*c**
8*d**2 + 30*a**2*b**5*c**9*d + a*b**6*c**10) + x**6*(42*a**7*c**5*d**5 + 24
5*a**6*b*c**6*d**4 + 420*a**5*b**2*c**7*d**3 + 525*a**4*b**3*c**8*d**2/2 +
175*a**3*b**4*c**9*d/3 + 7*a**2*b**5*c**10/2) + x**5*(42*a**7*c**6*d**4 + 1
68*a**6*b*c**7*d**3 + 189*a**5*b**2*c**8*d**2 + 70*a**4*b**3*c**9*d + 7*a**
3*b**4*c**10) + x**4*(30*a**7*c**7*d**3 + 315*a**6*b*c**8*d**2/4 + 105*a**5
*b**2*c**9*d/2 + 35*a**4*b**3*c**10/4) + x**3*(15*a**7*c**8*d**2 + 70*a**6*
b*c**9*d/3 + 7*a**5*b**2*c**10) + x**2*(5*a**7*c**9*d + 7*a**6*b*c**10/2)
```

Giac [B] time = 1.06255, size = 1758, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/18*b^7*d^10*x^18 + 10/17*b^7*c*d^9*x^17 + 7/17*a*b^6*d^10*x^17 + 45/16*b^
7*c^2*d^8*x^16 + 35/8*a*b^6*c*d^9*x^16 + 21/16*a^2*b^5*d^10*x^16 + 8*b^7*c^
3*d^7*x^15 + 21*a*b^6*c^2*d^8*x^15 + 14*a^2*b^5*c*d^9*x^15 + 7/3*a^3*b^4*d^
10*x^15 + 15*b^7*c^4*d^6*x^14 + 60*a*b^6*c^3*d^7*x^14 + 135/2*a^2*b^5*c^2*d
^8*x^14 + 25*a^3*b^4*c*d^9*x^14 + 5/2*a^4*b^3*d^10*x^14 + 252/13*b^7*c^5*d^
5*x^13 + 1470/13*a*b^6*c^4*d^6*x^13 + 2520/13*a^2*b^5*c^3*d^7*x^13 + 1575/1
3*a^3*b^4*c^2*d^8*x^13 + 350/13*a^4*b^3*c*d^9*x^13 + 21/13*a^5*b^2*d^10*x^1
3 + 35/2*b^7*c^6*d^4*x^12 + 147*a*b^6*c^5*d^5*x^12 + 735/2*a^2*b^5*c^4*d^6*
x^12 + 350*a^3*b^4*c^3*d^7*x^12 + 525/4*a^4*b^3*c^2*d^8*x^12 + 35/2*a^5*b^2
*c*d^9*x^12 + 7/12*a^6*b*d^10*x^12 + 120/11*b^7*c^7*d^3*x^11 + 1470/11*a*b^
6*c^6*d^4*x^11 + 5292/11*a^2*b^5*c^5*d^5*x^11 + 7350/11*a^3*b^4*c^4*d^6*x^1
1 + 4200/11*a^4*b^3*c^3*d^7*x^11 + 945/11*a^5*b^2*c^2*d^8*x^11 + 70/11*a^6*
b*c*d^9*x^11 + 1/11*a^7*d^10*x^11 + 9/2*b^7*c^8*d^2*x^10 + 84*a*b^6*c^7*d^3
*x^10 + 441*a^2*b^5*c^6*d^4*x^10 + 882*a^3*b^4*c^5*d^5*x^10 + 735*a^4*b^3*c
^4*d^6*x^10 + 252*a^5*b^2*c^3*d^7*x^10 + 63/2*a^6*b*c^2*d^8*x^10 + a^7*c*d^
9*x^10 + 10/9*b^7*c^9*d*x^9 + 35*a*b^6*c^8*d^2*x^9 + 280*a^2*b^5*c^7*d^3*x^
9 + 2450/3*a^3*b^4*c^6*d^4*x^9 + 980*a^4*b^3*c^5*d^5*x^9 + 490*a^5*b^2*c^4*
d^6*x^9 + 280/3*a^6*b*c^3*d^7*x^9 + 5*a^7*c^2*d^8*x^9 + 1/8*b^7*c^10*x^8 +
35/4*a*b^6*c^9*d*x^8 + 945/8*a^2*b^5*c^8*d^2*x^8 + 525*a^3*b^4*c^7*d^3*x^8
+ 3675/4*a^4*b^3*c^6*d^4*x^8 + 1323/2*a^5*b^2*c^5*d^5*x^8 + 735/4*a^6*b*c^4
*d^6*x^8 + 15*a^7*c^3*d^7*x^8 + a*b^6*c^10*x^7 + 30*a^2*b^5*c^9*d*x^7 + 225
*a^3*b^4*c^8*d^2*x^7 + 600*a^4*b^3*c^7*d^3*x^7 + 630*a^5*b^2*c^6*d^4*x^7 +
252*a^6*b*c^5*d^5*x^7 + 30*a^7*c^4*d^6*x^7 + 7/2*a^2*b^5*c^10*x^6 + 175/3*a
^3*b^4*c^9*d*x^6 + 525/2*a^4*b^3*c^8*d^2*x^6 + 420*a^5*b^2*c^7*d^3*x^6 + 24
5*a^6*b*c^6*d^4*x^6 + 42*a^7*c^5*d^5*x^6 + 7*a^3*b^4*c^10*x^5 + 70*a^4*b^3*
c^9*d*x^5 + 189*a^5*b^2*c^8*d^2*x^5 + 168*a^6*b*c^7*d^3*x^5 + 42*a^7*c^6*d^
4*x^5 + 35/4*a^4*b^3*c^10*x^4 + 105/2*a^5*b^2*c^9*d*x^4 + 315/4*a^6*b*c^8*d
^2*x^4 + 30*a^7*c^7*d^3*x^4 + 7*a^5*b^2*c^10*x^3 + 70/3*a^6*b*c^9*d*x^3 + 1
5*a^7*c^8*d^2*x^3 + 7/2*a^6*b*c^10*x^2 + 5*a^7*c^9*d*x^2 + a^7*c^10*x
```

3.1305 $\int (a+bx)^6(c+dx)^{10}\,dx$

Optimal. Leaf size=170

$$-\frac{3b^5(c+dx)^{16}(bc-ad)}{8d^7}+\frac{b^4(c+dx)^{15}(bc-ad)^2}{d^7}-\frac{10b^3(c+dx)^{14}(bc-ad)^3}{7d^7}+\frac{15b^2(c+dx)^{13}(bc-ad)^4}{13d^7}-\frac{b(c+dx)^{12}(bc}{2d^7}$$

```
[Out] ((b*c - a*d)^6*(c + d*x)^11)/(11*d^7) - (b*(b*c - a*d)^5*(c + d*x)^12)/(2*d
^7) + (15*b^2*(b*c - a*d)^4*(c + d*x)^13)/(13*d^7) - (10*b^3*(b*c - a*d)^3*
(c + d*x)^14)/(7*d^7) + (b^4*(b*c - a*d)^2*(c + d*x)^15)/d^7 - (3*b^5*(b*c
- a*d)*(c + d*x)^16)/(8*d^7) + (b^6*(c + d*x)^17)/(17*d^7)
```

Rubi [A] time = 0.672839, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^5(c+dx)^{16}(bc-ad)}{8d^7}+\frac{b^4(c+dx)^{15}(bc-ad)^2}{d^7}-\frac{10b^3(c+dx)^{14}(bc-ad)^3}{7d^7}+\frac{15b^2(c+dx)^{13}(bc-ad)^4}{13d^7}-\frac{b(c+dx)^{12}(bc}{2d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^6*(c + d*x)^11)/(11*d^7) - (b*(b*c - a*d)^5*(c + d*x)^12)/(2*d
^7) + (15*b^2*(b*c - a*d)^4*(c + d*x)^13)/(13*d^7) - (10*b^3*(b*c - a*d)^3*
(c + d*x)^14)/(7*d^7) + (b^4*(b*c - a*d)^2*(c + d*x)^15)/d^7 - (3*b^5*(b*c
- a*d)*(c + d*x)^16)/(8*d^7) + (b^6*(c + d*x)^17)/(17*d^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^6(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^6(c+dx)^{10}}{d^6}-\frac{6b(bc-ad)^5(c+dx)^{11}}{d^6}+\frac{15b^2(bc-ad)^4(c+dx)^{12}}{d^6}-\frac{20b^3(bc-a}{c}\right.\\ &= \frac{(bc-ad)^6(c+dx)^{11}}{11d^7}-\frac{b(bc-ad)^5(c+dx)^{12}}{2d^7}+\frac{15b^2(bc-ad)^4(c+dx)^{13}}{13d^7}-\frac{10b^3(bc-ad)^3(c+}{7d^7}\end{aligned}$$

Mathematica [B] time = 0.112031, size = 939, normalized size = 5.52

$$\frac{1}{17}b^6d^{10}x^{17}+\frac{1}{8}b^5d^9(5bc+3ad)x^{16}+b^4d^8\left(3b^2c^2+4abdc+a^2d^2\right)x^{15}+\frac{5}{7}b^3d^7\left(12b^3c^3+27ab^2dc^2+15a^2bd^2c+2a^3d^3\right)x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6*(c + d*x)^10,x]
```

```
[Out] a^6*c^10*x + a^5*c^9*(3*b*c + 5*a*d)*x^2 + 5*a^4*c^8*(b^2*c^2 + 4*a*b*c*d +
 3*a^2*d^2)*x^3 + (5*a^3*c^7*(2*b^3*c^3 + 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 +
```

```
12*a^3*d^3)*x^4)/2 + a^2*c^6*(3*b^4*c^4 + 40*a*b^3*c^3*d + 135*a^2*b^2*c^2
*d^2 + 144*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + a*c^5*(b^5*c^5 + 25*a*b^4*c^4*d
+ 150*a^2*b^3*c^3*d^2 + 300*a^3*b^2*c^2*d^3 + 210*a^4*b*c*d^4 + 42*a^5*d^5)
*x^6 + (c^4*(b^6*c^6 + 60*a*b^5*c^5*d + 675*a^2*b^4*c^4*d^2 + 2400*a^3*b^3*
c^3*d^3 + 3150*a^4*b^2*c^2*d^4 + 1512*a^5*b*c*d^5 + 210*a^6*d^6)*x^7)/7 + (
5*c^3*d*(b^6*c^6 + 27*a*b^5*c^5*d + 180*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d
^3 + 378*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 + 12*a^6*d^6)*x^8)/4 + 5*c^2*d^2
*(b^6*c^6 + 16*a*b^5*c^5*d + 70*a^2*b^4*c^4*d^2 + 112*a^3*b^3*c^3*d^3 + 70*
a^4*b^2*c^2*d^4 + 16*a^5*b*c*d^5 + a^6*d^6)*x^9 + c*d^3*(12*b^6*c^6 + 126*a
*b^5*c^5*d + 378*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^
4 + 27*a^5*b*c*d^5 + a^6*d^6)*x^10 + (d^4*(210*b^6*c^6 + 1512*a*b^5*c^5*d +
 3150*a^2*b^4*c^4*d^2 + 2400*a^3*b^3*c^3*d^3 + 675*a^4*b^2*c^2*d^4 + 60*a^5
*b*c*d^5 + a^6*d^6)*x^11)/11 + (b*d^5*(42*b^5*c^5 + 210*a*b^4*c^4*d + 300*a
^2*b^3*c^3*d^2 + 150*a^3*b^2*c^2*d^3 + 25*a^4*b*c*d^4 + a^5*d^5)*x^12)/2 +
(5*b^2*d^6*(42*b^4*c^4 + 144*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 + 40*a^3*b*c
*d^3 + 3*a^4*d^4)*x^13)/13 + (5*b^3*d^7*(12*b^3*c^3 + 27*a*b^2*c^2*d + 15*a
^2*b*c*d^2 + 2*a^3*d^3)*x^14)/7 + b^4*d^8*(3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)
*x^15 + (b^5*d^9*(5*b*c + 3*a*d)*x^16)/8 + (b^6*d^10*x^17)/17
```

Maple [B] time = 0.003, size = 991, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6*(d*x+c)^10,x)
```

```
[Out] 1/17*b^6*d^10*x^17+1/16*(6*a*b^5*d^10+10*b^6*c*d^9)*x^16+1/15*(15*a^2*b^4*d
^10+60*a*b^5*c*d^9+45*b^6*c^2*d^8)*x^15+1/14*(20*a^3*b^3*d^10+150*a^2*b^4*c
*d^9+270*a*b^5*c^2*d^8+120*b^6*c^3*d^7)*x^14+1/13*(15*a^4*b^2*d^10+200*a^3*
b^3*c*d^9+675*a^2*b^4*c^2*d^8+720*a*b^5*c^3*d^7+210*b^6*c^4*d^6)*x^13+1/12*
(6*a^5*b*d^10+150*a^4*b^2*c*d^9+900*a^3*b^3*c^2*d^8+1800*a^2*b^4*c^3*d^7+12
60*a*b^5*c^4*d^6+252*b^6*c^5*d^5)*x^12+1/11*(a^6*d^10+60*a^5*b*c*d^9+675*a^
4*b^2*c^2*d^8+2400*a^3*b^3*c^3*d^7+3150*a^2*b^4*c^4*d^6+1512*a*b^5*c^5*d^5+
210*b^6*c^6*d^4)*x^11+1/10*(10*a^6*c*d^9+270*a^5*b*c^2*d^8+1800*a^4*b^2*c^3
*d^7+4200*a^3*b^3*c^4*d^6+3780*a^2*b^4*c^5*d^5+1260*a*b^5*c^6*d^4+120*b^6*c
^7*d^3)*x^10+1/9*(45*a^6*c^2*d^8+720*a^5*b*c^3*d^7+3150*a^4*b^2*c^4*d^6+504
0*a^3*b^3*c^5*d^5+3150*a^2*b^4*c^6*d^4+720*a*b^5*c^7*d^3+45*b^6*c^8*d^2)*x^
9+1/8*(120*a^6*c^3*d^7+1260*a^5*b*c^4*d^6+3780*a^4*b^2*c^5*d^5+4200*a^3*b^3
*c^6*d^4+1800*a^2*b^4*c^7*d^3+270*a*b^5*c^8*d^2+10*b^6*c^9*d)*x^8+1/7*(210*
a^6*c^4*d^6+1512*a^5*b*c^5*d^5+3150*a^4*b^2*c^6*d^4+2400*a^3*b^3*c^7*d^3+67
5*a^2*b^4*c^8*d^2+60*a*b^5*c^9*d+b^6*c^10)*x^7+1/6*(252*a^6*c^5*d^5+1260*a^
5*b*c^6*d^4+1800*a^4*b^2*c^7*d^3+900*a^3*b^3*c^8*d^2+150*a^2*b^4*c^9*d+6*a*
b^5*c^10)*x^6+1/5*(210*a^6*c^6*d^4+720*a^5*b*c^7*d^3+675*a^4*b^2*c^8*d^2+20
0*a^3*b^3*c^9*d+15*a^2*b^4*c^10)*x^5+1/4*(120*a^6*c^7*d^3+270*a^5*b*c^8*d^2
+150*a^4*b^2*c^9*d+20*a^3*b^3*c^10)*x^4+1/3*(45*a^6*c^8*d^2+60*a^5*b*c^9*d+
15*a^4*b^2*c^10)*x^3+1/2*(10*a^6*c^9*d+6*a^5*b*c^10)*x^2+a^6*c^10*x
```

Maxima [B] time = 1.00393, size = 1319, normalized size = 7.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/17*b^6*d^10*x^17 + a^6*c^10*x + 1/8*(5*b^6*c*d^9 + 3*a*b^5*d^10)*x^16 + (
3*b^6*c^2*d^8 + 4*a*b^5*c*d^9 + a^2*b^4*d^10)*x^15 + 5/7*(12*b^6*c^3*d^7 +
27*a*b^5*c^2*d^8 + 15*a^2*b^4*c*d^9 + 2*a^3*b^3*d^10)*x^14 + 5/13*(42*b^6*c
^4*d^6 + 144*a*b^5*c^3*d^7 + 135*a^2*b^4*c^2*d^8 + 40*a^3*b^3*c*d^9 + 3*a^4
*b^2*d^10)*x^13 + 1/2*(42*b^6*c^5*d^5 + 210*a*b^5*c^4*d^6 + 300*a^2*b^4*c^3
*d^7 + 150*a^3*b^3*c^2*d^8 + 25*a^4*b^2*c*d^9 + a^5*b*d^10)*x^12 + 1/11*(21
0*b^6*c^6*d^4 + 1512*a*b^5*c^5*d^5 + 3150*a^2*b^4*c^4*d^6 + 2400*a^3*b^3*c^
3*d^7 + 675*a^4*b^2*c^2*d^8 + 60*a^5*b*c*d^9 + a^6*d^10)*x^11 + (12*b^6*c^7
*d^3 + 126*a*b^5*c^6*d^4 + 378*a^2*b^4*c^5*d^5 + 420*a^3*b^3*c^4*d^6 + 180*
a^4*b^2*c^3*d^7 + 27*a^5*b*c^2*d^8 + a^6*c*d^9)*x^10 + 5*(b^6*c^8*d^2 + 16*
a*b^5*c^7*d^3 + 70*a^2*b^4*c^6*d^4 + 112*a^3*b^3*c^5*d^5 + 70*a^4*b^2*c^4*d
^6 + 16*a^5*b*c^3*d^7 + a^6*c^2*d^8)*x^9 + 5/4*(b^6*c^9*d + 27*a*b^5*c^8*d^
2 + 180*a^2*b^4*c^7*d^3 + 420*a^3*b^3*c^6*d^4 + 378*a^4*b^2*c^5*d^5 + 126*a
^5*b*c^4*d^6 + 12*a^6*c^3*d^7)*x^8 + 1/7*(b^6*c^10 + 60*a*b^5*c^9*d + 675*a
^2*b^4*c^8*d^2 + 2400*a^3*b^3*c^7*d^3 + 3150*a^4*b^2*c^6*d^4 + 1512*a^5*b*c
^5*d^5 + 210*a^6*c^4*d^6)*x^7 + (a*b^5*c^10 + 25*a^2*b^4*c^9*d + 150*a^3*b^
3*c^8*d^2 + 300*a^4*b^2*c^7*d^3 + 210*a^5*b*c^6*d^4 + 42*a^6*c^5*d^5)*x^6 +
 (3*a^2*b^4*c^10 + 40*a^3*b^3*c^9*d + 135*a^4*b^2*c^8*d^2 + 144*a^5*b*c^7*d
^3 + 42*a^6*c^6*d^4)*x^5 + 5/2*(2*a^3*b^3*c^10 + 15*a^4*b^2*c^9*d + 27*a^5*
b*c^8*d^2 + 12*a^6*c^7*d^3)*x^4 + 5*(a^4*b^2*c^10 + 4*a^5*b*c^9*d + 3*a^6*c
^8*d^2)*x^3 + (3*a^5*b*c^10 + 5*a^6*c^9*d)*x^2
```

Fricas [B] time = 1.54228, size = 2484, normalized size = 14.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^10*b^6 + 5/8*x^16*d^9*c*b^6 + 3/8*x^16*d^10*b^5*a + 3*x^15*d^8*
c^2*b^6 + 4*x^15*d^9*c*b^5*a + x^15*d^10*b^4*a^2 + 60/7*x^14*d^7*c^3*b^6 +
135/7*x^14*d^8*c^2*b^5*a + 75/7*x^14*d^9*c*b^4*a^2 + 10/7*x^14*d^10*b^3*a^3
 + 210/13*x^13*d^6*c^4*b^6 + 720/13*x^13*d^7*c^3*b^5*a + 675/13*x^13*d^8*c^
2*b^4*a^2 + 200/13*x^13*d^9*c*b^3*a^3 + 15/13*x^13*d^10*b^2*a^4 + 21*x^12*d
^5*c^5*b^6 + 105*x^12*d^6*c^4*b^5*a + 150*x^12*d^7*c^3*b^4*a^2 + 75*x^12*d^
8*c^2*b^3*a^3 + 25/2*x^12*d^9*c*b^2*a^4 + 1/2*x^12*d^10*b*a^5 + 210/11*x^11
*d^4*c^6*b^6 + 1512/11*x^11*d^5*c^5*b^5*a + 3150/11*x^11*d^6*c^4*b^4*a^2 +
2400/11*x^11*d^7*c^3*b^3*a^3 + 675/11*x^11*d^8*c^2*b^2*a^4 + 60/11*x^11*d^9
*c*b*a^5 + 1/11*x^11*d^10*a^6 + 12*x^10*d^3*c^7*b^6 + 126*x^10*d^4*c^6*b^5*
a + 378*x^10*d^5*c^5*b^4*a^2 + 420*x^10*d^6*c^4*b^3*a^3 + 180*x^10*d^7*c^3*
b^2*a^4 + 27*x^10*d^8*c^2*b*a^5 + x^10*d^9*c*a^6 + 5*x^9*d^2*c^8*b^6 + 80*x
^9*d^3*c^7*b^5*a + 350*x^9*d^4*c^6*b^4*a^2 + 560*x^9*d^5*c^5*b^3*a^3 + 350*
x^9*d^6*c^4*b^2*a^4 + 80*x^9*d^7*c^3*b*a^5 + 5*x^9*d^8*c^2*a^6 + 5/4*x^8*d*
c^9*b^6 + 135/4*x^8*d^2*c^8*b^5*a + 225*x^8*d^3*c^7*b^4*a^2 + 525*x^8*d^4*c
^6*b^3*a^3 + 945/2*x^8*d^5*c^5*b^2*a^4 + 315/2*x^8*d^6*c^4*b*a^5 + 15*x^8*d
^7*c^3*a^6 + 1/7*x^7*c^10*b^6 + 60/7*x^7*d*c^9*b^5*a + 675/7*x^7*d^2*c^8*b^
4*a^2 + 2400/7*x^7*d^3*c^7*b^3*a^3 + 450*x^7*d^4*c^6*b^2*a^4 + 216*x^7*d^5*
c^5*b*a^5 + 30*x^7*d^6*c^4*a^6 + x^6*c^10*b^5*a + 25*x^6*d*c^9*b^4*a^2 + 15
0*x^6*d^2*c^8*b^3*a^3 + 300*x^6*d^3*c^7*b^2*a^4 + 210*x^6*d^4*c^6*b*a^5 + 4
2*x^6*d^5*c^5*a^6 + 3*x^5*c^10*b^4*a^2 + 40*x^5*d*c^9*b^3*a^3 + 135*x^5*d^2
*c^8*b^2*a^4 + 144*x^5*d^3*c^7*b*a^5 + 42*x^5*d^4*c^6*a^6 + 5*x^4*c^10*b^3*
a^3 + 75/2*x^4*d*c^9*b^2*a^4 + 135/2*x^4*d^2*c^8*b*a^5 + 30*x^4*d^3*c^7*a^6
 + 5*x^3*c^10*b^2*a^4 + 20*x^3*d*c^9*b*a^5 + 15*x^3*d^2*c^8*a^6 + 3*x^2*c^1
0*b*a^5 + 5*x^2*d*c^9*a^6 + x*c^10*a^6
```

Sympy [B] time = 0.187351, size = 1088, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6*(d*x+c)**10,x)
```

```
[Out] a**6*c**10*x + b**6*d**10*x**17/17 + x**16*(3*a*b**5*d**10/8 + 5*b**6*c*d**
9/8) + x**15*(a**2*b**4*d**10 + 4*a*b**5*c*d**9 + 3*b**6*c**2*d**8) + x**14
*(10*a**3*b**3*d**10/7 + 75*a**2*b**4*c*d**9/7 + 135*a*b**5*c**2*d**8/7 + 6
0*b**6*c**3*d**7/7) + x**13*(15*a**4*b**2*d**10/13 + 200*a**3*b**3*c*d**9/1
3 + 675*a**2*b**4*c**2*d**8/13 + 720*a*b**5*c**3*d**7/13 + 210*b**6*c**4*d*
*6/13) + x**12*(a**5*b*d**10/2 + 25*a**4*b**2*c*d**9/2 + 75*a**3*b**3*c**2*
d**8 + 150*a**2*b**4*c**3*d**7 + 105*a*b**5*c**4*d**6 + 21*b**6*c**5*d**5)
+ x**11*(a**6*d**10/11 + 60*a**5*b*c*d**9/11 + 675*a**4*b**2*c**2*d**8/11 +
 2400*a**3*b**3*c**3*d**7/11 + 3150*a**2*b**4*c**4*d**6/11 + 1512*a*b**5*c*
*5*d**5/11 + 210*b**6*c**6*d**4/11) + x**10*(a**6*c*d**9 + 27*a**5*b*c**2*d
**8 + 180*a**4*b**2*c**3*d**7 + 420*a**3*b**3*c**4*d**6 + 378*a**2*b**4*c**
5*d**5 + 126*a*b**5*c**6*d**4 + 12*b**6*c**7*d**3) + x**9*(5*a**6*c**2*d**8
 + 80*a**5*b*c**3*d**7 + 350*a**4*b**2*c**4*d**6 + 560*a**3*b**3*c**5*d**5
+ 350*a**2*b**4*c**6*d**4 + 80*a*b**5*c**7*d**3 + 5*b**6*c**8*d**2) + x**8*
(15*a**6*c**3*d**7 + 315*a**5*b*c**4*d**6/2 + 945*a**4*b**2*c**5*d**5/2 + 5
25*a**3*b**3*c**6*d**4 + 225*a**2*b**4*c**7*d**3 + 135*a*b**5*c**8*d**2/4 +
 5*b**6*c**9*d/4) + x**7*(30*a**6*c**4*d**6 + 216*a**5*b*c**5*d**5 + 450*a*
*4*b**2*c**6*d**4 + 2400*a**3*b**3*c**7*d**3/7 + 675*a**2*b**4*c**8*d**2/7
+ 60*a*b**5*c**9*d/7 + b**6*c**10/7) + x**6*(42*a**6*c**5*d**5 + 210*a**5*b
*c**6*d**4 + 300*a**4*b**2*c**7*d**3 + 150*a**3*b**3*c**8*d**2 + 25*a**2*b*
*4*c**9*d + a*b**5*c**10) + x**5*(42*a**6*c**6*d**4 + 144*a**5*b*c**7*d**3
+ 135*a**4*b**2*c**8*d**2 + 40*a**3*b**3*c**9*d + 3*a**2*b**4*c**10) + x**4
*(30*a**6*c**7*d**3 + 135*a**5*b*c**8*d**2/2 + 75*a**4*b**2*c**9*d/2 + 5*a*
*3*b**3*c**10) + x**3*(15*a**6*c**8*d**2 + 20*a**5*b*c**9*d + 5*a**4*b**2*c
**10) + x**2*(5*a**6*c**9*d + 3*a**5*b*c**10)
```

Giac [B] time = 1.05535, size = 1517, normalized size = 8.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/17*b^6*d^10*x^17 + 5/8*b^6*c*d^9*x^16 + 3/8*a*b^5*d^10*x^16 + 3*b^6*c^2*d
^8*x^15 + 4*a*b^5*c*d^9*x^15 + a^2*b^4*d^10*x^15 + 60/7*b^6*c^3*d^7*x^14 +
135/7*a*b^5*c^2*d^8*x^14 + 75/7*a^2*b^4*c*d^9*x^14 + 10/7*a^3*b^3*d^10*x^14
 + 210/13*b^6*c^4*d^6*x^13 + 720/13*a*b^5*c^3*d^7*x^13 + 675/13*a^2*b^4*c^2
*d^8*x^13 + 200/13*a^3*b^3*c*d^9*x^13 + 15/13*a^4*b^2*d^10*x^13 + 21*b^6*c^
5*d^5*x^12 + 105*a*b^5*c^4*d^6*x^12 + 150*a^2*b^4*c^3*d^7*x^12 + 75*a^3*b^3
*c^2*d^8*x^12 + 25/2*a^4*b^2*c*d^9*x^12 + 1/2*a^5*b*d^10*x^12 + 210/11*b^6*
c^6*d^4*x^11 + 1512/11*a*b^5*c^5*d^5*x^11 + 3150/11*a^2*b^4*c^4*d^6*x^11 +
2400/11*a^3*b^3*c^3*d^7*x^11 + 675/11*a^4*b^2*c^2*d^8*x^11 + 60/11*a^5*b*c*
d^9*x^11 + 1/11*a^6*d^10*x^11 + 12*b^6*c^7*d^3*x^10 + 126*a*b^5*c^6*d^4*x^1
0 + 378*a^2*b^4*c^5*d^5*x^10 + 420*a^3*b^3*c^4*d^6*x^10 + 180*a^4*b^2*c^3*d
^7*x^10 + 27*a^5*b*c^2*d^8*x^10 + a^6*c*d^9*x^10 + 5*b^6*c^8*d^2*x^9 + 80*a
*b^5*c^7*d^3*x^9 + 350*a^2*b^4*c^6*d^4*x^9 + 560*a^3*b^3*c^5*d^5*x^9 + 350*
a^4*b^2*c^4*d^6*x^9 + 80*a^5*b*c^3*d^7*x^9 + 5*a^6*c^2*d^8*x^9 + 5/4*b^6*c^
```

```
9*d*x^8 + 135/4*a*b^5*c^8*d^2*x^8 + 225*a^2*b^4*c^7*d^3*x^8 + 525*a^3*b^3*c
^6*d^4*x^8 + 945/2*a^4*b^2*c^5*d^5*x^8 + 315/2*a^5*b*c^4*d^6*x^8 + 15*a^6*c
^3*d^7*x^8 + 1/7*b^6*c^10*x^7 + 60/7*a*b^5*c^9*d*x^7 + 675/7*a^2*b^4*c^8*d^
2*x^7 + 2400/7*a^3*b^3*c^7*d^3*x^7 + 450*a^4*b^2*c^6*d^4*x^7 + 216*a^5*b*c^
5*d^5*x^7 + 30*a^6*c^4*d^6*x^7 + a*b^5*c^10*x^6 + 25*a^2*b^4*c^9*d*x^6 + 15
0*a^3*b^3*c^8*d^2*x^6 + 300*a^4*b^2*c^7*d^3*x^6 + 210*a^5*b*c^6*d^4*x^6 + 4
2*a^6*c^5*d^5*x^6 + 3*a^2*b^4*c^10*x^5 + 40*a^3*b^3*c^9*d*x^5 + 135*a^4*b^2
*c^8*d^2*x^5 + 144*a^5*b*c^7*d^3*x^5 + 42*a^6*c^6*d^4*x^5 + 5*a^3*b^3*c^10*
x^4 + 75/2*a^4*b^2*c^9*d*x^4 + 135/2*a^5*b*c^8*d^2*x^4 + 30*a^6*c^7*d^3*x^4
 + 5*a^4*b^2*c^10*x^3 + 20*a^5*b*c^9*d*x^3 + 15*a^6*c^8*d^2*x^3 + 3*a^5*b*c
^10*x^2 + 5*a^6*c^9*d*x^2 + a^6*c^10*x
```

3.1306 $\int (a+bx)^5(c+dx)^{10}\,dx$

Optimal. Leaf size=146

$$-\frac{b^4(c+dx)^{15}(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^{14}(bc-ad)^2}{7d^6}-\frac{10b^2(c+dx)^{13}(bc-ad)^3}{13d^6}+\frac{5b(c+dx)^{12}(bc-ad)^4}{12d^6}-\frac{(c+dx)^{11}(bc}{11d^6}$$

```
[Out] -((b*c - a*d)^5*(c + d*x)^11)/(11*d^6) + (5*b*(b*c - a*d)^4*(c + d*x)^12)/(
12*d^6) - (10*b^2*(b*c - a*d)^3*(c + d*x)^13)/(13*d^6) + (5*b^3*(b*c - a*d)
^2*(c + d*x)^14)/(7*d^6) - (b^4*(b*c - a*d)*(c + d*x)^15)/(3*d^6) + (b^5*(c
 + d*x)^16)/(16*d^6)
```

Rubi [A] time = 0.529386, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b^4(c+dx)^{15}(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^{14}(bc-ad)^2}{7d^6}-\frac{10b^2(c+dx)^{13}(bc-ad)^3}{13d^6}+\frac{5b(c+dx)^{12}(bc-ad)^4}{12d^6}-\frac{(c+dx)^{11}(bc}{11d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)^5*(c + d*x)^11)/(11*d^6) + (5*b*(b*c - a*d)^4*(c + d*x)^12)/(
12*d^6) - (10*b^2*(b*c - a*d)^3*(c + d*x)^13)/(13*d^6) + (5*b^3*(b*c - a*d)
^2*(c + d*x)^14)/(7*d^6) - (b^4*(b*c - a*d)*(c + d*x)^15)/(3*d^6) + (b^5*(c
 + d*x)^16)/(16*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^{10}\,dx=\int \left(\frac{(-bc+ad)^5(c+dx)^{10}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{11}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{12}}{d^5}+\frac{10b^3(bc-}{}\right.$$
$$=-\frac{(bc-ad)^5(c+dx)^{11}}{11d^6}+\frac{5b(bc-ad)^4(c+dx)^{12}}{12d^6}-\frac{10b^2(bc-ad)^3(c+dx)^{13}}{13d^6}+\frac{5b^3(bc-ad)^2}{7d^6}$$

Mathematica [B] time = 0.0827543, size = 811, normalized size = 5.55

$$\frac{1}{16}b^5d^{10}x^{16}+\frac{1}{3}b^4d^9(2bc+ad)x^{15}+\frac{5}{14}b^3d^8\left(9b^2c^2+10abdc+2a^2d^2\right)x^{14}+\frac{5}{13}b^2d^7\left(24b^3c^3+45ab^2dc^2+20a^2bd^2c+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^10,x]
```

```
[Out] a^5*c^10*x + (5*a^4*c^9*(b*c + 2*a*d)*x^2)/2 + (5*a^3*c^8*(2*b^2*c^2 + 10*a
*b*c*d + 9*a^2*d^2)*x^3)/3 + (5*a^2*c^7*(2*b^3*c^3 + 20*a*b^2*c^2*d + 45*a^
```

```
2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + a*c^6*(b^4*c^4 + 20*a*b^3*c^3*d + 90*a^2*b
^2*c^2*d^2 + 120*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + (c^5*(b^5*c^5 + 50*a*b^4*c
^4*d + 450*a^2*b^3*c^3*d^2 + 1200*a^3*b^2*c^2*d^3 + 1050*a^4*b*c*d^4 + 252*
a^5*d^5)*x^6)/6 + (5*c^4*d*(2*b^5*c^5 + 45*a*b^4*c^4*d + 240*a^2*b^3*c^3*d^
2 + 420*a^3*b^2*c^2*d^3 + 252*a^4*b*c*d^4 + 42*a^5*d^5)*x^7)/7 + (15*c^3*d^
2*(3*b^5*c^5 + 40*a*b^4*c^4*d + 140*a^2*b^3*c^3*d^2 + 168*a^3*b^2*c^2*d^3 +
 70*a^4*b*c*d^4 + 8*a^5*d^5)*x^8)/8 + (5*c^2*d^3*(8*b^5*c^5 + 70*a*b^4*c^4*
d + 168*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 40*a^4*b*c*d^4 + 3*a^5*d^5)
*x^9)/3 + (c*d^4*(42*b^5*c^5 + 252*a*b^4*c^4*d + 420*a^2*b^3*c^3*d^2 + 240*
a^3*b^2*c^2*d^3 + 45*a^4*b*c*d^4 + 2*a^5*d^5)*x^10)/2 + (d^5*(252*b^5*c^5 +
 1050*a*b^4*c^4*d + 1200*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 50*a^4*b*c
*d^4 + a^5*d^5)*x^11)/11 + (5*b*d^6*(42*b^4*c^4 + 120*a*b^3*c^3*d + 90*a^2*
b^2*c^2*d^2 + 20*a^3*b*c*d^3 + a^4*d^4)*x^12)/12 + (5*b^2*d^7*(24*b^3*c^3 +
 45*a*b^2*c^2*d + 20*a^2*b*c*d^2 + 2*a^3*d^3)*x^13)/13 + (5*b^3*d^8*(9*b^2*
c^2 + 10*a*b*c*d + 2*a^2*d^2)*x^14)/14 + (b^4*d^9*(2*b*c + a*d)*x^15)/3 + (
b^5*d^10*x^16)/16
```

Maple [B] time = 0.003, size = 841, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^10,x)
```

```
[Out] 1/16*b^5*d^10*x^16+1/15*(5*a*b^4*d^10+10*b^5*c*d^9)*x^15+1/14*(10*a^2*b^3*d
^10+50*a*b^4*c*d^9+45*b^5*c^2*d^8)*x^14+1/13*(10*a^3*b^2*d^10+100*a^2*b^3*c
*d^9+225*a*b^4*c^2*d^8+120*b^5*c^3*d^7)*x^13+1/12*(5*a^4*b*d^10+100*a^3*b^2
*c*d^9+450*a^2*b^3*c^2*d^8+600*a*b^4*c^3*d^7+210*b^5*c^4*d^6)*x^12+1/11*(a^
5*d^10+50*a^4*b*c*d^9+450*a^3*b^2*c^2*d^8+1200*a^2*b^3*c^3*d^7+1050*a*b^4*c
^4*d^6+252*b^5*c^5*d^5)*x^11+1/10*(10*a^5*c*d^9+225*a^4*b*c^2*d^8+1200*a^3*
b^2*c^3*d^7+2100*a^2*b^3*c^4*d^6+1260*a*b^4*c^5*d^5+210*b^5*c^6*d^4)*x^10+1
/9*(45*a^5*c^2*d^8+600*a^4*b*c^3*d^7+2100*a^3*b^2*c^4*d^6+2520*a^2*b^3*c^5*
d^5+1050*a*b^4*c^6*d^4+120*b^5*c^7*d^3)*x^9+1/8*(120*a^5*c^3*d^7+1050*a^4*b
*c^4*d^6+2520*a^3*b^2*c^5*d^5+2100*a^2*b^3*c^6*d^4+600*a*b^4*c^7*d^3+45*b^5
*c^8*d^2)*x^8+1/7*(210*a^5*c^4*d^6+1260*a^4*b*c^5*d^5+2100*a^3*b^2*c^6*d^4+
1200*a^2*b^3*c^7*d^3+225*a*b^4*c^8*d^2+10*b^5*c^9*d)*x^7+1/6*(252*a^5*c^5*d
^5+1050*a^4*b*c^6*d^4+1200*a^3*b^2*c^7*d^3+450*a^2*b^3*c^8*d^2+50*a*b^4*c^9
*d+b^5*c^10)*x^6+1/5*(210*a^5*c^6*d^4+600*a^4*b*c^7*d^3+450*a^3*b^2*c^8*d^2
+100*a^2*b^3*c^9*d+5*a*b^4*c^10)*x^5+1/4*(120*a^5*c^7*d^3+225*a^4*b*c^8*d^2
+100*a^3*b^2*c^9*d+10*a^2*b^3*c^10)*x^4+1/3*(45*a^5*c^8*d^2+50*a^4*b*c^9*d+
10*a^3*b^2*c^10)*x^3+1/2*(10*a^5*c^9*d+5*a^4*b*c^10)*x^2+a^5*c^10*x
```

Maxima [B] time = 0.987233, size = 1127, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/16*b^5*d^10*x^16 + a^5*c^10*x + 1/3*(2*b^5*c*d^9 + a*b^4*d^10)*x^15 + 5/1
4*(9*b^5*c^2*d^8 + 10*a*b^4*c*d^9 + 2*a^2*b^3*d^10)*x^14 + 5/13*(24*b^5*c^3
*d^7 + 45*a*b^4*c^2*d^8 + 20*a^2*b^3*c*d^9 + 2*a^3*b^2*d^10)*x^13 + 5/12*(4
```

```
2*b^5*c^4*d^6 + 120*a*b^4*c^3*d^7 + 90*a^2*b^3*c^2*d^8 + 20*a^3*b^2*c*d^9 +
 a^4*b*d^10)*x^12 + 1/11*(252*b^5*c^5*d^5 + 1050*a*b^4*c^4*d^6 + 1200*a^2*b
^3*c^3*d^7 + 450*a^3*b^2*c^2*d^8 + 50*a^4*b*c*d^9 + a^5*d^10)*x^11 + 1/2*(4
2*b^5*c^6*d^4 + 252*a*b^4*c^5*d^5 + 420*a^2*b^3*c^4*d^6 + 240*a^3*b^2*c^3*d
^7 + 45*a^4*b*c^2*d^8 + 2*a^5*c*d^9)*x^10 + 5/3*(8*b^5*c^7*d^3 + 70*a*b^4*c
^6*d^4 + 168*a^2*b^3*c^5*d^5 + 140*a^3*b^2*c^4*d^6 + 40*a^4*b*c^3*d^7 + 3*a
^5*c^2*d^8)*x^9 + 15/8*(3*b^5*c^8*d^2 + 40*a*b^4*c^7*d^3 + 140*a^2*b^3*c^6*
d^4 + 168*a^3*b^2*c^5*d^5 + 70*a^4*b*c^4*d^6 + 8*a^5*c^3*d^7)*x^8 + 5/7*(2*
b^5*c^9*d + 45*a*b^4*c^8*d^2 + 240*a^2*b^3*c^7*d^3 + 420*a^3*b^2*c^6*d^4 +
252*a^4*b*c^5*d^5 + 42*a^5*c^4*d^6)*x^7 + 1/6*(b^5*c^10 + 50*a*b^4*c^9*d +
450*a^2*b^3*c^8*d^2 + 1200*a^3*b^2*c^7*d^3 + 1050*a^4*b*c^6*d^4 + 252*a^5*c
^5*d^5)*x^6 + (a*b^4*c^10 + 20*a^2*b^3*c^9*d + 90*a^3*b^2*c^8*d^2 + 120*a^4
*b*c^7*d^3 + 42*a^5*c^6*d^4)*x^5 + 5/4*(2*a^2*b^3*c^10 + 20*a^3*b^2*c^9*d +
 45*a^4*b*c^8*d^2 + 24*a^5*c^7*d^3)*x^4 + 5/3*(2*a^3*b^2*c^10 + 10*a^4*b*c^
9*d + 9*a^5*c^8*d^2)*x^3 + 5/2*(a^4*b*c^10 + 2*a^5*c^9*d)*x^2
```

Fricas [B] time = 1.4991, size = 2132, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^10*b^5 + 2/3*x^15*d^9*c*b^5 + 1/3*x^15*d^10*b^4*a + 45/14*x^14*
d^8*c^2*b^5 + 25/7*x^14*d^9*c*b^4*a + 5/7*x^14*d^10*b^3*a^2 + 120/13*x^13*d
^7*c^3*b^5 + 225/13*x^13*d^8*c^2*b^4*a + 100/13*x^13*d^9*c*b^3*a^2 + 10/13*
x^13*d^10*b^2*a^3 + 35/2*x^12*d^6*c^4*b^5 + 50*x^12*d^7*c^3*b^4*a + 75/2*x^
12*d^8*c^2*b^3*a^2 + 25/3*x^12*d^9*c*b^2*a^3 + 5/12*x^12*d^10*b*a^4 + 252/1
1*x^11*d^5*c^5*b^5 + 1050/11*x^11*d^6*c^4*b^4*a + 1200/11*x^11*d^7*c^3*b^3*
a^2 + 450/11*x^11*d^8*c^2*b^2*a^3 + 50/11*x^11*d^9*c*b*a^4 + 1/11*x^11*d^10
*a^5 + 21*x^10*d^4*c^6*b^5 + 126*x^10*d^5*c^5*b^4*a + 210*x^10*d^6*c^4*b^3*
a^2 + 120*x^10*d^7*c^3*b^2*a^3 + 45/2*x^10*d^8*c^2*b*a^4 + x^10*d^9*c*a^5 +
 40/3*x^9*d^3*c^7*b^5 + 350/3*x^9*d^4*c^6*b^4*a + 280*x^9*d^5*c^5*b^3*a^2 +
 700/3*x^9*d^6*c^4*b^2*a^3 + 200/3*x^9*d^7*c^3*b*a^4 + 5*x^9*d^8*c^2*a^5 +
45/8*x^8*d^2*c^8*b^5 + 75*x^8*d^3*c^7*b^4*a + 525/2*x^8*d^4*c^6*b^3*a^2 + 3
15*x^8*d^5*c^5*b^2*a^3 + 525/4*x^8*d^6*c^4*b*a^4 + 15*x^8*d^7*c^3*a^5 + 10/
7*x^7*d*c^9*b^5 + 225/7*x^7*d^2*c^8*b^4*a + 1200/7*x^7*d^3*c^7*b^3*a^2 + 30
0*x^7*d^4*c^6*b^2*a^3 + 180*x^7*d^5*c^5*b*a^4 + 30*x^7*d^6*c^4*a^5 + 1/6*x^
6*c^10*b^5 + 25/3*x^6*d*c^9*b^4*a + 75*x^6*d^2*c^8*b^3*a^2 + 200*x^6*d^3*c^
7*b^2*a^3 + 175*x^6*d^4*c^6*b*a^4 + 42*x^6*d^5*c^5*a^5 + x^5*c^10*b^4*a + 2
0*x^5*d*c^9*b^3*a^2 + 90*x^5*d^2*c^8*b^2*a^3 + 120*x^5*d^3*c^7*b*a^4 + 42*x
^5*d^4*c^6*a^5 + 5/2*x^4*c^10*b^3*a^2 + 25*x^4*d*c^9*b^2*a^3 + 225/4*x^4*d^
2*c^8*b*a^4 + 30*x^4*d^3*c^7*a^5 + 10/3*x^3*c^10*b^2*a^3 + 50/3*x^3*d*c^9*b
*a^4 + 15*x^3*d^2*c^8*a^5 + 5/2*x^2*c^10*b*a^4 + 5*x^2*d*c^9*a^5 + x*c^10*a
^5
```

Sympy [B] time = 0.173309, size = 940, normalized size = 6.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**10,x)
```

```
[Out] a**5*c**10*x + b**5*d**10*x**16/16 + x**15*(a*b**4*d**10/3 + 2*b**5*c*d**9/
3) + x**14*(5*a**2*b**3*d**10/7 + 25*a*b**4*c*d**9/7 + 45*b**5*c**2*d**8/14
) + x**13*(10*a**3*b**2*d**10/13 + 100*a**2*b**3*c*d**9/13 + 225*a*b**4*c**
2*d**8/13 + 120*b**5*c**3*d**7/13) + x**12*(5*a**4*b*d**10/12 + 25*a**3*b**
2*c*d**9/3 + 75*a**2*b**3*c**2*d**8/2 + 50*a*b**4*c**3*d**7 + 35*b**5*c**4*
d**6/2) + x**11*(a**5*d**10/11 + 50*a**4*b*c*d**9/11 + 450*a**3*b**2*c**2*d
**8/11 + 1200*a**2*b**3*c**3*d**7/11 + 1050*a*b**4*c**4*d**6/11 + 252*b**5*
c**5*d**5/11) + x**10*(a**5*c*d**9 + 45*a**4*b*c**2*d**8/2 + 120*a**3*b**2*
c**3*d**7 + 210*a**2*b**3*c**4*d**6 + 126*a*b**4*c**5*d**5 + 21*b**5*c**6*d
**4) + x**9*(5*a**5*c**2*d**8 + 200*a**4*b*c**3*d**7/3 + 700*a**3*b**2*c**4
*d**6/3 + 280*a**2*b**3*c**5*d**5 + 350*a*b**4*c**6*d**4/3 + 40*b**5*c**7*d
**3/3) + x**8*(15*a**5*c**3*d**7 + 525*a**4*b*c**4*d**6/4 + 315*a**3*b**2*c
**5*d**5 + 525*a**2*b**3*c**6*d**4/2 + 75*a*b**4*c**7*d**3 + 45*b**5*c**8*d
**2/8) + x**7*(30*a**5*c**4*d**6 + 180*a**4*b*c**5*d**5 + 300*a**3*b**2*c**
6*d**4 + 1200*a**2*b**3*c**7*d**3/7 + 225*a*b**4*c**8*d**2/7 + 10*b**5*c**9
*d/7) + x**6*(42*a**5*c**5*d**5 + 175*a**4*b*c**6*d**4 + 200*a**3*b**2*c**7
*d**3 + 75*a**2*b**3*c**8*d**2 + 25*a*b**4*c**9*d/3 + b**5*c**10/6) + x**5*
(42*a**5*c**6*d**4 + 120*a**4*b*c**7*d**3 + 90*a**3*b**2*c**8*d**2 + 20*a**
2*b**3*c**9*d + a*b**4*c**10) + x**4*(30*a**5*c**7*d**3 + 225*a**4*b*c**8*d
**2/4 + 25*a**3*b**2*c**9*d + 5*a**2*b**3*c**10/2) + x**3*(15*a**5*c**8*d**
2 + 50*a**4*b*c**9*d/3 + 10*a**3*b**2*c**10/3) + x**2*(5*a**5*c**9*d + 5*a*
*4*b*c**10/2)
```

Giac [B] time = 1.07139, size = 1280, normalized size = 8.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/16*b^5*d^10*x^16 + 2/3*b^5*c*d^9*x^15 + 1/3*a*b^4*d^10*x^15 + 45/14*b^5*c
^2*d^8*x^14 + 25/7*a*b^4*c*d^9*x^14 + 5/7*a^2*b^3*d^10*x^14 + 120/13*b^5*c^
3*d^7*x^13 + 225/13*a*b^4*c^2*d^8*x^13 + 100/13*a^2*b^3*c*d^9*x^13 + 10/13*
a^3*b^2*d^10*x^13 + 35/2*b^5*c^4*d^6*x^12 + 50*a*b^4*c^3*d^7*x^12 + 75/2*a^
2*b^3*c^2*d^8*x^12 + 25/3*a^3*b^2*c*d^9*x^12 + 5/12*a^4*b*d^10*x^12 + 252/1
1*b^5*c^5*d^5*x^11 + 1050/11*a*b^4*c^4*d^6*x^11 + 1200/11*a^2*b^3*c^3*d^7*x
^11 + 450/11*a^3*b^2*c^2*d^8*x^11 + 50/11*a^4*b*c*d^9*x^11 + 1/11*a^5*d^10*
x^11 + 21*b^5*c^6*d^4*x^10 + 126*a*b^4*c^5*d^5*x^10 + 210*a^2*b^3*c^4*d^6*x
^10 + 120*a^3*b^2*c^3*d^7*x^10 + 45/2*a^4*b*c^2*d^8*x^10 + a^5*c*d^9*x^10 +
 40/3*b^5*c^7*d^3*x^9 + 350/3*a*b^4*c^6*d^4*x^9 + 280*a^2*b^3*c^5*d^5*x^9 +
 700/3*a^3*b^2*c^4*d^6*x^9 + 200/3*a^4*b*c^3*d^7*x^9 + 5*a^5*c^2*d^8*x^9 +
45/8*b^5*c^8*d^2*x^8 + 75*a*b^4*c^7*d^3*x^8 + 525/2*a^2*b^3*c^6*d^4*x^8 + 3
15*a^3*b^2*c^5*d^5*x^8 + 525/4*a^4*b*c^4*d^6*x^8 + 15*a^5*c^3*d^7*x^8 + 10/
7*b^5*c^9*d*x^7 + 225/7*a*b^4*c^8*d^2*x^7 + 1200/7*a^2*b^3*c^7*d^3*x^7 + 30
0*a^3*b^2*c^6*d^4*x^7 + 180*a^4*b*c^5*d^5*x^7 + 30*a^5*c^4*d^6*x^7 + 1/6*b^
5*c^10*x^6 + 25/3*a*b^4*c^9*d*x^6 + 75*a^2*b^3*c^8*d^2*x^6 + 200*a^3*b^2*c^
7*d^3*x^6 + 175*a^4*b*c^6*d^4*x^6 + 42*a^5*c^5*d^5*x^6 + a*b^4*c^10*x^5 + 2
0*a^2*b^3*c^9*d*x^5 + 90*a^3*b^2*c^8*d^2*x^5 + 120*a^4*b*c^7*d^3*x^5 + 42*a
^5*c^6*d^4*x^5 + 5/2*a^2*b^3*c^10*x^4 + 25*a^3*b^2*c^9*d*x^4 + 225/4*a^4*b*
c^8*d^2*x^4 + 30*a^5*c^7*d^3*x^4 + 10/3*a^3*b^2*c^10*x^3 + 50/3*a^4*b*c^9*d
*x^3 + 15*a^5*c^8*d^2*x^3 + 5/2*a^4*b*c^10*x^2 + 5*a^5*c^9*d*x^2 + a^5*c^10
*x
```

3.1307 $\int (a+bx)^4(c+dx)^{10}\,dx$

Optimal. Leaf size=119

$$-\frac{2b^3(c+dx)^{14}(bc-ad)}{7d^5}+\frac{6b^2(c+dx)^{13}(bc-ad)^2}{13d^5}-\frac{b(c+dx)^{12}(bc-ad)^3}{3d^5}+\frac{(c+dx)^{11}(bc-ad)^4}{11d^5}+\frac{b^4(c+dx)^{15}}{15d^5}$$

```
[Out] ((b*c - a*d)^4*(c + d*x)^11)/(11*d^5) - (b*(b*c - a*d)^3*(c + d*x)^12)/(3*d
^5) + (6*b^2*(b*c - a*d)^2*(c + d*x)^13)/(13*d^5) - (2*b^3*(b*c - a*d)*(c +
 d*x)^14)/(7*d^5) + (b^4*(c + d*x)^15)/(15*d^5)
```

Rubi [A] time = 0.437197, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b^3(c+dx)^{14}(bc-ad)}{7d^5}+\frac{6b^2(c+dx)^{13}(bc-ad)^2}{13d^5}-\frac{b(c+dx)^{12}(bc-ad)^3}{3d^5}+\frac{(c+dx)^{11}(bc-ad)^4}{11d^5}+\frac{b^4(c+dx)^{15}}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^4*(c + d*x)^11)/(11*d^5) - (b*(b*c - a*d)^3*(c + d*x)^12)/(3*d
^5) + (6*b^2*(b*c - a*d)^2*(c + d*x)^13)/(13*d^5) - (2*b^3*(b*c - a*d)*(c +
 d*x)^14)/(7*d^5) + (b^4*(c + d*x)^15)/(15*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^4(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^4(c+dx)^{10}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{11}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{12}}{d^4}-\frac{4b^3(bc-a}{}\right.$$

$$=\frac{(bc-ad)^4(c+dx)^{11}}{11d^5}-\frac{b(bc-ad)^3(c+dx)^{12}}{3d^5}+\frac{6b^2(bc-ad)^2(c+dx)^{13}}{13d^5}-\frac{2b^3(bc-ad)(c+}{7d^5}$$

Mathematica [B] time = 0.0749195, size = 660, normalized size = 5.55

$$\frac{1}{13}b^2d^8x^{13}\left(6a^2d^2+40abcd+45b^2c^2\right)+\frac{1}{3}bd^7x^{12}\left(15a^2bcd^2+a^3d^3+45ab^2c^2d+30b^3c^3\right)+\frac{1}{11}d^6x^{11}\left(270a^2b^2c^2d^2+4\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^10,x]
```

```
[Out] a^4*c^10*x + a^3*c^9*(2*b*c + 5*a*d)*x^2 + (a^2*c^8*(6*b^2*c^2 + 40*a*b*c*d
 + 45*a^2*d^2)*x^3)/3 + a*c^7*(b^3*c^3 + 15*a*b^2*c^2*d + 45*a^2*b*c*d^2 +
30*a^3*d^3)*x^4 + (c^6*(b^4*c^4 + 40*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 48
0*a^3*b*c*d^3 + 210*a^4*d^4)*x^5)/5 + (c^5*d*(5*b^4*c^4 + 90*a*b^3*c^3*d +
```

```
360*a^2*b^2*c^2*d^2 + 420*a^3*b*c*d^3 + 126*a^4*d^4)*x^6)/3 + (3*c^4*d^2*(1
5*b^4*c^4 + 160*a*b^3*c^3*d + 420*a^2*b^2*c^2*d^2 + 336*a^3*b*c*d^3 + 70*a^
4*d^4)*x^7)/7 + 3*c^3*d^3*(5*b^4*c^4 + 35*a*b^3*c^3*d + 63*a^2*b^2*c^2*d^2
+ 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^8 + (c^2*d^4*(70*b^4*c^4 + 336*a*b^3*c^3*d
+ 420*a^2*b^2*c^2*d^2 + 160*a^3*b*c*d^3 + 15*a^4*d^4)*x^9)/3 + (c*d^5*(126*
b^4*c^4 + 420*a*b^3*c^3*d + 360*a^2*b^2*c^2*d^2 + 90*a^3*b*c*d^3 + 5*a^4*d^
4)*x^10)/5 + (d^6*(210*b^4*c^4 + 480*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 40
*a^3*b*c*d^3 + a^4*d^4)*x^11)/11 + (b*d^7*(30*b^3*c^3 + 45*a*b^2*c^2*d + 15
*a^2*b*c*d^2 + a^3*d^3)*x^12)/3 + (b^2*d^8*(45*b^2*c^2 + 40*a*b*c*d + 6*a^2
*d^2)*x^13)/13 + (b^3*d^9*(5*b*c + 2*a*d)*x^14)/7 + (b^4*d^10*x^15)/15
```

Maple [B] time = 0.002, size = 691, normalized size = 5.8

$$\frac{b^4d^{10}x^{15}}{15}+\frac{\left(4\,ab^3d^{10}+10\,b^4cd^9\right)x^{14}}{14}+\frac{\left(6\,b^2a^2d^{10}+40\,ab^3cd^9+45\,b^4c^2d^8\right)x^{13}}{13}+\frac{\left(4\,a^3bd^{10}+60\,b^2a^2cd^9+180\,ab^3c^2d^8\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^10,x)
```

```
[Out] 1/15*b^4*d^10*x^15+1/14*(4*a*b^3*d^10+10*b^4*c*d^9)*x^14+1/13*(6*a^2*b^2*d^
10+40*a*b^3*c*d^9+45*b^4*c^2*d^8)*x^13+1/12*(4*a^3*b*d^10+60*a^2*b^2*c*d^9+
180*a*b^3*c^2*d^8+120*b^4*c^3*d^7)*x^12+1/11*(a^4*d^10+40*a^3*b*c*d^9+270*a
^2*b^2*c^2*d^8+480*a*b^3*c^3*d^7+210*b^4*c^4*d^6)*x^11+1/10*(10*a^4*c*d^9+1
80*a^3*b*c^2*d^8+720*a^2*b^2*c^3*d^7+840*a*b^3*c^4*d^6+252*b^4*c^5*d^5)*x^1
0+1/9*(45*a^4*c^2*d^8+480*a^3*b*c^3*d^7+1260*a^2*b^2*c^4*d^6+1008*a*b^3*c^5
*d^5+210*b^4*c^6*d^4)*x^9+1/8*(120*a^4*c^3*d^7+840*a^3*b*c^4*d^6+1512*a^2*b
^2*c^5*d^5+840*a*b^3*c^6*d^4+120*b^4*c^7*d^3)*x^8+1/7*(210*a^4*c^4*d^6+1008
*a^3*b*c^5*d^5+1260*a^2*b^2*c^6*d^4+480*a*b^3*c^7*d^3+45*b^4*c^8*d^2)*x^7+1
/6*(252*a^4*c^5*d^5+840*a^3*b*c^6*d^4+720*a^2*b^2*c^7*d^3+180*a*b^3*c^8*d^2
+10*b^4*c^9*d)*x^6+1/5*(210*a^4*c^6*d^4+480*a^3*b*c^7*d^3+270*a^2*b^2*c^8*d
^2+40*a*b^3*c^9*d+b^4*c^10)*x^5+1/4*(120*a^4*c^7*d^3+180*a^3*b*c^8*d^2+60*a
^2*b^2*c^9*d+4*a*b^3*c^10)*x^4+1/3*(45*a^4*c^8*d^2+40*a^3*b*c^9*d+6*a^2*b^2
*c^10)*x^3+1/2*(10*a^4*c^9*d+4*a^3*b*c^10)*x^2+a^4*c^10*x
```

Maxima [B] time = 1.24099, size = 926, normalized size = 7.78

$$\frac{1}{15}\,b^4d^{10}x^{15}+a^4c^{10}x+\frac{1}{7}\left(5\,b^4cd^9+2\,ab^3d^{10}\right)x^{14}+\frac{1}{13}\left(45\,b^4c^2d^8+40\,ab^3cd^9+6\,a^2b^2d^{10}\right)x^{13}+\frac{1}{3}\left(30\,b^4c^3d^7+45\,ab^3c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/15*b^4*d^10*x^15 + a^4*c^10*x + 1/7*(5*b^4*c*d^9 + 2*a*b^3*d^10)*x^14 + 1
/13*(45*b^4*c^2*d^8 + 40*a*b^3*c*d^9 + 6*a^2*b^2*d^10)*x^13 + 1/3*(30*b^4*c
^3*d^7 + 45*a*b^3*c^2*d^8 + 15*a^2*b^2*c*d^9 + a^3*b*d^10)*x^12 + 1/11*(210
*b^4*c^4*d^6 + 480*a*b^3*c^3*d^7 + 270*a^2*b^2*c^2*d^8 + 40*a^3*b*c*d^9 + a
^4*d^10)*x^11 + 1/5*(126*b^4*c^5*d^5 + 420*a*b^3*c^4*d^6 + 360*a^2*b^2*c^3*
d^7 + 90*a^3*b*c^2*d^8 + 5*a^4*c*d^9)*x^10 + 1/3*(70*b^4*c^6*d^4 + 336*a*b^
3*c^5*d^5 + 420*a^2*b^2*c^4*d^6 + 160*a^3*b*c^3*d^7 + 15*a^4*c^2*d^8)*x^9 +
 3*(5*b^4*c^7*d^3 + 35*a*b^3*c^6*d^4 + 63*a^2*b^2*c^5*d^5 + 35*a^3*b*c^4*d^
6 + 5*a^4*c^3*d^7)*x^8 + 3/7*(15*b^4*c^8*d^2 + 160*a*b^3*c^7*d^3 + 420*a^2*
b^2*c^6*d^4 + 336*a^3*b*c^5*d^5 + 70*a^4*c^4*d^6)*x^7 + 1/3*(5*b^4*c^9*d +
90*a*b^3*c^8*d^2 + 360*a^2*b^2*c^7*d^3 + 420*a^3*b*c^6*d^4 + 126*a^4*c^5*d^
```

```
5)*x^6 + 1/5*(b^4*c^10 + 40*a*b^3*c^9*d + 270*a^2*b^2*c^8*d^2 + 480*a^3*b*c
^7*d^3 + 210*a^4*c^6*d^4)*x^5 + (a*b^3*c^10 + 15*a^2*b^2*c^9*d + 45*a^3*b*c
^8*d^2 + 30*a^4*c^7*d^3)*x^4 + 1/3*(6*a^2*b^2*c^10 + 40*a^3*b*c^9*d + 45*a^
4*c^8*d^2)*x^3 + (2*a^3*b*c^10 + 5*a^4*c^9*d)*x^2
```

Fricas [B] time = 1.73123, size = 1689, normalized size = 14.19

$$\frac{1}{15}x^{15}d^{10}b^4 + \frac{5}{7}x^{14}d^9cb^4 + \frac{2}{7}x^{14}d^{10}b^3a + \frac{45}{13}x^{13}d^8c^2b^4 + \frac{40}{13}x^{13}d^9cb^3a + \frac{6}{13}x^{13}d^{10}b^2a^2 + 10x^{12}d^7c^3b^4 + 15x^{12}d^8c^2b^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*d^10*b^4 + 5/7*x^14*d^9*c*b^4 + 2/7*x^14*d^10*b^3*a + 45/13*x^13*
d^8*c^2*b^4 + 40/13*x^13*d^9*c*b^3*a + 6/13*x^13*d^10*b^2*a^2 + 10*x^12*d^7
*c^3*b^4 + 15*x^12*d^8*c^2*b^3*a + 5*x^12*d^9*c*b^2*a^2 + 1/3*x^12*d^10*b*a
^3 + 210/11*x^11*d^6*c^4*b^4 + 480/11*x^11*d^7*c^3*b^3*a + 270/11*x^11*d^8*
c^2*b^2*a^2 + 40/11*x^11*d^9*c*b*a^3 + 1/11*x^11*d^10*a^4 + 126/5*x^10*d^5*
c^5*b^4 + 84*x^10*d^6*c^4*b^3*a + 72*x^10*d^7*c^3*b^2*a^2 + 18*x^10*d^8*c^2
*b*a^3 + x^10*d^9*c*a^4 + 70/3*x^9*d^4*c^6*b^4 + 112*x^9*d^5*c^5*b^3*a + 14
0*x^9*d^6*c^4*b^2*a^2 + 160/3*x^9*d^7*c^3*b*a^3 + 5*x^9*d^8*c^2*a^4 + 15*x^
8*d^3*c^7*b^4 + 105*x^8*d^4*c^6*b^3*a + 189*x^8*d^5*c^5*b^2*a^2 + 105*x^8*d
^6*c^4*b*a^3 + 15*x^8*d^7*c^3*a^4 + 45/7*x^7*d^2*c^8*b^4 + 480/7*x^7*d^3*c^
7*b^3*a + 180*x^7*d^4*c^6*b^2*a^2 + 144*x^7*d^5*c^5*b*a^3 + 30*x^7*d^6*c^4*
a^4 + 5/3*x^6*d*c^9*b^4 + 30*x^6*d^2*c^8*b^3*a + 120*x^6*d^3*c^7*b^2*a^2 +
140*x^6*d^4*c^6*b*a^3 + 42*x^6*d^5*c^5*a^4 + 1/5*x^5*c^10*b^4 + 8*x^5*d*c^9
*b^3*a + 54*x^5*d^2*c^8*b^2*a^2 + 96*x^5*d^3*c^7*b*a^3 + 42*x^5*d^4*c^6*a^4
 + x^4*c^10*b^3*a + 15*x^4*d*c^9*b^2*a^2 + 45*x^4*d^2*c^8*b*a^3 + 30*x^4*d^
3*c^7*a^4 + 2*x^3*c^10*b^2*a^2 + 40/3*x^3*d*c^9*b*a^3 + 15*x^3*d^2*c^8*a^4
+ 2*x^2*c^10*b*a^3 + 5*x^2*d*c^9*a^4 + x*c^10*a^4
```

Sympy [B] time = 0.155678, size = 748, normalized size = 6.29

$$a^4c^{10}x + \frac{b^4d^{10}x^{15}}{15} + x^{14}\left(\frac{2ab^3d^{10}}{7} + \frac{5b^4cd^9}{7}\right) + x^{13}\left(\frac{6a^2b^2d^{10}}{13} + \frac{40ab^3cd^9}{13} + \frac{45b^4c^2d^8}{13}\right) + x^{12}\left(\frac{a^3bd^{10}}{3} + 5a^2b^2cd^9 + 1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**10,x)
```

```
[Out] a**4*c**10*x + b**4*d**10*x**15/15 + x**14*(2*a*b**3*d**10/7 + 5*b**4*c*d**
9/7) + x**13*(6*a**2*b**2*d**10/13 + 40*a*b**3*c*d**9/13 + 45*b**4*c**2*d**
8/13) + x**12*(a**3*b*d**10/3 + 5*a**2*b**2*c*d**9 + 15*a*b**3*c**2*d**8 +
10*b**4*c**3*d**7) + x**11*(a**4*d**10/11 + 40*a**3*b*c*d**9/11 + 270*a**2*
b**2*c**2*d**8/11 + 480*a*b**3*c**3*d**7/11 + 210*b**4*c**4*d**6/11) + x**1
0*(a**4*c*d**9 + 18*a**3*b*c**2*d**8 + 72*a**2*b**2*c**3*d**7 + 84*a*b**3*c
**4*d**6 + 126*b**4*c**5*d**5/5) + x**9*(5*a**4*c**2*d**8 + 160*a**3*b*c**3
*d**7/3 + 140*a**2*b**2*c**4*d**6 + 112*a*b**3*c**5*d**5 + 70*b**4*c**6*d**
4/3) + x**8*(15*a**4*c**3*d**7 + 105*a**3*b*c**4*d**6 + 189*a**2*b**2*c**5*
d**5 + 105*a*b**3*c**6*d**4 + 15*b**4*c**7*d**3) + x**7*(30*a**4*c**4*d**6
+ 144*a**3*b*c**5*d**5 + 180*a**2*b**2*c**6*d**4 + 480*a*b**3*c**7*d**3/7 +
 45*b**4*c**8*d**2/7) + x**6*(42*a**4*c**5*d**5 + 140*a**3*b*c**6*d**4 + 12
0*a**2*b**2*c**7*d**3 + 30*a*b**3*c**8*d**2 + 5*b**4*c**9*d/3) + x**5*(42*a
**4*c**6*d**4 + 96*a**3*b*c**7*d**3 + 54*a**2*b**2*c**8*d**2 + 8*a*b**3*c**
```

```
9*d + b**4*c**10/5) + x**4*(30*a**4*c**7*d**3 + 45*a**3*b*c**8*d**2 + 15*a*
*2*b**2*c**9*d + a*b**3*c**10) + x**3*(15*a**4*c**8*d**2 + 40*a**3*b*c**9*d
/3 + 2*a**2*b**2*c**10) + x**2*(5*a**4*c**9*d + 2*a**3*b*c**10)
```

Giac [B] time = 1.0532, size = 1041, normalized size = 8.75

$$\frac{1}{15}\,b^4d^{10}x^{15}+\frac{5}{7}\,b^4cd^9x^{14}+\frac{2}{7}\,ab^3d^{10}x^{14}+\frac{45}{13}\,b^4c^2d^8x^{13}+\frac{40}{13}\,ab^3cd^9x^{13}+\frac{6}{13}\,a^2b^2d^{10}x^{13}+10\,b^4c^3d^7x^{12}+15\,ab^3c^2d^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/15*b^4*d^10*x^15 + 5/7*b^4*c*d^9*x^14 + 2/7*a*b^3*d^10*x^14 + 45/13*b^4*c
^2*d^8*x^13 + 40/13*a*b^3*c*d^9*x^13 + 6/13*a^2*b^2*d^10*x^13 + 10*b^4*c^3*
d^7*x^12 + 15*a*b^3*c^2*d^8*x^12 + 5*a^2*b^2*c*d^9*x^12 + 1/3*a^3*b*d^10*x^
12 + 210/11*b^4*c^4*d^6*x^11 + 480/11*a*b^3*c^3*d^7*x^11 + 270/11*a^2*b^2*c
^2*d^8*x^11 + 40/11*a^3*b*c*d^9*x^11 + 1/11*a^4*d^10*x^11 + 126/5*b^4*c^5*d
^5*x^10 + 84*a*b^3*c^4*d^6*x^10 + 72*a^2*b^2*c^3*d^7*x^10 + 18*a^3*b*c^2*d^
8*x^10 + a^4*c*d^9*x^10 + 70/3*b^4*c^6*d^4*x^9 + 112*a*b^3*c^5*d^5*x^9 + 14
0*a^2*b^2*c^4*d^6*x^9 + 160/3*a^3*b*c^3*d^7*x^9 + 5*a^4*c^2*d^8*x^9 + 15*b^
4*c^7*d^3*x^8 + 105*a*b^3*c^6*d^4*x^8 + 189*a^2*b^2*c^5*d^5*x^8 + 105*a^3*b
*c^4*d^6*x^8 + 15*a^4*c^3*d^7*x^8 + 45/7*b^4*c^8*d^2*x^7 + 480/7*a*b^3*c^7*
d^3*x^7 + 180*a^2*b^2*c^6*d^4*x^7 + 144*a^3*b*c^5*d^5*x^7 + 30*a^4*c^4*d^6*
x^7 + 5/3*b^4*c^9*d*x^6 + 30*a*b^3*c^8*d^2*x^6 + 120*a^2*b^2*c^7*d^3*x^6 +
140*a^3*b*c^6*d^4*x^6 + 42*a^4*c^5*d^5*x^6 + 1/5*b^4*c^10*x^5 + 8*a*b^3*c^9
*d*x^5 + 54*a^2*b^2*c^8*d^2*x^5 + 96*a^3*b*c^7*d^3*x^5 + 42*a^4*c^6*d^4*x^5
 + a*b^3*c^10*x^4 + 15*a^2*b^2*c^9*d*x^4 + 45*a^3*b*c^8*d^2*x^4 + 30*a^4*c^
7*d^3*x^4 + 2*a^2*b^2*c^10*x^3 + 40/3*a^3*b*c^9*d*x^3 + 15*a^4*c^8*d^2*x^3
+ 2*a^3*b*c^10*x^2 + 5*a^4*c^9*d*x^2 + a^4*c^10*x
```

3.1308 $\int (a+bx)^3(c+dx)^{10}\,dx$

Optimal. Leaf size=92

$$-\frac{3b^2(c+dx)^{13}(bc-ad)}{13d^4}+\frac{b(c+dx)^{12}(bc-ad)^2}{4d^4}-\frac{(c+dx)^{11}(bc-ad)^3}{11d^4}+\frac{b^3(c+dx)^{14}}{14d^4}$$

```
[Out] -((b*c - a*d)^3*(c + d*x)^11)/(11*d^4) + (b*(b*c - a*d)^2*(c + d*x)^12)/(4*
d^4) - (3*b^2*(b*c - a*d)*(c + d*x)^13)/(13*d^4) + (b^3*(c + d*x)^14)/(14*d
^4)
```

Rubi [A] time = 0.348788, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^2(c+dx)^{13}(bc-ad)}{13d^4}+\frac{b(c+dx)^{12}(bc-ad)^2}{4d^4}-\frac{(c+dx)^{11}(bc-ad)^3}{11d^4}+\frac{b^3(c+dx)^{14}}{14d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)^3*(c + d*x)^11)/(11*d^4) + (b*(b*c - a*d)^2*(c + d*x)^12)/(4*
d^4) - (3*b^2*(b*c - a*d)*(c + d*x)^13)/(13*d^4) + (b^3*(c + d*x)^14)/(14*d
^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^3(c+dx)^{10}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{11}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{12}}{d^3}+\frac{b^3(c+dx)^{1}}{d^3}\right.$$
$$=-\frac{(bc-ad)^3(c+dx)^{11}}{11d^4}+\frac{b(bc-ad)^2(c+dx)^{12}}{4d^4}-\frac{3b^2(bc-ad)(c+dx)^{13}}{13d^4}+\frac{b^3(c+dx)^{14}}{14d^4}$$

Mathematica [B] time = 0.0609669, size = 511, normalized size = 5.55

$$\frac{1}{4}bd^8x^{12}\left(a^2d^2+10abcd+15b^2c^2\right)+\frac{1}{11}d^7x^{11}\left(30a^2bcd^2+a^3d^3+135ab^2c^2d+120b^3c^3\right)+\frac{1}{2}cd^6x^{10}\left(27a^2bcd^2+2a^3d^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^10,x]
```

```
[Out] a^3*c^10*x + (a^2*c^9*(3*b*c + 10*a*d)*x^2)/2 + a*c^8*(b^2*c^2 + 10*a*b*c*d
 + 15*a^2*d^2)*x^3 + (c^7*(b^3*c^3 + 30*a*b^2*c^2*d + 135*a^2*b*c*d^2 + 120
*a^3*d^3)*x^4)/4 + c^6*d*(2*b^3*c^3 + 27*a*b^2*c^2*d + 72*a^2*b*c*d^2 + 42*
a^3*d^3)*x^5 + (3*c^5*d^2*(5*b^3*c^3 + 40*a*b^2*c^2*d + 70*a^2*b*c*d^2 + 28
```

```
*a^3*d^3)*x^6)/2 + (6*c^4*d^3*(20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*d
^2 + 35*a^3*d^3)*x^7)/7 + (3*c^3*d^4*(35*b^3*c^3 + 126*a*b^2*c^2*d + 105*a^
2*b*c*d^2 + 20*a^3*d^3)*x^8)/4 + c^2*d^5*(28*b^3*c^3 + 70*a*b^2*c^2*d + 40*
a^2*b*c*d^2 + 5*a^3*d^3)*x^9 + (c*d^6*(42*b^3*c^3 + 72*a*b^2*c^2*d + 27*a^2
*b*c*d^2 + 2*a^3*d^3)*x^10)/2 + (d^7*(120*b^3*c^3 + 135*a*b^2*c^2*d + 30*a^
2*b*c*d^2 + a^3*d^3)*x^11)/11 + (b*d^8*(15*b^2*c^2 + 10*a*b*c*d + a^2*d^2)*
x^12)/4 + (b^2*d^9*(10*b*c + 3*a*d)*x^13)/13 + (b^3*d^10*x^14)/14
```

Maple [B] time = 0., size = 541, normalized size = 5.9

$$\frac{b^3d^{10}x^{14}}{14}+\frac{\left(3\,ab^2d^{10}+10\,b^3cd^9\right)x^{13}}{13}+\frac{\left(3\,a^2bd^{10}+30\,ab^2cd^9+45\,b^3c^2d^8\right)x^{12}}{12}+\frac{\left(a^3d^{10}+30\,a^2bcd^9+135\,ab^2c^2d^8+120\right.}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^10,x)
```

```
[Out] 1/14*b^3*d^10*x^14+1/13*(3*a*b^2*d^10+10*b^3*c*d^9)*x^13+1/12*(3*a^2*b*d^10
+30*a*b^2*c*d^9+45*b^3*c^2*d^8)*x^12+1/11*(a^3*d^10+30*a^2*b*c*d^9+135*a*b^
2*c^2*d^8+120*b^3*c^3*d^7)*x^11+1/10*(10*a^3*c*d^9+135*a^2*b*c^2*d^8+360*a*
b^2*c^3*d^7+210*b^3*c^4*d^6)*x^10+1/9*(45*a^3*c^2*d^8+360*a^2*b*c^3*d^7+630
*a*b^2*c^4*d^6+252*b^3*c^5*d^5)*x^9+1/8*(120*a^3*c^3*d^7+630*a^2*b*c^4*d^6+
756*a*b^2*c^5*d^5+210*b^3*c^6*d^4)*x^8+1/7*(210*a^3*c^4*d^6+756*a^2*b*c^5*d
^5+630*a*b^2*c^6*d^4+120*b^3*c^7*d^3)*x^7+1/6*(252*a^3*c^5*d^5+630*a^2*b*c^
6*d^4+360*a*b^2*c^7*d^3+45*b^3*c^8*d^2)*x^6+1/5*(210*a^3*c^6*d^4+360*a^2*b*
c^7*d^3+135*a*b^2*c^8*d^2+10*b^3*c^9*d)*x^5+1/4*(120*a^3*c^7*d^3+135*a^2*b*
c^8*d^2+30*a*b^2*c^9*d+b^3*c^10)*x^4+1/3*(45*a^3*c^8*d^2+30*a^2*b*c^9*d+3*a
*b^2*c^10)*x^3+1/2*(10*a^3*c^9*d+3*a^2*b*c^10)*x^2+a^3*c^10*x
```

Maxima [B] time = 0.974787, size = 722, normalized size = 7.85

$$\frac{1}{14}\,b^3d^{10}x^{14}+a^3c^{10}x+\frac{1}{13}\left(10\,b^3cd^9+3\,ab^2d^{10}\right)x^{13}+\frac{1}{4}\left(15\,b^3c^2d^8+10\,ab^2cd^9+a^2bd^{10}\right)x^{12}+\frac{1}{11}\left(120\,b^3c^3d^7+135\,ab^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/14*b^3*d^10*x^14 + a^3*c^10*x + 1/13*(10*b^3*c*d^9 + 3*a*b^2*d^10)*x^13 +
 1/4*(15*b^3*c^2*d^8 + 10*a*b^2*c*d^9 + a^2*b*d^10)*x^12 + 1/11*(120*b^3*c^
3*d^7 + 135*a*b^2*c^2*d^8 + 30*a^2*b*c*d^9 + a^3*d^10)*x^11 + 1/2*(42*b^3*c
^4*d^6 + 72*a*b^2*c^3*d^7 + 27*a^2*b*c^2*d^8 + 2*a^3*c*d^9)*x^10 + (28*b^3*
c^5*d^5 + 70*a*b^2*c^4*d^6 + 40*a^2*b*c^3*d^7 + 5*a^3*c^2*d^8)*x^9 + 3/4*(3
5*b^3*c^6*d^4 + 126*a*b^2*c^5*d^5 + 105*a^2*b*c^4*d^6 + 20*a^3*c^3*d^7)*x^8
 + 6/7*(20*b^3*c^7*d^3 + 105*a*b^2*c^6*d^4 + 126*a^2*b*c^5*d^5 + 35*a^3*c^4
*d^6)*x^7 + 3/2*(5*b^3*c^8*d^2 + 40*a*b^2*c^7*d^3 + 70*a^2*b*c^6*d^4 + 28*a
^3*c^5*d^5)*x^6 + (2*b^3*c^9*d + 27*a*b^2*c^8*d^2 + 72*a^2*b*c^7*d^3 + 42*a
^3*c^6*d^4)*x^5 + 1/4*(b^3*c^10 + 30*a*b^2*c^9*d + 135*a^2*b*c^8*d^2 + 120*
a^3*c^7*d^3)*x^4 + (a*b^2*c^10 + 10*a^2*b*c^9*d + 15*a^3*c^8*d^2)*x^3 + 1/2
*(3*a^2*b*c^10 + 10*a^3*c^9*d)*x^2
```

Fricas [B] time = 1.54085, size = 1318, normalized size = 14.33

$$\frac{1}{14}x^{14}d^{10}b^3+\frac{10}{13}x^{13}d^9cb^3+\frac{3}{13}x^{13}d^{10}b^2a+\frac{15}{4}x^{12}d^8c^2b^3+\frac{5}{2}x^{12}d^9cb^2a+\frac{1}{4}x^{12}d^{10}ba^2+\frac{120}{11}x^{11}d^7c^3b^3+\frac{135}{11}x^{11}d^8c^2b^2a+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*d^10*b^3 + 10/13*x^13*d^9*c*b^3 + 3/13*x^13*d^10*b^2*a + 15/4*x^1
2*d^8*c^2*b^3 + 5/2*x^12*d^9*c*b^2*a + 1/4*x^12*d^10*b*a^2 + 120/11*x^11*d^
7*c^3*b^3 + 135/11*x^11*d^8*c^2*b^2*a + 30/11*x^11*d^9*c*b*a^2 + 1/11*x^11*
d^10*a^3 + 21*x^10*d^6*c^4*b^3 + 36*x^10*d^7*c^3*b^2*a + 27/2*x^10*d^8*c^2*
b*a^2 + x^10*d^9*c*a^3 + 28*x^9*d^5*c^5*b^3 + 70*x^9*d^6*c^4*b^2*a + 40*x^9
*d^7*c^3*b*a^2 + 5*x^9*d^8*c^2*a^3 + 105/4*x^8*d^4*c^6*b^3 + 189/2*x^8*d^5*
c^5*b^2*a + 315/4*x^8*d^6*c^4*b*a^2 + 15*x^8*d^7*c^3*a^3 + 120/7*x^7*d^3*c^
7*b^3 + 90*x^7*d^4*c^6*b^2*a + 108*x^7*d^5*c^5*b*a^2 + 30*x^7*d^6*c^4*a^3 +
 15/2*x^6*d^2*c^8*b^3 + 60*x^6*d^3*c^7*b^2*a + 105*x^6*d^4*c^6*b*a^2 + 42*x
^6*d^5*c^5*a^3 + 2*x^5*d*c^9*b^3 + 27*x^5*d^2*c^8*b^2*a + 72*x^5*d^3*c^7*b*
a^2 + 42*x^5*d^4*c^6*a^3 + 1/4*x^4*c^10*b^3 + 15/2*x^4*d*c^9*b^2*a + 135/4*
x^4*d^2*c^8*b*a^2 + 30*x^4*d^3*c^7*a^3 + x^3*c^10*b^2*a + 10*x^3*d*c^9*b*a^
2 + 15*x^3*d^2*c^8*a^3 + 3/2*x^2*c^10*b*a^2 + 5*x^2*d*c^9*a^3 + x*c^10*a^3
```

Sympy [B] time = 0.1392, size = 586, normalized size = 6.37

$$a^3c^{10}x + \frac{b^3d^{10}x^{14}}{14} + x^{13}\left(\frac{3ab^2d^{10}}{13} + \frac{10b^3cd^9}{13}\right) + x^{12}\left(\frac{a^2bd^{10}}{4} + \frac{5ab^2cd^9}{2} + \frac{15b^3c^2d^8}{4}\right) + x^{11}\left(\frac{a^3d^{10}}{11} + \frac{30a^2bcd^9}{11} + \frac{135}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**10,x)
```

```
[Out] a**3*c**10*x + b**3*d**10*x**14/14 + x**13*(3*a*b**2*d**10/13 + 10*b**3*c*d
**9/13) + x**12*(a**2*b*d**10/4 + 5*a*b**2*c*d**9/2 + 15*b**3*c**2*d**8/4)
+ x**11*(a**3*d**10/11 + 30*a**2*b*c*d**9/11 + 135*a*b**2*c**2*d**8/11 + 12
0*b**3*c**3*d**7/11) + x**10*(a**3*c*d**9 + 27*a**2*b*c**2*d**8/2 + 36*a*b*
*2*c**3*d**7 + 21*b**3*c**4*d**6) + x**9*(5*a**3*c**2*d**8 + 40*a**2*b*c**3
*d**7 + 70*a*b**2*c**4*d**6 + 28*b**3*c**5*d**5) + x**8*(15*a**3*c**3*d**7
+ 315*a**2*b*c**4*d**6/4 + 189*a*b**2*c**5*d**5/2 + 105*b**3*c**6*d**4/4) +
 x**7*(30*a**3*c**4*d**6 + 108*a**2*b*c**5*d**5 + 90*a*b**2*c**6*d**4 + 120
*b**3*c**7*d**3/7) + x**6*(42*a**3*c**5*d**5 + 105*a**2*b*c**6*d**4 + 60*a*
b**2*c**7*d**3 + 15*b**3*c**8*d**2/2) + x**5*(42*a**3*c**6*d**4 + 72*a**2*b
*c**7*d**3 + 27*a*b**2*c**8*d**2 + 2*b**3*c**9*d) + x**4*(30*a**3*c**7*d**3
 + 135*a**2*b*c**8*d**2/4 + 15*a*b**2*c**9*d/2 + b**3*c**10/4) + x**3*(15*a
**3*c**8*d**2 + 10*a**2*b*c**9*d + a*b**2*c**10) + x**2*(5*a**3*c**9*d + 3*
a**2*b*c**10/2)
```

Giac [B] time = 1.06665, size = 802, normalized size = 8.72

$$\frac{1}{14}b^3d^{10}x^{14} + \frac{10}{13}b^3cd^9x^{13} + \frac{3}{13}ab^2d^{10}x^{13} + \frac{15}{4}b^3c^2d^8x^{12} + \frac{5}{2}ab^2cd^9x^{12} + \frac{1}{4}a^2bd^{10}x^{12} + \frac{120}{11}b^3c^3d^7x^{11} + \frac{135}{11}ab^2c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/14*b^3*d^10*x^14 + 10/13*b^3*c*d^9*x^13 + 3/13*a*b^2*d^10*x^13 + 15/4*b^3
*c^2*d^8*x^12 + 5/2*a*b^2*c*d^9*x^12 + 1/4*a^2*b*d^10*x^12 + 120/11*b^3*c^3
*d^7*x^11 + 135/11*a*b^2*c^2*d^8*x^11 + 30/11*a^2*b*c*d^9*x^11 + 1/11*a^3*d
```

```
^10*x^11 + 21*b^3*c^4*d^6*x^10 + 36*a*b^2*c^3*d^7*x^10 + 27/2*a^2*b*c^2*d^8
*x^10 + a^3*c*d^9*x^10 + 28*b^3*c^5*d^5*x^9 + 70*a*b^2*c^4*d^6*x^9 + 40*a^2
*b*c^3*d^7*x^9 + 5*a^3*c^2*d^8*x^9 + 105/4*b^3*c^6*d^4*x^8 + 189/2*a*b^2*c^
5*d^5*x^8 + 315/4*a^2*b*c^4*d^6*x^8 + 15*a^3*c^3*d^7*x^8 + 120/7*b^3*c^7*d^
3*x^7 + 90*a*b^2*c^6*d^4*x^7 + 108*a^2*b*c^5*d^5*x^7 + 30*a^3*c^4*d^6*x^7 +
 15/2*b^3*c^8*d^2*x^6 + 60*a*b^2*c^7*d^3*x^6 + 105*a^2*b*c^6*d^4*x^6 + 42*a
^3*c^5*d^5*x^6 + 2*b^3*c^9*d*x^5 + 27*a*b^2*c^8*d^2*x^5 + 72*a^2*b*c^7*d^3*
x^5 + 42*a^3*c^6*d^4*x^5 + 1/4*b^3*c^10*x^4 + 15/2*a*b^2*c^9*d*x^4 + 135/4*
a^2*b*c^8*d^2*x^4 + 30*a^3*c^7*d^3*x^4 + a*b^2*c^10*x^3 + 10*a^2*b*c^9*d*x^
3 + 15*a^3*c^8*d^2*x^3 + 3/2*a^2*b*c^10*x^2 + 5*a^3*c^9*d*x^2 + a^3*c^10*x
```

3.1309 $\int (a+bx)^2(c+dx)^{10}\,dx$

Optimal. Leaf size=65

$$-\frac{b(c+dx)^{12}(bc-ad)}{6d^3}+\frac{(c+dx)^{11}(bc-ad)^2}{11d^3}+\frac{b^2(c+dx)^{13}}{13d^3}$$

```
[Out] ((b*c - a*d)^2*(c + d*x)^11)/(11*d^3) - (b*(b*c - a*d)*(c + d*x)^12)/(6*d^3
) + (b^2*(c + d*x)^13)/(13*d^3)
```

Rubi [A] time = 0.250741, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b(c+dx)^{12}(bc-ad)}{6d^3}+\frac{(c+dx)^{11}(bc-ad)^2}{11d^3}+\frac{b^2(c+dx)^{13}}{13d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^11)/(11*d^3) - (b*(b*c - a*d)*(c + d*x)^12)/(6*d^3
) + (b^2*(c + d*x)^13)/(13*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{10}}{d^2}-\frac{2b(bc-ad)(c+dx)^{11}}{d^2}+\frac{b^2(c+dx)^{12}}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^{11}}{11d^3}-\frac{b(bc-ad)(c+dx)^{12}}{6d^3}+\frac{b^2(c+dx)^{13}}{13d^3}\end{aligned}$$

Mathematica [B] time = 0.0397498, size = 358, normalized size = 5.51

$$\frac{1}{11}d^8x^{11}\left(a^2d^2+20abcd+45b^2c^2\right)+cd^7x^{10}\left(a^2d^2+9abcd+12b^2c^2\right)+\frac{5}{3}c^2d^6x^9\left(3a^2d^2+16abcd+14b^2c^2\right)+\frac{3}{2}c^3d^5x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^10,x]
```

```
[Out] a^2*c^10*x + a*c^9*(b*c + 5*a*d)*x^2 + (c^8*(b^2*c^2 + 20*a*b*c*d + 45*a^2*
d^2)*x^3)/3 + (5*c^7*d*(b^2*c^2 + 9*a*b*c*d + 12*a^2*d^2)*x^4)/2 + 3*c^6*d^
2*(3*b^2*c^2 + 16*a*b*c*d + 14*a^2*d^2)*x^5 + 2*c^5*d^3*(10*b^2*c^2 + 35*a*
b*c*d + 21*a^2*d^2)*x^6 + 6*c^4*d^4*(5*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*x^
7 + (3*c^3*d^5*(21*b^2*c^2 + 35*a*b*c*d + 10*a^2*d^2)*x^8)/2 + (5*c^2*d^6*(
14*b^2*c^2 + 16*a*b*c*d + 3*a^2*d^2)*x^9)/3 + c*d^7*(12*b^2*c^2 + 9*a*b*c*d
```

```
 + a^2*d^2)*x^10 + (d^8*(45*b^2*c^2 + 20*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d
^9*(5*b*c + a*d)*x^12)/6 + (b^2*d^10*x^13)/13
```

Maple [B] time = 0.001, size = 391, normalized size = 6.

$$\frac{b^2d^{10}x^{13}}{13}+\frac{\left(2\,abd^{10}+10\,b^2cd^9\right)x^{12}}{12}+\frac{\left(a^2d^{10}+20\,abcd^9+45\,b^2c^2d^8\right)x^{11}}{11}+\frac{\left(10\,a^2cd^9+90\,abc^2d^8+120\,b^2c^3d^7\right)x^{10}}{10}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^10,x)
```

```
[Out] 1/13*b^2*d^10*x^13+1/12*(2*a*b*d^10+10*b^2*c*d^9)*x^12+1/11*(a^2*d^10+20*a*
b*c*d^9+45*b^2*c^2*d^8)*x^11+1/10*(10*a^2*c*d^9+90*a*b*c^2*d^8+120*b^2*c^3*
d^7)*x^10+1/9*(45*a^2*c^2*d^8+240*a*b*c^3*d^7+210*b^2*c^4*d^6)*x^9+1/8*(120
*a^2*c^3*d^7+420*a*b*c^4*d^6+252*b^2*c^5*d^5)*x^8+1/7*(210*a^2*c^4*d^6+504*
a*b*c^5*d^5+210*b^2*c^6*d^4)*x^7+1/6*(252*a^2*c^5*d^5+420*a*b*c^6*d^4+120*b
^2*c^7*d^3)*x^6+1/5*(210*a^2*c^6*d^4+240*a*b*c^7*d^3+45*b^2*c^8*d^2)*x^5+1/
4*(120*a^2*c^7*d^3+90*a*b*c^8*d^2+10*b^2*c^9*d)*x^4+1/3*(45*a^2*c^8*d^2+20*
a*b*c^9*d+b^2*c^10)*x^3+1/2*(10*a^2*c^9*d+2*a*b*c^10)*x^2+a^2*c^10*x
```

Maxima [B] time = 0.983961, size = 518, normalized size = 7.97

$$\frac{1}{13}\,b^2d^{10}x^{13}+a^2c^{10}x+\frac{1}{6}\left(5\,b^2cd^9+abd^{10}\right)x^{12}+\frac{1}{11}\left(45\,b^2c^2d^8+20\,abcd^9+a^2d^{10}\right)x^{11}+\left(12\,b^2c^3d^7+9\,abc^2d^8+a^2cd^9\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^10*x^13 + a^2*c^10*x + 1/6*(5*b^2*c*d^9 + a*b*d^10)*x^12 + 1/11*
(45*b^2*c^2*d^8 + 20*a*b*c*d^9 + a^2*d^10)*x^11 + (12*b^2*c^3*d^7 + 9*a*b*c
^2*d^8 + a^2*c*d^9)*x^10 + 5/3*(14*b^2*c^4*d^6 + 16*a*b*c^3*d^7 + 3*a^2*c^2
*d^8)*x^9 + 3/2*(21*b^2*c^5*d^5 + 35*a*b*c^4*d^6 + 10*a^2*c^3*d^7)*x^8 + 6*
(5*b^2*c^6*d^4 + 12*a*b*c^5*d^5 + 5*a^2*c^4*d^6)*x^7 + 2*(10*b^2*c^7*d^3 +
35*a*b*c^6*d^4 + 21*a^2*c^5*d^5)*x^6 + 3*(3*b^2*c^8*d^2 + 16*a*b*c^7*d^3 +
14*a^2*c^6*d^4)*x^5 + 5/2*(b^2*c^9*d + 9*a*b*c^8*d^2 + 12*a^2*c^7*d^3)*x^4
 + 1/3*(b^2*c^10 + 20*a*b*c^9*d + 45*a^2*c^8*d^2)*x^3 + (a*b*c^10 + 5*a^2*c^
9*d)*x^2
```

Fricas [B] time = 1.56089, size = 922, normalized size = 14.18

$$\frac{1}{13}x^{13}d^{10}b^2+\frac{5}{6}x^{12}d^9cb^2+\frac{1}{6}x^{12}d^{10}ba+\frac{45}{11}x^{11}d^8c^2b^2+\frac{20}{11}x^{11}d^9cba+\frac{1}{11}x^{11}d^{10}a^2+12x^{10}d^7c^3b^2+9x^{10}d^8c^2ba+x^{10}d^9ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^10*b^2 + 5/6*x^12*d^9*c*b^2 + 1/6*x^12*d^10*b*a + 45/11*x^11*d^
8*c^2*b^2 + 20/11*x^11*d^9*c*b*a + 1/11*x^11*d^10*a^2 + 12*x^10*d^7*c^3*b^2
 + 9*x^10*d^8*c^2*b*a + x^10*d^9*c*a^2 + 70/3*x^9*d^6*c^4*b^2 + 80/3*x^9*d^
7*c^3*b*a + 5*x^9*d^8*c^2*a^2 + 63/2*x^8*d^5*c^5*b^2 + 105/2*x^8*d^6*c^4*b*
```

```
a + 15*x^8*d^7*c^3*a^2 + 30*x^7*d^4*c^6*b^2 + 72*x^7*d^5*c^5*b*a + 30*x^7*d
^6*c^4*a^2 + 20*x^6*d^3*c^7*b^2 + 70*x^6*d^4*c^6*b*a + 42*x^6*d^5*c^5*a^2 +
 9*x^5*d^2*c^8*b^2 + 48*x^5*d^3*c^7*b*a + 42*x^5*d^4*c^6*a^2 + 5/2*x^4*d*c^
9*b^2 + 45/2*x^4*d^2*c^8*b*a + 30*x^4*d^3*c^7*a^2 + 1/3*x^3*c^10*b^2 + 20/3
*x^3*d*c^9*b*a + 15*x^3*d^2*c^8*a^2 + x^2*c^10*b*a + 5*x^2*d*c^9*a^2 + x*c^
10*a^2
```

Sympy [B] time = 0.122057, size = 415, normalized size = 6.38

$$a^2c^{10}x + \frac{b^2d^{10}x^{13}}{13} + x^{12}\left(\frac{abd^{10}}{6} + \frac{5b^2cd^9}{6}\right) + x^{11}\left(\frac{a^2d^{10}}{11} + \frac{20abcd^9}{11} + \frac{45b^2c^2d^8}{11}\right) + x^{10}\left(a^2cd^9 + 9abc^2d^8 + 12b^2c^3d^7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**10,x)
```

```
[Out] a**2*c**10*x + b**2*d**10*x**13/13 + x**12*(a*b*d**10/6 + 5*b**2*c*d**9/6)
+ x**11*(a**2*d**10/11 + 20*a*b*c*d**9/11 + 45*b**2*c**2*d**8/11) + x**10*(
a**2*c*d**9 + 9*a*b*c**2*d**8 + 12*b**2*c**3*d**7) + x**9*(5*a**2*c**2*d**8
 + 80*a*b*c**3*d**7/3 + 70*b**2*c**4*d**6/3) + x**8*(15*a**2*c**3*d**7 + 10
5*a*b*c**4*d**6/2 + 63*b**2*c**5*d**5/2) + x**7*(30*a**2*c**4*d**6 + 72*a*b
*c**5*d**5 + 30*b**2*c**6*d**4) + x**6*(42*a**2*c**5*d**5 + 70*a*b*c**6*d**
4 + 20*b**2*c**7*d**3) + x**5*(42*a**2*c**6*d**4 + 48*a*b*c**7*d**3 + 9*b**
2*c**8*d**2) + x**4*(30*a**2*c**7*d**3 + 45*a*b*c**8*d**2/2 + 5*b**2*c**9*d
/2) + x**3*(15*a**2*c**8*d**2 + 20*a*b*c**9*d/3 + b**2*c**10/3) + x**2*(5*a
**2*c**9*d + a*b*c**10)
```

Giac [B] time = 1.06227, size = 563, normalized size = 8.66

$$\frac{1}{13}b^2d^{10}x^{13} + \frac{5}{6}b^2cd^9x^{12} + \frac{1}{6}abd^{10}x^{12} + \frac{45}{11}b^2c^2d^8x^{11} + \frac{20}{11}abcd^9x^{11} + \frac{1}{11}a^2d^{10}x^{11} + 12\,b^2c^3d^7x^{10} + 9\,abc^2d^8x^{10} + a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^10*x^13 + 5/6*b^2*c*d^9*x^12 + 1/6*a*b*d^10*x^12 + 45/11*b^2*c^2
*d^8*x^11 + 20/11*a*b*c*d^9*x^11 + 1/11*a^2*d^10*x^11 + 12*b^2*c^3*d^7*x^10
 + 9*a*b*c^2*d^8*x^10 + a^2*c*d^9*x^10 + 70/3*b^2*c^4*d^6*x^9 + 80/3*a*b*c^
3*d^7*x^9 + 5*a^2*c^2*d^8*x^9 + 63/2*b^2*c^5*d^5*x^8 + 105/2*a*b*c^4*d^6*x^
8 + 15*a^2*c^3*d^7*x^8 + 30*b^2*c^6*d^4*x^7 + 72*a*b*c^5*d^5*x^7 + 30*a^2*c
^4*d^6*x^7 + 20*b^2*c^7*d^3*x^6 + 70*a*b*c^6*d^4*x^6 + 42*a^2*c^5*d^5*x^6 +
 9*b^2*c^8*d^2*x^5 + 48*a*b*c^7*d^3*x^5 + 42*a^2*c^6*d^4*x^5 + 5/2*b^2*c^9*
d*x^4 + 45/2*a*b*c^8*d^2*x^4 + 30*a^2*c^7*d^3*x^4 + 1/3*b^2*c^10*x^3 + 20/3
*a*b*c^9*d*x^3 + 15*a^2*c^8*d^2*x^3 + a*b*c^10*x^2 + 5*a^2*c^9*d*x^2 + a^2*
c^10*x
```

3.1310 $\int (a+bx)(c+dx)^{10}\,dx$

Optimal. Leaf size=38

$$\frac{b(c+dx)^{12}}{12d^2}-\frac{(c+dx)^{11}(bc-ad)}{11d^2}$$

```
[Out] -((b*c - a*d)*(c + d*x)^11)/(11*d^2) + (b*(c + d*x)^12)/(12*d^2)
```

Rubi [A] time = 0.0164524, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^{12}}{12d^2}-\frac{(c+dx)^{11}(bc-ad)}{11d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^11)/(11*d^2) + (b*(c + d*x)^12)/(12*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{10}\,dx&=\int \left(\frac{(-bc+ad)(c+dx)^{10}}{d}+\frac{b(c+dx)^{11}}{d}\right)dx\\&=-\frac{(bc-ad)(c+dx)^{11}}{11d^2}+\frac{b(c+dx)^{12}}{12d^2}\end{aligned}$$

Mathematica [B] time = 0.0276159, size = 220, normalized size = 5.79

$$\frac{5}{3}c^2d^7x^9(3ad+8bc)+\frac{15}{4}c^3d^6x^8(4ad+7bc)+6c^4d^5x^7(5ad+6bc)+7c^5d^4x^6(6ad+5bc)+6c^6d^3x^5(7ad+4bc)+\frac{15}{4}c^7d^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^10,x]
```

```
[Out] a*c^10*x + (c^9*(b*c + 10*a*d)*x^2)/2 + (5*c^8*d*(2*b*c + 9*a*d)*x^3)/3 + (
15*c^7*d^2*(3*b*c + 8*a*d)*x^4)/4 + 6*c^6*d^3*(4*b*c + 7*a*d)*x^5 + 7*c^5*d
^4*(5*b*c + 6*a*d)*x^6 + 6*c^4*d^5*(6*b*c + 5*a*d)*x^7 + (15*c^3*d^6*(7*b*c
 + 4*a*d)*x^8)/4 + (5*c^2*d^7*(8*b*c + 3*a*d)*x^9)/3 + (c*d^8*(9*b*c + 2*a*
d)*x^10)/2 + (d^9*(10*b*c + a*d)*x^11)/11 + (b*d^10*x^12)/12
```

Maple [B] time = 0.002, size = 241, normalized size = 6.3

$$\frac{bd^{10}x^{12}}{12}+\frac{\left(ad^{10}+10\,bcd^{9}\right)x^{11}}{11}+\frac{\left(10\,acd^{9}+45\,bc^{2}d^{8}\right)x^{10}}{10}+\frac{\left(45\,ac^{2}d^{8}+120\,bc^{3}d^{7}\right)x^{9}}{9}+\frac{\left(120\,ac^{3}d^{7}+210\,bc^{4}d^{6}\right)x^{8}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^10,x)
```

```
[Out] 1/12*b*d^10*x^12+1/11*(a*d^10+10*b*c*d^9)*x^11+1/10*(10*a*c*d^9+45*b*c^2*d^
8)*x^10+1/9*(45*a*c^2*d^8+120*b*c^3*d^7)*x^9+1/8*(120*a*c^3*d^7+210*b*c^4*d
^6)*x^8+1/7*(210*a*c^4*d^6+252*b*c^5*d^5)*x^7+1/6*(252*a*c^5*d^5+210*b*c^6*
d^4)*x^6+1/5*(210*a*c^6*d^4+120*b*c^7*d^3)*x^5+1/4*(120*a*c^7*d^3+45*b*c^8*
d^2)*x^4+1/3*(45*a*c^8*d^2+10*b*c^9*d)*x^3+1/2*(10*a*c^9*d+b*c^10)*x^2+a*c^
10*x
```

Maxima [B] time = 0.970548, size = 324, normalized size = 8.53

$$\frac{1}{12}\,bd^{10}x^{12}+ac^{10}x+\frac{1}{11}\left(10\,bcd^{9}+ad^{10}\right)x^{11}+\frac{1}{2}\left(9\,bc^{2}d^{8}+2\,acd^{9}\right)x^{10}+\frac{5}{3}\left(8\,bc^{3}d^{7}+3\,ac^{2}d^{8}\right)x^{9}+\frac{15}{4}\left(7\,bc^{4}d^{6}+4\,ac\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/12*b*d^10*x^12 + a*c^10*x + 1/11*(10*b*c*d^9 + a*d^10)*x^11 + 1/2*(9*b*c^
2*d^8 + 2*a*c*d^9)*x^10 + 5/3*(8*b*c^3*d^7 + 3*a*c^2*d^8)*x^9 + 15/4*(7*b*c
^4*d^6 + 4*a*c^3*d^7)*x^8 + 6*(6*b*c^5*d^5 + 5*a*c^4*d^6)*x^7 + 7*(5*b*c^6*
d^4 + 6*a*c^5*d^5)*x^6 + 6*(4*b*c^7*d^3 + 7*a*c^6*d^4)*x^5 + 15/4*(3*b*c^8*
d^2 + 8*a*c^7*d^3)*x^4 + 5/3*(2*b*c^9*d + 9*a*c^8*d^2)*x^3 + 1/2*(b*c^10 +
10*a*c^9*d)*x^2
```

Fricas [B] time = 1.51531, size = 554, normalized size = 14.58

$$\frac{1}{12}x^{12}d^{10}b+\frac{10}{11}x^{11}d^{9}cb+\frac{1}{11}x^{11}d^{10}a+\frac{9}{2}x^{10}d^{8}c^{2}b+x^{10}d^{9}ca+\frac{40}{3}x^{9}d^{7}c^{3}b+5x^{9}d^{8}c^{2}a+\frac{105}{4}x^{8}d^{6}c^{4}b+15x^{8}d^{7}c^{3}a+36$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*d^10*b + 10/11*x^11*d^9*c*b + 1/11*x^11*d^10*a + 9/2*x^10*d^8*c^2
*b + x^10*d^9*c*a + 40/3*x^9*d^7*c^3*b + 5*x^9*d^8*c^2*a + 105/4*x^8*d^6*c^
4*b + 15*x^8*d^7*c^3*a + 36*x^7*d^5*c^5*b + 30*x^7*d^6*c^4*a + 35*x^6*d^4*c
^6*b + 42*x^6*d^5*c^5*a + 24*x^5*d^3*c^7*b + 42*x^5*d^4*c^6*a + 45/4*x^4*d^
2*c^8*b + 30*x^4*d^3*c^7*a + 10/3*x^3*d*c^9*b + 15*x^3*d^2*c^8*a + 1/2*x^2*
c^10*b + 5*x^2*d*c^9*a + x*c^10*a
```

Sympy [B] time = 0.104946, size = 248, normalized size = 6.53

$$ac^{10}x+\frac{bd^{10}x^{12}}{12}+x^{11}\left(\frac{ad^{10}}{11}+\frac{10bcd^{9}}{11}\right)+x^{10}\left(acd^{9}+\frac{9bc^{2}d^{8}}{2}\right)+x^{9}\left(5ac^{2}d^{8}+\frac{40bc^{3}d^{7}}{3}\right)+x^{8}\left(15ac^{3}d^{7}+\frac{105bc^{4}d^{6}}{4}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**10,x)
```

```
[Out] a*c**10*x + b*d**10*x**12/12 + x**11*(a*d**10/11 + 10*b*c*d**9/11) + x**10*
(a*c*d**9 + 9*b*c**2*d**8/2) + x**9*(5*a*c**2*d**8 + 40*b*c**3*d**7/3) + x*
*8*(15*a*c**3*d**7 + 105*b*c**4*d**6/4) + x**7*(30*a*c**4*d**6 + 36*b*c**5*
d**5) + x**6*(42*a*c**5*d**5 + 35*b*c**6*d**4) + x**5*(42*a*c**6*d**4 + 24*
b*c**7*d**3) + x**4*(30*a*c**7*d**3 + 45*b*c**8*d**2/4) + x**3*(15*a*c**8*d
**2 + 10*b*c**9*d/3) + x**2*(5*a*c**9*d + b*c**10/2)
```

Giac [B] time = 1.07501, size = 325, normalized size = 8.55

$$\frac{1}{12}\,bd^{10}x^{12} + \frac{10}{11}\,bcd^{9}x^{11} + \frac{1}{11}\,ad^{10}x^{11} + \frac{9}{2}\,bc^{2}d^{8}x^{10} + acd^{9}x^{10} + \frac{40}{3}\,bc^{3}d^{7}x^{9} + 5\,ac^{2}d^{8}x^{9} + \frac{105}{4}\,bc^{4}d^{6}x^{8} + 15\,ac^{3}d^{7}x^{8} + 36$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/12*b*d^10*x^12 + 10/11*b*c*d^9*x^11 + 1/11*a*d^10*x^11 + 9/2*b*c^2*d^8*x^
10 + a*c*d^9*x^10 + 40/3*b*c^3*d^7*x^9 + 5*a*c^2*d^8*x^9 + 105/4*b*c^4*d^6*
x^8 + 15*a*c^3*d^7*x^8 + 36*b*c^5*d^5*x^7 + 30*a*c^4*d^6*x^7 + 35*b*c^6*d^4
*x^6 + 42*a*c^5*d^5*x^6 + 24*b*c^7*d^3*x^5 + 42*a*c^6*d^4*x^5 + 45/4*b*c^8*
d^2*x^4 + 30*a*c^7*d^3*x^4 + 10/3*b*c^9*d*x^3 + 15*a*c^8*d^2*x^3 + 1/2*b*c^
10*x^2 + 5*a*c^9*d*x^2 + a*c^10*x
```

3.1311 $\int (c+dx)^{10}\,dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^{11}}{11d}$$

```
[Out] (c + d*x)^11/(11*d)
```

Rubi [A] time = 0.0017762, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^{11}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10,x]
```

```
[Out] (c + d*x)^11/(11*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{10}\,dx = \frac{(c+dx)^{11}}{11d}$$

Mathematica [A] time = 0.0010003, size = 14, normalized size = 1.

$$\frac{(c+dx)^{11}}{11d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10,x]
```

```
[Out] (c + d*x)^11/(11*d)
```

Maple [A] time = 0., size = 13, normalized size = 0.9

$$\frac{(dx+c)^{11}}{11\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10,x)
```

```
[Out] 1/11*(d*x+c)^11/d
```

Maxima [A] time = 0.957012, size = 16, normalized size = 1.14

$$\frac{(dx+c)^{11}}{11\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/11*(d*x + c)^11/d
```

Fricas [B] time = 1.59102, size = 230, normalized size = 16.43

$$\frac{1}{11}x^{11}d^{10} + x^{10}d^9c + 5x^9d^8c^2 + 15x^8d^7c^3 + 30x^7d^6c^4 + 42x^6d^5c^5 + 42x^5d^4c^6 + 30x^4d^3c^7 + 15x^3d^2c^8 + 5x^2dc^9 + xc^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^10 + x^10*d^9*c + 5*x^9*d^8*c^2 + 15*x^8*d^7*c^3 + 30*x^7*d^6*c
^4 + 42*x^6*d^5*c^5 + 42*x^5*d^4*c^6 + 30*x^4*d^3*c^7 + 15*x^3*d^2*c^8 + 5*
x^2*d*c^9 + x*c^10
```

Sympy [B] time = 0.077456, size = 114, normalized size = 8.14

$$c^{10}x + 5c^9dx^2 + 15c^8d^2x^3 + 30c^7d^3x^4 + 42c^6d^4x^5 + 42c^5d^5x^6 + 30c^4d^6x^7 + 15c^3d^7x^8 + 5c^2d^8x^9 + cd^9x^{10} + \frac{d^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10,x)
```

```
[Out] c**10*x + 5*c**9*d*x**2 + 15*c**8*d**2*x**3 + 30*c**7*d**3*x**4 + 42*c**6*d
**4*x**5 + 42*c**5*d**5*x**6 + 30*c**4*d**6*x**7 + 15*c**3*d**7*x**8 + 5*c*
*2*d**8*x**9 + c*d**9*x**10 + d**10*x**11/11
```

Giac [A] time = 1.06738, size = 16, normalized size = 1.14

$$\frac{(dx+c)^{11}}{11\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/11*(d*x + c)^11/d
```

3.1312 $\int \frac{(c+dx)^{10}}{a+bx} \, dx$

Optimal. Leaf size=241

$$\frac{dx(bc-ad)^9}{b^{10}}+\frac{(c+dx)^2(bc-ad)^8}{2b^9}+\frac{(c+dx)^3(bc-ad)^7}{3b^8}+\frac{(c+dx)^4(bc-ad)^6}{4b^7}+\frac{(c+dx)^5(bc-ad)^5}{5b^6}+\frac{(c+dx)^6(bc-}{6b^5}$$

```
[Out] (d*(b*c - a*d)^9*x)/b^10 + ((b*c - a*d)^8*(c + d*x)^2)/(2*b^9) + ((b*c - a*
d)^7*(c + d*x)^3)/(3*b^8) + ((b*c - a*d)^6*(c + d*x)^4)/(4*b^7) + ((b*c - a
*d)^5*(c + d*x)^5)/(5*b^6) + ((b*c - a*d)^4*(c + d*x)^6)/(6*b^5) + ((b*c -
a*d)^3*(c + d*x)^7)/(7*b^4) + ((b*c - a*d)^2*(c + d*x)^8)/(8*b^3) + ((b*c -
 a*d)*(c + d*x)^9)/(9*b^2) + (c + d*x)^10/(10*b) + ((b*c - a*d)^10*Log[a +
b*x])/b^11
```

Rubi [A] time = 0.097776, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{dx(bc-ad)^9}{b^{10}}+\frac{(c+dx)^2(bc-ad)^8}{2b^9}+\frac{(c+dx)^3(bc-ad)^7}{3b^8}+\frac{(c+dx)^4(bc-ad)^6}{4b^7}+\frac{(c+dx)^5(bc-ad)^5}{5b^6}+\frac{(c+dx)^6(bc-}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^9*x)/b^10 + ((b*c - a*d)^8*(c + d*x)^2)/(2*b^9) + ((b*c - a*
d)^7*(c + d*x)^3)/(3*b^8) + ((b*c - a*d)^6*(c + d*x)^4)/(4*b^7) + ((b*c - a
*d)^5*(c + d*x)^5)/(5*b^6) + ((b*c - a*d)^4*(c + d*x)^6)/(6*b^5) + ((b*c -
a*d)^3*(c + d*x)^7)/(7*b^4) + ((b*c - a*d)^2*(c + d*x)^8)/(8*b^3) + ((b*c -
 a*d)*(c + d*x)^9)/(9*b^2) + (c + d*x)^10/(10*b) + ((b*c - a*d)^10*Log[a +
b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{a+bx}\,dx &= \int \left(\frac{d(bc-ad)^9}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)}+\frac{d(bc-ad)^8(c+dx)}{b^9}+\frac{d(bc-ad)^7(c+dx)^2}{b^8}+\frac{d(bc-ad)^6(c+dx)^3}{b^7}\right.\\ &= \frac{d(bc-ad)^9x}{b^{10}}+\frac{(bc-ad)^8(c+dx)^2}{2b^9}+\frac{(bc-ad)^7(c+dx)^3}{3b^8}+\frac{(bc-ad)^6(c+dx)^4}{4b^7}+\frac{(bc-ad)^5(c+d}{5b^6}\end{aligned}$$

Mathematica [B] time = 0.297758, size = 591, normalized size = 2.45

$$dx\left(45a^2b^7d^2\left(4704c^5d^2x^2+2940c^4d^3x^3+1344c^3d^4x^4+420c^2d^5x^5+5880c^6dx+6720c^7+80cd^6x^6+7d^7x^7\right)-120a$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x),x]
```

```
[Out] (d*x*(-2520*a^9*d^9 + 1260*a^8*b*d^8*(20*c + d*x) - 840*a^7*b^2*d^7*(135*c^
2 + 15*c*d*x + d^2*x^2) + 210*a^6*b^3*d^6*(1440*c^3 + 270*c^2*d*x + 40*c*d^
2*x^2 + 3*d^3*x^3) - 252*a^5*b^4*d^5*(2100*c^4 + 600*c^3*d*x + 150*c^2*d^2*
x^2 + 25*c*d^3*x^3 + 2*d^4*x^4) + 210*a^4*b^5*d^4*(3024*c^5 + 1260*c^4*d*x
+ 480*c^3*d^2*x^2 + 135*c^2*d^3*x^3 + 24*c*d^4*x^4 + 2*d^5*x^5) - 120*a^3*b
^6*d^3*(4410*c^6 + 2646*c^5*d*x + 1470*c^4*d^2*x^2 + 630*c^3*d^3*x^3 + 189*
c^2*d^4*x^4 + 35*c*d^5*x^5 + 3*d^6*x^6) + 45*a^2*b^7*d^2*(6720*c^7 + 5880*c
^6*d*x + 4704*c^5*d^2*x^2 + 2940*c^4*d^3*x^3 + 1344*c^3*d^4*x^4 + 420*c^2*d
^5*x^5 + 80*c*d^6*x^6 + 7*d^7*x^7) - 10*a*b^8*d*(11340*c^8 + 15120*c^7*d*x
+ 17640*c^6*d^2*x^2 + 15876*c^5*d^3*x^3 + 10584*c^4*d^4*x^4 + 5040*c^3*d^5*
x^5 + 1620*c^2*d^6*x^6 + 315*c*d^7*x^7 + 28*d^8*x^8) + b^9*(25200*c^9 + 567
00*c^8*d*x + 100800*c^7*d^2*x^2 + 132300*c^6*d^3*x^3 + 127008*c^5*d^4*x^4 +
 88200*c^4*d^5*x^5 + 43200*c^3*d^6*x^6 + 14175*c^2*d^7*x^7 + 2800*c*d^8*x^8
 + 252*d^9*x^9)))/(2520*b^10) + ((b*c - a*d)^10*Log[a + b*x])/b^11
```

Maple [B] time = 0.007, size = 1022, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a),x)
```

```
[Out] 1/8*d^10/b^3*x^8*a^2+45/8*d^8/b*x^8*c^2-1/7*d^10/b^4*x^7*a^3+120/7*d^7/b*x^
7*c^3+10*d/b*c^9*x-d^10/b^10*a^9*x+1/b^11*ln(b*x+a)*a^10*d^10+1/4*d^10/b^7*
x^4*a^6+105/2*d^4/b*x^4*c^6-1/3*d^10/b^8*x^3*a^7+40*d^3/b*x^3*c^7+1/2*d^10/
b^9*x^2*a^8+45/2*d^2/b*x^2*c^8+252/5*d^5/b*x^5*c^5+35*d^6/b*x^6*c^4-1/5*d^1
0/b^6*x^5*a^5+1/6*d^10/b^5*x^6*a^4-1/9*d^10/b^2*x^9*a+10/9*d^9/b*x^9*c-45*d
^8/b^8*a^7*c^2*x+120*d^7/b^7*a^6*c^3*x-45*d^2/b^2*a*c^8*x-60*d^3/b^2*x^2*a*
c^7-30*d^7/b^4*x^4*a^3*c^3+10*d^9/b^9*a^8*c*x+120*d^3/b^3*a^2*c^7*x-126*d^5
/b^4*x^2*a^3*c^5+105*d^4/b^3*x^2*a^2*c^6+45/2*d^8/b^7*x^2*a^6*c^2+45/4*d^8/
b^5*x^4*a^4*c^2+1/b*ln(b*x+a)*c^10+1/10*d^10/b*x^10-5/4*d^9/b^2*x^8*a*c-210
*d^6/b^6*a^5*c^4*x+252*d^5/b^5*a^4*c^5*x-210*d^4/b^4*a^3*c^6*x+105/2*d^6/b^
3*x^4*a^2*c^4-20*d^7/b^2*x^6*a*c^3-5/3*d^9/b^4*x^6*a^3*c+15/2*d^8/b^3*x^6*a
^2*c^2-9*d^8/b^4*x^5*a^3*c^2+10/7*d^9/b^3*x^7*a^2*c-45/7*d^8/b^2*x^7*a*c^2+
210/b^5*ln(b*x+a)*a^4*c^6*d^4-120/b^4*ln(b*x+a)*a^3*c^7*d^3+45/b^3*ln(b*x+a
)*a^2*c^8*d^2-10/b^2*ln(b*x+a)*a*c^9*d-60*d^7/b^6*x^2*a^5*c^3+105*d^6/b^5*x
^2*a^4*c^4-5*d^9/b^8*x^2*a^7*c+84*d^5/b^3*x^3*a^2*c^5-70*d^4/b^2*x^3*a*c^6-
5/2*d^9/b^6*x^4*a^5*c-63*d^5/b^2*x^4*a*c^5+10/3*d^9/b^7*x^3*a^6*c-15*d^8/b^
6*x^3*a^5*c^2+40*d^7/b^5*x^3*a^4*c^3-70*d^6/b^4*x^3*a^3*c^4+2*d^9/b^5*x^5*a
^4*c-42*d^6/b^2*x^5*a*c^4+24*d^7/b^3*x^5*a^2*c^3-252/b^6*ln(b*x+a)*a^5*c^5*
d^5-120/b^8*ln(b*x+a)*a^7*c^3*d^7+210/b^7*ln(b*x+a)*a^6*c^4*d^6+45/b^9*ln(b
*x+a)*a^8*c^2*d^8-10/b^10*ln(b*x+a)*a^9*c*d^9
```

Maxima [B] time = 0.978517, size = 1169, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2520*(252*b^9*d^10*x^10 + 280*(10*b^9*c*d^9 - a*b^8*d^10)*x^9 + 315*(45*b
^9*c^2*d^8 - 10*a*b^8*c*d^9 + a^2*b^7*d^10)*x^8 + 360*(120*b^9*c^3*d^7 - 45
*a*b^8*c^2*d^8 + 10*a^2*b^7*c*d^9 - a^3*b^6*d^10)*x^7 + 420*(210*b^9*c^4*d^
6 - 120*a*b^8*c^3*d^7 + 45*a^2*b^7*c^2*d^8 - 10*a^3*b^6*c*d^9 + a^4*b^5*d^1
0)*x^6 + 504*(252*b^9*c^5*d^5 - 210*a*b^8*c^4*d^6 + 120*a^2*b^7*c^3*d^7 - 4
5*a^3*b^6*c^2*d^8 + 10*a^4*b^5*c*d^9 - a^5*b^4*d^10)*x^5 + 630*(210*b^9*c^6
*d^4 - 252*a*b^8*c^5*d^5 + 210*a^2*b^7*c^4*d^6 - 120*a^3*b^6*c^3*d^7 + 45*a
^4*b^5*c^2*d^8 - 10*a^5*b^4*c*d^9 + a^6*b^3*d^10)*x^4 + 840*(120*b^9*c^7*d^
3 - 210*a*b^8*c^6*d^4 + 252*a^2*b^7*c^5*d^5 - 210*a^3*b^6*c^4*d^6 + 120*a^4
*b^5*c^3*d^7 - 45*a^5*b^4*c^2*d^8 + 10*a^6*b^3*c*d^9 - a^7*b^2*d^10)*x^3 +
1260*(45*b^9*c^8*d^2 - 120*a*b^8*c^7*d^3 + 210*a^2*b^7*c^6*d^4 - 252*a^3*b^
6*c^5*d^5 + 210*a^4*b^5*c^4*d^6 - 120*a^5*b^4*c^3*d^7 + 45*a^6*b^3*c^2*d^8
- 10*a^7*b^2*c*d^9 + a^8*b*d^10)*x^2 + 2520*(10*b^9*c^9*d - 45*a*b^8*c^8*d^
2 + 120*a^2*b^7*c^7*d^3 - 210*a^3*b^6*c^6*d^4 + 252*a^4*b^5*c^5*d^5 - 210*a
^5*b^4*c^4*d^6 + 120*a^6*b^3*c^3*d^7 - 45*a^7*b^2*c^2*d^8 + 10*a^8*b*c*d^9
- a^9*d^10)*x)/b^10 + (b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 12
0*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4
*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10
*d^10)*log(b*x + a)/b^11
```

Fricas [B] time = 1.76077, size = 1879, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2520*(252*b^10*d^10*x^10 + 280*(10*b^10*c*d^9 - a*b^9*d^10)*x^9 + 315*(45
*b^10*c^2*d^8 - 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 360*(120*b^10*c^3*d^7
- 45*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 420*(210*b^10*c
^4*d^6 - 120*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 - 10*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 504*(252*b^10*c^5*d^5 - 210*a*b^9*c^4*d^6 + 120*a^2*b^8*c^3*d
^7 - 45*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 630*(210*b
^10*c^6*d^4 - 252*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 - 120*a^3*b^7*c^3*d^7
 + 45*a^4*b^6*c^2*d^8 - 10*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 840*(120*b^1
0*c^7*d^3 - 210*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 - 210*a^3*b^7*c^4*d^6 +
 120*a^4*b^6*c^3*d^7 - 45*a^5*b^5*c^2*d^8 + 10*a^6*b^4*c*d^9 - a^7*b^3*d^10
)*x^3 + 1260*(45*b^10*c^8*d^2 - 120*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 - 2
52*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 120*a^5*b^5*c^3*d^7 + 45*a^6*b^4
*c^2*d^8 - 10*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 2520*(10*b^10*c^9*d - 45*
a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d^3 - 210*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5
*d^5 - 210*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 - 45*a^7*b^3*c^2*d^8 + 10*
a^8*b^2*c*d^9 - a^9*b*d^10)*x + 2520*(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b
^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^
5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9
*b*c*d^9 + a^10*d^10)*log(b*x + a))/b^11
```

Sympy [B] time = 1.43806, size = 772, normalized size = 3.2

$$\frac{d^{10}x^{10}}{10b} - \frac{x^9\left(ad^{10} - 10bcd^9\right)}{9b^2} + \frac{x^8\left(a^2d^{10} - 10abcd^9 + 45b^2c^2d^8\right)}{8b^3} - \frac{x^7\left(a^3d^{10} - 10a^2bcd^9 + 45ab^2c^2d^8 - 120b^3c^3d^7\right)}{7b^4} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a),x)
```

```
[Out] d**10*x**10/(10*b) - x**9*(a*d**10 - 10*b*c*d**9)/(9*b**2) + x**8*(a**2*d**
10 - 10*a*b*c*d**9 + 45*b**2*c**2*d**8)/(8*b**3) - x**7*(a**3*d**10 - 10*a*
*2*b*c*d**9 + 45*a*b**2*c**2*d**8 - 120*b**3*c**3*d**7)/(7*b**4) + x**6*(a*
*4*d**10 - 10*a**3*b*c*d**9 + 45*a**2*b**2*c**2*d**8 - 120*a*b**3*c**3*d**7
 + 210*b**4*c**4*d**6)/(6*b**5) - x**5*(a**5*d**10 - 10*a**4*b*c*d**9 + 45*
a**3*b**2*c**2*d**8 - 120*a**2*b**3*c**3*d**7 + 210*a*b**4*c**4*d**6 - 252*
b**5*c**5*d**5)/(5*b**6) + x**4*(a**6*d**10 - 10*a**5*b*c*d**9 + 45*a**4*b*
*2*c**2*d**8 - 120*a**3*b**3*c**3*d**7 + 210*a**2*b**4*c**4*d**6 - 252*a*b*
*5*c**5*d**5 + 210*b**6*c**6*d**4)/(4*b**7) - x**3*(a**7*d**10 - 10*a**6*b*
c*d**9 + 45*a**5*b**2*c**2*d**8 - 120*a**4*b**3*c**3*d**7 + 210*a**3*b**4*c
**4*d**6 - 252*a**2*b**5*c**5*d**5 + 210*a*b**6*c**6*d**4 - 120*b**7*c**7*d
**3)/(3*b**8) + x**2*(a**8*d**10 - 10*a**7*b*c*d**9 + 45*a**6*b**2*c**2*d**
8 - 120*a**5*b**3*c**3*d**7 + 210*a**4*b**4*c**4*d**6 - 252*a**3*b**5*c**5*
d**5 + 210*a**2*b**6*c**6*d**4 - 120*a*b**7*c**7*d**3 + 45*b**8*c**8*d**2)/
(2*b**9) - x*(a**9*d**10 - 10*a**8*b*c*d**9 + 45*a**7*b**2*c**2*d**8 - 120*
a**6*b**3*c**3*d**7 + 210*a**5*b**4*c**4*d**6 - 252*a**4*b**5*c**5*d**5 + 2
10*a**3*b**6*c**6*d**4 - 120*a**2*b**7*c**7*d**3 + 45*a*b**8*c**8*d**2 - 10
*b**9*c**9*d)/b**10 + (a*d - b*c)**10*log(a + b*x)/b**11
```

<u>Giac</u> [B] time = 1.05812, size = 1297, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2520*(252*b^9*d^10*x^10 + 2800*b^9*c*d^9*x^9 - 280*a*b^8*d^10*x^9 + 14175
*b^9*c^2*d^8*x^8 - 3150*a*b^8*c*d^9*x^8 + 315*a^2*b^7*d^10*x^8 + 43200*b^9*
c^3*d^7*x^7 - 16200*a*b^8*c^2*d^8*x^7 + 3600*a^2*b^7*c*d^9*x^7 - 360*a^3*b^
6*d^10*x^7 + 88200*b^9*c^4*d^6*x^6 - 50400*a*b^8*c^3*d^7*x^6 + 18900*a^2*b^
7*c^2*d^8*x^6 - 4200*a^3*b^6*c*d^9*x^6 + 420*a^4*b^5*d^10*x^6 + 127008*b^9*
c^5*d^5*x^5 - 105840*a*b^8*c^4*d^6*x^5 + 60480*a^2*b^7*c^3*d^7*x^5 - 22680*
a^3*b^6*c^2*d^8*x^5 + 5040*a^4*b^5*c*d^9*x^5 - 504*a^5*b^4*d^10*x^5 + 13230
0*b^9*c^6*d^4*x^4 - 158760*a*b^8*c^5*d^5*x^4 + 132300*a^2*b^7*c^4*d^6*x^4 -
 75600*a^3*b^6*c^3*d^7*x^4 + 28350*a^4*b^5*c^2*d^8*x^4 - 6300*a^5*b^4*c*d^9
*x^4 + 630*a^6*b^3*d^10*x^4 + 100800*b^9*c^7*d^3*x^3 - 176400*a*b^8*c^6*d^4
*x^3 + 211680*a^2*b^7*c^5*d^5*x^3 - 176400*a^3*b^6*c^4*d^6*x^3 + 100800*a^4
*b^5*c^3*d^7*x^3 - 37800*a^5*b^4*c^2*d^8*x^3 + 8400*a^6*b^3*c*d^9*x^3 - 840
*a^7*b^2*d^10*x^3 + 56700*b^9*c^8*d^2*x^2 - 151200*a*b^8*c^7*d^3*x^2 + 2646
00*a^2*b^7*c^6*d^4*x^2 - 317520*a^3*b^6*c^5*d^5*x^2 + 264600*a^4*b^5*c^4*d^
6*x^2 - 151200*a^5*b^4*c^3*d^7*x^2 + 56700*a^6*b^3*c^2*d^8*x^2 - 12600*a^7*
b^2*c*d^9*x^2 + 1260*a^8*b*d^10*x^2 + 25200*b^9*c^9*d*x - 113400*a*b^8*c^8*
d^2*x + 302400*a^2*b^7*c^7*d^3*x - 529200*a^3*b^6*c^6*d^4*x + 635040*a^4*b^
5*c^5*d^5*x - 529200*a^5*b^4*c^4*d^6*x + 302400*a^6*b^3*c^3*d^7*x - 113400*
a^7*b^2*c^2*d^8*x + 25200*a^8*b*c*d^9*x - 2520*a^9*d^10*x)/b^10 + (b^10*c^1
0 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6
*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7
+ 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)*log(abs(b*x + a))/b^11
```

3.1313 $\int \frac{(c+dx)^{10}}{(a+bx)^2}\,dx$

Optimal. Leaf size=258

$$\frac{5d^9(a+bx)^8(bc-ad)}{4b^{11}}+\frac{45d^8(a+bx)^7(bc-ad)^2}{7b^{11}}+\frac{20d^7(a+bx)^6(bc-ad)^3}{b^{11}}+\frac{42d^6(a+bx)^5(bc-ad)^4}{b^{11}}+\frac{63d^5(a+bx}{b}$$

```
[Out] (45*d^2*(b*c - a*d)^8*x)/b^10 - (b*c - a*d)^10/(b^11*(a + b*x)) + (60*d^3*(
b*c - a*d)^7*(a + b*x)^2)/b^11 + (70*d^4*(b*c - a*d)^6*(a + b*x)^3)/b^11 +
(63*d^5*(b*c - a*d)^5*(a + b*x)^4)/b^11 + (42*d^6*(b*c - a*d)^4*(a + b*x)^5
)/b^11 + (20*d^7*(b*c - a*d)^3*(a + b*x)^6)/b^11 + (45*d^8*(b*c - a*d)^2*(a
 + b*x)^7)/(7*b^11) + (5*d^9*(b*c - a*d)*(a + b*x)^8)/(4*b^11) + (d^10*(a +
 b*x)^9)/(9*b^11) + (10*d*(b*c - a*d)^9*Log[a + b*x])/b^11
```

Rubi [A] time = 0.472624, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{5d^9(a+bx)^8(bc-ad)}{4b^{11}}+\frac{45d^8(a+bx)^7(bc-ad)^2}{7b^{11}}+\frac{20d^7(a+bx)^6(bc-ad)^3}{b^{11}}+\frac{42d^6(a+bx)^5(bc-ad)^4}{b^{11}}+\frac{63d^5(a+bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^2,x]
```



```
[Out] (45*d^2*(b*c - a*d)^8*x)/b^10 - (b*c - a*d)^10/(b^11*(a + b*x)) + (60*d^3*(
b*c - a*d)^7*(a + b*x)^2)/b^11 + (70*d^4*(b*c - a*d)^6*(a + b*x)^3)/b^11 +
(63*d^5*(b*c - a*d)^5*(a + b*x)^4)/b^11 + (42*d^6*(b*c - a*d)^4*(a + b*x)^5
)/b^11 + (20*d^7*(b*c - a*d)^3*(a + b*x)^6)/b^11 + (45*d^8*(b*c - a*d)^2*(a
 + b*x)^7)/(7*b^11) + (5*d^9*(b*c - a*d)*(a + b*x)^8)/(4*b^11) + (d^10*(a +
 b*x)^9)/(9*b^11) + (10*d*(b*c - a*d)^9*Log[a + b*x])/b^11
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^2}\,dx &= \int \left(\frac{45d^2(bc-ad)^8}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^2}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)}+\frac{120d^3(bc-ad)^7(a+bx)}{b^{10}}+\frac{210d^4(bc-ad)^6(}{b^{10}}\right.\\ &= \frac{45d^2(bc-ad)^8x}{b^{10}}-\frac{(bc-ad)^{10}}{b^{11}(a+bx)}+\frac{60d^3(bc-ad)^7(a+bx)^2}{b^{11}}+\frac{70d^4(bc-ad)^6(a+bx)^3}{b^{11}}+\frac{63d^5(bc-}{b}\end{aligned}$$

Mathematica [B] time = 0.232005, size = 708, normalized size = 2.74

$$\frac{9a^2b^8d^2\left(11760c^6d^2x^2+5880c^5d^3x^3+2940c^4d^4x^4+1176c^3d^5x^5+336c^2d^6x^6-6720c^7dx-1260c^8+60cd^7x^7+5d^8x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^2,x]
```

```
[Out] (-252*a^10*d^10 + 252*a^9*b*d^9*(10*c + 9*d*x) + 1260*a^8*b^2*d^8*(-9*c^2 -
 16*c*d*x + d^2*x^2) - 420*a^7*b^3*d^7*(-72*c^3 - 189*c^2*d*x + 27*c*d^2*x^
2 + d^3*x^3) + 210*a^6*b^4*d^6*(-252*c^4 - 864*c^3*d*x + 216*c^2*d^2*x^2 +
18*c*d^3*x^3 + d^4*x^4) - 126*a^5*b^5*d^5*(-504*c^5 - 2100*c^4*d*x + 840*c^
3*d^2*x^2 + 120*c^2*d^3*x^3 + 15*c*d^4*x^4 + d^5*x^5) + 42*a^4*b^6*d^4*(-12
60*c^6 - 6048*c^5*d*x + 3780*c^4*d^2*x^2 + 840*c^3*d^3*x^3 + 180*c^2*d^4*x^
4 + 27*c*d^5*x^5 + 2*d^6*x^6) - 12*a^3*b^7*d^3*(-2520*c^7 - 13230*c^6*d*x +
 13230*c^5*d^2*x^2 + 4410*c^4*d^3*x^3 + 1470*c^3*d^4*x^4 + 378*c^2*d^5*x^5
+ 63*c*d^6*x^6 + 5*d^7*x^7) + 9*a^2*b^8*d^2*(-1260*c^8 - 6720*c^7*d*x + 117
60*c^6*d^2*x^2 + 5880*c^5*d^3*x^3 + 2940*c^4*d^4*x^4 + 1176*c^3*d^5*x^5 + 3
36*c^2*d^6*x^6 + 60*c*d^7*x^7 + 5*d^8*x^8) - a*b^9*d*(-2520*c^9 - 11340*c^8
*d*x + 45360*c^7*d^2*x^2 + 35280*c^6*d^3*x^3 + 26460*c^5*d^4*x^4 + 15876*c^
4*d^5*x^5 + 7056*c^3*d^6*x^6 + 2160*c^2*d^7*x^7 + 405*c*d^8*x^8 + 35*d^9*x^
9) + b^10*(-252*c^10 + 11340*c^8*d^2*x^2 + 15120*c^7*d^3*x^3 + 17640*c^6*d^
4*x^4 + 15876*c^5*d^5*x^5 + 10584*c^4*d^6*x^6 + 5040*c^3*d^7*x^7 + 1620*c^2
*d^8*x^8 + 315*c*d^9*x^9 + 28*d^10*x^10) - 2520*d*(-(b*c) + a*d)^9*(a + b*x
)*Log[a + b*x])/(252*b^11*(a + b*x))
```

Maple [B] time = 0.014, size = 1066, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^2,x)
```

```
[Out] -1/b/(b*x+a)*c^10-1/b^11/(b*x+a)*a^10*d^10-10/b^11*d^10*ln(b*x+a)*a^9+10/b^
2*d*ln(b*x+a)*c^9+9*d^10/b^10*a^8*x+70*d^4/b^2*x^3*c^6-4*d^10/b^9*x^2*a^7+6
0*d^3/b^2*x^2*c^7+5/4*d^9/b^2*x^8*c+3/7*d^10/b^4*x^7*a^2+45/7*d^8/b^2*x^7*c
^2-2/3*d^10/b^5*x^6*a^3+20*d^7/b^2*x^6*c^3+42*d^6/b^2*x^5*c^4-3/2*d^10/b^7*
x^4*a^5+63*d^5/b^2*x^4*c^5+7/3*d^10/b^8*x^3*a^6-1/4*d^10/b^3*x^8*a+45*d^2/b
^2*c^8*x+d^10/b^6*x^5*a^4-8*d^9/b^5*x^5*a^3*c-20/7*d^9/b^3*x^7*a*c+5*d^9/b^
4*x^6*a^2*c-15*d^8/b^3*x^6*a*c^2+10/b^10/(b*x+a)*a^9*c*d^9-45/b^9/(b*x+a)*a
^8*c^2*d^8+120/b^8/(b*x+a)*a^7*c^3*d^7-210/b^7/(b*x+a)*a^6*c^4*d^6+252/b^6/
(b*x+a)*a^5*c^5*d^5-210/b^5/(b*x+a)*a^4*c^6*d^4+120/b^4/(b*x+a)*a^3*c^7*d^3
-45/b^3/(b*x+a)*a^2*c^8*d^2-168*d^5/b^3*x^3*a*c^5+35*d^9/b^8*x^2*a^6*c+315*
d^8/b^8*a^6*c^2*x-720*d^7/b^7*a^5*c^3*x+1050*d^6/b^6*a^4*c^4*x-1008*d^5/b^5
*a^3*c^5*x+630*d^4/b^4*a^2*c^6*x-240*d^3/b^3*a*c^7*x+210*d^6/b^4*x^3*a^2*c^
4-20*d^9/b^7*x^3*a^5*c+75*d^8/b^6*x^3*a^4*c^2-160*d^7/b^5*x^3*a^3*c^3+90*d^
7/b^4*x^4*a^2*c^3-105*d^6/b^3*x^4*a*c^4-48*d^7/b^3*x^5*a*c^3+25/2*d^9/b^6*x
^4*a^4*c-45*d^8/b^5*x^4*a^3*c^2+27*d^8/b^4*x^5*a^2*c^2-360/b^9*d^8*ln(b*x+a
)*a^7*c^2+840/b^8*d^7*ln(b*x+a)*a^6*c^3-1260/b^7*d^6*ln(b*x+a)*a^5*c^4+1260
/b^6*d^5*ln(b*x+a)*a^4*c^5-840/b^5*d^4*ln(b*x+a)*a^3*c^6+360/b^4*d^3*ln(b*x
+a)*a^2*c^7-90/b^3*d^2*ln(b*x+a)*a*c^8-135*d^8/b^7*x^2*a^5*c^2+300*d^7/b^6*
x^2*a^4*c^3-420*d^6/b^5*x^2*a^3*c^4+378*d^5/b^4*x^2*a^2*c^5-210*d^4/b^3*x^2
*a*c^6-80*d^9/b^9*a^7*c*x+1/9*d^10/b^2*x^9+10/b^2/(b*x+a)*a*c^9*d+90/b^10*d
^9*ln(b*x+a)*a^8*c
```

Maxima [B] time = 0.985015, size = 1180, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 2
10*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^
3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)/(b^12*x + a*b^
11) + 1/252*(28*b^8*d^10*x^9 + 63*(5*b^8*c*d^9 - a*b^7*d^10)*x^8 + 36*(45*b
^8*c^2*d^8 - 20*a*b^7*c*d^9 + 3*a^2*b^6*d^10)*x^7 + 84*(60*b^8*c^3*d^7 - 45
*a*b^7*c^2*d^8 + 15*a^2*b^6*c*d^9 - 2*a^3*b^5*d^10)*x^6 + 252*(42*b^8*c^4*d
^6 - 48*a*b^7*c^3*d^7 + 27*a^2*b^6*c^2*d^8 - 8*a^3*b^5*c*d^9 + a^4*b^4*d^10
)*x^5 + 126*(126*b^8*c^5*d^5 - 210*a*b^7*c^4*d^6 + 180*a^2*b^6*c^3*d^7 - 90
*a^3*b^5*c^2*d^8 + 25*a^4*b^4*c*d^9 - 3*a^5*b^3*d^10)*x^4 + 84*(210*b^8*c^6
*d^4 - 504*a*b^7*c^5*d^5 + 630*a^2*b^6*c^4*d^6 - 480*a^3*b^5*c^3*d^7 + 225*
a^4*b^4*c^2*d^8 - 60*a^5*b^3*c*d^9 + 7*a^6*b^2*d^10)*x^3 + 252*(60*b^8*c^7*
d^3 - 210*a*b^7*c^6*d^4 + 378*a^2*b^6*c^5*d^5 - 420*a^3*b^5*c^4*d^6 + 300*a
^4*b^4*c^3*d^7 - 135*a^5*b^3*c^2*d^8 + 35*a^6*b^2*c*d^9 - 4*a^7*b*d^10)*x^2
 + 252*(45*b^8*c^8*d^2 - 240*a*b^7*c^7*d^3 + 630*a^2*b^6*c^6*d^4 - 1008*a^3
*b^5*c^5*d^5 + 1050*a^4*b^4*c^4*d^6 - 720*a^5*b^3*c^3*d^7 + 315*a^6*b^2*c^2
*d^8 - 80*a^7*b*c*d^9 + 9*a^8*d^10)*x)/b^10 + 10*(b^9*c^9*d - 9*a*b^8*c^8*d
^2 + 36*a^2*b^7*c^7*d^3 - 84*a^3*b^6*c^6*d^4 + 126*a^4*b^5*c^5*d^5 - 126*a^
5*b^4*c^4*d^6 + 84*a^6*b^3*c^3*d^7 - 36*a^7*b^2*c^2*d^8 + 9*a^8*b*c*d^9 - a
^9*d^10)*log(b*x + a)/b^11
```

Fricas [B] time = 1.95732, size = 2402, normalized size = 9.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^10*d^10*x^10 - 252*b^10*c^10 + 2520*a*b^9*c^9*d - 11340*a^2*b^8
*c^8*d^2 + 30240*a^3*b^7*c^7*d^3 - 52920*a^4*b^6*c^6*d^4 + 63504*a^5*b^5*c^
5*d^5 - 52920*a^6*b^4*c^4*d^6 + 30240*a^7*b^3*c^3*d^7 - 11340*a^8*b^2*c^2*d
^8 + 2520*a^9*b*c*d^9 - 252*a^10*d^10 + 35*(9*b^10*c*d^9 - a*b^9*d^10)*x^9
+ 45*(36*b^10*c^2*d^8 - 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 60*(84*b^10*c^3
*d^7 - 36*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 84*(126*b^1
0*c^4*d^6 - 84*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 - 9*a^3*b^7*c*d^9 + a^4*b
^6*d^10)*x^6 + 126*(126*b^10*c^5*d^5 - 126*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 - 36*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 210*(84*b^1
0*c^6*d^4 - 126*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 - 84*a^3*b^7*c^3*d^7 +
36*a^4*b^6*c^2*d^8 - 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 420*(36*b^10*c^7
*d^3 - 84*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 - 126*a^3*b^7*c^4*d^6 + 84*a^
4*b^6*c^3*d^7 - 36*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 +
1260*(9*b^10*c^8*d^2 - 36*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 - 126*a^3*b^7*
c^5*d^5 + 126*a^4*b^6*c^4*d^6 - 84*a^5*b^5*c^3*d^7 + 36*a^6*b^4*c^2*d^8 - 9
*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 252*(45*a*b^9*c^8*d^2 - 240*a^2*b^8*c^
7*d^3 + 630*a^3*b^7*c^6*d^4 - 1008*a^4*b^6*c^5*d^5 + 1050*a^5*b^5*c^4*d^6 -
 720*a^6*b^4*c^3*d^7 + 315*a^7*b^3*c^2*d^8 - 80*a^8*b^2*c*d^9 + 9*a^9*b*d^1
0)*x + 2520*(a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 + 36*a^3*b^7*c^7*d^3 - 84*a^4*
b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 - 126*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^
7 - 36*a^8*b^2*c^2*d^8 + 9*a^9*b*c*d^9 - a^10*d^10 + (b^10*c^9*d - 9*a*b^9*
c^8*d^2 + 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 - 1
26*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*
d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^12*x + a*b^11)
```

Sympy [B] time = 2.83658, size = 796, normalized size = 3.09

$$-\frac{a^{10}d^{10} - 10a^9bcd^9 + 45a^8b^2c^2d^8 - 120a^7b^3c^3d^7 + 210a^6b^4c^4d^6 - 252a^5b^5c^5d^5 + 210a^4b^6c^6d^4 - 120a^3b^7c^7d^3 + 45a^2b^8c^8d}{ab^{11} + b^{12}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**2,x)
```

```
[Out] -(a**10*d**10 - 10*a**9*b*c*d**9 + 45*a**8*b**2*c**2*d**8 - 120*a**7*b**3*c
**3*d**7 + 210*a**6*b**4*c**4*d**6 - 252*a**5*b**5*c**5*d**5 + 210*a**4*b**
6*c**6*d**4 - 120*a**3*b**7*c**7*d**3 + 45*a**2*b**8*c**8*d**2 - 10*a*b**9*
c**9*d + b**10*c**10)/(a*b**11 + b**12*x) + d**10*x**9/(9*b**2) - x**8*(a*d
**10 - 5*b*c*d**9)/(4*b**3) + x**7*(3*a**2*d**10 - 20*a*b*c*d**9 + 45*b**2*
c**2*d**8)/(7*b**4) - x**6*(2*a**3*d**10 - 15*a**2*b*c*d**9 + 45*a*b**2*c**
2*d**8 - 60*b**3*c**3*d**7)/(3*b**5) + x**5*(a**4*d**10 - 8*a**3*b*c*d**9 +
 27*a**2*b**2*c**2*d**8 - 48*a*b**3*c**3*d**7 + 42*b**4*c**4*d**6)/b**6 - x
**4*(3*a**5*d**10 - 25*a**4*b*c*d**9 + 90*a**3*b**2*c**2*d**8 - 180*a**2*b*
*3*c**3*d**7 + 210*a*b**4*c**4*d**6 - 126*b**5*c**5*d**5)/(2*b**7) + x**3*(
7*a**6*d**10 - 60*a**5*b*c*d**9 + 225*a**4*b**2*c**2*d**8 - 480*a**3*b**3*c
**3*d**7 + 630*a**2*b**4*c**4*d**6 - 504*a*b**5*c**5*d**5 + 210*b**6*c**6*d
**4)/(3*b**8) - x**2*(4*a**7*d**10 - 35*a**6*b*c*d**9 + 135*a**5*b**2*c**2*
d**8 - 300*a**4*b**3*c**3*d**7 + 420*a**3*b**4*c**4*d**6 - 378*a**2*b**5*c*
*5*d**5 + 210*a*b**6*c**6*d**4 - 60*b**7*c**7*d**3)/b**9 + x*(9*a**8*d**10
- 80*a**7*b*c*d**9 + 315*a**6*b**2*c**2*d**8 - 720*a**5*b**3*c**3*d**7 + 10
50*a**4*b**4*c**4*d**6 - 1008*a**3*b**5*c**5*d**5 + 630*a**2*b**6*c**6*d**4
 - 240*a*b**7*c**7*d**3 + 45*b**8*c**8*d**2)/b**10 - 10*d*(a*d - b*c)**9*lo
g(a + b*x)/b**11
```

Giac [B] time = 1.08716, size = 1366, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/252*(28*d^10 + 315*(b^2*c*d^9 - a*b*d^10)/((b*x + a)*b) + 1620*(b^4*c^2*d
^8 - 2*a*b^3*c*d^9 + a^2*b^2*d^10)/((b*x + a)^2*b^2) + 5040*(b^6*c^3*d^7 -
3*a*b^5*c^2*d^8 + 3*a^2*b^4*c*d^9 - a^3*b^3*d^10)/((b*x + a)^3*b^3) + 10584
*(b^8*c^4*d^6 - 4*a*b^7*c^3*d^7 + 6*a^2*b^6*c^2*d^8 - 4*a^3*b^5*c*d^9 + a^4
*b^4*d^10)/((b*x + a)^4*b^4) + 15876*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a
^2*b^8*c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)/((b*x
 + a)^5*b^5) + 17640*(b^12*c^6*d^4 - 6*a*b^11*c^5*d^5 + 15*a^2*b^10*c^4*d^6
 - 20*a^3*b^9*c^3*d^7 + 15*a^4*b^8*c^2*d^8 - 6*a^5*b^7*c*d^9 + a^6*b^6*d^10
)/((b*x + a)^6*b^6) + 15120*(b^14*c^7*d^3 - 7*a*b^13*c^6*d^4 + 21*a^2*b^12*
c^5*d^5 - 35*a^3*b^11*c^4*d^6 + 35*a^4*b^10*c^3*d^7 - 21*a^5*b^9*c^2*d^8 +
7*a^6*b^8*c*d^9 - a^7*b^7*d^10)/((b*x + a)^7*b^7) + 11340*(b^16*c^8*d^2 - 8
*a*b^15*c^7*d^3 + 28*a^2*b^14*c^6*d^4 - 56*a^3*b^13*c^5*d^5 + 70*a^4*b^12*c
^4*d^6 - 56*a^5*b^11*c^3*d^7 + 28*a^6*b^10*c^2*d^8 - 8*a^7*b^9*c*d^9 + a^8*
b^8*d^10)/((b*x + a)^8*b^8))*(b*x + a)^9/b^11 - 10*(b^9*c^9*d - 9*a*b^8*c^8
*d^2 + 36*a^2*b^7*c^7*d^3 - 84*a^3*b^6*c^6*d^4 + 126*a^4*b^5*c^5*d^5 - 126*
a^5*b^4*c^4*d^6 + 84*a^6*b^3*c^3*d^7 - 36*a^7*b^2*c^2*d^8 + 9*a^8*b*c*d^9 -
 a^9*d^10)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^11 - (b^19*c^10/(b*x +
a) - 10*a*b^18*c^9*d/(b*x + a) + 45*a^2*b^17*c^8*d^2/(b*x + a) - 120*a^3*b^
16*c^7*d^3/(b*x + a) + 210*a^4*b^15*c^6*d^4/(b*x + a) - 252*a^5*b^14*c^5*d^
5/(b*x + a) + 210*a^6*b^13*c^4*d^6/(b*x + a) - 120*a^7*b^12*c^3*d^7/(b*x +
```

```
a) + 45*a^8*b^11*c^2*d^8/(b*x + a) - 10*a^9*b^10*c*d^9/(b*x + a) + a^10*b^9
*d^10/(b*x + a))/b^20
```

3.1314 $\int \frac{(c+dx)^{10}}{(a+bx)^3}\,dx$

Optimal. Leaf size=262

$$\frac{10d^9(a+bx)^7(bc-ad)}{7b^{11}}+\frac{15d^8(a+bx)^6(bc-ad)^2}{2b^{11}}+\frac{24d^7(a+bx)^5(bc-ad)^3}{b^{11}}+\frac{105d^6(a+bx)^4(bc-ad)^4}{2b^{11}}+\frac{84d^5(a+bx)}{b^{11}}$$

```
[Out] (120*d^3*(b*c - a*d)^7*x)/b^10 - (b*c - a*d)^10/(2*b^11*(a + b*x)^2) - (10*
d*(b*c - a*d)^9)/(b^11*(a + b*x)) + (105*d^4*(b*c - a*d)^6*(a + b*x)^2)/b^1
1 + (84*d^5*(b*c - a*d)^5*(a + b*x)^3)/b^11 + (105*d^6*(b*c - a*d)^4*(a + b
*x)^4)/(2*b^11) + (24*d^7*(b*c - a*d)^3*(a + b*x)^5)/b^11 + (15*d^8*(b*c -
a*d)^2*(a + b*x)^6)/(2*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^7)/(7*b^11) +
(d^10*(a + b*x)^8)/(8*b^11) + (45*d^2*(b*c - a*d)^8*Log[a + b*x])/b^11
```

Rubi [A] time = 0.444295, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{10d^9(a+bx)^7(bc-ad)}{7b^{11}}+\frac{15d^8(a+bx)^6(bc-ad)^2}{2b^{11}}+\frac{24d^7(a+bx)^5(bc-ad)^3}{b^{11}}+\frac{105d^6(a+bx)^4(bc-ad)^4}{2b^{11}}+\frac{84d^5(a+bx)}{b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^3,x]
```

```
[Out] (120*d^3*(b*c - a*d)^7*x)/b^10 - (b*c - a*d)^10/(2*b^11*(a + b*x)^2) - (10*
d*(b*c - a*d)^9)/(b^11*(a + b*x)) + (105*d^4*(b*c - a*d)^6*(a + b*x)^2)/b^1
1 + (84*d^5*(b*c - a*d)^5*(a + b*x)^3)/b^11 + (105*d^6*(b*c - a*d)^4*(a + b
*x)^4)/(2*b^11) + (24*d^7*(b*c - a*d)^3*(a + b*x)^5)/b^11 + (15*d^8*(b*c -
a*d)^2*(a + b*x)^6)/(2*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^7)/(7*b^11) +
(d^10*(a + b*x)^8)/(8*b^11) + (45*d^2*(b*c - a*d)^8*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^3}\,dx &= \int \left(\frac{120d^3(bc-ad)^7}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^3}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^2}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)}+\frac{210d^4(bc-ad)^6(a+bx)}{b^{10}}+\right.\\ &= \frac{120d^3(bc-ad)^7x}{b^{10}}-\frac{(bc-ad)^{10}}{2b^{11}(a+bx)^2}-\frac{10d(bc-ad)^9}{b^{11}(a+bx)}+\frac{105d^4(bc-ad)^6(a+bx)^2}{b^{11}}+\frac{84d^5(bc-ad)^5(a+}{b^{11}}\end{aligned}$$

Mathematica [B] time = 0.235511, size = 708, normalized size = 2.7

$$3a^2b^8d^2\left(-21560c^6d^2x^2+15680c^5d^3x^3+4900c^4d^4x^4+1568c^3d^5x^5+392c^2d^6x^6-4480c^7dx+1260c^8+64cd^7x^7+5d^8x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^3,x]
```

```
[Out] (532*a^10*d^10 - 56*a^9*b*d^9*(85*c + 26*d*x) + 28*a^8*b^2*d^8*(675*c^2 + 3
80*c*d*x - 116*d^2*x^2) - 280*a^7*b^3*d^7*(156*c^3 + 117*c^2*d*x - 91*c*d^2
*x^2 + 3*d^3*x^3) + 210*a^6*b^4*d^6*(308*c^4 + 256*c^3*d*x - 414*c^2*d^2*x^
2 + 32*c*d^3*x^3 + d^4*x^4) - 84*a^5*b^5*d^5*(756*c^5 + 560*c^4*d*x - 2000*
c^3*d^2*x^2 + 280*c^2*d^3*x^3 + 20*c*d^4*x^4 + d^5*x^5) + 42*a^4*b^6*d^4*(9
80*c^6 + 336*c^5*d*x - 4760*c^4*d^2*x^2 + 1120*c^3*d^3*x^3 + 140*c^2*d^4*x^
4 + 16*c*d^5*x^5 + d^6*x^6) - 24*a^3*b^7*d^3*(700*c^7 - 490*c^6*d*x - 6174*
c^5*d^2*x^2 + 2450*c^4*d^3*x^3 + 490*c^3*d^4*x^4 + 98*c^2*d^5*x^5 + 14*c*d^
6*x^6 + d^7*x^7) + 3*a^2*b^8*d^2*(1260*c^8 - 4480*c^7*d*x - 21560*c^6*d^2*x
^2 + 15680*c^5*d^3*x^3 + 4900*c^4*d^4*x^4 + 1568*c^3*d^5*x^5 + 392*c^2*d^6*
x^6 + 64*c*d^7*x^7 + 5*d^8*x^8) - 2*a*b^9*d*(140*c^9 - 2520*c^8*d*x - 6720*
c^7*d^2*x^2 + 11760*c^6*d^3*x^3 + 5880*c^5*d^4*x^4 + 2940*c^4*d^5*x^5 + 117
6*c^3*d^6*x^6 + 336*c^2*d^7*x^7 + 60*c*d^8*x^8 + 5*d^9*x^9) + b^10*(-28*c^1
0 - 560*c^9*d*x + 6720*c^7*d^3*x^3 + 5880*c^6*d^4*x^4 + 4704*c^5*d^5*x^5 +
2940*c^4*d^6*x^6 + 1344*c^3*d^7*x^7 + 420*c^2*d^8*x^8 + 80*c*d^9*x^9 + 7*d^
10*x^10) + 2520*d^2*(b*c - a*d)^8*(a + b*x)^2*Log[a + b*x])/(56*b^11*(a + b
*x)^2)
```

Maple [B] time = 0.016, size = 1105, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^3,x)
```

```
[Out] -2520/b^8*d^7*ln(b*x+a)*a^5*c^3+3150/b^7*d^6*ln(b*x+a)*a^4*c^4-2520/b^6*d^5
*ln(b*x+a)*a^3*c^5+1260/b^5*d^4*ln(b*x+a)*a^2*c^6-360/b^4*d^3*ln(b*x+a)*a*c
^7-45/2/b^9/(b*x+a)^2*a^8*c^2*d^8+60/b^8/(b*x+a)^2*a^7*c^3*d^7-105/b^7/(b*x
+a)^2*a^6*c^4*d^6+126/b^6/(b*x+a)^2*a^5*c^5*d^5-105/b^5/(b*x+a)^2*a^4*c^6*d
^4+60/b^4/(b*x+a)^2*a^3*c^7*d^3-45/2/b^3/(b*x+a)^2*a^2*c^8*d^2+5/b^2/(b*x+a
)^2*a*c^9*d-1260/b^6*d^5/(b*x+a)*a^4*c^5+840/b^5*d^4/(b*x+a)*a^3*c^6-360/b^
4*d^3/(b*x+a)*a^2*c^7+90/b^3*d^2/(b*x+a)*a*c^8+5/b^10/(b*x+a)^2*a^9*c*d^9-6
30*d^4/b^4*a*c^6*x-150*d^8/b^6*x^3*a^3*c^2+240*d^7/b^5*x^3*a^2*c^3-210*d^6/
b^4*x^3*a*c^4-105*d^9/b^8*x^2*a^5*c+675/2*d^8/b^7*x^2*a^4*c^2+1260/b^9*d^8*
ln(b*x+a)*a^6*c^2-360/b^10*d^9*ln(b*x+a)*a^7*c-3/7*d^10/b^4*x^7*a+10/7*d^9/
b^3*x^7*c+15/2*d^8/b^3*x^6*c^2-2*d^10/b^6*x^5*a^3+120*d^3/b^3*c^7*x-36*d^10
/b^10*a^7*x+d^10/b^5*x^6*a^2+10/b^11*d^10/(b*x+a)*a^9-10/b^2*d/(b*x+a)*c^9-
1/2/b^11/(b*x+a)^2*a^10*d^10+45/b^11*d^10*ln(b*x+a)*a^8+45/b^3*d^2*ln(b*x+a
)*c^8+24*d^7/b^3*x^5*c^3+15/4*d^10/b^7*x^4*a^4+105/2*d^6/b^3*x^4*c^4-7*d^10
/b^8*x^3*a^5+84*d^5/b^3*x^3*c^5+14*d^10/b^9*x^2*a^6+105*d^4/b^3*x^2*c^6-1/2
/b/(b*x+a)^2*c^10+1/8*d^10/b^3*x^8-600*d^7/b^6*x^2*a^3*c^3+630*d^6/b^5*x^2*
a^2*c^4-378*d^5/b^4*x^2*a*c^5+280*d^9/b^9*a^6*c*x-90/b^10*d^9/(b*x+a)*a^8*c
+360/b^9*d^8/(b*x+a)*a^7*c^2-840/b^8*d^7/(b*x+a)*a^6*c^3+1260/b^7*d^6/(b*x+
a)*a^5*c^4-27*d^8/b^4*x^5*a*c^2-25*d^9/b^6*x^4*a^3*c+135/2*d^8/b^5*x^4*a^2*
c^2-90*d^7/b^4*x^4*a*c^3+50*d^9/b^7*x^3*a^4*c-945*d^8/b^8*a^5*c^2*x+1800*d^
7/b^7*a^4*c^3*x-2100*d^6/b^6*a^3*c^4*x+1512*d^5/b^5*a^2*c^5*x+12*d^9/b^5*x^
5*a^2*c-5*d^9/b^4*x^6*a*c
```

Maxima [B] time = 1.02342, size = 1189, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^10*c^10 + 10*a*b^9*c^9*d - 135*a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^
3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^5*b^5*c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 15
60*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c^2*d^8 + 170*a^9*b*c*d^9 - 19*a^10*d^10 +
 20*(b^10*c^9*d - 9*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4
 + 126*a^4*b^6*c^5*d^5 - 126*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*
b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 - a^9*b*d^10)*x)/(b^13*x^2 + 2*a*b^12*x + a^2
*b^11) + 1/56*(7*b^7*d^10*x^8 + 8*(10*b^7*c*d^9 - 3*a*b^6*d^10)*x^7 + 28*(1
5*b^7*c^2*d^8 - 10*a*b^6*c*d^9 + 2*a^2*b^5*d^10)*x^6 + 56*(24*b^7*c^3*d^7 -
 27*a*b^6*c^2*d^8 + 12*a^2*b^5*c*d^9 - 2*a^3*b^4*d^10)*x^5 + 70*(42*b^7*c^4
*d^6 - 72*a*b^6*c^3*d^7 + 54*a^2*b^5*c^2*d^8 - 20*a^3*b^4*c*d^9 + 3*a^4*b^3
*d^10)*x^4 + 56*(84*b^7*c^5*d^5 - 210*a*b^6*c^4*d^6 + 240*a^2*b^5*c^3*d^7 -
 150*a^3*b^4*c^2*d^8 + 50*a^4*b^3*c*d^9 - 7*a^5*b^2*d^10)*x^3 + 28*(210*b^7
*c^6*d^4 - 756*a*b^6*c^5*d^5 + 1260*a^2*b^5*c^4*d^6 - 1200*a^3*b^4*c^3*d^7
+ 675*a^4*b^3*c^2*d^8 - 210*a^5*b^2*c*d^9 + 28*a^6*b*d^10)*x^2 + 56*(120*b^
7*c^7*d^3 - 630*a*b^6*c^6*d^4 + 1512*a^2*b^5*c^5*d^5 - 2100*a^3*b^4*c^4*d^6
 + 1800*a^4*b^3*c^3*d^7 - 945*a^5*b^2*c^2*d^8 + 280*a^6*b*c*d^9 - 36*a^7*d^
10)*x)/b^10 + 45*(b^8*c^8*d^2 - 8*a*b^7*c^7*d^3 + 28*a^2*b^6*c^6*d^4 - 56*a
^3*b^5*c^5*d^5 + 70*a^4*b^4*c^4*d^6 - 56*a^5*b^3*c^3*d^7 + 28*a^6*b^2*c^2*d
^8 - 8*a^7*b*c*d^9 + a^8*d^10)*log(b*x + a)/b^11
```

Fricas [B] time = 1.94856, size = 2612, normalized size = 9.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^10*d^10*x^10 - 28*b^10*c^10 - 280*a*b^9*c^9*d + 3780*a^2*b^8*c^8*
d^2 - 16800*a^3*b^7*c^7*d^3 + 41160*a^4*b^6*c^6*d^4 - 63504*a^5*b^5*c^5*d^5
 + 64680*a^6*b^4*c^4*d^6 - 43680*a^7*b^3*c^3*d^7 + 18900*a^8*b^2*c^2*d^8 -
4760*a^9*b*c*d^9 + 532*a^10*d^10 + 10*(8*b^10*c*d^9 - a*b^9*d^10)*x^9 + 15*
(28*b^10*c^2*d^8 - 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 24*(56*b^10*c^3*d^7
- 28*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 42*(70*b^10*c^4*
d^6 - 56*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 - 8*a^3*b^7*c*d^9 + a^4*b^6*d^1
0)*x^6 + 84*(56*b^10*c^5*d^5 - 70*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 - 28*a
^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 210*(28*b^10*c^6*d^4
 - 56*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 - 56*a^3*b^7*c^3*d^7 + 28*a^4*b^6*
c^2*d^8 - 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 840*(8*b^10*c^7*d^3 - 28*a*
b^9*c^6*d^4 + 56*a^2*b^8*c^5*d^5 - 70*a^3*b^7*c^4*d^6 + 56*a^4*b^6*c^3*d^7
- 28*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 28*(480*a*b^9*
c^7*d^3 - 2310*a^2*b^8*c^6*d^4 + 5292*a^3*b^7*c^5*d^5 - 7140*a^4*b^6*c^4*d^
6 + 6000*a^5*b^5*c^3*d^7 - 3105*a^6*b^4*c^2*d^8 + 910*a^7*b^3*c*d^9 - 116*a
^8*b^2*d^10)*x^2 - 56*(10*b^10*c^9*d - 90*a*b^9*c^8*d^2 + 240*a^2*b^8*c^7*d
^3 - 210*a^3*b^7*c^6*d^4 - 252*a^4*b^6*c^5*d^5 + 840*a^5*b^5*c^4*d^6 - 960*
a^6*b^4*c^3*d^7 + 585*a^7*b^3*c^2*d^8 - 190*a^8*b^2*c*d^9 + 26*a^9*b*d^10)*
x + 2520*(a^2*b^8*c^8*d^2 - 8*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 - 56*a^5
*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 - 56*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8
 - 8*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8
*c^6*d^4 - 56*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 2
8*a^6*b^4*c^2*d^8 - 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 2*(a*b^9*c^8*d^2
- 8*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^4 - 56*a^4*b^6*c^5*d^5 + 70*a^5*b^5*
c^4*d^6 - 56*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 - 8*a^8*b^2*c*d^9 + a^9*b
```

```
*d^10)*x)*log(b*x + a))/(b^13*x^2 + 2*a*b^12*x + a^2*b^11)
```

Sympy [B] time = 6.57919, size = 828, normalized size = 3.16

$$\frac{19a^{10}d^{10} - 170a^9bcd^9 + 675a^8b^2c^2d^8 - 1560a^7b^3c^3d^7 + 2310a^6b^4c^4d^6 - 2268a^5b^5c^5d^5 + 1470a^4b^6c^6d^4 - 600a^3b^7c^7d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**3,x)
```

```
[Out] (19*a**10*d**10 - 170*a**9*b*c*d**9 + 675*a**8*b**2*c**2*d**8 - 1560*a**7*b
**3*c**3*d**7 + 2310*a**6*b**4*c**4*d**6 - 2268*a**5*b**5*c**5*d**5 + 1470*
a**4*b**6*c**6*d**4 - 600*a**3*b**7*c**7*d**3 + 135*a**2*b**8*c**8*d**2 - 1
0*a*b**9*c**9*d - b**10*c**10 + x*(20*a**9*b*d**10 - 180*a**8*b**2*c*d**9 +
 720*a**7*b**3*c**2*d**8 - 1680*a**6*b**4*c**3*d**7 + 2520*a**5*b**5*c**4*d
**6 - 2520*a**4*b**6*c**5*d**5 + 1680*a**3*b**7*c**6*d**4 - 720*a**2*b**8*c
**7*d**3 + 180*a*b**9*c**8*d**2 - 20*b**10*c**9*d))/(2*a**2*b**11 + 4*a*b**
12*x + 2*b**13*x**2) + d**10*x**8/(8*b**3) - x**7*(3*a*d**10 - 10*b*c*d**9)
/(7*b**4) + x**6*(2*a**2*d**10 - 10*a*b*c*d**9 + 15*b**2*c**2*d**8)/(2*b**5
) - x**5*(2*a**3*d**10 - 12*a**2*b*c*d**9 + 27*a*b**2*c**2*d**8 - 24*b**3*c
**3*d**7)/b**6 + x**4*(15*a**4*d**10 - 100*a**3*b*c*d**9 + 270*a**2*b**2*c*
*2*d**8 - 360*a*b**3*c**3*d**7 + 210*b**4*c**4*d**6)/(4*b**7) - x**3*(7*a**
5*d**10 - 50*a**4*b*c*d**9 + 150*a**3*b**2*c**2*d**8 - 240*a**2*b**3*c**3*d
**7 + 210*a*b**4*c**4*d**6 - 84*b**5*c**5*d**5)/b**8 + x**2*(28*a**6*d**10
- 210*a**5*b*c*d**9 + 675*a**4*b**2*c**2*d**8 - 1200*a**3*b**3*c**3*d**7 +
1260*a**2*b**4*c**4*d**6 - 756*a*b**5*c**5*d**5 + 210*b**6*c**6*d**4)/(2*b*
*9) - x*(36*a**7*d**10 - 280*a**6*b*c*d**9 + 945*a**5*b**2*c**2*d**8 - 1800
*a**4*b**3*c**3*d**7 + 2100*a**3*b**4*c**4*d**6 - 1512*a**2*b**5*c**5*d**5
+ 630*a*b**6*c**6*d**4 - 120*b**7*c**7*d**3)/b**10 + 45*d**2*(a*d - b*c)**8
*log(a + b*x)/b**11
```

Giac [B] time = 1.04931, size = 1247, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 45*(b^8*c^8*d^2 - 8*a*b^7*c^7*d^3 + 28*a^2*b^6*c^6*d^4 - 56*a^3*b^5*c^5*d^5
 + 70*a^4*b^4*c^4*d^6 - 56*a^5*b^3*c^3*d^7 + 28*a^6*b^2*c^2*d^8 - 8*a^7*b*c
*d^9 + a^8*d^10)*log(abs(b*x + a))/b^11 - 1/2*(b^10*c^10 + 10*a*b^9*c^9*d -
 135*a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^
5*b^5*c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 1560*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c
^2*d^8 + 170*a^9*b*c*d^9 - 19*a^10*d^10 + 20*(b^10*c^9*d - 9*a*b^9*c^8*d^2
+ 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 - 126*a^5*b
^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 - a^
9*b*d^10)*x)/((b*x + a)^2*b^11) + 1/56*(7*b^21*d^10*x^8 + 80*b^21*c*d^9*x^7
 - 24*a*b^20*d^10*x^7 + 420*b^21*c^2*d^8*x^6 - 280*a*b^20*c*d^9*x^6 + 56*a^
2*b^19*d^10*x^6 + 1344*b^21*c^3*d^7*x^5 - 1512*a*b^20*c^2*d^8*x^5 + 672*a^2
*b^19*c*d^9*x^5 - 112*a^3*b^18*d^10*x^5 + 2940*b^21*c^4*d^6*x^4 - 5040*a*b^
20*c^3*d^7*x^4 + 3780*a^2*b^19*c^2*d^8*x^4 - 1400*a^3*b^18*c*d^9*x^4 + 210*
a^4*b^17*d^10*x^4 + 4704*b^21*c^5*d^5*x^3 - 11760*a*b^20*c^4*d^6*x^3 + 1344
```

```
0*a^2*b^19*c^3*d^7*x^3 - 8400*a^3*b^18*c^2*d^8*x^3 + 2800*a^4*b^17*c*d^9*x^
3 - 392*a^5*b^16*d^10*x^3 + 5880*b^21*c^6*d^4*x^2 - 21168*a*b^20*c^5*d^5*x^
2 + 35280*a^2*b^19*c^4*d^6*x^2 - 33600*a^3*b^18*c^3*d^7*x^2 + 18900*a^4*b^1
7*c^2*d^8*x^2 - 5880*a^5*b^16*c*d^9*x^2 + 784*a^6*b^15*d^10*x^2 + 6720*b^21
*c^7*d^3*x - 35280*a*b^20*c^6*d^4*x + 84672*a^2*b^19*c^5*d^5*x - 117600*a^3
*b^18*c^4*d^6*x + 100800*a^4*b^17*c^3*d^7*x - 52920*a^5*b^16*c^2*d^8*x + 15
680*a^6*b^15*c*d^9*x - 2016*a^7*b^14*d^10*x)/b^24
```

3.1315 $\int \frac{(c+dx)^{10}}{(a+bx)^4} dx$

Optimal. Leaf size=258

$$\frac{5d^9(a+bx)^6(bc-ad)}{3b^{11}}+\frac{9d^8(a+bx)^5(bc-ad)^2}{b^{11}}+\frac{30d^7(a+bx)^4(bc-ad)^3}{b^{11}}+\frac{70d^6(a+bx)^3(bc-ad)^4}{b^{11}}+\frac{126d^5(a+bx}{b^1}$$

```
[Out] (210*d^4*(b*c - a*d)^6*x)/b^10 - (b*c - a*d)^10/(3*b^11*(a + b*x)^3) - (5*d
*(b*c - a*d)^9)/(b^11*(a + b*x)^2) - (45*d^2*(b*c - a*d)^8)/(b^11*(a + b*x)
) + (126*d^5*(b*c - a*d)^5*(a + b*x)^2)/b^11 + (70*d^6*(b*c - a*d)^4*(a + b
*x)^3)/b^11 + (30*d^7*(b*c - a*d)^3*(a + b*x)^4)/b^11 + (9*d^8*(b*c - a*d)^
2*(a + b*x)^5)/b^11 + (5*d^9*(b*c - a*d)*(a + b*x)^6)/(3*b^11) + (d^10*(a +
 b*x)^7)/(7*b^11) + (120*d^3*(b*c - a*d)^7*Log[a + b*x])/b^11
```

Rubi [A] time = 0.440824, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{5d^9(a+bx)^6(bc-ad)}{3b^{11}}+\frac{9d^8(a+bx)^5(bc-ad)^2}{b^{11}}+\frac{30d^7(a+bx)^4(bc-ad)^3}{b^{11}}+\frac{70d^6(a+bx)^3(bc-ad)^4}{b^{11}}+\frac{126d^5(a+bx}{b^1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^4,x]
```



```
[Out] (210*d^4*(b*c - a*d)^6*x)/b^10 - (b*c - a*d)^10/(3*b^11*(a + b*x)^3) - (5*d
*(b*c - a*d)^9)/(b^11*(a + b*x)^2) - (45*d^2*(b*c - a*d)^8)/(b^11*(a + b*x)
) + (126*d^5*(b*c - a*d)^5*(a + b*x)^2)/b^11 + (70*d^6*(b*c - a*d)^4*(a + b
*x)^3)/b^11 + (30*d^7*(b*c - a*d)^3*(a + b*x)^4)/b^11 + (9*d^8*(b*c - a*d)^
2*(a + b*x)^5)/b^11 + (5*d^9*(b*c - a*d)*(a + b*x)^6)/(3*b^11) + (d^10*(a +
 b*x)^7)/(7*b^11) + (120*d^3*(b*c - a*d)^7*Log[a + b*x])/b^11
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^4}dx=\int \left(\frac{210d^4(bc-ad)^6}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^4}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^3}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^2}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)}+\frac{252d^5}{}\right.$$
$$=\frac{210d^4(bc-ad)^6x}{b^{10}}-\frac{(bc-ad)^{10}}{3b^{11}(a+bx)^3}-\frac{5d(bc-ad)^9}{b^{11}(a+bx)^2}-\frac{45d^2(bc-ad)^8}{b^{11}(a+bx)}+\frac{126d^5(bc-ad)^5(a+bx)^2}{b^{11}}+$$

Mathematica [A] time = 0.180243, size = 427, normalized size = 1.66

$$21b^5d^8x^5\left(2a^2d^2-8abcd+9b^2c^2\right)+105b^4d^7x^4\left(5a^2bcd^2-a^3d^3-9ab^2c^2d+6b^3c^3\right)+35b^3d^6x^3\left(90a^2b^2c^2d^2-40a^3bc\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^4,x]
```

```
[Out] (21*b*d^4*(210*b^6*c^6 - 1008*a*b^5*c^5*d + 2100*a^2*b^4*c^4*d^2 - 2400*a^3
*b^3*c^3*d^3 + 1575*a^4*b^2*c^2*d^4 - 560*a^5*b*c*d^5 + 84*a^6*d^6)*x + 21*
b^2*d^5*(126*b^5*c^5 - 420*a*b^4*c^4*d + 600*a^2*b^3*c^3*d^2 - 450*a^3*b^2*
c^2*d^3 + 175*a^4*b*c*d^4 - 28*a^5*d^5)*x^2 + 35*b^3*d^6*(42*b^4*c^4 - 96*a
*b^3*c^3*d + 90*a^2*b^2*c^2*d^2 - 40*a^3*b*c*d^3 + 7*a^4*d^4)*x^3 + 105*b^4
*d^7*(6*b^3*c^3 - 9*a*b^2*c^2*d + 5*a^2*b*c*d^2 - a^3*d^3)*x^4 + 21*b^5*d^8
*(9*b^2*c^2 - 8*a*b*c*d + 2*a^2*d^2)*x^5 + 7*b^6*d^9*(5*b*c - 2*a*d)*x^6 +
3*b^7*d^10*x^7 - (7*(b*c - a*d)^10)/(a + b*x)^3 + (105*d*(-(b*c) + a*d)^9)/
(a + b*x)^2 - (945*d^2*(b*c - a*d)^8)/(a + b*x) + 2520*d^3*(b*c - a*d)^7*Lo
g[a + b*x])/(21*b^11)
```

Maple [B] time = 0.018, size = 1141, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^4,x)
```

```
[Out] -45/b^10*d^9/(b*x+a)^2*a^8*c+180/b^9*d^8/(b*x+a)^2*a^7*c^2-45/b^11*d^10/(b*
x+a)*a^8-1/3/b^11/(b*x+a)^3*a^10*d^10-120/b^11*d^10*ln(b*x+a)*a^7+120/b^4*d
^3*ln(b*x+a)*c^7+210*d^4/b^4*c^6*x+84*d^10/b^10*a^6*x-5*d^10/b^7*x^4*a^3+30
*d^7/b^4*x^4*c^3+35/3*d^10/b^8*x^3*a^4+70*d^6/b^4*x^3*c^4-28*d^10/b^9*x^2*a
^5+126*d^5/b^4*x^2*c^5-2/3*d^10/b^5*x^6*a+5/3*d^9/b^4*x^6*c+2*d^10/b^6*x^5*
a^2+9*d^8/b^4*x^5*c^2-1008*d^5/b^5*a*c^5*x+175*d^9/b^8*x^2*a^4*c-450*d^8/b^
7*x^2*a^3*c^2+600*d^7/b^6*x^2*a^2*c^3-420*d^6/b^5*x^2*a*c^4-560*d^9/b^9*a^5
*c*x-160*d^7/b^5*x^3*a*c^3+25*d^9/b^6*x^4*a^2*c-45*d^8/b^5*x^4*a*c^2-8*d^9/
b^5*x^5*a*c+360/b^10*d^9/(b*x+a)*a^7*c-1260/b^9*d^8/(b*x+a)*a^6*c^2+2520/b^
8*d^7/(b*x+a)*a^5*c^3-3150/b^7*d^6/(b*x+a)*a^4*c^4+2520/b^6*d^5/(b*x+a)*a^3
*c^5-1260/b^5*d^4/(b*x+a)*a^2*c^6+360/b^4*d^3/(b*x+a)*a*c^7-420/b^8*d^7/(b*
x+a)^2*a^6*c^3+630/b^7*d^6/(b*x+a)^2*a^5*c^4-630/b^6*d^5/(b*x+a)^2*a^4*c^5+
420/b^5*d^4/(b*x+a)^2*a^3*c^6-180/b^4*d^3/(b*x+a)^2*a^2*c^7+45/b^3*d^2/(b*x
+a)^2*a*c^8+10/3/b^10/(b*x+a)^3*a^9*c*d^9-15/b^9/(b*x+a)^3*a^8*c^2*d^8+40/b
^8/(b*x+a)^3*a^7*c^3*d^7-70/b^7/(b*x+a)^3*a^6*c^4*d^6+84/b^6/(b*x+a)^3*a^5*
c^5*d^5-70/b^5/(b*x+a)^3*a^4*c^6*d^4+40/b^4/(b*x+a)^3*a^3*c^7*d^3-15/b^3/(b
*x+a)^3*a^2*c^8*d^2+10/3/b^2/(b*x+a)^3*a*c^9*d+840/b^10*d^9*ln(b*x+a)*a^6*c
-200/3*d^9/b^7*x^3*a^3*c+150*d^8/b^6*x^3*a^2*c^2+1575*d^8/b^8*a^4*c^2*x-240
0*d^7/b^7*a^3*c^3*x-2520/b^9*d^8*ln(b*x+a)*a^5*c^2+4200/b^8*d^7*ln(b*x+a)*a
^4*c^3-4200/b^7*d^6*ln(b*x+a)*a^3*c^4+2520/b^6*d^5*ln(b*x+a)*a^2*c^5-840/b^
5*d^4*ln(b*x+a)*a*c^6+2100*d^6/b^6*a^2*c^4*x-1/3/b/(b*x+a)^3*c^10+1/7*d^10/
b^4*x^7-45/b^3*d^2/(b*x+a)*c^8+5/b^11*d^10/(b*x+a)^2*a^9-5/b^2*d/(b*x+a)^2*
c^9
```

Maxima [B] time = 1.11288, size = 1203, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(b^10*c^10 + 5*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 660*a^3*b^7*c^7*d^3
+ 2730*a^4*b^6*c^6*d^4 - 5922*a^5*b^5*c^5*d^5 + 7770*a^6*b^4*c^4*d^6 - 6420
```

```
*a^7*b^3*c^3*d^7 + 3285*a^8*b^2*c^2*d^8 - 955*a^9*b*c*d^9 + 121*a^10*d^10 +
 135*(b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 56*a^3*b^7*c^5*
d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c^2*d^8 - 8*a^7*
b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(b^10*c^9*d + 9*a*b^9*c^8*d^2 - 108*a^2*
b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 882*a^4*b^6*c^5*d^5 + 1134*a^5*b^5*c^4*
d^6 - 924*a^6*b^4*c^3*d^7 + 468*a^7*b^3*c^2*d^8 - 135*a^8*b^2*c*d^9 + 17*a^
9*b*d^10)*x)/(b^14*x^3 + 3*a*b^13*x^2 + 3*a^2*b^12*x + a^3*b^11) + 1/21*(3*
b^6*d^10*x^7 + 7*(5*b^6*c*d^9 - 2*a*b^5*d^10)*x^6 + 21*(9*b^6*c^2*d^8 - 8*a
*b^5*c*d^9 + 2*a^2*b^4*d^10)*x^5 + 105*(6*b^6*c^3*d^7 - 9*a*b^5*c^2*d^8 + 5
*a^2*b^4*c*d^9 - a^3*b^3*d^10)*x^4 + 35*(42*b^6*c^4*d^6 - 96*a*b^5*c^3*d^7
+ 90*a^2*b^4*c^2*d^8 - 40*a^3*b^3*c*d^9 + 7*a^4*b^2*d^10)*x^3 + 21*(126*b^6
*c^5*d^5 - 420*a*b^5*c^4*d^6 + 600*a^2*b^4*c^3*d^7 - 450*a^3*b^3*c^2*d^8 +
175*a^4*b^2*c*d^9 - 28*a^5*b*d^10)*x^2 + 21*(210*b^6*c^6*d^4 - 1008*a*b^5*c
^5*d^5 + 2100*a^2*b^4*c^4*d^6 - 2400*a^3*b^3*c^3*d^7 + 1575*a^4*b^2*c^2*d^8
 - 560*a^5*b*c*d^9 + 84*a^6*d^10)*x)/b^10 + 120*(b^7*c^7*d^3 - 7*a*b^6*c^6*
d^4 + 21*a^2*b^5*c^5*d^5 - 35*a^3*b^4*c^4*d^6 + 35*a^4*b^3*c^3*d^7 - 21*a^5
*b^2*c^2*d^8 + 7*a^6*b*c*d^9 - a^7*d^10)*log(b*x + a)/b^11
```

Fricas [B] time = 1.96918, size = 2786, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^10*d^10*x^10 - 7*b^10*c^10 - 35*a*b^9*c^9*d - 315*a^2*b^8*c^8*d^2
 + 4620*a^3*b^7*c^7*d^3 - 19110*a^4*b^6*c^6*d^4 + 41454*a^5*b^5*c^5*d^5 - 5
4390*a^6*b^4*c^4*d^6 + 44940*a^7*b^3*c^3*d^7 - 22995*a^8*b^2*c^2*d^8 + 6685
*a^9*b*c*d^9 - 847*a^10*d^10 + 5*(7*b^10*c*d^9 - a*b^9*d^10)*x^9 + 9*(21*b^
10*c^2*d^8 - 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 18*(35*b^10*c^3*d^7 - 21*a
*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 42*(35*b^10*c^4*d^6 -
35*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 - 7*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 126*(21*b^10*c^5*d^5 - 35*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 - 21*a^3*b^
7*c^2*d^8 + 7*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 630*(7*b^10*c^6*d^4 - 21*
a*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 - 35*a^3*b^7*c^3*d^7 + 21*a^4*b^6*c^2*d^
8 - 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 7*(1890*a*b^9*c^6*d^4 - 7938*a^2*
b^8*c^5*d^5 + 15330*a^3*b^7*c^4*d^6 - 16680*a^4*b^6*c^3*d^7 + 10575*a^5*b^5
*c^2*d^8 - 3665*a^6*b^4*c*d^9 + 539*a^7*b^3*d^10)*x^3 - 21*(45*b^10*c^8*d^2
 - 360*a*b^9*c^7*d^3 + 630*a^2*b^8*c^6*d^4 + 378*a^3*b^7*c^5*d^5 - 2730*a^4
*b^6*c^4*d^6 + 4080*a^5*b^5*c^3*d^7 - 3015*a^6*b^4*c^2*d^8 + 1145*a^7*b^3*c
*d^9 - 179*a^8*b^2*d^10)*x^2 - 21*(5*b^10*c^9*d + 45*a*b^9*c^8*d^2 - 540*a^
2*b^8*c^7*d^3 + 1890*a^3*b^7*c^6*d^4 - 3402*a^4*b^6*c^5*d^5 + 3570*a^5*b^5*
c^4*d^6 - 2220*a^6*b^4*c^3*d^7 + 765*a^7*b^3*c^2*d^8 - 115*a^8*b^2*c*d^9 +
a^9*b*d^10)*x + 2520*(a^3*b^7*c^7*d^3 - 7*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*
d^5 - 35*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 - 21*a^8*b^2*c^2*d^8 + 7*a^9*
b*c*d^9 - a^10*d^10 + (b^10*c^7*d^3 - 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5
- 35*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*d^7 - 21*a^5*b^5*c^2*d^8 + 7*a^6*b^4*
c*d^9 - a^7*b^3*d^10)*x^3 + 3*(a*b^9*c^7*d^3 - 7*a^2*b^8*c^6*d^4 + 21*a^3*b
^7*c^5*d^5 - 35*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 - 21*a^6*b^4*c^2*d^8 +
 7*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 3*(a^2*b^8*c^7*d^3 - 7*a^3*b^7*c^6*d
^4 + 21*a^4*b^6*c^5*d^5 - 35*a^5*b^5*c^4*d^6 + 35*a^6*b^4*c^3*d^7 - 21*a^7*
b^3*c^2*d^8 + 7*a^8*b^2*c*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^14*x^3 + 3*
a*b^13*x^2 + 3*a^2*b^12*x + a^3*b^11)
```

Sympy [B] time = 33.521, size = 853, normalized size = 3.31

$$-\frac{121a^{10}d^{10} - 955a^9bcd^9 + 3285a^8b^2c^2d^8 - 6420a^7b^3c^3d^7 + 7770a^6b^4c^4d^6 - 5922a^5b^5c^5d^5 + 2730a^4b^6c^6d^4 - 660a^3b^7c^7d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**4,x)
```

```
[Out] -(121*a**10*d**10 - 955*a**9*b*c*d**9 + 3285*a**8*b**2*c**2*d**8 - 6420*a**
7*b**3*c**3*d**7 + 7770*a**6*b**4*c**4*d**6 - 5922*a**5*b**5*c**5*d**5 + 27
30*a**4*b**6*c**6*d**4 - 660*a**3*b**7*c**7*d**3 + 45*a**2*b**8*c**8*d**2 +
 5*a*b**9*c**9*d + b**10*c**10 + x**2*(135*a**8*b**2*d**10 - 1080*a**7*b**3
*c*d**9 + 3780*a**6*b**4*c**2*d**8 - 7560*a**5*b**5*c**3*d**7 + 9450*a**4*b
**6*c**4*d**6 - 7560*a**3*b**7*c**5*d**5 + 3780*a**2*b**8*c**6*d**4 - 1080*
a*b**9*c**7*d**3 + 135*b**10*c**8*d**2) + x*(255*a**9*b*d**10 - 2025*a**8*b
**2*c*d**9 + 7020*a**7*b**3*c**2*d**8 - 13860*a**6*b**4*c**3*d**7 + 17010*a
**5*b**5*c**4*d**6 - 13230*a**4*b**6*c**5*d**5 + 6300*a**3*b**7*c**6*d**4 -
 1620*a**2*b**8*c**7*d**3 + 135*a*b**9*c**8*d**2 + 15*b**10*c**9*d))/(3*a**
3*b**11 + 9*a**2*b**12*x + 9*a*b**13*x**2 + 3*b**14*x**3) + d**10*x**7/(7*b
**4) - x**6*(2*a*d**10 - 5*b*c*d**9)/(3*b**5) + x**5*(2*a**2*d**10 - 8*a*b*
c*d**9 + 9*b**2*c**2*d**8)/b**6 - x**4*(5*a**3*d**10 - 25*a**2*b*c*d**9 + 4
5*a*b**2*c**2*d**8 - 30*b**3*c**3*d**7)/b**7 + x**3*(35*a**4*d**10 - 200*a*
*3*b*c*d**9 + 450*a**2*b**2*c**2*d**8 - 480*a*b**3*c**3*d**7 + 210*b**4*c**
4*d**6)/(3*b**8) - x**2*(28*a**5*d**10 - 175*a**4*b*c*d**9 + 450*a**3*b**2*
c**2*d**8 - 600*a**2*b**3*c**3*d**7 + 420*a*b**4*c**4*d**6 - 126*b**5*c**5*
d**5)/b**9 + x*(84*a**6*d**10 - 560*a**5*b*c*d**9 + 1575*a**4*b**2*c**2*d**
8 - 2400*a**3*b**3*c**3*d**7 + 2100*a**2*b**4*c**4*d**6 - 1008*a*b**5*c**5*
d**5 + 210*b**6*c**6*d**4)/b**10 - 120*d**3*(a*d - b*c)**7*log(a + b*x)/b**
11
```

Giac [B] time = 1.06662, size = 1224, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 120*(b^7*c^7*d^3 - 7*a*b^6*c^6*d^4 + 21*a^2*b^5*c^5*d^5 - 35*a^3*b^4*c^4*d^
6 + 35*a^4*b^3*c^3*d^7 - 21*a^5*b^2*c^2*d^8 + 7*a^6*b*c*d^9 - a^7*d^10)*log
(abs(b*x + a))/b^11 - 1/3*(b^10*c^10 + 5*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 -
 660*a^3*b^7*c^7*d^3 + 2730*a^4*b^6*c^6*d^4 - 5922*a^5*b^5*c^5*d^5 + 7770*a
^6*b^4*c^4*d^6 - 6420*a^7*b^3*c^3*d^7 + 3285*a^8*b^2*c^2*d^8 - 955*a^9*b*c*
d^9 + 121*a^10*d^10 + 135*(b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*
d^4 - 56*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 28*a^6
*b^4*c^2*d^8 - 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(b^10*c^9*d + 9*a*b
^9*c^8*d^2 - 108*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 882*a^4*b^6*c^5*d^
5 + 1134*a^5*b^5*c^4*d^6 - 924*a^6*b^4*c^3*d^7 + 468*a^7*b^3*c^2*d^8 - 135*
a^8*b^2*c*d^9 + 17*a^9*b*d^10)*x)/((b*x + a)^3*b^11) + 1/21*(3*b^24*d^10*x^
7 + 35*b^24*c*d^9*x^6 - 14*a*b^23*d^10*x^6 + 189*b^24*c^2*d^8*x^5 - 168*a*b
^23*c*d^9*x^5 + 42*a^2*b^22*d^10*x^5 + 630*b^24*c^3*d^7*x^4 - 945*a*b^23*c^
2*d^8*x^4 + 525*a^2*b^22*c*d^9*x^4 - 105*a^3*b^21*d^10*x^4 + 1470*b^24*c^4*
d^6*x^3 - 3360*a*b^23*c^3*d^7*x^3 + 3150*a^2*b^22*c^2*d^8*x^3 - 1400*a^3*b^
21*c*d^9*x^3 + 245*a^4*b^20*d^10*x^3 + 2646*b^24*c^5*d^5*x^2 - 8820*a*b^23*
c^4*d^6*x^2 + 12600*a^2*b^22*c^3*d^7*x^2 - 9450*a^3*b^21*c^2*d^8*x^2 + 3675
*a^4*b^20*c*d^9*x^2 - 588*a^5*b^19*d^10*x^2 + 4410*b^24*c^6*d^4*x - 21168*a
```

```
*b^23*c^5*d^5*x + 44100*a^2*b^22*c^4*d^6*x - 50400*a^3*b^21*c^3*d^7*x + 330
75*a^4*b^20*c^2*d^8*x - 11760*a^5*b^19*c*d^9*x + 1764*a^6*b^18*d^10*x)/b^28
```

3.1316 $\int \frac{(c+dx)^{10}}{(a+bx)^5} dx$

Optimal. Leaf size=262

$$\frac{2d^9(a+bx)^5(bc-ad)}{b^{11}} + \frac{45d^8(a+bx)^4(bc-ad)^2}{4b^{11}} + \frac{40d^7(a+bx)^3(bc-ad)^3}{b^{11}} + \frac{105d^6(a+bx)^2(bc-ad)^4}{b^{11}} + \frac{252d^5x(bc-a}{b^{10}}$$

```
[Out] (252*d^5*(b*c - a*d)^5*x)/b^10 - (b*c - a*d)^10/(4*b^11*(a + b*x)^4) - (10*
d*(b*c - a*d)^9)/(3*b^11*(a + b*x)^3) - (45*d^2*(b*c - a*d)^8)/(2*b^11*(a +
 b*x)^2) - (120*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)) + (105*d^6*(b*c - a*d)^
4*(a + b*x)^2)/b^11 + (40*d^7*(b*c - a*d)^3*(a + b*x)^3)/b^11 + (45*d^8*(b*
c - a*d)^2*(a + b*x)^4)/(4*b^11) + (2*d^9*(b*c - a*d)*(a + b*x)^5)/b^11 + (
d^10*(a + b*x)^6)/(6*b^11) + (210*d^4*(b*c - a*d)^6*Log[a + b*x])/b^11
```

Rubi [A] time = 0.421948, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2d^9(a+bx)^5(bc-ad)}{b^{11}} + \frac{45d^8(a+bx)^4(bc-ad)^2}{4b^{11}} + \frac{40d^7(a+bx)^3(bc-ad)^3}{b^{11}} + \frac{105d^6(a+bx)^2(bc-ad)^4}{b^{11}} + \frac{252d^5x(bc-a}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^5,x]
```

```
[Out] (252*d^5*(b*c - a*d)^5*x)/b^10 - (b*c - a*d)^10/(4*b^11*(a + b*x)^4) - (10*
d*(b*c - a*d)^9)/(3*b^11*(a + b*x)^3) - (45*d^2*(b*c - a*d)^8)/(2*b^11*(a +
 b*x)^2) - (120*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)) + (105*d^6*(b*c - a*d)^
4*(a + b*x)^2)/b^11 + (40*d^7*(b*c - a*d)^3*(a + b*x)^3)/b^11 + (45*d^8*(b*
c - a*d)^2*(a + b*x)^4)/(4*b^11) + (2*d^9*(b*c - a*d)*(a + b*x)^5)/b^11 + (
d^10*(a + b*x)^6)/(6*b^11) + (210*d^4*(b*c - a*d)^6*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^5} dx &= \int \left(\frac{252d^5(bc-ad)^5}{b^{10}} + \frac{(bc-ad)^{10}}{b^{10}(a+bx)^5} + \frac{10d(bc-ad)^9}{b^{10}(a+bx)^4} + \frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^3} + \frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^2} + \frac{210d^4(bc}{b^{10}(a+}\right.\\ &= \frac{252d^5(bc-ad)^5x}{b^{10}} - \frac{(bc-ad)^{10}}{4b^{11}(a+bx)^4} - \frac{10d(bc-ad)^9}{3b^{11}(a+bx)^3} - \frac{45d^2(bc-ad)^8}{2b^{11}(a+bx)^2} - \frac{120d^3(bc-ad)^7}{b^{11}(a+bx)} + \frac{105d^6(bc-}{}\end{aligned}$$

Mathematica [A] time = 0.196967, size = 359, normalized size = 1.37

$$15b^4d^8x^4\left(3a^2d^2-10abcd+9b^2c^2\right)+20b^3d^7x^3\left(30a^2bcd^2-7a^3d^3-45ab^2c^2d+24b^3c^3\right)+30b^2d^6x^2\left(135a^2b^2c^2d^2-70a^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^5,x]
```

```
[Out] (12*b*d^5*(252*b^5*c^5 - 1050*a*b^4*c^4*d + 1800*a^2*b^3*c^3*d^2 - 1575*a^3
*b^2*c^2*d^3 + 700*a^4*b*c*d^4 - 126*a^5*d^5)*x + 30*b^2*d^6*(42*b^4*c^4 -
120*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 - 70*a^3*b*c*d^3 + 14*a^4*d^4)*x^2 +
20*b^3*d^7*(24*b^3*c^3 - 45*a*b^2*c^2*d + 30*a^2*b*c*d^2 - 7*a^3*d^3)*x^3 +
 15*b^4*d^8*(9*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 12*b^5*d^9*(2*b*c -
a*d)*x^5 + 2*b^6*d^10*x^6 - (3*(b*c - a*d)^10)/(a + b*x)^4 + (40*d*(-(b*c)
+ a*d)^9)/(a + b*x)^3 - (270*d^2*(b*c - a*d)^8)/(a + b*x)^2 + (1440*d^3*(-(
b*c) + a*d)^7)/(a + b*x) + 2520*d^4*(b*c - a*d)^6*Log[a + b*x])/(12*b^11)
```

Maple [B] time = 0.018, size = 1172, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^5,x)
```

```
[Out] 5/2/b^2/(b*x+a)^4*a*c^9*d-105/2/b^7/(b*x+a)^4*a^6*c^4*d^6-105/2/b^5/(b*x+a)
^4*a^4*c^6*d^4+30/b^4/(b*x+a)^4*a^3*c^7*d^3-2520/b^6*d^5/(b*x+a)*a^2*c^5+84
0/b^5*d^4/(b*x+a)*a*c^6+180/b^10*d^9/(b*x+a)^2*a^7*c-630/b^9*d^8/(b*x+a)^2*
a^6*c^2+1260/b^8*d^7/(b*x+a)^2*a^5*c^3-1575/b^7*d^6/(b*x+a)^2*a^4*c^4+1260/
b^6*d^5/(b*x+a)^2*a^3*c^5-630/b^5*d^4/(b*x+a)^2*a^2*c^6+180/b^4*d^3/(b*x+a)
^2*a*c^7-30/b^10*d^9/(b*x+a)^3*a^8*c+120/b^9*d^8/(b*x+a)^3*a^7*c^2-280/b^8*
d^7/(b*x+a)^3*a^6*c^3+420/b^7*d^6/(b*x+a)^3*a^5*c^4-420/b^6*d^5/(b*x+a)^3*a
^4*c^5+280/b^5*d^4/(b*x+a)^3*a^3*c^6+63/b^6/(b*x+a)^4*a^5*c^5*d^5-120/b^4*d
^3/(b*x+a)*c^7-45/2/b^11*d^10/(b*x+a)^2*a^8-45/2/b^3*d^2/(b*x+a)^2*c^8+10/3
/b^11*d^10/(b*x+a)^3*a^9-10/3/b^2*d/(b*x+a)^3*c^9+210/b^11*d^10*ln(b*x+a)*a
^6+210/b^5*d^4*ln(b*x+a)*c^6-1/4/b^11/(b*x+a)^4*a^10*d^10-d^10/b^6*x^5*a+2*
d^9/b^5*x^5*c+15/4*d^10/b^7*x^4*a^2+45/4*d^8/b^5*x^4*c^2-35/3*d^10/b^8*x^3*
a^3+40*d^7/b^5*x^3*c^3+35*d^10/b^9*x^2*a^4+105*d^6/b^5*x^2*c^4-126*d^10/b^1
0*a^5*x+252*d^5/b^5*c^5*x+120/b^11*d^10/(b*x+a)*a^7-25/2*d^9/b^6*x^4*a*c+50
*d^9/b^7*x^3*a^2*c+1/6*d^10/b^5*x^6-1/4/b/(b*x+a)^4*c^10-75*d^8/b^6*x^3*a*c
^2-175*d^9/b^8*x^2*a^3*c+675/2*d^8/b^7*x^2*a^2*c^2-300*d^7/b^6*x^2*a*c^3+70
0*d^9/b^9*a^4*c*x-1575*d^8/b^8*a^3*c^2*x+1800*d^7/b^7*a^2*c^3*x-1050*d^6/b^
6*a*c^4*x-840/b^10*d^9/(b*x+a)*a^6*c+2520/b^9*d^8/(b*x+a)*a^5*c^2-4200/b^8*
d^7/(b*x+a)*a^4*c^3+4200/b^7*d^6/(b*x+a)*a^3*c^4+3150/b^9*d^8*ln(b*x+a)*a^4
*c^2-4200/b^8*d^7*ln(b*x+a)*a^3*c^3+3150/b^7*d^6*ln(b*x+a)*a^2*c^4-1260/b^6
*d^5*ln(b*x+a)*a*c^5+5/2/b^10/(b*x+a)^4*a^9*c*d^9-45/4/b^9/(b*x+a)^4*a^8*c^
2*d^8+30/b^8/(b*x+a)^4*a^7*c^3*d^7-45/4/b^3/(b*x+a)^4*a^2*c^8*d^2+30/b^3*d^
2/(b*x+a)^3*a*c^8-1260/b^10*d^9*ln(b*x+a)*a^5*c-120/b^4*d^3/(b*x+a)^3*a^2*c
^7
```

Maxima [B] time = 1.1744, size = 1219, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^10*c^10 + 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*
d^3 - 5250*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 - 35910*a^6*b^4*c^4*d^6
```

```
+ 38280*a^7*b^3*c^3*d^7 - 23985*a^8*b^2*c^2*d^8 + 8250*a^9*b*c*d^9 - 1207*a
^10*d^10 + 1440*(b^10*c^7*d^3 - 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 - 35*a
^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*d^7 - 21*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9
- a^7*b^3*d^10)*x^3 + 270*(b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 - 84*a^2*b^8*c^6*
d^4 + 280*a^3*b^7*c^5*d^5 - 490*a^4*b^6*c^4*d^6 + 504*a^5*b^5*c^3*d^7 - 308
*a^6*b^4*c^2*d^8 + 104*a^7*b^3*c*d^9 - 15*a^8*b^2*d^10)*x^2 + 20*(2*b^10*c^
9*d + 9*a*b^9*c^8*d^2 + 72*a^2*b^8*c^7*d^3 - 924*a^3*b^7*c^6*d^4 + 3276*a^4
*b^6*c^5*d^5 - 5922*a^5*b^5*c^4*d^6 + 6216*a^6*b^4*c^3*d^7 - 3852*a^7*b^3*c
^2*d^8 + 1314*a^8*b^2*c*d^9 - 191*a^9*b*d^10)*x)/(b^15*x^4 + 4*a*b^14*x^3 +
 6*a^2*b^13*x^2 + 4*a^3*b^12*x + a^4*b^11) + 1/12*(2*b^5*d^10*x^6 + 12*(2*b
^5*c*d^9 - a*b^4*d^10)*x^5 + 15*(9*b^5*c^2*d^8 - 10*a*b^4*c*d^9 + 3*a^2*b^3
*d^10)*x^4 + 20*(24*b^5*c^3*d^7 - 45*a*b^4*c^2*d^8 + 30*a^2*b^3*c*d^9 - 7*a
^3*b^2*d^10)*x^3 + 30*(42*b^5*c^4*d^6 - 120*a*b^4*c^3*d^7 + 135*a^2*b^3*c^2
*d^8 - 70*a^3*b^2*c*d^9 + 14*a^4*b*d^10)*x^2 + 12*(252*b^5*c^5*d^5 - 1050*a
*b^4*c^4*d^6 + 1800*a^2*b^3*c^3*d^7 - 1575*a^3*b^2*c^2*d^8 + 700*a^4*b*c*d^
9 - 126*a^5*d^10)*x)/b^10 + 210*(b^6*c^6*d^4 - 6*a*b^5*c^5*d^5 + 15*a^2*b^4
*c^4*d^6 - 20*a^3*b^3*c^3*d^7 + 15*a^4*b^2*c^2*d^8 - 6*a^5*b*c*d^9 + a^6*d^
10)*log(b*x + a)/b^11
```

Fricas [B] time = 1.92892, size = 2925, normalized size = 11.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^10*d^10*x^10 - 3*b^10*c^10 - 10*a*b^9*c^9*d - 45*a^2*b^8*c^8*d^2
- 360*a^3*b^7*c^7*d^3 + 5250*a^4*b^6*c^6*d^4 - 19404*a^5*b^5*c^5*d^5 + 3591
0*a^6*b^4*c^4*d^6 - 38280*a^7*b^3*c^3*d^7 + 23985*a^8*b^2*c^2*d^8 - 8250*a^
9*b*c*d^9 + 1207*a^10*d^10 + 4*(6*b^10*c*d^9 - a*b^9*d^10)*x^9 + 9*(15*b^10
*c^2*d^8 - 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 24*(20*b^10*c^3*d^7 - 15*a*b
^9*c^2*d^8 + 6*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 84*(15*b^10*c^4*d^6 - 20
*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 - 6*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 +
 504*(6*b^10*c^5*d^5 - 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 - 15*a^3*b^7*c
^2*d^8 + 6*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + (12096*a*b^9*c^5*d^5 - 42840
*a^2*b^8*c^4*d^6 + 66720*a^3*b^7*c^3*d^7 - 54765*a^4*b^6*c^2*d^8 + 23250*a^
5*b^5*c*d^9 - 4043*a^6*b^4*d^10)*x^4 - 4*(360*b^10*c^7*d^3 - 2520*a*b^9*c^6
*d^4 + 3024*a^2*b^8*c^5*d^5 + 5040*a^3*b^7*c^4*d^6 - 16320*a^4*b^6*c^3*d^7
+ 16965*a^5*b^5*c^2*d^8 - 8130*a^6*b^4*c*d^9 + 1523*a^7*b^3*d^10)*x^3 - 6*(
45*b^10*c^8*d^2 + 360*a*b^9*c^7*d^3 - 3780*a^2*b^8*c^6*d^4 + 10584*a^3*b^7*
c^5*d^5 - 13860*a^4*b^6*c^4*d^6 + 8880*a^5*b^5*c^3*d^7 - 1935*a^6*b^4*c^2*d
^8 - 570*a^7*b^3*c*d^9 + 263*a^8*b^2*d^10)*x^2 - 4*(10*b^10*c^9*d + 45*a*b^
9*c^8*d^2 + 360*a^2*b^8*c^7*d^3 - 4620*a^3*b^7*c^6*d^4 + 15624*a^4*b^6*c^5*
d^5 - 26460*a^5*b^5*c^4*d^6 + 25680*a^6*b^4*c^3*d^7 - 14535*a^7*b^3*c^2*d^8
 + 4470*a^8*b^2*c*d^9 - 577*a^9*b*d^10)*x + 2520*(a^4*b^6*c^6*d^4 - 6*a^5*b
^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 - 20*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 -
 6*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^6*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c
^4*d^6 - 20*a^3*b^7*c^3*d^7 + 15*a^4*b^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^
4*d^10)*x^4 + 4*(a*b^9*c^6*d^4 - 6*a^2*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 - 2
0*a^4*b^6*c^3*d^7 + 15*a^5*b^5*c^2*d^8 - 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^
3 + 6*(a^2*b^8*c^6*d^4 - 6*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 - 20*a^5*b^
5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 - 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 4*(a
^3*b^7*c^6*d^4 - 6*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 - 20*a^6*b^4*c^3*d^
7 + 15*a^7*b^3*c^2*d^8 - 6*a^8*b^2*c*d^9 + a^9*b*d^10)*x)*log(b*x + a))/(b^
15*x^4 + 4*a*b^14*x^3 + 6*a^2*b^13*x^2 + 4*a^3*b^12*x + a^4*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12658, size = 1577, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/12*(2*d^10 + 24*(b^2*c*d^9 - a*b*d^10)/((b*x + a)*b) + 135*(b^4*c^2*d^8 -
 2*a*b^3*c*d^9 + a^2*b^2*d^10)/((b*x + a)^2*b^2) + 480*(b^6*c^3*d^7 - 3*a*b
^5*c^2*d^8 + 3*a^2*b^4*c*d^9 - a^3*b^3*d^10)/((b*x + a)^3*b^3) + 1260*(b^8*
c^4*d^6 - 4*a*b^7*c^3*d^7 + 6*a^2*b^6*c^2*d^8 - 4*a^3*b^5*c*d^9 + a^4*b^4*d
^10)/((b*x + a)^4*b^4) + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*
c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)/((b*x + a)^5
*b^5))*(b*x + a)^6/b^11 - 210*(b^6*c^6*d^4 - 6*a*b^5*c^5*d^5 + 15*a^2*b^4*c
^4*d^6 - 20*a^3*b^3*c^3*d^7 + 15*a^4*b^2*c^2*d^8 - 6*a^5*b*c*d^9 + a^6*d^10
)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^11 - 1/12*(3*b^67*c^10/(b*x + a)
^4 + 40*b^66*c^9*d/(b*x + a)^3 - 30*a*b^66*c^9*d/(b*x + a)^4 + 270*b^65*c^8
*d^2/(b*x + a)^2 - 360*a*b^65*c^8*d^2/(b*x + a)^3 + 135*a^2*b^65*c^8*d^2/(b
*x + a)^4 + 1440*b^64*c^7*d^3/(b*x + a) - 2160*a*b^64*c^7*d^3/(b*x + a)^2 +
 1440*a^2*b^64*c^7*d^3/(b*x + a)^3 - 360*a^3*b^64*c^7*d^3/(b*x + a)^4 - 100
80*a*b^63*c^6*d^4/(b*x + a) + 7560*a^2*b^63*c^6*d^4/(b*x + a)^2 - 3360*a^3*
b^63*c^6*d^4/(b*x + a)^3 + 630*a^4*b^63*c^6*d^4/(b*x + a)^4 + 30240*a^2*b^6
2*c^5*d^5/(b*x + a) - 15120*a^3*b^62*c^5*d^5/(b*x + a)^2 + 5040*a^4*b^62*c^
5*d^5/(b*x + a)^3 - 756*a^5*b^62*c^5*d^5/(b*x + a)^4 - 50400*a^3*b^61*c^4*d
^6/(b*x + a) + 18900*a^4*b^61*c^4*d^6/(b*x + a)^2 - 5040*a^5*b^61*c^4*d^6/(
b*x + a)^3 + 630*a^6*b^61*c^4*d^6/(b*x + a)^4 + 50400*a^4*b^60*c^3*d^7/(b*x
 + a) - 15120*a^5*b^60*c^3*d^7/(b*x + a)^2 + 3360*a^6*b^60*c^3*d^7/(b*x + a
)^3 - 360*a^7*b^60*c^3*d^7/(b*x + a)^4 - 30240*a^5*b^59*c^2*d^8/(b*x + a) +
 7560*a^6*b^59*c^2*d^8/(b*x + a)^2 - 1440*a^7*b^59*c^2*d^8/(b*x + a)^3 + 13
5*a^8*b^59*c^2*d^8/(b*x + a)^4 + 10080*a^6*b^58*c*d^9/(b*x + a) - 2160*a^7*
b^58*c*d^9/(b*x + a)^2 + 360*a^8*b^58*c*d^9/(b*x + a)^3 - 30*a^9*b^58*c*d^9
/(b*x + a)^4 - 1440*a^7*b^57*d^10/(b*x + a) + 270*a^8*b^57*d^10/(b*x + a)^2
 - 40*a^9*b^57*d^10/(b*x + a)^3 + 3*a^10*b^57*d^10/(b*x + a)^4)/b^68
```

3.1317 $\int \frac{(c+dx)^{10}}{(a+bx)^6} dx$

Optimal. Leaf size=260

$$\frac{5d^9(a+bx)^4(bc-ad)}{2b^{11}}+\frac{15d^8(a+bx)^3(bc-ad)^2}{b^{11}}+\frac{60d^7(a+bx)^2(bc-ad)^3}{b^{11}}+\frac{210d^6x(bc-ad)^4}{b^{10}}-\frac{210d^4(bc-ad)^6}{b^{11}(a+bx)}-\frac{60d}{b^1}$$

```
[Out] (210*d^6*(b*c - a*d)^4*x)/b^10 - (b*c - a*d)^10/(5*b^11*(a + b*x)^5) - (5*d
*(b*c - a*d)^9)/(2*b^11*(a + b*x)^4) - (15*d^2*(b*c - a*d)^8)/(b^11*(a + b*
x)^3) - (60*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^2) - (210*d^4*(b*c - a*d)^6)
/(b^11*(a + b*x)) + (60*d^7*(b*c - a*d)^3*(a + b*x)^2)/b^11 + (15*d^8*(b*c
- a*d)^2*(a + b*x)^3)/b^11 + (5*d^9*(b*c - a*d)*(a + b*x)^4)/(2*b^11) + (d^
10*(a + b*x)^5)/(5*b^11) + (252*d^5*(b*c - a*d)^5*Log[a + b*x])/b^11
```

Rubi [A] time = 0.420905, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{5d^9(a+bx)^4(bc-ad)}{2b^{11}}+\frac{15d^8(a+bx)^3(bc-ad)^2}{b^{11}}+\frac{60d^7(a+bx)^2(bc-ad)^3}{b^{11}}+\frac{210d^6x(bc-ad)^4}{b^{10}}-\frac{210d^4(bc-ad)^6}{b^{11}(a+bx)}-\frac{60d}{b^1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^6,x]
```

```
[Out] (210*d^6*(b*c - a*d)^4*x)/b^10 - (b*c - a*d)^10/(5*b^11*(a + b*x)^5) - (5*d
*(b*c - a*d)^9)/(2*b^11*(a + b*x)^4) - (15*d^2*(b*c - a*d)^8)/(b^11*(a + b*
x)^3) - (60*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^2) - (210*d^4*(b*c - a*d)^6)
/(b^11*(a + b*x)) + (60*d^7*(b*c - a*d)^3*(a + b*x)^2)/b^11 + (15*d^8*(b*c
- a*d)^2*(a + b*x)^3)/b^11 + (5*d^9*(b*c - a*d)*(a + b*x)^4)/(2*b^11) + (d^
10*(a + b*x)^5)/(5*b^11) + (252*d^5*(b*c - a*d)^5*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^6} dx &= \int \left(\frac{210d^6(bc-ad)^4}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^6}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^5}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^4}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^3}+\frac{210d^4(bc}{b^{10}(a+}\right.\\ &= \frac{210d^6(bc-ad)^4x}{b^{10}}-\frac{(bc-ad)^{10}}{5b^{11}(a+bx)^5}-\frac{5d(bc-ad)^9}{2b^{11}(a+bx)^4}-\frac{15d^2(bc-ad)^8}{b^{11}(a+bx)^3}-\frac{60d^3(bc-ad)^7}{b^{11}(a+bx)^2}-\frac{210d^4(bc-a}{b^{11}(a+bx}\end{aligned}$$

Mathematica [A] time = 0.222811, size = 305, normalized size = 1.17

$$\frac{10b^3d^8x^3\left(7a^2d^2-20abcd+15b^2c^2\right)+10b^2d^7x^2\left(105a^2bcd^2-28a^3d^3-135ab^2c^2d+60b^3c^3\right)+10bd^6x\left(945a^2b^2c^2d^2-56\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^6,x]
```

```
[Out] (10*b*d^6*(210*b^4*c^4 - 720*a*b^3*c^3*d + 945*a^2*b^2*c^2*d^2 - 560*a^3*b*
c*d^3 + 126*a^4*d^4)*x + 10*b^2*d^7*(60*b^3*c^3 - 135*a*b^2*c^2*d + 105*a^2
*b*c*d^2 - 28*a^3*d^3)*x^2 + 10*b^3*d^8*(15*b^2*c^2 - 20*a*b*c*d + 7*a^2*d^
2)*x^3 + 5*b^4*d^9*(5*b*c - 3*a*d)*x^4 + 2*b^5*d^10*x^5 - (2*(b*c - a*d)^10
)/(a + b*x)^5 + (25*d*(-(b*c) + a*d)^9)/(a + b*x)^4 - (150*d^2*(b*c - a*d)^
8)/(a + b*x)^3 + (600*d^3*(-(b*c) + a*d)^7)/(a + b*x)^2 - (2100*d^4*(b*c -
a*d)^6)/(a + b*x) + 2520*d^5*(b*c - a*d)^5*Log[a + b*x])/(10*b^11)
```

Maple [B] time = 0.02, size = 1199, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^6,x)
```

```
[Out] -2520/b^9*d^8*ln(b*x+a)*a^3*c^2+2520/b^8*d^7*ln(b*x+a)*a^2*c^3-1260/b^7*d^6
*ln(b*x+a)*a*c^4-45/2/b^10*d^9/(b*x+a)^4*a^8*c+90/b^9*d^8/(b*x+a)^4*a^7*c^2
-210/b^8*d^7/(b*x+a)^4*a^6*c^3+315/b^7*d^6/(b*x+a)^4*a^5*c^4+2/b^2/(b*x+a)^
5*a*c^9*d+1260/b^10*d^9*ln(b*x+a)*a^4*c-9/b^3/(b*x+a)^5*a^2*c^8*d^2+45/2/b^
3*d^2/(b*x+a)^4*a*c^8-1260/b^6*d^5/(b*x+a)^2*a^2*c^5+420/b^5*d^4/(b*x+a)^2*
a*c^6+120/b^10*d^9/(b*x+a)^3*a^7*c-420/b^9*d^8/(b*x+a)^3*a^6*c^2+840/b^8*d^
7/(b*x+a)^3*a^5*c^3-1050/b^7*d^6/(b*x+a)^3*a^4*c^4+840/b^6*d^5/(b*x+a)^3*a^
3*c^5-315/b^6*d^5/(b*x+a)^4*a^4*c^5+210/b^5*d^4/(b*x+a)^4*a^3*c^6-90/b^4*d^
3/(b*x+a)^4*a^2*c^7-420/b^5*d^4/(b*x+a)^3*a^2*c^6+120/b^4*d^3/(b*x+a)^3*a*c
^7+1/5*d^10/b^6*x^5-1/5/b/(b*x+a)^5*c^10-3/2*d^10/b^7*x^4*a+5/2*d^9/b^6*x^4
*c+7*d^10/b^8*x^3*a^2+15*d^8/b^6*x^3*c^2-28*d^10/b^9*x^2*a^3+60*d^7/b^6*x^2
*c^3+126*d^10/b^10*a^4*x+210*d^6/b^6*c^4*x-210/b^11*d^10/(b*x+a)*a^6-210/b^
5*d^4/(b*x+a)*c^6+60/b^11*d^10/(b*x+a)^2*a^7-60/b^4*d^3/(b*x+a)^2*c^7-15/b^
11*d^10/(b*x+a)^3*a^8-15/b^3*d^2/(b*x+a)^3*c^8-1/5/b^11/(b*x+a)^5*a^10*d^10
-252/b^11*d^10*ln(b*x+a)*a^5+252/b^6*d^5*ln(b*x+a)*c^5+5/2/b^11*d^10/(b*x+a
)^4*a^9-5/2/b^2*d/(b*x+a)^4*c^9-2100/b^8*d^7/(b*x+a)^2*a^4*c^3+2100/b^7*d^6
/(b*x+a)^2*a^3*c^4+24/b^4/(b*x+a)^5*a^3*c^7*d^3-20*d^9/b^7*x^3*a*c+105*d^9/
b^8*x^2*a^2*c-135*d^8/b^7*x^2*a*c^2-560*d^9/b^9*a^3*c*x+945*d^8/b^8*a^2*c^2
*x-720*d^7/b^7*a*c^3*x+1260/b^10*d^9/(b*x+a)*a^5*c-3150/b^9*d^8/(b*x+a)*a^4
*c^2+4200/b^8*d^7/(b*x+a)*a^3*c^3-3150/b^7*d^6/(b*x+a)*a^2*c^4+1260/b^6*d^5
/(b*x+a)*a*c^5-420/b^10*d^9/(b*x+a)^2*a^6*c+1260/b^9*d^8/(b*x+a)^2*a^5*c^2+
2/b^10/(b*x+a)^5*a^9*c*d^9-9/b^9/(b*x+a)^5*a^8*c^2*d^8+24/b^8/(b*x+a)^5*a^7
*c^3*d^7-42/b^7/(b*x+a)^5*a^6*c^4*d^6+252/5/b^6/(b*x+a)^5*a^5*c^5*d^5-42/b^
5/(b*x+a)^5*a^4*c^6*d^4
```

Maxima [B] time = 1.27585, size = 1231, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/10*(2*b^10*c^10 + 5*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^
3 + 420*a^4*b^6*c^6*d^4 - 5754*a^5*b^5*c^5*d^5 + 18270*a^6*b^4*c^4*d^6 - 27
540*a^7*b^3*c^3*d^7 + 22290*a^8*b^2*c^2*d^8 - 9395*a^9*b*c*d^9 + 1627*a^10*
```

```
d^10 + 2100*(b^10*c^6*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 20*a^3*b
^7*c^3*d^7 + 15*a^4*b^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 600
*(b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 - 63*a^2*b^8*c^5*d^5 + 175*a^3*b^7*c^4*d^6
 - 245*a^4*b^6*c^3*d^7 + 189*a^5*b^5*c^2*d^8 - 77*a^6*b^4*c*d^9 + 13*a^7*b^
3*d^10)*x^3 + 150*(b^10*c^8*d^2 + 4*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 30
8*a^3*b^7*c^5*d^5 + 910*a^4*b^6*c^4*d^6 - 1316*a^5*b^5*c^3*d^7 + 1036*a^6*b
^4*c^2*d^8 - 428*a^7*b^3*c*d^9 + 73*a^8*b^2*d^10)*x^2 + 25*(b^10*c^9*d + 3*
a*b^9*c^8*d^2 + 12*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 - 1050*a^4*b^6*c^5*
d^5 + 3234*a^5*b^5*c^4*d^6 - 4788*a^6*b^4*c^3*d^7 + 3828*a^7*b^3*c^2*d^8 -
1599*a^8*b^2*c*d^9 + 275*a^9*b*d^10)*x)/(b^16*x^5 + 5*a*b^15*x^4 + 10*a^2*b
^14*x^3 + 10*a^3*b^13*x^2 + 5*a^4*b^12*x + a^5*b^11) + 1/10*(2*b^4*d^10*x^5
 + 5*(5*b^4*c*d^9 - 3*a*b^3*d^10)*x^4 + 10*(15*b^4*c^2*d^8 - 20*a*b^3*c*d^9
 + 7*a^2*b^2*d^10)*x^3 + 10*(60*b^4*c^3*d^7 - 135*a*b^3*c^2*d^8 + 105*a^2*b
^2*c*d^9 - 28*a^3*b*d^10)*x^2 + 10*(210*b^4*c^4*d^6 - 720*a*b^3*c^3*d^7 + 9
45*a^2*b^2*c^2*d^8 - 560*a^3*b*c*d^9 + 126*a^4*d^10)*x)/b^10 + 252*(b^5*c^5
*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d^8 + 5*a^4*b*
c*d^9 - a^5*d^10)*log(b*x + a)/b^11
```

Fricas [B] time = 1.84126, size = 2981, normalized size = 11.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^10*d^10*x^10 - 2*b^10*c^10 - 5*a*b^9*c^9*d - 15*a^2*b^8*c^8*d^2 -
 60*a^3*b^7*c^7*d^3 - 420*a^4*b^6*c^6*d^4 + 5754*a^5*b^5*c^5*d^5 - 18270*a^
6*b^4*c^4*d^6 + 27540*a^7*b^3*c^3*d^7 - 22290*a^8*b^2*c^2*d^8 + 9395*a^9*b*
c*d^9 - 1627*a^10*d^10 + 5*(5*b^10*c*d^9 - a*b^9*d^10)*x^9 + 15*(10*b^10*c^
2*d^8 - 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 60*(10*b^10*c^3*d^7 - 10*a*b^9*
c^2*d^8 + 5*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 420*(5*b^10*c^4*d^6 - 10*a*
b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 - 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + (1
0500*a*b^9*c^4*d^6 - 30000*a^2*b^8*c^3*d^7 + 35250*a^3*b^7*c^2*d^8 - 19375*
a^4*b^6*c*d^9 + 4127*a^5*b^5*d^10)*x^5 - 5*(420*b^10*c^6*d^4 - 2520*a*b^9*c
^5*d^5 + 2100*a^2*b^8*c^4*d^6 + 4800*a^3*b^7*c^3*d^7 - 10050*a^4*b^6*c^2*d^
8 + 6775*a^5*b^5*c*d^9 - 1607*a^6*b^4*d^10)*x^4 - 10*(60*b^10*c^7*d^3 + 420
*a*b^9*c^6*d^4 - 3780*a^2*b^8*c^5*d^5 + 8400*a^3*b^7*c^4*d^6 - 7800*a^4*b^6
*c^3*d^7 + 2550*a^5*b^5*c^2*d^8 + 475*a^6*b^4*c*d^9 - 347*a^7*b^3*d^10)*x^3
 - 10*(15*b^10*c^8*d^2 + 60*a*b^9*c^7*d^3 + 420*a^2*b^8*c^6*d^4 - 4620*a^3*
b^7*c^5*d^5 + 12600*a^4*b^6*c^4*d^6 - 16200*a^5*b^5*c^3*d^7 + 10950*a^6*b^4
*c^2*d^8 - 3725*a^7*b^3*c*d^9 + 493*a^8*b^2*d^10)*x^2 - 5*(5*b^10*c^9*d + 1
5*a*b^9*c^8*d^2 + 60*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 5250*a^4*b^6*c
^5*d^5 + 15750*a^5*b^5*c^4*d^6 - 22500*a^6*b^4*c^3*d^7 + 17250*a^7*b^3*c^2*
d^8 - 6875*a^8*b^2*c*d^9 + 1123*a^9*b*d^10)*x + 2520*(a^5*b^5*c^5*d^5 - 5*a
^6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 - 10*a^8*b^2*c^2*d^8 + 5*a^9*b*c*d^9 -
a^10*d^10 + (b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 - 10*a^3*b
^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 5*(a*b^9*c^5*d^5 - 5*a^2
*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 - 10*a^4*b^6*c^2*d^8 + 5*a^5*b^5*c*d^9 -
a^6*b^4*d^10)*x^4 + 10*(a^2*b^8*c^5*d^5 - 5*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^
3*d^7 - 10*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 10*(a^3*
b^7*c^5*d^5 - 5*a^4*b^6*c^4*d^6 + 10*a^5*b^5*c^3*d^7 - 10*a^6*b^4*c^2*d^8 +
 5*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 5*(a^4*b^6*c^5*d^5 - 5*a^5*b^5*c^4*d
^6 + 10*a^6*b^4*c^3*d^7 - 10*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 - a^9*b*d^10
)*x)*log(b*x + a))/(b^16*x^5 + 5*a*b^15*x^4 + 10*a^2*b^14*x^3 + 10*a^3*b^13
*x^2 + 5*a^4*b^12*x + a^5*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06812, size = 1192, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 252*(b^5*c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d^
8 + 5*a^4*b*c*d^9 - a^5*d^10)*log(abs(b*x + a))/b^11 - 1/10*(2*b^10*c^10 +
5*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d
^4 - 5754*a^5*b^5*c^5*d^5 + 18270*a^6*b^4*c^4*d^6 - 27540*a^7*b^3*c^3*d^7 +
 22290*a^8*b^2*c^2*d^8 - 9395*a^9*b*c*d^9 + 1627*a^10*d^10 + 2100*(b^10*c^6
*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 20*a^3*b^7*c^3*d^7 + 15*a^4*b
^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 600*(b^10*c^7*d^3 + 7*a*
b^9*c^6*d^4 - 63*a^2*b^8*c^5*d^5 + 175*a^3*b^7*c^4*d^6 - 245*a^4*b^6*c^3*d^
7 + 189*a^5*b^5*c^2*d^8 - 77*a^6*b^4*c*d^9 + 13*a^7*b^3*d^10)*x^3 + 150*(b^
10*c^8*d^2 + 4*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 308*a^3*b^7*c^5*d^5 + 9
10*a^4*b^6*c^4*d^6 - 1316*a^5*b^5*c^3*d^7 + 1036*a^6*b^4*c^2*d^8 - 428*a^7*
b^3*c*d^9 + 73*a^8*b^2*d^10)*x^2 + 25*(b^10*c^9*d + 3*a*b^9*c^8*d^2 + 12*a^
2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 - 1050*a^4*b^6*c^5*d^5 + 3234*a^5*b^5*c^
4*d^6 - 4788*a^6*b^4*c^3*d^7 + 3828*a^7*b^3*c^2*d^8 - 1599*a^8*b^2*c*d^9 +
275*a^9*b*d^10)*x)/((b*x + a)^5*b^11) + 1/10*(2*b^24*d^10*x^5 + 25*b^24*c*d
^9*x^4 - 15*a*b^23*d^10*x^4 + 150*b^24*c^2*d^8*x^3 - 200*a*b^23*c*d^9*x^3 +
 70*a^2*b^22*d^10*x^3 + 600*b^24*c^3*d^7*x^2 - 1350*a*b^23*c^2*d^8*x^2 + 10
50*a^2*b^22*c*d^9*x^2 - 280*a^3*b^21*d^10*x^2 + 2100*b^24*c^4*d^6*x - 7200*
a*b^23*c^3*d^7*x + 9450*a^2*b^22*c^2*d^8*x - 5600*a^3*b^21*c*d^9*x + 1260*a
^4*b^20*d^10*x)/b^30
```

3.1318 $\int \frac{(c+dx)^{10}}{(a+bx)^7} dx$

Optimal. Leaf size=262

$$\frac{10d^9(a+bx)^3(bc-ad)}{3b^{11}}+\frac{45d^8(a+bx)^2(bc-ad)^2}{2b^{11}}+\frac{120d^7x(bc-ad)^3}{b^{10}}-\frac{252d^5(bc-ad)^5}{b^{11}(a+bx)}-\frac{105d^4(bc-ad)^6}{b^{11}(a+bx)^2}-\frac{40d^3(bc-}{b^{11}(a+b}$$

```
[Out] (120*d^7*(b*c - a*d)^3*x)/b^10 - (b*c - a*d)^10/(6*b^11*(a + b*x)^6) - (2*d
*(b*c - a*d)^9)/(b^11*(a + b*x)^5) - (45*d^2*(b*c - a*d)^8)/(4*b^11*(a + b*
x)^4) - (40*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^3) - (105*d^4*(b*c - a*d)^6)
/(b^11*(a + b*x)^2) - (252*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)) + (45*d^8*(b
*c - a*d)^2*(a + b*x)^2)/(2*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^3)/(3*b^1
1) + (d^10*(a + b*x)^4)/(4*b^11) + (210*d^6*(b*c - a*d)^4*Log[a + b*x])/b^1
1
```

Rubi [A] time = 0.386594, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{10d^9(a+bx)^3(bc-ad)}{3b^{11}}+\frac{45d^8(a+bx)^2(bc-ad)^2}{2b^{11}}+\frac{120d^7x(bc-ad)^3}{b^{10}}-\frac{252d^5(bc-ad)^5}{b^{11}(a+bx)}-\frac{105d^4(bc-ad)^6}{b^{11}(a+bx)^2}-\frac{40d^3(bc-}{b^{11}(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^7,x]
```

```
[Out] (120*d^7*(b*c - a*d)^3*x)/b^10 - (b*c - a*d)^10/(6*b^11*(a + b*x)^6) - (2*d
*(b*c - a*d)^9)/(b^11*(a + b*x)^5) - (45*d^2*(b*c - a*d)^8)/(4*b^11*(a + b*
x)^4) - (40*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^3) - (105*d^4*(b*c - a*d)^6)
/(b^11*(a + b*x)^2) - (252*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)) + (45*d^8*(b
*c - a*d)^2*(a + b*x)^2)/(2*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^3)/(3*b^1
1) + (d^10*(a + b*x)^4)/(4*b^11) + (210*d^6*(b*c - a*d)^4*Log[a + b*x])/b^1
1
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^7}\,dx &= \int \left(\frac{120d^7(bc-ad)^3}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^7}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^6}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^5}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^4}+\frac{210d^4(bc}{b^{10}(a+}\right.\\ &= \frac{120d^7(bc-ad)^3x}{b^{10}}-\frac{(bc-ad)^{10}}{6b^{11}(a+bx)^6}-\frac{2d(bc-ad)^9}{b^{11}(a+bx)^5}-\frac{45d^2(bc-ad)^8}{4b^{11}(a+bx)^4}-\frac{40d^3(bc-ad)^7}{b^{11}(a+bx)^3}-\frac{105d^4(bc-ad}{b^{11}(a+bx)^2}\end{aligned}$$

Mathematica [A] time = 0.236312, size = 265, normalized size = 1.01

$$\frac{6b^2d^8x^2\left(28a^2d^2-70abcd+45b^2c^2\right)+12bd^7x\left(280a^2bcd^2-84a^3d^3-315ab^2c^2d+120b^3c^3\right)+4b^3d^9x^3(10bc-7ad)+\frac{302}{}}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^7,x]
```

```
[Out] (12*b*d^7*(120*b^3*c^3 - 315*a*b^2*c^2*d + 280*a^2*b*c*d^2 - 84*a^3*d^3)*x
+ 6*b^2*d^8*(45*b^2*c^2 - 70*a*b*c*d + 28*a^2*d^2)*x^2 + 4*b^3*d^9*(10*b*c
- 7*a*d)*x^3 + 3*b^4*d^10*x^4 - (2*(b*c - a*d)^10)/(a + b*x)^6 + (24*d*(-(b
*c) + a*d)^9)/(a + b*x)^5 - (135*d^2*(b*c - a*d)^8)/(a + b*x)^4 + (480*d^3*
(-(b*c) + a*d)^7)/(a + b*x)^3 - (1260*d^4*(b*c - a*d)^6)/(a + b*x)^2 + (302
4*d^5*(-(b*c) + a*d)^5)/(a + b*x) + 2520*d^6*(b*c - a*d)^4*Log[a + b*x])/(1
2*b^11)
```

Maple [B] time = 0.02, size = 1222, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^7,x)
```

```
[Out] -1400/b^8*d^7/(b*x+a)^3*a^4*c^3+1400/b^7*d^6/(b*x+a)^3*a^3*c^4-1575/b^9*d^8
/(b*x+a)^2*a^4*c^2+2100/b^8*d^7/(b*x+a)^2*a^3*c^3-1575/b^7*d^6/(b*x+a)^2*a^
2*c^4+42/b^6/(b*x+a)^6*a^5*c^5*d^5-35/b^5/(b*x+a)^6*a^4*c^6*d^4+18/b^3*d^2/
(b*x+a)^5*a*c^8-315*d^8/b^8*a*c^2*x-1260/b^10*d^9/(b*x+a)*a^4*c+2520/b^9*d^
8/(b*x+a)*a^3*c^2-2520/b^8*d^7/(b*x+a)*a^2*c^3+1260/b^7*d^6/(b*x+a)*a*c^4+5
/3/b^10/(b*x+a)^6*a^9*c*d^9-15/2/b^9/(b*x+a)^6*a^8*c^2*d^8+20/b^8/(b*x+a)^6
*a^7*c^3*d^7-35/b^7/(b*x+a)^6*a^6*c^4*d^6+20/b^4/(b*x+a)^6*a^3*c^7*d^3-15/2
/b^3/(b*x+a)^6*a^2*c^8*d^2+5/3/b^2/(b*x+a)^6*a*c^9*d+630/b^10*d^9/(b*x+a)^2
*a^5*c-840/b^6*d^5/(b*x+a)^3*a^2*c^5+280/b^5*d^4/(b*x+a)^3*a*c^6+630/b^6*d^
5/(b*x+a)^2*a*c^5-840/b^10*d^9*ln(b*x+a)*a^3*c+1260/b^9*d^8*ln(b*x+a)*a^2*c
^2-840/b^8*d^7*ln(b*x+a)*a*c^3-280/b^10*d^9/(b*x+a)^3*a^6*c+840/b^9*d^8/(b*
x+a)^3*a^5*c^2+1/4*d^10/b^7*x^4-1/6/b/(b*x+a)^6*c^10-315/b^9*d^8/(b*x+a)^4*
a^6*c^2+252/b^11*d^10/(b*x+a)*a^5-252/b^6*d^5/(b*x+a)*c^5-1/6/b^11/(b*x+a)^
6*a^10*d^10+90/b^10*d^9/(b*x+a)^4*a^7*c-105/b^11*d^10/(b*x+a)^2*a^6-105/b^5
*d^4/(b*x+a)^2*c^6+40/b^11*d^10/(b*x+a)^3*a^7-40/b^4*d^3/(b*x+a)^3*c^7-7/3*
d^10/b^8*x^3*a+10/3*d^9/b^7*x^3*c+14*d^10/b^9*x^2*a^2+45/2*d^8/b^7*x^2*c^2-
84*d^10/b^10*a^3*x+120*d^7/b^7*c^3*x+2/b^11*d^10/(b*x+a)^5*a^9-2/b^2*d/(b*x
+a)^5*c^9-45/4/b^11*d^10/(b*x+a)^4*a^8-45/4/b^3*d^2/(b*x+a)^4*c^8+210/b^11*
d^10*ln(b*x+a)*a^4+210/b^7*d^6*ln(b*x+a)*c^4+630/b^8*d^7/(b*x+a)^4*a^5*c^3-
1575/2/b^7*d^6/(b*x+a)^4*a^4*c^4+630/b^6*d^5/(b*x+a)^4*a^3*c^5-315/b^5*d^4/
(b*x+a)^4*a^2*c^6+90/b^4*d^3/(b*x+a)^4*a*c^7-35*d^9/b^8*x^2*a*c+280*d^9/b^9
*a^2*c*x-18/b^10*d^9/(b*x+a)^5*a^8*c+72/b^9*d^8/(b*x+a)^5*a^7*c^2-168/b^8*d
^7/(b*x+a)^5*a^6*c^3+252/b^7*d^6/(b*x+a)^5*a^5*c^4-252/b^6*d^5/(b*x+a)^5*a^
4*c^5+168/b^5*d^4/(b*x+a)^5*a^3*c^6-72/b^4*d^3/(b*x+a)^5*a^2*c^7
```

Maxima [B] time = 1.33374, size = 1249, normalized size = 4.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/12*(2*b^10*c^10 + 4*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3
 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 - 6174*a^6*b^4*c^4*d^6 + 16056*
```

```
a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 10036*a^9*b*c*d^9 - 2131*a^10*d^1
0 + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 - 10*a^3*b^7*
c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 1260*(b^10*c^6*d^4 + 6*a*b^
9*c^5*d^5 - 45*a^2*b^8*c^4*d^6 + 100*a^3*b^7*c^3*d^7 - 105*a^4*b^6*c^2*d^8
+ 54*a^5*b^5*c*d^9 - 11*a^6*b^4*d^10)*x^4 + 240*(2*b^10*c^7*d^3 + 7*a*b^9*c
^6*d^4 + 42*a^2*b^8*c^5*d^5 - 385*a^3*b^7*c^4*d^6 + 910*a^4*b^6*c^3*d^7 - 9
87*a^5*b^5*c^2*d^8 + 518*a^6*b^4*c*d^9 - 107*a^7*b^3*d^10)*x^3 + 45*(3*b^10
*c^8*d^2 + 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c^5*d^5 - 175
0*a^4*b^6*c^4*d^6 + 4312*a^5*b^5*c^3*d^7 - 4788*a^6*b^4*c^2*d^8 + 2552*a^7*
b^3*c*d^9 - 533*a^8*b^2*d^10)*x^2 + 6*(4*b^10*c^9*d + 9*a*b^9*c^8*d^2 + 24*
a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5 - 5754*a^5*b^5*c
^4*d^6 + 14616*a^6*b^4*c^3*d^7 - 16524*a^7*b^3*c^2*d^8 + 8916*a^8*b^2*c*d^9
 - 1879*a^9*b*d^10)*x)/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15*x^4 + 20*a^3*
b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11) + 1/12*(3*b^3*d^10*x^
4 + 4*(10*b^3*c*d^9 - 7*a*b^2*d^10)*x^3 + 6*(45*b^3*c^2*d^8 - 70*a*b^2*c*d^
9 + 28*a^2*b*d^10)*x^2 + 12*(120*b^3*c^3*d^7 - 315*a*b^2*c^2*d^8 + 280*a^2*
b*c*d^9 - 84*a^3*d^10)*x)/b^10 + 210*(b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2
*b^2*c^2*d^8 - 4*a^3*b*c*d^9 + a^4*d^10)*log(b*x + a)/b^11
```

Fricas [B] time = 1.94954, size = 2954, normalized size = 11.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*d^10*x^10 - 2*b^10*c^10 - 4*a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 -
24*a^3*b^7*c^7*d^3 - 84*a^4*b^6*c^6*d^4 - 504*a^5*b^5*c^5*d^5 + 6174*a^6*b^
4*c^4*d^6 - 16056*a^7*b^3*c^3*d^7 + 18414*a^8*b^2*c^2*d^8 - 10036*a^9*b*c*d
^9 + 2131*a^10*d^10 + 10*(4*b^10*c*d^9 - a*b^9*d^10)*x^9 + 45*(6*b^10*c^2*d
^8 - 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 360*(4*b^10*c^3*d^7 - 6*a*b^9*c^2*
d^8 + 4*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + (8640*a*b^9*c^3*d^7 - 18630*a^2
*b^8*c^2*d^8 + 14660*a^3*b^7*c*d^9 - 4043*a^4*b^6*d^10)*x^6 - 6*(504*b^10*c
^5*d^5 - 2520*a*b^9*c^4*d^6 + 1440*a^2*b^8*c^3*d^7 + 3510*a^3*b^7*c^2*d^8 -
 4580*a^4*b^6*c*d^9 + 1523*a^5*b^5*d^10)*x^5 - 15*(84*b^10*c^6*d^4 + 504*a*
b^9*c^5*d^5 - 3780*a^2*b^8*c^4*d^6 + 6480*a^3*b^7*c^3*d^7 - 4050*a^4*b^6*c^
2*d^8 + 460*a^5*b^5*c*d^9 + 263*a^6*b^4*d^10)*x^4 - 20*(24*b^10*c^7*d^3 + 8
4*a*b^9*c^6*d^4 + 504*a^2*b^8*c^5*d^5 - 4620*a^3*b^7*c^4*d^6 + 9840*a^4*b^6
*c^3*d^7 - 9090*a^5*b^5*c^2*d^8 + 3820*a^6*b^4*c*d^9 - 577*a^7*b^3*d^10)*x^
3 - 15*(9*b^10*c^8*d^2 + 24*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 504*a^3*b^
7*c^5*d^5 - 5250*a^4*b^6*c^4*d^6 + 12360*a^5*b^5*c^3*d^7 - 12870*a^6*b^4*c^
2*d^8 + 6340*a^7*b^3*c*d^9 - 1207*a^8*b^2*d^10)*x^2 - 6*(4*b^10*c^9*d + 9*a
*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^
5 - 5754*a^5*b^5*c^4*d^6 + 14376*a^6*b^4*c^3*d^7 - 15894*a^7*b^3*c^2*d^8 +
8356*a^8*b^2*c*d^9 - 1711*a^9*b*d^10)*x + 2520*(a^6*b^4*c^4*d^6 - 4*a^7*b^3
*c^3*d^7 + 6*a^8*b^2*c^2*d^8 - 4*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^4*d^6 -
4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 +
 6*(a*b^9*c^4*d^6 - 4*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 - 4*a^4*b^6*c*d^9
 + a^5*b^5*d^10)*x^5 + 15*(a^2*b^8*c^4*d^6 - 4*a^3*b^7*c^3*d^7 + 6*a^4*b^6*
c^2*d^8 - 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 20*(a^3*b^7*c^4*d^6 - 4*a^4
*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8 - 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 15
*(a^4*b^6*c^4*d^6 - 4*a^5*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 - 4*a^7*b^3*c*d^9
 + a^8*b^2*d^10)*x^2 + 6*(a^5*b^5*c^4*d^6 - 4*a^6*b^4*c^3*d^7 + 6*a^7*b^3*c
^2*d^8 - 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)*log(b*x + a))/(b^17*x^6 + 6*a*b^1
6*x^5 + 15*a^2*b^15*x^4 + 20*a^3*b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x
+ a^6*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.05499, size = 1185, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 210*(b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2*b^2*c^2*d^8 - 4*a^3*b*c*d^9 + a^
4*d^10)*log(abs(b*x + a))/b^11 - 1/12*(2*b^10*c^10 + 4*a*b^9*c^9*d + 9*a^2*
b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5
 - 6174*a^6*b^4*c^4*d^6 + 16056*a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 1
0036*a^9*b*c*d^9 - 2131*a^10*d^10 + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 +
10*a^2*b^8*c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x
^5 + 1260*(b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 - 45*a^2*b^8*c^4*d^6 + 100*a^3*b^
7*c^3*d^7 - 105*a^4*b^6*c^2*d^8 + 54*a^5*b^5*c*d^9 - 11*a^6*b^4*d^10)*x^4 +
 240*(2*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 42*a^2*b^8*c^5*d^5 - 385*a^3*b^7*c
^4*d^6 + 910*a^4*b^6*c^3*d^7 - 987*a^5*b^5*c^2*d^8 + 518*a^6*b^4*c*d^9 - 10
7*a^7*b^3*d^10)*x^3 + 45*(3*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6
*d^4 + 168*a^3*b^7*c^5*d^5 - 1750*a^4*b^6*c^4*d^6 + 4312*a^5*b^5*c^3*d^7 -
4788*a^6*b^4*c^2*d^8 + 2552*a^7*b^3*c*d^9 - 533*a^8*b^2*d^10)*x^2 + 6*(4*b^
10*c^9*d + 9*a*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*
a^4*b^6*c^5*d^5 - 5754*a^5*b^5*c^4*d^6 + 14616*a^6*b^4*c^3*d^7 - 16524*a^7*
b^3*c^2*d^8 + 8916*a^8*b^2*c*d^9 - 1879*a^9*b*d^10)*x)/((b*x + a)^6*b^11) +
 1/12*(3*b^21*d^10*x^4 + 40*b^21*c*d^9*x^3 - 28*a*b^20*d^10*x^3 + 270*b^21*
c^2*d^8*x^2 - 420*a*b^20*c*d^9*x^2 + 168*a^2*b^19*d^10*x^2 + 1440*b^21*c^3*
d^7*x - 3780*a*b^20*c^2*d^8*x + 3360*a^2*b^19*c*d^9*x - 1008*a^3*b^18*d^10*
x)/b^28
```

3.1319 $\int \frac{(c+dx)^{10}}{(a+bx)^8}\,dx$

Optimal. Leaf size=258

$$\frac{5d^9(a+bx)^2(bc-ad)}{b^{11}}+\frac{45d^8x(bc-ad)^2}{b^{10}}-\frac{210d^6(bc-ad)^4}{b^{11}(a+bx)}-\frac{126d^5(bc-ad)^5}{b^{11}(a+bx)^2}-\frac{70d^4(bc-ad)^6}{b^{11}(a+bx)^3}-\frac{30d^3(bc-ad)^7}{b^{11}(a+bx)^4}-\frac{9d^2}{b^{11}}$$

```
[Out] (45*d^8*(b*c - a*d)^2*x)/b^10 - (b*c - a*d)^10/(7*b^11*(a + b*x)^7) - (5*d*
(b*c - a*d)^9)/(3*b^11*(a + b*x)^6) - (9*d^2*(b*c - a*d)^8)/(b^11*(a + b*x)
^5) - (30*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^4) - (70*d^4*(b*c - a*d)^6)/(b
^11*(a + b*x)^3) - (126*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^2) - (210*d^6*(b
*c - a*d)^4)/(b^11*(a + b*x)) + (5*d^9*(b*c - a*d)*(a + b*x)^2)/b^11 + (d^1
0*(a + b*x)^3)/(3*b^11) + (120*d^7*(b*c - a*d)^3*Log[a + b*x])/b^11
```

Rubi [A] time = 0.3649, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{5d^9(a+bx)^2(bc-ad)}{b^{11}}+\frac{45d^8x(bc-ad)^2}{b^{10}}-\frac{210d^6(bc-ad)^4}{b^{11}(a+bx)}-\frac{126d^5(bc-ad)^5}{b^{11}(a+bx)^2}-\frac{70d^4(bc-ad)^6}{b^{11}(a+bx)^3}-\frac{30d^3(bc-ad)^7}{b^{11}(a+bx)^4}-\frac{9d^2}{b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^8,x]
```

```
[Out] (45*d^8*(b*c - a*d)^2*x)/b^10 - (b*c - a*d)^10/(7*b^11*(a + b*x)^7) - (5*d*
(b*c - a*d)^9)/(3*b^11*(a + b*x)^6) - (9*d^2*(b*c - a*d)^8)/(b^11*(a + b*x)
^5) - (30*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^4) - (70*d^4*(b*c - a*d)^6)/(b
^11*(a + b*x)^3) - (126*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^2) - (210*d^6*(b
*c - a*d)^4)/(b^11*(a + b*x)) + (5*d^9*(b*c - a*d)*(a + b*x)^2)/b^11 + (d^1
0*(a + b*x)^3)/(3*b^11) + (120*d^7*(b*c - a*d)^3*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^8}\,dx &= \int \left(\frac{45d^8(bc-ad)^2}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^8}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^7}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^6}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^5}+\frac{210d^4(bc-}{b^{10}(a+}\right.\\ &= \frac{45d^8(bc-ad)^2x}{b^{10}}-\frac{(bc-ad)^{10}}{7b^{11}(a+bx)^7}-\frac{5d(bc-ad)^9}{3b^{11}(a+bx)^6}-\frac{9d^2(bc-ad)^8}{b^{11}(a+bx)^5}-\frac{30d^3(bc-ad)^7}{b^{11}(a+bx)^4}-\frac{70d^4(bc-ad)^6}{b^{11}(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.265018, size = 239, normalized size = 0.93

$$\frac{21bd^8x\left(36a^2d^2-80abcd+45b^2c^2\right)+21b^2d^9x^2(5bc-4ad)-\frac{4410d^6(bc-ad)^4}{a+bx}+\frac{2646d^5(ad-bc)^5}{(a+bx)^2}-\frac{1470d^4(bc-ad)^6}{(a+bx)^3}+\frac{630d^3(ad-bc)^7}{(a+bx)^4}-}{21b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^8,x]
```

```
[Out] (21*b*d^8*(45*b^2*c^2 - 80*a*b*c*d + 36*a^2*d^2)*x + 21*b^2*d^9*(5*b*c - 4*
a*d)*x^2 + 7*b^3*d^10*x^3 - (3*(b*c - a*d)^10)/(a + b*x)^7 + (35*d*(-(b*c)
+ a*d)^9)/(a + b*x)^6 - (189*d^2*(b*c - a*d)^8)/(a + b*x)^5 + (630*d^3*(-(b
*c) + a*d)^7)/(a + b*x)^4 - (1470*d^4*(b*c - a*d)^6)/(a + b*x)^3 + (2646*d^
5*(-(b*c) + a*d)^5)/(a + b*x)^2 - (4410*d^6*(b*c - a*d)^4)/(a + b*x) + 2520
*d^7*(b*c - a*d)^3*Log[a + b*x])/(21*b^11)
```

Maple [B] time = 0.02, size = 1241, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^8,x)
```

```
[Out] 15/b^3*d^2/(b*x+a)^6*a*c^8-80*d^9/b^9*a*c*x+1400/b^8*d^7/(b*x+a)^3*a^3*c^3-
1050/b^7*d^6/(b*x+a)^3*a^2*c^4+420/b^6*d^5/(b*x+a)^3*a*c^5+72/b^10*d^9/(b*x
+a)^5*a^7*c-252/b^9*d^8/(b*x+a)^5*a^6*c^2+504/b^8*d^7/(b*x+a)^5*a^5*c^3-630
/b^7*d^6/(b*x+a)^5*a^4*c^4+1260/b^9*d^8/(b*x+a)^2*a^3*c^2-1260/b^8*d^7/(b*x
+a)^2*a^2*c^3+630/b^7*d^6/(b*x+a)^2*a*c^4+420/b^10*d^9/(b*x+a)^3*a^5*c-1050
/b^9*d^8/(b*x+a)^3*a^4*c^2+630/b^9*d^8/(b*x+a)^4*a^5*c^2-1050/b^8*d^7/(b*x+
a)^4*a^4*c^3+1050/b^7*d^6/(b*x+a)^4*a^3*c^4-45/7/b^3/(b*x+a)^7*a^2*c^8*d^2+
10/7/b^2/(b*x+a)^7*a*c^9*d+840/b^10*d^9/(b*x+a)*a^3*c-1260/b^9*d^8/(b*x+a)*
a^2*c^2+840/b^8*d^7/(b*x+a)*a*c^3-15/b^10*d^9/(b*x+a)^6*a^8*c+60/b^9*d^8/(b
*x+a)^6*a^7*c^2-140/b^8*d^7/(b*x+a)^6*a^6*c^3+210/b^7*d^6/(b*x+a)^6*a^5*c^4
-210/b^6*d^5/(b*x+a)^6*a^4*c^5+140/b^5*d^4/(b*x+a)^6*a^3*c^6-60/b^4*d^3/(b*
x+a)^6*a^2*c^7+30/b^11*d^10/(b*x+a)^4*a^7-30/b^4*d^3/(b*x+a)^4*c^7-30/b^7/(
b*x+a)^7*a^6*c^4*d^6+36/b^6/(b*x+a)^7*a^5*c^5*d^5-30/b^5/(b*x+a)^7*a^4*c^6*
d^4+120/7/b^4/(b*x+a)^7*a^3*c^7*d^3-210/b^10*d^9/(b*x+a)^4*a^6*c+120/7/b^8/
(b*x+a)^7*a^7*c^3*d^7+1/3*d^10/b^8*x^3-1/7/b/(b*x+a)^7*c^10-1/7/b^11/(b*x+a
)^7*a^10*d^10-120/b^11*d^10*ln(b*x+a)*a^3+120/b^8*d^7*ln(b*x+a)*c^3-210/b^1
1*d^10/(b*x+a)*a^4-210/b^7*d^6/(b*x+a)*c^4+5/3/b^11*d^10/(b*x+a)^6*a^9-5/3/
b^2*d/(b*x+a)^6*c^9-4*d^10/b^9*x^2*a+5*d^9/b^8*x^2*c+36*d^10/b^10*a^2*x+45*
d^8/b^8*c^2*x+126/b^11*d^10/(b*x+a)^2*a^5-126/b^6*d^5/(b*x+a)^2*c^5-9/b^11*
d^10/(b*x+a)^5*a^8-9/b^3*d^2/(b*x+a)^5*c^8-70/b^11*d^10/(b*x+a)^3*a^6-70/b^
5*d^4/(b*x+a)^3*c^6-630/b^6*d^5/(b*x+a)^4*a^2*c^5+210/b^5*d^4/(b*x+a)^4*a*c
^6+10/7/b^10/(b*x+a)^7*a^9*c*d^9-630/b^10*d^9/(b*x+a)^2*a^4*c-45/7/b^9/(b*x
+a)^7*a^8*c^2*d^8+504/b^6*d^5/(b*x+a)^5*a^3*c^5-252/b^5*d^4/(b*x+a)^5*a^2*c
^6+72/b^4*d^3/(b*x+a)^5*a*c^7+360/b^10*d^9*ln(b*x+a)*a^2*c-360/b^9*d^8*ln(b
*x+a)*a*c^2
```

Maxima [B] time = 1.34837, size = 1261, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/21*(3*b^10*c^10 + 5*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 18*a^3*b^7*c^7*d^3
 + 42*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 - 6534*a^
7*b^3*c^3*d^7 + 12987*a^8*b^2*c^2*d^8 - 10047*a^9*b*c*d^9 + 2761*a^10*d^10
```

```
+ 4410*(b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^
9 + a^4*b^6*d^10)*x^6 + 2646*(b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 - 30*a^2*b^8*c
^3*d^7 + 50*a^3*b^7*c^2*d^8 - 35*a^4*b^6*c*d^9 + 9*a^5*b^5*d^10)*x^5 + 1470
*(b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 110*a^3*b^7*c^3*d^7
 + 195*a^4*b^6*c^2*d^8 - 141*a^5*b^5*c*d^9 + 37*a^6*b^4*d^10)*x^4 + 210*(3*
b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 -
 875*a^4*b^6*c^3*d^7 + 1617*a^5*b^5*c^2*d^8 - 1197*a^6*b^4*c*d^9 + 319*a^7*
b^3*d^10)*x^3 + 63*(3*b^10*c^8*d^2 + 6*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*d^4 +
 42*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 1918*a^5*b^5*c^3*d^7 + 3654*a^6
*b^4*c^2*d^8 - 2754*a^7*b^3*c*d^9 + 743*a^8*b^2*d^10)*x^2 + 7*(5*b^10*c^9*d
 + 9*a*b^9*c^8*d^2 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 126*a^4*b^6*
c^5*d^5 + 630*a^5*b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 12042*a^7*b^3*c^2*d^
8 - 9207*a^8*b^2*c*d^9 + 2509*a^9*b*d^10)*x)/(b^18*x^7 + 7*a*b^17*x^6 + 21*
a^2*b^16*x^5 + 35*a^3*b^15*x^4 + 35*a^4*b^14*x^3 + 21*a^5*b^13*x^2 + 7*a^6*
b^12*x + a^7*b^11) + 1/3*(b^2*d^10*x^3 + 3*(5*b^2*c*d^9 - 4*a*b*d^10)*x^2 +
 3*(45*b^2*c^2*d^8 - 80*a*b*c*d^9 + 36*a^2*d^10)*x)/b^10 + 120*(b^3*c^3*d^7
 - 3*a*b^2*c^2*d^8 + 3*a^2*b*c*d^9 - a^3*d^10)*log(b*x + a)/b^11
```

Fricas [B] time = 1.95742, size = 2878, normalized size = 11.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^10*d^10*x^10 - 3*b^10*c^10 - 5*a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 -
18*a^3*b^7*c^7*d^3 - 42*a^4*b^6*c^6*d^4 - 126*a^5*b^5*c^5*d^5 - 630*a^6*b^4
*c^4*d^6 + 6534*a^7*b^3*c^3*d^7 - 12987*a^8*b^2*c^2*d^8 + 10047*a^9*b*c*d^9
 - 2761*a^10*d^10 + 35*(3*b^10*c*d^9 - a*b^9*d^10)*x^9 + 315*(3*b^10*c^2*d^
8 - 3*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 49*(135*a*b^9*c^2*d^8 - 195*a^2*b^8
*c*d^9 + 77*a^3*b^7*d^10)*x^7 - 49*(90*b^10*c^4*d^6 - 360*a*b^9*c^3*d^7 + 1
35*a^2*b^8*c^2*d^8 + 285*a^3*b^7*c*d^9 - 179*a^4*b^6*d^10)*x^6 - 147*(18*b^
10*c^5*d^5 + 90*a*b^9*c^4*d^6 - 540*a^2*b^8*c^3*d^7 + 675*a^3*b^7*c^2*d^8 -
 255*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 - 245*(6*b^10*c^6*d^4 + 18*a*b^9*c^5
*d^5 + 90*a^2*b^8*c^4*d^6 - 660*a^3*b^7*c^3*d^7 + 1035*a^4*b^6*c^2*d^8 - 61
5*a^5*b^5*c*d^9 + 121*a^6*b^4*d^10)*x^4 - 35*(18*b^10*c^7*d^3 + 42*a*b^9*c^
6*d^4 + 126*a^2*b^8*c^5*d^5 + 630*a^3*b^7*c^4*d^6 - 5250*a^4*b^6*c^3*d^7 +
9135*a^5*b^5*c^2*d^8 - 6195*a^6*b^4*c*d^9 + 1477*a^7*b^3*d^10)*x^3 - 21*(9*
b^10*c^8*d^2 + 18*a*b^9*c^7*d^3 + 42*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
+ 630*a^4*b^6*c^4*d^6 - 5754*a^5*b^5*c^3*d^7 + 10647*a^6*b^4*c^2*d^8 - 7707
*a^7*b^3*c*d^9 + 1981*a^8*b^2*d^10)*x^2 - 7*(5*b^10*c^9*d + 9*a*b^9*c^8*d^2
 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 630*a^5*
b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 11907*a^7*b^3*c^2*d^8 - 8967*a^8*b^2*c
*d^9 + 2401*a^9*b*d^10)*x + 2520*(a^7*b^3*c^3*d^7 - 3*a^8*b^2*c^2*d^8 + 3*a
^9*b*c*d^9 - a^10*d^10 + (b^10*c^3*d^7 - 3*a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9
- a^3*b^7*d^10)*x^7 + 7*(a*b^9*c^3*d^7 - 3*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^
9 - a^4*b^6*d^10)*x^6 + 21*(a^2*b^8*c^3*d^7 - 3*a^3*b^7*c^2*d^8 + 3*a^4*b^6
*c*d^9 - a^5*b^5*d^10)*x^5 + 35*(a^3*b^7*c^3*d^7 - 3*a^4*b^6*c^2*d^8 + 3*a^
5*b^5*c*d^9 - a^6*b^4*d^10)*x^4 + 35*(a^4*b^6*c^3*d^7 - 3*a^5*b^5*c^2*d^8 +
 3*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 21*(a^5*b^5*c^3*d^7 - 3*a^6*b^4*c^2*
d^8 + 3*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 7*(a^6*b^4*c^3*d^7 - 3*a^7*b^3*
c^2*d^8 + 3*a^8*b^2*c*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^18*x^7 + 7*a*b^
17*x^6 + 21*a^2*b^16*x^5 + 35*a^3*b^15*x^4 + 35*a^4*b^14*x^3 + 21*a^5*b^13*
x^2 + 7*a^6*b^12*x + a^7*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06426, size = 1177, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] 120*(b^3*c^3*d^7 - 3*a*b^2*c^2*d^8 + 3*a^2*b*c*d^9 - a^3*d^10)*log(abs(b*x
+ a))/b^11 - 1/21*(3*b^10*c^10 + 5*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 18*a^3
*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d
^6 - 6534*a^7*b^3*c^3*d^7 + 12987*a^8*b^2*c^2*d^8 - 10047*a^9*b*c*d^9 + 276
1*a^10*d^10 + 4410*(b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2646*(b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 -
30*a^2*b^8*c^3*d^7 + 50*a^3*b^7*c^2*d^8 - 35*a^4*b^6*c*d^9 + 9*a^5*b^5*d^10
)*x^5 + 1470*(b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 110*a^3
*b^7*c^3*d^7 + 195*a^4*b^6*c^2*d^8 - 141*a^5*b^5*c*d^9 + 37*a^6*b^4*d^10)*x
^4 + 210*(3*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 105*a^3*b
^7*c^4*d^6 - 875*a^4*b^6*c^3*d^7 + 1617*a^5*b^5*c^2*d^8 - 1197*a^6*b^4*c*d^
9 + 319*a^7*b^3*d^10)*x^3 + 63*(3*b^10*c^8*d^2 + 6*a*b^9*c^7*d^3 + 14*a^2*b
^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 1918*a^5*b^5*c^3*d^
7 + 3654*a^6*b^4*c^2*d^8 - 2754*a^7*b^3*c*d^9 + 743*a^8*b^2*d^10)*x^2 + 7*(
5*b^10*c^9*d + 9*a*b^9*c^8*d^2 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 +
126*a^4*b^6*c^5*d^5 + 630*a^5*b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 12042*a^
7*b^3*c^2*d^8 - 9207*a^8*b^2*c*d^9 + 2509*a^9*b*d^10)*x)/((b*x + a)^7*b^11)
 + 1/3*(b^16*d^10*x^3 + 15*b^16*c*d^9*x^2 - 12*a*b^15*d^10*x^2 + 135*b^16*c
^2*d^8*x - 240*a*b^15*c*d^9*x + 108*a^2*b^14*d^10*x)/b^24
```

3.1320 $\int \frac{(c+dx)^{10}}{(a+bx)^9} dx$

Optimal. Leaf size=258

$$\frac{d^9x(10bc-9ad)}{b^{10}} - \frac{120d^7(bc-ad)^3}{b^{11}(a+bx)} - \frac{105d^6(bc-ad)^4}{b^{11}(a+bx)^2} - \frac{84d^5(bc-ad)^5}{b^{11}(a+bx)^3} - \frac{105d^4(bc-ad)^6}{2b^{11}(a+bx)^4} - \frac{24d^3(bc-ad)^7}{b^{11}(a+bx)^5} - \frac{15d^2(bc-}{2b^{11}(a+}$$

```
[Out] (d^9*(10*b*c - 9*a*d)*x)/b^10 + (d^10*x^2)/(2*b^9) - (b*c - a*d)^10/(8*b^11
*(a + b*x)^8) - (10*d*(b*c - a*d)^9)/(7*b^11*(a + b*x)^7) - (15*d^2*(b*c -
a*d)^8)/(2*b^11*(a + b*x)^6) - (24*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^5) -
(105*d^4*(b*c - a*d)^6)/(2*b^11*(a + b*x)^4) - (84*d^5*(b*c - a*d)^5)/(b^11
*(a + b*x)^3) - (105*d^6*(b*c - a*d)^4)/(b^11*(a + b*x)^2) - (120*d^7*(b*c
- a*d)^3)/(b^11*(a + b*x)) + (45*d^8*(b*c - a*d)^2*Log[a + b*x])/b^11
```

Rubi [A] time = 0.340847, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^9x(10bc-9ad)}{b^{10}} - \frac{120d^7(bc-ad)^3}{b^{11}(a+bx)} - \frac{105d^6(bc-ad)^4}{b^{11}(a+bx)^2} - \frac{84d^5(bc-ad)^5}{b^{11}(a+bx)^3} - \frac{105d^4(bc-ad)^6}{2b^{11}(a+bx)^4} - \frac{24d^3(bc-ad)^7}{b^{11}(a+bx)^5} - \frac{15d^2(bc-}{2b^{11}(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^9,x]
```

```
[Out] (d^9*(10*b*c - 9*a*d)*x)/b^10 + (d^10*x^2)/(2*b^9) - (b*c - a*d)^10/(8*b^11
*(a + b*x)^8) - (10*d*(b*c - a*d)^9)/(7*b^11*(a + b*x)^7) - (15*d^2*(b*c -
a*d)^8)/(2*b^11*(a + b*x)^6) - (24*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^5) -
(105*d^4*(b*c - a*d)^6)/(2*b^11*(a + b*x)^4) - (84*d^5*(b*c - a*d)^5)/(b^11
*(a + b*x)^3) - (105*d^6*(b*c - a*d)^4)/(b^11*(a + b*x)^2) - (120*d^7*(b*c
- a*d)^3)/(b^11*(a + b*x)) + (45*d^8*(b*c - a*d)^2*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^9} dx &= \int \left(\frac{d^9(10bc-9ad)}{b^{10}} + \frac{d^{10}x}{b^9} + \frac{(bc-ad)^{10}}{b^{10}(a+bx)^9} + \frac{10d(bc-ad)^9}{b^{10}(a+bx)^8} + \frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^7} + \frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^6} + \frac{21}{b} \right.\\ &= \frac{d^9(10bc-9ad)x}{b^{10}} + \frac{d^{10}x^2}{2b^9} - \frac{(bc-ad)^{10}}{8b^{11}(a+bx)^8} - \frac{10d(bc-ad)^9}{7b^{11}(a+bx)^7} - \frac{15d^2(bc-ad)^8}{2b^{11}(a+bx)^6} - \frac{24d^3(bc-ad)^7}{b^{11}(a+bx)^5} - \frac{105d}{2b^{11}}\end{aligned}$$

Mathematica [B] time = 0.313807, size = 712, normalized size = 2.76

$$-a^2b^8d^2\left(1176c^6d^2x^2 + 4704c^5d^3x^3 + 14700c^4d^4x^4 + 47040c^3d^5x^5 - 105840c^2d^6x^6 + 192c^7dx + 15c^8 + 4480cd^7x^7 + 324\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^9,x]
```

```
[Out] (3601*a^10*d^10 + 2*a^9*b*d^9*(-4609*c + 13144*d*x) + a^8*b^2*d^8*(6849*c^2
 - 68704*c*d*x + 81928*d^2*x^2) + 8*a^7*b^3*d^7*(-105*c^3 + 6534*c^2*d*x -
27538*c*d^2*x^2 + 17542*d^3*x^3) + 14*a^6*b^4*d^6*(-15*c^4 - 480*c^3*d*x +
12348*c^2*d^2*x^2 - 28112*c*d^3*x^3 + 10010*d^4*x^4) - 28*a^5*b^5*d^5*(3*c^
5 + 60*c^4*d*x + 840*c^3*d^2*x^2 - 11508*c^2*d^3*x^3 + 15050*c*d^4*x^4 - 27
44*d^5*x^5) - 14*a^4*b^6*d^4*(3*c^6 + 48*c^5*d*x + 420*c^4*d^2*x^2 + 3360*c
^3*d^3*x^3 - 26250*c^2*d^4*x^4 + 19040*c*d^5*x^5 - 1064*d^6*x^6) - 8*a^3*b^
7*d^3*(3*c^7 + 42*c^6*d*x + 294*c^5*d^2*x^2 + 1470*c^4*d^3*x^3 + 7350*c^3*d
^4*x^4 - 32340*c^2*d^5*x^5 + 10780*c*d^6*x^6 + 728*d^7*x^7) - a^2*b^8*d^2*(
15*c^8 + 192*c^7*d*x + 1176*c^6*d^2*x^2 + 4704*c^5*d^3*x^3 + 14700*c^4*d^4*
x^4 + 47040*c^3*d^5*x^5 - 105840*c^2*d^6*x^6 + 4480*c*d^7*x^7 + 3248*d^8*x^
8) - 2*a*b^9*d*(5*c^9 + 60*c^8*d*x + 336*c^7*d^2*x^2 + 1176*c^6*d^3*x^3 + 2
940*c^5*d^4*x^4 + 5880*c^4*d^5*x^5 + 11760*c^3*d^6*x^6 - 10080*c^2*d^7*x^7
- 2240*c*d^8*x^8 + 140*d^9*x^9) - b^10*(7*c^10 + 80*c^9*d*x + 420*c^8*d^2*x
^2 + 1344*c^7*d^3*x^3 + 2940*c^6*d^4*x^4 + 4704*c^5*d^5*x^5 + 5880*c^4*d^6*
x^6 + 6720*c^3*d^7*x^7 - 560*c*d^9*x^9 - 28*d^10*x^10) + 2520*d^8*(b*c - a*
d)^2*(a + b*x)^8*Log[a + b*x])/(56*b^11*(a + b*x)^8)
```

Maple [B] time = 0.017, size = 1256, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^9,x)
```

```
[Out] 1/2*d^10*x^2/b^9-9*d^10/b^10*a*x+10*d^9/b^9*x*c+120/b^11*d^10/(b*x+a)*a^3-1
20/b^8*d^7/(b*x+a)*c^3-1/8/b^11/(b*x+a)^8*a^10*d^10-15/2/b^11*d^10/(b*x+a)^
6*a^8-15/2/b^3*d^2/(b*x+a)^6*c^8-105/b^11*d^10/(b*x+a)^2*a^4+10/7/b^11*d^10
/(b*x+a)^7*a^9-10/7/b^2*d/(b*x+a)^7*c^9+45/b^11*d^10*ln(b*x+a)*a^2+45/b^9*d
^8*ln(b*x+a)*c^2-105/2/b^11*d^10/(b*x+a)^4*a^6-105/2/b^5*d^4/(b*x+a)^4*c^6-
105/b^7*d^6/(b*x+a)^2*c^4+84/b^11*d^10/(b*x+a)^3*a^5-84/b^6*d^5/(b*x+a)^3*c
^5+24/b^11*d^10/(b*x+a)^5*a^7-24/b^4*d^3/(b*x+a)^5*c^7-180/b^6*d^5/(b*x+a)^
7*a^4*c^5+120/b^5*d^4/(b*x+a)^7*a^3*c^6-1/8/b/(b*x+a)^8*c^10+15/b^8/(b*x+a)
^8*a^7*c^3*d^7-105/4/b^7/(b*x+a)^8*a^6*c^4*d^6+63/2/b^6/(b*x+a)^8*a^5*c^5*d
^5-105/4/b^5/(b*x+a)^8*a^4*c^6*d^4+15/b^4/(b*x+a)^8*a^3*c^7*d^3-45/8/b^3/(b
*x+a)^8*a^2*c^8*d^2+5/4/b^2/(b*x+a)^8*a*c^9*d+60/b^10*d^9/(b*x+a)^6*a^7*c-2
10/b^9*d^8/(b*x+a)^6*a^6*c^2+360/7/b^9*d^8/(b*x+a)^7*a^7*c^2-120/b^8*d^7/(b
*x+a)^7*a^6*c^3+180/b^7*d^6/(b*x+a)^7*a^5*c^4-360/7/b^4*d^3/(b*x+a)^7*a^2*c
^7+90/7/b^3*d^2/(b*x+a)^7*a*c^8+420/b^8*d^7/(b*x+a)^6*a^5*c^3-525/b^7*d^6/(
b*x+a)^6*a^4*c^4+420/b^6*d^5/(b*x+a)^6*a^3*c^5-1575/2/b^9*d^8/(b*x+a)^4*a^4
*c^2+1050/b^8*d^7/(b*x+a)^4*a^3*c^3-1575/2/b^7*d^6/(b*x+a)^4*a^2*c^4+315/b^
10*d^9/(b*x+a)^4*a^5*c-90/b^10*d^9*ln(b*x+a)*a*c+315/b^6*d^5/(b*x+a)^4*a*c^
5-90/7/b^10*d^9/(b*x+a)^7*a^8*c+420/b^7*d^6/(b*x+a)^3*a*c^4-168/b^10*d^9/(b
*x+a)^5*a^6*c+504/b^9*d^8/(b*x+a)^5*a^5*c^2-840/b^8*d^7/(b*x+a)^5*a^4*c^3-3
60/b^10*d^9/(b*x+a)*a^2*c+360/b^9*d^8/(b*x+a)*a*c^2+5/4/b^10/(b*x+a)^8*a^9*
c*d^9-45/8/b^9/(b*x+a)^8*a^8*c^2*d^8+168/b^5*d^4/(b*x+a)^5*a*c^6-210/b^5*d^
4/(b*x+a)^6*a^2*c^6+60/b^4*d^3/(b*x+a)^6*a*c^7+420/b^10*d^9/(b*x+a)^2*a^3*c
-630/b^9*d^8/(b*x+a)^2*a^2*c^2+420/b^8*d^7/(b*x+a)^2*a*c^3-420/b^10*d^9/(b*
x+a)^3*a^4*c+840/b^9*d^8/(b*x+a)^3*a^3*c^2-840/b^8*d^7/(b*x+a)^3*a^2*c^3+84
0/b^7*d^6/(b*x+a)^5*a^3*c^4-504/b^6*d^5/(b*x+a)^5*a^2*c^5
```

Maxima [B] time = 1.37796, size = 1276, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/56*(7*b^10*c^10 + 10*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d
^3 + 42*a^4*b^6*c^6*d^4 + 84*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^
7*b^3*c^3*d^7 - 6849*a^8*b^2*c^2*d^8 + 9218*a^9*b*c*d^9 - 3601*a^10*d^10 +
6720*(b^10*c^3*d^7 - 3*a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7
+ 5880*(b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 - 18*a^2*b^8*c^2*d^8 + 20*a^3*b^7*c*
d^9 - 7*a^4*b^6*d^10)*x^6 + 2352*(2*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 20*a^2
*b^8*c^3*d^7 - 110*a^3*b^7*c^2*d^8 + 130*a^4*b^6*c*d^9 - 47*a^5*b^5*d^10)*x
^5 + 2940*(b^10*c^6*d^4 + 2*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 20*a^3*b^7*
c^3*d^7 - 125*a^4*b^6*c^2*d^8 + 154*a^5*b^5*c*d^9 - 57*a^6*b^4*d^10)*x^4 +
336*(4*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 14*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4
*d^6 + 140*a^4*b^6*c^3*d^7 - 959*a^5*b^5*c^2*d^8 + 1218*a^6*b^4*c*d^9 - 459
*a^7*b^3*d^10)*x^3 + 84*(5*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*
d^4 + 28*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 - 2058*
a^6*b^4*c^2*d^8 + 2676*a^7*b^3*c*d^9 - 1023*a^8*b^2*d^10)*x^2 + 8*(10*b^10*
c^9*d + 15*a*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4
*b^6*c^5*d^5 + 210*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2
*d^8 + 8658*a^8*b^2*c*d^9 - 3349*a^9*b*d^10)*x)/(b^19*x^8 + 8*a*b^18*x^7 +
28*a^2*b^17*x^6 + 56*a^3*b^16*x^5 + 70*a^4*b^15*x^4 + 56*a^5*b^14*x^3 + 28*
a^6*b^13*x^2 + 8*a^7*b^12*x + a^8*b^11) + 1/2*(b*d^10*x^2 + 2*(10*b*c*d^9 -
 9*a*d^10)*x)/b^10 + 45*(b^2*c^2*d^8 - 2*a*b*c*d^9 + a^2*d^10)*log(b*x + a)
/b^11
```

Fricas [B] time = 1.8687, size = 2763, normalized size = 10.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] 1/56*(28*b^10*d^10*x^10 - 7*b^10*c^10 - 10*a*b^9*c^9*d - 15*a^2*b^8*c^8*d^2
 - 24*a^3*b^7*c^7*d^3 - 42*a^4*b^6*c^6*d^4 - 84*a^5*b^5*c^5*d^5 - 210*a^6*b
^4*c^4*d^6 - 840*a^7*b^3*c^3*d^7 + 6849*a^8*b^2*c^2*d^8 - 9218*a^9*b*c*d^9
+ 3601*a^10*d^10 + 280*(2*b^10*c*d^9 - a*b^9*d^10)*x^9 + 112*(40*a*b^9*c*d^
9 - 29*a^2*b^8*d^10)*x^8 - 448*(15*b^10*c^3*d^7 - 45*a*b^9*c^2*d^8 + 10*a^2
*b^8*c*d^9 + 13*a^3*b^7*d^10)*x^7 - 392*(15*b^10*c^4*d^6 + 60*a*b^9*c^3*d^7
 - 270*a^2*b^8*c^2*d^8 + 220*a^3*b^7*c*d^9 - 38*a^4*b^6*d^10)*x^6 - 784*(6*
b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 60*a^2*b^8*c^3*d^7 - 330*a^3*b^7*c^2*d^8
+ 340*a^4*b^6*c*d^9 - 98*a^5*b^5*d^10)*x^5 - 980*(3*b^10*c^6*d^4 + 6*a*b^9*
c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 60*a^3*b^7*c^3*d^7 - 375*a^4*b^6*c^2*d^8 + 4
30*a^5*b^5*c*d^9 - 143*a^6*b^4*d^10)*x^4 - 112*(12*b^10*c^7*d^3 + 21*a*b^9*
c^6*d^4 + 42*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 420*a^4*b^6*c^3*d^7 -
2877*a^5*b^5*c^2*d^8 + 3514*a^6*b^4*c*d^9 - 1253*a^7*b^3*d^10)*x^3 - 28*(15
*b^10*c^8*d^2 + 24*a*b^9*c^7*d^3 + 42*a^2*b^8*c^6*d^4 + 84*a^3*b^7*c^5*d^5
+ 210*a^4*b^6*c^4*d^6 + 840*a^5*b^5*c^3*d^7 - 6174*a^6*b^4*c^2*d^8 + 7868*a
^7*b^3*c*d^9 - 2926*a^8*b^2*d^10)*x^2 - 8*(10*b^10*c^9*d + 15*a*b^9*c^8*d^2
 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4*b^6*c^5*d^5 + 210*a^5*b
^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2*d^8 + 8588*a^8*b^2*c*d^
9 - 3286*a^9*b*d^10)*x + 2520*(a^8*b^2*c^2*d^8 - 2*a^9*b*c*d^9 + a^10*d^10
```

```
+ (b^10*c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 8*(a*b^9*c^2*d^8 - 2*
a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 28*(a^2*b^8*c^2*d^8 - 2*a^3*b^7*c*d^9 +
 a^4*b^6*d^10)*x^6 + 56*(a^3*b^7*c^2*d^8 - 2*a^4*b^6*c*d^9 + a^5*b^5*d^10)*
x^5 + 70*(a^4*b^6*c^2*d^8 - 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 56*(a^5*b
^5*c^2*d^8 - 2*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 28*(a^6*b^4*c^2*d^8 - 2*
a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 8*(a^7*b^3*c^2*d^8 - 2*a^8*b^2*c*d^9 +
a^9*b*d^10)*x)*log(b*x + a))/(b^19*x^8 + 8*a*b^18*x^7 + 28*a^2*b^17*x^6 + 5
6*a^3*b^16*x^5 + 70*a^4*b^15*x^4 + 56*a^5*b^14*x^3 + 28*a^6*b^13*x^2 + 8*a^
7*b^12*x + a^8*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**9,x)

[Out] Timed out
```

Giac [B] time = 1.069, size = 1176, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="giac")

[Out] 45*(b^2*c^2*d^8 - 2*a*b*c*d^9 + a^2*d^10)*log(abs(b*x + a))/b^11 + 1/2*(b^9
*d^10*x^2 + 20*b^9*c*d^9*x - 18*a*b^8*d^10*x)/b^18 - 1/56*(7*b^10*c^10 + 10
*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4
 + 84*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 - 6849*a^
8*b^2*c^2*d^8 + 9218*a^9*b*c*d^9 - 3601*a^10*d^10 + 6720*(b^10*c^3*d^7 - 3*
a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 5880*(b^10*c^4*d^6 +
4*a*b^9*c^3*d^7 - 18*a^2*b^8*c^2*d^8 + 20*a^3*b^7*c*d^9 - 7*a^4*b^6*d^10)*x
^6 + 2352*(2*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 - 110*a^3*
b^7*c^2*d^8 + 130*a^4*b^6*c*d^9 - 47*a^5*b^5*d^10)*x^5 + 2940*(b^10*c^6*d^4
 + 2*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 - 125*a^4*b^6*c
^2*d^8 + 154*a^5*b^5*c*d^9 - 57*a^6*b^4*d^10)*x^4 + 336*(4*b^10*c^7*d^3 + 7
*a*b^9*c^6*d^4 + 14*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*
d^7 - 959*a^5*b^5*c^2*d^8 + 1218*a^6*b^4*c*d^9 - 459*a^7*b^3*d^10)*x^3 + 84
*(5*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*d^4 + 28*a^3*b^7*c^5*d^
5 + 70*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 - 2058*a^6*b^4*c^2*d^8 + 2676*
a^7*b^3*c*d^9 - 1023*a^8*b^2*d^10)*x^2 + 8*(10*b^10*c^9*d + 15*a*b^9*c^8*d^
2 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4*b^6*c^5*d^5 + 210*a^5*
b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2*d^8 + 8658*a^8*b^2*c*d
^9 - 3349*a^9*b*d^10)*x)/((b*x + a)^8*b^11)
```

3.1321 $\int \frac{(c+dx)^{10}}{(a+bx)^{10}} dx$

Optimal. Leaf size=257

$$-\frac{45d^8(bc-ad)^2}{b^{11}(a+bx)}-\frac{60d^7(bc-ad)^3}{b^{11}(a+bx)^2}-\frac{70d^6(bc-ad)^4}{b^{11}(a+bx)^3}-\frac{63d^5(bc-ad)^5}{b^{11}(a+bx)^4}-\frac{42d^4(bc-ad)^6}{b^{11}(a+bx)^5}-\frac{20d^3(bc-ad)^7}{b^{11}(a+bx)^6}-\frac{45d^2(bc-ad}{7b^{11}(a+bx}$$

```
[Out] (d^10*x)/b^10 - (b*c - a*d)^10/(9*b^11*(a + b*x)^9) - (5*d*(b*c - a*d)^9)/(
4*b^11*(a + b*x)^8) - (45*d^2*(b*c - a*d)^8)/(7*b^11*(a + b*x)^7) - (20*d^3
*(b*c - a*d)^7)/(b^11*(a + b*x)^6) - (42*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)
^5) - (63*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^4) - (70*d^6*(b*c - a*d)^4)/(b
^11*(a + b*x)^3) - (60*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^2) - (45*d^8*(b*c
 - a*d)^2)/(b^11*(a + b*x)) + (10*d^9*(b*c - a*d)*Log[a + b*x])/b^11
```

Rubi [A] time = 0.311267, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{45d^8(bc-ad)^2}{b^{11}(a+bx)}-\frac{60d^7(bc-ad)^3}{b^{11}(a+bx)^2}-\frac{70d^6(bc-ad)^4}{b^{11}(a+bx)^3}-\frac{63d^5(bc-ad)^5}{b^{11}(a+bx)^4}-\frac{42d^4(bc-ad)^6}{b^{11}(a+bx)^5}-\frac{20d^3(bc-ad)^7}{b^{11}(a+bx)^6}-\frac{45d^2(bc-ad}{7b^{11}(a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^10,x]
```

```
[Out] (d^10*x)/b^10 - (b*c - a*d)^10/(9*b^11*(a + b*x)^9) - (5*d*(b*c - a*d)^9)/(
4*b^11*(a + b*x)^8) - (45*d^2*(b*c - a*d)^8)/(7*b^11*(a + b*x)^7) - (20*d^3
*(b*c - a*d)^7)/(b^11*(a + b*x)^6) - (42*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)
^5) - (63*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^4) - (70*d^6*(b*c - a*d)^4)/(b
^11*(a + b*x)^3) - (60*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^2) - (45*d^8*(b*c
 - a*d)^2)/(b^11*(a + b*x)) + (10*d^9*(b*c - a*d)*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{10}} dx = \int \left(\frac{d^{10}}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{10}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^9}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^8}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^7}+\frac{210d^4(bc-ad)^6}{b^{10}(a+bx)^6}+\frac{2}{}\right.$$
$$=\frac{d^{10}x}{b^{10}}-\frac{(bc-ad)^{10}}{9b^{11}(a+bx)^9}-\frac{5d(bc-ad)^9}{4b^{11}(a+bx)^8}-\frac{45d^2(bc-ad)^8}{7b^{11}(a+bx)^7}-\frac{20d^3(bc-ad)^7}{b^{11}(a+bx)^6}-\frac{42d^4(bc-ad)^6}{b^{11}(a+bx)^5}-\frac{63d^5(bc}{b^{11}(a}$$

Mathematica [B] time = 0.411316, size = 708, normalized size = 2.75

$$-\frac{9a^2b^8d^2\left(336c^6d^2x^2+1176c^5d^3x^3+2940c^4d^4x^4+5880c^3d^5x^5+11760c^2d^6x^6+60c^7dx+5c^8-15120cd^7x^7+252d^8x^8\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^10,x]
```

```
[Out] -(4861*a^10*d^10 + a^9*b*d^9*(-7129*c + 41229*d*x) + 9*a^8*b^2*d^8*(140*c^2
 - 6849*c*d*x + 17064*d^2*x^2) + 12*a^7*b^3*d^7*(35*c^3 + 945*c^2*d*x - 196
02*c*d^2*x^2 + 27342*d^3*x^3) + 42*a^6*b^4*d^6*(5*c^4 + 90*c^3*d*x + 1080*c
^2*d^2*x^2 - 12348*c*d^3*x^3 + 10458*d^4*x^4) + 126*a^5*b^5*d^5*(c^5 + 15*c
^4*d*x + 120*c^3*d^2*x^2 + 840*c^2*d^3*x^3 - 5754*c*d^4*x^4 + 2982*d^5*x^5)
 + 42*a^4*b^6*d^4*(2*c^6 + 27*c^5*d*x + 180*c^4*d^2*x^2 + 840*c^3*d^3*x^3 +
 3780*c^2*d^4*x^4 - 15750*c*d^5*x^5 + 4704*d^6*x^6) + 12*a^3*b^7*d^3*(5*c^7
 + 63*c^6*d*x + 378*c^5*d^2*x^2 + 1470*c^4*d^3*x^3 + 4410*c^3*d^4*x^4 + 132
30*c^2*d^5*x^5 - 32340*c*d^6*x^6 + 4536*d^7*x^7) + 9*a^2*b^8*d^2*(5*c^8 + 6
0*c^7*d*x + 336*c^6*d^2*x^2 + 1176*c^5*d^3*x^3 + 2940*c^4*d^4*x^4 + 5880*c^
3*d^5*x^5 + 11760*c^2*d^6*x^6 - 15120*c*d^7*x^7 + 252*d^8*x^8) + a*b^9*d*(3
5*c^9 + 405*c^8*d*x + 2160*c^7*d^2*x^2 + 7056*c^6*d^3*x^3 + 15876*c^5*d^4*x
^4 + 26460*c^4*d^5*x^5 + 35280*c^3*d^6*x^6 + 45360*c^2*d^7*x^7 - 22680*c*d^
8*x^8 - 2268*d^9*x^9) + b^10*(28*c^10 + 315*c^9*d*x + 1620*c^8*d^2*x^2 + 50
40*c^7*d^3*x^3 + 10584*c^6*d^4*x^4 + 15876*c^5*d^5*x^5 + 17640*c^4*d^6*x^6
+ 15120*c^3*d^7*x^7 + 11340*c^2*d^8*x^8 - 252*d^10*x^10) + 2520*d^9*(-(b*c)
 + a*d)*(a + b*x)^9*Log[a + b*x])/(252*b^11*(a + b*x)^9)
```

Maple [B] time = 0.017, size = 1266, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^10,x)
```

```
[Out] -450/b^7*d^6/(b*x+a)^7*a^4*c^4+360/b^6*d^5/(b*x+a)^7*a^3*c^5-45/4/b^10*d^9/
(b*x+a)^8*a^8*c+45/b^9*d^8/(b*x+a)^8*a^7*c^2-105/b^8*d^7/(b*x+a)^8*a^6*c^3+
315/2/b^7*d^6/(b*x+a)^8*a^5*c^4-315/2/b^6*d^5/(b*x+a)^8*a^4*c^5+105/b^5*d^4
/(b*x+a)^8*a^3*c^6-45/b^4*d^3/(b*x+a)^8*a^2*c^7+45/4/b^3*d^2/(b*x+a)^8*a*c^
8-140/b^10*d^9/(b*x+a)^6*a^6*c+420/b^9*d^8/(b*x+a)^6*a^5*c^2-70/3/b^5/(b*x+
a)^9*a^4*c^6*d^4+40/3/b^4/(b*x+a)^9*a^3*c^7*d^3-5/b^3/(b*x+a)^9*a^2*c^8*d^2
+10/9/b^2/(b*x+a)^9*a*c^9*d+360/7/b^10*d^9/(b*x+a)^7*a^7*c-180/b^5*d^4/(b*x
+a)^7*a^2*c^6+360/7/b^4*d^3/(b*x+a)^7*a*c^7+315/b^7*d^6/(b*x+a)^4*a*c^4+10/
9/b^10/(b*x+a)^9*a^9*c*d^9-5/b^9/(b*x+a)^9*a^8*c^2*d^8+40/3/b^8/(b*x+a)^9*a
^7*c^3*d^7-70/3/b^7/(b*x+a)^9*a^6*c^4*d^6+28/b^6/(b*x+a)^9*a^5*c^5*d^5-700/
b^8*d^7/(b*x+a)^6*a^4*c^3+700/b^7*d^6/(b*x+a)^6*a^3*c^4+360/b^8*d^7/(b*x+a)
^7*a^5*c^3-180/b^9*d^8/(b*x+a)^7*a^6*c^2-45/b^9*d^8/(b*x+a)*c^2+5/4/b^11*d^
10/(b*x+a)^8*a^9-5/4/b^2*d/(b*x+a)^8*c^9+20/b^11*d^10/(b*x+a)^6*a^7-20/b^4*
d^3/(b*x+a)^6*c^7-45/7/b^11*d^10/(b*x+a)^7*a^8-45/7/b^3*d^2/(b*x+a)^7*c^8-4
2/b^5*d^4/(b*x+a)^5*c^6-10/b^11*d^10*ln(b*x+a)*a+10/b^10*d^9*ln(b*x+a)*c+63
/b^11*d^10/(b*x+a)^4*a^5-63/b^6*d^5/(b*x+a)^4*c^5-1/9/b^11/(b*x+a)^9*a^10*d
^10+60/b^11*d^10/(b*x+a)^2*a^3-60/b^8*d^7/(b*x+a)^2*c^3-70/b^11*d^10/(b*x+a
)^3*a^4-70/b^7*d^6/(b*x+a)^3*c^4-42/b^11*d^10/(b*x+a)^5*a^6-45/b^11*d^10/(b
*x+a)*a^2-1/9/b/(b*x+a)^9*c^10-420/b^6*d^5/(b*x+a)^6*a^2*c^5+140/b^5*d^4/(b
*x+a)^6*a*c^6-180/b^10*d^9/(b*x+a)^2*a^2*c+d^10*x/b^10+180/b^9*d^8/(b*x+a)^
2*a*c^2+280/b^10*d^9/(b*x+a)^3*a^3*c-420/b^9*d^8/(b*x+a)^3*a^2*c^2+280/b^8*
d^7/(b*x+a)^3*a*c^3+252/b^10*d^9/(b*x+a)^5*a^5*c-630/b^9*d^8/(b*x+a)^5*a^4*
c^2+840/b^8*d^7/(b*x+a)^5*a^3*c^3-630/b^7*d^6/(b*x+a)^5*a^2*c^4+252/b^6*d^5
/(b*x+a)^5*a*c^5-315/b^10*d^9/(b*x+a)^4*a^4*c+630/b^9*d^8/(b*x+a)^4*a^3*c^2
-630/b^8*d^7/(b*x+a)^4*a^2*c^3+90/b^10*d^9/(b*x+a)*a*c
```

Maxima [B] time = 1.30478, size = 1292, normalized size = 5.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] d^10*x/b^10 - 1/252*(28*b^10*c^10 + 35*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 6
0*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*
c^4*d^6 + 420*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 - 7129*a^9*b*c*d^9 + 4
861*a^10*d^10 + 11340*(b^10*c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1
5120*(b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 - 9*a^2*b^8*c*d^9 + 5*a^3*b^7*d^10)*x^
7 + 17640*(b^10*c^4*d^6 + 2*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 22*a^3*b^7*
c*d^9 + 13*a^4*b^6*d^10)*x^6 + 5292*(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*
a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d^8 - 125*a^4*b^6*c*d^9 + 77*a^5*b^5*d^10)
*x^5 + 5292*(2*b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*
b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^8 - 137*a^5*b^5*c*d^9 + 87*a^6*b^4*d^10)*x^4
 + 504*(10*b^10*c^7*d^3 + 14*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^
7*c^4*d^6 + 70*a^4*b^6*c^3*d^7 + 210*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 +
 669*a^7*b^3*d^10)*x^3 + 108*(15*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b
^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7
+ 420*a^6*b^4*c^2*d^8 - 2178*a^7*b^3*c*d^9 + 1443*a^8*b^2*d^10)*x^2 + 9*(35
*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 60*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 +
126*a^4*b^6*c^5*d^5 + 210*a^5*b^5*c^4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*
b^3*c^2*d^8 - 6849*a^8*b^2*c*d^9 + 4609*a^9*b*d^10)*x)/(b^20*x^9 + 9*a*b^19
*x^8 + 36*a^2*b^18*x^7 + 84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*
x^4 + 84*a^6*b^14*x^3 + 36*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11) + 10*(b*
c*d^9 - a*d^10)*log(b*x + a)/b^11
```

Fricas [B] time = 1.96615, size = 2597, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*d^10*x^10 + 2268*a*b^9*d^10*x^9 - 28*b^10*c^10 - 35*a*b^9*c
^9*d - 45*a^2*b^8*c^8*d^2 - 60*a^3*b^7*c^7*d^3 - 84*a^4*b^6*c^6*d^4 - 126*a
^5*b^5*c^5*d^5 - 210*a^6*b^4*c^4*d^6 - 420*a^7*b^3*c^3*d^7 - 1260*a^8*b^2*c
^2*d^8 + 7129*a^9*b*c*d^9 - 4861*a^10*d^10 - 2268*(5*b^10*c^2*d^8 - 10*a*b^
9*c*d^9 + a^2*b^8*d^10)*x^8 - 3024*(5*b^10*c^3*d^7 + 15*a*b^9*c^2*d^8 - 45*
a^2*b^8*c*d^9 + 18*a^3*b^7*d^10)*x^7 - 3528*(5*b^10*c^4*d^6 + 10*a*b^9*c^3*
d^7 + 30*a^2*b^8*c^2*d^8 - 110*a^3*b^7*c*d^9 + 56*a^4*b^6*d^10)*x^6 - 5292*
(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d^8
 - 125*a^4*b^6*c*d^9 + 71*a^5*b^5*d^10)*x^5 - 5292*(2*b^10*c^6*d^4 + 3*a*b^
9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^8 - 1
37*a^5*b^5*c*d^9 + 83*a^6*b^4*d^10)*x^4 - 504*(10*b^10*c^7*d^3 + 14*a*b^9*c
^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 70*a^4*b^6*c^3*d^7 + 210
*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 + 651*a^7*b^3*d^10)*x^3 - 108*(15*b^1
0*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 70
*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 - 2178*a^7*b^3
*c*d^9 + 1422*a^8*b^2*d^10)*x^2 - 9*(35*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 60*
a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 210*a^5*b^5*c^
4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 - 6849*a^8*b^2*c*d^9 + 4
581*a^9*b*d^10)*x + 2520*(a^9*b*c*d^9 - a^10*d^10 + (b^10*c*d^9 - a*b^9*d^1
```

```
0)*x^9 + 9*(a*b^9*c*d^9 - a^2*b^8*d^10)*x^8 + 36*(a^2*b^8*c*d^9 - a^3*b^7*d
^10)*x^7 + 84*(a^3*b^7*c*d^9 - a^4*b^6*d^10)*x^6 + 126*(a^4*b^6*c*d^9 - a^5
*b^5*d^10)*x^5 + 126*(a^5*b^5*c*d^9 - a^6*b^4*d^10)*x^4 + 84*(a^6*b^4*c*d^9
 - a^7*b^3*d^10)*x^3 + 36*(a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 9*(a^8*b^2*c
*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*
x^7 + 84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x
^3 + 36*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**10,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06507, size = 1170, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] d^10*x/b^10 + 10*(b*c*d^9 - a*d^10)*log(abs(b*x + a))/b^11 - 1/252*(28*b^10
*c^10 + 35*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^3 + 84*a^4*b
^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 420*a^7*b^3*c^3*d^
7 + 1260*a^8*b^2*c^2*d^8 - 7129*a^9*b*c*d^9 + 4861*a^10*d^10 + 11340*(b^10*
c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 15120*(b^10*c^3*d^7 + 3*a*b^9
*c^2*d^8 - 9*a^2*b^8*c*d^9 + 5*a^3*b^7*d^10)*x^7 + 17640*(b^10*c^4*d^6 + 2*
a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 22*a^3*b^7*c*d^9 + 13*a^4*b^6*d^10)*x^6
 + 5292*(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 30*a^3*b^7
*c^2*d^8 - 125*a^4*b^6*c*d^9 + 77*a^5*b^5*d^10)*x^5 + 5292*(2*b^10*c^6*d^4
+ 3*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2
*d^8 - 137*a^5*b^5*c*d^9 + 87*a^6*b^4*d^10)*x^4 + 504*(10*b^10*c^7*d^3 + 14
*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 70*a^4*b^6*c^3*d
^7 + 210*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 + 669*a^7*b^3*d^10)*x^3 + 108
*(15*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 42*a^3*b^7*c^5*
d^5 + 70*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 - 2178
*a^7*b^3*c*d^9 + 1443*a^8*b^2*d^10)*x^2 + 9*(35*b^10*c^9*d + 45*a*b^9*c^8*d
^2 + 60*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 210*a^
5*b^5*c^4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 - 6849*a^8*b^2*c
*d^9 + 4609*a^9*b*d^10)*x)/((b*x + a)^9*b^11)
```

3.1322 $\int \frac{(c+dx)^{10}}{(a+bx)^{11}}\,dx$

Optimal. Leaf size=271

$$-\frac{10d^9(bc-ad)}{b^{11}(a+bx)}-\frac{45d^8(bc-ad)^2}{2b^{11}(a+bx)^2}-\frac{40d^7(bc-ad)^3}{b^{11}(a+bx)^3}-\frac{105d^6(bc-ad)^4}{2b^{11}(a+bx)^4}-\frac{252d^5(bc-ad)^5}{5b^{11}(a+bx)^5}-\frac{35d^4(bc-ad)^6}{b^{11}(a+bx)^6}-\frac{120d^3(bc-}{7b^{11}(a+b}$$

```
[Out] -(b*c - a*d)^10/(10*b^11*(a + b*x)^10) - (10*d*(b*c - a*d)^9)/(9*b^11*(a +
b*x)^9) - (45*d^2*(b*c - a*d)^8)/(8*b^11*(a + b*x)^8) - (120*d^3*(b*c - a*d
)^7)/(7*b^11*(a + b*x)^7) - (35*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)^6) - (25
2*d^5*(b*c - a*d)^5)/(5*b^11*(a + b*x)^5) - (105*d^6*(b*c - a*d)^4)/(2*b^11
*(a + b*x)^4) - (40*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^3) - (45*d^8*(b*c -
a*d)^2)/(2*b^11*(a + b*x)^2) - (10*d^9*(b*c - a*d))/(b^11*(a + b*x)) + (d^1
0*Log[a + b*x])/b^11
```

Rubi [A] time = 0.286844, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{10d^9(bc-ad)}{b^{11}(a+bx)}-\frac{45d^8(bc-ad)^2}{2b^{11}(a+bx)^2}-\frac{40d^7(bc-ad)^3}{b^{11}(a+bx)^3}-\frac{105d^6(bc-ad)^4}{2b^{11}(a+bx)^4}-\frac{252d^5(bc-ad)^5}{5b^{11}(a+bx)^5}-\frac{35d^4(bc-ad)^6}{b^{11}(a+bx)^6}-\frac{120d^3(bc-}{7b^{11}(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^11,x]
```

```
[Out] -(b*c - a*d)^10/(10*b^11*(a + b*x)^10) - (10*d*(b*c - a*d)^9)/(9*b^11*(a +
b*x)^9) - (45*d^2*(b*c - a*d)^8)/(8*b^11*(a + b*x)^8) - (120*d^3*(b*c - a*d
)^7)/(7*b^11*(a + b*x)^7) - (35*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)^6) - (25
2*d^5*(b*c - a*d)^5)/(5*b^11*(a + b*x)^5) - (105*d^6*(b*c - a*d)^4)/(2*b^11
*(a + b*x)^4) - (40*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^3) - (45*d^8*(b*c -
a*d)^2)/(2*b^11*(a + b*x)^2) - (10*d^9*(b*c - a*d))/(b^11*(a + b*x)) + (d^1
0*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{11}}\,dx &= \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{11}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{10}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^9}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^8}+\frac{210d^4(bc-ad)^6}{b^{10}(a+bx)^7}+\frac{252d^5(b}{b^{10}(a\cdot}\right.\\ &= -\frac{(bc-ad)^{10}}{10b^{11}(a+bx)^{10}}-\frac{10d(bc-ad)^9}{9b^{11}(a+bx)^9}-\frac{45d^2(bc-ad)^8}{8b^{11}(a+bx)^8}-\frac{120d^3(bc-ad)^7}{7b^{11}(a+bx)^7}-\frac{35d^4(bc-ad)^6}{b^{11}(a+bx)^6}-\frac{252d^5(bc\cdot}{5b^{11}(a+}\end{aligned}$$

Mathematica [B] time = 0.358232, size = 591, normalized size = 2.18

$$\frac{d^{10}\log(a+bx)}{b^{11}}-\frac{(bc-ad)\left(a^2b^7d^2\left(49275c^5d^2x^2+154080c^4d^3x^3+326340c^3d^4x^4+497448c^2d^5x^5+9550c^6dx+847c^7\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^11,x]
```

```
[Out] -((b*c - a*d)*(7381*a^9*d^9 + a^8*b*d^8*(4861*c + 71290*d*x) + a^7*b^2*d^7*
(3601*c^2 + 46090*c*d*x + 308205*d^2*x^2) + a^6*b^3*d^6*(2761*c^3 + 33490*c
^2*d*x + 194805*c*d^2*x^2 + 784080*d^3*x^3) + a^5*b^4*d^5*(2131*c^4 + 25090
*c^3*d*x + 138105*c^2*d^2*x^2 + 481680*c*d^3*x^3 + 1296540*d^4*x^4) + a^4*b
^5*d^4*(1627*c^5 + 18790*c^4*d*x + 100305*c^3*d^2*x^2 + 330480*c^2*d^3*x^3
+ 767340*c*d^4*x^4 + 1450008*d^5*x^5) + a^3*b^6*d^3*(1207*c^6 + 13750*c^5*d
*x + 71955*c^4*d^2*x^2 + 229680*c^3*d^3*x^3 + 502740*c^2*d^4*x^4 + 814968*c
*d^5*x^5 + 1102500*d^6*x^6) + a^2*b^7*d^2*(847*c^7 + 9550*c^6*d*x + 49275*c
^5*d^2*x^2 + 154080*c^4*d^3*x^3 + 326340*c^3*d^4*x^4 + 497448*c^2*d^5*x^5 +
 573300*c*d^6*x^6 + 554400*d^7*x^7) + a*b^8*d*(532*c^8 + 5950*c^7*d*x + 303
75*c^6*d^2*x^2 + 93600*c^5*d^3*x^3 + 194040*c^4*d^4*x^4 + 285768*c^3*d^5*x^
5 + 308700*c^2*d^6*x^6 + 252000*c*d^7*x^7 + 170100*d^8*x^8) + b^9*(252*c^9
+ 2800*c^8*d*x + 14175*c^7*d^2*x^2 + 43200*c^6*d^3*x^3 + 88200*c^5*d^4*x^4
+ 127008*c^4*d^5*x^5 + 132300*c^3*d^6*x^6 + 100800*c^2*d^7*x^7 + 56700*c*d^
8*x^8 + 25200*d^9*x^9)))/(2520*b^11*(a + b*x)^10) + (d^10*Log[a + b*x])/b^1
1
```

Maple [B] time = 0.012, size = 1271, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^11,x)
```

```
[Out] 40*d^8/b^9/(b*x+a)^9*a^7*c^2+600*d^6/b^7/(b*x+a)^7*a^3*c^4-360*d^5/b^6/(b*x
+a)^7*a^2*c^5+120*d^4/b^5/(b*x+a)^7*a*c^6+140*d^6/b^7/(b*x+a)^9*a^5*c^4-140
*d^5/b^6/(b*x+a)^9*a^4*c^5-21/b^5/(b*x+a)^10*c^6*d^4*a^4+12/b^4/(b*x+a)^10*
a^3*c^7*d^3-9/2/b^3/(b*x+a)^10*c^8*d^2*a^2+1/b^2/(b*x+a)^10*c^9*d*a+12/b^8/
(b*x+a)^10*c^3*d^7*a^7-21/b^7/(b*x+a)^10*a^6*c^4*d^6+126/5/b^6/(b*x+a)^10*a
^5*c^5*d^5+1/b^10/(b*x+a)^10*c*d^9*a^9-9/2/b^9/(b*x+a)^10*c^2*d^8*a^8+210*d
^9/b^10/(b*x+a)^4*a^3*c-315*d^8/b^9/(b*x+a)^4*a^2*c^2+210*d^7/b^8/(b*x+a)^4
*a*c^3-10*d^9/b^10/(b*x+a)^9*a^8*c+10*d^2/b^3/(b*x+a)^9*a*c^8-120*d^9/b^10/
(b*x+a)^7*a^6*c+360*d^8/b^9/(b*x+a)^7*a^5*c^2-600*d^7/b^8/(b*x+a)^7*a^4*c^3
-45/8*d^10/b^11/(b*x+a)^8*a^8-45/8*d^2/b^3/(b*x+a)^8*c^8-35*d^10/b^11/(b*x+
a)^6*a^6-35*d^4/b^5/(b*x+a)^6*c^6-45/2*d^10/b^11/(b*x+a)^2*a^2-45/2*d^8/b^9
/(b*x+a)^2*c^2-1/10/b^11/(b*x+a)^10*a^10*d^10+10/b^11*d^10/(b*x+a)*a-10/b^1
0*d^9/(b*x+a)*c+120/7*d^10/b^11/(b*x+a)^7*a^7-120/7*d^3/b^4/(b*x+a)^7*c^7+1
0/9*d^10/b^11/(b*x+a)^9*a^9-10/9*d/b^2/(b*x+a)^9*c^9+40*d^10/b^11/(b*x+a)^3
*a^3-40*d^7/b^8/(b*x+a)^3*c^3+252/5*d^10/b^11/(b*x+a)^5*a^5-252/5*d^5/b^6/(
b*x+a)^5*c^5-105/2*d^10/b^11/(b*x+a)^4*a^4-105/2*d^6/b^7/(b*x+a)^4*c^4-1/10
/b/(b*x+a)^10*c^10+280/3*d^4/b^5/(b*x+a)^9*a^3*c^6-40*d^3/b^4/(b*x+a)^9*a^2
*c^7+d^10*ln(b*x+a)/b^11-280/3*d^7/b^8/(b*x+a)^9*a^6*c^3+45*d^9/b^10/(b*x+a
)^2*a*c-120*d^9/b^10/(b*x+a)^3*a^2*c+120*d^8/b^9/(b*x+a)^3*a*c^2-252*d^9/b^
10/(b*x+a)^5*a^4*c+504*d^8/b^9/(b*x+a)^5*a^3*c^2-504*d^7/b^8/(b*x+a)^5*a^2*
c^3+252*d^6/b^7/(b*x+a)^5*a*c^4+45*d^9/b^10/(b*x+a)^8*a^7*c-315/2*d^8/b^9/(
b*x+a)^8*a^6*c^2+315*d^7/b^8/(b*x+a)^8*a^5*c^3-1575/4*d^6/b^7/(b*x+a)^8*a^4
*c^4+315*d^5/b^6/(b*x+a)^8*a^3*c^5-315/2*d^4/b^5/(b*x+a)^8*a^2*c^6+45*d^3/b
^4/(b*x+a)^8*a*c^7+210*d^9/b^10/(b*x+a)^6*a^5*c-525*d^8/b^9/(b*x+a)^6*a^4*c
^2+700*d^7/b^8/(b*x+a)^6*a^3*c^3-525*d^6/b^7/(b*x+a)^6*a^2*c^4+210*d^5/b^6/
(b*x+a)^6*a*c^5
```

Maxima [B] time = 1.19226, size = 1316, normalized size = 4.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="maxima")
```

```
[Out] -1/2520*(252*b^10*c^10 + 280*a*b^9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^
7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6
 + 840*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^1
0*d^10 + 25200*(b^10*c*d^9 - a*b^9*d^10)*x^9 + 56700*(b^10*c^2*d^8 + 2*a*b^
9*c*d^9 - 3*a^2*b^8*d^10)*x^8 + 50400*(2*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 6
*a^2*b^8*c*d^9 - 11*a^3*b^7*d^10)*x^7 + 44100*(3*b^10*c^4*d^6 + 4*a*b^9*c^3
*d^7 + 6*a^2*b^8*c^2*d^8 + 12*a^3*b^7*c*d^9 - 25*a^4*b^6*d^10)*x^6 + 10584*
(12*b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d
^8 + 60*a^4*b^6*c*d^9 - 137*a^5*b^5*d^10)*x^5 + 8820*(10*b^10*c^6*d^4 + 12*
a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^
8 + 60*a^5*b^5*c*d^9 - 147*a^6*b^4*d^10)*x^4 + 720*(60*b^10*c^7*d^3 + 70*a*
b^9*c^6*d^4 + 84*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*d^
7 + 210*a^5*b^5*c^2*d^8 + 420*a^6*b^4*c*d^9 - 1089*a^7*b^3*d^10)*x^3 + 135*
(105*b^10*c^8*d^2 + 120*a*b^9*c^7*d^3 + 140*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c
^5*d^5 + 210*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 +
840*a^7*b^3*c*d^9 - 2283*a^8*b^2*d^10)*x^2 + 10*(280*b^10*c^9*d + 315*a*b^9
*c^8*d^2 + 360*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5
+ 630*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 + 2520*a
^8*b^2*c*d^9 - 7129*a^9*b*d^10)*x)/(b^21*x^10 + 10*a*b^20*x^9 + 45*a^2*b^19
*x^8 + 120*a^3*b^18*x^7 + 210*a^4*b^17*x^6 + 252*a^5*b^16*x^5 + 210*a^6*b^1
5*x^4 + 120*a^7*b^14*x^3 + 45*a^8*b^13*x^2 + 10*a^9*b^12*x + a^10*b^11) + d
^10*log(b*x + a)/b^11
```

Fricas [B] time = 2.01085, size = 2456, normalized size = 9.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="fricas")
```

```
[Out] -1/2520*(252*b^10*c^10 + 280*a*b^9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^
7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6
 + 840*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^1
0*d^10 + 25200*(b^10*c*d^9 - a*b^9*d^10)*x^9 + 56700*(b^10*c^2*d^8 + 2*a*b^
9*c*d^9 - 3*a^2*b^8*d^10)*x^8 + 50400*(2*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 6
*a^2*b^8*c*d^9 - 11*a^3*b^7*d^10)*x^7 + 44100*(3*b^10*c^4*d^6 + 4*a*b^9*c^3
*d^7 + 6*a^2*b^8*c^2*d^8 + 12*a^3*b^7*c*d^9 - 25*a^4*b^6*d^10)*x^6 + 10584*
(12*b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d
^8 + 60*a^4*b^6*c*d^9 - 137*a^5*b^5*d^10)*x^5 + 8820*(10*b^10*c^6*d^4 + 12*
a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^
8 + 60*a^5*b^5*c*d^9 - 147*a^6*b^4*d^10)*x^4 + 720*(60*b^10*c^7*d^3 + 70*a*
b^9*c^6*d^4 + 84*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*d^
7 + 210*a^5*b^5*c^2*d^8 + 420*a^6*b^4*c*d^9 - 1089*a^7*b^3*d^10)*x^3 + 135*
(105*b^10*c^8*d^2 + 120*a*b^9*c^7*d^3 + 140*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c
^5*d^5 + 210*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 +
840*a^7*b^3*c*d^9 - 2283*a^8*b^2*d^10)*x^2 + 10*(280*b^10*c^9*d + 315*a*b^9
```

```
*c^8*d^2 + 360*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5
+ 630*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 + 2520*a
^8*b^2*c*d^9 - 7129*a^9*b*d^10)*x - 2520*(b^10*d^10*x^10 + 10*a*b^9*d^10*x^
9 + 45*a^2*b^8*d^10*x^8 + 120*a^3*b^7*d^10*x^7 + 210*a^4*b^6*d^10*x^6 + 252
*a^5*b^5*d^10*x^5 + 210*a^6*b^4*d^10*x^4 + 120*a^7*b^3*d^10*x^3 + 45*a^8*b^
2*d^10*x^2 + 10*a^9*b*d^10*x + a^10*d^10)*log(b*x + a))/(b^21*x^10 + 10*a*b
^20*x^9 + 45*a^2*b^19*x^8 + 120*a^3*b^18*x^7 + 210*a^4*b^17*x^6 + 252*a^5*b
^16*x^5 + 210*a^6*b^15*x^4 + 120*a^7*b^14*x^3 + 45*a^8*b^13*x^2 + 10*a^9*b^
12*x + a^10*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**11,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.07657, size = 1180, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="giac")
```

```
[Out] d^10*log(abs(b*x + a))/b^11 - 1/2520*(25200*(b^9*c*d^9 - a*b^8*d^10)*x^9 +
56700*(b^9*c^2*d^8 + 2*a*b^8*c*d^9 - 3*a^2*b^7*d^10)*x^8 + 50400*(2*b^9*c^3
*d^7 + 3*a*b^8*c^2*d^8 + 6*a^2*b^7*c*d^9 - 11*a^3*b^6*d^10)*x^7 + 44100*(3*
b^9*c^4*d^6 + 4*a*b^8*c^3*d^7 + 6*a^2*b^7*c^2*d^8 + 12*a^3*b^6*c*d^9 - 25*a
^4*b^5*d^10)*x^6 + 10584*(12*b^9*c^5*d^5 + 15*a*b^8*c^4*d^6 + 20*a^2*b^7*c^
3*d^7 + 30*a^3*b^6*c^2*d^8 + 60*a^4*b^5*c*d^9 - 137*a^5*b^4*d^10)*x^5 + 882
0*(10*b^9*c^6*d^4 + 12*a*b^8*c^5*d^5 + 15*a^2*b^7*c^4*d^6 + 20*a^3*b^6*c^3*
d^7 + 30*a^4*b^5*c^2*d^8 + 60*a^5*b^4*c*d^9 - 147*a^6*b^3*d^10)*x^4 + 720*(
60*b^9*c^7*d^3 + 70*a*b^8*c^6*d^4 + 84*a^2*b^7*c^5*d^5 + 105*a^3*b^6*c^4*d^
6 + 140*a^4*b^5*c^3*d^7 + 210*a^5*b^4*c^2*d^8 + 420*a^6*b^3*c*d^9 - 1089*a^
7*b^2*d^10)*x^3 + 135*(105*b^9*c^8*d^2 + 120*a*b^8*c^7*d^3 + 140*a^2*b^7*c^
6*d^4 + 168*a^3*b^6*c^5*d^5 + 210*a^4*b^5*c^4*d^6 + 280*a^5*b^4*c^3*d^7 + 4
20*a^6*b^3*c^2*d^8 + 840*a^7*b^2*c*d^9 - 2283*a^8*b*d^10)*x^2 + 10*(280*b^9
*c^9*d + 315*a*b^8*c^8*d^2 + 360*a^2*b^7*c^7*d^3 + 420*a^3*b^6*c^6*d^4 + 50
4*a^4*b^5*c^5*d^5 + 630*a^5*b^4*c^4*d^6 + 840*a^6*b^3*c^3*d^7 + 1260*a^7*b^
2*c^2*d^8 + 2520*a^8*b*c*d^9 - 7129*a^9*d^10)*x + (252*b^10*c^10 + 280*a*b^
9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 +
 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 + 1260*a^8
*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^10*d^10)/b)/((b*x + a)^10*b^10)
```

3.1323 $\int \frac{(c+dx)^{10}}{(a+bx)^{12}} dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^{11}}{11(a+bx)^{11}(bc-ad)}$$

```
[Out] -(c + d*x)^11/(11*(b*c - a*d)*(a + b*x)^11)
```

Rubi [A] time = 0.0030368, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(c+dx)^{11}}{11(a+bx)^{11}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^12,x]
```

```
[Out] -(c + d*x)^11/(11*(b*c - a*d)*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{12}} dx = -\frac{(c+dx)^{11}}{11(bc-ad)(a+bx)^{11}}$$

Mathematica [B] time = 0.321978, size = 665, normalized size = 23.75

$$-\frac{a^2b^8d^2\left(55c^6d^2x^2+165c^5d^3x^3+330c^4d^4x^4+462c^3d^5x^5+462c^2d^6x^6+11c^7dx+c^8+330cd^7x^7+165d^8x^8\right)+a^3b^7d^3\left(5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^12,x]
```

```
[Out] -(a^10*d^10 + a^9*b*d^9*(c + 11*d*x) + a^8*b^2*d^8*(c^2 + 11*c*d*x + 55*d^2
*x^2) + a^7*b^3*d^7*(c^3 + 11*c^2*d*x + 55*c*d^2*x^2 + 165*d^3*x^3) + a^6*b
^4*d^6*(c^4 + 11*c^3*d*x + 55*c^2*d^2*x^2 + 165*c*d^3*x^3 + 330*d^4*x^4) +
a^5*b^5*d^5*(c^5 + 11*c^4*d*x + 55*c^3*d^2*x^2 + 165*c^2*d^3*x^3 + 330*c*d^
4*x^4 + 462*d^5*x^5) + a^4*b^6*d^4*(c^6 + 11*c^5*d*x + 55*c^4*d^2*x^2 + 165
*c^3*d^3*x^3 + 330*c^2*d^4*x^4 + 462*c*d^5*x^5 + 462*d^6*x^6) + a^3*b^7*d^3
*(c^7 + 11*c^6*d*x + 55*c^5*d^2*x^2 + 165*c^4*d^3*x^3 + 330*c^3*d^4*x^4 + 4
62*c^2*d^5*x^5 + 462*c*d^6*x^6 + 330*d^7*x^7) + a^2*b^8*d^2*(c^8 + 11*c^7*d
```

```
*x + 55*c^6*d^2*x^2 + 165*c^5*d^3*x^3 + 330*c^4*d^4*x^4 + 462*c^3*d^5*x^5 +
 462*c^2*d^6*x^6 + 330*c*d^7*x^7 + 165*d^8*x^8) + a*b^9*d*(c^9 + 11*c^8*d*x
 + 55*c^7*d^2*x^2 + 165*c^6*d^3*x^3 + 330*c^5*d^4*x^4 + 462*c^4*d^5*x^5 + 4
62*c^3*d^6*x^6 + 330*c^2*d^7*x^7 + 165*c*d^8*x^8 + 55*d^9*x^9) + b^10*(c^10
 + 11*c^9*d*x + 55*c^8*d^2*x^2 + 165*c^7*d^3*x^3 + 330*c^6*d^4*x^4 + 462*c^
5*d^5*x^5 + 462*c^4*d^6*x^6 + 330*c^3*d^7*x^7 + 165*c^2*d^8*x^8 + 55*c*d^9*
x^9 + 11*d^10*x^10))/(11*b^11*(a + b*x)^11)
```

Maple [B] time = 0.01, size = 866, normalized size = 30.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^12,x)
```

```
[Out] d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*
c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c
^8*d-b^9*c^9)/b^11/(b*x+a)^10-d^10/b^11/(b*x+a)+15*d^3*(a^7*d^7-7*a^6*b*c*d
^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*
d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^8+42*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*
a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^6+5*
d^9*(a*d-b*c)/b^11/(b*x+a)^2-1/11*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*
d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6
*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b
^11/(b*x+a)^11-15*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^3-42*d^6*(a^
4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^5
+30*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^4-5*d^2*
(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4
*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x
+a)^9-30*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+1
5*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^7
```

Maxima [B] time = 1.20962, size = 1242, normalized size = 44.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="maxima")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*
b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3
*c^3*d^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10 + 55*(b^10*c*d^9 + a*b
^9*d^10)*x^9 + 165*(b^10*c^2*d^8 + a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 330*(b
^10*c^3*d^7 + a*b^9*c^2*d^8 + a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 462*(b^10
*c^4*d^6 + a*b^9*c^3*d^7 + a^2*b^8*c^2*d^8 + a^3*b^7*c*d^9 + a^4*b^6*d^10)*
x^6 + 462*(b^10*c^5*d^5 + a*b^9*c^4*d^6 + a^2*b^8*c^3*d^7 + a^3*b^7*c^2*d^8
 + a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 330*(b^10*c^6*d^4 + a*b^9*c^5*d^5 +
a^2*b^8*c^4*d^6 + a^3*b^7*c^3*d^7 + a^4*b^6*c^2*d^8 + a^5*b^5*c*d^9 + a^6*b
^4*d^10)*x^4 + 165*(b^10*c^7*d^3 + a*b^9*c^6*d^4 + a^2*b^8*c^5*d^5 + a^3*b^
7*c^4*d^6 + a^4*b^6*c^3*d^7 + a^5*b^5*c^2*d^8 + a^6*b^4*c*d^9 + a^7*b^3*d^1
0)*x^3 + 55*(b^10*c^8*d^2 + a*b^9*c^7*d^3 + a^2*b^8*c^6*d^4 + a^3*b^7*c^5*d
^5 + a^4*b^6*c^4*d^6 + a^5*b^5*c^3*d^7 + a^6*b^4*c^2*d^8 + a^7*b^3*c*d^9 +
```

```
a^8*b^2*d^10)*x^2 + 11*(b^10*c^9*d + a*b^9*c^8*d^2 + a^2*b^8*c^7*d^3 + a^3*
b^7*c^6*d^4 + a^4*b^6*c^5*d^5 + a^5*b^5*c^4*d^6 + a^6*b^4*c^3*d^7 + a^7*b^3
*c^2*d^8 + a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^22*x^11 + 11*a*b^21*x^10 + 55*
a^2*b^20*x^9 + 165*a^3*b^19*x^8 + 330*a^4*b^18*x^7 + 462*a^5*b^17*x^6 + 462
*a^6*b^16*x^5 + 330*a^7*b^15*x^4 + 165*a^8*b^14*x^3 + 55*a^9*b^13*x^2 + 11*
a^10*b^12*x + a^11*b^11)
```

Fricas [B] time = 1.64626, size = 1858, normalized size = 66.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="fricas")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*
b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3
*c^3*d^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10 + 55*(b^10*c*d^9 + a*b
^9*d^10)*x^9 + 165*(b^10*c^2*d^8 + a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 330*(b
^10*c^3*d^7 + a*b^9*c^2*d^8 + a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 462*(b^10
*c^4*d^6 + a*b^9*c^3*d^7 + a^2*b^8*c^2*d^8 + a^3*b^7*c*d^9 + a^4*b^6*d^10)*
x^6 + 462*(b^10*c^5*d^5 + a*b^9*c^4*d^6 + a^2*b^8*c^3*d^7 + a^3*b^7*c^2*d^8
 + a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 330*(b^10*c^6*d^4 + a*b^9*c^5*d^5 +
a^2*b^8*c^4*d^6 + a^3*b^7*c^3*d^7 + a^4*b^6*c^2*d^8 + a^5*b^5*c*d^9 + a^6*b
^4*d^10)*x^4 + 165*(b^10*c^7*d^3 + a*b^9*c^6*d^4 + a^2*b^8*c^5*d^5 + a^3*b^
7*c^4*d^6 + a^4*b^6*c^3*d^7 + a^5*b^5*c^2*d^8 + a^6*b^4*c*d^9 + a^7*b^3*d^1
0)*x^3 + 55*(b^10*c^8*d^2 + a*b^9*c^7*d^3 + a^2*b^8*c^6*d^4 + a^3*b^7*c^5*d
^5 + a^4*b^6*c^4*d^6 + a^5*b^5*c^3*d^7 + a^6*b^4*c^2*d^8 + a^7*b^3*c*d^9 +
a^8*b^2*d^10)*x^2 + 11*(b^10*c^9*d + a*b^9*c^8*d^2 + a^2*b^8*c^7*d^3 + a^3*
b^7*c^6*d^4 + a^4*b^6*c^5*d^5 + a^5*b^5*c^4*d^6 + a^6*b^4*c^3*d^7 + a^7*b^3
*c^2*d^8 + a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^22*x^11 + 11*a*b^21*x^10 + 55*
a^2*b^20*x^9 + 165*a^3*b^19*x^8 + 330*a^4*b^18*x^7 + 462*a^5*b^17*x^6 + 462
*a^6*b^16*x^5 + 330*a^7*b^15*x^4 + 165*a^8*b^14*x^3 + 55*a^9*b^13*x^2 + 11*
a^10*b^12*x + a^11*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**12,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06017, size = 1284, normalized size = 45.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="giac")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + 55*b^10*c*d^9*x^9 + 55*a*b^9*d^10*x^9 + 165*b^10
*c^2*d^8*x^8 + 165*a*b^9*c*d^9*x^8 + 165*a^2*b^8*d^10*x^8 + 330*b^10*c^3*d^
7*x^7 + 330*a*b^9*c^2*d^8*x^7 + 330*a^2*b^8*c*d^9*x^7 + 330*a^3*b^7*d^10*x^
7 + 462*b^10*c^4*d^6*x^6 + 462*a*b^9*c^3*d^7*x^6 + 462*a^2*b^8*c^2*d^8*x^6
+ 462*a^3*b^7*c*d^9*x^6 + 462*a^4*b^6*d^10*x^6 + 462*b^10*c^5*d^5*x^5 + 462
*a*b^9*c^4*d^6*x^5 + 462*a^2*b^8*c^3*d^7*x^5 + 462*a^3*b^7*c^2*d^8*x^5 + 46
2*a^4*b^6*c*d^9*x^5 + 462*a^5*b^5*d^10*x^5 + 330*b^10*c^6*d^4*x^4 + 330*a*b
^9*c^5*d^5*x^4 + 330*a^2*b^8*c^4*d^6*x^4 + 330*a^3*b^7*c^3*d^7*x^4 + 330*a^
4*b^6*c^2*d^8*x^4 + 330*a^5*b^5*c*d^9*x^4 + 330*a^6*b^4*d^10*x^4 + 165*b^10
*c^7*d^3*x^3 + 165*a*b^9*c^6*d^4*x^3 + 165*a^2*b^8*c^5*d^5*x^3 + 165*a^3*b^
7*c^4*d^6*x^3 + 165*a^4*b^6*c^3*d^7*x^3 + 165*a^5*b^5*c^2*d^8*x^3 + 165*a^6
*b^4*c*d^9*x^3 + 165*a^7*b^3*d^10*x^3 + 55*b^10*c^8*d^2*x^2 + 55*a*b^9*c^7*
d^3*x^2 + 55*a^2*b^8*c^6*d^4*x^2 + 55*a^3*b^7*c^5*d^5*x^2 + 55*a^4*b^6*c^4*
d^6*x^2 + 55*a^5*b^5*c^3*d^7*x^2 + 55*a^6*b^4*c^2*d^8*x^2 + 55*a^7*b^3*c*d^
9*x^2 + 55*a^8*b^2*d^10*x^2 + 11*b^10*c^9*d*x + 11*a*b^9*c^8*d^2*x + 11*a^2
*b^8*c^7*d^3*x + 11*a^3*b^7*c^6*d^4*x + 11*a^4*b^6*c^5*d^5*x + 11*a^5*b^5*c
^4*d^6*x + 11*a^6*b^4*c^3*d^7*x + 11*a^7*b^3*c^2*d^8*x + 11*a^8*b^2*c*d^9*x
 + 11*a^9*b*d^10*x + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*b^7*c^
7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3*c^3*d
^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^11*b^11)
```

3.1324 $\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx$

Optimal. Leaf size=58

$$\frac{d(c+dx)^{11}}{132(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^{11}}{12(a+bx)^{12}(bc-ad)}$$

```
[Out] -(c + d*x)^11/(12*(b*c - a*d)*(a + b*x)^12) + (d*(c + d*x)^11)/(132*(b*c -
a*d)^2*(a + b*x)^11)
```

Rubi [A] time = 0.0101382, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d(c+dx)^{11}}{132(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^{11}}{12(a+bx)^{12}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^13,x]
```

```
[Out] -(c + d*x)^11/(12*(b*c - a*d)*(a + b*x)^12) + (d*(c + d*x)^11)/(132*(b*c -
a*d)^2*(a + b*x)^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx &= -\frac{(c+dx)^{11}}{12(bc-ad)(a+bx)^{12}}-\frac{d\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx}{12(bc-ad)}\\ &= -\frac{(c+dx)^{11}}{12(bc-ad)(a+bx)^{12}}+\frac{d(c+dx)^{11}}{132(bc-ad)^2(a+bx)^{11}}\end{aligned}$$

Mathematica [B] time = 0.300956, size = 684, normalized size = 11.79

$$-\frac{3a^2b^8d^2\left(154c^6d^2x^2+440c^5d^3x^3+825c^4d^4x^4+1056c^3d^5x^5+924c^2d^6x^6+32c^7dx+3c^8+528cd^7x^7+165d^8x^8\right)+4a^3\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^13,x]
```

```
[Out] -(a^10*d^10 + 2*a^9*b*d^9*(c + 6*d*x) + 3*a^8*b^2*d^8*(c^2 + 8*c*d*x + 22*d
^2*x^2) + 4*a^7*b^3*d^7*(c^3 + 9*c^2*d*x + 33*c*d^2*x^2 + 55*d^3*x^3) + a^6
*b^4*d^6*(5*c^4 + 48*c^3*d*x + 198*c^2*d^2*x^2 + 440*c*d^3*x^3 + 495*d^4*x^
4) + 6*a^5*b^5*d^5*(c^5 + 10*c^4*d*x + 44*c^3*d^2*x^2 + 110*c^2*d^3*x^3 + 1
65*c*d^4*x^4 + 132*d^5*x^5) + a^4*b^6*d^4*(7*c^6 + 72*c^5*d*x + 330*c^4*d^2
*x^2 + 880*c^3*d^3*x^3 + 1485*c^2*d^4*x^4 + 1584*c*d^5*x^5 + 924*d^6*x^6) +
 4*a^3*b^7*d^3*(2*c^7 + 21*c^6*d*x + 99*c^5*d^2*x^2 + 275*c^4*d^3*x^3 + 495
*c^3*d^4*x^4 + 594*c^2*d^5*x^5 + 462*c*d^6*x^6 + 198*d^7*x^7) + 3*a^2*b^8*d
^2*(3*c^8 + 32*c^7*d*x + 154*c^6*d^2*x^2 + 440*c^5*d^3*x^3 + 825*c^4*d^4*x^
4 + 1056*c^3*d^5*x^5 + 924*c^2*d^6*x^6 + 528*c*d^7*x^7 + 165*d^8*x^8) + 2*a
*b^9*d*(5*c^9 + 54*c^8*d*x + 264*c^7*d^2*x^2 + 770*c^6*d^3*x^3 + 1485*c^5*d
^4*x^4 + 1980*c^4*d^5*x^5 + 1848*c^3*d^6*x^6 + 1188*c^2*d^7*x^7 + 495*c*d^8
*x^8 + 110*d^9*x^9) + b^10*(11*c^10 + 120*c^9*d*x + 594*c^8*d^2*x^2 + 1760*
c^7*d^3*x^3 + 3465*c^6*d^4*x^4 + 4752*c^5*d^5*x^5 + 4620*c^4*d^6*x^6 + 3168
*c^3*d^7*x^7 + 1485*c^2*d^8*x^8 + 440*c*d^9*x^9 + 66*d^10*x^10))/(132*b^11*
(a + b*x)^12)
```

Maple [B] time = 0.011, size = 867, normalized size = 15.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^13,x)
```

```
[Out] -9/2*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^
4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/
b^11/(b*x+a)^10-105/4*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*
b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^8-35*d^6
*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+
a)^6-1/2*d^10/b^11/(b*x+a)^2+10/11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*
d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c
^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^11+10/3*d^9*(
a*d-b*c)/b^11/(b*x+a)^3+24*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3
)/b^11/(b*x+a)^5-45/4*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^4-1/12*(
a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4
*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2
*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^12+40/3*d^3*(a^7*d^7-7*
a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2
*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^9+36*d^5*(a^5*d^5-5*a^4*b*
c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*
x+a)^7
```

Maxima [B] time = 1.24292, size = 1331, normalized size = 22.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="maxima")
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 11*b^10*c^10 + 10*a*b^9*c^9*d + 9*a^2*b^8*c^8*d
^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*a^5*b^5*c^5*d^5 + 5*a^6*b^4*
c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9 + a^10*d^10
 + 220*(2*b^10*c*d^9 + a*b^9*d^10)*x^9 + 495*(3*b^10*c^2*d^8 + 2*a*b^9*c*d^
9 + a^2*b^8*d^10)*x^8 + 792*(4*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 2*a^2*b^8*c
*d^9 + a^3*b^7*d^10)*x^7 + 924*(5*b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 + 3*a^2*b^
8*c^2*d^8 + 2*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 792*(6*b^10*c^5*d^5 + 5*a
*b^9*c^4*d^6 + 4*a^2*b^8*c^3*d^7 + 3*a^3*b^7*c^2*d^8 + 2*a^4*b^6*c*d^9 + a^
5*b^5*d^10)*x^5 + 495*(7*b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6
 + 4*a^3*b^7*c^3*d^7 + 3*a^4*b^6*c^2*d^8 + 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*
x^4 + 220*(8*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 6*a^2*b^8*c^5*d^5 + 5*a^3*b^7
*c^4*d^6 + 4*a^4*b^6*c^3*d^7 + 3*a^5*b^5*c^2*d^8 + 2*a^6*b^4*c*d^9 + a^7*b^
3*d^10)*x^3 + 66*(9*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 7*a^2*b^8*c^6*d^4 + 6*
a^3*b^7*c^5*d^5 + 5*a^4*b^6*c^4*d^6 + 4*a^5*b^5*c^3*d^7 + 3*a^6*b^4*c^2*d^8
 + 2*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 12*(10*b^10*c^9*d + 9*a*b^9*c^8*d^
2 + 8*a^2*b^8*c^7*d^3 + 7*a^3*b^7*c^6*d^4 + 6*a^4*b^6*c^5*d^5 + 5*a^5*b^5*c
^4*d^6 + 4*a^6*b^4*c^3*d^7 + 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^23*x^12 + 12*a*b^22*x^11 + 66*a^2*b^21*x^10 + 220*a^3*b^20*x^9 +
495*a^4*b^19*x^8 + 792*a^5*b^18*x^7 + 924*a^6*b^17*x^6 + 792*a^7*b^16*x^5 +
 495*a^8*b^15*x^4 + 220*a^9*b^14*x^3 + 66*a^10*b^13*x^2 + 12*a^11*b^12*x +
a^12*b^11)
```

Fricas [B] time = 1.86512, size = 2041, normalized size = 35.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="fricas")
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 11*b^10*c^10 + 10*a*b^9*c^9*d + 9*a^2*b^8*c^8*d
^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*a^5*b^5*c^5*d^5 + 5*a^6*b^4*
c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9 + a^10*d^10
 + 220*(2*b^10*c*d^9 + a*b^9*d^10)*x^9 + 495*(3*b^10*c^2*d^8 + 2*a*b^9*c*d^
9 + a^2*b^8*d^10)*x^8 + 792*(4*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 2*a^2*b^8*c
*d^9 + a^3*b^7*d^10)*x^7 + 924*(5*b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 + 3*a^2*b^
8*c^2*d^8 + 2*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 792*(6*b^10*c^5*d^5 + 5*a
*b^9*c^4*d^6 + 4*a^2*b^8*c^3*d^7 + 3*a^3*b^7*c^2*d^8 + 2*a^4*b^6*c*d^9 + a^
5*b^5*d^10)*x^5 + 495*(7*b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6
 + 4*a^3*b^7*c^3*d^7 + 3*a^4*b^6*c^2*d^8 + 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*
x^4 + 220*(8*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 6*a^2*b^8*c^5*d^5 + 5*a^3*b^7
*c^4*d^6 + 4*a^4*b^6*c^3*d^7 + 3*a^5*b^5*c^2*d^8 + 2*a^6*b^4*c*d^9 + a^7*b^
3*d^10)*x^3 + 66*(9*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 7*a^2*b^8*c^6*d^4 + 6*
a^3*b^7*c^5*d^5 + 5*a^4*b^6*c^4*d^6 + 4*a^5*b^5*c^3*d^7 + 3*a^6*b^4*c^2*d^8
 + 2*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 12*(10*b^10*c^9*d + 9*a*b^9*c^8*d^
2 + 8*a^2*b^8*c^7*d^3 + 7*a^3*b^7*c^6*d^4 + 6*a^4*b^6*c^5*d^5 + 5*a^5*b^5*c
^4*d^6 + 4*a^6*b^4*c^3*d^7 + 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^23*x^12 + 12*a*b^22*x^11 + 66*a^2*b^21*x^10 + 220*a^3*b^20*x^9 +
495*a^4*b^19*x^8 + 792*a^5*b^18*x^7 + 924*a^6*b^17*x^6 + 792*a^7*b^16*x^5 +
 495*a^8*b^15*x^4 + 220*a^9*b^14*x^3 + 66*a^10*b^13*x^2 + 12*a^11*b^12*x +
a^12*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**13,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.07204, size = 1297, normalized size = 22.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="giac")
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 440*b^10*c*d^9*x^9 + 220*a*b^9*d^10*x^9 + 1485*
b^10*c^2*d^8*x^8 + 990*a*b^9*c*d^9*x^8 + 495*a^2*b^8*d^10*x^8 + 3168*b^10*c
^3*d^7*x^7 + 2376*a*b^9*c^2*d^8*x^7 + 1584*a^2*b^8*c*d^9*x^7 + 792*a^3*b^7*
d^10*x^7 + 4620*b^10*c^4*d^6*x^6 + 3696*a*b^9*c^3*d^7*x^6 + 2772*a^2*b^8*c^
2*d^8*x^6 + 1848*a^3*b^7*c*d^9*x^6 + 924*a^4*b^6*d^10*x^6 + 4752*b^10*c^5*d
^5*x^5 + 3960*a*b^9*c^4*d^6*x^5 + 3168*a^2*b^8*c^3*d^7*x^5 + 2376*a^3*b^7*c
^2*d^8*x^5 + 1584*a^4*b^6*c*d^9*x^5 + 792*a^5*b^5*d^10*x^5 + 3465*b^10*c^6*
d^4*x^4 + 2970*a*b^9*c^5*d^5*x^4 + 2475*a^2*b^8*c^4*d^6*x^4 + 1980*a^3*b^7*
c^3*d^7*x^4 + 1485*a^4*b^6*c^2*d^8*x^4 + 990*a^5*b^5*c*d^9*x^4 + 495*a^6*b^
4*d^10*x^4 + 1760*b^10*c^7*d^3*x^3 + 1540*a*b^9*c^6*d^4*x^3 + 1320*a^2*b^8*
c^5*d^5*x^3 + 1100*a^3*b^7*c^4*d^6*x^3 + 880*a^4*b^6*c^3*d^7*x^3 + 660*a^5*
b^5*c^2*d^8*x^3 + 440*a^6*b^4*c*d^9*x^3 + 220*a^7*b^3*d^10*x^3 + 594*b^10*c
^8*d^2*x^2 + 528*a*b^9*c^7*d^3*x^2 + 462*a^2*b^8*c^6*d^4*x^2 + 396*a^3*b^7*
c^5*d^5*x^2 + 330*a^4*b^6*c^4*d^6*x^2 + 264*a^5*b^5*c^3*d^7*x^2 + 198*a^6*b
^4*c^2*d^8*x^2 + 132*a^7*b^3*c*d^9*x^2 + 66*a^8*b^2*d^10*x^2 + 120*b^10*c^9
*d*x + 108*a*b^9*c^8*d^2*x + 96*a^2*b^8*c^7*d^3*x + 84*a^3*b^7*c^6*d^4*x +
72*a^4*b^6*c^5*d^5*x + 60*a^5*b^5*c^4*d^6*x + 48*a^6*b^4*c^3*d^7*x + 36*a^7
*b^3*c^2*d^8*x + 24*a^8*b^2*c*d^9*x + 12*a^9*b*d^10*x + 11*b^10*c^10 + 10*a
*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*
a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8
 + 2*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^12*b^11)
```

3.1325 $\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx$

Optimal. Leaf size=89

$$-\frac{d^2(c+dx)^{11}}{858(a+bx)^{11}(bc-ad)^3}+\frac{d(c+dx)^{11}}{78(a+bx)^{12}(bc-ad)^2}-\frac{(c+dx)^{11}}{13(a+bx)^{13}(bc-ad)}$$

```
[Out] -(c + d*x)^11/(13*(b*c - a*d)*(a + b*x)^13) + (d*(c + d*x)^11)/(78*(b*c - a
*d)^2*(a + b*x)^12) - (d^2*(c + d*x)^11)/(858*(b*c - a*d)^3*(a + b*x)^11)
```

Rubi [A] time = 0.0199106, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{d^2(c+dx)^{11}}{858(a+bx)^{11}(bc-ad)^3}+\frac{d(c+dx)^{11}}{78(a+bx)^{12}(bc-ad)^2}-\frac{(c+dx)^{11}}{13(a+bx)^{13}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^14,x]
```

```
[Out] -(c + d*x)^11/(13*(b*c - a*d)*(a + b*x)^13) + (d*(c + d*x)^11)/(78*(b*c - a
*d)^2*(a + b*x)^12) - (d^2*(c + d*x)^11)/(858*(b*c - a*d)^3*(a + b*x)^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx &= -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}}-\frac{(2d)\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx}{13(bc-ad)}\\ &= -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}}+\frac{d(c+dx)^{11}}{78(bc-ad)^2(a+bx)^{12}}+\frac{d^2\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx}{78(bc-ad)^2}\\ &= -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}}+\frac{d(c+dx)^{11}}{78(bc-ad)^2(a+bx)^{12}}-\frac{d^2(c+dx)^{11}}{858(bc-ad)^3(a+bx)^{11}}\end{aligned}$$

Mathematica [B] time = 0.287829, size = 690, normalized size = 7.75

$$-\frac{3a^2b^8d^2\left(728c^6d^2x^2+2002c^5d^3x^3+3575c^4d^4x^4+4290c^3d^5x^5+3432c^2d^6x^6+156c^7dx+15c^8+1716cd^7x^7+429d^8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^14,x]
```

```
[Out] -(a^10*d^10 + a^9*b*d^9*(3*c + 13*d*x) + 3*a^8*b^2*d^8*(2*c^2 + 13*c*d*x +
26*d^2*x^2) + 2*a^7*b^3*d^7*(5*c^3 + 39*c^2*d*x + 117*c*d^2*x^2 + 143*d^3*x
^3) + a^6*b^4*d^6*(15*c^4 + 130*c^3*d*x + 468*c^2*d^2*x^2 + 858*c*d^3*x^3 +
 715*d^4*x^4) + 3*a^5*b^5*d^5*(7*c^5 + 65*c^4*d*x + 260*c^3*d^2*x^2 + 572*c
^2*d^3*x^3 + 715*c*d^4*x^4 + 429*d^5*x^5) + a^4*b^6*d^4*(28*c^6 + 273*c^5*d
*x + 1170*c^4*d^2*x^2 + 2860*c^3*d^3*x^3 + 4290*c^2*d^4*x^4 + 3861*c*d^5*x^
5 + 1716*d^6*x^6) + 2*a^3*b^7*d^3*(18*c^7 + 182*c^6*d*x + 819*c^5*d^2*x^2 +
 2145*c^4*d^3*x^3 + 3575*c^3*d^4*x^4 + 3861*c^2*d^5*x^5 + 2574*c*d^6*x^6 +
858*d^7*x^7) + 3*a^2*b^8*d^2*(15*c^8 + 156*c^7*d*x + 728*c^6*d^2*x^2 + 2002
*c^5*d^3*x^3 + 3575*c^4*d^4*x^4 + 4290*c^3*d^5*x^5 + 3432*c^2*d^6*x^6 + 171
6*c*d^7*x^7 + 429*d^8*x^8) + a*b^9*d*(55*c^9 + 585*c^8*d*x + 2808*c^7*d^2*x
^2 + 8008*c^6*d^3*x^3 + 15015*c^5*d^4*x^4 + 19305*c^4*d^5*x^5 + 17160*c^3*d
^6*x^6 + 10296*c^2*d^7*x^7 + 3861*c*d^8*x^8 + 715*d^9*x^9) + b^10*(66*c^10
+ 715*c^9*d*x + 3510*c^8*d^2*x^2 + 10296*c^7*d^3*x^3 + 20020*c^6*d^4*x^4 +
27027*c^5*d^5*x^5 + 25740*c^4*d^6*x^6 + 17160*c^3*d^7*x^7 + 7722*c^2*d^8*x^
8 + 2145*c*d^9*x^9 + 286*d^10*x^10))/(858*b^11*(a + b*x)^13)
```

Maple [B] time = 0.008, size = 867, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^14,x)
```

```
[Out] 12*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*
b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^10+63/2*
d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^
4*d-b^5*c^5)/b^11/(b*x+a)^8+20*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3
*c^3)/b^11/(b*x+a)^6-45/11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56
*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8
*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^11-1/13*(a^10*d^10-10*a^9*b*c*d^9+45*a^8
*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+21
0*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^1
0*c^10)/b^11/(b*x+a)^13-1/3*d^10/b^11/(b*x+a)^3-9*d^8*(a^2*d^2-2*a*b*c*d+b^
2*c^2)/b^11/(b*x+a)^5+5/2*d^9*(a*d-b*c)/b^11/(b*x+a)^4+5/6*d*(a^9*d^9-9*a^8
*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*
b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^
11/(b*x+a)^12-70/3*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3
*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^9-30*d^6*(a
^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^
7
```

Maxima [B] time = 1.26755, size = 1346, normalized size = 15.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="maxima")
```

```
[Out] -1/858*(286*b^10*d^10*x^10 + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8
*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^
6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^
10*d^10 + 715*(3*b^10*c*d^9 + a*b^9*d^10)*x^9 + 1287*(6*b^10*c^2*d^8 + 3*a*
b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1716*(10*b^10*c^3*d^7 + 6*a*b^9*c^2*d^8 + 3
*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 1716*(15*b^10*c^4*d^6 + 10*a*b^9*c^3*d
^7 + 6*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 1287*(21*b^1
0*c^5*d^5 + 15*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 + 3*a
^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 715*(28*b^10*c^6*d^4 + 21*a*b^9*c^5*d^5
+ 15*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 6*a^4*b^6*c^2*d^8 + 3*a^5*b^5*c
*d^9 + a^6*b^4*d^10)*x^4 + 286*(36*b^10*c^7*d^3 + 28*a*b^9*c^6*d^4 + 21*a^2
*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8
+ 3*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 78*(45*b^10*c^8*d^2 + 36*a*b^9*c^7*
d^3 + 28*a^2*b^8*c^6*d^4 + 21*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 + 10*a^5
*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 + 3*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 13
*(55*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^
4 + 21*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 + 10*a^6*b^4*c^3*d^7 + 6*a^7*b^
3*c^2*d^8 + 3*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^24*x^13 + 13*a*b^23*x^12 +
78*a^2*b^22*x^11 + 286*a^3*b^21*x^10 + 715*a^4*b^20*x^9 + 1287*a^5*b^19*x^8
 + 1716*a^6*b^18*x^7 + 1716*a^7*b^17*x^6 + 1287*a^8*b^16*x^5 + 715*a^9*b^15
*x^4 + 286*a^10*b^14*x^3 + 78*a^11*b^13*x^2 + 13*a^12*b^12*x + a^13*b^11)
```

Fricas [B] time = 1.78129, size = 2126, normalized size = 23.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="fricas")
```

```
[Out] -1/858*(286*b^10*d^10*x^10 + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8
*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^
6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^
10*d^10 + 715*(3*b^10*c*d^9 + a*b^9*d^10)*x^9 + 1287*(6*b^10*c^2*d^8 + 3*a*
b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1716*(10*b^10*c^3*d^7 + 6*a*b^9*c^2*d^8 + 3
*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 1716*(15*b^10*c^4*d^6 + 10*a*b^9*c^3*d
^7 + 6*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 1287*(21*b^1
0*c^5*d^5 + 15*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 + 3*a
^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 715*(28*b^10*c^6*d^4 + 21*a*b^9*c^5*d^5
+ 15*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 6*a^4*b^6*c^2*d^8 + 3*a^5*b^5*c
*d^9 + a^6*b^4*d^10)*x^4 + 286*(36*b^10*c^7*d^3 + 28*a*b^9*c^6*d^4 + 21*a^2
*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8
+ 3*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 78*(45*b^10*c^8*d^2 + 36*a*b^9*c^7*
d^3 + 28*a^2*b^8*c^6*d^4 + 21*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 + 10*a^5
*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 + 3*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 13
*(55*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^
4 + 21*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 + 10*a^6*b^4*c^3*d^7 + 6*a^7*b^
3*c^2*d^8 + 3*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^24*x^13 + 13*a*b^23*x^12 +
78*a^2*b^22*x^11 + 286*a^3*b^21*x^10 + 715*a^4*b^20*x^9 + 1287*a^5*b^19*x^8
 + 1716*a^6*b^18*x^7 + 1716*a^7*b^17*x^6 + 1287*a^8*b^16*x^5 + 715*a^9*b^15
*x^4 + 286*a^10*b^14*x^3 + 78*a^11*b^13*x^2 + 13*a^12*b^12*x + a^13*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**14,x)

[Out] Timed out
```

Giac [B] time = 1.05961, size = 1297, normalized size = 14.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="giac")

[Out] -1/858*(286*b^10*d^10*x^10 + 2145*b^10*c*d^9*x^9 + 715*a*b^9*d^10*x^9 + 772
2*b^10*c^2*d^8*x^8 + 3861*a*b^9*c*d^9*x^8 + 1287*a^2*b^8*d^10*x^8 + 17160*b
^10*c^3*d^7*x^7 + 10296*a*b^9*c^2*d^8*x^7 + 5148*a^2*b^8*c*d^9*x^7 + 1716*a
^3*b^7*d^10*x^7 + 25740*b^10*c^4*d^6*x^6 + 17160*a*b^9*c^3*d^7*x^6 + 10296*
a^2*b^8*c^2*d^8*x^6 + 5148*a^3*b^7*c*d^9*x^6 + 1716*a^4*b^6*d^10*x^6 + 2702
7*b^10*c^5*d^5*x^5 + 19305*a*b^9*c^4*d^6*x^5 + 12870*a^2*b^8*c^3*d^7*x^5 +
7722*a^3*b^7*c^2*d^8*x^5 + 3861*a^4*b^6*c*d^9*x^5 + 1287*a^5*b^5*d^10*x^5 +
 20020*b^10*c^6*d^4*x^4 + 15015*a*b^9*c^5*d^5*x^4 + 10725*a^2*b^8*c^4*d^6*x
^4 + 7150*a^3*b^7*c^3*d^7*x^4 + 4290*a^4*b^6*c^2*d^8*x^4 + 2145*a^5*b^5*c*d
^9*x^4 + 715*a^6*b^4*d^10*x^4 + 10296*b^10*c^7*d^3*x^3 + 8008*a*b^9*c^6*d^4
*x^3 + 6006*a^2*b^8*c^5*d^5*x^3 + 4290*a^3*b^7*c^4*d^6*x^3 + 2860*a^4*b^6*c
^3*d^7*x^3 + 1716*a^5*b^5*c^2*d^8*x^3 + 858*a^6*b^4*c*d^9*x^3 + 286*a^7*b^3
*d^10*x^3 + 3510*b^10*c^8*d^2*x^2 + 2808*a*b^9*c^7*d^3*x^2 + 2184*a^2*b^8*c
^6*d^4*x^2 + 1638*a^3*b^7*c^5*d^5*x^2 + 1170*a^4*b^6*c^4*d^6*x^2 + 780*a^5*
b^5*c^3*d^7*x^2 + 468*a^6*b^4*c^2*d^8*x^2 + 234*a^7*b^3*c*d^9*x^2 + 78*a^8*
b^2*d^10*x^2 + 715*b^10*c^9*d*x + 585*a*b^9*c^8*d^2*x + 468*a^2*b^8*c^7*d^3
*x + 364*a^3*b^7*c^6*d^4*x + 273*a^4*b^6*c^5*d^5*x + 195*a^5*b^5*c^4*d^6*x
+ 130*a^6*b^4*c^3*d^7*x + 78*a^7*b^3*c^2*d^8*x + 39*a^8*b^2*c*d^9*x + 13*a^
9*b*d^10*x + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 36*a^3*b^
7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 +
10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^10*d^10)/((b*x +
 a)^13*b^11)
```

3.1326 $\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx$

Optimal. Leaf size=120

$$\frac{d^3(c+dx)^{11}}{4004(a+bx)^{11}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{364(a+bx)^{12}(bc-ad)^3}+\frac{3d(c+dx)^{11}}{182(a+bx)^{13}(bc-ad)^2}-\frac{(c+dx)^{11}}{14(a+bx)^{14}(bc-ad)}$$

```
[Out] -(c + d*x)^11/(14*(b*c - a*d)*(a + b*x)^14) + (3*d*(c + d*x)^11)/(182*(b*c
- a*d)^2*(a + b*x)^13) - (d^2*(c + d*x)^11)/(364*(b*c - a*d)^3*(a + b*x)^12
) + (d^3*(c + d*x)^11)/(4004*(b*c - a*d)^4*(a + b*x)^11)
```

Rubi [A] time = 0.029914, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d^3(c+dx)^{11}}{4004(a+bx)^{11}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{364(a+bx)^{12}(bc-ad)^3}+\frac{3d(c+dx)^{11}}{182(a+bx)^{13}(bc-ad)^2}-\frac{(c+dx)^{11}}{14(a+bx)^{14}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^15,x]
```

```
[Out] -(c + d*x)^11/(14*(b*c - a*d)*(a + b*x)^14) + (3*d*(c + d*x)^11)/(182*(b*c
- a*d)^2*(a + b*x)^13) - (d^2*(c + d*x)^11)/(364*(b*c - a*d)^3*(a + b*x)^12
) + (d^3*(c + d*x)^11)/(4004*(b*c - a*d)^4*(a + b*x)^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx &= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}}-\frac{(3d)\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx}{14(bc-ad)}\\
&= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}}+\frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}}+\frac{\left(3d^2\right)\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx}{91(bc-ad)^2}\\
&= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}}+\frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}}-\frac{d^2(c+dx)^{11}}{364(bc-ad)^3(a+bx)^{12}}-\frac{d^3\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx}{364(bc-ad)^3}\\
&= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}}+\frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}}-\frac{d^2(c+dx)^{11}}{364(bc-ad)^3(a+bx)^{12}}+\frac{d^3(c+dx)^{11}}{4004(bc-ad)^4(a+bx)^{11}}
\end{aligned}$$

Mathematica [B] time = 0.277874, size = 692, normalized size = 5.77

$$-\frac{a^2b^8d^2\left(7644c^6d^2x^2+20384c^5d^3x^3+35035c^4d^4x^4+40040c^3d^5x^5+30030c^2d^6x^6+1680c^7dx+165c^8+13728cd^7x^7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^15,x]
```

```
[Out] -(a^10*d^10 + 2*a^9*b*d^9*(2*c + 7*d*x) + a^8*b^2*d^8*(10*c^2 + 56*c*d*x +
91*d^2*x^2) + 4*a^7*b^3*d^7*(5*c^3 + 35*c^2*d*x + 91*c*d^2*x^2 + 91*d^3*x^3
) + 7*a^6*b^4*d^6*(5*c^4 + 40*c^3*d*x + 130*c^2*d^2*x^2 + 208*c*d^3*x^3 + 1
43*d^4*x^4) + 14*a^5*b^5*d^5*(4*c^5 + 35*c^4*d*x + 130*c^3*d^2*x^2 + 260*c^
2*d^3*x^3 + 286*c*d^4*x^4 + 143*d^5*x^5) + 7*a^4*b^6*d^4*(12*c^6 + 112*c^5*
d*x + 455*c^4*d^2*x^2 + 1040*c^3*d^3*x^3 + 1430*c^2*d^4*x^4 + 1144*c*d^5*x^
5 + 429*d^6*x^6) + 4*a^3*b^7*d^3*(30*c^7 + 294*c^6*d*x + 1274*c^5*d^2*x^2 +
 3185*c^4*d^3*x^3 + 5005*c^3*d^4*x^4 + 5005*c^2*d^5*x^5 + 3003*c*d^6*x^6 +
858*d^7*x^7) + a^2*b^8*d^2*(165*c^8 + 1680*c^7*d*x + 7644*c^6*d^2*x^2 + 203
84*c^5*d^3*x^3 + 35035*c^4*d^4*x^4 + 40040*c^3*d^5*x^5 + 30030*c^2*d^6*x^6
+ 13728*c*d^7*x^7 + 3003*d^8*x^8) + 2*a*b^9*d*(110*c^9 + 1155*c^8*d*x + 546
0*c^7*d^2*x^2 + 15288*c^6*d^3*x^3 + 28028*c^5*d^4*x^4 + 35035*c^4*d^5*x^5 +
 30030*c^3*d^6*x^6 + 17160*c^2*d^7*x^7 + 6006*c*d^8*x^8 + 1001*d^9*x^9) + b
^10*(286*c^10 + 3080*c^9*d*x + 15015*c^8*d^2*x^2 + 43680*c^7*d^3*x^3 + 8408
4*c^6*d^4*x^4 + 112112*c^5*d^5*x^5 + 105105*c^4*d^6*x^6 + 68640*c^3*d^7*x^7
 + 30030*c^2*d^8*x^8 + 8008*c*d^9*x^9 + 1001*d^10*x^10))/(4004*b^11*(a + b*
x)^14)
```

Maple [B] time = 0.01, size = 867, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^15,x)
```

```
[Out] -21*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2
*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^10-105/4*d^6*(a^4*d^4-4*a^
3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^8-15/2*d^8*
(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^6-1/14*(a^10*d^10-10*a^9*b*c*d^9+4
5*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d
^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*
d+b^10*c^10)/b^11/(b*x+a)^14+120/11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c
^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6
*d-b^7*c^7)/b^11/(b*x+a)^11+10/13*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d
^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^
6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^13+2*d^9*(a*d-
b*c)/b^11/(b*x+a)^5-1/4*d^10/b^11/(b*x+a)^4-15/4*d^2*(a^8*d^8-8*a^7*b*c*d^7
+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^
3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^12+28*d^5*(a^5*d^5
-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)
/b^11/(b*x+a)^9+120/7*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^1
1/(b*x+a)^7
```

Maxima [B] time = 1.26077, size = 1361, normalized size = 11.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="maxima")
```

```
[Out] -1/4004*(1001*b^10*d^10*x^10 + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165*a^2*b^
8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^5*d^5 +
 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^9*b*c*d
^9 + a^10*d^10 + 2002*(4*b^10*c*d^9 + a*b^9*d^10)*x^9 + 3003*(10*b^10*c^2*d
^8 + 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 3432*(20*b^10*c^3*d^7 + 10*a*b^9*c
^2*d^8 + 4*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 3003*(35*b^10*c^4*d^6 + 20*a
*b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 + 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2
002*(56*b^10*c^5*d^5 + 35*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 10*a^3*b^7*c
^2*d^8 + 4*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1001*(84*b^10*c^6*d^4 + 56*a
*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 10*a^4*b^6*c^2*d^8
 + 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 364*(120*b^10*c^7*d^3 + 84*a*b^9*c
^6*d^4 + 56*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 20*a^4*b^6*c^3*d^7 + 10*
a^5*b^5*c^2*d^8 + 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 91*(165*b^10*c^8*d^
2 + 120*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 56*a^3*b^7*c^5*d^5 + 35*a^4*b^
6*c^4*d^6 + 20*a^5*b^5*c^3*d^7 + 10*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9 + a^8
*b^2*d^10)*x^2 + 14*(220*b^10*c^9*d + 165*a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d
^3 + 84*a^3*b^7*c^6*d^4 + 56*a^4*b^6*c^5*d^5 + 35*a^5*b^5*c^4*d^6 + 20*a^6*
b^4*c^3*d^7 + 10*a^7*b^3*c^2*d^8 + 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^25*x
^14 + 14*a*b^24*x^13 + 91*a^2*b^23*x^12 + 364*a^3*b^22*x^11 + 1001*a^4*b^21
*x^10 + 2002*a^5*b^20*x^9 + 3003*a^6*b^19*x^8 + 3432*a^7*b^18*x^7 + 3003*a^
8*b^17*x^6 + 2002*a^9*b^16*x^5 + 1001*a^10*b^15*x^4 + 364*a^11*b^14*x^3 + 9
1*a^12*b^13*x^2 + 14*a^13*b^12*x + a^14*b^11)
```

Fricas [B] time = 1.89413, size = 2190, normalized size = 18.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="fricas")
```

```
[Out] -1/4004*(1001*b^10*d^10*x^10 + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165*a^2*b^
8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^5*d^5 +
 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^9*b*c*d
^9 + a^10*d^10 + 2002*(4*b^10*c*d^9 + a*b^9*d^10)*x^9 + 3003*(10*b^10*c^2*d
^8 + 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 3432*(20*b^10*c^3*d^7 + 10*a*b^9*c
^2*d^8 + 4*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 3003*(35*b^10*c^4*d^6 + 20*a
*b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 + 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2
002*(56*b^10*c^5*d^5 + 35*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 10*a^3*b^7*c
^2*d^8 + 4*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1001*(84*b^10*c^6*d^4 + 56*a
*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 10*a^4*b^6*c^2*d^8
 + 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 364*(120*b^10*c^7*d^3 + 84*a*b^9*c
^6*d^4 + 56*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 20*a^4*b^6*c^3*d^7 + 10*
a^5*b^5*c^2*d^8 + 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 91*(165*b^10*c^8*d^
2 + 120*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 56*a^3*b^7*c^5*d^5 + 35*a^4*b^
6*c^4*d^6 + 20*a^5*b^5*c^3*d^7 + 10*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9 + a^8
*b^2*d^10)*x^2 + 14*(220*b^10*c^9*d + 165*a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d
^3 + 84*a^3*b^7*c^6*d^4 + 56*a^4*b^6*c^5*d^5 + 35*a^5*b^5*c^4*d^6 + 20*a^6*
b^4*c^3*d^7 + 10*a^7*b^3*c^2*d^8 + 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^25*x
```

```
^14 + 14*a*b^24*x^13 + 91*a^2*b^23*x^12 + 364*a^3*b^22*x^11 + 1001*a^4*b^21
*x^10 + 2002*a^5*b^20*x^9 + 3003*a^6*b^19*x^8 + 3432*a^7*b^18*x^7 + 3003*a^
8*b^17*x^6 + 2002*a^9*b^16*x^5 + 1001*a^10*b^15*x^4 + 364*a^11*b^14*x^3 + 9
1*a^12*b^13*x^2 + 14*a^13*b^12*x + a^14*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**15,x)

[Out] Timed out
```

Giac [B] time = 1.06507, size = 1297, normalized size = 10.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="giac")

[Out] -1/4004*(1001*b^10*d^10*x^10 + 8008*b^10*c*d^9*x^9 + 2002*a*b^9*d^10*x^9 +
30030*b^10*c^2*d^8*x^8 + 12012*a*b^9*c*d^9*x^8 + 3003*a^2*b^8*d^10*x^8 + 68
640*b^10*c^3*d^7*x^7 + 34320*a*b^9*c^2*d^8*x^7 + 13728*a^2*b^8*c*d^9*x^7 +
3432*a^3*b^7*d^10*x^7 + 105105*b^10*c^4*d^6*x^6 + 60060*a*b^9*c^3*d^7*x^6 +
 30030*a^2*b^8*c^2*d^8*x^6 + 12012*a^3*b^7*c*d^9*x^6 + 3003*a^4*b^6*d^10*x^
6 + 112112*b^10*c^5*d^5*x^5 + 70070*a*b^9*c^4*d^6*x^5 + 40040*a^2*b^8*c^3*d
^7*x^5 + 20020*a^3*b^7*c^2*d^8*x^5 + 8008*a^4*b^6*c*d^9*x^5 + 2002*a^5*b^5*
d^10*x^5 + 84084*b^10*c^6*d^4*x^4 + 56056*a*b^9*c^5*d^5*x^4 + 35035*a^2*b^8
*c^4*d^6*x^4 + 20020*a^3*b^7*c^3*d^7*x^4 + 10010*a^4*b^6*c^2*d^8*x^4 + 4004
*a^5*b^5*c*d^9*x^4 + 1001*a^6*b^4*d^10*x^4 + 43680*b^10*c^7*d^3*x^3 + 30576
*a*b^9*c^6*d^4*x^3 + 20384*a^2*b^8*c^5*d^5*x^3 + 12740*a^3*b^7*c^4*d^6*x^3
+ 7280*a^4*b^6*c^3*d^7*x^3 + 3640*a^5*b^5*c^2*d^8*x^3 + 1456*a^6*b^4*c*d^9*
x^3 + 364*a^7*b^3*d^10*x^3 + 15015*b^10*c^8*d^2*x^2 + 10920*a*b^9*c^7*d^3*x
^2 + 7644*a^2*b^8*c^6*d^4*x^2 + 5096*a^3*b^7*c^5*d^5*x^2 + 3185*a^4*b^6*c^4
*d^6*x^2 + 1820*a^5*b^5*c^3*d^7*x^2 + 910*a^6*b^4*c^2*d^8*x^2 + 364*a^7*b^3
*c*d^9*x^2 + 91*a^8*b^2*d^10*x^2 + 3080*b^10*c^9*d*x + 2310*a*b^9*c^8*d^2*x
 + 1680*a^2*b^8*c^7*d^3*x + 1176*a^3*b^7*c^6*d^4*x + 784*a^4*b^6*c^5*d^5*x
+ 490*a^5*b^5*c^4*d^6*x + 280*a^6*b^4*c^3*d^7*x + 140*a^7*b^3*c^2*d^8*x + 5
6*a^8*b^2*c*d^9*x + 14*a^9*b*d^10*x + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165
*a^2*b^8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^
5*d^5 + 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^
9*b*c*d^9 + a^10*d^10)/((b*x + a)^14*b^11)
```

3.1327 $\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx$

Optimal. Leaf size=151

$$-\frac{d^4(c+dx)^{11}}{15015(a+bx)^{11}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{1365(a+bx)^{12}(bc-ad)^4}-\frac{2d^2(c+dx)^{11}}{455(a+bx)^{13}(bc-ad)^3}+\frac{2d(c+dx)^{11}}{105(a+bx)^{14}(bc-ad)^2}-\frac{(c+ ...}{15(a+b ...}$$

```
[Out] -(c + d*x)^11/(15*(b*c - a*d)*(a + b*x)^15) + (2*d*(c + d*x)^11)/(105*(b*c
- a*d)^2*(a + b*x)^14) - (2*d^2*(c + d*x)^11)/(455*(b*c - a*d)^3*(a + b*x)^
13) + (d^3*(c + d*x)^11)/(1365*(b*c - a*d)^4*(a + b*x)^12) - (d^4*(c + d*x)
^11)/(15015*(b*c - a*d)^5*(a + b*x)^11)
```

Rubi [A] time = 0.0444379, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{d^4(c+dx)^{11}}{15015(a+bx)^{11}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{1365(a+bx)^{12}(bc-ad)^4}-\frac{2d^2(c+dx)^{11}}{455(a+bx)^{13}(bc-ad)^3}+\frac{2d(c+dx)^{11}}{105(a+bx)^{14}(bc-ad)^2}-\frac{(c+ ...}{15(a+b ...}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^16,x]
```

```
[Out] -(c + d*x)^11/(15*(b*c - a*d)*(a + b*x)^15) + (2*d*(c + d*x)^11)/(105*(b*c
- a*d)^2*(a + b*x)^14) - (2*d^2*(c + d*x)^11)/(455*(b*c - a*d)^3*(a + b*x)^
13) + (d^3*(c + d*x)^11)/(1365*(b*c - a*d)^4*(a + b*x)^12) - (d^4*(c + d*x)
^11)/(15015*(b*c - a*d)^5*(a + b*x)^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx = -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} - \frac{(4d)\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx}{15(bc-ad)}$$
$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} + \frac{\left(2d^2\right)\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx}{35(bc-ad)^2}$$
$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} - \frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}} - \frac{\left(4d^3\right)\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx}{455(bc-ad)^3}$$
$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} - \frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}} + \frac{d^3(c+dx)^{11}}{1365(bc-ad)^4(a+$$
$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} - \frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}} + \frac{d^3(c+dx)^{11}}{1365(bc-ad)^4(a+$$

Mathematica [B] time = 0.282343, size = 690, normalized size = 4.57

$$-\frac{15a^2b^8d^2\left(1470c^6d^2x^2 + 3822c^5d^3x^3 + 6370c^4d^4x^4 + 7007c^3d^5x^5 + 5005c^2d^6x^6 + 330c^7dx + 33c^8 + 2145cd^7x^7 + 42$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^16,x]
```

```
[Out] -(a^10*d^10 + 5*a^9*b*d^9*(c + 3*d*x) + 15*a^8*b^2*d^8*(c^2 + 5*c*d*x + 7*d
^2*x^2) + 5*a^7*b^3*d^7*(7*c^3 + 45*c^2*d*x + 105*c*d^2*x^2 + 91*d^3*x^3) +
 35*a^6*b^4*d^6*(2*c^4 + 15*c^3*d*x + 45*c^2*d^2*x^2 + 65*c*d^3*x^3 + 39*d^
4*x^4) + 21*a^5*b^5*d^5*(6*c^5 + 50*c^4*d*x + 175*c^3*d^2*x^2 + 325*c^2*d^3
*x^3 + 325*c*d^4*x^4 + 143*d^5*x^5) + 35*a^4*b^6*d^4*(6*c^6 + 54*c^5*d*x +
210*c^4*d^2*x^2 + 455*c^3*d^3*x^3 + 585*c^2*d^4*x^4 + 429*c*d^5*x^5 + 143*d
^6*x^6) + 5*a^3*b^7*d^3*(66*c^7 + 630*c^6*d*x + 2646*c^5*d^2*x^2 + 6370*c^4
*d^3*x^3 + 9555*c^3*d^4*x^4 + 9009*c^2*d^5*x^5 + 5005*c*d^6*x^6 + 1287*d^7*
x^7) + 15*a^2*b^8*d^2*(33*c^8 + 330*c^7*d*x + 1470*c^6*d^2*x^2 + 3822*c^5*d
^3*x^3 + 6370*c^4*d^4*x^4 + 7007*c^3*d^5*x^5 + 5005*c^2*d^6*x^6 + 2145*c*d^
7*x^7 + 429*d^8*x^8) + 5*a*b^9*d*(143*c^9 + 1485*c^8*d*x + 6930*c^7*d^2*x^2
 + 19110*c^6*d^3*x^3 + 34398*c^5*d^4*x^4 + 42042*c^4*d^5*x^5 + 35035*c^3*d^
6*x^6 + 19305*c^2*d^7*x^7 + 6435*c*d^8*x^8 + 1001*d^9*x^9) + b^10*(1001*c^1
0 + 10725*c^9*d*x + 51975*c^8*d^2*x^2 + 150150*c^7*d^3*x^3 + 286650*c^6*d^4
*x^4 + 378378*c^5*d^5*x^5 + 350350*c^4*d^6*x^6 + 225225*c^3*d^7*x^7 + 96525
*c^2*d^8*x^8 + 25025*c*d^9*x^9 + 3003*d^10*x^10))/(15015*b^11*(a + b*x)^15)
```

Maple [B] time = 0.01, size = 867, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^16,x)
```

```
[Out] 126/5*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*
b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^10+15*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^
2*d-b^3*c^3)/b^11/(b*x+a)^8+5/3*d^9*(a*d-b*c)/b^11/(b*x+a)^6+5/7*d*(a^9*d^9
-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-12
```

```
6*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c
^9)/b^11/(b*x+a)^14-210/11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20
*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^11-
45/13*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a
^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)
/b^11/(b*x+a)^13-1/15*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*
b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120
*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^
15-1/5*d^10/b^11/(b*x+a)^5+10*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5
-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7
*c^7)/b^11/(b*x+a)^12-70/3*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^9-45/7*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^1
1/(b*x+a)^7
```

Maxima [B] time = 1.25357, size = 1376, normalized size = 9.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 1001*b^10*c^10 + 715*a*b^9*c^9*d + 495*a^2*
b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d
^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 + 5*a^9*b
*c*d^9 + a^10*d^10 + 5005*(5*b^10*c*d^9 + a*b^9*d^10)*x^9 + 6435*(15*b^10*c
^2*d^8 + 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 6435*(35*b^10*c^3*d^7 + 15*a*b
^9*c^2*d^8 + 5*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 5005*(70*b^10*c^4*d^6 +
35*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 + 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 3003*(126*b^10*c^5*d^5 + 70*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 + 15*a^3*
b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1365*(210*b^10*c^6*d^4
+ 126*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 + 35*a^3*b^7*c^3*d^7 + 15*a^4*b^6*
c^2*d^8 + 5*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 455*(330*b^10*c^7*d^3 + 210
*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 + 70*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*
d^7 + 15*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 105*(495*b
^10*c^8*d^2 + 330*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
 + 70*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 + 5*a^7*b^3
*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(715*b^10*c^9*d + 495*a*b^9*c^8*d^2 + 330*a
^2*b^8*c^7*d^3 + 210*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 70*a^5*b^5*c^4
*d^6 + 35*a^6*b^4*c^3*d^7 + 15*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^26*x^15 + 15*a*b^25*x^14 + 105*a^2*b^24*x^13 + 455*a^3*b^23*x^12
+ 1365*a^4*b^22*x^11 + 3003*a^5*b^21*x^10 + 5005*a^6*b^20*x^9 + 6435*a^7*b^
19*x^8 + 6435*a^8*b^18*x^7 + 5005*a^9*b^17*x^6 + 3003*a^10*b^16*x^5 + 1365*
a^11*b^15*x^4 + 455*a^12*b^14*x^3 + 105*a^13*b^13*x^2 + 15*a^14*b^12*x + a^
15*b^11)
```

Fricas [B] time = 1.9001, size = 2241, normalized size = 14.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="fricas")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 1001*b^10*c^10 + 715*a*b^9*c^9*d + 495*a^2*
b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d
^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 + 5*a^9*b
*c*d^9 + a^10*d^10 + 5005*(5*b^10*c*d^9 + a*b^9*d^10)*x^9 + 6435*(15*b^10*c
^2*d^8 + 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 6435*(35*b^10*c^3*d^7 + 15*a*b
^9*c^2*d^8 + 5*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 5005*(70*b^10*c^4*d^6 +
35*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 + 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 3003*(126*b^10*c^5*d^5 + 70*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 + 15*a^3*
b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1365*(210*b^10*c^6*d^4
+ 126*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 + 35*a^3*b^7*c^3*d^7 + 15*a^4*b^6*
c^2*d^8 + 5*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 455*(330*b^10*c^7*d^3 + 210
*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 + 70*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*
d^7 + 15*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 105*(495*b
^10*c^8*d^2 + 330*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
 + 70*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 + 5*a^7*b^3
*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(715*b^10*c^9*d + 495*a*b^9*c^8*d^2 + 330*a
^2*b^8*c^7*d^3 + 210*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 70*a^5*b^5*c^4
*d^6 + 35*a^6*b^4*c^3*d^7 + 15*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^26*x^15 + 15*a*b^25*x^14 + 105*a^2*b^24*x^13 + 455*a^3*b^23*x^12
+ 1365*a^4*b^22*x^11 + 3003*a^5*b^21*x^10 + 5005*a^6*b^20*x^9 + 6435*a^7*b^
19*x^8 + 6435*a^8*b^18*x^7 + 5005*a^9*b^17*x^6 + 3003*a^10*b^16*x^5 + 1365*
a^11*b^15*x^4 + 455*a^12*b^14*x^3 + 105*a^13*b^13*x^2 + 15*a^14*b^12*x + a^
15*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**16,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.08508, size = 1297, normalized size = 8.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="giac")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 25025*b^10*c*d^9*x^9 + 5005*a*b^9*d^10*x^9
+ 96525*b^10*c^2*d^8*x^8 + 32175*a*b^9*c*d^9*x^8 + 6435*a^2*b^8*d^10*x^8 +
225225*b^10*c^3*d^7*x^7 + 96525*a*b^9*c^2*d^8*x^7 + 32175*a^2*b^8*c*d^9*x^7
 + 6435*a^3*b^7*d^10*x^7 + 350350*b^10*c^4*d^6*x^6 + 175175*a*b^9*c^3*d^7*x
^6 + 75075*a^2*b^8*c^2*d^8*x^6 + 25025*a^3*b^7*c*d^9*x^6 + 5005*a^4*b^6*d^1
0*x^6 + 378378*b^10*c^5*d^5*x^5 + 210210*a*b^9*c^4*d^6*x^5 + 105105*a^2*b^8
*c^3*d^7*x^5 + 45045*a^3*b^7*c^2*d^8*x^5 + 15015*a^4*b^6*c*d^9*x^5 + 3003*a
^5*b^5*d^10*x^5 + 286650*b^10*c^6*d^4*x^4 + 171990*a*b^9*c^5*d^5*x^4 + 9555
0*a^2*b^8*c^4*d^6*x^4 + 47775*a^3*b^7*c^3*d^7*x^4 + 20475*a^4*b^6*c^2*d^8*x
^4 + 6825*a^5*b^5*c*d^9*x^4 + 1365*a^6*b^4*d^10*x^4 + 150150*b^10*c^7*d^3*x
^3 + 95550*a*b^9*c^6*d^4*x^3 + 57330*a^2*b^8*c^5*d^5*x^3 + 31850*a^3*b^7*c^
4*d^6*x^3 + 15925*a^4*b^6*c^3*d^7*x^3 + 6825*a^5*b^5*c^2*d^8*x^3 + 2275*a^6
```

```
*b^4*c*d^9*x^3 + 455*a^7*b^3*d^10*x^3 + 51975*b^10*c^8*d^2*x^2 + 34650*a*b^
9*c^7*d^3*x^2 + 22050*a^2*b^8*c^6*d^4*x^2 + 13230*a^3*b^7*c^5*d^5*x^2 + 735
0*a^4*b^6*c^4*d^6*x^2 + 3675*a^5*b^5*c^3*d^7*x^2 + 1575*a^6*b^4*c^2*d^8*x^2
 + 525*a^7*b^3*c*d^9*x^2 + 105*a^8*b^2*d^10*x^2 + 10725*b^10*c^9*d*x + 7425
*a*b^9*c^8*d^2*x + 4950*a^2*b^8*c^7*d^3*x + 3150*a^3*b^7*c^6*d^4*x + 1890*a
^4*b^6*c^5*d^5*x + 1050*a^5*b^5*c^4*d^6*x + 525*a^6*b^4*c^3*d^7*x + 225*a^7
*b^3*c^2*d^8*x + 75*a^8*b^2*c*d^9*x + 15*a^9*b*d^10*x + 1001*b^10*c^10 + 71
5*a*b^9*c^9*d + 495*a^2*b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6
*d^4 + 126*a^5*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a
^8*b^2*c^2*d^8 + 5*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^15*b^11)
```

3.1328 $\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx$

Optimal. Leaf size=182

$$\frac{d^5(c+dx)^{11}}{48048(a+bx)^{11}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{4368(a+bx)^{12}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{728(a+bx)^{13}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{168(a+bx)^{14}(bc-ad)^3}+\frac{d}{48(a+}$$

```
[Out] -(c + d*x)^11/(16*(b*c - a*d)*(a + b*x)^16) + (d*(c + d*x)^11)/(48*(b*c - a
*d)^2*(a + b*x)^15) - (d^2*(c + d*x)^11)/(168*(b*c - a*d)^3*(a + b*x)^14) +
 (d^3*(c + d*x)^11)/(728*(b*c - a*d)^4*(a + b*x)^13) - (d^4*(c + d*x)^11)/(
4368*(b*c - a*d)^5*(a + b*x)^12) + (d^5*(c + d*x)^11)/(48048*(b*c - a*d)^6*
(a + b*x)^11)
```

Rubi [A] time = 0.062674, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d^5(c+dx)^{11}}{48048(a+bx)^{11}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{4368(a+bx)^{12}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{728(a+bx)^{13}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{168(a+bx)^{14}(bc-ad)^3}+\frac{d}{48(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^17,x]
```


```
[Out] -(c + d*x)^11/(16*(b*c - a*d)*(a + b*x)^16) + (d*(c + d*x)^11)/(48*(b*c - a
*d)^2*(a + b*x)^15) - (d^2*(c + d*x)^11)/(168*(b*c - a*d)^3*(a + b*x)^14) +
 (d^3*(c + d*x)^11)/(728*(b*c - a*d)^4*(a + b*x)^13) - (d^4*(c + d*x)^11)/(
4368*(b*c - a*d)^5*(a + b*x)^12) + (d^5*(c + d*x)^11)/(48048*(b*c - a*d)^6*
(a + b*x)^11)
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx = -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} - \frac{(5d)\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx}{16(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} + \frac{d^2\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx}{12(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} - \frac{d^3\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx}{56(bc-ad)^3}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} + \frac{d^3(c+dx)^{11}}{728(bc-ad)^4(a+bx)^{13}}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} + \frac{d^3(c+dx)^{11}}{728(bc-ad)^4(a+bx)^{13}}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} + \frac{d^3(c+dx)^{11}}{728(bc-ad)^4(a+bx)^{13}}$$

Mathematica [B] time = 0.269272, size = 694, normalized size = 3.81

$$-\frac{3a^2b^8d^2\left(18480c^6d^2x^2 + 47040c^5d^3x^3 + 76440c^4d^4x^4 + 81536c^3d^5x^5 + 56056c^2d^6x^6 + 4224c^7dx + 429c^8 + 22880cd^7x^7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^17,x]
```

```
[Out] -(a^10*d^10 + 2*a^9*b*d^9*(3*c + 8*d*x) + 3*a^8*b^2*d^8*(7*c^2 + 32*c*d*x +
 40*d^2*x^2) + 8*a^7*b^3*d^7*(7*c^3 + 42*c^2*d*x + 90*c*d^2*x^2 + 70*d^3*x^
3) + 14*a^6*b^4*d^6*(9*c^4 + 64*c^3*d*x + 180*c^2*d^2*x^2 + 240*c*d^3*x^3 +
 130*d^4*x^4) + 84*a^5*b^5*d^5*(3*c^5 + 24*c^4*d*x + 80*c^3*d^2*x^2 + 140*c
^2*d^3*x^3 + 130*c*d^4*x^4 + 52*d^5*x^5) + 14*a^4*b^6*d^4*(33*c^6 + 288*c^5
*d*x + 1080*c^4*d^2*x^2 + 2240*c^3*d^3*x^3 + 2730*c^2*d^4*x^4 + 1872*c*d^5*
x^5 + 572*d^6*x^6) + 8*a^3*b^7*d^3*(99*c^7 + 924*c^6*d*x + 3780*c^5*d^2*x^2
 + 8820*c^4*d^3*x^3 + 12740*c^3*d^4*x^4 + 11466*c^2*d^5*x^5 + 6006*c*d^6*x^
6 + 1430*d^7*x^7) + 3*a^2*b^8*d^2*(429*c^8 + 4224*c^7*d*x + 18480*c^6*d^2*x
^2 + 47040*c^5*d^3*x^3 + 76440*c^4*d^4*x^4 + 81536*c^3*d^5*x^5 + 56056*c^2*
d^6*x^6 + 22880*c*d^7*x^7 + 4290*d^8*x^8) + 2*a*b^9*d*(1001*c^9 + 10296*c^8
*d*x + 47520*c^7*d^2*x^2 + 129360*c^6*d^3*x^3 + 229320*c^5*d^4*x^4 + 275184
*c^4*d^5*x^5 + 224224*c^3*d^6*x^6 + 120120*c^2*d^7*x^7 + 38610*c*d^8*x^8 +
5720*d^9*x^9) + b^10*(3003*c^10 + 32032*c^9*d*x + 154440*c^8*d^2*x^2 + 4435
20*c^7*d^3*x^3 + 840840*c^6*d^4*x^4 + 1100736*c^5*d^5*x^5 + 1009008*c^4*d^6
*x^6 + 640640*c^3*d^7*x^7 + 270270*c^2*d^8*x^8 + 68640*c*d^9*x^9 + 8008*d^1
0*x^10))/(48048*b^11*(a + b*x)^16)
```

Maple [B] time = 0.009, size = 867, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^17,x)
```

```
[Out] -21*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^1
1/(b*x+a)^10-45/8*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^8-1/6*d^10/b
^11/(b*x+a)^6-45/14*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^
3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*
c^7*d+b^8*c^8)/b^11/(b*x+a)^14-1/16*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^
2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b
^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)
/b^11/(b*x+a)^16+252/11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^
2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^11+120/13*d^3*(a^7*d^7-7*
a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2
*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^13+2/3*d*(a^9*d^9-9*a^8*b*
c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5
*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/
(b*x+a)^15-35/2*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^
3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^12+40/3*d^7*(a
^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^9+10/7*d^9*(a*d-b*
c)/b^11/(b*x+a)^7
```

Maxima [B] time = 1.26298, size = 1391, normalized size = 7.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="maxima")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 3003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^
2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5
*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^
9*b*c*d^9 + a^10*d^10 + 11440*(6*b^10*c*d^9 + a*b^9*d^10)*x^9 + 12870*(21*b
^10*c^2*d^8 + 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 11440*(56*b^10*c^3*d^7 +
21*a*b^9*c^2*d^8 + 6*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 8008*(126*b^10*c^4
*d^6 + 56*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 + 6*a^3*b^7*c*d^9 + a^4*b^6*d^
10)*x^6 + 4368*(252*b^10*c^5*d^5 + 126*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 +
 21*a^3*b^7*c^2*d^8 + 6*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1820*(462*b^10*
c^6*d^4 + 252*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 + 56*a^3*b^7*c^3*d^7 + 21
*a^4*b^6*c^2*d^8 + 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 560*(792*b^10*c^7*
d^3 + 462*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 + 126*a^3*b^7*c^4*d^6 + 56*a^
4*b^6*c^3*d^7 + 21*a^5*b^5*c^2*d^8 + 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 +
120*(1287*b^10*c^8*d^2 + 792*a*b^9*c^7*d^3 + 462*a^2*b^8*c^6*d^4 + 252*a^3*
b^7*c^5*d^5 + 126*a^4*b^6*c^4*d^6 + 56*a^5*b^5*c^3*d^7 + 21*a^6*b^4*c^2*d^8
 + 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 16*(2002*b^10*c^9*d + 1287*a*b^9*c
^8*d^2 + 792*a^2*b^8*c^7*d^3 + 462*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5*d^5 +
126*a^5*b^5*c^4*d^6 + 56*a^6*b^4*c^3*d^7 + 21*a^7*b^3*c^2*d^8 + 6*a^8*b^2*c
*d^9 + a^9*b*d^10)*x)/(b^27*x^16 + 16*a*b^26*x^15 + 120*a^2*b^25*x^14 + 560
*a^3*b^24*x^13 + 1820*a^4*b^23*x^12 + 4368*a^5*b^22*x^11 + 8008*a^6*b^21*x^
10 + 11440*a^7*b^20*x^9 + 12870*a^8*b^19*x^8 + 11440*a^9*b^18*x^7 + 8008*a^
10*b^17*x^6 + 4368*a^11*b^16*x^5 + 1820*a^12*b^15*x^4 + 560*a^13*b^14*x^3 +
 120*a^14*b^13*x^2 + 16*a^15*b^12*x + a^16*b^11)
```

Fricas [B] time = 1.83617, size = 2295, normalized size = 12.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="fricas")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 3003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^
2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5
*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^
9*b*c*d^9 + a^10*d^10 + 11440*(6*b^10*c*d^9 + a*b^9*d^10)*x^9 + 12870*(21*b
^10*c^2*d^8 + 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 11440*(56*b^10*c^3*d^7 +
21*a*b^9*c^2*d^8 + 6*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 8008*(126*b^10*c^4
*d^6 + 56*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 + 6*a^3*b^7*c*d^9 + a^4*b^6*d^
10)*x^6 + 4368*(252*b^10*c^5*d^5 + 126*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 +
 21*a^3*b^7*c^2*d^8 + 6*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1820*(462*b^10*
c^6*d^4 + 252*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 + 56*a^3*b^7*c^3*d^7 + 21
*a^4*b^6*c^2*d^8 + 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 560*(792*b^10*c^7*
d^3 + 462*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 + 126*a^3*b^7*c^4*d^6 + 56*a^
4*b^6*c^3*d^7 + 21*a^5*b^5*c^2*d^8 + 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 +
120*(1287*b^10*c^8*d^2 + 792*a*b^9*c^7*d^3 + 462*a^2*b^8*c^6*d^4 + 252*a^3*
b^7*c^5*d^5 + 126*a^4*b^6*c^4*d^6 + 56*a^5*b^5*c^3*d^7 + 21*a^6*b^4*c^2*d^8
 + 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 16*(2002*b^10*c^9*d + 1287*a*b^9*c
^8*d^2 + 792*a^2*b^8*c^7*d^3 + 462*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5*d^5 +
126*a^5*b^5*c^4*d^6 + 56*a^6*b^4*c^3*d^7 + 21*a^7*b^3*c^2*d^8 + 6*a^8*b^2*c
*d^9 + a^9*b*d^10)*x)/(b^27*x^16 + 16*a*b^26*x^15 + 120*a^2*b^25*x^14 + 560
*a^3*b^24*x^13 + 1820*a^4*b^23*x^12 + 4368*a^5*b^22*x^11 + 8008*a^6*b^21*x^
10 + 11440*a^7*b^20*x^9 + 12870*a^8*b^19*x^8 + 11440*a^9*b^18*x^7 + 8008*a^
10*b^17*x^6 + 4368*a^11*b^16*x^5 + 1820*a^12*b^15*x^4 + 560*a^13*b^14*x^3 +
 120*a^14*b^13*x^2 + 16*a^15*b^12*x + a^16*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**17,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06038, size = 1297, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="giac")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 68640*b^10*c*d^9*x^9 + 11440*a*b^9*d^10*x^9
 + 270270*b^10*c^2*d^8*x^8 + 77220*a*b^9*c*d^9*x^8 + 12870*a^2*b^8*d^10*x^8
 + 640640*b^10*c^3*d^7*x^7 + 240240*a*b^9*c^2*d^8*x^7 + 68640*a^2*b^8*c*d^9
*x^7 + 11440*a^3*b^7*d^10*x^7 + 1009008*b^10*c^4*d^6*x^6 + 448448*a*b^9*c^3
*d^7*x^6 + 168168*a^2*b^8*c^2*d^8*x^6 + 48048*a^3*b^7*c*d^9*x^6 + 8008*a^4*
b^6*d^10*x^6 + 1100736*b^10*c^5*d^5*x^5 + 550368*a*b^9*c^4*d^6*x^5 + 244608
*a^2*b^8*c^3*d^7*x^5 + 91728*a^3*b^7*c^2*d^8*x^5 + 26208*a^4*b^6*c*d^9*x^5
+ 4368*a^5*b^5*d^10*x^5 + 840840*b^10*c^6*d^4*x^4 + 458640*a*b^9*c^5*d^5*x^
4 + 229320*a^2*b^8*c^4*d^6*x^4 + 101920*a^3*b^7*c^3*d^7*x^4 + 38220*a^4*b^6
*c^2*d^8*x^4 + 10920*a^5*b^5*c*d^9*x^4 + 1820*a^6*b^4*d^10*x^4 + 443520*b^1
```

0*c^7*d^3*x^3 + 258720*a*b^9*c^6*d^4*x^3 + 141120*a^2*b^8*c^5*d^5*x^3 + 705
60*a^3*b^7*c^4*d^6*x^3 + 31360*a^4*b^6*c^3*d^7*x^3 + 11760*a^5*b^5*c^2*d^8*
x^3 + 3360*a^6*b^4*c*d^9*x^3 + 560*a^7*b^3*d^10*x^3 + 154440*b^10*c^8*d^2*x
^2 + 95040*a*b^9*c^7*d^3*x^2 + 55440*a^2*b^8*c^6*d^4*x^2 + 30240*a^3*b^7*c^
5*d^5*x^2 + 15120*a^4*b^6*c^4*d^6*x^2 + 6720*a^5*b^5*c^3*d^7*x^2 + 2520*a^6
*b^4*c^2*d^8*x^2 + 720*a^7*b^3*c*d^9*x^2 + 120*a^8*b^2*d^10*x^2 + 32032*b^1
0*c^9*d*x + 20592*a*b^9*c^8*d^2*x + 12672*a^2*b^8*c^7*d^3*x + 7392*a^3*b^7*
c^6*d^4*x + 4032*a^4*b^6*c^5*d^5*x + 2016*a^5*b^5*c^4*d^6*x + 896*a^6*b^4*c
^3*d^7*x + 336*a^7*b^3*c^2*d^8*x + 96*a^8*b^2*c*d^9*x + 16*a^9*b*d^10*x + 3
003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d
^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a
^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)
^16*b^11)

3.1329 $\int \frac{(c+dx)^{10}}{(a+bx)^{18}} dx$

Optimal. Leaf size=213

$$-\frac{d^6(c+dx)^{11}}{136136(a+bx)^{11}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{12376(a+bx)^{12}(bc-ad)^6}-\frac{3d^4(c+dx)^{11}}{6188(a+bx)^{13}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{476(a+bx)^{14}(bc-ad)^4}-136(\ldots$$

```
[Out] -(c + d*x)^11/(17*(b*c - a*d)*(a + b*x)^17) + (3*d*(c + d*x)^11)/(136*(b*c
 - a*d)^2*(a + b*x)^16) - (d^2*(c + d*x)^11)/(136*(b*c - a*d)^3*(a + b*x)^15
) + (d^3*(c + d*x)^11)/(476*(b*c - a*d)^4*(a + b*x)^14) - (3*d^4*(c + d*x)^
11)/(6188*(b*c - a*d)^5*(a + b*x)^13) + (d^5*(c + d*x)^11)/(12376*(b*c - a*
d)^6*(a + b*x)^12) - (d^6*(c + d*x)^11)/(136136*(b*c - a*d)^7*(a + b*x)^11)
```

Rubi [A] time = 0.0787245, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{d^6(c+dx)^{11}}{136136(a+bx)^{11}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{12376(a+bx)^{12}(bc-ad)^6}-\frac{3d^4(c+dx)^{11}}{6188(a+bx)^{13}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{476(a+bx)^{14}(bc-ad)^4}-136(\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^18,x]
```


```
[Out] -(c + d*x)^11/(17*(b*c - a*d)*(a + b*x)^17) + (3*d*(c + d*x)^11)/(136*(b*c
 - a*d)^2*(a + b*x)^16) - (d^2*(c + d*x)^11)/(136*(b*c - a*d)^3*(a + b*x)^15
) + (d^3*(c + d*x)^11)/(476*(b*c - a*d)^4*(a + b*x)^14) - (3*d^4*(c + d*x)^
11)/(6188*(b*c - a*d)^5*(a + b*x)^13) + (d^5*(c + d*x)^11)/(12376*(b*c - a*
d)^6*(a + b*x)^12) - (d^6*(c + d*x)^11)/(136136*(b*c - a*d)^7*(a + b*x)^11)
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx = -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} - \frac{(6d)\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx}{17(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} + \frac{(15d^2)\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx}{136(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} - \frac{d^3\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx}{34(bc-ad)^3}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3(c+dx)^{11}}{476(bc-ad)^4(a+}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3(c+dx)^{11}}{476(bc-ad)^4(a+}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3(c+dx)^{11}}{476(bc-ad)^4(a+}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3(c+dx)^{11}}{476(bc-ad)^4(a+}$$

Mathematica [B] time = 0.360073, size = 690, normalized size = 3.24

$$-\frac{a^2b^8d^2\left(125664c^6d^2x^2 + 314160c^5d^3x^3 + 499800c^4d^4x^4 + 519792c^3d^5x^5 + 346528c^2d^6x^6 + 29172c^7dx + 3003c^8 + 1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^18,x]
```

```
[Out] -(a^10*d^10 + a^9*b*d^9*(7*c + 17*d*x) + a^8*b^2*d^8*(28*c^2 + 119*c*d*x +
136*d^2*x^2) + 4*a^7*b^3*d^7*(21*c^3 + 119*c^2*d*x + 238*c*d^2*x^2 + 170*d^
3*x^3) + 14*a^6*b^4*d^6*(15*c^4 + 102*c^3*d*x + 272*c^2*d^2*x^2 + 340*c*d^3
*x^3 + 170*d^4*x^4) + 14*a^5*b^5*d^5*(33*c^5 + 255*c^4*d*x + 816*c^3*d^2*x^
2 + 1360*c^2*d^3*x^3 + 1190*c*d^4*x^4 + 442*d^5*x^5) + 14*a^4*b^6*d^4*(66*c
^6 + 561*c^5*d*x + 2040*c^4*d^2*x^2 + 4080*c^3*d^3*x^3 + 4760*c^2*d^4*x^4 +
 3094*c*d^5*x^5 + 884*d^6*x^6) + 4*a^3*b^7*d^3*(429*c^7 + 3927*c^6*d*x + 15
708*c^5*d^2*x^2 + 35700*c^4*d^3*x^3 + 49980*c^3*d^4*x^4 + 43316*c^2*d^5*x^5
 + 21658*c*d^6*x^6 + 4862*d^7*x^7) + a^2*b^8*d^2*(3003*c^8 + 29172*c^7*d*x
+ 125664*c^6*d^2*x^2 + 314160*c^5*d^3*x^3 + 499800*c^4*d^4*x^4 + 519792*c^3
*d^5*x^5 + 346528*c^2*d^6*x^6 + 136136*c*d^7*x^7 + 24310*d^8*x^8) + a*b^9*d
*(5005*c^9 + 51051*c^8*d*x + 233376*c^7*d^2*x^2 + 628320*c^6*d^3*x^3 + 1099
560*c^5*d^4*x^4 + 1299480*c^4*d^5*x^5 + 1039584*c^3*d^6*x^6 + 544544*c^2*d^
7*x^7 + 170170*c*d^8*x^8 + 24310*d^9*x^9) + b^10*(8008*c^10 + 85085*c^9*d*x
 + 408408*c^8*d^2*x^2 + 1166880*c^7*d^3*x^3 + 2199120*c^6*d^4*x^4 + 2858856
*c^5*d^5*x^5 + 2598960*c^4*d^6*x^6 + 1633632*c^3*d^7*x^7 + 680680*c^2*d^8*x
^8 + 170170*c*d^9*x^9 + 19448*d^10*x^10))/(136136*b^11*(a + b*x)^17)
```

Maple [B] time = 0.01, size = 867, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^18,x)
```

```
[Out] 12*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^10+5/4*d^
9*(a*d-b*c)/b^11/(b*x+a)^8+60/7*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d
^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b
^7*c^7)/b^11/(b*x+a)^14+5/8*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*
a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-
36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^16-210/11*d^6*(a^4*d
^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^11-1
/17*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^
6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+4
5*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^17-210/13*d^4*(a^6
*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2
-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^13-3*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6
*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^
2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^15+21*d^5*(a^5*d^5-5*a^4*
b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(
b*x+a)^12-5*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^9-1/7*d^10/b^11/(b
*x+a)^7
```

Maxima [B] time = 1.30187, size = 1405, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="maxima")
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*
a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*
c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7
*a^9*b*c*d^9 + a^10*d^10 + 24310*(7*b^10*c*d^9 + a*b^9*d^10)*x^9 + 24310*(2
8*b^10*c^2*d^8 + 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 19448*(84*b^10*c^3*d^7
 + 28*a*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 12376*(210*b^10
*c^4*d^6 + 84*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 + 7*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 6188*(462*b^10*c^5*d^5 + 210*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 + 28*a^3*b^7*c^2*d^8 + 7*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 2380*(924*b
^10*c^6*d^4 + 462*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 + 84*a^3*b^7*c^3*d^7
+ 28*a^4*b^6*c^2*d^8 + 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 680*(1716*b^10
*c^7*d^3 + 924*a*b^9*c^6*d^4 + 462*a^2*b^8*c^5*d^5 + 210*a^3*b^7*c^4*d^6 +
84*a^4*b^6*c^3*d^7 + 28*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x
^3 + 136*(3003*b^10*c^8*d^2 + 1716*a*b^9*c^7*d^3 + 924*a^2*b^8*c^6*d^4 + 46
2*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 + 84*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c
^2*d^8 + 7*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 17*(5005*b^10*c^9*d + 3003*a
*b^9*c^8*d^2 + 1716*a^2*b^8*c^7*d^3 + 924*a^3*b^7*c^6*d^4 + 462*a^4*b^6*c^5
*d^5 + 210*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 + 7*a^
8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^28*x^17 + 17*a*b^27*x^16 + 136*a^2*b^26*x^1
5 + 680*a^3*b^25*x^14 + 2380*a^4*b^24*x^13 + 6188*a^5*b^23*x^12 + 12376*a^6
*b^22*x^11 + 19448*a^7*b^21*x^10 + 24310*a^8*b^20*x^9 + 24310*a^9*b^19*x^8
+ 19448*a^10*b^18*x^7 + 12376*a^11*b^17*x^6 + 6188*a^12*b^16*x^5 + 2380*a^1
3*b^15*x^4 + 680*a^14*b^14*x^3 + 136*a^15*b^13*x^2 + 17*a^16*b^12*x + a^17*
b^11)
```

Fricas [B] time = 1.93375, size = 2338, normalized size = 10.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="fricas")
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*
a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*
c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7
*a^9*b*c*d^9 + a^10*d^10 + 24310*(7*b^10*c*d^9 + a*b^9*d^10)*x^9 + 24310*(2
8*b^10*c^2*d^8 + 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 19448*(84*b^10*c^3*d^7
 + 28*a*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 12376*(210*b^10
*c^4*d^6 + 84*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 + 7*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 6188*(462*b^10*c^5*d^5 + 210*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 + 28*a^3*b^7*c^2*d^8 + 7*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 2380*(924*b
^10*c^6*d^4 + 462*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 + 84*a^3*b^7*c^3*d^7
+ 28*a^4*b^6*c^2*d^8 + 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 680*(1716*b^10
*c^7*d^3 + 924*a*b^9*c^6*d^4 + 462*a^2*b^8*c^5*d^5 + 210*a^3*b^7*c^4*d^6 +
84*a^4*b^6*c^3*d^7 + 28*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x
^3 + 136*(3003*b^10*c^8*d^2 + 1716*a*b^9*c^7*d^3 + 924*a^2*b^8*c^6*d^4 + 46
2*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 + 84*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c
^2*d^8 + 7*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 17*(5005*b^10*c^9*d + 3003*a
*b^9*c^8*d^2 + 1716*a^2*b^8*c^7*d^3 + 924*a^3*b^7*c^6*d^4 + 462*a^4*b^6*c^5
*d^5 + 210*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 + 7*a^
8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^28*x^17 + 17*a*b^27*x^16 + 136*a^2*b^26*x^1
5 + 680*a^3*b^25*x^14 + 2380*a^4*b^24*x^13 + 6188*a^5*b^23*x^12 + 12376*a^6
*b^22*x^11 + 19448*a^7*b^21*x^10 + 24310*a^8*b^20*x^9 + 24310*a^9*b^19*x^8
+ 19448*a^10*b^18*x^7 + 12376*a^11*b^17*x^6 + 6188*a^12*b^16*x^5 + 2380*a^1
3*b^15*x^4 + 680*a^14*b^14*x^3 + 136*a^15*b^13*x^2 + 17*a^16*b^12*x + a^17*
b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**18,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06845, size = 1297, normalized size = 6.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="giac")
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 170170*b^10*c*d^9*x^9 + 24310*a*b^9*d^10*
x^9 + 680680*b^10*c^2*d^8*x^8 + 170170*a*b^9*c*d^9*x^8 + 24310*a^2*b^8*d^10
*x^8 + 1633632*b^10*c^3*d^7*x^7 + 544544*a*b^9*c^2*d^8*x^7 + 136136*a^2*b^8
*c*d^9*x^7 + 19448*a^3*b^7*d^10*x^7 + 2598960*b^10*c^4*d^6*x^6 + 1039584*a*
b^9*c^3*d^7*x^6 + 346528*a^2*b^8*c^2*d^8*x^6 + 86632*a^3*b^7*c*d^9*x^6 + 12
376*a^4*b^6*d^10*x^6 + 2858856*b^10*c^5*d^5*x^5 + 1299480*a*b^9*c^4*d^6*x^5
 + 519792*a^2*b^8*c^3*d^7*x^5 + 173264*a^3*b^7*c^2*d^8*x^5 + 43316*a^4*b^6*
```

c*d^9*x^5 + 6188*a^5*b^5*d^10*x^5 + 2199120*b^10*c^6*d^4*x^4 + 1099560*a*b^9*c^5*d^5*x^4 + 499800*a^2*b^8*c^4*d^6*x^4 + 199920*a^3*b^7*c^3*d^7*x^4 + 66640*a^4*b^6*c^2*d^8*x^4 + 16660*a^5*b^5*c*d^9*x^4 + 2380*a^6*b^4*d^10*x^4 + 1166880*b^10*c^7*d^3*x^3 + 628320*a*b^9*c^6*d^4*x^3 + 314160*a^2*b^8*c^5*d^5*x^3 + 142800*a^3*b^7*c^4*d^6*x^3 + 57120*a^4*b^6*c^3*d^7*x^3 + 19040*a^5*b^5*c^2*d^8*x^3 + 4760*a^6*b^4*c*d^9*x^3 + 680*a^7*b^3*d^10*x^3 + 408408*b^10*c^8*d^2*x^2 + 233376*a*b^9*c^7*d^3*x^2 + 125664*a^2*b^8*c^6*d^4*x^2 + 62832*a^3*b^7*c^5*d^5*x^2 + 28560*a^4*b^6*c^4*d^6*x^2 + 11424*a^5*b^5*c^3*d^7*x^2 + 3808*a^6*b^4*c^2*d^8*x^2 + 952*a^7*b^3*c*d^9*x^2 + 136*a^8*b^2*d^10*x^2 + 85085*b^10*c^9*d*x + 51051*a*b^9*c^8*d^2*x + 29172*a^2*b^8*c^7*d^3*x + 15708*a^3*b^7*c^6*d^4*x + 7854*a^4*b^6*c^5*d^5*x + 3570*a^5*b^5*c^4*d^6*x + 1428*a^6*b^4*c^3*d^7*x + 476*a^7*b^3*c^2*d^8*x + 119*a^8*b^2*c*d^9*x + 17*a^9*b*d^10*x + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^17*b^11)

3.1330 $\int \frac{(c+dx)^{10}}{(a+bx)^{19}}\,dx$

Optimal. Leaf size=244

$$\frac{d^7(c+dx)^{11}}{350064(a+bx)^{11}(bc-ad)^8}-\frac{d^6(c+dx)^{11}}{31824(a+bx)^{12}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{5304(a+bx)^{13}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{1224(a+bx)^{14}(bc-ad)^5}+\frac{}{24}$$

```
[Out] -(c + d*x)^11/(18*(b*c - a*d)*(a + b*x)^18) + (7*d*(c + d*x)^11)/(306*(b*c
- a*d)^2*(a + b*x)^17) - (7*d^2*(c + d*x)^11)/(816*(b*c - a*d)^3*(a + b*x)^
16) + (7*d^3*(c + d*x)^11)/(2448*(b*c - a*d)^4*(a + b*x)^15) - (d^4*(c + d*
x)^11)/(1224*(b*c - a*d)^5*(a + b*x)^14) + (d^5*(c + d*x)^11)/(5304*(b*c -
a*d)^6*(a + b*x)^13) - (d^6*(c + d*x)^11)/(31824*(b*c - a*d)^7*(a + b*x)^12
) + (d^7*(c + d*x)^11)/(350064*(b*c - a*d)^8*(a + b*x)^11)
```

Rubi [A] time = 0.104976, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d^7(c+dx)^{11}}{350064(a+bx)^{11}(bc-ad)^8}-\frac{d^6(c+dx)^{11}}{31824(a+bx)^{12}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{5304(a+bx)^{13}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{1224(a+bx)^{14}(bc-ad)^5}+\frac{}{24}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^19,x]
```


```
[Out] -(c + d*x)^11/(18*(b*c - a*d)*(a + b*x)^18) + (7*d*(c + d*x)^11)/(306*(b*c
- a*d)^2*(a + b*x)^17) - (7*d^2*(c + d*x)^11)/(816*(b*c - a*d)^3*(a + b*x)^
16) + (7*d^3*(c + d*x)^11)/(2448*(b*c - a*d)^4*(a + b*x)^15) - (d^4*(c + d*
x)^11)/(1224*(b*c - a*d)^5*(a + b*x)^14) + (d^5*(c + d*x)^11)/(5304*(b*c -
a*d)^6*(a + b*x)^13) - (d^6*(c + d*x)^11)/(31824*(b*c - a*d)^7*(a + b*x)^12
) + (d^7*(c + d*x)^11)/(350064*(b*c - a*d)^8*(a + b*x)^11)
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{19}}\,dx = -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} - \frac{(7d)\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx}{18(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} + \frac{(7d^2)\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx}{51(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} - \frac{(35d^3)\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx}{816(bc-ad)^3}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c+dx)^{11}}{2448(bc-ad)^4(a+bx}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c+dx)^{11}}{2448(bc-ad)^4(a+bx}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c+dx)^{11}}{2448(bc-ad)^4(a+bx}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c+dx)^{11}}{2448(bc-ad)^4(a+bx}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c+dx)^{11}}{2448(bc-ad)^4(a+bx}$$

Mathematica [B] time = 0.287515, size = 694, normalized size = 2.84

$$-\frac{9a^2b^8d^2\left(29172c^6d^2x^2+71808c^5d^3x^3+112200c^4d^4x^4+114240c^3d^5x^5+74256c^2d^6x^6+6864c^7dx+715c^8+28288cd^7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^19,x]
```

```
[Out] -(a^10*d^10 + 2*a^9*b*d^9*(4*c + 9*d*x) + 9*a^8*b^2*d^8*(4*c^2 + 16*c*d*x +
 17*d^2*x^2) + 24*a^7*b^3*d^7*(5*c^3 + 27*c^2*d*x + 51*c*d^2*x^2 + 34*d^3*x
^3) + 6*a^6*b^4*d^6*(55*c^4 + 360*c^3*d*x + 918*c^2*d^2*x^2 + 1088*c*d^3*x^
3 + 510*d^4*x^4) + 36*a^5*b^5*d^5*(22*c^5 + 165*c^4*d*x + 510*c^3*d^2*x^2 +
 816*c^2*d^3*x^3 + 680*c*d^4*x^4 + 238*d^5*x^5) + 6*a^4*b^6*d^4*(286*c^6 +
2376*c^5*d*x + 8415*c^4*d^2*x^2 + 16320*c^3*d^3*x^3 + 18360*c^2*d^4*x^4 + 1
1424*c*d^5*x^5 + 3094*d^6*x^6) + 24*a^3*b^7*d^3*(143*c^7 + 1287*c^6*d*x + 5
049*c^5*d^2*x^2 + 11220*c^4*d^3*x^3 + 15300*c^3*d^4*x^4 + 12852*c^2*d^5*x^5
 + 6188*c*d^6*x^6 + 1326*d^7*x^7) + 9*a^2*b^8*d^2*(715*c^8 + 6864*c^7*d*x +
 29172*c^6*d^2*x^2 + 71808*c^5*d^3*x^3 + 112200*c^4*d^4*x^4 + 114240*c^3*d^
5*x^5 + 74256*c^2*d^6*x^6 + 28288*c*d^7*x^7 + 4862*d^8*x^8) + 2*a*b^9*d*(57
20*c^9 + 57915*c^8*d*x + 262548*c^7*d^2*x^2 + 700128*c^6*d^3*x^3 + 1211760*
c^5*d^4*x^4 + 1413720*c^4*d^5*x^5 + 1113840*c^3*d^6*x^6 + 572832*c^2*d^7*x^
7 + 175032*c*d^8*x^8 + 24310*d^9*x^9) + b^10*(19448*c^10 + 205920*c^9*d*x +
 984555*c^8*d^2*x^2 + 2800512*c^7*d^3*x^3 + 5250960*c^6*d^4*x^4 + 6785856*c
^5*d^5*x^5 + 6126120*c^4*d^6*x^6 + 3818880*c^3*d^7*x^7 + 1575288*c^2*d^8*x^
8 + 388960*c*d^9*x^9 + 43758*d^10*x^10))/(350064*b^11*(a + b*x)^18)
```

Maple [B] time = 0.01, size = 867, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^19,x)
```

```
[Out] -9/2*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^10-1/8*d^10/b^11/(b*x+a)^
8-15*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^
2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^14-45/16*d^2*(a^8*d^8-8*a
^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*
b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^16+120/1
1*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^11+10/17*d
*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c
^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^
8*d-b^9*c^9)/b^11/(b*x+a)^17+252/13*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c
^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^13+8*d^3*(a^7
*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3
-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^15-1/18*(a^10*d^10-
10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-2
52*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d
^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^18-35/2*d^6*(a^4*d^4-4*a^3*b*c*d^
3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^12+10/9*d^9*(a*d-b*
c)/b^11/(b*x+a)^9
```

Maxima [B] time = 1.31229, size = 1420, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="maxima")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 643
5*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b
^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8
 + 8*a^9*b*c*d^9 + a^10*d^10 + 48620*(8*b^10*c*d^9 + a*b^9*d^10)*x^9 + 4375
8*(36*b^10*c^2*d^8 + 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 31824*(120*b^10*c^
3*d^7 + 36*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 18564*(330
*b^10*c^4*d^6 + 120*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 + 8*a^3*b^7*c*d^9 +
a^4*b^6*d^10)*x^6 + 8568*(792*b^10*c^5*d^5 + 330*a*b^9*c^4*d^6 + 120*a^2*b^
8*c^3*d^7 + 36*a^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 3060
*(1716*b^10*c^6*d^4 + 792*a*b^9*c^5*d^5 + 330*a^2*b^8*c^4*d^6 + 120*a^3*b^7
*c^3*d^7 + 36*a^4*b^6*c^2*d^8 + 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 816*(
3432*b^10*c^7*d^3 + 1716*a*b^9*c^6*d^4 + 792*a^2*b^8*c^5*d^5 + 330*a^3*b^7*
c^4*d^6 + 120*a^4*b^6*c^3*d^7 + 36*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 + a^7*
b^3*d^10)*x^3 + 153*(6435*b^10*c^8*d^2 + 3432*a*b^9*c^7*d^3 + 1716*a^2*b^8*
c^6*d^4 + 792*a^3*b^7*c^5*d^5 + 330*a^4*b^6*c^4*d^6 + 120*a^5*b^5*c^3*d^7 +
 36*a^6*b^4*c^2*d^8 + 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 18*(11440*b^10*
c^9*d + 6435*a*b^9*c^8*d^2 + 3432*a^2*b^8*c^7*d^3 + 1716*a^3*b^7*c^6*d^4 +
792*a^4*b^6*c^5*d^5 + 330*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 + 36*a^7*b^
3*c^2*d^8 + 8*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^29*x^18 + 18*a*b^28*x^17 +
153*a^2*b^27*x^16 + 816*a^3*b^26*x^15 + 3060*a^4*b^25*x^14 + 8568*a^5*b^24*
x^13 + 18564*a^6*b^23*x^12 + 31824*a^7*b^22*x^11 + 43758*a^8*b^21*x^10 + 48
620*a^9*b^20*x^9 + 43758*a^10*b^19*x^8 + 31824*a^11*b^18*x^7 + 18564*a^12*b
^17*x^6 + 8568*a^13*b^16*x^5 + 3060*a^14*b^15*x^4 + 816*a^15*b^14*x^3 + 153
*a^16*b^13*x^2 + 18*a^17*b^12*x + a^18*b^11)
```

Fricas [B] time = 1.94583, size = 2391, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="fricas")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 643
5*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b
^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8
 + 8*a^9*b*c*d^9 + a^10*d^10 + 48620*(8*b^10*c*d^9 + a*b^9*d^10)*x^9 + 4375
8*(36*b^10*c^2*d^8 + 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 31824*(120*b^10*c^
3*d^7 + 36*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 18564*(330
*b^10*c^4*d^6 + 120*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 + 8*a^3*b^7*c*d^9 +
a^4*b^6*d^10)*x^6 + 8568*(792*b^10*c^5*d^5 + 330*a*b^9*c^4*d^6 + 120*a^2*b^
8*c^3*d^7 + 36*a^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 3060
*(1716*b^10*c^6*d^4 + 792*a*b^9*c^5*d^5 + 330*a^2*b^8*c^4*d^6 + 120*a^3*b^7
*c^3*d^7 + 36*a^4*b^6*c^2*d^8 + 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 816*(
3432*b^10*c^7*d^3 + 1716*a*b^9*c^6*d^4 + 792*a^2*b^8*c^5*d^5 + 330*a^3*b^7*
c^4*d^6 + 120*a^4*b^6*c^3*d^7 + 36*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 + a^7*
b^3*d^10)*x^3 + 153*(6435*b^10*c^8*d^2 + 3432*a*b^9*c^7*d^3 + 1716*a^2*b^8*
c^6*d^4 + 792*a^3*b^7*c^5*d^5 + 330*a^4*b^6*c^4*d^6 + 120*a^5*b^5*c^3*d^7 +
 36*a^6*b^4*c^2*d^8 + 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 18*(11440*b^10*
c^9*d + 6435*a*b^9*c^8*d^2 + 3432*a^2*b^8*c^7*d^3 + 1716*a^3*b^7*c^6*d^4 +
792*a^4*b^6*c^5*d^5 + 330*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 + 36*a^7*b^
3*c^2*d^8 + 8*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^29*x^18 + 18*a*b^28*x^17 +
153*a^2*b^27*x^16 + 816*a^3*b^26*x^15 + 3060*a^4*b^25*x^14 + 8568*a^5*b^24*
x^13 + 18564*a^6*b^23*x^12 + 31824*a^7*b^22*x^11 + 43758*a^8*b^21*x^10 + 48
620*a^9*b^20*x^9 + 43758*a^10*b^19*x^8 + 31824*a^11*b^18*x^7 + 18564*a^12*b
^17*x^6 + 8568*a^13*b^16*x^5 + 3060*a^14*b^15*x^4 + 816*a^15*b^14*x^3 + 153
*a^16*b^13*x^2 + 18*a^17*b^12*x + a^18*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**19,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.05882, size = 1297, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="giac")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 388960*b^10*c*d^9*x^9 + 48620*a*b^9*d^10*
x^9 + 1575288*b^10*c^2*d^8*x^8 + 350064*a*b^9*c*d^9*x^8 + 43758*a^2*b^8*d^1
0*x^8 + 3818880*b^10*c^3*d^7*x^7 + 1145664*a*b^9*c^2*d^8*x^7 + 254592*a^2*b
```

^8*c*d^9*x^7 + 31824*a^3*b^7*d^10*x^7 + 6126120*b^10*c^4*d^6*x^6 + 2227680*a*b^9*c^3*d^7*x^6 + 668304*a^2*b^8*c^2*d^8*x^6 + 148512*a^3*b^7*c*d^9*x^6 + 18564*a^4*b^6*d^10*x^6 + 6785856*b^10*c^5*d^5*x^5 + 2827440*a*b^9*c^4*d^6*x^5 + 1028160*a^2*b^8*c^3*d^7*x^5 + 308448*a^3*b^7*c^2*d^8*x^5 + 68544*a^4*b^6*c*d^9*x^5 + 8568*a^5*b^5*d^10*x^5 + 5250960*b^10*c^6*d^4*x^4 + 2423520*a*b^9*c^5*d^5*x^4 + 1009800*a^2*b^8*c^4*d^6*x^4 + 367200*a^3*b^7*c^3*d^7*x^4 + 110160*a^4*b^6*c^2*d^8*x^4 + 24480*a^5*b^5*c*d^9*x^4 + 3060*a^6*b^4*d^10*x^4 + 2800512*b^10*c^7*d^3*x^3 + 1400256*a*b^9*c^6*d^4*x^3 + 646272*a^2*b^8*c^5*d^5*x^3 + 269280*a^3*b^7*c^4*d^6*x^3 + 97920*a^4*b^6*c^3*d^7*x^3 + 29376*a^5*b^5*c^2*d^8*x^3 + 6528*a^6*b^4*c*d^9*x^3 + 816*a^7*b^3*d^10*x^3 + 984555*b^10*c^8*d^2*x^2 + 525096*a*b^9*c^7*d^3*x^2 + 262548*a^2*b^8*c^6*d^4*x^2 + 121176*a^3*b^7*c^5*d^5*x^2 + 50490*a^4*b^6*c^4*d^6*x^2 + 18360*a^5*b^5*c^3*d^7*x^2 + 5508*a^6*b^4*c^2*d^8*x^2 + 1224*a^7*b^3*c*d^9*x^2 + 153*a^8*b^2*d^10*x^2 + 205920*b^10*c^9*d*x + 115830*a*b^9*c^8*d^2*x + 61776*a^2*b^8*c^7*d^3*x + 30888*a^3*b^7*c^6*d^4*x + 14256*a^4*b^6*c^5*d^5*x + 5940*a^5*b^5*c^4*d^6*x + 2160*a^6*b^4*c^3*d^7*x + 648*a^7*b^3*c^2*d^8*x + 144*a^8*b^2*c*d^9*x + 18*a^9*b*d^10*x + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 6435*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8 + 8*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^18*b^11)

3.1331 $\int \frac{(c+dx)^{10}}{(a+bx)^{20}}\,dx$

Optimal. Leaf size=273

$$-\frac{d^9(bc-ad)}{b^{11}(a+bx)^{10}}-\frac{45d^8(bc-ad)^2}{11b^{11}(a+bx)^{11}}-\frac{10d^7(bc-ad)^3}{b^{11}(a+bx)^{12}}-\frac{210d^6(bc-ad)^4}{13b^{11}(a+bx)^{13}}-\frac{18d^5(bc-ad)^5}{b^{11}(a+bx)^{14}}-\frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}}-\frac{15d^3(bc-ad}{2b^{11}(a+bx)}$$

```
[Out] -(b*c - a*d)^10/(19*b^11*(a + b*x)^19) - (5*d*(b*c - a*d)^9)/(9*b^11*(a + b
*x)^18) - (45*d^2*(b*c - a*d)^8)/(17*b^11*(a + b*x)^17) - (15*d^3*(b*c - a*
d)^7)/(2*b^11*(a + b*x)^16) - (14*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)^15) -
(18*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^14) - (210*d^6*(b*c - a*d)^4)/(13*b^
11*(a + b*x)^13) - (10*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^12) - (45*d^8*(b*
c - a*d)^2)/(11*b^11*(a + b*x)^11) - (d^9*(b*c - a*d))/(b^11*(a + b*x)^10)
- d^10/(9*b^11*(a + b*x)^9)
```

Rubi [A] time = 0.284117, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{d^9(bc-ad)}{b^{11}(a+bx)^{10}}-\frac{45d^8(bc-ad)^2}{11b^{11}(a+bx)^{11}}-\frac{10d^7(bc-ad)^3}{b^{11}(a+bx)^{12}}-\frac{210d^6(bc-ad)^4}{13b^{11}(a+bx)^{13}}-\frac{18d^5(bc-ad)^5}{b^{11}(a+bx)^{14}}-\frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}}-\frac{15d^3(bc-ad}{2b^{11}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^20,x]
```

```
[Out] -(b*c - a*d)^10/(19*b^11*(a + b*x)^19) - (5*d*(b*c - a*d)^9)/(9*b^11*(a + b
*x)^18) - (45*d^2*(b*c - a*d)^8)/(17*b^11*(a + b*x)^17) - (15*d^3*(b*c - a*
d)^7)/(2*b^11*(a + b*x)^16) - (14*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)^15) -
(18*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^14) - (210*d^6*(b*c - a*d)^4)/(13*b^
11*(a + b*x)^13) - (10*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^12) - (45*d^8*(b*
c - a*d)^2)/(11*b^11*(a + b*x)^11) - (d^9*(b*c - a*d))/(b^11*(a + b*x)^10)
- d^10/(9*b^11*(a + b*x)^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{20}}\,dx=\int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{20}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{19}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{18}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{17}}+\frac{210d^4(bc-ad)^6}{b^{10}(a+bx)^{16}}+\frac{252d^5(b}{b^{10}(a+}\right.$$
$$=-\frac{(bc-ad)^{10}}{19b^{11}(a+bx)^{19}}-\frac{5d(bc-ad)^9}{9b^{11}(a+bx)^{18}}-\frac{45d^2(bc-ad)^8}{17b^{11}(a+bx)^{17}}-\frac{15d^3(bc-ad)^7}{2b^{11}(a+bx)^{16}}-\frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}}-\frac{18d^5(bc-}{b^{11}(a+b}$$

Mathematica [B] time = 0.279299, size = 692, normalized size = 2.53

$$-\frac{9a^2b^8d^2\left(57057c^6d^2x^2+138567c^5d^3x^3+213180c^4d^4x^4+213180c^3d^5x^5+135660c^2d^6x^6+13585c^7dx+1430c^8+5038\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^20,x]
```

```
[Out] -(a^10*d^10 + a^9*b*d^9*(9*c + 19*d*x) + 9*a^8*b^2*d^8*(5*c^2 + 19*c*d*x +
19*d^2*x^2) + 3*a^7*b^3*d^7*(55*c^3 + 285*c^2*d*x + 513*c*d^2*x^2 + 323*d^3
*x^3) + 3*a^6*b^4*d^6*(165*c^4 + 1045*c^3*d*x + 2565*c^2*d^2*x^2 + 2907*c*d
^3*x^3 + 1292*d^4*x^4) + 9*a^5*b^5*d^5*(143*c^5 + 1045*c^4*d*x + 3135*c^3*d
^2*x^2 + 4845*c^2*d^3*x^3 + 3876*c*d^4*x^4 + 1292*d^5*x^5) + 3*a^4*b^6*d^4*
(1001*c^6 + 8151*c^5*d*x + 28215*c^4*d^2*x^2 + 53295*c^3*d^3*x^3 + 58140*c^
2*d^4*x^4 + 34884*c*d^5*x^5 + 9044*d^6*x^6) + 3*a^3*b^7*d^3*(2145*c^7 + 190
19*c^6*d*x + 73359*c^5*d^2*x^2 + 159885*c^4*d^3*x^3 + 213180*c^3*d^4*x^4 +
174420*c^2*d^5*x^5 + 81396*c*d^6*x^6 + 16796*d^7*x^7) + 9*a^2*b^8*d^2*(1430
*c^8 + 13585*c^7*d*x + 57057*c^6*d^2*x^2 + 138567*c^5*d^3*x^3 + 213180*c^4*
d^4*x^4 + 213180*c^3*d^5*x^5 + 135660*c^2*d^6*x^6 + 50388*c*d^7*x^7 + 8398*
d^8*x^8) + a*b^9*d*(24310*c^9 + 244530*c^8*d*x + 1100385*c^7*d^2*x^2 + 2909
907*c^6*d^3*x^3 + 4988412*c^5*d^4*x^4 + 5755860*c^4*d^5*x^5 + 4476780*c^3*d
^6*x^6 + 2267460*c^2*d^7*x^7 + 680238*c*d^8*x^8 + 92378*d^9*x^9) + b^10*(43
758*c^10 + 461890*c^9*d*x + 2200770*c^8*d^2*x^2 + 6235515*c^7*d^3*x^3 + 116
39628*c^6*d^4*x^4 + 14965236*c^5*d^5*x^5 + 13430340*c^4*d^6*x^6 + 8314020*c
^3*d^7*x^7 + 3401190*c^2*d^8*x^8 + 831402*c*d^9*x^9 + 92378*d^10*x^10))/(83
1402*b^11*(a + b*x)^19)
```

Maple [B] time = 0.009, size = 866, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^20,x)
```

```
[Out] d^9*(a*d-b*c)/b^11/(b*x+a)^10+18*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*
d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^14+15/2*d^3*(a^7
*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3
-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^16-45/11*d^8*(a^2*d
^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^11-45/17*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a
^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*
a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^17-210/13*d^6*(a^4*d^4-
4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^13-14*d
^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*
c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^15-1/19*(a^10*d^10-10*a^9*b*c*d
^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c
^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*
c^9*d+b^10*c^10)/b^11/(b*x+a)^19+5/9*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^
2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6
*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^18+10*d^7*(
a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^12-1/9*d^10/b^11/
(b*x+a)^9
```

Maxima [B] time = 1.3383, size = 1435, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="maxima")
```

```
[Out] -1/831402*(92378*b^10*d^10*x^10 + 43758*b^10*c^10 + 24310*a*b^9*c^9*d + 128
70*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 + 1287*a^5
*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d
^8 + 9*a^9*b*c*d^9 + a^10*d^10 + 92378*(9*b^10*c*d^9 + a*b^9*d^10)*x^9 + 75
582*(45*b^10*c^2*d^8 + 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 50388*(165*b^10*
c^3*d^7 + 45*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 27132*(4
95*b^10*c^4*d^6 + 165*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 + 9*a^3*b^7*c*d^9
+ a^4*b^6*d^10)*x^6 + 11628*(1287*b^10*c^5*d^5 + 495*a*b^9*c^4*d^6 + 165*a^
2*b^8*c^3*d^7 + 45*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 +
3876*(3003*b^10*c^6*d^4 + 1287*a*b^9*c^5*d^5 + 495*a^2*b^8*c^4*d^6 + 165*a^
3*b^7*c^3*d^7 + 45*a^4*b^6*c^2*d^8 + 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 +
969*(6435*b^10*c^7*d^3 + 3003*a*b^9*c^6*d^4 + 1287*a^2*b^8*c^5*d^5 + 495*a^
3*b^7*c^4*d^6 + 165*a^4*b^6*c^3*d^7 + 45*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9
+ a^7*b^3*d^10)*x^3 + 171*(12870*b^10*c^8*d^2 + 6435*a*b^9*c^7*d^3 + 3003*a
^2*b^8*c^6*d^4 + 1287*a^3*b^7*c^5*d^5 + 495*a^4*b^6*c^4*d^6 + 165*a^5*b^5*c
^3*d^7 + 45*a^6*b^4*c^2*d^8 + 9*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 19*(243
10*b^10*c^9*d + 12870*a*b^9*c^8*d^2 + 6435*a^2*b^8*c^7*d^3 + 3003*a^3*b^7*c
^6*d^4 + 1287*a^4*b^6*c^5*d^5 + 495*a^5*b^5*c^4*d^6 + 165*a^6*b^4*c^3*d^7 +
 45*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^30*x^19 + 19*a*b^
29*x^18 + 171*a^2*b^28*x^17 + 969*a^3*b^27*x^16 + 3876*a^4*b^26*x^15 + 1162
8*a^5*b^25*x^14 + 27132*a^6*b^24*x^13 + 50388*a^7*b^23*x^12 + 75582*a^8*b^2
2*x^11 + 92378*a^9*b^21*x^10 + 92378*a^10*b^20*x^9 + 75582*a^11*b^19*x^8 +
50388*a^12*b^18*x^7 + 27132*a^13*b^17*x^6 + 11628*a^14*b^16*x^5 + 3876*a^15
*b^15*x^4 + 969*a^16*b^14*x^3 + 171*a^17*b^13*x^2 + 19*a^18*b^12*x + a^19*b
^11)
```

Fricas [B] time = 1.95746, size = 2438, normalized size = 8.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="fricas")
```

```
[Out] -1/831402*(92378*b^10*d^10*x^10 + 43758*b^10*c^10 + 24310*a*b^9*c^9*d + 128
70*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 + 1287*a^5
*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d
^8 + 9*a^9*b*c*d^9 + a^10*d^10 + 92378*(9*b^10*c*d^9 + a*b^9*d^10)*x^9 + 75
582*(45*b^10*c^2*d^8 + 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 50388*(165*b^10*
c^3*d^7 + 45*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 27132*(4
95*b^10*c^4*d^6 + 165*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 + 9*a^3*b^7*c*d^9
+ a^4*b^6*d^10)*x^6 + 11628*(1287*b^10*c^5*d^5 + 495*a*b^9*c^4*d^6 + 165*a^
2*b^8*c^3*d^7 + 45*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 +
3876*(3003*b^10*c^6*d^4 + 1287*a*b^9*c^5*d^5 + 495*a^2*b^8*c^4*d^6 + 165*a^
3*b^7*c^3*d^7 + 45*a^4*b^6*c^2*d^8 + 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 +
969*(6435*b^10*c^7*d^3 + 3003*a*b^9*c^6*d^4 + 1287*a^2*b^8*c^5*d^5 + 495*a^
3*b^7*c^4*d^6 + 165*a^4*b^6*c^3*d^7 + 45*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9
+ a^7*b^3*d^10)*x^3 + 171*(12870*b^10*c^8*d^2 + 6435*a*b^9*c^7*d^3 + 3003*a
^2*b^8*c^6*d^4 + 1287*a^3*b^7*c^5*d^5 + 495*a^4*b^6*c^4*d^6 + 165*a^5*b^5*c
^3*d^7 + 45*a^6*b^4*c^2*d^8 + 9*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 19*(243
10*b^10*c^9*d + 12870*a*b^9*c^8*d^2 + 6435*a^2*b^8*c^7*d^3 + 3003*a^3*b^7*c
^6*d^4 + 1287*a^4*b^6*c^5*d^5 + 495*a^5*b^5*c^4*d^6 + 165*a^6*b^4*c^3*d^7 +
 45*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^30*x^19 + 19*a*b^
29*x^18 + 171*a^2*b^28*x^17 + 969*a^3*b^27*x^16 + 3876*a^4*b^26*x^15 + 1162
8*a^5*b^25*x^14 + 27132*a^6*b^24*x^13 + 50388*a^7*b^23*x^12 + 75582*a^8*b^2
```

```
2*x^11 + 92378*a^9*b^21*x^10 + 92378*a^10*b^20*x^9 + 75582*a^11*b^19*x^8 +
50388*a^12*b^18*x^7 + 27132*a^13*b^17*x^6 + 11628*a^14*b^16*x^5 + 3876*a^15
*b^15*x^4 + 969*a^16*b^14*x^3 + 171*a^17*b^13*x^2 + 19*a^18*b^12*x + a^19*b
^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**20,x)

[Out] Timed out
```

Giac [B] time = 1.07213, size = 1297, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="giac")

[Out] -1/831402*(92378*b^10*d^10*x^10 + 831402*b^10*c*d^9*x^9 + 92378*a*b^9*d^10*
x^9 + 3401190*b^10*c^2*d^8*x^8 + 680238*a*b^9*c*d^9*x^8 + 75582*a^2*b^8*d^1
0*x^8 + 8314020*b^10*c^3*d^7*x^7 + 2267460*a*b^9*c^2*d^8*x^7 + 453492*a^2*b
^8*c*d^9*x^7 + 50388*a^3*b^7*d^10*x^7 + 13430340*b^10*c^4*d^6*x^6 + 4476780
*a*b^9*c^3*d^7*x^6 + 1220940*a^2*b^8*c^2*d^8*x^6 + 244188*a^3*b^7*c*d^9*x^6
 + 27132*a^4*b^6*d^10*x^6 + 14965236*b^10*c^5*d^5*x^5 + 5755860*a*b^9*c^4*d
^6*x^5 + 1918620*a^2*b^8*c^3*d^7*x^5 + 523260*a^3*b^7*c^2*d^8*x^5 + 104652*
a^4*b^6*c*d^9*x^5 + 11628*a^5*b^5*d^10*x^5 + 11639628*b^10*c^6*d^4*x^4 + 49
88412*a*b^9*c^5*d^5*x^4 + 1918620*a^2*b^8*c^4*d^6*x^4 + 639540*a^3*b^7*c^3*
d^7*x^4 + 174420*a^4*b^6*c^2*d^8*x^4 + 34884*a^5*b^5*c*d^9*x^4 + 3876*a^6*b
^4*d^10*x^4 + 6235515*b^10*c^7*d^3*x^3 + 2909907*a*b^9*c^6*d^4*x^3 + 124710
3*a^2*b^8*c^5*d^5*x^3 + 479655*a^3*b^7*c^4*d^6*x^3 + 159885*a^4*b^6*c^3*d^7
*x^3 + 43605*a^5*b^5*c^2*d^8*x^3 + 8721*a^6*b^4*c*d^9*x^3 + 969*a^7*b^3*d^1
0*x^3 + 2200770*b^10*c^8*d^2*x^2 + 1100385*a*b^9*c^7*d^3*x^2 + 513513*a^2*b
^8*c^6*d^4*x^2 + 220077*a^3*b^7*c^5*d^5*x^2 + 84645*a^4*b^6*c^4*d^6*x^2 + 2
8215*a^5*b^5*c^3*d^7*x^2 + 7695*a^6*b^4*c^2*d^8*x^2 + 1539*a^7*b^3*c*d^9*x^
2 + 171*a^8*b^2*d^10*x^2 + 461890*b^10*c^9*d*x + 244530*a*b^9*c^8*d^2*x + 1
22265*a^2*b^8*c^7*d^3*x + 57057*a^3*b^7*c^6*d^4*x + 24453*a^4*b^6*c^5*d^5*x
 + 9405*a^5*b^5*c^4*d^6*x + 3135*a^6*b^4*c^3*d^7*x + 855*a^7*b^3*c^2*d^8*x
+ 171*a^8*b^2*c*d^9*x + 19*a^9*b*d^10*x + 43758*b^10*c^10 + 24310*a*b^9*c^9
*d + 12870*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 +
1287*a^5*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b
^2*c^2*d^8 + 9*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^19*b^11)
```

3.1332 $\int \frac{(c+dx)^{10}}{(a+bx)^{21}} dx$

Optimal. Leaf size=279

$$-\frac{10d^9(bc-ad)}{11b^{11}(a+bx)^{11}}-\frac{15d^8(bc-ad)^2}{4b^{11}(a+bx)^{12}}-\frac{120d^7(bc-ad)^3}{13b^{11}(a+bx)^{13}}-\frac{15d^6(bc-ad)^4}{b^{11}(a+bx)^{14}}-\frac{84d^5(bc-ad)^5}{5b^{11}(a+bx)^{15}}-\frac{105d^4(bc-ad)^6}{8b^{11}(a+bx)^{16}}-\frac{120d^3(bc-}{17b^{11}(a+}$$

```
[Out] -(b*c - a*d)^10/(20*b^11*(a + b*x)^20) - (10*d*(b*c - a*d)^9)/(19*b^11*(a +
 b*x)^19) - (5*d^2*(b*c - a*d)^8)/(2*b^11*(a + b*x)^18) - (120*d^3*(b*c - a
*d)^7)/(17*b^11*(a + b*x)^17) - (105*d^4*(b*c - a*d)^6)/(8*b^11*(a + b*x)^1
6) - (84*d^5*(b*c - a*d)^5)/(5*b^11*(a + b*x)^15) - (15*d^6*(b*c - a*d)^4)/
(b^11*(a + b*x)^14) - (120*d^7*(b*c - a*d)^3)/(13*b^11*(a + b*x)^13) - (15*
d^8*(b*c - a*d)^2)/(4*b^11*(a + b*x)^12) - (10*d^9*(b*c - a*d))/(11*b^11*(a
 + b*x)^11) - d^10/(10*b^11*(a + b*x)^10)
```

Rubi [A] time = 0.272436, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{10d^9(bc-ad)}{11b^{11}(a+bx)^{11}}-\frac{15d^8(bc-ad)^2}{4b^{11}(a+bx)^{12}}-\frac{120d^7(bc-ad)^3}{13b^{11}(a+bx)^{13}}-\frac{15d^6(bc-ad)^4}{b^{11}(a+bx)^{14}}-\frac{84d^5(bc-ad)^5}{5b^{11}(a+bx)^{15}}-\frac{105d^4(bc-ad)^6}{8b^{11}(a+bx)^{16}}-\frac{120d^3(bc-}{17b^{11}(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^21,x]
```


```
[Out] -(b*c - a*d)^10/(20*b^11*(a + b*x)^20) - (10*d*(b*c - a*d)^9)/(19*b^11*(a +
 b*x)^19) - (5*d^2*(b*c - a*d)^8)/(2*b^11*(a + b*x)^18) - (120*d^3*(b*c - a
*d)^7)/(17*b^11*(a + b*x)^17) - (105*d^4*(b*c - a*d)^6)/(8*b^11*(a + b*x)^1
6) - (84*d^5*(b*c - a*d)^5)/(5*b^11*(a + b*x)^15) - (15*d^6*(b*c - a*d)^4)/
(b^11*(a + b*x)^14) - (120*d^7*(b*c - a*d)^3)/(13*b^11*(a + b*x)^13) - (15*
d^8*(b*c - a*d)^2)/(4*b^11*(a + b*x)^12) - (10*d^9*(b*c - a*d))/(11*b^11*(a
 + b*x)^11) - d^10/(10*b^11*(a + b*x)^10)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{21}} dx = \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{21}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{20}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{19}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{18}}+\frac{210d^4(bc-ad)^6}{b^{10}(a+bx)^{17}}+\frac{252d^5(b}{b^{10}(a+}\right.$$
$$= -\frac{(bc-ad)^{10}}{20b^{11}(a+bx)^{20}}-\frac{10d(bc-ad)^9}{19b^{11}(a+bx)^{19}}-\frac{5d^2(bc-ad)^8}{2b^{11}(a+bx)^{18}}-\frac{120d^3(bc-ad)^7}{17b^{11}(a+bx)^{17}}-\frac{105d^4(bc-ad)^6}{8b^{11}(a+bx)^{16}}-\frac{84d^5(bc}{5b^{11}(a}$$

Mathematica [B] time = 0.270999, size = 692, normalized size = 2.48

$$-\frac{5a^2b^8d^2\left(190190c^6d^2x^2+456456c^5d^3x^3+692835c^4d^4x^4+682176c^3d^5x^5+426360c^2d^6x^6+45760c^7dx+4862c^8+155\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^21,x]
```

```
[Out] -(a^10*d^10 + 10*a^9*b*d^9*(c + 2*d*x) + 5*a^8*b^2*d^8*(11*c^2 + 40*c*d*x +
 38*d^2*x^2) + 20*a^7*b^3*d^7*(11*c^3 + 55*c^2*d*x + 95*c*d^2*x^2 + 57*d^3*
x^3) + 5*a^6*b^4*d^6*(143*c^4 + 880*c^3*d*x + 2090*c^2*d^2*x^2 + 2280*c*d^3
*x^3 + 969*d^4*x^4) + 2*a^5*b^5*d^5*(1001*c^5 + 7150*c^4*d*x + 20900*c^3*d^
2*x^2 + 31350*c^2*d^3*x^3 + 24225*c*d^4*x^4 + 7752*d^5*x^5) + 5*a^4*b^6*d^4
*(1001*c^6 + 8008*c^5*d*x + 27170*c^4*d^2*x^2 + 50160*c^3*d^3*x^3 + 53295*c
^2*d^4*x^4 + 31008*c*d^5*x^5 + 7752*d^6*x^6) + 20*a^3*b^7*d^3*(572*c^7 + 50
05*c^6*d*x + 19019*c^5*d^2*x^2 + 40755*c^4*d^3*x^3 + 53295*c^3*d^4*x^4 + 42
636*c^2*d^5*x^5 + 19380*c*d^6*x^6 + 3876*d^7*x^7) + 5*a^2*b^8*d^2*(4862*c^8
 + 45760*c^7*d*x + 190190*c^6*d^2*x^2 + 456456*c^5*d^3*x^3 + 692835*c^4*d^4
*x^4 + 682176*c^3*d^5*x^5 + 426360*c^2*d^6*x^6 + 155040*c*d^7*x^7 + 25194*d
^8*x^8) + 10*a*b^9*d*(4862*c^9 + 48620*c^8*d*x + 217360*c^7*d^2*x^2 + 57057
0*c^6*d^3*x^3 + 969969*c^5*d^4*x^4 + 1108536*c^4*d^5*x^5 + 852720*c^3*d^6*x
^6 + 426360*c^2*d^7*x^7 + 125970*c*d^8*x^8 + 16796*d^9*x^9) + b^10*(92378*c
^10 + 972400*c^9*d*x + 4618900*c^8*d^2*x^2 + 13041600*c^7*d^3*x^3 + 2424922
5*c^6*d^4*x^4 + 31039008*c^5*d^5*x^5 + 27713400*c^4*d^6*x^6 + 17054400*c^3*
d^7*x^7 + 6928350*c^2*d^8*x^8 + 1679600*c*d^9*x^9 + 184756*d^10*x^10))/(184
7560*b^11*(a + b*x)^20)
```

Maple [B] time = 0.01, size = 867, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^21,x)
```

```
[Out] -1/10*d^10/b^11/(b*x+a)^10-1/20*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^
8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c
^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^1
1/(b*x+a)^20-15*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+
b^4*c^4)/b^11/(b*x+a)^14-105/8*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^
4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)
^16+10/11*d^9*(a*d-b*c)/b^11/(b*x+a)^11+120/17*d^3*(a^7*d^7-7*a^6*b*c*d^6+2
1*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+
7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^17+120/13*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*
a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^13+84/5*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3
*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^15+10/1
9*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^
4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8
*c^8*d-b^9*c^9)/b^11/(b*x+a)^19-5/2*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c
^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*
c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^18-15/4*d^8*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/b^11/(b*x+a)^12
```

Maxima [B] time = 1.33424, size = 1450, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="maxima")

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 92378*b^10*c^10 + 48620*a*b^9*c^9*d + 2
4310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*a^4*b^6*c^6*d^4 + 2002*
a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^
2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10 + 167960*(10*b^10*c*d^9 + a*b^9*d^10)*x^
9 + 125970*(55*b^10*c^2*d^8 + 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 77520*(2
20*b^10*c^3*d^7 + 55*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 +
 38760*(715*b^10*c^4*d^6 + 220*a*b^9*c^3*d^7 + 55*a^2*b^8*c^2*d^8 + 10*a^3*
b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 15504*(2002*b^10*c^5*d^5 + 715*a*b^9*c^4*d^
6 + 220*a^2*b^8*c^3*d^7 + 55*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 + a^5*b^5*d
^10)*x^5 + 4845*(5005*b^10*c^6*d^4 + 2002*a*b^9*c^5*d^5 + 715*a^2*b^8*c^4*d
^6 + 220*a^3*b^7*c^3*d^7 + 55*a^4*b^6*c^2*d^8 + 10*a^5*b^5*c*d^9 + a^6*b^4*
d^10)*x^4 + 1140*(11440*b^10*c^7*d^3 + 5005*a*b^9*c^6*d^4 + 2002*a^2*b^8*c^
5*d^5 + 715*a^3*b^7*c^4*d^6 + 220*a^4*b^6*c^3*d^7 + 55*a^5*b^5*c^2*d^8 + 10
*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 190*(24310*b^10*c^8*d^2 + 11440*a*b^9*
c^7*d^3 + 5005*a^2*b^8*c^6*d^4 + 2002*a^3*b^7*c^5*d^5 + 715*a^4*b^6*c^4*d^6
 + 220*a^5*b^5*c^3*d^7 + 55*a^6*b^4*c^2*d^8 + 10*a^7*b^3*c*d^9 + a^8*b^2*d^
10)*x^2 + 20*(48620*b^10*c^9*d + 24310*a*b^9*c^8*d^2 + 11440*a^2*b^8*c^7*d^
3 + 5005*a^3*b^7*c^6*d^4 + 2002*a^4*b^6*c^5*d^5 + 715*a^5*b^5*c^4*d^6 + 220
*a^6*b^4*c^3*d^7 + 55*a^7*b^3*c^2*d^8 + 10*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(
b^31*x^20 + 20*a*b^30*x^19 + 190*a^2*b^29*x^18 + 1140*a^3*b^28*x^17 + 4845*
a^4*b^27*x^16 + 15504*a^5*b^26*x^15 + 38760*a^6*b^25*x^14 + 77520*a^7*b^24*
x^13 + 125970*a^8*b^23*x^12 + 167960*a^9*b^22*x^11 + 184756*a^10*b^21*x^10
+ 167960*a^11*b^20*x^9 + 125970*a^12*b^19*x^8 + 77520*a^13*b^18*x^7 + 38760
*a^14*b^17*x^6 + 15504*a^15*b^16*x^5 + 4845*a^16*b^15*x^4 + 1140*a^17*b^14*
x^3 + 190*a^18*b^13*x^2 + 20*a^19*b^12*x + a^20*b^11)
```

Fricas [B] time = 1.91626, size = 2504, normalized size = 8.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="fricas")

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 92378*b^10*c^10 + 48620*a*b^9*c^9*d + 2
4310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*a^4*b^6*c^6*d^4 + 2002*
a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^
2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10 + 167960*(10*b^10*c*d^9 + a*b^9*d^10)*x^
9 + 125970*(55*b^10*c^2*d^8 + 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 77520*(2
20*b^10*c^3*d^7 + 55*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 +
 38760*(715*b^10*c^4*d^6 + 220*a*b^9*c^3*d^7 + 55*a^2*b^8*c^2*d^8 + 10*a^3*
b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 15504*(2002*b^10*c^5*d^5 + 715*a*b^9*c^4*d^
6 + 220*a^2*b^8*c^3*d^7 + 55*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 + a^5*b^5*d
^10)*x^5 + 4845*(5005*b^10*c^6*d^4 + 2002*a*b^9*c^5*d^5 + 715*a^2*b^8*c^4*d
^6 + 220*a^3*b^7*c^3*d^7 + 55*a^4*b^6*c^2*d^8 + 10*a^5*b^5*c*d^9 + a^6*b^4*
d^10)*x^4 + 1140*(11440*b^10*c^7*d^3 + 5005*a*b^9*c^6*d^4 + 2002*a^2*b^8*c^
5*d^5 + 715*a^3*b^7*c^4*d^6 + 220*a^4*b^6*c^3*d^7 + 55*a^5*b^5*c^2*d^8 + 10
*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 190*(24310*b^10*c^8*d^2 + 11440*a*b^9*
c^7*d^3 + 5005*a^2*b^8*c^6*d^4 + 2002*a^3*b^7*c^5*d^5 + 715*a^4*b^6*c^4*d^6
 + 220*a^5*b^5*c^3*d^7 + 55*a^6*b^4*c^2*d^8 + 10*a^7*b^3*c*d^9 + a^8*b^2*d^
10)*x^2 + 20*(48620*b^10*c^9*d + 24310*a*b^9*c^8*d^2 + 11440*a^2*b^8*c^7*d^
3 + 5005*a^3*b^7*c^6*d^4 + 2002*a^4*b^6*c^5*d^5 + 715*a^5*b^5*c^4*d^6 + 220
*a^6*b^4*c^3*d^7 + 55*a^7*b^3*c^2*d^8 + 10*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(
b^31*x^20 + 20*a*b^30*x^19 + 190*a^2*b^29*x^18 + 1140*a^3*b^28*x^17 + 4845*
a^4*b^27*x^16 + 15504*a^5*b^26*x^15 + 38760*a^6*b^25*x^14 + 77520*a^7*b^24*
```

```
x^13 + 125970*a^8*b^23*x^12 + 167960*a^9*b^22*x^11 + 184756*a^10*b^21*x^10
+ 167960*a^11*b^20*x^9 + 125970*a^12*b^19*x^8 + 77520*a^13*b^18*x^7 + 38760
*a^14*b^17*x^6 + 15504*a^15*b^16*x^5 + 4845*a^16*b^15*x^4 + 1140*a^17*b^14*
x^3 + 190*a^18*b^13*x^2 + 20*a^19*b^12*x + a^20*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**21,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06218, size = 1297, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="giac")
```

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 1679600*b^10*c*d^9*x^9 + 167960*a*b^9*d
^10*x^9 + 6928350*b^10*c^2*d^8*x^8 + 1259700*a*b^9*c*d^9*x^8 + 125970*a^2*b
^8*d^10*x^8 + 17054400*b^10*c^3*d^7*x^7 + 4263600*a*b^9*c^2*d^8*x^7 + 77520
0*a^2*b^8*c*d^9*x^7 + 77520*a^3*b^7*d^10*x^7 + 27713400*b^10*c^4*d^6*x^6 +
8527200*a*b^9*c^3*d^7*x^6 + 2131800*a^2*b^8*c^2*d^8*x^6 + 387600*a^3*b^7*c*
d^9*x^6 + 38760*a^4*b^6*d^10*x^6 + 31039008*b^10*c^5*d^5*x^5 + 11085360*a*b
^9*c^4*d^6*x^5 + 3410880*a^2*b^8*c^3*d^7*x^5 + 852720*a^3*b^7*c^2*d^8*x^5 +
 155040*a^4*b^6*c*d^9*x^5 + 15504*a^5*b^5*d^10*x^5 + 24249225*b^10*c^6*d^4*
x^4 + 9699690*a*b^9*c^5*d^5*x^4 + 3464175*a^2*b^8*c^4*d^6*x^4 + 1065900*a^3
*b^7*c^3*d^7*x^4 + 266475*a^4*b^6*c^2*d^8*x^4 + 48450*a^5*b^5*c*d^9*x^4 + 4
845*a^6*b^4*d^10*x^4 + 13041600*b^10*c^7*d^3*x^3 + 5705700*a*b^9*c^6*d^4*x^
3 + 2282280*a^2*b^8*c^5*d^5*x^3 + 815100*a^3*b^7*c^4*d^6*x^3 + 250800*a^4*b
^6*c^3*d^7*x^3 + 62700*a^5*b^5*c^2*d^8*x^3 + 11400*a^6*b^4*c*d^9*x^3 + 1140
*a^7*b^3*d^10*x^3 + 4618900*b^10*c^8*d^2*x^2 + 2173600*a*b^9*c^7*d^3*x^2 +
950950*a^2*b^8*c^6*d^4*x^2 + 380380*a^3*b^7*c^5*d^5*x^2 + 135850*a^4*b^6*c^
4*d^6*x^2 + 41800*a^5*b^5*c^3*d^7*x^2 + 10450*a^6*b^4*c^2*d^8*x^2 + 1900*a^
7*b^3*c*d^9*x^2 + 190*a^8*b^2*d^10*x^2 + 972400*b^10*c^9*d*x + 486200*a*b^9
*c^8*d^2*x + 228800*a^2*b^8*c^7*d^3*x + 100100*a^3*b^7*c^6*d^4*x + 40040*a^
4*b^6*c^5*d^5*x + 14300*a^5*b^5*c^4*d^6*x + 4400*a^6*b^4*c^3*d^7*x + 1100*a
^7*b^3*c^2*d^8*x + 200*a^8*b^2*c*d^9*x + 20*a^9*b*d^10*x + 92378*b^10*c^10
+ 48620*a*b^9*c^9*d + 24310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*
a^4*b^6*c^6*d^4 + 2002*a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*
c^3*d^7 + 55*a^8*b^2*c^2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^20*b^
11)
```

3.1333 $\int \frac{(c+dx)^{10}}{(a+bx)^{22}} dx$

Optimal. Leaf size=279

$$-\frac{5d^9(bc-ad)}{6b^{11}(a+bx)^{12}}-\frac{45d^8(bc-ad)^2}{13b^{11}(a+bx)^{13}}-\frac{60d^7(bc-ad)^3}{7b^{11}(a+bx)^{14}}-\frac{14d^6(bc-ad)^4}{b^{11}(a+bx)^{15}}-\frac{63d^5(bc-ad)^5}{4b^{11}(a+bx)^{16}}-\frac{210d^4(bc-ad)^6}{17b^{11}(a+bx)^{17}}-\frac{20d^3(bc-a}{3b^{11}(a+b}$$

```
[Out] -(b*c - a*d)^10/(21*b^11*(a + b*x)^21) - (d*(b*c - a*d)^9)/(2*b^11*(a + b*x
)^20) - (45*d^2*(b*c - a*d)^8)/(19*b^11*(a + b*x)^19) - (20*d^3*(b*c - a*d)
^7)/(3*b^11*(a + b*x)^18) - (210*d^4*(b*c - a*d)^6)/(17*b^11*(a + b*x)^17)
- (63*d^5*(b*c - a*d)^5)/(4*b^11*(a + b*x)^16) - (14*d^6*(b*c - a*d)^4)/(b^
11*(a + b*x)^15) - (60*d^7*(b*c - a*d)^3)/(7*b^11*(a + b*x)^14) - (45*d^8*(
b*c - a*d)^2)/(13*b^11*(a + b*x)^13) - (5*d^9*(b*c - a*d))/(6*b^11*(a + b*x
)^12) - d^10/(11*b^11*(a + b*x)^11)
```

Rubi [A] time = 0.270568, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{5d^9(bc-ad)}{6b^{11}(a+bx)^{12}}-\frac{45d^8(bc-ad)^2}{13b^{11}(a+bx)^{13}}-\frac{60d^7(bc-ad)^3}{7b^{11}(a+bx)^{14}}-\frac{14d^6(bc-ad)^4}{b^{11}(a+bx)^{15}}-\frac{63d^5(bc-ad)^5}{4b^{11}(a+bx)^{16}}-\frac{210d^4(bc-ad)^6}{17b^{11}(a+bx)^{17}}-\frac{20d^3(bc-a}{3b^{11}(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^22,x]
```


```
[Out] -(b*c - a*d)^10/(21*b^11*(a + b*x)^21) - (d*(b*c - a*d)^9)/(2*b^11*(a + b*x
)^20) - (45*d^2*(b*c - a*d)^8)/(19*b^11*(a + b*x)^19) - (20*d^3*(b*c - a*d)
^7)/(3*b^11*(a + b*x)^18) - (210*d^4*(b*c - a*d)^6)/(17*b^11*(a + b*x)^17)
- (63*d^5*(b*c - a*d)^5)/(4*b^11*(a + b*x)^16) - (14*d^6*(b*c - a*d)^4)/(b^
11*(a + b*x)^15) - (60*d^7*(b*c - a*d)^3)/(7*b^11*(a + b*x)^14) - (45*d^8*(
b*c - a*d)^2)/(13*b^11*(a + b*x)^13) - (5*d^9*(b*c - a*d))/(6*b^11*(a + b*x
)^12) - d^10/(11*b^11*(a + b*x)^11)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{22}}dx &= \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{22}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{21}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{20}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{19}}+\frac{210d^4(bc-ad)^6}{b^{10}(a+bx)^{18}}+\frac{252d^5(b}{b^{10}(a+}\right.\\ &= -\frac{(bc-ad)^{10}}{21b^{11}(a+bx)^{21}}-\frac{d(bc-ad)^9}{2b^{11}(a+bx)^{20}}-\frac{45d^2(bc-ad)^8}{19b^{11}(a+bx)^{19}}-\frac{20d^3(bc-ad)^7}{3b^{11}(a+bx)^{18}}-\frac{210d^4(bc-ad)^6}{17b^{11}(a+bx)^{17}}-\frac{63d^5(bc}{4b^{11}(a+}\end{aligned}$$

Mathematica [B] time = 0.287279, size = 692, normalized size = 2.48

$$-\frac{3a^2b^8d^2\left(560560c^6d^2x^2+1331330c^5d^3x^3+1996995c^4d^4x^4+1939938c^3d^5x^5+1193808c^2d^6x^6+136136c^7dx+14586c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^22,x]
```

```
[Out] -(a^10*d^10 + a^9*b*d^9*(11*c + 21*d*x) + 3*a^8*b^2*d^8*(22*c^2 + 77*c*d*x
+ 70*d^2*x^2) + 2*a^7*b^3*d^7*(143*c^3 + 693*c^2*d*x + 1155*c*d^2*x^2 + 665
*d^3*x^3) + 7*a^6*b^4*d^6*(143*c^4 + 858*c^3*d*x + 1980*c^2*d^2*x^2 + 2090*
c*d^3*x^3 + 855*d^4*x^4) + 21*a^5*b^5*d^5*(143*c^5 + 1001*c^4*d*x + 2860*c^
3*d^2*x^2 + 4180*c^2*d^3*x^3 + 3135*c*d^4*x^4 + 969*d^5*x^5) + 7*a^4*b^6*d^
4*(1144*c^6 + 9009*c^5*d*x + 30030*c^4*d^2*x^2 + 54340*c^3*d^3*x^3 + 56430*
c^2*d^4*x^4 + 31977*c*d^5*x^5 + 7752*d^6*x^6) + 2*a^3*b^7*d^3*(9724*c^7 + 8
4084*c^6*d*x + 315315*c^5*d^2*x^2 + 665665*c^4*d^3*x^3 + 855855*c^3*d^4*x^4
 + 671517*c^2*d^5*x^5 + 298452*c*d^6*x^6 + 58140*d^7*x^7) + 3*a^2*b^8*d^2*(
14586*c^8 + 136136*c^7*d*x + 560560*c^6*d^2*x^2 + 1331330*c^5*d^3*x^3 + 199
6995*c^4*d^4*x^4 + 1939938*c^3*d^5*x^5 + 1193808*c^2*d^6*x^6 + 426360*c*d^7
*x^7 + 67830*d^8*x^8) + a*b^9*d*(92378*c^9 + 918918*c^8*d*x + 4084080*c^7*d
^2*x^2 + 10650640*c^6*d^3*x^3 + 17972955*c^5*d^4*x^4 + 20369349*c^4*d^5*x^5
 + 15519504*c^3*d^6*x^6 + 7674480*c^2*d^7*x^7 + 2238390*c*d^8*x^8 + 293930*
d^9*x^9) + b^10*(184756*c^10 + 1939938*c^9*d*x + 9189180*c^8*d^2*x^2 + 2586
5840*c^7*d^3*x^3 + 47927880*c^6*d^4*x^4 + 61108047*c^5*d^5*x^5 + 54318264*c
^4*d^6*x^6 + 33256080*c^3*d^7*x^7 + 13430340*c^2*d^8*x^8 + 3233230*c*d^9*x^
9 + 352716*d^10*x^10))/(3879876*b^11*(a + b*x)^21)
```

Maple [B] time = 0.009, size = 867, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^22,x)
```

```
[Out] 1/2*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*
b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b
^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^20+60/7*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c
^2*d-b^3*c^3)/b^11/(b*x+a)^14+63/4*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^
2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^16-1/11*d^10/b
^11/(b*x+a)^11-210/17*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*
b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^17-45/13
*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^13-14*d^6*(a^4*d^4-4*a^3*b*c*
d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^15-45/19*d^2*(a^8
*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4
-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^
19+20/3*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35
*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^18+
5/6*d^9*(a*d-b*c)/b^11/(b*x+a)^12-1/21*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2
*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^
4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^
10)/b^11/(b*x+a)^21
```

Maxima [B] time = 1.38266, size = 1465, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="maxima")
```

```
[Out] -1/3879876*(352716*b^10*d^10*x^10 + 184756*b^10*c^10 + 92378*a*b^9*c^9*d +
43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^3 + 8008*a^4*b^6*c^6*d^4 + 3003
*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*
c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10 + 293930*(11*b^10*c*d^9 + a*b^9*d^10)*
x^9 + 203490*(66*b^10*c^2*d^8 + 11*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 116280
*(286*b^10*c^3*d^7 + 66*a*b^9*c^2*d^8 + 11*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^
7 + 54264*(1001*b^10*c^4*d^6 + 286*a*b^9*c^3*d^7 + 66*a^2*b^8*c^2*d^8 + 11*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 20349*(3003*b^10*c^5*d^5 + 1001*a*b^9*c
^4*d^6 + 286*a^2*b^8*c^3*d^7 + 66*a^3*b^7*c^2*d^8 + 11*a^4*b^6*c*d^9 + a^5*
b^5*d^10)*x^5 + 5985*(8008*b^10*c^6*d^4 + 3003*a*b^9*c^5*d^5 + 1001*a^2*b^8
*c^4*d^6 + 286*a^3*b^7*c^3*d^7 + 66*a^4*b^6*c^2*d^8 + 11*a^5*b^5*c*d^9 + a^
6*b^4*d^10)*x^4 + 1330*(19448*b^10*c^7*d^3 + 8008*a*b^9*c^6*d^4 + 3003*a^2*
b^8*c^5*d^5 + 1001*a^3*b^7*c^4*d^6 + 286*a^4*b^6*c^3*d^7 + 66*a^5*b^5*c^2*d
^8 + 11*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 210*(43758*b^10*c^8*d^2 + 19448
*a*b^9*c^7*d^3 + 8008*a^2*b^8*c^6*d^4 + 3003*a^3*b^7*c^5*d^5 + 1001*a^4*b^6
*c^4*d^6 + 286*a^5*b^5*c^3*d^7 + 66*a^6*b^4*c^2*d^8 + 11*a^7*b^3*c*d^9 + a^
8*b^2*d^10)*x^2 + 21*(92378*b^10*c^9*d + 43758*a*b^9*c^8*d^2 + 19448*a^2*b^
8*c^7*d^3 + 8008*a^3*b^7*c^6*d^4 + 3003*a^4*b^6*c^5*d^5 + 1001*a^5*b^5*c^4*
d^6 + 286*a^6*b^4*c^3*d^7 + 66*a^7*b^3*c^2*d^8 + 11*a^8*b^2*c*d^9 + a^9*b*d
^10)*x)/(b^32*x^21 + 21*a*b^31*x^20 + 210*a^2*b^30*x^19 + 1330*a^3*b^29*x^1
8 + 5985*a^4*b^28*x^17 + 20349*a^5*b^27*x^16 + 54264*a^6*b^26*x^15 + 116280
*a^7*b^25*x^14 + 203490*a^8*b^24*x^13 + 293930*a^9*b^23*x^12 + 352716*a^10*
b^22*x^11 + 352716*a^11*b^21*x^10 + 293930*a^12*b^20*x^9 + 203490*a^13*b^19
*x^8 + 116280*a^14*b^18*x^7 + 54264*a^15*b^17*x^6 + 20349*a^16*b^16*x^5 + 5
985*a^17*b^15*x^4 + 1330*a^18*b^14*x^3 + 210*a^19*b^13*x^2 + 21*a^20*b^12*x
 + a^21*b^11)
```

Fricas [B] time = 1.96428, size = 2552, normalized size = 9.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="fricas")
```

```
[Out] -1/3879876*(352716*b^10*d^10*x^10 + 184756*b^10*c^10 + 92378*a*b^9*c^9*d +
43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^3 + 8008*a^4*b^6*c^6*d^4 + 3003
*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*
c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10 + 293930*(11*b^10*c*d^9 + a*b^9*d^10)*
x^9 + 203490*(66*b^10*c^2*d^8 + 11*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 116280
*(286*b^10*c^3*d^7 + 66*a*b^9*c^2*d^8 + 11*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^
7 + 54264*(1001*b^10*c^4*d^6 + 286*a*b^9*c^3*d^7 + 66*a^2*b^8*c^2*d^8 + 11*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 20349*(3003*b^10*c^5*d^5 + 1001*a*b^9*c
^4*d^6 + 286*a^2*b^8*c^3*d^7 + 66*a^3*b^7*c^2*d^8 + 11*a^4*b^6*c*d^9 + a^5*
b^5*d^10)*x^5 + 5985*(8008*b^10*c^6*d^4 + 3003*a*b^9*c^5*d^5 + 1001*a^2*b^8
*c^4*d^6 + 286*a^3*b^7*c^3*d^7 + 66*a^4*b^6*c^2*d^8 + 11*a^5*b^5*c*d^9 + a^
6*b^4*d^10)*x^4 + 1330*(19448*b^10*c^7*d^3 + 8008*a*b^9*c^6*d^4 + 3003*a^2*
b^8*c^5*d^5 + 1001*a^3*b^7*c^4*d^6 + 286*a^4*b^6*c^3*d^7 + 66*a^5*b^5*c^2*d
^8 + 11*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 210*(43758*b^10*c^8*d^2 + 19448
*a*b^9*c^7*d^3 + 8008*a^2*b^8*c^6*d^4 + 3003*a^3*b^7*c^5*d^5 + 1001*a^4*b^6
*c^4*d^6 + 286*a^5*b^5*c^3*d^7 + 66*a^6*b^4*c^2*d^8 + 11*a^7*b^3*c*d^9 + a^
8*b^2*d^10)*x^2 + 21*(92378*b^10*c^9*d + 43758*a*b^9*c^8*d^2 + 19448*a^2*b^
8*c^7*d^3 + 8008*a^3*b^7*c^6*d^4 + 3003*a^4*b^6*c^5*d^5 + 1001*a^5*b^5*c^4*
d^6 + 286*a^6*b^4*c^3*d^7 + 66*a^7*b^3*c^2*d^8 + 11*a^8*b^2*c*d^9 + a^9*b*d
^10)*x)/(b^32*x^21 + 21*a*b^31*x^20 + 210*a^2*b^30*x^19 + 1330*a^3*b^29*x^1
```

```
8 + 5985*a^4*b^28*x^17 + 20349*a^5*b^27*x^16 + 54264*a^6*b^26*x^15 + 116280
*a^7*b^25*x^14 + 203490*a^8*b^24*x^13 + 293930*a^9*b^23*x^12 + 352716*a^10*
b^22*x^11 + 352716*a^11*b^21*x^10 + 293930*a^12*b^20*x^9 + 203490*a^13*b^19
*x^8 + 116280*a^14*b^18*x^7 + 54264*a^15*b^17*x^6 + 20349*a^16*b^16*x^5 + 5
985*a^17*b^15*x^4 + 1330*a^18*b^14*x^3 + 210*a^19*b^13*x^2 + 21*a^20*b^12*x
 + a^21*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**22,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06939, size = 1297, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="giac")
```

```
[Out] -1/3879876*(352716*b^10*d^10*x^10 + 3233230*b^10*c*d^9*x^9 + 293930*a*b^9*d
^10*x^9 + 13430340*b^10*c^2*d^8*x^8 + 2238390*a*b^9*c*d^9*x^8 + 203490*a^2*
b^8*d^10*x^8 + 33256080*b^10*c^3*d^7*x^7 + 7674480*a*b^9*c^2*d^8*x^7 + 1279
080*a^2*b^8*c*d^9*x^7 + 116280*a^3*b^7*d^10*x^7 + 54318264*b^10*c^4*d^6*x^6
 + 15519504*a*b^9*c^3*d^7*x^6 + 3581424*a^2*b^8*c^2*d^8*x^6 + 596904*a^3*b^
7*c*d^9*x^6 + 54264*a^4*b^6*d^10*x^6 + 61108047*b^10*c^5*d^5*x^5 + 20369349
*a*b^9*c^4*d^6*x^5 + 5819814*a^2*b^8*c^3*d^7*x^5 + 1343034*a^3*b^7*c^2*d^8*
x^5 + 223839*a^4*b^6*c*d^9*x^5 + 20349*a^5*b^5*d^10*x^5 + 47927880*b^10*c^6
*d^4*x^4 + 17972955*a*b^9*c^5*d^5*x^4 + 5990985*a^2*b^8*c^4*d^6*x^4 + 17117
10*a^3*b^7*c^3*d^7*x^4 + 395010*a^4*b^6*c^2*d^8*x^4 + 65835*a^5*b^5*c*d^9*x
^4 + 5985*a^6*b^4*d^10*x^4 + 25865840*b^10*c^7*d^3*x^3 + 10650640*a*b^9*c^6
*d^4*x^3 + 3993990*a^2*b^8*c^5*d^5*x^3 + 1331330*a^3*b^7*c^4*d^6*x^3 + 3803
80*a^4*b^6*c^3*d^7*x^3 + 87780*a^5*b^5*c^2*d^8*x^3 + 14630*a^6*b^4*c*d^9*x^
3 + 1330*a^7*b^3*d^10*x^3 + 9189180*b^10*c^8*d^2*x^2 + 4084080*a*b^9*c^7*d^
3*x^2 + 1681680*a^2*b^8*c^6*d^4*x^2 + 630630*a^3*b^7*c^5*d^5*x^2 + 210210*a
^4*b^6*c^4*d^6*x^2 + 60060*a^5*b^5*c^3*d^7*x^2 + 13860*a^6*b^4*c^2*d^8*x^2
+ 2310*a^7*b^3*c*d^9*x^2 + 210*a^8*b^2*d^10*x^2 + 1939938*b^10*c^9*d*x + 91
8918*a*b^9*c^8*d^2*x + 408408*a^2*b^8*c^7*d^3*x + 168168*a^3*b^7*c^6*d^4*x
+ 63063*a^4*b^6*c^5*d^5*x + 21021*a^5*b^5*c^4*d^6*x + 6006*a^6*b^4*c^3*d^7*
x + 1386*a^7*b^3*c^2*d^8*x + 231*a^8*b^2*c*d^9*x + 21*a^9*b*d^10*x + 184756
*b^10*c^10 + 92378*a*b^9*c^9*d + 43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*
d^3 + 8008*a^4*b^6*c^6*d^4 + 3003*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 +
286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10)/((b*
x + a)^21*b^11)
```

3.1334 $\int \frac{(a+bx)^5}{c+dx}\,dx$

Optimal. Leaf size=122

$$\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}-\frac{(bc-ad)^5\log(c+dx)}{d^6}+\frac{(a+bx)^5}{5d}$$

```
[Out] (b*(b*c - a*d)^4*x)/d^5 - ((b*c - a*d)^3*(a + b*x)^2)/(2*d^4) + ((b*c - a*d
)^2*(a + b*x)^3)/(3*d^3) - ((b*c - a*d)*(a + b*x)^4)/(4*d^2) + (a + b*x)^5/
(5*d) - ((b*c - a*d)^5*Log[c + d*x])/d^6
```

Rubi [A] time = 0.0533321, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}-\frac{(bc-ad)^5\log(c+dx)}{d^6}+\frac{(a+bx)^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x),x]
```

```
[Out] (b*(b*c - a*d)^4*x)/d^5 - ((b*c - a*d)^3*(a + b*x)^2)/(2*d^4) + ((b*c - a*d
)^2*(a + b*x)^3)/(3*d^3) - ((b*c - a*d)*(a + b*x)^4)/(4*d^2) + (a + b*x)^5/
(5*d) - ((b*c - a*d)^5*Log[c + d*x])/d^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{c+dx}\,dx=\int \left(\frac{b(bc-ad)^4}{d^5}-\frac{b(bc-ad)^3(a+bx)}{d^4}+\frac{b(bc-ad)^2(a+bx)^2}{d^3}-\frac{b(bc-ad)(a+bx)^3}{d^2}+\frac{b(a+bx)^4}{d}+\frac{(-}{d^5}$$
$$=\frac{b(bc-ad)^4x}{d^5}-\frac{(bc-ad)^3(a+bx)^2}{2d^4}+\frac{(bc-ad)^2(a+bx)^3}{3d^3}-\frac{(bc-ad)(a+bx)^4}{4d^2}+\frac{(a+bx)^5}{5d}-\frac{(bc-ad)}{}$$

Mathematica [A] time = 0.0663722, size = 167, normalized size = 1.37

$$\frac{bdx\left(100a^2b^2d^2\left(6c^2-3cdx+2d^2x^2\right)+300a^3bd^3(dx-2c)+300a^4d^4+25ab^3d\left(6c^2dx-12c^3-4cd^2x^2+3d^3x^3\right)+b^4\left(20\right.\right.}{60d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x),x]
```

```
[Out] (b*d*x*(300*a^4*d^4 + 300*a^3*b*d^3*(-2*c + d*x) + 100*a^2*b^2*d^2*(6*c^2 -
 3*c*d*x + 2*d^2*x^2) + 25*a*b^3*d*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d
^3*x^3) + b^4*(60*c^4 - 30*c^3*d*x + 20*c^2*d^2*x^2 - 15*c*d^3*x^3 + 12*d^4
```

```
*x^4)) - 60*(b*c - a*d)^5*Log[c + d*x])/(60*d^6)
```

Maple [B] time = 0.004, size = 302, normalized size = 2.5

$$\frac{b^5x^5}{5d}+\frac{5ab^4x^4}{4d}-\frac{b^5x^4c}{4d^2}+\frac{10a^2b^3x^3}{3d}-\frac{5ab^4x^3c}{3d^2}+\frac{b^5x^3c^2}{3d^3}+5\frac{a^3b^2x^2}{d}-5\frac{a^2b^3x^2c}{d^2}+\frac{5ab^4x^2c^2}{2d^3}-\frac{b^5x^2c^3}{2d^4}+5\frac{a^4bx}{d}-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c),x)
```

```
[Out] 1/5*b^5/d*x^5+5/4*b^4/d*x^4*a-1/4*b^5/d^2*x^4*c+10/3*b^3/d*x^3*a^2-5/3*b^4/
d^2*x^3*a*c+1/3*b^5/d^3*x^3*c^2+5*b^2/d*x^2*a^3-5*b^3/d^2*x^2*a^2*c+5/2*b^4
/d^3*x^2*a*c^2-1/2*b^5/d^4*x^2*c^3+5*b/d*a^4*x-10*b^2/d^2*a^3*c*x+10*b^3/d^
3*a^2*c^2*x-5*b^4/d^4*a*c^3*x+b^5/d^5*c^4*x+1/d*ln(d*x+c)*a^5-5/d^2*ln(d*x+
c)*a^4*b*c+10/d^3*ln(d*x+c)*a^3*b^2*c^2-10/d^4*ln(d*x+c)*a^2*b^3*c^3+5/d^5*
ln(d*x+c)*a*b^4*c^4-1/d^6*ln(d*x+c)*b^5*c^5
```

Maxima [B] time = 0.974107, size = 348, normalized size = 2.85

$$\frac{12\,b^5d^4x^5-15\left(b^5cd^3-5\,ab^4d^4\right)x^4+20\left(b^5c^2d^2-5\,ab^4cd^3+10\,a^2b^3d^4\right)x^3-30\left(b^5c^3d-5\,ab^4c^2d^2+10\,a^2b^3cd^3-10\,a\right.}{60\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/60*(12*b^5*d^4*x^5 - 15*(b^5*c*d^3 - 5*a*b^4*d^4)*x^4 + 20*(b^5*c^2*d^2 -
 5*a*b^4*c*d^3 + 10*a^2*b^3*d^4)*x^3 - 30*(b^5*c^3*d - 5*a*b^4*c^2*d^2 + 10
*a^2*b^3*c*d^3 - 10*a^3*b^2*d^4)*x^2 + 60*(b^5*c^4 - 5*a*b^4*c^3*d + 10*a^2
*b^3*c^2*d^2 - 10*a^3*b^2*c*d^3 + 5*a^4*b*d^4)*x)/d^5 - (b^5*c^5 - 5*a*b^4*
c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*
log(d*x + c)/d^6
```

Fricas [B] time = 1.84718, size = 537, normalized size = 4.4

$$\frac{12\,b^5d^5x^5-15\left(b^5cd^4-5\,ab^4d^5\right)x^4+20\left(b^5c^2d^3-5\,ab^4cd^4+10\,a^2b^3d^5\right)x^3-30\left(b^5c^3d^2-5\,ab^4c^2d^3+10\,a^2b^3cd^4-10\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*d^5*x^5 - 15*(b^5*c*d^4 - 5*a*b^4*d^5)*x^4 + 20*(b^5*c^2*d^3 -
 5*a*b^4*c*d^4 + 10*a^2*b^3*d^5)*x^3 - 30*(b^5*c^3*d^2 - 5*a*b^4*c^2*d^3 +
10*a^2*b^3*c*d^4 - 10*a^3*b^2*d^5)*x^2 + 60*(b^5*c^4*d - 5*a*b^4*c^3*d^2 +
10*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x - 60*(b^5*c^5 - 5*a*
b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d
^5)*log(d*x + c))/d^6
```

Sympy [A] time = 0.662197, size = 202, normalized size = 1.66

$$\frac{b^5x^5}{5d} + \frac{x^4\left(5ab^4d - b^5c\right)}{4d^2} + \frac{x^3\left(10a^2b^3d^2 - 5ab^4cd + b^5c^2\right)}{3d^3} + \frac{x^2\left(10a^3b^2d^3 - 10a^2b^3cd^2 + 5ab^4c^2d - b^5c^3\right)}{2d^4} + \frac{x\left(5a^4bd^4 - 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c),x)
```

```
[Out] b**5*x**5/(5*d) + x**4*(5*a*b**4*d - b**5*c)/(4*d**2) + x**3*(10*a**2*b**3*
d**2 - 5*a*b**4*c*d + b**5*c**2)/(3*d**3) + x**2*(10*a**3*b**2*d**3 - 10*a*
*2*b**3*c*d**2 + 5*a*b**4*c**2*d - b**5*c**3)/(2*d**4) + x*(5*a**4*b*d**4 -
 10*a**3*b**2*c*d**3 + 10*a**2*b**3*c**2*d**2 - 5*a*b**4*c**3*d + b**5*c**4
)/d**5 + (a*d - b*c)**5*log(c + d*x)/d**6
```

Giac [B] time = 1.05788, size = 369, normalized size = 3.02

$$\frac{12\,b^5d^4x^5 - 15\,b^5cd^3x^4 + 75\,ab^4d^4x^4 + 20\,b^5c^2d^2x^3 - 100\,ab^4cd^3x^3 + 200\,a^2b^3d^4x^3 - 30\,b^5c^3dx^2 + 150\,ab^4c^2d^2x^2 - 300\,a^2}{60\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/60*(12*b^5*d^4*x^5 - 15*b^5*c*d^3*x^4 + 75*a*b^4*d^4*x^4 + 20*b^5*c^2*d^2
*x^3 - 100*a*b^4*c*d^3*x^3 + 200*a^2*b^3*d^4*x^3 - 30*b^5*c^3*d*x^2 + 150*a
*b^4*c^2*d^2*x^2 - 300*a^2*b^3*c*d^3*x^2 + 300*a^3*b^2*d^4*x^2 + 60*b^5*c^4
*x - 300*a*b^4*c^3*d*x + 600*a^2*b^3*c^2*d^2*x - 600*a^3*b^2*c*d^3*x + 300*
a^4*b*d^4*x)/d^5 - (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b
^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*log(abs(d*x + c))/d^6
```

3.1335 $\int \frac{(a+bx)^4}{c+dx}\,dx$

Optimal. Leaf size=98

$$-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(bc-ad)^4\log(c+dx)}{d^5}+\frac{(a+bx)^4}{4d}$$

```
[Out] -((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2*d^3) - ((b*c -
a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*d)^4*Log[c + d*x]
)/d^5
```

Rubi [A] time = 0.0383132, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(bc-ad)^4\log(c+dx)}{d^5}+\frac{(a+bx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x),x]
```

```
[Out] -((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2*d^3) - ((b*c -
a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*d)^4*Log[c + d*x]
)/d^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{c+dx}\,dx &= \int \left(-\frac{b(bc-ad)^3}{d^4}+\frac{b(bc-ad)^2(a+bx)}{d^3}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(a+bx)^3}{d}+\frac{(-bc+ad)^4}{d^4(c+dx)}\right)dx\\ &= -\frac{b(bc-ad)^3x}{d^4}+\frac{(bc-ad)^2(a+bx)^2}{2d^3}-\frac{(bc-ad)(a+bx)^3}{3d^2}+\frac{(a+bx)^4}{4d}+\frac{(bc-ad)^4\log(c+dx)}{d^5}\end{aligned}$$

Mathematica [A] time = 0.0417824, size = 115, normalized size = 1.17

$$\frac{bdx\left(36a^2bd^2(dx-2c)+48a^3d^3+8ab^2d\left(6c^2-3cdx+2d^2x^2\right)+b^3\left(6c^2dx-12c^3-4cd^2x^2+3d^3x^3\right)\right)+12(bc-ad)^4\,\mathrm{l}}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x),x]
```

```
[Out] (b*d*x*(48*a^3*d^3 + 36*a^2*b*d^2*(-2*c + d*x) + 8*a*b^2*d*(6*c^2 - 3*c*d*x
 + 2*d^2*x^2) + b^3*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^3*x^3)) + 12*(
```

```
b*c - a*d)^4*Log[c + d*x])/(12*d^5)
```

Maple [B] time = 0.003, size = 209, normalized size = 2.1

$$\frac{b^4x^4}{4d} + \frac{4x^3ab^3}{3d} - \frac{b^4x^3c}{3d^2} + 3\frac{x^2a^2b^2}{d} - 2\frac{b^3x^2ac}{d^2} + \frac{b^4x^2c^2}{2d^3} + 4\frac{bxa^3}{d} - 6\frac{b^2a^2cx}{d^2} + 4\frac{b^3ac^2x}{d^3} - \frac{b^4c^3x}{d^4} + \frac{\ln(dx+c)a^4}{d} - 4\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c),x)
```

```
[Out] 1/4*b^4/d*x^4+4/3*b^3/d*x^3*a-1/3*b^4/d^2*x^3*c+3*b^2/d*x^2*a^2-2*b^3/d^2*x
^2*a*c+1/2*b^4/d^3*x^2*c^2+4*b/d*a^3*x-6*b^2/d^2*a^2*c*x+4*b^3/d^3*a*c^2*x-
b^4/d^4*c^3*x+1/d*ln(d*x+c)*a^4-4/d^2*ln(d*x+c)*a^3*b*c+6/d^3*ln(d*x+c)*a^2
*b^2*c^2-4/d^4*ln(d*x+c)*a*b^3*c^3+1/d^5*ln(d*x+c)*b^4*c^4
```

Maxima [A] time = 0.994568, size = 239, normalized size = 2.44

$$\frac{3\,b^4d^3x^4 - 4\left(b^4cd^2 - 4\,ab^3d^3\right)x^3 + 6\left(b^4c^2d - 4\,ab^3cd^2 + 6\,a^2b^2d^3\right)x^2 - 12\left(b^4c^3 - 4\,ab^3c^2d + 6\,a^2b^2cd^2 - 4\,a^3bd^3\right)x}{12\,d^4} + \frac{(b^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^4*d^3*x^4 - 4*(b^4*c*d^2 - 4*a*b^3*d^3)*x^3 + 6*(b^4*c^2*d - 4*a*
b^3*c*d^2 + 6*a^2*b^2*d^3)*x^2 - 12*(b^4*c^3 - 4*a*b^3*c^2*d + 6*a^2*b^2*c*
d^2 - 4*a^3*b*d^3)*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c)/d^5
```

Fricas [A] time = 1.67524, size = 369, normalized size = 3.77

$$\frac{3\,b^4d^4x^4 - 4\left(b^4cd^3 - 4\,ab^3d^4\right)x^3 + 6\left(b^4c^2d^2 - 4\,ab^3cd^3 + 6\,a^2b^2d^4\right)x^2 - 12\left(b^4c^3d - 4\,ab^3c^2d^2 + 6\,a^2b^2cd^3 - 4\,a^3bd^4\right)x + 1}{12\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*d^4*x^4 - 4*(b^4*c*d^3 - 4*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 4*
a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 - 12*(b^4*c^3*d - 4*a*b^3*c^2*d^2 + 6*a^2*
b^2*c*d^3 - 4*a^3*b*d^4)*x + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c))/d^5
```

Sympy [A] time = 0.535401, size = 134, normalized size = 1.37

$$\frac{b^4x^4}{4d} + \frac{x^3\left(4ab^3d - b^4c\right)}{3d^2} + \frac{x^2\left(6a^2b^2d^2 - 4ab^3cd + b^4c^2\right)}{2d^3} + \frac{x\left(4a^3bd^3 - 6a^2b^2cd^2 + 4ab^3c^2d - b^4c^3\right)}{d^4} + \frac{(ad-bc)^4\log(c+}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c),x)
```

```
[Out] b**4*x**4/(4*d) + x**3*(4*a*b**3*d - b**4*c)/(3*d**2) + x**2*(6*a**2*b**2*d
**2 - 4*a*b**3*c*d + b**4*c**2)/(2*d**3) + x*(4*a**3*b*d**3 - 6*a**2*b**2*c
*d**2 + 4*a*b**3*c**2*d - b**4*c**3)/d**4 + (a*d - b*c)**4*log(c + d*x)/d**
5
```

Giac [A] time = 1.05425, size = 248, normalized size = 2.53

$$\frac{3\,b^4d^3x^4 - 4\,b^4cd^2x^3 + 16\,ab^3d^3x^3 + 6\,b^4c^2dx^2 - 24\,ab^3cd^2x^2 + 36\,a^2b^2d^3x^2 - 12\,b^4c^3x + 48\,ab^3c^2dx - 72\,a^2b^2cd^2x + 4}{12\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/12*(3*b^4*d^3*x^4 - 4*b^4*c*d^2*x^3 + 16*a*b^3*d^3*x^3 + 6*b^4*c^2*d*x^2
- 24*a*b^3*c*d^2*x^2 + 36*a^2*b^2*d^3*x^2 - 12*b^4*c^3*x + 48*a*b^3*c^2*d*x
 - 72*a^2*b^2*c*d^2*x + 48*a^3*b*d^3*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(d*x + c))/d^5
```

3.1336 $\int \frac{(a+bx)^3}{c+dx} \, dx$

Optimal. Leaf size=74

$$\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{(a+bx)^3}{3d}$$

```
[Out] (b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) + (a + b*x)^3/(
3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4
```

Rubi [A] time = 0.0298652, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{(a+bx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x),x]
```

```
[Out] (b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) + (a + b*x)^3/(
3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^3}{c+dx}\,dx &= \int \left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx\\
&= \frac{b(bc-ad)^2x}{d^3}-\frac{(bc-ad)(a+bx)^2}{2d^2}+\frac{(a+bx)^3}{3d}-\frac{(bc-ad)^3\log(c+dx)}{d^4}
\end{aligned}$$

Mathematica [A] time = 0.0284069, size = 74, normalized size = 1.

$$\frac{bdx\left(18a^2d^2+9abd(dx-2c)+b^2\left(6c^2-3cdx+2d^2x^2\right)\right)-6(bc-ad)^3\log(c+dx)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x),x]
```

```
[Out] (b*d*x*(18*a^2*d^2 + 9*a*b*d*(-2*c + d*x) + b^2*(6*c^2 - 3*c*d*x + 2*d^2*x^
2)) - 6*(b*c - a*d)^3*Log[c + d*x])/(6*d^4)
```

Maple [A] time = 0.002, size = 133, normalized size = 1.8

$$\frac{b^3x^3}{3\,d}+\frac{3\,ab^2x^2}{2\,d}-\frac{b^3x^2c}{2\,d^2}+3\,\frac{a^2bx}{d}-3\,\frac{ab^2cx}{d^2}+\frac{b^3c^2x}{d^3}+\frac{\ln(dx+c)\,a^3}{d}-3\,\frac{\ln(dx+c)\,a^2bc}{d^2}+3\,\frac{\ln(dx+c)\,ab^2c^2}{d^3}-\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/3*b^3/d*x^3+3/2*b^2/d*x^2*a-1/2*b^3/d^2*x^2*c+3*b/d*a^2*x-3*b^2/d^2*a*c*x
+b^3/d^3*c^2*x+1/d*ln(d*x+c)*a^3-3/d^2*ln(d*x+c)*a^2*b*c+3/d^3*ln(d*x+c)*a*
b^2*c^2-1/d^4*ln(d*x+c)*b^3*c^3
```

Maxima [A] time = 0.964813, size = 154, normalized size = 2.08

$$\frac{2\,b^3d^2x^3-3\left(b^3cd-3\,ab^2d^2\right)x^2+6\left(b^3c^2-3\,ab^2cd+3\,a^2bd^2\right)x}{6\,d^3}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*d^2*x^3 - 3*(b^3*c*d - 3*a*b^2*d^2)*x^2 + 6*(b^3*c^2 - 3*a*b^2*c
*d + 3*a^2*b*d^2)*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(d*x + c)/d^4
```

Fricas [A] time = 1.74385, size = 238, normalized size = 3.22

$$\frac{2\,b^3d^3x^3-3\left(b^3cd^2-3\,ab^2d^3\right)x^2+6\left(b^3c^2d-3\,ab^2cd^2+3\,a^2bd^3\right)x-6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log(dx+c}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*d^3*x^3 - 3*(b^3*c*d^2 - 3*a*b^2*d^3)*x^2 + 6*(b^3*c^2*d - 3*a*b
^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(d*x + c))/d^4
```

Sympy [A] time = 0.447796, size = 82, normalized size = 1.11

$$\frac{b^3x^3}{3d}+\frac{x^2\left(3ab^2d-b^3c\right)}{2d^2}+\frac{x\left(3a^2bd^2-3ab^2cd+b^3c^2\right)}{d^3}+\frac{(ad-bc)^3\log(c+dx)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c),x)
```

```
[Out] b**3*x**3/(3*d) + x**2*(3*a*b**2*d - b**3*c)/(2*d**2) + x*(3*a**2*b*d**2 -
3*a*b**2*c*d + b**3*c**2)/d**3 + (a*d - b*c)**3*log(c + d*x)/d**4
```

Giac [A] time = 1.05122, size = 157, normalized size = 2.12

$$\frac{2\,b^3d^2x^3 - 3\,b^3cdx^2 + 9\,ab^2d^2x^2 + 6\,b^3c^2x - 18\,ab^2cdx + 18\,a^2bd^2x}{6\,d^3} - \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(|dx + c|\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*d^2*x^3 - 3*b^3*c*d*x^2 + 9*a*b^2*d^2*x^2 + 6*b^3*c^2*x - 18*a*b
^2*c*d*x + 18*a^2*b*d^2*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(abs(d*x + c))/d^4
```

3.1337 $\int \frac{(a+bx)^2}{c+dx}\,dx$

Optimal. Leaf size=50

$$-\frac{bx(bc-ad)}{d^2}+\frac{(bc-ad)^2\log(c+dx)}{d^3}+\frac{(a+bx)^2}{2d}$$

```
[Out] -((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)^2*Log[c + d*x])
/d^3
```

Rubi [A] time = 0.0208861, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{bx(bc-ad)}{d^2}+\frac{(bc-ad)^2\log(c+dx)}{d^3}+\frac{(a+bx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x),x]
```

```
[Out] -((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)^2*Log[c + d*x])
/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{c+dx}\,dx &= \int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx\\ &= -\frac{b(bc-ad)x}{d^2}+\frac{(a+bx)^2}{2d}+\frac{(bc-ad)^2\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A] time = 0.0168373, size = 43, normalized size = 0.86

$$\frac{bdx(4ad-2bc+bdx)+2(bc-ad)^2\log(c+dx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x),x]
```

```
[Out] (b*d*x*(-2*b*c + 4*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[c + d*x])/(2*d^3)
```

Maple [A] time = 0.002, size = 74, normalized size = 1.5

$$\frac{b^{2}x^{2}}{2d}+2\frac{abx}{d}-\frac{b^{2}xc}{d^{2}}+\frac{\ln\left(dx+c\right)a^{2}}{d}-2\frac{\ln\left(dx+c\right)abc}{d^{2}}+\frac{\ln\left(dx+c\right)b^{2}c^{2}}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c),x)
```

```
[Out] 1/2*b^2/d*x^2+2*b/d*a*x-b^2/d^2*x*c+1/d*ln(d*x+c)*a^2-2/d^2*ln(d*x+c)*a*b*c
+1/d^3*ln(d*x+c)*b^2*c^2
```

Maxima [A] time = 0.976171, size = 81, normalized size = 1.62

$$\frac{b^{2}dx^{2}-2\left(b^{2}c-2\,abd\right)x}{2\,d^{2}}+\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(dx+c\right)}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*d*x^2 - 2*(b^2*c - 2*a*b*d)*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(d*x + c)/d^3
```

Fricas [A] time = 1.77896, size = 135, normalized size = 2.7

$$\frac{b^{2}d^{2}x^{2}-2\left(b^{2}cd-2\,abd^{2}\right)x+2\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(dx+c\right)}{2\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 - 2*(b^2*c*d - 2*a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(d*x + c))/d^3
```

Sympy [A] time = 0.36138, size = 44, normalized size = 0.88

$$\frac{b^{2}x^{2}}{2d}+\frac{x\left(2abd-b^{2}c\right)}{d^{2}}+\frac{(ad-bc)^{2}\log\left(c+dx\right)}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c),x)
```

```
[Out] b**2*x**2/(2*d) + x*(2*a*b*d - b**2*c)/d**2 + (a*d - b*c)**2*log(c + d*x)/d
**3
```

Giac [A] time = 1.05664, size = 81, normalized size = 1.62

$$\frac{b^2dx^2 - 2\,b^2cx + 4\,abdx}{2\,d^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(|dx + c|\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d*x^2 - 2*b^2*c*x + 4*a*b*d*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(abs(d*x + c))/d^3
```

3.1338 $\int \frac{a+bx}{c+dx} dx$

Optimal. Leaf size=26

$$\frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

```
[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Rubi [A] time = 0.0178974, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{c+dx} dx &= \int \left(\frac{b}{d} + \frac{-bc+ad}{d(c+dx)}\right) dx \\ &= \frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2} \end{aligned}$$

Mathematica [A] time = 0.0071804, size = 25, normalized size = 0.96

$$\frac{(ad-bc)\log(c+dx)}{d^2} + \frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x),x]
```

```
[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

Maple [A] time = 0.003, size = 32, normalized size = 1.2

$$\frac{bx}{d} + \frac{\ln(dx+c)\,a}{d} - \frac{\ln(dx+c)\,bc}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c),x)
```

```
[Out] b*x/d+1/d*ln(d*x+c)*a-1/d^2*ln(d*x+c)*b*c
```

Maxima [A] time = 0.968243, size = 35, normalized size = 1.35

$$\frac{bx}{d} - \frac{(bc - ad)\log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

Fricas [A] time = 1.75919, size = 54, normalized size = 2.08

$$\frac{bdx - (bc - ad)\log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

Sympy [A] time = 0.287584, size = 20, normalized size = 0.77

$$\frac{bx}{d} + \frac{(ad - bc)\log(c + dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x)
```

```
[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

Giac [A] time = 1.06185, size = 36, normalized size = 1.38

$$\frac{bx}{d} - \frac{(bc - ad)\log(|dx + c|)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

3.1339 $\int \frac{1}{c+dx} dx$

Optimal. Leaf size=10

$$\frac{\log(c+dx)}{d}$$

```
[Out] Log[c + d*x]/d
```

Rubi [A] time = 0.0014583, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-1),x]
```

```
[Out] Log[c + d*x]/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{c+dx} dx = \frac{\log(c+dx)}{d}$$

Mathematica [A] time = 0.000797, size = 10, normalized size = 1.

$$\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-1),x]
```

```
[Out] Log[c + d*x]/d
```

Maple [A] time = 0.001, size = 11, normalized size = 1.1

$$\frac{\ln(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c),x)
```

```
[Out] ln(d*x+c)/d
```

Maxima [A] time = 0.956083, size = 14, normalized size = 1.4

$$\frac{\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="maxima")
```

```
[Out] log(d*x + c)/d
```

Fricas [A] time = 1.73254, size = 22, normalized size = 2.2

$$\frac{\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="fricas")
```

```
[Out] log(d*x + c)/d
```

Sympy [A] time = 0.055457, size = 7, normalized size = 0.7

$$\frac{\log(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x)
```

```
[Out] log(c + d*x)/d
```

Giac [A] time = 1.05921, size = 15, normalized size = 1.5

$$\frac{\log(|dx+c|)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="giac")
```

```
[Out] log(abs(d*x + c))/d
```

3.1340 $\int \frac{1}{(a+bx)(c+dx)} \, dx$

Optimal. Leaf size=36

$$\frac{\log(a+bx)}{bc-ad} - \frac{\log(c+dx)}{bc-ad}$$

```
[Out] Log[a + b*x]/(b*c - a*d) - Log[c + d*x]/(b*c - a*d)
```

Rubi [A] time = 0.0076648, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {36, 31}

$$\frac{\log(a+bx)}{bc-ad} - \frac{\log(c+dx)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)),x]

[Out] Log[a + b*x]/(b*c - a*d) - Log[c + d*x]/(b*c - a*d)
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)} \, dx &= \frac{b \int \frac{1}{a+bx} \, dx}{bc-ad} - \frac{d \int \frac{1}{c+dx} \, dx}{bc-ad}\\ &= \frac{\log(a+bx)}{bc-ad} - \frac{\log(c+dx)}{bc-ad}\end{aligned}$$

Mathematica [A] time = 0.0107835, size = 26, normalized size = 0.72

$$\frac{\log(a+bx) - \log(c+dx)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)),x]

[Out] (Log[a + b*x] - Log[c + d*x])/(b*c - a*d)
```

Maple [A] time = 0.006, size = 37, normalized size = 1.

$$\frac{\ln(dx+c)}{ad-bc}-\frac{\ln(bx+a)}{ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c),x)
```

```
[Out] 1/(a*d-b*c)*ln(d*x+c)-1/(a*d-b*c)*ln(b*x+a)
```

Maxima [A] time = 0.980413, size = 49, normalized size = 1.36

$$\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)
```

Fricas [A] time = 1.71206, size = 58, normalized size = 1.61

$$\frac{\log(bx+a)-\log(dx+c)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] (log(b*x + a) - log(d*x + c))/(b*c - a*d)
```

Sympy [B] time = 0.306326, size = 128, normalized size = 3.56

$$\frac{\log\left(x+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}-\frac{\log\left(x+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x)
```

```
[Out] log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**2/(
a*d - b*c) + b*c)/(2*b*d))/(a*d - b*c) - log(x + (a**2*d**2/(a*d - b*c) - 2
*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))/(a*d - b
*c)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1341 $\int \frac{1}{(a+bx)^2(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
d*x])/(b*c - a*d)^2
```

Rubi [A] time = 0.0317131, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)),x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
d*x])/(b*c - a*d)^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)}\,dx &= \int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)dx\\ &= -\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.0245266, size = 53, normalized size = 0.93

$$\frac{d(a+bx)\log(c+dx)-d(a+bx)\log(a+bx)+ad-bc}{(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)),x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x)*Log[a + b*x] + d*(a + b*x)*Log[c + d*x])/((b*c
- a*d)^2*(a + b*x))
```

Maple [A] time = 0.014, size = 57, normalized size = 1.

$$\frac{d\ln(dx+c)}{(ad-bc)^2}+\frac{1}{(ad-bc)(bx+a)}-\frac{d\ln(bx+a)}{(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c),x)
```

```
[Out] d/(a*d-b*c)^2*ln(d*x+c)+1/(a*d-b*c)/(b*x+a)-d/(a*d-b*c)^2*ln(b*x+a)
```

Maxima [A] time = 0.970353, size = 124, normalized size = 2.18

$$-\frac{d\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2}+\frac{d\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2}-\frac{1}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + d*log(d*x + c)/(b^2*c^2 -
 2*a*b*c*d + a^2*d^2) - 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Fricas [A] time = 1.87019, size = 200, normalized size = 3.51

$$-\frac{bc-ad+(bdx+ad)\log(bx+a)-(bdx+ad)\log(dx+c)}{ab^2c^2-2\,a^2bcd+a^3d^2+\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*c - a*d + (b*d*x + a*d)*log(b*x + a) - (b*d*x + a*d)*log(d*x + c))/(a*b
^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)
```

Sympy [B] time = 0.778596, size = 233, normalized size = 4.09

$$\frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2}-\frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2}+\frac{1}{a^2d-abc+x\left(abc\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c),x)
```

```
[Out] d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a
*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*
d)/(2*b*d**2))/(a*d - b*c)**2 - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**
2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b*
*3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(a*d - b*c)**2 + 1/(a**2*d -
```

```
a*b*c + x*(a*b*d - b**2*c))
```

Giac [A] time = 1.07936, size = 105, normalized size = 1.84

$$\frac{bd\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{b}{\left(b^2c-abd\right)(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] b*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^3*c^2 - 2*a*b^2*c*d + a^
2*b*d^2) - b/((b^2*c - a*b*d)*(b*x + a))
```

3.1342 $\int \frac{1}{(a+bx)^3(c+dx)} \, dx$

Optimal. Leaf size=82

$$\frac{d^2 \log(a+bx)}{(bc-ad)^3} - \frac{d^2 \log(c+dx)}{(bc-ad)^3} + \frac{d}{(a+bx)(bc-ad)^2} - \frac{1}{2(a+bx)^2(bc-ad)}$$

```
[Out] -1/(2*(b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) + (d^2*Log[a +
 b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3
```

Rubi [A] time = 0.0454462, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{d^2 \log(a+bx)}{(bc-ad)^3} - \frac{d^2 \log(c+dx)}{(bc-ad)^3} + \frac{d}{(a+bx)(bc-ad)^2} - \frac{1}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)),x]
```

```
[Out] -1/(2*(b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) + (d^2*Log[a +
 b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+bx)^3(c+dx)} \, dx &= \int \left(\frac{b}{(bc-ad)(a+bx)^3} - \frac{bd}{(bc-ad)^2(a+bx)^2} + \frac{bd^2}{(bc-ad)^3(a+bx)} - \frac{d^3}{(bc-ad)^3(c+dx)}\right) dx \\ &= -\frac{1}{2(bc-ad)(a+bx)^2} + \frac{d}{(bc-ad)^2(a+bx)} + \frac{d^2 \log(a+bx)}{(bc-ad)^3} - \frac{d^2 \log(c+dx)}{(bc-ad)^3} \end{aligned}$$

Mathematica [A] time = 0.0639444, size = 67, normalized size = 0.82

$$\frac{\frac{(bc-ad)(3ad-bc+2bdx)}{(a+bx)^2} + 2d^2 \log(a+bx) - 2d^2 \log(c+dx)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)),x]
```

```
[Out] (((b*c - a*d)*(-(b*c) + 3*a*d + 2*b*d*x))/(a + b*x)^2 + 2*d^2*Log[a + b*x]
- 2*d^2*Log[c + d*x])/(2*(b*c - a*d)^3)
```

Maple [A] time = 0.007, size = 81, normalized size = 1.

$$\frac{d^2 \ln (dx+c)}{(ad-bc)^3}+\frac{1}{(2\,ad-2\,bc)\,(bx+a)^2}+\frac{d}{(ad-bc)^2\,(bx+a)}-\frac{d^2 \ln (bx+a)}{(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c),x)
```

```
[Out] d^2/(a*d-b*c)^3*ln(d*x+c)+1/2/(a*d-b*c)/(b*x+a)^2+d/(a*d-b*c)^2/(b*x+a)-d^2
/(a*d-b*c)^3*ln(b*x+a)
```

Maxima [B] time = 1.0096, size = 273, normalized size = 3.33

$$\frac{d^2 \log (bx+a)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}-\frac{d^2 \log (dx+c)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{2\,bdx-}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] d^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - d^2*
log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2*b
*d*x - b*c + 3*a*d)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b
^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Fricas [B] time = 1.83099, size = 491, normalized size = 5.99

$$-\frac{b^2c^2-4\,abcd+3\,a^2d^2-2\left(b^2cd-abd^2\right)x-2\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\log (bx+a)+2\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)}{2\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3+\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)x^2+2\left(ab^4c^3-3\,a^2b^3c^2d+3\,a^3b^2cd\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d^
2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x
+ a^2*d^2)*log(d*x + c))/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a
^5*d^3 + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^2 + 2*
(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*x)
```

Sympy [B] time = 1.21214, size = 381, normalized size = 4.65

$$\frac{d^2 \log \left(x+\frac{-\frac{a^4d^6}{(ad-bc)^3}+\frac{4a^3bcd^5}{(ad-bc)^3}-\frac{6a^2b^2c^2d^4}{(ad-bc)^3}+\frac{4ab^3c^3d^3}{(ad-bc)^3}+ad^3-\frac{b^4c^4d^2}{(ad-bc)^3}+bcd^2}{2bd^3}\right)}{(ad-bc)^3}-\frac{d^2 \log \left(x+\frac{\frac{a^4d^6}{(ad-bc)^3}-\frac{4a^3bcd^5}{(ad-bc)^3}+\frac{6a^2b^2c^2d^4}{(ad-bc)^3}-\frac{4ab^3c^3d^3}{(ad-bc)^3}+ad^3+\frac{b^4c^4d^2}{(ad-bc)^3}+bc}{2bd^3}\right.}{(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c),x)
```

```
[Out] d**2*log(x + (-a**4*d**6/(a*d - b*c)**3 + 4*a**3*b*c*d**5/(a*d - b*c)**3 -
6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 + 4*a*b**3*c**3*d**3/(a*d - b*c)**3 +
a*d**3 - b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a*d - b*c)*
*3 - d**2*log(x + (a**4*d**6/(a*d - b*c)**3 - 4*a**3*b*c*d**5/(a*d - b*c)**
3 + 6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 - 4*a*b**3*c**3*d**3/(a*d - b*c)**
3 + a*d**3 + b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a*d - b
*c)**3 + (3*a*d - b*c + 2*b*d*x)/(2*a**4*d**2 - 4*a**3*b*c*d + 2*a**2*b**2*
c**2 + x**2*(2*a**2*b**2*d**2 - 4*a*b**3*c*d + 2*b**4*c**2) + x*(4*a**3*b*d
**2 - 8*a**2*b**2*c*d + 4*a*b**3*c**2))
```

Giac [B] time = 1.06785, size = 223, normalized size = 2.72

$$\frac{bd^2\log(|bx+a|)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{d^3\log(|dx+c|)}{b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4}-\frac{b^2c^2-4\,abcd+3\,a^2d^2-2\left(b^2cd-abd^2\right)x}{2\,(bc-ad)^3(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] b*d^2*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*
d^3) - d^3*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4) - 1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x)
/((b*c - a*d)^3*(b*x + a)^2)
```

3.1343 $\int \frac{(a+bx)^5}{(c+dx)^2}\,dx$

Optimal. Leaf size=130

$$-\frac{5b^4(c+dx)^3(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^2(bc-ad)^2}{d^6}-\frac{10b^2x(bc-ad)^3}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b(bc-ad)^4\log(c+dx)}{d^6}+\frac{b^5(c+$$

```
[Out] (-10*b^2*(b*c - a*d)^3*x)/d^5 + (b*c - a*d)^5/(d^6*(c + d*x)) + (5*b^3*(b*c
 - a*d)^2*(c + d*x)^2)/d^6 - (5*b^4*(b*c - a*d)*(c + d*x)^3)/(3*d^6) + (b^5
*(c + d*x)^4)/(4*d^6) + (5*b*(b*c - a*d)^4*Log[c + d*x])/d^6
```

Rubi [A] time = 0.139165, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{5b^4(c+dx)^3(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^2(bc-ad)^2}{d^6}-\frac{10b^2x(bc-ad)^3}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b(bc-ad)^4\log(c+dx)}{d^6}+\frac{b^5(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^2,x]
```

```
[Out] (-10*b^2*(b*c - a*d)^3*x)/d^5 + (b*c - a*d)^5/(d^6*(c + d*x)) + (5*b^3*(b*c
 - a*d)^2*(c + d*x)^2)/d^6 - (5*b^4*(b*c - a*d)*(c + d*x)^3)/(3*d^6) + (b^5
*(c + d*x)^4)/(4*d^6) + (5*b*(b*c - a*d)^4*Log[c + d*x])/d^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^2}\,dx=\int \left(-\frac{10b^2(bc-ad)^3}{d^5}+\frac{(-bc+ad)^5}{d^5(c+dx)^2}+\frac{5b(bc-ad)^4}{d^5(c+dx)}+\frac{10b^3(bc-ad)^2(c+dx)}{d^5}-\frac{5b^4(bc-ad)(c+dx)}{d^5}\right.$$
$$=-\frac{10b^2(bc-ad)^3x}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b^3(bc-ad)^2(c+dx)^2}{d^6}-\frac{5b^4(bc-ad)(c+dx)^3}{3d^6}+\frac{b^5(c+dx)^4}{4d^6}+$$

Mathematica [A] time = 0.0717922, size = 228, normalized size = 1.75

$$\frac{60a^2b^3d^2\left(-4c^2dx+2c^3-3cd^2x^2+d^3x^3\right)+120a^3b^2d^3\left(-c^2+cdx+d^2x^2\right)+60a^4bcd^4-12a^5d^5+20ab^4d\left(6c^2d^2x^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^2,x]
```

```
[Out] (60*a^4*b*c*d^4 - 12*a^5*d^5 + 120*a^3*b^2*d^3*(-c^2 + c*d*x + d^2*x^2) + 6
0*a^2*b^3*d^2*(2*c^3 - 4*c^2*d*x - 3*c*d^2*x^2 + d^3*x^3) + 20*a*b^4*d*(-3*
```

```
c^4 + 9*c^3*d*x + 6*c^2*d^2*x^2 - 2*c*d^3*x^3 + d^4*x^4) + b^5*(12*c^5 - 48
*c^4*d*x - 30*c^3*d^2*x^2 + 10*c^2*d^3*x^3 - 5*c*d^4*x^4 + 3*d^5*x^5) + 60*
b*(b*c - a*d)^4*(c + d*x)*Log[c + d*x])/(12*d^6*(c + d*x))
```

Maple [B] time = 0.009, size = 326, normalized size = 2.5

$$\frac{b^5x^4}{4\,d^2}+\frac{5\,ab^4x^3}{3\,d^2}-\frac{2\,b^5x^3c}{3\,d^3}+5\,\frac{a^2b^3x^2}{d^2}-5\,\frac{ab^4x^2c}{d^3}+\frac{3\,b^5x^2c^2}{2\,d^4}+10\,\frac{a^3b^2x}{d^2}-20\,\frac{a^2b^3cx}{d^3}+15\,\frac{ab^4c^2x}{d^4}-4\,\frac{b^5c^3x}{d^5}-\frac{a^5}{d\,(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^2,x)
```

```
[Out] 1/4*b^5/d^2*x^4+5/3*b^4/d^2*x^3*a-2/3*b^5/d^3*x^3*c+5*b^3/d^2*x^2*a^2-5*b^4
/d^3*x^2*a*c+3/2*b^5/d^4*x^2*c^2+10*b^2/d^2*a^3*x-20*b^3/d^3*a^2*c*x+15*b^4
/d^4*a*c^2*x-4*b^5/d^5*c^3*x-1/d/(d*x+c)*a^5+5/d^2/(d*x+c)*a^4*b*c-10/d^3/(
d*x+c)*a^3*b^2*c^2+10/d^4/(d*x+c)*a^2*b^3*c^3-5/d^5/(d*x+c)*a*b^4*c^4+1/d^6
/(d*x+c)*b^5*c^5+5*b/d^2*ln(d*x+c)*a^4-20*b^2/d^3*ln(d*x+c)*a^3*c+30*b^3/d^
4*ln(d*x+c)*a^2*c^2-20*b^4/d^5*ln(d*x+c)*a*c^3+5*b^5/d^6*ln(d*x+c)*c^4
```

Maxima [B] time = 0.97263, size = 356, normalized size = 2.74

$$\frac{b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5}{d^7x+cd^6}+\frac{3\,b^5d^3x^4-4\left(2\,b^5cd^2-5\,ab^4d^3\right)x^3+6\left(3\,b^5c^2d-10\,ab^4c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*
b*c*d^4 - a^5*d^5)/(d^7*x + c*d^6) + 1/12*(3*b^5*d^3*x^4 - 4*(2*b^5*c*d^2 -
 5*a*b^4*d^3)*x^3 + 6*(3*b^5*c^2*d - 10*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^2 -
 12*(4*b^5*c^3 - 15*a*b^4*c^2*d + 20*a^2*b^3*c*d^2 - 10*a^3*b^2*d^3)*x)/d^5
 + 5*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b
*d^4)*log(d*x + c)/d^6
```

Fricas [B] time = 1.75834, size = 767, normalized size = 5.9

$$\frac{3\,b^5d^5x^5+12\,b^5c^5-60\,ab^4c^4d+120\,a^2b^3c^3d^2-120\,a^3b^2c^2d^3+60\,a^4bcd^4-12\,a^5d^5-5\left(b^5cd^4-4\,ab^4d^5\right)x^4+10\left(b^5c^2d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*d^5*x^5 + 12*b^5*c^5 - 60*a*b^4*c^4*d + 120*a^2*b^3*c^3*d^2 - 1
20*a^3*b^2*c^2*d^3 + 60*a^4*b*c*d^4 - 12*a^5*d^5 - 5*(b^5*c*d^4 - 4*a*b^4*d
^5)*x^4 + 10*(b^5*c^2*d^3 - 4*a*b^4*c*d^4 + 6*a^2*b^3*d^5)*x^3 - 30*(b^5*c^
3*d^2 - 4*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 - 4*a^3*b^2*d^5)*x^2 - 12*(4*b^5*
c^4*d - 15*a*b^4*c^3*d^2 + 20*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4)*x + 60*(b
^5*c^5 - 4*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3*b^2*c^2*d^3 + a^4*b*c*d^
```

```
4 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^
4*b*d^5)*x)*log(d*x + c))/(d^7*x + c*d^6)
```

Sympy [A] time = 1.14194, size = 224, normalized size = 1.72

$$\frac{b^5x^4}{4d^2} + \frac{5b\,(ad-bc)^4\log{(c+dx)}}{d^6} - \frac{a^5d^5 - 5a^4bcd^4 + 10a^3b^2c^2d^3 - 10a^2b^3c^3d^2 + 5ab^4c^4d - b^5c^5}{cd^6 + d^7x} + \frac{x^3\left(5ab^4d - 2b^5c\right)}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**2,x)
```

```
[Out] b**5*x**4/(4*d**2) + 5*b*(a*d - b*c)**4*log(c + d*x)/d**6 - (a**5*d**5 - 5*
a**4*b*c*d**4 + 10*a**3*b**2*c**2*d**3 - 10*a**2*b**3*c**3*d**2 + 5*a*b**4*
c**4*d - b**5*c**5)/(c*d**6 + d**7*x) + x**3*(5*a*b**4*d - 2*b**5*c)/(3*d**
3) + x**2*(10*a**2*b**3*d**2 - 10*a*b**4*c*d + 3*b**5*c**2)/(2*d**4) + x*(1
0*a**3*b**2*d**3 - 20*a**2*b**3*c*d**2 + 15*a*b**4*c**2*d - 4*b**5*c**3)/d*
*5
```

Giac [B] time = 1.07567, size = 458, normalized size = 3.52

$$\frac{\left(3\,b^5 - \frac{20\left(b^5cd-ab^4d^2\right)}{(dx+c)d} + \frac{60\left(b^5c^2d^2-2\,ab^4cd^3+a^2b^3d^4\right)}{(dx+c)^2d^2} - \frac{120\left(b^5c^3d^3-3\,ab^4c^2d^4+3\,a^2b^3cd^5-a^3b^2d^6\right)}{(dx+c)^3d^3}\right)(dx+c)^4}{12\,d^6} - \frac{5\left(b^5c^4 - 4\,ab^4c^3d + 6\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^5 - 20*(b^5*c*d - a*b^4*d^2)/((d*x + c)*d) + 60*(b^5*c^2*d^2 - 2*
a*b^4*c*d^3 + a^2*b^3*d^4)/((d*x + c)^2*d^2) - 120*(b^5*c^3*d^3 - 3*a*b^4*c
^2*d^4 + 3*a^2*b^3*c*d^5 - a^3*b^2*d^6)/((d*x + c)^3*d^3))*(d*x + c)^4/d^6
- 5*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*
d^4)*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 + (b^5*c^5*d^4/(d*x + c) -
5*a*b^4*c^4*d^5/(d*x + c) + 10*a^2*b^3*c^3*d^6/(d*x + c) - 10*a^3*b^2*c^2*d
^7/(d*x + c) + 5*a^4*b*c*d^8/(d*x + c) - a^5*d^9/(d*x + c))/d^10
```

3.1344 $\int \frac{(a+bx)^4}{(c+dx)^2} \, dx$

Optimal. Leaf size=104

$$-\frac{2b^3(c+dx)^2(bc-ad)}{d^5}+\frac{6b^2x(bc-ad)^2}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}+\frac{b^4(c+dx)^3}{3d^5}$$

```
[Out] (6*b^2*(b*c - a*d)^2*x)/d^4 - (b*c - a*d)^4/(d^5*(c + d*x)) - (2*b^3*(b*c -
 a*d)*(c + d*x)^2)/d^5 + (b^4*(c + d*x)^3)/(3*d^5) - (4*b*(b*c - a*d)^3*Log
[c + d*x])/d^5
```

Rubi [A] time = 0.100315, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b^3(c+dx)^2(bc-ad)}{d^5}+\frac{6b^2x(bc-ad)^2}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}+\frac{b^4(c+dx)^3}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^2,x]
```

```
[Out] (6*b^2*(b*c - a*d)^2*x)/d^4 - (b*c - a*d)^4/(d^5*(c + d*x)) - (2*b^3*(b*c -
 a*d)*(c + d*x)^2)/d^5 + (b^4*(c + d*x)^3)/(3*d^5) - (4*b*(b*c - a*d)^3*Log
[c + d*x])/d^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^2}\,dx &= \int \left(\frac{6b^2(bc-ad)^2}{d^4}+\frac{(-bc+ad)^4}{d^4(c+dx)^2}-\frac{4b(bc-ad)^3}{d^4(c+dx)}-\frac{4b^3(bc-ad)(c+dx)}{d^4}+\frac{b^4(c+dx)^2}{d^4}\right)dx\\ &= \frac{6b^2(bc-ad)^2x}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{2b^3(bc-ad)(c+dx)^2}{d^5}+\frac{b^4(c+dx)^3}{3d^5}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}\end{aligned}$$

Mathematica [A] time = 0.0564887, size = 165, normalized size = 1.59

$$\frac{18a^2b^2d^2\left(-c^2+cdx+d^2x^2\right)+12a^3bcd^3-3a^4d^4+6ab^3d\left(-4c^2dx+2c^3-3cd^2x^2+d^3x^3\right)-12b(c+dx)(bc-ad)^3\log(c+\ldots}{3d^5(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^2,x]
```

```
[Out] (12*a^3*b*c*d^3 - 3*a^4*d^4 + 18*a^2*b^2*d^2*(-c^2 + c*d*x + d^2*x^2) + 6*a
*b^3*d*(2*c^3 - 4*c^2*d*x - 3*c*d^2*x^2 + d^3*x^3) + b^4*(-3*c^4 + 9*c^3*d*
```

```
x + 6*c^2*d^2*x^2 - 2*c*d^3*x^3 + d^4*x^4) - 12*b*(b*c - a*d)^3*(c + d*x)*L
og[c + d*x])/(3*d^5*(c + d*x))
```

Maple [B] time = 0.009, size = 230, normalized size = 2.2

$$\frac{b^4x^3}{3\,d^2}+2\,\frac{b^3x^2a}{d^2}-\frac{b^4x^2c}{d^3}+6\,\frac{b^2a^2x}{d^2}-8\,\frac{ab^3cx}{d^3}+3\,\frac{b^4c^2x}{d^4}-\frac{a^4}{d\,(dx+c)}+4\,\frac{a^3bc}{d^2\,(dx+c)}-6\,\frac{b^2a^2c^2}{d^3\,(dx+c)}+4\,\frac{ab^3c^3}{d^4\,(dx+c)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^2,x)
```

```
[Out] 1/3*b^4/d^2*x^3+2*b^3/d^2*x^2*a-b^4/d^3*x^2*c+6*b^2/d^2*a^2*x-8*b^3/d^3*a*c
*x+3*b^4/d^4*c^2*x-1/d/(d*x+c)*a^4+4/d^2/(d*x+c)*a^3*b*c-6/d^3/(d*x+c)*a^2*
b^2*c^2+4/d^4/(d*x+c)*a*b^3*c^3-1/d^5/(d*x+c)*b^4*c^4+4*b/d^2*ln(d*x+c)*a^3
-12*b^2/d^3*ln(d*x+c)*a^2*c+12*b^3/d^4*ln(d*x+c)*a*c^2-4*b^4/d^5*ln(d*x+c)*
c^3
```

Maxima [A] time = 0.955658, size = 247, normalized size = 2.38

$$-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{d^6x+cd^5}+\frac{b^4d^2x^3-3\left(b^4cd-2\,ab^3d^2\right)x^2+3\left(3\,b^4c^2-8\,ab^3cd+6\,a^2b^2d^2\right)x}{3\,d^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(d
^6*x + c*d^5) + 1/3*(b^4*d^2*x^3 - 3*(b^4*c*d - 2*a*b^3*d^2)*x^2 + 3*(3*b^4
*c^2 - 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x)/d^4 - 4*(b^4*c^3 - 3*a*b^3*c^2*d + 3
*a^2*b^2*c*d^2 - a^3*b*d^3)*log(d*x + c)/d^5
```

Fricas [B] time = 1.72844, size = 540, normalized size = 5.19

$$\frac{b^4d^4x^4-3\,b^4c^4+12\,ab^3c^3d-18\,a^2b^2c^2d^2+12\,a^3bcd^3-3\,a^4d^4-2\left(b^4cd^3-3\,ab^3d^4\right)x^3+6\left(b^4c^2d^2-3\,ab^3cd^3+3\,a^2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*d^4*x^4 - 3*b^4*c^4 + 12*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 12*a^3
*b*c*d^3 - 3*a^4*d^4 - 2*(b^4*c*d^3 - 3*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 3
*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2 + 3*(3*b^4*c^3*d - 8*a*b^3*c^2*d^2 + 6*a^
2*b^2*c*d^3)*x - 12*(b^4*c^4 - 3*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 - a^3*b*c*
d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)*log(d*
x + c))/(d^6*x + c*d^5)
```

Sympy [A] time = 0.902879, size = 151, normalized size = 1.45

$$\frac{b^4x^3}{3d^2}+\frac{4b\,(ad-bc)^3\log(c+dx)}{d^5}-\frac{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}{cd^5+d^6x}+\frac{x^2\left(2ab^3d-b^4c\right)}{d^3}+\frac{x\left(6a^2b^2d^2-8ab\right.}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**2,x)
```

```
[Out] b**4*x**3/(3*d**2) + 4*b*(a*d - b*c)**3*log(c + d*x)/d**5 - (a**4*d**4 - 4*
a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(c*d**
5 + d**6*x) + x**2*(2*a*b**3*d - b**4*c)/d**3 + x*(6*a**2*b**2*d**2 - 8*a*b
**3*c*d + 3*b**4*c**2)/d**4
```

Giac [B] time = 1.0705, size = 331, normalized size = 3.18

$$\frac{\left(b^4-\frac{6\left(b^4cd-ab^3d^2\right)}{(dx+c)d}+\frac{18\left(b^4c^2d^2-2\,ab^3cd^3+a^2b^2d^4\right)}{(dx+c)^2d^2}\right)(dx+c)^3}{3\,d^5}+\frac{4\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^5}-\frac{\frac{b^4c^4d^3}{dx+c}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^4 - 6*(b^4*c*d - a*b^3*d^2)/((d*x + c)*d) + 18*(b^4*c^2*d^2 - 2*a*b^
3*c*d^3 + a^2*b^2*d^4)/((d*x + c)^2*d^2))*(d*x + c)^3/d^5 + 4*(b^4*c^3 - 3*
a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*log(abs(d*x + c)/((d*x + c)^2*ab
s(d)))/d^5 - (b^4*c^4*d^3/(d*x + c) - 4*a*b^3*c^3*d^4/(d*x + c) + 6*a^2*b^2
*c^2*d^5/(d*x + c) - 4*a^3*b*c*d^6/(d*x + c) + a^4*d^7/(d*x + c))/d^8
```

3.1345 $\int \frac{(a+bx)^3}{(c+dx)^2}\,dx$

Optimal. Leaf size=75

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}+\frac{b^3x^2}{2d^2}$$

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b*c - a*d)^3/(d^4*(c
+ d*x)) + (3*b*(b*c - a*d)^2*Log[c + d*x])/d^4
```

Rubi [A] time = 0.0621689, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}+\frac{b^3x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b*c - a*d)^3/(d^4*(c
+ d*x)) + (3*b*(b*c - a*d)^2*Log[c + d*x])/d^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^2}\,dx &= \int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{b^3x}{d^2}+\frac{(-bc+ad)^3}{d^3(c+dx)^2}+\frac{3b(bc-ad)^2}{d^3(c+dx)}\right)dx\\ &= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^2}{2d^2}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}\end{aligned}$$

Mathematica [A] time = 0.0343951, size = 114, normalized size = 1.52

$$\frac{3a^2bcd^2-a^3d^3-3ab^2c^2d+b^3c^3}{d^4(c+dx)}+\frac{3\left(a^2bd^2-2ab^2cd+b^3c^2\right)\log(c+dx)}{d^4}-\frac{b^2x(2bc-3ad)}{d^3}+\frac{b^3x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^4*(c + d*x)) + (3*(b^3*c^2 - 2*a*b^2*c*d +
 a^2*b*d^2)*Log[c + d*x])/d^4
```

Maple [B] time = 0.006, size = 149, normalized size = 2.

$$\frac{b^3x^2}{2\,d^2}+3\,\frac{ab^2x}{d^2}-2\,\frac{b^3xc}{d^3}-\frac{a^3}{d\,(dx+c)}+3\,\frac{a^2bc}{d^2\,(dx+c)}-3\,\frac{ab^2c^2}{d^3\,(dx+c)}+\frac{b^3c^3}{d^4\,(dx+c)}+3\,\frac{b\ln{(dx+c)}\,a^2}{d^2}-6\,\frac{b^2\ln{(dx+c)}\,ac}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/2*b^3*x^2/d^2+3*b^2/d^2*a*x-2*b^3/d^3*x*c-1/d/(d*x+c)*a^3+3/d^2/(d*x+c)*a
^2*b*c-3/d^3/(d*x+c)*a*b^2*c^2+1/d^4/(d*x+c)*b^3*c^3+3*b/d^2*ln(d*x+c)*a^2-
6*b^2/d^3*ln(d*x+c)*a*c+3*b^3/d^4*ln(d*x+c)*c^2
```

Maxima [A] time = 0.9742, size = 158, normalized size = 2.11

$$\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{d^5x+cd^4}+\frac{b^3dx^2-2\left(2\,b^3c-3\,ab^2d\right)x}{2\,d^3}+\frac{3\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)\log{(dx+c)}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^5*x + c*d^4) + 1/2*(
b^3*d*x^2 - 2*(2*b^3*c - 3*a*b^2*d)*x)/d^3 + 3*(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*log(d*x + c)/d^4
```

Fricas [B] time = 1.83058, size = 354, normalized size = 4.72

$$\frac{b^3d^3x^3+2\,b^3c^3-6\,ab^2c^2d+6\,a^2bcd^2-2\,a^3d^3-3\left(b^3cd^2-2\,ab^2d^3\right)x^2-2\left(2\,b^3c^2d-3\,ab^2cd^2\right)x+6\left(b^3c^3-2\,ab^2c^2d+a^2\cdots\right)}{2\left(d^5x+cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 -
3*(b^3*c*d^2 - 2*a*b^2*d^3)*x^2 - 2*(2*b^3*c^2*d - 3*a*b^2*c*d^2)*x + 6*(b^
3*c^3 - 2*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x)*log(d*x + c))/(d^5*x + c*d^4)
```

Sympy [A] time = 0.681938, size = 100, normalized size = 1.33

$$\frac{b^3x^2}{2d^2}+\frac{3b\,(ad-bc)^2\log{(c+dx)}}{d^4}-\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{cd^4+d^5x}+\frac{x\left(3ab^2d-2b^3c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] b**3*x**2/(2*d**2) + 3*b*(a*d - b*c)**2*log(c + d*x)/d**4 - (a**3*d**3 - 3*
a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(c*d**4 + d**5*x) + x*(3*a*b**
2*d - 2*b**3*c)/d**3
```

Giac [B] time = 1.07367, size = 224, normalized size = 2.99

$$\frac{\left(b^3-\frac{6\left(b^3cd-ab^2d^2\right)}{(dx+c)d}\right)(dx+c)^2}{2\,d^4}-\frac{3\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^4}+\frac{\frac{b^3c^3d^2}{dx+c}-\frac{3\,ab^2c^2d^3}{dx+c}+\frac{3\,a^2bcd^4}{dx+c}-\frac{a^3d^5}{dx+c}}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 6*(b^3*c*d - a*b^2*d^2)/((d*x + c)*d))*(d*x + c)^2/d^4 - 3*(b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^4 +
 (b^3*c^3*d^2/(d*x + c) - 3*a*b^2*c^2*d^3/(d*x + c) + 3*a^2*b*c*d^4/(d*x +
c) - a^3*d^5/(d*x + c))/d^6
```

3.1346 $\int \frac{(a+bx)^2}{(c+dx)^2} dx$

Optimal. Leaf size=51

$$-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}+\frac{b^2x}{d^2}$$

```
[Out] (b^2*x)/d^2 - (b*c - a*d)^2/(d^3*(c + d*x)) - (2*b*(b*c - a*d)*Log[c + d*x]
)/d^3
```

Rubi [A] time = 0.0385264, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^2,x]
```

```
[Out] (b^2*x)/d^2 - (b*c - a*d)^2/(d^3*(c + d*x)) - (2*b*(b*c - a*d)*Log[c + d*x]
)/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^2}\,dx &= \int \left(\frac{b^2}{d^2}+\frac{(-bc+ad)^2}{d^2(c+dx)^2}-\frac{2b(bc-ad)}{d^2(c+dx)}\right)dx\\ &= \frac{b^2x}{d^2}-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A] time = 0.0352948, size = 47, normalized size = 0.92

$$\frac{-\frac{(bc-ad)^2}{c+dx}+2b(ad-bc)\log(c+dx)+b^2dx}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^2,x]
```

```
[Out] (b^2*d*x - (b*c - a*d)^2/(c + d*x) + 2*b*(-(b*c) + a*d)*Log[c + d*x])/d^3
```

Maple [A] time = 0.005, size = 86, normalized size = 1.7

$$\frac{b^2x}{d^2} - \frac{a^2}{d\,(dx+c)} + 2\,\frac{abc}{d^2\,(dx+c)} - \frac{b^2c^2}{d^3\,(dx+c)} + 2\,\frac{b\ln(dx+c)\,a}{d^2} - 2\,\frac{b^2\ln(dx+c)\,c}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] b^2*x/d^2-1/d/(d*x+c)*a^2+2/d^2/(d*x+c)*a*b*c-1/d^3/(d*x+c)*b^2*c^2+2*b/d^2
*ln(d*x+c)*a-2*b^2/d^3*ln(d*x+c)*c
```

Maxima [A] time = 0.964595, size = 90, normalized size = 1.76

$$\frac{b^2x}{d^2} - \frac{b^2c^2 - 2\,abcd + a^2d^2}{d^4x + cd^3} - \frac{2\left(b^2c - abd\right)\log(dx+c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] b^2*x/d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(d^4*x + c*d^3) - 2*(b^2*c - a*
b*d)*log(d*x + c)/d^3
```

Fricas [A] time = 1.71685, size = 184, normalized size = 3.61

$$\frac{b^2d^2x^2 + b^2cdx - b^2c^2 + 2\,abcd - a^2d^2 - 2\left(b^2c^2 - abcd + \left(b^2cd - abd^2\right)x\right)\log(dx+c)}{d^4x + cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + b^2*c*d*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 - 2*(b^2*c^2 - a*b
*c*d + (b^2*c*d - a*b*d^2)*x)*log(d*x + c))/(d^4*x + c*d^3)
```

Sympy [A] time = 0.503073, size = 60, normalized size = 1.18

$$\frac{b^2x}{d^2} + \frac{2b\,(ad-bc)\log(c+dx)}{d^3} - \frac{a^2d^2 - 2abcd + b^2c^2}{cd^3 + d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] b**2*x/d**2 + 2*b*(a*d - b*c)*log(c + d*x)/d**3 - (a**2*d**2 - 2*a*b*c*d +
b**2*c**2)/(c*d**3 + d**4*x)
```

Giac [A] time = 1.0702, size = 132, normalized size = 2.59

$$\frac{(dx+c)b^2}{d^3}+\frac{2\left(b^2c-abd\right)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^3}-\frac{\frac{b^2c^2d}{dx+c}-\frac{2\,abcd^2}{dx+c}+\frac{a^2d^3}{dx+c}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (d*x + c)*b^2/d^3 + 2*(b^2*c - a*b*d)*log(abs(d*x + c)/((d*x + c)^2*abs(d))
)/d^3 - (b^2*c^2*d/(d*x + c) - 2*a*b*c*d^2/(d*x + c) + a^2*d^3/(d*x + c))/d
^4
```

3.1347 $\int \frac{a+bx}{(c+dx)^2} dx$

Optimal. Leaf size=31

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

```
[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

Rubi [A] time = 0.0208401, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^2,x]
```

```
[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^2}\,dx &= \int \left(\frac{-bc+ad}{d(c+dx)^2}+\frac{b}{d(c+dx)}\right)dx\\ &= \frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}\end{aligned}$$

Mathematica [A] time = 0.0101126, size = 31, normalized size = 1.

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^2,x]
```

```
[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

Maple [A] time = 0.004, size = 39, normalized size = 1.3

$$-\frac{a}{d\,(dx+c)}+\frac{bc}{d^2\,(dx+c)}+\frac{b\ln(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^2,x)
```

```
[Out] -1/d/(d*x+c)*a+1/d^2/(d*x+c)*b*c+b*ln(d*x+c)/d^2
```

Maxima [A] time = 0.943292, size = 46, normalized size = 1.48

$$\frac{bc-ad}{d^3x+cd^2}+\frac{b\log\left(dx+c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b*c - a*d)/(d^3*x + c*d^2) + b*log(d*x + c)/d^2
```

Fricas [A] time = 1.62261, size = 78, normalized size = 2.52

$$\frac{bc-ad+(bdx+bc)\log\left(dx+c\right)}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b*c - a*d + (b*d*x + b*c)*log(d*x + c))/(d^3*x + c*d^2)
```

Sympy [A] time = 0.336933, size = 27, normalized size = 0.87

$$\frac{b\log\left(c+dx\right)}{d^2}-\frac{ad-bc}{cd^2+d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**2,x)
```

```
[Out] b*log(c + d*x)/d**2 - (a*d - b*c)/(c*d**2 + d**3*x)
```

Giac [A] time = 1.0497, size = 77, normalized size = 2.48

$$-\frac{b\left(\frac{\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d}-\frac{c}{(dx+c)d}\right)}{d}-\frac{a}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -b*(log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*x + c)*d))/d - a/((d*x
 + c)*d)
```

3.1348 $\int \frac{1}{(c+dx)^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{d(c+dx)}$$

```
[Out] -(1/(d*(c + d*x)))
```

Rubi [A] time = 0.0016551, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-2),x]
```

```
[Out] -(1/(d*(c + d*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2} dx = -\frac{1}{d(c+dx)}$$

Mathematica [A] time = 0.0023961, size = 12, normalized size = 1.

$$-\frac{1}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-2),x]
```

```
[Out] -(1/(d*(c + d*x)))
```

Maple [A] time = 0.001, size = 13, normalized size = 1.1

$$-\frac{1}{d(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2,x)
```

```
[Out] -1/d/(d*x+c)
```

Maxima [A] time = 0.961277, size = 16, normalized size = 1.33

$$-\frac{1}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/((d*x + c)*d)
```

Fricas [A] time = 1.56419, size = 24, normalized size = 2.

$$-\frac{1}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/(d^2*x + c*d)
```

Sympy [A] time = 0.291893, size = 10, normalized size = 0.83

$$-\frac{1}{cd+d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2,x)
```

```
[Out] -1/(c*d + d**2*x)
```

Giac [A] time = 1.08254, size = 16, normalized size = 1.33

$$-\frac{1}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/((d*x + c)*d)
```

3.1349 $\int \frac{1}{(a+bx)(c+dx)^2} dx$

Optimal. Leaf size=56

$$\frac{1}{(c+dx)(bc-ad)} + \frac{b\log(a+bx)}{(bc-ad)^2} - \frac{b\log(c+dx)}{(bc-ad)^2}$$

```
[Out] 1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Log[c + d*x
])/(b*c - a*d)^2
```

Rubi [A] time = 0.0322105, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{1}{(c+dx)(bc-ad)} + \frac{b\log(a+bx)}{(bc-ad)^2} - \frac{b\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^2),x]
```

```
[Out] 1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Log[c + d*x
])/(b*c - a*d)^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^2} dx &= \int \left(\frac{b^2}{(bc-ad)^2(a+bx)} - \frac{d}{(bc-ad)(c+dx)^2} - \frac{bd}{(bc-ad)^2(c+dx)}\right) dx\\ &= \frac{1}{(bc-ad)(c+dx)} + \frac{b\log(a+bx)}{(bc-ad)^2} - \frac{b\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.0245609, size = 53, normalized size = 0.95

$$\frac{b(c+dx)\log(a+bx) - ad - b(c+dx)\log(c+dx) + bc}{(c+dx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^2),x]
```

```
[Out] (b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x])/((b*c - a
*d)^2*(c + d*x))
```

Maple [A] time = 0.01, size = 58, normalized size = 1.

$$-\frac{1}{(ad-bc)(dx+c)}-\frac{b\ln(dx+c)}{(ad-bc)^2}+\frac{b\ln(bx+a)}{(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^2,x)
```

```
[Out] -1/(a*d-b*c)/(d*x+c)-b/(a*d-b*c)^2*ln(d*x+c)+b/(a*d-b*c)^2*ln(b*x+a)
```

Maxima [A] time = 0.963627, size = 122, normalized size = 2.18

$$\frac{b\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2}-\frac{b\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2}+\frac{1}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - b*log(d*x + c)/(b^2*c^2 -
2*a*b*c*d + a^2*d^2) + 1/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Fricas [A] time = 1.79634, size = 198, normalized size = 3.54

$$\frac{bc-ad+(bdx+bc)\log(bx+a)-(bdx+bc)\log(dx+c)}{b^2c^3-2\,abc^2d+a^2cd^2+\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b*c - a*d + (b*d*x + b*c)*log(b*x + a) - (b*d*x + b*c)*log(d*x + c))/(b^2*
c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)
```

Sympy [B] time = 0.77394, size = 233, normalized size = 4.16

$$-\frac{b\log\left(x+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{(ad-bc)^2}+\frac{b\log\left(x+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{(ad-bc)^2}-\frac{1}{acd-bc^2+x\left(ad\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**2,x)
```

```
[Out] -b*log(x + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c)**2
 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 + b**2
*c)/(2*b**2*d))/(a*d - b*c)**2 + b*log(x + (a**3*b*d**3/(a*d - b*c)**2 - 3*
a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d -
b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(a*d - b*c)**2 - 1/(a*c*d -
```

```
b*c**2 + x*(a*d**2 - b*c*d))
```

Giac [A] time = 1.07192, size = 104, normalized size = 1.86

$$\frac{bd\log\left(\left|b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right|\right)}{b^2c^2d-2\,abcd^2+a^2d^3}+\frac{d}{\left(bcd-ad^2\right)(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] b*d*log(abs(b - b*c/(d*x + c) + a*d/(d*x + c)))/(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3) + d/((b*c*d - a*d^2)*(d*x + c))
```

3.1350 $\int \frac{1}{(a+bx)^2(c+dx)^2} dx$

Optimal. Leaf size=81

$$-\frac{b}{(a+bx)(bc-ad)^2}-\frac{d}{(c+dx)(bc-ad)^2}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}$$

```
[Out] -(b/((b*c - a*d)^2*(a + b*x))) - d/((b*c - a*d)^2*(c + d*x)) - (2*b*d*Log[a
 + b*x])/(b*c - a*d)^3 + (2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Rubi [A] time = 0.0496297, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{b}{(a+bx)(bc-ad)^2}-\frac{d}{(c+dx)(bc-ad)^2}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^2),x]
```

```
[Out] -(b/((b*c - a*d)^2*(a + b*x))) - d/((b*c - a*d)^2*(c + d*x)) - (2*b*d*Log[a
 + b*x])/(b*c - a*d)^3 + (2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)^2}\,dx &= \int \left(\frac{b^2}{(bc-ad)^2(a+bx)^2}-\frac{2b^2d}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^2}+\frac{2bd^2}{(bc-ad)^3(c+dx)}\right)dx\\ &= -\frac{b}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)^2(c+dx)}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}\end{aligned}$$

Mathematica [A] time = 0.0696383, size = 66, normalized size = 0.81

$$\frac{\frac{b(ad-bc)}{a+bx}+\frac{d(ad-bc)}{c+dx}-2bd\log(a+bx)+2bd\log(c+dx)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^2),x]
```

```
[Out] ((b*(-(b*c) + a*d))/(a + b*x) + (d*(-(b*c) + a*d))/(c + d*x) - 2*b*d*Log[a
+ b*x] + 2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Maple [A] time = 0.009, size = 82, normalized size = 1.

$$-\frac{d}{\left(ad-bc\right)^{2}\left(dx+c\right)}-2\frac{bd\ln\left(dx+c\right)}{\left(ad-bc\right)^{3}}-\frac{b}{\left(ad-bc\right)^{2}\left(bx+a\right)}+2\frac{bd\ln\left(bx+a\right)}{\left(ad-bc\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] -d/(a*d-b*c)^2/(d*x+c)-2*d/(a*d-b*c)^3*b*ln(d*x+c)-b/(a*d-b*c)^2/(b*x+a)+2*
d/(a*d-b*c)^3*b*ln(b*x+a)
```

Maxima [B] time = 0.985966, size = 281, normalized size = 3.47

$$-\frac{2\,bd\log\left(bx+a\right)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{2\,bd\log\left(dx+c\right)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}-\frac{2\,bd}{ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -2*b*d*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 2
*b*d*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - (2*
b*d*x + b*c + a*d)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*
a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d
^3)*x)
```

Fricas [B] time = 1.85811, size = 486, normalized size = 6.

$$-\frac{b^2c^2-a^2d^2+2\left(b^2cd-abd^2\right)x+2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)\log\left(bx+a\right)-2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right.\right.}{ab^3c^4-3\,a^2b^2c^3d+3\,a^3bc^2d^2-a^4cd^3+\left(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4\right)x^2+\left(b^4c^4-2\,ab^3c^3d+2\,a^3bcd\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*c^2 - a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2*x^2 + a*b*c*d +
(b^2*c*d + a*b*d^2)*x)*log(b*x + a) - 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d +
 a*b*d^2)*x)*log(d*x + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 -
 a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^
2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x)
```

Sympy [B] time = 1.23832, size = 405, normalized size = 5.

$$-\frac{2bd\log\left(x+\frac{-\frac{2a^4bd^5}{(ad-bc)^3}+\frac{8a^3b^2cd^4}{(ad-bc)^3}-\frac{12a^2b^3c^2d^3}{(ad-bc)^3}+\frac{8ab^4c^3d^2}{(ad-bc)^3}+2abd^2-\frac{2b^5c^4d}{(ad-bc)^3}+2b^2cd}{4b^2d^2}\right)}{(ad-bc)^3}+\frac{2bd\log\left(x+\frac{\frac{2a^4bd^5}{(ad-bc)^3}-\frac{8a^3b^2cd^4}{(ad-bc)^3}+\frac{12a^2b^3c^2d^3}{(ad-bc)^3}-\frac{8ab^4c^3d^2}{(ad-bc)^3}+2abd}{4b^2d^2}\right.}{(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] -2*b*d*log(x + (-2*a**4*b*d**5/(a*d - b*c)**3 + 8*a**3*b**2*c*d**4/(a*d - b
*c)**3 - 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 8*a*b**4*c**3*d**2/(a*d -
b*c)**3 + 2*a*b*d**2 - 2*b**5*c**4*d/(a*d - b*c)**3 + 2*b**2*c*d)/(4*b**2*d
**2))/(a*d - b*c)**3 + 2*b*d*log(x + (2*a**4*b*d**5/(a*d - b*c)**3 - 8*a**3
*b**2*c*d**4/(a*d - b*c)**3 + 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 - 8*a*b
**4*c**3*d**2/(a*d - b*c)**3 + 2*a*b*d**2 + 2*b**5*c**4*d/(a*d - b*c)**3 +
2*b**2*c*d)/(4*b**2*d**2))/(a*d - b*c)**3 - (a*d + b*c + 2*b*d*x)/(a**3*c*d
**2 - 2*a**2*b*c**2*d + a*b**2*c**3 + x**2*(a**2*b*d**3 - 2*a*b**2*c*d**2 +
 b**3*c**2*d) + x*(a**3*d**3 - a**2*b*c*d**2 - a*b**2*c**2*d + b**3*c**3))
```

Giac [A] time = 1.09522, size = 207, normalized size = 2.56

$$\frac{2\,b^2 d\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{b^3}{\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)(bx+a)}+\frac{bd^2}{(bc-ad)^3\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 2*b^2*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^4*c^3 - 3*a*b^3*c^2*
d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - b^3/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^
2)*(b*x + a)) + b*d^2/((b*c - a*d)^3*(b*c/(b*x + a) - a*d/(b*x + a) + d))
```

3.1351 $\int \frac{1}{(a+bx)^3(c+dx)^2} dx$

Optimal. Leaf size=109

$$\frac{d^2}{(c+dx)(bc-ad)^3}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bd^2\log(c+dx)}{(bc-ad)^4}+\frac{2bd}{(a+bx)(bc-ad)^3}-\frac{b}{2(a+bx)^2(bc-ad)^2}$$

```
[Out] -b/(2*(b*c - a*d)^2*(a + b*x)^2) + (2*b*d)/((b*c - a*d)^3*(a + b*x)) + d^2/
((b*c - a*d)^3*(c + d*x)) + (3*b*d^2*Log[a + b*x])/(b*c - a*d)^4 - (3*b*d^2
*Log[c + d*x])/(b*c - a*d)^4
```

Rubi [A] time = 0.0760537, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{d^2}{(c+dx)(bc-ad)^3}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bd^2\log(c+dx)}{(bc-ad)^4}+\frac{2bd}{(a+bx)(bc-ad)^3}-\frac{b}{2(a+bx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^2),x]
```

```
[Out] -b/(2*(b*c - a*d)^2*(a + b*x)^2) + (2*b*d)/((b*c - a*d)^3*(a + b*x)) + d^2/
((b*c - a*d)^3*(c + d*x)) + (3*b*d^2*Log[a + b*x])/(b*c - a*d)^4 - (3*b*d^2
*Log[c + d*x])/(b*c - a*d)^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^2}\,dx = \int \left(\frac{b^2}{(bc-ad)^2(a+bx)^3}-\frac{2b^2d}{(bc-ad)^3(a+bx)^2}+\frac{3b^2d^2}{(bc-ad)^4(a+bx)}-\frac{d^3}{(bc-ad)^3(c+dx)^2}-\right.$$
$$= -\frac{b}{2(bc-ad)^2(a+bx)^2}+\frac{2bd}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^3(c+dx)}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bc}{}$$

Mathematica [A] time = 0.0745736, size = 98, normalized size = 0.9

$$\frac{\frac{2d^2(bc-ad)}{c+dx}+\frac{4bd(bc-ad)}{a+bx}-\frac{b(bc-ad)^2}{(a+bx)^2}+6bd^2\log(a+bx)-6bd^2\log(c+dx)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^2),x]
```

```
[Out] (-((b*(b*c - a*d)^2)/(a + b*x)^2) + (4*b*d*(b*c - a*d))/(a + b*x) + (2*d^2*
(b*c - a*d))/(c + d*x) + 6*b*d^2*Log[a + b*x] - 6*b*d^2*Log[c + d*x])/(2*(b
```

```
*c - a*d)^4)
```

Maple [A] time = 0.01, size = 109, normalized size = 1.

$$-\frac{d^2}{(ad-bc)^3(dx+c)}-3\frac{d^2b\ln(dx+c)}{(ad-bc)^4}-\frac{b}{2(ad-bc)^2(bx+a)^2}+3\frac{d^2b\ln(bx+a)}{(ad-bc)^4}-2\frac{bd}{(ad-bc)^3(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] -d^2/(a*d-b*c)^3/(d*x+c)-3*d^2/(a*d-b*c)^4*b*ln(d*x+c)-1/2*b/(a*d-b*c)^2/(b
*x+a)^2+3*d^2/(a*d-b*c)^4*b*ln(b*x+a)-2*b/(a*d-b*c)^3*d/(b*x+a)
```

Maxima [B] time = 1.01499, size = 521, normalized size = 4.78

$$\frac{3\,bd^2\log(bx+a)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}-\frac{3\,bd^2\log(dx+c)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}+\frac{}{2\left(a^2b^3c^4-3\,a^3b^2c^3d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 3*b*d^2*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4) - 3*b*d^2*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b
^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 1/2*(6*b^2*d^2*x^2 - b^2*c^2 + 5*a*
b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^2)*x)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3
*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3
*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*
a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b
^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

Fricas [B] time = 1.88289, size = 991, normalized size = 9.09

$$-\frac{b^3c^3-6\,ab^2c^2d+3\,a^2bcd^2+2\,a^3d^3-6\left(b^3cd^2-ab^2d^3\right)x^2-3\left(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3\right)x-6\left(b^3d^3x^3+a^2bcd^2+(b^3\right.}{2\left(a^2b^4c^5-4\,a^3b^3c^4d+6\,a^4b^2c^3d^2-4\,a^5bc^2d^3+a^6cd^4+\left(b^6c^4d-4\,ab^5c^3d^2+6\,a^2b^4c^2d^3-4\,a^3b^3cd^4+a^4b^2d^5\right)x^3+(b^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*c^3 - 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3 - 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 - 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3*d^3
*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b
*d^3)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2
*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(d*x + c))/(a^2*b^4*c^5 - 4*a
^3*b^3*c^4*d + 6*a^4*b^2*c^3*d^2 - 4*a^5*b*c^2*d^3 + a^6*c*d^4 + (b^6*c^4*d
 - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^4*b^2*d^5)*x^3
 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2 + 8*a^3*b^3*c^2*d^3 - 7*a^4
*b^2*c*d^4 + 2*a^5*b*d^5)*x^2 + (2*a*b^5*c^5 - 7*a^2*b^4*c^4*d + 8*a^3*b^3*
```

```
c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*c*d^4 + a^6*d^5)*x)
```

Sympy [B] time = 1.94111, size = 632, normalized size = 5.8

$$-\frac{3bd^2\log\left(x+\frac{-\frac{3a^5bd^7}{(ad-bc)^4}+\frac{15a^4b^2cd^6}{(ad-bc)^4}-\frac{30a^3b^3c^2d^5}{(ad-bc)^4}+\frac{30a^2b^4c^3d^4}{(ad-bc)^4}-\frac{15ab^5c^4d^3}{(ad-bc)^4}+3abd^3+\frac{3b^6c^5d^2}{(ad-bc)^4}+3b^2cd^2}{6b^2d^3}\right)}{(ad-bc)^4}+\frac{3bd^2\log\left(x+\frac{\frac{3a^5bd^7}{(ad-bc)^4}-\frac{15a^4b^2cd^6}{(ad-bc)^4}+\frac{30a^3b^3c^2 \cdots}{(ad-bc) \cdots}\right.}{(ad \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] -3*b*d**2*log(x + (-3*a**5*b*d**7/(a*d - b*c)**4 + 15*a**4*b**2*c*d**6/(a*d
 - b*c)**4 - 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 + 30*a**2*b**4*c**3*d**4
/(a*d - b*c)**4 - 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 + 3*b**6*
c**5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**4 + 3
*b*d**2*log(x + (3*a**5*b*d**7/(a*d - b*c)**4 - 15*a**4*b**2*c*d**6/(a*d -
b*c)**4 + 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 - 30*a**2*b**4*c**3*d**4/(a
*d - b*c)**4 + 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 - 3*b**6*c**
5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**4 - (2*a
**2*d**2 + 5*a*b*c*d - b**2*c**2 + 6*b**2*d**2*x**2 + x*(9*a*b*d**2 + 3*b**
2*c*d))/(2*a**5*c*d**3 - 6*a**4*b*c**2*d**2 + 6*a**3*b**2*c**3*d - 2*a**2*b
**3*c**4 + x**3*(2*a**3*b**2*d**4 - 6*a**2*b**3*c*d**3 + 6*a*b**4*c**2*d**2
 - 2*b**5*c**3*d) + x**2*(4*a**4*b*d**4 - 10*a**3*b**2*c*d**3 + 6*a**2*b**3
*c**2*d**2 + 2*a*b**4*c**3*d - 2*b**5*c**4) + x*(2*a**5*d**4 - 2*a**4*b*c*d
**3 - 6*a**3*b**2*c**2*d**2 + 10*a**2*b**3*c**3*d - 4*a*b**4*c**4))
```

Giac [B] time = 1.09548, size = 292, normalized size = 2.68

$$\frac{3\,bd^3\log\left(\left|b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right|\right)}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}+\frac{d^5}{\left(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6\right)(dx+c)}+\frac{5\,b^3d^2-\frac{6\,(b \cdots}{\cdots}}{2\,(bc-ad)^4\left(b- \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 3*b*d^3*log(abs(b - b*c/(d*x + c) + a*d/(d*x + c)))/(b^4*c^4*d - 4*a*b^3*c^
3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) + d^5/((b^3*c^3*d^3 -
3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*(d*x + c)) + 1/2*(5*b^3*d^2 - 6*
(b^3*c*d^3 - a*b^2*d^4)/((d*x + c)*d))/((b*c - a*d)^4*(b - b*c/(d*x + c) +
a*d/(d*x + c))^2)
```

3.1352 $\int \frac{(a+bx)^6}{(c+dx)^3}\,dx$

Optimal. Leaf size=158

$$-\frac{2b^5(c+dx)^3(bc-ad)}{d^7}+\frac{15b^4(c+dx)^2(bc-ad)^2}{2d^7}-\frac{20b^3x(bc-ad)^3}{d^6}+\frac{15b^2(bc-ad)^4\log(c+dx)}{d^7}+\frac{6b(bc-ad)^5}{d^7(c+dx)}-\frac{(bc}{2d^7(}$$

```
[Out] (-20*b^3*(b*c - a*d)^3*x)/d^6 - (b*c - a*d)^6/(2*d^7*(c + d*x)^2) + (6*b*(b
*c - a*d)^5)/(d^7*(c + d*x)) + (15*b^4*(b*c - a*d)^2*(c + d*x)^2)/(2*d^7) -
 (2*b^5*(b*c - a*d)*(c + d*x)^3)/d^7 + (b^6*(c + d*x)^4)/(4*d^7) + (15*b^2*
(b*c - a*d)^4*Log[c + d*x])/d^7
```

Rubi [A] time = 0.20237, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b^5(c+dx)^3(bc-ad)}{d^7}+\frac{15b^4(c+dx)^2(bc-ad)^2}{2d^7}-\frac{20b^3x(bc-ad)^3}{d^6}+\frac{15b^2(bc-ad)^4\log(c+dx)}{d^7}+\frac{6b(bc-ad)^5}{d^7(c+dx)}-\frac{(bc}{2d^7(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(c + d*x)^3,x]
```

```
[Out] (-20*b^3*(b*c - a*d)^3*x)/d^6 - (b*c - a*d)^6/(2*d^7*(c + d*x)^2) + (6*b*(b
*c - a*d)^5)/(d^7*(c + d*x)) + (15*b^4*(b*c - a*d)^2*(c + d*x)^2)/(2*d^7) -
 (2*b^5*(b*c - a*d)*(c + d*x)^3)/d^7 + (b^6*(c + d*x)^4)/(4*d^7) + (15*b^2*
(b*c - a*d)^4*Log[c + d*x])/d^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^6}{(c+dx)^3}\,dx = \int \left(-\frac{20b^3(bc-ad)^3}{d^6}+\frac{(-bc+ad)^6}{d^6(c+dx)^3}-\frac{6b(bc-ad)^5}{d^6(c+dx)^2}+\frac{15b^2(bc-ad)^4}{d^6(c+dx)}+\frac{15b^4(bc-ad)^2(c+dx)}{d^6}-6b^5(b\right.$$

$$= -\frac{20b^3(bc-ad)^3x}{d^6}-\frac{(bc-ad)^6}{2d^7(c+dx)^2}+\frac{6b(bc-ad)^5}{d^7(c+dx)}+\frac{15b^4(bc-ad)^2(c+dx)^2}{2d^7}-\frac{2b^5(bc-ad)(c+dx)^3}{d^7}+$$

Mathematica [A] time = 0.111973, size = 303, normalized size = 1.92

$$\frac{30a^2b^4d^2\left(-11c^2d^2x^2+2c^3dx+7c^4-4cd^3x^3+d^4x^4\right)+40a^3b^3d^3\left(-4c^2dx-5c^3+4cd^2x^2+2d^3x^3\right)+30a^4b^2cd^4(3c+4dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(c + d*x)^3,x]
```

```
[Out] (-2*a^6*d^6 - 12*a^5*b*d^5*(c + 2*d*x) + 30*a^4*b^2*c*d^4*(3*c + 4*d*x) + 4
0*a^3*b^3*d^3*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + 30*a^2*b^4*d
^2*(7*c^4 + 2*c^3*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + 4*a*b^5*d
*(-27*c^5 + 6*c^4*d*x + 63*c^3*d^2*x^2 + 20*c^2*d^3*x^3 - 5*c*d^4*x^4 + 2*d
^5*x^5) + b^6*(22*c^6 - 16*c^5*d*x - 68*c^4*d^2*x^2 - 20*c^3*d^3*x^3 + 5*c^
2*d^4*x^4 - 2*c*d^5*x^5 + d^6*x^6) + 60*b^2*(b*c - a*d)^4*(c + d*x)^2*Log[c
 + d*x])/(4*d^7*(c + d*x)^2)
```

Maple [B] time = 0.01, size = 464, normalized size = 2.9

$$-\frac{a^{6}}{2\,d\,(dx+c)^{2}}+\frac{b^{6}x^{4}}{4\,d^{3}}+15\,\frac{b^{2}\ln(dx+c)\,a^{4}}{d^{3}}+15\,\frac{b^{6}\ln(dx+c)\,c^{4}}{d^{7}}-\frac{b^{6}c^{6}}{2\,d^{7}\,(dx+c)^{2}}-6\,\frac{a^{5}b}{d^{2}\,(dx+c)}+6\,\frac{b^{6}c^{5}}{d^{7}\,(dx+c)}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(d*x+c)^3,x)
```

```
[Out] -1/2/d/(d*x+c)^2*a^6+1/4*b^6/d^3*x^4+15*b^2/d^3*ln(d*x+c)*a^4+15*b^6/d^7*ln
(d*x+c)*c^4-1/2/d^7/(d*x+c)^2*b^6*c^6-6*b/d^2/(d*x+c)*a^5+6*b^6/d^7/(d*x+c)
*c^5+3*b^6/d^5*x^2*c^2-b^6/d^4*x^3*c+15/2*b^4/d^3*x^2*a^2+2*b^5/d^3*x^3*a+2
0*b^3/d^3*a^3*x-10*b^6/d^6*c^3*x-9*b^5/d^4*x^2*a*c-45*b^4/d^4*a^2*c*x+3/d^6
/(d*x+c)^2*a*b^5*c^5-60*b^3/d^4*ln(d*x+c)*a^3*c+90*b^4/d^5*ln(d*x+c)*a^2*c^
2-60*b^5/d^6*ln(d*x+c)*a*c^3+3/d^2/(d*x+c)^2*a^5*b*c-15/2/d^3/(d*x+c)^2*a^4
*b^2*c^2-60*b^3/d^4/(d*x+c)*a^3*c^2+10/d^4/(d*x+c)^2*a^3*b^3*c^3-15/2/d^5/(
d*x+c)^2*a^2*b^4*c^4-30*b^5/d^6/(d*x+c)*a*c^4+36*b^5/d^5*a*c^2*x+30*b^2/d^3
/(d*x+c)*a^4*c+60*b^4/d^5/(d*x+c)*a^2*c^3
```

Maxima [B] time = 0.986875, size = 491, normalized size = 3.11

$$\frac{11\,b^{6}c^{6}-54\,ab^{5}c^{5}d+105\,a^{2}b^{4}c^{4}d^{2}-100\,a^{3}b^{3}c^{3}d^{3}+45\,a^{4}b^{2}c^{2}d^{4}-6\,a^{5}bcd^{5}-a^{6}d^{6}+12\left(b^{6}c^{5}d-5\,ab^{5}c^{4}d^{2}+10\,a^{2}b^{4}c^{3}\right.}{2\left(d^{9}x^{2}+2\,cd^{8}x+c^{2}d^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(11*b^6*c^6 - 54*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2 - 100*a^3*b^3*c^3*d^
3 + 45*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 - a^6*d^6 + 12*(b^6*c^5*d - 5*a*b^5*
c^4*d^2 + 10*a^2*b^4*c^3*d^3 - 10*a^3*b^3*c^2*d^4 + 5*a^4*b^2*c*d^5 - a^5*b
*d^6)*x)/(d^9*x^2 + 2*c*d^8*x + c^2*d^7) + 1/4*(b^6*d^3*x^4 - 4*(b^6*c*d^2
- 2*a*b^5*d^3)*x^3 + 6*(2*b^6*c^2*d - 6*a*b^5*c*d^2 + 5*a^2*b^4*d^3)*x^2 -
4*(10*b^6*c^3 - 36*a*b^5*c^2*d + 45*a^2*b^4*c*d^2 - 20*a^3*b^3*d^3)*x)/d^6
+ 15*(b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b
^2*d^4)*log(d*x + c)/d^7
```

Fricas [B] time = 1.77956, size = 1110, normalized size = 7.03

$$\frac{b^{6}d^{6}x^{6}+22\,b^{6}c^{6}-108\,ab^{5}c^{5}d+210\,a^{2}b^{4}c^{4}d^{2}-200\,a^{3}b^{3}c^{3}d^{3}+90\,a^{4}b^{2}c^{2}d^{4}-12\,a^{5}bcd^{5}-2\,a^{6}d^{6}-2\left(b^{6}cd^{5}-4\,ab^{5}d^{6}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^6*d^6*x^6 + 22*b^6*c^6 - 108*a*b^5*c^5*d + 210*a^2*b^4*c^4*d^2 - 200
*a^3*b^3*c^3*d^3 + 90*a^4*b^2*c^2*d^4 - 12*a^5*b*c*d^5 - 2*a^6*d^6 - 2*(b^6
*c*d^5 - 4*a*b^5*d^6)*x^5 + 5*(b^6*c^2*d^4 - 4*a*b^5*c*d^5 + 6*a^2*b^4*d^6)
*x^4 - 20*(b^6*c^3*d^3 - 4*a*b^5*c^2*d^4 + 6*a^2*b^4*c*d^5 - 4*a^3*b^3*d^6)
*x^3 - 2*(34*b^6*c^4*d^2 - 126*a*b^5*c^3*d^3 + 165*a^2*b^4*c^2*d^4 - 80*a^3
*b^3*c*d^5)*x^2 - 4*(4*b^6*c^5*d - 6*a*b^5*c^4*d^2 - 15*a^2*b^4*c^3*d^3 + 4
0*a^3*b^3*c^2*d^4 - 30*a^4*b^2*c*d^5 + 6*a^5*b*d^6)*x + 60*(b^6*c^6 - 4*a*b
^5*c^5*d + 6*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4 + (b^6*c
^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^
6)*x^2 + 2*(b^6*c^5*d - 4*a*b^5*c^4*d^2 + 6*a^2*b^4*c^3*d^3 - 4*a^3*b^3*c^2
*d^4 + a^4*b^2*c*d^5)*x)*log(d*x + c))/(d^9*x^2 + 2*c*d^8*x + c^2*d^7)
```

Sympy [B] time = 2.63309, size = 335, normalized size = 2.12

$$\frac{b^6x^4}{4d^3} + \frac{15b^2\,(ad-bc)^4\log{(c+dx)}}{d^7} - \frac{a^6d^6 + 6a^5bcd^5 - 45a^4b^2c^2d^4 + 100a^3b^3c^3d^3 - 105a^2b^4c^4d^2 + 54ab^5c^5d - 11b^6c^6 + \cdots}{2c^2d^7 + 4cd^8x \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(d*x+c)**3,x)
```

```
[Out] b**6*x**4/(4*d**3) + 15*b**2*(a*d - b*c)**4*log(c + d*x)/d**7 - (a**6*d**6
+ 6*a**5*b*c*d**5 - 45*a**4*b**2*c**2*d**4 + 100*a**3*b**3*c**3*d**3 - 105*
a**2*b**4*c**4*d**2 + 54*a*b**5*c**5*d - 11*b**6*c**6 + x*(12*a**5*b*d**6 -
 60*a**4*b**2*c*d**5 + 120*a**3*b**3*c**2*d**4 - 120*a**2*b**4*c**3*d**3 +
60*a*b**5*c**4*d**2 - 12*b**6*c**5*d))/(2*c**2*d**7 + 4*c*d**8*x + 2*d**9*x
**2) + x**3*(2*a*b**5*d - b**6*c)/d**4 + x**2*(15*a**2*b**4*d**2 - 18*a*b**
5*c*d + 6*b**6*c**2)/(2*d**5) + x*(20*a**3*b**3*d**3 - 45*a**2*b**4*c*d**2
+ 36*a*b**5*c**2*d - 10*b**6*c**3)/d**6
```

Giac [B] time = 1.0626, size = 489, normalized size = 3.09

$$\frac{15\left(b^6c^4 - 4\,ab^5c^3d + 6\,a^2b^4c^2d^2 - 4\,a^3b^3cd^3 + a^4b^2d^4\right)\log\left(|dx+c|\right)}{d^7} + \frac{11\,b^6c^6 - 54\,ab^5c^5d + 105\,a^2b^4c^4d^2 - 100\,a^3b^3c^3d^3 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 15*(b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2
*d^4)*log(abs(d*x + c))/d^7 + 1/2*(11*b^6*c^6 - 54*a*b^5*c^5*d + 105*a^2*b^
4*c^4*d^2 - 100*a^3*b^3*c^3*d^3 + 45*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 - a^6*
d^6 + 12*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 10*a^2*b^4*c^3*d^3 - 10*a^3*b^3*c^2
*d^4 + 5*a^4*b^2*c*d^5 - a^5*b*d^6)*x)/((d*x + c)^2*d^7) + 1/4*(b^6*d^9*x^4
 - 4*b^6*c*d^8*x^3 + 8*a*b^5*d^9*x^3 + 12*b^6*c^2*d^7*x^2 - 36*a*b^5*c*d^8*
x^2 + 30*a^2*b^4*d^9*x^2 - 40*b^6*c^3*d^6*x + 144*a*b^5*c^2*d^7*x - 180*a^2
*b^4*c*d^8*x + 80*a^3*b^3*d^9*x)/d^12
```

3.1353 $\int \frac{(a+bx)^5}{(c+dx)^3}\,dx$

Optimal. Leaf size=133

$$-\frac{5b^4(c+dx)^2(bc-ad)}{2d^6}+\frac{10b^3x(bc-ad)^2}{d^5}-\frac{10b^2(bc-ad)^3\log(c+dx)}{d^6}-\frac{5b(bc-ad)^4}{d^6(c+dx)}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}+\frac{b^5(c+dx)^3}{3d^6}$$

```
[Out] (10*b^3*(b*c - a*d)^2*x)/d^5 + (b*c - a*d)^5/(2*d^6*(c + d*x)^2) - (5*b*(b*
c - a*d)^4)/(d^6*(c + d*x)) - (5*b^4*(b*c - a*d)*(c + d*x)^2)/(2*d^6) + (b^
5*(c + d*x)^3)/(3*d^6) - (10*b^2*(b*c - a*d)^3*Log[c + d*x])/d^6
```

Rubi [A] time = 0.120619, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{5b^4(c+dx)^2(bc-ad)}{2d^6}+\frac{10b^3x(bc-ad)^2}{d^5}-\frac{10b^2(bc-ad)^3\log(c+dx)}{d^6}-\frac{5b(bc-ad)^4}{d^6(c+dx)}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}+\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (10*b^3*(b*c - a*d)^2*x)/d^5 + (b*c - a*d)^5/(2*d^6*(c + d*x)^2) - (5*b*(b*
c - a*d)^4)/(d^6*(c + d*x)) - (5*b^4*(b*c - a*d)*(c + d*x)^2)/(2*d^6) + (b^
5*(c + d*x)^3)/(3*d^6) - (10*b^2*(b*c - a*d)^3*Log[c + d*x])/d^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^3}\,dx = \int \left(\frac{10b^3(bc-ad)^2}{d^5}+\frac{(-bc+ad)^5}{d^5(c+dx)^3}+\frac{5b(bc-ad)^4}{d^5(c+dx)^2}-\frac{10b^2(bc-ad)^3}{d^5(c+dx)}-\frac{5b^4(bc-ad)(c+dx)}{d^5}+\frac{b^5(c+\ldots}{d\ldots}\right.$$
$$=\frac{10b^3(bc-ad)^2x}{d^5}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}-\frac{5b(bc-ad)^4}{d^6(c+dx)}-\frac{5b^4(bc-ad)(c+dx)^2}{2d^6}+\frac{b^5(c+dx)^3}{3d^6}-\frac{10b^2(bc-\ldots}{}$$

Mathematica [A] time = 0.0700855, size = 230, normalized size = 1.73

$$30a^2b^3d^2\left(-4c^2dx-5c^3+4cd^2x^2+2d^3x^3\right)+30a^3b^2cd^3(3c+4dx)-15a^4bd^4(c+2dx)-3a^5d^5+15ab^4d\left(-11c^2d^2x^2+\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (-3*a^5*d^5 - 15*a^4*b*d^4*(c + 2*d*x) + 30*a^3*b^2*c*d^3*(3*c + 4*d*x) + 3
0*a^2*b^3*d^2*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + 15*a*b^4*d*(
```

```
7*c^4 + 2*c^3*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + b^5*(-27*c^5
+ 6*c^4*d*x + 63*c^3*d^2*x^2 + 20*c^2*d^3*x^3 - 5*c*d^4*x^4 + 2*d^5*x^5) -
60*b^2*(b*c - a*d)^3*(c + d*x)^2*Log[c + d*x])/(6*d^6*(c + d*x)^2)
```

Maple [B] time = 0.01, size = 346, normalized size = 2.6

$$\frac{b^5x^3}{3\,d^3}+\frac{5\,ab^4x^2}{2\,d^3}-\frac{3\,b^5x^2c}{2\,d^4}+10\,\frac{a^2b^3x}{d^3}-15\,\frac{ab^4cx}{d^4}+6\,\frac{b^5c^2x}{d^5}-5\,\frac{a^4b}{d^2\,(dx+c)}+20\,\frac{a^3b^2c}{d^3\,(dx+c)}-30\,\frac{a^2b^3c^2}{d^4\,(dx+c)}+20\,\frac{ab^4c}{d^5\,(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^3,x)
```

```
[Out] 1/3*b^5/d^3*x^3+5/2*b^4/d^3*x^2*a-3/2*b^5/d^4*x^2*c+10*b^3/d^3*a^2*x-15*b^4
/d^4*a*c*x+6*b^5/d^5*c^2*x-5*b/d^2/(d*x+c)*a^4+20*b^2/d^3/(d*x+c)*a^3*c-30*
b^3/d^4/(d*x+c)*a^2*c^2+20*b^4/d^5/(d*x+c)*a*c^3-5*b^5/d^6/(d*x+c)*c^4+10*b
^2/d^3*ln(d*x+c)*a^3-30*b^3/d^4*ln(d*x+c)*a^2*c+30*b^4/d^5*ln(d*x+c)*a*c^2-
10*b^5/d^6*ln(d*x+c)*c^3-1/2/d/(d*x+c)^2*a^5+5/2/d^2/(d*x+c)^2*a^4*b*c-5/d^
3/(d*x+c)^2*a^3*b^2*c^2+5/d^4/(d*x+c)^2*a^2*b^3*c^3-5/2/d^5/(d*x+c)^2*a*b^4
*c^4+1/2/d^6/(d*x+c)^2*b^5*c^5
```

Maxima [B] time = 0.996634, size = 366, normalized size = 2.75

$$-\frac{9\,b^5c^5-35\,ab^4c^4d+50\,a^2b^3c^3d^2-30\,a^3b^2c^2d^3+5\,a^4bcd^4+a^5d^5+10\left(b^5c^4d-4\,ab^4c^3d^2+6\,a^2b^3c^2d^3-4\,a^3b^2cd^4+a^4b\right.}{2\left(d^8x^2+2\,cd^7x+c^2d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3
+ 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2
*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/(d^8*x^2 + 2*c*d^7*x + c^2*d^6) + 1/
6*(2*b^5*d^2*x^3 - 3*(3*b^5*c*d - 5*a*b^4*d^2)*x^2 + 6*(6*b^5*c^2 - 15*a*b^
4*c*d + 10*a^2*b^3*d^2)*x)/d^5 - 10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*
d^2 - a^3*b^2*d^3)*log(d*x + c)/d^6
```

Fricas [B] time = 1.7846, size = 840, normalized size = 6.32

$$\frac{2\,b^5d^5x^5-27\,b^5c^5+105\,ab^4c^4d-150\,a^2b^3c^3d^2+90\,a^3b^2c^2d^3-15\,a^4bcd^4-3\,a^5d^5-5\left(b^5cd^4-3\,ab^4d^5\right)x^4+20\left(b^5c^2d^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*d^5*x^5 - 27*b^5*c^5 + 105*a*b^4*c^4*d - 150*a^2*b^3*c^3*d^2 + 9
0*a^3*b^2*c^2*d^3 - 15*a^4*b*c*d^4 - 3*a^5*d^5 - 5*(b^5*c*d^4 - 3*a*b^4*d^5
)*x^4 + 20*(b^5*c^2*d^3 - 3*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 3*(21*b^5*c^
3*d^2 - 55*a*b^4*c^2*d^3 + 40*a^2*b^3*c*d^4)*x^2 + 6*(b^5*c^4*d + 5*a*b^4*c
^3*d^2 - 20*a^2*b^3*c^2*d^3 + 20*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x - 60*(b^5*c
```

```
^5 - 3*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + (b^5*c^3*d^2 - 3
*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 - a^3*b^2*d^5)*x^2 + 2*(b^5*c^4*d - 3*a*b^
4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 - a^3*b^2*c*d^4)*x)*log(d*x + c))/(d^8*x^2 +
2*c*d^7*x + c^2*d^6)
```

Sympy [B] time = 2.01416, size = 253, normalized size = 1.9

$$\frac{b^5x^3}{3d^3} + \frac{10b^2\left(ad-bc\right)^3\log\left(c+dx\right)}{d^6} - \frac{a^5d^5 + 5a^4bcd^4 - 30a^3b^2c^2d^3 + 50a^2b^3c^3d^2 - 35ab^4c^4d + 9b^5c^5 + x\left(10a^4bd^5 - 4\right.}{2c^2d^6 + 4cd^7x + 2d^8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**3,x)
```

```
[Out] b**5*x**3/(3*d**3) + 10*b**2*(a*d - b*c)**3*log(c + d*x)/d**6 - (a**5*d**5
+ 5*a**4*b*c*d**4 - 30*a**3*b**2*c**2*d**3 + 50*a**2*b**3*c**3*d**2 - 35*a*
b**4*c**4*d + 9*b**5*c**5 + x*(10*a**4*b*d**5 - 40*a**3*b**2*c*d**4 + 60*a*
*2*b**3*c**2*d**3 - 40*a*b**4*c**3*d**2 + 10*b**5*c**4*d))/(2*c**2*d**6 + 4
*c*d**7*x + 2*d**8*x**2) + x**2*(5*a*b**4*d - 3*b**5*c)/(2*d**4) + x*(10*a*
*2*b**3*d**2 - 15*a*b**4*c*d + 6*b**5*c**2)/d**5
```

Giac [B] time = 1.07641, size = 356, normalized size = 2.68

$$-\frac{10\left(b^5c^3 - 3\,ab^4c^2d + 3\,a^2b^3cd^2 - a^3b^2d^3\right)\log\left(|dx+c|\right)}{d^6} - \frac{9\,b^5c^5 - 35\,ab^4c^4d + 50\,a^2b^3c^3d^2 - 30\,a^3b^2c^2d^3 + 5\,a^4bcd^4}{2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*log(abs(d*x +
 c))/d^6 - 1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*b^
2*c^2*d^3 + 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a
^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/((d*x + c)^2*d^6) + 1/6*(2
*b^5*d^6*x^3 - 9*b^5*c*d^5*x^2 + 15*a*b^4*d^6*x^2 + 36*b^5*c^2*d^4*x - 90*a
*b^4*c*d^5*x + 60*a^2*b^3*d^6*x)/d^9
```

3.1354 $\int \frac{(a+bx)^4}{(c+dx)^3} dx$

Optimal. Leaf size=103

$$-\frac{b^3x(3bc-4ad)}{d^4}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}+\frac{4b(bc-ad)^3}{d^5(c+dx)}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{b^4x^2}{2d^3}$$

```
[Out] -((b^3*(3*b*c - 4*a*d)*x)/d^4) + (b^4*x^2)/(2*d^3) - (b*c - a*d)^4/(2*d^5*(
c + d*x)^2) + (4*b*(b*c - a*d)^3)/(d^5*(c + d*x)) + (6*b^2*(b*c - a*d)^2*Lo
g[c + d*x])/d^5
```

Rubi [A] time = 0.0871533, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b^3x(3bc-4ad)}{d^4}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}+\frac{4b(bc-ad)^3}{d^5(c+dx)}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^3,x]
```

```
[Out] -((b^3*(3*b*c - 4*a*d)*x)/d^4) + (b^4*x^2)/(2*d^3) - (b*c - a*d)^4/(2*d^5*(
c + d*x)^2) + (4*b*(b*c - a*d)^3)/(d^5*(c + d*x)) + (6*b^2*(b*c - a*d)^2*Lo
g[c + d*x])/d^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^3}\,dx &= \int \left(-\frac{b^3(3bc-4ad)}{d^4}+\frac{b^4x}{d^3}+\frac{(-bc+ad)^4}{d^4(c+dx)^3}-\frac{4b(bc-ad)^3}{d^4(c+dx)^2}+\frac{6b^2(bc-ad)^2}{d^4(c+dx)}\right)dx\\ &= -\frac{b^3(3bc-4ad)x}{d^4}+\frac{b^4x^2}{2d^3}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{4b(bc-ad)^3}{d^5(c+dx)}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}\end{aligned}$$

Mathematica [A] time = 0.0530503, size = 167, normalized size = 1.62

$$\frac{6a^2b^2cd^2(3c+4dx)-4a^3bd^3(c+2dx)-a^4d^4+4ab^3d\left(-4c^2dx-5c^3+4cd^2x^2+2d^3x^3\right)+12b^2(c+dx)^2(bc-ad)^2\log(c+}{2d^5(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (-(a^4*d^4) - 4*a^3*b*d^3*(c + 2*d*x) + 6*a^2*b^2*c*d^2*(3*c + 4*d*x) + 4*a
*b^3*d*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + b^4*(7*c^4 + 2*c^3*
```

```
d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + 12*b^2*(b*c - a*d)^2*(c + d
*x)^2*Log[c + d*x])/(2*d^5*(c + d*x)^2)
```

Maple [B] time = 0.007, size = 245, normalized size = 2.4

$$\frac{b^4x^2}{2d^3}+4\frac{ab^3x}{d^3}-3\frac{b^4cx}{d^4}-4\frac{a^3b}{d^2(dx+c)}+12\frac{b^2a^2c}{d^3(dx+c)}-12\frac{ab^3c^2}{d^4(dx+c)}+4\frac{b^4c^3}{d^5(dx+c)}+6\frac{b^2\ln(dx+c)a^2}{d^3}-12\frac{b^3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^3,x)
```

```
[Out] 1/2*b^4*x^2/d^3+4*a*b^3*x/d^3-3*b^4*c*x/d^4-4*b/d^2/(d*x+c)*a^3+12*b^2/d^3/
(d*x+c)*a^2*c-12*b^3/d^4/(d*x+c)*a*c^2+4*b^4/d^5/(d*x+c)*c^3+6*b^2/d^3*ln(d
*x+c)*a^2-12*b^3/d^4*ln(d*x+c)*a*c+6*b^4/d^5*ln(d*x+c)*c^2-1/2/d/(d*x+c)^2*
a^4+2/d^2/(d*x+c)^2*a^3*b*c-3/d^3/(d*x+c)^2*a^2*b^2*c^2+2/d^4/(d*x+c)^2*a*b
^3*c^3-1/2/d^5/(d*x+c)^2*b^4*c^4
```

Maxima [A] time = 0.994885, size = 258, normalized size = 2.5

$$\frac{7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4+8\left(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4\right)x}{2\left(d^7x^2+2\,cd^6x+c^2d^5\right)}+\frac{b^4dx^2-2\left(3\,b^4c-\right.}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*
d^4 + 8*(b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/(d^7
*x^2 + 2*c*d^6*x + c^2*d^5) + 1/2*(b^4*d*x^2 - 2*(3*b^4*c - 4*a*b^3*d)*x)/d
^4 + 6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(d*x + c)/d^5
```

Fricas [B] time = 1.79979, size = 586, normalized size = 5.69

$$\frac{b^4d^4x^4+7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4-4\left(b^4cd^3-2\,ab^3d^4\right)x^3-\left(11\,b^4c^2d^2-16\,ab^3cd^3\right)x^2+2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 - a^4*d^4 - 4*(b^4*c*d^3 - 2*a*b^3*d^4)*x^3 - (11*b^4*c^2*d^2 - 16*
a*b^3*c*d^3)*x^2 + 2*(b^4*c^3*d - 8*a*b^3*c^2*d^2 + 12*a^2*b^2*c*d^3 - 4*a^
3*b*d^4)*x + 12*(b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2 + (b^4*c^2*d^2 -
 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(b^4*c^3*d - 2*a*b^3*c^2*d^2 + a^2*b^
2*c*d^3)*x)*log(d*x + c))/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)
```

Sympy [A] time = 1.43954, size = 184, normalized size = 1.79

$$\frac{b^4x^2}{2d^3} + \frac{6b^2(ad-bc)^2\log(c+dx)}{d^5} - \frac{a^4d^4 + 4a^3bcd^3 - 18a^2b^2c^2d^2 + 20ab^3c^3d - 7b^4c^4 + x\left(8a^3bd^4 - 24a^2b^2cd^3 + 24ab^3c^2\right.}{2c^2d^5 + 4cd^6x + 2d^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**3,x)
```

```
[Out] b**4*x**2/(2*d**3) + 6*b**2*(a*d - b*c)**2*log(c + d*x)/d**5 - (a**4*d**4 +
 4*a**3*b*c*d**3 - 18*a**2*b**2*c**2*d**2 + 20*a*b**3*c**3*d - 7*b**4*c**4
+ x*(8*a**3*b*d**4 - 24*a**2*b**2*c*d**3 + 24*a*b**3*c**2*d**2 - 8*b**4*c**
3*d))/(2*c**2*d**5 + 4*c*d**6*x + 2*d**7*x**2) + x*(4*a*b**3*d - 3*b**4*c)/
d**4
```

Giac [A] time = 1.05596, size = 247, normalized size = 2.4

$$\frac{6\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)\log\left(|dx+c|\right)}{d^5} + \frac{b^4d^3x^2 - 6\,b^4cd^2x + 8\,ab^3d^3x}{2\,d^6} + \frac{7\,b^4c^4 - 20\,ab^3c^3d + 18\,a^2b^2c^2d^2 - 4\,a^3bcd^3 -}{2\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(abs(d*x + c))/d^5 + 1/2*(b^4*d^
3*x^2 - 6*b^4*c*d^2*x + 8*a*b^3*d^3*x)/d^6 + 1/2*(7*b^4*c^4 - 20*a*b^3*c^3*
d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4 + 8*(b^4*c^3*d - 3*a*b^3*c
^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/((d*x + c)^2*d^5)
```

3.1355 $\int \frac{(a+bx)^3}{(c+dx)^3} dx$

Optimal. Leaf size=78

$$-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}-\frac{3b(bc-ad)^2}{d^4(c+dx)}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}+\frac{b^3x}{d^3}$$

```
[Out] (b^3*x)/d^3 + (b*c - a*d)^3/(2*d^4*(c + d*x)^2) - (3*b*(b*c - a*d)^2)/(d^4*
(c + d*x)) - (3*b^2*(b*c - a*d)*Log[c + d*x])/d^4
```

Rubi [A] time = 0.055235, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}-\frac{3b(bc-ad)^2}{d^4(c+dx)}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}+\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3 + (b*c - a*d)^3/(2*d^4*(c + d*x)^2) - (3*b*(b*c - a*d)^2)/(d^4*
(c + d*x)) - (3*b^2*(b*c - a*d)*Log[c + d*x])/d^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^3}\,dx &= \int \left(\frac{b^3}{d^3}+\frac{(-bc+ad)^3}{d^3(c+dx)^3}+\frac{3b(bc-ad)^2}{d^3(c+dx)^2}-\frac{3b^2(bc-ad)}{d^3(c+dx)}\right)dx\\ &= \frac{b^3x}{d^3}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}-\frac{3b(bc-ad)^2}{d^4(c+dx)}-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}\end{aligned}$$

Mathematica [A] time = 0.0392086, size = 114, normalized size = 1.46

$$\frac{-3a^2bd^2(c+2dx)-a^3d^3+3ab^2cd(3c+4dx)-6b^2(c+dx)^2(bc-ad)\log(c+dx)+b^3\left(-4c^2dx-5c^3+4cd^2x^2+2d^3x^3\right)}{2d^4(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (-(a^3*d^3) - 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*c*d*(3*c + 4*d*x) + b^3*(-5
*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) - 6*b^2*(b*c - a*d)*(c + d*x)^2
*Log[c + d*x])/(2*d^4*(c + d*x)^2)
```

Maple [B] time = 0.007, size = 160, normalized size = 2.1

$$\frac{b^3x}{d^3} - 3\frac{a^2b}{d^2(dx+c)} + 6\frac{ab^2c}{d^3(dx+c)} - 3\frac{b^3c^2}{d^4(dx+c)} + 3\frac{b^2\ln(dx+c)a}{d^3} - 3\frac{b^3\ln(dx+c)c}{d^4} - \frac{a^3}{2d(dx+c)^2} + \frac{3a^2bc}{2d^2(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] b^3*x/d^3-3*b/d^2/(d*x+c)*a^2+6*b^2/d^3/(d*x+c)*a*c-3*b^3/d^4/(d*x+c)*c^2+3
*b^2/d^3*ln(d*x+c)*a-3*b^3/d^4*ln(d*x+c)*c-1/2/d/(d*x+c)^2*a^3+3/2/d^2/(d*x
+c)^2*a^2*b*c-3/2/d^3/(d*x+c)^2*a*b^2*c^2+1/2/d^4/(d*x+c)^2*b^3*c^3
```

Maxima [A] time = 0.970939, size = 169, normalized size = 2.17

$$\frac{b^3x}{d^3} - \frac{5\,b^3c^3 - 9\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3 + 6\left(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3\right)x}{2\left(d^6x^2 + 2\,cd^5x + c^2d^4\right)} - \frac{3\left(b^3c - ab^2d\right)\log(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3 - 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(d^6*x^2 + 2*c*d^5*x + c^2*d^4) -
3*(b^3*c - a*b^2*d)*log(d*x + c)/d^4
```

Fricas [B] time = 1.79513, size = 375, normalized size = 4.81

$$\frac{2\,b^3d^3x^3 + 4\,b^3cd^2x^2 - 5\,b^3c^3 + 9\,ab^2c^2d - 3\,a^2bcd^2 - a^3d^3 - 2\left(2\,b^3c^2d - 6\,ab^2cd^2 + 3\,a^2bd^3\right)x - 6\left(b^3c^3 - ab^2c^2d + \left(b^3cd\right.\right.}{2\left(d^6x^2 + 2\,cd^5x + c^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 4*b^3*c*d^2*x^2 - 5*b^3*c^3 + 9*a*b^2*c^2*d - 3*a^2*b*
c*d^2 - a^3*d^3 - 2*(2*b^3*c^2*d - 6*a*b^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^3*
c^3 - a*b^2*c^2*d + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(b^3*c^2*d - a*b^2*c*d^
2)*x)*log(d*x + c))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)
```

Sympy [A] time = 1.0338, size = 128, normalized size = 1.64

$$\frac{b^3x}{d^3} + \frac{3b^2(ad-bc)\log(c+dx)}{d^4} - \frac{a^3d^3 + 3a^2bcd^2 - 9ab^2c^2d + 5b^3c^3 + x\left(6a^2bd^3 - 12ab^2cd^2 + 6b^3c^2d\right)}{2c^2d^4 + 4cd^5x + 2d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] b**3*x/d**3 + 3*b**2*(a*d - b*c)*log(c + d*x)/d**4 - (a**3*d**3 + 3*a**2*b*
c*d**2 - 9*a*b**2*c**2*d + 5*b**3*c**3 + x*(6*a**2*b*d**3 - 12*a*b**2*c*d**
2 + 6*b**3*c**2*d))/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2)
```

Giac [A] time = 1.06261, size = 151, normalized size = 1.94

$$\frac{b^3x}{d^3} - \frac{3\left(b^3c - ab^2d\right)\log\left(|dx + c|\right)}{d^4} - \frac{5\,b^3c^3 - 9\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3 + 6\left(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3\right)x}{2\,(dx + c)^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3 - 3*(b^3*c - a*b^2*d)*log(abs(d*x + c))/d^4 - 1/2*(5*b^3*c^3 - 9*
a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/((d*x + c)^2*d^4)
```

3.1356 $\int \frac{(a+bx)^2}{(c+dx)^3}\,dx$

Optimal. Leaf size=59

$$\frac{2b(bc-ad)}{d^3(c+dx)}-\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

```
[Out] -(b*c - a*d)^2/(2*d^3*(c + d*x)^2) + (2*b*(b*c - a*d))/(d^3*(c + d*x)) + (b
^2*Log[c + d*x])/d^3
```

Rubi [A] time = 0.0382741, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(bc-ad)}{d^3(c+dx)}-\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^3,x]
```

```
[Out] -(b*c - a*d)^2/(2*d^3*(c + d*x)^2) + (2*b*(b*c - a*d))/(d^3*(c + d*x)) + (b
^2*Log[c + d*x])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^3}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^3}-\frac{2b(bc-ad)}{d^2(c+dx)^2}+\frac{b^2}{d^2(c+dx)}\right)dx\\ &= -\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{2b(bc-ad)}{d^3(c+dx)}+\frac{b^2\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A] time = 0.0233252, size = 48, normalized size = 0.81

$$\frac{\frac{(bc-ad)(ad+3bc+4bdx)}{(c+dx)^2}+2b^2\log(c+dx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (((b*c - a*d)*(3*b*c + a*d + 4*b*d*x))/(c + d*x)^2 + 2*b^2*Log[c + d*x])/(2
*d^3)
```

Maple [A] time = 0.006, size = 92, normalized size = 1.6

$$-2\frac{ab}{d^2(dx+c)}+2\frac{b^2c}{d^3(dx+c)}+\frac{b^2\ln(dx+c)}{d^3}-\frac{a^2}{2d(dx+c)^2}+\frac{abc}{d^2(dx+c)^2}-\frac{b^2c^2}{2d^3(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] -2*b/d^2/(d*x+c)*a+2*b^2/d^3/(d*x+c)*c+b^2*ln(d*x+c)/d^3-1/2/d/(d*x+c)^2*a^
2+1/d^2/(d*x+c)^2*a*b*c-1/2/d^3/(d*x+c)^2*b^2*c^2
```

Maxima [A] time = 0.977537, size = 108, normalized size = 1.83

$$\frac{3b^2c^2-2abcd-a^2d^2+4\left(b^2cd-abd^2\right)x}{2\left(d^5x^2+2cd^4x+c^2d^3\right)}+\frac{b^2\log(dx+c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(d^5*x^2 +
2*c*d^4*x + c^2*d^3) + b^2*log(d*x + c)/d^3
```

Fricas [A] time = 1.86822, size = 205, normalized size = 3.47

$$\frac{3b^2c^2-2abcd-a^2d^2+4\left(b^2cd-abd^2\right)x+2\left(b^2d^2x^2+2b^2cdx+b^2c^2\right)\log(dx+c)}{2\left(d^5x^2+2cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [A] time = 0.605171, size = 80, normalized size = 1.36

$$\frac{b^2\log(c+dx)}{d^3}-\frac{a^2d^2+2abcd-3b^2c^2+x\left(4abd^2-4b^2cd\right)}{2c^2d^3+4cd^4x+2d^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] b**2*log(c + d*x)/d**3 - (a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2 + x*(4*a*b*d*
*2 - 4*b**2*c*d))/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2)
```

Giac [A] time = 1.05654, size = 93, normalized size = 1.58

$$\frac{b^2 \log \left(|dx+c|\right)}{d^3}+\frac{4 \left(b^2 c-abd\right) x+\frac{3 b^2 c^2-2 abcd-a^2 d^2}{d}}{2 \left(dx+c\right)^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^2*log(abs(d*x + c))/d^3 + 1/2*(4*(b^2*c - a*b*d)*x + (3*b^2*c^2 - 2*a*b*c
*d - a^2*d^2)/d)/((d*x + c)^2*d^2)
```

3.1357 $\int \frac{a+bx}{(c+dx)^3}\,dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^2}{2(c+dx)^2(bc-ad)}$$

```
[Out] (a + b*x)^2/(2*(b*c - a*d)*(c + d*x)^2)
```

Rubi [A] time = 0.0029068, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {37}

$$\frac{(a+bx)^2}{2(c+dx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^3,x]
```

```
[Out] (a + b*x)^2/(2*(b*c - a*d)*(c + d*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{a+bx}{(c+dx)^3}\,dx = \frac{(a+bx)^2}{2(bc-ad)(c+dx)^2}$$

Mathematica [A] time = 0.0086918, size = 26, normalized size = 0.93

$$-\frac{ad+b(c+2dx)}{2d^2(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^3,x]
```

```
[Out] -(a*d + b*(c + 2*d*x))/(2*d^2*(c + d*x)^2)
```

Maple [A] time = 0.004, size = 35, normalized size = 1.3

$$-\frac{b}{d^2\,(dx+c)}-\frac{ad-bc}{2\,d^2\,(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^3,x)
```

```
[Out] -b/d^2/(d*x+c)-1/2*(a*d-b*c)/d^2/(d*x+c)^2
```

Maxima [A] time = 0.958779, size = 51, normalized size = 1.82

$$-\frac{2\,bdx + bc + ad}{2\left(d^4x^2 + 2\,cd^3x + c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Fricas [A] time = 1.71051, size = 81, normalized size = 2.89

$$-\frac{2\,bdx + bc + ad}{2\left(d^4x^2 + 2\,cd^3x + c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [A] time = 0.407639, size = 39, normalized size = 1.39

$$-\frac{ad + bc + 2bdx}{2c^2d^2 + 4cd^3x + 2d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**3,x)
```

```
[Out] -(a*d + b*c + 2*b*d*x)/(2*c**2*d**2 + 4*c*d**3*x + 2*d**4*x**2)
```

Giac [A] time = 1.06412, size = 32, normalized size = 1.14

$$-\frac{2\,bdx + bc + ad}{2\,(dx + c)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/((d*x + c)^2*d^2)
```

3.1358 $\int \frac{1}{(c+dx)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2d(c+dx)^2}$$

```
[Out] -1/(2*d*(c + d*x)^2)
```

Rubi [A] time = 0.0017357, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-3),x]
```

```
[Out] -1/(2*d*(c + d*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3}\,dx = -\frac{1}{2d(c+dx)^2}$$

Mathematica [A] time = 0.0024733, size = 14, normalized size = 1.

$$-\frac{1}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-3),x]
```

```
[Out] -1/(2*d*(c + d*x)^2)
```

Maple [A] time = 0., size = 13, normalized size = 0.9

$$-\frac{1}{2\,d\,(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3,x)

[Out] -1/2/d/(d*x+c)^2
```

Maxima [A] time = 0.957684, size = 16, normalized size = 1.14

$$-\frac{1}{2(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="maxima")

[Out] -1/2/((d*x + c)^2*d)
```

Fricas [A] time = 1.62047, size = 49, normalized size = 3.5

$$-\frac{1}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="fricas")

[Out] -1/2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Sympy [B] time = 0.30866, size = 26, normalized size = 1.86

$$-\frac{1}{2c^2d+4cd^2x+2d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3,x)

[Out] -1/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2)
```

Giac [A] time = 1.07505, size = 16, normalized size = 1.14

$$-\frac{1}{2(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="giac")

[Out] -1/2/((d*x + c)^2*d)
```

3.1359 $\int \frac{1}{(a+bx)(c+dx)^3}\,dx$

Optimal. Leaf size=82

$$\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}$$

```
[Out] 1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b^2*Log[a +
b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3
```

Rubi [A] time = 0.0459267, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^3),x]
```

```
[Out] 1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b^2*Log[a +
b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^3}\,dx &= \int \left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{d}{(bc-ad)(c+dx)^3}-\frac{bd}{(bc-ad)^2(c+dx)^2}-\frac{b^2d}{(bc-ad)^3(c+dx)}\right)dx\\ &= \frac{1}{2(bc-ad)(c+dx)^2}+\frac{b}{(bc-ad)^2(c+dx)}+\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}\end{aligned}$$

Mathematica [A] time = 0.0477429, size = 67, normalized size = 0.82

$$\frac{2b^2\log(a+bx)+\frac{(bc-ad)(-ad+3bc+2bdx)}{(c+dx)^2}-2b^2\log(c+dx)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^3),x]
```

```
[Out] (((b*c - a*d)*(3*b*c - a*d + 2*b*d*x))/(c + d*x)^2 + 2*b^2*Log[a + b*x] - 2
*b^2*Log[c + d*x])/(2*(b*c - a*d)^3)
```

Maple [A] time = 0.009, size = 81, normalized size = 1.

$$-\frac{1}{(2\,ad-2\,bc)\,(dx+c)^{2}}+\frac{b^{2}\ln\left(dx+c\right)}{(ad-bc)^{3}}+\frac{b}{(ad-bc)^{2}\,(dx+c)}-\frac{b^{2}\ln\left(bx+a\right)}{(ad-bc)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^3,x)
```

```
[Out] -1/2/(a*d-b*c)/(d*x+c)^2+b^2/(a*d-b*c)^3*ln(d*x+c)+b/(a*d-b*c)^2/(d*x+c)-b^
2/(a*d-b*c)^3*ln(b*x+a)
```

Maxima [B] time = 0.974271, size = 273, normalized size = 3.33

$$\frac{b^{2}\log\left(bx+a\right)}{b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}}-\frac{b^{2}\log\left(dx+c\right)}{b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}}+\frac{2\,bdx+3\,b\cdots}{2\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}+\left(b^{2}c^{2}d^{2}-2\,abcd^{3}+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - b^2*
log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2*b
*d*x + 3*b*c - a*d)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2
*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

Fricas [B] time = 1.6942, size = 490, normalized size = 5.98

$$\frac{3\,b^{2}c^{2}-4\,abcd+a^{2}d^{2}+2\left(b^{2}cd-abd^{2}\right)x+2\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}\right)\log\left(bx+a\right)-2\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}\right)\log\left(\cdots\right.}{2\left(b^{3}c^{5}-3\,ab^{2}c^{4}d+3\,a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}+\left(b^{3}c^{3}d^{2}-3\,ab^{2}c^{2}d^{3}+3\,a^{2}bcd^{4}-a^{3}d^{5}\right)x^{2}+2\left(b^{3}c^{4}d-3\,ab^{2}c^{3}d^{2}+3\,a^{2}bc^{2}d^{3}-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*log(d*x + c))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^
2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^2 + 2*(
b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x)
```

Sympy [B] time = 1.21629, size = 381, normalized size = 4.65

$$\frac{b^{2}\log\left(x+\frac{-\frac{a^{4}b^{2}d^{4}}{(ad-bc)^{3}}+\frac{4a^{3}b^{3}cd^{3}}{(ad-bc)^{3}}-\frac{6a^{2}b^{4}c^{2}d^{2}}{(ad-bc)^{3}}+\frac{4ab^{5}c^{3}d}{(ad-bc)^{3}}+ab^{2}d-\frac{b^{6}c^{4}}{(ad-bc)^{3}}+b^{3}c}{2b^{3}d}\right)}{(ad-bc)^{3}}-\frac{b^{2}\log\left(x+\frac{\frac{a^{4}b^{2}d^{4}}{(ad-bc)^{3}}-\frac{4a^{3}b^{3}cd^{3}}{(ad-bc)^{3}}+\frac{6a^{2}b^{4}c^{2}d^{2}}{(ad-bc)^{3}}-\frac{4ab^{5}c^{3}d}{(ad-bc)^{3}}+ab^{2}d+\frac{b^{6}c^{4}}{(ad-bc)^{3}}+b^{3}c}{2b^{3}d}\right)}{(ad-bc)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**3,x)
```

```
[Out] b**2*log(x + (-a**4*b**2*d**4/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3/(a*d - b*
c)**3 - 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 + 4*a*b**5*c**3*d/(a*d - b*c)*
*3 + a*b**2*d - b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a*d - b*c)*
*3 - b**2*log(x + (a**4*b**2*d**4/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3/(a*d
- b*c)**3 + 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 - 4*a*b**5*c**3*d/(a*d - b
*c)**3 + a*b**2*d + b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a*d - b
*c)**3 + (-a*d + 3*b*c + 2*b*d*x)/(2*a**2*c**2*d**2 - 4*a*b*c**3*d + 2*b**2
*c**4 + x**2*(2*a**2*d**4 - 4*a*b*c*d**3 + 2*b**2*c**2*d**2) + x*(4*a**2*c*
d**3 - 8*a*b*c**2*d**2 + 4*b**2*c**3*d))
```

Giac [B] time = 1.07354, size = 223, normalized size = 2.72

$$\frac{b^3\log\left(|bx+a|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{b^2d\log\left(|dx+c|\right)}{b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4}+\frac{3\,b^2c^2-4\,abcd+a^2d^2+2\left(b^2cd-abd^2\right)}{2\,(bc-ad)^3(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^
3) - b^2*d*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4) + 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x)
/((b*c - a*d)^3*(d*x + c)^2)
```

3.1360 $\int \frac{1}{(a+bx)^2(c+dx)^3} \, dx$

Optimal. Leaf size=110

$$-\frac{b^2}{(a+bx)(bc-ad)^3}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log(c+dx)}{(bc-ad)^4}-\frac{2bd}{(c+dx)(bc-ad)^3}-\frac{d}{2(c+dx)^2(bc-ad)^2}$$

```
[Out] -(b^2/((b*c - a*d)^3*(a + b*x))) - d/(2*(b*c - a*d)^2*(c + d*x)^2) - (2*b*d
)/((b*c - a*d)^3*(c + d*x)) - (3*b^2*d*Log[a + b*x])/(b*c - a*d)^4 + (3*b^2
*d*Log[c + d*x])/(b*c - a*d)^4
```

Rubi [A] time = 0.0741628, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{b^2}{(a+bx)(bc-ad)^3}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log(c+dx)}{(bc-ad)^4}-\frac{2bd}{(c+dx)(bc-ad)^3}-\frac{d}{2(c+dx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -(b^2/((b*c - a*d)^3*(a + b*x))) - d/(2*(b*c - a*d)^2*(c + d*x)^2) - (2*b*d
)/((b*c - a*d)^3*(c + d*x)) - (3*b^2*d*Log[a + b*x])/(b*c - a*d)^4 + (3*b^2
*d*Log[c + d*x])/(b*c - a*d)^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)^3} \, dx &= \int \left(\frac{b^3}{(bc-ad)^3(a+bx)^2}-\frac{3b^3d}{(bc-ad)^4(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^3}+\frac{2bd^2}{(bc-ad)^3(c+dx)^2}+\frac{(b\right.\\ &= -\frac{b^2}{(bc-ad)^3(a+bx)}-\frac{d}{2(bc-ad)^2(c+dx)^2}-\frac{2bd}{(bc-ad)^3(c+dx)}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log}{(bc}\end{aligned}$$

Mathematica [A] time = 0.0970563, size = 97, normalized size = 0.88

$$-\frac{\frac{2b^2(bc-ad)}{a+bx}+6b^2d\log(a+bx)+\frac{4bd(bc-ad)}{c+dx}+\frac{d(bc-ad)^2}{(c+dx)^2}-6b^2d\log(c+dx)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -((2*b^2*(b*c - a*d))/(a + b*x) + (d*(b*c - a*d)^2)/(c + d*x)^2 + (4*b*d*(b
*c - a*d))/(c + d*x) + 6*b^2*d*Log[a + b*x] - 6*b^2*d*Log[c + d*x])/(2*(b*c
```

```
- a*d)^4)
```

Maple [A] time = 0.012, size = 108, normalized size = 1.

$$-\frac{d}{2\,(ad-bc)^2\,(dx+c)^2}+3\,\frac{b^2d\ln(dx+c)}{(ad-bc)^4}+2\,\frac{bd}{(ad-bc)^3\,(dx+c)}+\frac{b^2}{(ad-bc)^3\,(bx+a)}-3\,\frac{b^2d\ln(bx+a)}{(ad-bc)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] -1/2*d/(a*d-b*c)^2/(d*x+c)^2+3*d/(a*d-b*c)^4*b^2*ln(d*x+c)+2*d/(a*d-b*c)^3*
b/(d*x+c)+b^2/(a*d-b*c)^3/(b*x+a)-3*d/(a*d-b*c)^4*b^2*ln(b*x+a)
```

Maxima [B] time = 1.02901, size = 521, normalized size = 4.74

$$-\frac{3\,b^2d\log(bx+a)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}+\frac{3\,b^2d\log(dx+c)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}-\frac{}{2\left(ab^3c^5-3\,a^2b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -3*b^2*d*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4) + 3*b^2*d*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/2*(6*b^2*d^2*x^2 + 2*b^2*c^2 + 5
*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x)/(a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*
b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2
*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*
d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

Fricas [B] time = 1.84781, size = 991, normalized size = 9.01

$$-\frac{2\,b^3c^3+3\,ab^2c^2d-6\,a^2bcd^2+a^3d^3+6\left(b^3cd^2-ab^2d^3\right)x^2+3\left(3\,b^3c^2d-2\,ab^2cd^2-a^2bd^3\right)x+6\left(b^3d^3x^3+ab^2c^2d+\right.}{2\left(ab^4c^6-4\,a^2b^3c^5d+6\,a^3b^2c^4d^2-4\,a^4bc^3d^3+a^5c^2d^4+\left(b^5c^4d^2-4\,ab^4c^3d^3+6\,a^2b^3c^2d^4-4\,a^3b^2cd^5+a^4bd^6\right)x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*c^3 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 + 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x + 6*(b^3*d^3
*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c
*d^2)*x)*log(b*x + a) - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2
*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(d*x + c))/(a*b^4*c^6 - 4*a^2
*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5*c^4*d
^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^3
 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^4 -
 2*a^4*b*c*d^5 + a^5*d^6)*x^2 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^
```

```
2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x)
```

Sympy [B] time = 1.97404, size = 632, normalized size = 5.75

$$\frac{3b^2d\log\left(x+\frac{-\frac{3a^5b^2d^6}{(ad-bc)^4}+\frac{15a^4b^3cd^5}{(ad-bc)^4}-\frac{30a^3b^4c^2d^4}{(ad-bc)^4}+\frac{30a^2b^5c^3d^3}{(ad-bc)^4}-\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2+\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{(ad-bc)^4}-\frac{3b^2d\log\left(x+\frac{\frac{3a^5b^2d^6}{(ad-bc)^4}-\frac{15a^4b^3cd^5}{(ad-bc)^4}+\frac{30a^3b^4c^2d^4}{(ad-bc)^4}-30\ldots}{\ldots}\right)}{(ad-bc\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] 3*b**2*d*log(x + (-3*a**5*b**2*d**6/(a*d - b*c)**4 + 15*a**4*b**3*c*d**5/(a
*d - b*c)**4 - 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 + 30*a**2*b**5*c**3*d*
*3/(a*d - b*c)**4 - 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 + 3*
b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4 - 3*
b**2*d*log(x + (3*a**5*b**2*d**6/(a*d - b*c)**4 - 15*a**4*b**3*c*d**5/(a*d
- b*c)**4 + 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 - 30*a**2*b**5*c**3*d**3/
(a*d - b*c)**4 + 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 - 3*b**
7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4 + (-a**
2*d**2 + 5*a*b*c*d + 2*b**2*c**2 + 6*b**2*d**2*x**2 + x*(3*a*b*d**2 + 9*b**
2*c*d))/(2*a**4*c**2*d**3 - 6*a**3*b*c**3*d**2 + 6*a**2*b**2*c**4*d - 2*a*b
**3*c**5 + x**3*(2*a**3*b*d**5 - 6*a**2*b**2*c*d**4 + 6*a*b**3*c**2*d**3 -
2*b**4*c**3*d**2) + x**2*(2*a**4*d**5 - 2*a**3*b*c*d**4 - 6*a**2*b**2*c**2*
d**3 + 10*a*b**3*c**3*d**2 - 4*b**4*c**4*d) + x*(4*a**4*c*d**4 - 10*a**3*b*
c**2*d**3 + 6*a**2*b**2*c**3*d**2 + 2*a*b**3*c**4*d - 2*b**4*c**5))
```

Giac [B] time = 1.09483, size = 293, normalized size = 2.66

$$\frac{3\,b^3d\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}-\frac{b^5}{\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)(bx+a)}+\frac{5\,b^2d^3+\frac{6\left(b^4cd^2\ldots\right)}{(bx\ldots}}{2\,(bc-ad)^4\left(\frac{bc}{bx+a}-\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 3*b^3*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^5*c^4 - 4*a*b^4*c^3*
d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - b^5/((b^6*c^3 - 3*a*
b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*(b*x + a)) + 1/2*(5*b^2*d^3 + 6*
(b^4*c*d^2 - a*b^3*d^3)/((b*x + a)*b))/((b*c - a*d)^4*(b*c/(b*x + a) - a*d/
(b*x + a) + d)^2)
```

3.1361 $\int \frac{1}{(a+bx)^3(c+dx)^3}\,dx$

Optimal. Leaf size=143

$$\frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}-\frac{6b^2d^2\log(c+dx)}{(bc-ad)^5}+\frac{3b^2d}{(a+bx)(bc-ad)^4}-\frac{b^2}{2(a+bx)^2(bc-ad)^3}+\frac{3bd^2}{(c+dx)(bc-ad)^4}+\frac{d^2}{2(c+dx)^2(bc-ad)^3}$$

```
[Out] -b^2/(2*(b*c - a*d)^3*(a + b*x)^2) + (3*b^2*d)/((b*c - a*d)^4*(a + b*x)) +
d^2/(2*(b*c - a*d)^3*(c + d*x)^2) + (3*b*d^2)/((b*c - a*d)^4*(c + d*x)) + (
6*b^2*d^2*Log[a + b*x])/(b*c - a*d)^5 - (6*b^2*d^2*Log[c + d*x])/(b*c - a*d
)^5
```

Rubi [A] time = 0.101762, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}-\frac{6b^2d^2\log(c+dx)}{(bc-ad)^5}+\frac{3b^2d}{(a+bx)(bc-ad)^4}-\frac{b^2}{2(a+bx)^2(bc-ad)^3}+\frac{3bd^2}{(c+dx)(bc-ad)^4}+\frac{d^2}{2(c+dx)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -b^2/(2*(b*c - a*d)^3*(a + b*x)^2) + (3*b^2*d)/((b*c - a*d)^4*(a + b*x)) +
d^2/(2*(b*c - a*d)^3*(c + d*x)^2) + (3*b*d^2)/((b*c - a*d)^4*(c + d*x)) + (
6*b^2*d^2*Log[a + b*x])/(b*c - a*d)^5 - (6*b^2*d^2*Log[c + d*x])/(b*c - a*d
)^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3(c+dx)^3}\,dx &= \int \left(\frac{b^3}{(bc-ad)^3(a+bx)^3}-\frac{3b^3d}{(bc-ad)^4(a+bx)^2}+\frac{6b^3d^2}{(bc-ad)^5(a+bx)}-\frac{d^3}{(bc-ad)^3(c+dx)^3}-\right.\\ &= -\frac{b^2}{2(bc-ad)^3(a+bx)^2}+\frac{3b^2d}{(bc-ad)^4(a+bx)}+\frac{d^2}{2(bc-ad)^3(c+dx)^2}+\frac{3bd^2}{(bc-ad)^4(c+dx)}+\end{aligned}$$

Mathematica [A] time = 0.109488, size = 128, normalized size = 0.9

$$\frac{\frac{6b^2d(bc-ad)}{a+bx}-\frac{b^2(bc-ad)^2}{(a+bx)^2}+12b^2d^2\log(a+bx)+\frac{6bd^2(bc-ad)}{c+dx}+\frac{d^2(bc-ad)^2}{(c+dx)^2}-12b^2d^2\log(c+dx)}{2(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^3),x]
```

```
[Out] (-((b^2*(b*c - a*d)^2)/(a + b*x)^2) + (6*b^2*d*(b*c - a*d))/(a + b*x) + (d^
2*(b*c - a*d)^2)/(c + d*x)^2 + (6*b*d^2*(b*c - a*d))/(c + d*x) + 12*b^2*d^2
*Log[a + b*x] - 12*b^2*d^2*Log[c + d*x])/(2*(b*c - a*d)^5)
```

Maple [A] time = 0.011, size = 140, normalized size = 1.

$$-\frac{d^2}{2\,(ad-bc)^3\,(dx+c)^2}+6\,\frac{d^2b^2\ln(dx+c)}{(ad-bc)^5}+3\,\frac{d^2b}{(ad-bc)^4\,(dx+c)}+\frac{b^2}{2\,(ad-bc)^3\,(bx+a)^2}-6\,\frac{d^2b^2\ln(bx+a)}{(ad-bc)^5}+3\,\frac{}{(ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] -1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*d^2/(a*d-b*c)^5*b^2*ln(d*x+c)+3*d^2/(a*d-b
*c)^4*b/(d*x+c)+1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*d^2/(a*d-b*c)^5*b^2*ln(b*x+
a)+3*b^2/(a*d-b*c)^4*d/(b*x+a)
```

Maxima [B] time = 1.0205, size = 802, normalized size = 5.61

$$\frac{6\,b^2d^2\log(bx+a)}{b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5}-\frac{6\,b^2d^2\log(dx+c)}{b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 6*b^2*d^2*log(b*x + a)/(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a
^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) - 6*b^2*d^2*log(d*x + c)/(b^5*c^5
 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4
- a^5*d^5) + 1/2*(12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2
- a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 +
 a^2*b*d^3)*x)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b
*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4
 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*
a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b
^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d
^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^
3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*x)
```

Fricas [B] time = 1.98256, size = 1488, normalized size = 10.41

$$-\frac{b^4c^4-8\,ab^3c^3d+8\,a^3bcd^3-a^4d^4-12\left(b^4cd^3-ab^3d^4\right)x^3-18\left(b^4c^2d^2-a^2b^2d^4\right)x^2-4\left(b^4c^3d+6\,ab^3c^2d^2-6\,a^2b\right.}{2\left(a^2b^5c^7-5\,a^3b^4c^6d+10\,a^4b^3c^5d^2-10\,a^5b^2c^4d^3+5\,a^6bc^3d^4-a^7c^2d^5+\left(b^7c^5d^2-5\,ab^6c^4d^3+10\,a^2b^5c^3d^4-10\,a^3b^4c^2d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*c^4 - 8*a*b^3*c^3*d + 8*a^3*b*c*d^3 - a^4*d^4 - 12*(b^4*c*d^3 - a
*b^3*d^4)*x^3 - 18*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 - 4*(b^4*c^3*d + 6*a*b^3
*c^2*d^2 - 6*a^2*b^2*c*d^3 - a^3*b*d^4)*x - 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d
```

```
^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2
*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a) + 12*(b^4*d^4
*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a
*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(d*
x + c))/(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^
4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5 + (b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10
*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^4
+ 2*(b^7*c^6*d - 4*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3*c^2*d^5 +
4*a^5*b^2*c*d^6 - a^6*b*d^7)*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a^2*b^5*c^5*d
^2 + 25*a^3*b^4*c^4*d^3 - 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 + a^6*b*c*
d^6 - a^7*d^7)*x^2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b^4*c^5*d^2 - 5
*a^5*b^2*c^3*d^4 + 4*a^6*b*c^2*d^5 - a^7*c*d^6)*x)
```

Sympy [B] time = 2.71303, size = 881, normalized size = 6.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] 6*b**2*d**2*log(x + (-6*a**6*b**2*d**8/(a*d - b*c)**5 + 36*a**5*b**3*c*d**7
/(a*d - b*c)**5 - 90*a**4*b**4*c**2*d**6/(a*d - b*c)**5 + 120*a**3*b**5*c**
3*d**5/(a*d - b*c)**5 - 90*a**2*b**6*c**4*d**4/(a*d - b*c)**5 + 36*a*b**7*c
**5*d**3/(a*d - b*c)**5 + 6*a*b**2*d**3 - 6*b**8*c**6*d**2/(a*d - b*c)**5 +
 6*b**3*c*d**2)/(12*b**3*d**3))/(a*d - b*c)**5 - 6*b**2*d**2*log(x + (6*a**
6*b**2*d**8/(a*d - b*c)**5 - 36*a**5*b**3*c*d**7/(a*d - b*c)**5 + 90*a**4*b
**4*c**2*d**6/(a*d - b*c)**5 - 120*a**3*b**5*c**3*d**5/(a*d - b*c)**5 + 90*
a**2*b**6*c**4*d**4/(a*d - b*c)**5 - 36*a*b**7*c**5*d**3/(a*d - b*c)**5 + 6
*a*b**2*d**3 + 6*b**8*c**6*d**2/(a*d - b*c)**5 + 6*b**3*c*d**2)/(12*b**3*d*
*3))/(a*d - b*c)**5 + (-a**3*d**3 + 7*a**2*b*c*d**2 + 7*a*b**2*c**2*d - b**
3*c**3 + 12*b**3*d**3*x**3 + x**2*(18*a*b**2*d**3 + 18*b**3*c*d**2) + x*(4*
a**2*b*d**3 + 28*a*b**2*c*d**2 + 4*b**3*c**2*d))/(2*a**6*c**2*d**4 - 8*a**5
*b*c**3*d**3 + 12*a**4*b**2*c**4*d**2 - 8*a**3*b**3*c**5*d + 2*a**2*b**4*c*
*6 + x**4*(2*a**4*b**2*d**6 - 8*a**3*b**3*c*d**5 + 12*a**2*b**4*c**2*d**4 -
 8*a*b**5*c**3*d**3 + 2*b**6*c**4*d**2) + x**3*(4*a**5*b*d**6 - 12*a**4*b**
2*c*d**5 + 8*a**3*b**3*c**2*d**4 + 8*a**2*b**4*c**3*d**3 - 12*a*b**5*c**4*d
**2 + 4*b**6*c**5*d) + x**2*(2*a**6*d**6 - 18*a**4*b**2*c**2*d**4 + 32*a**3
*b**3*c**3*d**3 - 18*a**2*b**4*c**4*d**2 + 2*b**6*c**6) + x*(4*a**6*c*d**5
- 12*a**5*b*c**2*d**4 + 8*a**4*b**2*c**3*d**3 + 8*a**3*b**3*c**4*d**2 - 12*
a**2*b**4*c**5*d + 4*a*b**5*c**6))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1362 $\int \frac{(a+bx)^9}{(c+dx)^8}\,dx$

Optimal. Leaf size=232

$$-\frac{b^8x(8bc-9ad)}{d^9}+\frac{84b^6(bc-ad)^3}{d^{10}(c+dx)}-\frac{63b^5(bc-ad)^4}{d^{10}(c+dx)^2}+\frac{42b^4(bc-ad)^5}{d^{10}(c+dx)^3}-\frac{21b^3(bc-ad)^6}{d^{10}(c+dx)^4}+\frac{36b^2(bc-ad)^7}{5d^{10}(c+dx)^5}+\frac{36b^7(bc-ad)^2 \ldots}{d \ldots}$$

```
[Out] -((b^8*(8*b*c - 9*a*d)*x)/d^9) + (b^9*x^2)/(2*d^8) + (b*c - a*d)^9/(7*d^10*
(c + d*x)^7) - (3*b*(b*c - a*d)^8)/(2*d^10*(c + d*x)^6) + (36*b^2*(b*c - a*
d)^7)/(5*d^10*(c + d*x)^5) - (21*b^3*(b*c - a*d)^6)/(d^10*(c + d*x)^4) + (4
2*b^4*(b*c - a*d)^5)/(d^10*(c + d*x)^3) - (63*b^5*(b*c - a*d)^4)/(d^10*(c +
 d*x)^2) + (84*b^6*(b*c - a*d)^3)/(d^10*(c + d*x)) + (36*b^7*(b*c - a*d)^2*
Log[c + d*x])/d^10
```

Rubi [A] time = 0.356882, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b^8x(8bc-9ad)}{d^9}+\frac{84b^6(bc-ad)^3}{d^{10}(c+dx)}-\frac{63b^5(bc-ad)^4}{d^{10}(c+dx)^2}+\frac{42b^4(bc-ad)^5}{d^{10}(c+dx)^3}-\frac{21b^3(bc-ad)^6}{d^{10}(c+dx)^4}+\frac{36b^2(bc-ad)^7}{5d^{10}(c+dx)^5}+\frac{36b^7(bc-ad)^2 \ldots}{d \ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9/(c + d*x)^8,x]
```

```
[Out] -((b^8*(8*b*c - 9*a*d)*x)/d^9) + (b^9*x^2)/(2*d^8) + (b*c - a*d)^9/(7*d^10*
(c + d*x)^7) - (3*b*(b*c - a*d)^8)/(2*d^10*(c + d*x)^6) + (36*b^2*(b*c - a*
d)^7)/(5*d^10*(c + d*x)^5) - (21*b^3*(b*c - a*d)^6)/(d^10*(c + d*x)^4) + (4
2*b^4*(b*c - a*d)^5)/(d^10*(c + d*x)^3) - (63*b^5*(b*c - a*d)^4)/(d^10*(c +
 d*x)^2) + (84*b^6*(b*c - a*d)^3)/(d^10*(c + d*x)) + (36*b^7*(b*c - a*d)^2*
Log[c + d*x])/d^10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^9}{(c+dx)^8}\,dx &= \int \left(-\frac{b^8(8bc-9ad)}{d^9}+\frac{b^9x}{d^8}+\frac{(-bc+ad)^9}{d^9(c+dx)^8}+\frac{9b(bc-ad)^8}{d^9(c+dx)^7}-\frac{36b^2(bc-ad)^7}{d^9(c+dx)^6}+\frac{84b^3(bc-ad)^6}{d^9(c+dx)^5}-\frac{126b^4(\ldots}{d^9(c\ldots}\right.\\ &= -\frac{b^8(8bc-9ad)x}{d^9}+\frac{b^9x^2}{2d^8}+\frac{(bc-ad)^9}{7d^{10}(c+dx)^7}-\frac{3b(bc-ad)^8}{2d^{10}(c+dx)^6}+\frac{36b^2(bc-ad)^7}{5d^{10}(c+dx)^5}-\frac{21b^3(bc-ad)^6}{d^{10}(c+dx)^4}+\frac{42b^4(\ldots}{d^{10}(c\ldots}\end{aligned}$$

Mathematica [B] time = 0.25977, size = 584, normalized size = 2.52

$$-\frac{-6a^2b^7cd^2\left(20139c^4d^2x^2+30625c^3d^3x^3+26950c^2d^4x^4+7203c^5dx+1089c^6+13230cd^5x^5+2940d^6x^6\right)+840a^3b^6d^3\,(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9/(c + d*x)^8,x]
```

```
[Out] -(10*a^9*d^9 + 15*a^8*b*d^8*(c + 7*d*x) + 24*a^7*b^2*d^7*(c^2 + 7*c*d*x + 2
1*d^2*x^2) + 42*a^6*b^3*d^6*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3) +
 84*a^5*b^4*d^5*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x
^4) + 210*a^4*b^5*d^4*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x^3 +
35*c*d^4*x^4 + 21*d^5*x^5) + 840*a^3*b^6*d^3*(c^6 + 7*c^5*d*x + 21*c^4*d^2*
x^2 + 35*c^3*d^3*x^3 + 35*c^2*d^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6) - 6*a^2*b
^7*c*d^2*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*x^3 +
 26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6) + 6*a*b^8*d*(1443*c^8
+ 9261*c^7*d*x + 24843*c^6*d^2*x^2 + 35525*c^5*d^3*x^3 + 28175*c^4*d^4*x^4
+ 11025*c^3*d^5*x^5 + 735*c^2*d^6*x^6 - 735*c*d^7*x^7 - 105*d^8*x^8) - b^9*
(3349*c^9 + 20923*c^8*d*x + 53949*c^7*d^2*x^2 + 72275*c^6*d^3*x^3 + 50225*c
^5*d^4*x^4 + 12495*c^4*d^5*x^5 - 4655*c^3*d^6*x^6 - 3185*c^2*d^7*x^7 - 315*
c*d^8*x^8 + 35*d^9*x^9) - 2520*b^7*(b*c - a*d)^2*(c + d*x)^7*Log[c + d*x])/
(70*d^10*(c + d*x)^7)
```

Maple [B] time = 0.015, size = 1035, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9/(d*x+c)^8,x)
```

```
[Out] 1/2*b^9*x^2/d^8-72*b^8/d^9*ln(d*x+c)*a*c+9/7/d^2/(d*x+c)^7*a^8*b*c-36/7/d^3
/(d*x+c)^7*c^2*a^7*b^2+12/d^4/(d*x+c)^7*c^3*a^6*b^3-18/d^5/(d*x+c)^7*a^5*b^
4*c^4-315*b^5/d^6/(d*x+c)^4*a^4*c^2+420*b^6/d^7/(d*x+c)^4*a^3*c^3-315*b^7/d
^8/(d*x+c)^4*a^2*c^4+126*b^8/d^9/(d*x+c)^4*a*c^5+12*b^2/d^3/(d*x+c)^6*a^7*c
-42*b^3/d^4/(d*x+c)^6*a^6*c^2+84*b^4/d^5/(d*x+c)^6*a^5*c^3-105*b^5/d^6/(d*x
+c)^6*a^4*c^4+84*b^6/d^7/(d*x+c)^6*a^3*c^5-42*b^7/d^8/(d*x+c)^6*a^2*c^6+12*
b^8/d^9/(d*x+c)^6*a*c^7+252*b^6/d^7/(d*x+c)^2*a^3*c-378*b^7/d^8/(d*x+c)^2*a
^2*c^2+252*b^8/d^9/(d*x+c)^2*a*c^3+126*b^4/d^5/(d*x+c)^4*a^5*c-252*b^8/d^9/
(d*x+c)*a*c^2+252/5*b^3/d^4/(d*x+c)^5*a^6*c-756/5*b^4/d^5/(d*x+c)^5*a^5*c^2
+252*b^5/d^6/(d*x+c)^5*a^4*c^3-252*b^6/d^7/(d*x+c)^5*a^3*c^4+756/5*b^7/d^8/
(d*x+c)^5*a^2*c^5-252/5*b^8/d^9/(d*x+c)^5*a*c^6+9*b^8/d^8*a*x-8*b^9/d^9*x*c
-21*b^3/d^4/(d*x+c)^4*a^6-21*b^9/d^10/(d*x+c)^4*c^6+36/5*b^9/d^10/(d*x+c)^5
*c^7-63*b^5/d^6/(d*x+c)^2*a^4-63*b^9/d^10/(d*x+c)^2*c^4-42*b^4/d^5/(d*x+c)^
3*a^5+42*b^9/d^10/(d*x+c)^3*c^5-3/2*b/d^2/(d*x+c)^6*a^8-3/2*b^9/d^10/(d*x+c
)^6*c^8-36/5*b^2/d^3/(d*x+c)^5*a^7+1/7/d^10/(d*x+c)^7*b^9*c^9-84*b^6/d^7/(d
*x+c)*a^3+84*b^9/d^10/(d*x+c)*c^3+36*b^7/d^8*ln(d*x+c)*a^2+36*b^9/d^10*ln(d
*x+c)*c^2-1/7/d/(d*x+c)^7*a^9+210*b^5/d^6/(d*x+c)^3*a^4*c-420*b^6/d^7/(d*x+
c)^3*a^3*c^2+420*b^7/d^8/(d*x+c)^3*a^2*c^3-210*b^8/d^9/(d*x+c)^3*a*c^4+252*
b^7/d^8/(d*x+c)*a^2*c+18/d^6/(d*x+c)^7*a^4*b^5*c^5-12/d^7/(d*x+c)^7*a^3*b^6
*c^6+36/7/d^8/(d*x+c)^7*a^2*b^7*c^7-9/7/d^9/(d*x+c)^7*a*b^8*c^8
```

Maxima [B] time = 1.26394, size = 1061, normalized size = 4.57

$$\frac{3349\, b^9 c^9 - 8658\, a b^8 c^8 d + 6534\, a^2 b^7 c^7 d^2 - 840\, a^3 b^6 c^6 d^3 - 210\, a^4 b^5 c^5 d^4 - 84\, a^5 b^4 c^4 d^5 - 42\, a^6 b^3 c^3 d^6 - 24\, a^7 b^2 c^2 d^7 - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/70*(3349*b^9*c^9 - 8658*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d^2 - 840*a^3*b^6*
c^6*d^3 - 210*a^4*b^5*c^5*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^6*b^3*c^3*d^6 - 2
4*a^7*b^2*c^2*d^7 - 15*a^8*b*c*d^8 - 10*a^9*d^9 + 5880*(b^9*c^3*d^6 - 3*a*b
^8*c^2*d^7 + 3*a^2*b^7*c*d^8 - a^3*b^6*d^9)*x^6 + 4410*(7*b^9*c^4*d^5 - 20*
a*b^8*c^3*d^6 + 18*a^2*b^7*c^2*d^7 - 4*a^3*b^6*c*d^8 - a^4*b^5*d^9)*x^5 + 1
470*(47*b^9*c^5*d^4 - 130*a*b^8*c^4*d^5 + 110*a^2*b^7*c^3*d^6 - 20*a^3*b^6*
c^2*d^7 - 5*a^4*b^5*c*d^8 - 2*a^5*b^4*d^9)*x^4 + 1470*(57*b^9*c^6*d^3 - 154
*a*b^8*c^5*d^4 + 125*a^2*b^7*c^4*d^5 - 20*a^3*b^6*c^3*d^6 - 5*a^4*b^5*c^2*d
^7 - 2*a^5*b^4*c*d^8 - a^6*b^3*d^9)*x^3 + 126*(459*b^9*c^7*d^2 - 1218*a*b^8
*c^6*d^3 + 959*a^2*b^7*c^5*d^4 - 140*a^3*b^6*c^4*d^5 - 35*a^4*b^5*c^3*d^6 -
 14*a^5*b^4*c^2*d^7 - 7*a^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 + 21*(1023*b^9*c
^8*d - 2676*a*b^8*c^7*d^2 + 2058*a^2*b^7*c^6*d^3 - 280*a^3*b^6*c^5*d^4 - 70
*a^4*b^5*c^4*d^5 - 28*a^5*b^4*c^3*d^6 - 14*a^6*b^3*c^2*d^7 - 8*a^7*b^2*c*d^
8 - 5*a^8*b*d^9)*x)/(d^17*x^7 + 7*c*d^16*x^6 + 21*c^2*d^15*x^5 + 35*c^3*d^1
4*x^4 + 35*c^4*d^13*x^3 + 21*c^5*d^12*x^2 + 7*c^6*d^11*x + c^7*d^10) + 1/2*
(b^9*d*x^2 - 2*(8*b^9*c - 9*a*b^8*d)*x)/d^9 + 36*(b^9*c^2 - 2*a*b^8*c*d + a
^2*b^7*d^2)*log(d*x + c)/d^10
```

Fricas [B] time = 1.89062, size = 2298, normalized size = 9.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/70*(35*b^9*d^9*x^9 + 3349*b^9*c^9 - 8658*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d
^2 - 840*a^3*b^6*c^6*d^3 - 210*a^4*b^5*c^5*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^
6*b^3*c^3*d^6 - 24*a^7*b^2*c^2*d^7 - 15*a^8*b*c*d^8 - 10*a^9*d^9 - 315*(b^9
*c*d^8 - 2*a*b^8*d^9)*x^8 - 245*(13*b^9*c^2*d^7 - 18*a*b^8*c*d^8)*x^7 - 245
*(19*b^9*c^3*d^6 + 18*a*b^8*c^2*d^7 - 72*a^2*b^7*c*d^8 + 24*a^3*b^6*d^9)*x^
6 + 735*(17*b^9*c^4*d^5 - 90*a*b^8*c^3*d^6 + 108*a^2*b^7*c^2*d^7 - 24*a^3*b
^6*c*d^8 - 6*a^4*b^5*d^9)*x^5 + 245*(205*b^9*c^5*d^4 - 690*a*b^8*c^4*d^5 +
660*a^2*b^7*c^3*d^6 - 120*a^3*b^6*c^2*d^7 - 30*a^4*b^5*c*d^8 - 12*a^5*b^4*d
^9)*x^4 + 245*(295*b^9*c^6*d^3 - 870*a*b^8*c^5*d^4 + 750*a^2*b^7*c^4*d^5 -
120*a^3*b^6*c^3*d^6 - 30*a^4*b^5*c^2*d^7 - 12*a^5*b^4*c*d^8 - 6*a^6*b^3*d^9
)*x^3 + 21*(2569*b^9*c^7*d^2 - 7098*a*b^8*c^6*d^3 + 5754*a^2*b^7*c^5*d^4 -
840*a^3*b^6*c^4*d^5 - 210*a^4*b^5*c^3*d^6 - 84*a^5*b^4*c^2*d^7 - 42*a^6*b^3
*c*d^8 - 24*a^7*b^2*d^9)*x^2 + 7*(2989*b^9*c^8*d - 7938*a*b^8*c^7*d^2 + 617
4*a^2*b^7*c^6*d^3 - 840*a^3*b^6*c^5*d^4 - 210*a^4*b^5*c^4*d^5 - 84*a^5*b^4*
c^3*d^6 - 42*a^6*b^3*c^2*d^7 - 24*a^7*b^2*c*d^8 - 15*a^8*b*d^9)*x + 2520*(b
^9*c^9 - 2*a*b^8*c^8*d + a^2*b^7*c^7*d^2 + (b^9*c^2*d^7 - 2*a*b^8*c*d^8 + a
^2*b^7*d^9)*x^7 + 7*(b^9*c^3*d^6 - 2*a*b^8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 2
1*(b^9*c^4*d^5 - 2*a*b^8*c^3*d^6 + a^2*b^7*c^2*d^7)*x^5 + 35*(b^9*c^5*d^4 -
 2*a*b^8*c^4*d^5 + a^2*b^7*c^3*d^6)*x^4 + 35*(b^9*c^6*d^3 - 2*a*b^8*c^5*d^4
 + a^2*b^7*c^4*d^5)*x^3 + 21*(b^9*c^7*d^2 - 2*a*b^8*c^6*d^3 + a^2*b^7*c^5*d
^4)*x^2 + 7*(b^9*c^8*d - 2*a*b^8*c^7*d^2 + a^2*b^7*c^6*d^3)*x)*log(d*x + c)
)/(d^17*x^7 + 7*c*d^16*x^6 + 21*c^2*d^15*x^5 + 35*c^3*d^14*x^4 + 35*c^4*d^1
3*x^3 + 21*c^5*d^12*x^2 + 7*c^6*d^11*x + c^7*d^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06169, size = 976, normalized size = 4.21

$$\frac{36\left(b^9c^2 - 2\,ab^8cd + a^2b^7d^2\right)\log\left(|dx+c|\right)}{d^{10}} + \frac{b^9d^8x^2 - 16\,b^9cd^7x + 18\,ab^8d^8x}{2\,d^{16}} + \frac{3349\,b^9c^9 - 8658\,ab^8c^8d + 6534\,a^2b^7c^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 36*(b^9*c^2 - 2*a*b^8*c*d + a^2*b^7*d^2)*log(abs(d*x + c))/d^10 + 1/2*(b^9*
d^8*x^2 - 16*b^9*c*d^7*x + 18*a*b^8*d^8*x)/d^16 + 1/70*(3349*b^9*c^9 - 8658
*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d^2 - 840*a^3*b^6*c^6*d^3 - 210*a^4*b^5*c^5
*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^6*b^3*c^3*d^6 - 24*a^7*b^2*c^2*d^7 - 15*a^
8*b*c*d^8 - 10*a^9*d^9 + 5880*(b^9*c^3*d^6 - 3*a*b^8*c^2*d^7 + 3*a^2*b^7*c*
d^8 - a^3*b^6*d^9)*x^6 + 4410*(7*b^9*c^4*d^5 - 20*a*b^8*c^3*d^6 + 18*a^2*b^
7*c^2*d^7 - 4*a^3*b^6*c*d^8 - a^4*b^5*d^9)*x^5 + 1470*(47*b^9*c^5*d^4 - 130
*a*b^8*c^4*d^5 + 110*a^2*b^7*c^3*d^6 - 20*a^3*b^6*c^2*d^7 - 5*a^4*b^5*c*d^8
 - 2*a^5*b^4*d^9)*x^4 + 1470*(57*b^9*c^6*d^3 - 154*a*b^8*c^5*d^4 + 125*a^2*
b^7*c^4*d^5 - 20*a^3*b^6*c^3*d^6 - 5*a^4*b^5*c^2*d^7 - 2*a^5*b^4*c*d^8 - a^
6*b^3*d^9)*x^3 + 126*(459*b^9*c^7*d^2 - 1218*a*b^8*c^6*d^3 + 959*a^2*b^7*c^
5*d^4 - 140*a^3*b^6*c^4*d^5 - 35*a^4*b^5*c^3*d^6 - 14*a^5*b^4*c^2*d^7 - 7*a
^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 + 21*(1023*b^9*c^8*d - 2676*a*b^8*c^7*d^2
 + 2058*a^2*b^7*c^6*d^3 - 280*a^3*b^6*c^5*d^4 - 70*a^4*b^5*c^4*d^5 - 28*a^5
*b^4*c^3*d^6 - 14*a^6*b^3*c^2*d^7 - 8*a^7*b^2*c*d^8 - 5*a^8*b*d^9)*x)/((d*x
 + c)^7*d^10)
```

3.1363 $\int \frac{(a+bx)^8}{(c+dx)^8} dx$

Optimal. Leaf size=209

$$-\frac{28b^6(bc-ad)^2}{d^9(c+dx)}+\frac{28b^5(bc-ad)^3}{d^9(c+dx)^2}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}-\frac{8b^7(bc-ad)\log(c+dx)}{d^9}+\frac{4b(}{3d^9}$$

```
[Out] (b^8*x)/d^8 - (b*c - a*d)^8/(7*d^9*(c + d*x)^7) + (4*b*(b*c - a*d)^7)/(3*d^
9*(c + d*x)^6) - (28*b^2*(b*c - a*d)^6)/(5*d^9*(c + d*x)^5) + (14*b^3*(b*c
- a*d)^5)/(d^9*(c + d*x)^4) - (70*b^4*(b*c - a*d)^4)/(3*d^9*(c + d*x)^3) +
(28*b^5*(b*c - a*d)^3)/(d^9*(c + d*x)^2) - (28*b^6*(b*c - a*d)^2)/(d^9*(c +
 d*x)) - (8*b^7*(b*c - a*d)*Log[c + d*x])/d^9
```

Rubi [A] time = 0.277555, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{28b^6(bc-ad)^2}{d^9(c+dx)}+\frac{28b^5(bc-ad)^3}{d^9(c+dx)^2}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}-\frac{8b^7(bc-ad)\log(c+dx)}{d^9}+\frac{4b(}{3d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8/(c + d*x)^8,x]
```

```
[Out] (b^8*x)/d^8 - (b*c - a*d)^8/(7*d^9*(c + d*x)^7) + (4*b*(b*c - a*d)^7)/(3*d^
9*(c + d*x)^6) - (28*b^2*(b*c - a*d)^6)/(5*d^9*(c + d*x)^5) + (14*b^3*(b*c
- a*d)^5)/(d^9*(c + d*x)^4) - (70*b^4*(b*c - a*d)^4)/(3*d^9*(c + d*x)^3) +
(28*b^5*(b*c - a*d)^3)/(d^9*(c + d*x)^2) - (28*b^6*(b*c - a*d)^2)/(d^9*(c +
 d*x)) - (8*b^7*(b*c - a*d)*Log[c + d*x])/d^9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^8}{(c+dx)^8} dx = \int \left(\frac{b^8}{d^8}+\frac{(-bc+ad)^8}{d^8(c+dx)^8}-\frac{8b(bc-ad)^7}{d^8(c+dx)^7}+\frac{28b^2(bc-ad)^6}{d^8(c+dx)^6}-\frac{56b^3(bc-ad)^5}{d^8(c+dx)^5}+\frac{70b^4(bc-ad)^4}{d^8(c+dx)^4}-\frac{56b^5(bc-}{d^8(c+}\right.$$

$$=\frac{b^8x}{d^8}-\frac{(bc-ad)^8}{7d^9(c+dx)^7}+\frac{4b(bc-ad)^7}{3d^9(c+dx)^6}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}+\frac{28b^5(bc-}{d^9(c+dx}$$

Mathematica [B] time = 0.194985, size = 474, normalized size = 2.27

$$-\frac{420a^2b^6d^2\left(21c^4d^2x^2+35c^3d^3x^3+35c^2d^4x^4+7c^5dx+c^6+21cd^5x^5+7d^6x^6\right)+140a^3b^5d^3\left(21c^3d^2x^2+35c^2d^3x^3+7c^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8/(c + d*x)^8,x]
```

```
[Out] -(15*a^8*d^8 + 20*a^7*b*d^7*(c + 7*d*x) + 28*a^6*b^2*d^6*(c^2 + 7*c*d*x + 2
1*d^2*x^2) + 42*a^5*b^3*d^5*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3) +
 70*a^4*b^4*d^4*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x
^4) + 140*a^3*b^5*d^3*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x^3 +
35*c*d^4*x^4 + 21*d^5*x^5) + 420*a^2*b^6*d^2*(c^6 + 7*c^5*d*x + 21*c^4*d^2*
x^2 + 35*c^3*d^3*x^3 + 35*c^2*d^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6) - 2*a*b^7
*c*d*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*x^3 + 269
50*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6) + b^8*(1443*c^8 + 9261*c^7
*d*x + 24843*c^6*d^2*x^2 + 35525*c^5*d^3*x^3 + 28175*c^4*d^4*x^4 + 11025*c^
3*d^5*x^5 + 735*c^2*d^6*x^6 - 735*c*d^7*x^7 - 105*d^8*x^8) + 840*b^7*(b*c -
 a*d)*(c + d*x)^7*Log[c + d*x])/(105*d^9*(c + d*x)^7)
```

Maple [B] time = 0.013, size = 845, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8/(d*x+c)^8,x)
```

```
[Out] -1/7/d/(d*x+c)^7*a^8+b^8*x/d^8-28*b^5/d^6/(d*x+c)^2*a^3+28*b^8/d^9/(d*x+c)^
2*c^3-14*b^3/d^4/(d*x+c)^4*a^5+14*b^8/d^9/(d*x+c)^4*c^5-28/5*b^2/d^3/(d*x+c
)^5*a^6-28/5*b^8/d^9/(d*x+c)^5*c^6-4/3*b/d^2/(d*x+c)^6*a^7+4/3*b^8/d^9/(d*x
+c)^6*c^7-28*b^6/d^7/(d*x+c)*a^2-28*b^8/d^9/(d*x+c)*c^2+8*b^7/d^8*ln(d*x+c)
*a-8*b^8/d^9*ln(d*x+c)*c-70/3*b^4/d^5/(d*x+c)^3*a^4-70/3*b^8/d^9/(d*x+c)^3*
c^4-1/7/d^9/(d*x+c)^7*b^8*c^8+280/3*b^5/d^6/(d*x+c)^3*a^3*c-140*b^6/d^7/(d*
x+c)^3*a^2*c^2-140*b^5/d^6/(d*x+c)^4*a^3*c^2+28/3*b^2/d^3/(d*x+c)^6*a^6*c-2
8*b^3/d^4/(d*x+c)^6*a^5*c^2+140/3*b^4/d^5/(d*x+c)^6*a^4*c^3-140/3*b^5/d^6/(
d*x+c)^6*a^3*c^4+28*b^6/d^7/(d*x+c)^6*a^2*c^5-28/3*b^7/d^8/(d*x+c)^6*a*c^6+
56*b^7/d^8/(d*x+c)*a*c+140*b^6/d^7/(d*x+c)^4*a^2*c^3-70*b^7/d^8/(d*x+c)^4*a
*c^4+168/5*b^3/d^4/(d*x+c)^5*a^5*c-84*b^4/d^5/(d*x+c)^5*a^4*c^2+112*b^5/d^6
/(d*x+c)^5*a^3*c^3-84*b^6/d^7/(d*x+c)^5*a^2*c^4+168/5*b^7/d^8/(d*x+c)^5*a*c
^5+280/3*b^7/d^8/(d*x+c)^3*a*c^3+8/7/d^2/(d*x+c)^7*a^7*b*c-4/d^3/(d*x+c)^7*
a^6*b^2*c^2+8/d^4/(d*x+c)^7*a^5*b^3*c^3-10/d^5/(d*x+c)^7*a^4*b^4*c^4+8/d^6/
(d*x+c)^7*a^3*b^5*c^5-4/d^7/(d*x+c)^7*a^2*b^6*c^6+8/7/d^8/(d*x+c)^7*a*b^7*c
^7+84*b^6/d^7/(d*x+c)^2*a^2*c-84*b^7/d^8/(d*x+c)^2*a*c^2+70*b^4/d^5/(d*x+c)
^4*a^4*c
```

Maxima [B] time = 1.14791, size = 876, normalized size = 4.19

$$\frac{b^8x}{d^8} - \frac{1443\,b^8c^8 - 2178\,ab^7c^7d + 420\,a^2b^6c^6d^2 + 140\,a^3b^5c^5d^3 + 70\,a^4b^4c^4d^4 + 42\,a^5b^3c^3d^5 + 28\,a^6b^2c^2d^6 + 20\,a^7bcd^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] b^8*x/d^8 - 1/105*(1443*b^8*c^8 - 2178*a*b^7*c^7*d + 420*a^2*b^6*c^6*d^2 +
140*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 + 20*a^7*b*c*d^7 + 15*a^8*d^8 + 2940*(b^8*c^2*d^6 - 2*a*b^7*c*d^7 +
 a^2*b^6*d^8)*x^6 + 2940*(5*b^8*c^3*d^5 - 9*a*b^7*c^2*d^6 + 3*a^2*b^6*c*d^7
```

```
+ a^3*b^5*d^8)*x^5 + 2450*(13*b^8*c^4*d^4 - 22*a*b^7*c^3*d^5 + 6*a^2*b^6*c
^2*d^6 + 2*a^3*b^5*c*d^7 + a^4*b^4*d^8)*x^4 + 490*(77*b^8*c^5*d^3 - 125*a*b
^7*c^4*d^4 + 30*a^2*b^6*c^3*d^5 + 10*a^3*b^5*c^2*d^6 + 5*a^4*b^4*c*d^7 + 3*
a^5*b^3*d^8)*x^3 + 294*(87*b^8*c^6*d^2 - 137*a*b^7*c^5*d^3 + 30*a^2*b^6*c^4
*d^4 + 10*a^3*b^5*c^3*d^5 + 5*a^4*b^4*c^2*d^6 + 3*a^5*b^3*c*d^7 + 2*a^6*b^2
*d^8)*x^2 + 14*(669*b^8*c^7*d - 1029*a*b^7*c^6*d^2 + 210*a^2*b^6*c^5*d^3 +
70*a^3*b^5*c^4*d^4 + 35*a^4*b^4*c^3*d^5 + 21*a^5*b^3*c^2*d^6 + 14*a^6*b^2*c
*d^7 + 10*a^7*b*d^8)*x)/(d^16*x^7 + 7*c*d^15*x^6 + 21*c^2*d^14*x^5 + 35*c^3
*d^13*x^4 + 35*c^4*d^12*x^3 + 21*c^5*d^11*x^2 + 7*c^6*d^10*x + c^7*d^9) - 8
*(b^8*c - a*b^7*d)*log(d*x + c)/d^9
```

Fricas [B] time = 2.0194, size = 1770, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*b^8*d^8*x^8 + 735*b^8*c*d^7*x^7 - 1443*b^8*c^8 + 2178*a*b^7*c^7*
d - 420*a^2*b^6*c^6*d^2 - 140*a^3*b^5*c^5*d^3 - 70*a^4*b^4*c^4*d^4 - 42*a^5
*b^3*c^3*d^5 - 28*a^6*b^2*c^2*d^6 - 20*a^7*b*c*d^7 - 15*a^8*d^8 - 735*(b^8*
c^2*d^6 - 8*a*b^7*c*d^7 + 4*a^2*b^6*d^8)*x^6 - 735*(15*b^8*c^3*d^5 - 36*a*b
^7*c^2*d^6 + 12*a^2*b^6*c*d^7 + 4*a^3*b^5*d^8)*x^5 - 1225*(23*b^8*c^4*d^4 -
 44*a*b^7*c^3*d^5 + 12*a^2*b^6*c^2*d^6 + 4*a^3*b^5*c*d^7 + 2*a^4*b^4*d^8)*x
^4 - 245*(145*b^8*c^5*d^3 - 250*a*b^7*c^4*d^4 + 60*a^2*b^6*c^3*d^5 + 20*a^3
*b^5*c^2*d^6 + 10*a^4*b^4*c*d^7 + 6*a^5*b^3*d^8)*x^3 - 147*(169*b^8*c^6*d^2
 - 274*a*b^7*c^5*d^3 + 60*a^2*b^6*c^4*d^4 + 20*a^3*b^5*c^3*d^5 + 10*a^4*b^4
*c^2*d^6 + 6*a^5*b^3*c*d^7 + 4*a^6*b^2*d^8)*x^2 - 7*(1323*b^8*c^7*d - 2058*
a*b^7*c^6*d^2 + 420*a^2*b^6*c^5*d^3 + 140*a^3*b^5*c^4*d^4 + 70*a^4*b^4*c^3*
d^5 + 42*a^5*b^3*c^2*d^6 + 28*a^6*b^2*c*d^7 + 20*a^7*b*d^8)*x - 840*(b^8*c^
8 - a*b^7*c^7*d + (b^8*c*d^7 - a*b^7*d^8)*x^7 + 7*(b^8*c^2*d^6 - a*b^7*c*d^
7)*x^6 + 21*(b^8*c^3*d^5 - a*b^7*c^2*d^6)*x^5 + 35*(b^8*c^4*d^4 - a*b^7*c^3
*d^5)*x^4 + 35*(b^8*c^5*d^3 - a*b^7*c^4*d^4)*x^3 + 21*(b^8*c^6*d^2 - a*b^7*
c^5*d^3)*x^2 + 7*(b^8*c^7*d - a*b^7*c^6*d^2)*x)*log(d*x + c))/(d^16*x^7 + 7
*c*d^15*x^6 + 21*c^2*d^14*x^5 + 35*c^3*d^13*x^4 + 35*c^4*d^12*x^3 + 21*c^5*
d^11*x^2 + 7*c^6*d^10*x + c^7*d^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06049, size = 784, normalized size = 3.75

$$\frac{b^8x}{d^8} - \frac{8\left(b^8c - ab^7d\right)\log\left(|dx + c|\right)}{d^9} - \frac{1443\,b^8c^8 - 2178\,ab^7c^7d + 420\,a^2b^6c^6d^2 + 140\,a^3b^5c^5d^3 + 70\,a^4b^4c^4d^4 + 42\,a^5b^3c^3d^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] b^8*x/d^8 - 8*(b^8*c - a*b^7*d)*log(abs(d*x + c))/d^9 - 1/105*(1443*b^8*c^8
 - 2178*a*b^7*c^7*d + 420*a^2*b^6*c^6*d^2 + 140*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 + 20*a^7*b*c*d^7 + 15*a
^8*d^8 + 2940*(b^8*c^2*d^6 - 2*a*b^7*c*d^7 + a^2*b^6*d^8)*x^6 + 2940*(5*b^8
*c^3*d^5 - 9*a*b^7*c^2*d^6 + 3*a^2*b^6*c*d^7 + a^3*b^5*d^8)*x^5 + 2450*(13*
b^8*c^4*d^4 - 22*a*b^7*c^3*d^5 + 6*a^2*b^6*c^2*d^6 + 2*a^3*b^5*c*d^7 + a^4*
b^4*d^8)*x^4 + 490*(77*b^8*c^5*d^3 - 125*a*b^7*c^4*d^4 + 30*a^2*b^6*c^3*d^5
 + 10*a^3*b^5*c^2*d^6 + 5*a^4*b^4*c*d^7 + 3*a^5*b^3*d^8)*x^3 + 294*(87*b^8*
c^6*d^2 - 137*a*b^7*c^5*d^3 + 30*a^2*b^6*c^4*d^4 + 10*a^3*b^5*c^3*d^5 + 5*a
^4*b^4*c^2*d^6 + 3*a^5*b^3*c*d^7 + 2*a^6*b^2*d^8)*x^2 + 14*(669*b^8*c^7*d -
 1029*a*b^7*c^6*d^2 + 210*a^2*b^6*c^5*d^3 + 70*a^3*b^5*c^4*d^4 + 35*a^4*b^4
*c^3*d^5 + 21*a^5*b^3*c^2*d^6 + 14*a^6*b^2*c*d^7 + 10*a^7*b*d^8)*x)/((d*x +
 c)^7*d^9)
```

3.1364 $\int \frac{(a+bx)^7}{(c+dx)^8} dx$

Optimal. Leaf size=194

$$\frac{7b^6(bc-ad)}{d^8(c+dx)} - \frac{21b^5(bc-ad)^2}{2d^8(c+dx)^2} + \frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3} - \frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4} + \frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5} - \frac{7b(bc-ad)^6}{6d^8(c+dx)^6} + \frac{(bc-ad)^7}{7d^8(c+dx)^7} + \frac{b^7}{}$$

```
[Out] (b*c - a*d)^7/(7*d^8*(c + d*x)^7) - (7*b*(b*c - a*d)^6)/(6*d^8*(c + d*x)^6)
 + (21*b^2*(b*c - a*d)^5)/(5*d^8*(c + d*x)^5) - (35*b^3*(b*c - a*d)^4)/(4*d
^8*(c + d*x)^4) + (35*b^4*(b*c - a*d)^3)/(3*d^8*(c + d*x)^3) - (21*b^5*(b*c
 - a*d)^2)/(2*d^8*(c + d*x)^2) + (7*b^6*(b*c - a*d))/(d^8*(c + d*x)) + (b^7
*Log[c + d*x])/d^8
```

Rubi [A] time = 0.209351, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7b^6(bc-ad)}{d^8(c+dx)} - \frac{21b^5(bc-ad)^2}{2d^8(c+dx)^2} + \frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3} - \frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4} + \frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5} - \frac{7b(bc-ad)^6}{6d^8(c+dx)^6} + \frac{(bc-ad)^7}{7d^8(c+dx)^7} + \frac{b^7}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)^7/(7*d^8*(c + d*x)^7) - (7*b*(b*c - a*d)^6)/(6*d^8*(c + d*x)^6)
 + (21*b^2*(b*c - a*d)^5)/(5*d^8*(c + d*x)^5) - (35*b^3*(b*c - a*d)^4)/(4*d
^8*(c + d*x)^4) + (35*b^4*(b*c - a*d)^3)/(3*d^8*(c + d*x)^3) - (21*b^5*(b*c
 - a*d)^2)/(2*d^8*(c + d*x)^2) + (7*b^6*(b*c - a*d))/(d^8*(c + d*x)) + (b^7
*Log[c + d*x])/d^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{(c+dx)^8} dx &= \int \left(\frac{(-bc+ad)^7}{d^7(c+dx)^8} + \frac{7b(bc-ad)^6}{d^7(c+dx)^7} - \frac{21b^2(bc-ad)^5}{d^7(c+dx)^6} + \frac{35b^3(bc-ad)^4}{d^7(c+dx)^5} - \frac{35b^4(bc-ad)^3}{d^7(c+dx)^4} + \frac{21b^5(bc-ad)^2}{d^7(c+dx)^3}\right.\\ &= \frac{(bc-ad)^7}{7d^8(c+dx)^7} - \frac{7b(bc-ad)^6}{6d^8(c+dx)^6} + \frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5} - \frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4} + \frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3} - \frac{21b^5(bc-ad)^2}{2d^8(c+dx)^2} +\end{aligned}$$

Mathematica [A] time = 0.15939, size = 308, normalized size = 1.59

$$\frac{(bc-ad)\left(a^2b^4d^2\left(6909c^2d^2x^2+2793c^3dx+459c^4+8575cd^3x^3+4900d^4x^4\right)+a^3b^3d^3\left(1813c^2dx+319c^3+3969cd^2x^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/(c + d*x)^8,x]
```

```
[Out] ((b*c - a*d)*(60*a^6*d^6 + 10*a^5*b*d^5*(13*c + 49*d*x) + 2*a^4*b^2*d^4*(10
7*c^2 + 539*c*d*x + 882*d^2*x^2) + a^3*b^3*d^3*(319*c^3 + 1813*c^2*d*x + 39
69*c*d^2*x^2 + 3675*d^3*x^3) + a^2*b^4*d^2*(459*c^4 + 2793*c^3*d*x + 6909*c
^2*d^2*x^2 + 8575*c*d^3*x^3 + 4900*d^4*x^4) + a*b^5*d*(669*c^5 + 4263*c^4*d
*x + 11319*c^3*d^2*x^2 + 15925*c^2*d^3*x^3 + 12250*c*d^4*x^4 + 4410*d^5*x^5
) + b^6*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*x^3 +
26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6)))/(420*d^8*(c + d*x)^7)
 + (b^7*Log[c + d*x])/d^8
```

Maple [B] time = 0.008, size = 672, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/(d*x+c)^8,x)
```

```
[Out] -1/7/d/(d*x+c)^7*a^7+70/3*b^4/d^5/(d*x+c)^6*a^3*c^3-35/2*b^5/d^6/(d*x+c)^6*
a^2*c^4+7*b^2/d^3/(d*x+c)^6*a^5*c-35/2*b^3/d^4/(d*x+c)^6*a^4*c^2+21*b^6/d^7
/(d*x+c)^2*a*c+35*b^6/d^7/(d*x+c)^4*a*c^3-105/2*b^5/d^6/(d*x+c)^4*a^2*c^2-3
/d^3/(d*x+c)^7*a^5*b^2*c^2+5/d^4/(d*x+c)^7*a^4*b^3*c^3-5/d^5/(d*x+c)^7*a^3*
b^4*c^4+3/d^6/(d*x+c)^7*a^2*b^5*c^5-1/d^7/(d*x+c)^7*a*b^6*c^6+35*b^4/d^5/(d
*x+c)^4*a^3*c+1/d^2/(d*x+c)^7*a^6*b*c+b^7*ln(d*x+c)/d^8-35/4*b^7/d^8/(d*x+c
)^4*c^4-35/4*b^3/d^4/(d*x+c)^4*a^4+1/7/d^8/(d*x+c)^7*b^7*c^7-7*b^6/d^7/(d*x
+c)*a+7*b^7/d^8/(d*x+c)*c-35/3*b^4/d^5/(d*x+c)^3*a^3+35/3*b^7/d^8/(d*x+c)^3
*c^3-21/5*b^2/d^3/(d*x+c)^5*a^5+21/5*b^7/d^8/(d*x+c)^5*c^5-7/6*b/d^2/(d*x+c
)^6*a^6-7/6*b^7/d^8/(d*x+c)^6*c^6-21/2*b^5/d^6/(d*x+c)^2*a^2-21/2*b^7/d^8/(
d*x+c)^2*c^2+35*b^5/d^6/(d*x+c)^3*a^2*c-35*b^6/d^7/(d*x+c)^3*a*c^2+7*b^6/d^
7/(d*x+c)^6*a*c^5+21*b^3/d^4/(d*x+c)^5*a^4*c-42*b^4/d^5/(d*x+c)^5*a^3*c^2+4
2*b^5/d^6/(d*x+c)^5*a^2*c^3-21*b^6/d^7/(d*x+c)^5*a*c^4
```

Maxima [B] time = 1.04702, size = 722, normalized size = 3.72

$$\frac{1089\, b^7 c^7 - 420\, ab^6 c^6 d - 210\, a^2 b^5 c^5 d^2 - 140\, a^3 b^4 c^4 d^3 - 105\, a^4 b^3 c^3 d^4 - 84\, a^5 b^2 c^2 d^5 - 70\, a^6 bcd^6 - 60\, a^7 d^7 + 2940\, \big(b^7 c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/420*(1089*b^7*c^7 - 420*a*b^6*c^6*d - 210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c
^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7
*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(3*b^7*c^2*d^5 - 2*a*b^6*c*d
^6 - a^2*b^5*d^7)*x^5 + 2450*(11*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 - 3*a^2*b^5*
c*d^6 - 2*a^3*b^4*d^7)*x^4 + 1225*(25*b^7*c^4*d^3 - 12*a*b^6*c^3*d^4 - 6*a^
2*b^5*c^2*d^5 - 4*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 147*(137*b^7*c^5*d^2
 - 60*a*b^6*c^4*d^3 - 30*a^2*b^5*c^3*d^4 - 20*a^3*b^4*c^2*d^5 - 15*a^4*b^3*
c*d^6 - 12*a^5*b^2*d^7)*x^2 + 49*(147*b^7*c^6*d - 60*a*b^6*c^5*d^2 - 30*a^2
*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 - 15*a^4*b^3*c^2*d^5 - 12*a^5*b^2*c*d^6 -
 10*a^6*b*d^7)*x)/(d^15*x^7 + 7*c*d^14*x^6 + 21*c^2*d^13*x^5 + 35*c^3*d^12*
x^4 + 35*c^4*d^11*x^3 + 21*c^5*d^10*x^2 + 7*c^6*d^9*x + c^7*d^8) + b^7*log(
```

```
d*x + c)/d^8
```

Fricas [B] time = 1.82887, size = 1319, normalized size = 6.8

$$\frac{1089\, b^7c^7 - 420\, ab^6c^6d - 210\, a^2b^5c^5d^2 - 140\, a^3b^4c^4d^3 - 105\, a^4b^3c^3d^4 - 84\, a^5b^2c^2d^5 - 70\, a^6bcd^6 - 60\, a^7d^7 + 2940\left(b^7cd^6 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/420*(1089*b^7*c^7 - 420*a*b^6*c^6*d - 210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c
^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7
*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(3*b^7*c^2*d^5 - 2*a*b^6*c*d
^6 - a^2*b^5*d^7)*x^5 + 2450*(11*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 - 3*a^2*b^5*
c*d^6 - 2*a^3*b^4*d^7)*x^4 + 1225*(25*b^7*c^4*d^3 - 12*a*b^6*c^3*d^4 - 6*a^
2*b^5*c^2*d^5 - 4*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 147*(137*b^7*c^5*d^2
 - 60*a*b^6*c^4*d^3 - 30*a^2*b^5*c^3*d^4 - 20*a^3*b^4*c^2*d^5 - 15*a^4*b^3*
c*d^6 - 12*a^5*b^2*d^7)*x^2 + 49*(147*b^7*c^6*d - 60*a*b^6*c^5*d^2 - 30*a^2
*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 - 15*a^4*b^3*c^2*d^5 - 12*a^5*b^2*c*d^6 -
 10*a^6*b*d^7)*x + 420*(b^7*d^7*x^7 + 7*b^7*c*d^6*x^6 + 21*b^7*c^2*d^5*x^5
+ 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21*b^7*c^5*d^2*x^2 + 7*b^7*c^6*
d*x + b^7*c^7)*log(d*x + c))/(d^15*x^7 + 7*c*d^14*x^6 + 21*c^2*d^13*x^5 + 3
5*c^3*d^12*x^4 + 35*c^4*d^11*x^3 + 21*c^5*d^10*x^2 + 7*c^6*d^9*x + c^7*d^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.07333, size = 630, normalized size = 3.25

$$\frac{b^7\log\left(|dx+c|\right)}{d^8} + \frac{2940\left(b^7cd^5 - ab^6d^6\right)x^6 + 4410\left(3\,b^7c^2d^4 - 2\,ab^6cd^5 - a^2b^5d^6\right)x^5 + 2450\left(11\,b^7c^3d^3 - 6\,ab^6c^2d^4 - 3\,a^2b^5 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] b^7*log(abs(d*x + c))/d^8 + 1/420*(2940*(b^7*c*d^5 - a*b^6*d^6)*x^6 + 4410*
(3*b^7*c^2*d^4 - 2*a*b^6*c*d^5 - a^2*b^5*d^6)*x^5 + 2450*(11*b^7*c^3*d^3 -
6*a*b^6*c^2*d^4 - 3*a^2*b^5*c*d^5 - 2*a^3*b^4*d^6)*x^4 + 1225*(25*b^7*c^4*d
^2 - 12*a*b^6*c^3*d^3 - 6*a^2*b^5*c^2*d^4 - 4*a^3*b^4*c*d^5 - 3*a^4*b^3*d^6
)*x^3 + 147*(137*b^7*c^5*d - 60*a*b^6*c^4*d^2 - 30*a^2*b^5*c^3*d^3 - 20*a^3
*b^4*c^2*d^4 - 15*a^4*b^3*c*d^5 - 12*a^5*b^2*d^6)*x^2 + 49*(147*b^7*c^6 - 6
0*a*b^6*c^5*d - 30*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 - 15*a^4*b^3*c^2*d^
```

```
4 - 12*a^5*b^2*c*d^5 - 10*a^6*b*d^6)*x + (1089*b^7*c^7 - 420*a*b^6*c^6*d -
210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^
2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7*d^7)/d)/((d*x + c)^7*d^7)
```

3.1365 $\int \frac{(a+bx)^6}{(c+dx)^8} dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^7}{7(c+dx)^7(bc-ad)}$$

```
[Out] (a + b*x)^7/(7*(b*c - a*d)*(c + d*x)^7)
```

Rubi [A] time = 0.0029524, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{(a+bx)^7}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^7/(7*(b*c - a*d)*(c + d*x)^7)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^6}{(c+dx)^8} dx = \frac{(a+bx)^7}{7(bc-ad)(c+dx)^7}$$

Mathematica [B] time = 0.0892165, size = 271, normalized size = 9.68

$$-\frac{a^2b^4d^2\left(21c^2d^2x^2+7c^3dx+c^4+35cd^3x^3+35d^4x^4\right)+a^3b^3d^3\left(7c^2dx+c^3+21cd^2x^2+35d^3x^3\right)+a^4b^2d^4\left(c^2+7cdx+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(c + d*x)^8,x]
```

```
[Out] -(a^6*d^6 + a^5*b*d^5*(c + 7*d*x) + a^4*b^2*d^4*(c^2 + 7*c*d*x + 21*d^2*x^2
) + a^3*b^3*d^3*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3) + a^2*b^4*d^2
*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4) + a*b^5*d*(
c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x^3 + 35*c*d^4*x^4 + 21*d^5*x
^5) + b^6*(c^6 + 7*c^5*d*x + 21*c^4*d^2*x^2 + 35*c^3*d^3*x^3 + 35*c^2*d^4*x
^4 + 21*c*d^5*x^5 + 7*d^6*x^6))/(7*d^7*(c + d*x)^7)
```

Maple [B] time = 0.006, size = 357, normalized size = 12.8

$$-\frac{b^6}{d^7(dx+c)}-\frac{a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6}{7d^7(dx+c)^7}-3\frac{b^2\left(a^4d^4-4a^3bcd^3+\right.}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(d*x+c)^8,x)
```

```
[Out] -b^6/d^7/(d*x+c)-1/7*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c
^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/d^7/(d*x+c)^7-3*b^2*(a^4*d
^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/d^7/(d*x+c)^5-3*b
^5*(a*d-b*c)/d^7/(d*x+c)^2-5*b^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^7/(d*x+c)^3-
b*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*
d-b^5*c^5)/d^7/(d*x+c)^6-5*b^3*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3
)/d^7/(d*x+c)^4
```

Maxima [B] time = 1.05014, size = 537, normalized size = 19.18

$$-\frac{7b^6d^6x^6+b^6c^6+ab^5c^5d+a^2b^4c^4d^2+a^3b^3c^3d^3+a^4b^2c^2d^4+a^5bcd^5+a^6d^6+21\left(b^6cd^5+ab^5d^6\right)x^5+35\left(b^6c^2d^4+\right.}{7\left(d^{14}x^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + b^6*c^6 + a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3
*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c*d^5 + a^6*d^6 + 21*(b^6*c*d^5 + a*b^5*d^6)
*x^5 + 35*(b^6*c^2*d^4 + a*b^5*c*d^5 + a^2*b^4*d^6)*x^4 + 35*(b^6*c^3*d^3 +
 a*b^5*c^2*d^4 + a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^3 + 21*(b^6*c^4*d^2 + a*b^5
*c^3*d^3 + a^2*b^4*c^2*d^4 + a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^2 + 7*(b^6*c^5*
d + a*b^5*c^4*d^2 + a^2*b^4*c^3*d^3 + a^3*b^3*c^2*d^4 + a^4*b^2*c*d^5 + a^5
*b*d^6)*x)/(d^14*x^7 + 7*c*d^13*x^6 + 21*c^2*d^12*x^5 + 35*c^3*d^11*x^4 + 3
5*c^4*d^10*x^3 + 21*c^5*d^9*x^2 + 7*c^6*d^8*x + c^7*d^7)
```

Fricas [B] time = 1.69983, size = 787, normalized size = 28.11

$$-\frac{7b^6d^6x^6+b^6c^6+ab^5c^5d+a^2b^4c^4d^2+a^3b^3c^3d^3+a^4b^2c^2d^4+a^5bcd^5+a^6d^6+21\left(b^6cd^5+ab^5d^6\right)x^5+35\left(b^6c^2d^4+\right.}{7\left(d^{14}x^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + b^6*c^6 + a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3
*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c*d^5 + a^6*d^6 + 21*(b^6*c*d^5 + a*b^5*d^6)
*x^5 + 35*(b^6*c^2*d^4 + a*b^5*c*d^5 + a^2*b^4*d^6)*x^4 + 35*(b^6*c^3*d^3 +
 a*b^5*c^2*d^4 + a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^3 + 21*(b^6*c^4*d^2 + a*b^5
*c^3*d^3 + a^2*b^4*c^2*d^4 + a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^2 + 7*(b^6*c^5*
d + a*b^5*c^4*d^2 + a^2*b^4*c^3*d^3 + a^3*b^3*c^2*d^4 + a^4*b^2*c*d^5 + a^5
*b*d^6)*x)/(d^14*x^7 + 7*c*d^13*x^6 + 21*c^2*d^12*x^5 + 35*c^3*d^11*x^4 + 3
5*c^4*d^10*x^3 + 21*c^5*d^9*x^2 + 7*c^6*d^8*x + c^7*d^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.0643, size = 498, normalized size = 17.79

$$-\frac{7\,b^6d^6x^6 + 21\,b^6cd^5x^5 + 21\,ab^5d^6x^5 + 35\,b^6c^2d^4x^4 + 35\,ab^5cd^5x^4 + 35\,a^2b^4d^6x^4 + 35\,b^6c^3d^3x^3 + 35\,ab^5c^2d^4x^3 + 35\,a^2b^4c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + 21*b^6*c*d^5*x^5 + 21*a*b^5*d^6*x^5 + 35*b^6*c^2*d^4*
x^4 + 35*a*b^5*c*d^5*x^4 + 35*a^2*b^4*d^6*x^4 + 35*b^6*c^3*d^3*x^3 + 35*a*b
^5*c^2*d^4*x^3 + 35*a^2*b^4*c*d^5*x^3 + 35*a^3*b^3*d^6*x^3 + 21*b^6*c^4*d^2
*x^2 + 21*a*b^5*c^3*d^3*x^2 + 21*a^2*b^4*c^2*d^4*x^2 + 21*a^3*b^3*c*d^5*x^2
 + 21*a^4*b^2*d^6*x^2 + 7*b^6*c^5*d*x + 7*a*b^5*c^4*d^2*x + 7*a^2*b^4*c^3*d
^3*x + 7*a^3*b^3*c^2*d^4*x + 7*a^4*b^2*c*d^5*x + 7*a^5*b*d^6*x + b^6*c^6 +
a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c
*d^5 + a^6*d^6)/((d*x + c)^7*d^7)
```

3.1366 $\int \frac{(a+bx)^5}{(c+dx)^8}\,dx$

Optimal. Leaf size=58

$$\frac{b(a+bx)^6}{42(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^6}{7(c+dx)^7(bc-ad)}$$

```
[Out] (a + b*x)^6/(7*(b*c - a*d)*(c + d*x)^7) + (b*(a + b*x)^6)/(42*(b*c - a*d)^2
*(c + d*x)^6)
```

Rubi [A] time = 0.0112179, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{b(a+bx)^6}{42(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^6}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^6/(7*(b*c - a*d)*(c + d*x)^7) + (b*(a + b*x)^6)/(42*(b*c - a*d)^2
*(c + d*x)^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(c+dx)^8}\,dx &= \frac{(a+bx)^6}{7(bc-ad)(c+dx)^7}+\frac{b\int \frac{(a+bx)^5}{(c+dx)^7}\,dx}{7(bc-ad)}\\ &= \frac{(a+bx)^6}{7(bc-ad)(c+dx)^7}+\frac{b(a+bx)^6}{42(bc-ad)^2(c+dx)^6}\end{aligned}$$

Mathematica [B] time = 0.0577406, size = 205, normalized size = 3.53

$$-\frac{3a^2b^3d^2\left(7c^2dx+c^3+21cd^2x^2+35d^3x^3\right)+4a^3b^2d^3\left(c^2+7cdx+21d^2x^2\right)+5a^4bd^4(c+7dx)+6a^5d^5+2ab^4d\left(21c^2d\right.}{42d^6(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^8,x]
```

```
[Out] -(6*a^5*d^5 + 5*a^4*b*d^4*(c + 7*d*x) + 4*a^3*b^2*d^3*(c^2 + 7*c*d*x + 21*d
^2*x^2) + 3*a^2*b^3*d^2*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3) + 2*a
*b^4*d*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4) + b^5
*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x^3 + 35*c*d^4*x^4 + 21*d^5
*x^5))/(42*d^6*(c + d*x)^7)
```

Maple [B] time = 0.005, size = 265, normalized size = 4.6

$$-\frac{a^5d^5-5\,a^4bcd^4+10\,a^3b^2c^2d^3-10\,a^2b^3c^3d^2+5\,ab^4c^4d-b^5c^5}{7\,d^6\,(dx+c)^7}-\frac{b^5}{2\,d^6\,(dx+c)^2}-2\,\frac{b^2\left(a^3d^3-3\,a^2bcd^2+3\,ab^2c^2d-b^3c^3\right.}{d^6\,(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^8,x)
```

```
[Out] -1/7*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c
^4*d-b^5*c^5)/d^6/(d*x+c)^7-1/2*b^5/d^6/(d*x+c)^2-2*b^2*(a^3*d^3-3*a^2*b*c*
d^2+3*a*b^2*c^2*d-b^3*c^3)/d^6/(d*x+c)^5-5/2*b^3*(a^2*d^2-2*a*b*c*d+b^2*c^2
)/d^6/(d*x+c)^4-5/3*b^4*(a*d-b*c)/d^6/(d*x+c)^3-5/6*b*(a^4*d^4-4*a^3*b*c*d^
3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/d^6/(d*x+c)^6
```

Maxima [B] time = 1.01833, size = 440, normalized size = 7.59

$$-\frac{21\,b^5d^5x^5+b^5c^5+2\,ab^4c^4d+3\,a^2b^3c^3d^2+4\,a^3b^2c^2d^3+5\,a^4bcd^4+6\,a^5d^5+35\left(b^5cd^4+2\,ab^4d^5\right)x^4+35\left(b^5c^2d^3+2\,ab\right.}{42\left(d^{13}x^7+7\,cd^{12}x^6+21\,c^2d^{11}x^5+35\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3
*b^2*c^2*d^3 + 5*a^4*b*c*d^4 + 6*a^5*d^5 + 35*(b^5*c*d^4 + 2*a*b^4*d^5)*x^4
 + 35*(b^5*c^2*d^3 + 2*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 21*(b^5*c^3*d^2 +
 2*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 + 4*a^3*b^2*d^5)*x^2 + 7*(b^5*c^4*d + 2*
a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 4*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x)/(d^13
*x^7 + 7*c*d^12*x^6 + 21*c^2*d^11*x^5 + 35*c^3*d^10*x^4 + 35*c^4*d^9*x^3 +
21*c^5*d^8*x^2 + 7*c^6*d^7*x + c^7*d^6)
```

Fricas [B] time = 1.74847, size = 663, normalized size = 11.43

$$-\frac{21\,b^5d^5x^5+b^5c^5+2\,ab^4c^4d+3\,a^2b^3c^3d^2+4\,a^3b^2c^2d^3+5\,a^4bcd^4+6\,a^5d^5+35\left(b^5cd^4+2\,ab^4d^5\right)x^4+35\left(b^5c^2d^3+2\,ab\right.}{42\left(d^{13}x^7+7\,cd^{12}x^6+21\,c^2d^{11}x^5+35\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3
*b^2*c^2*d^3 + 5*a^4*b*c*d^4 + 6*a^5*d^5 + 35*(b^5*c*d^4 + 2*a*b^4*d^5)*x^4
 + 35*(b^5*c^2*d^3 + 2*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 21*(b^5*c^3*d^2 +
 2*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 + 4*a^3*b^2*d^5)*x^2 + 7*(b^5*c^4*d + 2*
a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 4*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x)/(d^13
*x^7 + 7*c*d^12*x^6 + 21*c^2*d^11*x^5 + 35*c^3*d^10*x^4 + 35*c^4*d^9*x^3 +
21*c^5*d^8*x^2 + 7*c^6*d^7*x + c^7*d^6)
```

Sympy [B] time = 52.1013, size = 348, normalized size = 6.

$$-\frac{6a^5d^5 + 5a^4bcd^4 + 4a^3b^2c^2d^3 + 3a^2b^3c^3d^2 + 2ab^4c^4d + b^5c^5 + 21b^5d^5x^5 + x^4\left(70ab^4d^5 + 35b^5cd^4\right) + x^3\left(105a^2b^3d^5 + \cdots\right.}{42c^7d^6 + 294c^6d^7x + 882c^5d^8x^2 + 14\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**8,x)
```

```
[Out] -(6*a**5*d**5 + 5*a**4*b*c*d**4 + 4*a**3*b**2*c**2*d**3 + 3*a**2*b**3*c**3*
d**2 + 2*a*b**4*c**4*d + b**5*c**5 + 21*b**5*d**5*x**5 + x**4*(70*a*b**4*d*
*5 + 35*b**5*c*d**4) + x**3*(105*a**2*b**3*d**5 + 70*a*b**4*c*d**4 + 35*b**
5*c**2*d**3) + x**2*(84*a**3*b**2*d**5 + 63*a**2*b**3*c*d**4 + 42*a*b**4*c*
*2*d**3 + 21*b**5*c**3*d**2) + x*(35*a**4*b*d**5 + 28*a**3*b**2*c*d**4 + 21
*a**2*b**3*c**2*d**3 + 14*a*b**4*c**3*d**2 + 7*b**5*c**4*d))/(42*c**7*d**6
+ 294*c**6*d**7*x + 882*c**5*d**8*x**2 + 1470*c**4*d**9*x**3 + 1470*c**3*d*
*10*x**4 + 882*c**2*d**11*x**5 + 294*c*d**12*x**6 + 42*d**13*x**7)
```

Giac [B] time = 1.06566, size = 366, normalized size = 6.31

$$-\frac{21\,b^5d^5x^5 + 35\,b^5cd^4x^4 + 70\,ab^4d^5x^4 + 35\,b^5c^2d^3x^3 + 70\,ab^4cd^4x^3 + 105\,a^2b^3d^5x^3 + 21\,b^5c^3d^2x^2 + 42\,ab^4c^2d^3x^2 + 63\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + 35*b^5*c*d^4*x^4 + 70*a*b^4*d^5*x^4 + 35*b^5*c^2*d^
3*x^3 + 70*a*b^4*c*d^4*x^3 + 105*a^2*b^3*d^5*x^3 + 21*b^5*c^3*d^2*x^2 + 42*
a*b^4*c^2*d^3*x^2 + 63*a^2*b^3*c*d^4*x^2 + 84*a^3*b^2*d^5*x^2 + 7*b^5*c^4*d
*x + 14*a*b^4*c^3*d^2*x + 21*a^2*b^3*c^2*d^3*x + 28*a^3*b^2*c*d^4*x + 35*a^
4*b*d^5*x + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3*b^2*c^2*d^3
 + 5*a^4*b*c*d^4 + 6*a^5*d^5)/((d*x + c)^7*d^6)
```

3.1367 $\int \frac{(a+bx)^4}{(c+dx)^8} dx$

Optimal. Leaf size=89

$$\frac{b^2(a+bx)^5}{105(c+dx)^5(bc-ad)^3}+\frac{b(a+bx)^5}{21(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^5}{7(c+dx)^7(bc-ad)}$$

```
[Out] (a + b*x)^5/(7*(b*c - a*d)*(c + d*x)^7) + (b*(a + b*x)^5)/(21*(b*c - a*d)^2
*(c + d*x)^6) + (b^2*(a + b*x)^5)/(105*(b*c - a*d)^3*(c + d*x)^5)
```

Rubi [A] time = 0.0192818, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{b^2(a+bx)^5}{105(c+dx)^5(bc-ad)^3}+\frac{b(a+bx)^5}{21(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^5}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^5/(7*(b*c - a*d)*(c + d*x)^7) + (b*(a + b*x)^5)/(21*(b*c - a*d)^2
*(c + d*x)^6) + (b^2*(a + b*x)^5)/(105*(b*c - a*d)^3*(c + d*x)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^4}{(c+dx)^8} dx &= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7}+\frac{(2b)\int \frac{(a+bx)^4}{(c+dx)^7} dx}{7(bc-ad)}\\
&= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7}+\frac{b(a+bx)^5}{21(bc-ad)^2(c+dx)^6}+\frac{b^2\int \frac{(a+bx)^4}{(c+dx)^6} dx}{21(bc-ad)^2}\\
&= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7}+\frac{b(a+bx)^5}{21(bc-ad)^2(c+dx)^6}+\frac{b^2(a+bx)^5}{105(bc-ad)^3(c+dx)^5}
\end{aligned}$$

Mathematica [A] time = 0.0478935, size = 144, normalized size = 1.62

$$-\frac{6a^2b^2d^2\left(c^2+7cdx+21d^2x^2\right)+10a^3bd^3(c+7dx)+15a^4d^4+3ab^3d\left(7c^2dx+c^3+21cd^2x^2+35d^3x^3\right)+b^4\left(21c^2d^2x^2\right.}{105d^5(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^8,x]
```

```
[Out] -(15*a^4*d^4 + 10*a^3*b*d^3*(c + 7*d*x) + 6*a^2*b^2*d^2*(c^2 + 7*c*d*x + 21
*d^2*x^2) + 3*a*b^3*d*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3) + b^4*(
c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4))/(105*d^5*(c
+ d*x)^7)
```

Maple [B] time = 0.005, size = 186, normalized size = 2.1

$$-\frac{6\,b^2\left(a^2d^2-2\,abcd+b^2c^2\right)}{5\,d^5\,(dx+c)^5}-\frac{a^4d^4-4\,a^3bcd^3+6\,a^2b^2c^2d^2-4\,c^3ab^3d+b^4c^4}{7\,d^5\,(dx+c)^7}-\frac{2\,b\left(a^3d^3-3\,a^2bcd^2+3\,ab^2c^2d-b^3c\right.}{3\,d^5\,(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^8,x)
```

```
[Out] -6/5*b^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^5/(d*x+c)^5-1/7*(a^4*d^4-4*a^3*b*c*d
^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/d^5/(d*x+c)^7-2/3*b*(a^3*d^3-3*
a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^5/(d*x+c)^6-1/3*b^4/d^5/(d*x+c)^3-b^3*
(a*d-b*c)/d^5/(d*x+c)^4
```

Maxima [B] time = 1.00428, size = 333, normalized size = 3.74

$$-\frac{35\,b^4d^4x^4+b^4c^4+3\,ab^3c^3d+6\,a^2b^2c^2d^2+10\,a^3bcd^3+15\,a^4d^4+35\left(b^4cd^3+3\,ab^3d^4\right)x^3+21\left(b^4c^2d^2+3\,ab^3cd^3+6\right.}{105\left(d^{12}x^7+7\,cd^{11}x^6+21\,c^2d^{10}x^5+35\,c^3d^9x^4+35\,c^4d^8x^3+21\,c^5d^7x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + b^4*c^4 + 3*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 10*a
^3*b*c*d^3 + 15*a^4*d^4 + 35*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 21*(b^4*c^2*d^
2 + 3*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 + 7*(b^4*c^3*d + 3*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3 + 10*a^3*b*d^4)*x)/(d^12*x^7 + 7*c*d^11*x^6 + 21*c^2*d^10*x^
5 + 35*c^3*d^9*x^4 + 35*c^4*d^8*x^3 + 21*c^5*d^7*x^2 + 7*c^6*d^6*x + c^7*d^
5)
```

Fricas [B] time = 1.48325, size = 512, normalized size = 5.75

$$-\frac{35\,b^4d^4x^4+b^4c^4+3\,ab^3c^3d+6\,a^2b^2c^2d^2+10\,a^3bcd^3+15\,a^4d^4+35\left(b^4cd^3+3\,ab^3d^4\right)x^3+21\left(b^4c^2d^2+3\,ab^3cd^3+6\right.}{105\left(d^{12}x^7+7\,cd^{11}x^6+21\,c^2d^{10}x^5+35\,c^3d^9x^4+35\,c^4d^8x^3+21\,c^5d^7x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + b^4*c^4 + 3*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 10*a
^3*b*c*d^3 + 15*a^4*d^4 + 35*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 21*(b^4*c^2*d^
2 + 3*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 + 7*(b^4*c^3*d + 3*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3 + 10*a^3*b*d^4)*x)/(d^12*x^7 + 7*c*d^11*x^6 + 21*c^2*d^10*x^
5 + 35*c^3*d^9*x^4 + 35*c^4*d^8*x^3 + 21*c^5*d^7*x^2 + 7*c^6*d^6*x + c^7*d^
5)
```

Sympy [B] time = 11.6131, size = 264, normalized size = 2.97

$$-\frac{15a^4d^4 + 10a^3bcd^3 + 6a^2b^2c^2d^2 + 3ab^3c^3d + b^4c^4 + 35b^4d^4x^4 + x^3\left(105ab^3d^4 + 35b^4cd^3\right) + x^2\left(126a^2b^2d^4 + 63ab^3cd^3 + \cdots\right.}{105c^7d^5 + 735c^6d^6x + 2205c^5d^7x^2 + 3675c^4d^8x^3 + 3675c^3d^9x^4 + 2205c^2d^{10}x^5 + \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**8,x)
```

```
[Out] -(15*a**4*d**4 + 10*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 + 3*a*b**3*c**3*d
 + b**4*c**4 + 35*b**4*d**4*x**4 + x**3*(105*a*b**3*d**4 + 35*b**4*c*d**3)
+ x**2*(126*a**2*b**2*d**4 + 63*a*b**3*c*d**3 + 21*b**4*c**2*d**2) + x*(70*
a**3*b*d**4 + 42*a**2*b**2*c*d**3 + 21*a*b**3*c**2*d**2 + 7*b**4*c**3*d))/(
105*c**7*d**5 + 735*c**6*d**6*x + 2205*c**5*d**7*x**2 + 3675*c**4*d**8*x**3
 + 3675*c**3*d**9*x**4 + 2205*c**2*d**10*x**5 + 735*c*d**11*x**6 + 105*d**1
2*x**7)
```

Giac [B] time = 1.06834, size = 248, normalized size = 2.79

$$-\frac{35\,b^4d^4x^4 + 35\,b^4cd^3x^3 + 105\,ab^3d^4x^3 + 21\,b^4c^2d^2x^2 + 63\,ab^3cd^3x^2 + 126\,a^2b^2d^4x^2 + 7\,b^4c^3dx + 21\,ab^3c^2d^2x + 42\,a^2b^2c\cdots}{105\,(dx+c)^7d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + 35*b^4*c*d^3*x^3 + 105*a*b^3*d^4*x^3 + 21*b^4*c^2*
d^2*x^2 + 63*a*b^3*c*d^3*x^2 + 126*a^2*b^2*d^4*x^2 + 7*b^4*c^3*d*x + 21*a*b
^3*c^2*d^2*x + 42*a^2*b^2*c*d^3*x + 70*a^3*b*d^4*x + b^4*c^4 + 3*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 + 10*a^3*b*c*d^3 + 15*a^4*d^4)/((d*x + c)^7*d^5)
```

3.1368 $\int \frac{(a+bx)^3}{(c+dx)^8} dx$

Optimal. Leaf size=92

$$\frac{3b^2(bc-ad)}{5d^4(c+dx)^5} - \frac{b(bc-ad)^2}{2d^4(c+dx)^6} + \frac{(bc-ad)^3}{7d^4(c+dx)^7} - \frac{b^3}{4d^4(c+dx)^4}$$

```
[Out] (b*c - a*d)^3/(7*d^4*(c + d*x)^7) - (b*(b*c - a*d)^2)/(2*d^4*(c + d*x)^6) +
 (3*b^2*(b*c - a*d))/(5*d^4*(c + d*x)^5) - b^3/(4*d^4*(c + d*x)^4)
```

Rubi [A] time = 0.057465, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3b^2(bc-ad)}{5d^4(c+dx)^5} - \frac{b(bc-ad)^2}{2d^4(c+dx)^6} + \frac{(bc-ad)^3}{7d^4(c+dx)^7} - \frac{b^3}{4d^4(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)^3/(7*d^4*(c + d*x)^7) - (b*(b*c - a*d)^2)/(2*d^4*(c + d*x)^6) +
 (3*b^2*(b*c - a*d))/(5*d^4*(c + d*x)^5) - b^3/(4*d^4*(c + d*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^8} dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^8} + \frac{3b(bc-ad)^2}{d^3(c+dx)^7} - \frac{3b^2(bc-ad)}{d^3(c+dx)^6} + \frac{b^3}{d^3(c+dx)^5}\right) dx \\ &= \frac{(bc-ad)^3}{7d^4(c+dx)^7} - \frac{b(bc-ad)^2}{2d^4(c+dx)^6} + \frac{3b^2(bc-ad)}{5d^4(c+dx)^5} - \frac{b^3}{4d^4(c+dx)^4}\end{aligned}$$

Mathematica [A] time = 0.0292335, size = 94, normalized size = 1.02

$$-\frac{10a^2bd^2(c+7dx) + 20a^3d^3 + 4ab^2d\left(c^2+7cdx+21d^2x^2\right) + b^3\left(7c^2dx+c^3+21cd^2x^2+35d^3x^3\right)}{140d^4(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^8,x]
```

```
[Out] -(20*a^3*d^3 + 10*a^2*b*d^2*(c + 7*d*x) + 4*a*b^2*d*(c^2 + 7*c*d*x + 21*d^2
*x^2) + b^3*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3))/(140*d^4*(c + d*
x)^7)
```

Maple [A] time = 0.005, size = 122, normalized size = 1.3

$$-\frac{a^3d^3-3\,a^2bcd^2+3\,ab^2c^2d-b^3c^3}{7\,d^4\left(dx+c\right)^7}-\frac{b\left(a^2d^2-2\,abcd+b^2c^2\right)}{2\,d^4\left(dx+c\right)^6}-\frac{3\,b^2\left(ad-bc\right)}{5\,d^4\left(dx+c\right)^5}-\frac{b^3}{4\,d^4\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^8,x)
```

```
[Out] -1/7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4/(d*x+c)^7-1/2*b*(a^2
*d^2-2*a*b*c*d+b^2*c^2)/d^4/(d*x+c)^6-3/5*b^2*(a*d-b*c)/d^4/(d*x+c)^5-1/4*b
^3/d^4/(d*x+c)^4
```

Maxima [B] time = 1.00277, size = 246, normalized size = 2.67

$$-\frac{35\,b^3d^3x^3+b^3c^3+4\,ab^2c^2d+10\,a^2bcd^2+20\,a^3d^3+21\,\left(b^3cd^2+4\,ab^2d^3\right)x^2+7\,\left(b^3c^2d+4\,ab^2cd^2+10\,a^2bd^3\right)x}{140\,\left(d^{11}x^7+7\,cd^{10}x^6+21\,c^2d^9x^5+35\,c^3d^8x^4+35\,c^4d^7x^3+21\,c^5d^6x^2+7\,c^6d^5x+c^7d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2 + 20*a^3*
d^3 + 21*(b^3*c*d^2 + 4*a*b^2*d^3)*x^2 + 7*(b^3*c^2*d + 4*a*b^2*c*d^2 + 10*
a^2*b*d^3)*x)/(d^11*x^7 + 7*c*d^10*x^6 + 21*c^2*d^9*x^5 + 35*c^3*d^8*x^4 +
35*c^4*d^7*x^3 + 21*c^5*d^6*x^2 + 7*c^6*d^5*x + c^7*d^4)
```

Fricas [B] time = 1.76952, size = 382, normalized size = 4.15

$$-\frac{35\,b^3d^3x^3+b^3c^3+4\,ab^2c^2d+10\,a^2bcd^2+20\,a^3d^3+21\,\left(b^3cd^2+4\,ab^2d^3\right)x^2+7\,\left(b^3c^2d+4\,ab^2cd^2+10\,a^2bd^3\right)x}{140\,\left(d^{11}x^7+7\,cd^{10}x^6+21\,c^2d^9x^5+35\,c^3d^8x^4+35\,c^4d^7x^3+21\,c^5d^6x^2+7\,c^6d^5x+c^7d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2 + 20*a^3*
d^3 + 21*(b^3*c*d^2 + 4*a*b^2*d^3)*x^2 + 7*(b^3*c^2*d + 4*a*b^2*c*d^2 + 10*
a^2*b*d^3)*x)/(d^11*x^7 + 7*c*d^10*x^6 + 21*c^2*d^9*x^5 + 35*c^3*d^8*x^4 +
35*c^4*d^7*x^3 + 21*c^5*d^6*x^2 + 7*c^6*d^5*x + c^7*d^4)
```

Sympy [B] time = 3.48897, size = 194, normalized size = 2.11

$$-\frac{20a^3d^3+10a^2bcd^2+4ab^2c^2d+b^3c^3+35b^3d^3x^3+x^2\left(84ab^2d^3+21b^3cd^2\right)+x\left(70a^2bd^3+28ab^2cd^2+7b^3c^2d\right)}{140c^7d^4+980c^6d^5x+2940c^5d^6x^2+4900c^4d^7x^3+4900c^3d^8x^4+2940c^2d^9x^5+980cd^{10}x^6+140d^{11}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**8,x)
```

```
[Out] -(20*a**3*d**3 + 10*a**2*b*c*d**2 + 4*a*b**2*c**2*d + b**3*c**3 + 35*b**3*d
**3*x**3 + x**2*(84*a*b**2*d**3 + 21*b**3*c*d**2) + x*(70*a**2*b*d**3 + 28*
a*b**2*c*d**2 + 7*b**3*c**2*d))/(140*c**7*d**4 + 980*c**6*d**5*x + 2940*c**
5*d**6*x**2 + 4900*c**4*d**7*x**3 + 4900*c**3*d**8*x**4 + 2940*c**2*d**9*x*
*5 + 980*c*d**10*x**6 + 140*d**11*x**7)
```

Giac [A] time = 1.07896, size = 154, normalized size = 1.67

$$-\frac{35\,b^3d^3x^3+21\,b^3cd^2x^2+84\,ab^2d^3x^2+7\,b^3c^2dx+28\,ab^2cd^2x+70\,a^2bd^3x+b^3c^3+4\,ab^2c^2d+10\,a^2bcd^2+20\,a^3d^3}{140\,(dx+c)^7d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 21*b^3*c*d^2*x^2 + 84*a*b^2*d^3*x^2 + 7*b^3*c^2*d*
x + 28*a*b^2*c*d^2*x + 70*a^2*b*d^3*x + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*
c*d^2 + 20*a^3*d^3)/((d*x + c)^7*d^4)
```

3.1369 $\int \frac{(a+bx)^2}{(c+dx)^8}\,dx$

Optimal. Leaf size=65

$$\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{(bc-ad)^2}{7d^3(c+dx)^7}-\frac{b^2}{5d^3(c+dx)^5}$$

```
[Out] -(b*c - a*d)^2/(7*d^3*(c + d*x)^7) + (b*(b*c - a*d))/(3*d^3*(c + d*x)^6) -
b^2/(5*d^3*(c + d*x)^5)
```

Rubi [A] time = 0.0398526, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{(bc-ad)^2}{7d^3(c+dx)^7}-\frac{b^2}{5d^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^8,x]
```

```
[Out] -(b*c - a*d)^2/(7*d^3*(c + d*x)^7) + (b*(b*c - a*d))/(3*d^3*(c + d*x)^6) -
b^2/(5*d^3*(c + d*x)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^8}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^8}-\frac{2b(bc-ad)}{d^2(c+dx)^7}+\frac{b^2}{d^2(c+dx)^6}\right)dx\\ &= -\frac{(bc-ad)^2}{7d^3(c+dx)^7}+\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{b^2}{5d^3(c+dx)^5}\end{aligned}$$

Mathematica [A] time = 0.0236939, size = 55, normalized size = 0.85

$$-\frac{15a^2d^2+5abd(c+7dx)+b^2\left(c^2+7cdx+21d^2x^2\right)}{105d^3(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^8,x]
```

```
[Out] -(15*a^2*d^2 + 5*a*b*d*(c + 7*d*x) + b^2*(c^2 + 7*c*d*x + 21*d^2*x^2))/(105
*d^3*(c + d*x)^7)
```

Maple [A] time = 0.003, size = 71, normalized size = 1.1

$$-\frac{a^2d^2-2\,abcd+c^2b^2}{7\,d^3\,(dx+c)^7}-\frac{b\,(ad-bc)}{3\,d^3\,(dx+c)^6}-\frac{b^2}{5\,d^3\,(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^8,x)
```

```
[Out] -1/7*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3/(d*x+c)^7-1/3*b*(a*d-b*c)/d^3/(d*x+c)^
6-1/5*b^2/d^3/(d*x+c)^5
```

Maxima [B] time = 0.982455, size = 177, normalized size = 2.72

$$-\frac{21\,b^2d^2x^2+b^2c^2+5\,abcd+15\,a^2d^2+7\left(b^2cd+5\,abd^2\right)x}{105\left(d^{10}x^7+7\,cd^9x^6+21\,c^2d^8x^5+35\,c^3d^7x^4+35\,c^4d^6x^3+21\,c^5d^5x^2+7\,c^6d^4x+c^7d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + 7*(b^2*c*d + 5*
a*b*d^2)*x)/(d^10*x^7 + 7*c*d^9*x^6 + 21*c^2*d^8*x^5 + 35*c^3*d^7*x^4 + 35*
c^4*d^6*x^3 + 21*c^5*d^5*x^2 + 7*c^6*d^4*x + c^7*d^3)
```

Fricas [B] time = 1.76736, size = 277, normalized size = 4.26

$$-\frac{21\,b^2d^2x^2+b^2c^2+5\,abcd+15\,a^2d^2+7\left(b^2cd+5\,abd^2\right)x}{105\left(d^{10}x^7+7\,cd^9x^6+21\,c^2d^8x^5+35\,c^3d^7x^4+35\,c^4d^6x^3+21\,c^5d^5x^2+7\,c^6d^4x+c^7d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + 7*(b^2*c*d + 5*
a*b*d^2)*x)/(d^10*x^7 + 7*c*d^9*x^6 + 21*c^2*d^8*x^5 + 35*c^3*d^7*x^4 + 35*
c^4*d^6*x^3 + 21*c^5*d^5*x^2 + 7*c^6*d^4*x + c^7*d^3)
```

Sympy [B] time = 1.51781, size = 139, normalized size = 2.14

$$-\frac{15a^2d^2+5abcd+b^2c^2+21b^2d^2x^2+x\left(35abd^2+7b^2cd\right)}{105c^7d^3+735c^6d^4x+2205c^5d^5x^2+3675c^4d^6x^3+3675c^3d^7x^4+2205c^2d^8x^5+735cd^9x^6+105d^{10}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**8,x)
```

```
[Out] -(15*a**2*d**2 + 5*a*b*c*d + b**2*c**2 + 21*b**2*d**2*x**2 + x*(35*a*b*d**2
 + 7*b**2*c*d))/(105*c**7*d**3 + 735*c**6*d**4*x + 2205*c**5*d**5*x**2 + 36
75*c**4*d**6*x**3 + 3675*c**3*d**7*x**4 + 2205*c**2*d**8*x**5 + 735*c*d**9*
```

```
x**6 + 105*d**10*x**7)
```

Giac [A] time = 1.05808, size = 82, normalized size = 1.26

$$-\frac{21\,b^2d^2x^2+7\,b^2cdx+35\,abd^2x+b^2c^2+5\,abcd+15\,a^2d^2}{105\,(dx+c)^7d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + 7*b^2*c*d*x + 35*a*b*d^2*x + b^2*c^2 + 5*a*b*c*d +
 15*a^2*d^2)/((d*x + c)^7*d^3)
```

3.1370 $\int \frac{a+bx}{(c+dx)^8} \, dx$

Optimal. Leaf size=38

$$\frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6}$$

```
[Out] (b*c - a*d)/(7*d^2*(c + d*x)^7) - b/(6*d^2*(c + d*x)^6)
```

Rubi [A] time = 0.0220319, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)/(7*d^2*(c + d*x)^7) - b/(6*d^2*(c + d*x)^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{(c+dx)^8} \, dx &= \int \left(\frac{-bc+ad}{d(c+dx)^8}+\frac{b}{d(c+dx)^7}\right) dx \\ &= \frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6} \end{aligned}$$

Mathematica [A] time = 0.009272, size = 27, normalized size = 0.71

$$-\frac{6ad+b(c+7dx)}{42d^2(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^8,x]
```

```
[Out] -(6*a*d + b*(c + 7*d*x))/(42*d^2*(c + d*x)^7)
```

Maple [A] time = 0.004, size = 35, normalized size = 0.9

$$-\frac{ad-bc}{7\,d^2\,(dx+c)^7}-\frac{b}{6\,d^2\,(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^8,x)
```

```
[Out] -1/7*(a*d-b*c)/d^2/(d*x+c)^7-1/6*b/d^2/(d*x+c)^6
```

Maxima [B] time = 0.975201, size = 127, normalized size = 3.34

$$-\frac{7\,bdx + bc + 6\,ad}{42\,\left(d^{9}x^{7} + 7\,cd^{8}x^{6} + 21\,c^{2}d^{7}x^{5} + 35\,c^{3}d^{6}x^{4} + 35\,c^{4}d^{5}x^{3} + 21\,c^{5}d^{4}x^{2} + 7\,c^{6}d^{3}x + c^{7}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/(d^9*x^7 + 7*c*d^8*x^6 + 21*c^2*d^7*x^5 + 35*
c^3*d^6*x^4 + 35*c^4*d^5*x^3 + 21*c^5*d^4*x^2 + 7*c^6*d^3*x + c^7*d^2)
```

Fricas [B] time = 1.78749, size = 198, normalized size = 5.21

$$-\frac{7\,bdx + bc + 6\,ad}{42\,\left(d^{9}x^{7} + 7\,cd^{8}x^{6} + 21\,c^{2}d^{7}x^{5} + 35\,c^{3}d^{6}x^{4} + 35\,c^{4}d^{5}x^{3} + 21\,c^{5}d^{4}x^{2} + 7\,c^{6}d^{3}x + c^{7}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/(d^9*x^7 + 7*c*d^8*x^6 + 21*c^2*d^7*x^5 + 35*
c^3*d^6*x^4 + 35*c^4*d^5*x^3 + 21*c^5*d^4*x^2 + 7*c^6*d^3*x + c^7*d^2)
```

Sympy [B] time = 0.880056, size = 100, normalized size = 2.63

$$-\frac{6ad + bc + 7bdx}{42c^{7}d^{2} + 294c^{6}d^{3}x + 882c^{5}d^{4}x^{2} + 1470c^{4}d^{5}x^{3} + 1470c^{3}d^{6}x^{4} + 882c^{2}d^{7}x^{5} + 294cd^{8}x^{6} + 42d^{9}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**8,x)
```

```
[Out] -(6*a*d + b*c + 7*b*d*x)/(42*c**7*d**2 + 294*c**6*d**3*x + 882*c**5*d**4*x*
*2 + 1470*c**4*d**5*x**3 + 1470*c**3*d**6*x**4 + 882*c**2*d**7*x**5 + 294*c
*d**8*x**6 + 42*d**9*x**7)
```

Giac [A] time = 1.06551, size = 34, normalized size = 0.89

$$-\frac{7\,bdx + bc + 6\,ad}{42\,(dx + c)^{7}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="giac")

[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/((d*x + c)^7*d^2)
```

3.1371 $\int \frac{1}{(c+dx)^8} dx$

Optimal. Leaf size=14

$$-\frac{1}{7d(c+dx)^7}$$

[Out] -1/(7*d*(c + d*x)^7)

Rubi [A] time = 0.0016125, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{7d(c+dx)^7}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(-8),x]

[Out] -1/(7*d*(c + d*x)^7)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^8} dx = -\frac{1}{7d(c+dx)^7}$$

Mathematica [A] time = 0.0026768, size = 14, normalized size = 1.

$$-\frac{1}{7d(c+dx)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(-8),x]

[Out] -1/(7*d*(c + d*x)^7)

Maple [A] time = 0., size = 13, normalized size = 0.9

$$-\frac{1}{7\,d\,(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^8,x)
```

```
[Out] -1/7/d/(d*x+c)^7
```

Maxima [A] time = 0.961186, size = 16, normalized size = 1.14

$$-\frac{1}{7\,(dx+c)^7 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/7/((d*x + c)^7*d)
```

Fricas [B] time = 1.75918, size = 162, normalized size = 11.57

$$-\frac{1}{7\left(d^8x^7+7\,cd^7x^6+21\,c^2d^6x^5+35\,c^3d^5x^4+35\,c^4d^4x^3+21\,c^5d^3x^2+7\,c^6d^2x+c^7d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(d^8*x^7 + 7*c*d^7*x^6 + 21*c^2*d^6*x^5 + 35*c^3*d^5*x^4 + 35*c^4*d^4*
x^3 + 21*c^5*d^3*x^2 + 7*c^6*d^2*x + c^7*d)
```

Sympy [B] time = 0.609726, size = 85, normalized size = 6.07

$$-\frac{1}{7c^7d+49c^6d^2x+147c^5d^3x^2+245c^4d^4x^3+245c^3d^5x^4+147c^2d^6x^5+49cd^7x^6+7d^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**8,x)
```

```
[Out] -1/(7*c**7*d + 49*c**6*d**2*x + 147*c**5*d**3*x**2 + 245*c**4*d**4*x**3 + 2
45*c**3*d**5*x**4 + 147*c**2*d**6*x**5 + 49*c*d**7*x**6 + 7*d**8*x**7)
```

Giac [A] time = 1.06536, size = 16, normalized size = 1.14

$$-\frac{1}{7\,(dx+c)^7 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/7/((d*x + c)^7*d)
```

3.1372 $\int \frac{1}{(a+bx)(c+dx)^8} dx$

Optimal. Leaf size=202

$$\frac{b^6}{(c+dx)(bc-ad)^7} + \frac{b^5}{2(c+dx)^2(bc-ad)^6} + \frac{b^4}{3(c+dx)^3(bc-ad)^5} + \frac{b^3}{4(c+dx)^4(bc-ad)^4} + \frac{b^2}{5(c+dx)^5(bc-ad)^3} + \frac{b^7 \log}{(bc}$$

```
[Out] 1/(7*(b*c - a*d)*(c + d*x)^7) + b/(6*(b*c - a*d)^2*(c + d*x)^6) + b^2/(5*(b
*c - a*d)^3*(c + d*x)^5) + b^3/(4*(b*c - a*d)^4*(c + d*x)^4) + b^4/(3*(b*c
- a*d)^5*(c + d*x)^3) + b^5/(2*(b*c - a*d)^6*(c + d*x)^2) + b^6/((b*c - a*d
)^7*(c + d*x)) + (b^7*Log[a + b*x])/(b*c - a*d)^8 - (b^7*Log[c + d*x])/(b*c
 - a*d)^8
```

Rubi [A] time = 0.169123, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{b^6}{(c+dx)(bc-ad)^7} + \frac{b^5}{2(c+dx)^2(bc-ad)^6} + \frac{b^4}{3(c+dx)^3(bc-ad)^5} + \frac{b^3}{4(c+dx)^4(bc-ad)^4} + \frac{b^2}{5(c+dx)^5(bc-ad)^3} + \frac{b^7 \log}{(bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^8),x]
```

```
[Out] 1/(7*(b*c - a*d)*(c + d*x)^7) + b/(6*(b*c - a*d)^2*(c + d*x)^6) + b^2/(5*(b
*c - a*d)^3*(c + d*x)^5) + b^3/(4*(b*c - a*d)^4*(c + d*x)^4) + b^4/(3*(b*c
- a*d)^5*(c + d*x)^3) + b^5/(2*(b*c - a*d)^6*(c + d*x)^2) + b^6/((b*c - a*d
)^7*(c + d*x)) + (b^7*Log[a + b*x])/(b*c - a*d)^8 - (b^7*Log[c + d*x])/(b*c
 - a*d)^8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^8} dx = \int \left(\frac{b^8}{(bc-ad)^8(a+bx)} - \frac{d}{(bc-ad)(c+dx)^8} - \frac{bd}{(bc-ad)^2(c+dx)^7} - \frac{b^2 d}{(bc-ad)^3(c+dx)^6} - \frac{}{(bc} \right.$$

$$= \frac{1}{7(bc-ad)(c+dx)^7} + \frac{b}{6(bc-ad)^2(c+dx)^6} + \frac{b^2}{5(bc-ad)^3(c+dx)^5} + \frac{b^3}{4(bc-ad)^4(c+dx)^4} + \frac{}{3}$$

Mathematica [A] time = 0.0928364, size = 196, normalized size = 0.97

$$\frac{84b^2(c+dx)^2(bc-ad)^5 + 105b^3(c+dx)^3(bc-ad)^4 + 140b^4(c+dx)^4(bc-ad)^3 + 210b^5(c+dx)^5(bc-ad)^2 + 420b^6(c+d}{420(c+dx)^7(bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^8),x]
```

```
[Out] (60*(b*c - a*d)^7 + 70*b*(b*c - a*d)^6*(c + d*x) + 84*b^2*(b*c - a*d)^5*(c
+ d*x)^2 + 105*b^3*(b*c - a*d)^4*(c + d*x)^3 + 140*b^4*(b*c - a*d)^3*(c + d
*x)^4 + 210*b^5*(b*c - a*d)^2*(c + d*x)^5 + 420*b^6*(b*c - a*d)*(c + d*x)^6
 + 420*b^7*(c + d*x)^7*Log[a + b*x] - 420*b^7*(c + d*x)^7*Log[c + d*x])/(42
0*(b*c - a*d)^8*(c + d*x)^7)
```

Maple [A] time = 0.016, size = 192, normalized size = 1.

$$-\frac{1}{(7\,ad-7\,bc)\,(dx+c)^{7}}-\frac{b^{2}}{5\,(ad-bc)^{3}\,(dx+c)^{5}}-\frac{b^{4}}{3\,(ad-bc)^{5}\,(dx+c)^{3}}-\frac{b^{6}}{(ad-bc)^{7}\,(dx+c)}+\frac{b}{6\,(ad-bc)^{2}\,(dx+c)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^8,x)
```

```
[Out] -1/7/(a*d-b*c)/(d*x+c)^7-1/5*b^2/(a*d-b*c)^3/(d*x+c)^5-1/3*b^4/(a*d-b*c)^5/
(d*x+c)^3-b^6/(a*d-b*c)^7/(d*x+c)+1/6*b/(a*d-b*c)^2/(d*x+c)^6+1/4*b^3/(a*d-
b*c)^4/(d*x+c)^4+1/2*b^5/(a*d-b*c)^6/(d*x+c)^2-b^7/(a*d-b*c)^8*ln(d*x+c)+b^
7/(a*d-b*c)^8*ln(b*x+a)
```

Maxima [B] time = 1.50427, size = 1914, normalized size = 9.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] b^7*log(b*x + a)/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8) - b^7*log(d*x + c)/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8) + 1/420*(420*b^6*d^6*x^6
+ 1089*b^6*c^6 - 1851*a*b^5*c^5*d + 2559*a^2*b^4*c^4*d^2 - 2341*a^3*b^3*c^3
*d^3 + 1334*a^4*b^2*c^2*d^4 - 430*a^5*b*c*d^5 + 60*a^6*d^6 + 210*(13*b^6*c*
d^5 - a*b^5*d^6)*x^5 + 70*(107*b^6*c^2*d^4 - 19*a*b^5*c*d^5 + 2*a^2*b^4*d^6
)*x^4 + 35*(319*b^6*c^3*d^3 - 101*a*b^5*c^2*d^4 + 25*a^2*b^4*c*d^5 - 3*a^3*
b^3*d^6)*x^3 + 21*(459*b^6*c^4*d^2 - 241*a*b^5*c^3*d^3 + 109*a^2*b^4*c^2*d^
4 - 31*a^3*b^3*c*d^5 + 4*a^4*b^2*d^6)*x^2 + 7*(669*b^6*c^5*d - 591*a*b^5*c^
4*d^2 + 459*a^2*b^4*c^3*d^3 - 241*a^3*b^3*c^2*d^4 + 74*a^4*b^2*c*d^5 - 10*a
^5*b*d^6)*x)/(b^7*c^14 - 7*a*b^6*c^13*d + 21*a^2*b^5*c^12*d^2 - 35*a^3*b^4*
c^11*d^3 + 35*a^4*b^3*c^10*d^4 - 21*a^5*b^2*c^9*d^5 + 7*a^6*b*c^8*d^6 - a^7
*c^7*d^7 + (b^7*c^7*d^7 - 7*a*b^6*c^6*d^8 + 21*a^2*b^5*c^5*d^9 - 35*a^3*b^4
*c^4*d^10 + 35*a^4*b^3*c^3*d^11 - 21*a^5*b^2*c^2*d^12 + 7*a^6*b*c*d^13 - a^
7*d^14)*x^7 + 7*(b^7*c^8*d^6 - 7*a*b^6*c^7*d^7 + 21*a^2*b^5*c^6*d^8 - 35*a^
3*b^4*c^5*d^9 + 35*a^4*b^3*c^4*d^10 - 21*a^5*b^2*c^3*d^11 + 7*a^6*b*c^2*d^1
2 - a^7*c*d^13)*x^6 + 21*(b^7*c^9*d^5 - 7*a*b^6*c^8*d^6 + 21*a^2*b^5*c^7*d^
7 - 35*a^3*b^4*c^6*d^8 + 35*a^4*b^3*c^5*d^9 - 21*a^5*b^2*c^4*d^10 + 7*a^6*b
*c^3*d^11 - a^7*c^2*d^12)*x^5 + 35*(b^7*c^10*d^4 - 7*a*b^6*c^9*d^5 + 21*a^2
*b^5*c^8*d^6 - 35*a^3*b^4*c^7*d^7 + 35*a^4*b^3*c^6*d^8 - 21*a^5*b^2*c^5*d^9
 + 7*a^6*b*c^4*d^10 - a^7*c^3*d^11)*x^4 + 35*(b^7*c^11*d^3 - 7*a*b^6*c^10*d
^4 + 21*a^2*b^5*c^9*d^5 - 35*a^3*b^4*c^8*d^6 + 35*a^4*b^3*c^7*d^7 - 21*a^5*
b^2*c^6*d^8 + 7*a^6*b*c^5*d^9 - a^7*c^4*d^10)*x^3 + 21*(b^7*c^12*d^2 - 7*a*
b^6*c^11*d^3 + 21*a^2*b^5*c^10*d^4 - 35*a^3*b^4*c^9*d^5 + 35*a^4*b^3*c^8*d^
```

```
6 - 21*a^5*b^2*c^7*d^7 + 7*a^6*b*c^6*d^8 - a^7*c^5*d^9)*x^2 + 7*(b^7*c^13*d
 - 7*a*b^6*c^12*d^2 + 21*a^2*b^5*c^11*d^3 - 35*a^3*b^4*c^10*d^4 + 35*a^4*b^
3*c^9*d^5 - 21*a^5*b^2*c^8*d^6 + 7*a^6*b*c^7*d^7 - a^7*c^6*d^8)*x)
```

Fricas [B] time = 2.20115, size = 3332, normalized size = 16.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/420*(1089*b^7*c^7 - 2940*a*b^6*c^6*d + 4410*a^2*b^5*c^5*d^2 - 4900*a^3*b^
4*c^4*d^3 + 3675*a^4*b^3*c^3*d^4 - 1764*a^5*b^2*c^2*d^5 + 490*a^6*b*c*d^6 -
 60*a^7*d^7 + 420*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 210*(13*b^7*c^2*d^5 - 14*a*
b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 70*(107*b^7*c^3*d^4 - 126*a*b^6*c^2*d^5 + 21
*a^2*b^5*c*d^6 - 2*a^3*b^4*d^7)*x^4 + 35*(319*b^7*c^4*d^3 - 420*a*b^6*c^3*d
^4 + 126*a^2*b^5*c^2*d^5 - 28*a^3*b^4*c*d^6 + 3*a^4*b^3*d^7)*x^3 + 21*(459*
b^7*c^5*d^2 - 700*a*b^6*c^4*d^3 + 350*a^2*b^5*c^3*d^4 - 140*a^3*b^4*c^2*d^5
 + 35*a^4*b^3*c*d^6 - 4*a^5*b^2*d^7)*x^2 + 7*(669*b^7*c^6*d - 1260*a*b^6*c^
5*d^2 + 1050*a^2*b^5*c^4*d^3 - 700*a^3*b^4*c^3*d^4 + 315*a^4*b^3*c^2*d^5 -
84*a^5*b^2*c*d^6 + 10*a^6*b*d^7)*x + 420*(b^7*d^7*x^7 + 7*b^7*c*d^6*x^6 + 2
1*b^7*c^2*d^5*x^5 + 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21*b^7*c^5*d^
2*x^2 + 7*b^7*c^6*d*x + b^7*c^7)*log(b*x + a) - 420*(b^7*d^7*x^7 + 7*b^7*c*
d^6*x^6 + 21*b^7*c^2*d^5*x^5 + 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21
*b^7*c^5*d^2*x^2 + 7*b^7*c^6*d*x + b^7*c^7)*log(d*x + c))/(b^8*c^15 - 8*a*b
^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4
 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8
 + (b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^1
0 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7
*b*c*d^14 + a^8*d^15)*x^7 + 7*(b^8*c^9*d^6 - 8*a*b^7*c^8*d^7 + 28*a^2*b^6*c
^7*d^8 - 56*a^3*b^5*c^6*d^9 + 70*a^4*b^4*c^5*d^10 - 56*a^5*b^3*c^4*d^11 + 2
8*a^6*b^2*c^3*d^12 - 8*a^7*b*c^2*d^13 + a^8*c*d^14)*x^6 + 21*(b^8*c^10*d^5
- 8*a*b^7*c^9*d^6 + 28*a^2*b^6*c^8*d^7 - 56*a^3*b^5*c^7*d^8 + 70*a^4*b^4*c^
6*d^9 - 56*a^5*b^3*c^5*d^10 + 28*a^6*b^2*c^4*d^11 - 8*a^7*b*c^3*d^12 + a^8*
c^2*d^13)*x^5 + 35*(b^8*c^11*d^4 - 8*a*b^7*c^10*d^5 + 28*a^2*b^6*c^9*d^6 -
56*a^3*b^5*c^8*d^7 + 70*a^4*b^4*c^7*d^8 - 56*a^5*b^3*c^6*d^9 + 28*a^6*b^2*c
^5*d^10 - 8*a^7*b*c^4*d^11 + a^8*c^3*d^12)*x^4 + 35*(b^8*c^12*d^3 - 8*a*b^7
*c^11*d^4 + 28*a^2*b^6*c^10*d^5 - 56*a^3*b^5*c^9*d^6 + 70*a^4*b^4*c^8*d^7 -
 56*a^5*b^3*c^7*d^8 + 28*a^6*b^2*c^6*d^9 - 8*a^7*b*c^5*d^10 + a^8*c^4*d^11)
*x^3 + 21*(b^8*c^13*d^2 - 8*a*b^7*c^12*d^3 + 28*a^2*b^6*c^11*d^4 - 56*a^3*b
^5*c^10*d^5 + 70*a^4*b^4*c^9*d^6 - 56*a^5*b^3*c^8*d^7 + 28*a^6*b^2*c^7*d^8
- 8*a^7*b*c^6*d^9 + a^8*c^5*d^10)*x^2 + 7*(b^8*c^14*d - 8*a*b^7*c^13*d^2 +
28*a^2*b^6*c^12*d^3 - 56*a^3*b^5*c^11*d^4 + 70*a^4*b^4*c^10*d^5 - 56*a^5*b^
3*c^9*d^6 + 28*a^6*b^2*c^8*d^7 - 8*a^7*b*c^7*d^8 + a^8*c^6*d^9)*x)
```

Sympy [B] time = 10.6435, size = 1776, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**8,x)
```

```
[Out] -b**7*log(x + (-a**9*b**7*d**9/(a*d - b*c)**8 + 9*a**8*b**8*c*d**8/(a*d - b
*c)**8 - 36*a**7*b**9*c**2*d**7/(a*d - b*c)**8 + 84*a**6*b**10*c**3*d**6/(a
*d - b*c)**8 - 126*a**5*b**11*c**4*d**5/(a*d - b*c)**8 + 126*a**4*b**12*c**
5*d**4/(a*d - b*c)**8 - 84*a**3*b**13*c**6*d**3/(a*d - b*c)**8 + 36*a**2*b*
*14*c**7*d**2/(a*d - b*c)**8 - 9*a*b**15*c**8*d/(a*d - b*c)**8 + a*b**7*d +
 b**16*c**9/(a*d - b*c)**8 + b**8*c)/(2*b**8*d))/(a*d - b*c)**8 + b**7*log(
x + (a**9*b**7*d**9/(a*d - b*c)**8 - 9*a**8*b**8*c*d**8/(a*d - b*c)**8 + 36
*a**7*b**9*c**2*d**7/(a*d - b*c)**8 - 84*a**6*b**10*c**3*d**6/(a*d - b*c)**
8 + 126*a**5*b**11*c**4*d**5/(a*d - b*c)**8 - 126*a**4*b**12*c**5*d**4/(a*d
 - b*c)**8 + 84*a**3*b**13*c**6*d**3/(a*d - b*c)**8 - 36*a**2*b**14*c**7*d*
*2/(a*d - b*c)**8 + 9*a*b**15*c**8*d/(a*d - b*c)**8 + a*b**7*d - b**16*c**9
/(a*d - b*c)**8 + b**8*c)/(2*b**8*d))/(a*d - b*c)**8 - (60*a**6*d**6 - 430*
a**5*b*c*d**5 + 1334*a**4*b**2*c**2*d**4 - 2341*a**3*b**3*c**3*d**3 + 2559*
a**2*b**4*c**4*d**2 - 1851*a*b**5*c**5*d + 1089*b**6*c**6 + 420*b**6*d**6*x
**6 + x**5*(-210*a*b**5*d**6 + 2730*b**6*c*d**5) + x**4*(140*a**2*b**4*d**6
 - 1330*a*b**5*c*d**5 + 7490*b**6*c**2*d**4) + x**3*(-105*a**3*b**3*d**6 +
875*a**2*b**4*c*d**5 - 3535*a*b**5*c**2*d**4 + 11165*b**6*c**3*d**3) + x**2
*(84*a**4*b**2*d**6 - 651*a**3*b**3*c*d**5 + 2289*a**2*b**4*c**2*d**4 - 506
1*a*b**5*c**3*d**3 + 9639*b**6*c**4*d**2) + x*(-70*a**5*b*d**6 + 518*a**4*b
**2*c*d**5 - 1687*a**3*b**3*c**2*d**4 + 3213*a**2*b**4*c**3*d**3 - 4137*a*b
**5*c**4*d**2 + 4683*b**6*c**5*d))/(420*a**7*c**7*d**7 - 2940*a**6*b*c**8*d
**6 + 8820*a**5*b**2*c**9*d**5 - 14700*a**4*b**3*c**10*d**4 + 14700*a**3*b*
*4*c**11*d**3 - 8820*a**2*b**5*c**12*d**2 + 2940*a*b**6*c**13*d - 420*b**7*
c**14 + x**7*(420*a**7*d**14 - 2940*a**6*b*c*d**13 + 8820*a**5*b**2*c**2*d*
*12 - 14700*a**4*b**3*c**3*d**11 + 14700*a**3*b**4*c**4*d**10 - 8820*a**2*b
**5*c**5*d**9 + 2940*a*b**6*c**6*d**8 - 420*b**7*c**7*d**7) + x**6*(2940*a*
*7*c*d**13 - 20580*a**6*b*c**2*d**12 + 61740*a**5*b**2*c**3*d**11 - 102900*
a**4*b**3*c**4*d**10 + 102900*a**3*b**4*c**5*d**9 - 61740*a**2*b**5*c**6*d*
*8 + 20580*a*b**6*c**7*d**7 - 2940*b**7*c**8*d**6) + x**5*(8820*a**7*c**2*d
**12 - 61740*a**6*b*c**3*d**11 + 185220*a**5*b**2*c**4*d**10 - 308700*a**4*
b**3*c**5*d**9 + 308700*a**3*b**4*c**6*d**8 - 185220*a**2*b**5*c**7*d**7 +
61740*a*b**6*c**8*d**6 - 8820*b**7*c**9*d**5) + x**4*(14700*a**7*c**3*d**11
 - 102900*a**6*b*c**4*d**10 + 308700*a**5*b**2*c**5*d**9 - 514500*a**4*b**3
*c**6*d**8 + 514500*a**3*b**4*c**7*d**7 - 308700*a**2*b**5*c**8*d**6 + 1029
00*a*b**6*c**9*d**5 - 14700*b**7*c**10*d**4) + x**3*(14700*a**7*c**4*d**10
- 102900*a**6*b*c**5*d**9 + 308700*a**5*b**2*c**6*d**8 - 514500*a**4*b**3*c
**7*d**7 + 514500*a**3*b**4*c**8*d**6 - 308700*a**2*b**5*c**9*d**5 + 102900
*a*b**6*c**10*d**4 - 14700*b**7*c**11*d**3) + x**2*(8820*a**7*c**5*d**9 - 6
1740*a**6*b*c**6*d**8 + 185220*a**5*b**2*c**7*d**7 - 308700*a**4*b**3*c**8*
d**6 + 308700*a**3*b**4*c**9*d**5 - 185220*a**2*b**5*c**10*d**4 + 61740*a*b
**6*c**11*d**3 - 8820*b**7*c**12*d**2) + x*(2940*a**7*c**6*d**8 - 20580*a**
6*b*c**7*d**7 + 61740*a**5*b**2*c**8*d**6 - 102900*a**4*b**3*c**9*d**5 + 10
2900*a**3*b**4*c**10*d**4 - 61740*a**2*b**5*c**11*d**3 + 20580*a*b**6*c**12
*d**2 - 2940*b**7*c**13*d))
```

Giac [B] time = 1.06759, size = 949, normalized size = 4.7

$$\frac{b^8 \log\left(|bx+a|\right)}{b^9c^8 - 8\,ab^8c^7d + 28\,a^2b^7c^6d^2 - 56\,a^3b^6c^5d^3 + 70\,a^4b^5c^4d^4 - 56\,a^5b^4c^3d^5 + 28\,a^6b^3c^2d^6 - 8\,a^7b^2cd^7 + a^8bd^8} - \frac{}{b^8c^8d -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] b^8*log(abs(b*x + a))/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^
3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^
6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8) - b^7*d*log(abs(d*x + c))/(b^8*c^8*d - 8*a
```

```
*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5
 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8 + a^8*d^9) + 1/4
20*(1089*b^7*c^7 - 2940*a*b^6*c^6*d + 4410*a^2*b^5*c^5*d^2 - 4900*a^3*b^4*c
^4*d^3 + 3675*a^4*b^3*c^3*d^4 - 1764*a^5*b^2*c^2*d^5 + 490*a^6*b*c*d^6 - 60
*a^7*d^7 + 420*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 210*(13*b^7*c^2*d^5 - 14*a*b^6
*c*d^6 + a^2*b^5*d^7)*x^5 + 70*(107*b^7*c^3*d^4 - 126*a*b^6*c^2*d^5 + 21*a^
2*b^5*c*d^6 - 2*a^3*b^4*d^7)*x^4 + 35*(319*b^7*c^4*d^3 - 420*a*b^6*c^3*d^4
+ 126*a^2*b^5*c^2*d^5 - 28*a^3*b^4*c*d^6 + 3*a^4*b^3*d^7)*x^3 + 21*(459*b^7
*c^5*d^2 - 700*a*b^6*c^4*d^3 + 350*a^2*b^5*c^3*d^4 - 140*a^3*b^4*c^2*d^5 +
35*a^4*b^3*c*d^6 - 4*a^5*b^2*d^7)*x^2 + 7*(669*b^7*c^6*d - 1260*a*b^6*c^5*d
^2 + 1050*a^2*b^5*c^4*d^3 - 700*a^3*b^4*c^3*d^4 + 315*a^4*b^3*c^2*d^5 - 84*
a^5*b^2*c*d^6 + 10*a^6*b*d^7)*x)/((b*c - a*d)^8*(d*x + c)^7)
```

3.1373 $\int \frac{1}{(a+bx)^2(c+dx)^8}\,dx$

Optimal. Leaf size=231

$$-\frac{b^7}{(a+bx)(bc-ad)^8}-\frac{7b^6d}{(c+dx)(bc-ad)^8}-\frac{3b^5d}{(c+dx)^2(bc-ad)^7}-\frac{5b^4d}{3(c+dx)^3(bc-ad)^6}-\frac{b^3d}{(c+dx)^4(bc-ad)^5}-\frac{}{5(c+d}$$

```
[Out] -(b^7/((b*c - a*d)^8*(a + b*x))) - d/(7*(b*c - a*d)^2*(c + d*x)^7) - (b*d)/
(3*(b*c - a*d)^3*(c + d*x)^6) - (3*b^2*d)/(5*(b*c - a*d)^4*(c + d*x)^5) - (
b^3*d)/((b*c - a*d)^5*(c + d*x)^4) - (5*b^4*d)/(3*(b*c - a*d)^6*(c + d*x)^3
) - (3*b^5*d)/((b*c - a*d)^7*(c + d*x)^2) - (7*b^6*d)/((b*c - a*d)^8*(c + d
*x)) - (8*b^7*d*Log[a + b*x])/(b*c - a*d)^9 + (8*b^7*d*Log[c + d*x])/(b*c -
 a*d)^9
```

Rubi [A] time = 0.270129, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{b^7}{(a+bx)(bc-ad)^8}-\frac{7b^6d}{(c+dx)(bc-ad)^8}-\frac{3b^5d}{(c+dx)^2(bc-ad)^7}-\frac{5b^4d}{3(c+dx)^3(bc-ad)^6}-\frac{b^3d}{(c+dx)^4(bc-ad)^5}-\frac{}{5(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^8),x]
```

```
[Out] -(b^7/((b*c - a*d)^8*(a + b*x))) - d/(7*(b*c - a*d)^2*(c + d*x)^7) - (b*d)/
(3*(b*c - a*d)^3*(c + d*x)^6) - (3*b^2*d)/(5*(b*c - a*d)^4*(c + d*x)^5) - (
b^3*d)/((b*c - a*d)^5*(c + d*x)^4) - (5*b^4*d)/(3*(b*c - a*d)^6*(c + d*x)^3
) - (3*b^5*d)/((b*c - a*d)^7*(c + d*x)^2) - (7*b^6*d)/((b*c - a*d)^8*(c + d
*x)) - (8*b^7*d*Log[a + b*x])/(b*c - a*d)^9 + (8*b^7*d*Log[c + d*x])/(b*c -
 a*d)^9
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)^8}\,dx&=\int\left(\frac{b^8}{(bc-ad)^8(a+bx)^2}-\frac{8b^8d}{(bc-ad)^9(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^8}+\frac{2bd^2}{(bc-ad)^3(c+dx)^7}+\right.\\&=-\frac{b^7}{(bc-ad)^8(a+bx)}-\frac{d}{7(bc-ad)^2(c+dx)^7}-\frac{bd}{3(bc-ad)^3(c+dx)^6}-\frac{3b^2d}{5(bc-ad)^4(c+dx)^5}\end{aligned}$$

Mathematica [A] time = 0.232436, size = 213, normalized size = 0.92

$$-\frac{\frac{105b^7(bc-ad)}{a+bx}+\frac{735b^6d(bc-ad)}{c+dx}+\frac{315b^5d(bc-ad)^2}{(c+dx)^2}+\frac{175b^4d(bc-ad)^3}{(c+dx)^3}+\frac{105b^3d(bc-ad)^4}{(c+dx)^4}+\frac{63b^2d(bc-ad)^5}{(c+dx)^5}+840b^7d\log(a+bx)+\frac{35bd(bc-}{(c+dx}}{105(bc-ad)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^8),x]
```

```
[Out] -((105*b^7*(b*c - a*d))/(a + b*x) - (15*d*(-(b*c) + a*d)^7)/(c + d*x)^7 + (
35*b*d*(b*c - a*d)^6)/(c + d*x)^6 + (63*b^2*d*(b*c - a*d)^5)/(c + d*x)^5 +
(105*b^3*d*(b*c - a*d)^4)/(c + d*x)^4 + (175*b^4*d*(b*c - a*d)^3)/(c + d*x)
^3 + (315*b^5*d*(b*c - a*d)^2)/(c + d*x)^2 + (735*b^6*d*(b*c - a*d))/(c + d
*x) + 840*b^7*d*Log[a + b*x] - 840*b^7*d*Log[c + d*x])/(105*(b*c - a*d)^9)
```

Maple [A] time = 0.02, size = 223, normalized size = 1.

$$-\frac{d}{7\,(ad-bc)^2\,(dx+c)^7}-8\,\frac{db^7\ln(dx+c)}{(ad-bc)^9}-7\,\frac{db^6}{(ad-bc)^8\,(dx+c)}+3\,\frac{db^5}{(ad-bc)^7\,(dx+c)^2}-\frac{5\,db^4}{3\,(ad-bc)^6\,(dx+c)^3}+\frac{}{(ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^8,x)
```

```
[Out] -1/7*d/(a*d-b*c)^2/(d*x+c)^7-8*d/(a*d-b*c)^9*b^7*ln(d*x+c)-7*d/(a*d-b*c)^8*
b^6/(d*x+c)+3*d/(a*d-b*c)^7*b^5/(d*x+c)^2-5/3*d/(a*d-b*c)^6*b^4/(d*x+c)^3+d
/(a*d-b*c)^5*b^3/(d*x+c)^4-3/5*d/(a*d-b*c)^4*b^2/(d*x+c)^5+1/3*d/(a*d-b*c)^
3*b/(d*x+c)^6-b^7/(a*d-b*c)^8/(b*x+a)+8*d/(a*d-b*c)^9*b^7*ln(b*x+a)
```

Maxima [B] time = 1.77341, size = 2539, normalized size = 10.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -8*b^7*d*log(b*x + a)/(b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^
3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*
d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9) + 8*b^7*d*log(d*x + c)/
(b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^
4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d
^7 + 9*a^8*b*c*d^8 - a^9*d^9) - 1/105*(840*b^7*d^7*x^7 + 105*b^7*c^7 + 1443
*a*b^6*c^6*d - 1497*a^2*b^5*c^5*d^2 + 1443*a^3*b^4*c^4*d^3 - 1007*a^4*b^3*c
^3*d^4 + 463*a^5*b^2*c^2*d^5 - 125*a^6*b*c*d^6 + 15*a^7*d^7 + 420*(13*b^7*c
*d^6 + a*b^6*d^7)*x^6 + 140*(107*b^7*c^2*d^5 + 20*a*b^6*c*d^6 - a^2*b^5*d^7
)*x^5 + 70*(319*b^7*c^3*d^4 + 113*a*b^6*c^2*d^5 - 13*a^2*b^5*c*d^6 + a^3*b^
4*d^7)*x^4 + 14*(1377*b^7*c^4*d^3 + 872*a*b^6*c^3*d^4 - 178*a^2*b^5*c^2*d^5
 + 32*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 14*(669*b^7*c^5*d^2 + 786*a*b^6*
c^4*d^3 - 264*a^2*b^5*c^3*d^4 + 86*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 2*a
^5*b^2*d^7)*x^2 + 2*(1089*b^7*c^6*d + 2832*a*b^6*c^5*d^2 - 1578*a^2*b^5*c^4
*d^3 + 872*a^3*b^4*c^3*d^4 - 353*a^4*b^3*c^2*d^5 + 88*a^5*b^2*c*d^6 - 10*a^
6*b*d^7)*x)/(a*b^8*c^15 - 8*a^2*b^7*c^14*d + 28*a^3*b^6*c^13*d^2 - 56*a^4*b
^5*c^12*d^3 + 70*a^5*b^4*c^11*d^4 - 56*a^6*b^3*c^10*d^5 + 28*a^7*b^2*c^9*d^
6 - 8*a^8*b*c^8*d^7 + a^9*c^7*d^8 + (b^9*c^8*d^7 - 8*a*b^8*c^7*d^8 + 28*a^2
*b^7*c^6*d^9 - 56*a^3*b^6*c^5*d^10 + 70*a^4*b^5*c^4*d^11 - 56*a^5*b^4*c^3*d
^12 + 28*a^6*b^3*c^2*d^13 - 8*a^7*b^2*c*d^14 + a^8*b*d^15)*x^8 + (7*b^9*c^9
*d^6 - 55*a*b^8*c^8*d^7 + 188*a^2*b^7*c^7*d^8 - 364*a^3*b^6*c^6*d^9 + 434*a
^4*b^5*c^5*d^10 - 322*a^5*b^4*c^4*d^11 + 140*a^6*b^3*c^3*d^12 - 28*a^7*b^2*
c^2*d^13 - a^8*b*c*d^14 + a^9*d^15)*x^7 + 7*(3*b^9*c^10*d^5 - 23*a*b^8*c^9*
d^6 + 76*a^2*b^7*c^8*d^7 - 140*a^3*b^6*c^7*d^8 + 154*a^4*b^5*c^6*d^9 - 98*a
```

```
^5*b^4*c^5*d^10 + 28*a^6*b^3*c^4*d^11 + 4*a^7*b^2*c^3*d^12 - 5*a^8*b*c^2*d^
13 + a^9*c*d^14)*x^6 + 7*(5*b^9*c^11*d^4 - 37*a*b^8*c^10*d^5 + 116*a^2*b^7*
c^9*d^6 - 196*a^3*b^6*c^8*d^7 + 182*a^4*b^5*c^7*d^8 - 70*a^5*b^4*c^6*d^9 -
28*a^6*b^3*c^5*d^10 + 44*a^7*b^2*c^4*d^11 - 19*a^8*b*c^3*d^12 + 3*a^9*c^2*d
^13)*x^5 + 35*(b^9*c^12*d^3 - 7*a*b^8*c^11*d^4 + 20*a^2*b^7*c^10*d^5 - 28*a
^3*b^6*c^9*d^6 + 14*a^4*b^5*c^8*d^7 + 14*a^5*b^4*c^7*d^8 - 28*a^6*b^3*c^6*d
^9 + 20*a^7*b^2*c^5*d^10 - 7*a^8*b*c^4*d^11 + a^9*c^3*d^12)*x^4 + 7*(3*b^9*
c^13*d^2 - 19*a*b^8*c^12*d^3 + 44*a^2*b^7*c^11*d^4 - 28*a^3*b^6*c^10*d^5 -
70*a^4*b^5*c^9*d^6 + 182*a^5*b^4*c^8*d^7 - 196*a^6*b^3*c^7*d^8 + 116*a^7*b^
2*c^6*d^9 - 37*a^8*b*c^5*d^10 + 5*a^9*c^4*d^11)*x^3 + 7*(b^9*c^14*d - 5*a*b
^8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 + 28*a^3*b^6*c^11*d^4 - 98*a^4*b^5*c^10*d^
5 + 154*a^5*b^4*c^9*d^6 - 140*a^6*b^3*c^8*d^7 + 76*a^7*b^2*c^7*d^8 - 23*a^8
*b*c^6*d^9 + 3*a^9*c^5*d^10)*x^2 + (b^9*c^15 - a*b^8*c^14*d - 28*a^2*b^7*c^
13*d^2 + 140*a^3*b^6*c^12*d^3 - 322*a^4*b^5*c^11*d^4 + 434*a^5*b^4*c^10*d^5
 - 364*a^6*b^3*c^9*d^6 + 188*a^7*b^2*c^8*d^7 - 55*a^8*b*c^7*d^8 + 7*a^9*c^6
*d^9)*x)
```

Fricas [B] time = 2.55013, size = 4788, normalized size = 20.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(105*b^8*c^8 + 1338*a*b^7*c^7*d - 2940*a^2*b^6*c^6*d^2 + 2940*a^3*b^
5*c^5*d^3 - 2450*a^4*b^4*c^4*d^4 + 1470*a^5*b^3*c^3*d^5 - 588*a^6*b^2*c^2*d
^6 + 140*a^7*b*c*d^7 - 15*a^8*d^8 + 840*(b^8*c*d^7 - a*b^7*d^8)*x^7 + 420*(
13*b^8*c^2*d^6 - 12*a*b^7*c*d^7 - a^2*b^6*d^8)*x^6 + 140*(107*b^8*c^3*d^5 -
 87*a*b^7*c^2*d^6 - 21*a^2*b^6*c*d^7 + a^3*b^5*d^8)*x^5 + 70*(319*b^8*c^4*d
^4 - 206*a*b^7*c^3*d^5 - 126*a^2*b^6*c^2*d^6 + 14*a^3*b^5*c*d^7 - a^4*b^4*d
^8)*x^4 + 14*(1377*b^8*c^5*d^3 - 505*a*b^7*c^4*d^4 - 1050*a^2*b^6*c^3*d^5 +
 210*a^3*b^5*c^2*d^6 - 35*a^4*b^4*c*d^7 + 3*a^5*b^3*d^8)*x^3 + 14*(669*b^8*
c^6*d^2 + 117*a*b^7*c^5*d^3 - 1050*a^2*b^6*c^4*d^4 + 350*a^3*b^5*c^3*d^5 -
105*a^4*b^4*c^2*d^6 + 21*a^5*b^3*c*d^7 - 2*a^6*b^2*d^8)*x^2 + 2*(1089*b^8*c
^7*d + 1743*a*b^7*c^6*d^2 - 4410*a^2*b^6*c^5*d^3 + 2450*a^3*b^5*c^4*d^4 - 1
225*a^4*b^4*c^3*d^5 + 441*a^5*b^3*c^2*d^6 - 98*a^6*b^2*c*d^7 + 10*a^7*b*d^8
)*x + 840*(b^8*d^8*x^8 + a*b^7*c^7*d + (7*b^8*c*d^7 + a*b^7*d^8)*x^7 + 7*(3
*b^8*c^2*d^6 + a*b^7*c*d^7)*x^6 + 7*(5*b^8*c^3*d^5 + 3*a*b^7*c^2*d^6)*x^5 +
 35*(b^8*c^4*d^4 + a*b^7*c^3*d^5)*x^4 + 7*(3*b^8*c^5*d^3 + 5*a*b^7*c^4*d^4)
*x^3 + 7*(b^8*c^6*d^2 + 3*a*b^7*c^5*d^3)*x^2 + (b^8*c^7*d + 7*a*b^7*c^6*d^2
)*x)*log(b*x + a) - 840*(b^8*d^8*x^8 + a*b^7*c^7*d + (7*b^8*c*d^7 + a*b^7*d
^8)*x^7 + 7*(3*b^8*c^2*d^6 + a*b^7*c*d^7)*x^6 + 7*(5*b^8*c^3*d^5 + 3*a*b^7*
c^2*d^6)*x^5 + 35*(b^8*c^4*d^4 + a*b^7*c^3*d^5)*x^4 + 7*(3*b^8*c^5*d^3 + 5*
a*b^7*c^4*d^4)*x^3 + 7*(b^8*c^6*d^2 + 3*a*b^7*c^5*d^3)*x^2 + (b^8*c^7*d + 7
*a*b^7*c^6*d^2)*x)*log(d*x + c))/(a*b^9*c^16 - 9*a^2*b^8*c^15*d + 36*a^3*b^
7*c^14*d^2 - 84*a^4*b^6*c^13*d^3 + 126*a^5*b^5*c^12*d^4 - 126*a^6*b^4*c^11*
d^5 + 84*a^7*b^3*c^10*d^6 - 36*a^8*b^2*c^9*d^7 + 9*a^9*b*c^8*d^8 - a^10*c^7
*d^9 + (b^10*c^9*d^7 - 9*a*b^9*c^8*d^8 + 36*a^2*b^8*c^7*d^9 - 84*a^3*b^7*c^
6*d^10 + 126*a^4*b^6*c^5*d^11 - 126*a^5*b^5*c^4*d^12 + 84*a^6*b^4*c^3*d^13
- 36*a^7*b^3*c^2*d^14 + 9*a^8*b^2*c*d^15 - a^9*b*d^16)*x^8 + (7*b^10*c^10*d
^6 - 62*a*b^9*c^9*d^7 + 243*a^2*b^8*c^8*d^8 - 552*a^3*b^7*c^7*d^9 + 798*a^4
*b^6*c^6*d^10 - 756*a^5*b^5*c^5*d^11 + 462*a^6*b^4*c^4*d^12 - 168*a^7*b^3*c
^3*d^13 + 27*a^8*b^2*c^2*d^14 + 2*a^9*b*c*d^15 - a^10*d^16)*x^7 + 7*(3*b^10
*c^11*d^5 - 26*a*b^9*c^10*d^6 + 99*a^2*b^8*c^9*d^7 - 216*a^3*b^7*c^8*d^8 +
294*a^4*b^6*c^7*d^9 - 252*a^5*b^5*c^6*d^10 + 126*a^6*b^4*c^5*d^11 - 24*a^7*
b^3*c^4*d^12 - 9*a^8*b^2*c^3*d^13 + 6*a^9*b*c^2*d^14 - a^10*c*d^15)*x^6 + 7
```

```
*(5*b^10*c^12*d^4 - 42*a*b^9*c^11*d^5 + 153*a^2*b^8*c^10*d^6 - 312*a^3*b^7*
c^9*d^7 + 378*a^4*b^6*c^8*d^8 - 252*a^5*b^5*c^7*d^9 + 42*a^6*b^4*c^6*d^10 +
 72*a^7*b^3*c^5*d^11 - 63*a^8*b^2*c^4*d^12 + 22*a^9*b*c^3*d^13 - 3*a^10*c^2
*d^14)*x^5 + 35*(b^10*c^13*d^3 - 8*a*b^9*c^12*d^4 + 27*a^2*b^8*c^11*d^5 - 4
8*a^3*b^7*c^10*d^6 + 42*a^4*b^6*c^9*d^7 - 42*a^6*b^4*c^7*d^9 + 48*a^7*b^3*c
^6*d^10 - 27*a^8*b^2*c^5*d^11 + 8*a^9*b*c^4*d^12 - a^10*c^3*d^13)*x^4 + 7*(
3*b^10*c^14*d^2 - 22*a*b^9*c^13*d^3 + 63*a^2*b^8*c^12*d^4 - 72*a^3*b^7*c^11
*d^5 - 42*a^4*b^6*c^10*d^6 + 252*a^5*b^5*c^9*d^7 - 378*a^6*b^4*c^8*d^8 + 31
2*a^7*b^3*c^7*d^9 - 153*a^8*b^2*c^6*d^10 + 42*a^9*b*c^5*d^11 - 5*a^10*c^4*d
^12)*x^3 + 7*(b^10*c^15*d - 6*a*b^9*c^14*d^2 + 9*a^2*b^8*c^13*d^3 + 24*a^3*
b^7*c^12*d^4 - 126*a^4*b^6*c^11*d^5 + 252*a^5*b^5*c^10*d^6 - 294*a^6*b^4*c^
9*d^7 + 216*a^7*b^3*c^8*d^8 - 99*a^8*b^2*c^7*d^9 + 26*a^9*b*c^6*d^10 - 3*a^
10*c^5*d^11)*x^2 + (b^10*c^16 - 2*a*b^9*c^15*d - 27*a^2*b^8*c^14*d^2 + 168*
a^3*b^7*c^13*d^3 - 462*a^4*b^6*c^12*d^4 + 756*a^5*b^5*c^11*d^5 - 798*a^6*b^
4*c^10*d^6 + 552*a^7*b^3*c^9*d^7 - 243*a^8*b^2*c^8*d^8 + 62*a^9*b*c^7*d^9 -
 7*a^10*c^6*d^10)*x)
```

Sympy [B] time = 22.2064, size = 2334, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**8,x)
```

```
[Out] -8*b**7*d*log(x + (-8*a**10*b**7*d**11/(a*d - b*c)**9 + 80*a**9*b**8*c*d**1
0/(a*d - b*c)**9 - 360*a**8*b**9*c**2*d**9/(a*d - b*c)**9 + 960*a**7*b**10*
c**3*d**8/(a*d - b*c)**9 - 1680*a**6*b**11*c**4*d**7/(a*d - b*c)**9 + 2016*
a**5*b**12*c**5*d**6/(a*d - b*c)**9 - 1680*a**4*b**13*c**6*d**5/(a*d - b*c)
**9 + 960*a**3*b**14*c**7*d**4/(a*d - b*c)**9 - 360*a**2*b**15*c**8*d**3/(a
*d - b*c)**9 + 80*a*b**16*c**9*d**2/(a*d - b*c)**9 + 8*a*b**7*d**2 - 8*b**1
7*c**10*d/(a*d - b*c)**9 + 8*b**8*c*d)/(16*b**8*d**2))/(a*d - b*c)**9 + 8*b
**7*d*log(x + (8*a**10*b**7*d**11/(a*d - b*c)**9 - 80*a**9*b**8*c*d**10/(a*
d - b*c)**9 + 360*a**8*b**9*c**2*d**9/(a*d - b*c)**9 - 960*a**7*b**10*c**3*
d**8/(a*d - b*c)**9 + 1680*a**6*b**11*c**4*d**7/(a*d - b*c)**9 - 2016*a**5*
b**12*c**5*d**6/(a*d - b*c)**9 + 1680*a**4*b**13*c**6*d**5/(a*d - b*c)**9 -
 960*a**3*b**14*c**7*d**4/(a*d - b*c)**9 + 360*a**2*b**15*c**8*d**3/(a*d -
b*c)**9 - 80*a*b**16*c**9*d**2/(a*d - b*c)**9 + 8*a*b**7*d**2 + 8*b**17*c**
10*d/(a*d - b*c)**9 + 8*b**8*c*d)/(16*b**8*d**2))/(a*d - b*c)**9 - (15*a**7
*d**7 - 125*a**6*b*c*d**6 + 463*a**5*b**2*c**2*d**5 - 1007*a**4*b**3*c**3*d
**4 + 1443*a**3*b**4*c**4*d**3 - 1497*a**2*b**5*c**5*d**2 + 1443*a*b**6*c**
6*d + 105*b**7*c**7 + 840*b**7*d**7*x**7 + x**6*(420*a*b**6*d**7 + 5460*b**
7*c*d**6) + x**5*(-140*a**2*b**5*d**7 + 2800*a*b**6*c*d**6 + 14980*b**7*c**
2*d**5) + x**4*(70*a**3*b**4*d**7 - 910*a**2*b**5*c*d**6 + 7910*a*b**6*c**2
*d**5 + 22330*b**7*c**3*d**4) + x**3*(-42*a**4*b**3*d**7 + 448*a**3*b**4*c*
d**6 - 2492*a**2*b**5*c**2*d**5 + 12208*a*b**6*c**3*d**4 + 19278*b**7*c**4*
d**3) + x**2*(28*a**5*b**2*d**7 - 266*a**4*b**3*c*d**6 + 1204*a**3*b**4*c**
2*d**5 - 3696*a**2*b**5*c**3*d**4 + 11004*a*b**6*c**4*d**3 + 9366*b**7*c**5
*d**2) + x*(-20*a**6*b*d**7 + 176*a**5*b**2*c*d**6 - 706*a**4*b**3*c**2*d**
5 + 1744*a**3*b**4*c**3*d**4 - 3156*a**2*b**5*c**4*d**3 + 5664*a*b**6*c**5*
d**2 + 2178*b**7*c**6*d))/(105*a**9*c**7*d**8 - 840*a**8*b*c**8*d**7 + 2940
*a**7*b**2*c**9*d**6 - 5880*a**6*b**3*c**10*d**5 + 7350*a**5*b**4*c**11*d**
4 - 5880*a**4*b**5*c**12*d**3 + 2940*a**3*b**6*c**13*d**2 - 840*a**2*b**7*c
**14*d + 105*a*b**8*c**15 + x**8*(105*a**8*b*d**15 - 840*a**7*b**2*c*d**14
+ 2940*a**6*b**3*c**2*d**13 - 5880*a**5*b**4*c**3*d**12 + 7350*a**4*b**5*c*
*4*d**11 - 5880*a**3*b**6*c**5*d**10 + 2940*a**2*b**7*c**6*d**9 - 840*a*b**
8*c**7*d**8 + 105*b**9*c**8*d**7) + x**7*(105*a**9*d**15 - 105*a**8*b*c*d**
```

```
14 - 2940*a**7*b**2*c**2*d**13 + 14700*a**6*b**3*c**3*d**12 - 33810*a**5*b*
*4*c**4*d**11 + 45570*a**4*b**5*c**5*d**10 - 38220*a**3*b**6*c**6*d**9 + 19
740*a**2*b**7*c**7*d**8 - 5775*a*b**8*c**8*d**7 + 735*b**9*c**9*d**6) + x**
6*(735*a**9*c*d**14 - 3675*a**8*b*c**2*d**13 + 2940*a**7*b**2*c**3*d**12 +
20580*a**6*b**3*c**4*d**11 - 72030*a**5*b**4*c**5*d**10 + 113190*a**4*b**5*
c**6*d**9 - 102900*a**3*b**6*c**7*d**8 + 55860*a**2*b**7*c**8*d**7 - 16905*
a*b**8*c**9*d**6 + 2205*b**9*c**10*d**5) + x**5*(2205*a**9*c**2*d**13 - 139
65*a**8*b*c**3*d**12 + 32340*a**7*b**2*c**4*d**11 - 20580*a**6*b**3*c**5*d*
*10 - 51450*a**5*b**4*c**6*d**9 + 133770*a**4*b**5*c**7*d**8 - 144060*a**3*
b**6*c**8*d**7 + 85260*a**2*b**7*c**9*d**6 - 27195*a*b**8*c**10*d**5 + 3675
*b**9*c**11*d**4) + x**4*(3675*a**9*c**3*d**12 - 25725*a**8*b*c**4*d**11 +
73500*a**7*b**2*c**5*d**10 - 102900*a**6*b**3*c**6*d**9 + 51450*a**5*b**4*c
**7*d**8 + 51450*a**4*b**5*c**8*d**7 - 102900*a**3*b**6*c**9*d**6 + 73500*a
**2*b**7*c**10*d**5 - 25725*a*b**8*c**11*d**4 + 3675*b**9*c**12*d**3) + x**
3*(3675*a**9*c**4*d**11 - 27195*a**8*b*c**5*d**10 + 85260*a**7*b**2*c**6*d*
*9 - 144060*a**6*b**3*c**7*d**8 + 133770*a**5*b**4*c**8*d**7 - 51450*a**4*b
**5*c**9*d**6 - 20580*a**3*b**6*c**10*d**5 + 32340*a**2*b**7*c**11*d**4 - 1
3965*a*b**8*c**12*d**3 + 2205*b**9*c**13*d**2) + x**2*(2205*a**9*c**5*d**10
 - 16905*a**8*b*c**6*d**9 + 55860*a**7*b**2*c**7*d**8 - 102900*a**6*b**3*c*
*8*d**7 + 113190*a**5*b**4*c**9*d**6 - 72030*a**4*b**5*c**10*d**5 + 20580*a
**3*b**6*c**11*d**4 + 2940*a**2*b**7*c**12*d**3 - 3675*a*b**8*c**13*d**2 +
735*b**9*c**14*d) + x*(735*a**9*c**6*d**9 - 5775*a**8*b*c**7*d**8 + 19740*a
**7*b**2*c**8*d**7 - 38220*a**6*b**3*c**9*d**6 + 45570*a**5*b**4*c**10*d**5
 - 33810*a**4*b**5*c**11*d**4 + 14700*a**3*b**6*c**12*d**3 - 2940*a**2*b**7
*c**13*d**2 - 105*a*b**8*c**14*d + 105*b**9*c**15))
```

Giac [B] time = 1.13522, size = 964, normalized size = 4.17

$$-\frac{b^{15}}{\left(b^{16}c^{8} - 8\,ab^{15}c^{7}d + 28\,a^{2}b^{14}c^{6}d^{2} - 56\,a^{3}b^{13}c^{5}d^{3} + 70\,a^{4}b^{12}c^{4}d^{4} - 56\,a^{5}b^{11}c^{3}d^{5} + 28\,a^{6}b^{10}c^{2}d^{6} - 8\,a^{7}b^{9}cd^{7} + a^{8}b^{8}d^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -b^15/((b^16*c^8 - 8*a*b^15*c^7*d + 28*a^2*b^14*c^6*d^2 - 56*a^3*b^13*c^5*d
^3 + 70*a^4*b^12*c^4*d^4 - 56*a^5*b^11*c^3*d^5 + 28*a^6*b^10*c^2*d^6 - 8*a^
7*b^9*c*d^7 + a^8*b^8*d^8)*(b*x + a)) + 8*b^8*d*log(abs(b*c/(b*x + a) - a*d
/(b*x + a) + d))/(b^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2 - 84*a^3*b^
7*c^6*d^3 + 126*a^4*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5 + 84*a^6*b^4*c^3*d^6
- 36*a^7*b^3*c^2*d^7 + 9*a^8*b^2*c*d^8 - a^9*b*d^9) + 1/105*(1443*b^7*d^8 +
 9366*(b^9*c*d^7 - a*b^8*d^8)/((b*x + a)*b) + 25578*(b^11*c^2*d^6 - 2*a*b^1
0*c*d^7 + a^2*b^9*d^8)/((b*x + a)^2*b^2) + 37730*(b^13*c^3*d^5 - 3*a*b^12*c
^2*d^6 + 3*a^2*b^11*c*d^7 - a^3*b^10*d^8)/((b*x + a)^3*b^3) + 31850*(b^15*c
^4*d^4 - 4*a*b^14*c^3*d^5 + 6*a^2*b^13*c^2*d^6 - 4*a^3*b^12*c*d^7 + a^4*b^1
1*d^8)/((b*x + a)^4*b^4) + 14700*(b^17*c^5*d^3 - 5*a*b^16*c^4*d^4 + 10*a^2*
b^15*c^3*d^5 - 10*a^3*b^14*c^2*d^6 + 5*a^4*b^13*c*d^7 - a^5*b^12*d^8)/((b*x
 + a)^5*b^5) + 2940*(b^19*c^6*d^2 - 6*a*b^18*c^5*d^3 + 15*a^2*b^17*c^4*d^4
- 20*a^3*b^16*c^3*d^5 + 15*a^4*b^15*c^2*d^6 - 6*a^5*b^14*c*d^7 + a^6*b^13*d
^8)/((b*x + a)^6*b^6))/((b*c - a*d)^9*(b*c/(b*x + a) - a*d/(b*x + a) + d)^7
)
```

3.1374 $\int \frac{1}{(a+bx)^3(c+dx)^8} dx$

Optimal. Leaf size=276

$$\frac{28b^6d^2}{(c+dx)(bc-ad)^9}+\frac{21b^5d^2}{2(c+dx)^2(bc-ad)^8}+\frac{5b^4d^2}{(c+dx)^3(bc-ad)^7}+\frac{5b^3d^2}{2(c+dx)^4(bc-ad)^6}+\frac{6b^2d^2}{5(c+dx)^5(bc-ad)^5}+\frac{36b^7d^2}{(bc}$$

```
[Out] -b^7/(2*(b*c - a*d)^8*(a + b*x)^2) + (8*b^7*d)/((b*c - a*d)^9*(a + b*x)) +
d^2/(7*(b*c - a*d)^3*(c + d*x)^7) + (b*d^2)/(2*(b*c - a*d)^4*(c + d*x)^6) +
 (6*b^2*d^2)/(5*(b*c - a*d)^5*(c + d*x)^5) + (5*b^3*d^2)/(2*(b*c - a*d)^6*(
c + d*x)^4) + (5*b^4*d^2)/((b*c - a*d)^7*(c + d*x)^3) + (21*b^5*d^2)/(2*(b*
c - a*d)^8*(c + d*x)^2) + (28*b^6*d^2)/((b*c - a*d)^9*(c + d*x)) + (36*b^7*
d^2*Log[a + b*x])/(b*c - a*d)^10 - (36*b^7*d^2*Log[c + d*x])/(b*c - a*d)^10
```

Rubi [A] time = 0.356038, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{28b^6d^2}{(c+dx)(bc-ad)^9}+\frac{21b^5d^2}{2(c+dx)^2(bc-ad)^8}+\frac{5b^4d^2}{(c+dx)^3(bc-ad)^7}+\frac{5b^3d^2}{2(c+dx)^4(bc-ad)^6}+\frac{6b^2d^2}{5(c+dx)^5(bc-ad)^5}+\frac{36b^7d^2}{(bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^8),x]
```



```
[Out] -b^7/(2*(b*c - a*d)^8*(a + b*x)^2) + (8*b^7*d)/((b*c - a*d)^9*(a + b*x)) +
d^2/(7*(b*c - a*d)^3*(c + d*x)^7) + (b*d^2)/(2*(b*c - a*d)^4*(c + d*x)^6) +
 (6*b^2*d^2)/(5*(b*c - a*d)^5*(c + d*x)^5) + (5*b^3*d^2)/(2*(b*c - a*d)^6*(
c + d*x)^4) + (5*b^4*d^2)/((b*c - a*d)^7*(c + d*x)^3) + (21*b^5*d^2)/(2*(b*
c - a*d)^8*(c + d*x)^2) + (28*b^6*d^2)/((b*c - a*d)^9*(c + d*x)) + (36*b^7*
d^2*Log[a + b*x])/(b*c - a*d)^10 - (36*b^7*d^2*Log[c + d*x])/(b*c - a*d)^10
```


Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^8} dx = \int \left(\frac{b^8}{(bc-ad)^8(a+bx)^3}-\frac{8b^8d}{(bc-ad)^9(a+bx)^2}+\frac{36b^8d^2}{(bc-ad)^{10}(a+bx)}-\frac{d^3}{(bc-ad)^3(c+dx)^8}-\right.$$

$$=-\frac{b^7}{2(bc-ad)^8(a+bx)^2}+\frac{8b^7d}{(bc-ad)^9(a+bx)}+\frac{d^2}{7(bc-ad)^3(c+dx)^7}+\frac{bd^2}{2(bc-ad)^4(c+dx)^6}+$$

Mathematica [A] time = 0.198945, size = 254, normalized size = 0.92

$$\frac{\frac{1960b^6d^2(bc-ad)}{c+dx}+\frac{735b^5d^2(bc-ad)^2}{(c+dx)^2}+\frac{350b^4d^2(bc-ad)^3}{(c+dx)^3}+\frac{175b^3d^2(bc-ad)^4}{(c+dx)^4}+\frac{84b^2d^2(bc-ad)^5}{(c+dx)^5}+\frac{560b^7d(bc-ad)}{a+bx}-\frac{35b^7(bc-ad)^2}{(a+bx)^2}+2520b^7d^2\log(}{70(bc-ad)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^8),x]
```

```
[Out] ((-35*b^7*(b*c - a*d)^2)/(a + b*x)^2 + (560*b^7*d*(b*c - a*d))/(a + b*x) +
(10*d^2*(b*c - a*d)^7)/(c + d*x)^7 + (35*b*d^2*(b*c - a*d)^6)/(c + d*x)^6 +
 (84*b^2*d^2*(b*c - a*d)^5)/(c + d*x)^5 + (175*b^3*d^2*(b*c - a*d)^4)/(c +
d*x)^4 + (350*b^4*d^2*(b*c - a*d)^3)/(c + d*x)^3 + (735*b^5*d^2*(b*c - a*d)
^2)/(c + d*x)^2 + (1960*b^6*d^2*(b*c - a*d))/(c + d*x) + 2520*b^7*d^2*Log[a
 + b*x] - 2520*b^7*d^2*Log[c + d*x])/(70*(b*c - a*d)^10)
```

Maple [A] time = 0.02, size = 265, normalized size = 1.

$$-\frac{d^2}{7\,(ad-bc)^3\,(dx+c)^7}-36\,\frac{d^2b^7\ln(dx+c)}{(ad-bc)^{10}}-28\,\frac{d^2b^6}{(ad-bc)^9\,(dx+c)}+\frac{21\,d^2b^5}{2\,(ad-bc)^8\,(dx+c)^2}-5\,\frac{d^2b^4}{(ad-bc)^7\,(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^8,x)
```

```
[Out] -1/7*d^2/(a*d-b*c)^3/(d*x+c)^7-36*d^2/(a*d-b*c)^10*b^7*ln(d*x+c)-28*d^2/(a*
d-b*c)^9*b^6/(d*x+c)+21/2*d^2/(a*d-b*c)^8*b^5/(d*x+c)^2-5*d^2/(a*d-b*c)^7*b
^4/(d*x+c)^3+5/2*d^2/(a*d-b*c)^6*b^3/(d*x+c)^4-6/5*d^2/(a*d-b*c)^5*b^2/(d*x
+c)^5+1/2*d^2/(a*d-b*c)^4*b/(d*x+c)^6-1/2*b^7/(a*d-b*c)^8/(b*x+a)^2+36*d^2/
(a*d-b*c)^10*b^7*ln(b*x+a)-8*b^7/(a*d-b*c)^9*d/(b*x+a)
```

Maxima [B] time = 2.18734, size = 3239, normalized size = 11.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 36*b^7*d^2*log(b*x + a)/(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 -
120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b
^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^
10*d^10) - 36*b^7*d^2*log(d*x + c)/(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8
*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5
+ 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b
*c*d^9 + a^10*d^10) + 1/70*(2520*b^8*d^8*x^8 - 35*b^8*c^8 + 595*a*b^7*c^7*d
 + 3349*a^2*b^6*c^6*d^2 - 2531*a^3*b^5*c^5*d^3 + 1879*a^4*b^4*c^4*d^4 - 106
1*a^5*b^3*c^3*d^5 + 409*a^6*b^2*c^2*d^6 - 95*a^7*b*c*d^7 + 10*a^8*d^8 + 126
0*(13*b^8*c*d^7 + 3*a*b^7*d^8)*x^7 + 420*(107*b^8*c^2*d^6 + 59*a*b^7*c*d^7
+ 2*a^2*b^6*d^8)*x^6 + 210*(319*b^8*c^3*d^5 + 327*a*b^7*c^2*d^6 + 27*a^2*b^
6*c*d^7 - a^3*b^5*d^8)*x^5 + 42*(1377*b^8*c^4*d^4 + 2467*a*b^7*c^3*d^5 + 38
7*a^2*b^6*c^2*d^6 - 33*a^3*b^5*c*d^7 + 2*a^4*b^4*d^8)*x^4 + 42*(669*b^8*c^5
*d^3 + 2163*a*b^7*c^4*d^4 + 608*a^2*b^6*c^3*d^5 - 92*a^3*b^5*c^2*d^6 + 13*a
^4*b^4*c*d^7 - a^5*b^3*d^8)*x^3 + 6*(1089*b^8*c^6*d^2 + 7515*a*b^7*c^5*d^3
+ 3924*a^2*b^6*c^4*d^4 - 976*a^3*b^5*c^3*d^5 + 249*a^4*b^4*c^2*d^6 - 45*a^5
*b^3*c*d^7 + 4*a^6*b^2*d^8)*x^2 + 3*(105*b^8*c^7*d + 3621*a*b^7*c^6*d^2 + 4
167*a^2*b^6*c^5*d^3 - 1713*a^3*b^5*c^4*d^4 + 737*a^4*b^4*c^3*d^5 - 243*a^5*
b^3*c^2*d^6 + 51*a^6*b^2*c*d^7 - 5*a^7*b*d^8)*x)/(a^2*b^9*c^16 - 9*a^3*b^8*
c^15*d + 36*a^4*b^7*c^14*d^2 - 84*a^5*b^6*c^13*d^3 + 126*a^6*b^5*c^12*d^4 -
 126*a^7*b^4*c^11*d^5 + 84*a^8*b^3*c^10*d^6 - 36*a^9*b^2*c^9*d^7 + 9*a^10*b
*c^8*d^8 - a^11*c^7*d^9 + (b^11*c^9*d^7 - 9*a*b^10*c^8*d^8 + 36*a^2*b^9*c^7
```

```
*d^9 - 84*a^3*b^8*c^6*d^10 + 126*a^4*b^7*c^5*d^11 - 126*a^5*b^6*c^4*d^12 +
84*a^6*b^5*c^3*d^13 - 36*a^7*b^4*c^2*d^14 + 9*a^8*b^3*c*d^15 - a^9*b^2*d^16
)*x^9 + (7*b^11*c^10*d^6 - 61*a*b^10*c^9*d^7 + 234*a^2*b^9*c^8*d^8 - 516*a^
3*b^8*c^7*d^9 + 714*a^4*b^7*c^6*d^10 - 630*a^5*b^6*c^5*d^11 + 336*a^6*b^5*c
^4*d^12 - 84*a^7*b^4*c^3*d^13 - 9*a^8*b^3*c^2*d^14 + 11*a^9*b^2*c*d^15 - 2*
a^10*b*d^16)*x^8 + (21*b^11*c^11*d^5 - 175*a*b^10*c^10*d^6 + 631*a^2*b^9*c^
9*d^7 - 1269*a^3*b^8*c^8*d^8 + 1506*a^4*b^7*c^7*d^9 - 966*a^5*b^6*c^6*d^10
+ 126*a^6*b^5*c^5*d^11 + 294*a^7*b^4*c^4*d^12 - 231*a^8*b^3*c^3*d^13 + 69*a
^9*b^2*c^2*d^14 - 5*a^10*b*c*d^15 - a^11*d^16)*x^7 + 7*(5*b^11*c^12*d^4 - 3
9*a*b^10*c^11*d^5 + 127*a^2*b^9*c^10*d^6 - 213*a^3*b^8*c^9*d^7 + 162*a^4*b^
7*c^8*d^8 + 42*a^5*b^6*c^7*d^9 - 210*a^6*b^5*c^6*d^10 + 198*a^7*b^4*c^5*d^1
1 - 87*a^8*b^3*c^4*d^12 + 13*a^9*b^2*c^3*d^13 + 3*a^10*b*c^2*d^14 - a^11*c*
d^15)*x^6 + 7*(5*b^11*c^13*d^3 - 35*a*b^10*c^12*d^4 + 93*a^2*b^9*c^11*d^5 -
 87*a^3*b^8*c^10*d^6 - 102*a^4*b^7*c^9*d^7 + 378*a^5*b^6*c^8*d^8 - 462*a^6*
b^5*c^7*d^9 + 282*a^7*b^4*c^6*d^10 - 63*a^8*b^3*c^5*d^11 - 23*a^9*b^2*c^4*d
^12 + 17*a^10*b*c^3*d^13 - 3*a^11*c^2*d^14)*x^5 + 7*(3*b^11*c^14*d^2 - 17*a
*b^10*c^13*d^3 + 23*a^2*b^9*c^12*d^4 + 63*a^3*b^8*c^11*d^5 - 282*a^4*b^7*c^
10*d^6 + 462*a^5*b^6*c^9*d^7 - 378*a^6*b^5*c^8*d^8 + 102*a^7*b^4*c^7*d^9 +
87*a^8*b^3*c^6*d^10 - 93*a^9*b^2*c^5*d^11 + 35*a^10*b*c^4*d^12 - 5*a^11*c^3
*d^13)*x^4 + 7*(b^11*c^15*d - 3*a*b^10*c^14*d^2 - 13*a^2*b^9*c^13*d^3 + 87*
a^3*b^8*c^12*d^4 - 198*a^4*b^7*c^11*d^5 + 210*a^5*b^6*c^10*d^6 - 42*a^6*b^5
*c^9*d^7 - 162*a^7*b^4*c^8*d^8 + 213*a^8*b^3*c^7*d^9 - 127*a^9*b^2*c^6*d^10
 + 39*a^10*b*c^5*d^11 - 5*a^11*c^4*d^12)*x^3 + (b^11*c^16 + 5*a*b^10*c^15*d
 - 69*a^2*b^9*c^14*d^2 + 231*a^3*b^8*c^13*d^3 - 294*a^4*b^7*c^12*d^4 - 126*
a^5*b^6*c^11*d^5 + 966*a^6*b^5*c^10*d^6 - 1506*a^7*b^4*c^9*d^7 + 1269*a^8*b
^3*c^8*d^8 - 631*a^9*b^2*c^7*d^9 + 175*a^10*b*c^6*d^10 - 21*a^11*c^5*d^11)*
x^2 + (2*a*b^10*c^16 - 11*a^2*b^9*c^15*d + 9*a^3*b^8*c^14*d^2 + 84*a^4*b^7*
c^13*d^3 - 336*a^5*b^6*c^12*d^4 + 630*a^6*b^5*c^11*d^5 - 714*a^7*b^4*c^10*d
^6 + 516*a^8*b^3*c^9*d^7 - 234*a^9*b^2*c^8*d^8 + 61*a^10*b*c^7*d^9 - 7*a^11
*c^6*d^10)*x)
```

Fricas [B] time = 3.05477, size = 6507, normalized size = 23.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/70*(35*b^9*c^9 - 630*a*b^8*c^8*d - 2754*a^2*b^7*c^7*d^2 + 5880*a^3*b^6*c
^6*d^3 - 4410*a^4*b^5*c^5*d^4 + 2940*a^5*b^4*c^4*d^5 - 1470*a^6*b^3*c^3*d^6
 + 504*a^7*b^2*c^2*d^7 - 105*a^8*b*c*d^8 + 10*a^9*d^9 - 2520*(b^9*c*d^8 - a
*b^8*d^9)*x^8 - 1260*(13*b^9*c^2*d^7 - 10*a*b^8*c*d^8 - 3*a^2*b^7*d^9)*x^7
- 420*(107*b^9*c^3*d^6 - 48*a*b^8*c^2*d^7 - 57*a^2*b^7*c*d^8 - 2*a^3*b^6*d^
9)*x^6 - 210*(319*b^9*c^4*d^5 + 8*a*b^8*c^3*d^6 - 300*a^2*b^7*c^2*d^7 - 28*
a^3*b^6*c*d^8 + a^4*b^5*d^9)*x^5 - 42*(1377*b^9*c^5*d^4 + 1090*a*b^8*c^4*d^
5 - 2080*a^2*b^7*c^3*d^6 - 420*a^3*b^6*c^2*d^7 + 35*a^4*b^5*c*d^8 - 2*a^5*b
^4*d^9)*x^4 - 42*(669*b^9*c^6*d^3 + 1494*a*b^8*c^5*d^4 - 1555*a^2*b^7*c^4*d
^5 - 700*a^3*b^6*c^3*d^6 + 105*a^4*b^5*c^2*d^7 - 14*a^5*b^4*c*d^8 + a^6*b^3
*d^9)*x^3 - 6*(1089*b^9*c^7*d^2 + 6426*a*b^8*c^6*d^3 - 3591*a^2*b^7*c^5*d^4
 - 4900*a^3*b^6*c^4*d^5 + 1225*a^4*b^5*c^3*d^6 - 294*a^5*b^4*c^2*d^7 + 49*a
^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 - 3*(105*b^9*c^8*d + 3516*a*b^8*c^7*d^2 +
 546*a^2*b^7*c^6*d^3 - 5880*a^3*b^6*c^5*d^4 + 2450*a^4*b^5*c^4*d^5 - 980*a^
5*b^4*c^3*d^6 + 294*a^6*b^3*c^2*d^7 - 56*a^7*b^2*c*d^8 + 5*a^8*b*d^9)*x - 2
520*(b^9*d^9*x^9 + a^2*b^7*c^7*d^2 + (7*b^9*c*d^8 + 2*a*b^8*d^9)*x^8 + (21*
b^9*c^2*d^7 + 14*a*b^8*c*d^8 + a^2*b^7*d^9)*x^7 + 7*(5*b^9*c^3*d^6 + 6*a*b^
8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 7*(5*b^9*c^4*d^5 + 10*a*b^8*c^3*d^6 + 3*a^
```

```
2*b^7*c^2*d^7)*x^5 + 7*(3*b^9*c^5*d^4 + 10*a*b^8*c^4*d^5 + 5*a^2*b^7*c^3*d^
6)*x^4 + 7*(b^9*c^6*d^3 + 6*a*b^8*c^5*d^4 + 5*a^2*b^7*c^4*d^5)*x^3 + (b^9*c
^7*d^2 + 14*a*b^8*c^6*d^3 + 21*a^2*b^7*c^5*d^4)*x^2 + (2*a*b^8*c^7*d^2 + 7*
a^2*b^7*c^6*d^3)*x)*log(b*x + a) + 2520*(b^9*d^9*x^9 + a^2*b^7*c^7*d^2 + (7
*b^9*c*d^8 + 2*a*b^8*d^9)*x^8 + (21*b^9*c^2*d^7 + 14*a*b^8*c*d^8 + a^2*b^7*
d^9)*x^7 + 7*(5*b^9*c^3*d^6 + 6*a*b^8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 7*(5*b
^9*c^4*d^5 + 10*a*b^8*c^3*d^6 + 3*a^2*b^7*c^2*d^7)*x^5 + 7*(3*b^9*c^5*d^4 +
 10*a*b^8*c^4*d^5 + 5*a^2*b^7*c^3*d^6)*x^4 + 7*(b^9*c^6*d^3 + 6*a*b^8*c^5*d
^4 + 5*a^2*b^7*c^4*d^5)*x^3 + (b^9*c^7*d^2 + 14*a*b^8*c^6*d^3 + 21*a^2*b^7*
c^5*d^4)*x^2 + (2*a*b^8*c^7*d^2 + 7*a^2*b^7*c^6*d^3)*x)*log(d*x + c))/(a^2*
b^10*c^17 - 10*a^3*b^9*c^16*d + 45*a^4*b^8*c^15*d^2 - 120*a^5*b^7*c^14*d^3
+ 210*a^6*b^6*c^13*d^4 - 252*a^7*b^5*c^12*d^5 + 210*a^8*b^4*c^11*d^6 - 120*
a^9*b^3*c^10*d^7 + 45*a^10*b^2*c^9*d^8 - 10*a^11*b*c^8*d^9 + a^12*c^7*d^10
+ (b^12*c^10*d^7 - 10*a*b^11*c^9*d^8 + 45*a^2*b^10*c^8*d^9 - 120*a^3*b^9*c^
7*d^10 + 210*a^4*b^8*c^6*d^11 - 252*a^5*b^7*c^5*d^12 + 210*a^6*b^6*c^4*d^13
 - 120*a^7*b^5*c^3*d^14 + 45*a^8*b^4*c^2*d^15 - 10*a^9*b^3*c*d^16 + a^10*b^
2*d^17)*x^9 + (7*b^12*c^11*d^6 - 68*a*b^11*c^10*d^7 + 295*a^2*b^10*c^9*d^8
- 750*a^3*b^9*c^8*d^9 + 1230*a^4*b^8*c^7*d^10 - 1344*a^5*b^7*c^6*d^11 + 966
*a^6*b^6*c^5*d^12 - 420*a^7*b^5*c^4*d^13 + 75*a^8*b^4*c^3*d^14 + 20*a^9*b^3
*c^2*d^15 - 13*a^10*b^2*c*d^16 + 2*a^11*b*d^17)*x^8 + (21*b^12*c^12*d^5 - 1
96*a*b^11*c^11*d^6 + 806*a^2*b^10*c^10*d^7 - 1900*a^3*b^9*c^9*d^8 + 2775*a^
4*b^8*c^8*d^9 - 2472*a^5*b^7*c^7*d^10 + 1092*a^6*b^6*c^6*d^11 + 168*a^7*b^5
*c^5*d^12 - 525*a^8*b^4*c^4*d^13 + 300*a^9*b^3*c^3*d^14 - 74*a^10*b^2*c^2*d
^15 + 4*a^11*b*c*d^16 + a^12*d^17)*x^7 + 7*(5*b^12*c^13*d^4 - 44*a*b^11*c^1
2*d^5 + 166*a^2*b^10*c^11*d^6 - 340*a^3*b^9*c^10*d^7 + 375*a^4*b^8*c^9*d^8
- 120*a^5*b^7*c^8*d^9 - 252*a^6*b^6*c^7*d^10 + 408*a^7*b^5*c^6*d^11 - 285*a
^8*b^4*c^5*d^12 + 100*a^9*b^3*c^4*d^13 - 10*a^10*b^2*c^3*d^14 - 4*a^11*b*c^
2*d^15 + a^12*c*d^16)*x^6 + 7*(5*b^12*c^14*d^3 - 40*a*b^11*c^13*d^4 + 128*a
^2*b^10*c^12*d^5 - 180*a^3*b^9*c^11*d^6 - 15*a^4*b^8*c^10*d^7 + 480*a^5*b^7
*c^9*d^8 - 840*a^6*b^6*c^8*d^9 + 744*a^7*b^5*c^7*d^10 - 345*a^8*b^4*c^6*d^1
1 + 40*a^9*b^3*c^5*d^12 + 40*a^10*b^2*c^4*d^13 - 20*a^11*b*c^3*d^14 + 3*a^1
2*c^2*d^15)*x^5 + 7*(3*b^12*c^15*d^2 - 20*a*b^11*c^14*d^3 + 40*a^2*b^10*c^1
3*d^4 + 40*a^3*b^9*c^12*d^5 - 345*a^4*b^8*c^11*d^6 + 744*a^5*b^7*c^10*d^7 -
 840*a^6*b^6*c^9*d^8 + 480*a^7*b^5*c^8*d^9 - 15*a^8*b^4*c^7*d^10 - 180*a^9*
b^3*c^6*d^11 + 128*a^10*b^2*c^5*d^12 - 40*a^11*b*c^4*d^13 + 5*a^12*c^3*d^14
)*x^4 + 7*(b^12*c^16*d - 4*a*b^11*c^15*d^2 - 10*a^2*b^10*c^14*d^3 + 100*a^3
*b^9*c^13*d^4 - 285*a^4*b^8*c^12*d^5 + 408*a^5*b^7*c^11*d^6 - 252*a^6*b^6*c
^10*d^7 - 120*a^7*b^5*c^9*d^8 + 375*a^8*b^4*c^8*d^9 - 340*a^9*b^3*c^7*d^10
+ 166*a^10*b^2*c^6*d^11 - 44*a^11*b*c^5*d^12 + 5*a^12*c^4*d^13)*x^3 + (b^12
*c^17 + 4*a*b^11*c^16*d - 74*a^2*b^10*c^15*d^2 + 300*a^3*b^9*c^14*d^3 - 525
*a^4*b^8*c^13*d^4 + 168*a^5*b^7*c^12*d^5 + 1092*a^6*b^6*c^11*d^6 - 2472*a^7
*b^5*c^10*d^7 + 2775*a^8*b^4*c^9*d^8 - 1900*a^9*b^3*c^8*d^9 + 806*a^10*b^2*
c^7*d^10 - 196*a^11*b*c^6*d^11 + 21*a^12*c^5*d^12)*x^2 + (2*a*b^11*c^17 - 1
3*a^2*b^10*c^16*d + 20*a^3*b^9*c^15*d^2 + 75*a^4*b^8*c^14*d^3 - 420*a^5*b^7
*c^13*d^4 + 966*a^6*b^6*c^12*d^5 - 1344*a^7*b^5*c^11*d^6 + 1230*a^8*b^4*c^1
0*d^7 - 750*a^9*b^3*c^9*d^8 + 295*a^10*b^2*c^8*d^9 - 68*a^11*b*c^7*d^10 + 7
*a^12*c^6*d^11)*x)
```

Sympy [B] time = 43.5253, size = 2914, normalized size = 10.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**8,x)
```

```
[Out] -36*b**7*d**2*log(x + (-36*a**11*b**7*d**13/(a*d - b*c)**10 + 396*a**10*b**
8*c*d**12/(a*d - b*c)**10 - 1980*a**9*b**9*c**2*d**11/(a*d - b*c)**10 + 594
```

```
0*a**8*b**10*c**3*d**10/(a*d - b*c)**10 - 11880*a**7*b**11*c**4*d**9/(a*d -
 b*c)**10 + 16632*a**6*b**12*c**5*d**8/(a*d - b*c)**10 - 16632*a**5*b**13*c
**6*d**7/(a*d - b*c)**10 + 11880*a**4*b**14*c**7*d**6/(a*d - b*c)**10 - 594
0*a**3*b**15*c**8*d**5/(a*d - b*c)**10 + 1980*a**2*b**16*c**9*d**4/(a*d - b
*c)**10 - 396*a*b**17*c**10*d**3/(a*d - b*c)**10 + 36*a*b**7*d**3 + 36*b**1
8*c**11*d**2/(a*d - b*c)**10 + 36*b**8*c*d**2)/(72*b**8*d**3))/(a*d - b*c)*
*10 + 36*b**7*d**2*log(x + (36*a**11*b**7*d**13/(a*d - b*c)**10 - 396*a**10
*b**8*c*d**12/(a*d - b*c)**10 + 1980*a**9*b**9*c**2*d**11/(a*d - b*c)**10 -
 5940*a**8*b**10*c**3*d**10/(a*d - b*c)**10 + 11880*a**7*b**11*c**4*d**9/(a
*d - b*c)**10 - 16632*a**6*b**12*c**5*d**8/(a*d - b*c)**10 + 16632*a**5*b**
13*c**6*d**7/(a*d - b*c)**10 - 11880*a**4*b**14*c**7*d**6/(a*d - b*c)**10 +
 5940*a**3*b**15*c**8*d**5/(a*d - b*c)**10 - 1980*a**2*b**16*c**9*d**4/(a*d
 - b*c)**10 + 396*a*b**17*c**10*d**3/(a*d - b*c)**10 + 36*a*b**7*d**3 - 36*
b**18*c**11*d**2/(a*d - b*c)**10 + 36*b**8*c*d**2)/(72*b**8*d**3))/(a*d - b
*c)**10 - (10*a**8*d**8 - 95*a**7*b*c*d**7 + 409*a**6*b**2*c**2*d**6 - 1061
*a**5*b**3*c**3*d**5 + 1879*a**4*b**4*c**4*d**4 - 2531*a**3*b**5*c**5*d**3
 + 3349*a**2*b**6*c**6*d**2 + 595*a*b**7*c**7*d - 35*b**8*c**8 + 2520*b**8*d
**8*x**8 + x**7*(3780*a*b**7*d**8 + 16380*b**8*c*d**7) + x**6*(840*a**2*b**
6*d**8 + 24780*a*b**7*c*d**7 + 44940*b**8*c**2*d**6) + x**5*(-210*a**3*b**5
*d**8 + 5670*a**2*b**6*c*d**7 + 68670*a*b**7*c**2*d**6 + 66990*b**8*c**3*d*
*5) + x**4*(84*a**4*b**4*d**8 - 1386*a**3*b**5*c*d**7 + 16254*a**2*b**6*c**
2*d**6 + 103614*a*b**7*c**3*d**5 + 57834*b**8*c**4*d**4) + x**3*(-42*a**5*b
**3*d**8 + 546*a**4*b**4*c*d**7 - 3864*a**3*b**5*c**2*d**6 + 25536*a**2*b**
6*c**3*d**5 + 90846*a*b**7*c**4*d**4 + 28098*b**8*c**5*d**3) + x**2*(24*a**
6*b**2*d**8 - 270*a**5*b**3*c*d**7 + 1494*a**4*b**4*c**2*d**6 - 5856*a**3*b
**5*c**3*d**5 + 23544*a**2*b**6*c**4*d**4 + 45090*a*b**7*c**5*d**3 + 6534*b
**8*c**6*d**2) + x*(-15*a**7*b*d**8 + 153*a**6*b**2*c*d**7 - 729*a**5*b**3*
c**2*d**6 + 2211*a**4*b**4*c**3*d**5 - 5139*a**3*b**5*c**4*d**4 + 12501*a**
2*b**6*c**5*d**3 + 10863*a*b**7*c**6*d**2 + 315*b**8*c**7*d))/(70*a**11*c**
7*d**9 - 630*a**10*b*c**8*d**8 + 2520*a**9*b**2*c**9*d**7 - 5880*a**8*b**3*
c**10*d**6 + 8820*a**7*b**4*c**11*d**5 - 8820*a**6*b**5*c**12*d**4 + 5880*a
**5*b**6*c**13*d**3 - 2520*a**4*b**7*c**14*d**2 + 630*a**3*b**8*c**15*d - 7
0*a**2*b**9*c**16 + x**9*(70*a**9*b**2*d**16 - 630*a**8*b**3*c*d**15 + 2520
*a**7*b**4*c**2*d**14 - 5880*a**6*b**5*c**3*d**13 + 8820*a**5*b**6*c**4*d**
12 - 8820*a**4*b**7*c**5*d**11 + 5880*a**3*b**8*c**6*d**10 - 2520*a**2*b**9
*c**7*d**9 + 630*a*b**10*c**8*d**8 - 70*b**11*c**9*d**7) + x**8*(140*a**10*
b*d**16 - 770*a**9*b**2*c*d**15 + 630*a**8*b**3*c**2*d**14 + 5880*a**7*b**4
*c**3*d**13 - 23520*a**6*b**5*c**4*d**12 + 44100*a**5*b**6*c**5*d**11 - 499
80*a**4*b**7*c**6*d**10 + 36120*a**3*b**8*c**7*d**9 - 16380*a**2*b**9*c**8*
d**8 + 4270*a*b**10*c**9*d**7 - 490*b**11*c**10*d**6) + x**7*(70*a**11*d**1
6 + 350*a**10*b*c*d**15 - 4830*a**9*b**2*c**2*d**14 + 16170*a**8*b**3*c**3*
d**13 - 20580*a**7*b**4*c**4*d**12 - 8820*a**6*b**5*c**5*d**11 + 67620*a**5
*b**6*c**6*d**10 - 105420*a**4*b**7*c**7*d**9 + 88830*a**3*b**8*c**8*d**8 -
 44170*a**2*b**9*c**9*d**7 + 12250*a*b**10*c**10*d**6 - 1470*b**11*c**11*d*
*5) + x**6*(490*a**11*c*d**15 - 1470*a**10*b*c**2*d**14 - 6370*a**9*b**2*c*
*3*d**13 + 42630*a**8*b**3*c**4*d**12 - 97020*a**7*b**4*c**5*d**11 + 102900
*a**6*b**5*c**6*d**10 - 20580*a**5*b**6*c**7*d**9 - 79380*a**4*b**7*c**8*d*
*8 + 104370*a**3*b**8*c**9*d**7 - 62230*a**2*b**9*c**10*d**6 + 19110*a*b**1
0*c**11*d**5 - 2450*b**11*c**12*d**4) + x**5*(1470*a**11*c**2*d**14 - 8330*
a**10*b*c**3*d**13 + 11270*a**9*b**2*c**4*d**12 + 30870*a**8*b**3*c**5*d**1
1 - 138180*a**7*b**4*c**6*d**10 + 226380*a**6*b**5*c**7*d**9 - 185220*a**5*
b**6*c**8*d**8 + 49980*a**4*b**7*c**9*d**7 + 42630*a**3*b**8*c**10*d**6 - 4
5570*a**2*b**9*c**11*d**5 + 17150*a*b**10*c**12*d**4 - 2450*b**11*c**13*d**
3) + x**4*(2450*a**11*c**3*d**13 - 17150*a**10*b*c**4*d**12 + 45570*a**9*b*
*2*c**5*d**11 - 42630*a**8*b**3*c**6*d**10 - 49980*a**7*b**4*c**7*d**9 + 18
5220*a**6*b**5*c**8*d**8 - 226380*a**5*b**6*c**9*d**7 + 138180*a**4*b**7*c*
*10*d**6 - 30870*a**3*b**8*c**11*d**5 - 11270*a**2*b**9*c**12*d**4 + 8330*a
*b**10*c**13*d**3 - 1470*b**11*c**14*d**2) + x**3*(2450*a**11*c**4*d**12 -
19110*a**10*b*c**5*d**11 + 62230*a**9*b**2*c**6*d**10 - 104370*a**8*b**3*c*
```

```
*7*d**9 + 79380*a**7*b**4*c**8*d**8 + 20580*a**6*b**5*c**9*d**7 - 102900*a*
*5*b**6*c**10*d**6 + 97020*a**4*b**7*c**11*d**5 - 42630*a**3*b**8*c**12*d**
4 + 6370*a**2*b**9*c**13*d**3 + 1470*a*b**10*c**14*d**2 - 490*b**11*c**15*d
) + x**2*(1470*a**11*c**5*d**11 - 12250*a**10*b*c**6*d**10 + 44170*a**9*b**
2*c**7*d**9 - 88830*a**8*b**3*c**8*d**8 + 105420*a**7*b**4*c**9*d**7 - 6762
0*a**6*b**5*c**10*d**6 + 8820*a**5*b**6*c**11*d**5 + 20580*a**4*b**7*c**12*
d**4 - 16170*a**3*b**8*c**13*d**3 + 4830*a**2*b**9*c**14*d**2 - 350*a*b**10
*c**15*d - 70*b**11*c**16) + x*(490*a**11*c**6*d**10 - 4270*a**10*b*c**7*d*
*9 + 16380*a**9*b**2*c**8*d**8 - 36120*a**8*b**3*c**9*d**7 + 49980*a**7*b**
4*c**10*d**6 - 44100*a**6*b**5*c**11*d**5 + 23520*a**5*b**6*c**12*d**4 - 58
80*a**4*b**7*c**13*d**3 - 630*a**3*b**8*c**14*d**2 + 770*a**2*b**9*c**15*d
- 140*a*b**10*c**16))
```

Giac [B] time = 1.08716, size = 1389, normalized size = 5.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 36*b^8*d^2*log(abs(b*x + a))/(b^11*c^10 - 10*a*b^10*c^9*d + 45*a^2*b^9*c^8*
d^2 - 120*a^3*b^8*c^7*d^3 + 210*a^4*b^7*c^6*d^4 - 252*a^5*b^6*c^5*d^5 + 210
*a^6*b^5*c^4*d^6 - 120*a^7*b^4*c^3*d^7 + 45*a^8*b^3*c^2*d^8 - 10*a^9*b^2*c*
d^9 + a^10*b*d^10) - 36*b^7*d^3*log(abs(d*x + c))/(b^10*c^10*d - 10*a*b^9*c
^9*d^2 + 45*a^2*b^8*c^8*d^3 - 120*a^3*b^7*c^7*d^4 + 210*a^4*b^6*c^6*d^5 - 2
52*a^5*b^5*c^5*d^6 + 210*a^6*b^4*c^4*d^7 - 120*a^7*b^3*c^3*d^8 + 45*a^8*b^2
*c^2*d^9 - 10*a^9*b*c*d^10 + a^10*d^11) - 1/70*(35*b^9*c^9 - 630*a*b^8*c^8*
d - 2754*a^2*b^7*c^7*d^2 + 5880*a^3*b^6*c^6*d^3 - 4410*a^4*b^5*c^5*d^4 + 29
40*a^5*b^4*c^4*d^5 - 1470*a^6*b^3*c^3*d^6 + 504*a^7*b^2*c^2*d^7 - 105*a^8*b
*c*d^8 + 10*a^9*d^9 - 2520*(b^9*c*d^8 - a*b^8*d^9)*x^8 - 1260*(13*b^9*c^2*d
^7 - 10*a*b^8*c*d^8 - 3*a^2*b^7*d^9)*x^7 - 420*(107*b^9*c^3*d^6 - 48*a*b^8*
c^2*d^7 - 57*a^2*b^7*c*d^8 - 2*a^3*b^6*d^9)*x^6 - 210*(319*b^9*c^4*d^5 + 8*
a*b^8*c^3*d^6 - 300*a^2*b^7*c^2*d^7 - 28*a^3*b^6*c*d^8 + a^4*b^5*d^9)*x^5 -
 42*(1377*b^9*c^5*d^4 + 1090*a*b^8*c^4*d^5 - 2080*a^2*b^7*c^3*d^6 - 420*a^3
*b^6*c^2*d^7 + 35*a^4*b^5*c*d^8 - 2*a^5*b^4*d^9)*x^4 - 42*(669*b^9*c^6*d^3
+ 1494*a*b^8*c^5*d^4 - 1555*a^2*b^7*c^4*d^5 - 700*a^3*b^6*c^3*d^6 + 105*a^4
*b^5*c^2*d^7 - 14*a^5*b^4*c*d^8 + a^6*b^3*d^9)*x^3 - 6*(1089*b^9*c^7*d^2 +
6426*a*b^8*c^6*d^3 - 3591*a^2*b^7*c^5*d^4 - 4900*a^3*b^6*c^4*d^5 + 1225*a^4
*b^5*c^3*d^6 - 294*a^5*b^4*c^2*d^7 + 49*a^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2
- 3*(105*b^9*c^8*d + 3516*a*b^8*c^7*d^2 + 546*a^2*b^7*c^6*d^3 - 5880*a^3*b^
6*c^5*d^4 + 2450*a^4*b^5*c^4*d^5 - 980*a^5*b^4*c^3*d^6 + 294*a^6*b^3*c^2*d^
7 - 56*a^7*b^2*c*d^8 + 5*a^8*b*d^9)*x)/((b*c - a*d)^10*(b*x + a)^2*(d*x + c
)^7)
```

3.1375 $\int (a+bx)^5\sqrt{c+dx}\,dx$

Optimal. Leaf size=156

$$-\frac{10b^4(c+dx)^{11/2}(bc-ad)}{11d^6}+\frac{20b^3(c+dx)^{9/2}(bc-ad)^2}{9d^6}-\frac{20b^2(c+dx)^{7/2}(bc-ad)^3}{7d^6}+\frac{2b(c+dx)^{5/2}(bc-ad)^4}{d^6}-\frac{2(c+dx)}{}$$

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(3/2))/(3*d^6) + (2*b*(b*c - a*d)^4*(c + d*x)^(
5/2))/d^6 - (20*b^2*(b*c - a*d)^3*(c + d*x)^(7/2))/(7*d^6) + (20*b^3*(b*c -
 a*d)^2*(c + d*x)^(9/2))/(9*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(11/2))/(1
1*d^6) + (2*b^5*(c + d*x)^(13/2))/(13*d^6)
```

Rubi [A] time = 0.0627983, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{10b^4(c+dx)^{11/2}(bc-ad)}{11d^6}+\frac{20b^3(c+dx)^{9/2}(bc-ad)^2}{9d^6}-\frac{20b^2(c+dx)^{7/2}(bc-ad)^3}{7d^6}+\frac{2b(c+dx)^{5/2}(bc-ad)^4}{d^6}-\frac{2(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(3/2))/(3*d^6) + (2*b*(b*c - a*d)^4*(c + d*x)^(
5/2))/d^6 - (20*b^2*(b*c - a*d)^3*(c + d*x)^(7/2))/(7*d^6) + (20*b^3*(b*c -
 a*d)^2*(c + d*x)^(9/2))/(9*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(11/2))/(1
1*d^6) + (2*b^5*(c + d*x)^(13/2))/(13*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5\sqrt{c+dx}\,dx &= \int \left(\frac{(-bc+ad)^5\sqrt{c+dx}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{3/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{5/2}}{d^5}+\frac{10b^3(bc-ad)^2}{d^5}\right.\\ &= -\frac{2(bc-ad)^5(c+dx)^{3/2}}{3d^6}+\frac{2b(bc-ad)^4(c+dx)^{5/2}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{7/2}}{7d^6}+\frac{20b^3(bc-ad)^2}{9d^6}\end{aligned}$$

Mathematica [A] time = 0.138097, size = 123, normalized size = 0.79

$$\frac{2(c+dx)^{3/2}\left(-12870b^2(c+dx)^2(bc-ad)^3+10010b^3(c+dx)^3(bc-ad)^2-4095b^4(c+dx)^4(bc-ad)+9009b(c+dx)(bc-\right.}{9009d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-3003*(b*c - a*d)^5 + 9009*b*(b*c - a*d)^4*(c + d*x) -
12870*b^2*(b*c - a*d)^3*(c + d*x)^2 + 10010*b^3*(b*c - a*d)^2*(c + d*x)^3 -
```

```
4095*b^4*(b*c - a*d)*(c + d*x)^4 + 693*b^5*(c + d*x)^5))/(9009*d^6)
```

Maple [B] time = 0.005, size = 273, normalized size = 1.8

$$\frac{1386\,b^5x^5d^5 + 8190\,ab^4d^5x^4 - 1260\,b^5cd^4x^4 + 20020\,a^2b^3d^5x^3 - 7280\,ab^4cd^4x^3 + 1120\,b^5c^2d^3x^3 + 25740\,a^3b^2d^5x^2 - 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(1/2),x)
```

```
[Out] 2/9009*(d*x+c)^(3/2)*(693*b^5*d^5*x^5+4095*a*b^4*d^5*x^4-630*b^5*c*d^4*x^4+
10010*a^2*b^3*d^5*x^3-3640*a*b^4*c*d^4*x^3+560*b^5*c^2*d^3*x^3+12870*a^3*b^
2*d^5*x^2-8580*a^2*b^3*c*d^4*x^2+3120*a*b^4*c^2*d^3*x^2-480*b^5*c^3*d^2*x^2
+9009*a^4*b*d^5*x-10296*a^3*b^2*c*d^4*x+6864*a^2*b^3*c^2*d^3*x-2496*a*b^4*c
^3*d^2*x+384*b^5*c^4*d*x+3003*a^5*d^5-6006*a^4*b*c*d^4+6864*a^3*b^2*c^2*d^3
-4576*a^2*b^3*c^3*d^2+1664*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

Maxima [A] time = 0.972099, size = 350, normalized size = 2.24

$$\frac{2\left(693\,(dx+c)^{\frac{13}{2}}b^5 - 4095\left(b^5c - ab^4d\right)(dx+c)^{\frac{11}{2}} + 10010\left(b^5c^2 - 2\,ab^4cd + a^2b^3d^2\right)(dx+c)^{\frac{9}{2}} - 12870\left(b^5c^3 - 3\,ab^4c^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9009*(693*(d*x + c)^(13/2)*b^5 - 4095*(b^5*c - a*b^4*d)*(d*x + c)^(11/2)
+ 10010*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(9/2) - 12870*(b^5*
c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(7/2) + 9009
*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4
)*(d*x + c)^(5/2) - 3003*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10
*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(3/2))/d^6
```

Fricas [B] time = 2.10166, size = 761, normalized size = 4.88

$$\frac{2\left(693\,b^5d^6x^6 - 256\,b^5c^6 + 1664\,ab^4c^5d - 4576\,a^2b^3c^4d^2 + 6864\,a^3b^2c^3d^3 - 6006\,a^4bc^2d^4 + 3003\,a^5cd^5 + 63\left(b^5cd^5 + 6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9009*(693*b^5*d^6*x^6 - 256*b^5*c^6 + 1664*a*b^4*c^5*d - 4576*a^2*b^3*c^4
*d^2 + 6864*a^3*b^2*c^3*d^3 - 6006*a^4*b*c^2*d^4 + 3003*a^5*c*d^5 + 63*(b^5
*c*d^5 + 65*a*b^4*d^6)*x^5 - 35*(2*b^5*c^2*d^4 - 13*a*b^4*c*d^5 - 286*a^2*b
^3*d^6)*x^4 + 10*(8*b^5*c^3*d^3 - 52*a*b^4*c^2*d^4 + 143*a^2*b^3*c*d^5 + 12
87*a^3*b^2*d^6)*x^3 - 3*(32*b^5*c^4*d^2 - 208*a*b^4*c^3*d^3 + 572*a^2*b^3*c
^2*d^4 - 858*a^3*b^2*c*d^5 - 3003*a^4*b*d^6)*x^2 + (128*b^5*c^5*d - 832*a*b
^4*c^4*d^2 + 2288*a^2*b^3*c^3*d^3 - 3432*a^3*b^2*c^2*d^4 + 3003*a^4*b*c*d^5
 + 3003*a^5*d^6)*x)*sqrt(d*x + c)/d^6
```

Sympy [B] time = 3.75216, size = 314, normalized size = 2.01

$$\frac{2\left(\frac{b^5(c+dx)^{\frac{13}{2}}}{13d^5}+\frac{(c+dx)^{\frac{11}{2}}\left(5ab^4d-5b^5c\right)}{11d^5}+\frac{(c+dx)^{\frac{9}{2}}\left(10a^2b^3d^2-20ab^4cd+10b^5c^2\right)}{9d^5}+\frac{(c+dx)^{\frac{7}{2}}\left(10a^3b^2d^3-30a^2b^3cd^2+30ab^4c^2d-10b^5c^3\right)}{7d^5}+\frac{(c+dx)^{\frac{5}{2}}\left(5a^4bd^4-\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**5*(c + d*x)**(13/2)/(13*d**5) + (c + d*x)**(11/2)*(5*a*b**4*d - 5*b**
5*c)/(11*d**5) + (c + d*x)**(9/2)*(10*a**2*b**3*d**2 - 20*a*b**4*c*d + 10*b
**5*c**2)/(9*d**5) + (c + d*x)**(7/2)*(10*a**3*b**2*d**3 - 30*a**2*b**3*c*d
**2 + 30*a*b**4*c**2*d - 10*b**5*c**3)/(7*d**5) + (c + d*x)**(5/2)*(5*a**4*
b*d**4 - 20*a**3*b**2*c*d**3 + 30*a**2*b**3*c**2*d**2 - 20*a*b**4*c**3*d +
5*b**5*c**4)/(5*d**5) + (c + d*x)**(3/2)*(a**5*d**5 - 5*a**4*b*c*d**4 + 10*
a**3*b**2*c**2*d**3 - 10*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d - b**5*c**5)
/(3*d**5))/d
```

Giac [B] time = 1.09713, size = 385, normalized size = 2.47

$$\frac{2\left(3003\,(dx+c)^{\frac{3}{2}}a^5+\frac{3003\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)a^4b}{d}+\frac{858\left(15\,(dx+c)^{\frac{7}{2}}-42\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2\right)a^3b^2}{d^2}+\frac{286\left(35\,(dx+c)^{\frac{9}{2}}-135\,(dx+c)^{\frac{7}{2}}c+189\,(\right.}{d^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9009*(3003*(d*x + c)^(3/2)*a^5 + 3003*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3
/2)*c)*a^4*b/d + 858*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2)*a^3*b^2/d^2 + 286*(35*(d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*
c + 189*(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)*a^2*b^3/d^3 + 13*(31
5*(d*x + c)^(11/2) - 1540*(d*x + c)^(9/2)*c + 2970*(d*x + c)^(7/2)*c^2 - 27
72*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4)*a*b^4/d^4 + (693*(d*x +
c)^(13/2) - 4095*(d*x + c)^(11/2)*c + 10010*(d*x + c)^(9/2)*c^2 - 12870*(d*
x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 3003*(d*x + c)^(3/2)*c^5)*b^5
/d^5)/d
```

3.1376 $\int (a+bx)^4\sqrt{c+dx}\,dx$

Optimal. Leaf size=129

$$-\frac{8b^3(c+dx)^{9/2}(bc-ad)}{9d^5}+\frac{12b^2(c+dx)^{7/2}(bc-ad)^2}{7d^5}-\frac{8b(c+dx)^{5/2}(bc-ad)^3}{5d^5}+\frac{2(c+dx)^{3/2}(bc-ad)^4}{3d^5}+\frac{2b^4(c+dx)}{11d^5}$$

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(3/2))/(3*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(5
/2))/(5*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(7/2))/(7*d^5) - (8*b^3*(b*c
 - a*d)*(c + d*x)^(9/2))/(9*d^5) + (2*b^4*(c + d*x)^(11/2))/(11*d^5)
```

Rubi [A] time = 0.0519554, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{9/2}(bc-ad)}{9d^5}+\frac{12b^2(c+dx)^{7/2}(bc-ad)^2}{7d^5}-\frac{8b(c+dx)^{5/2}(bc-ad)^3}{5d^5}+\frac{2(c+dx)^{3/2}(bc-ad)^4}{3d^5}+\frac{2b^4(c+dx)}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(3/2))/(3*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(5
/2))/(5*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(7/2))/(7*d^5) - (8*b^3*(b*c
 - a*d)*(c + d*x)^(9/2))/(9*d^5) + (2*b^4*(c + d*x)^(11/2))/(11*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^4\sqrt{c+dx}\,dx=\int\left(\frac{(-bc+ad)^4\sqrt{c+dx}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{3/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{5/2}}{d^4}-\frac{4b^3(bc-ad)}{d^4}\right.$$
$$=\frac{2(bc-ad)^4(c+dx)^{3/2}}{3d^5}-\frac{8b(bc-ad)^3(c+dx)^{5/2}}{5d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{7/2}}{7d^5}-\frac{8b^3(bc-ad)(}{9d^5}$$

Mathematica [A] time = 0.0867557, size = 101, normalized size = 0.78

$$\frac{2(c+dx)^{3/2}\left(2970b^2(c+dx)^2(bc-ad)^2-1540b^3(c+dx)^3(bc-ad)-2772b(c+dx)(bc-ad)^3+1155(bc-ad)^4+315b\right.}{3465d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(1155*(b*c - a*d)^4 - 2772*b*(b*c - a*d)^3*(c + d*x) + 2
970*b^2*(b*c - a*d)^2*(c + d*x)^2 - 1540*b^3*(b*c - a*d)*(c + d*x)^3 + 315*
```

```
b^4*(c + d*x)^4))/(3465*d^5)
```

Maple [A] time = 0.005, size = 186, normalized size = 1.4

$$\frac{630\,b^4x^4d^4+3080\,ab^3d^4x^3-560\,b^4cd^3x^3+5940\,a^2b^2d^4x^2-2640\,ab^3cd^3x^2+480\,b^4c^2d^2x^2+5544\,a^3bd^4x-4752\,a^2b^2cd}{3465\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(1/2),x)
```

```
[Out] 2/3465*(d*x+c)^(3/2)*(315*b^4*d^4*x^4+1540*a*b^3*d^4*x^3-280*b^4*c*d^3*x^3+
2970*a^2*b^2*d^4*x^2-1320*a*b^3*c*d^3*x^2+240*b^4*c^2*d^2*x^2+2772*a^3*b*d^
4*x-2376*a^2*b^2*c*d^3*x+1056*a*b^3*c^2*d^2*x-192*b^4*c^3*d*x+1155*a^4*d^4-
1848*a^3*b*c*d^3+1584*a^2*b^2*c^2*d^2-704*a*b^3*c^3*d+128*b^4*c^4)/d^5
```

Maxima [A] time = 0.956078, size = 244, normalized size = 1.89

$$\frac{2\left(315\,(dx+c)^{\frac{11}{2}}b^4-1540\left(b^4c-ab^3d\right)(dx+c)^{\frac{9}{2}}+2970\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)(dx+c)^{\frac{7}{2}}-2772\left(b^4c^3-3\,ab^3c^2d+3\,a\right.}{3465\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(d*x + c)^(11/2)*b^4 - 1540*(b^4*c - a*b^3*d)*(d*x + c)^(9/2) +
 2970*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(7/2) - 2772*(b^4*c^3
 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(5/2) + 1155*(b^4
*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*x +
c)^(3/2))/d^5
```

Fricas [B] time = 1.95661, size = 547, normalized size = 4.24

$$\frac{2\left(315\,b^4d^5x^5+128\,b^4c^5-704\,ab^3c^4d+1584\,a^2b^2c^3d^2-1848\,a^3bc^2d^3+1155\,a^4cd^4+35\left(b^4cd^4+44\,ab^3d^5\right)x^4-10\left(4\,b^4c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*b^4*d^5*x^5 + 128*b^4*c^5 - 704*a*b^3*c^4*d + 1584*a^2*b^2*c^3*
d^2 - 1848*a^3*b*c^2*d^3 + 1155*a^4*c*d^4 + 35*(b^4*c*d^4 + 44*a*b^3*d^5)*x
^4 - 10*(4*b^4*c^2*d^3 - 22*a*b^3*c*d^4 - 297*a^2*b^2*d^5)*x^3 + 6*(8*b^4*c
^3*d^2 - 44*a*b^3*c^2*d^3 + 99*a^2*b^2*c*d^4 + 462*a^3*b*d^5)*x^2 - (64*b^4
*c^4*d - 352*a*b^3*c^3*d^2 + 792*a^2*b^2*c^2*d^3 - 924*a^3*b*c*d^4 - 1155*a
^4*d^5)*x)*sqrt(d*x + c)/d^5
```

Sympy [A] time = 3.06919, size = 223, normalized size = 1.73

$$\frac{2\left(\frac{b^4(c+dx)^{\frac{11}{2}}}{11d^4}+\frac{(c+dx)^{\frac{9}{2}}\left(4ab^3d-4b^4c\right)}{9d^4}+\frac{(c+dx)^{\frac{7}{2}}\left(6a^2b^2d^2-12ab^3cd+6b^4c^2\right)}{7d^4}+\frac{(c+dx)^{\frac{5}{2}}\left(4a^3bd^3-12a^2b^2cd^2+12ab^3c^2d-4b^4c^3\right)}{5d^4}+\frac{(c+dx)^{\frac{3}{2}}\left(a^4d^4-4a^3b\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**4*(c + d*x)**(11/2)/(11*d**4) + (c + d*x)**(9/2)*(4*a*b**3*d - 4*b**4
*c)/(9*d**4) + (c + d*x)**(7/2)*(6*a**2*b**2*d**2 - 12*a*b**3*c*d + 6*b**4*
c**2)/(7*d**4) + (c + d*x)**(5/2)*(4*a**3*b*d**3 - 12*a**2*b**2*c*d**2 + 12
*a*b**3*c**2*d - 4*b**4*c**3)/(5*d**4) + (c + d*x)**(3/2)*(a**4*d**4 - 4*a*
*3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(3*d**4)
)/d
```

Giac [A] time = 1.09752, size = 278, normalized size = 2.16

$$\frac{2\left(1155\,(dx+c)^{\frac{3}{2}}a^4+\frac{924\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)a^3b}{d}+\frac{198\left(15\,(dx+c)^{\frac{7}{2}}-42\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2\right)a^2b^2}{d^2}+\frac{44\left(35\,(dx+c)^{\frac{9}{2}}-135\,(dx+c)^{\frac{7}{2}}c+189\right.}{d^3}\right.}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(1155*(d*x + c)^(3/2)*a^4 + 924*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/
2)*c)*a^3*b/d + 198*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x +
c)^(3/2)*c^2)*a^2*b^2/d^2 + 44*(35*(d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c
+ 189*(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)*a*b^3/d^3 + (315*(d*x
+ c)^(11/2) - 1540*(d*x + c)^(9/2)*c + 2970*(d*x + c)^(7/2)*c^2 - 2772*(d*x
 + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4)*b^4/d^4)/d
```

3.1377 $\int (a+bx)^3\sqrt{c+dx}\,dx$

Optimal. Leaf size=100

$$-\frac{6b^2(c+dx)^{7/2}(bc-ad)}{7d^4}+\frac{6b(c+dx)^{5/2}(bc-ad)^2}{5d^4}-\frac{2(c+dx)^{3/2}(bc-ad)^3}{3d^4}+\frac{2b^3(c+dx)^{9/2}}{9d^4}$$

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(3/2))/(3*d^4) + (6*b*(b*c - a*d)^2*(c + d*x)^(
5/2))/(5*d^4) - (6*b^2*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^4) + (2*b^3*(c + d
*x)^(9/2))/(9*d^4)
```

Rubi [A] time = 0.0355149, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{6b^2(c+dx)^{7/2}(bc-ad)}{7d^4}+\frac{6b(c+dx)^{5/2}(bc-ad)^2}{5d^4}-\frac{2(c+dx)^{3/2}(bc-ad)^3}{3d^4}+\frac{2b^3(c+dx)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(3/2))/(3*d^4) + (6*b*(b*c - a*d)^2*(c + d*x)^(
5/2))/(5*d^4) - (6*b^2*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^4) + (2*b^3*(c + d
*x)^(9/2))/(9*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3\sqrt{c+dx}\,dx &= \int \left(\frac{(-bc+ad)^3\sqrt{c+dx}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{3/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{5/2}}{d^3}+\frac{b^3(c+dx)^{7/2}}{d^3}\right)dx\\ &= -\frac{2(bc-ad)^3(c+dx)^{3/2}}{3d^4}+\frac{6b(bc-ad)^2(c+dx)^{5/2}}{5d^4}-\frac{6b^2(bc-ad)(c+dx)^{7/2}}{7d^4}+\frac{2b^3(c+dx)^{9/2}}{9d^4}\end{aligned}$$

Mathematica [A] time = 0.0604461, size = 79, normalized size = 0.79

$$\frac{2(c+dx)^{3/2}\left(-135b^2(c+dx)^2(bc-ad)+189b(c+dx)(bc-ad)^2-105(bc-ad)^3+35b^3(c+dx)^3\right)}{315d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-105*(b*c - a*d)^3 + 189*b*(b*c - a*d)^2*(c + d*x) - 13
5*b^2*(b*c - a*d)*(c + d*x)^2 + 35*b^3*(c + d*x)^3))/(315*d^4)
```

Maple [A] time = 0.004, size = 116, normalized size = 1.2

$$\frac{70\,b^3x^3d^3+270\,ab^2d^3x^2-60\,b^3cd^2x^2+378\,a^2bd^3x-216\,ab^2cd^2x+48\,b^3c^2dx+210\,a^3d^3-252\,a^2bcd^2+144\,ab^2c^2d-}{315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(1/2),x)
```

```
[Out] 2/315*(d*x+c)^(3/2)*(35*b^3*d^3*x^3+135*a*b^2*d^3*x^2-30*b^3*c*d^2*x^2+189*
a^2*b*d^3*x-108*a*b^2*c*d^2*x+24*b^3*c^2*d*x+105*a^3*d^3-126*a^2*b*c*d^2+72
*a*b^2*c^2*d-16*b^3*c^3)/d^4
```

Maxima [A] time = 0.967357, size = 159, normalized size = 1.59

$$\frac{2\left(35\,(dx+c)^{\frac{9}{2}}b^3-135\left(b^3c-ab^2d\right)(dx+c)^{\frac{7}{2}}+189\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)(dx+c)^{\frac{5}{2}}-105\left(b^3c^3-3\,ab^2c^2d+3\,a^2bc\right.}{315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(d*x + c)^(9/2)*b^3 - 135*(b^3*c - a*b^2*d)*(d*x + c)^(7/2) + 189
*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(5/2) - 105*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(3/2))/d^4
```

Fricas [A] time = 2.05845, size = 359, normalized size = 3.59

$$\frac{2\left(35\,b^3d^4x^4-16\,b^3c^4+72\,ab^2c^3d-126\,a^2bc^2d^2+105\,a^3cd^3+5\left(b^3cd^3+27\,ab^2d^4\right)x^3-3\left(2\,b^3c^2d^2-9\,ab^2cd^3-63\,a\right.\right.}{315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^3*d^4*x^4 - 16*b^3*c^4 + 72*a*b^2*c^3*d - 126*a^2*b*c^2*d^2 + 1
05*a^3*c*d^3 + 5*(b^3*c*d^3 + 27*a*b^2*d^4)*x^3 - 3*(2*b^3*c^2*d^2 - 9*a*b^
2*c*d^3 - 63*a^2*b*d^4)*x^2 + (8*b^3*c^3*d - 36*a*b^2*c^2*d^2 + 63*a^2*b*c*
d^3 + 105*a^3*d^4)*x)*sqrt(d*x + c)/d^4
```

Sympy [A] time = 2.36295, size = 146, normalized size = 1.46

$$\frac{2\left(\frac{b^3(c+dx)^{\frac{9}{2}}}{9d^3}+\frac{(c+dx)^{\frac{7}{2}}\left(3ab^2d-3b^3c\right)}{7d^3}+\frac{(c+dx)^{\frac{5}{2}}\left(3a^2bd^2-6ab^2cd+3b^3c^2\right)}{5d^3}+\frac{(c+dx)^{\frac{3}{2}}\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{3d^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**3*(c + d*x)**(9/2)/(9*d**3) + (c + d*x)**(7/2)*(3*a*b**2*d - 3*b**3*c
)/(7*d**3) + (c + d*x)**(5/2)*(3*a**2*b*d**2 - 6*a*b**2*c*d + 3*b**3*c**2)/
(5*d**3) + (c + d*x)**(3/2)*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
- b**3*c**3)/(3*d**3))/d
```

Giac [A] time = 1.06367, size = 188, normalized size = 1.88

$$\frac{2\left(105\,(dx+c)^{\frac{3}{2}}a^3+\frac{63\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)a^2b}{d}+\frac{9\left(15\,(dx+c)^{\frac{7}{2}}-42\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2\right)ab^2}{d^2}+\frac{\left(35\,(dx+c)^{\frac{9}{2}}-135\,(dx+c)^{\frac{7}{2}}c+189\,(dx+c)^{\frac{5}{2}}c^2-\cdots\right.}{d^3}\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(105*(d*x + c)^(3/2)*a^3 + 63*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*
c)*a^2*b/d + 9*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3
/2)*c^2)*a*b^2/d^2 + (35*(d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c + 189*(d*x
 + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)*b^3/d^3)/d
```

3.1378 $\int (a+bx)^2\sqrt{c+dx}\,dx$

Optimal. Leaf size=71

$$-\frac{4b(c+dx)^{5/2}(bc-ad)}{5d^3}+\frac{2(c+dx)^{3/2}(bc-ad)^2}{3d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}$$

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(3/2))/(3*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(5/2
))/(5*d^3) + (2*b^2*(c + d*x)^(7/2))/(7*d^3)
```

Rubi [A] time = 0.0247449, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b(c+dx)^{5/2}(bc-ad)}{5d^3}+\frac{2(c+dx)^{3/2}(bc-ad)^2}{3d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(3/2))/(3*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(5/2
))/(5*d^3) + (2*b^2*(c + d*x)^(7/2))/(7*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2\sqrt{c+dx}\,dx &= \int \left(\frac{(-bc+ad)^2\sqrt{c+dx}}{d^2}-\frac{2b(bc-ad)(c+dx)^{3/2}}{d^2}+\frac{b^2(c+dx)^{5/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2(c+dx)^{3/2}}{3d^3}-\frac{4b(bc-ad)(c+dx)^{5/2}}{5d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}\end{aligned}$$

Mathematica [A] time = 0.0353577, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{3/2}\left(35a^2d^2+14abd(3dx-2c)+b^2\left(8c^2-12cdx+15d^2x^2\right)\right)}{105d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-2*c + 3*d*x) + b^2*(8*c^2 - 12*
c*d*x + 15*d^2*x^2)))/(105*d^3)
```

Maple [A] time = 0.004, size = 63, normalized size = 0.9

$$\frac{30\,b^2x^2d^2+84\,abd^2x-24\,b^2cdx+70\,a^2d^2-56\,abcd+16\,b^2c^2}{105\,d^3}(dx+c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(1/2),x)
```

```
[Out] 2/105*(d*x+c)^(3/2)*(15*b^2*d^2*x^2+42*a*b*d^2*x-12*b^2*c*d*x+35*a^2*d^2-28
*a*b*c*d+8*b^2*c^2)/d^3
```

Maxima [A] time = 0.978978, size = 92, normalized size = 1.3

$$\frac{2\left(15\,(dx+c)^{\frac{7}{2}}b^2-42\left(b^2c-abd\right)(dx+c)^{\frac{5}{2}}+35\left(b^2c^2-2\,abcd+a^2d^2\right)(dx+c)^{\frac{3}{2}}\right)}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(d*x + c)^(7/2)*b^2 - 42*(b^2*c - a*b*d)*(d*x + c)^(5/2) + 35*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(3/2))/d^3
```

Fricas [A] time = 1.9933, size = 220, normalized size = 3.1

$$\frac{2\left(15\,b^2d^3x^3+8\,b^2c^3-28\,abc^2d+35\,a^2cd^2+3\left(b^2cd^2+14\,abd^3\right)x^2-\left(4\,b^2c^2d-14\,abcd^2-35\,a^2d^3\right)x\right)\sqrt{dx+c}}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*d^3*x^3 + 8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2 + 3*(b^2*c*
d^2 + 14*a*b*d^3)*x^2 - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*x)*sqrt(d
*x + c)/d^3
```

Sympy [A] time = 1.89997, size = 85, normalized size = 1.2

$$\frac{2\left(\frac{b^2(c+dx)^{\frac{7}{2}}}{7d^2}+\frac{(c+dx)^{\frac{5}{2}}\left(2abd-2b^2c\right)}{5d^2}+\frac{(c+dx)^{\frac{3}{2}}\left(a^2d^2-2abcd+b^2c^2\right)}{3d^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**2*(c + d*x)**(7/2)/(7*d**2) + (c + d*x)**(5/2)*(2*a*b*d - 2*b**2*c)/(
5*d**2) + (c + d*x)**(3/2)*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*d**2))/d
```

Giac [A] time = 1.06055, size = 113, normalized size = 1.59

$$\frac{2\left(35\,(dx+c)^{\frac{3}{2}}a^2+\frac{14\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)ab}{d}+\frac{\left(15\,(dx+c)^{\frac{7}{2}}-42\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2\right)b^2}{d^2}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(35*(d*x + c)^(3/2)*a^2 + 14*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c
)*a*b/d + (15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c
^2)*b^2/d^2)/d
```

3.1379 $\int (a+bx)\sqrt{c+dx}\,dx$

Optimal. Leaf size=42

$$\frac{2b(c+dx)^{5/2}}{5d^2}-\frac{2(c+dx)^{3/2}(bc-ad)}{3d^2}$$

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(3/2))/(3*d^2) + (2*b*(c + d*x)^(5/2))/(5*d^2)
```

Rubi [A] time = 0.0139514, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(c+dx)^{5/2}}{5d^2}-\frac{2(c+dx)^{3/2}(bc-ad)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(3/2))/(3*d^2) + (2*b*(c + d*x)^(5/2))/(5*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)\sqrt{c+dx}\,dx &= \int \left(\frac{(-bc+ad)\sqrt{c+dx}}{d}+\frac{b(c+dx)^{3/2}}{d}\right)dx\\ &= -\frac{2(bc-ad)(c+dx)^{3/2}}{3d^2}+\frac{2b(c+dx)^{5/2}}{5d^2}\end{aligned}$$

Mathematica [A] time = 0.0173893, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{3/2}(5ad-2bc+3bdx)}{15d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x))/(15*d^2)
```

Maple [A] time = 0.002, size = 27, normalized size = 0.6

$$\frac{6\,bdx+10\,ad-4\,bc}{15\,d^2}(dx+c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] 2/15*(d*x+c)^(3/2)*(3*b*d*x+5*a*d-2*b*c)/d^2
```

Maxima [A] time = 0.942432, size = 45, normalized size = 1.07

$$\frac{2\left(3\,(dx+c)^{\frac{5}{2}}b-5\,(bc-ad)(dx+c)^{\frac{3}{2}}\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(d*x + c)^(5/2)*b - 5*(b*c - a*d)*(d*x + c)^(3/2))/d^2
```

Fricas [A] time = 2.10574, size = 108, normalized size = 2.57

$$\frac{2\left(3\,bd^2x^2-2\,bc^2+5\,acd+\left(bcd+5\,ad^2\right)x\right)\sqrt{dx+c}}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b*d^2*x^2 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x)*sqrt(d*x + c)/
d^2
```

Sympy [A] time = 1.48833, size = 36, normalized size = 0.86

$$\frac{2\left(\frac{b(c+dx)^{\frac{5}{2}}}{5d}+\frac{(c+dx)^{\frac{3}{2}}(ad-bc)}{3d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b*(c + d*x)**(5/2)/(5*d) + (c + d*x)**(3/2)*(a*d - b*c)/(3*d))/d
```

Giac [A] time = 1.05429, size = 55, normalized size = 1.31

$$\frac{2\left(5\,(dx+c)^{\frac{3}{2}}a+\frac{\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)b}{d}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*(d*x + c)^(3/2)*a + (3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c)*b/d)/
d
```

3.1380 $\int \sqrt{c+dx}\,dx$

Optimal. Leaf size=16

$$\frac{2(c+dx)^{3/2}}{3d}$$

```
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

Rubi [A] time = 0.0015467, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{c+dx}\,dx = \frac{2(c+dx)^{3/2}}{3d}$$

Mathematica [A] time = 0.0036568, size = 16, normalized size = 1.

$$\frac{2(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

Maple [A] time = 0.001, size = 13, normalized size = 0.8

$$\frac{2}{3d}(dx+c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2),x)
```

[Out] 2/3*(d*x+c)^(3/2)/d

Maxima [A] time = 0.960383, size = 16, normalized size = 1.

$$\frac{2 (dx + c)^{\frac{3}{2}}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(d*x + c)^(3/2)/d
```

Fricas [A] time = 1.94815, size = 31, normalized size = 1.94

$$\frac{2 (dx + c)^{\frac{3}{2}}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(d*x + c)^(3/2)/d
```

Sympy [A] time = 0.052876, size = 12, normalized size = 0.75

$$\frac{2 (c + dx)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2),x)
```

```
[Out] 2*(c + d*x)**(3/2)/(3*d)
```

Giac [A] time = 1.09003, size = 16, normalized size = 1.

$$\frac{2 (dx + c)^{\frac{3}{2}}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(d*x + c)^(3/2)/d
```

3.1381 $\int \frac{\sqrt{c+dx}}{a+bx} dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

```
[Out] (2*Sqrt[c + d*x])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/b^(3/2)
```

Rubi [A] time = 0.0527786, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 63, 208}

$$\frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{a+bx} dx = \frac{2\sqrt{c+dx}}{b} + \frac{(bc-ad)\int \frac{1}{(a+bx)\sqrt{c+dx}} dx}{b}$$
$$= \frac{2\sqrt{c+dx}}{b} + \frac{(2(bc-ad)) \operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}} dx, x, \sqrt{c+dx}\right)}{bd}$$
$$= \frac{2\sqrt{c+dx}}{b} - \frac{2\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.0342759, size = 62, normalized size = 1.

$$\frac{2\sqrt{c+dx}}{b} - \frac{2\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/b^(3/2)
```

Maple [A] time = 0.01, size = 92, normalized size = 1.5

$$2\,\frac{\sqrt{dx+c}}{b} - 2\,\frac{ad}{b\sqrt{(ad-bc)\,b}} \arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right) + 2\,\frac{c}{\sqrt{(ad-bc)\,b}} \arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a),x)
```

```
[Out] 2*(d*x+c)^(1/2)/b-2/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)
*b)^(1/2))*a*d+2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(
1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.15195, size = 306, normalized size = 4.94

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,\sqrt{dx+c}}{b}, -\frac{2\left(\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx+c}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*
c - a*d)/b))/(b*x + a)) + 2*sqrt(d*x + c))/b, -2*(sqrt(-(b*c - a*d)/b)*arct
an(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqrt(d*x + c))/b]
```

Sympy [A] time = 3.06263, size = 61, normalized size = 0.98

$$\frac{2\left(\frac{d\sqrt{c+dx}}{b}-\frac{d(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x)/b - d*(a*d - b*c)*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b)
)/(b**2*sqrt((a*d - b*c)/b)))/d
```

Giac [A] time = 1.06447, size = 84, normalized size = 1.35

$$\frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}+\frac{2\,\sqrt{dx+c}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b*c - a*d)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*b) + 2*sqrt(d*x + c)/b
```

3.1382 $\int \frac{\sqrt{c+dx}}{(a+bx)^2} dx$

Optimal. Leaf size=70

$$-\frac{d \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx}}{b(a+bx)}$$

```
[Out] -(Sqrt[c + d*x]/(b*(a + b*x))) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*
c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.030299, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 63, 208}

$$-\frac{d \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[c + d*x]/(b*(a + b*x))) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*
c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx = -\frac{\sqrt{c+dx}}{b(a+bx)} + \frac{d\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2b}$$
$$= -\frac{\sqrt{c+dx}}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{b}$$
$$= -\frac{\sqrt{c+dx}}{b(a+bx)} - \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.0737274, size = 69, normalized size = 0.99

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{ad-bc}}\right)}{b^{3/2}\sqrt{ad-bc}} - \frac{\sqrt{c+dx}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[c + d*x]/(b*(a + b*x))) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b
*c) + a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d])
```

Maple [A] time = 0.011, size = 64, normalized size = 0.9

$$-\frac{d}{b\,(bdx+ad)}\sqrt{dx+c} + \frac{d}{b}\arctan\left(b\sqrt{dx+c}\frac{1}{\sqrt{(ad-bc)\,b}}\right)\frac{1}{\sqrt{(ad-bc)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^2,x)
```

```
[Out] -d/b*(d*x+c)^(1/2)/(b*d*x+a*d)+d/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/
2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.15632, size = 498, normalized size = 7.11

$$\left[\frac{\sqrt{b^2c-abd}(bdx+ad)\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right) - 2\left(b^2c-abd\right)\sqrt{dx+c}}{2\left(ab^3c-a^2b^2d+\left(b^4c-ab^3d\right)x\right)}, \frac{\sqrt{-b^2c+abd}(bdx+ad)\arctan\left(\frac{\sqrt{-b^2c}}{b}\right.}{ab^3c-a^2b^2d+\left(b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2*c - a*b*d)*(b*d*x + a*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b
^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(b^2*c - a*b*d)*sqrt(d*x + c))/
(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x), (sqrt(-b^2*c + a*b*d)*(b*d*x +
 a*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (b^2*c - a
*b*d)*sqrt(d*x + c))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x)]
```

Sympy [B] time = 28.5851, size = 573, normalized size = 8.19

$$-\frac{2ad^2\sqrt{c+dx}}{2a^2bd^2-2ab^2cd+2ab^2d^2x-2b^3cdx}+\frac{ad^2\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(-a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}+2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}-b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**2,x)
```

```
[Out] -2*a*d**2*sqrt(c + d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x - 2
*b**3*c*d*x) + a*d**2*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b
*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(
-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b) - a*d**2*sqrt(-1/(b*(a*d - b*
c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a
*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*
b) - c*d*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)*
*3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d -
b*c)**3)) + sqrt(c + d*x))/2 + c*d*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**
2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**
2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + 2*c*d*sqrt(c + d*x)
/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c*d*x) + 2*d*atan(sqrt(c
+ d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*d/b - c))
```

Giac [A] time = 1.08943, size = 97, normalized size = 1.39

$$\frac{d\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}-\frac{\sqrt{dx+c}d}{((dx+c)b-bc+ad)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b) - s
qrt(d*x + c)*d/(((d*x + c)*b - b*c + a*d)*b)
```

3.1383 $\int \frac{\sqrt{c+dx}}{(a+bx)^3} dx$

Optimal. Leaf size=110

$$\frac{d^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}} - \frac{d\sqrt{c+dx}}{4b(a+bx)(bc-ad)} - \frac{\sqrt{c+dx}}{2b(a+bx)^2}$$

```
[Out] -Sqrt[c + d*x]/(2*b*(a + b*x)^2) - (d*Sqrt[c + d*x])/(4*b*(b*c - a*d)*(a +
b*x)) + (d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(3/2)*(
b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0780258, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$\frac{d^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}} - \frac{d\sqrt{c+dx}}{4b(a+bx)(bc-ad)} - \frac{\sqrt{c+dx}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -Sqrt[c + d*x]/(2*b*(a + b*x)^2) - (d*Sqrt[c + d*x])/(4*b*(b*c - a*d)*(a +
b*x)) + (d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(3/2)*(
b*c - a*d)^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx &= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}+\frac{d\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{4b}\\
&= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}-\frac{d^2\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b(bc-ad)}\\
&= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}-\frac{d\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4b(bc-ad)}\\
&= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0172315, size = 52, normalized size = 0.47

$$\frac{2d^2(c+dx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^3,x]
```

```
[Out] (2*d^2*(c + d*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(3*(-(b*c) + a*d)^3)
```

Maple [A] time = 0.012, size = 111, normalized size = 1.

$$\frac{d^2}{4\,(bdx+ad)^2\,(ad-bc)}(dx+c)^{\frac{3}{2}}-\frac{d^2}{4\,(bdx+ad)^2\,b}\sqrt{dx+c}+\frac{d^2}{(4\,ad-4\,bc)\,b}\arctan\left(b\sqrt{dx+c}\frac{1}{\sqrt{(ad-bc)\,b}}\right)\frac{1}{\sqrt{(ad-bc)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^3,x)
```

```
[Out] 1/4*d^2/(b*d*x+a*d)^2/(a*d-b*c)*(d*x+c)^(3/2)-1/4*d^2/(b*d*x+a*d)^2/b*(d*x+
c)^(1/2)+1/4*d^2/(a*d-b*c)/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a
*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.16923, size = 949, normalized size = 8.63

$$\left[-\frac{\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\sqrt{b^2c-abd}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)+2\left(2\,b^3c^2-3\,ab^2cd+a^2bd^2+\left(b^3cd-ab^2d^2\right.\right.}{8\left(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2+\left(b^6c^2-2\,ab^5cd+a^2b^4d^2\right)x^2+2\left(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x
 + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(2*b^3
*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a
^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4
*d^2)*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x), -1/4*((b^2*d^2*
x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*
d)*sqrt(d*x + c)/(b*d*x + b*c)) + (2*b^3*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b
^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^
2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^
4*c*d + a^3*b^3*d^2)*x)]
```

Sympy [B] time = 92.7819, size = 1658, normalized size = 15.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**3,x)
```

```
[Out] -10*a**2*d**4*sqrt(c + d*x)/(8*a**4*b*d**4 - 16*a**3*b**2*c*d**3 + 16*a**3*
b**2*d**4*x - 48*a**2*b**3*c*d**3*x + 8*a**2*b**3*d**2*(c + d*x)**2 + 16*a*
b**4*c**3*d + 48*a*b**4*c**2*d**2*x - 16*a*b**4*c*d*(c + d*x)**2 - 8*b**5*c
**4 - 16*b**5*c**3*d*x + 8*b**5*c**2*(c + d*x)**2) + 20*a*c*d**3*sqrt(c + d
*x)/(8*a**4*d**4 - 16*a**3*b*c*d**3 + 16*a**3*b*d**4*x - 48*a**2*b**2*c*d**
3*x + 8*a**2*b**2*d**2*(c + d*x)**2 + 16*a*b**3*c**3*d + 48*a*b**3*c**2*d**
2*x - 16*a*b**3*c*d*(c + d*x)**2 - 8*b**4*c**4 - 16*b**4*c**3*d*x + 8*b**4*
c**2*(c + d*x)**2) - 6*a*d**3*(c + d*x)**(3/2)/(8*a**4*d**4 - 16*a**3*b*c*d
**3 + 16*a**3*b*d**4*x - 48*a**2*b**2*c*d**3*x + 8*a**2*b**2*d**2*(c + d*x)
**2 + 16*a*b**3*c**3*d + 48*a*b**3*c**2*d**2*x - 16*a*b**3*c*d*(c + d*x)**2
 - 8*b**4*c**4 - 16*b**4*c**3*d*x + 8*b**4*c**2*(c + d*x)**2) + 3*a*d**3*sq
rt(-1/(b*(a*d - b*c)**5))*log(-a**3*d**3*sqrt(-1/(b*(a*d - b*c)**5)) + 3*a*
*2*b*c*d**2*sqrt(-1/(b*(a*d - b*c)**5)) - 3*a*b**2*c**2*d*sqrt(-1/(b*(a*d -
 b*c)**5)) + b**3*c**3*sqrt(-1/(b*(a*d - b*c)**5)) + sqrt(c + d*x))/(8*b) -
 3*a*d**3*sqrt(-1/(b*(a*d - b*c)**5))*log(a**3*d**3*sqrt(-1/(b*(a*d - b*c)*
*5)) - 3*a**2*b*c*d**2*sqrt(-1/(b*(a*d - b*c)**5)) + 3*a*b**2*c**2*d*sqrt(-
1/(b*(a*d - b*c)**5)) - b**3*c**3*sqrt(-1/(b*(a*d - b*c)**5)) + sqrt(c + d*
x))/(8*b) - 10*b*c**2*d**2*sqrt(c + d*x)/(8*a**4*d**4 - 16*a**3*b*c*d**3 +
16*a**3*b*d**4*x - 48*a**2*b**2*c*d**3*x + 8*a**2*b**2*d**2*(c + d*x)**2 +
16*a*b**3*c**3*d + 48*a*b**3*c**2*d**2*x - 16*a*b**3*c*d*(c + d*x)**2 - 8*b
**4*c**4 - 16*b**4*c**3*d*x + 8*b**4*c**2*(c + d*x)**2) + 6*b*c*d**2*(c + d
*x)**(3/2)/(8*a**4*d**4 - 16*a**3*b*c*d**3 + 16*a**3*b*d**4*x - 48*a**2*b**
```

```
2*c*d**3*x + 8*a**2*b**2*d**2*(c + d*x)**2 + 16*a*b**3*c**3*d + 48*a*b**3*c
**2*d**2*x - 16*a*b**3*c*d*(c + d*x)**2 - 8*b**4*c**4 - 16*b**4*c**3*d*x +
8*b**4*c**2*(c + d*x)**2) - 3*c*d**2*sqrt(-1/(b*(a*d - b*c)**5))*log(-a**3*
d**3*sqrt(-1/(b*(a*d - b*c)**5)) + 3*a**2*b*c*d**2*sqrt(-1/(b*(a*d - b*c)**
5)) - 3*a*b**2*c**2*d*sqrt(-1/(b*(a*d - b*c)**5)) + b**3*c**3*sqrt(-1/(b*(a
*d - b*c)**5)) + sqrt(c + d*x))/8 + 3*c*d**2*sqrt(-1/(b*(a*d - b*c)**5))*lo
g(a**3*d**3*sqrt(-1/(b*(a*d - b*c)**5)) - 3*a**2*b*c*d**2*sqrt(-1/(b*(a*d -
 b*c)**5)) + 3*a*b**2*c**2*d*sqrt(-1/(b*(a*d - b*c)**5)) - b**3*c**3*sqrt(-
1/(b*(a*d - b*c)**5)) + sqrt(c + d*x))/8 + 2*d**2*sqrt(c + d*x)/(2*a**2*b*d
**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x - 2*b**3*c*d*x) - d**2*sqrt(-1/(b*(a*d
 - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1
/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x
))/(2*b) + d**2*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d -
 b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(
a*d - b*c)**3)) + sqrt(c + d*x))/(2*b)
```

Giac [A] time = 1.07411, size = 170, normalized size = 1.55

$$-\frac{d^2\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{4\left(b^2c-abd\right)\sqrt{-b^2c+abd}}-\frac{(dx+c)^{\frac{3}{2}}bd^2+\sqrt{dx+c}bcd^2-\sqrt{dx+c}ad^3}{4\left(b^2c-abd\right)((dx+c)b-bc+ad)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqrt
(-b^2*c + a*b*d)) - 1/4*((d*x + c)^(3/2)*b*d^2 + sqrt(d*x + c)*b*c*d^2 - sq
rt(d*x + c)*a*d^3)/((b^2*c - a*b*d)*((d*x + c)*b - b*c + a*d)^2)
```

3.1384 $\int \frac{\sqrt{c+dx}}{(a+bx)^4} dx$

Optimal. Leaf size=146

$$-\frac{d^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}+\frac{d^2\sqrt{c+dx}}{8b(a+bx)(bc-ad)^2}-\frac{d\sqrt{c+dx}}{12b(a+bx)^2(bc-ad)}-\frac{\sqrt{c+dx}}{3b(a+bx)^3}$$

```
[Out] -Sqrt[c + d*x]/(3*b*(a + b*x)^3) - (d*Sqrt[c + d*x])/(12*b*(b*c - a*d)*(a +
 b*x)^2) + (d^2*Sqrt[c + d*x])/(8*b*(b*c - a*d)^2*(a + b*x)) - (d^3*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(3/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.0978502, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{d^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}+\frac{d^2\sqrt{c+dx}}{8b(a+bx)(bc-ad)^2}-\frac{d\sqrt{c+dx}}{12b(a+bx)^2(bc-ad)}-\frac{\sqrt{c+dx}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^4,x]
```

```
[Out] -Sqrt[c + d*x]/(3*b*(a + b*x)^3) - (d*Sqrt[c + d*x])/(12*b*(b*c - a*d)*(a +
 b*x)^2) + (d^2*Sqrt[c + d*x])/(8*b*(b*c - a*d)^2*(a + b*x)) - (d^3*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(3/2)*(b*c - a*d)^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx &= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}+\frac{d\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{6b}\\ &= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}-\frac{d^2\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8b(bc-ad)}\\ &= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}+\frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)}+\frac{d^3\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b(bc-ad)^2}\\ &= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}+\frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)}+\frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{8b(bc-ad)^2}\\ &= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}+\frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)}-\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.0133593, size = 52, normalized size = 0.36

$$\frac{2d^3(c+dx)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^4,x]
```

```
[Out] (2*d^3*(c + d*x)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(3*(-(b*c) + a*d)^4)
```

Maple [A] time = 0.012, size = 170, normalized size = 1.2

$$\frac{d^3b}{8\,(bdx+ad)^3\left(a^2d^2-2\,abcd+b^2c^2\right)}(dx+c)^{\frac{5}{2}}+\frac{d^3}{3\,(bdx+ad)^3\,(ad-bc)}(dx+c)^{\frac{3}{2}}-\frac{d^3}{8\,(bdx+ad)^3\,b}\sqrt{dx+c}+\frac{}{8\,b\left(a^2d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^4,x)
```

```
[Out] 1/8*d^3/(b*d*x+a*d)^3*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(5/2)+1/3*d^3/(
b*d*x+a*d)^3/(a*d-b*c)*(d*x+c)^(3/2)-1/8*d^3/(b*d*x+a*d)^3/b*(d*x+c)^(1/2)+
1/8*d^3/b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^
(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.27726, size = 1581, normalized size = 10.83

$$\left[\frac{3\left(b^3d^3x^3 + 3\,ab^2d^3x^2 + 3\,a^2bd^3x + a^3d^3\right)\sqrt{b^2c - abd}\log\left(\frac{bdx + 2\,bc - ad - 2\,\sqrt{b^2c - abd}\sqrt{dx + c}}{bx + a}\right) - 2\left(8\,b^4c^3 - 22\,ab^3c^2d + 17\,a^2b\right.}{48\left(a^3b^5c^3 - 3\,a^4b^4c^2d + 3\,a^5b^3cd^2 - a^6b^2d^3 + \left(b^8c^3 - 3\,ab^7c^2d + 3\,a^2b^6cd^2 - a^3b^5d^3\right)x^3 + 3\left(ab^7c^3 - 3\,a^2b^6c^2d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^2
*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))
/(b*x + a)) - 2*(8*b^4*c^3 - 22*a*b^3*c^2*d + 17*a^2*b^2*c*d^2 - 3*a^3*b*d^
3 - 3*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^
2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 -
 a^6*b^2*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^
3 + 3*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^2 + 3
*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x), 1/24*(
3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a
*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (8*b^4*c^3
 - 22*a*b^3*c^2*d + 17*a^2*b^2*c*d^2 - 3*a^3*b*d^3 - 3*(b^4*c*d^2 - a*b^3*d
^3)*x^2 + 2*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(
a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3 + (b^8*c^3 -
3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^3 + 3*(a*b^7*c^3 - 3*a^2*b
^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^2 + 3*(a^2*b^6*c^3 - 3*a^3*b^5*
c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08041, size = 279, normalized size = 1.91

$$\frac{d^3\arctan\left(\frac{\sqrt{dx + cb}}{\sqrt{-b^2c + abd}}\right)}{8\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)\sqrt{-b^2c + abd}} + \frac{3\,(dx + c)^{\frac{5}{2}}b^2d^3 - 8\,(dx + c)^{\frac{3}{2}}b^2cd^3 - 3\,\sqrt{dx + c}b^2c^2d^3 + 8\,(dx + c)^{\frac{3}{2}}abd^4 + 6}{24\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)((dx + c)b - bc + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^2 - 2*a*b^2*c*
d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) + 1/24*(3*(d*x + c)^(5/2)*b^2*d^3 - 8*
(d*x + c)^(3/2)*b^2*c*d^3 - 3*sqrt(d*x + c)*b^2*c^2*d^3 + 8*(d*x + c)^(3/2)
*a*b*d^4 + 6*sqrt(d*x + c)*a*b*c*d^4 - 3*sqrt(d*x + c)*a^2*d^5)/((b^3*c^2 -
 2*a*b^2*c*d + a^2*b*d^2)*((d*x + c)*b - b*c + a*d)^3)
```

3.1385 $\int \frac{\sqrt{c+dx}}{(a+bx)^5}\,dx$

Optimal. Leaf size=182

$$\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{3/2}(bc-ad)^{7/2}}-\frac{5d^3\sqrt{c+dx}}{64b(a+bx)(bc-ad)^3}+\frac{5d^2\sqrt{c+dx}}{96b(a+bx)^2(bc-ad)^2}-\frac{d\sqrt{c+dx}}{24b(a+bx)^3(bc-ad)}-\frac{\sqrt{c+dx}}{4b(a+bx)^4}$$

```
[Out] -Sqrt[c + d*x]/(4*b*(a + b*x)^4) - (d*Sqrt[c + d*x])/(24*b*(b*c - a*d)*(a +
 b*x)^3) + (5*d^2*Sqrt[c + d*x])/(96*b*(b*c - a*d)^2*(a + b*x)^2) - (5*d^3*
Sqrt[c + d*x])/(64*b*(b*c - a*d)^3*(a + b*x)) + (5*d^4*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x])/Sqrt[b*c - a*d]])/(64*b^(3/2)*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.122998, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{3/2}(bc-ad)^{7/2}}-\frac{5d^3\sqrt{c+dx}}{64b(a+bx)(bc-ad)^3}+\frac{5d^2\sqrt{c+dx}}{96b(a+bx)^2(bc-ad)^2}-\frac{d\sqrt{c+dx}}{24b(a+bx)^3(bc-ad)}-\frac{\sqrt{c+dx}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^5,x]
```

```
[Out] -Sqrt[c + d*x]/(4*b*(a + b*x)^4) - (d*Sqrt[c + d*x])/(24*b*(b*c - a*d)*(a +
 b*x)^3) + (5*d^2*Sqrt[c + d*x])/(96*b*(b*c - a*d)^2*(a + b*x)^2) - (5*d^3*
Sqrt[c + d*x])/(64*b*(b*c - a*d)^3*(a + b*x)) + (5*d^4*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x])/Sqrt[b*c - a*d]])/(64*b^(3/2)*(b*c - a*d)^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^5}\,dx = -\frac{\sqrt{c+dx}}{4b(a+bx)^4} + \frac{d\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{8b}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4} - \frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3} - \frac{\left(5d^2\right)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{48b(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4} - \frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3} + \frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2} + \frac{\left(5d^3\right)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{64b(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4} - \frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3} + \frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx)} - \frac{\left(5d^4\right)\int \frac{}{(a+}}{128b(bc}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4} - \frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3} + \frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx)} - \frac{\left(5d^3\right)\text{Subs}}{}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4} - \frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3} + \frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx)} + \frac{5d^4\tanh^{-1}}{64b^{3/2}(bc}$$

Mathematica [C] time = 0.0142361, size = 52, normalized size = 0.29

$$\frac{2d^4(c+dx)^{3/2}\,{}_2F_1\left(\frac{3}{2},5;\frac{5}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(ad-bc)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^5,x]
```

```
[Out] (2*d^4*(c + d*x)^(3/2)*Hypergeometric2F1[3/2, 5, 5/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(3*(-(b*c) + a*d)^5)
```

Maple [A] time = 0.013, size = 248, normalized size = 1.4

$$\frac{5\,d^4b^2}{64\,(bdx+ad)^4\left(a^3d^3-3\,a^2bcd^2+3\,ab^2c^2d-b^3c^3\right)}(dx+c)^{\frac{7}{2}} + \frac{55\,d^4b}{192\,(bdx+ad)^4\left(a^2d^2-2\,abcd+b^2c^2\right)}(dx+c)^{\frac{5}{2}} + \frac{}{192\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^5,x)
```

```
[Out] 5/64*d^4/(b*d*x+a*d)^4*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(d
*x+c)^(7/2)+55/192*d^4/(b*d*x+a*d)^4*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^
(5/2)+73/192*d^4/(b*d*x+a*d)^4/(a*d-b*c)*(d*x+c)^(3/2)-5/64*d^4/(b*d*x+a*d)
^4/b*(d*x+c)^(1/2)+5/64*d^4/b/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.32994, size = 2404, normalized size = 13.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(48*b^5*c^4 - 184*a*b^4*c^3*d + 254*
a^2*b^3*c^2*d^2 - 133*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*
d^4)*x^3 - 5*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (8*b^5
*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqrt(d*x
 + c))/(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3
 + a^8*b^2*d^4 + (b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*
c*d^3 + a^4*b^6*d^4)*x^4 + 4*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d
^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*x^3 + 6*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d
+ 6*a^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^2 + 4*(a^3*b^7*c^4 -
 4*a^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x), -
1/192*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*
x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)
/(b*d*x + b*c)) + (48*b^5*c^4 - 184*a*b^4*c^3*d + 254*a^2*b^3*c^2*d^2 - 133
*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4)*x^3 - 5*(2*b^5*c
^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (8*b^5*c^3*d - 44*a*b^4*c^2
*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*b^6*c^4 -
 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4 + (b^1
0*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*
x^4 + 4*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3
+ a^5*b^5*d^4)*x^3 + 6*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 -
 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^2 + 4*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*
a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10069, size = 420, normalized size = 2.31

$$-\frac{5\,d^4\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{64\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)\sqrt{-b^2c+abd}}-\frac{15\,(dx+c)^{\frac{7}{2}}b^3d^4-55\,(dx+c)^{\frac{5}{2}}b^3cd^4+73\,(dx+c)^{\frac{3}{2}}b^3c^2d^4+15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -5/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^3 - 3*a*b^3*
c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(-b^2*c + a*b*d)) - 1/192*(15*(d*x
 + c)^(7/2)*b^3*d^4 - 55*(d*x + c)^(5/2)*b^3*c*d^4 + 73*(d*x + c)^(3/2)*b^3
*c^2*d^4 + 15*sqrt(d*x + c)*b^3*c^3*d^4 + 55*(d*x + c)^(5/2)*a*b^2*d^5 - 14
6*(d*x + c)^(3/2)*a*b^2*c*d^5 - 45*sqrt(d*x + c)*a*b^2*c^2*d^5 + 73*(d*x +
c)^(3/2)*a^2*b*d^6 + 45*sqrt(d*x + c)*a^2*b*c*d^6 - 15*sqrt(d*x + c)*a^3*d^
7)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*((d*x + c)*b -
b*c + a*d)^4)
```

3.1386 $\int \frac{\sqrt{c+dx}}{(a+bx)^6} dx$

Optimal. Leaf size=218

$$-\frac{7d^5 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{3/2}(bc-ad)^{9/2}}+\frac{7d^4\sqrt{c+dx}}{128b(a+bx)(bc-ad)^4}-\frac{7d^3\sqrt{c+dx}}{192b(a+bx)^2(bc-ad)^3}+\frac{7d^2\sqrt{c+dx}}{240b(a+bx)^3(bc-ad)^2}-\frac{d\sqrt{c}}{40b(a+bx)}$$

```
[Out] -Sqrt[c + d*x]/(5*b*(a + b*x)^5) - (d*Sqrt[c + d*x])/(40*b*(b*c - a*d)*(a +
 b*x)^4) + (7*d^2*Sqrt[c + d*x])/(240*b*(b*c - a*d)^2*(a + b*x)^3) - (7*d^3
*Sqrt[c + d*x])/(192*b*(b*c - a*d)^3*(a + b*x)^2) + (7*d^4*Sqrt[c + d*x])/(
128*b*(b*c - a*d)^4*(a + b*x)) - (7*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqr
t[b*c - a*d]])/(128*b^(3/2)*(b*c - a*d)^(9/2))
```

Rubi [A] time = 0.15143, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{7d^5 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{3/2}(bc-ad)^{9/2}}+\frac{7d^4\sqrt{c+dx}}{128b(a+bx)(bc-ad)^4}-\frac{7d^3\sqrt{c+dx}}{192b(a+bx)^2(bc-ad)^3}+\frac{7d^2\sqrt{c+dx}}{240b(a+bx)^3(bc-ad)^2}-\frac{d\sqrt{c}}{40b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^6,x]
```


```
[Out] -Sqrt[c + d*x]/(5*b*(a + b*x)^5) - (d*Sqrt[c + d*x])/(40*b*(b*c - a*d)*(a +
 b*x)^4) + (7*d^2*Sqrt[c + d*x])/(240*b*(b*c - a*d)^2*(a + b*x)^3) - (7*d^3
*Sqrt[c + d*x])/(192*b*(b*c - a*d)^3*(a + b*x)^2) + (7*d^4*Sqrt[c + d*x])/(
128*b*(b*c - a*d)^4*(a + b*x)) - (7*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqr
t[b*c - a*d]])/(128*b^(3/2)*(b*c - a*d)^(9/2))
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^6}\,dx = -\frac{\sqrt{c+dx}}{5b(a+bx)^5} + \frac{d\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx}{10b}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} - \frac{\left(7d^2\right)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{80b(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} + \frac{\left(7d^3\right)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{96b(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2} - \frac{\left(7d^4\right)\int}{128}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2} + \frac{7}{128b(b}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2} + \frac{7}{128b(b}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2} + \frac{7}{128b(b}$$

Mathematica [C] time = 0.0153353, size = 52, normalized size = 0.24

$$\frac{2d^5(c+dx)^{3/2}\,{}_2F_1\left(\frac{3}{2},6;\frac{5}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(ad-bc)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^6,x]
```

```
[Out] (2*d^5*(c + d*x)^(3/2)*Hypergeometric2F1[3/2, 6, 5/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(3*(-(b*c) + a*d)^6)
```

Maple [A] time = 0.016, size = 337, normalized size = 1.6

$$\frac{7\,d^5b^3}{128\,(bdx+ad)^5\left(a^4d^4-4\,a^3bcd^3+6\,a^2b^2c^2d^2-4\,ab^3c^3d+b^4c^4\right)}(dx+c)^{\frac{9}{2}} + \frac{49\,d^5b^2}{192\,(bdx+ad)^5\left(a^3d^3-3\,a^2bcd^2+3\,ab^2c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^6,x)
```

```
[Out] 7/128*d^5/(b*d*x+a*d)^5*b^3/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^
3*c^3*d+b^4*c^4)*(d*x+c)^(9/2)+49/192*d^5/(b*d*x+a*d)^5*b^2/(a^3*d^3-3*a^2*
b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(d*x+c)^(7/2)+7/15*d^5/(b*d*x+a*d)^5*b/(a^2*
d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(5/2)+79/192*d^5/(b*d*x+a*d)^5/(a*d-b*c)*(d*
x+c)^(3/2)-7/128*d^5/(b*d*x+a*d)^5/b*(d*x+c)^(1/2)+7/128*d^5/b/(a^4*d^4-4*a
^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/((a*d-b*c)*b)^(1/2)*arc
tan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.43228, size = 3452, normalized size = 15.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [1/3840*(105*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b
^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*
c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(384*b^6*c^5
- 1872*a*b^5*c^4*d + 3592*a^2*b^4*c^3*d^2 - 3314*a^3*b^3*c^2*d^3 + 1315*a^4
*b^2*c*d^4 - 105*a^5*b*d^5 - 105*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 70*(b^6*c^2*
d^3 - 8*a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 - 14*(4*b^6*c^3*d^2 - 27*a*b^5*c^2
*d^3 + 87*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(24*b^6*c^4*d - 152*a*b^5
*c^3*d^2 + 417*a^2*b^4*c^2*d^3 - 684*a^3*b^3*c*d^4 + 395*a^4*b^2*d^5)*x)*sq
rt(d*x + c))/(a^5*b^7*c^5 - 5*a^6*b^6*c^4*d + 10*a^7*b^5*c^3*d^2 - 10*a^8*b
^4*c^2*d^3 + 5*a^9*b^3*c*d^4 - a^10*b^2*d^5 + (b^12*c^5 - 5*a*b^11*c^4*d +
10*a^2*b^10*c^3*d^2 - 10*a^3*b^9*c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5*b^7*d^5)*x
^5 + 5*(a*b^11*c^5 - 5*a^2*b^10*c^4*d + 10*a^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2
*d^3 + 5*a^5*b^7*c*d^4 - a^6*b^6*d^5)*x^4 + 10*(a^2*b^10*c^5 - 5*a^3*b^9*c^
4*d + 10*a^4*b^8*c^3*d^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d
^5)*x^3 + 10*(a^3*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b
^6*c^2*d^3 + 5*a^7*b^5*c*d^4 - a^8*b^4*d^5)*x^2 + 5*(a^4*b^8*c^5 - 5*a^5*b^
7*c^4*d + 10*a^6*b^6*c^3*d^2 - 10*a^7*b^5*c^2*d^3 + 5*a^8*b^4*c*d^4 - a^9*b
^3*d^5)*x), 1/1920*(105*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3
 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(-b^2*c + a*b*d)*arcta
n(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (384*b^6*c^5 - 1872*a
*b^5*c^4*d + 3592*a^2*b^4*c^3*d^2 - 3314*a^3*b^3*c^2*d^3 + 1315*a^4*b^2*c*d
^4 - 105*a^5*b*d^5 - 105*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 70*(b^6*c^2*d^3 - 8*
a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 - 14*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 8
7*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(24*b^6*c^4*d - 152*a*b^5*c^3*d^2
 + 417*a^2*b^4*c^2*d^3 - 684*a^3*b^3*c*d^4 + 395*a^4*b^2*d^5)*x)*sqrt(d*x +
 c))/(a^5*b^7*c^5 - 5*a^6*b^6*c^4*d + 10*a^7*b^5*c^3*d^2 - 10*a^8*b^4*c^2*d
^3 + 5*a^9*b^3*c*d^4 - a^10*b^2*d^5 + (b^12*c^5 - 5*a*b^11*c^4*d + 10*a^2*b
^10*c^3*d^2 - 10*a^3*b^9*c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5*b^7*d^5)*x^5 + 5*(
a*b^11*c^5 - 5*a^2*b^10*c^4*d + 10*a^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2*d^3 + 5
```

```
*a^5*b^7*c*d^4 - a^6*b^6*d^5)*x^4 + 10*(a^2*b^10*c^5 - 5*a^3*b^9*c^4*d + 10
*a^4*b^8*c^3*d^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d^5)*x^3
+ 10*(a^3*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b^6*c^2*d
^3 + 5*a^7*b^5*c*d^4 - a^8*b^4*d^5)*x^2 + 5*(a^4*b^8*c^5 - 5*a^5*b^7*c^4*d
+ 10*a^6*b^6*c^3*d^2 - 10*a^7*b^5*c^2*d^3 + 5*a^8*b^4*c*d^4 - a^9*b^3*d^5)*
x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10144, size = 583, normalized size = 2.67

$$\frac{7\,d^5\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{128\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)\sqrt{-b^2c+abd}}+\frac{105\,(dx+c)^{\frac{9}{2}}b^4d^5-490\,(dx+c)^{\frac{7}{2}}b^4cd^5+896\,(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 7/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*sqrt(-b^2*c + a*b*
d)) + 1/1920*(105*(d*x + c)^(9/2)*b^4*d^5 - 490*(d*x + c)^(7/2)*b^4*c*d^5 +
 896*(d*x + c)^(5/2)*b^4*c^2*d^5 - 790*(d*x + c)^(3/2)*b^4*c^3*d^5 - 105*sq
rt(d*x + c)*b^4*c^4*d^5 + 490*(d*x + c)^(7/2)*a*b^3*d^6 - 1792*(d*x + c)^(5
/2)*a*b^3*c*d^6 + 2370*(d*x + c)^(3/2)*a*b^3*c^2*d^6 + 420*sqrt(d*x + c)*a*
b^3*c^3*d^6 + 896*(d*x + c)^(5/2)*a^2*b^2*d^7 - 2370*(d*x + c)^(3/2)*a^2*b^
2*c*d^7 - 630*sqrt(d*x + c)*a^2*b^2*c^2*d^7 + 790*(d*x + c)^(3/2)*a^3*b*d^8
 + 420*sqrt(d*x + c)*a^3*b*c*d^8 - 105*sqrt(d*x + c)*a^4*d^9)/((b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*((d*x + c)*
b - b*c + a*d)^5)
```

3.1387 $\int (a+bx)^5(c+dx)^{3/2}\,dx$

Optimal. Leaf size=158

$$-\frac{10b^4(c+dx)^{13/2}(bc-ad)}{13d^6}+\frac{20b^3(c+dx)^{11/2}(bc-ad)^2}{11d^6}-\frac{20b^2(c+dx)^{9/2}(bc-ad)^3}{9d^6}+\frac{10b(c+dx)^{7/2}(bc-ad)^4}{7d^6}-\frac{2(c}{}$$

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(5/2))/(5*d^6) + (10*b*(b*c - a*d)^4*(c + d*x)^
(7/2))/(7*d^6) - (20*b^2*(b*c - a*d)^3*(c + d*x)^(9/2))/(9*d^6) + (20*b^3*(
b*c - a*d)^2*(c + d*x)^(11/2))/(11*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(13
/2))/(13*d^6) + (2*b^5*(c + d*x)^(15/2))/(15*d^6)
```

Rubi [A] time = 0.0526392, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{10b^4(c+dx)^{13/2}(bc-ad)}{13d^6}+\frac{20b^3(c+dx)^{11/2}(bc-ad)^2}{11d^6}-\frac{20b^2(c+dx)^{9/2}(bc-ad)^3}{9d^6}+\frac{10b(c+dx)^{7/2}(bc-ad)^4}{7d^6}-\frac{2(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^(3/2),x]
```


```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(5/2))/(5*d^6) + (10*b*(b*c - a*d)^4*(c + d*x)^
(7/2))/(7*d^6) - (20*b^2*(b*c - a*d)^3*(c + d*x)^(9/2))/(9*d^6) + (20*b^3*(
b*c - a*d)^2*(c + d*x)^(11/2))/(11*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(13
/2))/(13*d^6) + (2*b^5*(c + d*x)^(15/2))/(15*d^6)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^{3/2}\,dx=\int \left(\frac{(-bc+ad)^5(c+dx)^{3/2}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{5/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{7/2}}{d^5}+\frac{10b^3(b\right.$$

$$=-\frac{2(bc-ad)^5(c+dx)^{5/2}}{5d^6}+\frac{10b(bc-ad)^4(c+dx)^{7/2}}{7d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{9/2}}{9d^6}+\frac{20b^3(bc}{}$$

Mathematica [A] time = 0.142616, size = 123, normalized size = 0.78

$$\frac{2(c+dx)^{5/2}\left(-50050b^2(c+dx)^2(bc-ad)^3+40950b^3(c+dx)^3(bc-ad)^2-17325b^4(c+dx)^4(bc-ad)+32175b(c+dx\right.}{45045d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-9009*(b*c - a*d)^5 + 32175*b*(b*c - a*d)^4*(c + d*x) -
 50050*b^2*(b*c - a*d)^3*(c + d*x)^2 + 40950*b^3*(b*c - a*d)^2*(c + d*x)^3
```

```
- 17325*b^4*(b*c - a*d)*(c + d*x)^4 + 3003*b^5*(c + d*x)^5))/(45045*d^6)
```

Maple [B] time = 0.006, size = 273, normalized size = 1.7

$$\frac{6006\,b^5x^5d^5+34650\,ab^4d^5x^4-4620\,b^5cd^4x^4+81900\,a^2b^3d^5x^3-25200\,ab^4cd^4x^3+3360\,b^5c^2d^3x^3+100100\,a^3b^2d^5x^2-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(3/2),x)
```

```
[Out] 2/45045*(d*x+c)^(5/2)*(3003*b^5*d^5*x^5+17325*a*b^4*d^5*x^4-2310*b^5*c*d^4*
x^4+40950*a^2*b^3*d^5*x^3-12600*a*b^4*c*d^4*x^3+1680*b^5*c^2*d^3*x^3+50050*
a^3*b^2*d^5*x^2-27300*a^2*b^3*c*d^4*x^2+8400*a*b^4*c^2*d^3*x^2-1120*b^5*c^3
*d^2*x^2+32175*a^4*b*d^5*x-28600*a^3*b^2*c*d^4*x+15600*a^2*b^3*c^2*d^3*x-48
00*a*b^4*c^3*d^2*x+640*b^5*c^4*d*x+9009*a^5*d^5-12870*a^4*b*c*d^4+11440*a^3
*b^2*c^2*d^3-6240*a^2*b^3*c^3*d^2+1920*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

Maxima [A] time = 0.962181, size = 350, normalized size = 2.22

$$\frac{2\left(3003\,(dx+c)^{\frac{15}{2}}b^5-17325\left(b^5c-ab^4d\right)(dx+c)^{\frac{13}{2}}+40950\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)(dx+c)^{\frac{11}{2}}-50050\left(b^5c^3-3\,ab^4c^2\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(d*x + c)^(15/2)*b^5 - 17325*(b^5*c - a*b^4*d)*(d*x + c)^(13/
2) + 40950*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(11/2) - 50050*(
b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(9/2) +
32175*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*
b*d^4)*(d*x + c)^(7/2) - 9009*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2
 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(5/2))/d^6
```

Fricas [B] time = 2.10938, size = 953, normalized size = 6.03

$$\frac{2\left(3003\,b^5d^7x^7-256\,b^5c^7+1920\,ab^4c^6d-6240\,a^2b^3c^5d^2+11440\,a^3b^2c^4d^3-12870\,a^4bc^3d^4+9009\,a^5c^2d^5+231\left(16\,b^5c\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^5*d^7*x^7 - 256*b^5*c^7 + 1920*a*b^4*c^6*d - 6240*a^2*b^3*c
^5*d^2 + 11440*a^3*b^2*c^4*d^3 - 12870*a^4*b*c^3*d^4 + 9009*a^5*c^2*d^5 + 2
31*(16*b^5*c*d^6 + 75*a*b^4*d^7)*x^6 + 63*(b^5*c^2*d^5 + 350*a*b^4*c*d^6 +
650*a^2*b^3*d^7)*x^5 - 35*(2*b^5*c^3*d^4 - 15*a*b^4*c^2*d^5 - 1560*a^2*b^3*
c*d^6 - 1430*a^3*b^2*d^7)*x^4 + 5*(16*b^5*c^4*d^3 - 120*a*b^4*c^3*d^4 + 390
*a^2*b^3*c^2*d^5 + 14300*a^3*b^2*c*d^6 + 6435*a^4*b*d^7)*x^3 - 3*(32*b^5*c^
5*d^2 - 240*a*b^4*c^4*d^3 + 780*a^2*b^3*c^3*d^4 - 1430*a^3*b^2*c^2*d^5 - 17
160*a^4*b*c*d^6 - 3003*a^5*d^7)*x^2 + (128*b^5*c^6*d - 960*a*b^4*c^5*d^2 +
3120*a^2*b^3*c^4*d^3 - 5720*a^3*b^2*c^3*d^4 + 6435*a^4*b*c^2*d^5 + 18018*a^
```

```
5*c*d^6)*x)*sqrt(d*x + c)/d^6
```

Sympy [A] time = 20.2502, size = 763, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(3/2),x)
```

```
[Out] a**5*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**5*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 10*a**4*b*c*(-c*(c
 + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 10*a**4*b*(c**2*(c + d*x)**(3
/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 20*a**3*b**2*c*
(c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**
3 + 20*a**3*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*
c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 20*a**2*b**3*c*(-c**3*(c
+ d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c +
 d*x)**(9/2)/9)/d**4 + 20*a**2*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c +
d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d
*x)**(11/2)/11)/d**4 + 10*a*b**4*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d
*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*
x)**(11/2)/11)/d**5 + 10*a*b**4*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)*
*(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c +
 d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5 + 2*b**5*c*(-c**5*(c + d*x)**
(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c +
 d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**6 + 2*
b**5*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4*(c + d*
x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2)/11 - 6
*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**6
```

Giac [B] time = 1.10514, size = 869, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/45045*(15015*(d*x + c)^(3/2)*a^5*c + 3003*(3*(d*x + c)^(5/2) - 5*(d*x + c
)^(3/2)*c)*a^5 + 15015*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c)*a^4*b*c/d
+ 4290*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)
*a^3*b^2*c/d^2 + 2145*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x
+ c)^(3/2)*c^2)*a^4*b/d + 1430*(35*(d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c
+ 189*(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)*a^2*b^3*c/d^3 + 1430*(
35*(d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c + 189*(d*x + c)^(5/2)*c^2 - 105*
(d*x + c)^(3/2)*c^3)*a^3*b^2/d^2 + 65*(315*(d*x + c)^(11/2) - 1540*(d*x + c
)^(9/2)*c + 2970*(d*x + c)^(7/2)*c^2 - 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x
 + c)^(3/2)*c^4)*a*b^4*c/d^4 + 130*(315*(d*x + c)^(11/2) - 1540*(d*x + c)^(
9/2)*c + 2970*(d*x + c)^(7/2)*c^2 - 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x +
c)^(3/2)*c^4)*a^2*b^3/d^3 + 5*(693*(d*x + c)^(13/2) - 4095*(d*x + c)^(11/2)
*c + 10010*(d*x + c)^(9/2)*c^2 - 12870*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)
^(5/2)*c^4 - 3003*(d*x + c)^(3/2)*c^5)*b^5*c/d^5 + 25*(693*(d*x + c)^(13/2)
 - 4095*(d*x + c)^(11/2)*c + 10010*(d*x + c)^(9/2)*c^2 - 12870*(d*x + c)^(7
```

```
/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 3003*(d*x + c)^(3/2)*c^5)*a*b^4/d^4 +
(3003*(d*x + c)^(15/2) - 20790*(d*x + c)^(13/2)*c + 61425*(d*x + c)^(11/2)*
c^2 - 100100*(d*x + c)^(9/2)*c^3 + 96525*(d*x + c)^(7/2)*c^4 - 54054*(d*x +
 c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6)*b^5/d^5)/d
```

3.1388 $\int (a+bx)^4 (c+dx)^{3/2}\,dx$

Optimal. Leaf size=129

$$-\frac{8b^3(c+dx)^{11/2}(bc-ad)}{11d^5}+\frac{4b^2(c+dx)^{9/2}(bc-ad)^2}{3d^5}-\frac{8b(c+dx)^{7/2}(bc-ad)^3}{7d^5}+\frac{2(c+dx)^{5/2}(bc-ad)^4}{5d^5}+\frac{2b^4(c+dx)}{13d^5}$$

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(5/2))/(5*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(7
/2))/(7*d^5) + (4*b^2*(b*c - a*d)^2*(c + d*x)^(9/2))/(3*d^5) - (8*b^3*(b*c
- a*d)*(c + d*x)^(11/2))/(11*d^5) + (2*b^4*(c + d*x)^(13/2))/(13*d^5)
```

Rubi [A] time = 0.0419296, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{11/2}(bc-ad)}{11d^5}+\frac{4b^2(c+dx)^{9/2}(bc-ad)^2}{3d^5}-\frac{8b(c+dx)^{7/2}(bc-ad)^3}{7d^5}+\frac{2(c+dx)^{5/2}(bc-ad)^4}{5d^5}+\frac{2b^4(c+dx)}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(5/2))/(5*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(7
/2))/(7*d^5) + (4*b^2*(b*c - a*d)^2*(c + d*x)^(9/2))/(3*d^5) - (8*b^3*(b*c
- a*d)*(c + d*x)^(11/2))/(11*d^5) + (2*b^4*(c + d*x)^(13/2))/(13*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^4(c+dx)^{3/2}\,dx=\int\left(\frac{(-bc+ad)^4(c+dx)^{3/2}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{5/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{7/2}}{d^4}-\frac{4b^3(bc-\right.$$

$$=\frac{2(bc-ad)^4(c+dx)^{5/2}}{5d^5}-\frac{8b(bc-ad)^3(c+dx)^{7/2}}{7d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{9/2}}{3d^5}-\frac{8b^3(bc-ad)}{11}$$

Mathematica [A] time = 0.0953262, size = 101, normalized size = 0.78

$$\frac{2(c+dx)^{5/2}\left(10010b^2(c+dx)^2(bc-ad)^2-5460b^3(c+dx)^3(bc-ad)-8580b(c+dx)(bc-ad)^3+3003(bc-ad)^4+115\right.}{15015d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(3003*(b*c - a*d)^4 - 8580*b*(b*c - a*d)^3*(c + d*x) + 1
0010*b^2*(b*c - a*d)^2*(c + d*x)^2 - 5460*b^3*(b*c - a*d)*(c + d*x)^3 + 115
5*b^4*(c + d*x)^4))/(15015*d^5)
```

Maple [A] time = 0.006, size = 186, normalized size = 1.4

$$\frac{2310\,b^4x^4d^4+10920\,ab^3d^4x^3-1680\,b^4cd^3x^3+20020\,a^2b^2d^4x^2-7280\,ab^3cd^3x^2+1120\,b^4c^2d^2x^2+17160\,a^3bd^4x-11440}{15015\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(3/2),x)
```

```
[Out] 2/15015*(d*x+c)^(5/2)*(1155*b^4*d^4*x^4+5460*a*b^3*d^4*x^3-840*b^4*c*d^3*x^
3+10010*a^2*b^2*d^4*x^2-3640*a*b^3*c*d^3*x^2+560*b^4*c^2*d^2*x^2+8580*a^3*b
*d^4*x-5720*a^2*b^2*c*d^3*x+2080*a*b^3*c^2*d^2*x-320*b^4*c^3*d*x+3003*a^4*d
^4-3432*a^3*b*c*d^3+2288*a^2*b^2*c^2*d^2-832*a*b^3*c^3*d+128*b^4*c^4)/d^5
```

Maxima [A] time = 0.977237, size = 244, normalized size = 1.89

$$\frac{2\left(1155\,(dx+c)^{\frac{13}{2}}b^4-5460\left(b^4c-ab^3d\right)(dx+c)^{\frac{11}{2}}+10010\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)(dx+c)^{\frac{9}{2}}-8580\left(b^4c^3-3\,ab^3c^2d+\right.\right.}{15015\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*(d*x + c)^(13/2)*b^4 - 5460*(b^4*c - a*b^3*d)*(d*x + c)^(11/2
) + 10010*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(9/2) - 8580*(b^4
*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(7/2) + 3003*
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*
x + c)^(5/2))/d^5
```

Fricas [B] time = 2.13225, size = 702, normalized size = 5.44

$$\frac{2\left(1155\,b^4d^6x^6+128\,b^4c^6-832\,ab^3c^5d+2288\,a^2b^2c^4d^2-3432\,a^3bc^3d^3+3003\,a^4c^2d^4+210\left(7\,b^4cd^5+26\,ab^3d^6\right)x^5+35\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^4*d^6*x^6 + 128*b^4*c^6 - 832*a*b^3*c^5*d + 2288*a^2*b^2*c^
4*d^2 - 3432*a^3*b*c^3*d^3 + 3003*a^4*c^2*d^4 + 210*(7*b^4*c*d^5 + 26*a*b^3
*d^6)*x^5 + 35*(b^4*c^2*d^4 + 208*a*b^3*c*d^5 + 286*a^2*b^2*d^6)*x^4 - 20*(
2*b^4*c^3*d^3 - 13*a*b^3*c^2*d^4 - 715*a^2*b^2*c*d^5 - 429*a^3*b*d^6)*x^3 +
 3*(16*b^4*c^4*d^2 - 104*a*b^3*c^3*d^3 + 286*a^2*b^2*c^2*d^4 + 4576*a^3*b*c
*d^5 + 1001*a^4*d^6)*x^2 - 2*(32*b^4*c^5*d - 208*a*b^3*c^4*d^2 + 572*a^2*b^
2*c^3*d^3 - 858*a^3*b*c^2*d^4 - 3003*a^4*c*d^5)*x)*sqrt(d*x + c)/d^5
```

Sympy [A] time = 15.5252, size = 559, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(3/2),x)
```

```
[Out] a**4*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**4*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 8*a**3*b*c*(-c*(c
+ d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 8*a**3*b*(c**2*(c + d*x)**(3/2
)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 12*a**2*b**2*c*(c
**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3
+ 12*a**2*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*
(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 8*a*b**3*c*(-c**3*(c + d*x)
**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)*
*(9/2)/9)/d**4 + 8*a*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2
)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2
)/11)/d**4 + 2*b**4*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5
+ 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11
)/d**5 + 2*b**4*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3
*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11
 + (c + d*x)**(13/2)/13)/d**5
```

Giac [B] time = 1.08234, size = 640, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/45045*(15015*(d*x + c)^(3/2)*a^4*c + 3003*(3*(d*x + c)^(5/2) - 5*(d*x + c
)^(3/2)*c)*a^4 + 12012*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c)*a^3*b*c/d
+ 2574*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)
*a^2*b^2*c/d^2 + 1716*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x
+ c)^(3/2)*c^2)*a^3*b/d + 572*(35*(d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c +
 189*(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)*a*b^3*c/d^3 + 858*(35*(
d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c + 189*(d*x + c)^(5/2)*c^2 - 105*(d*x
 + c)^(3/2)*c^3)*a^2*b^2/d^2 + 13*(315*(d*x + c)^(11/2) - 1540*(d*x + c)^(9
/2)*c + 2970*(d*x + c)^(7/2)*c^2 - 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c
)^(3/2)*c^4)*b^4*c/d^4 + 52*(315*(d*x + c)^(11/2) - 1540*(d*x + c)^(9/2)*c
+ 2970*(d*x + c)^(7/2)*c^2 - 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2
)*c^4)*a*b^3/d^3 + 5*(693*(d*x + c)^(13/2) - 4095*(d*x + c)^(11/2)*c + 1001
0*(d*x + c)^(9/2)*c^2 - 12870*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^
4 - 3003*(d*x + c)^(3/2)*c^5)*b^4/d^4)/d
```

3.1389 $\int (a+bx)^3(c+dx)^{3/2}\,dx$

Optimal. Leaf size=100

$$-\frac{2b^2(c+dx)^{9/2}(bc-ad)}{3d^4}+\frac{6b(c+dx)^{7/2}(bc-ad)^2}{7d^4}-\frac{2(c+dx)^{5/2}(bc-ad)^3}{5d^4}+\frac{2b^3(c+dx)^{11/2}}{11d^4}$$

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(5/2))/(5*d^4) + (6*b*(b*c - a*d)^2*(c + d*x)^(
7/2))/(7*d^4) - (2*b^2*(b*c - a*d)*(c + d*x)^(9/2))/(3*d^4) + (2*b^3*(c + d
*x)^(11/2))/(11*d^4)
```

Rubi [A] time = 0.0338238, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2b^2(c+dx)^{9/2}(bc-ad)}{3d^4}+\frac{6b(c+dx)^{7/2}(bc-ad)^2}{7d^4}-\frac{2(c+dx)^{5/2}(bc-ad)^3}{5d^4}+\frac{2b^3(c+dx)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(5/2))/(5*d^4) + (6*b*(b*c - a*d)^2*(c + d*x)^(
7/2))/(7*d^4) - (2*b^2*(b*c - a*d)*(c + d*x)^(9/2))/(3*d^4) + (2*b^3*(c + d
*x)^(11/2))/(11*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^{3/2}\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^{3/2}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{5/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{7/2}}{d^3}+\frac{b^3(c+dx)^{9/2}}{d^3}\right)dx\\ &= -\frac{2(bc-ad)^3(c+dx)^{5/2}}{5d^4}+\frac{6b(bc-ad)^2(c+dx)^{7/2}}{7d^4}-\frac{2b^2(bc-ad)(c+dx)^{9/2}}{3d^4}+\frac{2b^3(c+dx)^{11/2}}{11d^4}\end{aligned}$$

Mathematica [A] time = 0.0661977, size = 79, normalized size = 0.79

$$\frac{2(c+dx)^{5/2}\left(-385b^2(c+dx)^2(bc-ad)+495b(c+dx)(bc-ad)^2-231(bc-ad)^3+105b^3(c+dx)^3\right)}{1155d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-231*(b*c - a*d)^3 + 495*b*(b*c - a*d)^2*(c + d*x) - 38
5*b^2*(b*c - a*d)*(c + d*x)^2 + 105*b^3*(c + d*x)^3))/(1155*d^4)
```

Maple [A] time = 0.006, size = 116, normalized size = 1.2

$$\frac{210\,b^3x^3d^3+770\,ab^2d^3x^2-140\,b^3cd^2x^2+990\,a^2bd^3x-440\,ab^2cd^2x+80\,b^3c^2dx+462\,a^3d^3-396\,a^2bcd^2+176\,ab^2c^2}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(3/2),x)
```

```
[Out] 2/1155*(d*x+c)^(5/2)*(105*b^3*d^3*x^3+385*a*b^2*d^3*x^2-70*b^3*c*d^2*x^2+49
5*a^2*b*d^3*x-220*a*b^2*c*d^2*x+40*b^3*c^2*d*x+231*a^3*d^3-198*a^2*b*c*d^2+
88*a*b^2*c^2*d-16*b^3*c^3)/d^4
```

Maxima [A] time = 0.948524, size = 159, normalized size = 1.59

$$\frac{2\left(105\,(dx+c)^{\frac{11}{2}}b^3-385\left(b^3c-ab^2d\right)(dx+c)^{\frac{9}{2}}+495\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)(dx+c)^{\frac{7}{2}}-231\left(b^3c^3-3\,ab^2c^2d+3\,a^2\right.}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*(d*x + c)^(11/2)*b^3 - 385*(b^3*c - a*b^2*d)*(d*x + c)^(9/2) +
495*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(7/2) - 231*(b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(5/2))/d^4
```

Fricas [B] time = 1.98509, size = 474, normalized size = 4.74

$$2\left(105\,b^3d^5x^5-16\,b^3c^5+88\,ab^2c^4d-198\,a^2bc^3d^2+231\,a^3c^2d^3+35\left(4\,b^3cd^4+11\,ab^2d^5\right)x^4+5\left(b^3c^2d^3+110\,ab^2cd^4+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^3*d^5*x^5 - 16*b^3*c^5 + 88*a*b^2*c^4*d - 198*a^2*b*c^3*d^2 +
 231*a^3*c^2*d^3 + 35*(4*b^3*c*d^4 + 11*a*b^2*d^5)*x^4 + 5*(b^3*c^2*d^3 + 1
10*a*b^2*c*d^4 + 99*a^2*b*d^5)*x^3 - 3*(2*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3 -
264*a^2*b*c*d^4 - 77*a^3*d^5)*x^2 + (8*b^3*c^4*d - 44*a*b^2*c^3*d^2 + 99*a^
2*b*c^2*d^3 + 462*a^3*c*d^4)*x)*sqrt(d*x + c)/d^4
```

Sympy [A] time = 10.9798, size = 386, normalized size = 3.86

$$a^3c\left(\begin{cases}\sqrt{c}x & \text{for}\ d=0\\ \frac{2(c+dx)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+\frac{2a^3\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{6a^2bc\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}+\frac{6a^2b\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(3/2),x)
```

```
[Out] a**3*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**3*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 6*a**2*b*c*(-c*(c
+ d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 6*a**2*b*(c**2*(c + d*x)**(3/2
)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 6*a*b**2*c*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 6*
a*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x
)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 2*b**3*c*(-c**3*(c + d*x)**(3/2)/3
+ 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/
d**4 + 2*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2
*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11)/d**4
```

Giac [B] time = 1.06874, size = 440, normalized size = 4.4

$$2\left(1155\,(dx+c)^{\frac{3}{2}}a^3c + 231\left(3\,(dx+c)^{\frac{5}{2}} - 5\,(dx+c)^{\frac{3}{2}}c\right)a^3 + \frac{693\left(3\,(dx+c)^{\frac{5}{2}} - 5\,(dx+c)^{\frac{3}{2}}c\right)a^2bc}{d} + \frac{99\left(15\,(dx+c)^{\frac{7}{2}} - 42\,(dx+c)^{\frac{5}{2}}c + 35\,(dx+c)^{\frac{3}{2}}c\right.}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3465*(1155*(d*x + c)^(3/2)*a^3*c + 231*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(
3/2)*c)*a^3 + 693*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c)*a^2*b*c/d + 99*
(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)*a*b^2*
c/d^2 + 99*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*
c^2)*a^2*b/d + 11*(35*(d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c + 189*(d*x +
c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)*b^3*c/d^3 + 33*(35*(d*x + c)^(9/2)
- 135*(d*x + c)^(7/2)*c + 189*(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3
)*a*b^2/d^2 + (315*(d*x + c)^(11/2) - 1540*(d*x + c)^(9/2)*c + 2970*(d*x +
c)^(7/2)*c^2 - 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4)*b^3/d^3
)/d
```

3.1390 $\int (a+bx)^2(c+dx)^{3/2}\,dx$

Optimal. Leaf size=71

$$-\frac{4b(c+dx)^{7/2}(bc-ad)}{7d^3}+\frac{2(c+dx)^{5/2}(bc-ad)^2}{5d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}$$

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(5/2))/(5*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(7/2
))/(7*d^3) + (2*b^2*(c + d*x)^(9/2))/(9*d^3)
```

Rubi [A] time = 0.0237218, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b(c+dx)^{7/2}(bc-ad)}{7d^3}+\frac{2(c+dx)^{5/2}(bc-ad)^2}{5d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(5/2))/(5*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(7/2
))/(7*d^3) + (2*b^2*(c + d*x)^(9/2))/(9*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{3/2}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{3/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{5/2}}{d^2}+\frac{b^2(c+dx)^{7/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2(c+dx)^{5/2}}{5d^3}-\frac{4b(bc-ad)(c+dx)^{7/2}}{7d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}\end{aligned}$$

Mathematica [A] time = 0.0411658, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{5/2}\left(63a^2d^2+18abd(5dx-2c)+b^2\left(8c^2-20cdx+35d^2x^2\right)\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(63*a^2*d^2 + 18*a*b*d*(-2*c + 5*d*x) + b^2*(8*c^2 - 20*
c*d*x + 35*d^2*x^2)))/(315*d^3)
```

Maple [A] time = 0.005, size = 63, normalized size = 0.9

$$\frac{70\,b^2x^2d^2+180\,abd^2x-40\,b^2cdx+126\,a^2d^2-72\,abcd+16\,b^2c^2}{315\,d^3}(dx+c)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(3/2),x)
```

```
[Out] 2/315*(d*x+c)^(5/2)*(35*b^2*d^2*x^2+90*a*b*d^2*x-20*b^2*c*d*x+63*a^2*d^2-36
*a*b*c*d+8*b^2*c^2)/d^3
```

Maxima [A] time = 0.973623, size = 92, normalized size = 1.3

$$\frac{2\left(35\,(dx+c)^{\frac{9}{2}}b^2-90\left(b^2c-abd\right)(dx+c)^{\frac{7}{2}}+63\left(b^2c^2-2\,abcd+a^2d^2\right)(dx+c)^{\frac{5}{2}}\right)}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(d*x + c)^(9/2)*b^2 - 90*(b^2*c - a*b*d)*(d*x + c)^(7/2) + 63*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(5/2))/d^3
```

Fricas [B] time = 1.95459, size = 300, normalized size = 4.23

$$\frac{2\left(35\,b^2d^4x^4+8\,b^2c^4-36\,abc^3d+63\,a^2c^2d^2+10\left(5\,b^2cd^3+9\,abd^4\right)x^3+3\left(b^2c^2d^2+48\,abcd^3+21\,a^2d^4\right)x^2-2\left(2\,b^2c^3d-\right.\right.}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^2*d^4*x^4 + 8*b^2*c^4 - 36*a*b*c^3*d + 63*a^2*c^2*d^2 + 10*(5*b
^2*c*d^3 + 9*a*b*d^4)*x^3 + 3*(b^2*c^2*d^2 + 48*a*b*c*d^3 + 21*a^2*d^4)*x^2
 - 2*(2*b^2*c^3*d - 9*a*b*c^2*d^2 - 63*a^2*c*d^3)*x)*sqrt(d*x + c)/d^3
```

Sympy [A] time = 7.67253, size = 240, normalized size = 3.38

$$a^2c\left(\begin{cases}\sqrt{c}x & \text{for } d=0\\ \frac{2(c+dx)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+\frac{2a^2\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{4abc\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}+\frac{4ab\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx}{7}\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(3/2),x)
```

```
[Out] a**2*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 4*a*b*c*(-c*(c + d
*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 4*a*b*(c**2*(c + d*x)**(3/2)/3 -
```

```
2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 2*b**2*c*(c**2*(c + d*x
)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 2*b**2*(-c
**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7
 + (c + d*x)**(9/2)/9)/d**3
```

Giac [B] time = 1.06499, size = 274, normalized size = 3.86

$$\frac{2\left(105\,(dx+c)^{\frac{3}{2}}a^2c+21\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)a^2+\frac{42\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)abc}{d}+\frac{3\left(15\,(dx+c)^{\frac{7}{2}}-42\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2\right)b\cdots}{d^2}\cdots\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/315*(105*(d*x + c)^(3/2)*a^2*c + 21*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2
)*c)*a^2 + 42*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c)*a*b*c/d + 3*(15*(d*
x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)*b^2*c/d^2 + 6
*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)*a*b/d
 + (35*(d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c + 189*(d*x + c)^(5/2)*c^2 -
105*(d*x + c)^(3/2)*c^3)*b^2/d^2)/d
```

3.1391 $\int (a+bx)(c+dx)^{3/2}\,dx$

Optimal. Leaf size=42

$$\frac{2b(c+dx)^{7/2}}{7d^2}-\frac{2(c+dx)^{5/2}(bc-ad)}{5d^2}$$

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(5/2))/(5*d^2) + (2*b*(c + d*x)^(7/2))/(7*d^2)
```

Rubi [A] time = 0.0139126, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(c+dx)^{7/2}}{7d^2}-\frac{2(c+dx)^{5/2}(bc-ad)}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(5/2))/(5*d^2) + (2*b*(c + d*x)^(7/2))/(7*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{3/2}\,dx&=\int \left(\frac{(-bc+ad)(c+dx)^{3/2}}{d}+\frac{b(c+dx)^{5/2}}{d}\right)dx\\&=-\frac{2(bc-ad)(c+dx)^{5/2}}{5d^2}+\frac{2b(c+dx)^{7/2}}{7d^2}\end{aligned}$$

Mathematica [A] time = 0.0202293, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{5/2}(7ad-2bc+5bdx)}{35d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-2*b*c + 7*a*d + 5*b*d*x))/(35*d^2)
```

Maple [A] time = 0.002, size = 27, normalized size = 0.6

$$\frac{10\,bdx+14\,ad-4\,bc}{35\,d^2}(dx+c)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(3/2),x)
```

```
[Out] 2/35*(d*x+c)^(5/2)*(5*b*d*x+7*a*d-2*b*c)/d^2
```

Maxima [A] time = 0.948696, size = 45, normalized size = 1.07

$$\frac{2\left(5\,(dx+c)^{\frac{7}{2}}b-7\,(bc-ad)(dx+c)^{\frac{5}{2}}\right)}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*(d*x + c)^(7/2)*b - 7*(b*c - a*d)*(d*x + c)^(5/2))/d^2
```

Fricas [B] time = 1.94714, size = 155, normalized size = 3.69

$$\frac{2\left(5\,bd^3x^3-2\,bc^3+7\,ac^2d+\left(8\,bcd^2+7\,ad^3\right)x^2+\left(bc^2d+14\,acd^2\right)x\right)\sqrt{dx+c}}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b*d^3*x^3 - 2*b*c^3 + 7*a*c^2*d + (8*b*c*d^2 + 7*a*d^3)*x^2 + (b*c^
2*d + 14*a*c*d^2)*x)*sqrt(d*x + c)/d^2
```

Sympy [A] time = 0.681699, size = 146, normalized size = 3.48

$$\begin{cases} \frac{2ac^2\sqrt{c+dx}}{5d}+\frac{4acx\sqrt{c+dx}}{5}+\frac{2adx^2\sqrt{c+dx}}{5}-\frac{4bc^3\sqrt{c+dx}}{35d^2}+\frac{2bc^2x\sqrt{c+dx}}{35d}+\frac{16bcx^2\sqrt{c+dx}}{35}+\frac{2bdx^3\sqrt{c+dx}}{7} & \text{for } d\neq 0 \\ c^{\frac{3}{2}}\left(ax+\frac{bx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(3/2),x)
```

```
[Out] Piecewise((2*a*c**2*sqrt(c + d*x)/(5*d) + 4*a*c*x*sqrt(c + d*x)/5 + 2*a*d*x
**2*sqrt(c + d*x)/5 - 4*b*c**3*sqrt(c + d*x)/(35*d**2) + 2*b*c**2*x*sqrt(c
+ d*x)/(35*d) + 16*b*c*x**2*sqrt(c + d*x)/35 + 2*b*d*x**3*sqrt(c + d*x)/7,
Ne(d, 0)), (c**(3/2)*(a*x + b*x**2/2), True))
```

Giac [B] time = 1.04931, size = 140, normalized size = 3.33

$$\frac{2\left(35\,(dx+c)^{\frac{3}{2}}ac+7\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)a+\frac{7\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)bc}{d}+\frac{\left(15\,(dx+c)^{\frac{7}{2}}-42\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2\right)b}{d}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(35*(d*x + c)^(3/2)*a*c + 7*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c)
*a + 7*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c)*b*c/d + (15*(d*x + c)^(7/2
) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)*b/d)/d
```

3.1392 $\int (c+dx)^{3/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(c+dx)^{5/2}}{5d}$$

```
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

Rubi [A] time = 0.0014835, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{3/2}\,dx=\frac{2(c+dx)^{5/2}}{5d}$$

Mathematica [A] time = 0.0046322, size = 16, normalized size = 1.

$$\frac{2(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

Maple [A] time = 0.003, size = 13, normalized size = 0.8

$$\frac{2}{5d}(dx+c)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2),x)
```

```
[Out] 2/5*(d*x+c)^(5/2)/d
```

Maxima [A] time = 0.964246, size = 16, normalized size = 1.

$$\frac{2\,(dx+c)^{\frac{5}{2}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(d*x + c)^(5/2)/d
```

Fricas [B] time = 1.81845, size = 63, normalized size = 3.94

$$\frac{2\left(d^2x^2+2\,cdx+c^2\right)\sqrt{dx+c}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(d*x + c)/d
```

Sympy [A] time = 0.057442, size = 12, normalized size = 0.75

$$\frac{2\,(c+dx)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2),x)
```

```
[Out] 2*(c + d*x)**(5/2)/(5*d)
```

Giac [A] time = 1.06122, size = 16, normalized size = 1.

$$\frac{2\,(dx+c)^{\frac{5}{2}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*(d*x + c)^(5/2)/d
```

3.1393 $\int \frac{(c+dx)^{3/2}}{a+bx} dx$

Optimal. Leaf size=86

$$\frac{2\sqrt{c+dx}(bc-ad)}{b^2} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}} + \frac{2(c+dx)^{3/2}}{3b}$$

```
[Out] (2*(b*c - a*d)*Sqrt[c + d*x])/b^2 + (2*(c + d*x)^(3/2))/(3*b) - (2*(b*c - a
*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rubi [A] time = 0.0455595, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 63, 208}

$$\frac{2\sqrt{c+dx}(bc-ad)}{b^2} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}} + \frac{2(c+dx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[c + d*x])/b^2 + (2*(c + d*x)^(3/2))/(3*b) - (2*(b*c - a
*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{a+bx}\,dx = \frac{2(c+dx)^{3/2}}{3b} + \frac{(bc-ad)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{b}$$
$$= \frac{2(bc-ad)\sqrt{c+dx}}{b^2} + \frac{2(c+dx)^{3/2}}{3b} + \frac{(bc-ad)^2\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{b^2}$$
$$= \frac{2(bc-ad)\sqrt{c+dx}}{b^2} + \frac{2(c+dx)^{3/2}}{3b} + \frac{\left(2(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{b^2d}$$
$$= \frac{2(bc-ad)\sqrt{c+dx}}{b^2} + \frac{2(c+dx)^{3/2}}{3b} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.0716295, size = 77, normalized size = 0.9

$$\frac{2\sqrt{c+dx}(-3ad+4bc+bdx)}{3b^2} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*(4*b*c - 3*a*d + b*d*x))/(3*b^2) - (2*(b*c - a*d)^(3/2)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

Maple [B] time = 0.006, size = 167, normalized size = 1.9

$$\frac{2}{3\,b}(dx+c)^{\frac{3}{2}} - 2\,\frac{ad\sqrt{dx+c}}{b^2} + 2\,\frac{\sqrt{dx+c}c}{b} + 2\,\frac{a^2d^2}{b^2\sqrt{(ad-bc)\,b}}\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right) - 4\,\frac{acd}{b\sqrt{(ad-bc)\,b}}\arctan\left(\frac{b\sqrt{dx+}}{\sqrt{(ad-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a),x)
```

```
[Out] 2/3*(d*x+c)^(3/2)/b-2/b^2*a*d*(d*x+c)^(1/2)+2/b*(d*x+c)^(1/2)*c+2/b^2/((a*d
-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a^2*d^2-4/b/((a*
d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a*c*d+2/((a*d-b
*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8924, size = 424, normalized size = 4.93

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad+2\,\sqrt{dx+c}b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)-2\,(bdx+4\,bc-3\,ad)\sqrt{dx+c}}{3\,b^2}, -\frac{2\left(3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d + 2*sqrt(
d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) - 2*(b*d*x + 4*b*c - 3*a*d)*sqrt
(d*x + c))/b^2, -2/3*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x +
 c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (b*d*x + 4*b*c - 3*a*d)*sqrt(d*x
+ c))/b^2]
```

Sympy [A] time = 10.7802, size = 82, normalized size = 0.95

$$\frac{2\,(c+dx)^{\frac{3}{2}}}{3b}+\frac{\sqrt{c+dx}\,(-2ad+2bc)}{b^2}+\frac{2\,(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a),x)
```

```
[Out] 2*(c + d*x)**(3/2)/(3*b) + sqrt(c + d*x)*(-2*a*d + 2*b*c)/b**2 + 2*(a*d - b
*c)**2*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(b**3*sqrt((a*d - b*c)/b))
```

Giac [A] time = 1.06591, size = 142, normalized size = 1.65

$$\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}+\frac{2\left((dx+c)^{\frac{3}{2}}b^2+3\,\sqrt{dx+c}b^2c-3\,\sqrt{dx+c}abd\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*
d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/3*((d*x + c)^(3/2)*b^2 + 3*sqrt(d*x + c)
*b^2*c - 3*sqrt(d*x + c)*a*b*d)/b^3
```

3.1394 $\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{3d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{3d\sqrt{c+dx}}{b^2}$$

```
[Out] (3*d*Sqrt[c + d*x])/b^2 - (c + d*x)^(3/2)/(b*(a + b*x)) - (3*d*Sqrt[b*c - a
*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rubi [A] time = 0.0381649, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 63, 208}

$$-\frac{3d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{3d\sqrt{c+dx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x])/b^2 - (c + d*x)^(3/2)/(b*(a + b*x)) - (3*d*Sqrt[b*c - a
*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx &= -\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3d)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{2b}\\ &= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3d(bc-ad))\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2b^2}\\ &= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{b^2}\\ &= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}-\frac{3d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.0135204, size = 50, normalized size = 0.59

$$\frac{2d(c+dx)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{b(c+dx)}{ad-bc}\right)}{5(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (2*d*(c + d*x)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(5*(-(b*c) + a*d)^2)
```

Maple [B] time = 0.011, size = 148, normalized size = 1.7

$$2\,\frac{d\sqrt{dx+c}}{b^2}+\frac{ad^2}{b^2\,(bdx+ad)}\sqrt{dx+c}-\frac{dc}{b\,(bdx+ad)}\sqrt{dx+c}-3\,\frac{ad^2}{b^2\sqrt{(ad-bc)\,b}}\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)+3\,\frac{dc}{b\sqrt{(ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^2,x)
```

```
[Out] 2*d*(d*x+c)^(1/2)/b^2+1/b^2*(d*x+c)^(1/2)/(b*d*x+a*d)*a*d^2-d/b*(d*x+c)^(1/
2)/(b*d*x+a*d)*c-3/b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c
)*b)^(1/2))*a*d^2+3*d/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*
c)*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8096, size = 455, normalized size = 5.35

$$\left[\frac{3\,(bdx+ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,(2\,bdx-bc+3\,ad)\sqrt{dx+c}}{2\,(b^3x+ab^2)},-\frac{3\,(bdx+ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx-}}{}\right.}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*d*x + a*d)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt
(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(2*b*d*x - b*c + 3*a*d)*sqr
t(d*x + c))/(b^3*x + a*b^2), -(3*(b*d*x + a*d)*sqrt(-(b*c - a*d)/b)*arctan(
-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (2*b*d*x - b*c + 3*a*d
)*sqrt(d*x + c))/(b^3*x + a*b^2)]
```

Sympy [B] time = 83.9816, size = 923, normalized size = 10.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**2,x)
```

```
[Out] 2*a**2*d**3*sqrt(c + d*x)/(2*a**2*b**2*d**2 - 2*a*b**3*c*d + 2*a*b**3*d**2*
x - 2*b**4*c*d*x) - a**2*d**3*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sq
rt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c*
*2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b**2) + a**2*d**3*sqrt(-
1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d
*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt
(c + d*x))/(2*b**2) - 4*a*c*d**2*sqrt(c + d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*
d + 2*a*b**2*d**2*x - 2*b**3*c*d*x) + a*c*d**2*sqrt(-1/(b*(a*d - b*c)**3))*
log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*
c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/b - a*c*d*
*2*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) -
2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3
)) + sqrt(c + d*x))/b - 4*a*d**2*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**3*
sqrt(a*d/b - c)) - c**2*d*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-
1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*s
qrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + c**2*d*sqrt(-1/(b*(a*d - b*
c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a
*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 +
 2*c**2*d*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c*
d*x) + 4*c*d*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*d/b - c)) + 2
*d*sqrt(c + d*x)/b**2
```

Giac [A] time = 1.08262, size = 153, normalized size = 1.8

$$\frac{2\,\sqrt{dx+c}d}{b^2}+\frac{3\,\left(bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}-\frac{\sqrt{dx+c}bcd-\sqrt{dx+c}ad^2}{((dx+c)b-bc+ad)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*d/b^2 + 3*(b*c*d - a*d^2)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - (sqrt(d*x + c)*b*c*d - sqrt(d*x +
c)*a*d^2)/(((d*x + c)*b - b*c + a*d)*b^2)
```

3.1395 $\int \frac{(c+dx)^{3/2}}{(a+bx)^3} dx$

Optimal. Leaf size=100

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}$$

```
[Out] (-3*d*Sqrt[c + d*x])/(4*b^2*(a + b*x)) - (c + d*x)^(3/2)/(2*b*(a + b*x)^2)
- (3*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[
b*c - a*d])
```

Rubi [A] time = 0.0475181, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 63, 208}

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^3,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(4*b^2*(a + b*x)) - (c + d*x)^(3/2)/(2*b*(a + b*x)^2)
- (3*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[
b*c - a*d])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^3}\,dx = -\frac{(c+dx)^{3/2}}{2b(a+bx)^2} + \frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx}{4b}$$
$$= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)} - \frac{(c+dx)^{3/2}}{2b(a+bx)^2} + \frac{(3d^2)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b^2}$$
$$= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)} - \frac{(c+dx)^{3/2}}{2b(a+bx)^2} + \frac{(3d)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{4b^2}$$
$$= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)} - \frac{(c+dx)^{3/2}}{2b(a+bx)^2} - \frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.0977563, size = 90, normalized size = 0.9

$$\frac{3d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{ad-bc}}\right)}{4b^{5/2}\sqrt{ad-bc}} - \frac{\sqrt{c+dx}(3ad+2bc+5bdx)}{4b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^3,x]
```

```
[Out] -(Sqrt[c + d*x]*(2*b*c + 3*a*d + 5*b*d*x))/(4*b^2*(a + b*x)^2) + (3*d^2*Arc
Tan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(4*b^(5/2)*Sqrt[-(b*c) + a
*d])
```

Maple [A] time = 0.011, size = 121, normalized size = 1.2

$$-\frac{5\,d^2}{4\,(bdx+ad)^2\,b}(dx+c)^{\frac{3}{2}} - \frac{3\,d^3a}{4\,(bdx+ad)^2\,b^2}\sqrt{dx+c} + \frac{3\,d^2c}{4\,(bdx+ad)^2\,b}\sqrt{dx+c} + \frac{3\,d^2}{4\,b^2}\arctan\left(b\sqrt{dx+c}\frac{1}{\sqrt{(ad-bc)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^3,x)
```

```
[Out] -5/4*d^2/(b*d*x+a*d)^2/b*(d*x+c)^(3/2)-3/4*d^3/(b*d*x+a*d)^2/b^2*(d*x+c)^(1
/2)*a+3/4*d^2/(b*d*x+a*d)^2/b*(d*x+c)^(1/2)*c+3/4*d^2/b^2/((a*d-b*c)*b)^(1/
2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.81, size = 795, normalized size = 7.95

$$\left[\frac{3\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\sqrt{b^2c-abd}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)-2\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2+5\left(b^3cd-ab^2d^2\right)x\right)}{8\left(a^2b^4c-a^3b^3d+\left(b^6c-ab^5d\right)x^2+2\left(ab^5c-a^2b^4d\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*
x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(2*b^
3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 + 5*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))
/(a^2*b^4*c - a^3*b^3*d + (b^6*c - a*b^5*d)*x^2 + 2*(a*b^5*c - a^2*b^4*d)*x
), 1/4*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (2*b^3*c^2 + a*b^2*c*d
 - 3*a^2*b*d^2 + 5*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c - a^3
*b^3*d + (b^6*c - a*b^5*d)*x^2 + 2*(a*b^5*c - a^2*b^4*d)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09129, size = 146, normalized size = 1.46

$$\frac{3\,d^2\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{4\,\sqrt{-b^2c+abd}b^2}-\frac{5\,(dx+c)^{\frac{3}{2}}bd^2-3\,\sqrt{dx+c}bcd^2+3\,\sqrt{dx+c}ad^3}{4\,((dx+c)b-bc+ad)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b^2) - 1/4*(5*(d*x + c)^(3/2)*b*d^2 - 3*sqrt(d*x + c)*b*c*d^2 + 3*sqrt(d*x
+ c)*a*d^3)/(((d*x + c)*b - b*c + a*d)^2*b^2)
```

3.1396 $\int \frac{(c+dx)^{3/2}}{(a+bx)^4} dx$

Optimal. Leaf size=136

$$-\frac{d^2\sqrt{c+dx}}{8b^2(a+bx)(bc-ad)}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}-\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}$$

```
[Out] -(d*Sqrt[c + d*x])/(4*b^2*(a + b*x)^2) - (d^2*Sqrt[c + d*x])/(8*b^2*(b*c -
a*d)*(a + b*x)) - (c + d*x)^(3/2)/(3*b*(a + b*x)^3) + (d^3*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(5/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0571202, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{d^2\sqrt{c+dx}}{8b^2(a+bx)(bc-ad)}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}-\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^4,x]
```

```
[Out] -(d*Sqrt[c + d*x])/(4*b^2*(a + b*x)^2) - (d^2*Sqrt[c + d*x])/(8*b^2*(b*c -
a*d)*(a + b*x)) - (c + d*x)^(3/2)/(3*b*(a + b*x)^3) + (d^3*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^4}\,dx &= -\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx}{2b} \\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d^2\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8b^2} \\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}-\frac{d^3\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b^2(bc-ad)} \\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}-\frac{d^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{8b^2(bc-ad)} \\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0172427, size = 52, normalized size = 0.38

$$\frac{2d^3(c+dx)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};-\frac{b(c+dx)}{ad-bc}\right)}{5(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^4,x]
```

```
[Out] (2*d^3*(c + d*x)^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(5*(-(b*c) + a*d)^4)
```

Maple [A] time = 0.013, size = 163, normalized size = 1.2

$$\frac{d^3}{8\,(bdx+ad)^3\,(ad-bc)}(dx+c)^{\frac{5}{2}}-\frac{d^3}{3\,(bdx+ad)^3\,b}(dx+c)^{\frac{3}{2}}-\frac{d^4a}{8\,(bdx+ad)^3\,b^2}\sqrt{dx+c}+\frac{d^3c}{8\,(bdx+ad)^3\,b}\sqrt{dx+c}+\frac{}{(8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^4,x)
```

```
[Out] 1/8*d^3/(b*d*x+a*d)^3/(a*d-b*c)*(d*x+c)^(5/2)-1/3*d^3/(b*d*x+a*d)^3/b*(d*x+
c)^(3/2)-1/8*d^4/(b*d*x+a*d)^3/b^2*(d*x+c)^(1/2)*a+1/8*d^3/(b*d*x+a*d)^3/b*
(d*x+c)^(1/2)*c+1/8*d^3/(a*d-b*c)/b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^
(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.85628, size = 1368, normalized size = 10.06

$$\left[-\frac{3\left(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3\right)\sqrt{b^2c-abd}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)+2\left(8\,b^4c^3-10\,ab^3c^2d-a^2b^2c\right.}{48\left(a^3b^5c^2-2\,a^4b^4cd+a^5b^3d^2+\left(b^8c^2-2\,ab^7cd+a^2b^6d^2\right)x^3+3\left(ab^7c^2-2\,a^2b^6c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) + 2*(8*b^4*c^3 - 10*a*b^3*c^2*d - a^2*b^2*c*d^2 + 3*a^3*b*d^3
+ 3*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(7*b^4*c^2*d - 11*a*b^3*c*d^2 + 4*a^2*b
^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2 + (b^8
*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*
b^5*d^2)*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*b^4*d^2)*x), -1/24*(3*(
b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a*b*
d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (8*b^4*c^3 -
10*a*b^3*c^2*d - a^2*b^2*c*d^2 + 3*a^3*b*d^3 + 3*(b^4*c*d^2 - a*b^3*d^3)*x^
2 + 2*(7*b^4*c^2*d - 11*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3
*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2 + (b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d
^2)*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*x^2 + 3*(a^2*b^6*c^2
- 2*a^3*b^5*c*d + a^4*b^4*d^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08899, size = 250, normalized size = 1.84

$$-\frac{d^3\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{8\left(b^3c-ab^2d\right)\sqrt{-b^2c+abd}}-\frac{3\,(dx+c)^{\frac{5}{2}}b^2d^3+8\,(dx+c)^{\frac{3}{2}}b^2cd^3-3\,\sqrt{dx+c}b^2c^2d^3-8\,(dx+c)^{\frac{3}{2}}abd^4+6\,\sqrt{dx+c}a}{24\left(b^3c-ab^2d\right)((dx+c)b-bc+ad)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c - a*b^2*d)*sq
rt(-b^2*c + a*b*d)) - 1/24*(3*(d*x + c)^(5/2)*b^2*d^3 + 8*(d*x + c)^(3/2)*b
^2*c*d^3 - 3*sqrt(d*x + c)*b^2*c^2*d^3 - 8*(d*x + c)^(3/2)*a*b*d^4 + 6*sqrt
(d*x + c)*a*b*c*d^4 - 3*sqrt(d*x + c)*a^2*d^5)/((b^3*c - a*b^2*d)*((d*x + c
)*b - b*c + a*d)^3)
```

3.1397 $\int \frac{(c+dx)^{3/2}}{(a+bx)^5} dx$

Optimal. Leaf size=172

$$\frac{3d^3\sqrt{c+dx}}{64b^2(a+bx)(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{32b^2(a+bx)^2(bc-ad)}-\frac{3d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{5/2}(bc-ad)^{5/2}}-\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}$$

```
[Out] -(d*Sqrt[c + d*x])/(8*b^2*(a + b*x)^3) - (d^2*Sqrt[c + d*x])/(32*b^2*(b*c -
 a*d)*(a + b*x)^2) + (3*d^3*Sqrt[c + d*x])/(64*b^2*(b*c - a*d)^2*(a + b*x))
 - (c + d*x)^(3/2)/(4*b*(a + b*x)^4) - (3*d^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
])/Sqrt[b*c - a*d]])/(64*b^(5/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.0737483, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$\frac{3d^3\sqrt{c+dx}}{64b^2(a+bx)(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{32b^2(a+bx)^2(bc-ad)}-\frac{3d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{5/2}(bc-ad)^{5/2}}-\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^5,x]
```

```
[Out] -(d*Sqrt[c + d*x])/(8*b^2*(a + b*x)^3) - (d^2*Sqrt[c + d*x])/(32*b^2*(b*c -
 a*d)*(a + b*x)^2) + (3*d^3*Sqrt[c + d*x])/(64*b^2*(b*c - a*d)^2*(a + b*x))
 - (c + d*x)^(3/2)/(4*b*(a + b*x)^4) - (3*d^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
])/Sqrt[b*c - a*d]])/(64*b^(5/2)*(b*c - a*d)^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^5}\,dx = -\frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx}{8b}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{d^2\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{16b^2}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} - \frac{\left(3d^3\right)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{64b^2(bc-ad)}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} + \frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{\left(3d^4\right)\int \frac{1}{(a+bx)\sqrt{c+d\ldots}}}{128b^2(bc-ad\ldots}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} + \frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{\left(3d^3\right)\mathrm{Subst}\left(\int \ldots\right.}{64b^2\ldots}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} + \frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} - \frac{3d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c\ldots}}{\sqrt{bc-\ldots}}\right.}{64b^{5/2}(bc-ad)^{\ldots}}$$

Mathematica [C] time = 0.0169324, size = 52, normalized size = 0.3

$$\frac{2d^4(c+dx)^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};-\frac{b(c+dx)}{ad-bc}\right)}{5(ad-bc)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^5,x]
```

```
[Out] (2*d^4*(c + d*x)^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(5*(-(b*c) + a*d)^5)
```

Maple [A] time = 0.013, size = 222, normalized size = 1.3

$$\frac{3\,d^4b}{64\,(bdx+ad)^4\left(a^2d^2-2\,abcd+b^2c^2\right)}(dx+c)^{\frac{7}{2}} + \frac{11\,d^4}{64\,(bdx+ad)^4\,(ad-bc)}(dx+c)^{\frac{5}{2}} - \frac{11\,d^4}{64\,(bdx+ad)^4\,b}(dx+c)^{\frac{3}{2}} - \frac{\ldots}{64\,(\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^5,x)
```

```
[Out] 3/64*d^4/(b*d*x+a*d)^4*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(7/2)+11/64*d^
4/(b*d*x+a*d)^4/(a*d-b*c)*(d*x+c)^(5/2)-11/64*d^4/(b*d*x+a*d)^4/b*(d*x+c)^(
3/2)-3/64*d^5/(b*d*x+a*d)^4/b^2*(d*x+c)^(1/2)*a+3/64*d^4/(b*d*x+a*d)^4/b*(d
*x+c)^(1/2)*c+3/64*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/((a*d-b*c)*b)^(1/2)*
arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15403, size = 2101, normalized size = 12.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [1/128*(3*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*
x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c -
a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(16*b^5*c^4 - 40*a*b^4*c^3*d + 26*a^2*
b^3*c^2*d^2 + a^3*b^2*c*d^3 - 3*a^4*b*d^4 - 3*(b^5*c*d^3 - a*b^4*d^4)*x^3 +
 (2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (24*b^5*c^3*d - 68
*a*b^4*c^2*d^2 + 55*a^2*b^3*c*d^3 - 11*a^3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*
b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3 + (b^10*c^3 - 3*a
*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*x^4 + 4*(a*b^9*c^3 - 3*a^2*b^8*
c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*x^3 + 6*(a^2*b^8*c^3 - 3*a^3*b^7*c^2
*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*x^2 + 4*(a^3*b^7*c^3 - 3*a^4*b^6*c^2*d
+ 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*x), 1/64*(3*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3
 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (16*b^5*c^4 - 40*a*b^4
*c^3*d + 26*a^2*b^3*c^2*d^2 + a^3*b^2*c*d^3 - 3*a^4*b*d^4 - 3*(b^5*c*d^3 -
a*b^4*d^4)*x^3 + (2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (2
4*b^5*c^3*d - 68*a*b^4*c^2*d^2 + 55*a^2*b^3*c*d^3 - 11*a^3*b^2*d^4)*x)*sqrt
(d*x + c))/(a^4*b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3 +
 (b^10*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*x^4 + 4*(a*b^9*
c^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*x^3 + 6*(a^2*b^8*c^3
 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*x^2 + 4*(a^3*b^7*c^3 -
3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12818, size = 385, normalized size = 2.24

$$\frac{3\,d^4\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{64\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)\sqrt{-b^2c+abd}}+\frac{3\,(dx+c)^{\frac{7}{2}}b^3d^4-11\,(dx+c)^{\frac{5}{2}}b^3cd^4-11\,(dx+c)^{\frac{3}{2}}b^3c^2d^4+3\,\sqrt{dx+c}b^3c^3d^4}{64\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 3/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/64*(3*(d*x + c)^(7/2)*b^3*d^4 -
 11*(d*x + c)^(5/2)*b^3*c*d^4 - 11*(d*x + c)^(3/2)*b^3*c^2*d^4 + 3*sqrt(d*x
 + c)*b^3*c^3*d^4 + 11*(d*x + c)^(5/2)*a*b^2*d^5 + 22*(d*x + c)^(3/2)*a*b^2
*c*d^5 - 9*sqrt(d*x + c)*a*b^2*c^2*d^5 - 11*(d*x + c)^(3/2)*a^2*b*d^6 + 9*s
qrt(d*x + c)*a^2*b*c*d^6 - 3*sqrt(d*x + c)*a^3*d^7)/((b^4*c^2 - 2*a*b^3*c*d
 + a^2*b^2*d^2)*((d*x + c)*b - b*c + a*d)^4)
```

3.1398 $\int \frac{(c+dx)^{3/2}}{(a+bx)^6} dx$

Optimal. Leaf size=208

$$-\frac{3d^4\sqrt{c+dx}}{128b^2(a+bx)(bc-ad)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(a+bx)^2(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{80b^2(a+bx)^3(bc-ad)}+\frac{3d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{5/2}(bc-ad)^{7/2}}-\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}$$

```
[Out] (-3*d*Sqrt[c + d*x])/(40*b^2*(a + b*x)^4) - (d^2*Sqrt[c + d*x])/(80*b^2*(b*
c - a*d)*(a + b*x)^3) + (d^3*Sqrt[c + d*x])/(64*b^2*(b*c - a*d)^2*(a + b*x)
^2) - (3*d^4*Sqrt[c + d*x])/(128*b^2*(b*c - a*d)^3*(a + b*x)) - (c + d*x)^(
3/2)/(5*b*(a + b*x)^5) + (3*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c -
a*d]])/(128*b^(5/2)*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.093295, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{3d^4\sqrt{c+dx}}{128b^2(a+bx)(bc-ad)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(a+bx)^2(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{80b^2(a+bx)^3(bc-ad)}+\frac{3d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{5/2}(bc-ad)^{7/2}}-\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^6,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(40*b^2*(a + b*x)^4) - (d^2*Sqrt[c + d*x])/(80*b^2*(b*
c - a*d)*(a + b*x)^3) + (d^3*Sqrt[c + d*x])/(64*b^2*(b*c - a*d)^2*(a + b*x)
^2) - (3*d^4*Sqrt[c + d*x])/(128*b^2*(b*c - a*d)^3*(a + b*x)) - (c + d*x)^(
3/2)/(5*b*(a + b*x)^5) + (3*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c -
a*d]])/(128*b^(5/2)*(b*c - a*d)^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^6}\,dx = -\frac{(c+dx)^{3/2}}{5b(a+bx)^5} + \frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^5}\,dx}{10b}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{(c+dx)^{3/2}}{5b(a+bx)^5} + \frac{\left(3d^2\right)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{80b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} - \frac{(c+dx)^{3/2}}{5b(a+bx)^5} - \frac{d^3\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{32b^2(bc-ad)}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2} - \frac{(c+dx)^{3/2}}{5b(a+bx)^5} + \frac{\left(3d^4\right)\int \frac{1}{(a+bx)^2\sqrt{}}}{128b^2(bc-}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2} - \frac{3d^4\sqrt{c+dx}}{128b^2(bc-ad)^3(a+bx)} - \frac{(c}{5b(}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2} - \frac{3d^4\sqrt{c+dx}}{128b^2(bc-ad)^3(a+bx)} - \frac{(c}{5b(}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2} - \frac{3d^4\sqrt{c+dx}}{128b^2(bc-ad)^3(a+bx)} - \frac{(c}{5b(}$$

Mathematica [C] time = 0.0178282, size = 52, normalized size = 0.25

$$\frac{2d^5(c+dx)^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};-\frac{b(c+dx)}{ad-bc}\right)}{5(ad-bc)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^6,x]
```

```
[Out] (2*d^5*(c + d*x)^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(5*(-(b*c) + a*d)^6)
```

Maple [A] time = 0.015, size = 300, normalized size = 1.4

$$\frac{3\,d^5b^2}{128\,(bdx+ad)^5\left(a^3d^3-3\,a^2bcd^2+3\,ab^2c^2d-b^3c^3\right)}(dx+c)^{\frac{9}{2}} + \frac{7\,d^5b}{64\,(bdx+ad)^5\left(a^2d^2-2\,abcd+b^2c^2\right)}(dx+c)^{\frac{7}{2}} + \frac{}{5\,(bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^6,x)
```

```
[Out] 3/128*d^5/(b*d*x+a*d)^5*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(
d*x+c)^(9/2)+7/64*d^5/(b*d*x+a*d)^5*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(
7/2)+1/5*d^5/(b*d*x+a*d)^5/(a*d-b*c)*(d*x+c)^(5/2)-7/64*d^5/(b*d*x+a*d)^5/b
*(d*x+c)^(3/2)-3/128*d^6/(b*d*x+a*d)^5/b^2*(d*x+c)^(1/2)*a+3/128*d^5/(b*d*x
+a*d)^5/b*(d*x+c)^(1/2)*c+3/128*d^5/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)/b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.06007, size = 3065, normalized size = 14.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b
^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*
c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(128*b^6*c^5
- 464*a*b^5*c^4*d + 584*a^2*b^4*c^3*d^2 - 258*a^3*b^3*c^2*d^3 - 5*a^4*b^2*c
*d^4 + 15*a^5*b*d^5 + 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 - 10*(b^6*c^2*d^3 - 8*
a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 87
*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(88*b^6*c^4*d - 344*a*b^5*c^3*d^2
+ 489*a^2*b^4*c^2*d^3 - 268*a^3*b^3*c*d^4 + 35*a^4*b^2*d^5)*x)*sqrt(d*x + c
))/(a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a
^9*b^3*d^4 + (b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*
d^3 + a^4*b^8*d^4)*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d
^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*
d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*x^3 + 10*(a^3*b^9*c^
4 - 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*x^
2 + 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3
+ a^8*b^4*d^4)*x), -1/640*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d
^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(-b^2*c + a*b*d)
*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (128*b^6*c^5 -
464*a*b^5*c^4*d + 584*a^2*b^4*c^3*d^2 - 258*a^3*b^3*c^2*d^3 - 5*a^4*b^2*c*d
^4 + 15*a^5*b*d^5 + 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 - 10*(b^6*c^2*d^3 - 8*a*
b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 87*a
^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(88*b^6*c^4*d - 344*a*b^5*c^3*d^2 +
489*a^2*b^4*c^2*d^3 - 268*a^3*b^3*c*d^4 + 35*a^4*b^2*d^5)*x)*sqrt(d*x + c))
/(a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9
*b^3*d^4 + (b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*d^
3 + a^4*b^8*d^4)*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d^2
 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*d
+ 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*x^3 + 10*(a^3*b^9*c^4
- 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*x^2
+ 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3 +
```

```
a^8*b^4*d^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12158, size = 554, normalized size = 2.66

$$-\frac{3\,d^5\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{128\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)\sqrt{-b^2c+abd}}-\frac{15\,(dx+c)^{\frac{9}{2}}b^4d^5-70\,(dx+c)^{\frac{7}{2}}b^4cd^5+128\,(dx+c)^{\frac{5}{2}}b^4c^2d^5-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -3/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^3 - 3*a*b^4
*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt(-b^2*c + a*b*d)) - 1/640*(15*(
d*x + c)^(9/2)*b^4*d^5 - 70*(d*x + c)^(7/2)*b^4*c*d^5 + 128*(d*x + c)^(5/2)
*b^4*c^2*d^5 + 70*(d*x + c)^(3/2)*b^4*c^3*d^5 - 15*sqrt(d*x + c)*b^4*c^4*d^
5 + 70*(d*x + c)^(7/2)*a*b^3*d^6 - 256*(d*x + c)^(5/2)*a*b^3*c*d^6 - 210*(d
*x + c)^(3/2)*a*b^3*c^2*d^6 + 60*sqrt(d*x + c)*a*b^3*c^3*d^6 + 128*(d*x + c
)^(5/2)*a^2*b^2*d^7 + 210*(d*x + c)^(3/2)*a^2*b^2*c*d^7 - 90*sqrt(d*x + c)*
a^2*b^2*c^2*d^7 - 70*(d*x + c)^(3/2)*a^3*b*d^8 + 60*sqrt(d*x + c)*a^3*b*c*d
^8 - 15*sqrt(d*x + c)*a^4*d^9)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2
- a^3*b^2*d^3)*((d*x + c)*b - b*c + a*d)^5)
```

3.1399 $\int (a+bx)^5(c+dx)^{5/2}\,dx$

Optimal. Leaf size=158

$$-\frac{2b^4(c+dx)^{15/2}(bc-ad)}{3d^6}+\frac{20b^3(c+dx)^{13/2}(bc-ad)^2}{13d^6}-\frac{20b^2(c+dx)^{11/2}(bc-ad)^3}{11d^6}+\frac{10b(c+dx)^{9/2}(bc-ad)^4}{9d^6}-\frac{2(c}{}$$

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(7/2))/(7*d^6) + (10*b*(b*c - a*d)^4*(c + d*x)^
(9/2))/(9*d^6) - (20*b^2*(b*c - a*d)^3*(c + d*x)^(11/2))/(11*d^6) + (20*b^3
*(b*c - a*d)^2*(c + d*x)^(13/2))/(13*d^6) - (2*b^4*(b*c - a*d)*(c + d*x)^(1
5/2))/(3*d^6) + (2*b^5*(c + d*x)^(17/2))/(17*d^6)
```

Rubi [A] time = 0.050514, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2b^4(c+dx)^{15/2}(bc-ad)}{3d^6}+\frac{20b^3(c+dx)^{13/2}(bc-ad)^2}{13d^6}-\frac{20b^2(c+dx)^{11/2}(bc-ad)^3}{11d^6}+\frac{10b(c+dx)^{9/2}(bc-ad)^4}{9d^6}-\frac{2(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(7/2))/(7*d^6) + (10*b*(b*c - a*d)^4*(c + d*x)^
(9/2))/(9*d^6) - (20*b^2*(b*c - a*d)^3*(c + d*x)^(11/2))/(11*d^6) + (20*b^3
*(b*c - a*d)^2*(c + d*x)^(13/2))/(13*d^6) - (2*b^4*(b*c - a*d)*(c + d*x)^(1
5/2))/(3*d^6) + (2*b^5*(c + d*x)^(17/2))/(17*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^{5/2}\,dx=\int \left(\frac{(-bc+ad)^5(c+dx)^{5/2}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{7/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{9/2}}{d^5}+\frac{10b^3(b}{}\right.$$
$$=-\frac{2(bc-ad)^5(c+dx)^{7/2}}{7d^6}+\frac{10b(bc-ad)^4(c+dx)^{9/2}}{9d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{11/2}}{11d^6}+\frac{20b^3(bc}{}$$

Mathematica [A] time = 0.110852, size = 123, normalized size = 0.78

$$\frac{2(c+dx)^{7/2}\left(-139230b^2(c+dx)^2(bc-ad)^3+117810b^3(c+dx)^3(bc-ad)^2-51051b^4(c+dx)^4(bc-ad)+85085b(c+\right.}{153153d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-21879*(b*c - a*d)^5 + 85085*b*(b*c - a*d)^4*(c + d*x)
- 139230*b^2*(b*c - a*d)^3*(c + d*x)^2 + 117810*b^3*(b*c - a*d)^2*(c + d*x)
```

```
^3 - 51051*b^4*(b*c - a*d)*(c + d*x)^4 + 9009*b^5*(c + d*x)^5))/(153153*d^6
)
```

Maple [B] time = 0.005, size = 273, normalized size = 1.7

$$\frac{18018\,b^5x^5d^5+102102\,ab^4d^5x^4-12012\,b^5cd^4x^4+235620\,a^2b^3d^5x^3-62832\,ab^4cd^4x^3+7392\,b^5c^2d^3x^3+278460\,a^3b^2d^5x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(5/2),x)
```

```
[Out] 2/153153*(d*x+c)^(7/2)*(9009*b^5*d^5*x^5+51051*a*b^4*d^5*x^4-6006*b^5*c*d^4
*x^4+117810*a^2*b^3*d^5*x^3-31416*a*b^4*c*d^4*x^3+3696*b^5*c^2*d^3*x^3+1392
30*a^3*b^2*d^5*x^2-64260*a^2*b^3*c*d^4*x^2+17136*a*b^4*c^2*d^3*x^2-2016*b^5
*c^3*d^2*x^2+85085*a^4*b*d^5*x-61880*a^3*b^2*c*d^4*x+28560*a^2*b^3*c^2*d^3*
x-7616*a*b^4*c^3*d^2*x+896*b^5*c^4*d*x+21879*a^5*d^5-24310*a^4*b*c*d^4+1768
0*a^3*b^2*c^2*d^3-8160*a^2*b^3*c^3*d^2+2176*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

Maxima [A] time = 0.961839, size = 350, normalized size = 2.22

$$\frac{2\left(9009\,(dx+c)^{\frac{17}{2}}b^5-51051\,\left(b^5c-ab^4d\right)(dx+c)^{\frac{15}{2}}+117810\,\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)(dx+c)^{\frac{13}{2}}-139230\,\left(b^5c^3-3\,ab^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/153153*(9009*(d*x + c)^(17/2)*b^5 - 51051*(b^5*c - a*b^4*d)*(d*x + c)^(15
/2) + 117810*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(13/2) - 13923
0*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(11/2
) + 85085*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4)*(d*x + c)^(9/2) - 21879*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^
3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(7/2))/d^6
```

Fricas [B] time = 1.86077, size = 1148, normalized size = 7.27

$$\frac{2\left(9009\,b^5d^8x^8-256\,b^5c^8+2176\,ab^4c^7d-8160\,a^2b^3c^6d^2+17680\,a^3b^2c^5d^3-24310\,a^4bc^4d^4+21879\,a^5c^3d^5+3003\left(7\,b^5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/153153*(9009*b^5*d^8*x^8 - 256*b^5*c^8 + 2176*a*b^4*c^7*d - 8160*a^2*b^3*
c^6*d^2 + 17680*a^3*b^2*c^5*d^3 - 24310*a^4*b*c^4*d^4 + 21879*a^5*c^3*d^5 +
 3003*(7*b^5*c*d^7 + 17*a*b^4*d^8)*x^7 + 231*(55*b^5*c^2*d^6 + 527*a*b^4*c*
d^7 + 510*a^2*b^3*d^8)*x^6 + 63*(b^5*c^3*d^5 + 1207*a*b^4*c^2*d^6 + 4590*a^
2*b^3*c*d^7 + 2210*a^3*b^2*d^8)*x^5 - 35*(2*b^5*c^4*d^4 - 17*a*b^4*c^3*d^5
- 5406*a^2*b^3*c^2*d^6 - 10166*a^3*b^2*c*d^7 - 2431*a^4*b*d^8)*x^4 + (80*b^
5*c^5*d^3 - 680*a*b^4*c^4*d^4 + 2550*a^2*b^3*c^3*d^5 + 249730*a^3*b^2*c^2*d
^6 + 230945*a^4*b*c*d^7 + 21879*a^5*d^8)*x^3 - 3*(32*b^5*c^6*d^2 - 272*a*b^
```

```
4*c^5*d^3 + 1020*a^2*b^3*c^4*d^4 - 2210*a^3*b^2*c^3*d^5 - 60775*a^4*b*c^2*d
^6 - 21879*a^5*c*d^7)*x^2 + (128*b^5*c^7*d - 1088*a*b^4*c^6*d^2 + 4080*a^2*
b^3*c^5*d^3 - 8840*a^3*b^2*c^4*d^4 + 12155*a^4*b*c^3*d^5 + 65637*a^5*c^2*d^
6)*x)*sqrt(d*x + c)/d^6
```

Sympy [A] time = 32.8654, size = 1292, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(5/2),x)
```

```
[Out] a**5*c**2*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)
) + 4*a**5*c*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*a**5*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d + 10*a*
*4*b*c**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 20*a**4*b*c*(
c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2
 + 10*a**4*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c
 + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**2 + 20*a**3*b**2*c**2*(c**2*(c +
d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 40*a**3
*b**2*c*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*
x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 20*a**3*b**2*(c**4*(c + d*x)**(3/2
)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)
**(9/2)/9 + (c + d*x)**(11/2)/11)/d**3 + 20*a**2*b**3*c**2*(-c**3*(c + d*x)
**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)*
*(9/2)/9)/d**4 + 40*a**2*b**3*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)
**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)*
*(11/2)/11)/d**4 + 20*a**2*b**3*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)*
*(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c +
 d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**4 + 10*a*b**4*c**2*(c**4*(c + d
*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*
(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11)/d**5 + 20*a*b**4*c*(-c**5*(c + d
*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2
*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5
 + 10*a*b**4*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4
*(c + d*x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2
)/11 - 6*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**5 + 2*b**5*c**2*
(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2
)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(1
3/2)/13)/d**6 + 4*b**5*c*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)
/5 + 15*c**4*(c + d*x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c +
 d*x)**(11/2)/11 - 6*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**6 +
2*b**5*(-c**7*(c + d*x)**(3/2)/3 + 7*c**6*(c + d*x)**(5/2)/5 - 3*c**5*(c +
d*x)**(7/2) + 35*c**4*(c + d*x)**(9/2)/9 - 35*c**3*(c + d*x)**(11/2)/11 + 2
1*c**2*(c + d*x)**(13/2)/13 - 7*c*(c + d*x)**(15/2)/15 + (c + d*x)**(17/2)/
17)/d**6
```

Giac [B] time = 1.14182, size = 1469, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/765765*(255255*(d*x + c)^(3/2)*a^5*c^2 + 102102*(3*(d*x + c)^(5/2) - 5*(d
*x + c)^(3/2)*c)*a^5*c + 255255*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c)*a
^4*b*c^2/d + 7293*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)
^(3/2)*c^2)*a^5 + 72930*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*
x + c)^(3/2)*c^2)*a^3*b^2*c^2/d^2 + 72930*(15*(d*x + c)^(7/2) - 42*(d*x + c
)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)*a^4*b*c/d + 24310*(35*(d*x + c)^(9/2) -
 135*(d*x + c)^(7/2)*c + 189*(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)
*a^2*b^3*c^2/d^3 + 48620*(35*(d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c + 189*
(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)*a^3*b^2*c/d^2 + 12155*(35*(d
*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c + 189*(d*x + c)^(5/2)*c^2 - 105*(d*x
+ c)^(3/2)*c^3)*a^4*b/d + 1105*(315*(d*x + c)^(11/2) - 1540*(d*x + c)^(9/2)
*c + 2970*(d*x + c)^(7/2)*c^2 - 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(
3/2)*c^4)*a*b^4*c^2/d^4 + 4420*(315*(d*x + c)^(11/2) - 1540*(d*x + c)^(9/2)
*c + 2970*(d*x + c)^(7/2)*c^2 - 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(
3/2)*c^4)*a^2*b^3*c/d^3 + 2210*(315*(d*x + c)^(11/2) - 1540*(d*x + c)^(9/2)
*c + 2970*(d*x + c)^(7/2)*c^2 - 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(
3/2)*c^4)*a^3*b^2/d^2 + 85*(693*(d*x + c)^(13/2) - 4095*(d*x + c)^(11/2)*c
+ 10010*(d*x + c)^(9/2)*c^2 - 12870*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5
/2)*c^4 - 3003*(d*x + c)^(3/2)*c^5)*b^5*c^2/d^5 + 850*(693*(d*x + c)^(13/2)
 - 4095*(d*x + c)^(11/2)*c + 10010*(d*x + c)^(9/2)*c^2 - 12870*(d*x + c)^(7
/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 3003*(d*x + c)^(3/2)*c^5)*a*b^4*c/d^4
+ 850*(693*(d*x + c)^(13/2) - 4095*(d*x + c)^(11/2)*c + 10010*(d*x + c)^(9/
2)*c^2 - 12870*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 3003*(d*x +
 c)^(3/2)*c^5)*a^2*b^3/d^3 + 34*(3003*(d*x + c)^(15/2) - 20790*(d*x + c)^(1
3/2)*c + 61425*(d*x + c)^(11/2)*c^2 - 100100*(d*x + c)^(9/2)*c^3 + 96525*(d
*x + c)^(7/2)*c^4 - 54054*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6)*
b^5*c/d^5 + 85*(3003*(d*x + c)^(15/2) - 20790*(d*x + c)^(13/2)*c + 61425*(d
*x + c)^(11/2)*c^2 - 100100*(d*x + c)^(9/2)*c^3 + 96525*(d*x + c)^(7/2)*c^4
 - 54054*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6)*a*b^4/d^4 + 7*(64
35*(d*x + c)^(17/2) - 51051*(d*x + c)^(15/2)*c + 176715*(d*x + c)^(13/2)*c^
2 - 348075*(d*x + c)^(11/2)*c^3 + 425425*(d*x + c)^(9/2)*c^4 - 328185*(d*x
+ c)^(7/2)*c^5 + 153153*(d*x + c)^(5/2)*c^6 - 36465*(d*x + c)^(3/2)*c^7)*b^
5/d^5)/d
```

3.1400 $\int (a+bx)^4(c+dx)^{5/2}\,dx$

Optimal. Leaf size=129

$$-\frac{8b^3(c+dx)^{13/2}(bc-ad)}{13d^5}+\frac{12b^2(c+dx)^{11/2}(bc-ad)^2}{11d^5}-\frac{8b(c+dx)^{9/2}(bc-ad)^3}{9d^5}+\frac{2(c+dx)^{7/2}(bc-ad)^4}{7d^5}+\frac{2b^4(c+d}{15d^5}$$

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(7/2))/(7*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(9
/2))/(9*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(11/2))/(11*d^5) - (8*b^3*(b
*c - a*d)*(c + d*x)^(13/2))/(13*d^5) + (2*b^4*(c + d*x)^(15/2))/(15*d^5)
```

Rubi [A] time = 0.0427892, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{13/2}(bc-ad)}{13d^5}+\frac{12b^2(c+dx)^{11/2}(bc-ad)^2}{11d^5}-\frac{8b(c+dx)^{9/2}(bc-ad)^3}{9d^5}+\frac{2(c+dx)^{7/2}(bc-ad)^4}{7d^5}+\frac{2b^4(c+d}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(7/2))/(7*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(9
/2))/(9*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(11/2))/(11*d^5) - (8*b^3*(b
*c - a*d)*(c + d*x)^(13/2))/(13*d^5) + (2*b^4*(c + d*x)^(15/2))/(15*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)^4(c+dx)^{5/2}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{7/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{9/2}}{d^4}-\frac{4b^3(bc-\right.\\ &= \frac{2(bc-ad)^4(c+dx)^{7/2}}{7d^5}-\frac{8b(bc-ad)^3(c+dx)^{9/2}}{9d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{11/2}}{11d^5}-\frac{8b^3(bc-a}{1}\end{aligned}$$

Mathematica [A] time = 0.127086, size = 101, normalized size = 0.78

$$\frac{2(c+dx)^{7/2}\left(24570b^2(c+dx)^2(bc-ad)^2-13860b^3(c+dx)^3(bc-ad)-20020b(c+dx)(bc-ad)^3+6435(bc-ad)^4+\right.}{45045d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(6435*(b*c - a*d)^4 - 20020*b*(b*c - a*d)^3*(c + d*x) +
24570*b^2*(b*c - a*d)^2*(c + d*x)^2 - 13860*b^3*(b*c - a*d)*(c + d*x)^3 + 3
003*b^4*(c + d*x)^4))/(45045*d^5)
```

Maple [A] time = 0.005, size = 186, normalized size = 1.4

$$\frac{6006\,b^4x^4d^4+27720\,ab^3d^4x^3-3696\,b^4cd^3x^3+49140\,a^2b^2d^4x^2-15120\,ab^3cd^3x^2+2016\,b^4c^2d^2x^2+40040\,a^3bd^4x-218}{45045\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(5/2),x)
```

```
[Out] 2/45045*(d*x+c)^(7/2)*(3003*b^4*d^4*x^4+13860*a*b^3*d^4*x^3-1848*b^4*c*d^3*
x^3+24570*a^2*b^2*d^4*x^2-7560*a*b^3*c*d^3*x^2+1008*b^4*c^2*d^2*x^2+20020*a
^3*b*d^4*x-10920*a^2*b^2*c*d^3*x+3360*a*b^3*c^2*d^2*x-448*b^4*c^3*d*x+6435*
a^4*d^4-5720*a^3*b*c*d^3+3120*a^2*b^2*c^2*d^2-960*a*b^3*c^3*d+128*b^4*c^4)/
d^5
```

Maxima [A] time = 0.967286, size = 244, normalized size = 1.89

$$\frac{2\left(3003\,(dx+c)^{\frac{15}{2}}b^4-13860\left(b^4c-ab^3d\right)(dx+c)^{\frac{13}{2}}+24570\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)(dx+c)^{\frac{11}{2}}-20020\left(b^4c^3-3\,ab^3c^2\right.}{45045\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(d*x + c)^(15/2)*b^4 - 13860*(b^4*c - a*b^3*d)*(d*x + c)^(13/
2) + 24570*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(11/2) - 20020*(
b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(9/2) + 64
35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
(d*x + c)^(7/2))/d^5
```

Fricas [B] time = 1.94254, size = 855, normalized size = 6.63

$$2\left(3003\,b^4d^7x^7+128\,b^4c^7-960\,ab^3c^6d+3120\,a^2b^2c^5d^2-5720\,a^3bc^4d^3+6435\,a^4c^3d^4+231\left(31\,b^4cd^6+60\,ab^3d^7\right)x^6+6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^4*d^7*x^7 + 128*b^4*c^7 - 960*a*b^3*c^6*d + 3120*a^2*b^2*c^
5*d^2 - 5720*a^3*b*c^4*d^3 + 6435*a^4*c^3*d^4 + 231*(31*b^4*c*d^6 + 60*a*b^
3*d^7)*x^6 + 63*(71*b^4*c^2*d^5 + 540*a*b^3*c*d^6 + 390*a^2*b^2*d^7)*x^5 +
35*(b^4*c^3*d^4 + 636*a*b^3*c^2*d^5 + 1794*a^2*b^2*c*d^6 + 572*a^3*b*d^7)*x
^4 - 5*(8*b^4*c^4*d^3 - 60*a*b^3*c^3*d^4 - 8814*a^2*b^2*c^2*d^5 - 10868*a^3
*b*c*d^6 - 1287*a^4*d^7)*x^3 + 3*(16*b^4*c^5*d^2 - 120*a*b^3*c^4*d^3 + 390*
a^2*b^2*c^3*d^4 + 14300*a^3*b*c^2*d^5 + 6435*a^4*c*d^6)*x^2 - (64*b^4*c^6*d
 - 480*a*b^3*c^5*d^2 + 1560*a^2*b^2*c^4*d^3 - 2860*a^3*b*c^3*d^4 - 19305*a^
4*c^2*d^5)*x)*sqrt(d*x + c)/d^5
```

Sympy [A] time = 25.1616, size = 960, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(5/2),x)
```

```
[Out] a**4*c**2*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)
) + 4*a**4*c*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*a**4*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d + 8*a**
3*b*c**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 16*a**3*b*c*(c
**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2
+ 8*a**3*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c +
 d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**2 + 12*a**2*b**2*c**2*(c**2*(c + d*
x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 24*a**2*b
**2*c*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)
**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 12*a**2*b**2*(c**4*(c + d*x)**(3/2)/
3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**
(9/2)/9 + (c + d*x)**(11/2)/11)/d**3 + 8*a*b**3*c**2*(-c**3*(c + d*x)**(3/2
)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)
/9)/d**4 + 16*a*b**3*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5
 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/1
1)/d**4 + 8*a*b**3*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c
**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)
/11 + (c + d*x)**(13/2)/13)/d**4 + 2*b**4*c**2*(c**4*(c + d*x)**(3/2)/3 - 4
*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)
/9 + (c + d*x)**(11/2)/11)/d**5 + 4*b**4*c*(-c**5*(c + d*x)**(3/2)/3 + c**4
*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9
 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5 + 2*b**4*(c**6*(c
+ d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4*(c + d*x)**(7/2)/7 -
20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2)/11 - 6*c*(c + d*x)**
(13/2)/13 + (c + d*x)**(15/2)/15)/d**5
```

Giac [B] time = 1.1228, size = 1094, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/45045*(15015*(d*x + c)^(3/2)*a^4*c^2 + 6006*(3*(d*x + c)^(5/2) - 5*(d*x +
 c)^(3/2)*c)*a^4*c + 12012*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c)*a^3*b*
c^2/d + 429*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)
*c^2)*a^4 + 2574*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^
(3/2)*c^2)*a^2*b^2*c^2/d^2 + 3432*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*
c + 35*(d*x + c)^(3/2)*c^2)*a^3*b*c/d + 572*(35*(d*x + c)^(9/2) - 135*(d*x
+ c)^(7/2)*c + 189*(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)*a*b^3*c^2
/d^3 + 1716*(35*(d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c + 189*(d*x + c)^(5/
2)*c^2 - 105*(d*x + c)^(3/2)*c^3)*a^2*b^2*c/d^2 + 572*(35*(d*x + c)^(9/2) -
 135*(d*x + c)^(7/2)*c + 189*(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)
*a^3*b/d + 13*(315*(d*x + c)^(11/2) - 1540*(d*x + c)^(9/2)*c + 2970*(d*x +
c)^(7/2)*c^2 - 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4)*b^4*c^2
/d^4 + 104*(315*(d*x + c)^(11/2) - 1540*(d*x + c)^(9/2)*c + 2970*(d*x + c)^
(7/2)*c^2 - 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4)*a*b^3*c/d^
```

```
3 + 78*(315*(d*x + c)^(11/2) - 1540*(d*x + c)^(9/2)*c + 2970*(d*x + c)^(7/2
)*c^2 - 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4)*a^2*b^2/d^2 +
10*(693*(d*x + c)^(13/2) - 4095*(d*x + c)^(11/2)*c + 10010*(d*x + c)^(9/2)*
c^2 - 12870*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 3003*(d*x + c)
^(3/2)*c^5)*b^4*c/d^4 + 20*(693*(d*x + c)^(13/2) - 4095*(d*x + c)^(11/2)*c
+ 10010*(d*x + c)^(9/2)*c^2 - 12870*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5
/2)*c^4 - 3003*(d*x + c)^(3/2)*c^5)*a*b^3/d^3 + (3003*(d*x + c)^(15/2) - 20
790*(d*x + c)^(13/2)*c + 61425*(d*x + c)^(11/2)*c^2 - 100100*(d*x + c)^(9/2
)*c^3 + 96525*(d*x + c)^(7/2)*c^4 - 54054*(d*x + c)^(5/2)*c^5 + 15015*(d*x
+ c)^(3/2)*c^6)*b^4/d^4)/d
```

3.1401 $\int (a+bx)^3(c+dx)^{5/2}\,dx$

Optimal. Leaf size=100

$$-\frac{6b^2(c+dx)^{11/2}(bc-ad)}{11d^4}+\frac{2b(c+dx)^{9/2}(bc-ad)^2}{3d^4}-\frac{2(c+dx)^{7/2}(bc-ad)^3}{7d^4}+\frac{2b^3(c+dx)^{13/2}}{13d^4}$$

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(7/2))/(7*d^4) + (2*b*(b*c - a*d)^2*(c + d*x)^(
9/2))/(3*d^4) - (6*b^2*(b*c - a*d)*(c + d*x)^(11/2))/(11*d^4) + (2*b^3*(c +
 d*x)^(13/2))/(13*d^4)
```

Rubi [A] time = 0.0312856, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{6b^2(c+dx)^{11/2}(bc-ad)}{11d^4}+\frac{2b(c+dx)^{9/2}(bc-ad)^2}{3d^4}-\frac{2(c+dx)^{7/2}(bc-ad)^3}{7d^4}+\frac{2b^3(c+dx)^{13/2}}{13d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(7/2))/(7*d^4) + (2*b*(b*c - a*d)^2*(c + d*x)^(
9/2))/(3*d^4) - (6*b^2*(b*c - a*d)*(c + d*x)^(11/2))/(11*d^4) + (2*b^3*(c +
 d*x)^(13/2))/(13*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3(c+dx)^{5/2}\,dx=\int \left(\frac{(-bc+ad)^3(c+dx)^{5/2}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{7/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{9/2}}{d^3}+\frac{b^3(c+d}{d^3}\right.$$
$$=-\frac{2(bc-ad)^3(c+dx)^{7/2}}{7d^4}+\frac{2b(bc-ad)^2(c+dx)^{9/2}}{3d^4}-\frac{6b^2(bc-ad)(c+dx)^{11/2}}{11d^4}+\frac{2b^3(c+dx}{13d^4}$$

Mathematica [A] time = 0.069674, size = 79, normalized size = 0.79

$$\frac{2(c+dx)^{7/2}\left(-819b^2(c+dx)^2(bc-ad)+1001b(c+dx)(bc-ad)^2-429(bc-ad)^3+231b^3(c+dx)^3\right)}{3003d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-429*(b*c - a*d)^3 + 1001*b*(b*c - a*d)^2*(c + d*x) - 8
19*b^2*(b*c - a*d)*(c + d*x)^2 + 231*b^3*(c + d*x)^3))/(3003*d^4)
```

Maple [A] time = 0.005, size = 116, normalized size = 1.2

$$\frac{462\,b^3x^3d^3+1638\,ab^2d^3x^2-252\,b^3cd^2x^2+2002\,a^2bd^3x-728\,ab^2cd^2x+112\,b^3c^2dx+858\,a^3d^3-572\,a^2bcd^2+208\,ab^2c}{3003\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(5/2),x)
```

```
[Out] 2/3003*(d*x+c)^(7/2)*(231*b^3*d^3*x^3+819*a*b^2*d^3*x^2-126*b^3*c*d^2*x^2+1
001*a^2*b*d^3*x-364*a*b^2*c*d^2*x+56*b^3*c^2*d*x+429*a^3*d^3-286*a^2*b*c*d^
2+104*a*b^2*c^2*d-16*b^3*c^3)/d^4
```

Maxima [A] time = 0.965, size = 159, normalized size = 1.59

$$\frac{2\left(231\,(dx+c)^{\frac{13}{2}}b^3-819\left(b^3c-ab^2d\right)(dx+c)^{\frac{11}{2}}+1001\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)(dx+c)^{\frac{9}{2}}-429\left(b^3c^3-3\,ab^2c^2d+3\,a^2b\right.}{3003\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3003*(231*(d*x + c)^(13/2)*b^3 - 819*(b^3*c - a*b^2*d)*(d*x + c)^(11/2) +
 1001*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(9/2) - 429*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(7/2))/d^4
```

Fricas [B] time = 1.77024, size = 595, normalized size = 5.95

$$\frac{2\left(231\,b^3d^6x^6-16\,b^3c^6+104\,ab^2c^5d-286\,a^2bc^4d^2+429\,a^3c^3d^3+63\left(9\,b^3cd^5+13\,ab^2d^6\right)x^5+7\left(53\,b^3c^2d^4+299\,ab^2cd^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^3*d^6*x^6 - 16*b^3*c^6 + 104*a*b^2*c^5*d - 286*a^2*b*c^4*d^2
+ 429*a^3*c^3*d^3 + 63*(9*b^3*c*d^5 + 13*a*b^2*d^6)*x^5 + 7*(53*b^3*c^2*d^4
 + 299*a*b^2*c*d^5 + 143*a^2*b*d^6)*x^4 + (5*b^3*c^3*d^3 + 1469*a*b^2*c^2*d
^4 + 2717*a^2*b*c*d^5 + 429*a^3*d^6)*x^3 - 3*(2*b^3*c^4*d^2 - 13*a*b^2*c^3*
d^3 - 715*a^2*b*c^2*d^4 - 429*a^3*c*d^5)*x^2 + (8*b^3*c^5*d - 52*a*b^2*c^4*
d^2 + 143*a^2*b*c^3*d^3 + 1287*a^3*c^2*d^4)*x)*sqrt(d*x + c)/d^4
```

Sympy [A] time = 4.37279, size = 549, normalized size = 5.49

$$\begin{cases}\frac{2a^3c^3\sqrt{c+dx}}{7d}+\frac{6a^3c^2x\sqrt{c+dx}}{7}+\frac{6a^3cdx^2\sqrt{c+dx}}{7}+\frac{2a^3d^2x^3\sqrt{c+dx}}{7}-\frac{4a^2bc^4\sqrt{c+dx}}{21d^2}+\frac{2a^2bc^3x\sqrt{c+dx}}{21d}+\frac{10a^2bc^2x^2\sqrt{c+dx}}{7}+\frac{38a^2bcdx^3\sqrt{c+dx}}{21}+\frac{2a^2bc}{} \\ c^{\frac{5}{2}}\left(a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a**3*c**3*sqrt(c + d*x)/(7*d) + 6*a**3*c**2*x*sqrt(c + d*x)/7
+ 6*a**3*c*d*x**2*sqrt(c + d*x)/7 + 2*a**3*d**2*x**3*sqrt(c + d*x)/7 - 4*a*
*2*b*c**4*sqrt(c + d*x)/(21*d**2) + 2*a**2*b*c**3*x*sqrt(c + d*x)/(21*d) +
10*a**2*b*c**2*x**2*sqrt(c + d*x)/7 + 38*a**2*b*c*d*x**3*sqrt(c + d*x)/21 +
 2*a**2*b*d**2*x**4*sqrt(c + d*x)/3 + 16*a*b**2*c**5*sqrt(c + d*x)/(231*d**
3) - 8*a*b**2*c**4*x*sqrt(c + d*x)/(231*d**2) + 2*a*b**2*c**3*x**2*sqrt(c +
 d*x)/(77*d) + 226*a*b**2*c**2*x**3*sqrt(c + d*x)/231 + 46*a*b**2*c*d*x**4*
sqrt(c + d*x)/33 + 6*a*b**2*d**2*x**5*sqrt(c + d*x)/11 - 32*b**3*c**6*sqrt(
c + d*x)/(3003*d**4) + 16*b**3*c**5*x*sqrt(c + d*x)/(3003*d**3) - 4*b**3*c*
*4*x**2*sqrt(c + d*x)/(1001*d**2) + 10*b**3*c**3*x**3*sqrt(c + d*x)/(3003*d
) + 106*b**3*c**2*x**4*sqrt(c + d*x)/429 + 54*b**3*c*d*x**5*sqrt(c + d*x)/1
43 + 2*b**3*d**2*x**6*sqrt(c + d*x)/13, Ne(d, 0)), (c**(5/2)*(a**3*x + 3*a*
*2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4), True))
```

Giac [B] time = 1.08548, size = 770, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/45045*(15015*(d*x + c)^(3/2)*a^3*c^2 + 6006*(3*(d*x + c)^(5/2) - 5*(d*x +
 c)^(3/2)*c)*a^3*c + 9009*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c)*a^2*b*c
^2/d + 429*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*
c^2)*a^3 + 1287*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(
3/2)*c^2)*a*b^2*c^2/d^2 + 2574*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c +
 35*(d*x + c)^(3/2)*c^2)*a^2*b*c/d + 143*(35*(d*x + c)^(9/2) - 135*(d*x + c
)^(7/2)*c + 189*(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)*b^3*c^2/d^3
+ 858*(35*(d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c + 189*(d*x + c)^(5/2)*c^2
 - 105*(d*x + c)^(3/2)*c^3)*a*b^2*c/d^2 + 429*(35*(d*x + c)^(9/2) - 135*(d*
x + c)^(7/2)*c + 189*(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)*a^2*b/d
 + 26*(315*(d*x + c)^(11/2) - 1540*(d*x + c)^(9/2)*c + 2970*(d*x + c)^(7/2)
*c^2 - 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4)*b^3*c/d^3 + 39*
(315*(d*x + c)^(11/2) - 1540*(d*x + c)^(9/2)*c + 2970*(d*x + c)^(7/2)*c^2 -
 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4)*a*b^2/d^2 + 5*(693*(d
*x + c)^(13/2) - 4095*(d*x + c)^(11/2)*c + 10010*(d*x + c)^(9/2)*c^2 - 1287
0*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 3003*(d*x + c)^(3/2)*c^5
)*b^3/d^3)/d
```

3.1402 $\int (a+bx)^2(c+dx)^{5/2}\,dx$

Optimal. Leaf size=71

$$-\frac{4b(c+dx)^{9/2}(bc-ad)}{9d^3}+\frac{2(c+dx)^{7/2}(bc-ad)^2}{7d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}$$

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(7/2))/(7*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(9/2
))/(9*d^3) + (2*b^2*(c + d*x)^(11/2))/(11*d^3)
```

Rubi [A] time = 0.0229477, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b(c+dx)^{9/2}(bc-ad)}{9d^3}+\frac{2(c+dx)^{7/2}(bc-ad)^2}{7d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(7/2))/(7*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(9/2
))/(9*d^3) + (2*b^2*(c + d*x)^(11/2))/(11*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{5/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{7/2}}{d^2}+\frac{b^2(c+dx)^{9/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2(c+dx)^{7/2}}{7d^3}-\frac{4b(bc-ad)(c+dx)^{9/2}}{9d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}\end{aligned}$$

Mathematica [A] time = 0.0553781, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{7/2}\left(99a^2d^2+22abd(7dx-2c)+b^2\left(8c^2-28cdx+63d^2x^2\right)\right)}{693d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(99*a^2*d^2 + 22*a*b*d*(-2*c + 7*d*x) + b^2*(8*c^2 - 28*
c*d*x + 63*d^2*x^2)))/(693*d^3)
```

Maple [A] time = 0.006, size = 63, normalized size = 0.9

$$\frac{126\,b^2x^2d^2+308\,abd^2x-56\,b^2cdx+198\,a^2d^2-88\,abcd+16\,b^2c^2}{693\,d^3}\,(dx+c)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(5/2),x)
```

```
[Out] 2/693*(d*x+c)^(7/2)*(63*b^2*d^2*x^2+154*a*b*d^2*x-28*b^2*c*d*x+99*a^2*d^2-4
4*a*b*c*d+8*b^2*c^2)/d^3
```

Maxima [A] time = 0.952526, size = 92, normalized size = 1.3

$$\frac{2\left(63\,(dx+c)^{\frac{11}{2}}b^2-154\left(b^2c-abd\right)(dx+c)^{\frac{9}{2}}+99\left(b^2c^2-2\,abcd+a^2d^2\right)(dx+c)^{\frac{7}{2}}\right)}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*(d*x + c)^(11/2)*b^2 - 154*(b^2*c - a*b*d)*(d*x + c)^(9/2) + 99*(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(7/2))/d^3
```

Fricas [B] time = 1.84682, size = 382, normalized size = 5.38

$$\frac{2\left(63\,b^2d^5x^5+8\,b^2c^5-44\,abc^4d+99\,a^2c^3d^2+7\left(23\,b^2cd^4+22\,abd^5\right)x^4+\left(113\,b^2c^2d^3+418\,abcd^4+99\,a^2d^5\right)x^3+3\,(b}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^2*d^5*x^5 + 8*b^2*c^5 - 44*a*b*c^4*d + 99*a^2*c^3*d^2 + 7*(23*b
^2*c*d^4 + 22*a*b*d^5)*x^4 + (113*b^2*c^2*d^3 + 418*a*b*c*d^4 + 99*a^2*d^5)
*x^3 + 3*(b^2*c^3*d^2 + 110*a*b*c^2*d^3 + 99*a^2*c*d^4)*x^2 - (4*b^2*c^4*d
- 22*a*b*c^3*d^2 - 297*a^2*c^2*d^3)*x)*sqrt(d*x + c)/d^3
```

Sympy [A] time = 3.21486, size = 355, normalized size = 5.

$$\begin{cases}\frac{2a^2c^3\sqrt{c+dx}}{7d}+\frac{6a^2c^2x\sqrt{c+dx}}{7}+\frac{6a^2cdx^2\sqrt{c+dx}}{7}+\frac{2a^2d^2x^3\sqrt{c+dx}}{7}-\frac{8abc^4\sqrt{c+dx}}{63d^2}+\frac{4abc^3x\sqrt{c+dx}}{63d}+\frac{20abc^2x^2\sqrt{c+dx}}{21}+\frac{76abcdx^3\sqrt{c+dx}}{63}+\frac{4abd^2 \\ c^{\frac{5}{2}}\left(a^2x+abx^2+\frac{b^2x^3}{3}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a**2*c**3*sqrt(c + d*x)/(7*d) + 6*a**2*c**2*x*sqrt(c + d*x)/7
+ 6*a**2*c*d*x**2*sqrt(c + d*x)/7 + 2*a**2*d**2*x**3*sqrt(c + d*x)/7 - 8*a*
b*c**4*sqrt(c + d*x)/(63*d**2) + 4*a*b*c**3*x*sqrt(c + d*x)/(63*d) + 20*a*b
```

```
*c**2*x**2*sqrt(c + d*x)/21 + 76*a*b*c*d*x**3*sqrt(c + d*x)/63 + 4*a*b*d**2
*x**4*sqrt(c + d*x)/9 + 16*b**2*c**5*sqrt(c + d*x)/(693*d**3) - 8*b**2*c**4
*x*sqrt(c + d*x)/(693*d**2) + 2*b**2*c**3*x**2*sqrt(c + d*x)/(231*d) + 226*
b**2*c**2*x**3*sqrt(c + d*x)/693 + 46*b**2*c*d*x**4*sqrt(c + d*x)/99 + 2*b*
*2*d**2*x**5*sqrt(c + d*x)/11, Ne(d, 0)), (c**(5/2)*(a**2*x + a*b*x**2 + b*
*2*x**3/3), True))
```

Giac [B] time = 1.09028, size = 491, normalized size = 6.92

$$2\left(1155\,(dx+c)^{\frac{3}{2}}a^2c^2+462\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)a^2c+\frac{462\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)abc^2}{d}+33\left(15\,(dx+c)^{\frac{7}{2}}-42\,(dx+c)^{\frac{5}{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3465*(1155*(d*x + c)^(3/2)*a^2*c^2 + 462*(3*(d*x + c)^(5/2) - 5*(d*x + c)
^(3/2)*c)*a^2*c + 462*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c)*a*b*c^2/d +
 33*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)*a^
2 + 33*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)
*b^2*c^2/d^2 + 132*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c
)^(3/2)*c^2)*a*b*c/d + 22*(35*(d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c + 189
*(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)*b^2*c/d^2 + 22*(35*(d*x + c
)^(9/2) - 135*(d*x + c)^(7/2)*c + 189*(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(
3/2)*c^3)*a*b/d + (315*(d*x + c)^(11/2) - 1540*(d*x + c)^(9/2)*c + 2970*(d*
x + c)^(7/2)*c^2 - 2772*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4)*b^2
/d^2)/d
```

3.1403 $\int (a+bx)(c+dx)^{5/2}\,dx$

Optimal. Leaf size=42

$$\frac{2b(c+dx)^{9/2}}{9d^2}-\frac{2(c+dx)^{7/2}(bc-ad)}{7d^2}$$

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^2) + (2*b*(c + d*x)^(9/2))/(9*d^2)
```

Rubi [A] time = 0.0148131, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(c+dx)^{9/2}}{9d^2}-\frac{2(c+dx)^{7/2}(bc-ad)}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^2) + (2*b*(c + d*x)^(9/2))/(9*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^{5/2}}{d}+\frac{b(c+dx)^{7/2}}{d}\right)dx\\ &= -\frac{2(bc-ad)(c+dx)^{7/2}}{7d^2}+\frac{2b(c+dx)^{9/2}}{9d^2}\end{aligned}$$

Mathematica [A] time = 0.0224357, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{7/2}(9ad-2bc+7bdx)}{63d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-2*b*c + 9*a*d + 7*b*d*x))/(63*d^2)
```

Maple [A] time = 0.002, size = 27, normalized size = 0.6

$$\frac{14\,bdx+18\,ad-4\,bc}{63\,d^2}(dx+c)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(5/2),x)
```

```
[Out] 2/63*(d*x+c)^(7/2)*(7*b*d*x+9*a*d-2*b*c)/d^2
```

Maxima [A] time = 0.949677, size = 45, normalized size = 1.07

$$\frac{2\left(7\,(dx+c)^{\frac{9}{2}}b-9\,(bc-ad)(dx+c)^{\frac{7}{2}}\right)}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/63*(7*(d*x + c)^(9/2)*b - 9*(b*c - a*d)*(d*x + c)^(7/2))/d^2
```

Fricas [B] time = 1.75748, size = 205, normalized size = 4.88

$$\frac{2\left(7\,bd^4x^4-2\,bc^4+9\,ac^3d+\left(19\,bcd^3+9\,ad^4\right)x^3+3\left(5\,bc^2d^2+9\,acd^3\right)x^2+\left(bc^3d+27\,ac^2d^2\right)x\right)\sqrt{dx+c}}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b*d^4*x^4 - 2*b*c^4 + 9*a*c^3*d + (19*b*c*d^3 + 9*a*d^4)*x^3 + 3*(5
*b*c^2*d^2 + 9*a*c*d^3)*x^2 + (b*c^3*d + 27*a*c^2*d^2)*x)*sqrt(d*x + c)/d^2
```

Sympy [A] time = 2.17768, size = 194, normalized size = 4.62

$$\begin{cases}\frac{2ac^3\sqrt{c+dx}}{7d}+\frac{6ac^2x\sqrt{c+dx}}{7}+\frac{6acdx^2\sqrt{c+dx}}{7}+\frac{2ad^2x^3\sqrt{c+dx}}{7}-\frac{4bc^4\sqrt{c+dx}}{63d^2}+\frac{2bc^3x\sqrt{c+dx}}{63d}+\frac{10bc^2x^2\sqrt{c+dx}}{21}+\frac{38bcdx^3\sqrt{c+dx}}{63}+\frac{2bd^2x^4\sqrt{c+dx}}{9} & \text{f} \\ c^{\frac{5}{2}}\left(ax+\frac{bx^2}{2}\right) & \text{c}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a*c**3*sqrt(c + d*x)/(7*d) + 6*a*c**2*x*sqrt(c + d*x)/7 + 6*a*
c*d*x**2*sqrt(c + d*x)/7 + 2*a*d**2*x**3*sqrt(c + d*x)/7 - 4*b*c**4*sqrt(c
+ d*x)/(63*d**2) + 2*b*c**3*x*sqrt(c + d*x)/(63*d) + 10*b*c**2*x**2*sqrt(c
+ d*x)/21 + 38*b*c*d*x**3*sqrt(c + d*x)/63 + 2*b*d**2*x**4*sqrt(c + d*x)/9,
 Ne(d, 0)), (c**(5/2)*(a*x + b*x**2/2), True))
```

Giac [B] time = 1.07119, size = 263, normalized size = 6.26

$$\frac{2\left(105\,(dx+c)^{\frac{3}{2}}ac^2+42\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)ac+\frac{21\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)bc^2}{d}+3\left(15\,(dx+c)^{\frac{7}{2}}-42\,(dx+c)^{\frac{5}{2}}c+35\right.\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/315*(105*(d*x + c)^(3/2)*a*c^2 + 42*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2
)*c)*a*c + 21*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c)*b*c^2/d + 3*(15*(d*
x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)*a + 6*(15*(d*
x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)*b*c/d + (35*(
d*x + c)^(9/2) - 135*(d*x + c)^(7/2)*c + 189*(d*x + c)^(5/2)*c^2 - 105*(d*x
 + c)^(3/2)*c^3)*b/d)/d
```

3.1404 $\int (c+dx)^{5/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(c+dx)^{7/2}}{7d}$$

[Out] (2*(c + d*x)^(7/2))/(7*d)

Rubi [A] time = 0.0014248, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(5/2),x]

[Out] (2*(c + d*x)^(7/2))/(7*d)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{5/2}\,dx = \frac{2(c+dx)^{7/2}}{7d}$$

Mathematica [A] time = 0.006018, size = 16, normalized size = 1.

$$\frac{2(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(5/2),x]

[Out] (2*(c + d*x)^(7/2))/(7*d)

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$\frac{2}{7\,d}\left(dx+c\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2),x)

```
[Out] 2/7*(d*x+c)^(7/2)/d
```

Maxima [A] time = 0.952011, size = 16, normalized size = 1.

$$\frac{2\,(dx+c)^{\frac{7}{2}}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(d*x + c)^(7/2)/d
```

Fricas [B] time = 1.73007, size = 85, normalized size = 5.31

$$\frac{2\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)\sqrt{dx+c}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sqrt(d*x + c)/d
```

Sympy [A] time = 0.059051, size = 12, normalized size = 0.75

$$\frac{2\,(c+dx)^{\frac{7}{2}}}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2),x)
```

```
[Out] 2*(c + d*x)**(7/2)/(7*d)
```

Giac [B] time = 1.07311, size = 81, normalized size = 5.06

$$\frac{2\left(15\,(dx+c)^{\frac{7}{2}}-42\,(dx+c)^{\frac{5}{2}}c+70\,(dx+c)^{\frac{3}{2}}c^2+14\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/105*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5/2)*c + 70*(d*x + c)^(3/2)*c^2 +
 14*(3*(d*x + c)^(5/2) - 5*(d*x + c)^(3/2)*c)*c)/d
```

3.1405 $\int \frac{(c+dx)^{5/2}}{a+bx} \, dx$

Optimal. Leaf size=112

$$\frac{2\sqrt{c+dx}(bc-ad)^2}{b^3}+\frac{2(c+dx)^{3/2}(bc-ad)}{3b^2}-\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{2(c+dx)^{5/2}}{5b}$$

```
[Out] (2*(b*c - a*d)^2*Sqrt[c + d*x])/b^3 + (2*(b*c - a*d)*(c + d*x)^(3/2))/(3*b^
2) + (2*(c + d*x)^(5/2))/(5*b) - (2*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt
[c + d*x])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rubi [A] time = 0.0579813, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 63, 208}

$$\frac{2\sqrt{c+dx}(bc-ad)^2}{b^3}+\frac{2(c+dx)^{3/2}(bc-ad)}{3b^2}-\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{2(c+dx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x),x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[c + d*x])/b^3 + (2*(b*c - a*d)*(c + d*x)^(3/2))/(3*b^
2) + (2*(c + d*x)^(5/2))/(5*b) - (2*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt
[c + d*x])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{a+bx}\,dx = \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)\int \frac{(c+dx)^{3/2}}{a+bx}\,dx}{b}$$
$$= \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)^2\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{b^2}$$
$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)^3\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{b^3}$$
$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{\left(2(bc-ad)^3\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,\right.}{b^3d}$$
$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} - \frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

Mathematica [A] time = 0.150215, size = 105, normalized size = 0.94

$$\frac{2(bc-ad)\left(\sqrt{b}\sqrt{c+dx}(-3ad+4bc+bdx)-3(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\right)}{3b^{7/2}} + \frac{2(c+dx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x),x]
```

```
[Out] (2*(c + d*x)^(5/2))/(5*b) + (2*(b*c - a*d)*(Sqrt[b]*Sqrt[c + d*x]*(4*b*c -
3*a*d + b*d*x) - 3*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b
*c - a*d]]))/(3*b^(7/2))
```

Maple [B] time = 0.007, size = 263, normalized size = 2.4

$$\frac{2}{5\,b}\,(dx+c)^{\frac{5}{2}} - \frac{2\,ad}{3\,b^2}\,(dx+c)^{\frac{3}{2}} + \frac{2\,c}{3\,b}\,(dx+c)^{\frac{3}{2}} + 2\,\frac{a^2d^2\sqrt{dx+c}}{b^3} - 4\,\frac{acd\sqrt{dx+c}}{b^2} + 2\,\frac{c^2\sqrt{dx+c}}{b} - 2\,\frac{a^3d^3}{b^3\sqrt{(ad-bc)\,b}}\,\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a),x)
```

```
[Out] 2/5*(d*x+c)^(5/2)/b-2/3/b^2*(d*x+c)^(3/2)*a*d+2/3/b*(d*x+c)^(3/2)*c+2/b^3*a
^2*d^2*(d*x+c)^(1/2)-4/b^2*a*c*d*(d*x+c)^(1/2)+2/b*c^2*(d*x+c)^(1/2)-2/b^3/
((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a^3*d^3+6/b
^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a^2*c*d^
2-6/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a*c^2
*d+2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8973, size = 644, normalized size = 5.75

$$\left[\frac{15\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\left(3\,b^{2}d^{2}x^{2}+23\,b^{2}c^{2}-35\,abcd+15\,a^{2}d^{2}+\left(11\,b^{2}cd-\right.\right.}{15\,b^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d*x +
2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(3*b^2*
d^2*x^2 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d - 5*a*b*d^2)*x
)*sqrt(d*x + c))/b^3, -2/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^
2*d^2*x^2 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d - 5*a*b*d^2)
*x)*sqrt(d*x + c))/b^3]
```

Sympy [A] time = 19.9037, size = 121, normalized size = 1.08

$$\frac{2\left(c+dx\right)^{\frac{5}{2}}}{5b}+\frac{\left(c+dx\right)^{\frac{3}{2}}\left(-2ad+2bc\right)}{3b^{2}}+\frac{\sqrt{c+dx}\left(2a^{2}d^{2}-4abcd+2b^{2}c^{2}\right)}{b^{3}}-\frac{2\left(ad-bc\right)^{3}\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^{4}\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a),x)
```

```
[Out] 2*(c + d*x)**(5/2)/(5*b) + (c + d*x)**(3/2)*(-2*a*d + 2*b*c)/(3*b**2) + sqr
t(c + d*x)*(2*a**2*d**2 - 4*a*b*c*d + 2*b**2*c**2)/b**3 - 2*(a*d - b*c)**3*
atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(b**4*sqrt((a*d - b*c)/b))
```

Giac [A] time = 1.07464, size = 231, normalized size = 2.06

$$\frac{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}b^{3}}+\frac{2\left(3\left(dx+c\right)^{\frac{5}{2}}b^{4}+5\left(dx+c\right)^{\frac{3}{2}}b^{4}c+15\sqrt{dx+c}b^{4}c^{2}-5\left(dx+\right.\right.}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x + c)*
b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/15*(3*(d*x + c)^(5/2
)*b^4 + 5*(d*x + c)^(3/2)*b^4*c + 15*sqrt(d*x + c)*b^4*c^2 - 5*(d*x + c)^(3
```

```
/2)*a*b^3*d - 30*sqrt(d*x + c)*a*b^3*c*d + 15*sqrt(d*x + c)*a^2*b^2*d^2)/b^
5
```

3.1406 $\int \frac{(c+dx)^{5/2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=110

$$\frac{5d\sqrt{c+dx}(bc-ad)}{b^3}-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}-\frac{(c+dx)^{5/2}}{b(a+bx)}+\frac{5d(c+dx)^{3/2}}{3b^2}$$

```
[Out] (5*d*(b*c - a*d)*Sqrt[c + d*x])/b^3 + (5*d*(c + d*x)^(3/2))/(3*b^2) - (c +
d*x)^(5/2)/(b*(a + b*x)) - (5*d*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rubi [A] time = 0.0547527, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 63, 208}

$$\frac{5d\sqrt{c+dx}(bc-ad)}{b^3}-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}-\frac{(c+dx)^{5/2}}{b(a+bx)}+\frac{5d(c+dx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^2,x]
```

```
[Out] (5*d*(b*c - a*d)*Sqrt[c + d*x])/b^3 + (5*d*(c + d*x)^(3/2))/(3*b^2) - (c +
d*x)^(5/2)/(b*(a + b*x)) - (5*d*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^2}\,dx &= -\frac{(c+dx)^{5/2}}{b(a+bx)}+\frac{(5d)\int \frac{(c+dx)^{3/2}}{a+bx}\,dx}{2b} \\
&= \frac{5d(c+dx)^{3/2}}{3b^2}-\frac{(c+dx)^{5/2}}{b(a+bx)}+\frac{(5d(bc-ad))\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{2b^2} \\
&= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3}+\frac{5d(c+dx)^{3/2}}{3b^2}-\frac{(c+dx)^{5/2}}{b(a+bx)}+\frac{\left(5d(bc-ad)^2\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2b^3} \\
&= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3}+\frac{5d(c+dx)^{3/2}}{3b^2}-\frac{(c+dx)^{5/2}}{b(a+bx)}+\frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+d\right.}{b^3} \\
&= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3}+\frac{5d(c+dx)^{3/2}}{3b^2}-\frac{(c+dx)^{5/2}}{b(a+bx)}-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.015992, size = 50, normalized size = 0.45

$$\frac{2d(c+dx)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{b(c+dx)}{ad-bc}\right)}{7(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^2,x]
```

```
[Out] (2*d*(c + d*x)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(7*(-(b*c) + a*d)^2)
```

Maple [B] time = 0.011, size = 258, normalized size = 2.4

$$\frac{2d}{3b^2}(dx+c)^{\frac{3}{2}}-4\frac{ad^2\sqrt{dx+c}}{b^3}+4\frac{d\sqrt{dx+cc}}{b^2}-\frac{a^2d^3}{b^3(bdx+ad)}\sqrt{dx+c}+2\frac{\sqrt{dx+cacd^2}}{b^2(bdx+ad)}-\frac{dc^2}{b(bdx+ad)}\sqrt{dx+c}+5\frac{}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^2,x)
```

```
[Out] 2/3*d*(d*x+c)^(3/2)/b^2-4/b^3*a*d^2*(d*x+c)^(1/2)+4*d/b^2*(d*x+c)^(1/2)*c-1
/b^3*(d*x+c)^(1/2)/(b*d*x+a*d)*a^2*d^3+2/b^2*(d*x+c)^(1/2)/(b*d*x+a*d)*a*c*
d^2-d/b*(d*x+c)^(1/2)/(b*d*x+a*d)*c^2+5/b^3/((a*d-b*c)*b)^(1/2)*arctan(b*(d
*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a^2*d^3-10/b^2/((a*d-b*c)*b)^(1/2)*arctan(
b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a*c*d^2+5*d/b/((a*d-b*c)*b)^(1/2)*arct
an(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83364, size = 707, normalized size = 6.43

$$\left[-\frac{15\left(abcd-a^2d^2+\left(b^2cd-abd^2\right)x\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad+2\,\sqrt{dx+c}b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)-2\left(2\,b^2d^2x^2-3\,b^2c^2+20\,abcd-15\,a^2d^2+2\right.}{6\left(b^4x+ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x)*sqrt((b*c - a*d)/b)*l
og((b*d*x + 2*b*c - a*d + 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a))
 - 2*(2*b^2*d^2*x^2 - 3*b^2*c^2 + 20*a*b*c*d - 15*a^2*d^2 + 2*(7*b^2*c*d -
5*a*b*d^2)*x)*sqrt(d*x + c))/(b^4*x + a*b^3), -1/3*(15*(a*b*c*d - a^2*d^2 +
 (b^2*c*d - a*b*d^2)*x)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-
(b*c - a*d)/b)/(b*c - a*d)) - (2*b^2*d^2*x^2 - 3*b^2*c^2 + 20*a*b*c*d - 15*
a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x)*sqrt(d*x + c))/(b^4*x + a*b^3)]
```

Sympy [B] time = 148.11, size = 1312, normalized size = 11.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**2,x)
```

```
[Out] -2*a**3*d**4*sqrt(c + d*x)/(2*a**2*b**3*d**2 - 2*a*b**4*c*d + 2*a*b**4*d**2
*x - 2*b**5*c*d*x) + a**3*d**4*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*s
qrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c
**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b**3) - a**3*d**4*sqrt(
-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*
d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqr
t(c + d*x))/(2*b**3) + 6*a**2*c*d**3*sqrt(c + d*x)/(2*a**2*b**2*d**2 - 2*a*
b**3*c*d + 2*a*b**3*d**2*x - 2*b**4*c*d*x) - 3*a**2*c*d**3*sqrt(-1/(b*(a*d
- b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/
(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x)
)/(2*b**2) + 3*a**2*c*d**3*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-
1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*s
qrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b**2) + 6*a**2*d**3*atan(sqr
t(c + d*x)/sqrt(a*d/b - c))/(b**4*sqrt(a*d/b - c)) - 6*a*c**2*d**2*sqrt(c +
 d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x - 2*b**3*c*d*x) + 3*a
*c**2*d**2*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c
```

```
)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d
- b*c)**3)) + sqrt(c + d*x))/(2*b) - 3*a*c**2*d**2*sqrt(-1/(b*(a*d - b*c)**
3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d -
 b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b) -
 12*a*c*d**2*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**3*sqrt(a*d/b - c)) - 4
*a*d**2*sqrt(c + d*x)/b**3 - c**3*d*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d
**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b
**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + c**3*d*sqrt(-1/(b
*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqr
t(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c +
 d*x))/2 + 2*c**3*d*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x -
 2*b**2*c*d*x) + 6*c**2*d*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*
d/b - c)) + 4*c*d*sqrt(c + d*x)/b**2 + 2*d*(c + d*x)**(3/2)/(3*b**2)
```

Giac [A] time = 1.08169, size = 244, normalized size = 2.22

$$\frac{5\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^3}-\frac{\sqrt{dx+c}b^2c^2d-2\,\sqrt{dx+c}abcd^2+\sqrt{dx+c}a^2d^3}{((dx+c)b-bc+ad)b^3}+\frac{2\left((dx+c)^{\frac{3}{2}}b^4d+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - (sqrt(d*x + c)*b^2*c^2*d - 2*sqrt(d*x
+ c)*a*b*c*d^2 + sqrt(d*x + c)*a^2*d^3)/(((d*x + c)*b - b*c + a*d)*b^3) + 2
/3*((d*x + c)^(3/2)*b^4*d + 6*sqrt(d*x + c)*b^4*c*d - 6*sqrt(d*x + c)*a*b^3
*d^2)/b^6
```

3.1407 $\int \frac{(c+dx)^{5/2}}{(a+bx)^3}\,dx$

Optimal. Leaf size=119

$$-\frac{15d^2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{15d^2\sqrt{c+dx}}{4b^3}$$

```
[Out] (15*d^2*Sqrt[c + d*x])/(4*b^3) - (5*d*(c + d*x)^(3/2))/(4*b^2*(a + b*x)) -
(c + d*x)^(5/2)/(2*b*(a + b*x)^2) - (15*d^2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(7/2))
```

Rubi [A] time = 0.0493503, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 63, 208}

$$-\frac{15d^2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{15d^2\sqrt{c+dx}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x])/(4*b^3) - (5*d*(c + d*x)^(3/2))/(4*b^2*(a + b*x)) -
(c + d*x)^(5/2)/(2*b*(a + b*x)^2) - (15*d^2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^3}\,dx &= -\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{\left(15d^2\right)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{8b^2}\\
&= \frac{15d^2\sqrt{c+dx}}{4b^3}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{\left(15d^2(bc-ad)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b^3}\\
&= \frac{15d^2\sqrt{c+dx}}{4b^3}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{(15d(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4b^3}\\
&= \frac{15d^2\sqrt{c+dx}}{4b^3}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}-\frac{15d^2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0161051, size = 52, normalized size = 0.44

$$\frac{2d^2(c+dx)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};-\frac{b(c+dx)}{ad-bc}\right)}{7(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^3,x]
```

```
[Out] (2*d^2*(c + d*x)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(7*(-(b*c) + a*d)^3)
```

Maple [B] time = 0.015, size = 238, normalized size = 2.

$$2\,\frac{d^2\sqrt{dx+c}}{b^3}+\frac{9\,d^3a}{4\,b^2\,(bdx+ad)^2}(dx+c)^{\frac{3}{2}}-\frac{9\,d^2c}{4\,b\,(bdx+ad)^2}(dx+c)^{\frac{3}{2}}+\frac{7\,d^4a^2}{4\,b^3\,(bdx+ad)^2}\sqrt{dx+c}-\frac{7\,d^3ac}{2\,b^2\,(bdx+ad)^2}\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^3,x)
```

```
[Out] 2*d^2*(d*x+c)^(1/2)/b^3+9/4*d^3/b^2/(b*d*x+a*d)^2*(d*x+c)^(3/2)*a-9/4*d^2/b
/(b*d*x+a*d)^2*(d*x+c)^(3/2)*c+7/4*d^4/b^3/(b*d*x+a*d)^2*(d*x+c)^(1/2)*a^2-
7/2*d^3/b^2/(b*d*x+a*d)^2*(d*x+c)^(1/2)*a*c+7/4*d^2/b/(b*d*x+a*d)^2*(d*x+c)
^(1/2)*c^2-15/4*d^3/b^3/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*
c)*b)^(1/2))*a+15/4*d^2/b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*
d-b*c)*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8732, size = 728, normalized size = 6.12

$$\left[\frac{15\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\left(8\,b^2d^2x^2-2\,b^2c^2-5\,abcd+15\,a^2d^2-\left(9\,b^2cd-\right.\right.}{8\left(b^5x^2+2\,ab^4x+a^2b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d
*x + 2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(8
*b^2*d^2*x^2 - 2*b^2*c^2 - 5*a*b*c*d + 15*a^2*d^2 - (9*b^2*c*d - 25*a*b*d^2
)*x)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*d^2*x^2
+ 2*a*b*d^2*x + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(
-(b*c - a*d)/b)/(b*c - a*d)) - (8*b^2*d^2*x^2 - 2*b^2*c^2 - 5*a*b*c*d + 15*
a^2*d^2 - (9*b^2*c*d - 25*a*b*d^2)*x)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x +
 a^2*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28623, size = 231, normalized size = 1.94

$$\frac{2\,\sqrt{dx+c}d^2}{b^3}+\frac{15\left(bcd^2-ad^3\right)\arctan\left(\frac{\sqrt{dx+cb}}{\sqrt{-b^2c+abd}}\right)}{4\,\sqrt{-b^2c+abd}b^3}-\frac{9\,(dx+c)^{\frac{3}{2}}b^2cd^2-7\,\sqrt{dx+c}b^2c^2d^2-9\,(dx+c)^{\frac{3}{2}}abd^3+14\,\sqrt{dx+c}}{4\left((dx+c)b-bc+ad\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*d^2/b^3 + 15/4*(b*c*d^2 - a*d^3)*arctan(sqrt(d*x + c)*b/sqr
t(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - 1/4*(9*(d*x + c)^(3/2)*b^2*
c*d^2 - 7*sqrt(d*x + c)*b^2*c^2*d^2 - 9*(d*x + c)^(3/2)*a*b*d^3 + 14*sqrt(d
*x + c)*a*b*c*d^3 - 7*sqrt(d*x + c)*a^2*d^4)/(((d*x + c)*b - b*c + a*d)^2*b
^3)
```

3.1408 $\int \frac{(c+dx)^{5/2}}{(a+bx)^4} dx$

Optimal. Leaf size=126

$$-\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}$$

```
[Out] (-5*d^2*Sqrt[c + d*x])/(8*b^3*(a + b*x)) - (5*d*(c + d*x)^(3/2))/(12*b^2*(a
 + b*x)^2) - (c + d*x)^(5/2)/(3*b*(a + b*x)^3) - (5*d^3*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(7/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0501402, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 63, 208}

$$-\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^4,x]
```

```
[Out] (-5*d^2*Sqrt[c + d*x])/(8*b^3*(a + b*x)) - (5*d*(c + d*x)^(3/2))/(12*b^2*(a
 + b*x)^2) - (c + d*x)^(5/2)/(3*b*(a + b*x)^3) - (5*d^3*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(7/2)*Sqrt[b*c - a*d])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^4}\,dx = -\frac{(c+dx)^{5/2}}{3b(a+bx)^3} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^3}\,dx}{6b}$$
$$= -\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2} - \frac{(c+dx)^{5/2}}{3b(a+bx)^3} + \frac{\left(5d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx}{8b^2}$$
$$= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)} - \frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2} - \frac{(c+dx)^{5/2}}{3b(a+bx)^3} + \frac{\left(5d^3\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b^3}$$
$$= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)} - \frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2} - \frac{(c+dx)^{5/2}}{3b(a+bx)^3} + \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{8b^3}$$
$$= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)} - \frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2} - \frac{(c+dx)^{5/2}}{3b(a+bx)^3} - \frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.147758, size = 119, normalized size = 0.94

$$\frac{5d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{ad-bc}}\right)}{8b^{7/2}\sqrt{ad-bc}} - \frac{\sqrt{c+dx}\left(15a^2d^2+10abd(c+4dx)+b^2\left(8c^2+26cdx+33d^2x^2\right)\right)}{24b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^4,x]
```

```
[Out] -(Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(c + 4*d*x) + b^2*(8*c^2 + 26*c*d*x
+ 33*d^2*x^2)))/(24*b^3*(a + b*x)^3) + (5*d^3*ArcTan[(Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[-(b*c) + a*d]])/(8*b^(7/2)*Sqrt[-(b*c) + a*d])
```

Maple [A] time = 0.013, size = 204, normalized size = 1.6

$$-\frac{11\,d^3}{8\,(bdx+ad)^3\,b}(dx+c)^{\frac{5}{2}} - \frac{5\,d^4a}{3\,(bdx+ad)^3\,b^2}(dx+c)^{\frac{3}{2}} + \frac{5\,d^3c}{3\,(bdx+ad)^3\,b}(dx+c)^{\frac{3}{2}} - \frac{5\,d^5a^2}{8\,(bdx+ad)^3\,b^3}\sqrt{dx+c} + \frac{}{4\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^4,x)
```

```
[Out] -11/8*d^3/(b*d*x+a*d)^3/b*(d*x+c)^(5/2)-5/3*d^4/(b*d*x+a*d)^3/b^2*(d*x+c)^(
3/2)*a+5/3*d^3/(b*d*x+a*d)^3/b*(d*x+c)^(3/2)*c-5/8*d^5/(b*d*x+a*d)^3/b^3*(d
*x+c)^(1/2)*a^2+5/4*d^4/(b*d*x+a*d)^3/b^2*(d*x+c)^(1/2)*a*c-5/8*d^3/(b*d*x+
a*d)^3/b*(d*x+c)^(1/2)*c^2+5/8*d^3/b^3/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)
^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.91429, size = 1162, normalized size = 9.22

$$\left[\frac{15\left(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3\right)\sqrt{b^2c-abd}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)-2\left(8\,b^4c^3+2\,ab^3c^2d+5\,a^2b^2cd^2\right.}{48\left(a^3b^5c-a^4b^4d+\left(b^8c-ab^7d\right)x^3+3\left(ab^7c-a^2b^6d\right)x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) - 2*(8*b^4*c^3 + 2*a*b^3*c^2*d + 5*a^2*b^2*c*d^2 - 15*a^3*b*d^
3 + 33*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(13*b^4*c^2*d + 7*a*b^3*c*d^2 - 20*a
^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c - a^4*b^4*d + (b^8*c - a*b^7*d)*x^
3 + 3*(a*b^7*c - a^2*b^6*d)*x^2 + 3*(a^2*b^6*c - a^3*b^5*d)*x), 1/24*(15*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a*b*d
)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (8*b^4*c^3 + 2
*a*b^3*c^2*d + 5*a^2*b^2*c*d^2 - 15*a^3*b*d^3 + 33*(b^4*c*d^2 - a*b^3*d^3)*
x^2 + 2*(13*b^4*c^2*d + 7*a*b^3*c*d^2 - 20*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(
a^3*b^5*c - a^4*b^4*d + (b^8*c - a*b^7*d)*x^3 + 3*(a*b^7*c - a^2*b^6*d)*x^2
 + 3*(a^2*b^6*c - a^3*b^5*d)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0882, size = 217, normalized size = 1.72

$$\frac{5\,d^3\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{8\,\sqrt{-b^2c+abd}b^3}-\frac{33\,(dx+c)^{\frac{5}{2}}b^2d^3-40\,(dx+c)^{\frac{3}{2}}b^2cd^3+15\,\sqrt{dx+c}b^2c^2d^3+40\,(dx+c)^{\frac{3}{2}}abd^4-30\,\sqrt{dx+c}abc}{24\,((dx+c)b-bc+ad)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 5/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b^3) - 1/24*(33*(d*x + c)^(5/2)*b^2*d^3 - 40*(d*x + c)^(3/2)*b^2*c*d^3 + 15
*sqrt(d*x + c)*b^2*c^2*d^3 + 40*(d*x + c)^(3/2)*a*b*d^4 - 30*sqrt(d*x + c)*
a*b*c*d^4 + 15*sqrt(d*x + c)*a^2*d^5)/(((d*x + c)*b - b*c + a*d)^3*b^3)
```

3.1409 $\int \frac{(c+dx)^{5/2}}{(a+bx)^5}\,dx$

Optimal. Leaf size=162

$$-\frac{5d^3\sqrt{c+dx}}{64b^3(a+bx)(bc-ad)}-\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}+\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{7/2}(bc-ad)^{3/2}}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}$$

```
[Out] (-5*d^2*Sqrt[c + d*x])/(32*b^3*(a + b*x)^2) - (5*d^3*Sqrt[c + d*x])/(64*b^3
*(b*c - a*d)*(a + b*x)) - (5*d*(c + d*x)^(3/2))/(24*b^2*(a + b*x)^3) - (c +
 d*x)^(5/2)/(4*b*(a + b*x)^4) + (5*d^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[b*c - a*d]])/(64*b^(7/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0703069, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{5d^3\sqrt{c+dx}}{64b^3(a+bx)(bc-ad)}-\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}+\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{7/2}(bc-ad)^{3/2}}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^5,x]
```

```
[Out] (-5*d^2*Sqrt[c + d*x])/(32*b^3*(a + b*x)^2) - (5*d^3*Sqrt[c + d*x])/(64*b^3
*(b*c - a*d)*(a + b*x)) - (5*d*(c + d*x)^(3/2))/(24*b^2*(a + b*x)^3) - (c +
 d*x)^(5/2)/(4*b*(a + b*x)^4) + (5*d^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[b*c - a*d]])/(64*b^(7/2)*(b*c - a*d)^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^5}\,dx &= -\frac{(c+dx)^{5/2}}{4b(a+bx)^4}+\frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^4}\,dx}{8b}\\
&= -\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}+\frac{\left(5d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx}{16b^2}\\
&= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}+\frac{\left(5d^3\right)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{64b^3}\\
&= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}-\frac{\left(5d^4\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{128b^3(bc-ad)}\\
&= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}-\frac{\left(5d^3\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx\right.}{64b^3(bc-ad)}\\
&= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}+\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{7/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0177148, size = 52, normalized size = 0.32

$$\frac{2d^4(c+dx)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};-\frac{b(c+dx)}{ad-bc}\right)}{7(ad-bc)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^5,x]
```

```
[Out] (2*d^4*(c + d*x)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(7*(-(b*c) + a*d)^5)
```

Maple [A] time = 0.014, size = 246, normalized size = 1.5

$$\frac{5\,d^4}{64\,(bdx+ad)^4\,(ad-bc)}(dx+c)^{\frac{7}{2}}-\frac{73\,d^4}{192\,(bdx+ad)^4\,b}(dx+c)^{\frac{5}{2}}-\frac{55\,d^5a}{192\,(bdx+ad)^4\,b^2}(dx+c)^{\frac{3}{2}}+\frac{55\,d^4c}{192\,(bdx+ad)^4\,b}(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^5,x)
```

```
[Out] 5/64*d^4/(b*d*x+a*d)^4/(a*d-b*c)*(d*x+c)^(7/2)-73/192*d^4/(b*d*x+a*d)^4/b*(
d*x+c)^(5/2)-55/192*d^5/(b*d*x+a*d)^4/b^2*(d*x+c)^(3/2)*a+55/192*d^4/(b*d*x
+a*d)^4/b*(d*x+c)^(3/2)*c-5/64*d^6/(b*d*x+a*d)^4/b^3*(d*x+c)^(1/2)*a^2+5/32
*d^5/(b*d*x+a*d)^4/b^2*(d*x+c)^(1/2)*a*c-5/64*d^4/(b*d*x+a*d)^4/b*(d*x+c)^(
1/2)*c^2+5/64*d^4/(a*d-b*c)/b^3/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/
```

```
((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.06212, size = 1859, normalized size = 11.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="fricas")

[Out] [-1/384*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(48*b^5*c^4 - 56*a*b^4*c^3*d - 2*a^2
*b^3*c^2*d^2 - 5*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4)*
x^3 + (118*b^5*c^2*d^2 - 191*a*b^4*c*d^3 + 73*a^2*b^3*d^4)*x^2 + (136*b^5*c
^3*d - 172*a*b^4*c^2*d^2 - 19*a^2*b^3*c*d^3 + 55*a^3*b^2*d^4)*x)*sqrt(d*x +
 c))/(a^4*b^6*c^2 - 2*a^5*b^5*c*d + a^6*b^4*d^2 + (b^10*c^2 - 2*a*b^9*c*d +
 a^2*b^8*d^2)*x^4 + 4*(a*b^9*c^2 - 2*a^2*b^8*c*d + a^3*b^7*d^2)*x^3 + 6*(a^
2*b^8*c^2 - 2*a^3*b^7*c*d + a^4*b^6*d^2)*x^2 + 4*(a^3*b^7*c^2 - 2*a^4*b^6*c
*d + a^5*b^5*d^2)*x), -1/192*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2
*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c
 + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (48*b^5*c^4 - 56*a*b^4*c^3*d - 2*a
^2*b^3*c^2*d^2 - 5*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4
)*x^3 + (118*b^5*c^2*d^2 - 191*a*b^4*c*d^3 + 73*a^2*b^3*d^4)*x^2 + (136*b^5
*c^3*d - 172*a*b^4*c^2*d^2 - 19*a^2*b^3*c*d^3 + 55*a^3*b^2*d^4)*x)*sqrt(d*x
 + c))/(a^4*b^6*c^2 - 2*a^5*b^5*c*d + a^6*b^4*d^2 + (b^10*c^2 - 2*a*b^9*c*d
 + a^2*b^8*d^2)*x^4 + 4*(a*b^9*c^2 - 2*a^2*b^8*c*d + a^3*b^7*d^2)*x^3 + 6*(
a^2*b^8*c^2 - 2*a^3*b^7*c*d + a^4*b^6*d^2)*x^2 + 4*(a^3*b^7*c^2 - 2*a^4*b^6
*c*d + a^5*b^5*d^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**5,x)

[Out] Timed out
```

Giac [A] time = 1.11564, size = 350, normalized size = 2.16

$$-\frac{5\,d^{4}\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^{2}c+abd}}\right)}{64\left(b^{4}c-ab^{3}d\right)\sqrt{-b^{2}c+abd}}-\frac{15\,(dx+c)^{\frac{7}{2}}b^{3}d^{4}+73\,(dx+c)^{\frac{5}{2}}b^{3}cd^{4}-55\,(dx+c)^{\frac{3}{2}}b^{3}c^{2}d^{4}+15\,\sqrt{dx+c}b^{3}c^{3}d^{4}-73\,(dx}{192\,(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -5/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c - a*b^3*d)*s
qrt(-b^2*c + a*b*d)) - 1/192*(15*(d*x + c)^(7/2)*b^3*d^4 + 73*(d*x + c)^(5/
2)*b^3*c*d^4 - 55*(d*x + c)^(3/2)*b^3*c^2*d^4 + 15*sqrt(d*x + c)*b^3*c^3*d^
4 - 73*(d*x + c)^(5/2)*a*b^2*d^5 + 110*(d*x + c)^(3/2)*a*b^2*c*d^5 - 45*sqr
t(d*x + c)*a*b^2*c^2*d^5 - 55*(d*x + c)^(3/2)*a^2*b*d^6 + 45*sqrt(d*x + c)*
a^2*b*c*d^6 - 15*sqrt(d*x + c)*a^3*d^7)/((b^4*c - a*b^3*d)*((d*x + c)*b - b
*c + a*d)^4)
```

3.1410 $\int \frac{(c+dx)^{5/2}}{(a+bx)^6} dx$

Optimal. Leaf size=198

$$\frac{3d^4\sqrt{c+dx}}{128b^3(a+bx)(bc-ad)^2}-\frac{d^3\sqrt{c+dx}}{64b^3(a+bx)^2(bc-ad)}-\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{3d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{7/2}(bc-ad)^{5/2}}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx}{5b(a+b}$$

```
[Out] -(d^2*Sqrt[c + d*x])/(16*b^3*(a + b*x)^3) - (d^3*Sqrt[c + d*x])/(64*b^3*(b*
c - a*d)*(a + b*x)^2) + (3*d^4*Sqrt[c + d*x])/(128*b^3*(b*c - a*d)^2*(a + b
*x)) - (d*(c + d*x)^(3/2))/(8*b^2*(a + b*x)^4) - (c + d*x)^(5/2)/(5*b*(a +
b*x)^5) - (3*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(128*b^(
7/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.0922634, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$\frac{3d^4\sqrt{c+dx}}{128b^3(a+bx)(bc-ad)^2}-\frac{d^3\sqrt{c+dx}}{64b^3(a+bx)^2(bc-ad)}-\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{3d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{7/2}(bc-ad)^{5/2}}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx}{5b(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^6,x]
```

```
[Out] -(d^2*Sqrt[c + d*x])/(16*b^3*(a + b*x)^3) - (d^3*Sqrt[c + d*x])/(64*b^3*(b*
c - a*d)*(a + b*x)^2) + (3*d^4*Sqrt[c + d*x])/(128*b^3*(b*c - a*d)^2*(a + b
*x)) - (d*(c + d*x)^(3/2))/(8*b^2*(a + b*x)^4) - (c + d*x)^(5/2)/(5*b*(a +
b*x)^5) - (3*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(128*b^(
7/2)*(b*c - a*d)^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^6}\,dx = -\frac{(c+dx)^{5/2}}{5b(a+bx)^5}+\frac{d\int \frac{(c+dx)^{3/2}}{(a+bx)^5}\,dx}{2b}$$

$$= -\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}+\frac{\left(3d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx}{16b^2}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}+\frac{d^3\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{32b^3}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}-\frac{\left(3d^4\right)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{128b^3(bc-ad)}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2}+\frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}+$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2}+\frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}+$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2}+\frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}-$$

Mathematica [C] time = 0.0169155, size = 52, normalized size = 0.26

$$\frac{2d^5(c+dx)^{7/2}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};-\frac{b(c+dx)}{ad-bc}\right)}{7(ad-bc)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^6,x]
```

```
[Out] (2*d^5*(c + d*x)^(7/2)*Hypergeometric2F1[7/2, 6, 9/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(7*(-(b*c) + a*d)^6)
```

Maple [A] time = 0.015, size = 305, normalized size = 1.5

$$\frac{3\,d^5b}{128\,(bdx+ad)^5\left(a^2d^2-2\,abcd+b^2c^2\right)}(dx+c)^{\frac{9}{2}}+\frac{7\,d^5}{64\,(bdx+ad)^5\,(ad-bc)}(dx+c)^{\frac{7}{2}}-\frac{d^5}{5\,(bdx+ad)^5\,b}(dx+c)^{\frac{5}{2}}-\frac{}{64\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^6,x)
```

```
[Out] 3/128*d^5/(b*d*x+a*d)^5*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(9/2)+7/64*d^
5/(b*d*x+a*d)^5/(a*d-b*c)*(d*x+c)^(7/2)-1/5*d^5/(b*d*x+a*d)^5/b*(d*x+c)^(5/
2)-7/64*d^6/(b*d*x+a*d)^5/b^2*(d*x+c)^(3/2)*a+7/64*d^5/(b*d*x+a*d)^5/b*(d*x
+c)^(3/2)*c-3/128*d^7/(b*d*x+a*d)^5/b^3*(d*x+c)^(1/2)*a^2+3/64*d^6/(b*d*x+a
*d)^5/b^2*(d*x+c)^(1/2)*a*c-3/128*d^5/(b*d*x+a*d)^5/b*(d*x+c)^(1/2)*c^2+3/1
28*d^5/b^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)
^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.05082, size = 2743, normalized size = 13.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^
2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c
 - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(128*b^6*c^5 -
 304*a*b^5*c^4*d + 184*a^2*b^4*c^3*d^2 + 2*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^
4 - 15*a^5*b*d^5 - 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 10*(b^6*c^2*d^3 - 8*a*b
^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(124*b^6*c^3*d^2 - 357*a*b^5*c^2*d^3 + 29
7*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(168*b^6*c^4*d - 424*a*b^5*c^3*d^
2 + 279*a^2*b^4*c^2*d^3 + 12*a^3*b^3*c*d^4 - 35*a^4*b^2*d^5)*x)*sqrt(d*x +
c))/(a^5*b^7*c^3 - 3*a^6*b^6*c^2*d + 3*a^7*b^5*c*d^2 - a^8*b^4*d^3 + (b^12*
c^3 - 3*a*b^11*c^2*d + 3*a^2*b^10*c*d^2 - a^3*b^9*d^3)*x^5 + 5*(a*b^11*c^3
- 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - a^4*b^8*d^3)*x^4 + 10*(a^2*b^10*c^3
- 3*a^3*b^9*c^2*d + 3*a^4*b^8*c*d^2 - a^5*b^7*d^3)*x^3 + 10*(a^3*b^9*c^3 -
3*a^4*b^8*c^2*d + 3*a^5*b^7*c*d^2 - a^6*b^6*d^3)*x^2 + 5*(a^4*b^8*c^3 - 3*a
^5*b^7*c^2*d + 3*a^6*b^6*c*d^2 - a^7*b^5*d^3)*x), 1/640*(15*(b^5*d^5*x^5 +
5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x +
 a^5*d^5)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b
*d*x + b*c)) - (128*b^6*c^5 - 304*a*b^5*c^4*d + 184*a^2*b^4*c^3*d^2 + 2*a^3
*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - 15*a^5*b*d^5 - 15*(b^6*c*d^4 - a*b^5*d^5)*
x^4 + 10*(b^6*c^2*d^3 - 8*a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(124*b^6*c^3
*d^2 - 357*a*b^5*c^2*d^3 + 297*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(168
*b^6*c^4*d - 424*a*b^5*c^3*d^2 + 279*a^2*b^4*c^2*d^3 + 12*a^3*b^3*c*d^4 - 3
5*a^4*b^2*d^5)*x)*sqrt(d*x + c))/(a^5*b^7*c^3 - 3*a^6*b^6*c^2*d + 3*a^7*b^5
*c*d^2 - a^8*b^4*d^3 + (b^12*c^3 - 3*a*b^11*c^2*d + 3*a^2*b^10*c*d^2 - a^3*
b^9*d^3)*x^5 + 5*(a*b^11*c^3 - 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - a^4*b^8
*d^3)*x^4 + 10*(a^2*b^10*c^3 - 3*a^3*b^9*c^2*d + 3*a^4*b^8*c*d^2 - a^5*b^7*
d^3)*x^3 + 10*(a^3*b^9*c^3 - 3*a^4*b^8*c^2*d + 3*a^5*b^7*c*d^2 - a^6*b^6*d^
3)*x^2 + 5*(a^4*b^8*c^3 - 3*a^5*b^7*c^2*d + 3*a^6*b^6*c*d^2 - a^7*b^5*d^3)*
x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13512, size = 513, normalized size = 2.59

$$\frac{3\,d^5\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{128\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)\sqrt{-b^2c+abd}}+\frac{15\,(dx+c)^{\frac{9}{2}}b^4d^5-70\,(dx+c)^{\frac{7}{2}}b^4cd^5-128\,(dx+c)^{\frac{5}{2}}b^4c^2d^5+70\,(dx+c)^{\frac{3}{2}}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 3/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^2 - 2*a*b^4*
c*d + a^2*b^3*d^2)*sqrt(-b^2*c + a*b*d)) + 1/640*(15*(d*x + c)^(9/2)*b^4*d^
5 - 70*(d*x + c)^(7/2)*b^4*c*d^5 - 128*(d*x + c)^(5/2)*b^4*c^2*d^5 + 70*(d*
x + c)^(3/2)*b^4*c^3*d^5 - 15*sqrt(d*x + c)*b^4*c^4*d^5 + 70*(d*x + c)^(7/2
)*a*b^3*d^6 + 256*(d*x + c)^(5/2)*a*b^3*c*d^6 - 210*(d*x + c)^(3/2)*a*b^3*c
^2*d^6 + 60*sqrt(d*x + c)*a*b^3*c^3*d^6 - 128*(d*x + c)^(5/2)*a^2*b^2*d^7 +
 210*(d*x + c)^(3/2)*a^2*b^2*c*d^7 - 90*sqrt(d*x + c)*a^2*b^2*c^2*d^7 - 70*
(d*x + c)^(3/2)*a^3*b*d^8 + 60*sqrt(d*x + c)*a^3*b*c*d^8 - 15*sqrt(d*x + c)
*a^4*d^9)/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*((d*x + c)*b - b*c + a*d)^
5)
```

3.1411 $\int \frac{\sqrt{-1+x}}{(1+x)^2}\,dx$

Optimal. Leaf size=35

$$\frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\sqrt{x-1}}{x+1}$$

```
[Out] -(Sqrt[-1 + x]/(1 + x)) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/Sqrt[2]
```

Rubi [A] time = 0.0081189, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\sqrt{x-1}}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(1 + x)^2,x]
```

```
[Out] -(Sqrt[-1 + x]/(1 + x)) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/Sqrt[2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{-1+x}}{(1+x)^2}\,dx = -\frac{\sqrt{-1+x}}{1+x} + \frac{1}{2}\int \frac{1}{\sqrt{-1+x}(1+x)}\,dx$$
$$= -\frac{\sqrt{-1+x}}{1+x} + \text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{-1+x}\right)$$
$$= -\frac{\sqrt{-1+x}}{1+x} + \frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0277093, size = 51, normalized size = 1.46

$$\frac{-2x - \sqrt{2-2x}(x+1)\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + 2}{2\sqrt{x-1}(x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(1 + x)^2,x]
```

```
[Out] (2 - 2*x - Sqrt[2 - 2*x]*(1 + x)*ArcTanh[Sqrt[1 - x]/Sqrt[2]])/(2*Sqrt[-1 +
 x]*(1 + x))
```

Maple [A] time = 0.009, size = 30, normalized size = 0.9

$$\frac{\sqrt{2}}{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-1+x}\right) - \frac{1}{1+x}\sqrt{-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/2)/(1+x)^2,x)
```

```
[Out] 1/2*arctan(1/2*(-1+x)^(1/2)*2^(1/2))*2^(1/2)-(-1+x)^(1/2)/(1+x)
```

Maxima [A] time = 1.43599, size = 39, normalized size = 1.11

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right) - \frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - sqrt(x - 1)/(x + 1)
```

Fricas [A] time = 1.75722, size = 107, normalized size = 3.06

$$\frac{\sqrt{2}(x+1)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right) - 2\sqrt{x-1}}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*(x + 1)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - 2*sqrt(x - 1))/(x +
1)
```

Sympy [A] time = 1.48247, size = 104, normalized size = 2.97

$$\begin{cases} \frac{\sqrt{2}i\operatorname{acosh}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{2} + \frac{i}{\sqrt{-1+\frac{2}{x+1}}\sqrt{x+1}} - \frac{2i}{\sqrt{-1+\frac{2}{x+1}}(x+1)^{\frac{3}{2}}} & \text{for } \frac{2}{|x+1|} > 1 \\ -\frac{\sqrt{1-\frac{2}{x+1}}}{\sqrt{x+1}} - \frac{\sqrt{2}\operatorname{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/(1+x)**2,x)
```

```
[Out] Piecewise((sqrt(2)*I*acosh(sqrt(2)/sqrt(x + 1))/2 + I/(sqrt(-1 + 2/(x + 1))
*sqrt(x + 1)) - 2*I/(sqrt(-1 + 2/(x + 1))*(x + 1)**(3/2)), 2/Abs(x + 1) > 1
), (-sqrt(1 - 2/(x + 1))/sqrt(x + 1) - sqrt(2)*asin(sqrt(2)/sqrt(x + 1))/2,
 True))
```

Giac [A] time = 1.06907, size = 39, normalized size = 1.11

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right) - \frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - sqrt(x - 1)/(x + 1)
```

3.1412 $\int \frac{\sqrt{-1+x}}{(1+x)^3} dx$

Optimal. Leaf size=56

$$\frac{\sqrt{x-1}}{8(x+1)} - \frac{\sqrt{x-1}}{2(x+1)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] -Sqrt[-1 + x]/(2*(1 + x)^2) + Sqrt[-1 + x]/(8*(1 + x)) + ArcTan[Sqrt[-1 + x
]/Sqrt[2]]/(8*Sqrt[2])
```

Rubi [A] time = 0.0130528, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 203}

$$\frac{\sqrt{x-1}}{8(x+1)} - \frac{\sqrt{x-1}}{2(x+1)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(1 + x)^3,x]
```

```
[Out] -Sqrt[-1 + x]/(2*(1 + x)^2) + Sqrt[-1 + x]/(8*(1 + x)) + ArcTan[Sqrt[-1 + x
]/Sqrt[2]]/(8*Sqrt[2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x}}{(1+x)^3}\,dx &= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{1}{4}\int \frac{1}{\sqrt{-1+x}(1+x)^2}\,dx\\ &= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{1}{16}\int \frac{1}{\sqrt{-1+x}(1+x)}\,dx\\ &= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{1}{8}\mathrm{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\sqrt{-1+x}\right)\\ &= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{8\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.0052132, size = 28, normalized size = 0.5

$$\frac{1}{12}(x-1)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(1 + x)^3,x]

[Out] ((-1 + x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, (1 - x)/2])/12
```

Maple [A] time = 0.007, size = 40, normalized size = 0.7

$$2\,\frac{1/16\,(-1+x)^{3/2}-1/8\,\sqrt{-1+x}}{(1+x)^2}+\frac{\sqrt{2}}{16}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-1+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/2)/(1+x)^3,x)

[Out] 2*(1/16*(-1+x)^(3/2)-1/8*(-1+x)^(1/2))/(1+x)^2+1/16*arctan(1/2*(-1+x)^(1/2)
*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.43662, size = 58, normalized size = 1.04

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)+\frac{(x-1)^{\frac{3}{2}}-2\sqrt{x-1}}{8\left((x-1)^2+4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="maxima")

[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 1/8*((x - 1)^(3/2) - 2*sqrt(
x - 1))/((x - 1)^2 + 4*x)
```

Fricas [A] time = 1.83783, size = 140, normalized size = 2.5

$$\frac{\sqrt{2}\left(x^2+2\,x+1\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\sqrt{x-1}\right)+2\,\sqrt{x-1}(x-3)}{16\,\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(2)*(x^2 + 2*x + 1)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x -
1)*(x - 3))/(x^2 + 2*x + 1)
```

Sympy [A] time = 2.4935, size = 167, normalized size = 2.98

$$\begin{cases} \frac{\sqrt{2}i\,\mathrm{acosh}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{16} - \frac{i}{8\sqrt{-1+\frac{2}{x+1}}\sqrt{x+1}} + \frac{3i}{4\sqrt{-1+\frac{2}{x+1}}(x+1)^{\frac{3}{2}}} - \frac{i}{\sqrt{-1+\frac{2}{x+1}}(x+1)^{\frac{5}{2}}} & \text{for}\ \frac{2}{|x+1|} > 1 \\ -\frac{\sqrt{2}\,\mathrm{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{16} + \frac{1}{8\sqrt{1-\frac{2}{x+1}}\sqrt{x+1}} - \frac{3}{4\sqrt{1-\frac{2}{x+1}}(x+1)^{\frac{3}{2}}} + \frac{1}{\sqrt{1-\frac{2}{x+1}}(x+1)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/(1+x)**3,x)
```

```
[Out] Piecewise((sqrt(2)*I*acosh(sqrt(2)/sqrt(x + 1))/16 - I/(8*sqrt(-1 + 2/(x +
1))*sqrt(x + 1)) + 3*I/(4*sqrt(-1 + 2/(x + 1))*(x + 1)**(3/2)) - I/(sqrt(-1
 + 2/(x + 1))*(x + 1)**(5/2)), 2/Abs(x + 1) > 1), (-sqrt(2)*asin(sqrt(2)/sq
rt(x + 1))/16 + 1/(8*sqrt(1 - 2/(x + 1))*sqrt(x + 1)) - 3/(4*sqrt(1 - 2/(x
+ 1))*(x + 1)**(3/2)) + 1/(sqrt(1 - 2/(x + 1))*(x + 1)**(5/2)), True))
```

Giac [A] time = 1.08067, size = 50, normalized size = 0.89

$$\frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\sqrt{x-1}\right)+\frac{(x-1)^{\frac{3}{2}}-2\,\sqrt{x-1}}{8\,(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 1/8*((x - 1)^(3/2) - 2*sqrt(
x - 1))/(x + 1)^2
```

3.1413 $\int \frac{(a+bx)^5}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=154

$$-\frac{10b^4(c+dx)^{9/2}(bc-ad)}{9d^6}+\frac{20b^3(c+dx)^{7/2}(bc-ad)^2}{7d^6}-\frac{4b^2(c+dx)^{5/2}(bc-ad)^3}{d^6}+\frac{10b(c+dx)^{3/2}(bc-ad)^4}{3d^6}-\frac{2\sqrt{c+}}{}$$

```
[Out] (-2*(b*c - a*d)^5*Sqrt[c + d*x])/d^6 + (10*b*(b*c - a*d)^4*(c + d*x)^(3/2))
/(3*d^6) - (4*b^2*(b*c - a*d)^3*(c + d*x)^(5/2))/d^6 + (20*b^3*(b*c - a*d)^
2*(c + d*x)^(7/2))/(7*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(9/2))/(9*d^6) +
 (2*b^5*(c + d*x)^(11/2))/(11*d^6)
```

Rubi [A] time = 0.0507118, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{10b^4(c+dx)^{9/2}(bc-ad)}{9d^6}+\frac{20b^3(c+dx)^{7/2}(bc-ad)^2}{7d^6}-\frac{4b^2(c+dx)^{5/2}(bc-ad)^3}{d^6}+\frac{10b(c+dx)^{3/2}(bc-ad)^4}{3d^6}-\frac{2\sqrt{c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^5*Sqrt[c + d*x])/d^6 + (10*b*(b*c - a*d)^4*(c + d*x)^(3/2))
/(3*d^6) - (4*b^2*(b*c - a*d)^3*(c + d*x)^(5/2))/d^6 + (20*b^3*(b*c - a*d)^
2*(c + d*x)^(7/2))/(7*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(9/2))/(9*d^6) +
 (2*b^5*(c + d*x)^(11/2))/(11*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^5}{d^5\sqrt{c+dx}}+\frac{5b(bc-ad)^4\sqrt{c+dx}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{3/2}}{d^5}+\frac{10b^3(bc-ad)^2(c+dx)^{5/2}}{d^5}-\right.\\ &= -\frac{2(bc-ad)^5\sqrt{c+dx}}{d^6}+\frac{10b(bc-ad)^4(c+dx)^{3/2}}{3d^6}-\frac{4b^2(bc-ad)^3(c+dx)^{5/2}}{d^6}+\frac{20b^3(bc-ad)^2(c+d}{7d^6}\end{aligned}$$

Mathematica [A] time = 0.0822461, size = 123, normalized size = 0.8

$$\frac{2\sqrt{c+dx}\left(-1386b^2(c+dx)^2(bc-ad)^3+990b^3(c+dx)^3(bc-ad)^2-385b^4(c+dx)^4(bc-ad)+1155b(c+dx)(bc-ad)\right.}{693d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(-693*(b*c - a*d)^5 + 1155*b*(b*c - a*d)^4*(c + d*x) - 138
6*b^2*(b*c - a*d)^3*(c + d*x)^2 + 990*b^3*(b*c - a*d)^2*(c + d*x)^3 - 385*b
^4*(b*c - a*d)*(c + d*x)^4 + 63*b^5*(c + d*x)^5))/(693*d^6)
```

Maple [B] time = 0.006, size = 273, normalized size = 1.8

$$126\, b^5 x^5 d^5 + 770\, a b^4 d^5 x^4 - 140\, b^5 c d^4 x^4 + 1980\, a^2 b^3 d^5 x^3 - 880\, a b^4 c d^4 x^3 + 160\, b^5 c^2 d^3 x^3 + 2772\, a^3 b^2 d^5 x^2 - 2376\, a^2 b^3 c d^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(1/2),x)
```

```
[Out] 2/693*(d*x+c)^(1/2)*(63*b^5*d^5*x^5+385*a*b^4*d^5*x^4-70*b^5*c*d^4*x^4+990*
a^2*b^3*d^5*x^3-440*a*b^4*c*d^4*x^3+80*b^5*c^2*d^3*x^3+1386*a^3*b^2*d^5*x^2
-1188*a^2*b^3*c*d^4*x^2+528*a*b^4*c^2*d^3*x^2-96*b^5*c^3*d^2*x^2+1155*a^4*b
*d^5*x-1848*a^3*b^2*c*d^4*x+1584*a^2*b^3*c^2*d^3*x-704*a*b^4*c^3*d^2*x+128*
b^5*c^4*d*x+693*a^5*d^5-2310*a^4*b*c*d^4+3696*a^3*b^2*c^2*d^3-3168*a^2*b^3*
c^3*d^2+1408*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

Maxima [B] time = 0.983722, size = 382, normalized size = 2.48

$$2\left(693\,\sqrt{dx+c}\,a^5 + \frac{1155\left((dx+c)^{\frac{3}{2}} - 3\,\sqrt{dx+c}\,c\right)a^4 b}{d} + \frac{462\left(3\,(dx+c)^{\frac{5}{2}} - 10\,(dx+c)^{\frac{3}{2}}c + 15\,\sqrt{dx+c}\,c^2\right)a^3 b^2}{d^2} + \frac{198\left(5\,(dx+c)^{\frac{7}{2}} - 21\,(dx+c)^{\frac{5}{2}}c + 35\,(dx+c)^{\frac{3}{2}}c^2 - 3\right.}{d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(693*sqrt(d*x + c)*a^5 + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^4*b/d + 462*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a^3*b^2/d^2 + 198*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3/d^3 + 11*(35*(d*x + c)^(9/2)
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
 + 315*sqrt(d*x + c)*c^4)*a*b^4/d^4 + (63*(d*x + c)^(11/2) - 385*(d*x + c)^
(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x +
c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5/d^5)/d
```

Fricas [A] time = 1.82477, size = 586, normalized size = 3.81

$$2\left(63\, b^5 d^5 x^5 - 256\, b^5 c^5 + 1408\, a b^4 c^4 d - 3168\, a^2 b^3 c^3 d^2 + 3696\, a^3 b^2 c^2 d^3 - 2310\, a^4 b c d^4 + 693\, a^5 d^5 - 35\left(2\, b^5 c d^4 - 11\, a b^4 d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^5*d^5*x^5 - 256*b^5*c^5 + 1408*a*b^4*c^4*d - 3168*a^2*b^3*c^3*d
^2 + 3696*a^3*b^2*c^2*d^3 - 2310*a^4*b*c*d^4 + 693*a^5*d^5 - 35*(2*b^5*c*d^
4 - 11*a*b^4*d^5)*x^4 + 10*(8*b^5*c^2*d^3 - 44*a*b^4*c*d^4 + 99*a^2*b^3*d^5
```

```
)*x^3 - 6*(16*b^5*c^3*d^2 - 88*a*b^4*c^2*d^3 + 198*a^2*b^3*c*d^4 - 231*a^3*
b^2*d^5)*x^2 + (128*b^5*c^4*d - 704*a*b^4*c^3*d^2 + 1584*a^2*b^3*c^2*d^3 -
1848*a^3*b^2*c*d^4 + 1155*a^4*b*d^5)*x)*sqrt(d*x + c)/d^6
```

Sympy [A] time = 55.8994, size = 728, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise((-(2*a**5*c/sqrt(c + d*x) + 2*a**5*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) + 10*a**4*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d + 10*a**4*b*(c**2/s
qrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d + 20*a**3*b**2*c*(
c**2/sqrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 + 20*a**3
*b**2*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c
 + d*x)**(5/2)/5)/d**2 + 20*a**2*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(
c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 + 20*a**2*b**3*(c*
*4/sqrt(c + d*x) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c
+ d*x)**(5/2)/5 - (c + d*x)**(7/2)/7)/d**3 + 10*a*b**4*c*(c**4/sqrt(c + d*x
) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5
 - (c + d*x)**(7/2)/7)/d**4 + 10*a*b**4*(-c**5/sqrt(c + d*x) - 5*c**4*sqrt(
c + d*x) + 10*c**3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2) + 5*c*(c +
d*x)**(7/2)/7 - (c + d*x)**(9/2)/9)/d**4 + 2*b**5*c*(-c**5/sqrt(c + d*x) -
5*c**4*sqrt(c + d*x) + 10*c**3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2)
 + 5*c*(c + d*x)**(7/2)/7 - (c + d*x)**(9/2)/9)/d**5 + 2*b**5*(c**6/sqrt(c
+ d*x) + 6*c**5*sqrt(c + d*x) - 5*c**4*(c + d*x)**(3/2) + 4*c**3*(c + d*x)*
*(5/2) - 15*c**2*(c + d*x)**(7/2)/7 + 2*c*(c + d*x)**(9/2)/3 - (c + d*x)**(
11/2)/11)/d**5)/d, Ne(d, 0)), (Piecewise((a**5*x, Eq(b, 0)), ((a + b*x)**6/
(6*b), True))/sqrt(c), True))
```

Giac [B] time = 1.0831, size = 382, normalized size = 2.48

$$2\left(693\,\sqrt{dx+c}a^5 + \frac{1155\left((dx+c)^{\frac{3}{2}} - 3\,\sqrt{dx+c}c\right)a^4b}{d} + \frac{462\left(3\,(dx+c)^{\frac{5}{2}} - 10\,(dx+c)^{\frac{3}{2}}c + 15\,\sqrt{dx+c}c^2\right)a^3b^2}{d^2} + \frac{198\left(5\,(dx+c)^{\frac{7}{2}} - 21\,(dx+c)^{\frac{5}{2}}c + 35\,(dx+c)^{\frac{3}{2}}c\right.}{d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/693*(693*sqrt(d*x + c)*a^5 + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^4*b/d + 462*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a^3*b^2/d^2 + 198*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3/d^3 + 11*(35*(d*x + c)^(9/2)
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
 + 315*sqrt(d*x + c)*c^4)*a*b^4/d^4 + (63*(d*x + c)^(11/2) - 385*(d*x + c)^
(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x +
c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5/d^5)/d
```

3.1414 $\int \frac{(a+bx)^4}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=127

$$-\frac{8b^3(c+dx)^{7/2}(bc-ad)}{7d^5}+\frac{12b^2(c+dx)^{5/2}(bc-ad)^2}{5d^5}-\frac{8b(c+dx)^{3/2}(bc-ad)^3}{3d^5}+\frac{2\sqrt{c+dx}(bc-ad)^4}{d^5}+\frac{2b^4(c+dx)^{9/2}}{9d^5}$$

```
[Out] (2*(b*c - a*d)^4*Sqrt[c + d*x])/d^5 - (8*b*(b*c - a*d)^3*(c + d*x)^(3/2))/(
3*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(5/2))/(5*d^5) - (8*b^3*(b*c - a*d
)*(c + d*x)^(7/2))/(7*d^5) + (2*b^4*(c + d*x)^(9/2))/(9*d^5)
```

Rubi [A] time = 0.0410394, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{7/2}(bc-ad)}{7d^5}+\frac{12b^2(c+dx)^{5/2}(bc-ad)^2}{5d^5}-\frac{8b(c+dx)^{3/2}(bc-ad)^3}{3d^5}+\frac{2\sqrt{c+dx}(bc-ad)^4}{d^5}+\frac{2b^4(c+dx)^{9/2}}{9d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^4*Sqrt[c + d*x])/d^5 - (8*b*(b*c - a*d)^3*(c + d*x)^(3/2))/(
3*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(5/2))/(5*d^5) - (8*b^3*(b*c - a*d
)*(c + d*x)^(7/2))/(7*d^5) + (2*b^4*(c + d*x)^(9/2))/(9*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^4}{\sqrt{c+dx}}\,dx=\int \left(\frac{(-bc+ad)^4}{d^4\sqrt{c+dx}}-\frac{4b(bc-ad)^3\sqrt{c+dx}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{3/2}}{d^4}-\frac{4b^3(bc-ad)(c+dx)^{5/2}}{d^4}+\frac{b^4(c+\ldots}{d^4}\right.$$
$$=\frac{2(bc-ad)^4\sqrt{c+dx}}{d^5}-\frac{8b(bc-ad)^3(c+dx)^{3/2}}{3d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{5/2}}{5d^5}-\frac{8b^3(bc-ad)(c+dx)^{7/2}}{7d^5}+\ldots$$

Mathematica [A] time = 0.0827054, size = 101, normalized size = 0.8

$$\frac{2\sqrt{c+dx}\left(378b^2(c+dx)^2(bc-ad)^2-180b^3(c+dx)^3(bc-ad)-420b(c+dx)(bc-ad)^3+315(bc-ad)^4+35b^4(c+dx)^4\right)}{315d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(315*(b*c - a*d)^4 - 420*b*(b*c - a*d)^3*(c + d*x) + 378*b
^2*(b*c - a*d)^2*(c + d*x)^2 - 180*b^3*(b*c - a*d)*(c + d*x)^3 + 35*b^4*(c
```

+ d*x)^4))/(315*d^5)

Maple [A] time = 0.005, size = 186, normalized size = 1.5

$$\frac{70\,b^4x^4d^4+360\,ab^3d^4x^3-80\,b^4cd^3x^3+756\,a^2b^2d^4x^2-432\,ab^3cd^3x^2+96\,b^4c^2d^2x^2+840\,a^3bd^4x-1008\,a^2b^2cd^3x+5}{315\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(1/2),x)
```

```
[Out] 2/315*(d*x+c)^(1/2)*(35*b^4*d^4*x^4+180*a*b^3*d^4*x^3-40*b^4*c*d^3*x^3+378*
a^2*b^2*d^4*x^2-216*a*b^3*c*d^3*x^2+48*b^4*c^2*d^2*x^2+420*a^3*b*d^4*x-504*
a^2*b^2*c*d^3*x+288*a*b^3*c^2*d^2*x-64*b^4*c^3*d*x+315*a^4*d^4-840*a^3*b*c*
d^3+1008*a^2*b^2*c^2*d^2-576*a*b^3*c^3*d+128*b^4*c^4)/d^5
```

Maxima [A] time = 0.965529, size = 275, normalized size = 2.17

$$\frac{2\left(315\,\sqrt{dx+c}a^4+\frac{420\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}c\right)a^3b}{d}+\frac{126\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}c^2\right)a^2b^2}{d^2}+\frac{36\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-\right.}{d^3}\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^4 + 420*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^
3*b/d + 126*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b^2/d^2 + 36*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c
)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3/d^3 + (35*(d*x + c)^(9/2) - 180*(
d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*
sqrt(d*x + c)*c^4)*b^4/d^4)/d
```

Fricas [A] time = 1.79735, size = 405, normalized size = 3.19

$$\frac{2\left(35\,b^4d^4x^4+128\,b^4c^4-576\,ab^3c^3d+1008\,a^2b^2c^2d^2-840\,a^3bcd^3+315\,a^4d^4-20\left(2\,b^4cd^3-9\,ab^3d^4\right)x^3+6\left(8\,b^4c^2d\right.\right.}{315\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*d^4*x^4 + 128*b^4*c^4 - 576*a*b^3*c^3*d + 1008*a^2*b^2*c^2*d^
2 - 840*a^3*b*c*d^3 + 315*a^4*d^4 - 20*(2*b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 6*
(8*b^4*c^2*d^2 - 36*a*b^3*c*d^3 + 63*a^2*b^2*d^4)*x^2 - 4*(16*b^4*c^3*d - 7
2*a*b^3*c^2*d^2 + 126*a^2*b^2*c*d^3 - 105*a^3*b*d^4)*x)*sqrt(d*x + c)/d^5
```

Sympy [A] time = 39.6809, size = 532, normalized size = 4.19

$$\begin{cases} -\frac{\frac{2a^4c}{\sqrt{c+dx}}+2a^4\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)+\frac{8a^3bc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}+\frac{8a^3b\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}+\frac{12a^2b^2c\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{12a^2b^2\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\right.}{d^2} \\ \frac{\begin{cases} a^4x & \text{for } b=0 \\ \frac{(a+bx)^5}{5b} & \text{otherwise} \end{cases}}{\sqrt{c}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise((-(2*a**4*c/sqrt(c + d*x) + 2*a**4*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) + 8*a**3*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d + 8*a**3*b*(c**2/sqr
t(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d + 12*a**2*b**2*c*(c*
*2/sqrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 + 12*a**2*b
**2*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c +
 d*x)**(5/2)/5)/d**2 + 8*a*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*
x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 + 8*a*b**3*(c**4/sqrt(c
+ d*x) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5
/2)/5 - (c + d*x)**(7/2)/7)/d**3 + 2*b**4*c*(c**4/sqrt(c + d*x) + 4*c**3*sq
rt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5 - (c + d*x)*
*(7/2)/7)/d**4 + 2*b**4*(-c**5/sqrt(c + d*x) - 5*c**4*sqrt(c + d*x) + 10*c*
*3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2) + 5*c*(c + d*x)**(7/2)/7 -
(c + d*x)**(9/2)/9)/d**4)/d, Ne(d, 0)), (Piecewise((a**4*x, Eq(b, 0)), ((a
+ b*x)**5/(5*b), True))/sqrt(c), True))
```

Giac [A] time = 1.06405, size = 275, normalized size = 2.17

$$\frac{2\left(315\sqrt{dx+c}a^4+\frac{420\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}c\right)a^3b}{d}+\frac{126\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}c^2\right)a^2b^2}{d^2}+\frac{36\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{\right.}}{d^3}\right.}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^4 + 420*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^
3*b/d + 126*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b^2/d^2 + 36*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c
)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3/d^3 + (35*(d*x + c)^(9/2) - 180*(
d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*
sqrt(d*x + c)*c^4)*b^4/d^4)/d
```

3.1415 $\int \frac{(a+bx)^3}{\sqrt{c+dx}} dx$

Optimal. Leaf size=96

$$-\frac{6b^2(c+dx)^{5/2}(bc-ad)}{5d^4}+\frac{2b(c+dx)^{3/2}(bc-ad)^2}{d^4}-\frac{2\sqrt{c+dx}(bc-ad)^3}{d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}$$

```
[Out] (-2*(b*c - a*d)^3*Sqrt[c + d*x])/d^4 + (2*b*(b*c - a*d)^2*(c + d*x)^(3/2))/
d^4 - (6*b^2*(b*c - a*d)*(c + d*x)^(5/2))/(5*d^4) + (2*b^3*(c + d*x)^(7/2))
/(7*d^4)
```

Rubi [A] time = 0.0308993, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{6b^2(c+dx)^{5/2}(bc-ad)}{5d^4}+\frac{2b(c+dx)^{3/2}(bc-ad)^2}{d^4}-\frac{2\sqrt{c+dx}(bc-ad)^3}{d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^3*Sqrt[c + d*x])/d^4 + (2*b*(b*c - a*d)^2*(c + d*x)^(3/2))/
d^4 - (6*b^2*(b*c - a*d)*(c + d*x)^(5/2))/(5*d^4) + (2*b^3*(c + d*x)^(7/2))
/(7*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3\sqrt{c+dx}}+\frac{3b(bc-ad)^2\sqrt{c+dx}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{3/2}}{d^3}+\frac{b^3(c+dx)^{5/2}}{d^3}\right)dx\\ &= -\frac{2(bc-ad)^3\sqrt{c+dx}}{d^4}+\frac{2b(bc-ad)^2(c+dx)^{3/2}}{d^4}-\frac{6b^2(bc-ad)(c+dx)^{5/2}}{5d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}\end{aligned}$$

Mathematica [A] time = 0.0667105, size = 79, normalized size = 0.82

$$\frac{2\sqrt{c+dx}\left(-21b^2(c+dx)^2(bc-ad)+35b(c+dx)(bc-ad)^2-35(bc-ad)^3+5b^3(c+dx)^3\right)}{35d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(-35*(b*c - a*d)^3 + 35*b*(b*c - a*d)^2*(c + d*x) - 21*b^2
*(b*c - a*d)*(c + d*x)^2 + 5*b^3*(c + d*x)^3))/(35*d^4)
```

Maple [A] time = 0.005, size = 116, normalized size = 1.2

$$\frac{10\,b^3x^3d^3+42\,ab^2d^3x^2-12\,b^3cd^2x^2+70\,a^2bd^3x-56\,ab^2cd^2x+16\,b^3c^2dx+70\,a^3d^3-140\,a^2bcd^2+112\,ab^2c^2d-32\,b^3c^3}{35\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^(1/2),x)
```

```
[Out] 2/35*(d*x+c)^(1/2)*(5*b^3*d^3*x^3+21*a*b^2*d^3*x^2-6*b^3*c*d^2*x^2+35*a^2*b
*d^3*x-28*a*b^2*c*d^2*x+8*b^3*c^2*d*x+35*a^3*d^3-70*a^2*b*c*d^2+56*a*b^2*c^
2*d-16*b^3*c^3)/d^4
```

Maxima [A] time = 0.982013, size = 185, normalized size = 1.93

$$\frac{2\left(35\,\sqrt{dx+c}a^3+\frac{35\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}c\right)a^2b}{d}+\frac{7\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}c^2\right)ab^2}{d^2}+\frac{\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}c^3\right)}{d^3}\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(35*sqrt(d*x + c)*a^3 + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b
/d + 7*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*
b^2/d^2 + (5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^
2 - 35*sqrt(d*x + c)*c^3)*b^3/d^3)/d
```

Fricas [A] time = 1.78684, size = 251, normalized size = 2.61

$$\frac{2\left(5\,b^3d^3x^3-16\,b^3c^3+56\,ab^2c^2d-70\,a^2bcd^2+35\,a^3d^3-3\left(2\,b^3cd^2-7\,ab^2d^3\right)x^2+\left(8\,b^3c^2d-28\,ab^2cd^2+35\,a^2bd^3\right)x\right)\sqrt{c}}{35\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*d^3*x^3 - 16*b^3*c^3 + 56*a*b^2*c^2*d - 70*a^2*b*c*d^2 + 35*a^3
*d^3 - 3*(2*b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + (8*b^3*c^2*d - 28*a*b^2*c*d^2 +
35*a^2*b*d^3)*x)*sqrt(d*x + c)/d^4
```

Sympy [A] time = 25.7393, size = 366, normalized size = 3.81

$$\begin{cases} -\frac{\frac{2a^3c}{\sqrt{c+dx}}+2a^3\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)+\frac{6a^2bc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}+\frac{6a^2b\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}+\frac{6ab^2c\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{6ab^2\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\frac{(c+}{}\right.}{d^2}}{d} \\ \frac{\begin{cases} a^3x & \text{for } b=0 \\ \frac{(a+bx)^4}{4b} & \text{otherwise} \end{cases}}{\sqrt{c}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise((-(2*a**3*c/sqrt(c + d*x) + 2*a**3*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) + 6*a**2*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d + 6*a**2*b*(c**2/sqr
t(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d + 6*a*b**2*c*(c**2/s
qrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 + 6*a*b**2*(-c*
*3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(
5/2)/5)/d**2 + 2*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c
+ d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 + 2*b**3*(c**4/sqrt(c + d*x) + 4*c
**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5 - (c +
 d*x)**(7/2)/7)/d**3)/d, Ne(d, 0)), (Piecewise((a**3*x, Eq(b, 0)), ((a + b*
x)**4/(4*b), True))/sqrt(c), True))
```

Giac [A] time = 1.07964, size = 185, normalized size = 1.93

$$\frac{2\left(35\sqrt{dx+c}a^3+\frac{35\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}c\right)a^2b}{d}+\frac{7\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}c^2\right)ab^2}{d^2}+\frac{\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+\cdots}\right.}{d^3}\cdots\right)}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*sqrt(d*x + c)*a^3 + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b
/d + 7*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*
b^2/d^2 + (5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^
2 - 35*sqrt(d*x + c)*c^3)*b^3/d^3)/d
```

3.1416 $\int \frac{(a+bx)^2}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=69

$$-\frac{4b(c+dx)^{3/2}(bc-ad)}{3d^3}+\frac{2\sqrt{c+dx}(bc-ad)^2}{d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}$$

```
[Out] (2*(b*c - a*d)^2*Sqrt[c + d*x])/d^3 - (4*b*(b*c - a*d)*(c + d*x)^(3/2))/(3*
d^3) + (2*b^2*(c + d*x)^(5/2))/(5*d^3)
```

Rubi [A] time = 0.021249, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b(c+dx)^{3/2}(bc-ad)}{3d^3}+\frac{2\sqrt{c+dx}(bc-ad)^2}{d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[c + d*x])/d^3 - (4*b*(b*c - a*d)*(c + d*x)^(3/2))/(3*
d^3) + (2*b^2*(c + d*x)^(5/2))/(5*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2\sqrt{c+dx}}-\frac{2b(bc-ad)\sqrt{c+dx}}{d^2}+\frac{b^2(c+dx)^{3/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2\sqrt{c+dx}}{d^3}-\frac{4b(bc-ad)(c+dx)^{3/2}}{3d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}\end{aligned}$$

Mathematica [A] time = 0.0352896, size = 60, normalized size = 0.87

$$\frac{2\sqrt{c+dx}\left(15a^2d^2+10abd(dx-2c)+b^2\left(8c^2-4cdx+3d^2x^2\right)\right)}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x) + b^2*(8*c^2 - 4*c*d*x
 + 3*d^2*x^2)))/(15*d^3)
```

Maple [A] time = 0.004, size = 63, normalized size = 0.9

$$\frac{6\,b^{2}x^{2}d^{2}+20\,abd^{2}x-8\,b^{2}cdx+30\,a^{2}d^{2}-40\,abcd+16\,b^{2}c^{2}}{15\,d^{3}}\sqrt{dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] 2/15*(d*x+c)^(1/2)*(3*b^2*d^2*x^2+10*a*b*d^2*x-4*b^2*c*d*x+15*a^2*d^2-20*a*
b*c*d+8*b^2*c^2)/d^3
```

Maxima [A] time = 0.964189, size = 111, normalized size = 1.61

$$\frac{2\left(15\,\sqrt{dx+c}a^{2}+\frac{10\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}c\right)ab}{d}+\frac{\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}c^{2}\right)b^{2}}{d^{2}}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(d*x + c)*a^2 + 10*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b/d
 + (3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2/d^
2)/d
```

Fricas [A] time = 1.76244, size = 146, normalized size = 2.12

$$\frac{2\left(3\,b^{2}d^{2}x^{2}+8\,b^{2}c^{2}-20\,abcd+15\,a^{2}d^{2}-2\left(2\,b^{2}cd-5\,abd^{2}\right)x\right)\sqrt{dx+c}}{15\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*d^2*x^2 + 8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 - 2*(2*b^2*c*d -
5*a*b*d^2)*x)*sqrt(d*x + c)/d^3
```

Sympy [A] time = 14.4817, size = 231, normalized size = 3.35

$$\begin{cases} -\frac{\frac{2a^{2}c}{\sqrt{c+dx}}+2a^{2}\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)+\frac{4abc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}+\frac{4ab\left(\frac{c^{2}}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}+\frac{2b^{2}c\left(\frac{c^{2}}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^{2}}+\frac{2b^{2}\left(-\frac{c^{3}}{\sqrt{c+dx}}-3c^{2}\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\cdots\right.}{d^{2}}}{d} \\ \frac{\begin{cases} a^{2}x & \text{for}\ b=0 \\ \frac{(a+bx)^{3}}{3b} & \text{otherwise} \end{cases}}{\sqrt{c}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise((-(2*a**2*c/sqrt(c + d*x) + 2*a**2*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) + 4*a*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d + 4*a*b*(c**2/sqrt(c +
d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d + 2*b**2*c*(c**2/sqrt(c +
d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 + 2*b**2*(-c**3/sqrt(c
+ d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d*
*2)/d, Ne(d, 0)), (Piecewise((a**2*x, Eq(b, 0)), ((a + b*x)**3/(3*b), True)
)/sqrt(c), True))
```

Giac [A] time = 1.10357, size = 111, normalized size = 1.61

$$\frac{2\left(15\sqrt{dx+c}a^2+\frac{10\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}c\right)ab}{d}+\frac{\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}c^2\right)b^2}{d^2}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(d*x + c)*a^2 + 10*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b/d
 + (3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2/d^
2)/d
```

3.1417 $\int \frac{a+bx}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=40

$$\frac{2b(c+dx)^{3/2}}{3d^2}-\frac{2\sqrt{c+dx}\,(bc-ad)}{d^2}$$

```
[Out] (-2*(b*c - a*d)*Sqrt[c + d*x])/d^2 + (2*b*(c + d*x)^(3/2))/(3*d^2)
```

Rubi [A] time = 0.0129204, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(c+dx)^{3/2}}{3d^2}-\frac{2\sqrt{c+dx}\,(bc-ad)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)*Sqrt[c + d*x])/d^2 + (2*b*(c + d*x)^(3/2))/(3*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt{c+dx}}\,dx &= \int \left(\frac{-bc+ad}{d\sqrt{c+dx}}+\frac{b\sqrt{c+dx}}{d}\right)dx\\ &= -\frac{2(bc-ad)\sqrt{c+dx}}{d^2}+\frac{2b(c+dx)^{3/2}}{3d^2}\end{aligned}$$

Mathematica [A] time = 0.0171093, size = 29, normalized size = 0.72

$$\frac{2\sqrt{c+dx}\,(3ad-2bc+bdx)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(-2*b*c + 3*a*d + b*d*x))/(3*d^2)
```

Maple [A] time = 0.002, size = 26, normalized size = 0.7

$$\frac{2\,bdx+6\,ad-4\,bc}{3\,d^2}\sqrt{dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(1/2),x)
```

```
[Out] 2/3*(d*x+c)^(1/2)*(b*d*x+3*a*d-2*b*c)/d^2
```

Maxima [A] time = 0.955858, size = 53, normalized size = 1.32

$$\frac{2\left(3\sqrt{dx+c}a+\frac{\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}c\right)b}{d}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(d*x + c)*a + ((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b/d)/d
```

Fricas [A] time = 1.86197, size = 63, normalized size = 1.58

$$\frac{2\left(bdx-2bc+3ad\right)\sqrt{dx+c}}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*d*x - 2*b*c + 3*a*d)*sqrt(d*x + c)/d^2
```

Sympy [A] time = 3.27413, size = 121, normalized size = 3.02

$$\begin{cases} -\frac{\frac{2ac}{\sqrt{c+dx}}+2a\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)+\frac{2bc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}+\frac{2b\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}}{d} & \text{for } d \neq 0 \\ \frac{ax+\frac{bx^2}{2}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*c/sqrt(c + d*x) + 2*a*(-c/sqrt(c + d*x) - sqrt(c + d*x)) +
 2*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d + 2*b*(c**2/sqrt(c + d*x) + 2*c
*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d)/d, Ne(d, 0)), ((a*x + b*x**2/2)/sqr
t(c), True))
```

Giac [A] time = 1.05598, size = 53, normalized size = 1.32

$$\frac{2\left(3\sqrt{dx+c}a+\frac{\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}c\right)b}{d}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(d*x + c)*a + ((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b/d)/d
```

3.1418 $\int \frac{1}{\sqrt{c+dx}} dx$

Optimal. Leaf size=14

$$\frac{2\sqrt{c+dx}}{d}$$

```
[Out] (2*Sqrt[c + d*x])/d
```

Rubi [A] time = 0.0013672, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c + d*x],x]

[Out] (2*Sqrt[c + d*x])/d
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+dx}} dx = \frac{2\sqrt{c+dx}}{d}$$

Mathematica [A] time = 0.0033623, size = 14, normalized size = 1.

$$\frac{2\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c + d*x],x]

[Out] (2*Sqrt[c + d*x])/d
```

Maple [A] time = 0.002, size = 13, normalized size = 0.9

$$2\,\frac{\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(1/2),x)
```

```
[Out] 2*(d*x+c)^(1/2)/d
```

Maxima [A] time = 0.946391, size = 16, normalized size = 1.14

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(d*x + c)/d
```

Fricas [A] time = 1.77387, size = 26, normalized size = 1.86

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(d*x + c)/d
```

Sympy [A] time = 0.05371, size = 10, normalized size = 0.71

$$\frac{2\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(1/2),x)
```

```
[Out] 2*sqrt(c + d*x)/d
```

Giac [A] time = 1.0456, size = 16, normalized size = 1.14

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)/d
```

3.1419 $\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx$

Optimal. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a
*d])
```

Rubi [A] time = 0.0203073, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a
*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.0134536, size = 47, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a
*d])
```

Maple [A] time = 0.004, size = 37, normalized size = 0.8

$$2\frac{1}{\sqrt{(ad-bc)\,b}}\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2),x)
```

```
[Out] 2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86864, size = 266, normalized size = 5.66

$$\left[\frac{\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)}{\sqrt{b^2c-abd}},\ \frac{2\,\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{-b^2c+abd}\sqrt{dx+c}}{bdx+bc}\right)}{b^2c-abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a))
/sqrt(b^2*c - a*b*d), 2*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sq
rt(d*x + c)/(b*d*x + b*c))/(b^2*c - a*b*d)]
```

Sympy [A] time = 3.00064, size = 44, normalized size = 0.94

$$-\frac{2\operatorname{atan}\left(\frac{1}{\sqrt{\frac{b}{ad-bc}}\sqrt{c+dx}}\right)}{\sqrt{\frac{b}{ad-bc}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] -2*atan(1/(sqrt(b/(a*d - b*c))*sqrt(c + d*x)))/(sqrt(b/(a*d - b*c))*(a*d -
b*c))
```

Giac [A] time = 1.06016, size = 51, normalized size = 1.09

$$\frac{2\,\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

3.1420 $\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx$

Optimal. Leaf size=76

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx}}{(a+bx)(bc-ad)}$$

```
[Out] -(Sqrt[c + d*x]/((b*c - a*d)*(a + b*x))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0269979, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx}}{(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*Sqrt[c + d*x]),x]
```

```
[Out] -(Sqrt[c + d*x]/((b*c - a*d)*(a + b*x))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)} - \frac{d\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{bc-ad}$$

$$= -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.0685772, size = 76, normalized size = 1.

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{ad-bc}}\right)}{\sqrt{b}(ad-bc)^{3/2}} - \frac{\sqrt{c+dx}}{(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*Sqrt[c + d*x]),x]
```

```
[Out] -(Sqrt[c + d*x]/((b*c - a*d)*(a + b*x))) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

Maple [A] time = 0.008, size = 77, normalized size = 1.

$$\frac{d}{(ad-bc)(bdx+ad)}\sqrt{dx+c} + \frac{d}{ad-bc}\arctan\left(b\sqrt{dx+c}\frac{1}{\sqrt{(ad-bc)b}}\right)\frac{1}{\sqrt{(ad-bc)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] d*(d*x+c)^(1/2)/(a*d-b*c)/(b*d*x+a*d)+d/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arcta
n(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.1204, size = 603, normalized size = 7.93

$$\left[-\frac{\sqrt{b^2c-abd}(bdx+ad)\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)+2\left(b^2c-abd\right)\sqrt{dx+c}}{2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x\right)},-\frac{\sqrt{-b^2c+abd}(bdx+ad)\arctan\left(\frac{\sqrt{}}{}\right.}{ab^3c^2-2\,a^2b^2cd+a^3bd^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(b^2*c - a*b*d)*(b*d*x + a*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(
b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(b^2*c - a*b*d)*sqrt(d*x + c))
/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*
d^2)*x), -(sqrt(-b^2*c + a*b*d)*(b*d*x + a*d)*arctan(sqrt(-b^2*c + a*b*d)*s
qrt(d*x + c)/(b*d*x + b*c)) + (b^2*c - a*b*d)*sqrt(d*x + c))/(a*b^3*c^2 - 2
*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.07563, size = 117, normalized size = 1.54

$$-\frac{d\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}-\frac{\sqrt{dx+c}d}{((dx+c)b-bc+ad)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c
- a*d)) - sqrt(d*x + c)*d/(((d*x + c)*b - b*c + a*d)*(b*c - a*d))
```

3.1421 $\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx$

Optimal. Leaf size=114

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}(bc-ad)^{5/2}}+\frac{3d\sqrt{c+dx}}{4(a+bx)(bc-ad)^2}-\frac{\sqrt{c+dx}}{2(a+bx)^2(bc-ad)}$$

```
[Out] -Sqrt[c + d*x]/(2*(b*c - a*d)*(a + b*x)^2) + (3*d*Sqrt[c + d*x])/(4*(b*c -
a*d)^2*(a + b*x)) - (3*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]]
)/(4*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.0372377, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}(bc-ad)^{5/2}}+\frac{3d\sqrt{c+dx}}{4(a+bx)(bc-ad)^2}-\frac{\sqrt{c+dx}}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*Sqrt[c + d*x]),x]
```

```
[Out] -Sqrt[c + d*x]/(2*(b*c - a*d)*(a + b*x)^2) + (3*d*Sqrt[c + d*x])/(4*(b*c -
a*d)^2*(a + b*x)) - (3*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]]
)/(4*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2} - \frac{(3d)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{4(bc-ad)}$$
$$= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2} + \frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)} + \frac{(3d^2)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8(bc-ad)^2}$$
$$= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2} + \frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)} + \frac{(3d)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{4(bc-ad)^2}$$
$$= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2} + \frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)} - \frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.0110318, size = 50, normalized size = 0.44

$$\frac{2d^2\sqrt{c+dx}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};-\frac{b(c+dx)}{ad-bc}\right)}{(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*Sqrt[c + d*x]),x]
```

```
[Out] (2*d^2*Sqrt[c + d*x]*Hypergeometric2F1[1/2, 3, 3/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(-(b*c) + a*d)^3
```

Maple [A] time = 0.008, size = 115, normalized size = 1.

$$\frac{d^2}{(2\,ad-2\,bc)\,(bdx+ad)^2}\sqrt{dx+c} + \frac{3\,d^2}{4\,(ad-bc)^2\,(bdx+ad)}\sqrt{dx+c} + \frac{3\,d^2}{4\,(ad-bc)^2}\arctan\left(b\sqrt{dx+c}\frac{1}{\sqrt{(ad-bc)\,b}}\right)\frac{1}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(1/2),x)
```

```
[Out] 1/2*d^2*(d*x+c)^(1/2)/(a*d-b*c)/(b*d*x+a*d)^2+3/4*d^2/(a*d-b*c)^2*(d*x+c)^(
1/2)/(b*d*x+a*d)+3/4*d^2/(a*d-b*c)^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(
1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.35511, size = 1119, normalized size = 9.82

$$\left[\frac{3\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\sqrt{b^2c-abd}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)-2\left(2\,b^3c^2-7\,ab^2cd+5\,a^2bd^2-3\left(b^3cd-ab^2d^2\right.\right.}{8\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^2+2\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*
x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(2*b^
3*c^2 - 7*a*b^2*c*d + 5*a^2*b*d^2 - 3*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c
))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3 + (b^6*c^3
- 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^2 + 2*(a*b^5*c^3 - 3*a^2
*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x), 1/4*(3*(b^2*d^2*x^2 + 2*a*b
*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x
 + c)/(b*d*x + b*c)) - (2*b^3*c^2 - 7*a*b^2*c*d + 5*a^2*b*d^2 - 3*(b^3*c*d
- a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c
*d^2 - a^5*b*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3
)*x^2 + 2*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.0649, size = 200, normalized size = 1.75

$$\frac{3\,d^2\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{4\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{3\,(dx+c)^{\frac{3}{2}}bd^2-5\,\sqrt{dx+c}bcd^2+5\,\sqrt{dx+c}ad^3}{4\left(b^2c^2-2\,abcd+a^2d^2\right)\left((dx+c)b-bc+ad\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/4*(3*(d*x + c)^(3/2)*b*d^2 - 5*sqrt(d*
x + c)*b*c*d^2 + 5*sqrt(d*x + c)*a*d^3)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((
d*x + c)*b - b*c + a*d)^2)
```

3.1422 $\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx$

Optimal. Leaf size=147

$$-\frac{5d^2\sqrt{c+dx}}{8(a+bx)(bc-ad)^3}+\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}(bc-ad)^{7/2}}+\frac{5d\sqrt{c+dx}}{12(a+bx)^2(bc-ad)^2}-\frac{\sqrt{c+dx}}{3(a+bx)^3(bc-ad)}$$

```
[Out] -Sqrt[c + d*x]/(3*(b*c - a*d)*(a + b*x)^3) + (5*d*Sqrt[c + d*x])/(12*(b*c -
 a*d)^2*(a + b*x)^2) - (5*d^2*Sqrt[c + d*x])/(8*(b*c - a*d)^3*(a + b*x)) +
(5*d^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*Sqrt[b]*(b*c -
a*d)^(7/2))
```

Rubi [A] time = 0.0504401, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{5d^2\sqrt{c+dx}}{8(a+bx)(bc-ad)^3}+\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}(bc-ad)^{7/2}}+\frac{5d\sqrt{c+dx}}{12(a+bx)^2(bc-ad)^2}-\frac{\sqrt{c+dx}}{3(a+bx)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*Sqrt[c + d*x]),x]
```


```
[Out] -Sqrt[c + d*x]/(3*(b*c - a*d)*(a + b*x)^3) + (5*d*Sqrt[c + d*x])/(12*(b*c -
 a*d)^2*(a + b*x)^2) - (5*d^2*Sqrt[c + d*x])/(8*(b*c - a*d)^3*(a + b*x)) +
(5*d^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*Sqrt[b]*(b*c -
a*d)^(7/2))
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} - \frac{(5d)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{6(bc-ad)}$$
$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} + \frac{(5d^2)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8(bc-ad)^2}$$
$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} - \frac{(5d^3)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16(bc-ad)^3}$$
$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} - \frac{(5d^2)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{8(bc-ad}$$
$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} + \frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}(bc-ad)^{7/2}}$$

Mathematica [C] time = 0.0115655, size = 50, normalized size = 0.34

$$\frac{2d^3\sqrt{c+dx}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};-\frac{b(c+dx)}{ad-bc}\right)}{(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*Sqrt[c + d*x]),x]
```

```
[Out] (2*d^3*Sqrt[c + d*x]*Hypergeometric2F1[1/2, 4, 3/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(-(b*c) + a*d)^4
```

Maple [A] time = 0.007, size = 147, normalized size = 1.

$$\frac{d^3}{(3\,ad-3\,bc)\,(bdx+ad)^3}\sqrt{dx+c} + \frac{5\,d^3}{12\,(ad-bc)^2\,(bdx+ad)^2}\sqrt{dx+c} + \frac{5\,d^3}{8\,(ad-bc)^3\,(bdx+ad)}\sqrt{dx+c} + \frac{5\,d^3}{8\,(ad-bc)^3}\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*d^3*(d*x+c)^(1/2)/(a*d-b*c)/(b*d*x+a*d)^3+5/12*d^3/(a*d-b*c)^2*(d*x+c)^
(1/2)/(b*d*x+a*d)^2+5/8*d^3/(a*d-b*c)^3*(d*x+c)^(1/2)/(b*d*x+a*d)+5/8*d^3/(
a*d-b*c)^3/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.26014, size = 1805, normalized size = 12.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b
^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c
))/(b*x + a)) + 2*(8*b^4*c^3 - 34*a*b^3*c^2*d + 59*a^2*b^2*c*d^2 - 33*a^3*b
*d^3 + 15*(b^4*c*d^2 - a*b^3*d^3)*x^2 - 10*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a
^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*
c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^3 + 3*(a*b^7*c^4 - 4*a^2*b^6*c^3
*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^2 + 3*(a^2*b^6*c^
4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x)
, -1/24*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) +
(8*b^4*c^3 - 34*a*b^3*c^2*d + 59*a^2*b^2*c*d^2 - 33*a^3*b*d^3 + 15*(b^4*c*d
^2 - a*b^3*d^3)*x^2 - 10*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqr
t(d*x + c))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*
c*d^3 + a^7*b*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^
5*c*d^3 + a^4*b^4*d^4)*x^3 + 3*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2
*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^2 + 3*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*
d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.07195, size = 312, normalized size = 2.12

$$-\frac{5\,d^3\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{8\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}-\frac{15\,(dx+c)^{\frac{5}{2}}b^2d^3-40\,(dx+c)^{\frac{3}{2}}b^2cd^3+33\,\sqrt{dx+c}b^2c^2d^3+40}{24\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -5/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - 1/24*(15*(d*x + c)^
(5/2)*b^2*d^3 - 40*(d*x + c)^(3/2)*b^2*c*d^3 + 33*sqrt(d*x + c)*b^2*c^2*d^3
```

```
 + 40*(d*x + c)^(3/2)*a*b*d^4 - 66*sqrt(d*x + c)*a*b*c*d^4 + 33*sqrt(d*x +
c)*a^2*d^5)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)
*b - b*c + a*d)^3)
```

3.1423 $\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx$

Optimal. Leaf size=180

$$\frac{35d^3\sqrt{c+dx}}{64(a+bx)(bc-ad)^4}-\frac{35d^2\sqrt{c+dx}}{96(a+bx)^2(bc-ad)^3}-\frac{35d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64\sqrt{b}(bc-ad)^{9/2}}+\frac{7d\sqrt{c+dx}}{24(a+bx)^3(bc-ad)^2}-\frac{\sqrt{c+dx}}{4(a+bx)^4(bc-ad)}$$

```
[Out] -Sqrt[c + d*x]/(4*(b*c - a*d)*(a + b*x)^4) + (7*d*Sqrt[c + d*x])/(24*(b*c -
 a*d)^2*(a + b*x)^3) - (35*d^2*Sqrt[c + d*x])/(96*(b*c - a*d)^3*(a + b*x)^2
) + (35*d^3*Sqrt[c + d*x])/(64*(b*c - a*d)^4*(a + b*x)) - (35*d^4*ArcTanh[(
Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(64*Sqrt[b]*(b*c - a*d)^(9/2))
```

Rubi [A] time = 0.0647658, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{35d^3\sqrt{c+dx}}{64(a+bx)(bc-ad)^4}-\frac{35d^2\sqrt{c+dx}}{96(a+bx)^2(bc-ad)^3}-\frac{35d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64\sqrt{b}(bc-ad)^{9/2}}+\frac{7d\sqrt{c+dx}}{24(a+bx)^3(bc-ad)^2}-\frac{\sqrt{c+dx}}{4(a+bx)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^5*Sqrt[c + d*x]),x]
```


```
[Out] -Sqrt[c + d*x]/(4*(b*c - a*d)*(a + b*x)^4) + (7*d*Sqrt[c + d*x])/(24*(b*c -
 a*d)^2*(a + b*x)^3) - (35*d^2*Sqrt[c + d*x])/(96*(b*c - a*d)^3*(a + b*x)^2
) + (35*d^3*Sqrt[c + d*x])/(64*(b*c - a*d)^4*(a + b*x)) - (35*d^4*ArcTanh[(
Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(64*Sqrt[b]*(b*c - a*d)^(9/2))
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} - \frac{(7d)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{8(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} + \frac{\left(35d^2\right)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{48(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} - \frac{\left(35d^3\right)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}}{64(bc-ad)^3}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{35d^3\sqrt{c+dx}}{64(bc-ad)^4(a+bx)}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{35d^3\sqrt{c+dx}}{64(bc-ad)^4(a+bx)}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{35d^3\sqrt{c+dx}}{64(bc-ad)^4(a+bx)}$$

Mathematica [C] time = 0.0117945, size = 50, normalized size = 0.28

$$\frac{2d^4\sqrt{c+dx}\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};-\frac{b(c+dx)}{ad-bc}\right)}{(ad-bc)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^5*Sqrt[c + d*x]),x]
```

```
[Out] (2*d^4*Sqrt[c + d*x]*Hypergeometric2F1[1/2, 5, 3/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(-(b*c) + a*d)^5
```

Maple [A] time = 0.006, size = 179, normalized size = 1.

$$\frac{d^4}{(4ad-4bc)(bdx+ad)^4}\sqrt{dx+c} + \frac{7d^4}{24(ad-bc)^2(bdx+ad)^3}\sqrt{dx+c} + \frac{35d^4}{96(ad-bc)^3(bdx+ad)^2}\sqrt{dx+c} + \frac{35}{64(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^5/(d*x+c)^(1/2),x)
```

```
[Out] 1/4*d^4*(d*x+c)^(1/2)/(a*d-b*c)/(b*d*x+a*d)^4+7/24*d^4/(a*d-b*c)^2*(d*x+c)^
(1/2)/(b*d*x+a*d)^3+35/96*d^4/(a*d-b*c)^3*(d*x+c)^(1/2)/(b*d*x+a*d)^2+35/64
*d^4/(a*d-b*c)^4*(d*x+c)^(1/2)/(b*d*x+a*d)+35/64*d^4/(a*d-b*c)^4/((a*d-b*c)
*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39643, size = 2709, normalized size = 15.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(105*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(48*b^5*c^4 - 248*a*b^4*c^3*d + 526*
a^2*b^3*c^2*d^2 - 605*a^3*b^2*c*d^3 + 279*a^4*b*d^4 - 105*(b^5*c*d^3 - a*b^
4*d^4)*x^3 + 35*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 - 7*(
8*b^5*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqr
t(d*x + c))/(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*c^3*d^2 - 10*a^7*b^
3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5 + (b^10*c^5 - 5*a*b^9*c^4*d + 10*a^
2*b^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*x^4 + 4
*(a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5
*a^5*b^5*c*d^4 - a^6*b^4*d^5)*x^3 + 6*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a
^4*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*x^2 +
4*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d^3
+ 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*x), 1/192*(105*(b^4*d^4*x^4 + 4*a*b^3*d^4*
x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arc
tan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (48*b^5*c^4 - 248*a
*b^4*c^3*d + 526*a^2*b^3*c^2*d^2 - 605*a^3*b^2*c*d^3 + 279*a^4*b*d^4 - 105*
(b^5*c*d^3 - a*b^4*d^4)*x^3 + 35*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b
^3*d^4)*x^2 - 7*(8*b^5*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^
3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*c^
3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5 + (b^10*c^5 - 5*a*
b^9*c^4*d + 10*a^2*b^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5
*b^5*d^5)*x^4 + 4*(a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^
4*b^6*c^2*d^3 + 5*a^5*b^5*c*d^4 - a^6*b^4*d^5)*x^3 + 6*(a^2*b^8*c^5 - 5*a^3
*b^7*c^4*d + 10*a^4*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^
7*b^3*d^5)*x^2 + 4*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10
*a^6*b^4*c^2*d^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**5/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.07056, size = 447, normalized size = 2.48

$$\frac{35\,d^4\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{64\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{-b^2c+abd}}+\frac{105\,(dx+c)^{\frac{7}{2}}b^3d^4-385\,(dx+c)^{\frac{5}{2}}b^3cd^4+511\,(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 35/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d))
+ 1/192*(105*(d*x + c)^(7/2)*b^3*d^4 - 385*(d*x + c)^(5/2)*b^3*c*d^4 + 511*
(d*x + c)^(3/2)*b^3*c^2*d^4 - 279*sqrt(d*x + c)*b^3*c^3*d^4 + 385*(d*x + c)
^(5/2)*a*b^2*d^5 - 1022*(d*x + c)^(3/2)*a*b^2*c*d^5 + 837*sqrt(d*x + c)*a*b
^2*c^2*d^5 + 511*(d*x + c)^(3/2)*a^2*b*d^6 - 837*sqrt(d*x + c)*a^2*b*c*d^6
+ 279*sqrt(d*x + c)*a^3*d^7)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)*((d*x + c)*b - b*c + a*d)^4)
```

3.1424 $\int \frac{(a+bx)^5}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{10b^4(c+dx)^{7/2}(bc-ad)}{7d^6}+\frac{4b^3(c+dx)^{5/2}(bc-ad)^2}{d^6}-\frac{20b^2(c+dx)^{3/2}(bc-ad)^3}{3d^6}+\frac{10b\sqrt{c+dx}(bc-ad)^4}{d^6}+\frac{2(bc-ac}{d^6\sqrt{c+c}}$$

```
[Out] (2*(b*c - a*d)^5)/(d^6*Sqrt[c + d*x]) + (10*b*(b*c - a*d)^4*Sqrt[c + d*x])/
d^6 - (20*b^2*(b*c - a*d)^3*(c + d*x)^(3/2))/(3*d^6) + (4*b^3*(b*c - a*d)^2
*(c + d*x)^(5/2))/d^6 - (10*b^4*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^6) + (2*b
^5*(c + d*x)^(9/2))/(9*d^6)
```

Rubi [A] time = 0.0486974, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{10b^4(c+dx)^{7/2}(bc-ad)}{7d^6}+\frac{4b^3(c+dx)^{5/2}(bc-ad)^2}{d^6}-\frac{20b^2(c+dx)^{3/2}(bc-ad)^3}{3d^6}+\frac{10b\sqrt{c+dx}(bc-ad)^4}{d^6}+\frac{2(bc-ac}{d^6\sqrt{c+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^5)/(d^6*Sqrt[c + d*x]) + (10*b*(b*c - a*d)^4*Sqrt[c + d*x])/
d^6 - (20*b^2*(b*c - a*d)^3*(c + d*x)^(3/2))/(3*d^6) + (4*b^3*(b*c - a*d)^2
*(c + d*x)^(5/2))/d^6 - (10*b^4*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^6) + (2*b
^5*(c + d*x)^(9/2))/(9*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^{3/2}}\,dx=\int\left(\frac{(-bc+ad)^5}{d^5(c+dx)^{3/2}}+\frac{5b(bc-ad)^4}{d^5\sqrt{c+dx}}-\frac{10b^2(bc-ad)^3\sqrt{c+dx}}{d^5}+\frac{10b^3(bc-ad)^2(c+dx)^{3/2}}{d^5}-\frac{5b^4(bc-\right.$$

$$=\frac{2(bc-ad)^5}{d^6\sqrt{c+dx}}+\frac{10b(bc-ad)^4\sqrt{c+dx}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{3/2}}{3d^6}+\frac{4b^3(bc-ad)^2(c+dx)^{5/2}}{d^6}-\frac{10b}{}$$

Mathematica [A] time = 0.116278, size = 123, normalized size = 0.81

$$\frac{2\left(-210b^2(c+dx)^2(bc-ad)^3+126b^3(c+dx)^3(bc-ad)^2-45b^4(c+dx)^4(bc-ad)+315b(c+dx)(bc-ad)^4+63(bc-\right.}{63d^6\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^(3/2),x]
```

```
[Out] (2*(63*(b*c - a*d)^5 + 315*b*(b*c - a*d)^4*(c + d*x) - 210*b^2*(b*c - a*d)^
3*(c + d*x)^2 + 126*b^3*(b*c - a*d)^2*(c + d*x)^3 - 45*b^4*(b*c - a*d)*(c +
 d*x)^4 + 7*b^5*(c + d*x)^5))/(63*d^6*Sqrt[c + d*x])
```

Maple [B] time = 0.005, size = 273, normalized size = 1.8

$$-\frac{-14\,b^5x^5d^5-90\,ab^4d^5x^4+20\,b^5cd^4x^4-252\,a^2b^3d^5x^3+144\,ab^4cd^4x^3-32\,b^5c^2d^3x^3-420\,a^3b^2d^5x^2+504\,a^2b^3cd^4x^2-2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(3/2),x)
```

```
[Out] -2/63/(d*x+c)^(1/2)*(-7*b^5*d^5*x^5-45*a*b^4*d^5*x^4+10*b^5*c*d^4*x^4-126*a
^2*b^3*d^5*x^3+72*a*b^4*c*d^4*x^3-16*b^5*c^2*d^3*x^3-210*a^3*b^2*d^5*x^2+25
2*a^2*b^3*c*d^4*x^2-144*a*b^4*c^2*d^3*x^2+32*b^5*c^3*d^2*x^2-315*a^4*b*d^5*
x+840*a^3*b^2*c*d^4*x-1008*a^2*b^3*c^2*d^3*x+576*a*b^4*c^3*d^2*x-128*b^5*c^
4*d*x+63*a^5*d^5-630*a^4*b*c*d^4+1680*a^3*b^2*c^2*d^3-2016*a^2*b^3*c^3*d^2+
1152*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

Maxima [A] time = 0.969638, size = 360, normalized size = 2.37

$$\frac{2\left(\frac{7\,(dx+c)^{\frac{9}{2}}b^5-45\left(b^5c-ab^4d\right)(dx+c)^{\frac{7}{2}}+126\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)(dx+c)^{\frac{5}{2}}-210\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)(dx+c)^{\frac{3}{2}}+315\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-\right.}{d^5}\right.}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/63*((7*(d*x + c)^(9/2)*b^5 - 45*(b^5*c - a*b^4*d)*(d*x + c)^(7/2) + 126*(
b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(5/2) - 210*(b^5*c^3 - 3*a*b
^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(3/2) + 315*(b^5*c^4 -
4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*sqrt(d*x +
 c))/d^5 + 63*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^
2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)/(sqrt(d*x + c)*d^5))/d
```

Fricas [B] time = 2.112, size = 590, normalized size = 3.88

$$\frac{2\left(7\,b^5d^5x^5+256\,b^5c^5-1152\,ab^4c^4d+2016\,a^2b^3c^3d^2-1680\,a^3b^2c^2d^3+630\,a^4bcd^4-63\,a^5d^5-5\left(2\,b^5cd^4-9\,ab^4d^5\right)x^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^5*d^5*x^5 + 256*b^5*c^5 - 1152*a*b^4*c^4*d + 2016*a^2*b^3*c^3*d^2
 - 1680*a^3*b^2*c^2*d^3 + 630*a^4*b*c*d^4 - 63*a^5*d^5 - 5*(2*b^5*c*d^4 - 9
*a*b^4*d^5)*x^4 + 2*(8*b^5*c^2*d^3 - 36*a*b^4*c*d^4 + 63*a^2*b^3*d^5)*x^3 -
 2*(16*b^5*c^3*d^2 - 72*a*b^4*c^2*d^3 + 126*a^2*b^3*c*d^4 - 105*a^3*b^2*d^5
)*x^2 + (128*b^5*c^4*d - 576*a*b^4*c^3*d^2 + 1008*a^2*b^3*c^2*d^3 - 840*a^3
```

```
*b^2*c*d^4 + 315*a^4*b*d^5)*x)*sqrt(d*x + c)/(d^7*x + c*d^6)
```

Sympy [A] time = 30.5541, size = 243, normalized size = 1.6

$$\frac{2b^5(c+dx)^{\frac{9}{2}}}{9d^6}+\frac{(c+dx)^{\frac{7}{2}}\left(10ab^4d-10b^5c\right)}{7d^6}+\frac{(c+dx)^{\frac{5}{2}}\left(20a^2b^3d^2-40ab^4cd+20b^5c^2\right)}{5d^6}+\frac{(c+dx)^{\frac{3}{2}}\left(20a^3b^2d^3-60a\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**5*(c + d*x)**(9/2)/(9*d**6) + (c + d*x)**(7/2)*(10*a*b**4*d - 10*b**5*
c)/(7*d**6) + (c + d*x)**(5/2)*(20*a**2*b**3*d**2 - 40*a*b**4*c*d + 20*b**5
*c**2)/(5*d**6) + (c + d*x)**(3/2)*(20*a**3*b**2*d**3 - 60*a**2*b**3*c*d**2
 + 60*a*b**4*c**2*d - 20*b**5*c**3)/(3*d**6) + sqrt(c + d*x)*(10*a**4*b*d**
4 - 40*a**3*b**2*c*d**3 + 60*a**2*b**3*c**2*d**2 - 40*a*b**4*c**3*d + 10*b*
*5*c**4)/d**6 - 2*(a*d - b*c)**5/(d**6*sqrt(c + d*x))
```

Giac [B] time = 1.07866, size = 473, normalized size = 3.11

$$\frac{2\left(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5\right)}{\sqrt{dx+c}d^6}+\frac{2\left(7\,(dx+c)^{\frac{9}{2}}b^5d^{48}-45\,(dx+c)^{\frac{7}{2}}b^5cd^{48}+126\,(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^
4*b*c*d^4 - a^5*d^5)/(sqrt(d*x + c)*d^6) + 2/63*(7*(d*x + c)^(9/2)*b^5*d^48
 - 45*(d*x + c)^(7/2)*b^5*c*d^48 + 126*(d*x + c)^(5/2)*b^5*c^2*d^48 - 210*(
d*x + c)^(3/2)*b^5*c^3*d^48 + 315*sqrt(d*x + c)*b^5*c^4*d^48 + 45*(d*x + c)
^(7/2)*a*b^4*d^49 - 252*(d*x + c)^(5/2)*a*b^4*c*d^49 + 630*(d*x + c)^(3/2)*
a*b^4*c^2*d^49 - 1260*sqrt(d*x + c)*a*b^4*c^3*d^49 + 126*(d*x + c)^(5/2)*a^
2*b^3*d^50 - 630*(d*x + c)^(3/2)*a^2*b^3*c*d^50 + 1890*sqrt(d*x + c)*a^2*b^
3*c^2*d^50 + 210*(d*x + c)^(3/2)*a^3*b^2*d^51 - 1260*sqrt(d*x + c)*a^3*b^2*
c*d^51 + 315*sqrt(d*x + c)*a^4*b*d^52)/d^54
```

3.1425 $\int \frac{(a+bx)^4}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{8b^3(c+dx)^{5/2}(bc-ad)}{5d^5}+\frac{4b^2(c+dx)^{3/2}(bc-ad)^2}{d^5}-\frac{8b\sqrt{c+dx}(bc-ad)^3}{d^5}-\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}+\frac{2b^4(c+dx)^{7/2}}{7d^5}$$

```
[Out] (-2*(b*c - a*d)^4)/(d^5*Sqrt[c + d*x]) - (8*b*(b*c - a*d)^3*Sqrt[c + d*x])/
d^5 + (4*b^2*(b*c - a*d)^2*(c + d*x)^(3/2))/d^5 - (8*b^3*(b*c - a*d)*(c + d
*x)^(5/2))/(5*d^5) + (2*b^4*(c + d*x)^(7/2))/(7*d^5)
```

Rubi [A] time = 0.0371394, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{5/2}(bc-ad)}{5d^5}+\frac{4b^2(c+dx)^{3/2}(bc-ad)^2}{d^5}-\frac{8b\sqrt{c+dx}(bc-ad)^3}{d^5}-\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}+\frac{2b^4(c+dx)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^4)/(d^5*Sqrt[c + d*x]) - (8*b*(b*c - a*d)^3*Sqrt[c + d*x])/
d^5 + (4*b^2*(b*c - a*d)^2*(c + d*x)^(3/2))/d^5 - (8*b^3*(b*c - a*d)*(c + d
*x)^(5/2))/(5*d^5) + (2*b^4*(c + d*x)^(7/2))/(7*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^4}{d^4(c+dx)^{3/2}}-\frac{4b(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{6b^2(bc-ad)^2\sqrt{c+dx}}{d^4}-\frac{4b^3(bc-ad)(c+dx)^{3/2}}{d^4}+\frac{b^4(c+dx)^{5/2}}{d^4}\right) \\ &= -\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}-\frac{8b(bc-ad)^3\sqrt{c+dx}}{d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{3/2}}{d^5}-\frac{8b^3(bc-ad)(c+dx)^{5/2}}{5d^5}+\frac{2b^4(c+dx)^{7/2}}{7d^5}\end{aligned}$$

Mathematica [A] time = 0.0763483, size = 101, normalized size = 0.82

$$\frac{2\left(70b^2(c+dx)^2(bc-ad)^2-28b^3(c+dx)^3(bc-ad)-140b(c+dx)(bc-ad)^3-35(bc-ad)^4+5b^4(c+dx)^4\right)}{35d^5\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-35*(b*c - a*d)^4 - 140*b*(b*c - a*d)^3*(c + d*x) + 70*b^2*(b*c - a*d)^
2*(c + d*x)^2 - 28*b^3*(b*c - a*d)*(c + d*x)^3 + 5*b^4*(c + d*x)^4))/(35*d^
5*Sqrt[c + d*x])
```

Maple [A] time = 0.006, size = 186, normalized size = 1.5

$$-\frac{-10\,b^4x^4d^4-56\,ab^3d^4x^3+16\,b^4cd^3x^3-140\,a^2b^2d^4x^2+112\,ab^3cd^3x^2-32\,b^4c^2d^2x^2-280\,a^3bd^4x+560\,a^2b^2cd^3x-4}{35\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(3/2),x)
```

```
[Out] -2/35/(d*x+c)^(1/2)*(-5*b^4*d^4*x^4-28*a*b^3*d^4*x^3+8*b^4*c*d^3*x^3-70*a^2
*b^2*d^4*x^2+56*a*b^3*c*d^3*x^2-16*b^4*c^2*d^2*x^2-140*a^3*b*d^4*x+280*a^2*
b^2*c*d^3*x-224*a*b^3*c^2*d^2*x+64*b^4*c^3*d*x+35*a^4*d^4-280*a^3*b*c*d^3+5
60*a^2*b^2*c^2*d^2-448*a*b^3*c^3*d+128*b^4*c^4)/d^5
```

Maxima [A] time = 0.952235, size = 255, normalized size = 2.07

$$\frac{2\left(\frac{5\,(dx+c)^{\frac{7}{2}}b^4-28\left(b^4c-ab^3d\right)(dx+c)^{\frac{5}{2}}+70\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)(dx+c)^{\frac{3}{2}}-140\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)\sqrt{dx+c}}{d^4}-\frac{35\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2\right.}{\sqrt{dx+c}d^4}\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*((5*(d*x + c)^(7/2)*b^4 - 28*(b^4*c - a*b^3*d)*(d*x + c)^(5/2) + 70*(b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(3/2) - 140*(b^4*c^3 - 3*a*b^
3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(d*x + c))/d^4 - 35*(b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(sqrt(d*x + c)*
d^4))/d
```

Fricas [A] time = 2.01547, size = 412, normalized size = 3.35

$$\frac{2\left(5\,b^4d^4x^4-128\,b^4c^4+448\,ab^3c^3d-560\,a^2b^2c^2d^2+280\,a^3bcd^3-35\,a^4d^4-4\left(2\,b^4cd^3-7\,ab^3d^4\right)x^3+2\left(8\,b^4c^2d^2-2\right.\right.}{35\left(d^6x+cd^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*d^4*x^4 - 128*b^4*c^4 + 448*a*b^3*c^3*d - 560*a^2*b^2*c^2*d^2 +
 280*a^3*b*c*d^3 - 35*a^4*d^4 - 4*(2*b^4*c*d^3 - 7*a*b^3*d^4)*x^3 + 2*(8*b^
4*c^2*d^2 - 28*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^2 - 4*(16*b^4*c^3*d - 56*a*b
^3*c^2*d^2 + 70*a^2*b^2*c*d^3 - 35*a^3*b*d^4)*x)*sqrt(d*x + c)/(d^6*x + c*d
^5)
```

Sympy [A] time = 20.2228, size = 168, normalized size = 1.37

$$\frac{2b^4(c+dx)^{\frac{7}{2}}}{7d^5}+\frac{(c+dx)^{\frac{5}{2}}\left(8ab^3d-8b^4c\right)}{5d^5}+\frac{(c+dx)^{\frac{3}{2}}\left(12a^2b^2d^2-24ab^3cd+12b^4c^2\right)}{3d^5}+\frac{\sqrt{c+dx}\left(8a^3bd^3-24a^2b^2cd^2+\right.}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**4*(c + d*x)**(7/2)/(7*d**5) + (c + d*x)**(5/2)*(8*a*b**3*d - 8*b**4*c)
/(5*d**5) + (c + d*x)**(3/2)*(12*a**2*b**2*d**2 - 24*a*b**3*c*d + 12*b**4*c
**2)/(3*d**5) + sqrt(c + d*x)*(8*a**3*b*d**3 - 24*a**2*b**2*c*d**2 + 24*a*b
**3*c**2*d - 8*b**4*c**3)/d**5 - 2*(a*d - b*c)**4/(d**5*sqrt(c + d*x))
```

Giac [B] time = 1.08478, size = 324, normalized size = 2.63

$$-\frac{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)}{\sqrt{dx+c}d^5}+\frac{2\left(5\,(dx+c)^{\frac{7}{2}}b^4d^{30}-28\,(dx+c)^{\frac{5}{2}}b^4cd^{30}+70\,(dx+c)^{\frac{3}{2}}b^4c^2d^{30}-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/
(sqrt(d*x + c)*d^5) + 2/35*(5*(d*x + c)^(7/2)*b^4*d^30 - 28*(d*x + c)^(5/2)
*b^4*c*d^30 + 70*(d*x + c)^(3/2)*b^4*c^2*d^30 - 140*sqrt(d*x + c)*b^4*c^3*d
^30 + 28*(d*x + c)^(5/2)*a*b^3*d^31 - 140*(d*x + c)^(3/2)*a*b^3*c*d^31 + 42
0*sqrt(d*x + c)*a*b^3*c^2*d^31 + 70*(d*x + c)^(3/2)*a^2*b^2*d^32 - 420*sqrt
(d*x + c)*a^2*b^2*c*d^32 + 140*sqrt(d*x + c)*a^3*b*d^33)/d^35
```

3.1426 $\int \frac{(a+bx)^3}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{2b^2(c+dx)^{3/2}(bc-ad)}{d^4}+\frac{6b\sqrt{c+dx}(bc-ad)^2}{d^4}+\frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{2b^3(c+dx)^{5/2}}{5d^4}$$

```
[Out] (2*(b*c - a*d)^3)/(d^4*Sqrt[c + d*x]) + (6*b*(b*c - a*d)^2*Sqrt[c + d*x])/d
^4 - (2*b^2*(b*c - a*d)*(c + d*x)^(3/2))/d^4 + (2*b^3*(c + d*x)^(5/2))/(5*d
^4)
```

Rubi [A] time = 0.0297938, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2b^2(c+dx)^{3/2}(bc-ad)}{d^4}+\frac{6b\sqrt{c+dx}(bc-ad)^2}{d^4}+\frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{2b^3(c+dx)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^3)/(d^4*Sqrt[c + d*x]) + (6*b*(b*c - a*d)^2*Sqrt[c + d*x])/d
^4 - (2*b^2*(b*c - a*d)*(c + d*x)^(3/2))/d^4 + (2*b^3*(c + d*x)^(5/2))/(5*d
^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^{3/2}}+\frac{3b(bc-ad)^2}{d^3\sqrt{c+dx}}-\frac{3b^2(bc-ad)\sqrt{c+dx}}{d^3}+\frac{b^3(c+dx)^{3/2}}{d^3}\right)dx\\ &= \frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{6b(bc-ad)^2\sqrt{c+dx}}{d^4}-\frac{2b^2(bc-ad)(c+dx)^{3/2}}{d^4}+\frac{2b^3(c+dx)^{5/2}}{5d^4}\end{aligned}$$

Mathematica [A] time = 0.0544484, size = 78, normalized size = 0.83

$$\frac{2\left(-5b^2(c+dx)^2(bc-ad)+15b(c+dx)(bc-ad)^2+5(bc-ad)^3+b^3(c+dx)^3\right)}{5d^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^(3/2),x]
```

```
[Out] (2*(5*(b*c - a*d)^3 + 15*b*(b*c - a*d)^2*(c + d*x) - 5*b^2*(b*c - a*d)*(c +
 d*x)^2 + b^3*(c + d*x)^3))/(5*d^4*Sqrt[c + d*x])
```

Maple [A] time = 0.006, size = 116, normalized size = 1.2

$$-\frac{-2\,b^3x^3d^3-10\,ab^2d^3x^2+4\,b^3cd^2x^2-30\,a^2bd^3x+40\,ab^2cd^2x-16\,b^3c^2dx+10\,a^3d^3-60\,a^2bcd^2+80\,ab^2c^2d-32\,b^3c^3}{5\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] -2/5/(d*x+c)^(1/2)*(-b^3*d^3*x^3-5*a*b^2*d^3*x^2+2*b^3*c*d^2*x^2-15*a^2*b*d
^3*x+20*a*b^2*c*d^2*x-8*b^3*c^2*d*x+5*a^3*d^3-30*a^2*b*c*d^2+40*a*b^2*c^2*d
-16*b^3*c^3)/d^4
```

Maxima [A] time = 0.965714, size = 169, normalized size = 1.8

$$\frac{2\left(\frac{(dx+c)^{\frac{5}{2}}b^3-5\left(b^3c-ab^2d\right)(dx+c)^{\frac{3}{2}}+15\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)\sqrt{dx+c}}{d^3}+\frac{5\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}{\sqrt{dx+c}d^3}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(((d*x + c)^(5/2)*b^3 - 5*(b^3*c - a*b^2*d)*(d*x + c)^(3/2) + 15*(b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(d*x + c))/d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)/(sqrt(d*x + c)*d^3))/d
```

Fricas [A] time = 2.1117, size = 259, normalized size = 2.76

$$\frac{2\left(b^3d^3x^3+16\,b^3c^3-40\,ab^2c^2d+30\,a^2bcd^2-5\,a^3d^3-\left(2\,b^3cd^2-5\,ab^2d^3\right)x^2+\left(8\,b^3c^2d-20\,ab^2cd^2+15\,a^2bd^3\right)x\right)\sqrt{dx+}}{5\left(d^5x+cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*d^3*x^3 + 16*b^3*c^3 - 40*a*b^2*c^2*d + 30*a^2*b*c*d^2 - 5*a^3*d^3
 - (2*b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + (8*b^3*c^2*d - 20*a*b^2*c*d^2 + 15*a^2
*b*d^3)*x)*sqrt(d*x + c)/(d^5*x + c*d^4)
```

Sympy [A] time = 13.0578, size = 109, normalized size = 1.16

$$\frac{2b^3(c+dx)^{\frac{5}{2}}}{5d^4}+\frac{(c+dx)^{\frac{3}{2}}\left(6ab^2d-6b^3c\right)}{3d^4}+\frac{\sqrt{c+dx}\left(6a^2bd^2-12ab^2cd+6b^3c^2\right)}{d^4}-\frac{2\,(ad-bc)^3}{d^4\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**3*(c + d*x)**(5/2)/(5*d**4) + (c + d*x)**(3/2)*(6*a*b**2*d - 6*b**3*c)
/(3*d**4) + sqrt(c + d*x)*(6*a**2*b*d**2 - 12*a*b**2*c*d + 6*b**3*c**2)/d**
4 - 2*(a*d - b*c)**3/(d**4*sqrt(c + d*x))
```

Giac [A] time = 1.06752, size = 205, normalized size = 2.18

$$\frac{2\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)}{\sqrt{dx + c}d^4} + \frac{2\left((dx + c)^{\frac{5}{2}}b^3d^{16} - 5\,(dx + c)^{\frac{3}{2}}b^3cd^{16} + 15\,\sqrt{dx + c}b^3c^2d^{16} + 5\,(dx + c)^{\frac{3}{2}}ab^2d^{17}\right.}{5\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(sqrt(d*x + c)*d^4) +
 2/5*((d*x + c)^(5/2)*b^3*d^16 - 5*(d*x + c)^(3/2)*b^3*c*d^16 + 15*sqrt(d*x
 + c)*b^3*c^2*d^16 + 5*(d*x + c)^(3/2)*a*b^2*d^17 - 30*sqrt(d*x + c)*a*b^2*
c*d^17 + 15*sqrt(d*x + c)*a^2*b*d^18)/d^20
```

3.1427 $\int \frac{(a+bx)^2}{(c+dx)^{3/2}} dx$

Optimal. Leaf size=67

$$-\frac{4b\sqrt{c+dx}(bc-ad)}{d^3}-\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}+\frac{2b^2(c+dx)^{3/2}}{3d^3}$$

```
[Out] (-2*(b*c - a*d)^2)/(d^3*Sqrt[c + d*x]) - (4*b*(b*c - a*d)*Sqrt[c + d*x])/d^
3 + (2*b^2*(c + d*x)^(3/2))/(3*d^3)
```

Rubi [A] time = 0.0206038, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b\sqrt{c+dx}(bc-ad)}{d^3}-\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}+\frac{2b^2(c+dx)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^2)/(d^3*Sqrt[c + d*x]) - (4*b*(b*c - a*d)*Sqrt[c + d*x])/d^
3 + (2*b^2*(c + d*x)^(3/2))/(3*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{3/2}}-\frac{2b(bc-ad)}{d^2\sqrt{c+dx}}+\frac{b^2\sqrt{c+dx}}{d^2}\right)dx\\ &= -\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}-\frac{4b(bc-ad)\sqrt{c+dx}}{d^3}+\frac{2b^2(c+dx)^{3/2}}{3d^3}\end{aligned}$$

Mathematica [A] time = 0.0340529, size = 59, normalized size = 0.88

$$\frac{2\left(-3a^2d^2+6abd(2c+dx)+b^2\left(-8c^2-4cdx+d^2x^2\right)\right)}{3d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-3*a^2*d^2 + 6*a*b*d*(2*c + d*x) + b^2*(-8*c^2 - 4*c*d*x + d^2*x^2)))/(
3*d^3*Sqrt[c + d*x])
```

Maple [A] time = 0.005, size = 63, normalized size = 0.9

$$-\frac{-2\,b^2x^2d^2-12\,abd^2x+8\,b^2cdx+6\,a^2d^2-24\,abcd+16\,b^2c^2}{3\,d^3}\frac{1}{\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] -2/3/(d*x+c)^(1/2)*(-b^2*d^2*x^2-6*a*b*d^2*x+4*b^2*c*d*x+3*a^2*d^2-12*a*b*c
*d+8*b^2*c^2)/d^3
```

Maxima [A] time = 0.946276, size = 101, normalized size = 1.51

$$\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^2-6\left(b^2c-abd\right)\sqrt{dx+c}}{d^2}-\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)}{\sqrt{dx+c}d^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((d*x + c)^(3/2)*b^2 - 6*(b^2*c - a*b*d)*sqrt(d*x + c))/d^2 - 3*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x + c)*d^2))/d
```

Fricas [A] time = 2.02985, size = 157, normalized size = 2.34

$$\frac{2\left(b^2d^2x^2-8\,b^2c^2+12\,abcd-3\,a^2d^2-2\left(2\,b^2cd-3\,abd^2\right)x\right)\sqrt{dx+c}}{3\left(d^4x+cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*d^2*x^2 - 8*b^2*c^2 + 12*a*b*c*d - 3*a^2*d^2 - 2*(2*b^2*c*d - 3*a*
b*d^2)*x)*sqrt(d*x + c)/(d^4*x + c*d^3)
```

Sympy [A] time = 7.99504, size = 65, normalized size = 0.97

$$\frac{2b^2\left(c+dx\right)^{\frac{3}{2}}}{3d^3}+\frac{\sqrt{c+dx}\left(4abd-4b^2c\right)}{d^3}-\frac{2\left(ad-bc\right)^2}{d^3\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**2*(c + d*x)**(3/2)/(3*d**3) + sqrt(c + d*x)*(4*a*b*d - 4*b**2*c)/d**3
- 2*(a*d - b*c)**2/(d**3*sqrt(c + d*x))
```

Giac [A] time = 1.05313, size = 113, normalized size = 1.69

$$-\frac{2\left(b^2c^2 - 2\,abcd + a^2d^2\right)}{\sqrt{dx + c}d^3} + \frac{2\left((dx + c)^{\frac{3}{2}}b^2d^6 - 6\,\sqrt{dx + c}b^2cd^6 + 6\,\sqrt{dx + c}abd^7\right)}{3\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x + c)*d^3) + 2/3*((d*x + c)^(3/
2)*b^2*d^6 - 6*sqrt(d*x + c)*b^2*c*d^6 + 6*sqrt(d*x + c)*a*b*d^7)/d^9
```

3.1428 $\int \frac{a+bx}{(c+dx)^{3/2}} dx$

Optimal. Leaf size=38

$$\frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}$$

```
[Out] (2*(b*c - a*d))/(d^2*Sqrt[c + d*x]) + (2*b*Sqrt[c + d*x])/d^2
```

Rubi [A] time = 0.0142636, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d))/(d^2*Sqrt[c + d*x]) + (2*b*Sqrt[c + d*x])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{-bc+ad}{d(c+dx)^{3/2}}+\frac{b}{d\sqrt{c+dx}}\right)dx\\ &= \frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}\end{aligned}$$

Mathematica [A] time = 0.0193362, size = 27, normalized size = 0.71

$$\frac{2(-ad+2bc+bdx)}{d^2\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(2*b*c - a*d + b*d*x))/(d^2*Sqrt[c + d*x])
```

Maple [A] time = 0.003, size = 26, normalized size = 0.7

$$-2\frac{-bdx+ad-2bc}{\sqrt{dx+c}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(3/2),x)
```

```
[Out] -2/(d*x+c)^(1/2)*(-b*d*x+a*d-2*b*c)/d^2
```

Maxima [A] time = 0.946742, size = 50, normalized size = 1.32

$$\frac{2\left(\frac{\sqrt{dx+c}b}{d}+\frac{bc-ad}{\sqrt{dx+c}d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(d*x + c)*b/d + (b*c - a*d)/(sqrt(d*x + c)*d))/d
```

Fricas [A] time = 1.97996, size = 74, normalized size = 1.95

$$\frac{2\,(bdx+2\,bc-ad)\sqrt{dx+c}}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*d*x + 2*b*c - a*d)*sqrt(d*x + c)/(d^3*x + c*d^2)
```

Sympy [A] time = 0.577191, size = 60, normalized size = 1.58

$$\begin{cases} -\frac{2a}{d\sqrt{c+dx}}+\frac{4bc}{d^2\sqrt{c+dx}}+\frac{2bx}{d\sqrt{c+dx}} & \text{for } d\neq 0 \\ \frac{ax+\frac{bx^2}{2}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] Piecewise((-2*a/(d*sqrt(c + d*x)) + 4*b*c/(d**2*sqrt(c + d*x)) + 2*b*x/(d*s
qrt(c + d*x)), Ne(d, 0)), ((a*x + b*x**2/2)/c**(3/2), True))
```

Giac [A] time = 1.06666, size = 46, normalized size = 1.21

$$\frac{2\sqrt{dx+c}b}{d^2}+\frac{2\,(bc-ad)}{\sqrt{dx+c}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*b/d^2 + 2*(b*c - a*d)/(sqrt(d*x + c)*d^2)
```

3.1429 $\int \frac{1}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=14

$$-\frac{2}{d\sqrt{c+dx}}$$

```
[Out] -2/(d*Sqrt[c + d*x])
```

Rubi [A] time = 0.0015836, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-3/2),x]
```

```
[Out] -2/(d*Sqrt[c + d*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^{3/2}}\,dx = -\frac{2}{d\sqrt{c+dx}}$$

Mathematica [A] time = 0.0037861, size = 14, normalized size = 1.

$$-\frac{2}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-3/2),x]
```

```
[Out] -2/(d*Sqrt[c + d*x])
```

Maple [A] time = 0.002, size = 13, normalized size = 0.9

$$-2\,\frac{1}{d\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(3/2),x)
```

```
[Out] -2/d/(d*x+c)^(1/2)
```

Maxima [A] time = 0.931832, size = 16, normalized size = 1.14

$$-\frac{2}{\sqrt{dx+c}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(d*x + c)*d)
```

Fricas [A] time = 2.10794, size = 43, normalized size = 3.07

$$-\frac{2\sqrt{dx+c}}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(d*x + c)/(d^2*x + c*d)
```

Sympy [A] time = 0.056988, size = 12, normalized size = 0.86

$$-\frac{2}{d\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(3/2),x)
```

```
[Out] -2/(d*sqrt(c + d*x))
```

Giac [A] time = 1.06052, size = 16, normalized size = 1.14

$$-\frac{2}{\sqrt{dx+c}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(d*x + c)*d)
```

3.1430 $\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{2}{\sqrt{c+dx}(bc-ad)}-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

```
[Out] 2/((b*c - a*d)*Sqrt[c + d*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)
```

Rubi [A] time = 0.0273585, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{2}{\sqrt{c+dx}(bc-ad)}-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(3/2)),x]
```

```
[Out] 2/((b*c - a*d)*Sqrt[c + d*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx = \frac{2}{(bc-ad)\sqrt{c+dx}} + \frac{b\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{bc-ad}$$
$$= \frac{2}{(bc-ad)\sqrt{c+dx}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d(bc-ad)}$$
$$= \frac{2}{(bc-ad)\sqrt{c+dx}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.009799, size = 46, normalized size = 0.67

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b(c+dx)}{bc-ad}\right)}{\sqrt{c+dx}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x))/(b*c - a*d)])/((-(b*c) +
a*d)*Sqrt[c + d*x])
```

Maple [A] time = 0.009, size = 68, normalized size = 1.

$$-2\,{\frac{b}{(ad-bc)\sqrt{(ad-bc)\,b}}\arctan\left({\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}}\right)}-2\,{\frac{1}{(ad-bc)\sqrt{dx+c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(3/2),x)
```

```
[Out] -2*b/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/
2))-2/(a*d-b*c)/(d*x+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.10208, size = 456, normalized size = 6.61

$$\left[-\frac{(dx+c)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad+2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-2\,\sqrt{dx+c}}{bc^2-acd+\left(bcd-ad^2\right)x}, -\frac{2\left((dx+c)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-\right.}{bc^2-acd+\left(bcd-ad^2\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((d*x + c)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d + 2*(b*c - a*d)*s
qrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) - 2*sqrt(d*x + c))/(b*c^2 - a*
c*d + (b*c*d - a*d^2)*x), -2*((d*x + c)*sqrt(-b/(b*c - a*d))*arctan(-(b*c -
 a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - sqrt(d*x + c))/(b
*c^2 - a*c*d + (b*c*d - a*d^2)*x)]
```

Sympy [A] time = 5.06421, size = 60, normalized size = 0.87

$$-\frac{2}{\sqrt{c+dx}\,(ad-bc)}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] -2/(sqrt(c + d*x)*(a*d - b*c)) - 2*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/
(sqrt((a*d - b*c)/b)*(a*d - b*c))
```

Giac [A] time = 1.06208, size = 93, normalized size = 1.35

$$\frac{2\,b\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}+\frac{2}{(bc-ad)\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*b*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c
 - a*d)) + 2/((b*c - a*d)*sqrt(d*x + c))
```

3.1431 $\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{3d}{\sqrt{c+dx}(bc-ad)^2}-\frac{1}{(a+bx)\sqrt{c+dx}(bc-ad)}+\frac{3\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] (-3*d)/((b*c - a*d)^2*Sqrt[c + d*x]) - 1/((b*c - a*d)*(a + b*x)*Sqrt[c + d*
x]) + (3*Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c -
 a*d)^(5/2)
```

Rubi [A] time = 0.0384196, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{3d}{\sqrt{c+dx}(bc-ad)^2}-\frac{1}{(a+bx)\sqrt{c+dx}(bc-ad)}+\frac{3\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*d)/((b*c - a*d)^2*Sqrt[c + d*x]) - 1/((b*c - a*d)*(a + b*x)*Sqrt[c + d*
x]) + (3*Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c -
 a*d)^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx = -\frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3d)\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx}{2(bc-ad)}$$
$$= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3bd)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2(bc-ad)^2}$$
$$= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{(bc-ad)^2}$$
$$= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} + \frac{3\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.0121071, size = 48, normalized size = 0.48

$$-\frac{2d\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{\sqrt{c+dx}(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*d*Hypergeometric2F1[-1/2, 2, 1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/((-
(b*c) + a*d)^2*Sqrt[c + d*x])
```

Maple [A] time = 0.013, size = 101, normalized size = 1.

$$-2\,\frac{d}{(ad-bc)^2\sqrt{dx+c}} - \frac{bd}{(ad-bc)^2(bdx+ad)}\sqrt{dx+c} - 3\,\frac{bd}{(ad-bc)^2\sqrt{(ad-bc)\,b}}\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] -2*d/(a*d-b*c)^2/(d*x+c)^(1/2)-d*b/(a*d-b*c)^2*(d*x+c)^(1/2)/(b*d*x+a*d)-3*
d*b/(a*d-b*c)^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12702, size = 883, normalized size = 8.92

$$\left[\frac{3\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad+2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-2\,(3\,bdx+bc+2\,ad)\sqrt{dx+c}}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x\right)},\ \frac{3\left(bd^2x^2+\cdots\right.}{ab^2c^3\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)*sqrt(b/(b*c - a*d))*log((b*
d*x + 2*b*c - a*d + 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x +
 a)) - 2*(3*b*d*x + b*c + 2*a*d)*sqrt(d*x + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d
+ a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^
2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x), (3*(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2
)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c -
a*d))/(b*d*x + b*c)) - (3*b*d*x + b*c + 2*a*d)*sqrt(d*x + c))/(a*b^2*c^3 -
2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (
b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.05468, size = 193, normalized size = 1.95

$$-\frac{3\,bd\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}-\frac{3\,(dx+c)bd-2\,bcd+2\,ad^2}{\left(b^2c^2-2\,abcd+a^2d^2\right)\left((dx+c)^{\frac{3}{2}}b-\sqrt{dx+c}bc+\sqrt{dx+c}ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt(-b^2*c + a*b*d)) - (3*(d*x + c)*b*d - 2*b*c*d + 2*a*d^2)/((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x + c)^(3/2)*b - sqrt(d*x + c)*b*c + sqrt
(d*x + c)*a*d))
```

3.1432 $\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=140

$$\frac{15d^2}{4\sqrt{c+dx}(bc-ad)^3}-\frac{15\sqrt{b}d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{7/2}}+\frac{5d}{4(a+bx)\sqrt{c+dx}(bc-ad)^2}-\frac{1}{2(a+bx)^2\sqrt{c+dx}(bc-ad)}$$

```
[Out] (15*d^2)/(4*(b*c - a*d)^3*Sqrt[c + d*x]) - 1/(2*(b*c - a*d)*(a + b*x)^2*Sqr
t[c + d*x]) + (5*d)/(4*(b*c - a*d)^2*(a + b*x)*Sqrt[c + d*x]) - (15*Sqrt[b]
*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*(b*c - a*d)^(7/2)
)
```

Rubi [A] time = 0.0515628, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{15d^2}{4\sqrt{c+dx}(bc-ad)^3}-\frac{15\sqrt{b}d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{7/2}}+\frac{5d}{4(a+bx)\sqrt{c+dx}(bc-ad)^2}-\frac{1}{2(a+bx)^2\sqrt{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

```
[Out] (15*d^2)/(4*(b*c - a*d)^3*Sqrt[c + d*x]) - 1/(2*(b*c - a*d)*(a + b*x)^2*Sqr
t[c + d*x]) + (5*d)/(4*(b*c - a*d)^2*(a + b*x)*Sqrt[c + d*x]) - (15*Sqrt[b]
*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*(b*c - a*d)^(7/2)
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx = -\frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} - \frac{(5d)\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx}{4(bc-ad)}$$
$$= -\frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}} + \frac{(15d^2)\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx}{8(bc-ad)^2}$$
$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}} + \frac{(15b}{}$$
$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}} + \frac{(15b}{}$$
$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}} - \frac{15\sqrt{b}}{}$$

Mathematica [C] time = 0.0127035, size = 50, normalized size = 0.36

$$-\frac{2d^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{\sqrt{c+dx}(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*d^2*Hypergeometric2F1[-1/2, 3, 1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/(
(-(b*c) + a*d)^3*Sqrt[c + d*x])
```

Maple [A] time = 0.014, size = 179, normalized size = 1.3

$$-2\,\frac{d^2}{(ad-bc)^3\sqrt{dx+c}} - \frac{7\,d^2b^2}{4\,(ad-bc)^3\,(bdx+ad)^2}\,(dx+c)^{\frac{3}{2}} - \frac{9\,d^3ba}{4\,(ad-bc)^3\,(bdx+ad)^2}\sqrt{dx+c} + \frac{9\,d^2b^2c}{4\,(ad-bc)^3\,(bdx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] -2*d^2/(a*d-b*c)^3/(d*x+c)^(1/2)-7/4*d^2/(a*d-b*c)^3*b^2/(b*d*x+a*d)^2*(d*x
+c)^(3/2)-9/4*d^3/(a*d-b*c)^3*b/(b*d*x+a*d)^2*(d*x+c)^(1/2)*a+9/4*d^2/(a*d-
b*c)^3*b^2/(b*d*x+a*d)^2*(d*x+c)^(1/2)*c-15/4*d^2/(a*d-b*c)^3*b/((a*d-b*c)*
b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.25975, size = 1582, normalized size = 11.3

$$\left[-\frac{15\left(b^2d^3x^3+a^2cd^2+\left(b^2cd^2+2\,abd^3\right)x^2+\left(2\,abcd^2+a^2d^3\right)x\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad+2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-}{8\left(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+\left(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4\right)x^3+\left(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(b^2*d^3*x^3 + a^2*c*d^2 + (b^2*c*d^2 + 2*a*b*d^3)*x^2 + (2*a*b*c
*d^2 + a^2*d^3)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d + 2*(b*c -
a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) - 2*(15*b^2*d^2*x^2 - 2*
b^2*c^2 + 9*a*b*c*d + 8*a^2*d^2 + 5*(b^2*c*d + 5*a*b*d^2)*x)*sqrt(d*x + c))
/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d
- 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c
^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^
4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x), -1/4*(
15*(b^2*d^3*x^3 + a^2*c*d^2 + (b^2*c*d^2 + 2*a*b*d^3)*x^2 + (2*a*b*c*d^2 +
a^2*d^3)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/
(b*c - a*d))/(b*d*x + b*c)) - (15*b^2*d^2*x^2 - 2*b^2*c^2 + 9*a*b*c*d + 8*a
^2*d^2 + 5*(b^2*c*d + 5*a*b*d^2)*x)*sqrt(d*x + c))/(a^2*b^3*c^4 - 3*a^3*b^2
*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2
*b^3*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2
+ 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a
^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.08341, size = 316, normalized size = 2.26

$$\frac{15\,bd^2\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{4\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}+\frac{2\,d^2}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{dx+c}}+\frac{7\,(dx+c)^{\frac{3}{2}}b^2d^2-}{4\left(b^3c^3-3\,ab^2c^2d+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) + 2*d^2/((b^3*c^3 -
```

```
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(d*x + c)) + 1/4*(7*(d*x + c)
^(3/2)*b^2*d^2 - 9*sqrt(d*x + c)*b^2*c*d^2 + 9*sqrt(d*x + c)*a*b*d^3)/((b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)*b - b*c + a*d)^2
)
```

3.1433 $\int \frac{1}{(a+bx)^4(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{35d^3}{8\sqrt{c+dx}(bc-ad)^4}-\frac{35d^2}{24(a+bx)\sqrt{c+dx}(bc-ad)^3}+\frac{35\sqrt{b}d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{9/2}}+\frac{7d}{12(a+bx)^2\sqrt{c+dx}(bc-ad)^2}-\frac{1}{3(a}$$

```
[Out] (-35*d^3)/(8*(b*c - a*d)^4*Sqrt[c + d*x]) - 1/(3*(b*c - a*d)*(a + b*x)^3*Sq
rt[c + d*x]) + (7*d)/(12*(b*c - a*d)^2*(a + b*x)^2*Sqrt[c + d*x]) - (35*d^2
)/(24*(b*c - a*d)^3*(a + b*x)*Sqrt[c + d*x]) + (35*Sqrt[b]*d^3*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*(b*c - a*d)^(9/2))
```

Rubi [A] time = 0.0660012, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{35d^3}{8\sqrt{c+dx}(bc-ad)^4}-\frac{35d^2}{24(a+bx)\sqrt{c+dx}(bc-ad)^3}+\frac{35\sqrt{b}d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{9/2}}+\frac{7d}{12(a+bx)^2\sqrt{c+dx}(bc-ad)^2}-\frac{1}{3(a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*(c + d*x)^(3/2)),x]
```

```
[Out] (-35*d^3)/(8*(b*c - a*d)^4*Sqrt[c + d*x]) - 1/(3*(b*c - a*d)*(a + b*x)^3*Sq
rt[c + d*x]) + (7*d)/(12*(b*c - a*d)^2*(a + b*x)^2*Sqrt[c + d*x]) - (35*d^2
)/(24*(b*c - a*d)^3*(a + b*x)*Sqrt[c + d*x]) + (35*Sqrt[b]*d^3*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*(b*c - a*d)^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4(c+dx)^{3/2}}\,dx = -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} - \frac{(7d)\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx}{6(bc-ad)}$$
$$= -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}} + \frac{\left(35d^2\right)\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,c}{24(bc-ad)^2}$$
$$= -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}} - \frac{35d^2}{24(bc-ad)^3(a+bx)\sqrt{c}}$$
$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}} - \frac{}{2}$$
$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}} - \frac{}{2}$$
$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}} - \frac{}{2}$$

Mathematica [C] time = 0.0156973, size = 50, normalized size = 0.29

$$-\frac{2d^3\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{\sqrt{c+dx}(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*d^3*Hypergeometric2F1[-1/2, 4, 1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/(
(-(b*c) + a*d)^4*Sqrt[c + d*x])
```

Maple [B] time = 0.018, size = 292, normalized size = 1.7

$$-2\frac{d^3}{(ad-bc)^4\sqrt{dx+c}} - \frac{19\,d^3b^3}{8\,(ad-bc)^4\,(bdx+ad)^3}(dx+c)^{\frac{5}{2}} - \frac{17\,d^4b^2a}{3\,(ad-bc)^4\,(bdx+ad)^3}(dx+c)^{\frac{3}{2}} + \frac{17\,d^3b^3c}{3\,(ad-bc)^4\,(bdx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(d*x+c)^(3/2),x)
```

```
[Out] -2*d^3/(a*d-b*c)^4/(d*x+c)^(1/2)-19/8*d^3/(a*d-b*c)^4*b^3/(b*d*x+a*d)^3*(d*
x+c)^(5/2)-17/3*d^4/(a*d-b*c)^4*b^2/(b*d*x+a*d)^3*(d*x+c)^(3/2)*a+17/3*d^3/
(a*d-b*c)^4*b^3/(b*d*x+a*d)^3*(d*x+c)^(3/2)*c-29/8*d^5/(a*d-b*c)^4*b/(b*d*x
+a*d)^3*(d*x+c)^(1/2)*a^2+29/4*d^4/(a*d-b*c)^4*b^2/(b*d*x+a*d)^3*(d*x+c)^(1
/2)*a*c-29/8*d^3/(a*d-b*c)^4*b^3/(b*d*x+a*d)^3*(d*x+c)^(1/2)*c^2-35/8*d^3/(
a*d-b*c)^4*b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.37498, size = 2441, normalized size = 14.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^3*d^4*x^4 + a^3*c*d^3 + (b^3*c*d^3 + 3*a*b^2*d^4)*x^3 + 3*(a*
b^2*c*d^3 + a^2*b*d^4)*x^2 + (3*a^2*b*c*d^3 + a^3*d^4)*x)*sqrt(b/(b*c - a*d
))*log((b*d*x + 2*b*c - a*d + 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d
)))/(b*x + a)) - 2*(105*b^3*d^3*x^3 + 8*b^3*c^3 - 38*a*b^2*c^2*d + 87*a^2*b
*c*d^2 + 48*a^3*d^3 + 35*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 7*(2*b^3*c^2*d - 1
4*a*b^2*c*d^2 - 33*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^3*b^4*c^5 - 4*a^4*b^3*c^
4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^
6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c
^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^
4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2
 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 -
11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a
^7*d^5)*x), 1/24*(105*(b^3*d^4*x^4 + a^3*c*d^3 + (b^3*c*d^3 + 3*a*b^2*d^4)*
x^3 + 3*(a*b^2*c*d^3 + a^2*b*d^4)*x^2 + (3*a^2*b*c*d^3 + a^3*d^4)*x)*sqrt(-
b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*
x + b*c)) - (105*b^3*d^3*x^3 + 8*b^3*c^3 - 38*a*b^2*c^2*d + 87*a^2*b*c*d^2
+ 48*a^3*d^3 + 35*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 7*(2*b^3*c^2*d - 14*a*b^2
*c*d^2 - 33*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6
*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^6*c^3*d
^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c^5 - a*
b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a
^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^
4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 - 11*a^3*
b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)
*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.07874, size = 440, normalized size = 2.54

$$-\frac{35\, bd^3 \arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{8\left(b^4c^4 - 4\, ab^3c^3d + 6\, a^2b^2c^2d^2 - 4\, a^3bcd^3 + a^4d^4\right)\sqrt{-b^2c + abd}} - \frac{2\, d^3}{\left(b^4c^4 - 4\, ab^3c^3d + 6\, a^2b^2c^2d^2 - 4\, a^3bcd^3 + a^4d^4\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -35/8*b*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b^
3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d)
) - 2*d^3/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(d*x + c)) - 1/24*(57*(d*x + c)^(5/2)*b^3*d^3 - 136*(d*x + c)^(
3/2)*b^3*c*d^3 + 87*sqrt(d*x + c)*b^3*c^2*d^3 + 136*(d*x + c)^(3/2)*a*b^2*d
^4 - 174*sqrt(d*x + c)*a*b^2*c*d^4 + 87*sqrt(d*x + c)*a^2*b*d^5)/((b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*((d*x + c)*b
 - b*c + a*d)^3)
```

3.1434 $\int \frac{(a+bx)^5}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{2b^4(c+dx)^{5/2}(bc-ad)}{d^6}+\frac{20b^3(c+dx)^{3/2}(bc-ad)^2}{3d^6}-\frac{20b^2\sqrt{c+dx}(bc-ad)^3}{d^6}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}+\frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}+\frac{2b^5(c$$

```
[Out] (2*(b*c - a*d)^5)/(3*d^6*(c + d*x)^(3/2)) - (10*b*(b*c - a*d)^4)/(d^6*Sqrt[
c + d*x]) - (20*b^2*(b*c - a*d)^3*Sqrt[c + d*x])/d^6 + (20*b^3*(b*c - a*d)^
2*(c + d*x)^(3/2))/(3*d^6) - (2*b^4*(b*c - a*d)*(c + d*x)^(5/2))/d^6 + (2*b
^5*(c + d*x)^(7/2))/(7*d^6)
```

Rubi [A] time = 0.0483056, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2b^4(c+dx)^{5/2}(bc-ad)}{d^6}+\frac{20b^3(c+dx)^{3/2}(bc-ad)^2}{3d^6}-\frac{20b^2\sqrt{c+dx}(bc-ad)^3}{d^6}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}+\frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}+\frac{2b^5(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^(5/2),x]
```



```
[Out] (2*(b*c - a*d)^5)/(3*d^6*(c + d*x)^(3/2)) - (10*b*(b*c - a*d)^4)/(d^6*Sqrt[
c + d*x]) - (20*b^2*(b*c - a*d)^3*Sqrt[c + d*x])/d^6 + (20*b^3*(b*c - a*d)^
2*(c + d*x)^(3/2))/(3*d^6) - (2*b^4*(b*c - a*d)*(c + d*x)^(5/2))/d^6 + (2*b
^5*(c + d*x)^(7/2))/(7*d^6)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^{5/2}}\,dx=\int \left(\frac{(-bc+ad)^5}{d^5(c+dx)^{5/2}}+\frac{5b(bc-ad)^4}{d^5(c+dx)^{3/2}}-\frac{10b^2(bc-ad)^3}{d^5\sqrt{c+dx}}+\frac{10b^3(bc-ad)^2\sqrt{c+dx}}{d^5}-\frac{5b^4(bc-ad)(c+dx)^{3/}}{d^5}\right.$$
$$=\frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}-\frac{20b^2(bc-ad)^3\sqrt{c+dx}}{d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{3/2}}{3d^6}-\frac{2b^4(bc-ad)(}{d^6}$$

Mathematica [A] time = 0.112737, size = 123, normalized size = 0.81

$$\frac{2\left(-210b^2(c+dx)^2(bc-ad)^3+70b^3(c+dx)^3(bc-ad)^2-21b^4(c+dx)^4(bc-ad)-105b(c+dx)(bc-ad)^4+7(bc-ad)^5+\right.}{21d^6(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^(5/2),x]
```

```
[Out] (2*(7*(b*c - a*d)^5 - 105*b*(b*c - a*d)^4*(c + d*x) - 210*b^2*(b*c - a*d)^3
*(c + d*x)^2 + 70*b^3*(b*c - a*d)^2*(c + d*x)^3 - 21*b^4*(b*c - a*d)*(c + d
*x)^4 + 3*b^5*(c + d*x)^5))/(21*d^6*(c + d*x)^(3/2))
```

Maple [B] time = 0.006, size = 273, normalized size = 1.8

$$-\frac{-6\,b^5x^5d^5-42\,ab^4d^5x^4+12\,b^5cd^4x^4-140\,a^2b^3d^5x^3+112\,ab^4cd^4x^3-32\,b^5c^2d^3x^3-420\,a^3b^2d^5x^2+840\,a^2b^3cd^4x^2-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(5/2),x)
```

```
[Out] -2/21/(d*x+c)^(3/2)*(-3*b^5*d^5*x^5-21*a*b^4*d^5*x^4+6*b^5*c*d^4*x^4-70*a^2
*b^3*d^5*x^3+56*a*b^4*c*d^4*x^3-16*b^5*c^2*d^3*x^3-210*a^3*b^2*d^5*x^2+420*
a^2*b^3*c*d^4*x^2-336*a*b^4*c^2*d^3*x^2+96*b^5*c^3*d^2*x^2+105*a^4*b*d^5*x-
840*a^3*b^2*c*d^4*x+1680*a^2*b^3*c^2*d^3*x-1344*a*b^4*c^3*d^2*x+384*b^5*c^4
*d*x+7*a^5*d^5+70*a^4*b*c*d^4-560*a^3*b^2*c^2*d^3+1120*a^2*b^3*c^3*d^2-896*
a*b^4*c^4*d+256*b^5*c^5)/d^6
```

Maxima [A] time = 0.961379, size = 358, normalized size = 2.36

$$\frac{2\left(\frac{3\,(dx+c)^{\frac{7}{2}}b^5-21\left(b^5c-ab^4d\right)(dx+c)^{\frac{5}{2}}+70\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)(dx+c)^{\frac{3}{2}}-210\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)\sqrt{dx+c}}{d^5}+\frac{7\left(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3\right.}{}\right.}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*((3*(d*x + c)^(7/2)*b^5 - 21*(b^5*c - a*b^4*d)*(d*x + c)^(5/2) + 70*(b
^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(3/2) - 210*(b^5*c^3 - 3*a*b^
4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt(d*x + c))/d^5 + 7*(b^5*c^5 -
5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a
^5*d^5 - 15*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3
+ a^4*b*d^4)*(d*x + c))/((d*x + c)^(3/2)*d^5))/d
```

Fricas [B] time = 2.08844, size = 603, normalized size = 3.97

$$\frac{2\left(3\,b^5d^5x^5-256\,b^5c^5+896\,ab^4c^4d-1120\,a^2b^3c^3d^2+560\,a^3b^2c^2d^3-70\,a^4bcd^4-7\,a^5d^5-3\left(2\,b^5cd^4-7\,ab^4d^5\right)x^4+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b^5*d^5*x^5 - 256*b^5*c^5 + 896*a*b^4*c^4*d - 1120*a^2*b^3*c^3*d^2
+ 560*a^3*b^2*c^2*d^3 - 70*a^4*b*c*d^4 - 7*a^5*d^5 - 3*(2*b^5*c*d^4 - 7*a*b
^4*d^5)*x^4 + 2*(8*b^5*c^2*d^3 - 28*a*b^4*c*d^4 + 35*a^2*b^3*d^5)*x^3 - 6*(
16*b^5*c^3*d^2 - 56*a*b^4*c^2*d^3 + 70*a^2*b^3*c*d^4 - 35*a^3*b^2*d^5)*x^2
- 3*(128*b^5*c^4*d - 448*a*b^4*c^3*d^2 + 560*a^2*b^3*c^2*d^3 - 280*a^3*b^2*
```

```
c*d^4 + 35*a^4*b*d^5)*x)*sqrt(d*x + c)/(d^8*x^2 + 2*c*d^7*x + c^2*d^6)
```

Sympy [A] time = 38.8678, size = 196, normalized size = 1.29

$$\frac{2b^5(c+dx)^{\frac{7}{2}}}{7d^6} - \frac{10b(ad-bc)^4}{d^6\sqrt{c+dx}} + \frac{(c+dx)^{\frac{5}{2}}(10ab^4d - 10b^5c)}{5d^6} + \frac{(c+dx)^{\frac{3}{2}}(20a^2b^3d^2 - 40ab^4cd + 20b^5c^2)}{3d^6} + \frac{\sqrt{c+dx}\,(20a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(5/2),x)
```

```
[Out] 2*b**5*(c + d*x)**(7/2)/(7*d**6) - 10*b*(a*d - b*c)**4/(d**6*sqrt(c + d*x))
 + (c + d*x)**(5/2)*(10*a*b**4*d - 10*b**5*c)/(5*d**6) + (c + d*x)**(3/2)*(
20*a**2*b**3*d**2 - 40*a*b**4*c*d + 20*b**5*c**2)/(3*d**6) + sqrt(c + d*x)*
(20*a**3*b**2*d**3 - 60*a**2*b**3*c*d**2 + 60*a*b**4*c**2*d - 20*b**5*c**3)
/d**6 - 2*(a*d - b*c)**5/(3*d**6*(c + d*x)**(3/2))
```

Giac [B] time = 1.07182, size = 452, normalized size = 2.97

$$-\frac{2\left(15(dx+c)b^5c^4 - b^5c^5 - 60(dx+c)ab^4c^3d + 5ab^4c^4d + 90(dx+c)a^2b^3c^2d^2 - 10a^2b^3c^3d^2 - 60(dx+c)a^3b^2cd^3 + 10a\right.}{3(dx+c)^{\frac{3}{2}}d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(15*(d*x + c)*b^5*c^4 - b^5*c^5 - 60*(d*x + c)*a*b^4*c^3*d + 5*a*b^4*c
^4*d + 90*(d*x + c)*a^2*b^3*c^2*d^2 - 10*a^2*b^3*c^3*d^2 - 60*(d*x + c)*a^3
*b^2*c*d^3 + 10*a^3*b^2*c^2*d^3 + 15*(d*x + c)*a^4*b*d^4 - 5*a^4*b*c*d^4 +
a^5*d^5)/((d*x + c)^(3/2)*d^6) + 2/21*(3*(d*x + c)^(7/2)*b^5*d^36 - 21*(d*x
 + c)^(5/2)*b^5*c*d^36 + 70*(d*x + c)^(3/2)*b^5*c^2*d^36 - 210*sqrt(d*x + c
)*b^5*c^3*d^36 + 21*(d*x + c)^(5/2)*a*b^4*d^37 - 140*(d*x + c)^(3/2)*a*b^4*
c*d^37 + 630*sqrt(d*x + c)*a*b^4*c^2*d^37 + 70*(d*x + c)^(3/2)*a^2*b^3*d^38
 - 630*sqrt(d*x + c)*a^2*b^3*c*d^38 + 210*sqrt(d*x + c)*a^3*b^2*d^39)/d^42
```

3.1435 $\int \frac{(a+bx)^4}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{8b^3(c+dx)^{3/2}(bc-ad)}{3d^5}+\frac{12b^2\sqrt{c+dx}(bc-ad)^2}{d^5}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}-\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{2b^4(c+dx)^{5/2}}{5d^5}$$

```
[Out] (-2*(b*c - a*d)^4)/(3*d^5*(c + d*x)^(3/2)) + (8*b*(b*c - a*d)^3)/(d^5*Sqrt[
c + d*x]) + (12*b^2*(b*c - a*d)^2*Sqrt[c + d*x])/d^5 - (8*b^3*(b*c - a*d)*(
c + d*x)^(3/2))/(3*d^5) + (2*b^4*(c + d*x)^(5/2))/(5*d^5)
```

Rubi [A] time = 0.0385926, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{3/2}(bc-ad)}{3d^5}+\frac{12b^2\sqrt{c+dx}(bc-ad)^2}{d^5}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}-\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{2b^4(c+dx)^{5/2}}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^4)/(3*d^5*(c + d*x)^(3/2)) + (8*b*(b*c - a*d)^3)/(d^5*Sqrt[
c + d*x]) + (12*b^2*(b*c - a*d)^2*Sqrt[c + d*x])/d^5 - (8*b^3*(b*c - a*d)*(
c + d*x)^(3/2))/(3*d^5) + (2*b^4*(c + d*x)^(5/2))/(5*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^4}{d^4(c+dx)^{5/2}}-\frac{4b(bc-ad)^3}{d^4(c+dx)^{3/2}}+\frac{6b^2(bc-ad)^2}{d^4\sqrt{c+dx}}-\frac{4b^3(bc-ad)\sqrt{c+dx}}{d^4}+\frac{b^4(c+dx)^{3/2}}{d^4}\right)dx\\ &= -\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}+\frac{12b^2(bc-ad)^2\sqrt{c+dx}}{d^5}-\frac{8b^3(bc-ad)(c+dx)^{3/2}}{3d^5}+\frac{2b^4(c+dx)^{5/2}}{5d^5}\end{aligned}$$

Mathematica [A] time = 0.082142, size = 101, normalized size = 0.81

$$\frac{2\left(90b^2(c+dx)^2(bc-ad)^2-20b^3(c+dx)^3(bc-ad)+60b(c+dx)(bc-ad)^3-5(bc-ad)^4+3b^4(c+dx)^4\right)}{15d^5(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^(5/2),x]
```

```
[Out] (2*(-5*(b*c - a*d)^4 + 60*b*(b*c - a*d)^3*(c + d*x) + 90*b^2*(b*c - a*d)^2*
(c + d*x)^2 - 20*b^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4))/(15*d^5*
```

```
(c + d*x)^(3/2))
```

Maple [A] time = 0.005, size = 186, normalized size = 1.5

$$-\frac{-6\,b^4x^4d^4-40\,ab^3d^4x^3+16\,b^4cd^3x^3-180\,a^2b^2d^4x^2+240\,ab^3cd^3x^2-96\,b^4c^2d^2x^2+120\,a^3bd^4x-720\,a^2b^2cd^3x+960\,a}{15\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(5/2),x)
```

```
[Out] -2/15/(d*x+c)^(3/2)*(-3*b^4*d^4*x^4-20*a*b^3*d^4*x^3+8*b^4*c*d^3*x^3-90*a^2
*b^2*d^4*x^2+120*a*b^3*c*d^3*x^2-48*b^4*c^2*d^2*x^2+60*a^3*b*d^4*x-360*a^2*
b^2*c*d^3*x+480*a*b^3*c^2*d^2*x-192*b^4*c^3*d*x+5*a^4*d^4+40*a^3*b*c*d^3-24
0*a^2*b^2*c^2*d^2+320*a*b^3*c^3*d-128*b^4*c^4)/d^5
```

Maxima [A] time = 0.973523, size = 252, normalized size = 2.02

$$\frac{2\left(\frac{3\,(dx+c)^{\frac{5}{2}}b^4-20\,\left(b^4c-ab^3d\right)(dx+c)^{\frac{3}{2}}+90\,\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)\sqrt{dx+c}}{d^4}-\frac{5\,\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4-12\,\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right.}{(dx+c)^{\frac{3}{2}}d^4}\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(d*x + c)^(5/2)*b^4 - 20*(b^4*c - a*b^3*d)*(d*x + c)^(3/2) + 90*(b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*sqrt(d*x + c))/d^4 - 5*(b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4 - 12*(b^4*c^3 - 3*a*
b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c))/((d*x + c)^(3/2)*d^4))/
d
```

Fricas [A] time = 2.08512, size = 431, normalized size = 3.45

$$\frac{2\left(3\,b^4d^4x^4+128\,b^4c^4-320\,ab^3c^3d+240\,a^2b^2c^2d^2-40\,a^3bcd^3-5\,a^4d^4-4\left(2\,b^4cd^3-5\,ab^3d^4\right)x^3+6\left(8\,b^4c^2d^2-20\,ab^3c\right.\right.}{15\left(d^7x^2+2\,cd^6x+c^2d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*d^4*x^4 + 128*b^4*c^4 - 320*a*b^3*c^3*d + 240*a^2*b^2*c^2*d^2 -
 40*a^3*b*c*d^3 - 5*a^4*d^4 - 4*(2*b^4*c*d^3 - 5*a*b^3*d^4)*x^3 + 6*(8*b^4*
c^2*d^2 - 20*a*b^3*c*d^3 + 15*a^2*b^2*d^4)*x^2 + 12*(16*b^4*c^3*d - 40*a*b^
3*c^2*d^2 + 30*a^2*b^2*c*d^3 - 5*a^3*b*d^4)*x)*sqrt(d*x + c)/(d^7*x^2 + 2*c
*d^6*x + c^2*d^5)
```

Sympy [A] time = 28.054, size = 136, normalized size = 1.09

$$\frac{2b^4(c+dx)^{\frac{5}{2}}}{5d^5}-\frac{8b(ad-bc)^3}{d^5\sqrt{c+dx}}+\frac{(c+dx)^{\frac{3}{2}}\left(8ab^3d-8b^4c\right)}{3d^5}+\frac{\sqrt{c+dx}\left(12a^2b^2d^2-24ab^3cd+12b^4c^2\right)}{d^5}-\frac{2(ad-bc)^4}{3d^5(c+dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(5/2),x)
```

```
[Out] 2*b**4*(c + d*x)**(5/2)/(5*d**5) - 8*b*(a*d - b*c)**3/(d**5*sqrt(c + d*x))
+ (c + d*x)**(3/2)*(8*a*b**3*d - 8*b**4*c)/(3*d**5) + sqrt(c + d*x)*(12*a**
2*b**2*d**2 - 24*a*b**3*c*d + 12*b**4*c**2)/d**5 - 2*(a*d - b*c)**4/(3*d**5
*(c + d*x)**(3/2))
```

Giac [B] time = 1.07748, size = 309, normalized size = 2.47

$$\frac{2\left(12(dx+c)b^4c^3-b^4c^4-36(dx+c)ab^3c^2d+4ab^3c^3d+36(dx+c)a^2b^2cd^2-6a^2b^2c^2d^2-12(dx+c)a^3bd^3+4a^3bc\right.}{3(dx+c)^{\frac{3}{2}}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(12*(d*x + c)*b^4*c^3 - b^4*c^4 - 36*(d*x + c)*a*b^3*c^2*d + 4*a*b^3*c^
3*d + 36*(d*x + c)*a^2*b^2*c*d^2 - 6*a^2*b^2*c^2*d^2 - 12*(d*x + c)*a^3*b*d
^3 + 4*a^3*b*c*d^3 - a^4*d^4)/((d*x + c)^(3/2)*d^5) + 2/15*(3*(d*x + c)^(5/
2)*b^4*d^20 - 20*(d*x + c)^(3/2)*b^4*c*d^20 + 90*sqrt(d*x + c)*b^4*c^2*d^20
 + 20*(d*x + c)^(3/2)*a*b^3*d^21 - 180*sqrt(d*x + c)*a*b^3*c*d^21 + 90*sqrt
(d*x + c)*a^2*b^2*d^22)/d^25
```

3.1436 $\int \frac{(a+bx)^3}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{6b^2\sqrt{c+dx}(bc-ad)}{d^4}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}+\frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}+\frac{2b^3(c+dx)^{3/2}}{3d^4}$$

```
[Out] (2*(b*c - a*d)^3)/(3*d^4*(c + d*x)^(3/2)) - (6*b*(b*c - a*d)^2)/(d^4*Sqrt[c
 + d*x]) - (6*b^2*(b*c - a*d)*Sqrt[c + d*x])/d^4 + (2*b^3*(c + d*x)^(3/2))/
(3*d^4)
```

Rubi [A] time = 0.0309867, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{6b^2\sqrt{c+dx}(bc-ad)}{d^4}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}+\frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}+\frac{2b^3(c+dx)^{3/2}}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^3)/(3*d^4*(c + d*x)^(3/2)) - (6*b*(b*c - a*d)^2)/(d^4*Sqrt[c
 + d*x]) - (6*b^2*(b*c - a*d)*Sqrt[c + d*x])/d^4 + (2*b^3*(c + d*x)^(3/2))/
(3*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^{5/2}}+\frac{3b(bc-ad)^2}{d^3(c+dx)^{3/2}}-\frac{3b^2(bc-ad)}{d^3\sqrt{c+dx}}+\frac{b^3\sqrt{c+dx}}{d^3}\right)dx\\ &= \frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}-\frac{6b^2(bc-ad)\sqrt{c+dx}}{d^4}+\frac{2b^3(c+dx)^{3/2}}{3d^4}\end{aligned}$$

Mathematica [A] time = 0.0555299, size = 76, normalized size = 0.79

$$\frac{2\left(-9b^2(c+dx)^2(bc-ad)-9b(c+dx)(bc-ad)^2+(bc-ad)^3+b^3(c+dx)^3\right)}{3d^4(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^(5/2),x]
```

```
[Out] (2*((b*c - a*d)^3 - 9*b*(b*c - a*d)^2*(c + d*x) - 9*b^2*(b*c - a*d)*(c + d*
x)^2 + b^3*(c + d*x)^3))/(3*d^4*(c + d*x)^(3/2))
```

Maple [A] time = 0.005, size = 115, normalized size = 1.2

$$-\frac{-2\,b^3x^3d^3-18\,ab^2d^3x^2+12\,b^3cd^2x^2+18\,a^2bd^3x-72\,ab^2cd^2x+48\,b^3c^2dx+2\,a^3d^3+12\,a^2bcd^2-48\,ab^2c^2d+32\,b^3}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] -2/3/(d*x+c)^(3/2)*(-b^3*d^3*x^3-9*a*b^2*d^3*x^2+6*b^3*c*d^2*x^2+9*a^2*b*d^
3*x-36*a*b^2*c*d^2*x+24*b^3*c^2*d*x+a^3*d^3+6*a^2*b*c*d^2-24*a*b^2*c^2*d+16
*b^3*c^3)/d^4
```

Maxima [A] time = 0.961498, size = 165, normalized size = 1.72

$$\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^3-9\left(b^3c-ab^2d\right)\sqrt{dx+c}}{d^3}+\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3-9\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)(dx+c)}{(dx+c)^{\frac{3}{2}}d^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((d*x + c)^(3/2)*b^3 - 9*(b^3*c - a*b^2*d)*sqrt(d*x + c))/d^3 + (b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - 9*(b^3*c^2 - 2*a*b^2*c*d + a
^2*b*d^2)*(d*x + c))/((d*x + c)^(3/2)*d^3))/d
```

Fricas [A] time = 1.98774, size = 281, normalized size = 2.93

$$\frac{2\left(b^3d^3x^3-16\,b^3c^3+24\,ab^2c^2d-6\,a^2bcd^2-a^3d^3-3\left(2\,b^3cd^2-3\,ab^2d^3\right)x^2-3\left(8\,b^3c^2d-12\,ab^2cd^2+3\,a^2bd^3\right)x\right)\sqrt{dx}}{3\left(d^6x^2+2\,cd^5x+c^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*d^3*x^3 - 16*b^3*c^3 + 24*a*b^2*c^2*d - 6*a^2*b*c*d^2 - a^3*d^3 -
3*(2*b^3*c*d^2 - 3*a*b^2*d^3)*x^2 - 3*(8*b^3*c^2*d - 12*a*b^2*c*d^2 + 3*a^2
*b*d^3)*x)*sqrt(d*x + c)/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)
```

Sympy [A] time = 1.3615, size = 461, normalized size = 4.8

$$\begin{cases} -\frac{2a^3d^3}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}-\frac{12a^2bcd^2}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}-\frac{18a^2bd^3x}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}+\frac{48ab^2c^2d}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}+\frac{72ab^2cd^2x}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}+\frac{1}{3cd^4\sqrt{c}} \\ \frac{a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}}{c^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**3*d**3/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) -
 12*a**2*b*c*d**2/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 18*a*
*2*b*d**3*x/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 48*a*b**2*c
**2*d/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 72*a*b**2*c*d**2*
x/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 18*a*b**2*d**3*x**2/(
3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 32*b**3*c**3/(3*c*d**4*s
qrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 48*b**3*c**2*d*x/(3*c*d**4*sqrt(c
+ d*x) + 3*d**5*x*sqrt(c + d*x)) - 12*b**3*c*d**2*x**2/(3*c*d**4*sqrt(c + d
*x) + 3*d**5*x*sqrt(c + d*x)) + 2*b**3*d**3*x**3/(3*c*d**4*sqrt(c + d*x) +
3*d**5*x*sqrt(c + d*x)), Ne(d, 0)), ((a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**
3 + b**3*x**4/4)/c**(5/2), True))
```

Giac [A] time = 1.06457, size = 190, normalized size = 1.98

$$-\frac{2\left(9\,(dx+c)b^3c^2 - b^3c^3 - 18\,(dx+c)ab^2cd + 3\,ab^2c^2d + 9\,(dx+c)a^2bd^2 - 3\,a^2bcd^2 + a^3d^3\right)}{3\,(dx+c)^{\frac{3}{2}}d^4} + \frac{2\left((dx+c)^{\frac{3}{2}}b^3d^8 - 9\,\sqrt{dx\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(9*(d*x + c)*b^3*c^2 - b^3*c^3 - 18*(d*x + c)*a*b^2*c*d + 3*a*b^2*c^2*
d + 9*(d*x + c)*a^2*b*d^2 - 3*a^2*b*c*d^2 + a^3*d^3)/((d*x + c)^(3/2)*d^4)
+ 2/3*((d*x + c)^(3/2)*b^3*d^8 - 9*sqrt(d*x + c)*b^3*c*d^8 + 9*sqrt(d*x + c
)*a*b^2*d^9)/d^12
```

3.1437 $\int \frac{(a+bx)^2}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{4b(bc-ad)}{d^3\sqrt{c+dx}}-\frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}}+\frac{2b^2\sqrt{c+dx}}{d^3}$$

```
[Out] (-2*(b*c - a*d)^2)/(3*d^3*(c + d*x)^(3/2)) + (4*b*(b*c - a*d))/(d^3*Sqrt[c
+ d*x]) + (2*b^2*Sqrt[c + d*x])/d^3
```

Rubi [A] time = 0.0211512, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{4b(bc-ad)}{d^3\sqrt{c+dx}}-\frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}}+\frac{2b^2\sqrt{c+dx}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^2)/(3*d^3*(c + d*x)^(3/2)) + (4*b*(b*c - a*d))/(d^3*Sqrt[c
+ d*x]) + (2*b^2*Sqrt[c + d*x])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{5/2}}-\frac{2b(bc-ad)}{d^2(c+dx)^{3/2}}+\frac{b^2}{d^2\sqrt{c+dx}}\right)dx\\ &= -\frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}}+\frac{4b(bc-ad)}{d^3\sqrt{c+dx}}+\frac{2b^2\sqrt{c+dx}}{d^3}\end{aligned}$$

Mathematica [A] time = 0.0321098, size = 62, normalized size = 0.93

$$\frac{-2a^2d^2-4abd(2c+3dx)+2b^2\left(8c^2+12cdx+3d^2x^2\right)}{3d^3(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*a^2*d^2 - 4*a*b*d*(2*c + 3*d*x) + 2*b^2*(8*c^2 + 12*c*d*x + 3*d^2*x^2))
/(3*d^3*(c + d*x)^(3/2))
```

Maple [A] time = 0.004, size = 62, normalized size = 0.9

$$-\frac{-6\,b^2x^2d^2+12\,abd^2x-24\,b^2cdx+2\,a^2d^2+8\,abcd-16\,b^2c^2}{3\,d^3}\,(dx+c)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(5/2),x)
```

```
[Out] -2/3/(d*x+c)^(3/2)*(-3*b^2*d^2*x^2+6*a*b*d^2*x-12*b^2*c*d*x+a^2*d^2+4*a*b*c
*d-8*b^2*c^2)/d^3
```

Maxima [A] time = 0.956515, size = 97, normalized size = 1.45

$$\frac{2\left(\frac{3\,\sqrt{dx+c}b^2}{d^2}-\frac{b^2c^2-2\,abcd+a^2d^2-6\left(b^2c-abd\right)(dx+c)}{(dx+c)^{\frac{3}{2}}d^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(d*x + c)*b^2/d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 6*(b^2*c -
a*b*d)*(d*x + c))/((d*x + c)^(3/2)*d^2))/d
```

Fricas [A] time = 2.084, size = 174, normalized size = 2.6

$$\frac{2\left(3\,b^2d^2x^2+8\,b^2c^2-4\,abcd-a^2d^2+6\left(2\,b^2cd-abd^2\right)x\right)\sqrt{dx+c}}{3\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*d^2*x^2 + 8*b^2*c^2 - 4*a*b*c*d - a^2*d^2 + 6*(2*b^2*c*d - a*b*d
^2)*x)*sqrt(d*x + c)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [A] time = 1.20079, size = 265, normalized size = 3.96

$$\begin{cases} -\frac{2a^2d^2}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}}-\frac{8abcd}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}}-\frac{12abd^2x}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}}+\frac{16b^2c^2}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}}+\frac{24b^2cdx}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}}+\frac{6b^2}{3cd^3\sqrt{c+dx}} \\ \frac{a^2x+abx^2+\frac{b^2x^3}{3}}{c^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*d**2/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) -
 8*a*b*c*d/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) - 12*a*b*d**2*
```

```
x/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) + 16*b**2*c**2/(3*c*d**
3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) + 24*b**2*c*d*x/(3*c*d**3*sqrt(c
+ d*x) + 3*d**4*x*sqrt(c + d*x)) + 6*b**2*d**2*x**2/(3*c*d**3*sqrt(c + d*x)
 + 3*d**4*x*sqrt(c + d*x)), Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)/c
**(5/2), True))
```

Giac [A] time = 1.08627, size = 97, normalized size = 1.45

$$\frac{2\sqrt{dx+c}b^2}{d^3}+\frac{2\left(6\,(dx+c)b^2c-b^2c^2-6\,(dx+c)abd+2\,abcd-a^2d^2\right)}{3\,(dx+c)^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*b^2/d^3 + 2/3*(6*(d*x + c)*b^2*c - b^2*c^2 - 6*(d*x + c)*a*
b*d + 2*a*b*c*d - a^2*d^2)/((d*x + c)^(3/2)*d^3)
```

3.1438 $\int \frac{a+bx}{(c+dx)^{5/2}} \, dx$

Optimal. Leaf size=40

$$\frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}$$

```
[Out] (2*(b*c - a*d))/(3*d^2*(c + d*x)^(3/2)) - (2*b)/(d^2*Sqrt[c + d*x])
```

Rubi [A] time = 0.0139255, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d))/(3*d^2*(c + d*x)^(3/2)) - (2*b)/(d^2*Sqrt[c + d*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^{5/2}} \, dx &= \int \left(\frac{-bc+ad}{d(c+dx)^{5/2}}+\frac{b}{d(c+dx)^{3/2}}\right) dx\\ &= \frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0275788, size = 29, normalized size = 0.72

$$-\frac{2(ad+2bc+3bdx)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(2*b*c + a*d + 3*b*d*x))/(3*d^2*(c + d*x)^(3/2))
```

Maple [A] time = 0.003, size = 26, normalized size = 0.7

$$-\frac{6\,bdx+2\,ad+4\,bc}{3\,d^2}(dx+c)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3/(d*x+c)^(3/2)*(3*b*d*x+a*d+2*b*c)/d^2
```

Maxima [A] time = 0.949559, size = 38, normalized size = 0.95

$$-\frac{2\,(3\,(dx+c)b-bc+ad)}{3\,(dx+c)^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*(d*x + c)*b - b*c + a*d)/((d*x + c)^(3/2)*d^2)
```

Fricas [A] time = 2.00135, size = 103, normalized size = 2.58

$$-\frac{2\,(3\,bdx+2\,bc+ad)\sqrt{dx+c}}{3\,\left(d^4x^2+2\,cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b*d*x + 2*b*c + a*d)*sqrt(d*x + c)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [A] time = 1.07312, size = 124, normalized size = 3.1

$$\begin{cases} -\frac{2ad}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}} - \frac{4bc}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}} - \frac{6bdx}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}} & \text{for } d \neq 0 \\ \frac{ax+\frac{bx^2}{2}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a*d/(3*c*d**2*sqrt(c + d*x) + 3*d**3*x*sqrt(c + d*x)) - 4*b*c
/(3*c*d**2*sqrt(c + d*x) + 3*d**3*x*sqrt(c + d*x)) - 6*b*d*x/(3*c*d**2*sqrt
(c + d*x) + 3*d**3*x*sqrt(c + d*x)), Ne(d, 0)), ((a*x + b*x**2/2)/c**(5/2),
 True))
```

Giac [A] time = 1.05535, size = 38, normalized size = 0.95

$$-\frac{2\,(3\,(dx+c)b-bc+ad)}{3\,(dx+c)^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*(d*x + c)*b - b*c + a*d)/((d*x + c)^(3/2)*d^2)
```

3.1439 $\int \frac{1}{(c+dx)^{5/2}} dx$

Optimal. Leaf size=16

$$-\frac{2}{3d(c+dx)^{3/2}}$$

```
[Out] -2/(3*d*(c + d*x)^(3/2))
```

Rubi [A] time = 0.0013823, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-5/2),x]
```

```
[Out] -2/(3*d*(c + d*x)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^{5/2}} dx = -\frac{2}{3d(c+dx)^{3/2}}$$

Mathematica [A] time = 0.0067318, size = 16, normalized size = 1.

$$-\frac{2}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-5/2),x]
```

```
[Out] -2/(3*d*(c + d*x)^(3/2))
```

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$-\frac{2}{3d}(dx+c)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d/(d*x+c)^(3/2)
```

Maxima [A] time = 0.959591, size = 16, normalized size = 1.

$$-\frac{2}{3\,(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((d*x + c)^(3/2)*d)
```

Fricas [B] time = 2.04605, size = 68, normalized size = 4.25

$$-\frac{2\sqrt{dx+c}}{3\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(d*x + c)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Sympy [A] time = 0.06058, size = 14, normalized size = 0.88

$$-\frac{2}{3d\,(c+dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(5/2),x)
```

```
[Out] -2/(3*d*(c + d*x)**(3/2))
```

Giac [A] time = 1.08114, size = 16, normalized size = 1.

$$-\frac{2}{3\,(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/((d*x + c)^(3/2)*d)
```

3.1440 $\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{2b}{\sqrt{c+dx}(bc-ad)^2}+\frac{2}{3(c+dx)^{3/2}(bc-ad)}$$

```
[Out] 2/(3*(b*c - a*d)*(c + d*x)^(3/2)) + (2*b)/((b*c - a*d)^2*Sqrt[c + d*x]) - (
2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/
2)
```

Rubi [A] time = 0.0387108, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{2b}{\sqrt{c+dx}(bc-ad)^2}+\frac{2}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(5/2)),x]
```


```
[Out] 2/(3*(b*c - a*d)*(c + d*x)^(3/2)) + (2*b)/((b*c - a*d)^2*Sqrt[c + d*x]) - (
2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/
2)
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}}dx = \frac{2}{3(bc-ad)(c+dx)^{3/2}} + \frac{b\int \frac{1}{(a+bx)(c+dx)^{3/2}}dx}{bc-ad}$$
$$= \frac{2}{3(bc-ad)(c+dx)^{3/2}} + \frac{2b}{(bc-ad)^2\sqrt{c+dx}} + \frac{b^2\int \frac{1}{(a+bx)\sqrt{c+dx}}dx}{(bc-ad)^2}$$
$$= \frac{2}{3(bc-ad)(c+dx)^{3/2}} + \frac{2b}{(bc-ad)^2\sqrt{c+dx}} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}dx, x, \sqrt{c+dx}\right)}{d(bc-ad)^2}$$
$$= \frac{2}{3(bc-ad)(c+dx)^{3/2}} + \frac{2b}{(bc-ad)^2\sqrt{c+dx}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.011491, size = 48, normalized size = 0.52

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b(c+dx)}{bc-ad}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(5/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, (b*(c + d*x))/(b*c - a*d)])/(3*(b*c - a
*d)*(c + d*x)^(3/2))
```

Maple [A] time = 0.009, size = 90, normalized size = 1.

$$-\frac{2}{3\,ad-3\,bc}(dx+c)^{-\frac{3}{2}}+2\,\frac{b}{(ad-bc)^2\sqrt{dx+c}}+2\,\frac{b^2}{(ad-bc)^2\sqrt{(ad-bc)\,b}}\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3/(a*d-b*c)/(d*x+c)^(3/2)+2*b/(a*d-b*c)^2/(d*x+c)^(1/2)+2*b^2/(a*d-b*c)^
2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14546, size = 841, normalized size = 9.04

$$\left[\frac{3\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad-2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)+2\,(3\,bdx+4\,bc-ad)\sqrt{dx+c}}{3\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^2+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x\right)},-\frac{2\left(3\left(bd^2x^2+2\right.\right.}{3\left(b^2c^4-2\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*
b*c - a*d - 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2
*(3*b*d*x + 4*b*c - a*d)*sqrt(d*x + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^
2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^
2 + a^2*c*d^3)*x), -2/3*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-b/(b*c - a
*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c))
- (3*b*d*x + 4*b*c - a*d)*sqrt(d*x + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d
^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d
^2 + a^2*c*d^3)*x)]
```

Sympy [A] time = 6.62344, size = 83, normalized size = 0.89

$$\frac{2b}{\sqrt{c+dx}\,(ad-bc)^2}+\frac{2b\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)^2}-\frac{2}{3\,(c+dx)^{\frac{3}{2}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] 2*b/(sqrt(c + d*x)*(a*d - b*c)**2) + 2*b*atan(sqrt(c + d*x)/sqrt((a*d - b*c
)/b))/(sqrt((a*d - b*c)/b)*(a*d - b*c)**2) - 2/(3*(c + d*x)**(3/2)*(a*d - b
*c))
```

Giac [A] time = 1.07016, size = 153, normalized size = 1.65

$$\frac{2\,b^2\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{2\,(3\,(dx+c)b+bc-ad)}{3\left(b^2c^2-2\,abcd+a^2d^2\right)(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 2/3*(3*(d*x + c)*b + b*c - a*d)/((b^2*c^2
- 2*a*b*c*d + a^2*d^2)*(d*x + c)^(3/2))
```

3.1441 $\int \frac{1}{(a+bx)^2(c+dx)^{5/2}} dx$

Optimal. Leaf size=124

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}-\frac{5bd}{\sqrt{c+dx}(bc-ad)^3}-\frac{1}{(a+bx)(c+dx)^{3/2}(bc-ad)}-\frac{5d}{3(c+dx)^{3/2}(bc-ad)^2}$$

```
[Out] (-5*d)/(3*(b*c - a*d)^2*(c + d*x)^(3/2)) - 1/((b*c - a*d)*(a + b*x)*(c + d*
x)^(3/2)) - (5*b*d)/((b*c - a*d)^3*Sqrt[c + d*x]) + (5*b^(3/2)*d*ArcTanh[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2)
```

Rubi [A] time = 0.0497228, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}-\frac{5bd}{\sqrt{c+dx}(bc-ad)^3}-\frac{1}{(a+bx)(c+dx)^{3/2}(bc-ad)}-\frac{5d}{3(c+dx)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^(5/2)),x]
```


```
[Out] (-5*d)/(3*(b*c - a*d)^2*(c + d*x)^(3/2)) - 1/((b*c - a*d)*(a + b*x)*(c + d*
x)^(3/2)) - (5*b*d)/((b*c - a*d)^3*Sqrt[c + d*x]) + (5*b^(3/2)*d*ArcTanh[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2)
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx = -\frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{(5d)\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{(5bd)\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx}{2(bc-ad)^2}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}} - \frac{(5b^2d)\int \frac{1}{(a}{2(bc}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}} - \frac{(5b^2)\,\mathrm{Subs}}{}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}} + \frac{5b^{3/2}d\tan}{(bc}$$

Mathematica [C] time = 0.0143284, size = 50, normalized size = 0.4

$$-\frac{2d\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(c+dx)^{3/2}(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*d*Hypergeometric2F1[-3/2, 2, -1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/(3
*(-(b*c) + a*d)^2*(c + d*x)^(3/2))
```

Maple [A] time = 0.016, size = 125, normalized size = 1.

$$-\frac{2d}{3(ad-bc)^2}(dx+c)^{-\frac{3}{2}} + 4\frac{bd}{(ad-bc)^3\sqrt{dx+c}} + \frac{b^2d}{(ad-bc)^3(bdx+ad)}\sqrt{dx+c} + 5\frac{b^2d}{(ad-bc)^3\sqrt{(ad-bc)b}}\arctan\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*d/(a*d-b*c)^2/(d*x+c)^(3/2)+4*d/(a*d-b*c)^3*b/(d*x+c)^(1/2)+d*b^2/(a*d
-b*c)^3*(d*x+c)^(1/2)/(b*d*x+a*d)+5*d*b^2/(a*d-b*c)^3/((a*d-b*c)*b)^(1/2)*a
rctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.29637, size = 1592, normalized size = 12.84

$$\left[-\frac{15\left(b^2d^3x^3+abc^2d+\left(2\,b^2cd^2+abd^3\right)x^2+\left(b^2c^2d+2\,abcd^2\right)x\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad-2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)+2}{6\left(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5\right)x^3+\left(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(b^2*d^3*x^3 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^2 + (b^2*c^2
*d + 2*a*b*c*d^2)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c
- a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2*(15*b^2*d^2*x^2 +
3*b^2*c^2 + 14*a*b*c*d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x)*sqrt(d*x +
 c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^
3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d -
 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^
5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x), 1/
3*(15*(b^2*d^3*x^3 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^2 + (b^2*c^2*d +
 2*a*b*c*d^2)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqr
t(-b/(b*c - a*d))/(b*d*x + b*c)) - (15*b^2*d^2*x^2 + 3*b^2*c^2 + 14*a*b*c*d
 - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x)*sqrt(d*x + c))/(a*b^3*c^5 - 3*a^
2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^
3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a
^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^
2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.06954, size = 292, normalized size = 2.35

$$-\frac{5\,b^2d\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}-\frac{\sqrt{dx+c}b^2d}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)((dx+c)b-bc+ad)}-\frac{}{3\left(b^3c^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -5*b^2*d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - sqrt(d*x + c)*b^2*d
/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)*b - b*c +
a*d)) - 2/3*(6*(d*x + c)*b*d + b*c*d - a*d^2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(3/2))
```

3.1442 $\int \frac{1}{(a+bx)^3(c+dx)^{5/2}} \, dx$

Optimal. Leaf size=167

$$-\frac{35b^{3/2}d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{9/2}}+\frac{35bd^2}{4\sqrt{c+dx}(bc-ad)^4}+\frac{35d^2}{12(c+dx)^{3/2}(bc-ad)^3}+\frac{7d}{4(a+bx)(c+dx)^{3/2}(bc-ad)^2}-\frac{\cdots}{2(a+bx\cdots}$$

```
[Out] (35*d^2)/(12*(b*c - a*d)^3*(c + d*x)^(3/2)) - 1/(2*(b*c - a*d)*(a + b*x)^2*
(c + d*x)^(3/2)) + (7*d)/(4*(b*c - a*d)^2*(a + b*x)*(c + d*x)^(3/2)) + (35*
b*d^2)/(4*(b*c - a*d)^4*Sqrt[c + d*x]) - (35*b^(3/2)*d^2*ArcTanh[(Sqrt[b]*S
qrt[c + d*x])/Sqrt[b*c - a*d]])/(4*(b*c - a*d)^(9/2))
```

Rubi [A] time = 0.0616835, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{35b^{3/2}d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{9/2}}+\frac{35bd^2}{4\sqrt{c+dx}(bc-ad)^4}+\frac{35d^2}{12(c+dx)^{3/2}(bc-ad)^3}+\frac{7d}{4(a+bx)(c+dx)^{3/2}(bc-ad)^2}-\frac{\cdots}{2(a+bx\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

```
[Out] (35*d^2)/(12*(b*c - a*d)^3*(c + d*x)^(3/2)) - 1/(2*(b*c - a*d)*(a + b*x)^2*
(c + d*x)^(3/2)) + (7*d)/(4*(b*c - a*d)^2*(a + b*x)*(c + d*x)^(3/2)) + (35*
b*d^2)/(4*(b*c - a*d)^4*Sqrt[c + d*x]) - (35*b^(3/2)*d^2*ArcTanh[(Sqrt[b]*S
qrt[c + d*x])/Sqrt[b*c - a*d]])/(4*(b*c - a*d)^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^{5/2}}\,dx = -\frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} - \frac{(7d)\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx}{4(bc-ad)}$$

$$= -\frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+dx)^{3/2}} + \frac{(35d^2)\int \frac{1}{(a+bx)(c+dx)^{5/2}}}{8(bc-ad)^2}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+dx)^{3/2}}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+dx)^{3/2}}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+dx)^{3/2}}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+dx)^{3/2}}$$

Mathematica [C] time = 0.0157224, size = 52, normalized size = 0.31

$$-\frac{2d^2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(c+dx)^{3/2}(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*d^2*Hypergeometric2F1[-3/2, 3, -1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/
(3*(-(b*c) + a*d)^3*(c + d*x)^(3/2))
```

Maple [A] time = 0.016, size = 206, normalized size = 1.2

$$-\frac{2\,d^2}{3\,(ad-bc)^3}(dx+c)^{-\frac{3}{2}} + 6\,\frac{d^2b}{(ad-bc)^4\sqrt{dx+c}} + \frac{11\,d^2b^3}{4\,(ad-bc)^4\,(bdx+ad)^2}(dx+c)^{\frac{3}{2}} + \frac{13\,d^3b^2a}{4\,(ad-bc)^4\,(bdx+ad)^2}\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*d^2/(a*d-b*c)^3/(d*x+c)^(3/2)+6*d^2/(a*d-b*c)^4*b/(d*x+c)^(1/2)+11/4*d
^2/(a*d-b*c)^4*b^3/(b*d*x+a*d)^2*(d*x+c)^(3/2)+13/4*d^3/(a*d-b*c)^4*b^2/(b*
d*x+a*d)^2*(d*x+c)^(1/2)*a-13/4*d^2/(a*d-b*c)^4*b^3/(b*d*x+a*d)^2*(d*x+c)^(
1/2)*c+35/4*d^2/(a*d-b*c)^4*b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/
((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42103, size = 2472, normalized size = 14.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*d^4*x^4 + a^2*b*c^2*d^2 + 2*(b^3*c*d^3 + a*b^2*d^4)*x^3 + (
b^3*c^2*d^2 + 4*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(a*b^2*c^2*d^2 + a^2*b*c*d
^3)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c - a*d)*sqrt(d*
x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2*(105*b^3*d^3*x^3 - 6*b^3*c^3 + 3
9*a*b^2*c^2*d + 80*a^2*b*c*d^2 - 8*a^3*d^3 + 35*(4*b^3*c*d^2 + 5*a*b^2*d^3)
*x^2 + 7*(3*b^3*c^2*d + 34*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^
2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2
*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5
 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 +
2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b^6*c^6 - 9*a^2*b^4
*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*x^2 + 2*(a*b^5
*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^
2*d^4 + a^6*c*d^5)*x), -1/12*(105*(b^3*d^4*x^4 + a^2*b*c^2*d^2 + 2*(b^3*c*d
^3 + a*b^2*d^4)*x^3 + (b^3*c^2*d^2 + 4*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(a*
b^2*c^2*d^2 + a^2*b*c*d^3)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt
(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - (105*b^3*d^3*x^3 - 6*b^3*c^
3 + 39*a*b^2*c^2*d + 80*a^2*b*c*d^2 - 8*a^3*d^3 + 35*(4*b^3*c*d^2 + 5*a*b^2
*d^3)*x^2 + 7*(3*b^3*c^2*d + 34*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x + c)
)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^
6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*
c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d
^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b^6*c^6 - 9*a^
2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*x^2 + 2*(
a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5
*b*c^2*d^4 + a^6*c*d^5)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.08402, size = 402, normalized size = 2.41

$$\frac{35\,b^2d^2\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{4\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{-b^2c+abd}}+\frac{2\left(9\,(dx+c)bd^2+bcd^2-ad^3\right)}{3\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d
)) + 2/3*(9*(d*x + c)*b*d^2 + b*c*d^2 - a*d^3)/((b^4*c^4 - 4*a*b^3*c^3*d +
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*x + c)^(3/2)) + 1/4*(11*(d*
x + c)^(3/2)*b^3*d^2 - 13*sqrt(d*x + c)*b^3*c*d^2 + 13*sqrt(d*x + c)*a*b^2*
d^3)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^
4)*((d*x + c)*b - b*c + a*d)^2)
```

3.1443 $\int \frac{1}{(a+bx)^4(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=200

$$\frac{105b^{3/2}d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{11/2}}-\frac{105bd^3}{8\sqrt{c+dx}(bc-ad)^5}-\frac{35d^3}{8(c+dx)^{3/2}(bc-ad)^4}-\frac{21d^2}{8(a+bx)(c+dx)^{3/2}(bc-ad)^3}+\frac{}{4(a+bx)^2}$$

```
[Out] (-35*d^3)/(8*(b*c - a*d)^4*(c + d*x)^(3/2)) - 1/(3*(b*c - a*d)*(a + b*x)^3*
(c + d*x)^(3/2)) + (3*d)/(4*(b*c - a*d)^2*(a + b*x)^2*(c + d*x)^(3/2)) - (2
1*d^2)/(8*(b*c - a*d)^3*(a + b*x)*(c + d*x)^(3/2)) - (105*b*d^3)/(8*(b*c -
a*d)^5*Sqrt[c + d*x]) + (105*b^(3/2)*d^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/(8*(b*c - a*d)^(11/2))
```

Rubi [A] time = 0.134447, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{105b^{3/2}d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{11/2}}-\frac{105bd^3}{8\sqrt{c+dx}(bc-ad)^5}-\frac{35d^3}{8(c+dx)^{3/2}(bc-ad)^4}-\frac{21d^2}{8(a+bx)(c+dx)^{3/2}(bc-ad)^3}+\frac{}{4(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*(c + d*x)^(5/2)),x]
```

```
[Out] (-35*d^3)/(8*(b*c - a*d)^4*(c + d*x)^(3/2)) - 1/(3*(b*c - a*d)*(a + b*x)^3*
(c + d*x)^(3/2)) + (3*d)/(4*(b*c - a*d)^2*(a + b*x)^2*(c + d*x)^(3/2)) - (2
1*d^2)/(8*(b*c - a*d)^3*(a + b*x)*(c + d*x)^(3/2)) - (105*b*d^3)/(8*(b*c -
a*d)^5*Sqrt[c + d*x]) + (105*b^(3/2)*d^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/(8*(b*c - a*d)^(11/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4(c+dx)^{5/2}}\,dx = -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} - \frac{(3d)\int \frac{1}{(a+bx)^3(c+dx)^{5/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/2}} + \frac{\left(21d^2\right)\int \frac{1}{(a+bx)^2(c+dx)}}{8(bc-ad)^2}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/2}} - \frac{21d^2}{8(bc-ad)^3(a+bx)(c}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/}}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/}}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/}}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/}}$$

Mathematica [C] time = 0.0213188, size = 52, normalized size = 0.26

$$-\frac{2d^3\,{}_2F_1\left(-\frac{3}{2},4;-\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(c+dx)^{3/2}(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*d^3*Hypergeometric2F1[-3/2, 4, -1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/
(3*(-(b*c) + a*d)^4*(c + d*x)^(3/2))
```

Maple [A] time = 0.019, size = 319, normalized size = 1.6

$$-\frac{2\,d^3}{3\,(ad-bc)^4}(dx+c)^{-\frac{3}{2}} + 8\,\frac{d^3b}{(ad-bc)^5\sqrt{dx+c}} + \frac{41\,d^3b^4}{8\,(ad-bc)^5\,(bdx+ad)^3}(dx+c)^{\frac{5}{2}} + \frac{35\,d^4b^3a}{3\,(ad-bc)^5\,(bdx+ad)^3}(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*d^3/(a*d-b*c)^4/(d*x+c)^(3/2)+8*d^3/(a*d-b*c)^5*b/(d*x+c)^(1/2)+41/8*d
^3/(a*d-b*c)^5*b^4/(b*d*x+a*d)^3*(d*x+c)^(5/2)+35/3*d^4/(a*d-b*c)^5*b^3/(b*
d*x+a*d)^3*(d*x+c)^(3/2)*a-35/3*d^3/(a*d-b*c)^5*b^4/(b*d*x+a*d)^3*(d*x+c)^(
3/2)*c+55/8*d^5/(a*d-b*c)^5*b^2/(b*d*x+a*d)^3*(d*x+c)^(1/2)*a^2-55/4*d^4/(a
*d-b*c)^5*b^3/(b*d*x+a*d)^3*(d*x+c)^(1/2)*a*c+55/8*d^3/(a*d-b*c)^5*b^4/(b*d
*x+a*d)^3*(d*x+c)^(1/2)*c^2+105/8*d^3/(a*d-b*c)^5*b^2/((a*d-b*c)*b)^(1/2)*a
rctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.55919, size = 3717, normalized size = 18.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(315*(b^4*d^5*x^5 + a^3*b*c^2*d^3 + (2*b^4*c*d^4 + 3*a*b^3*d^5)*x^4
+ (b^4*c^2*d^3 + 6*a*b^3*c*d^4 + 3*a^2*b^2*d^5)*x^3 + (3*a*b^3*c^2*d^3 + 6*
a^2*b^2*c*d^4 + a^3*b*d^5)*x^2 + (3*a^2*b^2*c^2*d^3 + 2*a^3*b*c*d^4)*x)*sqr
t(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c - a*d)*sqrt(d*x + c)*sqr
t(b/(b*c - a*d)))/(b*x + a)) + 2*(315*b^4*d^4*x^4 + 8*b^4*c^4 - 50*a*b^3*c^
3*d + 165*a^2*b^2*c^2*d^2 + 208*a^3*b*c*d^3 - 16*a^4*d^4 + 420*(b^4*c*d^3 +
 2*a*b^3*d^4)*x^3 + 63*(b^4*c^2*d^2 + 18*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2
- 18*(b^4*c^3*d - 10*a*b^3*c^2*d^2 - 53*a^2*b^2*c*d^3 - 8*a^3*b*d^4)*x)*sqr
t(d*x + c))/(a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^
2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5*a*b^7*c^4*d^3
+ 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7)*
x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3*b^5*c^3*d
^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*x^4 + (b^8*c^7
+ a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^
4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (3*a*b^7*c^7 - 9
*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4
+ 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b^6*c^7 - 13*a^3
*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 +
7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*x), 1/24*(315*(b^4*d^5*x^5 + a^3*b*c^2*d^3 +
 (2*b^4*c*d^4 + 3*a*b^3*d^5)*x^4 + (b^4*c^2*d^3 + 6*a*b^3*c*d^4 + 3*a^2*b^2
*d^5)*x^3 + (3*a*b^3*c^2*d^3 + 6*a^2*b^2*c*d^4 + a^3*b*d^5)*x^2 + (3*a^2*b^
2*c^2*d^3 + 2*a^3*b*c*d^4)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt
(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - (315*b^4*d^4*x^4 + 8*b^4*c^
4 - 50*a*b^3*c^3*d + 165*a^2*b^2*c^2*d^2 + 208*a^3*b*c*d^3 - 16*a^4*d^4 + 4
20*(b^4*c*d^3 + 2*a*b^3*d^4)*x^3 + 63*(b^4*c^2*d^2 + 18*a*b^3*c*d^3 + 11*a^
2*b^2*d^4)*x^2 - 18*(b^4*c^3*d - 10*a*b^3*c^2*d^2 - 53*a^2*b^2*c*d^3 - 8*a^
3*b*d^4)*x)*sqrt(d*x + c))/(a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*
d^2 - 10*a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5
*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6
- a^5*b^3*d^7)*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 1
0*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*
x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25
*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (
3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a
^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b
^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6
*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.06702, size = 583, normalized size = 2.92

$$-\frac{105\, b^2 d^3 \arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{8\left(b^5c^5 - 5\, ab^4c^4d + 10\, a^2b^3c^3d^2 - 10\, a^3b^2c^2d^3 + 5\, a^4bcd^4 - a^5d^5\right)\sqrt{-b^2c + abd}} - \frac{315\, (dx+c)^4 b^4 d^3 - 840\, (dx+c)^3 b^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -105/8*b^2*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^5 - 5*a
*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*
d^5)*sqrt(-b^2*c + a*b*d)) - 1/24*(315*(d*x + c)^4*b^4*d^3 - 840*(d*x + c)^
3*b^4*c*d^3 + 693*(d*x + c)^2*b^4*c^2*d^3 - 144*(d*x + c)*b^4*c^3*d^3 - 16*
b^4*c^4*d^3 + 840*(d*x + c)^3*a*b^3*d^4 - 1386*(d*x + c)^2*a*b^3*c*d^4 + 43
2*(d*x + c)*a*b^3*c^2*d^4 + 64*a*b^3*c^3*d^4 + 693*(d*x + c)^2*a^2*b^2*d^5
- 432*(d*x + c)*a^2*b^2*c*d^5 - 96*a^2*b^2*c^2*d^5 + 144*(d*x + c)*a^3*b*d^
6 + 64*a^3*b*c*d^6 - 16*a^4*d^7)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3
*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*((d*x + c)^(3/2)*b - s
qrt(d*x + c)*b*c + sqrt(d*x + c)*a*d)^3)
```

3.1444 $\int (a+bx)^5(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{15/2}}{15bc^6}$$

```
[Out] (2*(a*c + b*c*x)^(15/2))/(15*b*c^6)
```

Rubi [A] time = 0.0045903, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{15/2}}{15bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(15/2))/(15*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^{3/2}\,dx &= \frac{\int (ac+bcx)^{13/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{15/2}}{15bc^6}\end{aligned}$$

Mathematica [A] time = 0.0204973, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6(c(a+bx))^{3/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^6*(c*(a + b*x))^(3/2))/(15*b)
```

Maple [A] time = 0.002, size = 23, normalized size = 1.1

$$\frac{2 (bx + a)^6}{15 b} (bcx + ac)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^(3/2),x)
```

```
[Out] 2/15*(b*x+a)^6*(b*c*x+a*c)^(3/2)/b
```

Maxima [A] time = 0.956037, size = 24, normalized size = 1.09

$$\frac{2 (bcx + ac)^{\frac{15}{2}}}{15 bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(b*c*x + a*c)^(15/2)/(b*c^6)
```

Fricas [B] time = 2.10389, size = 205, normalized size = 9.32

$$\frac{2 \left(b^7cx^7 + 7 ab^6cx^6 + 21 a^2b^5cx^5 + 35 a^3b^4cx^4 + 35 a^4b^3cx^3 + 21 a^5b^2cx^2 + 7 a^6bcx + a^7c\right) \sqrt{bcx + ac}}{15 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(b^7*c*x^7 + 7*a*b^6*c*x^6 + 21*a^2*b^5*c*x^5 + 35*a^3*b^4*c*x^4 + 35*
a^4*b^3*c*x^3 + 21*a^5*b^2*c*x^2 + 7*a^6*b*c*x + a^7*c)*sqrt(b*c*x + a*c)/b
```

Sympy [A] time = 1.09982, size = 66, normalized size = 3.

$$\begin{cases} \frac{2b^{\frac{13}{2}} c^{\frac{3}{2}} \left(\frac{a}{b}+x\right)^{\frac{15}{2}}}{15} & \text{for}\: \left|\frac{a}{b} + x\right| < 1 \\ b^{\frac{13}{2}} c^{\frac{3}{2}} G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{17}{2} \\ \frac{15}{2} & 0 \end{matrix} \middle| \frac{a}{b} + x \right) + b^{\frac{13}{2}} c^{\frac{3}{2}} G_{2,2}^{0,2}\left(\begin{matrix} \frac{17}{2}, 1 & \\ & \frac{15}{2}, 0 \end{matrix} \middle| \frac{a}{b} + x \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**(3/2),x)
```

```
[Out] Piecewise((2*b**(13/2)*c**(3/2)*(a/b + x)**(15/2)/15, Abs(a/b + x) < 1), (b
**(13/2)*c**(3/2)*meijerg(((1,), (17/2,)), ((15/2,), (0,)), a/b + x) + b**(
13/2)*c**(3/2)*meijerg(((17/2, 1), ()), ((), (15/2, 0)), a/b + x), True))
```

Giac [B] time = 1.09202, size = 668, normalized size = 30.36

$$2\left(15015\,(bcx+ac)^{\frac{3}{2}}a^6 - \frac{18018\left(5\,(bcx+ac)^{\frac{3}{2}}ac-3\,(bcx+ac)^{\frac{5}{2}}\right)a^5}{c} + \frac{6435\left(35\,(bcx+ac)^{\frac{3}{2}}a^2c^2-42\,(bcx+ac)^{\frac{5}{2}}ac+15\,(bcx+ac)^{\frac{7}{2}}\right)a^4}{c^2} - \frac{2860\left(105\,(bcx+ac)^{\frac{3}{2}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/45045*(15015*(b*c*x + a*c)^(3/2)*a^6 - 18018*(5*(b*c*x + a*c)^(3/2)*a*c -
 3*(b*c*x + a*c)^(5/2))*a^5/c + 6435*(35*(b*c*x + a*c)^(3/2)*a^2*c^2 - 42*(
b*c*x + a*c)^(5/2)*a*c + 15*(b*c*x + a*c)^(7/2))*a^4/c^2 - 2860*(105*(b*c*x
 + a*c)^(3/2)*a^3*c^3 - 189*(b*c*x + a*c)^(5/2)*a^2*c^2 + 135*(b*c*x + a*c)
^(7/2)*a*c - 35*(b*c*x + a*c)^(9/2))*a^3/c^3 + 195*(1155*(b*c*x + a*c)^(3/2
)*a^4*c^4 - 2772*(b*c*x + a*c)^(5/2)*a^3*c^3 + 2970*(b*c*x + a*c)^(7/2)*a^2
*c^2 - 1540*(b*c*x + a*c)^(9/2)*a*c + 315*(b*c*x + a*c)^(11/2))*a^2/c^4 - 3
0*(3003*(b*c*x + a*c)^(3/2)*a^5*c^5 - 9009*(b*c*x + a*c)^(5/2)*a^4*c^4 + 12
870*(b*c*x + a*c)^(7/2)*a^3*c^3 - 10010*(b*c*x + a*c)^(9/2)*a^2*c^2 + 4095*
(b*c*x + a*c)^(11/2)*a*c - 693*(b*c*x + a*c)^(13/2))*a/c^5 + (15015*(b*c*x
+ a*c)^(3/2)*a^6*c^6 - 54054*(b*c*x + a*c)^(5/2)*a^5*c^5 + 96525*(b*c*x + a
*c)^(7/2)*a^4*c^4 - 100100*(b*c*x + a*c)^(9/2)*a^3*c^3 + 61425*(b*c*x + a*c
)^(11/2)*a^2*c^2 - 20790*(b*c*x + a*c)^(13/2)*a*c + 3003*(b*c*x + a*c)^(15/
2))/c^6)/b
```

3.1445 $\int (a+bx)^5\sqrt{ac+bcx}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{13/2}}{13bc^6}$$

[Out] (2*(a*c + b*c*x)^(13/2))/(13*b*c^6)

Rubi [A] time = 0.0045631, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{13/2}}{13bc^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5*Sqrt[a*c + b*c*x],x]

[Out] (2*(a*c + b*c*x)^(13/2))/(13*b*c^6)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5\sqrt{ac+bcx}\,dx &= \frac{\int (ac+bcx)^{11/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{13/2}}{13bc^6}\end{aligned}$$

Mathematica [A] time = 0.0114057, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6\sqrt{c(a+bx)}}{13b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^5*Sqrt[a*c + b*c*x],x]

[Out] (2*(a + b*x)^6*Sqrt[c*(a + b*x)])/(13*b)

Maple [A] time = 0.003, size = 23, normalized size = 1.1

$$\frac{2\,(bx+a)^6}{13\,b}\sqrt{bcx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^(1/2),x)
```

```
[Out] 2/13*(b*x+a)^6*(b*c*x+a*c)^(1/2)/b
```

Maxima [A] time = 0.958328, size = 24, normalized size = 1.09

$$\frac{2\,(bcx+ac)^{\frac{13}{2}}}{13\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*c*x + a*c)^(13/2)/(b*c^6)
```

Fricas [B] time = 1.93621, size = 161, normalized size = 7.32

$$\frac{2\left(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6\right)\sqrt{bcx+ac}}{13\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/13*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*
x^2 + 6*a^5*b*x + a^6)*sqrt(b*c*x + a*c)/b
```

Sympy [A] time = 0.877335, size = 66, normalized size = 3.

$$\begin{cases} \frac{2b^{\frac{11}{2}}\sqrt{c}\left(\frac{a}{b}+x\right)^{\frac{13}{2}}}{13} & \text{for}\ \left|\frac{a}{b}+x\right|<1 \\ b^{\frac{11}{2}}\sqrt{c}G_{2,2}^{1,1}\left(\begin{matrix}1 & \frac{15}{2} \\ \frac{13}{2} & 0\end{matrix}\middle|\frac{a}{b}+x\right)+b^{\frac{11}{2}}\sqrt{c}G_{2,2}^{0,2}\left(\begin{matrix}\frac{15}{2},1 & \\ & \frac{13}{2},0\end{matrix}\middle|\frac{a}{b}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**(1/2),x)
```

```
[Out] Piecewise((2*b**(11/2)*sqrt(c)*(a/b + x)**(13/2)/13, Abs(a/b + x) < 1), (b*
*(11/2)*sqrt(c)*meijerg(((1,), (15/2,)), ((13/2,), (0,)), a/b + x) + b**(11
/2)*sqrt(c)*meijerg(((15/2, 1), ()), ((), (13/2, 0)), a/b + x), True))
```

Giac [B] time = 1.09542, size = 505, normalized size = 22.95

$$2\left(3003\,(bcx+ac)^{\frac{3}{2}}a^5-\frac{3003\left(5\,(bcx+ac)^{\frac{3}{2}}ac-3\,(bcx+ac)^{\frac{5}{2}}\right)a^4}{c}+\frac{858\left(35\,(bcx+ac)^{\frac{3}{2}}a^2c^2-42\,(bcx+ac)^{\frac{5}{2}}ac+15\,(bcx+ac)^{\frac{7}{2}}\right)a^3}{c^2}-\frac{286\left(105\,(bcx+ac)^{\frac{3}{2}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9009*(3003*(b*c*x + a*c)^(3/2)*a^5 - 3003*(5*(b*c*x + a*c)^(3/2)*a*c - 3*
(b*c*x + a*c)^(5/2))*a^4/c + 858*(35*(b*c*x + a*c)^(3/2)*a^2*c^2 - 42*(b*c*
x + a*c)^(5/2)*a*c + 15*(b*c*x + a*c)^(7/2))*a^3/c^2 - 286*(105*(b*c*x + a*
c)^(3/2)*a^3*c^3 - 189*(b*c*x + a*c)^(5/2)*a^2*c^2 + 135*(b*c*x + a*c)^(7/2
)*a*c - 35*(b*c*x + a*c)^(9/2))*a^2/c^3 + 13*(1155*(b*c*x + a*c)^(3/2)*a^4*
c^4 - 2772*(b*c*x + a*c)^(5/2)*a^3*c^3 + 2970*(b*c*x + a*c)^(7/2)*a^2*c^2 -
 1540*(b*c*x + a*c)^(9/2)*a*c + 315*(b*c*x + a*c)^(11/2))*a/c^4 - (3003*(b*
c*x + a*c)^(3/2)*a^5*c^5 - 9009*(b*c*x + a*c)^(5/2)*a^4*c^4 + 12870*(b*c*x
+ a*c)^(7/2)*a^3*c^3 - 10010*(b*c*x + a*c)^(9/2)*a^2*c^2 + 4095*(b*c*x + a*
c)^(11/2)*a*c - 693*(b*c*x + a*c)^(13/2))/c^5)/(b*c)
```

3.1446 $\int \frac{(a+bx)^5}{\sqrt{ac+bcx}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{11/2}}{11bc^6}$$

```
[Out] (2*(a*c + b*c*x)^(11/2))/(11*b*c^6)
```

Rubi [A] time = 0.0044284, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{11/2}}{11bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/Sqrt[a*c + b*c*x],x]
```

```
[Out] (2*(a*c + b*c*x)^(11/2))/(11*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\sqrt{ac+bcx}}\,dx &= \frac{\int (ac+bcx)^{9/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{11/2}}{11bc^6}\end{aligned}$$

Mathematica [A] time = 0.0134179, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{11b\sqrt{c(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/Sqrt[a*c + b*c*x],x]
```

```
[Out] (2*(a + b*x)^6)/(11*b*Sqrt[c*(a + b*x)])
```

Maple [A] time = 0.001, size = 23, normalized size = 1.1

$$\frac{2(bx+a)^6}{11b}\frac{1}{\sqrt{bcx+ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(1/2),x)
```

```
[Out] 2/11*(b*x+a)^6/b/(b*c*x+a*c)^(1/2)
```

Maxima [B] time = 0.964648, size = 505, normalized size = 22.95

$$2\left(693\sqrt{bcx+ac}a^5 - \frac{1155\left(3\sqrt{bcx+ac}ac-(bcx+ac)^{\frac{3}{2}}\right)a^4}{c} + \frac{462\left(15\sqrt{bcx+ac}a^2c^2-10(bcx+ac)^{\frac{3}{2}}ac+3(bcx+ac)^{\frac{5}{2}}\right)a^3}{c^2} - \frac{198\left(35\sqrt{bcx+ac}a^3c^3-35(bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(693*sqrt(b*c*x + a*c)*a^5 - 1155*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x +
 a*c)^(3/2))*a^4/c + 462*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(
3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^3/c^2 - 198*(35*sqrt(b*c*x + a*c)*a^3*c
^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x
 + a*c)^(7/2))*a^2/c^3 + 11*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a
*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/
2)*a*c + 35*(b*c*x + a*c)^(9/2))*a/c^4 - (693*sqrt(b*c*x + a*c)*a^5*c^5 - 1
155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3 - 990*(b
*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*x + a*c)^
(11/2))/c^5)/(b*c)
```

Fricas [B] time = 2.02566, size = 143, normalized size = 6.5

$$\frac{2\left(b^5x^5 + 5ab^4x^4 + 10a^2b^3x^3 + 10a^3b^2x^2 + 5a^4bx + a^5\right)\sqrt{bcx+ac}}{11bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/11*(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x +
 a^5)*sqrt(b*c*x + a*c)/(b*c)
```

Sympy [A] time = 1.29558, size = 73, normalized size = 3.32

$$\begin{cases} \frac{2b^{\frac{9}{2}}\left(\frac{a}{b}+x\right)^{\frac{11}{2}}}{11\sqrt{c}} & \text{for } \left|\frac{a}{b}+x\right| > 1 \vee \left|\frac{a}{b}+x\right| < 1 \\ \frac{b^{\frac{9}{2}}G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{13}{2} \\ \frac{11}{2} & 0 \end{matrix}\middle| \frac{a}{b}+x\right)}{\sqrt{c}} + \frac{b^{\frac{9}{2}}G_{2,2}^{0,2}\left(\begin{matrix} \frac{13}{2}, 1 & \\ & \frac{11}{2}, 0 \end{matrix}\middle| \frac{a}{b}+x\right)}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(1/2),x)
```

```
[Out] Piecewise((2*b**(9/2)*(a/b + x)**(11/2)/(11*sqrt(c)), (Abs(a/b + x) > 1) |
(Abs(a/b + x) < 1)), (b**(9/2)*meijerg(((1,), (13/2,)), ((11/2,), (0,)), a/
b + x)/sqrt(c) + b**(9/2)*meijerg(((13/2, 1), ()), ((), (11/2, 0)), a/b + x
)/sqrt(c), True))
```

Giac [B] time = 1.0683, size = 505, normalized size = 22.95

$$2\left(693\sqrt{bcx+ac}a^5 - \frac{1155\left(3\sqrt{bcx+ac}ac-(bcx+ac)^{\frac{3}{2}}\right)a^4}{c} + \frac{462\left(15\sqrt{bcx+ac}a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a^3}{c^2} - \frac{198\left(35\sqrt{bcx+ac}a^3c^3-35\,(bcx+ac\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/693*(693*sqrt(b*c*x + a*c)*a^5 - 1155*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x +
 a*c)^(3/2))*a^4/c + 462*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(
3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^3/c^2 - 198*(35*sqrt(b*c*x + a*c)*a^3*c
^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x
 + a*c)^(7/2))*a^2/c^3 + 11*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a
*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/
2)*a*c + 35*(b*c*x + a*c)^(9/2))*a/c^4 - (693*sqrt(b*c*x + a*c)*a^5*c^5 - 1
155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3 - 990*(b
*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*x + a*c)^
(11/2))/c^5)/(b*c)
```

3.1447 $\int \frac{(a+bx)^5}{(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{9/2}}{9bc^6}$$

```
[Out] (2*(a*c + b*c*x)^(9/2))/(9*b*c^6)
```

Rubi [A] time = 0.0043641, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{9/2}}{9bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(9/2))/(9*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{3/2}}\,dx &= \frac{\int (ac+bcx)^{7/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{9/2}}{9bc^6}\end{aligned}$$

Mathematica [A] time = 0.0148942, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{9b(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^6)/(9*b*(c*(a + b*x))^(3/2))
```

Maple [A] time = 0.001, size = 23, normalized size = 1.1

$$\frac{2\left(bx+a\right)^{6}}{9b}\left(bcx+ac\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(3/2),x)
```

```
[Out] 2/9*(b*x+a)^6/b/(b*c*x+a*c)^(3/2)
```

Maxima [A] time = 0.977158, size = 24, normalized size = 1.09

$$\frac{2\left(bcx+ac\right)^{\frac{9}{2}}}{9\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*c*x + a*c)^(9/2)/(b*c^6)
```

Fricas [B] time = 2.07865, size = 120, normalized size = 5.45

$$\frac{2\left(b^{4}x^{4}+4\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}+4\,a^{3}bx+a^{4}\right)\sqrt{bcx+ac}}{9\,bc^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/9*(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*sqrt(b*c*x +
a*c)/(b*c^2)
```

Sympy [A] time = 1.3438, size = 73, normalized size = 3.32

$$\begin{cases} \frac{2b^{\frac{7}{2}}\left(\frac{a}{b}+x\right)^{\frac{9}{2}}}{9c^{\frac{3}{2}}} & \text{for}\ \left|\frac{a}{b}+x\right|>1 \vee \left|\frac{a}{b}+x\right|<1 \\ \frac{b^{\frac{7}{2}}G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{11}{2} \\ \frac{9}{2} & 0 \end{matrix} \middle| \frac{a}{b}+x\right)}{c^{\frac{3}{2}}} + \frac{b^{\frac{7}{2}}G_{2,2}^{0,2}\left(\begin{matrix} \frac{11}{2}, 1 & \\ & \frac{9}{2}, 0 \end{matrix} \middle| \frac{a}{b}+x\right)}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(3/2),x)
```

```
[Out] Piecewise((2*b**(7/2)*(a/b + x)**(9/2)/(9*c**(3/2)), (Abs(a/b + x) > 1) | (
Abs(a/b + x) < 1)), (b**(7/2)*meijerg(((1,), (11/2,)), ((9/2,), (0,)), a/b
```

```
+ x)/c**(3/2) + b**(7/2)*meijerg(((11/2, 1), ()), ((), (9/2, 0)), a/b + x)/
c**(3/2), True))
```

Giac [B] time = 1.06055, size = 359, normalized size = 16.32

$$\frac{2\left(315\sqrt{bcx+ac}a^4 - \frac{420\left(3\sqrt{bcx+ac}ac-(bcx+ac)^{\frac{3}{2}}\right)a^3}{c} + \frac{126\left(15\sqrt{bcx+ac}a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a^2}{c^2} - \frac{36\left(35\sqrt{bcx+ac}a^3c^3-35\,(bcx+\right.}{}\right.}{315\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(b*c*x + a*c)*a^4 - 420*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x +
a*c)^(3/2))*a^3/c + 126*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3
/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^2/c^2 - 36*(35*sqrt(b*c*x + a*c)*a^3*c^3
 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x +
 a*c)^(7/2))*a/c^3 + (315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a*c)^(3/
2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/2)*a*c
+ 35*(b*c*x + a*c)^(9/2))/c^4)/(b*c^2)
```

3.1448 $\int \frac{(a+bx)^5}{(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{7/2}}{7bc^6}$$

```
[Out] (2*(a*c + b*c*x)^(7/2))/(7*b*c^6)
```

Rubi [A] time = 0.0042334, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{7/2}}{7bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(5/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(7/2))/(7*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{5/2}}\,dx &= \frac{\int (ac+bcx)^{5/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{7/2}}{7bc^6}\end{aligned}$$

Mathematica [A] time = 0.0132763, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{7b(c(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^6)/(7*b*(c*(a + b*x))^(5/2))
```

Maple [A] time = 0.002, size = 23, normalized size = 1.1

$$\frac{2\,(bx+a)^{6}}{7\,b}\,(bcx+ac)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(5/2),x)
```

```
[Out] 2/7*(b*x+a)^6/b/(b*c*x+a*c)^(5/2)
```

Maxima [A] time = 0.944802, size = 24, normalized size = 1.09

$$\frac{2\,(bcx+ac)^{\frac{7}{2}}}{7\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*c*x + a*c)^(7/2)/(b*c^6)
```

Fricas [B] time = 2.11416, size = 99, normalized size = 4.5

$$\frac{2\left(b^{3}x^{3}+3\,ab^{2}x^{2}+3\,a^{2}bx+a^{3}\right)\sqrt{bcx+ac}}{7\,bc^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*c*x + a*c)/(b*c^3)
```

Sympy [A] time = 1.38735, size = 73, normalized size = 3.32

$$\begin{cases} \frac{2b^{\frac{5}{2}}\left(\frac{a}{b}+x\right)^{\frac{7}{2}}}{7c^{\frac{5}{2}}} & \text{for } \left|\frac{a}{b}+x\right|>1 \vee \left|\frac{a}{b}+x\right|<1 \\ \frac{b^{\frac{5}{2}}G_{2,2}^{1,1}\left(\begin{matrix}1 & \frac{9}{2} \\ \frac{7}{2} & 0\end{matrix}\middle|\frac{a}{b}+x\right)}{c^{\frac{5}{2}}}+\frac{b^{\frac{5}{2}}G_{2,2}^{0,2}\left(\begin{matrix}\frac{9}{2},1 & \\ & \frac{7}{2},0\end{matrix}\middle|\frac{a}{b}+x\right)}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(5/2),x)
```

```
[Out] Piecewise((2*b**(5/2)*(a/b + x)**(7/2)/(7*c**(5/2)), (Abs(a/b + x) > 1) | (
Abs(a/b + x) < 1)), (b**(5/2)*meijerg(((1,), (9/2,)), ((7/2,), (0,)), a/b +
 x)/c**(5/2) + b**(5/2)*meijerg(((9/2, 1), ()), ((), (7/2, 0)), a/b + x)/c*
```

```
*(5/2), True))
```

Giac [B] time = 1.0773, size = 240, normalized size = 10.91

$$\frac{2\left(35\sqrt{bcx+ac}a^3 - \frac{35\left(3\sqrt{bcx+ac}ac-(bcx+ac)^{\frac{3}{2}}\right)a^2}{c} + \frac{7\left(15\sqrt{bcx+ac}a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a}{c^2} - \frac{35\sqrt{bcx+ac}a^3c^3-35\,(bcx+ac)^{\frac{3}{2}}a^2c^2+21\,(\cdots}{c^3}\right.}{35\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*sqrt(b*c*x + a*c)*a^3 - 35*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c
)^(3/2))*a^2/c + 7*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a
*c + 3*(b*c*x + a*c)^(5/2))*a/c^2 - (35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(b*c
*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^(7/2
))/c^3)/(b*c^3)
```

3.1449 $\int \frac{(a+bx)^5}{(ac+bcx)^{7/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{5/2}}{5bc^6}$$

```
[Out] (2*(a*c + b*c*x)^(5/2))/(5*b*c^6)
```

Rubi [A] time = 0.0042234, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{5/2}}{5bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(7/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(5/2))/(5*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{7/2}}\,dx &= \frac{\int (ac+bcx)^{3/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{5/2}}{5bc^6}\end{aligned}$$

Mathematica [A] time = 0.0126118, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{5b(c(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(7/2),x]
```

```
[Out] (2*(a + b*x)^6)/(5*b*(c*(a + b*x))^(7/2))
```

Maple [A] time = 0.002, size = 23, normalized size = 1.1

$$\frac{2\,(bx+a)^6}{5b}\,(bcx+ac)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(7/2),x)
```

```
[Out] 2/5*(b*x+a)^6/b/(b*c*x+a*c)^(7/2)
```

Maxima [A] time = 0.979997, size = 24, normalized size = 1.09

$$\frac{2\,(bcx+ac)^{\frac{5}{2}}}{5\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*c*x + a*c)^(5/2)/(b*c^6)
```

Fricas [A] time = 2.00158, size = 77, normalized size = 3.5

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{bcx+ac}}{5\,bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*c*x + a*c)/(b*c^4)
```

Sympy [A] time = 3.49026, size = 80, normalized size = 3.64

$$\begin{cases} \frac{2a^2\sqrt{ac+bcx}}{5bc^4} + \frac{4ax\sqrt{ac+bcx}}{5c^4} + \frac{2bx^2\sqrt{ac+bcx}}{5c^4} & \text{for } b \neq 0 \\ \frac{a^5x}{(ac)^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(7/2),x)
```

```
[Out] Piecewise((2*a**2*sqrt(a*c + b*c*x)/(5*b*c**4) + 4*a*x*sqrt(a*c + b*c*x)/(5
*c**4) + 2*b*x**2*sqrt(a*c + b*c*x)/(5*c**4), Ne(b, 0)), (a**5*x/(a*c)**(7/
2), True))
```

Giac [B] time = 1.07611, size = 143, normalized size = 6.5

$$\frac{2\left(15\sqrt{bcx+ac}a^2-\frac{10\left(3\sqrt{bcx+ac}ac-(bcx+ac)^{\frac{3}{2}}\right)a}{c}+\frac{15\sqrt{bcx+ac}a^2c^2-10(bcx+ac)^{\frac{3}{2}}ac+3(bcx+ac)^{\frac{5}{2}}}{c^2}\right)}{15\,bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(b*c*x + a*c)*a^2 - 10*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c
)^(3/2))*a/c + (15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c +
 3*(b*c*x + a*c)^(5/2))/c^2)/(b*c^4)
```

3.1450 $\int \frac{(a+bx)^5}{(ac+bcx)^{9/2}} dx$

Optimal. Leaf size=22

$$\frac{2(ac + bcx)^{3/2}}{3bc^6}$$

```
[Out] (2*(a*c + b*c*x)^(3/2))/(3*b*c^6)
```

Rubi [A] time = 0.0041991, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$\frac{2(ac + bcx)^{3/2}}{3bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(9/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(3/2))/(3*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a + bx)^5}{(ac + bcx)^{9/2}} dx &= \frac{\int \sqrt{ac + bcx}\, dx}{c^5}\\ &= \frac{2(ac + bcx)^{3/2}}{3bc^6}\end{aligned}$$

Mathematica [A] time = 0.0134821, size = 26, normalized size = 1.18

$$\frac{2(a + bx)\sqrt{c(a + bx)}}{3bc^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)*Sqrt[c*(a + b*x)])/(3*b*c^5)
```

Maple [A] time = 0.001, size = 23, normalized size = 1.1

$$\frac{2\left(bx+a\right)^{6}}{3b}\left(bcx+ac\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(9/2),x)
```

```
[Out] 2/3*(b*x+a)^6/b/(b*c*x+a*c)^(9/2)
```

Maxima [A] time = 0.966638, size = 24, normalized size = 1.09

$$\frac{2\left(bcx+ac\right)^{\frac{3}{2}}}{3\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*c*x + a*c)^(3/2)/(b*c^6)
```

Fricas [A] time = 1.96451, size = 55, normalized size = 2.5

$$\frac{2\sqrt{bcx+ac}(bx+a)}{3\,bc^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*c*x + a*c)*(b*x + a)/(b*c^5)
```

Sympy [A] time = 7.32454, size = 53, normalized size = 2.41

$$\begin{cases} \frac{2a\sqrt{ac+bcx}}{3bc^{5}}+\frac{2x\sqrt{ac+bcx}}{3c^{5}} & \text{for}\; b \neq 0 \\ \frac{a^{5}x}{(ac)^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(9/2),x)
```

```
[Out] Piecewise((2*a*sqrt(a*c + b*c*x)/(3*b*c**5) + 2*x*sqrt(a*c + b*c*x)/(3*c**5
), Ne(b, 0)), (a**5*x/(a*c)**(9/2), True))
```

Giac [B] time = 1.07177, size = 73, normalized size = 3.32

$$\frac{2\left(3\sqrt{bcx+ac}a-\frac{3\sqrt{bcx+ac}ac-(bcx+ac)^{\frac{3}{2}}}{c}\right)}{3\,bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(b*c*x + a*c)*a - (3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c)^(3/2)
)/c)/(b*c^5)
```

3.1451 $\int \frac{(a+bx)^5}{(ac+bcx)^{11/2}}\,dx$

Optimal. Leaf size=20

$$\frac{2\sqrt{ac+bcx}}{bc^6}$$

```
[Out] (2*Sqrt[a*c + b*c*x])/(b*c^6)
```

Rubi [A] time = 0.0041583, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$\frac{2\sqrt{ac+bcx}}{bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(11/2),x]
```

```
[Out] (2*Sqrt[a*c + b*c*x])/(b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{11/2}}\,dx &= \frac{\int \frac{1}{\sqrt{ac+bcx}}\,dx}{c^5}\\ &= \frac{2\sqrt{ac+bcx}}{bc^6}\end{aligned}$$

Mathematica [A] time = 0.0071777, size = 24, normalized size = 1.2

$$\frac{2(a+bx)}{bc^5\sqrt{c(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(11/2),x]
```

```
[Out] (2*(a + b*x))/(b*c^5*Sqrt[c*(a + b*x)])
```

Maple [A] time = 0.002, size = 23, normalized size = 1.2

$$2\frac{(bx+a)^6}{b(bcx+ac)^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(11/2),x)
```

```
[Out] 2*(b*x+a)^6/b/(b*c*x+a*c)^(11/2)
```

Maxima [A] time = 0.963576, size = 24, normalized size = 1.2

$$\frac{2\sqrt{bcx+ac}}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)
```

Fricas [A] time = 2.02837, size = 39, normalized size = 1.95

$$\frac{2\sqrt{bcx+ac}}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)
```

Sympy [A] time = 14.4151, size = 29, normalized size = 1.45

$$\begin{cases} \frac{2\sqrt{ac+bcx}}{bc^6} & \text{for } b \neq 0 \\ \frac{a^5x}{(ac)^{\frac{11}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(11/2),x)
```

```
[Out] Piecewise((2*sqrt(a*c + b*c*x)/(b*c**6), Ne(b, 0)), (a**5*x/(a*c)**(11/2),
True))
```

Giac [A] time = 1.05408, size = 24, normalized size = 1.2

$$\frac{2\sqrt{bcx + ac}}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)
```

3.1452 $\int \frac{(a+bx)^5}{(ac+bcx)^{13/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2}{bc^6\sqrt{ac+bcx}}$$

```
[Out] -2/(b*c^6*Sqrt[a*c + b*c*x])
```

Rubi [A] time = 0.0045802, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 32}

$$-\frac{2}{bc^6\sqrt{ac+bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(13/2),x]
```

```
[Out] -2/(b*c^6*Sqrt[a*c + b*c*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{13/2}}\,dx &= \frac{\int \frac{1}{(ac+bcx)^{3/2}}\,dx}{c^5}\\ &= -\frac{2}{bc^6\sqrt{ac+bcx}}\end{aligned}$$

Mathematica [A] time = 0.0091496, size = 24, normalized size = 1.2

$$-\frac{2(a+bx)}{bc^5(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(13/2),x]
```

```
[Out] (-2*(a + b*x))/(b*c^5*(c*(a + b*x))^(3/2))
```

Maple [A] time = 0.001, size = 23, normalized size = 1.2

$$-2\frac{(bx+a)^6}{b(bcx+ac)^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(13/2),x)
```

```
[Out] -2*(b*x+a)^6/b/(b*c*x+a*c)^(13/2)
```

Maxima [A] time = 0.955574, size = 24, normalized size = 1.2

$$-\frac{2}{\sqrt{bcx+ac}bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*c*x + a*c)*b*c^6)
```

Fricas [A] time = 1.97097, size = 59, normalized size = 2.95

$$-\frac{2\sqrt{bcx+ac}}{b^2c^7x+abc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*c*x + a*c)/(b^2*c^7*x + a*b*c^7)
```

Sympy [A] time = 39.472, size = 48, normalized size = 2.4

$$\begin{cases} -\frac{2\sqrt{ac+bcx}}{abc^7+b^2c^7x} & \text{for } a\neq 0 \\ -\frac{2}{b^{\frac{3}{2}}c^{\frac{13}{2}}\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(13/2),x)
```

```
[Out] Piecewise((-2*sqrt(a*c + b*c*x)/(a*b*c**7 + b**2*c**7*x), Ne(a, 0)), (-2/(b
**(3/2)*c**(13/2)*sqrt(x)), True))
```

<u>Giac</u> [A] time = 1.05479, size = 24, normalized size = 1.2

$$-\frac{2}{\sqrt{bcx+ac}bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(b*c*x + a*c)*b*c^6)
```

3.1453 $\int \frac{1}{(-2+x)\sqrt{2+x}}\,dx$

Optimal. Leaf size=14

$$-\tanh^{-1}\left(\frac{\sqrt{x+2}}{2}\right)$$

```
[Out] -ArcTanh[Sqrt[2 + x]/2]
```

Rubi [A] time = 0.0038494, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 207}

$$-\tanh^{-1}\left(\frac{\sqrt{x+2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)*Sqrt[2 + x]),x]
```

```
[Out] -ArcTanh[Sqrt[2 + x]/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-2+x)\sqrt{2+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx, x, \sqrt{2+x}\right)\\ &= -\tanh^{-1}\left(\frac{\sqrt{2+x}}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0025444, size = 14, normalized size = 1.

$$-\tanh^{-1}\left(\frac{\sqrt{x+2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + x)*Sqrt[2 + x]),x]
```

```
[Out] -ArcTanh[Sqrt[2 + x]/2]
```

Maple [B] time = 0.006, size = 22, normalized size = 1.6

$$-\frac{1}{2}\ln\left(\sqrt{2+x}+2\right)+\frac{1}{2}\ln\left(\sqrt{2+x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+x)/(2+x)^(1/2),x)
```

```
[Out] -1/2*ln((2+x)^(1/2)+2)+1/2*ln((2+x)^(1/2)-2)
```

Maxima [A] time = 0.941409, size = 28, normalized size = 2.

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(sqrt(x + 2) - 2)
```

Fricas [A] time = 2.02281, size = 73, normalized size = 5.21

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(sqrt(x + 2) - 2)
```

Sympy [A] time = 0.494461, size = 27, normalized size = 1.93

$$\begin{cases} -\operatorname{acoth}\left(\frac{\sqrt{x+2}}{2}\right) & \text{for}\: \frac{|x+2|}{4} > 1 \\ -\operatorname{atanh}\left(\frac{\sqrt{x+2}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)**(1/2),x)
```

```
[Out] Piecewise((-acoth(sqrt(x + 2)/2), Abs(x + 2)/4 > 1), (-atanh(sqrt(x + 2)/2)
, True))
```

Giac [B] time = 1.04679, size = 30, normalized size = 2.14

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\left|\sqrt{x+2}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(abs(sqrt(x + 2) - 2))
```

3.1454 $\int \frac{1}{(2+3x)\sqrt{1+5x}}\,dx$

Optimal. Leaf size=25

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{5x+1}\right)}{\sqrt{21}}$$

```
[Out] (2*ArcTan[Sqrt[3/7]*Sqrt[1 + 5*x]])/Sqrt[21]
```

Rubi [A] time = 0.0086591, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {63, 203}

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{5x+1}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)*Sqrt[1 + 5*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[3/7]*Sqrt[1 + 5*x]])/Sqrt[21]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+3x)\sqrt{1+5x}}\,dx &= \frac{2}{5}\text{Subst}\left(\int \frac{1}{\frac{7}{5}+\frac{3x^2}{5}}\,dx, x, \sqrt{1+5x}\right)\\ &= \frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{1+5x}\right)}{\sqrt{21}}\end{aligned}$$

Mathematica [A] time = 0.0088877, size = 25, normalized size = 1.

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{5x+1}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + 3*x)*Sqrt[1 + 5*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[3/7]*Sqrt[1 + 5*x]])/Sqrt[21]
```

Maple [A] time = 0.005, size = 19, normalized size = 0.8

$$\frac{2\sqrt{21}}{21}\arctan\left(\frac{\sqrt{21}}{7}\sqrt{1+5x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x)/(1+5*x)^(1/2),x)
```

```
[Out] 2/21*arctan(1/7*21^(1/2)*(1+5*x)^(1/2))*21^(1/2)
```

Maxima [A] time = 1.44277, size = 24, normalized size = 0.96

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

Fricas [A] time = 2.05966, size = 68, normalized size = 2.72

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

Sympy [A] time = 0.991192, size = 61, normalized size = 2.44

$$\begin{cases} \frac{2\sqrt{21}i\operatorname{acosh}\left(\frac{\sqrt{105}}{15\sqrt{x+\frac{2}{3}}}\right)}{21} & \text{for}\ \frac{7}{15\left|x+\frac{2}{3}\right|} > 1 \\ -\frac{2\sqrt{21}\operatorname{asin}\left(\frac{\sqrt{105}}{15\sqrt{x+\frac{2}{3}}}\right)}{21} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(21)*I*acosh(sqrt(105)/(15*sqrt(x + 2/3)))/21, 7/(15*Abs(x
 + 2/3)) > 1), (-2*sqrt(21)*asin(sqrt(105)/(15*sqrt(x + 2/3)))/21, True))
```

Giac [A] time = 1.07503, size = 24, normalized size = 0.96

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

3.1455 $\int \frac{\sqrt[3]{1-x}}{1+x} \, dx$

Optimal. Leaf size=84

$$3\sqrt[3]{1-x}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(x+1)}{2^{2/3}}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x}+1}{\sqrt{3}}\right)$$

```
[Out] 3*(1 - x)^(1/3) - 2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x)^(1/3))/Sqrt[3
]] + (3*Log[2^(1/3) - (1 - x)^(1/3)])/2^(2/3) - Log[1 + x]/2^(2/3)
```

Rubi [A] time = 0.0377397, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {50, 57, 617, 204, 31}

$$3\sqrt[3]{1-x}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(x+1)}{2^{2/3}}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/3)/(1 + x),x]
```

```
[Out] 3*(1 - x)^(1/3) - 2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x)^(1/3))/Sqrt[3
]] + (3*Log[2^(1/3) - (1 - x)^(1/3)])/2^(2/3) - Log[1 + x]/2^(2/3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1-x}}{1+x}\,dx &= 3\sqrt[3]{1-x}+2\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx\\
&= 3\sqrt[3]{1-x}-\frac{\log(1+x)}{2^{2/3}}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx,x,\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx,x,\sqrt[3]{1-x}\right)}{\sqrt[3]{2}}\\
&= 3\sqrt[3]{1-x}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(1+x)}{2^{2/3}}+\left(3\sqrt[3]{2}\right)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2^{2/3}\sqrt[3]{1-x}\right)\\
&= 3\sqrt[3]{1-x}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x}}{\sqrt{3}}\right)+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(1+x)}{2^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.047317, size = 104, normalized size = 1.24

$$3\sqrt[3]{1-x}+\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)-\frac{\log\left((1-x)^{2/3}+\sqrt[3]{2-2x}+2^{2/3}\right)}{2^{2/3}}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/3)/(1 + x),x]
```

```
[Out] 3*(1 - x)^(1/3) - 2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x)^(1/3))/Sqrt[3
]] + 2^(1/3)*Log[2^(1/3) - (1 - x)^(1/3)] - Log[2^(2/3) + (2 - 2*x)^(1/3) +
 (1 - x)^(2/3)]/2^(2/3)
```

Maple [A] time = 0.005, size = 84, normalized size = 1.

$$3\sqrt[3]{1-x}+\sqrt[3]{2}\ln\left(\sqrt[3]{1-x}-\sqrt[3]{2}\right)-\frac{\sqrt[3]{2}}{2}\ln\left((1-x)^{\frac{2}{3}}+\sqrt[3]{2}\sqrt[3]{1-x}+2^{\frac{2}{3}}\right)-\sqrt[3]{2}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2^{\frac{2}{3}}\sqrt[3]{1-x}\right)\right)\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/3)/(1+x),x)
```

```
[Out] 3*(1-x)^(1/3)+2^(1/3)*ln((1-x)^(1/3)-2^(1/3))-1/2*2^(1/3)*ln((1-x)^(2/3)+2^
(1/3)*(1-x)^(1/3)+2^(2/3))-2^(1/3)*arctan(1/3*(1+2^(2/3)*(1-x)^(1/3))*3^(1/
2))*3^(1/2)
```

Maxima [A] time = 1.46232, size = 116, normalized size = 1.38

$$-\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x+1)^{\frac{1}{3}}\right)\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+(-x+1)^{\frac{2}{3}}\right)+2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x+1)^{\frac{1}{3}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x + 1)^(1/3))) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(-2^(1/3) + (-x + 1)^(1/3)) + 3*(-x + 1)^(1/3)
```

Fricas [A] time = 2.19034, size = 275, normalized size = 3.27

$$-\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}2^{\frac{2}{3}}(-x+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+(-x+1)^{\frac{2}{3}}\right)+2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x+1)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/3*sqrt(3)*2^(2/3)*(-x + 1)^(1/3) + 1/3*sqrt(3)) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(-2^(1/3) + (-x + 1)^(1/3)) + 3*(-x + 1)^(1/3)
```

Sympy [C] time = 1.92376, size = 170, normalized size = 2.02

$$\frac{4\sqrt[3]{-1}\sqrt[3]{x-1}\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-2}e^{-\frac{i\pi}{3}}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}e^{\frac{i\pi}{3}}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}-\frac{4\sqrt[3]{-2}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}e^{i\pi}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-2}e^{\frac{i\pi}{3}}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{}}{}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/3)/(1+x),x)
```

```
[Out] 4*(-1)**(1/3)*(x - 1)**(1/3)*gamma(4/3)/gamma(7/3) + 4*(-2)**(1/3)*exp(-I*p
i/3)*log(-2**(2/3)*(x - 1)**(1/3)*exp_polar(I*pi/3)/2 + 1)*gamma(4/3)/(3*ga
mma(7/3)) - 4*(-2)**(1/3)*log(-2**(2/3)*(x - 1)**(1/3)*exp_polar(I*pi)/2 +
1)*gamma(4/3)/(3*gamma(7/3)) + 4*(-2)**(1/3)*exp(I*pi/3)*log(-2**(2/3)*(x -
 1)**(1/3)*exp_polar(5*I*pi/3)/2 + 1)*gamma(4/3)/(3*gamma(7/3))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1456 $\int \sqrt[3]{3-2x}(7+x)\,dx$

Optimal. Leaf size=27

$$\frac{3}{28}(3-2x)^{7/3}-\frac{51}{16}(3-2x)^{4/3}$$

```
[Out] (-51*(3 - 2*x)^(4/3))/16 + (3*(3 - 2*x)^(7/3))/28
```

Rubi [A] time = 0.0048718, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{28}(3-2x)^{7/3}-\frac{51}{16}(3-2x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x)^(1/3)*(7 + x),x]
```

```
[Out] (-51*(3 - 2*x)^(4/3))/16 + (3*(3 - 2*x)^(7/3))/28
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{3-2x}(7+x)\,dx &= \int \left(\frac{17}{2}\sqrt[3]{3-2x}-\frac{1}{2}(3-2x)^{4/3}\right)dx\\ &= -\frac{51}{16}(3-2x)^{4/3}+\frac{3}{28}(3-2x)^{7/3}\end{aligned}$$

Mathematica [A] time = 0.0082182, size = 18, normalized size = 0.67

$$-\frac{3}{112}(3-2x)^{4/3}(8x+107)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x)^(1/3)*(7 + x),x]
```

```
[Out] (-3*(3 - 2*x)^(4/3)*(107 + 8*x))/112
```

Maple [A] time = 0.002, size = 15, normalized size = 0.6

$$-\frac{24\,x+321}{112}(3-2\,x)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)^(1/3)*(7+x),x)
```

```
[Out] -3/112*(8*x+107)*(3-2*x)^(4/3)
```

Maxima [A] time = 0.954057, size = 26, normalized size = 0.96

$$\frac{3}{28}\,(-2\,x+3)^{\frac{7}{3}}-\frac{51}{16}\,(-2\,x+3)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="maxima")
```

```
[Out] 3/28*(-2*x + 3)^(7/3) - 51/16*(-2*x + 3)^(4/3)
```

Fricas [A] time = 1.99108, size = 63, normalized size = 2.33

$$\frac{3}{112}\left(16\,x^2+190\,x-321\right)(-2\,x+3)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="fricas")
```

```
[Out] 3/112*(16*x^2 + 190*x - 321)*(-2*x + 3)^(1/3)
```

Sympy [A] time = 0.888232, size = 114, normalized size = 4.22

$$\begin{cases}\frac{3(x+7)^2\sqrt[3]{2x-3}e^{\frac{i\pi}{3}}}{7}-\frac{51(x+7)\sqrt[3]{2x-3}e^{\frac{i\pi}{3}}}{56}-\frac{2601\sqrt[3]{2x-3}e^{\frac{i\pi}{3}}}{112} & \text{for } \frac{2|x+7|}{17}>1\\ \frac{3\sqrt[3]{3-2x}(x+7)^2}{7}-\frac{51\sqrt[3]{3-2x}(x+7)}{56}-\frac{2601\sqrt[3]{3-2x}}{112} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)**(1/3)*(7+x),x)
```

```
[Out] Piecewise((3*(x + 7)**2*(2*x - 3)**(1/3)*exp(I*pi/3)/7 - 51*(x + 7)*(2*x -
3)**(1/3)*exp(I*pi/3)/56 - 2601*(2*x - 3)**(1/3)*exp(I*pi/3)/112, 2*Abs(x +
 7)/17 > 1), (3*(3 - 2*x)**(1/3)*(x + 7)**2/7 - 51*(3 - 2*x)**(1/3)*(x + 7)
/56 - 2601*(3 - 2*x)**(1/3)/112, True))
```

Giac [A] time = 1.05487, size = 35, normalized size = 1.3

$$\frac{3}{28}\,(2\,x-3)^2(-2\,x+3)^{\frac{1}{3}}-\frac{51}{16}\,(-2\,x+3)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="giac")
```

```
[Out] 3/28*(2*x - 3)^2*(-2*x + 3)^(1/3) - 51/16*(-2*x + 3)^(4/3)
```

3.1457 $\int \sqrt[3]{1-x}(1+x)^2\,dx$

Optimal. Leaf size=38

$$-\frac{3}{10}(1-x)^{10/3}+\frac{12}{7}(1-x)^{7/3}-3(1-x)^{4/3}$$

```
[Out] -3*(1 - x)^(4/3) + (12*(1 - x)^(7/3))/7 - (3*(1 - x)^(10/3))/10
```

Rubi [A] time = 0.0068215, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3}{10}(1-x)^{10/3}+\frac{12}{7}(1-x)^{7/3}-3(1-x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/3)*(1 + x)^2,x]

[Out] -3*(1 - x)^(4/3) + (12*(1 - x)^(7/3))/7 - (3*(1 - x)^(10/3))/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{1-x}(1+x)^2\,dx &= \int \left(4\sqrt[3]{1-x}-4(1-x)^{4/3}+(1-x)^{7/3}\right)dx\\ &= -3(1-x)^{4/3}+\frac{12}{7}(1-x)^{7/3}-\frac{3}{10}(1-x)^{10/3}\end{aligned}$$

Mathematica [A] time = 0.0094329, size = 23, normalized size = 0.61

$$-\frac{3}{70}(1-x)^{4/3}\left(7x^2+26x+37\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/3)*(1 + x)^2,x]

[Out] (-3*(1 - x)^(4/3)*(37 + 26*x + 7*x^2))/70
```

Maple [A] time = 0.003, size = 20, normalized size = 0.5

$$-\frac{21\,x^2+78\,x+111}{70}\,(1-x)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/3)*(1+x)^2,x)
```

```
[Out] -3/70*(7*x^2+26*x+37)*(1-x)^(4/3)
```

Maxima [A] time = 0.953849, size = 38, normalized size = 1.

$$-\frac{3}{10}\,(-x+1)^{\frac{10}{3}}+\frac{12}{7}\,(-x+1)^{\frac{7}{3}}-3\,(-x+1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="maxima")
```

```
[Out] -3/10*(-x + 1)^(10/3) + 12/7*(-x + 1)^(7/3) - 3*(-x + 1)^(4/3)
```

Fricas [A] time = 2.04995, size = 68, normalized size = 1.79

$$\frac{3}{70}\left(7\,x^{3}+19\,x^{2}+11\,x-37\right)(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="fricas")
```

```
[Out] 3/70*(7*x^3 + 19*x^2 + 11*x - 37)*(-x + 1)^(1/3)
```

Sympy [A] time = 1.3091, size = 146, normalized size = 3.84

$$\begin{cases} -\frac{3\sqrt[3]{x-1}(x+1)^{3}e^{-\frac{2i\pi}{3}}}{10}+\frac{3\sqrt[3]{x-1}(x+1)^{2}e^{-\frac{2i\pi}{3}}}{35}+\frac{9\sqrt[3]{x-1}(x+1)e^{-\frac{2i\pi}{3}}}{35}+\frac{54\sqrt[3]{x-1}e^{-\frac{2i\pi}{3}}}{35} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{3\sqrt[3]{1-x}(x+1)^{3}}{10}-\frac{3\sqrt[3]{1-x}(x+1)^{2}}{35}-\frac{9\sqrt[3]{1-x}(x+1)}{35}-\frac{54\sqrt[3]{1-x}}{35} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/3)*(1+x)**2,x)
```

```
[Out] Piecewise((-3*(x - 1)**(1/3)*(x + 1)**3*exp(-2*I*pi/3)/10 + 3*(x - 1)**(1/3
)*(x + 1)**2*exp(-2*I*pi/3)/35 + 9*(x - 1)**(1/3)*(x + 1)*exp(-2*I*pi/3)/35
 + 54*(x - 1)**(1/3)*exp(-2*I*pi/3)/35, Abs(x + 1)/2 > 1), (3*(1 - x)**(1/3
)*(x + 1)**3/10 - 3*(1 - x)**(1/3)*(x + 1)**2/35 - 9*(1 - x)**(1/3)*(x + 1)
/35 - 54*(1 - x)**(1/3)/35, True))
```

Giac [A] time = 1.05786, size = 51, normalized size = 1.34

$$\frac{3}{10}\,(x-1)^{3}(-x+1)^{\frac{1}{3}}+\frac{12}{7}\,(x-1)^{2}(-x+1)^{\frac{1}{3}}-3\,(-x+1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="giac")
```

```
[Out] 3/10*(x - 1)^3*(-x + 1)^(1/3) + 12/7*(x - 1)^2*(-x + 1)^(1/3) - 3*(-x + 1)^
(4/3)
```

3.1458 $\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=139

$$-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(b^(2/3)*(b*c - a*d)^(1/3)) - Log[a + b*x]/(2*b^(2/3)*(b*c - a*d)^(1/3))
 + (3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(2/3)*(b*c - a
*d)^(1/3))
```

Rubi [A] time = 0.108445, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {55, 617, 204, 31}

$$-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(b^(2/3)*(b*c - a*d)^(1/3)) - Log[a + b*x]/(2*b^(2/3)*(b*c - a*d)^(1/3))
 + (3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(2/3)*(b*c - a
*d)^(1/3))
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx = -\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{c+dx}\right)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{b}}-x}\,dx,\right.}{2b^{2/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}\right)}{b^{2/3}\sqrt[3]{bc-ad}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}} - \frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [A] time = 0.0726313, size = 106, normalized size = 0.76

$$\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)-\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[
3]] - Log[a + b*x] + 3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2
*b^(2/3)*(b*c - a*d)^(1/3))
```

Maple [A] time = 0.008, size = 161, normalized size = 1.2

$$-\frac{1}{b}\ln\left(\sqrt[3]{dx+c}+\sqrt[3]{\frac{ad-bc}{b}}\right)\frac{1}{\sqrt[3]{\frac{ad-bc}{b}}}+\frac{1}{2b}\ln\left((dx+c)^{\frac{2}{3}}-\sqrt[3]{\frac{ad-bc}{b}}\sqrt[3]{dx+c}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{ad-bc}{b}}}+\frac{\sqrt{3}}{b}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/3),x)
```

```
[Out] -1/b/((a*d-b*c)/b)^(1/3)*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3))+1/2/b/((a*d-
b*c)/b)^(1/3)*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)
/b)^(2/3))+3^(1/2)/b/((a*d-b*c)/b)^(1/3)*arctan(1/3*3^(1/2)*(2/((a*d-b*c)/b
)^(1/3)*(d*x+c)^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.28262, size = 1289, normalized size = 9.27

$$\left[\frac{\sqrt{3}\left(b^{2}c-abd\right)\sqrt{-\frac{\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2\,b^{2}dx+3\,b^{2}c-abd-\sqrt{3}\left(\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}(bc-ad)+\left(b^{2}c-abd\right)(dx+c)^{\frac{1}{3}}-2\left(b^{3}c-ab^{2}d\right)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}\right)\sqrt{-\frac{\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}}{bc-ad}}-3\left(b^{3}c-\cdots\right.}{bx+a}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*(b^2*c - a*b*d)*sqrt(-(b^3*c - a*b^2*d)^(1/3)/(b*c - a*d))*lo
g((2*b^2*d*x + 3*b^2*c - a*b*d - sqrt(3)*((b^3*c - a*b^2*d)^(1/3)*(b*c - a*
d) + (b^2*c - a*b*d)*(d*x + c)^(1/3) - 2*(b^3*c - a*b^2*d)^(2/3)*(d*x + c)^
(2/3))*sqrt(-(b^3*c - a*b^2*d)^(1/3)/(b*c - a*d)) - 3*(b^3*c - a*b^2*d)^(2/
3)*(d*x + c)^(1/3))/(b*x + a)) - (b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(2/3
)*b^2 + (b^3*c - a*b^2*d)^(1/3)*(d*x + c)^(1/3)*b + (b^3*c - a*b^2*d)^(2/3)
) + 2*(b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(1/3)*b - (b^3*c - a*b^2*d)^(1/
3)))/(b^3*c - a*b^2*d), 1/2*(2*sqrt(3)*(b^2*c - a*b*d)*sqrt((b^3*c - a*b^2*
d)^(1/3)/(b*c - a*d))*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3)*b + (b^3*c - a*
b^2*d)^(1/3))*sqrt((b^3*c - a*b^2*d)^(1/3)/(b*c - a*d))/b) - (b^3*c - a*b^2
*d)^(2/3)*log((d*x + c)^(2/3)*b^2 + (b^3*c - a*b^2*d)^(1/3)*(d*x + c)^(1/3)
*b + (b^3*c - a*b^2*d)^(2/3)) + 2*(b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(1/
3)*b - (b^3*c - a*b^2*d)^(1/3)))/(b^3*c - a*b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x)**(1/3)), x)
```

Giac [A] time = 1.11485, size = 265, normalized size = 1.91

$$\frac{3\left(b^{3}c-ab^{2}d\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(dx+c)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^{3}c-\sqrt{3}ab^{2}d}-\frac{\log\left((dx+c)^{\frac{2}{3}}+(dx+c)^{\frac{1}{3}}\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\right)}{2\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}}+\frac{\left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\log\left(\left|(dx\right.\right.}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] 3*(b^3*c - a*b^2*d)^(2/3)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c - a
*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/(sqrt(3)*b^3*c - sqrt(3)*a*b^2*d) - 1/
2*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)/b)^(1/3) + ((b*c - a*d
)/b)^(2/3))/(b^3*c - a*b^2*d)^(1/3) + ((b*c - a*d)/b)^(2/3)*log(abs((d*x +
c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b*c - a*d)
```

3.1459 $\int \frac{1}{(a+bx)(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=140

$$-\frac{\log(a+bx)}{2\sqrt[3]{b}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}(bc-ad)^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}(bc-ad)^{2/3}}$$

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[
3]])/(b^(1/3)*(b*c - a*d)^(2/3))) - Log[a + b*x]/(2*b^(1/3)*(b*c - a*d)^(2/
3)) + (3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(1/3)*(b*c
- a*d)^(2/3))
```

Rubi [A] time = 0.0744044, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {57, 617, 204, 31}

$$-\frac{\log(a+bx)}{2\sqrt[3]{b}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}(bc-ad)^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(2/3)),x]
```


```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[
3]])/(b^(1/3)*(b*c - a*d)^(2/3))) - Log[a + b*x]/(2*b^(1/3)*(b*c - a*d)^(2/
3)) + (3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(1/3)*(b*c
- a*d)^(2/3))
```


Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{2/3}}\,dx = -\frac{\log(a+bx)}{2\sqrt[3]{b}(bc-ad)^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{b}}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}(bc-ad)^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{bc-adx}}{\sqrt[3]{b}}+x^2}\right.}{2b^{2/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{\log(a+bx)}{2\sqrt[3]{b}(bc-ad)^{2/3}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}(bc-ad)^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{b}\sqrt[3]{c+d}}{\sqrt[3]{bc-ad}}\right.}{\sqrt[3]{b}(bc-ad)^{2/3}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{b}(bc-ad)^{2/3}} - \frac{\log(a+bx)}{2\sqrt[3]{b}(bc-ad)^{2/3}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}(bc-ad)^{2/3}}$$

Mathematica [A] time = 0.0702584, size = 154, normalized size = 1.1

$$-\frac{\log\left(\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right.}{2\sqrt[3]{b}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(2/3)),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt
[3]] - 2*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)] + Log[(b*c - a*d)
^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3
)])/(2*b^(1/3)*(b*c - a*d)^(2/3))
```

Maple [A] time = 0.005, size = 160, normalized size = 1.1

$$\frac{1}{b}\ln\left(\sqrt[3]{dx+c}+\sqrt[3]{\frac{ad-bc}{b}}\right)\left(\frac{ad-bc}{b}\right)^{-\frac{2}{3}} - \frac{1}{2b}\ln\left((dx+c)^{\frac{2}{3}}-\sqrt[3]{\frac{ad-bc}{b}}\sqrt[3]{dx+c}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)\left(\frac{ad-bc}{b}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}}{b}\,\text{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(2/3),x)
```

```
[Out] 1/b/((a*d-b*c)/b)^(2/3)*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3))-1/2/b/((a*d-b
*c)/b)^(2/3)*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)/
b)^(2/3))+1/b/((a*d-b*c)/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((a*d-b*c)/
b)^(1/3)*(d*x+c)^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.4266, size = 1995, normalized size = 14.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(3)*(b^2*c - a*b*d)*sqrt(-(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1
/3)/b)*log(-(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + sq
rt(3)*(2*(b^2*c - a*b*d)*(d*x + c)^(2/3) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d
^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)
^(1/3))*sqrt(-(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b) - 3*(b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d)*(d*x + c)^(1/3))/(b*x + a)) + (b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(2/3)
 - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b
^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3)) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(1/3) + (b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)^(2/3)))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2), -1/2*(2*sqrt(3)*
(b^2*c - a*b*d)*sqrt((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b)*arctan(1/
3*sqrt(3)*((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) + 2*(b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3))*sqrt((b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)^(1/3)/b)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + (b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(2/3) - (b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b^2*c*d +
a^2*b*d^2)^(2/3)*(d*x + c)^(1/3)) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(
2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(1/3) + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*
d^2)^(2/3)))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x)**(2/3)), x)
```

Giac [A] time = 1.12254, size = 279, normalized size = 1.99

$$-\frac{3\left(b^3c-ab^2d\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2(dx+c)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c-\sqrt{3}abd}-\frac{\left(b^3c-ab^2d\right)^{\frac{1}{3}}\log\left((dx+c)^{\frac{2}{3}}+(dx+c)^{\frac{1}{3}}\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\right)}{2\left(b^2c-abd\right)}+-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] -3*(b^3*c - a*b^2*d)^(1/3)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c -
a*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/(sqrt(3)*b^2*c - sqrt(3)*a*b*d) - 1/2
*(b^3*c - a*b^2*d)^(1/3)*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)
/b)^(1/3) + ((b*c - a*d)/b)^(2/3))/(b^2*c - a*b*d) + ((b*c - a*d)/b)^(1/3)*
log(abs((d*x + c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b*c - a*d)
```

3.1460 $\int (a+bx)^{7/2}\sqrt{c+dx}\,dx$

Optimal. Leaf size=230

$$\frac{7(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{3/2}d^{9/2}}-\frac{7\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^4}{128bd^4}+\frac{7(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^3}{192bd^3}-\frac{7(a+bx)^{5/2}\sqrt{c+dx}(bc-}{240bd^2}$$

```
[Out] (-7*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b*d^4) + (7*(b*c - a*d)
^3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(192*b*d^3) - (7*(b*c - a*d)^2*(a + b*x)^
(5/2)*Sqrt[c + d*x])/(240*b*d^2) + ((b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*
x])/(40*b*d) + ((a + b*x)^(9/2)*Sqrt[c + d*x])/(5*b) + (7*(b*c - a*d)^5*Arc
Tanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(3/2)*d^(9/2)
)
```

Rubi [A] time = 0.160316, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$\frac{7(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{3/2}d^{9/2}}-\frac{7\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^4}{128bd^4}+\frac{7(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^3}{192bd^3}-\frac{7(a+bx)^{5/2}\sqrt{c+dx}(bc-}{240bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)*Sqrt[c + d*x],x]
```


```
[Out] (-7*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b*d^4) + (7*(b*c - a*d)
^3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(192*b*d^3) - (7*(b*c - a*d)^2*(a + b*x)^
(5/2)*Sqrt[c + d*x])/(240*b*d^2) + ((b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*
x])/(40*b*d) + ((a + b*x)^(9/2)*Sqrt[c + d*x])/(5*b) + (7*(b*c - a*d)^5*Arc
Tanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(3/2)*d^(9/2)
)
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^{7/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b} + \frac{(bc-ad)\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}\,dx}{10b}$$

$$= \frac{(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40bd} + \frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b} - \frac{\left(7(bc-ad)^2\right)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{80bd}$$

$$= -\frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{240bd^2} + \frac{(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40bd} + \frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b} + \frac{(7(}{}$$

$$= \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{240bd^2} + \frac{(bc-ad)(a+bx)^{7/2}\sqrt{c+}}{40bd}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c}}{240bd^2}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c}}{240bd^2}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c}}{240bd^2}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c}}{240bd^2}$$

Mathematica [A] time = 1.51989, size = 194, normalized size = 0.84

$$\frac{(a+bx)^{9/2}\sqrt{c+dx}\left(-\frac{70(bc-ad)^4}{d^4(a+bx)^4}+\frac{140(bc-ad)^3}{3d^3(a+bx)^3}-\frac{112(bc-ad)^2}{3d^2(a+bx)^2}+\frac{70(bc-ad)^{9/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{9/2}(a+bx)^{9/2}\sqrt{\frac{b(c+dx)}{bc-ad}}}+\frac{32bc-32ad}{ad+bdx}+256\right)}{1280b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)*Sqrt[c + d*x],x]
```

```
[Out] ((a + b*x)^(9/2)*Sqrt[c + d*x]*(256 - (70*(b*c - a*d)^4)/(d^4*(a + b*x)^4)
+ (140*(b*c - a*d)^3)/(3*d^3*(a + b*x)^3) - (112*(b*c - a*d)^2)/(3*d^2*(a +
 b*x)^2) + (32*b*c - 32*a*d)/(a*d + b*d*x) + (70*(b*c - a*d)^(9/2)*ArcSinh[
(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(9/2)*(a + b*x)^(9/2)*Sqrt[(b*
(c + d*x))/(b*c - a*d)]])))/(1280*b)
```

Maple [B] time = 0.008, size = 858, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/5/d*(b*x+a)^(7/2)*(d*x+c)^(3/2)-7/32/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3*c+
7/128/d^4*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^4*b^3+7/128/b*(d*x+c)^(1/2)*(b*x+a)
^(1/2)*a^4-7/40/d^2*(b*x+a)^(5/2)*(d*x+c)^(3/2)*b*c+7/48/d^3*(b*x+a)^(3/2)*
(d*x+c)^(3/2)*b^2*c^2-7/64/d^4*(b*x+a)^(1/2)*(d*x+c)^(3/2)*b^3*c^3-7/32/d^3
*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^3*b^2-7/24/d^2*(b*x+a)^(3/2)*(d*x+c)^(3/2)
*a*b*c-21/64/d^2*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a^2*b*c+21/64/d^3*(b*x+a)^(1/2
)*(d*x+c)^(3/2)*a*b^2*c^2+21/64/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^2*c^2*b+7
/40/d*(b*x+a)^(5/2)*(d*x+c)^(3/2)*a+7/48/d*(b*x+a)^(3/2)*(d*x+c)^(3/2)*a^2+
7/64/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a^3+35/128/d^2*((b*x+a)*(d*x+c))^(1/2)/(
d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+
(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^2*c^3*b^2-35/256/d^3*((b*x+a)*(d*x+c)
)^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+
(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a*c^4*b^3+35/256*((b*x+a)*(d*x
+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/
2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^4*c-35/128/d*((b*x+a)*(d*
x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1
/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^3*c^2*b+7/256/d^4*((b*x+
a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b
*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*c^5*b^4-7/256*d/b*((
b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x
)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.60415, size = 1573, normalized size = 6.84

$$\left[-\frac{105\left(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5\right)\sqrt{bd}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 - 4\,(2\,bd\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(105*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^
2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*
a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*
x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(384*b^5*d^5*x^4 - 105*b^5*c^4*d + 49
0*a*b^4*c^3*d^2 - 896*a^2*b^3*c^2*d^3 + 790*a^3*b^2*c*d^4 + 105*a^4*b*d^5 +
 48*(b^5*c*d^4 + 31*a*b^4*d^5)*x^3 - 8*(7*b^5*c^2*d^3 - 32*a*b^4*c*d^4 - 26
3*a^2*b^3*d^5)*x^2 + 2*(35*b^5*c^3*d^2 - 161*a*b^4*c^2*d^3 + 289*a^2*b^3*c*
d^4 + 605*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^5), -1/3840*(
105*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
```

```
a^4*b*c*d^4 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*
d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)
*x)) - 2*(384*b^5*d^5*x^4 - 105*b^5*c^4*d + 490*a*b^4*c^3*d^2 - 896*a^2*b^3
*c^2*d^3 + 790*a^3*b^2*c*d^4 + 105*a^4*b*d^5 + 48*(b^5*c*d^4 + 31*a*b^4*d^5
)*x^3 - 8*(7*b^5*c^2*d^3 - 32*a*b^4*c*d^4 - 263*a^2*b^3*d^5)*x^2 + 2*(35*b^
5*c^3*d^2 - 161*a*b^4*c^2*d^3 + 289*a^2*b^3*c*d^4 + 605*a^3*b^2*d^5)*x)*sqr
t(b*x + a)*sqrt(d*x + c))/(b^2*d^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38353, size = 1335, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*(30*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(
6*(b*x + a)/b^2 + (b^7*c*d^5 - 17*a*b^6*d^6)/(b^8*d^6)) - (5*b^8*c^2*d^4 +
6*a*b^7*c*d^5 - 59*a^2*b^6*d^6)/(b^8*d^6)) + 3*(5*b^9*c^3*d^3 + a*b^8*c^2*d
^4 - a^2*b^7*c*d^5 - 5*a^3*b^6*d^6)/(b^8*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4
 - 4*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + 5*a^4*d^4)*log(abs(-
sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*
b*d^3))*a*abs(b) + 20*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2
*(b*x + a)/(b^4*d^2) + (b*c*d - a*d^2)/(b^4*d^4)) + (b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*
b*d)))/(sqrt(b*d)*b^3*d^3))*a^3*abs(b)/b^2 + (sqrt(b^2*c + (b*x + a)*b*d -
a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^3 + (b^13*c*d^7 - 31*a*b
^12*d^8)/(b^15*d^8)) - (7*b^14*c^2*d^6 + 16*a*b^13*c*d^7 - 263*a^2*b^12*d^8
)/(b^15*d^8)) + 5*(7*b^15*c^3*d^5 + 9*a*b^14*c^2*d^6 + 9*a^2*b^13*c*d^7 - 1
21*a^3*b^12*d^8)/(b^15*d^8))*(b*x + a) - 15*(7*b^16*c^4*d^4 + 2*a*b^15*c^3*
d^5 - 2*a^3*b^13*c*d^7 - 7*a^4*b^12*d^8)/(b^15*d^8))*sqrt(b*x + a) - 15*(7*
b^5*c^5 - 5*a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3 - 5*a^4*b*c
*d^4 + 7*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)
*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^4))*b*abs(b) + 3*(sqrt(b^2*c + (b*x + a)*b
*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/(b^6*d^2) + (b*c*d^3 -
7*a*d^4)/(b^6*d^6)) - 3*(b^2*c^2*d^2 - a^2*d^4)/(b^6*d^6)) - 3*(b^3*c^3 - a
*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt
(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^5*d^4))*a^2*abs(b)/b^2)/b
```

3.1461 $\int (a+bx)^{5/2}\sqrt{c+dx}\,dx$

Optimal. Leaf size=192

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{3/2}d^{7/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^3}{64bd^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^2}{96bd^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}(bc-}{24bd}$$

```
[Out] (5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b*d^3) - (5*(b*c - a*d)^2
*(a + b*x)^(3/2)*Sqrt[c + d*x])/(96*b*d^2) + ((b*c - a*d)*(a + b*x)^(5/2)*S
qrt[c + d*x])/(24*b*d) + ((a + b*x)^(7/2)*Sqrt[c + d*x])/(4*b) - (5*(b*c -
a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(3/2
)*d^(7/2))
```

Rubi [A] time = 0.0946061, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{3/2}d^{7/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^3}{64bd^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^2}{96bd^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}(bc-}{24bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*Sqrt[c + d*x],x]
```


```
[Out] (5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b*d^3) - (5*(b*c - a*d)^2
*(a + b*x)^(3/2)*Sqrt[c + d*x])/(96*b*d^2) + ((b*c - a*d)*(a + b*x)^(5/2)*S
qrt[c + d*x])/(24*b*d) + ((a + b*x)^(7/2)*Sqrt[c + d*x])/(4*b) - (5*(b*c -
a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(3/2
)*d^(7/2))
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^{5/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b} + \frac{(bc-ad)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{8b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24bd} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b} - \frac{\left(5(bc-ad)^2\right)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{48bd}$$

$$= -\frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24bd} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b} + \frac{(5(}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+d}}{24bd}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+d}}{24bd}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+d}}{24bd}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+d}}{24bd}$$

Mathematica [A] time = 0.5712, size = 190, normalized size = 0.99

$$\frac{b\sqrt{d}\sqrt{a+bx}(c+dx)\left(a^2bd^2(73c+118dx)+15a^3d^3+ab^2d\left(-55c^2+36cdx+136d^2x^2\right)+b^3\left(-10c^2dx+15c^3+8cd^2x^2\right.\right.}{192b^2d^{7/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(15*a^3*d^3 + a^2*b*d^2*(73*c + 118*d*x)
 + a*b^2*d*(-55*c^2 + 36*c*d*x + 136*d^2*x^2) + b^3*(15*c^3 - 10*c^2*d*x +
8*c*d^2*x^2 + 48*d^3*x^3)) - 15*(b*c - a*d)^(9/2)*Sqrt[(b*(c + d*x))/(b*c -
 a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(192*b^2*d^(7/2)*S
qrt[c + d*x])
```

Maple [B] time = 0.006, size = 645, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*(b*x+a)^(5/2)*(d*x+c)^(3/2)+5/24/d*(b*x+a)^(3/2)*(d*x+c)^(3/2)*a-5/24
/d^2*(b*x+a)^(3/2)*(d*x+c)^(3/2)*b*c+5/32/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a^2
```

```
-5/16/d^2*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a*b*c+5/32/d^3*(b*x+a)^(1/2)*(d*x+c)^
(3/2)*b^2*c^2+5/64/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3-15/64/d*(d*x+c)^(1/2)*
(b*x+a)^(1/2)*a^2*c+15/64/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^2*b-5/64/d^3*
(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^3*b^2-5/128*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d
+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^4+5/32*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1
/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)
*x+a*c)^(1/2))/(b*d)^(1/2)*a^3*c-15/64/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1
/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)
*x+a*c)^(1/2))/(b*d)^(1/2)*a^2*c^2*b+5/32/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d
+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a*c^3*b^2-5/128/d^3*((b*x+a)*(d*x+c))^(1/2)
/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*
b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*c^4*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.36695, size = 1203, normalized size = 6.27

$$\left[\frac{15\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{\ }\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*
b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b
*d^2)*x) + 4*(48*b^4*d^4*x^3 + 15*b^4*c^3*d - 55*a*b^3*c^2*d^2 + 73*a^2*b^2
*c*d^3 + 15*a^3*b*d^4 + 8*(b^4*c*d^3 + 17*a*b^3*d^4)*x^2 - 2*(5*b^4*c^2*d^2
 - 18*a*b^3*c*d^3 - 59*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^
4), 1/384*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
+ a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(48
*b^4*d^4*x^3 + 15*b^4*c^3*d - 55*a*b^3*c^2*d^2 + 73*a^2*b^2*c*d^3 + 15*a^3*
b*d^4 + 8*(b^4*c*d^3 + 17*a*b^3*d^4)*x^2 - 2*(5*b^4*c^2*d^2 - 18*a*b^3*c*d^
3 - 59*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35062, size = 842, normalized size = 4.39

$$5\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{6\,(bx+a)}{b^2}+\frac{b^7cd^5-17\,ab^6d^6}{b^8d^6}\right)-\frac{5\,b^8c^2d^4+6\,ab^7cd^5-59\,a^2b^6d^6}{b^8d^6}\right)+\frac{3\left(5\,b^9c^3d^3+ab^8c^2d^4-\cdots\right.}{b^8c\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/960*(5*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*
(b*x + a)/b^2 + (b^7*c*d^5 - 17*a*b^6*d^6)/(b^8*d^6)) - (5*b^8*c^2*d^4 + 6*
a*b^7*c*d^5 - 59*a^2*b^6*d^6)/(b^8*d^6)) + 3*(5*b^9*c^3*d^3 + a*b^8*c^2*d^4
 - a^2*b^7*c*d^5 - 5*a^3*b^6*d^6)/(b^8*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 -
 4*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + 5*a^4*d^4)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*
d^3))*abs(b) + 10*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*
x + a)/(b^4*d^2) + (b*c*d - a*d^2)/(b^4*d^4)) + (b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*b^3*d^3))*a^2*abs(b)/b^2 + (sqrt(b^2*c + (b*x + a)*b*d - a*b*
d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/(b^6*d^2) + (b*c*d^3 - 7*a*d^4)/
(b^6*d^6)) - 3*(b^2*c^2*d^2 - a^2*d^4)/(b^6*d^6)) - 3*(b^3*c^3 - a*b^2*c^2*
d - a^2*b*c*d^2 + a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^5*d^4))*a*abs(b)/b^2)/b
```

3.1462 $\int (a+bx)^{3/2}\sqrt{c+dx}\,dx$

Optimal. Leaf size=154

$$\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8bd^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{12bd}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

```
[Out] -((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b*d^2) + ((b*c - a*d)*(a +
b*x)^(3/2)*Sqrt[c + d*x])/(12*b*d) + ((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*b)
+ ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/
(8*b^(3/2)*d^(5/2))
```

Rubi [A] time = 0.0718098, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8bd^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{12bd}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*Sqrt[c + d*x],x]
```

```
[Out] -((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b*d^2) + ((b*c - a*d)*(a +
b*x)^(3/2)*Sqrt[c + d*x])/(12*b*d) + ((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*b)
+ ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/
(8*b^(3/2)*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} + \frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6b}$$

$$= \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} - \frac{(bc-ad)^2\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{8bd}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} + \frac{(bc-a}{}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} + \frac{(bc-a}{}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} + \frac{(bc-a}{}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} + \frac{(bc-a}{}$$

Mathematica [A] time = 0.418966, size = 151, normalized size = 0.98

$$\frac{b\sqrt{d}\sqrt{a+bx}(c+dx)\left(3a^2d^2+2abd(4c+7dx)+b^2\left(-3c^2+2cdx+8d^2x^2\right)\right)+3(bc-ad)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{24b^2d^{5/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(3*a^2*d^2 + 2*a*b*d*(4*c + 7*d*x) + b^2
*(-3*c^2 + 2*c*d*x + 8*d^2*x^2)) + 3*(b*c - a*d)^(7/2)*Sqrt[(b*(c + d*x))/(
b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(24*b^2*d^(5/
2)*Sqrt[c + d*x])
```

Maple [B] time = 0.006, size = 460, normalized size = 3.

$$\frac{1}{3d}(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}} + \frac{a}{4d}\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}} - \frac{bc}{4d^2}\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}} + \frac{a^2}{8b}\sqrt{bx+a}\sqrt{dx+c} - \frac{ac}{4d}\sqrt{bx+a}\sqrt{dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/3/d*(b*x+a)^(3/2)*(d*x+c)^(3/2)+1/4/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a-1/4/d
^2*(b*x+a)^(1/2)*(d*x+c)^(3/2)*b*c+1/8/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^2-1/
4/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c+1/8/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^2
*b-1/16*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d
+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^
3+3/16*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*
b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^2*c-3
/16/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b
```

```
*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a*c^2*b+
1/16/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/
2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*c^3*b
^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.36129, size = 919, normalized size = 5.97

$$\left[-\frac{3\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{bd}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 - 4\,(2\,bdx + bc + ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+\cdots}\right.}{96\,b^2d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sq
rt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d
^3*x^2 - 3*b^3*c^2*d + 8*a*b^2*c*d^2 + 3*a^2*b*d^3 + 2*(b^3*c*d^2 + 7*a*b^2
*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^3), -1/48*(3*(b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a
*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*
d + a*b*d^2)*x)) - 2*(8*b^3*d^3*x^2 - 3*b^3*c^2*d + 8*a*b^2*c*d^2 + 3*a^2*b
*d^3 + 2*(b^3*c*d^2 + 7*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^3
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23307, size = 456, normalized size = 2.96

$$\frac{\frac{20\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(\frac{2\,(bx+a)}{b^4d^2}+\frac{bcd-ad^2}{b^4d^4}\right)+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,b^3d^3}\right)a|b|}{b^2}+\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{b^6d^2}\cdots\right.\right.\right.}{\cdots}}{1920\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*(20*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)/
(b^4*d^2) + (b*c*d - a*d^2)/(b^4*d^4)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqr
t(b*d)*b^3*d^3))*a*abs(b)/b^2 + (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b
*x + a)*(2*(b*x + a)*(4*(b*x + a)/(b^6*d^2) + (b*c*d^3 - 7*a*d^4)/(b^6*d^6)
) - 3*(b^2*c^2*d^2 - a^2*d^4)/(b^6*d^6)) - 3*(b^3*c^3 - a*b^2*c^2*d - a^2*b
*c*d^2 + a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)
*b*d - a*b*d)))/(sqrt(b*d)*b^5*d^4))*abs(b)/b^2)/b
```

3.1463 $\int \sqrt{a+bx}\sqrt{c+dx}\,dx$

Optimal. Leaf size=116

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

```
[Out] ((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b*d) + ((a + b*x)^(3/2)*Sqrt[c
 + d*x])/(2*b) - ((b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sq
rt[c + d*x])])/(4*b^(3/2)*d^(3/2))
```

Rubi [A] time = 0.0545892, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] ((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b*d) + ((a + b*x)^(3/2)*Sqrt[c
 + d*x])/(2*b) - ((b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sq
rt[c + d*x])])/(4*b^(3/2)*d^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx = \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}dx}{4b}$$
$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{8bd}$$
$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}dx, x, \sqrt{a+\right.}{4b^2d}$$
$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{4b^2d}$$
$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}$$

Mathematica [A] time = 0.268607, size = 118, normalized size = 1.02

$$\frac{b\sqrt{d}\sqrt{a+bx}(c+dx)(ad+b(c+2dx)) - (bc-ad)^{5/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4b^2d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(a*d + b*(c + 2*d*x)) - (b*c - a*d)^(5/2
)*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c
- a*d]])/(4*b^2*d^(3/2)*Sqrt[c + d*x])
```

Maple [B] time = 0.004, size = 305, normalized size = 2.6

$$\frac{1}{2d}\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}} + \frac{a}{4b}\sqrt{bx+a}\sqrt{dx+c} - \frac{c}{4d}\sqrt{bx+a}\sqrt{dx+c} - \frac{da^2}{8b}\sqrt{(bx+a)(dx+c)}\ln\left(\left(\frac{ad}{2}+\frac{bc}{2}+bdx\right)\frac{1}{\sqrt{bd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)+1/4/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a-1/4/d
*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c-1/8*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x
+a*c)^(1/2))/(b*d)^(1/2)*a^2+1/4*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x
+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^
(1/2))/(b*d)^(1/2)*a*c-1/8/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^
(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2
))/(b*d)^(1/2)*c^2*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27849, size = 699, normalized size = 6.03

$$\left[\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{bd}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 - 4\,(2\,bdx + bc + ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c} + 8\,\left(b^2cd + a\right.\right.}{16\,b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^
2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*s
qrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + b^2*c*d + a*b*d^
2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2), 1/8*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*s
qrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(2*b^2*d^
2*x + b^2*c*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x), x)
```

Giac [A] time = 1.11055, size = 189, normalized size = 1.63

$$\frac{\left(\sqrt{b^2c + (bx+a)bd - abd}\sqrt{bx+a}\left(\frac{2\,(bx+a)}{b^4d^2} + \frac{bcd - ad^2}{b^4d^4}\right) + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a} + \sqrt{b^2c + (bx+a)bd - abd}\right|\right)}{\sqrt{bd}b^3d^3}\right)|b|}{96\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)/(b^4*d
^2) + (b*c*d - a*d^2)/(b^4*d^4)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(
-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)
*b^3*d^3))*abs(b)/b^3
```

3.1464 $\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}} dx$

Optimal. Leaf size=72

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}$$

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])
```

Rubi [A] time = 0.0377266, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}\sqrt{c+dx}}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{2b}$$

$$= \frac{\sqrt{a+bx}\sqrt{c+dx}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b^2}$$

$$= \frac{\sqrt{a+bx}\sqrt{c+dx}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^2}$$

$$= \frac{\sqrt{a+bx}\sqrt{c+dx}}{b} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}$$

Mathematica [A] time = 0.100161, size = 117, normalized size = 1.62

$$\frac{\sqrt{c+dx}\left(\sqrt{d}\sqrt{a+bx}\sqrt{\frac{b(c+dx)}{bc-ad}} + \sqrt{bc-ad}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)}{b\sqrt{d}\sqrt{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(Sqrt[d]*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)] + Sqr
t[b*c - a*d]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(b*Sqrt[d]*
Sqrt[(b*(c + d*x))/(b*c - a*d)])
```

Maple [A] time = 0.004, size = 107, normalized size = 1.5

$$\frac{1}{b}\sqrt{bx+a}\sqrt{dx+c} - \frac{ad-bc}{2b}\sqrt{(bx+a)(dx+c)}\ln\left(\left(\frac{ad}{2}+\frac{bc}{2}+bdx\right)\frac{1}{\sqrt{bd}} + \sqrt{dx^2b+(ad+bc)x+ac}\right)\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] (b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d
+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.21051, size = 558, normalized size = 7.75

$$\left[\frac{4\sqrt{bx+a}\sqrt{dx+c}bd-(bc-ad)\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}-\right.}{4\,b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*b^2*d
^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*
sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x))/(b^2*d), 1/2*(2*sqr
t(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(-b*d)*arctan(1/2*(2*b*d*x +
 b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d +
 (b^2*c*d + a*b*d^2)*x)))/(b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/sqrt(a + b*x), x)
```

Giac [A] time = 1.0982, size = 126, normalized size = 1.75

$$-\frac{\left(\frac{(b^2c-abd)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)|b|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)
*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x +
a))*abs(b)/b^3
```

3.1465 $\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}} dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}} - \frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}$$

```
[Out] (-2*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(3/2)
```

Rubi [A] time = 0.0332162, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {47, 63, 217, 206}

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}} - \frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx &= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{d\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{b}\\
&= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^2}\\
&= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^2}\\
&= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.217975, size = 99, normalized size = 1.5

$$
\frac{2\left(\sqrt{d}\sqrt{bc-ad}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)-\frac{b(c+dx)}{\sqrt{a+bx}}\right)}{b^2\sqrt{c+dx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-((b*(c + d*x))/Sqrt[a + b*x]) + Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[(b*(c + d
*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(b^2*S
qrt[c + d*x])
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$
\int \sqrt{dx+c}\,(bx+a)^{-\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(1/2)/(b*x+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.61006, size = 551, normalized size = 8.35

$$\left[\frac{(bx+a)\sqrt{\frac{d}{b}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,b^2dx+b^2c+abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}}+8\left(b^2cd+abd^2\right)x\right)-4\,\sqrt{}}{2\left(b^2x+ab\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*x + a)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*x + a*b), -((b
*x + a)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
+ c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) + 2*sqrt(b*x + a)*
sqrt(d*x + c))/(b^2*x + a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c+dx}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*x)**(3/2), x)
```

Giac [B] time = 1.12738, size = 177, normalized size = 2.68

$$-\frac{\left(\frac{\sqrt{bd}\log\left(\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{b}+\frac{4\left(\sqrt{bd}bc-\sqrt{bd}ad\right)}{b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(b*d)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^2)/b + 4*(sqrt(b*d)*b*c - sqrt(b*d)*a*d)/(b^2*c - a*b*d - (sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2))*abs(b)/b^2
```

3.1466 $\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2(c+dx)^{3/2}}{3(a+bx)^{3/2}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(3/2))/(3*(b*c - a*d)*(a + b*x)^(3/2))
```

Rubi [A] time = 0.0029979, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{2(c+dx)^{3/2}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(3*(b*c - a*d)*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{3/2}}{3(bc-ad)(a+bx)^{3/2}}$$

Mathematica [A] time = 0.011371, size = 32, normalized size = 1.

$$-\frac{2(c+dx)^{3/2}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(3*(b*c - a*d)*(a + b*x)^(3/2))
```

Maple [A] time = 0.002, size = 27, normalized size = 0.8

$$\frac{2}{3\,ad-3\,bc}\,(dx+c)^{\frac{3}{2}}\,(bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(5/2),x)
```

```
[Out] 2/3/(b*x+a)^(3/2)*(d*x+c)^(3/2)/(a*d-b*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3436, size = 140, normalized size = 4.38

$$-\frac{2\sqrt{bx+a}(dx+c)^{\frac{3}{2}}}{3\left(a^2bc-a^3d+\left(b^3c-ab^2d\right)x^2+2\left(ab^2c-a^2bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x + a)*(d*x + c)^(3/2)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x^2
 + 2*(a*b^2*c - a^2*b*d)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*x)**(5/2), x)
```

Giac [B] time = 1.13717, size = 205, normalized size = 6.41

$$-\frac{4\left(\sqrt{bd}b^4c^2d-2\sqrt{bd}ab^3cd^2+\sqrt{bd}a^2b^2d^3+3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4d\right)|b|}{3\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(sqrt(b*d)*b^4*c^2*d - 2*sqrt(b*d)*a*b^3*c*d^2 + sqrt(b*d)*a^2*b^2*d^3
 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*d)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2)^3*b^2)
```

3.1467 $\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=66

$$\frac{4d(c+dx)^{3/2}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{5(a+bx)^{5/2}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(3/2))/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (4*d*(c + d*x)^(3/2)
)/(15*(b*c - a*d)^2*(a + b*x)^(3/2))
```

Rubi [A] time = 0.0086535, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4d(c+dx)^{3/2}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (4*d*(c + d*x)^(3/2)
)/(15*(b*c - a*d)^2*(a + b*x)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx &= -\frac{2(c+dx)^{3/2}}{5(bc-ad)(a+bx)^{5/2}}-\frac{(2d)\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx}{5(bc-ad)}\\ &= -\frac{2(c+dx)^{3/2}}{5(bc-ad)(a+bx)^{5/2}}+\frac{4d(c+dx)^{3/2}}{15(bc-ad)^2(a+bx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0201384, size = 46, normalized size = 0.7

$$\frac{2(c+dx)^{3/2}(5ad-3bc+2bdx)}{15(a+bx)^{5/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(7/2),x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-3*b*c + 5*a*d + 2*b*d*x))/(15*(b*c - a*d)^2*(a + b*x)^
(5/2))
```

Maple [A] time = 0.005, size = 54, normalized size = 0.8

$$\frac{4\,bdx+10\,ad-6\,bc}{15\,a^2d^2-30\,abcd+15\,b^2c^2}\,(dx+c)^{\frac{3}{2}}\,(bx+a)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(7/2),x)
```

```
[Out] 2/15*(d*x+c)^(3/2)*(2*b*d*x+5*a*d-3*b*c)/(b*x+a)^(5/2)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.31271, size = 365, normalized size = 5.53

$$\frac{2\left(2\,bd^2x^2-3\,bc^2+5\,acd-\left(bcd-5\,ad^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{15\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^3+3\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)x^2+3\left(a^2b^3c^2-2\,a^3b^2cd+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b*d^2*x^2 - 3*b*c^2 + 5*a*c*d - (b*c*d - 5*a*d^2)*x)*sqrt(b*x + a)*
sqrt(d*x + c)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4*c*d
 + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 + 3*(
a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22328, size = 603, normalized size = 9.14

$$8\left(\sqrt{bd}b^7c^3d^2 - 3\sqrt{bd}ab^6c^2d^3 + 3\sqrt{bd}a^2b^5cd^4 - \sqrt{bd}a^3b^4d^5 - 5\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^2 b^5c^2d^2 + 1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] 8/15*(sqrt(b*d)*b^7*c^3*d^2 - 3*sqrt(b*d)*a*b^6*c^2*d^3 + 3*sqrt(b*d)*a^2*b
^5*c*d^4 - sqrt(b*d)*a^3*b^4*d^5 - 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^5*c^2*d^2 + 10*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^4*c*d^3 - 5*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^
2*b^3*d^4 - 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^4*b^3*c*d^2 + 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*a*b^2*d^3 - 15*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b*d^2)*abs(b)/((b^2*c - a*b*d
 - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*b^2
)
```

3.1468 $\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{3/2}(bc-ad)^3}+\frac{8d(c+dx)^{3/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{7(a+bx)^{7/2}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(3/2))/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (8*d*(c + d*x)^(3/2)
)/(35*(b*c - a*d)^2*(a + b*x)^(5/2)) - (16*d^2*(c + d*x)^(3/2))/(105*(b*c -
 a*d)^3*(a + b*x)^(3/2))
```

Rubi [A] time = 0.016475, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{3/2}(bc-ad)^3}+\frac{8d(c+dx)^{3/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (8*d*(c + d*x)^(3/2)
)/(35*(b*c - a*d)^2*(a + b*x)^(5/2)) - (16*d^2*(c + d*x)^(3/2))/(105*(b*c -
 a*d)^3*(a + b*x)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx &= -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}}-\frac{(4d)\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx}{7(bc-ad)}\\
&= -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}}+\frac{8d(c+dx)^{3/2}}{35(bc-ad)^2(a+bx)^{5/2}}+\frac{\left(8d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx}{35(bc-ad)^2}\\
&= -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}}+\frac{8d(c+dx)^{3/2}}{35(bc-ad)^2(a+bx)^{5/2}}-\frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0387565, size = 77, normalized size = 0.76

$$-\frac{2(c+dx)^{3/2}\left(35a^2d^2+14abd(2dx-3c)+b^2\left(15c^2-12cdx+8d^2x^2\right)\right)}{105(a+bx)^{7/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-3*c + 2*d*x) + b^2*(15*c^2 - 1
2*c*d*x + 8*d^2*x^2)))/(105*(b*c - a*d)^3*(a + b*x)^(7/2))
```

Maple [A] time = 0.006, size = 105, normalized size = 1.

$$\frac{16\,b^2d^2x^2+56\,abd^2x-24\,b^2cdx+70\,a^2d^2-84\,abcd+30\,b^2c^2}{105\,a^3d^3-315\,a^2bcd^2+315\,ab^2c^2d-105\,b^3c^3}\,(dx+c)^{\frac{3}{2}}\,(bx+a)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(9/2),x)
```

```
[Out] 2/105*(d*x+c)^(3/2)*(8*b^2*d^2*x^2+28*a*b*d^2*x-12*b^2*c*d*x+35*a^2*d^2-42*
a*b*c*d+15*b^2*c^2)/(b*x+a)^(7/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*
c^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 10.1854, size = 682, normalized size = 6.75

$$-\frac{2\left(8\,b^2d^3x^3+15\,b^2c^3-42\,abc^2d+35\,a^2cd^2-4\left(b^2cd^2-7\,abd^3\right)\right.}{105\left(a^4b^3c^3-3\,a^5b^2c^2d+3\,a^6bcd^2-a^7d^3+\left(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3\right)x^4+4\left(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*b^2*d^3*x^3 + 15*b^2*c^3 - 42*a*b*c^2*d + 35*a^2*c*d^2 - 4*(b^2*c
*d^2 - 7*a*b*d^3)*x^2 + (3*b^2*c^2*d - 14*a*b*c*d^2 + 35*a^2*d^3)*x)*sqrt(b
*x + a)*sqrt(d*x + c)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*
d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 + 4*(a*
b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^3 + 6*(a^2*b^5
```

```
*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^2 + 4*(a^3*b^4*c^
3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28461, size = 930, normalized size = 9.21

$$-\frac{32\left(\sqrt{bd}b^{10}c^4d^3 - 4\sqrt{bd}ab^9c^3d^4 + 6\sqrt{bd}a^2b^8c^2d^5 - 4\sqrt{bd}a^3b^7cd^6 + \sqrt{bd}a^4b^6d^7 - 7\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c+(bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] -32/105*(sqrt(b*d)*b^10*c^4*d^3 - 4*sqrt(b*d)*a*b^9*c^3*d^4 + 6*sqrt(b*d)*a
^2*b^8*c^2*d^5 - 4*sqrt(b*d)*a^3*b^7*c*d^6 + sqrt(b*d)*a^4*b^6*d^7 - 7*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^8
*c^3*d^3 + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a*b^7*c^2*d^4 - 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^6*c*d^5 + 7*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^5*d^6 + 21*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^6
*c^2*d^3 - 42*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^4*a*b^5*c*d^4 + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b
^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^4*d^5 + 35*sqrt(b*d)*(sqrt(b*d)*sqrt
(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^4*c*d^3 - 35*sqrt(b*d)
*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^3*d^
4 + 70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^8*b^2*d^3)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b
^2*c + (b*x + a)*b*d - a*b*d))^2)^7*b^2)
```

3.1469 $\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx$

Optimal. Leaf size=136

$$\frac{32d^3(c+dx)^{3/2}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{4d(c+dx)^{3/2}}{21(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{9(a+bx)^{9/2}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(3/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (4*d*(c + d*x)^(3/2)
)/(21*(b*c - a*d)^2*(a + b*x)^(7/2)) - (16*d^2*(c + d*x)^(3/2))/(105*(b*c -
 a*d)^3*(a + b*x)^(5/2)) + (32*d^3*(c + d*x)^(3/2))/(315*(b*c - a*d)^4*(a +
 b*x)^(3/2))
```

Rubi [A] time = 0.0285565, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32d^3(c+dx)^{3/2}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{4d(c+dx)^{3/2}}{21(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (4*d*(c + d*x)^(3/2)
)/(21*(b*c - a*d)^2*(a + b*x)^(7/2)) - (16*d^2*(c + d*x)^(3/2))/(105*(b*c -
 a*d)^3*(a + b*x)^(5/2)) + (32*d^3*(c + d*x)^(3/2))/(315*(b*c - a*d)^4*(a +
 b*x)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx = -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(2d)\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx}{3(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}} + \frac{\left(8d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx}{21(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}} - \frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{5/2}} - \frac{\left(16d^3\right)\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,d}{105(bc-ad)^3}$$

$$= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}} - \frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{5/2}} + \frac{32d^3(c+dx)^3}{315(bc-ad)^4(a+}$$

Mathematica [A] time = 0.0528143, size = 118, normalized size = 0.87

$$\frac{2(c+dx)^{3/2}\left(63a^2bd^2(2dx-3c)+105a^3d^3+9ab^2d\left(15c^2-12cdx+8d^2x^2\right)+b^3\left(30c^2dx-35c^3-24cd^2x^2+16d^3x^3\right)\right)}{315(a+bx)^{9/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(11/2),x]
```

```
[Out] (2*(c + d*x)^(3/2)*(105*a^3*d^3 + 63*a^2*b*d^2*(-3*c + 2*d*x) + 9*a*b^2*d*(
15*c^2 - 12*c*d*x + 8*d^2*x^2) + b^3*(-35*c^3 + 30*c^2*d*x - 24*c*d^2*x^2 +
 16*d^3*x^3)))/(315*(b*c - a*d)^4*(a + b*x)^(9/2))
```

Maple [A] time = 0.009, size = 171, normalized size = 1.3

$$\frac{32\,b^3d^3x^3+144\,ab^2d^3x^2-48\,b^3cd^2x^2+252\,a^2bd^3x-216\,ab^2cd^2x+60\,b^3c^2dx+210\,a^3d^3-378\,a^2bcd^2+270\,ab^2c^2d}{315\,d^4a^4-1260\,bd^3ca^3+1890\,b^2d^2c^2a^2-1260\,b^3dc^3a+315\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(11/2),x)
```

```
[Out] 2/315*(d*x+c)^(3/2)*(16*b^3*d^3*x^3+72*a*b^2*d^3*x^2-24*b^3*c*d^2*x^2+126*a
^2*b*d^3*x-108*a*b^2*c*d^2*x+30*b^3*c^2*d*x+105*a^3*d^3-189*a^2*b*c*d^2+135
*a*b^2*c^2*d-35*b^3*c^3)/(b*x+a)^(9/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2
*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 42.6591, size = 1080, normalized size = 7.94

$$\frac{2\left(16\,b^3d^4x^4 - 35\,b^3c^4 + 135\,ab^2c^3d - \right.}{315\left(a^5b^4c^4 - 4\,a^6b^3c^3d + 6\,a^7b^2c^2d^2 - 4\,a^8bcd^3 + a^9d^4 + \left(b^9c^4 - 4\,ab^8c^3d + 6\,a^2b^7c^2d^2 - 4\,a^3b^6cd^3 + a^4b^5d^4\right)x^5 + 5\left(ab^8c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(16*b^3*d^4*x^4 - 35*b^3*c^4 + 135*a*b^2*c^3*d - 189*a^2*b*c^2*d^2 +
105*a^3*c*d^3 - 8*(b^3*c*d^3 - 9*a*b^2*d^4)*x^3 + 6*(b^3*c^2*d^2 - 6*a*b^2*
c*d^3 + 21*a^2*b*d^4)*x^2 - (5*b^3*c^3*d - 27*a*b^2*c^2*d^2 + 63*a^2*b*c*d^
3 - 105*a^3*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^5*b^4*c^4 - 4*a^6*b^3*c^
3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4 + (b^9*c^4 - 4*a*b^8*c^3*
d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^5 + 5*(a*b^8*c^4 -
 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^4 +
 10*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 +
a^6*b^3*d^4)*x^3 + 10*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 -
4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x^2 + 5*(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a
^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.41645, size = 1335, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="giac")
```

```
[Out] 64/315*(sqrt(b*d)*b^13*c^5*d^4 - 5*sqrt(b*d)*a*b^12*c^4*d^5 + 10*sqrt(b*d)*
a^2*b^11*c^3*d^6 - 10*sqrt(b*d)*a^3*b^10*c^2*d^7 + 5*sqrt(b*d)*a^4*b^9*c*d^
8 - sqrt(b*d)*a^5*b^8*d^9 - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*b^11*c^4*d^4 + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^10*c^3*d^5 - 54*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*
b^9*c^2*d^6 + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a^3*b^8*c*d^7 - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^7*d^8 + 36*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^9*c^3*d^4 - 108*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a
*b^8*c^2*d^5 + 108*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^4*a^2*b^7*c*d^6 - 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^6*d^7 - 84*sqrt(b*d)*(sqrt(b*
```

```
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^7*c^2*d^4 + 168
*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
6*a*b^6*c*d^5 - 84*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^6*a^2*b^5*d^6 - 189*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^5*c*d^4 + 189*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^4*d^5 - 315*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b
^3*d^4)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2)^9*b^2)
```

3.1470 $\int \frac{\sqrt{c+dx}}{(a+bx)^{13/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{256d^4(c+dx)^{3/2}}{3465(a+bx)^{3/2}(bc-ad)^5}+\frac{128d^3(c+dx)^{3/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{32d^2(c+dx)^{3/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{16d(c+dx)^{3/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c}{11(a+bx}$$

```
[Out] (-2*(c + d*x)^(3/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (16*d*(c + d*x)^(3
/2))/(99*(b*c - a*d)^2*(a + b*x)^(9/2)) - (32*d^2*(c + d*x)^(3/2))/(231*(b*
c - a*d)^3*(a + b*x)^(7/2)) + (128*d^3*(c + d*x)^(3/2))/(1155*(b*c - a*d)^4
*(a + b*x)^(5/2)) - (256*d^4*(c + d*x)^(3/2))/(3465*(b*c - a*d)^5*(a + b*x)
^(3/2))
```

Rubi [A] time = 0.0414111, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{256d^4(c+dx)^{3/2}}{3465(a+bx)^{3/2}(bc-ad)^5}+\frac{128d^3(c+dx)^{3/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{32d^2(c+dx)^{3/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{16d(c+dx)^{3/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c}{11(a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (16*d*(c + d*x)^(3
/2))/(99*(b*c - a*d)^2*(a + b*x)^(9/2)) - (32*d^2*(c + d*x)^(3/2))/(231*(b*
c - a*d)^3*(a + b*x)^(7/2)) + (128*d^3*(c + d*x)^(3/2))/(1155*(b*c - a*d)^4
*(a + b*x)^(5/2)) - (256*d^4*(c + d*x)^(3/2))/(3465*(b*c - a*d)^5*(a + b*x)
^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{13/2}}\,dx = -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} - \frac{(8d)\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx}{11(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} + \frac{(16d^2)\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx}{33(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}} - \frac{(64d^3)\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}}{231(bc-ad)^3}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}} + \frac{128d^3(c+d}{1155(bc-ad)^4(a}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}} + \frac{128d^3(c+d}{1155(bc-ad)^4(a}$$

Mathematica [A] time = 0.0776076, size = 170, normalized size = 0.99

$$-\frac{2(c+dx)^{3/2}\left(198a^2b^2d^2\left(15c^2-12cdx+8d^2x^2\right)+924a^3bd^3(2dx-3c)+1155a^4d^4+44ab^3d\left(30c^2dx-35c^3-24cd^2x\right.\right.}{3465(a+bx)^{11/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(1155*a^4*d^4 + 924*a^3*b*d^3*(-3*c + 2*d*x) + 198*a^2*
b^2*d^2*(15*c^2 - 12*c*d*x + 8*d^2*x^2) + 44*a*b^3*d*(-35*c^3 + 30*c^2*d*x
- 24*c*d^2*x^2 + 16*d^3*x^3) + b^4*(315*c^4 - 280*c^3*d*x + 240*c^2*d^2*x^2
 - 192*c*d^3*x^3 + 128*d^4*x^4)))/(3465*(b*c - a*d)^5*(a + b*x)^(11/2))
```

Maple [A] time = 0.009, size = 256, normalized size = 1.5

$$\frac{256\,b^4d^4x^4+1408\,ab^3d^4x^3-384\,b^4cd^3x^3+3168\,a^2b^2d^4x^2-2112\,ab^3cd^3x^2+480\,b^4c^2d^2x^2+3696\,a^3bd^4x-4752\,a^2b^2}{3465\,a^5d^5-17325\,a^4bcd^4+34650\,a^3b^2c^2d^3-3465}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(13/2),x)
```

```
[Out] 2/3465*(d*x+c)^(3/2)*(128*b^4*d^4*x^4+704*a*b^3*d^4*x^3-192*b^4*c*d^3*x^3+1
584*a^2*b^2*d^4*x^2-1056*a*b^3*c*d^3*x^2+240*b^4*c^2*d^2*x^2+1848*a^3*b*d^4
*x-2376*a^2*b^2*c*d^3*x+1320*a*b^3*c^2*d^2*x-280*b^4*c^3*d*x+1155*a^4*d^4-2
772*a^3*b*c*d^3+2970*a^2*b^2*c^2*d^2-1540*a*b^3*c^3*d+315*b^4*c^4)/(b*x+a)^
(11/2)/(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4
*c^4*d-b^5*c^5)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 90.3644, size = 1613, normalized size = 9.43

$$-\frac{2}{3465\left(a^6b^5c^5 - 5\,a^7b^4c^4d + 10\,a^8b^3c^3d^2 - 10\,a^9b^2c^2d^3 + 5\,a^{10}bcd^4 - a^{11}d^5 + \left(b^{11}c^5 - 5\,ab^{10}c^4d + 10\,a^2b^9c^3d^2 - 10\,a^3b^8c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] -2/3465*(128*b^4*d^5*x^5 + 315*b^4*c^5 - 1540*a*b^3*c^4*d + 2970*a^2*b^2*c^
3*d^2 - 2772*a^3*b*c^2*d^3 + 1155*a^4*c*d^4 - 64*(b^4*c*d^4 - 11*a*b^3*d^5)
*x^4 + 16*(3*b^4*c^2*d^3 - 22*a*b^3*c*d^4 + 99*a^2*b^2*d^5)*x^3 - 8*(5*b^4*
c^3*d^2 - 33*a*b^3*c^2*d^3 + 99*a^2*b^2*c*d^4 - 231*a^3*b*d^5)*x^2 + (35*b^
4*c^4*d - 220*a*b^3*c^3*d^2 + 594*a^2*b^2*c^2*d^3 - 924*a^3*b*c*d^4 + 1155*
a^4*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b^5*c^5 - 5*a^7*b^4*c^4*d + 10
*a^8*b^3*c^3*d^2 - 10*a^9*b^2*c^2*d^3 + 5*a^10*b*c*d^4 - a^11*d^5 + (b^11*c
^5 - 5*a*b^10*c^4*d + 10*a^2*b^9*c^3*d^2 - 10*a^3*b^8*c^2*d^3 + 5*a^4*b^7*c
*d^4 - a^5*b^6*d^5)*x^6 + 6*(a*b^10*c^5 - 5*a^2*b^9*c^4*d + 10*a^3*b^8*c^3*
d^2 - 10*a^4*b^7*c^2*d^3 + 5*a^5*b^6*c*d^4 - a^6*b^5*d^5)*x^5 + 15*(a^2*b^9
*c^5 - 5*a^3*b^8*c^4*d + 10*a^4*b^7*c^3*d^2 - 10*a^5*b^6*c^2*d^3 + 5*a^6*b^
5*c*d^4 - a^7*b^4*d^5)*x^4 + 20*(a^3*b^8*c^5 - 5*a^4*b^7*c^4*d + 10*a^5*b^6
*c^3*d^2 - 10*a^6*b^5*c^2*d^3 + 5*a^7*b^4*c*d^4 - a^8*b^3*d^5)*x^3 + 15*(a^
4*b^7*c^5 - 5*a^5*b^6*c^4*d + 10*a^6*b^5*c^3*d^2 - 10*a^7*b^4*c^2*d^3 + 5*a
^8*b^3*c*d^4 - a^9*b^2*d^5)*x^2 + 6*(a^5*b^6*c^5 - 5*a^6*b^5*c^4*d + 10*a^7
*b^4*c^3*d^2 - 10*a^8*b^3*c^2*d^3 + 5*a^9*b^2*c*d^4 - a^10*b*d^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(13/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.58973, size = 1816, normalized size = 10.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="giac")
```

```
[Out] -512/3465*(sqrt(b*d)*b^16*c^6*d^5 - 6*sqrt(b*d)*a*b^15*c^5*d^6 + 15*sqrt(b*
d)*a^2*b^14*c^4*d^7 - 20*sqrt(b*d)*a^3*b^13*c^3*d^8 + 15*sqrt(b*d)*a^4*b^12
*c^2*d^9 - 6*sqrt(b*d)*a^5*b^11*c*d^10 + sqrt(b*d)*a^6*b^10*d^11 - 11*sqrt(
```

```
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^14
*c^5*d^5 + 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a*b^13*c^4*d^6 - 110*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^12*c^3*d^7 + 110*sqrt(b*d)*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^11*c^2*d
^8 - 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a
*b*d))^2*a^4*b^10*c*d^9 + 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^2*a^5*b^9*d^10 + 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^12*c^4*d^5 - 220*sqrt(b*
d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^11
*c^3*d^6 + 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*a^2*b^10*c^2*d^7 - 220*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^9*c*d^8 + 55*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^8*d^9 - 1
65*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^6*b^10*c^3*d^5 + 495*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a*b^9*c^2*d^6 - 495*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^8*c*d^7 + 165*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^7*d
^8 + 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^8*b^8*c^2*d^5 - 660*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^8*a*b^7*c*d^6 + 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^6*d^7 + 924*sqrt(b*d)
*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^6*c*d
^5 - 924*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^10*a*b^5*d^6 + 1386*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^12*b^4*d^5)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^11*b^2)
```

3.1471 $\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx$

Optimal. Leaf size=227

$$\frac{3\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^4}{128b^2d^3}-\frac{(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^3}{64b^2d^2}-\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{5/2}d^{7/2}}+\frac{(a+bx)^{5/2}\sqrt{c+dx}(bc-a}{80b^2d}$$

```
[Out] (3*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b^2*d^3) - ((b*c - a*d)^
3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(64*b^2*d^2) + ((b*c - a*d)^2*(a + b*x)^(5
/2)*Sqrt[c + d*x])/(80*b^2*d) + (3*(b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*x
])/(40*b^2) + ((a + b*x)^(7/2)*(c + d*x)^(3/2))/(5*b) - (3*(b*c - a*d)^5*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(5/2)*d^(7/2
))
```

Rubi [A] time = 0.130224, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$\frac{3\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^4}{128b^2d^3}-\frac{(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^3}{64b^2d^2}-\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{5/2}d^{7/2}}+\frac{(a+bx)^{5/2}\sqrt{c+dx}(bc-a}{80b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(c + d*x)^(3/2),x]
```

```
[Out] (3*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b^2*d^3) - ((b*c - a*d)^
3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(64*b^2*d^2) + ((b*c - a*d)^2*(a + b*x)^(5
/2)*Sqrt[c + d*x])/(80*b^2*d) + (3*(b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*x
])/(40*b^2) + ((a + b*x)^(7/2)*(c + d*x)^(3/2))/(5*b) - (3*(b*c - a*d)^5*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(5/2)*d^(7/2
))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx = \frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b} + \frac{(3(bc-ad))\int (a+bx)^{5/2}\sqrt{c+dx}\,dx}{10b}$$

$$= \frac{3(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40b^2} + \frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b} + \frac{\left(3(bc-ad)^2\right)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{80b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{80b^2d} + \frac{3(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40b^2} + \frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b} -$$

$$= -\frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{80b^2d} + \frac{3(bc-ad)(a+bx)^{7/2}\sqrt{c}}{40b^2}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+}}{80b^2d}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+}}{80b^2d}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+}}{80b^2d}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+}}{80b^2d}$$

Mathematica [A] time = 1.75577, size = 187, normalized size = 0.82

$$\frac{(a+bx)^{7/2}\sqrt{c+dx}\left(\frac{15(bc-ad)^4}{d^3(a+bx)^3}+\frac{10(ad-bc)^3}{d^2(a+bx)^2}-\frac{15(bc-ad)^{9/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{7/2}(a+bx)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}}+\frac{8(bc-ad)^2}{d(a+bx)}+48(bc-ad)+128b(c+dx)\right)}{640b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(c + d*x)^(3/2),x]
```

```
[Out] ((a + b*x)^(7/2)*Sqrt[c + d*x]*(48*(b*c - a*d) + (15*(b*c - a*d)^4)/(d^3*(a
 + b*x)^3) + (10*(-(b*c) + a*d)^3)/(d^2*(a + b*x)^2) + (8*(b*c - a*d)^2)/(d
*(a + b*x)) + 128*b*(c + d*x) - (15*(b*c - a*d)^(9/2)*ArcSinh[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[b*c - a*d]])/(d^(7/2)*(a + b*x)^(7/2)*Sqrt[(b*(c + d*x))/(b
*c - a*d)])))/(640*b^2)
```

Maple [B] time = 0.007, size = 853, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(3/2),x)
```

```
[Out] 1/5/d*(b*x+a)^(5/2)*(d*x+c)^(5/2)+1/8/d*(b*x+a)^(3/2)*(d*x+c)^(5/2)*a+1/16/
d*(b*x+a)^(1/2)*(d*x+c)^(5/2)*a^2-1/8/d^2*(b*x+a)^(3/2)*(d*x+c)^(5/2)*b*c+1
/16/d^3*(b*x+a)^(1/2)*(d*x+c)^(5/2)*b^2*c^2+1/64/b*(d*x+c)^(3/2)*(b*x+a)^(1
/2)*a^3-3/64/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^2*c-1/64/d^3*(d*x+c)^(3/2)*(b*
x+a)^(1/2)*c^3*b^2-3/128*d/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^4-9/64/d*(d*x+
c)^(1/2)*(b*x+a)^(1/2)*a^2*c^2-3/128/d^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^4*b^
2+3/32/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^3*b-1/8/d^2*(b*x+a)^(1/2)*(d*x+c
)^(5/2)*a*b*c+3/64/d^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*c^2*b+3/32/b*(d*x+c)^(
1/2)*(b*x+a)^(1/2)*a^3*c-15/256*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(
b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*
c)^(1/2))/(b*d)^(1/2)*a^4*c+15/128*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b
*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c
)^(1/2))/(b*d)^(1/2)*a^3*c^2+15/256/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/
2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*
x+a*c)^(1/2))/(b*d)^(1/2)*a*c^4*b^2+3/256*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(
d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+
(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^5-15/128/d*((b*x+a)*(d*x+c))^(1/2)/(d
*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(
a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^2*c^3*b-3/256/d^3*((b*x+a)*(d*x+c))^(1
/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x
^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*c^5*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.18923, size = 1547, normalized size = 6.81

$$\left[-\frac{15\left(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5\right)\sqrt{bd}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 + 4\,(2\,bdx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2
*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a
*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x
 + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(128*b^5*d^5*x^4 + 15*b^5*c^4*d - 70*a
*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 + 70*a^3*b^2*c*d^4 - 15*a^4*b*d^5 + 16*(
11*b^5*c*d^4 + 21*a*b^4*d^5)*x^3 + 8*(b^5*c^2*d^3 + 64*a*b^4*c*d^4 + 31*a^2
*b^3*d^5)*x^2 - 2*(5*b^5*c^3*d^2 - 23*a*b^4*c^2*d^3 - 233*a^2*b^3*c*d^4 - 5
*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^4), 1/1280*(15*(b^5*c^
5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4
```

```
 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(1
28*b^5*d^5*x^4 + 15*b^5*c^4*d - 70*a*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 + 70
*a^3*b^2*c*d^4 - 15*a^4*b*d^5 + 16*(11*b^5*c*d^4 + 21*a*b^4*d^5)*x^3 + 8*(b
^5*c^2*d^3 + 64*a*b^4*c*d^4 + 31*a^2*b^3*d^5)*x^2 - 2*(5*b^5*c^3*d^2 - 23*a
*b^4*c^2*d^3 - 233*a^2*b^3*c*d^4 - 5*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x
 + c))/(b^3*d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)*(c + d*x)**(3/2), x)
```

Giac [B] time = 1.43241, size = 1987, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1920*(10*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(
6*(b*x + a)/b^2 + (b^7*c*d^5 - 17*a*b^6*d^6)/(b^8*d^6)) - (5*b^8*c^2*d^4 +
6*a*b^7*c*d^5 - 59*a^2*b^6*d^6)/(b^8*d^6)) + 3*(5*b^9*c^3*d^3 + a*b^8*c^2*d
^4 - a^2*b^7*c*d^5 - 5*a^3*b^6*d^6)/(b^8*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4
 - 4*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + 5*a^4*d^4)*log(abs(-
sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*
b*d^3))*c*abs(b) + 20*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2
*(b*x + a)/(b^4*d^2) + (b*c*d - a*d^2)/(b^4*d^4)) + (b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*
b*d)))/(sqrt(b*d)*b^3*d^3))*a^2*c*abs(b)/b^2 + (sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^3 + (b^13*c*d^7 - 31*a
*b^12*d^8)/(b^15*d^8)) - (7*b^14*c^2*d^6 + 16*a*b^13*c*d^7 - 263*a^2*b^12*d
^8)/(b^15*d^8)) + 5*(7*b^15*c^3*d^5 + 9*a*b^14*c^2*d^6 + 9*a^2*b^13*c*d^7 -
 121*a^3*b^12*d^8)/(b^15*d^8))*(b*x + a) - 15*(7*b^16*c^4*d^4 + 2*a*b^15*c^
3*d^5 - 2*a^3*b^13*c*d^7 - 7*a^4*b^12*d^8)/(b^15*d^8))*sqrt(b*x + a) - 15*(
7*b^5*c^5 - 5*a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3 - 5*a^4*b
*c*d^4 + 7*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^4))*d*abs(b) + 20*(sqrt(b^2*c + (b*x + a
)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^2 + (b^7*c*d^5 - 17
*a*b^6*d^6)/(b^8*d^6)) - (5*b^8*c^2*d^4 + 6*a*b^7*c*d^5 - 59*a^2*b^6*d^6)/(
b^8*d^6)) + 3*(5*b^9*c^3*d^3 + a*b^8*c^2*d^4 - a^2*b^7*c*d^5 - 5*a^3*b^6*d^
6)/(b^8*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 - 4*a*b^3*c^3*d - 2*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + 5*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^
2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^3))*a*d*abs(b)/b + 2*(sqrt(b^
2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/(b^6*d
^2) + (b*c*d^3 - 7*a*d^4)/(b^6*d^6)) - 3*(b^2*c^2*d^2 - a^2*d^4)/(b^6*d^6))
 - 3*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*log(abs(-sqrt(b*d)*sqr
t(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^5*d^4))*a*c
```

```
*abs(b)/b^2 + (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x +
a)*(4*(b*x + a)/(b^6*d^2) + (b*c*d^3 - 7*a*d^4)/(b^6*d^6)) - 3*(b^2*c^2*d^2
 - a^2*d^4)/(b^6*d^6)) - 3*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*
log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(s
qrt(b*d)*b^5*d^4))*a^2*d*abs(b)/b^3)/b
```

3.1472 $\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx$

Optimal. Leaf size=189

$$-\frac{3\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^3}{64b^2d^2}+\frac{3(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{5/2}d^{5/2}}+\frac{(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^2}{32b^2d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}(b}{8b^2}$$

```
[Out] (-3*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b^2*d^2) + ((b*c - a*d)^
2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(32*b^2*d) + ((b*c - a*d)*(a + b*x)^(5/2)*
Sqrt[c + d*x])/(8*b^2) + ((a + b*x)^(5/2)*(c + d*x)^(3/2))/(4*b) + (3*(b*c
- a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(5
/2)*d^(5/2))
```

Rubi [A] time = 0.0898839, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$-\frac{3\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^3}{64b^2d^2}+\frac{3(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{5/2}d^{5/2}}+\frac{(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^2}{32b^2d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}(b}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(3/2),x]
```

```
[Out] (-3*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b^2*d^2) + ((b*c - a*d)^
2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(32*b^2*d) + ((b*c - a*d)*(a + b*x)^(5/2)*
Sqrt[c + d*x])/(8*b^2) + ((a + b*x)^(5/2)*(c + d*x)^(3/2))/(4*b) + (3*(b*c
- a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(5
/2)*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx = \frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b} + \frac{(3(bc-ad))\int (a+bx)^{3/2}\sqrt{c+dx}\,dx}{8b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b} + \frac{(bc-ad)^2\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{16b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b} - \frac{(3(bc$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2}$$

Mathematica [A] time = 0.536016, size = 193, normalized size = 1.02

$$\frac{3(bc-ad)^{9/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right) - b\sqrt{d}\sqrt{a+bx}(c+dx)\left(-a^2bd^2(11c+2dx)+3a^3d^3-ab^2d\left(11c^2+44cdx+24d^2x\right.\right.}{64b^3d^{5/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(3/2),x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(3*a^3*d^3 - a^2*b*d^2*(11*c + 2*d*x)
- a*b^2*d*(11*c^2 + 44*c*d*x + 24*d^2*x^2) + b^3*(3*c^3 - 2*c^2*d*x - 24*c*
d^2*x^2 - 16*d^3*x^3))) + 3*(b*c - a*d)^(9/2)*Sqrt[(b*(c + d*x))/(b*c - a*d
)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(64*b^3*d^(5/2)*Sqrt[c
 + d*x])
```

Maple [B] time = 0.004, size = 640, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(3/2),x)
```

```
[Out] 1/4/d*(b*x+a)^(3/2)*(d*x+c)^(5/2)+1/8/d*(b*x+a)^(1/2)*(d*x+c)^(5/2)*a-1/8/d
^2*(b*x+a)^(1/2)*(d*x+c)^(5/2)*b*c+1/32/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^2-1
/16/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*c+1/32/d^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*
```

```
c^2*b-3/64*d/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3+9/64/b*(d*x+c)^(1/2)*(b*x+
a)^(1/2)*a^2*c-9/64/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^2+3/64/d^2*(d*x+c)^(1
/2)*(b*x+a)^(1/2)*c^3*b+3/128*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)
/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+
a*c)^(1/2))/(b*d)^(1/2)*a^4-3/32*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/
(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a
*c)^(1/2))/(b*d)^(1/2)*a^3*c+9/64*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*
x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)
^(1/2))/(b*d)^(1/2)*a^2*c^2-3/32/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b
*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c
)^(1/2))/(b*d)^(1/2)*a*c^3*b+3/128/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x
+a*c)^(1/2))/(b*d)^(1/2)*c^4*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.17454, size = 1177, normalized size = 6.23

$$\left[\frac{3\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,bdx+bc+ad\right)\sqrt{bd}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b
*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*
d^2)*x) + 4*(16*b^4*d^4*x^3 - 3*b^4*c^3*d + 11*a*b^3*c^2*d^2 + 11*a^2*b^2*c
*d^3 - 3*a^3*b*d^4 + 24*(b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(b^4*c^2*d^2 + 22*a
*b^3*c*d^3 + a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^3), -1/128
*(3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(
d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(16*b^4*d^4*x
^3 - 3*b^4*c^3*d + 11*a*b^3*c^2*d^2 + 11*a^2*b^2*c*d^3 - 3*a^3*b*d^4 + 24*(
b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(b^4*c^2*d^2 + 22*a*b^3*c*d^3 + a^2*b^2*d^4)
*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(3/2), x)
```

Giac [B] time = 1.28398, size = 1107, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1920*(20*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)/
(b^4*d^2) + (b*c*d - a*d^2)/(b^4*d^4)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqr
t(b*d)*b^3*d^3))*a*c*abs(b)/b^2 + 10*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(
2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^2 + (b^7*c*d^5 - 17*a*b^6*d^6)/(b^8
*d^6)) - (5*b^8*c^2*d^4 + 6*a*b^7*c*d^5 - 59*a^2*b^6*d^6)/(b^8*d^6)) + 3*(5
*b^9*c^3*d^3 + a*b^8*c^2*d^4 - a^2*b^7*c*d^5 - 5*a^3*b^6*d^6)/(b^8*d^6))*sq
rt(b*x + a) + 3*(5*b^4*c^4 - 4*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 - 4*a^3*b*c*
d^3 + 5*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*
b*d - a*b*d)))/(sqrt(b*d)*b*d^3))*d*abs(b)/b + (sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/(b^6*d^2) + (b*c*d^3 - 7*a
*d^4)/(b^6*d^6)) - 3*(b^2*c^2*d^2 - a^2*d^4)/(b^6*d^6)) - 3*(b^3*c^3 - a*b^
2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^
2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^5*d^4))*c*abs(b)/b^2 + (sqrt(b^
2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/(b^6*d
^2) + (b*c*d^3 - 7*a*d^4)/(b^6*d^6)) - 3*(b^2*c^2*d^2 - a^2*d^4)/(b^6*d^6))
 - 3*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*log(abs(-sqrt(b*d)*sqr
t(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^5*d^4))*a*d
*abs(b)/b^3)/b
```

3.1473 $\int \sqrt{a+bx}(c+dx)^{3/2}\,dx$

Optimal. Leaf size=151

$$-\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8b^2d}+\frac{(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{4b^2}+\frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}$$

```
[Out] ((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b^2*d) + ((b*c - a*d)*(a + b
*x)^(3/2)*Sqrt[c + d*x])/(4*b^2) + ((a + b*x)^(3/2)*(c + d*x)^(3/2))/(3*b)
- ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/
(8*b^(5/2)*d^(3/2))
```

Rubi [A] time = 0.0676236, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$-\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8b^2d}+\frac{(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{4b^2}+\frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(3/2),x]
```


```
[Out] ((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b^2*d) + ((b*c - a*d)*(a + b
*x)^(3/2)*Sqrt[c + d*x])/(4*b^2) + ((a + b*x)^(3/2)*(c + d*x)^(3/2))/(3*b)
- ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/
(8*b^(5/2)*d^(3/2))
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx}(c+dx)^{3/2}\,dx = \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} + \frac{(bc-ad)\int \sqrt{a+bx}\sqrt{c+dx}\,dx}{2b}$$

$$= \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} + \frac{(bc-ad)^2\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{8b^2}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} - \frac{(bc-ad)}{}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} - \frac{(bc-ad)}{}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} - \frac{(bc-ad)}{}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} - \frac{(bc-ad)}{}$$

Mathematica [A] time = 0.4281, size = 152, normalized size = 1.01

$$\frac{-b\sqrt{d}\sqrt{a+bx}(c+dx)\left(3a^2d^2-2abd(4c+dx)+b^2\left(-\left(3c^2+14cdx+8d^2x^2\right)\right)\right)-3(bc-ad)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{24b^3d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(3/2),x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(3*a^2*d^2 - 2*a*b*d*(4*c + d*x) - b^2
*(3*c^2 + 14*c*d*x + 8*d^2*x^2))) - 3*(b*c - a*d)^(7/2)*Sqrt[(b*(c + d*x))/
(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(24*b^3*d^(3
/2)*Sqrt[c + d*x])
```

Maple [B] time = 0.006, size = 459, normalized size = 3.

$$\frac{1}{3d}\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}} + \frac{a}{12b}\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}} - \frac{c}{12d}\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}} - \frac{da^2}{8b^2}\sqrt{bx+a}\sqrt{dx+c} + \frac{ac}{4b}\sqrt{bx+a}\sqrt{dx+c} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(3/2),x)
```

```
[Out] 1/3/d*(b*x+a)^(1/2)*(d*x+c)^(5/2)+1/12/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a-1/12
/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*c-1/8*d/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^2+
1/4/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c-1/8/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^2
+1/16*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a
*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*
a^3-3/16*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*
d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a
^2*c+3/16*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1
```

```
/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a*c^
2-1/16/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/
2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*c^3*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01597, size = 918, normalized size = 6.08

$$\left[-\frac{3\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{bd}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 + 4\,(2\,bdx + bc + ad)\sqrt{bd}\sqrt{bx+a}\sqrt{d}\right.}{96\,b^3c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sq
rt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d
^3*x^2 + 3*b^3*c^2*d + 8*a*b^2*c*d^2 - 3*a^2*b*d^3 + 2*(7*b^3*c*d^2 + a*b^2
*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^2), 1/48*(3*(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*
d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d
 + a*b*d^2)*x)) + 2*(8*b^3*d^3*x^2 + 3*b^3*c^2*d + 8*a*b^2*c*d^2 - 3*a^2*b*
d^3 + 2*(7*b^3*c*d^2 + a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^2)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17705, size = 458, normalized size = 3.03

$$\frac{\frac{20\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(\frac{2\,(bx+a)}{b^4d^2}+\frac{bcd-ad^2}{b^4d^4}\right)+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}b^3d^3}\right)c|b|}{b^2}+\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(b}{b}\right.\right.\right.}{}}{1920\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1920*(20*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)/
(b^4*d^2) + (b*c*d - a*d^2)/(b^4*d^4)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqr
t(b*d)*b^3*d^3))*c*abs(b)/b^2 + (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b
*x + a)*(2*(b*x + a)*(4*(b*x + a)/(b^6*d^2) + (b*c*d^3 - 7*a*d^4)/(b^6*d^6)
) - 3*(b^2*c^2*d^2 - a^2*d^4)/(b^6*d^6)) - 3*(b^3*c^3 - a*b^2*c^2*d - a^2*b
*c*d^2 + a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)
*b*d - a*b*d)))/(sqrt(b*d)*b^5*d^4))*d*abs(b)/b^3)/b
```

3.1474 $\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=113

$$\frac{3\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4b^2}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}+\frac{\sqrt{a+bx}(c+dx)^{3/2}}{2b}$$

```
[Out] (3*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b^2) + (Sqrt[a + b*x]*(c + d
*x)^(3/2))/(2*b) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b
]*Sqrt[c + d*x])])/(4*b^(5/2)*Sqrt[d])
```

Rubi [A] time = 0.0510619, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$\frac{3\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4b^2}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}+\frac{\sqrt{a+bx}(c+dx)^{3/2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (3*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b^2) + (Sqrt[a + b*x]*(c + d
*x)^(3/2))/(2*b) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b
]*Sqrt[c + d*x])])/(4*b^(5/2)*Sqrt[d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}(c+dx)^{3/2}}{2b} + \frac{(3(bc-ad))\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{4b}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}(c+dx)^{3/2}}{2b} + \frac{\left(3(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{8b^2}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}(c+dx)^{3/2}}{2b} + \frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{4b^3}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}(c+dx)^{3/2}}{2b} + \frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{4b^3}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}(c+dx)^{3/2}}{2b} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}$$

Mathematica [A] time = 0.28947, size = 109, normalized size = 0.96

$$\frac{\sqrt{c+dx}\left(\sqrt{a+bx}(-3ad+5bc+2bdx)+\frac{3(bc-ad)^{3/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{\frac{b(c+dx)}{bc-ad}}}\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(Sqrt[a + b*x]*(5*b*c - 3*a*d + 2*b*d*x) + (3*(b*c - a*d)^(3
/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[(b*(c +
 d*x))/(b*c - a*d)])))/(4*b^2)
```

Maple [B] time = 0.007, size = 308, normalized size = 2.7

$$\frac{1}{2b}\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}} - \frac{3\,ad}{4\,b^2}\sqrt{bx+a}\sqrt{dx+c} + \frac{3\,c}{4\,b}\sqrt{bx+a}\sqrt{dx+c} + \frac{3\,a^2d^2}{8\,b^2}\sqrt{(bx+a)\,(dx+c)}\ln\left(\left(\frac{ad}{2}+\frac{bc}{2}+bdx\right)\frac{1}{\sqrt{bd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*d+3
/4/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c+3/8/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^
(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*
c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^2*d^2-3/4/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^
(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*
c)*x+a*c)^(1/2))/(b*d)^(1/2)*a*d*c+3/8*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x
+a*c)^(1/2))/(b*d)^(1/2)*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91523, size = 711, normalized size = 6.29

$$\left[\frac{3\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{bd}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 + 4\,(2\,bdx + bc + ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c} + 8\,\left(b^2cd\right.\right.}{16\,b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)
*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + 5*b^2*c*d - 3*
a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d), -1/8*(3*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(2
*b^2*d^2*x + 5*b^2*c*d - 3*a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c + dx)^{\frac{3}{2}}}{\sqrt{a + bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/sqrt(a + b*x), x)
```

Giac [B] time = 1.1496, size = 327, normalized size = 2.89

$$-\frac{\frac{48\left(\frac{\left(b^2c - abd\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a} + \sqrt{b^2c + (bx+a)bd - abd}\right|\right)}{\sqrt{bd}} - \sqrt{b^2c + (bx+a)bd - abd}\sqrt{bx+a}\right)c|b|}{b^2} - \frac{\left(\sqrt{b^2c + (bx+a)bd - abd}\sqrt{bx+a}\left(\frac{2\,(bx+a)}{b^4d^2} + \frac{bcd - 5\,ad^2}{b^4d^4}\right) + \frac{\left(b^2c^2 + 2\,abcd - 3\,a^2d^2\right.}{}\right.}{b^3}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*(48*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sq
rt(b*x + a))*c*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x +
 a)*(2*(b*x + a)/(b^4*d^2) + (b*c*d - 5*a*d^2)/(b^4*d^4)) + (b^2*c^2 + 2*a*
b*c*d - 3*a^2*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a
)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^3))*d*abs(b)/b^3)/b
```

3.1475 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}} \, dx$

Optimal. Leaf size=98

$$\frac{3d\sqrt{a+bx}\sqrt{c+dx}}{b^2}+\frac{3\sqrt{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}$$

```
[Out] (3*d*Sqrt[a + b*x]*Sqrt[c + d*x])/b^2 - (2*(c + d*x)^(3/2))/(b*Sqrt[a + b*x
]) + (3*Sqrt[d]*(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c
 + d*x])])/b^(5/2)
```

Rubi [A] time = 0.046969, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{3d\sqrt{a+bx}\sqrt{c+dx}}{b^2}+\frac{3\sqrt{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(3/2),x]
```


```
[Out] (3*d*Sqrt[a + b*x]*Sqrt[c + d*x])/b^2 - (2*(c + d*x)^(3/2))/(b*Sqrt[a + b*x
]) + (3*Sqrt[d]*(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c
 + d*x])])/b^(5/2)
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{b}\\ &= \frac{3d\sqrt{a+bx}\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{2b^2}\\ &= \frac{3d\sqrt{a+bx}\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^3}\\ &= \frac{3d\sqrt{a+bx}\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^3}\\ &= \frac{3d\sqrt{a+bx}\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.0521282, size = 71, normalized size = 0.72

$$-\frac{2(c+dx)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*Hypergeometric2F1[-3/2, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(3/2))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{2}}(bx+a)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(3/2)/(b*x+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.67628, size = 703, normalized size = 7.17

$$\left[-\frac{3\left(abc-a^2d+\left(b^2c-abd\right)x\right)\sqrt{\frac{d}{b}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\left(2\,b^2dx+b^2c+abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}}\right.}{4\left(b^3x+ab^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(a*b*c - a^2*d + (b^2*c - a*b*d)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 +
b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(b*d*x - 2*b*c + 3*
a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*x + a*b^2), -1/2*(3*(a*b*c - a^2*d +
 (b^2*c - a*b*d)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x +
a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) - 2*(b
*d*x - 2*b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*x + a*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(c+dx)^{\frac{3}{2}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/(a + b*x)**(3/2), x)
```

Giac [B] time = 1.21896, size = 275, normalized size = 2.81

$$\frac{\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}d|b|}{b^4}-\frac{3\left(\sqrt{bd}bc|b|-\sqrt{bd}ad|b|\right)\log\left(\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{2\,b^4}-\frac{}{\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*d*abs(b)/b^4 - 3/2*(sqrt(
b*d)*b*c*abs(b) - sqrt(b*d)*a*d*abs(b))*log((sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^2)/b^4 - 4*(sqrt(b*d)*b^2*c^2*abs(b) - 2*s
qrt(b*d)*a*b*c*d*abs(b) + sqrt(b*d)*a^2*d^2*abs(b))/((b^2*c - a*b*d - (sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*b^3)
```

3.1476 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=92

$$\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}-\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}$$

```
[Out] (-2*d*Sqrt[c + d*x])/(b^2*Sqrt[a + b*x]) - (2*(c + d*x)^(3/2))/(3*b*(a + b*
x)^(3/2)) + (2*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/b^(5/2)
```

Rubi [A] time = 0.0397725, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {47, 63, 217, 206}

$$\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}-\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*d*Sqrt[c + d*x])/(b^2*Sqrt[a + b*x]) - (2*(c + d*x)^(3/2))/(3*b*(a + b*
x)^(3/2)) + (2*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}}\,dx &= -\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{d\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx}{b}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{d^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{b^2}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^3}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^3}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0446405, size = 73, normalized size = 0.79

$$-\frac{2(c+dx)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*Hypergeometric2F1[-3/2, -3/2, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(3/2))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{2}} (bx+a)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(3/2)/(b*x+a)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.5806, size = 734, normalized size = 7.98

$$\left[\frac{3\left(b^2dx^2+2\,abdx+a^2d\right)\sqrt{\frac{d}{b}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,b^2dx+b^2c+abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}}+8\left(\right.\right.}{6\left(b^4x^2+2\,ab^3x+a^2b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*d*x^2 + 2*a*b*d*x + a^2*d)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c
^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt
(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(4*b*d*x + b*c + 3*a*d)*
sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2), -1/3*(3*(b^2*
d*x^2 + 2*a*b*d*x + a^2*d)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt
(b*x + a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x))
 + 2*(4*b*d*x + b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x^2 + 2*a*b^
3*x + a^2*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/(a + b*x)**(5/2), x)
```

Giac [B] time = 1.30569, size = 614, normalized size = 6.67

$$-\frac{\sqrt{bd}d|b|\log\left(\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{b^4}-\frac{8\left(2\sqrt{bd}b^5c^3d|b|-6\sqrt{bd}ab^4c^2d^2|b|+6\sqrt{bd}a^2b^3cd^3|b|-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*d*abs(b)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2)/b^4 - 8/3*(2*sqrt(b*d)*b^5*c^3*d*abs(b) - 6*sqrt(b*d)*a*b^4
*c^2*d^2*abs(b) + 6*sqrt(b*d)*a^2*b^3*c*d^3*abs(b) - 2*sqrt(b*d)*a^3*b^2*d^
4*abs(b) - 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*b^3*c^2*d*abs(b) + 6*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^2*c*d^2*abs(b) - 3*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b*d^3*abs(b
```

```
) + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^4*b*c*d*abs(b) - 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^4*a*d^2*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3*b^3)
```

3.1477 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2(c+dx)^{5/2}}{5(a+bx)^{5/2}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(5/2))/(5*(b*c - a*d)*(a + b*x)^(5/2))
```

Rubi [A] time = 0.0030631, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{2(c+dx)^{5/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(5*(b*c - a*d)*(a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{5/2}}{5(bc-ad)(a+bx)^{5/2}}$$

Mathematica [A] time = 0.0140612, size = 32, normalized size = 1.

$$-\frac{2(c+dx)^{5/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(5*(b*c - a*d)*(a + b*x)^(5/2))
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$\frac{2}{5\,ad-5\,bc}\,(dx+c)^{\frac{5}{2}}\,(bx+a)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(7/2),x)
```

```
[Out] 2/5/(b*x+a)^(5/2)*(d*x+c)^(5/2)/(a*d-b*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.3366, size = 216, normalized size = 6.75

$$-\frac{2\left(d^2x^2+2\,cdx+c^2\right)\sqrt{bx+a}\sqrt{dx+c}}{5\left(a^3bc-a^4d+\left(b^4c-ab^3d\right)x^3+3\left(ab^3c-a^2b^2d\right)x^2+3\left(a^2b^2c-a^3bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b*c - a^4*d
 + (b^4*c - a*b^3*d)*x^3 + 3*(a*b^3*c - a^2*b^2*d)*x^2 + 3*(a^2*b^2*c - a^3
*b*d)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.53208, size = 505, normalized size = 15.78

$$-\frac{4\left(\sqrt{bd}b^8c^4d^2|b|-4\sqrt{bd}ab^7c^3d^3|b|+6\sqrt{bd}a^2b^6c^2d^4|b|-4\sqrt{bd}a^3b^5cd^5|b|+\sqrt{bd}a^4b^4d^6|b|+10\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] -4/5*(sqrt(b*d)*b^8*c^4*d^2*abs(b) - 4*sqrt(b*d)*a*b^7*c^3*d^3*abs(b) + 6*s
qrt(b*d)*a^2*b^6*c^2*d^4*abs(b) - 4*sqrt(b*d)*a^3*b^5*c*d^5*abs(b) + sqrt(b
*d)*a^4*b^4*d^6*abs(b) + 10*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^4*b^4*c^2*d^2*abs(b) - 20*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^3*c*d^3*abs(b) +
10*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^4*a^2*b^2*d^4*abs(b) + 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^8*d^2*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*b^3)
```

3.1478 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx$

Optimal. Leaf size=66

$$\frac{4d(c+dx)^{5/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{7(a+bx)^{7/2}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(5/2))/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (4*d*(c + d*x)^(5/2)
)/(35*(b*c - a*d)^2*(a + b*x)^(5/2))
```

Rubi [A] time = 0.0082446, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4d(c+dx)^{5/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (4*d*(c + d*x)^(5/2)
)/(35*(b*c - a*d)^2*(a + b*x)^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx &= -\frac{2(c+dx)^{5/2}}{7(bc-ad)(a+bx)^{7/2}}-\frac{(2d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx}{7(bc-ad)}\\ &= -\frac{2(c+dx)^{5/2}}{7(bc-ad)(a+bx)^{7/2}}+\frac{4d(c+dx)^{5/2}}{35(bc-ad)^2(a+bx)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0250242, size = 46, normalized size = 0.7

$$\frac{2(c+dx)^{5/2}(7ad-5bc+2bdx)}{35(a+bx)^{7/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(9/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-5*b*c + 7*a*d + 2*b*d*x))/(35*(b*c - a*d)^2*(a + b*x)^
(7/2))
```

Maple [A] time = 0.005, size = 54, normalized size = 0.8

$$\frac{4\,bdx+14\,ad-10\,bc}{35\,a^{2}d^{2}-70\,abcd+35\,b^{2}c^{2}}\left(dx+c\right)^{\frac{5}{2}}\left(bx+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(9/2),x)
```

```
[Out] 2/35*(d*x+c)^(5/2)*(2*b*d*x+7*a*d-5*b*c)/(b*x+a)^(7/2)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 11.8059, size = 483, normalized size = 7.32

$$\frac{2\left(2\,bd^{3}x^{3}-5\,bc^{3}+7\,ac^{2}d-\left(bcd^{2}-7\,ad^{3}\right)x^{2}-2\left(4\,bc^{2}d-7\,acd^{2}\right)x\right)\sqrt{bx+a}\sqrt{}}{35\left(a^{4}b^{2}c^{2}-2\,a^{5}bcd+a^{6}d^{2}+\left(b^{6}c^{2}-2\,ab^{5}cd+a^{2}b^{4}d^{2}\right)x^{4}+4\left(ab^{5}c^{2}-2\,a^{2}b^{4}cd+a^{3}b^{3}d^{2}\right)x^{3}+6\left(a^{2}b^{4}c^{2}-2\,a^{3}b^{3}cd+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/35*(2*b*d^3*x^3 - 5*b*c^3 + 7*a*c^2*d - (b*c*d^2 - 7*a*d^3)*x^2 - 2*(4*b*
c^2*d - 7*a*c*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^2*c^2 - 2*a^5*b*c*
d + a^6*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 4*(a*b^5*c^2 - 2*
a^2*b^4*c*d + a^3*b^3*d^2)*x^3 + 6*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d
^2)*x^2 + 4*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.53585, size = 1382, normalized size = 20.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 8/35*(sqrt(b*d)*b^10*c^5*d^3*abs(b) - 5*sqrt(b*d)*a*b^9*c^4*d^4*abs(b) + 10
*sqrt(b*d)*a^2*b^8*c^3*d^5*abs(b) - 10*sqrt(b*d)*a^3*b^7*c^2*d^6*abs(b) + 5
*sqrt(b*d)*a^4*b^6*c*d^7*abs(b) - sqrt(b*d)*a^5*b^5*d^8*abs(b) - 7*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^8*c^4
*d^3*abs(b) + 28*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a*b^7*c^3*d^4*abs(b) - 42*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^6*c^2*d^5*abs(b) + 28*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a
^3*b^5*c*d^6*abs(b) - 7*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2*a^4*b^4*d^7*abs(b) - 14*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^6*c^3*d^3*abs(b) + 42*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
a*b^5*c^2*d^4*abs(b) - 42*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^4*a^2*b^4*c*d^5*abs(b) + 14*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^3*d^6*abs(b) -
70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^6*b^4*c^2*d^3*abs(b) + 140*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^6*a*b^3*c*d^4*abs(b) - 70*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^2*d^5*abs(b) -
 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^8*b^2*c*d^3*abs(b) + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^8*a*b*d^4*abs(b) - 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*d^3*abs(b))/((b^2*c - a*b
*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^7*b
^2)
```

3.1479 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}} dx$

Optimal. Leaf size=101

$$-\frac{16d^2(c+dx)^{5/2}}{315(a+bx)^{5/2}(bc-ad)^3}+\frac{8d(c+dx)^{5/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{9(a+bx)^{9/2}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(5/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (8*d*(c + d*x)^(5/2)
)/(63*(b*c - a*d)^2*(a + b*x)^(7/2)) - (16*d^2*(c + d*x)^(5/2))/(315*(b*c -
 a*d)^3*(a + b*x)^(5/2))
```

Rubi [A] time = 0.015951, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d^2(c+dx)^{5/2}}{315(a+bx)^{5/2}(bc-ad)^3}+\frac{8d(c+dx)^{5/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (8*d*(c + d*x)^(5/2)
)/(63*(b*c - a*d)^2*(a + b*x)^(7/2)) - (16*d^2*(c + d*x)^(5/2))/(315*(b*c -
 a*d)^3*(a + b*x)^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}} dx &= -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}}-\frac{(4d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}} dx}{9(bc-ad)}\\
&= -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{8d(c+dx)^{5/2}}{63(bc-ad)^2(a+bx)^{7/2}}+\frac{\left(8d^2\right)\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}} dx}{63(bc-ad)^2}\\
&= -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{8d(c+dx)^{5/2}}{63(bc-ad)^2(a+bx)^{7/2}}-\frac{16d^2(c+dx)^{5/2}}{315(bc-ad)^3(a+bx)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0572788, size = 77, normalized size = 0.76

$$-\frac{2(c+dx)^{5/2}\left(63a^2d^2+18abd(2dx-5c)+b^2\left(35c^2-20cdx+8d^2x^2\right)\right)}{315(a+bx)^{9/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*(63*a^2*d^2 + 18*a*b*d*(-5*c + 2*d*x) + b^2*(35*c^2 - 2
0*c*d*x + 8*d^2*x^2)))/(315*(b*c - a*d)^3*(a + b*x)^(9/2))
```

Maple [A] time = 0.006, size = 105, normalized size = 1.

$$\frac{16\,b^2d^2x^2+72\,abd^2x-40\,b^2cdx+126\,a^2d^2-180\,abcd+70\,b^2c^2}{315\,a^3d^3-945\,a^2bcd^2+945\,ab^2c^2d-315\,b^3c^3}\,(dx+c)^{\frac{5}{2}}\,(bx+a)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(11/2),x)
```

```
[Out] 2/315*(d*x+c)^(5/2)*(8*b^2*d^2*x^2+36*a*b*d^2*x-20*b^2*c*d*x+63*a^2*d^2-90*
a*b*c*d+35*b^2*c^2)/(b*x+a)^(9/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*
c^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 48.1185, size = 860, normalized size = 8.51

$$-\frac{2\left(8\,b^2d^4x^4+35\,b^2c^4-90\,abc^3d+63\,a^2c^2d^2-4\left(b^2cd^3-9\right.\right.}{315\left(a^5b^3c^3-3\,a^6b^2c^2d+3\,a^7bcd^2-a^8d^3+\left(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3\right)x^5+5\left(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] -2/315*(8*b^2*d^4*x^4 + 35*b^2*c^4 - 90*a*b*c^3*d + 63*a^2*c^2*d^2 - 4*(b^2
*c*d^3 - 9*a*b*d^4)*x^3 + 3*(b^2*c^2*d^2 - 6*a*b*c*d^3 + 21*a^2*d^4)*x^2 +
2*(25*b^2*c^3*d - 72*a*b*c^2*d^2 + 63*a^2*c*d^3)*x)*sqrt(b*x + a)*sqrt(d*x
+ c)/(a^5*b^3*c^3 - 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3 + (b^8*c^3 -
3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^5 + 5*(a*b^7*c^3 - 3*a^2*b
```

```
^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^4 + 10*(a^2*b^6*c^3 - 3*a^3*b^5
*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^3 + 10*(a^3*b^5*c^3 - 3*a^4*b^4*c
^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x^2 + 5*(a^4*b^4*c^3 - 3*a^5*b^3*c^2*
d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.7454, size = 1882, normalized size = 18.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="giac")
```

```
[Out] -32/315*(sqrt(b*d)*b^12*c^6*d^4*abs(b) - 6*sqrt(b*d)*a*b^11*c^5*d^5*abs(b)
+ 15*sqrt(b*d)*a^2*b^10*c^4*d^6*abs(b) - 20*sqrt(b*d)*a^3*b^9*c^3*d^7*abs(b
) + 15*sqrt(b*d)*a^4*b^8*c^2*d^8*abs(b) - 6*sqrt(b*d)*a^5*b^7*c*d^9*abs(b)
+ sqrt(b*d)*a^6*b^6*d^10*abs(b) - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^10*c^5*d^4*abs(b) + 45*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^9*c^4*d^
5*abs(b) - 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a^2*b^8*c^3*d^6*abs(b) + 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^7*c^2*d^7*abs(b) - 45*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^
4*b^6*c*d^8*abs(b) + 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2*a^5*b^5*d^9*abs(b) + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^8*c^4*d^4*abs(b) - 144*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
a*b^7*c^3*d^5*abs(b) + 216*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^4*a^2*b^6*c^2*d^6*abs(b) - 144*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^5*c*d^7*abs
(b) + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^4*a^4*b^4*d^8*abs(b) + 126*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^6*c^3*d^4*abs(b) - 378*sqrt(b*d)*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^5*c^2*d^5
*abs(b) + 378*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^6*a^2*b^4*c*d^6*abs(b) - 126*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^3*d^7*abs(b) + 441*sqrt(b*
d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^4*c^
2*d^4*abs(b) - 882*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^8*a*b^3*c*d^5*abs(b) + 441*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^2*d^6*abs(b) + 315*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^
2*c*d^4*abs(b) - 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d))^10*a*b*d^5*abs(b) + 210*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
```

```
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*d^4*abs(b))/((b^2*c - a*b*d -
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^9*b)
```

3.1480 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{13/2}} dx$

Optimal. Leaf size=136

$$\frac{32d^3(c+dx)^{5/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{5/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{4d(c+dx)^{5/2}}{33(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{11(a+bx)^{11/2}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(5/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (4*d*(c + d*x)^(5/
2))/(33*(b*c - a*d)^2*(a + b*x)^(9/2)) - (16*d^2*(c + d*x)^(5/2))/(231*(b*c
 - a*d)^3*(a + b*x)^(7/2)) + (32*d^3*(c + d*x)^(5/2))/(1155*(b*c - a*d)^4*(
a + b*x)^(5/2))
```

Rubi [A] time = 0.0283103, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32d^3(c+dx)^{5/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{5/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{4d(c+dx)^{5/2}}{33(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(13/2),x]
```


```
[Out] (-2*(c + d*x)^(5/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (4*d*(c + d*x)^(5/
2))/(33*(b*c - a*d)^2*(a + b*x)^(9/2)) - (16*d^2*(c + d*x)^(5/2))/(231*(b*c
 - a*d)^3*(a + b*x)^(7/2)) + (32*d^3*(c + d*x)^(5/2))/(1155*(b*c - a*d)^4*(
a + b*x)^(5/2))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{13/2}}dx = -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} - \frac{(6d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}}dx}{11(bc-ad)}$$
$$= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}} + \frac{\left(8d^2\right)\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}dx}{33(bc-ad)^2}$$
$$= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}} - \frac{16d^2(c+dx)^{5/2}}{231(bc-ad)^3(a+bx)^{7/2}} - \frac{\left(16d^3\right)\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}dx}{231(bc-ad)^3}$$
$$= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}} - \frac{16d^2(c+dx)^{5/2}}{231(bc-ad)^3(a+bx)^{7/2}} + \frac{32d^3(c+dx)^{5/2}}{1155(bc-ad)^4(a+}$$

Mathematica [A] time = 0.0634928, size = 118, normalized size = 0.87

$$\frac{2(c+dx)^{5/2}\left(99a^2bd^2(2dx-5c)+231a^3d^3+11ab^2d\left(35c^2-20cdx+8d^2x^2\right)+b^3\left(70c^2dx-105c^3-40cd^2x^2+16d^3x^3\right)\right)}{1155(a+bx)^{11/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(13/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(231*a^3*d^3 + 99*a^2*b*d^2*(-5*c + 2*d*x) + 11*a*b^2*d*
(35*c^2 - 20*c*d*x + 8*d^2*x^2) + b^3*(-105*c^3 + 70*c^2*d*x - 40*c*d^2*x^2
 + 16*d^3*x^3)))/(1155*(b*c - a*d)^4*(a + b*x)^(11/2))
```

Maple [A] time = 0.009, size = 171, normalized size = 1.3

$$\frac{32\,b^3d^3x^3+176\,ab^2d^3x^2-80\,b^3cd^2x^2+396\,a^2bd^3x-440\,ab^2cd^2x+140\,b^3c^2dx+462\,a^3d^3-990\,a^2bcd^2+770\,ab^2c^2d-}{1155\,d^4a^4-4620\,bd^3ca^3+6930\,b^2d^2c^2a^2-4620\,b^3dc^3a+1155\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(13/2),x)
```

```
[Out] 2/1155*(d*x+c)^(5/2)*(16*b^3*d^3*x^3+88*a*b^2*d^3*x^2-40*b^3*c*d^2*x^2+198*
a^2*b*d^3*x-220*a*b^2*c*d^2*x+70*b^3*c^2*d*x+231*a^3*d^3-495*a^2*b*c*d^2+38
5*a*b^2*c^2*d-105*b^3*c^3)/(b*x+a)^(11/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*
c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 100.004, size = 1328, normalized size = 9.76

$$\frac{2\left(16\,b^3d^5x^5 - 105\,b^3c^5 + 385\,ab^2c\right.}{1155\left(a^6b^4c^4 - 4\,a^7b^3c^3d + 6\,a^8b^2c^2d^2 - 4\,a^9bcd^3 + a^{10}d^4 + \left(b^{10}c^4 - 4\,ab^9c^3d + 6\,a^2b^8c^2d^2 - 4\,a^3b^7cd^3 + a^4b^6d^4\right)x^6 + 6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*b^3*d^5*x^5 - 105*b^3*c^5 + 385*a*b^2*c^4*d - 495*a^2*b*c^3*d^2
+ 231*a^3*c^2*d^3 - 8*(b^3*c*d^4 - 11*a*b^2*d^5)*x^4 + 2*(3*b^3*c^2*d^3 - 2
2*a*b^2*c*d^4 + 99*a^2*b*d^5)*x^3 - (5*b^3*c^3*d^2 - 33*a*b^2*c^2*d^3 + 99*
a^2*b*c*d^4 - 231*a^3*d^5)*x^2 - 2*(70*b^3*c^4*d - 275*a*b^2*c^3*d^2 + 396*
a^2*b*c^2*d^3 - 231*a^3*c*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b^4*c^4
- 4*a^7*b^3*c^3*d + 6*a^8*b^2*c^2*d^2 - 4*a^9*b*c*d^3 + a^10*d^4 + (b^10*c^
4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*x^6
+ 6*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^
5*b^5*d^4)*x^5 + 15*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*
a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^4 + 20*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a^
5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x^3 + 15*(a^4*b^6*c^4 - 4*a^
5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*x^2 + 6*(a
^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 + a^9*b*
d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(13/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.09559, size = 2461, normalized size = 18.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="giac")
```

```
[Out] 64/1155*(sqrt(b*d)*b^14*c^7*d^5*abs(b) - 7*sqrt(b*d)*a*b^13*c^6*d^6*abs(b)
+ 21*sqrt(b*d)*a^2*b^12*c^5*d^7*abs(b) - 35*sqrt(b*d)*a^3*b^11*c^4*d^8*abs(
b) + 35*sqrt(b*d)*a^4*b^10*c^3*d^9*abs(b) - 21*sqrt(b*d)*a^5*b^9*c^2*d^10*a
bs(b) + 7*sqrt(b*d)*a^6*b^8*c*d^11*abs(b) - sqrt(b*d)*a^7*b^7*d^12*abs(b) -
 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^2*b^12*c^6*d^5*abs(b) + 66*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*a*b^11*c^5*d^6*abs(b) - 165*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^10*c^4*d^
7*abs(b) + 220*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^2*a^3*b^9*c^3*d^8*abs(b) - 165*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^8*c^2*d^9*abs(b) + 66*s
```

```
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*
a^5*b^7*c*d^10*abs(b) - 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^2*a^6*b^6*d^11*abs(b) + 55*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^10*c^5*d^5*abs(b) -
 275*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*a*b^9*c^4*d^6*abs(b) + 550*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^8*c^3*d^7*abs(b) - 550*sqrt(b*d)*(s
qrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^7*c^2
*d^8*abs(b) + 275*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
a)*b*d - a*b*d))^4*a^4*b^6*c*d^9*abs(b) - 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^5*d^10*abs(b) - 165*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^
8*c^4*d^5*abs(b) + 660*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a*b^7*c^3*d^6*abs(b) - 990*sqrt(b*d)*(sqrt(b*d)*sqr
t(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^6*c^2*d^7*abs(b)
+ 660*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^6*a^3*b^5*c*d^8*abs(b) - 165*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^4*b^4*d^9*abs(b) - 825*sqrt(b*d)*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^6*c^3*d^5*ab
s(b) + 2475*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d))^8*a*b^5*c^2*d^6*abs(b) - 2475*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^4*c*d^7*abs(b) + 825*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b
^3*d^8*abs(b) - 2541*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d))^10*b^4*c^2*d^5*abs(b) + 5082*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^3*c*d^6*abs(b) - 254
1*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
^10*a^2*b^2*d^7*abs(b) - 2079*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^12*b^2*c*d^5*abs(b) + 2079*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b*d^6*abs(b) -
1155*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^14*d^5*abs(b))/(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2)^11
```

3.1481 $\int (a+bx)^{5/2}(c+dx)^{5/2}\,dx$

Optimal. Leaf size=262

$$\frac{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^5}{512b^3d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^4}{768b^3d^2}-\frac{5(bc-ad)^6\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{512b^{7/2}d^{7/2}}+\frac{(a+bx)^{5/2}\sqrt{c+dx}(b}{192b^3d}$$

```
[Out] (5*(b*c - a*d)^5*Sqrt[a + b*x]*Sqrt[c + d*x])/(512*b^3*d^3) - (5*(b*c - a*d
)^4*(a + b*x)^(3/2)*Sqrt[c + d*x])/(768*b^3*d^2) + ((b*c - a*d)^3*(a + b*x)
^(5/2)*Sqrt[c + d*x])/(192*b^3*d) + ((b*c - a*d)^2*(a + b*x)^(7/2)*Sqrt[c +
 d*x])/(32*b^3) + ((b*c - a*d)*(a + b*x)^(7/2)*(c + d*x)^(3/2))/(12*b^2) +
((a + b*x)^(7/2)*(c + d*x)^(5/2))/(6*b) - (5*(b*c - a*d)^6*ArcTanh[(Sqrt[d]
*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(512*b^(7/2)*d^(7/2))
```

Rubi [A] time = 0.1493, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$\frac{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^5}{512b^3d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^4}{768b^3d^2}-\frac{5(bc-ad)^6\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{512b^{7/2}d^{7/2}}+\frac{(a+bx)^{5/2}\sqrt{c+dx}(b}{192b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(c + d*x)^(5/2),x]
```

```
[Out] (5*(b*c - a*d)^5*Sqrt[a + b*x]*Sqrt[c + d*x])/(512*b^3*d^3) - (5*(b*c - a*d
)^4*(a + b*x)^(3/2)*Sqrt[c + d*x])/(768*b^3*d^2) + ((b*c - a*d)^3*(a + b*x)
^(5/2)*Sqrt[c + d*x])/(192*b^3*d) + ((b*c - a*d)^2*(a + b*x)^(7/2)*Sqrt[c +
 d*x])/(32*b^3) + ((b*c - a*d)*(a + b*x)^(7/2)*(c + d*x)^(3/2))/(12*b^2) +
((a + b*x)^(7/2)*(c + d*x)^(5/2))/(6*b) - (5*(b*c - a*d)^6*ArcTanh[(Sqrt[d]
*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(512*b^(7/2)*d^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^{5/2}(c+dx)^{5/2}\,dx = \frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b}+\frac{(5(bc-ad))\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx}{12b}$$
$$=\frac{(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}}{12b^2}+\frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b}+\frac{(bc-ad)^2\int (a+bx)^{5/2}\sqrt{c+dx}\,dx}{8b^2}$$
$$=\frac{(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}}{32b^3}+\frac{(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}}{12b^2}+\frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b}+\ldots$$
$$=\frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}{192b^3d}+\frac{(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}}{32b^3}+\frac{(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}}{12b^2}\ldots$$
$$=-\frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2}+\frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}{192b^3d}+\frac{(bc-ad)^2(a+bx)^{7/2}\sqrt{c+\ldots}}{32b^3}$$
$$=\frac{5(bc-ad)^5\sqrt{a+bx}\sqrt{c+dx}}{512b^3d^3}-\frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2}+\frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+d\ldots}}{192b^3d}$$
$$=\frac{5(bc-ad)^5\sqrt{a+bx}\sqrt{c+dx}}{512b^3d^3}-\frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2}+\frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+d\ldots}}{192b^3d}$$
$$=\frac{5(bc-ad)^5\sqrt{a+bx}\sqrt{c+dx}}{512b^3d^3}-\frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2}+\frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+d\ldots}}{192b^3d}$$
$$=\frac{5(bc-ad)^5\sqrt{a+bx}\sqrt{c+dx}}{512b^3d^3}-\frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2}+\frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+d\ldots}}{192b^3d}$$

Mathematica [A] time = 2.5292, size = 209, normalized size = 0.8

$$\frac{(a+bx)^{7/2}\sqrt{c+dx}\left(\frac{15(bc-ad)^5}{d^3(a+bx)^3}-\frac{10(bc-ad)^4}{d^2(a+bx)^2}-\frac{15(bc-ad)^{11/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{7/2}(a+bx)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}}+\frac{8(bc-ad)^3}{d(a+bx)}+128b(c+dx)(bc-ad)+48(bc-ad)^2+\ldots\right)}{1536b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(c + d*x)^(5/2),x]
```

```
[Out] ((a + b*x)^(7/2)*Sqrt[c + d*x]*(48*(b*c - a*d)^2 + (15*(b*c - a*d)^5)/(d^3*
(a + b*x)^3) - (10*(b*c - a*d)^4)/(d^2*(a + b*x)^2) + (8*(b*c - a*d)^3)/(d*
(a + b*x)) + 128*b*(b*c - a*d)*(c + d*x) + 256*b^2*(c + d*x)^2 - (15*(b*c -
 a*d)^(11/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(7/2)*(a
+ b*x)^(7/2)*Sqrt[(b*(c + d*x))/(b*c - a*d)])))/(1536*b^3)
```

Maple [B] time = 0.006, size = 1089, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(5/2),x)
```

```
[Out] 5/512*d^2/b^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^5-5/512/d^3*(d*x+c)^(1/2)*(b*x+
a)^(1/2)*c^5*b^2-1/64/d*(d*x+c)^(5/2)*(b*x+a)^(1/2)*a^2*c-1/192/d^3*(d*x+c)
^(5/2)*(b*x+a)^(1/2)*c^3*b^2-5/768*d/b^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^4-5/
768/d^3*(d*x+c)^(3/2)*(b*x+a)^(1/2)*c^4*b^2-1/12/d^2*(b*x+a)^(3/2)*(d*x+c)^
(7/2)*b*c-25/256/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^2*c^3-5/128/d*(d*x+c)^(3/2
)*(b*x+a)^(1/2)*a^2*c^2+1/32/d^3*(b*x+a)^(1/2)*(d*x+c)^(7/2)*b^2*c^2+1/32/d
*(b*x+a)^(1/2)*(d*x+c)^(7/2)*a^2+1/192/b*(d*x+c)^(5/2)*(b*x+a)^(1/2)*a^3+1/
12/d*(b*x+a)^(3/2)*(d*x+c)^(7/2)*a+25/256/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3
*c^2+5/192/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^3*c+25/256*((b*x+a)*(d*x+c))^(1/
2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^
2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^3*c^3+1/6/d*(b*x+a)^(5/2)*(d*x+c)
^(7/2)-25/512*d/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^4*c-1/16/d^2*(b*x+a)^(1/2
)*(d*x+c)^(7/2)*a*b*c+1/64/d^2*(d*x+c)^(5/2)*(b*x+a)^(1/2)*a*c^2*b+5/192/d^
2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*c^3*b+25/512/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2
)*a*c^4*b-5/1024/d^3*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln
((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)
^(1/2)*c^6*b^3-5/1024*d^3/b^3*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)
^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/
2))/(b*d)^(1/2)*a^6+15/512/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1
/2))/(b*d)^(1/2)*a*c^5*b^2-75/1024*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x
+a*c)^(1/2))/(b*d)^(1/2)*a^4*c^2-75/1024/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^
(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*
c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^2*c^4*b+15/512*d^2/b^2*((b*x+a)*(d*x+c))^(1/
2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^
2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^5*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.50246, size = 1924, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6144*(15*(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*
d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(b*d)*log(8*b^2*d^2
*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sq
rt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(256*b^6*d^6*x^5 +
```

```
15*b^6*c^5*d - 85*a*b^5*c^4*d^2 + 198*a^2*b^4*c^3*d^3 + 198*a^3*b^3*c^2*d^
4 - 85*a^4*b^2*c*d^5 + 15*a^5*b*d^6 + 640*(b^6*c*d^5 + a*b^5*d^6)*x^4 + 16*
(27*b^6*c^2*d^4 + 106*a*b^5*c*d^5 + 27*a^2*b^4*d^6)*x^3 + 8*(b^6*c^3*d^3 +
159*a*b^5*c^2*d^4 + 159*a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^2 - 2*(5*b^6*c^4*d^2
 - 28*a*b^5*c^3*d^3 - 594*a^2*b^4*c^2*d^4 - 28*a^3*b^3*c*d^5 + 5*a^4*b^2*d^
6)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^4), 1/3072*(15*(b^6*c^6 - 6*a*b^5
*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a
^5*b*c*d^5 + a^6*d^6)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d
)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*
x)) + 2*(256*b^6*d^6*x^5 + 15*b^6*c^5*d - 85*a*b^5*c^4*d^2 + 198*a^2*b^4*c^
3*d^3 + 198*a^3*b^3*c^2*d^4 - 85*a^4*b^2*c*d^5 + 15*a^5*b*d^6 + 640*(b^6*c*
d^5 + a*b^5*d^6)*x^4 + 16*(27*b^6*c^2*d^4 + 106*a*b^5*c*d^5 + 27*a^2*b^4*d^
6)*x^3 + 8*(b^6*c^3*d^3 + 159*a*b^5*c^2*d^4 + 159*a^2*b^4*c*d^5 + a^3*b^3*d
^6)*x^2 - 2*(5*b^6*c^4*d^2 - 28*a*b^5*c^3*d^3 - 594*a^2*b^4*c^2*d^4 - 28*a^
3*b^3*c*d^5 + 5*a^4*b^2*d^6)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.65402, size = 3542, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/7680*(40*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(
6*(b*x + a)/b^2 + (b^7*c*d^5 - 17*a*b^6*d^6)/(b^8*d^6)) - (5*b^8*c^2*d^4 +
6*a*b^7*c*d^5 - 59*a^2*b^6*d^6)/(b^8*d^6)) + 3*(5*b^9*c^3*d^3 + a*b^8*c^2*d
^4 - a^2*b^7*c*d^5 - 5*a^3*b^6*d^6)/(b^8*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4
 - 4*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + 5*a^4*d^4)*log(abs(-
sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*
b*d^3))*c^2*abs(b) + 80*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*
(2*(b*x + a)/(b^4*d^2) + (b*c*d - a*d^2)/(b^4*d^4)) + (b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d -
a*b*d)))/(sqrt(b*d)*b^3*d^3))*a^2*c^2*abs(b)/b^2 + 8*(sqrt(b^2*c + (b*x + a
)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^3 + (b^13*c*d^7
- 31*a*b^12*d^8)/(b^15*d^8)) - (7*b^14*c^2*d^6 + 16*a*b^13*c*d^7 - 263*a^2*
b^12*d^8)/(b^15*d^8)) + 5*(7*b^15*c^3*d^5 + 9*a*b^14*c^2*d^6 + 9*a^2*b^13*c
*d^7 - 121*a^3*b^12*d^8)/(b^15*d^8))*(b*x + a) - 15*(7*b^16*c^4*d^4 + 2*a*b
^15*c^3*d^5 - 2*a^3*b^13*c*d^7 - 7*a^4*b^12*d^8)/(b^15*d^8))*sqrt(b*x + a)
- 15*(7*b^5*c^5 - 5*a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3 - 5
*a^4*b*c*d^4 + 7*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^4))*c*d*abs(b) + 160*(sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^2 + (b^7*c
*d^5 - 17*a*b^6*d^6)/(b^8*d^6)) - (5*b^8*c^2*d^4 + 6*a*b^7*c*d^5 - 59*a^2*b
```

```
^6*d^6)/(b^8*d^6)) + 3*(5*b^9*c^3*d^3 + a*b^8*c^2*d^4 - a^2*b^7*c*d^5 - 5*a
^3*b^6*d^6)/(b^8*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 - 4*a*b^3*c^3*d - 2*a^2
*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + 5*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a)
+ sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^3))*a*c*d*abs(b)/b +
 (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(2*(b*x + a)*(8*(b*x + a)*(10*(
b*x + a)/b^4 + (b^21*c*d^9 - 49*a*b^20*d^10)/(b^24*d^10)) - 3*(3*b^22*c^2*d
^8 + 10*a*b^21*c*d^9 - 253*a^2*b^20*d^10)/(b^24*d^10)) + (21*b^23*c^3*d^7 +
 49*a*b^22*c^2*d^8 + 79*a^2*b^21*c*d^9 - 1429*a^3*b^20*d^10)/(b^24*d^10))*(
b*x + a) - 5*(21*b^24*c^4*d^6 + 28*a*b^23*c^3*d^7 + 30*a^2*b^22*c^2*d^8 + 2
8*a^3*b^21*c*d^9 - 491*a^4*b^20*d^10)/(b^24*d^10))*(b*x + a) + 15*(21*b^25*
c^5*d^5 + 7*a*b^24*c^4*d^6 + 2*a^2*b^23*c^3*d^7 - 2*a^3*b^22*c^2*d^8 - 7*a^
4*b^21*c*d^9 - 21*a^5*b^20*d^10)/(b^24*d^10))*sqrt(b*x + a) + 15*(21*b^6*c^
6 - 14*a*b^5*c^5*d - 5*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 - 5*a^4*b^2*c^2*
d^4 - 14*a^5*b*c*d^5 + 21*a^6*d^6)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(
b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^5))*d^2*abs(b) + 40*(sqrt
(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^2
+ (b^7*c*d^5 - 17*a*b^6*d^6)/(b^8*d^6)) - (5*b^8*c^2*d^4 + 6*a*b^7*c*d^5 -
59*a^2*b^6*d^6)/(b^8*d^6)) + 3*(5*b^9*c^3*d^3 + a*b^8*c^2*d^4 - a^2*b^7*c*d
^5 - 5*a^3*b^6*d^6)/(b^8*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 - 4*a*b^3*c^3*d
 - 2*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + 5*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b
*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^3))*a^2*d^2*
abs(b)/b^2 + 8*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x
 + a)*(8*(b*x + a)/b^3 + (b^13*c*d^7 - 31*a*b^12*d^8)/(b^15*d^8)) - (7*b^14
*c^2*d^6 + 16*a*b^13*c*d^7 - 263*a^2*b^12*d^8)/(b^15*d^8)) + 5*(7*b^15*c^3*
d^5 + 9*a*b^14*c^2*d^6 + 9*a^2*b^13*c*d^7 - 121*a^3*b^12*d^8)/(b^15*d^8))*(
b*x + a) - 15*(7*b^16*c^4*d^4 + 2*a*b^15*c^3*d^5 - 2*a^3*b^13*c*d^7 - 7*a^4
*b^12*d^8)/(b^15*d^8))*sqrt(b*x + a) - 15*(7*b^5*c^5 - 5*a*b^4*c^4*d - 2*a^
2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 + 7*a^5*d^5)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2
*d^4))*a*d^2*abs(b)/b + 8*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a
)*(2*(b*x + a)*(4*(b*x + a)/(b^6*d^2) + (b*c*d^3 - 7*a*d^4)/(b^6*d^6)) - 3*
(b^2*c^2*d^2 - a^2*d^4)/(b^6*d^6)) - 3*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2
 + a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d)))/(sqrt(b*d)*b^5*d^4))*a*c^2*abs(b)/b^2 + 8*(sqrt(b^2*c + (b*x + a)
*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/(b^6*d^2) + (b*c*d^3
- 7*a*d^4)/(b^6*d^6)) - 3*(b^2*c^2*d^2 - a^2*d^4)/(b^6*d^6)) - 3*(b^3*c^3 -
 a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sq
rt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^5*d^4))*a^2*c*d*abs(b)/b^3
)/b
```

3.1482 $\int (a+bx)^{3/2}(c+dx)^{5/2}\,dx$

Optimal. Leaf size=224

$$-\frac{3\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^4}{128b^3d^2}+\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{7/2}d^{5/2}}+\frac{(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^3}{64b^3d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}(bc-$$

```
[Out] (-3*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b^3*d^2) + ((b*c - a*d)
^3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(64*b^3*d) + ((b*c - a*d)^2*(a + b*x)^(5/
2)*Sqrt[c + d*x])/(16*b^3) + ((b*c - a*d)*(a + b*x)^(5/2)*(c + d*x)^(3/2))/
(8*b^2) + ((a + b*x)^(5/2)*(c + d*x)^(5/2))/(5*b) + (3*(b*c - a*d)^5*ArcTan
h[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(7/2)*d^(5/2))
```

Rubi [A] time = 0.119448, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$-\frac{3\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^4}{128b^3d^2}+\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{7/2}d^{5/2}}+\frac{(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^3}{64b^3d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}(bc-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(5/2),x]
```

```
[Out] (-3*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b^3*d^2) + ((b*c - a*d)
^3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(64*b^3*d) + ((b*c - a*d)^2*(a + b*x)^(5/
2)*Sqrt[c + d*x])/(16*b^3) + ((b*c - a*d)*(a + b*x)^(5/2)*(c + d*x)^(3/2))/
(8*b^2) + ((a + b*x)^(5/2)*(c + d*x)^(5/2))/(5*b) + (3*(b*c - a*d)^5*ArcTan
h[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(7/2)*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{5/2}\,dx = \frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b} + \frac{(bc-ad)\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx}{2b}$$
$$= \frac{(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b} + \frac{\left(3(bc-ad)^2\right)\int (a+bx)^{3/2}\sqrt{c+}}{16b^2}$$
$$= \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{16b^3} + \frac{(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b} +$$
$$= \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{16b^3} + \frac{(bc-ad)(a+bx)^{5/2}(c+}{8b^2}$$
$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c}}{16b^3}$$
$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c}}{16b^3}$$
$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c}}{16b^3}$$
$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c}}{16b^3}$$

Mathematica [A] time = 1.57511, size = 187, normalized size = 0.83

$$\frac{(a+bx)^{5/2}\sqrt{c+dx}\left(-\frac{15(bc-ad)^4}{d^2(a+bx)^2} + \frac{15(bc-ad)^{9/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{5/2}(a+bx)^{5/2}\sqrt{\frac{b(c+dx)}{bc-ad}}} + \frac{10(bc-ad)^3}{d(a+bx)} + 80b(c+dx)(bc-ad) + 40(bc-ad)^2 + 128b^2(c\right.}{640b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/2),x]
```

```
[Out] ((a + b*x)^(5/2)*Sqrt[c + d*x]*(40*(b*c - a*d)^2 - (15*(b*c - a*d)^4)/(d^2*
(a + b*x)^2) + (10*(b*c - a*d)^3)/(d*(a + b*x)) + 80*b*(b*c - a*d)*(c + d*x
) + 128*b^2*(c + d*x)^2 + (15*(b*c - a*d)^(9/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b
*x])/Sqrt[b*c - a*d]])/(d^(5/2)*(a + b*x)^(5/2)*Sqrt[(b*(c + d*x))/(b*c - a
*d)])))/(640*b^3)
```

Maple [B] time = 0.005, size = 848, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(5/2),x)
```

```
[Out] 1/5/d*(b*x+a)^(3/2)*(d*x+c)^(7/2)-15/128*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c
)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+
b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^3*c^2+15/128*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d
+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^2*c^3+15/256*d^2/b^2*((b*x+a)*(d*x+c))^(1
/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x
^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^4*c-3/32*d/b^2*(d*x+c)^(1/2)*(b*
x+a)^(1/2)*a^3*c+9/64/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^2*c^2+3/64/b*(d*x+c)^
(3/2)*(b*x+a)^(1/2)*a^2*c+1/64/d^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*c^3*b+3/128*
d^2/b^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^4-3/32/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*
a*c^3+3/128/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^4*b+3/40/d*(b*x+a)^(1/2)*(d*x
+c)^(7/2)*a-3/40/d^2*(b*x+a)^(1/2)*(d*x+c)^(7/2)*b*c+1/80/b*(d*x+c)^(5/2)*(
b*x+a)^(1/2)*a^2-1/40/d*(d*x+c)^(5/2)*(b*x+a)^(1/2)*a*c+1/80/d^2*(d*x+c)^(5
/2)*(b*x+a)^(1/2)*c^2*b-1/64*d/b^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^3-3/64/d*(
d*x+c)^(3/2)*(b*x+a)^(1/2)*a*c^2-15/256/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(
1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c
)*x+a*c)^(1/2))/(b*d)^(1/2)*a*c^4*b+3/256/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d
+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*c^5*b^2-3/256*d^3/b^3*((b*x+a)*(d*x+c))^(1/
2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^
2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.20935, size = 1548, normalized size = 6.91

$$\left[-\frac{15\left(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5\right)\sqrt{bd}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 - 4\,(2\,bdx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2
*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a
*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x
 + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(128*b^5*d^5*x^4 - 15*b^5*c^4*d + 70*a
*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 - 70*a^3*b^2*c*d^4 + 15*a^4*b*d^5 + 16*(
21*b^5*c*d^4 + 11*a*b^4*d^5)*x^3 + 8*(31*b^5*c^2*d^3 + 64*a*b^4*c*d^4 + a^2
*b^3*d^5)*x^2 + 2*(5*b^5*c^3*d^2 + 233*a*b^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 5
*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^3), -1/1280*(15*(b^5*c
^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^
4 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*
x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(
128*b^5*d^5*x^4 - 15*b^5*c^4*d + 70*a*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 - 7
```

```
0*a^3*b^2*c*d^4 + 15*a^4*b*d^5 + 16*(21*b^5*c*d^4 + 11*a*b^4*d^5)*x^3 + 8*(
31*b^5*c^2*d^3 + 64*a*b^4*c*d^4 + a^2*b^3*d^5)*x^2 + 2*(5*b^5*c^3*d^2 + 233
*a*b^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 5*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*
x + c))/(b^4*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^{\frac{3}{2}} (c+dx)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(5/2), x)
```

Giac [B] time = 1.46129, size = 2002, normalized size = 8.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1920*(20*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)/
(b^4*d^2) + (b*c*d - a*d^2)/(b^4*d^4)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqr
t(b*d)*b^3*d^3))*a*c^2*abs(b)/b^2 + 20*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^2 + (b^7*c*d^5 - 17*a*b^6*d^6)/(b
^8*d^6)) - (5*b^8*c^2*d^4 + 6*a*b^7*c*d^5 - 59*a^2*b^6*d^6)/(b^8*d^6)) + 3*
(5*b^9*c^3*d^3 + a*b^8*c^2*d^4 - a^2*b^7*c*d^5 - 5*a^3*b^6*d^6)/(b^8*d^6))*
sqrt(b*x + a) + 3*(5*b^4*c^4 - 4*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + 5*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a
)*b*d - a*b*d)))/(sqrt(b*d)*b*d^3))*c*d*abs(b)/b + 10*(sqrt(b^2*c + (b*x +
a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^2 + (b^7*c*d^5 - 1
7*a*b^6*d^6)/(b^8*d^6)) - (5*b^8*c^2*d^4 + 6*a*b^7*c*d^5 - 59*a^2*b^6*d^6)/
(b^8*d^6)) + 3*(5*b^9*c^3*d^3 + a*b^8*c^2*d^4 - a^2*b^7*c*d^5 - 5*a^3*b^6*d
^6)/(b^8*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 - 4*a*b^3*c^3*d - 2*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + 5*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b
^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^3))*a*d^2*abs(b)/b^2 + (sqrt
(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b
^3 + (b^13*c*d^7 - 31*a*b^12*d^8)/(b^15*d^8)) - (7*b^14*c^2*d^6 + 16*a*b^13
*c*d^7 - 263*a^2*b^12*d^8)/(b^15*d^8)) + 5*(7*b^15*c^3*d^5 + 9*a*b^14*c^2*d
^6 + 9*a^2*b^13*c*d^7 - 121*a^3*b^12*d^8)/(b^15*d^8))*(b*x + a) - 15*(7*b^1
6*c^4*d^4 + 2*a*b^15*c^3*d^5 - 2*a^3*b^13*c*d^7 - 7*a^4*b^12*d^8)/(b^15*d^8
))*sqrt(b*x + a) - 15*(7*b^5*c^5 - 5*a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 - 2*a^
3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 + 7*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x + a)
 + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^4))*d^2*abs(b)/b
+ (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x +
 a)/(b^6*d^2) + (b*c*d^3 - 7*a*d^4)/(b^6*d^6)) - 3*(b^2*c^2*d^2 - a^2*d^4)/
(b^6*d^6)) - 3*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^5
*d^4))*c^2*abs(b)/b^2 + 2*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a
)*(2*(b*x + a)*(4*(b*x + a)/(b^6*d^2) + (b*c*d^3 - 7*a*d^4)/(b^6*d^6)) - 3*
(b^2*c^2*d^2 - a^2*d^4)/(b^6*d^6)) - 3*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2
```

```
+ a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d -
a*b*d)))/(sqrt(b*d)*b^5*d^4))*a*c*d*abs(b)/b^3)/b
```

3.1483 $\int \sqrt{a+bx}(c+dx)^{5/2}\,dx$

Optimal. Leaf size=186

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{7/2}d^{3/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^3}{64b^3d}+\frac{5(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^2}{32b^3}+\frac{5(a+bx)^{3/2}(c+dx)}{24b^2}$$

```
[Out] (5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b^3*d) + (5*(b*c - a*d)^2
*(a + b*x)^(3/2)*Sqrt[c + d*x])/(32*b^3) + (5*(b*c - a*d)*(a + b*x)^(3/2)*(
c + d*x)^(3/2))/(24*b^2) + ((a + b*x)^(3/2)*(c + d*x)^(5/2))/(4*b) - (5*(b*
c - a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^
(7/2)*d^(3/2))
```

Rubi [A] time = 0.0863186, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{7/2}d^{3/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^3}{64b^3d}+\frac{5(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^2}{32b^3}+\frac{5(a+bx)^{3/2}(c+dx)}{24b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(5/2),x]
```

```
[Out] (5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b^3*d) + (5*(b*c - a*d)^2
*(a + b*x)^(3/2)*Sqrt[c + d*x])/(32*b^3) + (5*(b*c - a*d)*(a + b*x)^(3/2)*(
c + d*x)^(3/2))/(24*b^2) + ((a + b*x)^(3/2)*(c + d*x)^(5/2))/(4*b) - (5*(b*
c - a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^
(7/2)*d^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx}(c+dx)^{5/2}\,dx = \frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b} + \frac{(5(bc-ad))\int \sqrt{a+bx}(c+dx)^{3/2}\,dx}{8b}$$
$$= \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^2} + \frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b} + \frac{\left(5(bc-ad)^2\right)\int \sqrt{a+bx}\sqrt{c+dx}\,dx}{16b^2}$$
$$= \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^2} + \frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b} + \frac{(5}{}$$
$$= \frac{5(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3}}{24b^2}$$
$$= \frac{5(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3}}{24b^2}$$
$$= \frac{5(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3}}{24b^2}$$
$$= \frac{5(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3}}{24b^2}$$

Mathematica [A] time = 0.56046, size = 191, normalized size = 1.03

$$\frac{b\sqrt{d}\sqrt{a+bx}(c+dx)\left(-5a^2bd^2(11c+2dx)+15a^3d^3+ab^2d\left(73c^2+36cdx+8d^2x^2\right)+b^3\left(118c^2dx+15c^3+136cd^2x^2+4\right.\right.}{192b^4d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(5/2),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(15*a^3*d^3 - 5*a^2*b*d^2*(11*c + 2*d*x)
 + a*b^2*d*(73*c^2 + 36*c*d*x + 8*d^2*x^2) + b^3*(15*c^3 + 118*c^2*d*x + 13
6*c*d^2*x^2 + 48*d^3*x^3)) - 15*(b*c - a*d)^(9/2)*Sqrt[(b*(c + d*x))/(b*c -
 a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(192*b^4*d^(3/2)*S
qrt[c + d*x])
```

Maple [B] time = 0.005, size = 641, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(5/2),x)
```

```
[Out] 1/4/d*(b*x+a)^(1/2)*(d*x+c)^(7/2)+1/24/b*(d*x+c)^(5/2)*(b*x+a)^(1/2)*a-1/24
/d*(d*x+c)^(5/2)*(b*x+a)^(1/2)*c-5/96*d/b^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^2
+5/48/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*c-5/96/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*
```

```
c^2+5/64*d^2/b^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3-15/64*d/b^2*(d*x+c)^(1/2)*
(b*x+a)^(1/2)*a^2*c+15/64/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^2-5/64/d*(d*x+c
)^(1/2)*(b*x+a)^(1/2)*c^3-5/128*d^3/b^3*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/
2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*
x+a*c)^(1/2))/(b*d)^(1/2)*a^4+5/32*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^
(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*
c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^3*c-15/64*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c
)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+
b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^2*c^2+5/32*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)
^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b
*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a*c^3-5/128/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)
^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b
*c)*x+a*c)^(1/2))/(b*d)^(1/2)*c^4*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07672, size = 1203, normalized size = 6.47

$$\left[\frac{15 \left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)\sqrt{bd}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 - 4\,(2\,bdx + bc + ad)\sqrt{b}\right.}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*
b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b
*d^2)*x) + 4*(48*b^4*d^4*x^3 + 15*b^4*c^3*d + 73*a*b^3*c^2*d^2 - 55*a^2*b^2
*c*d^3 + 15*a^3*b*d^4 + 8*(17*b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(59*b^4*c^2*d^
2 + 18*a*b^3*c*d^3 - 5*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^
2), 1/384*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
+ a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(48
*b^4*d^4*x^3 + 15*b^4*c^3*d + 73*a*b^3*c^2*d^2 - 55*a^2*b^2*c*d^3 + 15*a^3*
b*d^4 + 8*(17*b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(59*b^4*c^2*d^2 + 18*a*b^3*c*d
^3 - 5*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34565, size = 852, normalized size = 4.58

$$\frac{\frac{10\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(\frac{2(bx+a)}{b^4d^2}+\frac{bcd-ad^2}{b^4d^4}\right)+\frac{(b^2c^2-2abcd+a^2d^2)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}b^3d^3}\right)c^2|b|}{b^2}+\frac{5\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2(bx+a)\left(4(bx+a)\left(6\cdots\right.\right.\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/960*(10*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)/(
b^4*d^2) + (b*c*d - a*d^2)/(b^4*d^4)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt
(b*d)*b^3*d^3))*c^2*abs(b)/b^2 + 5*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*
(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^2 + (b^7*c*d^5 - 17*a*b^6*d^6)/(b^8*d
^6)) - (5*b^8*c^2*d^4 + 6*a*b^7*c*d^5 - 59*a^2*b^6*d^6)/(b^8*d^6)) + 3*(5*b
^9*c^3*d^3 + a*b^8*c^2*d^4 - a^2*b^7*c*d^5 - 5*a^3*b^6*d^6)/(b^8*d^6))*sqrt
(b*x + a) + 3*(5*b^4*c^4 - 4*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + 5*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)))/(sqrt(b*d)*b*d^3))*d^2*abs(b)/b^2 + (sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/(b^6*d^2) + (b*c*d^3 - 7
*a*d^4)/(b^6*d^6)) - 3*(b^2*c^2*d^2 - a^2*d^4)/(b^6*d^6)) - 3*(b^3*c^3 - a*
b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(
b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^5*d^4))*c*d*abs(b)/b^3)/b
```

3.1484 $\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}} dx$

Optimal. Leaf size=148

$$\frac{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8b^3}+\frac{5\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}{12b^2}+\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{7/2}\sqrt{d}}+\frac{\sqrt{a+bx}(c+dx)^{5/2}}{3b}$$

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b^3) + (5*(b*c - a*d)*Sqrt
[a + b*x]*(c + d*x)^(3/2))/(12*b^2) + (Sqrt[a + b*x]*(c + d*x)^(5/2))/(3*b)
 + (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(8*b^(7/2)*Sqrt[d])
```

Rubi [A] time = 0.0663875, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$\frac{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8b^3}+\frac{5\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}{12b^2}+\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{7/2}\sqrt{d}}+\frac{\sqrt{a+bx}(c+dx)^{5/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b^3) + (5*(b*c - a*d)*Sqrt
[a + b*x]*(c + d*x)^(3/2))/(12*b^2) + (Sqrt[a + b*x]*(c + d*x)^(5/2))/(3*b)
 + (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(8*b^(7/2)*Sqrt[d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}(c+dx)^{5/2}}{3b} + \frac{(5(bc-ad))\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx}{6b}$$

$$= \frac{5(bc-ad)\sqrt{a+bx}(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}(c+dx)^{5/2}}{3b} + \frac{\left(5(bc-ad)^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{8b^2}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}(c+dx)^{5/2}}{3b} + \frac{\left(5(bc-ad)^3\right)\int}{16b}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}(c+dx)^{5/2}}{3b} + \frac{\left(5(bc-ad)^3\right)\text{Su}}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}(c+dx)^{5/2}}{3b} + \frac{\left(5(bc-ad)^3\right)\text{Su}}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}(c+dx)^{5/2}}{3b} + \frac{5(bc-ad)^3\,\text{tanl}}{8b^{7/2}}$$

Mathematica [A] time = 0.405914, size = 139, normalized size = 0.94

$$\frac{\sqrt{c+dx}\left(\sqrt{a+bx}\left(15a^2d^2-10abd(4c+dx)+b^2\left(33c^2+26cdx+8d^2x^2\right)\right)+\frac{15(bc-ad)^{5/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{\frac{b(c+dx)}{bc-ad}}}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(Sqrt[a + b*x]*(15*a^2*d^2 - 10*a*b*d*(4*c + d*x) + b^2*(33*
c^2 + 26*c*d*x + 8*d^2*x^2)) + (15*(b*c - a*d)^(5/2)*ArcSinh[(Sqrt[d]*Sqrt[
a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[(b*(c + d*x))/(b*c - a*d)])))/(24
*b^3)
```

Maple [B] time = 0.006, size = 465, normalized size = 3.1

$$\frac{1}{3b}\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}-\frac{5\,ad}{12\,b^2}\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}+\frac{5\,c}{12\,b}\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}+\frac{5\,a^2d^2}{8\,b^3}\sqrt{bx+a}\sqrt{dx+c}-\frac{5\,adc}{4\,b^2}\sqrt{bx+a}\sqrt{dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/3*(d*x+c)^(5/2)*(b*x+a)^(1/2)/b-5/12/b^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*d+
5/12/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*c+5/8/b^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^
2*d^2-5/4/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*d*c+5/8/b*(d*x+c)^(1/2)*(b*x+a)
```

```
^(1/2)*c^2-5/16/b^3*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln(
(1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^
(1/2)*a^3*d^3+15/16/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)
*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b
*d)^(1/2)*a^2*d^2*c-15/16/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(
1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2)
)/(b*d)^(1/2)*a*d*c^2+5/16*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1
/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))
/(b*d)^(1/2)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9694, size = 933, normalized size = 6.3

$$\left[-\frac{15\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{bd}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 - 4\,(2\,bdx + bc + ad)\sqrt{bd}\sqrt{bx+a}\sqrt{}\right.}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*s
qrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*
d^3*x^2 + 33*b^3*c^2*d - 40*a*b^2*c*d^2 + 15*a^2*b*d^3 + 2*(13*b^3*c*d^2 -
5*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d), -1/48*(15*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b
*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (
b^2*c*d + a*b*d^2)*x)) - 2*(8*b^3*d^3*x^2 + 33*b^3*c^2*d - 40*a*b^2*c*d^2 +
 15*a^2*b*d^3 + 2*(13*b^3*c*d^2 - 5*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x +
c))/(b^4*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{5}{2}}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(5/2)/sqrt(a + b*x), x)
```

Giac [B] time = 1.32708, size = 614, normalized size = 4.15

$$-\frac{\frac{24\left(\frac{(b^2c-abd)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)c^2|b|}{b^2}-\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{b^2}+\frac{b^6cd^3-13\,ab^5d^4}{b^7d^4}\right)-\frac{3\,(b^7c^2}{}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(24*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sq
rt(b*x + a))*c^2*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x
 + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4))
 - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3
 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a
) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*d^2*abs(b)/b^2
 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)/(b^4*d^2
) + (b*c*d - 5*a*d^2)/(b^4*d^4)) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*log(ab
s(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*
d)*b^3*d^3))*c*d*abs(b)/b^3)/b
```

3.1485 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{5d\sqrt{a+bx}(c+dx)^{3/2}}{2b^2}+\frac{15d\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4b^3}+\frac{15\sqrt{d}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{7/2}}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}$$

```
[Out] (15*d*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b^3) + (5*d*Sqrt[a + b*x]
*(c + d*x)^(3/2))/(2*b^2) - (2*(c + d*x)^(5/2))/(b*Sqrt[a + b*x]) + (15*Sqr
t[d]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])]
)/(4*b^(7/2))
```

Rubi [A] time = 0.0628883, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{5d\sqrt{a+bx}(c+dx)^{3/2}}{2b^2}+\frac{15d\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4b^3}+\frac{15\sqrt{d}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{7/2}}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (15*d*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b^3) + (5*d*Sqrt[a + b*x]
*(c + d*x)^(3/2))/(2*b^2) - (2*(c + d*x)^(5/2))/(b*Sqrt[a + b*x]) + (15*Sqr
t[d]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])]
)/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}}\,dx = -\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx}{b}$$

$$= \frac{5d\sqrt{a+bx}(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{(15d(bc-ad))\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{4b^2}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4b^3} + \frac{5d\sqrt{a+bx}(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{\left(15d(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}}{8b^3}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4b^3} + \frac{5d\sqrt{a+bx}(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{\left(15d(bc-ad)^2\right)\text{Subst}\left(\int \frac{}{\sqrt{c-}}\right.}{4b^4}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4b^3} + \frac{5d\sqrt{a+bx}(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{\left(15d(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{d}{}}\right.}{4b^4}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4b^3} + \frac{5d\sqrt{a+bx}(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{15\sqrt{d}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a}}{\sqrt{b}\sqrt{c}}\right.}{4b^{7/2}}$$

Mathematica [C] time = 0.06509, size = 71, normalized size = 0.51

$$-\frac{2(c+dx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*Hypergeometric2F1[-5/2, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/2))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{2}}(bx+a)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.27286, size = 971, normalized size = 7.04

$$\left[\frac{15\left(ab^2c^2 - 2a^2bcd + a^3d^2 + \left(b^3c^2 - 2ab^2cd + a^2bd^2\right)x\right)\sqrt{\frac{d}{b}}\log\left(8b^2d^2x^2 + b^2c^2 + 6abcd + a^2d^2 + 4\left(2b^2dx + b^2c + \right.\right.}{16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*
(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*
c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x^2 - 8*b^2*c^2 + 25*a*b*c*d - 15*a^2*d^2
+ (9*b^2*c*d - 5*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x + a*b^3),
-1/8*(15*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*
b*d^2)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*
x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) - 2*(2*b^2*d^2*x
^2 - 8*b^2*c^2 + 25*a*b*c*d - 15*a^2*d^2 + (9*b^2*c*d - 5*a*b*d^2)*x)*sqrt(
b*x + a)*sqrt(d*x + c))/(b^4*x + a*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c + dx)^{\frac{5}{2}}}{(a + bx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(5/2)/(a + b*x)**(3/2), x)
```

Giac [B] time = 1.31625, size = 387, normalized size = 2.8

$$\frac{1}{4}\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(\frac{2(bx+a)d^2|b|}{b^5}+\frac{9\left(b^{10}cd^3|b|-ab^9d^4|b|\right)}{b^{14}d^2}\right)-\frac{15\left(\sqrt{bd}b^2c^2|b|-2\sqrt{bd}abcd|b|+\sqrt{bd}a^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*d^2*abs(
b)/b^5 + 9*(b^10*c*d^3*abs(b) - a*b^9*d^4*abs(b))/(b^14*d^2)) - 15/8*(sqrt(
b*d)*b^2*c^2*abs(b) - 2*sqrt(b*d)*a*b*c*d*abs(b) + sqrt(b*d)*a^2*d^2*abs(b)
)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/b^
5 - 4*(sqrt(b*d)*b^3*c^3*abs(b) - 3*sqrt(b*d)*a*b^2*c^2*d*abs(b) + 3*sqrt(b
*d)*a^2*b*c*d^2*abs(b) - sqrt(b*d)*a^3*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*b^4)
```

3.1486 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{5d^2\sqrt{a+bx}\sqrt{c+dx}}{b^3}+\frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}$$

```
[Out] (5*d^2*Sqrt[a + b*x]*Sqrt[c + d*x])/b^3 - (10*d*(c + d*x)^(3/2))/(3*b^2*Sqr
t[a + b*x]) - (2*(c + d*x)^(5/2))/(3*b*(a + b*x)^(3/2)) + (5*d^(3/2)*(b*c -
 a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(7/2)
```

Rubi [A] time = 0.0615877, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{5d^2\sqrt{a+bx}\sqrt{c+dx}}{b^3}+\frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (5*d^2*Sqrt[a + b*x]*Sqrt[c + d*x])/b^3 - (10*d*(c + d*x)^(3/2))/(3*b^2*Sqr
t[a + b*x]) - (2*(c + d*x)^(5/2))/(3*b*(a + b*x)^(3/2)) + (5*d^(3/2)*(b*c -
 a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^{5/2}}\,dx &= -\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}}\,dx}{3b}\\
&= -\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{\left(5d^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{b^2}\\
&= \frac{5d^2\sqrt{a+bx}\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{\left(5d^2(bc-ad)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{2b^3}\\
&= \frac{5d^2\sqrt{a+bx}\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{\left(5d^2(bc-ad)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a}\right.}{b^4}\\
&= \frac{5d^2\sqrt{a+bx}\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{\left(5d^2(bc-ad)\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^4}\\
&= \frac{5d^2\sqrt{a+bx}\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.065952, size = 73, normalized size = 0.57

$$-\frac{2(c+dx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*Hypergeometric2F1[-5/2, -3/2, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/2))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{2}}(bx+a)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.81133, size = 1034, normalized size = 8.08

$$\left[-\frac{15\left(a^2bcd - a^3d^2 + \left(b^3cd - ab^2d^2\right)x^2 + 2\left(ab^2cd - a^2bd^2\right)x\right)\sqrt{\frac{d}{b}}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 - 4\left(2\,b^2dx + b\right.\right.}{12\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(15*(a^2*b*c*d - a^3*d^2 + (b^3*c*d - a*b^2*d^2)*x^2 + 2*(a*b^2*c*d
- a^2*b*d^2)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 - 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*(3*b^2*d^2*x^2 - 2*b^2*c^2 - 10*a*b*c*d + 15*a^2
*d^2 - 2*(7*b^2*c*d - 10*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/6*(15*(a^2*b*c*d - a^3*d^2 + (b^3*c*d - a*b^2*d^
2)*x^2 + 2*(a*b^2*c*d - a^2*b*d^2)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c
+ a*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d +
 a*d^2)*x)) - 2*(3*b^2*d^2*x^2 - 2*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 2*(7
*b^2*c*d - 10*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x
 + a^2*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.47049, size = 878, normalized size = 6.86

$$\frac{\sqrt{b^2c + (bx+a)bd - abd}\sqrt{bx+a}\,ad^2|b|}{b^5} - \frac{5\left(\sqrt{bd}bcd|b| - \sqrt{bd}ad^2|b|\right)\log\left(\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^2\right)}{2\,b^5} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*d^2*abs(b)/b^5 - 5/2*(sqr
t(b*d)*b*c*d*abs(b) - sqrt(b*d)*a*d^2*abs(b))*log((sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/b^5 - 4/3*(7*sqrt(b*d)*b^6*c^4*d*
abs(b) - 28*sqrt(b*d)*a*b^5*c^3*d^2*abs(b) + 42*sqrt(b*d)*a^2*b^4*c^2*d^3*a
bs(b) - 28*sqrt(b*d)*a^3*b^3*c*d^4*abs(b) + 7*sqrt(b*d)*a^4*b^2*d^5*abs(b)
- 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^2*b^4*c^3*d*abs(b) + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^2*a*b^3*c^2*d^2*abs(b) - 36*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^2*c*d^3*abs(b
) + 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^2*a^3*b*d^4*abs(b) + 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*b^2*c^2*d*abs(b) - 18*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b*c*d^2*abs(b) + 9*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
a^2*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2)^3*b^4)
```

3.1487 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}$$

```
[Out] (-2*d^2*Sqrt[c + d*x])/(b^3*Sqrt[a + b*x]) - (2*d*(c + d*x)^(3/2))/(3*b^2*(
a + b*x)^(3/2)) - (2*(c + d*x)^(5/2))/(5*b*(a + b*x)^(5/2)) + (2*d^(5/2)*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(7/2)
```

Rubi [A] time = 0.0519115, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {47, 63, 217, 206}

$$-\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(7/2),x]
```


```
[Out] (-2*d^2*Sqrt[c + d*x])/(b^3*Sqrt[a + b*x]) - (2*d*(c + d*x)^(3/2))/(3*b^2*(
a + b*x)^(3/2)) - (2*(c + d*x)^(5/2))/(5*b*(a + b*x)^(5/2)) + (2*d^(5/2)*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(7/2)
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^{7/2}}\,dx &= -\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}}\,dx}{b}\\
&= -\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d^2\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx}{b^2}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d^3\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{b^3}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{\left(2d^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^4}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{\left(2d^3\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^4}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0686998, size = 73, normalized size = 0.61

$$
-\frac{2(c+dx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*Hypergeometric2F1[-5/2, -5/2, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/2))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$
\int (dx+c)^{\frac{5}{2}}(bx+a)^{-\frac{7}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.97312, size = 1015, normalized size = 8.46

$$\left[\frac{15\left(b^3d^2x^3+3\,ab^2d^2x^2+3\,a^2bd^2x+a^3d^2\right)\sqrt{\frac{d}{b}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,b^2dx+b^2c+abd\right)\sqrt{bx+a}\right.}{30\left(b^6x^3+3\,ab^5x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^3*d^2*x^3 + 3*a*b^2*d^2*x^2 + 3*a^2*b*d^2*x + a^3*d^2)*sqrt(d/
b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b^2*d*x + b^2*c
 + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x)
- 4*(23*b^2*d^2*x^2 + 3*b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d + 35
*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^
4*x + a^3*b^3), -1/15*(15*(b^3*d^2*x^3 + 3*a*b^2*d^2*x^2 + 3*a^2*b*d^2*x +
a^3*d^2)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
 + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) + 2*(23*b^2*d^2*x
^2 + 3*b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d + 35*a*b*d^2)*x)*sqrt
(b*x + a)*sqrt(d*x + c))/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.71627, size = 1384, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*d^2*abs(b)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)
*b*d - a*b*d))^2)/b^5 - 4/15*(23*sqrt(b*d)*b^9*c^5*d^2*abs(b) - 115*sqrt(b*
d)*a*b^8*c^4*d^3*abs(b) + 230*sqrt(b*d)*a^2*b^7*c^3*d^4*abs(b) - 230*sqrt(b
*d)*a^3*b^6*c^2*d^5*abs(b) + 115*sqrt(b*d)*a^4*b^5*c*d^6*abs(b) - 23*sqrt(b
```

*d)*a^5*b^4*d^7*abs(b) - 70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^7*c^4*d^2*abs(b) + 280*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^6*c^3*d^3*abs(b) - 420*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^5*c^2*d^4*abs(b) + 280*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^4*c*d^5*abs(b) - 70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^3*d^6*abs(b) + 140*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^5*c^3*d^2*abs(b) - 420*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^4*c^2*d^3*abs(b) + 420*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^3*c*d^4*abs(b) - 140*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^2*d^5*abs(b) - 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^3*c^2*d^2*abs(b) + 180*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^2*c*d^3*abs(b) - 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b*d^4*abs(b) + 45*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b*c*d^2*abs(b) - 45*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*b^4)

3.1488 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}} dx$

Optimal. Leaf size=32

$$-\frac{2(c+dx)^{7/2}}{7(a+bx)^{7/2}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(7/2))/(7*(b*c - a*d)*(a + b*x)^(7/2))
```

Rubi [A] time = 0.0028235, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{2(c+dx)^{7/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(9/2),x]

[Out] (-2*(c + d*x)^(7/2))/(7*(b*c - a*d)*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}} dx = -\frac{2(c+dx)^{7/2}}{7(bc-ad)(a+bx)^{7/2}}$$

Mathematica [A] time = 0.019864, size = 32, normalized size = 1.

$$-\frac{2(c+dx)^{7/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(9/2),x]

[Out] (-2*(c + d*x)^(7/2))/(7*(b*c - a*d)*(a + b*x)^(7/2))
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$\frac{2}{7\,ad-7\,bc} (dx+c)^{\frac{7}{2}} (bx+a)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(9/2),x)
```

```
[Out] 2/7/(b*x+a)^(7/2)*(d*x+c)^(7/2)/(a*d-b*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 10.8826, size = 281, normalized size = 8.78

$$-\frac{2\left(d^{3}x^{3}+3\,cd^{2}x^{2}+3\,c^{2}dx+c^{3}\right)\sqrt{bx+a}\sqrt{dx+c}}{7\left(a^{4}bc-a^{5}d+\left(b^{5}c-ab^{4}d\right)x^{4}+4\left(ab^{4}c-a^{2}b^{3}d\right)x^{3}+6\left(a^{2}b^{3}c-a^{3}b^{2}d\right)x^{2}+4\left(a^{3}b^{2}c-a^{4}bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/7*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sqrt(b*x + a)*sqrt(d*x + c)/
(a^4*b*c - a^5*d + (b^5*c - a*b^4*d)*x^4 + 4*(a*b^4*c - a^2*b^3*d)*x^3 + 6*
(a^2*b^3*c - a^3*b^2*d)*x^2 + 4*(a^3*b^2*c - a^4*b*d)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.93681, size = 953, normalized size = 29.78

$$-\frac{4\left(\sqrt{bd}b^{12}c^{6}d^{3}|b|-6\sqrt{bd}ab^{11}c^{5}d^{4}|b|+15\sqrt{bd}a^{2}b^{10}c^{4}d^{5}|b|-20\sqrt{bd}a^{3}b^{9}c^{3}d^{6}|b|+15\sqrt{bd}a^{4}b^{8}c^{2}d^{7}|b|-6\sqrt{bd}a^{5}b^{7}cd^{8}|b|+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] -4/7*(sqrt(b*d)*b^12*c^6*d^3*abs(b) - 6*sqrt(b*d)*a*b^11*c^5*d^4*abs(b) + 1
5*sqrt(b*d)*a^2*b^10*c^4*d^5*abs(b) - 20*sqrt(b*d)*a^3*b^9*c^3*d^6*abs(b) +
 15*sqrt(b*d)*a^4*b^8*c^2*d^7*abs(b) - 6*sqrt(b*d)*a^5*b^7*c*d^8*abs(b) + s
qrt(b*d)*a^6*b^6*d^9*abs(b) + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*b^8*c^4*d^3*abs(b) - 84*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^7*c^3*d^4*ab
s(b) + 126*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^4*a^2*b^6*c^2*d^5*abs(b) - 84*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^5*c*d^6*abs(b) + 21*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^4
*d^7*abs(b) + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^8*b^4*c^2*d^3*abs(b) - 70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^3*c*d^4*abs(b) + 35*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^2
*d^5*abs(b) + 7*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)
*b*d - a*b*d))^12*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^7*b^4)
```

3.1489 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx$

Optimal. Leaf size=66

$$\frac{4d(c+dx)^{7/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{9(a+bx)^{9/2}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(7/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (4*d*(c + d*x)^(7/2)
)/(63*(b*c - a*d)^2*(a + b*x)^(7/2))
```

Rubi [A] time = 0.0090831, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4d(c+dx)^{7/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (4*d*(c + d*x)^(7/2)
)/(63*(b*c - a*d)^2*(a + b*x)^(7/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx &= -\frac{2(c+dx)^{7/2}}{9(bc-ad)(a+bx)^{9/2}}-\frac{(2d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx}{9(bc-ad)}\\ &= -\frac{2(c+dx)^{7/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{4d(c+dx)^{7/2}}{63(bc-ad)^2(a+bx)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0314782, size = 46, normalized size = 0.7

$$\frac{2(c+dx)^{7/2}(9ad-7bc+2bdx)}{63(a+bx)^{9/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(11/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-7*b*c + 9*a*d + 2*b*d*x))/(63*(b*c - a*d)^2*(a + b*x)^
(9/2))
```

Maple [A] time = 0.005, size = 54, normalized size = 0.8

$$\frac{4\,bdx+18\,ad-14\,bc}{63\,a^{2}d^{2}-126\,abcd+63\,b^{2}c^{2}}\left(dx+c\right)^{\frac{7}{2}}\left(bx+a\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(11/2),x)
```

```
[Out] 2/63*(d*x+c)^(7/2)*(2*b*d*x+9*a*d-7*b*c)/(b*x+a)^(9/2)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 44.933, size = 605, normalized size = 9.17

$$\frac{2\left(2\,bd^{4}x^{4}-7\,bc^{4}+9\,ac^{3}d-\left(bcd^{3}-9\,ad^{4}\right)x^{3}-3\left(5\,bc^{2}d^{2}-9\,acd^{3}\right)x^{2}-\left(\right.}{63\left(a^{5}b^{2}c^{2}-2\,a^{6}bcd+a^{7}d^{2}+\left(b^{7}c^{2}-2\,ab^{6}cd+a^{2}b^{5}d^{2}\right)x^{5}+5\left(ab^{6}c^{2}-2\,a^{2}b^{5}cd+a^{3}b^{4}d^{2}\right)x^{4}+10\left(a^{2}b^{5}c^{2}-2\,a^{3}b^{4}cd-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/63*(2*b*d^4*x^4 - 7*b*c^4 + 9*a*c^3*d - (b*c*d^3 - 9*a*d^4)*x^3 - 3*(5*b*
c^2*d^2 - 9*a*c*d^3)*x^2 - (19*b*c^3*d - 27*a*c^2*d^2)*x)*sqrt(b*x + a)*sqr
t(d*x + c)/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2 + (b^7*c^2 - 2*a*b^6*c*d +
a^2*b^5*d^2)*x^5 + 5*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*x^4 + 10*(a^
2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*x^3 + 10*(a^3*b^4*c^2 - 2*a^4*b^3*
c*d + a^5*b^2*d^2)*x^2 + 5*(a^4*b^3*c^2 - 2*a^5*b^2*c*d + a^6*b*d^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.15476, size = 2465, normalized size = 37.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="giac")
```

```
[Out] 8/63*(sqrt(b*d)*b^14*c^7*d^4*abs(b) - 7*sqrt(b*d)*a*b^13*c^6*d^5*abs(b) + 2
1*sqrt(b*d)*a^2*b^12*c^5*d^6*abs(b) - 35*sqrt(b*d)*a^3*b^11*c^4*d^7*abs(b)
+ 35*sqrt(b*d)*a^4*b^10*c^3*d^8*abs(b) - 21*sqrt(b*d)*a^5*b^9*c^2*d^9*abs(b
) + 7*sqrt(b*d)*a^6*b^8*c*d^10*abs(b) - sqrt(b*d)*a^7*b^7*d^11*abs(b) - 9*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*
b^12*c^6*d^4*abs(b) + 54*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^2*a*b^11*c^5*d^5*abs(b) - 135*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^10*c^4*d^6*abs
(b) + 180*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2*a^3*b^9*c^3*d^7*abs(b) - 135*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^8*c^2*d^8*abs(b) + 54*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b
^7*c*d^9*abs(b) - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
+ a)*b*d - a*b*d))^2*a^6*b^6*d^10*abs(b) - 27*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^10*c^5*d^4*abs(b) + 135*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a
*b^9*c^4*d^5*abs(b) - 270*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^4*a^2*b^8*c^3*d^6*abs(b) + 270*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^7*c^2*d^7*ab
s(b) - 135*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^4*a^4*b^6*c*d^8*abs(b) + 27*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^5*d^9*abs(b) - 189*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^8*c^4*d^
4*abs(b) + 756*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^6*a*b^7*c^3*d^5*abs(b) - 1134*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^6*c^2*d^6*abs(b) + 756*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*
a^3*b^5*c*d^7*abs(b) - 189*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^6*a^4*b^4*d^8*abs(b) - 189*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^6*c^3*d^4*abs(b) +
567*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^8*a*b^5*c^2*d^5*abs(b) - 567*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b
^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^4*c*d^6*abs(b) + 189*sqrt(b*d)*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^3*d^7*ab
s(b) - 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^10*b^4*c^2*d^4*abs(b) + 630*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^3*c*d^5*abs(b) - 315*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^2*b^2*
d^6*abs(b) - 105*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^12*b^2*c*d^4*abs(b) + 105*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b*d^5*abs(b) - 63*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*d^4*abs(b
))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
```

```
- a*b*d))^2)^9*b^3)
```

3.1490 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{16d^2(c+dx)^{7/2}}{693(a+bx)^{7/2}(bc-ad)^3}+\frac{8d(c+dx)^{7/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{11(a+bx)^{11/2}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(7/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (8*d*(c + d*x)^(7/
2))/(99*(b*c - a*d)^2*(a + b*x)^(9/2)) - (16*d^2*(c + d*x)^(7/2))/(693*(b*c
 - a*d)^3*(a + b*x)^(7/2))
```

Rubi [A] time = 0.0162974, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d^2(c+dx)^{7/2}}{693(a+bx)^{7/2}(bc-ad)^3}+\frac{8d(c+dx)^{7/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (8*d*(c + d*x)^(7/
2))/(99*(b*c - a*d)^2*(a + b*x)^(9/2)) - (16*d^2*(c + d*x)^(7/2))/(693*(b*c
 - a*d)^3*(a + b*x)^(7/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}}\,dx &= -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}}-\frac{(4d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx}{11(bc-ad)}\\
&= -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{8d(c+dx)^{7/2}}{99(bc-ad)^2(a+bx)^{9/2}}+\frac{\left(8d^2\right)\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx}{99(bc-ad)^2}\\
&= -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{8d(c+dx)^{7/2}}{99(bc-ad)^2(a+bx)^{9/2}}-\frac{16d^2(c+dx)^{7/2}}{693(bc-ad)^3(a+bx)^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0613316, size = 77, normalized size = 0.76

$$-\frac{2(c+dx)^{7/2}\left(99a^2d^2+22abd(2dx-7c)+b^2\left(63c^2-28cdx+8d^2x^2\right)\right)}{693(a+bx)^{11/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2)*(99*a^2*d^2 + 22*a*b*d*(-7*c + 2*d*x) + b^2*(63*c^2 - 2
8*c*d*x + 8*d^2*x^2)))/(693*(b*c - a*d)^3*(a + b*x)^(11/2))
```

Maple [A] time = 0.007, size = 105, normalized size = 1.

$$\frac{16\,b^2d^2x^2+88\,abd^2x-56\,b^2cdx+198\,a^2d^2-308\,abcd+126\,b^2c^2}{693\,a^3d^3-2079\,a^2bcd^2+2079\,ab^2c^2d-693\,b^3c^3}(dx+c)^{\frac{7}{2}}(bx+a)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(13/2),x)
```

```
[Out] 2/693*(d*x+c)^(7/2)*(8*b^2*d^2*x^2+44*a*b*d^2*x-28*b^2*c*d*x+99*a^2*d^2-154
*a*b*c*d+63*b^2*c^2)/(b*x+a)^(11/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 112.612, size = 1045, normalized size = 10.35

$$-\frac{2\left(8\,b^2d^5x^5+63\,b^2c^5-154\,abc^4d+99\,a^2c^3d^2-4\left(b^2cd^4\right.\right.}{693\left(a^6b^3c^3-3\,a^7b^2c^2d+3\,a^8bcd^2-a^9d^3+\left(b^9c^3-3\,ab^8c^2d+3\,a^2b^7cd^2-a^3b^6d^3\right)x^6+6\left(ab^8c^3-3\,a^2b^7c^2d+3\,a^3b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] -2/693*(8*b^2*d^5*x^5 + 63*b^2*c^5 - 154*a*b*c^4*d + 99*a^2*c^3*d^2 - 4*(b^
2*c*d^4 - 11*a*b*d^5)*x^4 + (3*b^2*c^2*d^3 - 22*a*b*c*d^4 + 99*a^2*d^5)*x^3
 + (113*b^2*c^3*d^2 - 330*a*b*c^2*d^3 + 297*a^2*c*d^4)*x^2 + (161*b^2*c^4*d
 - 418*a*b*c^3*d^2 + 297*a^2*c^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b
^3*c^3 - 3*a^7*b^2*c^2*d + 3*a^8*b*c*d^2 - a^9*d^3 + (b^9*c^3 - 3*a*b^8*c^2
```

```
*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*x^6 + 6*(a*b^8*c^3 - 3*a^2*b^7*c^2*d +
3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*x^5 + 15*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*
a^4*b^5*c*d^2 - a^5*b^4*d^3)*x^4 + 20*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^
5*b^4*c*d^2 - a^6*b^3*d^3)*x^3 + 15*(a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*
b^3*c*d^2 - a^7*b^2*d^3)*x^2 + 6*(a^5*b^4*c^3 - 3*a^6*b^3*c^2*d + 3*a^7*b^2
*c*d^2 - a^8*b*d^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(13/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.65298, size = 3127, normalized size = 30.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="giac")
```

```
[Out] -32/693*(sqrt(b*d)*b^16*c^8*d^5*abs(b) - 8*sqrt(b*d)*a*b^15*c^7*d^6*abs(b)
+ 28*sqrt(b*d)*a^2*b^14*c^6*d^7*abs(b) - 56*sqrt(b*d)*a^3*b^13*c^5*d^8*abs(
b) + 70*sqrt(b*d)*a^4*b^12*c^4*d^9*abs(b) - 56*sqrt(b*d)*a^5*b^11*c^3*d^10*
abs(b) + 28*sqrt(b*d)*a^6*b^10*c^2*d^11*abs(b) - 8*sqrt(b*d)*a^7*b^9*c*d^12
*abs(b) + sqrt(b*d)*a^8*b^8*d^13*abs(b) - 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^14*c^7*d^5*abs(b) + 77*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b
^13*c^6*d^6*abs(b) - 231*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^2*a^2*b^12*c^5*d^7*abs(b) + 385*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^11*c^4*d^8*a
bs(b) - 385*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d))^2*a^4*b^10*c^3*d^9*abs(b) + 231*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^9*c^2*d^10*abs(b) - 77*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a
^6*b^8*c*d^11*abs(b) + 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2*a^7*b^7*d^12*abs(b) + 55*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^12*c^6*d^5*abs(b) -
330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^4*a*b^11*c^5*d^6*abs(b) + 825*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^10*c^4*d^7*abs(b) - 1100*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^9*c
^3*d^8*abs(b) + 825*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
+ a)*b*d - a*b*d))^4*a^4*b^8*c^2*d^9*abs(b) - 330*sqrt(b*d)*(sqrt(b*d)*sqrt
(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^7*c*d^10*abs(b) +
55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^4*a^6*b^6*d^11*abs(b) + 297*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^6*b^10*c^5*d^5*abs(b) - 1485*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^9*c^4*d^6*ab
s(b) + 2970*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
```

- a*b*d))^6*a^2*b^8*c^3*d^7*abs(b) - 2970*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^7*c^2*d^8*abs(b) + 1485*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^4*b^6*c*d^9*abs(b) - 297*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^5*b^5*d^10*abs(b) + 1485*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^8*c^4*d^5*abs(b) - 5940*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^7*c^3*d^6*abs(b) + 8910*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^6*c^2*d^7*abs(b) - 5940*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^5*c*d^8*abs(b) + 1485*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^4*b^4*d^9*abs(b) + 2079*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^6*c^3*d^5*abs(b) - 6237*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^5*c^2*d^6*abs(b) + 6237*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^2*b^4*c*d^7*abs(b) - 2079*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^3*b^3*d^8*abs(b) + 2541*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^4*c^2*d^5*abs(b) - 5082*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b^3*c*d^6*abs(b) + 2541*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a^2*b^2*d^7*abs(b) + 1155*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*b^2*c*d^5*abs(b) - 1155*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a*b*d^6*abs(b) + 462*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16*d^5*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^11*b^2)

3.1491 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{15/2}}\,dx$

Optimal. Leaf size=136

$$\frac{32d^3(c+dx)^{7/2}}{3003(a+bx)^{7/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{7/2}}{429(a+bx)^{9/2}(bc-ad)^3}+\frac{12d(c+dx)^{7/2}}{143(a+bx)^{11/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{13(a+bx)^{13/2}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(7/2))/(13*(b*c - a*d)*(a + b*x)^(13/2)) + (12*d*(c + d*x)^(7
/2))/(143*(b*c - a*d)^2*(a + b*x)^(11/2)) - (16*d^2*(c + d*x)^(7/2))/(429*(
b*c - a*d)^3*(a + b*x)^(9/2)) + (32*d^3*(c + d*x)^(7/2))/(3003*(b*c - a*d)^
4*(a + b*x)^(7/2))
```

Rubi [A] time = 0.0278713, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32d^3(c+dx)^{7/2}}{3003(a+bx)^{7/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{7/2}}{429(a+bx)^{9/2}(bc-ad)^3}+\frac{12d(c+dx)^{7/2}}{143(a+bx)^{11/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{13(a+bx)^{13/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(15/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(13*(b*c - a*d)*(a + b*x)^(13/2)) + (12*d*(c + d*x)^(7
/2))/(143*(b*c - a*d)^2*(a + b*x)^(11/2)) - (16*d^2*(c + d*x)^(7/2))/(429*(
b*c - a*d)^3*(a + b*x)^(9/2)) + (32*d^3*(c + d*x)^(7/2))/(3003*(b*c - a*d)^
4*(a + b*x)^(7/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{15/2}}\,dx = -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} - \frac{(6d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}}\,dx}{13(bc-ad)}$$
$$= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} + \frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}} + \frac{\left(24d^2\right)\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx}{143(bc-ad)^2}$$
$$= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} + \frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}} - \frac{16d^2(c+dx)^{7/2}}{429(bc-ad)^3(a+bx)^{9/2}} - \frac{\left(16d^3\right)\int \frac{(c+dx}{(a+bx}}{429(bc-ad}$$
$$= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} + \frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}} - \frac{16d^2(c+dx)^{7/2}}{429(bc-ad)^3(a+bx)^{9/2}} + \frac{32d^3(c+}{3003(bc-ad)}$$

Mathematica [A] time = 0.0786368, size = 118, normalized size = 0.87

$$\frac{2(c+dx)^{7/2}\left(143a^2bd^2(2dx-7c)+429a^3d^3+13ab^2d\left(63c^2-28cdx+8d^2x^2\right)+b^3\left(126c^2dx-231c^3-56cd^2x^2+16d\right.\right.}{3003(a+bx)^{13/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(15/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(429*a^3*d^3 + 143*a^2*b*d^2*(-7*c + 2*d*x) + 13*a*b^2*d
*(63*c^2 - 28*c*d*x + 8*d^2*x^2) + b^3*(-231*c^3 + 126*c^2*d*x - 56*c*d^2*x
^2 + 16*d^3*x^3)))/(3003*(b*c - a*d)^4*(a + b*x)^(13/2))
```

Maple [A] time = 0.008, size = 171, normalized size = 1.3

$$\frac{32\,b^3d^3x^3+208\,ab^2d^3x^2-112\,b^3cd^2x^2+572\,a^2bd^3x-728\,ab^2cd^2x+252\,b^3c^2dx+858\,a^3d^3-2002\,a^2bcd^2+1638\,ab}{3003\,d^4a^4-12012\,bd^3ca^3+18018\,b^2d^2c^2a^2-12012\,b^3dc^3a+3003\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(15/2),x)
```

```
[Out] 2/3003*(d*x+c)^(7/2)*(16*b^3*d^3*x^3+104*a*b^2*d^3*x^2-56*b^3*c*d^2*x^2+286
*a^2*b*d^3*x-364*a*b^2*c*d^2*x+126*b^3*c^2*d*x+429*a^3*d^3-1001*a^2*b*c*d^2
+819*a*b^2*c^2*d-231*b^3*c^3)/(b*x+a)^(13/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b
^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(15/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.24609, size = 3872, normalized size = 28.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="giac")
```

```
[Out] 64/3003*(sqrt(b*d)*b^18*c^9*d^6*abs(b) - 9*sqrt(b*d)*a*b^17*c^8*d^7*abs(b)
+ 36*sqrt(b*d)*a^2*b^16*c^7*d^8*abs(b) - 84*sqrt(b*d)*a^3*b^15*c^6*d^9*abs(
b) + 126*sqrt(b*d)*a^4*b^14*c^5*d^10*abs(b) - 126*sqrt(b*d)*a^5*b^13*c^4*d^
11*abs(b) + 84*sqrt(b*d)*a^6*b^12*c^3*d^12*abs(b) - 36*sqrt(b*d)*a^7*b^11*c
^2*d^13*abs(b) + 9*sqrt(b*d)*a^8*b^10*c*d^14*abs(b) - sqrt(b*d)*a^9*b^9*d^1
5*abs(b) - 13*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*b^16*c^8*d^6*abs(b) + 104*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^15*c^7*d^7*abs(b) - 364*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*
b^14*c^6*d^8*abs(b) + 728*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2*a^3*b^13*c^5*d^9*abs(b) - 910*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^12*c^4*d^10
*abs(b) + 728*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a^5*b^11*c^3*d^11*abs(b) - 364*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^6*b^10*c^2*d^12*abs(b) + 1
04*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^2*a^7*b^9*c*d^13*abs(b) - 13*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^
2*c + (b*x + a)*b*d - a*b*d))^2*a^8*b^8*d^14*abs(b) + 78*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^14*c^7*d^6*abs(
b) - 546*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^4*a*b^13*c^6*d^7*abs(b) + 1638*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^12*c^5*d^8*abs(b) - 2730*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3
*b^11*c^4*d^9*abs(b) + 2730*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^4*a^4*b^10*c^3*d^10*abs(b) - 1638*sqrt(b*d)*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^9*c^2*d
```

^11*abs(b) + 546*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^6*b^8*c*d^12*abs(b) - 78*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^7*b^7*d^13*abs(b) - 286*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^12*c^6*d^6*abs(b) + 1716*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^11*c^5*d^7*abs(b) - 4290*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^10*c^4*d^8*abs(b) + 5720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^9*c^3*d^9*abs(b) - 4290*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^4*b^8*c^2*d^10*abs(b) + 1716*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^5*b^7*c*d^11*abs(b) - 286*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^6*b^6*d^12*abs(b) - 2288*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^10*c^5*d^6*abs(b) + 11440*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^9*c^4*d^7*abs(b) - 22880*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^8*c^3*d^8*abs(b) + 22880*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^7*c^2*d^9*abs(b) - 11440*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^4*b^6*c*d^10*abs(b) + 2288*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^5*b^5*d^11*abs(b) - 10296*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^8*c^4*d^6*abs(b) + 41184*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^7*c^3*d^7*abs(b) - 61776*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^2*b^6*c^2*d^8*abs(b) + 41184*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^3*b^5*c*d^9*abs(b) - 10296*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^4*b^4*d^10*abs(b) - 16302*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^6*c^3*d^6*abs(b) + 48906*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b^5*c^2*d^7*abs(b) - 48906*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a^2*b^4*c*d^8*abs(b) + 16302*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a^3*b^3*d^9*abs(b) - 18018*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*b^4*c^2*d^6*abs(b) + 36036*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a*b^3*c*d^7*abs(b) - 18018*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a^2*b^2*d^8*abs(b) - 9009*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16*b^2*c*d^6*abs(b) + 9009*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16*a*b*d^7*abs(b) - 3003*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^18*d^6*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^13*b)

3.1492 $\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}} dx$

Optimal. Leaf size=183

$$-\frac{35\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^3}{64d^4}+\frac{35(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^2}{96d^3}-\frac{7(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)}{24d^2}+\frac{35(bc-ad)^4\tanh^{-1}}{64\sqrt{b}d^{9/2}}$$

```
[Out] (-35*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*d^4) + (35*(b*c - a*d)^
2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(96*d^3) - (7*(b*c - a*d)*(a + b*x)^(5/2)*
Sqrt[c + d*x])/(24*d^2) + ((a + b*x)^(7/2)*Sqrt[c + d*x])/(4*d) + (35*(b*c
- a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*Sqrt
[b]*d^(9/2))
```

Rubi [A] time = 0.0967502, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$-\frac{35\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^3}{64d^4}+\frac{35(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^2}{96d^3}-\frac{7(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)}{24d^2}+\frac{35(bc-ad)^4\tanh^{-1}}{64\sqrt{b}d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/Sqrt[c + d*x],x]
```

```
[Out] (-35*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*d^4) + (35*(b*c - a*d)^
2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(96*d^3) - (7*(b*c - a*d)*(a + b*x)^(5/2)*
Sqrt[c + d*x])/(24*d^2) + ((a + b*x)^(7/2)*Sqrt[c + d*x])/(4*d) + (35*(b*c
- a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*Sqrt
[b]*d^(9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}dx = \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}dx}{8d}$$

$$= -\frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d} + \frac{\left(35(bc-ad)^2\right)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}dx}{48d^2}$$

$$= \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d} - \frac{\left(35(bc-\right.}{}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2}$$

Mathematica [A] time = 0.650824, size = 189, normalized size = 1.03

$$\frac{\sqrt{d}\sqrt{a+bx}(c+dx)\left(a^2bd^2(326dx-511c)+279a^3d^3+ab^2d\left(385c^2-252cdx+200d^2x^2\right)+b^3\left(70c^2dx-105c^3-56cd\right.\right.}{192d^{9/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(279*a^3*d^3 + a^2*b*d^2*(-511*c + 326*d*x
) + a*b^2*d*(385*c^2 - 252*c*d*x + 200*d^2*x^2) + b^3*(-105*c^3 + 70*c^2*d*
x - 56*c*d^2*x^2 + 48*d^3*x^3)) + (105*(b*c - a*d)^(9/2)*Sqrt[(b*(c + d*x))
/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/b)/(192*d^(
9/2)*Sqrt[c + d*x])
```

Maple [B] time = 0.005, size = 650, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/4*(b*x+a)^(7/2)*(d*x+c)^(1/2)/d+7/24/d*(b*x+a)^(5/2)*(d*x+c)^(1/2)*a-7/24
/d^2*(b*x+a)^(5/2)*(d*x+c)^(1/2)*b*c+35/96/d*(b*x+a)^(3/2)*(d*x+c)^(1/2)*a^
2-35/48/d^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)*a*b*c+35/96/d^3*(b*x+a)^(3/2)*(d*x+
c)^(1/2)*b^2*c^2+35/64/d*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a^3-105/64/d^2*(b*x+a)
^(1/2)*(d*x+c)^(1/2)*a^2*b*c+105/64/d^3*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a*b^2*c
^2-35/64/d^4*(b*x+a)^(1/2)*(d*x+c)^(1/2)*b^3*c^3+35/128*((b*x+a)*(d*x+c))^(
1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*
x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^4-35/32/d*((b*x+a)*(d*x+c))^(1/
2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^
2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^3*b*c+105/64/d^2*((b*x+a)*(d*x+c)
)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+
(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^2*b^2*c^2-35/32/d^3*((b*x+a)
*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d
)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a*b^3*c^3+35/128/d^4*(
(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*
x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*b^4*c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.96458, size = 1226, normalized size = 6.7

$$\left[\frac{105\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)\sqrt{bd}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 + 4\,(2\,bdx + bc + ad)\sqrt{bd}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2
*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*
b*d^2)*x) + 4*(48*b^4*d^4*x^3 - 105*b^4*c^3*d + 385*a*b^3*c^2*d^2 - 511*a^2
*b^2*c*d^3 + 279*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 25*a*b^3*d^4)*x^2 + 2*(35*b^4
*c^2*d^2 - 126*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c
))/(b*d^5), -1/384*(105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)
*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x
)) - 2*(48*b^4*d^4*x^3 - 105*b^4*c^3*d + 385*a*b^3*c^2*d^2 - 511*a^2*b^2*c*
d^3 + 279*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 25*a*b^3*d^4)*x^2 + 2*(35*b^4*c^2*d^
2 - 126*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d
^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12165, size = 362, normalized size = 1.98

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{6\,(bx+a)}{bd}-\frac{7\left(bcd^5-ad^6\right)}{bd^7}\right)+\frac{35\left(b^2c^2d^4-2\,abcd^5+a^2d^6\right)}{bd^7}\right)-\frac{105\left(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2b\right.}{bd^7}\right.}{192\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*
x + a)/(b*d) - 7*(b*c*d^5 - a*d^6)/(b*d^7)) + 35*(b^2*c^2*d^4 - 2*a*b*c*d^5
 + a^2*d^6)/(b*d^7)) - 105*(b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 -
 a^3*d^6)/(b*d^7))*sqrt(b*x + a) - 105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2
*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt
(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^4))*b/abs(b)
```

3.1493 $\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=148

$$\frac{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{12d^2}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8\sqrt{b}d^{7/2}}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}$$

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*d^3) - (5*(b*c - a*d)*(a +
 b*x)^(3/2)*Sqrt[c + d*x])/(12*d^2) + ((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d)
 - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(8*Sqrt[b]*d^(7/2))
```

Rubi [A] time = 0.0723475, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$\frac{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{12d^2}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8\sqrt{b}d^{7/2}}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/Sqrt[c + d*x],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*d^3) - (5*(b*c - a*d)*(a +
 b*x)^(3/2)*Sqrt[c + d*x])/(12*d^2) + ((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d)
 - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(8*Sqrt[b]*d^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \frac{(5(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6d}$$

$$= -\frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} + \frac{\left(5(bc-ad)^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{8d^2}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \frac{\left(5(bc-ad)^3\right)}{1}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \frac{\left(5(bc-ad)^3\right)}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \frac{\left(5(bc-ad)^3\right)}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \frac{5(bc-ad)^3\,\mathrm{t}}{8}$$

Mathematica [A] time = 0.497898, size = 150, normalized size = 1.01

$$\frac{\sqrt{d}\sqrt{a+bx}(c+dx)\left(33a^2d^2+2abd(13dx-20c)+b^2\left(15c^2-10cdx+8d^2x^2\right)\right)-\frac{15(bc-ad)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b}}{24d^{7/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(33*a^2*d^2 + 2*a*b*d*(-20*c + 13*d*x) + b
^2*(15*c^2 - 10*c*d*x + 8*d^2*x^2)) - (15*(b*c - a*d)^(7/2)*Sqrt[(b*(c + d*
x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/b)/(24*d
^(7/2)*Sqrt[c + d*x])
```

Maple [B] time = 0.007, size = 465, normalized size = 3.1

$$\frac{1}{3d}(bx+a)^{\frac{5}{2}}\sqrt{dx+c}+\frac{5a}{12d}(bx+a)^{\frac{3}{2}}\sqrt{dx+c}-\frac{5bc}{12d^2}(bx+a)^{\frac{3}{2}}\sqrt{dx+c}+\frac{5a^2}{8d}\sqrt{bx+a}\sqrt{dx+c}-\frac{5abc}{4d^2}\sqrt{bx+a}\sqrt{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*(b*x+a)^(5/2)*(d*x+c)^(1/2)/d+5/12/d*(b*x+a)^(3/2)*(d*x+c)^(1/2)*a-5/12
/d^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)*b*c+5/8/d*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a^2-
5/4/d^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a*b*c+5/8/d^3*(b*x+a)^(1/2)*(d*x+c)^(1/
2)*b^2*c^2+5/16*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2
```

```
*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2
)*a^3-15/16/d*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a
*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*
a^2*b*c+15/16/d^2*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1
/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1
/2)*a*b^2*c^2-5/16/d^3*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*
ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*
d)^(1/2)*b^3*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.52327, size = 932, normalized size = 6.3

$$\left[-\frac{15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx}\right.}{96\,bc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*s
qrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*
d^3*x^2 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 1
3*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^4), 1/48*(15*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*
c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b
^2*c*d + a*b*d^2)*x)) + 2*(8*b^3*d^3*x^2 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 +
33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 13*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c
))/(b*d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{5}{2}}}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)/sqrt(c + d*x), x)
```

Giac [A] time = 1.13627, size = 267, normalized size = 1.8

$$\frac{\left(\sqrt{b^2c + (bx+a)bd - abd}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{bd} - \frac{5\,\left(bcd^3-ad^4\right)}{bd^5}\right) + \frac{15\,\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)}{bd^5}\right) + \frac{15\,\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3\right.}{}\right.}{24\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*
x + a)/(b*d) - 5*(b*c*d^3 - a*d^4)/(b*d^5)) + 15*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)/(b*d^5)) + 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*d^3))*b/abs(b)
```

3.1494 $\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}} dx$

Optimal. Leaf size=113

$$-\frac{3\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4d^2}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4\sqrt{b}d^{5/2}}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}$$

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*d^2) + ((a + b*x)^(3/2)*Sqr
t[c + d*x])/(2*d) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[
b]*Sqrt[c + d*x])])/(4*Sqrt[b]*d^(5/2))
```

Rubi [A] time = 0.0500458, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$-\frac{3\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4d^2}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4\sqrt{b}d^{5/2}}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/Sqrt[c + d*x],x]
```

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*d^2) + ((a + b*x)^(3/2)*Sqr
t[c + d*x])/(2*d) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[
b]*Sqrt[c + d*x])])/(4*Sqrt[b]*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} - \frac{(3(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4d}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{\left(3(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{8d^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+\right.}{4bd^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{4bd^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4\sqrt{b}d^{5/2}}$$

Mathematica [A] time = 0.362573, size = 119, normalized size = 1.05

$$\frac{\sqrt{d}\sqrt{a+bx}(c+dx)(5ad-3bc+2bdx)+\frac{3(bc-ad)^{5/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b}}{4d^{5/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(-3*b*c + 5*a*d + 2*b*d*x) + (3*(b*c - a*d
)^(5/2)*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqr
t[b*c - a*d]])/b)/(4*d^(5/2)*Sqrt[c + d*x])
```

Maple [B] time = 0.004, size = 308, normalized size = 2.7

$$\frac{1}{2d}(bx+a)^{\frac{3}{2}}\sqrt{dx+c} + \frac{3a}{4d}\sqrt{bx+a}\sqrt{dx+c} - \frac{3bc}{4d^2}\sqrt{bx+a}\sqrt{dx+c} + \frac{3a^2}{8}\sqrt{(bx+a)(dx+c)}\ln\left(\left(\frac{ad}{2}+\frac{bc}{2}+bdx\right)\frac{1}{\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d+3/4/d*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a-3/4/d
^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)*b*c+3/8*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2
)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x
+a*c)^(1/2))/(b*d)^(1/2)*a^2-3/4/d*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d
*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c
)^(1/2))/(b*d)^(1/2)*a*b*c+3/8/d^2*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d
*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c
)^(1/2))/(b*d)^(1/2)*b^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.50645, size = 711, normalized size = 6.29

$$\left[\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+\right.\right.}{16\,bd^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)
*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x - 3*b^2*c*d + 5*
a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^3), -1/8*(3*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(2
*b^2*d^2*x - 3*b^2*c*d + 5*a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/sqrt(c + d*x), x)
```

Giac [A] time = 1.09693, size = 188, normalized size = 1.66

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(\frac{2\,(bx+a)}{bd}-\frac{3\left(bcd-ad^2\right)}{bd^3}\right)-\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}d^2}\right)b}{4\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)/(b*d) -
 3*(b*c*d - a*d^2)/(b*d^3)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^
2))*b/abs(b)
```

3.1495 $\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}$$

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))
```

Rubi [A] time = 0.0352477, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/Sqrt[c + d*x],x]
```



```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx = \frac{\sqrt{a+bx}\sqrt{c+dx}}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{2d}$$

$$= \frac{\sqrt{a+bx}\sqrt{c+dx}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{bd}$$

$$= \frac{\sqrt{a+bx}\sqrt{c+dx}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{bd}$$

$$= \frac{\sqrt{a+bx}\sqrt{c+dx}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}$$

Mathematica [A] time = 0.256086, size = 103, normalized size = 1.41

$$\frac{b\sqrt{d}\sqrt{a+bx}(c+dx)-(bc-ad)^{3/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{bd^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x) - (b*c - a*d)^(3/2)*Sqrt[(b*(c + d*x))/(
b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*d^(3/2)*Sq
rt[c + d*x])
```

Maple [A] time = 0.005, size = 107, normalized size = 1.5

$$\frac{1}{d}\sqrt{bx+a}\sqrt{dx+c}-\frac{-ad+bc}{2d}\sqrt{(bx+a)(dx+c)}\ln\left(\left(\frac{ad}{2}+\frac{bc}{2}+bdx\right)\frac{1}{\sqrt{bd}}+\sqrt{dx^2b+(ad+bc)x+ac}\right)\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] (b*x+a)^(1/2)*(d*x+c)^(1/2)/d-1/2*(-a*d+b*c)/d*((b*x+a)*(d*x+c))^(1/2)/(b*x
+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*
d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.59864, size = 558, normalized size = 7.64

$$\left[\frac{4\sqrt{bx+a}\sqrt{dx+c}bd - (bc-ad)\sqrt{bd}\log\left(8b^2d^2x^2 + b^2c^2 + 6abcd + a^2d^2 + 4(2bdx + bc + ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c} + 8\right.}{4bd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*b^2*d
^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*
sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x))/(b*d^2), 1/2*(2*sqr
t(b*x + a)*sqrt(d*x + c)*b*d + (b*c - a*d)*sqrt(-b*d)*arctan(1/2*(2*b*d*x +
 b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d +
 (b^2*c*d + a*b*d^2)*x)))/(b*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/sqrt(c + d*x), x)
```

Giac [A] time = 1.16533, size = 131, normalized size = 1.79

$$\frac{b\left(\frac{(bc-ad)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}d} + \frac{\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}}{bd}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] b*((b*c - a*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x +
 a)/(b*d))/abs(b)
```

3.1496 $\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=42

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[
d])
```

Rubi [A] time = 0.0260562, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[
d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [A] time = 0.0607072, size = 77, normalized size = 1.83

$$\frac{2\sqrt{c+dx}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}\sqrt{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[d]
*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x))/(b*c - a*d)])
```

Maple [B] time = 0.003, size = 76, normalized size = 1.8

$$\sqrt{(bx+a)(dx+c)}\ln\left(\left(\frac{ad}{2}+\frac{bc}{2}+bdx\right)\frac{1}{\sqrt{bd}}+\sqrt{dx^2b+(ad+bc)x+ac}\right)\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{dx+c}}\frac{1}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] ((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d
*x)/(b*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.36794, size = 413, normalized size = 9.83

$$\left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)}{2\,bd},-\frac{\sqrt{-bd}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d
*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^
2)*x)/(b*d), -sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b
*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x))/(b*d
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*sqrt(c + d*x)), x)
```

Giac [A] time = 1.09406, size = 68, normalized size = 1.62

$$-\frac{2\,b\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*abs(b))
```

3.1497 $\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)}$$

```
[Out] (-2*Sqrt[c + d*x])/((b*c - a*d)*Sqrt[a + b*x])
```

Rubi [A] time = 0.0028276, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{2\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/((b*c - a*d)*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{(bc-ad)\sqrt{a+bx}}$$

Mathematica [A] time = 0.0078147, size = 30, normalized size = 1.

$$\frac{2\sqrt{c+dx}}{\sqrt{a+bx}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[c + d*x])/((-(b*c) + a*d)*Sqrt[a + b*x])
```

Maple [A] time = 0.004, size = 27, normalized size = 0.9

$$2\,\frac{\sqrt{dx+c}}{\sqrt{bx+a}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/(b*x+a)^(1/2)*(d*x+c)^(1/2)/(a*d-b*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2934, size = 92, normalized size = 3.07

$$-\frac{2\sqrt{bx+a}\sqrt{dx+c}}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)*sqrt(d*x + c)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*sqrt(c + d*x)), x)
```

Giac [B] time = 1.07062, size = 89, normalized size = 2.97

$$-\frac{4\sqrt{bd}b}{\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(b*d)*b/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2)*abs(b))
```

3.1498 $\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=66

$$\frac{4d\sqrt{c+dx}}{3\sqrt{a+bx}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

```
[Out] (-2*Sqrt[c + d*x])/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (4*d*Sqrt[c + d*x])/(3
*(b*c - a*d)^2*Sqrt[a + b*x])
```

Rubi [A] time = 0.0083011, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4d\sqrt{c+dx}}{3\sqrt{a+bx}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (4*d*Sqrt[c + d*x])/(3
*(b*c - a*d)^2*Sqrt[a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx &= -\frac{2\sqrt{c+dx}}{3(bc-ad)(a+bx)^{3/2}}-\frac{(2d)\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx}{3(bc-ad)}\\ &= -\frac{2\sqrt{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{4d\sqrt{c+dx}}{3(bc-ad)^2\sqrt{a+bx}}\end{aligned}$$

Mathematica [A] time = 0.0147802, size = 46, normalized size = 0.7

$$\frac{2\sqrt{c+dx}(3ad-bc+2bdx)}{3(a+bx)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*(-(b*c) + 3*a*d + 2*b*d*x))/(3*(b*c - a*d)^2*(a + b*x)^(3/
2))
```

Maple [A] time = 0.005, size = 54, normalized size = 0.8

$$\frac{4\,bdx+6\,ad-2\,bc}{3\,a^{2}d^{2}-6\,abcd+3\,b^{2}c^{2}}\sqrt{dx+c}\,(bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/3*(d*x+c)^(1/2)*(2*b*d*x+3*a*d-b*c)/(b*x+a)^(3/2)/(a^2*d^2-2*a*b*c*d+b^2*
c^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.04411, size = 250, normalized size = 3.79

$$\frac{2\,(2\,bdx-bc+3\,ad)\sqrt{bx+a}\sqrt{dx+c}}{3\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}+\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}\right)x^{2}+2\left(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*d*x - b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^2*c^2 - 2*a^
3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^
2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+bx)^{\frac{5}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*sqrt(c + d*x)), x)
```

Giac [B] time = 1.09907, size = 163, normalized size = 2.47

$$\frac{8\left(b^{2}c-abd-3\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}\right)\sqrt{bd}b^{2}d}{3\left(b^{2}c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}\right)^{3}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(b^2*c - a*b*d - 3*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2)*sqrt(b*d)*b^2*d/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3*abs(b))
```

3.1499 $\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=101

$$-\frac{16d^2\sqrt{c+dx}}{15\sqrt{a+bx}(bc-ad)^3}+\frac{8d\sqrt{c+dx}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{5(a+bx)^{5/2}(bc-ad)}$$

```
[Out] (-2*Sqrt[c + d*x])/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (8*d*Sqrt[c + d*x])/(1
5*(b*c - a*d)^2*(a + b*x)^(3/2)) - (16*d^2*Sqrt[c + d*x])/(15*(b*c - a*d)^3
*Sqrt[a + b*x])
```

Rubi [A] time = 0.016225, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d^2\sqrt{c+dx}}{15\sqrt{a+bx}(bc-ad)^3}+\frac{8d\sqrt{c+dx}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*Sqrt[c + d*x]),x]
```


```
[Out] (-2*Sqrt[c + d*x])/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (8*d*Sqrt[c + d*x])/(1
5*(b*c - a*d)^2*(a + b*x)^(3/2)) - (16*d^2*Sqrt[c + d*x])/(15*(b*c - a*d)^3
*Sqrt[a + b*x])
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx &= -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}}-\frac{(4d)\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx}{5(bc-ad)}\\
&= -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}}+\frac{8d\sqrt{c+dx}}{15(bc-ad)^2(a+bx)^{3/2}}+\frac{\left(8d^2\right)\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx}{15(bc-ad)^2}\\
&= -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}}+\frac{8d\sqrt{c+dx}}{15(bc-ad)^2(a+bx)^{3/2}}-\frac{16d^2\sqrt{c+dx}}{15(bc-ad)^3\sqrt{a+bx}}
\end{aligned}$$

Mathematica [A] time = 0.0304289, size = 75, normalized size = 0.74

$$-\frac{2\sqrt{c+dx}\left(15a^2d^2-10abd(c-2dx)+b^2\left(3c^2-4cdx+8d^2x^2\right)\right)}{15(a+bx)^{5/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(15*a^2*d^2 - 10*a*b*d*(c - 2*d*x) + b^2*(3*c^2 - 4*c*d*x
 + 8*d^2*x^2)))/(15*(b*c - a*d)^3*(a + b*x)^(5/2))
```

Maple [A] time = 0.005, size = 105, normalized size = 1.

$$\frac{16\,b^2d^2x^2+40\,abd^2x-8\,b^2cdx+30\,a^2d^2-20\,abcd+6\,b^2c^2}{15\,a^3d^3-45\,a^2bcd^2+45\,ab^2c^2d-15\,b^3c^3}\sqrt{dx+c}\,(bx+a)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/15*(d*x+c)^(1/2)*(8*b^2*d^2*x^2+20*a*b*d^2*x-4*b^2*c*d*x+15*a^2*d^2-10*a*
b*c*d+3*b^2*c^2)/(b*x+a)^(5/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.72415, size = 509, normalized size = 5.04

$$-\frac{2\left(8\,b^2d^2x^2+3\,b^2c^2-10\,abcd+15\,a^2d^2-4\left(b^2cd-5\,abd^2\right)x\right)\sqrt{bx+a}}{15\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*b^2*d^2*x^2 + 3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 4*(b^2*c*d - 5
*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3
*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b
^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^
3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{7}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(7/2)*sqrt(c + d*x)), x)
```

Giac [B] time = 1.14124, size = 306, normalized size = 3.03

$$-\frac{32\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2 - 5\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^2 b^2c + 5\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd}\right.\right.}{15\left(b^2c - abd - \left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - a}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -32/15*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 5*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^2*c + 5*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b*d + 10*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4)*sqrt(b*d)*b^3*d^2/((b^2*c - a*b*
d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*ab
s(b))
```

3.1500 $\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=136

$$\frac{32d^3\sqrt{c+dx}}{35\sqrt{a+bx}(bc-ad)^4}-\frac{16d^2\sqrt{c+dx}}{35(a+bx)^{3/2}(bc-ad)^3}+\frac{12d\sqrt{c+dx}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{7(a+bx)^{7/2}(bc-ad)}$$

```
[Out] (-2*Sqrt[c + d*x])/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (12*d*Sqrt[c + d*x])/(
35*(b*c - a*d)^2*(a + b*x)^(5/2)) - (16*d^2*Sqrt[c + d*x])/(35*(b*c - a*d)^
3*(a + b*x)^(3/2)) + (32*d^3*Sqrt[c + d*x])/(35*(b*c - a*d)^4*Sqrt[a + b*x]
)
```

Rubi [A] time = 0.0280244, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32d^3\sqrt{c+dx}}{35\sqrt{a+bx}(bc-ad)^4}-\frac{16d^2\sqrt{c+dx}}{35(a+bx)^{3/2}(bc-ad)^3}+\frac{12d\sqrt{c+dx}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*Sqrt[c + d*x]),x]
```


```
[Out] (-2*Sqrt[c + d*x])/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (12*d*Sqrt[c + d*x])/(
35*(b*c - a*d)^2*(a + b*x)^(5/2)) - (16*d^2*Sqrt[c + d*x])/(35*(b*c - a*d)^
3*(a + b*x)^(3/2)) + (32*d^3*Sqrt[c + d*x])/(35*(b*c - a*d)^4*Sqrt[a + b*x]
)
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}dx = -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} - \frac{(6d)\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}dx}{7(bc-ad)}$$

$$= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} + \frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}} + \frac{(24d^2)\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}dx}{35(bc-ad)^2}$$

$$= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} + \frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}} - \frac{16d^2\sqrt{c+dx}}{35(bc-ad)^3(a+bx)^{3/2}} - \frac{(16d^3)\int \frac{}{(a+}}{35(bc}$$

$$= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} + \frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}} - \frac{16d^2\sqrt{c+dx}}{35(bc-ad)^3(a+bx)^{3/2}} + \frac{32d^3\sqrt{c}}{35(bc-ad)^{}}$$

Mathematica [A] time = 0.0456081, size = 116, normalized size = 0.85

$$\frac{2\sqrt{c+dx}\left(-35a^2bd^2(c-2dx)+35a^3d^3+7ab^2d\left(3c^2-4cdx+8d^2x^2\right)+b^3\left(6c^2dx-5c^3-8cd^2x^2+16d^3x^3\right)\right)}{35(a+bx)^{7/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*(35*a^3*d^3 - 35*a^2*b*d^2*(c - 2*d*x) + 7*a*b^2*d*(3*c^2
- 4*c*d*x + 8*d^2*x^2) + b^3*(-5*c^3 + 6*c^2*d*x - 8*c*d^2*x^2 + 16*d^3*x^3
)))/(35*(b*c - a*d)^4*(a + b*x)^(7/2))
```

Maple [A] time = 0.008, size = 171, normalized size = 1.3

$$\frac{32\,b^3d^3x^3+112\,ab^2d^3x^2-16\,b^3cd^2x^2+140\,a^2bd^3x-56\,ab^2cd^2x+12\,b^3c^2dx+70\,a^3d^3-70\,a^2bcd^2+42\,ab^2c^2d-10\,b}{35\,d^4a^4-140\,bd^3ca^3+210\,b^2d^2c^2a^2-140\,b^3dc^3a+35\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/35*(d*x+c)^(1/2)*(16*b^3*d^3*x^3+56*a*b^2*d^3*x^2-8*b^3*c*d^2*x^2+70*a^2*
b*d^3*x-28*a*b^2*c*d^2*x+6*b^3*c^2*d*x+35*a^3*d^3-35*a^2*b*c*d^2+21*a*b^2*c
^2*d-5*b^3*c^3)/(b*x+a)^(7/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*
b^3*c^3*d+b^4*c^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 12.3486, size = 848, normalized size = 6.24

$$\frac{2\left(16\,b^3d^3x^3 - 5\,b^3c^3 + 21\,ab^2c^2d - 35\,a^2b\right.}{35\left(a^4b^4c^4 - 4\,a^5b^3c^3d + 6\,a^6b^2c^2d^2 - 4\,a^7bcd^3 + a^8d^4 + \left(b^8c^4 - 4\,ab^7c^3d + 6\,a^2b^6c^2d^2 - 4\,a^3b^5cd^3 + a^4b^4d^4\right)x^4 + 4\left(ab^7c^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*d^3*x^3 - 5*b^3*c^3 + 21*a*b^2*c^2*d - 35*a^2*b*c*d^2 + 35*a^3
*d^3 - 8*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 2*(3*b^3*c^2*d - 14*a*b^2*c*d^2 +
35*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d
 + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d +
 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^7*c^4 - 4*
a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^3 + 6*
(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*
b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6
*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18414, size = 521, normalized size = 3.83

$$\frac{64\left(b^6c^3 - 3\,ab^5c^2d + 3\,a^2b^4cd^2 - a^3b^3d^3 - 7\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^2 b^4c^2 + 14\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c +}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 64/35*(b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3 - 7*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^4*c^2 + 14*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^3*c*d - 7
*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^2*
d^2 + 21*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
b^2*c - 21*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
4*a*b*d - 35*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^6)*sqrt(b*d)*b^4*d^3/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^
2*c + (b*x + a)*b*d - a*b*d))^2)^7*abs(b))
```

3.1501 $\int \frac{1}{(a+bx)^{11/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=171

$$-\frac{256d^4\sqrt{c+dx}}{315\sqrt{a+bx}(bc-ad)^5}+\frac{128d^3\sqrt{c+dx}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{32d^2\sqrt{c+dx}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{16d\sqrt{c+dx}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2\sqrt{c}}{9(a+bx)^{9}}$$

```
[Out] (-2*Sqrt[c + d*x])/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (16*d*Sqrt[c + d*x])/(
63*(b*c - a*d)^2*(a + b*x)^(7/2)) - (32*d^2*Sqrt[c + d*x])/(105*(b*c - a*d)
^3*(a + b*x)^(5/2)) + (128*d^3*Sqrt[c + d*x])/(315*(b*c - a*d)^4*(a + b*x)^
(3/2)) - (256*d^4*Sqrt[c + d*x])/(315*(b*c - a*d)^5*Sqrt[a + b*x])
```

Rubi [A] time = 0.0409882, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{256d^4\sqrt{c+dx}}{315\sqrt{a+bx}(bc-ad)^5}+\frac{128d^3\sqrt{c+dx}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{32d^2\sqrt{c+dx}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{16d\sqrt{c+dx}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2\sqrt{c}}{9(a+bx)^{9}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (16*d*Sqrt[c + d*x])/(
63*(b*c - a*d)^2*(a + b*x)^(7/2)) - (32*d^2*Sqrt[c + d*x])/(105*(b*c - a*d)
^3*(a + b*x)^(5/2)) + (128*d^3*Sqrt[c + d*x])/(315*(b*c - a*d)^4*(a + b*x)^
(3/2)) - (256*d^4*Sqrt[c + d*x])/(315*(b*c - a*d)^5*Sqrt[a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/2}\sqrt{c+dx}}dx = -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(8d)\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}dx}{9(bc-ad)}$$

$$= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} + \frac{(16d^2)\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}dx}{21(bc-ad)^2}$$

$$= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}} - \frac{(64d^3)\int \frac{}{(a+b}}{105(bc}$$

$$= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}} + \frac{128d^3\sqrt{}}{315(bc-ad)^4}$$

$$= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}} + \frac{128d^3\sqrt{}}{315(bc-ad)^4}$$

Mathematica [A] time = 0.0631599, size = 168, normalized size = 0.98

$$-\frac{2\sqrt{c+dx}\left(126a^2b^2d^2\left(3c^2-4cdx+8d^2x^2\right)-420a^3bd^3(c-2dx)+315a^4d^4+36ab^3d\left(6c^2dx-5c^3-8cd^2x^2+16d^3x^3\right)+\right.}{315(a+bx)^{9/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(315*a^4*d^4 - 420*a^3*b*d^3*(c - 2*d*x) + 126*a^2*b^2*d^
2*(3*c^2 - 4*c*d*x + 8*d^2*x^2) + 36*a*b^3*d*(-5*c^3 + 6*c^2*d*x - 8*c*d^2*
x^2 + 16*d^3*x^3) + b^4*(35*c^4 - 40*c^3*d*x + 48*c^2*d^2*x^2 - 64*c*d^3*x^
3 + 128*d^4*x^4)))/(315*(b*c - a*d)^5*(a + b*x)^(9/2))
```

Maple [A] time = 0.009, size = 256, normalized size = 1.5

$$\frac{256\,b^4d^4x^4+1152\,ab^3d^4x^3-128\,b^4cd^3x^3+2016\,a^2b^2d^4x^2-576\,ab^3cd^3x^2+96\,b^4c^2d^2x^2+1680\,a^3bd^4x-1008\,a^2b^2cd^3x}{315\,a^5d^5-1575\,a^4bcd^4+3150\,a^3b^2c^2d^3-3150\,a^2b^3c^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/315*(d*x+c)^(1/2)*(128*b^4*d^4*x^4+576*a*b^3*d^4*x^3-64*b^4*c*d^3*x^3+100
8*a^2*b^2*d^4*x^2-288*a*b^3*c*d^3*x^2+48*b^4*c^2*d^2*x^2+840*a^3*b*d^4*x-50
4*a^2*b^2*c*d^3*x+216*a*b^3*c^2*d^2*x-40*b^4*c^3*d*x+315*a^4*d^4-420*a^3*b*
c*d^3+378*a^2*b^2*c^2*d^2-180*a*b^3*c^3*d+35*b^4*c^4)/(b*x+a)^(9/2)/(a^5*d^
5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 48.8145, size = 1301, normalized size = 7.61

$$-\frac{}{315\left(a^5b^5c^5-5\,a^6b^4c^4d+10\,a^7b^3c^3d^2-10\,a^8b^2c^2d^3+5\,a^9bcd^4-a^{10}d^5+\left(b^{10}c^5-5\,ab^9c^4d+10\,a^2b^8c^3d^2-10\,a^3b^7c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(128*b^4*d^4*x^4 + 35*b^4*c^4 - 180*a*b^3*c^3*d + 378*a^2*b^2*c^2*d^
2 - 420*a^3*b*c*d^3 + 315*a^4*d^4 - 64*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 48*(
b^4*c^2*d^2 - 6*a*b^3*c*d^3 + 21*a^2*b^2*d^4)*x^2 - 8*(5*b^4*c^3*d - 27*a*b
^3*c^2*d^2 + 63*a^2*b^2*c*d^3 - 105*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x +
c)/(a^5*b^5*c^5 - 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*c^2*d^3
 + 5*a^9*b*c*d^4 - a^10*d^5 + (b^10*c^5 - 5*a*b^9*c^4*d + 10*a^2*b^8*c^3*d^
2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*x^5 + 5*(a*b^9*c^5
- 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*a^5*b^5*c*d
^4 - a^6*b^4*d^5)*x^4 + 10*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a^4*b^6*c^3*
d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*x^3 + 10*(a^3*b^7
*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d^3 + 5*a^7*b^
3*c*d^4 - a^8*b^2*d^5)*x^2 + 5*(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*
c^3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26576, size = 805, normalized size = 4.71

$$-\frac{512\left(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4-9\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^6c^3+27\left(\sqrt{bd}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -512/315*(b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a
^4*b^4*d^4 - 9*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^2*b^6*c^3 + 27*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
```

```
a*b*d))^2*a*b^5*c^2*d - 27*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a^2*b^4*c*d^2 + 9*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2*a^3*b^3*d^3 + 36*(sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^4*c^2 - 72*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^3*c*d + 36*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^2*d^2 - 84*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^2*c + 84*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b*d + 126*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8)*sqrt(b*d)*b^5*d^
4/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2)^9*abs(b))
```

3.1502 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2}-\frac{35b(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{12d^3}+\frac{35b\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8d^4}-\frac{35\sqrt{b}(bc-ad)^3\tanh^{-1}\left(\right.}{8d^{9/2}}$$

```
[Out] (-2*(a + b*x)^(7/2))/(d*Sqrt[c + d*x]) + (35*b*(b*c - a*d)^2*Sqrt[a + b*x]*
Sqrt[c + d*x])/(8*d^4) - (35*b*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x])/(
12*d^3) + (7*b*(a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d^2) - (35*Sqrt[b]*(b*c -
a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(9/2)
)
```

Rubi [A] time = 0.0891534, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2}-\frac{35b(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{12d^3}+\frac{35b\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8d^4}-\frac{35\sqrt{b}(bc-ad)^3\tanh^{-1}\left(\right.}{8d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(3/2),x]
```


```
[Out] (-2*(a + b*x)^(7/2))/(d*Sqrt[c + d*x]) + (35*b*(b*c - a*d)^2*Sqrt[a + b*x]*
Sqrt[c + d*x])/(8*d^4) - (35*b*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x])/(
12*d^3) + (7*b*(a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d^2) - (35*Sqrt[b]*(b*c -
a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(9/2)
)
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx = -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{(7b)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2} - \frac{(35b(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6d^2}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2} + \frac{\left(35b(bc-ad)^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,d}{8d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a+bx)^{5/2}\sqrt{c}}{3d^2}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a+bx)^{5/2}\sqrt{c}}{3d^2}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a+bx)^{5/2}\sqrt{c}}{3d^2}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a+bx)^{5/2}\sqrt{c}}{3d^2}$$

Mathematica [C] time = 0.0671263, size = 73, normalized size = 0.42

$$\frac{2(a+bx)^{9/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{d(a+bx)}{ad-bc}\right)}{9b(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(9/2)*((b*(c + d*x))/(b*c - a*d))^(3/2)*Hypergeometric2F1[3/2,
 9/2, 11/2, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(3/2))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{2}} (dx+c)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.12697, size = 1312, normalized size = 7.54

$$\left[-\frac{105 \left(b^3c^4 - 3\,ab^2c^3d + 3\,a^2bc^2d^2 - a^3cd^3 + \left(b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3d^4\right)x\right)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abc\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(105*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3 + (b^3*c
^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*x)*sqrt(b/d)*log(8*b^2*d^
2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(
b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d^3*
x^3 + 105*b^3*c^3 - 280*a*b^2*c^2*d + 231*a^2*b*c*d^2 - 48*a^3*d^3 - 2*(7*b
^3*c*d^2 - 19*a*b^2*d^3)*x^2 + (35*b^3*c^2*d - 98*a*b^2*c*d^2 + 87*a^2*b*d^
3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x + c*d^4), 1/48*(105*(b^3*c^4 - 3*
a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3 + (b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqr
t(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)
) + 2*(8*b^3*d^3*x^3 + 105*b^3*c^3 - 280*a*b^2*c^2*d + 231*a^2*b*c*d^2 - 48
*a^3*d^3 - 2*(7*b^3*c*d^2 - 19*a*b^2*d^3)*x^2 + (35*b^3*c^2*d - 98*a*b^2*c*
d^2 + 87*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x + c*d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{7}{2}}}{(c+dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(7/2)/(c + d*x)**(3/2), x)
```

Giac [B] time = 1.16745, size = 423, normalized size = 2.43

$$\frac{\left(\left(2\left(\frac{4\,(bx+a)b^2d^6}{b^{10}cd^8-ab^9d^9}-\frac{7\left(b^3cd^5-ab^2d^6\right)}{b^{10}cd^8-ab^9d^9}\right)(bx+a)+\frac{35\left(b^4c^2d^4-2\,ab^3cd^5+a^2b^2d^6\right)}{b^{10}cd^8-ab^9d^9}\right)(bx+a)+\frac{105\left(b^5c^3d^3-3\,ab^4c^2d^4+3\,a^2b^3cd^5-a^3b^2d^6\right)}{b^{10}cd^8-ab^9d^9}\right)\sqrt{bx+a}}{184320\,\sqrt{b^2c+(bx+a)bd-abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/184320*((2*(4*(b*x + a)*b^2*d^6/(b^10*c*d^8 - a*b^9*d^9) - 7*(b^3*c*d^5 -
 a*b^2*d^6)/(b^10*c*d^8 - a*b^9*d^9))*(b*x + a) + 35*(b^4*c^2*d^4 - 2*a*b^3
*c*d^5 + a^2*b^2*d^6)/(b^10*c*d^8 - a*b^9*d^9))*(b*x + a) + 105*(b^5*c^3*d^
3 - 3*a*b^4*c^2*d^4 + 3*a^2*b^3*c*d^5 - a^3*b^2*d^6)/(b^10*c*d^8 - a*b^9*d^
9))*sqrt(b*x + a)/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) + 7/12288*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d)))/(sqrt(b*d)*b^7*d^5)
```

3.1503 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}-\frac{15b\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4d^3}+\frac{15\sqrt{b}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4d^{7/2}}-\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}$$

```
[Out] (-2*(a + b*x)^(5/2))/(d*Sqrt[c + d*x]) - (15*b*(b*c - a*d)*Sqrt[a + b*x]*Sq
rt[c + d*x])/(4*d^3) + (5*b*(a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d^2) + (15*Sq
rt[b]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(4*d^(7/2))
```

Rubi [A] time = 0.0666129, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}-\frac{15b\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4d^3}+\frac{15\sqrt{b}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4d^{7/2}}-\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(3/2),x]
```


```
[Out] (-2*(a + b*x)^(5/2))/(d*Sqrt[c + d*x]) - (15*b*(b*c - a*d)*Sqrt[a + b*x]*Sq
rt[c + d*x])/(4*d^3) + (5*b*(a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d^2) + (15*Sq
rt[b]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(4*d^(7/2))
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx = -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} + \frac{(5b)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} - \frac{(15b(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4d^2}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{\left(15b(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+d}}}{8d^3}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{\left(15(bc-ad)^2\right)\text{Subst}\left(\int \frac{}{\sqrt{c}}\right.}{4d^3}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{\left(15(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-}\right.}{4d^3}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{15\sqrt{b}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{b}}\right.}{4d^{7/2}}$$

Mathematica [C] time = 0.0554737, size = 73, normalized size = 0.53

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(3/2)*Hypergeometric2F1[3/2,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(3/2))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{2}}(dx+c)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.45612, size = 971, normalized size = 7.04

$$\left[\frac{15\left(b^2c^3 - 2\,abc^2d + a^2cd^2 + \left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x\right)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 + 4\left(2\,bd^2x + bcd + \right.\right.}{16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*
(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*
c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x^2 - 15*b^2*c^2 + 25*a*b*c*d - 8*a^2*d^2
- (5*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x + c*d^3),
-1/8*(15*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^
2*d^3)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*
x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) - 2*(2*b^2*d^2*x
^2 - 15*b^2*c^2 + 25*a*b*c*d - 8*a^2*d^2 - (5*b^2*c*d - 9*a*b*d^2)*x)*sqrt(
b*x + a)*sqrt(d*x + c))/(d^4*x + c*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx)^{\frac{5}{2}}}{(c + dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(3/2), x)
```

Giac [B] time = 1.17737, size = 302, normalized size = 2.19

$$\frac{\left(\left(\frac{2\,(bx+a)b^2d^4}{b^8cd^6-ab^7d^7}-\frac{5\left(b^3cd^3-ab^2d^4\right)}{b^8cd^6-ab^7d^7}\right)(bx+a)-\frac{15\left(b^4c^2d^2-2\,ab^3cd^3+a^2b^2d^4\right)}{b^8cd^6-ab^7d^7}\right)\sqrt{bx+a}}{1536\,\sqrt{b^2c+(bx+a)bd-abd}}-\frac{5\,(bc-ad)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+\cdots}\right.\right.}{512\,\sqrt{bd}b^5d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1536*((2*(b*x + a)*b^2*d^4/(b^8*c*d^6 - a*b^7*d^7) - 5*(b^3*c*d^3 - a*b^2
*d^4)/(b^8*c*d^6 - a*b^7*d^7))*(b*x + a) - 15*(b^4*c^2*d^2 - 2*a*b^3*c*d^3
+ a^2*b^2*d^4)/(b^8*c*d^6 - a*b^7*d^7))*sqrt(b*x + a)/sqrt(b^2*c + (b*x + a
)*b*d - a*b*d) - 5/512*(b*c - a*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(
b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^5*d^4)
```

3.1504 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}} dx$

Optimal. Leaf size=98

$$\frac{3b\sqrt{a+bx}\sqrt{c+dx}}{d^2}-\frac{3\sqrt{b}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{5/2}}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}$$

```
[Out] (-2*(a + b*x)^(3/2))/(d*Sqrt[c + d*x]) + (3*b*Sqrt[a + b*x]*Sqrt[c + d*x])/
d^2 - (3*Sqrt[b]*(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[
c + d*x])])/d^(5/2)
```

Rubi [A] time = 0.0471672, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{3b\sqrt{a+bx}\sqrt{c+dx}}{d^2}-\frac{3\sqrt{b}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{5/2}}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(d*Sqrt[c + d*x]) + (3*b*Sqrt[a + b*x]*Sqrt[c + d*x])/
d^2 - (3*Sqrt[b]*(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[
c + d*x])])/d^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{(3b)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{d}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\sqrt{c+dx}}{d^2}-\frac{(3b(bc-ad))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{2d^2}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\sqrt{c+dx}}{d^2}-\frac{(3(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\sqrt{c+dx}}{d^2}-\frac{(3(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\sqrt{c+dx}}{d^2}-\frac{3\sqrt{b}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0482684, size = 73, normalized size = 0.74

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(3/2)*Hypergeometric2F1[3/2,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(3/2))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.50091, size = 702, normalized size = 7.16

$$\left[-\frac{3\left(bc^2-acd+\left(bcd-ad^2\right)x\right)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,bd^2x+bcd+ad^2\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right.}{4\left(d^3x+cd^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 +
b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(b*d*x + 3*b*c - 2*
a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^3*x + c*d^2), 1/2*(3*(b*c^2 - a*c*d +
(b*c*d - a*d^2)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a
)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*(b*
d*x + 3*b*c - 2*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^3*x + c*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(3/2), x)
```

Giac [A] time = 1.13762, size = 207, normalized size = 2.11

$$\frac{\left(\frac{(bx+a)b^2d^2}{b^6cd^4-ab^5d^5}+\frac{3\left(b^3cd-ab^2d^2\right)}{b^6cd^4-ab^5d^5}\right)\sqrt{bx+a}}{32\sqrt{b^2c+(bx+a)bd-abd}}+\frac{3\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{32\sqrt{bd}b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*((b*x + a)*b^2*d^2/(b^6*c*d^4 - a*b^5*d^5) + 3*(b^3*c*d - a*b^2*d^2)/(
b^6*c*d^4 - a*b^5*d^5))*sqrt(b*x + a)/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) +
 3/32*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)))/(sqrt(b*d)*b^3*d^3)
```

3.1505 $\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}$$

```
[Out] (-2*Sqrt[a + b*x])/(d*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(3/2)
```

Rubi [A] time = 0.0318719, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {47, 63, 217, 206}

$$\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(d*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx = -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}} + \frac{b\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{d}$$

$$= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d}$$

$$= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}} + \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}$$

Mathematica [A] time = 0.333918, size = 95, normalized size = 1.44

$$\frac{2\sqrt{bc-ad}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right) - 2\sqrt{d}\sqrt{a+bx}}{d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[a + b*x] + 2*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x))/(b*c - a*d
)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(3/2)*Sqrt[c + d*x]
)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\,(dx+c)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.57288, size = 551, normalized size = 8.35

$$\left[\frac{(dx+c)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,bd^2x+bcd+ad^2\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}}+8\left(b^2cd+abd^2\right)x\right)-}{2\left(d^2x+cd\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((d*x + c)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*sqrt(b*x + a)*sqrt(d*x + c))/(d^2*x + c*d), -((d
*x + c)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
+ c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*sqrt(b*x + a)*
sqrt(d*x + c))/(d^2*x + c*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(3/2), x)
```

Giac [A] time = 1.13503, size = 130, normalized size = 1.97

$$-\frac{2\,b^2\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}d|b|}-\frac{2\,\sqrt{bx+a}b^2}{\sqrt{b^2c+(bx+a)bd-abd}d|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*b^2*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*
d)))/(sqrt(b*d)*d*abs(b)) - 2*sqrt(b*x + a)*b^2/(sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)*d*abs(b))
```

3.1506 $\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}} dx$

Optimal. Leaf size=30

$$\frac{2\sqrt{a+bx}}{\sqrt{c+dx}(bc-ad)}$$

```
[Out] (2*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x])
```

Rubi [A] time = 0.0027923, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{2\sqrt{a+bx}}{\sqrt{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}} dx = \frac{2\sqrt{a+bx}}{(bc-ad)\sqrt{c+dx}}$$

Mathematica [A] time = 0.0073985, size = 30, normalized size = 1.

$$\frac{2\sqrt{a+bx}}{\sqrt{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x])
```

Maple [A] time = 0.003, size = 27, normalized size = 0.9

$$-2\frac{\sqrt{bx+a}}{\sqrt{dx+c}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2*(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*d-b*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.87163, size = 90, normalized size = 3.

$$\frac{2\sqrt{bx+a}\sqrt{dx+c}}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)*sqrt(d*x + c)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(3/2)), x)
```

Giac [A] time = 1.07261, size = 63, normalized size = 2.1

$$\frac{2\sqrt{bx+a}b^2}{\sqrt{b^2c+(bx+a)bd-abd}(bc|b|-ad|b|)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)*b^2/(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(b*c*abs(b) - a*d*
abs(b)))
```

3.1507 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{4d\sqrt{a+bx}}{\sqrt{c+dx}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}$$

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x]) - (4*d*Sqrt[a + b*x])/((b*c -
a*d)^2*Sqrt[c + d*x])
```

Rubi [A] time = 0.0090855, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{4d\sqrt{a+bx}}{\sqrt{c+dx}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x]) - (4*d*Sqrt[a + b*x])/((b*c -
a*d)^2*Sqrt[c + d*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx &= -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}-\frac{(2d)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}}\,dx}{bc-ad}\\ &= -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}-\frac{4d\sqrt{a+bx}}{(bc-ad)^2\sqrt{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0178148, size = 42, normalized size = 0.68

$$-\frac{2(ad+b(c+2dx))}{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(a*d + b*(c + 2*d*x)))/((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])
```

Maple [A] time = 0.004, size = 52, normalized size = 0.8

$$-2\,{\frac {2\,bdx+ad+bc}{\sqrt {bx+a}\sqrt {dx+c} \left({a}^{2}{d}^{2}-2\,abcd+{b}^{2}{c}^{2} \right) }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2*(2*b*d*x+a*d+b*c)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a^2*d^2-2*a*b*c*d+b^2*c^2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.99294, size = 259, normalized size = 4.18

$$-\frac{2\,{\left(2\,b d x + b c + a d\right)} \sqrt{b x + a} \sqrt{d x + c}}{a b^{2} c^{3} - 2\,a^{2} b c^{2} d + a^{3} c d^{2} + {\left(b^{3} c^{2} d - 2\,a b^{2} c d^{2} + a^{2} b d^{3}\right)} x^{2} + {\left(b^{3} c^{3} - a b^{2} c^{2} d - a^{2} b c d^{2} + a^{3} d^{3}\right)} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(a*b^2*c^3 - 2*a^2*b*c
^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 -
 a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b x\right)^{\frac{3}{2}} \left(c + d x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(3/2)), x)
```

Giac [B] time = 1.13544, size = 192, normalized size = 3.1

$$-\frac{2\sqrt{bx+a}b^2d}{\left(b^2c^2|b|-2\,abcd|b|+a^2d^2|b|\right)\sqrt{b^2c+(bx+a)bd-abd}}-\frac{4\sqrt{bd}b^2}{\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)(bc|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2*d/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b) + a^2*d^2*abs(b)
)*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) - 4*sqrt(b*d)*b^2/((b^2*c - a*b*d -
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*(b*c*abs
(b) - a*d*abs(b)))
```

3.1508 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{16d^2\sqrt{a+bx}}{3\sqrt{c+dx}(bc-ad)^3}+\frac{8d}{3\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}$$

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x]) + (8*d)/(3*(b*c - a*d)^2*S
qrt[a + b*x]*Sqrt[c + d*x]) + (16*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^3*Sqrt[
c + d*x])
```

Rubi [A] time = 0.0178873, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{16d^2\sqrt{a+bx}}{3\sqrt{c+dx}(bc-ad)^3}+\frac{8d}{3\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x]) + (8*d)/(3*(b*c - a*d)^2*S
qrt[a + b*x]*Sqrt[c + d*x]) + (16*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^3*Sqrt[
c + d*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx &= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}-\frac{(4d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx}{3(bc-ad)}\\
&= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}+\frac{8d}{3(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}+\frac{\left(8d^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}}\,dx}{3(bc-ad)^2}\\
&= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}+\frac{8d}{3(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}+\frac{16d^2\sqrt{a+bx}}{3(bc-ad)^3\sqrt{c+dx}}
\end{aligned}$$

Mathematica [A] time = 0.029055, size = 75, normalized size = 0.74

$$\frac{2\left(3a^2d^2+6abd(c+2dx)+b^2\left(-c^2+4cdx+8d^2x^2\right)\right)}{3(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (2*(3*a^2*d^2 + 6*a*b*d*(c + 2*d*x) + b^2*(-c^2 + 4*c*d*x + 8*d^2*x^2)))/(3
*(b*c - a*d)^3*(a + b*x)^(3/2)*Sqrt[c + d*x])
```

Maple [A] time = 0.006, size = 105, normalized size = 1.

$$-\frac{16\,b^2d^2x^2+24\,abd^2x+8\,b^2cdx+6\,a^2d^2+12\,abcd-2\,b^2c^2}{3\,a^3d^3-9\,a^2bcd^2+9\,ab^2c^2d-3\,b^3c^3}(bx+a)^{-\frac{3}{2}}\frac{1}{\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(8*b^2*d^2*x^2+12*a*b*d^2*x+4*b^2*c*d*x+3*a^2*d^2+6*a*b*c*d-b^2*c^2)/(
b*x+a)^(3/2)/(d*x+c)^(1/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.70748, size = 544, normalized size = 5.39

$$\frac{2\left(8\,b^2d^2x^2-b^2c^2+6\,abcd+3\,a^2d^2+4\left(b^2cd+3\,abd^2\right)x\right)\sqrt{}}{3\left(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+\left(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4\right)x^3+\left(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d^2+5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*b^2*d^2*x^2 - b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2 + 4*(b^2*c*d + 3*a*b*d
^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b
*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3
*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^
3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 +
```

```
a^4*b*c*d^3 - a^5*d^4)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(3/2)), x)
```

Giac [B] time = 1.32288, size = 497, normalized size = 4.92

$$\frac{2\sqrt{bx+a}b^2d^2}{\left(b^3c^3|b| - 3\,ab^2c^2d|b| + 3\,a^2bcd^2|b| - a^3d^3|b|\right)\sqrt{b^2c + (bx+a)bd - abd}} + \frac{4\left(5\,\sqrt{bd}b^6c^2d - 10\,\sqrt{bd}ab^5cd^2 + 5\,\sqrt{bd}a^2b^4d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)*b^2*d^2/((b^3*c^3*abs(b) - 3*a*b^2*c^2*d*abs(b) + 3*a^2*b*c
*d^2*abs(b) - a^3*d^3*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) + 4/3*(5
*sqrt(b*d)*b^6*c^2*d - 10*sqrt(b*d)*a*b^5*c*d^2 + 5*sqrt(b*d)*a^2*b^4*d^3 -
 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^2*b^4*c*d + 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
a)*b*d - a*b*d))^2*a*b^3*d^2 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*b^2*d)/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b
) + a^2*d^2*abs(b))*(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^2)^3)
```

3.1509 $\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{32d^3\sqrt{a+bx}}{5\sqrt{c+dx}(bc-ad)^4}-\frac{16d^2}{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^3}+\frac{4d}{5(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^2}-\frac{2}{5(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)}$$

```
[Out] -2/(5*(b*c - a*d)*(a + b*x)^(5/2)*Sqrt[c + d*x]) + (4*d)/(5*(b*c - a*d)^2*(
a + b*x)^(3/2)*Sqrt[c + d*x]) - (16*d^2)/(5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqr
t[c + d*x]) - (32*d^3*Sqrt[a + b*x])/(5*(b*c - a*d)^4*Sqrt[c + d*x])
```

Rubi [A] time = 0.0270072, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{32d^3\sqrt{a+bx}}{5\sqrt{c+dx}(bc-ad)^4}-\frac{16d^2}{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^3}+\frac{4d}{5(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^2}-\frac{2}{5(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(5*(b*c - a*d)*(a + b*x)^(5/2)*Sqrt[c + d*x]) + (4*d)/(5*(b*c - a*d)^2*(
a + b*x)^(3/2)*Sqrt[c + d*x]) - (16*d^2)/(5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqr
t[c + d*x]) - (32*d^3*Sqrt[a + b*x])/(5*(b*c - a*d)^4*Sqrt[c + d*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx = -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} - \frac{(6d)\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx}{5(bc-ad)}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} + \frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}} + \frac{\left(8d^2\right)\int \frac{1}{(a+bx)^{3/2}(c+dx)}}{5(bc-ad)^2}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} + \frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}} - \frac{16d^2}{5(bc-ad)^3\sqrt{a+bx}\sqrt{}}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} + \frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}} - \frac{16d^2}{5(bc-ad)^3\sqrt{a+bx}\sqrt{}}$$

Mathematica [A] time = 0.04155, size = 114, normalized size = 0.84

$$-\frac{2\left(15a^2bd^2(c+2dx)+5a^3d^3+5ab^2d\left(-c^2+4cdx+8d^2x^2\right)+b^3\left(-2c^2dx+c^3+8cd^2x^2+16d^3x^3\right)\right)}{5(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(5*a^3*d^3 + 15*a^2*b*d^2*(c + 2*d*x) + 5*a*b^2*d*(-c^2 + 4*c*d*x + 8*d
^2*x^2) + b^3*(c^3 - 2*c^2*d*x + 8*c*d^2*x^2 + 16*d^3*x^3)))/(5*(b*c - a*d)
^4*(a + b*x)^(5/2)*Sqrt[c + d*x])
```

Maple [A] time = 0.009, size = 170, normalized size = 1.3

$$-\frac{32\,b^3d^3x^3+80\,ab^2d^3x^2+16\,b^3cd^2x^2+60\,a^2bd^3x+40\,ab^2cd^2x-4\,b^3c^2dx+10\,a^3d^3+30\,a^2bcd^2-10\,ab^2c^2d+2\,b^3c^3}{5\,d^4a^4-20\,bd^3ca^3+30\,b^2d^2c^2a^2-20\,b^3dc^3a+5\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2/5*(16*b^3*d^3*x^3+40*a*b^2*d^3*x^2+8*b^3*c*d^2*x^2+30*a^2*b*d^3*x+20*a*b
^2*c*d^2*x-2*b^3*c^2*d*x+5*a^3*d^3+15*a^2*b*c*d^2-5*a*b^2*c^2*d+b^3*c^3)/(b
*x+a)^(5/2)/(d*x+c)^(1/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*
c^3*d+b^4*c^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 9.54481, size = 915, normalized size = 6.73

$$-\frac{2\left(16\,b^3d^3x^3+b^3c^3\right.}{5\left(a^3b^4c^5-4\,a^4b^3c^4d+6\,a^5b^2c^3d^2-4\,a^6bc^2d^3+a^7cd^4+\left(b^7c^4d-4\,ab^6c^3d^2+6\,a^2b^5c^2d^3-4\,a^3b^4cd^4+a^4b^3d^5\right)x^4+\left(b^7\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 5*a^3*d^3
 + 8*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 10*a*b^2*c*d^2 - 15*a^2
*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a
^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^6*c^3*d^2
 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c^5 - a*b^
6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5
*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*
b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^
4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{7}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(7/2)*(c + d*x)**(3/2)), x)
```

Giac [B] time = 1.82799, size = 1121, normalized size = 8.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2*d^3/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b) + 6*a^2*b^
2*c^2*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d)) - 4/5*(11*sqrt(b*d)*b^10*c^4*d^2 - 44*sqrt(b*d)*a*b^9*c
^3*d^3 + 66*sqrt(b*d)*a^2*b^8*c^2*d^4 - 44*sqrt(b*d)*a^3*b^7*c*d^5 + 11*sqr
t(b*d)*a^4*b^6*d^6 - 50*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2*b^8*c^3*d^2 + 150*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^7*c^2*d^3 - 150*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^6*c*
d^4 + 50*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a^3*b^5*d^5 + 80*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^4*b^6*c^2*d^2 - 160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^5*c*d^3 + 80*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^4*d^
```

```
4 - 30*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^6*b^4*c*d^2 + 30*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a*b^3*d^3 + 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^2*d^2)/((b^3*c^3*abs(b) - 3*a*b^2*
c^2*d*abs(b) + 3*a^2*b*c*d^2*abs(b) - a^3*d^3*abs(b))*(b^2*c - a*b*d - (sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5)
```

3.1510 $\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}} dx$

Optimal. Leaf size=171

$$\frac{256d^4\sqrt{a+bx}}{35\sqrt{c+dx}(bc-ad)^5}+\frac{128d^3}{35\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^4}-\frac{32d^2}{35(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^3}+\frac{16d}{35(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)^2}$$

```
[Out] -2/(7*(b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*x]) + (16*d)/(35*(b*c - a*d)^2
*(a + b*x)^(5/2)*Sqrt[c + d*x]) - (32*d^2)/(35*(b*c - a*d)^3*(a + b*x)^(3/2
)*Sqrt[c + d*x]) + (128*d^3)/(35*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])
 + (256*d^4*Sqrt[a + b*x])/(35*(b*c - a*d)^5*Sqrt[c + d*x])
```

Rubi [A] time = 0.0432695, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{256d^4\sqrt{a+bx}}{35\sqrt{c+dx}(bc-ad)^5}+\frac{128d^3}{35\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^4}-\frac{32d^2}{35(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^3}+\frac{16d}{35(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(7*(b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*x]) + (16*d)/(35*(b*c - a*d)^2
*(a + b*x)^(5/2)*Sqrt[c + d*x]) - (32*d^2)/(35*(b*c - a*d)^3*(a + b*x)^(3/2
)*Sqrt[c + d*x]) + (128*d^3)/(35*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])
 + (256*d^4*Sqrt[a + b*x])/(35*(b*c - a*d)^5*Sqrt[c + d*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx = -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} - \frac{(8d)\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx}{7(bc-ad)}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} + \frac{\left(48d^2\right)\int \frac{1}{(a+bx)^{5/2}(c+}}{35(bc-ad)^2}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} - \frac{32d^2}{35(bc-ad)^3(a+bx}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} - \frac{32d^2}{35(bc-ad)^3(a+bx}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} - \frac{32d^2}{35(bc-ad)^3(a+bx}$$

Mathematica [A] time = 0.0578715, size = 166, normalized size = 0.97

$$\frac{2\left(70a^2b^2d^2\left(-c^2+4cdx+8d^2x^2\right)+140a^3bd^3(c+2dx)+35a^4d^4+28ab^3d\left(-2c^2dx+c^3+8cd^2x^2+16d^3x^3\right)+b^4\left(-1\right.}{35(a+bx)^{7/2}\sqrt{c+dx}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (2*(35*a^4*d^4 + 140*a^3*b*d^3*(c + 2*d*x) + 70*a^2*b^2*d^2*(-c^2 + 4*c*d*x
 + 8*d^2*x^2) + 28*a*b^3*d*(c^3 - 2*c^2*d*x + 8*c*d^2*x^2 + 16*d^3*x^3) + b
^4*(-5*c^4 + 8*c^3*d*x - 16*c^2*d^2*x^2 + 64*c*d^3*x^3 + 128*d^4*x^4)))/(35
*(b*c - a*d)^5*(a + b*x)^(7/2)*Sqrt[c + d*x])
```

Maple [A] time = 0.009, size = 256, normalized size = 1.5

$$-\frac{256\,b^4d^4x^4+896\,ab^3d^4x^3+128\,b^4cd^3x^3+1120\,a^2b^2d^4x^2+448\,ab^3cd^3x^2-32\,b^4c^2d^2x^2+560\,a^3bd^4x+560\,a^2b^2cd^3x}{35\,a^5d^5-175\,a^4bcd^4+350\,a^3b^2c^2d^3-350\,a^2b^3c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2/35*(128*b^4*d^4*x^4+448*a*b^3*d^4*x^3+64*b^4*c*d^3*x^3+560*a^2*b^2*d^4*x
^2+224*a*b^3*c*d^3*x^2-16*b^4*c^2*d^2*x^2+280*a^3*b*d^4*x+280*a^2*b^2*c*d^3
*x-56*a*b^3*c^2*d^2*x+8*b^4*c^3*d*x+35*a^4*d^4+140*a^3*b*c*d^3-70*a^2*b^2*c
^2*d^2+28*a*b^3*c^3*d-5*b^4*c^4)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(a^5*d^5-5*a^4
*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 28.4017, size = 1400, normalized size = 8.19

$$\frac{}{35\left(a^4b^5c^6 - 5\,a^5b^4c^5d + 10\,a^6b^3c^4d^2 - 10\,a^7b^2c^3d^3 + 5\,a^8bc^2d^4 - a^9cd^5 + \left(b^9c^5d - 5\,ab^8c^4d^2 + 10\,a^2b^7c^3d^3 - 10\,a^3b^6c^2d^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(128*b^4*d^4*x^4 - 5*b^4*c^4 + 28*a*b^3*c^3*d - 70*a^2*b^2*c^2*d^2 + 1
40*a^3*b*c*d^3 + 35*a^4*d^4 + 64*(b^4*c*d^3 + 7*a*b^3*d^4)*x^3 - 16*(b^4*c^
2*d^2 - 14*a*b^3*c*d^3 - 35*a^2*b^2*d^4)*x^2 + 8*(b^4*c^3*d - 7*a*b^3*c^2*d
^2 + 35*a^2*b^2*c*d^3 + 35*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b
^5*c^6 - 5*a^5*b^4*c^5*d + 10*a^6*b^3*c^4*d^2 - 10*a^7*b^2*c^3*d^3 + 5*a^8*
b*c^2*d^4 - a^9*c*d^5 + (b^9*c^5*d - 5*a*b^8*c^4*d^2 + 10*a^2*b^7*c^3*d^3 -
 10*a^3*b^6*c^2*d^4 + 5*a^4*b^5*c*d^5 - a^5*b^4*d^6)*x^5 + (b^9*c^6 - a*b^8
*c^5*d - 10*a^2*b^7*c^4*d^2 + 30*a^3*b^6*c^3*d^3 - 35*a^4*b^5*c^2*d^4 + 19*
a^5*b^4*c*d^5 - 4*a^6*b^3*d^6)*x^4 + 2*(2*a*b^8*c^6 - 7*a^2*b^7*c^5*d + 5*a
^3*b^6*c^4*d^2 + 10*a^4*b^5*c^3*d^3 - 20*a^5*b^4*c^2*d^4 + 13*a^6*b^3*c*d^5
 - 3*a^7*b^2*d^6)*x^3 + 2*(3*a^2*b^7*c^6 - 13*a^3*b^6*c^5*d + 20*a^4*b^5*c^
4*d^2 - 10*a^5*b^4*c^3*d^3 - 5*a^6*b^3*c^2*d^4 + 7*a^7*b^2*c*d^5 - 2*a^8*b*
d^6)*x^2 + (4*a^3*b^6*c^6 - 19*a^4*b^5*c^5*d + 35*a^5*b^4*c^4*d^2 - 30*a^6*
b^3*c^3*d^3 + 10*a^7*b^2*c^2*d^4 + a^8*b*c*d^5 - a^9*d^6)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.83831, size = 2049, normalized size = 11.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)*b^2*d^4/((b^5*c^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^
3*c^3*d^2*abs(b) - 10*a^3*b^2*c^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d
^5*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) + 4/35*(93*sqrt(b*d)*b^14*c
^6*d^3 - 558*sqrt(b*d)*a*b^13*c^5*d^4 + 1395*sqrt(b*d)*a^2*b^12*c^4*d^5 - 1
860*sqrt(b*d)*a^3*b^11*c^3*d^6 + 1395*sqrt(b*d)*a^4*b^10*c^2*d^7 - 558*sqrt
```

```
(b*d)*a^5*b^9*c*d^8 + 93*sqrt(b*d)*a^6*b^8*d^9 - 616*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^12*c^5*d^3 + 3080*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*
a*b^11*c^4*d^4 - 6160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*
x + a)*b*d - a*b*d))^2*a^2*b^10*c^3*d^5 + 6160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^9*c^2*d^6 - 3080*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4
*b^8*c*d^7 + 616*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a^5*b^7*d^8 + 1673*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^10*c^4*d^3 - 6692*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^9*c^3*d^4 +
10038*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^4*a^2*b^8*c^2*d^5 - 6692*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^7*c*d^6 + 1673*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^6*d^7 - 2240*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^
8*c^3*d^3 + 6720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^6*a*b^7*c^2*d^4 - 6720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^6*c*d^5 + 2240*sqrt(b*d)*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^5*d^6 +
 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^8*b^6*c^2*d^3 - 2030*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^8*a*b^5*c*d^4 + 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^4*d^5 - 280*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^4*c*d^
3 + 280*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a
*b*d))^10*a*b^3*d^4 + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^12*b^2*d^3)/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b)
 + 6*a^2*b^2*c^2*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*(b^2*c
 - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
2)^7)
```

3.1511 $\int \frac{1}{(a+bx)^{11/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{512d^5\sqrt{a+bx}}{63\sqrt{c+dx}(bc-ad)^6}-\frac{256d^4}{63\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^5}+\frac{64d^3}{63(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^4}-\frac{32d^2}{63(a+bx)^{5/2}\sqrt{c+dx}(bc-ad}$$

```
[Out] -2/(9*(b*c - a*d)*(a + b*x)^(9/2)*Sqrt[c + d*x]) + (20*d)/(63*(b*c - a*d)^2
*(a + b*x)^(7/2)*Sqrt[c + d*x]) - (32*d^2)/(63*(b*c - a*d)^3*(a + b*x)^(5/2
)*Sqrt[c + d*x]) + (64*d^3)/(63*(b*c - a*d)^4*(a + b*x)^(3/2)*Sqrt[c + d*x]
) - (256*d^4)/(63*(b*c - a*d)^5*Sqrt[a + b*x]*Sqrt[c + d*x]) - (512*d^5*Sqr
t[a + b*x])/(63*(b*c - a*d)^6*Sqrt[c + d*x])
```

Rubi [A] time = 0.0565066, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{512d^5\sqrt{a+bx}}{63\sqrt{c+dx}(bc-ad)^6}-\frac{256d^4}{63\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^5}+\frac{64d^3}{63(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^4}-\frac{32d^2}{63(a+bx)^{5/2}\sqrt{c+dx}(bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(9*(b*c - a*d)*(a + b*x)^(9/2)*Sqrt[c + d*x]) + (20*d)/(63*(b*c - a*d)^2
*(a + b*x)^(7/2)*Sqrt[c + d*x]) - (32*d^2)/(63*(b*c - a*d)^3*(a + b*x)^(5/2
)*Sqrt[c + d*x]) + (64*d^3)/(63*(b*c - a*d)^4*(a + b*x)^(3/2)*Sqrt[c + d*x]
) - (256*d^4)/(63*(b*c - a*d)^5*Sqrt[a + b*x]*Sqrt[c + d*x]) - (512*d^5*Sqr
t[a + b*x])/(63*(b*c - a*d)^6*Sqrt[c + d*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/2}(c+dx)^{3/2}}\,dx = -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} - \frac{(10d)\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx}{9(bc-ad)}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} + \frac{\left(80d^2\right)\int \frac{1}{(a+bx)^{7/2}(c\cdot}}{63(bc-ad)}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{32d^2}{63(bc-ad)^3(a+b}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{32d^2}{63(bc-ad)^3(a+b}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{32d^2}{63(bc-ad)^3(a+b}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{32d^2}{63(bc-ad)^3(a+b}$$

Mathematica [A] time = 0.0781708, size = 226, normalized size = 1.1

$$\frac{512d^5\sqrt{a+bx}}{63\sqrt{c+dx}(bc-ad)^5(ad-bc)} + \frac{256d^4}{63\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^4(ad-bc)} + \frac{64d^3}{63(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)^4} - \frac{}{63(a+bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(9*(b*c - a*d)*(a + b*x)^(9/2)*Sqrt[c + d*x]) + (20*d)/(63*(b*c - a*d)^2
*(a + b*x)^(7/2)*Sqrt[c + d*x]) - (32*d^2)/(63*(b*c - a*d)^3*(a + b*x)^(5/2
)*Sqrt[c + d*x]) + (64*d^3)/(63*(b*c - a*d)^4*(a + b*x)^(3/2)*Sqrt[c + d*x]
) + (256*d^4)/(63*(b*c - a*d)^4*(-(b*c) + a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])
 + (512*d^5*Sqrt[a + b*x])/(63*(b*c - a*d)^5*(-(b*c) + a*d)*Sqrt[c + d*x])
```

Maple [B] time = 0.011, size = 356, normalized size = 1.7

$$-\frac{512\,b^5d^5x^5 + 2304\,ab^4d^5x^4 + 256\,b^5cd^4x^4 + 4032\,a^2b^3d^5x^3 + 1152\,ab^4cd^4x^3 - 64\,b^5c^2d^3x^3 + 3360\,a^3b^2d^5x^2 + 2016\,a}{63\,d^6a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2/63*(256*b^5*d^5*x^5+1152*a*b^4*d^5*x^4+128*b^5*c*d^4*x^4+2016*a^2*b^3*d^
5*x^3+576*a*b^4*c*d^4*x^3-32*b^5*c^2*d^3*x^3+1680*a^3*b^2*d^5*x^2+1008*a^2*
b^3*c*d^4*x^2-144*a*b^4*c^2*d^3*x^2+16*b^5*c^3*d^2*x^2+630*a^4*b*d^5*x+840*
a^3*b^2*c*d^4*x-252*a^2*b^3*c^2*d^3*x+72*a*b^4*c^3*d^2*x-10*b^5*c^4*d*x+63*
a^5*d^5+315*a^4*b*c*d^4-210*a^3*b^2*c^2*d^3+126*a^2*b^3*c^3*d^2-45*a*b^4*c^
4*d+7*b^5*c^5)/(b*x+a)^(9/2)/(d*x+c)^(1/2)/(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^
2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 88.2424, size = 1960, normalized size = 9.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(256*b^5*d^5*x^5 + 7*b^5*c^5 - 45*a*b^4*c^4*d + 126*a^2*b^3*c^3*d^2 -
 210*a^3*b^2*c^2*d^3 + 315*a^4*b*c*d^4 + 63*a^5*d^5 + 128*(b^5*c*d^4 + 9*a*
b^4*d^5)*x^4 - 32*(b^5*c^2*d^3 - 18*a*b^4*c*d^4 - 63*a^2*b^3*d^5)*x^3 + 16*
(b^5*c^3*d^2 - 9*a*b^4*c^2*d^3 + 63*a^2*b^3*c*d^4 + 105*a^3*b^2*d^5)*x^2 -
2*(5*b^5*c^4*d - 36*a*b^4*c^3*d^2 + 126*a^2*b^3*c^2*d^3 - 420*a^3*b^2*c*d^4
 - 315*a^4*b*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^5*b^6*c^7 - 6*a^6*b^5*c
^6*d + 15*a^7*b^4*c^5*d^2 - 20*a^8*b^3*c^4*d^3 + 15*a^9*b^2*c^3*d^4 - 6*a^1
0*b*c^2*d^5 + a^11*c*d^6 + (b^11*c^6*d - 6*a*b^10*c^5*d^2 + 15*a^2*b^9*c^4*
d^3 - 20*a^3*b^8*c^3*d^4 + 15*a^4*b^7*c^2*d^5 - 6*a^5*b^6*c*d^6 + a^6*b^5*d
^7)*x^6 + (b^11*c^7 - a*b^10*c^6*d - 15*a^2*b^9*c^5*d^2 + 55*a^3*b^8*c^4*d^
3 - 85*a^4*b^7*c^3*d^4 + 69*a^5*b^6*c^2*d^5 - 29*a^6*b^5*c*d^6 + 5*a^7*b^4*
d^7)*x^5 + 5*(a*b^10*c^7 - 4*a^2*b^9*c^6*d + 3*a^3*b^8*c^5*d^2 + 10*a^4*b^7
*c^4*d^3 - 25*a^5*b^6*c^3*d^4 + 24*a^6*b^5*c^2*d^5 - 11*a^7*b^4*c*d^6 + 2*a
^8*b^3*d^7)*x^4 + 10*(a^2*b^9*c^7 - 5*a^3*b^8*c^6*d + 9*a^4*b^7*c^5*d^2 - 5
*a^5*b^6*c^4*d^3 - 5*a^6*b^5*c^3*d^4 + 9*a^7*b^4*c^2*d^5 - 5*a^8*b^3*c*d^6
+ a^9*b^2*d^7)*x^3 + 5*(2*a^3*b^8*c^7 - 11*a^4*b^7*c^6*d + 24*a^5*b^6*c^5*d
^2 - 25*a^6*b^5*c^4*d^3 + 10*a^7*b^4*c^3*d^4 + 3*a^8*b^3*c^2*d^5 - 4*a^9*b^
2*c*d^6 + a^10*b*d^7)*x^2 + (5*a^4*b^7*c^7 - 29*a^5*b^6*c^6*d + 69*a^6*b^5*
c^5*d^2 - 85*a^7*b^4*c^4*d^3 + 55*a^8*b^3*c^3*d^4 - 15*a^9*b^2*c^2*d^5 - a^
10*b*c*d^6 + a^11*d^7)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.49973, size = 3291, normalized size = 15.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2*d^5/((b^6*c^6*abs(b) - 6*a*b^5*c^5*d*abs(b) + 15*a^2*b
^4*c^4*d^2*abs(b) - 20*a^3*b^3*c^3*d^3*abs(b) + 15*a^4*b^2*c^2*d^4*abs(b) -
 6*a^5*b*c*d^5*abs(b) + a^6*d^6*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
) - 4/63*(193*sqrt(b*d)*b^18*c^8*d^4 - 1544*sqrt(b*d)*a*b^17*c^7*d^5 + 5404
*sqrt(b*d)*a^2*b^16*c^6*d^6 - 10808*sqrt(b*d)*a^3*b^15*c^5*d^7 + 13510*sqrt
(b*d)*a^4*b^14*c^4*d^8 - 10808*sqrt(b*d)*a^5*b^13*c^3*d^9 + 5404*sqrt(b*d)*
a^6*b^12*c^2*d^10 - 1544*sqrt(b*d)*a^7*b^11*c*d^11 + 193*sqrt(b*d)*a^8*b^10
*d^12 - 1674*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*b^16*c^7*d^4 + 11718*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^15*c^6*d^5 - 35154*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^14*c^5*d^
6 + 58590*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2*a^3*b^13*c^4*d^7 - 58590*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^12*c^3*d^8 + 35154*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^11*c^2
*d^9 - 11718*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*a^6*b^10*c*d^10 + 1674*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^7*b^9*d^11 + 6318*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^14*c^6*d^4 - 3
7908*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*a*b^13*c^5*d^5 + 94770*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*a^2*b^12*c^4*d^6 - 126360*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^11*c^3*d^7 +
 94770*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^4*a^4*b^10*c^2*d^8 - 37908*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^9*c*d^9 + 6318*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^6*b^8*d^10 - 1331
4*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
^6*b^12*c^5*d^4 + 66570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^6*a*b^11*c^4*d^5 - 133140*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^10*c^3*d^6 + 133140
*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
6*a^3*b^9*c^2*d^7 - 66570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^6*a^4*b^8*c*d^8 + 13314*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^5*b^7*d^9 + 16128*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^10
*c^4*d^4 - 64512*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^8*a*b^9*c^3*d^5 + 96768*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^8*c^2*d^6 - 64512*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^7*c
*d^7 + 16128*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^8*a^4*b^6*d^8 - 8190*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^10*b^8*c^3*d^4 + 24570*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^7*c^2*d^5 - 24
570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^10*a^2*b^6*c*d^6 + 8190*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^10*a^3*b^5*d^7 + 2898*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^6*c^2*d^4 - 5796*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b^
5*c*d^5 + 2898*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^12*a^2*b^4*d^6 - 630*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^14*b^4*c*d^4 + 630*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a*b^3*d^5 + 63*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16*b^2
*d^4)/((b^5*c^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^3*c^3*d^2*abs(b) -
```

```
10*a^3*b^2*c^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d^5*abs(b))*(b^2*c
- a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2
)^9)
```

3.1512 $\int \frac{(a+bx)^{9/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=204

$$\frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3}-\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{4d^4}+\frac{105b^2\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8d^5}-\frac{105b^{3/2}(bc-ad)^3\tan}{8d^{11/2}}$$

```
[Out] (-2*(a + b*x)^(9/2))/(3*d*(c + d*x)^(3/2)) - (6*b*(a + b*x)^(7/2))/(d^2*Sqr
t[c + d*x]) + (105*b^2*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*d^5) -
 (35*b^2*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x])/(4*d^4) + (7*b^2*(a + b
*x)^(5/2)*Sqrt[c + d*x])/d^3 - (105*b^(3/2)*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*
Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(11/2))
```

Rubi [A] time = 0.108661, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3}-\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{4d^4}+\frac{105b^2\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8d^5}-\frac{105b^{3/2}(bc-ad)^3\tan}{8d^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(9/2))/(3*d*(c + d*x)^(3/2)) - (6*b*(a + b*x)^(7/2))/(d^2*Sqr
t[c + d*x]) + (105*b^2*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*d^5) -
 (35*b^2*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x])/(4*d^4) + (7*b^2*(a + b
*x)^(5/2)*Sqrt[c + d*x])/d^3 - (105*b^(3/2)*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*
Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(11/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{(c+dx)^{5/2}}\,dx = -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} + \frac{(3b)\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{d^2}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3} - \frac{\left(35b^2(bc-ad)\right)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{2d^3}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4d^4} + \frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3} + \frac{(105b^2 \cdots}{\cdots}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4d^4} \cdots$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4d^4} \cdots$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4d^4} \cdots$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4d^4} \cdots$$

Mathematica [C] time = 0.096833, size = 73, normalized size = 0.36

$$\frac{2(a+bx)^{11/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2} {}_2F_1\left(\frac{5}{2},\frac{11}{2};\frac{13}{2};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*((b*(c + d*x))/(b*c - a*d))^(5/2)*Hypergeometric2F1[5/2
, 11/2, 13/2, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(5/2))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{9}{2}} (dx+c)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 16.1911, size = 1874, normalized size = 9.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(315*(b^4*c^5 - 3*a*b^3*c^4*d + 3*a^2*b^2*c^3*d^2 - a^3*b*c^2*d^3 +
(b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^2 + 2*(b^4*
c^4*d - 3*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 - a^3*b*c*d^4)*x)*sqrt(b/d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d
^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8
*b^4*d^4*x^4 + 315*b^4*c^4 - 840*a*b^3*c^3*d + 693*a^2*b^2*c^2*d^2 - 144*a^
3*b*c*d^3 - 16*a^4*d^4 - 2*(9*b^4*c*d^3 - 25*a*b^3*d^4)*x^3 + 3*(21*b^4*c^2
*d^2 - 60*a*b^3*c*d^3 + 55*a^2*b^2*d^4)*x^2 + 2*(210*b^4*c^3*d - 567*a*b^3*
c^2*d^2 + 477*a^2*b^2*c*d^3 - 104*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)
)/(d^7*x^2 + 2*c*d^6*x + c^2*d^5), 1/48*(315*(b^4*c^5 - 3*a*b^3*c^4*d + 3*a
^2*b^2*c^3*d^2 - a^3*b*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2
*c*d^4 - a^3*b*d^5)*x^2 + 2*(b^4*c^4*d - 3*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^
3 - a^3*b*c*d^4)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x +
a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*(8
*b^4*d^4*x^4 + 315*b^4*c^4 - 840*a*b^3*c^3*d + 693*a^2*b^2*c^2*d^2 - 144*a^
3*b*c*d^3 - 16*a^4*d^4 - 2*(9*b^4*c*d^3 - 25*a*b^3*d^4)*x^3 + 3*(21*b^4*c^2
*d^2 - 60*a*b^3*c*d^3 + 55*a^2*b^2*d^4)*x^2 + 2*(210*b^4*c^3*d - 567*a*b^3*
c^2*d^2 + 477*a^2*b^2*c*d^3 - 104*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)
)/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25007, size = 675, normalized size = 3.31

$$\frac{\left(\left(\left(2\,(bx+a)\left(\frac{4\left(b^6cd^8-ab^5d^9\right)(bx+a)}{b^2cd^9|b|-abd^{10}|b|}-\frac{9\left(b^7c^2d^7-2\,ab^6cd^8+a^2b^5d^9\right)}{b^2cd^9|b|-abd^{10}|b|}\right)+\frac{63\left(b^8c^3d^6-3\,ab^7c^2d^7+3\,a^2b^6cd^8-a^3b^5d^9\right)}{b^2cd^9|b|-abd^{10}|b|}\right)(bx+a)+\frac{420\left(b^9c^4d^5-4\,ab^8c^3d^6\right.}{b^2c}}{24\left(b^2c+(bx+a)bd-ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(((2*(b*x + a)*(4*(b^6*c*d^8 - a*b^5*d^9)*(b*x + a)/(b^2*c*d^9*abs(b)
- a*b*d^10*abs(b)) - 9*(b^7*c^2*d^7 - 2*a*b^6*c*d^8 + a^2*b^5*d^9)/(b^2*c*d
^9*abs(b) - a*b*d^10*abs(b))) + 63*(b^8*c^3*d^6 - 3*a*b^7*c^2*d^7 + 3*a^2*b
^6*c*d^8 - a^3*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*(b*x + a) + 4
20*(b^9*c^4*d^5 - 4*a*b^8*c^3*d^6 + 6*a^2*b^7*c^2*d^7 - 4*a^3*b^6*c*d^8 + a
^4*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*(b*x + a) + 315*(b^10*c^5
*d^4 - 5*a*b^9*c^4*d^5 + 10*a^2*b^8*c^3*d^6 - 10*a^3*b^7*c^2*d^7 + 5*a^4*b^
6*c*d^8 - a^5*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*sqrt(b*x + a)/
(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) + 105/8*(b^6*c^3 - 3*a*b^5*c^2*d + 3*
a^2*b^4*c*d^2 - a^3*b^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^5*abs(b))
```

3.1513 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=170

$$\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}-\frac{35b^2\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4d^4}+\frac{35b^{3/2}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4d^{9/2}}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}-\frac{2(a}{3d(}$$

```
[Out] (-2*(a + b*x)^(7/2))/(3*d*(c + d*x)^(3/2)) - (14*b*(a + b*x)^(5/2))/(3*d^2*
Sqrt[c + d*x]) - (35*b^2*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*d^4) +
 (35*b^2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(6*d^3) + (35*b^(3/2)*(b*c - a*d)^2
*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*d^(9/2))
```

Rubi [A] time = 0.0788944, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}-\frac{35b^2\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4d^4}+\frac{35b^{3/2}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4d^{9/2}}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}-\frac{2(a}{3d(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(5/2),x]
```



```
[Out] (-2*(a + b*x)^(7/2))/(3*d*(c + d*x)^(3/2)) - (14*b*(a + b*x)^(5/2))/(3*d^2*
Sqrt[c + d*x]) - (35*b^2*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*d^4) +
 (35*b^2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(6*d^3) + (35*b^(3/2)*(b*c - a*d)^2
*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*d^(9/2))
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{5/2}}\,dx = -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} + \frac{(7b)\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx}{3d}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} + \frac{\left(35b^2\right)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} + \frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3} - \frac{\left(35b^2(bc-ad)\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3} + \frac{\left(35b^2(\ldots\right.}{\ldots}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3} + \frac{\left(35b(\ldots\right.}{\ldots}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3} + \frac{\left(35b(\ldots\right.}{\ldots}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3} + \frac{35b^{3/2}\ldots}{\ldots}$$

Mathematica [C] time = 0.0788387, size = 73, normalized size = 0.43

$$\frac{2(a+bx)^{9/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2} {}_2F_1\left(\frac{5}{2},\frac{9}{2};\frac{11}{2};\frac{d(a+bx)}{ad-bc}\right)}{9b(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(9/2)*((b*(c + d*x))/(b*c - a*d))^(5/2)*Hypergeometric2F1[5/2,
 9/2, 11/2, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(5/2))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{2}} (dx+c)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 9.68811, size = 1423, normalized size = 8.37

$$\left[\frac{105\left(b^3c^4 - 2\,ab^2c^3d + a^2bc^2d^2 + \left(b^3c^2d^2 - 2\,ab^2cd^3 + a^2bd^4\right)x^2 + 2\left(b^3c^3d - 2\,ab^2c^2d^2 + a^2bcd^3\right)x\right)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 + (b^3*c^2*d^2 - 2*a*b^
2*c*d^3 + a^2*b*d^4)*x^2 + 2*(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3)*x)
*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x
 + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*
d^2)*x) + 4*(6*b^3*d^3*x^3 - 105*b^3*c^3 + 175*a*b^2*c^2*d - 56*a^2*b*c*d^2
 - 8*a^3*d^3 - 3*(7*b^3*c*d^2 - 13*a*b^2*d^3)*x^2 - 2*(70*b^3*c^2*d - 119*a
*b^2*c*d^2 + 40*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^6*x^2 + 2*c*d
^5*x + c^2*d^4), -1/24*(105*(b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 + (b^3
*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(b^3*c^3*d - 2*a*b^2*c^2*d^2
+ a^2*b*c*d^3)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) - 2*(6*b
^3*d^3*x^3 - 105*b^3*c^3 + 175*a*b^2*c^2*d - 56*a^2*b*c*d^2 - 8*a^3*d^3 - 3
*(7*b^3*c*d^2 - 13*a*b^2*d^3)*x^2 - 2*(70*b^3*c^2*d - 119*a*b^2*c*d^2 + 40*
a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(5/2),x)
```

[Out] Timed out

Giac [B] time = 1.22686, size = 513, normalized size = 3.02

$$\frac{\left(\left(3\left(bx+a\right)\left(\frac{2\left(b^{6}cd^{6}-ab^{5}d^{7}\right)(bx+a)}{b^{2}cd^{7}|b|-abd^{8}|b|}-\frac{7\left(b^{7}c^{2}d^{5}-2\,ab^{6}cd^{6}+a^{2}b^{5}d^{7}\right)}{b^{2}cd^{7}|b|-abd^{8}|b|}\right)-\frac{140\left(b^{8}c^{3}d^{4}-3\,ab^{7}c^{2}d^{5}+3\,a^{2}b^{6}cd^{6}-a^{3}b^{5}d^{7}\right)}{b^{2}cd^{7}|b|-abd^{8}|b|}\right)(bx+a)-\frac{105\left(b^{9}c^{4}d^{3}-4\,ab^{8}c^{3}d^{4}\cdots\right.}{b^{2}c\cdots}\cdots}{12\left(b^{2}c+(bx+a)bd-abd\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*((3*(b*x + a)*(2*(b^6*c*d^6 - a*b^5*d^7)*(b*x + a)/(b^2*c*d^7*abs(b) -
 a*b*d^8*abs(b)) - 7*(b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)/(b^2*c*d^7
*abs(b) - a*b*d^8*abs(b))) - 140*(b^8*c^3*d^4 - 3*a*b^7*c^2*d^5 + 3*a^2*b^6
*c*d^6 - a^3*b^5*d^7)/(b^2*c*d^7*abs(b) - a*b*d^8*abs(b)))*(b*x + a) - 105*
(b^9*c^4*d^3 - 4*a*b^8*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 4*a^3*b^6*c*d^6 + a^4*
b^5*d^7)/(b^2*c*d^7*abs(b) - a*b*d^8*abs(b)))*sqrt(b*x + a)/(b^2*c + (b*x +
 a)*b*d - a*b*d)^(3/2) - 35/4*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*log(abs
(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d
)*d^4*abs(b))
```

3.1514 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{5b^2\sqrt{a+bx}\sqrt{c+dx}}{d^3}-\frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{7/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}$$

```
[Out] (-2*(a + b*x)^(5/2))/(3*d*(c + d*x)^(3/2)) - (10*b*(a + b*x)^(3/2))/(3*d^2*
Sqrt[c + d*x]) + (5*b^2*Sqrt[a + b*x]*Sqrt[c + d*x])/d^3 - (5*b^(3/2)*(b*c
- a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(7/2)
```

Rubi [A] time = 0.0583784, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{5b^2\sqrt{a+bx}\sqrt{c+dx}}{d^3}-\frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{7/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(5/2),x]
```


```
[Out] (-2*(a + b*x)^(5/2))/(3*d*(c + d*x)^(3/2)) - (10*b*(a + b*x)^(3/2))/(3*d^2*
Sqrt[c + d*x]) + (5*b^2*Sqrt[a + b*x]*Sqrt[c + d*x])/d^3 - (5*b^(3/2)*(b*c
- a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(7/2)
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/2}}\,dx &= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}+\frac{(5b)\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx}{3d}\\
&= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{\left(5b^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{d^2}\\
&= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\sqrt{c+dx}}{d^3}-\frac{\left(5b^2(bc-ad)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{2d^3}\\
&= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\sqrt{c+dx}}{d^3}-\frac{(5b(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a}\right.}{d^3}\\
&= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\sqrt{c+dx}}{d^3}-\frac{(5b(bc-ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d^3}\\
&= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\sqrt{c+dx}}{d^3}-\frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0713854, size = 73, normalized size = 0.57

$$
\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(5/2)*Hypergeometric2F1[5/2,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/2))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$
\int (bx+a)^{\frac{5}{2}}(dx+c)^{-\frac{5}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.47217, size = 1033, normalized size = 8.07

$$\left[-\frac{15\left(b^2c^3 - abc^2d + \left(b^2cd^2 - abd^3\right)x^2 + 2\left(b^2c^2d - abcd^2\right)x\right)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 + 4\left(2\,bd^2x + bc\right.\right.}{12\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(15*(b^2*c^3 - a*b*c^2*d + (b^2*c*d^2 - a*b*d^3)*x^2 + 2*(b^2*c^2*d
 - a*b*c*d^2)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*(3*b^2*d^2*x^2 + 15*b^2*c^2 - 10*a*b*c*d - 2*a^2
*d^2 + 2*(10*b^2*c*d - 7*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x^2
+ 2*c*d^4*x + c^2*d^3), 1/6*(15*(b^2*c^3 - a*b*c^2*d + (b^2*c*d^2 - a*b*d^3
)*x^2 + 2*(b^2*c^2*d - a*b*c*d^2)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c +
 a*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c +
a*b*d)*x)) + 2*(3*b^2*d^2*x^2 + 15*b^2*c^2 - 10*a*b*c*d - 2*a^2*d^2 + 2*(10
*b^2*c*d - 7*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x^2 + 2*c*d^4*x
+ c^2*d^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18898, size = 373, normalized size = 2.91

$$\frac{\left((bx+a)\left(\frac{3\left(b^6cd^4-ab^5d^5\right)(bx+a)}{b^2cd^5|b|-abd^6|b|}+\frac{20\left(b^7c^2d^3-2\,ab^6cd^4+a^2b^5d^5\right)}{b^2cd^5|b|-abd^6|b|}\right)+\frac{15\left(b^8c^3d^2-3\,ab^7c^2d^3+3\,a^2b^6cd^4-a^3b^5d^5\right)}{b^2cd^5|b|-abd^6|b|}\right)\sqrt{bx+a}}{3\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}+\frac{5\left(b^4c-ab^3d\right)\text{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x + a)*(3*(b^6*c*d^4 - a*b^5*d^5)*(b*x + a)/(b^2*c*d^5*abs(b) - a*b
*d^6*abs(b)) + 20*(b^7*c^2*d^3 - 2*a*b^6*c*d^4 + a^2*b^5*d^5)/(b^2*c*d^5*ab
s(b) - a*b*d^6*abs(b))) + 15*(b^8*c^3*d^2 - 3*a*b^7*c^2*d^3 + 3*a^2*b^6*c*d
^4 - a^3*b^5*d^5)/(b^2*c*d^5*abs(b) - a*b*d^6*abs(b)))*sqrt(b*x + a)/(b^2*c
 + (b*x + a)*b*d - a*b*d)^(3/2) + 5*(b^4*c - a*b^3*d)*log(abs(-sqrt(b*d)*sq
rt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^3*abs(b))
```

3.1515 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}} dx$

Optimal. Leaf size=92

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{5/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}$$

```
[Out] (-2*(a + b*x)^(3/2))/(3*d*(c + d*x)^(3/2)) - (2*b*Sqrt[a + b*x])/(d^2*Sqrt[
c + d*x]) + (2*b^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/d^(5/2)
```

Rubi [A] time = 0.0398938, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {47, 63, 217, 206}

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{5/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*d*(c + d*x)^(3/2)) - (2*b*Sqrt[a + b*x])/(d^2*Sqrt[
c + d*x]) + (2*b^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/d^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}+\frac{b\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx}{d}\\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{b^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{(2b)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{(2b)\operatorname{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.521885, size = 111, normalized size = 1.21

$$
\frac{6(bc-ad)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)-2\sqrt{d}\sqrt{a+bx}(ad+3bc+4bdx)}{3d^{5/2}(c+dx)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[a + b*x]*(3*b*c + a*d + 4*b*d*x) + 6*(b*c - a*d)^(3/2)*((b
*(c + d*x))/(b*c - a*d))^(3/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a
*d]])/(3*d^(5/2)*(c + d*x)^(3/2))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$
\int (bx+a)^{\frac{3}{2}}(dx+c)^{-\frac{5}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.14876, size = 734, normalized size = 7.98

$$\left[\frac{3\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,bd^2x+bcd+ad^2\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}}+8\left(b\right.\right.}{6\left(d^4x^2+2\,cd^3x+c^2d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c
^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt
(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(4*b*d*x + 3*b*c + a*d)*
sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2), -1/3*(3*(b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt
(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x))
 + 2*(4*b*d*x + 3*b*c + a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^
3*x + c^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(5/2), x)
```

Giac [B] time = 1.15123, size = 296, normalized size = 3.22

$$\frac{\sqrt{bd}\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{16\left(b^5cd^4-ab^4d^5\right)}+\frac{\sqrt{bx+a}\left(\frac{4\left(b^5cd^2-ab^4d^3\right)(bx+a)}{b^8c^2d^4-2\,ab^7cd^5+a^2b^6d^6}+\frac{3\left(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3\right)}{b^8c^2d^4-2\,ab^7cd^5+a^2b^6d^6}\right)}{48\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)))/(b^5*c*d^4 - a*b^4*d^5) + 1/48*sqrt(b*x + a)*(4*(b^5*c*d^2 - a
*b^4*d^3)*(b*x + a)/(b^8*c^2*d^4 - 2*a*b^7*c*d^5 + a^2*b^6*d^6) + 3*(b^6*c^
2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)/(b^8*c^2*d^4 - 2*a*b^7*c*d^5 + a^2*b^6*d
^6))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2)
```

3.1516 $\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}} dx$

Optimal. Leaf size=32

$$\frac{2(a+bx)^{3/2}}{3(c+dx)^{3/2}(bc-ad)}$$

```
[Out] (2*(a + b*x)^(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2))
```

Rubi [A] time = 0.0027823, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{2(a+bx)^{3/2}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}} dx = \frac{2(a+bx)^{3/2}}{3(bc-ad)(c+dx)^{3/2}}$$

Mathematica [A] time = 0.0094239, size = 32, normalized size = 1.

$$\frac{2(a+bx)^{3/2}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2))
```

Maple [A] time = 0.003, size = 27, normalized size = 0.8

$$-\frac{2}{3\,ad-3\,bc}(bx+a)^{\frac{3}{2}}(dx+c)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(b*x+a)^(3/2)/(d*x+c)^(3/2)/(a*d-b*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.19569, size = 139, normalized size = 4.34

$$\frac{2\,(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{3\,\left(bc^3-ac^2d+\left(bcd^2-ad^3\right)x^2+2\,\left(bc^2d-acd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)*sqrt(d*x + c)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*x^2
+ 2*(b*c^2*d - a*c*d^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(5/2), x)
```

Giac [B] time = 1.14782, size = 90, normalized size = 2.81

$$-\frac{(bx+a)^{\frac{3}{2}}b^4d}{24\,\left(b^8c^2d^4-2\,ab^7cd^5+a^2b^6d^6\right)\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*(b*x + a)^(3/2)*b^4*d/((b^8*c^2*d^4 - 2*a*b^7*c*d^5 + a^2*b^6*d^6)*(b
^2*c + (b*x + a)*b*d - a*b*d)^(3/2))
```

3.1517 $\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=66

$$\frac{4b\sqrt{a+bx}}{3\sqrt{c+dx}(bc-ad)^2}+\frac{2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)}$$

```
[Out] (2*Sqrt[a + b*x])/(3*(b*c - a*d)*(c + d*x)^(3/2)) + (4*b*Sqrt[a + b*x])/(3*
(b*c - a*d)^2*Sqrt[c + d*x])
```

Rubi [A] time = 0.0082581, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4b\sqrt{a+bx}}{3\sqrt{c+dx}(bc-ad)^2}+\frac{2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/(3*(b*c - a*d)*(c + d*x)^(3/2)) + (4*b*Sqrt[a + b*x])/(3*
(b*c - a*d)^2*Sqrt[c + d*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}}\,dx &= \frac{2\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/2}}+\frac{(2b)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}}\,dx}{3(bc-ad)}\\ &= \frac{2\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/2}}+\frac{4b\sqrt{a+bx}}{3(bc-ad)^2\sqrt{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0153541, size = 46, normalized size = 0.7

$$\frac{2\sqrt{a+bx}(-ad+3bc+2bdx)}{3(c+dx)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[a + b*x]*(3*b*c - a*d + 2*b*d*x))/(3*(b*c - a*d)^2*(c + d*x)^(3/2))
```

Maple [A] time = 0.006, size = 53, normalized size = 0.8

$$-\frac{-4\,bdx+2\,ad-6\,bc}{3\,a^2d^2-6\,abcd+3\,b^2c^2}\sqrt{bx+a}\,(dx+c)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(-2*b*d*x+a*d-3*b*c)/(d*x+c)^(3/2)/(a^2*d^2-2*a*b*c*d+b^
2*c^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.65885, size = 250, normalized size = 3.79

$$\frac{2\,(2\,bdx+3\,bc-ad)\sqrt{bx+a}\sqrt{dx+c}}{3\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^2+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*d*x + 3*b*c - a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*c^4 - 2*a*b*c^
3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*
d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(5/2)), x)
```

Giac [B] time = 1.08413, size = 173, normalized size = 2.62

$$-\frac{\left(\frac{2\,(bx+a)b^4d^2}{b^8c^2d^4-2\,ab^7cd^5+a^2b^6d^6}+\frac{3\,\left(b^5cd-ab^4d^2\right)}{b^8c^2d^4-2\,ab^7cd^5+a^2b^6d^6}\right)\sqrt{bx+a}}{24\,\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*(2*(b*x + a)*b^4*d^2/(b^8*c^2*d^4 - 2*a*b^7*c*d^5 + a^2*b^6*d^6) + 3*
(b^5*c*d - a*b^4*d^2)/(b^8*c^2*d^4 - 2*a*b^7*c*d^5 + a^2*b^6*d^6))*sqrt(b*x
 + a)/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2)
```

3.1518 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{16bd\sqrt{a+bx}}{3\sqrt{c+dx}(bc-ad)^3}-\frac{8d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/2)) - (8*d*Sqrt[a + b*x])/(3*(b*
c - a*d)^2*(c + d*x)^(3/2)) - (16*b*d*Sqrt[a + b*x])/(3*(b*c - a*d)^3*Sqrt[
c + d*x])
```

Rubi [A] time = 0.0172845, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16bd\sqrt{a+bx}}{3\sqrt{c+dx}(bc-ad)^3}-\frac{8d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/2)) - (8*d*Sqrt[a + b*x])/(3*(b*
c - a*d)^2*(c + d*x)^(3/2)) - (16*b*d*Sqrt[a + b*x])/(3*(b*c - a*d)^3*Sqrt[
c + d*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx &= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{3/2}}-\frac{(4d)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}}\,dx}{bc-ad}\\
&= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{3/2}}-\frac{8d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/2}}-\frac{(8bd)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}}\,dx}{3(bc-ad)^2}\\
&= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{3/2}}-\frac{8d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/2}}-\frac{16bd\sqrt{a+bx}}{3(bc-ad)^3\sqrt{c+dx}}
\end{aligned}$$

Mathematica [A] time = 0.0322252, size = 78, normalized size = 0.8

$$\frac{2a^2d^2 - 4abd(3c + 2dx) - 2b^2\left(3c^2 + 12cdx + 8d^2x^2\right)}{3\sqrt{a + bx}(c + dx)^{3/2}(bc - ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (2*a^2*d^2 - 4*a*b*d*(3*c + 2*d*x) - 2*b^2*(3*c^2 + 12*c*d*x + 8*d^2*x^2))/
(3*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(3/2))
```

Maple [A] time = 0.006, size = 104, normalized size = 1.1

$$-\frac{-16\,b^2d^2x^2 - 8\,abd^2x - 24\,b^2cdx + 2\,a^2d^2 - 12\,abcd - 6\,b^2c^2}{3\,a^3d^3 - 9\,a^2bcd^2 + 9\,ab^2c^2d - 3\,b^3c^3}\frac{1}{\sqrt{bx + a}}(dx + c)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(-8*b^2*d^2*x^2-4*a*b*d^2*x-12*b^2*c*d*x+a^2*d^2-6*a*b*c*d-3*b^2*c^2)/
(b*x+a)^(1/2)/(d*x+c)^(3/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.79127, size = 545, normalized size = 5.56

$$-\frac{2\left(8\,b^2d^2x^2 + 3\,b^2c^2 + 6\,abcd - a^2d^2 + 4\left(3\,b^2cd + abd^2\right.\right.}{3\left(ab^3c^5 - 3\,a^2b^2c^4d + 3\,a^3bc^3d^2 - a^4c^2d^3 + \left(b^4c^3d^2 - 3\,ab^3c^2d^3 + 3\,a^2b^2cd^4 - a^3bd^5\right)x^3 + \left(2\,b^4c^4d - 5\,ab^3c^3d^2 + 3\,a^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*d^2*x^2 + 3*b^2*c^2 + 6*a*b*c*d - a^2*d^2 + 4*(3*b^2*c*d + a*b*
d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*
c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 -
a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b
*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*
```

```
b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(5/2)), x)
```

Giac [B] time = 1.19274, size = 393, normalized size = 4.01

$$-\frac{4\sqrt{bd}b^3}{\left(b^2c^2|b|-2\,abcd|b|+a^2d^2|b|\right)\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}+\frac{\sqrt{bx+a}\left(\frac{5\left(b^6c^2d^3|b|-2\,ab^5cd^4|b|+a^2\right.}{b^8c^2d^4-2\,ab^7cd^5+a}\right.}{24\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(b*d)*b^3/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b) + a^2*d^2*abs(b))*(b^2
*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^2)) + 1/24*sqrt(b*x + a)*(5*(b^6*c^2*d^3*abs(b) - 2*a*b^5*c*d^4*abs(b) +
a^2*b^4*d^5*abs(b))*(b*x + a)/(b^8*c^2*d^4 - 2*a*b^7*c*d^5 + a^2*b^6*d^6) +
 6*(b^7*c^3*d^2*abs(b) - 3*a*b^6*c^2*d^3*abs(b) + 3*a^2*b^5*c*d^4*abs(b) -
a^3*b^4*d^5*abs(b))/(b^8*c^2*d^4 - 2*a*b^7*c*d^5 + a^2*b^6*d^6))/(b^2*c + (
b*x + a)*b*d - a*b*d)^(3/2)
```

3.1519 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{32bd^2\sqrt{a+bx}}{3\sqrt{c+dx}(bc-ad)^4}+\frac{16d^2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^3}+\frac{4d}{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(3/2)) + (4*d)/((b*c - a*d)^2*S
qrt[a + b*x]*(c + d*x)^(3/2)) + (16*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^3*(c
+ d*x)^(3/2)) + (32*b*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^4*Sqrt[c + d*x])
```

Rubi [A] time = 0.0284987, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32bd^2\sqrt{a+bx}}{3\sqrt{c+dx}(bc-ad)^4}+\frac{16d^2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^3}+\frac{4d}{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(3/2)) + (4*d)/((b*c - a*d)^2*S
qrt[a + b*x]*(c + d*x)^(3/2)) + (16*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^3*(c
+ d*x)^(3/2)) + (32*b*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^4*Sqrt[c + d*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} - \frac{(2d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx}{bc-ad}$$
$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{4d}{(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/2}} + \frac{\left(8d^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}}\,d}{(bc-ad)^2}$$
$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{4d}{(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/2}} + \frac{16d^2\sqrt{a+bx}}{3(bc-ad)^3(c+dx)^{3/2}}$$
$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{4d}{(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/2}} + \frac{16d^2\sqrt{a+bx}}{3(bc-ad)^3(c+dx)^{3/2}}$$

Mathematica [A] time = 0.0506458, size = 118, normalized size = 0.87

$$\frac{6a^2bd^2(3c+2dx) - 2a^3d^3 + 6ab^2d\left(3c^2+12cdx+8d^2x^2\right) + b^3\left(12c^2dx-2c^3+48cd^2x^2+32d^3x^3\right)}{3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*a^3*d^3 + 6*a^2*b*d^2*(3*c + 2*d*x) + 6*a*b^2*d*(3*c^2 + 12*c*d*x + 8*d
^2*x^2) + b^3*(-2*c^3 + 12*c^2*d*x + 48*c*d^2*x^2 + 32*d^3*x^3))/(3*(b*c -
a*d)^4*(a + b*x)^(3/2)*(c + d*x)^(3/2))
```

Maple [A] time = 0.007, size = 169, normalized size = 1.3

$$-\frac{-32\,b^3d^3x^3 - 48\,ab^2d^3x^2 - 48\,b^3cd^2x^2 - 12\,a^2bd^3x - 72\,ab^2cd^2x - 12\,b^3c^2dx + 2\,a^3d^3 - 18\,a^2bcd^2 - 18\,ab^2c^2d + 2\,b^3c^3}{3\,d^4a^4 - 12\,bd^3ca^3 + 18\,b^2d^2c^2a^2 - 12\,b^3dc^3a + 3\,b^4c^4} (b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(-16*b^3*d^3*x^3-24*a*b^2*d^3*x^2-24*b^3*c*d^2*x^2-6*a^2*b*d^3*x-36*a*
b^2*c*d^2*x-6*b^3*c^2*d*x+a^3*d^3-9*a^2*b*c*d^2-9*a*b^2*c^2*d+b^3*c^3)/(b*x
+a)^(3/2)/(d*x+c)^(3/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^
3*d+b^4*c^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 11.7611, size = 886, normalized size = 6.56

$$\frac{2\left(16\,b^3d^3x^3 - b^3c^3\right.}{3\left(a^2b^4c^6 - 4\,a^3b^3c^5d + 6\,a^4b^2c^4d^2 - 4\,a^5bc^3d^3 + a^6c^2d^4 + \left(b^6c^4d^2 - 4\,ab^5c^3d^3 + 6\,a^2b^4c^2d^4 - 4\,a^3b^3cd^5 + a^4b^2d^6\right)x^4 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*d^3*x^3 - b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 - a^3*d^3 + 2
4*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(b^3*c^2*d + 6*a*b^2*c*d^2 + a^2*b*d^3)*x
)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^
4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*
a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b
^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*
b*d^6)*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*
c^2*d^4 + a^6*d^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2
 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(5/2)), x)
```

Giac [B] time = 1.77136, size = 718, normalized size = 5.32

$$-\frac{\sqrt{bx+a}\left(\frac{8\left(b^7c^3d^4|b|-3\,ab^6c^2d^5|b|+3\,a^2b^5cd^6|b|-a^3b^4d^7|b|\right)(bx+a)}{b^8c^2d^4-2\,ab^7cd^5+a^2b^6d^6}+\frac{9\left(b^8c^4d^3|b|-4\,ab^7c^3d^4|b|+6\,a^2b^6c^2d^5|b|-4\,a^3b^5cd^6|b|+a^4b^4d^7|b|\right)}{b^8c^2d^4-2\,ab^7cd^5+a^2b^6d^6}\right)}{24\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}+\frac{8\left(4\sqrt{bdb}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(b*x + a)*(8*(b^7*c^3*d^4*abs(b) - 3*a*b^6*c^2*d^5*abs(b) + 3*a^2
*b^5*c*d^6*abs(b) - a^3*b^4*d^7*abs(b))*(b*x + a)/(b^8*c^2*d^4 - 2*a*b^7*c*
d^5 + a^2*b^6*d^6) + 9*(b^8*c^4*d^3*abs(b) - 4*a*b^7*c^3*d^4*abs(b) + 6*a^2
*b^6*c^2*d^5*abs(b) - 4*a^3*b^5*c*d^6*abs(b) + a^4*b^4*d^7*abs(b))/(b^8*c^2
*d^4 - 2*a*b^7*c*d^5 + a^2*b^6*d^6))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2)
+ 8/3*(4*sqrt(b*d)*b^7*c^2*d - 8*sqrt(b*d)*a*b^6*c*d^2 + 4*sqrt(b*d)*a^2*b^
5*d^3 - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2*b^5*c*d + 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2*a*b^4*d^2 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^3*d)/((b^3*c^3*abs(b) - 3*a*b^2*c^
2*d*abs(b) + 3*a^2*b*c*d^2*abs(b) - a^3*d^3*abs(b))*(b^2*c - a*b*d - (sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3)
```

3.1520 $\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{256bd^3\sqrt{a+bx}}{15\sqrt{c+dx}(bc-ad)^5}-\frac{128d^3\sqrt{a+bx}}{15(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{5\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)^3}+\frac{16d}{15(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^2}-$$

```
[Out] -2/(5*(b*c - a*d)*(a + b*x)^(5/2)*(c + d*x)^(3/2)) + (16*d)/(15*(b*c - a*d)
^2*(a + b*x)^(3/2)*(c + d*x)^(3/2)) - (32*d^2)/(5*(b*c - a*d)^3*Sqrt[a + b*
x]*(c + d*x)^(3/2)) - (128*d^3*Sqrt[a + b*x])/(15*(b*c - a*d)^4*(c + d*x)^(
3/2)) - (256*b*d^3*Sqrt[a + b*x])/(15*(b*c - a*d)^5*Sqrt[c + d*x])
```

Rubi [A] time = 0.0441092, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{256bd^3\sqrt{a+bx}}{15\sqrt{c+dx}(bc-ad)^5}-\frac{128d^3\sqrt{a+bx}}{15(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{5\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)^3}+\frac{16d}{15(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(5*(b*c - a*d)*(a + b*x)^(5/2)*(c + d*x)^(3/2)) + (16*d)/(15*(b*c - a*d)
^2*(a + b*x)^(3/2)*(c + d*x)^(3/2)) - (32*d^2)/(5*(b*c - a*d)^3*Sqrt[a + b*
x]*(c + d*x)^(3/2)) - (128*d^3*Sqrt[a + b*x])/(15*(b*c - a*d)^4*(c + d*x)^(
3/2)) - (256*b*d^3*Sqrt[a + b*x])/(15*(b*c - a*d)^5*Sqrt[c + d*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx = -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{(8d)\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx}{5(bc-ad)}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{(16d^2)\int \frac{}{(a+bx)^{}}}{5(bc-}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} - \frac{32}{5(bc-ad)^3\sqrt{a}}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} - \frac{32}{5(bc-ad)^3\sqrt{a}}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} - \frac{32}{5(bc-ad)^3\sqrt{a}}$$

Mathematica [A] time = 0.0622627, size = 170, normalized size = 0.99

$$-\frac{2\left(30a^2b^2d^2\left(3c^2+12cdx+8d^2x^2\right)+20a^3bd^3(3c+2dx)-5a^4d^4+20ab^3d\left(6c^2dx-c^3+24cd^2x^2+16d^3x^3\right)+b^4\left(48\right.}{15(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(-5*a^4*d^4 + 20*a^3*b*d^3*(3*c + 2*d*x) + 30*a^2*b^2*d^2*(3*c^2 + 12*c
*d*x + 8*d^2*x^2) + 20*a*b^3*d*(-c^3 + 6*c^2*d*x + 24*c*d^2*x^2 + 16*d^3*x^
3) + b^4*(3*c^4 - 8*c^3*d*x + 48*c^2*d^2*x^2 + 192*c*d^3*x^3 + 128*d^4*x^4)
))/(15*(b*c - a*d)^5*(a + b*x)^(5/2)*(c + d*x)^(3/2))
```

Maple [A] time = 0.01, size = 256, normalized size = 1.5

$$-\frac{-256\,b^4d^4x^4-640\,ab^3d^4x^3-384\,b^4cd^3x^3-480\,a^2b^2d^4x^2-960\,ab^3cd^3x^2-96\,b^4c^2d^2x^2-80\,a^3bd^4x-720\,a^2b^2cd^3x}{15\,a^5d^5-75\,a^4bcd^4+150\,a^3b^2c^2d^3-150\,a^2b^3c^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/15*(-128*b^4*d^4*x^4-320*a*b^3*d^4*x^3-192*b^4*c*d^3*x^3-240*a^2*b^2*d^4
*x^2-480*a*b^3*c*d^3*x^2-48*b^4*c^2*d^2*x^2-40*a^3*b*d^4*x-360*a^2*b^2*c*d^
3*x-120*a*b^3*c^2*d^2*x+8*b^4*c^3*d*x+5*a^4*d^4-60*a^3*b*c*d^3-90*a^2*b^2*c
^2*d^2+20*a*b^3*c^3*d-3*b^4*c^4)/(b*x+a)^(5/2)/(d*x+c)^(3/2)/(a^5*d^5-5*a^4
*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 43.6118, size = 1440, normalized size = 8.37

$$-\frac{}{15\left(a^3b^5c^7 - 5\,a^4b^4c^6d + 10\,a^5b^3c^5d^2 - 10\,a^6b^2c^4d^3 + 5\,a^7bc^3d^4 - a^8c^2d^5 + \left(b^8c^5d^2 - 5\,ab^7c^4d^3 + 10\,a^2b^6c^3d^4 - 10\,a^3b^5c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(128*b^4*d^4*x^4 + 3*b^4*c^4 - 20*a*b^3*c^3*d + 90*a^2*b^2*c^2*d^2 +
60*a^3*b*c*d^3 - 5*a^4*d^4 + 64*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 48*(b^4*c
^2*d^2 + 10*a*b^3*c*d^3 + 5*a^2*b^2*d^4)*x^2 - 8*(b^4*c^3*d - 15*a*b^3*c^2*
d^2 - 45*a^2*b^2*c*d^3 - 5*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b
^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^2*c^4*d^3 + 5*a^7*
b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d
^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7)*x^5 + (2*b^8*c^6*d
 - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^
2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17
*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^
5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a
^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d
^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^
4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 -
2*a^8*c*d^6)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.22649, size = 1362, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x + a)*(11*(b^8*c^4*d^5*abs(b) - 4*a*b^7*c^3*d^6*abs(b) + 6*a^2
*b^6*c^2*d^7*abs(b) - 4*a^3*b^5*c*d^8*abs(b) + a^4*b^4*d^9*abs(b))*(b*x + a
)/(b^8*c^2*d^4 - 2*a*b^7*c*d^5 + a^2*b^6*d^6) + 12*(b^9*c^5*d^4*abs(b) - 5*
a*b^8*c^4*d^5*abs(b) + 10*a^2*b^7*c^3*d^6*abs(b) - 10*a^3*b^6*c^2*d^7*abs(b
) + 5*a^4*b^5*c*d^8*abs(b) - a^5*b^4*d^9*abs(b))/(b^8*c^2*d^4 - 2*a*b^7*c*d
^5 + a^2*b^6*d^6))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) - 4/15*(73*sqrt(b*
```

```
d)*b^11*c^4*d^2 - 292*sqrt(b*d)*a*b^10*c^3*d^3 + 438*sqrt(b*d)*a^2*b^9*c^2*
d^4 - 292*sqrt(b*d)*a^3*b^8*c*d^5 + 73*sqrt(b*d)*a^4*b^7*d^6 - 320*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^9*c^3
*d^2 + 960*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^2*a*b^8*c^2*d^3 - 960*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b
^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^7*c*d^4 + 320*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^6*d^5 + 490*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^
7*c^2*d^2 - 980*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)
*b*d - a*b*d))^4*a*b^6*c*d^3 + 490*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^5*d^4 - 240*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^5*c*d^2 + 240*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b
^4*d^3 + 45*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d))^8*b^3*d^2)/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b) + 6*a^2*b^2*c
^2*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*(b^2*c - a*b*d - (sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5)
```

3.1521 $\int \frac{1}{(a+bx)^{9/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=207

$$\frac{512bd^4\sqrt{a+bx}}{21\sqrt{c+dx}(bc-ad)^6}+\frac{256d^4\sqrt{a+bx}}{21(c+dx)^{3/2}(bc-ad)^5}+\frac{64d^3}{7\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{21(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^3}+\frac{}{7(}$$

```
[Out] -2/(7*(b*c - a*d)*(a + b*x)^(7/2)*(c + d*x)^(3/2)) + (4*d)/(7*(b*c - a*d)^2
*(a + b*x)^(5/2)*(c + d*x)^(3/2)) - (32*d^2)/(21*(b*c - a*d)^3*(a + b*x)^(3
/2)*(c + d*x)^(3/2)) + (64*d^3)/(7*(b*c - a*d)^4*Sqrt[a + b*x]*(c + d*x)^(3
/2)) + (256*d^4*Sqrt[a + b*x])/(21*(b*c - a*d)^5*(c + d*x)^(3/2)) + (512*b*
d^4*Sqrt[a + b*x])/(21*(b*c - a*d)^6*Sqrt[c + d*x])
```

Rubi [A] time = 0.0610089, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{512bd^4\sqrt{a+bx}}{21\sqrt{c+dx}(bc-ad)^6}+\frac{256d^4\sqrt{a+bx}}{21(c+dx)^{3/2}(bc-ad)^5}+\frac{64d^3}{7\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{21(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^3}+\frac{}{7(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(7*(b*c - a*d)*(a + b*x)^(7/2)*(c + d*x)^(3/2)) + (4*d)/(7*(b*c - a*d)^2
*(a + b*x)^(5/2)*(c + d*x)^(3/2)) - (32*d^2)/(21*(b*c - a*d)^3*(a + b*x)^(3
/2)*(c + d*x)^(3/2)) + (64*d^3)/(7*(b*c - a*d)^4*Sqrt[a + b*x]*(c + d*x)^(3
/2)) + (256*d^4*Sqrt[a + b*x])/(21*(b*c - a*d)^5*(c + d*x)^(3/2)) + (512*b*
d^4*Sqrt[a + b*x])/(21*(b*c - a*d)^6*Sqrt[c + d*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}(c+dx)^{5/2}}\,dx = -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} - \frac{(10d)\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx}{7(bc-ad)}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{(16d^2)\int \frac{1}{(a+bx)^{5/2}\cdots}}{7(bc-a\cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{32\cdots}{21(bc-ad)^3(a+\cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{32\cdots}{21(bc-ad)^3(a+\cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{32\cdots}{21(bc-ad)^3(a+\cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{32\cdots}{21(bc-ad)^3(a+\cdots}$$

Mathematica [A] time = 0.0785279, size = 233, normalized size = 1.13

$$\frac{2\left(70a^2b^3d^2\left(6c^2dx-c^3+24cd^2x^2+16d^3x^3\right)+70a^3b^2d^3\left(3c^2+12cdx+8d^2x^2\right)+35a^4bd^4(3c+2dx)-7a^5d^5+7ab^4\cdots\right.}{21(a+bx)^{7/2}(c+\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (2*(-7*a^5*d^5 + 35*a^4*b*d^4*(3*c + 2*d*x) + 70*a^3*b^2*d^3*(3*c^2 + 12*c*
d*x + 8*d^2*x^2) + 70*a^2*b^3*d^2*(-c^3 + 6*c^2*d*x + 24*c*d^2*x^2 + 16*d^3
*x^3) + 7*a*b^4*d*(3*c^4 - 8*c^3*d*x + 48*c^2*d^2*x^2 + 192*c*d^3*x^3 + 128
*d^4*x^4) + b^5*(-3*c^5 + 6*c^4*d*x - 16*c^3*d^2*x^2 + 96*c^2*d^3*x^3 + 384
*c*d^4*x^4 + 256*d^5*x^5)))/(21*(b*c - a*d)^6*(a + b*x)^(7/2)*(c + d*x)^(3/
2))
```

Maple [B] time = 0.011, size = 356, normalized size = 1.7

$$-\frac{-512\,b^5d^5x^5-1792\,ab^4d^5x^4-768\,b^5cd^4x^4-2240\,a^2b^3d^5x^3-2688\,ab^4cd^4x^3-192\,b^5c^2d^3x^3-1120\,a^3b^2d^5x^2-3360\cdots}{21\,d^6a^6\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/21*(-256*b^5*d^5*x^5-896*a*b^4*d^5*x^4-384*b^5*c*d^4*x^4-1120*a^2*b^3*d^
5*x^3-1344*a*b^4*c*d^4*x^3-96*b^5*c^2*d^3*x^3-560*a^3*b^2*d^5*x^2-1680*a^2*
b^3*c*d^4*x^2-336*a*b^4*c^2*d^3*x^2+16*b^5*c^3*d^2*x^2-70*a^4*b*d^5*x-840*a
^3*b^2*c*d^4*x-420*a^2*b^3*c^2*d^3*x+56*a*b^4*c^3*d^2*x-6*b^5*c^4*d*x+7*a^5
*d^5-105*a^4*b*c*d^4-210*a^3*b^2*c^2*d^3+70*a^2*b^3*c^3*d^2-21*a*b^4*c^4*d+
3*b^5*c^5)/(b*x+a)^(7/2)/(d*x+c)^(3/2)/(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^
2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 97.4435, size = 2045, normalized size = 9.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(256*b^5*d^5*x^5 - 3*b^5*c^5 + 21*a*b^4*c^4*d - 70*a^2*b^3*c^3*d^2 + 2
10*a^3*b^2*c^2*d^3 + 105*a^4*b*c*d^4 - 7*a^5*d^5 + 128*(3*b^5*c*d^4 + 7*a*b
^4*d^5)*x^4 + 32*(3*b^5*c^2*d^3 + 42*a*b^4*c*d^4 + 35*a^2*b^3*d^5)*x^3 - 16
*(b^5*c^3*d^2 - 21*a*b^4*c^2*d^3 - 105*a^2*b^3*c*d^4 - 35*a^3*b^2*d^5)*x^2
+ 2*(3*b^5*c^4*d - 28*a*b^4*c^3*d^2 + 210*a^2*b^3*c^2*d^3 + 420*a^3*b^2*c*d
^4 + 35*a^4*b*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^6*c^8 - 6*a^5*b^5*
c^7*d + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4 - 6*a^
9*b*c^3*d^5 + a^10*c^2*d^6 + (b^10*c^6*d^2 - 6*a*b^9*c^5*d^3 + 15*a^2*b^8*c
^4*d^4 - 20*a^3*b^7*c^3*d^5 + 15*a^4*b^6*c^2*d^6 - 6*a^5*b^5*c*d^7 + a^6*b^
4*d^8)*x^6 + 2*(b^10*c^7*d - 4*a*b^9*c^6*d^2 + 3*a^2*b^8*c^5*d^3 + 10*a^3*b
^7*c^4*d^4 - 25*a^4*b^6*c^3*d^5 + 24*a^5*b^5*c^2*d^6 - 11*a^6*b^4*c*d^7 + 2
*a^7*b^3*d^8)*x^5 + (b^10*c^8 + 2*a*b^9*c^7*d - 27*a^2*b^8*c^6*d^2 + 64*a^3
*b^7*c^5*d^3 - 55*a^4*b^6*c^4*d^4 - 6*a^5*b^5*c^3*d^5 + 43*a^6*b^4*c^2*d^6
- 28*a^7*b^3*c*d^7 + 6*a^8*b^2*d^8)*x^4 + 4*(a*b^9*c^8 - 3*a^2*b^8*c^7*d -
2*a^3*b^7*c^6*d^2 + 19*a^4*b^6*c^5*d^3 - 30*a^5*b^5*c^4*d^4 + 19*a^6*b^4*c^
3*d^5 - 2*a^7*b^3*c^2*d^6 - 3*a^8*b^2*c*d^7 + a^9*b*d^8)*x^3 + (6*a^2*b^8*c
^8 - 28*a^3*b^7*c^7*d + 43*a^4*b^6*c^6*d^2 - 6*a^5*b^5*c^5*d^3 - 55*a^6*b^4
*c^4*d^4 + 64*a^7*b^3*c^3*d^5 - 27*a^8*b^2*c^2*d^6 + 2*a^9*b*c*d^7 + a^10*d
^8)*x^2 + 2*(2*a^3*b^7*c^8 - 11*a^4*b^6*c^7*d + 24*a^5*b^5*c^6*d^2 - 25*a^6
*b^4*c^5*d^3 + 10*a^7*b^3*c^4*d^4 + 3*a^8*b^2*c^3*d^5 - 4*a^9*b*c^2*d^6 + a
^10*c*d^7)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.44747, size = 2314, normalized size = 11.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(b*x + a)*(14*(b^9*c^5*d^6*abs(b) - 5*a*b^8*c^4*d^7*abs(b) + 10*a
^2*b^7*c^3*d^8*abs(b) - 10*a^3*b^6*c^2*d^9*abs(b) + 5*a^4*b^5*c*d^10*abs(b)
 - a^5*b^4*d^11*abs(b))*(b*x + a)/(b^8*c^2*d^4 - 2*a*b^7*c*d^5 + a^2*b^6*d^
6) + 15*(b^10*c^6*d^5*abs(b) - 6*a*b^9*c^5*d^6*abs(b) + 15*a^2*b^8*c^4*d^7*
abs(b) - 20*a^3*b^7*c^3*d^8*abs(b) + 15*a^4*b^6*c^2*d^9*abs(b) - 6*a^5*b^5*
c*d^10*abs(b) + a^6*b^4*d^11*abs(b))/(b^8*c^2*d^4 - 2*a*b^7*c*d^5 + a^2*b^6
*d^6))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) + 8/21*(79*sqrt(b*d)*b^15*c^6*
d^3 - 474*sqrt(b*d)*a*b^14*c^5*d^4 + 1185*sqrt(b*d)*a^2*b^13*c^4*d^5 - 1580
*sqrt(b*d)*a^3*b^12*c^3*d^6 + 1185*sqrt(b*d)*a^4*b^11*c^2*d^7 - 474*sqrt(b*
d)*a^5*b^10*c*d^8 + 79*sqrt(b*d)*a^6*b^9*d^9 - 511*sqrt(b*d)*(sqrt(b*d)*sqr
t(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^13*c^5*d^3 + 2555*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*
b^12*c^4*d^4 - 5110*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
+ a)*b*d - a*b*d))^2*a^2*b^11*c^3*d^5 + 5110*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^10*c^2*d^6 - 2555*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*
b^9*c*d^7 + 511*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)
*b*d - a*b*d))^2*a^5*b^8*d^8 + 1344*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^11*c^4*d^3 - 5376*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^10*c^3*d^4 +
8064*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*a^2*b^9*c^2*d^5 - 5376*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*a^3*b^8*c*d^6 + 1344*sqrt(b*d)*(sqrt(b*d)*sqr
t(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^7*d^7 - 1750*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^9
*c^3*d^3 + 5250*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)
*b*d - a*b*d))^6*a*b^8*c^2*d^4 - 5250*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^7*c*d^5 + 1750*sqrt(b*d)*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^6*d^6 +
1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^8*b^7*c^2*d^3 - 2030*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^8*a*b^6*c*d^4 + 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^5*d^5 - 315*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^5*c*d^3
 + 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^10*a*b^4*d^4 + 42*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^12*b^3*d^3)/((b^5*c^5*abs(b) - 5*a*b^4*c^4*d*abs(b)
+ 10*a^2*b^3*c^3*d^2*abs(b) - 10*a^3*b^2*c^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs
(b) - a^5*d^5*abs(b))*(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^2)^7)
```

3.1522 $\int \frac{1}{\sqrt{a+bx}\sqrt{4+a+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}$$

```
[Out] (2*ArcSinh[Sqrt[a + b*x]/2])/b
```

Rubi [A] time = 0.0061986, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[4 + a + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[a + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx}\sqrt{4+a+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{a+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0089983, size = 19, normalized size = 1.

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[4 + a + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[a + b*x]/2])/b
```

Maple [B] time = 0.008, size = 86, normalized size = 4.5

$$\sqrt{(bx+a)(bx+a+4)}\ln\left(\left(\frac{ab}{2}+\frac{b(a+4)}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+(ab+b(a+4))x+a(a+4)}\right)\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{bx+a+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x)
```

```
[Out] ((b*x+a)*(b*x+a+4))^(1/2)/(b*x+a)^(1/2)/(b*x+a+4)^(1/2)*ln((1/2*a*b+1/2*b*(
a+4)+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*(a+4))*x+a*(a+4))^(1/2))/(b^2)^(1/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.10401, size = 76, normalized size = 4.

$$-\frac{\log\left(-bx+\sqrt{bx+a+4}\sqrt{bx+a}-a-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + a + 4)*sqrt(b*x + a) - a - 2)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+bx}\sqrt{a+bx+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(b*x+a+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*sqrt(a + b*x + 4)), x)
```

Giac [A] time = 1.24562, size = 34, normalized size = 1.79

$$-\frac{2\,\log\left(\left|-\sqrt{bx+a+4}+\sqrt{bx+a}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + a + 4) + sqrt(b*x + a)))/b
```

3.1523 $\int \frac{1}{\sqrt{2+bx}\sqrt{6+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+2}\right)}{b}$$

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]/2])/b
```

Rubi [A] time = 0.0049453, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + b*x]*Sqrt[6 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+bx}\sqrt{6+bx}}\,dx &= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{2+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{2+bx}\right)}{b}\end{aligned}$$

Mathematica [B] time = 0.0105344, size = 39, normalized size = 2.05

$$\frac{2\sqrt{bx+2}\sin^{-1}\left(\frac{1}{2}\sqrt{-bx-2}\right)}{b\sqrt{-bx-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + b*x]*Sqrt[6 + b*x]),x]
```

```
[Out] (2*Sqrt[2 + b*x]*ArcSin[Sqrt[-2 - b*x]/2])/(b*Sqrt[-2 - b*x])
```

Maple [B] time = 0.006, size = 66, normalized size = 3.5

$$\sqrt{(bx+2)(bx+6)}\ln\left((b^2x+4b)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+8bx+12}\right)\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{bx+6}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x)
```

```
[Out] ((b*x+2)*(b*x+6))^(1/2)/(b*x+2)^(1/2)/(b*x+6)^(1/2)*ln((b^2*x+4*b)/(b^2)^(1
/2)+(b^2*x^2+8*b*x+12)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05807, size = 65, normalized size = 3.42

$$-\frac{\log\left(-bx+\sqrt{bx+6}\sqrt{bx+2}-4\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 6)*sqrt(b*x + 2) - 4)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx+2}\sqrt{bx+6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(1/2)/(b*x+6)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 2)*sqrt(b*x + 6)), x)
```

Giac [A] time = 1.14252, size = 32, normalized size = 1.68

$$-\frac{2 \log\left(\left|-\sqrt{bx+6}+\sqrt{bx+2}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + 6) + sqrt(b*x + 2)))/b
```

3.1524 $\int \frac{1}{\sqrt{1+bx}\sqrt{5+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+1}\right)}{b}$$

```
[Out] (2*ArcSinh[Sqrt[1 + b*x]/2])/b
```

Rubi [A] time = 0.0049356, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + b*x]*Sqrt[5 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[1 + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+bx}\sqrt{5+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{1+bx}\right)}{b}\end{aligned}$$

Mathematica [B] time = 0.0113841, size = 39, normalized size = 2.05

$$\frac{2\sqrt{bx+1}\sin^{-1}\left(\frac{1}{2}\sqrt{-bx-1}\right)}{b\sqrt{-bx-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x]*Sqrt[5 + b*x]),x]
```

```
[Out] (2*Sqrt[1 + b*x]*ArcSin[Sqrt[-1 - b*x]/2])/(b*Sqrt[-1 - b*x])
```

Maple [B] time = 0.007, size = 66, normalized size = 3.5

$$\sqrt{(bx+1)(bx+5)}\ln\left((b^2x+3b)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+6bx+5}\right)\frac{1}{\sqrt{bx+1}}\frac{1}{\sqrt{bx+5}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x)
```

```
[Out] ((b*x+1)*(b*x+5))^(1/2)/(b*x+1)^(1/2)/(b*x+5)^(1/2)*ln((b^2*x+3*b)/(b^2)^(1
/2)+(b^2*x^2+6*b*x+5)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13267, size = 65, normalized size = 3.42

$$-\frac{\log\left(-bx+\sqrt{bx+5}\sqrt{bx+1}-3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 5)*sqrt(b*x + 1) - 3)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx+1}\sqrt{bx+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)**(1/2)/(b*x+5)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 1)*sqrt(b*x + 5)), x)
```

Giac [A] time = 1.14361, size = 32, normalized size = 1.68

$$-\frac{2 \log\left(\left|-\sqrt{bx+5}+\sqrt{bx+1}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + 5) + sqrt(b*x + 1)))/b
```

3.1525 $\int \frac{1}{\sqrt{bx}\sqrt{4+bx}}\,dx$

Optimal. Leaf size=17

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}$$

```
[Out] (2*ArcSinh[Sqrt[b*x]/2])/b
```

Rubi [A] time = 0.0038164, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx}\sqrt{4+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0100814, size = 34, normalized size = 2.

$$\frac{2\sqrt{x}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}\sqrt{bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (2*Sqrt[x]*ArcSinh[(Sqrt[b]*Sqrt[x])/2])/(Sqrt[b]*Sqrt[b*x])
```

Maple [B] time = 0.006, size = 60, normalized size = 3.5

$$\sqrt{bx\,(bx+4)}\ln\left((b^2x+2\,b)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+4\,bx}\right)\frac{1}{\sqrt{bx}}\frac{1}{\sqrt{bx+4}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/2)/(b*x+4)^(1/2),x)
```

```
[Out] (b*x*(b*x+4))^(1/2)/(b*x)^(1/2)/(b*x+4)^(1/2)*ln((b^2*x+2*b)/(b^2)^(1/2)+(b
^2*x^2+4*b*x)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11496, size = 59, normalized size = 3.47

$$-\frac{\log\left(-bx+\sqrt{bx+4}\sqrt{bx}-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 4)*sqrt(b*x) - 2)/b
```

Sympy [A] time = 1.36611, size = 15, normalized size = 0.88

$$\frac{2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)**(1/2)/(b*x+4)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/2)/b
```

Giac [A] time = 1.1591, size = 30, normalized size = 1.76

$$-\frac{2 \log\left(\left|-\sqrt{bx+4}+\sqrt{bx}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + 4) + sqrt(b*x)))/b
```

3.1526 $\int \frac{1}{\sqrt{-1+bx}\sqrt{3+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-1}\right)}{b}$$

```
[Out] (2*ArcSinh[Sqrt[-1 + b*x]/2])/b
```

Rubi [A] time = 0.0050002, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + b*x]*Sqrt[3 + b*x]),x]

[Out] (2*ArcSinh[Sqrt[-1 + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+bx}\sqrt{3+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{-1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-1+bx}\right)}{b}\end{aligned}$$

Mathematica [B] time = 0.0115028, size = 39, normalized size = 2.05

$$\frac{2\sqrt{bx-1}\sin^{-1}\left(\frac{1}{2}\sqrt{1-bx}\right)}{b\sqrt{1-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*Sqrt[-1 + b*x]*ArcSin[Sqrt[1 - b*x]/2])/(b*Sqrt[1 - b*x])
```

Maple [B] time = 0.006, size = 64, normalized size = 3.4

$$\sqrt{(bx-1)(bx+3)}\ln\left((b^2x+b)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+2bx-3}\right)\frac{1}{\sqrt{bx-1}}\frac{1}{\sqrt{bx+3}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x)
```

```
[Out] ((b*x-1)*(b*x+3))^(1/2)/(b*x-1)^(1/2)/(b*x+3)^(1/2)*ln((b^2*x+b)/(b^2)^(1/2
)+(b^2*x^2+2*b*x-3)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95455, size = 65, normalized size = 3.42

$$-\frac{\log\left(-bx+\sqrt{bx+3}\sqrt{bx-1}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 3)*sqrt(b*x - 1) - 1)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx-1}\sqrt{bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)**(1/2)/(b*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 1)*sqrt(b*x + 3)), x)
```

Giac [A] time = 1.14172, size = 32, normalized size = 1.68

$$-\frac{2 \log \left(\left|-\sqrt{bx+3}+\sqrt{bx-1}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + 3) + sqrt(b*x - 1)))/b
```

3.1527 $\int \frac{1}{\sqrt{-2+bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

```
[Out] ArcCosh[(b*x)/2]/b
```

Rubi [A] time = 0.0026196, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {52}

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcCosh[(b*x)/2]/b
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+bx}\sqrt{2+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Mathematica [B] time = 0.0044345, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx-2}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[-2 + b*x]/Sqrt[2 + b*x]])/b
```

Maple [B] time = 0.004, size = 57, normalized size = 5.2

$$\sqrt{(bx-2)(bx+2)}\ln\left(b^2x\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2-4}\right)\frac{1}{\sqrt{bx-2}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((b*x-2)*(b*x+2))^(1/2)/(b*x-2)^(1/2)/(b*x+2)^(1/2)*ln(b^2*x/(b^2)^(1/2)+(b
^2*x^2-4)^(1/2))/(b^2)^(1/2)
```

Maxima [B] time = 0.956742, size = 43, normalized size = 3.91

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-4}\sqrt{b^2}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*sqrt(b^2))/sqrt(b^2)
```

Fricas [B] time = 2.00157, size = 59, normalized size = 5.36

$$-\frac{\log\left(-bx+\sqrt{bx+2}\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x - 2))/b
```

Sympy [C] time = 3.14773, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{4e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4*e
xp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4/(b**2*x**2))/(4*pi**(3
/2)*b)
```

Giac [B] time = 1.10963, size = 32, normalized size = 2.91

$$-\frac{2\,\log\left(\left|-\sqrt{bx+2}+\sqrt{bx-2}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + 2) + sqrt(b*x - 2)))/b
```

3.1528 $\int \frac{1}{\sqrt{-3+bx}\sqrt{1+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-3}\right)}{b}$$

```
[Out] (2*ArcSinh[Sqrt[-3 + b*x]/2])/b
```

Rubi [A] time = 0.0049623, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-3}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 + b*x]*Sqrt[1 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-3 + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+bx}\sqrt{1+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{-3+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-3+bx}\right)}{b}\end{aligned}$$

Mathematica [B] time = 0.0100298, size = 39, normalized size = 2.05

$$\frac{2\sqrt{bx-3}\sin^{-1}\left(\frac{1}{2}\sqrt{3-bx}\right)}{b\sqrt{3-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + b*x]*Sqrt[1 + b*x]),x]
```

```
[Out] (2*Sqrt[-3 + b*x]*ArcSin[Sqrt[3 - b*x]/2])/(b*Sqrt[3 - b*x])
```

Maple [B] time = 0.007, size = 66, normalized size = 3.5

$$\sqrt{(bx-3)(bx+1)}\ln\left((b^2x-b)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2-2\,bx-3}\right)\frac{1}{\sqrt{bx-3}}\frac{1}{\sqrt{bx+1}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x)
```

```
[Out] ((b*x-3)*(b*x+1))^(1/2)/(b*x-3)^(1/2)/(b*x+1)^(1/2)*ln((b^2*x-b)/(b^2)^(1/2
)+(b^2*x^2-2*b*x-3)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97952, size = 65, normalized size = 3.42

$$-\frac{\log\left(-bx+\sqrt{bx+1}\sqrt{bx-3}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 1)*sqrt(b*x - 3) + 1)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx-3}\sqrt{bx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)**(1/2)/(b*x+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 3)*sqrt(b*x + 1)), x)
```

Giac [A] time = 1.15403, size = 32, normalized size = 1.68

$$-\frac{2\,\log\left(\left|-\sqrt{bx+1}+\sqrt{bx-3}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + 1) + sqrt(b*x - 3)))/b
```

3.1529 $\int \frac{1}{\sqrt{2+bx}\sqrt{3+bx}}\,dx$

Optimal. Leaf size=15

$$\frac{2\sinh^{-1}\left(\sqrt{bx+2}\right)}{b}$$

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]])/b
```

Rubi [A] time = 0.004099, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\sqrt{bx+2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+bx}\sqrt{3+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\sqrt{2+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0043638, size = 15, normalized size = 1.

$$\frac{2\sinh^{-1}\left(\sqrt{bx+2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]])/b
```

Maple [B] time = 0.006, size = 66, normalized size = 4.4

$$\sqrt{(bx+2)(bx+3)}\ln\left(\left(\frac{5b}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+5bx+6}\right)\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{bx+3}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x)
```

```
[Out] ((b*x+2)*(b*x+3))^(1/2)/(b*x+2)^(1/2)/(b*x+3)^(1/2)*ln((5/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+5*b*x+6)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.02501, size = 70, normalized size = 4.67

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+3}\sqrt{bx+2}-5\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b*x + 3)*sqrt(b*x + 2) - 5)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx+2}\sqrt{bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(1/2)/(b*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 2)*sqrt(b*x + 3)), x)
```

Giac [A] time = 1.13573, size = 32, normalized size = 2.13

$$-\frac{2 \log\left(\left|-\sqrt{bx+3}+\sqrt{bx+2}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + 3) + sqrt(b*x + 2)))/b
```

3.1530 $\int \frac{1}{2+bx} dx$

Optimal. Leaf size=10

$$\frac{\log(bx+2)}{b}$$

```
[Out] Log[2 + b*x]/b
```

Rubi [A] time = 0.0013725, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(bx+2)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(-1),x]
```

```
[Out] Log[2 + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+bx} dx = \frac{\log(2+bx)}{b}$$

Mathematica [A] time = 0.0009751, size = 10, normalized size = 1.

$$\frac{\log(bx+2)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(-1),x]
```

```
[Out] Log[2 + b*x]/b
```

Maple [A] time = 0.001, size = 11, normalized size = 1.1

$$\frac{\ln(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2),x)
```

```
[Out] ln(b*x+2)/b
```

Maxima [A] time = 0.947636, size = 14, normalized size = 1.4

$$\frac{\log(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="maxima")
```

```
[Out] log(b*x + 2)/b
```

Fricas [A] time = 1.90311, size = 22, normalized size = 2.2

$$\frac{\log(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="fricas")
```

```
[Out] log(b*x + 2)/b
```

Sympy [A] time = 0.058025, size = 7, normalized size = 0.7

$$\frac{\log(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x)
```

```
[Out] log(b*x + 2)/b
```

Giac [A] time = 1.06089, size = 15, normalized size = 1.5

$$\frac{\log(|bx+2|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="giac")
```

```
[Out] log(abs(b*x + 2))/b
```

3.1531 $\int \frac{1}{\sqrt{1+bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=15

$$\frac{2\sinh^{-1}\left(\sqrt{bx+1}\right)}{b}$$

```
[Out] (2*ArcSinh[Sqrt[1 + b*x]])/b
```

Rubi [A] time = 0.0040925, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\sqrt{bx+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[1 + b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+bx}\sqrt{2+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\sqrt{1+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0042996, size = 15, normalized size = 1.

$$\frac{2\sinh^{-1}\left(\sqrt{bx+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[1 + b*x]])/b
```

Maple [B] time = 0.005, size = 66, normalized size = 4.4

$$\sqrt{(bx+1)(bx+2)}\ln\left(\left(\frac{3b}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+3bx+2}\right)\frac{1}{\sqrt{bx+1}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((b*x+1)*(b*x+2))^(1/2)/(b*x+1)^(1/2)/(b*x+2)^(1/2)*ln((3/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+3*b*x+2)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03242, size = 70, normalized size = 4.67

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+2}\sqrt{bx+1}-3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x + 1) - 3)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx+1}\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 1)*sqrt(b*x + 2)), x)
```

Giac [A] time = 1.12913, size = 32, normalized size = 2.13

$$-\frac{2 \log\left(\left|-\sqrt{bx+2}+\sqrt{bx+1}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + 2) + sqrt(b*x + 1)))/b
```

3.1532 $\int \frac{1}{\sqrt{bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}$$

```
[Out] (2*ArcSinh[Sqrt[b*x]/Sqrt[2]])/b
```

Rubi [A] time = 0.004641, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[b*x]/Sqrt[2]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx}\sqrt{2+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \sqrt{bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0078384, size = 36, normalized size = 1.89

$$\frac{2\sqrt{x}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}\sqrt{bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*Sqrt[x]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(Sqrt[b]*Sqrt[b*x])
```

Maple [B] time = 0.005, size = 58, normalized size = 3.1

$$\sqrt{bx\,(bx+2)}\ln\left((b^2x+b)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+2\,bx}\right)\frac{1}{\sqrt{bx}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] (b*x*(b*x+2))^(1/2)/(b*x)^(1/2)/(b*x+2)^(1/2)*ln((b^2*x+b)/(b^2)^(1/2)+(b^2
*x^2+2*b*x)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96446, size = 59, normalized size = 3.11

$$-\frac{\log\left(-bx+\sqrt{bx+2}\sqrt{bx}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x) - 1)/b
```

Sympy [A] time = 1.4215, size = 20, normalized size = 1.05

$$\frac{2\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b
```

Giac [A] time = 1.13851, size = 30, normalized size = 1.58

$$-\frac{2 \log \left(\left|-\sqrt{bx+2}+\sqrt{bx}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + 2) + sqrt(b*x)))/b
```

3.1533 $\int \frac{1}{\sqrt{-1+bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-1}}{\sqrt{3}}\right)}{b}$$

```
[Out] (2*ArcSinh[Sqrt[-1 + b*x]/Sqrt[3]])/b
```

Rubi [A] time = 0.0062222, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-1}}{\sqrt{3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-1 + b*x]/Sqrt[3]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+bx}\sqrt{2+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{-1+bx}}{\sqrt{3}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0089457, size = 41, normalized size = 1.95

$$\frac{2\sqrt{bx-1}\sin^{-1}\left(\frac{\sqrt{1-bx}}{\sqrt{3}}\right)}{b\sqrt{1-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*Sqrt[-1 + b*x]*ArcSin[Sqrt[1 - b*x]/Sqrt[3]])/(b*Sqrt[1 - b*x])
```

Maple [B] time = 0.006, size = 65, normalized size = 3.1

$$\sqrt{(bx-1)(bx+2)}\ln\left(\left(\frac{b}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+bx-2}\right)\frac{1}{\sqrt{bx-1}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((b*x-1)*(b*x+2))^(1/2)/(b*x-1)^(1/2)/(b*x+2)^(1/2)*ln((1/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+b*x-2)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95051, size = 70, normalized size = 3.33

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+2}\sqrt{bx-1}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x - 1) - 1)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx-1}\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 1)*sqrt(b*x + 2)), x)
```

Giac [A] time = 1.15204, size = 32, normalized size = 1.52

$$-\frac{2 \log \left(\left|-\sqrt{bx+2}+\sqrt{bx-1}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + 2) + sqrt(b*x - 1)))/b
```

3.1534 $\int \frac{1}{\sqrt{-2+bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

```
[Out] ArcCosh[(b*x)/2]/b
```

Rubi [A] time = 0.0022657, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {52}

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcCosh[(b*x)/2]/b
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+bx}\sqrt{2+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Mathematica [B] time = 0.002413, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx-2}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[-2 + b*x]/Sqrt[2 + b*x]])/b
```

Maple [B] time = 0., size = 57, normalized size = 5.2

$$\sqrt{(bx-2)(bx+2)}\ln\left(b^2x\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2-4}\right)\frac{1}{\sqrt{bx-2}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((b*x-2)*(b*x+2))^(1/2)/(b*x-2)^(1/2)/(b*x+2)^(1/2)*ln(b^2*x/(b^2)^(1/2)+(b
^2*x^2-4)^(1/2))/(b^2)^(1/2)
```

Maxima [B] time = 0.98159, size = 43, normalized size = 3.91

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-4}\sqrt{b^2}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*sqrt(b^2))/sqrt(b^2)
```

Fricas [B] time = 2.04784, size = 59, normalized size = 5.36

$$-\frac{\log\left(-bx+\sqrt{bx+2}\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x - 2))/b
```

Sympy [C] time = 3.18175, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{4e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4*e
xp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4/(b**2*x**2))/(4*pi**(3
/2)*b)
```

Giac [B] time = 1.10346, size = 32, normalized size = 2.91

$$-\frac{2\,\log\left(\left|-\sqrt{bx+2}+\sqrt{bx-2}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + 2) + sqrt(b*x - 2)))/b
```

3.1535 $\int \frac{1}{\sqrt{-3+bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-3}}{\sqrt{5}}\right)}{b}$$

```
[Out] (2*ArcSinh[Sqrt[-3 + b*x]/Sqrt[5]])/b
```

Rubi [A] time = 0.0057734, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-3}}{\sqrt{5}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-3 + b*x]/Sqrt[5]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+bx}\sqrt{2+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, \sqrt{-3+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{-3+bx}}{\sqrt{5}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0097141, size = 41, normalized size = 1.95

$$\frac{2\sqrt{bx-3}\sin^{-1}\left(\frac{\sqrt{3-bx}}{\sqrt{5}}\right)}{b\sqrt{3-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*Sqrt[-3 + b*x]*ArcSin[Sqrt[3 - b*x]/Sqrt[5]])/(b*Sqrt[3 - b*x])
```

Maple [B] time = 0.007, size = 66, normalized size = 3.1

$$\sqrt{(bx-3)(bx+2)}\,\ln\left(\left(-\frac{b}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2-bx-6}\right)\frac{1}{\sqrt{bx-3}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((b*x-3)*(b*x+2))^(1/2)/(b*x-3)^(1/2)/(b*x+2)^(1/2)*ln((-1/2*b+b^2*x)/(b^2)
^(1/2)+(b^2*x^2-b*x-6)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98939, size = 70, normalized size = 3.33

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+2}\sqrt{bx-3}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x - 3) + 1)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx-3}\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 3)*sqrt(b*x + 2)), x)
```

Giac [A] time = 1.14834, size = 32, normalized size = 1.52

$$-\frac{2\log\left(\left|-\sqrt{bx+2}+\sqrt{bx-3}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + 2) + sqrt(b*x - 3)))/b
```

3.1536 $\int \frac{1}{\sqrt{3-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}$$

```
[Out] -(ArcSin[(1 - 2*b*x)/5]/b)
```

Rubi [A] time = 0.0137068, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {53, 619, 216}

$$-\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - b*x]*Sqrt[2 + b*x]),x]

[Out] -(ArcSin[(1 - 2*b*x)/5]/b)
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3-bx}\sqrt{2+bx}}\,dx &= \int \frac{1}{\sqrt{6+bx-b^2x^2}}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{25b^2}}}\,dx, x, b-2b^2x\right)}{5b^2}\\
&= -\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.01148, size = 22, normalized size = 1.38

$$-\frac{2\sin^{-1}\left(\frac{\sqrt{3-bx}}{\sqrt{5}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcSin[Sqrt[3 - b*x]/Sqrt[5]])/b
```

Maple [B] time = 0.008, size = 65, normalized size = 4.1

$$\sqrt{(-bx+3)(bx+2)}\arctan\left(\sqrt{b^2}\left(x-\frac{1}{2b}\right)\frac{1}{\sqrt{-b^2x^2+bx+6}}\right)\frac{1}{\sqrt{-bx+3}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((-b*x+3)*(b*x+2))^(1/2)/(-b*x+3)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x-1/2/b)/(-b^2*x^2+b*x+6)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.01378, size = 104, normalized size = 6.5

$$-\frac{\arctan\left(\frac{(2\,bx-1)\sqrt{bx+2}\sqrt{-bx+3}}{2\left(b^2x^2-bx-6\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x - 1)*sqrt(b*x + 2)*sqrt(-b*x + 3)/(b^2*x^2 - b*x - 6))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-bx+3}\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 3)*sqrt(b*x + 2)), x)
```

Giac [A] time = 1.07897, size = 24, normalized size = 1.5

$$\frac{2 \arcsin\left(\frac{1}{5}\sqrt{5}\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/5*sqrt(5)*sqrt(b*x + 2))/b
```

3.1537 $\int \frac{1}{\sqrt{2-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}$$

```
[Out] ArcSin[(b*x)/2]/b
```

Rubi [A] time = 0.0033367, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {41, 216}

$$\frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[(b*x)/2]/b
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-bx}\sqrt{2+bx}}\,dx &= \int \frac{1}{\sqrt{4-b^2x^2}}\,dx\\ &= \frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0076968, size = 11, normalized size = 1.

$$\frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[(b*x)/2]/b
```

Maple [B] time = 0.007, size = 56, normalized size = 5.1

$$\sqrt{(-bx+2)(bx+2)}\arctan\left(x\sqrt{b^2}\frac{1}{\sqrt{-b^2x^2+4}}\right)\frac{1}{\sqrt{-bx+2}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((-b*x+2)*(b*x+2))^(1/2)/(-b*x+2)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*x/(-b^2*x^2+4)^(1/2))
```

Maxima [A] time = 1.429, size = 24, normalized size = 2.18

$$\frac{\arcsin\left(\frac{b^2x}{2\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/2*b^2*x/sqrt(b^2))/sqrt(b^2)
```

Fricas [B] time = 2.02964, size = 74, normalized size = 6.73

$$-\frac{2\arctan\left(\frac{\sqrt{bx+2}\sqrt{-bx+2}-2}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(b*x + 2)*sqrt(-b*x + 2) - 2)/(b*x))/b
```

Sympy [C] time = 3.18199, size = 76, normalized size = 6.91

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{4e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
4/(b**2*x**2))/(4*pi**(3/2)*b) + meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ())
```

```
, ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**
(3/2)*b)
```

Giac [A] time = 1.07294, size = 20, normalized size = 1.82

$$\frac{2\arcsin\left(\frac{1}{2}\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/2*sqrt(b*x + 2))/b
```

3.1538 $\int \frac{1}{\sqrt{1-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sin^{-1}\left(\frac{1}{3}(-2bx-1)\right)}{b}$$

```
[Out] -(ArcSin[(-1 - 2*b*x)/3]/b)
```

Rubi [A] time = 0.0133358, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {53, 619, 216}

$$-\frac{\sin^{-1}\left(\frac{1}{3}(-2bx-1)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] -(ArcSin[(-1 - 2*b*x)/3]/b)
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-bx}\sqrt{2+bx}}\,dx &= \int \frac{1}{\sqrt{2-bx-b^2x^2}}\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9b^2}}}\,dx, x, -b-2b^2x\right)}{3b^2}\\ &= -\frac{\sin^{-1}\left(\frac{1}{3}(-1-2bx)\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0124177, size = 22, normalized size = 1.38

$$-\frac{2\sin^{-1}\left(\frac{\sqrt{1-bx}}{\sqrt{3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcSin[Sqrt[1 - b*x]/Sqrt[3]])/b
```

Maple [B] time = 0.007, size = 66, normalized size = 4.1

$$\sqrt{(-bx+1)(bx+2)}\arctan\left(\sqrt{b^2}\left(x+\frac{1}{2b}\right)\frac{1}{\sqrt{-b^2x^2-bx+2}}\right)\frac{1}{\sqrt{-bx+1}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((-b*x+1)*(b*x+2))^(1/2)/(-b*x+1)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+1/2/b)/(-b^2*x^2-b*x+2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.01138, size = 104, normalized size = 6.5

$$-\frac{\arctan\left(\frac{(2\,bx+1)\sqrt{bx+2}\sqrt{-bx+1}}{2\left(b^2x^2+bx-2\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x + 1)*sqrt(b*x + 2)*sqrt(-b*x + 1)/(b^2*x^2 + b*x - 2))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx+1}\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 1)*sqrt(b*x + 2)), x)
```

Giac [A] time = 1.07573, size = 24, normalized size = 1.5

$$\frac{2 \arcsin\left(\frac{1}{3} \sqrt{3}\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/3*sqrt(3)*sqrt(b*x + 2))/b
```

3.1539 $\int \frac{1}{\sqrt{-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=10

$$\frac{\sin^{-1}(bx+1)}{b}$$

```
[Out] ArcSin[1 + b*x]/b
```

Rubi [A] time = 0.009602, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {53, 619, 216}

$$\frac{\sin^{-1}(bx+1)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-(b*x)]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[1 + b*x]/b
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-bx}\sqrt{2+bx}}\,dx &= \int \frac{1}{\sqrt{-2bx-b^2x^2}}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2b-2b^2x\right)}{2b^2}\\
&= \frac{\sin^{-1}(1+bx)}{b}
\end{aligned}$$

Mathematica [B] time = 0.0121286, size = 51, normalized size = 5.1

$$\frac{2\sqrt{x}\sqrt{bx+2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}\sqrt{-bx(bx+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-(b*x)]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[2 + b*x]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(Sqrt[b]*Sqrt[
-(b*x*(2 + b*x))])
```

Maple [B] time = 0.005, size = 58, normalized size = 5.8

$$\sqrt{-bx\,(bx+2)}\arctan\left((x+b^{-1})\sqrt{b^2}\frac{1}{\sqrt{-b^2x^2-2\,bx}}\right)\frac{1}{\sqrt{-bx}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] (-b*x*(b*x+2))^(1/2)/(-b*x)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b^2)^(1
/2)*(x+1/b)/(-b^2*x^2-2*b*x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.00147, size = 61, normalized size = 6.1

$$-\frac{2\,\arctan\left(\frac{\sqrt{bx+2}\sqrt{-bx}}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(b*x + 2)*sqrt(-b*x)/(b*x))/b
```

Sympy [C] time = 1.41673, size = 24, normalized size = 2.4

$$-\frac{2i\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] -2*I*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b
```

Giac [A] time = 1.06147, size = 24, normalized size = 2.4

$$\frac{2 \arcsin\left(\frac{1}{2} \sqrt{2}\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/2*sqrt(2)*sqrt(b*x + 2))/b
```

3.1540 $\int \frac{1}{\sqrt{-1-bx}\sqrt{2+bx}} \, dx$

Optimal. Leaf size=11

$$\frac{\sin^{-1}(2bx+3)}{b}$$

```
[Out] ArcSin[3 + 2*b*x]/b
```

Rubi [A] time = 0.0100564, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {53, 619, 216}

$$\frac{\sin^{-1}(2bx+3)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[3 + 2*b*x]/b
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1-bx}\sqrt{2+bx}} \, dx &= \int \frac{1}{\sqrt{-2-3bx-b^2x^2}} \, dx \\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{b^2}}} \, dx, x, -3b-2b^2x\right)}{b^2} \\
&= \frac{\sin^{-1}(3+2bx)}{b}
\end{aligned}$$

Mathematica [B] time = 0.0138349, size = 49, normalized size = 4.45

$$\frac{2\sqrt{bx+1}\sqrt{bx+2}\sinh^{-1}\left(\sqrt{bx+1}\right)}{b\sqrt{-(bx+1)(bx+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*Sqrt[1 + b*x]*Sqrt[2 + b*x]*ArcSinh[Sqrt[1 + b*x]])/(b*Sqrt[-((1 + b*x)*
(2 + b*x))])
```

Maple [B] time = 0.006, size = 66, normalized size = 6.

$$\sqrt{(-bx-1)(bx+2)}\arctan\left(\sqrt{b^2}\left(x+\frac{3}{2b}\right)\frac{1}{\sqrt{-b^2x^2-3bx-2}}\right)\frac{1}{\sqrt{-bx-1}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((-b*x-1)*(b*x+2))^(1/2)/(-b*x-1)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+3/2/b)/(-b^2*x^2-3*b*x-2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.05474, size = 107, normalized size = 9.73

$$-\frac{\arctan\left(\frac{(2bx+3)\sqrt{bx+2}\sqrt{-bx-1}}{2(b^2x^2+3bx+2)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x + 3)*sqrt(b*x + 2)*sqrt(-b*x - 1)/(b^2*x^2 + 3*b*x + 2))
/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx-1}\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 1)*sqrt(b*x + 2)), x)
```

Giac [A] time = 1.0648, size = 18, normalized size = 1.64

$$\frac{2\arcsin\left(\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(sqrt(b*x + 2))/b
```

3.1541 $\int \frac{1}{\sqrt{-2-bx}\sqrt{2+bx}} dx$

Optimal. Leaf size=29

$$\frac{\sqrt{bx+2}\log(bx+2)}{b\sqrt{-bx-2}}$$

```
[Out] (Sqrt[2 + b*x]*Log[2 + b*x])/(b*Sqrt[-2 - b*x])
```

Rubi [A] time = 0.0042249, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {23, 31}

$$\frac{\sqrt{bx+2}\log(bx+2)}{b\sqrt{-bx-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (Sqrt[2 + b*x]*Log[2 + b*x])/(b*Sqrt[-2 - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-2-bx}\sqrt{2+bx}} dx &= \frac{\sqrt{2+bx}\int \frac{1}{2+bx} dx}{\sqrt{-2-bx}} \\ &= \frac{\sqrt{2+bx}\log(2+bx)}{b\sqrt{-2-bx}} \end{aligned}$$

Mathematica [A] time = 0.0083102, size = 28, normalized size = 0.97

$$\frac{(bx+2)\log(bx+2)}{b\sqrt{-(bx+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ((2 + b*x)*Log[2 + b*x])/(b*Sqrt[-(2 + b*x)^2])
```

Maple [A] time = 0.003, size = 26, normalized size = 0.9

$$\frac{\ln \left(bx+2\right)}{b}\sqrt{bx+2}\frac{1}{\sqrt{-bx-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ln(b*x+2)*(b*x+2)^(1/2)/b/(-b*x-2)^(1/2)
```

Maxima [A] time = 0.928029, size = 22, normalized size = 0.76

$$\sqrt{-\frac{1}{b^2}}\log\left(x+\frac{2}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-1/b^2)*log(x + 2/b)
```

Fricas [A] time = 2.04833, size = 4, normalized size = 0.14

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [C] time = 1.87655, size = 53, normalized size = 1.83

$$\begin{cases} -\frac{i\log\left(x+\frac{2}{b}\right)}{b} & \text{for}\ \left|x+\frac{2}{b}\right|<1 \\ \frac{i\log\left(\frac{1}{x+\frac{2}{b}}\right)}{b} & \text{for}\ \frac{1}{\left|x+\frac{2}{b}\right|}<1 \\ \frac{i G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x+\frac{2}{b}\right)}{b}-\frac{i G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x+\frac{2}{b}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*log(x + 2/b)/b, Abs(x + 2/b) < 1), (I*log(1/(x + 2/b))/b, 1/A
bs(x + 2/b) < 1), (I*meijerg(((), (1, 1)), ((0, 0), ()), x + 2/b)/b - I*mei
jerg(((1, 1), ()), ((), (0, 0)), x + 2/b)/b, True))
```

Giac [C] time = 1.05501, size = 16, normalized size = 0.55

$$-\frac{i\log(|bx+2|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -I*log(abs(b*x + 2))/b
```

3.1542 $\int \frac{1}{\sqrt{-3-bx}\sqrt{2+bx}} dx$

Optimal. Leaf size=26

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{bx+2}}\right)}{b}$$

```
[Out] (-2*ArcTan[Sqrt[-3 - b*x]/Sqrt[2 + b*x]])/b
```

Rubi [A] time = 0.0137541, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {63, 217, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - b*x]*Sqrt[2 + b*x]),x]

[Out] (-2*ArcTan[Sqrt[-3 - b*x]/Sqrt[2 + b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-3-bx}\sqrt{2+bx}} dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}} dx, x, \sqrt{-3-bx}\right)}{b}\\
&= -\frac{2\,\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}\\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}
\end{aligned}$$

Mathematica [B] time = 0.0153942, size = 53, normalized size = 2.04

$$-\frac{2\sqrt{-bx-3}\sqrt{-bx-2}\sin^{-1}\left(\sqrt{bx+3}\right)}{b\sqrt{bx+2}\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*Sqrt[-3 - b*x]*Sqrt[-2 - b*x]*ArcSin[Sqrt[3 + b*x]])/(b*Sqrt[2 + b*x]*S
qrt[3 + b*x])
```

Maple [B] time = 0.006, size = 66, normalized size = 2.5

$$\sqrt{(-bx-3)(bx+2)}\arctan\left(\sqrt{b^2}\left(x+\frac{5}{2b}\right)\frac{1}{\sqrt{-b^2x^2-5bx-6}}\right)\frac{1}{\sqrt{-bx-3}}\frac{1}{\sqrt{bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((-b*x-3)*(b*x+2))^(1/2)/(-b*x-3)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+5/2/b)/(-b^2*x^2-5*b*x-6)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04065, size = 107, normalized size = 4.12

$$-\frac{\arctan\left(\frac{(2bx+5)\sqrt{bx+2}\sqrt{-bx-3}}{2(b^2x^2+5bx+6)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x + 5)*sqrt(b*x + 2)*sqrt(-b*x - 3)/(b^2*x^2 + 5*b*x + 6))
/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx-3}\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 3)*sqrt(b*x + 2)), x)
```

Giac [C] time = 1.06312, size = 20, normalized size = 0.77

$$-\frac{2i\arcsin\left(i\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*I*arcsin(I*sqrt(b*x + 2))/b
```

3.1543 $\int \frac{1}{\sqrt{2-bx}\sqrt{3-bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}$$

```
[Out] (-2*ArcSinh[Sqrt[2 - b*x]])/b
```

Rubi [A] time = 0.0054359, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - b*x]*Sqrt[3 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[2 - b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-bx}\sqrt{3-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0071046, size = 16, normalized size = 1.

$$-\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - b*x]*Sqrt[3 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[2 - b*x]])/b
```

Maple [B] time = 0.005, size = 70, normalized size = 4.4

$$\sqrt{(-bx+2)(-bx+3)}\ln\left(\left(-\frac{5b}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2-5bx+6}\right)\frac{1}{\sqrt{-bx+2}}\frac{1}{\sqrt{-bx+3}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x)
```

```
[Out] ((-b*x+2)*(-b*x+3))^(1/2)/(-b*x+2)^(1/2)/(-b*x+3)^(1/2)*ln((-5/2*b+b^2*x)/(
b^2)^(1/2)+(b^2*x^2-5*b*x+6)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.02101, size = 73, normalized size = 4.56

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+3}\sqrt{-bx+2}+5\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 3)*sqrt(-b*x + 2) + 5)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-bx+2}\sqrt{-bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(1/2)/(-b*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 2)*sqrt(-b*x + 3)), x)
```

Giac [A] time = 1.14565, size = 35, normalized size = 2.19

$$\frac{2 \log\left(\left|-\sqrt{-bx+3}+\sqrt{-bx+2}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(abs(-sqrt(-b*x + 3) + sqrt(-b*x + 2)))/b
```

3.1544 $\int \frac{1}{2-bx} \, dx$

Optimal. Leaf size=12

$$-\frac{\log(2-bx)}{b}$$

```
[Out] -(Log[2 - b*x]/b)
```

Rubi [A] time = 0.0014485, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {31}

$$-\frac{\log(2-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(-1),x]
```

```
[Out] -(Log[2 - b*x]/b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2-bx} \, dx = -\frac{\log(2-bx)}{b}$$

Mathematica [A] time = 0.0011097, size = 12, normalized size = 1.

$$-\frac{\log(2-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(-1),x]
```

```
[Out] -(Log[2 - b*x]/b)
```

Maple [A] time = 0.001, size = 13, normalized size = 1.1

$$-\frac{\ln(-bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2),x)
```

```
[Out] -ln(-b*x+2)/b
```

Maxima [A] time = 0.954928, size = 15, normalized size = 1.25

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="maxima")
```

```
[Out] -log(b*x - 2)/b
```

Fricas [A] time = 1.93477, size = 23, normalized size = 1.92

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="fricas")
```

```
[Out] -log(b*x - 2)/b
```

Sympy [A] time = 0.060902, size = 8, normalized size = 0.67

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x)
```

```
[Out] -log(b*x - 2)/b
```

Giac [A] time = 1.05433, size = 16, normalized size = 1.33

$$-\frac{\log(|bx-2|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="giac")
```

```
[Out] -log(abs(b*x - 2))/b
```

3.1545 $\int \frac{1}{\sqrt{1-bx}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}$$

```
[Out] (-2*ArcSinh[Sqrt[1 - b*x]])/b
```

Rubi [A] time = 0.0052831, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[1 - b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-bx}\sqrt{2-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{1-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0065096, size = 16, normalized size = 1.

$$-\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[1 - b*x]])/b
```

Maple [B] time = 0.007, size = 70, normalized size = 4.4

$$\sqrt{(-bx+1)(-bx+2)}\ln\left(\left(-\frac{3b}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2-3bx+2}\right)\frac{1}{\sqrt{-bx+1}}\frac{1}{\sqrt{-bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] ((-b*x+1)*(-b*x+2))^(1/2)/(-b*x+1)^(1/2)/(-b*x+2)^(1/2)*ln((-3/2*b+b^2*x)/(
b^2)^(1/2)+(b^2*x^2-3*b*x+2)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.94618, size = 73, normalized size = 4.56

$$-\frac{\log\left(-2bx+2\sqrt{-bx+2}\sqrt{-bx+1}+3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x + 1) + 3)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-bx+1}\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 1)*sqrt(-b*x + 2)), x)
```

<u>Giac</u> [A] time = 1.14366, size = 35, normalized size = 2.19

$$\frac{2 \log \left(\left|-\sqrt{-bx+2}+\sqrt{-bx+1}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(abs(-sqrt(-b*x + 2) + sqrt(-b*x + 1)))/b
```

3.1546 $\int \frac{1}{\sqrt{-bx}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=20

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}$$

```
[Out] (-2*ArcSinh[Sqrt[-(b*x)]/Sqrt[2]])/b
```

Rubi [A] time = 0.0053592, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-(b*x)]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[-(b*x)]/Sqrt[2]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-bx}\sqrt{2-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \sqrt{-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0085961, size = 37, normalized size = 1.85

$$\frac{2\sqrt{x}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}\sqrt{-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-(b*x)]*Sqrt[2 - b*x]),x]
```

```
[Out] (2*Sqrt[x]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(Sqrt[b]*Sqrt[-(b*x)])
```

Maple [B] time = 0.004, size = 64, normalized size = 3.2

$$\sqrt{-bx\left(-bx+2\right)}\ln\left((b^{2}x-b)\frac{1}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}-2\,bx}\right)\frac{1}{\sqrt{-bx}}\frac{1}{\sqrt{-bx+2}}\frac{1}{\sqrt{b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] (-b*x*(-b*x+2))^(1/2)/(-b*x)^(1/2)/(-b*x+2)^(1/2)*ln((b^2*x-b)/(b^2)^(1/2)+
(b^2*x^2-2*b*x)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99129, size = 62, normalized size = 3.1

$$-\frac{\log\left(-bx+\sqrt{-bx+2}\sqrt{-bx}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(-b*x + 2)*sqrt(-b*x) + 1)/b
```

Sympy [A] time = 1.48838, size = 53, normalized size = 2.65

$$\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b} & \text{for}\ \frac{|bx|}{2}>1\\ -\frac{2i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b, Abs(b*x)/2 > 1), (-2*I*as
in(sqrt(2)*sqrt(b)*sqrt(x)/2)/b, True))
```

Giac [A] time = 1.1284, size = 32, normalized size = 1.6

$$\frac{2 \log \left(\left| -\sqrt{-bx+2} + \sqrt{-bx} \right| \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(abs(-sqrt(-b*x + 2) + sqrt(-b*x)))/b
```

3.1547 $\int \frac{1}{\sqrt{-1-bx}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=22

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-1}}{\sqrt{3}}\right)}{b}$$

```
[Out] (-2*ArcSinh[Sqrt[-1 - b*x]/Sqrt[3]])/b
```

Rubi [A] time = 0.0071565, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-1}}{\sqrt{3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[-1 - b*x]/Sqrt[3]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1-bx}\sqrt{2-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-1-bx}\right)}{b}\\
&= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-1-bx}}{\sqrt{3}}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0103784, size = 40, normalized size = 1.82

$$-\frac{2\sqrt{-bx-1}\sin^{-1}\left(\frac{\sqrt{bx+1}}{\sqrt{3}}\right)}{b\sqrt{bx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*Sqrt[-1 - b*x]*ArcSin[Sqrt[1 + b*x]/Sqrt[3]])/(b*Sqrt[1 + b*x])
```

Maple [B] time = 0.006, size = 70, normalized size = 3.2

$$\sqrt{(-bx-1)(-bx+2)}\ln\left(\left(-\frac{b}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2-bx-2}\right)\frac{1}{\sqrt{-bx-1}}\frac{1}{\sqrt{-bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] ((-b*x-1)*(-b*x+2))^(1/2)/(-b*x-1)^(1/2)/(-b*x+2)^(1/2)*ln((-1/2*b+b^2*x)/(
b^2)^(1/2)+(b^2*x^2-b*x-2)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98493, size = 73, normalized size = 3.32

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+2}\sqrt{-bx-1}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x - 1) + 1)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-bx-1}\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 1)*sqrt(-b*x + 2)), x)
```

Giac [A] time = 1.15079, size = 35, normalized size = 1.59

$$\frac{2\log\left(\left|-\sqrt{-bx+2}+\sqrt{-bx-1}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(abs(-sqrt(-b*x + 2) + sqrt(-b*x - 1)))/b
```

3.1548 $\int \frac{1}{\sqrt{-2-bx}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=12

$$-\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

```
[Out] -(ArcCosh[-(b*x)/2]/b)
```

Rubi [A] time = 0.003148, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {52}

$$-\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - b*x]*Sqrt[2 - b*x]),x]

[Out] -(ArcCosh[-(b*x)/2]/b)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-bx}\sqrt{2-bx}}\,dx = -\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

Mathematica [B] time = 0.0045568, size = 27, normalized size = 2.25

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{-bx-2}}{\sqrt{2-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - b*x]*Sqrt[2 - b*x]),x]

[Out] (-2*ArcTanh[Sqrt[-2 - b*x]/Sqrt[2 - b*x]])/b
```

Maple [B] time = 0.005, size = 61, normalized size = 5.1

$$\sqrt{(-bx-2)(-bx+2)}\ln\left(b^2x\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2-4}\right)\frac{1}{\sqrt{-bx-2}}\frac{1}{\sqrt{-bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] ((-b*x-2)*(-b*x+2))^(1/2)/(-b*x-2)^(1/2)/(-b*x+2)^(1/2)*ln(b^2*x/(b^2)^(1/2
)+(b^2*x^2-4)^(1/2))/(b^2)^(1/2)
```

Maxima [B] time = 0.951835, size = 43, normalized size = 3.58

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-4}\sqrt{b^2}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*sqrt(b^2))/sqrt(b^2)
```

Fricas [B] time = 2.01459, size = 62, normalized size = 5.17

$$-\frac{\log\left(-bx+\sqrt{-bx+2}\sqrt{-bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(-b*x + 2)*sqrt(-b*x - 2))/b
```

Sympy [C] time = 3.25108, size = 78, normalized size = 6.5

$$-\frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{4e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] -meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4/
(b**2*x**2))/(4*pi**(3/2)*b) - I*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ())
, ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**
(3/2)*b)
```

Giac [B] time = 1.10542, size = 35, normalized size = 2.92

$$\frac{2\,\log\left(\left|-\sqrt{-bx+2}+\sqrt{-bx-2}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(abs(-sqrt(-b*x + 2) + sqrt(-b*x - 2)))/b
```

3.1549 $\int \frac{1}{\sqrt{-3-bx}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=22

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{5}}\right)}{b}$$

```
[Out] (-2*ArcSinh[Sqrt[-3 - b*x]/Sqrt[5]])/b
```

Rubi [A] time = 0.0068181, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{5}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[-3 - b*x]/Sqrt[5]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3-bx}\sqrt{2-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, \sqrt{-3-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{5}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0097834, size = 40, normalized size = 1.82

$$-\frac{2\sqrt{-bx-3}\sin^{-1}\left(\frac{\sqrt{bx+3}}{\sqrt{5}}\right)}{b\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*Sqrt[-3 - b*x]*ArcSin[Sqrt[3 + b*x]/Sqrt[5]])/(b*Sqrt[3 + b*x])
```

Maple [B] time = 0.005, size = 69, normalized size = 3.1

$$\sqrt{(-bx-3)(-bx+2)}\ln\left(\left(\frac{b}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+bx-6}\right)\frac{1}{\sqrt{-bx-3}}\frac{1}{\sqrt{-bx+2}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] ((-b*x-3)*(-b*x+2))^(1/2)/(-b*x-3)^(1/2)/(-b*x+2)^(1/2)*ln((1/2*b+b^2*x)/(b
^2)^(1/2)+(b^2*x^2+b*x-6)^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99002, size = 73, normalized size = 3.32

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+2}\sqrt{-bx-3}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x - 3) - 1)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx-3}\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 3)*sqrt(-b*x + 2)), x)
```

Giac [A] time = 1.15213, size = 35, normalized size = 1.59

$$\frac{2 \log\left(\left|-\sqrt{-bx+2}+\sqrt{-bx-3}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(abs(-sqrt(-b*x + 2) + sqrt(-b*x - 3)))/b
```

3.1550 $\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

[Out] ArcCosh[(b*x)/4]/b

Rubi [A] time = 0.0022748, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {52}

$$\frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]

[Out] ArcCosh[(b*x)/4]/b

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

Mathematica [B] time = 0.0045976, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx-4}}{\sqrt{bx+4}}\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]

[Out] (2*ArcTanh[Sqrt[-4 + b*x]/Sqrt[4 + b*x]])/b

Maple [B] time = 0.005, size = 57, normalized size = 5.2

$$\sqrt{(bx-4)(bx+4)}\ln\left(b^2x\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2-16}\right)\frac{1}{\sqrt{bx-4}}\frac{1}{\sqrt{bx+4}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x)
```

```
[Out] ((b*x-4)*(b*x+4))^(1/2)/(b*x-4)^(1/2)/(b*x+4)^(1/2)*ln(b^2*x/(b^2)^(1/2)+(b
^2*x^2-16)^(1/2))/(b^2)^(1/2)
```

Maxima [B] time = 0.950375, size = 43, normalized size = 3.91

$$\frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2 - 16}\sqrt{b^2}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 16)*sqrt(b^2))/sqrt(b^2)
```

Fricas [B] time = 1.988, size = 59, normalized size = 5.36

$$-\frac{\log\left(-bx + \sqrt{bx + 4}\sqrt{bx - 4}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 4)*sqrt(b*x - 4))/b
```

Sympy [C] time = 3.21102, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| {\frac{16e^{2i\pi}}{b^2x^2}} \right)}{4\pi^{\frac{3}{2}}b} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| {\frac{16}{b^2x^2}} \right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)**(1/2)/(b*x+4)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 16*
exp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 16/(b**2*x**2))/(4*pi**
(3/2)*b)
```

Giac [B] time = 1.10343, size = 32, normalized size = 2.91

$$-\frac{2\,\log\left(\left|-\sqrt{bx + 4} + \sqrt{bx - 4}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + 4) + sqrt(b*x - 4)))/b
```

3.1551 $\int \frac{1}{\sqrt{\frac{-b+bc}{d}+bx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{bx-\frac{b(1-c)}{d}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] (2*ArcSinh[(Sqrt[d]*Sqrt[-((b*(1 - c))/d) + b*x])/Sqrt[b]])/(Sqrt[b]*Sqrt[d
])
```

Rubi [A] time = 0.0153134, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{bx-\frac{b(1-c)}{d}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[(-b + b*c)/d + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcSinh[(Sqrt[d]*Sqrt[-((b*(1 - c))/d) + b*x])/Sqrt[b]])/(Sqrt[b]*Sqrt[d
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{-b+bc}{d}+bx}\sqrt{c+dx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{-b+bc}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{\frac{-b+bc}{d}+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{-\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}\end{aligned}$$

Mathematica [A] time = 0.0250398, size = 41, normalized size = 0.95

$$\frac{2\sqrt{c+dx-1}\sinh^{-1}\left(\sqrt{c+dx-1}\right)}{d\sqrt{\frac{b(c+dx-1)}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[(-b + b*c)/d + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[-1 + c + d*x]*ArcSinh[Sqrt[-1 + c + d*x]])/(d*Sqrt[(b*(-1 + c + d*x
))/d])
```

Maple [B] time = 0.014, size = 100, normalized size = 2.3

$$\sqrt{\left(bx+\frac{b(c-1)}{d}\right)(dx+c)}\ln\left(\left(\frac{b(c-1)}{2}+\frac{bc}{2}+bdx\right)\frac{1}{\sqrt{bd}}+\sqrt{dx^2b+(b(c-1)+bc)x+\frac{b(c-1)c}{d}}\right)\frac{1}{\sqrt{bx+\frac{b(c-1)}{d}}}\frac{1}{\sqrt{dx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] ((b*x+b*(c-1)/d)*(d*x+c))^(1/2)/(b*x+b*(c-1)/d)^(1/2)/(d*x+c)^(1/2)*ln((1/2
*b*(c-1)+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(b*(c-1)+b*c)*x+b*(c-1)/d*c)^(
1/2))/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.10981, size = 405, normalized size = 9.42

$$\left[\frac{\sqrt{bd}\log\left(8\,bd^2x^2+8\,bc^2+8\,(2\,bc-b)dx+4\,\sqrt{bd}(2\,dx+2\,c-1)\sqrt{dx+c}\sqrt{\frac{bdx+bc-b}{d}}-8\,bc+b\right)}{2\,bd},-\frac{\sqrt{-bd}\arctan\left(\frac{\sqrt{-bd}(2\,d}{2\,(bd^2}\right.}{bc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b*d^2*x^2 + 8*b*c^2 + 8*(2*b*c - b)*d*x + 4*sqrt(b*d)*
(2*d*x + 2*c - 1)*sqrt(d*x + c)*sqrt((b*d*x + b*c - b)/d) - 8*b*c + b)/(b*d
), -sqrt(-b*d)*arctan(1/2*sqrt(-b*d)*(2*d*x + 2*c - 1)*sqrt(d*x + c)*sqrt((
```

```
b*d*x + b*c - b)/d)/(b*d^2*x^2 + b*c^2 + (2*b*c - b)*d*x - b*c))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\left(\frac{c}{d} + x - \frac{1}{d}\right)}\sqrt{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*(c/d + x - 1/d))*sqrt(c + d*x)), x)
```

Giac [B] time = 1.06311, size = 84, normalized size = 1.95

$$-\frac{2\, b \log\left(-\sqrt{bd}\sqrt{bx + \frac{bc-b}{d}} + \sqrt{\left(bx + \frac{bc-b}{d}\right)bd + b^2}\right)}{\sqrt{bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*log(-sqrt(b*d)*sqrt(b*x + (b*c - b)/d) + sqrt((b*x + (b*c - b)/d)*b*d
+ b^2))/(sqrt(b*d)*abs(b))
```

3.1552 $\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx$

Optimal. Leaf size=22

$$\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-3}}{\sqrt{3}}\right)$$

```
[Out] Sqrt[2]*ArcSinh[Sqrt[-3 + 2*x]/Sqrt[3]]
```

Rubi [A] time = 0.0040966, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {54, 215}

$$\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-3}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]

[Out] Sqrt[2]*ArcSinh[Sqrt[-3 + 2*x]/Sqrt[3]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-3+2x}\right)\\
&= \sqrt{2}\sinh^{-1}\left(\frac{\sqrt{-3+2x}}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0083228, size = 31, normalized size = 1.41

$$\frac{\sqrt{4x-6}\sin^{-1}\left(\sqrt{1-\frac{2x}{3}}\right)}{\sqrt{3-2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]
```

```
[Out] (Sqrt[-6 + 4*x]*ArcSin[Sqrt[1 - (2*x)/3]])/Sqrt[3 - 2*x]
```

Maple [B] time = 0.005, size = 48, normalized size = 2.2

$$\frac{\sqrt{2}}{2}\sqrt{x(-3+2x)}\ln\left(\frac{\sqrt{2}}{2}\left(-\frac{3}{2}+2x\right)+\sqrt{2x^2-3x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-3+2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-3+2*x)^(1/2),x)
```

```
[Out] 1/2*(x*(-3+2*x))^(1/2)/x^(1/2)/(-3+2*x)^(1/2)*ln(1/2*(-3/2+2*x)*2^(1/2)+(2*
x^2-3*x)^(1/2))*2^(1/2)
```

Maxima [B] time = 1.46487, size = 55, normalized size = 2.5

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2x-3}}{\sqrt{x}}}{\sqrt{2}+\frac{\sqrt{2x-3}}{\sqrt{x}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sqrt(2*x - 3)/sqrt(x))/(sqrt(2) + sqrt(2*x - 3
)/sqrt(x)))
```

Fricas [A] time = 1.9871, size = 82, normalized size = 3.73

$$\frac{1}{2}\sqrt{2}\log\left(-2\sqrt{2}\sqrt{2x-3}\sqrt{x}-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-2*sqrt(2)*sqrt(2*x - 3)*sqrt(x) - 4*x + 3)
```

Sympy [A] time = 1.05414, size = 44, normalized size = 2.

$$\begin{cases}\sqrt{2}\operatorname{acosh}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{for}\ \frac{2|x|}{3}>1\\ -\sqrt{2}i\operatorname{asin}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-3+2*x)**(1/2),x)
```

```
[Out] Piecewise((sqrt(2)*acosh(sqrt(6)*sqrt(x)/3), 2*Abs(x)/3 > 1), (-sqrt(2)*I*a
sin(sqrt(6)*sqrt(x)/3), True))
```

Giac [A] time = 1.06913, size = 31, normalized size = 1.41

$$-\sqrt{2}\log\left(\sqrt{2}\sqrt{x}-\sqrt{2x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*log(sqrt(2)*sqrt(x) - sqrt(2*x - 3))
```

3.1553 $\int \frac{1}{\sqrt{-3+2x}\sqrt{2+3x}}\,dx$

Optimal. Leaf size=26

$$\sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{2x-3}\right)$$

```
[Out] Sqrt[2/3]*ArcSinh[Sqrt[3/13]*Sqrt[-3 + 2*x]]
```

Rubi [A] time = 0.0080107, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {54, 215}

$$\sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{2x-3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 + 2*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] Sqrt[2/3]*ArcSinh[Sqrt[3/13]*Sqrt[-3 + 2*x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+2x}\sqrt{2+3x}}\,dx &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{13+3x^2}}\,dx, x, \sqrt{-3+2x}\right)\\ &= \sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{-3+2x}\right)\end{aligned}$$

Mathematica [A] time = 0.0083718, size = 26, normalized size = 1.

$$\sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{2x-3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + 2*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] Sqrt[2/3]*ArcSinh[Sqrt[3/13]*Sqrt[-3 + 2*x]]
```

Maple [B] time = 0.005, size = 57, normalized size = 2.2

$$\frac{\sqrt{6}}{6}\sqrt{(-3+2x)(2+3x)}\ln\left(\frac{\sqrt{6}}{6}\left(-\frac{5}{2}+6x\right)+\sqrt{6x^2-5x-6}\right)\frac{1}{\sqrt{-3+2x}}\frac{1}{\sqrt{2+3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x)
```

```
[Out] 1/6*((-3+2*x)*(2+3*x))^(1/2)/(-3+2*x)^(1/2)/(2+3*x)^(1/2)*ln(1/6*(-5/2+6*x)
*6^(1/2)+(6*x^2-5*x-6)^(1/2))*6^(1/2)
```

Maxima [A] time = 1.44174, size = 38, normalized size = 1.46

$$\frac{1}{6}\sqrt{6}\log\left(2\sqrt{6}\sqrt{6x^2-5x-6}+12x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(6)*log(2*sqrt(6)*sqrt(6*x^2 - 5*x - 6) + 12*x - 5)
```

Fricas [B] time = 1.98652, size = 146, normalized size = 5.62

$$\frac{1}{12}\sqrt{3}\sqrt{2}\log\left(4\sqrt{3}\sqrt{2}(12x-5)\sqrt{3x+2}\sqrt{2x-3}+288x^2-240x-119\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*sqrt(2)*log(4*sqrt(3)*sqrt(2)*(12*x - 5)*sqrt(3*x + 2)*sqrt(2*
x - 3) + 288*x^2 - 240*x - 119)
```

Sympy [A] time = 1.0987, size = 58, normalized size = 2.23

$$\begin{cases} \frac{\sqrt{6}\operatorname{acosh}\left(\frac{\sqrt{78}\sqrt{x+\frac{2}{3}}}{13}\right)}{3} & \text{for}\ \frac{6\left|x+\frac{2}{3}\right|}{13} > 1 \\ -\frac{\sqrt{6}i\operatorname{asin}\left(\frac{\sqrt{78}\sqrt{x+\frac{2}{3}}}{13}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((sqrt(6)*acosh(sqrt(78)*sqrt(x + 2/3)/13)/3, 6*Abs(x + 2/3)/13 >
1), (-sqrt(6)*I*asin(sqrt(78)*sqrt(x + 2/3)/13)/3, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1554 $\int \frac{1}{\sqrt{\frac{b-bc}{d}+bx}\sqrt{c-dx}}\,dx$

Optimal. Leaf size=42

$$\frac{2\sin^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] (2*ArcSin[(Sqrt[d]*Sqrt[(b*(1 - c))/d + b*x])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

Rubi [A] time = 0.0161989, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {63, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[(b - b*c)/d + b*x]*Sqrt[c - d*x]),x]
```

```
[Out] (2*ArcSin[(Sqrt[d]*Sqrt[(b*(1 - c))/d + b*x])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{b-bc}{d}+bx}\sqrt{c-dx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{b-bc}{b}-\frac{dx^2}{b}}}\,dx, x, \sqrt{\frac{b-bc}{d}+bx}\right)}{b}\\
&= \frac{2\sin^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.0501009, size = 67, normalized size = 1.6

$$\frac{2\sqrt{-d}\sqrt{-c+dx+1}\sinh^{-1}\left(\frac{\sqrt{-d}\sqrt{c-dx}}{\sqrt{d}}\right)}{d^{3/2}\sqrt{\frac{b(-c+dx+1)}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[(b - b*c)/d + b*x]*Sqrt[c - d*x]),x]
```

```
[Out] (2*Sqrt[-d]*Sqrt[1 - c + d*x]*ArcSinh[(Sqrt[-d]*Sqrt[c - d*x])/Sqrt[d]])/(d
^(3/2)*Sqrt[(b*(1 - c + d*x))/d])
```

Maple [B] time = 0.023, size = 118, normalized size = 2.8

$$\sqrt{\left(\frac{b(1-c)}{d}+bx\right)(-dx+c)}\arctan\left(\sqrt{bd}\left(x-\frac{-b(1-c)+bc}{2bd}\right)\frac{1}{\sqrt{-dx^2b+(-b(1-c)+bc)x+\frac{b(1-c)c}{d}}}\right)\frac{1}{\sqrt{\frac{b(1-c)}{d}+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x)
```

```
[Out] ((b*(1-c)/d+b*x)*(-d*x+c))^(1/2)/(b*(1-c)/d+b*x)^(1/2)/(-d*x+c)^(1/2)/(b*d)
^(1/2)*arctan((b*d)^(1/2)*(x-1/2*(-b*(1-c)+b*c)/b/d)/(-d*x^2*b+(-b*(1-c)+b*
c)*x+b*(1-c)/d*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.11584, size = 409, normalized size = 9.74

$$\left[-\frac{\sqrt{-bd}\log\left(8bd^2x^2+8bc^2-8(2bc-b)dx-4\sqrt{-bd}(2dx-2c+1)\sqrt{-dx+c}\sqrt{\frac{bdx-bc+b}{d}}-8bc+b\right)}{2bd}, -\frac{\sqrt{bd}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b*d^2*x^2 + 8*b*c^2 - 8*(2*b*c - b)*d*x - 4*sqrt(-b*
d)*(2*d*x - 2*c + 1)*sqrt(-d*x + c)*sqrt((b*d*x - b*c + b)/d) - 8*b*c + b)/
```

```
(b*d), -sqrt(b*d)*arctan(1/2*sqrt(b*d)*(2*d*x - 2*c + 1)*sqrt(-d*x + c)*sqr
t((b*d*x - b*c + b)/d)/(b*d^2*x^2 + b*c^2 - (2*b*c - b)*d*x - b*c))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\left(-\frac{c}{d}+x+\frac{1}{d}\right)}\sqrt{c-dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)**(1/2)/(-d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*(-c/d + x + 1/d))*sqrt(c - d*x)), x)
```

Giac [B] time = 1.0759, size = 90, normalized size = 2.14

$$-\frac{2\,b\log\left(-\sqrt{-bd}\sqrt{bx-\frac{bc-b}{d}}+\sqrt{-\left(bx-\frac{bc-b}{d}\right)bd+b^2}\right)}{\sqrt{-bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*log(-sqrt(-b*d)*sqrt(b*x - (b*c - b)/d) + sqrt(-(b*x - (b*c - b)/d)*b*
d + b^2))/(sqrt(-b*d)*abs(b))
```

3.1555 $\int \frac{1}{\sqrt{4-x}\sqrt{x}} dx$

Optimal. Leaf size=10

$$-\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] -ArcSin[1 - x/2]
```

Rubi [A] time = 0.0074331, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {53, 619, 216}

$$-\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[4 - x]*Sqrt[x]),x]

[Out] -ArcSin[1 - x/2]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{4-x}\sqrt{x}} dx &= \int \frac{1}{\sqrt{4x-x^2}} dx \\
&= -\left(\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}} dx, x, 4-2x\right)\right) \\
&= -\sin^{-1}\left(1-\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0099107, size = 14, normalized size = 1.4

$$-2\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[4 - x]*Sqrt[x]),x]
```

```
[Out] -2*ArcSin[Sqrt[1 - x/4]]
```

Maple [B] time = 0.004, size = 27, normalized size = 2.7

$$\sqrt{(-x+4)\,x}\arcsin\left(-1+\frac{x}{2}\right)\frac{1}{\sqrt{-x+4}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+4)^(1/2)/x^(1/2),x)
```

```
[Out] ((-x+4)*x)^(1/2)/(-x+4)^(1/2)/x^(1/2)*arcsin(-1+1/2*x)
```

Maxima [B] time = 1.42676, size = 19, normalized size = 1.9

$$-2\arctan\left(\frac{\sqrt{-x+4}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-x + 4)/sqrt(x))
```

Fricas [B] time = 2.02496, size = 45, normalized size = 4.5

$$-2\arctan\left(\frac{\sqrt{-x+4}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-x + 4)/sqrt(x))
```

Sympy [A] time = 1.00913, size = 26, normalized size = 2.6

$$\begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{x}}{2}\right) & \text{for}\ \frac{|x|}{4}>1 \\ 2\operatorname{asin}\left(\frac{\sqrt{x}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(x)/2), Abs(x)/4 > 1), (2*asin(sqrt(x)/2), True))
```

Giac [A] time = 1.06964, size = 11, normalized size = 1.1

$$2 \arcsin\left(\frac{1}{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/2*sqrt(x))
```

3.1556 $\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)$$

```
[Out] Sqrt[2]*ArcSin[Sqrt[2/3]*Sqrt[x]]
```

Rubi [A] time = 0.0067617, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {54, 216}

$$\sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]

[Out] Sqrt[2]*ArcSin[Sqrt[2/3]*Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{3-2x^2}}\,dx, x, \sqrt{x}\right)\\ &= \sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0043195, size = 20, normalized size = 1.

$$\sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]
```

```
[Out] Sqrt[2]*ArcSin[Sqrt[2/3]*Sqrt[x]]
```

Maple [B] time = 0.005, size = 31, normalized size = 1.6

$$\frac{\sqrt{2}}{2}\sqrt{(3-2x)\,x}\,\arcsin\left(\frac{4x}{3}-1\right)\frac{1}{\sqrt{3-2x}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(1/2)/x^(1/2),x)
```

```
[Out] 1/2*((3-2*x)*x)^(1/2)/(3-2*x)^(1/2)/x^(1/2)*2^(1/2)*arcsin(4/3*x-1)
```

Maxima [A] time = 1.43391, size = 28, normalized size = 1.4

$$-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2x+3}}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*x + 3)/sqrt(x))
```

Fricas [A] time = 1.98882, size = 72, normalized size = 3.6

$$-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2x+3}}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*x + 3)/sqrt(x))
```

Sympy [A] time = 1.0215, size = 44, normalized size = 2.2

$$\begin{cases} -\sqrt{2}i\operatorname{acosh}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{for}\ \frac{2|x|}{3}>1 \\ \sqrt{2}\operatorname{asin}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-sqrt(2)*I*acosh(sqrt(6)*sqrt(x)/3), 2*Abs(x)/3 > 1), (sqrt(2)*a
sin(sqrt(6)*sqrt(x)/3), True))
```

Giac [A] time = 1.06279, size = 18, normalized size = 0.9

$$\sqrt{2}\arcsin\left(\frac{1}{3}\sqrt{6}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arcsin(1/3*sqrt(6)*sqrt(x))
```

3.1557 $\int \frac{1}{\sqrt{3-2x}\sqrt{3+5x}}\,dx$

Optimal. Leaf size=26

$$\sqrt{\frac{2}{5}}\sin^{-1}\left(\sqrt{\frac{2}{21}}\sqrt{5x+3}\right)$$

```
[Out] Sqrt[2/5]*ArcSin[Sqrt[2/21]*Sqrt[3 + 5*x]]
```

Rubi [A] time = 0.0089491, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {54, 216}

$$\sqrt{\frac{2}{5}}\sin^{-1}\left(\sqrt{\frac{2}{21}}\sqrt{5x+3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*x]*Sqrt[3 + 5*x]),x]
```

```
[Out] Sqrt[2/5]*ArcSin[Sqrt[2/21]*Sqrt[3 + 5*x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2x}\sqrt{3+5x}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{21-2x^2}}\,dx, x, \sqrt{3+5x}\right)}{\sqrt{5}}\\ &= \sqrt{\frac{2}{5}}\sin^{-1}\left(\sqrt{\frac{2}{21}}\sqrt{3+5x}\right)\end{aligned}$$

Mathematica [A] time = 0.0090089, size = 27, normalized size = 1.04

$$-\sqrt{\frac{2}{5}}\sin^{-1}\left(\sqrt{\frac{5}{21}}\sqrt{3-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*x]*Sqrt[3 + 5*x]),x]
```

```
[Out] -(Sqrt[2/5]*ArcSin[Sqrt[5/21]*Sqrt[3 - 2*x]])
```

Maple [B] time = 0.007, size = 39, normalized size = 1.5

$$\frac{\sqrt{10}}{10}\sqrt{(3-2x)(3+5x)}\arcsin\left(\frac{20x}{21}-\frac{3}{7}\right)\frac{1}{\sqrt{3-2x}}\frac{1}{\sqrt{3+5x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x)
```

```
[Out] 1/10*((3-2*x)*(3+5*x))^(1/2)/(3-2*x)^(1/2)/(3+5*x)^(1/2)*10^(1/2)*arcsin(20
/21*x-3/7)
```

Maxima [A] time = 1.42553, size = 15, normalized size = 0.58

$$-\frac{1}{10}\sqrt{10}\arcsin\left(-\frac{20}{21}x+\frac{3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(10)*arcsin(-20/21*x + 3/7)
```

Fricas [B] time = 1.98971, size = 140, normalized size = 5.38

$$-\frac{1}{5}\sqrt{5}\sqrt{2}\arctan\left(\frac{\sqrt{5}\sqrt{2}\sqrt{5x+3}\sqrt{-2x+3}-3\sqrt{5}\sqrt{2}}{10x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(5)*sqrt(2)*arctan(1/10*(sqrt(5)*sqrt(2)*sqrt(5*x + 3)*sqrt(-2*x +
 3) - 3*sqrt(5)*sqrt(2))/x)
```

Sympy [A] time = 1.0972, size = 58, normalized size = 2.23

$$\begin{cases} -\frac{\sqrt{10}i\operatorname{acosh}\left(\frac{\sqrt{210}\sqrt{x+\frac{3}{5}}}{21}\right)}{5} & \text{for}\ \frac{10\left|x+\frac{3}{5}\right|}{21}>1 \\ \frac{\sqrt{10}\operatorname{asin}\left(\frac{\sqrt{210}\sqrt{x+\frac{3}{5}}}{21}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(1/2)/(3+5*x)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(10)*I*acosh(sqrt(210)*sqrt(x + 3/5)/21)/5, 10*Abs(x + 3/5)
/21 > 1), (sqrt(10)*asin(sqrt(210)*sqrt(x + 3/5)/21)/5, True))
```

Giac [A] time = 1.06205, size = 28, normalized size = 1.08

$$\frac{1}{5}\sqrt{5}\sqrt{2}\arcsin\left(\frac{1}{21}\sqrt{42}\sqrt{5x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*sqrt(2)*arcsin(1/21*sqrt(42)*sqrt(5*x + 3))
```

3.1558 $\int \frac{1}{\sqrt{a-bx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=43

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] (-2*ArcTan[(Sqrt[d]*Sqrt[a - b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[
d])
```

Rubi [A] time = 0.0277071, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {63, 217, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[d]*Sqrt[a - b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[
d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx}\sqrt{c+dx}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{ad}{b}-\frac{dx^2}{b}}}\,dx, x, \sqrt{a-bx}\right)}{b}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a-bx}}{\sqrt{c+dx}}\right)}{b}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [B] time = 0.0787922, size = 103, normalized size = 2.4

$$\frac{2\sqrt{-b}\sqrt{-ad-bc}\sqrt{\frac{b(c+dx)}{ad+bc}}\sin^{-1}\left(\frac{\sqrt{-b}\sqrt{d}\sqrt{a-bx}}{\sqrt{b}\sqrt{-ad-bc}}\right)}{b^{3/2}\sqrt{d}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[-b]*Sqrt[-(b*c) - a*d]*Sqrt[(b*(c + d*x))/(b*c + a*d)]*ArcSin[(Sqrt
[-b]*Sqrt[d]*Sqrt[a - b*x])/(Sqrt[b]*Sqrt[-(b*c) - a*d])])/(b^(3/2)*Sqrt[d]
*Sqrt[c + d*x])
```

Maple [B] time = 0.007, size = 84, normalized size = 2.

$$\sqrt{(-bx+a)(dx+c)}\arctan\left(\sqrt{bd}\left(x-\frac{ad-bc}{2bd}\right)\frac{1}{\sqrt{-dx^2b+(ad-bc)x+ac}}\right)\frac{1}{\sqrt{-bx+a}}\frac{1}{\sqrt{dx+c}}\frac{1}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] ((-b*x+a)*(d*x+c))^(1/2)/(-b*x+a)^(1/2)/(d*x+c)^(1/2)/(b*d)^(1/2)*arctan((b
*d)^(1/2)*(x-1/2*(a*d-b*c)/b/d)/(-d*x^2*b+(a*d-b*c)*x+a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12206, size = 417, normalized size = 9.7

$$\left[-\frac{\sqrt{-bd}\log\left(8\,b^2d^2x^2+b^2c^2-6\,abcd+a^2d^2-4\,(2\,bdx+bc-ad)\sqrt{-bd}\sqrt{-bx+a}\sqrt{dx+c}+8\left(b^2cd-abd^2\right)x\right)}{2\,bd},-\frac{\sqrt{bd}\,\mathrm{ar}\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 - 4*(2*b
*d*x + b*c - a*d)*sqrt(-b*d)*sqrt(-b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d - a*
b*d^2)*x)/(b*d), -sqrt(b*d)*arctan(1/2*(2*b*d*x + b*c - a*d)*sqrt(b*d)*sqrt
(-b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x))/(
b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-bx}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - b*x)*sqrt(c + d*x)), x)
```

Giac [A] time = 1.0829, size = 73, normalized size = 1.7

$$\frac{2\,b\log\left(\left|-\sqrt{-bd}\sqrt{-bx+a}+\sqrt{b^2c+(bx-a)bd+abd}\right|\right)}{\sqrt{-bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*b*log(abs(-sqrt(-b*d)*sqrt(-b*x + a) + sqrt(b^2*c + (b*x - a)*b*d + a*b*d
)))/(sqrt(-b*d)*abs(b))
```

3.1559 $\int (a+bx)^{3/2}\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=457

$$-\frac{108\,3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)^3\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{935b^{4/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-108*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/3))/(935*b*d^2) + (12*(b*c -
 a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/3))/(187*b*d) + (6*(a + b*x)^(5/2)*(c +
d*x)^(1/3))/(17*b) - (108*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^3*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
 - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3]
)*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(935*b^(4/3)*d^3*Sqrt[a
+ b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/
3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.627175, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {50, 63, 219}

$$-\frac{108\,3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)^3\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}}\right.\right.}{935b^{4/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(1/3),x]
```


```
[Out] (-108*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/3))/(935*b*d^2) + (12*(b*c -
 a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/3))/(187*b*d) + (6*(a + b*x)^(5/2)*(c +
d*x)^(1/3))/(17*b) - (108*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^3*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
 - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3]
)*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(935*b^(4/3)*d^3*Sqrt[a
+ b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/
3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt[3]{c+dx}\,dx = \frac{6(a+bx)^{5/2}\sqrt[3]{c+dx}}{17b}+\frac{(2(bc-ad))\int \frac{(a+bx)^{3/2}}{(c+dx)^{2/3}}\,dx}{17b}$$

$$=\frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd}+\frac{6(a+bx)^{5/2}\sqrt[3]{c+dx}}{17b}-\frac{\left(18(bc-ad)^2\right)\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}}\,dx}{187bd}$$

$$=-\frac{108(bc-ad)^2\sqrt{a+bx}\sqrt[3]{c+dx}}{935bd^2}+\frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd}+\frac{6(a+bx)^{5/2}\sqrt[3]{c+dx}}{17b}+\frac{(5}{}$$

$$=-\frac{108(bc-ad)^2\sqrt{a+bx}\sqrt[3]{c+dx}}{935bd^2}+\frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd}+\frac{6(a+bx)^{5/2}\sqrt[3]{c+dx}}{17b}+\frac{(1}{}$$

$$=-\frac{108(bc-ad)^2\sqrt{a+bx}\sqrt[3]{c+dx}}{935bd^2}+\frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd}+\frac{6(a+bx)^{5/2}\sqrt[3]{c+dx}}{17b}-\frac{10}{}$$

Mathematica [C] time = 0.0381882, size = 73, normalized size = 0.16

$$\frac{2(a+bx)^{5/2}\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(1/3),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}\sqrt[3]{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(3/2)*(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^{\frac{3}{2}}\sqrt[3]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/3), x)
```

3.1560 $\int \sqrt{a+bx}\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=419

$$\frac{12\,3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-}\right.\right.}{55b^{4/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (12*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/3))/(55*b*d) + (6*(a + b*x)^(3/2
)*(c + d*x)^(1/3))/(11*b) + (12*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^2*((b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3
)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3
])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(55*b^(4/3)*d^2*Sq
rt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.373558, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {50, 63, 219}

$$\frac{12\,3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\right.}{55b^{4/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(1/3),x]
```

```
[Out] (12*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/3))/(55*b*d) + (6*(a + b*x)^(3/2
)*(c + d*x)^(1/3))/(11*b) + (12*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^2*((b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3
)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3
])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(55*b^(4/3)*d^2*Sq
rt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt[3]{c+dx}\,dx = \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b} + \frac{(2(bc-ad))\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}}\,dx}{11b}$$
$$= \frac{12(bc-ad)\sqrt{a+bx}\sqrt[3]{c+dx}}{55bd} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b} - \frac{\left(6(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{2/3}}\,dx}{55bd}$$
$$= \frac{12(bc-ad)\sqrt{a+bx}\sqrt[3]{c+dx}}{55bd} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b} - \frac{\left(18(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx\right.}{55bd^2}$$
$$= \frac{12(bc-ad)\sqrt{a+bx}\sqrt[3]{c+dx}}{55bd} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b} + \frac{12\ 3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)^2\left(\sqrt[3]{bc-ad}\right.}{}$$

Mathematica [C] time = 0.0257057, size = 73, normalized size = 0.17

$$\frac{2(a+bx)^{3/2}\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(1/3),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\sqrt[3]{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx+a}(dx+c)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+bx}\sqrt[3]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/3),x)
```

```
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/3), x)
```

3.1561 $\int \frac{\sqrt[3]{c+dx}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=381

$$\frac{6\sqrt{a+bx}\sqrt[3]{c+dx}}{5b} - \frac{4\,3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}}{5b^{4/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left((1-\sqrt{3})\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (6*Sqrt[a + b*x]*(c + d*x)^(1/3))/(5*b) - (4*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c
 - a*d)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/
3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/(
(1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^(4
/3)*d*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*
(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
))^2)])
```

Rubi [A] time = 0.271655, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {50, 63, 219}

$$\frac{6\sqrt{a+bx}\sqrt[3]{c+dx}}{5b} - \frac{4\,3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\Bigl(\sin^{-1}\Bigl(}{5b^{4/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left((1-\sqrt{3})\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/Sqrt[a + b*x],x]
```

```
[Out] (6*Sqrt[a + b*x]*(c + d*x)^(1/3))/(5*b) - (4*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c
 - a*d)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/
3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/(
(1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^(4
/3)*d*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*
(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{\sqrt{a+bx}}\,dx = \frac{6\sqrt{a+bx}\sqrt[3]{c+dx}}{5b} + \frac{(2(bc-ad))\int \frac{1}{\sqrt{a+bx}(c+dx)^{2/3}}\,dx}{5b}$$

$$= \frac{6\sqrt{a+bx}\sqrt[3]{c+dx}}{5b} + \frac{(6(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{5bd}$$

$$= \frac{6\sqrt{a+bx}\sqrt[3]{c+dx}}{5b} - \frac{4\,3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(c+\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}{5b^{4/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{\cdots}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c\cdots}\right.}}}$$

Mathematica [C] time = 0.0217836, size = 71, normalized size = 0.19

$$\frac{2\sqrt{a+bx}\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 1/2, 3/2, (d*(a +
b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \sqrt[3]{dx+c}\frac{1}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^{\frac{1}{3}}}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(1/3)/sqrt(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c+dx}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(1/3)/sqrt(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/sqrt(b*x + a), x)
```

3.1562 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=366

$$-\frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right),4\sqrt{3}\right.}{\sqrt[4]{3}b^{4/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-2*(c + d*x)^(1/3))/(b*Sqrt[a + b*x]) - (4*Sqrt[2 - Sqrt[3]]*((b*c - a*d)^
(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a
*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3)
 - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(4/3)*Sqrt[a + b*
x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))
/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.261813, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {47, 63, 219}

$$-\frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{4/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(3/2),x]
```


```
[Out] (-2*(c + d*x)^(1/3))/(b*Sqrt[a + b*x]) - (4*Sqrt[2 - Sqrt[3]]*((b*c - a*d)^
(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a
*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3)
 - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(4/3)*Sqrt[a + b*
x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))
/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{3/2}}\,dx = -\frac{2\sqrt[3]{c+dx}}{b\sqrt{a+bx}} + \frac{(2d)\int \frac{1}{\sqrt{a+bx}(c+dx)^{2/3}}\,dx}{3b}$$

$$= -\frac{2\sqrt[3]{c+dx}}{b\sqrt{a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{b}$$

$$= -\frac{2\sqrt[3]{c+dx}}{b\sqrt{a+bx}} - \frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\cdots\right.\right.}{\sqrt[4]{3}b^{4/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Mathematica [C] time = 0.0227167, size = 71, normalized size = 0.19

$$-\frac{2\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(3/2),x]

[Out] (-2*(c + d*x)^(1/3)*Hypergeometric2F1[-1/2, -1/3, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \sqrt[3]{dx+c}\,(bx+a)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(3/2),x)

[Out] int((d*x+c)^(1/3)/(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{3}}}{b^2x^2+2\,abx+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(3/2), x)
```

3.1563 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=417

$$\frac{4\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right),4}{9\sqrt[4]{3}b^{4/3}\sqrt{a+bx}(bc-ad)\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-2*(c + d*x)^(1/3))/(3*b*(a + b*x)^(3/2)) - (4*d*(c + d*x)^(1/3))/(9*b*(b*
c - a*d)*Sqrt[a + b*x]) + (4*Sqrt[2 - Sqrt[3]]*d*((b*c - a*d)^(1/3) - b^(1/
3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c
+ d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*b^(4/3)*(b*c - a*d)*Sqrt[a + b
*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
)/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.365754, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {47, 51, 63, 219}

$$\frac{4\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{}\right.}{9\sqrt[4]{3}b^{4/3}\sqrt{a+bx}(bc-ad)\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/3))/(3*b*(a + b*x)^(3/2)) - (4*d*(c + d*x)^(1/3))/(9*b*(b*
c - a*d)*Sqrt[a + b*x]) + (4*Sqrt[2 - Sqrt[3]]*d*((b*c - a*d)^(1/3) - b^(1/
3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c
+ d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*b^(4/3)*(b*c - a*d)*Sqrt[a + b
*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
)/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{5/2}}\,dx = -\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}} + \frac{(2d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx}{9b}$$

$$= -\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}} - \frac{4d\sqrt[3]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} - \frac{\left(2d^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{2/3}}\,dx}{27b(bc-ad)}$$

$$= -\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}} - \frac{4d\sqrt[3]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} - \frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{9b(bc-ad)}$$

$$= -\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}} - \frac{4d\sqrt[3]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} + \frac{4\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\cdots\right.}}}{9\sqrt[4]{3}b^{4/3}(bc-ad)\sqrt{a+bx}\cdots}$$

Mathematica [C] time = 0.0232628, size = 73, normalized size = 0.18

$$-\frac{2\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/3)*Hypergeometric2F1[-3/2, -1/3, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \sqrt[3]{dx+c}\,(bx+a)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{3}}}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(5/2), x)
```

3.1564 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=457

$$-\frac{28\sqrt{2-\sqrt{3}}d^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right.\right.}{135\sqrt[4]{3}b^{4/3}\sqrt{a+bx}(bc-ad)^2\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-2*(c + d*x)^(1/3))/(5*b*(a + b*x)^(5/2)) - (4*d*(c + d*x)^(1/3))/(45*b*(b
*c - a*d)*(a + b*x)^(3/2)) + (28*d^2*(c + d*x)^(1/3))/(135*b*(b*c - a*d)^2*
Sqrt[a + b*x]) - (28*Sqrt[2 - Sqrt[3]]*d^2*((b*c - a*d)^(1/3) - b^(1/3)*(c
+ d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)
^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3
)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b
^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3))], -7 + 4*Sqrt[3]])/(135*3^(1/4)*b^(4/3)*(b*c - a*d)^2*Sqrt[a + b*x
]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/
((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.447022, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {47, 51, 63, 219}

$$-\frac{28\sqrt{2-\sqrt{3}}d^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\right|-7+}{135\sqrt[4]{3}b^{4/3}\sqrt{a+bx}(bc-ad)^2\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(7/2),x]
```


```
[Out] (-2*(c + d*x)^(1/3))/(5*b*(a + b*x)^(5/2)) - (4*d*(c + d*x)^(1/3))/(45*b*(b
*c - a*d)*(a + b*x)^(3/2)) + (28*d^2*(c + d*x)^(1/3))/(135*b*(b*c - a*d)^2*
Sqrt[a + b*x]) - (28*Sqrt[2 - Sqrt[3]]*d^2*((b*c - a*d)^(1/3) - b^(1/3)*(c
+ d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)
^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3
)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b
^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3))], -7 + 4*Sqrt[3]])/(135*3^(1/4)*b^(4/3)*(b*c - a*d)^2*Sqrt[a + b*x
]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/
((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/2}}\,dx &= -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}}+\frac{(2d)\int \frac{1}{(a+bx)^{5/2}(c+dx)^{2/3}}\,dx}{15b}\\
&= -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}}-\frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}}-\frac{\left(14d^2\right)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx}{135b(bc-ad)}\\
&= -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}}-\frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}}+\frac{28d^2\sqrt[3]{c+dx}}{135b(bc-ad)^2\sqrt{a+bx}}+\frac{\left(14d^3\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{2/3}}\,dx}{405b(bc-ad)^2}\\
&= -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}}-\frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}}+\frac{28d^2\sqrt[3]{c+dx}}{135b(bc-ad)^2\sqrt{a+bx}}+\frac{\left(14d^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx,\right.}{135b(bc-ad)^2}\\
&= -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}}-\frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}}+\frac{28d^2\sqrt[3]{c+dx}}{135b(bc-ad)^2\sqrt{a+bx}}-\frac{28\sqrt{2-\sqrt{3}}d^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.0259664, size = 73, normalized size = 0.16

$$-\frac{2\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{3};-\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/2}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(1/3)*Hypergeometric2F1[-5/2, -1/3, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \sqrt[3]{dx+c}\,(bx+a)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{3}}}{b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(7/2),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/2), x)
```

3.1565 $\int \frac{(a+bx)^{3/2}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=839

$$\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{(1-\sqrt{3})\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\right|-7+}{91b^{2/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left((1-\sqrt{3})\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-54*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(2/3))/(91*d^2) + (6*(a + b*x)^(3/
2)*(c + d*x)^(2/3))/(13*d) - (162*(b*c - a*d)^2*Sqrt[a + b*x])/(91*b^(2/3)*
d^2*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))) + (81*3^(1
/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^(7/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1
/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1
/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))], -7 + 4*Sqrt[3]])/(91*b^(2/3)*d^3*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^
(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) - (54*Sqrt[2]*3^(3/4)*(b*c - a*d
)^(7/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/
3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/(
(1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(91*b^(
2/3)*d^3*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/
3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))^2)])
```

Rubi [A] time = 0.937686, antiderivative size = 839, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {50, 63, 304, 219, 1879}

$$\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{(1-\sqrt{3})\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\right|-7+}{91b^{2/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left((1-\sqrt{3})\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(1/3),x]
```

```
[Out] (-54*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(2/3))/(91*d^2) + (6*(a + b*x)^(3/
2)*(c + d*x)^(2/3))/(13*d) - (162*(b*c - a*d)^2*Sqrt[a + b*x])/(91*b^(2/3)*
d^2*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))) + (81*3^(1
/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^(7/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1
/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1
/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))], -7 + 4*Sqrt[3]])/(91*b^(2/3)*d^3*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^
(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) - (54*Sqrt[2]*3^(3/4)*(b*c - a*d
```

```
)^(7/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/
3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/(
(1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(91*b^(
2/3)*d^3*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/
3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt[3]{c+dx}}\,dx = \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} - \frac{(9(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[3]{c+dx}}\,dx}{13d}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}(c+dx)^{2/3}}{91d^2} + \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} + \frac{\left(27(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}\sqrt[3]{c+dx}}\,dx}{91d^2}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}(c+dx)^{2/3}}{91d^2} + \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} + \frac{\left(81(bc-ad)^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx\right.}{91d^3}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}(c+dx)^{2/3}}{91d^2} + \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} - \frac{\left(81(bc-ad)^2\right)\text{Subst}\left(\int \frac{(1+\sqrt{3})\sqrt[3]{bc-ad-}}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\right.}{91\sqrt[3]{b}d^3}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}(c+dx)^{2/3}}{91d^2} + \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} - \frac{162(bc-ad)^2\sqrt{a+bx}}{91b^{2/3}d^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c}\right.}$$

Mathematica [C] time = 0.0284428, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{5/2}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/3),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/3))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}\frac{1}{\sqrt[3]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt[3]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/3), x)
```

3.1566 $\int \frac{\sqrt{a+bx}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=804

$$-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+\right.}{7b^{2/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (6*Sqrt[a + b*x]*(c + d*x)^(2/3))/(7*d) + (18*(b*c - a*d)*Sqrt[a + b*x])/(7
*b^(2/3)*d*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))) - (
9*3^(1/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^(4/3)*((b*c - a*d)^(1/3) - b^(1/3)*
(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d
*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(
1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3))], -7 + 4*Sqrt[3]])/(7*b^(2/3)*d^2*Sqrt[a + b*x]*Sqrt[-(((b*c -
a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) + (6*Sqrt[2]*3^(3/4)*(b*c -
 a*d)^(4/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)
^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3
))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF
[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - S
qrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(7*
b^(2/3)*d^2*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^
(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3))^2)])
```

Rubi [A] time = 0.667953, antiderivative size = 804, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {50, 63, 304, 219, 1879}

$$-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+\right.}{7b^{2/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(1/3),x]
```

```
[Out] (6*Sqrt[a + b*x]*(c + d*x)^(2/3))/(7*d) + (18*(b*c - a*d)*Sqrt[a + b*x])/(7
*b^(2/3)*d*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))) - (
9*3^(1/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^(4/3)*((b*c - a*d)^(1/3) - b^(1/3)*
(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d
*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(
1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3))], -7 + 4*Sqrt[3]])/(7*b^(2/3)*d^2*Sqrt[a + b*x]*Sqrt[-(((b*c -
a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) + (6*Sqrt[2]*3^(3/4)*(b*c -
 a*d)^(4/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)
^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3
```

```
))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF
[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - S
qrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(7*
b^(2/3)*d^2*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^
(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt[3]{c+dx}}\,dx = \frac{6\sqrt{a+bx}(c+dx)^{2/3}}{7d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}\sqrt[3]{c+dx}}\,dx}{7d}$$

$$= \frac{6\sqrt{a+bx}(c+dx)^{2/3}}{7d} - \frac{(9(bc-ad))\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{7d^2}$$

$$= \frac{6\sqrt{a+bx}(c+dx)^{2/3}}{7d} + \frac{(9(bc-ad))\,\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{7\sqrt[3]{b}d^2} - \frac{\left(9\sqrt{2\left(2+\sqrt{3}\right)}(bc\right.}{}$$

$$= \frac{6\sqrt{a+bx}(c+dx)^{2/3}}{7d} + \frac{18(bc-ad)\sqrt{a+bx}}{7b^{2/3}d\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)} - \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}(bc-ad)^{4/3}\left(\sqrt[3]{bc-\right.}{}$$

Mathematica [C] time = 0.0250908, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{3/2}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(1/3),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/3))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\frac{1}{\sqrt[3]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/3), x)
```

3.1567 $\int \frac{1}{\sqrt{a+bx}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=762

$$-\frac{2\sqrt{2}3^{3/4}\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c\cdots}}\cdots\right)\right.}{b^{2/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-6*Sqrt[a + b*x])/(b^(2/3)*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c +
 d*x)^(1/3))) + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^(1/3)*((b*c - a*d)
^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c -
a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c -
 a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(b^(2/3)*d*Sqrt[a + b*x]*Sq
rt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1
- Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) - (2*Sqrt[2]*3
^(3/4)*(b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt
[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(
c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*
Sqrt[3]])/(b^(2/3)*d*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)
*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.578089, antiderivative size = 762, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {63, 304, 219, 1879}

$$-\frac{6\sqrt{a+bx}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{2\sqrt{2}3^{3/4}\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\cdots}{b^{2/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\cdots\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\cdots\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(1/3)),x]
```

```
[Out] (-6*Sqrt[a + b*x])/(b^(2/3)*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c +
 d*x)^(1/3))) + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^(1/3)*((b*c - a*d)
^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c -
a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c -
 a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(b^(2/3)*d*Sqrt[a + b*x]*Sq
rt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1
- Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) - (2*Sqrt[2]*3
^(3/4)*(b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt
[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(
c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
```

```
1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*
Sqrt[3]])/(b^(2/3)*d*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)
*(c + d*x)^(1/3))^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt[3]{c+dx}}\,dx = \frac{3\,\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= -\frac{3\,\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{\sqrt[3]{b}d} + \frac{\left(3\sqrt{2\left(2+\sqrt{3}\right)}\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\right.}{\sqrt[3]{b}d}$$

$$= -\frac{6\sqrt{a+bx}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)} + \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\ }}{b^{2/3}d\sqrt{\ }}$$

Mathematica [C] time = 0.0200082, size = 71, normalized size = 0.09

$$\frac{2\sqrt{a+bx}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3, 1
/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/3))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt[3]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{2}{3}}}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(2/3)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/3)), x)
```

3.1568 $\int \frac{1}{(a+bx)^{3/2}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=796

$$-\frac{2\sqrt{a+bx}d}{b^{2/3}(bc-ad)\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c}\right.}}}{b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}}{\left((1-\right.}}}$$

```
[Out] (-2*(c + d*x)^(2/3))/((b*c - a*d)*Sqrt[a + b*x]) - (2*d*Sqrt[a + b*x])/(b^(
2/3)*(b*c - a*d)*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^
(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)
^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c +
 d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))],
-7 + 4*Sqrt[3]])/(b^(2/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*
d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c
 - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) - (2*Sqrt[2]*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d
)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c
- a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*(b*c - a*d)^(2
/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
^2)])
```

Rubi [A] time = 0.689837, antiderivative size = 796, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {51, 63, 304, 219, 1879}

$$-\frac{2\sqrt{a+bx}d}{b^{2/3}(bc-ad)\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c}\right.}}}{b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}}{\left((1-\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(1/3)),x]
```

```
[Out] (-2*(c + d*x)^(2/3))/((b*c - a*d)*Sqrt[a + b*x]) - (2*d*Sqrt[a + b*x])/(b^(
2/3)*(b*c - a*d)*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^
(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)
^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c +
 d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))],
-7 + 4*Sqrt[3]])/(b^(2/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*
d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c
 - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) - (2*Sqrt[2]*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d
```

```
)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c
- a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*(b*c - a*d)^(2
/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
^2)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt[3]{c+dx}}\,dx = -\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}} + \frac{d\int \frac{1}{\sqrt{a+bx}\sqrt[3]{c+dx}}\,dx}{3(bc-ad)}$$

$$= -\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}} - \frac{\text{Subst}\left(\int \frac{(1+\sqrt{3})\sqrt[3]{bc-ad}-\sqrt[3]{b}x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{\sqrt[3]{b}(bc-ad)} + \frac{\sqrt{2\left(2+\sqrt{3}\right)}\,\text{Subst}\Big(}{\sqrt[3]{b}(}$$

$$= -\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}} - \frac{2d\sqrt{a+bx}}{b^{2/3}(bc-ad)\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}\right.}{}$$

Mathematica [C] time = 0.0227276, size = 71, normalized size = 0.09

$$-\frac{2\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/3)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-1/2, 1/3, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(1/3))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{3}{2}}\frac{1}{\sqrt[3]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{2}{3}}}{b^2dx^3+a^2c+\left(b^2c+2\,abd\right)x^2+\left(2\,abc+a^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(2/3)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+bx)^{\frac{3}{2}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/3)), x)
```

3.1569 $\int \frac{1}{(a+bx)^{5/2}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=842

$$\frac{10\sqrt{a+bx}d^2}{9b^{2/3}(bc-ad)^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{5\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^2\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+\cdots}\right.}}}{3\,3^{3/4}b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\cdots}}$$

```
[Out] (-2*(c + d*x)^(2/3))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (10*d*(c + d*x)^(2/3
))/(9*(b*c - a*d)^2*Sqrt[a + b*x]) + (10*d^2*Sqrt[a + b*x])/(9*b^(2/3)*(b*c
 - a*d)^2*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))) - (5
*Sqrt[2 + Sqrt[3]]*d*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b
*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c +
d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*
EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
)/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt
[3]])/(3*3^(3/4)*b^(2/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d
)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c
- a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) + (10*Sqrt[2]*d*((b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*
d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*b^(2/3)*(b*c - a*d
)^(5/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3
)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3))^2)])
```

Rubi [A] time = 0.833726, antiderivative size = 842, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {51, 63, 304, 219, 1879}

$$\frac{10\sqrt{a+bx}d^2}{9b^{2/3}(bc-ad)^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{5\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^2\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+\cdots}\right.}}}{3\,3^{3/4}b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(1/3)),x]
```

```
[Out] (-2*(c + d*x)^(2/3))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (10*d*(c + d*x)^(2/3
))/(9*(b*c - a*d)^2*Sqrt[a + b*x]) + (10*d^2*Sqrt[a + b*x])/(9*b^(2/3)*(b*c
 - a*d)^2*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))) - (5
*Sqrt[2 + Sqrt[3]]*d*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b
*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c +
d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*
EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
)/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt
[3]])/(3*3^(3/4)*b^(2/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d
)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c
- a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) + (10*Sqrt[2]*d*((b*c - a*d)^(
```

```
1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*
d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3)
 - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*b^(2/3)*(b*c - a*d
)^(5/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3
)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3))^2)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt[3]{c+dx}}\,dx = -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(5d)\int \frac{1}{(a+bx)^{3/2}\sqrt[3]{c+dx}}\,dx}{9(bc-ad)}$$

$$= -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{(5d^2)\int \frac{1}{\sqrt{a+bx}\sqrt[3]{c+dx}}\,dx}{27(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{(5d)\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{9(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(5d)\,\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c}\right.}{9\sqrt[3]{b}(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{10d^2\sqrt{a+bx}}{9b^{2/3}(bc-ad)^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\right.}$$

Mathematica [C] time = 0.0218393, size = 73, normalized size = 0.09

$$-\frac{2\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/3)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-3/2, 1/3, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/3))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{2}}\frac{1}{\sqrt[3]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{2}{3}}}{b^3dx^4+a^3c+\left(b^3c+3\,ab^2d\right)x^3+3\left(ab^2c+a^2bd\right)x^2+\left(3\,a^2bc+a^3d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(2/3)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+bx)^{\frac{5}{2}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/3)), x)
```

3.1570 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=416

$$-\frac{54\,3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc}}\right.\right.}{55\sqrt[3]{b}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-54*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/3))/(55*d^2) + (6*(a + b*x)^(3/
2)*(c + d*x)^(1/3))/(11*d) - (54*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^2*((
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/
3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c -
 a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(55*b^(1/3)*d^3*S
qrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.385129, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {50, 63, 219}

$$-\frac{54\,3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{}}\right.\right.}{55\sqrt[3]{b}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(2/3),x]
```

```
[Out] (-54*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/3))/(55*d^2) + (6*(a + b*x)^(3/
2)*(c + d*x)^(1/3))/(11*d) - (54*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^2*((
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/
3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c -
 a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(55*b^(1/3)*d^3*S
qrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{2/3}}\,dx = \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d} - \frac{(9(bc-ad))\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}}\,dx}{11d}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\sqrt[3]{c+dx}}{55d^2} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d} + \frac{\left(27(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{2/3}}\,dx}{55d^2}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\sqrt[3]{c+dx}}{55d^2} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d} + \frac{\left(81(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c}\right.}{55d^3}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\sqrt[3]{c+dx}}{55d^2} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d} - \frac{54\ 3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c}\right.}{}$$

Mathematica [C] time = 0.0358568, size = 73, normalized size = 0.18

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(2/3),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(2/3))
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(2/3),x)

[Out] int((b*x+a)^(3/2)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="maxima")

[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="fricas")

[Out] integral((b*x + a)^(3/2)/(d*x + c)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(2/3),x)

[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="giac")

[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(2/3), x)
```

3.1571 $\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=381

$$\frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}}\right.\right.}{5\sqrt[3]{b}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (6*Sqrt[a + b*x]*(c + d*x)^(1/3))/(5*d) + (6*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c
 - a*d)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/
3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/(
(1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^(1
/3)*d^2*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3
)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3))^2)])
```

Rubi [A] time = 0.29645, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {50, 63, 219}

$$\frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\right|}{5\sqrt[3]{b}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(2/3),x]
```

```
[Out] (6*Sqrt[a + b*x]*(c + d*x)^(1/3))/(5*d) + (6*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c
 - a*d)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/
3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/(
(1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^(1
/3)*d^2*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3
)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}}\,dx = \frac{6\sqrt{a+bx}\sqrt[3]{c+dx}}{5d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}(c+dx)^{2/3}}\,dx}{5d}$$

$$= \frac{6\sqrt{a+bx}\sqrt[3]{c+dx}}{5d} - \frac{(9(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{5d^2}$$

$$= \frac{6\sqrt{a+bx}\sqrt[3]{c+dx}}{5d} + \frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c\cdots}\right.}}}{5\sqrt[3]{b}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad\cdots}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad\cdots}\right.}}}$$

Mathematica [C] time = 0.0253615, size = 73, normalized size = 0.19

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(2/3),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(2/3))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\,(dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}}{(dx+c)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(2/3), x)
```

3.1572 $\int \frac{1}{\sqrt{a+bx}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=345

$$-\frac{2\, 3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-2*3^(3/4)*Sqrt[2 - Sqrt[3]]*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2
/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7
 + 4*Sqrt[3]])/(b^(1/3)*d*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^
(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.223403, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 219}

$$-\frac{2\, 3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(2/3)),x]
```

```
[Out] (-2*3^(3/4)*Sqrt[2 - Sqrt[3]]*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2
/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7
 + 4*Sqrt[3]])/(b^(1/3)*d*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^
(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
```

```
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{2/3}}\,dx = \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= -\frac{2\;3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{bc-ad}\,\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1+\cdots}{(1-\cdots}\right.\right.}{\sqrt[3]{b}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}$$

Mathematica [C] time = 0.0217865, size = 71, normalized size = 0.21

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(2/3)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[1/2, 2
/3, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(2/3))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}}(dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{3}}}{bdx^2+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(2/3)), x)
```

3.1573 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=383

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right),4\sqrt{3}\cdots\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{a+bx}(bc-ad)\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-2*(c + d*x)^(1/3))/((b*c - a*d)*Sqrt[a + b*x]) + (2*Sqrt[2 - Sqrt[3]]*((b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3
)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3
])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*(
b*c - a*d)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(
1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)
^(1/3))^2)])
```

Rubi [A] time = 0.293588, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {51, 63, 219}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{a+bx}(bc-ad)\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(2/3)),x]
```

```
[Out] (-2*(c + d*x)^(1/3))/((b*c - a*d)*Sqrt[a + b*x]) + (2*Sqrt[2 - Sqrt[3]]*((b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3
)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3
])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*(
b*c - a*d)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(
1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)
^(1/3))^2)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx = -\frac{2\sqrt[3]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{d\int \frac{1}{\sqrt{a+bx}(c+dx)^{2/3}}\,dx}{3(bc-ad)}$$

$$= -\frac{2\sqrt[3]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2\sqrt[3]{c+dx}}{(bc-ad)\sqrt{a+bx}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+d\cdots}\right.}}}{\sqrt[4]{3}\sqrt[3]{b}(bc-ad)\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc\cdots}}{\left(\left(1-\cdots\right.\right.}}}$$

Mathematica [C] time = 0.0215687, size = 71, normalized size = 0.19

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(2/3)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-1/2, 2/3, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(2/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{3}{2}} (dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{3}}}{b^2dx^3+a^2c+\left(b^2c+2\,abd\right)x^2+\left(2\,abc+a^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(2/3)), x)
```

3.1574 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=421

$$-\frac{14\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\right.}{9\sqrt[4]{3}\sqrt[3]{b}\sqrt{a+bx}(bc-ad)^2\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-2*(c + d*x)^(1/3))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (14*d*(c + d*x)^(1/3
))/(9*(b*c - a*d)^2*Sqrt[a + b*x]) - (14*Sqrt[2 - Sqrt[3]]*d*((b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*
d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*b^(1/3)*(b*c - a*d
)^2*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
^2)])
```

Rubi [A] time = 0.368191, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {51, 63, 219}

$$-\frac{14\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\middle|-7+4\right.}{9\sqrt[4]{3}\sqrt[3]{b}\sqrt{a+bx}(bc-ad)^2\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(2/3)),x]
```



```
[Out] (-2*(c + d*x)^(1/3))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (14*d*(c + d*x)^(1/3
))/(9*(b*c - a*d)^2*Sqrt[a + b*x]) - (14*Sqrt[2 - Sqrt[3]]*d*((b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*
d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*b^(1/3)*(b*c - a*d
)^2*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
^2)])
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{2/3}}\,dx = -\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(7d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx}{9(bc-ad)}$$

$$= -\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{14d\sqrt[3]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{\left(7d^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{2/3}}\,dx}{27(bc-ad)^2}$$

$$= -\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{14d\sqrt[3]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(7d)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{9(bc-ad)^2}$$

$$= -\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{14d\sqrt[3]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{14\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{(\ldots}}{9\sqrt[4]{3}\sqrt[3]{b}(\ldots}$$

Mathematica [C] time = 0.0223231, size = 73, normalized size = 0.17

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(2/3)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-3/2, 2/3, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(2/3))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{2}}(dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{3}}}{b^3dx^4+a^3c+\left(b^3c+3\,ab^2d\right)x^3+3\left(ab^2c+a^2bd\right)x^2+\left(3\,a^2bc+a^3d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(2/3)), x)
```

3.1575 $\int (a+bx)^{2/3}\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=219

$$\frac{(bc-ad)^2\log(c+dx)}{18b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{6b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}(bc-a}{6bd}$$

```
[Out] ((b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3))/(6*b*d) + ((a + b*x)^(5/3)*(c
 + d*x)^(1/3))/(2*b) + ((b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*
x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(3*Sqrt[3]*b^(4/3)*d^(5/3)) +
 ((b*c - a*d)^2*Log[c + d*x])/(18*b^(4/3)*d^(5/3)) + ((b*c - a*d)^2*Log[-1
+ (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(6*b^(4/3)*d^(5/3))
```

Rubi [A] time = 0.0875222, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$\frac{(bc-ad)^2\log(c+dx)}{18b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{6b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}(bc-a}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)*(c + d*x)^(1/3),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3))/(6*b*d) + ((a + b*x)^(5/3)*(c
 + d*x)^(1/3))/(2*b) + ((b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*
x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(3*Sqrt[3]*b^(4/3)*d^(5/3)) +
 ((b*c - a*d)^2*Log[c + d*x])/(18*b^(4/3)*d^(5/3)) + ((b*c - a*d)^2*Log[-1
+ (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(6*b^(4/3)*d^(5/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int (a+bx)^{2/3}\sqrt[3]{c+dx}\,dx = \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}dx}{6b}$$
$$= \frac{(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6bd} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b} - \frac{(bc-ad)^2\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{2/3}}dx}{9bd}$$
$$= \frac{(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6bd} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b} + \frac{(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{3\sqrt{3}b^{4/3}d^{5/3}} +$$

Mathematica [C] time = 0.0314631, size = 73, normalized size = 0.33

$$\frac{3(a+bx)^{5/3}\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{3};\frac{8}{3};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)*(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 5/3, 8/3, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{2}{3}}\sqrt[3]{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)*(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(2/3)*(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(2/3)*(d*x + c)^(1/3), x)
```

Fricas [A] time = 2.06942, size = 1800, normalized size = 8.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(b*d^2)^(1
/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 + 3*(b*d^2)^(1/3)*(b*x + a)^(2/3)*(
d*x + c)^(1/3)*d + 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (b*
d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^2)^(1/3)*(b*d*x + a*d))*s
qrt(-(b*d^2)^(1/3)/b)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*lo
g(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a)
) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x
 + c)^(2/3)*b*d + (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(
1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(3*b^2*d^3*x + b^2*c*d^2 + 2*a*b*d^3)*(b
*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*d^3), -1/18*(6*sqrt(1/3)*(b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*(2*(b*d^2)
^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))*sqrt(
(b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*
d^2)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x +
a))/(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*log(((b*x +
a)^(1/3)*(d*x + c)^(2/3)*b*d + (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3
) + (b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) - 3*(3*b^2*d^3*x + b^2*c*d^2 +
2*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^{\frac{2}{3}}\sqrt[3]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)*(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(2/3)*(c + d*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(2/3)*(d*x + c)^(1/3), x)
```

3.1576 $\int \frac{\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=172

$$-\frac{(bc-ad)\log(c+dx)}{6b^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d^{2/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b}$$

```
[Out] ((a + b*x)^(2/3)*(c + d*x)^(1/3))/b - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^
(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(Sqrt[3]*b^(4/3)
*d^(2/3)) - ((b*c - a*d)*Log[c + d*x])/(6*b^(4/3)*d^(2/3)) - ((b*c - a*d)*L
og[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(4/3)*d^
(2/3))
```

Rubi [A] time = 0.0455278, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$-\frac{(bc-ad)\log(c+dx)}{6b^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d^{2/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(2/3)*(c + d*x)^(1/3))/b - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^
(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(Sqrt[3]*b^(4/3)
*d^(2/3)) - ((b*c - a*d)*Log[c + d*x])/(6*b^(4/3)*d^(2/3)) - ((b*c - a*d)*L
og[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(4/3)*d^
(2/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\,dx = \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{2/3}}\,dx}{3b}$$

$$= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b} - \frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}b^{4/3}d^{2/3}} - \frac{(bc-ad)\log(c+dx)}{6b^{4/3}d^{2/3}} - \frac{(bc-ad)\log\left(-1+\cdot\right.}{2b^{4/3}d^{2/3}}$$

Mathematica [C] time = 0.0235564, size = 73, normalized size = 0.42

$$\frac{3(a+bx)^{2/3}\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{d(a+bx)}{ad-bc}\right)}{2b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, (d*(a
+ b*x))/(-(b*c) + a*d)])/(2*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \sqrt[3]{dx+c}\frac{1}{\sqrt[3]{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(1/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(1/3), x)
```

Fricas [B] time = 2.00917, size = 1547, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt(-(b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 + 3*(b*d^2)^(1
/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d + 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x
 + c)^(2/3)*b*d - (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^2)^(
1/3)*(b*d*x + a*d))*sqrt(-(b*d^2)^(1/3)/b)) - 2*(b*d^2)^(2/3)*(b*c - a*d)*l
og(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a
)) + (b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (
b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))
/(b*x + a)))/(b^2*d^2), 1/6*(6*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 + 6*sq
rt(1/3)*(b^2*c*d - a*b*d^2)*sqrt((b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*(2*(b*d^
2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))*sqr
t((b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(b*d^2)^(2/3)*(b*c - a*d)*log(((b
*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a)) + (
b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (b*d^2)
^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))/(b*x
+ a)))/(b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c + dx}}{\sqrt[3]{a + bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(1/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^{\frac{1}{3}}}{(bx + a)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(1/3), x)
```

3.1577 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=149

$$-\frac{3\sqrt[3]{d}\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}}-\frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{4/3}}-\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}-\frac{\sqrt[3]{d}\log(c+dx)}{2b^{4/3}}$$

```
[Out] (-3*(c + d*x)^(1/3))/(b*(a + b*x)^(1/3)) - (Sqrt[3]*d^(1/3)*ArcTan[1/Sqrt[3
] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))]])/b^(4/3)
 - (d^(1/3)*Log[c + d*x])/(2*b^(4/3)) - (3*d^(1/3)*Log[-1 + (d^(1/3)*(a + b
*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(4/3))
```

Rubi [A] time = 0.032848, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 59}

$$-\frac{3\sqrt[3]{d}\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}}-\frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{4/3}}-\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}-\frac{\sqrt[3]{d}\log(c+dx)}{2b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(4/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(b*(a + b*x)^(1/3)) - (Sqrt[3]*d^(1/3)*ArcTan[1/Sqrt[3
] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))]])/b^(4/3)
 - (d^(1/3)*Log[c + d*x])/(2*b^(4/3)) - (3*d^(1/3)*Log[-1 + (d^(1/3)*(a + b
*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(4/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{4/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}} + \frac{d\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{2/3}}\,dx}{b}$$
$$= -\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}} - \frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{b^{4/3}} - \frac{\sqrt[3]{d}\log(c+dx)}{2b^{4/3}} - \frac{3\sqrt[3]{d}\log\left(-1+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{2b^{4/3}}$$

Mathematica [C] time = 0.0272532, size = 71, normalized size = 0.48

$$-\frac{3\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[3]{a+bx}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(4/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/3)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \sqrt[3]{dx+c}\,(bx+a)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(4/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(4/3), x)
```

Fricas [B] time = 1.8196, size = 603, normalized size = 4.05

$$-\frac{2\sqrt{3}(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}b\left(-\frac{d}{b}\right)^{\frac{2}{3}}+\sqrt{3}(bdx+ad)}{3(bdx+ad)}\right) + (bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\log\left(\frac{(bx+a)\left(-\frac{d}{b}\right)^{\frac{2}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\left(-\frac{d}{b}\right)^{\frac{1}{3}}}{bx+a}\right.}{2\left(b^2x+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(b*x + a)*(-d/b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(2/3
)*(d*x + c)^(1/3)*b*(-d/b)^(2/3) + sqrt(3)*(b*d*x + a*d))/(b*d*x + a*d)) +
(b*x + a)*(-d/b)^(1/3)*log(((b*x + a)*(-d/b)^(2/3) - (b*x + a)^(2/3)*(d*x +
 c)^(1/3)*(-d/b)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(b*x + a)) - 2*(b
*x + a)*(-d/b)^(1/3)*log(((b*x + a)*(-d/b)^(1/3) + (b*x + a)^(2/3)*(d*x + c
)^(1/3))/(b*x + a)) + 6*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*x + a*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(4/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(4/3), x)
```

3.1578 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx$

Optimal. Leaf size=32

$$-\frac{3(c+dx)^{4/3}}{4(a+bx)^{4/3}(bc-ad)}$$

```
[Out] (-3*(c + d*x)^(4/3))/(4*(b*c - a*d)*(a + b*x)^(4/3))
```

Rubi [A] time = 0.0029945, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{3(c+dx)^{4/3}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(4*(b*c - a*d)*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx = -\frac{3(c+dx)^{4/3}}{4(bc-ad)(a+bx)^{4/3}}$$

Mathematica [A] time = 0.0117813, size = 32, normalized size = 1.

$$-\frac{3(c+dx)^{4/3}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(4*(b*c - a*d)*(a + b*x)^(4/3))
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$\frac{3}{4\,ad-4\,bc}\,(dx+c)^{\frac{4}{3}}\,(bx+a)^{-\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(7/3),x)
```

```
[Out] 3/4/(b*x+a)^(4/3)*(d*x+c)^(4/3)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/3), x)
```

Fricas [B] time = 1.72426, size = 143, normalized size = 4.47

$$-\frac{3\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}}{4\left(a^2bc-a^3d+\left(b^3c-ab^2d\right)x^2+2\left(ab^2c-a^2bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] -3/4*(b*x + a)^(2/3)*(d*x + c)^(4/3)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x
^2 + 2*(a*b^2*c - a^2*b*d)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(7/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(7/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="giac")

[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/3), x)
```

3.1579 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}} dx$

Optimal. Leaf size=66

$$\frac{9d(c+dx)^{4/3}}{28(a+bx)^{4/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{7(a+bx)^{7/3}(bc-ad)}$$

```
[Out] (-3*(c + d*x)^(4/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (9*d*(c + d*x)^(4/3)
)/(28*(b*c - a*d)^2*(a + b*x)^(4/3))
```

Rubi [A] time = 0.0086674, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{9d(c+dx)^{4/3}}{28(a+bx)^{4/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{7(a+bx)^{7/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(10/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (9*d*(c + d*x)^(4/3)
)/(28*(b*c - a*d)^2*(a + b*x)^(4/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx &= -\frac{3(c+dx)^{4/3}}{7(bc-ad)(a+bx)^{7/3}}-\frac{(3d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx}{7(bc-ad)}\\ &= -\frac{3(c+dx)^{4/3}}{7(bc-ad)(a+bx)^{7/3}}+\frac{9d(c+dx)^{4/3}}{28(bc-ad)^2(a+bx)^{4/3}}\end{aligned}$$

Mathematica [A] time = 0.0229644, size = 46, normalized size = 0.7

$$\frac{3(c+dx)^{4/3}(7ad-4bc+3bdx)}{28(a+bx)^{7/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(10/3),x]
```

```
[Out] (3*(c + d*x)^(4/3)*(-4*b*c + 7*a*d + 3*b*d*x))/(28*(b*c - a*d)^2*(a + b*x)^
(7/3))
```

Maple [A] time = 0.005, size = 54, normalized size = 0.8

$$\frac{9\,bdx+21\,ad-12\,bc}{28\,a^{2}d^{2}-56\,abcd+28\,b^{2}c^{2}}(dx+c)^{\frac{4}{3}}(bx+a)^{-\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(10/3),x)
```

```
[Out] 3/28*(d*x+c)^(4/3)*(3*b*d*x+7*a*d-4*b*c)/(b*x+a)^(7/3)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(10/3), x)
```

Fricas [B] time = 1.71984, size = 370, normalized size = 5.61

$$\frac{3\left(3\,bd^{2}x^{2}-4\,bc^{2}+7\,acd-\left(bcd-7\,ad^{2}\right)x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{28\left(a^{3}b^{2}c^{2}-2\,a^{4}bcd+a^{5}d^{2}+\left(b^{5}c^{2}-2\,ab^{4}cd+a^{2}b^{3}d^{2}\right)x^{3}+3\left(ab^{4}c^{2}-2\,a^{2}b^{3}cd+a^{3}b^{2}d^{2}\right)x^{2}+3\left(a^{2}b^{3}c^{2}-2\,a^{3}b^{2}cd+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 3/28*(3*b*d^2*x^2 - 4*b*c^2 + 7*a*c*d - (b*c*d - 7*a*d^2)*x)*(b*x + a)^(2/3
)*(d*x + c)^(1/3)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4
*c*d + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 +
 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(10/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(10/3), x)
```

3.1580 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx$

Optimal. Leaf size=101

$$-\frac{27d^2(c+dx)^{4/3}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{9d(c+dx)^{4/3}}{35(a+bx)^{7/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{10(a+bx)^{10/3}(bc-ad)}$$

```
[Out] (-3*(c + d*x)^(4/3))/(10*(b*c - a*d)*(a + b*x)^(10/3)) + (9*d*(c + d*x)^(4/
3))/(35*(b*c - a*d)^2*(a + b*x)^(7/3)) - (27*d^2*(c + d*x)^(4/3))/(140*(b*c
 - a*d)^3*(a + b*x)^(4/3))
```

Rubi [A] time = 0.0173515, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{27d^2(c+dx)^{4/3}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{9d(c+dx)^{4/3}}{35(a+bx)^{7/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{10(a+bx)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(13/3),x]
```


```
[Out] (-3*(c + d*x)^(4/3))/(10*(b*c - a*d)*(a + b*x)^(10/3)) + (9*d*(c + d*x)^(4/
3))/(35*(b*c - a*d)^2*(a + b*x)^(7/3)) - (27*d^2*(c + d*x)^(4/3))/(140*(b*c
 - a*d)^3*(a + b*x)^(4/3))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx &= -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}}-\frac{(3d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx}{5(bc-ad)}\\ &= -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}}+\frac{9d(c+dx)^{4/3}}{35(bc-ad)^2(a+bx)^{7/3}}+\frac{\left(9d^2\right)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx}{35(bc-ad)^2}\\ &= -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}}+\frac{9d(c+dx)^{4/3}}{35(bc-ad)^2(a+bx)^{7/3}}-\frac{27d^2(c+dx)^{4/3}}{140(bc-ad)^3(a+bx)^{4/3}}\end{aligned}$$

Mathematica [A] time = 0.0400891, size = 77, normalized size = 0.76

$$-\frac{3(c+dx)^{4/3}\left(35a^2d^2+10abd(3dx-4c)+b^2\left(14c^2-12cdx+9d^2x^2\right)\right)}{140(a+bx)^{10/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(13/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3)*(35*a^2*d^2 + 10*a*b*d*(-4*c + 3*d*x) + b^2*(14*c^2 - 1
2*c*d*x + 9*d^2*x^2)))/(140*(b*c - a*d)^3*(a + b*x)^(10/3))
```

Maple [A] time = 0.005, size = 105, normalized size = 1.

$$\frac{27\,b^2d^2x^2+90\,abd^2x-36\,b^2cdx+105\,a^2d^2-120\,abcd+42\,b^2c^2}{140\,a^3d^3-420\,a^2cbd^2+420\,ab^2c^2d-140\,b^3c^3}(dx+c)^{\frac{4}{3}}(bx+a)^{-\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(13/3),x)
```

```
[Out] 3/140*(d*x+c)^(4/3)*(9*b^2*d^2*x^2+30*a*b*d^2*x-12*b^2*c*d*x+35*a^2*d^2-40*
a*b*c*d+14*b^2*c^2)/(b*x+a)^(10/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3
*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(13/3), x)
```

Fricas [B] time = 1.75163, size = 689, normalized size = 6.82

$$-\frac{3\left(9\,b^2d^3x^3+14\,b^2c^3-40\,abc^2d+35\,a^2cd^2-3\left(b^2cd^2-10\,abd^3\right)x\right.}{140\left(a^4b^3c^3-3\,a^5b^2c^2d+3\,a^6bcd^2-a^7d^3+\left(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3\right)x^4+4\left(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="fricas")
```

```
[Out] -3/140*(9*b^2*d^3*x^3 + 14*b^2*c^3 - 40*a*b*c^2*d + 35*a^2*c*d^2 - 3*(b^2*c
*d^2 - 10*a*b*d^3)*x^2 + (2*b^2*c^2*d - 10*a*b*c*d^2 + 35*a^2*d^3)*x)*(b*x
+ a)^(2/3)*(d*x + c)^(1/3)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 -
```

```
 a^7*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 +
4*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^3 + 6*(a^
2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^2 + 4*(a^3*b
^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(13/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(13/3), x)
```

3.1581 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{16/3}}\,dx$

Optimal. Leaf size=136

$$\frac{243d^3(c+dx)^{4/3}}{1820(a+bx)^{4/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{4/3}}{455(a+bx)^{7/3}(bc-ad)^3}+\frac{27d(c+dx)^{4/3}}{130(a+bx)^{10/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{13(a+bx)^{13/3}(bc-ad)}$$

```
[Out] (-3*(c + d*x)^(4/3))/(13*(b*c - a*d)*(a + b*x)^(13/3)) + (27*d*(c + d*x)^(4
/3))/(130*(b*c - a*d)^2*(a + b*x)^(10/3)) - (81*d^2*(c + d*x)^(4/3))/(455*(
b*c - a*d)^3*(a + b*x)^(7/3)) + (243*d^3*(c + d*x)^(4/3))/(1820*(b*c - a*d)
^4*(a + b*x)^(4/3))
```

Rubi [A] time = 0.0283204, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{243d^3(c+dx)^{4/3}}{1820(a+bx)^{4/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{4/3}}{455(a+bx)^{7/3}(bc-ad)^3}+\frac{27d(c+dx)^{4/3}}{130(a+bx)^{10/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{13(a+bx)^{13/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(16/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(13*(b*c - a*d)*(a + b*x)^(13/3)) + (27*d*(c + d*x)^(4
/3))/(130*(b*c - a*d)^2*(a + b*x)^(10/3)) - (81*d^2*(c + d*x)^(4/3))/(455*(
b*c - a*d)^3*(a + b*x)^(7/3)) + (243*d^3*(c + d*x)^(4/3))/(1820*(b*c - a*d)
^4*(a + b*x)^(4/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{16/3}}\,dx = -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} - \frac{(9d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx}{13(bc-ad)}$$

$$= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} + \frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}} + \frac{\left(27d^2\right)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx}{65(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} + \frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}} - \frac{81d^2(c+dx)^{4/3}}{455(bc-ad)^3(a+bx)^{7/3}} - \frac{\left(81d^3\right)\int \frac{\sqrt[3]{c+}}{(a+bx}}{455(bc-a}$$

$$= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} + \frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}} - \frac{81d^2(c+dx)^{4/3}}{455(bc-ad)^3(a+bx)^{7/3}} + \frac{243d^3(c+}{1820(bc-ad)}$$

Mathematica [A] time = 0.0594977, size = 118, normalized size = 0.87

$$\frac{3(c+dx)^{4/3}\left(195a^2bd^2(3dx-4c)+455a^3d^3+39ab^2d\left(14c^2-12cdx+9d^2x^2\right)+b^3\left(126c^2dx-140c^3-108cd^2x^2+81d\right.\right.}{1820(a+bx)^{13/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(16/3),x]
```

```
[Out] (3*(c + d*x)^(4/3)*(455*a^3*d^3 + 195*a^2*b*d^2*(-4*c + 3*d*x) + 39*a*b^2*d
*(14*c^2 - 12*c*d*x + 9*d^2*x^2) + b^3*(-140*c^3 + 126*c^2*d*x - 108*c*d^2*
x^2 + 81*d^3*x^3)))/(1820*(b*c - a*d)^4*(a + b*x)^(13/3))
```

Maple [A] time = 0.009, size = 171, normalized size = 1.3

$$\frac{243\,b^3d^3x^3+1053\,ab^2d^3x^2-324\,b^3cd^2x^2+1755\,a^2bd^3x-1404\,ab^2cd^2x+378\,b^3c^2dx+1365\,a^3d^3-2340\,a^2cbd^2+16}{1820\,d^4a^4-7280\,bd^3ca^3+10920\,b^2d^2c^2a^2-7280\,b^3dc^3a+1820\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(16/3),x)
```

```
[Out] 3/1820*(d*x+c)^(4/3)*(81*b^3*d^3*x^3+351*a*b^2*d^3*x^2-108*b^3*c*d^2*x^2+58
5*a^2*b*d^3*x-468*a*b^2*c*d^2*x+126*b^3*c^2*d*x+455*a^3*d^3-780*a^2*b*c*d^2
+546*a*b^2*c^2*d-140*b^3*c^3)/(b*x+a)^(13/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b
^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(16/3), x)
```

Fricas [B] time = 1.77431, size = 1098, normalized size = 8.07

$$\frac{3\left(81\,b^3d^4x^4 - 140\,b^3c^4 + 546\,ab^2c^3d - 780\right.}{1820\left(a^5b^4c^4 - 4\,a^6b^3c^3d + 6\,a^7b^2c^2d^2 - 4\,a^8bcd^3 + a^9d^4 + \left(b^9c^4 - 4\,ab^8c^3d + 6\,a^2b^7c^2d^2 - 4\,a^3b^6cd^3 + a^4b^5d^4\right)x^5 + 5\left(ab^8\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 3/1820*(81*b^3*d^4*x^4 - 140*b^3*c^4 + 546*a*b^2*c^3*d - 780*a^2*b*c^2*d^2
+ 455*a^3*c*d^3 - 27*(b^3*c*d^3 - 13*a*b^2*d^4)*x^3 + 9*(2*b^3*c^2*d^2 - 13
*a*b^2*c*d^3 + 65*a^2*b*d^4)*x^2 - (14*b^3*c^3*d - 78*a*b^2*c^2*d^2 + 195*a
^2*b*c*d^3 - 455*a^3*d^4)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^5*b^4*c^4 -
 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4 + (b^9*c^4 -
 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^5 + 5
*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b
^4*d^4)*x^4 + 10*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5
*b^4*c*d^3 + a^6*b^3*d^4)*x^3 + 10*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b
^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x^2 + 5*(a^4*b^5*c^4 - 4*a^5*b^
4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(16/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(16/3), x)
```

3.1582 $\int (a+bx)^{4/3}\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=655

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)^3((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d \cdots}{\cdots}}}{10\,2^{2/3}b^{4/3}d^{7/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(b\cdots}{(\cdots}}}$$

```
[Out] (-3*(b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(20*b*d^2) + (3*(b*c - a
*d)*(a + b*x)^(4/3)*(c + d*x)^(1/3))/(40*b*d) + (3*(a + b*x)^(7/3)*(c + d*x
)^(1/3))/(8*b) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^3*((a + b*x)*(c + d
*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b
^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^
(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(10*2^(2/3)*b^(4/3)*d^(7/3)*(
a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3
)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))
)/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 1.41646, antiderivative size = 655, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 62, 623, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)^3((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d \cdots}{\cdots}}}{10\,2^{2/3}b^{4/3}d^{7/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad\cdots}{\left(2^{2/3}\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)*(c + d*x)^(1/3),x]
```


```
[Out] (-3*(b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(20*b*d^2) + (3*(b*c - a
*d)*(a + b*x)^(4/3)*(c + d*x)^(1/3))/(40*b*d) + (3*(a + b*x)^(7/3)*(c + d*x
)^(1/3))/(8*b) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^3*((a + b*x)*(c + d
*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b
^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^
(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(10*2^(2/3)*b^(4/3)*d^(7/3)*(
a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3
)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))
)/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int (a+bx)^{4/3}\sqrt[3]{c+dx}\,dx = \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b} + \frac{(bc-ad)\int \frac{(a+bx)^{4/3}}{(c+dx)^{2/3}}\,dx}{8b}$$

$$= \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b} - \frac{(bc-ad)^2\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}\,dx}{10bd}$$

$$= -\frac{3(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20bd^2} + \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b} + \frac{(bc -}{}$$

$$= -\frac{3(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20bd^2} + \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b} + \frac{((bc -}{}$$

$$= -\frac{3(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20bd^2} + \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b} + \frac{(3(bc}{}$$

$$= -\frac{3(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20bd^2} + \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b} + \frac{3^{3/4}\sqrt{}}{}$$

Mathematica [C] time = 0.0300274, size = 73, normalized size = 0.11

$$\frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{1}{3},\frac{7}{3};\frac{10}{3};\frac{d(a+bx)}{ad-bc}\right)}{7b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)*(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 7/3, 10/3, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{4}{3}}\sqrt[3]{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)*(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(4/3)*(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{4}{3}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)*(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(4/3)*(d*x + c)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx+a)^{\frac{4}{3}}(dx+c)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)*(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(4/3)*(d*x + c)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^{\frac{4}{3}}\sqrt[3]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)*(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(4/3)*(c + d*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{4}{3}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)*(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(4/3)*(d*x + c)^(1/3), x)
```

3.1583 $\int \sqrt[3]{a+bx}\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=617

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)^2((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}\cdots}{\cdots}}}{5\,2^{2/3}b^{4/3}d^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\cdots}}$$

```
[Out] (3*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(10*b*d) + (3*(a + b*x)^(4/
3)*(c + d*x)^(1/3))/(5*b) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^2*((a +
b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3)
- 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2
*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)
*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*2^(2/3)*b^(4/3)
*d^(4/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
 a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.849056, antiderivative size = 617, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 62, 623, 218}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)^2((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}\cdots}{\cdots}}}{5\,2^{2/3}b^{4/3}d^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-\cdots}{\left(2^{2/3}\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)*(c + d*x)^(1/3),x]
```


```
[Out] (3*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(10*b*d) + (3*(a + b*x)^(4/
3)*(c + d*x)^(1/3))/(5*b) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^2*((a +
b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3)
- 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2
*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)
*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*2^(2/3)*b^(4/3)
*d^(4/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
 a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \sqrt[3]{a+bx}\sqrt[3]{c+dx}\,dx = \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} + \frac{(bc-ad)\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}\,dx}{5b}$$

$$= \frac{3(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{10bd} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} - \frac{(bc-ad)^2\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{10bd}$$

$$= \frac{3(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{10bd} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} - \frac{\left((bc-ad)^2((a+bx)(c+dx))^{2/3}\right)\int \frac{}{(ac+(bc+\ldots}}{10bd(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= \frac{3(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{10bd} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} - \frac{\left(3(bc-ad)^2((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad\ldots}\right.}{10bd(a+\ldots}$$

$$= \frac{3(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{10bd} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)^2((a+bx)(c+dx))^{2/\ldots}}{\ldots}$$

Mathematica [C] time = 0.0285014, size = 73, normalized size = 0.12

$$\frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{7}{3};\frac{d(a+bx)}{ad-bc}\right)}{4b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)*(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, (d*(a
+ b*x))/(-(b*c) + a*d)])/(4*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx+a}\,\sqrt[3]{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)*(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(1/3)*(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)*(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/3)*(d*x + c)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)*(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)*(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(1/3)*(c + d*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)*(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/3)*(d*x + c)^(1/3), x)
```

3.1584 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=576

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/}\cdots}{\cdots}}}{2^{2/3}b^{4/3}\sqrt[3]{d}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-\cdots}{(2^{2/}\cdots}}}$$

```
[Out] (3*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(2*b) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c
 - a*d)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c
 - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d
*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(
2/3)*b^(4/3)*d^(1/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*
Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*
d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.562596, antiderivative size = 576, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 62, 623, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/}\cdots}{\cdots}}}{2^{2/3}b^{4/3}\sqrt[3]{d}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\cdots}{(2^{2/3}\sqrt[3]{b}\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(2*b) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c
 - a*d)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c
 - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d
*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(
2/3)*b^(4/3)*d^(1/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*
Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*
d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{2b}$$

$$= \frac{3\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{2b} + \frac{\left((bc-ad)((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^{2/3}}\,dx}{2b(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= \frac{3\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{2b} + \frac{\left(3(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+ad)^2+4b}}\right.}{2b(a+bx)^{2/3}(c+dx)^{2/3}(bc+ad+2bdx)}$$

$$= \frac{3\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{2b} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/}\right.}{2^{2/3}b^{4/3}\sqrt[3]{d}(a+bx)^{2/}}$$

Mathematica [C] time = 0.0239834, size = 71, normalized size = 0.12

$$\frac{3\sqrt[3]{a+bx}\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \sqrt[3]{dx+c}\,(bx+a)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(2/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(1/3)/(b*x + a)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(2/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(2/3), x)
```

3.1585 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{5/3}}\,dx$

Optimal. Leaf size=568

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+b\cdots}{(2\cdots}}}{2^{2/3}b^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\cdots}{(2^{2/3}\sqrt[3]{b}\cdots}}}$$

```
[Out] (-3*(c + d*x)^(1/3))/(2*b*(a + b*x)^(2/3)) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*d^(
2/3)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c -
a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x)
)^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(2/3
)*b^(4/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c
- a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d
*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.594428, antiderivative size = 568, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {47, 62, 623, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+b\cdots}{(2\cdots}}}{2^{2/3}b^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}(2^{2/3}\cdots}{(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(\cdots}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(5/3),x]
```


```
[Out] (-3*(c + d*x)^(1/3))/(2*b*(a + b*x)^(2/3)) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*d^(
2/3)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c -
a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x)
)^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(2/3
)*b^(4/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c
- a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d
*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{5/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}} + \frac{d\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{2b}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}} + \frac{\left(d((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^{2/3}}\,dx}{2b(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}} + \frac{\left(3d((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+ad)^2+4bdx^3}}\,dx,x,\sqrt[3]{\ldots}\right.}{2b(a+bx)^{2/3}(c+dx)^{2/3}(bc+ad+2bdx)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{\ldots}\right.}{2^{2/3}b^{4/3}(a+bx)^{2/3}(c+dx\ldots}$$

Mathematica [C] time = 0.0236049, size = 73, normalized size = 0.13

$$-\frac{3\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{1}{3};\frac{d(a+bx)}{ad-bc}\right)}{2b(a+bx)^{2/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(5/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Hypergeometric2F1[-2/3, -1/3, 1/3, (d*(a + b*x))/(-(b*c
) + a*d)])/(2*b*(a + b*x)^(2/3)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \sqrt[3]{dx+c}\,(bx+a)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(5/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}}{b^2x^2+2\,abx+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(5/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(5/3), x)
```

3.1586 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{8/3}}\,dx$

Optimal. Leaf size=617

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+}{(}}}{5\,2^{2/3}b^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)(ad+bc+2bdx)\sqrt{\frac{(}{}}}$$

```
[Out] (-3*(c + d*x)^(1/3))/(5*b*(a + b*x)^(5/3)) - (3*d*(c + d*x)^(1/3))/(10*b*(b
*c - a*d)*(a + b*x)^(2/3)) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*d^(5/3)*((a + b*x)*
(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(
2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1
/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x
)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*2^(2/3)*b^(4/3)*(b*c
 - a*d)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x
))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
 b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.843378, antiderivative size = 617, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 51, 62, 623, 218}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+}{(}}}{5\,2^{2/3}b^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)(ad+bc+2bdx)\sqrt{\frac{(bc-a}{(2^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(8/3),x]
```


```
[Out] (-3*(c + d*x)^(1/3))/(5*b*(a + b*x)^(5/3)) - (3*d*(c + d*x)^(1/3))/(10*b*(b
*c - a*d)*(a + b*x)^(2/3)) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*d^(5/3)*((a + b*x)*
(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(
2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1
/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x
)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*2^(2/3)*b^(4/3)*(b*c
 - a*d)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x
))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
 b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{8/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} + \frac{d\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}\,dx}{5b}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} - \frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}} - \frac{d^2\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{10b(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} - \frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}} - \frac{\left(d^2((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^{2/3}}\,dx}{10b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} - \frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}} - \frac{\left(3d^2((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\right.}{10b(bc-ad)(a+bx)^{2/3}(c+}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} - \frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bd}}{}$$

5

Mathematica [C] time = 0.0246114, size = 73, normalized size = 0.12

$$-\frac{3\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{5}{3},-\frac{1}{3};-\frac{2}{3};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(8/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Hypergeometric2F1[-5/3, -1/3, -2/3, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/3)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \sqrt[3]{dx+c}\,(bx+a)^{-\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(8/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(8/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(8/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x
 + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(8/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(8/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(8/3), x)
```

3.1587 $\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=216

$$-\frac{(bc-ad)^2\log(a+bx)}{9b^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{3b^{2/3}d^{7/3}}-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}d^{7/3}}-\frac{2\sqrt[3]{a+bx}(c+dx)^{2/3}}{3d^2}$$

```
[Out] (-2*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(2/3))/(3*d^2) + ((a + b*x)^(4/3)
*(c + d*x)^(2/3))/(2*d) - (2*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c
 + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(3*Sqrt[3]*b^(2/3)*d^(7/
3)) - ((b*c - a*d)^2*Log[a + b*x])/(9*b^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log
[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(3*b^(2/3)*d^(7
/3))
```

Rubi [A] time = 0.0903724, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$-\frac{(bc-ad)^2\log(a+bx)}{9b^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{3b^{2/3}d^{7/3}}-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}d^{7/3}}-\frac{2\sqrt[3]{a+bx}(c+dx)^{2/3}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/(c + d*x)^(1/3),x]
```

```
[Out] (-2*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(2/3))/(3*d^2) + ((a + b*x)^(4/3)
*(c + d*x)^(2/3))/(2*d) - (2*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c
 + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(3*Sqrt[3]*b^(2/3)*d^(7/
3)) - ((b*c - a*d)^2*Log[a + b*x])/(9*b^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log
[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(3*b^(2/3)*d^(7
/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx = \frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d} - \frac{(2(bc-ad))\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{3d}$$

$$= -\frac{2(bc-ad)\sqrt[3]{a+bx}(c+dx)^{2/3}}{3d^2} + \frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d} + \frac{\left(2(bc-ad)^2\right)\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{9d^2}$$

$$= -\frac{2(bc-ad)\sqrt[3]{a+bx}(c+dx)^{2/3}}{3d^2} + \frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d} - \frac{2(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{3\sqrt{3}b^{2/3}d^{7/3}} - (bc$$

Mathematica [C] time = 0.0299672, size = 73, normalized size = 0.34

$$\frac{3(a+bx)^{7/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{7}{3};\frac{10}{3};\frac{d(a+bx)}{ad-bc}\right)}{7b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 7/3, 10/3, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/3))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{4}{3}}\frac{1}{\sqrt[3]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(1/3), x)
```

Fricas [B] time = 2.00156, size = 1835, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((-b^2*d)^(1
/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d + 3*(-b^2*d)^(1/3)*(b*x + a)^(1/3)*(
d*x + c)^(2/3)*b + 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b
^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (-b^2*d)^(1/3)*(b*d*x + b*c))
*sqrt((-b^2*d)^(1/3)/d)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)
*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*
(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 4*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d
 - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)) + 3*(3*b^3*d^2*x - 4*b^3*c*d + 7*a*
b^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(b^2*d^3), 1/18*(12*sqrt(1/3)*(b^
3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(-b^2*d)^(1/3)/d)*arctan(sqrt(1/
3)*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*
x + b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) + 2*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b
^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))
/(d*x + c)) - 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x +
a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)) + 3*(3*
b^3*d^2*x - 4*b^3*c*d + 7*a*b^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(b^2*
d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{4}{3}}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(4/3)/(c + d*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(1/3), x)
```

3.1588 $\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=171

$$\frac{(bc-ad)\log(a+bx)}{6b^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d^{4/3}}+\frac{\sqrt[3]{a+bx}(c+dx)^{2/3}}{d}$$

```
[Out] ((a + b*x)^(1/3)*(c + d*x)^(2/3))/d + ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*b^
(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(Sqrt[3]*b^(2/3)
*d^(4/3)) + ((b*c - a*d)*Log[a + b*x])/(6*b^(2/3)*d^(4/3)) + ((b*c - a*d)*L
og[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*b^(2/3)*d^
(4/3))
```

Rubi [A] time = 0.040657, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$\frac{(bc-ad)\log(a+bx)}{6b^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d^{4/3}}+\frac{\sqrt[3]{a+bx}(c+dx)^{2/3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/(c + d*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(1/3)*(c + d*x)^(2/3))/d + ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*b^
(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(Sqrt[3]*b^(2/3)
*d^(4/3)) + ((b*c - a*d)*Log[a + b*x])/(6*b^(2/3)*d^(4/3)) + ((b*c - a*d)*L
og[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*b^(2/3)*d^
(4/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx = \frac{\sqrt[3]{a+bx}(c+dx)^{2/3}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{3d}$$

$$= \frac{\sqrt[3]{a+bx}(c+dx)^{2/3}}{d} + \frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{\sqrt{3}b^{2/3}d^{4/3}} + \frac{(bc-ad)\log(a+bx)}{6b^{2/3}d^{4/3}} + \frac{(bc-ad)\log\left(-1\right.}{2b^{2/3}d}$$

Mathematica [C] time = 0.0256927, size = 73, normalized size = 0.43

$$\frac{3(a+bx)^{4/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{4}{3};\frac{7}{3};\frac{d(a+bx)}{ad-bc}\right)}{4b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 4/3, 7/3, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*(c + d*x)^(1/3))
```

Maple [F] time = 0.012, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx+a}\frac{1}{\sqrt[3]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(1/3), x)
```

Fricas [B] time = 1.99174, size = 1571, normalized size = 9.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2*d - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt((-b^2*d)^(1/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d + 3*(-b^2*d)^(1
/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b + 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*(d*x
 + c)^(1/3)*b*d - (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (-b^2*d)
^(1/3)*(b*d*x + b*c))*sqrt((-b^2*d)^(1/3)/d)) - (-b^2*d)^(2/3)*(b*c - a*d)*
log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(
d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) + 2*(-b^2*d)^(2/3
)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*
x + c))/(d*x + c)))/(b^2*d^2), 1/6*(6*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2*d
 - 6*sqrt(1/3)*(b^2*c*d - a*b*d^2)*sqrt(-(-b^2*d)^(1/3)/d)*arctan(sqrt(1/3)
*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x
+ b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) - (-b^2*d)^(2/3)*(b*c -
a*d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1
/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) + 2*(-b^2*d)
^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3
)*(d*x + c))/(d*x + c)))/(b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a + bx}}{\sqrt[3]{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(1/3)/(c + d*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^{\frac{1}{3}}}{(dx + c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(1/3), x)
```

3.1589 $\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=126

$$-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}$$

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*
(a + b*x)^(1/3))])/(b^(2/3)*d^(1/3))) - Log[a + b*x]/(2*b^(2/3)*d^(1/3)) -
(3*Log[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*b^(2/3
)*d^(1/3))
```

Rubi [A] time = 0.014312, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {59}

$$-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*
(a + b*x)^(1/3))])/(b^(2/3)*d^(1/3))) - Log[a + b*x]/(2*b^(2/3)*d^(1/3)) -
(3*Log[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*b^(2/3
)*d^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}-\frac{3\log\left(-1+\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{2b^{2/3}\sqrt[3]{d}}$$

Mathematica [C] time = 0.0266001, size = 71, normalized size = 0.56

$$\frac{3\sqrt[3]{a+bx}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(a + b*x)^(1/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 1/3, 4/3, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/3))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{2}{3}} \frac{1}{\sqrt[3]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(1/3)), x)
```

Fricas [B] time = 1.91845, size = 1334, normalized size = 10.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*b*d*sqrt((-b^2*d)^(1/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d +
3*(-b^2*d)^(1/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b + sqrt(3)*(2*(b*x + a)^(
2/3)*(d*x + c)^(1/3)*b*d - (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) +
 (-b^2*d)^(1/3)*(b*d*x + b*c))*sqrt((-b^2*d)^(1/3)/d)) + (-b^2*d)^(2/3)*log
(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x
 + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 2*(-b^2*d)^(2/3)*l
og(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x +
c)))/(b^2*d), 1/2*(2*sqrt(3)*b*d*sqrt(-(-b^2*d)^(1/3)/d)*arctan(1/3*sqrt(3)
*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x
+ b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) + (-b^2*d)^(2/3)*log(((b
*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c
)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 2*(-b^2*d)^(2/3)*log((
(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)))
/(b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{2}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(2/3)*(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(1/3)), x)
```

3.1590 $\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=32

$$-\frac{3(c+dx)^{2/3}}{2(a+bx)^{2/3}(bc-ad)}$$

[Out] (-3*(c + d*x)^(2/3))/(2*(b*c - a*d)*(a + b*x)^(2/3))

Rubi [A] time = 0.0029671, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{3(c+dx)^{2/3}}{2(a+bx)^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x]

[Out] (-3*(c + d*x)^(2/3))/(2*(b*c - a*d)*(a + b*x)^(2/3))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{2(bc-ad)(a+bx)^{2/3}}$$

Mathematica [A] time = 0.0121046, size = 32, normalized size = 1.

$$-\frac{3(c+dx)^{2/3}}{2(a+bx)^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x]

[Out] (-3*(c + d*x)^(2/3))/(2*(b*c - a*d)*(a + b*x)^(2/3))

Maple [A] time = 0.005, size = 27, normalized size = 0.8

$$\frac{3}{2\,ad-2\,bc}(dx+c)^{\frac{2}{3}}(bx+a)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x)
```

```
[Out] 3/2/(b*x+a)^(2/3)*(d*x+c)^(2/3)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(1/3)), x)
```

Fricas [A] time = 1.76267, size = 100, normalized size = 3.12

$$-\frac{3\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{2\left(abc-a^2d+\left(b^2c-abd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/2*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/3)*(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(1/3)), x)
```

3.1591 $\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=66

$$\frac{9d(c+dx)^{2/3}}{10(a+bx)^{2/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{5(a+bx)^{5/3}(bc-ad)}$$

```
[Out] (-3*(c + d*x)^(2/3))/(5*(b*c - a*d)*(a + b*x)^(5/3)) + (9*d*(c + d*x)^(2/3)
)/(10*(b*c - a*d)^2*(a + b*x)^(2/3))
```

Rubi [A] time = 0.0090983, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{9d(c+dx)^{2/3}}{10(a+bx)^{2/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{5(a+bx)^{5/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(5*(b*c - a*d)*(a + b*x)^(5/3)) + (9*d*(c + d*x)^(2/3)
)/(10*(b*c - a*d)^2*(a + b*x)^(2/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx &= -\frac{3(c+dx)^{2/3}}{5(bc-ad)(a+bx)^{5/3}}-\frac{(3d)\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx}{5(bc-ad)}\\ &= -\frac{3(c+dx)^{2/3}}{5(bc-ad)(a+bx)^{5/3}}+\frac{9d(c+dx)^{2/3}}{10(bc-ad)^2(a+bx)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.0171238, size = 46, normalized size = 0.7

$$\frac{3(c+dx)^{2/3}(5ad-2bc+3bdx)}{10(a+bx)^{5/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(c + d*x)^(2/3)*(-2*b*c + 5*a*d + 3*b*d*x))/(10*(b*c - a*d)^2*(a + b*x)^
(5/3))
```

Maple [A] time = 0.006, size = 54, normalized size = 0.8

$$\frac{9\,bdx+15\,ad-6\,bc}{10\,a^2d^2-20\,abcd+10\,b^2c^2}\,(dx+c)^{\frac{2}{3}}\,(bx+a)^{-\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x)
```

```
[Out] 3/10*(d*x+c)^(2/3)*(3*b*d*x+5*a*d-2*b*c)/(b*x+a)^(5/3)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(1/3)), x)
```

Fricas [B] time = 1.8712, size = 259, normalized size = 3.92

$$\frac{3\,(3\,bdx-2\,bc+5\,ad)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{10\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(3*b*d*x - 2*b*c + 5*a*d)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a
*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{8}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(8/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(8/3)*(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(1/3)), x)
```

3.1592 $\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=101

$$-\frac{27d^2(c+dx)^{2/3}}{40(a+bx)^{2/3}(bc-ad)^3}+\frac{9d(c+dx)^{2/3}}{20(a+bx)^{5/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{8(a+bx)^{8/3}(bc-ad)}$$

```
[Out] (-3*(c + d*x)^(2/3))/(8*(b*c - a*d)*(a + b*x)^(8/3)) + (9*d*(c + d*x)^(2/3)
)/(20*(b*c - a*d)^2*(a + b*x)^(5/3)) - (27*d^2*(c + d*x)^(2/3))/(40*(b*c -
a*d)^3*(a + b*x)^(2/3))
```

Rubi [A] time = 0.0176758, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{27d^2(c+dx)^{2/3}}{40(a+bx)^{2/3}(bc-ad)^3}+\frac{9d(c+dx)^{2/3}}{20(a+bx)^{5/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{8(a+bx)^{8/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(8*(b*c - a*d)*(a + b*x)^(8/3)) + (9*d*(c + d*x)^(2/3)
)/(20*(b*c - a*d)^2*(a + b*x)^(5/3)) - (27*d^2*(c + d*x)^(2/3))/(40*(b*c -
a*d)^3*(a + b*x)^(2/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx &= -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}}-\frac{(3d)\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx}{4(bc-ad)}\\
&= -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}}+\frac{9d(c+dx)^{2/3}}{20(bc-ad)^2(a+bx)^{5/3}}+\frac{\left(9d^2\right)\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx}{20(bc-ad)^2}\\
&= -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}}+\frac{9d(c+dx)^{2/3}}{20(bc-ad)^2(a+bx)^{5/3}}-\frac{27d^2(c+dx)^{2/3}}{40(bc-ad)^3(a+bx)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0339313, size = 77, normalized size = 0.76

$$-\frac{3(c+dx)^{2/3}\left(20a^2d^2+8abd(3dx-2c)+b^2\left(5c^2-6cdx+9d^2x^2\right)\right)}{40(a+bx)^{8/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3)*(20*a^2*d^2 + 8*a*b*d*(-2*c + 3*d*x) + b^2*(5*c^2 - 6*c
*d*x + 9*d^2*x^2)))/(40*(b*c - a*d)^3*(a + b*x)^(8/3))
```

Maple [A] time = 0.005, size = 105, normalized size = 1.

$$\frac{27\,b^2d^2x^2+72\,abd^2x-18\,b^2cdx+60\,a^2d^2-48\,abcd+15\,b^2c^2}{40\,a^3d^3-120\,a^2cbd^2+120\,ab^2c^2d-40\,b^3c^3}(dx+c)^{\frac{2}{3}}(bx+a)^{-\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x)
```

```
[Out] 3/40*(d*x+c)^(2/3)*(9*b^2*d^2*x^2+24*a*b*d^2*x-6*b^2*c*d*x+20*a^2*d^2-16*a*
b*c*d+5*b^2*c^2)/(b*x+a)^(8/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(1/3)), x)
```

Fricas [B] time = 2.09935, size = 514, normalized size = 5.09

$$-\frac{3\left(9\,b^2d^2x^2+5\,b^2c^2-16\,abcd+20\,a^2d^2-6\left(b^2cd-4\,abd^2\right)x\right)(bx+a)}{40\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/40*(9*b^2*d^2*x^2 + 5*b^2*c^2 - 16*a*b*c*d + 20*a^2*d^2 - 6*(b^2*c*d - 4
*a*b*d^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d
 + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a
^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^
```

```
2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3
)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/3)/(d*x+c)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(1/3)), x)
```

3.1593 $\int \frac{1}{(a+bx)^{14/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=136

$$\frac{243d^3(c+dx)^{2/3}}{440(a+bx)^{2/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{2/3}}{220(a+bx)^{5/3}(bc-ad)^3}+\frac{27d(c+dx)^{2/3}}{88(a+bx)^{8/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{11(a+bx)^{11/3}(bc-ad)}$$

```
[Out] (-3*(c + d*x)^(2/3))/(11*(b*c - a*d)*(a + b*x)^(11/3)) + (27*d*(c + d*x)^(2
/3))/(88*(b*c - a*d)^2*(a + b*x)^(8/3)) - (81*d^2*(c + d*x)^(2/3))/(220*(b*
c - a*d)^3*(a + b*x)^(5/3)) + (243*d^3*(c + d*x)^(2/3))/(440*(b*c - a*d)^4*
(a + b*x)^(2/3))
```

Rubi [A] time = 0.0282084, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{243d^3(c+dx)^{2/3}}{440(a+bx)^{2/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{2/3}}{220(a+bx)^{5/3}(bc-ad)^3}+\frac{27d(c+dx)^{2/3}}{88(a+bx)^{8/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{11(a+bx)^{11/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(11*(b*c - a*d)*(a + b*x)^(11/3)) + (27*d*(c + d*x)^(2
/3))/(88*(b*c - a*d)^2*(a + b*x)^(8/3)) - (81*d^2*(c + d*x)^(2/3))/(220*(b*
c - a*d)^3*(a + b*x)^(5/3)) + (243*d^3*(c + d*x)^(2/3))/(440*(b*c - a*d)^4*
(a + b*x)^(2/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{14/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} - \frac{(9d)\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx}{11(bc-ad)}$$

$$= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} + \frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}} + \frac{(27d^2)\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx}{44(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} + \frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}} - \frac{81d^2(c+dx)^{2/3}}{220(bc-ad)^3(a+bx)^{5/3}} - \frac{(81d^3)\int}{22}$$

$$= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} + \frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}} - \frac{81d^2(c+dx)^{2/3}}{220(bc-ad)^3(a+bx)^{5/3}} + \frac{243}{440(bc}$$

Mathematica [A] time = 0.0468981, size = 118, normalized size = 0.87

$$\frac{3(c+dx)^{2/3}\left(132a^2bd^2(3dx-2c)+220a^3d^3+33ab^2d\left(5c^2-6cdx+9d^2x^2\right)+b^3\left(45c^2dx-40c^3-54cd^2x^2+81d^3x^3\right)\right)}{440(a+bx)^{11/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(c + d*x)^(2/3)*(220*a^3*d^3 + 132*a^2*b*d^2*(-2*c + 3*d*x) + 33*a*b^2*d
*(5*c^2 - 6*c*d*x + 9*d^2*x^2) + b^3*(-40*c^3 + 45*c^2*d*x - 54*c*d^2*x^2 +
 81*d^3*x^3)))/(440*(b*c - a*d)^4*(a + b*x)^(11/3))
```

Maple [A] time = 0.007, size = 171, normalized size = 1.3

$$\frac{243\,b^3d^3x^3+891\,ab^2d^3x^2-162\,b^3cd^2x^2+1188\,a^2bd^3x-594\,ab^2cd^2x+135\,b^3c^2dx+660\,a^3d^3-792\,a^2cbd^2+495\,ab^2}{440\,d^4a^4-1760\,bd^3ca^3+2640\,b^2d^2c^2a^2-1760\,b^3dc^3a+440\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x)
```

```
[Out] 3/440*(d*x+c)^(2/3)*(81*b^3*d^3*x^3+297*a*b^2*d^3*x^2-54*b^3*c*d^2*x^2+396*
a^2*b*d^3*x-198*a*b^2*c*d^2*x+45*b^3*c^2*d*x+220*a^3*d^3-264*a^2*b*c*d^2+16
5*a*b^2*c^2*d-40*b^3*c^3)/(b*x+a)^(11/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c
^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{14}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(14/3)*(d*x + c)^(1/3)), x)
```

Fricas [B] time = 2.747, size = 865, normalized size = 6.36

$$\frac{3\left(81\,b^3d^3x^3-40\,b^3c^3+165\,ab^2c^2d-264\,a^2bc\cdots\right.}{440\left(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+\left(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4\right)x^4+4\left(ab^7c\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(81*b^3*d^3*x^3 - 40*b^3*c^3 + 165*a*b^2*c^2*d - 264*a^2*b*c*d^2 + 22
0*a^3*d^3 - 27*(2*b^3*c*d^2 - 11*a*b^2*d^3)*x^2 + 9*(5*b^3*c^2*d - 22*a*b^2
*c*d^2 + 44*a^2*b*d^3)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^4*b^4*c^4 - 4*
a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*
a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a
*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*
d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3
*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(14/3)/(d*x+c)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{14}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(14/3)*(d*x + c)^(1/3)), x)
```

3.1594 $\int \frac{(a+bx)^{8/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1365

result too large to display

```
[Out] (3*(b*c - a*d)^2*(a + b*x)^(2/3)*(c + d*x)^(2/3))/(7*d^3) - (12*(b*c - a*d)
*(a + b*x)^(5/3)*(c + d*x)^(2/3))/(35*d^2) + (3*(a + b*x)^(8/3)*(c + d*x)^(
2/3))/(10*d) - (3*2^(2/3)*(b*c - a*d)^3*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b
*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(7*b^(2/3)*d^(11/3)*(
a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c -
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))) + (3*3^(
1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^(11/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[
(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
 b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(
b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((
a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((
1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3))], -7 - 4*Sqrt[3]])/(7*2^(1/3)*b^(2/3)*d^(11/3)*(a + b*x)^(1/3)*(
c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(
2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])
*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2
]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (2*2^(1/6)*3^(3/4)*(b*c - a*d)^(11/3)*((
a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3)
+ 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4
/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)
 + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b
*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*b^(2/3)*d^(
11/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*
d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))
^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 2.80328, antiderivative size = 1365, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 62, 623, 303, 218, 1877}

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(\cdots}}{\left(\left(1+\sqrt{3}\right)(\cdots\right.}}}{7\sqrt[3]{2}b^{2/3}d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/\cdots}}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(8/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(b*c - a*d)^2*(a + b*x)^(2/3)*(c + d*x)^(2/3))/(7*d^3) - (12*(b*c - a*d)
*(a + b*x)^(5/3)*(c + d*x)^(2/3))/(35*d^2) + (3*(a + b*x)^(8/3)*(c + d*x)^(
2/3))/(10*d) - (3*2^(2/3)*(b*c - a*d)^3*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b
```

```
*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(7*b^(2/3)*d^(11/3)*(
a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c -
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))) + (3*3^(
1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^(11/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[
(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
 b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(
b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((
a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((
1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3))], -7 - 4*Sqrt[3]])/(7*2^(1/3)*b^(2/3)*d^(11/3)*(a + b*x)^(1/3)*(
c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(
2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])
*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2
]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (2*2^(1/6)*3^(3/4)*(b*c - a*d)^(11/3)*((
a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3)
+ 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4
/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)
 + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b
*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*b^(2/3)*d^(
11/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*
d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))
^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{8/3}}{\sqrt[3]{c+dx}}\,dx = \frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d} - \frac{(4(bc-ad))\int \frac{(a+bx)^{5/3}}{\sqrt[3]{c+dx}}\,dx}{5d}$$

$$= -\frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d} + \frac{\left(4(bc-ad)^2\right)\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx}{7d^2}$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d} - \cdots$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d} - \cdots$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d} - \cdots$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d} - \cdots$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d} - \cdots$$

Mathematica [C] time = 0.0322891, size = 73, normalized size = 0.05

$$\frac{3(a+bx)^{11/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{14}{3};\frac{d(a+bx)}{ad-bc}\right)}{11b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(8/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(11/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3
, 11/3, 14/3, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(1/3))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{8}{3}} \frac{1}{\sqrt[3]{dx+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(8/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(8/3)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{8}{3}}}{(dx+c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(8/3)/(d*x + c)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(2/3)/(d*x + c)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(8/3)/(d*x+c)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{8}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(8/3)/(d*x + c)^(1/3), x)
```

3.1595 $\int \frac{(a+bx)^{5/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1330

result too large to display

```
[Out] (-15*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(2/3))/(28*d^2) + (3*(a + b*x)^(
5/3)*(c + d*x)^(2/3))/(7*d) + (15*(b*c - a*d)^2*((a + b*x)*(c + d*x))^(1/3)
*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(14*2^(1/3)*b
^(2/3)*d^(8/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 +
Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^
(1/3))) - (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^(8/3)*((a + b*x)*(c + d
*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b
^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^
(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin
[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(28*2^(1/3)*b^(2/3)*d^(8/3)*(
a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3
)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))
)/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (5*3^(3/4)*(b*c - a*d)^(
8/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c -
a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x)
)^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*2^(5
/6)*b^(2/3)*d^(8/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*S
qrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 2.00991, antiderivative size = 1330, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 62, 623, 303, 218, 1877}

$$-\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(\cdots}}{\left((1+\sqrt{3})(\cdots\right.}}}{28\sqrt[3]{2}b^{2/3}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}(\cdots}{\left((1+\sqrt{3})(\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/3)/(c + d*x)^(1/3),x]
```

```
[Out] (-15*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(2/3))/(28*d^2) + (3*(a + b*x)^(
5/3)*(c + d*x)^(2/3))/(7*d) + (15*(b*c - a*d)^2*((a + b*x)*(c + d*x))^(1/3)
*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(14*2^(1/3)*b
^(2/3)*d^(8/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 +
```

```
Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^
(1/3))) - (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^(8/3)*((a + b*x)*(c + d
*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b
^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^
(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin
[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(28*2^(1/3)*b^(2/3)*d^(8/3)*(
a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3
)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))
)/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (5*3^(3/4)*(b*c - a*d)^(
8/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c -
a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x)
)^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*2^(5
/6)*b^(2/3)*d^(8/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*S
qrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/3}}{\sqrt[3]{c+dx}}\,dx = \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} - \frac{(5(bc-ad))\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx}{7d}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{\left(5(bc-ad)^2\right)\int \frac{1}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}\,dx}{14d^2}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{\left(5(bc-ad)^2\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{}{\sqrt[3]{ac+}}}{14d^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{\left(15(bc-ad)^2\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+}\right.}{14d}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{\left(15(bc-ad)^2\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+}\right.}{14\ 2^{2/3}}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{15(bc-ad)^2\sqrt[3]{(a+bx)(}}{14\sqrt[3]{2}b^{2/3}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad}$$

Mathematica [C] time = 0.0291162, size = 73, normalized size = 0.05

$$\frac{3(a+bx)^{8/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{11}{3};\frac{d(a+bx)}{ad-bc}\right)}{8b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(8/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 8/3, 11/3, (d*(a + b*x))/(-(b*c) + a*d)])/(8*b*(c + d*x)^(1/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{3}} \frac{1}{\sqrt[3]{dx+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(5/3)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/3)/(d*x + c)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{5}{3}}}{\sqrt[3]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(5/3)/(c + d*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(1/3), x)
```

3.1596 $\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1293

result too large to display

```
[Out] (3*(a + b*x)^(2/3)*(c + d*x)^(2/3))/(4*d) - (3*(b*c - a*d)*((a + b*x)*(c +
d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2
*2^(1/3)*b^(2/3)*d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d
*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^(5/3)*((a + b*
x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) -
2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2
^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellipti
cE[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*b^(2/3)*d
^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a
*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (3^(3/4)*(b*c -
a*d)^(5/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c
- a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((
b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c
+ d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1
+ Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
+ 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(
2^(5/6)*b^(2/3)*d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*
x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 1.56308, antiderivative size = 1293, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 62, 623, 303, 218, 1877}

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(\cdots}}{\left(\left(1+\sqrt{3}\right)(\cdots}}}{4\sqrt[3]{2}b^{2/3}d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}(\cdots}{\left(\left(1+\sqrt{3}\right)(\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(c + d*x)^(2/3))/(4*d) - (3*(b*c - a*d)*((a + b*x)*(c +
d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2
*2^(1/3)*b^(2/3)*d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d
*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^(5/3)*((a + b*
```

```
x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) -
2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2
^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellipti
cE[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*b^(2/3)*d
^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a
*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (3^(3/4)*(b*c -
a*d)^(5/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c
- a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((
b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c
+ d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1
+ Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
+ 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(
2^(5/6)*b^(2/3)*d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*
x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx = \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{2d}$$

$$= \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{\left((bc-ad)\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}\,dx}{2d\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{\left(3(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-4abcd+(bc+\dots}}\right.}{2d\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)}$$

$$= \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{\left(3(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{(1-\sqrt{3})(bc-ad)^{2\dots}}{\sqrt{-4abcd+(bc\dots}}\right.}{2\,2^{2/3}\sqrt[3]{b}d^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+\dots}$$

$$= \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{3(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b\dots}}{2\sqrt[3]{2}b^{2/3}d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2\dots}\right.}$$

Mathematica [C] time = 0.0277515, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{5/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 5/3, 8/3, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/3))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{2}{3}}\frac{1}{\sqrt[3]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)/(d*x + c)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{2}{3}}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(2/3)/(c + d*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(1/3), x)
```

3.1597 $\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1257

result too large to display

```
[Out] (3*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*
(c + 2*d*x))^2])/(2^(1/3)*b^(2/3)*d^(2/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(
b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*
d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c -
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*
c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c +
d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 +
Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^
(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) +
2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*
2^(1/3)*b^(2/3)*d^(2/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*
x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) +
 (2^(1/6)*3^(3/4)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c +
 a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*
(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c -
a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*
x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sq
rt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1
/3))], -7 - 4*Sqrt[3]])/(b^(2/3)*d^(2/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b
*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^
(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3
) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(
c + 2*d*x))^2])
```

Rubi [A] time = 1.17851, antiderivative size = 1257, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {62, 623, 303, 218, 1877}

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2\cdots}{\cdots}}}{2\sqrt[3]{2}b^{2/3}d^{2/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(b\cdots\right.}{\left(\left(1+\sqrt{3}\right)(bc-\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*
(c + 2*d*x))^2])/(2^(1/3)*b^(2/3)*d^(2/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(
b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*
d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c -
```

```
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*
c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c +
d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 +
Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^
(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) +
2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*
2^(1/3)*b^(2/3)*d^(2/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*
x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) +
 (2^(1/6)*3^(3/4)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c +
 a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*
(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c -
a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*
x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sq
rt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1
/3))], -7 - 4*Sqrt[3]])/(b^(2/3)*d^(2/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b
*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^
(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3
) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(
c + 2*d*x))^2])
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
```

```
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx = \frac{\sqrt[3]{(a+bx)(c+dx)}\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}\,dx}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= \frac{\left(3\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{-4abcd+(bc+ad)^2+4bdx^3}}\,dx, x, \sqrt[3]{(a+bx)(c+dx)}\right.}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)}$$

$$= \frac{\left(3\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\mathrm{Subst}\left(\int \frac{(1-\sqrt{3})(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}x}{\sqrt{-4abcd+(bc+ad)^2+4bdx^3}}\,dx, x, \sqrt[3]{(a+bx)(c-\right.}{2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)}$$

$$= \frac{3\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt[3]{2}b^{2/3}d^{2/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+d\right.}$$

Mathematica [C] time = 0.0231144, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{2/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{d(a+bx)}{ad-bc}\right)}{2b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(a + b*x)^(2/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 2/3, 5/3, (d*(a + b*x))/(-(b*c) + a*d)])/(2*b*(c + d*x)^(1/3))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{bx+a}}\frac{1}{\sqrt[3]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(1/3)*(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(1/3)), x)
```

3.1598 $\int \frac{1}{(a+bx)^{4/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1297

result too large to display

```
[Out] (-3*(c + d*x)^(2/3))/((b*c - a*d)*(a + b*x)^(1/3)) + (3*d^(1/3)*((a + b*x)*
(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2
])/(2^(1/3)*b^(2/3)*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d
+ 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
 b*x)*(c + d*x))^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*((a + b*x)*
(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(
2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1
/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x
)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*2^(1/3)*b^(2/3)*(b*c
 - a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((
b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
 + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)
*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2^(1/6)*
3^(3/4)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sq
rt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x
)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))
/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3
)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(
2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3
]])/(b^(2/3)*(b*c - a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d +
 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3
)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x)
)^2])
```

Rubi [A] time = 1.58642, antiderivative size = 1297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {51, 62, 623, 303, 218, 1877}

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\cdots}{\left((1+\sqrt{3}\cdots\right.}}}{2\sqrt[3]{2}b^{2/3}\sqrt[3]{bc-ad}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-\cdots\right.}{\left((1+\sqrt{3})(bc-a\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(4/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/((b*c - a*d)*(a + b*x)^(1/3)) + (3*d^(1/3)*((a + b*x)*
(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2
])/(2^(1/3)*b^(2/3)*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d
+ 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
```

```
 b*x)*(c + d*x))^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*((a + b*x)*
(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(
2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1
/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x
)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*2^(1/3)*b^(2/3)*(b*c
 - a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((
b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
 + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)
*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2^(1/6)*
3^(3/4)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sq
rt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x
)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))
/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3
)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(
2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3
]])/(b^(2/3)*(b*c - a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d +
 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3
)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x)
)^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}}+\frac{d\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{bc-ad}$$

$$= -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}}+\frac{\left(d\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}\,dx}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}}+\frac{\left(3d\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{-4abcd+(bc+ad)^2+4b}}\right.}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)}$$

$$= -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}}+\frac{\left(3d^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\mathrm{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2}{\sqrt{-4abcd+(bc+ad)}}\right.}{2^{2/3}\sqrt[3]{b}(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+}$$

$$= -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}}+\frac{3\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+}}{\sqrt[3]{2}b^{2/3}(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right.}$$

Mathematica [C] time = 0.0226675, size = 71, normalized size = 0.05

$$-\frac{3\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-1/3, 1/3, 2/3, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/3)*(c + d*x)^(1/3))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (bx + a)^{-\frac{4}{3}} \frac{1}{\sqrt[3]{dx + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx + a)^{\frac{4}{3}}(dx + c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx + a)^{\frac{2}{3}}(dx + c)^{\frac{2}{3}}}{b^2dx^3 + a^2c + \left(b^2c + 2\,abd\right)x^2 + \left(2\,abc + a^2d\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx)^{\frac{4}{3}}\sqrt[3]{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(4/3)*(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx + a)^{\frac{4}{3}}(dx + c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(1/3)), x)
```

3.1599 $\int \frac{1}{(a+bx)^{7/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1335

result too large to display

```
[Out] (-3*(c + d*x)^(2/3))/(4*(b*c - a*d)*(a + b*x)^(4/3)) + (3*d*(c + d*x)^(2/3)
)/(2*(b*c - a*d)^2*(a + b*x)^(1/3)) - (3*d^(4/3)*((a + b*x)*(c + d*x))^(1/3
)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2*2^(1/3)*b
^(2/3)*(b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*
((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*((a + b*x)*(c + d*x))^
(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^
(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3
)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)
*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1
- Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*b^(2/3)*(b*c - a*d)^(4/
3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^
(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1
/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (3^(3/4)*d^(4/3)*((a
 + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) +
 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/
3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)
+ 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*
c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*(
(a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1
/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*b^(2/3
)*(b*c - a*d)^(4/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*S
qrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 2.01247, antiderivative size = 1335, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {51, 62, 623, 303, 218, 1877}

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(\cdots}{\left((1+\sqrt{3})(\cdots\right.}}}{4\sqrt[3]{2}b^{2/3}(bc-ad)^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}(\cdots}{\left((1+\sqrt{3})(\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(4*(b*c - a*d)*(a + b*x)^(4/3)) + (3*d*(c + d*x)^(2/3)
)/(2*(b*c - a*d)^2*(a + b*x)^(1/3)) - (3*d^(4/3)*((a + b*x)*(c + d*x))^(1/3
)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2*2^(1/3)*b
^(2/3)*(b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*
```

```
((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*((a + b*x)*(c + d*x))^
(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^
(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3
)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)
*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1
- Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*b^(2/3)*(b*c - a*d)^(4/
3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^
(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1
/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (3^(3/4)*d^(4/3)*((a
 + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) +
 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/
3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)
+ 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*
c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*(
(a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1
/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*b^(2/3
)*(b*c - a*d)^(4/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*S
qrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} - \frac{d\int \frac{1}{(a+bx)^{4/3}\sqrt[3]{c+dx}}\,dx}{2(bc-ad)}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{d^2\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{2(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{\left(d^2\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}}{2(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{\left(3d^2\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)}\right.}{2(bc-ad)^2\sqrt[3]{a}}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{\left(3d^{5/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx}\right.}{2\,2^{2/3}\sqrt[3]{b}(bc-a}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{3d^{4/3}\sqrt[3]{(a+bx)(c+a}}{2\sqrt[3]{2}b^{2/3}(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+a}$$

Mathematica [C] time = 0.0221073, size = 73, normalized size = 0.05

$$-\frac{3\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};-\frac{1}{3};\frac{d(a+bx)}{ad-bc}\right)}{4b(a+bx)^{4/3}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-4/3, 1/3, -1/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(4*b*(a + b*x)^(4/3)*(c + d*x)^(1/3))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{7}{3}} \frac{1}{\sqrt[3]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{b^3dx^4+a^3c+\left(b^3c+3\,ab^2d\right)x^3+3\left(ab^2c+a^2bd\right)x^2+\left(3\,a^2bc+a^3d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*
b^2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{7}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(7/3)*(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(1/3)), x)
```

3.1600 $\int \frac{1}{(a+bx)^{10/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1372

result too large to display

```
[Out] (-3*(c + d*x)^(2/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (15*d*(c + d*x)^(2/3
))/(28*(b*c - a*d)^2*(a + b*x)^(4/3)) - (15*d^2*(c + d*x)^(2/3))/(14*(b*c -
 a*d)^3*(a + b*x)^(1/3)) + (15*d^(7/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*
c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(14*2^(1/3)*b^(2/3)*(b
*c - a*d)^3*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqr
t[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/
3))) - (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(7/3)*((a + b*x)*(c + d*x))^(1/3)*Sq
rt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3
)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)
*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^
(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))
/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))], -7 - 4*Sqrt[3]])/(28*2^(1/3)*b^(2/3)*(b*c - a*d)^(7/3)*(a +
 b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*(
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/(
(1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d
*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (5*3^(3/4)*d^(7/3)*((a + b*
x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) -
2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2
^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*2^(5/6)*b^(2/3)*(
b*c - a*d)^(7/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt
[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 2.44206, antiderivative size = 1372, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {51, 62, 623, 303, 218, 1877}

$$-\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(\cdots}}{\left((1+\sqrt{3})(\cdots\right.}}\cdots}{28\sqrt[3]{2}b^{2/3}(bc-ad)^{7/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}(\cdots}{\left((1+\sqrt{3})(b\cdots\right.}}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(10/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (15*d*(c + d*x)^(2/3
))/(28*(b*c - a*d)^2*(a + b*x)^(4/3)) - (15*d^2*(c + d*x)^(2/3))/(14*(b*c -
 a*d)^3*(a + b*x)^(1/3)) + (15*d^(7/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*
```

```
c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(14*2^(1/3)*b^(2/3)*(b
*c - a*d)^3*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqr
t[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/
3))) - (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(7/3)*((a + b*x)*(c + d*x))^(1/3)*Sq
rt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3
)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)
*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^
(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))
/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))], -7 - 4*Sqrt[3]])/(28*2^(1/3)*b^(2/3)*(b*c - a*d)^(7/3)*(a +
 b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*(
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/(
(1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d
*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (5*3^(3/4)*d^(7/3)*((a + b*
x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) -
2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2
^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*2^(5/6)*b^(2/3)*(
b*c - a*d)^(7/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt
[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{10/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} - \frac{(5d)\int \frac{1}{(a+bx)^{7/3}\sqrt[3]{c+dx}}\,dx}{7(bc-ad)}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} + \frac{\left(5d^2\right)\int \frac{1}{(a+bx)^{4/3}\sqrt[3]{c+dx}}\,dx}{14(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}} + \frac{\left(5d^3\right)\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c}}}{14(bc-ad)}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}} + \frac{\left(5d^3\sqrt[3]{(a+bx)(c}\right.}{14(bc-}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}} + \frac{\left(15d^3\sqrt[3]{(a+bx)(}\right.}{}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}} + \frac{\left(15d^{8/3}\sqrt[3]{(a+bx}\right.}{}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}} + \frac{}{14\sqrt[3]{2}b^{2/3}(bc-a}$$

Mathematica [C] time = 0.0252353, size = 73, normalized size = 0.05

$$-\frac{3\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};-\frac{4}{3};\frac{d(a+bx)}{ad-bc}\right)}{7b(a+bx)^{7/3}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(10/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-7/3, 1/3, -4/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(7*b*(a + b*x)^(7/3)*(c + d*x)^(1/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{10}{3}} \frac{1}{\sqrt[3]{dx+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{10}{3}}(dx+c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{b^4dx^5 + a^4c + \left(b^4c + 4\,ab^3d\right)x^4 + 2\left(2\,ab^3c + 3\,a^2b^2d\right)x^3 + 2\left(3\,a^2b^2c + 2\,a^3bd\right)x^2 + \left(4\,a^3bc + a^4d\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^4*d*x^5 + a^4*c + (b^4*c + 4*a*
b^3*d)*x^4 + 2*(2*a*b^3*c + 3*a^2*b^2*d)*x^3 + 2*(3*a^2*b^2*c + 2*a^3*b*d)*
x^2 + (4*a^3*b*c + a^4*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(10/3)/(d*x+c)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{10}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(1/3)), x)
```

3.1601 $\int \frac{(a+bx)^{5/3}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=216

$$-\frac{5(a+bx)^{2/3}\sqrt[3]{c+dx}(bc-ad)}{6d^2}-\frac{5(bc-ad)^2\log(c+dx)}{18\sqrt[3]{b}d^{8/3}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{6\sqrt[3]{b}d^{8/3}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{\cdots}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{\cdots}}\right.}{3\sqrt{3}\sqrt[3]{b}d^{8/3}}$$

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3))/(6*d^2) + ((a + b*x)^(5/3)
*(c + d*x)^(1/3))/(2*d) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a
 + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(3*Sqrt[3]*b^(1/3)*d^(8/
3)) - (5*(b*c - a*d)^2*Log[c + d*x])/(18*b^(1/3)*d^(8/3)) - (5*(b*c - a*d)^
2*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(6*b^(1/3)
*d^(8/3))
```

Rubi [A] time = 0.080201, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$-\frac{5(a+bx)^{2/3}\sqrt[3]{c+dx}(bc-ad)}{6d^2}-\frac{5(bc-ad)^2\log(c+dx)}{18\sqrt[3]{b}d^{8/3}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{6\sqrt[3]{b}d^{8/3}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{\cdots}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{\cdots}}\right.}{3\sqrt{3}\sqrt[3]{b}d^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/3)/(c + d*x)^(2/3),x]
```

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3))/(6*d^2) + ((a + b*x)^(5/3)
*(c + d*x)^(1/3))/(2*d) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a
 + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(3*Sqrt[3]*b^(1/3)*d^(8/
3)) - (5*(b*c - a*d)^2*Log[c + d*x])/(18*b^(1/3)*d^(8/3)) - (5*(b*c - a*d)^
2*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(6*b^(1/3)
*d^(8/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{5/3}}{(c+dx)^{2/3}}\,dx = \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d} - \frac{(5(bc-ad))\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx}{6d}$$

$$= -\frac{5(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6d^2} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d} + \frac{\left(5(bc-ad)^2\right)\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{2/3}}\,dx}{9d^2}$$

$$= -\frac{5(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6d^2} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d} - \frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{3\sqrt{3}\sqrt[3]{b}d^{8/3}} - \frac{5(bc-}{}$$

Mathematica [C] time = 0.0345999, size = 73, normalized size = 0.34

$$\frac{3(a+bx)^{8/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{8}{3};\frac{11}{3};\frac{d(a+bx)}{ad-bc}\right)}{8b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(8/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 8/3, 11/3, (d*(a + b*x))/(-(b*c) + a*d)])/(8*b*(c + d*x)^(2/3))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{3}}(dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(2/3), x)
```

Fricas [B] time = 2.09284, size = 1835, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((-b*d^2)^(
1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 - 3*(-b*d^2)^(1/3)*(b*x + a)^(2/3)
*(d*x + c)^(1/3)*d - 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (
-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (-b*d^2)^(1/3)*(b*d*x + a*d
))*sqrt((-b*d^2)^(1/3)/b)) - 10*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b*d^2)^(2
/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b
*x + a)) + 5*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b*d^2)^(2/3)*log(((b*x + a)^
(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
- (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(3*b^2*d^3*x - 5*b^2*c*d^2 +
 8*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*d^4), 1/18*(30*sqrt(1/3)*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(-b*d^2)^(1/3)/b)*arctan(sqrt(1
/3)*(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d
*x + a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 10*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(-b*d^2)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (
-b*d^2)^(2/3)*(b*x + a))/(b*x + a)) + 5*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b
*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)*(b*x
+ a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(
3*b^2*d^3*x - 5*b^2*c*d^2 + 8*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*
d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{5}{3}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral((a + b*x)**(5/3)/(c + d*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(2/3), x)
```

3.1602 $\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=169

$$\frac{(bc-ad)\log(c+dx)}{3\sqrt[3]{b}d^{5/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{\sqrt[3]{b}d^{5/3}}+\frac{2(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d}$$

```
[Out] ((a + b*x)^(2/3)*(c + d*x)^(1/3))/d + (2*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*
d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(Sqrt[3]*b^(1/
3)*d^(5/3)) + ((b*c - a*d)*Log[c + d*x])/(3*b^(1/3)*d^(5/3)) + ((b*c - a*d)
*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(b^(1/3)*d^
(5/3))
```

Rubi [A] time = 0.0407949, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$\frac{(bc-ad)\log(c+dx)}{3\sqrt[3]{b}d^{5/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{\sqrt[3]{b}d^{5/3}}+\frac{2(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/(c + d*x)^(2/3),x]
```

```
[Out] ((a + b*x)^(2/3)*(c + d*x)^(1/3))/d + (2*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*
d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(Sqrt[3]*b^(1/
3)*d^(5/3)) + ((b*c - a*d)*Log[c + d*x])/(3*b^(1/3)*d^(5/3)) + ((b*c - a*d)
*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(b^(1/3)*d^
(5/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx = \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d} - \frac{(2(bc-ad))\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{2/3}}\,dx}{3d}$$

$$= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d} + \frac{2(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}\sqrt[3]{b}d^{5/3}} + \frac{(bc-ad)\log(c+dx)}{3\sqrt[3]{b}d^{5/3}} + \frac{(bc-ad)\log}{\sqrt[3]{}}$$

Mathematica [C] time = 0.0255701, size = 73, normalized size = 0.43

$$\frac{3(a+bx)^{5/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{5}{3};\frac{8}{3};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 5/3, 8/3, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(2/3))
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{2}{3}} (dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(2/3), x)
```

Fricas [B] time = 2.2609, size = 1567, normalized size = 9.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt((-b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 - 3*(-b*d^2)^(
1/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d - 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*
x + c)^(2/3)*b*d - (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (-b*d^2
)^(1/3)*(b*d*x + a*d))*sqrt((-b*d^2)^(1/3)/b)) + 2*(-b*d^2)^(2/3)*(b*c - a*
d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*
x + a)) - (-b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b
*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x
 + a*d))/(b*x + a)))/(b*d^3), 1/3*(3*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2
- 6*sqrt(1/3)*(b^2*c*d - a*b*d^2)*sqrt(-(-b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*
(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x +
 a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) + 2*(-b*d^2)^(2/3)*(b*c -
 a*d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/
(b*x + a)) - (-b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3
)*b*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*
d*x + a*d))/(b*x + a)))/(b*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{2}{3}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral((a + b*x)**(2/3)/(c + d*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(2/3), x)
```

3.1603 $\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=126

$$-\frac{3\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2\sqrt[3]{b}d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}d^{2/3}}$$

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*
(c + d*x)^(1/3))])/(b^(1/3)*d^(2/3))) - Log[c + d*x]/(2*b^(1/3)*d^(2/3)) -
(3*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(1/3
)*d^(2/3))
```

Rubi [A] time = 0.0151174, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {59}

$$-\frac{3\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2\sqrt[3]{b}d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*
(c + d*x)^(1/3))])/(b^(1/3)*d^(2/3))) - Log[c + d*x]/(2*b^(1/3)*d^(2/3)) -
(3*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(1/3
)*d^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{2/3}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt[3]{b}d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}d^{2/3}}-\frac{3\log\left(-1+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{2\sqrt[3]{b}d^{2/3}}$$

Mathematica [C] time = 0.0276266, size = 73, normalized size = 0.58

$$\frac{3(a+bx)^{2/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{d(a+bx)}{ad-bc}\right)}{2b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(a + b*x)^(2/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 2/3, 5/3, (d*(a + b*x))/(-(b*c) + a*d)])/(2*b*(c + d*x)^(2/3))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{bx+a}}\,(dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(2/3)), x)
```

Fricas [B] time = 2.39415, size = 1335, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*b*d*sqrt((-b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 -
 3*(-b*d^2)^(1/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d - sqrt(3)*(2*(b*x + a)^
(1/3)*(d*x + c)^(2/3)*b*d - (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
+ (-b*d^2)^(1/3)*(b*d*x + a*d))*sqrt((-b*d^2)^(1/3)/b)) - 2*(-b*d^2)^(2/3)*
log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*x +
 a)) + (-b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(
2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x +
 a)))/(b*d^2), 1/2*(2*sqrt(3)*b*d*sqrt(-(-b*d^2)^(1/3)/b)*arctan(1/3*sqrt(3
)*(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x
 + a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(-b*d^2)^(2/3)*log(
((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*x + a))
 + (-b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)
*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a))
)/(b*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(1/3)*(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(2/3)), x)
```

3.1604 $\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=30

$$-\frac{3\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}(bc-ad)}$$

[Out] (-3*(c + d*x)^(1/3))/((b*c - a*d)*(a + b*x)^(1/3))

Rubi [A] time = 0.0030281, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{3\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x]

[Out] (-3*(c + d*x)^(1/3))/((b*c - a*d)*(a + b*x)^(1/3))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{(bc-ad)\sqrt[3]{a+bx}}$$

Mathematica [A] time = 0.008687, size = 30, normalized size = 1.

$$\frac{3\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}(ad-bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x]

[Out] (3*(c + d*x)^(1/3))/((-(b*c) + a*d)*(a + b*x)^(1/3))

Maple [A] time = 0.004, size = 27, normalized size = 0.9

$$3\,\frac{\sqrt[3]{dx+c}}{\sqrt[3]{bx+a}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x)
```

```
[Out] 3/(b*x+a)^(1/3)*(d*x+c)^(1/3)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{4}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(2/3)), x)
```

Fricas [A] time = 2.47829, size = 97, normalized size = 3.23

$$-\frac{3\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] -3*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{4}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(4/3)*(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{4}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(2/3)), x)
```

3.1605 $\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=66

$$\frac{9d\sqrt[3]{c+dx}}{4\sqrt[3]{a+bx}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{4(a+bx)^{4/3}(bc-ad)}$$

```
[Out] (-3*(c + d*x)^(1/3))/(4*(b*c - a*d)*(a + b*x)^(4/3)) + (9*d*(c + d*x)^(1/3)
)/(4*(b*c - a*d)^2*(a + b*x)^(1/3))
```

Rubi [A] time = 0.0095416, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{9d\sqrt[3]{c+dx}}{4\sqrt[3]{a+bx}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(4*(b*c - a*d)*(a + b*x)^(4/3)) + (9*d*(c + d*x)^(1/3)
)/(4*(b*c - a*d)^2*(a + b*x)^(1/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx &= -\frac{3\sqrt[3]{c+dx}}{4(bc-ad)(a+bx)^{4/3}}-\frac{(3d)\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx}{4(bc-ad)}\\ &= -\frac{3\sqrt[3]{c+dx}}{4(bc-ad)(a+bx)^{4/3}}+\frac{9d\sqrt[3]{c+dx}}{4(bc-ad)^2\sqrt[3]{a+bx}}\end{aligned}$$

Mathematica [A] time = 0.0165527, size = 46, normalized size = 0.7

$$\frac{3\sqrt[3]{c+dx}(4ad-bc+3bdx)}{4(a+bx)^{4/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(c + d*x)^(1/3)*(-(b*c) + 4*a*d + 3*b*d*x))/(4*(b*c - a*d)^2*(a + b*x)^(
4/3))
```

Maple [A] time = 0.003, size = 54, normalized size = 0.8

$$\frac{9\,bdx+12\,ad-3\,bc}{4\,a^2d^2-8\,abcd+4\,b^2c^2}\sqrt[3]{dx+c}\,(bx+a)^{-\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x)
```

```
[Out] 3/4*(d*x+c)^(1/3)*(3*b*d*x+4*a*d-b*c)/(b*x+a)^(4/3)/(a^2*d^2-2*a*b*c*d+b^2*
c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(2/3)), x)
```

Fricas [B] time = 2.36251, size = 255, normalized size = 3.86

$$\frac{3\,(3\,bdx-bc+4\,ad)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{4\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/4*(3*b*d*x - b*c + 4*a*d)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^2*b^2*c^2 -
2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^
3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{7}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(7/3)*(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(2/3)), x)
```

3.1606 $\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=101

$$-\frac{27d^2\sqrt[3]{c+dx}}{14\sqrt[3]{a+bx}(bc-ad)^3}+\frac{9d\sqrt[3]{c+dx}}{14(a+bx)^{4/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{7(a+bx)^{7/3}(bc-ad)}$$

```
[Out] (-3*(c + d*x)^(1/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (9*d*(c + d*x)^(1/3)
)/(14*(b*c - a*d)^2*(a + b*x)^(4/3)) - (27*d^2*(c + d*x)^(1/3))/(14*(b*c -
a*d)^3*(a + b*x)^(1/3))
```

Rubi [A] time = 0.017218, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{27d^2\sqrt[3]{c+dx}}{14\sqrt[3]{a+bx}(bc-ad)^3}+\frac{9d\sqrt[3]{c+dx}}{14(a+bx)^{4/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{7(a+bx)^{7/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (9*d*(c + d*x)^(1/3)
)/(14*(b*c - a*d)^2*(a + b*x)^(4/3)) - (27*d^2*(c + d*x)^(1/3))/(14*(b*c -
a*d)^3*(a + b*x)^(1/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx &= -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}}-\frac{(6d)\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx}{7(bc-ad)}\\
&= -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}}+\frac{9d\sqrt[3]{c+dx}}{14(bc-ad)^2(a+bx)^{4/3}}+\frac{\left(9d^2\right)\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx}{14(bc-ad)^2}\\
&= -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}}+\frac{9d\sqrt[3]{c+dx}}{14(bc-ad)^2(a+bx)^{4/3}}-\frac{27d^2\sqrt[3]{c+dx}}{14(bc-ad)^3\sqrt[3]{a+bx}}
\end{aligned}$$

Mathematica [A] time = 0.0308928, size = 75, normalized size = 0.74

$$-\frac{3\sqrt[3]{c+dx}\left(14a^2d^2-7abd(c-3dx)+b^2\left(2c^2-3cdx+9d^2x^2\right)\right)}{14(a+bx)^{7/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(14*a^2*d^2 - 7*a*b*d*(c - 3*d*x) + b^2*(2*c^2 - 3*c*d*
x + 9*d^2*x^2)))/(14*(b*c - a*d)^3*(a + b*x)^(7/3))
```

Maple [A] time = 0.006, size = 105, normalized size = 1.

$$\frac{27\,b^2d^2x^2+63\,abd^2x-9\,b^2cdx+42\,a^2d^2-21\,abcd+6\,b^2c^2}{14\,a^3d^3-42\,a^2cbd^2+42\,ab^2c^2d-14\,b^3c^3}\sqrt[3]{dx+c}\,(bx+a)^{-\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x)
```

```
[Out] 3/14*(d*x+c)^(1/3)*(9*b^2*d^2*x^2+21*a*b*d^2*x-3*b^2*c*d*x+14*a^2*d^2-7*a*b
*c*d+2*b^2*c^2)/(b*x+a)^(7/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{10}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(2/3)), x)
```

Fricas [B] time = 2.44345, size = 513, normalized size = 5.08

$$-\frac{3\left(9\,b^2d^2x^2+2\,b^2c^2-7\,abcd+14\,a^2d^2-3\left(b^2cd-7\,abd^2\right)x\right)(bx+a)^{\frac{2}{3}}}{14\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] -3/14*(9*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 14*a^2*d^2 - 3*(b^2*c*d - 7*
a*b*d^2)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d
+ 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^
3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2
*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)
```

```
*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(10/3)/(d*x+c)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{10}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(2/3)), x)
```

3.1607 $\int \frac{1}{(a+bx)^{13/3}(c+dx)^{2/3}} dx$

Optimal. Leaf size=136

$$\frac{243d^3\sqrt[3]{c+dx}}{140\sqrt[3]{a+bx}(bc-ad)^4}-\frac{81d^2\sqrt[3]{c+dx}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{27d\sqrt[3]{c+dx}}{70(a+bx)^{7/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{10(a+bx)^{10/3}(bc-ad)}$$

```
[Out] (-3*(c + d*x)^(1/3))/(10*(b*c - a*d)*(a + b*x)^(10/3)) + (27*d*(c + d*x)^(1
/3))/(70*(b*c - a*d)^2*(a + b*x)^(7/3)) - (81*d^2*(c + d*x)^(1/3))/(140*(b*
c - a*d)^3*(a + b*x)^(4/3)) + (243*d^3*(c + d*x)^(1/3))/(140*(b*c - a*d)^4*
(a + b*x)^(1/3))
```

Rubi [A] time = 0.0295051, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{243d^3\sqrt[3]{c+dx}}{140\sqrt[3]{a+bx}(bc-ad)^4}-\frac{81d^2\sqrt[3]{c+dx}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{27d\sqrt[3]{c+dx}}{70(a+bx)^{7/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{10(a+bx)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x]
```


```
[Out] (-3*(c + d*x)^(1/3))/(10*(b*c - a*d)*(a + b*x)^(10/3)) + (27*d*(c + d*x)^(1
/3))/(70*(b*c - a*d)^2*(a + b*x)^(7/3)) - (81*d^2*(c + d*x)^(1/3))/(140*(b*
c - a*d)^3*(a + b*x)^(4/3)) + (243*d^3*(c + d*x)^(1/3))/(140*(b*c - a*d)^4*
(a + b*x)^(1/3))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{13/3}(c+dx)^{2/3}}\,dx &= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} - \frac{(9d)\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx}{10(bc-ad)} \\ &= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} + \frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}} + \frac{(27d^2)\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx}{35(bc-ad)^2} \\ &= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} + \frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}} - \frac{81d^2\sqrt[3]{c+dx}}{140(bc-ad)^3(a+bx)^{4/3}} - (81d^3 \\ &= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} + \frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}} - \frac{81d^2\sqrt[3]{c+dx}}{140(bc-ad)^3(a+bx)^{4/3}} + \frac{24}{140(b}\end{aligned}$$

Mathematica [A] time = 0.0484734, size = 116, normalized size = 0.85

$$\frac{3\sqrt[3]{c+dx}\left(-105a^2bd^2(c-3dx)+140a^3d^3+30ab^2d\left(2c^2-3cdx+9d^2x^2\right)+b^3\left(18c^2dx-14c^3-27cd^2x^2+81d^3x^3\right)\right)}{140(a+bx)^{10/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(c + d*x)^(1/3)*(140*a^3*d^3 - 105*a^2*b*d^2*(c - 3*d*x) + 30*a*b^2*d*(2
*c^2 - 3*c*d*x + 9*d^2*x^2) + b^3*(-14*c^3 + 18*c^2*d*x - 27*c*d^2*x^2 + 81
*d^3*x^3)))/(140*(b*c - a*d)^4*(a + b*x)^(10/3))
```

Maple [A] time = 0.005, size = 171, normalized size = 1.3

$$\frac{243\,b^3d^3x^3+810\,ab^2d^3x^2-81\,b^3cd^2x^2+945\,a^2bd^3x-270\,ab^2cd^2x+54\,b^3c^2dx+420\,a^3d^3-315\,a^2cbd^2+180\,ab^2c^2d}{140\,d^4a^4-560\,bd^3ca^3+840\,b^2d^2c^2a^2-560\,b^3dc^3a+140\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x)
```

```
[Out] 3/140*(d*x+c)^(1/3)*(81*b^3*d^3*x^3+270*a*b^2*d^3*x^2-27*b^3*c*d^2*x^2+315*
a^2*b*d^3*x-90*a*b^2*c*d^2*x+18*b^3*c^2*d*x+140*a^3*d^3-105*a^2*b*c*d^2+60*
a*b^2*c^2*d-14*b^3*c^3)/(b*x+a)^(10/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2
*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{13}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(13/3)*(d*x + c)^(2/3)), x)
```

Fricas [B] time = 2.23929, size = 861, normalized size = 6.33

$$\frac{3\left(81\,b^{3}d^{3}x^{3}-14\,b^{3}c^{3}+60\,ab^{2}c^{2}d-105\,a^{2}bc\right.}{140\left(a^{4}b^{4}c^{4}-4\,a^{5}b^{3}c^{3}d+6\,a^{6}b^{2}c^{2}d^{2}-4\,a^{7}bcd^{3}+a^{8}d^{4}+\left(b^{8}c^{4}-4\,ab^{7}c^{3}d+6\,a^{2}b^{6}c^{2}d^{2}-4\,a^{3}b^{5}cd^{3}+a^{4}b^{4}d^{4}\right)x^{4}+4\left(ab^{7}c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/140*(81*b^3*d^3*x^3 - 14*b^3*c^3 + 60*a*b^2*c^2*d - 105*a^2*b*c*d^2 + 140
*a^3*d^3 - 27*(b^3*c*d^2 - 10*a*b^2*d^3)*x^2 + 9*(2*b^3*c^2*d - 10*a*b^2*c*
d^2 + 35*a^2*b*d^3)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^4*b^4*c^4 - 4*a^5
*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b
^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^
7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4
)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*
d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d
^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(13/3)/(d*x+c)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{13}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(13/3)*(d*x + c)^(2/3)), x)
```

3.1608 $\int \frac{(a+bx)^{7/3}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=649

$$-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)^3((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\cdots}{\cdots}}}{10\,2^{2/3}\sqrt[3]{b}d^{10/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\cdots}$$

```
[Out] (21*(b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(20*d^3) - (21*(b*c - a*
d)*(a + b*x)^(4/3)*(c + d*x)^(1/3))/(40*d^2) + (3*(a + b*x)^(7/3)*(c + d*x)
^(1/3))/(8*d) - (7*3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^3*((a + b*x)*(c +
d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1
/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*
b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b
^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3
) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*(
(a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(10*2^(2/3)*b^(1/3)*d^(10/3)
*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2
/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 1.11785, antiderivative size = 649, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 62, 623, 218}

$$-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)^3((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\cdots}{\cdots}}}{10\,2^{2/3}\sqrt[3]{b}d^{10/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc\cdots}{(2\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/3)/(c + d*x)^(2/3),x]
```

```
[Out] (21*(b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(20*d^3) - (21*(b*c - a*
d)*(a + b*x)^(4/3)*(c + d*x)^(1/3))/(40*d^2) + (3*(a + b*x)^(7/3)*(c + d*x)
^(1/3))/(8*d) - (7*3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^3*((a + b*x)*(c +
d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1
/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*
b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b
^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3
) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*(
(a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(10*2^(2/3)*b^(1/3)*d^(10/3)
*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2
/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
```

```
c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{(a+bx)^{7/3}}{(c+dx)^{2/3}}\,dx = \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{4/3}}{(c+dx)^{2/3}}\,dx}{8d}$$

$$= -\frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d} + \frac{\left(7(bc-ad)^2\right)\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}\,dx}{10d^2}$$

$$= \frac{21(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20d^3} - \frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d} - \frac{\left(7(bc-ad)^3\right.}{}$$

$$= \frac{21(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20d^3} - \frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d} - \frac{\left(7(bc-ad)^3\right.}{}$$

$$= \frac{21(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20d^3} - \frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d} - \frac{\left(21(bc-ad)\right.}{}$$

$$= \frac{21(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20d^3} - \frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d} - \frac{7\,3^{3/4}\sqrt{2+}}{}$$

Mathematica [C] time = 0.0314303, size = 73, normalized size = 0.11

$$\frac{3(a+bx)^{10/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{10}{3};\frac{13}{3};\frac{d(a+bx)}{ad-bc}\right)}{10b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(10/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3
, 10/3, 13/3, (d*(a + b*x))/(-(b*c) + a*d)])/(10*b*(c + d*x)^(2/3))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{3}}(dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/3)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(7/3)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/3)/(d*x + c)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{7}{3}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral((a + b*x)**(7/3)/(c + d*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(2/3), x)
```

3.1609 $\int \frac{(a+bx)^{4/3}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=614

$$\frac{2\sqrt[3]{2}3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)^2((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\cdots}}{5\sqrt[3]{b}d^{7/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\cdots}}$$

```
[Out] (-6*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(5*d^2) + (3*(a + b*x)^(4/
3)*(c + d*x)^(1/3))/(5*d) + (2*2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d
)^2*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^
(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a
*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))
^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[
3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^
(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/
3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*b^(1/
3)*d^(7/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c
 - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.817457, antiderivative size = 614, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 62, 623, 218}

$$\frac{2\sqrt[3]{2}3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)^2((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\cdots}}{5\sqrt[3]{b}d^{7/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)\cdots}{\left(2^{2/3}\sqrt[3]{\cdots}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/(c + d*x)^(2/3),x]
```


```
[Out] (-6*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(5*d^2) + (3*(a + b*x)^(4/
3)*(c + d*x)^(1/3))/(5*d) + (2*2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d
)^2*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^
(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a
*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))
^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[
3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^
(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/
3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*b^(1/
3)*d^(7/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c
 - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{(c+dx)^{2/3}}\,dx = \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} - \frac{(4(bc-ad))\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}\,dx}{5d}$$

$$= -\frac{6(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{5d^2} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} + \frac{\left(2(bc-ad)^2\right)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{5d^2}$$

$$= -\frac{6(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{5d^2} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} + \frac{\left(2(bc-ad)^2((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{\left(ac+(bc+ad)x\right.}}{5d^2(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= -\frac{6(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{5d^2} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} + \frac{\left(6(bc-ad)^2((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2}\right.}{5d^2(a+bx)^{2}}$$

$$= -\frac{6(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{5d^2} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} + \frac{2\sqrt[3]{2}3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)^2((a+bx)(c+dx))^{2}}{}$$

Mathematica [C] time = 0.028695, size = 73, normalized size = 0.12

$$\frac{3(a+bx)^{7/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{7}{3};\frac{10}{3};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 7/3, 10/3, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(2/3))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{4}{3}} (dx+c)^{-\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(4/3)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(4/3)/(d*x + c)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{4}{3}}}{(c+dx)^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral((a + b*x)**(4/3)/(c + d*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(2/3), x)
```

3.1610 $\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=577

$$\frac{3\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{2d} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc\right.}{2^{2/3}\sqrt[3]{b}d^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(a}$$

```
[Out] (3*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(2*d) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c
 - a*d)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c
 - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d
*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(
2/3)*b^(1/3)*d^(4/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*
Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*
d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.547072, antiderivative size = 577, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 62, 623, 218}

$$\frac{3\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{2d} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc\right.}{2^{2/3}\sqrt[3]{b}d^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/(c + d*x)^(2/3),x]
```



```
[Out] (3*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(2*d) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c
 - a*d)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c
 - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d
*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(
2/3)*b^(1/3)*d^(4/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*
Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*
d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{2d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{2d}$$

$$= \frac{3\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{2d} - \frac{\left((bc-ad)((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^{2/3}}\,dx}{2d(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= \frac{3\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{2d} - \frac{\left(3(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+ad)^2+4b}}\right.}{2d(a+bx)^{2/3}(c+dx)^{2/3}(bc+ad+2bdx)}$$

$$= \frac{3\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{2d} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/}\right.}{2^{2/3}\sqrt[3]{b}d^{4/3}(a+bx)^{2/3}}$$

Mathematica [C] time = 0.0258259, size = 73, normalized size = 0.13

$$\frac{3(a+bx)^{4/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};\frac{d(a+bx)}{ad-bc}\right)}{4b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 4/3, 7/3, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*(c + d*x)^(2/3))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx+a}\,(dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)/(d*x + c)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral((a + b*x)**(1/3)/(c + d*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(2/3), x)
```

3.1611 $\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=542

$$\frac{\sqrt[3]{2}3^{3/4}\sqrt{2+\sqrt{3}}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx}{(2^2}}}{\sqrt[3]{b}\sqrt[3]{d}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}(}{(2^{2/3}\sqrt[3]{b}\sqrt[3]{}}}}$$

```
[Out] (2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c +
a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a
*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x
)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c -
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqr
t[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/
3))], -7 - 4*Sqrt[3]])/(b^(1/3)*d^(1/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*
c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c
 + 2*d*x))^2])
```

Rubi [A] time = 0.429822, antiderivative size = 542, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {62, 623, 218}

$$\frac{\sqrt[3]{2}3^{3/4}\sqrt{2+\sqrt{3}}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx}{(2^2}}}{\sqrt[3]{b}\sqrt[3]{d}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}(2^{2/3}}{(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a}}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(2/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c +
a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a
*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x
)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c -
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqr
t[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/
3))], -7 - 4*Sqrt[3]])/(b^(1/3)*d^(1/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*
c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c
 + 2*d*x))^2])
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
```

```
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx &= \frac{((a+bx)(c+dx))^{2/3}\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^{2/3}}\,dx}{(a+bx)^{2/3}(c+dx)^{2/3}}\\ &= \frac{\left(3((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+ad)^2+4bdx^3}}\,dx, x, \sqrt[3]{(a+bx)}\right.}{(a+bx)^{2/3}(c+dx)^{2/3}(bc+ad+2bdx)}\\ &= \frac{\sqrt[3]{2}3^{3/4}\sqrt{2+\sqrt{3}}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx}\right.}{\sqrt[3]{b}\sqrt[3]{d}(a+bx)^{2/3}(c+dx)^{2/3}(bc+}\end{aligned}$$

Mathematica [C] time = 0.0233321, size = 71, normalized size = 0.13

$$\frac{3\sqrt[3]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{d(a+bx)}{ad-bc}\right)}{b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(a + b*x)^(1/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[1/3,
 2/3, 4/3, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(2/3))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{2}{3}}(dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{2}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(2/3)*(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(2/3)), x)
```

3.1612 $\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=586

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)\cdots}{\left(2^{2/3}\cdots\right.}}}{2^{2/3}\sqrt[3]{b}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)(ad+bc+2bdx)\sqrt{\frac{(bc-ad\cdots}{\left(2^{2/3}\cdots\right.}}}$$

```
[Out] (-3*(c + d*x)^(1/3))/(2*(b*c - a*d)*(a + b*x)^(2/3)) - (3^(3/4)*Sqrt[2 + Sq
rt[3]]*d^(2/3)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((
b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqr
t[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)
*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/
((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2
/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]
])/(2^(2/3)*b^(1/3)*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d
+ 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/
3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x
))^2])
```

Rubi [A] time = 0.549742, antiderivative size = 586, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {51, 62, 623, 218}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)\cdots}{\left(2^{2/3}\cdots\right.}}}{2^{2/3}\sqrt[3]{b}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(\cdots\right.}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/3)*(c + d*x)^(2/3)),x]
```


```
[Out] (-3*(c + d*x)^(1/3))/(2*(b*c - a*d)*(a + b*x)^(2/3)) - (3^(3/4)*Sqrt[2 + Sq
rt[3]]*d^(2/3)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((
b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqr
t[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)
*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/
((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2
/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]
])/(2^(2/3)*b^(1/3)*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d
+ 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/
3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x
))^2])
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}} - \frac{d\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{2(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}} - \frac{\left(d((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^{2/3}}\,dx}{2(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}} - \frac{\left(3d((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{-4abcd\cdots}}\right.}{2(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}(bc+\cdots}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\left((bc\cdots\right.}{2^{2/3}\sqrt[3]{b}(\cdots}$$

Mathematica [C] time = 0.0244417, size = 73, normalized size = 0.12

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{2}{3};\frac{1}{3};\frac{d(a+bx)}{ad-bc}\right)}{2b(a+bx)^{2/3}(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-2/3, 2/3, 1/3, (d*
(a + b*x))/(-(b*c) + a*d)])/(2*b*(a + b*x)^(2/3)*(c + d*x)^(2/3))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{3}}(dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}}{b^2dx^3+a^2c+\left(b^2c+2\,abd\right)x^2+\left(2\,abc+a^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{3}}(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/3)*(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(2/3)), x)
```

3.1613 $\int \frac{1}{(a+bx)^{8/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=621

$$\frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+\cdots}{(\cdots}}}{5\sqrt[3]{b}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)^2(ad+bc+2bdx)\sqrt{\frac{(bc-ad\cdots}{\left(2^{2/3}\sqrt[3]{\cdots}\right.}}}$$

```
[Out] (-3*(c + d*x)^(1/3))/(5*(b*c - a*d)*(a + b*x)^(5/3)) + (6*d*(c + d*x)^(1/3)
)/(5*(b*c - a*d)^2*(a + b*x)^(2/3)) + (2*2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*
d^(5/3)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c
 - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d
*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*b
^(1/3)*(b*c - a*d)^2*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*
Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*
d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.812154, antiderivative size = 621, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {51, 62, 623, 218}

$$\frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+\cdots}{(\cdots}}}{5\sqrt[3]{b}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)^2(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(\cdots\right.}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{\cdots}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(8/3)*(c + d*x)^(2/3)),x]
```


```
[Out] (-3*(c + d*x)^(1/3))/(5*(b*c - a*d)*(a + b*x)^(5/3)) + (6*d*(c + d*x)^(1/3)
)/(5*(b*c - a*d)^2*(a + b*x)^(2/3)) + (2*2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*
d^(5/3)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c
 - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d
*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*b
^(1/3)*(b*c - a*d)^2*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*
Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*
d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{8/3}(c+dx)^{2/3}}\,dx &= -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} - \frac{(4d)\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}\,dx}{5(bc-ad)} \\
&= -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} + \frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}} + \frac{\left(2d^2\right)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{5(bc-ad)^2} \\
&= -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} + \frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}} + \frac{\left(2d^2((a+bx)(c+dx))^{2/3}\right)\int \frac{}{\left(ac+(bc+ad \cdots\right.}}{5(bc-ad)^2(a+bx)^{2/3}(c+dx \cdots} \\
&= -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} + \frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}} + \frac{\left(6d^2((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+\cdots}\right.}{5(bc-ad)\cdots} \\
&= -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} + \frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}} + \frac{2\sqrt[3]{2}3^{3/4}\sqrt{2+\sqrt{3}}d^{5/3}((a+bx)(c+dx \cdots}{\cdots}
\end{aligned}$$

Mathematica [C] time = 0.0229708, size = 73, normalized size = 0.12

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}\,{}_2F_1\left(-\frac{5}{3},\frac{2}{3};-\frac{2}{3};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/3}(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-5/3, 2/3, -2/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(5*b*(a + b*x)^(5/3)*(c + d*x)^(2/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{8}{3}} (dx+c)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}}{b^3dx^4 + a^3c + \left(b^3c + 3\,ab^2d\right)x^3 + 3\left(ab^2c + a^2bd\right)x^2 + \left(3\,a^2bc + a^3d\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*
b^2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{8}{3}}\,(c+dx)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(8/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(8/3)*(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(2/3)), x)
```

3.1614 $\int \frac{1}{(a+bx)^{11/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=656

$$-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{8/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+b\cdots}{(2\cdots}}}{10\,2^{2/3}\sqrt[3]{b}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)^3(ad+bc+2bdx)\sqrt{\frac{(bc\cdots}{(2\cdots}}}$$

```
[Out] (-3*(c + d*x)^(1/3))/(8*(b*c - a*d)*(a + b*x)^(8/3)) + (21*d*(c + d*x)^(1/3
))/(40*(b*c - a*d)^2*(a + b*x)^(5/3)) - (21*d^2*(c + d*x)^(1/3))/(20*(b*c -
 a*d)^3*(a + b*x)^(2/3)) - (7*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^(8/3)*((a + b*x)*
(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(
2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1
/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x
)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(10*2^(2/3)*b^(1/3)*(b*
c - a*d)^3*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c
 - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 1.09015, antiderivative size = 656, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {51, 62, 623, 218}

$$-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{8/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+b\cdots}{(2\cdots}}}{10\,2^{2/3}\sqrt[3]{b}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)^3(ad+bc+2bdx)\sqrt{\frac{(bc-ad)\cdots}{(2^{2/3}\sqrt[3]{\cdots}}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/3)*(c + d*x)^(2/3)),x]
```


```
[Out] (-3*(c + d*x)^(1/3))/(8*(b*c - a*d)*(a + b*x)^(8/3)) + (21*d*(c + d*x)^(1/3
))/(40*(b*c - a*d)^2*(a + b*x)^(5/3)) - (21*d^2*(c + d*x)^(1/3))/(20*(b*c -
 a*d)^3*(a + b*x)^(2/3)) - (7*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^(8/3)*((a + b*x)*
(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(
2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1
/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x
)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(10*2^(2/3)*b^(1/3)*(b*
c - a*d)^3*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c
 - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} - \frac{(7d)\int \frac{1}{(a+bx)^{8/3}(c+dx)^{2/3}}\,dx}{8(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} + \frac{\left(7d^2\right)\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}\,dx}{10(bc-ad)^2}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} - \frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/3}} - \frac{\left(7d^3\right)\int \cdots}{20 \cdots}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} - \frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/3}} - \frac{\left(7d^3((a \cdots\right.}{2 \cdots}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} - \frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/3}} - \frac{\left(21d^3((a \cdots\right.}{\cdots}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} - \frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/3}} - \frac{7\,3^{3/4}\sqrt{2 \cdots}}{\cdots}$$

Mathematica [C] time = 0.0227554, size = 73, normalized size = 0.11

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(-\frac{8}{3},\frac{2}{3};-\frac{5}{3};\frac{d(a+bx)}{ad-bc}\right)}{8b(a+bx)^{8/3}(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-8/3, 2/3, -5/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(8*b*(a + b*x)^(8/3)*(c + d*x)^(2/3))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{11}{3}}(dx+c)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}}{b^4dx^5+a^4c+\left(b^4c+4\,ab^3d\right)x^4+2\left(2\,ab^3c+3\,a^2b^2d\right)x^3+2\left(3\,a^2b^2c+2\,a^3bd\right)x^2+\left(4\,a^3bc+a^4d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^4*d*x^5 + a^4*c + (b^4*c + 4*a*
b^3*d)*x^4 + 2*(2*a*b^3*c + 3*a^2*b^2*d)*x^3 + 2*(3*a^2*b^2*c + 2*a^3*b*d)*
x^2 + (4*a^3*b*c + a^4*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/3)/(d*x+c)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(2/3)), x)
```

3.1615 $\int \frac{(a+bx)^{7/3}}{(c+dx)^{4/3}} \, dx$

Optimal. Leaf size=241

$$\frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} - \frac{14b\sqrt[3]{a+bx}(c+dx)^{2/3}(bc-ad)}{3d^3} - \frac{7\sqrt[3]{b}(bc-ad)^2\log(a+bx)}{9d^{10/3}} - \frac{7\sqrt[3]{b}(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}} - \right.}{3d^{10/3}}$$

```
[Out] (-3*(a + b*x)^(7/3))/(d*(c + d*x)^(1/3)) - (14*b*(b*c - a*d)*(a + b*x)^(1/3
)*(c + d*x)^(2/3))/(3*d^3) + (7*b*(a + b*x)^(4/3)*(c + d*x)^(2/3))/(2*d^2)
- (14*b^(1/3)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/
(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(3*Sqrt[3]*d^(10/3)) - (7*b^(1/3)*(b*c
- a*d)^2*Log[a + b*x])/(9*d^(10/3)) - (7*b^(1/3)*(b*c - a*d)^2*Log[-1 + (b^
(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(3*d^(10/3))
```

Rubi [A] time = 0.106996, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {47, 50, 59}

$$\frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} - \frac{14b\sqrt[3]{a+bx}(c+dx)^{2/3}(bc-ad)}{3d^3} - \frac{7\sqrt[3]{b}(bc-ad)^2\log(a+bx)}{9d^{10/3}} - \frac{7\sqrt[3]{b}(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}} - \right.}{3d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(7/3))/(d*(c + d*x)^(1/3)) - (14*b*(b*c - a*d)*(a + b*x)^(1/3
)*(c + d*x)^(2/3))/(3*d^3) + (7*b*(a + b*x)^(4/3)*(c + d*x)^(2/3))/(2*d^2)
- (14*b^(1/3)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/
(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(3*Sqrt[3]*d^(10/3)) - (7*b^(1/3)*(b*c
- a*d)^2*Log[a + b*x])/(9*d^(10/3)) - (7*b^(1/3)*(b*c - a*d)^2*Log[-1 + (b^
(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(3*d^(10/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
```

```
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{7/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} + \frac{(7b)\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} - \frac{(14b(bc-ad))\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{3d^2}$$

$$= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} - \frac{14b(bc-ad)\sqrt[3]{a+bx}(c+dx)^{2/3}}{3d^3} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} + \frac{\left(14b(bc-ad)^2\right)\int}{9d^3}$$

$$= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} - \frac{14b(bc-ad)\sqrt[3]{a+bx}(c+dx)^{2/3}}{3d^3} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} - \frac{14\sqrt[3]{b}(bc-ad)^2\tan}{3\sqrt{3}}$$

Mathematica [C] time = 0.0584074, size = 73, normalized size = 0.3

$$\frac{3(a+bx)^{10/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},\frac{10}{3};\frac{13}{3};\frac{d(a+bx)}{ad-bc}\right)}{10b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(10/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3
, 10/3, 13/3, (d*(a + b*x))/(-(b*c) + a*d)])/(10*b*(c + d*x)^(4/3))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{3}}(dx+c)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(7/3)/(d*x+c)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(4/3), x)
```

Fricas [B] time = 2.45291, size = 995, normalized size = 4.13

$$-\frac{28\sqrt{3}\left(b^2c^3 - 2\,abc^2d + a^2cd^2 + \left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}(bdx+bc)}{3\,(bdx+bc)}\right) + 14\,(b^2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/18*(28*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*x)*(-b/d)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*(d*x
+ c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) + 14*(b^2
*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*(-b
/d)^(1/3)*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*(-b
/d)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 28*(b^2*c^3 - 2*a
*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*(-b/d)^(1/3)*
log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c)) -
 3*(3*b^2*d^2*x^2 - 28*b^2*c^2 + 49*a*b*c*d - 18*a^2*d^2 - (7*b^2*c*d - 13*
a*b*d^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(d^4*x + c*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx)^{\frac{7}{3}}}{(c + dx)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(7/3)/(c + d*x)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^{\frac{7}{3}}}{(dx + c)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(4/3), x)
```

3.1616 $\int \frac{(a+bx)^{4/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=195

$$\frac{4b\sqrt[3]{a+bx}(c+dx)^{2/3}}{d^2}+\frac{2\sqrt[3]{b}(bc-ad)\log(a+bx)}{3d^{7/3}}+\frac{2\sqrt[3]{b}(bc-ad)\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{d^{7/3}}+\frac{4\sqrt[3]{b}(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+d}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+}}\right.}{\sqrt{3}d^{7/3}}$$

```
[Out] (-3*(a + b*x)^(4/3))/(d*(c + d*x)^(1/3)) + (4*b*(a + b*x)^(1/3)*(c + d*x)^(
2/3))/d^2 + (4*b^(1/3)*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^
(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(Sqrt[3]*d^(7/3)) + (2*b^(1/3)*(
b*c - a*d)*Log[a + b*x])/(3*d^(7/3)) + (2*b^(1/3)*(b*c - a*d)*Log[-1 + (b^(
1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/d^(7/3)
```

Rubi [A] time = 0.0706522, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {47, 50, 59}

$$\frac{4b\sqrt[3]{a+bx}(c+dx)^{2/3}}{d^2}+\frac{2\sqrt[3]{b}(bc-ad)\log(a+bx)}{3d^{7/3}}+\frac{2\sqrt[3]{b}(bc-ad)\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{d^{7/3}}+\frac{4\sqrt[3]{b}(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+d}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+}}\right.}{\sqrt{3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(4/3))/(d*(c + d*x)^(1/3)) + (4*b*(a + b*x)^(1/3)*(c + d*x)^(
2/3))/d^2 + (4*b^(1/3)*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^
(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(Sqrt[3]*d^(7/3)) + (2*b^(1/3)*(
b*c - a*d)*Log[a + b*x])/(3*d^(7/3)) + (2*b^(1/3)*(b*c - a*d)*Log[-1 + (b^(
1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/d^(7/3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
```

```
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}} + \frac{(4b)\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}} + \frac{4b\sqrt[3]{a+bx}(c+dx)^{2/3}}{d^2} - \frac{(4b(bc-ad))\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{3d^2}$$

$$= -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}} + \frac{4b\sqrt[3]{a+bx}(c+dx)^{2/3}}{d^2} + \frac{4\sqrt[3]{b}(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{\sqrt{3}d^{7/3}} + \frac{2\sqrt[3]{b}(bc-ad)\log(}{3d^{7/3}}$$

Mathematica [C] time = 0.0482111, size = 73, normalized size = 0.37

$$\frac{3(a+bx)^{7/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3} {}_2F_1\left(\frac{4}{3},\frac{7}{3};\frac{10}{3};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3,
 7/3, 10/3, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(4/3))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{4}{3}}(dx+c)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(4/3), x)
```

Fricas [A] time = 2.39372, size = 745, normalized size = 3.82

$$\frac{4\sqrt{3}\left(bc^2-acd+\left(bcd-ad^2\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}(bdx+bc)}{3\,(bdx+bc)}\right)+2\left(bc^2-acd+\left(bcd-ad^2\right)x\right)\left(-\frac{b}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/3*(4*sqrt(3)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*(-b/d)^(1/3)*arctan(1/3*
(2*sqrt(3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x
+ b*c))/(b*d*x + b*c)) + 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*(-b/d)^(1/3)
*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*(-b/d)^(1/3)
 + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 4*(b*c^2 - a*c*d + (b*c*d
- a*d^2)*x)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x
 + c)^(2/3))/(d*x + c)) + 3*(b*d*x + 4*b*c - 3*a*d)*(b*x + a)^(1/3)*(d*x +
c)^(2/3))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{4}{3}}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(4/3)/(c + d*x)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(4/3), x)
```

3.1617 $\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=149

$$-\frac{3\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2d^{4/3}}-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{d^{4/3}}-\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}-\frac{\sqrt[3]{b}\log(a+bx)}{2d^{4/3}}$$

```
[Out] (-3*(a + b*x)^(1/3))/(d*(c + d*x)^(1/3)) - (Sqrt[3]*b^(1/3)*ArcTan[1/Sqrt[3
] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/d^(4/3)
 - (b^(1/3)*Log[a + b*x])/(2*d^(4/3)) - (3*b^(1/3)*Log[-1 + (b^(1/3)*(c + d
*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*d^(4/3))
```

Rubi [A] time = 0.0308381, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 59}

$$-\frac{3\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2d^{4/3}}-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{d^{4/3}}-\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}-\frac{\sqrt[3]{b}\log(a+bx)}{2d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(1/3))/(d*(c + d*x)^(1/3)) - (Sqrt[3]*b^(1/3)*ArcTan[1/Sqrt[3
] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/d^(4/3)
 - (b^(1/3)*Log[a + b*x])/(2*d^(4/3)) - (3*b^(1/3)*Log[-1 + (b^(1/3)*(c + d
*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*d^(4/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{4/3}}\,dx = -\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}} + \frac{b\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}} - \frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{d^{4/3}} - \frac{\sqrt[3]{b}\log(a+bx)}{2d^{4/3}} - \frac{3\sqrt[3]{b}\log\left(-1+\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{2d^{4/3}}$$

Mathematica [C] time = 0.0444046, size = 73, normalized size = 0.49

$$\frac{3(a+bx)^{4/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3} {}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{d(a+bx)}{ad-bc}\right)}{4b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3,
 4/3, 7/3, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*(c + d*x)^(4/3))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx+a}\,(dx+c)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(4/3), x)
```

Fricas [B] time = 2.30863, size = 603, normalized size = 4.05

$$-\frac{2\sqrt{3}(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}(bdx+bc)}{3\,(bdx+bc)}\right)+(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-\frac{b}{d}\right)^{\frac{1}{3}}\cdots}{dx+c}\right.}{2\left(d^2x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(d*x + c)*(-b/d)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3
)*(d*x + c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) +
(d*x + c)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x +
 c)^(2/3)*(-b/d)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 2*(d
*x + c)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x + c
)^(2/3))/(d*x + c)) + 6*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(d^2*x + c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(1/3)/(c + d*x)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(4/3), x)
```

3.1618 $\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=30

$$\frac{3\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}(bc-ad)}$$

```
[Out] (3*(a + b*x)^(1/3))/((b*c - a*d)*(c + d*x)^(1/3))
```

Rubi [A] time = 0.0031594, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{3\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x]

[Out] (3*(a + b*x)^(1/3))/((b*c - a*d)*(c + d*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,dx = \frac{3\sqrt[3]{a+bx}}{(bc-ad)\sqrt[3]{c+dx}}$$

Mathematica [A] time = 0.0080146, size = 30, normalized size = 1.

$$\frac{3\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x]

[Out] (3*(a + b*x)^(1/3))/((b*c - a*d)*(c + d*x)^(1/3))
```

Maple [A] time = 0.004, size = 27, normalized size = 0.9

$$-3\,\frac{\sqrt[3]{bx+a}}{\sqrt[3]{dx+c}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x)
```

```
[Out] -3*(b*x+a)^(1/3)/(d*x+c)^(1/3)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(4/3)), x)
```

Fricas [A] time = 2.2954, size = 96, normalized size = 3.2

$$\frac{3\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 3*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{2}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(2/3)*(c + d*x)**(4/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(4/3)), x)
```

3.1619 $\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=66

$$-\frac{9d\sqrt[3]{a+bx}}{2\sqrt[3]{c+dx}(bc-ad)^2}-\frac{3}{2(a+bx)^{2/3}\sqrt[3]{c+dx}(bc-ad)}$$

```
[Out] -3/(2*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3)) - (9*d*(a + b*x)^(1/3))/
(2*(b*c - a*d)^2*(c + d*x)^(1/3))
```

Rubi [A] time = 0.0086929, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{9d\sqrt[3]{a+bx}}{2\sqrt[3]{c+dx}(bc-ad)^2}-\frac{3}{2(a+bx)^{2/3}\sqrt[3]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(2*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3)) - (9*d*(a + b*x)^(1/3))/
(2*(b*c - a*d)^2*(c + d*x)^(1/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx &= -\frac{3}{2(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}-\frac{(3d)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,dx}{2(bc-ad)}\\ &= -\frac{3}{2(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}-\frac{9d\sqrt[3]{a+bx}}{2(bc-ad)^2\sqrt[3]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0161732, size = 45, normalized size = 0.68

$$-\frac{3(2ad+b(c+3dx))}{2(a+bx)^{2/3}\sqrt[3]{c+dx}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(2*a*d + b*(c + 3*d*x)))/(2*(b*c - a*d)^2*(a + b*x)^(2/3)*(c + d*x)^(1/
3))
```

Maple [A] time = 0.004, size = 53, normalized size = 0.8

$$-\frac{9\,bdx+6\,ad+3\,bc}{2\,a^2d^2-4\,abcd+2\,b^2c^2}(bx+a)^{-\frac{2}{3}}\frac{1}{\sqrt[3]{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x)
```

```
[Out] -3/2*(3*b*d*x+2*a*d+b*c)/(b*x+a)^(2/3)/(d*x+c)^(1/3)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(4/3)), x)
```

Fricas [B] time = 2.31814, size = 270, normalized size = 4.09

$$-\frac{3\,(3\,bdx+bc+2\,ad)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -3/2*(3*b*d*x + b*c + 2*a*d)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/3)*(c + d*x)**(4/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(4/3)), x)
```

3.1620 $\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=101

$$\frac{27d^2\sqrt[3]{a+bx}}{5\sqrt[3]{c+dx}(bc-ad)^3}+\frac{9d}{5(a+bx)^{2/3}\sqrt[3]{c+dx}(bc-ad)^2}-\frac{3}{5(a+bx)^{5/3}\sqrt[3]{c+dx}(bc-ad)}$$

```
[Out] -3/(5*(b*c - a*d)*(a + b*x)^(5/3)*(c + d*x)^(1/3)) + (9*d)/(5*(b*c - a*d)^2
*(a + b*x)^(2/3)*(c + d*x)^(1/3)) + (27*d^2*(a + b*x)^(1/3))/(5*(b*c - a*d)
^3*(c + d*x)^(1/3))
```

Rubi [A] time = 0.0180775, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{27d^2\sqrt[3]{a+bx}}{5\sqrt[3]{c+dx}(bc-ad)^3}+\frac{9d}{5(a+bx)^{2/3}\sqrt[3]{c+dx}(bc-ad)^2}-\frac{3}{5(a+bx)^{5/3}\sqrt[3]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(5*(b*c - a*d)*(a + b*x)^(5/3)*(c + d*x)^(1/3)) + (9*d)/(5*(b*c - a*d)^2
*(a + b*x)^(2/3)*(c + d*x)^(1/3)) + (27*d^2*(a + b*x)^(1/3))/(5*(b*c - a*d)
^3*(c + d*x)^(1/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx &= -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}}-\frac{(6d)\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx}{5(bc-ad)}\\
&= -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}}+\frac{9d}{5(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}}+\frac{\left(9d^2\right)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,d}{5(bc-ad)^2}\\
&= -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}}+\frac{9d}{5(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}}+\frac{27d^2\sqrt[3]{a+bx}}{5(bc-ad)^3\sqrt[3]{c+dx}}
\end{aligned}$$

Mathematica [A] time = 0.0292202, size = 75, normalized size = 0.74

$$\frac{3\left(5a^2d^2+5abd(c+3dx)+b^2\left(-c^2+3cdx+9d^2x^2\right)\right)}{5(a+bx)^{5/3}\sqrt[3]{c+dx}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(5*a^2*d^2 + 5*a*b*d*(c + 3*d*x) + b^2*(-c^2 + 3*c*d*x + 9*d^2*x^2)))/(5
*(b*c - a*d)^3*(a + b*x)^(5/3)*(c + d*x)^(1/3))
```

Maple [A] time = 0.004, size = 105, normalized size = 1.

$$-\frac{27\,b^2d^2x^2+45\,abd^2x+9\,b^2cdx+15\,a^2d^2+15\,abcd-3\,b^2c^2}{5\,a^3d^3-15\,a^2cbd^2+15\,ab^2c^2d-5\,b^3c^3}(bx+a)^{-\frac{5}{3}}\frac{1}{\sqrt[3]{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x)
```

```
[Out] -3/5*(9*b^2*d^2*x^2+15*a*b*d^2*x+3*b^2*c*d*x+5*a^2*d^2+5*a*b*c*d-b^2*c^2)/(
b*x+a)^(5/3)/(d*x+c)^(1/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(4/3)), x)
```

Fricas [B] time = 2.37148, size = 549, normalized size = 5.44

$$\frac{3\left(9\,b^2d^2x^2-b^2c^2+5\,abcd+5\,a^2d^2+3\left(b^2cd+5\,abd^2\right)x}{5\left(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+\left(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4\right)x^3+\left(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/5*(9*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 5*a^2*d^2 + 3*(b^2*c*d + 5*a*b*d
^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a
^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 -
 a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*
```

```
c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d
^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{8}{3}}(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(8/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(8/3)*(c + d*x)**(4/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(4/3)), x)
```

3.1621 $\int \frac{1}{(a+bx)^{11/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=136

$$-\frac{243d^3\sqrt[3]{a+bx}}{40\sqrt[3]{c+dx}(bc-ad)^4}-\frac{81d^2}{40(a+bx)^{2/3}\sqrt[3]{c+dx}(bc-ad)^3}+\frac{27d}{40(a+bx)^{5/3}\sqrt[3]{c+dx}(bc-ad)^2}-\frac{3}{8(a+bx)^{8/3}\sqrt[3]{c+dx}(bc$$

```
[Out] -3/(8*(b*c - a*d)*(a + b*x)^(8/3)*(c + d*x)^(1/3)) + (27*d)/(40*(b*c - a*d)
^2*(a + b*x)^(5/3)*(c + d*x)^(1/3)) - (81*d^2)/(40*(b*c - a*d)^3*(a + b*x)^
(2/3)*(c + d*x)^(1/3)) - (243*d^3*(a + b*x)^(1/3))/(40*(b*c - a*d)^4*(c + d
*x)^(1/3))
```

Rubi [A] time = 0.0296401, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{243d^3\sqrt[3]{a+bx}}{40\sqrt[3]{c+dx}(bc-ad)^4}-\frac{81d^2}{40(a+bx)^{2/3}\sqrt[3]{c+dx}(bc-ad)^3}+\frac{27d}{40(a+bx)^{5/3}\sqrt[3]{c+dx}(bc-ad)^2}-\frac{3}{8(a+bx)^{8/3}\sqrt[3]{c+dx}(bc$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x]
```


```
[Out] -3/(8*(b*c - a*d)*(a + b*x)^(8/3)*(c + d*x)^(1/3)) + (27*d)/(40*(b*c - a*d)
^2*(a + b*x)^(5/3)*(c + d*x)^(1/3)) - (81*d^2)/(40*(b*c - a*d)^3*(a + b*x)^
(2/3)*(c + d*x)^(1/3)) - (243*d^3*(a + b*x)^(1/3))/(40*(b*c - a*d)^4*(c + d
*x)^(1/3))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/3}(c+dx)^{4/3}}\,dx = -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} - \frac{(9d)\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx}{8(bc-ad)}$$

$$= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} + \frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}} + \frac{(27d^2)\int \frac{1}{(a+bx)^{5/3}(c+dx)}}{20(bc-ad)^2}$$

$$= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} + \frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}} - \frac{81d^2}{40(bc-ad)^3(a+bx)^{2/}}$$

$$= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} + \frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}} - \frac{81d^2}{40(bc-ad)^3(a+bx)^{2/}}$$

Mathematica [A] time = 0.0469954, size = 116, normalized size = 0.85

$$-\frac{3\left(60a^2bd^2(c+3dx)+40a^3d^3+24ab^2d\left(-c^2+3cdx+9d^2x^2\right)+b^3\left(-9c^2dx+5c^3+27cd^2x^2+81d^3x^3\right)\right)}{40(a+bx)^{8/3}\sqrt[3]{c+dx}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(40*a^3*d^3 + 60*a^2*b*d^2*(c + 3*d*x) + 24*a*b^2*d*(-c^2 + 3*c*d*x + 9
*d^2*x^2) + b^3*(5*c^3 - 9*c^2*d*x + 27*c*d^2*x^2 + 81*d^3*x^3)))/(40*(b*c
- a*d)^4*(a + b*x)^(8/3)*(c + d*x)^(1/3))
```

Maple [A] time = 0.008, size = 171, normalized size = 1.3

$$-\frac{243\,b^3d^3x^3+648\,ab^2d^3x^2+81\,b^3cd^2x^2+540\,a^2bd^3x+216\,ab^2cd^2x-27\,b^3c^2dx+120\,a^3d^3+180\,a^2cbd^2-72\,ab^2c^2d+}{40\,d^4a^4-160\,bd^3ca^3+240\,b^2d^2c^2a^2-160\,b^3dc^3a+40\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x)
```

```
[Out] -3/40*(81*b^3*d^3*x^3+216*a*b^2*d^3*x^2+27*b^3*c*d^2*x^2+180*a^2*b*d^3*x+72
*a*b^2*c*d^2*x-9*b^3*c^2*d*x+40*a^3*d^3+60*a^2*b*c*d^2-24*a*b^2*c^2*d+5*b^3
*c^3)/(b*x+a)^(8/3)/(d*x+c)^(1/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-
4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(4/3)), x)
```

Fricas [B] time = 2.95757, size = 927, normalized size = 6.82

$$-\frac{3\left(81\,b^3d^3x^3+5\,b^3\cdots\right.}{40\left(a^3b^4c^5-4\,a^4b^3c^4d+6\,a^5b^2c^3d^2-4\,a^6bc^2d^3+a^7cd^4+\left(b^7c^4d-4\,ab^6c^3d^2+6\,a^2b^5c^2d^3-4\,a^3b^4cd^4+a^4b^3d^5\right)x^4+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -3/40*(81*b^3*d^3*x^3 + 5*b^3*c^3 - 24*a*b^2*c^2*d + 60*a^2*b*c*d^2 + 40*a^
3*d^3 + 27*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 9*(b^3*c^2*d - 8*a*b^2*c*d^2 - 2
0*a^2*b*d^3)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^3*b^4*c^5 - 4*a^4*b^3*c^
4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^
6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c
^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^
4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2
 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 -
11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a
^7*d^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/3)/(d*x+c)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(4/3)), x)
```

3.1622 $\int \frac{(a+bx)^{8/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1355

result too large to display

```
[Out] (-3*(a + b*x)^(8/3))/(d*(c + d*x)^(1/3)) - (30*b*(b*c - a*d)*(a + b*x)^(2/3
)*(c + d*x)^(2/3))/(7*d^3) + (24*b*(a + b*x)^(5/3)*(c + d*x)^(2/3))/(7*d^2)
 + (30*2^(2/3)*b^(1/3)*(b*c - a*d)^2*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c
+ a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(7*d^(11/3)*(a + b*x)^(1
/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3)
+ 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))) - (15*2^(2/3)*3^(1/
4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c - a*d)^(8/3)*((a + b*x)*(c + d*x))^(1/3)*
Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1
/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/
3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*
b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
 + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*d^(11/3)*(a + b*x)^(1/3)*(c + d*x)^(1
/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d
)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d
 + b*(c + 2*d*x))^2]) + (20*2^(1/6)*3^(3/4)*b^(1/3)*(b*c - a*d)^(8/3)*((a +
 b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3)
 - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) +
2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c
- a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*d^(11/3)*(a +
b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((
b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((
1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 2.51064, antiderivative size = 1355, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {47, 50, 62, 623, 303, 218, 1877}

$$-\frac{15\, 2^{2/3}\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\cdots}{\cdots}}}{7d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}(\cdots}{\left((1+\sqrt{3})(\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(8/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(8/3))/(d*(c + d*x)^(1/3)) - (30*b*(b*c - a*d)*(a + b*x)^(2/3
)*(c + d*x)^(2/3))/(7*d^3) + (24*b*(a + b*x)^(5/3)*(c + d*x)^(2/3))/(7*d^2)
 + (30*2^(2/3)*b^(1/3)*(b*c - a*d)^2*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c
```

```
+ a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(7*d^(11/3)*(a + b*x)^(1
/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3)
+ 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))) - (15*2^(2/3)*3^(1/
4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c - a*d)^(8/3)*((a + b*x)*(c + d*x))^(1/3)*
Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1
/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/
3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*
b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
 + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*d^(11/3)*(a + b*x)^(1/3)*(c + d*x)^(1
/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d
)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d
 + b*(c + 2*d*x))^2]) + (20*2^(1/6)*3^(3/4)*b^(1/3)*(b*c - a*d)^(8/3)*((a +
 b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3)
 - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) +
2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c
- a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*d^(11/3)*(a +
b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((
b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((
1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
```

```
<= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{8/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} + \frac{(8b)\int \frac{(a+bx)^{5/3}}{\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} - \frac{(40b(bc-ad))\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx}{7d^2}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{\left(20b(bc-ad)^2\right)\int \frac{}{\sqrt[3]{}}}{7d^3}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{\left(20b(bc-ad)^2\sqrt[3]{(a+}\right.}{7d}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{\left(60b(bc-ad)^2\sqrt[3]{(a+}\right.}{}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{\left(30\sqrt[3]{2}b^{2/3}(bc-ad)^2\right.}{}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{30\,2^{2/3}\sqrt[3]{b}(b}{7d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+}}$$

Mathematica [C] time = 0.0579846, size = 73, normalized size = 0.05

$$\frac{3(a+bx)^{11/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}{}_2F_1\left(\frac{4}{3},\frac{11}{3};\frac{14}{3};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(8/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(11/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3
, 11/3, 14/3, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(4/3))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{8}{3}}(dx+c)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(8/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(8/3)/(d*x+c)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{8}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(8/3)/(d*x + c)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(2/3)*(d*x + c)^(2/3)/(d^2*x^2
 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(8/3)/(d*x+c)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{8}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(8/3)/(d*x + c)^(4/3), x)
```

3.1623 $\int \frac{(a+bx)^{5/3}}{(c+dx)^{4/3}}\, dx$

Optimal. Leaf size=1317

result too large to display

```
[Out] (-3*(a + b*x)^(5/3))/(d*(c + d*x)^(1/3)) + (15*b*(a + b*x)^(2/3)*(c + d*x)^
(2/3))/(4*d^2) - (15*b^(1/3)*(b*c - a*d)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(
b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2*2^(1/3)*d^(8/3)*(
a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c -
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))) + (15*3^
(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c - a*d)^(5/3)*((a + b*x)*(c + d*x))^(1/
3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/
3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d
^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^
(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/
3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*d^(8/3)*(a + b*x)^(1/3)*(c
 + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2
/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]
*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (5*3^(3/4)*b^(1/3)*(b*c - a*d)^(5/3)*((a
+ b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) +
2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3
) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) +
 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c
 - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*d^(8/3)
*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2
/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 1.99805, antiderivative size = 1317, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {47, 50, 62, 623, 303, 218, 1877}

$$\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\cdots}{\left((1-\cdots\right.}}\cdots}{4\sqrt[3]{2}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(5/3))/(d*(c + d*x)^(1/3)) + (15*b*(a + b*x)^(2/3)*(c + d*x)^
(2/3))/(4*d^2) - (15*b^(1/3)*(b*c - a*d)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(
b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2*2^(1/3)*d^(8/3)*(
a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c -
```

```
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))) + (15*3^
(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c - a*d)^(5/3)*((a + b*x)*(c + d*x))^(1/
3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/
3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d
^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^
(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/
3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*d^(8/3)*(a + b*x)^(1/3)*(c
 + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2
/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]
*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (5*3^(3/4)*b^(1/3)*(b*c - a*d)^(5/3)*((a
+ b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) +
2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3
) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) +
 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c
 - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*d^(8/3)
*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2
/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

<u>Rule 47</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 50</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 62</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

<u>Rule 623</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}}+\frac{(5b)\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}}+\frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2}-\frac{(5b(bc-ad))\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{2d^2}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}}+\frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2}-\frac{\left(5b(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}\,dx}{2d^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}}+\frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2}-\frac{\left(15b(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\,S}{2d^2\sqrt[3]{a+bx}\sqrt[3]{c+}}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}}+\frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2}-\frac{\left(15b^{2/3}(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right.}{2\ 2^{2/3}d^{7/3}\sqrt[3]{a+bx}}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}}+\frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2}-\frac{15\sqrt[3]{b}(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad}}{2\sqrt[3]{2}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(b\right.}$$

Mathematica [C] time = 0.048206, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{8/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},\frac{8}{3};\frac{11}{3};\frac{d(a+bx)}{ad-bc}\right)}{8b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(8/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3,
 8/3, 11/3, (d*(a + b*x))/(-(b*c) + a*d)])/(8*b*(c + d*x)^(4/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{3}} (dx+c)^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(5/3)/(d*x+c)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{2}{3}}}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/3)*(d*x + c)^(2/3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{5}{3}}}{(c+dx)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(5/3)/(c + d*x)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(4/3), x)
```

3.1624 $\int \frac{(a+bx)^{2/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1279

result too large to display

```
[Out] (-3*(a + b*x)^(2/3))/(d*(c + d*x)^(1/3)) + (3*2^(2/3)*b^(1/3)*((a + b*x)*(c
 + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
/(d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[
3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)
)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c +
 d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)
*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*
b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/
3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
 + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(1/3)*d^(5/3)*(a + b*x)^
(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2*2^(1/6)*3^(3/4)*b^(1/3)*(b*c -
 a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c
 - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[(
(b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c
 + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1
 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x
))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^
(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/
(d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
 a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 1.54122, antiderivative size = 1279, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {47, 62, 623, 303, 218, 1877}

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4}\cdots}{\cdots}}\cdots}{\sqrt[3]{2}d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc\cdots\right)}{\left(\left(1+\sqrt{3}\right)(bc-\cdots\right)}}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(2/3))/(d*(c + d*x)^(1/3)) + (3*2^(2/3)*b^(1/3)*((a + b*x)*(c
 + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
/(d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[
3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)
)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c +
```

```
d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)
*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*
b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/
3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
 + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(1/3)*d^(5/3)*(a + b*x)^
(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2*2^(1/6)*3^(3/4)*b^(1/3)*(b*c -
 a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c
 - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[(
(b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c
 + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1
 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x
))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^
(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/
(d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
 a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{(2b)\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{\left(2b\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}\,dx}{d\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{\left(6b\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-4abcd+(bc+ad)^2+4bdx^3}}\,dx, x, \sqrt[3]{(a\right.}{d\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)}$$

$$= -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{\left(3\sqrt[3]{2}b^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}x}{\sqrt{-4abcd+(bc+ad)^2+4bdx^3}}\,d\right.}{d^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)}$$

$$= -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{3\,2^{2/3}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c}\right.}$$

Mathematica [C] time = 0.0406978, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{5/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},\frac{5}{3};\frac{8}{3};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3,
 5/3, 8/3, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(4/3))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{2}{3}}(dx+c)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(2/3)/(d*x+c)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{2}{3}}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(2/3)/(c + d*x)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(4/3), x)
```

3.1625 $\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1298

result too large to display

```
[Out] (3*(a + b*x)^(2/3))/((b*c - a*d)*(c + d*x)^(1/3)) - (3*b^(1/3)*((a + b*x)*(
c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]
)/(2^(1/3)*d^(2/3)*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d +
 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*((a + b*x)*(
c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*
b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2
/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/
3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^
(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/
3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*2^(1/3)*d^(2/3)*(b*c
- a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b
*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (2^(1/6)*3
^(3/4)*b^(1/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((
b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqr
t[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)
*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/
((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2
/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]
])/(d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d +
2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/
3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))
^2])
```

Rubi [A] time = 1.52365, antiderivative size = 1298, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {51, 62, 623, 303, 218, 1877}

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{\cdots}}{\left((1+\cdots\right.}}}{2\sqrt[3]{2}d^{2/3}\sqrt[3]{bc-ad}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(\cdots\right.}{\left((1+\sqrt{3})(bc\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(a + b*x)^(2/3))/((b*c - a*d)*(c + d*x)^(1/3)) - (3*b^(1/3)*((a + b*x)*(
c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]
)/(2^(1/3)*d^(2/3)*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d +
 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
```

```
b*x)*(c + d*x))^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*((a + b*x)*(
c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*
b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2
/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/
3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^
(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/
3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*2^(1/3)*d^(2/3)*(b*c
- a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b
*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (2^(1/6)*3
^(3/4)*b^(1/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((
b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqr
t[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)
*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/
((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2
/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]
])/(d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d +
2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/
3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))
^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{4/3}}\,dx = \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{b\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{bc-ad}$$

$$= \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{\left(b\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}\,dx}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{\left(3b\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-4abcd+(bc+ad)^2+4}}\right.}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx}$$

$$= \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{\left(3b^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}}{\sqrt{-4abcd+(bc+a}}\right.}{2^{2/3}\sqrt[3]{d}(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad}$$

$$= \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{3\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+}}{\sqrt[3]{2}d^{2/3}(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)\right.}$$

Mathematica [C] time = 0.0346799, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{2/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};\frac{d(a+bx)}{ad-bc}\right)}{2b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(a + b*x)^(2/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[2/3,
 4/3, 5/3, (d*(a + b*x))/(-(b*c) + a*d)])/(2*b*(c + d*x)^(4/3))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{bx+a}}(dx+c)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

```
[Out] int(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(4/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{bd^2x^3+ac^2+\left(2\,bcd+ad^2\right)x^2+\left(bc^2+2\,acd\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*
d^2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(1/3)*(c + d*x)**(4/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(4/3)), x)
```

3.1626 $\int \frac{1}{(a+bx)^{4/3}(c+dx)^{4/3}} dx$

Optimal. Leaf size=1327

result too large to display

```
[Out] -3/((b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3)) - (6*d*(a + b*x)^(2/3))/((
b*c - a*d)^2*(c + d*x)^(1/3)) + (3*2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/((
b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sq
rt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1
/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^
(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^
(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3
)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)
*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1
- Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(1/3)*(b*c - a*d)^(4/3)*(a + b*
x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*
c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1
+ Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2*2^(1/6)*3^(3/4)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c -
a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x)
)^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/((b*c -
 a*d)^(4/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*
c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*(
(a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 1.95846, antiderivative size = 1327, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {51, 62, 623, 303, 218, 1877}

$$-\frac{6(a+bx)^{2/3}d}{(bc-ad)^2\sqrt[3]{c+dx}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right.}{\sqrt[3]{2}(bc-ad)^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(4/3)*(c + d*x)^(4/3)),x]
```


```
[Out] -3/((b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3)) - (6*d*(a + b*x)^(2/3))/((
b*c - a*d)^2*(c + d*x)^(1/3)) + (3*2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/((
b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sq
rt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1
```

```
/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^
(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^
(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3
)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)
*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1
- Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(1/3)*(b*c - a*d)^(4/3)*(a + b*
x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*
c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1
+ Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2*2^(1/6)*3^(3/4)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c -
a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x)
)^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/((b*c -
 a*d)^(4/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*
c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*(
(a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
```

```
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}(c+dx)^{4/3}}\,dx = -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}} - \frac{(2d)\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{4/3}}\,dx}{bc-ad}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{(2bd)\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{(bc-ad)^2}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{\left(2bd\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+}}}{(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{\left(6bd\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bd}\right.}{(bc-ad)^2}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{\left(3\sqrt[3]{2}b^{2/3}d^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+}\right.}{(bc-}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{3\,2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c-}}{(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2}$$

Mathematica [C] time = 0.0351939, size = 71, normalized size = 0.05

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}{}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{2}{3};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[3]{a+bx}(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[-1/3, 4/3, 2/3, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/3)*(c + d*x)^(4/3))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{4}{3}} (dx+c)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

```
[Out] int(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{4}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(4/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{b^2d^2x^4+a^2c^2+2\left(b^2cd+abd^2\right)x^3+\left(b^2c^2+4\,abcd+a^2d^2\right)x^2+2\left(abc^2+a^2cd\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*
d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c
*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{4}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(4/3)*(c + d*x)**(4/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{4}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(4/3)), x)
```

3.1627 $\int \frac{1}{(a+bx)^{7/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1370

result too large to display

```
[Out] -3/(4*(b*c - a*d)*(a + b*x)^(4/3)*(c + d*x)^(1/3)) + (15*d)/(4*(b*c - a*d)^
2*(a + b*x)^(1/3)*(c + d*x)^(1/3)) + (15*d^2*(a + b*x)^(2/3))/(2*(b*c - a*d
)^3*(c + d*x)^(1/3)) - (15*b^(1/3)*d^(4/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt
[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2*2^(1/3)*(b*c -
a*d)^3*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])
*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))
+ (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*d^(4/3)*((a + b*x)*(c + d*x))^(1/3)
*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)
*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(
1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2
/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/
3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*(b*c - a*d)^(7/3)*(a + b*x)^
(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (5*3^(3/4)*b^(1/3)*d^(4/3)*((a +
b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3)
- 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2
*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)
*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*(b*c - a*
d)^(7/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
 a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 2.41796, antiderivative size = 1370, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {51, 62, 623, 303, 218, 1877}

$$\frac{15(a+bx)^{2/3}d^2}{2(bc-ad)^3\sqrt[3]{c+dx}}+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+\right.}{4\sqrt[3]{2}(bc-ad)^{7/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/3)*(c + d*x)^(4/3)),x]
```


```
[Out] -3/(4*(b*c - a*d)*(a + b*x)^(4/3)*(c + d*x)^(1/3)) + (15*d)/(4*(b*c - a*d)^
2*(a + b*x)^(1/3)*(c + d*x)^(1/3)) + (15*d^2*(a + b*x)^(2/3))/(2*(b*c - a*d
)^3*(c + d*x)^(1/3)) - (15*b^(1/3)*d^(4/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt
[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2*2^(1/3)*(b*c -
```

```
a*d)^3*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])
*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))
+ (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*d^(4/3)*((a + b*x)*(c + d*x))^(1/3)
*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)
*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(
1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2
/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/
3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*(b*c - a*d)^(7/3)*(a + b*x)^
(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (5*3^(3/4)*b^(1/3)*d^(4/3)*((a +
b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3)
- 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2
*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)
*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*(b*c - a*
d)^(7/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
 a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/3}(c+dx)^{4/3}}\,dx = -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} - \frac{(5d)\int \frac{1}{(a+bx)^{4/3}(c+dx)^{4/3}}\,dx}{4(bc-ad)}$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}} + \frac{\left(5d^2\right)\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{4/3}}\,d}{2(bc-ad)^2}$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}} + \frac{15d^2(a+bx)^{2/3}}{2(bc-ad)^3\sqrt[3]{c+dx}} -$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}} + \frac{15d^2(a+bx)^{2/3}}{2(bc-ad)^3\sqrt[3]{c+dx}} -$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}} + \frac{15d^2(a+bx)^{2/3}}{2(bc-ad)^3\sqrt[3]{c+dx}} -$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}} + \frac{15d^2(a+bx)^{2/3}}{2(bc-ad)^3\sqrt[3]{c+dx}} -$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}} + \frac{15d^2(a+bx)^{2/3}}{2(bc-ad)^3\sqrt[3]{c+dx}} -$$

Mathematica [C] time = 0.0379122, size = 73, normalized size = 0.05

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}\,{}_2F_1\left(-\frac{4}{3},\frac{4}{3};-\frac{1}{3};\frac{d(a+bx)}{ad-bc}\right)}{4b(a+bx)^{4/3}(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(4/3)),x]
```

[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[-4/3, 4/3, -1/3, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*(a + b*x)^(4/3)*(c + d*x)^(4/3))

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{7}{3}} (dx+c)^{-\frac{4}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x)

[Out] int(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{3}} (dx+c)^{\frac{4}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="maxima")

[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(4/3)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{2}{3}} (dx+c)^{\frac{2}{3}}}{b^3d^2x^5 + a^3c^2 + \left(2\,b^3cd + 3\,ab^2d^2\right)x^4 + \left(b^3c^2 + 6\,ab^2cd + 3\,a^2bd^2\right)x^3 + \left(3\,ab^2c^2 + 6\,a^2bcd + a^3d^2\right)x^2 + \left(3\,a^2bc^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="fricas")

[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*d + 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{7}{3}} (c+dx)^{\frac{4}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(7/3)/(d*x+c)**(4/3),x)

[Out] Integral(1/((a + b*x)**(7/3)*(c + d*x)**(4/3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(4/3)), x)
```

3.1628 $\int \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}}\,dx$

Optimal. Leaf size=77

$$\sqrt[3]{x-1}(x+1)^{2/3}+\frac{1}{3}\log(x-1)+\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}-1\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (-1 + x)^(1/3)*(1 + x)^(2/3) + (2*ArcTan[1/Sqrt[3] + (2*(1 + x)^(1/3))/(Sqr
t[3]*(-1 + x)^(1/3))])/Sqrt[3] + Log[-1 + x]/3 + Log[-1 + (1 + x)^(1/3)/(-1
 + x)^(1/3)]
```

Rubi [A] time = 0.0138222, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {50, 59}

$$\sqrt[3]{x-1}(x+1)^{2/3}+\frac{1}{3}\log(x-1)+\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}-1\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)^(1/3)/(1 + x)^(1/3),x]
```

```
[Out] (-1 + x)^(1/3)*(1 + x)^(2/3) + (2*ArcTan[1/Sqrt[3] + (2*(1 + x)^(1/3))/(Sqr
t[3]*(-1 + x)^(1/3))])/Sqrt[3] + Log[-1 + x]/3 + Log[-1 + (1 + x)^(1/3)/(-1
 + x)^(1/3)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}}\,dx &= \sqrt[3]{-1+x}(1+x)^{2/3}-\frac{2}{3}\int \frac{1}{(-1+x)^{2/3}\sqrt[3]{1+x}}\,dx\\
&= \sqrt[3]{-1+x}(1+x)^{2/3}+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{-1+x}}\right)}{\sqrt{3}}+\frac{1}{3}\log(-1+x)+\log\left(-1+\frac{\sqrt[3]{1+x}}{\sqrt[3]{-1+x}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0187196, size = 48, normalized size = 0.62

$$\frac{3\left(\frac{x-1}{x+1}\right)^{4/3}(x+1)^{4/3}\,{}_2F_1\left(\frac{1}{3},\frac{4}{3};\frac{7}{3};\frac{1-x}{2}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)^(1/3)/(1 + x)^(1/3),x]
```

```
[Out] (3*((-1 + x)/(1 + x))^(4/3)*(1 + x)^(4/3)*Hypergeometric2F1[1/3, 4/3, 7/3,
(1 - x)/2])/(4*2^(1/3))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \sqrt[3]{-1+x}\,\frac{1}{\sqrt[3]{1+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/3)/(1+x)^(1/3),x)
```

```
[Out] int((-1+x)^(1/3)/(1+x)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)^{\frac{1}{3}}}{(x+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)^(1/3)/(x + 1)^(1/3), x)
```

Fricas [A] time = 2.0401, size = 359, normalized size = 4.66

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(x+1)+2\sqrt{3}(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}}{3(x+1)}\right)+(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}-\frac{1}{3}\log\left(\frac{(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}+(x+1)^{\frac{1}{3}}(x-1)^{\frac{2}{3}}+}{x+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*(sqrt(3)*(x + 1) + 2*sqrt(3)*(x + 1)^(2/3)*(x - 1)^
(1/3))/(x + 1)) + (x + 1)^(2/3)*(x - 1)^(1/3) - 1/3*log(((x + 1)^(2/3)*(x -
 1)^(1/3) + (x + 1)^(1/3)*(x - 1)^(2/3) + x + 1)/(x + 1)) + 2/3*log(((x + 1
)^(2/3)*(x - 1)^(1/3) - x - 1)/(x + 1))
```

Sympy [C] time = 2.48307, size = 39, normalized size = 0.51

$$\frac{2^{\frac{2}{3}}(x-1)^{\frac{4}{3}}\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{(x-1)e^{i\pi}}{2}\right)}{2\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/3)/(1+x)**(1/3),x)
```

```
[Out] 2**(2/3)*(x - 1)**(4/3)*gamma(4/3)*hyper((1/3, 4/3), (7/3,), (x - 1)*exp_po
lar(I*pi)/2)/(2*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)^{\frac{1}{3}}}{(x+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)^(1/3)/(x + 1)^(1/3), x)
```

3.1629 $\int (a+bx)^{3/2}\sqrt[4]{c+dx}\,dx$

Optimal. Leaf size=185

$$\frac{16(bc-ad)^{13/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{77b^{5/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)^2}{77bd^2}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}(bc-\ldots}{77bd}$$

```
[Out] (-8*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/4))/(77*b*d^2) + (4*(b*c - a*d
)*(a + b*x)^(3/2)*(c + d*x)^(1/4))/(77*b*d) + (4*(a + b*x)^(5/2)*(c + d*x)^
(1/4))/(11*b) + (16*(b*c - a*d)^(13/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(77*b^(5
/4)*d^3*Sqrt[a + b*x])
```

Rubi [A] time = 0.212465, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 224, 221}

$$\frac{16(bc-ad)^{13/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{77b^{5/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)^2}{77bd^2}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}(bc-ad)}{77bd}+\frac{4\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(1/4),x]
```

```
[Out] (-8*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/4))/(77*b*d^2) + (4*(b*c - a*d
)*(a + b*x)^(3/2)*(c + d*x)^(1/4))/(77*b*d) + (4*(a + b*x)^(5/2)*(c + d*x)^
(1/4))/(11*b) + (16*(b*c - a*d)^(13/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(77*b^(5
/4)*d^3*Sqrt[a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt[4]{c+dx}\,dx = \frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b} + \frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}}\,dx}{11b}$$

$$= \frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd} + \frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b} - \frac{\left(6(bc-ad)^2\right)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx}{77bd}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{77bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd} + \frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b} + \frac{(4(bc}{}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{77bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd} + \frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b} + \frac{(16(b}{}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{77bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd} + \frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b} + \frac{(16(b}{}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{77bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd} + \frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b} + \frac{16(bc}{}$$

Mathematica [C] time = 0.0363875, size = 73, normalized size = 0.39

$$\frac{2(a+bx)^{5/2}\sqrt[4]{c+dx}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(1/4)*Hypergeometric2F1[-1/4, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}\sqrt[4]{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(3/2)*(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^{\frac{3}{2}}\sqrt[4]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/4), x)
```

3.1630 $\int \sqrt{a+bx}\sqrt[4]{c+dx}\,dx$

Optimal. Leaf size=147

$$-\frac{8(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{21b^{5/4}d^2\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)}{21bd}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b}$$

```
[Out] (4*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4))/(21*b*d) + (4*(a + b*x)^(3/2)
*(c + d*x)^(1/4))/(7*b) - (8*(b*c - a*d)^(9/4)*Sqrt[-((d*(a + b*x))/(b*c -
a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/
(21*b^(5/4)*d^2*Sqrt[a + b*x])
```

Rubi [A] time = 0.0888444, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 224, 221}

$$-\frac{8(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{21b^{5/4}d^2\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)}{21bd}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(1/4),x]
```

```
[Out] (4*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4))/(21*b*d) + (4*(a + b*x)^(3/2)
*(c + d*x)^(1/4))/(7*b) - (8*(b*c - a*d)^(9/4)*Sqrt[-((d*(a + b*x))/(b*c -
a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/
(21*b^(5/4)*d^2*Sqrt[a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt[4]{c+dx}\,dx = \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx}{7b}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{21bd} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} - \frac{\left(2(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{21bd}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{21bd} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} - \frac{\left(8(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x\right.}{21bd^2}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{21bd} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} - \frac{\left(8(bc-ad)^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{b}{(a-}}}\right.}{21bd^2\sqrt{a+bx}}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{21bd} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} - \frac{8(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+}}{\sqrt[4]{bc-a}}\right.\right.}{21b^{5/4}d^2\sqrt{a+bx}}$$

Mathematica [C] time = 0.0263748, size = 73, normalized size = 0.5

$$\frac{2(a+bx)^{3/2}\sqrt[4]{c+dx}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(1/4)*Hypergeometric2F1[-1/4, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\sqrt[4]{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}(dx+c)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx+a}(dx+c)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+bx}\sqrt[4]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/4),x)
```

```
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}(dx+c)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/4), x)
```

3.1631 $\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=111

$$\frac{4(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{3b^{5/4}d\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\sqrt[4]{c+dx}}{3b}$$

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(1/4))/(3*b) + (4*(b*c - a*d)^(5/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(3*b^(5/4)*d*Sqrt[a + b*x])
```

Rubi [A] time = 0.0673849, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 224, 221}

$$\frac{4(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}d\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\sqrt[4]{c+dx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/4)/Sqrt[a + b*x],x]
```

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(1/4))/(3*b) + (4*(b*c - a*d)^(5/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(3*b^(5/4)*d*Sqrt[a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

b/a] && GtQ[a, 0]

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx &= \frac{4\sqrt{a+bx}\sqrt[4]{c+dx}}{3b}+\frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{3b}\\
&= \frac{4\sqrt{a+bx}\sqrt[4]{c+dx}}{3b}+\frac{(4(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3bd}\\
&= \frac{4\sqrt{a+bx}\sqrt[4]{c+dx}}{3b}+\frac{\left(4(bc-ad)\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3bd\sqrt{a+bx}}\\
&= \frac{4\sqrt{a+bx}\sqrt[4]{c+dx}}{3b}+\frac{4(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}d\sqrt{a+bx}}
\end{aligned}
$$

Mathematica [C] time = 0.0228263, size = 71, normalized size = 0.64

$$
\frac{2\sqrt{a+bx}\sqrt[4]{c+dx}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/4)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, (d*(a +
b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$
\int \sqrt[4]{dx+c}\frac{1}{\sqrt{bx+a}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/4)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(1/4)/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{(dx+c)^{\frac{1}{4}}}{\sqrt{bx+a}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/4)/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^{\frac{1}{4}}}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(1/4)/sqrt(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/4)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(1/4)/sqrt(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{4}}}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/4)/sqrt(b*x + a), x)
```

3.1632 $\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=104

$$\frac{2\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{b^{5/4}\sqrt{a+bx}}-\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}$$

```
[Out] (-2*(c + d*x)^(1/4))/(b*Sqrt[a + b*x]) + (2*(b*c - a*d)^(1/4)*Sqrt[-((d*(a
+ b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d
)^(1/4)], -1])/(b^(5/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.0642513, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {47, 63, 224, 221}

$$\frac{2\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{5/4}\sqrt{a+bx}}-\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/4)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4))/(b*Sqrt[a + b*x]) + (2*(b*c - a*d)^(1/4)*Sqrt[-((d*(a
+ b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d
)^(1/4)], -1])/(b^(5/4)*Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/2}}\,dx &= -\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{d\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{2b}\\
&= -\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{b}\\
&= -\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{\left(2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{b\sqrt{a+bx}}\\
&= -\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{2\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{5/4}\sqrt{a+bx}}
\end{aligned}
$$

Mathematica [C] time = 0.0229519, size = 71, normalized size = 0.68

$$
-\frac{2\sqrt[4]{c+dx}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/4)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$
\int \sqrt[4]{dx+c}\,(bx+a)^{-\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/4)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(1/4)/(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/4)/(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{4}}}{b^2x^2+2\,abx+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/4)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(1/4)/(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/4)/(b*x + a)^(3/2), x)
```

3.1633 $\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{3b^{5/4}\sqrt{a+bx}(bc-ad)^{3/4}}-\frac{d\sqrt[4]{c+dx}}{3b\sqrt{a+bx}(bc-ad)}-\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}}$$

```
[Out] (-2*(c + d*x)^(1/4))/(3*b*(a + b*x)^(3/2)) - (d*(c + d*x)^(1/4))/(3*b*(b*c
- a*d)*Sqrt[a + b*x]) - (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(5/4)*(b*c - a*
d)^(3/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.0942337, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 51, 63, 224, 221}

$$-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}\sqrt{a+bx}(bc-ad)^{3/4}}-\frac{d\sqrt[4]{c+dx}}{3b\sqrt{a+bx}(bc-ad)}-\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4))/(3*b*(a + b*x)^(3/2)) - (d*(c + d*x)^(1/4))/(3*b*(b*c
- a*d)*Sqrt[a + b*x]) - (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(5/4)*(b*c - a*
d)^(3/4)*Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/2}}\,dx &= -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}}+\frac{d\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx}{6b}\\
&= -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}}-\frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}}-\frac{d^2\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{12b(bc-ad)}\\
&= -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}}-\frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}}-\frac{d\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3b(bc-ad)}\\
&= -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}}-\frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}}-\frac{\left(d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3b(bc-ad)\sqrt{a+bx}}\\
&= -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}}-\frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}}-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}(bc-ad)^{3/4}\sqrt{a+bx}}
\end{aligned}
$$

Mathematica [C] time = 0.0240671, size = 73, normalized size = 0.5

$$
-\frac{2\sqrt[4]{c+dx}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$
\int \sqrt[4]{dx+c}\,(bx+a)^{-\frac{5}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/4)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(1/4)/(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/4)/(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{4}}}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/4)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(1/4)/(a + b*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/4)/(b*x + a)^(5/2), x)
```

3.1634 $\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=185

$$\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{6b^{5/4}\sqrt{a+bx}(bc-ad)^{7/4}}+\frac{d^2\sqrt[4]{c+dx}}{6b\sqrt{a+bx}(bc-ad)^2}-\frac{d\sqrt[4]{c+dx}}{15b(a+bx)^{3/2}(bc-ad)}-\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}}$$

```
[Out] (-2*(c + d*x)^(1/4))/(5*b*(a + b*x)^(5/2)) - (d*(c + d*x)^(1/4))/(15*b*(b*c
 - a*d)*(a + b*x)^(3/2)) + (d^2*(c + d*x)^(1/4))/(6*b*(b*c - a*d)^2*Sqrt[a
+ b*x]) + (d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)
*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(6*b^(5/4)*(b*c - a*d)^(7/4)*Sqr
t[a + b*x])
```

Rubi [A] time = 0.128342, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 51, 63, 224, 221}

$$\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{6b^{5/4}\sqrt{a+bx}(bc-ad)^{7/4}}+\frac{d^2\sqrt[4]{c+dx}}{6b\sqrt{a+bx}(bc-ad)^2}-\frac{d\sqrt[4]{c+dx}}{15b(a+bx)^{3/2}(bc-ad)}-\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/4)/(a + b*x)^(7/2),x]
```


```
[Out] (-2*(c + d*x)^(1/4))/(5*b*(a + b*x)^(5/2)) - (d*(c + d*x)^(1/4))/(15*b*(b*c
 - a*d)*(a + b*x)^(3/2)) + (d^2*(c + d*x)^(1/4))/(6*b*(b*c - a*d)^2*Sqrt[a
+ b*x]) + (d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)
*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(6*b^(5/4)*(b*c - a*d)^(7/4)*Sqr
t[a + b*x])
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{7/2}}\,dx = -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}}+\frac{d\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/4}}\,dx}{10b}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}}-\frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}}-\frac{d^2\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx}{12b(bc-ad)}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}}-\frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}}+\frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}}+\frac{d^3\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{24b(bc-ad)^2}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}}-\frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}}+\frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}}+\frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{\right.}{6b(bc-ad)^2}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}}-\frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}}+\frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}}+\frac{\left(d^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{b}{(a-}}}\right.}{6b(bc-ad)^2\sqrt{a}}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}}-\frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}}+\frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}}+\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+d}}{\sqrt[4]{bc-ad}}\right.\right.}{6b^{5/4}(bc-ad)^{7/4}\sqrt{a+b}}$$

Mathematica [C] time = 0.0242917, size = 73, normalized size = 0.39

$$-\frac{2\sqrt[4]{c+dx}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{4};-\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \sqrt[4]{dx+c}\,(bx+a)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/4)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(1/4)/(b*x+a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/4)/(b*x + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{4}}}{b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/4)/(b*x+a)**(7/2),x)
```

```
[Out] Integral((c + d*x)**(1/4)/(a + b*x)**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/4)/(b*x + a)^(7/2), x)
```

3.1635 $\int (a+bx)^{3/2}(c+dx)^{3/4}\,dx$

Optimal. Leaf size=270

$$-\frac{16(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{65b^{7/4}d^3\sqrt{a+bx}}+\frac{16(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{65b^{7/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a\cdots}}{}$$

```
[Out] (-8*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(3/4))/(65*b*d^2) + (4*(b*c - a*d
)*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(39*b*d) + (4*(a + b*x)^(5/2)*(c + d*x)^
(3/4))/(13*b) + (16*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(65*b^(7
/4)*d^3*Sqrt[a + b*x]) - (16*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(65*b^(7/4)*d^3*Sqrt[a + b*x])
```

Rubi [A] time = 0.364743, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {50, 63, 307, 224, 221, 1200, 1199, 424}

$$-\frac{16(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{65b^{7/4}d^3\sqrt{a+bx}}+\frac{16(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{65b^{7/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}(c+\cdots}{6\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(3/4),x]
```


```
[Out] (-8*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(3/4))/(65*b*d^2) + (4*(b*c - a*d
)*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(39*b*d) + (4*(a + b*x)^(5/2)*(c + d*x)^
(3/4))/(13*b) + (16*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(65*b^(7
/4)*d^3*Sqrt[a + b*x]) - (16*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(65*b^(7/4)*d^3*Sqrt[a + b*x])
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
```

```
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{3/4}\,dx = \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b} + \frac{(3(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}dx}{13b}$$

$$= \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b} - \frac{\left(2(bc-ad)^2\right)\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}dx}{13bd}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b}$$

Mathematica [C] time = 0.0359712, size = 73, normalized size = 0.27

$$\frac{2(a+bx)^{5/2}(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(3/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(3/4)*Hypergeometric2F1[-3/4, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(3/4))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(3/2)*(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(3/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(3/4), x)
```

3.1636 $\int \sqrt{a+bx}(c+dx)^{3/4}\,dx$

Optimal. Leaf size=232

$$\frac{8(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{15b^{7/4}d^2\sqrt{a+bx}}-\frac{8(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{7/4}d^2\sqrt{a+bx}}+4\sqrt{a+bx}$$

```
[Out] (4*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4))/(15*b*d) + (4*(a + b*x)^(3/2)
*(c + d*x)^(3/4))/(9*b) - (8*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(15*b^(7/4)*d^2*Sqrt[a + b*x]) + (8*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x)
)/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4
)], -1])/(15*b^(7/4)*d^2*Sqrt[a + b*x])
```

Rubi [A] time = 0.245906, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {50, 63, 307, 224, 221, 1200, 1199, 424}

$$\frac{8(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{7/4}d^2\sqrt{a+bx}}-\frac{8(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{7/4}d^2\sqrt{a+bx}}+\frac{4\sqrt{a+bx}(c+dx}{15bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(3/4),x]
```

```
[Out] (4*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4))/(15*b*d) + (4*(a + b*x)^(3/2)
*(c + d*x)^(3/4))/(9*b) - (8*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(15*b^(7/4)*d^2*Sqrt[a + b*x]) + (8*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x)
)/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4
)], -1])/(15*b^(7/4)*d^2*Sqrt[a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}(c+dx)^{3/4}\,dx = \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}dx}{3b}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} - \frac{\left(2(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}dx}{15bd}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} - \frac{\left(8(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}d\right.}{15bd^2}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} + \frac{\left(8(bc-ad)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\right.}{15b^{3/2}d^2}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} + \frac{\left(8(bc-ad)^{5/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int -\right.}{15b^{3/2}d^2\sqrt{a+}}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} + \frac{8(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\right.\right.}{15b^{7/4}d^2\sqrt{a+bx}}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} - \frac{8(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\right.\right.}{15b^{7/4}d^2\sqrt{a+bx}}$$

Mathematica [C] time = 0.0265635, size = 73, normalized size = 0.31

$$\frac{2(a+bx)^{3/2}(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(3/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(3/4)*Hypergeometric2F1[-3/4, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(3/4))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\,(dx+c)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}(dx+c)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{bx+a}(dx+c)^{\frac{3}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+bx}\,(c+dx)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(3/4),x)
```

```
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(3/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}(dx+c)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(3/4), x)
```

3.1637 $\int \frac{(c+dx)^{3/4}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=196

$$-\frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{5b^{7/4}d\sqrt{a+bx}}+\frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{7/4}d\sqrt{a+bx}}+\frac{4\sqrt{a+}}{}$$

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(3/4))/(5*b) + (12*(b*c - a*d)^(7/4)*Sqrt[-((d*(
a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a
*d)^(1/4)], -1])/(5*b^(7/4)*d*Sqrt[a + b*x]) - (12*(b*c - a*d)^(7/4)*Sqrt[-
((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*
c - a*d)^(1/4)], -1])/(5*b^(7/4)*d*Sqrt[a + b*x])
```

Rubi [A] time = 0.221909, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {50, 63, 307, 224, 221, 1200, 1199, 424}

$$-\frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{7/4}d\sqrt{a+bx}}+\frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{7/4}d\sqrt{a+bx}}+\frac{4\sqrt{a+bx}(c+}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/4)/Sqrt[a + b*x],x]
```

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(3/4))/(5*b) + (12*(b*c - a*d)^(7/4)*Sqrt[-((d*(
a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a
*d)^(1/4)], -1])/(5*b^(7/4)*d*Sqrt[a + b*x]) - (12*(b*c - a*d)^(7/4)*Sqrt[-
((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*
c - a*d)^(1/4)], -1])/(5*b^(7/4)*d*Sqrt[a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/4}}{\sqrt{a+bx}}\,dx = \frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5b} + \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{5b}$$

$$= \frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5b} + \frac{(12(bc-ad))\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5bd}$$

$$= \frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5b} - \frac{\left(12(bc-ad)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5b^{3/2}d} + \frac{\left(12(bc-ad)^{3/2}\right)\operatorname{Sul}}{}$$

$$= \frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5b} - \frac{\left(12(bc-ad)^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5b^{3/2}d\sqrt{a+bx}} + \frac{\left(12(bc-a\right.}{}$$

$$= \frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5b} - \frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{7/4}d\sqrt{a+bx}} + \frac{\left(12(bc-ad)^{3/2}\sqrt{\frac{d(a+bx}{-bc+a}}\right.}{}$$

$$= \frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5b} + \frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{7/4}d\sqrt{a+bx}} - \frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+b}{bc-a}}}{5b^{7/4}}$$

Mathematica [C] time = 0.0225947, size = 71, normalized size = 0.36

$$\frac{2\sqrt{a+bx}(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/4)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, (d*(a +
b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(3/4))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{4}}\frac{1}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/4)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(3/4)/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{3}{4}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(3/4)/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx+c)^{\frac{3}{4}}}{\sqrt{bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(3/4)/sqrt(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{3}{4}}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/4)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(3/4)/sqrt(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{3}{4}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/4)/sqrt(b*x + a), x)
```

3.1638 $\int \frac{(c+dx)^{3/4}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{b^{7/4}\sqrt{a+bx}}+\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}}-\frac{2(c+dx)^{3/4}}{b\sqrt{a+b}}$$

```
[Out] (-2*(c + d*x)^(3/4))/(b*Sqrt[a + b*x]) + (6*(b*c - a*d)^(3/4)*Sqrt[-((d*(a
+ b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d
)^(1/4)], -1])/(b^(7/4)*Sqrt[a + b*x]) - (6*(b*c - a*d)^(3/4)*Sqrt[-((d*(a
+ b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d
)^(1/4)], -1])/(b^(7/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.214829, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {47, 63, 307, 224, 221, 1200, 1199, 424}

$$-\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}}+\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}}-\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/4)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4))/(b*Sqrt[a + b*x]) + (6*(b*c - a*d)^(3/4)*Sqrt[-((d*(a
+ b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d
)^(1/4)], -1])/(b^(7/4)*Sqrt[a + b*x]) - (6*(b*c - a*d)^(3/4)*Sqrt[-((d*(a
+ b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d
)^(1/4)], -1])/(b^(7/4)*Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/4}}{(a+bx)^{3/2}}\,dx = -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}}+\frac{(3d)\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{2b}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}}+\frac{6\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{b}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}}-\frac{\left(6\sqrt{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{b^{3/2}}+\frac{\left(6\sqrt{bc-ad}\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{b}\cdots}{\sqrt{bc-\cdots}}}{\sqrt{a-\frac{bc}{d}+\cdots}}\cdots\right)}{b^{3/2}}\cdots$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}}-\frac{\left(6\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{b^{3/2}\sqrt{a+bx}}+\frac{\left(6\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\cdots\right.}{\cdots}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}}-\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}}+\frac{\left(6\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\cdots\right.}{b^{3/2}\sqrt{a\cdots}}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}}+\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}}-\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\cdots\right.}{b^{7/4}\sqrt{a+bx}}$$

Mathematica [C] time = 0.0220727, size = 71, normalized size = 0.39

$$-\frac{2(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/4)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(3/4))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{4}}\,(bx+a)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/4)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(3/4)/(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{3}{4}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{b^2x^2+2\,abx+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{3}{4}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/4)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(3/4)/(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{3}{4}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(3/2), x)
```

3.1639 $\int \frac{(c+dx)^{3/4}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=221

$$-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{b^{7/4}\sqrt{a+bx}\sqrt[4]{bc-ad}}+\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}\sqrt[4]{bc-ad}}-\frac{d(c+dx)^{3/4}}{b\sqrt{a+bx}(bc-ad)}-\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}}$$

```
[Out] (-2*(c + d*x)^(3/4))/(3*b*(a + b*x)^(3/2)) - (d*(c + d*x)^(3/4))/(b*(b*c -
a*d)*Sqrt[a + b*x]) + (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSi
n[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(7/4)*(b*c - a*d)^(
1/4)*Sqrt[a + b*x]) - (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSi
n[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(7/4)*(b*c - a*d)^(
1/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.235718, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 51, 63, 307, 224, 221, 1200, 1199, 424}

$$-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}\sqrt[4]{bc-ad}}+\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}\sqrt[4]{bc-ad}}-\frac{d(c+dx)^{3/4}}{b\sqrt{a+bx}(bc-ad)}-\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4))/(3*b*(a + b*x)^(3/2)) - (d*(c + d*x)^(3/4))/(b*(b*c -
a*d)*Sqrt[a + b*x]) + (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSi
n[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(7/4)*(b*c - a*d)^(
1/4)*Sqrt[a + b*x]) - (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSi
n[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(7/4)*(b*c - a*d)^(
1/4)*Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/4}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}}+\frac{d\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}dx}{2b}$$
$$=-\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}}-\frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}}+\frac{d^2\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}dx}{4b(bc-ad)}$$
$$=-\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}}-\frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}}+\frac{d\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}dx,x,\sqrt[4]{c+dx}\right)}{b(bc-ad)}$$
$$=-\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}}-\frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}}-\frac{d\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}dx,x,\sqrt[4]{c+dx}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{d\,\mathrm{Subst}\left(\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{bc-ad}}}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\right.}{b^{3/2}\sqrt{bc-}}$$
$$=-\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}}-\frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}}-\frac{\left(d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}dx,x,\sqrt[4]{c+dx}\right)}{b^{3/2}\sqrt{bc-ad}\sqrt{a+bx}}+\frac{\left(d\sqrt{\frac{d(a+bx}{-bc+ad}}\right.}{}$$
$$=-\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}}-\frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}}-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt[4]{bc-ad}\sqrt{a+bx}}+\frac{\left(d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int\right.}{b^{3/2}\sqrt{bc-}}$$
$$=-\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}}-\frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}}+\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt[4]{bc-ad}\sqrt{a+bx}}-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\right.\right.}{b^{7/4}\sqrt[4]{bc-ad}\sqrt{}}$$

Mathematica [C] time = 0.0239278, size = 73, normalized size = 0.33

$$-\frac{2(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(3/4))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{4}}(bx+a)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/4)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(3/4)/(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{3}{4}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{3}{4}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/4)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(3/4)/(a + b*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{3}{4}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(5/2), x)
```

3.1640 $\int \frac{(c+dx)^{3/4}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=270

$$\frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{10b^{7/4}\sqrt{a+bx}(bc-ad)^{5/4}}-\frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{10b^{7/4}\sqrt{a+bx}(bc-ad)^{5/4}}+\frac{3d^2(c+dx)^{3/4}}{10b\sqrt{a+bx}(bc-ad)^2}-\frac{}{5b(a}$$

```
[Out] (-2*(c + d*x)^(3/4))/(5*b*(a + b*x)^(5/2)) - (d*(c + d*x)^(3/4))/(5*b*(b*c
- a*d)*(a + b*x)^(3/2)) + (3*d^2*(c + d*x)^(3/4))/(10*b*(b*c - a*d)^2*Sqrt[
a + b*x]) - (3*d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(
1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(10*b^(7/4)*(b*c - a*d)^(5/4
)*Sqrt[a + b*x]) + (3*d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(10*b^(7/4)*(b*c - a*
d)^(5/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.284775, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 51, 63, 307, 224, 221, 1200, 1199, 424}

$$\frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{10b^{7/4}\sqrt{a+bx}(bc-ad)^{5/4}}-\frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{10b^{7/4}\sqrt{a+bx}(bc-ad)^{5/4}}+\frac{3d^2(c+dx)^{3/4}}{10b\sqrt{a+bx}(bc-ad)^2}-\frac{d(c+d}{5b(a+bx)^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4))/(5*b*(a + b*x)^(5/2)) - (d*(c + d*x)^(3/4))/(5*b*(b*c
- a*d)*(a + b*x)^(3/2)) + (3*d^2*(c + d*x)^(3/4))/(10*b*(b*c - a*d)^2*Sqrt[
a + b*x]) - (3*d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(
1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(10*b^(7/4)*(b*c - a*d)^(5/4
)*Sqrt[a + b*x]) + (3*d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(10*b^(7/4)*(b*c - a*
d)^(5/4)*Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/4}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} + \frac{(3d)\int \frac{1}{(a+bx)^{5/2}\sqrt[4]{c+dx}}\,dx}{10b}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} - \frac{\left(3d^2\right)\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}\,dx}{20b(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} - \frac{\left(3d^3\right)\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{40b(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} - \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt{}\right.}{10b(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} + \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt{}\right.}{10b^{3/2}(bc-ad)^{3/2}}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} + \frac{\left(3d^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right.}}}\right.}{10b^{3/2}(bc-ad)^{3/2}\sqrt{a}}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} + \frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right.}{10b^{7/4}(bc-ad)^{5/4}\sqrt{a+bx}}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} - \frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right.}{10b^{7/4}(bc-ad)^{5/4}\sqrt{a+bx}}$$

Mathematica [C] time = 0.0269819, size = 73, normalized size = 0.27

$$-\frac{2(c+dx)^{3/4}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{4};-\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(3/4))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{4}}(bx+a)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/4)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(3/4)/(b*x+a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{3}{4}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{3}{4}}}{(a+bx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/4)/(b*x+a)**(7/2),x)
```

```
[Out] Integral((c + d*x)**(3/4)/(a + b*x)**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{3}{4}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(7/2), x)
```

3.1641 $\int (a+bx)^{3/2}(c+dx)^{5/4}\,dx$

Optimal. Leaf size=220

$$\frac{16(bc-ad)^{17/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{231b^{9/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)^3}{231b^2d^2}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}(bc-ad)^2}{231b^2d}$$

```
[Out] (-8*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(1/4))/(231*b^2*d^2) + (4*(b*c -
a*d)^2*(a + b*x)^(3/2)*(c + d*x)^(1/4))/(231*b^2*d) + (4*(b*c - a*d)*(a + b
*x)^(5/2)*(c + d*x)^(1/4))/(33*b^2) + (4*(a + b*x)^(5/2)*(c + d*x)^(5/4))/(
15*b) + (16*(b*c - a*d)^(17/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF
[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(231*b^(9/4)*d^3
*Sqrt[a + b*x])
```

Rubi [A] time = 0.168037, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 224, 221}

$$-\frac{8\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)^3}{231b^2d^2}+\frac{16(bc-ad)^{17/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{231b^{9/4}d^3\sqrt{a+bx}}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}(bc-ad)^2}{231b^2d}+4($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(5/4),x]
```


```
[Out] (-8*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(1/4))/(231*b^2*d^2) + (4*(b*c -
a*d)^2*(a + b*x)^(3/2)*(c + d*x)^(1/4))/(231*b^2*d) + (4*(b*c - a*d)*(a + b
*x)^(5/2)*(c + d*x)^(1/4))/(33*b^2) + (4*(a + b*x)^(5/2)*(c + d*x)^(5/4))/(
15*b) + (16*(b*c - a*d)^(17/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF
[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(231*b^(9/4)*d^3
*Sqrt[a + b*x])
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{5/4}\,dx = \frac{4(a+bx)^{5/2}(c+dx)^{5/4}}{15b} + \frac{(bc-ad)\int (a+bx)^{3/2}\sqrt[4]{c+dx}\,dx}{3b}$$

$$= \frac{4(bc-ad)(a+bx)^{5/2}\sqrt[4]{c+dx}}{33b^2} + \frac{4(a+bx)^{5/2}(c+dx)^{5/4}}{15b} + \frac{(bc-ad)^2\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}}\,dx}{33b^2}$$

$$= \frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d} + \frac{4(bc-ad)(a+bx)^{5/2}\sqrt[4]{c+dx}}{33b^2} + \frac{4(a+bx)^{5/2}(c+dx)^{5/4}}{15b}$$

$$= -\frac{8(bc-ad)^3\sqrt{a+bx}\sqrt[4]{c+dx}}{231b^2d^2} + \frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d} + \frac{4(bc-ad)(a+bx)^{5/2}\sqrt[4]{}}{33b^2}$$

$$= -\frac{8(bc-ad)^3\sqrt{a+bx}\sqrt[4]{c+dx}}{231b^2d^2} + \frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d} + \frac{4(bc-ad)(a+bx)^{5/2}\sqrt[4]{}}{33b^2}$$

$$= -\frac{8(bc-ad)^3\sqrt{a+bx}\sqrt[4]{c+dx}}{231b^2d^2} + \frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d} + \frac{4(bc-ad)(a+bx)^{5/2}\sqrt[4]{}}{33b^2}$$

$$= -\frac{8(bc-ad)^3\sqrt{a+bx}\sqrt[4]{c+dx}}{231b^2d^2} + \frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d} + \frac{4(bc-ad)(a+bx)^{5/2}\sqrt[4]{}}{33b^2}$$

Mathematica [C] time = 0.0680344, size = 73, normalized size = 0.33

$$\frac{2(a+bx)^{5/2}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(3/2)*(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bdx^2+ac+(bc+ad)x\right)\sqrt{bx+a}(dx+c)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^2 + a*c + (b*c + a*d)*x)*sqrt(b*x + a)*(d*x + c)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(5/4), x)
```

3.1642 $\int \sqrt{a+bx}(c+dx)^{5/4}\,dx$

Optimal. Leaf size=182

$$-\frac{40(bc-ad)^{13/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{231b^{9/4}d^2\sqrt{a+bx}}+\frac{20\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)^2}{231b^2d}+\frac{20(a+bx)^{3/2}\sqrt[4]{c+dx}(b$$

```
[Out] (20*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/4))/(231*b^2*d) + (20*(b*c - a
*d)*(a + b*x)^(3/2)*(c + d*x)^(1/4))/(77*b^2) + (4*(a + b*x)^(3/2)*(c + d*x
)^(5/4))/(11*b) - (40*(b*c - a*d)^(13/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]
*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(231*b
^(9/4)*d^2*Sqrt[a + b*x])
```

Rubi [A] time = 0.104016, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 224, 221}

$$-\frac{40(bc-ad)^{13/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{231b^{9/4}d^2\sqrt{a+bx}}+\frac{20\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)^2}{231b^2d}+\frac{20(a+bx)^{3/2}\sqrt[4]{c+dx}(bc-ad)}{77b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(5/4),x]
```

```
[Out] (20*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/4))/(231*b^2*d) + (20*(b*c - a
*d)*(a + b*x)^(3/2)*(c + d*x)^(1/4))/(77*b^2) + (4*(a + b*x)^(3/2)*(c + d*x
)^(5/4))/(11*b) - (40*(b*c - a*d)^(13/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]
*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(231*b
^(9/4)*d^2*Sqrt[a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}(c+dx)^{5/4}\,dx = \frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b}+\frac{(5(bc-ad))\int \sqrt{a+bx}\sqrt[4]{c+dx}\,dx}{11b}$$

$$=\frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2}+\frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b}+\frac{\left(5(bc-ad)^2\right)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx}{77b^2}$$

$$=\frac{20(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{231b^2d}+\frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2}+\frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b}-\frac{(1}{}$$

$$=\frac{20(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{231b^2d}+\frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2}+\frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b}-\frac{(4}{}$$

$$=\frac{20(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{231b^2d}+\frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2}+\frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b}-\frac{(4}{}$$

$$=\frac{20(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{231b^2d}+\frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2}+\frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b}-\frac{40}{}$$

Mathematica [C] time = 0.046882, size = 73, normalized size = 0.4

$$\frac{2(a+bx)^{3/2}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\,(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx+a}(dx+c)^{\frac{5}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4), x)
```

Sympy [A] time = 70.5203, size = 218, normalized size = 1.2

$$-\frac{2ad\,(a+bx)^{\frac{3}{2}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{ade^{i\pi}}{b\,\text{polar\_lift}\left(-\frac{ad}{b}+c\right)}+\frac{dxe^{i\pi}}{\text{polar\_lift}\left(-\frac{ad}{b}+c\right)}\right)\sqrt[4]{\text{polar\_lift}\left(-\frac{ad}{b}+c\right)}}{3b^2}+\frac{2c\,(a+bx)^{\frac{3}{2}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{}{b\,\text{pola}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(5/4),x)
```

```
[Out] -2*a*d*(a + b*x)**(3/2)*hyper((-1/4, 3/2), (5/2,), a*d*exp_polar(I*pi)/(b*p
olar_lift(-a*d/b + c)) + d*x*exp_polar(I*pi)/polar_lift(-a*d/b + c))*polar_
lift(-a*d/b + c)**(1/4)/(3*b**2) + 2*c*(a + b*x)**(3/2)*hyper((-1/4, 3/2),
(5/2,), a*d*exp_polar(I*pi)/(b*polar_lift(-a*d/b + c)) + d*x*exp_polar(I*pi
)/polar_lift(-a*d/b + c))*polar_lift(-a*d/b + c)**(1/4)/(3*b) + 2*d*(a + b*
x)**(5/2)*hyper((-1/4, 5/2), (7/2,), a*d*exp_polar(I*pi)/(b*polar_lift(-a*d
/b + c)) + d*x*exp_polar(I*pi)/polar_lift(-a*d/b + c))*polar_lift(-a*d/b +
c)**(1/4)/(5*b**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(5/4), x)
```

3.1643 $\int \frac{(c+dx)^{5/4}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=144

$$\frac{20(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{21b^{9/4}d\sqrt{a+bx}}+\frac{20\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)}{21b^2}+\frac{4\sqrt{a+bx}(c+dx)^{5/4}}{7b}$$

```
[Out] (20*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4))/(21*b^2) + (4*Sqrt[a + b*x]*
(c + d*x)^(5/4))/(7*b) + (20*(b*c - a*d)^(9/4)*Sqrt[-((d*(a + b*x))/(b*c -
a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/
(21*b^(9/4)*d*Sqrt[a + b*x])
```

Rubi [A] time = 0.0835383, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 224, 221}

$$\frac{20\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)}{21b^2}+\frac{20(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{21b^{9/4}d\sqrt{a+bx}}+\frac{4\sqrt{a+bx}(c+dx)^{5/4}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/Sqrt[a + b*x],x]
```

```
[Out] (20*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4))/(21*b^2) + (4*Sqrt[a + b*x]*
(c + d*x)^(5/4))/(7*b) + (20*(b*c - a*d)^(9/4)*Sqrt[-((d*(a + b*x))/(b*c -
a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/
(21*b^(9/4)*d*Sqrt[a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{\sqrt{a+bx}}\,dx = \frac{4\sqrt{a+bx}(c+dx)^{5/4}}{7b} + \frac{(5(bc-ad))\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx}{7b}$$

$$= \frac{20(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{21b^2} + \frac{4\sqrt{a+bx}(c+dx)^{5/4}}{7b} + \frac{\left(5(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{21b^2}$$

$$= \frac{20(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{21b^2} + \frac{4\sqrt{a+bx}(c+dx)^{5/4}}{7b} + \frac{\left(20(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{\right.}{21b^2d}$$

$$= \frac{20(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{21b^2} + \frac{4\sqrt{a+bx}(c+dx)^{5/4}}{7b} + \frac{\left(20(bc-ad)^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\right.}{21b^2d\sqrt{a+bx}}$$

$$= \frac{20(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{21b^2} + \frac{4\sqrt{a+bx}(c+dx)^{5/4}}{7b} + \frac{20(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right.}{21b^{9/4}d\sqrt{a+bx}}$$

Mathematica [C] time = 0.038504, size = 71, normalized size = 0.49

$$\frac{2\sqrt{a+bx}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, (d*(a +
b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{4}}\frac{1}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^{\frac{5}{4}}}{\sqrt{bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(5/4)/sqrt(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{5}{4}}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(5/4)/sqrt(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/sqrt(b*x + a), x)
```

3.1644 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=132

$$\frac{10(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{3b^{9/4}\sqrt{a+bx}}+\frac{10d\sqrt{a+bx}\sqrt[4]{c+dx}}{3b^2}-\frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}}$$

```
[Out] (10*d*Sqrt[a + b*x]*(c + d*x)^(1/4))/(3*b^2) - (2*(c + d*x)^(5/4))/(b*Sqrt[
a + b*x]) + (10*(b*c - a*d)^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellipt
icF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(9/4)*Sq
rt[a + b*x])
```

Rubi [A] time = 0.0783782, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 224, 221}

$$\frac{10d\sqrt{a+bx}\sqrt[4]{c+dx}}{3b^2}+\frac{10(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{9/4}\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(3/2),x]
```

```
[Out] (10*d*Sqrt[a + b*x]*(c + d*x)^(1/4))/(3*b^2) - (2*(c + d*x)^(5/4))/(b*Sqrt[
a + b*x]) + (10*(b*c - a*d)^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellipt
icF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(9/4)*Sq
rt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/4}}{(a+bx)^{3/2}}\,dx &= -\frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx}{2b} \\
&= \frac{10d\sqrt{a+bx}\sqrt[4]{c+dx}}{3b^2} - \frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{(5d(bc-ad))\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{6b^2} \\
&= \frac{10d\sqrt{a+bx}\sqrt[4]{c+dx}}{3b^2} - \frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{(10(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3b^2} \\
&= \frac{10d\sqrt{a+bx}\sqrt[4]{c+dx}}{3b^2} - \frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{\left(10(bc-ad)\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3b^2\sqrt{a+bx}} \\
&= \frac{10d\sqrt{a+bx}\sqrt[4]{c+dx}}{3b^2} - \frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{10(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{9/4}\sqrt{a+bx}}
\end{aligned}$$

Mathematica [C] time = 0.0384983, size = 71, normalized size = 0.54

$$-\frac{2(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{4}}(bx+a)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{4}}}{b^2x^2+2\,abx+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(3/2), x)
```

3.1645 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{5d\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{3b^{9/4}\sqrt{a+bx}}-\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}}$$

```
[Out] (-5*d*(c + d*x)^(1/4))/(3*b^2*Sqrt[a + b*x]) - (2*(c + d*x)^(5/4))/(3*b*(a
+ b*x)^(3/2)) + (5*d*(b*c - a*d)^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(9/
4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.076054, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {47, 63, 224, 221}

$$-\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}}+\frac{5d\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{9/4}\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-5*d*(c + d*x)^(1/4))/(3*b^2*Sqrt[a + b*x]) - (2*(c + d*x)^(5/4))/(3*b*(a
+ b*x)^(3/2)) + (5*d*(b*c - a*d)^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(9/
4)*Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/2}}\,dx &= -\frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}}+\frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/2}}\,dx}{6b}\\
&= -\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}}+\frac{\left(5d^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{12b^2}\\
&= -\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}}+\frac{(5d)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3b^2}\\
&= -\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}}+\frac{\left(5d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3b^2\sqrt{a+bx}}\\
&= -\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}}+\frac{5d\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{9/4}\sqrt{a+bx}}
\end{aligned}$$

Mathematica [C] time = 0.0411444, size = 73, normalized size = 0.54

$$-\frac{2(c+dx)^{5/4}\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(5/4)*Hypergeometric2F1[-3/2, -5/4, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{4}}(bx+a)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{4}}}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/2), x)
```

3.1646 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=175

$$-\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{6b^{9/4}\sqrt{a+bx}(bc-ad)^{3/4}}-\frac{d^2\sqrt[4]{c+dx}}{6b^2\sqrt{a+bx}(bc-ad)}-\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}}$$

```
[Out] -(d*(c + d*x)^(1/4))/(3*b^2*(a + b*x)^(3/2)) - (d^2*(c + d*x)^(1/4))/(6*b^2
*(b*c - a*d)*Sqrt[a + b*x]) - (2*(c + d*x)^(5/4))/(5*b*(a + b*x)^(5/2)) - (
d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^
(1/4))/(b*c - a*d)^(1/4)], -1])/(6*b^(9/4)*(b*c - a*d)^(3/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.103906, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 51, 63, 224, 221}

$$-\frac{d^2\sqrt[4]{c+dx}}{6b^2\sqrt{a+bx}(bc-ad)}-\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{6b^{9/4}\sqrt{a+bx}(bc-ad)^{3/4}}-\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(7/2),x]
```

```
[Out] -(d*(c + d*x)^(1/4))/(3*b^2*(a + b*x)^(3/2)) - (d^2*(c + d*x)^(1/4))/(6*b^2
*(b*c - a*d)*Sqrt[a + b*x]) - (2*(c + d*x)^(5/4))/(5*b*(a + b*x)^(5/2)) - (
d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^
(1/4))/(b*c - a*d)^(1/4)], -1])/(6*b^(9/4)*(b*c - a*d)^(3/4)*Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/4}}{(a+bx)^{7/2}}\,dx &= -\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}}+\frac{d\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/2}}\,dx}{2b}\\
&= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}}+\frac{d^2\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx}{12b^2}\\
&= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}}-\frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}}-\frac{d^3\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{24b^2(bc-ad)}\\
&= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}}-\frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}}-\frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{6b^2(bc-ad)}\\
&= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}}-\frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}}-\frac{\left(d^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{6b^2(bc-ad)\sqrt{a+bx}}\\
&= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}}-\frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}}-\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{6b^{9/4}(bc-ad)^{3/4}\sqrt{a+bx}}
\end{aligned}$$

Mathematica [C] time = 0.046409, size = 73, normalized size = 0.42

$$-\frac{2(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{4};-\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/4)*Hypergeometric2F1[-5/2, -5/4, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{4}}(bx+a)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{4}}}{b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(7/2), x)
```

3.1647 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/2}}\,dx$

Optimal. Leaf size=213

$$\frac{5d^3\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{84b^{9/4}\sqrt{a+bx}(bc-ad)^{7/4}}+\frac{5d^3\sqrt[4]{c+dx}}{84b^2\sqrt{a+bx}(bc-ad)^2}-\frac{d^2\sqrt[4]{c+dx}}{42b^2(a+bx)^{3/2}(bc-ad)}-\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}}-$$

```
[Out] -(d*(c + d*x)^(1/4))/(7*b^2*(a + b*x)^(5/2)) - (d^2*(c + d*x)^(1/4))/(42*b^
2*(b*c - a*d)*(a + b*x)^(3/2)) + (5*d^3*(c + d*x)^(1/4))/(84*b^2*(b*c - a*d
)^2*Sqrt[a + b*x]) - (2*(c + d*x)^(5/4))/(7*b*(a + b*x)^(7/2)) + (5*d^3*Sqr
t[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/
(b*c - a*d)^(1/4)], -1])/(84*b^(9/4)*(b*c - a*d)^(7/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.120252, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 51, 63, 224, 221}

$$\frac{5d^3\sqrt[4]{c+dx}}{84b^2\sqrt{a+bx}(bc-ad)^2}-\frac{d^2\sqrt[4]{c+dx}}{42b^2(a+bx)^{3/2}(bc-ad)}+\frac{5d^3\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{84b^{9/4}\sqrt{a+bx}(bc-ad)^{7/4}}-\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}}-\frac{2(c+dx}{7b(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(9/2),x]
```

```
[Out] -(d*(c + d*x)^(1/4))/(7*b^2*(a + b*x)^(5/2)) - (d^2*(c + d*x)^(1/4))/(42*b^
2*(b*c - a*d)*(a + b*x)^(3/2)) + (5*d^3*(c + d*x)^(1/4))/(84*b^2*(b*c - a*d
)^2*Sqrt[a + b*x]) - (2*(c + d*x)^(5/4))/(7*b*(a + b*x)^(7/2)) + (5*d^3*Sqr
t[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/
(b*c - a*d)^(1/4)], -1])/(84*b^(9/4)*(b*c - a*d)^(7/4)*Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/2}}\,dx = -\frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{7/2}}\,dx}{14b}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}} + \frac{d^2\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/4}}\,dx}{28b^2}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}} - \frac{\left(5d^3\right)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx}{168b^2(bc-ad)}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} + \frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}} + \frac{\left(5d^4\right)\int -}{336b^2}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} + \frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}} + \frac{\left(5d^3\right)\text{Sub}}{}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} + \frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}} + \frac{\left(5d^3\sqrt{\frac{d(a}{-bc}}\right.}{}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} + \frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}} + \frac{5d^3\sqrt{-\frac{d(a}{bc}}}{84b}$$

Mathematica [C] time = 0.0499101, size = 73, normalized size = 0.34

$$-\frac{2(c+dx)^{5/4}\,{}_2F_1\left(-\frac{7}{2},-\frac{5}{4};-\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(5/4)*Hypergeometric2F1[-7/2, -5/4, -5/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(7*b*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{4}} (bx+a)^{-\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(9/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{4}}}{b^5x^5+5\,ab^4x^4+10\,a^2b^3x^3+10\,a^3b^2x^2+5\,a^4bx+a^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4)/(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*
x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/2), x)
```

3.1648 $\int \frac{(a+bx)^{5/2}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=264

$$\frac{32(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{39b^{3/4}d^4\sqrt{a+bx}}-\frac{32(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{39b^{3/4}d^4\sqrt{a+bx}}+\frac{16\sqrt{a\cdots}}{}$$

```
[Out] (16*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(3/4))/(39*d^3) - (40*(b*c - a*d)
*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(117*d^2) + (4*(a + b*x)^(5/2)*(c + d*x)^
(3/4))/(13*d) - (32*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(39*b^(3
/4)*d^4*Sqrt[a + b*x]) + (32*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(39*b^(3/4)*d^4*Sqrt[a + b*x])
```

Rubi [A] time = 0.298464, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {50, 63, 307, 224, 221, 1200, 1199, 424}

$$\frac{32(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{39b^{3/4}d^4\sqrt{a+bx}}-\frac{32(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{39b^{3/4}d^4\sqrt{a+bx}}+\frac{16\sqrt{a+bx}(c+\cdots}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(1/4),x]
```


```
[Out] (16*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(3/4))/(39*d^3) - (40*(b*c - a*d)
*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(117*d^2) + (4*(a + b*x)^(5/2)*(c + d*x)^
(3/4))/(13*d) - (32*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(39*b^(3
/4)*d^4*Sqrt[a + b*x]) + (32*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(39*b^(3/4)*d^4*Sqrt[a + b*x])
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt[4]{c+dx}}\,dx = \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d} - \frac{(10(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx}{13d}$$

$$= -\frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d} + \frac{\left(20(bc-ad)^2\right)\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{39d^2}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d} - \frac{(8(bc}{}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d} - \frac{(32(bc}{}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d} + \frac{(32(bc}{}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d} + \frac{(32(bc}{}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d} + \frac{32(bc}{}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d} - \frac{32(bc}{}$$

Mathematica [C] time = 0.0333047, size = 73, normalized size = 0.28

$$\frac{2(a+bx)^{7/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/4))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{2}}\frac{1}{\sqrt[4]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx+a}}{(dx+c)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(d*x + c)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{5}{2}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(1/4), x)
```

3.1649 $\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}} dx$

Optimal. Leaf size=229

$$-\frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{15b^{3/4}d^3\sqrt{a+bx}}+\frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{3/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a\cdots}}{}$$

```
[Out] (-8*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4))/(15*d^2) + (4*(a + b*x)^(3/2
)*(c + d*x)^(3/4))/(9*d) + (16*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c
 - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1
])/(15*b^(3/4)*d^3*Sqrt[a + b*x]) - (16*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b
*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(
1/4)], -1])/(15*b^(3/4)*d^3*Sqrt[a + b*x])
```

Rubi [A] time = 0.244781, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {50, 63, 307, 224, 221, 1200, 1199, 424}

$$-\frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{3/4}d^3\sqrt{a+bx}}+\frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{3/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}(c+\cdots}{1\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(1/4),x]
```

```
[Out] (-8*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4))/(15*d^2) + (4*(a + b*x)^(3/2
)*(c + d*x)^(3/4))/(9*d) + (16*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c
 - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1
])/(15*b^(3/4)*d^3*Sqrt[a + b*x]) - (16*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b
*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(
1/4)], -1])/(15*b^(3/4)*d^3*Sqrt[a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
```

```
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}dx = \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} - \frac{(2(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}dx}{3d}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} + \frac{\left(4(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}dx}{15d^2}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} + \frac{\left(16(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}dx, x, \right.}{15d^3}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} - \frac{\left(16(bc-ad)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}dx, x, \right.}{15\sqrt{b}d^3}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} - \frac{\left(16(bc-ad)^{5/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{b}{(a-}}}\right.}{15\sqrt{b}d^3\sqrt{a+bx}}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} - \frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+d}}{\sqrt[4]{bc-ad}}\right.\right.}{15b^{3/4}d^3\sqrt{a+bx}}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} + \frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+d}}{\sqrt[4]{bc-ad}}\right.\right.}{15b^{3/4}d^3\sqrt{a+bx}}$$

Mathematica [C] time = 0.0282069, size = 73, normalized size = 0.32

$$\frac{2(a+bx)^{5/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/4))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}\frac{1}{\sqrt[4]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/4), x)
```

3.1650 $\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=196

$$\frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{5b^{3/4}d^2\sqrt{a+bx}}-\frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}}+\frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5d}$$

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(3/4))/(5*d) - (8*(b*c - a*d)^(7/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(5*b^(3/4)*d^2*Sqrt[a + b*x]) + (8*(b*c - a*d)^(7/4)*Sqrt[-
((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*
c - a*d)^(1/4)], -1])/(5*b^(3/4)*d^2*Sqrt[a + b*x])
```

Rubi [A] time = 0.21557, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {50, 63, 307, 224, 221, 1200, 1199, 424}

$$\frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}}-\frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}}+\frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(1/4),x]
```

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(3/4))/(5*d) - (8*(b*c - a*d)^(7/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(5*b^(3/4)*d^2*Sqrt[a + b*x]) + (8*(b*c - a*d)^(7/4)*Sqrt[-
((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*
c - a*d)^(1/4)], -1])/(5*b^(3/4)*d^2*Sqrt[a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx = \frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5d} - \frac{(2(bc-ad))\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{5d}$$

$$= \frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5d} - \frac{(8(bc-ad))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d^2}$$

$$= \frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5d} + \frac{\left(8(bc-ad)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5\sqrt{b}d^2} - \frac{\left(8(bc-ad)^{3/2}\right)\mathrm{Subst}\left(\ldots\right.}{5\ldots}$$

$$= \frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5d} + \frac{\left(8(bc-ad)^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5\sqrt{b}d^2\sqrt{a+bx}} - \frac{\left(8(bc-ad)^{3/2}\ldots\right.}{\ldots}$$

$$= \frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5d} + \frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}} - \frac{\left(8(bc-ad)^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{S}\ldots}{5\sqrt{\ldots}}$$

$$= \frac{4\sqrt{a+bx}(c+dx)^{3/4}}{5d} - \frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}} + \frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\ldots\right.}{5b^{3/4}d^2\sqrt{\ldots}}$$

Mathematica [C] time = 0.0253037, size = 73, normalized size = 0.37

$$\frac{2(a+bx)^{3/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/4))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\frac{1}{\sqrt[4]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx}}{\sqrt[4]{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx + a}}{(dx + c)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/4), x)
```

3.1651 $\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=167

$$\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}}-\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{b^{3/4}d\sqrt{a+bx}}$$

```
[Out] (4*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b
^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*d*Sqrt[a + b*x])
- (4*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[
(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*d*Sqrt[a + b*x]
)
```

Rubi [A] time = 0.194074, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {63, 307, 224, 221, 1200, 1199, 424}

$$\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}}-\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b
^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*d*Sqrt[a + b*x])
- (4*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[
(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*d*Sqrt[a + b*x]
)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx = \frac{4\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d}$$

$$= -\frac{\left(4\sqrt{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{\sqrt{b}d} + \frac{\left(4\sqrt{bc-ad}\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{bc-ad}}}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x\right.}{\sqrt{b}d}$$

$$= -\frac{\left(4\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{\sqrt{b}d\sqrt{a+bx}} + \frac{\left(4\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}}{\sqrt{b}d\sqrt{a}}$$

$$= -\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}} + \frac{\left(4\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{\sqrt{b}}{\sqrt{bc-}}}}{\sqrt{1-\frac{\sqrt{b}}{\sqrt{bc-}}}}\right.}{\sqrt{b}d\sqrt{a+bx}}$$

$$= \frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}} - \frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right.\right.}{b^{3/4}d\sqrt{a+bx}}$$

Mathematica [C] time = 0.0215496, size = 71, normalized size = 0.43

$$\frac{2\sqrt{a+bx}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(1/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4, 1
/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/4))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt[4]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{bdx^2+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/4)), x)
```

3.1652 $\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=191

$$-\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{b^{3/4}\sqrt{a+bx}\sqrt[4]{bc-ad}}+\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}\sqrt[4]{bc-ad}}-\frac{2(c+dx)^{3/4}}{\sqrt{a+bx}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(3/4))/((b*c - a*d)*Sqrt[a + b*x]) + (2*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(b^(3/4)*(b*c - a*d)^(1/4)*Sqrt[a + b*x]) - (2*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(b^(3/4)*(b*c - a*d)^(1/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.224496, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {51, 63, 307, 224, 221, 1200, 1199, 424}

$$-\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}\sqrt[4]{bc-ad}}+\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}\sqrt[4]{bc-ad}}-\frac{2(c+dx)^{3/4}}{\sqrt{a+bx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(1/4)),x]
```


```
[Out] (-2*(c + d*x)^(3/4))/((b*c - a*d)*Sqrt[a + b*x]) + (2*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(b^(3/4)*(b*c - a*d)^(1/4)*Sqrt[a + b*x]) - (2*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(b^(3/4)*(b*c - a*d)^(1/4)*Sqrt[a + b*x])
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}\,dx = -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} + \frac{d\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{2(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{\sqrt{b}\sqrt{bc-ad}} + \frac{2\,\text{Subst}\left(\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{bc-ad}}}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{\ldots}\right.}{\sqrt{b}\sqrt{bc-ad}}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} - \frac{\left(2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{\sqrt{b}\sqrt{bc-ad}\sqrt{a+bx}} + \frac{\left(2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\ldots}{\sqrt{b}\sqrt{\ldots}}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} - \frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt[4]{bc-ad}\sqrt{a+bx}} + \frac{\left(2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{\sqrt{b}x^2}{\sqrt{bc-a\ldots}}}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{bc-a\ldots}}}}\ldots\right.}{\sqrt{b}\sqrt{bc-ad}\sqrt{a+\ldots}}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} + \frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt[4]{bc-ad}\sqrt{a+bx}} - \frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\ldots\right.}{b^{3/4}\sqrt[4]{bc-ad}\sqrt{a+bx}}$$

Mathematica [C] time = 0.0216798, size = 71, normalized size = 0.37

$$-\frac{2\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(1/4))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{3}{2}}\frac{1}{\sqrt[4]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{b^2dx^3+a^2c+\left(b^2c+2\,abd\right)x^2+\left(2\,abc+a^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/4)), x)
```

3.1653 $\int \frac{1}{(a+bx)^{5/2}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=224

$$\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{b^{3/4}\sqrt{a+bx}(bc-ad)^{5/4}}-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}(bc-ad)^{5/4}}+\frac{d(c+dx)^{3/4}}{\sqrt{a+bx}(bc-ad)^2}-\frac{2(c+d}{3(a+bx)^{3/2}(}$$

```
[Out] (-2*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (d*(c + d*x)^(3/4))/
((b*c - a*d)^2*Sqrt[a + b*x]) - (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellip
ticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*(b*
c - a*d)^(5/4)*Sqrt[a + b*x]) + (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellip
ticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*(b*
c - a*d)^(5/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.262006, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {51, 63, 307, 224, 221, 1200, 1199, 424}

$$\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}(bc-ad)^{5/4}}-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}(bc-ad)^{5/4}}+\frac{d(c+dx)^{3/4}}{\sqrt{a+bx}(bc-ad)^2}-\frac{2(c+dx)^{3/4}}{3(a+bx)^{3/2}(bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(1/4)),x]
```


```
[Out] (-2*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (d*(c + d*x)^(3/4))/
((b*c - a*d)^2*Sqrt[a + b*x]) - (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellip
ticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*(b*
c - a*d)^(5/4)*Sqrt[a + b*x]) + (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellip
ticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*(b*
c - a*d)^(5/4)*Sqrt[a + b*x])
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
```

```
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt[4]{c+dx}}\,dx = -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} - \frac{d\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}dx}{2(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} - \frac{d^2\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}dx}{4(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} - \frac{d\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}dx, x, \sqrt[4]{c+dx}\right)}{(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} + \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}dx, x, \sqrt[4]{c+dx}\right)}{\sqrt{b}(bc-ad)^{3/2}} - \frac{d\,\text{Sul}}{}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} + \frac{\left(d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}dx, x, \sqrt[4]{c+a}\right.}{\sqrt{b}(bc-ad)^{3/2}\sqrt{a+bx}}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} + \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}(bc-ad)^{5/4}\sqrt{a+bx}} - \frac{\left(d\sqrt{\frac{d(}{-}}\right.}{}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} - \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}(bc-ad)^{5/4}\sqrt{a+bx}} + \frac{d\sqrt{-\frac{a}{}}}{}$$

Mathematica [C] time = 0.0210228, size = 73, normalized size = 0.33

$$-\frac{2\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/4))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{2}}\frac{1}{\sqrt[4]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{b^3dx^4+a^3c+\left(b^3c+3\,ab^2d\right)x^3+3\left(ab^2c+a^2bd\right)x^2+\left(3\,a^2bc+a^3d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/4)), x)
```

3.1654 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=144

$$\frac{16(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{7\sqrt[4]{b}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)}{7d^2}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d}$$

```
[Out] (-8*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4))/(7*d^2) + (4*(a + b*x)^(3/2)
*(c + d*x)^(1/4))/(7*d) + (16*(b*c - a*d)^(9/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(7*b^(1/4)*d^3*Sqrt[a + b*x])
```

Rubi [A] time = 0.083312, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 224, 221}

$$-\frac{8\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)}{7d^2}+\frac{16(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{7\sqrt[4]{b}d^3\sqrt{a+bx}}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(3/4),x]
```


```
[Out] (-8*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4))/(7*d^2) + (4*(a + b*x)^(3/2)
*(c + d*x)^(1/4))/(7*d) + (16*(b*c - a*d)^(9/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(7*b^(1/4)*d^3*Sqrt[a + b*x])
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}}\,dx = \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} - \frac{(6(bc-ad))\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx}{7d}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{7d^2} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} + \frac{\left(4(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{7d^2}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{7d^2} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} + \frac{\left(16(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{\ldots}\right)}{7d^3}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{7d^2} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} + \frac{\left(16(bc-ad)^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)\ldots}}}\ldots\right)}{7d^3\sqrt{a+bx}}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{7d^2} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} + \frac{16(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\ldots\right)}{7\sqrt[4]{b}d^3\sqrt{a+bx}}$$

Mathematica [C] time = 0.0303692, size = 73, normalized size = 0.51

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(3/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(3/4))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(3/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(3/4), x)
```

3.1655 $\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=111

$$\frac{4\sqrt{a+bx}\sqrt[4]{c+dx}}{3d}-\frac{8(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{3\sqrt[4]{b}d^2\sqrt{a+bx}}$$

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(1/4))/(3*d) - (8*(b*c - a*d)^(5/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(3*b^(1/4)*d^2*Sqrt[a + b*x])
```

Rubi [A] time = 0.0672476, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 224, 221}

$$\frac{4\sqrt{a+bx}\sqrt[4]{c+dx}}{3d}-\frac{8(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}d^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(3/4),x]
```

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(1/4))/(3*d) - (8*(b*c - a*d)^(5/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(3*b^(1/4)*d^2*Sqrt[a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx &= \frac{4\sqrt{a+bx}\sqrt[4]{c+dx}}{3d} - \frac{(2(bc-ad))\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{3d}\\
&= \frac{4\sqrt{a+bx}\sqrt[4]{c+dx}}{3d} - \frac{(8(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3d^2}\\
&= \frac{4\sqrt{a+bx}\sqrt[4]{c+dx}}{3d} - \frac{\left(8(bc-ad)\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3d^2\sqrt{a+bx}}\\
&= \frac{4\sqrt{a+bx}\sqrt[4]{c+dx}}{3d} - \frac{8(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}d^2\sqrt{a+bx}}
\end{aligned}$$

Mathematica [C] time = 0.0262108, size = 73, normalized size = 0.66

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(3/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(3/4))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\,(dx+c)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}}{(dx+c)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(3/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(3/4), x)
```

3.1656 $\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=83

$$\frac{4\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{\sqrt[4]{b}d\sqrt{a+bx}}$$

```
[Out] (4*(b*c - a*d)^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b
^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(1/4)*d*Sqrt[a + b*x])
```

Rubi [A] time = 0.0510196, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {63, 224, 221}

$$\frac{4\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}d\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(b*c - a*d)^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b
^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(1/4)*d*Sqrt[a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx = \frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d}$$

$$= \frac{\left(4\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d\sqrt{a+bx}}$$

$$= \frac{4\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}d\sqrt{a+bx}}$$

Mathematica [C] time = 0.0225044, size = 71, normalized size = 0.86

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(3/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[1/2, 3
/4, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(3/4))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}}(dx+c)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(3/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{4}}}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(3/4)), x)
```

3.1657 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=111

$$-\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a+bx}(bc-ad)^{3/4}}-\frac{2\sqrt[4]{c+dx}}{\sqrt{a+bx}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(1/4))/((b*c - a*d)*Sqrt[a + b*x]) - (2*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(b^(1/4)*(b*c - a*d)^(3/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.0602431, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {51, 63, 224, 221}

$$-\frac{2\sqrt[4]{c+dx}}{\sqrt{a+bx}(bc-ad)}-\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{a+bx}(bc-ad)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/4)),x]
```


```
[Out] (-2*(c + d*x)^(1/4))/((b*c - a*d)*Sqrt[a + b*x]) - (2*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(b^(1/4)*(b*c - a*d)^(3/4)*Sqrt[a + b*x])
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx &= -\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{d\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{2(bc-ad)} \\
&= -\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{bc-ad} \\
&= -\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{\left(2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{(bc-ad)\sqrt{a+bx}} \\
&= -\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}(bc-ad)^{3/4}\sqrt{a+bx}}
\end{aligned}
$$

Mathematica [C] time = 0.0219049, size = 71, normalized size = 0.64

$$
-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}(c+dx)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(3/4))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$
\int (bx+a)^{-\frac{3}{2}}(dx+c)^{-\frac{3}{4}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{4}}}{b^2dx^3+a^2c+\left(b^2c+2\,abd\right)x^2+\left(2\,abc+a^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/4)), x)
```

3.1658 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=149

$$\frac{5d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{3\sqrt[4]{b}\sqrt{a+bx}(bc-ad)^{7/4}}+\frac{5d\sqrt[4]{c+dx}}{3\sqrt{a+bx}(bc-ad)^2}-\frac{2\sqrt[4]{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

```
[Out] (-2*(c + d*x)^(1/4))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (5*d*(c + d*x)^(1/4)
)/(3*(b*c - a*d)^2*Sqrt[a + b*x]) + (5*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]
*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(
1/4)*(b*c - a*d)^(7/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.0745647, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {51, 63, 224, 221}

$$\frac{5d\sqrt[4]{c+dx}}{3\sqrt{a+bx}(bc-ad)^2}-\frac{2\sqrt[4]{c+dx}}{3(a+bx)^{3/2}(bc-ad)}+\frac{5d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}\sqrt{a+bx}(bc-ad)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*(c + d*x)^(1/4))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (5*d*(c + d*x)^(1/4)
)/(3*(b*c - a*d)^2*Sqrt[a + b*x]) + (5*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]
*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(
1/4)*(b*c - a*d)^(7/4)*Sqrt[a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/4}}\,dx &= -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}}-\frac{(5d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx}{6(bc-ad)}\\
&= -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}}+\frac{\left(5d^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{12(bc-ad)^2}\\
&= -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}}+\frac{(5d)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3(bc-ad)^2}\\
&= -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}}+\frac{\left(5d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3(bc-ad)^2\sqrt{a+bx}}\\
&= -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}}+\frac{5d\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}(bc-ad)^{7/4}\sqrt{a+bx}}
\end{aligned}$$

Mathematica [C] time = 0.0244902, size = 73, normalized size = 0.49

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(3/4))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{2}}(dx+c)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(3/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{4}}}{b^3dx^4+a^3c+\left(b^3c+3\,ab^2d\right)x^3+3\left(ab^2c+a^2bd\right)x^2+\left(3\,a^2bc+a^3d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(3/4)), x)
```

3.1659 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=254

$$-\frac{32\sqrt[4]{b}(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{3d^4\sqrt{a+bx}}-\frac{16b\sqrt{a+bx}(c+dx)^{3/4}(bc-ad)}{3d^3}+\frac{40b(a+bx)^{3/2}(c+}{9d^2}$$

```
[Out] (-4*(a + b*x)^(5/2))/(d*(c + d*x)^(1/4)) - (16*b*(b*c - a*d)*Sqrt[a + b*x]*
(c + d*x)^(3/4))/(3*d^3) + (40*b*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(9*d^2) +
 (32*b^(1/4)*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Elliptic
E[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*d^4*Sqrt[a +
 b*x]) - (32*b^(1/4)*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*
EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*d^4*
Sqrt[a + b*x])
```

Rubi [A] time = 0.262013, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 50, 63, 307, 224, 221, 1200, 1199, 424}

$$-\frac{16b\sqrt{a+bx}(c+dx)^{3/4}(bc-ad)}{3d^3}+\frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2}-\frac{32\sqrt[4]{b}(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^4\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(5/2))/(d*(c + d*x)^(1/4)) - (16*b*(b*c - a*d)*Sqrt[a + b*x]*
(c + d*x)^(3/4))/(3*d^3) + (40*b*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(9*d^2) +
 (32*b^(1/4)*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Elliptic
E[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*d^4*Sqrt[a +
 b*x]) - (32*b^(1/4)*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*
EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*d^4*
Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} + \frac{(10b)\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} - \frac{(20b(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{3d^2}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} + \frac{\left(8b(bc-ad)^2\right)\int \frac{}{\sqrt{a}}}{3d^3}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} + \frac{\left(32b(bc-ad)^2\right)\text{Su}}{}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} - \frac{\left(32\sqrt{b}(bc-ad)^{5/2}\right)}{}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} - \frac{\left(32\sqrt{b}(bc-ad)^{5/2}\sqrt{}\right.}{}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} - \frac{32\sqrt[4]{b}(bc-ad)^{11/4}\sqrt{}}{}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} + \frac{32\sqrt[4]{b}(bc-ad)^{11/4}\sqrt{}}{}$$

Mathematica [C] time = 0.0627819, size = 73, normalized size = 0.29

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(\frac{5}{4},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/4))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{2}}(dx+c)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*(d*x + c)^(3/4)/(d^2*x^2 +
 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(5/4), x)
```

3.1660 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=220

$$\frac{48\sqrt[4]{b}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{5d^3\sqrt{a+bx}}+\frac{24b\sqrt{a+bx}(c+dx)^{3/4}}{5d^2}-\frac{48\sqrt[4]{b}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E}{5d^3\sqrt{a+}}$$

```
[Out] (-4*(a + b*x)^(3/2))/(d*(c + d*x)^(1/4)) + (24*b*Sqrt[a + b*x]*(c + d*x)^(3
/4))/(5*d^2) - (48*b^(1/4)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b*c - a*
d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5
*d^3*Sqrt[a + b*x]) + (48*b^(1/4)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b
*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)],
-1])/(5*d^3*Sqrt[a + b*x])
```

Rubi [A] time = 0.234948, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 50, 63, 307, 224, 221, 1200, 1199, 424}

$$\frac{24b\sqrt{a+bx}(c+dx)^{3/4}}{5d^2}+\frac{48\sqrt[4]{b}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}-\frac{48\sqrt[4]{b}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\right.}{5d^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(5/4),x]
```


```
[Out] (-4*(a + b*x)^(3/2))/(d*(c + d*x)^(1/4)) + (24*b*Sqrt[a + b*x]*(c + d*x)^(3
/4))/(5*d^2) - (48*b^(1/4)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b*c - a*
d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5
*d^3*Sqrt[a + b*x]) + (48*b^(1/4)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b
*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)],
-1])/(5*d^3*Sqrt[a + b*x])
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{(6b)\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}(c+dx)^{3/4}}{5d^2} - \frac{(12b(bc-ad))\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{5d^2}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}(c+dx)^{3/4}}{5d^2} - \frac{(48b(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d^3}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}(c+dx)^{3/4}}{5d^2} + \frac{\left(48\sqrt{b}(bc-ad)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d^3}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}(c+dx)^{3/4}}{5d^2} + \frac{\left(48\sqrt{b}(bc-ad)^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x,\right.}{5d^3\sqrt{a+bx}}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}(c+dx)^{3/4}}{5d^2} + \frac{48\sqrt[4]{b}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}} -$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}(c+dx)^{3/4}}{5d^2} - \frac{48\sqrt[4]{b}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}} +$$

Mathematica [C] time = 0.0517911, size = 73, normalized size = 0.33

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{5}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(5/4))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{4}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(3/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(5/4), x)
```

3.1661 $\int \frac{\sqrt{a+bx}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=190

$$-\frac{8\sqrt[4]{b}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{d^2\sqrt{a+bx}}+\frac{8\sqrt[4]{b}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}}-$$

```
[Out] (-4*Sqrt[a + b*x])/(d*(c + d*x)^(1/4)) + (8*b^(1/4)*(b*c - a*d)^(3/4)*Sqrt[
-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b
*c - a*d)^(1/4)], -1])/(d^2*Sqrt[a + b*x]) - (8*b^(1/4)*(b*c - a*d)^(3/4)*S
qrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4)
)/(b*c - a*d)^(1/4)], -1])/(d^2*Sqrt[a + b*x])
```

Rubi [A] time = 0.207322, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {47, 63, 307, 224, 221, 1200, 1199, 424}

$$-\frac{8\sqrt[4]{b}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}}+\frac{8\sqrt[4]{b}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}}-\frac{4\sqrt{a+b}}{d\sqrt[4]{c+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(5/4),x]
```

```
[Out] (-4*Sqrt[a + b*x])/(d*(c + d*x)^(1/4)) + (8*b^(1/4)*(b*c - a*d)^(3/4)*Sqrt[
-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b
*c - a*d)^(1/4)], -1])/(d^2*Sqrt[a + b*x]) - (8*b^(1/4)*(b*c - a*d)^(3/4)*S
qrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4)
)/(b*c - a*d)^(1/4)], -1])/(d^2*Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/4}}\,dx = -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} + \frac{(2b)\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} + \frac{(8b)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d^2}$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} - \frac{\left(8\sqrt{b}\sqrt{bc-ad}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d^2} + \frac{\left(8\sqrt{b}\sqrt{bc-ad}\right)\mathrm{Subst}\left(\int \frac{1+\frac{\sqrt{b}\cdots}{\sqrt{bc}\cdots}}{\sqrt{a-\frac{bc}{d}\cdots}}\cdots\right.}{d^2}$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} - \frac{\left(8\sqrt{b}\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d^2\sqrt{a+bx}} + \frac{\left(8\sqrt{b}\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\cdots\right.}{\cdots}$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} - \frac{8\sqrt[4]{b}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}} + \frac{\left(8\sqrt{b}\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\cdots\right.}{d^2\sqrt{a+\cdots}}$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} + \frac{8\sqrt[4]{b}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}} - \frac{8\sqrt[4]{b}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin\cdots\right.}{d^2\sqrt{a+bx}}$$

Mathematica [C] time = 0.041427, size = 73, normalized size = 0.38

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{5}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(5/4))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\,(dx+c)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(5/4), x)
```

3.1662 $\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=197

$$\frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{d\sqrt{a+bx}\sqrt[4]{bc-ad}}+\frac{4\sqrt{a+bx}}{\sqrt[4]{c+dx}(bc-ad)}-\frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt{a+bx}\sqrt[4]{bc-ad}}$$

```
[Out] (4*Sqrt[a + b*x])/((b*c - a*d)*(c + d*x)^(1/4)) - (4*b^(1/4)*Sqrt[-((d*(a +
 b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)
^(1/4)], -1])/(d*(b*c - a*d)^(1/4)*Sqrt[a + b*x]) + (4*b^(1/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(d*(b*c - a*d)^(1/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.211094, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {51, 63, 307, 224, 221, 1200, 1199, 424}

$$\frac{4\sqrt{a+bx}}{\sqrt[4]{c+dx}(bc-ad)}+\frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt{a+bx}\sqrt[4]{bc-ad}}-\frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt{a+bx}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(5/4)),x]
```

```
[Out] (4*Sqrt[a + b*x])/((b*c - a*d)*(c + d*x)^(1/4)) - (4*b^(1/4)*Sqrt[-((d*(a +
 b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)
^(1/4)], -1])/(d*(b*c - a*d)^(1/4)*Sqrt[a + b*x]) + (4*b^(1/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(d*(b*c - a*d)^(1/4)*Sqrt[a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/4}}\,dx = \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{b\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{bc-ad}$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{(4b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d(bc-ad)}$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} + \frac{\left(4\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d\sqrt{bc-ad}} - \frac{\left(4\sqrt{b}\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{b}\cdots}{\sqrt{bc-\cdots}}}{\sqrt{a-\frac{bc}{d}+\cdots}}\cdots\right.}{d\sqrt{bc-\cdots}}$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} + \frac{\left(4\sqrt{b}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d\sqrt{bc-ad}\sqrt{a+bx}} - \frac{\left(4\sqrt{b}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\cdots}{\cdots}$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} + \frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt[4]{bc-ad}\sqrt{a+bx}} - \frac{\left(4\sqrt{b}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \cdots\right.}{d\sqrt{bc-a\cdots}}$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt[4]{bc-ad}\sqrt{a+bx}} + \frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\cdots\right.\right.}{d\sqrt[4]{bc-ad}\sqrt{a+\cdots}}$$

Mathematica [C] time = 0.0348903, size = 71, normalized size = 0.36

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(5/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[1/2, 5
/4, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(5/4))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}}(dx+c)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(5/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{bd^2x^3+ac^2+\left(2\,bcd+ad^2\right)x^2+\left(bc^2+2\,acd\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*d^
2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(5/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(5/4)), x)
```

3.1663 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=222

$$-\frac{6\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{\sqrt{a+bx}(bc-ad)^{5/4}}-\frac{6d\sqrt{a+bx}}{\sqrt[4]{c+dx}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)}+\frac{6\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E}{\sqrt{a+}}$$

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4)) - (6*d*Sqrt[a + b*x])/((b*c
- a*d)^2*(c + d*x)^(1/4)) + (6*b^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c -
a*d)^(5/4)*Sqrt[a + b*x]) - (6*b^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c -
a*d)^(5/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.231177, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {51, 63, 307, 224, 221, 1200, 1199, 424}

$$-\frac{6d\sqrt{a+bx}}{\sqrt[4]{c+dx}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)}-\frac{6\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt{a+bx}(bc-ad)^{5/4}}+\frac{6\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\right.}{\sqrt{a+bx}(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(5/4)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4)) - (6*d*Sqrt[a + b*x])/((b*c
- a*d)^2*(c + d*x)^(1/4)) + (6*b^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c -
a*d)^(5/4)*Sqrt[a + b*x]) - (6*b^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c -
a*d)^(5/4)*Sqrt[a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
```

```
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/4}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}} - \frac{(3d)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{(3bd)\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{2(bc-ad)^2}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{(6b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+\right.}{(bc-ad)^2}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} - \frac{\left(6\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c\right.}{(bc-ad)^{3/2}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} - \frac{\left(6\sqrt{b}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\right.}{(bc-ad)^{3/2}\sqrt{a+bx}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} - \frac{6\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|}{(bc-ad)^{5/4}\sqrt{a+bx}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{6\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|}{(bc-ad)^{5/4}\sqrt{a+bx}}$$

Mathematica [C] time = 0.0383637, size = 71, normalized size = 0.32

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(-\frac{1}{2},\frac{5}{4};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(5/4))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{3}{2}}(dx+c)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(5/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{b^2d^2x^4+a^2c^2+2\left(b^2cd+abd^2\right)x^3+\left(b^2c^2+4\,abcd+a^2d^2\right)x^2+2\left(abc^2+a^2cd\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*d
+ a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c*d
)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(5/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(5/4)), x)
```

3.1664 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=261

$$\frac{7\sqrt[4]{b}d\sqrt{-\frac{d(a+bx)}{bc-ad}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{\sqrt{a+bx}(bc-ad)^{9/4}}+\frac{7d^2\sqrt{a+bx}}{\sqrt[4]{c+dx}(bc-ad)^3}+\frac{7d}{3\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)^2}-\frac{}{3(a+bx)^{3/2}\sqrt[4]{}}$$

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/4)) + (7*d)/(3*(b*c - a*d)^2
*Sqrt[a + b*x]*(c + d*x)^(1/4)) + (7*d^2*Sqrt[a + b*x])/((b*c - a*d)^3*(c +
 d*x)^(1/4)) - (7*b^(1/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[Ar
cSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c - a*d)^(9/4)*
Sqrt[a + b*x]) + (7*b^(1/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[
ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c - a*d)^(9/4
)*Sqrt[a + b*x])
```

Rubi [A] time = 0.26017, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {51, 63, 307, 224, 221, 1200, 1199, 424}

$$\frac{7d^2\sqrt{a+bx}}{\sqrt[4]{c+dx}(bc-ad)^3}+\frac{7d}{3\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}\sqrt[4]{c+dx}(bc-ad)}+\frac{7\sqrt[4]{b}d\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+d}}{\sqrt[4]{bc-ad}}\right.\right.}{\sqrt{a+bx}(bc-ad)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(5/4)),x]
```


```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/4)) + (7*d)/(3*(b*c - a*d)^2
*Sqrt[a + b*x]*(c + d*x)^(1/4)) + (7*d^2*Sqrt[a + b*x])/((b*c - a*d)^3*(c +
 d*x)^(1/4)) - (7*b^(1/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[Ar
cSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c - a*d)^(9/4)*
Sqrt[a + b*x]) + (7*b^(1/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[
ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c - a*d)^(9/4
)*Sqrt[a + b*x])
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/4}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} - \frac{(7d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/4}}\,dx}{6(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}} + \frac{(7d^2)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/4}}\,d}{4(bc-ad)^2}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}} + \frac{7d^2\sqrt{a+bx}}{(bc-ad)^3\sqrt[4]{c+dx}} -$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}} + \frac{7d^2\sqrt{a+bx}}{(bc-ad)^3\sqrt[4]{c+dx}} -$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}} + \frac{7d^2\sqrt{a+bx}}{(bc-ad)^3\sqrt[4]{c+dx}} +$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}} + \frac{7d^2\sqrt{a+bx}}{(bc-ad)^3\sqrt[4]{c+dx}} +$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}} + \frac{7d^2\sqrt{a+bx}}{(bc-ad)^3\sqrt[4]{c+dx}} +$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}} + \frac{7d^2\sqrt{a+bx}}{(bc-ad)^3\sqrt[4]{c+dx}} -$$

Mathematica [C] time = 0.0375448, size = 73, normalized size = 0.28

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(-\frac{3}{2},\frac{5}{4};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(5/4))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{2}}(dx+c)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x)
```

[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="maxima")

[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(5/4)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{b^3d^2x^5 + a^3c^2 + \left(2\,b^3cd + 3\,ab^2d^2\right)x^4 + \left(b^3c^2 + 6\,ab^2cd + 3\,a^2bd^2\right)x^3 + \left(3\,ab^2c^2 + 6\,a^2bcd + a^3d^2\right)x^2 + \left(3\,a^2bc^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="fricas")

[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*d + 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(5/4),x)

[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(5/4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="giac")

[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(5/4)), x)

3.1665 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=207

$$-\frac{320b^{3/4}(bc-ad)^{13/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{33d^5\sqrt{a+bx}}+\frac{56b(a+bx)^{5/2}\sqrt[4]{c+dx}}{33d^2}-\frac{80b(a+bx)^{3/2}\sqrt[4]{c+dx}(bc}{33d^3}$$

```
[Out] (-4*(a + b*x)^(7/2))/(3*d*(c + d*x)^(3/4)) + (160*b*(b*c - a*d)^2*Sqrt[a +
b*x]*(c + d*x)^(1/4))/(33*d^4) - (80*b*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x
)^(1/4))/(33*d^3) + (56*b*(a + b*x)^(5/2)*(c + d*x)^(1/4))/(33*d^2) - (320*
b^(3/4)*(b*c - a*d)^(13/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(33*d^5*Sqrt[a + b*x
])
```

Rubi [A] time = 0.136313, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 224, 221}

$$-\frac{320b^{3/4}(bc-ad)^{13/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{33d^5\sqrt{a+bx}}+\frac{56b(a+bx)^{5/2}\sqrt[4]{c+dx}}{33d^2}-\frac{80b(a+bx)^{3/2}\sqrt[4]{c+dx}(bc-ad)}{33d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(7/4),x]
```

```
[Out] (-4*(a + b*x)^(7/2))/(3*d*(c + d*x)^(3/4)) + (160*b*(b*c - a*d)^2*Sqrt[a +
b*x]*(c + d*x)^(1/4))/(33*d^4) - (80*b*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x
)^(1/4))/(33*d^3) + (56*b*(a + b*x)^(5/2)*(c + d*x)^(1/4))/(33*d^2) - (320*
b^(3/4)*(b*c - a*d)^(13/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(33*d^5*Sqrt[a + b*x
])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{7/4}}\,dx = -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{(14b)\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/4}}\,dx}{3d}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{56b(a+bx)^{5/2}\sqrt[4]{c+dx}}{33d^2} - \frac{(140b(bc-ad))\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}}\,dx}{33d^2}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{33d^3} + \frac{56b(a+bx)^{5/2}\sqrt[4]{c+dx}}{33d^2} + \frac{\left(40b(bc-ad)^2\right)\int \frac{\sqrt{a+}}{(c+dx)}}{11d^3}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{160b(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{33d^4} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{33d^3} + \frac{56b(a+bx)^{5/2}}{33d^2}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{160b(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{33d^4} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{33d^3} + \frac{56b(a+bx)^{5/2}}{33d^2}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{160b(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{33d^4} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{33d^3} + \frac{56b(a+bx)^{5/2}}{33d^2}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{160b(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{33d^4} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{33d^3} + \frac{56b(a+bx)^{5/2}}{33d^2}$$

Mathematica [C] time = 0.0705959, size = 73, normalized size = 0.35

$$\frac{2(a+bx)^{9/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4} {}_2F_1\left(\frac{7}{4},\frac{9}{2};\frac{11}{2};\frac{d(a+bx)}{ad-bc}\right)}{9b(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(7/4),x]
```

```
[Out] (2*(a + b*x)^(9/2)*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[7/4,
 9/2, 11/2, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(7/4))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{2}} (dx+c)^{-\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(7/4),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/2)/(d*x + c)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\sqrt{bx+a}(dx+c)^{\frac{1}{4}}}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)*(d*x + c)^
(1/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/2)/(d*x + c)^(7/4), x)
```

3.1666 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=137

$$-\frac{16b^{3/4}(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{3d^3\sqrt{a+bx}}+\frac{8b\sqrt{a+bx}\sqrt[4]{c+dx}}{3d^2}-\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}}$$

```
[Out] (-4*(a + b*x)^(3/2))/(3*d*(c + d*x)^(3/4)) + (8*b*Sqrt[a + b*x]*(c + d*x)^(
1/4))/(3*d^2) - (16*b^(3/4)*(b*c - a*d)^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a
*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(
3*d^3*Sqrt[a + b*x])
```

Rubi [A] time = 0.0804734, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 224, 221}

$$-\frac{16b^{3/4}(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^3\sqrt{a+bx}}+\frac{8b\sqrt{a+bx}\sqrt[4]{c+dx}}{3d^2}-\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(7/4),x]
```

```
[Out] (-4*(a + b*x)^(3/2))/(3*d*(c + d*x)^(3/4)) + (8*b*Sqrt[a + b*x]*(c + d*x)^(
1/4))/(3*d^2) - (16*b^(3/4)*(b*c - a*d)^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a
*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(
3*d^3*Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/4}}\,dx &= -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}}+\frac{(2b)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx}{d}\\ &= -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}}+\frac{8b\sqrt{a+bx}\sqrt[4]{c+dx}}{3d^2}-\frac{(4b(bc-ad))\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{3d^2}\\ &= -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}}+\frac{8b\sqrt{a+bx}\sqrt[4]{c+dx}}{3d^2}-\frac{(16b(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3d^3}\\ &= -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}}+\frac{8b\sqrt{a+bx}\sqrt[4]{c+dx}}{3d^2}-\frac{\left(16b(bc-ad)\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3d^3\sqrt{a+bx}}\\ &= -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}}+\frac{8b\sqrt{a+bx}\sqrt[4]{c+dx}}{3d^2}-\frac{16b^{3/4}(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^3\sqrt{a+bx}}\end{aligned}$$

Mathematica [C] time = 0.0488485, size = 73, normalized size = 0.53

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4}\,{}_2F_1\left(\frac{7}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(7/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[7/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(7/4))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}\,(dx+c)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(7/4),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{4}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(1/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(7/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(7/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(7/4), x)
```

3.1667 $\int \frac{\sqrt{a+bx}}{(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=111

$$\frac{8b^{3/4}\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{3d^2\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}}$$

```
[Out] (-4*Sqrt[a + b*x])/(3*d*(c + d*x)^(3/4)) + (8*b^(3/4)*(b*c - a*d)^(1/4)*Sqr
t[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/
(b*c - a*d)^(1/4)], -1])/(3*d^2*Sqrt[a + b*x])
```

Rubi [A] time = 0.0637834, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {47, 63, 224, 221}

$$\frac{8b^{3/4}\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^2\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(7/4),x]
```

```
[Out] (-4*Sqrt[a + b*x])/(3*d*(c + d*x)^(3/4)) + (8*b^(3/4)*(b*c - a*d)^(1/4)*Sqr
t[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/
(b*c - a*d)^(1/4)], -1])/(3*d^2*Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx}}{(c+dx)^{7/4}}\,dx &= -\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}} + \frac{(2b)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{3d} \\
&= -\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}} + \frac{(8b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3d^2} \\
&= -\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}} + \frac{\left(8b\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3d^2\sqrt{a+bx}} \\
&= -\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}} + \frac{8b^{3/4}\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^2\sqrt{a+bx}}
\end{aligned}
$$

Mathematica [C] time = 0.044321, size = 73, normalized size = 0.66

$$
\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4} {}_2F_1\left(\frac{3}{2},\frac{7}{4};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(c+dx)^{7/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(7/4),x]

[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[3/2,
 7/4, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(7/4))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$
\int \sqrt{bx+a}\,(dx+c)^{-\frac{7}{4}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(7/4),x)

[Out] int((b*x+a)^(1/2)/(d*x+c)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{7}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{4}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+bx}}{(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(7/4),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(7/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx+a}}{(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(7/4), x)
```

3.1668 $\int \frac{1}{\sqrt{a+bx}(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=118

$$\frac{4b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{3d\sqrt{a+bx}(bc-ad)^{3/4}}+\frac{4\sqrt{a+bx}}{3(c+dx)^{3/4}(bc-ad)}$$

```
[Out] (4*Sqrt[a + b*x])/(3*(b*c - a*d)*(c + d*x)^(3/4)) + (4*b^(3/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(3*d*(b*c - a*d)^(3/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.0700133, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {51, 63, 224, 221}

$$\frac{4b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d\sqrt{a+bx}(bc-ad)^{3/4}}+\frac{4\sqrt{a+bx}}{3(c+dx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(7/4)),x]
```

```
[Out] (4*Sqrt[a + b*x])/(3*(b*c - a*d)*(c + d*x)^(3/4)) + (4*b^(3/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(3*d*(b*c - a*d)^(3/4)*Sqrt[a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a+bx}(c+dx)^{7/4}}\,dx &= \frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}}+\frac{b\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{3(bc-ad)}\\
&= \frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}}+\frac{(4b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3d(bc-ad)}\\
&= \frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}}+\frac{\left(4b\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3d(bc-ad)\sqrt{a+bx}}\\
&= \frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}}+\frac{4b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d(bc-ad)^{3/4}\sqrt{a+bx}}
\end{aligned}
$$

Mathematica [C] time = 0.0369033, size = 71, normalized size = 0.6

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4}{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(7/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[1/2, 7
/4, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(7/4))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}}(dx+c)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(7/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{4}}}{bd^2x^3+ac^2+\left(2\,bcd+ad^2\right)x^2+\left(bc^2+2\,acd\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*d^
2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(7/4),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(7/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(7/4)), x)
```

3.1669 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=146

$$-\frac{10b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{3\sqrt{a+bx}(bc-ad)^{7/4}}-\frac{10d\sqrt{a+bx}}{3(c+dx)^{3/4}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}(c+dx)^{3/4}(bc-ad)}$$

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4)) - (10*d*Sqrt[a + b*x])/(3*(b
*c - a*d)^2*(c + d*x)^(3/4)) - (10*b^(3/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d)
)]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*(
b*c - a*d)^(7/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.0784827, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {51, 63, 224, 221}

$$-\frac{10b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt{a+bx}(bc-ad)^{7/4}}-\frac{10d\sqrt{a+bx}}{3(c+dx)^{3/4}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}(c+dx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(7/4)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4)) - (10*d*Sqrt[a + b*x])/(3*(b
*c - a*d)^2*(c + d*x)^(3/4)) - (10*b^(3/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d)
)]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*(
b*c - a*d)^(7/4)*Sqrt[a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/4}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}} - \frac{(5d)\int \frac{1}{\sqrt{a+bx}(c+dx)^{7/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}} - \frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}} - \frac{(5bd)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}}\,dx}{6(bc-ad)^2}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}} - \frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}} - \frac{(10b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x \ldots\right)}{3(bc-ad)^2}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}} - \frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}} - \frac{\left(10b\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\ldots}} \ldots\right)}{3(bc-ad)^2\sqrt{a \ldots}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}} - \frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}} - \frac{10b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\ldots}{\sqrt[4]{bc\ldots}}\ldots\right)\right)}{3(bc-ad)^{7/4}\sqrt{a+b\ldots}}$$

Mathematica [C] time = 0.0355567, size = 71, normalized size = 0.49

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4} {}_2F_1\left(-\frac{1}{2},\frac{7}{4};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(7/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[-1/2, 7/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(7/4))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{3}{2}}(dx+c)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(7/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{4}}}{b^2d^2x^4+a^2c^2+2\left(b^2cd+abd^2\right)x^3+\left(b^2c^2+4\,abcd+a^2d^2\right)x^2+2\left(abc^2+a^2cd\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*d
+ a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c*d
)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(7/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(7/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(7/4)), x)
```

3.1670 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=178

$$\frac{5b^{3/4}d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{\sqrt{a+bx}(bc-ad)^{11/4}}+\frac{5d^2\sqrt{a+bx}}{(c+dx)^{3/4}(bc-ad)^3}+\frac{3d}{\sqrt{a+bx}(c+dx)^{3/4}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+dx)^{3/4}(bc-ad)}$$

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(3/4)) + (3*d)/((b*c - a*d)^2*S
qrt[a + b*x]*(c + d*x)^(3/4)) + (5*d^2*Sqrt[a + b*x])/((b*c - a*d)^3*(c + d
*x)^(3/4)) + (5*b^(3/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c - a*d)^(11/4)*S
qrt[a + b*x])
```

Rubi [A] time = 0.103164, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {51, 63, 224, 221}

$$\frac{5b^{3/4}d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt{a+bx}(bc-ad)^{11/4}}+\frac{5d^2\sqrt{a+bx}}{(c+dx)^{3/4}(bc-ad)^3}+\frac{3d}{\sqrt{a+bx}(c+dx)^{3/4}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(7/4)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(3/4)) + (3*d)/((b*c - a*d)^2*S
qrt[a + b*x]*(c + d*x)^(3/4)) + (5*d^2*Sqrt[a + b*x])/((b*c - a*d)^3*(c + d
*x)^(3/4)) + (5*b^(3/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c - a*d)^(11/4)*S
qrt[a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{7/4}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} - \frac{(3d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}} + \frac{\left(15d^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{7/4}}\,d}{4(bc-ad)^2}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}} + \frac{5d^2\sqrt{a+bx}}{(bc-ad)^3(c+dx)^{3/4}} +$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}} + \frac{5d^2\sqrt{a+bx}}{(bc-ad)^3(c+dx)^{3/4}} +$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}} + \frac{5d^2\sqrt{a+bx}}{(bc-ad)^3(c+dx)^{3/4}} +$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}} + \frac{5d^2\sqrt{a+bx}}{(bc-ad)^3(c+dx)^{3/4}} +$$

Mathematica [C] time = 0.0382015, size = 73, normalized size = 0.41

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4} {}_2F_1\left(-\frac{3}{2},\frac{7}{4};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(7/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[-3/2, 7/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(7/4))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{2}} (dx+c)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x)
```

[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x, algorithm="maxima")

[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(7/4)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{4}}}{b^3d^2x^5+a^3c^2+\left(2\,b^3cd+3\,ab^2d^2\right)x^4+\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^3+\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^2+\left(3\,a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x, algorithm="fricas")

[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*d + 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(7/4),x)

[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(7/4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x, algorithm="giac")

[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(7/4)), x)

3.1671 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=286

$$-\frac{448b^{5/4}(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{15d^5\sqrt{a+bx}}+\frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3}-\frac{224b^2\sqrt{a+bx}(c+dx)^{3/4}}{15d^4}$$

```
[Out] (-4*(a + b*x)^(7/2))/(5*d*(c + d*x)^(5/4)) - (56*b*(a + b*x)^(5/2))/(5*d^2*
(c + d*x)^(1/4)) - (224*b^2*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4))/(15*
d^4) + (112*b^2*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(9*d^3) + (448*b^(5/4)*(b*
c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4
)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(15*d^5*Sqrt[a + b*x]) - (448*b
^(5/4)*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(15*d^5*Sqrt[a + b*x]
)
```

Rubi [A] time = 0.296989, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 50, 63, 307, 224, 221, 1200, 1199, 424}

$$\frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3}-\frac{224b^2\sqrt{a+bx}(c+dx)^{3/4}(bc-ad)}{15d^4}-\frac{448b^{5/4}(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15d^5\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(9/4),x]
```

```
[Out] (-4*(a + b*x)^(7/2))/(5*d*(c + d*x)^(5/4)) - (56*b*(a + b*x)^(5/2))/(5*d^2*
(c + d*x)^(1/4)) - (224*b^2*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4))/(15*
d^4) + (112*b^2*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(9*d^3) + (448*b^(5/4)*(b*
c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4
)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(15*d^5*Sqrt[a + b*x]) - (448*b
^(5/4)*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(15*d^5*Sqrt[a + b*x]
)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{9/4}}\,dx = -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} + \frac{(14b)\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/4}}\,dx}{5d}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} + \frac{(28b^2)\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx}{d^2}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} + \frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3} - \frac{(56b^2(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{3d^3}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3} +$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3} +$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3} -$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3} -$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3} -$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3} +$$

Mathematica [C] time = 0.0877278, size = 73, normalized size = 0.26

$$\frac{2(a+bx)^{9/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4} {}_2F_1\left(\frac{9}{4},\frac{9}{2};\frac{11}{2};\frac{d(a+bx)}{ad-bc}\right)}{9b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(9/4),x]
```

```
[Out] (2*(a + b*x)^(9/2)*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[9/4,
 9/2, 11/2, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(9/4))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{2}} (dx+c)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(9/4),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/2)/(d*x + c)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)*(d*x + c)^
(3/4)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/2)/(d*x + c)^(9/4), x)
```

3.1672 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{9/4}} dx$

Optimal. Leaf size=248

$$\frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{5d^4\sqrt{a+bx}}+\frac{48b^2\sqrt{a+bx}(c+dx)^{3/4}}{5d^3}-\frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\right.}{5d^4\sqrt{a+b}}$$

```
[Out] (-4*(a + b*x)^(5/2))/(5*d*(c + d*x)^(5/4)) - (8*b*(a + b*x)^(3/2))/(d^2*(c
+ d*x)^(1/4)) + (48*b^2*Sqrt[a + b*x]*(c + d*x)^(3/4))/(5*d^3) - (96*b^(5/4
)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^
(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^4*Sqrt[a + b*x]) + (96
*b^(5/4)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^4*Sqrt[a + b*x]
)
```

Rubi [A] time = 0.257393, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 50, 63, 307, 224, 221, 1200, 1199, 424}

$$\frac{48b^2\sqrt{a+bx}(c+dx)^{3/4}}{5d^3}+\frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^4\sqrt{a+bx}}-\frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\right.\right.}{5d^4\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(9/4),x]
```


```
[Out] (-4*(a + b*x)^(5/2))/(5*d*(c + d*x)^(5/4)) - (8*b*(a + b*x)^(3/2))/(d^2*(c
+ d*x)^(1/4)) + (48*b^2*Sqrt[a + b*x]*(c + d*x)^(3/4))/(5*d^3) - (96*b^(5/4
)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^
(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^4*Sqrt[a + b*x]) + (96
*b^(5/4)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^4*Sqrt[a + b*x]
)
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{9/4}}\,dx = -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} + \frac{(2b)\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/4}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{(12b^2)\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{d^2}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}(c+dx)^{3/4}}{5d^3} - \frac{(24b^2(bc-ad))\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{5d^3}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}(c+dx)^{3/4}}{5d^3} - \frac{(96b^2(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x\right.}{5d^4}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}(c+dx)^{3/4}}{5d^3} + \frac{(96b^{3/2}(bc-ad)^{3/2})\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\right.}{5d^4}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}(c+dx)^{3/4}}{5d^3} + \frac{\left(96b^{3/2}(bc-ad)^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int -\right.}{5d^4\sqrt{a+bx}}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}(c+dx)^{3/4}}{5d^3} + \frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.}{5d^4\sqrt{a+bx}}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}(c+dx)^{3/4}}{5d^3} - \frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.}{5d^4\sqrt{a+bx}}$$

Mathematica [C] time = 0.0734947, size = 73, normalized size = 0.29

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}\,{}_2F_1\left(\frac{9}{4},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(9/4),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[9/4,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(9/4))
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{2}}(dx+c)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(9/4),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*(d*x + c)^(3/4)/(d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(9/4), x)
```

3.1673 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=222

$$-\frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{5d^3\sqrt{a+bx}}+\frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}-$$

```
[Out] (-4*(a + b*x)^(3/2))/(5*d*(c + d*x)^(5/4)) - (24*b*Sqrt[a + b*x])/(5*d^2*(c
 + d*x)^(1/4)) + (48*b^(5/4)*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/(b*c -
a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/
(5*d^3*Sqrt[a + b*x]) - (48*b^(5/4)*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(5*d^3*Sqrt[a + b*x])
```

Rubi [A] time = 0.23486, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {47, 63, 307, 224, 221, 1200, 1199, 424}

$$-\frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}+\frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}-\frac{24b\sqrt{a}}{5d^2\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(9/4),x]
```

```
[Out] (-4*(a + b*x)^(3/2))/(5*d*(c + d*x)^(5/4)) - (24*b*Sqrt[a + b*x])/(5*d^2*(c
 + d*x)^(1/4)) + (48*b^(5/4)*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/(b*c -
a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/
(5*d^3*Sqrt[a + b*x]) - (48*b^(5/4)*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(5*d^3*Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
```

```
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{9/4}}\,dx = -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} + \frac{(6b)\int \frac{\sqrt{a+bx}}{(c+dx)^{5/4}}\,dx}{5d}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} + \frac{\left(12b^2\right)\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{5d^2}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} + \frac{\left(48b^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d^3}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} - \frac{\left(48b^{3/2}\sqrt{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d^3} + \frac{\left(48b^{3/2}\sqrt{bc \cdots\right.}{\cdots}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} - \frac{\left(48b^{3/2}\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d^3\sqrt{a+bx}} + \frac{(4 \cdots}{\cdots}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} - \frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}} + \frac{\left(48b^{3/2}\sqrt{bc \cdots\right.}{\cdots}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} + \frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}} - \frac{48b^{5/4}(bc - \cdots}{\cdots}$$

Mathematica [C] time = 0.0641297, size = 73, normalized size = 0.33

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4} {}_2F_1\left(\frac{9}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(9/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[9/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(9/4))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}} (dx+c)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(9/4),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{4}}}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(3/4)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(9/4), x)
```

3.1674 $\int \frac{\sqrt{a+bx}}{(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=232

$$\frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{5d^2\sqrt{a+bx}\sqrt[4]{bc-ad}}-\frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^2\sqrt{a+bx}\sqrt[4]{bc-ad}}+\frac{8b\sqrt{a+bx}}{5d\sqrt[4]{c+dx}(bc-ad)}-\frac{4}{5d(}$$

```
[Out] (-4*Sqrt[a + b*x])/(5*d*(c + d*x)^(5/4)) + (8*b*Sqrt[a + b*x])/(5*d*(b*c -
a*d)*(c + d*x)^(1/4)) - (8*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellip
ticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^2*(b*c
- a*d)^(1/4)*Sqrt[a + b*x]) + (8*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]
*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^2
*(b*c - a*d)^(1/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.233056, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 51, 63, 307, 224, 221, 1200, 1199, 424}

$$\frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^2\sqrt{a+bx}\sqrt[4]{bc-ad}}-\frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^2\sqrt{a+bx}\sqrt[4]{bc-ad}}+\frac{8b\sqrt{a+bx}}{5d\sqrt[4]{c+dx}(bc-ad)}-\frac{4\sqrt{a+bx}}{5d(c+dx)^{5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(9/4),x]
```

```
[Out] (-4*Sqrt[a + b*x])/(5*d*(c + d*x)^(5/4)) + (8*b*Sqrt[a + b*x])/(5*d*(b*c -
a*d)*(c + d*x)^(1/4)) - (8*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellip
ticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^2*(b*c
- a*d)^(1/4)*Sqrt[a + b*x]) + (8*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]
*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^2
*(b*c - a*d)^(1/4)*Sqrt[a + b*x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{9/4}}\,dx = -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{(2b)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/4}}\,dx}{5d}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} - \frac{(2b^2)\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{5d(bc-ad)}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} - \frac{(8b^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d^2(bc-ad)}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} + \frac{(8b^{3/2})\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d^2\sqrt{bc-ad}} - \frac{(8b^{3/2})\,\text{Subst}}{}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} + \frac{\left(8b^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d^2\sqrt{bc-ad}\sqrt{a+bx}} - \frac{(8b^{3/}}{}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} + \frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^2\sqrt[4]{bc-ad}\sqrt{a+bx}} - \frac{\left(8b^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{S}}{5c}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} - \frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^2\sqrt[4]{bc-ad}\sqrt{a+bx}} + \frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F}{5d^2\sqrt[4]{bc}}$$

Mathematica [C] time = 0.0538914, size = 73, normalized size = 0.31

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}{}_2F_1\left(\frac{3}{2},\frac{9}{4};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(9/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[3/2,
 9/4, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(9/4))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\,(dx+c)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(9/4),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
 c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(9/4),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(9/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(9/4), x)
```

3.1675 $\int \frac{1}{\sqrt{a+bx}(c+dx)^{9/4}} \, dx$

Optimal. Leaf size=236

$$\frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{5d\sqrt{a+bx}(bc-ad)^{5/4}}-\frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d\sqrt{a+bx}(bc-ad)^{5/4}}+\frac{12b\sqrt{a+bx}}{5\sqrt[4]{c+dx}(bc-ad)^2}+\frac{}{5}$$

```
[Out] (4*Sqrt[a + b*x])/(5*(b*c - a*d)*(c + d*x)^(5/4)) + (12*b*Sqrt[a + b*x])/(5
*(b*c - a*d)^2*(c + d*x)^(1/4)) - (12*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a
*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(
5*d*(b*c - a*d)^(5/4)*Sqrt[a + b*x]) + (12*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*
c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -
1])/(5*d*(b*c - a*d)^(5/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.235882, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {51, 63, 307, 224, 221, 1200, 1199, 424}

$$\frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d\sqrt{a+bx}(bc-ad)^{5/4}}-\frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d\sqrt{a+bx}(bc-ad)^{5/4}}+\frac{12b\sqrt{a+bx}}{5\sqrt[4]{c+dx}(bc-ad)^2}+\frac{4\sqrt{}}{5(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(9/4)),x]
```

```
[Out] (4*Sqrt[a + b*x])/(5*(b*c - a*d)*(c + d*x)^(5/4)) + (12*b*Sqrt[a + b*x])/(5
*(b*c - a*d)^2*(c + d*x)^(1/4)) - (12*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a
*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(
5*d*(b*c - a*d)^(5/4)*Sqrt[a + b*x]) + (12*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*
c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -
1])/(5*d*(b*c - a*d)^(5/4)*Sqrt[a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
```

```
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{9/4}}\,dx = \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{(3b)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/4}}\,dx}{5(bc-ad)}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} - \frac{(3b^2)\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{5(bc-ad)^2}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} - \frac{(12b^2)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d(bc-ad)^2}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} + \frac{(12b^{3/2})\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d(bc-ad)^{3/2}} -$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} + \frac{\left(12b^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{\right.}{\ldots}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} + \frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d(bc-ad)^{5/4}\sqrt{a+bx}} - \ldots$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} - \frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d(bc-ad)^{5/4}\sqrt{a+bx}} + \ldots$$

Mathematica [C] time = 0.0469273, size = 71, normalized size = 0.3

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(9/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[1/2, 9
/4, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(9/4))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}}(dx+c)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(9/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{bd^3x^4+ac^3+\left(3\,bcd^2+ad^3\right)x^3+3\left(bc^2d+acd^2\right)x^2+\left(bc^3+3\,ac^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b*d^3*x^4 + a*c^3 + (3*b*c*d^2 + a*
d^3)*x^3 + 3*(b*c^2*d + a*c*d^2)*x^2 + (b*c^3 + 3*a*c^2*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(9/4),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(9/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(9/4)), x)
```

3.1676 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=262

$$-\frac{42b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{5\sqrt{a+bx}(bc-ad)^{9/4}}+\frac{42b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}(bc-ad)^{9/4}}-\frac{42bd\sqrt{a+bx}}{5\sqrt[4]{c+dx}(bc-ad)^3}-$$

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(5/4)) - (14*d*Sqrt[a + b*x])/(5*(b
*c - a*d)^2*(c + d*x)^(5/4)) - (42*b*d*Sqrt[a + b*x])/(5*(b*c - a*d)^3*(c +
 d*x)^(1/4)) + (42*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(9/4)
*Sqrt[a + b*x]) - (42*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[
ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(9
/4)*Sqrt[a + b*x])
```

Rubi [A] time = 0.256844, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {51, 63, 307, 224, 221, 1200, 1199, 424}

$$-\frac{42b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}(bc-ad)^{9/4}}+\frac{42b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}(bc-ad)^{9/4}}-\frac{42bd\sqrt{a+bx}}{5\sqrt[4]{c+dx}(bc-ad)^3}-\frac{14}{5(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(9/4)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(5/4)) - (14*d*Sqrt[a + b*x])/(5*(b
*c - a*d)^2*(c + d*x)^(5/4)) - (42*b*d*Sqrt[a + b*x])/(5*(b*c - a*d)^3*(c +
 d*x)^(1/4)) + (42*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(9/4)
*Sqrt[a + b*x]) - (42*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[
ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(9
/4)*Sqrt[a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{9/4}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{5/4}} - \frac{(7d)\int \frac{1}{\sqrt{a+bx}(c+dx)^{9/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{(21bd)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/4}}\,dx}{10(bc-ad)^2}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+bx}}{5(bc-ad)^3\sqrt[4]{c+dx}} + \frac{(21b^2d)\int}{10(b}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+bx}}{5(bc-ad)^3\sqrt[4]{c+dx}} + \frac{(42b^2)\,\mathrm{Sub}}{}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+bx}}{5(bc-ad)^3\sqrt[4]{c+dx}} - \frac{(42b^{3/2})\,\mathrm{Su}}{}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+bx}}{5(bc-ad)^3\sqrt[4]{c+dx}} - \frac{\left(42b^{3/2}\sqrt{\frac{d(}{-}}\right.}{}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+bx}}{5(bc-ad)^3\sqrt[4]{c+dx}} - \frac{42b^{5/4}\sqrt{-\frac{d}{}}}{5}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+bx}}{5(bc-ad)^3\sqrt[4]{c+dx}} + \frac{42b^{5/4}\sqrt{-\frac{d}{}}}{5}$$

Mathematica [C] time = 0.046052, size = 71, normalized size = 0.27

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4} {}_2F_1\left(-\frac{1}{2},\frac{9}{4};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(9/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[-1/2, 9/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(9/4))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{3}{2}}(dx+c)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x)
```

[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="maxima")

[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(9/4)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{b^2d^3x^5+a^2c^3+\left(3\,b^2cd^2+2\,abd^3\right)x^4+\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^3+\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^2+\left(2\,ab\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="fricas")

[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^2*d^3*x^5 + a^2*c^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*x^4 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^3 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^2 + (2*a*b*c^3 + 3*a^2*c^2*d)*x), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(9/4),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="giac")

[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(9/4)), x)

3.1677 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=303

$$\frac{77b^{5/4}d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right),-1\right)}{5\sqrt{a+bx}(bc-ad)^{13/4}}-\frac{77b^{5/4}d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}(bc-ad)^{13/4}}+\frac{77bd^2\sqrt{a+bx}}{5\sqrt[4]{c+dx}(bc-ad)^4}+$$

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(5/4)) + (11*d)/(3*(b*c - a*d)^
2*Sqrt[a + b*x]*(c + d*x)^(5/4)) + (77*d^2*Sqrt[a + b*x])/(15*(b*c - a*d)^3
*(c + d*x)^(5/4)) + (77*b*d^2*Sqrt[a + b*x])/(5*(b*c - a*d)^4*(c + d*x)^(1/
4)) - (77*b^(5/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^
(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(13/4)*Sqrt[
a + b*x]) + (77*b^(5/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(13/4)
*Sqrt[a + b*x])
```

Rubi [A] time = 0.289997, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {51, 63, 307, 224, 221, 1200, 1199, 424}

$$\frac{77b^{5/4}d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}(bc-ad)^{13/4}}-\frac{77b^{5/4}d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}(bc-ad)^{13/4}}+\frac{77bd^2\sqrt{a+bx}}{5\sqrt[4]{c+dx}(bc-ad)^4}+\frac{7}{15(c+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(9/4)),x]
```


```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(5/4)) + (11*d)/(3*(b*c - a*d)^
2*Sqrt[a + b*x]*(c + d*x)^(5/4)) + (77*d^2*Sqrt[a + b*x])/(15*(b*c - a*d)^3
*(c + d*x)^(5/4)) + (77*b*d^2*Sqrt[a + b*x])/(5*(b*c - a*d)^4*(c + d*x)^(1/
4)) - (77*b^(5/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^
(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(13/4)*Sqrt[
a + b*x]) + (77*b^(5/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(13/4)
*Sqrt[a + b*x])
```


Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{9/4}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} - \frac{(11d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{9/4}}\,dx}{6(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/4}} + \frac{\left(77d^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{9/4}}}{12(bc-ad)^2}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/4}} + \frac{77d^2\sqrt{a+bx}}{15(bc-ad)^3(c+dx)^{5/}}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/4}} + \frac{77d^2\sqrt{a+bx}}{15(bc-ad)^3(c+dx)^{5/}}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/4}} + \frac{77d^2\sqrt{a+bx}}{15(bc-ad)^3(c+dx)^{5/}}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/4}} + \frac{77d^2\sqrt{a+bx}}{15(bc-ad)^3(c+dx)^{5/}}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/4}} + \frac{77d^2\sqrt{a+bx}}{15(bc-ad)^3(c+dx)^{5/}}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/4}} + \frac{77d^2\sqrt{a+bx}}{15(bc-ad)^3(c+dx)^{5/}}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/4}} + \frac{77d^2\sqrt{a+bx}}{15(bc-ad)^3(c+dx)^{5/}}$$

Mathematica [C] time = 0.0478869, size = 73, normalized size = 0.24

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}{}_2F_1\left(-\frac{3}{2},\frac{9}{4};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(9/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[-3/2, 9/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(9/4))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{2}}(dx+c)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(9/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{4}}}{b^3d^3x^6+a^3c^3+3\left(b^3cd^2+ab^2d^3\right)x^5+3\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)x^4+\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^3*d^3*x^6 + a^3*c^3 + 3*(b^3*c*d^
2 + a*b^2*d^3)*x^5 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c
^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^3 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*x^2 + 3*(a^2*b*c^3 + a^3*c^2*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(9/4)), x)
```

3.1678 $\int (a+bx)^{3/4}(c+dx)^{5/4}\,dx$

Optimal. Leaf size=205

$$\frac{5(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}(bc-ad)^2}{96b^2d}+\frac{5(a+bx)^{7/4}\sqrt[4]{c+dx}(bc}{24b^2}$$

```
[Out] (5*(b*c - a*d)^2*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(96*b^2*d) + (5*(b*c - a*
d)*(a + b*x)^(7/4)*(c + d*x)^(1/4))/(24*b^2) + ((a + b*x)^(7/4)*(c + d*x)^(
5/4))/(3*b) + (5*(b*c - a*d)^3*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c
 + d*x)^(1/4))])/(64*b^(9/4)*d^(7/4)) - (5*(b*c - a*d)^3*ArcTanh[(d^(1/4)*(
a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(64*b^(9/4)*d^(7/4))
```

Rubi [A] time = 0.13922, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 331, 298, 205, 208}

$$\frac{5(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}(bc-ad)^2}{96b^2d}+\frac{5(a+bx)^{7/4}\sqrt[4]{c+dx}(bc}{24b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)*(c + d*x)^(5/4),x]
```

```
[Out] (5*(b*c - a*d)^2*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(96*b^2*d) + (5*(b*c - a*
d)*(a + b*x)^(7/4)*(c + d*x)^(1/4))/(24*b^2) + ((a + b*x)^(7/4)*(c + d*x)^(
5/4))/(3*b) + (5*(b*c - a*d)^3*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c
 + d*x)^(1/4))])/(64*b^(9/4)*d^(7/4)) - (5*(b*c - a*d)^3*ArcTanh[(d^(1/4)*(
a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(64*b^(9/4)*d^(7/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (a+bx)^{3/4}(c+dx)^{5/4}\,dx = \frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b}+\frac{(5(bc-ad))\int (a+bx)^{3/4}\sqrt[4]{c+dx}\,dx}{12b}$$

$$= \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2}+\frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b}+\frac{\left(5(bc-ad)^2\right)\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx}{96b^2}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d}+\frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2}+\frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b}-$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d}+\frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2}+\frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b}-$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d}+\frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2}+\frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b}-$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d}+\frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2}+\frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b}-$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d}+\frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2}+\frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b}+$$

Mathematica [C] time = 0.0599047, size = 73, normalized size = 0.36

$$\frac{4(a+bx)^{7/4}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{d(a+bx)}{ad-bc}\right)}{7b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)*(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(7/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 7/4, 11/4, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{4}} (dx+c)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(3/4)*(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{4}} (dx+c)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)*(d*x + c)^(5/4), x)
```

Fricas [B] time = 4.42256, size = 4761, normalized size = 23.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/384*(60*b^2*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 22
0*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6
*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4)*arc
tan(((b^10*c^3*d^5 - 3*a*b^9*c^2*d^6 + 3*a^2*b^8*c*d^7 - a^3*b^7*d^8)*(b*x
+ a)^(3/4)*(d*x + c)^(1/4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^1
0*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 9
24*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^
3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))
^(3/4) + (b^8*d^5*x + a*b^7*d^5)*sqrt(((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^
4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d
^6)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*d^4*x + a*b^4*d^4)*sqrt((b^12*c^12 -
 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^
8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^
11*b*c*d^11 + a^12*d^12)/(b^9*d^7)))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(b^9*d^7))^(3/4))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^1
0*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^
5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a
^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^
13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 +
495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b
```

```
^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^
10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) + 15*b^2*d*((b^12*c^12 - 12*a*b^
11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^
4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a
^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d
^11 + a^12*d^12)/(b^9*d^7))^(1/4)*log(-5*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*d^2*x + a*b^2*d^2
)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d
^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*
a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c
^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4))/(b*x + a)) - 15*b
^2*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4)*log(-5*((b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x + a)^(3/4)*(d*x + c)^(1/4
) - (b^3*d^2*x + a*b^2*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^
10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7)
)^(1/4))/(b*x + a)) - 4*(32*b^2*d^2*x^2 + 5*b^2*c^2 + 42*a*b*c*d - 15*a^2*d
^2 + 4*(13*b^2*c*d + 3*a*b*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + bx)^{\frac{3}{4}} (c + dx)^{\frac{5}{4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)*(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(3/4)*(c + d*x)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^{\frac{3}{4}} (dx + c)^{\frac{5}{4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/4)*(d*x + c)^(5/4), x)
```

3.1679 $\int \frac{(c+dx)^{5/4}}{\sqrt[4]{a+bx}} \, dx$

Optimal. Leaf size=167

$$-\frac{5(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(bc-ad)^2 \tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}(bc-ad)}{8b^2}+\frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}$$

```
[Out] (5*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(8*b^2) + ((a + b*x)^(3/4)*
(c + d*x)^(5/4))/(2*b) - (5*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/
(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(9/4)*d^(3/4)) + (5*(b*c - a*d)^2*ArcTanh
[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(9/4)*d^(3/4))
```

Rubi [A] time = 0.102389, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 331, 298, 205, 208}

$$-\frac{5(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(bc-ad)^2 \tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}(bc-ad)}{8b^2}+\frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(1/4),x]
```

```
[Out] (5*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(8*b^2) + ((a + b*x)^(3/4)*
(c + d*x)^(5/4))/(2*b) - (5*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/
(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(9/4)*d^(3/4)) + (5*(b*c - a*d)^2*ArcTanh
[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(9/4)*d^(3/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/4}}{\sqrt[4]{a+bx}}\,dx &= \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}+\frac{(5(bc-ad))\int \frac{\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\,dx}{8b}\\
&= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2}+\frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}+\frac{\left(5(bc-ad)^2\right)\int \frac{1}{\sqrt[4]{a+bx}(c+dx)^{3/4}}\,dx}{32b^2}\\
&= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2}+\frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}+\frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx,x,\right.}{8b^3}\\
&= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2}+\frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}+\frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right.}{8b^3}\\
&= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2}+\frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}+\frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}x^2}\,dx,x,\frac{\sqrt[4]{a+}}{\sqrt[4]{c+}}\right.}{16b^2\sqrt{d}}\\
&= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2}+\frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(bc-ad}{}
\end{aligned}$$

Mathematica [C] time = 0.0438787, size = 73, normalized size = 0.44

$$\frac{4(a+bx)^{3/4}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{d(a+bx)}{ad-bc}\right)}{3b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(3/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 3/4, 7/4, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{4}} \frac{1}{\sqrt[4]{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(1/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(1/4), x)
```

Fricas [B] time = 3.64042, size = 3090, normalized size = 18.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/32*(20*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*arctan(-((b^9*c^2*d^2 - 2*a*b^8*c*d^
3 + a^2*b^7*d^4)*(b*x + a)^(3/4)*(d*x + c)^(1/4)*((b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3
*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(3/4) -
 (b^8*d^2*x + a*b^7*d^2)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*d^2*x + a*b^
4*d^2)*sqrt((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*
b*c*d^7 + a^8*d^8)/(b^9*d^3)))/(b*x + a))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(3/4))/(a*b^8*
c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^
4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d
^8 + (b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 7
0*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d
^7 + a^8*b*d^8)*x)) - 5*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*log(5*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*d*x + a*b^2*d)*((b^
8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
```

```
^8)/(b^9*d^3))^(1/4))/(b*x + a)) + 5*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*log(5*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b^3*d*x +
a*b^2*d)*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^
3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4))/(b*x + a)) - 4*(4*b*d*x + 9*b*c - 5*a*d)
*(b*x + a)^(3/4)*(d*x + c)^(1/4))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c + dx)^{\frac{5}{4}}}{\sqrt[4]{a + bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(1/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^{\frac{5}{4}}}{(bx + a)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(1/4), x)
```

3.1680 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/4}}\,dx$

Optimal. Leaf size=152

$$\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{5\sqrt[4]{d}(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5\sqrt[4]{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{9/4}}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}$$

```
[Out] (5*d*(a + b*x)^(3/4)*(c + d*x)^(1/4))/b^2 - (4*(c + d*x)^(5/4))/(b*(a + b*x
)^(1/4)) - (5*d^(1/4)*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)
*(c + d*x)^(1/4))])/(2*b^(9/4)) + (5*d^(1/4)*(b*c - a*d)*ArcTanh[(d^(1/4)*(
a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(9/4))
```

Rubi [A] time = 0.0975301, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {47, 50, 63, 331, 298, 205, 208}

$$\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{5\sqrt[4]{d}(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5\sqrt[4]{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{9/4}}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(5/4),x]
```

```
[Out] (5*d*(a + b*x)^(3/4)*(c + d*x)^(1/4))/b^2 - (4*(c + d*x)^(5/4))/(b*(a + b*x
)^(1/4)) - (5*d^(1/4)*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)
*(c + d*x)^(1/4))])/(2*b^(9/4)) + (5*d^(1/4)*(b*c - a*d)*ArcTanh[(d^(1/4)*(
a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(9/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/4}}\,dx = -\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}+\frac{(5d)\int \frac{\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\,dx}{b}$$

$$=\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}+\frac{(5d(bc-ad))\int \frac{1}{\sqrt[4]{a+bx}(c+dx)^{3/4}}\,dx}{4b^2}$$

$$=\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}+\frac{(5d(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx,x,\sqrt[4]{a+bx}\right)}{b^3}$$

$$=\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}+\frac{(5d(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^3}$$

$$=\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}+\frac{\left(5\sqrt{d}(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}x^2}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2b^2}-\frac{\left(5\sqrt{d}(bc\right.}{}$$

$$=\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}-\frac{5\sqrt[4]{d}(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5\sqrt[4]{d}(bc-ad)\tanh^{-1}\left(\right.}{2b^{9/4}}$$

Mathematica [C] time = 0.0438857, size = 71, normalized size = 0.47

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{4};\frac{3}{4};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[4]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(5/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -1/4, 3/4, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{4}} (bx+a)^{-\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(5/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/4), x)
```

Fricas [B] time = 3.10835, size = 1817, normalized size = 11.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/4*(20*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 -
 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*arctan(((b^8*c - a*b^7*d)*(b*x + a)^(3
/4)*(d*x + c)^(1/4)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a
^3*b*c*d^4 + a^4*d^5)/b^9)^(3/4) + (b^8*x + a*b^7)*sqrt(((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*x + a*b^4)*sqrt((b^4*c^4*d
 - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9))/(b*
x + a))*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 +
 a^4*d^5)/b^9)^(3/4))/(a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 + 6*a^3*b^2*c^2*d^3
- 4*a^4*b*c*d^4 + a^5*d^5 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d^
3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)) + 5*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*
b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*log(-
5*((b*c - a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*x + a*b^2)*((b^4*c^4*
d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/
4))/(b*x + a)) - 5*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^
2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*log(-5*((b*c - a*d)*(b*x +
a)^(3/4)*(d*x + c)^(1/4) - (b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 +
```

```
6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4))/(b*x + a)) + 4*(b*
d*x - 4*b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^3*x + a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(5/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/4), x)
```

3.1681 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/4}} dx$

Optimal. Leaf size=134

$$-\frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}+\frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}-\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}$$

```
[Out] (-4*d*(c + d*x)^(1/4))/(b^2*(a + b*x)^(1/4)) - (4*(c + d*x)^(5/4))/(5*b*(a
+ b*x)^(5/4)) - (2*d^(5/4)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d
*x)^(1/4))])/b^(9/4) + (2*d^(5/4)*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4
)*(c + d*x)^(1/4))])/b^(9/4)
```

Rubi [A] time = 0.0876632, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {47, 63, 331, 298, 205, 208}

$$-\frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}+\frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}-\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(9/4),x]
```

```
[Out] (-4*d*(c + d*x)^(1/4))/(b^2*(a + b*x)^(1/4)) - (4*(c + d*x)^(5/4))/(5*b*(a
+ b*x)^(5/4)) - (2*d^(5/4)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d
*x)^(1/4))])/b^(9/4) + (2*d^(5/4)*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4
)*(c + d*x)^(1/4))])/b^(9/4)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/4}}\,dx &= -\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}+\frac{d\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/4}}\,dx}{b}\\
&= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}+\frac{d^2\int \frac{1}{\sqrt[4]{a+bx}(c+dx)^{3/4}}\,dx}{b^2}\\
&= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}+\frac{\left(4d^2\right)\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx,x,\sqrt[4]{a+bx}\right)}{b^3}\\
&= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}+\frac{\left(4d^2\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^3}\\
&= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}+\frac{\left(2d^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}x^2}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^2}-\frac{\left(2d^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}x^2}\,dx
\right)}{b^2}\\
&= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}-\frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}+\frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0580159, size = 73, normalized size = 0.54

$$
-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{5}{4};-\frac{1}{4};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(9/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -5/4, -1/4, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{4}} (bx+a)^{-\frac{9}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(9/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/4), x)
```

Fricas [B] time = 2.78767, size = 842, normalized size = 6.28

$$-\frac{20\left(b^4x^2+2\,ab^3x+a^2b^2\right)\left(\frac{d^5}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}b^7d\left(\frac{d^5}{b^9}\right)^{\frac{3}{4}}-\left(b^8x+ab^7\right)\sqrt{\frac{\sqrt{bx+a}\sqrt{dx+c}d^2+\left(b^5x+ab^4\right)\sqrt{\frac{d^5}{b^9}}}{bx+a}}\left(\frac{d^5}{b^9}\right)^{\frac{3}{4}}}{bd^5x+ad^5}\right)-5\left(b^4x^2+2\,ab^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="fricas")
```

```
[Out] -1/5*(20*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9)^(1/4)*arctan(-((b*x + a)
^(3/4)*(d*x + c)^(1/4)*b^7*d*(d^5/b^9)^(3/4) - (b^8*x + a*b^7)*sqrt((sqrt(b
*x + a)*sqrt(d*x + c)*d^2 + (b^5*x + a*b^4)*sqrt(d^5/b^9))/(b*x + a))*(d^5/
b^9)^(3/4))/(b*d^5*x + a*d^5)) - 5*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9
)^(1/4)*log(((b*x + a)^(3/4)*(d*x + c)^(1/4)*d + (b^3*x + a*b^2)*(d^5/b^9)^
(1/4))/(b*x + a)) + 5*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9)^(1/4)*log((
(b*x + a)^(3/4)*(d*x + c)^(1/4)*d - (b^3*x + a*b^2)*(d^5/b^9)^(1/4))/(b*x +
 a)) + 4*(6*b*d*x + b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^4*x^2
+ 2*a*b^3*x + a^2*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/4), x)
```

3.1682 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx$

Optimal. Leaf size=32

$$-\frac{4(c+dx)^{9/4}}{9(a+bx)^{9/4}(bc-ad)}$$

```
[Out] (-4*(c + d*x)^(9/4))/(9*(b*c - a*d)*(a + b*x)^(9/4))
```

Rubi [A] time = 0.002907, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{4(c+dx)^{9/4}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(13/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(9*(b*c - a*d)*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx = -\frac{4(c+dx)^{9/4}}{9(bc-ad)(a+bx)^{9/4}}$$

Mathematica [A] time = 0.0149801, size = 32, normalized size = 1.

$$-\frac{4(c+dx)^{9/4}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(13/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(9*(b*c - a*d)*(a + b*x)^(9/4))
```

Maple [A] time = 0.005, size = 27, normalized size = 0.8

$$\frac{4}{9\,ad-9\,bc}(dx+c)^{\frac{9}{4}}(bx+a)^{-\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(13/4),x)
```

```
[Out] 4/9/(b*x+a)^(9/4)*(d*x+c)^(9/4)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(13/4), x)
```

Fricas [B] time = 2.63168, size = 221, normalized size = 6.91

$$-\frac{4\left(d^2x^2+2\,cdx+c^2\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{9\left(a^3bc-a^4d+\left(b^4c-ab^3d\right)x^3+3\left(ab^3c-a^2b^2d\right)x^2+3\left(a^2b^2c-a^3bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="fricas")
```

```
[Out] -4/9*(d^2*x^2 + 2*c*d*x + c^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^3*b*c - a
^4*d + (b^4*c - a*b^3*d)*x^3 + 3*(a*b^3*c - a^2*b^2*d)*x^2 + 3*(a^2*b^2*c -
 a^3*b*d)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(13/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(13/4), x)
```

3.1683 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}} dx$

Optimal. Leaf size=66

$$\frac{16d(c+dx)^{9/4}}{117(a+bx)^{9/4}(bc-ad)^2} - \frac{4(c+dx)^{9/4}}{13(a+bx)^{13/4}(bc-ad)}$$

```
[Out] (-4*(c + d*x)^(9/4))/(13*(b*c - a*d)*(a + b*x)^(13/4)) + (16*d*(c + d*x)^(9
/4))/(117*(b*c - a*d)^2*(a + b*x)^(9/4))
```

Rubi [A] time = 0.0093824, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{16d(c+dx)^{9/4}}{117(a+bx)^{9/4}(bc-ad)^2} - \frac{4(c+dx)^{9/4}}{13(a+bx)^{13/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(17/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(13*(b*c - a*d)*(a + b*x)^(13/4)) + (16*d*(c + d*x)^(9
/4))/(117*(b*c - a*d)^2*(a + b*x)^(9/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}} dx &= -\frac{4(c+dx)^{9/4}}{13(bc-ad)(a+bx)^{13/4}} - \frac{(4d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}} dx}{13(bc-ad)} \\
&= -\frac{4(c+dx)^{9/4}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d(c+dx)^{9/4}}{117(bc-ad)^2(a+bx)^{9/4}}
\end{aligned}$$

Mathematica [A] time = 0.0259256, size = 46, normalized size = 0.7

$$\frac{4(c+dx)^{9/4}(13ad-9bc+4bdx)}{117(a+bx)^{13/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(17/4),x]
```

```
[Out] (4*(c + d*x)^(9/4)*(-9*b*c + 13*a*d + 4*b*d*x))/(117*(b*c - a*d)^2*(a + b*x
)^(13/4))
```

Maple [A] time = 0.005, size = 54, normalized size = 0.8

$$\frac{16\,bdx+52\,ad-36\,bc}{117\,a^2d^2-234\,abcd+117\,b^2c^2}\,(dx+c)^{\frac{9}{4}}\,(bx+a)^{-\frac{13}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(17/4),x)
```

```
[Out] 4/117*(d*x+c)^(9/4)*(4*b*d*x+13*a*d-9*b*c)/(b*x+a)^(13/4)/(a^2*d^2-2*a*b*c*
d+b^2*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{17}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(17/4), x)
```

Fricas [B] time = 2.98466, size = 494, normalized size = 7.48

$$\frac{4\left(4\,bd^3x^3-9\,bc^3+13\,ac^2d-\left(bcd^2-13\,ad^3\right)x^2-2\left(7\,bc^2d-13\,acd^2\right)x\right)(bx+a)^{\frac{3}{4}}(dx}{117\left(a^4b^2c^2-2\,a^5bcd+a^6d^2+\left(b^6c^2-2\,ab^5cd+a^2b^4d^2\right)x^4+4\left(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2\right)x^3+6\left(a^2b^4c^2-2\,a^3b^3cd+a^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="fricas")
```

```
[Out] 4/117*(4*b*d^3*x^3 - 9*b*c^3 + 13*a*c^2*d - (b*c*d^2 - 13*a*d^3)*x^2 - 2*(7
*b*c^2*d - 13*a*c*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^4*b^2*c^2 - 2*
a^5*b*c*d + a^6*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 4*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^3 + 6*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a
^4*b^2*d^2)*x^2 + 4*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(17/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{17}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(17/4), x)
```

3.1684 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}}\,dx$

Optimal. Leaf size=101

$$-\frac{128d^2(c+dx)^{9/4}}{1989(a+bx)^{9/4}(bc-ad)^3}+\frac{32d(c+dx)^{9/4}}{221(a+bx)^{13/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{17(a+bx)^{17/4}(bc-ad)}$$

```
[Out] (-4*(c + d*x)^(9/4))/(17*(b*c - a*d)*(a + b*x)^(17/4)) + (32*d*(c + d*x)^(9
/4))/(221*(b*c - a*d)^2*(a + b*x)^(13/4)) - (128*d^2*(c + d*x)^(9/4))/(1989
*(b*c - a*d)^3*(a + b*x)^(9/4))
```

Rubi [A] time = 0.0173348, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{128d^2(c+dx)^{9/4}}{1989(a+bx)^{9/4}(bc-ad)^3}+\frac{32d(c+dx)^{9/4}}{221(a+bx)^{13/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{17(a+bx)^{17/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(21/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(17*(b*c - a*d)*(a + b*x)^(17/4)) + (32*d*(c + d*x)^(9
/4))/(221*(b*c - a*d)^2*(a + b*x)^(13/4)) - (128*d^2*(c + d*x)^(9/4))/(1989
*(b*c - a*d)^3*(a + b*x)^(9/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}}\,dx &= -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}}-\frac{(8d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx}{17(bc-ad)}\\ &= -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}}+\frac{32d(c+dx)^{9/4}}{221(bc-ad)^2(a+bx)^{13/4}}+\frac{\left(32d^2\right)\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx}{221(bc-ad)^2}\\ &= -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}}+\frac{32d(c+dx)^{9/4}}{221(bc-ad)^2(a+bx)^{13/4}}-\frac{128d^2(c+dx)^{9/4}}{1989(bc-ad)^3(a+bx)^{9/4}}\end{aligned}$$

Mathematica [A] time = 0.0468354, size = 77, normalized size = 0.76

$$-\frac{4(c+dx)^{9/4}\left(221a^2d^2+34abd(4dx-9c)+b^2\left(117c^2-72cdx+32d^2x^2\right)\right)}{1989(a+bx)^{17/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(21/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4)*(221*a^2*d^2 + 34*a*b*d*(-9*c + 4*d*x) + b^2*(117*c^2 -
 72*c*d*x + 32*d^2*x^2)))/(1989*(b*c - a*d)^3*(a + b*x)^(17/4))
```

Maple [A] time = 0.006, size = 105, normalized size = 1.

$$\frac{128\,b^2d^2x^2+544\,abd^2x-288\,b^2cdx+884\,a^2d^2-1224\,abcd+468\,b^2c^2}{1989\,a^3d^3-5967\,a^2cbd^2+5967\,ab^2c^2d-1989\,b^3c^3}(dx+c)^{\frac{9}{4}}(bx+a)^{-\frac{17}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(21/4),x)
```

```
[Out] 4/1989*(d*x+c)^(9/4)*(32*b^2*d^2*x^2+136*a*b*d^2*x-72*b^2*c*d*x+221*a^2*d^2
-306*a*b*c*d+117*b^2*c^2)/(b*x+a)^(17/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2
*d-b^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{21}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(21/4), x)
```

Fricas [B] time = 3.57569, size = 879, normalized size = 8.7

$$-\frac{4\left(32\,b^2d^4x^4+117\,b^2c^4-306\,abc^3d+221\,a^2c^2d^2-8\left(b^2cd^3-\right.\right.}{1989\left(a^5b^3c^3-3\,a^6b^2c^2d+3\,a^7bcd^2-a^8d^3+\left(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3\right)x^5+5\left(ab^7c^3-3\,a^2b^6c^2d+3\,a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="fricas")
```

```
[Out] -4/1989*(32*b^2*d^4*x^4 + 117*b^2*c^4 - 306*a*b*c^3*d + 221*a^2*c^2*d^2 - 8
*(b^2*c*d^3 - 17*a*b*d^4)*x^3 + (5*b^2*c^2*d^2 - 34*a*b*c*d^3 + 221*a^2*d^4
)*x^2 + 2*(81*b^2*c^3*d - 238*a*b*c^2*d^2 + 221*a^2*c*d^3)*x)*(b*x + a)^(3/
```

```
4)*(d*x + c)^(1/4)/(a^5*b^3*c^3 - 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3
 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^5 + 5*(a*b^7
*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^4 + 10*(a^2*b^6*c
^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^3 + 10*(a^3*b^5*c^3
 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x^2 + 5*(a^4*b^4*c^3 -
3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(21/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{21}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(21/4), x)
```

3.1685 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{25/4}} dx$

Optimal. Leaf size=136

$$\frac{512d^3(c+dx)^{9/4}}{13923(a+bx)^{9/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{9/4}}{1547(a+bx)^{13/4}(bc-ad)^3}+\frac{16d(c+dx)^{9/4}}{119(a+bx)^{17/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{21(a+bx)^{21/4}(bc-ad)}$$

```
[Out] (-4*(c + d*x)^(9/4))/(21*(b*c - a*d)*(a + b*x)^(21/4)) + (16*d*(c + d*x)^(9
/4))/(119*(b*c - a*d)^2*(a + b*x)^(17/4)) - (128*d^2*(c + d*x)^(9/4))/(1547
*(b*c - a*d)^3*(a + b*x)^(13/4)) + (512*d^3*(c + d*x)^(9/4))/(13923*(b*c -
a*d)^4*(a + b*x)^(9/4))
```

Rubi [A] time = 0.0289592, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{512d^3(c+dx)^{9/4}}{13923(a+bx)^{9/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{9/4}}{1547(a+bx)^{13/4}(bc-ad)^3}+\frac{16d(c+dx)^{9/4}}{119(a+bx)^{17/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{21(a+bx)^{21/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(25/4),x]
```


```
[Out] (-4*(c + d*x)^(9/4))/(21*(b*c - a*d)*(a + b*x)^(21/4)) + (16*d*(c + d*x)^(9
/4))/(119*(b*c - a*d)^2*(a + b*x)^(17/4)) - (128*d^2*(c + d*x)^(9/4))/(1547
*(b*c - a*d)^3*(a + b*x)^(13/4)) + (512*d^3*(c + d*x)^(9/4))/(13923*(b*c -
a*d)^4*(a + b*x)^(9/4))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{25/4}}\,dx = -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} - \frac{(4d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}}\,dx}{7(bc-ad)}$$

$$= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} + \frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}} + \frac{(32d^2)\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx}{119(bc-ad)^2}$$

$$= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} + \frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}} - \frac{128d^2(c+dx)^{9/4}}{1547(bc-ad)^3(a+bx)^{13/4}} - \frac{(128d^3)\int \frac{(c+dx}{(a+bx}}{1547(bc-a}$$

$$= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} + \frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}} - \frac{128d^2(c+dx)^{9/4}}{1547(bc-ad)^3(a+bx)^{13/4}} + \frac{512d^3(c+}{13923(bc-ad)}$$

Mathematica [A] time = 0.0638237, size = 118, normalized size = 0.87

$$\frac{4(c+dx)^{9/4}\left(357a^2bd^2(4dx-9c)+1547a^3d^3+21ab^2d\left(117c^2-72cdx+32d^2x^2\right)+b^3\left(468c^2dx-663c^3-288cd^2x^2+12\right.\right.}{13923(a+bx)^{21/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(25/4),x]
```

```
[Out] (4*(c + d*x)^(9/4)*(1547*a^3*d^3 + 357*a^2*b*d^2*(-9*c + 4*d*x) + 21*a*b^2*
d*(117*c^2 - 72*c*d*x + 32*d^2*x^2) + b^3*(-663*c^3 + 468*c^2*d*x - 288*c*d
^2*x^2 + 128*d^3*x^3)))/(13923*(b*c - a*d)^4*(a + b*x)^(21/4))
```

Maple [A] time = 0.008, size = 171, normalized size = 1.3

$$\frac{512\,b^3d^3x^3+2688\,ab^2d^3x^2-1152\,b^3cd^2x^2+5712\,a^2bd^3x-6048\,ab^2cd^2x+1872\,b^3c^2dx+6188\,a^3d^3-12852\,a^2cbd^2+98}{13923\,a^4d^4-55692\,a^3bcd^3+83538\,a^2c^2b^2d^2-55692\,ab^3c^3d+13923\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(25/4),x)
```

```
[Out] 4/13923*(d*x+c)^(9/4)*(128*b^3*d^3*x^3+672*a*b^2*d^3*x^2-288*b^3*c*d^2*x^2+
1428*a^2*b*d^3*x-1512*a*b^2*c*d^2*x+468*b^3*c^2*d*x+1547*a^3*d^3-3213*a^2*b
*c*d^2+2457*a*b^2*c^2*d-663*b^3*c^3)/(b*x+a)^(21/4)/(a^4*d^4-4*a^3*b*c*d^3+
6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{25}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(25/4), x)
```

Fricas [B] time = 4.62618, size = 1354, normalized size = 9.96

$$\frac{4\left(128\,b^3d^5x^5-663\,b^3c^5+2457\,ab^2c^4d-3\right.}{13923\left(a^6b^4c^4-4\,a^7b^3c^3d+6\,a^8b^2c^2d^2-4\,a^9bcd^3+a^{10}d^4+\left(b^{10}c^4-4\,ab^9c^3d+6\,a^2b^8c^2d^2-4\,a^3b^7cd^3+a^4b^6d^4\right)x^6+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="fricas")
```

```
[Out] 4/13923*(128*b^3*d^5*x^5 - 663*b^3*c^5 + 2457*a*b^2*c^4*d - 3213*a^2*b*c^3*
d^2 + 1547*a^3*c^2*d^3 - 32*(b^3*c*d^4 - 21*a*b^2*d^5)*x^4 + 4*(5*b^3*c^2*d
^3 - 42*a*b^2*c*d^4 + 357*a^2*b*d^5)*x^3 - (15*b^3*c^3*d^2 - 105*a*b^2*c^2*
d^3 + 357*a^2*b*c*d^4 - 1547*a^3*d^5)*x^2 - 2*(429*b^3*c^4*d - 1701*a*b^2*c
^3*d^2 + 2499*a^2*b*c^2*d^3 - 1547*a^3*c*d^4)*x)*(b*x + a)^(3/4)*(d*x + c)^
(1/4)/(a^6*b^4*c^4 - 4*a^7*b^3*c^3*d + 6*a^8*b^2*c^2*d^2 - 4*a^9*b*c*d^3 +
a^10*d^4 + (b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3
+ a^4*b^6*d^4)*x^6 + 6*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4
*a^4*b^6*c*d^3 + a^5*b^5*d^4)*x^5 + 15*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a
^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^4 + 20*(a^3*b^7*c^4 - 4*a
^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x^3 + 15*
(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*
b^2*d^4)*x^2 + 6*(a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8
*b^2*c*d^3 + a^9*b*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(25/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{25}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(25/4), x)
```

3.1686 $\int (a+bx)^{5/4}(c+dx)^{5/4}\,dx$

Optimal. Leaf size=408

$$\frac{5(bc-ad)^{9/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{EllipticF}\left(2\tan\right.}{168\sqrt{2}b^{9/4}d^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (-5*(b*c - a*d)^3*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(84*b^2*d^2) + ((b*c - a
*d)^2*(a + b*x)^(5/4)*(c + d*x)^(1/4))/(42*b^2*d) + ((b*c - a*d)*(a + b*x)^
(9/4)*(c + d*x)^(1/4))/(7*b^2) + (2*(a + b*x)^(9/4)*(c + d*x)^(5/4))/(7*b)
+ (5*(b*c - a*d)^(9/2)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*
x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[
(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*
x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)
*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(168*Sqrt[2]*b^(9/4)*
d^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b
*(c + 2*d*x))^2])
```

Rubi [A] time = 0.590231, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 62, 623, 220}

$$-\frac{5\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)^3}{84b^2d^2}+\frac{5(bc-ad)^{9/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad\ldots}{(bc-ad)^2\left(2\ldots\right.}}}{168\sqrt{2}b^{9/4}d^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)*(c + d*x)^(5/4),x]
```

```
[Out] (-5*(b*c - a*d)^3*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(84*b^2*d^2) + ((b*c - a
*d)^2*(a + b*x)^(5/4)*(c + d*x)^(1/4))/(42*b^2*d) + ((b*c - a*d)*(a + b*x)^
(9/4)*(c + d*x)^(1/4))/(7*b^2) + (2*(a + b*x)^(9/4)*(c + d*x)^(5/4))/(7*b)
+ (5*(b*c - a*d)^(9/2)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*
x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[
(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*
x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)
*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(168*Sqrt[2]*b^(9/4)*
d^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b
*(c + 2*d*x))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
```

```
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (a+bx)^{5/4}(c+dx)^{5/4}\,dx = \frac{2(a+bx)^{9/4}(c+dx)^{5/4}}{7b}+\frac{(5(bc-ad))\int (a+bx)^{5/4}\sqrt[4]{c+dx}\,dx}{14b}$$

$$= \frac{(bc-ad)(a+bx)^{9/4}\sqrt[4]{c+dx}}{7b^2}+\frac{2(a+bx)^{9/4}(c+dx)^{5/4}}{7b}+\frac{(bc-ad)^2\int \frac{(a+bx)^{5/4}}{(c+dx)^{3/4}}\,dx}{28b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d}+\frac{(bc-ad)(a+bx)^{9/4}\sqrt[4]{c+dx}}{7b^2}+\frac{2(a+bx)^{9/4}(c+dx)^{5/4}}{7b}-$$

$$= -\frac{5(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{84b^2d^2}+\frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d}+\frac{(bc-ad)(a+bx)^{9/4}\sqrt[4]{c+}}{7b^2}$$

$$= -\frac{5(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{84b^2d^2}+\frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d}+\frac{(bc-ad)(a+bx)^{9/4}\sqrt[4]{c+}}{7b^2}$$

$$= -\frac{5(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{84b^2d^2}+\frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d}+\frac{(bc-ad)(a+bx)^{9/4}\sqrt[4]{c+}}{7b^2}$$

$$= -\frac{5(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{84b^2d^2}+\frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d}+\frac{(bc-ad)(a+bx)^{9/4}\sqrt[4]{c+}}{7b^2}$$

Mathematica [C] time = 0.0583903, size = 73, normalized size = 0.18

$$\frac{4(a+bx)^{9/4}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{ad-bc}\right)}{9b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)*(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 9/4, 13/4, (d*(a
 + b*x))/(-(b*c) + a*d)])/(9*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{4}} (dx+c)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(5/4)*(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{4}} (dx+c)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/4)*(d*x + c)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bdx^2+ac+(bc+ad)x\right)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(1/4)*(d*x + c)^(1/4), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)*(d*x+c)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)*(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1687 $\int \sqrt[4]{a+bx}(c+dx)^{5/4}\,dx$

Optimal. Leaf size=370

$$-\frac{(bc-ad)^{7/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,\text{EllipticF}\left(2\right.}{12\sqrt{2}b^{9/4}d^{5/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] ((b*c - a*d)^2*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(6*b^2*d) + ((b*c - a*d)*(a
 + b*x)^(5/4)*(c + d*x)^(1/4))/(3*b^2) + (2*(a + b*x)^(5/4)*(c + d*x)^(5/4)
)/(5*b) - ((b*c - a*d)^(7/2)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d +
2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))
*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(
a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d
^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(12*Sqrt[2]*b^(
9/4)*d^(5/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*
d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.378294, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 62, 623, 220}

$$-\frac{(bc-ad)^{7/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{12\sqrt{2}b^{9/4}d^{5/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)*(c + d*x)^(5/4),x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(6*b^2*d) + ((b*c - a*d)*(a
 + b*x)^(5/4)*(c + d*x)^(1/4))/(3*b^2) + (2*(a + b*x)^(5/4)*(c + d*x)^(5/4)
)/(5*b) - ((b*c - a*d)^(7/2)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d +
2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))
*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(
a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d
^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(12*Sqrt[2]*b^(
9/4)*d^(5/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*
d + b*(c + 2*d*x))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
```

```
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \sqrt[4]{a+bx}(c+dx)^{5/4}\,dx = \frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b} + \frac{(bc-ad)\int \sqrt[4]{a+bx}\sqrt[4]{c+dx}\,dx}{2b}$$

$$= \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b} + \frac{(bc-ad)^2\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{3/4}}\,dx}{12b^2}$$

$$= \frac{(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{6b^2d} + \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b} - \frac{(bc-a\ldots}{\ldots}$$

$$= \frac{(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{6b^2d} + \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b} - \frac{((bc-\ldots}{\ldots}$$

$$= \frac{(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{6b^2d} + \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b} - \frac{((bc-\ldots}{\ldots}$$

$$= \frac{(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{6b^2d} + \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b} - \frac{(bc-a\ldots}{\ldots}$$

Mathematica [C] time = 0.0496794, size = 73, normalized size = 0.2

$$\frac{4(a+bx)^{5/4}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{d(a+bx)}{ad-bc}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)*(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(5/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 5/4, 9/4, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \sqrt[4]{bx+a}\,(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(1/4)*(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{1}{4}}(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/4)*(d*x + c)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^{\frac{1}{4}}(dx+c)^{\frac{5}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[4]{a+bx}\,(c+dx)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)*(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(1/4)*(c + d*x)**(5/4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)*(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1688 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{3/4}} dx$

Optimal. Leaf size=332

$$\frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{EllipticF}\left(2\tan\right.}{6\sqrt{2}b^{9/4}\sqrt[4]{d}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(3*b^2) + (2*(a + b*x)^(1/4
)*(c + d*x)^(5/4))/(3*b) + (5*(b*c - a*d)^(5/2)*((a + b*x)*(c + d*x))^(3/4)
*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqr
t[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan
[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/
2])/(6*Sqrt[2]*b^(9/4)*d^(1/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d +
 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.287217, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 62, 623, 220}

$$\frac{5\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)}{3b^2}+\frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{}}{}\right.}}}{6\sqrt{2}b^{9/4}\sqrt[4]{d}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(3/4),x]
```

```
[Out] (5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(3*b^2) + (2*(a + b*x)^(1/4
)*(c + d*x)^(5/4))/(3*b) + (5*(b*c - a*d)^(5/2)*((a + b*x)*(c + d*x))^(3/4)
*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqr
t[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan
[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/
2])/(6*Sqrt[2]*b^(9/4)*d^(1/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d +
 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{3/4}}\,dx = \frac{2\sqrt[4]{a+bx}(c+dx)^{5/4}}{3b} + \frac{(5(bc-ad))\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/4}}\,dx}{6b}$$

$$= \frac{5(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{3b^2} + \frac{2\sqrt[4]{a+bx}(c+dx)^{5/4}}{3b} + \frac{\left(5(bc-ad)^2\right)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{12b^2}$$

$$= \frac{5(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{3b^2} + \frac{2\sqrt[4]{a+bx}(c+dx)^{5/4}}{3b} + \frac{\left(5(bc-ad)^2((a+bx)(c+dx))^{3/4}\right)\int \frac{}{(ac+(bc+ac}}{12b^2(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= \frac{5(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{3b^2} + \frac{2\sqrt[4]{a+bx}(c+dx)^{5/4}}{3b} + \frac{\left(5(bc-ad)^2((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+}\right.}{3b^2(a+bx}$$

$$= \frac{5(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{3b^2} + \frac{2\sqrt[4]{a+bx}(c+dx)^{5/4}}{3b} + \frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+}}{6\sqrt{2}b^{9/4}}$$

Mathematica [C] time = 0.044489, size = 71, normalized size = 0.21

$$\frac{4\sqrt[4]{a+bx}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};\frac{5}{4};\frac{d(a+bx)}{ad-bc}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(3/4),x]
```

```
[Out] (4*(a + b*x)^(1/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 1/4, 5/4, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{4}}(bx+a)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(3/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(5/4)/(b*x + a)^(3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(3/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(3/4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1689 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{7/4}}\,dx$

Optimal. Leaf size=325

$$\frac{5d^{3/4}(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{Elliptic}}{3\sqrt{2}b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (10*d*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(3*b^2) - (4*(c + d*x)^(5/4))/(3*b*(
a + b*x)^(3/4)) + (5*d^(3/4)*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(3/4)*
Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d
*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt
[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[
(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2
])/(3*Sqrt[2]*b^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)
*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.27846, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 62, 623, 220}

$$\frac{5d^{3/4}(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan\right.}{3\sqrt{2}b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(7/4),x]
```

```
[Out] (10*d*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(3*b^2) - (4*(c + d*x)^(5/4))/(3*b*(
a + b*x)^(3/4)) + (5*d^(3/4)*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(3/4)*
Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d
*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt
[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[
(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2
])/(3*Sqrt[2]*b^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)
*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{5/4}}{(a+bx)^{7/4}}\,dx &= -\frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}}+\frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/4}}\,dx}{3b}\\
&= \frac{10d\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{3b^2}-\frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}}+\frac{(5d(bc-ad))\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{6b^2}\\
&= \frac{10d\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{3b^2}-\frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}}+\frac{\left(5d(bc-ad)((a+bx)(c+dx))^{3/4}\right)\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^{3/4}}\,dx}{6b^2(a+bx)^{3/4}(c+dx)^{3/4}}\\
&= \frac{10d\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{3b^2}-\frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}}+\frac{\left(10d(bc-ad)((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}}{3b^2(a+bx)^{3/4}(c+dx)^{3/4}(b}\\
&= \frac{10d\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{3b^2}-\frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}}+\frac{5d^{3/4}(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\,\Big(1+}{3\sqrt{2}b^{9/4}(a+bx)^{3/4}(c+}
\end{aligned}
$$

Mathematica [C] time = 0.040524, size = 73, normalized size = 0.22

$$
-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{3}{4};\frac{1}{4};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(7/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -3/4, 1/4, (d*(a + b*x))/(-(b*c
) + a*d)])/(3*b*(a + b*x)^(3/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{4}} (bx+a)^{-\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(7/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{5}{4}}}{b^2x^2+2\,abx+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(7/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(7/4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1690 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{11/4}} dx$

Optimal. Leaf size=325

$$\frac{5\sqrt{2}d^{7/4}\sqrt{bc-ad}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{Elliptic}}{21b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (-20*d*(c + d*x)^(1/4))/(21*b^2*(a + b*x)^(3/4)) - (4*(c + d*x)^(5/4))/(7*b
*(a + b*x)^(7/4)) + (5*Sqrt[2]*d^(7/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x)
)^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x
)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 +
 (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2
*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*
d]], 1/2])/(21*b^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x
)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.299479, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {47, 62, 623, 220}

$$\frac{5\sqrt{2}d^{7/4}\sqrt{bc-ad}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan\right.}{21b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(11/4),x]
```


```
[Out] (-20*d*(c + d*x)^(1/4))/(21*b^2*(a + b*x)^(3/4)) - (4*(c + d*x)^(5/4))/(7*b
*(a + b*x)^(7/4)) + (5*Sqrt[2]*d^(7/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x)
)^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x
)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 +
 (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2
*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*
d]], 1/2])/(21*b^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x
)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{11/4}}\,dx = -\frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{7/4}}\,dx}{7b}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}} + \frac{\left(5d^2\right)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{21b^2}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}} + \frac{\left(5d^2((a+bx)(c+dx))^{3/4}\right)\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^{3/4}}\,dx}{21b^2(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}} + \frac{\left(20d^2((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{-4abcd}}\right.}{21b^2(a+bx)^{3/4}(c+dx)^{3/4}(bc+ad}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}} + \frac{5\sqrt{2}d^{7/4}\sqrt{bc-ad}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\left(1+\frac{2}{}\right.}{21b^{9/4}(a+bx)^{3/4}(c+dx}$$

Mathematica [C] time = 0.0458935, size = 73, normalized size = 0.22

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{7}{4},-\frac{5}{4};-\frac{3}{4};\frac{d(a+bx)}{ad-bc}\right)}{7b(a+bx)^{7/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(11/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-7/4, -5/4, -3/4, (d*(a + b*x))/(-(b*
c) + a*d)])/(7*b*(a + b*x)^(7/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{4}}(bx+a)^{-\frac{11}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(11/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(11/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(11/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(11/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{5}{4}}}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(11/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x
 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(11/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(11/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1691 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{15/4}}\,dx$

Optimal. Leaf size=363

$$-\frac{10\sqrt{2}d^{11/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{EllipticF}\left(2\tan\right.}{231b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}\sqrt{bc-ad}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (-20*d*(c + d*x)^(1/4))/(77*b^2*(a + b*x)^(7/4)) - (20*d^2*(c + d*x)^(1/4))
/(231*b^2*(b*c - a*d)*(a + b*x)^(3/4)) - (4*(c + d*x)^(5/4))/(11*b*(a + b*x
)^(11/4)) - (10*Sqrt[2]*d^(11/4)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*
d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a
*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sq
rt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/
4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(231*b^(9/4
)*Sqrt[b*c - a*d]*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqr
t[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.352148, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 51, 62, 623, 220}

$$-\frac{20d^2\sqrt[4]{c+dx}}{231b^2(a+bx)^{3/4}(bc-ad)}-\frac{10\sqrt{2}d^{11/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+\ldots}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(\ldots}{bc\ldots}\right.}}}{231b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}\sqrt{bc-ad}(ad+bc+2bdx)\sqrt{(a\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(15/4),x]
```

```
[Out] (-20*d*(c + d*x)^(1/4))/(77*b^2*(a + b*x)^(7/4)) - (20*d^2*(c + d*x)^(1/4))
/(231*b^2*(b*c - a*d)*(a + b*x)^(3/4)) - (4*(c + d*x)^(5/4))/(11*b*(a + b*x
)^(11/4)) - (10*Sqrt[2]*d^(11/4)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*
d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a
*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sq
rt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/
4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(231*b^(9/4
)*Sqrt[b*c - a*d]*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqr
t[(a*d + b*(c + 2*d*x))^2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{15/4}}\,dx = -\frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{11/4}}\,dx}{11b}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} + \frac{\left(5d^2\right)\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}\,dx}{77b^2}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} - \frac{\left(10d^3\right)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{231b^2(bc-ad)}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} - \frac{\left(10d^3((a+bx)(c+dx))^{3/4}\right)\int \cdots}{231b^2(bc-ad)(a+bx)^{3}\cdots}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} - \frac{\left(40d^3((a+bx)(c+dx))^{3/4}\sqrt{(bc\cdots}\right.}{231b^2\cdots}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} - \frac{10\sqrt{2}d^{11/4}((a+bx)(c+dx))^{3/4}\cdots}{\cdots}$$

Mathematica [C] time = 0.0526712, size = 73, normalized size = 0.2

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{11}{4},-\frac{5}{4};-\frac{7}{4};\frac{d(a+bx)}{ad-bc}\right)}{11b(a+bx)^{11/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(15/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-11/4, -5/4, -7/4, (d*(a + b*x))/(-(b
*c) + a*d)])/(11*b*(a + b*x)^(11/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{4}} (bx+a)^{-\frac{15}{4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(15/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(15/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{15}{4}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(15/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(15/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{5}{4}}}{b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(15/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2
*x^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(15/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(15/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1692 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{19/4}} dx$

Optimal. Leaf size=401

$$\frac{4\sqrt{2}d^{15/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\right.}{231b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(bc-ad)^{3/2}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (-4*d*(c + d*x)^(1/4))/(33*b^2*(a + b*x)^(11/4)) - (4*d^2*(c + d*x)^(1/4))/
(231*b^2*(b*c - a*d)*(a + b*x)^(7/4)) + (8*d^3*(c + d*x)^(1/4))/(231*b^2*(b
*c - a*d)^2*(a + b*x)^(3/4)) - (4*(c + d*x)^(5/4))/(15*b*(a + b*x)^(15/4))
+ (4*Sqrt[2]*d^(15/4)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x
)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(
a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x
)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*
((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(231*b^(9/4)*(b*c - a*
d)^(3/2)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d +
b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.425196, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 51, 62, 623, 220}

$$\frac{8d^3\sqrt[4]{c+dx}}{231b^2(a+bx)^{3/4}(bc-ad)^2}-\frac{4d^2\sqrt[4]{c+dx}}{231b^2(a+bx)^{7/4}(bc-ad)}+\frac{4\sqrt{2}d^{15/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(}{bc-ad}\right.}{231b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(19/4),x]
```


```
[Out] (-4*d*(c + d*x)^(1/4))/(33*b^2*(a + b*x)^(11/4)) - (4*d^2*(c + d*x)^(1/4))/
(231*b^2*(b*c - a*d)*(a + b*x)^(7/4)) + (8*d^3*(c + d*x)^(1/4))/(231*b^2*(b
*c - a*d)^2*(a + b*x)^(3/4)) - (4*(c + d*x)^(5/4))/(15*b*(a + b*x)^(15/4))
+ (4*Sqrt[2]*d^(15/4)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x
)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(
a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x
)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*
((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(231*b^(9/4)*(b*c - a*
d)^(3/2)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d +
b*(c + 2*d*x))^2])
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
```

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{19/4}}\,dx = -\frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}}+\frac{d\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{15/4}}\,dx}{3b}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}}-\frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}}+\frac{d^2\int \frac{1}{(a+bx)^{11/4}(c+dx)^{3/4}}\,dx}{33b^2}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}}-\frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}}-\frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}}-\frac{\left(2d^3\right)\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}\,dx}{77b^2(bc-ad)}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}}-\frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}}+\frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}}+$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}}-\frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}}+\frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}}+$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}}-\frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}}+\frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}}+$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}}-\frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}}+\frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}}+$$

Mathematica [C] time = 0.0522909, size = 73, normalized size = 0.18

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{15}{4},-\frac{5}{4};-\frac{11}{4};\frac{d(a+bx)}{ad-bc}\right)}{15b(a+bx)^{15/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(19/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-15/4, -5/4, -11/4, (d*(a + b*x))/(-(
b*c) + a*d)])/(15*b*(a + b*x)^(15/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{4}} (bx+a)^{-\frac{19}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(19/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(19/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{19}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(19/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(19/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{5}{4}}}{b^5x^5+5\,ab^4x^4+10\,a^2b^3x^3+10\,a^3b^2x^2+5\,a^4bx+a^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(19/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4)/(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^
3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(19/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(19/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1693 $\int \frac{(a+bx)^{5/4}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=167

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}-\frac{5\sqrt[4]{a+bx}(c+dx)^{3/4}(bc-ad)}{8d^2}+\frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d}$$

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(3/4))/(8*d^2) + ((a + b*x)^(5/4)
*(c + d*x)^(3/4))/(2*d) + (5*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))
/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4)) + (5*(b*c - a*d)^2*ArcTan
h[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4)
)
```

Rubi [A] time = 0.102244, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 240, 212, 208, 205}

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}-\frac{5\sqrt[4]{a+bx}(c+dx)^{3/4}(bc-ad)}{8d^2}+\frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)/(c + d*x)^(1/4),x]
```

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(3/4))/(8*d^2) + ((a + b*x)^(5/4)
*(c + d*x)^(3/4))/(2*d) + (5*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))
/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4)) + (5*(b*c - a*d)^2*ArcTan
h[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4)
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{5/4}}{\sqrt[4]{c+dx}}\,dx = \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} - \frac{(5(bc-ad))\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx}{8d}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{\left(5(bc-ad)^2\right)\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{32d^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x,$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+b}}{\sqrt[4]{c+d}}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}x^2}\,dx, x,$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}} + \frac{5(bc-a}{}$$

Mathematica [C] time = 0.0288506, size = 73, normalized size = 0.44

$$\frac{4(a+bx)^{9/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{ad-bc}\right)}{9b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)/(c + d*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 9/4, 13/4, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(1/4))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{4}} \frac{1}{\sqrt[4]{dx+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(1/4), x)
```

Fricas [B] time = 3.19403, size = 3090, normalized size = 18.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/32*(20*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*arctan(-((b^4*c^2*d^7 - 2*a*b^3*c*d^
8 + a^2*b^2*d^9)*(b*x + a)^(1/4)*(d*x + c)^(3/4)*((b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3
*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(3/4) -
 (b^2*d^8*x + b^2*c*d^7)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b^2*d^5*x + b^2*
c*d^4)*sqrt((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*
b*c*d^7 + a^8*d^8)/(b^3*d^9)))/(d*x + c))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(3/4))/(b^8*c^
9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^
5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d
^8 + (b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4
 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c
*d^8 + a^8*d^9)*x)) - 5*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*log(5*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (b*d^3*x + b*c*d^2)*((b^
8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
```

```
^8)/(b^3*d^9))^(1/4))/(d*x + c)) + 5*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*log(5*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/4)*(d*x + c)^(3/4) - (b*d^3*x +
b*c*d^2)*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^
3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4))/(d*x + c)) - 4*(4*b*d*x - 5*b*c + 9*a*d)
*(b*x + a)^(1/4)*(d*x + c)^(3/4))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{5}{4}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(5/4)/(c + d*x)**(1/4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1694 $\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=127

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}+\frac{\sqrt[4]{a+bx}(c+dx)^{3/4}}{d}$$

```
[Out] ((a + b*x)^(1/4)*(c + d*x)^(3/4))/d - ((b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x
)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(3/4)*d^(5/4)) - ((b*c - a*d)*Arc
Tanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(3/4)*d^(5/
4))
```

Rubi [A] time = 0.0738602, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 240, 212, 208, 205}

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}+\frac{\sqrt[4]{a+bx}(c+dx)^{3/4}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)/(c + d*x)^(1/4),x]
```

```
[Out] ((a + b*x)^(1/4)*(c + d*x)^(3/4))/d - ((b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x
)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(3/4)*d^(5/4)) - ((b*c - a*d)*Arc
Tanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(3/4)*d^(5/
4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx = \frac{\sqrt[4]{a+bx}(c+dx)^{3/4}}{d}-\frac{(bc-ad)\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{4d}$$

$$=\frac{\sqrt[4]{a+bx}(c+dx)^{3/4}}{d}-\frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx,x,\sqrt[4]{a+bx}\right)}{bd}$$

$$=\frac{\sqrt[4]{a+bx}(c+dx)^{3/4}}{d}-\frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{bd}$$

$$=\frac{\sqrt[4]{a+bx}(c+dx)^{3/4}}{d}-\frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}x^2}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2\sqrt{b}d}-\frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}x^2}\,dx,x,\right.}{2\sqrt{b}d}$$

$$=\frac{\sqrt[4]{a+bx}(c+dx)^{3/4}}{d}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}$$

Mathematica [C] time = 0.0260649, size = 73, normalized size = 0.57

$$\frac{4(a+bx)^{5/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)/(c + d*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(5/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 5/4, 9/4, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/4))
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \sqrt[4]{bx+a}\frac{1}{\sqrt[4]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(1/4), x)
```

Fricas [B] time = 2.58844, size = 1744, normalized size = 13.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(b^3*d^5))^(1/4)*arctan(((b^3*c*d^4 - a*b^2*d^5)*(b*x + a)^(1/4)*(d
*x + c)^(3/4)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)/(b^3*d^5))^(3/4) + (b^2*d^5*x + b^2*c*d^4)*sqrt(((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b^2*d^3*x + b^2*c*d^2)*sqr
t((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(
b^3*d^5)))/(d*x + c))*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(3/4))/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*
c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4 + (b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^
2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*x)) + d*((b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4)*log(-((b*c
- a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (b*d^2*x + b*c*d)*((b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4))/
(d*x + c)) - d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)/(b^3*d^5))^(1/4)*log(-((b*c - a*d)*(b*x + a)^(1/4)*(d*x + c)^(
3/4) - (b*d^2*x + b*c*d)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4))/(d*x + c)) - 4*(b*x + a)^(1/4)*(d*
x + c)^(3/4))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(1/4)/(c + d*x)**(1/4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1695 $\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

```
[Out] (2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(3/4)*d^
(1/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(
b^(3/4)*d^(1/4))
```

Rubi [A] time = 0.0583946, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {63, 240, 212, 208, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(3/4)*d^
(1/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(
b^(3/4)*d^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx = \frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x, \sqrt[4]{a+bx}\right)}{b}$$
$$= \frac{4\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{b}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{b}}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

Mathematica [C] time = 0.0286975, size = 71, normalized size = 0.84

$$\frac{4\sqrt[4]{a+bx}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 1/4, 5/4, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/4))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{3}{4}}\frac{1}{\sqrt[4]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(1/4)), x)
```

Fricas [B] time = 2.17117, size = 581, normalized size = 6.84

$$-4\left(\frac{1}{b^3d}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}b^2d\left(\frac{1}{b^3d}\right)^{\frac{3}{4}}-\left(b^2d^2x+b^2cd\right)\sqrt{\frac{\left(b^2dx+b^2c\right)\sqrt{\frac{1}{b^3d}}+\sqrt{bx+a}\sqrt{dx+c}}{dx+c}}\left(\frac{1}{b^3d}\right)^{\frac{3}{4}}}{dx+c}\right)+\left(\frac{1}{b^3d}\right)^{\frac{1}{4}}\log\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -4*(1/(b^3*d))^(1/4)*arctan(-((b*x + a)^(1/4)*(d*x + c)^(3/4)*b^2*d*(1/(b^3
*d))^(3/4) - (b^2*d^2*x + b^2*c*d)*sqrt(((b^2*d*x + b^2*c)*sqrt(1/(b^3*d))
+ sqrt(b*x + a)*sqrt(d*x + c))/(d*x + c))*(1/(b^3*d))^(3/4))/(d*x + c)) + (
1/(b^3*d))^(1/4)*log(((b*d*x + b*c)*(1/(b^3*d))^(1/4) + (b*x + a)^(1/4)*(d*
x + c)^(3/4))/(d*x + c)) - (1/(b^3*d))^(1/4)*log(-((b*d*x + b*c)*(1/(b^3*d)
)^(1/4) - (b*x + a)^(1/4)*(d*x + c)^(3/4))/(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{3}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/4)*(c + d*x)**(1/4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1696 $\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=32

$$-\frac{4(c+dx)^{3/4}}{3(a+bx)^{3/4}(bc-ad)}$$

```
[Out] (-4*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/4))
```

Rubi [A] time = 0.0032892, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{4(c+dx)^{3/4}}{3(a+bx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/4}}$$

Mathematica [A] time = 0.0110246, size = 32, normalized size = 1.

$$-\frac{4(c+dx)^{3/4}}{3(a+bx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/4))
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$\frac{4}{3\,ad-3\,bc}\,(dx+c)^{\frac{3}{4}}\,(bx+a)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x)
```

```
[Out] 4/3/(b*x+a)^(3/4)*(d*x+c)^(3/4)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(1/4)), x)
```

Fricas [A] time = 1.93565, size = 100, normalized size = 3.12

$$-\frac{4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{3\left(abc-a^{2}d+\left(b^{2}c-abd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/3*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{7}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(7/4)*(c + d*x)**(1/4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1697 $\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=66

$$\frac{16d(c+dx)^{3/4}}{21(a+bx)^{3/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{7(a+bx)^{7/4}(bc-ad)}$$

```
[Out] (-4*(c + d*x)^(3/4))/(7*(b*c - a*d)*(a + b*x)^(7/4)) + (16*d*(c + d*x)^(3/4
))/(21*(b*c - a*d)^2*(a + b*x)^(3/4))
```

Rubi [A] time = 0.0090494, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{16d(c+dx)^{3/4}}{21(a+bx)^{3/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{7(a+bx)^{7/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(7*(b*c - a*d)*(a + b*x)^(7/4)) + (16*d*(c + d*x)^(3/4
))/(21*(b*c - a*d)^2*(a + b*x)^(3/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx &= -\frac{4(c+dx)^{3/4}}{7(bc-ad)(a+bx)^{7/4}}-\frac{(4d)\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx}{7(bc-ad)}\\ &= -\frac{4(c+dx)^{3/4}}{7(bc-ad)(a+bx)^{7/4}}+\frac{16d(c+dx)^{3/4}}{21(bc-ad)^2(a+bx)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.0160321, size = 46, normalized size = 0.7

$$\frac{4(c+dx)^{3/4}(7ad-3bc+4bdx)}{21(a+bx)^{7/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(c + d*x)^(3/4)*(-3*b*c + 7*a*d + 4*b*d*x))/(21*(b*c - a*d)^2*(a + b*x)^
(7/4))
```

Maple [A] time = 0.003, size = 54, normalized size = 0.8

$$\frac{16\,bdx+28\,ad-12\,bc}{21\,a^2d^2-42\,abcd+21\,b^2c^2}\,(dx+c)^{\frac{3}{4}}\,(bx+a)^{-\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x)
```

```
[Out] 4/21*(d*x+c)^(3/4)*(4*b*d*x+7*a*d-3*b*c)/(b*x+a)^(7/4)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(1/4)), x)
```

Fricas [B] time = 3.1938, size = 259, normalized size = 3.92

$$\frac{4\,(4\,bdx-3\,bc+7\,ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{21\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/21*(4*b*d*x - 3*b*c + 7*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a
*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/4)/(d*x+c)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1698 $\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=101

$$-\frac{128d^2(c+dx)^{3/4}}{231(a+bx)^{3/4}(bc-ad)^3}+\frac{32d(c+dx)^{3/4}}{77(a+bx)^{7/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{11(a+bx)^{11/4}(bc-ad)}$$

```
[Out] (-4*(c + d*x)^(3/4))/(11*(b*c - a*d)*(a + b*x)^(11/4)) + (32*d*(c + d*x)^(3
/4))/(77*(b*c - a*d)^2*(a + b*x)^(7/4)) - (128*d^2*(c + d*x)^(3/4))/(231*(b
*c - a*d)^3*(a + b*x)^(3/4))
```

Rubi [A] time = 0.0171828, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{128d^2(c+dx)^{3/4}}{231(a+bx)^{3/4}(bc-ad)^3}+\frac{32d(c+dx)^{3/4}}{77(a+bx)^{7/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{11(a+bx)^{11/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(11*(b*c - a*d)*(a + b*x)^(11/4)) + (32*d*(c + d*x)^(3
/4))/(77*(b*c - a*d)^2*(a + b*x)^(7/4)) - (128*d^2*(c + d*x)^(3/4))/(231*(b
*c - a*d)^3*(a + b*x)^(3/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx &= -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}}-\frac{(8d)\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx}{11(bc-ad)}\\
&= -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}}+\frac{32d(c+dx)^{3/4}}{77(bc-ad)^2(a+bx)^{7/4}}+\frac{\left(32d^2\right)\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx}{77(bc-ad)^2}\\
&= -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}}+\frac{32d(c+dx)^{3/4}}{77(bc-ad)^2(a+bx)^{7/4}}-\frac{128d^2(c+dx)^{3/4}}{231(bc-ad)^3(a+bx)^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.0310133, size = 77, normalized size = 0.76

$$-\frac{4(c+dx)^{3/4}\left(77a^2d^2+22abd(4dx-3c)+b^2\left(21c^2-24cdx+32d^2x^2\right)\right)}{231(a+bx)^{11/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4)*(77*a^2*d^2 + 22*a*b*d*(-3*c + 4*d*x) + b^2*(21*c^2 - 2
4*c*d*x + 32*d^2*x^2)))/(231*(b*c - a*d)^3*(a + b*x)^(11/4))
```

Maple [A] time = 0.005, size = 105, normalized size = 1.

$$\frac{128\,b^2d^2x^2+352\,abd^2x-96\,b^2cdx+308\,a^2d^2-264\,abcd+84\,b^2c^2}{231\,a^3d^3-693\,a^2cbd^2+693\,ab^2c^2d-231\,b^3c^3}(dx+c)^{\frac{3}{4}}(bx+a)^{-\frac{11}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x)
```

```
[Out] 4/231*(d*x+c)^(3/4)*(32*b^2*d^2*x^2+88*a*b*d^2*x-24*b^2*c*d*x+77*a^2*d^2-66
*a*b*c*d+21*b^2*c^2)/(b*x+a)^(11/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx+a)^{\frac{15}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(1/4)), x)
```

Fricas [B] time = 6.77432, size = 522, normalized size = 5.17

$$-\frac{4\left(32\,b^2d^2x^2+21\,b^2c^2-66\,abcd+77\,a^2d^2-8\left(3\,b^2cd-11\,abd^2\right)x\right)(b}{231\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/231*(32*b^2*d^2*x^2 + 21*b^2*c^2 - 66*a*b*c*d + 77*a^2*d^2 - 8*(3*b^2*c*
d - 11*a*b*d^2)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^3*b^3*c^3 - 3*a^4*b^2
*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d
^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 -
```

```
a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5
*b*d^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(15/4)/(d*x+c)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1699 $\int \frac{1}{(a+bx)^{19/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=136

$$\frac{512d^3(c+dx)^{3/4}}{1155(a+bx)^{3/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{3/4}}{385(a+bx)^{7/4}(bc-ad)^3}+\frac{16d(c+dx)^{3/4}}{55(a+bx)^{11/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{15(a+bx)^{15/4}(bc-ad)}$$

```
[Out] (-4*(c + d*x)^(3/4))/(15*(b*c - a*d)*(a + b*x)^(15/4)) + (16*d*(c + d*x)^(3
/4))/(55*(b*c - a*d)^2*(a + b*x)^(11/4)) - (128*d^2*(c + d*x)^(3/4))/(385*(
b*c - a*d)^3*(a + b*x)^(7/4)) + (512*d^3*(c + d*x)^(3/4))/(1155*(b*c - a*d)
^4*(a + b*x)^(3/4))
```

Rubi [A] time = 0.0305414, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{512d^3(c+dx)^{3/4}}{1155(a+bx)^{3/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{3/4}}{385(a+bx)^{7/4}(bc-ad)^3}+\frac{16d(c+dx)^{3/4}}{55(a+bx)^{11/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{15(a+bx)^{15/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x]
```


```
[Out] (-4*(c + d*x)^(3/4))/(15*(b*c - a*d)*(a + b*x)^(15/4)) + (16*d*(c + d*x)^(3
/4))/(55*(b*c - a*d)^2*(a + b*x)^(11/4)) - (128*d^2*(c + d*x)^(3/4))/(385*(
b*c - a*d)^3*(a + b*x)^(7/4)) + (512*d^3*(c + d*x)^(3/4))/(1155*(b*c - a*d)
^4*(a + b*x)^(3/4))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{19/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} - \frac{(4d)\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx}{5(bc-ad)}$$

$$= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} + \frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}} + \frac{\left(32d^2\right)\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx}{55(bc-ad)^2}$$

$$= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} + \frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}} - \frac{128d^2(c+dx)^{3/4}}{385(bc-ad)^3(a+bx)^{7/4}} - \frac{\left(128d^3\right)\int}{385}$$

$$= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} + \frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}} - \frac{128d^2(c+dx)^{3/4}}{385(bc-ad)^3(a+bx)^{7/4}} + \frac{512d^3}{1155(bc-}$$

Mathematica [A] time = 0.047585, size = 118, normalized size = 0.87

$$\frac{4(c+dx)^{3/4}\left(165a^2bd^2(4dx-3c)+385a^3d^3+15ab^2d\left(21c^2-24cdx+32d^2x^2\right)+b^3\left(84c^2dx-77c^3-96cd^2x^2+128d^3x^3\right.\right.}{1155(a+bx)^{15/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(c + d*x)^(3/4)*(385*a^3*d^3 + 165*a^2*b*d^2*(-3*c + 4*d*x) + 15*a*b^2*d
*(21*c^2 - 24*c*d*x + 32*d^2*x^2) + b^3*(-77*c^3 + 84*c^2*d*x - 96*c*d^2*x^
2 + 128*d^3*x^3)))/(1155*(b*c - a*d)^4*(a + b*x)^(15/4))
```

Maple [A] time = 0.008, size = 171, normalized size = 1.3

$$\frac{512\,x^3b^3d^3+1920\,ab^2d^3x^2-384\,b^3cd^2x^2+2640\,a^2bd^3x-1440\,ab^2cd^2x+336\,b^3c^2dx+1540\,a^3d^3-1980\,a^2cbd^2+1260}{1155\,a^4d^4-4620\,a^3bcd^3+6930\,a^2c^2b^2d^2-4620\,ab^3c^3d+1155\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x)
```

```
[Out] 4/1155*(d*x+c)^(3/4)*(128*b^3*d^3*x^3+480*a*b^2*d^3*x^2-96*b^3*c*d^2*x^2+66
0*a^2*b*d^3*x-360*a*b^2*c*d^2*x+84*b^3*c^2*d*x+385*a^3*d^3-495*a^2*b*c*d^2+
315*a*b^2*c^2*d-77*b^3*c^3)/(b*x+a)^(15/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2
*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{19}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(19/4)*(d*x + c)^(1/4)), x)
```

Fricas [B] time = 17.7949, size = 865, normalized size = 6.36

$$\frac{4\left(128\,b^{3}d^{3}x^{3}-77\,b^{3}c^{3}+315\,ab^{2}c^{2}d-495\right.}{1155\left(a^{4}b^{4}c^{4}-4\,a^{5}b^{3}c^{3}d+6\,a^{6}b^{2}c^{2}d^{2}-4\,a^{7}bcd^{3}+a^{8}d^{4}+\left(b^{8}c^{4}-4\,ab^{7}c^{3}d+6\,a^{2}b^{6}c^{2}d^{2}-4\,a^{3}b^{5}cd^{3}+a^{4}b^{4}d^{4}\right)x^{4}+4\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/1155*(128*b^3*d^3*x^3 - 77*b^3*c^3 + 315*a*b^2*c^2*d - 495*a^2*b*c*d^2 +
385*a^3*d^3 - 96*(b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + 12*(7*b^3*c^2*d - 30*a*b^2
*c*d^2 + 55*a^2*b*d^3)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^4*b^4*c^4 - 4*
a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*
a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a
*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*
d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3
*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(19/4)/(d*x+c)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1700 $\int \frac{(a+bx)^{7/4}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=751

$$\frac{7(bc-ad)^{7/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,\text{EllipticF}\left(2\tan^{-}\right.}{20\sqrt{2}b^{3/4}d^{11/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (-7*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(3/4))/(15*d^2) + (2*(a + b*x)^(7
/4)*(c + d*x)^(3/4))/(5*d) + (7*(b*c - a*d)*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[
(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(10*Sqrt[b]*d^(5/2)
*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt
[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (7*(b*c - a*d)^(7/2)*((a + b
*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*
Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c
- a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)
]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/
Sqrt[b*c - a*d]], 1/2])/(10*Sqrt[2]*b^(3/4)*d^(11/4)*(a + b*x)^(1/4)*(c + d
*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (7*(b*c -
a*d)^(7/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (
2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c
+ 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)
])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*
(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(20*Sqrt[2]*b^(3/4)*d^(11/4)*(a +
 b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x)
)^2])
```

Rubi [A] time = 0.788887, antiderivative size = 751, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 62, 623, 305, 220, 1196}

$$\frac{7(bc-ad)^{7/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt{2}\sqrt[4]{b}\right.\right.}{20\sqrt{2}b^{3/4}d^{11/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/4)/(c + d*x)^(1/4),x]
```


```
[Out] (-7*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(3/4))/(15*d^2) + (2*(a + b*x)^(7
/4)*(c + d*x)^(3/4))/(5*d) + (7*(b*c - a*d)*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[
(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(10*Sqrt[b]*d^(5/2)
*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt
[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (7*(b*c - a*d)^(7/2)*((a + b
*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*
Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c
- a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)
]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/
Sqrt[b*c - a*d]], 1/2])/(10*Sqrt[2]*b^(3/4)*d^(11/4)*(a + b*x)^(1/4)*(c + d
*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (7*(b*c -
a*d)^(7/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (
2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c
+ 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)
```

```
])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*
(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(20*Sqrt[2]*b^(3/4)*d^(11/4)*(a +
 b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x)
)^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{7/4}}{\sqrt[4]{c+dx}}\,dx = \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx}{10d}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{\left(7(bc-ad)^2\right)\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx}{20d^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{\left(7(bc-ad)^2\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{}{\sqrt[4]{ac+(b}}}{20d^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{\left(7(bc-ad)^2\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad}\right.}{5d^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{\left(7(bc-ad)^3\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad}\right.}{10\sqrt{b}d^{5}}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{7(bc-ad)\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+}}{10\sqrt{b}d^{5/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2b}$$

Mathematica [C] time = 0.0294736, size = 73, normalized size = 0.1

$$\frac{4(a+bx)^{11/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{15}{4};\frac{d(a+bx)}{ad-bc}\right)}{11b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/4)/(c + d*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(11/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4
, 11/4, 15/4, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(1/4))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{4}}\frac{1}{\sqrt[4]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/4)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(7/4)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(7/4)/(d*x + c)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{7}{4}}}{\sqrt[4]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(7/4)/(c + d*x)**(1/4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1701 $\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=705

$$-\frac{(bc-ad)^{5/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{EllipticF}\left(2\tan^{-}\right.}{2\sqrt{2}b^{3/4}d^{7/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (2*(a + b*x)^(3/4)*(c + d*x)^(3/4))/(3*d) - (Sqrt[(a + b*x)*(c + d*x)]*Sqrt
[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(Sqrt[b]*d^(3/2)*(
a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d
]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + ((b*c - a*d)^(5/2)*((a + b*x)*
(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt
[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*
d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*El
lipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt
[b*c - a*d]], 1/2])/(Sqrt[2]*b^(3/4)*d^(7/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4
)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - ((b*c - a*d)^(5/2)
*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*
Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^
2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c -
a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))
^(1/4))/Sqrt[b*c - a*d]], 1/2])/(2*Sqrt[2]*b^(3/4)*d^(7/4)*(a + b*x)^(1/4)*
(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.616858, antiderivative size = 705, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 62, 623, 305, 220, 1196}

$$-\frac{(bc-ad)^{5/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}}{}\right.\right.}{2\sqrt{2}b^{3/4}d^{7/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)/(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(3/4)*(c + d*x)^(3/4))/(3*d) - (Sqrt[(a + b*x)*(c + d*x)]*Sqrt
[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(Sqrt[b]*d^(3/2)*(
a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d
]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + ((b*c - a*d)^(5/2)*((a + b*x)*
(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt
[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*
d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*El
lipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt
[b*c - a*d]], 1/2])/(Sqrt[2]*b^(3/4)*d^(7/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4
)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - ((b*c - a*d)^(5/2)
*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*
Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^
2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c -
a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))
^(1/4))/Sqrt[b*c - a*d]], 1/2])/(2*Sqrt[2]*b^(3/4)*d^(7/4)*(a + b*x)^(1/4)*
```

```
(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx = \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{(bc-ad)\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx}{2d}$$

$$= \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{\left((bc-ad)\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}\,dx}{2d\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

$$= \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{\left(2(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-4abcd+(bc+ad)^2-}}\right.}{d\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)}$$

$$= \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{\left((bc-ad)^2\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+ad)^2+}}\right.}{\sqrt{b}d^{3/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)}$$

$$= \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}d^{3/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)} + \frac{(bc-ad}{}$$

Mathematica [C] time = 0.0255792, size = 73, normalized size = 0.1

$$\frac{4(a+bx)^{7/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{d(a+bx)}{ad-bc}\right)}{7b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)/(c + d*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(7/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 7/4, 11/4, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/4))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{4}}\frac{1}{\sqrt[4]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)/(d*x + c)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{4}}}{\sqrt[4]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(3/4)/(c + d*x)**(1/4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1702 $\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=688

$$\frac{(bc-ad)^{3/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\right.}{\sqrt{2}b^{3/4}d^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (2*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c
 + 2*d*x))^2])/(Sqrt[b]*Sqrt[d]*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4)
*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(
b*c - a*d))) - (Sqrt[2]*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[
(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])
/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*S
qrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt
[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b
^(3/4)*d^(3/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(
a*d + b*(c + 2*d*x))^2]) + ((b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*S
qrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*
x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[
b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(
Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2]
)/(Sqrt[2]*b^(3/4)*d^(3/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b
*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.495464, antiderivative size = 688, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {62, 623, 305, 220, 1196}

$$\frac{(bc-ad)^{3/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\right.\right.}{\sqrt{2}b^{3/4}d^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (2*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c
 + 2*d*x))^2])/(Sqrt[b]*Sqrt[d]*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4)
*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(
b*c - a*d))) - (Sqrt[2]*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[
(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])
/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*S
qrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt
[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b
^(3/4)*d^(3/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(
a*d + b*(c + 2*d*x))^2]) + ((b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*S
qrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*
x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[
b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(
Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2]
)/(Sqrt[2]*b^(3/4)*d^(3/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b
```

```
*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx = \frac{\sqrt[4]{(a+bx)(c+dx)}\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}\,dx}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

$$= \frac{\left(4\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{-4abcd+(bc+ad)^2+4bdx^4}}\,dx, x, \sqrt[4]{(a+bx)(c+\right.}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)}$$

$$= \frac{\left(2(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+ad)^2+4bdx^4}}\,dx, x, \sqrt[4]{(a\right.}{\sqrt{b}\sqrt{d}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)}$$

$$= \frac{2\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\sqrt{d}(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)} - \frac{\sqrt{2}(bc-ad)^{3/2}\sqrt[4]{}}{}$$

Mathematica [C] time = 0.0217177, size = 73, normalized size = 0.11

$$\frac{4(a+bx)^{3/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{d(a+bx)}{ad-bc}\right)}{3b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(a + b*x)^(3/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 3/4, 7/4, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/4))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{bx+a}}\frac{1}{\sqrt[4]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(1/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{3}{4}}}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(1/4)*(c + d*x)**(1/4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1703 $\int \frac{1}{(a+bx)^{5/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=718

$$\frac{\sqrt{2}\sqrt[4]{d}\sqrt{bc-ad}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{EllipticF}\left(2\,\text{ta}\right.}{b^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (-4*(c + d*x)^(3/4))/((b*c - a*d)*(a + b*x)^(1/4)) + (4*Sqrt[d]*Sqrt[(a + b
*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
/(Sqrt[b]*(b*c - a*d)^2*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*
x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (2*Sq
rt[2]*d^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d +
 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d)
)*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[
(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*
d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b^(3/4)*(a +
b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))
^2]) + (Sqrt[2]*d^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b
*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(
b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqr
t[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2
]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b^(
3/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c
 + 2*d*x))^2])
```

Rubi [A] time = 0.609157, antiderivative size = 718, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {51, 62, 623, 305, 220, 1196}

$$\frac{\sqrt{2}\sqrt[4]{d}\sqrt{bc-ad}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{2}\right.\right.}{b^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/((b*c - a*d)*(a + b*x)^(1/4)) + (4*Sqrt[d]*Sqrt[(a + b
*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
/(Sqrt[b]*(b*c - a*d)^2*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*
x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (2*Sq
rt[2]*d^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d +
 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d)
)*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[
(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*
d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b^(3/4)*(a +
b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))
^2]) + (Sqrt[2]*d^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b
*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(
b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqr
t[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2
]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b^(
```

```
3/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c
 + 2*d*x))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{(2d)\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx}{bc-ad}$$

$$= -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{\left(2d\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}\,dx}{(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

$$= -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{\left(8d\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-4abcd+(bc+ad)^2+4b\ldots}}\right.}{(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)}$$

$$= -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{\left(4\sqrt{d}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+ad)^2+\ldots}}\right.}{\sqrt{b}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)}$$

$$= -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{4\sqrt{d}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)}$$

Mathematica [C] time = 0.023055, size = 71, normalized size = 0.1

$$-\frac{4\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{4}}\frac{1}{\sqrt[4]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(1/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{3}{4}}}{b^2dx^3+a^2c+\left(b^2c+2\,abd\right)x^2+\left(2\,abc+a^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+bx)^{\frac{5}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/4)*(c + d*x)**(1/4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1704 $\int \frac{1}{(a+bx)^{9/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=760

$$-\frac{2\sqrt{2}d^{5/4}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}(\cdots\right.}{5b^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}\sqrt{bc-ad}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (-4*(c + d*x)^(3/4))/(5*(b*c - a*d)*(a + b*x)^(5/4)) + (8*d*(c + d*x)^(3/4)
)/(5*(b*c - a*d)^2*(a + b*x)^(1/4)) - (8*d^(3/2)*Sqrt[(a + b*x)*(c + d*x)]*
Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(5*Sqrt[b]*(b*
c - a*d)^3*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sq
rt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + (4*Sqrt[2]*d^(5/4)
*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*
Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^
2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c -
a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))
^(1/4))/Sqrt[b*c - a*d]], 1/2]])/(5*b^(3/4)*Sqrt[b*c - a*d]*(a + b*x)^(1/4)*
(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (2*S
qrt[2]*d^(5/4)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1
 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b
*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b
*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2]])/(5*b^(3/4)*Sqrt[b*c - a*d]*(a
 + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*
x))^2])
```

Rubi [A] time = 0.726072, antiderivative size = 760, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {51, 62, 623, 305, 220, 1196}

$$-\frac{2\sqrt{2}d^{5/4}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\cdots}{\sqrt{b\cdots}}\right.\right.}{5b^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}\sqrt{bc-ad}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(5*(b*c - a*d)*(a + b*x)^(5/4)) + (8*d*(c + d*x)^(3/4)
)/(5*(b*c - a*d)^2*(a + b*x)^(1/4)) - (8*d^(3/2)*Sqrt[(a + b*x)*(c + d*x)]*
Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(5*Sqrt[b]*(b*
c - a*d)^3*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sq
rt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + (4*Sqrt[2]*d^(5/4)
*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*
Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^
2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c -
a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))
^(1/4))/Sqrt[b*c - a*d]], 1/2]])/(5*b^(3/4)*Sqrt[b*c - a*d]*(a + b*x)^(1/4)*
(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (2*S
qrt[2]*d^(5/4)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1
 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b
*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
```

```
d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b
*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(5*b^(3/4)*Sqrt[b*c - a*d]*(a
 + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*
x))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} - \frac{(2d)\int \frac{1}{(a+bx)^{5/4}\sqrt[4]{c+dx}}\,dx}{5(bc-ad)}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{\left(4d^2\right)\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx}{5(bc-ad)^2}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{\left(4d^2\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}\,d}{5(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{\left(16d^2\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)}{5(bc-ad)^2\sqrt[4]{a+}}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{\left(8d^{3/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)}{5\sqrt{b}(bc-ad)\sqrt[4]{a}}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{8d^{3/2}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bd}}{5\sqrt{b}(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2b}$$

Mathematica [C] time = 0.0250803, size = 73, normalized size = 0.1

$$-\frac{4\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/4}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, (d
*(a + b*x))/(-(b*c) + a*d)])/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{9}{4}}\frac{1}{\sqrt[4]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{9}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(1/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{3}{4}}}{b^3dx^4+a^3c+\left(b^3c+3\,ab^2d\right)x^3+3\left(ab^2c+a^2bd\right)x^2+\left(3\,a^2bc+a^3d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*
b^2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{9}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(9/4)*(c + d*x)**(1/4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1705 $\int \frac{(a+bx)^{7/4}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=167

$$-\frac{7(a+bx)^{3/4}\sqrt[4]{c+dx}(bc-ad)}{8d^2}-\frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}d^{11/4}}+\frac{21(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}d^{11/4}}+\frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d}$$

```
[Out] (-7*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(8*d^2) + ((a + b*x)^(7/4)
*(c + d*x)^(1/4))/(2*d) - (21*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4)
)/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(1/4)*d^(11/4)) + (21*(b*c - a*d)^2*Arc
Tanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(1/4)*d^(1
1/4))
```

Rubi [A] time = 0.105018, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 331, 298, 205, 208}

$$-\frac{7(a+bx)^{3/4}\sqrt[4]{c+dx}(bc-ad)}{8d^2}-\frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}d^{11/4}}+\frac{21(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}d^{11/4}}+\frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/4)/(c + d*x)^(3/4),x]
```

```
[Out] (-7*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(8*d^2) + ((a + b*x)^(7/4)
*(c + d*x)^(1/4))/(2*d) - (21*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4)
)/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(1/4)*d^(11/4)) + (21*(b*c - a*d)^2*Arc
Tanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(1/4)*d^(1
1/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{7/4}}{(c+dx)^{3/4}}\,dx = \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx}{8d}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{\left(21(bc-ad)^2\right)\int \frac{1}{\sqrt[4]{a+bx}(c+dx)^{3/4}}\,dx}{32d^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{\left(21(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x}{8bd^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{\left(21(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}}{8bd^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{\left(21(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}x^2}\,dx, x, \frac{\sqrt[4]{a}}{\sqrt[4]{c}}}{16d^{5/2}}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} - \frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}d^{11/4}} + \frac{21(bc-a}{}$$

Mathematica [C] time = 0.0324898, size = 73, normalized size = 0.44

$$\frac{4(a+bx)^{11/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(\frac{3}{4},\frac{11}{4};\frac{15}{4};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/4)/(c + d*x)^(3/4),x]
```

```
[Out] (4*(a + b*x)^(11/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4
, 11/4, 15/4, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(3/4))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{4}} (dx+c)^{-\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(3/4), x)
```

Fricas [B] time = 3.75017, size = 3069, normalized size = 18.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/32*(84*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*arctan(-((b^3*c^2*d^8 - 2*a*b^2*c*d^9
 + a^2*b*d^10)*(b*x + a)^(3/4)*(d*x + c)^(1/4)*((b^8*c^8 - 8*a*b^7*c^7*d +
28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c
^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(3/4) - (b
^2*d^8*x + a*b*d^8)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b*d^6*x + a*d^6)*sqrt
((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(b*d^11)))/(b*x + a))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^
2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b
^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(3/4))/(a*b^8*c^8 - 8*a^2*b
^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 5
6*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8 + (b^9*c^8
 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4
*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^
8)*x)) - 21*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*log(21*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b*d^3*x + a*d^3)*((b^8*c^8 - 8*a*b^
7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56
*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^
(1/4))/(b*x + a)) + 21*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 -
```

```
 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*log(21*((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b*d^3*x + a*d^3)*((b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(b*d^11))^(1/4))/(b*x + a)) - 4*(4*b*d*x - 7*b*c + 11*a*d)*(b*x + a)^(3/4)
*(d*x + c)^(1/4))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{7}{4}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(7/4)/(c + d*x)**(3/4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1706 $\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}} dx$

Optimal. Leaf size=127

$$\frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}d^{7/4}}-\frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}d^{7/4}}+\frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d}$$

```
[Out] ((a + b*x)^(3/4)*(c + d*x)^(1/4))/d + (3*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b
*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(1/4)*d^(7/4)) - (3*(b*c - a*d)
*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(1/4)*d
^(7/4))
```

Rubi [A] time = 0.080577, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 331, 298, 205, 208}

$$\frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}d^{7/4}}-\frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}d^{7/4}}+\frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)/(c + d*x)^(3/4),x]
```

```
[Out] ((a + b*x)^(3/4)*(c + d*x)^(1/4))/d + (3*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b
*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(1/4)*d^(7/4)) - (3*(b*c - a*d)
*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(1/4)*d
^(7/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx = \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt[4]{a+bx}(c+dx)^{3/4}}\,dx}{4d}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{bd}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{bd}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2d^{3/2}} + \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}x^2}\right.}{2d^{3/2}}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} + \frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}d^{7/4}} - \frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}d^{7/4}}$$

Mathematica [C] time = 0.027152, size = 73, normalized size = 0.57

$$\frac{4(a+bx)^{7/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)/(c + d*x)^(3/4),x]
```

```
[Out] (4*(a + b*x)^(7/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 7/4, 11/4, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(3/4))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{4}}(dx+c)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(3/4), x)
```

Fricas [B] time = 3.11552, size = 1721, normalized size = 13.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/4*(12*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(b*d^7))^(1/4)*arctan(((b^2*c*d^5 - a*b*d^6)*(b*x + a)^(3/4)*(d*x
+ c)^(1/4)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(b*d^7))^(3/4) + (b^2*d^5*x + a*b*d^5)*sqrt(((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b*d^4*x + a*d^4)*sqrt((b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7)))/(b*x
 + a))*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*
d^4)/(b*d^7))^(3/4))/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a
^4*b*c*d^3 + a^5*d^4 + (b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3
*b^2*c*d^3 + a^4*b*d^4)*x)) + 3*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7))^(1/4)*log(-3*((b*c - a*d)*(b*x + a
)^(3/4)*(d*x + c)^(1/4) + (b*d^2*x + a*d^2)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7))^(1/4))/(b*x + a)) - 3*d*
((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b
*d^7))^(1/4)*log(-3*((b*c - a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b*d^2*x
 + a*d^2)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(b*d^7))^(1/4))/(b*x + a)) - 4*(b*x + a)^(3/4)*(d*x + c)^(1/4))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{4}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(3/4)/(c + d*x)**(3/4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1707 $\int \frac{1}{\sqrt[4]{a+bx}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}d^{3/4}}$$

```
[Out] (-2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(1/4)*d
^(3/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/
(b^(1/4)*d^(3/4))
```

Rubi [A] time = 0.0665139, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {63, 331, 298, 205, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(1/4)*d
^(3/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/
(b^(1/4)*d^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx}(c+dx)^{3/4}}\,dx = \frac{4\,\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{b}$$

$$= \frac{4\,\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{d}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{d}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}d^{3/4}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}d^{3/4}}$$

Mathematica [C] time = 0.029191, size = 73, normalized size = 0.86

$$\frac{4(a+bx)^{3/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{d(a+bx)}{ad-bc}\right)}{3b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(a + b*x)^(3/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 3/4, 7/4, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(3/4))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{bx+a}}(dx+c)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(3/4)), x)
```

Fricas [B] time = 2.43529, size = 581, normalized size = 6.84

$$-4\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}bd^2\left(\frac{1}{bd^3}\right)^{\frac{3}{4}}-\left(b^2d^2x+abd^2\right)\sqrt{\frac{\left(bd^2x+ad^2\right)\sqrt{\frac{1}{bd^3}}+\sqrt{bx+a}\sqrt{dx+c}}{bx+a}}\left(\frac{1}{bd^3}\right)^{\frac{3}{4}}}{bx+a}\right)+\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -4*(1/(b*d^3))^(1/4)*arctan(-((b*x + a)^(3/4)*(d*x + c)^(1/4)*b*d^2*(1/(b*d
^3))^(3/4) - (b^2*d^2*x + a*b*d^2)*sqrt(((b*d^2*x + a*d^2)*sqrt(1/(b*d^3))
+ sqrt(b*x + a)*sqrt(d*x + c))/(b*x + a))*(1/(b*d^3))^(3/4))/(b*x + a)) + (
1/(b*d^3))^(1/4)*log(((b*d*x + a*d)*(1/(b*d^3))^(1/4) + (b*x + a)^(3/4)*(d*
x + c)^(1/4))/(b*x + a)) - (1/(b*d^3))^(1/4)*log(-((b*d*x + a*d)*(1/(b*d^3)
)^(1/4) - (b*x + a)^(3/4)*(d*x + c)^(1/4))/(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(1/4)*(c + d*x)**(3/4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1708 $\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=30

$$-\frac{4\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}(bc-ad)}$$

```
[Out] (-4*(c + d*x)^(1/4))/((b*c - a*d)*(a + b*x)^(1/4))
```

Rubi [A] time = 0.0030378, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{4\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x]

[Out] (-4*(c + d*x)^(1/4))/((b*c - a*d)*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{(bc-ad)\sqrt[4]{a+bx}}$$

Mathematica [A] time = 0.0070169, size = 30, normalized size = 1.

$$\frac{4\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x]

[Out] (4*(c + d*x)^(1/4))/((-(b*c) + a*d)*(a + b*x)^(1/4))
```

Maple [A] time = 0.004, size = 27, normalized size = 0.9

$$4\,\frac{\sqrt[4]{dx+c}}{\sqrt[4]{bx+a}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x)
```

```
[Out] 4/(b*x+a)^(1/4)*(d*x+c)^(1/4)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(3/4)), x)
```

Fricas [A] time = 2.31905, size = 97, normalized size = 3.23

$$-\frac{4\,(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -4*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{4}}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/4)*(c + d*x)**(3/4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1709 $\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=66

$$\frac{16d\sqrt[4]{c+dx}}{5\sqrt[4]{a+bx}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{5(a+bx)^{5/4}(bc-ad)}$$

```
[Out] (-4*(c + d*x)^(1/4))/(5*(b*c - a*d)*(a + b*x)^(5/4)) + (16*d*(c + d*x)^(1/4
))/(5*(b*c - a*d)^2*(a + b*x)^(1/4))
```

Rubi [A] time = 0.0093289, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{16d\sqrt[4]{c+dx}}{5\sqrt[4]{a+bx}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{5(a+bx)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(5*(b*c - a*d)*(a + b*x)^(5/4)) + (16*d*(c + d*x)^(1/4
))/(5*(b*c - a*d)^2*(a + b*x)^(1/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx &= -\frac{4\sqrt[4]{c+dx}}{5(bc-ad)(a+bx)^{5/4}}-\frac{(4d)\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx}{5(bc-ad)}\\ &= -\frac{4\sqrt[4]{c+dx}}{5(bc-ad)(a+bx)^{5/4}}+\frac{16d\sqrt[4]{c+dx}}{5(bc-ad)^2\sqrt[4]{a+bx}}\end{aligned}$$

Mathematica [A] time = 0.0145438, size = 46, normalized size = 0.7

$$\frac{4\sqrt[4]{c+dx}(5ad-bc+4bdx)}{5(a+bx)^{5/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(c + d*x)^(1/4)*(-(b*c) + 5*a*d + 4*b*d*x))/(5*(b*c - a*d)^2*(a + b*x)^(
5/4))
```

Maple [A] time = 0.005, size = 54, normalized size = 0.8

$$\frac{16\,bdx+20\,ad-4\,bc}{5\,a^{2}d^{2}-10\,abcd+5\,b^{2}c^{2}}\sqrt[4]{dx+c}\,(bx+a)^{-\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x)
```

```
[Out] 4/5*(d*x+c)^(1/4)*(4*b*d*x+5*a*d-b*c)/(b*x+a)^(5/4)/(a^2*d^2-2*a*b*c*d+b^2*
c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{9}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(3/4)), x)
```

Fricas [B] time = 2.23628, size = 255, normalized size = 3.86

$$\frac{4\,(4\,bdx-bc+5\,ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{5\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}+\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}\right)x^{2}+2\left(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/5*(4*b*d*x - b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^2*b^2*c^2 -
2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^
3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{9}{4}}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(9/4)*(c + d*x)**(3/4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1710 $\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=101

$$-\frac{128d^2\sqrt[4]{c+dx}}{45\sqrt[4]{a+bx}(bc-ad)^3}+\frac{32d\sqrt[4]{c+dx}}{45(a+bx)^{5/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{9(a+bx)^{9/4}(bc-ad)}$$

```
[Out] (-4*(c + d*x)^(1/4))/(9*(b*c - a*d)*(a + b*x)^(9/4)) + (32*d*(c + d*x)^(1/4
))/(45*(b*c - a*d)^2*(a + b*x)^(5/4)) - (128*d^2*(c + d*x)^(1/4))/(45*(b*c
- a*d)^3*(a + b*x)^(1/4))
```

Rubi [A] time = 0.0174494, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{128d^2\sqrt[4]{c+dx}}{45\sqrt[4]{a+bx}(bc-ad)^3}+\frac{32d\sqrt[4]{c+dx}}{45(a+bx)^{5/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(9*(b*c - a*d)*(a + b*x)^(9/4)) + (32*d*(c + d*x)^(1/4
))/(45*(b*c - a*d)^2*(a + b*x)^(5/4)) - (128*d^2*(c + d*x)^(1/4))/(45*(b*c
- a*d)^3*(a + b*x)^(1/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx &= -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}}-\frac{(8d)\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx}{9(bc-ad)}\\
&= -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}}+\frac{32d\sqrt[4]{c+dx}}{45(bc-ad)^2(a+bx)^{5/4}}+\frac{\left(32d^2\right)\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx}{45(bc-ad)^2}\\
&= -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}}+\frac{32d\sqrt[4]{c+dx}}{45(bc-ad)^2(a+bx)^{5/4}}-\frac{128d^2\sqrt[4]{c+dx}}{45(bc-ad)^3\sqrt[4]{a+bx}}
\end{aligned}$$

Mathematica [A] time = 0.0316454, size = 75, normalized size = 0.74

$$-\frac{4\sqrt[4]{c+dx}\left(45a^2d^2-18abd(c-4dx)+b^2\left(5c^2-8cdx+32d^2x^2\right)\right)}{45(a+bx)^{9/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4)*(45*a^2*d^2 - 18*a*b*d*(c - 4*d*x) + b^2*(5*c^2 - 8*c*d
*x + 32*d^2*x^2)))/(45*(b*c - a*d)^3*(a + b*x)^(9/4))
```

Maple [A] time = 0.005, size = 105, normalized size = 1.

$$\frac{128\,b^2d^2x^2+288\,abd^2x-32\,b^2cdx+180\,a^2d^2-72\,abcd+20\,b^2c^2}{45\,a^3d^3-135\,a^2cbd^2+135\,ab^2c^2d-45\,b^3c^3}\sqrt[4]{dx+c}\,(bx+a)^{-\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x)
```

```
[Out] 4/45*(d*x+c)^(1/4)*(32*b^2*d^2*x^2+72*a*b*d^2*x-8*b^2*c*d*x+45*a^2*d^2-18*a
*b*c*d+5*b^2*c^2)/(b*x+a)^(9/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^
3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{13}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(13/4)*(d*x + c)^(3/4)), x)
```

Fricas [B] time = 2.36675, size = 516, normalized size = 5.11

$$-\frac{4\left(32\,b^2d^2x^2+5\,b^2c^2-18\,abcd+45\,a^2d^2-8\left(b^2cd-9\,abd^2\right)x\right)(bx+}{45\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -4/45*(32*b^2*d^2*x^2 + 5*b^2*c^2 - 18*a*b*c*d + 45*a^2*d^2 - 8*(b^2*c*d -
9*a*b*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*
d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 -
a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b
```

```
^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^
3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(13/4)/(d*x+c)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1711 $\int \frac{1}{(a+bx)^{17/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=136

$$\frac{512d^3\sqrt[4]{c+dx}}{195\sqrt[4]{a+bx}(bc-ad)^4}-\frac{128d^2\sqrt[4]{c+dx}}{195(a+bx)^{5/4}(bc-ad)^3}+\frac{16d\sqrt[4]{c+dx}}{39(a+bx)^{9/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{13(a+bx)^{13/4}(bc-ad)}$$

```
[Out] (-4*(c + d*x)^(1/4))/(13*(b*c - a*d)*(a + b*x)^(13/4)) + (16*d*(c + d*x)^(1
/4))/(39*(b*c - a*d)^2*(a + b*x)^(9/4)) - (128*d^2*(c + d*x)^(1/4))/(195*(b
*c - a*d)^3*(a + b*x)^(5/4)) + (512*d^3*(c + d*x)^(1/4))/(195*(b*c - a*d)^4
*(a + b*x)^(1/4))
```

Rubi [A] time = 0.0281114, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{512d^3\sqrt[4]{c+dx}}{195\sqrt[4]{a+bx}(bc-ad)^4}-\frac{128d^2\sqrt[4]{c+dx}}{195(a+bx)^{5/4}(bc-ad)^3}+\frac{16d\sqrt[4]{c+dx}}{39(a+bx)^{9/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{13(a+bx)^{13/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(13*(b*c - a*d)*(a + b*x)^(13/4)) + (16*d*(c + d*x)^(1
/4))/(39*(b*c - a*d)^2*(a + b*x)^(9/4)) - (128*d^2*(c + d*x)^(1/4))/(195*(b
*c - a*d)^3*(a + b*x)^(5/4)) + (512*d^3*(c + d*x)^(1/4))/(195*(b*c - a*d)^4
*(a + b*x)^(1/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{17/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} - \frac{(12d)\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx}{13(bc-ad)}$$
$$= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}} + \frac{\left(32d^2\right)\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx}{39(bc-ad)^2}$$
$$= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}} - \frac{128d^2\sqrt[4]{c+dx}}{195(bc-ad)^3(a+bx)^{5/4}} - \frac{\left(128d^3\right)\int}{19}$$
$$= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}} - \frac{128d^2\sqrt[4]{c+dx}}{195(bc-ad)^3(a+bx)^{5/4}} + \frac{512d}{195(bc-}$$

Mathematica [A] time = 0.0469024, size = 116, normalized size = 0.85

$$\frac{4\sqrt[4]{c+dx}\left(-117a^2bd^2(c-4dx)+195a^3d^3+13ab^2d\left(5c^2-8cdx+32d^2x^2\right)+b^3\left(20c^2dx-15c^3-32cd^2x^2+128d^3x^3\right)\right)}{195(a+bx)^{13/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(c + d*x)^(1/4)*(195*a^3*d^3 - 117*a^2*b*d^2*(c - 4*d*x) + 13*a*b^2*d*(5
*c^2 - 8*c*d*x + 32*d^2*x^2) + b^3*(-15*c^3 + 20*c^2*d*x - 32*c*d^2*x^2 + 1
28*d^3*x^3)))/(195*(b*c - a*d)^4*(a + b*x)^(13/4))
```

Maple [A] time = 0.007, size = 171, normalized size = 1.3

$$\frac{512\,x^3b^3d^3+1664\,ab^2d^3x^2-128\,b^3cd^2x^2+1872\,a^2bd^3x-416\,ab^2cd^2x+80\,b^3c^2dx+780\,a^3d^3-468\,a^2cbd^2+260\,ab^2c^2d}{195\,a^4d^4-780\,a^3bcd^3+1170\,a^2c^2b^2d^2-780\,ab^3c^3d+195\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x)
```

```
[Out] 4/195*(d*x+c)^(1/4)*(128*b^3*d^3*x^3+416*a*b^2*d^3*x^2-32*b^3*c*d^2*x^2+468
*a^2*b*d^3*x-104*a*b^2*c*d^2*x+20*b^3*c^2*d*x+195*a^3*d^3-117*a^2*b*c*d^2+6
5*a*b^2*c^2*d-15*b^3*c^3)/(b*x+a)^(13/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c
^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{17}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(17/4)*(d*x + c)^(3/4)), x)
```

Fricas [B] time = 2.59598, size = 864, normalized size = 6.35

$$\frac{4\left(128\,b^3d^3x^3 - 15\,b^3c^3 + 65\,ab^2c^2d - 117\,a^2\cdots\right.}{195\left(a^4b^4c^4 - 4\,a^5b^3c^3d + 6\,a^6b^2c^2d^2 - 4\,a^7bcd^3 + a^8d^4 + \left(b^8c^4 - 4\,ab^7c^3d + 6\,a^2b^6c^2d^2 - 4\,a^3b^5cd^3 + a^4b^4d^4\right)x^4 + 4\left(a\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/195*(128*b^3*d^3*x^3 - 15*b^3*c^3 + 65*a*b^2*c^2*d - 117*a^2*b*c*d^2 + 19
5*a^3*d^3 - 32*(b^3*c*d^2 - 13*a*b^2*d^3)*x^2 + 4*(5*b^3*c^2*d - 26*a*b^2*c
*d^2 + 117*a^2*b*d^3)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^4*b^4*c^4 - 4*a
^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a
*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*
b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d
^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*
c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2
*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(17/4)/(d*x+c)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1712 $\int \frac{(a+bx)^{5/4}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=332

$$\frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,\text{EllipticF}\Bigl(2\,\text{ta}}{6\sqrt{2}\sqrt[4]{b}d^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(3*d^2) + (2*(a + b*x)^(5/
4)*(c + d*x)^(1/4))/(3*d) + (5*(b*c - a*d)^(5/2)*((a + b*x)*(c + d*x))^(3/4
)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
 d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sq
rt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTa
n[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1
/2])/(6*Sqrt[2]*b^(1/4)*d^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d
+ 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.28617, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 62, 623, 220}

$$-\frac{5\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)}{3d^2}+\frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+\cdots}{(bc-ad)^2\bigl(2\sqrt{b}\cdots}}}{6\sqrt{2}\sqrt[4]{b}d^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(a\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)/(c + d*x)^(3/4),x]
```


```
[Out] (-5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(3*d^2) + (2*(a + b*x)^(5/
4)*(c + d*x)^(1/4))/(3*d) + (5*(b*c - a*d)^(5/2)*((a + b*x)*(c + d*x))^(3/4
)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
 d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sq
rt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTa
n[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1
/2])/(6*Sqrt[2]*b^(1/4)*d^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d
+ 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+bx)^{5/4}}{(c+dx)^{3/4}}\,dx = \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} - \frac{(5(bc-ad))\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{3/4}}\,dx}{6d}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{3d^2} + \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} + \frac{\left(5(bc-ad)^2\right)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{12d^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{3d^2} + \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} + \frac{\left(5(bc-ad)^2((a+bx)(c+dx))^{3/4}\right)\int \frac{}{(ac+(bc+}}{12d^2(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{3d^2} + \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} + \frac{\left(5(bc-ad)^2((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad}}{3d^2(a+}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{3d^2} + \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} + \frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad}}{6\sqrt{2}\sqrt[4]{}}$$

Mathematica [C] time = 0.0308594, size = 73, normalized size = 0.22

$$\frac{4(a+bx)^{9/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{ad-bc}\right)}{9b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)/(c + d*x)^(3/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 9/4, 13/4, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(3/4))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{4}}(dx+c)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(5/4)/(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/4)/(d*x + c)^(3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{5}{4}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(5/4)/(c + d*x)**(3/4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1713 $\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=295

$$\frac{2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{d}-\frac{(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c}}{bc-ad}\right.}}}{\sqrt{2}\sqrt[4]{b}d^{5/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b}}$$

```
[Out] (2*(a + b*x)^(1/4)*(c + d*x)^(1/4))/d - ((b*c - a*d)^(3/2)*((a + b*x)*(c +
d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a +
 b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*
(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*Ellipti
cF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c
- a*d]], 1/2])/(Sqrt[2]*b^(1/4)*d^(5/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*
c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.223518, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 62, 623, 220}

$$\frac{2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{d}-\frac{(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c}}{bc-ad}\right.}}}{\sqrt{2}\sqrt[4]{b}d^{5/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)/(c + d*x)^(3/4),x]
```

```
[Out] (2*(a + b*x)^(1/4)*(c + d*x)^(1/4))/d - ((b*c - a*d)^(3/2)*((a + b*x)*(c +
d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a +
 b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*
(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*Ellipti
cF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c
- a*d]], 1/2])/(Sqrt[2]*b^(1/4)*d^(5/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*
c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{3/4}}\,dx = \frac{2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{2d}$$

$$= \frac{2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{d} - \frac{\left((bc-ad)((a+bx)(c+dx))^{3/4}\right)\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^{3/4}}\,dx}{2d(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= \frac{2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{d} - \frac{\left(2(bc-ad)((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+ad)^2+4b}}\right.}{d(a+bx)^{3/4}(c+dx)^{3/4}(bc+ad+2bdx)}$$

$$= \frac{2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{d} - \frac{(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)\sqrt{\frac{}{(bc}}}{\sqrt{2}\sqrt[4]{b}d^{5/4}(a+bx)^{3/4}(c+dx)^{3/4}(bc+ad+2b}$$

Mathematica [C] time = 0.0253697, size = 73, normalized size = 0.25

$$\frac{4(a+bx)^{5/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)/(c + d*x)^(3/4),x]
```

```
[Out] (4*(a + b*x)^(5/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 5/4, 9/4, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(3/4))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \sqrt[4]{bx+a}\,(dx+c)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(1/4)/(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)/(d*x + c)^(3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(1/4)/(c + d*x)**(3/4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1714 $\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=270

$$\frac{\sqrt{2}\sqrt{bc-ad}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,\text{EllipticF}\left(2\tan\right.}{\sqrt[4]{b}\sqrt[4]{d}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (Sqrt[2]*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*
d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqr
t[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a +
b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/
4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b^(1/4)*d^(1/4)*(a
 + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*
x))^2])
```

Rubi [A] time = 0.175063, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {62, 623, 220}

$$\frac{\sqrt{2}\sqrt{bc-ad}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt{2}\right.\right.}{\sqrt[4]{b}\sqrt[4]{d}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*
d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqr
t[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a +
b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/
4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b^(1/4)*d^(1/4)*(a
 + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*
x))^2])
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx = \frac{((a+bx)(c+dx))^{3/4}\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^{3/4}}\,dx}{(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= \frac{\left(4((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+ad)^2+4bdx^4}}\,dx, x, \sqrt[4]{(a+\right.}{(a+bx)^{3/4}(c+dx)^{3/4}(bc+ad+2bdx)}$$

$$= \frac{\sqrt{2}\sqrt{bc-ad}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)\sqrt{\frac{(a}{(bc-ad)^2(1}}}{\sqrt[4]{b}\sqrt[4]{d}(a+bx)^{3/4}(c+dx)^{3/4}(bc+ad+2bdx)\sqrt{(ad}}$$

Mathematica [C] time = 0.0224537, size = 71, normalized size = 0.26

$$\frac{4\sqrt[4]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{d(a+bx)}{ad-bc}\right)}{b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[1/4,
 3/4, 5/4, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(3/4))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{3}{4}}(dx+c)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(3/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{1}{4}}}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(1/4)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+bx)^{\frac{3}{4}}(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/4)*(c + d*x)**(3/4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1715 $\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=306

$$-\frac{2\sqrt{2}d^{3/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{EllipticF}\left(2\tan\right.}{3\sqrt[4]{b}(a+bx)^{3/4}(c+dx)^{3/4}\sqrt{bc-ad}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (-4*(c + d*x)^(1/4))/(3*(b*c - a*d)*(a + b*x)^(3/4)) - (2*Sqrt[2]*d^(3/4)*(
(a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sq
rt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/
((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*
d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(
1/4))/Sqrt[b*c - a*d]], 1/2])/(3*b^(1/4)*Sqrt[b*c - a*d]*(a + b*x)^(3/4)*(c
 + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.21517, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {51, 62, 623, 220}

$$-\frac{2\sqrt{2}d^{3/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{}}{}\right.\right.}{3\sqrt[4]{b}(a+bx)^{3/4}(c+dx)^{3/4}\sqrt{bc-ad}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(3*(b*c - a*d)*(a + b*x)^(3/4)) - (2*Sqrt[2]*d^(3/4)*(
(a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sq
rt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/
((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*
d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(
1/4))/Sqrt[b*c - a*d]], 1/2])/(3*b^(1/4)*Sqrt[b*c - a*d]*(a + b*x)^(3/4)*(c
 + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}\,dx &= -\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}}-\frac{(2d)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{3(bc-ad)}\\
&= -\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}}-\frac{\left(2d((a+bx)(c+dx))^{3/4}\right)\int \frac{1}{\left(ac+(bc+ad)x+bdx^2\right)^{3/4}}\,dx}{3(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}\\
&= -\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}}-\frac{\left(8d((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc}\ldots\right)}{3(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}(bc+ac\ldots}\\
&= -\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}}-\frac{2\sqrt{2}d^{3/4}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)}\ldots}{bc-ad}\ldots\right.}{3\sqrt[4]{b}\sqrt{bc-ad}(a+bx)^{3/4}(c+dx)^{3}\ldots}\end{aligned}$$

Mathematica [C] time = 0.0243104, size = 73, normalized size = 0.24

$$-\frac{4\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/4}(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, (d*
(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/4)*(c + d*x)^(3/4))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{7}{4}}(dx+c)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(3/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{1}{4}}}{b^2dx^3+a^2c+\left(b^2c+2\,abd\right)x^2+\left(2\,abc+a^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(1/4)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{7}{4}}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(7/4)*(c + d*x)**(3/4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1716 $\int \frac{1}{(a+bx)^{11/4}(c+dx)^{3/4}} dx$

Optimal. Leaf size=339

$$\frac{4\sqrt{2}d^{7/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\right.\right.}{7\sqrt[4]{b}(a+bx)^{3/4}(c+dx)^{3/4}(bc-ad)^{3/2}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (-4*(c + d*x)^(1/4))/(7*(b*c - a*d)*(a + b*x)^(7/4)) + (8*d*(c + d*x)^(1/4)
)/(7*(b*c - a*d)^2*(a + b*x)^(3/4)) + (4*Sqrt[2]*d^(7/4)*((a + b*x)*(c + d*
x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b
*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1
 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF
[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c -
a*d]], 1/2])/(7*b^(1/4)*(b*c - a*d)^(3/2)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(
b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.279612, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {51, 62, 623, 220}

$$\frac{4\sqrt{2}d^{7/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}{\sqrt{}}\right.\right.}{7\sqrt[4]{b}(a+bx)^{3/4}(c+dx)^{3/4}(bc-ad)^{3/2}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(7*(b*c - a*d)*(a + b*x)^(7/4)) + (8*d*(c + d*x)^(1/4)
)/(7*(b*c - a*d)^2*(a + b*x)^(3/4)) + (4*Sqrt[2]*d^(7/4)*((a + b*x)*(c + d*
x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b
*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1
 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF
[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c -
a*d]], 1/2])/(7*b^(1/4)*(b*c - a*d)^(3/2)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(
b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} - \frac{(6d)\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}\,dx}{7(bc-ad)}$$

$$= -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} + \frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}} + \frac{\left(4d^2\right)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{7(bc-ad)^2}$$

$$= -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} + \frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}} + \frac{\left(4d^2((a+bx)(c+dx))^{3/4}\right)\int \frac{}{\left(ac+(bc+a\right.}}{7(bc-ad)^2(a+bx)^{3/4}(c+d}$$

$$= -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} + \frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}} + \frac{\left(16d^2((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad}\right.}{7(bc-ad}$$

$$= -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} + \frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}} + \frac{4\sqrt{2}d^{7/4}((a+bx)(c+dx))^{3/4}\sqrt{(bc+}}{7\sqrt[4]{b}}$$

Mathematica [C] time = 0.0250698, size = 73, normalized size = 0.22

$$-\frac{4\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};\frac{d(a+bx)}{ad-bc}\right)}{7b(a+bx)^{7/4}(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, (d
*(a + b*x))/(-(b*c) + a*d)])/(7*b*(a + b*x)^(7/4)*(c + d*x)^(3/4))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{11}{4}} (dx+c)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(3/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{1}{4}}}{b^3dx^4+a^3c+\left(b^3c+3\,ab^2d\right)x^3+3\left(ab^2c+a^2bd\right)x^2+\left(3\,a^2bc+a^3d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(1/4)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*
b^2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/4)/(d*x+c)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1717 $\int \frac{(a+bx)^{5/4}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=152

$$\frac{5b\sqrt[4]{a+bx}(c+dx)^{3/4}}{d^2}-\frac{5\sqrt[4]{b}(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{5\sqrt[4]{b}(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}$$

```
[Out] (-4*(a + b*x)^(5/4))/(d*(c + d*x)^(1/4)) + (5*b*(a + b*x)^(1/4)*(c + d*x)^(
3/4))/d^2 - (5*b^(1/4)*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4
)*(c + d*x)^(1/4))])/(2*d^(9/4)) - (5*b^(1/4)*(b*c - a*d)*ArcTanh[(d^(1/4)*
(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*d^(9/4))
```

Rubi [A] time = 0.0871252, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {47, 50, 63, 240, 212, 208, 205}

$$\frac{5b\sqrt[4]{a+bx}(c+dx)^{3/4}}{d^2}-\frac{5\sqrt[4]{b}(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{5\sqrt[4]{b}(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(5/4))/(d*(c + d*x)^(1/4)) + (5*b*(a + b*x)^(1/4)*(c + d*x)^(
3/4))/d^2 - (5*b^(1/4)*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4
)*(c + d*x)^(1/4))])/(2*d^(9/4)) - (5*b^(1/4)*(b*c - a*d)*ArcTanh[(d^(1/4)*
(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*d^(9/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/4}}{(c+dx)^{5/4}}\,dx &= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{(5b)\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx}{d}\\
&= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{5b\sqrt[4]{a+bx}(c+dx)^{3/4}}{d^2}-\frac{(5b(bc-ad))\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{4d^2}\\
&= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{5b\sqrt[4]{a+bx}(c+dx)^{3/4}}{d^2}-\frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx,x,\sqrt[4]{a+bx}\right)}{d^2}\\
&= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{5b\sqrt[4]{a+bx}(c+dx)^{3/4}}{d^2}-\frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d^2}\\
&= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{5b\sqrt[4]{a+bx}(c+dx)^{3/4}}{d^2}-\frac{\left(5\sqrt{b}(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}x^2}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2d^2}-\frac{\left(5\sqrt{b}(bc-\right.}{}\\
&= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{5b\sqrt[4]{a+bx}(c+dx)^{3/4}}{d^2}-\frac{5\sqrt[4]{b}(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{5\sqrt[4]{b}(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}}{\sqrt[4]{b}}\right.}{2d^{9/4}}
\end{aligned}$$

Mathematica [C] time = 0.0528923, size = 73, normalized size = 0.48

$$\frac{4(a+bx)^{9/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{ad-bc}\right)}{9b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 9/4, 13/4, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(5/4))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int (bx + a)^{\frac{5}{4}} (dx + c)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(5/4)/(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^{\frac{5}{4}}}{(dx + c)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(5/4), x)
```

Fricas [B] time = 3.15997, size = 1818, normalized size = 11.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/4*(20*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*arctan(((b*c*d^7 - a*d^8)*(b*x + a)^(
1/4)*(d*x + c)^(3/4)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*
b^2*c*d^3 + a^4*b*d^4)/d^9)^(3/4) + (d^8*x + c*d^7)*sqrt(((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (d^5*x + c*d^4)*sqrt((b^5*c^4
- 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9))/(d
*x + c))*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4)/d^9)^(3/4))/(b^5*c^5 - 4*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3
*b^2*c^2*d^3 + a^4*b*c*d^4 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d
^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)) + 5*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b
^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*log(
-5*((b*c - a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (d^3*x + c*d^2)*((b^5*c^4
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1
/4))/(d*x + c)) - 5*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c
^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*log(-5*((b*c - a*d)*(b*x +
 a)^(1/4)*(d*x + c)^(3/4) - (d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a
```

```
^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4))/(d*x + c)) - 4*(b
*d*x + 5*b*c - 4*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{5}{4}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(5/4)/(c + d*x)**(5/4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1718 $\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=108

$$\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}-\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}$$

```
[Out] (-4*(a + b*x)^(1/4))/(d*(c + d*x)^(1/4)) + (2*b^(1/4)*ArcTan[(d^(1/4)*(a +
b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4) + (2*b^(1/4)*ArcTanh[(d^(1/
4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4)
```

Rubi [A] time = 0.0652558, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {47, 63, 240, 212, 208, 205}

$$\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}-\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(1/4))/(d*(c + d*x)^(1/4)) + (2*b^(1/4)*ArcTan[(d^(1/4)*(a +
b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4) + (2*b^(1/4)*ArcTanh[(d^(1/
4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{5/4}}\,dx &= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{b\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{4\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx,x,\sqrt[4]{a+bx}\right)}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{4\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{\left(2\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}x^2}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d}+\frac{\left(2\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}x^2}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}
\end{aligned}
$$

Mathematica [C] time = 0.0434086, size = 73, normalized size = 0.68

$$\frac{4(a+bx)^{5/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(5/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 5/4, 9/4, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(5/4))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \sqrt[4]{bx+a}\,(dx+c)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(5/4), x)
```

Fricas [B] time = 2.47309, size = 656, normalized size = 6.07

$$-\frac{4\left(d^2x+cd\right)\left(\frac{b}{d^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}d^4\left(\frac{b}{d^5}\right)^{\frac{3}{4}}-\left(d^5x+cd^4\right)\sqrt{\frac{\left(d^3x+cd^2\right)\sqrt{\frac{b}{d^5}}+\sqrt{bx+a}\sqrt{dx+c}}{dx+c}}\left(\frac{b}{d^5}\right)^{\frac{3}{4}}}{bdx+bc}\right)-\left(d^2x+cd\right)\left(\frac{b}{d^5}\right)^{\frac{1}{4}}\log\left(\frac{\left(d^2x+c\right.}{}\right.}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -(4*(d^2*x + c*d)*(b/d^5)^(1/4)*arctan(-((b*x + a)^(1/4)*(d*x + c)^(3/4)*d^
4*(b/d^5)^(3/4) - (d^5*x + c*d^4)*sqrt(((d^3*x + c*d^2)*sqrt(b/d^5) + sqrt(
b*x + a)*sqrt(d*x + c))/(d*x + c))*(b/d^5)^(3/4))/(b*d*x + b*c)) - (d^2*x +
 c*d)*(b/d^5)^(1/4)*log(((d^2*x + c*d)*(b/d^5)^(1/4) + (b*x + a)^(1/4)*(d*x
 + c)^(3/4))/(d*x + c)) + (d^2*x + c*d)*(b/d^5)^(1/4)*log(-((d^2*x + c*d)*(
b/d^5)^(1/4) - (b*x + a)^(1/4)*(d*x + c)^(3/4))/(d*x + c)) + 4*(b*x + a)^(1
/4)*(d*x + c)^(3/4))/(d^2*x + c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(1/4)/(c + d*x)**(5/4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1719 $\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=30

$$\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}(bc-ad)}$$

```
[Out] (4*(a + b*x)^(1/4))/((b*c - a*d)*(c + d*x)^(1/4))
```

Rubi [A] time = 0.002851, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4))/((b*c - a*d)*(c + d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx = \frac{4\sqrt[4]{a+bx}}{(bc-ad)\sqrt[4]{c+dx}}$$

Mathematica [A] time = 0.0070774, size = 30, normalized size = 1.

$$\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4))/((b*c - a*d)*(c + d*x)^(1/4))
```

Maple [A] time = 0.005, size = 27, normalized size = 0.9

$$-4\,{\frac{\sqrt[4]{bx+a}}{\sqrt[4]{dx+c}\,(ad-bc)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x)
```

```
[Out] -4*(b*x+a)^(1/4)/(d*x+c)^(1/4)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(5/4)), x)
```

Fricas [A] time = 2.19313, size = 96, normalized size = 3.2

$$\frac{4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] 4*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{3}{4}}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/4)*(c + d*x)**(5/4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1720 $\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=66

$$-\frac{16d\sqrt[4]{a+bx}}{3\sqrt[4]{c+dx}(bc-ad)^2}-\frac{4}{3(a+bx)^{3/4}\sqrt[4]{c+dx}(bc-ad)}$$

```
[Out] -4/(3*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4)) - (16*d*(a + b*x)^(1/4))
/(3*(b*c - a*d)^2*(c + d*x)^(1/4))
```

Rubi [A] time = 0.008442, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d\sqrt[4]{a+bx}}{3\sqrt[4]{c+dx}(bc-ad)^2}-\frac{4}{3(a+bx)^{3/4}\sqrt[4]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(3*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4)) - (16*d*(a + b*x)^(1/4))
/(3*(b*c - a*d)^2*(c + d*x)^(1/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}}\,dx &= -\frac{4}{3(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}-\frac{(4d)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx}{3(bc-ad)}\\ &= -\frac{4}{3(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}-\frac{16d\sqrt[4]{a+bx}}{3(bc-ad)^2\sqrt[4]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0170147, size = 45, normalized size = 0.68

$$-\frac{4(3ad+b(c+4dx))}{3(a+bx)^{3/4}\sqrt[4]{c+dx}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(3*a*d + b*(c + 4*d*x)))/(3*(b*c - a*d)^2*(a + b*x)^(3/4)*(c + d*x)^(1/
4))
```

Maple [A] time = 0.005, size = 53, normalized size = 0.8

$$-\frac{16\,bdx+12\,ad+4\,bc}{3\,a^2d^2-6\,abcd+3\,b^2c^2}(bx+a)^{-\frac{3}{4}}\frac{1}{\sqrt[4]{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x)
```

```
[Out] -4/3*(4*b*d*x+3*a*d+b*c)/(b*x+a)^(3/4)/(d*x+c)^(1/4)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(5/4)), x)
```

Fricas [B] time = 2.68122, size = 270, normalized size = 4.09

$$-\frac{4\,(4\,bdx+bc+3\,ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{3\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -4/3*(4*b*d*x + b*c + 3*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{7}{4}}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(7/4)*(c + d*x)**(5/4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1721 $\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=101

$$\frac{128d^2\sqrt[4]{a+bx}}{21\sqrt[4]{c+dx}(bc-ad)^3}+\frac{32d}{21(a+bx)^{3/4}\sqrt[4]{c+dx}(bc-ad)^2}-\frac{4}{7(a+bx)^{7/4}\sqrt[4]{c+dx}(bc-ad)}$$

```
[Out] -4/(7*(b*c - a*d)*(a + b*x)^(7/4)*(c + d*x)^(1/4)) + (32*d)/(21*(b*c - a*d)
^2*(a + b*x)^(3/4)*(c + d*x)^(1/4)) + (128*d^2*(a + b*x)^(1/4))/(21*(b*c -
a*d)^3*(c + d*x)^(1/4))
```

Rubi [A] time = 0.0165107, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{128d^2\sqrt[4]{a+bx}}{21\sqrt[4]{c+dx}(bc-ad)^3}+\frac{32d}{21(a+bx)^{3/4}\sqrt[4]{c+dx}(bc-ad)^2}-\frac{4}{7(a+bx)^{7/4}\sqrt[4]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x]
```


```
[Out] -4/(7*(b*c - a*d)*(a + b*x)^(7/4)*(c + d*x)^(1/4)) + (32*d)/(21*(b*c - a*d)
^2*(a + b*x)^(3/4)*(c + d*x)^(1/4)) + (128*d^2*(a + b*x)^(1/4))/(21*(b*c -
a*d)^3*(c + d*x)^(1/4))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx &= -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}-\frac{(8d)\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}}\,dx}{7(bc-ad)}\\
&= -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}+\frac{32d}{21(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}+\frac{\left(32d^2\right)\int \frac{1}{(a+bx)^{3/4}(c+dx)}}{21(bc-ad)^2}\\
&= -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}+\frac{32d}{21(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}+\frac{128d^2\sqrt[4]{a+bx}}{21(bc-ad)^3\sqrt[4]{c+dx}}
\end{aligned}$$

Mathematica [A] time = 0.0314335, size = 76, normalized size = 0.75

$$\frac{84a^2d^2+56abd(c+4dx)+4b^2\left(-3c^2+8cdx+32d^2x^2\right)}{21(a+bx)^{7/4}\sqrt[4]{c+dx}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (84*a^2*d^2 + 56*a*b*d*(c + 4*d*x) + 4*b^2*(-3*c^2 + 8*c*d*x + 32*d^2*x^2))
/(21*(b*c - a*d)^3*(a + b*x)^(7/4)*(c + d*x)^(1/4))
```

Maple [A] time = 0.006, size = 105, normalized size = 1.

$$-\frac{128\,b^2d^2x^2+224\,abd^2x+32\,b^2cdx+84\,a^2d^2+56\,abcd-12\,b^2c^2}{21\,a^3d^3-63\,a^2cbd^2+63\,ab^2c^2d-21\,b^3c^3}(bx+a)^{-\frac{7}{4}}\frac{1}{\sqrt[4]{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x)
```

```
[Out] -4/21*(32*b^2*d^2*x^2+56*a*b*d^2*x+8*b^2*c*d*x+21*a^2*d^2+14*a*b*c*d-3*b^2*
c^2)/(b*x+a)^(7/4)/(d*x+c)^(1/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c
^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(5/4)), x)
```

Fricas [B] time = 4.1968, size = 558, normalized size = 5.52

$$\frac{4\left(32\,b^2d^2x^2-3\,b^2c^2+14\,abcd+21\,a^2d^2+8\left(b^2cd+7\,abd\right.\right.}{21\left(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+\left(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4\right)x^3+\left(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] 4/21*(32*b^2*d^2*x^2 - 3*b^2*c^2 + 14*a*b*c*d + 21*a^2*d^2 + 8*(b^2*c*d + 7
*a*b*d^2)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d
 + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c
*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^
```

```
3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2
*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/4)/(d*x+c)**(5/4),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="giac")

[Out] Timed out
```

3.1722 $\int \frac{1}{(a+bx)^{15/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=136

$$-\frac{512d^3\sqrt[4]{a+bx}}{77\sqrt[4]{c+dx}(bc-ad)^4}-\frac{128d^2}{77(a+bx)^{3/4}\sqrt[4]{c+dx}(bc-ad)^3}+\frac{48d}{77(a+bx)^{7/4}\sqrt[4]{c+dx}(bc-ad)^2}-\frac{4}{11(a+bx)^{11/4}\sqrt[4]{c+dx}(b}$$

```
[Out] -4/(11*(b*c - a*d)*(a + b*x)^(11/4)*(c + d*x)^(1/4)) + (48*d)/(77*(b*c - a*
d)^2*(a + b*x)^(7/4)*(c + d*x)^(1/4)) - (128*d^2)/(77*(b*c - a*d)^3*(a + b*
x)^(3/4)*(c + d*x)^(1/4)) - (512*d^3*(a + b*x)^(1/4))/(77*(b*c - a*d)^4*(c
+ d*x)^(1/4))
```

Rubi [A] time = 0.027462, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{512d^3\sqrt[4]{a+bx}}{77\sqrt[4]{c+dx}(bc-ad)^4}-\frac{128d^2}{77(a+bx)^{3/4}\sqrt[4]{c+dx}(bc-ad)^3}+\frac{48d}{77(a+bx)^{7/4}\sqrt[4]{c+dx}(bc-ad)^2}-\frac{4}{11(a+bx)^{11/4}\sqrt[4]{c+dx}(b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(11*(b*c - a*d)*(a + b*x)^(11/4)*(c + d*x)^(1/4)) + (48*d)/(77*(b*c - a*
d)^2*(a + b*x)^(7/4)*(c + d*x)^(1/4)) - (128*d^2)/(77*(b*c - a*d)^3*(a + b*
x)^(3/4)*(c + d*x)^(1/4)) - (512*d^3*(a + b*x)^(1/4))/(77*(b*c - a*d)^4*(c
+ d*x)^(1/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{15/4}(c+dx)^{5/4}}\,dx = -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} - \frac{(12d)\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx}{11(bc-ad)}$$
$$= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} + \frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}} + \frac{\left(96d^2\right)\int \frac{1}{(a+bx)^{7/4}(c+}}{77(bc-ad)^2}$$
$$= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} + \frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}} - \frac{128d^2}{77(bc-ad)^3(a+bx}$$
$$= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} + \frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}} - \frac{128d^2}{77(bc-ad)^3(a+bx}$$

Mathematica [A] time = 0.0422317, size = 116, normalized size = 0.85

$$-\frac{4\left(77a^2bd^2(c+4dx)+77a^3d^3+11ab^2d\left(-3c^2+8cdx+32d^2x^2\right)+b^3\left(-12c^2dx+7c^3+32cd^2x^2+128d^3x^3\right)\right)}{77(a+bx)^{11/4}\sqrt[4]{c+dx}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(77*a^3*d^3 + 77*a^2*b*d^2*(c + 4*d*x) + 11*a*b^2*d*(-3*c^2 + 8*c*d*x +
 32*d^2*x^2) + b^3*(7*c^3 - 12*c^2*d*x + 32*c*d^2*x^2 + 128*d^3*x^3)))/(77*
(b*c - a*d)^4*(a + b*x)^(11/4)*(c + d*x)^(1/4))
```

Maple [A] time = 0.008, size = 171, normalized size = 1.3

$$-\frac{512\,x^3b^3d^3+1408\,ab^2d^3x^2+128\,b^3cd^2x^2+1232\,a^2bd^3x+352\,ab^2cd^2x-48\,b^3c^2dx+308\,a^3d^3+308\,a^2cbd^2-132\,ab^2c}{77\,a^4d^4-308\,a^3bcd^3+462\,a^2c^2b^2d^2-308\,ab^3c^3d+77\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x)
```

```
[Out] -4/77*(128*b^3*d^3*x^3+352*a*b^2*d^3*x^2+32*b^3*c*d^2*x^2+308*a^2*b*d^3*x+8
8*a*b^2*c*d^2*x-12*b^3*c^2*d*x+77*a^3*d^3+77*a^2*b*c*d^2-33*a*b^2*c^2*d+7*b
^3*c^3)/(b*x+a)^(11/4)/(d*x+c)^(1/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d
^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{15}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(5/4)), x)
```

Fricas [B] time = 7.96956, size = 934, normalized size = 6.87

$$-\frac{4\left(128\,b^3d^3x^3+7\,b^3c^{\cdots}\right.}{77\left(a^3b^4c^5-4\,a^4b^3c^4d+6\,a^5b^2c^3d^2-4\,a^6bc^2d^3+a^7cd^4+\left(b^7c^4d-4\,ab^6c^3d^2+6\,a^2b^5c^2d^3-4\,a^3b^4cd^4+a^4b^3d^5\right)x^4+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -4/77*(128*b^3*d^3*x^3 + 7*b^3*c^3 - 33*a*b^2*c^2*d + 77*a^2*b*c*d^2 + 77*a
^3*d^3 + 32*(b^3*c*d^2 + 11*a*b^2*d^3)*x^2 - 4*(3*b^3*c^2*d - 22*a*b^2*c*d^
2 - 77*a^2*b*d^3)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^3*b^4*c^5 - 4*a^4*b
^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4
*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (
b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3
*c*d^4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^
3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c
^5 - 11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^
4 + a^7*d^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(15/4)/(d*x+c)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1723 $\int \frac{(a+bx)^{11/4}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=776

$$\frac{77\sqrt[4]{b}(bc-ad)^{7/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{EllipticF}\left(2\,\mathrm{t}\right.}{20\sqrt{2}d^{15/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (-4*(a + b*x)^(11/4))/(d*(c + d*x)^(1/4)) - (77*b*(b*c - a*d)*(a + b*x)^(3/
4)*(c + d*x)^(3/4))/(15*d^3) + (22*b*(a + b*x)^(7/4)*(c + d*x)^(3/4))/(5*d^
2) + (77*Sqrt[b]*(b*c - a*d)*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*
b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(10*d^(7/2)*(a + b*x)^(1/4)*(c + d
*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
 d*x)])/(b*c - a*d))) - (77*b^(1/4)*(b*c - a*d)^(7/2)*((a + b*x)*(c + d*x))
^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)
*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 +
(2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*
ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d
]], 1/2])/(10*Sqrt[2]*d^(15/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d +
 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (77*b^(1/4)*(b*c - a*d)^(7/2)*((
a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqr
t[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/(
(b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d
))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1
/4))/Sqrt[b*c - a*d]], 1/2])/(20*Sqrt[2]*d^(15/4)*(a + b*x)^(1/4)*(c + d*x)
^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.879976, antiderivative size = 776, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {47, 50, 62, 623, 305, 220, 1196}

$$-\frac{77b(a+bx)^{3/4}(c+dx)^{3/4}(bc-ad)}{15d^3}+\frac{77\sqrt{b}(bc-ad)\sqrt{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{10d^{7/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{22b(a+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(11/4)/(c + d*x)^(5/4),x]
```


```
[Out] (-4*(a + b*x)^(11/4))/(d*(c + d*x)^(1/4)) - (77*b*(b*c - a*d)*(a + b*x)^(3/
4)*(c + d*x)^(3/4))/(15*d^3) + (22*b*(a + b*x)^(7/4)*(c + d*x)^(3/4))/(5*d^
2) + (77*Sqrt[b]*(b*c - a*d)*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*
b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(10*d^(7/2)*(a + b*x)^(1/4)*(c + d
*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
 d*x)])/(b*c - a*d))) - (77*b^(1/4)*(b*c - a*d)^(7/2)*((a + b*x)*(c + d*x))
^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)
*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 +
(2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*
ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d
]], 1/2])/(10*Sqrt[2]*d^(15/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d +
 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (77*b^(1/4)*(b*c - a*d)^(7/2)*((
a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqr
t[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/(
```

```
(b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d
))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1
/4))/Sqrt[b*c - a*d]], 1/2])/(20*Sqrt[2]*d^(15/4)*(a + b*x)^(1/4)*(c + d*x)
^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{11/4}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} + \frac{(11b)\int \frac{(a+bx)^{7/4}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} - \frac{(77b(bc-ad))\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx}{10d^2}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \frac{\left(77b(bc-ad)^2\right)\int}{20d^3}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \frac{\left(77b(bc-ad)^2\sqrt[4]{(a}\right.}{2}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \frac{\left(77b(bc-ad)^2\sqrt[4]{(a}\right.}{}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \frac{\left(77\sqrt{b}(bc-ad)^3\sqrt[4]{}\right.}{}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \frac{77\sqrt{b}(bc-ad)\sqrt{(a}}{10d^{7/2}\sqrt[4]{a+b}}$$

Mathematica [C] time = 0.0606517, size = 73, normalized size = 0.09

$$\frac{4(a+bx)^{15/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{5}{4},\frac{15}{4};\frac{19}{4};\frac{d(a+bx)}{ad-bc}\right)}{15b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(11/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(15/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4
, 15/4, 19/4, (d*(a + b*x))/(-(b*c) + a*d)])/(15*b*(c + d*x)^(5/4))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{11}{4}}(dx+c)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(11/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(11/4)/(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{11}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(11/4)/(d*x + c)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2 + 2\,abx + a^2\right)(bx + a)^{\frac{3}{4}}(dx + c)^{\frac{3}{4}}}{d^2x^2 + 2\,cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(3/4)*(d*x + c)^(3/4)/(d^2*x^2
 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(11/4)/(d*x+c)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^{\frac{11}{4}}}{(dx + c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(11/4)/(d*x + c)^(5/4), x)
```

3.1724 $\int \frac{(a+bx)^{7/4}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=730

$$-\frac{7\sqrt[4]{b}(bc-ad)^{5/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{EllipticF}\left(2\right.}{2\sqrt{2}d^{11/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (-4*(a + b*x)^(7/4))/(d*(c + d*x)^(1/4)) + (14*b*(a + b*x)^(3/4)*(c + d*x)^
(3/4))/(3*d^2) - (7*Sqrt[b]*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b
*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(d^(5/2)*(a + b*x)^(1/4)*(c + d*x)^
(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x
)])/(b*c - a*d))) + (7*b^(1/4)*(b*c - a*d)^(5/2)*((a + b*x)*(c + d*x))^(1/4
)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
 d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sq
rt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTa
n[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1
/2])/(Sqrt[2]*d^(11/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x
)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (7*b^(1/4)*(b*c - a*d)^(5/2)*((a + b*x)*
(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt
[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*
d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*El
lipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt
[b*c - a*d]], 1/2])/(2*Sqrt[2]*d^(11/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*
c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.725212, antiderivative size = 730, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {47, 50, 62, 623, 305, 220, 1196}

$$\frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2}-\frac{7\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{d^{5/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}-\frac{7\sqrt[4]{b}(bc-ad)^{5/2}\sqrt[4]{(a+bx)(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/4)/(c + d*x)^(5/4),x]
```


```
[Out] (-4*(a + b*x)^(7/4))/(d*(c + d*x)^(1/4)) + (14*b*(a + b*x)^(3/4)*(c + d*x)^
(3/4))/(3*d^2) - (7*Sqrt[b]*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b
*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(d^(5/2)*(a + b*x)^(1/4)*(c + d*x)^
(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x
)])/(b*c - a*d))) + (7*b^(1/4)*(b*c - a*d)^(5/2)*((a + b*x)*(c + d*x))^(1/4
)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
 d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sq
rt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTa
n[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1
/2])/(Sqrt[2]*d^(11/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x
)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (7*b^(1/4)*(b*c - a*d)^(5/2)*((a + b*x)*
(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt
[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*
d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*El
lipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt
```

```
[b*c - a*d]], 1/2])/(2*Sqrt[2]*d^(11/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*
c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{7/4}}{(c+dx)^{5/4}}dx = -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{(7b)\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}dx}{d}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{(7b(bc-ad))\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}dx}{2d^2}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{\left(7b(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}dx}{2d^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{\left(14b(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Sub}}{d^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}(b}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{\left(7\sqrt{b}(bc-ad)^2\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Su}}{d^{5/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{7\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2}}{d^{5/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c}}{bc-ad}\right.}$$

Mathematica [C] time = 0.0498698, size = 73, normalized size = 0.1

$$\frac{4(a+bx)^{11/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{15}{4};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(11/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4
, 11/4, 15/4, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(5/4))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{4}}(dx+c)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(7/4)/(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{5}{4}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{3}{4}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(7/4)*(d*x + c)^(3/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{7}{4}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(7/4)/(c + d*x)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(5/4), x)
```

3.1725 $\int \frac{(a+bx)^{3/4}}{(c+dx)^{5/4}} dx$

Optimal. Leaf size=712

$$\frac{3\sqrt[4]{b}(bc-ad)^{3/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{EllipticF}\left(2\,\text{ta}\right.}{\sqrt{2}d^{7/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (-4*(a + b*x)^(3/4))/(d*(c + d*x)^(1/4)) + (6*Sqrt[b]*Sqrt[(a + b*x)*(c + d
*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(d^(3/2)*
(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*S
qrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (3*Sqrt[2]*b^(1/4
)*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^
2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*
d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*
(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((
a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(d^(7/4)*(a + b*x)^(1/4)
*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (3*
b^(1/4)*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b
*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sq
rt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a +
 b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1
/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(Sqrt[2]*d^(7/4)*(
a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d
*x))^2])
```

Rubi [A] time = 0.608653, antiderivative size = 712, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {47, 62, 623, 305, 220, 1196}

$$\frac{6\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{d^{3/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{3\sqrt[4]{b}(bc-ad)^{3/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2}}{\sqrt{2}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)/(c + d*x)^(5/4),x]
```


```
[Out] (-4*(a + b*x)^(3/4))/(d*(c + d*x)^(1/4)) + (6*Sqrt[b]*Sqrt[(a + b*x)*(c + d
*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(d^(3/2)*
(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*S
qrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (3*Sqrt[2]*b^(1/4
)*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^
2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*
d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*
(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((
a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(d^(7/4)*(a + b*x)^(1/4)
*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (3*
b^(1/4)*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b
*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sq
rt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a +
 b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1
/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(Sqrt[2]*d^(7/4)*(
```

```
a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d
*x))^2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{3/4}}{(c+dx)^{5/4}}dx = -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{(3b)\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}dx}{d}$$
$$= -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{\left(3b\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}dx}{d\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$
$$= -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{\left(12b\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-4abcd+(bc+ad)^2+4bdx^4}}dx, x, \sqrt[4]{(a\right.}{d\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)}$$
$$= -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{\left(6\sqrt{b}(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+ad)^2+4bdx^4}}\right.}{d^{3/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)}$$
$$= -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{6\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{d^{3/2}(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)} - \frac{3\sqrt{2}\sqrt[4]{b}(bc}{}$$

Mathematica [C] time = 0.0421649, size = 73, normalized size = 0.1

$$\frac{4(a+bx)^{7/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(7/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 7/4, 11/4, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/4))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{4}}(dx+c)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(3/4)/(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{5}{4}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{3}{4}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{4}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(3/4)/(c + d*x)**(5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(5/4), x)
```

3.1726 $\int \frac{1}{\sqrt[4]{a+bx}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=719

$$-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{bc-ad}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,\text{EllipticF}\left(2\,\mathrm{t}\right.}{d^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] (4*(a + b*x)^(3/4))/((b*c - a*d)*(c + d*x)^(1/4)) - (4*Sqrt[b]*Sqrt[(a + b*
x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/
(Sqrt[d]*(b*c - a*d)^2*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x
)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + (2*Sqr
t[2]*b^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d +
2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))
*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(
a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d
^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(d^(3/4)*(a + b
*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^
2]) - (Sqrt[2]*b^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*
c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b
*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt
[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]
*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(d^(3
/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c
+ 2*d*x))^2])
```

Rubi [A] time = 0.611607, antiderivative size = 719, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {51, 62, 623, 305, 220, 1196}

$$-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{bc-ad}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{d^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(3/4))/((b*c - a*d)*(c + d*x)^(1/4)) - (4*Sqrt[b]*Sqrt[(a + b*
x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/
(Sqrt[d]*(b*c - a*d)^2*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x
)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + (2*Sqr
t[2]*b^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d +
2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))
*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(
a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d
^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(d^(3/4)*(a + b
*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^
2]) - (Sqrt[2]*b^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*
c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b
*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt
[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]
*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(d^(3
```

```
/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c
+ 2*d*x))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx}(c+dx)^{5/4}}\,dx = \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{(2b)\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx}{bc-ad}$$

$$= \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{\left(2b\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}\,dx}{(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

$$= \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{\left(8b\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-4abcd+(bc+ad)^2+4bdx}}\right.}{(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)}$$

$$= \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{\left(4\sqrt{b}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+ad)^2+4b}}\right.}{\sqrt{d}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)}$$

$$= \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{4\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt{d}(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)} +$$

Mathematica [C] time = 0.0345654, size = 73, normalized size = 0.1

$$\frac{4(a+bx)^{3/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};\frac{d(a+bx)}{ad-bc}\right)}{3b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(3/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[3/4,
 5/4, 7/4, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(5/4))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{bx+a}}\,(dx+c)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(5/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{3}{4}}}{bd^2x^3+ac^2+\left(2\,bcd+ad^2\right)x^2+\left(bc^2+2\,acd\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*
d^2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(1/4)*(c + d*x)**(5/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(5/4)), x)
```

3.1727 $\int \frac{1}{(a+bx)^{5/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=750

$$\frac{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\right.}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}\sqrt{bc-ad}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

```
[Out] -4/((b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4)) - (8*d*(a + b*x)^(3/4))/((
b*c - a*d)^2*(c + d*x)^(1/4)) + (8*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)
]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/((b*c - a*d)
^3*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sq
rt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (4*Sqrt[2]*b^(1/4)*d^(1/4)
*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*
Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^
2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c -
a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))
^(1/4))/Sqrt[b*c - a*d]], 1/2])/(Sqrt[b*c - a*d]*(a + b*x)^(1/4)*(c + d*x)^
(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2*Sqrt[2]*b^(
1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 +
 (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(
c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*
x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x
)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(Sqrt[b*c - a*d]*(a + b*x)^(1/4
)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.739789, antiderivative size = 750, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {51, 62, 623, 305, 220, 1196}

$$-\frac{8d(a+bx)^{3/4}}{\sqrt[4]{c+dx}(bc-ad)^2}+\frac{8\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)^3(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}-\frac{4}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/((b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4)) - (8*d*(a + b*x)^(3/4))/((
b*c - a*d)^2*(c + d*x)^(1/4)) + (8*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)
]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/((b*c - a*d)
^3*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sq
rt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (4*Sqrt[2]*b^(1/4)*d^(1/4)
*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*
Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^
2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c -
a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))
^(1/4))/Sqrt[b*c - a*d]], 1/2])/(Sqrt[b*c - a*d]*(a + b*x)^(1/4)*(c + d*x)^
(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2*Sqrt[2]*b^(
1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 +
 (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(
c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*
x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x
```

```
)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(Sqrt[b*c - a*d]*(a + b*x)^(1/4
)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/4}(c+dx)^{5/4}}\,dx = -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}} - \frac{(2d)\int \frac{1}{\sqrt[4]{a+bx}(c+dx)^{5/4}}\,dx}{bc-ad}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{(4bd)\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx}{(bc-ad)^2}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{\left(4bd\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+}}}{(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{\left(16bd\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2b}\right.}{(bc-ad)^2}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{\left(8\sqrt{b}\sqrt{d}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+}\right.}{(bc-ad}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{8\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+}}{(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2}$$

Mathematica [C] time = 0.0410293, size = 71, normalized size = 0.09

$$-\frac{4\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[4]{a+bx}(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/4)*(c + d*x)^(5/4))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{4}}(dx+c)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(5/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{3}{4}}}{b^2d^2x^4+a^2c^2+2\left(b^2cd+abd^2\right)x^3+\left(b^2c^2+4\,abcd+a^2d^2\right)x^2+2\left(abc^2+a^2cd\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*
d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c
*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{4}}(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/4)*(c + d*x)**(5/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(5/4)), x)
```

3.1728 $\int \frac{1}{(a+bx)^{9/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=795

$$\frac{48(a+bx)^{3/4}d^2}{5(bc-ad)^3\sqrt[4]{c+dx}}-\frac{48\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}d^{3/2}}{5(bc-ad)^4\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{24\sqrt{2}\sqrt[4]{b}\sqrt[4]{(a+bx)(c+dx)}}{}$$

```
[Out] -4/(5*(b*c - a*d)*(a + b*x)^(5/4)*(c + d*x)^(1/4)) + (24*d)/(5*(b*c - a*d)^
2*(a + b*x)^(1/4)*(c + d*x)^(1/4)) + (48*d^2*(a + b*x)^(3/4))/(5*(b*c - a*d
)^3*(c + d*x)^(1/4)) - (48*Sqrt[b]*d^(3/2)*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(
b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(5*(b*c - a*d)^4*(a
+ b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*
Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + (24*Sqrt[2]*b^(1/4)*d^(5/4)*((a
+ b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[
d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b
*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))
^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4
))/Sqrt[b*c - a*d]], 1/2])/(5*(b*c - a*d)^(3/2)*(a + b*x)^(1/4)*(c + d*x)^(
1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (12*Sqrt[2]*b^(
1/4)*d^(5/4)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 +
 (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(
c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*
x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x
)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(5*(b*c - a*d)^(3/2)*(a + b*x)^
(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rubi [A] time = 0.864912, antiderivative size = 795, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {51, 62, 623, 305, 220, 1196}

$$\frac{48(a+bx)^{3/4}d^2}{5(bc-ad)^3\sqrt[4]{c+dx}}-\frac{48\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}d^{3/2}}{5(bc-ad)^4\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{24\sqrt{2}\sqrt[4]{b}\sqrt[4]{(a+bx)(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/4)*(c + d*x)^(5/4)),x]
```


```
[Out] -4/(5*(b*c - a*d)*(a + b*x)^(5/4)*(c + d*x)^(1/4)) + (24*d)/(5*(b*c - a*d)^
2*(a + b*x)^(1/4)*(c + d*x)^(1/4)) + (48*d^2*(a + b*x)^(3/4))/(5*(b*c - a*d
)^3*(c + d*x)^(1/4)) - (48*Sqrt[b]*d^(3/2)*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(
b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(5*(b*c - a*d)^4*(a
+ b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*
Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + (24*Sqrt[2]*b^(1/4)*d^(5/4)*((a
+ b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[
d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b
*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))
^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4
))/Sqrt[b*c - a*d]], 1/2])/(5*(b*c - a*d)^(3/2)*(a + b*x)^(1/4)*(c + d*x)^(
1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (12*Sqrt[2]*b^(
1/4)*d^(5/4)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 +
 (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(
```

```
c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*
x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x
)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(5*(b*c - a*d)^(3/2)*(a + b*x)^
(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/((a + b*x)*(c + d*x))^m, Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 623

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[(d*Sqrt[(b + 2*c*x)^2])/(b + 2*c*x), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/4}(c+dx)^{5/4}}\,dx = -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} - \frac{(6d)\int \frac{1}{(a+bx)^{5/4}(c+dx)^{5/4}}\,dx}{5(bc-ad)}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}} + \frac{\left(12d^2\right)\int \frac{1}{\sqrt[4]{a+bx}(c+dx)^{5/4}}\,dx}{5(bc-ad)^2}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}} + \frac{48d^2(a+bx)^{3/4}}{5(bc-ad)^3\sqrt[4]{c+dx}} - \frac{(24}{}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}} + \frac{48d^2(a+bx)^{3/4}}{5(bc-ad)^3\sqrt[4]{c+dx}} - \frac{(24}{}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}} + \frac{48d^2(a+bx)^{3/4}}{5(bc-ad)^3\sqrt[4]{c+dx}} - \frac{(96}{}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}} + \frac{48d^2(a+bx)^{3/4}}{5(bc-ad)^3\sqrt[4]{c+dx}} - \frac{(48}{}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}} + \frac{48d^2(a+bx)^{3/4}}{5(bc-ad)^3\sqrt[4]{c+dx}} - \frac{48}{5(b}$$

Mathematica [C] time = 0.0401408, size = 73, normalized size = 0.09

$$-\frac{4\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{4};-\frac{1}{4};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/4}(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, (d
*(a + b*x))/(-(b*c) + a*d)])/(5*b*(a + b*x)^(5/4)*(c + d*x)^(5/4))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{9}{4}}(dx+c)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{9}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(5/4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{3}{4}}}{b^3d^2x^5+a^3c^2+\left(2\,b^3cd+3\,ab^2d^2\right)x^4+\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^3+\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^2+\left(3\,a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*
d + 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2
*c^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/4)/(d*x+c)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{9}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(5/4)), x)
```

3.1729 $\int \frac{1}{\sqrt[4]{1-ax}(1+bx)^{3/4}}\,dx$

Optimal. Leaf size=279

$$-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}+\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{bx+1}}\right)}{\sqrt[4]{a}b^{3/4}}-\sqrt{2}\tan^{-1}$$

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*(1 - a*x)^(1/4))/(a^(1/4)*(1 + b*x)^(1
/4))])/(a^(1/4)*b^(3/4)) - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*(1 - a*x)^(
1/4))/(a^(1/4)*(1 + b*x)^(1/4))])/(a^(1/4)*b^(3/4)) - Log[Sqrt[a] + (Sqrt[b
]*Sqrt[1 - a*x])/Sqrt[1 + b*x] - (Sqrt[2]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4))/
(1 + b*x)^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)) + Log[Sqrt[a] + (Sqrt[b]*Sqrt[1
- a*x])/Sqrt[1 + b*x] + (Sqrt[2]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4))/(1 + b*x)
^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4))
```

Rubi [A] time = 0.303988, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}+\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{bx+1}}\right)}{\sqrt[4]{a}b^{3/4}}-\sqrt{2}\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a*x)^(1/4)*(1 + b*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*(1 - a*x)^(1/4))/(a^(1/4)*(1 + b*x)^(1
/4))])/(a^(1/4)*b^(3/4)) - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*(1 - a*x)^(
1/4))/(a^(1/4)*(1 + b*x)^(1/4))])/(a^(1/4)*b^(3/4)) - Log[Sqrt[a] + (Sqrt[b
]*Sqrt[1 - a*x])/Sqrt[1 + b*x] - (Sqrt[2]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4))/
(1 + b*x)^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)) + Log[Sqrt[a] + (Sqrt[b]*Sqrt[1
- a*x])/Sqrt[1 + b*x] + (Sqrt[2]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4))/(1 + b*x)
^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1-ax}(1+bx)^{3/4}}\,dx = -\frac{4\,\text{Subst}\left(\int \frac{x^2}{\left(1+\frac{b}{a}-\frac{bx^4}{a}\right)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{a}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{a\sqrt{b}} - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{a\sqrt{b}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{b} - \frac{\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{1+bx}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{1+bx}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\sqrt{2}\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}b^{3/4}} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}b^{3/4}} - \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{1+bx}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-a\ldots}}{\sqrt[4]{1+bx}}\right.}{\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

Mathematica [C] time = 0.0353814, size = 65, normalized size = 0.23

$$-\frac{4(1-ax)^{3/4}\left(\frac{abx+a}{a+b}\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{b-abx}{a+b}\right)}{3a(bx+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a*x)^(1/4)*(1 + b*x)^(3/4)),x]
```

```
[Out] (-4*(1 - a*x)^(3/4)*((a + a*b*x)/(a + b))^(3/4)*Hypergeometric2F1[3/4, 3/4,
 7/4, (b - a*b*x)/(a + b)])/(3*a*(1 + b*x)^(3/4))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{-ax+1}}(bx+1)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x)
```

```
[Out] int(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-ax+1)^{\frac{1}{4}}(bx+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*x + 1)^(1/4)*(b*x + 1)^(3/4)), x)
```

Fricas [A] time = 2.56453, size = 586, normalized size = 2.1

$$-4\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{(-ax+1)^{\frac{3}{4}}(bx+1)^{\frac{1}{4}}ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}-\left(a^2b^2x-ab^2\right)\sqrt{\frac{\left(ab^2x-b^2\right)\sqrt{-\frac{1}{ab^3}}-\sqrt{-ax+1}\sqrt{bx+1}}{ax-1}}\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}}{ax-1}\right)-\left(-\frac{1}{a\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="fricas")
```

```
[Out] -4*(-1/(a*b^3))^(1/4)*arctan(-((-a*x + 1)^(3/4)*(b*x + 1)^(1/4)*a*b^2*(-1/(
a*b^3))^(3/4) - (a^2*b^2*x - a*b^2)*sqrt(((a*b^2*x - b^2)*sqrt(-1/(a*b^3))
- sqrt(-a*x + 1)*sqrt(b*x + 1))/(a*x - 1))*(-1/(a*b^3))^(3/4))/(a*x - 1)) -
 (-1/(a*b^3))^(1/4)*log(((a*b*x - b)*(-1/(a*b^3))^(1/4) + (-a*x + 1)^(3/4)*
(b*x + 1)^(1/4))/(a*x - 1)) + (-1/(a*b^3))^(1/4)*log(-((a*b*x - b)*(-1/(a*b
^3))^(1/4) - (-a*x + 1)^(3/4)*(b*x + 1)^(1/4))/(a*x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt[4]{-ax+1}\,(bx+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/4)/(b*x+1)**(3/4),x)
```

```
[Out] Integral(1/((-a*x + 1)**(1/4)*(b*x + 1)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(-ax+1)^{\frac{1}{4}}(bx+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-a*x + 1)^(1/4)*(b*x + 1)^(3/4)), x)
```

3.1730 $\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx$

Optimal. Leaf size=193

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{a}$$

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - (Sqrt[2
]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - Log[1 + Sqrt[1
 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(Sqrt[2]
*a) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
a*x)^(1/4)]/(Sqrt[2]*a)
```

Rubi [A] time = 0.138051, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a*x)^(1/4)*(1 + a*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - (Sqrt[2
]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - Log[1 + Sqrt[1
 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(Sqrt[2]
*a) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
a*x)^(1/4)]/(Sqrt[2]*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx = -\frac{4\,\mathrm{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$

$$= -\frac{4\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{2\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-}\right.}{\sqrt{2}}$$

$$= -\frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{\sqrt{2}\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\right.}{a}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} + \frac{\log\left(\right.}{}$$

Mathematica [C] time = 0.0077021, size = 42, normalized size = 0.22

$$-\frac{2\sqrt[4]{2}(1-ax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-ax)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a*x)^(1/4)*(1 + a*x)^(3/4)),x]
```

```
[Out] (-2*2^(1/4)*(1 - a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (1 - a*x)/2])/
(3*a)
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{-ax+1}}\,(ax+1)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x)
```

```
[Out] int(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax+1)^{\frac{3}{4}}(-ax+1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + 1)^(3/4)*(-a*x + 1)^(1/4)), x)
```

Fricas [B] time = 2.5946, size = 1235, normalized size = 6.4

$$2\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a^3\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-\sqrt{2}\left(a^4x-a^3\right)\sqrt{\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+a}{ax-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*(a^(-4))^(1/4)*arctan(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*
a^3*(a^(-4))^(3/4) - sqrt(2)*(a^4*x - a^3)*sqrt((sqrt(2)*(a*x + 1)^(1/4)*(-
a*x + 1)^(3/4)*a*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt(a^(-4)) - sqrt(a*x + 1
)*sqrt(-a*x + 1))/(a*x - 1))*(a^(-4))^(3/4) + a*x - 1)/(a*x - 1)) + 2*sqrt(
2)*(a^(-4))^(1/4)*arctan(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*a^3*(a^
```

```
(-4))^(3/4) - sqrt(2)*(a^4*x - a^3)*sqrt(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x +
1)^(3/4)*a*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(a*x + 1)*sqrt
(-a*x + 1))/(a*x - 1))*(a^(-4))^(3/4) - a*x + 1)/(a*x - 1)) - 1/2*sqrt(2)*(
a^(-4))^(1/4)*log((sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*a*(a^(-4))^(1/4
) + (a^3*x - a^2)*sqrt(a^(-4)) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*x - 1)) +
 1/2*sqrt(2)*(a^(-4))^(1/4)*log(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*
a*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(a*x + 1)*sqrt(-a*x + 1
))/(a*x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{-ax+1}\,(ax+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/4)/(a*x+1)**(3/4),x)
```

```
[Out] Integral(1/((-a*x + 1)**(1/4)*(a*x + 1)**(3/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax+1)^{\frac{3}{4}}(-ax+1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + 1)^(3/4)*(-a*x + 1)^(1/4)), x)
```

3.1731 $\int \frac{(a+bx)^{3/2}}{\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=74

$$\frac{2(a+bx)^{5/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{5}{2};\frac{7}{2};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[5]{c+dx}}$$

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5,
 5/2, 7/2, -((d*(a + b*x))/(b*c - a*d))])/(5*b*(c + d*x)^(1/5))
```

Rubi [A] time = 0.0290308, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{2(a+bx)^{5/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{5}{2};\frac{7}{2};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(1/5),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5,
 5/2, 7/2, -((d*(a + b*x))/(b*c - a*d))])/(5*b*(c + d*x)^(1/5))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{\sqrt[5]{c+dx}}\,dx &= \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{(a+bx)^{3/2}}{\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}\\ &= \frac{2(a+bx)^{5/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{5}{2};\frac{7}{2};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[5]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0320213, size = 73, normalized size = 0.99

$$\frac{2(a+bx)^{5/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/5),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/5))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}\frac{1}{\sqrt[5]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/5),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(1/5),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{5}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(1/5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt[5]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/5),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(1/5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/5), x)
```

3.1732 $\int \frac{\sqrt{a+bx}}{\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=74

$$\frac{2(a+bx)^{3/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{3}{2};\frac{5}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b\sqrt[5]{c+dx}}$$

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5,
 3/2, 5/2, -((d*(a + b*x))/(b*c - a*d))])/(3*b*(c + d*x)^(1/5))
```

Rubi [A] time = 0.019703, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{2(a+bx)^{3/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{3}{2};\frac{5}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(1/5),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5,
 3/2, 5/2, -((d*(a + b*x))/(b*c - a*d))])/(3*b*(c + d*x)^(1/5))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{\sqrt[5]{c+dx}}\,dx &= \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{\sqrt{a+bx}}{\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}\\
&= \frac{2(a+bx)^{3/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{3}{2};\frac{5}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b\sqrt[5]{c+dx}}
\end{aligned}$$

Mathematica [A] time = 0.0264569, size = 73, normalized size = 0.99

$$\frac{2(a+bx)^{3/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(1/5),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/5))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\frac{1}{\sqrt[5]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/5),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(1/5),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{5}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(1/5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}}{\sqrt[5]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/5),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(1/5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/5), x)
```

3.1733 $\int \frac{1}{\sqrt{a+bx}\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{a+bx}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{3}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[5]{c+dx}}$$

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5, 1
/2, 3/2, -((d*(a + b*x))/(b*c - a*d))])/(b*(c + d*x)^(1/5))
```

Rubi [A] time = 0.0196033, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{2\sqrt{a+bx}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{3}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(1/5)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5, 1
/2, 3/2, -((d*(a + b*x))/(b*c - a*d))])/(b*(c + d*x)^(1/5))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx}\sqrt[5]{c+dx}}\,dx &= \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{\sqrt{a+bx}\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}\\ &= \frac{2\sqrt{a+bx}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{3}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[5]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0205918, size = 71, normalized size = 0.99

$$\frac{2\sqrt{a+bx}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(1/5)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5, 1
/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/5))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt[5]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{4}{5}}}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(4/5)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx}\sqrt[5]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/5),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(1/5)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1734 $\int \frac{1}{(a+bx)^{3/2}\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=72

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{5};\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt{a+bx}\sqrt[5]{c+dx}}$$

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-1/2, 1/5, 1/2, -((
d*(a + b*x))/(b*c - a*d))])/(b*Sqrt[a + b*x]*(c + d*x)^(1/5))
```

Rubi [A] time = 0.0198808, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{5};\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt{a+bx}\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(1/5)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-1/2, 1/5, 1/2, -((
d*(a + b*x))/(b*c - a*d))])/(b*Sqrt[a + b*x]*(c + d*x)^(1/5))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{3/2}\sqrt[5]{c+dx}}\,dx &= \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{(a+bx)^{3/2}\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}\\
&= -\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{5};\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt{a+bx}\sqrt[5]{c+dx}}
\end{aligned}$$

Mathematica [A] time = 0.02144, size = 71, normalized size = 0.99

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{5};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/5)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-1/2, 1/5, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(1/5))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{3}{2}}\frac{1}{\sqrt[5]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{4}{5}}}{b^2dx^3+a^2c+\left(b^2c+2\,abd\right)x^2+\left(2\,abc+a^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(4/5)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt[5]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/5),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(1/5)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/5)), x)
```

3.1735 $\int \frac{1}{(a+bx)^{5/2}\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{5};-\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b(a+bx)^{3/2}\sqrt[5]{c+dx}}$$

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-3/2, 1/5, -1/2, -(
(d*(a + b*x))/(b*c - a*d))])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/5))
```

Rubi [A] time = 0.0205612, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{5};-\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b(a+bx)^{3/2}\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(1/5)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-3/2, 1/5, -1/2, -(
(d*(a + b*x))/(b*c - a*d))])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/5))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{5/2}\sqrt[5]{c+dx}}\,dx &= \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{(a+bx)^{5/2}\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}\\
&= -\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{5};-\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b(a+bx)^{3/2}\sqrt[5]{c+dx}}
\end{aligned}$$

Mathematica [A] time = 0.0246489, size = 73, normalized size = 0.99

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{5};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/5)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-3/2, 1/5, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/5))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{2}}\frac{1}{\sqrt[5]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/5)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{4}{5}}}{b^3dx^4+a^3c+\left(b^3c+3\,ab^2d\right)x^3+3\left(ab^2c+a^2bd\right)x^2+\left(3\,a^2bc+a^3d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(4/5)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\sqrt[5]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/5),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(1/5)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/5)), x)
```

3.1736 $\int (a+bx)^{5/2}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=487

$$-\frac{81\,3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{11/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad-}}{\sqrt[3]{bc-ad-}}\right.\right.}{2816bd^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (81*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(1408*b*d^3) - (9*(b*c - a
*d)^2*(a + b*x)^(3/2)*(c + d*x)^(1/6))/(352*b*d^2) + (3*(b*c - a*d)*(a + b*
x)^(5/2)*(c + d*x)^(1/6))/(176*b*d) + (3*(a + b*x)^(7/2)*(c + d*x)^(1/6))/(
11*b) - (81*3^(3/4)*(b*c - a*d)^(11/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) -
 b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/
3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqr
t[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1
- Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1
/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(2816*b*d^4*Sqrt[a + b*x]*Sqrt[-((
b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*
c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.522135, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {50, 63, 225}

$$-\frac{81\,3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{11/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}}\right.\right.}{2816bd^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(c + d*x)^(1/6),x]
```


```
[Out] (81*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(1408*b*d^3) - (9*(b*c - a
*d)^2*(a + b*x)^(3/2)*(c + d*x)^(1/6))/(352*b*d^2) + (3*(b*c - a*d)*(a + b*
x)^(5/2)*(c + d*x)^(1/6))/(176*b*d) + (3*(a + b*x)^(7/2)*(c + d*x)^(1/6))/(
11*b) - (81*3^(3/4)*(b*c - a*d)^(11/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) -
 b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/
3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqr
t[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1
- Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1
/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(2816*b*d^4*Sqrt[a + b*x]*Sqrt[-((
b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*
c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (a+bx)^{5/2}\sqrt[6]{c+dx}\,dx = \frac{3(a+bx)^{7/2}\sqrt[6]{c+dx}}{11b}+\frac{(bc-ad)\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/6}}\,dx}{22b}$$

$$= \frac{3(bc-ad)(a+bx)^{5/2}\sqrt[6]{c+dx}}{176bd}+\frac{3(a+bx)^{7/2}\sqrt[6]{c+dx}}{11b}-\frac{\left(15(bc-ad)^2\right)\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}}\,dx}{352bd}$$

$$= -\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-ad)(a+bx)^{5/2}\sqrt[6]{c+dx}}{176bd}+\frac{3(a+bx)^{7/2}\sqrt[6]{c+dx}}{11b}+\frac{(27}{}$$

$$= \frac{81(bc-ad)^3\sqrt{a+bx}\sqrt[6]{c+dx}}{1408bd^3}-\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-ad)(a+bx)^{5/2}\sqrt[6]{c+dx}}{176bd}$$

$$= \frac{81(bc-ad)^3\sqrt{a+bx}\sqrt[6]{c+dx}}{1408bd^3}-\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-ad)(a+bx)^{5/2}\sqrt[6]{c+dx}}{176bd}$$

$$= \frac{81(bc-ad)^3\sqrt{a+bx}\sqrt[6]{c+dx}}{1408bd^3}-\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-ad)(a+bx)^{5/2}\sqrt[6]{c+dx}}{176bd}$$

Mathematica [C] time = 0.042876, size = 73, normalized size = 0.15

$$\frac{2(a+bx)^{7/2}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 7/2, 9/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{2}} \sqrt[6]{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(5/2)*(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)*(d*x + c)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx+a}(dx+c)^{\frac{1}{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*(d*x + c)^(1/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^{\frac{5}{2}} \sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(5/2)*(c + d*x)**(1/6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/6),x, algorithm="giac")

[Out] integrate((b*x + a)^(5/2)*(d*x + c)^(1/6), x)
```

3.1737 $\int (a+bx)^{3/2}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=449

$$\frac{27\,3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{8/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\operatorname{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-}{\sqrt[3]{bc-ad}-(1+}\right.\right.}{640bd^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-27*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/6))/(320*b*d^2) + (3*(b*c - a
*d)*(a + b*x)^(3/2)*(c + d*x)^(1/6))/(80*b*d) + (3*(a + b*x)^(5/2)*(c + d*x
)^(1/6))/(8*b) + (27*3^(3/4)*(b*c - a*d)^(8/3)*(c + d*x)^(1/6)*((b*c - a*d)
^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c -
a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) -
(1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/
3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[
3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(640*b*d^3*Sqrt[a + b*x]*S
qrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.336353, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {50, 63, 225}

$$\frac{27\,3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{8/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c}}\right.\right.}{640bd^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(1/6),x]
```

```
[Out] (-27*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/6))/(320*b*d^2) + (3*(b*c - a
*d)*(a + b*x)^(3/2)*(c + d*x)^(1/6))/(80*b*d) + (3*(a + b*x)^(5/2)*(c + d*x
)^(1/6))/(8*b) + (27*3^(3/4)*(b*c - a*d)^(8/3)*(c + d*x)^(1/6)*((b*c - a*d)
^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c -
a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) -
(1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/
3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[
3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(640*b*d^3*Sqrt[a + b*x]*S
qrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt[6]{c+dx}\,dx = \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8b}+\frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}}\,dx}{16b}$$

$$= \frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8b}-\frac{\left(9(bc-ad)^2\right)\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}\,dx}{160bd}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}}{320bd^2}+\frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8b}+\frac{(27(\cdots}{\cdots}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}}{320bd^2}+\frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8b}+\frac{(81(\cdots}{\cdots}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}}{320bd^2}+\frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8b}+\frac{27\ 3\cdots}{\cdots}$$

Mathematica [C] time = 0.0300574, size = 73, normalized size = 0.16

$$\frac{2(a+bx)^{5/2}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}\sqrt[6]{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(3/2)*(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^{\frac{3}{2}}(dx + c)^{\frac{1}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx + a)^{\frac{3}{2}}(dx + c)^{\frac{1}{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(1/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + bx)^{\frac{3}{2}} \sqrt[6]{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(1/6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^{\frac{3}{2}}(dx + c)^{\frac{1}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/6), x)
```

3.1738 $\int \sqrt{a+bx}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=411

$$-\frac{3\,3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{5/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3}}{\sqrt[3]{bc-ad}-(1+\sqrt{3}}\right.\right.}{40bd^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (3*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/6))/(20*b*d) + (3*(a + b*x)^(3/2)
*(c + d*x)^(1/6))/(5*b) - (3*3^(3/4)*(b*c - a*d)^(5/3)*(c + d*x)^(1/6)*((b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)
*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^
(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c -
a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1
 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(40*b*d^2*Sqrt[a +
 b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.28383, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {50, 63, 225}

$$-\frac{3\,3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{5/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right.\right.}{40bd^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(1/6),x]
```

```
[Out] (3*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/6))/(20*b*d) + (3*(a + b*x)^(3/2)
*(c + d*x)^(1/6))/(5*b) - (3*3^(3/4)*(b*c - a*d)^(5/3)*(c + d*x)^(1/6)*((b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)
*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^
(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c -
a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1
 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(40*b*d^2*Sqrt[a +
 b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt[6]{c+dx}\,dx = \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}\,dx}{10b}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}{20bd} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b} - \frac{\left(3(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/6}}\,dx}{40bd}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}{20bd} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b} - \frac{\left(9(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x\right.}{20bd^2}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}{20bd} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b} - \frac{3\,3^{3/4}(bc-ad)^{5/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\right.}{}$$

Mathematica [C] time = 0.025405, size = 73, normalized size = 0.18

$$\frac{2(a+bx)^{3/2}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\sqrt[6]{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{bx+a}(dx+c)^{\frac{1}{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+bx}\sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/6),x)
```

```
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(1/6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/6), x)
```

3.1739 $\int \frac{\sqrt[6]{c+dx}}{\sqrt{a+bx}} dx$

Optimal. Leaf size=375

$$\frac{3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{2/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})}{\sqrt[3]{bc-ad}-(1+\sqrt{3})}\right.\right.}{4bd\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(2*b) + (3^(3/4)*(b*c - a*d)^(2/3)*(c + d
*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(
2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))
/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[A
rcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a
*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(4*b
*d*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/
3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))^2)])
```

Rubi [A] time = 0.232912, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {50, 63, 225}

$$\frac{3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{2/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\right.\right.}{4bd\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/Sqrt[a + b*x],x]
```


```
[Out] (3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(2*b) + (3^(3/4)*(b*c - a*d)^(2/3)*(c + d
*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(
2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))
/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[A
rcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a
*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(4*b
*d*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/
3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))^2)])
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt{a+bx}}\,dx = \frac{3\sqrt{a+bx}\sqrt[6]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/6}}\,dx}{4b}$$

$$= \frac{3\sqrt{a+bx}\sqrt[6]{c+dx}}{2b} + \frac{(3(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{2bd}$$

$$= \frac{3\sqrt{a+bx}\sqrt[6]{c+dx}}{2b} + \frac{3^{3/4}(bc-ad)^{2/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(c+d}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}{4bd\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c}\right.}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+}\right.}}}$$

Mathematica [C] time = 0.0214657, size = 71, normalized size = 0.19

$$\frac{2\sqrt{a+bx}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, (d*(a +
b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int \sqrt[6]{dx+c}\frac{1}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(1/6)/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{6}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/6)/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^{\frac{1}{6}}}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(1/6)/sqrt(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(1/6)/sqrt(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{6}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/6)/sqrt(b*x + a), x)
```

3.1740 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=367

$$\frac{\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right),\frac{1}{4}\left(2+\right.\right.}{\sqrt[4]{3}\,b\sqrt{a+bx}\sqrt[3]{bc-ad}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-2*(c + d*x)^(1/6))/(b*Sqrt[a + b*x]) + ((c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(3^(1/4)*b*(b*c - a*d)^(1/3)*Sq
rt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))
^2)])
```

Rubi [A] time = 0.229241, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {47, 63, 225}

$$\frac{\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,b\sqrt{a+bx}\sqrt[3]{bc-ad}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/6))/(b*Sqrt[a + b*x]) + ((c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(3^(1/4)*b*(b*c - a*d)^(1/3)*Sq
rt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))
^2)])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{3/2}}\,dx &= -\frac{2\sqrt[6]{c+dx}}{b\sqrt{a+bx}}+\frac{d\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/6}}\,dx}{3b}\\
&= -\frac{2\sqrt[6]{c+dx}}{b\sqrt{a+bx}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{b}\\
&= -\frac{2\sqrt[6]{c+dx}}{b\sqrt{a+bx}}+\frac{\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\cdots\right.\right.}{\sqrt[4]{3}b\sqrt[3]{bc-ad}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}
\end{aligned}$$

Mathematica [C] time = 0.0221659, size = 71, normalized size = 0.19

$$-\frac{2\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/6)*Hypergeometric2F1[-1/2, -1/6, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \sqrt[6]{dx+c}\,(bx+a)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(1/6)/(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{6}}}{b^2x^2+2\,abx+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(3/2), x)
```

3.1741 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{5/2}} dx$

Optimal. Leaf size=409

$$-\frac{2d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\right),}{9\sqrt[4]{3}b\sqrt{a+bx}(bc-ad)^{4/3}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-2*(c + d*x)^(1/6))/(3*b*(a + b*x)^(3/2)) - (2*d*(c + d*x)^(1/6))/(9*b*(b*
c - a*d)*Sqrt[a + b*x]) - (2*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)
*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c +
d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c +
 d*x)^(1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4)*b*(b*c - a*d)^(4/3)*Sqrt[a + b*
x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.279606, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {47, 51, 63, 225}

$$-\frac{2d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{\right.}\right.}{9\sqrt[4]{3}b\sqrt{a+bx}(bc-ad)^{4/3}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/6))/(3*b*(a + b*x)^(3/2)) - (2*d*(c + d*x)^(1/6))/(9*b*(b*
c - a*d)*Sqrt[a + b*x]) - (2*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)
*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c +
d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c +
 d*x)^(1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4)*b*(b*c - a*d)^(4/3)*Sqrt[a + b*
x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{5/2}}\,dx = -\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}} + \frac{d\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/6}}\,dx}{9b}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}} - \frac{2d\sqrt[6]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} - \frac{\left(2d^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/6}}\,dx}{27b(bc-ad)}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}} - \frac{2d\sqrt[6]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} - \frac{(4d)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{9b(bc-ad)}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}} - \frac{2d\sqrt[6]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} - \frac{2d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+\cdots}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}\cdots}{9\sqrt[4]{3}b(bc-ad)^{4/3}\sqrt{a+bx}\sqrt{\cdots}}$$

Mathematica [C] time = 0.0234319, size = 73, normalized size = 0.18

$$-\frac{2\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{6};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/6)*Hypergeometric2F1[-3/2, -1/6, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \sqrt[6]{dx+c}\,(bx+a)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(1/6)/(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{6}}}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(5/2), x)
```

3.1742 $\int (a+bx)^{3/2}(c+dx)^{5/6}\,dx$

Optimal. Leaf size=896

$$-\frac{81\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}(2+\cdots}{448b^{5/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-27*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(5/6))/(224*b*d^2) + (3*(b*c - a
*d)*(a + b*x)^(3/2)*(c + d*x)^(5/6))/(28*b*d) + (3*(a + b*x)^(5/2)*(c + d*x
)^(5/6))/(10*b) - (81*(1 + Sqrt[3])*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(
1/6))/(448*b^(5/3)*d^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)
^(1/3))) - (81*3^(1/4)*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(448*b^(5/3)*d^3*Sqrt[a + b*x]*
Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (27*
3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
 - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(
1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (
1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(896*b^(5/3)*d^3*Sqrt[a + b*x]*S
qrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 1.07839, antiderivative size = 896, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {50, 63, 308, 225, 1881}

$$-\frac{81\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}(2+\cdots}{448b^{5/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(5/6),x]
```

```
[Out] (-27*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(5/6))/(224*b*d^2) + (3*(b*c - a
*d)*(a + b*x)^(3/2)*(c + d*x)^(5/6))/(28*b*d) + (3*(a + b*x)^(5/2)*(c + d*x
)^(5/6))/(10*b) - (81*(1 + Sqrt[3])*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(
1/6))/(448*b^(5/3)*d^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)
^(1/3))) - (81*3^(1/4)*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(448*b^(5/3)*d^3*Sqrt[a + b*x]*
Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
```

```
)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (27*
3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
 - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(
1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (
1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(896*b^(5/3)*d^3*Sqrt[a + b*x]*S
qrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{5/6}\,dx = \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10b} + \frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx}{4b}$$

$$= \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10b} - \frac{\left(9(bc-ad)^2\right)\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx}{56bd}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/6}}{224bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + \frac{3(a+bx)^{5/2}(c+d}{10b}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/6}}{224bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + \frac{3(a+bx)^{5/2}(c+d}{10b}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/6}}{224bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + \frac{3(a+bx)^{5/2}(c+d}{10b}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/6}}{224bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + \frac{3(a+bx)^{5/2}(c+d}{10b}$$

Mathematica [C] time = 0.0366744, size = 73, normalized size = 0.08

$$\frac{2(a+bx)^{5/2}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(3/2)*(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(5/6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(5/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1743 $\int \sqrt{a+bx}(c+dx)^{5/6}\,dx$

Optimal. Leaf size=858

$$\frac{45\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\Big(\cos^{-1}\Big(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\Big)|\frac{1}{4}\Big(2+}{112b^{5/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (15*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(5/6))/(56*b*d) + (3*(a + b*x)^(3/2
)*(c + d*x)^(5/6))/(7*b) + (45*(1 + Sqrt[3])*(b*c - a*d)^2*Sqrt[a + b*x]*(c
 + d*x)^(1/6))/(112*b^(5/3)*d*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))) + (45*3^(1/4)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d
)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c -
 a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) -
 (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1
/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(112*b^(5/3)*d^2*Sqrt[a +
 b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
+ (15*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d)^
(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a
*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (
1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3
) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(224*b^(5/3)*d^2*Sqrt[a + b
*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^
(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.822589, antiderivative size = 858, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {50, 63, 308, 225, 1881}

$$\frac{45\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\Big(\cos^{-1}\Big(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\Big)|\frac{1}{4}\Big(2+}{112b^{5/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(5/6),x]
```

```
[Out] (15*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(5/6))/(56*b*d) + (3*(a + b*x)^(3/2
)*(c + d*x)^(5/6))/(7*b) + (45*(1 + Sqrt[3])*(b*c - a*d)^2*Sqrt[a + b*x]*(c
 + d*x)^(1/6))/(112*b^(5/3)*d*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))) + (45*3^(1/4)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d
)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c -
 a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) -
 (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1
/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(112*b^(5/3)*d^2*Sqrt[a +
 b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

```
+ (15*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d)^
(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a
*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (
1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3
) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(224*b^(5/3)*d^2*Sqrt[a + b
*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^
(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \sqrt{a+bx}(c+dx)^{5/6}\,dx = \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} + \frac{(5(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx}{14b}$$

$$= \frac{15(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56bd} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} - \frac{\left(15(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{112bd}$$

$$= \frac{15(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56bd} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} - \frac{\left(45(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+}}\right.}{56bd^2}$$

$$= \frac{15(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56bd} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} + \frac{\left(45(bc-ad)^2\right)\text{Subst}\left(\int \frac{(-1+\sqrt{3}}{}\right.}{112b}$$

$$= \frac{15(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56bd} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} + \frac{45\left(1+\sqrt{3}\right)(bc-ad)^2\sqrt{a+b}}{112b^{5/3}d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\right.}$$

Mathematica [C] time = 0.0307956, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{3/2}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\,(dx+c)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx+a}(dx+c)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/6),x, algorithm="maxima")

[Out] integrate(sqrt(b*x + a)*(d*x + c)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{bx+a}(dx+c)^{\frac{5}{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/6),x, algorithm="fricas")

[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(5/6),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/6),x, algorithm="giac")

[Out] Timed out
```

3.1744 $\int \frac{(c+dx)^{5/6}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=817

$$-\frac{15\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\right.\right.}{8b^{5/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (3*Sqrt[a + b*x]*(c + d*x)^(5/6))/(4*b) - (15*(1 + Sqrt[3])*(b*c - a*d)*Sqr
t[a + b*x]*(c + d*x)^(1/6))/(8*b^(5/3)*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))) - (15*3^(1/4)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/
3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d
)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c
- a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) -
(1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(8*b^(5/3)*d*Sqr
t[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
+ d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^
2)]) - (5*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
 - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3)
 - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^
(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(16*b^(5/3)*d*Sqrt[a +
b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)
^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.70952, antiderivative size = 817, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {50, 63, 308, 225, 1881}

$$-\frac{15\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\right.\right.}{8b^{5/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/Sqrt[a + b*x],x]
```


```
[Out] (3*Sqrt[a + b*x]*(c + d*x)^(5/6))/(4*b) - (15*(1 + Sqrt[3])*(b*c - a*d)*Sqr
t[a + b*x]*(c + d*x)^(1/6))/(8*b^(5/3)*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))) - (15*3^(1/4)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/
3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d
)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c
- a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) -
(1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(8*b^(5/3)*d*Sqr
t[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
+ d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^
2)]) - (5*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
```

```
- a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3)
 - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^
(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(16*b^(5/3)*d*Sqrt[a +
b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)
^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{\sqrt{a+bx}}\,dx = \frac{3\sqrt{a+bx}(c+dx)^{5/6}}{4b} + \frac{(5(bc-ad))\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{8b}$$

$$= \frac{3\sqrt{a+bx}(c+dx)^{5/6}}{4b} + \frac{(15(bc-ad))\,\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{4bd}$$

$$= \frac{3\sqrt{a+bx}(c+dx)^{5/6}}{4b} - \frac{(15(bc-ad))\,\mathrm{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{8b^{5/3}d} - \frac{\left(15\left(1-\sqrt{3}\right.\right.}{}$$

$$= \frac{3\sqrt{a+bx}(c+dx)^{5/6}}{4b} - \frac{15\left(1+\sqrt{3}\right)(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}{8b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)} - \frac{15\sqrt[4]{3}(bc-ad)^{4/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc\right.}{}$$

Mathematica [C] time = 0.021917, size = 71, normalized size = 0.09

$$\frac{2\sqrt{a+bx}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 1/2, 3/2, (d*(a +
b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{6}}\frac{1}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{6}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/sqrt(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^{\frac{5}{6}}}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(5/6)/sqrt(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{5}{6}}}{\sqrt{a+bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(5/6)/sqrt(a + b*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1745 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=798

$$-\frac{5\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}\,d}{b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{5\sqrt[4]{3}\sqrt[3]{bc-ad}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+\cdots}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c}\cdots\right.}}}{b^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-\cdots}\right.}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\cdots\right.\right.}}}$$

```
[Out] (-2*(c + d*x)^(5/6))/(b*Sqrt[a + b*x]) - (5*(1 + Sqrt[3])*d*Sqrt[a + b*x]*(
c + d*x)^(1/6))/(b^(5/3)*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*
x)^(1/3))) - (5*3^(1/4)*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(b^(5/3)*Sqrt[a + b*x]*Sqrt[-((
b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*
c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (5*(1 - Sqrt
[3])*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/
3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/
3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1
/3))], (2 + Sqrt[3])/4])/(2*3^(1/4)*b^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(
c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^
(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.683955, antiderivative size = 798, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 63, 308, 225, 1881}

$$-\frac{5\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}\,d}{b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{5\sqrt[4]{3}\sqrt[3]{bc-ad}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+\cdots}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c}\cdots\right.}}}{b^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-\cdots}\right.}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\cdots\right.\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/(b*Sqrt[a + b*x]) - (5*(1 + Sqrt[3])*d*Sqrt[a + b*x]*(
c + d*x)^(1/6))/(b^(5/3)*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*
x)^(1/3))) - (5*3^(1/4)*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(b^(5/3)*Sqrt[a + b*x]*Sqrt[-((
b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*
c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (5*(1 - Sqrt
[3])*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/
```

```
3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/
3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1
/3))], (2 + Sqrt[3])/4])/(2*3^(1/4)*b^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(
c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^
(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{3/2}}\,dx = -\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}} + \frac{(5d)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{3b}$$

$$= -\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}} + \frac{10\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{b}$$

$$= -\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}} - \frac{5\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{b^{5/3}} - \frac{\left(5\left(1-\sqrt{3}\right)(bc-ad)^{2/3}\right)\text{Sub}}{b}$$

$$= -\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}} - \frac{5\left(1+\sqrt{3}\right)d\sqrt{a+bx}\sqrt[6]{c+dx}}{b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)} - \frac{5\sqrt[4]{3}\sqrt[3]{bc-ad}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{}\right.}{}$$

Mathematica [C] time = 0.0227163, size = 71, normalized size = 0.09

$$-\frac{2(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{6}}(bx+a)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{6}}}{b^2x^2+2\,abx+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(3/2), x)
```

3.1746 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=854

$$-\frac{10\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}d^2}{9b^{5/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{10\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+\right.}}}}{3\,3^{3/4}b^{5/3}(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-}}$$

```
[Out] (-2*(c + d*x)^(5/6))/(3*b*(a + b*x)^(3/2)) - (10*d*(c + d*x)^(5/6))/(9*b*(b
*c - a*d)*Sqrt[a + b*x]) - (10*(1 + Sqrt[3])*d^2*Sqrt[a + b*x]*(c + d*x)^(1
/6))/(9*b^(5/3)*(b*c - a*d)*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c +
 d*x)^(1/3))) - (10*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3
) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
+ d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3
)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/
3))], (2 + Sqrt[3])/4])/(3*3^(3/4)*b^(5/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]*
Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (5*(
1 - Sqrt[3])*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
)*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(
2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c +
d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (
2 + Sqrt[3])/4])/(9*3^(1/4)*b^(5/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]*Sqrt[-(
(b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.797578, antiderivative size = 854, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {47, 51, 63, 308, 225, 1881}

$$-\frac{10\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}d^2}{9b^{5/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{10\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+\right.}}}}{3\,3^{3/4}b^{5/3}(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(5/2),x]
```


```
[Out] (-2*(c + d*x)^(5/6))/(3*b*(a + b*x)^(3/2)) - (10*d*(c + d*x)^(5/6))/(9*b*(b
*c - a*d)*Sqrt[a + b*x]) - (10*(1 + Sqrt[3])*d^2*Sqrt[a + b*x]*(c + d*x)^(1
/6))/(9*b^(5/3)*(b*c - a*d)*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c +
 d*x)^(1/3))) - (10*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3
) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
+ d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3
)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/
3))], (2 + Sqrt[3])/4])/(3*3^(3/4)*b^(5/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]*
Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (5*(
```

```
1 - Sqrt[3])*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
)*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(
2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c +
d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (
2 + Sqrt[3])/4])/(9*3^(1/4)*b^(5/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]*Sqrt[-(
(b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
```

```
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} + \frac{(5d)\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx}{9b}$$

$$= -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} - \frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}} + \frac{(10d^2)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{27b(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} - \frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}} + \frac{(20d)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{9b(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} - \frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}} - \frac{(10d)\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{9b^{5/3}(bc-ad)} - \cdots$$

$$= -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} - \frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}} - \frac{10\left(1+\sqrt{3}\right)d^2\sqrt{a+bx}\sqrt[6]{c+dx}}{9b^{5/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)} - \frac{10d\sqrt[6]{\cdots}}{\cdots}$$

Mathematica [C] time = 0.0251925, size = 73, normalized size = 0.09

$$-\frac{2(c+dx)^{5/6}\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{6};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6)*Hypergeometric2F1[-3/2, -5/6, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{6}}(bx+a)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{6}}}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(5/2), x)
```

3.1747 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=896

$$\frac{8\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}d^3}{27b^{5/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{8\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+}\right.}}}{9\,3^{3/4}b^{5/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{\cdot}}$$

```
[Out] (-2*(c + d*x)^(5/6))/(5*b*(a + b*x)^(5/2)) - (2*d*(c + d*x)^(5/6))/(9*b*(b*
c - a*d)*(a + b*x)^(3/2)) + (8*d^2*(c + d*x)^(5/6))/(27*b*(b*c - a*d)^2*Sqr
t[a + b*x]) + (8*(1 + Sqrt[3])*d^3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(27*b^(5/
3)*(b*c - a*d)^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)) + (8*d^2*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*S
qrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3
)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/
3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x
)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 +
 Sqrt[3])/4])/(9*3^(3/4)*b^(5/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^
(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c
- a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (4*(1 - Sqrt[3
])*d^2*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[(
(b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c
+ d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2
]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/
3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt
[3])/4])/(27*3^(1/4)*b^(5/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3
)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*
d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.90147, antiderivative size = 896, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {47, 51, 63, 308, 225, 1881}

$$\frac{8\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}d^3}{27b^{5/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{8\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+}\right.}}}{9\,3^{3/4}b^{5/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{\cdot}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/(5*b*(a + b*x)^(5/2)) - (2*d*(c + d*x)^(5/6))/(9*b*(b*
c - a*d)*(a + b*x)^(3/2)) + (8*d^2*(c + d*x)^(5/6))/(27*b*(b*c - a*d)^2*Sqr
t[a + b*x]) + (8*(1 + Sqrt[3])*d^3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(27*b^(5/
3)*(b*c - a*d)^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)) + (8*d^2*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*S
qrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3
)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/
3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x
)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 +
 Sqrt[3])/4])/(9*3^(3/4)*b^(5/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^
```

```
(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c
- a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (4*(1 - Sqrt[3
])*d^2*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[(
(b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c
+ d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2
]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/
3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt
[3])/4])/(27*3^(1/4)*b^(5/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3
)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*
d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
```

```
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} + \frac{d\int \frac{1}{(a+bx)^{5/2}\sqrt[6]{c+dx}}\,dx}{3b}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} - \frac{\left(4d^2\right)\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx}{27b(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} + \frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}} - \frac{\left(8d^3\right)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{81b(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} + \frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}} - \frac{\left(16d^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx\right.}{27b(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} + \frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}} + \frac{\left(8d^2\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2}}{\sqrt{a-\frac{bc}{d}+\frac{b}{}}}\right.}{27b^{5/3}(bc-}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} + \frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}} + \frac{8\left(1+\sqrt{3}\right)d^3\sqrt{a+}}{27b^{5/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\right.}$$

Mathematica [C] time = 0.0254339, size = 73, normalized size = 0.08

$$-\frac{2(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{6};-\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6)*Hypergeometric2F1[-5/2, -5/6, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{6}}\,(bx+a)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{6}}}{b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(7/2), x)
```

3.1748 $\int \frac{(a+bx)^{5/2}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=890

$$\frac{243\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\right.\right.}{448b^{2/3}d^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (81*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(5/6))/(224*d^3) - (9*(b*c - a*d)
*(a + b*x)^(3/2)*(c + d*x)^(5/6))/(28*d^2) + (3*(a + b*x)^(5/2)*(c + d*x)^(
5/6))/(10*d) + (243*(1 + Sqrt[3])*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(1/
6))/(448*b^(2/3)*d^3*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))) + (243*3^(1/4)*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
 - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(
1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (
1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(448*b^(2/3)*d^4*Sqrt[a + b*x]*S
qrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (81*3
^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1
/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1
 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(
1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(896*b^(2/3)*d^4*Sqrt[a + b*x]*Sq
rt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
)/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.952239, antiderivative size = 890, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {50, 63, 308, 225, 1881}

$$\frac{243\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\right.\right.}{448b^{2/3}d^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(1/6),x]
```

```
[Out] (81*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(5/6))/(224*d^3) - (9*(b*c - a*d)
*(a + b*x)^(3/2)*(c + d*x)^(5/6))/(28*d^2) + (3*(a + b*x)^(5/2)*(c + d*x)^(
5/6))/(10*d) + (243*(1 + Sqrt[3])*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(1/
6))/(448*b^(2/3)*d^3*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))) + (243*3^(1/4)*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
 - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(
1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (
1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(448*b^(2/3)*d^4*Sqrt[a + b*x]*S
```

```
qrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (81*3
^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1
/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1
 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(
1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(896*b^(2/3)*d^4*Sqrt[a + b*x]*Sq
rt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
)/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt[6]{c+dx}}\,dx = \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d} - \frac{(3(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx}{4d}$$

$$= -\frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d} + \frac{\left(27(bc-ad)^2\right)\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx}{56d^2}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/6}}{224d^3} - \frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d} - \frac{(81(}{}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/6}}{224d^3} - \frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d} - \frac{(243}{}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/6}}{224d^3} - \frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d} + \frac{(243}{}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}(c+dx)^{5/6}}{224d^3} - \frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d} + \frac{2}{448}$$

Mathematica [C] time = 0.0351842, size = 73, normalized size = 0.08

$$\frac{2(a+bx)^{7/2}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/6))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{2}}\frac{1}{\sqrt[6]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2 + 2\,abx + a^2\right)\sqrt{bx + a}}{(dx + c)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(d*x + c)^(1/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx)^{\frac{5}{2}}}{\sqrt[6]{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(1/6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^{\frac{5}{2}}}{(dx + c)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(1/6), x)
```

3.1749 $\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=855

$$-\frac{81\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}(2+\right.}{112b^{2/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-27*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(5/6))/(56*d^2) + (3*(a + b*x)^(3/
2)*(c + d*x)^(5/6))/(7*d) - (81*(1 + Sqrt[3])*(b*c - a*d)^2*Sqrt[a + b*x]*(
c + d*x)^(1/6))/(112*b^(2/3)*d^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)
*(c + d*x)^(1/3))) - (81*3^(1/4)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*
c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3
) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)
^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(112*b^(2/3)*d^3*Sqrt[
a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)
]) - (27*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*
d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
- a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3)
- (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(
1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqr
t[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(224*b^(2/3)*d^3*Sqrt[a
+ b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.790495, antiderivative size = 855, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {50, 63, 308, 225, 1881}

$$-\frac{81\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}(2+\right.}{112b^{2/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(1/6),x]
```


```
[Out] (-27*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(5/6))/(56*d^2) + (3*(a + b*x)^(3/
2)*(c + d*x)^(5/6))/(7*d) - (81*(1 + Sqrt[3])*(b*c - a*d)^2*Sqrt[a + b*x]*(
c + d*x)^(1/6))/(112*b^(2/3)*d^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)
*(c + d*x)^(1/3))) - (81*3^(1/4)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*
c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3
) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)
^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(112*b^(2/3)*d^3*Sqrt[
a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)
```

```
]) - (27*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*
d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
- a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3)
- (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(
1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqr
t[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(224*b^(2/3)*d^3*Sqrt[a
+ b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx = \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} - \frac{(9(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}dx}{14d}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56d^2} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} + \frac{\left(27(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}dx}{112d^2}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56d^2} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} + \frac{\left(81(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}dx\right.}{56d^3}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56d^2} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} - \frac{\left(81(bc-ad)^2\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)}{\sqrt{a-\frac{bc}{d}+}}\right.}{112b^{2/3}d^3}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56d^2} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} - \frac{81\left(1+\sqrt{3}\right)(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c-}}{112b^{2/3}d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c}\right.}$$

Mathematica [C] time = 0.0278385, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{5/2}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/6))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}\frac{1}{\sqrt[6]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(1/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(1/6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/6), x)
```

3.1750 $\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=820

$$\frac{9\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{\right.\right.}{8b^{2/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (3*Sqrt[a + b*x]*(c + d*x)^(5/6))/(4*d) + (9*(1 + Sqrt[3])*(b*c - a*d)*Sqrt
[a + b*x]*(c + d*x)^(1/6))/(8*b^(2/3)*d*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))) + (9*3^(1/4)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/
3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d
)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c
- a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) -
(1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(8*b^(2/3)*d^2*S
qrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)^2)]) + (3*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c -
 a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b
*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/
3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d
)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(16*b^(2/3)*d^2*Sqrt[
a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)
])
```

Rubi [A] time = 0.689719, antiderivative size = 820, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {50, 63, 308, 225, 1881}

$$\frac{9\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{\right.\right.}{8b^{2/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(1/6),x]
```

```
[Out] (3*Sqrt[a + b*x]*(c + d*x)^(5/6))/(4*d) + (9*(1 + Sqrt[3])*(b*c - a*d)*Sqrt
[a + b*x]*(c + d*x)^(1/6))/(8*b^(2/3)*d*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))) + (9*3^(1/4)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/
3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d
)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c
- a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) -
(1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(8*b^(2/3)*d^2*S
qrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)^2)]) + (3*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c -
```

```
 a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b
*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/
3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d
)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(16*b^(2/3)*d^2*Sqrt[
a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)
])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx = \frac{3\sqrt{a+bx}(c+dx)^{5/6}}{4d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{8d}$$

$$= \frac{3\sqrt{a+bx}(c+dx)^{5/6}}{4d} - \frac{(9(bc-ad))\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{4d^2}$$

$$= \frac{3\sqrt{a+bx}(c+dx)^{5/6}}{4d} + \frac{(9(bc-ad))\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{8b^{2/3}d^2} + \frac{\left(9\left(1-\sqrt{3}\right)(b\right.}{}$$

$$= \frac{3\sqrt{a+bx}(c+dx)^{5/6}}{4d} + \frac{9\left(1+\sqrt{3}\right)(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}{8b^{2/3}d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)} + \frac{9\sqrt[4]{3}(bc-ad)^{4/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-\right.}{}$$

Mathematica [C] time = 0.0237868, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{3/2}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/6))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\frac{1}{\sqrt[6]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(1/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/6),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(1/6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/6), x)
```

3.1751 $\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=780

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-\cdots}}{\sqrt[3]{bc-\cdots}}\right)\cdots\right)}{2b^{2/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-3*(1 + Sqrt[3])*Sqrt[a + b*x]*(c + d*x)^(1/6))/(b^(2/3)*((b*c - a*d)^(1/3
) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))) - (3*3^(1/4)*(b*c - a*d)^(1/3)*
(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c -
a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^
(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*Ellip
ticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4]
)/(b^(2/3)*d*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(
c + d*x)^(1/3))^2)]) - (3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(1/3)*(c + d*x)^(
1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3)
+ b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*
c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos
[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(
1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(2*b^(2/3
)*d*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1
/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))^2)])
```

Rubi [A] time = 0.581985, antiderivative size = 780, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {63, 308, 225, 1881}

$$-\frac{3\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}}{b^{2/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+\cdots}{\left(\sqrt[3]{bc-ad}-(1\cdots\right)}}\cdots}{2b^{2/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\cdots}{\left(\sqrt[3]{bc-ad}\cdots\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(1/6)),x]
```

```
[Out] (-3*(1 + Sqrt[3])*Sqrt[a + b*x]*(c + d*x)^(1/6))/(b^(2/3)*((b*c - a*d)^(1/3
) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))) - (3*3^(1/4)*(b*c - a*d)^(1/3)*
(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c -
a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^
(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*Ellip
ticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4]
)/(b^(2/3)*d*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(
c + d*x)^(1/3))^2)]) - (3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(1/3)*(c + d*x)^(
1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3)
+ b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*
```

```
c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos
[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(
1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(2*b^(2/3
)*d*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1
/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx = \frac{6\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d}$$

$$= -\frac{3\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{b^{2/3}d} - \frac{\left(3\left(1-\sqrt{3}\right)(bc-ad)^{2/3}\right)\text{Subst}\left(\int \cdots\right.}{b^{2/3}d}$$

$$= -\frac{3\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}}{b^{2/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)} - \frac{3\sqrt[4]{3}\sqrt[3]{bc-ad}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\cdots}{b^{2/3}\cdots}$$

Mathematica [C] time = 0.0211029, size = 71, normalized size = 0.09

$$\frac{2\sqrt{a+bx}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(1/6)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6, 1
/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/6))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt[6]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{6}}}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/6),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(1/6)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1752 $\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=813

$$-\frac{2\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}d}{b^{2/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{2\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c}\right.}}}{b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}}{\left(\sqrt[3]{\right.}}}}$$

```
[Out] (-2*(c + d*x)^(5/6))/((b*c - a*d)*Sqrt[a + b*x]) - (2*(1 + Sqrt[3])*d*Sqrt[
a + b*x]*(c + d*x)^(1/6))/(b^(2/3)*(b*c - a*d)*((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))) - (2*3^(1/4)*(c + d*x)^(1/6)*((b*c - a*d)^
(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a
*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (
1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3
) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(b^(2/3)*(b*c - a*d)^(2/3)*
Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*
(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3
))^2)]) - ((1 - Sqrt[3])*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(
1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*
(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(
1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))], (2 + Sqrt[3])/4])/(3^(1/4)*b^(2/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]
*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/
3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.676843, antiderivative size = 813, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {51, 63, 308, 225, 1881}

$$-\frac{2\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}d}{b^{2/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{2\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c}\right.}}}{b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}}{\left(\sqrt[3]{\right.}}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(1/6)),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/((b*c - a*d)*Sqrt[a + b*x]) - (2*(1 + Sqrt[3])*d*Sqrt[
a + b*x]*(c + d*x)^(1/6))/(b^(2/3)*(b*c - a*d)*((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))) - (2*3^(1/4)*(c + d*x)^(1/6)*((b*c - a*d)^
(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a
*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (
1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3
) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(b^(2/3)*(b*c - a*d)^(2/3)*
Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*
(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3
))^2)]) - ((1 - Sqrt[3])*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(
```

```
1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*
(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(
1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))], (2 + Sqrt[3])/4])/(3^(1/4)*b^(2/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]
*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/
3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx = -\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}} + \frac{(2d)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{3(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}} + \frac{4\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}} - \frac{2\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{b^{2/3}(bc-ad)} - \frac{\left(2\left(1-\sqrt{3}\right)\right)\text{Su}}{}$$

$$= -\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}} - \frac{2\left(1+\sqrt{3}\right)d\sqrt{a+bx}\sqrt[6]{c+dx}}{b^{2/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)} - \frac{2\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{b}\right.}{}$$

Mathematica [C] time = 0.0227394, size = 71, normalized size = 0.09

$$-\frac{2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/2, 1/6, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(1/6))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{3}{2}}\frac{1}{\sqrt[6]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{6}}}{b^2dx^3+a^2c+\left(b^2c+2\,abd\right)x^2+\left(2\,abc+a^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/6),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(1/6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/6)), x)
```

3.1753 $\int \frac{1}{(a+bx)^{5/2}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=858

$$\frac{8\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}d^2}{9b^{2/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{8\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/\cdots}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+d\cdots}\right.}}}{3\ 3^{3/4}b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\cdots}}$$

```
[Out] (-2*(c + d*x)^(5/6))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (8*d*(c + d*x)^(5/6)
)/(9*(b*c - a*d)^2*Sqrt[a + b*x]) + (8*(1 + Sqrt[3])*d^2*Sqrt[a + b*x]*(c +
 d*x)^(1/6))/(9*b^(2/3)*(b*c - a*d)^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))) + (8*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)
*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c +
d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c +
 d*x)^(1/3))], (2 + Sqrt[3])/4])/(3*3^(3/4)*b^(2/3)*(b*c - a*d)^(5/3)*Sqrt[
a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)
]) + (4*(1 - Sqrt[3])*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1
/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(
c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1
/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4)*b^(2/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x
]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.752202, antiderivative size = 858, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {51, 63, 308, 225, 1881}

$$\frac{8\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}d^2}{9b^{2/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{8\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/\cdots}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+d\cdots}\right.}}}{3\ 3^{3/4}b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(1/6)),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (8*d*(c + d*x)^(5/6)
)/(9*(b*c - a*d)^2*Sqrt[a + b*x]) + (8*(1 + Sqrt[3])*d^2*Sqrt[a + b*x]*(c +
 d*x)^(1/6))/(9*b^(2/3)*(b*c - a*d)^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))) + (8*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)
*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c +
d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c +
 d*x)^(1/3))], (2 + Sqrt[3])/4])/(3*3^(3/4)*b^(2/3)*(b*c - a*d)^(5/3)*Sqrt[
a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)
```

```
]) + (4*(1 - Sqrt[3])*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1
/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(
c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1
/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4)*b^(2/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x
]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt[6]{c+dx}}\,dx = -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(4d)\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx}{9(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} + \frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{(8d^2)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{27(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} + \frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{(16d)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{9(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} + \frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(8d)\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x\right.}{9b^{2/3}(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} + \frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{8\left(1+\sqrt{3}\right)d^2\sqrt{a+bx}\sqrt[6]{c+dx}}{9b^{2/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\right.}$$

Mathematica [C] time = 0.0214173, size = 73, normalized size = 0.09

$$-\frac{2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{6};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-3/2, 1/6, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/6))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{2}}\frac{1}{\sqrt[6]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{6}}}{b^3dx^4+a^3c+\left(b^3c+3\,ab^2d\right)x^3+3\left(ab^2c+a^2bd\right)x^2+\left(3\,a^2bc+a^3d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(1/6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/6)), x)
```

3.1754 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=440

$$-\frac{81\,3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{8/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(}{\sqrt[3]{bc-ad}-(}\right.\right.}{128d^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (81*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/6))/(64*d^3) - (9*(b*c - a*d)*
(a + b*x)^(3/2)*(c + d*x)^(1/6))/(16*d^2) + (3*(a + b*x)^(5/2)*(c + d*x)^(1
/6))/(8*d) - (81*3^(3/4)*(b*c - a*d)^(8/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
 (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(128*d^4*Sqrt[a + b*x]*Sqrt[-(
(b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.32723, antiderivative size = 440, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {50, 63, 225}

$$-\frac{81\,3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{8/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}}\right.\right.}{128d^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(5/6),x]
```


```
[Out] (81*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/6))/(64*d^3) - (9*(b*c - a*d)*
(a + b*x)^(3/2)*(c + d*x)^(1/6))/(16*d^2) + (3*(a + b*x)^(5/2)*(c + d*x)^(1
/6))/(8*d) - (81*3^(3/4)*(b*c - a*d)^(8/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
 (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(128*d^4*Sqrt[a + b*x]*Sqrt[-(
(b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/6}}\,dx = \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8d} - \frac{(15(bc-ad))\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}}\,dx}{16d}$$

$$= -\frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8d} + \frac{\left(27(bc-ad)^2\right)\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}\,dx}{32d^2}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}}{64d^3} - \frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8d} - \frac{\left(81(bc-ad)^3\right.}{}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}}{64d^3} - \frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8d} - \frac{\left(243(bc-ad)\right.}{}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}}{64d^3} - \frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8d} - \frac{81\,3^{3/4}(bc-a}{}$$

Mathematica [C] time = 0.0387945, size = 73, normalized size = 0.17

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(\frac{5}{6},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/6))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{2}}(dx+c)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx+a}}{(dx+c)^{\frac{5}{6}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(d*x + c)^(5/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(5/6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(5/6), x)
```

3.1755 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=405

$$\frac{27\,3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{5/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-}{\sqrt[3]{bc-ad}-(1+}\right.\right.}{40d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-27*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/6))/(20*d^2) + (3*(a + b*x)^(3/
2)*(c + d*x)^(1/6))/(5*d) + (27*3^(3/4)*(b*c - a*d)^(5/3)*(c + d*x)^(1/6)*(
(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1
/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*
d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c
 - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) -
 (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(40*d^3*Sqrt[a
+ b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.279166, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {50, 63, 225}

$$\frac{27\,3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{5/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c}}\right.\right.}{40d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(5/6),x]
```


```
[Out] (-27*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/6))/(20*d^2) + (3*(a + b*x)^(3/
2)*(c + d*x)^(1/6))/(5*d) + (27*3^(3/4)*(b*c - a*d)^(5/3)*(c + d*x)^(1/6)*(
(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1
/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*
d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c
 - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) -
 (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(40*d^3*Sqrt[a
+ b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}}\,dx = \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d} - \frac{(9(bc-ad))\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}\,dx}{10d}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}{20d^2} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d} + \frac{\left(27(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/6}}\,dx}{40d^2}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}{20d^2} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d} + \frac{\left(81(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c}\right.}{20d^3}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}{20d^2} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d} + \frac{27\ 3^{3/4}(bc-ad)^{5/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{}\right.}{}$$

Mathematica [C] time = 0.0310273, size = 73, normalized size = 0.18

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(\frac{5}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(5/6))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(5/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(5/6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(5/6), x)
```

3.1756 $\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=372

$$\frac{3\sqrt{a+bx}\sqrt[6]{c+dx}}{2d}-\frac{3\,3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{2/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\text{EllipticF}\left(\cdots\right)}{4d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(2*d) - (3*3^(3/4)*(b*c - a*d)^(2/3)*(c +
 d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)
^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF
[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c -
 a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(4
*d^2*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(
1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)
^(1/3))^2)])
```

Rubi [A] time = 0.235514, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {50, 63, 225}

$$\frac{3\sqrt{a+bx}\sqrt[6]{c+dx}}{2d}-\frac{3\,3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{2/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}F\left(\cos^{-1}\left(\cdots\right)\right)}{4d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(5/6),x]
```

```
[Out] (3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(2*d) - (3*3^(3/4)*(b*c - a*d)^(2/3)*(c +
 d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)
^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF
[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c -
 a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(4
*d^2*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(
1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)
^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}\,dx = \frac{3\sqrt{a+bx}\sqrt[6]{c+dx}}{2d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/6}}\,dx}{4d}$$

$$= \frac{3\sqrt{a+bx}\sqrt[6]{c+dx}}{2d} - \frac{(9(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{2d^2}$$

$$= \frac{3\sqrt{a+bx}\sqrt[6]{c+dx}}{2d} - \frac{3\,3^{3/4}(bc-ad)^{2/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{}\right.}}}{4d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}\right.}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\right.}}}$$

Mathematica [C] time = 0.0246816, size = 73, normalized size = 0.2

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(5/6))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\,(dx+c)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}}{(dx+c)^{\frac{5}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(5/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(5/6),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(5/6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(5/6), x)
```

3.1757 $\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=343

$$\frac{3^{3/4}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right),\right.}{d\sqrt{a+bx}\sqrt[3]{bc-ad}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (3^(3/4)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt
[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(
c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))
^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sq
rt[3])/4])/(d*(b*c - a*d)^(1/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/
3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.194058, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 225}

$$\frac{3^{3/4}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{\right.\right.}{d\sqrt{a+bx}\sqrt[3]{bc-ad}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(5/6)),x]
```

```
[Out] (3^(3/4)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt
[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(
c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))
^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sq
rt[3])/4])/(d*(b*c - a*d)^(1/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/
3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
```

```
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/6}}\,dx = \frac{6\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d}$$

$$= \frac{3^{3/4}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad-}}{\sqrt[3]{bc-ad-}}\right.\right.}{d\sqrt[3]{bc-ad}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Mathematica [C] time = 0.0228293, size = 71, normalized size = 0.21

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(5/6)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/2, 5
/6, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(5/6))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}}(dx+c)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(5/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{6}}}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(5/6),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(5/6)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1758 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=372

$$-\frac{2\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right),\frac{1}{4}(2}{\sqrt[4]{3}\sqrt{a+bx}(bc-ad)^{4/3}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-2*(c + d*x)^(1/6))/((b*c - a*d)*Sqrt[a + b*x]) - (2*(c + d*x)^(1/6)*((b*c
 - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*
(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(
1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a
*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1
+ Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(3^(1/4)*(b*c - a*d
)^(4/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d
*x)^(1/3))^2)])
```

Rubi [A] time = 0.230609, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {51, 63, 225}

$$-\frac{2\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{a+bx}(bc-ad)^{4/3}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(5/6)),x]
```

```
[Out] (-2*(c + d*x)^(1/6))/((b*c - a*d)*Sqrt[a + b*x]) - (2*(c + d*x)^(1/6)*((b*c
 - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*
(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(
1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a
*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1
+ Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(3^(1/4)*(b*c - a*d
)^(4/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d
*x)^(1/3))^2)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/6}}\,dx = -\frac{2\sqrt[6]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{(2d)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/6}}\,dx}{3(bc-ad)}$$

$$= -\frac{2\sqrt[6]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2\sqrt[6]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{2\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(\ldots}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\ldots\right.}}\ldots}{\sqrt[4]{3}(bc-ad)^{4/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c\ldots}}{\left(\sqrt[3]{bc-\ldots}\right.}}}$$

Mathematica [C] time = 0.0226778, size = 71, normalized size = 0.19

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(-\frac{1}{2},\frac{5}{6};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[-1/2, 5/6, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(5/6))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{3}{2}}(dx+c)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(5/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{6}}}{b^2dx^3+a^2c+\left(b^2c+2\,abd\right)x^2+\left(2\,abc+a^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(5/6),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(5/6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(5/6)), x)
```

3.1759 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=410

$$\frac{16d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right),\right.}{9\sqrt[4]{3}\sqrt{a+bx}(bc-ad)^{7/3}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-2*(c + d*x)^(1/6))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (16*d*(c + d*x)^(1/6
))/(9*(b*c - a*d)^2*Sqrt[a + b*x]) + (16*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
 (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4)*(b*c - a*d)^(7/3)*S
qrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)^2)])
```

Rubi [A] time = 0.268404, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {51, 63, 225}

$$\frac{16d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{\right.\right.}{9\sqrt[4]{3}\sqrt{a+bx}(bc-ad)^{7/3}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(5/6)),x]
```

```
[Out] (-2*(c + d*x)^(1/6))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (16*d*(c + d*x)^(1/6
))/(9*(b*c - a*d)^2*Sqrt[a + b*x]) + (16*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
 (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4)*(b*c - a*d)^(7/3)*S
qrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)^2)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/6}}\,dx = -\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(8d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/6}}\,dx}{9(bc-ad)}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{16d\sqrt[6]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{\left(16d^2\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/6}}\,dx}{27(bc-ad)^2}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{16d\sqrt[6]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(32d)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{9(bc-ad)^2}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{16d\sqrt[6]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{16d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{(b\cdots}}{9\sqrt[4]{3}(bc-\cdots}$$

Mathematica [C] time = 0.0235246, size = 73, normalized size = 0.18

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{6};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[-3/2, 5/6, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(5/6))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{2}}(dx+c)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(5/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{1}{6}}}{b^3dx^4+a^3c+\left(b^3c+3\,ab^2d\right)x^3+3\left(ab^2c+a^2bd\right)x^2+\left(3\,a^2bc+a^3d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(5/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(5/6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(5/6)), x)
```

3.1760 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=880

$$-\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}}+\frac{45b(c+dx)^{5/6}(a+bx)^{3/2}}{7d^2}-\frac{405b(bc-ad)(c+dx)^{5/6}\sqrt{a+bx}}{56d^3}-\frac{1215\left(1+\sqrt{3}\right)\sqrt[3]{b}(bc-ad)^2\sqrt[6]{c+dx}\sqrt{a+}}{112d^3\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right.}$$

```
[Out] (-6*(a + b*x)^(5/2))/(d*(c + d*x)^(1/6)) - (405*b*(b*c - a*d)*Sqrt[a + b*x]
*(c + d*x)^(5/6))/(56*d^3) + (45*b*(a + b*x)^(3/2)*(c + d*x)^(5/6))/(7*d^2)
 - (1215*(1 + Sqrt[3])*b^(1/3)*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/6))
/(112*d^3*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))) - (1
215*3^(1/4)*b^(1/3)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3
)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 -
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/
3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(112*d^4*Sqrt[a + b*x]*Sqrt[-((b^(1
/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c -
a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (405*3^(3/4)*(1
- Sqrt[3])*b^(1/3)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b
^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)
*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[
3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 -
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3
)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(224*d^4*Sqrt[a + b*x]*Sqrt[-((b^(1/
3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a
*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.901428, antiderivative size = 880, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {47, 50, 63, 308, 225, 1881}

$$-\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}}+\frac{45b(c+dx)^{5/6}(a+bx)^{3/2}}{7d^2}-\frac{405b(bc-ad)(c+dx)^{5/6}\sqrt{a+bx}}{56d^3}-\frac{1215\left(1+\sqrt{3}\right)\sqrt[3]{b}(bc-ad)^2\sqrt[6]{c+dx}\sqrt{a+}}{112d^3\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(7/6),x]
```


```
[Out] (-6*(a + b*x)^(5/2))/(d*(c + d*x)^(1/6)) - (405*b*(b*c - a*d)*Sqrt[a + b*x]
*(c + d*x)^(5/6))/(56*d^3) + (45*b*(a + b*x)^(3/2)*(c + d*x)^(5/6))/(7*d^2)
 - (1215*(1 + Sqrt[3])*b^(1/3)*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/6))
/(112*d^3*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))) - (1
215*3^(1/4)*b^(1/3)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3
)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 -
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/
3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(112*d^4*Sqrt[a + b*x]*Sqrt[-((b^(1
```

```
/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c -
a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (405*3^(3/4)*(1
- Sqrt[3])*b^(1/3)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b
^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)
*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[
3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 -
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3
)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(224*d^4*Sqrt[a + b*x]*Sqrt[-((b^(1/
3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a
*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
```

```
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{7/6}}\,dx = -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} + \frac{(15b)\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} - \frac{(135b(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx}{14d^2}$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} - \frac{405b(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56d^3} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} + \frac{\left(405b(bc-ad)^2\right)\int \ldots}{112d^3}$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} - \frac{405b(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56d^3} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} + \frac{\left(1215b(bc-ad)^2\right)\mathrm{Su}\ldots}{\ldots}$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} - \frac{405b(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56d^3} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} - \frac{\left(1215\sqrt[3]{b}(bc-ad)^2\right)\mathrm{S}\ldots}{\ldots}$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} - \frac{405b(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56d^3} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} - \frac{1215\left(1+\sqrt{3}\right)\sqrt[3]{b}(bc\ldots}{112d^3\left(\sqrt[3]{bc-ad}-\ldots\right.}$$

Mathematica [C] time = 0.0607014, size = 73, normalized size = 0.08

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6} {}_2F_1\left(\frac{7}{6},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(7/6),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(7/6))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{2}} (dx+c)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(7/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx+a}(dx+c)^{\frac{5}{6}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*(d*x + c)^(5/6)/(d^2*x^2 +
 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(7/6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(7/6), x)
```

3.1761 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=844

$$-\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}}+\frac{27b(c+dx)^{5/6}\sqrt{a+bx}}{4d^2}+\frac{81\left(1+\sqrt{3}\right)\sqrt[3]{b}(bc-ad)\sqrt[6]{c+dx}\sqrt{a+bx}}{8d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{81\sqrt[4]{3}\sqrt[3]{b}(bc-ad)^{4/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc\cdots}\right.}{\cdots}$$

```
[Out] (-6*(a + b*x)^(3/2))/(d*(c + d*x)^(1/6)) + (27*b*Sqrt[a + b*x]*(c + d*x)^(5
/6))/(4*d^2) + (81*(1 + Sqrt[3])*b^(1/3)*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x
)^(1/6))/(8*d^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))
) + (81*3^(1/4)*b^(1/3)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(8*d^3*Sqrt[a + b*x]*Sqrt[-((b^
(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c
- a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (27*3^(3/4)*(1
 - Sqrt[3])*b^(1/3)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3
)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 -
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/
3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(16*d^3*Sqrt[a + b*x]*Sqrt[-((b^(1/
3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a
*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.77254, antiderivative size = 844, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {47, 50, 63, 308, 225, 1881}

$$-\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}}+\frac{27b(c+dx)^{5/6}\sqrt{a+bx}}{4d^2}+\frac{81\left(1+\sqrt{3}\right)\sqrt[3]{b}(bc-ad)\sqrt[6]{c+dx}\sqrt{a+bx}}{8d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{81\sqrt[4]{3}\sqrt[3]{b}(bc-ad)^{4/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(3/2))/(d*(c + d*x)^(1/6)) + (27*b*Sqrt[a + b*x]*(c + d*x)^(5
/6))/(4*d^2) + (81*(1 + Sqrt[3])*b^(1/3)*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x
)^(1/6))/(8*d^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))
) + (81*3^(1/4)*b^(1/3)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(8*d^3*Sqrt[a + b*x]*Sqrt[-((b^
(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c
- a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (27*3^(3/4)*(1
```

```
 - Sqrt[3])*b^(1/3)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3
)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 -
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/
3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(16*d^3*Sqrt[a + b*x]*Sqrt[-((b^(1/
3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a
*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
```

```
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/6}}\,dx = -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{(9b)\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{27b\sqrt{a+bx}(c+dx)^{5/6}}{4d^2} - \frac{(27b(bc-ad))\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{8d^2}$$

$$= -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{27b\sqrt{a+bx}(c+dx)^{5/6}}{4d^2} - \frac{(81b(bc-ad))\,\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{4d^3}$$

$$= -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{27b\sqrt{a+bx}(c+dx)^{5/6}}{4d^2} + \frac{\left(81\sqrt[3]{b}(bc-ad)\right)\mathrm{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c\cdot\right.}{8d^3}$$

$$= -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{27b\sqrt{a+bx}(c+dx)^{5/6}}{4d^2} + \frac{81\left(1+\sqrt{3}\right)\sqrt[3]{b}(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}{8d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)} + \frac{81\sqrt[4]{3}\sqrt[3]{b}(bc-\cdot}{\cdot}$$

Mathematica [C] time = 0.0477265, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{7}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(7/6),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(7/6))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(7/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{6}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(5/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(7/6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(7/6), x)
```

3.1762 $\int \frac{\sqrt{a+bx}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=806

$$-\frac{9\sqrt[4]{3}\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\right.\right.}{d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (-6*Sqrt[a + b*x])/(d*(c + d*x)^(1/6)) - (9*(1 + Sqrt[3])*b^(1/3)*Sqrt[a +
b*x]*(c + d*x)^(1/6))/(d*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*
x)^(1/3))) - (9*3^(1/4)*b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
 - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3)
 - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^
(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(d^2*Sqrt[a + b*x]*Sqrt
[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/
((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (3*3^(3/
4)*(1 - Sqrt[3])*b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
 (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(2*d^2*Sqrt[a + b*x]*Sqrt[-((b
^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c
 - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.667921, antiderivative size = 806, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 63, 308, 225, 1881}

$$-\frac{9\sqrt[4]{3}\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-(1-\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}}\right.\right.}{d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-(1+\sqrt{3})\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(7/6),x]
```

```
[Out] (-6*Sqrt[a + b*x])/(d*(c + d*x)^(1/6)) - (9*(1 + Sqrt[3])*b^(1/3)*Sqrt[a +
b*x]*(c + d*x)^(1/6))/(d*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*
x)^(1/3))) - (9*3^(1/4)*b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
 - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3)
 - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^
(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(d^2*Sqrt[a + b*x]*Sqrt
[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/
((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (3*3^(3/
4)*(1 - Sqrt[3])*b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
```

```
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
 (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(2*d^2*Sqrt[a + b*x]*Sqrt[-((b
^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c
 - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{7/6}}\,dx = -\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}} + \frac{(3b)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}} + \frac{(18b)\,\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d^2}$$

$$= -\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}} - \frac{\left(9\sqrt[3]{b}\right)\mathrm{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d^2} - \frac{\left(9\left(1-\sqrt{3}\right)\sqrt[3]{b}(bc-ad)^{2/3}\right)\mathrm{S}}{}$$

$$= -\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}} - \frac{9\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{a+bx}\sqrt[6]{c+dx}}{d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)} - \frac{9\sqrt[4]{3}\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+}\right.}{}$$

Mathematica [C] time = 0.0406142, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(7/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(7/6))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \sqrt{bx+a}\,(dx+c)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(7/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{6}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(7/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1763 $\int \frac{1}{\sqrt{a+bx}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=817

$$\frac{6\sqrt[4]{3}\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\right.\right.}{d(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

```
[Out] (6*Sqrt[a + b*x])/((b*c - a*d)*(c + d*x)^(1/6)) + (6*(1 + Sqrt[3])*b^(1/3)*
Sqrt[a + b*x]*(c + d*x)^(1/6))/((b*c - a*d)*((b*c - a*d)^(1/3) - (1 + Sqrt[
3])*b^(1/3)*(c + d*x)^(1/3))) + (6*3^(1/4)*b^(1/3)*(c + d*x)^(1/6)*((b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*
c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3
) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)
^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(d*(b*c - a*d)^(2/3)*S
qrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)^2]) + (3^(3/4)*(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1
/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1
 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(
1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(d*(b*c - a*d)^(2/3)*Sqrt[a + b*x
]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2])
```

Rubi [A] time = 0.671677, antiderivative size = 817, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {51, 63, 308, 225, 1881}

$$\frac{6\sqrt[4]{3}\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\right.\right.}{d(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(7/6)),x]
```


```
[Out] (6*Sqrt[a + b*x])/((b*c - a*d)*(c + d*x)^(1/6)) + (6*(1 + Sqrt[3])*b^(1/3)*
Sqrt[a + b*x]*(c + d*x)^(1/6))/((b*c - a*d)*((b*c - a*d)^(1/3) - (1 + Sqrt[
3])*b^(1/3)*(c + d*x)^(1/3))) + (6*3^(1/4)*b^(1/3)*(c + d*x)^(1/6)*((b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*
c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3
) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)
^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(d*(b*c - a*d)^(2/3)*S
qrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)^2]) + (3^(3/4)*(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1
```

```
/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1
 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(
1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(d*(b*c - a*d)^(2/3)*Sqrt[a + b*x
]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{7/6}}\,dx = \frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}} - \frac{(2b)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{bc-ad}$$

$$= \frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}} - \frac{(12b)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d(bc-ad)}$$

$$= \frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}} + \frac{\left(6\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d(bc-ad)} + \frac{\left(6\left(1-\sqrt{3}\right)\sqrt[3]{b}\right.}{}$$

$$= \frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}} + \frac{6\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{a+bx}\sqrt[6]{c+dx}}{(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)} + \frac{6\sqrt[4]{3}\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-a\right.}{}$$

Mathematica [C] time = 0.0332696, size = 71, normalized size = 0.09

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(7/6)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[1/2, 7
/6, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(7/6))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}}(dx+c)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(7/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{6}}}{bd^2x^3+ac^2+\left(2\,bcd+ad^2\right)x^2+\left(bc^2+2\,acd\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*d^
2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(7/6)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1764 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=844

$$-\frac{8\sqrt{a+bx}d}{(bc-ad)^2\sqrt[6]{c+dx}}-\frac{8\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{a+bx}\sqrt[6]{c+dx}d}{(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{8\sqrt[4]{3}\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)}{}}}{(bc-ad)^{5/3}}$$

[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/6)) - (8*d*Sqrt[a + b*x])/((b*c - a*d)^2*(c + d*x)^(1/6)) - (8*(1 + Sqrt[3])*b^(1/3)*d*Sqrt[a + b*x]*(c + d*x)^(1/6))/((b*c - a*d)^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))) - (8*3^(1/4)*b^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/((b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (4*(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(3^(1/4)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])

Rubi [A] time = 0.758512, antiderivative size = 844, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {51, 63, 308, 225, 1881}

$$-\frac{8\sqrt{a+bx}d}{(bc-ad)^2\sqrt[6]{c+dx}}-\frac{8\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{a+bx}\sqrt[6]{c+dx}d}{(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{8\sqrt[4]{3}\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)}{}}}{(bc-ad)^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(7/6)),x]

[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/6)) - (8*d*Sqrt[a + b*x])/((b*c - a*d)^2*(c + d*x)^(1/6)) - (8*(1 + Sqrt[3])*b^(1/3)*d*Sqrt[a + b*x]*(c + d*x)^(1/6))/((b*c - a*d)^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))) - (8*3^(1/4)*b^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/((b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (4*(1 - Sqr

```
t[3])*b^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2
/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d
*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2
 + Sqrt[3])/4])/(3^(1/4)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c
 + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(
1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/6}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}} - \frac{(4d)\int \frac{1}{\sqrt{a+bx}(c+dx)^{7/6}}\,dx}{3(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}} - \frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}} + \frac{(8bd)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{3(bc-ad)^2}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}} - \frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}} + \frac{(16b)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+d}\right.}{(bc-ad)^2}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}} - \frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}} - \frac{\left(8\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\right.}{(bc-ad)^2}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}} - \frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}} - \frac{8\left(1+\sqrt{3}\right)\sqrt[3]{b}d\sqrt{a+bx}\sqrt[6]{c+dx}}{(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c}\right.}$$

Mathematica [C] time = 0.0348741, size = 71, normalized size = 0.08

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6} {}_2F_1\left(-\frac{1}{2},\frac{7}{6};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(7/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[-1/2, 7/6, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(7/6))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{3}{2}}(dx+c)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(7/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{6}}}{b^2d^2x^4+a^2c^2+2\left(b^2cd+abd^2\right)x^3+\left(b^2c^2+4\,abcd+a^2d^2\right)x^2+2\left(abc^2+a^2cd\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*d
+ a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c*d
)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(7/6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(7/6)), x)
```

3.1765 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=893

$$\frac{80\sqrt{a+bx}d^2}{9(bc-ad)^3\sqrt[6]{c+dx}}+\frac{80\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{a+bx}\sqrt[6]{c+dx}d^2}{9(bc-ad)^3\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{80\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2}\cdots}{(\cdots}}}{3\,3^{3/4}(bc-ad\cdots}$$

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/6)) + (20*d)/(9*(b*c - a*d)^
2*Sqrt[a + b*x]*(c + d*x)^(1/6)) + (80*d^2*Sqrt[a + b*x])/(9*(b*c - a*d)^3*
(c + d*x)^(1/6)) + (80*(1 + Sqrt[3])*b^(1/3)*d^2*Sqrt[a + b*x]*(c + d*x)^(1
/6))/(9*(b*c - a*d)^3*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))) + (80*b^(1/3)*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1
/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(
c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1
/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))], (2 + Sqrt[3])/4])/(3*3^(3/4)*(b*c - a*d)^(8/3)*Sqrt[a + b*x]*Sqrt[-
((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((
b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (40*(1 - S
qrt[3])*b^(1/3)*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) +
b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*
x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))]
, (2 + Sqrt[3])/4])/(9*3^(1/4)*(b*c - a*d)^(8/3)*Sqrt[a + b*x]*Sqrt[-((b^(1
/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c -
a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rubi [A] time = 0.868424, antiderivative size = 893, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {51, 63, 308, 225, 1881}

$$\frac{80\sqrt{a+bx}d^2}{9(bc-ad)^3\sqrt[6]{c+dx}}+\frac{80\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{a+bx}\sqrt[6]{c+dx}d^2}{9(bc-ad)^3\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{80\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2}\cdots}{(\cdots}}}{3\,3^{3/4}(bc-ad\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(7/6)),x]
```


```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/6)) + (20*d)/(9*(b*c - a*d)^
2*Sqrt[a + b*x]*(c + d*x)^(1/6)) + (80*d^2*Sqrt[a + b*x])/(9*(b*c - a*d)^3*
(c + d*x)^(1/6)) + (80*(1 + Sqrt[3])*b^(1/3)*d^2*Sqrt[a + b*x]*(c + d*x)^(1
/6))/(9*(b*c - a*d)^3*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))) + (80*b^(1/3)*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1
/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(
c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1
/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))], (2 + Sqrt[3])/4])/(3*3^(3/4)*(b*c - a*d)^(8/3)*Sqrt[a + b*x]*Sqrt[-
```

```
((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((
b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (40*(1 - S
qrt[3])*b^(1/3)*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) +
b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*
x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))]
, (2 + Sqrt[3])/4])/(9*3^(1/4)*(b*c - a*d)^(8/3)*Sqrt[a + b*x]*Sqrt[-((b^(1
/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c -
a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{7/6}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} - \frac{(10d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/6}}\,dx}{9(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}} + \frac{(40d^2)\int \frac{1}{\sqrt{a+bx}(c+dx)^{7/6}}\,dx}{27(bc-ad)^2}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}} + \frac{80d^2\sqrt{a+bx}}{9(bc-ad)^3\sqrt[6]{c+dx}} - \frac{(80}{}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}} + \frac{80d^2\sqrt{a+bx}}{9(bc-ad)^3\sqrt[6]{c+dx}} - \frac{(16}{}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}} + \frac{80d^2\sqrt{a+bx}}{9(bc-ad)^3\sqrt[6]{c+dx}} + \frac{(80}{}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}} + \frac{80d^2\sqrt{a+bx}}{9(bc-ad)^3\sqrt[6]{c+dx}} + \frac{}{9(b}$$

Mathematica [C] time = 0.0362682, size = 73, normalized size = 0.08

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6} {}_2F_1\left(-\frac{3}{2},\frac{7}{6};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(7/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[-3/2, 7/6, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(7/6))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{2}}(dx+c)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(7/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx+a}(dx+c)^{\frac{5}{6}}}{b^3d^2x^5+a^3c^2+\left(2\,b^3cd+3\,ab^2d^2\right)x^4+\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^3+\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^2+\left(3\,a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*d
+ 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2*c
^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(7/6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(7/6)), x)
```

3.1766 $\int \sqrt[6]{a+bx}(c+dx)^{13/6}\,dx$

Optimal. Leaf size=84

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}(bc-ad)^2\,{}_2F_1\left(-\frac{13}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] (6*(b*c - a*d)^2*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-13/6, 7
/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*b^3*((b*(c + d*x))/(b*c - a*d))
^(1/6))
```

Rubi [A] time = 0.0211692, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}(bc-ad)^2\,{}_2F_1\left(-\frac{13}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(13/6),x]
```


```
[Out] (6*(b*c - a*d)^2*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-13/6, 7
/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*b^3*((b*(c + d*x))/(b*c - a*d))
^(1/6))
```


Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \sqrt[6]{a+bx}(c+dx)^{13/6}\,dx &= \frac{\left((bc-ad)^2\sqrt[6]{c+dx}\right)\int \sqrt[6]{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}dx}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\\ &= \frac{6(bc-ad)^2(a+bx)^{7/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{13}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\end{aligned}$$

Mathematica [A] time = 0.0887131, size = 73, normalized size = 0.87

$$\frac{6(a+bx)^{7/6}(c+dx)^{13/6}\,{}_2F_1\left(-\frac{13}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(13/6)*Hypergeometric2F1[-13/6, 7/6, 13/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(13/6))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \sqrt[6]{bx+a}\,(dx+c)^{\frac{13}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(13/6),x)
```

```
[Out] int((b*x+a)^(1/6)*(d*x+c)^(13/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{1}{6}}(dx+c)^{\frac{13}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(13/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d^2x^2+2\,cdx+c^2\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{1}{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*(b*x + a)^(1/6)*(d*x + c)^(1/6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1767 $\int \sqrt[6]{a+bx}(c+dx)^{7/6}\,dx$

Optimal. Leaf size=82

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}(bc-ad)\,{}_2F_1\left(-\frac{7}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] (6*(b*c - a*d)*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-7/6, 7/6,
 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*b^2*((b*(c + d*x))/(b*c - a*d))^(1
/6))
```

Rubi [A] time = 0.0241791, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}(bc-ad)\,{}_2F_1\left(-\frac{7}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(7/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-7/6, 7/6,
 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*b^2*((b*(c + d*x))/(b*c - a*d))^(1
/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \sqrt[6]{a+bx}(c+dx)^{7/6}\,dx &= \frac{\left((bc-ad)\sqrt[6]{c+dx}\right)\int \sqrt[6]{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}\,dx}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\\ &= \frac{6(bc-ad)(a+bx)^{7/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{7}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\end{aligned}$$

Mathematica [A] time = 0.0494231, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{7/6}(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, 7/6, 13/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \sqrt[6]{bx+a}\,(dx+c)^{\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(1/6)*(d*x+c)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{1}{6}}(dx+c)^{\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(7/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^{\frac{1}{6}}(dx+c)^{\frac{7}{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(7/6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(7/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1768 $\int \sqrt[6]{a+bx}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 7/6, 13/6, -((d*
(a + b*x))/(b*c - a*d))])/(7*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Rubi [A] time = 0.0204714, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 7/6, 13/6, -((d*
(a + b*x))/(b*c - a*d))])/(7*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \sqrt[6]{a+bx}\sqrt[6]{c+dx}\,dx &= \frac{\sqrt[6]{c+dx}\int \sqrt[6]{a+bx}\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\,dx}{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\\ &= \frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\end{aligned}$$

Mathematica [A] time = 0.026476, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 7/6, 13/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \sqrt[6]{bx+a}\sqrt[6]{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(1/6)*(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{1}{6}}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx+a)^{\frac{1}{6}}(dx+c)^{\frac{1}{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[6]{a+bx}\sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)*(c + d*x)**(1/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1769 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b(c+dx)^{5/6}}$$

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 7/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*b*(c + d*x)^(5/6))
```

Rubi [A] time = 0.0201391, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 7/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*b*(c + d*x)^(5/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{5/6}}\,dx &= \frac{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\int \frac{\sqrt[6]{a+bx}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}\,dx}{(c+dx)^{5/6}}\\ &= \frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b(c+dx)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.026093, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 7/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/6))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int \sqrt[6]{bx+a}\,(dx+c)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{5}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)/(d*x + c)^(5/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(5/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1770 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=81

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(c+dx)^{5/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[7/6,
 11/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*(b*c - a*d)*(c + d*x)^(5/6))
```

Rubi [A] time = 0.0205303, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[7/6,
 11/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*(b*c - a*d)*(c + d*x)^(5/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{11/6}}\,dx &= \frac{\left(b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{\sqrt[6]{a+bx}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}\,dx}{(bc-ad)(c+dx)^{5/6}}\\
&= \frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(bc-ad)(c+dx)^{5/6}}
\end{aligned}$$

Mathematica [A] time = 0.0476979, size = 73, normalized size = 0.9

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(11/6)*Hypergeometric2F1[7/6
, 11/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(11/6))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \sqrt[6]{bx+a}\,(dx+c)^{-\frac{11}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(11/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(11/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(11/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{1}{6}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(11/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1771 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6b(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{7}{6},\frac{17}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(c+dx)^{5/6}(bc-ad)^2}$$

```
[Out] (6*b*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[7/
6, 17/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*(b*c - a*d)^2*(c + d*x)^(5
/6))
```

Rubi [A] time = 0.0212505, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6b(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{7}{6},\frac{17}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*b*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[7/
6, 17/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*(b*c - a*d)^2*(c + d*x)^(5
/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{17/6}}\,dx &= \frac{\left(b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{\sqrt[6]{a+bx}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{17/6}}\,dx}{(bc-ad)^2(c+dx)^{5/6}}\\ &= \frac{6b(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{7}{6},\frac{17}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(bc-ad)^2(c+dx)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.0311649, size = 81, normalized size = 0.99

$$\frac{6b(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{17}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*b*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[7/
6, 17/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*(b*c - a*d)^2*(c + d*x)^(5
/6))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \sqrt[6]{bx+a}\,(dx+c)^{-\frac{17}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(17/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(17/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(17/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{1}{6}}}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(17/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1772 $\int \sqrt[6]{a+bx}(c+dx)^{5/6}\,dx$

Optimal. Leaf size=427

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}+\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{24\sqrt{3}b^{11/6}}$$

```
[Out] (5*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*b*d) + ((a + b*x)^(7/6)
*(c + d*x)^(5/6))/(2*b) + (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
 + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(
7/6)) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sq
rt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(7/6)) - (5*(b*c -
a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^
(11/6)*d^(7/6)) + (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*
b^(11/6)*d^(7/6)) - (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(14
4*b^(11/6)*d^(7/6))
```

Rubi [A] time = 0.643188, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}+\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{24\sqrt{3}b^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(5/6),x]
```


```
[Out] (5*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*b*d) + ((a + b*x)^(7/6)
*(c + d*x)^(5/6))/(2*b) + (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
 + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(
7/6)) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sq
rt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(7/6)) - (5*(b*c -
a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^
(11/6)*d^(7/6)) + (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*
b^(11/6)*d^(7/6)) - (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(14
4*b^(11/6)*d^(7/6))
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt[6]{a+bx}(c+dx)^{5/6}\,dx = \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} + \frac{(5(bc-ad))\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{12b}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{\left(5(bc-ad)^2\right)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{72bd}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx\right.}{12b^2d}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{}}{\sqrt[6]{}}\right.}{12b^2d}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}}\right.}{36b^{11/6}d}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{11/6}d^{7/6}} + \frac{(5(}{}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{11/6}d^{7/6}} + \frac{5(b}{}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} + \frac{5(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{24\sqrt{3}b^{11/6}d^{7/6}} - \frac{5(b}{}$$

Mathematica [C] time = 0.0303143, size = 73, normalized size = 0.17

$$\frac{6(a+bx)^{7/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 7/6, 13/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \sqrt[6]{bx+a}\,(dx+c)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(1/6)*(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(5/6), x)
```

Fricas [B] time = 2.89224, size = 12593, normalized size = 29.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] 1/144*(20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^
6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3
*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/
6)*arctan(-1/3*(2*sqrt(3)*(b^11*c^2*d^6 - 2*a*b^10*c*d^7 + a^2*b^9*d^8)*(b*
x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^
7))^(5/6) - 2*sqrt(3)*(b^9*d^7*x + b^9*c*d^6)*sqrt(((b^4*c^2*d - 2*a*b^3*c*
d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)/(b^11*d^7))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d^3*x
 + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/3))/(d*x
 + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(5/6) + sqrt(3)*(b
^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 +
 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*
b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d
^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2
+ 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^
5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4
*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^1
2*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*
b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*
d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*
a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*
b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^
8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 4
```

95*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*
b*c*d^12 + a^12*d^13)*x)) + 20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^11*d^7))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^11*c^2*d^6 - 2*a*b^10*c*d
^7 + a^2*b^9*d^8)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c
^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(b^11*d^7))^(5/6) - 2*sqrt(3)*(b^9*d^7*x + b^9*c*d^6)*sqrt(-((
b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6) - (b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x +
c)^(2/3) - (b^4*d^3*x + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
b^11*d^7))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^
7))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 -
220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*
b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d
^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d
- 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^
4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5
*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 -
12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^
10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*
d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220
*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12
+ (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^
9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 7
92*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b
^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) - 5*b*d*((b^12*c^12 - 12*a*
b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495
*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c
*d^11 + a^12*d^12)/(b^11*d^7))^(1/6)*log(25*((b^4*c^2*d - 2*a*b^3*c*d^2 + a
^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^11*d^7))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d^3*x + b^4*
c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/3))/(d*x + c))
+ 5*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9
*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
- 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10
*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6)*log(-25*((b
^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b

```
^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^
10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6) - (b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x +
c)^(2/3) - (b^4*d^3*x + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*
b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7
*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b
^11*d^7))^(1/3))/(d*x + c)) - 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/
(b^11*d^7))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/6)*(d
*x + c)^(5/6) + (b^2*d^2*x + b^2*c*d)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a
^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*
c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)
/(b^11*d^7))^(1/6))/(d*x + c)) + 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^
7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^1
2)/(b^11*d^7))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/6)
*(d*x + c)^(5/6) - (b^2*d^2*x + b^2*c*d)*((b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(b^11*d^7))^(1/6))/(d*x + c)) + 12*(6*b*d*x + 5*b*c + a*d)*(b*x + a)^(1
/6)*(d*x + c)^(5/6))/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[6]{a + bx}\,(c + dx)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)*(c + d*x)**(5/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1773 $\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=378

$$\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}-\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{\cdots}}{\sqrt{3}\sqrt[6]{\cdots}}\right)}{2\sqrt{3}b^{5/6}d^{7/6}}$$

```
[Out] ((a + b*x)^(1/6)*(c + d*x)^(5/6))/d + ((b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^
(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(5/
6)*d^(7/6)) - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(
Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(5/6)*d^(7/6)) - ((b*c - a*
d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(5/6)
*d^(7/6)) + ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(5/6)*d^(
7/6)) - ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3
) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(5/6)*d^(7/6)
)
```

Rubi [A] time = 0.497057, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}-\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{\cdots}}{\sqrt{3}\sqrt[6]{\cdots}}\right)}{2\sqrt{3}b^{5/6}d^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(1/6),x]
```


```
[Out] ((a + b*x)^(1/6)*(c + d*x)^(5/6))/d + ((b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^
(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(5/
6)*d^(7/6)) - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(
Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(5/6)*d^(7/6)) - ((b*c - a*
d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(5/6)
*d^(7/6)) + ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(5/6)*d^(
7/6)) - ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3
) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(5/6)*d^(7/6)
)
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}dx = \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}dx}{6d}$$

$$= \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}dx, x, \sqrt[6]{a+bx}\right)}{bd}$$

$$= \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{bd}$$

$$= \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{5/6}d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{\sqrt[6]{b}+\frac{\sqrt[6]{d}}{2}}{\sqrt[3]{b}+\sqrt[6]{b}\sqrt[6]{d}x-}\right.}{3b^{5/6}d}$$

$$= \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}} + \frac{(bc-ad)\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{12b^{5/6}d^{7/6}} -$$

$$= \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}} + \frac{(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{12b^{5/6}d^{7/6}} - \frac{(bc}{}$$

$$= \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{d} + \frac{(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}b^{5/6}d^{7/6}} - \frac{(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}b^{5/6}d^{7/6}} - \frac{(bc-ad)\tanh^{-1}\left(\right.}{3b^{5/6}d^{7/6}}$$

Mathematica [C] time = 0.0251235, size = 73, normalized size = 0.19

$$\frac{6(a+bx)^{7/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 7/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/6))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \sqrt[6]{bx+a}\frac{1}{\sqrt[6]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(1/6), x)
```

Fricas [B] time = 2.40401, size = 6461, normalized size = 17.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*
b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6
)*arctan(1/3*(2*sqrt(3)*(b^5*c*d^6 - a*b^4*d^7)*(b*x + a)^(1/6)*(d*x + c)^(
5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(5/6) + 2*sqrt(3)*
(b^4*d^7*x + b^4*c*d^6)*sqrt(((b^2*c*d - a*b*d^2)*(b*x + a)^(1/6)*(d*x + c)
^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6) + (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d^3*x + b^2*
c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/3))/(d*x + c)
)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*
a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(5/6) + sqrt(3)*(b^6*
c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*
c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a
^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 +
a^6*d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4
*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*
b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5
- 6*a^5*b*c*d^6 + a^6*d^7)*x)) + 4*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15
*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
+ a^6*d^6)/(b^5*d^7))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^5*c*d^6 - a*b^4*d^7)*(
b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d
^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^
5*d^7))^(5/6) + 2*sqrt(3)*(b^4*d^7*x + b^4*c*d^6)*sqrt(-((b^2*c*d - a*b*d^2
)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^
4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/
(b^5*d^7))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c
)^(2/3) - (b^2*d^3*x + b^2*c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^
4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/
(b^5*d^7))^(1/3))/(d*x + c))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2
 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*
d^7))^(5/6) - sqrt(3)*(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^
3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6
*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2
*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b
^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^
```

```
6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^
3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x)) + d*((b^6*c^6 - 6
*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(((b^2*c*d - a*b*d^2)*(b*x
+ a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^
7))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3)
 + (b^2*d^3*x + b^2*c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^
7))^(1/3))/(d*x + c)) - d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7
))^(1/6)*log(-((b^2*c*d - a*b*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6) - (b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^2*d^3*x + b^2*c*d^2)*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/3))/(d*x + c)) + 2*d*((b^6*c
^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c
^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(-((b*c - a*d)*(b*x +
 a)^(1/6)*(d*x + c)^(5/6) + (b*d^2*x + b*c*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
 + a^6*d^6)/(b^5*d^7))^(1/6))/(d*x + c)) - 2*d*((b^6*c^6 - 6*a*b^5*c^5*d +
15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^
5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(-((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(
5/6) - (b*d^2*x + b*c*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7)
)^(1/6))/(d*x + c)) - 12*(b*x + a)^(1/6)*(d*x + c)^(5/6))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(1/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1774 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=332

$$-\frac{\sqrt[6]{b}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}+\frac{\sqrt[6]{b}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}-\frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}}\cdot$$

```
[Out] (-6*(a + b*x)^(1/6))/(d*(c + d*x)^(1/6)) - (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6)
 + (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) + (2*b^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) - (b^(1/6)*Log[b^(1/3) + (d^(1/
3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c
+ d*x)^(1/6)])/(2*d^(7/6)) + (b^(1/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2
*d^(7/6))
```

Rubi [A] time = 0.487119, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{\sqrt[6]{b}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}+\frac{\sqrt[6]{b}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}-\frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(1/6))/(d*(c + d*x)^(1/6)) - (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6)
 + (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) + (2*b^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) - (b^(1/6)*Log[b^(1/3) + (d^(1/
3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c
+ d*x)^(1/6)])/(2*d^(7/6)) + (b^(1/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2
*d^(7/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx = -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{b\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{6\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{6\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{\left(2\sqrt[6]{b}\right)\mathrm{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d} + \frac{\left(2\sqrt[6]{b}\right)\mathrm{Subst}\left(\int \frac{\sqrt[6]{b}+\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}+\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,d\right.}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}} - \frac{\sqrt[6]{b}\,\mathrm{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2d^{7/6}} + \frac{\sqrt[6]{b}\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}} - \frac{\sqrt[6]{b}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2d^{7/6}} + \frac{\sqrt[6]{b}\log\left(\sqrt[3]{b}+\right.}{}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} - \frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{7/6}} + \frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{7/6}} + \frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}} -$$

Mathematica [C] time = 0.0414696, size = 73, normalized size = 0.22

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{7}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6,
 7/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(7/6))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \sqrt[6]{bx+a}\,(dx+c)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(7/6), x)
```

Fricas [B] time = 1.80483, size = 1704, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(3)*(d^2*x + c*d)*(b/d^7)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a
)^(1/6)*(d*x + c)^(5/6)*d^6*(b/d^7)^(5/6) - 2*sqrt(3)*(d^7*x + c*d^6)*sqrt(
((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6) + (d^3*x + c*d^2)*(b/d^7)^
(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c))*(b/d^7)^(5/6) + sqrt(3)
*(b*d*x + b*c))/(b*d*x + b*c)) + 4*sqrt(3)*(d^2*x + c*d)*(b/d^7)^(1/6)*arct
an(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*d^6*(b/d^7)^(5/6) - 2*sq
rt(3)*(d^7*x + c*d^6)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6
) - (d^3*x + c*d^2)*(b/d^7)^(1/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x +
 c))*(b/d^7)^(5/6) - sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) - (d^2*x + c*d)*
(b/d^7)^(1/6)*log(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6) + (d^3
*x + c*d^2)*(b/d^7)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c)) + (
d^2*x + c*d)*(b/d^7)^(1/6)*log(-4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7
)^(1/6) - (d^3*x + c*d^2)*(b/d^7)^(1/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3))/
(d*x + c)) - 2*(d^2*x + c*d)*(b/d^7)^(1/6)*log(((d^2*x + c*d)*(b/d^7)^(1/6)
 + (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d*x + c)) + 2*(d^2*x + c*d)*(b/d^7)^(1
/6)*log(-((d^2*x + c*d)*(b/d^7)^(1/6) - (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d
*x + c)) + 12*(b*x + a)^(1/6)*(d*x + c)^(5/6))/(d^2*x + c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(7/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(7/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1775 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{7/6}}{7(c+dx)^{7/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(7/6))/(7*(b*c - a*d)*(c + d*x)^(7/6))
```

Rubi [A] time = 0.0032505, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6(a+bx)^{7/6}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(7*(b*c - a*d)*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx = \frac{6(a+bx)^{7/6}}{7(bc-ad)(c+dx)^{7/6}}$$

Mathematica [A] time = 0.0095994, size = 32, normalized size = 1.

$$\frac{6(a+bx)^{7/6}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(7*(b*c - a*d)*(c + d*x)^(7/6))
```

Maple [A] time = 0.003, size = 27, normalized size = 0.8

$$-\frac{6}{7\,ad-7\,bc}\,(bx+a)^{\frac{7}{6}}\,(dx+c)^{-\frac{7}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(13/6),x)
```

```
[Out] -6/7*(b*x+a)^(7/6)/(d*x+c)^(7/6)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(13/6), x)
```

Fricas [B] time = 1.49147, size = 142, normalized size = 4.44

$$\frac{6\,(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{5}{6}}}{7\left(bc^3-ac^2d+\left(bcd^2-ad^3\right)x^2+2\left(bc^2d-acd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] 6/7*(b*x + a)^(7/6)*(d*x + c)^(5/6)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*x^
2 + 2*(b*c^2*d - a*c*d^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1776 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=66

$$\frac{36b(a+bx)^{7/6}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{13(c+dx)^{13/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(7/6))/(13*(b*c - a*d)*(c + d*x)^(13/6)) + (36*b*(a + b*x)^(7/
6))/(91*(b*c - a*d)^2*(c + d*x)^(7/6))
```

Rubi [A] time = 0.0090521, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b(a+bx)^{7/6}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(13*(b*c - a*d)*(c + d*x)^(13/6)) + (36*b*(a + b*x)^(7/
6))/(91*(b*c - a*d)^2*(c + d*x)^(7/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx &= \frac{6(a+bx)^{7/6}}{13(bc-ad)(c+dx)^{13/6}}+\frac{(6b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx}{13(bc-ad)}\\ &= \frac{6(a+bx)^{7/6}}{13(bc-ad)(c+dx)^{13/6}}+\frac{36b(a+bx)^{7/6}}{91(bc-ad)^2(c+dx)^{7/6}}\end{aligned}$$

Mathematica [A] time = 0.0220619, size = 46, normalized size = 0.7

$$\frac{6(a+bx)^{7/6}(-7ad+13bc+6bdx)}{91(c+dx)^{13/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(13*b*c - 7*a*d + 6*b*d*x))/(91*(b*c - a*d)^2*(c + d*x)^
(13/6))
```

Maple [A] time = 0.005, size = 54, normalized size = 0.8

$$-\frac{-36\,bdx+42\,ad-78\,bc}{91\,a^2d^2-182\,abcd+91\,b^2c^2}\,(bx+a)^{\frac{7}{6}}\,(dx+c)^{-\frac{13}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(19/6),x)
```

```
[Out] -6/91*(b*x+a)^(7/6)*(-6*b*d*x+7*a*d-13*b*c)/(d*x+c)^(13/6)/(a^2*d^2-2*a*b*c
*d+b^2*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(19/6), x)
```

Fricas [B] time = 1.55797, size = 373, normalized size = 5.65

$$\frac{6\left(6\,b^2dx^2+13\,abc-7\,a^2d+\left(13\,b^2c-abd\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{91\left(b^2c^5-2\,abc^4d+a^2c^3d^2+\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)x^3+3\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2+3\left(b^2c^4d-2\,abc^3d^2+a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] 6/91*(6*b^2*d*x^2 + 13*a*b*c - 7*a^2*d + (13*b^2*c - a*b*d)*x)*(b*x + a)^(1
/6)*(d*x + c)^(5/6)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^2*d^3 - 2
*a*b*c*d^4 + a^2*d^5)*x^3 + 3*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2
 + 3*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1777 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}} dx$

Optimal. Leaf size=101

$$\frac{432b^2(a+bx)^{7/6}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{72b(a+bx)^{7/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{19(c+dx)^{19/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(7/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (72*b*(a + b*x)^(7/
6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6)) + (432*b^2*(a + b*x)^(7/6))/(1729*
(b*c - a*d)^3*(c + d*x)^(7/6))
```

Rubi [A] time = 0.0209129, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2(a+bx)^{7/6}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{72b(a+bx)^{7/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (72*b*(a + b*x)^(7/
6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6)) + (432*b^2*(a + b*x)^(7/6))/(1729*
(b*c - a*d)^3*(c + d*x)^(7/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}} dx &= \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{(12b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}} dx}{19(bc-ad)}\\ &= \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{72b(a+bx)^{7/6}}{247(bc-ad)^2(c+dx)^{13/6}}+\frac{\left(72b^2\right)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}} dx}{247(bc-ad)^2}\\ &= \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{72b(a+bx)^{7/6}}{247(bc-ad)^2(c+dx)^{13/6}}+\frac{432b^2(a+bx)^{7/6}}{1729(bc-ad)^3(c+dx)^{7/6}}\end{aligned}$$

Mathematica [A] time = 0.0416167, size = 77, normalized size = 0.76

$$\frac{6(a+bx)^{7/6}\left(91a^2d^2-14abd(19c+6dx)+b^2\left(247c^2+228cdx+72d^2x^2\right)\right)}{1729(c+dx)^{19/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(91*a^2*d^2 - 14*a*b*d*(19*c + 6*d*x) + b^2*(247*c^2 + 2
28*c*d*x + 72*d^2*x^2)))/(1729*(b*c - a*d)^3*(c + d*x)^(19/6))
```

Maple [A] time = 0.007, size = 105, normalized size = 1.

$$-\frac{432\,b^2d^2x^2-504\,abd^2x+1368\,b^2cdx+546\,a^2d^2-1596\,abcd+1482\,b^2c^2}{1729\,a^3d^3-5187\,a^2cbd^2+5187\,ab^2c^2d-1729\,b^3c^3}(bx+a)^{\frac{7}{6}}(dx+c)^{-\frac{19}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(25/6),x)
```

```
[Out] -6/1729*(b*x+a)^(7/6)*(72*b^2*d^2*x^2-84*a*b*d^2*x+228*b^2*c*d*x+91*a^2*d^2
-266*a*b*c*d+247*b^2*c^2)/(d*x+c)^(19/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2
*d-b^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(25/6), x)
```

Fricas [B] time = 1.50056, size = 695, normalized size = 6.88

$$\frac{6\left(72\,b^3d^2x^3+247\,ab^2c^2-266\,a^2bcd+91\,a^3d^2+12\left(19\,b^3cd-ab^2d^2\right)\right.}{1729\left(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3+\left(b^3c^3d^4-3\,ab^2c^2d^5+3\,a^2bcd^6-a^3d^7\right)x^4+4\left(b^3c^4d^3-3\,ab^2c^3d^4+3\,a^2bc^2d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/1729*(72*b^3*d^2*x^3 + 247*a*b^2*c^2 - 266*a^2*b*c*d + 91*a^3*d^2 + 12*(1
9*b^3*c*d - a*b^2*d^2)*x^2 + (247*b^3*c^2 - 38*a*b^2*c*d + 7*a^2*b*d^2)*x)*
(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2
```

```
- a^3*c^4*d^3 + (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*x
^4 + 4*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*x^3 +
6*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^2 + 4*(
b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(25/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1778 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{31/6}}\,dx$

Optimal. Leaf size=136

$$\frac{7776b^3(a+bx)^{7/6}}{43225(c+dx)^{7/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{7/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{108b(a+bx)^{7/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{25(c+dx)^{25/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(7/6))/(25*(b*c - a*d)*(c + d*x)^(25/6)) + (108*b*(a + b*x)^(7
/6))/(475*(b*c - a*d)^2*(c + d*x)^(19/6)) + (1296*b^2*(a + b*x)^(7/6))/(617
5*(b*c - a*d)^3*(c + d*x)^(13/6)) + (7776*b^3*(a + b*x)^(7/6))/(43225*(b*c
- a*d)^4*(c + d*x)^(7/6))
```

Rubi [A] time = 0.0339152, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3(a+bx)^{7/6}}{43225(c+dx)^{7/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{7/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{108b(a+bx)^{7/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{25(c+dx)^{25/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(25*(b*c - a*d)*(c + d*x)^(25/6)) + (108*b*(a + b*x)^(7
/6))/(475*(b*c - a*d)^2*(c + d*x)^(19/6)) + (1296*b^2*(a + b*x)^(7/6))/(617
5*(b*c - a*d)^3*(c + d*x)^(13/6)) + (7776*b^3*(a + b*x)^(7/6))/(43225*(b*c
- a*d)^4*(c + d*x)^(7/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{31/6}}\,dx = \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{(18b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}}\,dx}{25(bc-ad)}$$

$$= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{(216b^2)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx}{475(bc-ad)^2}$$

$$= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{1296b^2(a+bx)^{7/6}}{6175(bc-ad)^3(c+dx)^{13/6}} + \frac{(1296b^3)\int \frac{}{(c}}{6175(bc-}$$

$$= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{1296b^2(a+bx)^{7/6}}{6175(bc-ad)^3(c+dx)^{13/6}} + \frac{7776b^3(a}{43225(bc-a}$$

Mathematica [A] time = 0.0604482, size = 118, normalized size = 0.87

$$\frac{6(a+bx)^{7/6}\left(273a^2bd^2(25c+6dx)-1729a^3d^3-21ab^2d\left(475c^2+300cdx+72d^2x^2\right)+b^3\left(8550c^2dx+6175c^3+5400c\right.\right.}{43225(c+dx)^{25/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(-1729*a^3*d^3 + 273*a^2*b*d^2*(25*c + 6*d*x) - 21*a*b^2
*d*(475*c^2 + 300*c*d*x + 72*d^2*x^2) + b^3*(6175*c^3 + 8550*c^2*d*x + 5400
*c*d^2*x^2 + 1296*d^3*x^3)))/(43225*(b*c - a*d)^4*(c + d*x)^(25/6))
```

Maple [A] time = 0.008, size = 171, normalized size = 1.3

$$-\frac{-7776\,x^3b^3d^3+9072\,ab^2d^3x^2-32400\,b^3cd^2x^2-9828\,a^2bd^3x+37800\,ab^2cd^2x-51300\,b^3c^2dx+10374\,a^3d^3-4095}{43225\,a^4d^4-172900\,a^3bcd^3+259350\,a^2c^2b^2d^2-172900\,b^3dc^3a+43225\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(31/6),x)
```

```
[Out] -6/43225*(b*x+a)^(7/6)*(-1296*b^3*d^3*x^3+1512*a*b^2*d^3*x^2-5400*b^3*c*d^2
*x^2-1638*a^2*b*d^3*x+6300*a*b^2*c*d^2*x-8550*b^3*c^2*d*x+1729*a^3*d^3-6825
*a^2*b*c*d^2+9975*a*b^2*c^2*d-6175*b^3*c^3)/(d*x+c)^(25/6)/(a^4*d^4-4*a^3*b
*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{31}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(31/6), x)
```

Fricas [B] time = 1.64057, size = 1115, normalized size = 8.2

$$\frac{6\left(1296\,b^4d^3x^4+6175\,ab^3c^3-9975\,a^2b^2c^2d+68\cdots\right.}{43225\left(b^4c^9-4\,ab^3c^8d+6\,a^2b^2c^7d^2-4\,a^3bc^6d^3+a^4c^5d^4+\left(b^4c^4d^5-4\,ab^3c^3d^6+6\,a^2b^2c^2d^7-4\,a^3bcd^8+a^4d^9\right)x^5+5\left(b^4\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="fricas")
```

```
[Out] 6/43225*(1296*b^4*d^3*x^4 + 6175*a*b^3*c^3 - 9975*a^2*b^2*c^2*d + 6825*a^3*
b*c*d^2 - 1729*a^4*d^3 + 216*(25*b^4*c*d^2 - a*b^3*d^3)*x^3 + 18*(475*b^4*c
^2*d - 50*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + (6175*b^4*c^3 - 1425*a*b^3*c^2
*d + 525*a^2*b^2*c*d^2 - 91*a^3*b*d^3)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
 + (b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4
*d^9)*x^5 + 5*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b*
c^2*d^7 + a^4*c*d^8)*x^4 + 10*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^
4*d^5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*x^3 + 10*(b^4*c^7*d^2 - 4*a*b^3*c^6*
d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*x^2 + 5*(b^4*c^8*d
 - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(31/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1779 $\int (a+bx)^{5/6}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=427

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\tan^{-1}(}{24\sqrt{3}b}$$

```
[Out] ((b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b*d) + ((a + b*x)^(11/6)*
(c + d*x)^(1/6))/(2*b) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
+ b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11
/6)) + (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqr
t[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11/6)) - (5*(b*c - a
*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(
7/6)*d^(11/6)) + (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(
c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b
^(7/6)*d^(11/6)) - (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144
*b^(7/6)*d^(11/6))
```

Rubi [A] time = 0.642347, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 331, 296, 634, 618, 204, 628, 208}

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\tan^{-1}(}{24\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)*(c + d*x)^(1/6),x]
```


```
[Out] ((b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b*d) + ((a + b*x)^(11/6)*
(c + d*x)^(1/6))/(2*b) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
+ b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11
/6)) + (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqr
t[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11/6)) - (5*(b*c - a
*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(
7/6)*d^(11/6)) + (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(
c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b
^(7/6)*d^(11/6)) - (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144
*b^(7/6)*d^(11/6))
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (a+bx)^{5/6}\sqrt[6]{c+dx}\,dx = \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}dx}{12b}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{\left(5(bc-ad)^2\right)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{5/6}}dx}{72bd}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}} \ldots\right.}{12b^2d}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}dx, x, \ldots\right.}{12b^2d}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}\ldots}\right.}{36b^{7/6}d^{5/3}}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{7/6}d^{11/6}} + \frac{(5(\ldots}{\ldots}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{7/6}d^{11/6}} + \frac{5(b\ldots}{\ldots}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{5(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{24\sqrt{3}b^{7/6}d^{11/6}} + \frac{5(b\ldots}{\ldots}$$

Mathematica [C] time = 0.0326985, size = 73, normalized size = 0.17

$$\frac{6(a+bx)^{11/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)*(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 11/6, 17/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(11*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{6}}\sqrt[6]{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)*(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(5/6)*(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(1/6), x)
```

Fricas [B] time = 3.30471, size = 12593, normalized size = 29.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] 1/144*(20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^
6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3
*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/
6)*arctan(-1/3*(2*sqrt(3)*(b^8*c^2*d^9 - 2*a*b^7*c*d^10 + a^2*b^6*d^11)*(b*
x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^1
1))^(5/6) - 2*sqrt(3)*(b^7*d^9*x + a*b^6*d^9)*sqrt(((b^3*c^2*d^2 - 2*a*b^2*
c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)/(b^7*d^11))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^4*x
 + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/3))/(b*x
 + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(5/6) + sqrt(3)*(a
*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^
3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a
^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c
^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66
*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b
^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^
8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*
b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*
a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c
^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 +
 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^
12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d
^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*
```

a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) + 20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^8*c^2*d^9 - 2*a*b^7*c*d^10 + a^2*b^6*d^11)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(5/6) - 2*sqrt(3)*(b^7*d^9*x + a*b^6*d^9)*sqrt(-((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^3*d^4*x + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/3))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(5/6) - sqrt(3)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) - 5*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6)*log(25*((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^4*x + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/3))/(b*x + a)) + 5*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6)*log(-25*((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b

```
^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^
10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6) - (b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x +
c)^(1/3) - (b^3*d^4*x + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*
b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7
*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b
^7*d^11))^(1/3))/(b*x + a)) - 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/
(b^7*d^11))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(5/6)*(d
*x + c)^(1/6) + (b^2*d^2*x + a*b*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a
^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*
c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)
/(b^7*d^11))^(1/6))/(b*x + a)) + 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^
7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^1
2)/(b^7*d^11))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(5/6)
*(d*x + c)^(1/6) - (b^2*d^2*x + a*b*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(b^7*d^11))^(1/6))/(b*x + a)) + 12*(6*b*d*x + b*c + 5*a*d)*(b*x + a)^(5
/6)*(d*x + c)^(1/6))/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^{\frac{5}{6}}\sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)*(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(5/6)*(c + d*x)**(1/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1780 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=378

$$\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}d^{11/6}}-\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}d^{11/6}}-\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{2\sqrt{3}\sqrt[6]{b}d}$$

```
[Out] ((a + b*x)^(5/6)*(c + d*x)^(1/6))/d - (5*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(
1/6)*d^(11/6)) + (5*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/
6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(1/6)*d^(11/6)) - (5*(
b*c - a*d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3
*b^(1/6)*d^(11/6)) + (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*
b^(1/6)*d^(11/6)) - (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b
^(1/6)*d^(11/6))
```

Rubi [A] time = 0.55679, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 331, 296, 634, 618, 204, 628, 208}

$$\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}d^{11/6}}-\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}d^{11/6}}-\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{2\sqrt{3}\sqrt[6]{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(5/6),x]
```

```
[Out] ((a + b*x)^(5/6)*(c + d*x)^(1/6))/d - (5*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(
1/6)*d^(11/6)) + (5*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/
6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(1/6)*d^(11/6)) - (5*(
b*c - a*d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3
*b^(1/6)*d^(11/6)) + (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*
b^(1/6)*d^(11/6)) - (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b
^(1/6)*d^(11/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}\,dx = \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{5/6}}\,dx}{6d}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{bd}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{bd}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3d^{5/3}} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{-\ldots}{\sqrt[3]{b}-\sqrt[6]{b}\ldots}\ldots\right)}{3\sqrt[6]{b}d^{5/\ldots}}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}d^{11/6}} + \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \ldots\right)}{12\sqrt[6]{b}d^{11/6}}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}d^{11/6}} + \frac{5(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\ldots\right)}{12\sqrt[6]{b}d^{11/6}}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\sqrt[6]{b}d^{11/6}} + \frac{5(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\sqrt[6]{b}d^{11/6}} - \frac{5(bc-ad)\ldots}{\ldots}$$

Mathematica [C] time = 0.0289052, size = 73, normalized size = 0.19

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6
, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(5/6))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(5/6), x)
```

Fricas [B] time = 2.82932, size = 6407, normalized size = 16.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] -1/12*(20*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3
*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6
)*arctan(1/3*(2*sqrt(3)*(b^2*c*d^9 - a*b*d^10)*(b*x + a)^(5/6)*(d*x + c)^(1
/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + 2*sqrt(3)*(b
^2*d^9*x + a*b*d^9)*sqrt(((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)
*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a
^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) + (b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^4*x + a*d^4)*((b^6*
c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*
c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a))*((b^6*c^6 -
6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^
4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + sqrt(3)*(a*b^6*c^6 - 6*a^2*b
^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6
*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20
*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a
*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a
^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^
2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 +
 a^6*b*d^6)*x)) + 20*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d
^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*
d^11))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^2*c*d^9 - a*b*d^10)*(b*x + a)^(5/6)*(
d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*
c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + 2
*sqrt(3)*(b^2*d^9*x + a*b*d^9)*sqrt(-((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*
x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^
3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) - (b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^4*x + a
*d^4)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a))*
((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^
4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) - sqrt(3)*(a*b^6*c
^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2
*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*
c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b
*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^
3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c
```

```
^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5
*b^2*c*d^5 + a^6*b*d^6)*x)) + 5*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^
4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/
(b*d^11))^(1/6)*log(25*((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*(
(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) + (b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^4*x + a*d^4)*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a)) - 5*d*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*log(-25*((b*c*d^2 - a*d^3)
*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(
b*d^11))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^
(1/3) - (b*d^4*x + a*d^4)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11)
)^(1/3))/(b*x + a)) + 10*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11
))^(1/6)*log(-5*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b*d^2*x + a
*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6))/(b*x + a))
- 10*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*log(-5*((b*
c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b*d^2*x + a*d^2)*((b^6*c^6 - 6*
a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
- 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6))/(b*x + a)) - 12*(b*x + a)^(5/6)
*(d*x + c)^(1/6))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{5}{6}}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(5/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1781 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=334

$$-\frac{b^{5/6}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{b^{5/6}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{\sqrt{3}b^{5/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}}-$$

```
[Out] (-6*(a + b*x)^(5/6))/(5*d*(c + d*x)^(5/6)) + (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt
[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(11
/6) - (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt
[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(11/6) + (2*b^(5/6)*ArcTanh[(d^(1/6)*(a +
b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(11/6) - (b^(5/6)*Log[b^(1/3) + (
d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6)
)/(c + d*x)^(1/6)])/(2*d^(11/6)) + (b^(5/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x
)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6
)])/(2*d^(11/6))
```

Rubi [A] time = 0.560891, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{b^{5/6}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{b^{5/6}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{\sqrt{3}b^{5/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(11/6),x]
```

```
[Out] (-6*(a + b*x)^(5/6))/(5*d*(c + d*x)^(5/6)) + (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt
[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(11
/6) - (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt
[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(11/6) + (2*b^(5/6)*ArcTanh[(d^(1/6)*(a +
b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(11/6) - (b^(5/6)*Log[b^(1/3) + (
d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6)
)/(c + d*x)^(1/6)])/(2*d^(11/6)) + (b^(5/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x
)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6
)])/(2*d^(11/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{11/6}}\,dx = -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{b\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{5/6}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{6\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{6\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{\left(2b^{5/6}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^{5/3}} + \frac{\left(2b^{5/6}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}+\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}+\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\right.}{d^{5/3}}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}} - \frac{b^{5/6}\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2d^{11/6}} + \frac{b^{5/6}\,\text{Subs}}{}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}} - \frac{b^{5/6}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2d^{11/6}} + \frac{b^{5/6}\log\left(\sqrt[3]{b}+\right.}{}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{\sqrt{3}b^{5/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{11/6}} - \frac{\sqrt{3}b^{5/6}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{11/6}} + \frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}}$$

Mathematica [C] time = 0.0505406, size = 73, normalized size = 0.22

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6} {}_2F_1\left(\frac{11}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(11/6)*Hypergeometric2F1[11
/6, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(11/6))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{-\frac{11}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(11/6), x)
```

Fricas [B] time = 2.18966, size = 1887, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] -1/10*(20*sqrt(3)*(d^2*x + c*d)*(b^5/d^11)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*
x + a)^(5/6)*(d*x + c)^(1/6)*b*d^9*(b^5/d^11)^(5/6) - 2*sqrt(3)*(b*d^9*x +
a*d^9)*sqrt(((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) + (b*x
+ a)^(2/3)*(d*x + c)^(1/3)*b^2 + (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x +
 a))*(b^5/d^11)^(5/6) + sqrt(3)*(b^6*x + a*b^5))/(b^6*x + a*b^5)) + 20*sqrt
(3)*(d^2*x + c*d)*(b^5/d^11)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(
d*x + c)^(1/6)*b*d^9*(b^5/d^11)^(5/6) - 2*sqrt(3)*(b*d^9*x + a*d^9)*sqrt(-(
(b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) - (b*x + a)^(2/3)*(d
*x + c)^(1/3)*b^2 - (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x + a))*(b^5/d^1
1)^(5/6) - sqrt(3)*(b^6*x + a*b^5))/(b^6*x + a*b^5)) - 5*(d^2*x + c*d)*(b^5
/d^11)^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6)
+ (b*x + a)^(2/3)*(d*x + c)^(1/3)*b^2 + (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))
/(b*x + a)) + 5*(d^2*x + c*d)*(b^5/d^11)^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x
 + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) - (b*x + a)^(2/3)*(d*x + c)^(1/3)*b^2 -
(b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x + a)) - 10*(d^2*x + c*d)*(b^5/d^11
)^(1/6)*log(((b*x + a)^(5/6)*(d*x + c)^(1/6)*b + (b*d^2*x + a*d^2)*(b^5/d^1
1)^(1/6))/(b*x + a)) + 10*(d^2*x + c*d)*(b^5/d^11)^(1/6)*log(((b*x + a)^(5/
6)*(d*x + c)^(1/6)*b - (b*d^2*x + a*d^2)*(b^5/d^11)^(1/6))/(b*x + a)) + 12*
(b*x + a)^(5/6)*(d*x + c)^(1/6))/(d^2*x + c*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(11/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(11/6), x)
```

3.1782 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{11/6}}{11(c+dx)^{11/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(11/6))/(11*(b*c - a*d)*(c + d*x)^(11/6))
```

Rubi [A] time = 0.0032335, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6(a+bx)^{11/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(17/6),x]

[Out] (6*(a + b*x)^(11/6))/(11*(b*c - a*d)*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}}\,dx = \frac{6(a+bx)^{11/6}}{11(bc-ad)(c+dx)^{11/6}}$$

Mathematica [A] time = 0.0107152, size = 32, normalized size = 1.

$$\frac{6(a+bx)^{11/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(17/6),x]

[Out] (6*(a + b*x)^(11/6))/(11*(b*c - a*d)*(c + d*x)^(11/6))
```

Maple [A] time = 0.005, size = 27, normalized size = 0.8

$$-\frac{6}{11\,ad-11\,bc}(bx+a)^{\frac{11}{6}}(dx+c)^{-\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(17/6),x)
```

```
[Out] -6/11*(b*x+a)^(11/6)/(d*x+c)^(11/6)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(17/6), x)
```

Fricas [B] time = 1.81663, size = 144, normalized size = 4.5

$$\frac{6\,(bx+a)^{\frac{11}{6}}(dx+c)^{\frac{1}{6}}}{11\left(bc^3-ac^2d+\left(bcd^2-ad^3\right)x^2+2\left(bc^2d-acd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] 6/11*(b*x + a)^(11/6)*(d*x + c)^(1/6)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*
x^2 + 2*(b*c^2*d - a*c*d^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(17/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(17/6), x)
```

3.1783 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}} dx$

Optimal. Leaf size=66

$$\frac{36b(a+bx)^{11/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{17(c+dx)^{17/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(11/6))/(17*(b*c - a*d)*(c + d*x)^(17/6)) + (36*b*(a + b*x)^(1
1/6))/(187*(b*c - a*d)^2*(c + d*x)^(11/6))
```

Rubi [A] time = 0.0090605, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b(a+bx)^{11/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{17(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(23/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(17*(b*c - a*d)*(c + d*x)^(17/6)) + (36*b*(a + b*x)^(1
1/6))/(187*(b*c - a*d)^2*(c + d*x)^(11/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}} dx &= \frac{6(a+bx)^{11/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{(6b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}} dx}{17(bc-ad)}\\ &= \frac{6(a+bx)^{11/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{36b(a+bx)^{11/6}}{187(bc-ad)^2(c+dx)^{11/6}}\end{aligned}$$

Mathematica [A] time = 0.0229957, size = 46, normalized size = 0.7

$$\frac{6(a+bx)^{11/6}(-11ad+17bc+6bdx)}{187(c+dx)^{17/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(23/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(17*b*c - 11*a*d + 6*b*d*x))/(187*(b*c - a*d)^2*(c + d*
x)^(17/6))
```

Maple [A] time = 0.003, size = 54, normalized size = 0.8

$$-\frac{-36\,bdx+66\,ad-102\,bc}{187\,a^2d^2-374\,abcd+187\,b^2c^2}\,(bx+a)^{\frac{11}{6}}\,(dx+c)^{-\frac{17}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(23/6),x)
```

```
[Out] -6/187*(b*x+a)^(11/6)*(-6*b*d*x+11*a*d-17*b*c)/(d*x+c)^(17/6)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(23/6), x)
```

Fricas [B] time = 1.83813, size = 378, normalized size = 5.73

$$\frac{6\left(6\,b^2dx^2+17\,abc-11\,a^2d+\left(17\,b^2c-5\,abd\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{187\left(b^2c^5-2\,abc^4d+a^2c^3d^2+\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)x^3+3\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2+3\left(b^2c^4d-2\,abc^3d^2+a^2c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] 6/187*(6*b^2*d*x^2 + 17*a*b*c - 11*a^2*d + (17*b^2*c - 5*a*b*d)*x)*(b*x + a
)^(5/6)*(d*x + c)^(1/6)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^2*d^3
 - 2*a*b*c*d^4 + a^2*d^5)*x^3 + 3*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)
*x^2 + 3*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(23/6), x)
```

3.1784 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}}\,dx$

Optimal. Leaf size=101

$$\frac{432b^2(a+bx)^{11/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{72b(a+bx)^{11/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{23(c+dx)^{23/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(11/6))/(23*(b*c - a*d)*(c + d*x)^(23/6)) + (72*b*(a + b*x)^(1
1/6))/(391*(b*c - a*d)^2*(c + d*x)^(17/6)) + (432*b^2*(a + b*x)^(11/6))/(43
01*(b*c - a*d)^3*(c + d*x)^(11/6))
```

Rubi [A] time = 0.0197437, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2(a+bx)^{11/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{72b(a+bx)^{11/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{23(c+dx)^{23/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(29/6),x]
```



```
[Out] (6*(a + b*x)^(11/6))/(23*(b*c - a*d)*(c + d*x)^(23/6)) + (72*b*(a + b*x)^(1
1/6))/(391*(b*c - a*d)^2*(c + d*x)^(17/6)) + (432*b^2*(a + b*x)^(11/6))/(43
01*(b*c - a*d)^3*(c + d*x)^(11/6))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}}\,dx &= \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{(12b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}}\,dx}{23(bc-ad)}\\
&= \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{72b(a+bx)^{11/6}}{391(bc-ad)^2(c+dx)^{17/6}}+\frac{\left(72b^2\right)\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}}\,dx}{391(bc-ad)^2}\\
&= \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{72b(a+bx)^{11/6}}{391(bc-ad)^2(c+dx)^{17/6}}+\frac{432b^2(a+bx)^{11/6}}{4301(bc-ad)^3(c+dx)^{11/6}}
\end{aligned}$$

Mathematica [A] time = 0.0431869, size = 77, normalized size = 0.76

$$\frac{6(a+bx)^{11/6}\left(187a^2d^2-22abd(23c+6dx)+b^2\left(391c^2+276cdx+72d^2x^2\right)\right)}{4301(c+dx)^{23/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(29/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(187*a^2*d^2 - 22*a*b*d*(23*c + 6*d*x) + b^2*(391*c^2 +
 276*c*d*x + 72*d^2*x^2)))/(4301*(b*c - a*d)^3*(c + d*x)^(23/6))
```

Maple [A] time = 0.005, size = 105, normalized size = 1.

$$-\frac{432\,b^2d^2x^2-792\,abd^2x+1656\,b^2cdx+1122\,a^2d^2-3036\,abcd+2346\,b^2c^2}{4301\,a^3d^3-12903\,a^2cbd^2+12903\,ab^2c^2d-4301\,b^3c^3}(bx+a)^{\frac{11}{6}}(dx+c)^{-\frac{23}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(29/6),x)
```

```
[Out] -6/4301*(b*x+a)^(11/6)*(72*b^2*d^2*x^2-132*a*b*d^2*x+276*b^2*c*d*x+187*a^2*
d^2-506*a*b*c*d+391*b^2*c^2)/(d*x+c)^(23/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*
c^2*d-b^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{29}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(29/6), x)
```

Fricas [B] time = 1.90572, size = 702, normalized size = 6.95

$$\frac{6\left(72\,b^3d^2x^3+391\,ab^2c^2-506\,a^2bcd+187\,a^3d^2+12\left(23\,b^3cd-5\,ab^2\right.\right.}{4301\left(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3+\left(b^3c^3d^4-3\,ab^2c^2d^5+3\,a^2bcd^6-a^3d^7\right)x^4+4\left(b^3c^4d^3-3\,ab^2c^3d^4+3\,a^2b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="fricas")
```

```
[Out] 6/4301*(72*b^3*d^2*x^3 + 391*a*b^2*c^2 - 506*a^2*b*c*d + 187*a^3*d^2 + 12*(
23*b^3*c*d - 5*a*b^2*d^2)*x^2 + (391*b^3*c^2 - 230*a*b^2*c*d + 55*a^2*b*d^2
)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5
```

```
*d^2 - a^3*c^4*d^3 + (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d
^7)*x^4 + 4*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*x
^3 + 6*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^2
+ 4*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(29/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{29}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(29/6), x)
```

3.1785 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{35/6}}\,dx$

Optimal. Leaf size=136

$$\frac{7776b^3(a+bx)^{11/6}}{124729(c+dx)^{11/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{11/6}}{11339(c+dx)^{17/6}(bc-ad)^3}+\frac{108b(a+bx)^{11/6}}{667(c+dx)^{23/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{29(c+dx)^{29/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(11/6))/(29*(b*c - a*d)*(c + d*x)^(29/6)) + (108*b*(a + b*x)^(
11/6))/(667*(b*c - a*d)^2*(c + d*x)^(23/6)) + (1296*b^2*(a + b*x)^(11/6))/(
11339*(b*c - a*d)^3*(c + d*x)^(17/6)) + (7776*b^3*(a + b*x)^(11/6))/(124729
*(b*c - a*d)^4*(c + d*x)^(11/6))
```

Rubi [A] time = 0.0334923, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3(a+bx)^{11/6}}{124729(c+dx)^{11/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{11/6}}{11339(c+dx)^{17/6}(bc-ad)^3}+\frac{108b(a+bx)^{11/6}}{667(c+dx)^{23/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{29(c+dx)^{29/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(35/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(29*(b*c - a*d)*(c + d*x)^(29/6)) + (108*b*(a + b*x)^(
11/6))/(667*(b*c - a*d)^2*(c + d*x)^(23/6)) + (1296*b^2*(a + b*x)^(11/6))/(
11339*(b*c - a*d)^3*(c + d*x)^(17/6)) + (7776*b^3*(a + b*x)^(11/6))/(124729
*(b*c - a*d)^4*(c + d*x)^(11/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{35/6}}\,dx = \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{(18b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}}\,dx}{29(bc-ad)}$$
$$= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}} + \frac{(216b^2)\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}}\,dx}{667(bc-ad)^2}$$
$$= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}} + \frac{1296b^2(a+bx)^{11/6}}{11339(bc-ad)^3(c+dx)^{17/6}} + \frac{(1296b^3)\int \frac{(a+l}{(c+d}}{11339(bc-}$$
$$= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}} + \frac{1296b^2(a+bx)^{11/6}}{11339(bc-ad)^3(c+dx)^{17/6}} + \frac{7776b^3(a}{124729(bc-ad}$$

Mathematica [A] time = 0.0604629, size = 118, normalized size = 0.87

$$\frac{6(a+bx)^{11/6}\left(561a^2bd^2(29c+6dx)-4301a^3d^3-33ab^2d\left(667c^2+348cdx+72d^2x^2\right)+b^3\left(12006c^2dx+11339c^3+6264c\right.\right.}{124729(c+dx)^{29/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(35/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(-4301*a^3*d^3 + 561*a^2*b*d^2*(29*c + 6*d*x) - 33*a*b^
2*d*(667*c^2 + 348*c*d*x + 72*d^2*x^2) + b^3*(11339*c^3 + 12006*c^2*d*x + 6
264*c*d^2*x^2 + 1296*d^3*x^3)))/(124729*(b*c - a*d)^4*(c + d*x)^(29/6))
```

Maple [A] time = 0.008, size = 171, normalized size = 1.3

$$-\frac{-7776\,x^3b^3d^3+14256\,ab^2d^3x^2-37584\,b^3cd^2x^2-20196\,a^2bd^3x+68904\,ab^2cd^2x-72036\,b^3c^2dx+25806\,a^3d^3-97614}{124729\,a^4d^4-498916\,a^3bcd^3+748374\,a^2c^2b^2d^2-498916\,b^3dc^3a+124729\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(35/6),x)
```

```
[Out] -6/124729*(b*x+a)^(11/6)*(-1296*b^3*d^3*x^3+2376*a*b^2*d^3*x^2-6264*b^3*c*d
^2*x^2-3366*a^2*b*d^3*x+11484*a*b^2*c*d^2*x-12006*b^3*c^2*d*x+4301*a^3*d^3-
16269*a^2*b*c*d^2+22011*a*b^2*c^2*d-11339*b^3*c^3)/(d*x+c)^(29/6)/(a^4*d^4-
4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{35}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(35/6), x)
```

Fricas [B] time = 1.90939, size = 1131, normalized size = 8.32

$$\frac{6\left(1296\,b^4d^3x^4+11339\,ab^3c^3-22011\,a^2b^2c^2d+162\right.}{124729\left(b^4c^9-4\,ab^3c^8d+6\,a^2b^2c^7d^2-4\,a^3bc^6d^3+a^4c^5d^4+\left(b^4c^4d^5-4\,ab^3c^3d^6+6\,a^2b^2c^2d^7-4\,a^3bcd^8+a^4d^9\right)x^5+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="fricas")
```

```
[Out] 6/124729*(1296*b^4*d^3*x^4 + 11339*a*b^3*c^3 - 22011*a^2*b^2*c^2*d + 16269*
a^3*b*c*d^2 - 4301*a^4*d^3 + 216*(29*b^4*c*d^2 - 5*a*b^3*d^3)*x^3 + 18*(667
*b^4*c^2*d - 290*a*b^3*c*d^2 + 55*a^2*b^2*d^3)*x^2 + (11339*b^4*c^3 - 10005
*a*b^3*c^2*d + 4785*a^2*b^2*c*d^2 - 935*a^3*b*d^3)*x)*(b*x + a)^(5/6)*(d*x
+ c)^(1/6)/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 +
 a^4*c^5*d^4 + (b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b
*c*d^8 + a^4*d^9)*x^5 + 5*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^
6 - 4*a^3*b*c^2*d^7 + a^4*c*d^8)*x^4 + 10*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 +
6*a^2*b^2*c^4*d^5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*x^3 + 10*(b^4*c^7*d^2 -
4*a*b^3*c^6*d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*x^2 +
5*(b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*
c^4*d^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(35/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{35}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(35/6), x)
```

3.1786 $\int (a+bx)^{5/6}(c+dx)^{11/6}\,dx$

Optimal. Leaf size=82

$$\frac{6(a+bx)^{11/6}(c+dx)^{5/6}(bc-ad)\,{}_2F_1\left(-\frac{11}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] (6*(b*c - a*d)*(a + b*x)^(11/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-11/6, 11
/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*b^2*((b*(c + d*x))/(b*c - a*d)
)^(5/6))
```

Rubi [A] time = 0.0262025, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{11/6}(c+dx)^{5/6}(bc-ad)\,{}_2F_1\left(-\frac{11}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)*(c + d*x)^(11/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(11/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-11/6, 11
/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*b^2*((b*(c + d*x))/(b*c - a*d)
)^(5/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{5/6}(c+dx)^{11/6}\,dx &= \frac{\left((bc-ad)(c+dx)^{5/6}\right)\int (a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}\,dx}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}\\ &= \frac{6(bc-ad)(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.0583176, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{11/6}(c+dx)^{11/6}\,{}_2F_1\left(-\frac{11}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)*(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(c + d*x)^(11/6)*Hypergeometric2F1[-11/6, 11/6, 17/6, (
d*(a + b*x))/(-(b*c) + a*d)])/(11*b*((b*(c + d*x))/(b*c - a*d))^(11/6))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{11}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)*(d*x+c)^(11/6),x)
```

```
[Out] int((b*x+a)^(5/6)*(d*x+c)^(11/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{11}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(11/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{11}{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(11/6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)*(d*x+c)**(11/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(11/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1787 $\int (a+bx)^{5/6}(c+dx)^{5/6}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] (6*(a + b*x)^(11/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 11/6, 17/6, -((
d*(a + b*x))/(b*c - a*d))])/(11*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Rubi [A] time = 0.0205306, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)*(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 11/6, 17/6, -((
d*(a + b*x))/(b*c - a*d))])/(11*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int (a+bx)^{5/6}(c+dx)^{5/6}\,dx &= \frac{(c+dx)^{5/6}\int (a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}dx}{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}\\
&= \frac{6(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}
\end{aligned}$$

Mathematica [A] time = 0.0274251, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)*(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 11/6, 17/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(11*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)*(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(5/6)*(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)*(d*x+c)**(5/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1788 $\int \frac{(a+bx)^{5/6}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\sqrt[6]{c+dx}}$$

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6
, 11/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*b*(c + d*x)^(1/6))
```

Rubi [A] time = 0.0198072, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6
, 11/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*b*(c + d*x)^(1/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{5/6}}{\sqrt[6]{c+dx}}\,dx &= \frac{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\int \frac{(a+bx)^{5/6}}{\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[6]{c+dx}}\\ &= \frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\sqrt[6]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.026051, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6
, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(1/6))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{6}}\frac{1}{\sqrt[6]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{1}{6}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)/(d*x + c)^(1/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^{\frac{5}{6}}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(1/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1789 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=81

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11\sqrt[6]{c+dx}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[7/6
, 11/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*(b*c - a*d)*(c + d*x)^(1/6
))
```

Rubi [A] time = 0.0203077, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11\sqrt[6]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[7/6
, 11/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*(b*c - a*d)*(c + d*x)^(1/6
))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{5/6}}{(c+dx)^{7/6}}\,dx &= \frac{\left(b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{(a+bx)^{5/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}}\,dx}{(bc-ad)\sqrt[6]{c+dx}}\\ &= \frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)\sqrt[6]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0402007, size = 73, normalized size = 0.9

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6
, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(7/6))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(7/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(7/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(7/6), x)
```

3.1790 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6b(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11\sqrt[6]{c+dx}(bc-ad)^2}$$

```
[Out] (6*b*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1
1/6, 13/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*(b*c - a*d)^2*(c + d*x)
^(1/6))
```

Rubi [A] time = 0.0215203, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6b(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11\sqrt[6]{c+dx}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(13/6),x]
```


```
[Out] (6*b*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1
1/6, 13/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*(b*c - a*d)^2*(c + d*x)
^(1/6))
```


Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{5/6}}{(c+dx)^{13/6}}\,dx &= \frac{\left(b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{(a+bx)^{5/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}}\,dx}{(bc-ad)^2\sqrt[6]{c+dx}}\\ &= \frac{6b(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)^2\sqrt[6]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0552378, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(13/6)*Hypergeometric2F1[11
/6, 13/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(13/6))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{-\frac{13}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(13/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(13/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(13/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(13/6), x)
```

3.1791 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{19/6}} dx$

Optimal. Leaf size=84

$$\frac{6b^2(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{11}{6},\frac{19}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11\sqrt[6]{c+dx}(bc-ad)^3}$$

```
[Out] (6*b^2*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1
[11/6, 19/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*(b*c - a*d)^3*(c + d*
x)^(1/6))
```

Rubi [A] time = 0.0219392, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6b^2(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{11}{6},\frac{19}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11\sqrt[6]{c+dx}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*b^2*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1
[11/6, 19/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*(b*c - a*d)^3*(c + d*
x)^(1/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{5/6}}{(c+dx)^{19/6}}\,dx &= \frac{\left(b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{(a+bx)^{5/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{19/6}}\,dx}{(bc-ad)^3\sqrt[6]{c+dx}}\\ &= \frac{6b^2(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{11}{6},\frac{19}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)^3\sqrt[6]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0387846, size = 81, normalized size = 0.96

$$\frac{6b(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{11}{6},\frac{19}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11(c+dx)^{7/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*b*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[1
1/6, 19/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*(b*c - a*d)^2*(c + d*x)
^(7/6))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{-\frac{19}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(19/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(19/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(19/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}{d^4x^4+4\,cd^3x^3+6\,c^2d^2x^2+4\,c^3dx+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2
*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(19/6), x)
```

3.1792 $\int (a+bx)^{7/6}(c+dx)^{13/6}\,dx$

Optimal. Leaf size=84

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}(bc-ad)^2\,{}_2F_1\left(-\frac{13}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] (6*(b*c - a*d)^2*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-13/6,
13/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*b^3*((b*(c + d*x))/(b*c - a*
d))^(1/6))
```

Rubi [A] time = 0.02099, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}(bc-ad)^2\,{}_2F_1\left(-\frac{13}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)*(c + d*x)^(13/6),x]
```

```
[Out] (6*(b*c - a*d)^2*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-13/6,
13/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*b^3*((b*(c + d*x))/(b*c - a*
d))^(1/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{7/6}(c+dx)^{13/6}\,dx &= \frac{\left((bc-ad)^2\sqrt[6]{c+dx}\right)\int (a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}dx}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\\ &= \frac{6(bc-ad)^2(a+bx)^{13/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{13}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\end{aligned}$$

Mathematica [A] time = 0.0859782, size = 73, normalized size = 0.87

$$\frac{6(a+bx)^{13/6}(c+dx)^{13/6}\,{}_2F_1\left(-\frac{13}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)*(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(c + d*x)^(13/6)*Hypergeometric2F1[-13/6, 13/6, 19/6, (
d*(a + b*x))/(-(b*c) + a*d)])/(13*b*((b*(c + d*x))/(b*c - a*d))^(13/6))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{\frac{13}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)*(d*x+c)^(13/6),x)
```

```
[Out] int((b*x+a)^(7/6)*(d*x+c)^(13/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{\frac{13}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)*(d*x + c)^(13/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bd^2x^3+ac^2+\left(2\,bcd+ad^2\right)x^2+\left(bc^2+2\,acd\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{1}{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] integral((b*d^2*x^3 + a*c^2 + (2*b*c*d + a*d^2)*x^2 + (b*c^2 + 2*a*c*d)*x)*
(b*x + a)^(1/6)*(d*x + c)^(1/6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)*(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1793 $\int (a+bx)^{7/6}(c+dx)^{7/6}\,dx$

Optimal. Leaf size=82

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}(bc-ad)\,{}_2F_1\left(-\frac{7}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] (6*(b*c - a*d)*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-7/6, 13/
6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*b^2*((b*(c + d*x))/(b*c - a*d))
^(1/6))
```

Rubi [A] time = 0.0222514, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}(bc-ad)\,{}_2F_1\left(-\frac{7}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)*(c + d*x)^(7/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-7/6, 13/
6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*b^2*((b*(c + d*x))/(b*c - a*d))
^(1/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{7/6}(c+dx)^{7/6}\,dx &= \frac{\left((bc-ad)\sqrt[6]{c+dx}\right)\int (a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}dx}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\\ &= \frac{6(bc-ad)(a+bx)^{13/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{7}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\end{aligned}$$

Mathematica [A] time = 0.0566748, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{13/6}(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)*(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, 13/6, 19/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(13*b*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)*(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(7/6)*(d*x+c)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)*(d*x + c)^(7/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bdx^2+ac+(bc+ad)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{1}{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(1/6)*(d*x + c)^(1/6), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)*(d*x+c)**(7/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1794 $\int (a+bx)^{7/6}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] (6*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 13/6, 19/6, -((
d*(a + b*x))/(b*c - a*d))])/(13*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Rubi [A] time = 0.0196854, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)*(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 13/6, 19/6, -((
d*(a + b*x))/(b*c - a*d))])/(13*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{7/6}\sqrt[6]{c+dx}\,dx &= \frac{\sqrt[6]{c+dx}\int (a+bx)^{7/6}\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\,dx}{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\\ &= \frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\end{aligned}$$

Mathematica [A] time = 0.0300639, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)*(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 13/6, 19/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(13*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{6}}\sqrt[6]{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)*(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(7/6)*(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)*(d*x + c)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx+a)^{\frac{7}{6}}(dx+c)^{\frac{1}{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(7/6)*(d*x + c)^(1/6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)*(d*x+c)**(1/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1795 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b(c+dx)^{5/6}}$$

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6
, 13/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*b*(c + d*x)^(5/6))
```

Rubi [A] time = 0.0184608, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6
, 13/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*b*(c + d*x)^(5/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{7/6}}{(c+dx)^{5/6}}\,dx &= \frac{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\int \frac{(a+bx)^{7/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}\,dx}{(c+dx)^{5/6}}\\ &= \frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b(c+dx)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.0307045, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6
, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(5/6))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{5}{6}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(7/6)/(d*x + c)^(5/6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(5/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1796 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=81

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(c+dx)^{5/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[11/
6, 13/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*(b*c - a*d)*(c + d*x)^(5/
6))
```

Rubi [A] time = 0.0193788, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[11/
6, 13/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*(b*c - a*d)*(c + d*x)^(5/
6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{7/6}}{(c+dx)^{11/6}}\,dx &= \frac{\left(b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{(a+bx)^{7/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}\,dx}{(bc-ad)(c+dx)^{5/6}}\\ &= \frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(bc-ad)(c+dx)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.0503754, size = 73, normalized size = 0.9

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(11/6)*Hypergeometric2F1[11
/6, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(11/6))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{-\frac{11}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(11/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(11/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(11/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{1}{6}}}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(7/6)*(d*x + c)^(1/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(11/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1797 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6b(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{13}{6},\frac{17}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(c+dx)^{5/6}(bc-ad)^2}$$

```
[Out] (6*b*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1
3/6, 17/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*(b*c - a*d)^2*(c + d*x)
^(5/6))
```

Rubi [A] time = 0.0201316, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6b(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{13}{6},\frac{17}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*b*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1
3/6, 17/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*(b*c - a*d)^2*(c + d*x)
^(5/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{7/6}}{(c+dx)^{17/6}}\,dx &= \frac{\left(b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{(a+bx)^{7/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{17/6}}\,dx}{(bc-ad)^2(c+dx)^{5/6}}\\ &= \frac{6b(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{13}{6},\frac{17}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(bc-ad)^2(c+dx)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.0347163, size = 81, normalized size = 0.99

$$\frac{6b(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{13}{6},\frac{17}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*b*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1
3/6, 17/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*(b*c - a*d)^2*(c + d*x)
^(5/6))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{-\frac{17}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(17/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(17/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(17/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{1}{6}}}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(7/6)*(d*x + c)^(1/6)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(17/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1798 $\int \frac{(a+bx)^{7/6}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=424

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}-\frac{7(bc-ad)^2\tan^{-1}\left(\cdots\right.}{24\sqrt{3}b^{5/6}\cdots}$$

```
[Out] (-7*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*d^2) + ((a + b*x)^(7/6
)*(c + d*x)^(5/6))/(2*d) - (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(
a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(
13/6)) + (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(S
qrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(13/6)) + (7*(b*c -
 a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b
^(5/6)*d^(13/6)) - (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144
*b^(5/6)*d^(13/6)) + (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(1
44*b^(5/6)*d^(13/6))
```

Rubi [A] time = 0.552235, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}-\frac{7(bc-ad)^2\tan^{-1}\left(\cdots\right.}{24\sqrt{3}b^{5/6}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(1/6),x]
```


```
[Out] (-7*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*d^2) + ((a + b*x)^(7/6
)*(c + d*x)^(5/6))/(2*d) - (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(
a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(
13/6)) + (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(S
qrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(13/6)) + (7*(b*c -
 a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b
^(5/6)*d^(13/6)) - (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144
*b^(5/6)*d^(13/6)) + (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(1
44*b^(5/6)*d^(13/6))
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{\sqrt[6]{c+dx}}\,dx = \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} - \frac{(7(bc-ad))\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}dx}{12d}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{\left(7(bc-ad)^2\right)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}dx}{72d^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}dx, x, \sqrt[6]{a\cdots}\right.}{12bd^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{12bd^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}dx, \cdots\right.}{36b^{5/6}d^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{5/6}d^{13/6}} - \frac{\left(7(bc-a\cdots\right.}{\cdots}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{5/6}d^{13/6}} - \frac{7(bc-ad\cdots}{\cdots}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} - \frac{7(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{24\sqrt{3}b^{5/6}d^{13/6}} + \frac{7(bc-ad\cdots}{2\cdots}$$

Mathematica [C] time = 0.0281494, size = 73, normalized size = 0.17

$$\frac{6(a+bx)^{13/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6
, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(1/6))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{6}}\frac{1}{\sqrt[6]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(1/6), x)
```

Fricas [B] time = 4.27837, size = 12598, normalized size = 29.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] -1/144*(28*sqrt(3)*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1
/6)*arctan(-1/3*(2*sqrt(3)*(b^6*c^2*d^11 - 2*a*b^5*c*d^12 + a^2*b^4*d^13)*(
b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10
*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^
9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d
^13))^(5/6) - 2*sqrt(3)*(b^4*d^12*x + b^4*c*d^11)*sqrt(((b^3*c^2*d^2 - 2*a*
b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(b^5*d^13))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d
^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/3))/
(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*
b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(5/6) + sqrt(3
)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d
^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*
a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c
^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*
d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 79
2*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4
*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 +
 a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*
a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*
c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 +
 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 1
```

2*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) + 28*sqrt(3)*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^6*c^2*d^11 - 2*a*b^5*c*d^12 + a^2*b^4*d^13)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(5/6) - 2*sqrt(3)*(b^4*d^12*x + b^4*c*d^11)*sqrt(-((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^2*d^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) - 7*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*log(49*((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/3))/(d*x + c)) + 7*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +

```
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*log(
-49*((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5
/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9
*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 79
2*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2
*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6) - (b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*
(d*x + c)^(2/3) - (b^2*d^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*
b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d
^12)/(b^5*d^13))^(1/3))/(d*x + c)) - 14*d^2*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^5*d^13))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(
1/6)*(d*x + c)^(5/6) + (b*d^3*x + b*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^5*d^13))^(1/6))/(d*x + c)) + 14*d^2*((b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(b^5*d^13))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^
(1/6)*(d*x + c)^(5/6) - (b*d^3*x + b*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^5*d^13))^(1/6))/(d*x + c)) - 12*(6*b*d*x - 7*b*c + 13*a*d)*(b*x +
 a)^(1/6)*(d*x + c)^(5/6))/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(1/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1799 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{7/6}} dx$

Optimal. Leaf size=403

$$\frac{7b\sqrt[6]{a+bx}(c+dx)^{5/6}}{d^2} + \frac{7\sqrt[6]{b}(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}} + \frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}} + \sqrt[3]{b}\right)}{12d^{13/6}} - \frac{7\sqrt[6]{b}(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}} + \frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}} + \right.}{12d^{13/6}}$$

```
[Out] (-6*(a + b*x)^(7/6))/(d*(c + d*x)^(1/6)) + (7*b*(a + b*x)^(1/6)*(c + d*x)^(
5/6))/d^2 + (7*b^(1/6)*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^
(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]])/(2*Sqrt[3]*d^(13/6)) - (7*b^(1/6
)*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/
6)*(c + d*x)^(1/6))]])/(2*Sqrt[3]*d^(13/6)) - (7*b^(1/6)*(b*c - a*d)*ArcTanh
[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))]])/(3*d^(13/6)) + (7*b^
(1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) -
 (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]])/(12*d^(13/6)) - (7*b^(
1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) +
(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]])/(12*d^(13/6))
```

Rubi [A] time = 0.526033, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {47, 50, 63, 240, 210, 634, 618, 204, 628, 208}

$$\frac{7b\sqrt[6]{a+bx}(c+dx)^{5/6}}{d^2} + \frac{7\sqrt[6]{b}(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}} + \frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}} + \sqrt[3]{b}\right)}{12d^{13/6}} - \frac{7\sqrt[6]{b}(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}} + \frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}} + \right.}{12d^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(7/6))/(d*(c + d*x)^(1/6)) + (7*b*(a + b*x)^(1/6)*(c + d*x)^(
5/6))/d^2 + (7*b^(1/6)*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^
(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]])/(2*Sqrt[3]*d^(13/6)) - (7*b^(1/6
)*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/
6)*(c + d*x)^(1/6))]])/(2*Sqrt[3]*d^(13/6)) - (7*b^(1/6)*(b*c - a*d)*ArcTanh
[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))]])/(3*d^(13/6)) + (7*b^
(1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) -
 (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]])/(12*d^(13/6)) - (7*b^(
1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) +
(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]])/(12*d^(13/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 240

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/n]

Rule 210

Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{7/6}}\,dx = -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{(7b)\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}(c+dx)^{5/6}}{d^2} - \frac{(7b(bc-ad))\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}(c+dx)^{5/6}}{d^2} - \frac{(7(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}(c+dx)^{5/6}}{d^2} - \frac{(7(bc-ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}(c+dx)^{5/6}}{d^2} - \frac{\left(7\sqrt[6]{b}(bc-ad)\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3d^2} - \cdots (7$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}(c+dx)^{5/6}}{d^2} - \frac{7\sqrt[6]{b}(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3d^{13/6}} + \frac{\left(7\sqrt[6]{b}(bc-ad)\right)\text{Subst}\left(\int \cdots\right.}{1\cdots}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}(c+dx)^{5/6}}{d^2} - \frac{7\sqrt[6]{b}(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3d^{13/6}} + \frac{7\sqrt[6]{b}(bc-ad)\log\left(\sqrt[3]{b}+\cdots\right.}{12d^{1\cdots}}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}(c+dx)^{5/6}}{d^2} + \frac{7\sqrt[6]{b}(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}d^{13/6}} - \frac{7\sqrt[6]{b}(bc-ad)\tan^{-1}\left(\frac{1+\cdots}{\cdots}\right.}{2\sqrt{3}d^{13/6}}$$

Mathematica [C] time = 0.0497565, size = 73, normalized size = 0.18

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{7}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6
, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(7/6))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(7/6), x)
```

Fricas [B] time = 3.76755, size = 6580, normalized size = 16.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/12*(28*sqrt(3)*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^
4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d
^6)/d^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b*c*d^11 - a*d^12)*(b*x + a)^(1/6)*(
d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*
c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(5/6) + 2
*sqrt(3)*(d^12*x + c*d^11)*sqrt(((b*c*d^2 - a*d^3)*(b*x + a)^(1/6)*(d*x + c
)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3
 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6) + (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (d^5*x + c*d^4)*(
(b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4
*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/3))/(d*x + c))*((b^7*c
^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c
^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(5/6) + sqrt(3)*(b^7*c^7 - 6*a*
b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 -
6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5
*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*
b*d^7)*x))/(b^7*c^7 - 6*a*b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d
^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*
a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^
5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 28*sqrt(3)*(d^3*x + c*d^2)*((b^7*c^6
 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2
*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b*c*
d^11 - a*d^12)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d +
15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*
d^5 + a^6*b*d^6)/d^13)^(5/6) + 2*sqrt(3)*(d^12*x + c*d^11)*sqrt(-((b*c*d^2
- a*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2
*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 +
a^6*b*d^6)/d^13)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d
*x + c)^(2/3) - (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*
d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6
)/d^13)^(1/3))/(d*x + c))*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 -
20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13
)^(5/6) - sqrt(3)*(b^7*c^7 - 6*a*b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^
```

```
4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6
*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3
*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x))/(b^7*c^7 - 6*a*b^6*c^6*d + 15*a
^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^
5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^
3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 7*(
d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*
c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*log
(49*((b*c*d^2 - a*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*
c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^
5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x
 + a)^(1/3)*(d*x + c)^(2/3) + (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 1
5*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d
^5 + a^6*b*d^6)/d^13)^(1/3))/(d*x + c)) - 7*(d^3*x + c*d^2)*((b^7*c^6 - 6*a
*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 -
 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*log(-49*((b*c*d^2 - a*d^3)*(b*x +
 a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 -
20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13
)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) -
 (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^
4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/3))/
(d*x + c)) + 14*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*
d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6
)/d^13)^(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) + (d^3*x
+ c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^
3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6))/(d*x + c
)) - 14*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20
*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^
(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) - (d^3*x + c*d^2)
*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a
^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6))/(d*x + c)) - 12*
(b*d*x + 7*b*c - 6*a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6))/(d^3*x + c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(7/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1800 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=358

$$-\frac{b^{7/6}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}+\frac{b^{7/6}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}-\frac{\sqrt{3}b^{7/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{13/6}}$$

```
[Out] (-6*(a + b*x)^(7/6))/(7*d*(c + d*x)^(7/6)) - (6*b*(a + b*x)^(1/6))/(d^2*(c
+ d*x)^(1/6)) - (Sqrt[3]*b^(7/6)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1
/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(13/6) + (Sqrt[3]*b^(7/6)*ArcTan
[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]
)/d^(13/6) + (2*b^(7/6)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x
)^(1/6))])/d^(13/6) - (b^(7/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c +
 d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2*d^(13/
6)) + (b^(7/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b
^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2*d^(13/6))
```

Rubi [A] time = 0.501067, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{b^{7/6}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}+\frac{b^{7/6}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}-\frac{\sqrt{3}b^{7/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(13/6),x]
```


```
[Out] (-6*(a + b*x)^(7/6))/(7*d*(c + d*x)^(7/6)) - (6*b*(a + b*x)^(1/6))/(d^2*(c
+ d*x)^(1/6)) - (Sqrt[3]*b^(7/6)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1
/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(13/6) + (Sqrt[3]*b^(7/6)*ArcTan
[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]
)/d^(13/6) + (2*b^(7/6)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x
)^(1/6))])/d^(13/6) - (b^(7/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c +
 d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2*d^(13/
6)) + (b^(7/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b
^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2*d^(13/6))
```


Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{13/6}}\,dx = -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} + \frac{b\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} + \frac{b^2\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} + \frac{(6b)\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} + \frac{(6b)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} + \frac{\left(2b^{7/6}\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^2} + \frac{\left(2b^{7/6}\right)\text{Subst}\Big(\int}{}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} + \frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{13/6}} - \frac{b^{7/6}\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \right.}{2d^{13/6}}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} + \frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{13/6}} - \frac{b^{7/6}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2d^{13/6}}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} - \frac{\sqrt{3}b^{7/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{13/6}} + \frac{\sqrt{3}b^{7/6}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{13/6}} + \frac{2b^{7/6}\,\text{t}}{}$$

Mathematica [C] time = 0.063971, size = 73, normalized size = 0.2

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}{}_2F_1\left(\frac{13}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(13/6)*Hypergeometric2F1[13
/6, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(13/6))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{-\frac{13}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(13/6), x)
```

Fricas [B] time = 2.2691, size = 2087, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] -1/14*(28*sqrt(3)*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*arctan(-
1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^11*(b^7/d^13)^(5/6) - 2*
sqrt(3)*(d^12*x + c*d^11)*sqrt(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^2*(b^7/
d^13)^(1/6) + (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 + (d^5*x + c*d^4)*(b^7/d^
13)^(1/3))/(d*x + c))*(b^7/d^13)^(5/6) + sqrt(3)*(b^7*d*x + b^7*c))/(b^7*d*
x + b^7*c)) + 28*sqrt(3)*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*a
rctan(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^11*(b^7/d^13)^(5/
6) - 2*sqrt(3)*(d^12*x + c*d^11)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d
^2*(b^7/d^13)^(1/6) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 - (d^5*x + c*d^4)
*(b^7/d^13)^(1/3))/(d*x + c))*(b^7/d^13)^(5/6) - sqrt(3)*(b^7*d*x + b^7*c))
/(b^7*d*x + b^7*c)) - 7*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*lo
g(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^2*(b^7/d^13)^(1/6) + (b*x + a)^(1/
3)*(d*x + c)^(2/3)*b^2 + (d^5*x + c*d^4)*(b^7/d^13)^(1/3))/(d*x + c)) + 7*(
d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*log(-4*((b*x + a)^(1/6)*(d*
x + c)^(5/6)*b*d^2*(b^7/d^13)^(1/6) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 -
 (d^5*x + c*d^4)*(b^7/d^13)^(1/3))/(d*x + c)) - 14*(d^4*x^2 + 2*c*d^3*x + c
^2*d^2)*(b^7/d^13)^(1/6)*log(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b + (d^3*x +
c*d^2)*(b^7/d^13)^(1/6))/(d*x + c)) + 14*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b
^7/d^13)^(1/6)*log(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b - (d^3*x + c*d^2)*(b^
7/d^13)^(1/6))/(d*x + c)) + 12*(8*b*d*x + 7*b*c + a*d)*(b*x + a)^(1/6)*(d*x
 + c)^(5/6))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1801 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{13/6}}{13(c+dx)^{13/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(13/6))/(13*(b*c - a*d)*(c + d*x)^(13/6))
```

Rubi [A] time = 0.0030103, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6(a+bx)^{13/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(13*(b*c - a*d)*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx = \frac{6(a+bx)^{13/6}}{13(bc-ad)(c+dx)^{13/6}}$$

Mathematica [A] time = 0.0129569, size = 32, normalized size = 1.

$$\frac{6(a+bx)^{13/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(13*(b*c - a*d)*(c + d*x)^(13/6))
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$-\frac{6}{13\,ad-13\,bc}\,(bx+a)^{\frac{13}{6}}\,(dx+c)^{-\frac{13}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(19/6),x)
```

```
[Out] -6/13*(b*x+a)^(13/6)/(d*x+c)^(13/6)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(19/6), x)
```

Fricas [B] time = 1.76069, size = 221, normalized size = 6.91

$$\frac{6\left(b^2x^2+2\,abx+a^2\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{13\left(bc^4-ac^3d+\left(bcd^3-ad^4\right)x^3+3\left(bc^2d^2-acd^3\right)x^2+3\left(bc^3d-ac^2d^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] 6/13*(b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b*c^4 - a*c
^3*d + (b*c*d^3 - a*d^4)*x^3 + 3*(b*c^2*d^2 - a*c*d^3)*x^2 + 3*(b*c^3*d - a
*c^2*d^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1802 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}} dx$

Optimal. Leaf size=66

$$\frac{36b(a+bx)^{13/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{19(c+dx)^{19/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(13/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (36*b*(a + b*x)^(1
3/6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6))
```

Rubi [A] time = 0.0092703, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b(a+bx)^{13/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (36*b*(a + b*x)^(1
3/6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx &= \frac{6(a+bx)^{13/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{(6b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx}{19(bc-ad)}\\ &= \frac{6(a+bx)^{13/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{36b(a+bx)^{13/6}}{247(bc-ad)^2(c+dx)^{13/6}}\end{aligned}$$

Mathematica [A] time = 0.027283, size = 46, normalized size = 0.7

$$\frac{6(a+bx)^{13/6}(-13ad+19bc+6bdx)}{247(c+dx)^{19/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(19*b*c - 13*a*d + 6*b*d*x))/(247*(b*c - a*d)^2*(c + d*
x)^(19/6))
```

Maple [A] time = 0.005, size = 54, normalized size = 0.8

$$-\frac{-36\,bdx+78\,ad-114\,bc}{247\,a^2d^2-494\,abcd+247\,b^2c^2}\,(bx+a)^{\frac{13}{6}}\,(dx+c)^{-\frac{19}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(25/6),x)
```

```
[Out] -6/247*(b*x+a)^(13/6)*(-6*b*d*x+13*a*d-19*b*c)/(d*x+c)^(19/6)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(25/6), x)
```

Fricas [B] time = 1.92627, size = 497, normalized size = 7.53

$$\frac{6\left(6\,b^3dx^3+19\,a^2bc-13\,a^3d+\left(19\,b^3c-ab^2d\right)x^2+2\left(19\,ab^2c-10\,a^2bd\right)x\right)(bx+a)}{247\left(b^2c^6-2\,abc^5d+a^2c^4d^2+\left(b^2c^2d^4-2\,abcd^5+a^2d^6\right)x^4+4\left(b^2c^3d^3-2\,abc^2d^4+a^2cd^5\right)x^3+6\left(b^2c^4d^2-2\,abc^3d^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/247*(6*b^3*d*x^3 + 19*a^2*b*c - 13*a^3*d + (19*b^3*c - a*b^2*d)*x^2 + 2*(
19*a*b^2*c - 10*a^2*b*d)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^2*c^6 - 2*a*
b*c^5*d + a^2*c^4*d^2 + (b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*x^4 + 4*(b^2*
c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*x^3 + 6*(b^2*c^4*d^2 - 2*a*b*c^3*d^3 +
 a^2*c^2*d^4)*x^2 + 4*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(25/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1803 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}} dx$

Optimal. Leaf size=101

$$\frac{432b^2(a+bx)^{13/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{72b(a+bx)^{13/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{25(c+dx)^{25/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(13/6))/(25*(b*c - a*d)*(c + d*x)^(25/6)) + (72*b*(a + b*x)^(1
3/6))/(475*(b*c - a*d)^2*(c + d*x)^(19/6)) + (432*b^2*(a + b*x)^(13/6))/(61
75*(b*c - a*d)^3*(c + d*x)^(13/6))
```

Rubi [A] time = 0.0182365, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2(a+bx)^{13/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{72b(a+bx)^{13/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{25(c+dx)^{25/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(25*(b*c - a*d)*(c + d*x)^(25/6)) + (72*b*(a + b*x)^(1
3/6))/(475*(b*c - a*d)^2*(c + d*x)^(19/6)) + (432*b^2*(a + b*x)^(13/6))/(61
75*(b*c - a*d)^3*(c + d*x)^(13/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}}\,dx &= \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{(12b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx}{25(bc-ad)}\\ &= \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{72b(a+bx)^{13/6}}{475(bc-ad)^2(c+dx)^{19/6}}+\frac{\left(72b^2\right)\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx}{475(bc-ad)^2}\\ &= \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{72b(a+bx)^{13/6}}{475(bc-ad)^2(c+dx)^{19/6}}+\frac{432b^2(a+bx)^{13/6}}{6175(bc-ad)^3(c+dx)^{13/6}}\end{aligned}$$

Mathematica [A] time = 0.0440663, size = 77, normalized size = 0.76

$$\frac{6(a+bx)^{13/6}\left(247a^2d^2-26abd(25c+6dx)+b^2\left(475c^2+300cdx+72d^2x^2\right)\right)}{6175(c+dx)^{25/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(247*a^2*d^2 - 26*a*b*d*(25*c + 6*d*x) + b^2*(475*c^2 +
 300*c*d*x + 72*d^2*x^2)))/(6175*(b*c - a*d)^3*(c + d*x)^(25/6))
```

Maple [A] time = 0.006, size = 105, normalized size = 1.

$$-\frac{432\,b^2d^2x^2-936\,abd^2x+1800\,b^2cdx+1482\,a^2d^2-3900\,abcd+2850\,b^2c^2}{6175\,a^3d^3-18525\,a^2cbd^2+18525\,ab^2c^2d-6175\,b^3c^3}(bx+a)^{\frac{13}{6}}(dx+c)^{-\frac{25}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(31/6),x)
```

```
[Out] -6/6175*(b*x+a)^(13/6)*(72*b^2*d^2*x^2-156*a*b*d^2*x+300*b^2*c*d*x+247*a^2*
d^2-650*a*b*c*d+475*b^2*c^2)/(d*x+c)^(25/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*
c^2*d-b^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{31}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(31/6), x)
```

Fricas [B] time = 1.86186, size = 880, normalized size = 8.71

$$\frac{6\left(72\,b^4d^2x^4+475\,a^2b^2c^2-650\,a^3bcd+247\,a^4d^2+12\left(25\,b^4cd-ab^3\right.\right.}{6175\left(b^3c^8-3\,ab^2c^7d+3\,a^2bc^6d^2-a^3c^5d^3+\left(b^3c^3d^5-3\,ab^2c^2d^6+3\,a^2bcd^7-a^3d^8\right)x^5+5\left(b^3c^4d^4-3\,ab^2c^3d^5+3\,a^2bc^2d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="fricas")
```

```
[Out] 6/6175*(72*b^4*d^2*x^4 + 475*a^2*b^2*c^2 - 650*a^3*b*c*d + 247*a^4*d^2 + 12
*(25*b^4*c*d - a*b^3*d^2)*x^3 + (475*b^4*c^2 - 50*a*b^3*c*d + 7*a^2*b^2*d^2
)*x^2 + 2*(475*a*b^3*c^2 - 500*a^2*b^2*c*d + 169*a^3*b*d^2)*x)*(b*x + a)^(1
```

```
/6)*(d*x + c)^(5/6)/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c^5*d^
3 + (b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8)*x^5 + 5*(b^3*
c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*x^4 + 10*(b^3*c^5*
d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^3 + 10*(b^3*c^6*d^
2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x^2 + 5*(b^3*c^7*d - 3
*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(31/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1804 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{37/6}}\,dx$

Optimal. Leaf size=136

$$\frac{7776b^3(a+bx)^{13/6}}{191425(c+dx)^{13/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{13/6}}{14725(c+dx)^{19/6}(bc-ad)^3}+\frac{108b(a+bx)^{13/6}}{775(c+dx)^{25/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{31(c+dx)^{31/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(13/6))/(31*(b*c - a*d)*(c + d*x)^(31/6)) + (108*b*(a + b*x)^(
13/6))/(775*(b*c - a*d)^2*(c + d*x)^(25/6)) + (1296*b^2*(a + b*x)^(13/6))/(
14725*(b*c - a*d)^3*(c + d*x)^(19/6)) + (7776*b^3*(a + b*x)^(13/6))/(191425
*(b*c - a*d)^4*(c + d*x)^(13/6))
```

Rubi [A] time = 0.0296088, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3(a+bx)^{13/6}}{191425(c+dx)^{13/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{13/6}}{14725(c+dx)^{19/6}(bc-ad)^3}+\frac{108b(a+bx)^{13/6}}{775(c+dx)^{25/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{31(c+dx)^{31/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(37/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(31*(b*c - a*d)*(c + d*x)^(31/6)) + (108*b*(a + b*x)^(
13/6))/(775*(b*c - a*d)^2*(c + d*x)^(25/6)) + (1296*b^2*(a + b*x)^(13/6))/(
14725*(b*c - a*d)^3*(c + d*x)^(19/6)) + (7776*b^3*(a + b*x)^(13/6))/(191425
*(b*c - a*d)^4*(c + d*x)^(13/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{37/6}}\,dx = \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{(18b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}}\,dx}{31(bc-ad)}$$
$$= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}} + \frac{(216b^2)\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx}{775(bc-ad)^2}$$
$$= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}} + \frac{1296b^2(a+bx)^{13/6}}{14725(bc-ad)^3(c+dx)^{19/6}} + \frac{(1296b^3)\int}{14725(bc}$$
$$= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}} + \frac{1296b^2(a+bx)^{13/6}}{14725(bc-ad)^3(c+dx)^{19/6}} + \frac{7776b^3}{191425(bc-}$$

Mathematica [A] time = 0.0672193, size = 118, normalized size = 0.87

$$\frac{6(a+bx)^{13/6}\left(741a^2bd^2(31c+6dx)-6175a^3d^3-39ab^2d\left(775c^2+372cdx+72d^2x^2\right)+b^3\left(13950c^2dx+14725c^3+66\right.\right.}{191425(c+dx)^{31/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(37/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(-6175*a^3*d^3 + 741*a^2*b*d^2*(31*c + 6*d*x) - 39*a*b^
2*d*(775*c^2 + 372*c*d*x + 72*d^2*x^2) + b^3*(14725*c^3 + 13950*c^2*d*x + 6
696*c*d^2*x^2 + 1296*d^3*x^3)))/(191425*(b*c - a*d)^4*(c + d*x)^(31/6))
```

Maple [A] time = 0.008, size = 171, normalized size = 1.3

$$-\frac{-7776\,x^3b^3d^3+16848\,ab^2d^3x^2-40176\,b^3cd^2x^2-26676\,a^2bd^3x+87048\,ab^2cd^2x-83700\,b^3c^2dx+37050\,a^3d^3-13}{191425\,a^4d^4-765700\,a^3bcd^3+1148550\,a^2c^2b^2d^2-765700\,ab^3c^3d+191425}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(37/6),x)
```

```
[Out] -6/191425*(b*x+a)^(13/6)*(-1296*b^3*d^3*x^3+2808*a*b^2*d^3*x^2-6696*b^3*c*d
^2*x^2-4446*a^2*b*d^3*x+14508*a*b^2*c*d^2*x-13950*b^3*c^2*d*x+6175*a^3*d^3-
22971*a^2*b*c*d^2+30225*a*b^2*c^2*d-14725*b^3*c^3)/(d*x+c)^(31/6)/(a^4*d^4-
4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{37}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(37/6), x)
```

Fricas [B] time = 1.88865, size = 1373, normalized size = 10.1

$$\frac{6\left(1296\,b^5d^3x^5 + 14725\,a^2b^3c^3 - 30225\,a^3b^2c^2d + 229\right.}{191425\left(b^4c^{10} - 4\,ab^3c^9d + 6\,a^2b^2c^8d^2 - 4\,a^3bc^7d^3 + a^4c^6d^4 + \left(b^4c^4d^6 - 4\,ab^3c^3d^7 + 6\,a^2b^2c^2d^8 - 4\,a^3bcd^9 + a^4d^{10}\right)x^6 + 6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="fricas")
```

```
[Out] 6/191425*(1296*b^5*d^3*x^5 + 14725*a^2*b^3*c^3 - 30225*a^3*b^2*c^2*d + 2297
1*a^4*b*c*d^2 - 6175*a^5*d^3 + 216*(31*b^5*c*d^2 - a*b^4*d^3)*x^4 + 18*(775
*b^5*c^2*d - 62*a*b^4*c*d^2 + 7*a^2*b^3*d^3)*x^3 + (14725*b^5*c^3 - 2325*a*
b^4*c^2*d + 651*a^2*b^3*c*d^2 - 91*a^3*b^2*d^3)*x^2 + 2*(14725*a*b^4*c^3 -
23250*a^2*b^3*c^2*d + 15717*a^3*b^2*c*d^2 - 3952*a^4*b*d^3)*x)*(b*x + a)^(1
/6)*(d*x + c)^(5/6)/(b^4*c^10 - 4*a*b^3*c^9*d + 6*a^2*b^2*c^8*d^2 - 4*a^3*b
*c^7*d^3 + a^4*c^6*d^4 + (b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2*b^2*c^2*d^8
 - 4*a^3*b*c*d^9 + a^4*d^10)*x^6 + 6*(b^4*c^5*d^5 - 4*a*b^3*c^4*d^6 + 6*a^2
*b^2*c^3*d^7 - 4*a^3*b*c^2*d^8 + a^4*c*d^9)*x^5 + 15*(b^4*c^6*d^4 - 4*a*b^3
*c^5*d^5 + 6*a^2*b^2*c^4*d^6 - 4*a^3*b*c^3*d^7 + a^4*c^2*d^8)*x^4 + 20*(b^4
*c^7*d^3 - 4*a*b^3*c^6*d^4 + 6*a^2*b^2*c^5*d^5 - 4*a^3*b*c^4*d^6 + a^4*c^3*
d^7)*x^3 + 15*(b^4*c^8*d^2 - 4*a*b^3*c^7*d^3 + 6*a^2*b^2*c^6*d^4 - 4*a^3*b*
c^5*d^5 + a^4*c^4*d^6)*x^2 + 6*(b^4*c^9*d - 4*a*b^3*c^8*d^2 + 6*a^2*b^2*c^7
*d^3 - 4*a^3*b*c^6*d^4 + a^4*c^5*d^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(37/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1805 $\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx$

Optimal. Leaf size=424

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tan^{-}}{24\sqrt{3}}$$

```
[Out] (7*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b^2) + ((a + b*x)^(5/6)
*(c + d*x)^(7/6))/(2*b) + (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
 + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(
5/6)) - (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sq
rt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(5/6)) + (7*(b*c -
a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^
(13/6)*d^(5/6)) - (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*
b^(13/6)*d^(5/6)) + (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(14
4*b^(13/6)*d^(5/6))
```

Rubi [A] time = 0.609888, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tan^{-}}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(7/6)/(a + b*x)^(1/6),x]
```


```
[Out] (7*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b^2) + ((a + b*x)^(5/6)
*(c + d*x)^(7/6))/(2*b) + (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
 + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(
5/6)) - (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sq
rt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(5/6)) + (7*(b*c -
a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^
(13/6)*d^(5/6)) - (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*
b^(13/6)*d^(5/6)) + (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(14
4*b^(13/6)*d^(5/6))
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx = \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{(7(bc-ad))\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}dx}{12b}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{5/6}}dx}{72b^2}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}dx, x, \right.}{12b^3}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right.}{12b^3}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}d\right.}{36b^{13/6}d^{2/3}}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{13/6}d^{5/6}} - \frac{(7(bc-}{}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{13/6}d^{5/6}} - \frac{7(bc-a}{}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{24\sqrt{3}b^{13/6}d^{5/6}} - \frac{7(bc-}{}$$

Mathematica [C] time = 0.0397547, size = 73, normalized size = 0.17

$$\frac{6(a+bx)^{5/6}(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(7/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, 5/6, 11/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{7}{6}}\frac{1}{\sqrt[6]{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(7/6)/(b*x+a)^(1/6),x)
```

[Out] int((d*x+c)^(7/6)/(b*x+a)^(1/6),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="maxima")

[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(1/6), x)

Fricas [B] time = 3.46043, size = 12598, normalized size = 29.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="fricas")

```
[Out] -1/144*(28*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1
/6)*arctan(-1/3*(2*sqrt(3)*(b^13*c^2*d^4 - 2*a*b^12*c*d^5 + a^2*b^11*d^6)*(
b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10
*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^
9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*
d^5))^(5/6) - 2*sqrt(3)*(b^12*d^4*x + a*b^11*d^4)*sqrt(((b^4*c^2*d - 2*a*b^
3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(b^13*d^5))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*d
^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/3))/
(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*
b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(5/6) + sqrt(3
)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^
9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 7
92*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b
^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d
+ 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a
^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^
4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a
^12*b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 -
220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b
```

^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d
^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*
a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c
^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 +
495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*
b^2*c*d^11 + a^12*b*d^12)*x)) + 28*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(b^13*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^13*c^2*d^4 - 2*a*b^12
*c*d^5 + a^2*b^11*d^6)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(b^13*d^5))^(5/6) - 2*sqrt(3)*(b^12*d^4*x + a*b^11*d^4)*s
qrt(-((b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(
1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6) - (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)
*(d*x + c)^(1/3) - (b^5*d^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^13*d^5))^(1/3))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
b^13*d^5))^(5/6) - sqrt(3)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*
c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5
+ 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^1
0*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13
*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 49
5*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6
*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10
- 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d
+ 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^
6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4
*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^1
3*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^1
0*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6
- 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^1
0*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) - 7*b^2*((b^12*c^12
- 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b
^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^
7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a
^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6)*log(49*((b^4*c^2*d - 2*a*b^3*c*
d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(b^13*d^5))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*d^2*x
+ a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/3))/(b*x
+ a)) + 7*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*

```
a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c
^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6)*log(
-49*((b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1
/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9
*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 79
2*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2
*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6) - (b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*
(d*x + c)^(1/3) - (b^5*d^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*
b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d
^12)/(b^13*d^5))^(1/3))/(b*x + a)) - 14*b^2*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^13*d^5))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(
5/6)*(d*x + c)^(1/6) + (b^3*d*x + a*b^2*d)*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^13*d^5))^(1/6))/(b*x + a)) + 14*b^2*((b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(b^13*d^5))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^
(5/6)*(d*x + c)^(1/6) - (b^3*d*x + a*b^2*d)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^13*d^5))^(1/6))/(b*x + a)) - 12*(6*b*d*x + 13*b*c - 7*a*d)*(b*x +
 a)^(5/6)*(d*x + c)^(1/6))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(7/6)/(b*x+a)**(1/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1806 $\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx$

Optimal. Leaf size=378

$$-\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{2\sqrt{3}b^{7/6}d^{5}}$$

```
[Out] ((a + b*x)^(5/6)*(c + d*x)^(1/6))/b + ((b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^
(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(7/
6)*d^(5/6)) - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(
Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(7/6)*d^(5/6)) + ((b*c - a*
d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(7/6)
*d^(5/6)) - ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(7/6)*d^(
5/6)) + ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3
) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(7/6)*d^(5/6)
)
```

Rubi [A] time = 0.559539, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{2\sqrt{3}b^{7/6}d^{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(1/6),x]
```


```
[Out] ((a + b*x)^(5/6)*(c + d*x)^(1/6))/b + ((b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^
(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(7/
6)*d^(5/6)) - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(
Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(7/6)*d^(5/6)) + ((b*c - a*
d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(7/6)
*d^(5/6)) - ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(7/6)*d^(
5/6)) + ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3
) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(7/6)*d^(5/6)
)
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx = \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{5/6}}\,dx}{6b}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^2}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{2/3}} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}\cdots}{\sqrt[3]{b}+\sqrt[6]{b}\cdots}\right)}{3b^{7/6}d\cdots}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\cdots\right)}{12b^{7/6}d^{5/6}}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}} - \frac{(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{12b^{7/6}d^{5/6}} + \cdots$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}b^{7/6}d^{5/6}} - \frac{(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}b^{7/6}d^{5/6}} + \frac{(bc-ad)\tanh\cdots}{3b^{7/6}\cdots}$$

Mathematica [C] time = 0.0223731, size = 73, normalized size = 0.19

$$\frac{6(a+bx)^{5/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 5/6, 11/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \sqrt[6]{dx+c}\frac{1}{\sqrt[6]{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(1/6),x)
```

```
[Out] int((d*x+c)^(1/6)/(b*x+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(1/6), x)
```

Fricas [B] time = 2.65001, size = 6460, normalized size = 17.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b
^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)
*arctan(1/3*(2*sqrt(3)*(b^7*c*d^4 - a*b^6*d^5)*(b*x + a)^(5/6)*(d*x + c)^(1
/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) + 2*sqrt(3)*(
b^7*d^4*x + a*b^6*d^4)*sqrt(((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^
(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) + (b^2*c^2
- 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^2*x + a*b^2
*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a))
*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a
^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) + sqrt(3)*(a*b^6
*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b
^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^
5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6
*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*
c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6
*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a
^5*b^2*c*d^5 + a^6*b*d^6)*x)) + 4*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*
a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 +
 a^6*d^6)/(b^7*d^5))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^7*c*d^4 - a*b^6*d^5)*(b
*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^
2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7
*d^5))^(5/6) + 2*sqrt(3)*(b^7*d^4*x + a*b^6*d^4)*sqrt(-((b^2*c*d - a*b*d^2)
*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(
b^7*d^5))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)
^(1/3) - (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(
b^7*d^5))^(1/3))/(b*x + a))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2
- 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d
^5))^(5/6) - sqrt(3)*(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20
*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6
- 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*
d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^
3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a
```

```
^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3
 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x)) + b*((b^6*c^6 - 6*
a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
- 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*log(((b^2*c*d - a*b*d^2)*(b*x +
 a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5
))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
+ (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5
))^(1/3))/(b*x + a)) - b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5)
)^(1/6)*log(-((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a)) + 2*b*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*log(-((b*c - a*d)*(b*x +
a)^(5/6)*(d*x + c)^(1/6) + (b^2*d*x + a*b*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15
*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
+ a^6*d^6)/(b^7*d^5))^(1/6))/(b*x + a)) - 2*b*((b^6*c^6 - 6*a*b^5*c^5*d + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
 + a^6*d^6)/(b^7*d^5))^(1/6)*log(-((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1
/6) - (b^2*d*x + a*b*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))
^(1/6))/(b*x + a)) + 12*(b*x + a)^(5/6)*(d*x + c)^(1/6))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(1/6),x)
```

```
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(1/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1807 $\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=309

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}d^{5/6}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\right.}{}$$

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c
 + d*x)^(1/6))])/(b^(1/6)*d^(5/6)) - (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/6)
*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(1/6)*d^(5/6)) + (
2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(1/6)*d^
(5/6)) - Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)
*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(1/6)*d^(5/6)) + Log[b^(1/3
) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^
(1/6))/(c + d*x)^(1/6)]/(2*b^(1/6)*d^(5/6))
```

Rubi [A] time = 0.509348, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}d^{5/6}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c
 + d*x)^(1/6))])/(b^(1/6)*d^(5/6)) - (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/6)
*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(1/6)*d^(5/6)) + (
2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(1/6)*d^
(5/6)) - Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)
*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(1/6)*d^(5/6)) + Log[b^(1/3
) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^
(1/6))/(c + d*x)^(1/6)]/(2*b^(1/6)*d^(5/6))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{5/6}}\,dx = \frac{6\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b}$$

$$= \frac{6\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^{2/3}} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}d^{2/3}} + \frac{2\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}d^{5/6}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2\sqrt[6]{b}d^{5/6}} + \frac{\text{Subst}\left(\int \frac{\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}x}{\sqrt[3]{b}+\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,d\right.}{2\sqrt[6]{b}d^{5/6}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}d^{5/6}} - \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2\sqrt[6]{b}d^{5/6}} + \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+b}}{\sqrt[6]{c+dx}}\right.}{2\sqrt[6]{b}d^{5/6}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{b}d^{5/6}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{b}d^{5/6}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}d^{5/6}} - \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+}}{\sqrt[3]{c+dx}}\right.}{2\sqrt[6]{b}}$$

Mathematica [C] time = 0.0271541, size = 73, normalized size = 0.24

$$\frac{6(a+bx)^{5/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 5/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(5/6))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[6]{bx+a}}(dx+c)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(5/6)), x)
```

Fricas [B] time = 2.05292, size = 1638, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*(1/(b*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x +
c)^(1/6)*b*d^4*(1/(b*d^5))^(5/6) - 2*sqrt(3)*(b^2*d^4*x + a*b*d^4)*sqrt(((b
*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d^5))^(1/6) + (b*d^2*x + a*d^2)*(1/(b
*d^5))^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a))*(1/(b*d^5))^(5/6
) + sqrt(3)*(b*x + a))/(b*x + a)) - 2*sqrt(3)*(1/(b*d^5))^(1/6)*arctan(-1/3
*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^4*(1/(b*d^5))^(5/6) - 2*sqr
t(3)*(b^2*d^4*x + a*b*d^4)*sqrt(-((b*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d
^5))^(1/6) - (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) - (b*x + a)^(2/3)*(d*x + c
)^(1/3))/(b*x + a))*(1/(b*d^5))^(5/6) - sqrt(3)*(b*x + a))/(b*x + a)) + 1/2
*(1/(b*d^5))^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d^5))^(1/
6) + (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))
/(b*x + a)) - 1/2*(1/(b*d^5))^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x + c)^(1/6)
*d*(1/(b*d^5))^(1/6) - (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) - (b*x + a)^(2/3
)*(d*x + c)^(1/3))/(b*x + a)) + (1/(b*d^5))^(1/6)*log(((b*d*x + a*d)*(1/(b*
d^5))^(1/6) + (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b*x + a)) - (1/(b*d^5))^(1/
6)*log(-((b*d*x + a*d)*(1/(b*d^5))^(1/6) - (b*x + a)^(5/6)*(d*x + c)^(1/6))
/(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[6]{a + bx}\,(c + dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(5/6),x)
```

```
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(5/6)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1808 $\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(5/6))/(5*(b*c - a*d)*(c + d*x)^(5/6))
```

Rubi [A] time = 0.0031016, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(5*(b*c - a*d)*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{11/6}}\,dx = \frac{6(a+bx)^{5/6}}{5(bc-ad)(c+dx)^{5/6}}$$

Mathematica [A] time = 0.0107222, size = 32, normalized size = 1.

$$\frac{6(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(5*(b*c - a*d)*(c + d*x)^(5/6))
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$-\frac{6}{5\,ad-5\,bc}(bx+a)^{\frac{5}{6}}(dx+c)^{-\frac{5}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x)
```

```
[Out] -6/5*(b*x+a)^(5/6)/(d*x+c)^(5/6)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(11/6)), x)
```

Fricas [A] time = 1.86014, size = 99, normalized size = 3.09

$$\frac{6\,(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{5\left(bc^2-acd+\left(bcd-ad^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] 6/5*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(11/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(11/6)), x)
```

3.1809 $\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=66

$$\frac{36b(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(5/6))/(11*(b*c - a*d)*(c + d*x)^(11/6)) + (36*b*(a + b*x)^(5/
6))/(55*(b*c - a*d)^2*(c + d*x)^(5/6))
```

Rubi [A] time = 0.0104578, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(11*(b*c - a*d)*(c + d*x)^(11/6)) + (36*b*(a + b*x)^(5/
6))/(55*(b*c - a*d)^2*(c + d*x)^(5/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{17/6}}\,dx&=\frac{6(a+bx)^{5/6}}{11(bc-ad)(c+dx)^{11/6}}+\frac{(6b)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{11/6}}\,dx}{11(bc-ad)}\\&=\frac{6(a+bx)^{5/6}}{11(bc-ad)(c+dx)^{11/6}}+\frac{36b(a+bx)^{5/6}}{55(bc-ad)^2(c+dx)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.0167769, size = 46, normalized size = 0.7

$$\frac{6(a+bx)^{5/6}(-5ad+11bc+6bdx)}{55(c+dx)^{11/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(11*b*c - 5*a*d + 6*b*d*x))/(55*(b*c - a*d)^2*(c + d*x)^
(11/6))
```

Maple [A] time = 0.006, size = 54, normalized size = 0.8

$$-\frac{-36\,bdx+30\,ad-66\,bc}{55\,a^2d^2-110\,abcd+55\,b^2c^2}\left(bx+a\right)^{\frac{5}{6}}\left(dx+c\right)^{-\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x)
```

```
[Out] -6/55*(b*x+a)^(5/6)*(-6*b*d*x+5*a*d-11*b*c)/(d*x+c)^(11/6)/(a^2*d^2-2*a*b*c
*d+b^2*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(17/6)), x)
```

Fricas [B] time = 1.87032, size = 261, normalized size = 3.95

$$\frac{6\,(6\,bdx+11\,bc-5\,ad)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{55\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^2+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] 6/55*(6*b*d*x + 11*b*c - 5*a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^2*c^4 -
2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(
b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(17/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(17/6)), x)
```

3.1810 $\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{23/6}}\,dx$

Optimal. Leaf size=101

$$\frac{432b^2(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^3}+\frac{72b(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(5/6))/(17*(b*c - a*d)*(c + d*x)^(17/6)) + (72*b*(a + b*x)^(5/
6))/(187*(b*c - a*d)^2*(c + d*x)^(11/6)) + (432*b^2*(a + b*x)^(5/6))/(935*(
b*c - a*d)^3*(c + d*x)^(5/6))
```

Rubi [A] time = 0.0201421, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^3}+\frac{72b(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(17*(b*c - a*d)*(c + d*x)^(17/6)) + (72*b*(a + b*x)^(5/
6))/(187*(b*c - a*d)^2*(c + d*x)^(11/6)) + (432*b^2*(a + b*x)^(5/6))/(935*(
b*c - a*d)^3*(c + d*x)^(5/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{23/6}}\,dx &= \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{(12b)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{17/6}}\,dx}{17(bc-ad)}\\ &= \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{72b(a+bx)^{5/6}}{187(bc-ad)^2(c+dx)^{11/6}}+\frac{\left(72b^2\right)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{11/6}}\,dx}{187(bc-ad)^2}\\ &= \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{72b(a+bx)^{5/6}}{187(bc-ad)^2(c+dx)^{11/6}}+\frac{432b^2(a+bx)^{5/6}}{935(bc-ad)^3(c+dx)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.033556, size = 77, normalized size = 0.76

$$\frac{6(a+bx)^{5/6}\left(55a^2d^2-10abd(17c+6dx)+b^2\left(187c^2+204cdx+72d^2x^2\right)\right)}{935(c+dx)^{17/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(55*a^2*d^2 - 10*a*b*d*(17*c + 6*d*x) + b^2*(187*c^2 + 2
04*c*d*x + 72*d^2*x^2)))/(935*(b*c - a*d)^3*(c + d*x)^(17/6))
```

Maple [A] time = 0.006, size = 105, normalized size = 1.

$$-\frac{432\,b^2d^2x^2-360\,abd^2x+1224\,b^2cdx+330\,a^2d^2-1020\,abcd+1122\,b^2c^2}{935\,a^3d^3-2805\,a^2cbd^2+2805\,ab^2c^2d-935\,b^3c^3}(bx+a)^{\frac{5}{6}}(dx+c)^{-\frac{17}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x)
```

```
[Out] -6/935*(b*x+a)^(5/6)*(72*b^2*d^2*x^2-60*a*b*d^2*x+204*b^2*c*d*x+55*a^2*d^2-
170*a*b*c*d+187*b^2*c^2)/(d*x+c)^(17/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*
d-b^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(23/6)), x)
```

Fricas [B] time = 1.84383, size = 525, normalized size = 5.2

$$\frac{6\left(72\,b^2d^2x^2+187\,b^2c^2-170\,abcd+55\,a^2d^2+12\left(17\,b^2cd-5\,abd^2\right)x\right)(bx+}{935\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3+\left(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6\right)x^3+3\left(b^3c^4d^2-3\,ab^2c^3d^3+3\,a^2bc^2d^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] 6/935*(72*b^2*d^2*x^2 + 187*b^2*c^2 - 170*a*b*c*d + 55*a^2*d^2 + 12*(17*b^2
*c*d - 5*a*b*d^2)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^3*c^6 - 3*a*b^2*c^5
*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2
*b*c*d^5 - a^3*d^6)*x^3 + 3*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^
```

```
4 - a^3*c*d^5)*x^2 + 3*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3
*c^2*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(23/6)), x)
```

3.1811 $\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{29/6}}\,dx$

Optimal. Leaf size=136

$$\frac{7776b^3(a+bx)^{5/6}}{21505(c+dx)^{5/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{5/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{108b(a+bx)^{5/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{23(c+dx)^{23/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(5/6))/(23*(b*c - a*d)*(c + d*x)^(23/6)) + (108*b*(a + b*x)^(5
/6))/(391*(b*c - a*d)^2*(c + d*x)^(17/6)) + (1296*b^2*(a + b*x)^(5/6))/(430
1*(b*c - a*d)^3*(c + d*x)^(11/6)) + (7776*b^3*(a + b*x)^(5/6))/(21505*(b*c
- a*d)^4*(c + d*x)^(5/6))
```

Rubi [A] time = 0.0301452, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3(a+bx)^{5/6}}{21505(c+dx)^{5/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{5/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{108b(a+bx)^{5/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{23(c+dx)^{23/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(23*(b*c - a*d)*(c + d*x)^(23/6)) + (108*b*(a + b*x)^(5
/6))/(391*(b*c - a*d)^2*(c + d*x)^(17/6)) + (1296*b^2*(a + b*x)^(5/6))/(430
1*(b*c - a*d)^3*(c + d*x)^(11/6)) + (7776*b^3*(a + b*x)^(5/6))/(21505*(b*c
- a*d)^4*(c + d*x)^(5/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{29/6}}\,dx = \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{(18b)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{23/6}}\,dx}{23(bc-ad)}$$
$$= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{\left(216b^2\right)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{17/6}}\,dx}{391(bc-ad)^2}$$
$$= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{1296b^2(a+bx)^{5/6}}{4301(bc-ad)^3(c+dx)^{11/6}} + \frac{(129\ldots}{\ldots}$$
$$= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{1296b^2(a+bx)^{5/6}}{4301(bc-ad)^3(c+dx)^{11/6}} + \frac{7\ldots}{2150\ldots}$$

Mathematica [A] time = 0.0508207, size = 118, normalized size = 0.87

$$\frac{6(a+bx)^{5/6}\left(165a^2bd^2(23c+6dx)-935a^3d^3-15ab^2d\left(391c^2+276cdx+72d^2x^2\right)+b^3\left(7038c^2dx+4301c^3+4968cd\ldots\right.\right.}{21505(c+dx)^{23/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(-935*a^3*d^3 + 165*a^2*b*d^2*(23*c + 6*d*x) - 15*a*b^2*
d*(391*c^2 + 276*c*d*x + 72*d^2*x^2) + b^3*(4301*c^3 + 7038*c^2*d*x + 4968*
c*d^2*x^2 + 1296*d^3*x^3)))/(21505*(b*c - a*d)^4*(c + d*x)^(23/6))
```

Maple [A] time = 0.007, size = 171, normalized size = 1.3

$$-\frac{-7776\,x^3b^3d^3+6480\,ab^2d^3x^2-29808\,b^3cd^2x^2-5940\,a^2bd^3x+24840\,ab^2cd^2x-42228\,b^3c^2dx+5610\,a^3d^3-22770\ldots}{21505\,a^4d^4-86020\,a^3bcd^3+129030\,a^2c^2b^2d^2-86020\,ab^3c^3d+21505\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x)
```

```
[Out] -6/21505*(b*x+a)^(5/6)*(-1296*b^3*d^3*x^3+1080*a*b^2*d^3*x^2-4968*b^3*c*d^2
*x^2-990*a^2*b*d^3*x+4140*a*b^2*c*d^2*x-7038*b^3*c^2*d*x+935*a^3*d^3-3795*a
^2*b*c*d^2+5865*a*b^2*c^2*d-4301*b^3*c^3)/(d*x+c)^(23/6)/(a^4*d^4-4*a^3*b*c
*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{29}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(29/6)), x)
```

Fricas [B] time = 1.91764, size = 883, normalized size = 6.49

$$\frac{6\left(1296\,b^3d^3x^3+4301\,b^3c^3-5865\,ab^2c^2d+3795\,a^2b\cdots\right.}{21505\left(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4+\left(b^4c^4d^4-4\,ab^3c^3d^5+6\,a^2b^2c^2d^6-4\,a^3bcd^7+a^4d^8\right)x^4+4\left(b^4\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="fricas")
```

```
[Out] 6/21505*(1296*b^3*d^3*x^3 + 4301*b^3*c^3 - 5865*a*b^2*c^2*d + 3795*a^2*b*c*
d^2 - 935*a^3*d^3 + 216*(23*b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + 18*(391*b^3*c^2*
d - 230*a*b^2*c*d^2 + 55*a^2*b*d^3)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^4
*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4 +
(b^4*c^4*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^
8)*x^4 + 4*(b^4*c^5*d^3 - 4*a*b^3*c^4*d^4 + 6*a^2*b^2*c^3*d^5 - 4*a^3*b*c^2
*d^6 + a^4*c*d^7)*x^3 + 6*(b^4*c^6*d^2 - 4*a*b^3*c^5*d^3 + 6*a^2*b^2*c^4*d^
4 - 4*a^3*b*c^3*d^5 + a^4*c^2*d^6)*x^2 + 4*(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6
*a^2*b^2*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(29/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{29}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(29/6)), x)
```

3.1812 $\int \frac{(c+dx)^{11/6}}{\sqrt[6]{a+bx}}\,dx$

Optimal. Leaf size=82

$$\frac{6(a+bx)^{5/6}(c+dx)^{5/6}(bc-ad)\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] (6*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-11/6, 5/6
, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*b^2*((b*(c + d*x))/(b*c - a*d))^(
5/6))
```

Rubi [A] time = 0.0235723, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{5/6}(c+dx)^{5/6}(bc-ad)\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(11/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-11/6, 5/6
, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*b^2*((b*(c + d*x))/(b*c - a*d))^(
5/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(c+dx)^{11/6}}{\sqrt[6]{a+bx}}\,dx = \frac{\left((bc-ad)(c+dx)^{5/6}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}{\sqrt[6]{a+bx}}dx}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$
$$= \frac{6(bc-ad)(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Mathematica [A] time = 0.0498552, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{5/6}(c+dx)^{11/6}\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(11/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(11/6)*Hypergeometric2F1[-11/6, 5/6, 11/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(11/6))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{11}{6}}\frac{1}{\sqrt[6]{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(11/6)/(b*x+a)^(1/6),x)
```

```
[Out] int((d*x+c)^(11/6)/(b*x+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(11/6)/(b*x + a)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{1}{6}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(11/6)/(b*x + a)^(1/6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(11/6)/(b*x+a)**(1/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1813 $\int \frac{(c+dx)^{5/6}}{\sqrt[6]{a+bx}} \, dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 5/6, 11/6, -((d*
(a + b*x))/(b*c - a*d))])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Rubi [A] time = 0.0202515, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 5/6, 11/6, -((d*
(a + b*x))/(b*c - a*d))])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{5/6}}{\sqrt[6]{a+bx}} \, dx &= \frac{(c+dx)^{5/6} \int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}{\sqrt[6]{a+bx}} \, dx}{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}\\ &= \frac{6(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.0245478, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 5/6, 11/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{6}}\frac{1}{\sqrt[6]{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(1/6),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(1/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{1}{6}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(5/6)/(b*x + a)^(1/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{5}{6}}}{\sqrt[6]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(1/6),x)
```

```
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(1/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1814 $\int \frac{1}{\sqrt[6]{a+bx}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[6]{c+dx}}$$

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 5/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*b*(c + d*x)^(1/6))
```

Rubi [A] time = 0.0205361, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 5/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*b*(c + d*x)^(1/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[6]{a+bx}\sqrt[6]{c+dx}}\,dx &= \frac{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{\sqrt[6]{a+bx}\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[6]{c+dx}}\\ &= \frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[6]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0220886, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 5/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/6))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[6]{bx+a}}\frac{1}{\sqrt[6]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(1/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[6]{a+bx}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(1/6),x)

[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(1/6)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="giac")

[Out] Timed out
```

3.1815 $\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=81

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5\sqrt[6]{c+dx}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[5/6,
 7/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*(b*c - a*d)*(c + d*x)^(1/6))
```

Rubi [A] time = 0.0204251, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5\sqrt[6]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[5/6,
 7/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*(b*c - a*d)*(c + d*x)^(1/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{7/6}}\,dx &= \frac{\left(b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt[6]{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}}\,dx}{(bc-ad)\sqrt[6]{c+dx}}\\ &= \frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)\sqrt[6]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0337345, size = 73, normalized size = 0.9

$$\frac{6(a+bx)^{5/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[5/6,
 7/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(7/6))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[6]{bx+a}}(dx+c)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(7/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}{bd^2x^3+ac^2+\left(2\,bcd+ad^2\right)x^2+\left(bc^2+2\,acd\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*
d^2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(7/6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(7/6)), x)
```

3.1816 $\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{13/6}} dx$

Optimal. Leaf size=82

$$\frac{6b(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5\sqrt[6]{c+dx}(bc-ad)^2}$$

```
[Out] (6*b*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[5/
6, 13/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*(b*c - a*d)^2*(c + d*x)^(1
/6))
```

Rubi [A] time = 0.0201833, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6b(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5\sqrt[6]{c+dx}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*b*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[5/
6, 13/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*(b*c - a*d)^2*(c + d*x)^(1
/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{13/6}}\,dx &= \frac{\left(b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt[6]{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}}\,dx}{(bc-ad)^2\sqrt[6]{c+dx}}\\ &= \frac{6b(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)^2\sqrt[6]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0471512, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{5/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(13/6)*Hypergeometric2F1[5/6
, 13/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(13/6))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[6]{bx+a}}(dx+c)^{-\frac{13}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(13/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}{bd^3x^4+ac^3+\left(3\,bcd^2+ad^3\right)x^3+3\left(bc^2d+acd^2\right)x^2+\left(bc^3+3\,ac^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b*d^3*x^4 + a*c^3 + (3*b*c*d^2 +
a*d^3)*x^3 + 3*(b*c^2*d + a*c*d^2)*x^2 + (b*c^3 + 3*a*c^2*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(13/6)), x)
```

3.1817 $\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=84

$$\frac{6b^2(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{5}{6},\frac{19}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5\sqrt[6]{c+dx}(bc-ad)^3}$$

```
[Out] (6*b^2*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[
5/6, 19/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*(b*c - a*d)^3*(c + d*x)^
(1/6))
```

Rubi [A] time = 0.0204141, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6b^2(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{5}{6},\frac{19}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5\sqrt[6]{c+dx}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*b^2*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[
5/6, 19/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*(b*c - a*d)^3*(c + d*x)^
(1/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{19/6}}\,dx &= \frac{\left(b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt[6]{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{19/6}}\,dx}{(bc-ad)^3\sqrt[6]{c+dx}}\\ &= \frac{6b^2(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{5}{6},\frac{19}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)^3\sqrt[6]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0328864, size = 81, normalized size = 0.96

$$\frac{6b(a+bx)^{5/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{5}{6},\frac{19}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5(c+dx)^{7/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*b*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[5/
6, 19/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*(b*c - a*d)^2*(c + d*x)^(7
/6))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[6]{bx+a}}(dx+c)^{-\frac{19}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(19/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}{bd^4x^5+ac^4+\left(4\,bcd^3+ad^4\right)x^4+2\left(3\,bc^2d^2+2\,acd^3\right)x^3+2\left(2\,bc^3d+3\,ac^2d^2\right)x^2+\left(bc^4+4\,ac^3d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b*d^4*x^5 + a*c^4 + (4*b*c*d^3 +
a*d^4)*x^4 + 2*(3*b*c^2*d^2 + 2*a*c*d^3)*x^3 + 2*(2*b*c^3*d + 3*a*c^2*d^2)*
x^2 + (b*c^4 + 4*a*c^3*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(19/6)), x)
```

3.1818 $\int \frac{(c+dx)^{13/6}}{(a+bx)^{5/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6\sqrt[6]{a+bx}\sqrt[6]{c+dx}(bc-ad)^2\,{}_2F_1\left(-\frac{13}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] (6*(b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-13/6, 1
/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/(b^3*((b*(c + d*x))/(b*c - a*d))^(1
/6))
```

Rubi [A] time = 0.0198021, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6\sqrt[6]{a+bx}\sqrt[6]{c+dx}(bc-ad)^2\,{}_2F_1\left(-\frac{13}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(13/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-13/6, 1
/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/(b^3*((b*(c + d*x))/(b*c - a*d))^(1
/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{13/6}}{(a+bx)^{5/6}}\,dx &= \frac{\left((bc-ad)^2\sqrt[6]{c+dx}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}}{(a+bx)^{5/6}}\,dx}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\\ &= \frac{6(bc-ad)^2\sqrt[6]{a+bx}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{13}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\end{aligned}$$

Mathematica [A] time = 0.0635564, size = 71, normalized size = 0.87

$$\frac{6\sqrt[6]{a+bx}(c+dx)^{13/6}\,{}_2F_1\left(-\frac{13}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(13/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(13/6)*Hypergeometric2F1[-13/6, 1/6, 7/6, (d*(
a + b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(13/6))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{13}{6}}(bx+a)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(13/6)/(b*x+a)^(5/6),x)
```

```
[Out] int((d*x+c)^(13/6)/(b*x+a)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{13}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(13/6)/(b*x + a)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^2x^2+2\,cdx+c^2\right)(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{5}{6}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*(d*x + c)^(1/6)/(b*x + a)^(5/6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(13/6)/(b*x+a)**(5/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1819 $\int \frac{(c+dx)^{7/6}}{(a+bx)^{5/6}}\,dx$

Optimal. Leaf size=80

$$\frac{6\sqrt[6]{a+bx}\sqrt[6]{c+dx}(bc-ad)\,_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] (6*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-7/6, 1/6,
 7/6, -((d*(a + b*x))/(b*c - a*d))])/(b^2*((b*(c + d*x))/(b*c - a*d))^(1/6)
)
```

Rubi [A] time = 0.0215323, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6\sqrt[6]{a+bx}\sqrt[6]{c+dx}(bc-ad)\,_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(7/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-7/6, 1/6,
 7/6, -((d*(a + b*x))/(b*c - a*d))])/(b^2*((b*(c + d*x))/(b*c - a*d))^(1/6)
)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{7/6}}{(a+bx)^{5/6}}\,dx &= \frac{\left((bc-ad)\sqrt[6]{c+dx}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}}{(a+bx)^{5/6}}\,dx}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\\ &= \frac{6(bc-ad)\sqrt[6]{a+bx}\sqrt[6]{c+dx}\,_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\end{aligned}$$

Mathematica [A] time = 0.0388363, size = 71, normalized size = 0.89

$$\frac{6\sqrt[6]{a+bx}(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(7/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, 1/6, 7/6, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{7}{6}}(bx+a)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(7/6)/(b*x+a)^(5/6),x)
```

```
[Out] int((d*x+c)^(7/6)/(b*x+a)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{5}{6}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(7/6)/(b*x + a)^(5/6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(7/6)/(b*x+a)**(5/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1820 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{5/6}}\,dx$

Optimal. Leaf size=72

$$\frac{6\sqrt[6]{a+bx}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 1/6, 7/6, -((d*(
a + b*x))/(b*c - a*d))])/(b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Rubi [A] time = 0.0193598, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6\sqrt[6]{a+bx}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 1/6, 7/6, -((d*(
a + b*x))/(b*c - a*d))])/(b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{5/6}}\,dx &= \frac{\sqrt[6]{c+dx}\int \frac{\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}{(a+bx)^{5/6}}\,dx}{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\\ &= \frac{6\sqrt[6]{a+bx}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}\end{aligned}$$

Mathematica [A] time = 0.0238663, size = 71, normalized size = 0.99

$$\frac{6\sqrt[6]{a+bx}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 1/6, 7/6, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \sqrt[6]{dx+c}\,(bx+a)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(5/6),x)
```

```
[Out] int((d*x+c)^(1/6)/(b*x+a)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(5/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{5}{6}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(1/6)/(b*x + a)^(5/6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[6]{c + dx}}{(a + bx)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(5/6),x)
```

```
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(5/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1821 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{5/6}} dx$

Optimal. Leaf size=72

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b(c+dx)^{5/6}}$$

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/6,
 5/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/(b*(c + d*x)^(5/6))
```

Rubi [A] time = 0.0199112, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/6,
 5/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/(b*(c + d*x)^(5/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/6}(c+dx)^{5/6}}\,dx &= \frac{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\int \frac{1}{(a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}\,dx}{(c+dx)^{5/6}}\\ &= \frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b(c+dx)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.0229782, size = 71, normalized size = 0.99

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/6,
 5/6, 7/6, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(5/6))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{6}}(dx+c)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

```
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(5/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{1}{6}}}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{6}}(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(5/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(5/6)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1822 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=79

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(c+dx)^{5/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/6,
 11/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)*(c + d*x)^(5/6))
```

Rubi [A] time = 0.018723, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/6,
 11/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)*(c + d*x)^(5/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/6}(c+dx)^{11/6}}\,dx &= \frac{\left(b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{1}{(a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}\,dx}{(bc-ad)(c+dx)^{5/6}}\\ &= \frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(c+dx)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.0384578, size = 71, normalized size = 0.9

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(11/6)*Hypergeometric2F1[1/6
, 11/6, 7/6, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(11/6))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{6}}(dx+c)^{-\frac{11}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

```
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(11/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{1}{6}}}{bd^2x^3+ac^2+\left(2\,bcd+ad^2\right)x^2+\left(bc^2+2\,acd\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*
d^2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(11/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1823 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=80

$$\frac{6b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(c+dx)^{5/6}(bc-ad)^2}$$

```
[Out] (6*b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/
6, 17/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^2*(c + d*x)^(5/6)
)
```

Rubi [A] time = 0.0200252, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{6b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/
6, 17/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^2*(c + d*x)^(5/6)
)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{17/6}}\,dx = \frac{\left(b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{1}{(a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{17/6}}\,dx}{(bc-ad)^2(c+dx)^{5/6}}$$

$$= \frac{6b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2(c+dx)^{5/6}}$$

Mathematica [A] time = 0.0247523, size = 79, normalized size = 0.99

$$\frac{6b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/
6, 17/6, 7/6, (d*(a + b*x))/(-(b*c) + a*d)])/((b*c - a*d)^2*(c + d*x)^(5/6)
)
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{6}}(dx+c)^{-\frac{17}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x)
```

```
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(17/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{1}{6}}}{bd^3x^4+ac^3+\left(3\,bcd^2+ad^3\right)x^3+3\left(bc^2d+acd^2\right)x^2+\left(bc^3+3\,ac^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(b*d^3*x^4 + a*c^3 + (3*b*c*d^2 +
a*d^3)*x^3 + 3*(b*c^2*d + a*c*d^2)*x^2 + (b*c^3 + 3*a*c^2*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(17/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1824 $\int \frac{(c+dx)^{11/6}}{(a+bx)^{5/6}} dx$

Optimal. Leaf size=424

$$\frac{11\sqrt[6]{a+bx}(c+dx)^{5/6}(bc-ad)}{12b^2}-\frac{55(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\cdots\right)}{144b^{17/6}\sqrt[6]{d}}$$

```
[Out] (11*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*b^2) + ((a + b*x)^(1/6
)*(c + d*x)^(11/6))/(2*b) - (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)
*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(17/6)*
d^(1/6)) + (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))
/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(17/6)*d^(1/6)) + (55*(b
*c - a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(
36*b^(17/6)*d^(1/6)) - (55*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(
1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])
/(144*b^(17/6)*d^(1/6)) + (55*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x
)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6
)])/(144*b^(17/6)*d^(1/6))
```

Rubi [A] time = 0.549184, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$\frac{11\sqrt[6]{a+bx}(c+dx)^{5/6}(bc-ad)}{12b^2}-\frac{55(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\cdots\right)}{144b^{17/6}\sqrt[6]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(11/6)/(a + b*x)^(5/6),x]
```


```
[Out] (11*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*b^2) + ((a + b*x)^(1/6
)*(c + d*x)^(11/6))/(2*b) - (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)
*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(17/6)*
d^(1/6)) + (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))
/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(17/6)*d^(1/6)) + (55*(b
*c - a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(
36*b^(17/6)*d^(1/6)) - (55*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(
1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])
/(144*b^(17/6)*d^(1/6)) + (55*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x
)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6
)])/(144*b^(17/6)*d^(1/6))
```


Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{11/6}}{(a+bx)^{5/6}}\,dx = \frac{\sqrt[6]{a+bx}(c+dx)^{11/6}}{2b} + \frac{(11(bc-ad))\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx}{12b}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}(c+dx)^{11/6}}{2b} + \frac{\left(55(bc-ad)^2\right)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{72b^2}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}(c+dx)^{11/6}}{2b} + \frac{\left(55(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x\ldots\right.}{12b^3}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}(c+dx)^{11/6}}{2b} + \frac{\left(55(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+\ldots}}{\sqrt[6]{c+\ldots}}\right.}{12b^3}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}(c+dx)^{11/6}}{2b} + \frac{\left(55(bc-ad)^2\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\ldots\right.}{36b^{17/6}}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}(c+dx)^{11/6}}{2b} + \frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{17/6}\sqrt[6]{d}} + \frac{(55(bc\ldots}{\ldots}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}(c+dx)^{11/6}}{2b} + \frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{17/6}\sqrt[6]{d}} - \frac{55(bc\ldots}{\ldots}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}(c+dx)^{11/6}}{2b} - \frac{55(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{24\sqrt{3}b^{17/6}\sqrt[6]{d}} + \frac{55(bc\ldots}{\ldots}$$

Mathematica [C] time = 0.0427112, size = 71, normalized size = 0.17

$$\frac{6\sqrt[6]{a+bx}(c+dx)^{11/6}\,{}_2F_1\left(-\frac{11}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(11/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(11/6)*Hypergeometric2F1[-11/6, 1/6, 7/6, (d*(
a + b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(11/6))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{11}{6}} (bx+a)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(11/6)/(b*x+a)^(5/6),x)
```

```
[Out] int((d*x+c)^(11/6)/(b*x+a)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(11/6)/(b*x + a)^(5/6), x)
```

Fricas [B] time = 2.9361, size = 12492, normalized size = 29.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="fricas")
```

```
[Out] -1/144*(220*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d
^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*
a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c
^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/
6)*arctan(-1/3*(2*sqrt(3)*(b^16*c^2*d - 2*a*b^15*c*d^2 + a^2*b^14*d^3)*(b*x
 + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^
10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))
^(5/6) - 2*sqrt(3)*(b^14*d^2*x + b^14*c*d)*sqrt(((b^5*c^2 - 2*a*b^4*c*d + a
^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^17*d))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^6*d*x + b^6*c)*(
(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
+ 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7
*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*
d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/3))/(d*x + c))*((b^12*c^1
2 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4
*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*
d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12
*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(5/6) + sqrt(3)*(b^12*c^13 - 12*a*b^1
1*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^
4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a
^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2
*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*
d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924
*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*
c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c
^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*
a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c
^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 -
 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*
```

a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7
*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9
- 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^1
3)*x)) + 220*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*
d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924
*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*
c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1
/6)*arctan(-1/3*(2*sqrt(3)*(b^16*c^2*d - 2*a*b^15*c*d^2 + a^2*b^14*d^3)*(b*
x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d)
)^(5/6) - 2*sqrt(3)*(b^14*d^2*x + b^14*c*d)*sqrt(-((b^5*c^2 - 2*a*b^4*c*d +
 a^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*
d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*
a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c
^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^
12*d^12)/(b^17*d))^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^6*d*x + b^6*c)
*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^
3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a
^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^
2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/3))/(d*x + c))*((b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b
^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*
d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495
*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c
^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^1
0*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 9
24*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^
3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12
*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 49
5*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5
*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10
 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 6
6*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b
^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^
9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d
^13)*x)) - 55*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6)*log
(3025*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6
)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d
^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*
a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c
^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6) + (b^4*c^4 - 4*a*b^
3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x
 + c)^(2/3) + (b^6*d*x + b^6*c)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^1
0*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a
^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17
*d))^(1/3))/(d*x + c)) + 55*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^1
0*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a
^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17

```
*d))^(1/6)*log(-3025*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(1/6)
*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 22
0*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6
*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6) - (b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x
+ a)^(1/3)*(d*x + c)^(2/3) - (b^6*d*x + b^6*c)*((b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(b^17*d))^(1/3))/(d*x + c)) - 110*b^2*((b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(b^17*d))^(1/6)*log(55*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a
)^(1/6)*(d*x + c)^(5/6) + (b^3*d*x + b^3*c)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^17*d))^(1/6))/(d*x + c)) + 110*b^2*((b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(b^17*d))^(1/6)*log(55*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(
1/6)*(d*x + c)^(5/6) - (b^3*d*x + b^3*c)*((b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(b^17*d))^(1/6))/(d*x + c)) - 12*(6*b*d*x + 17*b*c - 11*a*d)*(b*x + a)^
(1/6)*(d*x + c)^(5/6))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(11/6)/(b*x+a)**(5/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1825 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx$

Optimal. Leaf size=378

$$-\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}-\frac{5(bc-ad)\tan^{-1}\left(\cdots\right.}{2\sqrt{3}b^{11\cdots}}$$

```
[Out] ((a + b*x)^(1/6)*(c + d*x)^(5/6))/b - (5*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(
11/6)*d^(1/6)) + (5*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/
6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(11/6)*d^(1/6)) + (5*(
b*c - a*d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3
*b^(11/6)*d^(1/6)) - (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*
b^(11/6)*d^(1/6)) + (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b
^(11/6)*d^(1/6))
```

Rubi [A] time = 0.479407, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}-\frac{5(bc-ad)\tan^{-1}\left(\cdots\right.}{2\sqrt{3}b^{11\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(5/6),x]
```

```
[Out] ((a + b*x)^(1/6)*(c + d*x)^(5/6))/b - (5*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(
11/6)*d^(1/6)) + (5*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/
6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(11/6)*d^(1/6)) + (5*(
b*c - a*d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3
*b^(11/6)*d^(1/6)) - (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*
b^(11/6)*d^(1/6)) + (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b
^(11/6)*d^(1/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx = \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6b}$$

$$= \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^2}$$

$$= \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}$$

$$= \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\,\mathrm{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{11/6}} + \frac{(5(bc-ad))\,\mathrm{Subst}\left(\int \right.}{}$$

$$= \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{b} + \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}} + \frac{(5(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x\right.}{4b^{5/3}}$$

$$= \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{b} + \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}} - \frac{5(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right.}{12b^{11/6}\sqrt[6]{d}}$$

$$= \frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{b} - \frac{5(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}b^{11/6}\sqrt[6]{d}} + \frac{5(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}b^{11/6}\sqrt[6]{d}} + \frac{5(bc-ad}{}$$

Mathematica [C] time = 0.0235483, size = 71, normalized size = 0.19

$$\frac{6\sqrt[6]{a+bx}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 1/6, 7/6, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{6}}(bx+a)^{-\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(5/6),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(5/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(5/6), x)
```

Fricas [B] time = 2.35376, size = 6406, normalized size = 16.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="fricas")
```

```
[Out] 1/12*(20*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*
b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)
*arctan(1/3*(2*sqrt(3)*(b^10*c*d - a*b^9*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/
6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15
*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + 2*sqrt(3)*(b^
9*d^2*x + b^9*c*d)*sqrt(((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*
((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^
4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) + (b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d*x + b^4*c)*((b^6*c
^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c
^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c))*((b^6*c^6 - 6
*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + sqrt(3)*(b^6*c^7 - 6*a*b^5*c^
6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*
b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 -
 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x))/(b^
6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^
2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15
*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6
+ a^6*d^7)*x)) + 20*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^
2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^1
1*d))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^10*c*d - a*b^9*d^2)*(b*x + a)^(1/6)*(d
*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c
^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + 2*
sqrt(3)*(b^9*d^2*x + b^9*c*d)*sqrt(-((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x
 + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3
*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) - (b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^4*d*x + b^
4*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c))*(
(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) - sqrt(3)*(b^6*c^7
- 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*
d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b
^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*
d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3
 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*
```

```
c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*
a^5*b*c*d^6 + a^6*d^7)*x)) + 5*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(
b^11*d))^(1/6)*log(25*((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((
b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) + (b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d*x + b^4*c)*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c)) - 5*b*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*log(-25*((b^3*c - a*b^2*d)*
(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*
d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b
^11*d))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(
2/3) - (b^4*d*x + b^4*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))
^(1/3))/(d*x + c)) + 10*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d)
)^(1/6)*log(-5*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) + (b^2*d*x + b^
2*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6))/(d*x + c)) -
 10*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*log(-5*((b*c
 - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) - (b^2*d*x + b^2*c)*((b^6*c^6 - 6*a
*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6))/(d*x + c)) + 12*(b*x + a)^(1/6)*
(d*x + c)^(5/6))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(5/6),x)
```

```
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(5/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1826 $\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=309

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\right.}{b}$$

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*
(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1
/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))
+ (2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)
*d^(1/6)) - Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1
/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(5/6)*d^(1/6)) + Log[b^(
1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*
x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(5/6)*d^(1/6))
```

Rubi [A] time = 0.443833, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x]
```


```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*
(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1
/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))
+ (2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)
*d^(1/6)) - Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1
/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(5/6)*d^(1/6)) + Log[b^(
1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*
x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(5/6)*d^(1/6))
```


Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx = \frac{6\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{b}$$

$$= \frac{6\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^{5/6}} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt[6]{b}+\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}+\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^{5/6}} + \text{2 Su}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{a}}\right.}{2b^{2/3}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} - \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{5/6}\sqrt[6]{d}} + \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+b}}{\sqrt[6]{c+dx}}\right.}{2b^{5/6}\sqrt[6]{d}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} - \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a}}{\sqrt[3]{c+}}\right.}{2b}$$

Mathematica [C] time = 0.0265806, size = 71, normalized size = 0.23

$$\frac{6\sqrt[6]{a+bx}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 1/6, 7/6, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/6))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{5}{6}}\frac{1}{\sqrt[6]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(1/6)), x)
```

Fricas [B] time = 1.7933, size = 1638, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*(1/(b^5*d))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x +
c)^(5/6)*b^4*d*(1/(b^5*d))^(5/6) - 2*sqrt(3)*(b^4*d^2*x + b^4*c*d)*sqrt(((b
*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5*d))^(1/6) + (b^2*d*x + b^2*c)*(1/(b
^5*d))^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c))*(1/(b^5*d))^(5/6
) + sqrt(3)*(d*x + c))/(d*x + c)) - 2*sqrt(3)*(1/(b^5*d))^(1/6)*arctan(-1/3
*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b^4*d*(1/(b^5*d))^(5/6) - 2*sqr
t(3)*(b^4*d^2*x + b^4*c*d)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5
*d))^(1/6) - (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) - (b*x + a)^(1/3)*(d*x + c
)^(2/3))/(d*x + c))*(1/(b^5*d))^(5/6) - sqrt(3)*(d*x + c))/(d*x + c)) + 1/2
*(1/(b^5*d))^(1/6)*log(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5*d))^(1/
6) + (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))
/(d*x + c)) - 1/2*(1/(b^5*d))^(1/6)*log(-4*((b*x + a)^(1/6)*(d*x + c)^(5/6)
*b*(1/(b^5*d))^(1/6) - (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) - (b*x + a)^(1/3
)*(d*x + c)^(2/3))/(d*x + c)) + (1/(b^5*d))^(1/6)*log(((b*d*x + b*c)*(1/(b^
5*d))^(1/6) + (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d*x + c)) - (1/(b^5*d))^(1/
6)*log(-((b*d*x + b*c)*(1/(b^5*d))^(1/6) - (b*x + a)^(1/6)*(d*x + c)^(5/6))
/(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{6}}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(1/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(1/6)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1827 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=30

$$\frac{6\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(1/6))/((b*c - a*d)*(c + d*x)^(1/6))
```

Rubi [A] time = 0.0029776, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x]

[Out] (6*(a + b*x)^(1/6))/((b*c - a*d)*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{(bc-ad)\sqrt[6]{c+dx}}$$

Mathematica [A] time = 0.0072219, size = 30, normalized size = 1.

$$\frac{6\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x]

[Out] (6*(a + b*x)^(1/6))/((b*c - a*d)*(c + d*x)^(1/6))
```

Maple [A] time = 0.004, size = 27, normalized size = 0.9

$$-6\,\frac{\sqrt[6]{bx+a}}{\sqrt[6]{dx+c}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x)
```

```
[Out] -6*(b*x+a)^(1/6)/(d*x+c)^(1/6)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(7/6)), x)
```

Fricas [A] time = 1.46144, size = 96, normalized size = 3.2

$$\frac{6\,(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] 6*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{5}{6}}\,(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(7/6)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1828 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}} dx$

Optimal. Leaf size=66

$$\frac{36b\sqrt[6]{a+bx}}{7\sqrt[6]{c+dx}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{7(c+dx)^{7/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(1/6))/(7*(b*c - a*d)*(c + d*x)^(7/6)) + (36*b*(a + b*x)^(1/6)
)/(7*(b*c - a*d)^2*(c + d*x)^(1/6))
```

Rubi [A] time = 0.0092258, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b\sqrt[6]{a+bx}}{7\sqrt[6]{c+dx}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(7*(b*c - a*d)*(c + d*x)^(7/6)) + (36*b*(a + b*x)^(1/6)
)/(7*(b*c - a*d)^2*(c + d*x)^(1/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}} dx &= \frac{6\sqrt[6]{a+bx}}{7(bc-ad)(c+dx)^{7/6}}+\frac{(6b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}} dx}{7(bc-ad)}\\ &= \frac{6\sqrt[6]{a+bx}}{7(bc-ad)(c+dx)^{7/6}}+\frac{36b\sqrt[6]{a+bx}}{7(bc-ad)^2\sqrt[6]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0156081, size = 46, normalized size = 0.7

$$\frac{6\sqrt[6]{a+bx}(-ad+7bc+6bdx)}{7(c+dx)^{7/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(7*b*c - a*d + 6*b*d*x))/(7*(b*c - a*d)^2*(c + d*x)^(7/6
))
```

Maple [A] time = 0.004, size = 53, normalized size = 0.8

$$-\frac{-36\,bdx+6\,ad-42\,bc}{7\,a^2d^2-14\,abcd+7\,b^2c^2}\sqrt[6]{bx+a}\,(dx+c)^{-\frac{7}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x)
```

```
[Out] -6/7*(b*x+a)^(1/6)*(-6*b*d*x+a*d-7*b*c)/(d*x+c)^(7/6)/(a^2*d^2-2*a*b*c*d+b^
2*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(13/6)), x)
```

Fricas [B] time = 1.52248, size = 255, normalized size = 3.86

$$\frac{6\,(6\,bdx+7\,bc-ad)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{7\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^2+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] 6/7*(6*b*d*x + 7*b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^2*c^4 - 2*a*
b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*
c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1829 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}} \, dx$

Optimal. Leaf size=101

$$\frac{432b^2\sqrt[6]{a+bx}}{91\sqrt[6]{c+dx}(bc-ad)^3}+\frac{72b\sqrt[6]{a+bx}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{13(c+dx)^{13/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(1/6))/(13*(b*c - a*d)*(c + d*x)^(13/6)) + (72*b*(a + b*x)^(1/
6))/(91*(b*c - a*d)^2*(c + d*x)^(7/6)) + (432*b^2*(a + b*x)^(1/6))/(91*(b*c
 - a*d)^3*(c + d*x)^(1/6))
```

Rubi [A] time = 0.0175022, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2\sqrt[6]{a+bx}}{91\sqrt[6]{c+dx}(bc-ad)^3}+\frac{72b\sqrt[6]{a+bx}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(13*(b*c - a*d)*(c + d*x)^(13/6)) + (72*b*(a + b*x)^(1/
6))/(91*(b*c - a*d)^2*(c + d*x)^(7/6)) + (432*b^2*(a + b*x)^(1/6))/(91*(b*c
 - a*d)^3*(c + d*x)^(1/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}} \, dx &= \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}}+\frac{(12b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}} \, dx}{13(bc-ad)}\\
&= \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}}+\frac{72b\sqrt[6]{a+bx}}{91(bc-ad)^2(c+dx)^{7/6}}+\frac{\left(72b^2\right)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}} \, dx}{91(bc-ad)^2}\\
&= \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}}+\frac{72b\sqrt[6]{a+bx}}{91(bc-ad)^2(c+dx)^{7/6}}+\frac{432b^2\sqrt[6]{a+bx}}{91(bc-ad)^3\sqrt[6]{c+dx}}
\end{aligned}$$

Mathematica [A] time = 0.0319607, size = 77, normalized size = 0.76

$$\frac{6\sqrt[6]{a+bx}\left(7a^2d^2-2abd(13c+6dx)+b^2\left(91c^2+156cdx+72d^2x^2\right)\right)}{91(c+dx)^{13/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(7*a^2*d^2 - 2*a*b*d*(13*c + 6*d*x) + b^2*(91*c^2 + 156*
c*d*x + 72*d^2*x^2)))/(91*(b*c - a*d)^3*(c + d*x)^(13/6))
```

Maple [A] time = 0.006, size = 105, normalized size = 1.

$$-\frac{432\,b^2d^2x^2-72\,abd^2x+936\,b^2cdx+42\,a^2d^2-156\,abcd+546\,b^2c^2}{91\,a^3d^3-273\,a^2cbd^2+273\,ab^2c^2d-91\,b^3c^3}\sqrt[6]{bx+a}\,(dx+c)^{-\frac{13}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x)
```

```
[Out] -6/91*(b*x+a)^(1/6)*(72*b^2*d^2*x^2-12*a*b*d^2*x+156*b^2*c*d*x+7*a^2*d^2-26
*a*b*c*d+91*b^2*c^2)/(d*x+c)^(13/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(19/6)), x)
```

Fricas [B] time = 1.5599, size = 517, normalized size = 5.12

$$\frac{6\left(72\,b^2d^2x^2+91\,b^2c^2-26\,abcd+7\,a^2d^2+12\left(13\,b^2cd-abd^2\right)x\right)(bx+a)}{91\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3+\left(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6\right)x^3+3\left(b^3c^4d^2-3\,ab^2c^3d^3+3\,a^2bc^2d^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] 6/91*(72*b^2*d^2*x^2 + 91*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2 + 12*(13*b^2*c*d
 - a*b*d^2)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^3*c^6 - 3*a*b^2*c^5*d + 3
*a^2*b*c^4*d^2 - a^3*c^3*d^3 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d
^5 - a^3*d^6)*x^3 + 3*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^
```

```
3*c*d^5)*x^2 + 3*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d
^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1830 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{25/6}} dx$

Optimal. Leaf size=136

$$\frac{7776b^3\sqrt[6]{a+bx}}{1729\sqrt[6]{c+dx}(bc-ad)^4}+\frac{1296b^2\sqrt[6]{a+bx}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{108b\sqrt[6]{a+bx}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{19(c+dx)^{19/6}(bc-ad)}$$

```
[Out] (6*(a + b*x)^(1/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (108*b*(a + b*x)^(1
/6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6)) + (1296*b^2*(a + b*x)^(1/6))/(172
9*(b*c - a*d)^3*(c + d*x)^(7/6)) + (7776*b^3*(a + b*x)^(1/6))/(1729*(b*c -
a*d)^4*(c + d*x)^(1/6))
```

Rubi [A] time = 0.030887, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3\sqrt[6]{a+bx}}{1729\sqrt[6]{c+dx}(bc-ad)^4}+\frac{1296b^2\sqrt[6]{a+bx}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{108b\sqrt[6]{a+bx}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (108*b*(a + b*x)^(1
/6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6)) + (1296*b^2*(a + b*x)^(1/6))/(172
9*(b*c - a*d)^3*(c + d*x)^(7/6)) + (7776*b^3*(a + b*x)^(1/6))/(1729*(b*c -
a*d)^4*(c + d*x)^(1/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{25/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{(18b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}}\,dx}{19(bc-ad)}$$
$$= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{(216b^2)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,dx}{247(bc-ad)^2}$$
$$= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{1296b^2\sqrt[6]{a+bx}}{1729(bc-ad)^3(c+dx)^{7/6}} + \frac{(12\cdots}{\cdots}$$
$$= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{1296b^2\sqrt[6]{a+bx}}{1729(bc-ad)^3(c+dx)^{7/6}} + \frac{\cdots}{172\cdots}$$

Mathematica [A] time = 0.0475552, size = 118, normalized size = 0.87

$$\frac{6\sqrt[6]{a+bx}\left(21a^2bd^2(19c+6dx)-91a^3d^3-3ab^2d\left(247c^2+228cdx+72d^2x^2\right)+b^3\left(4446c^2dx+1729c^3+4104cd^2x^2+\cdots\right.\right.}{1729(c+dx)^{19/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(-91*a^3*d^3 + 21*a^2*b*d^2*(19*c + 6*d*x) - 3*a*b^2*d*(
247*c^2 + 228*c*d*x + 72*d^2*x^2) + b^3*(1729*c^3 + 4446*c^2*d*x + 4104*c*d
^2*x^2 + 1296*d^3*x^3)))/(1729*(b*c - a*d)^4*(c + d*x)^(19/6))
```

Maple [A] time = 0.008, size = 171, normalized size = 1.3

$$-\frac{-7776\,x^3b^3d^3+1296\,ab^2d^3x^2-24624\,b^3cd^2x^2-756\,a^2bd^3x+4104\,ab^2cd^2x-26676\,b^3c^2dx+546\,a^3d^3-2394\,a^2cb\cdots}{1729\,a^4d^4-6916\,a^3bcd^3+10374\,a^2c^2b^2d^2-6916\,ab^3c^3d+1729\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x)
```

```
[Out] -6/1729*(b*x+a)^(1/6)*(-1296*b^3*d^3*x^3+216*a*b^2*d^3*x^2-4104*b^3*c*d^2*x
^2-126*a^2*b*d^3*x+684*a*b^2*c*d^2*x-4446*b^3*c^2*d*x+91*a^3*d^3-399*a^2*b*
c*d^2+741*a*b^2*c^2*d-1729*b^3*c^3)/(d*x+c)^(19/6)/(a^4*d^4-4*a^3*b*c*d^3+6
*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(25/6)), x)
```

Fricas [B] time = 1.622, size = 872, normalized size = 6.41

$$\frac{6\left(1296\,b^3d^3x^3+1729\,b^3c^3-741\,ab^2c^2d+399\,a^2\right.}{1729\left(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4+\left(b^4c^4d^4-4\,ab^3c^3d^5+6\,a^2b^2c^2d^6-4\,a^3bcd^7+a^4d^8\right)x^4+4\left(b^4c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/1729*(1296*b^3*d^3*x^3 + 1729*b^3*c^3 - 741*a*b^2*c^2*d + 399*a^2*b*c*d^2
 - 91*a^3*d^3 + 216*(19*b^3*c*d^2 - a*b^2*d^3)*x^2 + 18*(247*b^3*c^2*d - 38
*a*b^2*c*d^2 + 7*a^2*b*d^3)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^4*c^8 - 4
*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (b^4*c^4
*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^8)*x^4 +
 4*(b^4*c^5*d^3 - 4*a*b^3*c^4*d^4 + 6*a^2*b^2*c^3*d^5 - 4*a^3*b*c^2*d^6 + a
^4*c*d^7)*x^3 + 6*(b^4*c^6*d^2 - 4*a*b^3*c^5*d^3 + 6*a^2*b^2*c^4*d^4 - 4*a^
3*b*c^3*d^5 + a^4*c^2*d^6)*x^2 + 4*(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6*a^2*b^2
*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(25/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1831 $\int \frac{(c+dx)^{13/6}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=449

$$\frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2}+\frac{91d(a+bx)^{5/6}\sqrt[6]{c+dx}(bc-ad)}{12b^3}-\frac{91\sqrt[6]{d}(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{19/6}}+\cdots$$

```
[Out] (91*d*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b^3) + (13*d*(a + b*
x)^(5/6)*(c + d*x)^(7/6))/(2*b^2) - (6*(c + d*x)^(13/6))/(b*(a + b*x)^(1/6)
) + (91*d^(1/6)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6)
)/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(19/6)) - (91*d^(1/6)*(
b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6
)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(19/6)) + (91*d^(1/6)*(b*c - a*d)^2*ArcT
anh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(19/6)) - (
91*d^(1/6)*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b^(19/6))
+ (91*d^(1/6)*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*
x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b^(19/6
))
```

Rubi [A] time = 0.660012, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {47, 50, 63, 331, 296, 634, 618, 204, 628, 208}

$$\frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2}+\frac{91d(a+bx)^{5/6}\sqrt[6]{c+dx}(bc-ad)}{12b^3}-\frac{91\sqrt[6]{d}(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{19/6}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(13/6)/(a + b*x)^(7/6),x]
```

```
[Out] (91*d*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b^3) + (13*d*(a + b*
x)^(5/6)*(c + d*x)^(7/6))/(2*b^2) - (6*(c + d*x)^(13/6))/(b*(a + b*x)^(1/6)
) + (91*d^(1/6)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6)
)/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(19/6)) - (91*d^(1/6)*(
b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6
)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(19/6)) + (91*d^(1/6)*(b*c - a*d)^2*ArcT
anh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(19/6)) - (
91*d^(1/6)*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b^(19/6))
+ (91*d^(1/6)*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*
x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b^(19/6
))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{13/6}}{(a+bx)^{7/6}}\,dx = -\frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(13d)\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx}{b}$$

$$= \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(91d(bc-ad))\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{12b^2}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\left(91d(bc-ad)^2\right)\int}{72b^3}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\left(91d(bc-ad)^2\right)\mathrm{Su}}{}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\left(91d(bc-ad)^2\right)\mathrm{Su}}{}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\left(91\sqrt[3]{d}(bc-ad)^2\right)\mathrm{S}}{}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{d}(bc-ad)^2\tan}{36b^{19}}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{d}(bc-ad)^2\tan}{36b^{19}}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{d}(bc-ad)^2\tan}{24\sqrt{3}b}$$

Mathematica [C] time = 0.0599391, size = 71, normalized size = 0.16

$$-\frac{6(c+dx)^{13/6}\,{}_2F_1\left(-\frac{13}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(13/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(13/6)*Hypergeometric2F1[-13/6, -1/6, 5/6, (d*(a + b*x))/(-(b
*c) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(13/6))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{13}{6}} (bx+a)^{-\frac{7}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(13/6)/(b*x+a)^(7/6),x)
```

```
[Out] int((d*x+c)^(13/6)/(b*x+a)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{13}{6}}}{(bx+a)^{\frac{7}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(13/6)/(b*x + a)^(7/6), x)
```

Fricas [B] time = 2.98162, size = 12721, normalized size = 28.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/144*(364*sqrt(3)*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66
*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^
7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9
 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^
13)/b^19)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^18*c^2 - 2*a*b^17*c*d + a^2*b^16*
d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 6
6*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b
^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^
9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d
^13)/b^19)^(5/6) - 2*sqrt(3)*(b^17*x + a*b^16)*sqrt(((b^5*c^2 - 2*a*b^4*c*d
 + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c
^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5
- 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8
*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^
12 + a^12*d^13)/b^19)^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^7*x + a*b^6
)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c
^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 -
792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*
b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/3))/(b*x + a))*((b^12
*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 +
 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*
b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*
d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) + sqrt(3)*(a*b^12*c^12*d -
```

12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x))/(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x)) + 364*sqrt(3)*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^18*c^2 - 2*a*b^17*c*d + a^2*b^16*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) - 2*sqrt(3)*(b^17*x + a*b^16)*sqrt(-((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/3))/(b*x + a))*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) - sqrt(3)*(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x))/(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x)) - 91*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*log(8281*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495

```
*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*
c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11
 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
 + (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^
3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a
^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^
3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/3))/
(b*x + a)) + 91*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2
*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^
7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 2
20*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/
b^19)^(1/6)*log(-8281*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6
)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3
 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^
6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3
*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) -
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*
x + a)^(2/3)*(d*x + c)^(1/3) - (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^
11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 -
 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*
b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^1
2 + a^12*d^13)/b^19)^(1/3))/(b*x + a)) - 182*(b^4*x + a*b^3)*((b^12*c^12*d
- 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4
*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*
d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 1
2*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*log(91*((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b^4*x + a*b^3)*((b^12*c^12*d - 12*
a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*
c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 +
 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^1
1*b*c*d^12 + a^12*d^13)/b^19)^(1/6))/(b*x + a)) + 182*(b^4*x + a*b^3)*((b^1
2*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4
+ 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7
*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2
*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*log(91*((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b^4*x + a*b^3)*((b^12*c^1
2*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495
*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*
c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11
 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6))/(b*x + a)) - 12*(6*b^2*d^2*x^
2 - 72*b^2*c^2 + 169*a*b*c*d - 91*a^2*d^2 + (25*b^2*c*d - 13*a*b*d^2)*x)*(b
*x + a)^(5/6)*(d*x + c)^(1/6))/(b^4*x + a*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(13/6)/(b*x+a)**(7/6),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{13}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(13/6)/(b*x + a)^(7/6), x)
```

3.1832 $\int \frac{(c+dx)^{7/6}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=403

$$\frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2}-\frac{7\sqrt[6]{d}(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{d}(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\right.}{12b^{13/6}}$$

```
[Out] (7*d*(a + b*x)^(5/6)*(c + d*x)^(1/6))/b^2 - (6*(c + d*x)^(7/6))/(b*(a + b*x
)^(1/6)) + (7*d^(1/6)*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(
1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]])/(2*Sqrt[3]*b^(13/6)) - (7*d^(1/6)
*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6
)*(c + d*x)^(1/6))]])/(2*Sqrt[3]*b^(13/6)) + (7*d^(1/6)*(b*c - a*d)*ArcTanh[
(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))]])/(3*b^(13/6)) - (7*d^(
1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) -
(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]])/(12*b^(13/6)) + (7*d^(1
/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (
b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]])/(12*b^(13/6))
```

Rubi [A] time = 0.595627, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {47, 50, 63, 331, 296, 634, 618, 204, 628, 208}

$$\frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2}-\frac{7\sqrt[6]{d}(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{d}(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\right.}{12b^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(7/6)/(a + b*x)^(7/6),x]
```

```
[Out] (7*d*(a + b*x)^(5/6)*(c + d*x)^(1/6))/b^2 - (6*(c + d*x)^(7/6))/(b*(a + b*x
)^(1/6)) + (7*d^(1/6)*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(
1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]])/(2*Sqrt[3]*b^(13/6)) - (7*d^(1/6)
*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6
)*(c + d*x)^(1/6))]])/(2*Sqrt[3]*b^(13/6)) + (7*d^(1/6)*(b*c - a*d)*ArcTanh[
(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))]])/(3*b^(13/6)) - (7*d^(
1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) -
(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]])/(12*b^(13/6)) + (7*d^(1
/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (
b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]])/(12*b^(13/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{7/6}}{(a+bx)^{7/6}}\,dx = -\frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d)\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{b}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{5/6}}\,dx}{6b^2}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^3}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^3}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{\left(7\sqrt[3]{d}(bc-ad)\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{13/6}} + \frac{(7\sqrt[3]{}}{}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{13/6}} - \frac{\left(7\sqrt[6]{d}(bc-ad)\right)\text{Subst}\left(\int\right.}{12}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{13/6}} - \frac{7\sqrt[6]{d}(bc-ad)\log\left(\sqrt[3]{b}+\right.}{12b^{13/}}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}b^{13/6}} - \frac{7\sqrt[6]{d}(bc-ad)\tan^{-1}\left(\frac{1+2}{}\right.}{2\sqrt{3}b^{13/6}}$$

Mathematica [C] time = 0.0412782, size = 71, normalized size = 0.18

$$-\frac{6(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(7/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, -1/6, 5/6, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{7}{6}}(bx+a)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(7/6)/(b*x+a)^(7/6),x)

[Out] int((d*x+c)^(7/6)/(b*x+a)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="maxima")

[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(7/6), x)
```

Fricas [B] time = 2.50115, size = 6579, normalized size = 16.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="fricas")

[Out] 1/12*(28*sqrt(3)*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4
*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^
7)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b^12*c - a*b^11*d)*(b*x + a)^(5/6)*(d
*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b
^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(5/6) + 2*
sqrt(3)*(b^12*x + a*b^11)*sqrt(((b^3*c - a*b^2*d)*(b*x + a)^(5/6)*(d*x + c)
^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*
d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6) + (b^2*c^2
- 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*x + a*b^4)*((
b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*
a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/3))/(b*x + a))*((b^6*c^
6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^
2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(5/6) + sqrt(3)*(a*b^6*c^6*d - 6
*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2
*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*
c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b
*d^7)*x))/(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^
3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a
*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5
 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 28*sqrt(3)*(b^3*x + a*b^2)*((b^6*c^6*
d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*
c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b^12*
c - a*b^11*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2
 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c
*d^6 + a^6*d^7)/b^13)^(5/6) + 2*sqrt(3)*(b^12*x + a*b^11)*sqrt(-((b^3*c - a
*b^2*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*
a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 +
 a^6*d^7)/b^13)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*
x + c)^(1/3) - (b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c
^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)
```

```
/b^13)^(1/3))/(b*x + a))*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3
 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)
^(5/6) - sqrt(3)*(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20
*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*
d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*
c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x))/(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2
 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c
*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3
*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 7*(b
^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b
^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*log(
49*((b^3*c - a*b^2*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5
*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6
*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x
+ a)^(2/3)*(d*x + c)^(1/3) + (b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2
+ 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*
d^6 + a^6*d^7)/b^13)^(1/3))/(b*x + a)) - 7*(b^3*x + a*b^2)*((b^6*c^6*d - 6*
a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^
5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*log(-49*((b^3*c - a*b^2*d)*(b*x +
a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3
 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)
^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) -
(b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3
*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/3))/(
b*x + a)) + 14*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c
^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)
/b^13)^(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b^3*x +
 a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3
*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6))/(b*x + a)
) - 14*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 -
 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(
1/6)*log(-7*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b^3*x + a*b^2)*
((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 1
5*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6))/(b*x + a)) + 12*(
b*d*x - 6*b*c + 7*a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6))/(b^3*x + a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(7/6)/(b*x+a)**(7/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(7/6), x)
```

3.1833 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=332

$$-\frac{\sqrt[6]{d}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt[6]{d}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\cdots$$

```
[Out] (-6*(c + d*x)^(1/6))/(b*(a + b*x)^(1/6)) + (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]])/b^(7/6)
 - (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))]])/b^(7/6) + (2*d^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))]])/b^(7/6) - (d^(1/6)*Log[b^(1/3) + (d^(1/
3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c
+ d*x)^(1/6)]])/(2*b^(7/6)) + (d^(1/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]])/(2
*b^(7/6))
```

Rubi [A] time = 0.540582, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{\sqrt[6]{d}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt[6]{d}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(1/6))/(b*(a + b*x)^(1/6)) + (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]])/b^(7/6)
 - (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))]])/b^(7/6) + (2*d^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))]])/b^(7/6) - (d^(1/6)*Log[b^(1/3) + (d^(1/
3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c
+ d*x)^(1/6)]])/(2*b^(7/6)) + (d^(1/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]])/(2
*b^(7/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{7/6}}\,dx = -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{d\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{5/6}}\,dx}{b}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{(6d)\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^2}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{(6d)\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{\left(2\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^{7/6}} + \frac{\left(2\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}+\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}+\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x\right.}{b^{7/6}}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}} - \frac{\sqrt[6]{d}\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{7/6}} + \frac{\sqrt[6]{d}\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}} - \frac{\sqrt[6]{d}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{7/6}} + \frac{\sqrt[6]{d}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a}}{\sqrt[3]{c+}}\right.}{2b}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{7/6}} - \frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{7/6}} + \frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}} - \cdots$$

Mathematica [C] time = 0.0264023, size = 71, normalized size = 0.21

$$-\frac{6\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, -1/6, 5/6, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \sqrt[6]{dx+c}\,(bx+a)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(7/6),x)
```

```
[Out] int((d*x+c)^(1/6)/(b*x+a)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(7/6), x)
```

Fricas [B] time = 1.88771, size = 1704, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(3)*(b^2*x + a*b)*(d/b^7)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a
)^(5/6)*(d*x + c)^(1/6)*b^6*(d/b^7)^(5/6) - 2*sqrt(3)*(b^7*x + a*b^6)*sqrt(
((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6) + (b^3*x + a*b^2)*(d/b^7)^
(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a))*(d/b^7)^(5/6) + sqrt(3)
*(b*d*x + a*d))/(b*d*x + a*d)) + 4*sqrt(3)*(b^2*x + a*b)*(d/b^7)^(1/6)*arct
an(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*b^6*(d/b^7)^(5/6) - 2*sq
rt(3)*(b^7*x + a*b^6)*sqrt(-((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6
) - (b^3*x + a*b^2)*(d/b^7)^(1/3) - (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x +
 a))*(d/b^7)^(5/6) - sqrt(3)*(b*d*x + a*d))/(b*d*x + a*d)) - (b^2*x + a*b)*
(d/b^7)^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6) + (b^3
*x + a*b^2)*(d/b^7)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a)) + (
b^2*x + a*b)*(d/b^7)^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7
)^(1/6) - (b^3*x + a*b^2)*(d/b^7)^(1/3) - (b*x + a)^(2/3)*(d*x + c)^(1/3))/
(b*x + a)) - 2*(b^2*x + a*b)*(d/b^7)^(1/6)*log(((b^2*x + a*b)*(d/b^7)^(1/6)
 + (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b*x + a)) + 2*(b^2*x + a*b)*(d/b^7)^(1
/6)*log(-((b^2*x + a*b)*(d/b^7)^(1/6) - (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b
*x + a)) + 12*(b*x + a)^(5/6)*(d*x + c)^(1/6))/(b^2*x + a*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(7/6),x)
```

```
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(7/6), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1834 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=30

$$-\frac{6\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}(bc-ad)}$$

```
[Out] (-6*(c + d*x)^(1/6))/((b*c - a*d)*(a + b*x)^(1/6))
```

Rubi [A] time = 0.0033471, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{6\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (-6*(c + d*x)^(1/6))/((b*c - a*d)*(a + b*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{5/6}}\,dx = -\frac{6\sqrt[6]{c+dx}}{(bc-ad)\sqrt[6]{a+bx}}$$

Mathematica [A] time = 0.0075832, size = 30, normalized size = 1.

$$\frac{6\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (6*(c + d*x)^(1/6))/((-(b*c) + a*d)*(a + b*x)^(1/6))
```

Maple [A] time = 0.003, size = 27, normalized size = 0.9

$$6\,\frac{\sqrt[6]{dx+c}}{\sqrt[6]{bx+a}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x)
```

```
[Out] 6/(b*x+a)^(1/6)*(d*x+c)^(1/6)/(a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(5/6)), x)
```

Fricas [A] time = 1.47153, size = 97, normalized size = 3.23

$$-\frac{6\,(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] -6*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(5/6),x)
```

```
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(5/6)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1835 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=64

$$-\frac{36d(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}(c+dx)^{5/6}(bc-ad)}$$

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6)) - (36*d*(a + b*x)^(5/6))/(
5*(b*c - a*d)^2*(c + d*x)^(5/6))
```

Rubi [A] time = 0.0105572, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{36d(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x]
```

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6)) - (36*d*(a + b*x)^(5/6))/(
5*(b*c - a*d)^2*(c + d*x)^(5/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/6}(c+dx)^{11/6}}\,dx &= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}-\frac{(6d)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{11/6}}\,dx}{bc-ad}\\ &= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}(c+dx)^{5/6}}-\frac{36d(a+bx)^{5/6}}{5(bc-ad)^2(c+dx)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.015811, size = 45, normalized size = 0.7

$$-\frac{6(ad+5bc+6bdx)}{5\sqrt[6]{a+bx}(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (-6*(5*b*c + a*d + 6*b*d*x))/(5*(b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(5/
6))
```

Maple [A] time = 0.005, size = 53, normalized size = 0.8

$$-\frac{36\,bdx + 6\,ad + 30\,bc}{5\,a^2d^2 - 10\,abcd + 5\,b^2c^2}\frac{1}{\sqrt[6]{bx + a}}(dx + c)^{-\frac{5}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x)
```

```
[Out] -6/5*(6*b*d*x+a*d+5*b*c)/(b*x+a)^(1/6)/(d*x+c)^(5/6)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx + a)^{\frac{7}{6}}(dx + c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(11/6)), x)
```

Fricas [B] time = 1.60743, size = 270, normalized size = 4.22

$$-\frac{6\,(6\,bdx + 5\,bc + ad)(bx + a)^{\frac{5}{6}}(dx + c)^{\frac{1}{6}}}{5\left(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + \left(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3\right)x^2 + \left(b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] -6/5*(6*b*d*x + 5*b*c + a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(11/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(11/6)), x)
```

3.1836 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=98

$$-\frac{432bd(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^3}-\frac{72d(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}(c+dx)^{11/6}(bc-ad)}$$

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(11/6)) - (72*d*(a + b*x)^(5/6))/
(11*(b*c - a*d)^2*(c + d*x)^(11/6)) - (432*b*d*(a + b*x)^(5/6))/(55*(b*c -
a*d)^3*(c + d*x)^(5/6))
```

Rubi [A] time = 0.0202185, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{432bd(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^3}-\frac{72d(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x]
```


```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(11/6)) - (72*d*(a + b*x)^(5/6))/
(11*(b*c - a*d)^2*(c + d*x)^(11/6)) - (432*b*d*(a + b*x)^(5/6))/(55*(b*c -
a*d)^3*(c + d*x)^(5/6))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/6}(c+dx)^{17/6}}\,dx &= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}(c+dx)^{11/6}}-\frac{(12d)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{17/6}}\,dx}{bc-ad}\\ &= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}(c+dx)^{11/6}}-\frac{72d(a+bx)^{5/6}}{11(bc-ad)^2(c+dx)^{11/6}}-\frac{(72bd)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{11/6}}\,dx}{11(bc-ad)^2}\\ &= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}(c+dx)^{11/6}}-\frac{72d(a+bx)^{5/6}}{11(bc-ad)^2(c+dx)^{11/6}}-\frac{432bd(a+bx)^{5/6}}{55(bc-ad)^3(c+dx)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.0299008, size = 77, normalized size = 0.79

$$-\frac{6\left(-5a^2d^2+2abd(11c+6dx)+b^2\left(55c^2+132cdx+72d^2x^2\right)\right)}{55\sqrt[6]{a+bx}(c+dx)^{11/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (-6*(-5*a^2*d^2 + 2*a*b*d*(11*c + 6*d*x) + b^2*(55*c^2 + 132*c*d*x + 72*d^2
*x^2)))/(55*(b*c - a*d)^3*(a + b*x)^(1/6)*(c + d*x)^(11/6))
```

Maple [A] time = 0.006, size = 105, normalized size = 1.1

$$-\frac{-432\,b^2d^2x^2-72\,abd^2x-792\,b^2cdx+30\,a^2d^2-132\,abcd-330\,b^2c^2}{55\,a^3d^3-165\,a^2cbd^2+165\,ab^2c^2d-55\,b^3c^3}\frac{1}{\sqrt[6]{bx+a}}(dx+c)^{-\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x)
```

```
[Out] -6/55*(-72*b^2*d^2*x^2-12*a*b*d^2*x-132*b^2*c*d*x+5*a^2*d^2-22*a*b*c*d-55*b
^2*c^2)/(b*x+a)^(1/6)/(d*x+c)^(11/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b
^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(17/6)), x)
```

Fricas [B] time = 1.67105, size = 562, normalized size = 5.73

$$-\frac{6\left(72\,b^2d^2x^2+55\,b^2c^2+22\,abcd-5\,a^2d^2+12\left(11\,b^2cd+a\right.\right.}{55\left(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5\right)x^3+\left(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] -6/55*(72*b^2*d^2*x^2 + 55*b^2*c^2 + 22*a*b*c*d - 5*a^2*d^2 + 12*(11*b^2*c*
d + a*b*d^2)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*
```

```
b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2
*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*
d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(17/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(17/6)), x)
```

3.1837 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{23/6}}\,dx$

Optimal. Leaf size=134

$$-\frac{7776b^2d(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^4}-\frac{1296bd(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^3}-\frac{108d(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}(c+dx)^{17/6}(bc-ad)}$$

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(17/6)) - (108*d*(a + b*x)^(5/6))
/(17*(b*c - a*d)^2*(c + d*x)^(17/6)) - (1296*b*d*(a + b*x)^(5/6))/(187*(b*c
 - a*d)^3*(c + d*x)^(11/6)) - (7776*b^2*d*(a + b*x)^(5/6))/(935*(b*c - a*d)
^4*(c + d*x)^(5/6))
```

Rubi [A] time = 0.0327841, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{7776b^2d(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^4}-\frac{1296bd(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^3}-\frac{108d(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x]
```


```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(17/6)) - (108*d*(a + b*x)^(5/6))
/(17*(b*c - a*d)^2*(c + d*x)^(17/6)) - (1296*b*d*(a + b*x)^(5/6))/(187*(b*c
 - a*d)^3*(c + d*x)^(11/6)) - (7776*b^2*d*(a + b*x)^(5/6))/(935*(b*c - a*d)
^4*(c + d*x)^(5/6))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{23/6}}\,dx = -\frac{6}{(bc-ad)\sqrt[6]{a+bx}(c+dx)^{17/6}} - \frac{(18d)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{23/6}}\,dx}{bc-ad}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}(c+dx)^{17/6}} - \frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}} - \frac{(216bd)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{17/6}}\,dx}{17(bc-ad)^2}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}(c+dx)^{17/6}} - \frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}} - \frac{1296bd(a+bx)^{5/6}}{187(bc-ad)^3(c+dx)^{11/6}} - \frac{(1}{}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}(c+dx)^{17/6}} - \frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}} - \frac{1296bd(a+bx)^{5/6}}{187(bc-ad)^3(c+dx)^{11/6}} - \frac{}{93}$$

Mathematica [A] time = 0.0467412, size = 118, normalized size = 0.88

$$-\frac{6\left(-15a^2bd^2(17c+6dx)+55a^3d^3+3ab^2d\left(187c^2+204cdx+72d^2x^2\right)+b^3\left(3366c^2dx+935c^3+3672cd^2x^2+1296d^3x^3\right)\right)}{935\sqrt[6]{a+bx}(c+dx)^{17/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (-6*(55*a^3*d^3 - 15*a^2*b*d^2*(17*c + 6*d*x) + 3*a*b^2*d*(187*c^2 + 204*c*
d*x + 72*d^2*x^2) + b^3*(935*c^3 + 3366*c^2*d*x + 3672*c*d^2*x^2 + 1296*d^3
*x^3)))/(935*(b*c - a*d)^4*(a + b*x)^(1/6)*(c + d*x)^(17/6))
```

Maple [A] time = 0.007, size = 171, normalized size = 1.3

$$-\frac{7776\,x^3b^3d^3+1296\,ab^2d^3x^2+22032\,b^3cd^2x^2-540\,a^2bd^3x+3672\,ab^2cd^2x+20196\,b^3c^2dx+330\,a^3d^3-1530\,a^2cbd^2+\cdots}{935\,a^4d^4-3740\,a^3bcd^3+5610\,b^2d^2c^2a^2-3740\,ab^3c^3d+935\,b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x)
```

```
[Out] -6/935*(1296*b^3*d^3*x^3+216*a*b^2*d^3*x^2+3672*b^3*c*d^2*x^2-90*a^2*b*d^3*
x+612*a*b^2*c*d^2*x+3366*b^3*c^2*d*x+55*a^3*d^3-255*a^2*b*c*d^2+561*a*b^2*c
^2*d+935*b^3*c^3)/(b*x+a)^(1/6)/(d*x+c)^(17/6)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2
*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(23/6)), x)
```

Fricas [B] time = 1.67966, size = 946, normalized size = 7.06

$$-\frac{6\left(1296\,b^3d^3x^3+935\,b^3c^3\right.}{935\left(ab^4c^7-4\,a^2b^3c^6d+6\,a^3b^2c^5d^2-4\,a^4bc^4d^3+a^5c^3d^4+\left(b^5c^4d^3-4\,ab^4c^3d^4+6\,a^2b^3c^2d^5-4\,a^3b^2cd^6+a^4bd^7\right)x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] -6/935*(1296*b^3*d^3*x^3 + 935*b^3*c^3 + 561*a*b^2*c^2*d - 255*a^2*b*c*d^2
+ 55*a^3*d^3 + 216*(17*b^3*c*d^2 + a*b^2*d^3)*x^2 + 18*(187*b^3*c^2*d + 34*
a*b^2*c*d^2 - 5*a^2*b*d^3)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^4*c^7 -
4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*
c^4*d^3 - 4*a*b^4*c^3*d^4 + 6*a^2*b^3*c^2*d^5 - 4*a^3*b^2*c*d^6 + a^4*b*d^7
)*x^4 + (3*b^5*c^5*d^2 - 11*a*b^4*c^4*d^3 + 14*a^2*b^3*c^3*d^4 - 6*a^3*b^2*
c^2*d^5 - a^4*b*c*d^6 + a^5*d^7)*x^3 + 3*(b^5*c^6*d - 3*a*b^4*c^5*d^2 + 2*a
^2*b^3*c^4*d^3 + 2*a^3*b^2*c^3*d^4 - 3*a^4*b*c^2*d^5 + a^5*c*d^6)*x^2 + (b^
5*c^7 - a*b^4*c^6*d - 6*a^2*b^3*c^5*d^2 + 14*a^3*b^2*c^4*d^3 - 11*a^4*b*c^3
*d^4 + 3*a^5*c^2*d^5)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(23/6)), x)
```

3.1838 $\int \frac{(c+dx)^{11/6}}{(a+bx)^{7/6}} dx$

Optimal. Leaf size=80

$$-\frac{6(c+dx)^{5/6}(bc-ad)\,{}_2F_1\left(-\frac{11}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] (-6*(b*c - a*d)*(c + d*x)^(5/6)*Hypergeometric2F1[-11/6, -1/6, 5/6, -((d*(a
 + b*x))/(b*c - a*d))])/(b^2*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5
/6))
```

Rubi [A] time = 0.0238371, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$-\frac{6(c+dx)^{5/6}(bc-ad)\,{}_2F_1\left(-\frac{11}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(11/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(b*c - a*d)*(c + d*x)^(5/6)*Hypergeometric2F1[-11/6, -1/6, 5/6, -((d*(a
 + b*x))/(b*c - a*d))])/(b^2*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5
/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(c+dx)^{11/6}}{(a+bx)^{7/6}}\,dx = \frac{\left((bc-ad)(c+dx)^{5/6}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}{(a+bx)^{7/6}}dx}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$
$$= -\frac{6(bc-ad)(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Mathematica [A] time = 0.0482562, size = 71, normalized size = 0.89

$$-\frac{6(c+dx)^{11/6}\,{}_2F_1\left(-\frac{11}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(11/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(11/6)*Hypergeometric2F1[-11/6, -1/6, 5/6, (d*(a + b*x))/(-(b
*c) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(11/6))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{11}{6}}(bx+a)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(11/6)/(b*x+a)^(7/6),x)
```

```
[Out] int((d*x+c)^(11/6)/(b*x+a)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(11/6)/(b*x + a)^(7/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{11}{6}}}{b^2x^2+2\,abx+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(11/6)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(11/6)/(b*x+a)**(7/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(11/6)/(b*x + a)^(7/6), x)
```

3.1839 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=72

$$-\frac{6(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] (-6*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, -1/6, 5/6, -((d*(a + b*x))/(b*c
 - a*d))])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Rubi [A] time = 0.020786, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$-\frac{6(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, -1/6, 5/6, -((d*(a + b*x))/(b*c
 - a*d))])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/6}}{(a+bx)^{7/6}}\,dx &= \frac{(c+dx)^{5/6}\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}{(a+bx)^{7/6}}\,dx}{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}\\
&= -\frac{6(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}
\end{aligned}$$

Mathematica [A] time = 0.0221944, size = 71, normalized size = 0.99

$$-\frac{6(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, -1/6, 5/6, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{6}} (bx+a)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(7/6),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(7/6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}{b^2x^2+2\,abx+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(7/6),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1840 $\int \frac{1}{(a+bx)^{7/6}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=72

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\sqrt[6]{c+dx}}$$

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -((
d*(a + b*x))/(b*c - a*d))])/(b*(a + b*x)^(1/6)*(c + d*x)^(1/6))
```

Rubi [A] time = 0.0205284, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -((
d*(a + b*x))/(b*c - a*d))])/(b*(a + b*x)^(1/6)*(c + d*x)^(1/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{7/6}\sqrt[6]{c+dx}}\,dx &= \frac{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{(a+bx)^{7/6}\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[6]{c+dx}}\\
&= -\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\sqrt[6]{c+dx}}
\end{aligned}$$

Mathematica [A] time = 0.0228953, size = 71, normalized size = 0.99

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/6)*(c + d*x)^(1/6))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{7}{6}}\frac{1}{\sqrt[6]{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(1/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}{b^2dx^3+a^2c+\left(b^2c+2\,abd\right)x^2+\left(2\,abc+a^2d\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(1/6),x)
```

```
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(1/6)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1841 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{7/6}} dx$

Optimal. Leaf size=79

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{\sqrt[6]{a+bx}\sqrt[6]{c+dx}(bc-ad)}$$

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 7/6, 5/6, -((
d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(1/6))
```

Rubi [A] time = 0.0206713, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{\sqrt[6]{a+bx}\sqrt[6]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 7/6, 5/6, -((
d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(1/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{7/6}(c+dx)^{7/6}}\,dx &= \frac{\left(b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{(a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}}\,dx}{(bc-ad)\sqrt[6]{c+dx}}\\
&= -\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)\sqrt[6]{a+bx}\sqrt[6]{c+dx}}
\end{aligned}$$

Mathematica [A] time = 0.0351487, size = 71, normalized size = 0.9

$$-\frac{6\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[-1/6, 7/6, 5/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/6)*(c + d*x)^(7/6))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{7}{6}}(dx+c)^{-\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(7/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}{b^2d^2x^4+a^2c^2+2\left(b^2cd+abd^2\right)x^3+\left(b^2c^2+4\,abcd+a^2d^2\right)x^2+2\left(abc^2+a^2cd\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*
d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c
*d)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}\,(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(7/6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(7/6)), x)
```

3.1842 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{13/6}} dx$

Optimal. Leaf size=80

$$-\frac{6b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{\sqrt[6]{a+bx}\sqrt[6]{c+dx}(bc-ad)^2}$$

```
[Out] (-6*b*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 13/6, 5/6,
-((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(1/
6))
```

Rubi [A] time = 0.0209196, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$-\frac{6b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{\sqrt[6]{a+bx}\sqrt[6]{c+dx}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (-6*b*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 13/6, 5/6,
-((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(1/
6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/6}(c+dx)^{13/6}}\,dx &= \frac{\left(b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{(a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}}\,dx}{(bc-ad)^2\sqrt[6]{c+dx}}\\ &= -\frac{6b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2\sqrt[6]{a+bx}\sqrt[6]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0447769, size = 71, normalized size = 0.89

$$-\frac{6\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(13/6)*Hypergeometric2F1[-1/6, 13/6, 5/6, (
d*(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/6)*(c + d*x)^(13/6))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{7}{6}}(dx+c)^{-\frac{13}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

```
[Out] int(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(13/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}{b^2d^3x^5+a^2c^3+\left(3\,b^2cd^2+2\,abd^3\right)x^4+\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^3+\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^2+\left(2\,ab\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*d^3*x^5 + a^2*c^3 + (3*b^2*c*
d^2 + 2*a*b*d^3)*x^4 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^3 + (b^2*c^3
 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^2 + (2*a*b*c^3 + 3*a^2*c^2*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(13/6)), x)
```

3.1843 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=82

$$-\frac{6b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{\sqrt[6]{a+bx}\sqrt[6]{c+dx}(bc-ad)^3}$$

```
[Out] (-6*b^2*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 19/6, 5/6
, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^3*(a + b*x)^(1/6)*(c + d*x)^(
1/6))
```

Rubi [A] time = 0.0223072, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$-\frac{6b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{\sqrt[6]{a+bx}\sqrt[6]{c+dx}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (-6*b^2*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 19/6, 5/6
, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^3*(a + b*x)^(1/6)*(c + d*x)^(
1/6))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/6}(c+dx)^{19/6}}\,dx &= \frac{\left(b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{(a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{19/6}}\,dx}{(bc-ad)^3\sqrt[6]{c+dx}}\\ &= -\frac{6b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3\sqrt[6]{a+bx}\sqrt[6]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0344366, size = 79, normalized size = 0.96

$$-\frac{6b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{\sqrt[6]{a+bx}(c+dx)^{7/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (-6*b*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[-1/6, 19/6, 5/6,
(d*(a + b*x))/(-(b*c) + a*d)])/((b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(7/
6))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{7}{6}}(dx+c)^{-\frac{19}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x)
```

```
[Out] int(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(19/6)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}{b^2d^4x^6+a^2c^4+2\left(2\,b^2cd^3+abd^4\right)x^5+\left(6\,b^2c^2d^2+8\,abcd^3+a^2d^4\right)x^4+4\left(b^2c^3d+3\,abc^2d^2+a^2cd^3\right)x^3+\left(b^2c^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*d^4*x^6 + a^2*c^4 + 2*(2*b^2*
c*d^3 + a*b*d^4)*x^5 + (6*b^2*c^2*d^2 + 8*a*b*c*d^3 + a^2*d^4)*x^4 + 4*(b^2
*c^3*d + 3*a*b*c^2*d^2 + a^2*c*d^3)*x^3 + (b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^
2*d^2)*x^2 + 2*(a*b*c^4 + 2*a^2*c^3*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(19/6)), x)
```

3.1844 $\int (a+bx)^m(a+b(2+m)x)\,dx$

Optimal. Leaf size=11

$$x(a+bx)^{m+1}$$

```
[Out] x*(a + b*x)^(1 + m)
```

Rubi [A] time = 0.0022903, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {34}

$$x(a+bx)^{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a + b*(2 + m)*x),x]
```

```
[Out] x*(a + b*x)^(1 + m)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\int (a+bx)^m(a+b(2+m)x)\,dx = x(a+bx)^{1+m}$$

Mathematica [A] time = 0.0108006, size = 11, normalized size = 1.

$$x(a+bx)^{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a + b*(2 + m)*x),x]
```

```
[Out] x*(a + b*x)^(1 + m)
```

Maple [A] time = 0.003, size = 12, normalized size = 1.1

$$x\,(bx+a)^{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(a+b*(2+m)*x),x)
```

```
[Out] x*(b*x+a)^(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71111, size = 36, normalized size = 3.27

$$(bx^2 + ax)(bx + a)^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^m
```

Sympy [B] time = 0.251909, size = 20, normalized size = 1.82

$$ax(a + bx)^m + bx^2(a + bx)^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(a+b*(2+m)*x),x)
```

```
[Out] a*x*(a + b*x)**m + b*x**2*(a + b*x)**m
```

Giac [B] time = 1.06955, size = 31, normalized size = 2.82

$$(bx + a)^m bx^2 + (bx + a)^m ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="giac")
```

```
[Out] (b*x + a)^m*b*x^2 + (b*x + a)^m*a*x
```

3.1845 $\int (a+bx)^m (c+dx)^n \, dx$

Optimal. Leaf size=61

$$-\frac{(a+bx)^{m+1}(c+dx)^{n+1} {}_2F_1\left(1,m+n+2;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)}$$

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(1 + n)*Hypergeometric2F1[1, 2 + m + n, 2 +
n, (b*(c + d*x))/(b*c - a*d)])/((b*c - a*d)*(1 + n)))
```

Rubi [A] time = 0.0256269, antiderivative size = 74, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {70, 69}

$$\frac{(a+bx)^{m+1}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a
+ b*x))/(b*c - a*d))])/(b*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int (a+bx)^m (c+dx)^n \, dx &= \left((c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right) \int (a+bx)^m \left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n \, dx \\
&= \frac{(a+bx)^{1+m}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{b(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.0264761, size = 73, normalized size = 1.2

$$\frac{(a+bx)^{m+1}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} {}_2F_1\left(m+1,-n;m+2;\frac{d(a+bx)}{ad-bc}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, (d*(a +
b*x))/(-(b*c) + a*d)])/(b*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int (bx+a)^m (dx+c)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n,x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^m (dx+c)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m*(d*x + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^m (dx+c)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^m (c+dx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n,x)
```

```
[Out] Integral((a + b*x)**m*(c + d*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^m (dx + c)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^m*(d*x + c)^n, x)
```

3.1846 $\int (a+bx)^m (c+dx)^3\,dx$

Optimal. Leaf size=110

$$\frac{3d^2(bc-ad)(a+bx)^{m+3}}{b^4(m+3)}+\frac{(bc-ad)^3(a+bx)^{m+1}}{b^4(m+1)}+\frac{3d(bc-ad)^2(a+bx)^{m+2}}{b^4(m+2)}+\frac{d^3(a+bx)^{m+4}}{b^4(m+4)}$$

```
[Out] ((b*c - a*d)^3*(a + b*x)^(1 + m))/(b^4*(1 + m)) + (3*d*(b*c - a*d)^2*(a + b
*x)^(2 + m))/(b^4*(2 + m)) + (3*d^2*(b*c - a*d)*(a + b*x)^(3 + m))/(b^4*(3
+ m)) + (d^3*(a + b*x)^(4 + m))/(b^4*(4 + m))
```

Rubi [A] time = 0.0545642, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3d^2(bc-ad)(a+bx)^{m+3}}{b^4(m+3)}+\frac{(bc-ad)^3(a+bx)^{m+1}}{b^4(m+1)}+\frac{3d(bc-ad)^2(a+bx)^{m+2}}{b^4(m+2)}+\frac{d^3(a+bx)^{m+4}}{b^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^(1 + m))/(b^4*(1 + m)) + (3*d*(b*c - a*d)^2*(a + b
*x)^(2 + m))/(b^4*(2 + m)) + (3*d^2*(b*c - a*d)*(a + b*x)^(3 + m))/(b^4*(3
+ m)) + (d^3*(a + b*x)^(4 + m))/(b^4*(4 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^m(c+dx)^3\,dx=\int \left(\frac{(bc-ad)^3(a+bx)^m}{b^3}+\frac{3d(bc-ad)^2(a+bx)^{1+m}}{b^3}+\frac{3d^2(bc-ad)(a+bx)^{2+m}}{b^3}+\frac{d^3(a+bx)^{3+m}}{b^3}\right)dx$$
$$=\frac{(bc-ad)^3(a+bx)^{1+m}}{b^4(1+m)}+\frac{3d(bc-ad)^2(a+bx)^{2+m}}{b^4(2+m)}+\frac{3d^2(bc-ad)(a+bx)^{3+m}}{b^4(3+m)}+\frac{d^3(a+bx)^{4+m}}{b^4(4+m)}$$

Mathematica [A] time = 0.0752514, size = 94, normalized size = 0.85

$$\frac{(a+bx)^{m+1}\left(\frac{3d^2(a+bx)^2(bc-ad)}{m+3}+\frac{3d(a+bx)(bc-ad)^2}{m+2}+\frac{(bc-ad)^3}{m+1}+\frac{d^3(a+bx)^3}{m+4}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^3,x]
```

```
[Out] ((a + b*x)^(1 + m)*((b*c - a*d)^3/(1 + m) + (3*d*(b*c - a*d)^2*(a + b*x))/(
2 + m) + (3*d^2*(b*c - a*d)*(a + b*x)^2)/(3 + m) + (d^3*(a + b*x)^3)/(4 + m
```

```
)))/b^4
```

Maple [B] time = 0.009, size = 389, normalized size = 3.5

$$-\frac{(bx+a)^{1+m}\left(-b^3d^3m^3x^3-3\,b^3cd^2m^3x^2-6\,b^3d^3m^2x^3+3\,ab^2d^3m^2x^2-3\,b^3c^2dm^3x-21\,b^3cd^2m^2x^2-11\,b^3d^3mx^3+6\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^3,x)
```

```
[Out] -(b*x+a)^(1+m)*(-b^3*d^3*m^3*x^3-3*b^3*c*d^2*m^3*x^2-6*b^3*d^3*m^2*x^3+3*a*
b^2*d^3*m^2*x^2-3*b^3*c^2*d*m^3*x-21*b^3*c*d^2*m^2*x^2-11*b^3*d^3*m*x^3+6*a
*b^2*c*d^2*m^2*x+9*a*b^2*d^3*m*x^2-b^3*c^3*m^3-24*b^3*c^2*d*m^2*x-42*b^3*c*
d^2*m*x^2-6*b^3*d^3*x^3-6*a^2*b*d^3*m*x+3*a*b^2*c^2*d*m^2+30*a*b^2*c*d^2*m*
x+6*a*b^2*d^3*x^2-9*b^3*c^3*m^2-57*b^3*c^2*d*m*x-24*b^3*c*d^2*x^2-6*a^2*b*c
*d^2*m-6*a^2*b*d^3*x+21*a*b^2*c^2*d*m+24*a*b^2*c*d^2*x-26*b^3*c^3*m-36*b^3*
c^2*d*x+6*a^3*d^3-24*a^2*b*c*d^2+36*a*b^2*c^2*d-24*b^3*c^3)/b^4/(m^4+10*m^3
+35*m^2+50*m+24)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.95845, size = 1011, normalized size = 9.19

$$\frac{\left(ab^3c^3m^3+24\,ab^3c^3-36\,a^2b^2c^2d+24\,a^3bcd^2-6\,a^4d^3+\left(b^4d^3m^3+6\,b^4d^3m^2+11\,b^4d^3m+6\,b^4d^3\right)x^4+\left(24\,b^4cd^2+\left(3\,b^4\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^3*c^3*m^3 + 24*a*b^3*c^3 - 36*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 6*a^4*d
^3 + (b^4*d^3*m^3 + 6*b^4*d^3*m^2 + 11*b^4*d^3*m + 6*b^4*d^3)*x^4 + (24*b^4
*c*d^2 + (3*b^4*c*d^2 + a*b^3*d^3)*m^3 + 3*(7*b^4*c*d^2 + a*b^3*d^3)*m^2 +
2*(21*b^4*c*d^2 + a*b^3*d^3)*m)*x^3 + 3*(3*a*b^3*c^3 - a^2*b^2*c^2*d)*m^2 +
 3*(12*b^4*c^2*d + (b^4*c^2*d + a*b^3*c*d^2)*m^3 + (8*b^4*c^2*d + 5*a*b^3*c
*d^2 - a^2*b^2*d^3)*m^2 + (19*b^4*c^2*d + 4*a*b^3*c*d^2 - a^2*b^2*d^3)*m)*x
^2 + (26*a*b^3*c^3 - 21*a^2*b^2*c^2*d + 6*a^3*b*c*d^2)*m + (24*b^4*c^3 + (b
^4*c^3 + 3*a*b^3*c^2*d)*m^3 + 3*(3*b^4*c^3 + 7*a*b^3*c^2*d - 2*a^2*b^2*c*d^
2)*m^2 + 2*(13*b^4*c^3 + 18*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 3*a^3*b*d^3)*m
)*x)*(b*x + a)^m/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)
```

Sympy [A] time = 4.21127, size = 4056, normalized size = 36.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**3,x)
```

```
[Out] Piecewise((a**m*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(
b, 0)), (6*a**5*d**3*log(a/b + x)/(6*a**5*b**4 + 18*a**4*b**5*x + 18*a**3*b
**6*x**2 + 6*a**2*b**7*x**3) + 2*a**5*d**3/(6*a**5*b**4 + 18*a**4*b**5*x +
18*a**3*b**6*x**2 + 6*a**2*b**7*x**3) + 18*a**4*b*d**3*x*log(a/b + x)/(6*a*
*5*b**4 + 18*a**4*b**5*x + 18*a**3*b**6*x**2 + 6*a**2*b**7*x**3) + 18*a**3*
b**2*d**3*x**2*log(a/b + x)/(6*a**5*b**4 + 18*a**4*b**5*x + 18*a**3*b**6*x*
*2 + 6*a**2*b**7*x**3) - 9*a**3*b**2*d**3*x**2/(6*a**5*b**4 + 18*a**4*b**5*
x + 18*a**3*b**6*x**2 + 6*a**2*b**7*x**3) - 2*a**2*b**3*c**3/(6*a**5*b**4 +
 18*a**4*b**5*x + 18*a**3*b**6*x**2 + 6*a**2*b**7*x**3) + 6*a**2*b**3*d**3*
x**3*log(a/b + x)/(6*a**5*b**4 + 18*a**4*b**5*x + 18*a**3*b**6*x**2 + 6*a**
2*b**7*x**3) - 9*a**2*b**3*d**3*x**3/(6*a**5*b**4 + 18*a**4*b**5*x + 18*a**
3*b**6*x**2 + 6*a**2*b**7*x**3) + 9*a*b**4*c**2*d*x**2/(6*a**5*b**4 + 18*a*
*4*b**5*x + 18*a**3*b**6*x**2 + 6*a**2*b**7*x**3) + 6*a*b**4*c*d**2*x**3/(6
*a**5*b**4 + 18*a**4*b**5*x + 18*a**3*b**6*x**2 + 6*a**2*b**7*x**3) + 3*b**
5*c**2*d*x**3/(6*a**5*b**4 + 18*a**4*b**5*x + 18*a**3*b**6*x**2 + 6*a**2*b*
*7*x**3), Eq(m, -4)), (-6*a**4*d**3*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5
*x + 2*a*b**6*x**2) - 3*a**4*d**3/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x
**2) + 6*a**3*b*c*d**2*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6
*x**2) + 3*a**3*b*c*d**2/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) - 12
*a**3*b*d**3*x*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) +
 12*a**2*b**2*c*d**2*x*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6
*x**2) - 6*a**2*b**2*d**3*x**2*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x +
2*a*b**6*x**2) + 6*a**2*b**2*d**3*x**2/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b
**6*x**2) - a*b**3*c**3/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + 6*a
*b**3*c*d**2*x**2*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2
) - 6*a*b**3*c*d**2*x**2/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + 2*
a*b**3*d**3*x**3/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) + 3*b**4*c**
2*d*x**2/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2), Eq(m, -3)), (6*a**3
*d**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3*d**3/(2*a*b**4 + 2*b**5*x
) - 12*a**2*b*c*d**2*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 12*a**2*b*c*d**2/
(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d**3*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
+ 6*a*b**2*c**2*d*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a*b**2*c**2*d/(2*a
*b**4 + 2*b**5*x) - 12*a*b**2*c*d**2*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) -
 3*a*b**2*d**3*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c**3/(2*a*b**4 + 2*b**5*
x) + 6*b**3*c**2*d*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*b**3*c*d**2*x**
2/(2*a*b**4 + 2*b**5*x) + b**3*d**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(m, -2)),
 (-a**3*d**3*log(a/b + x)/b**4 + 3*a**2*c*d**2*log(a/b + x)/b**3 + a**2*d**
3*x/b**3 - 3*a*c**2*d*log(a/b + x)/b**2 - 3*a*c*d**2*x/b**2 - a*d**3*x**2/(
2*b**2) + c**3*log(a/b + x)/b + 3*c**2*d*x/b + 3*c*d**2*x**2/(2*b) + d**3*x
**3/(3*b), Eq(m, -1)), (-6*a**4*d**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 6*a**3*b*c*d**2*m*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 24*a**3*b*c*
d**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24
*b**4) + 6*a**3*b*d**3*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4
*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*c**2*d*m**2*(a + b*x)**m/(b**4*m
**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 21*a**2*b**2*c**
2*d*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 2
4*b**4) - 36*a**2*b**2*c**2*d*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b
**4*m**2 + 50*b**4*m + 24*b**4) - 6*a**2*b**2*c*d**2*m**2*x*(a + b*x)**m/(b
**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 24*a**2*b**
2*c*d**2*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**
```

```
4*m + 24*b**4) - 3*a**2*b**2*d**3*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b*
*4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*d**3*m*x**2*(a
+ b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) +
 a*b**3*c**3*m**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 5
0*b**4*m + 24*b**4) + 9*a*b**3*c**3*m**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*
m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 26*a*b**3*c**3*m*(a + b*x)**m/
(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 24*a*b**3
*c**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 2
4*b**4) + 3*a*b**3*c**2*d*m**3*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 3
5*b**4*m**2 + 50*b**4*m + 24*b**4) + 21*a*b**3*c**2*d*m**2*x*(a + b*x)**m/(
b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 36*a*b**3*
c**2*d*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*
m + 24*b**4) + 3*a*b**3*c*d**2*m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*
m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 15*a*b**3*c*d**2*m**2*x**2*(a
+ b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) +
 12*a*b**3*c*d**2*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m
**2 + 50*b**4*m + 24*b**4) + a*b**3*d**3*m**3*x**3*(a + b*x)**m/(b**4*m**4
+ 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 3*a*b**3*d**3*m**2*x
**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) + 2*a*b**3*d**3*m*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b*
*4*m**2 + 50*b**4*m + 24*b**4) + b**4*c**3*m**3*x*(a + b*x)**m/(b**4*m**4 +
 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 9*b**4*c**3*m**2*x*(a
 + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4)
+ 26*b**4*c**3*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 +
50*b**4*m + 24*b**4) + 24*b**4*c**3*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**
3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 3*b**4*c**2*d*m**3*x**2*(a + b*x)
**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 24*b*
*4*c**2*d*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 +
 50*b**4*m + 24*b**4) + 57*b**4*c**2*d*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*
b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 36*b**4*c**2*d*x**2*(a +
b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 3
*b**4*c*d**2*m**3*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**
2 + 50*b**4*m + 24*b**4) + 21*b**4*c*d**2*m**2*x**3*(a + b*x)**m/(b**4*m**4
 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 42*b**4*c*d**2*m*x*
*3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b
**4) + 24*b**4*c*d**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4
*m**2 + 50*b**4*m + 24*b**4) + b**4*d**3*m**3*x**4*(a + b*x)**m/(b**4*m**4
+ 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 6*b**4*d**3*m**2*x**
4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b*
*4) + 11*b**4*d**3*m*x**4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*
m**2 + 50*b**4*m + 24*b**4) + 6*b**4*d**3*x**4*(a + b*x)**m/(b**4*m**4 + 10
*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4), True))
```

Giac [B] time = 1.07578, size = 1125, normalized size = 10.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^4*d^3*m^3*x^4 + 3*(b*x + a)^m*b^4*c*d^2*m^3*x^3 + (b*x + a)^
m*a*b^3*d^3*m^3*x^3 + 6*(b*x + a)^m*b^4*d^3*m^2*x^4 + 3*(b*x + a)^m*b^4*c^2
*d*m^3*x^2 + 3*(b*x + a)^m*a*b^3*c*d^2*m^3*x^2 + 21*(b*x + a)^m*b^4*c*d^2*m
^2*x^3 + 3*(b*x + a)^m*a*b^3*d^3*m^2*x^3 + 11*(b*x + a)^m*b^4*d^3*m*x^4 + (
b*x + a)^m*b^4*c^3*m^3*x + 3*(b*x + a)^m*a*b^3*c^2*d*m^3*x + 24*(b*x + a)^m
*b^4*c^2*d*m^2*x^2 + 15*(b*x + a)^m*a*b^3*c*d^2*m^2*x^2 - 3*(b*x + a)^m*a^2
```

```
*b^2*d^3*m^2*x^2 + 42*(b*x + a)^m*b^4*c*d^2*m*x^3 + 2*(b*x + a)^m*a*b^3*d^3
*m*x^3 + 6*(b*x + a)^m*b^4*d^3*x^4 + (b*x + a)^m*a*b^3*c^3*m^3 + 9*(b*x + a
)^m*b^4*c^3*m^2*x + 21*(b*x + a)^m*a*b^3*c^2*d*m^2*x - 6*(b*x + a)^m*a^2*b^
2*c*d^2*m^2*x + 57*(b*x + a)^m*b^4*c^2*d*m*x^2 + 12*(b*x + a)^m*a*b^3*c*d^2
*m*x^2 - 3*(b*x + a)^m*a^2*b^2*d^3*m*x^2 + 24*(b*x + a)^m*b^4*c*d^2*x^3 + 9
*(b*x + a)^m*a*b^3*c^3*m^2 - 3*(b*x + a)^m*a^2*b^2*c^2*d*m^2 + 26*(b*x + a)
^m*b^4*c^3*m*x + 36*(b*x + a)^m*a*b^3*c^2*d*m*x - 24*(b*x + a)^m*a^2*b^2*c*
d^2*m*x + 6*(b*x + a)^m*a^3*b*d^3*m*x + 36*(b*x + a)^m*b^4*c^2*d*x^2 + 26*(
b*x + a)^m*a*b^3*c^3*m - 21*(b*x + a)^m*a^2*b^2*c^2*d*m + 6*(b*x + a)^m*a^3
*b*c*d^2*m + 24*(b*x + a)^m*b^4*c^3*x + 24*(b*x + a)^m*a*b^3*c^3 - 36*(b*x
+ a)^m*a^2*b^2*c^2*d + 24*(b*x + a)^m*a^3*b*c*d^2 - 6*(b*x + a)^m*a^4*d^3)/
(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)
```

3.1847 $\int (a+bx)^m (c+dx)^2 \, dx$

Optimal. Leaf size=78

$$\frac{(bc-ad)^2(a+bx)^{m+1}}{b^3(m+1)}+\frac{2d(bc-ad)(a+bx)^{m+2}}{b^3(m+2)}+\frac{d^2(a+bx)^{m+3}}{b^3(m+3)}$$

```
[Out] ((b*c - a*d)^2*(a + b*x)^(1 + m))/(b^3*(1 + m)) + (2*d*(b*c - a*d)*(a + b*x
)^(2 + m))/(b^3*(2 + m)) + (d^2*(a + b*x)^(3 + m))/(b^3*(3 + m))
```

Rubi [A] time = 0.0318711, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{(bc-ad)^2(a+bx)^{m+1}}{b^3(m+1)}+\frac{2d(bc-ad)(a+bx)^{m+2}}{b^3(m+2)}+\frac{d^2(a+bx)^{m+3}}{b^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^(1 + m))/(b^3*(1 + m)) + (2*d*(b*c - a*d)*(a + b*x
)^(2 + m))/(b^3*(2 + m)) + (d^2*(a + b*x)^(3 + m))/(b^3*(3 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^m}{b^2}+\frac{2d(bc-ad)(a+bx)^{1+m}}{b^2}+\frac{d^2(a+bx)^{2+m}}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^{1+m}}{b^3(1+m)}+\frac{2d(bc-ad)(a+bx)^{2+m}}{b^3(2+m)}+\frac{d^2(a+bx)^{3+m}}{b^3(3+m)}\end{aligned}$$

Mathematica [A] time = 0.073742, size = 67, normalized size = 0.86

$$\frac{(a+bx)^{m+1}\left(\frac{2d(a+bx)(bc-ad)}{m+2}+\frac{(bc-ad)^2}{m+1}+\frac{d^2(a+bx)^2}{m+3}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^2,x]
```

```
[Out] ((a + b*x)^(1 + m)*((b*c - a*d)^2/(1 + m) + (2*d*(b*c - a*d)*(a + b*x))/(2
+ m) + (d^2*(a + b*x)^2)/(3 + m)))/b^3
```

Maple [B] time = 0.006, size = 159, normalized size = 2.

$$\frac{(bx+a)^{1+m}\left(b^2d^2m^2x^2+2\,b^2cdm^2x+3\,b^2d^2mx^2-2\,abd^2mx+b^2c^2m^2+8\,b^2cdmx+2\,b^2d^2x^2-2\,abcdm-2\,abd^2x+\right.}{b^3\left(m^3+6\,m^2+11\,m+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^2,x)
```

```
[Out] (b*x+a)^(1+m)*(b^2*d^2*m^2*x^2+2*b^2*c*d*m^2*x+3*b^2*d^2*m*x^2-2*a*b*d^2*m*
x+b^2*c^2*m^2+8*b^2*c*d*m*x+2*b^2*d^2*x^2-2*a*b*c*d*m-2*a*b*d^2*x+5*b^2*c^2
*m+6*b^2*c*d*x+2*a^2*d^2-6*a*b*c*d+6*b^2*c^2)/b^3/(m^3+6*m^2+11*m+6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.84051, size = 478, normalized size = 6.13

$$\frac{\left(ab^2c^2m^2+6\,ab^2c^2-6\,a^2bcd+2\,a^3d^2+\left(b^3d^2m^2+3\,b^3d^2m+2\,b^3d^2\right)x^3+\left(6\,b^3cd+\left(2\,b^3cd+ab^2d^2\right)m^2+\left(8\,b^3cd+\right.\right.\right.}{b^3m^3+6\,b^3m^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^2*c^2*m^2 + 6*a*b^2*c^2 - 6*a^2*b*c*d + 2*a^3*d^2 + (b^3*d^2*m^2 + 3*b
^3*d^2*m + 2*b^3*d^2)*x^3 + (6*b^3*c*d + (2*b^3*c*d + a*b^2*d^2)*m^2 + (8*b
^3*c*d + a*b^2*d^2)*m)*x^2 + (5*a*b^2*c^2 - 2*a^2*b*c*d)*m + (6*b^3*c^2 + (
b^3*c^2 + 2*a*b^2*c*d)*m^2 + (5*b^3*c^2 + 6*a*b^2*c*d - 2*a^2*b*d^2)*m)*x)*
(b*x + a)^m/(b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)
```

Sympy [A] time = 1.87896, size = 1504, normalized size = 19.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**2,x)
```

```
[Out] Piecewise((a**m*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(b, 0)), (2*a**3*d**2*
log(a/b + x)/(2*a**3*b**3 + 4*a**2*b**4*x + 2*a*b**5*x**2) + a**3*d**2/(2*a
**3*b**3 + 4*a**2*b**4*x + 2*a*b**5*x**2) + 4*a**2*b*d**2*x*log(a/b + x)/(2
*a**3*b**3 + 4*a**2*b**4*x + 2*a*b**5*x**2) - a*b**2*c**2/(2*a**3*b**3 + 4*
a**2*b**4*x + 2*a*b**5*x**2) + 2*a*b**2*d**2*x**2*log(a/b + x)/(2*a**3*b**3
```

```
 + 4*a**2*b**4*x + 2*a*b**5*x**2) - 2*a*b**2*d**2*x**2/(2*a**3*b**3 + 4*a**
2*b**4*x + 2*a*b**5*x**2) + 2*b**3*c*d*x**2/(2*a**3*b**3 + 4*a**2*b**4*x +
2*a*b**5*x**2), Eq(m, -3)), (-2*a**2*d**2*log(a/b + x)/(a*b**3 + b**4*x) -
2*a**2*d**2/(a*b**3 + b**4*x) + 2*a*b*c*d*log(a/b + x)/(a*b**3 + b**4*x) +
2*a*b*c*d/(a*b**3 + b**4*x) - 2*a*b*d**2*x*log(a/b + x)/(a*b**3 + b**4*x) -
 b**2*c**2/(a*b**3 + b**4*x) + 2*b**2*c*d*x*log(a/b + x)/(a*b**3 + b**4*x)
+ b**2*d**2*x**2/(a*b**3 + b**4*x), Eq(m, -2)), (a**2*d**2*log(a/b + x)/b**
3 - 2*a*c*d*log(a/b + x)/b**2 - a*d**2*x/b**2 + c**2*log(a/b + x)/b + 2*c*d
*x/b + d**2*x**2/(2*b), Eq(m, -1)), (2*a**3*d**2*(a + b*x)**m/(b**3*m**3 +
6*b**3*m**2 + 11*b**3*m + 6*b**3) - 2*a**2*b*c*d*m*(a + b*x)**m/(b**3*m**3
+ 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 6*a**2*b*c*d*(a + b*x)**m/(b**3*m**3
+ 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 2*a**2*b*d**2*m*x*(a + b*x)**m/(b**3*
m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + a*b**2*c**2*m**2*(a + b*x)**m/(b
**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 5*a*b**2*c**2*m*(a + b*x)**m
/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 6*a*b**2*c**2*(a + b*x)**
m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 2*a*b**2*c*d*m**2*x*(a +
 b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 6*a*b**2*c*d*m*x*
(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + a*b**2*d**2*m
**2*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + a*b*
*2*d**2*m*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3)
+ b**3*c**2*m**2*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b*
*3) + 5*b**3*c**2*m*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6
*b**3) + 6*b**3*c**2*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m +
6*b**3) + 2*b**3*c*d*m**2*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b
**3*m + 6*b**3) + 8*b**3*c*d*m*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 +
 11*b**3*m + 6*b**3) + 6*b**3*c*d*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**
2 + 11*b**3*m + 6*b**3) + b**3*d**2*m**2*x**3*(a + b*x)**m/(b**3*m**3 + 6*b
**3*m**2 + 11*b**3*m + 6*b**3) + 3*b**3*d**2*m*x**3*(a + b*x)**m/(b**3*m**3
 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 2*b**3*d**2*x**3*(a + b*x)**m/(b**3*
m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3), True))
```

Giac [B] time = 1.05417, size = 520, normalized size = 6.67

$$(bx+a)^m b^3 d^2 m^2 x^3 + 2\,(bx+a)^m b^3 c d m^2 x^2 + (bx+a)^m a b^2 d^2 m^2 x^2 + 3\,(bx+a)^m b^3 d^2 m x^3 + (bx+a)^m b^3 c^2 m^2 x + 2\,(bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^3*d^2*m^2*x^3 + 2*(b*x + a)^m*b^3*c*d*m^2*x^2 + (b*x + a)^m*
a*b^2*d^2*m^2*x^2 + 3*(b*x + a)^m*b^3*d^2*m*x^3 + (b*x + a)^m*b^3*c^2*m^2*x
 + 2*(b*x + a)^m*a*b^2*c*d*m^2*x + 8*(b*x + a)^m*b^3*c*d*m*x^2 + (b*x + a)^
m*a*b^2*d^2*m*x^2 + 2*(b*x + a)^m*b^3*d^2*x^3 + (b*x + a)^m*a*b^2*c^2*m^2 +
 5*(b*x + a)^m*b^3*c^2*m*x + 6*(b*x + a)^m*a*b^2*c*d*m*x - 2*(b*x + a)^m*a^
2*b*d^2*m*x + 6*(b*x + a)^m*b^3*c*d*x^2 + 5*(b*x + a)^m*a*b^2*c^2*m - 2*(b*
x + a)^m*a^2*b*c*d*m + 6*(b*x + a)^m*b^3*c^2*x + 6*(b*x + a)^m*a*b^2*c^2 -
6*(b*x + a)^m*a^2*b*c*d + 2*(b*x + a)^m*a^3*d^2)/(b^3*m^3 + 6*b^3*m^2 + 11*
b^3*m + 6*b^3)
```

3.1848 $\int (a+bx)^m(c+dx)\,dx$

Optimal. Leaf size=46

$$\frac{(bc-ad)(a+bx)^{m+1}}{b^2(m+1)}+\frac{d(a+bx)^{m+2}}{b^2(m+2)}$$

```
[Out] ((b*c - a*d)*(a + b*x)^(1 + m))/(b^2*(1 + m)) + (d*(a + b*x)^(2 + m))/(b^2*
(2 + m))
```

Rubi [A] time = 0.0182928, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(bc-ad)(a+bx)^{m+1}}{b^2(m+1)}+\frac{d(a+bx)^{m+2}}{b^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^(1 + m))/(b^2*(1 + m)) + (d*(a + b*x)^(2 + m))/(b^2*
(2 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^m}{b}+\frac{d(a+bx)^{1+m}}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^{1+m}}{b^2(1+m)}+\frac{d(a+bx)^{2+m}}{b^2(2+m)}\end{aligned}$$

Mathematica [A] time = 0.0301487, size = 41, normalized size = 0.89

$$\frac{(a+bx)^{m+1}(-ad+bc(m+2)+bd(m+1)x)}{b^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(-(a*d) + b*c*(2 + m) + b*d*(1 + m)*x))/(b^2*(1 + m)*(2
+ m))
```

Maple [A] time = 0.002, size = 49, normalized size = 1.1

$$-\frac{(bx+a)^{1+m}(-bdmx-bcm-bdx+ad-2\,bc)}{b^2\left(m^2+3\,m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c),x)
```

```
[Out] -(b*x+a)^(1+m)*(-b*d*m*x-b*c*m-b*d*x+a*d-2*b*c)/b^2/(m^2+3*m+2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79118, size = 171, normalized size = 3.72

$$\frac{\left(abcm+2\,abc-a^2d+\left(b^2dm+b^2d\right)x^2+\left(2\,b^2c+\left(b^2c+abd\right)m\right)x\right)(bx+a)^m}{b^2m^2+3\,b^2m+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*b*c*m + 2*a*b*c - a^2*d + (b^2*d*m + b^2*d)*x^2 + (2*b^2*c + (b^2*c + a*
b*d)*m)*x)*(b*x + a)^m/(b^2*m^2 + 3*b^2*m + 2*b^2)
```

Sympy [A] time = 0.804482, size = 377, normalized size = 8.2

$$\begin{cases} a^m\left(cx+\frac{dx^2}{2}\right) \\ \frac{ad\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x}+\frac{ad}{ab^2+b^3x}-\frac{bc}{ab^2+b^3x}+\frac{bdx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} \\ -\frac{ad\log\left(\frac{a}{b}+x\right)}{b^2}+\frac{c\log\left(\frac{a}{b}+x\right)}{b}+\frac{dx}{b} \\ -\frac{a^2d(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{abcm(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{2abc(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{abdmx(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{b^2cmx(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{2b^2cx(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{b^2dmx^2(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c),x)
```

```
[Out] Piecewise((a**m*(c*x + d*x**2/2), Eq(b, 0)), (a*d*log(a/b + x)/(a*b**2 + b*
*3*x) + a*d/(a*b**2 + b**3*x) - b*c/(a*b**2 + b**3*x) + b*d*x*log(a/b + x)/
(a*b**2 + b**3*x), Eq(m, -2)), (-a*d*log(a/b + x)/b**2 + c*log(a/b + x)/b +
 d*x/b, Eq(m, -1)), (-a**2*d*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) +
 a*b*c*m*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + 2*a*b*c*(a + b*x)**
```

```
m/(b**2*m**2 + 3*b**2*m + 2*b**2) + a*b*d*m*x*(a + b*x)**m/(b**2*m**2 + 3*b
**2*m + 2*b**2) + b**2*c*m*x*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) +
 2*b**2*c*x*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + b**2*d*m*x**2*(a
 + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + b**2*d*x**2*(a + b*x)**m/(b**2
*m**2 + 3*b**2*m + 2*b**2), True))
```

Giac [B] time = 1.08224, size = 178, normalized size = 3.87

$$\frac{(bx+a)^m b^2 dmx^2 + (bx+a)^m b^2 cmx + (bx+a)^m abdmx + (bx+a)^m b^2 dx^2 + (bx+a)^m abcm + 2\,(bx+a)^m b^2 cx + 2\,(bx}{b^2m^2 + 3\,b^2m + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^2*d*m*x^2 + (b*x + a)^m*b^2*c*m*x + (b*x + a)^m*a*b*d*m*x +
(b*x + a)^m*b^2*d*x^2 + (b*x + a)^m*a*b*c*m + 2*(b*x + a)^m*b^2*c*x + 2*(b*
x + a)^m*a*b*c - (b*x + a)^m*a^2*d)/(b^2*m^2 + 3*b^2*m + 2*b^2)
```

3.1849 $\int \frac{(a+bx)^m}{c+dx}\,dx$

Optimal. Leaf size=51

$$\frac{(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)}$$

```
[Out] ((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/(b*c
 - a*d))])/((b*c - a*d)*(1 + m))
```

Rubi [A] time = 0.0106833, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {68}

$$\frac{(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/(c + d*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/(b*c
 - a*d))])/((b*c - a*d)*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^m}{c+dx}\,dx=\frac{(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(1+m)}$$

Mathematica [A] time = 0.0101233, size = 51, normalized size = 1.

$$-\frac{(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{d(a+bx)}{ad-bc}\right)}{(m+1)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/(c + d*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*x))/(-(b*
c) + a*d)])/((-(b*c) + a*d)*(1 + m)))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c),x)
```

```
[Out] int((b*x+a)^m/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^m}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^m}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c),x)
```

```
[Out] Integral((a + b*x)**m/(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="giac")

[Out] integrate((b*x + a)^m/(d*x + c), x)
```

3.1850 $\int \frac{(a+bx)^m}{(c+dx)^2} dx$

Optimal. Leaf size=52

$$\frac{b(a+bx)^{m+1} {}_2F_1\left(2,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)^2}$$

```
[Out] (b*(a + b*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(a + b*x))/(b*
c - a*d))])/((b*c - a*d)^2*(1 + m))
```

Rubi [A] time = 0.0108203, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {68}

$$\frac{b(a+bx)^{m+1} {}_2F_1\left(2,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/(c + d*x)^2,x]
```

```
[Out] (b*(a + b*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(a + b*x))/(b*
c - a*d))])/((b*c - a*d)^2*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^m}{(c+dx)^2} dx = \frac{b(a+bx)^{1+m} {}_2F_1\left(2,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2(1+m)}$$

Mathematica [A] time = 0.0136869, size = 52, normalized size = 1.

$$\frac{b(a+bx)^{m+1} {}_2F_1\left(2,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/(c + d*x)^2,x]
```

```
[Out] (b*(a + b*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(a + b*x))/(b*
c - a*d))])/((b*c - a*d)^2*(1 + m))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c)^2,x)
```

```
[Out] int((b*x+a)^m/(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^m}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^m}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*x)**m/(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^m/(d*x + c)^2, x)
```

3.1851 $\int \frac{(a+bx)^m}{(c+dx)^3} dx$

Optimal. Leaf size=54

$$\frac{b^2(a+bx)^{m+1} {}_2F_1\left(3,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)^3}$$

```
[Out] (b^2*(a + b*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(a + b*x))/(
b*c - a*d))])/((b*c - a*d)^3*(1 + m))
```

Rubi [A] time = 0.0131582, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {68}

$$\frac{b^2(a+bx)^{m+1} {}_2F_1\left(3,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/(c + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(a + b*x))/(
b*c - a*d))])/((b*c - a*d)^3*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^m}{(c+dx)^3} dx = \frac{b^2(a+bx)^{1+m} {}_2F_1\left(3,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3(1+m)}$$

Mathematica [A] time = 0.0138603, size = 54, normalized size = 1.

$$\frac{b^2(a+bx)^{m+1} {}_2F_1\left(3,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/(c + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(a + b*x))/(
b*c - a*d))])/((b*c - a*d)^3*(1 + m))
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c)^3,x)
```

```
[Out] int((b*x+a)^m/(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^m}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^m}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c)**3,x)
```

```
[Out] Integral((a + b*x)**m/(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^m/(d*x + c)^3, x)
```

3.1852 $\int (a+bx)^3(c+dx)^n\,dx$

Optimal. Leaf size=111

$$-\frac{3b^2(bc-ad)(c+dx)^{n+3}}{d^4(n+3)}-\frac{(bc-ad)^3(c+dx)^{n+1}}{d^4(n+1)}+\frac{3b(bc-ad)^2(c+dx)^{n+2}}{d^4(n+2)}+\frac{b^3(c+dx)^{n+4}}{d^4(n+4)}$$

```
[Out] -(((b*c - a*d)^3*(c + d*x)^(1 + n))/(d^4*(1 + n))) + (3*b*(b*c - a*d)^2*(c
+ d*x)^(2 + n))/(d^4*(2 + n)) - (3*b^2*(b*c - a*d)*(c + d*x)^(3 + n))/(d^4*
(3 + n)) + (b^3*(c + d*x)^(4 + n))/(d^4*(4 + n))
```

Rubi [A] time = 0.0566002, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^2(bc-ad)(c+dx)^{n+3}}{d^4(n+3)}-\frac{(bc-ad)^3(c+dx)^{n+1}}{d^4(n+1)}+\frac{3b(bc-ad)^2(c+dx)^{n+2}}{d^4(n+2)}+\frac{b^3(c+dx)^{n+4}}{d^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^n,x]
```

```
[Out] -(((b*c - a*d)^3*(c + d*x)^(1 + n))/(d^4*(1 + n))) + (3*b*(b*c - a*d)^2*(c
+ d*x)^(2 + n))/(d^4*(2 + n)) - (3*b^2*(b*c - a*d)*(c + d*x)^(3 + n))/(d^4*
(3 + n)) + (b^3*(c + d*x)^(4 + n))/(d^4*(4 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^n\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^n}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{1+n}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{2+n}}{d^3}+\frac{b^3(c+dx)^{3+n}}{d^3}\right)dx\\ &= -\frac{(bc-ad)^3(c+dx)^{1+n}}{d^4(1+n)}+\frac{3b(bc-ad)^2(c+dx)^{2+n}}{d^4(2+n)}-\frac{3b^2(bc-ad)(c+dx)^{3+n}}{d^4(3+n)}+\frac{b^3(c+dx)^{4+n}}{d^4(4+n)}\end{aligned}$$

Mathematica [A] time = 0.0745138, size = 95, normalized size = 0.86

$$\frac{(c+dx)^{n+1}\left(-\frac{3b^2(c+dx)^2(bc-ad)}{n+3}+\frac{3b(c+dx)(bc-ad)^2}{n+2}-\frac{(bc-ad)^3}{n+1}+\frac{b^3(c+dx)^3}{n+4}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*(-((b*c - a*d)^3/(1 + n)) + (3*b*(b*c - a*d)^2*(c + d*x)
)/(2 + n) - (3*b^2*(b*c - a*d)*(c + d*x)^2)/(3 + n) + (b^3*(c + d*x)^3)/(4
```

```
+ n)))/d^4
```

Maple [B] time = 0.008, size = 386, normalized size = 3.5

$$\frac{(dx+c)^{1+n}\left(b^3d^3n^3x^3+3\,ab^2d^3n^3x^2+6\,b^3d^3n^2x^3+3\,a^2bd^3n^3x+21\,ab^2d^3n^2x^2-3\,b^3cd^2n^2x^2+11\,b^3d^3nx^3+a^3d^3n^3+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^n,x)
```

```
[Out] (d*x+c)^(1+n)*(b^3*d^3*n^3*x^3+3*a*b^2*d^3*n^3*x^2+6*b^3*d^3*n^2*x^3+3*a^2*
b*d^3*n^3*x+21*a*b^2*d^3*n^2*x^2-3*b^3*c*d^2*n^2*x^2+11*b^3*d^3*n*x^3+a^3*d
^3*n^3+24*a^2*b*d^3*n^2*x-6*a*b^2*c*d^2*n^2*x+42*a*b^2*d^3*n*x^2-9*b^3*c*d^
2*n*x^2+6*b^3*d^3*x^3+9*a^3*d^3*n^2-3*a^2*b*c*d^2*n^2+57*a^2*b*d^3*n*x-30*a
*b^2*c*d^2*n*x+24*a*b^2*d^3*x^2+6*b^3*c^2*d*n*x-6*b^3*c*d^2*x^2+26*a^3*d^3*
n-21*a^2*b*c*d^2*n+36*a^2*b*d^3*x+6*a*b^2*c^2*d*n-24*a*b^2*c*d^2*x+6*b^3*c^
2*d*x+24*a^3*d^3-36*a^2*b*c*d^2+24*a*b^2*c^2*d-6*b^3*c^3)/d^4/(n^4+10*n^3+3
5*n^2+50*n+24)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.76685, size = 1011, normalized size = 9.11

$$\frac{\left(a^3cd^3n^3-6\,b^3c^4+24\,ab^2c^3d-36\,a^2bc^2d^2+24\,a^3cd^3+\left(b^3d^4n^3+6\,b^3d^4n^2+11\,b^3d^4n+6\,b^3d^4\right)x^4+\left(24\,ab^2d^4+\left(b^3cd^3\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a^3*c*d^3*n^3 - 6*b^3*c^4 + 24*a*b^2*c^3*d - 36*a^2*b*c^2*d^2 + 24*a^3*c*d
^3 + (b^3*d^4*n^3 + 6*b^3*d^4*n^2 + 11*b^3*d^4*n + 6*b^3*d^4)*x^4 + (24*a*b
^2*d^4 + (b^3*c*d^3 + 3*a*b^2*d^4)*n^3 + 3*(b^3*c*d^3 + 7*a*b^2*d^4)*n^2 +
2*(b^3*c*d^3 + 21*a*b^2*d^4)*n)*x^3 - 3*(a^2*b*c^2*d^2 - 3*a^3*c*d^3)*n^2 +
 3*(12*a^2*b*d^4 + (a*b^2*c*d^3 + a^2*b*d^4)*n^3 - (b^3*c^2*d^2 - 5*a*b^2*c
*d^3 - 8*a^2*b*d^4)*n^2 - (b^3*c^2*d^2 - 4*a*b^2*c*d^3 - 19*a^2*b*d^4)*n)*x
^2 + (6*a*b^2*c^3*d - 21*a^2*b*c^2*d^2 + 26*a^3*c*d^3)*n + (24*a^3*d^4 + (3
*a^2*b*c*d^3 + a^3*d^4)*n^3 - 3*(2*a*b^2*c^2*d^2 - 7*a^2*b*c*d^3 - 3*a^3*d^
4)*n^2 + 2*(3*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 18*a^2*b*c*d^3 + 13*a^3*d^4)*n
)*x)*(d*x + c)^n/(d^4*n^4 + 10*d^4*n^3 + 35*d^4*n^2 + 50*d^4*n + 24*d^4)
```

Sympy [A] time = 4.44555, size = 4056, normalized size = 36.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4), Eq(
d, 0)), (-2*a**3*c**2*d**3/(6*c**5*d**4 + 18*c**4*d**5*x + 18*c**3*d**6*x**
2 + 6*c**2*d**7*x**3) + 9*a**2*b*c*d**4*x**2/(6*c**5*d**4 + 18*c**4*d**5*x
+ 18*c**3*d**6*x**2 + 6*c**2*d**7*x**3) + 3*a**2*b*d**5*x**3/(6*c**5*d**4 +
 18*c**4*d**5*x + 18*c**3*d**6*x**2 + 6*c**2*d**7*x**3) + 6*a*b**2*c*d**4*x
**3/(6*c**5*d**4 + 18*c**4*d**5*x + 18*c**3*d**6*x**2 + 6*c**2*d**7*x**3) +
 6*b**3*c**5*log(c/d + x)/(6*c**5*d**4 + 18*c**4*d**5*x + 18*c**3*d**6*x**2
 + 6*c**2*d**7*x**3) + 2*b**3*c**5/(6*c**5*d**4 + 18*c**4*d**5*x + 18*c**3*
d**6*x**2 + 6*c**2*d**7*x**3) + 18*b**3*c**4*d*x*log(c/d + x)/(6*c**5*d**4
+ 18*c**4*d**5*x + 18*c**3*d**6*x**2 + 6*c**2*d**7*x**3) + 18*b**3*c**3*d**
2*x**2*log(c/d + x)/(6*c**5*d**4 + 18*c**4*d**5*x + 18*c**3*d**6*x**2 + 6*c
**2*d**7*x**3) - 9*b**3*c**3*d**2*x**2/(6*c**5*d**4 + 18*c**4*d**5*x + 18*c
**3*d**6*x**2 + 6*c**2*d**7*x**3) + 6*b**3*c**2*d**3*x**3*log(c/d + x)/(6*c
**5*d**4 + 18*c**4*d**5*x + 18*c**3*d**6*x**2 + 6*c**2*d**7*x**3) - 9*b**3*
c**2*d**3*x**3/(6*c**5*d**4 + 18*c**4*d**5*x + 18*c**3*d**6*x**2 + 6*c**2*d
**7*x**3), Eq(n, -4)), (-a**3*c*d**3/(2*c**3*d**4 + 4*c**2*d**5*x + 2*c*d**
6*x**2) + 3*a**2*b*d**4*x**2/(2*c**3*d**4 + 4*c**2*d**5*x + 2*c*d**6*x**2)
+ 6*a*b**2*c**3*d*log(c/d + x)/(2*c**3*d**4 + 4*c**2*d**5*x + 2*c*d**6*x**2
) + 3*a*b**2*c**3*d/(2*c**3*d**4 + 4*c**2*d**5*x + 2*c*d**6*x**2) + 12*a*b*
*2*c**2*d**2*x*log(c/d + x)/(2*c**3*d**4 + 4*c**2*d**5*x + 2*c*d**6*x**2) +
 6*a*b**2*c*d**3*x**2*log(c/d + x)/(2*c**3*d**4 + 4*c**2*d**5*x + 2*c*d**6*
x**2) - 6*a*b**2*c*d**3*x**2/(2*c**3*d**4 + 4*c**2*d**5*x + 2*c*d**6*x**2)
- 6*b**3*c**4*log(c/d + x)/(2*c**3*d**4 + 4*c**2*d**5*x + 2*c*d**6*x**2) -
3*b**3*c**4/(2*c**3*d**4 + 4*c**2*d**5*x + 2*c*d**6*x**2) - 12*b**3*c**3*d*
x*log(c/d + x)/(2*c**3*d**4 + 4*c**2*d**5*x + 2*c*d**6*x**2) - 6*b**3*c**2*
d**2*x**2*log(c/d + x)/(2*c**3*d**4 + 4*c**2*d**5*x + 2*c*d**6*x**2) + 6*b*
*3*c**2*d**2*x**2/(2*c**3*d**4 + 4*c**2*d**5*x + 2*c*d**6*x**2) + 2*b**3*c*
d**3*x**3/(2*c**3*d**4 + 4*c**2*d**5*x + 2*c*d**6*x**2), Eq(n, -3)), (-2*a*
*3*d**3/(2*c*d**4 + 2*d**5*x) + 6*a**2*b*c*d**2*log(c/d + x)/(2*c*d**4 + 2*
d**5*x) + 6*a**2*b*c*d**2/(2*c*d**4 + 2*d**5*x) + 6*a**2*b*d**3*x*log(c/d +
 x)/(2*c*d**4 + 2*d**5*x) - 12*a*b**2*c**2*d*log(c/d + x)/(2*c*d**4 + 2*d**
5*x) - 12*a*b**2*c**2*d/(2*c*d**4 + 2*d**5*x) - 12*a*b**2*c*d**2*x*log(c/d
+ x)/(2*c*d**4 + 2*d**5*x) + 6*a*b**2*d**3*x**2/(2*c*d**4 + 2*d**5*x) + 6*b
**3*c**3*log(c/d + x)/(2*c*d**4 + 2*d**5*x) + 6*b**3*c**3/(2*c*d**4 + 2*d**
5*x) + 6*b**3*c**2*d*x*log(c/d + x)/(2*c*d**4 + 2*d**5*x) - 3*b**3*c*d**2*x
**2/(2*c*d**4 + 2*d**5*x) + b**3*d**3*x**3/(2*c*d**4 + 2*d**5*x), Eq(n, -2)
), (a**3*log(c/d + x)/d - 3*a**2*b*c*log(c/d + x)/d**2 + 3*a**2*b*x/d + 3*a
*b**2*c**2*log(c/d + x)/d**3 - 3*a*b**2*c*x/d**2 + 3*a*b**2*x**2/(2*d) - b*
*3*c**3*log(c/d + x)/d**4 + b**3*c**2*x/d**3 - b**3*c*x**2/(2*d**2) + b**3*
x**3/(3*d), Eq(n, -1)), (a**3*c*d**3*n**3*(c + d*x)**n/(d**4*n**4 + 10*d**4
*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 9*a**3*c*d**3*n**2*(c + d*x)*
*n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 26*a**
3*c*d**3*n*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*
n + 24*d**4) + 24*a**3*c*d**3*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d
**4*n**2 + 50*d**4*n + 24*d**4) + a**3*d**4*n**3*x*(c + d*x)**n/(d**4*n**4
+ 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 9*a**3*d**4*n**2*x*(
c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4)
 + 26*a**3*d**4*n*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 +
 50*d**4*n + 24*d**4) + 24*a**3*d**4*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n*
*3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 3*a**2*b*c**2*d**2*n**2*(c + d*x
)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 21*a
```

```
**2*b*c**2*d**2*n*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 5
0*d**4*n + 24*d**4) - 36*a**2*b*c**2*d**2*(c + d*x)**n/(d**4*n**4 + 10*d**4
*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a**2*b*c*d**3*n**3*x*(c + d
*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 21
*a**2*b*c*d**3*n**2*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2
 + 50*d**4*n + 24*d**4) + 36*a**2*b*c*d**3*n*x*(c + d*x)**n/(d**4*n**4 + 10
*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a**2*b*d**4*n**3*x**2*
(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4
) + 24*a**2*b*d**4*n**2*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d*
*4*n**2 + 50*d**4*n + 24*d**4) + 57*a**2*b*d**4*n*x**2*(c + d*x)**n/(d**4*n
**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 36*a**2*b*d**4*x
**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*
d**4) + 6*a*b**2*c**3*d*n*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*
n**2 + 50*d**4*n + 24*d**4) + 24*a*b**2*c**3*d*(c + d*x)**n/(d**4*n**4 + 10
*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 6*a*b**2*c**2*d**2*n**2*
x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d*
*4) - 24*a*b**2*c**2*d**2*n*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d
**4*n**2 + 50*d**4*n + 24*d**4) + 3*a*b**2*c*d**3*n**3*x**2*(c + d*x)**n/(d
**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 15*a*b**2*c
*d**3*n**2*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*
d**4*n + 24*d**4) + 12*a*b**2*c*d**3*n*x**2*(c + d*x)**n/(d**4*n**4 + 10*d*
*4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a*b**2*d**4*n**3*x**3*(c
+ d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) +
 21*a*b**2*d**4*n**2*x**3*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*
n**2 + 50*d**4*n + 24*d**4) + 42*a*b**2*d**4*n*x**3*(c + d*x)**n/(d**4*n**4
 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 24*a*b**2*d**4*x**3
*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**
4) - 6*b**3*c**4*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50
*d**4*n + 24*d**4) + 6*b**3*c**3*d*n*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n*
*3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 3*b**3*c**2*d**2*n**2*x**2*(c +
d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 3
*b**3*c**2*d**2*n*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**
2 + 50*d**4*n + 24*d**4) + b**3*c*d**3*n**3*x**3*(c + d*x)**n/(d**4*n**4 +
10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*b**3*c*d**3*n**2*x**
3*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d*
*4) + 2*b**3*c*d**3*n*x**3*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4
*n**2 + 50*d**4*n + 24*d**4) + b**3*d**4*n**3*x**4*(c + d*x)**n/(d**4*n**4
+ 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 6*b**3*d**4*n**2*x**
4*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d*
*4) + 11*b**3*d**4*n*x**4*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*
n**2 + 50*d**4*n + 24*d**4) + 6*b**3*d**4*x**4*(c + d*x)**n/(d**4*n**4 + 10
*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4), True))
```

Giac [B] time = 1.0581, size = 1125, normalized size = 10.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*b^3*d^4*n^3*x^4 + (d*x + c)^n*b^3*c*d^3*n^3*x^3 + 3*(d*x + c)^
n*a*b^2*d^4*n^3*x^3 + 6*(d*x + c)^n*b^3*d^4*n^2*x^4 + 3*(d*x + c)^n*a*b^2*c
*d^3*n^3*x^2 + 3*(d*x + c)^n*a^2*b*d^4*n^3*x^2 + 3*(d*x + c)^n*b^3*c*d^3*n^
2*x^3 + 21*(d*x + c)^n*a*b^2*d^4*n^2*x^3 + 11*(d*x + c)^n*b^3*d^4*n*x^4 + 3
*(d*x + c)^n*a^2*b*c*d^3*n^3*x + (d*x + c)^n*a^3*d^4*n^3*x - 3*(d*x + c)^n*
b^3*c^2*d^2*n^2*x^2 + 15*(d*x + c)^n*a*b^2*c*d^3*n^2*x^2 + 24*(d*x + c)^n*a
```

```
^2*b*d^4*n^2*x^2 + 2*(d*x + c)^n*b^3*c*d^3*n*x^3 + 42*(d*x + c)^n*a*b^2*d^4
*n*x^3 + 6*(d*x + c)^n*b^3*d^4*x^4 + (d*x + c)^n*a^3*c*d^3*n^3 - 6*(d*x + c
)^n*a*b^2*c^2*d^2*n^2*x + 21*(d*x + c)^n*a^2*b*c*d^3*n^2*x + 9*(d*x + c)^n*
a^3*d^4*n^2*x - 3*(d*x + c)^n*b^3*c^2*d^2*n*x^2 + 12*(d*x + c)^n*a*b^2*c*d^
3*n*x^2 + 57*(d*x + c)^n*a^2*b*d^4*n*x^2 + 24*(d*x + c)^n*a*b^2*d^4*x^3 - 3
*(d*x + c)^n*a^2*b*c^2*d^2*n^2 + 9*(d*x + c)^n*a^3*c*d^3*n^2 + 6*(d*x + c)^
n*b^3*c^3*d*n*x - 24*(d*x + c)^n*a*b^2*c^2*d^2*n*x + 36*(d*x + c)^n*a^2*b*c
*d^3*n*x + 26*(d*x + c)^n*a^3*d^4*n*x + 36*(d*x + c)^n*a^2*b*d^4*x^2 + 6*(d
*x + c)^n*a*b^2*c^3*d*n - 21*(d*x + c)^n*a^2*b*c^2*d^2*n + 26*(d*x + c)^n*a
^3*c*d^3*n + 24*(d*x + c)^n*a^3*d^4*x - 6*(d*x + c)^n*b^3*c^4 + 24*(d*x + c
)^n*a*b^2*c^3*d - 36*(d*x + c)^n*a^2*b*c^2*d^2 + 24*(d*x + c)^n*a^3*c*d^3)/
(d^4*n^4 + 10*d^4*n^3 + 35*d^4*n^2 + 50*d^4*n + 24*d^4)
```

3.1853 $\int (a+bx)^2(c+dx)^n\,dx$

Optimal. Leaf size=78

$$\frac{(bc-ad)^2(c+dx)^{n+1}}{d^3(n+1)}-\frac{2b(bc-ad)(c+dx)^{n+2}}{d^3(n+2)}+\frac{b^2(c+dx)^{n+3}}{d^3(n+3)}$$

```
[Out] ((b*c - a*d)^2*(c + d*x)^(1 + n))/(d^3*(1 + n)) - (2*b*(b*c - a*d)*(c + d*x
)^(2 + n))/(d^3*(2 + n)) + (b^2*(c + d*x)^(3 + n))/(d^3*(3 + n))
```

Rubi [A] time = 0.0320649, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{(bc-ad)^2(c+dx)^{n+1}}{d^3(n+1)}-\frac{2b(bc-ad)(c+dx)^{n+2}}{d^3(n+2)}+\frac{b^2(c+dx)^{n+3}}{d^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^n,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^(1 + n))/(d^3*(1 + n)) - (2*b*(b*c - a*d)*(c + d*x
)^(2 + n))/(d^3*(2 + n)) + (b^2*(c + d*x)^(3 + n))/(d^3*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^n\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^n}{d^2}-\frac{2b(bc-ad)(c+dx)^{1+n}}{d^2}+\frac{b^2(c+dx)^{2+n}}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^{1+n}}{d^3(1+n)}-\frac{2b(bc-ad)(c+dx)^{2+n}}{d^3(2+n)}+\frac{b^2(c+dx)^{3+n}}{d^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.0677709, size = 67, normalized size = 0.86

$$\frac{(c+dx)^{n+1}\left(-\frac{2b(c+dx)(bc-ad)}{n+2}+\frac{(bc-ad)^2}{n+1}+\frac{b^2(c+dx)^2}{n+3}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*((b*c - a*d)^2/(1 + n) - (2*b*(b*c - a*d)*(c + d*x))/(2
+ n) + (b^2*(c + d*x)^2)/(3 + n)))/d^3
```

Maple [B] time = 0.007, size = 159, normalized size = 2.

$$\frac{(dx+c)^{1+n}\left(b^2d^2n^2x^2+2\,abd^2n^2x+3\,b^2d^2nx^2+a^2d^2n^2+8\,abd^2nx-2\,b^2cdnx+2\,b^2d^2x^2+5\,a^2d^2n-2\,abcdn+6\,ab\right.}{d^3\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^n,x)
```

```
[Out] (d*x+c)^(1+n)*(b^2*d^2*n^2*x^2+2*a*b*d^2*n^2*x+3*b^2*d^2*n*x^2+a^2*d^2*n^2+
8*a*b*d^2*n*x-2*b^2*c*d*n*x+2*b^2*d^2*x^2+5*a^2*d^2*n-2*a*b*c*d*n+6*a*b*d^2
*x-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/d^3/(n^3+6*n^2+11*n+6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.93191, size = 478, normalized size = 6.13

$$\frac{\left(a^2cd^2n^2+2\,b^2c^3-6\,abc^2d+6\,a^2cd^2+\left(b^2d^3n^2+3\,b^2d^3n+2\,b^2d^3\right)x^3+\left(6\,abd^3+\left(b^2cd^2+2\,abd^3\right)n^2+\left(b^2cd^2+8\,ab\right.\right.\right.}{d^3n^3+6\,d^3n^2+11\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a^2*c*d^2*n^2 + 2*b^2*c^3 - 6*a*b*c^2*d + 6*a^2*c*d^2 + (b^2*d^3*n^2 + 3*b
^2*d^3*n + 2*b^2*d^3)*x^3 + (6*a*b*d^3 + (b^2*c*d^2 + 2*a*b*d^3)*n^2 + (b^2
*c*d^2 + 8*a*b*d^3)*n)*x^2 - (2*a*b*c^2*d - 5*a^2*c*d^2)*n + (6*a^2*d^3 + (
2*a*b*c*d^2 + a^2*d^3)*n^2 - (2*b^2*c^2*d - 6*a*b*c*d^2 - 5*a^2*d^3)*n)*x)*
(d*x + c)^n/(d^3*n^3 + 6*d^3*n^2 + 11*d^3*n + 6*d^3)
```

Sympy [A] time = 1.99613, size = 1504, normalized size = 19.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a**2*x + a*b*x**2 + b**2*x**3/3), Eq(d, 0)), (-a**2*c*d**2
/(2*c**3*d**3 + 4*c**2*d**4*x + 2*c*d**5*x**2) + 2*a*b*d**3*x**2/(2*c**3*d*
*3 + 4*c**2*d**4*x + 2*c*d**5*x**2) + 2*b**2*c**3*log(c/d + x)/(2*c**3*d**3
 + 4*c**2*d**4*x + 2*c*d**5*x**2) + b**2*c**3/(2*c**3*d**3 + 4*c**2*d**4*x
+ 2*c*d**5*x**2) + 4*b**2*c**2*d*x*log(c/d + x)/(2*c**3*d**3 + 4*c**2*d**4*
```

```
x + 2*c*d**5*x**2) + 2*b**2*c*d**2*x**2*log(c/d + x)/(2*c**3*d**3 + 4*c**2*
d**4*x + 2*c*d**5*x**2) - 2*b**2*c*d**2*x**2/(2*c**3*d**3 + 4*c**2*d**4*x +
 2*c*d**5*x**2), Eq(n, -3)), (-a**2*d**2/(c*d**3 + d**4*x) + 2*a*b*c*d*log(
c/d + x)/(c*d**3 + d**4*x) + 2*a*b*c*d/(c*d**3 + d**4*x) + 2*a*b*d**2*x*log
(c/d + x)/(c*d**3 + d**4*x) - 2*b**2*c**2*log(c/d + x)/(c*d**3 + d**4*x) -
2*b**2*c**2/(c*d**3 + d**4*x) - 2*b**2*c*d*x*log(c/d + x)/(c*d**3 + d**4*x)
 + b**2*d**2*x**2/(c*d**3 + d**4*x), Eq(n, -2)), (a**2*log(c/d + x)/d - 2*a
*b*c*log(c/d + x)/d**2 + 2*a*b*x/d + b**2*c**2*log(c/d + x)/d**3 - b**2*c*x
/d**2 + b**2*x**2/(2*d), Eq(n, -1)), (a**2*c*d**2*n**2*(c + d*x)**n/(d**3*n
**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 5*a**2*c*d**2*n*(c + d*x)**n/(d**
3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a**2*c*d**2*(c + d*x)**n/(d*
*3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + a**2*d**3*n**2*x*(c + d*x)**n
/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 5*a**2*d**3*n*x*(c + d*x)
**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a**2*d**3*x*(c + d*x
)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) - 2*a*b*c**2*d*n*(c + d
*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) - 6*a*b*c**2*d*(c + d
*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*a*b*c*d**2*n**2*x
*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a*b*c*d**2
*n*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*a*b*d*
*3*n**2*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) +
8*a*b*d**3*n*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**
3) + 6*a*b*d**3*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*
d**3) + 2*b**2*c**3*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d
**3) - 2*b**2*c**2*d*n*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n
+ 6*d**3) + b**2*c*d**2*n**2*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 1
1*d**3*n + 6*d**3) + b**2*c*d**2*n*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n*
*2 + 11*d**3*n + 6*d**3) + b**2*d**3*n**2*x**3*(c + d*x)**n/(d**3*n**3 + 6*
d**3*n**2 + 11*d**3*n + 6*d**3) + 3*b**2*d**3*n*x**3*(c + d*x)**n/(d**3*n**
3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*b**2*d**3*x**3*(c + d*x)**n/(d**3
*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3), True))
```

Giac [B] time = 1.07337, size = 520, normalized size = 6.67

$$\frac{(dx+c)^n b^2 d^3 n^2 x^3 + (dx+c)^n b^2 c d^2 n^2 x^2 + 2\,(dx+c)^n a b d^3 n^2 x^2 + 3\,(dx+c)^n b^2 d^3 n x^3 + 2\,(dx+c)^n a b c d^2 n^2 x + (dx+c)^n a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*b^2*d^3*n^2*x^3 + (d*x + c)^n*b^2*c*d^2*n^2*x^2 + 2*(d*x + c)^
n*a*b*d^3*n^2*x^2 + 3*(d*x + c)^n*b^2*d^3*n*x^3 + 2*(d*x + c)^n*a*b*c*d^2*n
^2*x + (d*x + c)^n*a^2*d^3*n^2*x + (d*x + c)^n*b^2*c*d^2*n*x^2 + 8*(d*x + c
)^n*a*b*d^3*n*x^2 + 2*(d*x + c)^n*b^2*d^3*x^3 + (d*x + c)^n*a^2*c*d^2*n^2 -
 2*(d*x + c)^n*b^2*c^2*d*n*x + 6*(d*x + c)^n*a*b*c*d^2*n*x + 5*(d*x + c)^n*
a^2*d^3*n*x + 6*(d*x + c)^n*a*b*d^3*x^2 - 2*(d*x + c)^n*a*b*c^2*d*n + 5*(d*
x + c)^n*a^2*c*d^2*n + 6*(d*x + c)^n*a^2*d^3*x + 2*(d*x + c)^n*b^2*c^3 - 6*
(d*x + c)^n*a*b*c^2*d + 6*(d*x + c)^n*a^2*c*d^2)/(d^3*n^3 + 6*d^3*n^2 + 11*
d^3*n + 6*d^3)
```

3.1854 $\int (a+bx)(c+dx)^n\,dx$

Optimal. Leaf size=47

$$\frac{b(c+dx)^{n+2}}{d^2(n+2)}-\frac{(bc-ad)(c+dx)^{n+1}}{d^2(n+1)}$$

```
[Out] -(((b*c - a*d)*(c + d*x)^(1 + n))/(d^2*(1 + n))) + (b*(c + d*x)^(2 + n))/(d
^2*(2 + n))
```

Rubi [A] time = 0.018252, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^{n+2}}{d^2(n+2)}-\frac{(bc-ad)(c+dx)^{n+1}}{d^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^n,x]
```

```
[Out] -(((b*c - a*d)*(c + d*x)^(1 + n))/(d^2*(1 + n))) + (b*(c + d*x)^(2 + n))/(d
^2*(2 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^n\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^n}{d}+\frac{b(c+dx)^{1+n}}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^{1+n}}{d^2(1+n)}+\frac{b(c+dx)^{2+n}}{d^2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.028029, size = 41, normalized size = 0.87

$$\frac{(c+dx)^{n+1}(ad(n+2)-bc+bd(n+1)x)}{d^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*(-(b*c) + a*d*(2 + n) + b*d*(1 + n)*x))/(d^2*(1 + n)*(2
+ n))
```

Maple [A] time = 0.002, size = 46, normalized size = 1.

$$\frac{(dx+c)^{1+n}\,(bdnx+adn+bdx+2\,ad-bc)}{d^2\left(n^2+3\,n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^n,x)
```

```
[Out] (d*x+c)^(1+n)*(b*d*n*x+a*d*n+b*d*x+2*a*d-b*c)/d^2/(n^2+3*n+2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88476, size = 171, normalized size = 3.64

$$\frac{\left(acdn-bc^2+2\,acd+\left(bd^2n+bd^2\right)x^2+\left(2\,ad^2+\left(bcd+ad^2\right)n\right)x\right)(dx+c)^n}{d^2n^2+3\,d^2n+2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a*c*d*n - b*c^2 + 2*a*c*d + (b*d^2*n + b*d^2)*x^2 + (2*a*d^2 + (b*c*d + a*
d^2)*n)*x)*(d*x + c)^n/(d^2*n^2 + 3*d^2*n + 2*d^2)
```

Sympy [A] time = 0.841468, size = 377, normalized size = 8.02

$$\begin{cases} c^n\left(ax+\frac{bx^2}{2}\right) \\ -\frac{ad}{cd^2+d^3x}+\frac{bc\log\left(\frac{c}{d}+x\right)}{cd^2+d^3x}+\frac{bc}{cd^2+d^3x}+\frac{bdx\log\left(\frac{c}{d}+x\right)}{cd^2+d^3x} \\ \frac{a\log\left(\frac{c}{d}+x\right)}{d}-\frac{bc\log\left(\frac{c}{d}+x\right)}{d^2}+\frac{bx}{d} \\ \frac{acdn(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{2acd(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{ad^2nx(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{2ad^2x(c+dx)^n}{d^2n^2+3d^2n+2d^2}-\frac{bc^2(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{bcdnx(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{bd^2nx^2(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{bd^2x^2(c+dx)}{d^2n^2+3d^2n+2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a*x + b*x**2/2), Eq(d, 0)), (-a*d/(c*d**2 + d**3*x) + b*c*
log(c/d + x)/(c*d**2 + d**3*x) + b*c/(c*d**2 + d**3*x) + b*d*x*log(c/d + x)
/(c*d**2 + d**3*x), Eq(n, -2)), (a*log(c/d + x)/d - b*c*log(c/d + x)/d**2 +
 b*x/d, Eq(n, -1)), (a*c*d*n*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) +
 2*a*c*d*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + a*d**2*n*x*(c + d*x
```

```
)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + 2*a*d**2*x*(c + d*x)**n/(d**2*n**2 +
 3*d**2*n + 2*d**2) - b*c**2*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) +
 b*c*d*n*x*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + b*d**2*n*x**2*(c
+ d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + b*d**2*x**2*(c + d*x)**n/(d**2*
n**2 + 3*d**2*n + 2*d**2), True))
```

Giac [B] time = 1.06168, size = 178, normalized size = 3.79

$$\frac{(dx+c)^n bd^2nx^2 + (dx+c)^n bcdnx + (dx+c)^n ad^2nx + (dx+c)^n bd^2x^2 + (dx+c)^n acdn + 2\,(dx+c)^n ad^2x - (dx+c)^n b}{d^2n^2 + 3\,d^2n + 2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*b*d^2*n*x^2 + (d*x + c)^n*b*c*d*n*x + (d*x + c)^n*a*d^2*n*x +
(d*x + c)^n*b*d^2*x^2 + (d*x + c)^n*a*c*d*n + 2*(d*x + c)^n*a*d^2*x - (d*x
+ c)^n*b*c^2 + 2*(d*x + c)^n*a*c*d)/(d^2*n^2 + 3*d^2*n + 2*d^2)
```

3.1855 $\int (c+dx)^n\,dx$

Optimal. Leaf size=18

$$\frac{(c+dx)^{n+1}}{d(n+1)}$$

```
[Out] (c + d*x)^(1 + n)/(d*(1 + n))
```

Rubi [A] time = 0.0029468, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n,x]
```

```
[Out] (c + d*x)^(1 + n)/(d*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^n\,dx=\frac{(c+dx)^{1+n}}{d(1+n)}$$

Mathematica [A] time = 0.0088098, size = 17, normalized size = 0.94

$$\frac{(c+dx)^{n+1}}{dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n,x]
```

```
[Out] (c + d*x)^(1 + n)/(d + d*n)
```

Maple [A] time = 0.001, size = 19, normalized size = 1.1

$$\frac{(dx+c)^{1+n}}{d\,(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n,x)
```

```
[Out] (d*x+c)^(1+n)/d/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85336, size = 45, normalized size = 2.5

$$\frac{(dx+c)(dx+c)^n}{dn+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (d*x + c)*(d*x + c)^n/(d*n + d)
```

Sympy [A] time = 0.069606, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(c+dx)^{n+1}}{n+1} & \text{for}\ n \neq -1 \\ \log{(c+dx)} & \text{otherwise} \end{cases}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n,x)
```

```
[Out] Piecewise(((c + d*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(c + d*x), True))/d
```

Giac [A] time = 1.04885, size = 24, normalized size = 1.33

$$\frac{(dx+c)^{n+1}}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="giac")
```

```
[Out] (d*x + c)^(n + 1)/(d*(n + 1))
```

3.1856 $\int \frac{(c+dx)^n}{a+bx}\,dx$

Optimal. Leaf size=51

$$-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)}$$

```
[Out] -(((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(c + d*x))/(b*c
- a*d)])/((b*c - a*d)*(1 + n)))
```

Rubi [A] time = 0.0116944, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {68}

$$-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n/(a + b*x),x]
```

```
[Out] -(((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(c + d*x))/(b*c
- a*d)])/((b*c - a*d)*(1 + n)))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(c+dx)^n}{a+bx}\,dx = -\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)(1+n)}$$

Mathematica [A] time = 0.0084255, size = 51, normalized size = 1.

$$-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n/(a + b*x),x]
```

```
[Out] -(((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(c + d*x))/(b*c
- a*d)])/((b*c - a*d)*(1 + n)))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n/(b*x+a),x)
```

```
[Out] int((d*x+c)^n/(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n/(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^n}{bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n/(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^n}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n/(b*x+a),x)
```

```
[Out] Integral((c + d*x)**n/(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^n/(b*x + a), x)
```

3.1857 $\int \frac{(c+dx)^n}{(a+bx)^2} dx$

Optimal. Leaf size=51

$$\frac{d(c+dx)^{n+1} {}_2F_1\left(2,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)^2}$$

```
[Out] (d*(c + d*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (b*(c + d*x))/(b*c
- a*d)])/((b*c - a*d)^2*(1 + n))
```

Rubi [A] time = 0.0111316, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {68}

$$\frac{d(c+dx)^{n+1} {}_2F_1\left(2,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n/(a + b*x)^2,x]
```

```
[Out] (d*(c + d*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (b*(c + d*x))/(b*c
- a*d)])/((b*c - a*d)^2*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(c+dx)^n}{(a+bx)^2} dx = \frac{d(c+dx)^{1+n} {}_2F_1\left(2,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)^2(1+n)}$$

Mathematica [A] time = 0.0134705, size = 52, normalized size = 1.02

$$\frac{d(c+dx)^{n+1} {}_2F_1\left(2,n+1;n+2;-\frac{b(c+dx)}{ad-bc}\right)}{(n+1)(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n/(a + b*x)^2,x]
```

```
[Out] (d*(c + d*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, -((b*(c + d*x))/(-(
b*c) + a*d))])/((-(b*c) + a*d)^2*(1 + n))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n/(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^n/(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n/(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^n}{b^2x^2+2\,abx+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^n}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n/(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**n/(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^n/(b*x + a)^2, x)
```

3.1858 $\int \frac{(c+dx)^n}{(a+bx)^3} dx$

Optimal. Leaf size=54

$$-\frac{d^2(c+dx)^{n+1} {}_2F_1\left(3,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)^3}$$

```
[Out] -((d^2*(c + d*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, (b*(c + d*x))/(
b*c - a*d)])/((b*c - a*d)^3*(1 + n)))
```

Rubi [A] time = 0.0153957, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {68}

$$-\frac{d^2(c+dx)^{n+1} {}_2F_1\left(3,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n/(a + b*x)^3,x]
```

```
[Out] -((d^2*(c + d*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, (b*(c + d*x))/(
b*c - a*d)])/((b*c - a*d)^3*(1 + n)))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(c+dx)^n}{(a+bx)^3} dx = -\frac{d^2(c+dx)^{1+n} {}_2F_1\left(3,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)^3(1+n)}$$

Mathematica [A] time = 0.0140079, size = 54, normalized size = 1.

$$\frac{d^2(c+dx)^{n+1} {}_2F_1\left(3,n+1;n+2;-\frac{b(c+dx)}{ad-bc}\right)}{(n+1)(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n/(a + b*x)^3,x]
```

```
[Out] (d^2*(c + d*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, -((b*(c + d*x))/(
-(b*c) + a*d))])/((-(b*c) + a*d)^3*(1 + n))
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n/(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^n/(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n/(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^n}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^n}{(a+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n/(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**n/(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^n/(b*x + a)^3, x)
```

3.1859 $\int (a+bx)^{-4+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=143

$$-\frac{2d^2(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(3-n)(bc-ad)^3}-\frac{(a+bx)^{n-3}(c+dx)^{1-n}}{(3-n)(bc-ad)}+\frac{2d(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(3-n)(bc-ad)^2}$$

```
[Out] -(((a + b*x)^(-3 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(3 - n))) + (2*d*(a +
 b*x)^(-2 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^2*(2 - n)*(3 - n)) - (2*d^2*
(a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^3*(1 - n)*(2 - n)*(3 - n
))
```

Rubi [A] time = 0.0627202, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{2d^2(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(3-n)(bc-ad)^3}-\frac{(a+bx)^{n-3}(c+dx)^{1-n}}{(3-n)(bc-ad)}+\frac{2d(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(3-n)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(3 - n))) + (2*d*(a +
 b*x)^(-2 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^2*(2 - n)*(3 - n)) - (2*d^2*
(a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^3*(1 - n)*(2 - n)*(3 - n
))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-4+n}(c+dx)^{-n}\,dx &= -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)}-\frac{(2d)\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx}{(bc-ad)(3-n)}\\ &= -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)}+\frac{2d(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)^2(2-n)(3-n)}+\frac{\left(2d^2\right)\int (a+bx)^{-2+n}(c+dx)^{-}}{(bc-ad)^2(2-n)(3-n)}\\ &= -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)}+\frac{2d(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)^2(2-n)(3-n)}-\frac{2d^2(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)^3(1-n)(2-n)(3-r}\end{aligned}$$

Mathematica [A] time = 0.0647185, size = 112, normalized size = 0.78

$$\frac{(a+bx)^{n-3}(c+dx)^{1-n}\left(a^2d^2\left(n^2-5n+6\right)-2abd(n-3)(c(n-1)+dx)+b^2\left(c^2\left(n^2-3n+2\right)+2cd(n-1)x+2d^2x^2\right)\right)}{(n-3)(n-2)(n-1)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-3 + n)*(c + d*x)^(1 - n)*(a^2*d^2*(6 - 5*n + n^2) - 2*a*b*d*(-
3 + n)*(c*(-1 + n) + d*x) + b^2*(c^2*(2 - 3*n + n^2) + 2*c*d*(-1 + n)*x + 2
*d^2*x^2)))/((b*c - a*d)^3*(-3 + n)*(-2 + n)*(-1 + n))
```

Maple [B] time = 0.006, size = 322, normalized size = 2.3

$$-\frac{(bx+a)^{-3+n}(dx+c)\left(a^2d^2n^2-2\,abcdn^2-2\,abd^2nx+b^2c^2n^2+2\,b^2cdnx+2\,b^2d^2x^2-5\,a^2d^2n+8\,abc\right.}{\left(a^3d^3n^3-3\,a^2bcd^2n^3+3\,ab^2c^2dn^3-b^3c^3n^3-6\,a^3d^3n^2+18\,a^2bcd^2n^2-18\,ab^2c^2dn^2+6\,b^3c^3n^2+11\,a^3d^3n-33\,a^2bcd^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-4+n)/((d*x+c)^n),x)
```

```
[Out] -(b*x+a)^(-3+n)*(d*x+c)*(a^2*d^2*n^2-2*a*b*c*d*n^2-2*a*b*d^2*n*x+b^2*c^2*n^
2+2*b^2*c*d*n*x+2*b^2*d^2*x^2-5*a^2*d^2*n+8*a*b*c*d*n+6*a*b*d^2*x-3*b^2*c^2
*n-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*d^2*n^
3+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3-6*a^3*d^3*n^2+18*a^2*b*c*d^2*n^2-18*a*b^2*c
^2*d*n^2+6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n-11*b^
3*c^3*n-6*a^3*d^3+18*a^2*b*c*d^2-18*a*b^2*c^2*d+6*b^3*c^3)/((d*x+c)^n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n-4}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 4)/(d*x + c)^n, x)
```

Fricas [B] time = 2.04388, size = 1023, normalized size = 7.15

$$-\frac{\left(2\,b^3d^3x^4+2\,ab^2c^3-6\,a^2bc^2d+6\,a^3cd^2+2\left(4\,ab^2d^3+\left(b^3cd^2-ab^2d^3\right)n\right)x^3+\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)n^2+\left(12\,a^2bd\right.\right.}{\left(6\,b^3c^3-18\,ab^2c^2d+18\,a^2bc\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] -(2*b^3*d^3*x^4 + 2*a*b^2*c^3 - 6*a^2*b*c^2*d + 6*a^3*c*d^2 + 2*(4*a*b^2*d^
3 + (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*n^2 + (12*a^2*b*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 - (b^3*
c^2*d - 8*a*b^2*c*d^2 + 7*a^2*b*d^3)*n)*x^2 - (3*a*b^2*c^3 - 8*a^2*b*c^2*d
+ 5*a^3*c*d^2)*n + (2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 6*a^3*d^3 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 - (3*b^3*c^3 - 7*a*b^2
*c^2*d - a^2*b*c*d^2 + 5*a^3*d^3)*n)*x)*(b*x + a)^(n - 4)/((6*b^3*c^3 - 18*
a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*n^2 - 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n)*(d*x +
 c)^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-4+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n-4}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 4)/(d*x + c)^n, x)
```

3.1860 $\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=86

$$\frac{d(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(bc-ad)^2}-\frac{(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(bc-ad)}$$

```
[Out] -(((a + b*x)^(-2 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(2 - n))) + (d*(a + b
*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^2*(1 - n)*(2 - n))
```

Rubi [A] time = 0.0124284, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{d(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(bc-ad)^2}-\frac{(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-2 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(2 - n))) + (d*(a + b
*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^2*(1 - n)*(2 - n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx &= -\frac{(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)(2-n)}-\frac{d\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx}{(bc-ad)(2-n)}\\ &= -\frac{(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)(2-n)}+\frac{d(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)^2(1-n)(2-n)}\end{aligned}$$

Mathematica [A] time = 0.0311377, size = 59, normalized size = 0.69

$$\frac{(a+bx)^{n-2}(c+dx)^{1-n}(-ad(n-2)+bc(n-1)+bdx)}{(n-2)(n-1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-2 + n)*(c + d*x)^(1 - n)*(-(a*d*(-2 + n)) + b*c*(-1 + n) + b*d
*x))/((b*c - a*d)^2*(-2 + n)*(-1 + n))
```

Maple [A] time = 0.006, size = 127, normalized size = 1.5

$$-\frac{(bx+a)^{-2+n}(dx+c)(adn-bcn-bdx-2ad+bc)}{\left(a^2d^2n^2-2abcdn^2+b^2c^2n^2-3a^2d^2n+6abcdn-3b^2c^2n+2a^2d^2-4abcd+2b^2c^2\right)(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-3+n)/((d*x+c)^n),x)
```

```
[Out] -(b*x+a)^(-2+n)*(d*x+c)*(a*d*n-b*c*n-b*d*x-2*a*d+b*c)/(a^2*d^2*n^2-2*a*b*c*
d*n^2+b^2*c^2*n^2-3*a^2*d^2*n+6*a*b*c*d*n-3*b^2*c^2*n+2*a^2*d^2-4*a*b*c*d+2
*b^2*c^2)/((d*x+c)^n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n-3}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 3)/(d*x + c)^n, x)
```

Fricas [B] time = 1.95155, size = 417, normalized size = 4.85

$$\frac{\left(b^2d^2x^3-abc^2+2a^2cd+\left(3abd^2+\left(b^2cd-abd^2\right)n\right)x^2+\left(abc^2-a^2cd\right)n-\left(b^2c^2-2abcd-2a^2d^2-\left(b^2c^2-a^2d^2\right)n\right)x}{\left(2b^2c^2-4abcd+2a^2d^2+\left(b^2c^2-2abcd+a^2d^2\right)n^2-3\left(b^2c^2-2abcd+a^2d^2\right)n\right)(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 - a*b*c^2 + 2*a^2*c*d + (3*a*b*d^2 + (b^2*c*d - a*b*d^2)*n)*x^
2 + (a*b*c^2 - a^2*c*d)*n - (b^2*c^2 - 2*a*b*c*d - 2*a^2*d^2 - (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^(n - 3)/((2*b^2*c^2 - 4*a*b*c*d + 2*a^2*d^2 + (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*n^2 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n)*(d*x
+ c)^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-3+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n-3}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 3)/(d*x + c)^n, x)
```

3.1861 $\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=39

$$-\frac{(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(bc-ad)}$$

```
[Out] -(((a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(1 - n)))
```

Rubi [A] time = 0.0041875, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(1 - n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx = -\frac{(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)(1-n)}$$

Mathematica [A] time = 0.0120888, size = 36, normalized size = 0.92

$$\frac{(a+bx)^{n-1}(c+dx)^{1-n}}{(n-1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(-1 + n))
```

Maple [A] time = 0.002, size = 45, normalized size = 1.2

$$-\frac{(bx+a)^{-1+n}(dx+c)}{(adn-bcn-ad+bc)(dx+c)^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-2+n)/((d*x+c)^n),x)
```

```
[Out] -(b*x+a)^(-1+n)*(d*x+c)/(a*d*n-b*c*n-a*d+b*c)/((d*x+c)^n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n-2}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 2)/(d*x + c)^n, x)
```

Fricas [A] time = 1.82054, size = 127, normalized size = 3.26

$$-\frac{\left(bdx^2+ac+(bc+ad)x\right)(bx+a)^{n-2}}{(bc-ad-(bc-ad)n)(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(n - 2)/((b*c - a*d - (b*c - a*d
)*n)*(d*x + c)^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-2+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n-2}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 2)/(d*x + c)^n, x)
```

3.1862 $\int (a+bx)^{-1+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=66

$$\frac{(a+bx)^n(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n;n+1;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

```
[Out] ((a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, n, 1 + n, -
((d*(a + b*x))/(b*c - a*d))])/(b*n*(c + d*x)^n)
```

Rubi [A] time = 0.0237162, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{(a+bx)^n(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n;n+1;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, n, 1 + n, -
((d*(a + b*x))/(b*c - a*d))])/(b*n*(c + d*x)^n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-1+n}(c+dx)^{-n}\,dx &= \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n\right)\int (a+bx)^{-1+n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}\,dx\\ &= \frac{(a+bx)^n(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n;1+n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.0203116, size = 65, normalized size = 0.98

$$\frac{(a+bx)^n(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n;n+1;\frac{d(a+bx)}{ad-bc}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, n, 1 + n, (
d*(a + b*x))/(-(b*c) + a*d)])/(b*n*(c + d*x)^n)
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{-1+n}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-1+n)/((d*x+c)^n),x)
```

```
[Out] int((b*x+a)^(-1+n)/((d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n-1}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 1)/(d*x + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(bx+a)^{n-1}}{(dx+c)^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n - 1)/(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-1+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n-1}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 1)/(d*x + c)^n, x)
```

3.1863 $\int (a+bx)^n(c+dx)^{-n}\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{n+1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+1;n+2;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+1)}$$

```
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(1 + n)*(c + d*x)^n)
```

Rubi [A] time = 0.0232951, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {70, 69}

$$\frac{(a+bx)^{n+1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+1;n+2;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(1 + n)*(c + d*x)^n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n(c+dx)^{-n}\,dx &= \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n\right)\int (a+bx)^n\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}dx\\ &= \frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0177964, size = 71, normalized size = 0.99

$$\frac{(a+bx)^{n+1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+1;n+2;\frac{d(a+bx)}{ad-bc}\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(1 + n)*(c + d*x)^n)
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/((d*x+c)^n),x)
```

```
[Out] int((b*x+a)^n/((d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/(d*x + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^n}{(dx+c)^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/((d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^n}{(dx + c)^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/(d*x + c)^n, x)
```

3.1864 $\int (a+bx)^{1+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{n+2}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+2;n+3;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+2)}$$

```
[Out] ((a + b*x)^(2 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 2 + n
, 3 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(2 + n)*(c + d*x)^n)
```

Rubi [A] time = 0.0229974, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{(a+bx)^{n+2}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+2;n+3;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(2 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 2 + n
, 3 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(2 + n)*(c + d*x)^n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{1+n}(c+dx)^{-n}\,dx &= \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n\right)\int (a+bx)^{1+n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}dx\\ &= \frac{(a+bx)^{2+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,2+n;3+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(2+n)}\end{aligned}$$

Mathematica [A] time = 0.0430342, size = 89, normalized size = 1.24

$$\frac{(bc-ad)(a+bx)^n(c+dx)^{1-n}\left(\frac{d(a+bx)}{ad-bc}\right)^{-n} {}_2F_1\left(-n-1,1-n;2-n;\frac{b(c+dx)}{bc-ad}\right)}{d^2(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1 + n)/(c + d*x)^n,x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^n*(c + d*x)^(1 - n)*Hypergeometric2F1[-1 - n, 1 - n,
 2 - n, (b*(c + d*x))/(b*c - a*d)])/(d^2*(-1 + n)*((d*(a + b*x))/(-(b*c) +
a*d))^n)
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{1+n}}{(dx+c)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1+n)/((d*x+c)^n),x)
```

```
[Out] int((b*x+a)^(1+n)/((d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n+1}}{(dx+c)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n + 1)/(d*x + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^{n+1}}{(dx+c)^{n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n + 1)/(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n+1}}{(dx+c)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n + 1)/(d*x + c)^n, x)
```

3.1865 $\int (a+bx)^{2+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{n+3}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+3;n+4;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+3)}$$

```
[Out] ((a + b*x)^(3 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 3 + n
, 4 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(3 + n)*(c + d*x)^n)
```

Rubi [A] time = 0.0227955, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$\frac{(a+bx)^{n+3}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+3;n+4;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(3 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 3 + n
, 4 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(3 + n)*(c + d*x)^n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{2+n}(c+dx)^{-n}\,dx &= \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n\right)\int (a+bx)^{2+n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}dx\\ &= \frac{(a+bx)^{3+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,3+n;4+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(3+n)}\end{aligned}$$

Mathematica [A] time = 0.0373166, size = 92, normalized size = 1.28

$$-\frac{(bc-ad)^2(a+bx)^n(c+dx)^{1-n}\left(\frac{d(a+bx)}{ad-bc}\right)^{-n} {}_2F_1\left(-n-2,1-n;2-n;\frac{b(c+dx)}{bc-ad}\right)}{d^3(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2 + n)/(c + d*x)^n,x]
```

```
[Out] -(((b*c - a*d)^2*(a + b*x)^n*(c + d*x)^(1 - n)*Hypergeometric2F1[-2 - n, 1
- n, 2 - n, (b*(c + d*x))/(b*c - a*d)])/(d^3*(-1 + n)*((d*(a + b*x))/(-(b*c
) + a*d))^n))
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{2+n}}{(dx+c)^{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2+n)/((d*x+c)^n),x)
```

```
[Out] int((b*x+a)^(2+n)/((d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n+2}}{(dx+c)^{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n + 2)/(d*x + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)^{n+2}}{(dx+c)^{n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n + 2)/(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{n+2}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n + 2)/(d*x + c)^n, x)
```

3.1866 $\int (a+bx)^{-n}(c+dx)^n\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{-n}(c+dx)^{n+1}\left(-\frac{d(a+bx)}{bc-ad}\right)^n {}_2F_1\left(n,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{d(n+1)}$$

```
[Out] ((-((d*(a + b*x))/(b*c - a*d)))^n*(c + d*x)^(1 + n)*Hypergeometric2F1[n, 1
+ n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/(d*(1 + n)*(a + b*x)^n)
```

Rubi [A] time = 0.0305638, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {70, 69}

$$\frac{(a+bx)^{-n}(c+dx)^{n+1}\left(-\frac{d(a+bx)}{bc-ad}\right)^n {}_2F_1\left(n,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n/(a + b*x)^n,x]
```

```
[Out] ((-((d*(a + b*x))/(b*c - a*d)))^n*(c + d*x)^(1 + n)*Hypergeometric2F1[n, 1
+ n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/(d*(1 + n)*(a + b*x)^n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-n}(c+dx)^n\,dx &= \left((a+bx)^{-n}\left(\frac{d(a+bx)}{-bc+ad}\right)^n\right)\int (c+dx)^n\left(-\frac{ad}{bc-ad}-\frac{bdx}{bc-ad}\right)^{-n}\,dx\\ &= \frac{(a+bx)^{-n}\left(-\frac{d(a+bx)}{bc-ad}\right)^n (c+dx)^{1+n}\,{}_2F_1\left(n,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{d(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0197348, size = 71, normalized size = 0.99

$$\frac{(a+bx)^{-n}(c+dx)^{n+1}\left(\frac{d(a+bx)}{ad-bc}\right)^n {}_2F_1\left(n,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n/(a + b*x)^n,x]
```

```
[Out] (((d*(a + b*x))/(-(b*c) + a*d))^n*(c + d*x)^(1 + n)*Hypergeometric2F1[n, 1
+ n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/(d*(1 + n)*(a + b*x)^n)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n/((b*x+a)^n),x)
```

```
[Out] int((d*x+c)^n/((b*x+a)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n/(b*x + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^n}{(bx+a)^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n/(b*x + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^{-n}(c+dx)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n/((b*x+a)**n),x)
```

```
[Out] Integral((a + b*x)**(-n)*(c + d*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^n/(b*x + a)^n, x)
```

3.1867 $\int (a+bx)^{-1-n}(c+dx)^n\,dx$

Optimal. Leaf size=75

$$-\frac{(a+bx)^{-n}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

```
[Out] -(((c + d*x)^n*Hypergeometric2F1[-n, -n, 1 - n, -((d*(a + b*x))/(b*c - a*d)
)])/(b*n*(a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n))
```

Rubi [A] time = 0.0238464, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$-\frac{(a+bx)^{-n}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1 - n)*(c + d*x)^n,x]
```

```
[Out] -(((c + d*x)^n*Hypergeometric2F1[-n, -n, 1 - n, -((d*(a + b*x))/(b*c - a*d)
)])/(b*n*(a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-1-n}(c+dx)^n\,dx &= \left((c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{-1-n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n dx\\ &= -\frac{(a+bx)^{-n}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.0198404, size = 74, normalized size = 0.99

$$-\frac{(a+bx)^{-n}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{d(a+bx)}{ad-bc}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1 - n)*(c + d*x)^n,x]
```

```
[Out] -(((c + d*x)^n*Hypergeometric2F1[-n, -n, 1 - n, (d*(a + b*x))/(-(b*c) + a*d
)])/(b*n*(a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int (bx + a)^{-1-n} (dx + c)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-1-n)*(d*x+c)^n,x)
```

```
[Out] int((b*x+a)^(-1-n)*(d*x+c)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^{-n-1}(dx + c)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 1)*(d*x + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx + a)^{-n-1}(dx + c)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(-n - 1)*(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-1-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{-n-1}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 1)*(d*x + c)^n, x)
```

3.1868 $\int (a+bx)^{-2-n}(c+dx)^n\,dx$

Optimal. Leaf size=37

$$-\frac{(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(bc-ad)}$$

```
[Out] -(((a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(1 + n)))
```

Rubi [A] time = 0.0059839, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(1 + n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^{-2-n}(c+dx)^n\,dx = -\frac{(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)(1+n)}$$

Mathematica [A] time = 0.0127175, size = 38, normalized size = 1.03

$$\frac{(a+bx)^{-n-1}(c+dx)^{n+1}}{(-n-1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2 - n)*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(-1 - n))
```

Maple [A] time = 0.003, size = 41, normalized size = 1.1

$$\frac{(bx+a)^{-1-n}(dx+c)^{1+n}}{adn-bcn+ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-2-n)*(d*x+c)^n,x)
```

```
[Out] (b*x+a)^(-1-n)*(d*x+c)^(1+n)/(a*d*n-b*c*n+a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{-n-2}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 2)*(d*x + c)^n, x)
```

Fricas [A] time = 1.91859, size = 126, normalized size = 3.41

$$-\frac{\left(bdx^2+ac+(bc+ad)x\right)(bx+a)^{-n-2}(dx+c)^n}{bc-ad+(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(-n - 2)*(d*x + c)^n/(b*c - a*d
+ (b*c - a*d)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-2-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{-n-2}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 2)*(d*x + c)^n, x)
```

3.1869 $\int (a+bx)^{-3-n}(c+dx)^n\,dx$

Optimal. Leaf size=80

$$\frac{d(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(bc-ad)^2}-\frac{(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(bc-ad)}$$

```
[Out] -(((a + b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(2 + n))) + (d*(a + b
*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Rubi [A] time = 0.0197497, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{d(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(bc-ad)^2}-\frac{(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(2 + n))) + (d*(a + b
*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-3-n}(c+dx)^n\,dx &= -\frac{(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)(2+n)}-\frac{d\int (a+bx)^{-2-n}(c+dx)^n\,dx}{(bc-ad)(2+n)}\\ &= -\frac{(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)(2+n)}+\frac{d(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A] time = 0.0287897, size = 60, normalized size = 0.75

$$\frac{(a+bx)^{-n-2}(c+dx)^{n+1}(ad(n+2)-b(cn+c-dx))}{(n+1)(n+2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3 - n)*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-2 - n)*(c + d*x)^(1 + n)*(a*d*(2 + n) - b*(c + c*n - d*x)))/((
b*c - a*d)^2*(1 + n)*(2 + n))
```

Maple [A] time = 0.004, size = 123, normalized size = 1.5

$$\frac{(bx+a)^{-2-n}(dx+c)^{1+n}(adn-bcn+bdx+2ad-bc)}{a^2d^2n^2-2abcdn^2+b^2c^2n^2+3a^2d^2n-6abcdn+3b^2c^2n+2a^2d^2-4abcd+2b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-3-n)*(d*x+c)^n,x)
```

```
[Out] (b*x+a)^(-2-n)*(d*x+c)^(1+n)*(a*d*n-b*c*n+b*d*x+2*a*d-b*c)/(a^2*d^2*n^2-2*a
*b*c*d*n^2+b^2*c^2*n^2+3*a^2*d^2*n-6*a*b*c*d*n+3*b^2*c^2*n+2*a^2*d^2-4*a*b*
c*d+2*b^2*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{-n-3}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 3)*(d*x + c)^n, x)
```

Fricas [B] time = 2.26891, size = 416, normalized size = 5.2

$$\frac{\left(b^2d^2x^3-abc^2+2a^2cd+\left(3abd^2-\left(b^2cd-abd^2\right)n\right)x^2-\left(abc^2-a^2cd\right)n-\left(b^2c^2-2abcd-2a^2d^2+\left(b^2c^2-a^2d^2\right)n\right)x\right)(b\cdots}{2b^2c^2-4abcd+2a^2d^2+\left(b^2c^2-2abcd+a^2d^2\right)n^2+3\left(b^2c^2-2abcd+a^2d^2\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 - a*b*c^2 + 2*a^2*c*d + (3*a*b*d^2 - (b^2*c*d - a*b*d^2)*n)*x^
2 - (a*b*c^2 - a^2*c*d)*n - (b^2*c^2 - 2*a*b*c*d - 2*a^2*d^2 + (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^(-n - 3)*(d*x + c)^n/(2*b^2*c^2 - 4*a*b*c*d + 2*a^2
*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-3-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{-n-3}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 3)*(d*x + c)^n, x)
```

3.1870 $\int (a+bx)^{-4-n}(c+dx)^n\,dx$

Optimal. Leaf size=131

$$-\frac{2d^2(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(bc-ad)^3}-\frac{(a+bx)^{-n-3}(c+dx)^{n+1}}{(n+3)(bc-ad)}+\frac{2d(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(bc-ad)^2}$$

```
[Out] -(((a + b*x)^(-3 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(3 + n))) + (2*d*(a +
 b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(2 + n)*(3 + n)) - (2*d^2*
(a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n
))
```

Rubi [A] time = 0.046199, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{2d^2(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(bc-ad)^3}-\frac{(a+bx)^{-n-3}(c+dx)^{n+1}}{(n+3)(bc-ad)}+\frac{2d(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(3 + n))) + (2*d*(a +
 b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(2 + n)*(3 + n)) - (2*d^2*
(a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n
))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-4-n}(c+dx)^n\,dx&=-\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)}-\frac{(2d)\int (a+bx)^{-3-n}(c+dx)^n\,dx}{(bc-ad)(3+n)}\\&=-\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)}+\frac{2d(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^2(2+n)(3+n)}+\frac{\left(2d^2\right)\int (a+bx)^{-2-n}(c+dx)^n\,dx}{(bc-ad)^2(2+n)(3+n)}\\&=-\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)}+\frac{2d(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^2(2+n)(3+n)}-\frac{2d^2(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)^3(1+n)(2+n)(3+n)}\end{aligned}$$

Mathematica [A] time = 0.0565999, size = 113, normalized size = 0.86

$$-\frac{(a+bx)^{-n-3}(c+dx)^{n+1}\left(a^2d^2\left(n^2+5n+6\right)-2abd(n+3)(cn+c-dx)+b^2\left(c^2\left(n^2+3n+2\right)-2cd(n+1)x+2d^2x^2\right)\right)}{(n+1)(n+2)(n+3)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 - n)*(c + d*x)^(1 + n)*(a^2*d^2*(6 + 5*n + n^2) - 2*a*b*d*
(3 + n)*(c + c*n - d*x) + b^2*(c^2*(2 + 3*n + n^2) - 2*c*d*(1 + n)*x + 2*d^
2*x^2)))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n)))
```

Maple [B] time = 0.006, size = 318, normalized size = 2.4

$$\frac{(bx+a)^{-3-n}(dx+c)^{1+n}\left(a^2d^2n^2-2\,abcdn^2+2\,abd^2nx+b^2c^2n^2-2\,b^2cdnx+2\,b^2d^2x^2+5\,a^2d^2n-8\,abcdn\right.}{a^3d^3n^3-3\,a^2bcd^2n^3+3\,ab^2c^2dn^3-b^3c^3n^3+6\,a^3d^3n^2-18\,a^2bcd^2n^2+18\,ab^2c^2dn^2-6\,b^3c^3n^2+11\,a^3d^3n-33\,a^2bcd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-4-n)*(d*x+c)^n,x)
```

```
[Out] (b*x+a)^(-3-n)*(d*x+c)^(1+n)*(a^2*d^2*n^2-2*a*b*c*d*n^2+2*a*b*d^2*n*x+b^2*c
^2*n^2-2*b^2*c*d*n*x+2*b^2*d^2*x^2+5*a^2*d^2*n-8*a*b*c*d*n+6*a*b*d^2*x+3*b^
2*c^2*n-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*d
^2*n^3+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3+6*a^3*d^3*n^2-18*a^2*b*c*d^2*n^2+18*a*
b^2*c^2*d*n^2-6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n-
11*b^3*c^3*n+6*a^3*d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{-n-4}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 4)*(d*x + c)^n, x)
```

Fricas [B] time = 2.23917, size = 1022, normalized size = 7.8

$$-\frac{\left(2\,b^3d^3x^4+2\,ab^2c^3-6\,a^2bc^2d+6\,a^3cd^2+2\left(4\,ab^2d^3-\left(b^3cd^2-ab^2d^3\right)n\right)x^3+\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)n^2+\left(12\,a^2\right.}{6\,b^3c^3-18\,ab^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] -(2*b^3*d^3*x^4 + 2*a*b^2*c^3 - 6*a^2*b*c^2*d + 6*a^3*c*d^2 + 2*(4*a*b^2*d^
3 - (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*n^2 + (12*a^2*b*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 + (b^3*
c^2*d - 8*a*b^2*c*d^2 + 7*a^2*b*d^3)*n)*x^2 + (3*a*b^2*c^3 - 8*a^2*b*c^2*d
+ 5*a^3*c*d^2)*n + (2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 6*a^3*d^3 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 + (3*b^3*c^3 - 7*a*b^2
*c^2*d - a^2*b*c*d^2 + 5*a^3*d^3)*n)*x)*(b*x + a)^(-n - 4)*(d*x + c)^n/(6*b
^3*c^3 - 18*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*n^2 + 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-4-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{-n-4}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 4)*(d*x + c)^n, x)
```

3.1871 $\int (a+bx)^{-5-n}(c+dx)^n\,dx$

Optimal. Leaf size=186

$$-\frac{6d^2(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(n+4)(bc-ad)^3}+\frac{6d^3(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}-\frac{(a+bx)^{-n-4}(c+dx)^{n+1}}{(n+4)(bc-ad)}+\frac{3d(a+bx)^{-n-}}{(n+3)(n+4}$$

```
[Out] -(((a + b*x)^(-4 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(4 + n))) + (3*d*(a +
 b*x)^(-3 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(3 + n)*(4 + n)) - (6*d^2*
(a + b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^3*(2 + n)*(3 + n)*(4 + n
)) + (6*d^3*(a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^4*(1 + n)*(2
 + n)*(3 + n)*(4 + n))
```

Rubi [A] time = 0.0862265, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{6d^2(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(n+4)(bc-ad)^3}+\frac{6d^3(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}-\frac{(a+bx)^{-n-4}(c+dx)^{n+1}}{(n+4)(bc-ad)}+\frac{3d(a+bx)^{-n-}}{(n+3)(n+4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-5 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-4 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(4 + n))) + (3*d*(a +
 b*x)^(-3 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(3 + n)*(4 + n)) - (6*d^2*
(a + b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^3*(2 + n)*(3 + n)*(4 + n
)) + (6*d^3*(a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^4*(1 + n)*(2
 + n)*(3 + n)*(4 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^{-5-n}(c+dx)^n\,dx = -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} - \frac{(3d)\int (a+bx)^{-4-n}(c+dx)^n\,dx}{(bc-ad)(4+n)}$$
$$= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} + \frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)} + \frac{(6d^2)\int (a+bx)^{-3-n}(c+dx)^n\,dx}{(bc-ad)^2(3+n)(4+n)}$$
$$= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} + \frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)} - \frac{6d^2(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^3(2+n)(3+n)(4+n)} -$$
$$= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} + \frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)} - \frac{6d^2(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^3(2+n)(3+n)(4+n)} +$$

Mathematica [A] time = 0.0870107, size = 195, normalized size = 1.05

$$\frac{(a+bx)^{-n-4}(c+dx)^{n+1}\left(-3a^2bd^2\left(n^2+7n+12\right)(cn+c-dx)+a^3d^3\left(n^3+9n^2+26n+24\right)+3ab^2d(n+4)\left(c^2\left(n^2+3n\right.\right.\right.}{(n+1)(n+2)(n+3)(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-5 - n)*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-4 - n)*(c + d*x)^(1 + n)*(a^3*d^3*(24 + 26*n + 9*n^2 + n^3) -
3*a^2*b*d^2*(12 + 7*n + n^2)*(c + c*n - d*x) + 3*a*b^2*d*(4 + n)*(c^2*(2 +
3*n + n^2) - 2*c*d*(1 + n)*x + 2*d^2*x^2) - b^3*(c^3*(6 + 11*n + 6*n^2 + n^
3) - 3*c^2*d*(2 + 3*n + n^2)*x + 6*c*d^2*(1 + n)*x^2 - 6*d^3*x^3)))/((b*c -
 a*d)^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

Maple [B] time = 0.008, size = 661, normalized size = 3.6

$$\frac{(bx+a)^{-4-n}(dx+c)^{1+n}\left(a^3d^3n^3-3a^2bcd^2n^3+3a^2bd^3n^2x+3ab^2c^2dn^3-6ab^2cd^2n^2x+6ab^2d^3nx^2-b^3c^3n^3+3b^3c^2d\right.}{a^4d^4n^4-4a^3bcd^3n^4+6a^2b^2c^2d^2n^4-4ab^3c^3dn^4+b^4c^4n^4+10a^4d^4n^3-40a^3bcd^3n^3+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-5-n)*(d*x+c)^n,x)
```

```
[Out] (b*x+a)^(-4-n)*(d*x+c)^(1+n)*(a^3*d^3*n^3-3*a^2*b*c*d^2*n^3+3*a^2*b*d^3*n^2
*x+3*a*b^2*c^2*d*n^3-6*a*b^2*c*d^2*n^2*x+6*a*b^2*d^3*n*x^2-b^3*c^3*n^3+3*b^
3*c^2*d*n^2*x-6*b^3*c*d^2*n*x^2+6*b^3*d^3*x^3+9*a^3*d^3*n^2-24*a^2*b*c*d^2*
n^2+21*a^2*b*d^3*n*x+21*a*b^2*c^2*d*n^2-30*a*b^2*c*d^2*n*x+24*a*b^2*d^3*x^2
-6*b^3*c^3*n^2+9*b^3*c^2*d*n*x-6*b^3*c*d^2*x^2+26*a^3*d^3*n-57*a^2*b*c*d^2*
n+36*a^2*b*d^3*x+42*a*b^2*c^2*d*n-24*a*b^2*c*d^2*x-11*b^3*c^3*n+6*b^3*c^2*d
*x+24*a^3*d^3-36*a^2*b*c*d^2+24*a*b^2*c^2*d-6*b^3*c^3)/(a^4*d^4*n^4-4*a^3*b
*c*d^3*n^4+6*a^2*b^2*c^2*d^2*n^4-4*a*b^3*c^3*d*n^4+b^4*c^4*n^4+10*a^4*d^4*n
^3-40*a^3*b*c*d^3*n^3+60*a^2*b^2*c^2*d^2*n^3-40*a*b^3*c^3*d*n^3+10*b^4*c^4*
n^3+35*a^4*d^4*n^2-140*a^3*b*c*d^3*n^2+210*a^2*b^2*c^2*d^2*n^2-140*a*b^3*c^
3*d*n^2+35*b^4*c^4*n^2+50*a^4*d^4*n-200*a^3*b*c*d^3*n+300*a^2*b^2*c^2*d^2*n
-200*a*b^3*c^3*d*n+50*b^4*c^4*n+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*d
^2-96*a*b^3*c^3*d+24*b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{-n-5}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 5)*(d*x + c)^n, x)
```

Fricas [B] time = 2.36492, size = 1945, normalized size = 10.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (6*b^4*d^4*x^5 - 6*a*b^3*c^4 + 24*a^2*b^2*c^3*d - 36*a^3*b*c^2*d^2 + 24*a^4
*c*d^3 + 6*(5*a*b^3*d^4 - (b^4*c*d^3 - a*b^3*d^4)*n)*x^4 - (a*b^3*c^4 - 3*a
^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*n^3 + 3*(20*a^2*b^2*d^4 + (b^4*
c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*n^2 + (b^4*c^2*d^2 - 10*a*b^3*c*d^3
+ 9*a^2*b^2*d^4)*n)*x^3 - 3*(2*a*b^3*c^4 - 7*a^2*b^2*c^3*d + 8*a^3*b*c^2*d^
2 - 3*a^4*c*d^3)*n^2 + (60*a^3*b*d^4 - (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2
*b^2*c*d^3 - a^3*b*d^4)*n^3 - 3*(b^4*c^3*d - 6*a*b^3*c^2*d^2 + 9*a^2*b^2*c*
d^3 - 4*a^3*b*d^4)*n^2 - (2*b^4*c^3*d - 15*a*b^3*c^2*d^2 + 60*a^2*b^2*c*d^3
 - 47*a^3*b*d^4)*n)*x^2 - (11*a*b^3*c^4 - 42*a^2*b^2*c^3*d + 57*a^3*b*c^2*d
^2 - 26*a^4*c*d^3)*n - (6*b^4*c^4 - 24*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 - 2
4*a^3*b*c*d^3 - 24*a^4*d^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4
*d^4)*n^3 + 3*(2*b^4*c^4 - 6*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^
3 - 3*a^4*d^4)*n^2 + (11*b^4*c^4 - 40*a*b^3*c^3*d + 45*a^2*b^2*c^2*d^2 + 10
*a^3*b*c*d^3 - 26*a^4*d^4)*n)*x)*(b*x + a)^(-n - 5)*(d*x + c)^n/(24*b^4*c^4
 - 96*a*b^3*c^3*d + 144*a^2*b^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^
4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*n^4 +
10*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
n^3 + 35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*n^2 + 50*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-5-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{-n-5}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 5)*(d*x + c)^n, x)
```

3.1872 $\int (a+bx)^n (c+dx)^{-n}\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{n+1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+1;n+2;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+1)}$$

```
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(1 + n)*(c + d*x)^n)
```

Rubi [A] time = 0.0226195, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {70, 69}

$$\frac{(a+bx)^{n+1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+1;n+2;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(1 + n)*(c + d*x)^n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n(c+dx)^{-n}\,dx &= \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n\right)\int (a+bx)^n\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}dx\\ &= \frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0043694, size = 71, normalized size = 0.99

$$\frac{(a+bx)^{n+1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+1;n+2;\frac{d(a+bx)}{ad-bc}\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(1 + n)*(c + d*x)^n)
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^n}{(dx + c)^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/((d*x+c)^n),x)
```

```
[Out] int((b*x+a)^n/((d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^n}{(dx + c)^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/(d*x + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx + a)^n}{(dx + c)^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/((d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/(d*x + c)^n, x)
```

3.1873 $\int (a+bx)^n (c+dx)^{-1-n}\,dx$

Optimal. Leaf size=75

$$-\frac{(a+bx)^n (c+dx)^{-n} \left(-\frac{d(a+bx)}{bc-ad}\right)^{-n} {}_2F_1\left(-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{dn}$$

```
[Out] -(((a + b*x)^n*Hypergeometric2F1[-n, -n, 1 - n, (b*(c + d*x))/(b*c - a*d)])
/(d*n*(-((d*(a + b*x))/(b*c - a*d)))^n*(c + d*x)^n))
```

Rubi [A] time = 0.0262994, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {70, 69}

$$-\frac{(a+bx)^n (c+dx)^{-n} \left(-\frac{d(a+bx)}{bc-ad}\right)^{-n} {}_2F_1\left(-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-1 - n),x]
```

```
[Out] -(((a + b*x)^n*Hypergeometric2F1[-n, -n, 1 - n, (b*(c + d*x))/(b*c - a*d)])
/(d*n*(-((d*(a + b*x))/(b*c - a*d)))^n*(c + d*x)^n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int (a+bx)^n (c+dx)^{-1-n}\,dx &= \left((a+bx)^n \left(\frac{d(a+bx)}{-bc+ad}\right)^{-n}\right) \int (c+dx)^{-1-n} \left(-\frac{ad}{bc-ad}-\frac{bdx}{bc-ad}\right)^n dx\\
&= -\frac{(a+bx)^n \left(-\frac{d(a+bx)}{bc-ad}\right)^{-n} (c+dx)^{-n}\, {}_2F_1\left(-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{dn}
\end{aligned}$$

Mathematica [A] time = 0.0231484, size = 74, normalized size = 0.99

$$-\frac{(a+bx)^n (c+dx)^{-n} \left(\frac{d(a+bx)}{ad-bc}\right)^{-n} {}_2F_1\left(-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-1 - n),x]
```

```
[Out] -(((a + b*x)^n*Hypergeometric2F1[-n, -n, 1 - n, (b*(c + d*x))/(b*c - a*d)])
/(d*n*((d*(a + b*x))/(-(b*c) + a*d))^n*(c + d*x)^n))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int (bx + a)^{n} (dx + c)^{-1-n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-1-n),x)
```

```
[Out] int((b*x+a)^n*(d*x+c)^(-1-n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^{n} (dx + c)^{-n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-1-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx + a)^{n} (dx + c)^{-n-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-1-n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n*(d*x + c)^(-n - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-1-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n (dx+c)^{-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-1-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 1), x)
```

3.1874 $\int (a+bx)^n (c+dx)^{-2-n}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(bc-ad)}$$

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)*(1 + n))
```

Rubi [A] time = 0.0045702, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-2 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)*(1 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^n (c+dx)^{-2-n}\,dx = \frac{(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)(1+n)}$$

Mathematica [A] time = 0.0097371, size = 36, normalized size = 1.

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-2 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)*(1 + n))
```

Maple [A] time = 0.004, size = 42, normalized size = 1.2

$$-\frac{(bx+a)^{1+n}(dx+c)^{-1-n}}{adn-bcn+ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-2-n),x)
```

```
[Out] -(b*x+a)^(1+n)*(d*x+c)^(-1-n)/(a*d*n-b*c*n+a*d-b*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n(dx+c)^{-n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 2), x)
```

Fricas [A] time = 2.25502, size = 124, normalized size = 3.44

$$\frac{\left(bdx^2+ac+(bc+ad)x\right)(bx+a)^n(dx+c)^{-n-2}}{bc-ad+(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="fricas")
```

```
[Out] (b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^n*(d*x + c)^(-n - 2)/(b*c - a*d +
 (b*c - a*d)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-2-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n(dx+c)^{-n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 2), x)
```

3.1875 $\int (a+bx)^n (c+dx)^{-3-n}\,dx$

Optimal. Leaf size=79

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(bc-ad)}+\frac{b(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(bc-ad)^2}$$

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)*(2 + n)) + (b*(a + b*x)
^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Rubi [A] time = 0.0169357, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(bc-ad)}+\frac{b(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-3 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)*(2 + n)) + (b*(a + b*x)
^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n(c+dx)^{-3-n}\,dx &= \frac{(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)(2+n)}+\frac{b\int (a+bx)^n(c+dx)^{-2-n}\,dx}{(bc-ad)(2+n)}\\ &= \frac{(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)(2+n)}+\frac{b(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A] time = 0.0292703, size = 59, normalized size = 0.75

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-2}(-ad(n+1)+bc(n+2)+bdx)}{(n+1)(n+2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-3 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-2 - n)*(-(a*d*(1 + n)) + b*c*(2 + n) + b*d*x
))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Maple [A] time = 0.003, size = 124, normalized size = 1.6

$$-\frac{(bx+a)^{1+n}(dx+c)^{-2-n}(adn-bcn-bdx+ad-2bc)}{a^2d^2n^2-2abcdn^2+b^2c^2n^2+3a^2d^2n-6abcdn+3b^2c^2n+2a^2d^2-4abcd+2b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-3-n),x)
```

```
[Out] -(b*x+a)^(1+n)*(d*x+c)^(-2-n)*(a*d*n-b*c*n-b*d*x+a*d-2*b*c)/(a^2*d^2*n^2-2*
a*b*c*d*n^2+b^2*c^2*n^2+3*a^2*d^2*n-6*a*b*c*d*n+3*b^2*c^2*n+2*a^2*d^2-4*a*b
*c*d+2*b^2*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n(dx+c)^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 3), x)
```

Fricas [B] time = 2.19001, size = 416, normalized size = 5.27

$$\frac{\left(b^2d^2x^3+2abc^2-a^2cd+\left(3b^2cd+\left(b^2cd-abd^2\right)n\right)x^2+\left(abc^2-a^2cd\right)n+\left(2b^2c^2+2abcd-a^2d^2+\left(b^2c^2-a^2d^2\right)n\right)x\right)(b}{2b^2c^2-4abcd+2a^2d^2+\left(b^2c^2-2abcd+a^2d^2\right)n^2+3\left(b^2c^2-2abcd+a^2d^2\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 + 2*a*b*c^2 - a^2*c*d + (3*b^2*c*d + (b^2*c*d - a*b*d^2)*n)*x^
2 + (a*b*c^2 - a^2*c*d)*n + (2*b^2*c^2 + 2*a*b*c*d - a^2*d^2 + (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 3)/(2*b^2*c^2 - 4*a*b*c*d + 2*a^2
*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-3-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n(dx+c)^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 3), x)
```

3.1876 $\int (a+bx)^n(c+dx)^{-4-n}\,dx$

Optimal. Leaf size=130

$$\frac{2b^2(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(bc-ad)^3}+\frac{(a+bx)^{n+1}(c+dx)^{-n-3}}{(n+3)(bc-ad)}+\frac{2b(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(bc-ad)^2}$$

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-3 - n))/((b*c - a*d)*(3 + n)) + (2*b*(a + b*
x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)^2*(2 + n)*(3 + n)) + (2*b^2*(a
+ b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n))
```

Rubi [A] time = 0.0372081, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{2b^2(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(bc-ad)^3}+\frac{(a+bx)^{n+1}(c+dx)^{-n-3}}{(n+3)(bc-ad)}+\frac{2b(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-4 - n),x]
```


```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-3 - n))/((b*c - a*d)*(3 + n)) + (2*b*(a + b*
x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)^2*(2 + n)*(3 + n)) + (2*b^2*(a
+ b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n))
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n(c+dx)^{-4-n}\,dx &= \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)}+\frac{(2b)\int (a+bx)^n(c+dx)^{-3-n}\,dx}{(bc-ad)(3+n)}\\ &= \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)}+\frac{2b(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^2(2+n)(3+n)}+\frac{\left(2b^2\right)\int (a+bx)^n(c+dx)^{-2-n}\,dx}{(bc-ad)^2(2+n)(3+n)}\\ &= \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)}+\frac{2b(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^2(2+n)(3+n)}+\frac{2b^2(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)^3(1+n)(2+n)(3+n)}\end{aligned}$$

Mathematica [A] time = 0.0581298, size = 112, normalized size = 0.86

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-3}\left(a^2d^2\left(n^2+3n+2\right)-2abd(n+1)(c(n+3)+dx)+b^2\left(c^2\left(n^2+5n+6\right)+2cd(n+3)x+2d^2x\right.\right.}{(n+1)(n+2)(n+3)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-4 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-3 - n)*(a^2*d^2*(2 + 3*n + n^2) - 2*a*b*d*(1
 + n)*(c*(3 + n) + d*x) + b^2*(c^2*(6 + 5*n + n^2) + 2*c*d*(3 + n)*x + 2*d^
2*x^2)))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n))
```

Maple [B] time = 0.006, size = 319, normalized size = 2.5

$$-\frac{(bx+a)^{1+n}(dx+c)^{-3-n}\left(a^2d^2n^2-2\,abcdn^2-2\,abd^2nx+b^2c^2n^2+2\,b^2cdnx+2\,b^2d^2x^2+3\,a^2d^2n-8\,abcd\right.}{a^3d^3n^3-3\,a^2bcd^2n^3+3\,ab^2c^2dn^3-b^3c^3n^3+6\,a^3d^3n^2-18\,a^2bcd^2n^2+18\,ab^2c^2dn^2-6\,b^3c^3n^2+11\,a^3d^3n-33\,a^2bcd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-4-n),x)
```

```
[Out] -(b*x+a)^(1+n)*(d*x+c)^(-3-n)*(a^2*d^2*n^2-2*a*b*c*d*n^2-2*a*b*d^2*n*x+b^2*
c^2*n^2+2*b^2*c*d*n*x+2*b^2*d^2*x^2+3*a^2*d^2*n-8*a*b*c*d*n-2*a*b*d^2*x+5*b
^2*c^2*n+6*b^2*c*d*x+2*a^2*d^2-6*a*b*c*d+6*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*
d^2*n^3+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3+6*a^3*d^3*n^2-18*a^2*b*c*d^2*n^2+18*a
*b^2*c^2*d*n^2-6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n
-11*b^3*c^3*n+6*a^3*d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n(dx+c)^{-n-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 4), x)
```

Fricas [B] time = 2.26154, size = 1021, normalized size = 7.85

$$\frac{\left(2\,b^3d^3x^4+6\,ab^2c^3-6\,a^2bc^2d+2\,a^3cd^2+2\left(4\,b^3cd^2+\left(b^3cd^2-ab^2d^3\right)n\right)x^3+\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)n^2+\left(12\,b^3c^2\right.\right.}{6\,b^3c^3-18\,ab^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="fricas")
```

```
[Out] (2*b^3*d^3*x^4 + 6*a*b^2*c^3 - 6*a^2*b*c^2*d + 2*a^3*c*d^2 + 2*(4*b^3*c*d^2
 + (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)
```

```
*n^2 + (12*b^3*c^2*d + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 + (7*b^3
*c^2*d - 8*a*b^2*c*d^2 + a^2*b*d^3)*n)*x^2 + (5*a*b^2*c^3 - 8*a^2*b*c^2*d +
 3*a^3*c*d^2)*n + (6*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 + (5*b^3*c^3 - a*b^2*c^
2*d - 7*a^2*b*c*d^2 + 3*a^3*d^3)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 4)/(6*b^
3*c^3 - 18*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*n^2 + 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-4-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n (dx+c)^{-n-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 4), x)
```

3.1877 $\int (a+bx)^n (c+dx)^{-5-n}\,dx$

Optimal. Leaf size=185

$$\frac{6b^2(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(n+4)(bc-ad)^3}+\frac{6b^3(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}+\frac{(a+bx)^{n+1}(c+dx)^{-n-4}}{(n+4)(bc-ad)}+\frac{3b(a+bx)^{n+1}(c}{(n+3)(n+4)}$$

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-4 - n))/((b*c - a*d)*(4 + n)) + (3*b*(a + b*
x)^(1 + n)*(c + d*x)^(-3 - n))/((b*c - a*d)^2*(3 + n)*(4 + n)) + (6*b^2*(a
+ b*x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)^3*(2 + n)*(3 + n)*(4 + n))
+ (6*b^3*(a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^4*(1 + n)*(2 +
n)*(3 + n)*(4 + n))
```

Rubi [A] time = 0.0616625, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{6b^2(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(n+4)(bc-ad)^3}+\frac{6b^3(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}+\frac{(a+bx)^{n+1}(c+dx)^{-n-4}}{(n+4)(bc-ad)}+\frac{3b(a+bx)^{n+1}(c}{(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-5 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-4 - n))/((b*c - a*d)*(4 + n)) + (3*b*(a + b*
x)^(1 + n)*(c + d*x)^(-3 - n))/((b*c - a*d)^2*(3 + n)*(4 + n)) + (6*b^2*(a
+ b*x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)^3*(2 + n)*(3 + n)*(4 + n))
+ (6*b^3*(a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^4*(1 + n)*(2 +
n)*(3 + n)*(4 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^n(c+dx)^{-5-n}\,dx = \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{(3b)\int (a+bx)^n(c+dx)^{-4-n}\,dx}{(bc-ad)(4+n)}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)} + \frac{(6b^2)\int (a+bx)^n(c+dx)^{-3-n}\,dx}{(bc-ad)^2(3+n)(4+n)}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)} + \frac{6b^2(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^3(2+n)(3+n)(4+n)} + \ldots$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)} + \frac{6b^2(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^3(2+n)(3+n)(4+n)} + \ldots$$

Mathematica [A] time = 0.0910045, size = 195, normalized size = 1.05

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-4}\left(3a^2bd^2\left(n^2+3n+2\right)(c(n+4)+dx)-a^3d^3\left(n^3+6n^2+11n+6\right)-3ab^2d(n+1)\left(c^2\left(n^2+7n+\ldots\right.\right.\right.}{(n+1)(n+2)(n+3)(n \ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-5 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-4 - n)*(-(a^3*d^3*(6 + 11*n + 6*n^2 + n^3))
+ 3*a^2*b*d^2*(2 + 3*n + n^2)*(c*(4 + n) + d*x) - 3*a*b^2*d*(1 + n)*(c^2*(1
2 + 7*n + n^2) + 2*c*d*(4 + n)*x + 2*d^2*x^2) + b^3*(c^3*(24 + 26*n + 9*n^2
 + n^3) + 3*c^2*d*(12 + 7*n + n^2)*x + 6*c*d^2*(4 + n)*x^2 + 6*d^3*x^3)))/(
(b*c - a*d)^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

Maple [B] time = 0.007, size = 662, normalized size = 3.6

$$-\frac{(bx+a)^{1+n}(dx+c)^{-4-n}\left(a^3d^3n^3-3a^2bcd^2n^3-3a^2bd^3n^2x+3ab^2c^2dn^3+6ab^2cd^2n^2x+6ab^2d^3nx^2-b^3c^3n^3-3b^3c^2\ldots\right.}{a^4d^4n^4-4a^3bcd^3n^4+6a^2b^2c^2d^2n^4-4ab^3c^3dn^4+b^4c^4n^4+10a^4d^4n^3-40a^3bcd^3n^3+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-5-n),x)
```

```
[Out] -(b*x+a)^(1+n)*(d*x+c)^(-4-n)*(a^3*d^3*n^3-3*a^2*b*c*d^2*n^3-3*a^2*b*d^3*n^
2*x+3*a*b^2*c^2*d*n^3+6*a*b^2*c*d^2*n^2*x+6*a*b^2*d^3*n*x^2-b^3*c^3*n^3-3*b
^3*c^2*d*n^2*x-6*b^3*c*d^2*n*x^2-6*b^3*d^3*x^3+6*a^3*d^3*n^2-21*a^2*b*c*d^2
*n^2-9*a^2*b*d^3*n*x+24*a*b^2*c^2*d*n^2+30*a*b^2*c*d^2*n*x+6*a*b^2*d^3*x^2-
9*b^3*c^3*n^2-21*b^3*c^2*d*n*x-24*b^3*c*d^2*x^2+11*a^3*d^3*n-42*a^2*b*c*d^2
*n-6*a^2*b*d^3*x+57*a*b^2*c^2*d*n+24*a*b^2*c*d^2*x-26*b^3*c^3*n-36*b^3*c^2*
d*x+6*a^3*d^3-24*a^2*b*c*d^2+36*a*b^2*c^2*d-24*b^3*c^3)/(a^4*d^4*n^4-4*a^3*
b*c*d^3*n^4+6*a^2*b^2*c^2*d^2*n^4-4*a*b^3*c^3*d*n^4+b^4*c^4*n^4+10*a^4*d^4*
n^3-40*a^3*b*c*d^3*n^3+60*a^2*b^2*c^2*d^2*n^3-40*a*b^3*c^3*d*n^3+10*b^4*c^4
*n^3+35*a^4*d^4*n^2-140*a^3*b*c*d^3*n^2+210*a^2*b^2*c^2*d^2*n^2-140*a*b^3*c
^3*d*n^2+35*b^4*c^4*n^2+50*a^4*d^4*n-200*a^3*b*c*d^3*n+300*a^2*b^2*c^2*d^2*
n-200*a*b^3*c^3*d*n+50*b^4*c^4*n+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*
d^2-96*a*b^3*c^3*d+24*b^4*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^n(dx+c)^{-n-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 5), x)
```

Fricas [B] time = 2.41156, size = 1945, normalized size = 10.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="fricas")
```

```
[Out] (6*b^4*d^4*x^5 + 24*a*b^3*c^4 - 36*a^2*b^2*c^3*d + 24*a^3*b*c^2*d^2 - 6*a^4
*c*d^3 + 6*(5*b^4*c*d^3 + (b^4*c*d^3 - a*b^3*d^4)*n)*x^4 + (a*b^3*c^4 - 3*a
^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*n^3 + 3*(20*b^4*c^2*d^2 + (b^4*
c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*n^2 + (9*b^4*c^2*d^2 - 10*a*b^3*c*d^
3 + a^2*b^2*d^4)*n)*x^3 + 3*(3*a*b^3*c^4 - 8*a^2*b^2*c^3*d + 7*a^3*b*c^2*d^
2 - 2*a^4*c*d^3)*n^2 + (60*b^4*c^3*d + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2
*b^2*c*d^3 - a^3*b*d^4)*n^3 + 3*(4*b^4*c^3*d - 9*a*b^3*c^2*d^2 + 6*a^2*b^2*
c*d^3 - a^3*b*d^4)*n^2 + (47*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 15*a^2*b^2*c*d^
3 - 2*a^3*b*d^4)*n)*x^2 + (26*a*b^3*c^4 - 57*a^2*b^2*c^3*d + 42*a^3*b*c^2*d
^2 - 11*a^4*c*d^3)*n + (24*b^4*c^4 + 24*a*b^3*c^3*d - 36*a^2*b^2*c^2*d^2 +
24*a^3*b*c*d^3 - 6*a^4*d^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4
*d^4)*n^3 + 3*(3*b^4*c^4 - 4*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 6*a^3*b*c*d^
3 - 2*a^4*d^4)*n^2 + (26*b^4*c^4 - 10*a*b^3*c^3*d - 45*a^2*b^2*c^2*d^2 + 40
*a^3*b*c*d^3 - 11*a^4*d^4)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 5)/(24*b^4*c^4
 - 96*a*b^3*c^3*d + 144*a^2*b^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^
4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*n^4 +
10*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
n^3 + 35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*n^2 + 50*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-5-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^n (dx + c)^{-n-5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="giac")

[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 5), x)
```

3.1878 $\int (a+bx)^{-2+n}(c+dx)^{1-n}\,dx$

Optimal. Leaf size=83

$$-\frac{(bc-ad)(a+bx)^{n-1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n-1,n-1;n;-\frac{d(a+bx)}{bc-ad}\right)}{b^2(1-n)}$$

```
[Out] -(((b*c - a*d)*(a + b*x)^(-1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeomet
ric2F1[-1 + n, -1 + n, n, -((d*(a + b*x))/(b*c - a*d))])/(b^2*(1 - n)*(c +
d*x)^n))
```

Rubi [A] time = 0.0287925, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {70, 69}

$$-\frac{(bc-ad)(a+bx)^{n-1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n-1,n-1;n;-\frac{d(a+bx)}{bc-ad}\right)}{b^2(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2 + n)*(c + d*x)^(1 - n),x]
```

```
[Out] -(((b*c - a*d)*(a + b*x)^(-1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeomet
ric2F1[-1 + n, -1 + n, n, -((d*(a + b*x))/(b*c - a*d))])/(b^2*(1 - n)*(c +
d*x)^n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-2+n}(c+dx)^{1-n}\,dx &= \frac{\left((bc-ad)(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n\right)\int (a+bx)^{-2+n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{1-n}dx}{b}\\ &= -\frac{(bc-ad)(a+bx)^{-1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(-1+n,-1+n;n;-\frac{d(a+bx)}{bc-ad}\right)}{b^2(1-n)}\end{aligned}$$

Mathematica [A] time = 0.0420604, size = 75, normalized size = 0.9

$$\frac{(a+bx)^{n-1}(c+dx)^{1-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n-1}{}_2F_1\left(n-1,n-1;n;\frac{d(a+bx)}{ad-bc}\right)}{b(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2 + n)*(c + d*x)^(1 - n),x]
```

```
[Out] ((a + b*x)^(-1 + n)*(c + d*x)^(1 - n)*((b*(c + d*x))/(b*c - a*d))^(-1 + n)*
Hypergeometric2F1[-1 + n, -1 + n, n, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(-1
+ n))
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int (bx+a)^{-2+n}(dx+c)^{1-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-2+n)*(d*x+c)^(1-n),x)
```

```
[Out] int((b*x+a)^(-2+n)*(d*x+c)^(1-n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{n-2}(dx+c)^{-n+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)*(d*x+c)^(1-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 2)*(d*x + c)^(-n + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^{n-2}(dx+c)^{-n+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)*(d*x+c)^(1-n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n - 2)*(d*x + c)^(-n + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-2+n)*(d*x+c)**(1-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{n-2}(dx+c)^{-n+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)*(d*x+c)^(1-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 2)*(d*x + c)^(-n + 1), x)
```

3.1879 $\int (a+bx)^{1+n}(c+dx)^{-1-n}\,dx$

Optimal. Leaf size=84

$$\frac{(bc-ad)(a+bx)^n(c+dx)^{-n}\left(-\frac{d(a+bx)}{bc-ad}\right)^{-n}{}_2F_1\left(-n-1,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{d^2n}$$

```
[Out] ((b*c - a*d)*(a + b*x)^n*Hypergeometric2F1[-1 - n, -n, 1 - n, (b*(c + d*x))
/(b*c - a*d)])/(d^2*n*(-((d*(a + b*x))/(b*c - a*d)))^n*(c + d*x)^n)
```

Rubi [A] time = 0.0297747, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {70, 69}

$$\frac{(bc-ad)(a+bx)^n(c+dx)^{-n}\left(-\frac{d(a+bx)}{bc-ad}\right)^{-n}{}_2F_1\left(-n-1,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{d^2n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1 + n)*(c + d*x)^(-1 - n),x]
```


```
[Out] ((b*c - a*d)*(a + b*x)^n*Hypergeometric2F1[-1 - n, -n, 1 - n, (b*(c + d*x))
/(b*c - a*d)])/(d^2*n*(-((d*(a + b*x))/(b*c - a*d)))^n*(c + d*x)^n)
```


Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{1+n}(c+dx)^{-1-n}\,dx &= \frac{\left((-bc+ad)(a+bx)^n\left(\frac{d(a+bx)}{-bc+ad}\right)^{-n}\right)\int (c+dx)^{-1-n}\left(-\frac{ad}{bc-ad}-\frac{bdx}{bc-ad}\right)^{1+n}dx}{d}\\ &= \frac{(bc-ad)(a+bx)^n\left(-\frac{d(a+bx)}{bc-ad}\right)^{-n}(c+dx)^{-n}{}_2F_1\left(-1-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{d^2n}\end{aligned}$$

Mathematica [A] time = 0.0319157, size = 83, normalized size = 0.99

$$\frac{(bc-ad)(a+bx)^n(c+dx)^{-n}\left(\frac{d(a+bx)}{ad-bc}\right)^{-n}{}_2F_1\left(-n-1,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{d^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1 + n)*(c + d*x)^(-1 - n),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^n*Hypergeometric2F1[-1 - n, -n, 1 - n, (b*(c + d*x))
/(b*c - a*d)])/(d^2*n*((d*(a + b*x))/(-(b*c) + a*d))^n*(c + d*x)^n)
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int (bx+a)^{1+n} (dx+c)^{-1-n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1+n)*(d*x+c)^(-1-n),x)
```

```
[Out] int((b*x+a)^(1+n)*(d*x+c)^(-1-n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{n+1}(dx+c)^{-n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)*(d*x+c)^(-1-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n + 1)*(d*x + c)^(-n - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx+a)^{n+1}(dx+c)^{-n-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)*(d*x+c)^(-1-n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n + 1)*(d*x + c)^(-n - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1+n)*(d*x+c)**(-1-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{n+1}(dx+c)^{-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)*(d*x+c)^(-1-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n + 1)*(d*x + c)^(-n - 1), x)
```

3.1880 $\int (a+bx)^m (c+dx)^{1+2n-2(1+n)}\,dx$

Optimal. Leaf size=51

$$\frac{(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)}$$

```
[Out] ((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/(b*c
- a*d))])/((b*c - a*d)*(1 + m))
```

Rubi [A] time = 0.0091459, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7, 68}

$$\frac{(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(1 + 2*n - 2*(1 + n)),x]
```

```
[Out] ((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/(b*c
- a*d))])/((b*c - a*d)*(1 + m))
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m (c+dx)^{1+2n-2(1+n)}\,dx &= \int \frac{(a+bx)^m}{c+dx}\,dx\\ &= \frac{(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0039276, size = 51, normalized size = 1.

$$-\frac{(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{d(a+bx)}{ad-bc}\right)}{(m+1)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(1 + 2*n - 2*(1 + n)),x]
```

```
[Out] -(((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*x))/(-(b*
c) + a*d)])/((-(b*c) + a*d)*(1 + m)))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c),x)
```

```
[Out] int((b*x+a)^m/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)^m}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^m}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c),x)
```

```
[Out] Integral((a + b*x)**m/(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^m/(d*x + c), x)
```

3.1881 $\int \frac{(c+dx)^{1+2n-2(1+n)}}{(a+bx)^2} dx$

Optimal. Leaf size=57

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
d*x])/(b*c - a*d)^2
```

Rubi [A] time = 0.0301914, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7, 44}

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2,x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
d*x])/(b*c - a*d)^2
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{1+2n-2(1+n)}}{(a+bx)^2}\,dx &= \int \frac{1}{(a+bx)^2(c+dx)}\,dx\\
&= \int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)dx\\
&= -\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}
\end{aligned}$$

Mathematica [A] time = 0.0256779, size = 53, normalized size = 0.93

$$\frac{d(a+bx)\log(c+dx)-d(a+bx)\log(a+bx)+ad-bc}{(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2,x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x)*Log[a + b*x] + d*(a + b*x)*Log[c + d*x])/((b*c
 - a*d)^2*(a + b*x))
```

Maple [A] time = 0.007, size = 57, normalized size = 1.

$$\frac{d\ln\left(dx+c\right)}{\left(ad-bc\right)^{2}}+\frac{1}{\left(ad-bc\right)\left(bx+a\right)}-\frac{d\ln\left(bx+a\right)}{\left(ad-bc\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c),x)
```

```
[Out] d/(a*d-b*c)^2*ln(d*x+c)+1/(a*d-b*c)/(b*x+a)-d/(a*d-b*c)^2*ln(b*x+a)
```

Maxima [A] time = 0.990379, size = 124, normalized size = 2.18

$$-\frac{d\log\left(bx+a\right)}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}+\frac{d\log\left(dx+c\right)}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}-\frac{1}{abc-a^{2}d+\left(b^{2}c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + d*log(d*x + c)/(b^2*c^2 -
 2*a*b*c*d + a^2*d^2) - 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Fricas [A] time = 2.28794, size = 200, normalized size = 3.51

$$-\frac{bc-ad+\left(bdx+ad\right)\log\left(bx+a\right)-\left(bdx+ad\right)\log\left(dx+c\right)}{ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2}+\left(b^{3}c^{2}-2\,ab^{2}cd+a^{2}bd^{2}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*c - a*d + (b*d*x + a*d)*log(b*x + a) - (b*d*x + a*d)*log(d*x + c))/(a*b
^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)
```

Sympy [B] time = 0.814668, size = 233, normalized size = 4.09

$$\frac{d\log\left(x+\frac{-\frac{a^{3}d^{4}}{(ad-bc)^{2}}+\frac{3a^{2}bcd^{3}}{(ad-bc)^{2}}-\frac{3ab^{2}c^{2}d^{2}}{(ad-bc)^{2}}+ad^{2}+\frac{b^{3}c^{3}d}{(ad-bc)^{2}}+bcd}{2bd^{2}}\right)}{\left(ad-bc\right)^{2}}-\frac{d\log\left(x+\frac{\frac{a^{3}d^{4}}{(ad-bc)^{2}}-\frac{3a^{2}bcd^{3}}{(ad-bc)^{2}}+\frac{3ab^{2}c^{2}d^{2}}{(ad-bc)^{2}}+ad^{2}-\frac{b^{3}c^{3}d}{(ad-bc)^{2}}+bcd}{2bd^{2}}\right)}{\left(ad-bc\right)^{2}}+\frac{1}{a^{2}d-abc+x\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c),x)
```

```
[Out] d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a
*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*
d)/(2*b*d**2))/(a*d - b*c)**2 - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**
2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b*
*3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(a*d - b*c)**2 + 1/(a**2*d -
a*b*c + x*(a*b*d - b**2*c))
```

Giac [A] time = 1.06661, size = 105, normalized size = 1.84

$$\frac{bd\log\left(\left|\frac{bc}{bx+a} - \frac{ad}{bx+a} + d\right|\right)}{b^3c^2 - 2\,ab^2cd + a^2bd^2} - \frac{b}{\left(b^2c - abd\right)(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] b*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^3*c^2 - 2*a*b^2*c*d + a^
2*b*d^2) - b/((b^2*c - a*b*d)*(b*x + a))
```

3.1882 $\int (a+bx)^m(ac(1+m)+bc(2+m)x)^{-3-m}\,dx$

Optimal. Leaf size=95

$$\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-1}}{a^2bc^2(m+1)(m+2)}-\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-2}}{abc(m+2)}$$

```
[Out] -(((a + b*x)^(1 + m)*(a*c*(1 + m) + b*c*(2 + m)*x)^(-2 - m))/(a*b*c*(2 + m)
)) + ((a + b*x)^(1 + m)*(a*c*(1 + m) + b*c*(2 + m)*x)^(-1 - m))/(a^2*b*c^2*
(1 + m)*(2 + m))
```

Rubi [A] time = 0.0360218, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {45, 37}

$$\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-1}}{a^2bc^2(m+1)(m+2)}-\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-2}}{abc(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(a*c*(1 + m) + b*c*(2 + m)*x)^(-2 - m))/(a*b*c*(2 + m)
)) + ((a + b*x)^(1 + m)*(a*c*(1 + m) + b*c*(2 + m)*x)^(-1 - m))/(a^2*b*c^2*
(1 + m)*(2 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(ac(1+m)+bc(2+m)x)^{-3-m}\,dx &= -\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-2-m}}{abc(2+m)}-\frac{\int (a+bx)^m(ac(1+m)+\ldots}{ac(2+\ldots}\\ &= -\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-2-m}}{abc(2+m)}+\frac{(a+bx)^{1+m}(ac(1+m)\ldots}{a^2bc^2(1+m)\ldots}\end{aligned}$$

Mathematica [A] time = 0.0516635, size = 54, normalized size = 0.57

$$\frac{x(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m}}{a^2c^3(m+1)(a(m+1)+b(m+2)x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m),x]
```

```
[Out] (x*(a + b*x)^(1 + m))/(a^2*c^3*(1 + m)*(a*(1 + m) + b*(2 + m)*x)^2*(a*c*(1
+ m) + b*c*(2 + m)*x)^m)
```

Maple [A] time = 0.005, size = 57, normalized size = 0.6

$$\frac{(bx+a)^{1+m}(bxm+am+2bx+a)x(bcxm+acm+2bcx+ac)^{-3-m}}{a^2(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x)
```

```
[Out] (b*x+a)^(1+m)*(b*m*x+a*m+2*b*x+a)/a^2/(1+m)*x*(b*c*m*x+a*c*m+2*b*c*x+a*c)^(
-3-m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bc(m+2)x+ac(m+1))^{-m-3}(bx+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="maxima")
```

```
[Out] integrate((b*c*(m + 2)*x + a*c*(m + 1))^(-m - 3)*(b*x + a)^m, x)
```

Fricas [A] time = 2.61231, size = 181, normalized size = 1.91

$$\frac{\left((b^2m+2b^2)x^3+(2abm+3ab)x^2+\left(a^2m+a^2\right)x\right)(acm+ac+(bcm+2bc)x)^{-m-3}(bx+a)^m}{a^2m+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="fricas")
```

```
[Out] ((b^2*m + 2*b^2)*x^3 + (2*a*b*m + 3*a*b)*x^2 + (a^2*m + a^2)*x)*(a*c*m + a*
c + (b*c*m + 2*b*c)*x)^(-m - 3)*(b*x + a)^m/(a^2*m + a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(a*c*(1+m)+b*c*(2+m)*x)**(-3-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bc(m+2)x + ac(m+1))^{-m-3}(bx+a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="giac")
```

```
[Out] integrate((b*c*(m + 2)*x + a*c*(m + 1))^(-m - 3)*(b*x + a)^m, x)
```

3.1883 $\int (a+bx)^{-1-\frac{bc}{bc-ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx$

Optimal. Leaf size=97

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

```
[Out] -((c + d*x)^((a*d)/(b*c - a*d))/(b*c*(a + b*x)^((b*c)/(b*c - a*d)))) + (c +
 d*x)^((a*d)/(b*c - a*d))/(a*b*c*(a + b*x)^((a*d)/(b*c - a*d)))
```

Rubi [A] time = 0.0221451, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {45, 37}

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1 + (a*d)/(b*c - a*d)),x
]
```

```
[Out] -((c + d*x)^((a*d)/(b*c - a*d))/(b*c*(a + b*x)^((b*c)/(b*c - a*d)))) + (c +
 d*x)^((a*d)/(b*c - a*d))/(a*b*c*(a + b*x)^((a*d)/(b*c - a*d)))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-1-\frac{bc}{bc-ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx &= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}-\frac{d\int (a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx}{bc}\\ &= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}+\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}\end{aligned}$$

Mathematica [A] time = 0.0285467, size = 46, normalized size = 0.47

$$\frac{x(a+bx)^{\frac{bc}{ad-bc}}(c+dx)^{\frac{ad}{bc-ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1 + (a*d)/(b*c - a
*d)),x]
```

```
[Out] (x*(a + b*x)^((b*c)/(-(b*c) + a*d))*(c + d*x)^((a*d)/(b*c - a*d)))/(a*c)
```

Maple [A] time = 0.004, size = 66, normalized size = 0.7

$$\frac{x}{ac}(bx+a)^{1-\frac{ad-2bc}{ad-bc}}(dx+c)^{1-\frac{2ad-bc}{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x)
```

```
[Out] (b*x+a)^(1-(a*d-2*b*c)/(a*d-b*c))*(d*x+c)^(1-(2*a*d-b*c)/(a*d-b*c))/a/c*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{bc}{bc-ad}-1}(dx+c)^{\frac{ad}{bc-ad}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="maxima")
```

```
[Out] integrate((b*x + a)^(-b*c/(b*c - a*d) - 1)*(d*x + c)^(a*d/(b*c - a*d) - 1),
 x)
```

Fricas [A] time = 2.52583, size = 161, normalized size = 1.66

$$\frac{bdx^3+acx+(bc+ad)x^2}{(bx+a)^{\frac{2bc-ad}{bc-ad}}(dx+c)^{\frac{bc-2ad}{bc-ad}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="fricas")
```

```
[Out] (b*d*x^3 + a*c*x + (b*c + a*d)*x^2)/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*
(d*x + c)^((b*c - 2*a*d)/(b*c - a*d))*a*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-1-b*c/(-a*d+b*c))*(d*x+c)**(-1+a*d/(-a*d+b*c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^{-\frac{bc}{bc-ad}-1}(dx+c)^{\frac{ad}{bc-ad}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="giac")
```

```
[Out] integrate((b*x + a)^(-b*c/(b*c - a*d) - 1)*(d*x + c)^(a*d/(b*c - a*d) - 1),
 x)
```

3.1884 $\int (a+bx)^{\frac{-2bc+ad}{bc-ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx$

Optimal. Leaf size=97

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

```
[Out] -((c + d*x)^((a*d)/(b*c - a*d))/(b*c*(a + b*x)^((b*c)/(b*c - a*d)))) + (c +
 d*x)^((a*d)/(b*c - a*d))/(a*b*c*(a + b*x)^((a*d)/(b*c - a*d)))
```

Rubi [A] time = 0.018449, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {45, 37}

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^((b*c - 2*a*d)/(-(b*c)
 + a*d)),x]
```

```
[Out] -((c + d*x)^((a*d)/(b*c - a*d))/(b*c*(a + b*x)^((b*c)/(b*c - a*d)))) + (c +
 d*x)^((a*d)/(b*c - a*d))/(a*b*c*(a + b*x)^((a*d)/(b*c - a*d)))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{\frac{-2bc+ad}{bc-ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx &= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}-\frac{d\int (a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx}{bc}\\ &= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}+\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}\end{aligned}$$

Mathematica [A] time = 0.0457103, size = 46, normalized size = 0.47

$$\frac{x(a+bx)^{\frac{bc}{ad-bc}}(c+dx)^{\frac{ad}{bc-ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^((b*c - 2*a*d)/(
-(b*c) + a*d)),x]
```

```
[Out] (x*(a + b*x)^((b*c)/(-(b*c) + a*d))*(c + d*x)^((a*d)/(b*c - a*d)))/(a*c)
```

Maple [A] time = 0.004, size = 66, normalized size = 0.7

$$\frac{x}{ac}(bx+a)^{1-\frac{ad-2bc}{ad-bc}}(dx+c)^{1-\frac{2ad-bc}{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c)),x)
```

```
[Out] (b*x+a)^(1-(a*d-2*b*c)/(a*d-b*c))*(d*x+c)^(1-(2*a*d-b*c)/(a*d-b*c))/a/c*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{2bc-ad}{bc-ad}}(dx+c)^{\frac{bc-2ad}{bc-ad}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*(d*x + c)^((b*c - 2*a*d)
/(b*c - a*d))), x)
```

Fricas [A] time = 2.43558, size = 161, normalized size = 1.66

$$\frac{bdx^3+acx+(bc+ad)x^2}{(bx+a)^{\frac{2bc-ad}{bc-ad}}(dx+c)^{\frac{bc-2ad}{bc-ad}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="fricas")
```

```
[Out] (b*d*x^3 + a*c*x + (b*c + a*d)*x^2)/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*
(d*x + c)^((b*c - 2*a*d)/(b*c - a*d))*a*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)**((-2*a*d+b*c)/(a*d-b*c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^{\frac{2bc-ad}{bc-ad}}(dx+c)^{\frac{bc-2ad}{bc-ad}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*(d*x + c)^((b*c - 2*a*d)
/(b*c - a*d))), x)
```

3.1885 $\int \frac{(1-x)^n}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=30

$$2^{n+1}\sqrt{x+1}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{x+1}{2}\right)$$

```
[Out] 2^(1 + n)*Sqrt[1 + x]*Hypergeometric2F1[1/2, -n, 3/2, (1 + x)/2]
```

Rubi [A] time = 0.0067607, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {69}

$$2^{n+1}\sqrt{x+1}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^n/Sqrt[1 + x],x]
```

```
[Out] 2^(1 + n)*Sqrt[1 + x]*Hypergeometric2F1[1/2, -n, 3/2, (1 + x)/2]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(1-x)^n}{\sqrt{1+x}}\,dx = 2^{1+n}\sqrt{1+x}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1+x}{2}\right)$$

Mathematica [A] time = 0.0058675, size = 30, normalized size = 1.

$$2^{n+1}\sqrt{x+1}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^n/Sqrt[1 + x],x]
```

```
[Out] 2^(1 + n)*Sqrt[1 + x]*Hypergeometric2F1[1/2, -n, 3/2, (1 + x)/2]
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int (1-x)^n \frac{1}{\sqrt{1+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^n/(1+x)^(1/2),x)
```

```
[Out] int((1-x)^n/(1+x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x+1)^n}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x + 1)^n/sqrt(x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-x+1)^n}{\sqrt{x+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-x + 1)^n/sqrt(x + 1), x)
```

Sympy [C] time = 2.01301, size = 29, normalized size = 0.97

$$2 \cdot 2^n \sqrt{x+1}\, {}_2F_1\left(\begin{matrix} \frac{1}{2}, -n \\ \frac{3}{2} \end{matrix}\middle| \frac{(x+1)\, e^{2i\pi}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**n/(1+x)**(1/2),x)
```

```
[Out] 2*2**n*sqrt(x + 1)*hyper((1/2, -n), (3/2,), (x + 1)*exp_polar(2*I*pi)/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x+1)^n}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-x + 1)^n/sqrt(x + 1), x)
```

3.1886 $\int \frac{(1+x)^n}{\sqrt{1-x}} dx$

Optimal. Leaf size=35

$$-2^{n+1}\sqrt{1-x}\,_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1-x}{2}\right)$$

```
[Out] -(2^(1 + n)*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, (1 - x)/2])
```

Rubi [A] time = 0.0055015, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {69}

$$-2^{n+1}\sqrt{1-x}\,_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^n/Sqrt[1 - x],x]
```

```
[Out] -(2^(1 + n)*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, (1 - x)/2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(1+x)^n}{\sqrt{1-x}}\,dx = -2^{1+n}\sqrt{1-x}\,_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1-x}{2}\right)$$

Mathematica [A] time = 0.0063314, size = 35, normalized size = 1.

$$-2^{n+1}\sqrt{1-x}\,_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^n/Sqrt[1 - x],x]
```

```
[Out] -(2^(1 + n)*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, (1 - x)/2])
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int (1+x)^n \frac{1}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^n/(1-x)^(1/2),x)
```

```
[Out] int((1+x)^n/(1-x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^n}{\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^n/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^n/sqrt(-x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(x+1)^n\sqrt{-x+1}}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^n/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(x + 1)^n*sqrt(-x + 1)/(x - 1), x)
```

Sympy [C] time = 2.04889, size = 31, normalized size = 0.89

$$-2\cdot 2^n i\sqrt{x-1}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-n\\ \frac{3}{2}\end{matrix}\middle|\frac{(x-1)\,e^{i\pi}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**n/(1-x)**(1/2),x)
```

```
[Out] -2*2**n*I*sqrt(x - 1)*hyper((1/2, -n), (3/2,), (x - 1)*exp_polar(I*pi)/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^n}{\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^n/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)^n/sqrt(-x + 1), x)
```

3.1887 $\int (1-x)^n (1+x)^{7/3}\,dx$

Optimal. Leaf size=33

$$\frac{3}{5}2^{n-1}(x+1)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{x+1}{2}\right)$$

```
[Out] (3*2^(-1 + n)*(1 + x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 + x)/2])/
5
```

Rubi [A] time = 0.007115, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {69}

$$\frac{3}{5}2^{n-1}(x+1)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^n*(1 + x)^(7/3),x]
```

```
[Out] (3*2^(-1 + n)*(1 + x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 + x)/2])/
5
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (1-x)^n(1+x)^{7/3}\,dx=\frac{3}{5}2^{-1+n}(1+x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1+x}{2}\right)$$

Mathematica [A] time = 0.0119043, size = 33, normalized size = 1.

$$\frac{3}{5}2^{n-1}(x+1)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^n*(1 + x)^(7/3),x]
```

```
[Out] (3*2^(-1 + n)*(1 + x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 + x)/2])/
5
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int (1-x)^n (1+x)^{\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^n*(1+x)^(7/3),x)
```

```
[Out] int((1-x)^n*(1+x)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x+1)^{\frac{7}{3}}(-x+1)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n*(1+x)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^(7/3)*(-x + 1)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^2+2\,x+1\right)(x+1)^{\frac{1}{3}}(-x+1)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n*(1+x)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((x^2 + 2*x + 1)*(x + 1)^(1/3)*(-x + 1)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**n*(1+x)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x+1)^{\frac{7}{3}}(-x+1)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n*(1+x)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)^(7/3)*(-x + 1)^n, x)
```

3.1888 $\int (1-x)^{7/3}(1+x)^n\,dx$

Optimal. Leaf size=37

$$-\frac{3}{5}2^{n-1}(1-x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1-x}{2}\right)$$

```
[Out] (-3*2^(-1 + n)*(1 - x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 - x)/2])
/5
```

Rubi [A] time = 0.0052561, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {69}

$$-\frac{3}{5}2^{n-1}(1-x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/3)*(1 + x)^n,x]
```

```
[Out] (-3*2^(-1 + n)*(1 - x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 - x)/2])
/5
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (1-x)^{7/3}(1+x)^n\,dx=-\frac{3}{5}2^{-1+n}(1-x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1-x}{2}\right)$$

Mathematica [A] time = 0.0119005, size = 37, normalized size = 1.

$$-\frac{3}{5}2^{n-1}(1-x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/3)*(1 + x)^n,x]
```

```
[Out] (-3*2^(-1 + n)*(1 - x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 - x)/2])
/5
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int (1-x)^{\frac{7}{3}} (1+x)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/3)*(1+x)^n,x)
```

```
[Out] int((1-x)^(7/3)*(1+x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x+1)^n(-x+1)^{\frac{7}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/3)*(1+x)^n,x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^n*(-x + 1)^(7/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^2-2\,x+1\right)(x+1)^n(-x+1)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/3)*(1+x)^n,x, algorithm="fricas")
```

```
[Out] integral((x^2 - 2*x + 1)*(x + 1)^n*(-x + 1)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/3)*(1+x)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x+1)^n(-x+1)^{\frac{7}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/3)*(1+x)^n,x, algorithm="giac")
```

```
[Out] integrate((x + 1)^n*(-x + 1)^(7/3), x)
```

3.1889 $\int (1+2x)^{-m}(2+3x)^m \, dx$

Optimal. Leaf size=47

$$\frac{2^{-m-1}(2x+1)^{1-m}\,{}_2F_1(1-m,-m;2-m;-3(2x+1))}{1-m}$$

```
[Out] (2^(-1 - m)*(1 + 2*x)^(1 - m)*Hypergeometric2F1[1 - m, -m, 2 - m, -3*(1 + 2
*x)])/(1 - m)
```

Rubi [A] time = 0.0144975, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {69}

$$\frac{2^{-m-1}(2x+1)^{1-m}\,{}_2F_1(1-m,-m;2-m;-3(2x+1))}{1-m}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^m/(1 + 2*x)^m,x]
```

```
[Out] (2^(-1 - m)*(1 + 2*x)^(1 - m)*Hypergeometric2F1[1 - m, -m, 2 - m, -3*(1 + 2
*x)])/(1 - m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (1+2x)^{-m}(2+3x)^m \, dx = \frac{2^{-1-m}(1+2x)^{1-m}\,{}_2F_1(1-m,-m;2-m;-3(1+2x))}{1-m}$$

Mathematica [A] time = 0.012871, size = 47, normalized size = 1.

$$\frac{2^{-m-1}(2x+1)^{1-m}\,{}_2F_1(1-m,-m;2-m;-3(2x+1))}{1-m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^m/(1 + 2*x)^m,x]
```

```
[Out] (2^(-1 - m)*(1 + 2*x)^(1 - m)*Hypergeometric2F1[1 - m, -m, 2 - m, -3*(1 + 2
*x)])/(1 - m)
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{(2+3x)^m}{(1+2x)^m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^m/((1+2*x)^m),x)
```

```
[Out] int((2+3*x)^m/((1+2*x)^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^m}{(2x+1)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^m/((1+2*x)^m),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)^m/(2*x + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(3x+2)^m}{(2x+1)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^m/((1+2*x)^m),x, algorithm="fricas")
```

```
[Out] integral((3*x + 2)^m/(2*x + 1)^m, x)
```

Sympy [C] time = 61.1545, size = 42, normalized size = 0.89

$$\frac{3^{2m}\left(x+\frac{2}{3}\right)\left(x+\frac{2}{3}\right)^m e^{-i\pi m}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix} m, m+1 \\ m+2 \end{matrix}\middle| 6x+4\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**m/((1+2*x)**m),x)
```

```
[Out] 3**(2*m)*(x + 2/3)*(x + 2/3)**m*exp(-I*pi*m)*gamma(m + 1)*hyper((m, m + 1),
 (m + 2,), 6*x + 4)/gamma(m + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^m}{(2x+1)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^m/((1+2*x)^m),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)^m/(2*x + 1)^m, x)
```

3.1890 $\int \left(\frac{d(a+bx)}{-bc+ad}\right)^m (c+dx)^n\,dx$

Optimal. Leaf size=45

$$\frac{(c+dx)^{n+1}\,{}_2F_1\left(-m,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{d(n+1)}$$

```
[Out] ((c + d*x)^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, (b*(c + d*x))/(b*c -
 a*d)])/(d*(1 + n))
```

Rubi [A] time = 0.01852, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {186, 69}

$$\frac{(c+dx)^{n+1}\,{}_2F_1\left(-m,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d*(a + b*x))/(-(b*c) + a*d))^m*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, (b*(c + d*x))/(b*c -
 a*d)])/(d*(1 + n))
```

Rule 186

```
Int[(u_)^(m_.)*(v_)^(n_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^n, x] /; FreeQ[{m, n}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u
, v}, x]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \left(\frac{d(a+bx)}{-bc+ad}\right)^m (c+dx)^n\,dx &= \int (c+dx)^n \left(-\frac{ad}{bc-ad}-\frac{bdx}{bc-ad}\right)^m\,dx\\ &= \frac{(c+dx)^{1+n}\,{}_2F_1\left(-m,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{d(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0327576, size = 88, normalized size = 1.96

$$\frac{(a+bx)(c+dx)^n\left(\frac{d(a+bx)}{ad-bc}\right)^m\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\,{}_2F_1\left(m+1,-n;m+2;\frac{d(a+bx)}{ad-bc}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*(a + b*x))/(-(b*c) + a*d))^m*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)*((d*(a + b*x))/(-(b*c) + a*d))^m*(c + d*x)^n*Hypergeometric2F1[1
 + m, -n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(1 + m)*((b*(c + d*x))/(
b*c - a*d))^n)
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int \left(\frac{d\,(bx+a)}{ad-bc}\right)^{m}(dx+c)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x)
```

```
[Out] int((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{n}\left(-\frac{(bx+a)d}{bc-ad}\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n*(-(b*x + a)*d/(b*c - a*d))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^{n}\left(-\frac{bdx+ad}{bc-ad}\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n*(-(b*d*x + a*d)/(b*c - a*d))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{d\,(a+bx)}{ad-bc}\right)^{m}(c+dx)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(b*x+a)/(a*d-b*c))**m*(d*x+c)**n,x)
```

```
[Out] Integral((d*(a + b*x)/(a*d - b*c))**m*(c + d*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^n \left(-\frac{(bx+a)d}{bc-ad}\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^n*(-(b*x + a)*d/(b*c - a*d))^m, x)
```

3.1891 $\int (a + bx + cx^2 + dx^3) \, dx$

Optimal. Leaf size=28

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

Rubi [A] time = 0.0046075, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x + c*x^2 + d*x^3,x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

Rubi steps

$$\int (a + bx + cx^2 + dx^3) \, dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

Mathematica [A] time = 0.0000379, size = 28, normalized size = 1.

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x + c*x^2 + d*x^3,x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

Maple [A] time = 0., size = 23, normalized size = 0.8

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x^3+c*x^2+b*x+a,x)
```

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/4*d*x^4
```

Maxima [A] time = 0.990462, size = 30, normalized size = 1.07

$$\frac{1}{4}dx^4 + \frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="maxima")
```

```
[Out] 1/4*d*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Fricas [A] time = 1.66902, size = 55, normalized size = 1.96

$$\frac{1}{4}x^4d + \frac{1}{3}x^3c + \frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*d + 1/3*x^3*c + 1/2*x^2*b + x*a
```

Sympy [A] time = 0.05424, size = 22, normalized size = 0.79

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x**3+c*x**2+b*x+a,x)
```

```
[Out] a*x + b*x**2/2 + c*x**3/3 + d*x**4/4
```

Giac [A] time = 1.05218, size = 30, normalized size = 1.07

$$\frac{1}{4}dx^4 + \frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="giac")
```

```
[Out] 1/4*d*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

3.1892 $\int \left(-x^3+x^4\right) dx$

Optimal. Leaf size=15

$$\frac{x^5}{5}-\frac{x^4}{4}$$

```
[Out] -x^4/4 + x^5/5
```

Rubi [A] time = 0.0019756, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{x^5}{5}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[-x^3 + x^4,x]
```

```
[Out] -x^4/4 + x^5/5
```

Rubi steps

$$\int \left(-x^3+x^4\right) dx=-\frac{x^4}{4}+\frac{x^5}{5}$$

Mathematica [A] time = 0.0000316, size = 15, normalized size = 1.

$$\frac{x^5}{5}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[-x^3 + x^4,x]
```

```
[Out] -x^4/4 + x^5/5
```

Maple [A] time = 0.002, size = 12, normalized size = 0.8

$$-\frac{x^4}{4}+\frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4-x^3,x)
```

```
[Out] -1/4*x^4+1/5*x^5
```

Maxima [A] time = 0.971731, size = 15, normalized size = 1.

$$\frac{1}{5}x^5 - \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="maxima")
```

```
[Out] 1/5*x^5 - 1/4*x^4
```

Fricas [A] time = 1.75352, size = 26, normalized size = 1.73

$$\frac{1}{5}x^5 - \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="fricas")
```

```
[Out] 1/5*x^5 - 1/4*x^4
```

Sympy [A] time = 0.050116, size = 8, normalized size = 0.53

$$\frac{x^5}{5} - \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4-x**3,x)
```

```
[Out] x**5/5 - x**4/4
```

Giac [A] time = 1.06505, size = 15, normalized size = 1.

$$\frac{1}{5}x^5 - \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="giac")
```

```
[Out] 1/5*x^5 - 1/4*x^4
```

3.1893 $\int \left(-1+x^5\right) dx$

Optimal. Leaf size=11

$$\frac{x^6}{6}-x$$

[Out] -x + x^6/6

Rubi [A] time = 0.00155, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{x^6}{6}-x$$

Antiderivative was successfully verified.

[In] Int[-1 + x^5,x]

[Out] -x + x^6/6

Rubi steps

$$\int \left(-1+x^5\right) dx=-x+\frac{x^6}{6}$$

Mathematica [A] time = 0.0000415, size = 11, normalized size = 1.

$$\frac{x^6}{6}-x$$

Antiderivative was successfully verified.

[In] Integrate[-1 + x^5,x]

[Out] -x + x^6/6

Maple [A] time = 0., size = 10, normalized size = 0.9

$$-x+\frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5-1,x)

[Out] -x+1/6*x^6

Maxima [A] time = 0.984466, size = 12, normalized size = 1.09

$$\frac{1}{6}x^6 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="maxima")
```

```
[Out] 1/6*x^6 - x
```

Fricas [A] time = 1.69048, size = 18, normalized size = 1.64

$$\frac{1}{6}x^6 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="fricas")
```

```
[Out] 1/6*x^6 - x
```

Sympy [A] time = 0.048918, size = 5, normalized size = 0.45

$$\frac{x^6}{6} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5-1,x)
```

```
[Out] x**6/6 - x
```

Giac [A] time = 1.07457, size = 12, normalized size = 1.09

$$\frac{1}{6}x^6 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="giac")
```

```
[Out] 1/6*x^6 - x
```

3.1894 $\int (7+4x)\,dx$

Optimal. Leaf size=9

$$2x^2+7x$$

```
[Out] 7*x + 2*x^2
```

Rubi [A] time = 0.0014703, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$2x^2+7x$$

Antiderivative was successfully verified.

```
[In] Int[7 + 4*x,x]
```

```
[Out] 7*x + 2*x^2
```

Rubi steps

$$\int (7+4x)\,dx = 7x+2x^2$$

Mathematica [A] time = 0.0000427, size = 9, normalized size = 1.

$$2x^2+7x$$

Antiderivative was successfully verified.

```
[In] Integrate[7 + 4*x,x]
```

```
[Out] 7*x + 2*x^2
```

Maple [A] time = 0.002, size = 10, normalized size = 1.1

$$2x^2+7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(7+4*x,x)
```

```
[Out] 2*x^2+7*x
```

Maxima [A] time = 0.988286, size = 12, normalized size = 1.33

$$2x^2+7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="maxima")
```

```
[Out] 2*x^2 + 7*x
```

Fricas [A] time = 1.72032, size = 18, normalized size = 2.

$$2x^2 + 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="fricas")
```

```
[Out] 2*x^2 + 7*x
```

Sympy [A] time = 0.048471, size = 7, normalized size = 0.78

$$2x^2 + 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x)
```

```
[Out] 2*x**2 + 7*x
```

Giac [A] time = 1.07415, size = 12, normalized size = 1.33

$$2\,x^2 + 7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="giac")
```

```
[Out] 2*x^2 + 7*x
```

3.1895 $\int (4x + \pi x^3) \, dx$

Optimal. Leaf size=14

$$\frac{\pi x^4}{4} + 2x^2$$

[Out] 2*x^2 + (Pi*x^4)/4

Rubi [A] time = 0.002349, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{\pi x^4}{4} + 2x^2$$

Antiderivative was successfully verified.

[In] Int[4*x + Pi*x^3,x]

[Out] 2*x^2 + (Pi*x^4)/4

Rubi steps

$$\int (4x + \pi x^3) \, dx = 2x^2 + \frac{\pi x^4}{4}$$

Mathematica [A] time = 0.0000272, size = 14, normalized size = 1.

$$\frac{\pi x^4}{4} + 2x^2$$

Antiderivative was successfully verified.

[In] Integrate[4*x + Pi*x^3,x]

[Out] 2*x^2 + (Pi*x^4)/4

Maple [A] time = 0., size = 13, normalized size = 0.9

$$2x^2 + \frac{\pi x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(Pi*x^3+4*x,x)

[Out] 2*x^2+1/4*Pi*x^4

Maxima [A] time = 0.985601, size = 16, normalized size = 1.14

$$\frac{1}{4}\,\pi x^4 + 2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="maxima")
```

```
[Out] 1/4*pi*x^4 + 2*x^2
```

Fricas [A] time = 2.01498, size = 27, normalized size = 1.93

$$\frac{1}{4}\,\pi x^4 + 2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="fricas")
```

```
[Out] 1/4*pi*x^4 + 2*x^2
```

Sympy [A] time = 0.052748, size = 10, normalized size = 0.71

$$\frac{\pi x^4}{4} + 2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x**3+4*x,x)
```

```
[Out] pi*x**4/4 + 2*x**2
```

Giac [A] time = 1.05804, size = 16, normalized size = 1.14

$$\frac{1}{4}\,\pi x^4 + 2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="giac")
```

```
[Out] 1/4*pi*x^4 + 2*x^2
```

3.1896 $\int (2x+5x^2) \, dx$

Optimal. Leaf size=11

$$\frac{5x^3}{3}+x^2$$

```
[Out] x^2 + (5*x^3)/3
```

Rubi [A] time = 0.0014909, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{5x^3}{3}+x^2$$

Antiderivative was successfully verified.

```
[In] Int[2*x + 5*x^2,x]
```

```
[Out] x^2 + (5*x^3)/3
```

Rubi steps

$$\int (2x+5x^2) \, dx = x^2+\frac{5x^3}{3}$$

Mathematica [A] time = 0.0000475, size = 11, normalized size = 1.

$$\frac{5x^3}{3}+x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[2*x + 5*x^2,x]
```

```
[Out] x^2 + (5*x^3)/3
```

Maple [A] time = 0.001, size = 10, normalized size = 0.9

$$x^2+\frac{5x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5*x^2+2*x,x)
```

```
[Out] x^2+5/3*x^3
```

Maxima [A] time = 0.969249, size = 12, normalized size = 1.09

$$\frac{5}{3}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 + x^2
```

Fricas [A] time = 1.9637, size = 20, normalized size = 1.82

$$\frac{5}{3}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="fricas")
```

```
[Out] 5/3*x^3 + x^2
```

Sympy [A] time = 0.048115, size = 8, normalized size = 0.73

$$\frac{5x^3}{3} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x**2+2*x,x)
```

```
[Out] 5*x**3/3 + x**2
```

Giac [A] time = 1.05408, size = 12, normalized size = 1.09

$$\frac{5}{3}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="giac")
```

```
[Out] 5/3*x^3 + x^2
```

3.1897 $\int \left(\frac{x^2}{2}+\frac{x^3}{3}\right) dx$

Optimal. Leaf size=15

$$\frac{x^4}{12}+\frac{x^3}{6}$$

```
[Out] x^3/6 + x^4/12
```

Rubi [A] time = 0.0019564, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{x^4}{12}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/2 + x^3/3,x]
```

```
[Out] x^3/6 + x^4/12
```

Rubi steps

$$\int \left(\frac{x^2}{2}+\frac{x^3}{3}\right) dx=\frac{x^3}{6}+\frac{x^4}{12}$$

Mathematica [A] time = 0.0000454, size = 15, normalized size = 1.

$$\frac{x^4}{12}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/2 + x^3/3,x]
```

```
[Out] x^3/6 + x^4/12
```

Maple [A] time = 0.001, size = 12, normalized size = 0.8

$$\frac{x^3}{6}+\frac{x^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^2+1/3*x^3,x)
```

```
[Out] 1/6*x^3+1/12*x^4
```

Maxima [A] time = 0.974128, size = 15, normalized size = 1.

$$\frac{1}{12}x^4 + \frac{1}{6}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="maxima")
```

```
[Out] 1/12*x^4 + 1/6*x^3
```

Fricas [A] time = 1.89879, size = 27, normalized size = 1.8

$$\frac{1}{12}x^4 + \frac{1}{6}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^4 + 1/6*x^3
```

Sympy [A] time = 0.050445, size = 8, normalized size = 0.53

$$\frac{x^4}{12} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**2+1/3*x**3,x)
```

```
[Out] x**4/12 + x**3/6
```

Giac [A] time = 1.09694, size = 15, normalized size = 1.

$$\frac{1}{12}x^4 + \frac{1}{6}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="giac")
```

```
[Out] 1/12*x^4 + 1/6*x^3
```

3.1898 $\int (3-5x+2x^2)\,dx$

Optimal. Leaf size=18

$$\frac{2x^3}{3}-\frac{5x^2}{2}+3x$$

```
[Out] 3*x - (5*x^2)/2 + (2*x^3)/3
```

Rubi [A] time = 0.0025386, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{2x^3}{3}-\frac{5x^2}{2}+3x$$

Antiderivative was successfully verified.

```
[In] Int[3 - 5*x + 2*x^2,x]
```

```
[Out] 3*x - (5*x^2)/2 + (2*x^3)/3
```

Rubi steps

$$\int (3-5x+2x^2)\,dx=3x-\frac{5x^2}{2}+\frac{2x^3}{3}$$

Mathematica [A] time = 0.0000442, size = 18, normalized size = 1.

$$\frac{2x^3}{3}-\frac{5x^2}{2}+3x$$

Antiderivative was successfully verified.

```
[In] Integrate[3 - 5*x + 2*x^2,x]
```

```
[Out] 3*x - (5*x^2)/2 + (2*x^3)/3
```

Maple [A] time = 0.001, size = 15, normalized size = 0.8

$$3x-\frac{5x^2}{2}+\frac{2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x^2-5*x+3,x)
```

```
[Out] 3*x-5/2*x^2+2/3*x^3
```

Maxima [A] time = 0.949765, size = 19, normalized size = 1.06

$$\frac{2}{3}x^3 - \frac{5}{2}x^2 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="maxima")
```

```
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

Fricas [A] time = 1.86253, size = 34, normalized size = 1.89

$$\frac{2}{3}x^3 - \frac{5}{2}x^2 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="fricas")
```

```
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

Sympy [A] time = 0.051925, size = 15, normalized size = 0.83

$$\frac{2x^3}{3} - \frac{5x^2}{2} + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x**2-5*x+3,x)
```

```
[Out] 2*x**3/3 - 5*x**2/2 + 3*x
```

Giac [A] time = 1.0569, size = 19, normalized size = 1.06

$$\frac{2}{3}x^3 - \frac{5}{2}x^2 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="giac")
```

```
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

3.1899 $\int \left(-2x+x^2+x^3\right) dx$

Optimal. Leaf size=20

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

```
[Out] -x^2 + x^3/3 + x^4/4
```

Rubi [A] time = 0.002542, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

Antiderivative was successfully verified.

```
[In] Int[-2*x + x^2 + x^3,x]
```

```
[Out] -x^2 + x^3/3 + x^4/4
```

Rubi steps

$$\int \left(-2x+x^2+x^3\right) dx=-x^2+\frac{x^3}{3}+\frac{x^4}{4}$$

Mathematica [A] time = 0.0000514, size = 20, normalized size = 1.

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[-2*x + x^2 + x^3,x]
```

```
[Out] -x^2 + x^3/3 + x^4/4
```

Maple [A] time = 0., size = 17, normalized size = 0.9

$$-x^2+\frac{x^3}{3}+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3+x^2-2*x,x)
```

```
[Out] -x^2+1/3*x^3+1/4*x^4
```

Maxima [A] time = 0.984933, size = 22, normalized size = 1.1

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Fricas [A] time = 1.67551, size = 34, normalized size = 1.7

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Sympy [A] time = 0.05078, size = 12, normalized size = 0.6

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3+x**2-2*x,x)
```

```
[Out] x**4/4 + x**3/3 - x**2
```

Giac [A] time = 1.04924, size = 22, normalized size = 1.1

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

3.1900 $\int \left(1-x^2-3x^5\right) dx$

Optimal. Leaf size=16

$$-\frac{x^6}{2}-\frac{x^3}{3}+x$$

```
[Out] x - x^3/3 - x^6/2
```

Rubi [A] time = 0.0018788, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$-\frac{x^6}{2}-\frac{x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In] Int[1 - x^2 - 3*x^5,x]
```

```
[Out] x - x^3/3 - x^6/2
```

Rubi steps

$$\int \left(1-x^2-3x^5\right) dx = x-\frac{x^3}{3}-\frac{x^6}{2}$$

Mathematica [A] time = 0.0000448, size = 16, normalized size = 1.

$$-\frac{x^6}{2}-\frac{x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - x^2 - 3*x^5,x]
```

```
[Out] x - x^3/3 - x^6/2
```

Maple [A] time = 0.001, size = 13, normalized size = 0.8

$$x-\frac{x^3}{3}-\frac{x^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3*x^5-x^2+1,x)
```

```
[Out] x-1/3*x^3-1/2*x^6
```

Maxima [A] time = 0.98298, size = 16, normalized size = 1.

$$-\frac{1}{2}\,x^6 - \frac{1}{3}\,x^3 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="maxima")
```

```
[Out] -1/2*x^6 - 1/3*x^3 + x
```

Fricas [A] time = 1.69309, size = 32, normalized size = 2.

$$-\frac{1}{2}x^6 - \frac{1}{3}x^3 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="fricas")
```

```
[Out] -1/2*x^6 - 1/3*x^3 + x
```

Sympy [A] time = 0.052106, size = 10, normalized size = 0.62

$$-\frac{x^6}{2} - \frac{x^3}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x**5-x**2+1,x)
```

```
[Out] -x**6/2 - x**3/3 + x
```

Giac [A] time = 1.05034, size = 16, normalized size = 1.

$$-\frac{1}{2}\,x^6 - \frac{1}{3}\,x^3 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="giac")
```

```
[Out] -1/2*x^6 - 1/3*x^3 + x
```

3.1901 $\int (5 + 2x + 3x^2 + 4x^3) \, dx$

Optimal. Leaf size=13

$$x^4 + x^3 + x^2 + 5x$$

```
[Out] 5*x + x^2 + x^3 + x^4
```

Rubi [A] time = 0.0018286, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$x^4 + x^3 + x^2 + 5x$$

Antiderivative was successfully verified.

```
[In] Int[5 + 2*x + 3*x^2 + 4*x^3,x]
```

```
[Out] 5*x + x^2 + x^3 + x^4
```

Rubi steps

$$\int (5 + 2x + 3x^2 + 4x^3) \, dx = 5x + x^2 + x^3 + x^4$$

Mathematica [A] time = 0.0000453, size = 13, normalized size = 1.

$$x^4 + x^3 + x^2 + 5x$$

Antiderivative was successfully verified.

```
[In] Integrate[5 + 2*x + 3*x^2 + 4*x^3,x]
```

```
[Out] 5*x + x^2 + x^3 + x^4
```

Maple [A] time = 0., size = 14, normalized size = 1.1

$$x^4 + x^3 + x^2 + 5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x^3+3*x^2+2*x+5,x)
```

```
[Out] x^4+x^3+x^2+5*x
```

Maxima [A] time = 0.984751, size = 18, normalized size = 1.38

$$x^4 + x^3 + x^2 + 5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="maxima")
```

```
[Out] x^4 + x^3 + x^2 + 5*x
```

Fricas [A] time = 1.68933, size = 31, normalized size = 2.38

$$x^4 + x^3 + x^2 + 5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="fricas")
```

```
[Out] x^4 + x^3 + x^2 + 5*x
```

Sympy [A] time = 0.053149, size = 12, normalized size = 0.92

$$x^4 + x^3 + x^2 + 5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x**3+3*x**2+2*x+5,x)
```

```
[Out] x**4 + x**3 + x**2 + 5*x
```

Giac [A] time = 1.07394, size = 18, normalized size = 1.38

$$x^4 + x^3 + x^2 + 5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="giac")
```

```
[Out] x^4 + x^3 + x^2 + 5*x
```

3.1902 $\int \left(a+\frac{d}{x^3}+\frac{c}{x^2}+\frac{b}{x}\right) dx$

Optimal. Leaf size=22

$$ax + b\log(x) - \frac{c}{x} - \frac{d}{2x^2}$$

```
[Out] -d/(2*x^2) - c/x + a*x + b*Log[x]
```

Rubi [A] time = 0.0039866, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$ax + b\log(x) - \frac{c}{x} - \frac{d}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[a + d/x^3 + c/x^2 + b/x,x]
```

```
[Out] -d/(2*x^2) - c/x + a*x + b*Log[x]
```

Rubi steps

$$\int \left(a+\frac{d}{x^3}+\frac{c}{x^2}+\frac{b}{x}\right) dx = -\frac{d}{2x^2} - \frac{c}{x} + ax + b\log(x)$$

Mathematica [A] time = 0.0063733, size = 22, normalized size = 1.

$$ax + b\log(x) - \frac{c}{x} - \frac{d}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + d/x^3 + c/x^2 + b/x,x]
```

```
[Out] -d/(2*x^2) - c/x + a*x + b*Log[x]
```

Maple [A] time = 0.001, size = 21, normalized size = 1.

$$-\frac{d}{2\,x^2} - \frac{c}{x} + ax + b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+d/x^3+c/x^2+b/x,x)
```

```
[Out] -1/2*d/x^2-c/x+a*x+b*ln(x)
```

Maxima [A] time = 0.983506, size = 27, normalized size = 1.23

$$ax + b\log(x) - \frac{c}{x} - \frac{d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="maxima")
```

```
[Out] a*x + b*log(x) - c/x - 1/2*d/x^2
```

Fricas [A] time = 2.0407, size = 65, normalized size = 2.95

$$\frac{2\,ax^3 + 2\,bx^2\log(x) - 2\,cx - d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*x^3 + 2*b*x^2*log(x) - 2*c*x - d)/x^2
```

Sympy [A] time = 0.305357, size = 19, normalized size = 0.86

$$ax + b\log(x) - \frac{2cx + d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x**3+c/x**2+b/x,x)
```

```
[Out] a*x + b*log(x) - (2*c*x + d)/(2*x**2)
```

Giac [A] time = 1.07752, size = 28, normalized size = 1.27

$$ax + b\log(|x|) - \frac{c}{x} - \frac{d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="giac")
```

```
[Out] a*x + b*log(abs(x)) - c/x - 1/2*d/x^2
```

3.1903 $\int \left(\frac{1}{x^5}+x+x^5\right) dx$

Optimal. Leaf size=22

$$\frac{x^6}{6}+\frac{x^2}{2}-\frac{1}{4x^4}$$

```
[Out] -1/(4*x^4) + x^2/2 + x^6/6
```

Rubi [A] time = 0.0026291, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{x^6}{6}+\frac{x^2}{2}-\frac{1}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5) + x + x^5,x]
```

```
[Out] -1/(4*x^4) + x^2/2 + x^6/6
```

Rubi steps

$$\int \left(\frac{1}{x^5}+x+x^5\right) dx=-\frac{1}{4x^4}+\frac{x^2}{2}+\frac{x^6}{6}$$

Mathematica [A] time = 0.0012878, size = 22, normalized size = 1.

$$\frac{x^6}{6}+\frac{x^2}{2}-\frac{1}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5) + x + x^5,x]
```

```
[Out] -1/(4*x^4) + x^2/2 + x^6/6
```

Maple [A] time = 0.001, size = 17, normalized size = 0.8

$$-\frac{1}{4\,x^4}+\frac{x^2}{2}+\frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5+x+x^5,x)
```

```
[Out] -1/4/x^4+1/2*x^2+1/6*x^6
```

Maxima [A] time = 1.0322, size = 22, normalized size = 1.

$$\frac{1}{6}\,x^{6}+\frac{1}{2}\,x^{2}-\frac{1}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="maxima")
```

```
[Out] 1/6*x^6 + 1/2*x^2 - 1/4/x^4
```

Fricas [A] time = 1.8937, size = 42, normalized size = 1.91

$$\frac{2\,x^{10}+6\,x^{6}-3}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*x^10 + 6*x^6 - 3)/x^4
```

Sympy [A] time = 0.077435, size = 15, normalized size = 0.68

$$\frac{x^{6}}{6}+\frac{x^{2}}{2}-\frac{1}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5+x+x**5,x)
```

```
[Out] x**6/6 + x**2/2 - 1/(4*x**4)
```

Giac [A] time = 1.05989, size = 22, normalized size = 1.

$$\frac{1}{6}\,x^{6}+\frac{1}{2}\,x^{2}-\frac{1}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="giac")
```

```
[Out] 1/6*x^6 + 1/2*x^2 - 1/4/x^4
```

3.1904 $\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right) dx$

Optimal. Leaf size=15

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

```
[Out] -1/(2*x^2) - x^(-1) + Log[x]
```

Rubi [A] time = 0.0020757, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3) + x^(-2) + x^(-1),x]
```

```
[Out] -1/(2*x^2) - x^(-1) + Log[x]
```

Rubi steps

$$\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right) dx = -\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Mathematica [A] time = 0.0022421, size = 15, normalized size = 1.

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3) + x^(-2) + x^(-1),x]
```

```
[Out] -1/(2*x^2) - x^(-1) + Log[x]
```

Maple [A] time = 0.001, size = 14, normalized size = 0.9

$$-\frac{1}{2\,x^2}-x^{-1}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3+1/x^2+1/x,x)
```

```
[Out] -1/2/x^2-1/x+ln(x)
```

Maxima [A] time = 0.95597, size = 18, normalized size = 1.2

$$-\frac{1}{x} - \frac{1}{2\,x^2} + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="maxima")
```

```
[Out] -1/x - 1/2/x^2 + log(x)
```

Fricas [A] time = 1.94769, size = 46, normalized size = 3.07

$$\frac{2\,x^2\log\left(x\right) - 2\,x - 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^2*log(x) - 2*x - 1)/x^2
```

Sympy [A] time = 0.080251, size = 12, normalized size = 0.8

$$\log\left(x\right) - \frac{2x+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3+1/x**2+1/x,x)
```

```
[Out] log(x) - (2*x + 1)/(2*x**2)
```

Giac [A] time = 1.05113, size = 19, normalized size = 1.27

$$-\frac{1}{x} - \frac{1}{2\,x^2} + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="giac")
```

```
[Out] -1/x - 1/2/x^2 + log(abs(x))
```

3.1905 $\int \left(-\frac{2}{x^2}+\frac{3}{x}\right) dx$

Optimal. Leaf size=10

$$\frac{2}{x}+3\log(x)$$

[Out] 2/x + 3*Log[x]

Rubi [A] time = 0.0017358, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{2}{x}+3\log(x)$$

Antiderivative was successfully verified.

[In] Int[-2/x^2 + 3/x,x]

[Out] 2/x + 3*Log[x]

Rubi steps

$$\int \left(-\frac{2}{x^2}+\frac{3}{x}\right) dx=\frac{2}{x}+3\log(x)$$

Mathematica [A] time = 0.0013866, size = 10, normalized size = 1.

$$\frac{2}{x}+3\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[-2/x^2 + 3/x,x]

[Out] 2/x + 3*Log[x]

Maple [A] time = 0.001, size = 11, normalized size = 1.1

$$2\,x^{-1}+3\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-2/x^2+3/x,x)

[Out] 2/x+3*ln(x)

Maxima [A] time = 0.948475, size = 14, normalized size = 1.4

$$\frac{2}{x} + 3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="maxima")
```

```
[Out] 2/x + 3*log(x)
```

Fricas [A] time = 1.95024, size = 27, normalized size = 2.7

$$\frac{3 x \log(x) + 2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="fricas")
```

```
[Out] (3*x*log(x) + 2)/x
```

Sympy [A] time = 0.075587, size = 7, normalized size = 0.7

$$3 \log(x) + \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x**2+3/x,x)
```

```
[Out] 3*log(x) + 2/x
```

Giac [A] time = 1.05383, size = 15, normalized size = 1.5

$$\frac{2}{x} + 3 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="giac")
```

```
[Out] 2/x + 3*log(abs(x))
```

3.1906 $\int \left(-\frac{1}{7x^6}+x^6\right) dx$

Optimal. Leaf size=15

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

```
[Out] 1/(35*x^5) + x^7/7
```

Rubi [A] time = 0.0018043, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

Antiderivative was successfully verified.

```
[In] Int[-1/(7*x^6) + x^6,x]
```

```
[Out] 1/(35*x^5) + x^7/7
```

Rubi steps

$$\int \left(-\frac{1}{7x^6}+x^6\right) dx=\frac{1}{35x^5}+\frac{x^7}{7}$$

Mathematica [A] time = 0.0013016, size = 15, normalized size = 1.

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/(7*x^6) + x^6,x]
```

```
[Out] 1/(35*x^5) + x^7/7
```

Maple [A] time = 0.002, size = 12, normalized size = 0.8

$$\frac{1}{35\,x^5}+\frac{x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/7/x^6+x^6,x)
```

```
[Out] 1/35/x^5+1/7*x^7
```

Maxima [A] time = 0.956039, size = 15, normalized size = 1.

$$\frac{1}{7}x^7 + \frac{1}{35x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="maxima")
```

```
[Out] 1/7*x^7 + 1/35/x^5
```

Fricas [A] time = 1.8777, size = 31, normalized size = 2.07

$$\frac{5x^{12}+1}{35x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="fricas")
```

```
[Out] 1/35*(5*x^12 + 1)/x^5
```

Sympy [A] time = 0.080995, size = 10, normalized size = 0.67

$$\frac{x^7}{7} + \frac{1}{35x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x**6+x**6,x)
```

```
[Out] x**7/7 + 1/(35*x**5)
```

Giac [A] time = 1.05916, size = 15, normalized size = 1.

$$\frac{1}{7}x^7 + \frac{1}{35x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="giac")
```

```
[Out] 1/7*x^7 + 1/35/x^5
```

3.1907 $\int \left(1+\frac{1}{x}+x\right) dx$

Optimal. Leaf size=11

$$\frac{x^2}{2}+x+\log(x)$$

```
[Out] x + x^2/2 + Log[x]
```

Rubi [A] time = 0.0013859, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{x^2}{2}+x+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1 + x^(-1) + x,x]
```

```
[Out] x + x^2/2 + Log[x]
```

Rubi steps

$$\int \left(1+\frac{1}{x}+x\right) dx = x+\frac{x^2}{2}+\log(x)$$

Mathematica [A] time = 0.0005127, size = 11, normalized size = 1.

$$\frac{x^2}{2}+x+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + x^(-1) + x,x]
```

```
[Out] x + x^2/2 + Log[x]
```

Maple [A] time = 0., size = 10, normalized size = 0.9

$$x+\frac{x^2}{2}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1+1/x+x,x)
```

```
[Out] x+1/2*x^2+ln(x)
```

Maxima [A] time = 0.952602, size = 12, normalized size = 1.09

$$\frac{1}{2}x^2 + x + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x + log(x)
```

Fricas [A] time = 1.95136, size = 30, normalized size = 2.73

$$\frac{1}{2}x^2 + x + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x + log(x)
```

Sympy [A] time = 0.070122, size = 8, normalized size = 0.73

$$\frac{x^2}{2} + x + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x)
```

```
[Out] x**2/2 + x + log(x)
```

Giac [A] time = 1.04252, size = 14, normalized size = 1.27

$$\frac{1}{2}x^2 + x + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x + log(abs(x))
```

3.1908 $\int \left(-\frac{3}{x^3}+\frac{4}{x^2}\right) dx$

Optimal. Leaf size=13

$$\frac{3}{2x^2}-\frac{4}{x}$$

```
[Out] 3/(2*x^2) - 4/x
```

Rubi [A] time = 0.0019179, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{3}{2x^2}-\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Int[-3/x^3 + 4/x^2,x]
```

```
[Out] 3/(2*x^2) - 4/x
```

Rubi steps

$$\int \left(-\frac{3}{x^3}+\frac{4}{x^2}\right) dx=\frac{3}{2x^2}-\frac{4}{x}$$

Mathematica [A] time = 0.0013716, size = 13, normalized size = 1.

$$\frac{3}{2x^2}-\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/x^3 + 4/x^2,x]
```

```
[Out] 3/(2*x^2) - 4/x
```

Maple [A] time = 0., size = 12, normalized size = 0.9

$$\frac{3}{2\,x^2}-4\,x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/x^3+4/x^2,x)
```

```
[Out] 3/2/x^2-4/x
```

Maxima [A] time = 0.989355, size = 15, normalized size = 1.15

$$-\frac{4}{x}+\frac{3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x^3+4/x^2,x, algorithm="maxima")
```

```
[Out] -4/x + 3/2/x^2
```

Fricas [A] time = 1.82569, size = 27, normalized size = 2.08

$$-\frac{8\,x-3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x^3+4/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(8*x - 3)/x^2
```

Sympy [A] time = 0.074432, size = 10, normalized size = 0.77

$$-\frac{8x-3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x**3+4/x**2,x)
```

```
[Out] -(8*x - 3)/(2*x**2)
```

Giac [A] time = 1.05104, size = 15, normalized size = 1.15

$$-\frac{4}{x}+\frac{3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x^3+4/x^2,x, algorithm="giac")
```

```
[Out] -4/x + 3/2/x^2
```

3.1909 $\int \left(\frac{1}{x} + 2x + x^2\right) dx$

Optimal. Leaf size=13

$$\frac{x^3}{3} + x^2 + \log(x)$$

```
[Out] x^2 + x^3/3 + Log[x]
```

Rubi [A] time = 0.0016842, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{x^3}{3} + x^2 + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1) + 2*x + x^2,x]
```

```
[Out] x^2 + x^3/3 + Log[x]
```

Rubi steps

$$\int \left(\frac{1}{x} + 2x + x^2\right) dx = x^2 + \frac{x^3}{3} + \log(x)$$

Mathematica [A] time = 0.0008609, size = 13, normalized size = 1.

$$\frac{x^3}{3} + x^2 + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1) + 2*x + x^2,x]
```

```
[Out] x^2 + x^3/3 + Log[x]
```

Maple [A] time = 0.001, size = 12, normalized size = 0.9

$$x^2 + \frac{x^3}{3} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x+2*x+x^2,x)
```

```
[Out] x^2+1/3*x^3+ln(x)
```

Maxima [A] time = 0.986445, size = 15, normalized size = 1.15

$$\frac{1}{3}x^3 + x^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + x^2 + log(x)
```

Fricas [A] time = 1.97332, size = 32, normalized size = 2.46

$$\frac{1}{3}x^3 + x^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x^2 + log(x)
```

Sympy [A] time = 0.073692, size = 10, normalized size = 0.77

$$\frac{x^3}{3} + x^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x**2,x)
```

```
[Out] x**3/3 + x**2 + log(x)
```

Giac [A] time = 1.056, size = 16, normalized size = 1.23

$$\frac{1}{3}x^3 + x^2 + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + x^2 + log(abs(x))
```

3.1910 $\int \left(x^{5/6}-x^3\right) dx$

Optimal. Leaf size=17

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

```
[Out] (6*x^(11/6))/11 - x^4/4
```

Rubi [A] time = 0.0018104, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/6) - x^3,x]
```

```
[Out] (6*x^(11/6))/11 - x^4/4
```

Rubi steps

$$\int \left(x^{5/6}-x^3\right) dx=\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Mathematica [A] time = 0.0019205, size = 17, normalized size = 1.

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/6) - x^3,x]
```

```
[Out] (6*x^(11/6))/11 - x^4/4
```

Maple [A] time = 0., size = 12, normalized size = 0.7

$$\frac{6}{11}x^{\frac{11}{6}}-\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/6)-x^3,x)
```

```
[Out] 6/11*x^(11/6)-1/4*x^4
```

Maxima [A] time = 0.969834, size = 15, normalized size = 0.88

$$-\frac{1}{4}x^4 + \frac{6}{11}x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

Fricas [A] time = 1.89175, size = 35, normalized size = 2.06

$$-\frac{1}{4}x^4 + \frac{6}{11}x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x, algorithm="fricas")
```

```
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

Sympy [A] time = 0.055255, size = 12, normalized size = 0.71

$$\frac{6x^{\frac{11}{6}}}{11} - \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/6)-x**3,x)
```

```
[Out] 6*x**(11/6)/11 - x**4/4
```

Giac [A] time = 1.05014, size = 15, normalized size = 0.88

$$-\frac{1}{4}x^4 + \frac{6}{11}x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x, algorithm="giac")
```

```
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

3.1911 $\int \left(33+\sqrt[33]{x}\right) dx$

Optimal. Leaf size=13

$$\frac{33x^{34/33}}{34}+33x$$

[Out] 33*x + (33*x^(34/33))/34

Rubi [A] time = 0.0016099, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{33x^{34/33}}{34}+33x$$

Antiderivative was successfully verified.

[In] Int[33 + x^(1/33),x]

[Out] 33*x + (33*x^(34/33))/34

Rubi steps

$$\int \left(33+\sqrt[33]{x}\right) dx = 33x+\frac{33x^{34/33}}{34}$$

Mathematica [A] time = 0.0013808, size = 13, normalized size = 1.

$$\frac{33x^{34/33}}{34}+33x$$

Antiderivative was successfully verified.

[In] Integrate[33 + x^(1/33),x]

[Out] 33*x + (33*x^(34/33))/34

Maple [A] time = 0.001, size = 10, normalized size = 0.8

$$33\,x+\frac{33}{34}x^{\frac{34}{33}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(33+x^(1/33),x)

[Out] 33*x+33/34*x^(34/33)

Maxima [A] time = 0.95642, size = 12, normalized size = 0.92

$$\frac{33}{34}\,x^{\frac{34}{33}} + 33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="maxima")
```

```
[Out] 33/34*x^(34/33) + 33*x
```

Fricas [A] time = 1.91538, size = 32, normalized size = 2.46

$$\frac{33}{34}\,x^{\frac{34}{33}} + 33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="fricas")
```

```
[Out] 33/34*x^(34/33) + 33*x
```

Sympy [A] time = 0.052312, size = 10, normalized size = 0.77

$$\frac{33x^{\frac{34}{33}}}{34} + 33x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x**(1/33),x)
```

```
[Out] 33*x**(34/33)/34 + 33*x
```

Giac [A] time = 1.07811, size = 12, normalized size = 0.92

$$\frac{33}{34}\,x^{\frac{34}{33}} + 33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="giac")
```

```
[Out] 33/34*x^(34/33) + 33*x
```

3.1912 $\int \left(\frac{1}{2\sqrt{x}} + 2\sqrt{x}\right) dx$

Optimal. Leaf size=15

$$\frac{4x^{3/2}}{3} + \sqrt{x}$$

```
[Out] Sqrt[x] + (4*x^(3/2))/3
```

Rubi [A] time = 0.0013879, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{4x^{3/2}}{3} + \sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(2*Sqrt[x]) + 2*Sqrt[x],x]
```

```
[Out] Sqrt[x] + (4*x^(3/2))/3
```

Rubi steps

$$\int \left(\frac{1}{2\sqrt{x}} + 2\sqrt{x}\right) dx = \sqrt{x} + \frac{4x^{3/2}}{3}$$

Mathematica [A] time = 0.0034131, size = 14, normalized size = 0.93

$$\frac{1}{3}\sqrt{x}(4x + 3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(2*Sqrt[x]) + 2*Sqrt[x],x]
```

```
[Out] (Sqrt[x]*(3 + 4*x))/3
```

Maple [A] time = 0.002, size = 11, normalized size = 0.7

$$\frac{4x + 3}{3}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/x^(1/2)+2*x^(1/2),x)
```

```
[Out] 1/3*x^(1/2)*(4*x+3)
```

Maxima [A] time = 0.965711, size = 12, normalized size = 0.8

$$\frac{4}{3}x^{\frac{3}{2}} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*x^(3/2) + sqrt(x)
```

Fricas [A] time = 2.12502, size = 31, normalized size = 2.07

$$\frac{1}{3}(4x+3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(4*x + 3)*sqrt(x)
```

Sympy [A] time = 0.054219, size = 12, normalized size = 0.8

$$\frac{4x^{\frac{3}{2}}}{3} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x**(1/2)+2*x**(1/2),x)
```

```
[Out] 4*x**(3/2)/3 + sqrt(x)
```

Giac [A] time = 1.04754, size = 12, normalized size = 0.8

$$\frac{4}{3}x^{\frac{3}{2}} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*x^(3/2) + sqrt(x)
```

3.1913 $\int \left(-\frac{1}{x^2}+\frac{10}{x}+6\sqrt{x}\right) dx$

Optimal. Leaf size=15

$$4x^{3/2}+\frac{1}{x}+10\log(x)$$

[Out] x^(-1) + 4*x^(3/2) + 10*Log[x]

Rubi [A] time = 0.002104, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$4x^{3/2}+\frac{1}{x}+10\log(x)$$

Antiderivative was successfully verified.

[In] Int[-x^(-2) + 10/x + 6*Sqrt[x],x]

[Out] x^(-1) + 4*x^(3/2) + 10*Log[x]

Rubi steps

$$\int \left(-\frac{1}{x^2}+\frac{10}{x}+6\sqrt{x}\right) dx = \frac{1}{x}+4x^{3/2}+10\log(x)$$

Mathematica [A] time = 0.0091254, size = 15, normalized size = 1.

$$4x^{3/2}+\frac{1}{x}+10\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[-x^(-2) + 10/x + 6*Sqrt[x],x]

[Out] x^(-1) + 4*x^(3/2) + 10*Log[x]

Maple [A] time = 0.001, size = 14, normalized size = 0.9

$$x^{-1}+4\,x^{3/2}+10\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/x^2+10/x+6*x^(1/2),x)

[Out] 1/x+4*x^(3/2)+10*ln(x)

Maxima [A] time = 0.960703, size = 18, normalized size = 1.2

$$4\,x^{\frac{3}{2}} + \frac{1}{x} + 10\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="maxima")
```

```
[Out] 4*x^(3/2) + 1/x + 10*log(x)
```

Fricas [A] time = 2.36892, size = 53, normalized size = 3.53

$$\frac{4\,x^{\frac{5}{2}} + 20\,x\log\left(\sqrt{x}\right) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="fricas")
```

```
[Out] (4*x^(5/2) + 20*x*log(sqrt(x)) + 1)/x
```

Sympy [A] time = 0.056319, size = 14, normalized size = 0.93

$$4x^{\frac{3}{2}} + 10\log\left(x\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x**2+10/x+6*x**(1/2),x)
```

```
[Out] 4*x**(3/2) + 10*log(x) + 1/x
```

Giac [A] time = 1.05302, size = 19, normalized size = 1.27

$$4\,x^{\frac{3}{2}} + \frac{1}{x} + 10\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="giac")
```

```
[Out] 4*x^(3/2) + 1/x + 10*log(abs(x))
```

3.1914 $\int \left(\frac{1}{x^{3/2}}+x^{3/2}\right) dx$

Optimal. Leaf size=17

$$\frac{2x^{5/2}}{5}-\frac{2}{\sqrt{x}}$$

[Out] -2/Sqrt[x] + (2*x^(5/2))/5

Rubi [A] time = 0.0018102, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{2x^{5/2}}{5}-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[x^(-3/2) + x^(3/2),x]

[Out] -2/Sqrt[x] + (2*x^(5/2))/5

Rubi steps

$$\int \left(\frac{1}{x^{3/2}}+x^{3/2}\right) dx=-\frac{2}{\sqrt{x}}+\frac{2x^{5/2}}{5}$$

Mathematica [A] time = 0.0041351, size = 14, normalized size = 0.82

$$\frac{2\left(x^3-5\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-3/2) + x^(3/2),x]

[Out] (2*(-5 + x^3))/(5*Sqrt[x])

Maple [A] time = 0.003, size = 11, normalized size = 0.7

$$\frac{2x^3-10}{5}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)+x^(3/2),x)

[Out] 2/5*(x^3-5)/x^(1/2)

Maxima [A] time = 0.969533, size = 15, normalized size = 0.88

$$\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) - 2/sqrt(x)
```

Fricas [A] time = 2.18133, size = 31, normalized size = 1.82

$$\frac{2\left(x^3 - 5\right)}{5\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(x^3 - 5)/sqrt(x)
```

Sympy [A] time = 0.053506, size = 14, normalized size = 0.82

$$\frac{2x^{\frac{5}{2}}}{5} - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)+x**(3/2),x)
```

```
[Out] 2*x**(5/2)/5 - 2/sqrt(x)
```

Giac [A] time = 1.04802, size = 15, normalized size = 0.88

$$\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) - 2/sqrt(x)
```

3.1915 $\int \left(-5x^{3/2}+7x^{5/2}\right) dx$

Optimal. Leaf size=15

$$2x^{7/2}-2x^{5/2}$$

```
[Out] -2*x^(5/2) + 2*x^(7/2)
```

Rubi [A] time = 0.0018477, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$2x^{7/2}-2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[-5*x^(3/2) + 7*x^(5/2),x]
```

```
[Out] -2*x^(5/2) + 2*x^(7/2)
```

Rubi steps

$$\int \left(-5x^{3/2}+7x^{5/2}\right) dx=-2x^{5/2}+2x^{7/2}$$

Mathematica [A] time = 0.0031643, size = 10, normalized size = 0.67

$$2(x-1)x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-5*x^(3/2) + 7*x^(5/2),x]
```

```
[Out] 2*(-1 + x)*x^(5/2)
```

Maple [A] time = 0.004, size = 9, normalized size = 0.6

$$2\,x^{5/2}\left(-1+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-5*x^(3/2)+7*x^(5/2),x)
```

```
[Out] 2*x^(5/2)*(-1+x)
```

Maxima [A] time = 0.973274, size = 15, normalized size = 1.

$$2\,x^{\frac{7}{2}}-2\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2*x^(7/2) - 2*x^(5/2)
```

Fricas [A] time = 2.20989, size = 31, normalized size = 2.07

$$2\left(x^3 - x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2*(x^3 - x^2)*sqrt(x)
```

Sympy [A] time = 0.054393, size = 12, normalized size = 0.8

$$2x^{\frac{7}{2}} - 2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x**(3/2)+7*x**(5/2),x)
```

```
[Out] 2*x**(7/2) - 2*x**(5/2)
```

Giac [A] time = 1.0534, size = 15, normalized size = 1.

$$2\,x^{\frac{7}{2}} - 2\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="giac")
```

```
[Out] 2*x^(7/2) - 2*x^(5/2)
```

3.1916 $\int \left(\frac{2}{\sqrt{x}}+\sqrt{x}-\frac{x}{2}\right) dx$

Optimal. Leaf size=24

$$-\frac{x^2}{4}+\frac{2x^{3/2}}{3}+4\sqrt{x}$$

```
[Out] 4*Sqrt[x] + (2*x^(3/2))/3 - x^2/4
```

Rubi [A] time = 0.0024671, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$-\frac{x^2}{4}+\frac{2x^{3/2}}{3}+4\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[2/Sqrt[x] + Sqrt[x] - x/2,x]
```

```
[Out] 4*Sqrt[x] + (2*x^(3/2))/3 - x^2/4
```

Rubi steps

$$\int \left(\frac{2}{\sqrt{x}}+\sqrt{x}-\frac{x}{2}\right) dx = 4\sqrt{x}+\frac{2x^{3/2}}{3}-\frac{x^2}{4}$$

Mathematica [A] time = 0.0043234, size = 24, normalized size = 1.

$$-\frac{x^2}{4}+\frac{2x^{3/2}}{3}+4\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/Sqrt[x] + Sqrt[x] - x/2,x]
```

```
[Out] 4*Sqrt[x] + (2*x^(3/2))/3 - x^2/4
```

Maple [A] time = 0.001, size = 17, normalized size = 0.7

$$\frac{2}{3}x^{\frac{3}{2}}-\frac{x^2}{4}+4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*x+2/x^(1/2)+x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)-1/4*x^2+4*x^(1/2)
```

Maxima [A] time = 0.972828, size = 22, normalized size = 0.92

$$-\frac{1}{4}x^2 + \frac{2}{3}x^{\frac{3}{2}} + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 2/3*x^(3/2) + 4*sqrt(x)
```

Fricas [A] time = 2.34832, size = 43, normalized size = 1.79

$$-\frac{1}{4}x^2 + \frac{2}{3}(x+6)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*x^2 + 2/3*(x + 6)*sqrt(x)
```

Sympy [A] time = 0.056591, size = 19, normalized size = 0.79

$$\frac{2x^{\frac{3}{2}}}{3} + 4\sqrt{x} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x**(1/2)+x**(1/2),x)
```

```
[Out] 2*x**(3/2)/3 + 4*sqrt(x) - x**2/4
```

Giac [A] time = 1.05495, size = 22, normalized size = 0.92

$$-\frac{1}{4}x^2 + \frac{2}{3}x^{\frac{3}{2}} + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 2/3*x^(3/2) + 4*sqrt(x)
```

3.1917 $\int \left(-\frac{2}{x}+\frac{\sqrt{x}}{5}+x^{3/2}\right) dx$

Optimal. Leaf size=23

$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{15}-2\log(x)$$

```
[Out] (2*x^(3/2))/15 + (2*x^(5/2))/5 - 2*Log[x]
```

Rubi [A] time = 0.0025672, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{15}-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-2/x + Sqrt[x]/5 + x^(3/2),x]
```

```
[Out] (2*x^(3/2))/15 + (2*x^(5/2))/5 - 2*Log[x]
```

Rubi steps

$$\int \left(-\frac{2}{x}+\frac{\sqrt{x}}{5}+x^{3/2}\right) dx=\frac{2x^{3/2}}{15}+\frac{2x^{5/2}}{5}-2\log(x)$$

Mathematica [A] time = 0.0074627, size = 23, normalized size = 1.

$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{15}-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-2/x + Sqrt[x]/5 + x^(3/2),x]
```

```
[Out] (2*x^(3/2))/15 + (2*x^(5/2))/5 - 2*Log[x]
```

Maple [A] time = 0., size = 16, normalized size = 0.7

$$\frac{2}{15}x^{\frac{3}{2}}+\frac{2}{5}x^{\frac{5}{2}}-2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/x+x^(3/2)+1/5*x^(1/2),x)
```

```
[Out] 2/15*x^(3/2)+2/5*x^(5/2)-2*ln(x)
```

Maxima [A] time = 0.961844, size = 20, normalized size = 0.87

$$\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{15}x^{\frac{3}{2}} - 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) + 2/15*x^(3/2) - 2*log(x)
```

Fricas [A] time = 2.16926, size = 58, normalized size = 2.52

$$\frac{2}{15}\left(3x^2 + x\right)\sqrt{x} - 4\log\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + x)*sqrt(x) - 4*log(sqrt(x))
```

Sympy [A] time = 0.058089, size = 20, normalized size = 0.87

$$\frac{2x^{\frac{5}{2}}}{5} + \frac{2x^{\frac{3}{2}}}{15} - 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x**(3/2)+1/5*x**(1/2),x)
```

```
[Out] 2*x**(5/2)/5 + 2*x**(3/2)/15 - 2*log(x)
```

Giac [A] time = 1.06504, size = 22, normalized size = 0.96

$$\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{15}x^{\frac{3}{2}} - 2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) + 2/15*x^(3/2) - 2*log(abs(x))
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
    not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                          #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```